Towards an Affordable Multi-DOF Force Feedback Motion Control Input Device

by

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Abstract

A novel multi-degree-of-freedom force feedback motion control input device design has been proposed for 3D human-machine applications. This haptic device utilizes a new electro-mechanical design to achieve a large translation range for each axis of motion, while remaining suitable for mass production at low cost.

Parallelogram linkages have been used to obtain displacements along each axis. The current device prototype uses an orthogonal arrangement of three parallelogram linkages to obtain displacements along all axes in Cartesian space. Springs have been used to center the device and a slot-and-tab hinge structure has been designed and used as a practical joint.

An affordable and compact microelectronic sensor that is based on a grayscale with varying reflectance has been employed in order to sense the end-effector position. The nonlinearity of the sensor has been addressed and linearly compensated.

A Lorentz force-based linear actuator design has been proposed. The actuator consists of a stator and a slider. The equivalent magnetic circuit model has been derived to assist the design computations. Experimental results show that the magnetic flux density along the air gap is approximately uniform and that the actuating force, although its level needs to be increased for use in the commercial product, is a linear function of the current applied to the coil windings on the slider.

The device kinematics and dynamics have been derived and simulations have been performed to investigate the relationship between joint trajectories and work space actuating forces.
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Chapter 1

Introduction

An "input device" is any interface used to introduce data into a computer. The most common input devices are the keyboard, mouse and track ball. A keyboard enables text entry, and directional cursor movement but no direct position input is allowed. A mouse is used to specify absolute position. It can select an object and control its 2D motion. A track ball is used most often with a laptop computer. The user controls the cursor by rolling the ball with the thumb. All these input devices are 2D interfaces.

For manipulating and viewing 3D objects, a 3D input device is desirable [41]. Several 3D input devices, including mouse-based 6 degree of freedom (dof or DOF) devices, free-moving isotonic position control devices, desktop isometric and elastic devices, and multi-dof armature-based devices have been investigated in [40,41]. For example, the Magellan$^{TM}$ 3D Controller made by Logitech is a commercial desktop 6-DOF elastic pointing device with a small motion range [1].

These 2D and 3D interfaces are uni-lateral, passive devices [21]. They transfer energy from the operator to the computer while only vision and/or auditory feedback is directed to the user [12,39].

In the interaction between human and computer via input devices, especially when dealing with tasks that involve contact, one would expect the relationship between the hand and the controlled machine to be bi-directional [12]. The interface would not only receive mechanical inputs from the hand, but would also deliver stimuli to the force and displacement receptors of the hand. This
tactile and kinesthetic information is very useful to an operator who manipulates an object [3]. While a passive uni-lateral input device can't achieve this goal, a force-feedback or haptic device can [21].

According to the Merriam-Webster dictionary, the word “haptic” refers to “relating to or based on the sense of touch”. A haptic device by providing haptic feedback allows a user to interact with a computer. Haptic feedback has two cognitive senses: (i) the tactile sense giving an awareness of surface, and (ii) the kinesthetic sense providing information on body position and movement. Thus, a haptic device is a machine that is controlled by the human hand and can be programmed to give the human operator a sensation of forces associated with various arbitrary maneuvers [12].

Haptic or force-feedback input devices can be widely used in the areas of CAD design, tele-operation, training, medical simulation and even entertainment. For example, such a device can be used as a hand (master) controller that gives the operator a force or load feeling when he is remotely maneuvering a mass, just as if he were working on the site [20,22].

Over the past decades, research in this area has led to a number of device products. The present research is concerned with the design of a new haptic input device.

1.1 Motivation and Objectives

Haptic device designs vary from planar 2-DOF to spatial 6-DOF with a specified motion range. As explained before, a planar 2-DOF haptic device is not suitable for 3D applications no matter how large the workspace is. Most desktop 6-DOF haptic devices have very limited translation and/or rotation ranges. The three translations and three rotations provided by these 6-DOF devices are not fully decoupled. This is due to the complex electro-mechanical design of the device. For applications where only pure, larger translations along three axes in Cartesian space are required, a 6-DOF haptic device is not necessary.

The cost for making a haptic device depends mostly upon the selected sensors, actuators and the mechanical design. Position sensors have been widely used in haptic devices to detect the end-effector positions and/or orientations. Due to the required resolution, the cost of these sensors
varies tremendously, from a few dollars to hundreds of dollars apiece. In most cases, however, high-cost and high-fidelity sensors may not be necessary [10]. Most current haptic devices are usually research-based and not designed for mass production. As long as the cost is not a big problem, any haptic device could be designed and built at will. For a product that is to be attractive to consumers, not only performance but also cost are of particular importance!

The force feedback of a haptic device comes from installed actuators. Simply, a brushed dc motor can be used as an actuator, however, nonlinearity and friction may render the motor control non-trivial. Most haptic devices are driven directly, thus a linear actuator that is easy to control is highly desirable.

In summary, an affordable force-feedback device that provides three larger, axial translations in Cartesian space would be welcomed by the consumer market. The design of such an interface is the goal of this research project.

The main objectives of the thesis are: (i) to design a mechanical device based on ease of use, suitability for mass production and achieving three orthogonal translations in Cartesian space with large motion ranges; (ii) to select an affordable, but effective position sensor; (iii) to design a linear actuator for force-feedback; and (iv) to derive and simulate kinematic and dynamic models of the device.

1.2 Typical Force-Feedback Input Devices

A force-feedback input device has bi-directionality as a distinguishing feature. It “reads and writes” to and from the human hand [12]. In the following, the electro-mechanical design of some typical force-feedback input devices including: pantograph-based devices, string-based devices, PHANToM™ and maglev-based devices, are reviewed.

1.2.1 Pantograph-based Haptic Devices

The first pantograph-based haptic interface was built by Ramstein and Hayward at McGill
1.2 Typical Force-Feedback Input Devices

University [11]. It is a 2-DOF planar five-bar linkage device with a workspace of 100x160mm$^2$. Two 2-bar serial linkages, the left and right chains, are connected to a ground-mounted bar. Their distal ends are connected to a handle. Two grounded dc motors are installed on the ground-mounted bar, to drive the handle in a plane. Two optical encoders are installed co-axially with each of the two motors in order to measure the two active joint angles. The two chains are identical, thus providing a well conditioned and symmetrical work space for the pantograph.

The Hayward's 5-bar pantograph is shown in Figure 1.1. This picture is reproduced from http://www.cim.mcgill.ca/~haptic/devices/pantograph.html.

![Figure 1.1: The Hayward's 5-bar linkage.](image)

Based on the pantograph, a number of similar haptic devices have been designed and built, such as the UBC 3-DOF twin-pantograph haptic mouse [35] and 5-DOF twin-pantograph haptic pen [31], the 6-DOF parallel platform master hand controller [23], the 6-DOF desktop force display [17] and the 6-DOF master arm [25].

1.2.2 String-based Haptic Devices

The first string-based haptic device was the SPace Interface Device for Artificial Reality, or SPIDAR, developed by Ishii and Sato at the Precision and Intelligence Laboratory of the Tokyo Institute of Technology [19]. It is a 3-DOF wire-driven interface that measures the 3D motion of an operator's finger tip [28].
The SPIDAR consists of a cap, into which one inserts the index finger. The cap is held by four strings from the corners of a cube frame. The string is wound around a pulley, to which an electrical motor is attached. The string tension is controlled by the motor. The finger motion is measured using rotary encoders mounted to each motor.

A stereoscopic vision system provides visual feedback. Through red and green glasses, the operator sees a 3D wireframe object. When the finger reaches a position occupied by the object, the wire is restrained and the finger motion is restricted. The user feels as if he has touched the object.

As an extension of SPIDAR, SPIDAR II uses two finger-caps, each of them held by four strings from four separate corners of a cube frame. It can be used for virtual pick-and-place tasks [15,16]. Figure 1.2 depicts the schematic of SPIDAR II. This drawing is reproduced from [15]. Other string-based devices have also been presented [38].

![Figure 1.2: The schematic of SPIDAR II.](image)

1.2.3 PHANToM™

The PHANToM is a multi-DOF (three active and three passive) desktop force-reflecting interface developed by Massie and Salisbury at the Massachusetts Institute of Technology [24]. It tracks finger motion and can exert a controlled force on the finger tip, creating compelling illusions of interaction with solid objects.
1.2 Typical Force-Feedback Input Devices

The current version of PHANToM is shown in Figure 1.3. This picture is reproduced from http://www.sensible.com/products/premium.htm.

![Figure 1.3: The current version of PHANToM.](image)

The x, y and z finger tip coordinates are recorded by three encoders, while three decoupled brushed dc motors control the x, y and z forces exerted upon the user. Motor torques are transmitted through pre-tensioned cables to a stiff, lightweight aluminum linkage. A passive 3-DOF gimbal is attached to a thimble at the end of the linkage. Since the three passive rotational axes of the gimbal coincide at a point, no torque is applied on the user, but allows any orientation of the finger tip.

The PHANToM is commercially produced by SensAble Technologies, Inc. and has gained acceptance in the haptic research community. For example, see Mor [26] for an arthroscopic surgery simulation using PHANToM.

1.2.4 Maglev-based Haptic Devices

Maglev is an abbreviation of magnetic levitation. Such levitation is generated by a Lorentz force acting on a current-carrying linear conductor in a static magnetic field [14]. Well known maglev devices include the Magic Wrist, Magic Mouse, UBC Wrist, UBC PowerMouse, and CMU Haptic Device.
1. The Magic Wrist

The Magic Wrist was the first maglev-based haptic device that was designed and built by the IBM Research Division at Thomas J. Watson Research Center [13]. It consists of a stator and a flotor. The stator is a rigid support structure with mounted magnet assemblies and three narrow-beam LEDs. The flotor is a hexagonal box structure containing position-sensing photodiodes or position-sensing device (PSD) and flat copper coils that are nested between the stator’s inner and outer rings. This flotor is levitated by Lorentz forces that are generated by driving controlled currents through the coils in the magnetic fields. A ball grip, the end-effector, is attached on top of the flotor.

The position and orientation of the flotor are calculated as follows: a triplet of narrow, coplanar and radial light beams generated by the LEDs impinge on the three two-dimensional lateral effect PSDs. The centroids of the light spots projected on the active areas of the PSDs are obtained by measuring the current through the PSD’s electrodes.

The Magic Wrist allows a fine motion range of ±5mm for translation and ±4° for rotation. It can lift a weight of 20N in addition to the 9.4N weight of its flotor body. The maximum torque about the vertical axis is 1.7Nm [6].

Figure 1.4 depicts the Magic Wrist schematic. This drawing is reproduced from [6].

![Figure 1.4: Schematic of the Magic Wrist Assembly.](image)
2. The Magic Mouse

The Magic Mouse is a 2-DOF haptic interface, designed and built by Kelley and Salcudean [21]. Its main structure is a stationary unit, incorporating a lightweight 2-DOF moving coil plate. The coil plate consists of a hand-wound voice-coil that is sandwiched between two sheets of aluminum. A suspension system, made up of two perpendicular linear ball slides and a single low-friction Teflon leg, constrains the plate to translational motion within a plane.

An infrared LED has been hidden inside the handle. The LED light is directed onto a two-dimensional position sensitive detector (PSD) mounted onto the base plate. The position of the Magic Mouse handle is detected by the PSD. The Magic Mouse has a motion range of 17x17mm².

Two permanent magnet stators have been anchored above each coil by an L-shaped aluminum plate, while another two matching permanent magnet stators have been anchored to the base plate below the coils. Thus, two electromagnetic flat coil actuators are formed to provide force feedback to the user.

The Magic Mouse schematic is depicted in Figure 1.5. This drawing is reproduced from [21].

3. The UBC Wrist

The UBC Wrist was designed and built in the Robotics and Control Laboratory at the University of British Columbia [32]. Similar to the Magic Wrist, it consists of a stator and a flotor in parallel, and uses the same optical position sensors and similar actuators.
1.2 Typical Force-Feedback Input Devices

The UBC Wrist fits within a small cylinder. Three horizontal and three vertical flat coils are imbedded in the flotor. Each coil fits in the gap of a matching magnetic pair attached to the stator. The flotor's horizontal plate has holes to allow supporting posts to hold the stator as well as the magnetic pairs. The wrist flotor has a motion range of roughly ±4.5mm for translation and ±6° for rotation.

The UBC Wrist is substantially smaller than the Magic Wrist, since it uses a star configuration for all the flat coils. No center volume is left unused! Figure 1.6 depicts the schematic of the UBC Wrist. This drawing is reproduced from [32].

Figure 1.6: Schematic of the UBC Wrist Assembly.
4. The UBC PowerMouse

The UBC PowerMouse is another haptic device built at the UBC Robotics and Control Laboratory [29]. Compared to other maglev devices, it has utilized a novel geometry and a novel optical sensor.

The PowerMouse is a desktop mouse-like device. A disk-shaped handle with two buttons is attached to a cubic flotor with the flat coils of six Lorentz actuators embedded in its faces. Twenty-four permanent magnets on the stator generate the six magnetic fields crossing the coils. The stator is attached by three mounting posts to a plastic base.

A printed circuit board, located at the base of the stator, carries the position sensors, power electronics and a microcontroller. The wide magnetic gaps of the actuators allow spatial flotor motion with a translation of ±3mm and a rotation of ±5° from a nominal center.

The optical sensing system is designed to detect flotor motion with respect to the stator. The system uses three LED-generated infrared light planes projected in sequence onto three linear one-dimensional position sensing diodes, one tenth of the price of the two-dimensional sensor used in the Magic and UBC Wrist and mounted as an equilateral triangle on the circuit board. Each light plane crosses two of these diodes. Thus, six light-plane intersections with the diodes are obtained, allowing for the solution of the handle location.

The PowerMouse actuator design maximizes the force-to-power-consumption ratio. The device provides a peak force of 34N and a maximum continuous force of 16N. The PowerMouse schematic is depicted in Figure 1.7. This drawing is reproduced from [30].

5. The CMU Haptic Device

The CMU Haptic Device was designed and built in the Microdynamic System Laboratory at Carnegie Mellon University [5,18]. It features a large decoupled motion range and comfortable form for hand manipulation.

The device flotor is a hemispherical shell with a handle at center. This shape results in decoupled translation and rotation, since the flotor can be rotated about its center without colliding with the stator.
The main flotor body is a thin hemispherical aluminum shell with large oval cutouts for the actuator coils. The coils wound from ribbon wire on spherical forms fit together in a densely packed configuration to maximize the flotor area used to generate actuating forces.

The free space around the flotor, the magnet assemblies, and the position sensing system in the stator have been completely redesigned to give the desired motion range and also to conform to the spheric shape.

The CMU Haptic Device has a motion range ±25mm for translation and ±20° for rotation. The maximum force is 60N and the maximum torque 3Nm. Figure 1.8 depicts the schematic of the CMU Haptic Device assembly. This drawing is reproduced from [18].

1.3 Scope and Contributions

The scope of the thesis project was to design a low-cost large workspace motion control input device. This includes the mechanical design of the device, the selection and testing of the position sensor, and the design and validation of the linear actuator.
Due to the limitations of the available facilities, the motion control input device prototype is not yet complete. The thesis contributions are: (i) completion of the device mechanical design; (ii) completion of the selection, testing, and linear compensation of the position sensor; (iii) completion of the design and validation of the linear actuator; (iv) completion of the installation of the sensor, actuator, and other accessories into the device prototype, (v) completion of the derivation and simulation of the device kinematics and dynamics.

1.4 Thesis Outline

Following is the outline of the thesis:

Chapter 1, Introduction: This chapter introduces the background of the new motion control input device design. The need for a large translational motion range and low cost motivate the present research. The objectives of the research and an outline of the thesis are given thereafter.

Chapter 2, The Mechanical Design: The detailed mechanical design of this haptic device is presented, including the design of a parallelogram linkage mechanism, a centering mechanism and a slot-and-tab hinge structure as a joint.
Chapter 3, The Position Transducer: The design of a position transducer used for the device is presented. An infrared reflective opto-sensor was chosen and tested for the position sensing. The nonlinearity between voltage output and measured position is addressed and calibrated using a One Shot Compensation algorithm. A flexible scaling factor is introduced to improve the convergence rate of this algorithm.

Chapter 4, The Linear Actuator: The design of a linear actuator is presented. The actuator's magnetic circuit model is derived and necessary design computations are performed. An experimental platform was built for testing the magnetic field uniformity along the length of the air gap, as well as the dependency between the actuating Lorentz force and the applied dc current.

Chapter 5, Interface Kinematics and Dynamics: The kinematics and dynamics of the motion control input device are derived. Simulations of the inverse and direct dynamics are performed and motion coupling is investigated through these simulations.

Chapter 6, Conclusions and Recommendations: Research results, contributions and recommendations for future work are summarized in this final chapter.
Chapter 2

The Mechanical Design

This chapter describes the mechanical design of the motion control input device. The resulting prototype aims at achieving low friction 3-DOF translation with a large motion range in Cartesian space, at an affordable price.

The design goals are discussed first. Then, the 1-DOF motion mechanism, the parallelogram linkage, is detailed. Based on this structure, motion along all three axes is achieved by appropriately designing and arranging three parallelograms, the X-, Y- and Z-Parallelograms. Finally, the centering mechanism and the machinable joint design are presented. Necessary calculations are performed to assist these designs.

2.1 Design Goals

This haptic device is designed to achieve the following goals:

1. It should provide 3-DOF translation in Cartesian space with a motion range of $[-25.4, +25.4]$mm along each axis.

2. It should have a zero/center position to which the input device should return when there is no force applied to it. This position should also be used to calibrate the device before usage.
There must be a centering mechanism in order to achieve this zero position.

3. It should have installed position transducers to sense the handle position in Cartesian space and actuators to give the operator a feeling of the contact and/or load.

4. It should be designed to have as little friction as possible, i.e. all joints should be designed carefully.

2.2 The Parallelogram Linkage - A Mechanism for 1-DOF Motion

Large motion along each axis is achieved by a parallelogram linkage. Such a linkage is depicted in Figure 2.1(a) and its corresponding 3D view is shown in Figure 2.1(b).

![Parallelogram Linkage Schematic](image)

(a) The parallelogram linkage schematic.

![3D View of Parallelogram Linkage](image)

(b) A 3D view of the parallelogram linkage.

Figure 2.1: The parallelogram schematic and its 3D view.

The parallelogram linkage comprises four sides, as shown in Figure 2.1(a):

- one *fixed* side, segment AD, the edges of which, A and D, form the fixed joints.
- one *translating* side, segment BC, which translates relative to the fixed side AD.
2.3 The 3-DOF Motion Schematics

- two rotating sides, segments AB and DC with identical length \( l \), that connect the fixed side to the translating side, by rotating about the fixed joints A and D, in the plane of the parallelogram.

The displacements of the translating side BC can be decomposed into two orthogonal components: (i) \( d_p \), parallel to the fixed side AD, and (ii) \( d_c \), perpendicular to AD. Both \( d_p \) and \( d_c \) are in the same plane of the parallelogram.

From Figure 2.1(a), \( d_p \) and \( d_c \) can be expressed by following equations:

\[
\begin{align*}
    d_p &= l \sin \theta, \\
    d_c &= l - l \cos \theta,
\end{align*}
\]

where \( \theta \) is the rotation angle for segments AB and DC.

Then, the relationship between \( d_c \) and \( d_p \) can be solved as:

\[
d_c = d_p \tan \frac{\theta}{2}, \tag{2.3}
\]

According to the design goals, the maximum value for \( d_p \) would be:

\[
d_{p_{\text{max}}} = 25.4\,\text{mm}. \tag{2.4}
\]

If \( \theta \) is small, \( d_c \) will be much smaller than \( d_p \). For example, if \( \theta = 0.2593\,\text{rad} \) or \( 14.86^\circ \), then \( d_c = 3.31\,\text{mm} \ll d_{p_{\text{max}}} = 25.4\,\text{mm} \). Thus, the displacement of BC is predominantly in the direction parallel to the fixed side AD. Perpendicular direction of motion, \( d_c \), is comparatively small and can be neglected.

2.3 The 3-DOF Motion Schematics

The input device uses an arrangement of three orthogonal parallelograms, the X-, Y- and Z-
2.3 The 3-DOF Motion Schematics

Parallelograms, to restrict displacement to translation in Cartesian space. The major difference between them lies in the size of plates, the position and orientation of the hinge.

Specifically, motion along the X axis is provided by the X-Parallelogram, which forms the base for the other two parallelograms. Figure 2.2 illustrates the X-Parallelogram schematic.

![Figure 2.2: The X-Parallelogram schematic.](image)

Note in Figure 2.2, the partial hinge structure on the X-TranslatingPlate is used for hinging two rotating plates of the Y-Parallelogram.

Motion along the Y axis is achieved by hinging two rotating plates of the Y-Parallelogram onto the translating plate of the X-Parallelogram, as depicted in Figure 2.3. Note that a vertical plate, Y-VerticalPlate, welded perpendicularly to the translating plate of the Y-Parallelogram, forms the fixed plate of the Z-Parallelogram.

For motion along the Z axis, the two rotating plates of the Z-Parallelogram are hinged onto the vertical plate of the Y-Parallelogram. Figure 2.4 illustrates this assembly.

These three parallelograms are designed such that the X-Parallelogram forms the base on which the Y-Parallelogram hinges and the Z-Parallelogram hinges on the Y-Parallelogram. The device...
2.3 The 3-DOF Motion Schematics

Figure 2.3: The X- and Y-Parallelograms.

Figure 2.4: The X-, Y- and Z-Parallelograms.
handle is then attached to the translating plate of the Z-Parallelogram.

2.4 The Centering Mechanism

The three parallelogram linkages provide translations predominantly along each axis in Cartesian space; however, without an appropriate centering mechanism, the plates would lean or fall to one side when there is no actuating force applied. Another reason why the parallelogram linkages should be centered is that the input device needs a zero position that can be used for calibration. A third reason for this is the offset actuator force required to compensate for gravity. The centering force means that zero force needs to be applied at the “home/center” position.

When centered, each parallelogram linkage must be in its neutral position, i.e. when each side is perpendicular to its neighbour. Coupled piano springs are used as the centering mechanism. For the X-Parallelogram, four identical springs formed into two pairs are hooked between the X-TranslatingPlates and the device case. During motion along the X axis, one pair of springs compress whereas the other pair extend. Forces from both springs will bring the X-Parallelogram to its zero position.

For the Y-Parallelogram, two identical piano springs are hooked between the two Y-RotatingPlates and the center of the X-TranslatingPlate of the X-Parallelogram. Similarly, forces from the springs will bring the Y-Parallelogram back to its zero position.

For the Z-Parallelogram, two identical piano springs are mounted in parallel between the two Z-RotatingPlates of the Z-Parallelogram. Unlike the springs used in the X- and Y-Parallelograms, these two springs have initial deformations that balance the gravity along Z axis. So the two Z-RotatingPlates are initially perpendicular to the Z-TranslatingPlate as well as the Y-VerticalPlate.

The elastic constants of these springs are derived through experiments, as shown in Appendix B.
2.5 The Slot-and-Tab Joint Structure

In Figure 2.4, hinges are used to connect one plate to the other. However, due to the limitations of facilities at hand, such moulded structure is difficult to machine. Instead, a slot-and-tab hinge structure is proposed to achieve a practical joint used in the haptic device.

Figure 2.5 shows such a slot-and-tab hinge. If two plates A and B need to be hinged together, a slot is cut in plate A and a tab is machined on plate B. By inserting plate B into plate A, a hinge is obtained. The contact between plate A and B is a “line” which can change due to the thickness of the plate - there are two pivot points. However, for a slot-and-tab hinge between two thin plates, insignificant friction occurs and can be neglected.

The plate with a tab is the translating plate, whereas the plate with a slot is the rotating plate. The slot width is equal to the tab width, thus no side displacement between two plates occurs. The computation of the slot height is depicted in Figure 2.6. Here, three plates with the identical thickness $l_0$, plate 1, 2 and 3 are required to be hinged together. Two slot-and-tab structures are
2.5 The Slot-and-Tab Joint Structure

designed: (i) the upper slot-and-tab hinge between plates 1 and 2, and (ii) the lower slot-and-tab hinge between plates 2 and 3. The height of the two slots, $h_1$, and $h_2$, are computed as follows.

When plate 2 is rotating counter-clockwise, its pivot points are $O$ and $B$. However, the pivot points are changed into $O'$ and $B'$ when plate 2 is rotating clockwise. Without loss of generality, plate 2 rotates counter-clockwise about the origin $O$ by an angle of $\theta$. Plate 1 stays at its original position while plate 3 is displaced. For the upper slot, the distance from the lowest point $C'$ on
plate 2 to the surface of plate 1, $x_1$, satisfies:

$$x_1 = h_1 \cos \theta - l_0 - l_0 \sin \theta, \quad \theta \in [-\theta_{\text{max}}, \theta_{\text{max}}],$$  \hspace{1cm} (2.5)$$

where $\theta_{\text{max}}$ is the maximum rotation angle for plate 2. Height $h_1$ is chosen such that $x_1 > 0$ when $\theta = \theta_{\text{max}}$, thus guaranteeing that plate 2 will never touch plate 1.

For the lower slot, the distance between the lowest point $A'$ on plate 2 and the surface of plate 3, $x_2$, satisfies:

$$x_2 = h_2 \cos \theta - l_0 - l_0 \sin \theta, \quad \theta \in [-\theta_{\text{max}}, \theta_{\text{max}}].$$  \hspace{1cm} (2.6)$$

Similarly, $h_2$ is chosen such that $x_2 > 0$ when $\theta = \theta_{\text{max}}$. This guarantees that plate 2 will never touch plate 3.

$\theta_{\text{max}}$ is computed via:

$$\theta_{\text{max}} = \sin^{-1} \frac{d_{\text{max}}}{l},$$  \hspace{1cm} (2.7)$$

where $d_{\text{max}} = 25.4\text{mm}$, or half of the translational range along one axis, and $l$, the length of plate 2, is determined by the parallelogram linkage design.

Substituting $\theta_{\text{max}}$ for $\theta$ in (2.5) and (2.6), and also letting $x_1$ and $x_2$ be:

$$x_1 > 0,$$

$$x_2 > 0.$$

Then,

$$h_1 \cos \theta_{\text{max}} - l_0 - l_0 \sin \theta_{\text{max}} > 0, \hspace{1cm} (2.8)$$

$$h_2 \cos \theta_{\text{max}} - l_0 - l_0 \sin \theta_{\text{max}} > 0. \hspace{1cm} (2.9)$$
2.6 Summary

The resulting constrains on slot heights $h_1$ and $h_2$ are:

\[
\begin{align*}
    h_1 &> \frac{l_0 + l_0 \sin \theta_{\text{max}}}{\cos \theta_{\text{max}}}, & \text{and} \\
    h_2 &> \frac{l_0 + l_0 \sin \theta_{\text{max}}}{\cos \theta_{\text{max}}}.
\end{align*}
\]  

(2.10) (2.11)

2.6 Summary

In this chapter, the mechanical design of the motion control input device has been detailed. A parallelogram linkage is used as the basic mechanism to generate a displacement along one axis. Three parallelogram linkages have been coupled such that one is mounted on top of the other.

Piano springs have been used for centering the mechanism. A slot-and-tab hinge structure has been selected to implement a machinable low friction joint. The width of the slot on one plate is equal to that of the tab on the other plate. The height of the slot is solely dependent upon the plate thickness and the maximum rotation angle.

A prototype of the interface mechanism has been built, as shown in Figure 2.7. All of the plates have been machined and assembled to configure the three parallelogram linkages. Spaces are reserved for the installation of the position transducer and the actuator along each axis in Cartesian space.
Figure 2.7: The designed prototype.
Chapter 3

The Position Transducer

This chapter describes the design of a position transducer for the haptic device. The position of its end-effector is prescribed in Cartesian space and is sensed along each axis by a microelectronic sensor. This sensed information is converted into a voltage with range of [0, 5]V.

It is desirable that the position transducer achieves linearity between end-effector position and transducer’s voltage output. The transducer consists of: (i) a position sensor; and (ii) electronic circuitry for converting sensed information into a voltage signal.

In the following, the characteristics of the HOA0149 sensor are presented and its control circuit is designed. Then the grayscale with varying reflectance as the object for HOA0149 to sense, is detailed. Finally, compensation of the sensor’s nonlinearity is carried out, and an algorithm called the One Shot Compensation is introduced.

3.1 HOA0149 and its Control Circuit

A variety of proprioceptive sensors such as potentiometers, encoders and resolvers, can be used as position sensors [33]. However, the objectives of this project require that the sensor be affordable, have good performance, be compact and light weight so that it may easily be installed within the interface mechanism. It should not introduce additional dynamics. Moreover, the sensor should
provide a voltage output within a suitable range.

According to these criteria, an optoelectronic sensor, the Honeywell HOA0149, was chosen for position sensing. This microelectronic sensor consists of (i) a transmitter - an infrared emitting diode (IRED) and (ii) a receiver - a focused NPN silicon phototransistor. These components are contained in a low profile as shown in Figure 3.1, and a schematic of the HOA0149 is depicted in Figure 3.2.

![Figure 3.1: HOA0149 and its package dimensions in millimeters.](image)

![Figure 3.2: A schematic of the HOA0149.](image)

Usually, HOA0149 is used to detect the presence of an object within its field of view. When
powered by the control circuit, its transmitter projects a beam of infrared light along its optical axes. If there is no object at the point at which the two optical axes converge, then there is no reflection and the receiver’s phototransistor works in the cutoff (OFF) region: no current is passing from the collector to the emitter.

On the other hand, if there is an object on the converging optical axes, the receiver receives reflected light from the object’s surface. The phototransistor works in the active (ON) region: current $I_E$ in Figure 3.2 is occurred and flowing from the collector to the emitter. Hence, the collector-emitter current $I_E$ indicates whether or not an object is in front of the sensor.

In this project, the use of HOA0149 has been extended by letting the receiver’s phototransistor always work in its linear region. By varying the reflectance of the object surface, varied outputs are obtained. The magnitude of the phototransistor’s emitter current $I_E$ depends solely on the reflectance of the object surface: the brighter the surface, the more reflection and the larger the current. Figure 3.3 shows the control circuitry designed for HOA0149 that is used in this project [34].

![Figure 3.3: The control circuitry for HOA0149.](image-url)

Since the power source is easily influenced by disturbances, a voltage regulator is used to apply a constant voltage to the HOA0149. Thus, different values for $I_F$ are obtained by varying the value of the forward resistor, $R_F$.

It is not very convenient to use the emitter current as receiver output. By connecting a load resistor, $R_L$, between emitter and ground, this current is converted into a load voltage, $V_L$. 
3.2 Grayscale Design and Preliminary Tests

The capacitors and resistors used in Figure 3.3 are tuned as follows: $C_1 = 100\mu f$, $C_2 = 0.1\mu f$, $C_3 = 0.001\mu f$, $R_F = 115\Omega$, $R_L = 1.925k\Omega$.

3.2 Grayscale Design and Preliminary Tests

As discussed previously, a surface with varying reflectance is required. When this object is placed at the point of convergence of the transmitter and receiver axes and sensed by a moving HOA0149, different voltage readings corresponding to varying reflectance are obtained. If the reflectance versus position relationship is known, the voltage reading versus position can be derived. The design of such a surface, called a grayscale, is addressed in this section. Moreover, three initial tests on the position sensor and on the grayscale are performed.

3.2.1 The Grayscale

A black surface has minimum light reflectance while a white surface has maximum light reflectance. In between black and white are shades of gray with reflectance ranging between the minimum and maximum values. A grayscale is a surface with varying reflectance expressed by black, shades of gray and white.

The grayscale was generated using Matlab™ functions to associate varying reflectance with gray-level values ranging from 0 (black) to 255 (white) and printed onto a mask. An exemplary grayscale generated by specifying linearly increasing gray-level values is shown in Figure 3.4. The specified gray-level values are plotted in the top figure while the corresponding grayscale image is shown in the bottom figure. The grayscale reflectance clearly increases as the gray-level values increase.

3.2.2 Preliminary Tests

In this section, the performance of the HOA0149 is investigated. A Newport™ Positioning Platform is used in the experimental setup. This platform provides a 25mm translational range.
3.2 Grayscale Design and Preliminary Tests

along a horizontal axis. The HOA0149 is fixed onto this platform such that it moves along the horizontal axis. The grayscale is attached to a stationary L-shaped platform facing the sensor. A box covers the sensor and the grayscale in order to eliminate disturbances from other light sources, such as fluorescent light. This experimental setup is illustrated in Figure 3.5.

1. The Sensor Resolution

Before the preliminary tests, the resolution of the sensor is investigated using the oscilloscope. A grayscale is generated in Matlab\textsuperscript{TM} by specifying gray-level values to be linearly increasing from 6 to 255 and printed onto a mask. Not only the dc component but also the ac component of the sensor outputs have been recorded and plotted in Figure 3.6.

As Figure 3.6 shows, the peak-to-peak value from the envelope of the ac component is approximately 6mV. Thus, the resultant resolution of the sensor is found to be 0.044mm.
3.2 Grayscale Design and Preliminary Tests

Figure 3.5: The sensor and grayscale experimental setup.

Figure 3.6: Voltage readings versus displacements for sensor's resolution test.
2. The Optimal Distance between the HOA0149 and the Grayscale

According to the specification, the distance between the HOA0149 and the grayscale plays an important role in the sensing. Therefore, this effect is investigated now.

The same grayscale used in the previous sensor resolution test is sensed again by the HOA0149. Voltage readings are taken at positions of 1.8mm, 5.0mm, 10.0mm, 15.0mm, 20.0mm, 25.0mm and 26.7mm when the distances are varying from 0 to 18.0mm. The results from sensing this grayscale are plotted in Figure 3.7.

![Figure 3.7: Voltage readings versus distances for different positions.](image)

As Figure 3.7 shows, the optimal distance between the HOA0149 and the grayscale for the testing setup is 4.5mm. At this distance, the sensor will have a maximum output. Thus, in the following tests, the distance between the sensor and the grayscale is always set to be this optimal value.

3. Linearity Test

This test investigates the linearity between voltage readings and various specified gray-level values. Several uniform gray patches were generated and sensed by the HOA0149. The gray-level
values that were chosen for these patches were 0, 30, 60, 90, 120, 150, 180, 210, 240 and 255, respectively. The experimental results are plotted together in Figure 3.8.

![Graph showing voltage readings versus positions for different gray patches.](image)

Figure 3.8: Voltage readings versus positions for different gray patches.

As seen from Figure 3.8, the voltage readings sensed from these gray patches increase as the gray-level values increase from 0 to 255. For positions less than 3.9mm, the voltage readings are not stable. This is because that not all the grayscale is within the field view of the HOA0149. Hence, the effective position range, or the stable reading range, is chosen as [3.9, 26.7]mm.

The averaged voltage reading over the effective position range for each gray patch can be computed and plotted with respect to the chosen gray-level value in Figure 3.9. It is clear that this averaged voltage value is not a linear function of the gray-level value. This is due to nonlinearities such as the sensor itself, the printer and the quality of the paper on which the grayscale is printed.

4. Repeatability Test

This test investigates the repeatability of position sensing using the HOA0149. The grayscale used here is generated by specifying linearly increasing gray-level values from 6 to 255. The same grayscale is sensed four times. All voltage readings are plotted in Figure 3.10.
Figure 3.9: Averaged voltage readings versus specified gray-level values.

Figure 3.10: The experimental results for the repeatability test.
As Figure 3.10 illustrates, the voltage readings are almost the same for all tests on the same grayscale. The errors between these four tests are very small, below 5mV. This shows that the sensing of a grayscale is highly repeatable.

3.3 The Sensor Calibration and the OSC Algorithm

The initial tests have shown that the voltage output is not a linear function of the gray-level value. Hence, if the gray-level values vary linearly with the device end-effector position, the relationship between voltage reading and position is not linear. However, a linear position transducer is our objective. To achieve this objective, the nonlinearity between voltage readings and gray-level values must be compensated.

Various methods can be used to compensate for this nonlinearity. One technique compensates by using a lookup table in software. Whenever a new voltage reading is obtained, the corresponding position value is found by referring to the table. The basis for doing this is the repeatability of position sensing. However, this method would rely on the computer since it requires a lot of resources from the computer.

Another option consists in modifying the grayscale itself in terms of the difference between voltage readings and desired linear voltages. The resultant compensated grayscale is independent from other resources such as the computer. This forms what we call the One Shot Compensation (OSC) algorithm.

3.3.1 The OSC Algorithm

The OSC algorithm can be summarized as follows:

1. Choose position values to construct a vector $P$:

$$P_i \in [P_{min}, P_{max}], \quad i = 1, \ldots, N,$$  \hspace{1cm} (3.1)
3.3 The Sensor Calibration and the OSC Algorithm

where $N$ is the total number of positions.

2. Specify a vector $C$ of linearly increasing gray-level values corresponding to the position vector $P$:

$$C_i \in [C_{\text{min}}, C_{\text{max}}], \quad i = 1, \ldots, N.$$  \hfill (3.2)

Use vector $C$ to generate a grayscale in Matlab with increasing reflectance.

3. Sense the grayscale and construct the vector $V$ from voltage readings:

$$V_i \in [V_{\text{min}}, V_{\text{max}}], \quad i = 1, \ldots, N.$$  \hfill (3.3)

where $V_{\text{min}}$ is the minimum value and $V_{\text{max}}$ is the maximum value.

Plot results in the following two graphs:

- Graph $C$ versus $P$: gray-level value versus position.
- Graph $V$ versus $P$: voltage reading versus position.

4. Draw a straight line between points $(P_{\text{min}}, V_{\text{min}})$ and $(P_{\text{max}}, V_{\text{max}})$. This line forms the vector $V_d$, the objective of the compensation, which can be expressed as:

$$V_{di} = \frac{(V_{\text{max}} - V_{\text{min}})}{(P_{\text{max}} - P_{\text{min}})} (P_i - P_{\text{min}}) + V_{\text{min}}, \quad i = 1, \ldots, N.$$  \hfill (3.4)

5. Compute the voltage difference vector $\Delta V$:

$$\Delta V = V_d - V.$$  \hfill (3.5)

6. Compute the Maximum Deviation from Linearity in percentage ($MDLP$) for $\Delta V$.

$$MDLP_{\Delta V} = \frac{\max_{i=1,\ldots,N} |\Delta V_i|}{\max_{i=1,\ldots,N} V_i} \times 100\%. \hfill (3.6)$$
3.3 The Sensor Calibration and the OSC Algorithm

If the value is smaller than the prescribed threshold, skip the rest of OSC steps and terminate the compensation. Otherwise,

7. Compute the ratio vector $R$:

$$R_i = \frac{\Delta V_i}{V_i}, \quad i = 1, \ldots, N.$$  \hfill (3.7)

8. Compute the gray-level compensation vector $\Delta C$,

$$\Delta C_i = C_i R_i, \quad i = 1, \ldots, N.$$  \hfill (3.8)

9. Compute the compensated gray-level vector $C_d$,

$$C_d = C + \Delta C.$$  \hfill (3.9)

10. Based on compensated gray-level values $C_d$, generate and print a new compensated grayscale. Go back to Step 3.

Direct application of the ratio vector $R$ may result in overcompensation. This is analogous to a feedback system with an overly high loop gain. To avoid this problem, a new dimensionless variable, called the Scaling Factor $S$, is introduced in the algorithm, where:

$$S \in [0, 1].$$ \hfill (3.10)

Then Equation (3.8) becomes:

$$\Delta C_i = C_i R_i S, \quad i = 1, \ldots, N.$$ \hfill (3.11)

If $S = 0$, there is no compensation. If $S = 1$, then there will be full gray-level compensation as before, since $\Delta C_i = C_i \cdot R_i$. For $S \in (0, 1)$, there will be partial gray-level compensation. Therefore, the scaling factor gives one degree of freedom to adjust the compensation and controls the speed of convergence.
Two examples of applying the OSC algorithm are presented in the following sections. The experimental setup is the same as before and the forward current of the transmitter, $I_F$, has been adjusted to 30.0mA. Initial gray-level values for both examples are specified linearly increasing from 6 to 255. The only difference of these two examples is that the scaling factor in the first example is chosen to be a small constant whereas in the second example it is chosen manually according to the previous compensation result.

### 3.3.2 Example 1: Compensation with a Fixed Scaling Factor

In this example, the scaling factor $S$ is chosen to be a small constant in order to reduce and smooth the gray-level compensation for each OSC iteration:

$$S = 10\%.$$  

The compensation converges after sixteen iterations. Due to length considerations, only portion of the compensation results are plotted in Figure 3.11 and Figure 3.12. The voltage-position relationships are plotted on the left side, while the relationships between previous and new compensated gray-level values are lotted on the right side.

The $V$ versus $P$ graphs show that the voltage difference magnitude satisfies $|\Delta V_1| > |\Delta V_2| > \ldots > |\Delta V_{16}|$. This means that nonlinearities are significant during initial stages. The corresponding gray-level compensations for these stages are larger as well, i.e., $|\Delta C_1| > |\Delta C_2| > \ldots > |\Delta C_{16}|$.

The MDLP values for all iterations are summarized in Table 3.1 and are also plotted in Figure 3.13. After sixteen compensation iterations, the voltage reading is almost a linear function of position.

However, as shown in Figure 3.13, the MDLP value for the $9^{th}$ compensation iteration is larger than that of the $8^{th}$. Also, the MDLP values for the $12^{th}$ and $13^{th}$ compensation iterations are larger than that of the $11^{th}$. These may be due to printer and paper nonlinearities. For example, if the paper quality is not uniform, even if the gray-level values are the same, the printed grayscales are different.
3.3 The Sensor Calibration and the OSC Algorithm

(a) Measured voltages $V_1$ from sensing the $0^{th}$ original grayscale and computed linearized voltages $V_{d1}$.

(b) Gray-level values $C_1$ specified for the $0^{th}$ original and $C_{d1}$ for the $1^{st}$ compensated grayscale.

(c) Measured voltages $V_4$ from sensing the $3^{rd}$ compensated grayscale and computed linearized voltages $V_{d4}$.

(d) Gray-level values $C_3(=C_{d2})$ for the $3^{rd}$ and $C_{d4}$ for the $4^{th}$ compensated grayscale.

(e) Measured voltages $V_7$ from sensing the $6^{th}$ compensated grayscale and computed linearized voltages $V_{d7}$.

(f) Gray-level values $C_5(=C_{d6})$ for the $6^{th}$ and $C_{d7}$ for the $7^{th}$ compensated grayscale.

Figure 3.11: Results from sensing and compensating the $0^{th}$, $3^{rd}$ and $6^{th}$ grayscales for example 1.
3.3 The Sensor Calibration and the OSC Algorithm

(a) Measured voltages $V_{10}$ from sensing the 9th compensated grayscale and computed linearized voltages $V_{d10}$.

(b) Gray-level values $C_{10} (=C_{d9})$ for the 9th and $C_{d10}$ for the 10th compensated grayscale.

(c) Measured voltages $V_{13}$ from sensing the 12th compensated grayscale and computed linearized voltages $V_{d13}$.

(d) Gray-level values $C_{13} (=C_{d12})$ for the 12th and $C_{d13}$ for the 13th compensated grayscale.

(e) Measured voltages $V_{16}$ from sensing the 15th compensated grayscale and computed linearized voltages $V_{d16}$.

(f) Gray-level values $C_{16} (=C_{d15})$ for the 15th and $C_{d16}$ for the 16th compensated grayscale.

Figure 3.12: Results from sensing and compensating the 9th, 12th and 15th grayscales for example 1.
3.3 The Sensor Calibration and the OSC Algorithm

| Number of Iterations $i$ | $\max |\Delta V_i | \, [\text{V}]$ | $\text{MDLP}_{\Delta V_i} \, [%]$ |
|-------------------------|-----------------|------------------|
| 1                      | 0.5562          | 15.8101          |
| 2                      | 0.5088          | 14.6207          |
| 3                      | 0.4746          | 13.5755          |
| 4                      | 0.4326          | 12.4525          |
| 5                      | 0.4171          | 11.8226          |
| 6                      | 0.3345          | 9.5653           |
| 7                      | 0.2388          | 6.9177           |
| 8                      | 0.2197          | 6.3848           |
| 9                      | 0.2635          | 7.6710           |
| 10                     | 0.1937          | 5.65054          |
| 11                     | 0.1161          | 3.3077           |
| 12                     | 0.1157          | 3.3497           |
| 13                     | 0.1196          | 3.4859           |
| 14                     | 0.0862          | 2.4906           |
| 15                     | 0.0833          | 2.4229           |
| 16                     | 0.0634          | 1.8522           |

Figure 3.13: The MDLP values of $\Delta V = V_d - V$ for example 1.
3.3.3 Example 2: Compensation with a Flexible Scaling Factor

In this example, a new initial grayscale is generated similarly by specifying gray-level values to be linearly increasing from 6 to 255. The scaling factor for each step is chosen visually, depending on the result of the previous iteration. If an overcompensated result is obtained, the scaling factor for the current step should decrease and vice versa. The initial value for $S$ is chosen:

$$ S_1 = 100\%.$$

The compensation converges after six iterations. Compensation results are illustrated in Figure 3.14 and Figure 3.15. Again, $V, V_d$ versus $P$ relationships are plotted on the left side, while $C, C_d$ versus $P$ relationships are plotted on the right side.

$V$ versus $P$ graphs show that $|\Delta V_1| > |\Delta V_2| > \ldots > |\Delta V_6|$. Hence, the gray-level compensation satisfies $|\Delta C_1| > |\Delta C_2| > \ldots > |\Delta C_6|$. After five iterations, color compensation becomes very small and a practically linear voltage reading is achieved.

Table 3.2 summarizes the MDLP value for each OSC iteration. Also, the statistical results are plotted in Figure 3.16.

| Number of Iterations $i$ | max $|\Delta V_i|$ [V] | $MDLP_{\Delta V_i}$ [%] | Scaling Factor $S_i$ |
|--------------------------|-------------------------|-------------------------|---------------------|
| 1                        | 0.2582                  | 7.2325                  | 100%                |
| 2                        | 0.1740                  | 4.8293                  | 100%                |
| 3                        | 0.0967                  | 2.7378                  | 100%                |
| 4                        | 0.1079                  | 3.0515                  | 50%                 |
| 5                        | 0.0717                  | 2.0186                  | 50%                 |
| 6                        | 0.0556                  | 1.5751                  | 50%                 |

3.4 Summary

The design of the position transducer has been presented in this chapter. The transducer consists of a sensor with its control circuit and a grayscale. In accordance with the project objectives,
3.4 Summary

(a) Measured voltages $V_1$ from sensing the $0^{th}$ original grayscale and computed linearized voltages $V_{d1}$.

(b) Gray-level values $C_1$ for the $0^{th}$ original and $C_{d1}$ for the $1^{st}$ compensated grayscale when $S_1 = 100\%$.

(c) Measured voltages $V_2$ from sensing the $1^{st}$ compensated grayscale and computed linearized voltages $V_{d2}$.

(d) Gray-level values $C_2 (= C_{d1})$ for the $1^{st}$ and $C_{d2}$ for the $2^{nd}$ compensated grayscale when $S_2 = 100\%$.

(e) Measured voltages $V_3$ from sensing the $2^{nd}$ compensated grayscale and computed linearized voltages $V_{d3}$.

(f) Gray-level values $C_3 (= C_{d2})$ for the $2^{nd}$ and $C_{d3}$ for the $3^{rd}$ compensated grayscale when $S_3 = 100\%$.

Figure 3.14: Results from sensing and compensating the $0^{th}$, $1^{st}$ and $2^{nd}$ grayscales for example 2.
3.4 Summary

(a) Measured voltages $V_4$ from sensing the $3^{rd}$ compensated grayscale and computed linearized voltages $V_{d4}$.

(b) Gray-level values $C_4(=C_{d3})$ for the $3^{rd}$ and $C_{d4}$ for the $4^{th}$ compensated grayscale when $S_4 = 50\%$.

(c) Measured voltages $V_5$ from sensing the $4^{th}$ compensated grayscale and computed linearized voltages $V_{d5}$.

(d) Gray-level values $C_5(=C_{d4})$ for the $4^{th}$ and $C_{d5}$ for the $5^{th}$ compensated grayscale when $S_5 = 50\%$.

(e) Measured voltages $V_6$ from sensing the $5^{th}$ compensated grayscale and computed linearized voltages $V_{d6}$.

(f) Gray-level values $C_6(=C_{d5})$ for the $5^{th}$ and $C_{d6}$ for the $6^{th}$ compensated grayscale when $S_6 = 50\%$.

Figure 3.15: Results from sensing and compensating the $3^{rd}$, $4^{th}$ and $5^{th}$ grayscales for example 2.
Honeywell’s infrared reflective sensor HOA0149 has been chosen as the position sensor. A grayscale is generated in Matlab\textsuperscript{TM}, printed onto a mask and used as an object with varying reflectance for the HOA0149 to sense.

The performance of the sensor on the grayscale has been tested. The nonlinearity between voltage reading and position has been addressed and calibrated using the OSC algorithm. The effectiveness of the OSC algorithm has been illustrated by two examples. A flexible scaling factor was introduced to improve the convergence rate of the OSC algorithm.

Some other preliminary tests on the sensor have been performed, such as how fast the sensor responds to a reflectance change and to determine the effective reflectance sensing area. However, due to the limitations in the experimental instruments, these tests have not been completed. Hence, the results obtained are not discussed in this thesis.
Chapter 4

The Linear Actuator

For controlling the end-effector motion in Cartesian space, a linear actuator has been designed for each axis. The actuator input is a current passing through a moving coil placed in a constant magnetic field. Its output is a force that is parallel to one axis. Actuator design is based on a permanent magnetic system, and the actuating force is a Lorentz force.

In this chapter, the actuator design is presented. A typical permanent magnetic system with an air gap is reviewed. Then, the actuator mechanical design is presented and its characteristics such as the flux density along the air gap and the generated Lorentz force, are computed. Finally, the actuator prototype is validated through two experiments.

4.1 Actuator Design Requirements

For a haptic device, actuators drive the end-effector within its work space. An actuation force is generated to give an operator the feeling of force or load. This is called “force feedback” or “haptic feedback”.

The actuator design is based on energy conversion from electrical to mechanical. For the actuators used in the motion control input device, the requirements are as follows:
1. The motion range of an actuator should be no less than \([-25.4, 25.4]\)mm since the end-effector of the input device can translate within that range along each axis in Cartesian space.

2. The actuator body should be stationary whereas its moving part should be installed such that it does not touch the body and therefore not generate friction.

3. The actuator should be easy to control. The actuating force should vary linearly with the control signal.

4. The actuator should be compact so as to easily fit within the device.

5. The actuator should consume as little power as possible since three actuators are required for translation along the \(X, Y\) and \(Z\) axes.

A permanent magnetic system best fits these requirements, since a permanent magnet can produce flux in an air gap without exciting coils and, thus no electric power is dissipated [8]. It can also be small so that an actuator with limited dimensions is feasible. Furthermore, a current-carrying conductor exposed to a uniform magnetic field produced by a permanent magnet generates a force which varies linearly with the current. These features initiate the actuator design.

### 4.2 A Typical Magnetic Circuit

A typical magnetic circuit consists of a permanent magnet, an air gap and two pieces of soft iron [7, 9], as depicted in Figure 4.1. The magnetic flux path of the whole system, the dashed line in the figure, is generated by the magnet and is completed by the two soft iron pieces and an air gap.

The permanent magnet is the only source of energy in the system. This energy is dissipated by the air gap, the soft iron and the magnet itself. The magnet's performance depends solely on its *residual flux* \((B_r)\) and recoil permeability \((\mu_{rec})\), which are determined by the *recoil line* on the magnet's *demagnetization curve* [2, 4, 27].
4.2 A Typical Magnetic Circuit

The reluctances of the magnet, $R_m$, the air gap, $R_g$, and the soft iron, $R_s$, are defined as \([2,4,27]\):

$$R_m = \frac{l_m}{\mu_{rec}A_m}, \quad R_s = \frac{l_s}{\mu_sA_s}, \quad R_g = \frac{l_g}{\mu_0A_g}.$$  \(4.1, 4.2, 4.3\)

where,

- the lengths of the magnet, the air gap, the soft iron in the flux path are $l_m$, $l_g$ and $l_s$ respectively. Usually the length of each element is computed by choosing its center line \([9,36]\). For example, the length of the soft iron in the path, $l_s$, could be found by:

$$l_s = L - l_m - l_g.$$  \(4.4\)

where $L$ is the total length of the flux path.

- the corresponding cross-sectional areas of the magnet, the air gap, the soft iron are $A_m$, $A_g$ and $A_s$ respectively, as depicted in Figure 4.1;
4.2 A Typical Magnetic Circuit

- the permeability of the air gap (free space) is $\mu_0$, and $\mu_0 = 4\pi \cdot 10^{-7}\text{H/m}$;

- the permeability of the soft iron is $\mu_s$, and it is very high, e.g. $\mu_s = 5000\mu_0$.

The analysis of such magnetic circuit is complex, therefore, an approximating model is most often used [36]. Two equivalent magnetic circuit models, depicted in Figure 4.2, can be used here.

![Diagram of magnetic circuit models](image)

Figure 4.2: Two equivalent magnetic circuit models.

In the flux source model, Figure 4.2(a), $\phi_{r1}$ is the total flux produced by the permanent magnet where $\phi_{r1} = B_{r1}A_m$. $\phi_g$ is the flux flowing through the air gap. In the mmf source model, Figure 4.2(b), $F_m = \phi_{r1}R_m$ is the corresponding magnetomotive force (mmf) [9].

These two models are analogous to the current and voltage source models commonly encountered in electric circuit analysis with reluctance corresponding to resistance. Hence, all electric circuit
4.3 The Actuator Design

Theorems can be applied here. For example, \( \phi_g \), the magnetic flux in the air gap, is computed via:

\[
\phi_g = \frac{R_m}{R_m + R_s + R_g} \phi_{r1}. \tag{4.5}
\]

For the magnetic circuit as depicted in Figure 4.1, it is the magnetic field along the air gap that is of great interest to users. Since \( \phi_{r1} = B_{r1} A_m \) and \( \phi_g = B_g A_g \), the magnetic flux density in the air gap, \( B_g \), is found by:

\[
B_g = \frac{R_m}{R_m + R_s + R_g} \frac{A_m}{A_g} B_{r1}. \tag{4.6}
\]

4.3 The Actuator Design

A stationary part, the stator, and a moving part, the slider, are the two main actuator components. Their design schematics are presented in the following section.

4.3.1 The Stator

The stator is composed of a trunk, permanent magnets and two air gaps. The stator trunk is constructed from five highly permeable soft iron bars. Four of them form a rectangular shape while a fifth, the center bar, is placed lengthwise down the center of the stator.

Two rows of fourteen neodymium-iron-boron permanent magnets are attached on the inner sides of the two long side iron bars, and thus two long thin air gaps are formed between the center bar and long side iron bars in the trunk. The stator is depicted in Figure 4.3.

All magnets face the center bar with same polarities, such that a flux path is formed through the stator. This flux path can be decomposed into the following sub-paths: upper-left, upper-right, lower-left and lower-right, as shown in Figure 4.4.
4.3 The Actuator Design

Figure 4.3: The stator design.

Figure 4.4: The path of the magnetic flux flowing in the stator.
4.3.2 The Slider

The slider is composed of two aluminum brackets, a copper core and a coil winding. The brackets fix the slider onto a movable plate. Connecting these two brackets is the copper core, on which the coil winding is tightly wound, as shown in Figure 4.5.

When a current-carrying unconstrained conductor is placed in a magnetic field, it moves due to a Lorentz force. Thus, one way of assembling the slider and the stator is to install the slider inside the stator along the center bar as shown in Figure 4.6.

Appendix C illustrates how to install the three actuators onto the 3-DOF parallelogram linkages for the haptic device.

4.4 Design Computations

Three key issues are discussed in this section: (i) the recoil line of the permanent magnet, (ii) the induced flux density along the air gap, and (iii) the Lorentz force exerted upon the current-carrying winding in the air gap.
4.4 Design Computations

4.4.1 Recoil Line of TRI-NEO-30

The chosen permanent magnets are TRI-NEO-30 (neodymium-iron-boron). They are small, as depicted in Figure 4.7(a), however, their energy product $B_{\text{max}}H_{\text{max}}$ is quite large, as up to 27/30MGOe. The demagnetization curves, under various working temperatures, are shown in Figure 4.7(b).

Since the actuator works at room temperature, the demagnetization curve at 20°C is used to approximate the recoil line:

$$B_m = 1.15 + 1.43 \cdot 10^{-6} H_m,$$  \hspace{1cm} (4.7)

where the residual flux, $B_{r1}$, is 1.15T and the recoil permeability, $\mu_{\text{rec}}$, is $1.43 \cdot 10^{-6}$H/m.

Thus, for such a permanent magnet with length $l_m$ and cross-sectional area $A_m$, its reluctance
4.4 Design Computations

(a) TRI-NEO-30 dimensions in millimeters.

(b) Demagnetization curves of TRI-NEO-30.

Figure 4.7: Dimensions and demagnetization curves of TRI-NEO-30.

could be computed by:

\[ R_m = \frac{l_m}{1.43 \cdot 10^{-6} A_m}. \]  \(4.8\)

4.4.2 The Stator Equivalent Circuit Model

Since a magnet is equivalent to a flux source in parallel with its reluctance, all twenty-eight permanent magnets used in one actuator are equivalent to twenty-eight flux sources: \(\phi_{r,1}, \phi_{r,2}, \ldots, \phi_{r,28}\), in parallel with their corresponding reluctances \(R_{m,1}, R_{m,2}, \ldots, R_{m,28}\).

The soft iron and two air gaps can be segmented into small pieces according to the permanent magnet’s dimension. Thus, twenty-eight air gap segments with reluctance \(R_{g,1}, R_{g,2}, \ldots, R_{g,28}\), and forty-five soft iron segments with reluctance \(R_{i,1}, R_{i,2}, \ldots, R_{i,45}\) have been formed. The equivalent circuit model for the stator is then depicted in Figure 4.8 and the corresponding dimensions are
illustrated in Figure 4.9.

According to the actuator design, \( l_r = 7.5\text{mm}, b_m = 5.08\text{mm}, l_c = 7.5\text{mm}, l_m = 2.54\text{mm}, l_g = 2.5\text{mm}, \) and \( l_0 = 3.75\text{mm}. \) If the fringing flux is neglected, the cross-sectional areas of the magnet, the air gap segments and the soft iron segments are the same, \( A_m = A_g = A_i. \)
All magnets have the same demagnetization curve, and their cross-sectional area are all the same, e.g. \( A_m = 10.16 \times 5.08 \text{mm}^2 \), thus each of them produces the same flux:

\[
\phi_{r,k} = B_{r1} A_m, \quad k = 1, \ldots, 28, \tag{4.9}
\]

and has the same reluctance:

\[
R_{m,k} = \frac{l_m}{\mu_{rec} A_m}, \quad k = 1, \ldots, 28. \tag{4.10}
\]

The reluctance of each air gap segment is

\[
R_{g,k} = \frac{l_g}{\mu_0 A_g}, \quad k = 1, \ldots, 28. \tag{4.11}
\]

The soft iron segments have different reluctances since they have different lengths. Without loss of generality, the lengths of the soft iron segments have been chosen as shown in Figure 4.9. Therefore,

1. for segments with a length of \( l_0 \), i.e. segment \( AB \):

\[
R_0 = \frac{l_0}{\mu_i A_i}, \tag{4.12}
\]

2. for segments with a length of \( l_{i,31} = \frac{1}{2} b_m + \frac{1}{2} l_r \), i.e. segment \( BC \):

\[
R_{i,31} = R_{i,45} = \frac{l_{i,31}}{\mu_i A_i}, \tag{4.13}
\]

3. for segments with a length of \( l_{i,1} = l_0 + l_g + l_m + l_c + \frac{1}{2} l_r + \frac{1}{2} b_m \), i.e. segment \( CDEF \):

\[
R_{i,1} = R_{i,15} = R_{i,16} = R_{i,28} = \frac{l_{i,1}}{\mu_i A_i}, \tag{4.14}
\]
(4) for segments with a length of \( l_{1,2} = b_m + \frac{1}{2}l_c \), i.e. segment \( EGH \):

\[
R_{1,2} = \ldots = R_{4,14} = R_{4,32} = \ldots = R_{4,44} = R_{4,17} = \ldots = R_{4,29} = \frac{l_{1,2}}{\mu_1 A_1}.
\] (4.15)

Hence, the equivalent flux source model is symmetrical with respect to the vertical and horizontal center lines, the dash-dotted lines, as shown in Figure 4.8. The entire magnetic circuit model for the stator can be decomposed into four identical sub-circuits: the upper-left, the lower-left, the upper-right and the lower-right sub-circuits.

In the following computation, the upper-left and lower-left sub-circuits are combined into a left-half sub-circuit. This is due to the fact that all the flux from those two rows of permanent magnets are flowing through the center iron bar. Figure 4.10 illustrates the equivalent model of the left-half sub-circuit.

4.4.3 The Flux and Flux Density Computation

Kirchhoff's law is applied to compute the mmf at each labeled node: \( F_1, \ldots, F_{13}, F_{29}, \ldots, F_{35}, F_{43}, \ldots, F_{50}, \) and \( F_{73}, \ldots, F_{85} \) in Figure 4.10. Then, the magnetic flux, \( \psi_{g,1}, \ldots, \psi_{g,7}, \psi_{g,22}, \ldots, \psi_{g,28} \), and the flux density, \( B_{g,1}, \ldots, B_{g,7}, B_{g,22}, \ldots, B_{g,28} \), for each small air gap segment are
derived (See Appendix A).

For example, the magnetic flux $\phi_{g,1}$ is:

$$\phi_{g,1} = \frac{F_{20} - F_{44}}{R_{g,1}},$$

and the corresponding flux density $B_{g,1}$ is:

$$B_{g,1} = \frac{\phi_{g,1}}{A_g} = \frac{F_{20} - F_{44}}{R_{g,1}A_g}.$$  \hspace{1cm} (4.17)

As discussed before, the upper-left and upper-right sub-circuits are symmetrical. Thus, the flux densities for air gap segments labeled 1, 2, ..., 7, 8, 9, ..., 14 satisfy the following relationships:

$$B_{g,1} = B_{g,14},$$  \hspace{1cm} (4.18)
$$B_{g,2} = B_{g,13},$$  \hspace{1cm} (4.19)
$$B_{g,3} = B_{g,12},$$  \hspace{1cm} (4.20)
$$B_{g,4} = B_{g,11},$$  \hspace{1cm} (4.21)
$$B_{g,5} = B_{g,10},$$  \hspace{1cm} (4.22)
$$B_{g,6} = B_{g,9},$$  \hspace{1cm} (4.23)
$$B_{g,7} = B_{g,8}.$$  \hspace{1cm} (4.24)

Let $d$ be the displacement from the air gap segment to the center line as shown in Figure 4.9.

The computed flux densities $B_{g,1}, ..., B_{g,7}$, and $B_{g,8}, ..., B_{g,14}$ as functions of $d$, are plotted in Figure 4.11.

As Figure 4.11 shows, the magnetic flux densities on both ends of the air gap are slightly greater than those of the center. This is due to the fact that the magnetic flux takes the shortest path. More flux is flowing through both left and right soft iron ends. The averaged magnetic flux density in the air gap ($\bar{B}_g$) is 0.53T.
4.4 Design Computations

Figure 4.11: The flux density distribution along the air gap.

4.4.4 The Lorentz Force

Figure 4.12(a) illustrates the magnetic field that is present in the air gap, with respect to the current applied to the coil windings. Lorentz forces exert upon segments $AD$ and $BC$ of the windings, to push the core away from reader (see Figure 4.12(b)).

The Lorentz force for an $N$-turn coil winding that is conducting a current, $I$, in a uniform magnetic field $B_g$ is:

$$F = NIB_gL_{eff},$$

(4.25)

where the effective length of the winding that is exposed to the magnetic field is $L_{eff}$. For our actuator prototype, $L_{eff}$ is 0.010mm, determined by the structure of the stator and the slider.

Note that Equation (4.25) is valid only if the direction of the current $I$ is perpendicular to that of the magnetic field $B_g$. 
4.5 The Actuator Design Validation

In this section, the actuator design is validated through two experiments performed using a specially designed testing mechanism. The first experiment investigates the linearity of the Lorentz force with respect to an applied dc current. The second experiment investigates the distribution of the magnetic flux density along the length of the air gap. A nearly uniform magnetic field is expected.

4.5.1 The Experimental Mechanism

An experimental mechanism, consisting of a base plate, a supporting pillar, a cross-shaped rotating arm and two plastic weighing cups, is designed for validating the actuator prototype. The supporting pillar has been screwed onto the base plate. The center of the cross-shaped arm is connected to the tip of the pillar by a pair of bearings. The entire cross-shaped arm is supported by the pillar and can rotate about a pivot point that is formed by the two bearings.

The two cups are used for adjusting weights on the cross-shaped arm. The weight difference between the two cups drives the cross-shaped arm to rotate clockwise or counter-clockwise.
4.5 The Actuator Design Validation

A pointer is attached to the vertical part of the cross-shaped arm and a scale plate is attached to the pillar. Thus, when the cross-shaped arm is rotating about the pivot point, the angle and direction of the rotation can be read from the scale. Figure 4.13 illustrates the experimental mechanism.

![Experimental mechanism](image)

Figure 4.13: The experimental mechanism for the actuator validation.

The stator has been screwed onto the base plate, and the slider is attached to the vertical end of the cross-shaped arm, as illustrated in Figure 4.14. Therefore, when the slider is actuated by a dc current, it pushes the cross-shaped arm to rotate around the pivot point. Again, the angle and rotation direction can be obtained from the scale reading. By design, the rotational range of the cross-shaped arm is \([-7.3^\circ, 8.3^\circ]\), or \([-15.5, 17.6]\) mm.

This experimental mechanism can be used to validate the actuator design since the Lorentz force can be balanced by adjusting the weights in two cups at each location within the motion range of the cross-shaped arm. If the dc current applied to the coil windings and the weights in the cups are known, and the rotating angle is read, the Lorentz force as well as the magnetic field density at any location can be computed. The relationship between the Lorentz force and the weights in the cups, the rotating angle is derived in the next sub-section.
4.5.2 Actuating Force Computation

A schematic of the cross-shaped arm and all forces exerting upon it are depicted in Figure 4.15. The vertical part of the cross-shaped arm is composed of segments \( PO \) and \( CO \), whereas its horizontal part is composed of segments \( LO \) and \( RO \). The corresponding masses of these segments are shown at the center of mass (COM). A coordinate system is chosen with its origin \( O \) at the pivot point.

The parameters of the cross-shaped arm are measured and the associated torques are computed symbolically. These results are summarized in Table 4.1.

The resultant torque about the pivot point is:

\[
\sum \tau = \frac{1}{2} m_p g a_p \sin \theta + \frac{1}{2} m_l g a_l \cos \theta + W_l g a_l \cos \theta - \frac{1}{2} m_s g a_c \sin \theta \\
- m_c g a_c \sin \theta + F_a a_c \cos \theta - \frac{1}{2} m_r g a_r \cos \theta - W_r g a_r \cos \theta. \tag{4.26}
\]

For \( m_p \approx 0 \), \( a_l = a_r = a \) and \( m_l = m_r \), Equation (4.26) becomes:

\[
\sum \tau = W_l g a_l \cos \theta - \frac{1}{2} m_s g a_c \sin \theta - m_c g a_c \sin \theta + F_a a_c \cos \theta - W_r g a_r \cos \theta. \tag{4.27}
\]
4.5 The Actuator Design Validation

Figure 4.15: The schematic of the cross-shaped arm.

Table 4.1: Parameters of the cross-shaped arm.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
<th>Value</th>
<th>Unit</th>
<th>Torque [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_p )</td>
<td>mass of segment PO</td>
<td>( \approx 0 )</td>
<td>kg</td>
<td>( \frac{1}{2} m_p g a_p \sin \theta )</td>
</tr>
<tr>
<td>( a_p )</td>
<td>length of PO</td>
<td>116.0</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>( m_l )</td>
<td>mass of segment LO</td>
<td>0.040</td>
<td>kg</td>
<td>( \frac{1}{2} m_l g a_l \cos \theta )</td>
</tr>
<tr>
<td>( W_l )</td>
<td>weights added to the left cup</td>
<td></td>
<td>kg</td>
<td>( W_l g a_l \cos \theta )</td>
</tr>
<tr>
<td>( a_l )</td>
<td>length of LO</td>
<td>136.0</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>( m_s )</td>
<td>mass of segment CO</td>
<td>0.0447</td>
<td>kg</td>
<td>( -\frac{1}{2} m_s g a_s \sin \theta )</td>
</tr>
<tr>
<td>( m_c )</td>
<td>mass of the slider</td>
<td>0.0074</td>
<td>kg</td>
<td>( -m_c g a_c \sin \theta )</td>
</tr>
<tr>
<td>( a_c )</td>
<td>length of CO</td>
<td>122.0</td>
<td>mm</td>
<td></td>
</tr>
<tr>
<td>( F_a )</td>
<td>actuating force</td>
<td></td>
<td>N</td>
<td>( F_a a_c \cos \theta )</td>
</tr>
<tr>
<td>( m_r )</td>
<td>mass of segment RO</td>
<td>0.040</td>
<td>kg</td>
<td>( -\frac{1}{2} m_r g a_r \cos \theta )</td>
</tr>
<tr>
<td>( W_r )</td>
<td>weights added to the right cup</td>
<td></td>
<td>N</td>
<td>( -W_r g a_r \cos \theta )</td>
</tr>
<tr>
<td>( a_r )</td>
<td>length of RO</td>
<td>136.0</td>
<td>mm</td>
<td></td>
</tr>
</tbody>
</table>
4.5 The Actuator Design Validation

By setting \( \sum \tau = 0 \) in Equation (4.27), the actuating force, \( F_a \), becomes:

\[
F_a = (W_r - W_t)g\frac{a}{a_c} + \frac{1}{2}(m_s + 2m_c)g\tan \theta.
\]  
(4.28)

4.5.3 Test 1: Actuating Force versus Applied DC Current

In this experiment, the force-current dependency is investigated. For convenience and also to eliminate possible non-uniformity in the air gap, the equilibrium is chosen at \( \theta = 0 \). By setting \( \theta = 0 \) in Equation (4.28), the actuating force \( F_a \) becomes:

\[
F_a = (W_r - W_t)g\frac{a}{a_c},
\]  
(4.29)

Thus, the flux density at \( \theta = 0 \) is computed from Equations (4.29) and (4.25):

\[
B|\theta=0 = \frac{F_a}{I\frac{1}{N L_{eff}}}
\]  
(4.30)

When a dc current is applied to the slider winding and the swinging arm is driven by the slider to a position where \( \theta \neq 0 \), the weights in the two cups are adjusted so that the swinging arm is forced to back to \( \theta = 0 \). The values of \( W_t \) and \( W_r \) are used to compute \( F_a \).

Four tests have been performed, the averaged results of which are plotted in Figure 4.16.

As Figure 4.16 shows, the force varies linearly with the current. The slope of the line \((F-I)\) is 1.444N/A. Since the winding has \( N = 260 \) turns, the magnetic flux density at \( \theta = 0 \) is 0.555T. This experimental result is roughly consistent with the design computations where \( B_{g,r} = 0.5333T \), as illustrated in the previous section.

4.5.4 Test 2: Uniformity of Magnetic Field along Air Gap

In this experiment, a constant dc current is applied; therefore, the actuation force should be a constant, provided that \( B_g \) is constant.
From Equations (4.25) and (4.28), the flux density along the air gap is computed as:

$$B = k_w (W_r - W_l) + k_\theta \tan \theta,$$  \hspace{1cm} (4.31)

where $k_w = \frac{g_0}{N_{\text{eff}}} I$ and $k_\theta = \frac{1}{2} \frac{m_1 + 2m_2}{N_{\text{eff}}} I$ are both constants if the current $I$ is constant.

The experiments are carried out by adjusting the weights in the two cups so that measurements can be collected at several positions along the length of the air gap. Two tests are performed. First, the swinging arm is driven to rotate counter-clockwise ($\theta \geq 0$) by a negative current:

$$\begin{cases} I_- = -224.6 \text{mA}, \\ k_{w_-} = -0.01873 \text{T/g}, \\ k_{\theta_-} = -0.4998 \text{T}, \end{cases}$$ \hspace{1cm} (4.32)

and $F_a$ is along the $-X$ axis.
Second, the swinging arm is rotated clockwise ($\theta \leq 0$) by a positive current:

$$\begin{align*}
I_+ &= 262.2\text{mA}, \\
k_{w+} &= 0.01604\text{T/g}, \\
k_{\theta+} &= 0.4281\text{T}.
\end{align*}$$

(4.33)

In this case, $F_a$ is along the $+X$ axis.

The results from these two experiments are combined and plotted in Figure 4.17.

![Magnetic field density versus the length of the air gap.](image)

**Figure 4.17**: Magnetic field density versus the length of the air gap.

As Figure 4.17 shows, the magnetic field along the air gap is almost uniform, as predicted.

### 4.6 Position Transducer - Actuator Assembly

As mentioned in Chapter 2, spaces are reserved for installing the position transducer and the actuator into the device. Due to the fact that they are all related to the motion along one axis,
they can be assembled into one integrated assembly, called position transducer - actuator assembly.

The stator is fixed with respect to one direction of motion, and originally the slider is screwed onto the translating plate along that direction. In order to install the position transducer together with the actuator, two new mounting brackets are designed to attach this assembly onto the translating plate. The slider, however, is now mounted onto these two new brackets rather than the translating plate.

The HOA0149 sensor is mounted onto one of the new brackets, facing towards the outside of the stator trunk on which a compensated grayscale is glued. The other new bracket is available for other component installation in the future.

The schematic of the assembly is depicted in Figure 4.18, while Figure 4.19 shows the resultant assemblies installed along the $X$, $Y$ and $Z$ axes in the device prototype.

![Schematic of Position Transducer - Actuator Assembly](image)

Figure 4.18: The schematic of position transducer - actuator assembly.
4.7 Summary

This chapter has presented the design of a linear actuator prototype. The actuator consists of a slider and a stator. TRI-NEO-30 permanent magnets are installed on the stator trunk and produce the flux flowing into the air gap. An $N$-turn core winding is wound around the slider, which is placed in the air gap. When a dc-current flows through the winding, a Lorentz force is generated and pushes the slider.

The actuator magnetic circuit model has been derived and the design computations have been performed using this model. An experimental mechanism has been designed and built for evaluating the actuator prototype. The results show that the force is a linear function of the applied current, and the magnetic field along the air gap is nearly uniform.

Finally, the position transducer and actuator for motion along one axis have been grouped into an integrated assembly by virtue of the actuator design.
Chapter 5

Interface Kinematics and Dynamics

In this chapter, motion along the $X$, $Y$ and $Z$ axes in Cartesian space is described. The center of mass of each plate is computed and the mechanism kinematics are derived. Then, the device joint space dynamics are computed using the *Lagrangian* formulation. Finally, inverse and forward dynamics simulations are performed.

5.1 The General Motion Description

The general motion of the device in Cartesian space can be decomposed into motions along the $X$, $Y$ and $Z$ axes:

1. for motion along the $X$ axis, the two $X$-RotatingPlates, and $X$-TranslatingPlate onto which the $Y$-Actuator is installed are involved. Under actuation, both $X$-RotatingPlates rotate by an angle of $\theta_1$ about an axis parallel to the $Y$ axis, whereas $X$-TranslatingPlate translates along the $X$ axis.

2. for motion along the $Y$ axis, the two $Y$-RotatingPlates, and $Y$-TranslatingPlate onto which the $Z$-Actuator is installed are involved. Under actuation, two $Y$-RotatingPlates rotate about an axis parallel to the $X$ axis, by an angle of $\theta_2$. Meanwhile, $Y$-TranslatingPlate translates...
5.1 The General Motion Description

along the Y axis. Since Y-Plates are hinged onto the X-Plat, motion along the Y axis is coupled with motion along the X axis.

3. for motion along the Z axis, two Z-RotatingPlates, and Z-TranslatingPlate to which a handle is attached are involved. Both Z-RotatingPlates are designed such that Z-TranslatingPlate translates along the Z axis when they rotate about an axis parallel to the Y axis by an angle of \( \theta_3 \). Since Z-Top and Z-Bottom are hinged onto Y-Plat, motion along the Z axis is coupled with motion along the Y axis. Hence, it is also coupled with motion along the X axis.

Five springs are installed in the device: two X-Springs with elastic constants \( k_{x1} \) and \( k_{x2} \) for motion along the X axis, two Y-Springs with elastic constants \( k_{y1} \) and \( k_{y2} \) for motion along the Y axis and one Z-Spring with elastic constant \( k_z \) for motion along the Z axis. These springs bring the moving plates to the center position if there is no actuating or hand force.

As mentioned in Chapter 2, the X-Springs and Y-Springs are installed with one end hooked onto a moving plate and the other end hooked onto a plate stationary with respect to that direction of motion. For motion along the Z axis, Z-Spring is hooked onto the two Z-RotatingPlates.

All plates are rigid, thus each of them is equivalent to a simple link. The equivalent link model for the motion control input device is illustrated in Figure 5.1, and Table 5.1 lists its equivalent link parameters.

<table>
<thead>
<tr>
<th>Plate</th>
<th>Equivalent Link Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mass</td>
</tr>
<tr>
<td>Front X-Plates(X-RotatingPlates)</td>
<td>( m_{x1} )</td>
</tr>
<tr>
<td>Rear X-Plates(X-RotatingPlates)</td>
<td>( m_{x2} )</td>
</tr>
<tr>
<td>X-Plat(X-TranslatingPlate)+Y-Actuator</td>
<td>( m_{x3} )</td>
</tr>
<tr>
<td>Left Y-Plates(Y-RotatingPlates)</td>
<td>( m_{y1} )</td>
</tr>
<tr>
<td>Right Y-Plates(Y-RotatingPlates)</td>
<td>( m_{y2} )</td>
</tr>
<tr>
<td>Y-Plat(Y-TranslatingPlate)+Z-Actuator</td>
<td>( m_{y3} )</td>
</tr>
<tr>
<td>Z-Bottom(Z-RotatingPlates)</td>
<td>( m_{z1} )</td>
</tr>
<tr>
<td>Z-Top(Z-RotatingPlates)</td>
<td>( m_{z2} )</td>
</tr>
<tr>
<td>Z-Side(Z-TranslatingPlate)+Handle</td>
<td>( m_{z3} )</td>
</tr>
</tbody>
</table>
Figure 5.1: The equivalent link model of the motion control input device.
5.2 The Center of Mass for Each Plate

The joint variables are chosen as:

\[ q = [\theta_1, \theta_2, \theta_3]^T. \]  \hspace{1cm} (5.1)

and

\[ \theta_k \in [-\theta_{k,\text{max}}, \theta_{k,\text{max}}], \quad k = 1, 2, 3, \]  \hspace{1cm} (5.2)

where \( \theta_{k,\text{max}} \) is determined by the maximum motion range of the translating plate and the length of each rotating plate.

According to the specification, the maximum motion range of the translating plate along one axis, \( d_{\text{max}} \), is specified to be 25.4mm. If the lengths of X-RotatingPlates, Y-RotatingPlates and Z-RotatingPlates are \( l_x \), \( l_y \) and \( l_z \) respectively, then \( \theta_{1,\text{max}}, \theta_{2,\text{max}} \) and \( \theta_{3,\text{max}} \) can be computed by:

\[
\begin{align*}
\theta_{1,\text{max}} &= \sin^{-1}\left(\frac{d_{\text{max}}}{l_x}\right) = \sin^{-1}\left(\frac{25.4}{l_x}\right) \\
\theta_{2,\text{max}} &= \sin^{-1}\left(\frac{d_{\text{max}}}{l_y}\right) = \sin^{-1}\left(\frac{25.4}{l_y}\right) \\
\theta_{3,\text{max}} &= \sin^{-1}\left(\frac{d_{\text{max}}}{l_z}\right) = \sin^{-1}\left(\frac{25.4}{l_z}\right)
\end{align*}
\]  \hspace{1cm} (5.3)

The results of the maximum rotating angles are summarized in Table 5.2.

<table>
<thead>
<tr>
<th>Joint Angle</th>
<th>Length [mm]</th>
<th>Maximum Value [rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_1 )</td>
<td>( l_x = 99.06 )</td>
<td>( \theta_{1,\text{max}} = 0.2593 )</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>( l_y = 117.39 )</td>
<td>( \theta_{2,\text{max}} = 0.2181 )</td>
</tr>
<tr>
<td>( \theta_3 )</td>
<td>( l_z = 64.5 )</td>
<td>( \theta_{3,\text{max}} = 0.4048 )</td>
</tr>
</tbody>
</table>

5.2 The Center of Mass for Each Plate

The input device motion can be decomposed into motion along three orthogonal axes. The equivalent single axis model is separated from the general link model to facilitate analysis.
1. The Equivalent X-axis Model

![Figure 5.2: The equivalent X-axis model.](image)

Using Figure 5.2, the COM positions for the equivalent X-axis model are computed as:

\[
\begin{align*}
\begin{bmatrix}
x_{cx1} \\
y_{cx1} \\
z_{cx1}
\end{bmatrix}
&= 
\begin{bmatrix}
(1 - \lambda_{x0})l_{x3} - \lambda_{x1}l_{x1}\sin\theta_1 \\
0 \\
l_x(1 - \lambda_{x1}\cos\theta_1)
\end{bmatrix}, & (5.4) \\
\begin{bmatrix}
x_{cx2} \\
y_{cx2} \\
z_{cx2}
\end{bmatrix}
&= 
\begin{bmatrix}
-\lambda_{x0}l_{x3} - \lambda_{x2}l_{x2}\sin\theta_1 \\
0 \\
l_x(1 - \lambda_{x2}\cos\theta_1)
\end{bmatrix}, & (5.5) \\
\begin{bmatrix}
x_{cx3} \\
y_{cx3} \\
z_{cx3}
\end{bmatrix}
&= 
\begin{bmatrix}
-l_x\sin\theta_1 \\
0 \\
l_x(1 - \cos\theta_1)
\end{bmatrix}. & (5.6)
\end{align*}
\]
2. The Equivalent Y-axis Model

Using Figure 5.3, the COM positions for the equivalent Y-axis model are computed as:

\[
\begin{bmatrix}
x_{cy1} \\
y_{cy1} \\
z_{cy1}
\end{bmatrix} =
\begin{bmatrix}
-l_x \sin \theta_1 \\
-a_y - \lambda_y l_y \sin \theta_2 \\
l_x (1 - \cos \theta_1) + \lambda_y l_y \cos \theta_2
\end{bmatrix},
\]

(5.7)

\[
\begin{bmatrix}
x_{cy2} \\
y_{cy2} \\
z_{cy2}
\end{bmatrix} =
\begin{bmatrix}
-l_x \sin \theta_1 \\
a_y - \lambda_y l_y \sin \theta_2 \\
l_x (1 - \cos \theta_1) + \lambda_y l_y \cos \theta_2
\end{bmatrix},
\]

(5.8)

\[
\begin{bmatrix}
x_{cy3} \\
y_{cy3} \\
z_{cy3}
\end{bmatrix} =
\begin{bmatrix}
-l_x \sin \theta_1 \\
-l_y \sin \theta_2 \\
l_x (1 - \cos \theta_1) + l_y \cos \theta_2 + (\lambda_y l_y - \lambda_y l_y) l_y
\end{bmatrix}
\]

(5.9)
3. The Equivalent Z-axis Model

Figure 5.4: The equivalent Z-axis model.
5.3 The Interface Kinematics

Using Figure 5.4, the COM positions for the equivalent Z-axis model are computed as:

\[
\begin{bmatrix}
x_{cz1} \\
y_{cz1} \\
z_{cz1}
\end{bmatrix} =
\begin{bmatrix}
-l_x \sin \theta_1 + \lambda_{z1} l_z \cos \theta_3 \\
-l_y \sin \theta_2 \\
l_x (1 - \cos \theta_1) + l_y \cos \theta_2 - \lambda_{z1} l_z \sin \theta_3 - \lambda_y l_y 3
\end{bmatrix}, \quad (5.10)
\]

\[
\begin{bmatrix}
x_{cz2} \\
y_{cz2} \\
z_{cz2}
\end{bmatrix} =
\begin{bmatrix}
-l_x \sin \theta_1 + \lambda_{z2} l_z \cos \theta_3 \\
-l_y \sin \theta_2 \\
l_x (1 - \cos \theta_1) + l_y \cos \theta_2 - \lambda_{z2} l_z \sin \theta_3 + (1 - \lambda_y) l_y 3
\end{bmatrix}, \quad (5.11)
\]

\[
\begin{bmatrix}
x_{cz3} \\
y_{cz3} \\
z_{cz3}
\end{bmatrix} =
\begin{bmatrix}
-l_x \sin \theta_1 + l_z h \cos \theta_3 \\
-l_y \sin \theta_2 \\
l_x (1 - \cos \theta_1) + l_y \cos \theta_2 - l_z h \sin \theta_3 - \lambda_y l_y 3 + \lambda_{z3} l_z 3
\end{bmatrix}, \quad (5.12)
\]

5.3 The Interface Kinematics

The kinematics relates joint variables and to end-effector variables. Forward kinematics maps joint positions \( q \) to end-effector positions in work space \( X \).

Let the end-effector position be:

\[ X = [p_x, p_y, p_z]^T. \quad (5.13) \]

From Figures 5.2, 5.3 and 5.4, the forward kinematics is:

\[
X =
\begin{bmatrix}
p_x \\
p_y \\
p_z
\end{bmatrix} =
\begin{bmatrix}
-l_x \sin \theta_1 + l_z h \cos \theta_3 \\
-l_y \sin \theta_2 \\
l_x (1 - \cos \theta_1) + l_y \cos \theta_2 - l_z h \sin \theta_3 - \lambda_y l_y 3 + l_z 3
\end{bmatrix}. \quad (5.14)
\]

Thus, the workspace of the haptic device can be computed by specifying the values for \( \theta_1, \theta_2 \) and \( \theta_3 \), and the result is plotted in Figure 5.5.
The velocity mapping from joint velocity $\dot{q}$ to end-effector velocity $\dot{X}$ and acceleration mapping from joint acceleration $\ddot{q}$ to end-effector acceleration $\ddot{X}$ are derived by computing the first and second time derivatives of Equation (5.14).

*Inverse kinematics* maps end-effector positions to joint positions. From Equation (5.14), end-effector position $X$ is mapped to joint space as:

$$
\begin{align*}
\theta_2 &= \arctan 2(-p_y, \sqrt{l_y^2 - p_y^2}) & \in [-0.2181, 0.2181]\text{rad}, \\
\theta_1 &= \arcsin\left(\frac{p_x^2 + l_x^2 - z_1^2}{2l_x \sqrt{l_x^2 + z_1^2}}\right) - \arctan 2(z_1, p_x) & \in [-0.2593, 0.2593]\text{rad}, \\
\theta_3 &= \arcsin\left(\frac{l_x^2 + l_z^2 + z_1^2 - l_x^2}{2l_x \sqrt{l_x^2 + z_1^2}}\right) - \arctan 2(p_x, z_1) & \in [-0.4048, 0.4048]\text{rad},
\end{align*}
$$

(5.15)

where $z_1 = l_x + l_y \cos \theta_2 - \lambda y l_y 3 + l_z - p_z$ after $\theta_2$ is solved first.
5.4 The Potential Energy

The potential energy has two components: (i) gravitational potential energy, and (ii) elastic potential energy.

1. The gravitational potential energy for a plate with mass \( m_i \) is:

\[
V_{g,mi} = -m_i g^T r_{cog,mi},
\]

where \( g = [0, 0, -g]^T \) is the acceleration due to gravity, and \( r_{cog,mi} \) is the displacement from the center of mass \( C_{mi} \) to the corresponding pivot point.

According to Equation (5.16), the gravitational potential energy of the device can be obtained by (i) computing the gravitational potential energies of the X-Parallelogram, \( V_{g,x} \), the Y-Parallelogram, \( V_{g,y} \), and Z-Parallelogram, \( V_{g,z} \), and (ii) summing these results into \( V_g \):

\[
V_g = V_{g,x} + V_{g,y} + V_{g,z},
\]

where:

\[
V_{g,x} = (m_{x1} \lambda_{x1} + m_{x2} \lambda_{x2} + m_{x3})gl_x(1 - \cos \theta_1),
\]

\[
V_{g,y} = -(m_{y1} \lambda_{y1} + m_{y2} \lambda_{y2} + m_{y3})gl_y(1 - \cos \theta_2),
\]

\[
V_{g,z} = (m_{z1} + m_{z2} + m_{z3})gl_z(1 - \cos \theta_1) - (m_{z1} + m_{z2} + m_{z3})gl_y(1 - \cos \theta_2)
\]

\[
-(m_{z1} \lambda_{z1} + m_{z2} \lambda_{z2} + m_{z3})g \sin \theta_3.
\]

Note that all the gravitational potential energies are computed with respect to the same reference point so that they can be summed together.
2. The elastic potential energy for a spring with constant \( k_i \) and deformation \( \delta_{ki} \) is:

\[
V_{e,ki} = \frac{1}{2} k_i \delta_{ki}^2,
\]  

(5.21)

where \( \delta_{ki} \) is the deformation of the spring.

The deformations for all the springs used in the haptic device are computed and summarized as follows:

- for the two X-Springs:

\[
\delta_{x1} = \sqrt{a_x^2 + b_x^2 + 2a_x b_x \sin \theta_1} - l_{sx0},
\]

(5.22)

\[
\delta_{x2} = \sqrt{a_x^2 + b_x^2 - 2a_x b_x \sin \theta_1} - l_{sx0},
\]

(5.23)

where \( l_{sx0} \) is the free length of the spring.

- for the two Y-Springs:

\[
\delta_{y1} = \sqrt{a_y^2 + b_y^2 + 2a_y b_y \sin \theta_2} - l_{sy0},
\]

(5.24)

\[
\delta_{y2} = \sqrt{a_y^2 + b_y^2 - 2a_y b_y \sin \theta_2} - l_{sy0},
\]

(5.25)

where \( l_{sy0} \) is the free length of the spring.

- for the Z-Spring:

\[
\delta_z = \sqrt{a_z^2 + b_z^2 + 2a_z b_z \sin \theta_3} - l_{sz0},
\]

(5.26)

where \( l_{sz0} \) is the free length of the spring.

Thus, the elastic potential energy of the device is computed as:

\[
V_e = V_{e,x} + V_{e,y} + V_{e,z},
\]

(5.27)
where:

\[ V_{e,x} = \frac{1}{2} k_x (\sqrt{a_x^2 + b_x^2} + 2a_x b_x \sin \theta_1 - l_{x0})^2 \]

\[ + \frac{1}{2} k_x (\sqrt{a_x^2 + b_x^2} - 2a_x b_x \sin \theta_1 - l_{x0})^2, \quad (5.28) \]

\[ V_{e,y} = \frac{1}{2} k_y (\sqrt{a_y^2 + b_y^2} + 2a_y b_y \sin \theta_2 - l_{y0})^2 \]

\[ + \frac{1}{2} k_y (\sqrt{a_y^2 + b_y^2} - 2a_y b_y \sin \theta_2 - l_{y0})^2, \quad (5.29) \]

\[ V_{e,z} = \frac{1}{2} k_z (\sqrt{a_z^2 + b_z^2} + 2a_z b_z \sin \theta_3 - l_{z0})^2. \quad (5.30) \]

Note that \( V_{e,x}, V_{e,y} \) and \( V_{e,z} \) are the elastic potential energies of the X-Parallelogram, Y-Parallelogram and Z-Parallelogram respectively.

\section*{5.5 The Kinetic Energy}

The kinetic energy for a plate with mass \( m_i \) and inertia \( I_i \) is:

\[ T_i = \frac{1}{2} m_i \dot{\mathbf{v}}_{cmi,o}^T \mathbf{v}_{cmi,o} + \frac{1}{2} \omega_{cmi,o}^T I_i \omega_{cmi,o}, \quad (5.31) \]

where \( \mathbf{v}_{cmi,o} \) is its COM linear velocity, and \( \omega_{cmi,o} \) is the plate angular velocity.

According to Equation (5.31), the X-Plates and X-Plat kinetic energies are:

\[ T_{x1} = \frac{1}{2} (m_{x1} \lambda_{x1}^2 l_x^2 + I_{x1}) \dot{\theta}_1^2, \quad (5.32) \]

\[ T_{x2} = \frac{1}{2} (m_{x2} \lambda_{x2}^2 l_x^2 + I_{x2}) \dot{\theta}_1^2, \quad (5.33) \]

\[ T_{x3} = \frac{1}{2} m_{x3} l_x^2 \dot{\theta}_1^2. \quad (5.34) \]
The Y-Plates and Y-Plat kinetic energies are:

$$
T_{y1} = \frac{1}{2} m_{y1} l_2^2 \dot{\theta}_1^2 + \frac{1}{2} (m_{y1} \lambda_{y1} l_3^2 + I_{y1}) \dot{\theta}_2^2 - m_{y1} l_2 \lambda_{y1} l_y \sin \theta_1 \sin \theta_2 \dot{\theta}_1 \dot{\theta}_2, \quad (5.35)
$$

$$
T_{y2} = \frac{1}{2} m_{y2} l_2^2 \dot{\theta}_1^2 + \frac{1}{2} (m_{y2} \lambda_{y2} l_3^2 + I_{y2}) \dot{\theta}_2^2 - m_{y2} l_2 \lambda_{y2} l_y \sin \theta_1 \sin \theta_2 \dot{\theta}_1 \dot{\theta}_2, \quad (5.36)
$$

$$
T_{y3} = \frac{1}{2} m_{y3} l_2^2 \dot{\theta}_1^2 + \frac{1}{2} m_{y3} l_2^2 \dot{\theta}_2^2 - m_{y3} l_3 \lambda_y l_y \sin \theta_1 \sin \theta_2 \dot{\theta}_1 \dot{\theta}_2. \quad (5.37)
$$

The Z-Top, Z-Bottom and Z-Side kinetic energies are:

$$
T_{z1} = \frac{1}{2} m_{z1} l_2^2 \dot{\theta}_1^2 + \frac{1}{2} m_{z1} l_2^2 \dot{\theta}_2^2 + \frac{1}{2} (m_{z1} \lambda_{z1} l_3^2 + I_{z1}) \dot{\theta}_3^2
- m_{z1} l_3 \lambda_{z1} l_y \sin \theta_1 \sin \theta_2 \dot{\theta}_1 \dot{\theta}_2 + m_{z1} l_3 \lambda_{z1} l_y \sin \theta_2 \cos \theta_3 \dot{\theta}_2 \dot{\theta}_3
- m_{z1} l_3 \lambda_{z1} l_z \sin (\theta_1 - \theta_3) \dot{\theta}_1 \dot{\theta}_3, \quad (5.38)
$$

$$
T_{z2} = \frac{1}{2} m_{z2} l_2^2 \dot{\theta}_1^2 + \frac{1}{2} m_{z2} l_2^2 \dot{\theta}_2^2 + \frac{1}{2} (m_{z2} \lambda_{z2} l_3^2 + I_{z2}) \dot{\theta}_3^2
- m_{z2} l_3 \lambda_{z2} l_y \sin \theta_1 \sin \theta_2 \dot{\theta}_1 \dot{\theta}_2 + m_{z2} l_3 \lambda_{z2} l_y \sin \theta_2 \cos \theta_3 \dot{\theta}_2 \dot{\theta}_3
- m_{z2} l_3 \lambda_{z2} l_z \sin (\theta_1 - \theta_3) \dot{\theta}_1 \dot{\theta}_3, \quad (5.39)
$$

$$
T_{z3} = \frac{1}{2} m_{z3} l_2^2 \dot{\theta}_1^2 + \frac{1}{2} m_{z3} l_2^2 \dot{\theta}_2^2 + \frac{1}{2} m_{z3} l_2^2 \dot{\theta}_3^2
- m_{z3} l_3 \lambda_{z3} l_y \sin \theta_1 \sin \theta_2 \dot{\theta}_1 \dot{\theta}_2 + m_{z3} l_3 \lambda_{z3} l_y \sin \theta_2 \cos \theta_3 \dot{\theta}_2 \dot{\theta}_3
- m_{z3} l_3 \lambda_{z3} l_z \sin (\theta_1 - \theta_3) \dot{\theta}_1 \dot{\theta}_3. \quad (5.40)
$$

The kinetic energy for the MCID is then given by:

$$
T = \sum_{i=1}^{3} T_{zi} + \sum_{i=1}^{3} T_{yi} + \sum_{i=1}^{3} T_{zi}. \quad (5.41)
$$
5.6 The Interface Dynamics

Set

\[ M_{11} = m_1 \lambda_1^2 l_x^2 + I_x + m_2 \lambda_2^2 l_x^2 + I_x + m_3 \lambda_3^2 \]
\[ M_{12} = (m_1 \lambda_1 + m_2 \lambda_2 + m_3)(l_x l_y) \]
\[ M_{13} = (m_1 \lambda_1 l_x + m_2 \lambda_2 l_x + m_3 l_x) \]
\[ M_{22} = m_1 \lambda_1^2 l_y^2 + I_y + m_2 \lambda_2^2 l_y^2 + I_y + m_3 \lambda_3^2 \]
\[ M_{23} = (m_1 \lambda_1 l_x + m_2 \lambda_2 l_x + m_3 l_x) \]
\[ M_{33} = m_1 \lambda_1^2 l_z^2 + I_z + m_2 \lambda_2^2 l_z^2 + I_z + m_3 \lambda_3^2 \]
\[ G_1 = (m_1 \lambda_1 + m_2 \lambda_2 + m_3 + m_4 + m_5 + m_6 + m_7 + m_8) l_x g \]
\[ G_2 = (m_1 \lambda_1 + m_2 \lambda_2 + m_3 + m_4 + m_5 + m_6 + m_7 + m_8) l_y g \]
\[ G_3 = (m_1 \lambda_1 l_x + m_2 \lambda_2 l_x + m_3 l_x) g \]
\[ p_1 = a_x^2 + b_x^2, \quad q_1 = a_x b_x \]
\[ p_2 = a_y^2 + b_y^2, \quad q_2 = a_y b_y \]
\[ p_3 = a_z^2 + b_z^2, \quad q_3 = a_z b_z \]
\[ k_x = k_x \]
\[ k_y = k_y \]

The Lagrangian of the haptic device, \( L \), is defined as [33, 37]:

\[ L = T - (V_g + V_e) \]

and the Lagrangian equation is expressed by:

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_{k}} - \frac{\partial L}{\partial \theta_{k}} = \tau_{k}, \quad k = 1, 2, 3. \]
Thus, the joint space dynamic model of the device is derived as:

\[ D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + S(q) = \tau, \]  

(5.57)

where:

1. the *inertia matrix* \( D(q) \) is:

\[
D(q) = \begin{bmatrix}
    M_{11} & -M_{12}\sin\theta_1\sin\theta_2 & -M_{13}\sin(\theta_1 - \theta_3) \\
    -M_{12}\sin\theta_1\sin\theta_2 & M_{22} & M_{23}\sin\theta_2\cos\theta_3 \\
    -M_{13}\sin(\theta_1 - \theta_3) & M_{23}\sin\theta_2\cos\theta_3 & M_{33}
\end{bmatrix},
\]

(5.58)

and \( D(q) \) is a symmetrical 3-by-3 matrix;

2. the *centrifugal and coriolis terms* are:

\[
C(q, \dot{q}) = \begin{bmatrix}
    0 & -M_{12}\sin\theta_1\cos\theta_2\dot{\theta}_2 & M_{13}\cos(\theta_1 - \theta_3)\dot{\theta}_3 \\
    -M_{12}\cos\theta_1\sin\theta_2\dot{\theta}_1 & 0 & -M_{23}\sin\theta_2\sin\theta_3\dot{\theta}_3 \\
    -M_{13}\cos(\theta_1 - \theta_3)\dot{\theta}_1 & M_{23}\sin\theta_2\cos\theta_3\dot{\theta}_2 & 0
\end{bmatrix},
\]

(5.59)

and \( D(q) - 2C(q, \dot{q}) \) is a skew-symmetrical 3-by-3 matrix;

3. the *gravitational force vector* is:

\[
G(q) = \begin{bmatrix}
    G_1\sin\theta_1 \\
    -G_2\sin\theta_2 \\
    -G_3\cos\theta_3
\end{bmatrix},
\]

(5.60)
(4) the spring force vector is:

\[
S(q) = \begin{bmatrix}
    k_xq_1 \left( \frac{\sqrt{p_1+2q_1 \sin \theta_1 - l_{x0}}}{\sqrt{p_1+2q_1 \sin \theta_1}} - \frac{\sqrt{p_1-2q_1 \sin \theta_1 - l_{x0}}}{\sqrt{p_1-2q_1 \sin \theta_1}} \right) \cos \theta_1 \\
    k_yq_2 \left( \frac{\sqrt{p_2+2q_2 \sin \theta_2 - l_{y0}}}{\sqrt{p_2+2q_2 \sin \theta_2}} - \frac{\sqrt{p_2-2q_2 \sin \theta_2 - l_{y0}}}{\sqrt{p_2-2q_2 \sin \theta_2}} \right) \cos \theta_2 \\
    k_zq_3 \left( \frac{\sqrt{p_3+2q_3 \sin \theta_3 - l_{z0}}}{\sqrt{p_3+2q_3 \sin \theta_3}} - \frac{\sqrt{p_3-2q_3 \sin \theta_3 - l_{z0}}}{\sqrt{p_3-2q_3 \sin \theta_3}} \right) \cos \theta_3
\end{bmatrix}
\]  

(5.61)

(5) the joint torques are:

\[
\tau(q) = \tau_a - J^T(q)F_e,
\]

(5.62)

where \(J^T(q)\) is the transposed velocity Jacobian. The actuating torque vector, \(\tau_a\), is defined as:

\[
\tau_a = [\tau_{a1}, \tau_{a2}, \tau_{a3}]^T,
\]

(5.63)

and the contact force vector, \(F_e\), is defined as:

\[
F_e = [f_{ex}, f_{ey}, f_{ez}]^T.
\]

(5.64)

5.7 The Velocity and Actuation Jacobians

Two Jacobian matrices are required for the kinematics and dynamics of the haptic device: (i) the velocity Jacobian \(J(q)\), and (ii) the actuation Jacobian \(J_a(q)\). The velocity Jacobian maps joint velocities \(\dot{q}\) to end-effector velocities \(\dot{X}\). Since the contact force, \(F_e\), is acting upon the end-effector, the velocity Jacobian also maps the work space contact force \(F_e\) to joint torques.

The actuation Jacobian maps the work space actuating force, \(F_a\), to actuating joint torques \(\tau_a\). Since the actuator's slider has been screwed onto one translating plate such as X-Plat, Y-Plat, and Z-Side, the actuation Jacobian is derived based on the COM velocities of these plates.
5.7 The Velocity and Actuation Jacobians

5.7.1 The Velocity Jacobian $J(q)$

The end-effector velocity $\dot{X}$:

$$\dot{X} = [v_x, v_y, v_z]^T,$$

is derived by taking the first time derivative of Equation (5.14):

$$\dot{X} = J(q)\dot{q},$$

where $J(q)$, the velocity Jacobian, is given by:

$$J(q) = \begin{bmatrix}
-l_x \cos \theta_1 & 0 & -l_{zh} \sin \theta_3 \\
0 & -l_y \cos \theta_2 & 0 \\
l_x \sin \theta_1 & -l_y \sin \theta_2 & -l_{zh} \cos \theta_3
\end{bmatrix}.$$  \hfill (5.67)

The determinant of the velocity Jacobian, $J(q)$, is:

$$det[J(q)] = -l_x l_y l_{zh} \cos(\theta_1 - \theta_3) \cos \theta_2.$$  \hfill (5.68)

Due to the effective ranges of joint variables, $det[J(q)]$ is never zero. Thus, the velocity Jacobian $J(q)$ is never singular and its inverse $J^{-1}(q)$ always exists.

5.7.2 The Actuation Jacobian $J_a(q)$

The velocities $\dot{X}_{xz3}$ of the X-Plat, $\dot{X}_{yz3}$ of the Y-Plate, and $\dot{X}_{cz3}$ of the Z-Side are derived by computing the first time derivatives of Equations (5.6), (5.9) and (5.12), respectively:
5.7 The Velocity and Actuation Jacobians

\[
\dot{\mathbf{X}}_{x3} = \begin{bmatrix}
-l_x \cos \theta_1 & 0 & 0 \\
0 & 0 & 0 \\
l_x \sin \theta_1 & 0 & 0
\end{bmatrix} \dot{q},
\] (5.69)

\[
\dot{\mathbf{X}}_{y3} = \begin{bmatrix}
-l_x \cos \theta_1 & 0 & 0 \\
0 & -l_y \cos \theta_2 & 0 \\
l_x \sin \theta_1 & -l_y \sin \theta_2 & 0
\end{bmatrix} \dot{q},
\] (5.70)

\[
\dot{\mathbf{X}}_{z3} = \begin{bmatrix}
-l_x \cos \theta_1 & 0 & -l_z h \sin \theta_3 \\
0 & -l_y \cos \theta_2 & 0 \\
l_x \sin \theta_1 & -l_y \sin \theta_2 & -l_z h \cos \theta_3
\end{bmatrix} \dot{q},
\] (5.71)

Thus, \( \tau_{a1}, \tau_{a2}, \tau_{a3} \), the joint torques generated by the X-Actuator, Y-Actuator and Z-Actuator respectively, are computed as:

\[
\tau_{a1} = \begin{bmatrix}
-l_x \cos \theta_1 f_{a,x} \\
0 \\
0
\end{bmatrix},
\] (5.72)

\[
\tau_{a2} = \begin{bmatrix}
0 \\
-l_y \cos \theta_2 f_{a,y} \\
0
\end{bmatrix},
\] (5.73)

\[
\tau_{a3} = \begin{bmatrix}
l_x \sin \theta_1 f_{a,x} \\
-l_y \sin \theta_2 f_{a,y} \\
-l_z h \cos \theta_3 f_{a,z}
\end{bmatrix},
\] (5.74)
and the joint actuating torques are summed into:

\[ \tau_a = \sum_{i=1}^{3} \tau_{ai} = J_a^T(q) F_a, \tag{5.75} \]

where \( F_a = [f_{ax}, f_{ay}, f_{az}]^T \) is the actuating force and \( J_a(q) \), the actuation Jacobian, is given by:

\[
J_a(q) = \begin{bmatrix}
-l_x \cos \theta_1 & 0 & 0 \\
0 & -l_y \cos \theta_2 & 0 \\
l_x \sin \theta_1 & -l_y \sin \theta_2 & -l_z h \cos \theta_3
\end{bmatrix}. \tag{5.76}
\]

The determinant of the actuation Jacobian, \( J_a(q) \), is then:

\[ \text{det}[J_a(q)] = -l_x l_y l_z h \cos \theta_1 \cos \theta_2 \cos \theta_3. \tag{5.77} \]

Due to the effective ranges of joint variables, \( \text{det}[J_a(q)] \) is never zero. Thus, the actuation Jacobian \( J_a(q) \) is never singular and its inverse \( J_a^{-1}(q) \) always exists.

Inserting Equations (5.67) and (5.76) into the dynamic model (5.57), the dynamics of the motion control input device become:

\[ D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + S(q) = J_a^T(q) F_a - J_a^T(q) F_e. \tag{5.78} \]

5.8 Dynamic Model Simulations

In this section, simulations of the dynamic model are performed. The relationship between the specified joint positions and resultant actuating forces, and the relationship between the applied actuating forces and resultant joint positions and end-effector positions are illustrated.
5.8 Dynamic Model Simulations

5.8.1 Parameter Measurements and Calculations

The masses and lengths of the main plates including X-RotatingPlates, X-TranslatingPlate, Y-RotatingPlates, Y-TranslatingPlate, Z-RotatingPlates, Z-TranslatingPlate and the actuators, were measured. The inertia and the COM position for each rotating plate (X-RotatingPlates, Y-RotatingPlates, Z-RotatingPlates) were computed based on design dimensions. The results are listed in Table 5.3,

<table>
<thead>
<tr>
<th>Plate</th>
<th>Mass [kg]</th>
<th>Inertia ×10⁻³[Nm]</th>
<th>Length [m]</th>
<th>COM</th>
<th>Other [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front X-Plates</td>
<td>m₁ = 0.1062</td>
<td>I₁ = 0.3977</td>
<td>l₁ = 0.09906</td>
<td>λ₁ = 0.5</td>
<td></td>
</tr>
<tr>
<td>Rear X-Plates</td>
<td>m₂ = 0.1062</td>
<td>I₂ = 0.3977</td>
<td></td>
<td>λ₂ = 0.5</td>
<td></td>
</tr>
<tr>
<td>X-Plat+Y-Actuator</td>
<td>m₃ = 0.3195</td>
<td>I₃ = 0.1158</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Left Y-Plates</td>
<td>m₄ = 0.1375</td>
<td>I₄ = 0.2666</td>
<td>l₄ = 0.11739</td>
<td>λ₄ = 0.4068</td>
<td></td>
</tr>
<tr>
<td>Right Y-Plates</td>
<td>m₅ = 0.1375</td>
<td>I₅ = 0.7266</td>
<td></td>
<td>λ₅ = 0.4068</td>
<td></td>
</tr>
<tr>
<td>Y-Plat+Z-Actuator</td>
<td>m₆ = 0.2968</td>
<td>I₆ = 0.09357</td>
<td></td>
<td>λ₆ = 0.48</td>
<td></td>
</tr>
<tr>
<td>Z-Bottom</td>
<td>m₇ = 0.1285</td>
<td>I₇ = 0.6442</td>
<td>l₇ = 0.1176</td>
<td>λ₇ = 0.5</td>
<td></td>
</tr>
<tr>
<td>Z-Top</td>
<td>m₈ = 0.0767</td>
<td>I₈ = 0.2552</td>
<td></td>
<td>λ₈ = 0.3684</td>
<td></td>
</tr>
<tr>
<td>Z-Side+Handle</td>
<td>m₉ = 0.2348</td>
<td>I₉ = 0.19357</td>
<td>l₉ = 0.19357</td>
<td>λ₉ = 0.48</td>
<td></td>
</tr>
</tbody>
</table>

The elastic constants of X-Springs, Y-Springs and Z-Spring were tested by a series of experiments (See Appendix B) and the final results are summarized in Table 5.4.

<table>
<thead>
<tr>
<th>Spring</th>
<th>Elastic Constant [N/m]</th>
<th>Undeformed Length [m]</th>
<th>Hooking Positions [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-Springs</td>
<td>kₓ = 867</td>
<td>lₓ₀ = 0.0138</td>
<td>aₓ = 0.0275, bₓ = 0.0242</td>
</tr>
<tr>
<td>Y-Springs</td>
<td>kᵧ = 2011</td>
<td>lᵧ₀ = 0.0280</td>
<td>aᵧ = 0.0686, bᵧ = 0.0220</td>
</tr>
<tr>
<td>Z-Spring</td>
<td>kₜ = 217</td>
<td>lₜ₀ = 0.0175</td>
<td>aₜ = 0.0588, bₜ = 0.0925</td>
</tr>
</tbody>
</table>

5.8.2 Inverse Dynamic Model Simulations

In the inverse dynamic model simulation, joint positions, velocities, accelerations are specified, and working space actuating forces are computed, assuming no contact. According to Equation
(5.78), the required actuating forces for a given joint trajectory ($\ddot{q}$, $\dot{q}$ and $q$) are:

$$F_a = J_a^{-T}(q)[D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + S(q)].$$

(5.79)

Three simulations were performed. In each of these, only one joint trajectory is specified and driven by a sinusoidal signal at a time, whereas the other two are set to zero.

1. Simulation by driving $\theta_1$ only

In this simulation, $\theta_2 = \theta_3 = 0$ and $\theta_1$ is specified as:

$$\theta_1 = 0.2593\sin(2\pi t), \quad t = 0, \ldots, 20s.$$  

The simulation results are shown in Figure 5.6.

![Figure 5.6: Simulation results of driving $\theta_1$ only.](image-url)
As Figure 5.6 shows, only actuating forces along the \( X \) and \( Z \) axes are required to achieve the given trajectory. No actuating force along the \( Y \) axis is required. Also, the force generated by the \( X \)-Actuator, \( f_{ax} \) is predominant, compared to the \( Z \)-Actuator force, \( f_{az} \).

2. Simulation by driving \( \theta_2 \) only

In this simulation, \( \theta_1 = \theta_3 = 0 \) and \( \theta_2 \) is specified as:

\[
\theta_2 = 0.2181\sin(2\pi t), \quad t = 0, \ldots, 20 \text{s}.
\]

The simulation results are shown in Figure 5.7.

As Figure 5.7 shows, actuating forces along both \( Y \) and \( Z \) axes are required to achieve the given trajectory. No actuating force along the \( X \) axis is required. Note that the force generated by the \( Y \) axis, \( f_{ay} \) is predominant, compared to the \( Z \)-Actuator force, \( f_{az} \).
3. Simulation by driving $\theta_3$ only

In this simulation, $\theta_1 = \theta_2 = 0$ and $\theta_3$ is specified as:

$$\theta_3 = 0.4048\sin(2\pi t), \quad t = 0, \ldots, 20s.$$ 

The simulation results are shown in Figure 5.8.

As Figure 5.8 shows, actuating forces along both $X$ and $Z$ axes are required to achieve the given trajectory. No actuating force along the $Y$ axis is required. Also, the force generated by the $Z$-Actuator, $f_{az}$, is predominant, compared to the $X$-Actuator force, $f_{ax}$. 
5.8.3 Forward Dynamic Model Simulations

In the forward dynamic model simulation, the actuating forces are specified, and joint positions or end-effector positions are computed, assuming no contact. According to Equation (5.78), for the given actuating force $F_a = [f_{ax}, f_{ay}, f_{az}]^T$, the joint accelerations are:

$$\ddot{q} = D^{-1}(q)[J_a^T(q)F_a - C(q, \dot{q})\dot{q} - G(q) - S(q)].$$  \hfill (5.80)

Then, the joint velocities $\dot{q}$ can be computed by:

$$\dot{q} = \int \ddot{q}(\tau) d\tau, \quad \text{initial } \dot{q}(0),$$  \hfill (5.81)

and the joint positions $q$ are:

$$q = \int \dot{q}(\tau) d\tau, \quad \text{initial } q(0).$$  \hfill (5.82)

Three simulations are performed. Only one actuating force is specified as a sinusoidal signal at a time, whereas the other two forces are zero. The initial joint position $q(0)$ and velocity $\dot{q}(0)$ are chosen to be zero.
1. Applying actuating force along the $X$ axis only

The actuating force generated by the X-Actuator $f_{ax}$ is chosen as a sinusoidal input. The simulation results are shown in Figure 5.9.

As Figure 5.9 shows, applying $f_{ax} \neq 0$ and $f_{ay} = f_{az} = 0$, motions along both $X$ and $Z$ axes are generated. This means that motion along the $X$ axis is coupled with motion along the $Z$ axis.
2. Applying actuating force along the $Y$ axis only

The actuating force generated by the Y-Actuator $f_{ay}$ is specified as a sinusoidal input. The simulation results are shown in Figure 5.10.

![Figure 5.10: Results of forward dynamics simulation for applying the actuating force along the Y axis only.](image)

As Figure 5.10 shows, a force along the $Y$ axis generates motions along all three axes. This means that motion along the $Y$ axis is coupled with motions along both the $X$ and $Z$ axes.
3. Applying actuating force along the Z axis only

The actuating force generated by the Z-Actuator, $f_{az}$ is specified as a sinusoidal input. The simulation results are shown in Figure 5.11.

As Figure 5.11 shows, a force applied along the Z axis generates motions along X and Z directions, but not with motion along the Y axis.

5.9 Summary

The derivation of the haptic device kinematics and dynamics have been presented in this chapter.
5.9 Summary

The joint variables have been chosen to be three angles: $\theta_1$, $\theta_2$, and $\theta_3$, formed by the rotation of the three parallelograms.

The dynamics have been computed using the Lagrangian formulation, after the potential and kinetic energies of all plates were derived. Two Jacobian matrices were derived in order to map work space actuating and contact forces into corresponding joint torques.

Inverse and forward dynamics simulations have been performed to illustrate the relationships between joint trajectories and work space actuating forces. Also, motion coupling issues have been investigated through these simulations.
Chapter 6

Conclusions and Recommendations

6.1 Conclusions

The work presented in this thesis describes the design of a new haptic device with the following main features:

- low component costs (see Appendix D for details),
- desktop dimensions - 160mm x 180mm x 260mm,
- 3 degree of freedom in Cartesian space,
- translation predominantly along each axis,
- large motion range of ±25.4mm along each axis, and
- force-feedback by actuators for all axes, but the force magnitude needs to be improved.

Although the device prototype has not yet been completed, component design, machining, installation and testing, as well as model derivation and simulation have been carried out individually. The thesis contributions and conclusions can be summarized as follows:

1. in the device and component design:
6.1 Conclusions

- A parallelogram linkage was employed as a basic mechanism for generating displacement predominantly along one axis in Cartesian space. Three parallelogram linkages have been designed and installed such that the X-Parallelogram forms a base on top of which the Y-Parallelogram sits; the Z-Parallelogram in turn, is mounted on top of the Y-Parallelogram. The device uses an arrangement of these orthogonal parallelograms to restrict displacements to three rectangular axes. However, the device motion is not decoupled.

- An arrangement of piano springs was used as a centering mechanism to bring all parallelograms to the device zero/center position. This position is also used for device calibration.

- A slot-and-tab structure was adopted to produce a machinable low friction joint. The width of the slot on one plate should be equal to that of the tab on the other plate, whereas the height of the slot is solely dependent upon the plate thickness and the maximum rotation angle.

- An assembly of the position sensor, grayscale and actuator was integrated for each axis.

2. in the position transducer:

- The HOA0149, an affordable and high performance microelectronic infrared reflective sensor, was selected for position sensing.

- A grayscale with varying reflectance was generated using Matlab™ for sensing by the HOA0149.

- Investigation of the HOA0149 characteristics for grayscale sensing, such as the repeatability and linearity, was performed.

- An OSC algorithm was presented to compensate for the sensor's nonlinearity. A flexible scaling factor was introduced in the algorithm to improve the convergence rate. The effectiveness of the algorithm was illustrated and compared by two examples.

3. in the linear actuator:
6.1 Conclusions

- An actuator consisting of a stator and a slider was designed to provide force feedback to the operator. Two rows of fourteen TRI-NEO-30 permanent magnets have been installed on the stator trunk, generating magnetic fields within two long and narrow air gaps. An N-turn coil is wound around the slider, which is placed along the center bar. A Lorentz force is generated and exerting upon the slider when a dc current is applied to the winding.

- An equivalent magnetic circuit model was designed to compute the field density along the length of the air gap.

- An experimental platform was built to validate the actuator design. Test results, which closely approximate the theoretical computations, show that the force is a linear function of the applied current and that the magnetic field along the length of the air gap is nearly uniform. Also, the level of actuating force suggests that the current actuator is suitable for generating a force feedback to a finger motion input device rather than a hand-held device.

- Further investigation on the effect of change of actuator dimensions has been performed and the corresponding result can be referred to Appendix E.

4. in the device kinematics and dynamics:

- The device forward and inverse kinematic equations were derived by separating motion along each axis in Cartesian space. The joint variables were chosen to be three rotational angles and the center of mass for each plate was computed.

- The device dynamic equations were derived using the Lagrangian formulation, after the potential and kinetic energies of all plates were computed. The velocity and actuation Jacobian matrices were derived in order to map work space actuating and contact forces into corresponding joint torques.

- Simulations on the device inverse and forward dynamic models were performed to illustrate the relationships between joint trajectories and actuating forces. Motion coupling issues were also investigated and illustrated by the simulations.
6.2 Recommendations for Future Work

The component design and validation have been completed in the thesis. However, there is still a lot of work to be done before a workable device prototype is achieved. Recommendations for possible future work are summarized according to (i) device mechanical design, (ii) position transducer and (iii) linear actuator.

1. Device Mechanical Design

- The weight of the haptic device should be reduced, while remaining rigidity and stiffness.
- The parallelogram linkages as well as slots machined on the main plates should be made by advanced cutting machines rather than by heavy millers, for improved precision.
- The slot-and-tab hinge structure could be further improved. The problem with the original design, shown in Figure 2.5, still lies in the friction. When plate $A$ is not rotating, the contact between plates $A$ and $B$ is a surface contact rather than an edge contact. In order to minimize the contact area, both edges of the slot have been sharpened. Thus, plates $A$ and $B$ have an edge contact all the time. Due to the intense labour and because the joint are easily worn out, a new design using pre-made plastic inserts (pieces $S_1$, $S_2$ and $S_3$) is presented and depicted in Figure 6.1.

![Figure 6.1: A new slot-and-tab design schematic.](image)
• In order to give the operator a feeling of the interface zero position, a detent mechanism should be added to the mechanical design. A possible design schematic, based on the fact that a mild steel plate is attracted by a permanent magnet, is depicted in Figure 6.2. The distance between the steel plate and magnet, \( d \), can be adjusted to achieve a desirable detent effect.

Figure 6.2: The detent mechanism schematic.

• The installation of the sensor, grayscale and actuator is still labour intensive, for example, the slider should be aligned without touching the center bar or the permanent magnets. The current installation distance between the slider and the center bar is specified to be 0.1in. A better assembly design and alignment technique is required.

2. Position Transducer Design and Compensation

• The current sensor testing setup only provides a motion range up to 26.7mm. However, the actual range for the end-effector to move along one axis is specified as \( \pm25.4 \)mm. Thus, this testing setup needs to be expanded to at least accommodate that maximum motion range.

• The motion of the translating plate in the parallelogram linkage is illustrated in Figure 2.1(a). It has two orthogonal components, one is parallel \((d_p)\) and the other perpendicular \((d_c)\) to the fixed side. Although the motion along the perpendicular direction is small, compensation of the sensing in that direction should be also performed. A new
6.2 Recommendations for Future Work

sensor compensation setup that exactly simulate the motion of the translating plate is required, and the OSC algorithm may be modified accordingly.

- Several preliminary tests on the HOA0149 sensor have been performed in the thesis. The size of the sensor infrared light spot on the grayscale has been roughly tested. In order to improve the sensing resolution, special optical instruments are required in the future to precisely determine this size.

- During the sensing, the grayscale is fixed while the HOA0149 sensor is moving with the translating plate along one axis in the Cartesian space. No experiment on how fast the sensor would respond to a reflectance change on the grayscale has been carried out. This is of great importance if the position information sensed by the sensor is used for estimating the motion velocity.

3. Linear Actuator Design

- The current actuator is still too weak, even though the magnetic field density along the air gap has been increased nearly six times over that of the previous design. In order to generate enough force-feedback to the human hand, the actuating force needs to be increased. Since the Lorentz force is computed by \( F_a = NIBgLeff \), there are four parameters which would affect the force magnitude:

  (a) the turns of the coil winding, \( N \), determined by the width of the air gap and the thickness of the wire;

  (b) the applied dc current, \( I \), limited by the maximum current without overheating, for the current wire (AWG33), the maximum current is less than 250mA;

  (c) the magnetic field density along the air gap, \( Bg \), determined by the layout and dimension of the actuator as well as the choice of permanent magnets;

  (d) the effective length of the coil winding immersed in the magnetic field, \( Leff \), is dependent upon the dimensions of the permanent magnet and of the actuator.

Compared to the previous maglev-based haptic devices [5,13,21,29,32], \( Bg \) in our actuator design is almost the same. Thus, a possible solution to increase the actuating
force would be (i) selecting an appropriate wire which can afford a high current $I$, (ii) efficiently packing the wire to achieve a high value of $L_{eff}$.

- We have not got the best materials for the actuator design. Thus a permanent magnet with a high $B_{max}H_{max}$ value, and a soft iron with a high permeability value are still required to generate a high flux density $B_g$ along the air gap.

- The dimensions of the stator and slider play an important role in the actuator design. Further investigations on the selection and optimization on the configuration of the stator components are required.

In this project, component design and testing have been completed. The next step of the project would be: (i) the improvement of the actuator design. As mentioned previously, there are a few ways to modify the current design. For example, a type of flat wire giving a high packing ratio and conducting a high current could be selected for the slider’s coil winding; (ii) the installation of the position transducers into the input device to verify end-effector position sensing and the nonlinearity compensation of the HOA0149 sensor.
Bibliography


[25] Nobuto Matsuhira, Makoto Asakura, Hiroyuki Bamba, and Michihiro Uenohara. Development of an Advanced Master-Slave Manipulator Using a Pantograph Master Arm and a Redundant


Appendix A

The Actuator Design Computations

Kirchhoff’s law is applied at each labeled node: 1, 3, ..., 13, 29, 30, ..., 35, 43, 44, ..., 50, 59, 60, ..., 65, 73, 75, ..., 85 in Figure 4.10. The fluxes flowing into and out of the node are summed to zero and the following equations are obtained:

1. At node 1:

\[ \frac{F_1 - F_{43}}{R_{i1}} + \phi_{r1} + \frac{F_1 - F_{29}}{R_{m1}} + \frac{F_1 - F_3}{R_{i2}} = 0. \]  \hspace{1cm} (A-1)

2. At nodes (3, 5, 7, 9, 11):

\[ \frac{F_{2k+1} - F_{2k-1}}{R_{i,k+1}} + \phi_{r,k+1} + \frac{F_{2k+1} - F_{k+29}}{R_{m,k+1}} + \frac{F_{2k+1} - F_{2k+3}}{R_{i,k+2}} = 0, \quad k = 1, \ldots, 5. \]  \hspace{1cm} (A-2)

3. At node 13:

\[ \frac{F_{13} - F_{11}}{R_{i7}} + \phi_{r7} + \frac{F_{13} - F_{35}}{R_{m7}} = 0. \]  \hspace{1cm} (A-3)

4. At nodes (29, 30, 31, 32, 33, 34, 35):

\[ \phi_{r,k} + \frac{F_{2k-1} - F_{k+28}}{R_{m,k}} + \frac{F_{k+43} - F_{k+28}}{R_{g,k}} = 0, \quad k = 1, \ldots, 7. \]  \hspace{1cm} (A-4)
5. At node 43:
\[ \frac{F_1 - F_{43}}{R_{i1}} + \frac{F_{44} - F_{43}}{R_{i31}} + \frac{F_{73} - F_{43}}{R_{i30}} = 0. \] \hspace{1cm} (A-5)

6. At nodes (44, 45, 46, 47, 48, 49):
\[ \frac{F_{k+42} - F_{k+43}}{R_{i,k+30}} + \frac{F_{k+28} - F_{k+43}}{R_{g,k}} + \frac{F_{k+44} - F_{k+43}}{R_{i,k+31}} + \frac{F_{k+58} - F_{k+43}}{R_{g,29-k}} = 0, \quad k = 1, \ldots, 6. \] \hspace{1cm} (A-6)

7. At node 50:
\[ \frac{F_{49} - F_{50}}{R_{i37}} + \frac{F_{35} - F_{50}}{R_{g7}} + \frac{F_{65} - F_{50}}{R_{g22}} = 0. \] \hspace{1cm} (A-7)

8. At nodes (59, 60, 61, 62, 63, 64, 65):
\[ \frac{F_{k+43} - F_{k+58}}{R_{g,29-k}} + \phi_{r,29-k} + \frac{F_{2k+71} - F_{k+58}}{R_{m,29-k}} = 0, \quad k = 1, \ldots, 7. \] \hspace{1cm} (A-8)

9. At node 73:
\[ \frac{F_{73} - F_{43}}{R_{i30}} + \phi_{r28} + \frac{F_{73} - F_{59}}{R_{m28}} + \frac{F_{73} - F_{75}}{R_{i29}} = 0. \] \hspace{1cm} (A-9)

10. At nodes (75, 77, 79, 81, 83):
\[ \frac{F_{2k+73} - F_{2k+71}}{R_{i,30-k}} + \phi_{r,28-k} + \frac{F_{2k+73} - F_{k+59}}{R_{m,28-k}} + \frac{F_{2k+73} - F_{2k+75}}{R_{i,29-k}} = 0, \quad k = 1, \ldots, 5. \] \hspace{1cm} (A-10)

11. At node 85:
\[ \frac{F_{85} - F_{83}}{R_{i24}} + \phi_{r22} + \frac{F_{85} - F_{65}}{R_{m22}} = 0. \] \hspace{1cm} (A-11)

Among these equations, only $F_1, \ldots, F_{13}, F_{29}, \ldots, F_{35}, F_{43}, \ldots, F_{50}, F_{73}, \ldots, F_{85}$ are unknown mmfs, while the permanent magnetic flux and reluctances can be computed using the magnet, air
gap and soft iron dimensions listed in Table A-1.

Table A-1: Parameters for the actuator design computations.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{r,k}$</td>
<td>the flux of each permanent magnet</td>
<td>$59.3547 \times 10^{-6} \text{[Wb]}$</td>
</tr>
<tr>
<td>$R_{m,k}$</td>
<td>the reluctance of each magnet</td>
<td>$34.4144 \times 10^6 \text{[A/Wb]}$</td>
</tr>
<tr>
<td>$R_{g,k}$</td>
<td>the reluctance of each air gap segment</td>
<td>$38.5454 \times 10^6 \text{[A/Wb]}$</td>
</tr>
<tr>
<td>$R_0$</td>
<td>the reluctance of the soft iron segment</td>
<td>$0.01156 \times 10^6 \text{[A/Wb]}$</td>
</tr>
<tr>
<td>$R_{i,1} = R_{i,30}$</td>
<td>the reluctance of the iron segment</td>
<td>$0.06963 \times 10^6 \text{[A/Wb]}$</td>
</tr>
<tr>
<td>$R_{i,k}$</td>
<td>the reluctance of the iron segment</td>
<td>$0.02723 \times 10^6 \text{[A/Wb]}$</td>
</tr>
<tr>
<td>$R_{i,k}$</td>
<td>the reluctance of the iron segment</td>
<td>$0.02723 \times 10^6 \text{[A/Wb]}$</td>
</tr>
<tr>
<td>$R_{i,k}$</td>
<td>the reluctance of the iron segment</td>
<td>$0.01940 \times 10^6 \text{[A/Wb]}$</td>
</tr>
</tbody>
</table>

Therefore, equations (A-1), (A-2), ..., and (A-11) can be lumped into:

$$\mathbf{PF} = \Phi.$$  \hspace{1cm} (A-12)

where $\mathbf{P}$ is a 36-by-36 coefficient matrix of inverses of reluctances, $\Phi$ is a 36-by-1 vector of magnetic fluxes, and $\mathbf{F}$ is a 36-by-1 vector of unknown mmfs.

Then, $\mathbf{F}$ results as:

$$\mathbf{F} = \mathbf{P}^{-1}\Phi.$$  \hspace{1cm} (A-13)

The magnetic flux for each air gap segment shown in Figure ?? can be computed by:

$$\phi_{g,k} = \begin{cases}  
\frac{F_{28+k} - F_{23+k}}{R_{g,k} + R_0}, & k = 1, \ldots, 7, \\
\frac{F_{27-k} - F_{22-k}}{R_{g,21+k} + R_0}, & k = 22, \ldots, 28, 
\end{cases}$$ \hspace{1cm} (A-14)

and the corresponding flux density is:

$$B_{g,k} = \frac{\phi_{g,k}}{A_g}, \quad k = 1, \ldots, 7 \text{ and } k = 22, \ldots, 28.$$ \hspace{1cm} (A-15)
Appendix B

Experimentally Determined Spring Constants

In order to simulate device motion in Cartesian space, the spring constants for the X-Springs $k_x$, Y-Springs $k_y$ and Z-Spring $k_z$ need be determined. A schematic of a simple experimental setup is illustrated in Figure B.1.

![Diagram of spring constant experiment setup](Image)

Figure B.1: The spring constant experiment setup.

The spring that is to be tested, is hooked such that one end is attached to a fixed wooden block while the other end is attached to a known weight over an almost frictionless pulley. By changing
the weight (force), the spring length changes. The slope of the force-deformation characteristic is the spring constant.

1. Spring constant for X-Springs $k_x$

Two experiments were performed to determine the X-Springs' constants. The experimental force-deformation dependency is plotted in Figure B.2. Matlab's `polyfit` command is used to fit the experimental data and to compute the slope.

![Graph](image)

(a) Test 1: Force versus X-Spring Deformation. (b) Test 2: Force versus X-Spring Deformation.

Figure B.2: Experimental result for the X-Spring constant.

The slope of the line in Figure B.2(a) is $432.2 \text{N/m}$, whereas the slope of the line in Figure B.2(b) is $434.8 \text{N/m}$. Thus, the average value of them is $433.5 \text{N/m}$.

2. Spring constant for Y-Springs $k_y$

Two experiments were performed to determine the Y-Springs' constants. Figure B.3 shows the measured force-deformation relationship.

The slope is $997.6 \text{N/m}$ in Figure B.3(a) and $1013.4 \text{N/m}$ in Figure B.3(b). Thus, the average value of them is $1005.5 \text{N/m}$.
3. Spring constant for Z-Springs $k_z$

Two experiments were performed to determine the Z-Springs' constants. The measured force-deformation relationship is plotted in Figure B.4.

(a) Test 1: Force versus Z-Spring Deformation.  
(b) Test 2: Force versus Z-Spring Deformation.

Figure B.4: Experimental result for the Z-Spring constant.
The slope is 108.2N/m in Figure B.4(a) and 108.8N/m in Figure B.4(b). Thus, the average of them is 108.5N/m.

According to the mechanical design, each X-Spring, Y-Spring and Z-Spring consists of two identical springs in parallel. Thus, the resulting spring constants are double the values obtained in the experiments:

\[
\begin{align*}
  k_x &= 433.5 \times 2 = 867 \text{ N/m}, \\
  k_y &= 1005.5 \times 2 = 2011 \text{ N/m}, \\
  k_z &= 108.5 \times 2 = 217 \text{ N/m}.
\end{align*}
\] (B-1)
Appendix C

The Installation of Actuators

Figure C.1 shows the X-, Y- and Z-Actuators installed onto the X-, Y- and Z-Parallelograms.

Figure C.1: The X-, Y- and Z-Parallelograms and X-, Y- and Z-Actuators.
Appendix D

The Component Cost Breakdown

The costs of components used in building this new haptic device are summarized in Table A-1.

Table A-1: The component cost breakdown.

<table>
<thead>
<tr>
<th>Item</th>
<th>Unit Price (C$)</th>
<th>Quantity</th>
<th>Cost (C$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel, aluminum sheets for motion plates, actuators, case, etc.</td>
<td>20.00</td>
<td></td>
<td>20.00</td>
</tr>
<tr>
<td>Piano springs for centering mechanism</td>
<td>1.00</td>
<td>7</td>
<td>7.00</td>
</tr>
<tr>
<td>Position sensors HOA0149</td>
<td>1.00</td>
<td>3</td>
<td>3.00</td>
</tr>
<tr>
<td>Permanent magnets NdFeB (27MGOe)</td>
<td>1.00</td>
<td>84</td>
<td>84.00</td>
</tr>
<tr>
<td>AWG33 Copper wire</td>
<td></td>
<td></td>
<td>3.00</td>
</tr>
<tr>
<td>Total cost:</td>
<td></td>
<td></td>
<td>117.00</td>
</tr>
</tbody>
</table>
Appendix E

The Effect of Change of Actuator Dimensions

The effect of change of actuator dimensions is investigated in this appendix. Particularly, the change of actuator dimension is specified as the change of the motion range of slider. According to actuator design, the motion range of slider is solely dependent upon the number of permanent magnets per row. This can be depicted in Figure E.1. For example, the slider motion range for a configuration of fourteen magnets per row is 71.12mm.

Figure E.1: The actuator and corresponding permanent magnet row.
Thus, referring to the flux source equivalent model discussed before, a simulation program has been developed in Matlab\textsuperscript{TM} for computing the magnetic field density along the air gap for various numbers of magnets per row. The resultant field density distribution for different numbers of magnets per row is shown in Figure E.2. Note that only one measurement is taken for each magnet and displacements are computed with respect to the actuator center line.

![Flux Density Distribution for 1 to 20 PMs per Row](image)

Figure E.2: Magnetic field density distribution along the air gap for different number of magnets per row.

As Figure E.2 shows, the maximum field density is achieved when there is only one permanent magnet per row. As the number of the magnets per row increases, the field density along the air gap decreases. The outer curve in Figure E.2 corresponds to a configuration of twenty permanent magnets per row.