FINITE STRIP ANALYSIS OF SANDWICH PANELS

by

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Abstract

A finite strip analysis of sandwich wood panels is presented. The panels consist of upper and lower plates stiffened by beams (joists) in one direction only. The analysis considers a Fourier series expansion in the stiffeners' direction and a polynomial finite element approximation in the direction normal to the stiffeners. The number of degrees of freedom is 34, which includes also the lateral and torsional deformation of the joists. This permits consideration of the effect of joist bridging on the maximum deflection and bending stresses.

A maximum of 20 joists per panel can be analysed. The modulus of elasticity ($E$) of the joists may be selected randomly from a distribution, and controlled to be within a given range. The upper and lower plates may possess orthotropic properties. Nails connect the plates to the joists. Nailing may be considered either as a continuous or as discrete connectors. The loading may be in the form of an uniformly distributed load spread over the entire panel or over a maximum of 20 smaller (concentrated) areas of the top plate.

Numerical investigations have been carried out to verify the program. Parametric studies have been done to understand the behavior of the model. Lastly, the formula for shear lag given by CSA Standard CAN3-086.1-M84 is checked against the shear lag obtained from the current computer program.
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Notation

\( u \) \hspace{1cm} \text{axial displacement of plate}

\( v \) \hspace{1cm} \text{lateral displacement of plate}

\( w \) \hspace{1cm} \text{vertical displacement of plate}

\( U \) \hspace{1cm} \text{axial displacement of joist}

\( V \) \hspace{1cm} \text{lateral displacement of joist}

\( W \) \hspace{1cm} \text{vertical displacement of joist.}

\( \theta \) \hspace{1cm} \text{angle of rotation of the joist}

\( u_{1n} \) \hspace{1cm} \text{axial displacement at node 1 of the finite-strip element}

\hspace{1cm} \text{for the } n \text{th term of the Fourier series}

\( v_{1n} \) \hspace{1cm} \text{lateral displacement at node 1 of the finite-strip element}

\hspace{1cm} \text{for the } n \text{th term of the Fourier series}

\( w_{1n} \) \hspace{1cm} \text{vertical displacement at node 1 of the finite-strip element}

\hspace{1cm} \text{for the } n \text{th term of the Fourier series}

\( u'_{1n} \) \hspace{1cm} \text{derivative of } u_{1n} \text{ with respect to } x.

\hspace{1cm} \text{Similarly } u'_{3n}, u'_{4n} \text{ & } u'_{6n}

\( v'_{1n} \) \hspace{1cm} \text{derivative of } v_{1n} \text{ with respect to } x.

\hspace{1cm} \text{Similarly } v'_{3n}, v'_{4n} \text{ & } v'_{6n}

\( w'_{1n} \) \hspace{1cm} \text{derivative of } w_{1n} \text{ with respect to } x.

\hspace{1cm} \text{Similarly } w'_{2n}, w'_{3n}, w'_{4n} \text{ & } w'_{5n}, w'_{6n}

\( \xi, \eta \) \hspace{1cm} \text{normalised coordinates}

\( M_0 \) \hspace{1cm} \text{shape function. Similarly } M_1, M_2, \ldots, M_{13}

\( F_{1n} \) \hspace{1cm} \text{product of shape function and deformation vector.}

\hspace{1cm} \text{Similarly } F_{2n}, F_{3n}, \ldots, F_{6n}
$K_x, K_{x1}$ flexural stiffness in the $x$ direction for upper & lower plates

$K_y, K_{y1}$ flexural stiffness in the $y$ direction for upper & lower plates

$K_{\nu}, K_{\nu1}$ coefficient associated with Poisson's ratio for upper & lower plates

$D_x, D_{x1}$ axial stiffness in $x$ direction for upper & lower plates

$D_y, D_{y1}$ axial stiffness in $y$ direction for upper & lower plates

$D_{\nu}, D_{\nu1}$ coefficient associated with Poisson's ratio for upper & lower plates

$D_G, D_{G1}$ in plane shear stiffness for upper and lower plates

$E_x, E_{x1}$ modulus of elasticity of upper & lower plates in $x$ direction

$E_y, E_{y1}$ modulus of elasticity of upper & lower plates in $y$ direction

$\nu_{xy}$ Poisson's ratio, strain in $x$ direction when stress is applied in $y$ direction (for upper plate)

$\nu_{yx}$ Poisson's ratio, strain in $y$ direction when stress is applied in $y$ direction (for upper plate)

$G, G1$ shear modulus in $x - y$ plane for upper & lower plates

$\nu_{x\nu1}$ Poisson's ratio, strain in $x$ direction when stress is applied in $y$ direction (for lower plate)

$\nu_{y\nu1}$ Poisson's ratio, strain in $y$ direction when stress is applied in $y$ direction (for lower plate)
$U_U$ strain energy in the upper plate of one strip element

$U_L$ strain energy in the lower plate of one strip element

$U_J$ strain energy in the joist

$U_{NC}$ strain energy in the upper plate connectors

$U_{NL}$ strain energy in the lower plate connectors

$U$ total strain energy in one T-beam strip element

$\delta_n$ deformation vector for one T-beam strip element corresponding to the $n$-th Fourier term

$\Delta_k$ global deformation vector for all the elements corresponding to the $k$-th Fourier term

$R_k$ load vector corresponding to the $k$-th Fourier term

$\epsilon$ tolerance factor

$NJT$ number of joists

$d$ thickness of the upper plate

$HJT$ depth of the joist

$s$ spacing between the joists

$L_p$ panel span

$X_G$ panel geometry reduction factor to account for shear lag.
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CHAPTER 1

Introduction and Literature Review

1.1 General Remarks

In the early ages of civilization the existence of stiffened structural forms was perhaps learnt from the great book of nature. Sea shells, leaves and trees - all of these are, in fact stiffened structures. The observation of such structures created by nature indicate that strength and rigidity depend not only on the material but also on the form. This was realised by the Egyptians at least around 3000 B.C. and they developed a craft made of planks fastened around a wooden framework. Also the ancient Viking ships were made of planks which were tied on the inside to ribs.

However, due to limited number of strong materials and limited knowledge about them, the development of stiffened structural elements was very much restricted. Their wide use began in the nineteenth century, mainly with the application of the steel plates for hulls of ships and with the development of the steel bridges and aircraft structures. In addition to the applications above, stiffened plates are also widely used in the shape of ribbed, waffle and sandwich type slabs for floor and roof construction in buildings. The rigidity of the plates is obtained either by various types of stiffeners reinforcing thin sheets or by the geometry of the form of these sheets, as in the stressed skin panels for the walls of buildings. Very often such sheets derive their stiffness from a corrugated geometry.
1.2 The Model and The Existing Methods

1.2.1 The Model

The type of wood-panel system considered in the present study is shown in Fig. 1.1. Two covers, also called sheathing, are fastened (nailed) to the wood joist at the top and bottom. This assembly behaves as a stiffened plate under the action of load.

Figure 1.1: Wood Panel Assembly
The cover may be considered as an orthotropic plate because it may have different modulus of elasticity in the directions parallel and perpendicular to the joist. The panels making up the cover may have gaps between them which will influence the behavior of the structure. Each of the joists may have different moduli of elasticity because of wood’s natural variability, and the fastener’s properties will depend on a large number of parameters, some of which are the type of the fastener, nail diameter, type of the nail coating, depth of penetration of the nails and direction of nail penetration in relation to the grain direction in the joist.

A simple way of analysing this structure is to assume that the applied load is carried by the joists only, thereby not taking into account any composite action in the assembly. This approach may be correct if the bending stiffness of the covers is very small relative to that of the joists and the nail interval is large. However in realistic conditions, such analysis cannot be proved justified.

The model presented here considers the structure as a single/multi cell box-girder deck. Let us at first look into the analytical finite element and finite-strip methods available for solving the problem.

1.2.2 The Existing Methods:

1.2.2.1 Analysis of Box-Girders as Thin-Walled Beams

In this method, the actual thin walled space structure is regarded as a single beam. The application of the method is advantageous if the girder cross-section is not too complicated (single or double box cell) and if the cross-sectional dimensions are small in relation to the span. The girder may have a variable cross-section (such as variable thickness of webs, or also the height), the various parts of the structure may be made of materials having different properties, and the static system may be fairly
complex (continuous beam, framework). The solution is based on a number of simplifying assumptions and belongs therefore into the group of methods based on the ordinary theory of beams. In accordance with the accepted assumption concerning the small dimensions of the girder cross section in relation to the length of the span, the transverse bending and the torsional effects of the separate walls of the structure may be neglected (one-way action) and a simple longitudinal bending of cross-section assumed.

1.2.2.2 Analysis of Box-girders as Folded Plate Structures

If the box-girder is of constant cross-section and if its end cross-sections, perpendicular to the girder axis, are transversely stiffened against the deformation of their shape, it is very advantageous to analyse it as a folded plate structure. Constant cross-section of the structure is the basic prerequisite for the direct application of this method. In some cases of exceptional arrangements, continuous structures of variable cross-section can be analysed also; for the solution of general folded-plate structures of variable cross-section, the iterative procedure is suitable. In this procedure the complexity of the cross-section as well as the ratio of the cross-sectional dimensions to the span are nearly immaterial, because the solution is not based on ordinary girder principles, but on the elasticity plane stress and the plate bending theory. Within the scope of assumptions of the theory of elasticity, the folded plate theory represents an exact method, because it considers the structure in its actual form as an assembly of rectangular plate-shaped parts forming together a real spatial system. Hence the solution does not make any difference between open and closed cross-sections on the multi-cell structures, nor does differentiate the various kinds of stresses (bending, shear, torsion); the loading of the structure may be quite general (horizontal, vertical, longitudinal, arbitrarily distributed on any surface and/or linear, continuous moment loading, concentrated forces etc.).
1.2.2.3 Plate Idealization of Box Structure

Box structures, particularly when they are of a massive character represent a transition to plate structures with longitudinal hollows between them, and for this reason the plate model may be used for an approximate calculation.

The simplest way of modelling the actual structure is to substitute it by a solid isotropic plate whose thickness can be determined so that the flexural rigidity of plate corresponds to that of the actual structure. Thus the bending behavior of the structure as a whole is relatively well expressed, but the torsional behavior less precisely, and the transverse distribution of the load is very much over estimated in a more thin-walled structure without diaphragms, because the transverse rigidity of the actual structure is formed almost solely by the frame stiffness of the cross-section. Hence this substitution is suitable particularly for the analysis of the effects of a not pronouncedly concentrated loading.

Better results are obtained by using an orthotropic plate for substitution, or, better still, a sandwich plate composed of three layers: the marginal layers representing the effect of the lower and the upper flanges, and the inner (core) layer transferring only the shear effects which represent the shear and transverse frame behavior of the structure.

1.2.2.4 Grillage Idealization

For structures of a greater width composed of a large number of girders, which by themselves do not have a pronouncedly thin-walled character, which may be connected by a slab, and which are moreover often skew, a solution according to the folded plate theory or by other methods may be too complicated. In this case very good results are obtained by regarding the actual structure as a grillage. Good results can also be arrived at by using Hamberg's proceeding (Ref. 13) where the individual box-girders, on the basis of their stiffnesses in bending and torsion, are
assigned distributing numbers which are then used in the analysis of the grillage by
the harmonic-load method.

1.2.2.5 Finite Element Methods

For the type of problem we are dealing with, either a 2D or a 3D finite element model
can be used.

A simpler form of two dimensional analysis is the grillage analogy and attempts
have been made to employ this method incorporating the effects of the transverse cell
distortion in a cellular structure (Ref. 1,14,16,23). A formulation with shear-weak
plate elements is also available (Ref. 7). In this case the shear stiffness of the plate
is assumed to represent the distortional stiffness of the actual cell. But this approach
does seem suitable for single-cell or twin-cell structures (Ref. 8,25).

An alternate way in which an attempt has been made to utilise the advantages
of the three-dimensional approach whilst at the same time reducing the number of
degrees of freedom per element (and thus reducing the overall size of the problem to
an equivalent two-dimensional model) is by making suitable simplifying assumptions,
for example, omitting the local bending deformations at each node \((\theta_x, \theta_y, \theta_z)\) and
retaining only the three translational degrees as freedom \((u, v, w)\) (Ref. 24).

The use of shear web element coupled with the elimination of the rotational degrees
of freedom for nodes associated with the flanges can also lead to an equivalent two-
dimensional model of a cellular structure (Ref. 12).

To formulate a three-dimensional model, quadrilateral or traingular elements are
used (Ref. 10, 15, 26, 29)

1.2.2.6 Finite Strip Method

The Finite element method is known as the most powerful and versatile tool of so-
lution in structural analysis. However, for many structures having regular geometric
plans and simple boundary conditions, a full finite element analysis is very often unnecessary. For this reason, the finite strip method was developed. In this method the structure is divided into two dimensional (strips) or three-dimensional (prisms, layers) subdomains in which one opposite pair of sides (2D) or one or more opposite pairs of faces (3D) of such a subdomain are in coincidence with the boundaries of the structure.

The finite strip method can be considered as a special form of the finite element procedure using the displacement approach. Unlike the standard finite element method, which uses polynomial displacement functions in all directions, the finite strip method calls for the use of simple polynomials in some directions and continuously differentiable smooth series in the other directions, with the stipulation that the series should satisfy a priori the boundary conditions at the end of the strips or prisms. The general form of a displacement function is given as a product of polynomials and series. Thus for a strip, in which a two dimensional problem is reduced to a one-dimensional problem,

\[ w = \sum_{m=1}^{n} f_m(y)X_m \]  

where, \( f_m(y) \) is the polynomial expression with undetermined constants for the mth term of the series and \( X_m \) is the series which satisfies the end conditions and deflected shape in the x-direction.

Cheung (Ref. 3, 4) formed a rectangular flat strip by combining a bending strip and a plane stress strip. There are four displacement components (u, v, w, \( \theta \)) at each end. He analyzed a right box-girder bridge with this element.

Loo and Cusens (Ref. 18) have demonstrated that improved accuracy can be obtained by using the auxiliary nodal line (ANL) procedure in analyzing different internal stresses in box-girder structures. They also suggested that there is no justification in using more than one ANL per strip. The ANL technique allows higher order functions to be incorporated in the finite-strip formulation but leaves the bandwidth
of the overall matrix equation unchanged.

The 'plate-strip elements' developed by the authors mentioned above were based on Kirchhoff's plate theory. However, when the members of the structure cannot be classified anymore as 'thin' within the context of Kirchhoff's hypothesis, the Kirchhoff's plate theory may produce inaccurate numerical results. Mindlin's plate theory (Ref. 20) which takes into account the effect of transverse shear deformation (thus making it valid for thick plate cases) has been extended by Benson and Hinton (Ref. 2) in the context of the finite strip method for the static and dynamic analysis of plates using the 3-noded strip element. Onate (Ref. 21) extended the same to deal with straight box-girder bridges. Onate and Suarez (Ref. 22) have shown that plates, bridges and axisymmetric shells of uniform transverse cross-section can be treated in a simple and unified form using two-noded Mindlin linear strip elements with a single quadrature integrating point.

Cheung and Fan (Ref. 5) have done static analysis of right box-girder bridges by the spline finite-strip method, in which the function series which satisfies the end conditions a priori in the longitudinal direction has been replaced by a spline function. The converged answers are nearly correct when compared with experimental results and finite element solutions.

Thomson, Goodman and Vanderbilt (Ref. 27) presented a paper which considers a model with a top cover and joists underneath. The paper takes into account the composite action between the cover and the joists. The floor is assumed as a system of crossing beams, with a series of T-beams and sheathing strips perpendicular to these T-beams. Each of the T-beams represent a joist with a portion of 'contributory' cover, which may contain a few layers of sheathing. A finite element analysis is done by further subdividing the T-beams and the sheathing strips into smaller elements and satisfying the compatibility requirement in the form of deformation at the points of intersection. A computer program FEAFLO based on this model has been widely used
in simulating the floor behavior under different levels of variability in joist properties (Ref. 28). The program takes into account the slip between the cover and the joists due to the nail deformation.

In the paper mentioned above, the width of the T-beam flange, which is similar to the idea of effective width, is difficult to estimate beforehand. This is due to the fact that it is dependent upon the ratio of floor span to joist separation. When the ratio is large, the effective width tends to become equal to the joist spacing and this is the value often used in the FEAFLO model. This approach may not be accurate for short spans with comparatively wide opening for joists. Moreover an effective width is always defined with an objective, for example, to equate the deflection of the T-beam with the actual deflection of the stiffened structure. In general, a different effective width would be obtained if the objective is changed to equality of maximum bending stresses. In view of these setbacks, Foschi (Ref. 11) has proposed an improvement in modelling by using the finite strip model implemented in the program FAP. A linear behavior for fastener's has to be assumed.

In the present study Foschi's model has been taken into consideration except that, as mentioned in Art. 1.2.1, a bottom cover has been fastened to the joists.

1.2.3 Objectives of the Present Study

The objective of the study is to look into the behavior of the stiffened structure, with double (top and bottom) covers, and to compare the design provisions for Stressed Skin Panels as given in the CSA Standard CAN3-086.1-M84 "Engineering Design in Wood (Limit States Design)". In this context the verification of the expression for the shear lag effect, as given by Clause 8.6.3.2 of that Code, is to be done and its dependence on the number of longitudinal ribs studied.
CHAPTER 2

Theoretical Formulation

2.1 The Model

A short general description of the finite strip method has been given in the previous chapter.

The panel is modelled as consisting of I-beam finite strips in the x-direction. The nodes considered for each finite strip element are shown in the Fig. 2.1. The deformations of the plate in the y-direction are approximated by a one-dimensional finite element using the nodes shown in the Fig. 2.1. In the x-direction (longitudinal), a Fourier series approximation is employed. Using this model, the deflections of the plates and the joist can be expressed as:

**Upper plate displacements** (middle surface):

Vertical displacement: \[ w(x, y) = \sum_{n=1}^{N} F_{1n}(y) \sin \frac{n\pi x}{L} \] (2.1)

(z-direction)

Axial displacement: \[ u(x, y) = \sum_{n=1}^{N} F_{2n}(y) \cos \frac{n\pi x}{L} \] (2.2)

(x-direction)

Lateral displacement: \[ v(x, y) = \sum_{n=1}^{N} F_{3n}(y) \sin \frac{n\pi x}{L} \] (2.3)

(y-direction)
where \( N \) is the number of terms taken in the Fourier series. Since \( w = v = 0 \) at \( x = L \), the assumed functions imply simply supported boundary conditions at the ends. The functions \( F_{1n}(y) \), \( F_{2n}(y) \) and \( F_{3n}(y) \) are determined from a finite element polynomial approximation in \( y \)-direction. These functions are expressed in terms of the nodal degrees of freedom at points 1, 2 and 3 in Fig. 2.1.

**Joist displacements:**

The displacement functions for joists are expressed as follows

Vertical displacement: \( W(x) = \sum_{n=1}^{N} W_n \sin \frac{n\pi x}{L} \) \( \text{ (in } z \text{- direction) } \) \( (2.4) \)

Axial displacement: \( U(x) = \sum_{n=1}^{N} U_n \cos \frac{n\pi x}{L} \) \( \text{ (in } x \text{- direction) } \) \( (2.5) \)

Lateral displacement: \( V(x) = \sum_{n=1}^{N} V_n \sin \frac{n\pi x}{L} \) \( \text{ (in } y \text{- direction) } \) \( (2.6) \)
Rotation: \[ \theta(x) = \sum_{n=1}^{N} \theta_n \sin \frac{n\pi x}{L} \] (2.7)

where \( W_n, U_n, V_n \) and \( \theta_n \) are joist degrees of freedom as shown in Fig. 2.2.

Lower plate displacement (middle surface):

Vertical displacement: \[ w(x, y) = \sum_{n=1}^{N} F_{4n}(y) \sin \frac{n\pi x}{L} \] (2.8)

Axial displacement: \[ u(x, y) = \sum_{n=1}^{N} F_{5n}(y) \cos \frac{n\pi x}{L} \] (2.9)

Lateral displacement: \[ v(x, y) = \sum_{n=1}^{N} F_{6n}(y) \sin \frac{n\pi x}{L} \] (2.10)
Here, again, the functions $F_{4n}(y)$, $F_{5n}(y)$ and $F_{6n}(y)$ are determined from a finite element polynomial approximation in $y$ - direction. These functions are expressed in terms of the nodal degrees of freedom at points 4, 5 and 6 in Fig. 2.1. For each strip, a total of 34 degrees of freedom are identified associated with each $n$ (term in Fourier series). The elemental nodal degrees of freedom vector is shown below. For the $n$-th Fourier term there are six degrees of freedom at each of the nodes 1, 3, 4 and 6, three at node 2 and 5 and four node B. For example, $w_{1n}'$ is the component denoting the derivative of $w(x,y)$ with respect to $y$ at node 1, multiplied by the joist spacing ‘s’, to make the vector dimensionally consistent.

Using the shape functions shown in the Appendix A, and the non-dimensional variable $\xi = 2y/s$, the functions $F_{1n}(y)$, $F_{2n}(y)$, $F_{3n}(y)$, $F_{4n}(y)$, $F_{5n}$ and $F_{6n}(y)$ can be written in terms of $\{\delta_n\}$:

$$F_{1n}(\xi) = \{M_0(\xi)\}^T\{\delta_n\} \quad (2.11)$$

$$F_{2n}(\xi) = \{M_3(\xi)\}^T\{\delta_n\} \quad (2.12)$$

$$F_{3n}(\xi) = \{M_5(\xi)\}^T\{\delta_n\} \quad (2.13)$$

$$F_{4n}(\xi) = \{M_7(\xi)\}^T\{\delta_n\} \quad (2.14)$$

$$F_{5n}(\xi) = \{M_{10}(\xi)\}^T\{\delta_n\} \quad (2.15)$$
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\[ F_{6n}(\xi) = \{M_{12}(\xi)\}^T \{\delta_n\} \quad (2.16) \]

Furthermore, these equations permit us to obtain the following derivatives:

\[ \frac{dF_{1n}(\xi)}{d\xi} = \{M_1(\xi)\}^T \{\delta_n\} \quad (2.17) \]

\[ \frac{d^2F_{1n}(\xi)}{d\xi} = \{M_2(\xi)\}^T \{\delta_n\} \quad (2.18) \]

\[ \frac{dF_{2n}(\xi)}{d\xi} = \{M_4(\xi)\}^T \{\delta_n\} \quad (2.19) \]

\[ \frac{dF_{3n}(\xi)}{d\xi} = \{M_6(\xi)\}^T \{\delta_n\} \quad (2.20) \]

\[ \frac{dF_{4n}(\xi)}{d\xi} = \{M_8(\xi)\}^T \{\delta_n\} \quad (2.21) \]

\[ \frac{d^2F_{4n}(\xi)}{d\xi} = \{M_9(\xi)\}^T \{\delta_n\} \quad (2.22) \]

\[ \frac{dF_{5n}(\xi)}{d\xi} = \{M_{11}(\xi)\}^T \{\delta_n\} \quad (2.23) \]

\[ \frac{dF_{6n}(\xi)}{d\xi} = \{M_{13}(\xi)\}^T \{\delta_n\} \quad (2.24) \]

With the displacement functions and their derivatives thus defined, the strain energy of a strip element can be computed and the stiffness matrix derived as explained in the next section.

2.2 Derivation of the Stiffness Matrix

The stiffness matrix is obtained from the total strain energy of the system. In order to obtain the total energy, we have to consider contributions from the various components of the floor system i.e. the plates, the joists and the connectors between the joist and the plates.
2.2.1 Strain Energy in the Upper Plate

Using the small deflection orthotropic plate theory, the strain energy per unit area of the upper plate can be expressed as:

\[
\Delta U_u = \left(\frac{K_x}{2}\right)\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{K_y}{2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + \left(\frac{\partial^2 w}{\partial x \partial y}\right) \left(\frac{\partial^2 w}{\partial y \partial x}\right) + 2K_G\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 + \left(\frac{D_x}{2}\right)\left(\frac{\partial^2 u}{\partial x^2}\right)^2 + \left(\frac{D_y}{2}\right)\left(\frac{\partial^2 v}{\partial y^2}\right)^2 + D_y\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial y}\right) + D_G/2 \left(\frac{\partial u}{\partial y} + \left(\frac{\partial v}{\partial x}\right)^2\right)
\]

(2.25)

\[
K_x = \text{flexural stiffness in the } x \text{- direction per unit length} = \frac{E_x d^3}{12(1 - \nu_{xy}\nu_{yx})}
\]

\[
K_y = \text{flexural stiffness in the } y \text{- direction per unit length} = K_x \left(\frac{E_y}{E_x}\right)
\]

\[
K_\nu = \text{coefficient associated with Poisson’s ratio} = \nu_{xy}K_x
\]

\[
K_G = \text{torsional stiffness} = \frac{G d^3}{12}
\]

\[
D_x = \text{axial stiffness in } x \text{ direction} = \frac{E_x d}{(1 - \nu_{xy}\nu_{yx})}
\]

\[
D_y = \text{axial stiffness in } y \text{- direction} = D_x \left(\frac{E_y}{E_x}\right)
\]

\[
D_\nu = \text{coefficient associated with Poisson’s ratio} = \nu_{xy}D_x
\]

\[
D_G = \text{In plane shear stiffness} = Gd
\]

\[
E_x = \text{Modulus of elasticity in } x \text{- direction}
\]

\[
E_y = \text{Modulus of elasticity in } y \text{- direction}
\]

\[
\nu_{xy} = \text{Poisson’s ratio, strain in } x \text{ direction when stress is applied in } y \text{ direction}
\]

\[
\nu_{yx} = \text{Poisson’s ratio, strain in } y \text{ direction when stress is applied in } x \text{ direction}
\]
\( G \) = Shear modulus in x-y plane

The total strain energy, \( U_u \) in the upper plate finite strip can be obtained by integrating over the area as shown Fig. 2.3. Thus the upper plate strain energy can be expressed as:

\[
U_u = \int_{\frac{L}{2}}^{\frac{L}{2}} \int_{0}^{L} \Delta u \, dx \, dy \quad (2.26)
\]

On substituting the expressions for displacements given by eqns. (2.1) to (2.3) into eqn. (2.25) and integrating over the span length, we obtain,

\[
U_{U1} = \int_{0}^{L} (K_x/2) \left( \frac{\partial^2 u}{\partial x^2} \right)^2 dx = (K_x/2) \sum_{n=1}^{N} \frac{F_{1n}^2(y)}{2} \frac{n^4 \pi^4}{2L^3} \quad (2.27)
\]

a result in which the orthogonality of the trigonometric functions has been used.

Similarly,

\[
U_{U2} = \int_{0}^{L} (K_y/2) \left( \frac{\partial^2 u}{\partial y^2} \right)^2 dx = (K_y/2) \sum_{n=1}^{N} \frac{(F_{1n}^r(y))^2}{2L} \quad (2.28)
\]

\[
U_{U3} = \int_{0}^{L} K_\nu \left( \frac{\partial^2 w}{\partial x^2} \right) \left( \frac{\partial^2 w}{\partial y^2} \right) dx = -K_\nu \sum_{n=1}^{N} F_{1n}(y) F_{1n}^r(y) \frac{n^2 \pi^2}{2L} \quad (2.29)
\]

\[
U_{U4} = 2 K_G \int_{0}^{L} (\frac{\partial^2 w}{\partial x \partial y})^2 dx = 2 K_G \sum_{n=1}^{N} (F_{1n}(y))^2 \frac{n^2 \pi^2}{2L} \quad (2.30)
\]

\[
U_{U5} = \int_{0}^{L} (D_x/2) \left( \frac{\partial u}{\partial x} \right)^2 dx = (D_x/2) \sum_{n=1}^{N} (F_{2n}(y))^2 \frac{n^2 \pi^2}{2L} \quad (2.31)
\]

\[
U_{U6} = \int_{0}^{L} (D_y/2) \left( \frac{\partial v}{\partial y} \right)^2 dx = (D_y/2) \sum_{n=1}^{N} (F_{3n}^r(y))^2 (L/2) \quad (2.32)
\]

\[
U_{U7} = \int_{0}^{L} D_\nu \left( \frac{\partial u}{\partial x} \right) \left( \frac{\partial v}{\partial y} \right) dx = -D_\nu \sum_{n=1}^{N} F_{2n}(y) F_{3n}(y) \left( \frac{n \pi}{2} \right) \quad (2.33)
\]

\[
U_{U8} = \int_{0}^{L} (D_G/2) \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 dx

= (D_G/2) \sum_{n=1}^{N} \left[ F_{2n}(y) + (n \pi/L) F_{3n}(y) \right]^2 L/2 \quad (2.34)
\]
Substituting the expressions for the functions $F_1(y), F_2(y)$ and $F_3(y)$ and their derivatives, as given by eqns.(2.11) to (2.13) and eqns. (2.17) to (2.20), integrate over the joist spacing, and we get

$$U_{U1} = \sum_{n=1}^{N} \left( \frac{K_x}{2} \right) \frac{n^4 \pi^4}{4 L^3} s \int_{-1}^{1} \{\delta_n\}^T \{M_0(\xi)\} \{M_0(\xi)\}^T \{\delta_n\} \, d\xi \quad (2.35)$$

$$U_{U2} = \sum_{n=1}^{N} \left( \frac{K_y}{2} \right) \frac{4 L}{s^3} \int_{-1}^{1} \{\delta_n\}^T \{M_2(\xi)\} \{M_2(\xi)\}^T \{\delta_n\} \, d\xi \quad (2.36)$$
\[ U_{U3} = \sum_{n=1}^{N} \left( -K_x/2 \right) \frac{2n^2 \pi^2}{s L} \int_{-1}^{1} \{\delta_n\}^T \{M_0(\xi)\} \{M_2(\xi)\}^T \{\delta_n\} \, d\xi \]  
(2.37)

\[ U_{U4} = \sum_{n=1}^{N} 2K_G \frac{n^2 \pi^2 s}{4L} \int_{-1}^{1} \{\delta_n\}^T \{M_1(\xi)\} \{M_1(\xi)\}^T \{\delta_n\} \, d\xi \]  
(2.38)

\[ U_{U5} = \sum_{n=1}^{N} (D_\varepsilon/2) \frac{n^2 \pi^2 s}{L} \int_{-1}^{1} \{\delta_n\}^T \{M_3(\xi)\} \{M_3(\xi)\}^T \{\delta_n\} \, d\xi \]  
(2.39)

\[ U_{U6} = \sum_{n=1}^{N} (D_\varepsilon/2) L/s \int_{-1}^{1} \{\delta_n\}^T \{M_6(\xi)\} \{M_6(\xi)\}^T \{\delta_n\} \, d\xi \]  
(2.40)

\[ U_{U7} = \sum_{n=1}^{N} (-D_\varepsilon n \pi/2) \int_{-1}^{1} \{\delta_n\}^T \{M_3(\xi)\} \{M_3(\xi)\}^T \{\delta_n\} \, d\xi \]  
(2.41)

\[ U_{U8} = \sum_{n=1}^{N} (D_G/2) (s L/4) \int_{-1}^{1} \{\delta_n\}^T \left( 2/s \{M_4(\xi)\} + (n \pi/L) \{M_5(\xi)\} \right) \} \{\delta_n\} \, d\xi \]  
(2.42)

Finally, the strain energy in the upper plate of one strip element is:

\[ U_U = U_{U1} + U_{U2} + U_{U3} + U_{U4} + U_{U5} + U_{U6} + U_{U7} + U_{U8} \]  
(2.43)

### 2.2.2 Strain energy in the lower plate

Using the same small deflection orthotropic plate theory, the strain energy of the lower plate per unit area can be expressed as:

\[ \Delta U_L = (K_{x1}/2) \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + (K_{y1}/2) \left( \frac{\partial^2 w}{\partial y^2} \right)^2 + (K_{ad}) \left( \frac{\partial^2 w}{\partial x^2} \right) \left( \frac{\partial^2 w}{\partial y^2} \right) \]  
\[ + 2K_G1 \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 + (D_{x1}/2) \left( \frac{\partial u}{\partial x} \right)^2 + (D_{y1}/2) \left( \frac{\partial v}{\partial y} \right)^2 + D_{v1} (\partial u / \partial x)(\partial v / \partial y) \]  
\[ + D_{G1}/2 \left( (\partial u / \partial y) + (\partial v / \partial x) \right)^2 \]  
(2.44)

where:

\[ K_{x1} = \text{flexural stiffness in the x - direction per unit length} \]  
\[ = \frac{E_{x1}d^3}{12(1 - \nu_{yx1}\nu_{xy1})} \]  

\[ K_{y1} = \text{flexural stiffness in the y - direction per unit length} \]  
\[ = K_{x1}(\frac{E_{y1}}{E_{x1}}) \]
\[ K_{v1} = \text{coefficient associated with Poisson's ratio} \quad (2.45) \]
\[ = \nu_{xy1} K_{x1} \]

\[ K_{G1} = \text{torsional stiffness} = G1 \frac{d_1^3}{12} \]
\[ D_{x1} = \text{axial stiffness in x direction} = \frac{E_{x1} d_1}{(1 - \nu_{xy1} \nu_{xx1})} \]
\[ D_{y1} = \text{axial stiffness in y direction} = D_{x1} \frac{E_{y1}}{E_{x1}} \]
\[ D_{\nu1} = \text{coefficient associated with Poisson's ratio} = \nu_{xy1} D_{x1} \]

\[ D_{G1} = \text{in plane shear stiffness} = G1 d_1 \]
\[ E_{x1} = \text{Modulus of elasticity in x direction} \]
\[ E_{y1} = \text{Modulus of elasticity in y direction} \]
\[ \nu_{xy1} = \text{Poisson's ratio, strain in x direction when stress is applied in y direction} \]
\[ \nu_{yx1} = \text{Poisson's ratio, strain in y direction when stress is applied in x direction} \]
\[ G_1 = \text{Shear modulus in x-y plane} \]

Now the total strain energy in the lower plate finite strip can be obtained by integrating over the same area as shown in Fig. 2.3 for the upper plate. Thus the strain energy in the lower plate can be expressed as:

\[ U_L = \int_{-s/2}^{s/2} \int_0^L \Delta U_L \ dx \ dy \quad (2.46) \]

On substituting the expressions for displacements as given by eqns. (2.8) to (2.10) into eqn. (2.44) and integrating over the span length and then after substituting the expressions for functions \( F_{4n} (y) \), \( F_{6n} (y) \) and \( F_{6n} (y) \) and their derivatives as given by eqns. (2.14) to (2.16) and eqns. (2.21) to (2.24) and at last integrating over the
joist spacing, we get,

\[ U_{L1} = \sum_{n=1}^{N} \left( K_{z1}/2 \right) \frac{n^4 \pi^4}{4 L^5} s \int_{-1}^{1} \{\delta_n\}^T \{M_7(\xi)\} \{M_7(\xi)\}^T \{\delta_n\} \, d\xi \] (2.47)

\[ U_{L2} = \sum_{n=1}^{N} \left( K_{y1}/2 \right) \frac{4 L}{s^3} \int_{-1}^{1} \{\delta_n\}^T \{M_8(\xi)\} \{M_8(\xi)\}^T \{\delta_n\} \, d\xi \] (2.48)

\[ U_{L3} = \sum_{n=1}^{N} \left( -K_{v1}/2 \right) \frac{2 n^2 \pi^2}{s L} \int_{-1}^{1} \{\delta_n\}^T \{M_9(\xi)\} \{M_9(\xi)\}^T \{\delta_n\} \, d\xi \] (2.49)

\[ U_{L4} = \sum_{n=1}^{N} 2 K_G \frac{n^2 \pi^2}{s L} \int_{-1}^{1} \{\delta_n\}^T \{M_{10}(\xi)\} \{M_{10}(\xi)\}^T \{\delta_n\} \, d\xi \] (2.50)

\[ U_{L5} = \sum_{n=1}^{N} \left( D_{x1}/2 \right) \frac{n^2 \pi^2}{4 L} \int_{-1}^{1} \{\delta_n\}^T \{M_{11}(\xi)\} \{M_{11}(\xi)\}^T \{\delta_n\} \, d\xi \] (2.51)

\[ U_{L6} = \sum_{n=1}^{N} \left( D_{y1}/2 \right) \frac{n \pi}{s} \int_{-1}^{1} \{\delta_n\}^T \{M_{13}(\xi)\} \{M_{13}(\xi)\}^T \{\delta_n\} \, d\xi \] (2.52)

\[ U_{L7} = \sum_{n=1}^{N} \left( -D_{v1} \pi/2 \right) \int_{-1}^{1} \{\delta_n\}^T \{M_{10}(\xi)\} \{M_{10}(\xi)\}^T \{\delta_n\} \, d\xi \] (2.53)

\[ U_{L8} = \sum_{n=1}^{N} \left( D_{G1}/2 \right) \frac{(s L/4)}{s} \int_{-1}^{1} \{\delta_n\}^T \left( 2/s \{M_{11}(\xi)\} + (n \pi/L) \{M_{12}(\xi)\} \right) \] (2.54)

Thus the strain energy in the lower sheathing of one element of one element is:

\[ U_L = U_{L1} + U_{L2} + U_{L3} + U_{L4} + U_{L5} + U_{L6} + U_{L7} + U_{L8} \] (2.55)

### 2.2.3 Strain Energy in the Joist

Strain energy in the joist consists of two flexural components (one being bending in the vertical plane and the other, lateral bending in the horizontal plane), an axial component and a torsional component. The strain energy is thus expressed as

\[ U_J = E I_y/2 \int_0^L \left( \frac{d^2 W}{dx^2} \right)^2 \, dx + E I_z/2 \int_0^L \left( \frac{d^2 V}{dx^2} \right)^2 \, dx \]

\[ + E A/2 \int_0^L \left( \frac{dU}{dx} \right)^2 \, dx + \frac{GI_T}{2} \int_0^L \left( \frac{d\theta}{dx} \right)^2 \, dx \] (2.56)
where, for a rectangular cross-section,

\[ I_y = \frac{bh^3}{12} \]
\[ I_z = \frac{hb^3}{12} \]
\[ A = bh \]
\[ I_t = \beta \frac{(h/b)hc}{b} \]

and \( B \) being the joist width, \( h \) the joist depth and \( \beta \) the torsional constant function of the ratio \( h/b \).

Using the expressions for \( W(x), U(x), V(x) \) and \( \theta(x) \) as given by eqns. 2.4 to 2.7, the strain energy in the joist is written as:

\[ U_J = E I_y/2 \sum_{n=1}^{N} W_n^2 \frac{n^4 \pi^2}{2L^3} + E I_z/2 \sum_{n=1}^{N} V_n^2 \frac{n^4 \pi^4}{2L^3} \]
\[ + E A/2 \sum_{n=1}^{N} U_n^2 \frac{n^2 \pi^2}{2L} + G I_t/2 \sum_{n=1}^{N} \theta_n^2 \frac{n^2 \pi^2}{2L} \]

(2.57)

### 2.2.4 Strain Energy in the Upper Plate Connectors

It is assumed here that the upper plate is connected to the joist by uniformly placed non-rigid fasteners along the centerline of the top surface of the joist. The model assumes three types of slips between the plate and the joist. These are:

1. Slip parallel to the joist \( = \Delta u \)
   \[ = [u_0 - d/2(\frac{dW}{dx})] - [U + h/2(\frac{dW}{dx})] \]

(2.58)

2. Slip perpendicular to the joist \( = \Delta v \)
   \[ = [v_0 - d/2(\frac{\partial w_0}{\partial y})] - [V + (h/2)\theta] \]

(2.59)

3. Rotational Slip \( = \phi \)
   \[ = (\frac{\partial w_0}{\partial y}) - \theta \]

(2.60)
where \( d \) is the thickness of the upper plate, and the displacements \( u_0, v_0 \) and \( w_0 \) correspond to node 2 in the upper plate’s mid-plane directly over the joist, as shown in the Fig. 2.1. In the above eqns. (2.57) to (2.59) it has been assumed that the vertical displacement of the upper plate \( w_0 \) and the displacement \( W \) of the joist are identical. Letting \( K_{nx}, K_{ny} \) represent single connector stiffnesses corresponding to the \( x \) and \( y \) directions, \( NA \) is the number of nails per joist, \( (\Delta u)_i \) is the slip in the \( x \) direction for the \( i \)-th connector, \( (\Delta v)_i \) is the slip in the \( y \) direction for the \( i \)-th connector and \( K_{n\theta} \) the single nail stiffness against the rotational slip, then the strain energy accumulated in the connectors can be expressed as

\[
U_{NU} = \sum_{i=1}^{NA} \left[ \frac{(K_{nx}/2)}{2e} (\Delta u)_i^2 + \frac{(K_{ny}/2)}{2e} (\Delta v)_i^2 + \frac{(K_{n\theta}/2)}{2e} (\phi)_i^2 \right] \tag{2.61}
\]

Eqn. (2.60) considers the fasteners in a discrete manner and may be replaced by using an equivalent continuous connector by taking the slips \( \Delta u, \Delta v \) and \( \phi \) as continuous functions. In this case, the strain energy, is expressed as follows:

\[
U_{NU} = \frac{K_{nx}}{2e} \int_0^L (\Delta u)^2 \, dx + \frac{K_{ny}}{2e} \int_0^L (\Delta v)^2 \, dx + \frac{K_{n\theta}}{2e} (\phi)^2 \, dx \tag{2.62}
\]

where \( e \) is the connector spacing.

On substituting the expressions for plate and joist displacements from eqns. (2.1) to (2.7) one obtains:

\[
U_{NC} = \sum_{n=1}^{N} \left\{ \frac{L}{2} \frac{K_{nx}}{2e} (F_{2n}(\xi = 0) - U_n - W_n \frac{n \pi}{2 L} (h + d))^2 \\
+ \frac{L}{2} \frac{K_{ny}}{2e} (F_{3n}(\xi = 0) - V_n - \theta_n (h/2 d/2))^2 \\
+ \frac{L}{2} \frac{K_{n\theta}}{2e} (w'_{2n} - \theta_n)^2 \right\} \tag{2.63}
\]

Eqn. (2.62) can be written in terms of the vector \( \{\delta_n\} \) as given in the pp. 13. To this end, we define the following vectors:

\[
\{e_{13}\}^T = (0 \ldots \ldots 1 \ldots \ldots 0 \ldots \ldots 0), \text{where the thirteenth element is unity}
\]
\{\epsilon_{17}^T\} = (0 \ldots 1 \ldots 0 0), where the seventeenth element is unity and

Similarly, vectors \{\epsilon_{16}, \epsilon_{14}, \epsilon_{18}\} and \{\epsilon_{19}\} are defined. Thus, the strain energy in the upper connectors is given by:

\[ U_{NC} = \sum_{n=1}^{N} \left\{ \frac{L}{2} \frac{K_{nx}}{2e} \{\delta_n\}^T (e_{13} - e_{17} - e_{16} \frac{n\pi(h + d)}{2L}) \right\} \]
\[ + \frac{L}{2} \frac{K_{ny}}{2e} \{\delta_n\}^T (e_{14} - e_{18} - e_{19} \frac{h}{2s} - e_{15} \frac{d}{2s}) \]
\[ + \frac{L}{2} \frac{K_{n\phi}}{2e} \{\delta_n\}^T (\frac{e_{15}}{s} - \frac{e_{19}}{s})(\frac{e_{15}}{s} - \frac{e_{19}}{s})^T \{\delta_n\} \]

(2.64)

### 2.2.5 Strain Energy in Lower Plate Connectors

Here also we assume that the lower plate is connected to the joist by uniformly spaced fasteners along the center line of the bottom surface of the joist. The model also assumes three types of slips between the plate and the joist.

1. Slip parallel to the joist = $\Delta u$
   \[ = [u_0 + d_1/2(\frac{dW}{dx})] - [U - h/2(\frac{dW}{dx})] \]

2. Slip perpendicular to the joist = $\Delta v$
   \[ = [v_0 + d_1/2(\frac{\partial w_0}{\partial y})] - [V - (h/2) \theta] \]

3. Rotational Slip = $\phi$
   \[ = \theta - \left( \frac{\partial w_0}{\partial y} \right) \]

where $d_1$ = thickness of the lower plate; and the displacements $u_0$, $v_0$ and $w_0$ correspond to node 5 in the lower plate's mid-plane directly under the joist, as shown in the Fig. 2.1. In the above eqns. (2.64) to (2.66) it has been assumed that
the vertical displacement of the lower plate, \( w_0 \), and the displacement, \( W \), of the joist are identical.

Following a similar development as previously described for the upper connectors, we obtain the strain energy in the lower plate connectors as follows:

\[
U_{NL} = \sum_{n=1}^{N} \left[ \frac{L}{2e} K_{n+1} \{\delta_n\}^T \left( e_{20} - e_{17} - e_{16} \frac{n\pi(h + d)}{2L} \right) \right.
\]
\[
+ \frac{L}{2e} \frac{K_n}{L} \{\delta_n\}^T \left( e_{21} - e_{18} - e_{19} \frac{h}{2s} - e_{22} \frac{d}{2s} \right) \}
\]
\[
\left. + \frac{L}{2e} \frac{K_{n+1}}{L} \{\delta_n\}^T \left( e_{23} - e_{18} - e_{19} \frac{h}{2s} - e_{22} \frac{d}{2s} \right) \{\delta_n\} \right]
\]

where

\[
\{e_{20}\}^T = (0 0 \ldots 1 \ldots \ldots 0 0), \text{ where the twentieth element is unity}
\]

\[
\{e_{21}\}^T = (0 0 \ldots 1 \ldots \ldots 0 0), \text{ where the twentyfirst element is unity}
\]

Similarly, vectors \( \{e_{17}\}, \{e_{18}\}, \{e_{19}\} \) and \( \{e_{22}\} \) are defined. Finally the total strain energy in one \( T \)-beam strip element is given by,

\[
U(n) = U_U(n) + U_L(n) + U_J(n) + U_{NU}(n) + U_{NL}(n)
\]

### 2.2.6 Stiffness Matrix and the Solution of the System of Equations

On taking the first variation of \( U(n) \) with respect to \( \{\delta_n\} \) we get an expression of the form \( [K_{n+1}]\{\delta_n\} \) in which \( [K_{n+1}] \) is the element stiffness matrix of size \( 34 \times 34 \). There are 34 unknowns per element, but 12 of them are shared with the adjoining element (6 at each of the nodes 3 and 6 of Fig.2.1). The global stiffness matrix for
Each Fourier term is obtained by assembling the stiffness matrix of each element as shown in Fig. 2.4 below.

By considering the variation of $U$ with respect to different $\{\delta_n\}$, the overall global stiffness matrix can be obtained. This global system has a structure as shown in eqn. 2.69 in which $\{\Delta_K\}$ are the global vectors and the submatrices $[B(i,j)]$ are the global submatrices. Each global vector $\{\Delta_K\}$ has $22 \times NJT + 12$ components, and each submatrix, $[B(i,j)]$, is a symmetric matrix of size $(22 \times NJT + 12)$. $[B(i,j)]$ is a banded matrix, however, with a bandwidth of 34.

The vectors $\{R_k\}(k = 1, 2, \ldots, N)$ correspond to the first variation of the load potential, $U_L$. $U_L$ is the load potential per joist corresponding to the applied load function $p(x, y)$ and is given by,
Chapter 2: Theoretical Formulation

\[ U_L = \int_{-s/2}^{s/2} \int_0^L \rho(x,y) w \, dx \, dy \]

\[
\begin{bmatrix}
[B(1,1)] & [B(1,2)] & \ldots & [B(1,K)] & \ldots & [B(1,N)] \\
[B(2,1)] & [B(2,2)] & \ldots & [B(2,K)] & \ldots & [B(2,N)] \\
& \ddots & \ddots & \ddots & \ddots & \ddots \\
&B(K,1) & [B(K,2)] & \ldots & [B(K,K)] & \ldots & [B(K,N)] \\
&B(N,1) & [B(N,2)] & \ldots & [B(N,K)] & \ldots & [B(N,N)]
\end{bmatrix}
\begin{bmatrix}
\{\Delta_1\} \\
\{\Delta_2\} \\
\vdots \\
\{\Delta_K\} \\
\vdots \\
\{\Delta_N\}
\end{bmatrix}
= 
\begin{bmatrix}
\{R_1\} \\
\{R_2\} \\
\vdots \\
\{R_K\} \\
\vdots \\
\{R_N\}
\end{bmatrix}
\tag{2.70}
\]

The global system can easily be solved by an iterative procedure as follows:

\[
\{\Delta_K\}_i = [B(K,K)]^{-1} \left\{ \{R_K\} - \sum_{n=1}^{N} [B(K,n)] \{\Delta_n\}^{i-1} \right\} \tag{2.71}
\]

\[
(i = 1, 2, \ldots N), \quad (n \neq K) \& (K = 1, 2, \ldots N)
\]

with starting vectors

\[
\{\Delta_K\}_0 = [B(K,K)]^{-1} \{R_K\} \tag{2.72}
\]
Chapter 2: Theoretical Formulation

The iterations can be stopped when, for example

\[ \text{mod}(\{\delta_K\}^i - \{\delta_K\}^{i-1})_2 < \epsilon (\text{mod}(\{\Delta_K\}^{i-1}))_2 \]

for \( K = 1, 2, \ldots, N \) and \( \epsilon \) a small number (e.g., 0.001) and

\[ \text{mod}(X)_2 = \|X\|_2 = \left( \sqrt{X_1^2 + X_2^2 + \cdots + X_n^2} \right) \]
CHAPTER 3

The Computer Program

3.1 Program Features

The Computer Program performs a static analysis of the stiffened structure with equidistant stiffeners in one direction. Nonlinearities in the fasteners or inelastic joist or cover behavior have not been included.

The program output gives information about

- maximum joist deflection for each joist in the floor
- maximum joist deflection in the floor
- maximum bending stress for each joist
- maximum joist bending stress in the whole floor
- load sharing factor for bending stress for each joist
- maximum load sharing factor for bending stress for all the joists
- deflections of the cover at nodes 1 and 4.
- maximum bending stress in the upper and the lower plate for each element
- maximum bending stress in the upper plate for the whole floor
- maximum bending stress in the lower plate for the whole floor
- maximum upper plate deflection between joists for each joist
- maximum lower plate deflection between joists for each joist
- maximum upper plate deflection between joists for the whole floor
- maximum upper plate deflection between joists for the whole floor
A few important features of the program are described below:

- The modulus of elasticity of the joists can be directly put into the datafile or can be selected from a distribution representing the population of joists' EI (joist stiffness). The distribution may be 3-parameter Weibull, 2-parameter Lognormal or Normal. Lower and Upper limits of the interval within which the modulus of elasticity of the joists should lie can also be specified.
- A minimum strength (or Proof level) accepted for the joists can be specified.
- There is a provision for gaps in the cover.
- The nailing pattern may be treated as discrete or as continuous throughout the length of the connection between the joist and the cover.
- Load on the floor is uniformly distributed, either over the whole area or over smaller, individual areas, simulating concentrated loads.
- Boundary conditions may be imposed on the problem.

The dimension statements in the program impose the following limitations:

1. The number of joists in the floor is limited to 20.
2. The number of Fourier series terms that can be considered for the solution are limited to a maximum of five terms, so that for symmetrical problems in the direction of the joists the five terms will be of order 1, 3, 5, 7 and 9. For non-symmetrical problems the five terms will be 1, 2, 3, 4 and 5.
3. There can be a maximum of twenty loaded areas on the floor.
4. The number of gaps in the cover is limited to 5.

These limitations can be easily overcome by changing the dimension statements for the matrices and the vectors.
3.2 Program Structure

The program consists of a number of subroutines which find the modulus of elasticity of the joist, compute the strength of the joist, evaluate the shape functions, construct the stiffness matrix and the load vector and, finally, solve the system of equations. A brief description of the main subroutines and the operations they carry out is presented here.

Subroutine DISTR: This subroutine assigns the modulus of elasticity of the joist depending upon whether the distribution representing the population of joists' stiffness has been assumed Weibull, Lognormal or Normal.

Subroutine STREND: This routine assigns the strength of the joist, based on a correlation with its stiffness.

Subroutine GENMTX: This subroutine generates the shape functions and then utilise them to evaluate the required integrals in the development of the stiffness matrix. The routine computes the integrals numerically using a six point Gauss Quadrature rule, which will be exact for the degree of polynomial utilised for shape functions in the y-direction.

Subroutine STIF: In this subroutine, the stiffness matrix is developed by considering only the contribution of the upper and the lower plates and the connections. The contribution of the joist to the stiffness matrix is added later on in the main program. The load vector is also computed.

Subroutine DECMP and SOLV: The subroutine DECMP decomposes the stiffness matrix by Cholesky's decomposition method and stores it columnwise. Since the stiffness matrix is symmetrical and banded, only the lower triangular portion of the band is taken into consideration. The subroutine SOLV performs the forward and backward substitution to determine the solution vector.
CHAPTER 4

Verification and Numerical Results

4.1 Introduction

In this chapter, the finite-strip element computer program developed in the previous chapters is verified with the floor analysis program, FAP (Ref. 11). Next a parametric study is taken up to observe the behavior of the model under various conditions.

The program has been implemented and tested on a mainframe IBM 3081K and an AST 286 Premium (IBM PC-AT compatible) at the University of British Columbia. Double Precision (Real * 8) arithmetic is used only in the calculation of the stiffness matrix. This results in savings in terms of computer memory utilised and CPU time.

A detailed discussion about the features of the program and its limitations is given earlier in Chapter 3.

4.2 Verification

Two verifications of the present program have been carried out as explained below:

4.2.1 Verification for flange stresses in the longitudinal direction:

The values of the maximum deflection and the longitudinal normal stress at the top layer of the upper plate obtained from the present computer program are sought to be verified against the known solutions for them from the elementary beam theory.
Chapter 4: Verification and Numerical Results

A finite-strip element with dimensions as shown in Fig. 4.1 is considered. Other properties are -

- Panel span \( (L_p) \): 3800 mm.
- Uniformly distributed load on the upper plate: 0.001916 MPa
- Nail load-slip modulus (parallel to the joist): 17500000 N/mm.
- Nail load-slip modulus (perpendicular to the joist): 17500000 N/mm.
- Nail rotation modulus: 44500000 N mm/radian
- Boundary conditions: no rotation in the z-y plane
- Modulus of elasticity of plate in x-direction = 12000 MPa
- Modulus of elasticity of plate in y-direction = 12000 MPa
- Modulus of elasticity of joist = 12000 MPa
- Poisson’s ratio = \( \nu_{xy} = \nu_{yx} = 0.20 \)
- Bending stiffness of plates in x and y directions = 3515625 N mm.
- Axial stiffness of plates in x and y directions = 187500 N/mm.

Here very high values of nail load-slip modulus and nail rotation modulus have been considered so that the connections may be rendered almost fixed. The study was done by considering the first three terms (i.e. \( n = 1, 3 \) and 5) of the Fourier series.

For the section shown in Fig. 4.1, following the elementary bending theory,

\[
EI = \left\{ \frac{40 \times 190^3}{12} \right\} \times 12000 + 2((3375000 \times 400) + (180000 \times 400) \times 102.5^2)
\]

\[
= 1.7899 \times 10^{12} N \cdot mm^2
\]

Moment = \( ((0.001916 \times 400) \times 3800^2)/8 = 1383352N = mm. \)

Therefore, the average longitudinal normal stress at top layer of upper plate

\[
\sigma_{av} = \frac{1383352}{1.7899 \times 10^{12} \times 12000 \times 110} = 1.02\text{MPa}
\]
Chapter 4: Verification and Numerical Results

Maximum deflection, $\Delta = \frac{(5/384) \times (0.001916 \times 400) \times 3800^4}{1.7899 \times 10^{12}}$

$= 1.162$ mm.

From the computer program,
Longitudinal normal stress at point $A = \sigma_{max} = 1.06$ MPa
and Maximum deflection = $1.193$ mm

It may be observed that the program computed stress is very close to $\sigma_{av}$. The case of deflection is also similar. $\sigma_{max} > \sigma_{av}$ is due to shear lag effect.

4.2.2 Verification by comparison with FAP:

In the present program, the model consists of an upper and a lower plate with a joist in between as shown in Fig. 2.1. The plates are connected to the joist by nail connectors. On the other hand, in the program FAP, the model is the same as
described above except that there is no lower plate.

In order to compare the results from the two programs, the thickness of the lower plate is reduced to 0.001 mm. and the nail stiffnesses for the connection between the lower plate and the joist made sufficiently small. As a result, the lower plate, which is extremely thin, is very loosely connected to the joist and has almost no effect on the behavior of the structure.

The datafile for the FAP and the present program have the following properties in common:

panel span ($L_p$): 3800 mm.

spacing between the joists ($s$): 400 mm.

depth of the joist ($HJT$): 190 mm.

thickness of upper plate ($d$) : 15mm.

uniformly distributed load on the upper plate: 0.001916 MPa

nail load-slip modulus (parallel to the joist): 1750 N/mm.

nail load-slip modulus (perpendicular to the joist): 1750 N/mm.

nail rotation modulus : 4450000 N mm/radian

boundary conditions: no rotation in the z-y plane

modulus of elasticity of top layer of the plate in x - direction = 12000 MPa

modulus of elasticity of joist = 12000 MPa

bending stiffness of plates in x direction = 1017270 N mm.

bending stiffness of plates in y direction = 3164840 N mm.

axial stiffness of plates in x direction = 59567 N/mm.

axial stiffness of plates in y direction = 75335 N/mm.

The comparison was done by considering the first three terms (i.e. $n = 1, 3$ and 5) of the Fourier series. The results were obtained for panels with 1 or 4 joists, and were as follows:
**CASE I:** 1 Panel with 1 Joist

<table>
<thead>
<tr>
<th>Present Work</th>
<th>FAP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Joist:</strong></td>
<td></td>
</tr>
<tr>
<td>Deflection</td>
<td>4.5747 mm.</td>
</tr>
<tr>
<td>Bending stress</td>
<td>4.2488 MPa</td>
</tr>
<tr>
<td><strong>Upper Cover:</strong></td>
<td></td>
</tr>
<tr>
<td>Deflection</td>
<td>4.7027 mm.</td>
</tr>
</tbody>
</table>

**CASE II:** 1 Panel with 4 Joists

<table>
<thead>
<tr>
<th>Present Work</th>
<th>FAP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Element # 1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Joist:</strong></td>
<td></td>
</tr>
<tr>
<td>Deflection</td>
<td>4.5641 mm.</td>
</tr>
<tr>
<td>Bending stress</td>
<td>4.2415 MPa</td>
</tr>
<tr>
<td><strong>Upper Cover:</strong></td>
<td></td>
</tr>
<tr>
<td>Deflection</td>
<td>4.6801 mm.</td>
</tr>
<tr>
<td><strong>Element # 2</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Joist:</strong></td>
<td></td>
</tr>
<tr>
<td>Deflection</td>
<td>4.5114 mm.</td>
</tr>
<tr>
<td>Bending stress</td>
<td>4.1866 MPa</td>
</tr>
<tr>
<td><strong>Upper Cover:</strong></td>
<td></td>
</tr>
<tr>
<td>Deflection</td>
<td>4.5944 mm.</td>
</tr>
<tr>
<td><strong>Element # 3</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Joist:</strong></td>
<td></td>
</tr>
<tr>
<td>Deflection</td>
<td>4.5076 mm.</td>
</tr>
<tr>
<td>Bending stress</td>
<td>4.1756 MPa</td>
</tr>
</tbody>
</table>
Chapter 4: Verification and Numerical Results

4.3 Parametric Study

A brief parametric study has been undertaken to investigate the behavior of the model due to the changes in -

- the nail stiffness
- the ratio of the plate thickness to the depth of the joist
- the ratio of the spacing between the joists to the panel span.

For the purpose of investigation a panel with 10 joists is considered. The stiffness of the joists are represented by a 3-parameter Weibull distribution. The lower and the upper limits of the modulus of elasticity of the joists are set at 8000 MPa and 14000 MPa respectively. All the joists are selected at random but within these upper and the lower limits. The rotation, in the z-y plane, of the nodes 2, B and 5 in the 1st and the 10th element (see fig. C.3) are restricted. For the purpose of comparison, the highest values of the stresses and the deflections in the floor are selected. The conditions mentioned above are kept constant during the whole parametric study as described in Art. 4.3.1, 4.3.2 and 4.3.3.
4.3.1 Effect due to change in nail stiffness

A structure with
(i) panel span \((L_p)\) : 3800 mm.
(ii) spacing between joists \((s)\) : 400 mm.
(iii) thickness of the plates : 15mm.
(iv) u.d.l. on the upper plate : 0.001916 MPa
and (v) depth of the joists \((H_JT)\) : 190 mm.

has been considered. The nail load-slip modulii in the directions parallel and perpendicular to the joist, \(RKPAL\) and \(RKPER\) respectively, are kept equal. All the assumptions made in Art. 4.3 have also been incorporated.

\(RKPAL\) (and \(RKPER\)) is increased from 0.0175 N/mm. to 17500000 N/mm. in six steps (see Figs. 4.2, 4.3 and 4.4) keeping the nail rotation modulus, \(RKROT\), fixed at 44.5 N mm./radian. In the second and third steps, \(RKROT\) is changed to 44500 N mm./radian and 44500000 N mm./radian respectively and \(RKPAL\) (and \(RKPER\) also) is gradually increased as before. The resulting change in the joist deflection, joist bending stress and the stress in the longitudinal direction in the upper plate is shown in Fig. 4.2, Fig. 4.3 and Fig. 4.4.

We can observe from the figures that:
(i) as the nail stiffness is increased the joist deflection and the joist bending stress decreases, and the stress in the upper plate increases. However this rate of change decreases with increase in the stiffness of the connector.
(ii) beyond the value of 17500 N/mm. for \(RKPAL\), there is very small change in the joist deflection, joist bending stress and the stress in the upper plate.
(iii) initially the joist bending stress is very high and the upper cover stress in the longitudinal direction is very small. Thenafter there is a gradual decrease in the joist bending stress and an increase in and the upper plate stress. At the end, the stress in the upper plate in the longitudinal direction is slightly higher than the joist bending
stress.

(iv) the effect of increase of $RK_{ROT}$ is much less than that of the increase of $RKP_{AL}$.

(v) there is no use to increase $RK_{ROT}$ beyond 44500 N mm./radian.

It is clear that as the connector stiffness is increased, more and more load gets transferred to the plates. A limiting case is reached when $RKP_{AL}$ (and $RKP_{ER}$) is 17500 N/mm. and $RK_{ROT}$ is 44500 N mm./radian. The effect of increase of $RKP_{AL}$ is much more visible than that of $RK_{ROT}$.

### 4.3.2 Effect due to the change in the ratio of plate thickness to the depth of the joist:

The assumptions made in the Art. 4.3 prevail here too. Other properties of the panel considered are as described below:

(i) thickness of the plates: 15mm.

(ii) spacing between the joists: 380 mm.

(iii) panel span: 3800 mm.

(iv) uniformly distributed load on the upper plate: 0.0024 MPa.

While keeping the thickness of the plate constant at 15mm., the depth of the joist is increased from 150 mm. to 190 mm., 250 mm., 300 mm. and 375 mm. in four steps. This leads to a decrease in the $d/HJT$ ratio. For each ratio, five cases of nail load-slip modulus ($RKP_{AL}$ and $RKP_{ER}$) and nail rotation modulus ($RK_{ROT}$) were considered as described below:

**CASE 1**: 0.0175 N/mm. , 0.0175 N/mm. , 44500 N mm/radian

**CASE 2**: 550.0 N/mm. , 550.0 N/mm. , 44500 N mm/radian

**CASE 3**: 1750.0 N/mm. , 1750.0 N/mm. , 44500 N mm/radian

**CASE 4**: 17500 N/mm. , 17500 N/mm. , 44500 N mm/radian

**CASE 5**: 17500000 N/mm. , 17500000 N/mm. , 44500 N mm/radian

The change in the joist deflection, joist bending stress and the maximum stress
in the topmost layer in the upper plate (above NODE 2) in the x - direction, due to the variation in the $d/HJT$ ratio and the nail stiffness, is shown in Fig. 4.5, Fig. 4.6 and Fig. 4.7. In these figures, the values obtained at $d/HJT = 0.04$ in each of the cases mentioned above are assumed equal to unity and values at higher $d/HJT$ ratios normalised on this basis.

A few observations can be made as follows:

- when the nailing is minimal, the applied load is carried almost entirely by the joists and, as a result, the joist deflection and the joist bending stress are quite high while the stress in the x-direction at the topmost fibre of the upper plate is very small.
- If the nail stiffnesses are assigned even a small value, the composite action of the plates and the joist comes into play. This is reflected by the sudden increase in the stress in the upper plate.
- It can be observed from Case 4 and Case 5 that beyond a particular value of nail stiffness, the change in the deflection and the bending stress in the joist and the stress in the upper plate is very small, which shows that the connections are almost rigid.
- When the $d/HJT$ ratio is increased, the rate of increase of joist deflection is found to be more rapid for Case 1 than for the other cases. This is very obvious because as $d/HJT$ increases, $HJT$ decreases since we have kept $d$ (thickness of the plate) constant.
- Unlike in the case of joist deflection (Fig.4.5) the rate of increase of joist bending stress for all cases except for Case 1 is almost linear. In Case 1, however, a rapid rate of increase is observed. The increase in the upper cover stress follows the same pattern.
4.3.3 Effect due to the change in the ratio of spacing between the joists to the panel span:

A study has been done to investigate the variation in three parameters, namely, joist deflection, joist bending stress and the maximum stress in the top fibre of the upper plate in x-direction, due to a gradual change in $d/HJT$ ratio ($d$ being the plate thickness and $HJT$ the depth of the joist) when the $s/L_p$ ratio is increased in steps from 0.05 to 0.25. The $s/L_p$ ratio is increased by keeping $L_p$ constant and gradually raising the value of $s$.

Figs. 4.8, 4.9 and 4.10 are drawn assuming the nail load-slip modulii in directions both parallel and perpendicular to the joist as 1750 N/mm and the nail rotation modulus as 4450000 N mm/radian. The assumptions made in Art. 4.3 have also been incorporated. Other properties of the structure are as follows:

- thickness of the plate: 15mm.
- panel span: 3800 mm.
- uniformly distributed load on the upper plate: 0.0024 MPa

It may be observed that at higher $s/L_p$ ratios, the joist deflection, joist bending stress and the stress in the upper cover in the longitudinal direction is larger. Fig. 4.9 and 4.10 show that increase in the joist bending stress and the upper cover stress is almost linear with increase in $d/HJT$ ratio.

Furthermore we can see from Fig. 4.8 that for a particular value of $d/HJT$ and $s/L_p$ ratios if the corresponding joist deflection is found to be excessive, then in order to achieve a lower level of joist deflection either the $s/L_p$ ratio may be decreased (by decreasing $s$) or the $d/HJT$ ratio may be decreased (preferably by increasing $HJT$). Similar inferences can be drawn from Figs. 4.9 and 4.10.
Figure 4.2: Joist deflection vs. Nail stiffness

RKROT = 44.5 N mm./ radian

RKROT = 44500 and 44500000 N mm./ radian
Figure 4.3: Joist Bending stress vs. Nail stiffness
Figure 4.4: Stress at top layer of upper plate in x-direction at varying nail stiffnesses
Figure 4.5: \( \frac{d}{HJT} \) vs. Joist Deflection at varying nail stiffnesses
Figure 4.6: d/HJT vs. Joist Bending stress at varying nail stiffnesses
Figure 4.7: $d/HJT$ vs. Stress at top layer of upper plate in x-direction at varying nail stiffnesses
Figure 4.8: d/HJT vs. Joist Deflection at varying $s/L_p$
Figure 4.9: $d/HJT$ vs. Joist Bending stress at varying $s/L_p$
RKPAL = RKPAL1 = 1750 N/mm
RKPER = RKPER1 = 1750 N/mm
RKROT = RKROT1 = 4450000 N mm/radian

Figure 4.10: d/HJT vs. Stress at top layer of upper plate in x-direction at varying s/Lp

Chapter 4: Verification and Numerical Results
4.4 Study of the Shear Lag Effect

According to the Clause 8.6.3.1 of CSA Standard CAN3-086.1-M84 'Engineering Design in Wood (Limit State Design)', the factored bending moment resistance along the direction of the webs of a stressed skin panel shall be the least of the factored resistances of the tension or the compression flanges or the web determined as follows:

(a) tension flange

\[
M_r = \phi t_p (K_D K'_S K_T) X_J X_G \frac{(EI)_c}{B_a K'_S c_t}
\]

(b) compression flange

\[
M_r = \phi p_p (K_D K'_S K_T) X_J X_G \frac{(EI)_c}{B_a K'_S c_c}
\]

(c) web

\[
M_r = \phi F_b K_{Zb} K_H K_L X_G \frac{(EI)_c}{E K_{SE} c_w}
\]

where

\[
\phi = 0.8
\]

\[
t_p = \text{specified strength in tension, N/mm (Tables 7.3.A and 7.3.C)}
\]

\[
p_p = \text{specified strength in compression, N/mm (tables 7.3.A and 7.3.C)}
\]

\[
B_a = \text{specified axial stiffness, N/mm (Tables 7.3.B and 7.3.D)}
\]

\[
F_b = f_b (K_D K_{Sb} K_T)
\]

\[
f_b = \text{specified strength in bending of webs, MPa (Tables 5.3.1.1, 5.3.1.2, 5.3.1.3, 5.3.1.4 and 5.3.2 for Sawn Lumber and Table 6.3 for Glulam.)}
\]

\[
X_J = \text{stress joint factor (Clause 8.3)}
\]

\[
X_G = \text{panel geometry reduction factor (Clause 8.6.3.2)}
\]
Chapter 4: Verification and Numerical Results

\( E = \) modulus of elasticity of web, MPa (Tables 5.3.1.1, 5.3.1.2, 5.3.1.3, 5.3.1.4 and 5.3.2 for Sawn Lumber and Table 6.3 for Glulam.)

\( c_w = \) greatest distance from neutral axis to outer edge of web, mm.

In the formulae above, \( X_G \) accounts for shear lag. Clause 8.6.3.2 of the Code defines factor \( X_G \) as:

\[
X_G = 1 - 4.8 (s/L_p)^2
\]

where, \( s = \) clear spacing between the joists (in mm.)

\( L_p = \) panel span (in mm.)

This formula for \( X_G \) is valid for values of \( s/L_p \) ranging from 0.05 to 0.25.

In order to verify the formula given by the Code, a study was undertaken to find the values of \( X_G \) due to a gradual change in the \( d/HJT \) ratio when \( s/L_p \) is increased in steps from 0.05 to 0.25. The stiffness of the joists is represented by a 3 parameter Weibull distribution. A limiting condition, given by \( EMIN \) and \( EMAX \), is also imposed on the lower and the upper limits of the modulus of elasticity of joists. The study considered 50 panels with 4 joists each. Other properties are -

- panel span: 3800 mm.
- thickness of the plates: 15mm.
- uniformly distributed load on the upper plate: 0.0024 MPa
- boundary conditions: none (Panel simply supported)

The spacing between the joists, \( s \), is varied from 190 mm. to 570 mm. to 950 mm. Similarly, the depth of the joist, \( HJT \), is increased from 100 mm. to 200 mm. and 300 mm. The first three Fourier terms (\( n = 1, 3 \) and 5) have been considered.

For each combination in \( d/HJT \) and \( s/L_p \) ratios, the stress at the topmost fibre of the upper plate in the x-direction, \( \sigma_{av} \), is found following the elementary bending theory as in Art. 4.2.1, by assuming the average modulus of elasticity of joists as \( (EMIN + EMAX)/2 \). The stress at the same level, \( \sigma_{max} \), is also determined from the computer program. Next, \( \frac{\sigma_{max}}{\sigma_{max}} \) gives the value of \( X_G \) for each element. The
minimum value of $X_G$ at each of the 50 panels is found out and then arranged in an ascending order. $X_G$ is plotted against the probability of not being conservative for varying $s/L_p$ and $d/HJT$ ratios in Fig. 4.11, Fig. 4.12 and Fig. 4.13.

A few comments can be made on the basis of the figures mentioned above.

(a) At $s/L_p = 0.05$, $X_G$ according to the Code is 0.988. It may be observed in Fig. 4.11 that corresponding to $X_G = 0.988$, the probability of being not conservative may range from 0.525 to 0.620 depending upon the $d/HJT$ ratio.

(b) At $s/L_p = 0.15$, $X_G$ as per Code is 0.892, which may be termed quite satisfactory as may be observed in Fig. 4.12.

(c) In Fig. 4.13 for $s/L_p = 0.25$ we can see that the minimum $X_G$ at zero probability is approximately 0.85, whereas the Code value of $X_G$ is 0.7.

To facilitate the study of the shear lag effect in greater detail, an investigation was done to observe the effect of the number of longitudinal ribs (joists).

4.4.1 Effect of the number of longitudinal ribs

To observe the effect of the number of longitudinal ribs, a structure with the following properties has been considered.

Number of panels : 50
Panel span ($L_p$) : 3800 mm
Width of each panel : 4000 mm
Thickness of the plates (d) : 15 mm
U.D.L. on the upper plate of each panel : 0.0024 MPa
Nail load - slip modulus (parallel to the joist) : 17500000 N / mm
Nail load - slip modulus (perpendicular to the joist) : 17500000 N / mm
Nail rotation modulus : 44500000 N mm/radian
Joist stiffness : represented by a 3 parameter Weibull distribution
Joist modulus of elasticity : selected randomly between 8000 and 14000 MPa
Chapter 4: Verification and Numerical Results

Boundary condition: none.

The nail stiffness considered here are very high and thus render the connections almost fixed. The number of the longitudinal ribs (joists) considered are 4, 5, 6, 7 and 8. At the same time the spacing between the joists gets reduced because the total width of the panel remains constant at 4000 mm. Two cases are considered, in the first case the depth of the joist ($HJT$) is 200 mm. and in the second 300 mm.

For each variation in the number of joists, the value of $X_Q$ is calculated in the computer program as mentioned earlier in Art. 4.4. for each element in the structure. The minimum value of $X_Q$ for each panel is found. After being arranged in an ascending order, these $X_Q$ values are plotted against the probability of being not conservative.

From the Fig. 4.14 and Fig. 4.15 we may observe that:

(a) increasing the number of joists causes an increase in $X_G$;
(b) when $d/HJT = 0.075$ and the number of joists per panel is 6, $X_G$ at zero probability of being non-conservative is 0.91. On the other hand if $d/HJT = 0.05$ and the number of joists be the same, at 5 percent probability $X_G$ will be 0.895. Though $d/HJT$ has been changed, at zero probability $X_G$ is 0.89. Thus there is a 2 percent change in $X_G$ as a result of the change in the $d/HJT$ ratio.

Similar phenomenon is seen if the number of joists is changed.

(c) When the number of joists are 4, 5 and 6, the $s/L_p$ ratios are given by 0.25, 0.20 and 0.167 respectively. The proximity of the curve corresponding to 5 joists to the one representing 6 joists in comparison to the curve representing 4 joists may be attributed to the fact that 0.20 is nearer to 0.167 than to 0.25.

Thus it may be concluded that:

(a) an increase in the number of joists decreases the shear lag;
(b) $X_G$ is not entirely independent of the $d/HJT$ ratio.
(c) $X_G$ depends on the $s/L_p$ ratio.
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We may observe from Art. 4.4 and Art. 4.4.1 that we should recommend the $X_G$ values to be adapted on the basis of 4 joists being used and the $d/HJT$ ratio giving the minimum $X_G$. This would be on the conservative side because the maximum shear lag occurs when the number of joists is 4. Furthermore, there is small practical chance of using less than 4 joists per panel. The $X_G$ values which may be recommended, assuming 5 percent probability of being not conservative to be acceptable, are as described below:

At $s/L_p = 0.05$, the value of $X_G$ to be adopted is 0.92. Similarly at $s/L_p = 0.15$ and 0.25, $X_G$ should be taken as 0.91 and 0.86 respectively.

A plot based on this recommendation is shown in Fig. 4.16. Two formulae can be put forward from this plot.

\[
X_G = 0.925 - 0.1 \frac{s}{L_p}, \text{ for } 0.05 \leq s/L_p \leq 0.15 \tag{4.1}
\]

\[
X_G = 0.985 - 0.5 \frac{s}{L_p}, \text{ for } 0.15 \leq s/L_p \leq 0.25 \tag{4.2}
\]
Figure 4.11: Probability of non-conservative $X_G$ at $s/L_p = 0.05$
Figure 4.12: Probability of non-conservative $X_p$ at $s/L = 0.15$
Figure 4.13: Probability of non-conservative $X_G$ at $s/L_p = 0.25$
Figure 4.14: Probability levels of $X_G$ at $d/HJT = 0.075$
Figure 4.15: Probability levels of $X_G$ at $d/HJT = 0.05$
Figure 4.16: Panel geometry reduction factor $X_G$ at varying $s/L_p$ ratios
5.1 Conclusion

Stiffness of the connectors have a great impact on the behavior of the stiffened plates. The composite behavior is highly dependent on the nail stiffness. Nail stiffness for rotational slip has considerably less significance in comparison to the nail stiffness corresponding to the x and y directions. Even small amount of nail stiffness can affect the stress resultants appreciably.

At general levels of connector stiffnesses, the relation between the $d/HJT$ ratio ($d$ being the thickness of the plates and $HJT$ the depth of the joist) and the joist bending stress has been found to be almost linear. The relation between the $d/HJT$ ratio and the upper cover stress in the longitudinal direction is also similar. However, the rate of change of joist deflection increases with increase in the $d/HJT$ ratio. The rate of change is higher for higher $s/L_p$ ratios.

The shear lag factor $X_G$ may be assumed independent of the $d/HJT$ ratio. The Code (CSA Standard CAN3-086.1-M84) value for $X_G$ has been found to be, for $NJT = 4$, underconservative and overconservative at $s/L_p = 0.05$ and 0.25 respectively. At $s/L_p = 0.15$ the Code gives an acceptable value of $X_G$.

$X_G$ is dependent on the $s/L_p$ ratio. An increase in the number of joists (i.e. the number of stiffeners) decreases the resulting shear lag.
5.2 Scope for further research

In the study for ‘effect due to change in the \( d/HJ \) ratio’, 15 mm. thick upper and lower plates and their modulus of elasticity have been selected from experimental data. The plates are made of plywood. The nominal plywood thickness, number of plies constituting them, type of timber from which the plies have originated would obviously affect its bending and axial stiffness and other related properties. Hence further investigation is needed by changing the plywood thicknesses, the number of plies, type of timber constituting them, the depth of the joists, panel spans and the different types of connections between the plate and the joist.

The Art. 4.3.3 discusses the effect of changing the \( s/L_p \) ratio. The study has been done based on a particular set of nail stiffnesses and only three different \( s/L_p \) ratios have been looked into. Furthermore, the same type of plate as described above has been used here too. Hence, a more comprehensive study on this topic should be done with

(i) different set of nail stiffness values
(ii) a few extra \( s/L_p \) ratios, and
(iii) by changing the properties of plate (e.g. plywood thickness, number of plies, type of timber constituting the plywood etc.)
Bibliography


A.1 Shape Functions:

A.1.1 Vector \{M_0\}

\[ M_0 (1) = \xi^2 - \frac{5}{4} \xi^3 - \frac{1}{2} \xi^4 + \frac{3}{4} \xi^5 \]

\[ M_0 (2) = (\xi^2 - \xi^3 - \xi^4 + \xi^5) / 8.0 \]

\[ M_0 (16) = 1.0 - 2.0 \xi^2 + \xi^4 \]

\[ M_0 (15) = (\xi - 2.0 \xi^3 + \xi^5) / 2.0 \]

\[ M_0 (23) = \xi^2 + \frac{5}{4} \xi^3 - \frac{1}{2} \xi^4 - \frac{3}{4} \xi^5 \]

\[ M_0 (24) = (-\xi^2 - \xi^3 + \xi^4 + \xi^5) / 8.0 \]

All other components, \(M_0 (k) = 0.0, k = 1, 2, ..., 34.\)

A.1.2 Vector \{M_3\}

\[ M_3 (3) = \frac{1}{4} (-3 \xi + 4 \xi^2 + \xi^3 - 2 \xi^4) \]

\[ M_3 (4) = (-\xi + \xi^2 + \xi^3 - \xi^4) / 8.0 \]

\[ M_3 (13) = 1.0 - 2.0 \xi^2 + \xi^4 \]

\[ M_3 (25) = \frac{1}{4} (3 \xi + 4 \xi^2 - \xi^3 - \xi^4) \]

\[ M_3 (26) = (-\xi - \xi^2 + \xi^3 + \xi^4) / 8.0 \]

All other components, \(M_3 (k) = 0.0, k = 1, 2, ..., 34.\)
A.1.3 Vector \( \{M_5\} \)

\[ M_5(5) = M_3(3); \ M_5(6) = M_3(4); \ M_5(14) = M_3(13); \]
\[ M_5(27) = M_3(25); \ M_5(28) = M_3(26) \]

All other components, \( M_5(k) = 0.0, k = 1,2, \ldots, 34. \)

A.1.4 Vectors \( \{M_1\}, \{M_2\}, \{M_4\}, \text{ and } \{M_6\} \)

\[ M_1(k) = \frac{dM_0(k)}{d\xi}; \quad M_2(k) = \frac{d^2M_0(k)}{d\xi^2}; \]
\[ M_4(k) = \frac{dM_2(k)}{d\xi}; \quad M_6(k) = \frac{dM_2(k)}{d\xi} \]

in which: \( k = 1,2, \ldots, 34 \)
Appendix B

B.1 Shape Functions:

B.1.1 Vector \( \{M_7\} \)

\[
M_7(7) = \xi^2 - 5/4\xi^3 - 1/2\xi^4 + 3/4\xi^5
\]
\[
M_7(8) = (\xi^2 - \xi^3 - \xi^4 + \xi^5)/8.0
\]
\[
M_7(16) = 1.0 - 2.0\xi^2 + \xi^4
\]
\[
M_7(22) = (\xi - 2.0\xi^3 + \xi^5)/2.0
\]
\[
M_7(29) = \xi^2 + 5/4\xi^3 - 1/2\xi^4 - 3/4\xi^5
\]
\[
M_7(30) = (-\xi^2 - \xi^3 + \xi^4 + \xi^5)/8.0
\]

All other components, \(M_7(k) = 0.0, k = 1,2,\ldots,34.\)

B.1.2 Vector \( \{M_{10}\} \)

\[
M_{10}(9) = 1/4 ( -3\xi + 4\xi^2 + \xi^3 - 2\xi^4 )
\]
\[
M_{10}(10) = ( -\xi + \xi^2 + \xi^3 - \xi^4 )/8.0
\]
\[
M_{10}(20) = 1.0 - 2.0\xi^2 + \xi^4
\]
\[
M_{10}(31) = 1/4 ( 3\xi + 4\xi^2 - \xi^3 - \xi^4 )
\]
\[
M_{10}(32) = ( -\xi - \xi^2 + \xi^3 + \xi^4 )/8.0
\]

All other components, \(M_{10}(k) = 0.0, k = 1,2,\ldots,34.\)
Appendix B:

B.1.3 Vector \{M_{12}\}

\[ M_{12}(11) = M_{10}(9); M_{12}(12) = M_{10}(10); M_{12}(21) = M_{10}(20); \]
\[ M_{12}(33) = M_{10}(31); M_{12}(34) = M_{10}(32) \]

All other components, \(M_{12}(k) = 0.0, k = 1, 2, ..., 34.\)

B.1.4 Vectors \{M_8\}, \{M_9\}, \{M_{11}\}, and \{M_{13}\}

\[ M_8(k) = \frac{dM_7(k)}{d\xi}; M_9(k) = \frac{d^2M_7(k)}{d\xi^2}; \]
\[ M_{11}(k) = \frac{dM_{10}(k)}{d\xi}; M_{13}(k) = \frac{dM_{12}(k)}{d\xi}; \]

in which: \(k = 1, 2, ..., 34\)
C.1 Input

The input file should be set up as follows:

1. Enter: on free format

**NM, NJT, ISYM, INPTE, NFLOR, IPRNT, TOL**

where **NM** = maximum order of sine/cosine terms in the Fourier series. There can be a maximum of 5 terms, so that for symmetrical problems in the direction of the joists, **NM** can be up to 9 (the five terms will be of the order 1, 3, 5, 7, and 9). If **NM** = 3, there will be two terms, those for order 1 and 3. For non-symmetrical problems, **NM** is just the maximum order of sine/cosines and agrees with the number of terms: thus, **NM** = 5 means terms with order 1, 2, 3, 4, 5.

**NJT** = number of joists, with a maximum of 20.

**ISYM** = 0 if non-symmetrical problem along the joists;

1 for symmetrical problems.

**INPTE** = 4 to pick $E$ - values from a distribution;

2 to enter joist's $E$ - values one by one;

3 to enter joist's $E$-values and strengths one by one.

**NFLOR** = numbers of floors to be run.

**IPRNT** = output parameter. If 0, output is just a summary. If 1, output is comprehensive, including detailed stresses and deflections joist by joist.

**TOL** = Tolerance to stop iterations in the solution of the system of equations.
This is needed only if $NM$ exceeds 1, and the value $TOL = 0.001$ may be used.

2. If $INPTE \neq 4$ skip to 9.

Otherwise, when $INPTE = 4$ (Case of Simulations)

Enter: $Z1, CVS$

where $Z1$ = random number initializer (any value can be entered) for the random number generator used by the system.

$CVS$ = a number used with the minimum selected value within which the $E$ for all the joists in the floor will fall. Thus, if $CVS = 0.10$, all the joists’ $E$ will fall within 1.0 and 1.1 times the minimum selected $E$.

The use of $CVS \neq 0.0$ allows the selection of floors with joists of similar properties, but with the minimum $E$ within a floor still randomly selected from the entire population. Thus, in the same simulation one is mixing floors with joists all nearly equally stiff and strong with floors utilizing joists all nearly equally flexible and weak. This procedure can be used to simulate the ‘lot sampling’ technique, with $CVS$ related to the within-lot variability.

When $CVS$ is a large number, all floors will have $E$ picked at random from the entire population. This is a way of doing a normal simulation, without introducing the ‘lot’ concept.

When $CVS = 0.0$, all joists within a floor will be picked at random but within a certain interval for $E$.

3. If $CVS \neq 0.0$ skip to 4.

When $CVS = 0.0$,

Enter: $EMIN$, $EMAX$

where $EMIN$, $EMAX$ = are the lower and upper limits of the interval within which all the $E$’s in the floor must lie. This option is useful to study the effect of controlling the $E$ of the joists. (MSR application, for example).

In order to do a normal simulation, $EMIN$ and $EMAX$ can be choosen as the
Appendix C: USER'S MANUAL - FAPP

lowest and the highest $E$ in the population.

4. Enter: **KDISTR, EO, EM, EK**
   
   where **KDISTR** = identifier for the type of distribution representing the population of joists' $EI$ (joist stiffness).
   
   KDISTR = 1 if the distribution is a three - parameter Weibull;
   
   KDISTR = 2 if the distribution is a two - parameter Lognormal;
   
   KDISTR = 3 if the distribution is a Normal.
   
   **EO, EM, EK** = parameters for the corresponding distributions. $EO$ is the minimum value of $EI$ for the three parameter Weibull, and $EO = 0.0$ otherwise.

5. Enter: **QA , QB**
   
   where **QA , QB** = intercept and slope, respectively, of the regression line between M.O.R. and $E$ for the population of joists considered.

6. Enter: **EOO , E11 , CV1 , CV2 , CV3**
   
   where **EOO** = minimum value of $E$ in the population;
   
   $E11$ = maximum value of $E$ in the population;
   
   **CV1 , CV2 , CV3** = coefficients that permit the calculation of the variability in M.O.R. for a given value of $E$. These coefficients are obtained by expressing the variability in Fig. C.1 according to the following equation:

   $$CV = CV1 * (E - EOO)^{CV2} * (E11 - E)^{CV3}$$

   where **CV** is the coefficient of variation estimated from Fig. C.1 for the value $E$.

7. Enter: **SPROOF**
   
   where **SPROOF** = Minimum strength (or Proof level) accepted for the joists. This strength refers to 'short term' strength.

8. Enter: **S , RL , D , D1**
   
   where **S** = joist spacing (in)
   
   **RL** = joist span (in)
Appendix C: USER'S MANUAL - FAPP

D = upper cover (sheathing) thickness (in)
Dl = lower cover (sheathing) thickness (in)


10. Enter: RKX, RKY, RKV, RKG, DX, DY, DV, DG, ECOV, NFACE & TV.

where RKX = bending stiffness of the upper cover in the x-direction, parallel to the joists. For plywood, this is usually parallel to the face grain.

RKY = bending stiffness of the upper cover in the y-direction, perpendicular to the joists. For plywood, this is usually parallel to the face grain.

RKV, RKG = parameters for plate bending, connected respectively to Poisson’s effect and shear by torsion.

DX = in-plane (axial) stiffness if the upper cover in x-direction.

DY = in-plane (axial) stiffness if the upper cover in y-direction.

DV, DG = parameters for in-plane axial stiffness, related respectively to Poisson’s effects and shear.

ECOV = modulus of elasticity for the outermost veneer (for plywood) with grain perpendicular to the joists. For a non-layered panel, ECOV is the E of the panel.

NFACE = 1 if cover face grain (plywood) is parallel to the joists;
= 2 if cover face grain is perpendicular to the joists.

NFACE = 1 for non-layered panels.

TV = thickness of outermost veneer (in) in the plywood cover. In non-layered panels, TV is the thickness of an arbitrarily defined outer layer.

11. Enter: RKX1, RKY1, RKV1, RKG1, DX1, DY1, DV1, DG1, ECOV1, NFACE1 & TV1.

where RKX1, RKY1, RKV1, RKG1, DX1, DY1, DV1, DG1, ECOV1, NFACE1, & TV1 are the same properties as mentioned above in 10 but for the lower
Appendix C: USER'S MANUAL - FAPP

cover.

12. If INPTE ≠ 4 skip to 13.

When INPTE = 4 (case of Simulations),

Enter: BJT, HJT, BETA, ALPHA, REG

where BJT = width of joists (in)
HJT = depth of joists (in)
BETA = torsion constant which is a function of the ratio $HJT/BJT$, for rectangular cross-sections. It can be found, for example, in pp. 36.7, Handbook of Engineering Mechanics, W. Flugge, ed.

ALPHA = shear deflection constant for rectangular cross-sections, (for parabolic distribution of shear stresses, this constant is 1.2)

REG = ratio of $E$ of joist to the shear modulus ($E/G$ ratio).

Skip to 16.

13. Enter: for each joist, and consecutively,

BJ(I) , I = 1, NJT (width of joists, (in))
HJ(I) , I = 1, NJT (depth of joists, (in))
STORE(I, 1), I = 1, NJT (E for each of the joists)

14. If INPTE ≠ 3 skip to 15.

Otherwise, when INPTE = 3

Enter: STREN(I) , I = 1, NJT (strength of joists)

15. ONLY for the first floor,

Enter: BETA , ALPHA , REG

where these constants are defined in Step 12.

DATA ON NAILING: (Entered only for the first floor)

16. Enter: ENL, XIN, RKPAL, RKPER, RKROT, NDISCR

where ENL = nail spacing (in) along the joist span.
XIN = distance between support and first nail along the joist span (in).
RKPAL , RKPER = nail load-slip modulii , respectively, in the directions parallel and perpendicular to the joist.
RKROT = nail rotation modulus.
NDSCR = 0 if the actual nailing pattern is transformed to an equivalent continuous connector for the purpose of analysis;
= 1 if actual (discrete) nailing pattern is treated as actually is (discrete).

The program uses the same type of nailing (nail and pattern) for all the joists.

16. Enter: ENL1, XIN1, RKPAL1, RKPER1, RKROT1, NDSCR1
    where ENL1, XIN1, RKPAL1, RKPER1, RKROT1, RKROT1 & NDSCR1 are the same properties as described above but for the connection between the lower cover and the bottom of the joist.

DATA ON GAPS IN THE COVER (SHEETING):
(Entered only for the first floor)

17. Enter: NGAPS
    NGAPS = number of gaps present in the upper cover. These are gaps perpendicular to the joist direction. If NGAPS = 0 skip to 23.

18. Enter: NGT
    where NGT = 0 if the gaps are as entered in 19.
    = 1 if the gaps are adjusted , automatically, to the nailing pattern.

19. Enter: from I = 1 until NGAPS, (for each gap),
    GAPX(I) , GAP(I)
    where GAPX(I) = x - coordinate of the beginning of the i\textsuperscript{th} -gap (in)
    GAP(I) = gap width of the i\textsuperscript{th} - gap (in)

20. Enter: NGAPS1
where \( \text{NGAPS} \) = number of gaps present in the lower cover. These are gaps perpendicular to the joist direction. If \( \text{NGAPS} = 0 \) skip to 23.

21. Enter: \( \text{NGT1} \)
   where \( \text{NGT1} = 0 \) if the gaps are as entered in 22.
   \( = 1 \) if the gaps are adjusted, automatically, to the nailing pattern.

22. Enter: from \( I = 1 \) until \( \text{NGAPS1} \) (for each gap),
\[
\text{GAPX1}(I), \text{GAP1}(I)
\]
where \( \text{GAPX1}(I) \) = \( x \)-coordinate of the beginning of the \( i^{th} \)-gap (in)
\( \text{GAP1}(I) \) = gap width of the \( i^{th} \)-gap (in)

**APPLIED LOADS:** (Entered only for the first floor)

23. Enter: \( \text{NLOAD, NLU, PLOAD, SREF, WREF} \)
   where \( \text{NLOAD} \) = number of loaded areas, entered one by one (max. 12). If \( \text{NLOAD} = 0 \) then the load is uniformly distributed over the entire floor and equal to \( \text{PLOAD} \).

\( \text{NLU} = 1 \) if all the joists have the same load distribution, in which case one can have upto 12 loaded areas per joist;
\( = 0 \) if different joists have different loadings.

In the case of \( \text{NLOAD} = 0 \), \( \text{NLU} \) is used to distinguish the cases of loaded or unloaded outer flanges for the outside joists. If \( \text{NLU} = 1 \), all joists are equally loaded, meaning that even the outside joists receive the same load and thus their outer flanges are also loaded. If \( \text{NLU} = 0 \), the outside joists have their outer flanges unloaded.

In the case of \( \text{NLOAD} \neq 0 \), the outside joists have the outer flanges unloaded automatically, only if \( \text{NLU} = 1 \).

\( \text{PLOAD} \) = magnitude of the uniformly distributed load.

\( \text{SREF} \) = calculated stress on a joist (acting alone, with no sheathing contribution) under the load \( \text{PLOAD} \).
\( WREF \) = calculated deflection of a joist (acting alone, with no sheathing contribution) under the load \( PLOAD \) and for an assumed modulus of elasticity \( E \).

\( SREF \) and \( WREF \) are used to compute load sharing factors for stresses and deflections, relating these as occurring in the floor system to those occurring in a single joist with no sheathing contribution.

If \( NLOAD = 0 \) (Uniformly distributed load) skip to 25.

24. When \( NLOAD \neq 0 \),
   Enter: for each loaded area, \((I = 1, NLOAD)\)
   \[
   NLJO(I), \text{ STORE}(I,7)
   \]
   then \( \text{STORE}(I,3), \text{ STORE}(I,4), \text{ STORE}(I,5), \text{ STORE}(I,5) \)
   where \( NLJO(I) \) = number of joist loaded;
   \( \text{STORE}(I,7) \) = load, uniformly distributed, over the loaded area;
   \( \text{STORE}(I,3) \) to \( \text{STORE}(I,6) \) = coordinate of the loaded area, respectively,
   \( X1, X2, Y1, Y2 \) are as shown in Fig. C.2. \( X1 \) and \( X2 \) are global coordinates, while \( Y1 \) and \( Y2 \) are local to the joist element.

BOUNDARY CONDITIONS: (Entered only for the first floor)

25. Enter: \( NBC \)
   where \( NBC \) = number of boundary conditions imposed on the problem.
   If \( NBC = 0 \) skip to 27.

26. When \( NBC \neq 0 \),
   Enter: for each boundary condition \((I = 1, NBC)\),
   \[
   IBC(I,1), IBC(I,2)
   \]
   where \( IBC(I,1) \) = joist number where the boundary condition exists;
   \( IBC(I,2) \) = code number for the boundary condition imposed. See Fig. C.3 and the Table C.1.
   All boundary conditions, when imposed, restrict the corresponding displacement
or rotation to zero.

27. If INPTE = 4 (Case of Simulations) this is the end of the file.

For cases INPTE ≠ 0, Data for another floor must be entered unless only one floor is being analyzed (NFLOR = 1).

Data for the subsequent floors must be entered starting again from step 13.

Units: The program works with any set of consistent units. That is, if inches are used, and loads are in pounds, stresses are given in psi and deflections in inches.

If millimeters are used for dimensions, Newtons for loads, deflections will be in millimeters and stresses in MPa.

Figure C.1: Correlation between joist M.O.R. and its Modulus of Elasticity
Figure C.2: Loading Area Coordinates
Figure C.3: The Nodes of an element
Table C.1: Table of Boundary Condition Codes

<table>
<thead>
<tr>
<th>POINT</th>
<th>CONDITION</th>
<th>CODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>VERTICAL DISPLACEMENT</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>ROTATION IN (z,y) PLANE</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>DISPLACEMENT IN x-DIRECTION</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>DERIVATIVE OF DISPLACEMENT IN x-DIRECTION w.r.t. x</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>DISPLACEMENT IN y-DIRECTION</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>DERIVATIVE OF DISPLACEMENT IN y-DIRECTION w.r.t. y</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
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<tr>
<td>4</td>
<td>ROTATION IN (z,y) PLANE</td>
<td>8</td>
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<td>DISPLACEMENT IN x-DIRECTION</td>
<td>9</td>
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<td>DISPLACEMENT IN y-DIRECTION</td>
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<td>DISPLACEMENT IN y-DIRECTION</td>
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<td>ROTATION IN (z,y) PLANE</td>
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<td>DERIVATIVE OF DISPLACEMENT IN x-DIRECTION w.r.t. x</td>
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<td>DISPLACEMENT IN y-DIRECTION</td>
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<td>DERIVATIVE OF DISPLACEMENT IN y-DIRECTION w.r.t. y</td>
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<td>ROTATION IN (z,y) PLANE</td>
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<td>DISPLACEMENT IN x-DIRECTION</td>
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<td>DERIVATIVE OF DISPLACEMENT IN x-DIRECTION w.r.t. x</td>
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<td>DISPLACEMENT IN y-DIRECTION</td>
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<td>DERIVATIVE OF DISPLACEMENT IN y-DIRECTION w.r.t. y</td>
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A listing of the program FAPP is presented in this appendix. The Computer Language FORTRAN is used to develop the program. The user's manual describing the data file to run the program is described in Appendix C.
**FLOOR ANALYSIS PROGRAM, F.A.P**

**WITH GAPS IN ELEMENTS**

**20 JOISTS LIMIT**

DOUBLE PRECISION TEMP, TEMP1, TEMP2, TEMP3, TEMP4, STIFF
COMMON/B2/RX1, RXY, RKG, DX, DY, DV, RL, FACTOR,
1 RX1,RXY, RKG, DX, DY, DV, RL
COMMON/B3/E(6), H(6), S, NM, NSTEP, NMAX
COMMON/B4/D, D1, E(5), GJ,/community/ENL, ENL1, RKPAL, RKPAL1, RKP, RKP1,
1 PLD, X1, X2, Y1, Y2, BJ(20), HJ(20),
2 HJTM, ALPHA, RKRO, RKRO1
COMMON/B5/STORE(20,7), ECOV, ECOV1, NFACE, NFACE1, WP, TV, TV1
COMMON/B6/GAPX0(5), GAP0(5), GAPX(5), GAP(5), XIN, NGAPS, NAI, NDSCR,
1 NGAPS1, GAPX1(5), GAP1(5), GAPX01(5), GAP1(5), XIN1, NDSCR1, NAI1
COMMON/B8/STIFF(34,34)
COMMON/B9/SVEC(6,452)

DOUBLE PRECISION PI = 4.0D0 * DATAN(1.0D0)
ETA(1) = 0.932469514203152D0
ETA(2) = 0.661209386466265D0
ETA(3) = 0.238619186083319D0
H(1) = 0.171324492379170D0
H(2) = 0.360761573048139D0
H(3) = 0.467913934572691D0
DO 1 I = 1, 3
ETA(7-I) = -ETA(I)
H(7-I) = H(I)
1 CONTINUE
CALL GENMTX

**Z1 IS A RANDOM NUMBER GENERATION SEED**
CVS = VARIABILITY IN JOISTS' E: IN THIS CASE, E WILL BE
RANDOMLY CHOSEN BUT WITHIN + OR - CVS(%) OF THE FIRST
E. THIS FIRST E IS RANDOMLY CHOSEN FROM THE ENTIRE
DISTRIBUTION
CVS = 0.0 FOR RANDOM SELECTION FROM DISTRIBUTION:
IN THIS CASE, E WILL BE RANDOMLY SELECTED BETWEEN
THE INPUT LIMITS EMIN AND EMAX
NM IS THE MAXIMUM ORDER FOR THE SINE & COSINE SERIES
NM CAN BE UP TO 11 FOR SYMMETRIC PROBLEMS (ISYM = 1)
NM CAN ONLY BE UP TO 6 FOR NON-SYMMETRIC PROBLEMS
NJT IS NUMBER OF JOISTS
SYM IS 1 FOR SYMMETRIC CASE
INPTE IS 2 FOR INPUTING PROPERTIES OF JOISTS ONE BY ONE
INPTE IS 3 FOR INPUTING PROPERTIES OF JOISTS ONE BY ONE
PLUS THEIR CORRESPONDING STRENGTHS
INPTE IS 4 TO SELECT PROPERTIES OF JOISTS FROM DISTRIBUTION
NFLOR IS THE NUMBER OF FLOORS TO BE RUN
C IPRNT IS 0 FOR SUMMARY OUTPUT
C NGAPS IS NUMBER OF GAPS IN EACH ELEMENT IN UPPER COVER. THESE GAPS
C ARE PERPENDICULAR TO JOIST DIRECTION
C GAPX(I) IS X-COORD OF BEGINNING OF I-TH GAP IN UPPER COVER
C GAP(I) IS WIDTH OF I-TH GAP (NGT=0) IN UPPER COVER
C NGAPS1 IS NUMBER OF GAPS IN EACH ELEMENT IN LOWER COVER. THESE GAPS
C ARE PERPENDICULAR TO JOIST DIRECTION
C GAPX(I) IS X-COORD OF BEGINNING OF I-TH GAP IN LOWER COVER
C GAP(I) IS WIDTH OF I-TH GAP (NGT=0) IN LOWER COVER
C NGT = 0 IF GAPS ARE ENTERED BY GAPX(I) AND GAP(I)
C = 1 IF GAPS ARE ADJUSTED AUTOMATICALLY TO THE NAILING PATTERN
C NGT1=  0 IF GAPS ARE ENTERED BY GAPX(I) AND GAP(I)
C = 1 IF GAPS ARE ADJUSTED AUTOMATICALLY TO THE NAILING PATTERN
C EO,EM,Ek PARAMETERS FOR JOISTS' EI DISTRIBUTION
C EO = MINIMUM EI FOR THE DISTRIBUTION (LOCATION FOR 3-P WEIBULL
C OR ASSUMED 0.0 FOR 2-P LOGNORMAL OR NORMAL DISTRIBUTIONS)
C QA, QB ARE PARAMETERS FOR STRENGTH / E CORRELATION
C QA = INTERCEPT OF REGRESSION LINE M.O.R / M.O.E
C QB = SLOPE OF REGRESSION LINE M.O.R / M.O.E
C CV1, CV2, CV3 ARE COEFFICIENTS OF EQUATION TO COMPUTE THE
C COEFFICIENT OF VARIATION OF M.O.R AS A FUNCTION OF M.O.E
C EOO AND E11 ARE, RESPECTIVELY, THE MINIMUM AND MAXIMUM M.O.E
C USED TO DETERMINE THE COEFFICIENTS CV1, CV2 AND CV3
C WARNING : EMIN MUST NOT BE SMALLER THAN EOO
C EMAX MUST NOT BE LARGER THAN E11
C KDISTR = 1 IF THE EI DISTRIBUTION IS 3-P WEIBULL
C = 2 IF THE EI DISTRIBUTION IS 2-P LOGNORMAL
C = 3 IF THE EI DISTRIBUTION IS NORMAL
C STORE(I,1) = E FOR JOIST I
C STORE(I,2) = G FOR JOIST I
C STORE(I,3) = X1 FOR LOADED AREA I
C STORE(I,4) = X2 FOR LOADED AREA I
C STORE(I,5) = Y1 FOR LOADED AREA I
C STORE(I,6) = Y2 FOR LOADED AREA I
C STORE(I,7) = DISTRIBUTED LOAD ON AREA I FROM X1 TO X2 AND Y1 TO Y2
C ECOV = E FOR THE UPPER COVER, PARALLEL TO GRAIN (VENNEER)
C EC0V1= E FOR THE LOWER COVER, PARALLEL TO GRAIN (VENNEER)
C NFACE = 1 IF UPPER COVER GRAIN PARALLEL TO JOISTS
C = 2 IF UPPER COVER GRAIN PERPENDICULAR TO JOISTS
C NFACE1= 1 IF LOWER COVER GRAIN PARALLEL TO JOISTS
C = 2 IF LOWER COVER GRAIN PERPENDICULAR TO JOISTS
C TV = THICKNESS OF OUTERMOST UPPER VENNER
C TV1= THICKNESS OF OUTERMOST LOWER VENNER
C COVER STRESSES ALL PERPENDICULAR TO THE JOISTS
C SPROOF = PROOF LEVEL (OR MINIMUM) STRENGTH FOR JOISTS
C NAI =NO. OF NAILS ON EACH JOIST NAILED FROM UPPER COVER
C NAI1=NO. OF NAILS ON EACH JOIST NAILED FROM LOWER COVER
C TOL = TOLERANCE FOR CONVERGENCE IF NM GREATER THAN 1
C XIN =DIST. BETWEEN SUPPORT AND FIRST NAIL ALONG THE JOIST SPAN
C ON THE UPPER COVER SIDE
C XIN1=DIST. BETWEEN SUPPORT AND FIRST NAIL ALONG THE JOIST SPAN
C ON THE UPPER COVER SIDE
C NDISCR=0 IF THE ACTUAL NAILING PATTERN IN UPPER COVER IS
C TRANSFORMED TO AN EQUIVALENT CONTINUOUS CONNECTOR
C FOR THE PURPOSE OF ANALYSIS
C =1 IF ACTUAL (DISCRETE) NAILING PATTERN IS TREATED AS
C ACTUALLY IS
C NDSCR1=0 IF THE ACTUAL NAILING PATTERN IN LOWER COVER IS
C TRANSFORMED TO AN EQUIVALENT CONTINUOUS CONNECTOR
C FOR THE PURPOSE OF ANALYSIS
C =1 IF ACTUAL (DISCRETE) NAILING PATTERN IS TREATED AS
C ACTUALLY IS
C================================================================:
C
C READ(1,116) (TITLE(I), I=1,20)
READ(1,1000) NM, NJT, ISYM, INPTE, CFLOR, IPRNT, TOL
IF (INPTE.NE.4) GO TO 700
READ(1,1000) ZI, CVS
IF (CVS.NE.0.0) GO TO 706
READ(1,1000) EMIN, EMAX
706 READ(1,1000) KDISTR, EO, EM, EK
READ(1,1000) QA, QB
READ(1,1000) EOO, Ell, CV1, CV2, CV3
READ(1,1000) SPROOF
700 CONTINUE
READ(1,1000) S, RL, D, D1
READ(1,1000) FACTOR
READ(1,1000) RX, RY, RK, RX1, DX, DY, DG
READ(1,1000) EC0V, NFACE, TV
READM, 1000) QA, QB
READ(1,1000) EOO, Ell, CV1, CV2, CV3
READ(1,1000) SPROOF
711 IF (INPTE.NE.4) GO TO 701
READ(1,1000) BJT, HJT, BETA, ALPHA, REG
701 IF (II.GT.1) GO TO 889
READ(1,1000) (BJ(I), I = 1, NJT)
READ(1,1000) (HJ(I), I = 1, NJT)
889 READM, 1000) (STORE(I,1), I = 1, NJT)
IF (INPTE.NE.3) GO TO 780
READ(1,1000) (BETA, ALPHA, REG
702 IF (II.GT.1) GO TO 115
READ(1,1000) S, RL, D, D1
DO 702 I = 1, NJT
Bj(I) = BJT
RJ(I) = HJT
GO TO 702
701 IF (II.GT.1) GO TO 889
READ(1,1000) (Bj(I), I = 1, NJT)
READ(1,1000) (Hj(I), I = 1, NJT)
889 READM, 1000) (STORE(I,1), I = 1, NJT)
IF (INPTE.NE.3) GO TO 780
READ(1,1000) (BETA, ALPHA, REG
702 IF (II.GT.1) GO TO 115
READ(1,1000) S, RL, D, D1
DO 702 I = 1, NJT
HjTM = HJT / NJT
READ(1,1000) ENL, XIN, RKPAL, RKPER, RKROT, NDSCR
READ(1,1000) ENL1, XIN1, RKPAL1, RKPER1, RKROT1, NDSCR1
NAI = (RL - 2.0*XIN) / ENL
NAI = NAI + 1
NAI1 = NAI1 + 1
READ(1,1000) NGAPS
IF (NGAPS.EQ.0) GO TO 121
READ(1,1000) NGT
DO 123 I=1,NGAPS
READ(1,1000)GAPX(I),GAP(I)
GAPX0(I) = GAPX(I)
GAP0(I) = GAP(I)
IF (NGT.EQ.0) GO TO 123
NG = (GAPX(I) - XIN)/ENL
XG1 = XIN + NG * ENL
XG2 = XG1 + ENL
IF (XG1.EQ.GAPX(I)) GO TO 824
XG3 = GAPX(I) + GAP(I)
GAPX(I) = XG1
GAP(I) = 2.0 * ENL
IF (XG2.GT.XG3) GAP(I) = ENL
GO TO 123
824
GAPX(I) = XG1 - ENL
GAP(I) = 2.0 * ENL
123 CONTINUE
121 CONTINUE
READ(1,1000) NGAPS1
IF(NGAPS1.EQ.0)GO TO 421
READ(1,1000)NGT1
DO 423 1=1,NGAPS1
READ(1,1000)GAPX1(I),GAP 1(I)
GAPX01(I)=GAPX1(I)
GAP01(I)=GAP 1(I)
IF(NGT1.EQ.0)GO TO 423
NG1=(GAPX1(I)-XIN1)/ENL1
XG4=XIN1+NG1*ENL1
XG5=XG4+ENL1
IF(XG4.EQ.GAPX1(I))GO TO 825
XG6=GAPX1(I)+GAP 1(I)
GAPX1(I)=XG4
GAP1(I)=2.0*ENL1
IF(XG5.GT.XG6)GAP1(I)=ENL1
GO TO 423
825
GAPX1(I)=XG4-ENL1
GAP 1(I) =2.0*ENL1
423 CONTINUE
421 CONTINUE
READ(1,1000) NLOAD, NLU, PLOAD, SREF, WREF
C PAUSE'STOPPED AT 211'
C PLOAD = CHARACTERISTIC LOAD (E.G., THE MAXIMUM DISTRIBUTED LOAD)
C IF NLOAD = 0 , U.D.L OVER ENTIRE FLOOR (PLOAD PSI)
C IF NLU = 0 , OUTSIDE FLANGES ARE UNLOADED
C IF NLU = 1 , OUTSIDE FLANGES ARE LOADED (ALL JOISTS
C EQUALLY LOADED)
C IF NLOAD DIFFERENT FROM ZERO, DEFINE
C NLOAD = NUMBER OF LOADED AREAS, ENTERED ONE BY ONE (MAX. 20)
C NLU = 0 IF DIFFERENT JOISTS HAVE DIFFERENT LOADING AS ENTERED
C NLU = 1 IF ALL JOISTS ARE EQUALLY LOADED (IN THIS CASE, THERE
C CAN BE UP TO 20 LOADED AREAS ON A JOIST).
C SREF = JOIST ALONE BENDING STRESS UNDER PLOAD (TO COMPUTE LOAD SHARING
C FACTOR GAMS(I))
C WREF = JOIST ALONE DEFLECTION UNDER PLOAD (TO COMPUTE LOAD SHARING
C FACTOR GAMW(I))
C IF (NLOAD.NE.0) GO TO 11
DO 19 I = 1, NJT
STORE(I,3) = 0.0
STORE(I,4) = RL
STORE(I,5) = -S/2.0
STORE(I,6) = S/2.0
STORE(I,7) = PLOAD
19  CONTINUE
GO TO 22
11  DO 5 I = 1, NLOAD
READ(1,1000) NLJO(I), STORE(I,7)
READ(1,1000) (STORE(I,J), J=3,6)
5  CONTINUE
22  CONTINUE
C INPUT BOUNDARY CONDITIONS
C NBC = NUMBER OF B.C.
C IBC(I,1) = JOIST NUMBER (ELEMENT NUMBER)
C IBC(I,2) = NUMBER OF THE CONSTRAINT (1-34) IN THE UNKNOWNS' VECTOR
READ(1,1000) NBC
IF (NBC.EQ.0) GO TO 9
DO 6 I = 1, NBC
6  READ(1,1000) (IBC(I,J), J=1,2)
9  CONTINUE
DO 704 I = 1, NJT
AJT(I) = BJ(I) * HJ(I)
RIY(I) = BJ(I) * HJ(I)**3 /12.0
RIZ(I) = HJ(I) * BJ(I)**3 /12.0
RIT(I) = BETA * HJ(I) * BJ(I)**3
704  CONTINUE
PIN2 = PI**2/(2.0*RL)
PIN4 = PI**4/(2.0*RL**3)
NEQ = NJT + 22
LHB = 34
NA = 34 + NEQ
NSTEP = 1
IF (ISYM.EQ.1) NSTEP = 2
NMAX = NM
IF (NSTEP.EQ.2) NMAX = (NM + 1)/2
IF(IPRNT.EQ.0) GO TO 130
116  FORMAT(20A4)
C WRITE(2,102)(TITLE(I),I=1,20)
C102  FORMAT(1H1,' FLOOR ANALYSIS',5X,20A4,/.2X,16(1H*),///)
WRITE(2,103)NJT,NM
103  FORMAT(' NUMBER OF ELEMENTS =',I5,'5X','N =',I5,///
WRITE(2,550)NGAPS,NGAPS1
550  FORMAT(//,'NO.OF GAPS(UPPER) =',15,5X,'NO.OF GAPS(LOWER) =',15,///
WRITE(2,551)S,RL
551  FORMAT('JOIST SPACING=',E13.6,5X,'JOIST SPAN=',E13.6,///
WRITE(2,104)
104  FORMAT(' PROPERTIES AND DIMENSIONS FOR UPPER FLANGE:',///
WRITE(2,105)D,ECOV,NFACE
105  FORMAT('UPPER COVER THICKNESS=',E13.6,///
1  'MOD. OF ELASTICITY OF OUTERMOST VENEER=',E13.6,///
2  'DIRECTION OF COVER GRAIN=',I3,///
WRITE(2.106)RKX,RKY,RKV,RKG
106  FORMAT( 'K(X) =',E13.6,10X,'K(Y) =',E13.6,///
&'K(V) =',E13.6,10X,'K(G) =',E13.6)
WRITE(2,107)DX,DY,DV,DG
107  FORMAT( 'D(X) =',E13.6,10X,'D(Y) =',E13.6,///
&D(V) =',E13.6,10X,'D(G) =',E13.6)
WRITE(2,552)
CONTINUE
IF (IPRNT.EQ.0) GO TO 838
WRITE(2,113) II
FORMAT(1H1,' FLOOR NO.=',I3,/.'  JOIST  PROPERTIES'/)
838 CONTINUE
IF(INPTE.EQ.4)GO TO 622
GO TO 833
DO 648 I8 = 1, NJT
625 IF(INPTE.EQ.4) THEN
   Z1 = RAND(Z1)
ELSE
   Z1 = RAND(IZ1,IZ2)
ENDIF
C IF (Z1.LE.0.0.OR.Z1.GE.1.0) GO TO 625
CALL DISTR(Z1,KDISTR,Q1,E0,EM,EK,STORE(I8,1))
C WRITE(2,4639) I8, STORE(I8,1)
C4639 FORMAT('ELAS(',I3,')=', 4X,F13.7,/) IF (CVS.EQ.0.0) GO TO 707
IF (I8.EQ.1) GO TO 626
IF (STORE(I8,1).*GE.EMIN.AND.STORE(I8,1).LE.EMAX) GO TO 624
GO TO 625
626 IF (STORE(I8,1).LT.E00.OR.STORE(I8,1).GT.E11) GO TO 625
ESMIN = STORE(I8,1)*(1.0 - CVS)
IF (ESMIN.LT.E00)  ESMIN  = E00
ESMAX =  STORE(I8,1)*(1.0  + CVS)
IF (ESMAX.GT.E11) ESMAX = E11
GO TO 624
707 IF (STORE(I8,1).GE.EMIN.AND.STORE(I8,1).LE.EMAX) GO TO 624
GO TO 625
624 CONTINUE
C
C IF (Z1.LE.0.0.OR.Z1.GE.1.0) GO TO 708
CALL STREND(STREN(I8),STORE(I8,1),QA,QB,CV1,CV2,CV3,E00,E11,Z1, 1 KOUT,SPROOF)
C WRITE(2,4621) I8 , STREN(I8)
C4621 FORMAT( 'STREN(',I3,') =  ',4X,F13.7,/) IF (KOUT.EQ.1) GO TO 625
IF (STREN(I8).GT.SPROOF)  GO TO 648
GO TO 708
648 CONTINUE
833 CONTINUE
DO 834 I8 = 1, NJT
STORE(I8,1) = STORE(I8,1)*10.0**6
STORE(I8,2) = STORE(I8,1)/REG
IF (IPRNT.EQ.0) GO TO 834
WRITE(2,114) I8, (STORE(I8,J), J = 1,2), BJ(I,J), HJ(I8)
C834 CONTINUE
90
114 FORMAT(' JOIST ',I2,10X,'E = ',E13.6,10X,'G = ',E13.6,
110X,'B = ',F7.3,10X,'H = ',F7.3)
834 CONTINUE
C
C
IF (II .GT. 1) GO TO 770
RKX = RKX * FACTOR
RKY = RKY * FACTOR
RKV = RKV * FACTOR
RKG = RKG * FACTOR
DX = DX * FACTOR
DY = DY * FACTOR
DV = DV * FACTOR
DG = DG * FACTOR
RKPAL = RKPAL * FACTOR
RKPER = RKPER * FACTOR
RKROT = RKROT * FACTOR
770 CONTINUE
IF (II .GT. 1) GO TO 563
RKX1 = RKX1 * FACTOR
RKY1 = RKY1 * FACTOR
RKV1 = RKV1 * FACTOR
RKG1 = RKG1 * FACTOR
DX1 = DX1 * FACTOR
DY1 = DY1 * FACTOR
DV1 = DV1 * FACTOR
DG1 = DG1 * FACTOR
RKPAL1 = RKPAL1 * FACTOR
RKPER1 = RKPER1 * FACTOR
RKROT1 = RKROT1 * FACTOR
563 CONTINUE
DO 410 I = 1, NJT
STORE(I,1) = STORE(I,1) * FACTOR
410 STORE(I,2) = STORE(I,2) * FACTOR
C PAUSE 'STOPPED AT 413'
C * ASSEMBLY OF GLOBAL MATRICES *
DO 300 IK = 1, NM, NSTEP
DO 300 IN = 1, IK, NSTEP
IKM = IK
INM = IN
IF (NSTEP .EQ. 1) GO TO 3000
IKM = (IK + D/2)
INM = (IN + 1)/2
3000 IF (I .EQ. IK) GO TO 735
IF (II .GT. 1) GO TO 300
IKO = (INM-1)*NMAX + IKM - INM*(INM+1)/2
DO 3 I = 1, NA
3 ASTIF(IKO,I) = 0.0
GO TO 260
735 IKO = IKM
DO 3001 I = 1, NA
3001 DK(I) = 0.0
DO 4 I = 1, NEQ
4 FORCE(IKO,I) = 0.0
DO 250 IE = 1, NJT
EJT = STORE(IE,1)
GJT = STORE(IE,2)
KLO = 0
250
ILO = 1
IF (NLOAD.NE.0) GO TO 737
X1 = STORE(IE,3)
X2 = STORE(IE,4)
Y1 = STORE(IE,5)
Y2 = STORE(IE,6)
IF (NLU.EQ.1) GO TO 736
IF (IE.EQ.1) Y1 = 0.0
IF (IE.EQ.NJT) Y2 = 0.0
736 PLD = STORE(IE,7) * FACTOR
GO TO 740
737 IF (NLUO(ILO).EQ.IE.OR NLU.EQ.1) GO TO 739
ILO = ILO + 1
IF (ILO.GT.NLOAD) GO TO 742
GO TO 737
739 X1 = STORE(ILO,3)
X2 = STORE(ILO,4)
Y1 = STORE(ILO,5)
Y2 = STORE(ILO,6)
738 PLD = STORE(ILO,7) * FACTOR
GO TO 740
740 CALL STIF(VECTR,IE,KLO,IN,IK,PI)
GO TO 741
742 IF (KLO.EQ.1) GO TO 745
PLD = 0.0
CALL STIF(VECTR,IE,KLO,IN,IK,PI)
741 IJ = (IE-1)*22
DO 204 J=1,34
IF (KLO.EQ.1) GO TO 746
DO 203 K=1,J
JK=LHB*(IJ+K-1)+J-K+1
IF (K.NE.J) GO TO 208
IF (J.EQ.19) GO TO 204
IF (J.EQ.18) GO TO 203
IF (J.EQ.17) GO TO 202
IF (J.EQ.16) GO TO 201
GO TO 203
401 TEMP1 = DK(JK) + STIFF(J,K)
GO TO 203
402 TEMP2 = DK(JK) + STIFF(J,K)
GO TO 203
403 TEMP3 = DK(JK) + STIFF(J,K)
GO TO 203
404 TEMP4 = DK(JK) + STIFF(J,K)
GO TO 203
408 DK(JK) = DK(JK) + STIFF(J,K)
203 CONTINUE
746 FORCE(IKO,IJ+J) = FORCE(IKO,IJ+J) + VECTR(J)
204 CONTINUE
KLO = 1
IF (NLOAD.EQ.0) GO TO 745
ILO = ILO + 1
IF (ILO.GT.NLOAD) GO TO 745
GO TO 737
745 JK = LHB*(IJ+15) + 1
DK(JK)=TEMP1+EJT*RIY(IE)*PIN4*IK**4
JK=LHB*(IJ+16)+1
DK(JK)=TEMP2+EJT*AJT(IE)*PIN2*IK**2
JK=LHB*(IJ+17)+1
DK(JK)=TEMP3+EJT*RIZ(IE)*PIN4*IK**4
JK=LHB*(IJ+18)+1
DK(JK)=TEMP4+GJT*RIT(IE)*PIN2*IK**2/(S**2)

C CONTINUE
C PAUSE 'STOPPED AT 497.021'
C
C WRITE(2,929) (STIFF(17,K),K=1,34)
929 FORMAT(E15.6)
C WRITE(2,930) DK(545)
930 FORMAT(E15.6)
C
C IF(NBC.EQ.0)GO TO 206
DO 205 I=1,NBC
NE=IBC(I,1)
NDOF=IBC(I,2)
M=(NE-1)**2+2+NDOF
J1=1
MM=M-1
IF(M.GE.34)J1=M-33
DO 17 J=J1,MM
JK=(LHB-1)*(J-1)+M
17 DK(JK)=0.0
M1=M+1
M8=M+33
IF (M8.GT.NEQ) M8 = NEQ
DO 18 J=M1,M8
JK=(LHB-1)*(M-1)+J
18 DK(JK)=0.0
JK=(LHB-1)*(M-1)+M
DK(JK)=1.0
FORCE(IK0,M)=0.0
205 CONTINUE
206 CONTINUE
C CALL DECMPI(NEQ,LHB,DK)
C
C DO 3002 I = 1, NA
3002 GSTIF(IK0,I) = DK(I)
GO TO 300
C
260 CONTINUE
DO 280 IE = 1, NJT
CALL STIF(VECTR,IE,0,IN,IK,PI)
IJ=(IE-1)**22
DO 264 J=1,34
DO 264 K=1,34
JK = (IJ+K-1)*(2*LHB-1)+LHB+J*K
ASTIF(IK0JK)=ASTIF(IK0JK)+STIFF(J,K)
264 CONTINUE
280 CONTINUE
C IF(NBC.EQ.0)GO TO 300
DO 288 I=1,NBC
NE=IBC(I,1)
NDOF=IBC(I,2)
M=(NE-1)**2+2+NDOF
M1=M-33
M2=M+33
IF(M1.LE.0)M1=1
IF(M2.GT.NEQ)M2=NEQ
DO 285 J=M1,M2
JK=(M-1)*(2*LHB-1)+J-M+LHB
ASTIF(IKO,JK)=0.0
JK=(J-1)*(2*LHB-1)+M-J+LHB
ASTIF(IKO,JK)=0.0
CONTINUE

C *SOLUTION OF SYSTEM*
C
DO 320 IK = 1, NMAX
   DO 3003 I = 1, NA
      DK(I) = GSTIFdK(I)
   DO 3004 I = 1, NEQ
      X(I) = FORCE(IK,I)
3004 X(I) = FORCE(IK,I)
C
   CALL SOLV(NEQ,LHB,DK,X)
C
   WRITE(2,958) IK
   FORMAT(/,'IK=','I2.2X,'DEFORMATION VECTOR',/)
C
   WRITE(2,959) (X(I),I=1,NEQ)
C
3005 DO 3005 I = 1, NEQ
      SVEC(IK,I) = X(I)
3005 SVEC(IK,I) = X(I)
CONTINUE
C
C PAUSE 'AFTER THE BRANCHING CONDITION AT 595`
C
C IF (NM.EQ.1) GO TO 400
   IF(NGAPS.EQ.0.AND.NGAPS1.EQ.0)GO TO 400
   NFLAG = 1
   DO 350 IK = 1, NM, NSTEP
      IKM = IK
      IF (NSTEP.EQ.2) IKM = (IK+1)/2
   DO 870 I = 1, NEQ
      XX(I) = FORCE(IKM,I)
C
   PAUSE 'STOPPED AFTER LINE 603`
C
   DO 340 IN = 1, NM, NSTEP
      IF (IN.EQ.IK) GO TO 340
         INM = IN
      IF (NSTEP.EQ.2) INM = (IN+1)/2
      IF (IN.GT.IK) GO TO 3260
         IKO = (INM-1)*NMAX + IKM - INM*(INM+1)/2
   DO 3263 DO 3263 I = 1, NEQ
      J1 = I - 33
      J2 = I + 33
      IF (J1.LE.0) J1 = 1
      IF (J2.GT.NEQ) J2 = NEQ
      TEMP = 0.0
   DO 330 J = J1, J2
      TEMP1 = SVEC(INM,J)
   DO 330 TEMP1 = SVEC(INM,J)
      IF (IN.GT.IK) GO TO 3265
         IJ = (J-1)*(2*LHB-1) + LHB + I - J
   DO 3265 IJ = (J-1)*(2*LHB-1) + LHB + I - J
GO TO 3266
C
$I_J = (I - 1) \cdot (2 \cdot LHB - 1) + LHB + J - I$

$TEMP = TEMP + \text{ASTIF}(IKO, IJ) \cdot \text{TEMP}^1$

CONTINUE

$XX(I) = XX(I) - TEMP$

CONTINUE

PAUSE 'STOPPED AFTER LINE 628'

DO 3013 $I = 1, NA$

$DK(I) = \text{GSTIF}(IKM, I)$

DO 3014 $I = 1, NEQ$

$X(I) = XX(I)$

CALL SOLV(NEQ, LHB, DK, X)

TEMP = 0.0

TEMP1 = 0.0

DO 3015 $I = 1, NEQ$

TEMP = TEMP + $X(I)^2$

TEMP1 = TEMP1 + $SVEC(IKM, I) \cdot 2$

TEMP = DSQRT(TEMP)

TEMPI = DSQRT(TEMPI)

EPS = DABS((TEMP - TEMPI)/TEMP1)

NAC = 1

IF (EPS.GT.TOL) NAC = 0

NFLAG = NFLAG'NAC

DO 873 $I = 1, NEQ$

$SVEC(IKM, I) = X(I)$

350 CONTINUE

IF (NFLAG.EQ.1) GO TO 400

GO TO 322

"OUTPUT"

NLOAD = NO. OF LOADED AREAS, ENTERED ONE BY ONE (MAX. 12)

IF NLOAD=0 THEN THE LOAD IS UNIFORMLY DISTRIBUTED

OVER THE ENTIRE FLOOR AND EQUAL TO PLOAD

NLU = 1 IF ALL JOISTS HAVE THE SAME LOAD DISTRIBUTION, IN

WHICH CASE ONE CAN HAVE UPTO 12 LOADED AREAS PER JOIST

=0 IF DIFFERENT JOISTS HAVE DIFFERENT LOADINGS

PLOAD = MAGNITUDE OF THE UNIFORMLY DISTRIBUTED LOAD

SREF = STRESS ON A JOIST (ACTING ALONG WITH NO SHEATHING CONTRIBUTION) UNDER THE LOAD PLOAD

$=6 \cdot w \cdot S \cdot L^4 / 8 \cdot B \cdot H^2$, $w$ - UNIFORMLY DISTRIBUTED LOAD

$S$ - SPACING BETWEEN JOISTS

$B \cdot H^2$ - SECTION MODULUS

$=6 \cdot P \cdot L^4 / 4 \cdot B \cdot H^2$, $P$ - LOCALISED LOAD

WREF = DEFLECTION OF A JOIST (ACTING ALONG WITH NO SHEATHING CONTRIBUTION) UNDER THE LOAD PLOAD AND FOR AN ASSUMED MOD. OF ELASTICITY E

$=(5/384) \cdot (w \cdot S \cdot L^4 / 4 \cdot B \cdot H^2$)

GAMS(I) = LOAD SHARING FACTOR FOR MAX. BENDING STRESS OF I-TH JOIST IN THE FLOOR

GAMS = MAX. LOAD FACTOR IN THE CASE ABOVE AMONG ALL THE JOISTS IN THE FLOOR

GAMW(I) = LOAD SHARING FACTOR FOR MAX. DEFLECTION OF I-TH JOIST
C IN THE FLOOR

C GAMWM=MAX. LOAD SHARING FACTOR IN THE CASE ABOVE AMONG ALL THE
C JOISTS IN THE FLOOR

C PAUSE 'BEFORE CALL OUT'

CALL OUT(NEQ,NJT,WMAX,SMAX,SF,WFT,WCOV,WCOV1,WCM,WCM1,CMA,CMA1,
& IPRNT,PI,WSM)

C PAUSE 'AFTER CALL OUT'

WRITE(3,627) II, WMAX, SMAX, WSM, (SF(I),I=1,NJT)

627 FORMAT(/, 'FLOOR NO. *****', I8, //, 
1 'MAX. JOIST DEFLN. IN FLOOR=', G20.10, //, 
2 'MAX. JOIST BENDING STRESS IN FLOOR=', G20.10, //, 
3 'MAX. STRESS=', G20.10, ' (NODE 2 - XDIRECTION)', //, 
4 'MAX. BENDING STRESS FOR EACH JOIST=', 20G20.10, //)

IF(ABS(WMAX).GE.ABS(WMAFL)) WMAFL = WMAX
IF(ABS(SMAX).GE.ABS(SMAFL)) SMAFL = SMAX
IF(ABS(WSM).GE.ABS(WSMFL)) WSMFL = WSM
IF(II.EQ.NFLOR) THEN
WRITE(5,575) WMAFL, SMAFL, WSMFL

575 FORMAT(/, 'JOIST DEFLECTION=', G20.10, //, 
1 'JOIST BENDING STRESS=', G20.10, //, 
2 'UPPER COVER STRESS (X - DIRECTION)', G20.10)
ELSE
GO TO 580
ENDIF

580 CONTINUE

C WSTAT = WMAX * SIN(PI*(0.5 * 24.0/RL))
WSTAT = 25.4 * WSTAT

888 FORMAT(F6.3)

C WRITE(3,632) (WFT(I), I=1,NJT)

632 FORMAT(/, 'MAX. DEFLN. FOR EACH JOIST IN FLOOR=', 20G20.10, //)

GAMS = 0.0
GAMMM = 0.0
DO 784 I = 1, NJT
GAMS(I) = SF(I) / SREF
GAMM(I) = WFT(I) / WREF
IF (ABS(GAMS(I)).GT.GAMSM) GAMSM = ABS(GAMS(I))
IF (ABS(GAMM(I)).GT.GAMWM) GAMWM = ABS(GAMM(I))

784 CONTINUE

C PAUSE 'AFTER DIVISION BY SREF AND WREF'

C WRITE(3,639) GAMSM, (GAMS(I),I=1,NJT)

C639 FORMAT(/, 'MAX. LOAD SHARING FACTOR FOR BENDING STRESS FOR ALL
C THE JOISTS=', 4X,G20.10, //, 'LOAD SHARING FACTOR FOR BENDING
C STRESS FOR EACH JOIST=', 4X,20G20.9, //)

C WRITE(3,640) GAMM, (GAMM(I), I=1,NJT)

C640 FORMAT(/, 'MAX. LOAD SHARING FACTOR FOR JOIST DEFLECT=', 4X,G20.10, //,
C 'LOAD SHARING FACTOR FOR BENDING STRESS FOR EACH JOIST=', 
C 4X, 20G20.10, //)

C WRITE(3,641) CMA,CMA1, WCM,WCM1, (WCOV(I), I=1, NJT)

C641 FORMAT(/, 'MAX. BENDING STRESS IN UPPER SHEATHING=', 4X,G20.10, //,
C 'MAX. BENDING STRESS IN LOWER SHEATHING=', 2X,G20.10, //,
C 'MAX. UPPER SHEATHING DEFLN. BETWN. JOISTS=', 2X,G20.10, //,
C 'MAX. LOWER SHEATHING DEFLN. BETWN. JOISTS=', 2X,G20.10, //,
C 'MAX. UP. SHEATH. DEFLN. BET. JOISTS FOR EACH JOIST=', 
C 5 20G15.6)
IF (ABS(WCM) .GE. ABS(WCMFL))  WCMFL = WCM
IF (ABS(WCM1) .GE. ABS(WCM1FL))  WCM1FL = WCM1
IF (II.EQ.NFLOR)  THEN
WRITE(5,585)  WCMFL , WCM1FL
585  FORMAT(//, 'MAX. UPPER COVER DEFLN. BET. JOISTS=',2X,G20.10,//,
      'MAX. LOWER COVER DEFLN. BET. JOISTS=',2X,G20.10)
ELSE
GO TO 590
ENDIF
590  CONTINUE

C
C WRITE(3.643)  (WCOVKI).  I= 1, NJT)
C643  FORMAT(//,'MAX. LR. SHEATH. DEFLN.BET. JOISTS FOR EACH JOIST=','.
C  2  20G15.6)
    IF (INPTE.EQ.2)  GO TO 782
    SFMAX = 0.0
    DO 781  I = 1, NJT
    SF(I) = SF(I) / STREN(I)
    781  IF  (SF(I).GT.SFMAX)  SFMAX = SF(I)
    C PAUSE'AFTER LOOP ENDING @ 781; SFMAX FOUND'
    PMAX = PLOAD / SFMAX
    C WRITE(3,B42)  (SF(I),  I = 1, NJT)
    C642  FORMAT(/,'STRESS RATIO FOR EACH J0IST='.4X. 20G20.10.//)
    C WRITE(3,655)  PMAX
    C655  FORMAT (/.'LOAD PRODUCING  FIRST  JOIST  FAILURE='.4X,G20.10,//)
782  CONTINUE
II = II + 1
    IF (II.GT.NFLOR)  GO TO 629
    IF (INPTE.NE.4)  GO TO 711
    GO TO 115
629  CONTINUE
    IF (IPRNT.EQ.O)  GO TO 771
    GO TO 772
771  CONTINUE
772  STOP
END
C============================================================
C
SUBROUTINE DISTR(Z1,KDISTR,QI,EO,EM,EK,ELAS)
IF (KDISTR.EQ.3) GO TO 20
IF (KDISTR.EQ.2) GO TO 10
ELAS = EO + EM * (1.0/EK) * RNORM(1.0-Z1)**(1.0/EK)
ELAS = ELAS / QI
RETURN
10  ELAS = EM * EK * RNORM(Z1)
ELAS = EXP(ELAS) / QI
RETURN
20  ELAS = EM * EK * RNORM(Z1)
ELAS = ELAS / QI
RETURN
END
C=====================================================================
C
SUBROUTINE STREND(S, E, A, B, CV1,CV2,CV3,E00,E11,Z1,KOUT,SPROOF)
KOUT = 0
CV = CV1 * ((E - E00)**CV2) * ((E11 - E)**CV3)
WRITE(*,*) 'CV=', CV

SMAX = ( A + B*E) * ( 1.0 + 2.0*CV)

WRITE(*,*) 'SMAX=', SMAX

IF (SPROOF .GE. SMAX) GO TO 10
S = (A + B*E) * (1.0 + CV * RNORM(Z1))
RETURN
10 KOUT = 1
RETURN
END

FUNCTION RNORM(Z1)
RMEAN = 0.0
RDT = 1.0
IP = 0
IF (Z1.GT.0.5) GO TO 310
GO TO 311
310 IP = 1
Z1 = 1.0 - Z1
311 TEM = SORT(-2.0*ALOG(Z1))
A1 = 2.51557 + TEM * (0.802853 + TEM*0.010328)
A2 = 1.0+TEM*(1.432788+TEM*(0.189269+TEM*1.308E-3))
VV = TEM - A1/A2
IF (IP.EQ.1) GO TO 312
GO TO 313
312 VV = -VV
313 RNORM = VV * RDT + RMEAN
RETURN
END

SUBROUTINE DMAT(II)
DOUBLE PRECISION A
COMMON/B8/A(34,34)
COMMON/B3/ETA(6),H(6)
COMMON/B10/RM(6,34),RMK(6,34)
DO 1 I=1,34
DO 1 J=1,34
A(I,J)=0.DO
1 CONTINUE
DO 2 I=1,6
DO 101 J=1,34
F1 = RM(I,J)
IF (II.EQ.2.OR.II.EQ.4) F1 = RM1(I,J)
IF (F1.EQ.0.0) GO TO 101
DO 100 K=1,34
F2 = RM1(I,K)
IF (II.EQ.1.OR.II.EQ.4) F2 = RM(I,K)
IF (F2.EQ.0.0) GO TO 100
A(J,K) = A(J,K) + F1*F2*H(I)
100 CONTINUE
101 CONTINUE
2 CONTINUE
RETURN
SUBROUTINE DECMP(N,LHB,A)
DOUBLE PRECISION TEMP, SUM
DIMENSION A(15368)
C 'A IS STORED COLUMNWISE'
KB=LHB-1
C 'DECOMPOSITION'
TEMP=A(1)
TEMP=DSQRT(TEMP)
A(1)=TEMP
DO 1 I=2,LHB
A(I)=A(I)/TEMP
DO 20 J=2,N
J1=J-1
IJD=LHB*J-KB
SUM=A(IJD)
K0=1
IF(J.GT.LHB) K0=J-KB
DO 5 K=K0,J1
JK=KB*K+J-KB
TEMP=A(JK)
5 SUM=SUM-TEMP**2
A(IJD)=DSQRT(SUM)
DO 18 1=1,KB
II=J+1
K0=1
IF(II.GT.LHB) K0=II-KB
SUM=A(IJD+I)
IF(I.EQ.KB)GO TO 15
DO 10 K=K0,J1
IK=KB*K+II-KB
TEMP=A(IK)
10 SUM=SUM-A(IK)*TEMP
15 A(IJD+I)=SUM/A(IJD)
18 CONTINUE
20 CONTINUE
RETURN
END

SUBROUTINE SOLV(N,LHB,A,B)
DOUBLE PRECISION TEMP, SUM
DIMENSION A(15368), B(452)
C 'FORWARD SUBSTITUTION'
KB=LHB-1
TEMP=A(1)
B(1)=B(1)/TEMP
DO 30 I=2,N
I1=I-1
K0=1
IF(I1.GT.LHB) K0=I1-KB
SUM=B(I1)
II=LHB*I1-KB
DO 25 K=K0,I1
IK=KB*K+I1-KB
TEMP = A(IK)

SUM = SUM - TEMP * B(K)

B(I) = SUM / A(II)

CONTINUE

C *BACKWARD SUBSTITUTION*

N1 = N - 1
LB = LHB * N - KB
TEMP = A(LB)
B(N) = B(N) / TEMP
DO 50 I = 1, N1
I = N - I + 1
N = N - I
IF (I .GT. KB) KO = NI + KB
SUM = B(NI)
II = LHB * NI - KB
DO 40 K = I, KO
IK = KB * NI - K - KB
TEMP = A(IK)

40 SUM = SUM - TEMP * B(K)

B(NI) = SUM / A(II)

CONTINUE

RETURN

END

C===============================================================

C SUBROUTINE STRESS(N1, STR, WJ, WS, WJT, WC, WCB, SC1, SC2, SC3, SC4, & CO1, CO2, SM1, SM2, IE, PI, WC1)

DIMENSION ST(34), ST1(34), W(34), W1(34), WS(34), WS1(34)

C DIMENSION F(452), CK1(19), CK2(19), CL1(19), CL2(19)

COMMON/B2/RKX, RKY, RKV, RKG, DX, DY, DV, DL, RL, FACTOR.
& RKX1, RKY1, RKV1, RKG1, DX1, DY1, DV1, DL1
COMMON/B3/ETA(6), H(6), S, NM, NSTEP, NMAX

COMMON/B4/D, D1, DJT, BJT, ETA, ENL, ENL1, RKPAL, RKPAL1, RKPER, RKPER1,
& PLD, X1, X2, Y1, Y2, BJ(20), HJ(20), HJTM, ALPHA, RKROT, RKROT1

COMMON/B5/STORE(20,7), ECOV, ECOV1, NFACE, NFACE1, WP, TV, TV1

COMMON/B6/GAPX0(5), GAP0(5), GAPX(5), GAP(5), XIN, NGAPS, NAI,
& NDSCR, NGAPS1, GAPX1(5), GAP1(5), GAP01(5), GAPX01(5), XIN1,
& NDSCR1, NAI1

COMMON/B9/SVEC(6, 452)

C DO 1 I = 1, 19
W(I) = 0.0
WS(I) = 0.0
1 ST(I) = 0.0

C EJT = STORE(IE, 1)
GJT = STORE(IE, 2)
BJT = BJ(IE)
HJT = HJ(IE)

C DO 10 N = 1, NM, NSTEP
PIN = N * PI
PINL = PIN / RL
IKO = N
IF (NSTEP .EQ. 2) IKO = (N+1)/2
DO 60 J = 1, N1
F(J) = SVEC(IKO, J)
\begin{verbatim}
J = (IE - 1)*22 + 16
FAC = (-F(J+1)*PINL*HJT*F(J)*(PINL**2)/2.0)*STORE(IE,1)
DO 92 I = 1, 19
   XL = SIN(0.05*I*PIN)
   W(I) = W(I) + F(J)*XL
   ST(I) = ST(I) + FAC*XL
92 CONTINUE
10 CONTINUE
C
   STR = 0.0
   WJT = 0.0
DO 12 I = 1, 19
   IF (ABS(ST(I)).GE.ABS(STR))  STR = ST(I)
   IF (ABS(W(I)).GE.ABS(WJT))  WJT = W(I)
12 CONTINUE
   STR = STR / FACTOR
   DO 16 I = 1,19
      W(I) = 0.0
      W1(I)= 0.0
      WS(I) = 0.0
      CK1(I)=0.0
      CK2(I)=0.0
      CL1(I)=0.0
      CL2(I)=0.0
      ST(I) = 0.0
      WS1(I)= 0.0
16 ST1(I)= 0.0
C
   ECC = ECOV /(1.0 - 0.02*0.40)
   ECC1=ECO1V/(1.0-0.02*0.40)
   EXC=ECC
C
   DO 30 N = 1, NM, NSTEP
      PIN = N*PI
      PINL = PIN /RL
      IKO = N
      IF (NSTEP.EQ.2) IKO = (N+D/2
       DO 62 J = 1, N1
       62 F(J) =  SVEC(IKO,J)
        IF(NFACE.EQ.2.AND.NFACE1.EQ.2)GO  TO 13
        UYX=0.02
        UYX1=0.02
        HD=D/2.0 -TV
        HD1=D1/2.0-TV1
        GO TO 15
13 UYX=0.02
       UYX1=0.02
       HD=D/2.0
       HD1=D1/2.0
       15 CONTINUE
       J=(IE-1)*22
       FAC1 = F(J+6)/S - UYX*PINL*F(J+3) - HD*(-46.0*F(J+1)
       1 -14.0*F(J+23)+32.0*F(J+16)-12.0*F(J+2)-2.0*F(J+24)
       2 -16.0*F(J+15))/(S**2) + UYX*HD*F(J+1)*PINL**2
C
       FAC2 = (-1.5*F(J+5)+1.5*F(J+27)-0.25*F(J+6)-0.25*F(J+28))/S
       1 - UYX*PINL*F(J+13) + HD*(8.0*F(J+1)+8.0*F(J+23)-16.0*F(J+16)
\end{verbatim}
101

2 \*F\((J+2)\*F\((J+24))/(S**2) - UYX*HD*F(J+16)*PINL''2

C

FAC3=-PINL*F(J+13)-(D/2)-TV)*(PINL''2)*F(J+16)
1 +((0.04/S)*(-0.75*F(J+5)+0.75*F(J+27)-0.125*F(J+6)\*0.125*F(J+28))
2 +((D/2)*TV)*((0.08/S/S)*F(J+1)+2.0*F(J+23)+4.0*F(J+16)
3+0.25*F(J+2*-0.25*F(J+24))

C

FACB=-PINL*F(J+3)-(D/2)-TV)*(PINL''2)*F(J+1)+0.02*F(J+6)/S
1 +(D/2)*TV)*((0.08/S/S)*(3.5*F(J+23)-11.5*F(J+1)+8.0*F(J+16)
2 +3.0*F(J+2)*0.5*F(J+24)-4.0*F(J+15))

C

FAC5=-PINL*F(J+20)-(D/2)-TV)*(PINL''2)*F(J+16)
1 +((0.04/S)*(-0.75*F(J+11)+0.75*F(J+33)-0.125*F(J+12)-0.125*F(J+34))
2 +((D/2)*TV)*((0.08/S/S)*(2.0*F(J+7)+2.0*F(J+29)+4.0*F(J+16)
3 +0.25*F(J+8)-0.25*F(J+30))

C

FAC6=-PINL*F(J+31)-(D/2)-TV)*(PINL''2)*F(J+29)+0.02*F(J+34)/S
1 -(D/2)*TV)*((0.08/S/S)*(3.5*F(J+7)-11.5*F(J+29)+8.0*F(J+16)
2 +0.5*F(J+8)+3.0*F(J+30)+4.0*F(J+22))

C

J1=(IE-1)*22+6

FAC3=F(J+16)/S-UYX*PINL*F(J+3)-HD1*(-46.0*F(J+1)
1 +14.0*F(J+1)+32*F(J+10)-12.0*F(J+2)-2.0*F(J+24)
2 -16.0*F(J+16))\*S*2)+UYX1*HD1*F(J+1)*PINL''2

C

FAC4=(-1.5*F(J+5)+1.5*F(J+27)-0.25*F(J+6)-0.25*F(J+28))/S
1 -UYX*PINL*F(J+14)+HD1*(8.0*F(J+1)+8.0*F(J+23)-16.0*F(J+10)
2 +F(J+2)+F(J+24))/\*S**2)-UYX1*HD1*F(J+10)*PINL''2

C

FAC1 = FAC1 * ECC
FAC2 = FAC2 * ECC
FAC3 = FAC3 * ECC
FAC4 = FAC4 * ECC
FAC5 = FAC5 * EXC
FAC6 = FAC6 * EXC
FAC7 = FAC7 * EXC
FAC8 = FAC8 * EXC

C

DO 20 I= 1, 19
XL = I*0.05*RL
IF (NGAPS.EQ.0) GO TO 19
DO 18 K= 1, NGAPS
ZO = GAPX(K)
Z1 = ZO + GAP(K)
IF (XL.GT.ZO.AND.XL.LT.ZD) GO TO 20
CONTINUE
18
IF (NGAPS1.EQ.0) GO TO 564
DO 565 K= 1, NGAPS1
Z2=GAPX1(K)
Z3=Z2+GAP1(K)
IF (XL.GT.Z2.AND.XL.LT.Z3) GO TO 20
CONTINUE
565
XL = SIN(0.05*I*PIN)
W(I) = W(I) + F(J+1)*XL
W1(I)= W1(I) + F(J+1)*XL
WS(I) = WS(I) + FAC1 * XL
ST(I) = ST(I) + FAC2 * XL
WS1(I) = WS1(I) + FAC3 * XL
ST1(I) = ST1(I) + FAC4 * XL
CK1(I) = CK1(I) + FAC5 * XL
CK2(I) = CK2(I) + FAC6 * XL
CL1(I) = CL1(I) + FAC7 * XL
CL2(I) = CL2(I) + FAC8 * XL
20 CONTINUE
30 CONTINUE
C
C RAM = FAC5 * SIN(NM * PI * RL / 4)
C RAM1 = FAC6 * SIN(NM * PI * RL / 4)
C WRITE('*, '*), 'RAM=', RAM, 'RAM1=', RAM1
C
WC = 0.0
WC1 = 0.0
SC1 = 0.0
SC2 = 0.0
SC3 = 0.0
SC4 = 0.0
C01 = 0.0
C02 = 0.0
SM1 = 0.0
SM2 = 0.0
C
DO 40 I = 1, 19
IF (ABS(W(I)).GT.ABS(WC)) WC = W(I)
IF (ABS(W1(I)).GT.ABS(WC1)) WC1 = W1(I)
IF (ABS(WS(I)).GT.ABS(SC1)) SC1 = ABS(WS(I))
IF (ABS(ST(I)).GT.ABS(SC2)) SC2 = ABS(ST(I))
IF (ABS(WS(I)).GT.ABS(SC3)) SC3 = ABS(WS(I))
IF (ABS(WS(I)).GT.ABS(SC4)) SC4 = ABS(ST(I))
IF (ABS(CK1(I)).GT.ABS(CQ1)) CQ1 = ABS(CK1(I))
IF (ABS(CK2(I)).GT.ABS(CQ2)) CQ2 = ABS(CK2(I))
IF (ABS(CL1(I)).GT.ABS(SM1)) SM1 = ABS(CL1(I))
IF (ABS(CL2(I)).GT.ABS(SM2)) SM2 = ABS(CL2(I))
40 CONTINUE
C
WRITE('*, '*), 'WC=', WC
C
WCB = 0.0
WCB1 = 0.0
IF (IE.EQ.1) GO TO 45
WCB = WC - (WJT + WP)/2.0
WCB1 = WC1 - (WJT + WP)/2.0
45 WF = WJT
RETURN
END
SUBROUTINE OUT(N1, N2, WMA, SMA, SF, WFT, WCOV, WCOV1, WCM, & WCM1, CMA, CMA1, IPRNT, PI, WSM)
COMMON/B3/ETA(6), H(6), S, NM, NSTEP, NMAX
COMMON/B9/SEVC(6, 452)
DIMENSION SF(20), WFT(20), WCOV(20), WCOV1(20)
DIMENSION F(452)
C
WMA IS THE MAX JOIST DEFLECTION
C SMA IS THE JOIST BENDING STRESS IN THE FLOOR
C CMA IS THE MAX BENDING STRESS IN THE UPPER SHEATHING
- THE DIRECTION PERPENDICULAR TO THE JOIST
- THE DIRECTION PERPENDICULAR TO THE JOIST
- THE DIRECTION PERPENDICULAR TO THE JOIST
- THE DIRECTION PERPENDICULAR TO THE JOIST
- THE DIRECTION PERPENDICULAR TO THE JOIST
- IN THE ENTIRE UPPER FLOOR
- IN THE ENTIRE LOWER FLOOR
- IN THE ENTIRE UPPER FLOOR
- IN THE ENTIRE LOWER FLOOR
- IN THE ENTIRE UPPER FLOOR
- IN THE ENTIRE LOWER FLOOR

WMA = 0.0
WCM = 0.0
WCM1= 0.0
SMA = 0.0
CMA = 0.0
CMA1=0.0
DO 2 IE=1,N2
CALL STRESS(N1,ST,WJ,WJS,WJT,WC,WCB,WCB1,SC1,SC2,SC3,SC4,
1 C01,C02,SM1,SM2,IE,PI,W1)
IF (IPRNT.EQ.0) GO TO 202
WRITE(2,3) IE
3 FORMAT(/, ' * ELEMENT ',I2, ' *', /
&9X, 'U(F)', 11X, 'V(F>', 11X, 'W(F)', 11X, 'W(J)', 11X, 'U(J)', 11X, 'V(J)', 11X, 'O(F)'
&)
WRITE(2,8)
8 FORMAT(' 1H+.98X,'-',14X, '-')
DO 20 I = 1, NM, NSTEP
IKO=I
IF(NSTEP.EQ.2)IKO=(1+1)/2
DO 40 J = 1, N1
40 F(J) = SVEC(IKO,J)
J = (IE - 1)*22
WRITE(2,18)1
18 FORMAT('  N =•,13)
RADN=F(J+19)/S
RADF = F(J+15)/S
RADF1=F(J+22)/S
WRITE(2,4)F(J+3),F(J+5),F(J+1)
4 FORMAT(' 1'.3E15.6)
WRITE(2,5)F(J+13),F(J+14),F(J+16),F(J+16),F(J+17),F(J+18),
1 RADN, RADF
5 FORMAT(' 2'.8E15.6)
WRITE(2,6)F(J+25),F(J+27),F(J+23)
6 FORMAT(' 3'.3E15.6)
WRITE(2,566) F(J+9),F(J+11),F(J+7)
566 FORMAT(' 4'.3E15.6)
WRITE(2,567) F(J+20),F(J+21),F(J+16),F(J+16),F(J+17),F(J+18),
1 RADN,RADF1
567 FORMAT(' 5'.8E15.6)
WRITE(2,568)F(J+31),F(J+33),F(J+29)
568 FORMAT(' 6'.3E15.6)
20 CONTINUE
WRITE(2,7) WJT,ST,WC,SC1,SC2,SM1,SM2,WC1,SC3,SC4,C02,C01
7 FORMAT(/, ' JOIST:', '/', 'DEFLECTION=', 'E13.6,' (BENDING)'), /
1 'BENDING STRESS=', 'E13.6,/',
& 'UPPER COVER:', '/',
2 'DEFLECTION=', 'E13.6,' '(NODE 1)',/,
'MAX. STRESS=' ,E13.6, ' (NODE 1 - Y DIRECTION)' ./
3 'MAX. STRESS=' ,E13.6, ' (NODE 2 - Y DIRECTION)' ,/
4 'MAX. STRESS=' ,E13.6, ' (NODE 2 - X DIRECTION)' ,/
5 'MAX. STRESS=' ,E13.6, ' (NODE 1 - X DIRECTION)' ,/
6 'LOWER COVER:' ,/
7 'DEFLECTION=' ,E13.6, ' (NODE 4)' ,/
8 'MAX. STRESS=' ,E13.6, ' (NODE 4 - Y DIRECTION)' ,/
9 'MAX. STRESS=' ,E13.6, ' (NODE 5 - Y DIRECTION)' ,/
10 'MAX. STRESS=' ,E13.6, ' (NODE 6 - X DIRECTION)' ,/
11 'MAX. STRESS=' ,E13.6, ' (NODE 5 - X DIRECTION)' ,/
12 IF (ABS(WJT).GE.ABS(WMA)) WMA = WJT
13 IF (ABS(STR).GE.ABS(SMA)) SMA = STR
14 IF (ABS(SC1).GE.ABS(CMA)) CMA = SC1
15 IF (ABS(SC2).GE.ABS(CMA)) CMA = SC2
16 IF (ABS(SC3).GE.ABS(CMA1)) CMA1=SC3
17 IF (ABS(SC4).GE.ABS(CMA1)) CMA1=SC4
18 IF (ABS(WCB).GE.ABS(WCM)) WCM = WCB
19 IF (ABS(WCB1).GE.ABS(WCM1)) WCM1 = WCB1
20 IF (ABS(SM1).GE.ABS(WSM)) WSM = SM1
21 SF(IE)=STR
22 WFT(IE)  = WJT
23 WCOV(IE) =  WCB
24 WCOV1(IE)= WCB1
25 CONTINUE
26 IF(IPRNT.EQ.0)GO TO 204
27 WRITE(2,10)SMA,WMA,CMA,CMA1,WCM,WCM1
28 10 FORMAT(/,' MAX. JOIST STRESS',E15.6,/,'MAX. JOIST DEFLECTION',
29   'MAX. BENDING STRESS IN THE UPPER SHEATHING',E15.6,/.
30   'MAX. BENDING STRESS IN THE LOWER SHEATHING',E15.6,/.
31   'MAX. COVER DEFLN BETWN JOISTS IN ENTIRE UPPER FLOOR',E15.6,/.
32   'MAX COVER DEFLN BETWN JOISTS IN ENTIRE LOWER FLOOR',E15.6,/)
33 204 CONTINUE
34 RETURN
35 END
36 C========================================================================
37 C
38 SUBROUTINE ZERO
39 COMMON/B10/RM(6,34), RM1(6,34)
40 DO 1 I = 1, 6
41 DO 1 J = 1, 34
42 RM(I,J)  = 0.0
43 1 RM1(I,J) = 0.0
44 RETURN
45 END
46 C========================================================================
47 C
48 SUBROUTINE GENMTX
49 DOUBLE PRECISION AA
50 COMMON/B1/RMO(34,34), RM02(34,34), RMO2(34,34), RM0(34,34),
51 RM22(34,34), RM20(34,34), RM20(34,34), RM66(34,34), RM63(34,34),
52 RM44(34,34), RM45(34,34), RM54(34,34), RM55(34,34), RM11(34,34),
53 RM77(34,34), RM99(34,34), RM97(34,34), RM97(34,34), RM1010(34,34),
54 RM1313(34,34), RM1013(34,34), RM1310(34,34), RM1111(34,34),
55 RM1112(34,34), RM1211(34,34), RM1212(34,34), RM88(34,34)
56 COMMON/B8/AA(34,34)
57 COMMON/B3/E1A(6),H(6)
58 COMMON/B10/RM(6,34), RM1(6,34)
CALL ZERO

C MO AND M2 MATRICES
DO 1 I = 1, 6
ETA2 = ETA(I)**2
ETA3 = ETA(I)**3
ETA4 = ETA(I)**4
ETA5 = ETA(I)**5
RM(I,1) = ETA2 - 5.0*ETA3/4.0 - ETA4/2.0 + 3.0*ETA5/4.0
RM(I,23) = ETA2 + 5.0*ETA3/4.0 - ETA4/2.0 - 3.0*ETA5/4.0
RM(I,16) = 1.0 - 2.0*ETA2 + ETA4
RM(I,2) = (ETA2 - ETA3 + ETA4 + ETA5) / 8.0
RM(I,23) = (-ETA2 + ETA3 + ETA4 + ETA5) / 8.0
RM(I,24) = ETA2 + 5.0*ETA3/4.0 - ETA4/2.0 - 15.0*ETA5/8.0
RM(I,23) = 2.0 - 15.0*ETA2/8.0 - 5.0*ETA3/8.0
RM(I,24) = 1.0 - 2.0*ETA2 + ETA4
RM(I,23) = (-ETA2 + ETA3 - ETA4 + ETA5) / 8.0
RM(I,24) = 1.0 + ETA2 - ETA3 - 2.0*ETA4/8.0
1  CONTINUE

CALL DMAT(1)
DO 4500 I = 1, 34
DO 4500 J = 1, 34
4500 RMOO(I,J) = AA(I,J)
CALL DMAT(2)
DO 4500 I = 1, 34
DO 4500 J = 1, 34
4500 RM22(I,J) = AA(I,J)
CALL DMAT(3)
DO 4502 I = 1, 34
DO 4502 J = 1, 34
4502 RM02(I,J) = AA(I,J)
CALL DMAT(4)
DO 4503 I = 1, 34
DO 4503 J = 1, 34
4503 RM20(I,J) = AA(I,J)
CALL ZERO

C M3 AND M6 MATRICES
DO 2 I = 1, 6
ETA2 = ETA(I)**2
ETA3 = ETA(I)**3
ETA4 = ETA(I)**4
ETA5 = ETA(I)**5
RM(I,3) = -3.0*ETA(I)/4.0 + ETA2 + ETA3/4.0 + ETA4/2.0
RM(I,25) = 3.0*ETA(I)/4.0 + ETA2 - ETA3/4.0 - ETA4/2.0
RM(I,13) = 1.0 - 2.0*ETA2 + ETA4
RM(I,14) = (-ETA(I) + ETA2 + ETA3 - ETA4) / 8.0
RM(I,26) = (-ETA(I) - ETA2 + ETA3 + ETA4) / 8.0
RM(I,27) = 3.0*ETA2 + ETA3 + ETA4/2.0
RM(I,28) = 3.0*ETA2 + ETA3 - ETA4/2.0
RM(I,14) = -4.0*ETA(I)/4.0 - ETA3
RM(I,25) = 3.0*ETA2 + ETA3 - 4.0*ETA3/8.0
RM(I,28) = (1.0 - 2.0*ETA(I) + 3.0*ETA2 + 4.0*ETA3)/8.0
2  CONTINUE

CALL DMAT(1)
DO 4504 I = 1, 34
DO 4504 J = 1, 34
4504 RM33(I,J) = AA(I,J)
CALL DMAT(2)
DO 4505 I = 1, 34
DO 4505 J = 1, 34
4505 RM66(I,J) = AA(I,J)
CALL DMAT(3)
DO 4506 I = 1, 34
DO 4506 J = 1, 34
4506 RM36(I,J) = AA(I,J)
CALL DMAT(4)
DO 4507 I = 1, 34
DO 4507 J = 1, 34
4507 RM63(I,J) = AA(I,J)

C M5 AND M4 MATRICES
DO 3 I = 1, 6
RM(I,5) = RM(I,3)
RM(I,3) = 0.0
RM(I,27) = RM(I,25)
RM(I,25) = 0.0
RM(I,14) = RM(I,13)
RM(I,13) = 0.0
RM(I,6) = RM(I,4)
RM(I,4) = 0.0
RM(I,28) = RM(I,26)
RM(I,26) = 0.0
RM(I,1) = RM(I,5)
RM(I,5) = 0.0
RM(I,25) = RM(I,27)
RM(I,27) = 0.0
RM(I,13) = RM(I,14)
RM(I,14) = 0.0
RM(I,6) = RM(I,4)
RM(I,4) = 0.0
RM(I,28) = RM(I,26)
RM(I,26) = 0.0

3 CONTINUE
CALL DMAT(2)
DO 4510 I = 1, 34
DO 4510 J = 1, 34
4510 RM44(I,J) = AA(I,J)
CALL DMAT(4)
DO 4511 I = 1, 34
DO 4511 J = 1, 34
4511 RM45(I,J) = AA(I,J)
CALL DMAT(3)
DO 4512 I = 1, 34
DO 4512 J = 1, 34
4512 RM54(I,J) = AA(I,J)
CALL DMAT(1)
DO 4513 I = 1, 34
DO 4513 J = 1, 34
4513 RM55(I,J) = AA(I,J)
CALL ZERO

C M1 MATRIX
DO 5 I = 1, 6
ETA2 = ETA(I)**2
ETA3 = ETA(I)**3
ETA4 = ETA(I)**4
RM(I,1) = 2.0*ETA(I) - 15.0*ETA2/4.0 - 2.0*ETA3 + 15.0*ETA4/4.0
RM(I,23) = 2.0*ETA(I) + 15.0*ETA2/4.0 - 2.0*ETA3 - 15.0*ETA4/4.0
\( RM(I,16) = -4.0 \cdot ETA(I) + 4.0 \cdot ETA3 \)
\( RM(I,2) = (2.0 \cdot ETA(I) - 3.0 \cdot ETA2 - 4.0 \cdot ETA3 + 5.0 \cdot ETA4) / 8.0 \)
\( RM(I,24) = (-2.0 \cdot ETA(I) - 3.0 \cdot ETA2 + 4.0 \cdot ETA3 + 5.0 \cdot ETA4) / 8.0 \)
\( RM(I,15) = (1.0 - 6.0 \cdot ETA2 + 5.0 \cdot ETA4) / 2.0 \)

5 CONTINUE

CALL DMAT(1)
DO 4515 I = 1, 34
DO 4515 J = 1, 34
4515 RM11(I,J) = AA(I,J)
CALL ZERO

C M7 AND M9 MATRICES
DO 58 I = 1, 6
ETA2 = ETA(I)**2
ETA3 = ETA(I)**3
ETA4 = ETA(I)**4
ETA5 = ETA(I)**5
RM(I,7) = ETA2 - 5.0 \cdot ETA3 / 4.0 - ETA4 / 2.0 + 3.0 \cdot ETA5 / 4.0
RM(I,29) = ETA2 + 5.0 \cdot ETA3 / 4.0 - ETA4 / 2.0 - 3.0 \cdot ETA5 / 4.0
RM(I,18) = ETA2 - ETA3 + ETA4 / 2.0 - ETA5 / 2.0
RM(I,30) = ETA2 - ETA3 + ETA4 + ETA5 / 2.0
RM(I,22) = ETA2 - ETA3 - ETA4 - ETA5 / 2.0
RM(I,16) = ETA2 - ETA3 - ETA4 + ETA5 / 2.0
RM(I,8) = ETA2 - ETA3 + ETA4 + ETA5 / 2.0
RM(I,30) = ETA2 - ETA3 - ETA4 + ETA5 / 2.0
RM(I,22) = ETA2 - ETA3 + ETA4 + ETA5 / 2.0
RM(I,7) = ETA2 - ETA3 - ETA4 + ETA5 / 2.0
RM(I,29) = ETA2 + ETA3 - ETA4 - ETA5 / 2.0
RM(I,18) = ETA2 + ETA3 - ETA4 + ETA5 / 2.0
RM(I,30) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,22) = ETA2 + ETA3 + ETA4 + ETA5 / 2.0
RM(I,16) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,8) = ETA2 - ETA3 - ETA4 - ETA5 / 2.0
RM(I,30) = ETA2 + ETA3 + ETA4 + ETA5 / 2.0
RM(I,22) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,7) = ETA2 + ETA3 - ETA4 - ETA5 / 2.0
RM(I,29) = ETA2 - ETA3 + ETA4 + ETA5 / 2.0
RM(I,18) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,30) = ETA2 + ETA3 + ETA4 + ETA5 / 2.0
RM(I,22) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,16) = ETA2 - ETA3 + ETA4 + ETA5 / 2.0
RM(I,8) = ETA2 - ETA3 - ETA4 - ETA5 / 2.0
RM(I,30) = ETA2 + ETA3 + ETA4 + ETA5 / 2.0
RM(I,22) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,7) = ETA2 + ETA3 - ETA4 - ETA5 / 2.0
RM(I,29) = ETA2 - ETA3 + ETA4 + ETA5 / 2.0
RM(I,18) = ETA2 + ETA3 - ETA4 + ETA5 / 2.0
RM(I,30) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,22) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,16) = ETA2 - ETA3 - ETA4 + ETA5 / 2.0
RM(I,8) = ETA2 - ETA3 + ETA4 + ETA5 / 2.0
RM(I,30) = ETA2 + ETA3 + ETA4 + ETA5 / 2.0
RM(I,22) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,7) = ETA2 - ETA3 + ETA4 + ETA5 / 2.0
RM(I,29) = ETA2 + ETA3 - ETA4 - ETA5 / 2.0
RM(I,18) = ETA2 + ETA3 - ETA4 + ETA5 / 2.0
RM(I,30) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,22) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,16) = ETA2 - ETA3 + ETA4 + ETA5 / 2.0
RM(I,8) = ETA2 - ETA3 - ETA4 - ETA5 / 2.0
RM(I,30) = ETA2 + ETA3 + ETA4 + ETA5 / 2.0
RM(I,22) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,7) = ETA2 + ETA3 - ETA4 - ETA5 / 2.0
RM(I,29) = ETA2 - ETA3 + ETA4 + ETA5 / 2.0
RM(I,18) = ETA2 + ETA3 - ETA4 + ETA5 / 2.0
RM(I,30) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,22) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,16) = ETA2 - ETA3 - ETA4 + ETA5 / 2.0
RM(I,8) = ETA2 - ETA3 + ETA4 + ETA5 / 2.0
RM(I,30) = ETA2 + ETA3 + ETA4 + ETA5 / 2.0
RM(I,22) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,7) = ETA2 + ETA3 - ETA4 - ETA5 / 2.0
RM(I,29) = ETA2 - ETA3 + ETA4 + ETA5 / 2.0
RM(I,18) = ETA2 + ETA3 - ETA4 + ETA5 / 2.0
RM(I,30) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,22) = ETA2 + ETA3 + ETA4 - ETA5 / 2.0
RM(I,16) = ETA2 - ETA3 + ETA4 + ETA5 / 2.0
5 CONTINUE

CALL DMAT(1)
DO 4600 I = 1, 34
DO 4600 J = 1, 34
4600 RM77(I,J) = AA(I,J)
CALL DMAT(2)
DO 4601 I = 1, 34
DO 4601 J = 1, 34
4601 RM99(I,J) = AA(I,J)
CALL DMAT(3)
DO 4602 I = 1, 34
DO 4602 J = 1, 34
4602 RM97(I,J) = AA(I,J)
CALL DMAT(4)
DO 4603 I = 1, 34
DO 4603 J = 1, 34
4603 RM97(I,J) = AA(I,J)
CALL ZERO

C M10 AND M13 MATRICES
DO 59 I = 1, 6
ETA2 = ETA(I)**2
ETA3 = ETA(I)**3
ETA4 = ETA(I)**4
RM(I,9) = -3.0 \cdot ETA(I)/4.0 + ETA2 + ETA3/4.0 - ETA4/2.0
RM(I,31) = 3.0 \cdot ETA(I)/4.0 + ETA2 - ETA3/4.0 + ETA4/2.0
RM(I,20) = 1.0 - 2.0 \cdot ETA2 + ETA4
RM(I,10) = (ETA(I) + ETA2 + ETA3 - ETA4) / 8.0
RM(I,32) = (ETA(I) - ETA2 + ETA3 + ETA4) / 8.0
RM(I,11) = 3.0 / 4.0 + 2.0 \cdot ETA(I) + 3.0 \cdot ETA2/4.0 + 2.0 \cdot ETA3
RM(I,33) = 3.0 / 4.0 + 2.0 \cdot ETA(I) - 3.0 \cdot ETA2/4.0 - 2.0 \cdot ETA3
RM1(I,21)=-4.0*ETA(I)+4.0*ETA3
RM1(I,12)=(-1.0+2.0*ETA(I)+3.0*ETA2-4.0*ETA3)/8.0
RM1(I,34)=(-1.0-2.0*ETA(I)+3.0*ETA2+4.0*ETA3)/8.0

59 CONTINUE

CALL DMAT(1)
DO 4604 I=1,34
   DO 4604 J=1,34
5604 RM110(I,J)=AA(I,J)
   CALL DMAT(2)
   DO 4605 I=1,34
      DO 4605 J=1,34
4605 RM113(I,J)=AA(I,J)
   CALL DMAT(3)
   DO 4606 I=1,34
      DO 4606 J=1,34
4606 RM1103(I,J)=AA(I,J)
   CALL DMAT(4)
   DO 4607 I=1,34
      DO 4607 J=1,34
4607 RM1130(I,J)=AA(I,J)
C M12 AND M11 MATRICES
DO 60 I=1,6
   RM(I,11)=RM(I,9)
   RM(I,9)=0.0
   RM(I,33)=RM(I,31)
   RM(I,31)=0.0
   RM(I,21)=RM(I,20)
   RM(I,20)=0.0
   RM(I,12)=RM(I,10)
   RM(I,10)=0.0
   RM(I,34)=RM(I,32)
   RM(I,32)=0.0
   RM(I,6)=RM(I,11)
   RM(I,11)=0.0
   RM(I,31)=RM(I,33)
   RM(I,33)=0.0
   RM(I,21)=RM(I,20)
   RM(I,20)=0.0
   RM(I,10)=RM(I,12)
   RM(I,12)=0.0
   RM(I,34)=RM(I,32)
   RM(I,32)=0.0
60 CONTINUE

CALL DMAT(2)
DO 4610 I=1,34
   DO 4610 J=1,34
4610 RM1111(I,J)=AA(I,J)
   CALL DMAT(4)
   DO 4611 I=1,34
      DO 4611 J=1,34
4611 RM1112(I,J)=AA(I,J)
   CALL DMAT(3)
   DO 4612 I=1,34
      DO 4612 J=1,34
CALL DMAT(1)
DO 4613 I=1,34
DO 4613 J=1,34
4613 RM1212(I,J)=AA(I,J)
CALL ZERO
C M8 MATRIX
DO 61 I=1,6
ETA2=ETA(I)**2
ETA3=ETA(I)**3
ETA4=ETA(I)**4
RM(I,7)=2.0*ETA(I)-15.0*ETA2/4.0-2.0*ETA3+15.0*ETA4/4.0
RM(I,29)=2.0*ETA(I)+15.0*ETA2/4.0-2.0*ETA3-15.0*ETA4/4.0
RM(I,16)=-4.0*ETA(I)+4.0*ETA3
RM(I,8)=(2.0*ETA(I)-3.0*ETA2-4.0*ETA3+5.0*ETA4)/8.0
RM(I,30)=(-2.0*ETA(I)-3.0*ETA2+4.0*ETA3+5.0*ETA4)/8.0
RM(I,22)=(1.0-6.0*ETA2+5.0*ETA4)/2.0
61 CONTINUE
CALL DMAT(1)
DO 4615 I=1,34
DO 4615 J=1,34
4615 RM88(I,J)=AA(I,J)
RETURN
END
FUNCTION Z(N,M,Z1,Z0,RL,NX,PI)
IF(N.EQ.M)GO TO 10
S1=SIN((N-M)*PI*Z1/RL)
S2=SIN((N-M)*PI*Z0/RL)
S3=SIN((N+M)*PI*Z1/RL)
S4=SIN((N+M)*PI*Z0/RL)
D1=(S1-S2)/(2.0*(N-M))
D2=(S3-S4)/(2.0*(N+M))
IF(NX.EQ.1)GO TO 5
Z=RL*(D1-D2)/PI
RETURN
5 Z=RL*(D1+D2)/PI
RETURN
10 S1=SIN(2.0*N*PI*Z1/RL)
S2=SIN(2.0*N*PI*Z0/RL)
D1=PI*(Z1-Z0)/(2.0*RL)
D2=(S1-S2)/(4.0*N)
IF(NX.EQ.1)GO TO 12
Z=RL*(D1-D2)/PI
RETURN
12 Z=RL*(D1+D2)/PI
RETURN
END
C
SUBROUTINE STIF(V,IE,KLO,IN,IK,PI)
DOUBLE PRECISION STIFF
DIMENSION V(34), AD(34,34), AD1(34,34),
COMMON/B1/RMOO(34,34),RM22(34,34),RM02(34,34),
RM33(34,34),RM66(34,34),RM3F(34,34),RM63(34,34),
RM45(34,34),RM54(34,34),RM55(34,34),RM11(34,34),
RM77(34,34),RM99(34,34),RM79(34,34),RM97(34,34),
RM1010(34,34),
4 RM1313(34,34), RM1013(34,34), RM1310(34,34), RM1111(34,34),
5 RM1112(34,34), RM1211(34,34), RM1212(34,34), RM88(34,34)
COMMON/B8/STIFF(34,34)
COMMON/B2/RKX,RKY, RKV, RKG, DX, DY, DV, DG, RL, FACTOR, RKX1, RKY1,
1 RKV1, RKG1, DX1, DY1, DV1, DG1
COMMON/B3/ETA(6), H(6), S, NM, NSTEP, NMAX
COMMON/B6/GAPX0(5), GAPC(5), GAPX(5), GAP(5), XIN, NGAPS, NAI, NDISCR,
1 NGAPS1, GAPX1(5), GAP1(5), GAP01(5), GAPX01(5), XIN1, NDISCR1, NAI1
COMMON/B4/D1,E1T, GJT, RETA, ENL, ENL1, RKPAL, RKPAL1, RKPER, RKPER1,
1 PLD, X1, X2, X1, Y2, B1(20), HJ(20), HJTM, ALPHA, RKROT, RKROT1
IF (IE.GT.1) GC TO 30
IF (KLO.EQ.1) GO TO 30
P2=PI**2
P4=PI**4
C * UPPER AND LOWER COVER *
DO 1 I=1,34
DO 1 J=1,34
1 STIFF(I,J)=0.0
FAC=0.0
IF(IN.EQ.IK)FAC=RKX*(IK**4)*P4*S/(4.0*RL**3)
IF(IN.EQ.IK)FAC1=RKX1*(IK**4)*P4*S/(4.0*RL**3)
IF(NGAPS.EQ.0)GO TO 3
IF(NGAPS1.EQ.0)GO TO 3
DO 2 I=1,NGAPS
Z0 = GAPX(I)
Z1 = Z0 + GAP(I)
R = Z(IN,IK,Z1,Z0,RL,0,PI)
2 FAC=FAC-RKX**P4*S*R**(IN*IK)**2/(2.0*RL**4)
DO 202 I=1,NGAPS
Z2 = GAPX1(I)
Z3 = Z2 + GAP1(I)
R = Z(IN,IK,Z3,Z2,RL,0,PI)
202 FAC1 = FAC1 - RKX1**P4*S*R**(IN*IK)**2/(2.0*RL**4)
3 DO 3500 I = 1,34
3500 DO 3500 J = 1,34
3500 AD(I,J) = RM00(I,J)
CALL ADD(AD,FAC)
DO 3520 I=1,34
3520 AD(I,J) = RM77(I,J)
CALL ADD(AD,FAC1)
FAC=0.0
IF(IN.EQ.IK)FAC=RKY*4.0*RL/(S**3)
IF(IN.EQ.IK)FAC1=RKY1*4.0*RL/(S**3)
IF(NGAPS.EQ.0)GO TO 5
IF(NGAPS1.EQ.0)GO TO 5
DO 4 I=1,NGAPS
Z0 = GAPX0(I)
Z1 = Z0 + GAP0(I)
R = Z(IN,IK,Z1,Z0,RL,0,PI)
4 FAC=FAC-RKY**8.0*R/(S**3)
DO 204 I=1,NGAPS
Z2 = GAPX01(I)
Z3 = Z2 + GAP01(I)
R = Z(IN,IK,Z3,Z2,RL,0,PI)
204 FAC1 = FAC1 - RKY1**8.0*R/(S**3)
5 DO 3501 I = 1,34
3501 DO 3501 J = 1,34
3501  \( AD(I,J) = RM22(I,J) \)
CALL ADD(AD,FAC)
DO 3521  I=1,34
DO 3521  J=1,34
3521  \( AD(I,J) = RM99(I,J) \)
CALL ADD(AD,FAC1)
FAC=0.0
IF(IN.EQ.IK)FAC=−RKV*P2/(S*RL)
IF(IN.EQ.IK)FAC2=−RKV1*P2/(S*RL)
IF(NGAPS.EQ.0)GO TO 7
IF(NGAPS1.EQ.0)GO TO 7
DO 6  I=1,NGAPS
Z0=GAPX(I)
Z1=Z0+GAP(I)
R=Z(IN,IK,Z1,Z0,RL,0,PI)
6  FAC=FAC+RKV*2.0*P2*R/(S*RL**2)
DO 206  I=1,NGAPS1
Z2=GAPX1(I)
Z3=Z2+GAP1(I)
R=Z(IN,IK,Z3,Z2,RL,0,PI)
206  FAC2=FAC2+RKV1*2.0*P2*R/(S*RL**2)
7  CONTINUE
FAC1=FAC*(IK**2)
FAC3=FAC2*(IK**2)
DO 3502  I=1,34
DO 3502  J=1,34
3502  \( AD(I,J) = RM02(I,J) \)
CALL ADD(AD,FAC1)
DO 3522  I=1,34
DO 3522  J=1,34
3522  \( AD(I,J) = RM79(I,J) \)
CALL ADD(AD,FAC3)
FAC1=FAC1*(IN**2)
FAC3=FAC2*(IN**2)
DO 3503  I=1,34
DO 3503  J=1,34
3503  \( AD(I,J) = RM20(I,J) \)
CALL ADD(AD,FAC1)
DO 3523  I=1,34
DO 3523  J=1,34
3523  \( AD(I,J) = RM97(I,J) \)
CALL ADD(AD,FAC3)
FAC=0.0
IF(IN.EQ.IK)FAC=RKG*4.0*P2*IK**2/(S*RL)
IF(IN.EQ.IK)FAC1=RKG1*4.0*P2*IK**2/(S*RL)
IF(NGAPS.EQ.0)GO TO 9
IF(NGAPS1.EQ.0)GO TO 9
DO 8  I=1,NGAPS
Z0=GAPX(I)
Z1=Z0+GAP(I)
R=Z(IN,IK,Z1,Z0,RL,1,PI)
8  FAC=FAC−RKG*8.0*P2*R*IK*IN/(S*RL**2)
DO 208  I=1,NGAPS1
Z2=GAPX1(I)
Z3=Z2+GAP1(I)
R=Z(IN,IK,Z3,Z2,RL,1,PI)
208  FAC1=FAC1−RKG1*8.0*P2*R*IK1*IN/(S*RL**2)
9  CONTINUE
DO 3505 I = 1, 34
DO 3505 J = 1, 34
3505 AD(I, J) = RM11(I, J)
CALL ADD(AD, FAC)
DO 3524 I = 1, 34
DO 3524 J = 1, 34
3524 AD(I, J) = RM88(I, J)
CALL ADD(AD, FAC1)
FAC = 0.0
IF(IN.EQ.IK) FAC = DX*S*P2*IK**2/(4.0*RL)
IF(IN.EQ.IK) FAC1 = DX1*S*P2*IK**2/(4.0*RL)
IF(NGAPS.EQ.0) GO TO 11
IF(NGAPS1.EQ.0) GO TO 11
DO 10 I = 1, NGAPS
Z0 = GAPX(I)
Z1 = Z0 + GAP(I)
R = Z(IN, IK, Z1, Z0, RL, O, PI)
10 FAC = FAC - DX*S*P2*R*IK*IN/(2.0*RL**2)
DO 210 I = 1, NGAPS1
Z2 = GAPX1(I)
Z3 = Z2 + GAP1(I)
R = Z(IN, IK, Z2, Z3, Z1, Z0, RL, O, PI)
210 FAC1 = FAC1 - DX1*S*P2*R*IK*IN/(2.0*RL**2)
CONTINUE
DO 3506 I = 1, 34
DO 3506 J = 1, 34
3506 AD(I, J) = RM33(I, J)
CALL ADD(AD, FAC)
DO 3525 I = 1, 34
DO 3525 J = 1, 34
3525 AD(I, J) = RM1010(I, J)
CALL ADD(AD, FAC1)
FAC = 0.0
IF(IN.EQ.IK) FAC = DY*RL/S
IF(IN.EQ.IK) FAC1 = DY1*RL/S
IF(NGAPS.EQ.0) GO TO 13
IF(NGAPS1.EQ.0) GO TO 13
DO 12 I = 1, NGAPS
Z0 = GAPX(I)
Z1 = Z0 + GAP(I)
R = Z(IN, IK, Z1, Z0, RL, O, PI)
12 FAC = FAC - DY*2.0*R/S
DO 212 I = 1, NGAPS1
Z2 = GAPX1(I)
Z3 = Z2 + GAP1(I)
R = Z(IN, IK, Z2, Z3, Z1, Z0, RL, O, PI)
212 FAC1 = FAC1 - DY1*2.0*R/S
CONTINUE
DO 3508 I = 1, 34
DO 3508 J = 1, 34
3508 AD(I, J) = RM66(I, J)
CALL ADD(AD, FAC)
DO 3526 I = 1, 34
DO 3526 J = 1, 34
3526 AD(I, J) = RM1313(I, J)
CALL ADD(AD, FAC1)
FAC = 0.0
IF(IN.EQ.IK) FAC = -DV*PI/2.0
IF(IN.EQ.IK) FAC2=-DV1*PI/2.0
IF(NGAPS.EQ.0) GO TO 15
IF(NGAPS1.EQ.0) GO TO 15
DO 14 I=1,NGAPS
ZO = GAPX(I)
Z1 = ZO + GAP(I)
R=Z(IN,IK,Z1,ZO,RL,0,PI)
14 FAC=FAC+DV*PI*R/RL
DO 214 I=1,NGAPS1
Z2=GAPX1(I)
Z3=Z2+GAP1(I)
R=Z(IN,IK,Z3,Z2,RL,0,PI)
214 FAC2=FAC2+DV1*PI*R/RL
15 CONTINUE
FAC1=FAC*IK
FAC3=FAC2*IK
DO 3509 I = 1,34
DO 3509 J = 1,34
3509 AD(I,J) = RM36(I,J)
CALL ADD(AD,FAC1)
DO 3527 I=1,34
DO 3527 J=1,34
3527 AD(I,J)=RM1013(I,J)
CALL ADD(AD,FAC3)
FAC1=FAC*IN
FAC3=FAC2*IN
DO 3511 I = 1,34
DO 3511 J = 1,34
3511 AD(I,J) = RM63(I,J)
CALL ADD(AD,FAC1)
DO 3528 I = 1,34
DO 3528 J = 1,34
3528 AD(I,J)=RM1310(I,J)
CALL ADD(AD,FAC3)
FAC=0.0
IF(IN.EQ.IK) FAC=DG*RL*S/4.0
IF(IN.EQ.IK) FAC2=DG1*RL*S/4.0
IF(NGAPS.EQ.0) GO TO 17
IF(NGAPS1.EQ.0) GO TO 17
DO 16 I=1,NGAPS
ZO = GAPX(I)
Z1 = ZO + GAP(I)
R=Z(IN,IK,Z1,ZO,RL,1,PI)
16 FAC=FAC-DG*S*R/2.0
DO 216 I=1,NGAPS1
Z2=GAPX1(I)
Z3=Z2+GAP1(I)
R=Z(IN,IK,Z3,Z2,RL,1,PI)
216 FAC2=FAC2-DG1*S*R/2.0
17 CONTINUE
FAC1=FAC*4.0/(S**2)
FAC3=FAC2*4.0/(S**2)
DO 3515 I = 1,34
DO 3515 J = 1,34
3515 AD(I,J) = RM44(I,J)
CALL ADD(AD,FAC1)
DO 3529 I=1,34
DO 3529 J=1,34
AD(I,J) = RM1111(I,J)
CALL ADD(AD,FAC3)
FAC1 = FAC2 * 2.0 * IN * PI / (S * RL)
FAC3 = FAC2 * 2.0 * IN * PI / (S * RL)
DO 3516 I = 1, 34
DO 3516 J = 1, 34
3516 AD(I,J) = RM45(I,J)
CALL ADD(AD,FAC1)
DO 3530 I = 1, 34
DO 3530 J = 1, 34
AD(I,J) = RM1112(I,J)
CALL ADD(AD,FAC3)
FAC1 = FAC2 * 2.0 * IN * PI / (S * RL)
FAC3 = FAC2 * 2.0 * IN * PI / (S * RL)
DO 3518 I = 1, 34
DO 3518 J = 1, 34
3518 AD(I,J) = RM54(I,J)
CALL ADD(AD,FAC1)
DO 3531 I = 1, 34
DO 3531 J = 1, 34
AD(I,J) = RM1211(I,J)
CALL ADD(AD,FAC3)
FAC1 = FAC2 * 2.0 * IN * PI / (RL**2)
FAC3 = FAC2 * 2.0 * IN * PI / (RL**2)
DO 3519 I = 1, 34
DO 3519 J = 1, 34
3519 AD(I,J) = RM55(I,J)
CALL ADD(AD,FAC1)
DO 3532 I = 1, 34
DO 3532 J = 1, 34
AD(I,J) = RM1212(I,J)
CALL ADD(AD,FAC3)

C * NAILING ON UPPER AND LOWER COVER *
DO 18 I = 1, 34
AD(1,I) = 0.0
AD(2,I) = 0.0
AD(3,I) = 0.0
AD(4,I) = 0.0
AD(5,I) = 0.0
AD(6,I) = 0.0

18 AD(6,1) = 0.0
AD(1,13) = 1.0
AD(1,17) = -1.0
AD(1,18) = -D/(2.0 * RL)
AD(2,13) = 1.0
AD(2,17) = -1.0
AD(2,18) = -D/(2.0 * RL)
AD(3,14) = 1.0
AD(3,18) = -1.0
AD(3,15) = -D/(2.0 * S)
AD(3,19) = -HJTM/(2.0 * S)
AD(4,14) = 1.0
AD(4,18) = -1.0
AD(4,15) = -D/(2.0 * S)
AD(4,19) = -HJTM/(2.0 * S)
AD(5,15) = 1.0/S
AD(5,19) = -1.0/S
AD(6,15) = 1.0/S
AD(6,19) = -1.0/S
IF (NDISCR.EQ.0) GO TO 423
SPAR=0.0
SPER=0.0
DO 20 I=1,NAI
    X = XIN + (I-1)*ENL
    IF (NGAPS.EQ.0) GO TO 420
    DO 415 J = 1, NGAPS
        X11 = GAPX(J)
        X12 = GAPX(J) + GAP(J)
        IF (X.GE.X11.AND.X.LE.X12) GO TO 418
    415 CONTINUE
    GO TO 420
418 X = XIN + (I-1)*ENL
    X = X*PI/RL
    SPAR=SPAR+COS(IN*X)*COS(IK*X)
    SPER=SPER+SIN(IN*X)*SIN(IK*X)
20 CONTINUE
    GO TO 426
423 IF (IN.NE.IK) GO TO 428
    SPAR = RL /(2.0*ENL)
    SPER = RL /(2.0*ENL)
    GO TO 426
428 SPAR =0.0
    SPER =0.0
    SPAR = SPAR * RKPAL
    SPER=SPER*RKPER
    SROT = SPER * RKROT
    C
    DO 518 J=1,6
    DO 518 I=1,34
        AD1(J,I)=0.0
        AD1(1,20)=1.0
        AD1(1,17)=-1.0
        AD1(1,16)=IK*PI*(HJTM+D1)/(2.0*RL)
        AD1(2,20)=1.0
        AD1(2,17)=-1.0
        AD1(2,16)=IN*PI*(HJTM+D1)/(2.0*RL)
        AD1(3,18)=-1.0
        AD1(3,19)=HJTM/(2.0*S)
        AD1(3,21)=1.0
        AD1(3,22)=D1/(2.0*S)
        AD1(4,18)=-1.0
        AD1(4,19)=HJTM/(2.0*S)
        AD1(4,21)=1.0
        AD1(4,22)=D1/(2.0*S)
        AD1(5,19)=-1.0/S
        AD1(5,22)=1.0/S
        AD1(6,19)=-1.0/S
        AD1(6,22)=1.0/S
        IF(NDSCR1.EQ.0) GO TO 923
        SPAR1=0.0
        SPER1=0.0
    DO 29 I=1,NAI
        X = XIN1 + (I-1) * ENL1
        IF (NGAPS1.EQ.0) GO TO 920
        DO 915 J = 1, NGAPS1
            X11 = GAPX1(J)
            X12 = GAPX1(J) + GAP1(J)
        915 CONTINUE
        GO TO 920
923 IF (IN1.NE.IK1) GO TO 928
        SPAR1 = RL1 /(2.0*ENL1)
        SPER1 = RL1 /(2.0*ENL1)
        GO TO 926
928 SPAR1 =0.0
        SPER1 =0.0
        SPAR1 = SPAR1 * RKPAL1
        SPER1=SPER1*RKPER1
        SROT1 = SPER1 * RKROT1
    C
    DO 5181 J=1,6
    DO 5181 I=1,34
        AD11(J,I)=0.0
        AD11(1,20)=1.0
        AD11(1,17)=-1.0
        AD11(1,16)=IK1*PI1*(HJTM1+D11)/(2.01*RL1)
        AD11(2,20)=1.0
        AD11(2,17)=-1.0
        AD11(2,16)=IN1*PI1*(HJTM1+D11)/(2.01*RL1)
        AD11(3,18)=-1.0
        AD11(3,19)=HJTM1/(2.0*S1)
        AD11(3,21)=1.0
        AD11(3,22)=D11/(2.0*S1)
        AD11(4,18)=-1.0
        AD11(4,19)=HJTM1/(2.0*S1)
        AD11(4,21)=1.0
        AD11(4,22)=D11/(2.0*S1)
        AD11(5,19)=-1.0/S1
        AD11(5,22)=1.0/S1
        AD11(6,19)=-1.0/S1
        AD11(6,22)=1.0/S1
        IF(NDSCR11.EQ.0) GO TO 9231
        SPAR1=0.0
        SPER1=0.0
IF(X.GE.X11.AND.X.LE.X12)GO TO 918
915   CONTINUE
   GO TO 920
918   GO TO 29
920   X=X*PI/RL
      SPAR1=SPAR1+COS(IN*X)*COS(IK*X)
      SPER1=SPER1+SIN(IN*X)*SIN(IK*X)
   CONTINUE
   GO TO 926
923   IF(IN.NE.IK)GO TO 928
      SPAR1= RL/(2.0*ENL1)
      SPER1= RL/(.  .0*ENL1)
   GO TO 926
928   SPAR1=0.0
      SPER1=0.0
926   SPAR1=SPAR1*RKPAL1
      SPER1=SPER1*RKPER1
      SROT1=SPER1*RKROT1
C
   DO 25  I=1,34
   DO 25  J=1,34
      STIFF(I,J)=STIFF(I,J)+SPAR*AD(1,I)*AD(2,J)+SPER*AD(3,I)*AD(4,J)
   1 + SROT*AD(5,I)*AD(6,J)
   2 +SPER1*AD(3,I) *AD(4.J) +SROT1 *AD(5,I) *AD(6,J)
   3 +SPAR1 *AD(1,I) *AD(2,J)
   CONTINUE
C
C   * LOAD VECTOR *
30   IF(IN.NE.IK)RETURN
   DO 35 I=1,34
      V(I)=0.0
   IF(PLD.EQ.O.O)RETURN
   PINL=PI*IK/RL
   FACTR=PLD*(Y2-Y1)*(C0S(PINL*X1)-C0S(PINL*X2))/(2.0*PINL)
   XIX=2.0*Y1/5
   XIX=2.0*Y2/5
   DO 37 I=1,6
      XI=XIX+(XIY-XIX)*(1.0+ETA(I))/2.0
      XI2=XI**2
      XI3=XI**3
      XI4=XI**4
      X15=XI**5
      V(1)=V(1)+(XI2-5.0*X13/4.0-X14/2.0+3.0*X15/4.0)*H(I)
      V(23)=V(23)+(XI2+5*X13/4.0-X14/2.0-3.0*X15/4.0)*H(I)
      V(16)=V(16)+(1.0-2.0*X12+X14)*H(I)
      V(2)=V(2)+(-XI2-X13+X14+X15)*H(I)/8.0
      V(24)=V(24)+(-XI2-X13+X14+X15)*H(I)/8.0
37   V(15)=V(15)+(-XI2-X13+X14+X15)*H(I)/2.0
   V(1)=V(1)*FACTR
   V(23)=V(23)*FACTR
   V(16)=V(16)*FACTR
   V(2)=V(2)*FACTR
   V(24)=V(24)*FACTR
   V(15)=V(15)*FACTR
   RETURN
C
C
SUBROUTINE ADD(AD,FAC)
DOUBLE PRECISION STIFF
DIMENSION AD(34,34)
COMMON/B8/STIFF(34,34)
IF(FAC.EQ.0.0)RETURN
DO 20 I=1,34
DO 20 J=1,34
    STIFF(I,J) = STIFF(I,J) + FAC*AD(I,J)
20  RETURN
END