SLENDERNESS EFFECTS IN PRESTRESSED CONCRETE COLUMNS

by

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We accept this thesis as conforming to the required standard

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ABSTRACT.
The purpose of this investigation was to compare the behaviour of real prestressed concrete columns to the predictions of a mathematical model. A previously developed computer programme, based on the mathematical model, had suggested several problems which needed examination. The programme indicated that, in some circumstances, an unstable equilibrium configuration could occur. The existence of this unstable loading path meant that a snap-through type of buckling was a possibility.

To check these hypotheses, six T-shaped prestressed concrete columns were constructed and tested at the University of British Columbia. In most instances, experimental observations closely matched the predictions of the mathematical model. The computed and observed peak loads compared well and the presence of an unstable equilibrium path was confirmed. Unfortunately, attempts to measure curvatures and to compare them with mathematically obtained values were unsuccessful. No satisfactory explanation for this problem was found.

Having established the validity of the mathematical model through the experimental programme, an examination of snap-through buckling was made. It was concluded that prestressed concrete columns are not prone to snap-through buckling, although sufficient additional energy applied to a column might result in a jump from a stable equilibrium configuration to an unstable one.
Acknowledgements.

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1. Introduction

In recent years, the lack of a rational method for designing for slenderness effects in prestressed concrete columns and load bearing wall panels has caused some difficulty to engineers. In general, it has probably been the practice to apply the A.C.I. procedure, although that method was derived for reinforced concrete members and is not necessarily applicable to prestressed elements of the types commonly in use. With the advent of efficient computers, engineers no longer need to resort to design procedures based on doubtful analogies.

Since Euler's time, considerable effort has been devoted to column analysis and design. One of the most noteworthy of papers on column buckling was presented by Von Karman (1) in 1910. Von Karman's theory was based on the actual stress-strain relation of the column material and used numerical integration of angular rotations along the column length, as well as moment-curvature relationships, to obtain a column deflection curve (CDC). Being both general and non-linear, it was, until recently, a laborious method of obtaining CDC's, for that reason, attempts to use simple deflected shapes to approximate the real CDC have prevailed for some time.

Many of the difficulties encountered in determining the buckling load of concrete columns arose from the highly non-linear concrete stress-strain curve and the cracking of concrete. For steel, the problem was somewhat simpler. For example, Ketter, Kaminsky and Beedle (2) obtained moment-curvature relationships for steel wide-flange columns by assuming a bi-linear steel stress-strain curve. Selecting
various stress distributions for strains beyond the elastic limit, combinations of load, moment and curvature were derived. From these combinations, graphs of moment-curvature and load-curvature were plotted for cases when the steel had:

i) reached the elastic limit
ii) yielded through the flange
iii) yielded to 1/4 depth
iv) yielded to 1/2 depth
v) yielded to 3/4 depth

For a selected load, one could then plot the moment-curvature relationship. A trial-and-error numerical approach was employed to determine column deflection curves for a pin-ended column: a deflected shape was assumed and moments computed at points along the column. From the moment-curvature curves, curvature values were interpolated and these integrated to get a new deflected shape. If the assumed curve was correct, the deflection values would be identical to the assumed values.

Comparison of the results was made with a sine curve assumption and a partial cosine curve suggested by Osgood and Westergaard (3) for steel columns.

In 1958 Broms and Veest (4) used the assumption that the deflection of reinforced concrete columns could be approximated by part of a cosine curve of wavelength 2L, such that the deflection shape was \( y = y_m \cos \frac{\pi x}{L} \). For a column of length \( l = l_1 + l_2 \) the end eccentricities would be \( e_1 = y_m \cos \frac{\pi l_1}{L} \) and \( e_2 = y_m \cos \frac{\pi l_2}{L} \).

Then the curvature would be

\[
\frac{1}{\xi} = \frac{d^2 y}{dx^2} = y_m \left( \frac{\pi}{L} \right)^2 \cos \frac{\pi x}{L} \quad (A)
\]

which at maximum moment yields:
Equilibrium equations which solve for $P$ and $M$ in terms of $\varepsilon_4 - \varepsilon_1$ were established. $P$ and $\varepsilon_4 - \varepsilon_1$ values were selected ($\varepsilon_4$ varying) and the moment at mid-span $M$ determined. Then $y_m = \frac{M}{P}$ was found and substituted in (B) to solve for $\frac{\pi}{L}$.

Finally, the terms for $y$, $e_1$ and $e_2$ were arranged to obtain:

$$\frac{1}{y_m} = y_m \left(\frac{\pi}{L}\right)^2 = \varepsilon_4 - \varepsilon_1$$

which could be differentiated to get:

$$\frac{y_m}{L} \frac{d}{dy_m} \left(\frac{\pi}{L}\right) = \cot \frac{\pi_1}{L} + \cot \frac{\pi_2}{L}$$

Solution of the equation (D) yielded the critical slenderness lengths $l_1$ and corresponding values of $l_1$ and $l_2$.

Broms and Veest used a graphical solution to solve the equation.

With the advancement of computers, attempts to use approximating curves based on sine or cosine functions have been succeeded by numerical methods. Chang and Ferguson (5) for instance, began with Hognestad's (6) stress-strain relation for concrete in flexure and an idealized bi-linear stress-strain curve for steel to obtain column deflection curves for reinforced concrete columns. They derived equilibrium equations for load and moment in terms of edge strains for rectangular columns and developed moment-curvature relations for various loads. For convenience in computer programming, the non-linearity of the moment-curvature curves was overcome by dividing the curves into small segments to which first and second order polynomials were fitted. Knowing the curvature of a column section, it was
possible to integrate, first to obtain the slope, then the
deflection of the column. Integration was performed using
Simpson's rule and the trapezoidal rule. For large scale
application or for non-rectangular sections, Chang and
Ferguson's technique might prove impractical. Nevertheless,
they did observe reasonable comparison between theoretical and
tested column deflection curves.

Since 1965, numerous methods have been suggested for
determining the strength of prestressed concrete columns.
R. Itaya (7) reported a simple approach based on static equilib­
rium and geometrical compatability and discussed modifications
to the 1963 A.C.I. code based on the code provisions for
reinforced concrete. Stability effects were not discussed.
K.J. Brown (8) used equilibrium equations and a parabolic concrete
stress-strain relation for $f''_c = 0.85f'_c$ and $\xi_{ult} = 180 \times 10^{-5}$
to obtain the ultimate strength load-moment interaction curve
for prestressed concrete columns. He tested 73 rectangular
prestressed sections himself, and used results of 3 prestressed
and 162 regular reinforced concrete columns tested by others to
compare with his theoretical predictions. Since most of the
columns failed above or just below the balanced point on the
ultimate strength interaction curve, moment magnification was
discussed but instability effects were not encountered. This
will be enlarged upon below.

Kabaila and Hall (9) investigated buckling of prestressed
concrete columns using a mathematical model applicable to
rectangular sections only. Using a concrete stress-strain
relation reported by Rüsch (10), an equation for the prestressing
steel stress-strain curve, and a linear strain distribution, equations for load and moment \((P,M)\) at any section were found. The column was divided into segments of finite length and a central difference formula used to obtain the column deflection curve. Results from this mathematical model were compared with tests on 30 prestressed columns tested by H.R. Brown and A.S. Hall (11) and with two cosine assumptions for the deflected shape of the column. Good comparisons were found with experimental results for prestress up to 0.30 \(f'_c\). It was also found that the cosine assumption \((y = y_m \cos \frac{\pi x}{L})\) always underestimated the buckling load while \(y = e+(y_m-e)\cos\frac{\pi x}{L}\) overestimated the buckling load. As an interesting side effect, it was also discovered that the initial tangent modulus of elasticity of concrete was dependent on the prestressing force sustained by the concrete, probably due to creep effects. This phenomenon was investigated by Brettle (12).

T.Y. Lin and F.R. Lakhwara (13) modelled slenderness effects in partially prestressed concrete columns assuming both bi-linear concrete and bi-linear prestressing steel stress-strain curves. By fixing the ultimate concrete strain at 0.003 and varying the depth to the neutral axis, a closed form solution was obtained for the load-moment combinations to cause failure. Knowing the strain distribution at failure, a mid-height curvature was obtained. This mid-height curvature was then used to fit a cosine wave curve as the deflected shape of the column.

A technique similar to the mathematical model used in this thesis was proposed by Zia and Moreadith (14). Assuming a
linear strain distribution and using equilibrium equations for forces and moments on that section, moment curvature curves were derived by varying the values of the concrete edge strains (and the load). After construction of these moment curvature curves, numerical integration was carried out with two known boundary conditions:

i) the slope must vanish at mid-height

ii) the deflection at the ends must vanish

Beginning with an assumed deflection and known slope \( \frac{dy}{dx} = 0 \) at midheight, the slope and deflection at various points along the column were calculated successively. Calculations continued until the deflection was equal to zero. This was done for various assumed mid-span deflections, and the longest column length obtained was selected as the critical column length.

An attempt to determine the ultimate strength interaction curve using the equivalent rectangular stress block permitted by the A.C.I. Building Code (318 - 63) and using \( \varepsilon_{ult} = 0.003 \) was made by Zia and Guillermo (15). However, Nathan, (16) pointed out that this method can lead to anomalies when the area of concrete on the compression side of the neutral axis is of irregular shape. For this reason, as Nathan suggested, it would be a mistake to introduce an irrational approach to the design of prestressed concrete columns when the opportunity of using a rational method existed at the outset.

In 1968, S.Aroni (17) reported tests on 36 prestressed pin-ended rectangular columns in which effects of varying the eccentricity of load, initial prestress and slenderness ratio
were examined. Aroni's method of analysis was basically similar to previously reported analyses. A column was first divided into "finite elements" of chosen segment length. Two equations of equilibrium were written to handle each of seven combinations of concrete strain distribution with four unknowns $(P, \varepsilon, \Delta \varepsilon, y)$. An initial deflected shape of the column was assumed from which $\Delta \varepsilon$ at mid-height and hence curvature could obtained. Concrete stresses were found from the strains using Hognestad's stress-strain curve. Knowing the curvature at mid-span, a recurrence formula (actually a Taylor series expansion) was then used for numerical integration along the length of the column to get the column deflection curve. Finally the required boundary condition for the last segment was checked, that is, $y_m = e$, the end eccentricity. If this was not satisfied the assumed mid-height $\Delta \varepsilon$ was changed and the whole procedure repeated. In many ways Aroni's iterative technique was similar to that suggested by Ketter, Kaminsky, and Beedle (2).

Aroni's method also shows some likeness to the mathematical model employed in this thesis. However, instead of determining moment-curvature relationships for various load values before using the recurrence formula, Aroni calculated them at each stage in the numerical integration. If his initial assumption for the deflected shape of the column was wrong, all the other curvature values along the column would also be incorrect, leading to mis-matched end eccentricities (between assumed and calculated CDC's). Hence he was forced to re-evaluate the deflected shape by trial-and-error. In this thesis, the curvature at mid-span for a known deflection is correct from the
outset; thereafter all other combinations of moment and curvature calculated along the column are also correct within the accuracy of the initial assumptions. Therefore, no trial-and-error or iteration procedures are necessary.

Development of the computer model used in this thesis originated with Chandwani and Nathan (18). A programme was initially devised to compute the interaction curve for prestressed concrete sections of any shape with any arrangement of prestressing and mild steel. Results from the computer model were compared with Whitney's theory (19) and it was found that the "equivalent" rectangular sections were not successful in representing interaction curves for T-sections.

Later, the computer programme was extended by Nathan (20) to calculate load-moment-curvature relationships and column deflection curves. Some limited experimental verification with tests on rectangular reinforced concrete sections by Chang and Ferguson (5) was observed.

The mathematical model was also used for comparison with the current A.C.I. Building Code (318-71) specifications for prestressed concrete columns by Nathan (21). The A.C.I. design procedure uses moment magnifiers to handle slenderness effects. The magnification factor is:

\[ \epsilon = \frac{1}{1 - \frac{P_u}{P_c}} \]

where

\[ P_c = \frac{n^2EI}{(kl_u)^2} \]

The question immediately arises: "What does one use for EI?" Formulas 10.7 or 10.8 in the A.C.I. code imply a linear moment-curvature relationship, (as, indeed, does the basic form of the
magnification factor) which is quite reasonable for loads above the "balanced value", when failure is governed by compression. This is generally the case for concrete sections with conventional reinforcing where the "balanced load" is at a relatively low percentage of the maximum pure axial load. However, for prestressed concrete sections with a wide compression flange, the "balanced load" is a much higher percentage of the maximum load, and the range of tensile failure, and, therefore, ductile behavior, is greatly increased.

Using the analytical model, it has been found that for slender prestressed concrete sections with wide compression flanges, the ductility can lead to an instability type of failure before material failure occurs if the end moments are sufficiently large. In certain circumstances this instability failure might occur under load and end moment conditions which should have been safe according to the A.C.I. formulae.

Nathan (21) also pointed out that under these conditions, there exists a definite possibility that snap-through type buckling can occur even before the predicted point of instability, so that the A.C.I. formulae may be doubly unconservative in this region. It is difficult to predict the point at which snap-through may occur since it depends upon accidental disturbances and imperfections, or small unanticipated lateral loads; nevertheless, some indication of the potential danger of snap-through may be obtained by studying the differences in the potential energies associated with the stable and unstable loading paths.

Since all the previous conjectures rely on a mathematical
model, establishing the validity of the model is essential. With some minor modifications, and the addition of subroutines to calculate the potential energy of the column, the computer programme used in this thesis is the same as that developed by Nathan and Chandwani.

The purpose of this thesis is, therefore, to choose a case which severely stretches the assumptions of the mathematical model and to compare the analytical solution with tests on real columns. Having determined the reliability of the mathematical model, comments can then be made on implications of the latter for the stability of prestressed concrete columns. In particular, the question of snap-through buckling can be examined with somewhat more confidence.
2. The Mathematical Model.

2.1 Assumptions:

i) Bernoulli's assumption: plane sections remain plane and normal to the neutral surface.

ii) a bi-linear mild steel stress-strain curve.

iii) a well defined concrete stress-strain curve applicable to both axial load and flexural behavior.

iv) a known stress-strain curve for the prestressing steel.

v) perfect bond between mild or prestressing steel and concrete.

2.2 Load-Moment Interaction Curve: Short Column.

The computation begins with determination of the material failure limits for a short column (that is, with no slenderness effects). The extreme fibre in compression is given the failure value of concrete strain ($\varepsilon_{ult}$). A neutral axis depth is selected and the linear strain diagram is then completely defined. (Fig 1a) The concrete stress-strain curve is constructed by fitting a smooth curve to a collection of experimentally obtained points, or, alternatively, the programme contains a default polynomial which may be adjusted in terms of chosen values of cylinder strength ($f'_c$), peak strain ($\varepsilon_0$), and ultimate strain ($\varepsilon_{ult}$). The concrete stress distribution implied by the previously described strain diagram is then known; the resultant force and moment about the plastic centroid are numerically integrated by subdividing the compression area into narrow strips parallel to
the neutral axis, to each of which is attributed a constant stress. Likewise the strains in the prestressed and/or mild steel reinforcement can be obtained from the strain diagram. The prestressing steel stress-strain curve can also be fed in using point values or by using a default polynomial included in the programme. Summing all the loads and moments across the section gives the failure conditions for that section. For example, point 1 on the interaction curve of Fig. 1b might be determined by the strain configuration 0-1 of Fig. 1a with rotation \( \phi_1 \), or point 5 of Fig. 1b by 0-5 of Fig. 1a with rotation \( \phi_5 \). Discrete limits for load, moment and rotation are thus acquired and a smooth curve can be fitted to them. The curve so generated is referred to as the "Short Column Interaction Curve".

2.3 Potential Energy of Load.

For each strain distribution associated with the load-moment interaction curve, the strain at the plastic centroid can be readily found. When the centroidal strain is multiplied by the total load on the section, the loss of potential energy per unit length due to compressive shortening of the column is known.

Contributions to potential energy due to curvature of the axis, rotation of the end moments, and strain energy are obtained separately for a member of given length, once the deflections, slopes and curvatures are completely defined for a given column deflection curve.
2.4 Load Moment Curvature Relationships, Energy of Load Contours.

The moment curvature relationships can be determined by a similar procedure. It will be realized that the curvature is given by the extreme fibre strain divided by the neutral axis depth, i.e. it is the angle of the strain diagram as shown in Fig. 2a. Thus for a given value of curvature, each value of neutral axis depth defines a possible strain diagram. When the stresses are computed and integrated as in section 2.2, each neutral axis depth yields a pair of corresponding values of load and moment for the given curvature.

When these values are plotted on the load moment diagram, a smooth curve can be drawn through them to give a curvature contour as on Fig. 2b. Only the points falling within the short column interaction curve are valid. Pfrang, Siess, and Sozen (22) obtained contours of curvature in a similar manner for reinforced concrete columns.

From these contours, the moment-curvature relation for any value of the load can be extracted as shown in Fig. 2c.

As described in section 2.3, it is possible to calculate the loss of potential energy per unit length caused by compressive shortening of the column. Thus, for a chosen curvature, each value of neutral axis depth defines a strain distribution from which the strain at the plastic centroid can be determined. Each neutral axis depth, therefore, yields a pair of corresponding values of load and load energy per unit length for the given curvature.

Again, when these values are plotted on the load-load energy diagram, smooth contours are obtained (see Fig. 2d).
2.5 Column Deflection Curves.

An infinitely long column under a given axial load may occupy an infinite number of equilibrium configurations. A linear solution yields waves whose amplitude is arbitrary but whose length depends on the load. When material non-linearities are included, both the wave length and amplitude are arbitrary but related. The wave forms are symmetrical about the point of maximum deflection from the thrust line so that the shape of a quarter wave-length is representative of the entire configuration. This configuration is called a "Column Deflection Curve" (CDC).

Galombos (23) suggested the following numerical method for the determination of CDC's:

Let the distance along the thrust line be \( x \).
Let the displacement of the CDC from the thrust line be \( y \).
Expanding \( y(x) \) in a Taylor series about a point \( x_0 \):
\[
y(x_0 + \Delta x) = y(x_0) + y'(x_0)\Delta x + \frac{1}{2}y''(x_0)\Delta x^2 + \ldots
\]

Similarly, expanding \( y'(x) \) in a Taylor series about \( x_0 \):
\[
y'(x_0 + \Delta x) = y'(x_0) + y''(x_0)\Delta x + \frac{1}{2}y'''(x_0)\Delta x^2 + \ldots
\]
Recalling that \( y' \) is the slope (\( \alpha \)) of the CDC and \( y'' \) is a good approximation to the curvature (\( \phi \)), one obtains, by truncating the series after the second derivative of \( y \):
\[
y(x_0 + \Delta x) = y(x_0) + \alpha(x_0)\Delta x + \frac{1}{2}\phi(x_0)\Delta x^2 \quad (A)
\]
\[
(x_0 + \Delta x) = \alpha(x_0) + \phi(x_0)\Delta x \quad (B)
\]
Truncation of the approximation at the second derivative is equivalent to assuming constant (circular) curvature over \( \Delta x \).

With relationships (A) and (B), one can proceed to find the CDC for a pin-ended column bent in single curvature as follows:
i) pick a value of the axial load for which the CDC is sought.

ii) enter the graph of the short column interaction curve and obtain the maximum moment (which will occur at mid-length) and the corresponding rotation, and energy of loading (per unit length). Material failure occurs at this combination of load and moment.

iii) the mid-length displacement \( y \) of the column is just the mid-length moment divided by the axial load; the slope \( (\alpha) \) at mid-length is zero by symmetry of the deflected shape.

iv) now calculate \( y(x_0 + \Delta x) \) and \( \alpha(x_0 + \Delta x) \) for a new point a distance \( \Delta x \) along the thrust line from the mid-length using the Taylor series expansion.

v) knowing the deflection at this new point, calculate the moment, enter the contours of:
   a) curvature to obtain a new value of \( \phi \).
   b) load energy to obtain the load energy per unit length at that location.

vi) calculate a new \( y \) and \( \alpha \) further along the thrust line. Continue in this way along the thrust line until the half-length of the column is exceeded or the CDC crosses over the thrust line.

vii) establish a family of CDC's by decreasing the mid-height moment incrementally from the maximum value and determining a new CDC for each increment. Note that for each segment along the CDC's, values of decreased potential energy of load per unit length
due to axial shortening, are now available.

In reality, the deflected shape of a column increases with load in a process reversed from that described above. For example, referring to Fig. 3b for a column of length L with load P at an end eccentricity $e_1$, the mid-length moment would be $P_y_1$. For a slightly higher end moment, $P_{e_2}$, the mid-length moment would be $P_{y_2}$. At $e_2$, the maximum end eccentricity possible has been reached, although the mid-length moment $P_{y_3}$ is less than the moment $P_{y_5}$ which would result in material failure. In fact, if the end moment could be backed down to $P_{e_4}$, the mid-length moment would still increase to $P_{y_4}$. Material failure would occur when the mid-height moment reached $P_{y_5}$, even though the end moment would be only $P_{e_5}$.

From the preceding discussion on CDC's, it can be seen that for a given load, and end moment, there may exist two values of the mid-length moment for which the column is in equilibrium. For example, end moments $P_{e_2}$ and $P_{e_4}$ (having the same numerical value) lead to two different mid-length moments $P_{y_2}$ and $P_{y_4}$.

The energy of load per unit length due to axial compression is now available at each segment of the CDC. When this is integrated numerically along the length of the column, the total energy of loading at the plastic centroid is obtained. Finally it is necessary to determine the strain energy in the column, loss of energy due to the applied end moment rotating through the end slope, and energy losses from the shortening of the column due to bending. These last three terms necessary to determine the potential energy of the column will be discussed.
2.6 **End Moment vs. Mid-height Moment Curves.**

By plotting the end moment versus mid-length moment for a selected axial load (say $P_2$ of section 2.5), curves such as those of Fig. 4 are obtained for a column of particular length. Other axial loads yield similar curves. Generally, the higher the axial load, the higher is the peak end moment on each curve.

A column whose eccentricity of loading is known can now be examined. Let that eccentricity be $e$. For an axial load $P_1$, the end moment would be $P_1e$. Entering $P_1e$ as the ordinate on Fig. 4 one finds only one corresponding value of mid-length moment from the curve for $P_1$. Selecting a slightly higher load say $P_2$, the end moment would be $P_2e$, for which two corresponding mid-height moments would be obtained. By increasing the axial load, one will eventually find a load for which only one mid-length moment is possible. Increasing the axial load also increases the slenderness effect so that the mid-length moment grows at a faster rate than the end moment. For loads above the peak value no mid-height moments can be found. Hence, the maximum load and end moment that this particular column could sustain would be $P_4$ and $P_4e$ respectively.

2.7 **Load-Mid-height Moment Curves.**

Returning to the load-moment diagram on which the short column interaction curve is shown, one may now use the data from section 2.6 to plot a graph of load vs. mid-length moment, on the same axes. In the previous discussion, it was found that
only one value of mid-height moment occurred for load \( P_1 \), two for loads \( P_2 \) and \( P_3 \), and one for \( P_4 \). If these and values for other loads up to \( P_4 \) are plotted on this diagram, a curve such as OB of Fig. 5 is obtained. The "line of no slenderness effect" simply gives the end moment conditions for corresponding axial loads. Arrows on the curve follow the direction of the curve as the column is compressed.

The same column with a much smaller end eccentricity might produce the curve OA with no associated downward path. Slenderness effects resulting from the smaller end eccentricity are much reduced; for this case, failure occurs above the "balanced point" by compression.

Note that for the column whose cross-section, length and eccentricity of loading produce the curve OB, there is a rather long descending branch of the load-moment curve beyond the peak load, indicating an instability type of failure. Material failure of the section would not occur until the load-mid-height moment curve intersected the short column interaction curve at B.

Since the instability case is one which severely stretches the capabilities of the mathematical model, the experimental portion of this thesis was aimed at duplicating this type of failure with tests on real columns.
3. The Experimental Programme.

3.1 Design Constraints.

To obtain a prestressed concrete column suitable for testing within the constraints of the structural laboratory at the University of British Columbia, several factors were considered:

i) the column required a large compression flange in order to produce a relatively high balanced point on the short column interaction curve, so that the instability failure mode could be ensured.

ii) the column had to be sufficiently long to develop the prestressing force of a 3/8 inch diameter 7 wire strand which was the smallest strand available at the Con-Force Products Ltd. precast concrete plant where the columns would be cast. Since the computer model does not handle loss of prestress due to transfer at ends of the column, these effects had to be minimized.

iii) the web had to be sufficiently thick to prevent spalling of the concrete due to transfer of the prestressing force.

iv) the tallest mechanical screw type testing machine in the structural laboratory could accommodate specimens not exceeding 12 feet in length.

3.2 Testing Machine Calibration.

For columns subject to an instability type of failure, the mathematical model predicts that the column will continue to deflect laterally, once the peak load has been reached, even
Further, if the slope of the descending branch of the load-axial deflection curve for the column became equal to the testing machine load-deflection curve, premature failure of the column might occur. To ensure that this would not happen, a load deflection curve for the 200,000 lb. Tinius Olsen testing machine was obtained as follows. The testing machine loading head was set 10 feet above the weighing table (10 feet being the anticipated length of column to be tested). A hydraulic jack was positioned on the weighing table so that load could be applied to the loading head of the testing machine by the jack. As the load was applied, the deflection between the weighing table and the head was measured. The load-deflection curve obtained is shown in Fig. 6 along with a load-axial deflection curve calculated for the columns prior to testing.

3.3 Column Dimensions; Details of End Bearings.

Dimensions and details of the 12 inch wide, 10 foot long T-shaped pin-ended column selected for testing are illustrated in Figs. 7a to 7c. The single prestressing strand was positioned 5 inches from the bottom edge of the web, approximately 0.125 inches above the calculated position of the plastic centroid. The axis of loading was through the flange, 1.75 inches above the plastic centroid.

Each column was cast with ½ inch thick machined steel plates built in at the ends. Three number 3 reinforcing bars were welded to the inside of the plate and extended 6 inches longitudinally into the flange of the column; one number 4 also extended 15 inches into the web to prevent longitudinal cracking
caused by stress transfer from the ends of the prestressing strand. Closed stirrups of number 7 reinforcing wire, hooked around the longitudinal reinforcing within the first 6 inches of the column ends, were also used to prevent spalling due to the stress transfer.

The built in steel end plates were used as mounting blocks for additional $\frac{1}{2}$ inch thick machined steel bearing plates. A $\frac{1}{4}$ inch deep groove was machined into this additional plate to accommodate a 12 inch long, 1 inch diameter roller bearing. The second plate was then bolted to the first and carefully adjusted in position so that the axis of the roller would be at the desired 1.75 inch eccentricity from the calculated plastic centroid of the section.

Bearing seats on both the weighing table and loading head consisted of 4 inch wide, 3 inch deep, 13 inch long machined steel blocks. A $\frac{1}{4}$ inch deep groove was machined longitudinally on each block to closely fit the one inch diameter roller. The lower bearing seat was bolted to a $\frac{3}{4}$ inch thick plate which could be adjusted in position to seat the column correctly by adjusting the lower bearing seat on the weighing table.

3.4 Formwork, Casting and Material Testing.

The six T-shaped columns used in the testing programme were all made at the ConForce Products Ltd. precasting plant in Richmond B.C. Formwork for the column was constructed of $\frac{3}{4}$ inch thick plywood with a dimensional tolerance of $\pm \frac{1}{16}$ inch. Dimensional accuracy of the forms was checked prior to casting. The columns were cast two at a time, laid end-to-end in a 28
foot 8½ inch stressing bed. After the concrete had been placed, each column was cured with heat for approximately 15 hours before the forms were removed. Standard 6 inch diameter, 12 inch long concrete cylinders were made at the same time as the concrete was placed; cylinders were kept with the columns and cured at the same time. After the initial curing, the forms were stripped and the prestressing strand released from the bulkheads. Cylinders were tested to ensure that the concrete had sufficient strength to sustain the prestress. Both the columns and the cylinders were stored under cover but open to the air for the duration of the period before testing.

Compressive strength tests were carried out on the cylinders at as close a time as possible to the date of the column test, usually within a day. The first column was tested 57 days after casting and the last after 123 days. Cylinders could not be tested on the same day as the columns as the same testing machine was required for both tests. A mechanical screw-type testing machine was necessary in order to obtain the downward slope of the concrete stress-strain curve. Measured cylinder strengths were used in the computer programme to simulate the behaviour of the column as closely as possible.

Stress-strain curves for the prestressing steel were determined in the testing laboratories of Wire Rope Industries Ltd., 3185 Grandview Hwy., Burnaby, B.C. (see Fig. 8).

3.5 Measurement of Prestress.

Computed load-mid-length moment curves for the column are shown in Fig. 9 for a 1.75 inch eccentricity and expected final
prestress forces of 11, 12 and 13 kips assuming $f'_c = 5$ ksi, $\varepsilon_0 = 0.002$ and $\varepsilon_{ult} = 0.003$. At the time of casting, the prestress force was determined from measured elongations. No instrumentation was subsequently used to measure loss of prestress in the strand prior to testing, but estimates of losses were made using methods suggested by Libby (24). A preliminary estimate showed the final prestressing force in the columns would be about 12 kips.

Referring to Fig. 9, it can be seen that an error of $\pm 1$ kip from the estimated 12 kips prestress force results in an error of only $\pm 2.8\%$ in the prediction of the peak load. Since the primary purpose of testing the columns was to establish the existence of the unstable downward path of the load-mid-length moment curve, such an error was considered of minor importance.

3.6 Instrumentation.

3.6.1 Curvature Measurement.

It will be recalled that the analytical procedure begins with the determination of the moment-curvature relationship for various load levels and then goes on to predict the column behaviour. Thus, it was desired to check the computed moment-curvature variation and then to compare the overall behaviour of the column with the predicted performance. Two independent methods of curvature measurement were employed on the test columns.

The first system measured curvature assuming circular curvature over the same segment length as was used in the Taylor series expansion of the mathematical model. This consisted of
a series of aluminium cantilevers arranged in a straight line down one side of the web, parallel to the column axis. The cantilevers were fastened to aluminium brackets, which, in turn, were bolted to short aluminium bars epoxied to the concrete surface at $7\frac{1}{2}$ inch spacing, this being the segment length used in the computer programme. The tip deflection for each cantilever was measured by a linearly varying displacement transducer mounted on the aluminium bracket adjacent to it, Figs. 10a and 10b illustrate this system. Since the magnitude of the rotations were extremely small, differences in deflection measurement due to rotation of the transducer holder were neglected. A calibration model was constructed to simulate rotations on the column and it was found that small changes in the angle at which the transducer core entered the transducer had little effect. Loose fitting cores were used to prevent binding of the core in the transducer.

Referring to Fig. 10c, calculation of curvature using these devices proceeds as follows:

i) the curved segment of the column is represented by the line OA', assuming circular curvature.

ii) the deflection measured by the linear varying displacement transducer in the adjacent bracket is the distance A'B, where A'B is normal to the curve at A'. Because the rotation between adjacent segments on the curve is always small, one may argue that A'B is very closely approximated by AB.

iv) using a Taylor series expansion truncated after the third term, one obtains:
\[ y(x + \Delta x) = y_0 + \frac{dy}{dx} \Delta x + \frac{1}{2} \frac{d^2y}{dx^2} \Delta x^2 \]

but: \[ CD = y_0 \]

and: \[ BC = \frac{dy}{dx} \Delta x \]

hence: \[ AB = \frac{1}{2} \frac{d^2y}{dx^2} \Delta x^2 \]

and since \[ \frac{d^2y}{dx^2} \] is a good approximation to the curvature

then: \[ \phi = \frac{2AB}{x^2} \]

Thus the curvature over \( \Delta x \) is twice the cantilever tip
deflection divided by the square of the segment length.

The second system was intended to measure the deflected
shape of the column. Attempts to measure deflections were made
on the first column tested using dial gauges equally spaced 7\( \frac{1}{2} \)

inches apart down the length of the column. To avoid observing
each gauge at every load stage, photographs were taken of the
gauges. This proved unsatisfactory for several reasons:

i) it was difficult to read the gauges from the photos
despite use of accurate equipment.

ii) it was time consuming to interpret the readings for
each gauge position on the column.

iii) the range of the gauges was only 1 inch, but the
columns deflected as much as 2 inches at midspan.
Therefore, the gauges had to be repositioned half
way through the test at a critical stage when the
column was becoming unstable.

iv) the results of the gauge readings were not immediately
available during the testing.
For the second and subsequent column tests, another method of deflection measurement was employed. The upper end of a 2-inch square rectangular steel tube was fixed to the loading head of the testing machine; the lower end was connected to the weighing table with Schneeberger bearings which permitted only vertical motion of the steel tube (Fig. 11a). An aluminium carriage with roller bearings could be propelled up and down the steel column with a motorized pulley system. A linear varying displacement transducer with a ±1 inch range was attached to the carriage and the core of the transducer spring loaded so that it was bearing upon and normal to the centre line of concrete column flange. The tip of the transducer core was equipped with a teflon runner. To reduce the roughness of the concrete surface, a masking tape track ran along the centre line of the flange (Fig. 11b). When the carriage was run up and down the column, the displacement transducer produced a continuous voltage output which could be calibrated to plot the deflection of the column as the abscissa on an x-y plotter, with the position of the carriage along the column being used as the ordinate.

To determine the position of the carriage, the top pulley was equipped with a threaded axle extension. As the pulley rotated, a threaded socket was driven in, or out, by the turning axle. The displacement of the socket could then be measured with a linear varying displacement transducer fixed to the steel column. Output voltage from the transducer was calibrated as the ordinate on the x-y plot. Fig. 11c illustrates the pulley transducer mounted at the top of the steel tube.
Prior to testing, short wires were attached at known locations transverse to the path of the carriage on the flange of the concrete column, to produce calibrating marks on the x-y plot. These wires were removed before the undeflected shape of the column was obtained. Typically, it took about 3 seconds to run the carriage up and down the column to produce an x-y plot. Fig. 11d is a photograph of the x-y plotter and typical curves measured by the device. The transducer measuring the column deflections was accurate to within ± 0.005 inches.

Using the experimentally obtained deflection graphs, curvatures were calculated as follows. The ordinate of the graph was divided into the same segment lengths as were used in the mathematical model; deflections were then obtained at discrete points along the column for each load stage. It was found that numerical differentiation of deflections at discrete points would require measurement accuracy of ± 0.00001 inches to obtain accurate curvatures. Therefore, instead of numerical differentiation, a computer library subroutine was used to fit a curve to the deflection measurements. Estimates of the accuracy associated with each measurement could be included in the programme and the curve selected which best fitted the experimental values within the specified tolerances. The computer programme also provided first and second derivatives for the curve so fitted; the second derivative was used as a good approximation to the curvature of the column. Curves fitted to experimental values are illustrated in Figs. 12a to 12e for loads ranging from 5 to 35 kips, on the five column tests reported.
3.6.2 Calibration of Linearly Varying Displacement Transducers.

Linear varying displacement transducers with a $\pm$ 0.1 inch deflection range were used to measure the cantilever tip deflections. For calibration, each transducer, complete with its connecting wires, was mounted in a brass calibration stand. The transducer core was fastened to a threaded crankshaft on the calibration stand. One complete turn of the crank displaced the core 0.025 inches. A Hewlett Packard power supply with digital voltage controls capable of producing six volts continuous DC power accurate to within $\pm$ 0.0001 volts supplied the input voltage to the transducers. A digital voltmeter with four significant digits was used to monitor both the input and output voltages.

To calibrate the transducer, the core was moved to positions

i) $+0.025$

ii) $+0.050$

iii) $+0.075$

iv) $-0.25$

v) $-0.050$

vi) $-0.075$ inches from the position of approximately neutral output voltage. The output voltages were recorded at each position of the core. This procedure was repeated three times for each transducer and the average output voltage determined for each position. Later, a computer programme was used to fit a smooth curve through the calibration values.


The stability condition of the column could be observed on
an axial load vs. axial deflection graph plotted on a second x-y recorder. A linearly varying displacement transducer measured the axial shortening by recording the vertical motion of the steel tube (on which carriage travelled) relative to the weighing table. A second transducer mounted inside the Tinius Olsen testing machine was calibrated to read the applied load. Output from both these transducers produced load-deflection plots such as that of Fig. 13. Neutral equilibrium of the columns was reached when the slope of the curve became zero; unstable equilibrium was observed when the slope became negative.

3.6.4 Data Recording System.

At each load stage, output voltages from the displacement transducers were scanned by a Vidar digital voltmeter, scanner and recording system. Data stored by the system included;

i) clock time (day, hour, minutes, seconds).

ii) output voltage from the load measuring transducer.

iii) seven output voltages from cantilever tip deflection measurement.

iv) output voltage from the axial deflection measuring transducer.

v) the input voltage to each transducer.

The accuracy of the Vidar digital voltmeter was checked by comparison with an equally accurate voltmeter. No error was observed.

A computer programme was developed to convert the voltages recorded on magnetic tape to the desired information using the calibration data for each transducer.
For several tests, a Digital PDP11 mini-computer was connected to the Vidar recording system and used to convert the transducer voltages to curvatures and deflections, while testing was in progress. The mini-computer tended to slow down the rate of testing since its printer was not sufficiently fast. Library programmes on the university's main IBM 370 computer were also more accurate than those used on the mini-computer.

3.7. **Conduct of the Tests.**

3.7.1 **Prior to Loading.**

The procedure followed for each column is described below:

i) all electrical equipment was switched on 4 hours in advance of testing to ensure that the linear varying displacement transducers had reached a stationary voltage output while the core remained in a fixed position.

ii) the transducers were positioned in their holders to make maximum use of their range of deflection measurement.

iii) the axial load and axial deflection transducers were calibrated.

iv) the transducers mounted on the carriage and pulley system were calibrated and the initial undeflected shape of the column was determined using the x-y plotter.

v) a small load (typically 5 kips) was applied to the column then removed, to ensure correct seating of the end bearings.
3.7.2. **Loading Procedures:**

i) a slow rate of deflection, in the range of 0.0025 to 0.005 in. per min., of the loading head was selected. Load stages for instrument readings were chosen at every 5 kips from 0 kips to 35 kips; readings were taken without interrupting the loading rate.

ii) at loads beyond 35 kips, readings were taken at every additional 2 kips or whenever practical, until the peak load had been reached. Thereafter, as many readings as possible were made as the load descended and the deflections increased.

iii) once the lateral deflection of the columns had increased to the point that sufficient data had been acquired, the test was stopped and the loading head of the testing machine raised until there was no load on the column. Several columns were tested to material failure. A typical column test took about one hour to complete.
4. Results from the Testing Program.


With the exception of the first column test, the behavior of the columns showed remarkable similarity. In testing the first column, cracking originated on the web about three feet from the top of the column. Although other cracks appeared throughout the length of the specimen, the first crack grew most rapidly; thereafter, all the curvature tended to be concentrated at that section. It would appear that this was an unusually weak section on that particular column so that it reached a peak load of 35 kips while the remaining columns peaked at loads between 40 and 45 kips. For this reason, and because of difficulties in interpretation of the deflection guages from photographs, results from the first test have not been included in this thesis.

The remaining five columns all developed reasonably symmetrical crack distributions throughout their central regions. Cracks did not begin to appear until the peak load was approached within one kip. These began as fine hair-line cracks approximately equally spaced eight to twelve inches apart. As the lateral deflections increased, these cracks grew quickly while further cracking was initiated toward the ends of the column. All the cracks began at the outer fibre of the web and travelled inward normal to the axis of the column. About three inches from the outer fibre of the web, the cracks branched diagonally and developed a Y shape. This pattern can be seen in Fig. 13a. Fig. 13b and 13c illustrate the symmetrical crack distribution. Fig. 13d gives some indication of the amount of curvature.
sustained by the columns before material failure occurred.

Material failure was usually preceded by one of the cracks in the central regions of the column opening wide. Failure of the concrete by compression in the flange adjacent to the wide crack would soon follow. Fig. 14a. and 14b. illustrate failure of the concrete in the flange.

During the course of the experimental programme, it was fortunate that only one crack passed beneath the epoxied transducer holders; that occurred on the second column beneath a transducer located 22\(\frac{1}{2}\) inches below mid-height.

A typical load-axial deflection curve recorded for the columns is shown in Fig. 15. From this graph it is evident that the shape of the unstable load deflection curve for the column did indeed approach that of the testing machine curve. Fig. 6 obtained during the preparatory stages was incorrect as it measured only the centroidal shoretening and neglected rotational deflection due to the eccentricity of the load. Furthermore, Fig. 6 was calculated using the approximate relation \(\Delta_1 = \frac{PL}{AE}\). This was subsequently refined by using the strains at the plastic centroid of each column segment in the computer programme. This result is also shown on Fig. 15.

4.2. **Concrete Stress-Strain Curves.**

The concrete stress-strain curves obtained in the laboratory on standard 6 inch diameter, 12 inch long cylinders, and subsequently used in the computer model are shown in Figs. 16a to 16e. The peak load ranged from approximately 5.04 to 5.50 kips, with \(\varepsilon_0\) values of 0.0028 to 0.0031 and failure strains
\( \varepsilon_{\text{max}} \), from 0.0035 to 0.0040 for compression tests of fifteen to thirty minute duration. Failure of the cylinders occurred when the slope of the concrete load deflection curve became equal to the slope of the testing machine unloading curve. Fig. 16b shows a modified concrete stress-strain curve the use of which will be discussed in section 4.5.

4.3. Load-Moment Interaction Curve.

Using the method discussed in Section 2.6, end moment vs. mid-length moment curves were plotted for columns 2, 3, 4, 5 and 6 using the experimentally obtained concrete and steel stress-strain curves. These graphs (Figs. 17a to 17e) were subsequently used to produce graphs of load vs. mid-length moment for the mathematical model for a 1.75 inch end eccentricity. The load-mid-length moment curves are shown in Figs. 18a to 18d for columns 3, 4, 5, 6; the curves for column 2 are discussed in section 4.5.

The corresponding experimentally acquired load-mid-length moment curves are also plotted on Figs. 18a to 18d, for comparison with the computer models. The experimental curves were determined by measuring the total midspan eccentricity (which included the 1.75 inch end eccentricity plus the measured midspan deflection) for various loads, using the experimental column deflection curves (Figs. 12b to 12e). The mid-length moment was then simply the total midspan eccentricity multiplied by the load for a particular stage of the test.

The experimental and mathematically obtained load-mid-height moment curves are closely correlated, although the
mathematical curves underestimate the peak load and curvature slightly. More importantly, the experimental curves exhibit the same downward path after the peak load has been reached, confirming the existence of the unstable condition predicted by the computer model.

4.4 Moment-Curvature Relationships.

In the mathematical model, combinations of moment and curvature for a chosen load can be found by inserting the load as the ordinate on the load-moment-curvature graph (Fig. 2b) to produce a curve such as Fig. 2c by finding intersection points on the contours of curvature. The computer model can output these values for selected loads. Thus, knowing the load, and entering the midheight moment as the ordinate on a curve such as Fig. 2c, the corresponding curvature is obtained.

Experimentally, curvatures were observed from the seven cantilever devices spanning the central portion of the column from 22\frac{1}{2} inches above to 30 inches below mid-height. In addition, experimental values of curvature were found by differentiating the experimental column deflection curves at the same seven corresponding positions along the column by the procedure described in section 3.6.1. These values are plotted for each column in Figs. 19a to 19e, showing the range of experimental values and their average, as well as the curvatures predicted for the same moments by the computer model.

The experimental attempts to measure curvature do not compare well with predicted values, despite good agreement in the overall behaviour of the columns. In view of the poor
agreement, a study was made of column 2 to determine how factors such as the shape of the concrete stress-strain curve, the value of the prestressing force, and built-in deformities might affect the moment-curvature relationships. These effects are discussed in the next section.

4.5 Case Study: Column 2: Factors Affecting Moment-Curvature Relationships.

4.5.1 Concrete Stress Strain Curves.

Examination of Figs. 19a to 19e suggests that the mathematically obtained moment-curvature relationships lead to larger curvatures than those measured experimentally. This would imply that the mathematical model was not as stiff as the real one. Since the only factors involved in determining the stiffness of the column were the steel and concrete stress-strain curves, it was suspected that the strains obtained in the concrete cylinder tests were excessively high, especially since the commonly quoted strain at peak stress is $\varepsilon_o = 0.0020$. To study the effect of reducing $\varepsilon_o$ to 0.0020, the strains of the original concrete stress-strain curve were reduced proportionally so that the peak strain became 0.0020 and $\varepsilon_{ult}$ became 0.0025. When this was done, the agreement between experimental and computed values was much improved (Fig. 20a) compared to the original values (Fig. 20b). Unfortunately, changing the concrete stress-strain curve resulted in a computed load-mid-moment curve with a much higher peak load (Fig. 21a). The original load mid-moment curve is illustrated in Fig. 21b for comparison.
At first it was thought that snap through buckling might account for the lower peak load obtained for the experimental model as opposed to the computer model with the modified concrete curve. Then agreement between computed and predicted moment-curvature relationships, could be obtained, and at the same time the discrepancy between curves could be accounted for. However, for reasons to be discussed in section 5.4, this proved unlikely to be the case, so snap-through buckling was discarded as a possibility and the modified stress-strain curve assumption was abandoned.

To study the effect of varying the concrete failure strains, the short column interaction curves were re-computed based on the modified concrete stress-strain curve having \( \varepsilon_0 = 0.0020 \) but using values of \( \varepsilon_{ult} \) from 0.22% to 0.34%. The results were similar regardless of whether the original or modified curves were used. The curves obtained are shown in Fig. 22. The experimental loads at material failure for the six test columns were found to range from 15 to 22 kips. For that load range, it is evident that the failure strain of the concrete would have little significance since all the short column interaction curves are nearly co-incident. It is interesting that for T sections, the failure strain \( \varepsilon_{ult} \), which produces the largest envelope of possible load-moment combinations, (which would presumably be reached by a real short column) is not much greater the concrete strain \( \varepsilon_0 \) at peak stress. This fact was also observed by H. Rusch (25), who pointed out that the failure strain of concrete in flexure is highly dependent on the shape of the section. Rusch's observations are illustrated in Fig 23.
Another factor which might affect the concrete stress-strain distribution is related to the fact that stress-strain relations in axial compression and flexure are not the same, partly due to the influence of the stress gradient in flexure, and even more due to differences in the time rate of application of strain. When the time rate of application of strain is considered, the effect is to increase the strain at peak stress, while peak stress is reduced slightly, relative to the short duration cylinder test. Fig. 24 shows schematically how this might happen according to Rusch. Rusch also found that stress gradients effects were negligible.

The above discussion suggests that the concrete stress-strain curves should be modified by increasing $\varepsilon_o$, however, this would serve to make the computer model less stiff than before and the correlation of moment and curvature would become worse.

4.5.2 Prestressing Force.

Fig. 25 shows the computed load-mid-height moment curve for column 2 using both an 8 kip and a 16 kip final prestressing force. The correlation between these curves and the experimentally obtained curve is not as good as that in which the prestress force was calculated to be 13.35 kips, based on the original estimate of the prestress losses. In addition, there is poor agreement between experimental and calculated curvatures (Fig. 26) especially if an 8 kip prestress force is used. It is unlikely, therefore, that a gross error in estimating the final prestress force would account for the unsatisfactory agreement between observed and predicted curvatures.
4.5.3 Effect of Built in Deformation.

No column can be constructed without built-in imperfections or non-homogeneity. Despite attempts to manufacture the columns used in the test programme as accurately as possible, it is not unlikely that lateral deformations in the plane of the web existed prior to testing. Such deformations could be caused by inaccuracies in the formwork, and in placing the concrete in the forms, slight misplacement of the prestress, or perhaps by additional curvatures resulting from creep effects due to the slight eccentricity on the prestress. Any known eccentricities would be taken into account in the programme.

To study the effect of built-in deformations, the computer model was modified as follows:

i) The moment-curvature relationships were calculated as before, assuming no change in the material and sectional properties of the column.

ii) An estimate was made of built-in deflections along the column.

iii) For a particular load, the mid-height moment was due to the sum of the built-in deformation and lateral deflection caused by bending of the column. The curvatures in the next segment, a distance \( \Delta x \) along the column, were calculated using the Taylor series expansion as described in section 2.5 with the total moment at mid-height.

iv) The deflection of a point a distance \( \Delta x \) along the column from mid-height was calculated using the previously computed curvature. This was the deflection
at the point due to bending of the column alone; to this was added the built-in deformation at the point. The total deflection could then be used to determine the moment sustained by the column at that cross-section.

v) The curvature in the next segment was based on the total moment obtained at the previous point on the column.

vi) The calculation was continued in this way along the length of the column until the thrust line was crossed or the column length exceeded.

The modified computer programme could now produce end moment vs. mid-length moment curves which included initial deformations. From these, new load-mid-height moment curves could also be obtained.

By examining the column deflection curves, slopes and curvatures, it was found that the curvatures in the midspan regions of the column were only slightly affected by small built-in deformations, provided the load was stable, and less than about 90% of the peak load.

This can be understood by studying the moment curvature curves such as Fig. 2c for various loads. Under stable load conditions it is found that one is always dealing with the steepest portion of the moment curvature curve. Thus, a small additional moment due to an imperfection in the column straightness does not significantly change the curvature obtained. The curvatures only begin to increase rapidly with small changes in the moment when one is approaching the peak load. It is in this range
that built in deformations have the greatest effect. A built-in deformation, in the plane of bending will result in a reduced peak load if it is in the same direction as the bending. However, in the lower range of stable loads, it will not affect the curvatures to any great extent.

On the above basis, it is, therefore, believed that built-in deformations cannot account for the discrepancy between observed and predicted curvatures. For further confirmation, a study of column 2 was made using a 16 kip prestressing force and varying the built-in deflection. For a built-in midspan deflection of 0.1 inches in the same direction as bending occurred, there was excellent agreement between observed and calculated load-mid-height curves (Fig. 27a); the same correlation was not found with moment curvature relationships (Fig. 28a). A good correlation with moments and curvatures could be found assuming 0.2 inches maximum built in deflection which opposed the direction of bending, (Fig. 28b), but this led to poor agreement with load-mid-length moment curves (Fig. 27b).

4.6 Summary.

The over-all behavior of the concrete columns closely matched the predicted behavior as witnessed by the load mid-height moment curves. Attempts to compare the observed curvatures with calculated values proved unsuccessful despite examination of various factors which might have affected the moment-curvature relationships. It can only be concluded that either:

i) The attempts to measure curvatures in the laboratory were unsuccessful.
ii) The over-all behavior of the columns was not significantly affected by wide variations in moment curvature relationships while the columns remained stable.
5. **Investigations into Snap-Through Buckling.**

5.1 **Potential Energy of the Column.**

The computer programme used to obtain column deflection curves was easily adapted to determine the potential energy of the column. It will be recalled that the method of determining the column deflection curves involved a Taylor series expansion in which the slopes and curvatures were calculated for equally spaced segments along the length of a column. For each column length studied, the end moment and slope were also determined.

For uni-axial bending, there are four terms necessary (neglecting higher order effects) in the potential energy expression for a column. They are:

i) Loss of energy of the load caused by shortening of the column due to compressive strain along the plastic centroid.

ii) Loss of energy of the load resulting from the column ends moving closer together as the column bends laterally.

iii) Loss of energy of the end moments (about the plastic centroid) rotating through the end slopes.

iv) Strain energy in the column (energy gain).

In the computer model, each of the above terms is resolved as follows:

i) The energy loss per unit length due to compressive shortening of the column is determined by the method described in sections 2.3 and 2.4 from the strain distribution of each segment along the column length.
By integrating the energy loss per unit length for each segment along the length of the column, the total energy loss due to compressive shortening is obtained.

ii) Referring to Fig. 29, energy loss of the load due to bending of the column is found:

\[ dl = \sqrt{dx^2 + dy^2} \]

\[ = \left[ 1 + \frac{1}{2} \left( \frac{dy}{dx} \right)^2 - \frac{1}{8} \left( \frac{dy}{dx} \right)^4 + \ldots \right] \, dx \]

(from Binomial Expansion.)

approximating yields:

\[ dl - dx \approx \frac{1}{2} \left( \frac{dy}{dx} \right)^2 \, dx \]

from which the total shortening along the column length would be:

\[ \Delta l = \int_0^1 \frac{1}{2} \left( \frac{dy}{dx} \right)^2 \, dx \]

The loss of potential energy of the load would, therefore, be:

\[ \text{P.E.} = P \int_0^1 \frac{1}{2} \left( \frac{dy}{dx} \right)^2 \, dx \]

Since the slopes \( \frac{dy}{dx} \) are known for each segment along the column length, the above integral can be easily solved numerically.

iii) The energy loss due to the end moments rotating through the end slopes is a simple multiplication performed after each column deflection curve is obtained.

iv) Strain energy of the column:
At any cross-section along the length of the column, the strain distribution determines the curvature at that location as well as the resisting forces and moments on the section. The strain energy per unit length at each section is, therefore, the area under the moment-curvature curve with the curvature ranging from zero to the value determined by the strain distribution. Expressing the above in integral form, the strain energy for a segment of length 'dx' is obtained:

\[ dU = \int_0^\phi M(\phi) \, d\phi \, dx \]

(where \( \phi_0 \) is the total curvature at the section under consideration).

The above expression for \( dU \) can then be integrated along the length of the column to obtain the total strain energy:

\[ U = \int_0^{\phi_0} M(\phi) \, d(\phi) \, dx \]

Addition of the above four terms yields the total potential energy of the column for a given load and column deflection curve.

When several column deflection curves are studied for a column of given length and applied load, a graph of the potential energy of the column can be made. From Fig. 30 it can be seen that only one column deflection curve leads to a relative minimum of potential energy corresponding to stable equilibrium. Likewise, the column deflection curve which produces an unstable configuration corresponds to a relative maximum on the potential energy diagram.
5.2 End Moment vs. Potential Energy Curves.

In the computer model, the first column deflection curve is found for a selected load using the failure moment at mid-height. Thereafter, the mid-height moment is decreased incrementally, resulting in a new column deflection curve for each increment. Each column deflection curve produces a unique combination of end and mid-height moments which can be plotted to obtain a smooth curve as described in section 2.6.

Similarly, each column deflection curve produces a single combination of end moment and potential energy for a given column length. A plot of these values results in curves such as those of Fig. 31.

As in section 2.6, a column whose eccentricity of loading is known may now be studied. The end eccentricity and load determines the end moment to be used as the ordinate on Fig. 31. The curves in Fig. 31 were derived for column 4. As an example, for a 1.75 inch end eccentricity of a 25 kip load, the end moment would be 43.75 inch-kips or 3.65 foot-kips. A horizontal line through 3.65 ft-kips on Fig. 31 intersects the potential energy curve for the 25 kip load at two locations (-1.25 inch-kips and +2.25 inch-kips). Following the above procedure the potential energy of the column under various loading conditions can be found.

Fig. 31 was based on a concrete stress-strain curve having $\varepsilon_0 = 0.0028$ and $\varepsilon_{ult} = 0.0035$. Similar curves were derived for column 4 having concrete with $\varepsilon_0 = 0.0020$ and $\varepsilon_{ult} = 0.0025$ (see Fig. 32).
5.3 Load vs. Potential Energy Curves.

The combinations of load and potential energy obtained in section 5.2 may now be plotted in the same way as load-mid-height moment curves were found (section 2.7).

Fig. 33a illustrates the load-mid-height moment curves for column 4 for concretes having $\varepsilon_0 = 0.0028$ and $\varepsilon_0 = 0.0020$ as well as the experimental results. Fig. 33b shows the corresponding computed load-potential energy curves. Arrows indicate the direction of the potential energy curve as the column is loaded, reaches neutral equilibrium, and is then unloaded down the unstable equilibrium path. In the stable range, the potential energy decreases with increasing load. Once the peak load has been reached, the load decreases while the potential energy increases rapidly along the unstable path.

5.4 Load vs. Energy Difference Curves.

If one considers the difference in potential energy between the stable and unstable equilibrium conditions under a given load, some indication as to the amount of energy required to cause snap-through buckling may be found. These energy difference curves are plotted in Fig. 34b for comparison with load-mid-height moment curves for column 4 (Fig. 34a).

At the outset of the experimental programme, it was believed that such curves would slope downward, convex when viewed from the origin. This would mean the energy difference curve would approach the zero energy position asymptotically, when nearing the peak load. If this were the case, a very small amount of externally applied energy might cause a snap-through
to unstable equilibrium at a somewhat lower load than the theoretical peak.

Fig. 34b, however, shows that the potential energy difference curves do not approach zero energy difference asymptotically as surmised. In fact, for loads just below the peak load, there is a substantial difference in the potential energy between the stable and unstable equilibrium paths. The shape of the load-energy difference curves would, therefore, suggest that the columns are not prone to snap-through buckling. Of course, if sufficient additional energy was provided to jump from the stable to the unstable path, a snap-through might still occur.

Finally, the energy difference curves computed for column 4 for concrete with $\epsilon_o = 0.0020$ indicate that considerable energy would be necessary to cause snap-through buckling at a load equal to the experimentally observed value. For this reason the hypothesis that the concrete stress-strain curves were in error, that the observed moment-curvature relationships were correct, and that the observed load-moment relationship reflected snap-through buckling, was abandoned.
CONCLUSIONS.

The objective of this thesis was to test the validity of a mathematical model which attempted to predict the stability of prestressed concrete columns. Six real prestressed concrete columns were tested in the structural laboratories at the University of British Columbia and their behaviour was compared to the predictions of a computer programme based on the mathematical model. The computed behaviour was closely matched by the real columns. In particular, load versus mid-height moment curves determined by computer approximated the same curves obtained on the test columns favourably. The existence of a downward sloping unstable loading path on the load-mid-height moment curve was found in both the mathematical model and on the test columns.

Having confirmed that the mathematical model did indeed approximate real behaviour satisfactorily, the possibility of a snap-through type of buckling was examined. Differences in the potential energies associated with the stable and unstable loading paths for the column were determined, and a graph of the energy difference as a function of load was prepared. Initially it was thought that the energy difference between the two equilibrium paths would approach zero asymptotically as the critical load was approached. However, results from the computer programme indicate that the columns were not prone to snap-through buckling. It should be noted, though, that sufficient additional energy applied to the column near the critical load might still cause a snap-through.
In general, the instrumentation used to monitor the conditions of the test columns behaved well. Attempts to measure the curvatures of the columns during the testing, and then to compare the curvatures to computer-predicted values were unfortunately, unsuccessful. No satisfactory explanation for the poor comparison could be found, although the following two reasons were suggested:

i) the deflection measurements necessary to obtain the curvatures were too small to measure accurately despite use of linearly varying displacement transducers and precise monitoring equipment.

ii) the curvatures measured were perhaps correct but the overall behaviour of the column was not significantly affected by wide variations in moment-curvature relationships while the column remained stable.

Further research in this field might be directed toward adapting the mathematical model to handle various conditions of end fixity in prestressed columns. Development of an accurate method of measuring curvature on real concrete members would also be useful in both reinforced and prestressed concrete research.
Each value on curve is determined by different neutral axis depth.

SHORT COLUMN INTERACTION CURVE

Fig.1b.
STRAIN DIAGRAMS

Fig. 2a.

LOAD-MOMENT-CURVATURE RELATIONSHIPS

Each value on contour line is determined by different neutral axis depth.

Fig. 2b.
Curvature

MOMENT-CURVATURE RELATION FOR LOAD $P_0$

Fig. 2c.
Fig. 2d. LOAD ENERGY CONTOURS DUE TO COMPRESSIVE SHORTENING OF THE PLASTIC CENTROID
COLUMN DEFLECTION CURVE
Fig. 3a.

COLUMN DEFLECTION CURVES FOR VARIOUS MID-HEIGHT DEFLECTIONS
Fig. 3b.
Note: $P_1 < P_2 < P_3 < P_4$

Single value of end moment may give two values of mid-height moment.
LOAD vs. MID-HEIGHT MOMENT CURVES

Fig. 5.
Axial Deflection (inches)

LOAD-DEFLECTION CURVES

Fig. 6.
Note: all dimensions in inches.

1 inch diameter roller bearing

½ inch thick bearing plate

½ inch thick steel end block cast into column.

Fig. 7a. COLUMN DIMENSIONS
Fig. 7b. END PLATES and END REINFORCING
Fig. 7c. END BEARINGS
Cross-sectional area: 0.080 in.²
Failure stress: 265 k.s.i.
Failure strain: 6.0%
LOAD-MID-HEIGHT MOMENT INTERACTION CURVES

Fig. 9.
Fig. 10a. Curvature measurement using cantilever system. Each transducer mounted in aluminum holder measures tip deflection of adjacent cantilever.

Fig. 10b. Arrangement of cantilevers in straight line along web of column. Cantilevers measured curvature in column from 22½ inches above to 30 inches below midheight. (The fifth transducer from the bottom is at midheight.)
SCHEMATIC DIAGRAM: CANTILEVER CURVATURE MEASURING DEVICES
(not to scale)

APPROXIMATION USED TO DETERMINE CURVATURE OF TYPICAL SEGMENT O-A

Fig. 10c.
Fig. 11a. Illustrates device for obtaining column deflection curves. Lower end of steel tube is connected to concrete column with Schneeberger bearings. Transducer core is elastically loaded to bear upon masking tape track. Pulley system is powered by electric drill. Second transducer measures motion of cantilever connected to steel tube, thereby determining axial deflection of the concrete column.

Fig. 11b. This photograph shows the carriage at the bottom end of the steel tube rail. The masking tape track along the centre line of the flange of the concrete column is clearly visible.
Fig. 11c. The 2-inch square steel tube rail and pulley system can be seen just to the right of the concrete column. The circular object at the top is a mounting bracket for the transducer monitoring the rotation of the top pulley.

Fig. 11d. Illustrates use of x-y plotter to obtain column deflection curves. Curves are bunched together under stable load conditions but spread apart as lateral deflections increase rapidly under unstable load conditions.
Fig. 12a.

COLUMN DEFLECTION CURVES

COLUMN 2

LOAD (kips)

- 25.0
- 32.5
- 41.0
- 41.2
- 40.0
- 35.0
- 30.0
- 25.0
- 20.0
- 15.0
- 10.0

ECCENTRICITY (INCHES)

THRUST LINE (INCHES)
COLUMN DEFLECTION CURVES

COLUMN 3

LOAD (kips)

20.0
22.0
25.0
29.0
41.0
42.5
40.0
35.0
30.0
25.0
20.0
15.0
10.0

THRUST LINE (INCHES)

Fig. 12b.
COLUMN DEFLECTION CURVES

COLUMN 4

LOAD (kips)

THRUST LINE (INCHES)

ECCENTRICITY (INCHES)

Fig. 12c.
COLUMN DEFLECTION CURVES
COLUMN 5

LOAD (kips)
20.5
26.0
28.5
42.0
40.0
35.0
30.0
25.0
20.0
15.0
10.0
5.0
2.0

Fig. 12d.
COLUMN DEFLECTION CURVES
COLUMN 6

LOAD (kips)

THRUST LINE (INCHES)

ECCENTRICITY (INCHES)

Fig. 12e.
Fig. 13a. Illustrates crack pattern developed by column after peak load has been reached. Notice branch of 'Y'-shape beginning about 3 inches from edge of web.

Fig. 13b. Illustrates the symmetrical crack distribution in central portions of the column. Crack spacing ranges from 8 to 12 inches. Transducers are spaced 7½ inches apart.
Fig. 13c. This photograph also shows the equally spaced crack pattern in central regions of the column.

Fig. 13d. The view from the bottom of the column shows the amount of curvature the column was capable of sustaining before failure of the concrete occurred in compression.
Figs. 14a. and 14b. show compression failure of the concrete flange at the ultimate lateral deflection. These are photographs of the second column tested.
LOAD-AXIAL SHORTENING CURVE FOR COLUMN 4

- experimental
- computed

Fig. 15
Fig. 16a. and 16b. Concrete Stress Strain Curves

COLUMN 1

COLUMN 2

\[ \varepsilon_0 = 0.0028 \]
\[ \varepsilon_{\text{max}} = 0.0033 \]

\[ \varepsilon_0 = 0.0020 \]
\[ \varepsilon_0 = 0.0031 \]
\[ \varepsilon_{\text{max}} = 0.0025 \]
\[ \varepsilon_{\text{max}} = 0.0038 \]
Figures 16c. and 16d. CONCRETE STRESS STRAIN CURVES
Figures 16e. and 16f. CONCRETE STRESS STRAIN CURVES
Figs. 17a. and 17b. END vs. MID-HEIGHT MOMENT CURVES
Figs. 17c. and 17d. END vs. MID-HEIGHT MOMENT CURVES
Fig. 17e. END vs. MID-HEIGHT MOMENT CURVE
Figs. 18a. and 18b. LOAD-MID-HEIGHT MOMENT CURVES
Figs. 18c. and 18d. LOAD-MID-HEIGHT MOMENT CURVES
Fig. 19a. OBSERVED and PREDICTED CURVATURES
Fig. 19b. OBSERVED and PREDICTED CURVATURES
Average (Cantilever Devices)
Average (Observed Deflection Method)
Range of Observed Values for Each System
Computer Predicted Curvatures

Fig. 19c. OBSERVED AND PREDICTED CURVATURES
Fig. 19d. OBSERVED and PREDICTED CURVATURES
Fig. 19e. OBSERVED and PREDICTED CURVATURES
MOMENT-CURVATURE RELATIONSHIPS
for COLUMN 2

- prestress force: $13.35^k$
- max. built-in
defl'n. = 0 in.
- concrete:
  $\varepsilon_0 = 0.0020$

$\Delta$ Cantilever system
○ Measured CDC system
□ Computer prediction

Fig. 20a.

- prestress force: $13.35^k$
- max. built-in
defl'n. = 0 in.
- concrete:
  $\varepsilon_0 = 0.0031$

$\Delta$ Cantilever system
○ Measured CDC system
□ Computer prediction

Fig. 20b.
Figs. 21a. and 21b. LOAD-MID-HEIGHT MOMENT CURVES

---

**Fig. 21a.**

- **Experimental**
- **Computed using:**
  - Prestress force = 13.35 k
  - Max. built-in defl'n. = 0 in.
  - Concrete:
    - $\varepsilon_0 = 0.0020$

**Fig. 21b.**

- **Experimental**
- **Computed using:**
  - Prestress force = 13.35 k
  - Max. built-in defl'n = 0 in.
  - Concrete:
    - $\varepsilon_0 = 0.0031$
Fig. 22. LOAD-MOMENT INTERACTION CURVES FOR VARIOUS CONCRETE FAILURE STRAINS
Fig. 23a. Strain and stress distribution at ultimate strength after 1 hr, $f'_c = 3000$ psi at 56 days.

Fig. 23b. Ultimate strain as a function of cross section and position of neutral axis.
**Fig. 24a.** Stress-strain curves for various strain rates of concentric loading

**Fig. 24b.** Determination of stress-strain relationship in flexure (schematic only) 

*left* Stress-strain curves for concentric compression and various strain rates

*right* Stress-strain relationship for eccentric compression after 1 hr of loading at constant strain rates

![Graph showing stress-strain relationship](image-url)
Fig. 25. LOAD-MID-HEIGHT MOMENT CURVES

Fig. 26. MOMENT-CURVATURE RELATIONSHIPS
Figs. 27a. and 27b. LOAD-MID-HEIGHT MOMENT CURVES
MOMENT-CURVATURE RELATIONSHIPS
for COLUMN 2

- Prestress force = 16
- Max. built-in defl'n = +0.1 in.
- Concrete:
  \( \varepsilon_0 = 0.0031 \)

Cantilever system
○ Measured CDC system
□ Computer prediction

Fig. 28a.

- Prestress force = 16
- Max. built-in defl'n = -0.2 in.
- Concrete:
  \( \varepsilon_0 = 0.0031 \)

Cantilever system
○ Measured CDC system
□ Computer prediction

Fig. 28b.
Fig. 29. SHORTENING ALONG THE LINE OF LOADING of the COLUMN DUE TO BENDING
load-mid-height curve (typical)

Line of No Slenderness Effect

Short Column Interaction Curve

stable equilibrium

unstable equilibrium

*Fig. 30.*
Fig. 31.

Potential Energy (kips-in.)

COLUMN 4
concrete: $\varepsilon_0 = 0.0028$
$\varepsilon_{ult} = 0.0035$
COLUMN 4
concrete: $\varepsilon_0 = 0.0020$
$\varepsilon_{ult} = 0.0025$

Fig. 32.
Fig. 33a. LOAD-MID-HEIGHT MOMENT CURVES

A: experimental curve
B: computed curve for $\epsilon_0 = 0.0028$
C: computed curve for $\epsilon_0 = 0.0020$

Fig. 33b. LOAD-POTENTIAL ENERGY CURVES

B: computed curve for $\epsilon_0 = 0.0028$
C: computed curve for $\epsilon_0 = 0.0020$
Fig. 34a. LOAD-MID-HEIGHT MOMENT CURVES

Fig. 34b. LOAD-ENERGY DIFFERENCE CURVES
BIBLIOGRAPHY.


7) Itaya, R., "Design and Uses of Prestressed Concrete Columns," Journal of the Prestressed Concrete Institute, June, 1965, p. 69.


