

cl

THERMOPHORESIS IN SOLS

by

GORDON SPENCER McNAB

B.A.Sc., University of British Columbia, 1970

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF  
THE REQUIREMENTS FOR THE DEGREE OF  
MASTER OF APPLIED SCIENCE

in the Department  
of  
Chemical Engineering

We accept this thesis as conforming to the  
required standard

THE UNIVERSITY OF BRITISH COLUMBIA

August, 1972

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the Head of my Department or by his representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

---

Gordon S. McNab

Department of Chemical Engineering

The University of British Columbia  
Vancouver 8, Canada

Date

Sept 8/72

## Abstract

Experiments were conducted on the motion of micron-size, spherical latex particles in stagnant liquids due to applied temperature gradients. The phenomenon, called thermophoresis, has previously been reported for gases but not for liquids.

Dilute suspensions of particles in water or n-hexane were trapped between two horizontal, parallel disks. The top disk was heated whereas the lower one was cooled, thus creating a temperature gradient in the liquid. The particle motion was measured by observing light reflected from the spheres with a low power microscope.

The particle thermophoretic velocity,  $\underline{v}_{th}$ , was found to be a function of temperature gradient,  $\underline{\nabla}T$ , absolute temperature,  $T_K$ , particle thermal conductivity,  $k_p$ , fluid thermal conductivity,  $k_f$ , viscosity,  $\mu$ , and density,  $\rho$ :

$$\underline{v}_{th} = - 0.26 \frac{k_f}{2k_f + k_p} \frac{\mu}{\rho T_K} \underline{\nabla}T$$

This equation was tested in the ranges  $284^\circ K \leq T_K \leq 343^\circ K$  and  $7,000^\circ K m^{-1} \leq \underline{\nabla}T \leq 30,000^\circ K m^{-1}$ . No dependence on particle diameter was noted.

Thermophoresis in liquids is a weak effect and impractical for liquid-particle separation. It may, however, be important in engineering situations where large temperature gradients occur.

## Table of Contents

	Page
LIST OF TABLES . . . . .	vii
LIST OF FIGURES . . . . .	ix
 Chapter	
I. INTRODUCTION . . . . .	1
II. PREVIOUS WORK . . . . .	4
THERMOPHORESIS IN GASES . . . . .	4
The Small Particle Regime ( $Kn \gg 1$ ) . . . . .	5
The Large Particle Regime ( $Kn \ll 1$ ) . . . . .	6
THERMOPHORESIS IN LIQUIDS . . . . .	10
III. APPARATUS . . . . .	16
INTRODUCTION . . . . .	16
THE CELL . . . . .	17
The Optical Tube . . . . .	19
The Base . . . . .	19
The Bottom Disk . . . . .	21
The Spacer Ring . . . . .	21
The Top Disk . . . . .	24
THE HEATING OF THE TOP DISK . . . . .	24
The Electric Heating Coil . . . . .	24
Associated Electrical Apparatus . . . . .	26

	iv
Chapter	Page
THE COOLING OF THE BOTTOM DISK . . . . .	28
TEMPERATURE MEASUREMENT . . . . .	28
OPTICAL EQUIPMENT . . . . .	29
The Light Source . . . . .	29
The Microscope . . . . .	30
IV. PHYSICAL PROPERTIES OF THE EXPERIMENTAL MATERIALS . .	32
THE PARTICLES . . . . .	32
THE LIQUIDS . . . . .	32
Water . . . . .	33
Viscosity . . . . .	33
Density . . . . .	33
Thermal Conductivity . . . . .	34
Hexane . . . . .	23
Viscosity . . . . .	35
Density . . . . .	35
Thermal Conductivity . . . . .	35
V. EXPERIMENTAL METHOD . . . . .	43
PREPARATIONS FOR A RUN . . . . .	43
THE RUN . . . . .	45
VI. METHODS OF DATA ANALYSIS . . . . .	48
PRELIMINARY DATA TRANSFORMATIONS . . . . .	48
Disk Temperatures . . . . .	48

Chapter	Page
Top Disk . . . . .	48
Bottom Disk . . . . .	49
Local Temperature and Temperature Gradient . . .	50
Thermophoretic Velocity . . . . .	53
ANALYSIS OF TRANSFORMED DATA . . . . .	54
The Empirical Equation for $v_{th}$ . . . . .	54
Block Values of $\alpha$ . . . . .	55
Distribution of the Coefficients in Each Block . .	56
Modified t Test . . . . .	57
VII. RESULTS AND DISCUSSIONS . . . . .	59
THE EXISTENCE OF THERMOPHORESIS IN LIQUIDS . . . .	59
AN EMPIRICAL EQUATION FOR THE THERMOPHORETIC VELOCITY . . . . .	59
Analysis of the Thermophoretic Coefficient . . .	59
Summary of the Block $\bar{\alpha}$ 's . . . . .	59
Distribution of the $\alpha$ 's in Each Block . . . .	60
The Effect of Particle Diameter on $\alpha$ . . . .	61
The Effect of Liquid Thermal Conductivity on $\alpha$ . . . . .	63
The Final Empirical Equation for $v_{th}$ . . . .	65
THERMOPHORETIC FORCE . . . . .	66
THE EFFECT OF GRAVITY . . . . .	67
CELL WALL EFFECTS ON THE LOCAL TEMPERATURE AND TEMPERATURE GRADIENT . . . . .	67
A THEORETICAL MODEL OF THERMOPHORESIS IN LIQUIDS . .	68

Chapter	Page
VIII. CONCLUSIONS AND RECOMMENDATIONS . . . . .	81
CONCLUSIONS . . . . .	81
RECOMMENDATIONS . . . . .	82
NOMENCLATURE . . . . .	83
REFERENCES . . . . .	91
APPENDICES . . . . .	94
I. PROPERTIES OF WATER AND n-HEXANE . . . . .	95
II. EXPERIMENTAL DATA . . . . .	100
III. SAMPLE AND ERROR CALCULATIONS . . . . .	120
CALCULATION OF $\alpha$ AND ERROR ESTIMATION . . . . .	120
Water . . . . .	121
Hexane . . . . .	127
Observations . . . . .	131
THE UPPER VELOCITY LIMIT OF STOKES LAW . . . . .	131
THE TEMPERATURE DROP ACROSS EACH DISK . . . . .	132
THE TEMPERATURE RISE OF COOLING WATER IN PASSING THROUGH THE BOTTOM DISK . . . . .	133
THE EFFECT OF TEMPERATURE DEPENDENT $k_f$ ON $T_{PC}$ AND $dT/dx$ . . . . .	133
Water . . . . .	134
Hexane . . . . .	134
IV. THERMOPHORETIC FORCE AND VELOCITY FOR THE LARGE PARTICLE REGIME . . . . .	136

## List of Tables

Table	Page
1. Thermal Conductivities of the Apparatus Materials . . .	17
2. Statistical Data for Each Block . . . . .	60
3. Comparison of Experimental and Predicted $\bar{\alpha}$ 's . . . . .	64
4. The Thermal Conductivity of Water . . . . .	96
5. The Viscosity of Hexane . . . . .	97
6. The Density of Hexane . . . . .	98
7. The Thermal Conductivity of Hexane . . . . .	99
8. Code Letters for Blocks . . . . .	101
9. Experimental Data of Block A, $(T_T - T_B) = 23.1^\circ\text{C}$ . . . .	102
10. Experimental Data of Block A, $(T_T - T_B) = 32.7^\circ\text{C}$ . . . .	103
11. Experimental Data of Block A, $(T_T - T_B) = 39.7^\circ\text{C}$ . . . .	104
12. Experimental Data of Block A, $(T_T - T_B) = 49.2^\circ\text{C}$ . . . .	105
13. Experimental Data of Block A, $(T_T - T_B) = 50.2^\circ\text{C}$ . . . .	106
14. Experimental Data of Block A, $(T_T - T_B) = 55.7^\circ\text{C}$ . . . .	107
15. Experimental Data of Block A, $(T_T - T_B) = 57.3^\circ\text{C}$ . . . .	108
16. Experimental Data of Block A, $(T_T - T_B) = 69.0^\circ\text{C}$ . . . .	109
17. Experimental Data of Block A, $(T_T - T_B) = 78.0^\circ\text{C}$ . . . .	110
18. Experimental Data of Block A, $(T_T - T_B) = 81.7^\circ\text{C}$ . . . .	111
19. Experimental Data of Block B, $(T_T - T_B) = 37.7^\circ\text{C}$ . . . .	112
20. Experimental Data of Block B, $(T_T - T_B) = 39.5^\circ\text{C}$ . . . .	113
21. Experimental Data of Block B, $(T_T - T_B) = 44.9^\circ\text{C}$ . . . .	114



## Table

## Page

22. Experimental Data of Block C, $(T_T - T_B) = 34.6^\circ\text{C}$	115
23. Experimental Data of Block C, $(T_T - T_B) = 56.0^\circ\text{C}$	116
24. Experimental Data of Block C, $(T_T - T_B) = 68.9^\circ\text{C}$	117
25. Experimental Data of Block D, $(T_T - T_B) = 37.8^\circ\text{C}$	118
26. Experimental Data of Block D, $(T_T - T_B) = 54.9^\circ\text{C}$	119
27. Sample and Error Calculations--Water	123
28. Sample and Error Calculations--Hexane	128
29. The Upper Velocity Limit of Stokes Law	132

## List of Figures

Figure	Page
1. An Overall View of the Cell . . . . .	18
2. The Base . . . . .	20
3. The Bottom Disk . . . . .	22
4. The Spacer Ring . . . . .	23
5. The Top Disk . . . . .	25
6. The Electrical Apparatus . . . . .	27
7. The Microscope . . . . .	30
8. Viscosity of Water as a Function of Temperature . . . .	37
9. Density of Water as a Function of Temperature . . . .	38
10. Thermal Conductivity of Water as a Function of Temperature . . . . .	39
11. Viscosity of n-Hexane as a Function of Temperature . .	40
12. Density of n-Hexane as a Function of Temperature . . .	41
13. Thermal Conductivity of n-Hexane as a Function of Temperature . . . . .	42
14. The c.d.f. of Block A (Water, $d_p = 1.011$ microns) . . .	70
15. The c.d.f. of Block B (n-Hexane, $d_p = 1.011$ microns) . . . . .	71
16. The c.d.f. of Block C (Water, $d_p = 0.790$ microns) . . .	72
17. The c.d.f. of Block D (n-Hexane, $d_p = 0.790$ microns) . . . . .	73
18. The c.d.f. of Block E (all Water) . . . . .	74
19. The c.d.f. of Block F (all n-Hexane) . . . . .	75

Figure	Page
20. $\mu/\rho$ $T_{PK}$ as a Function of Temperature for Water and n-Hexane . . . . .	76
21. Thermophoretic Velocity in Water as a Function of Temperature Gradient . . . . .	77
22. Thermophoretic Velocity in n-Hexane as a Function of Temperature Gradient . . . . .	78
23. Gravitational Terminal Velocity in Water as a Function of Temperature . . . . .	79
24. Gravitational Terminal Velocity in n-Hexane as a Function of Temperature . . . . .	80
25. Co-ordinate System and Variables for the Large Particle Regime . . . . .	137

## Acknowledgment

The author wishes to express sincere appreciation for the efforts of his supervisor, Dr. A. Meisen, without whose encouragement and determination this project would never have been completed.

The assistance of Dr. J. Leja from the Department of Mineral Engineering and Dr. A. Kozac of the Faculty of Forestry are also gratefully acknowledged.

Thanks are also due to the entire workshop staff for all their help, advice, and patience.

Ms. K. L. Idler assisted the author in presenting this thesis in a recognizable form of English. Without her efforts, this work would have taken considerably longer.

The work reported in this thesis was supported financially by the National Research Council in the form of a Research Assistantship.

## Chapter I

### Introduction

When a temperature gradient is imposed on a stagnant gas which contains micron-size ( $10^{-6}$ m) particles, the particles move down the gradient towards regions of lower temperature. This phenomenon, known as *Thermophoresis*, is not a result of thermal convection currents in the gas but is due entirely to the presence of a temperature gradient. This gradient causes a net imbalance in the average momentum transferred to a particle by the fluid molecules which surround it. This imbalance gives rise to a momentum flux, also known as the *Thermophoretic force*, which moves the particle down the gradient at a terminal velocity called the *Thermophoretic velocity*.

The author considered it likely that thermophoresis existed in liquids as well as gases because of the many similarities between the two types of fluids. However, because of certain differences between these states, there was no assurance that the phenomenon would be present. This work, therefore, was an attempt to extend the concept of thermophoresis to liquids.

The present project was undertaken because, to the author's knowledge, no experimental evidence of thermophoresis in liquids has been reported in the literature. One theoretical study was found but its results are inconclusive. In addition, it was hoped that the thermophoretic velocity might be large enough to be important in

engineering problems where a liquid has a large temperature gradient. An example of this type of situation is the particulate fouling of sensible heat exchangers where large gradients may exist in the laminar sub-layer. If the particle velocities should prove to be very large, the thermophoretic effect might provide a novel method for the removal of micron-size particles from liquids.

The initial objective of this work was to prove or disprove the existence of thermophoresis in liquids by experimental measurement of the particle thermophoretic velocities. The major effort was experimental because the molecular theory of the structure of liquids, and the effect of temperature gradients upon it, is insufficiently developed to provide an exact analytical approach. Nevertheless, a number of attempts were made to derive a theoretical expression for the thermophoretic velocity. These, unfortunately, proved to be unsuccessful and are not described in the present thesis.

The second objective was to find an empirical relationship between the thermophoretic velocity and the viscosity, fluid density and thermal conductivity, absolute temperature, temperature gradient, and particle diameter. These parameters were considered to be important because they govern thermophoresis in gases.

Micron-size, spherical particles were suspended in a stagnant liquid to form a sol (1) with a void fraction in excess of 0.999. A temperature gradient was created in the liquid by trapping it between two horizontal, parallel disks maintained at different temperatures. The formation of natural convection currents was suppressed by heating

the upper disk and cooling the bottom one.

The experimental method used for measuring thermophoretic velocities was direct visual observation (with a low power microscope) of scattered light reflected from the particles. The resulting specks of light indicated the particle positions even though the spheres themselves were too small to be directly visible. The motion of the particles in the presence of a temperature gradient compared with their motion in its absence was considered to be a measure of the thermophoretic velocity.

Since the equation for thermophoretic velocity was empirical, it had to be based on a statistical analysis of the effect of each relevant parameter. An effort was therefore made to vary these parameters over large ranges in order to obtain as valid an equation as possible.

The spherical particles which were used in this study were polystyrene as this was the only material in which uniform, micron-size particles were readily available. Two average particle diameters were used, 1.011 microns and 0.790 microns. Two different liquids, water and n-hexane, were selected primarily to give a large variation in liquid thermal conductivity. The other liquid properties varied according to the liquid and the absolute temperature. Different temperature gradients were applied, ranging from 7,000 to 30,000  $^{\circ}\text{K m}^{-1}$ .

## Chapter II

### Previous Work

Although the present work is concerned only with thermophoresis in liquids, a brief review of this phenomenon in gases is provided. This is done for two reasons. First, the terminology and many important concepts, which have been well developed for gases, are also applicable to liquids. Second, the existing literature on thermophoresis in liquids fails to provide an adequate understanding of the phenomenon.

#### 1. Thermophoresis in Gases

Due to the aforementioned reasons, this section is restricted to a discussion of the most important papers on thermophoresis in gases. More comprehensive accounts have been prepared by Waldmann and Schmitt (2), Springer (3), and Fuchs (4).

Basically, only one mechanism causes thermophoresis in gases. This mechanism can be readily understood once it is recognized that the mean velocity and momentum of fluid molecules are increasing functions of temperature. Therefore, molecules which originate in the hotter region of the fluid strike a particle with greater mean momentum than those which come from the cooler region. As a result, the particle experiences a net force which moves it down the temperature gradient.

Two regimes have been defined for the convenient analysis of



thermophoresis. Their definition is based on the limits of a single parameter, the *Knudsen Number* (3), which is defined as:

$$Kn = \frac{2L}{d_p} \quad [1]$$

where  $d_p$  is the particle diameter and  $L$  is the mean free path of the fluid molecules. The limits of  $Kn$  correspond to the *small particle regime* ( $Kn \gg 1$ ), sometimes called the free molecule region, and the *large particle regime* ( $Kn \ll 1$ ), also known as the continuum region. A zone exists between these two limits which ranges from approximately  $0.1 < Kn < 10$  and is sometimes referred to as the *transition region*. The two regimes differ in the model used to describe the transfer of momentum between molecules and particles.

a. The Small Particle Regime ( $Kn \gg 1$ )

In this regime, the particles are considered to be so minute that they do not affect conditions in the gas. Molecule-particle collisions are infrequent and thus the number of reflected molecules in the vicinity of any one particle is very low. The likelihood of the reflected molecules interacting with incident molecules is negligible. Consequently, molecules impinge on a particle with momenta corresponding to conditions prevalent in the bulk of the gas.

Although this regime was analysed in the early part of the twentieth century (2,5,6), a more rigorous solution has recently become available and is outlined here. The equation predicting thermophoretic

velocity was obtained from Kinetic Theory. The net momentum transfer was computed by summing the momenta of the incident molecules minus the momenta of the reflected molecules over the entire particle surface. Several models have been postulated for calculating the momenta of the reflected molecules but the principle ones are based on elastic and diffuse collisions.

Waldmann (7), Derjaguin and Bakanov (8), and Mason and Chapman (9) appear to have independently derived the same equation for the thermophoretic velocity,  $v_{th}$ , in the small particle regime:

$$v_{th} = - \frac{6}{(8+a\pi)} \frac{\mu}{\rho T_K} \nabla T \quad [2]$$

where  $T_K$  and  $\nabla T$  denote the absolute temperature and the fluid temperature gradient far away from the particle, respectively. By definition, the coefficient "a" equals zero for elastic (specular) molecule-particle collisions and one when the directions of the reflected molecules are random and the speeds can be described by a Maxwellian distribution function. There are sufficient experimental data to suggest that Equation [2] predicts a thermophoretic velocity within 5 per cent of the experimental value as  $Kn \rightarrow \infty$  (10).

#### b. The Large Particle Regime ( $Kn \ll 1$ )

In this regime, the molecules which are reflected by the particle tend to interact with the incident ones since the number of reflected molecules in the vicinity of a particle is very high.

Therefore, the incident molecules are not only influenced by conditions in the bulk of the fluid but also by those near the particle surface. Since it is very difficult to account for the simultaneous occurrence of these effects, the Kinetic Theory approach employed for  $Kn \gg 1$  cannot be used. As a result, a new model for the momentum transfer, partly based on continuum mechanics, has been developed and is sometimes called the *thermal creep* (or slip) model.

In order to understand this model it is necessary to realize that a temperature gradient in the fluid induces a similar gradient along the particle surface. Furthermore, fluid molecules colliding with a particular point on the particle rebound with momentum related to the surface temperature at that point. The mean momentum of the reflected molecules is thus primarily a function of the particle surface temperature and not the region of the fluid where they originated. Hence, upon collision, a molecule coming from the hot region gives momentum to the particle whereas a molecule from the cold region acquires momentum. Therefore, a net momentum flux exists and forces the particle down the temperature gradient. If the particle were held stationary, the fluid adjacent to the surface would flow up the gradient with a velocity called the thermal creep velocity. This flow is only due to the temperature gradient tangential to the particle surface (2).

Maxwell (11) derived an equation for the thermal creep velocity,

$\underline{v}_s$ :

$$\underline{v}_s = \frac{3}{4} \frac{\mu}{\rho T_K} \frac{dT}{ds} \quad [3]$$

where  $\frac{dT}{ds}$  is the tangential temperature gradient.

Epstein (12) derived an expression for the thermophoretic velocity in the large particle regime. He solved the Fourier heat conduction equation to obtain the tangential temperature gradient along the surface of a large particle and substituted this result into Equation [3] to obtain the thermal creep velocity. This velocity served as a boundary condition in the solution of the Navier-Stokes equation for creeping flow past a sphere. Epstein's final equation is:

$$\underline{v}_{th} = - \frac{3}{2} \frac{k_f}{2k_f + k_p} \frac{\mu}{\rho T_K} \nabla T \quad [4]$$

where  $k_f$  and  $k_p$  are the fluid and particle thermal conductivities, respectively. Both conductivities are present in Equation [4] because the temperature distribution in the particle affects that of the fluid and vice versa. This effect only becomes important at low Knudsen numbers.

Epstein's equation predicts thermophoretic velocities within an order of magnitude of the experimental results provided  $Kn < 0.1$  and  $k_p/k_f \leq 10$  (13). However, as  $k_p/k_f \rightarrow \infty$ , Equation [4] predicts  $\underline{v}_{th} \rightarrow 0$  whereas in reality, the velocity does not vanish (14).

Brock (15) refined Epstein's derivation by including corrections for temperature jump (a function of the type of molecule-particle collision) and hydrodynamic slip. His equation is:

$$\underline{v}_{th} = - \frac{3}{2} \frac{(k_f + C_T k_p Kn)}{(1 + 3C_M Kn)(2k_f + k_p + 2C_T k_p Kn)} \frac{\mu}{\rho T_K} \nabla T \quad [5]$$

where  $C_T$  and  $C_M$  are dimensionless constants accounting for the temperature jump and hydrodynamic slip, respectively. The usefulness of Equation [5] is limited because the constants can only be determined experimentally and are dependent on the chemical composition of the particle and fluid. Equation [5] reduces to Equation [4] when  $Kn = 0$ .

Brock's equation gives velocities that are correct to within an order of magnitude provided  $Kn < 0.1$  (13). It does not predict zero thermophoretic velocities at high  $k_p/k_f$  unless  $Kn = 0$ .

Derjaguin and Bakanov (16) claimed that Equation [3] is incorrect and instead included an additional heat flux term in the Fourier equation. This flux is poorly defined and Waldmann and Schmidt (2) state that neither their justification for this term nor their resulting derivation are well understood. Their final equation is:

$$\underline{v}_{th} = - \frac{1}{2} \frac{8k_f + k_p}{2k_f + k_p} \frac{\mu}{\rho T_K} \underline{\nabla T} \quad [6]$$

Derjaguin and Yalamov (17) included the temperature jump at the particle surface in the derivation of Equation [6] and obtained:

$$\underline{v}_{th} = - \frac{1}{2} \frac{8k_f + k_p + 2C_T k_p Kn}{2k_f + k_p + 2C_T k_p Kn} \frac{\mu}{\rho T_K} \underline{\nabla T} \quad [7]$$

This equation has the same weakness regarding the experimental constant,  $C_T$ , as previously noted in the case of Equation [5]. When  $Kn = 0$ , Equation [7] reduces to Equation [6].

Both Equations [6] and [7] predict velocities which are within an order of magnitude of the experimental values at  $Kn < 0.1$  (13).

Neither equation predicts zero velocities at high  $k_p/k_f$ . Equations [4] to [7] have been verified for particle diameters as large as 30 microns (18).

There are two key observations that can be made regarding all these theoretical equations for thermophoretic velocity. In all cases,  $v_{th}$  is proportional to the temperature gradient and, if  $Kn$  is approximately zero (as it is in liquids) or very large,  $v_{th}$  is independent of particle diameter.

## 2. Thermophoresis in Liquids

As noted previously, no experimental data on liquid thermophoresis have yet been published. However, Dwyer (19) attempted to construct a theoretical model for this phenomenon by modifying the derivation of the isothermal equations for Brownian motion to include variable fluid temperatures. His analysis is restricted to freely moving particles which exhibit a large degree of Brownian motion. This limits the applicability of his work to sols with particle diameters much less than a micron (20).

Based on Newton's Second Law, Dwyer postulated that the one-dimensional particle acceleration is equal to a fluid drag term (which he assumes to be Stokesian) plus a rapidly fluctuating force term. The latter is independent of particle velocity and accounts for the irregular molecular bombardment which causes Brownian motion. This particular form of Newton's Second Law is sometimes called the Langevin Equation.

Dwyer considered a small interval of time,  $\Delta\theta$ , during which only the fluctuating term was allowed to vary. The interval was chosen to be so small that any space or time variations in temperature during this period were regarded as negligible. This model can be understood by imagining that the fluid is divided into a large number of very small regions, each of which is at a different, but internally constant, temperature. The particle jumps from one region to another at the end of each  $\Delta\theta$ . When the particle is in a region, it is affected only by the fluid in that particular region. Therefore, the particle is in a constant temperature fluid during each  $\Delta\theta$ , although the value of this temperature may change between time intervals.

Dwyer then modified Chandrasekhar's solution (21) of Langevin's equation, which is valid for the isothermal case, and derived a "displacement probability distribution" for the particles after time  $\theta$ . A "displacement probability distribution" is an equation which gives the probability of finding a particle at a certain displacement (final position minus initial position) from its initial point. Dwyer's distribution contains temperature as a function of position and time, but does not include the temperature gradient because of the assumption of constant fluid temperature in each  $\Delta\theta$ . He used this distribution to obtain an equation for the "mean square displacement,"  $\langle\Lambda^2\rangle$ , in one dimension after time  $\theta$ . The latter is the mean of the square of the displacements of an infinite number of particles evaluated after time  $\theta$ . Dwyer's equation is:

$$\langle\Lambda^2\rangle = \left\langle \frac{2k}{\beta m_p} \int_0^\theta T_K(\Lambda(\xi)) (1 - \exp(\beta(\xi - \theta)))^2 d\xi \right\rangle \quad [8]$$

where

$$\beta = \frac{3\pi d_p \mu}{m_p} \quad [9]$$

and  $k$  is the Boltzmann's Constant and  $m_p$  denotes the particle mass. The "probability averaging" operator, symbolized by  $\langle \rangle$ , takes the mean of the enclosed function over an infinite number of identical particles all starting at the same initial point, i.e., all at the same initial temperature. Dwyer never makes this definition clear.

The following particular cases of varying temperature were considered by Dwyer.

Case A: The temperature is a linear function of displacement.

$$T_K = T_{K0} + N\Lambda \quad [10]$$

where

$$\Lambda = x' - x'_0 \quad [11]$$

$T_{K0}$  is the temperature at the initial point,  $x'_0$ , and  $N$  is a constant temperature gradient.

Dwyer substituted Equation [10] into Equation [8] and found that a constant temperature gradient does not change the mean square displacement from that of the isothermal ( $T_{K0}$ ) value, i.e., there is no thermophoresis for a constant temperature gradient. He explained this result by claiming that the square of the particle displacement is increased by the same amount when the particle goes up the gradient



as it is decreased when the particle goes down the gradient. Since he postulated that the same number of particles go up the gradient as down, this difference vanishes in the averaging procedure.

Case B: The temperature is a linear function of the square of the displacement.

$$T_K = T_{K0} + M\Lambda^2 \quad [12]$$

Although Equation [12] is mathematically valid, it does not describe any physically realizable temperature distribution. When Equation [12] is used in Equation [8], the mean square displacement will increase if  $M$  is positive and decrease if  $M$  is negative. Dwyer explained this by asserting that a positive  $M$  indicates that the particle will always move from the initial point into regions of higher temperature no matter what direction it takes, whereas a negative  $M$  indicates that the particle will always travel into colder regions. Again, this shows that Equation [12] has no physical meaning.

Equation [8] describes the mean square displacement as a function of temperature. If it is correct, it should reduce to Einstein's Equation under isothermal conditions (22). With  $T_K = T_{K0}$ , Equation [8] becomes upon integration:

$$\langle \Lambda^2 \rangle = \frac{2kT_{K0}}{m_p \beta^2} \left( \beta\theta - \frac{3}{2} + \frac{\exp(-\beta\theta)}{2} (4 - \exp(-\beta\theta)) \right) \quad [13]$$

As  $\theta$  approaches infinity, Equation [13] tends to

$$\langle \Lambda^2 \rangle = \frac{2kT_{K0} \theta}{3\pi d_p \mu} \quad [14]$$

Equation [14] is identical to Einstein's Equation for the one-dimensional mean square displacement in a uniform fluid. This indicates, therefore, that the steady state component of Equation [13] is correct. However, the validity of the transients in Dwyer's equation has not been established.

Although Dwyer uses Equation [8] to support conclusions that have a bearing on liquid thermophoresis, each step in his derivation is equally valid for gases. Therefore, his analysis and his final equation should hold for both types of fluid. However, thermophoresis of particles which exhibit a large degree of Brownian motion exists in gases, even for constant temperature gradients. Dwyer's equation, therefore, is incompatible with experimental observations.

Dwyer's mathematical analysis appears to be correct although his variables are poorly defined and some of the steps in his derivation are difficult to follow. However, a major conceptual error was found in his work.

As stated previously, Dwyer assumed that a particle was affected only by fluid at a constant temperature during each time interval,  $\Delta\theta$ . However, if the particle lies in an isothermal fluid, no net imbalance can arise in the average momentum transferred to the particle from the fluid molecules, and, as a result, no thermophoretic force is possible. What Dwyer has calculated, therefore, is the

effect of a non-uniform fluid temperature upon the mean square displacement of particles under the assumption that they are only influenced by a series of isothermal fluids.

## Chapter III

### Apparatus

#### 1. Introduction

The apparatus consisted basically of a cylindrical cell in which a small volume of liquid, containing the particles, was trapped between two horizontal, parallel disks. The top disk was heated electrically whereas the bottom one was kept at a lower temperature by circulating cooling water through it. This established an axial (vertical) temperature gradient in the stagnant sol without creating convection currents. The remainder of the equipment fulfilled measurement and support functions and included a light source, microscope, and controls for the disk temperatures.

The particles were made "visible" by illuminating them and observing the scattered light through a low-power microscope. This arrangement is sometimes called an Ultramicroscope (23) and is suitable for viewing particles ranging from approximately 0.5 to 1.5 microns in diameter. The Ultramicroscope was ideal for the present study because its large focal depth made it possible to observe particles in the centre of the region between the disks. Cell wall effects could thus be minimized.

The particle velocity was measured by incorporating a grid reticle into the microscope eyepiece. Since the distance between the

grid lines was known, the velocity could be calculated from the time required for a particle to travel halfway between them.

## 2. The Cell

An overall view of the cell is given in Figure 1. The two disks and the base were made from leaded brass. The optical tube and spacer ring consisted of borosilicate pyrex glass and plexiglass, respectively.

Table 1 gives the thermal conductivities of these materials and representative values for water and hexane. These conductivities are given for completeness and for later use in estimating the radial heat flux.

Table 1

Thermal conductivities of the Apparatus Materials

Material	Thermal Conductivity $\text{J m}^{-1} \text{s}^{-1} \text{K}^{-1}$
Brass	119 (24)
Glass	0.880 (25)
Plexiglass	0.208 (24)
Water	0.600
Hexane	0.130

The optical tube rested firmly upon the base and fitted tightly over a rubber O-ring. The latter was inset in the bottom disk and provided a tight seal. The top disk rested on the spacer ring. This

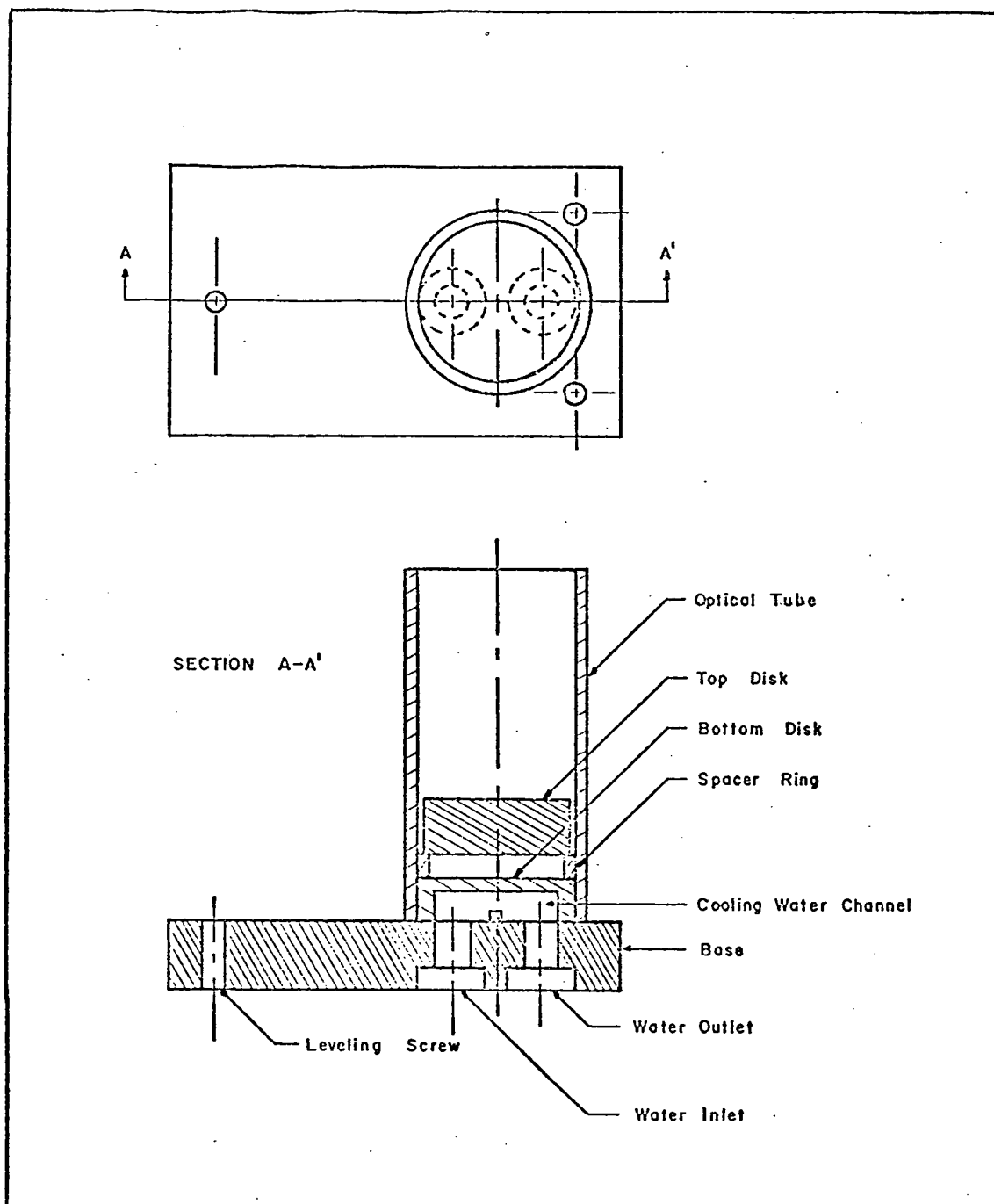


Figure 1

An Overall View of the Cell

ring, which was not physically attached to either disk, kept the two parallel and separated by a known distance. The face of the bottom disk was parallel to the base. Hence, both disk faces could be positioned horizontally by levelling the base.

A priori specification of the cell dimensions was difficult. Problems were encountered particularly in selecting a suitable disk diameter. This diameter had to be large enough to minimize wall effects and to provide space for the heating and cooling equipment. However, it also had to be sufficiently small to prevent excessive absorption of the scattered light by the liquid. A number of apparatuses were built before a reasonable balance between these requirements could be achieved. Details of the successful apparatus are presented in the following subsections.

#### a. The Optical Tube

The optical tube had an outside diameter of 1" and an inside diameter of  $7/8$ ". The ends were ground smooth, parallel, and at right angles to the tube axis which allowed the tube to sit flat on the base.

#### b. The Base

The base, which provided a stable foundation for the cell, is shown in Figure 2 with the bottom disk outlined to indicate the cooling water channel. The baffle was added to ensure turbulent flow of cooling water which entered and left the base through the two holes shown. Approximately one inch lengths of  $3/8$ " O.D.,  $1/4$ " I.D. copper tubing

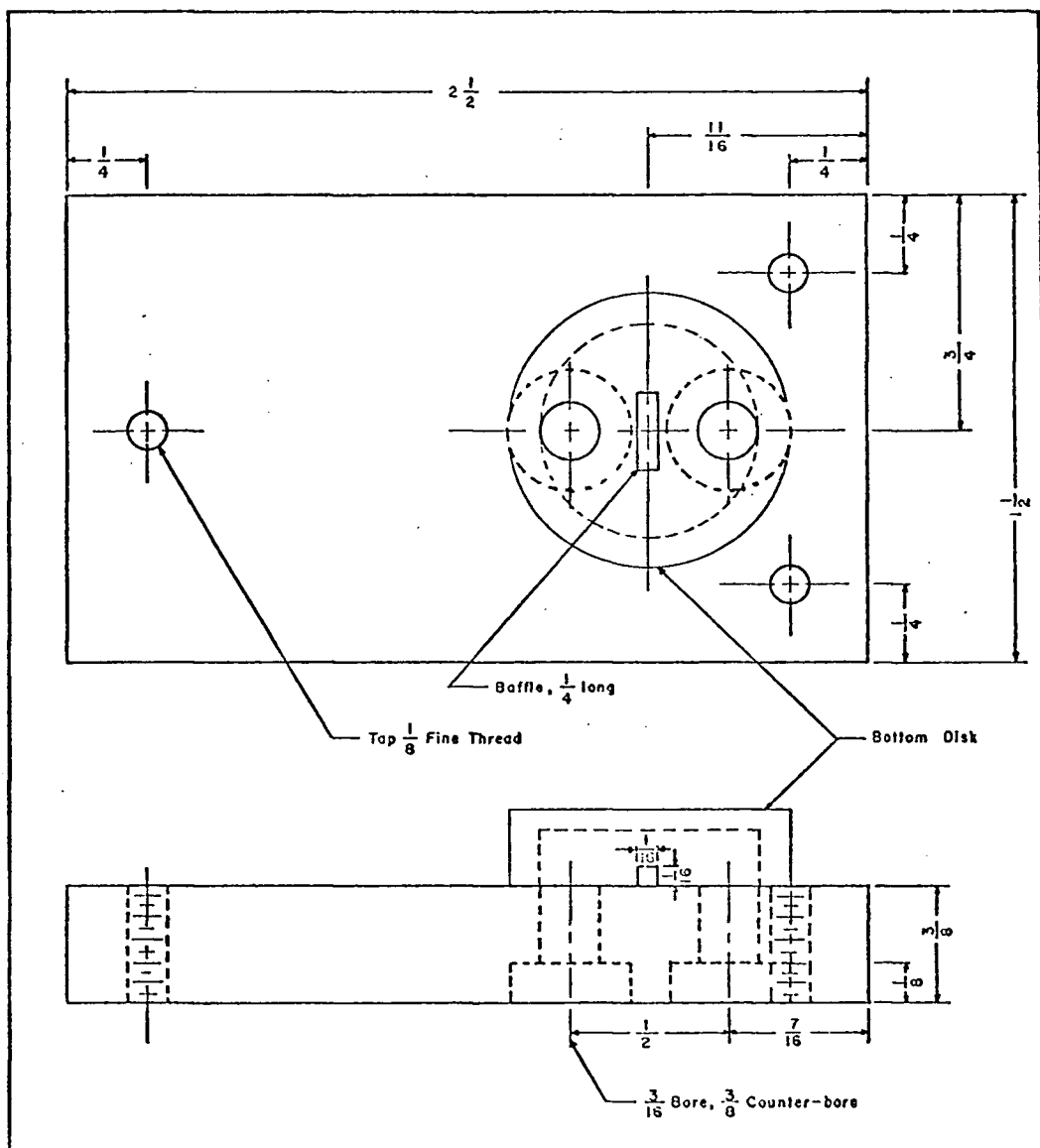


Figure 2  
The Base



were soldered into these holes and connected to the water bath by two feet of polyethylene tubing (1/2" O.D., 3/8" I.D.).

The three holes set in the triangular pattern accommodated levelling screws. This was found to be the most stable arrangement for levelling the base.

#### c. The Bottom Disk

The bottom disk is depicted in Figure 3. Its face was polished to a tolerance of less than 0.001" to ensure an even axial heat flux. The disk was soldered to the base in such a manner that its face was parallel with the base surface to within 0.1° of arc.

#### d. The Spacer Ring

Figure 4 shows the spacer ring whose function was to keep the disks parallel and separated by a constant distance of 0.118" (0.003m). The ring was made from plexiglass because this material could withstand temperatures in excess of 100°C without softening noticeably. In addition, its coefficient of thermal expansion,  $8.1 \times 10^{-5} \text{ } ^\circ\text{K}^{-1}$  (24), was sufficiently low to prevent radial or axial thermal expansion problems. For ease of assembly, the ring's outside diameter was 0.01" less than the inside diameter of the optical tube. The faces of the ring were machined at right angles to its axis, smooth, and parallel to a tolerance of less than 0.001".

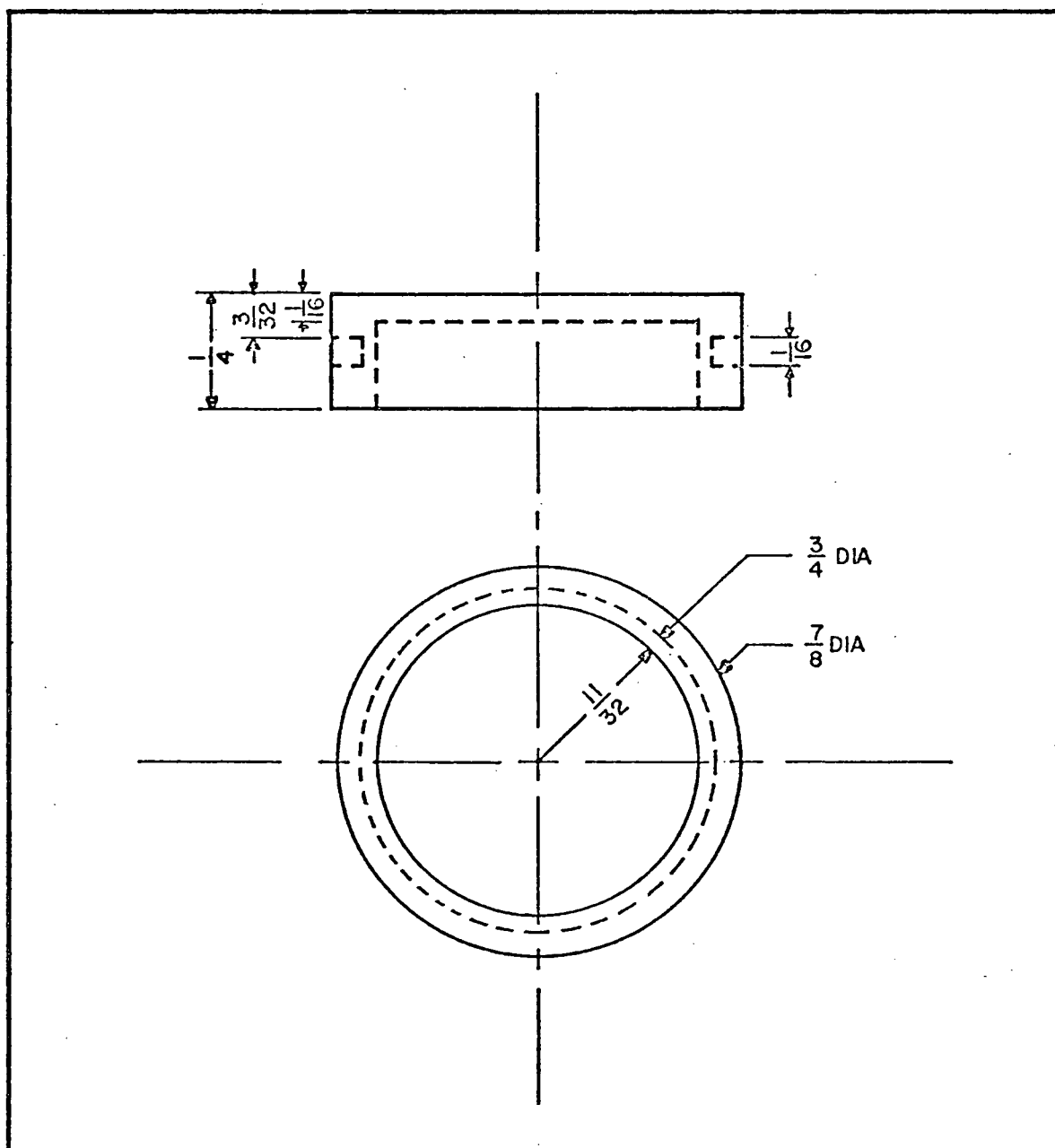


Figure 3  
The Bottom Disk

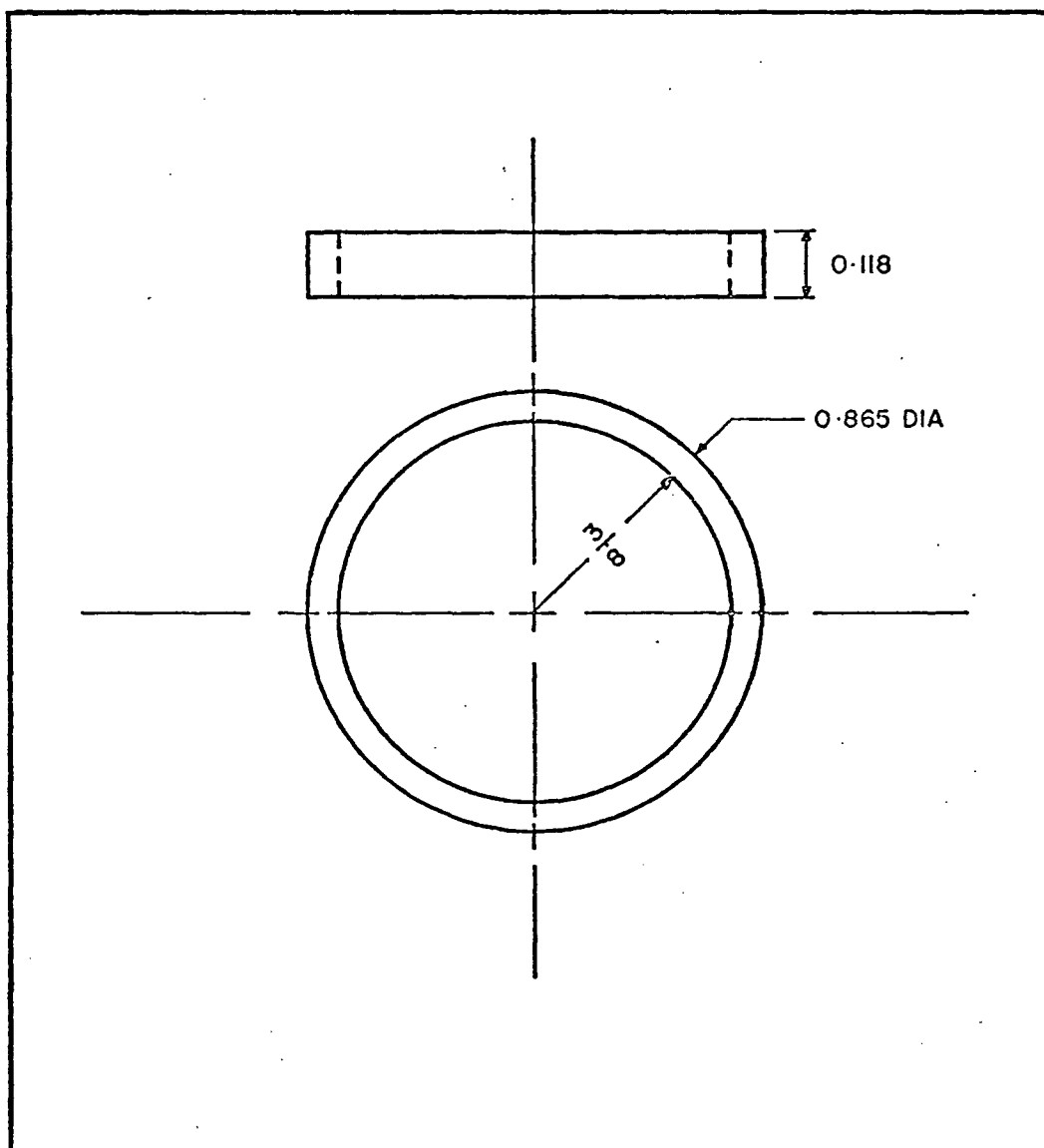


Figure 4  
The Spacer Ring

### e. The Top Disk

The last part of the cell was the top disk which is shown in Figure 5. The face of this disk was also polished to a tolerance of less than 0.001". Its O.D. was 1/16" smaller than the optical tube's I.D. to allow for thermal expansion and ease of assembly.

The hollow part of the disk contained an electrical heating coil and a thermocouple, which are not shown in Figure 5 but which are discussed in detail later. After all electrical parts had been inserted into the top disk, it was filled with "Easypoxy," manufactured by CONAP Incorporated. This epoxy glue provided an adequate seal against water and hexane and could withstand temperatures in excess of 100°C.

## 3. The Heating of the Top Disk

### a. The Electric Heating Coil

Since high heat fluxes were desired, it was necessary for the top disk to reach temperatures of nearly 100°C. This requirement, combined with the small space inside the disk, made the use of an electrical heating wire mandatory. The wire chosen was pyrotenax 24K with an O.D. of 0.093".

Pyrotenax wire has a resistance alloy core (with a specific resistance approximately twenty-five times that of copper), surrounded by an electrically insulating layer of compressed magnesium oxide. This, in turn, is coated with a thin layer of copper. Although, the 24K wire has a maximum power rating of 17 watts per foot, it was found

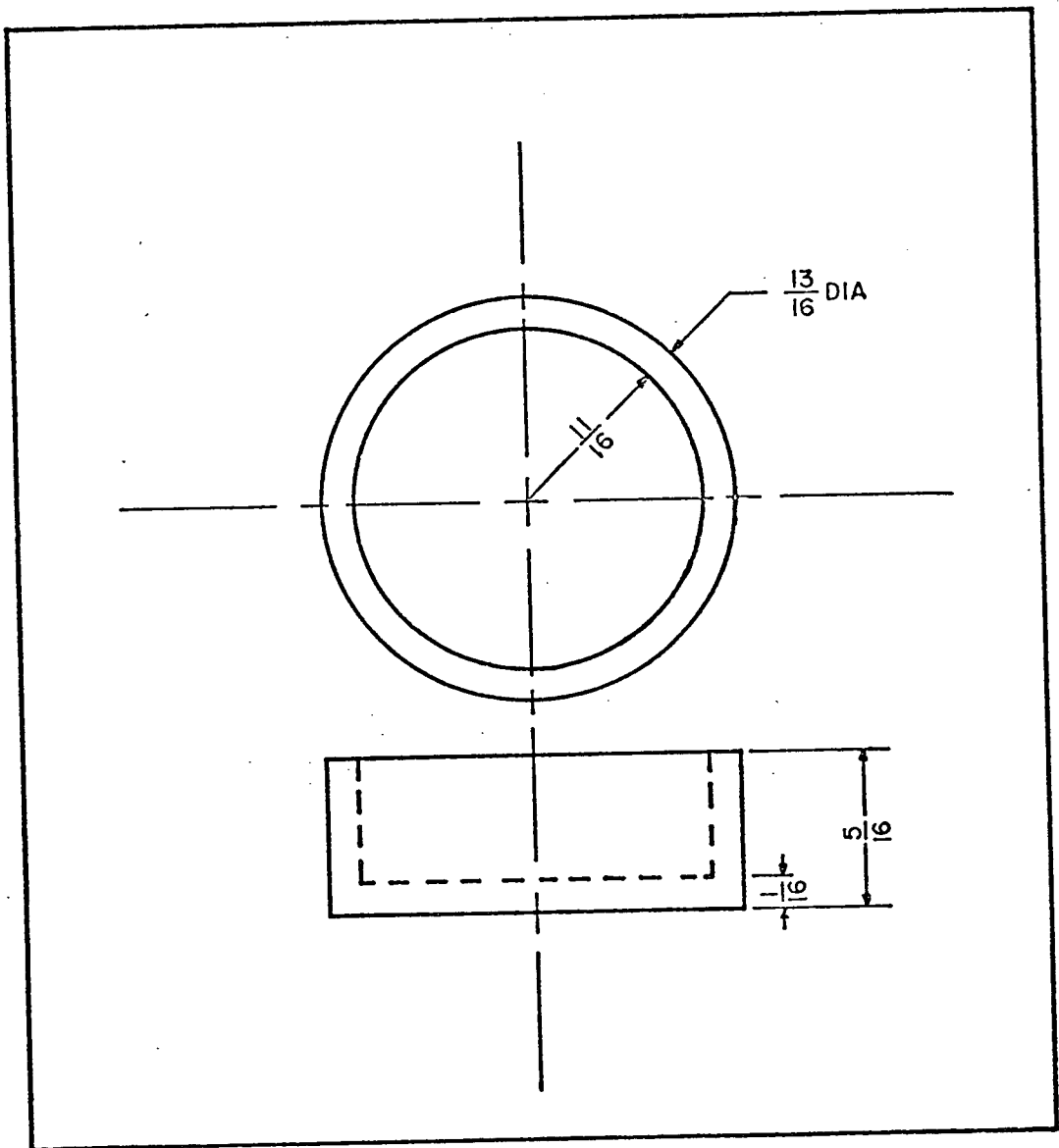


Figure 5  
The Top Disk

that, with care, this figure could be exceeded.

A flat coil, containing approximately six inches of pyrotenax wire, was soldered flush to the inside face of the top disk. This coil covered almost the entire face, thereby producing an even, axial heat flux. The ends of the coil were bent upwards out of the disk and connected to the power supply. A small space was left in the centre of the coil for insertion of a thermocouple.

#### b. Associated Electrical Apparatus

The circuit used for powering the heating coil is depicted in Figure 6. It consisted basically of a Variac, a transformer, an ammeter, and a voltmeter. The latter two were incorporated into the circuit for safety reasons. The mains current (117 volts, 60 hertz) was delivered to the transformer and controlled by a Superior Electric Company Variac whose range was 0 to 117 volts. This Variac controlled the top disk temperature by adjusting the power dissipated in the coil.

It was necessary to pass the primary current through a transformer to reduce the voltage to a level the coil could tolerate. This transformer was a Hammond 167P6 with maximum secondary ratings of 6.3 volts and 5 amps.

Since the coil could easily be overloaded, even with the transformer present, an AC voltmeter (0 to 10 volts) and ammeter (0 to 10 amps) were incorporated into the circuit. They were employed as monitoring devices to provide a measure of the power used by the coil.

This circuit had the advantage of easy control over the power

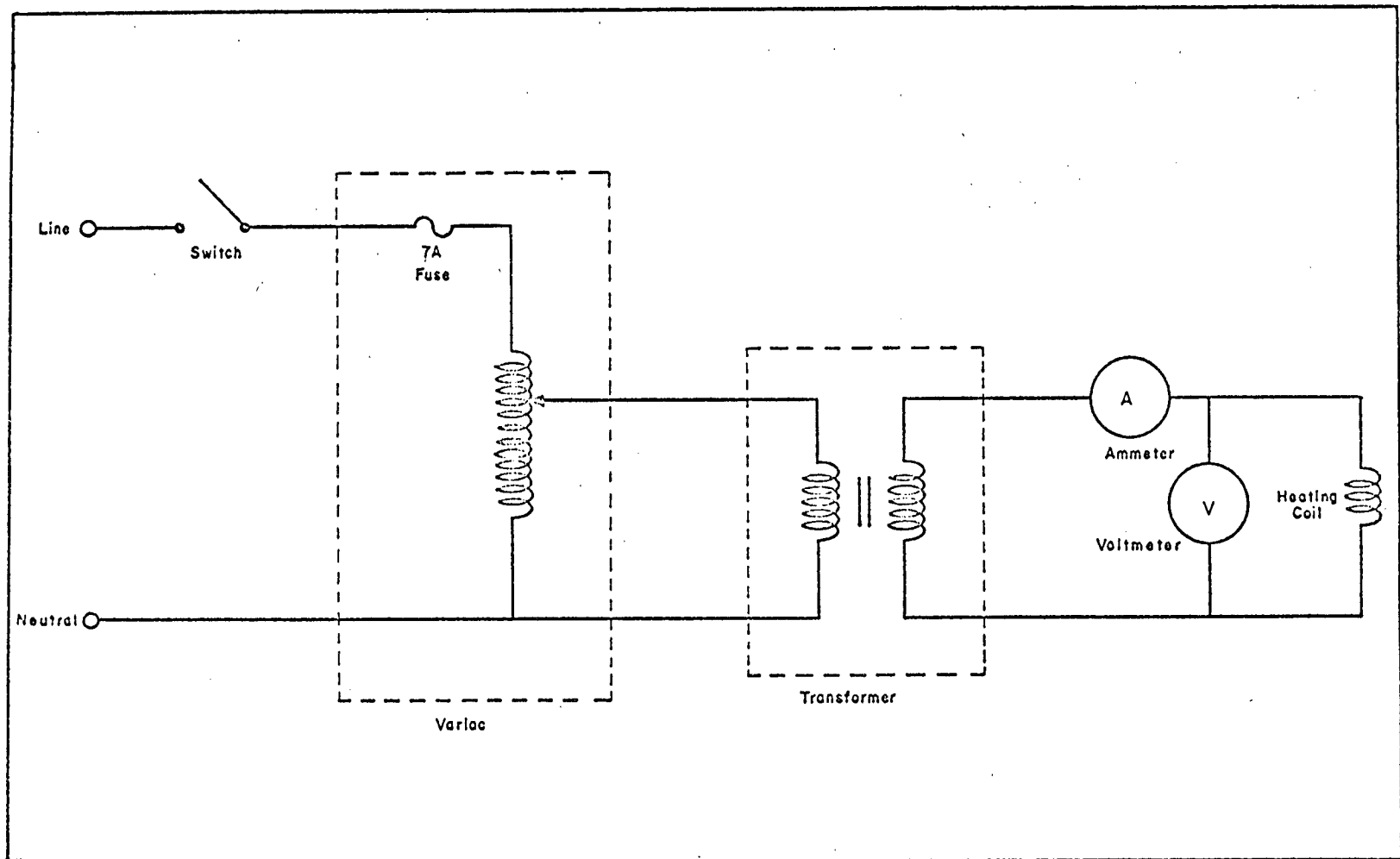


Figure 6  
The Electrical Apparatus

dissipated in the coil. Varying the 117 volt primary current was considerably simpler and more accurate than adjusting the much smaller secondary current.

#### 4. The Cooling of the Bottom Disk

The bottom disk had to function as a heat sink and this was accomplished by circulating cooling water through it from a reservoir. The latter was a 10 litre Lauda NB-515 Constant Temperature Bath with an internal heating coil. Tap water was passed through this coil and the bath water was thereby maintained at a temperature near 7°C. An electric pump, which formed part of the bath, served to mix the bath water and to circulate it to the bottom disk at a rate of approximately three litres per minute.

#### 5. Temperature Measurement

The top disk temperature was measured by means of a 28 gauge (0.040" O.D.) Copper-Constantan thermocouple. Such a small size was selected because of space limitations within the top disk. After welding and testing, the hot junction was soldered to the centre of the inside face (in the space left in the heating coil). The cold junction was placed in a thermos flask filled with ice water and the resulting EMF was measured with a Leeds and Northrup 8690 Millivolt Potentiometer. Since the current-carrying pyrotenax wire was electrically insulated, the thermocouple EMF was a direct indication of the top disk temperature.



It was impossible to place a thermocouple in the bottom disk since it was necessary to maximize the flow of cooling water. The bottom disk temperature was estimated from that of the bath water and will be discussed later.

## 6. Optical Equipment

### a. The Light Source

The particles were observed visually and a light source of the correct intensity was therefore required. An underpowered source rendered the particles invisible whereas an excessively strong one obscured the scattered light and created spurious convection currents in the cell. By experimentation, it was furthermore found that white light gave the best particle illumination. The best source that satisfied all these criteria was a small "Coollite" florescent lamp (Model F6T5-CW-HH) manufactured by the Western Electric Company.

The lamp's luminescent section was 7-1/2" long, had an O.D. of 1/2", and a power consumption of 6 watts. In order to give the correct illumination, the bulb was covered with black insulating tape except for a small section 1" long and 125° of arc (approximately 4-1/2 per cent of the available surface area). Since the initial lamp output was 260 lumens, the luminous flux available after taping was approximately 12 lumens. The uncovered area was placed approximately one inch from the outside of the optical tube.

## b. The Microscope

The microscope, which is shown in Figure 7, was assembled from parts supplied by the Gaerther Scientific Company. It was a monocular arrangement based on an adjustable rack and pinion focusing M101A microscope tube with an M239 eyepiece and M226 objective lens. The eyepiece was mounted on an M194 90° erecting prism with an M193F upper adapter. This arrangement was listed by Gaerther as providing a total magnification power of 27 and a focal length of 0.1374m between the subject and the objective lens. These values are for an optical path in air but they did not seem to change greatly when the subject lay behind a small layer of liquid.



Figure 7  
The Microscope

The microscope was mounted on an M309 support stand by means of an M330 Rod and Collar which allowed the microscope to move in all three directions. The entire construction weighed less than twenty pounds and, as a result, could be easily moved.

The grid reticle in the eyepiece was custom-made by Gaerther and had fifteen parallel, equispaced lines which covered almost the entire field of vision. The distance between these lines was experimentally found to be equivalent to a distance of 0.0004m at the point of focus. The reticle and the eyepiece could be rotated about their axis so that the lines could be placed parallel to the edges of the disks.

It was found by experiment that the particles were most clearly "visible" when the horizontal angle between the incident light to the cell and the microscope's optical path was approximately  $110^\circ$ .

## Chapter IV

### Physical Properties of the Experimental Materials

#### 1. The Particles

The spherical particles used in the present study were polystyrene latex and supplied by the Dow Chemical Company. Their size was extremely uniform and two diameters were selected for this work,  $d_p = 1.011 \pm 0.001$  microns (Lot Number LS-1138-B) and  $d_p = 0.790 \pm 0.001$  microns (Lot Number LS-1117-B). The error bounds correspond to the 95 per cent confidence limits and were calculated from data provided by Dow.

According to Dow, the specific gravity of the particles is 1.05 ( $1048 \text{ kg m}^{-3}$ ) at room temperature. Unfortunately, no indication of the accuracy of this value was given. Furthermore, the supplier did not specify the particle thermal conductivity but Perry (24) gives a value of  $0.128 \text{ J m}^{-1} \text{ s}^{-1} \text{ }^\circ\text{K}^{-1}$ .

#### 2. The Liquids

Water and n-hexane ( $\text{C}_6\text{H}_{14}$ ) were used in this study for a number of reasons. Basically, they were chosen because of their very different thermal conductivities, as is evident from Table 1 (page 17). In addition, their viscosities and densities are similar, they will not damage the construction materials, and they are non-toxic.

Measurements of particle velocity were taken at different

points within the cell and thus at different temperatures. It was therefore necessary to obtain equations for viscosity, density, and thermal conductivity as functions of temperature. When no simple equation could be found in the literature, an empirical equation was fitted to the available data by means of a regression programme written and made available by Dr. A. Kozak of the Faculty of Forestry at The University of British Columbia (26). All graphs of the equations are presented at the end of the "Physical Properties" chapter, pages 37 to 42, to avoid unnecessary interruption of the text.

#### a. Water

##### i. Viscosity

Three equations predicting the viscosity of water were found in the literature (24,25,27). Although the predicted viscosities agreed to within 1 per cent in the temperature range of interest ( $5^{\circ}\text{C} \leq T_C \leq 90^{\circ}\text{C}$ ), the equations were greatly different in complexity. The simplest equation is given by Perry (24) and was adopted for this work:

$$\frac{1}{\mu} = 21.482((T_C - 8.435) + (8078.4 + (T_C - 8.435)^2)^{1/2}) - 1200 \quad [15]$$

where  $T_C$  is the temperature expressed in degrees Celsius. The curve corresponding to Equation [15] is shown in Figure 8.

##### ii. Density

Data for the density of water were found in the Chemical Rubber Company Handbook (25). By means of the previously mentioned programme

an equation was fitted to these data for one degree intervals in the range  $15^{\circ}\text{C} \leq T_C \leq 95^{\circ}\text{C}$  giving:

$$\rho = 999.168 - 0.00426 T_C^2 \quad [16]$$

The average deviations between the results from this equation and the above-mentioned data are less than 1 per cent. The curve (without the data) is shown in Figure 9.

### iii. Thermal Conductivity

Data were taken from three sources (24,25,28) and are listed in Appendix I. They were fitted by the regression programme which produced the following empirical equation:

$$k_f = 0.567558 + 0.001862289 T_C - (7.15006 \times 10^{-6}) T_C^2 \quad [17]$$

The deviations between the calculated thermal conductivities and the data were on the average less than 1 per cent. The data and the curve produced from Equation [17] are shown in Figure 10.

### b. Hexane

A commercial grade of hexane was used whose greatest impurity was water. This amounted to less than 0.05 per cent while all other impurities were present in the parts per billion range.

### i. Viscosity

Data for hexane viscosity were taken from three sources (25, 29,30) and are listed in Appendix I. The regression programme gave the equation:

$$\mu = 0.00117 - 0.0000029 T_K \quad [18]$$

where  $T_K$  is the temperature expressed in degrees Kelvin. Deviations (as previously defined) were found to average less than 1 per cent. The data and Equation [18] are shown in Figure 11.

### ii. Density

These data were tabulated by Timmermans (29) and are also listed in Appendix I. The programme defined the equation:

$$\rho = 931.5 - 0.928 T_K \quad [19]$$

The differences between the predicted density and the physical data averaged less than 1 per cent. Figure 12 provides a comparison between the data and Equation [19].

### iii. Thermal Conductivity

It was very difficult to obtain consistent data from the literature for the thermal conductivity of hexane. However, Reid and Sherwood (31) gave some experimental values which agree reasonably well with those used by Sakiadis and Coates (32) who obtained an empirical

equation for the alkane series. Both groups of data are recorded in Appendix I (with a value from the International Critical Tables (33)) and can be described by an equation derived by the programme:

$$k_f = 0.140 - 0.00044 T_c \quad [20]$$

The deviations averaged less than 2 per cent. Figure 13 shows Equation [20] and the data. According to Equation [20], hexane has a thermal conductivity that is approximately 1/5 to 1/4 that of water.



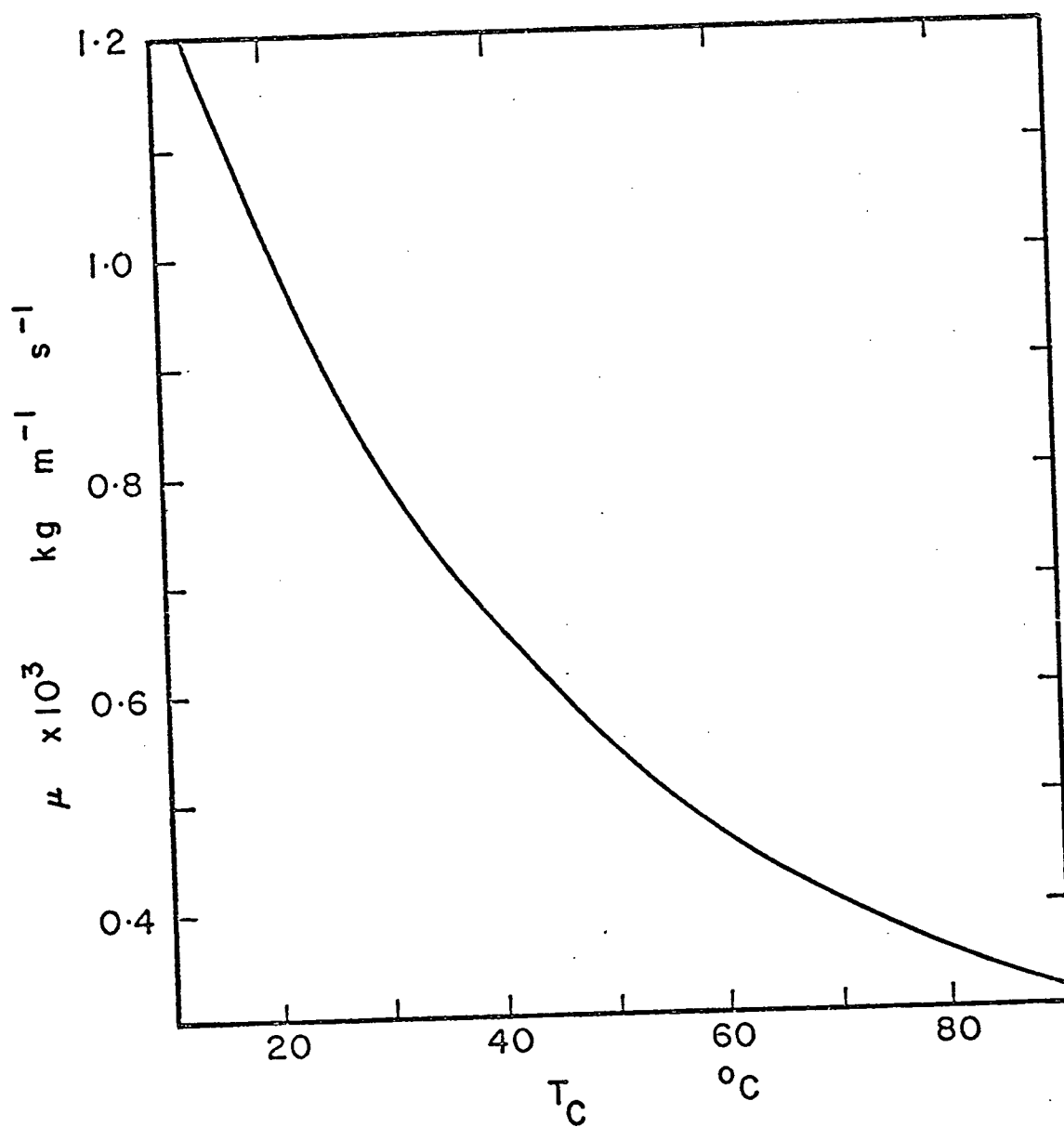


Figure 8

Viscosity of Water as a Function of Temperature

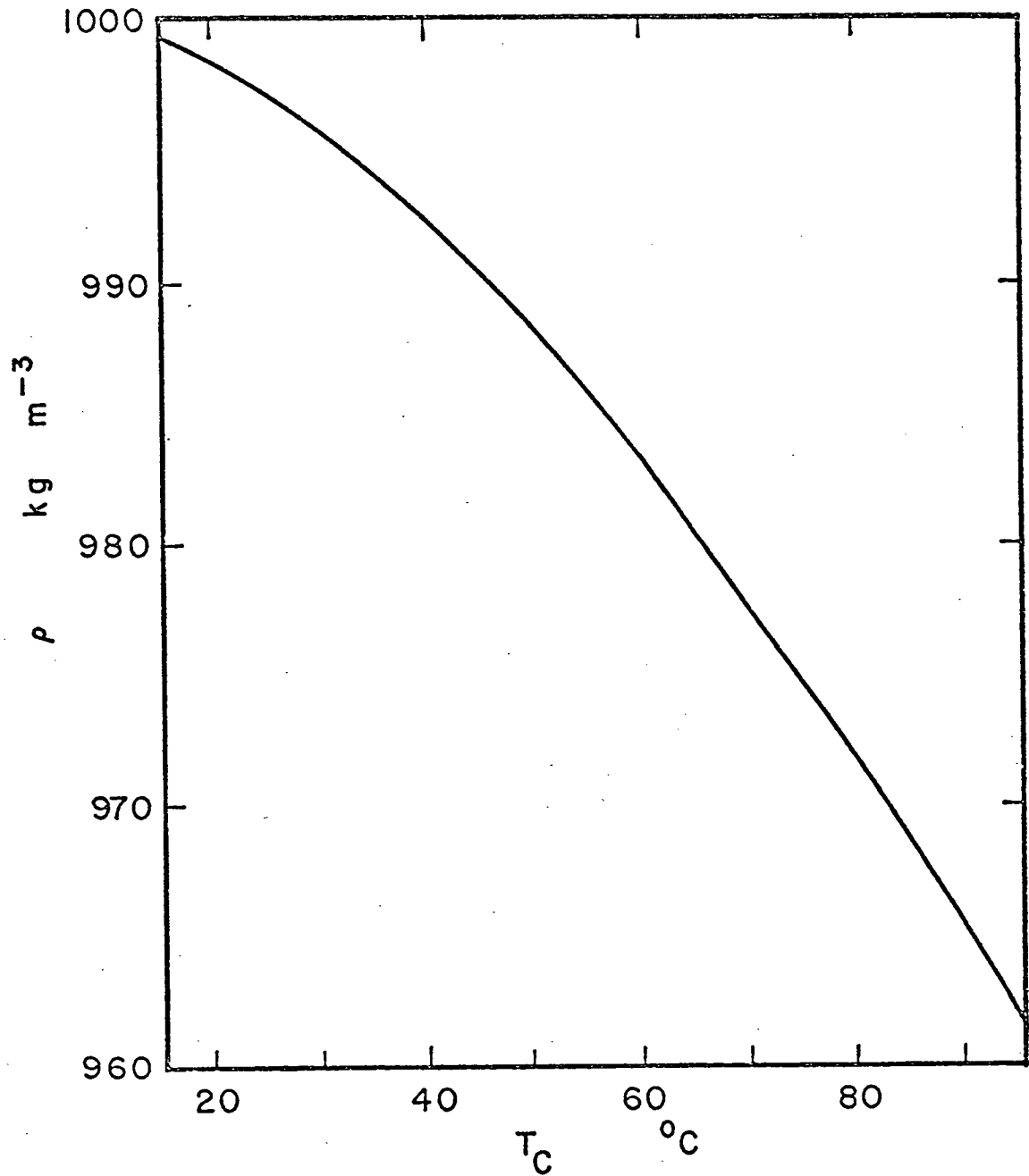


Figure 9

Density of Water as a Function of Temperature

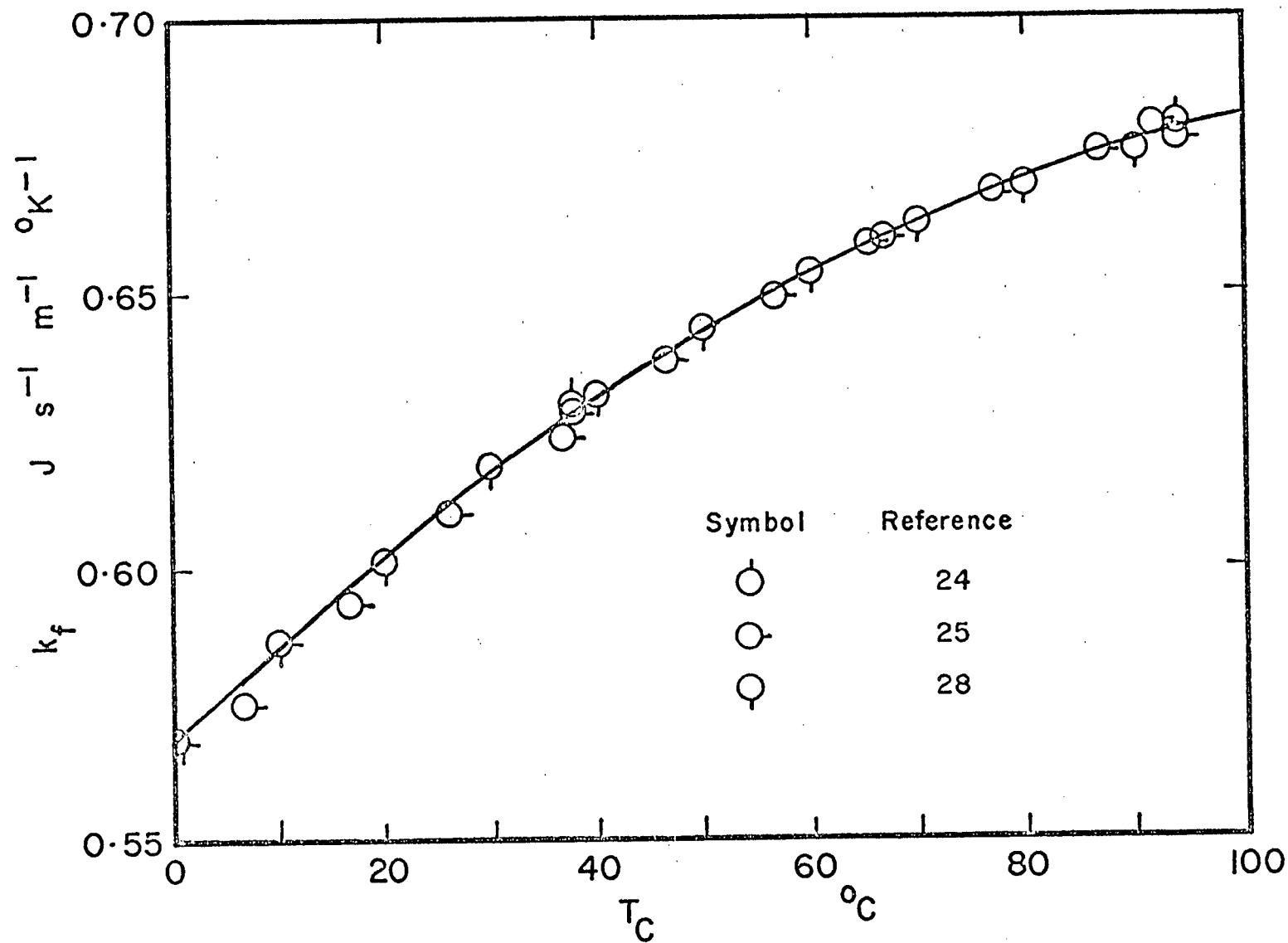


Figure 10  
Thermal Conductivity of Water as a Function of Temperature

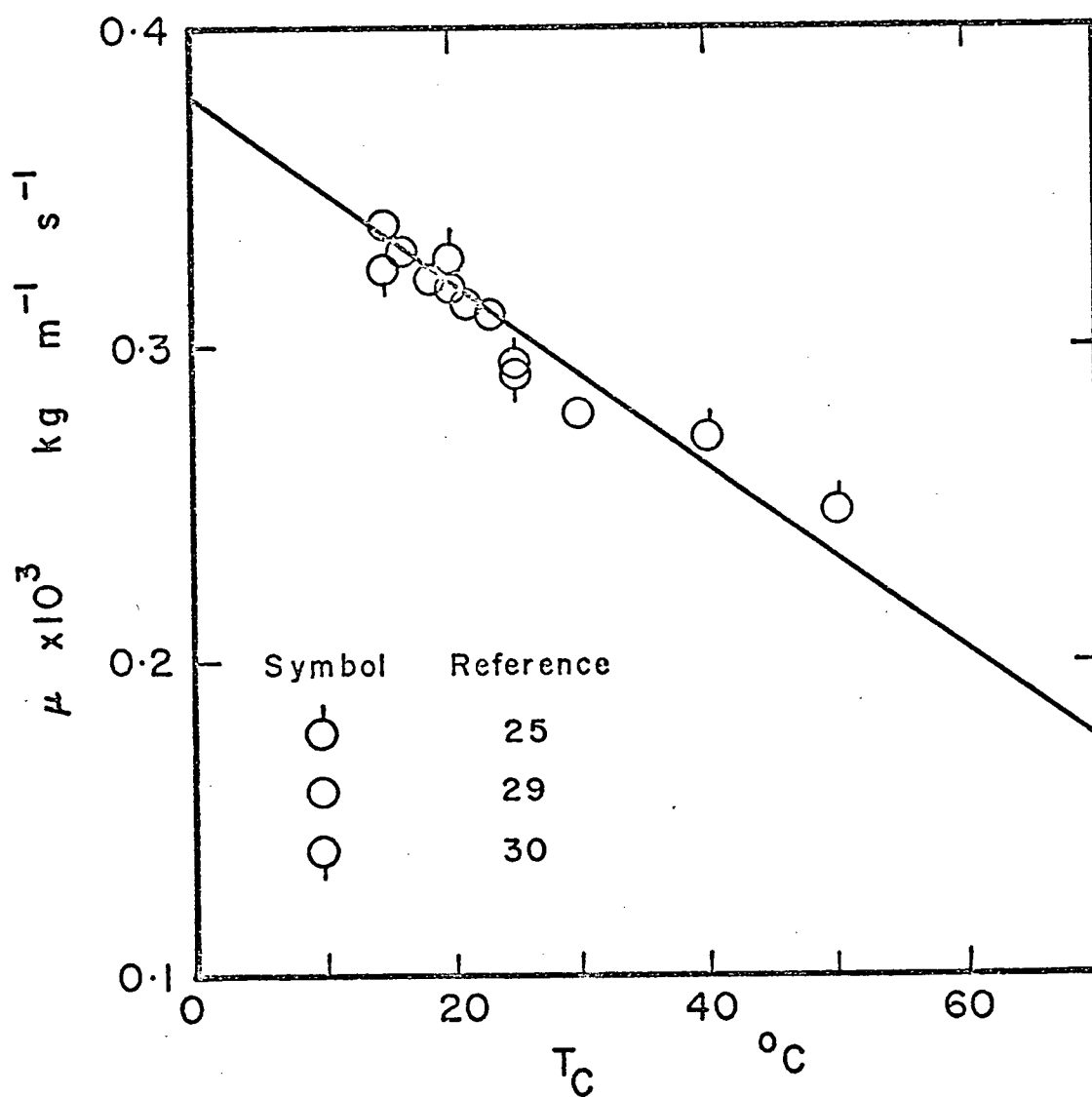


Figure 11

Viscosity of n-Hexane as a Function of Temperature

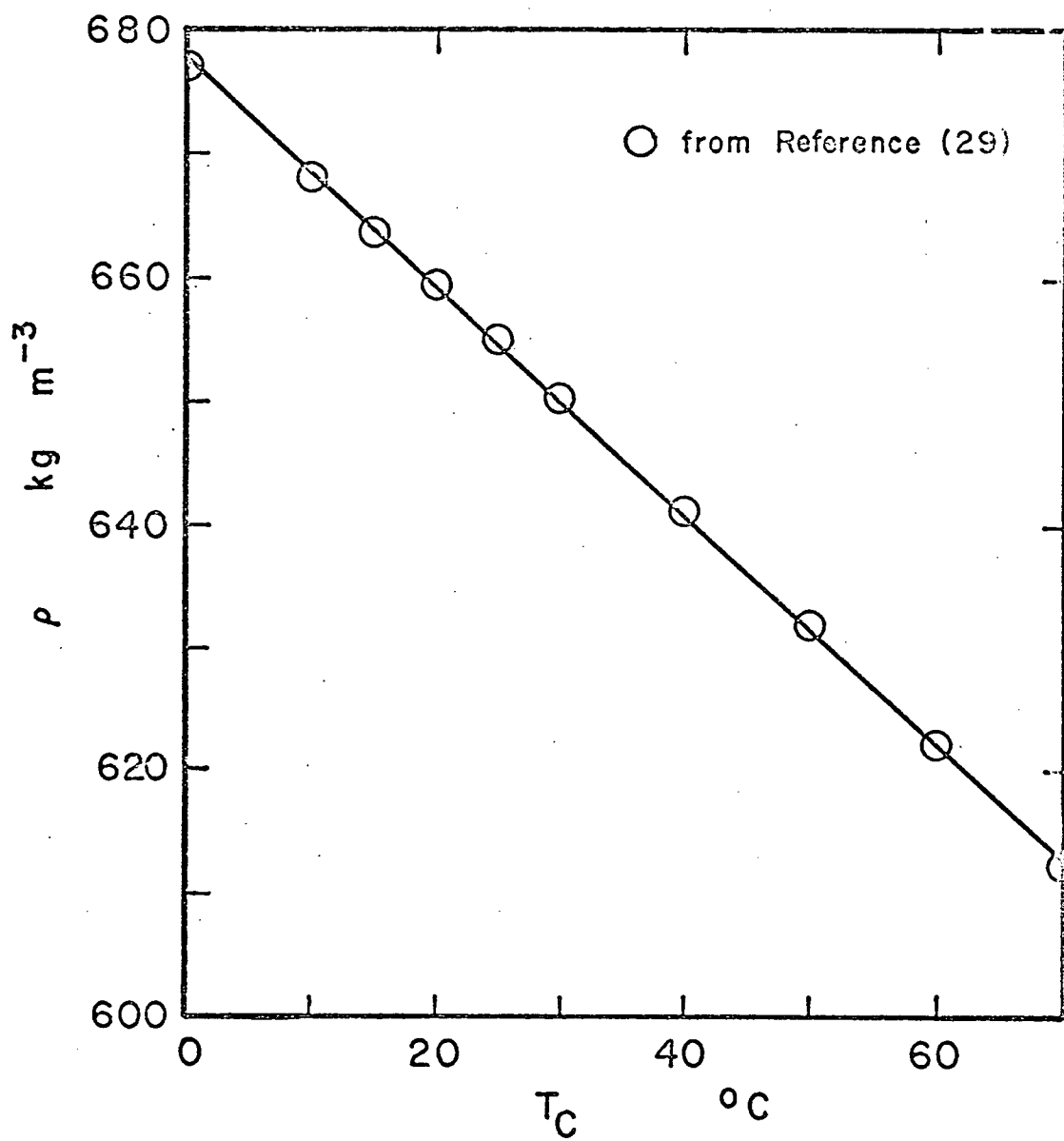


Figure 12

Density of n-Hexane as a Function of Temperature

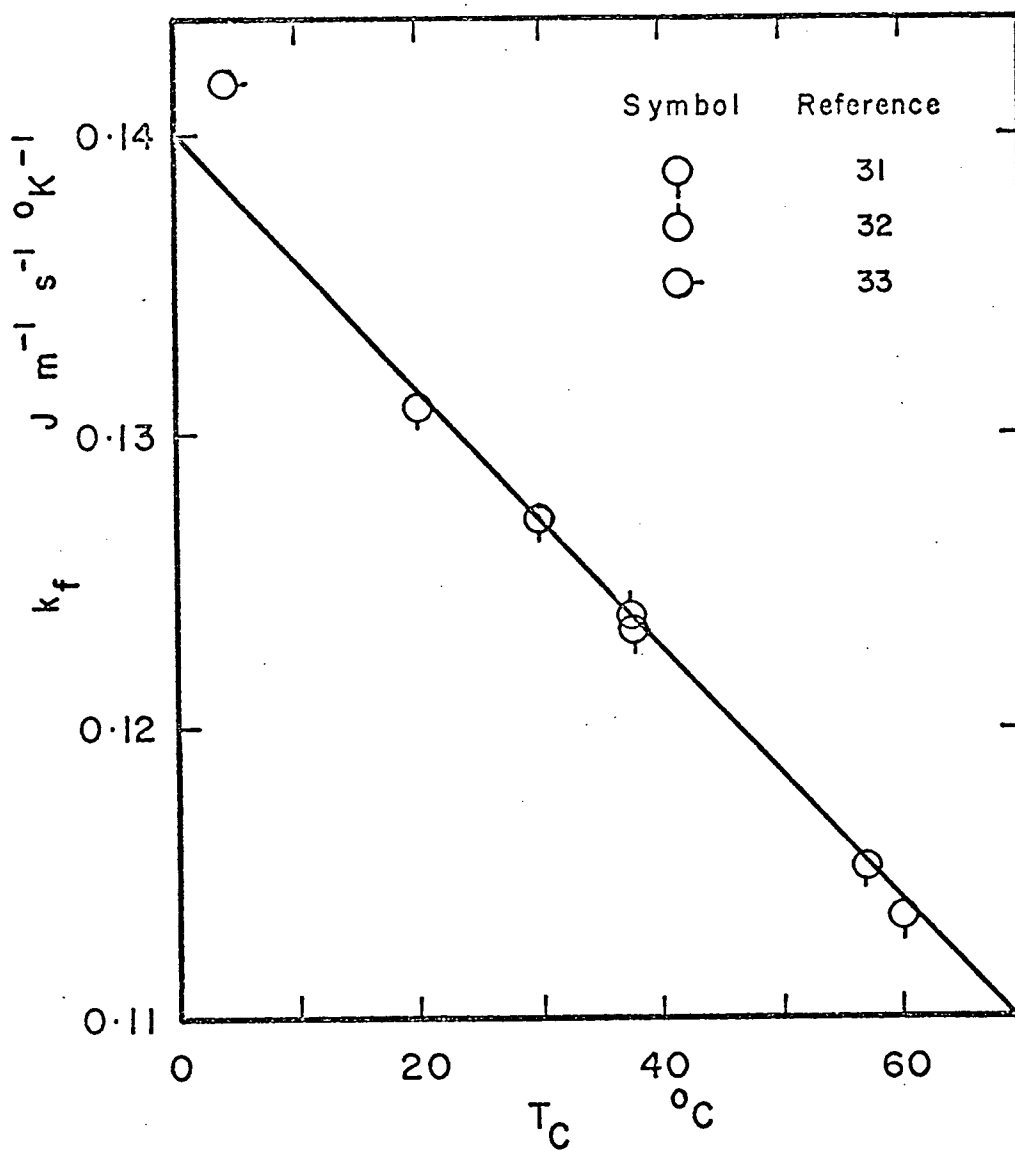


Figure 13

Thermal Conductivity of n-Hexane as a Function of Temperature

## Chapter V

### Experimental Method

#### 1. Preparations for a Run

The sol was prepared as follows. One drop of the concentrated Dow suspension, which consisted of latex particles in water, was placed in one litre of either water or hexane and well mixed. When the former was used, it had been previously distilled, heated to a brisk boil, and allowed to cool under cover. This procedure expelled most of the dissolved gases which otherwise tended to form bubbles on the underside of the top disk and effect the axial heat flux. Hexane was experimentally found not to require degassing.

One drop of Dow suspension had a volume of approximately  $5 \times 10^{-5}$  l. Since the volume fraction of particles per drop was given by Dow as 0.1, the actual volume of particles per drop was  $5 \times 10^{-6}$  l. Therefore, one drop per litre of water produced a sol with a void fraction between 0.999990 and 0.999999. In hexane, the void fraction was even closer to unity because water and hexane are insoluble so particle transfer was inhibited.

After the sol had been prepared it was set aside and covered to keep out airborne dust. With proper care, this sol lasted a number of runs before it became contaminated. A small syringe was filled with sol from this covered container to provide make-up in the cell if necessary during a run.

After the top disk was removed, the cell was filled with sol. The top disk was then eased into position, thereby displacing some sol which was drawn off with a second syringe and discarded. This procedure ensured that no air bubbles were trapped under the top disk which was correctly placed when seated flat on the spacer ring.

The liquid level was kept approximately  $3/4$  inch above the disk's upper surface and acted as a reservoir to compensate for any loss of liquid by evaporation.

The base of the cell was placed horizontally by means of the levelling screws and a Fisher 12-000 circular level (sensitive to within one degree) located on its surface. The disks' parallel alignment was checked with the reticle grid lines through the microscope. Extra weight was added to the top disk to increase stability. This entire procedure was repeated often to ensure that the apparatus remained in its correct position.

Three to four hours before a run started, the flows of tap water to the bath and cooling water to the bottom disk were begun. Approximately fifteen minutes before the start, the Variac was set to a pre-selected reading and the power was fed to the pyrotenax heating coil. An experimental plot of the top disk temperature versus time indicated that the cell reached essentially steady state approximately fifteen minutes after the power was switched on.

Finally, the microscope was focused on the region inside the spacer ring and one of the grid lines was aligned with the edge of the bottom disk. The run could then commence at any time.



## 2. The Run

The fluorescent lamp was turned on. It was used only for short periods of time to minimize the radiant heating of the liquid and particles.

The experiments were conducted only at night since the light scattered by the particles was too faint to be visible in daylight. Some time had to be allowed for the eye to become accustomed to the darkness and to locate the particles. A small piece of carbon paper was used to provide a black background for easier viewing.

When seen through the microscope, the individual latex particles appeared as small, shapeless specks of light. Occasionally larger specks with definite shapes were also observed and since these represented airborne dust particles or agglomerates they were ignored. On account of their size, these larger particles had a higher terminal velocity due to gravity (34) which might have interfered with the determination of the thermophoretic velocity. Similar precautions were not necessary for particles whose diameter was less than 0.5 microns because, as previously noted, they did not scatter light and thus were invisible.

When some liquid from the reservoir evaporated, more was added with the make-up syringe. The sample in the cell was replaced whenever it became badly contaminated with airborne dust or particle agglomerates. The cell was emptied, rinsed several times with distilled water, and refilled with fresh sol before every run or more often, if necessary.

For each datum point, a stopwatch was used to measure the *transit time*, i.e., the time a particle took to travel one-half of a grid space. Also, the initial and final particle grid positions, expressed in grid spaces from the bottom disk, were recorded. The measurements were restricted to such small distances in order to minimize the variations in absolute temperature during particle movement.

Velocities were measured only for those particles which were more than 1/8 inch from the inner surface of the spacer ring. This reduced the thermal and hydrodynamic edge effects. It was noted experimentally that particles more than 1/8 inch from the ring behaved similarly regardless of radial position but those closer to the edge moved erratically. In addition, no measurements were taken within one grid space of either disk to eliminate any hydrodynamic effect.

When the concentration had become so low that particles were difficult to locate, the top disk was gently moved up and down to remix the sol and to add fresh sol from the reservoir. After agitation, no measurements were made for approximately three minutes to allow very large particles to settle out and to let the system again reach steady state. It was found experimentally that three minutes was sufficient time for this to occur.

In the course of a run it was necessary to check periodically for the presence of thermal convection currents. These currents manifested themselves in the movement of the particles near the surfaces of the upper and lower disks. If the particles near the upper disk moved horizontally in one direction while those near the lower disk

moved in the opposite direction, convection currents were thought to be present. They could be eliminated by relevening the apparatus.

After 30 to 35 velocity measurements had been taken, the run was ended due to excessive operator eye strain. The temperature of the cooling water bath and the thermocouple EMF, which had been checked periodically during a run, were recorded when the experiment was over.

Although no definite shutdown procedure was established, the heating coil was always shut off prior to stopping the flow of cooling water. All remaining devices were switched off in a random order.

The terminal velocity due to gravity was measured for comparison with the thermophoretic velocity. It was obtained by duplicating the above procedures but without applying a temperature gradient.

## Chapter VI

### Methods of Data Analysis

#### 1. Preliminary Data Transformations

The experimental data were in the form of particle positions, transit times, thermocouple readings, and cooling water bath temperatures. They could not be used in this form and had to be transformed before any further analysis was possible. This section outlines the techniques used for these initial transformations.

##### a. Disk Temperatures

##### i. Top Disk

As mentioned previously, the top disk temperature was measured with a thermocouple whose calibration is briefly discussed here.

This procedure consisted of two steps. First, the EMF values given in reference (35) for copper-Constantan thermocouples were correlated with temperature by the regression programme. The resulting empirical equation for  $0 \leq T_C \leq 100^\circ\text{C}$  is:

$$T_C = 0.140687 + 25.5802(\text{EMF}) - 0.529102(\text{EMF})^2 \quad [21]$$

Second, the differences between the experimental temperatures and the values predicted by Equation [21] were found. Corrections could then be made in Equation [21] so that it predicted the experimental temperatures.

The latter were obtained using the following method. The top disk was submerged in the cooling water bath until steady state was achieved. The EMF value and the corresponding bath temperature were recorded. This procedure was repeated at different temperatures.

It was found that Equation [21] predicted temperatures which were approximately 1.1 per cent lower than the experimental values. Therefore, the equation was adjusted to give correspondingly higher results, i.e.:

$$T_C = 0.142235 + 25.8616(\text{EMF}) - 0.534922(\text{EMF})^2 \quad [22]$$

The value predicted by Equation [22] was considered to be the top disk temperature,  $T_T$ , since the temperature drop across the brass face was found to be negligibly small. A numerical estimate of this drop is given in Appendix III.

#### ii. Bottom Disk

The bottom disk face temperature was assumed to equal that of the water in the bath,  $T_W$ , plus a small correction factor of  $0.1^\circ\text{C}$ . This value accounted for the temperature rise between the bath and the bottom disk. The equation for the bottom disk temperature,  $T_B$ , is:

$$T_B = T_W + 0.1 \quad [23]$$

Since the flow of cooling water was rapid (approximately 3 litres/minute), the theoretical temperature rise of the water between

the inlet and the outlet of the bottom disk was less than  $0.03^{\circ}\text{C}$  (see Appendix III). As a check, thermometers were placed at the ends of the copper inlet and outlet tubes and indicated no noticeable difference. The temperature drop across the film resistance in the water channel could not be accurately calculated and was not included in the correction. An estimate of this drop is given in Appendix III, where it is also shown that a fairly large error in the bottom disk temperature does not significantly affect the final results.

#### b. Local Temperature and Temperature Gradient

This subsection outlines the techniques used for calculating the local temperature and gradient at a particle position from the two disk temperatures and thermal conductivity.

The average position,  $P_{os}$ , of a particle during a measurement, was defined as:

$$P_{os} = \frac{S_T + E_D}{2} \quad [24]$$

where the initial position,  $S_T$ , and the final position,  $E_D$ , were measured in grid spaces from the bottom disk. Since the region between the disks was 7.5 grid spaces wide, the average particle position could be expressed in terms of the dimensionless distance from the top disk,  $\frac{x}{h}$ , as follows:

$$\frac{x}{h} = 1.0 - \frac{P_{os}}{7.5} \quad [25]$$

where  $x = 0$  and  $x = h$  at the top and bottom disk, respectively.

The local temperature at  $\frac{x}{h}$  was calculated from a derivation based on Fourier's Law which, for one dimension and steady state is:

$$q = - k_f \frac{dT}{dx} \quad [26]$$

The constant axial heat flux is denoted by  $q$ . The equations for thermal conductivity used in the present work were of the form:

$$k_f = A + B T_C + C T_C^2 \quad [27]$$

Combining Equations [26] and [27] results in:

$$q = - (A + B T_C + C T_C^2) \frac{dT}{dx} \quad [28]$$

Integrating Equation [28] gives

$$\int_0^x q dx = - \int_{T_T}^{T_C} (A + B T_C + C T_C^2) dT \quad [29]$$

or

$$q x = A(T_T - T_C) + \frac{B}{2}(T_T^2 - T_C^2) + \frac{C}{3}(T_T^3 - T_C^3) \quad [30]$$

At  $x = h$ ,  $T_C = T_B$ , so that

$$q h = A(T_T - T_B) + \frac{B}{2}(T_T^2 - T_B^2) + \frac{C}{3}(T_T^3 - T_B^3) \quad [31]$$

Equations [30] and [31] can be combined to yield:

$$\frac{C}{3}T_C^3 + \frac{B}{2}T_C^2 + AT_C + [qh(\frac{x}{h}) - (AT_T + \frac{B}{2}T_T^2 + \frac{C}{3}T_T^3)] = 0 \quad [32]$$

It can be seen that there are three temperatures which satisfy Equation [32] at  $x/h$ . However, only one of these,  $T_{PC}$ , was physically meaningful and was found to fall within 10 per cent of the temperature,  $T_{LIN}$ , calculated on the basis of a constant temperature gradient. This temperature is defined by:

$$T_{LIN} = T_T - (T_T - T_B)(\frac{x}{h}) \quad [33]$$

Equation [32] reduces to Equation [33] when  $k_f$  is independent of temperature ( $B = C = 0$ ).

Although Equation [32] can be solved analytically (36), it was simpler to find the solution by an iterative procedure. The following method was adopted.

Let the left-hand side of Equation [32] equal  $\phi$ . Denote the values obtained for the  $(i-1)$ th and  $i$ th iteration by  $T_{C(i-1)}$ ,  $\phi_{(i-1)}$  and  $T_{C(i)}$ ,  $\phi_{(i)}$ . If the differences between these values are small, a linear approximation can be made for  $T_C$ :

$$T_C = D\phi + E_0 \quad [34]$$

where the slope,  $D$ , and intercept,  $E_0$ , are defined by

$$D = \frac{T_{C(i)} - T_{C(i-1)}}{\phi_{(i)} - \phi_{(i-1)}} \quad [35]$$



$$E_o = T_{C(i)} - D\phi_{(i)} \quad [36]$$

Since the intercept is the numerical value of  $T_C$  when  $\phi$  in Equation [34] is equal to zero, the next  $T_C$  in the series,  $T_{C(i+1)}$ , is set equal to  $E_o$  and  $\phi_{(i+1)}$  is calculated by substituting  $T_{C(i+1)}$  in Equation [32]. These new values are substituted in Equations [35] and [36] to continue the iteration. The process is stopped when  $|\phi_{(i)}|$  reaches a preselected value close to zero.  $T_{PC}$  is then set equal to  $T_{C(i)}$ .

The initial values for the iteration were chosen to give rapid and stable convergence. They were  $T_{C(1)} = T_{LIN}$ ,  $\phi_{(1)} = -0.1$ ,  $T_{C(2)} = T_{UN} + 1.0$ , and a value for  $\phi_{(2)}$  calculated by substituting  $T_{C(2)}$  in Equation [32]. The terminal limit for  $|\phi_{(i)}|$  was chosen to be  $10^{-5}$  since further reduction did not alter  $T_{PC}$  noticeably.

The local temperature,  $T_{PC}$ , was used to calculate  $\mu$ ,  $\rho$ , and  $k_f$  at each  $\frac{x}{h}$ . When Equations [27] and [31] were combined and rearranged, the local temperature gradient was given by the equation:

$$\frac{dT}{dx} = - \frac{qh}{hk_f} \quad [37]$$

### c. Thermophoretic Velocity

The particle velocity,  $v_{ex}$ , which was determined experimentally, resulted from the thermal and gravitational forces. Since these forces are additive and acted in the same direction, the thermophoretic velocity is given by:

$$v_{th} = v_{ex} - v_g \quad [38]$$

where  $v_g$  denotes the terminal settling velocity due to gravity, i.e.:

$$v_g = \frac{(\rho_p - \rho)d_p^2 g}{18\mu} \quad [39]$$

Equation [39] is valid provided the particle Reynolds Number does not exceed 0.1 (34), see Appendix III.

The experimental velocity was calculated from the transit time,  $\tau$ , i.e., the time required by the particle to travel one-half of a grid space, 0.0002m. Hence:

$$v_{ex} = \frac{0.0002}{\tau} \quad [40]$$

## 2. Analysis of Transformed Data

The previous section outlined the techniques used to transform the raw data into  $T_T$ ,  $T_B$ ,  $v_{th}$ ,  $T_{PC}$ ,  $\frac{dT}{dx}$ ,  $\mu$ ,  $\rho$ , and  $k_f$ . These resulting data were then analysed to give as much information as possible about the thermophoresis of sols. This section presents the methods used in this analysis and is predicated upon the existence of thermophoresis in liquids. The latter is discussed in the next chapter.

### a. The Empirical Equation for $v_{th}$

An empirical equation for  $v_{th}$  was obtained using the regression programme and postulating numerous possible relationships including some derived by the Rayleigh Method of dimensional analysis (24).

Since an empirical equation is statistically meaningful only if obtained on the basis of many different data points, such an equation could only express  $v_{th}$  in terms of  $T_{PC}$ ,  $dT/dx$ ,  $\mu$ , and  $\rho$ . The equation which best described the data was:

$$v_{th} = - \alpha \frac{\mu}{\rho T_{PK}} \frac{dT}{dx} \quad [41]$$

where

$$T_{PK} = T_{PC} + 273.2 \quad [42]$$

The dimensionless coefficient,  $\alpha$ , is defined by rearranging Equation [41]:

$$\alpha = - \frac{v_{th}}{\frac{\mu}{\rho T_{PK}} \frac{dT}{dx}} \quad [43]$$

#### b. Block Values of $\alpha$

Experiments were conducted with two liquids and particle diameters. For the sake of convenience each combination, e.g., water 1.011 microns, is referred to as a block.

The  $n$  values of  $\alpha$  in each block were expressed in a more usable form for further analysis by:

$$\bar{\alpha} = \frac{1}{n} \sum_{i=1}^n \alpha_i \quad [44]$$

$$s^2 = \frac{1}{n-1} \left( \sum_{i=1}^n \alpha_i^2 - \left( \sum_{i=1}^n \alpha_i \right)^2 / n \right) \quad [45]$$

and

$$s = (s^2)^{1/2} \quad [46]$$

where  $\bar{\alpha}$  is the mean of all the individual coefficients in a block and  $s^2$  and  $s$  are the variance and standard deviations of the  $\alpha$ 's, respectively. The value of  $\alpha_i$  was calculated from the  $i$ th data point by Equation [43]. If  $v_{th}$  was a function of  $d_p$  or  $k_f$ , the mean coefficients of the blocks would differ significantly.

### c. Distribution of the Coefficients in Each Block

However, before any further statistical analysis could be carried out on the  $\bar{\alpha}$ 's, information had to be obtained on the distributions of the individual coefficients about their block mean. If these distributions could be described by a normal distribution function, the statistical analysis would be greatly simplified. It would also have the added benefit of exposing any operator bias in the data collection. The distributions were found by the following method.

The data from each block was transformed into a variable,  $z$ , by the following equation:

$$z_i = \frac{\alpha_i - \bar{\alpha}}{s} \quad [47]$$

This variable has the same distributions as the coefficients and it facilitates comparison of these with the normal distribution.

The cumulative distribution function (c.d.f.) for the  $z$ 's was calculated for each block. The c.d.f. at a value of  $z$  is the fraction

of the total number of observations which have a  $z_i$  less than or equal to that value of  $z$ . The c.d.f. is, therefore, a function of  $z$  (37). The c.d.f. for each block was plotted on normal probability paper for comparison with that of the normal distribution function. The latter is a straight line in this co-ordinate system.

#### d. Modified t Test

Since, as will appear later, the variances are not the same for each block, the normal t test of the hypothesis  $\bar{\alpha}_1 = \bar{\alpha}_2$  is not possible. The subscripts 1 and 2 represent two different blocks of data. However, Wetherill (38) gives a technique which can be applied under these circumstances. His t values for the above hypothesis are calculated using the following equation:

$$t = \frac{|\bar{\alpha}_1 - \bar{\alpha}_2|}{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^{1/2}} \quad [48]$$

Furthermore, the degrees of freedom,  $f$ , were defined by:

$$\frac{1}{f} = \frac{s_1^4}{k_0^2 n_1^2 (n_1 - 1)} + \frac{s_2^4}{k_0^2 n_2^2 (n_2 - 1)} \quad [49]$$

where

$$k_0 = \frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \quad [50]$$

The value of  $t$  calculated from Equation [48] was compared with  $t_{95}$ , i.e., the  $t$  for a 95 per cent confidence limit (with  $f$  degrees of freedom) taken from a statistical  $t$  table. The 95 per cent limit was chosen because it is generally used in experimental work. When the  $t$  calculated from Equation [48] was less than  $t_{95}$ , the two  $\bar{\alpha}$ 's were considered to be equal. Conversely, when the experimental  $t$  value was greater, the two  $\bar{\alpha}$ 's could not be regarded as equal. This test should never be applied without some appreciation of the physical situation.

## Chapter VII

### Results and Discussions

#### 1. The Existence of Thermophoresis in Liquids

The first objective of this work was achieved when it was discovered that particles possessed a measurable velocity towards the cooler regions of the liquid over and above that due to gravity alone. Therefore, *thermophoresis does exist in liquids*. The thermophoretic velocity, and the other transformed data, can be found in Appendix II.

#### 2. An Empirical Equation for the Thermophoretic Velocity

As mentioned previously, it was possible to obtain an empirical equation for the thermophoretic velocity. When the regression programme was used with the 342 points of the water-1.011 micron block, an expression in the form of Equation [41] best described the data. This was also found to be true when data from the other basic blocks were used.

##### a. Analysis of the Thermophoretic Coefficient

##### i. Summary of the Block $\bar{\alpha}$ 's

Equations [44], [45], and [46] were used to reduce the data in each block to a form more convenient for interpretation. The results are shown in Table 2. The capital letters A to F are a code to

identify each block.

Table 2  
Statistical Data for Each Block

$d_p$ (microns)	Fluid	
	H <sub>2</sub> O	Hexane
1.011	Block A	Block B
	$n = 342$ $\bar{\alpha} = 0.1148$ $s^2 = 0.000303$ $s = 0.0174$	$n = 94$ $\bar{\alpha} = 0.0856$ $s^2 = 0.000066$ $s = 0.0081$
0.790	Block C	Block D
	$n = 107$ $\bar{\alpha} = 0.1149$ $s^2 = 0.000268$ $s = 0.0164$	$n = 57$ $\bar{\alpha} = 0.0903$ $s^2 = 0.000141$ $s = 0.0119$
Both	Block E	Block F
	$n = 449$ $\bar{\alpha} = 0.1148$ $s^2 = 0.000294$ $s = 0.0171$	$n = 151$ $\bar{\alpha} = 0.0874$ $s^2 = 0.000099$ $s = 0.0099$

ii. Distribution of the  $\alpha$ 's in  
Each Block

The cumulative distribution function of the  $\alpha$ 's in each block was obtained by transforming  $\alpha_i$  to  $z_i$  using Equation [47]. The resulting c.d.f.'s are shown in Figures 14 to 19 inclusive (these can be found at



the end of Chapter VII). The straight line on each graph corresponds to the c.d.f. of a normal distribution function. It can be seen from these figures that the experimental data can be approximated by the line except for some deviation at very negative  $z$  and very positive  $z$ .

These deviations are attributable to a small operator bias. A negative  $z$ , as can be seen from Equation [47], implies that  $\alpha_i < \bar{\alpha}$ . This means the experimental  $v_{th}$  is lower than the value predicted by Equation [41] when  $\alpha$  is replaced by  $\bar{\alpha}$ . Conversely, a positive  $z$  indicates a higher experimental value of  $v_{th}$ . Since both deviations were such that the experimental c.d.f. was less than the normal c.d.f. at extreme values of  $z$ , it is apparent that more observations were taken of particles with a very high  $v_{th}$ , and less with very low  $v_{th}$ , than the normal distribution function would predict. Although the operator made a conscious effort to avoid this tendency, it is obvious that some bias was present. However, since less than 5 per cent of the observations in any one block were affected by this proclivity, the thermophoretic coefficients can be regarded as normally distributed about  $\bar{\alpha}$ .

### iii. The Effect of Particle Diameter on $\alpha$

The effect of particle diameter on  $\alpha$  can be found by comparing Block A with Block C, and Block B with Block D. The first hypothesis tested was  $\bar{\alpha}_A = \bar{\alpha}_C$ . When the values from Table 2 were substituted into Equations [48] to [50],  $f$  was found to be approximately infinity and  $t = 0.054$ . Since the latter value is much less than  $t_{95} = 1.96$ ,

the hypothesis is confirmed and particle diameter has no effect on thermophoretic velocity in water.

The same test was carried out on the hypothesis  $\bar{\alpha}_B = \bar{\alpha}_D$ . In this case,  $f = 88$  and  $t = 2.63$ , slightly larger than  $t_{95} = 1.99$ . Although the  $t$  test indicates that  $\bar{\alpha}_B$  and  $\bar{\alpha}_D$  are unequal, results from tests where  $t \approx t_{95}$  cannot be used as a basis for definitive conclusions (38). In addition, the results must be interpreted with care because the variances of the two blocks are different, as shown subsequently.

An  $F$  test (38) was carried out on the hypothesis  $s_B^2 = s_D^2$ . The experimental value,  $F_{ex}$ , and degrees of freedom,  $\nu_D$  and  $\nu_B$  are defined by:

$$F_{ex} = \frac{s_D^2}{s_B^2} \quad [51]$$

$$\nu_D = n_D - 1 \quad [52]$$

$$\nu_B = n_B - 1 \quad [53]$$

Using the values in Table 2,  $F_{ex} = 2.14$ , and from a statistical  $F$  table, the 95 per cent value is approximately 1.4. Since the experimental value is larger than the tabulated one, the variances are unequal.

Different variances signify that some error is present in one block and absent in the other. As can be seen by comparing Figures 15 and 17, the data of Block D deviate more from the normal distribution

than those of Block B. This indicates that the significantly larger variance in Block D may be due to the previously mentioned operator bias.

There is only an approximately five per cent difference between  $\bar{\alpha}_B$  and  $\bar{\alpha}_D$  which is best explained by the extra error. In addition, the water data show no particle diameter effect. Thus, it is unlikely that the thermophoretic velocity in hexane is a function of particle size.

Consequently, it was concluded that the thermophoretic velocity is independent of particle diameter. Other workers (17) found this to be true in gases for small Knudsen Numbers, i.e., the same condition which can be considered to exist in liquids.

#### iv. The Effect of Liquid Thermal Conductivity on $\alpha$

Since particle diameter does not affect thermophoretic velocity, all the  $\alpha$ 's for water (Block E) can be compared with all those for hexane (Block F). If thermophoretic velocity is dependent upon liquid thermal conductivity, it should become apparent since there is a 400 to 500 per cent difference in  $k_f$  between the two liquids.

The t test was applied to the hypothesis  $\bar{\alpha}_E = \bar{\alpha}_F$ . Equations [48] to [50] gave a  $f \approx \infty$  and  $t = 23.9$ . This is very much larger than  $t_{95} = 1.96$  and therefore a significant difference exists between  $\bar{\alpha}_E$  and  $\bar{\alpha}_F$ . Thus, thermophoretic velocity must be regarded as a function of liquid thermal conductivity.

This effect can be compared with that found in gases. The

ratio  $\bar{\alpha}_F/\bar{\alpha}_E$  can be contrasted with the values predicted by Epstein (Equation [4]) and Derjaguin (Equation [6]) for  $Kn = 0$ :

$$\text{Epstein:} \quad \bar{\alpha} = \frac{3}{2} \frac{k_f}{2k_f + k_p} \quad [54]$$

$$\text{Derjaguin:} \quad \bar{\alpha} = \frac{1}{2} \frac{8k_f + k_p}{2k_f + k_p} \quad [55]$$

The  $\bar{\alpha}$ 's for hexane and water and their ratios are presented in Table 3. The values were calculated using average thermal conductivities in Equations [54] and [55]. As can be seen, the ratio for liquids is very

Table 3

Comparison of Experimental and Predicted  $\bar{\alpha}$ 's

$$k_f \text{ (water)} = 0.600 \text{ J s}^{-1} \text{ m}^{-1} \text{ }^\circ\text{K}^{-1}$$

$$k_f \text{ (hexane)} = 0.130 \text{ J s}^{-1} \text{ m}^{-1} \text{ }^\circ\text{K}^{-1}$$

$$k_p = 0.128 \text{ J s}^{-1} \text{ m}^{-1} \text{ }^\circ\text{K}^{-1}$$

Author	$\bar{\alpha}$ water	$\bar{\alpha}$ hexane	$\bar{\alpha}$ hexane/ $\bar{\alpha}$ water (%)
Epstein	0.6777	0.5026	74.2
Derjaguin	1.855	1.505	81.2
McNab	0.1148	0.0874	76.1

close to that predicted for gases by the Epstein relationship. Although essentially only two values of liquid thermal conductivity have been studied, it appears that an expression similar to Epstein's

relation holds for both types of fluid.

b. The Final Empirical Equation for  $v_{th}$

The data obtained in this work can definitely be described by an equation of the form:

$$v_{th} = - \bar{\alpha} \frac{\mu}{\rho T_{PK}} \nabla T \quad [56]$$

where the more general  $\nabla T$  replaces  $dT/dx$ . The large variances and the error analysis in Appendix III indicate that only two significant figures are justified for  $\bar{\alpha}$ . Therefore,  $\bar{\alpha}$  equals 0.11 for water and 0.087 for hexane.

However, as previously noted, the functional relationship between  $\bar{\alpha}$  and  $k_f$  may be described by a form of Equation [54]. The coefficient  $\bar{\alpha}$  can hence be written as:

$$\bar{\alpha} = \gamma \frac{k_f}{2k_f + k_p} \quad [57]$$

When the numerical values from Table 3 are substituted into Equation [57], the coefficient  $\gamma$  equals 0.254 and 0.261 for water and hexane, respectively. Since only two figures can be regarded as significant,  $\gamma$  becomes 0.26 for both fluids. Thus:

$$v_{th} = - 0.26 \frac{k_f}{2k_f + k_p} \frac{\mu}{\rho T_{PK}} \nabla T \quad [58]$$

In addition, the thermophoretic velocity for liquids is  $100 (0.26/1.5) = 17\%$  that predicted by the Epstein equation.

The fact that the Epstein gas equation, albeit with different coefficients, appears to hold for both types of fluid suggests the existence of thermal creep in liquids. A more detailed discussion of thermal creep is given in Appendix IV.

The lower value of  $v_{th}$  for hexane than for water at the same  $T_{PC}$  and temperature gradient is explained by Figure 20, where it is shown that  $\frac{\mu}{\rho T_{PK}}$  is lower for hexane than water at the same temperature. The difference in thermophoretic velocity is also apparent in Figures 21 and 22 which depict the magnitude of  $v_{th}$  as calculated by Equation [58]. Note that, due to the effect of  $\frac{\mu}{\rho T_{PK}}$ ,  $v_{th}$  drops with rising temperature.

There is one further piece of evidence which supports the existence of thermophoresis in liquids. When Equation [56] was applied to a case of particulate fouling in a sensible heat exchanger, it was found to explain the failure of small particles to deposit on a hot wall (39).

### 3. Thermophoretic Force

The thermophoretic force,  $F_{th}$ , can be obtained from the velocity by means of Stokes Law. The Stokes drag force,  $F_D$ , is:

$$F_D = - 3 \pi \mu d_p v_p \quad [59]$$

where  $v_p$  is the speed of the particle relative to the stagnant fluid.

At steady state, this particle velocity is equal to  $v_{th}$  and

$$F_{th} = - F_D \quad [60]$$

Therefore, combining Equations [58], [59], and [60]:

$$F_{th} = - 0.78 \pi \frac{k_f}{2k_f + k_p} d_p \frac{\mu^2}{\rho T_{PK}} \nabla T \quad [61]$$

#### 4. The Effect of Gravity

As mentioned in Chapter VI, the measured velocity is the sum of  $v_{th}$  and the gravitational terminal velocity,  $v_g$ . Figures 23 and 24 give the magnitude of  $v_g$  as a function of  $T_C$  as calculated by Equation [39]. Since  $v_g$  for water is very much smaller than  $v_{ex}$ , it can be neglected. However, for hexane,  $v_g$  is of the same order as  $v_{th}$  and must be taken into account. The gravitational terminal velocity for a 1.011 micron particle was measured experimentally in both liquids at approximately 20°C and found to be within 5 per cent of the values predicted by Stokes Law.

#### 5. Cell Wall Effects on the Local Temperature and Temperature Gradient

A computer simulation was based on the numerical solution of Fourier's Equation:

$$\Delta^2 T = 0 \quad [62]$$

This was used to study the effect of radial heat losses through the cell wall upon the vertical temperature gradient and local temperature. Heat transfer to the air outside the cell was considered to be by natural convection. The computer was supplied with the cell dimensions and representative values of physical properties from Table 1. The simulation indicated that the heat loss did not effect the temperature gradient and temperature significantly. This, combined with the precautions mentioned in Chapter V, assured that the radial heat flux did not affect the experimental measurements.

## 6. A Theoretical Model of Thermophoresis in Liquids

Several attempts were made to develop a theoretical model to predict the liquid thermophoretic velocity. However, when physical data were used, they failed. Nevertheless, one model showed promise.

This was based on the concept of thermal creep. However, Equation [3] was modified by an application of the Enskog theory of dense systems (40). Maxwell's thermal creep equation thus became:

$$\frac{v_s}{s} = \Gamma \frac{3}{4} \frac{\mu}{\rho T_K} \frac{dT}{ds} \quad [63]$$

Where  $\Gamma$  is a rather complicated function of the viscosity and thermal conductivity of the liquid and of the equivalent properties for the substance in the gaseous state at zero pressure. A functional relationship, which is still imprecise in its concept of the physical



situation, was derived. However, depending upon the values of the fluid properties,  $r$  could be either imaginary or real with a value less than one. The latter result is required to explain the data obtained in this work.

When physical data for water and hexane were used,  $r$  was imaginary but very close to the point where it would become real. Therefore, some corrections in the derivation and better data might allow this model to apply in the liquid case.

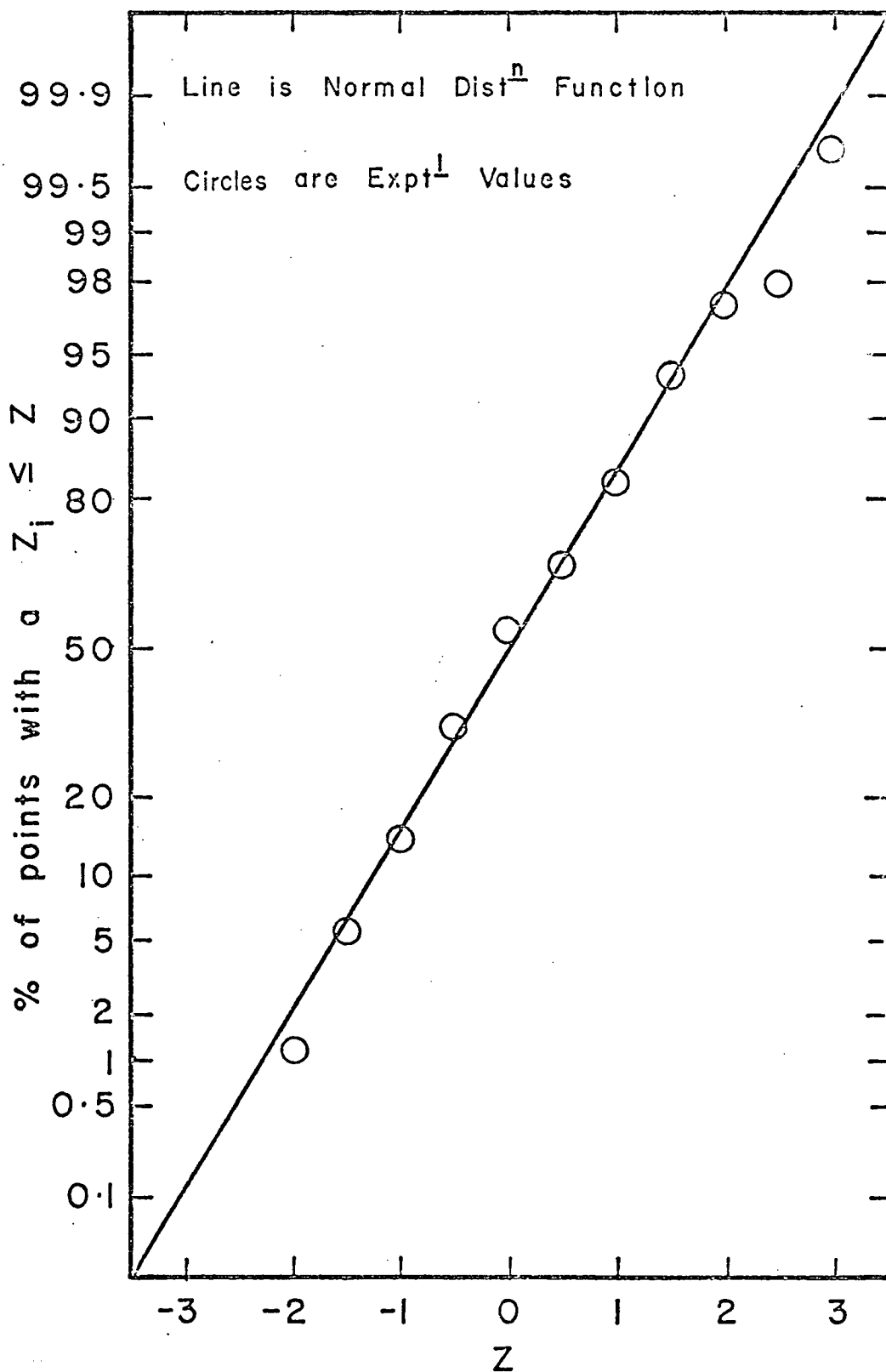


Figure 14

The c.d.f. of Block A (Water,  $d_p = 1.011$  microns)

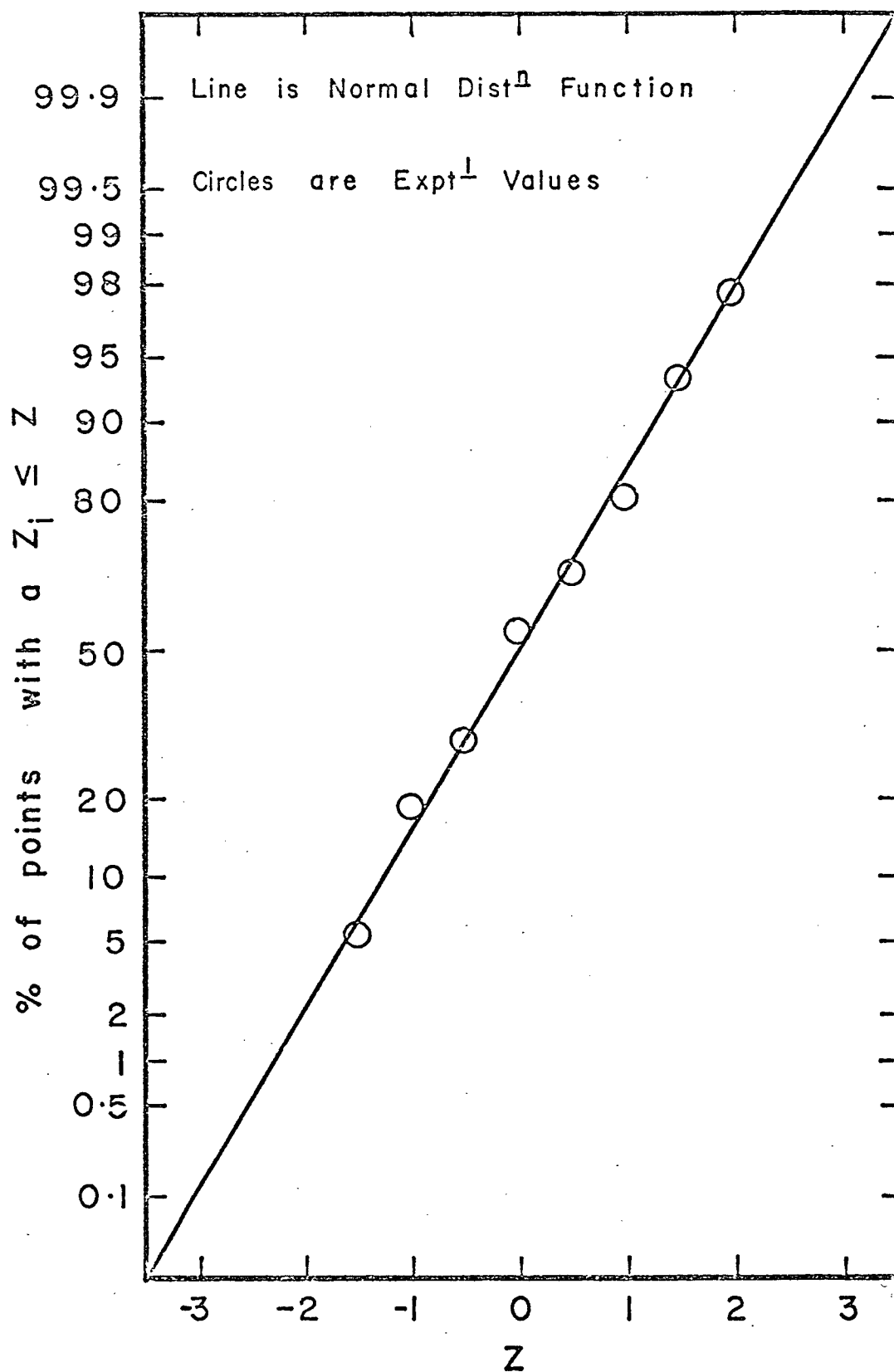


Figure 15

The c.d.f. of Block B (n-Hexane,  $d_p = 1.011$  microns)

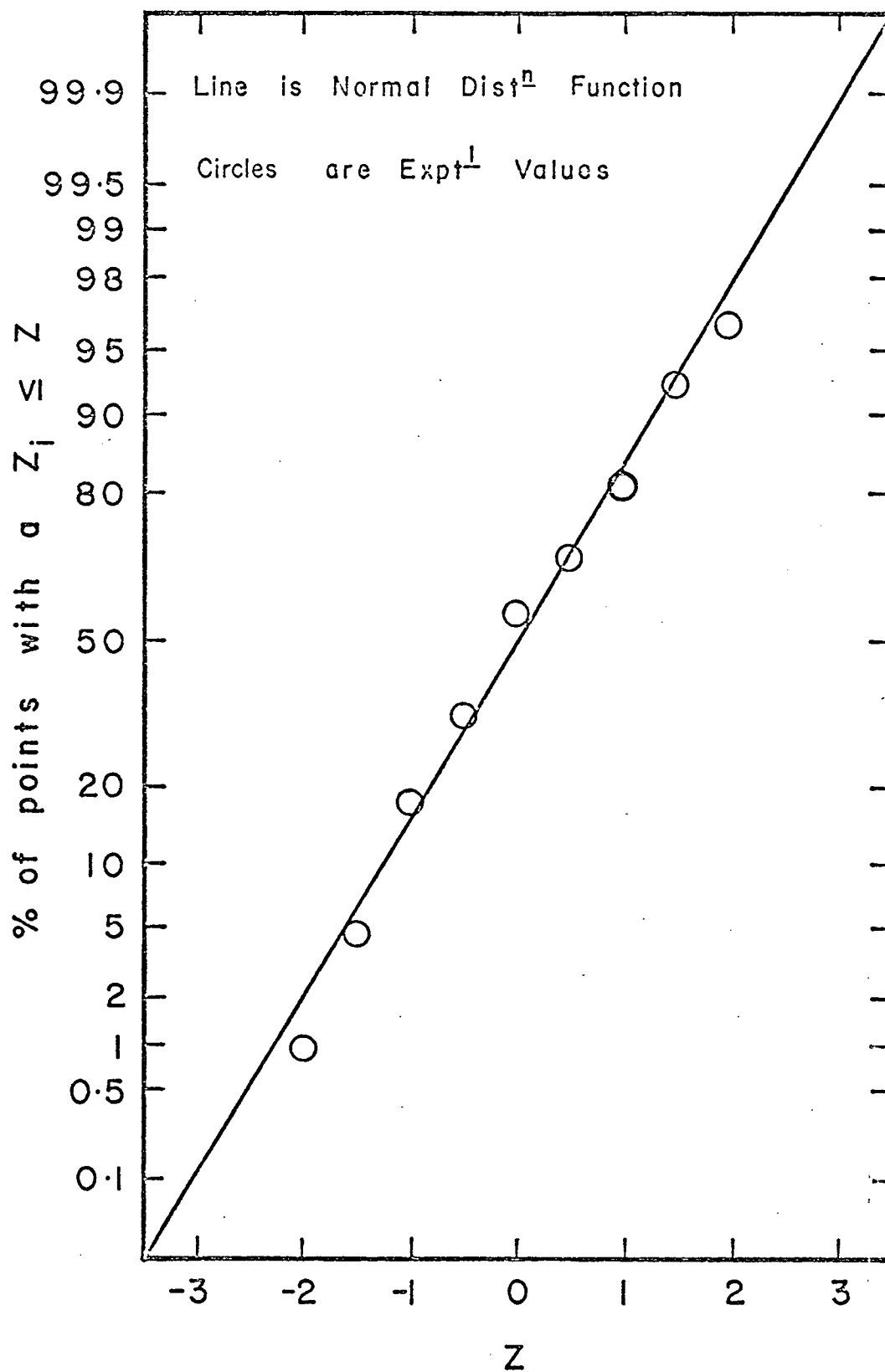


Figure 16

The c.d.f. of Block C (Water,  $d_p = 0.790$  microns)

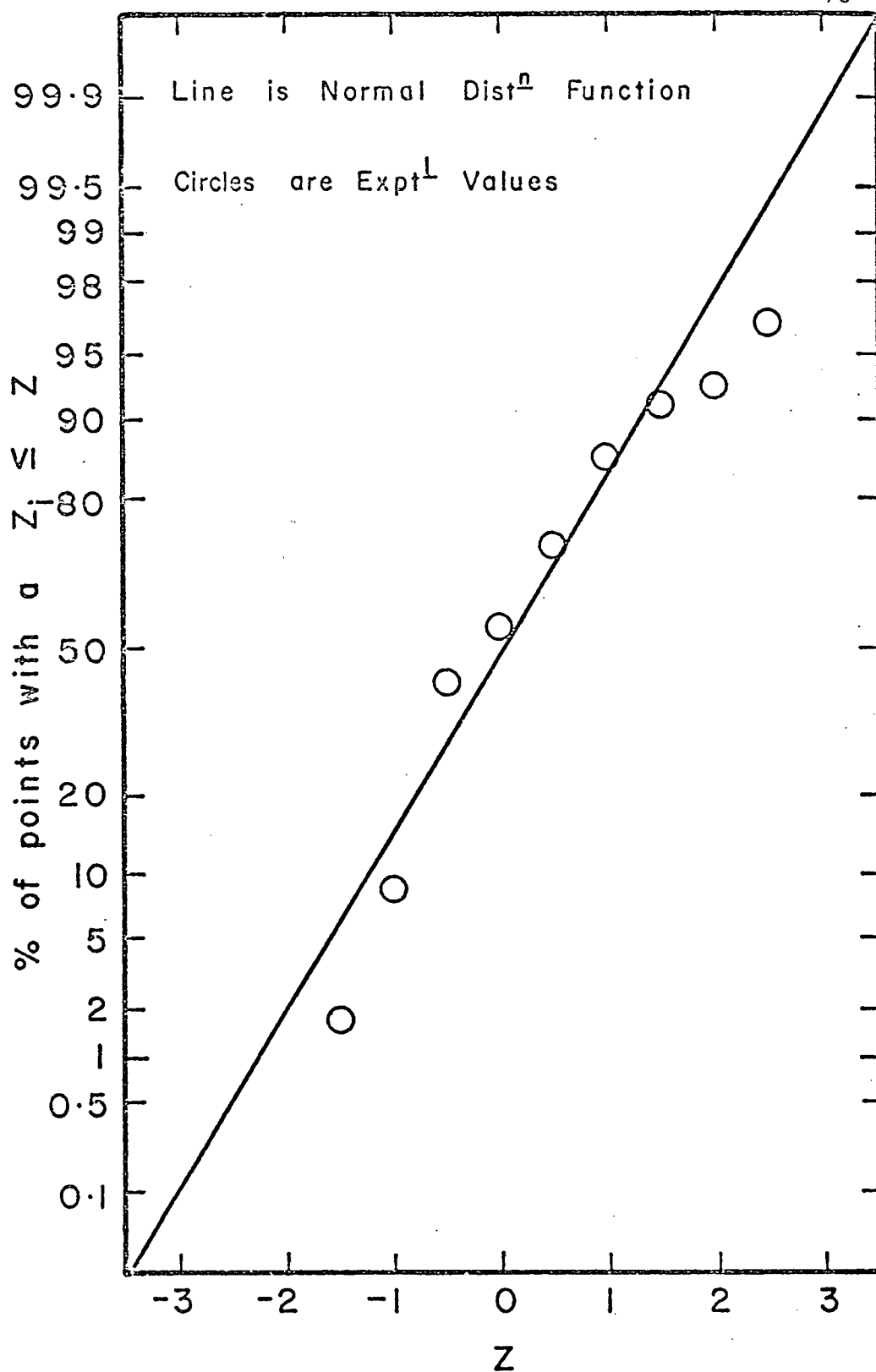


Figure 17

The c.d.f. of Block D (n-Hexane,  $d_p = 0.790$  microns)

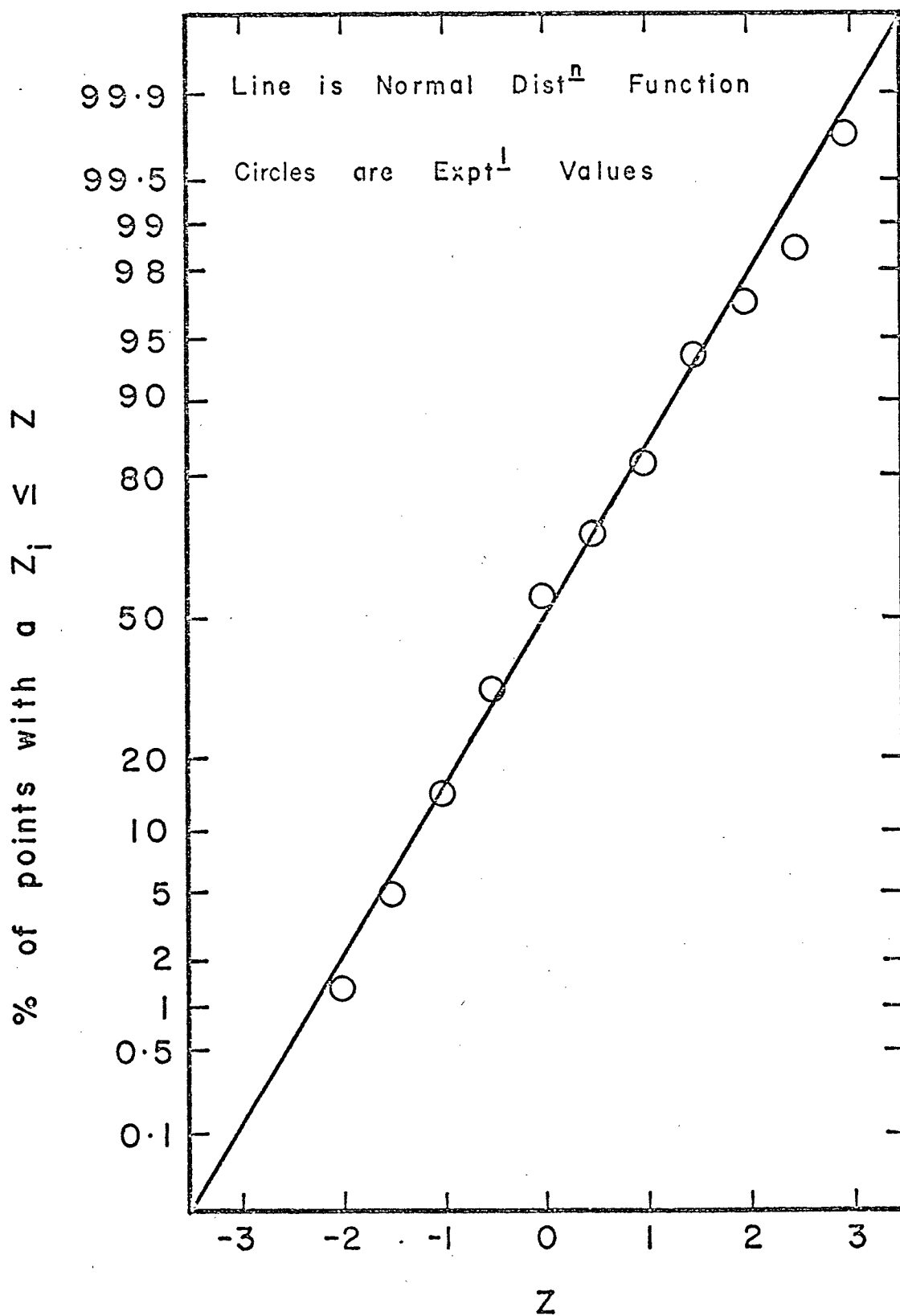


Figure 18

The c.d.f. of Block E (all Water)

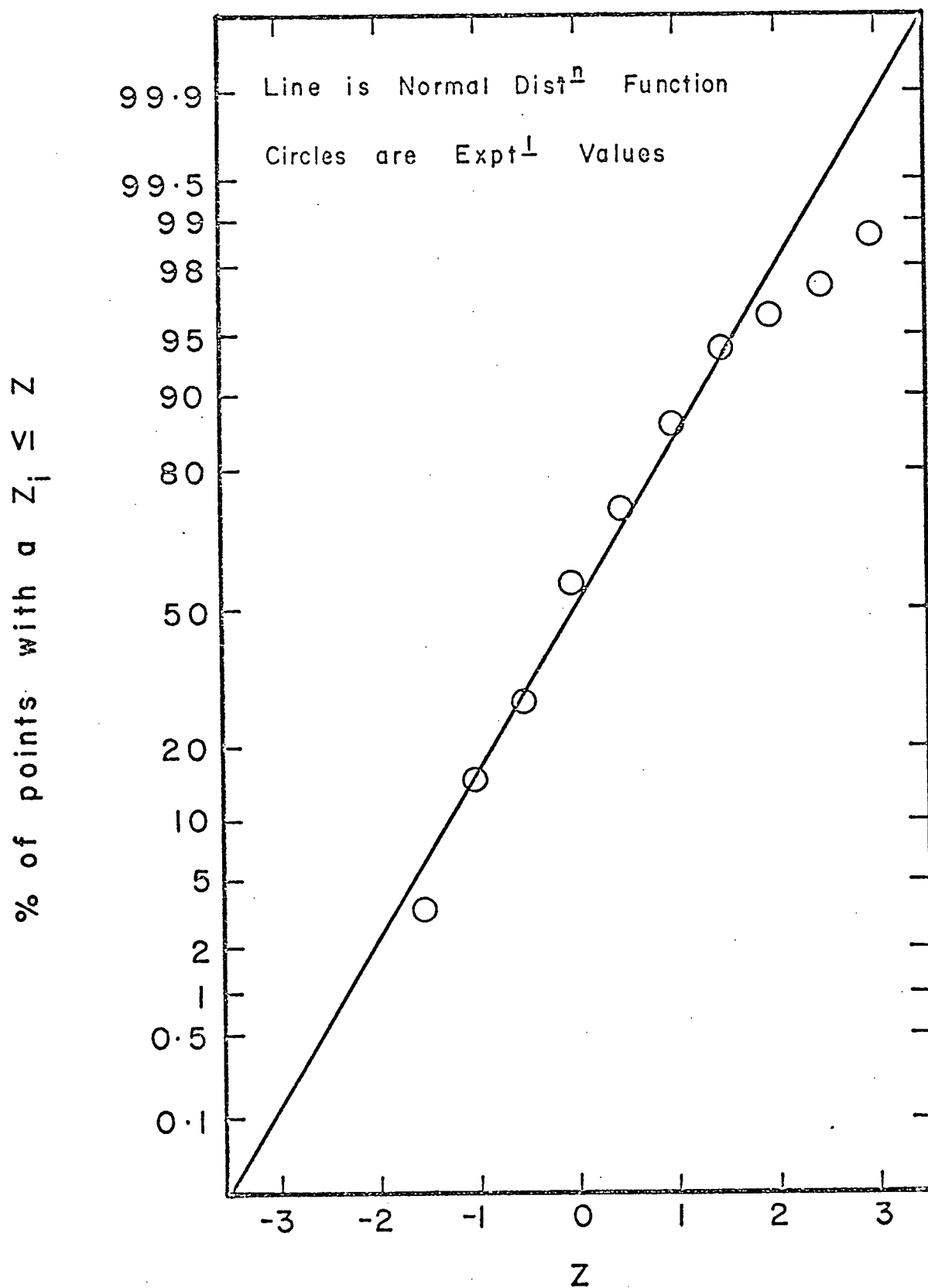


Figure 19

The c.d.f. of Block F (all n-Hexane)

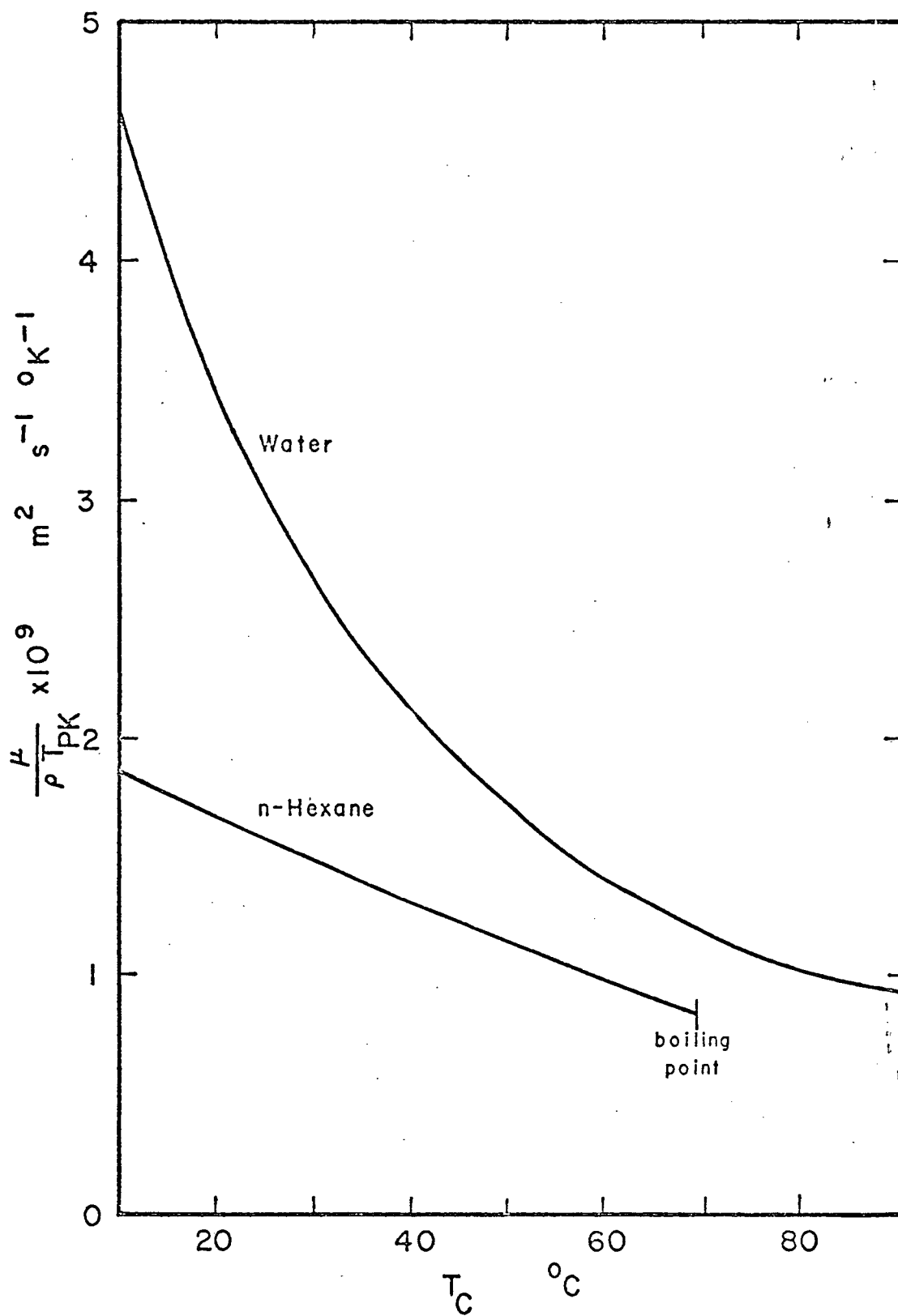


Figure 20

$\mu/p T_{PK}$  as a Function of Temperature for Water and n-Hexane



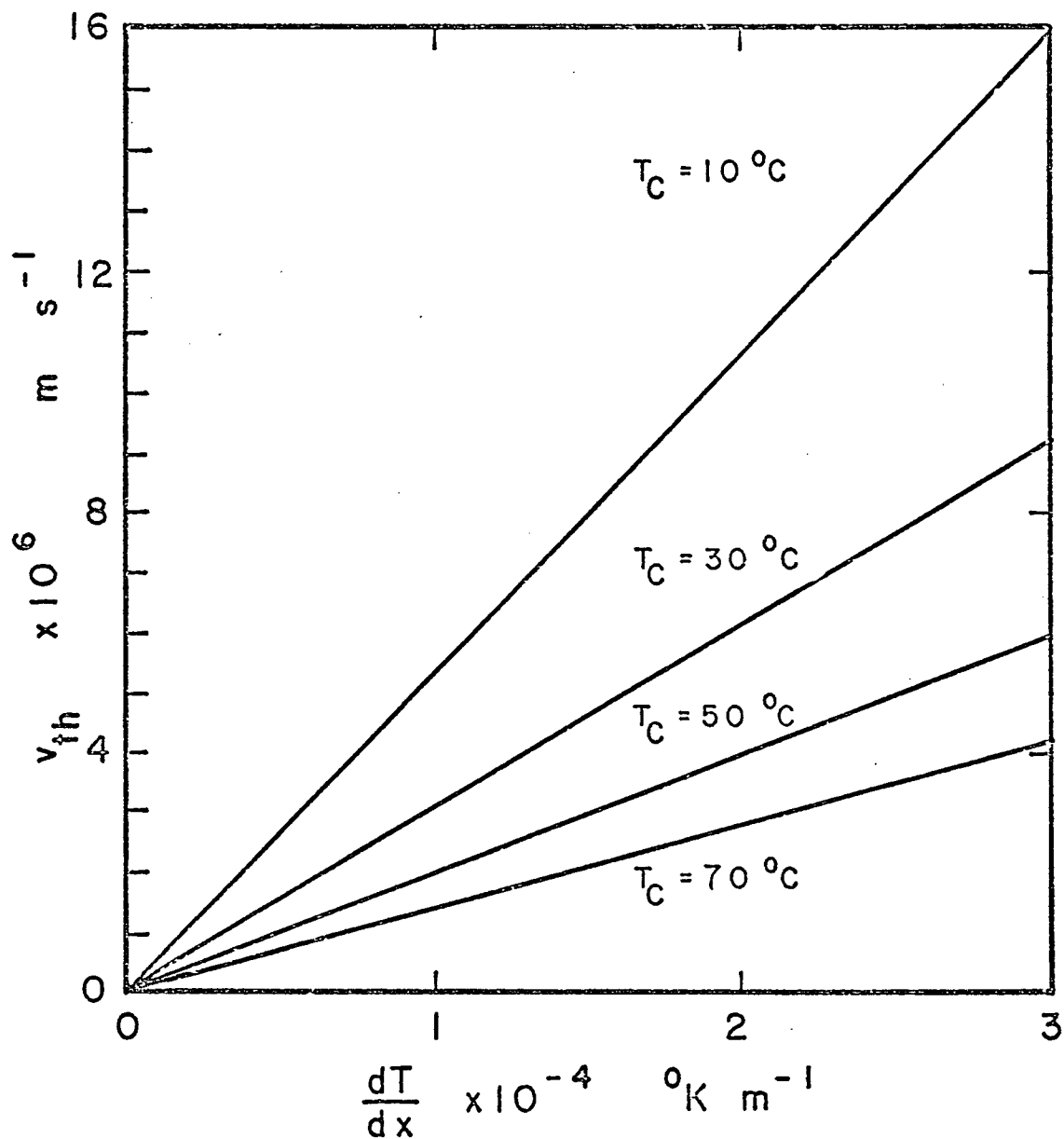


Figure 21

Thermophoretic Velocity in Water as a Function of Temperature Gradient

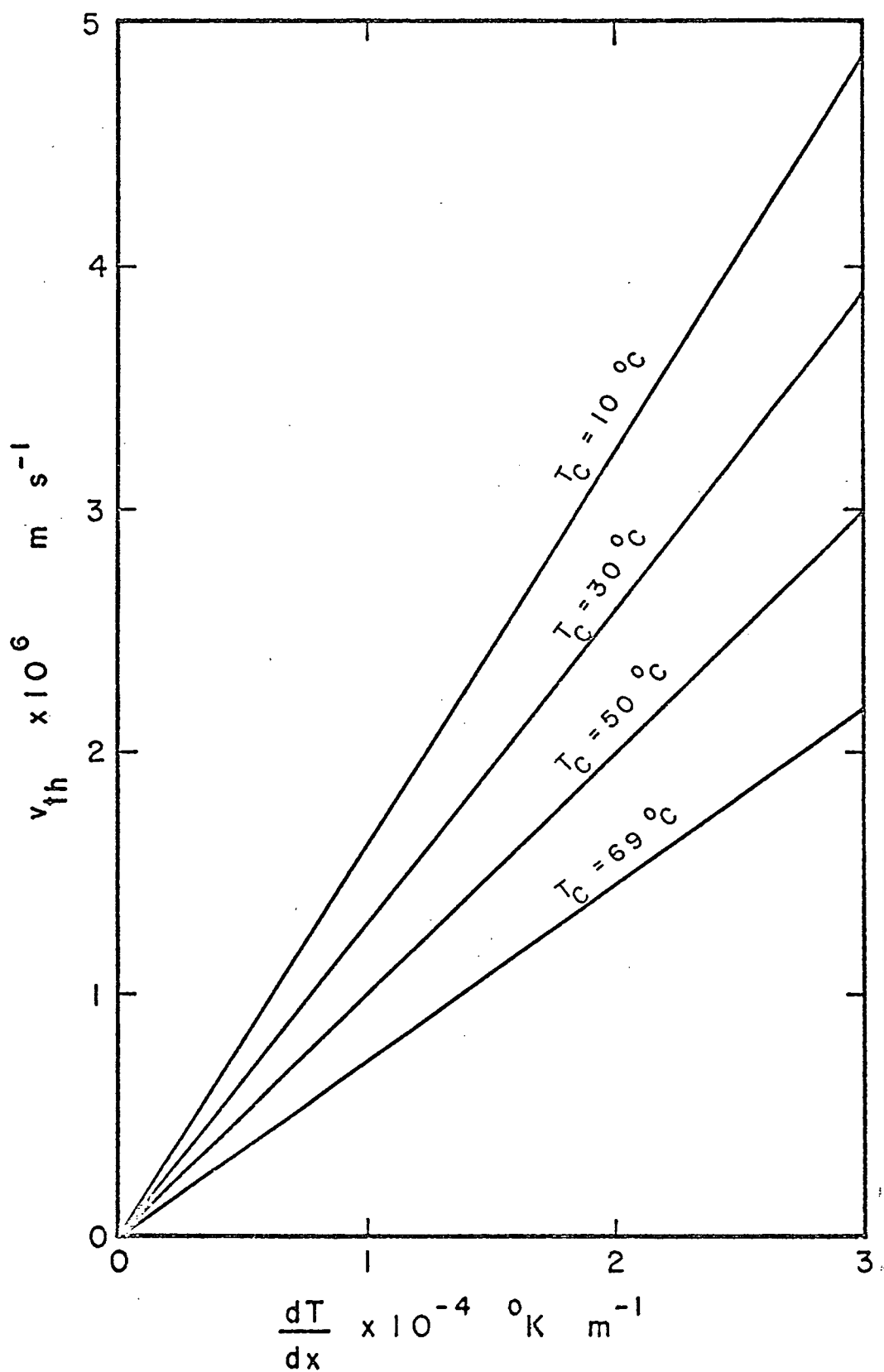


Figure 22

Thermophoretic Velocity in n-Hexane as a Function of Temperature Gradient

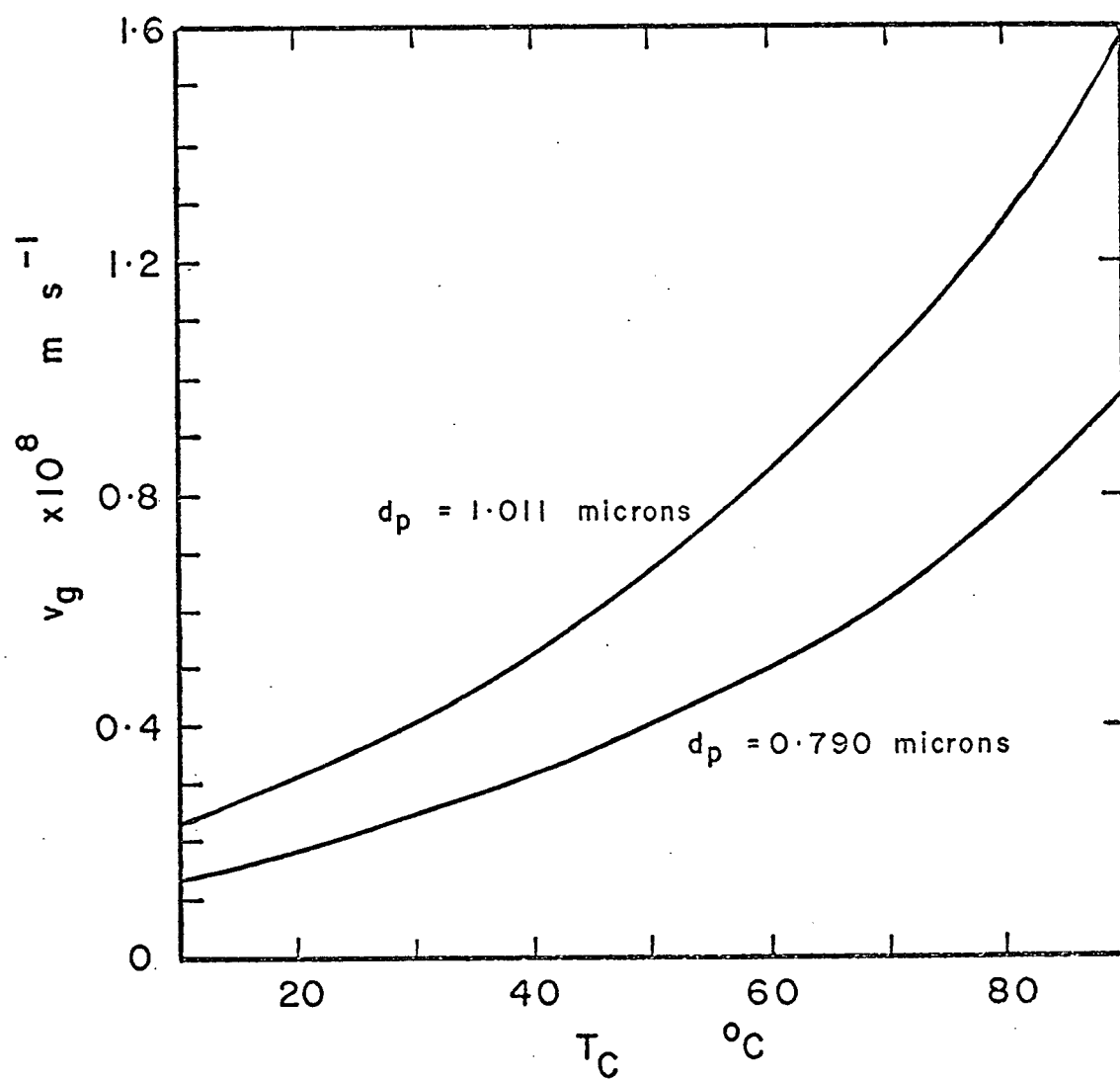


Figure 23

Gravitational Terminal Velocity in Water as a Function of Temperature

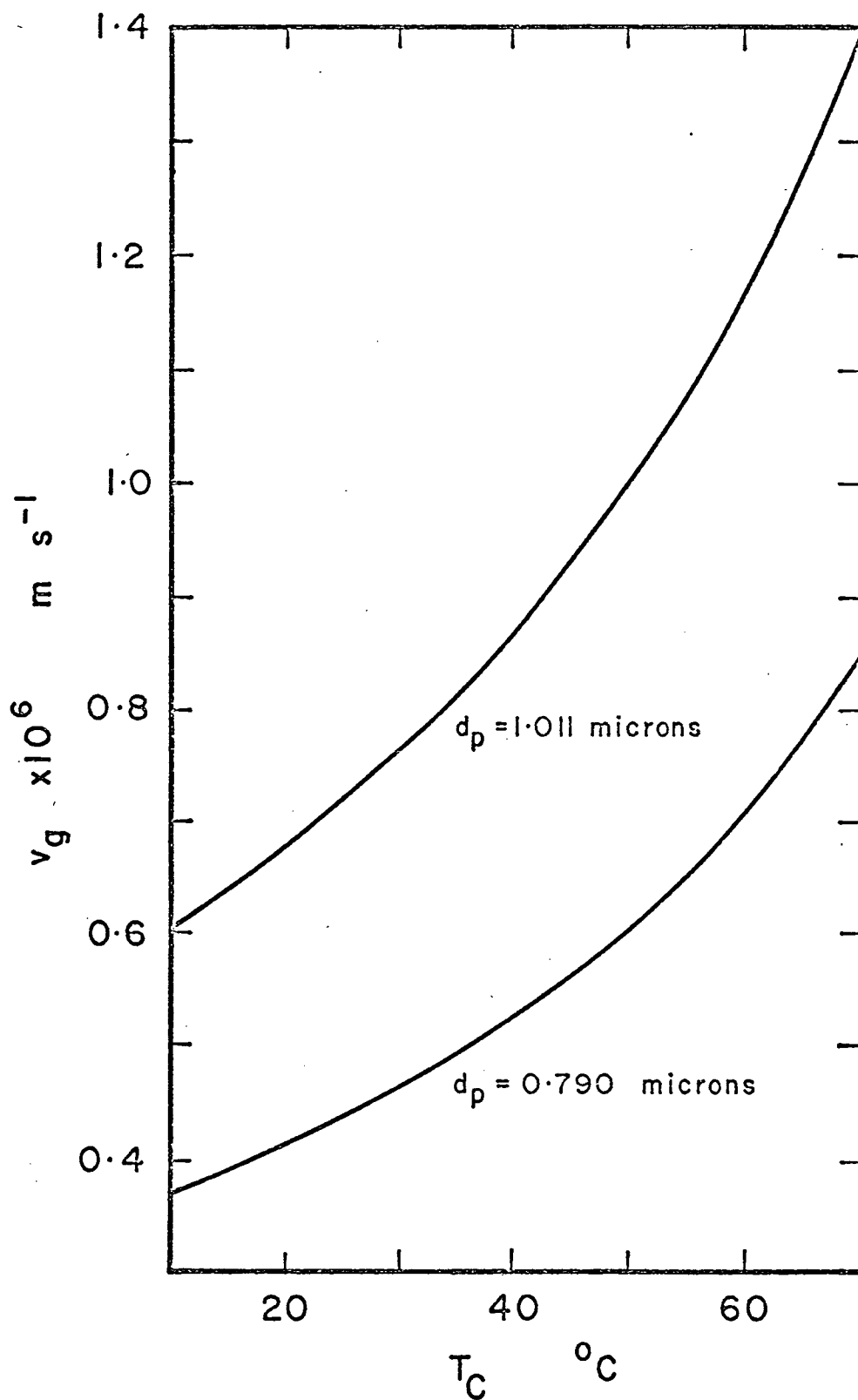


Figure 24

Gravitational Terminal Velocity in n-Hexane as a Function of Temperature

## Chapter VIII

### Conclusions and Recommendations

#### 1. Conclusions

- Thermophoresis was found to occur in liquids.
- The particle thermophoretic velocity in a liquid can be described by the equation:

$$\underline{v}_{th} = - \bar{\alpha} \frac{\mu}{\rho T_K} \nabla T$$

where  $\bar{\alpha}$  equals 0.11 for water and 0.087 for n-hexane.

- Thermophoretic velocity is affected by liquid thermal conductivity.
- Evidence exists that the equation for velocity can be rewritten to account for the thermal conductivity effect:

$$\underline{v}_{th} = - 0.26 \frac{k_f}{2k_f + k_p} \frac{\mu}{\rho T_K} \nabla T$$

- Particle diameter did not effect the thermophoretic velocity.
- The thermophoretic force on a particle in a liquid can be described by the equation:

$$\underline{F}_{th} = - 0.78 \pi \frac{k_f}{2k_f + k_p} d_p \frac{\mu^2}{\rho T_{PK}} \nabla T$$

- The form of the equations for liquids is similar to Epstein's expressions used to describe thermophoresis in gases. This lends credence to the existence of thermal creep in liquids.
- Both thermophoretic velocity and force in liquids are 17 per cent of the values predicted by the Epstein relationships.
- The thermophoretic effect in liquids is very small and is important only in situations where very high temperature gradients exist. It is therefore impractical for use in liquid-particle separations.

## 2. Recommendations

Although the velocity and force equations derived for thermophoresis in liquids may be significant in some engineering problems, the thermophoretic effect is too small to warrant further experimental study by engineers. However, the development of a useful theoretical model for the effect may be worthwhile.

## Nomenclature

- $a$  = dimensionless coefficient used in Equation [3]
- $A$  = coefficient used in Equation [27],  $J\ s^{-1}\ m^{-1}\ ^{\circ}K^{-1}$
- $A_o$  = constant used in Equation [IV-19]
- $B$  = coefficient used in Equation [27],  $J\ s^{-1}\ m^{-1}\ ^{\circ}K^{-2}$
- $B_o$  = constant used in Equation [IV-19]
- $C$  = coefficient used in Equation [27],  $J\ s^{-1}\ m^{-1}\ ^{\circ}K^{-3}$
- c.d.f. = cumulative distribution function, defined in Chapter IV, sub-section 2(c)
- $C_o$  = constant used in Equation [IV-19]
- $C_M$  = experimental dimensionless constant used in Equation [5]
- $C_T$  = experimental dimensionless constant used in Equations [5] and [7]
- $C_w$  = specific heat of water,  $4200\ J\ kg^{-1}\ ^{\circ}C^{-1}$
- $D$  = slope of Equation [34],  $s\ m\ ^{\circ}C\ J^{-1}$
- $D_o$  = constant used in Equation [IV-19]
- $d_p$  = latex particle diameter, microns

$dT/ds$  = temperature gradient tangential to particle surface,  $^{\circ}\text{K m}^{-1}$

$dT/dx$  = one-dimensional temperature gradient in Cartesian coordinates,  $^{\circ}\text{K m}^{-1}$  or  $^{\circ}\text{C m}^{-1}$

$E$  = operator defined by Equation [II-7]

$E_D$  = final position of a particle during an observation, grid spaces from the bottom disk

EMF = top disk thermocouple reading, mV

$E_o$  = intercept of Equation [34],  $^{\circ}\text{C}$

exp = symbol for exponent of e

$f$  = degrees of freedom for experimental  $t$  value, defined by Equation [49]

$F_{ex}$  = experimental  $f$  test value, defined by Equation [51]

$F_f$  = force exerted by fluid on particle, defined by Equation [IV-28],  $\text{kg m s}^{-2}$

$F_D$  = Stokes drag force on particle, defined by Equation [59],  $\text{kg m s}^{-2}$

$F_{th}$  = thermophoretic force on particle, defined by Equation [60],  $\text{kg m s}^{-2}$

$\underline{F}_{th}$  = vectorial form of thermophoretic force,  $\text{kg m s}^{-2}$



- $g$  = acceleration of gravity,  $9.8 \text{ m s}^{-2}$
- $h$  = distance between the disks,  $0.003 \text{ m}$
- $k$  = Boltzman's constant,  $1.38 \times 10^{-23} \text{ J } ^\circ\text{K}^{-1}$
- $k_B$  = thermal conductivity of disk brass,  $\text{J s}^{-1} \text{ m}^{-1} ^\circ\text{K}^{-1}$
- $k_f$  = fluid thermal conductivity,  $\text{J s}^{-1} \text{ m}^{-1} ^\circ\text{K}^{-1}$
- $Kn$  = Knudsen Number, defined by Equation [1]
- $k_o$  = dimensionless variable used in Equation [49], defined by Equation [50]
- $k_p$  = particle thermal conductivity,  $\text{J s}^{-1} \text{ m}^{-1} ^\circ\text{K}^{-1}$
- $L$  = mean free path of fluid molecules,  $\text{m}$
- $M$  = coefficient used in Equation [12],  $^\circ\text{K m}^{-2}$
- $m_p$  = spherical particle mass,  $\text{kg}$
- $m_w$  = mass flux of water through cooling water channel,  $\text{kg s}^{-1}$
- $n$  = number of observations
- $N$  = coefficient used in Equation [10],  $^\circ\text{K m}^{-1}$
- $P_{os}$  = average particle position during an observation, defined by Equation [24], grid spaces from the bottom disk

- $q$  = axial heat flux, defined by Equation [26],  $\text{J s}^{-1} \text{m}^{-2}$
- $r$  = position along the radial axis, defined on Figure 25, m
- $Re$  = particle Reynolds Number, defined in Appendix III, section 2
- $s$  = i) as a variable: standard deviation of the  $\alpha$ 's about  $\bar{\alpha}$  in a block of data, defined by Equation [46]  
 = ii) as a dimension: seconds
- $s^2$  = variance of the  $\alpha$ 's about  $\bar{\alpha}$  in a block of data, defined by Equation [45]
- $S_T$  = initial position of a particle during an observation, grid spaces from the bottom disk
- $t$  = dimensionless variable for t test, defined by Equation [48]
- $t_{95}$  = value from a 95 per cent statistical t table for f degrees of freedom
- $T_B$  = bottom disk temperature, defined by Equation [23],  $^{\circ}\text{C}$
- $T_C$  = temperature,  $^{\circ}\text{C}$
- $T_K$  = temperature,  $^{\circ}\text{K}$
- $T_{K0}$  = temperature at Dwyer's initial point,  $^{\circ}\text{K}$
- $T_{PC}$  = local temperature, determined from Equation [32],  $^{\circ}\text{C}$

- $T_{PK}$  = local temperature, defined by Equation [42], °K
- $T_T$  = top disk temperature, defined by Equation [22], °C
- $T_W$  = cooling water temperature in the bath, °C
- $v_{ex}$  = measured velocity, defined by Equation [40],  $m\ s^{-1}$
- $v_f$  = flow of fluid at  $r = \infty$  with respect to particle,  $m\ s^{-1}$
- $v_g$  = gravitational terminal velocity, defined by Equation [39],  
 $m\ s^{-1}$
- $v_\theta$  = tangential component of fluid velocity, defined on Figure 25,  
 $m\ s^{-1}$
- $v_m$  = upper velocity limit of Stokes Law, defined in Appendix III,  
section 2,  $m\ s^{-1}$
- $v_p$  = velocity of particle relative to stagnant fluid, used in  
Equation [59],  $m\ s^{-1}$
- $v_r$  = radial component of fluid velocity, defined on Figure 25,  
 $m\ s^{-1}$
- $v_s$  = thermal creep velocity, defined by Equation [3],  $m\ s^{-1}$
- $v_{th}$  = thermophoretic velocity,  $m\ s^{-1}$
- $\underline{v}_{th}$  = vectorial form of thermophoretic velocity,  $m\ s^{-1}$

- $x$  = distance from the top disk, m
- $x'$  = Dwyer's particle position, m
- $x'_0$  = Dwyer's initial particle position, m
- $z$  = transformed dimensionless variable for  $\alpha$ , defined by Equation [47]

#### Greek Letters

- $\alpha$  = dimensionless thermophoretic coefficient defined by Equation [43]
- $\bar{\alpha}$  = mean value of  $\alpha$  for a block of data, defined by Equation [44]
- $\beta$  = variable defined by Equation [9],  $s^{-1}$
- $\gamma$  = dimensionless coefficient used in Equation [57]
- $r$  = dimensionless variable used in Equation [63] to modify Equation [3]
- $\underline{\Delta}^2$  = vector operator " $\text{del}^2$ "
- $\Delta T$  = temperature change used in Appendix III, sections 3 and 4,  $^{\circ}\text{C}$
- $\Delta x$  = thickness of disk face, 1/16"
- $\Delta \theta$  = small interval of time defined by Dwyer, s

- $\xi$  = dummy variable used for time in Equation [8], s
- $\theta$  = angular position as defined in Figure 25
- $\Theta$  = time used in Dwyer's work, s
- $\Lambda$  = displacement of particle, defined by Equation [11], m
- $\langle \Lambda^2 \rangle$  = mean square displacement, defined by Equation [8],  $m^2$
- $\mu$  = fluid viscosity,  $kg\ m^{-1}\ s^{-1}$
- $\nu$  = degrees of freedom used in F tests, defined by Equations [52] and 53
- $\pi$  = 3.1416
- $\rho$  = fluid density,  $kg\ m^{-3}$
- $\rho_p$  = particle density,  $kg\ m^{-3}$
- $\Sigma$  = summation operator
- $\tau$  = transit time for particle to move one-half grid space, s
- $\phi$  = left-hand side of Equation [32],  $J\ s^{-1}\ m^{-1}$
- $\psi$  = stream function, defined by Equations [IV-8], [IV-9], and [IV-19]
- $\omega$  = coefficient defined by Equation [IV-5],  $m\ s^{-1}$

## Subscripts

A to F = code letters for each block of data, defined in Table 2

$i$  =  $i^{\text{th}}$  value in a collection of values

1,2 = dummy variables identifying two different blocks of data

LIN = case of temperature independent  $k_f$

## Miscellaneous

$< >$  = operator used in Equation [8]

$\underline{\nabla}$  = vector operator "grad" or "gradient"

## References

1. Sienko, M. J. and R. A. Plane. *Chemistry*, Canadian Edition New York: McGraw-Hill Co. Ltd., 1961.
2. Waldmann, L. and K. H. Schmitt in *Aerosol Science*, ed. C. N. Davies. London: Academic Press, 1966.
3. Springer, G. S. *J. Colloid Sci.*, **34**, 215 (1970).
4. Fuchs, N. A. *The Mechanics of Aerosols*. New York: The MacMillan Co., 1964.
5. Einstein, A. *Z. Phys.*, **27**, 1 (1924).
6. Clusius, K. *Z. Ver. dt. Ing. Beiheft Verfahrenstechnik*, **23** (1941).
7. Waldmann, L. *Z. Naturf.*, **14a**, 589 (1959).
8. Deryaguin, B. V. and S. P. Bakanov. *Dokl. Akad. Nauk. SSSR (Phys.)*, **117**, 959 (1957).
9. Mason, E. and S. Chapman. *J. Chem. Phys.*, **36**, 627 (1962).
10. Brock, J. R. *J. Colloid Sci.*, **23**, 448 (1967).
11. Maxwell, J. C. *Phil. Trans. Roy. Soc. London*, **170**, **1**, 231 (1879).
12. Epstein, P. S. *Z. Physik*, **54**, 537 (1929).
13. Derjaguin, B. V., A. I. Storozhilova, Ya. I. Rabinovich. *J. Colloid Sci.*, **21**, 35 (1966).
14. Keng, E. Y. H. and C. Orr, Jr. *J. Colloid Sci.*, **22**, 107 (1966).
15. Brock, J. R. *J. Colloid Sci.*, **17**, 768 (1962).
16. Derjaguin, B. V. and S. P. Bakanov. *Dokl. Akad. Nauk. SSSR (Phys. Chem.)* **147**, 139 (1962).
17. Derjaguin, B. V. and Yu. Yalamov. *J. Colloid Sci.*, **20**, 555 (1965).
18. Fulford, G. D., M. Moo-Young, and M. Babu. *Can. J. Chem. Eng.*, **49**, 553 (1971).
19. Dwyer, H. A. AEC Tech. Report 107-ME-F, February 1966.

20. Happel, J. and H. Brenner. *Low Reynolds Number Hydrodynamics*. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1965.
21. Chandrasekhar, S. *Reviews of Modern Physics*, 15, 1 (1943).
22. Glasstone, S. *Textbook of Physical Chemistry*, 2nd ed. London: The MacMillan Co., 1960.
23. Zsigmundy, R. A. *Colloids and the Ultramicroscope*. New York: John Wiley and Sons, 1909.
24. Perry, R. H. (ed.). *Chemical Engineers' Handbook*, 4th ed. New York: McGraw-Hill Book Co., 1963.
25. Weast, R. C. (ed.). *Chemical Rubber Company Handbook of Chemistry and Physics*, 51st ed. Cleveland: Chemical Rubber Company, 1970.
26. Kozak, A, University of British Columbia, personal communication, (1971).
27. Engineering Sciences Data, Item 68009. London: The Institution of Mechanical Engineers, 1968.
28. Engineering Sciences Data, Item 67031. London: The Institution of Mechanical Engineers, 1967.
29. Timmermans, J. *Physico-chemical Constants of Pure Organic Compounds*, Vol. I. New York: Elsevier Publishing Co., 1950.
30. Wiessburger, A. (ed.). *Organic Solvents*, 2nd ed. New York: Interscience Publishers, 1955.
31. Reid, R. C. and T. K. Sherwood. *The Properties of Gases and Liquids*, 2nd ed. New York: McGraw-Hill Book Co., 1966.
32. Sakiadis, B. C. and J. Coates. *A.I.Ch.E. Jl.*, 1, 275 (1955).
33. Washburn, E. W. (ed.). *International Critical Tables*. New York: McGraw-Hill Book Co., 1929.
34. Foust, A. S., L. A. Wenzel, C. W. Clump, L. Maus, and L. B. Andersen. *Principles of Unit Operations*. New York: John Wiley and Sons, 1960.
35. *Conversion Tables for Thermocouples*, Issue 4. Philadelphia: Leeds and Northrup Co.
36. Uspensky, J. V. *Theory of Equations*. New York: McGraw-Hill Book Co., 1948.



37. Guttman, I. and S. S. Wilks. *Introductory Engineering Statistics*. New York: John Wiley and Sons, 1965.
38. Wetherill, G. B. *Elementary Statistical Methods*. London: Methuen and Co., 1967.
39. Hopkins, R. M., University of British Columbia, personal communication (1972).
40. Hirschfelder, J. O., C. F. Curtiss, and R. B. Bird. *Molecular Theory of Gases and Liquids*. New York: John Wiley, 1964.
41. Bennett, C. O. and J. E. Myers. *Momentum, Heat, and Mass Transfer*. New York: McGraw-Hill Book Co., 1962.

## APPENDICES

## Appendix I

### Properties of Water and n-Hexane

This Appendix lists most of the data used to obtain the empirical relationships between fluid properties and temperature. Each table gives the experimental data in order of increasing temperature and identifies the source as a reference number. The empirical equations are given in the chapter entitled "Physical Properties of the Experimental Materials," page 32.

Table 4  
The Thermal Conductivity of Water

$T_C$ (°C)	$k_f$ (J s <sup>-1</sup> m <sup>-1</sup> °C <sup>-1</sup> )	Source
0.00	0.569	(25)
0.00	0.569	(28)
6.84	0.575	(25)
10.00	0.587	(25)
10.00	0.587	(28)
16.84	0.593	(25)
20.00	0.604	(28)
24.84	0.610	(25)
30.00	0.619	(28)
36.84	0.624	(25)
37.78	0.629	(24)
37.78	0.630	(25)
40.00	0.632	(28)
46.84	0.638	(25)
50.00	0.644	(21)
56.84	0.649	(25)
60.00	0.654	(28)
65.56	0.659	(25)
66.84	0.660	(25)
70.00	0.663	(28)
76.84	0.669	(25)
80.00	0.670	(28)
86.84	0.676	(25)
90.00	0.676	(28)
93.33	0.681	(24)
93.33	0.678	(25)
96.84	0.681	(25)

Table 5  
The Viscosity of Hexane

$T_C$ (°C)	$T_K$ (°K)	$\mu$ (kg m <sup>-1</sup> s <sup>-1</sup> ) $\times 10^3$	Source
0.00	273.20	0.381	(31)
15.00	288.20	0.337	(29)
15.00	288.20	0.324	(30)
16.35	289.55	0.328	(29)
18.55	291.75	0.320	(29)
20.00	293.20	0.318	(29)
20.00	293.20	0.326	(25)
21.40	294.60	0.313	(29)
23.22	296.42	0.310	(29)
25.00	298.20	0.294	(29)
25.00	298.20	0.292	(30)
25.00	298.20	0.294	(25)
30.00	303.20	0.278	(29)
40.00	313.20	0.271	(25)
40.00	313.20	0.262	(31)
50.00	323.20	0.248	(25)

Table 6  
The Density of Hexane

$T_C$ (°C)	$T_K$ (°K)	$\rho$ (kg m <sup>-3</sup> )	Source
0.0	273.2	676.9	(29)
10.0	283.2	668.3	(29)
15.0	288.2	663.9	(29)
20.0	293.2	659.5	(29)
25.0	298.2	654.9	(29)
30.0	303.2	650.5	(29)
40.0	313.2	641.2	(29)
50.0	323.2	631.8	(29)
60.0	333.2	622.1	(29)
70.0	343.2	612.2	(29)

Table 7  
The Thermal Conductivity of Hexane

$T_C$ (°C)	$k_f$ (J s <sup>-1</sup> m <sup>-1</sup> °C <sup>-1</sup> )	Source
4.0	0.1420	(33) <sup>*</sup>
20.0	0.1310	(31)
30.0	0.1270	(31)
37.8	0.1240	(32) <sup>**</sup>
38.0	0.1235	(31)
57.0	0.1150	(31)
60.0	0.1135	(31)

<sup>\*</sup> Value claimed to have large possible variation.

<sup>\*\*</sup> In addition, slope =  $-0.000436 \text{ J s}^{-1} \text{ m}^{-1} \text{ °C}^{-2}$  for range  
 $T_C = 32.8^\circ\text{C}$  to  $57.2^\circ\text{C}$ .

## Appendix II

### Experimental Data

This appendix contains the experimental data obtained in the course of the present work. The code letters for each block of data are presented in Table 8.



Table 8  
Code Letters for Blocks

Letter	Liquid	dp (microns)
A	Water	1.011
B	Hexane	1.011
C	Water	0.790
D	Hexane	0.790

Table 9  
Experimental Data of Block A,  $(T_T - T_B) = 23.1^\circ\text{C}$

LIQUID = Water									
$d_p$ (microns) = 1.011									
$T_T$ ( $^{\circ}\text{C}$ ) = 30.2									
$T_B$ ( $^{\circ}\text{C}$ ) = 7.1									
$n = 36$									
$P_{Os}$	$\tau$ (s)	$T_{PC}$ ( $^{\circ}\text{C}$ )	$dT/dx$ ( $^{\circ}\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1}\text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\nu/\rho T_{PK}$ ( $\text{m}^2\text{s}^{-1}\text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.25	50.	11.1	7860.	0.587	998.6	12.66	4.00	4.46	0.1141
1.75	43.	12.6	7825.	0.590	998.5	12.12	4.65	4.25	0.1399
1.75	62.	12.6	7825.	0.590	998.5	12.12	3.23	4.25	0.0972
2.25	56.	14.2	7790.	0.593	998.3	11.62	3.57	4.05	0.1131
2.25	48.	14.2	7790.	0.593	998.3	11.62	4.17	4.05	0.1321
2.25	65.	14.2	7790.	0.593	998.3	11.62	3.08	4.05	0.0976
2.25	52.	14.2	7790.	0.593	998.3	11.62	3.85	4.05	0.1220
2.75	76.	15.7	7757.	0.595	998.1	11.16	2.63	3.87	0.0376
2.75	50.	15.7	7757.	0.595	998.1	11.16	4.00	3.87	0.1333
2.75	62.	15.7	7757.	0.595	998.1	11.16	3.23	3.87	0.1076
2.75	56.	15.7	7757.	0.595	998.1	11.16	3.57	3.87	0.1189
2.75	57.	15.7	7757.	0.595	998.1	11.16	3.51	3.87	0.1169
2.75	68.	15.7	7757.	0.595	998.1	11.16	2.94	3.87	0.0980
3.25	56.	17.3	7724.	0.598	997.9	10.73	3.57	3.70	0.1249
3.25	79.	17.3	7724.	0.598	997.9	10.73	2.53	3.70	0.0885
3.25	65.	17.3	7724.	0.598	997.9	10.73	3.08	3.70	0.1078
3.25	70.	17.3	7724.	0.598	997.9	10.73	2.86	3.70	0.1001
3.25	58.	17.3	7724.	0.598	997.9	10.73	3.45	3.70	0.1207
3.25	60.	17.3	7724.	0.598	997.9	10.73	3.33	3.70	0.1165
3.75	50.	18.8	7692.	0.600	997.7	10.32	4.00	3.54	0.1468
3.75	58.	18.8	7692.	0.600	997.7	10.32	3.45	3.54	0.1266
3.75	78.	18.8	7692.	0.600	997.7	10.32	2.56	3.54	0.0940
3.75	67.	18.8	7692.	0.600	997.7	10.32	2.99	3.54	0.1097
3.75	68.	18.8	7692.	0.600	997.7	10.32	2.94	3.54	0.1073
3.75	59.	18.8	7692.	0.600	997.7	10.32	3.39	3.54	0.1244
4.25	69.	20.4	7661.	0.603	997.4	9.94	2.90	3.39	0.1115
4.25	59.	20.4	7661.	0.603	997.4	9.94	3.39	3.39	0.1304
4.25	73.	20.4	7661.	0.603	997.4	9.94	2.74	3.39	0.1054
4.25	60.	20.4	7661.	0.603	997.4	9.94	3.33	3.39	0.1280
4.75	79.	21.9	7631.	0.605	997.1	9.58	2.53	3.26	0.1018
4.75	73.	21.9	7631.	0.605	997.1	9.58	2.74	3.26	0.1103
5.25	70.	23.4	7602.	0.607	996.8	9.25	2.86	3.13	0.1203
5.25	59.	23.4	7602.	0.607	996.8	9.25	3.39	3.13	0.1426
5.25	68.	23.4	7602.	0.607	996.8	9.25	2.94	3.13	0.1237
5.75	82.	24.9	7573.	0.610	996.5	8.93	2.44	3.01	0.1072
5.75	76.	24.9	7573.	0.610	996.5	8.93	2.63	3.01	0.1155

Table 10

Experimental Data of Block A,  $(T_T - T_B) = 32.7^\circ\text{C}$ 

LIQUID = Water  $d_p$  (microns) = 1.011  
 $T_T$  ( $^\circ\text{C}$ ) = 40.0  $T_B$  ( $^\circ\text{C}$ ) = 7.3  
 $n = 34$

$P_{os}$	$\tau$ (s)	$T_{PC}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$u$ ( $\text{kg m}^{-1}\text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\mu/\rho T_{PK}$ ( $\text{m}^2\text{s}^{-1}\text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.75	36.	15.2	11135.	0.594	998.2	11.32	5.56	3.93	0.1270
1.75	36.	15.2	11135.	0.594	998.2	11.32	5.56	3.93	0.1270
1.75	51.	15.2	11135.	0.594	998.2	11.32	3.92	3.93	0.0895
2.25	40.	17.4	11067.	0.598	997.9	10.69	5.00	3.69	0.1225
2.25	49.	17.4	11067.	0.598	997.9	10.69	4.08	3.69	0.1000
2.25	49.	17.4	11067.	0.598	997.9	10.69	4.08	3.69	0.1000
2.75	55.	19.6	11002.	0.601	997.5	10.12	3.64	3.47	0.0955
2.75	50.	19.6	11002.	0.601	997.5	10.12	4.00	3.47	0.1049
2.75	44.	19.6	11002.	0.601	997.5	10.12	4.55	3.47	0.1193
2.75	40.	19.6	11002.	0.601	997.5	10.12	5.00	3.47	0.1311
2.75	58.	19.6	11002.	0.601	997.5	10.12	3.45	3.47	0.0905
2.75	45.	19.6	11002.	0.601	997.5	10.12	4.44	3.47	0.1164
2.75	52.	19.6	11002.	0.601	997.5	10.12	3.85	3.47	0.1010
3.25	60.	21.8	10940.	0.605	997.1	9.60	3.33	3.26	0.0933
3.25	57.	21.8	10940.	0.605	997.1	9.60	3.51	3.26	0.0983
3.25	45.	21.8	10940.	0.605	997.1	9.60	4.44	3.26	0.1243
3.25	41.	21.8	10940.	0.605	997.1	9.60	4.88	3.26	0.1367
3.25	62.	21.8	10940.	0.605	997.1	9.60	3.23	3.26	0.0905
3.75	60.	24.0	10879.	0.608	996.7	9.13	3.33	3.08	0.0993
3.75	49.	24.0	10879.	0.608	996.7	9.13	4.08	3.08	0.1217
3.75	45.	24.0	10879.	0.608	996.7	9.13	4.44	3.08	0.1325
3.75	60.	24.0	10879.	0.608	996.7	9.13	3.33	3.08	0.0993
4.25	63.	26.2	10821.	0.611	996.3	8.69	3.17	2.91	0.1006
4.25	56.	26.2	10821.	0.611	996.3	8.69	3.57	2.91	0.1132
4.25	60.	26.2	10821.	0.611	996.3	8.69	3.33	2.91	0.1056
4.25	61.	26.2	10821.	0.611	996.3	8.69	3.20	2.91	0.1040
4.75	65.	28.3	10765.	0.615	995.8	8.29	3.08	2.76	0.1037
4.75	50.	28.3	10765.	0.615	995.8	8.29	4.00	2.76	0.1346
4.75	65.	28.3	10765.	0.615	995.8	8.29	3.08	2.76	0.1037
4.75	58.	28.3	10765.	0.615	995.8	8.29	3.45	2.76	0.1161
4.75	61.	28.3	10765.	0.615	995.8	8.29	3.28	2.76	0.1104
5.25	58.	30.5	10711.	0.618	995.2	7.91	3.45	2.62	0.1230
5.25	70.	30.5	10711.	0.618	995.2	7.91	2.86	2.62	0.1020
5.25	65.	30.5	10711.	0.618	995.2	7.91	3.08	2.62	0.1098

Table 11

Experimental Data of Block A,  $(T_T - T_B) = 39.7^\circ\text{C}$ 

LIQUID = Water  
 $d_p$  (microns) = 1.011  
 $T_T$  ( $^\circ\text{C}$ ) = 47.0  
 $T_B$  ( $^\circ\text{C}$ ) = 7.3  
 $n = 35$

$P_{os}$	$\tau$ (s)	$T_{PC}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1} \text{m}^{-1} \text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1} \text{s}^{-1}$ ) $\times 10^4$	$V_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\nu/\rho T_{PK}$ ( $\text{m}^2 \text{s}^{-1} \text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.75	30.	16.9	13563.	0.597	997.9	10.82	6.67	3.74	0.1315
2.25	48.	19.6	13466.	0.601	997.5	10.12	4.17	3.46	0.0894
2.25	31.	19.6	13466.	0.601	997.5	10.12	6.45	3.46	0.1383
2.25	39.	19.6	13466.	0.601	997.5	10.12	5.13	3.46	0.1100
2.75	57.	22.3	13372.	0.606	997.0	9.49	3.51	3.22	0.0815
2.75	57.	22.3	13372.	0.606	997.0	9.49	3.51	3.22	0.0815
2.75	33.	22.3	13372.	0.606	997.0	9.49	6.06	3.22	0.1407
2.75	36.	22.3	13372.	0.606	997.0	9.49	5.56	3.22	0.1291
2.75	51.	22.3	13372.	0.606	997.0	9.49	3.92	3.22	0.0910
2.75	42.	22.3	13372.	0.606	997.0	9.49	4.76	3.22	0.1105
2.75	40.	22.3	13372.	0.606	997.0	9.49	5.00	3.22	0.1161
3.25	42.	25.0	13283.	0.610	996.5	8.92	4.76	3.00	0.1193
3.75	39.	27.6	13198.	0.614	995.9	8.41	5.13	2.81	0.1384
3.75	56.	27.6	13198.	0.614	995.9	8.41	3.57	2.81	0.0963
3.75	41.	27.6	13198.	0.614	995.9	8.41	4.88	2.81	0.1317
3.75	61.	27.6	13198.	0.614	995.9	8.41	3.28	2.81	0.0885
3.75	39.	27.6	13198.	0.614	995.9	8.41	5.13	2.81	0.1384
3.75	36.	27.6	13198.	0.614	995.9	8.41	5.56	2.81	0.1500
3.75	48.	27.6	13198.	0.614	995.9	8.41	4.17	2.81	0.1125
4.25	46.	30.3	13117.	0.617	995.3	7.95	4.35	2.63	0.1260
4.25	64.	30.3	13117.	0.617	995.3	7.95	3.13	2.63	0.0907
4.25	67.	30.3	13117.	0.617	995.3	7.95	2.99	2.63	0.0866
4.25	46.	30.3	13117.	0.617	995.3	7.95	4.35	2.63	0.1260
4.25	61.	30.3	13117.	0.617	995.3	7.95	3.28	2.63	0.0950
4.75	65.	32.9	13039.	0.621	994.6	7.53	3.08	2.47	0.0955
4.75	61.	32.9	13039.	0.621	994.6	7.53	3.28	2.47	0.1017
4.75	48.	32.9	13039.	0.621	994.6	7.53	4.17	2.47	0.1293
4.75	56.	32.9	13039.	0.621	994.6	7.53	3.57	2.47	0.1107
4.75	61.	32.9	13039.	0.621	994.6	7.53	3.28	2.47	0.1017
4.75	56.	32.9	13039.	0.621	994.6	7.53	3.57	2.47	0.1107
5.25	63.	35.5	12964.	0.625	993.8	7.15	3.17	2.33	0.1050
5.25	69.	35.5	12964.	0.625	993.8	7.15	2.90	2.33	0.0960
5.25	45.	35.5	12964.	0.625	993.8	7.15	4.44	2.33	0.1470
5.25	71.	35.5	12964.	0.625	993.8	7.15	2.82	2.33	0.0934
5.75	67.	38.1	12893.	0.628	993.0	6.79	2.99	2.20	0.1055

Table 12

Experimental Data of Block A,  $(T_T - T_B) = 49.2^\circ\text{C}$ 

LIQUID = Water       $d_p$  (microns) = 1.011  
 $T_T$  ( $^\circ\text{C}$ ) = 56.5       $T_B$  ( $^\circ\text{C}$ ) = 7.3  
 $n = 36$

$P_{os}$	$\tau$ (s)	$T_{PC}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1}\text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\nu/\rho T_{PK}$ ( $\text{m}^2\text{s}^{-1}\text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.75	29.	19.3	16875.	0.601	997.6	10.20	6.90	3.49	0.1170
1.75	28.	19.3	16875.	0.601	997.6	10.20	7.14	3.49	0.1211
2.25	29.	22.7	16729.	0.606	997.0	9.41	6.90	3.19	0.1293
2.25	36.	22.7	16729.	0.606	997.0	9.41	5.56	3.19	0.1042
2.25	39.	22.7	16729.	0.606	997.0	9.41	5.13	3.19	0.0961
2.75	51.	26.0	16591.	0.611	996.3	8.72	3.77	2.92	0.0777
2.75	26.	26.0	16591.	0.611	996.3	8.72	7.69	2.92	0.1585
2.75	30.	26.0	16591.	0.611	996.3	8.72	6.67	2.92	0.1375
2.75	31.	26.0	16591.	0.611	996.3	8.72	6.45	2.92	0.1329
2.75	39.	26.0	16591.	0.611	996.3	8.72	5.13	2.92	0.1057
2.75	44.	26.0	16591.	0.611	996.3	8.72	4.55	2.92	0.0938
2.75	35.	26.0	16591.	0.611	996.3	8.72	5.71	2.92	0.1177
3.25	36.	29.3	16460.	0.616	995.5	8.11	5.56	2.69	0.1254
3.25	49.	29.3	16460.	0.616	995.5	8.11	4.08	2.69	0.0920
3.25	31.	29.3	16460.	0.616	995.5	8.11	6.45	2.69	0.1455
3.25	44.	29.3	16460.	0.616	995.5	8.11	4.55	2.69	0.1026
3.75	52.	32.6	16336.	0.621	994.6	7.57	3.85	2.49	0.0947
3.75	37.	32.6	16336.	0.621	994.6	7.57	5.41	2.49	0.1330
3.75	45.	32.6	16336.	0.621	994.6	7.57	4.44	2.49	0.1092
3.75	39.	32.6	16336.	0.621	994.6	7.57	5.13	2.49	0.1261
4.25	59.	35.8	16220.	0.625	993.7	7.09	3.39	2.31	0.0905
4.25	44.	35.8	16220.	0.625	993.7	7.09	4.55	2.31	0.1215
4.25	39.	35.8	16220.	0.625	993.7	7.09	5.13	2.31	0.1369
4.75	45.	39.1	16109.	0.629	992.7	6.66	4.44	2.15	0.1202
4.75	58.	39.1	16109.	0.629	992.7	6.66	3.45	2.15	0.0996
4.75	43.	39.1	16109.	0.629	992.7	6.66	4.65	2.15	0.1343
4.75	63.	39.1	16109.	0.629	992.7	6.66	3.17	2.15	0.0915
4.75	42.	39.1	16109.	0.629	992.7	6.66	4.76	2.15	0.1375
4.75	58.	39.1	16109.	0.629	992.7	6.66	3.45	2.15	0.0996
5.25	61.	42.3	16004.	0.634	991.6	6.28	3.28	2.01	0.1021
5.25	63.	42.3	16004.	0.634	991.6	6.28	3.17	2.01	0.0987
5.25	63.	42.3	16004.	0.634	991.6	6.28	3.17	2.01	0.0987
5.75	68.	45.5	15905.	0.637	990.4	5.93	2.94	1.88	0.0984
5.75	45.	45.5	15905.	0.637	990.4	5.93	4.44	1.88	0.1486
5.75	65.	45.5	15905.	0.637	990.4	5.93	3.08	1.88	0.1031
5.75	50.	45.5	15905.	0.637	990.4	5.93	4.00	1.88	0.1339

Table 13

Experimental Data of Block A,  $(T_T - T_B) = 50.2^\circ\text{C}$ 

LIQUID = Water  
 $d_p$  (microns) = 1.011  
 $T_T$  ( $^\circ\text{C}$ ) = 57.4  
 $T_B$  ( $^\circ\text{C}$ ) = 7.2  
 $n = 33$

$P_{os}$	$\tau$ (s)	$T_{pc}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1} \text{m}^{-1} \text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1} \text{s}^{-1}$ ) $\times 10^{-4}$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\nu/\rho T_{pk}$ ( $\text{m}^2 \text{s}^{-1} \text{K}^{-1}$ ) $\times 10^9$	$\alpha$
2.75	50.	22.9	17073.	0.606	996.9	9.36	4.00	3.17	0.0739
2.25	46.	22.9	17071.	0.606	996.9	9.36	4.35	3.17	0.0804
2.75	53.	26.3	16930.	0.612	996.2	8.66	3.77	2.90	0.0767
2.75	46.	26.3	16930.	0.612	996.2	8.66	4.35	2.90	0.0885
2.75	36.	26.3	16930.	0.612	996.2	8.66	5.56	2.90	0.1131
3.25	51.	29.7	16794.	0.617	995.4	8.05	3.92	2.67	0.0874
3.25	32.	29.7	16794.	0.617	995.4	8.05	6.25	2.67	0.1394
3.25	51.	29.7	16794.	0.617	995.4	8.05	3.92	2.67	0.0874
3.25	33.	29.7	16794.	0.617	995.4	8.05	6.06	2.67	0.1362
3.75	48.	33.0	16666.	0.621	994.5	7.51	4.17	2.47	0.1015
3.75	36.	33.0	16666.	0.621	994.5	7.51	5.56	2.47	0.1353
3.75	30.	33.0	16666.	0.621	994.5	7.51	6.67	2.47	0.1673
3.75	41.	33.0	16666.	0.621	994.5	7.51	4.88	2.47	0.1188
3.75	49.	33.0	16666.	0.621	994.5	7.51	4.08	2.47	0.0993
3.75	32.	33.0	16666.	0.621	994.5	7.51	6.25	2.47	0.1521
3.75	32.	33.0	16666.	0.621	994.5	7.51	6.25	2.47	0.1521
3.75	59.	33.0	16666.	0.621	994.5	7.51	3.39	2.47	0.0825
3.75	42.	33.0	16666.	0.621	994.5	7.51	4.76	2.47	0.1159
4.25	41.	36.3	16545.	0.626	993.5	7.03	5.26	2.28	0.1291
4.25	38.	36.3	16545.	0.626	993.5	7.03	3.70	2.28	0.1392
4.75	54.	39.6	16431.	0.630	992.5	6.59	4.55	2.12	0.1304
4.75	44.	39.6	16431.	0.630	992.5	6.59	5.71	2.12	0.1636
4.75	35.	39.6	16431.	0.630	992.5	6.59	3.92	2.12	0.1123
4.75	51.	39.6	16431.	0.630	992.5	6.59	5.71	2.12	0.1636
4.75	35.	39.6	16431.	0.630	992.5	6.59	5.71	2.12	0.1636
5.25	79.	42.9	16323.	0.634	991.3	6.21	2.53	1.98	0.0782
5.25	62.	42.9	16323.	0.634	991.3	6.21	3.23	1.98	0.0999
5.25	45.	42.9	16323.	0.634	991.3	6.21	4.44	1.98	0.1373
5.25	39.	42.9	16323.	0.634	991.3	6.21	5.13	1.98	0.1507
5.75	46.	46.2	16221.	0.638	990.1	5.86	4.35	1.85	0.1345
5.75	45.	46.2	16221.	0.638	990.1	5.86	4.44	1.85	0.1477
5.75	62.	46.2	16221.	0.638	990.1	5.86	3.23	1.85	0.1075
5.75	50.	46.2	16221.	0.638	990.1	5.86	4.00	1.85	0.1331

Table 14

Experimental Data of Block A,  $(T_T - T_B) = 55.7^\circ\text{C}$ 

LIQUID = Water  
 $d_p$  (microns) = 1.011  
 $T_T$  ( $^\circ\text{C}$ ) = 63.3  
 $T_B$  ( $^\circ\text{C}$ ) = 7.6  
 $n = 34$

$P_{os}$	$\tau$ (s)	$T_{PC}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1} \text{m}^{-1} \text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1} \text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\nu/\rho$ $T_{PK}$ ( $\text{m}^2 \text{s}^{-1} \text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.75	19.	21.3	19146.	0.604	997.2	9.73	10.53	3.31	0.1660
1.75	29.	21.3	19146.	0.604	997.2	9.73	6.90	3.31	0.1088
2.25	21.	25.1	18963.	0.610	996.5	8.90	9.52	3.00	0.1676
2.25	38.	25.1	18963.	0.610	996.5	8.90	5.26	3.00	0.0926
2.25	33.	25.1	18963.	0.610	996.5	8.90	6.06	3.00	0.1067
2.25	28.	25.1	18963.	0.610	996.5	8.90	7.14	3.00	0.1257
2.25	32.	25.1	18963.	0.610	996.5	8.90	6.25	3.00	0.1100
2.25	41.	25.1	18963.	0.610	996.5	8.90	4.88	3.00	0.0859
2.75	45.	28.8	18791.	0.615	995.6	8.19	4.44	2.72	0.0867
2.75	34.	28.8	18791.	0.615	995.6	8.19	5.88	2.72	0.1149
2.75	35.	28.8	18791.	0.615	995.6	8.19	5.71	2.72	0.1115
2.75	35.	28.8	18791.	0.615	995.6	8.19	5.71	2.72	0.1115
3.25	36.	32.6	18630.	0.621	994.6	7.57	5.56	2.49	0.1199
3.25	44.	32.6	18630.	0.621	994.6	7.57	4.55	2.49	0.0981
3.25	34.	32.6	18630.	0.621	994.6	7.57	5.88	2.49	0.1268
3.25	29.	32.6	18630.	0.621	994.6	7.57	6.90	2.49	0.1487
3.75	43.	36.3	18478.	0.626	993.6	7.03	4.65	2.29	0.1101
3.75	45.	36.3	18478.	0.626	993.6	7.03	4.44	2.29	0.1051
3.75	35.	36.3	18478.	0.626	993.6	7.03	5.71	2.29	0.1351
3.75	31.	36.3	18478.	0.626	993.6	7.03	6.06	2.29	0.1434
3.75	47.	36.3	18478.	0.626	993.6	7.03	4.26	2.29	0.1008
4.25	39.	40.0	18336.	0.631	992.4	6.55	5.13	2.11	0.1327
4.25	45.	40.0	18336.	0.631	992.4	6.55	4.44	2.11	0.1149
4.25	45.	40.0	18336.	0.631	992.4	6.55	4.44	2.11	0.1149
4.75	41.	43.6	18203.	0.635	991.1	6.13	4.88	1.95	0.1374
4.75	42.	43.6	18203.	0.635	991.1	6.13	4.76	1.95	0.1340
4.75	43.	43.6	18203.	0.635	991.1	6.13	4.65	1.95	0.1309
4.75	42.	43.6	18203.	0.635	991.1	6.13	4.76	1.95	0.1340
4.75	58.	43.6	18203.	0.635	991.1	6.13	3.45	1.95	0.0971
5.25	59.	47.3	18077.	0.640	989.7	5.75	3.39	1.81	0.1035
5.25	45.	47.3	18077.	0.640	989.7	5.75	4.44	1.81	0.1355
5.25	59.	47.3	18077.	0.640	989.7	5.75	3.39	1.81	0.1035
5.75	72.	50.9	17960.	0.644	988.1	5.41	2.78	1.69	0.0917
5.75	51.	50.9	17960.	0.644	988.1	5.41	3.92	1.69	0.1252

Table 15  
Experimental Data of Block A,  $(T_T - T_B) = 57.3^\circ\text{C}$

LIQUID = Water											
$d_p$ (microns) = 1.011											
$T_T$ ( $^{\circ}\text{C}$ ) = 64.5											
$T_B$ ( $^{\circ}\text{C}$ ) = 7.2											
$n = 29$											
$P_{05}$	$\tau$ (s)	$T_{PC}$ ( $^{\circ}\text{C}$ )	$dT/dx$ ( $^{\circ}\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1}\text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\mu/\rho T_{PK}$ ( $\text{m}^2\text{s}^{-1}\text{K}^{-1}$ ) $\times 10^9$	$\alpha$		
1.75	31.	21.3	19709.	0.604	997.2	9.73	6.45	3.31	0.0988		
1.75	33.	21.3	19709.	0.604	997.2	9.73	6.06	3.31	0.0928		
1.75	28.	21.3	19709.	0.604	997.2	9.73	7.14	3.31	0.1094		
2.25	31.	25.2	19515.	0.610	996.5	8.88	6.45	2.99	0.1107		
2.25	31.	25.2	19515.	0.610	996.5	8.88	6.45	2.99	0.1107		
2.25	32.	25.2	19515.	0.610	996.5	8.88	6.25	2.99	0.1072		
2.75	36.	29.1	19334.	0.616	995.6	8.15	5.56	2.71	0.1062		
2.75	36.	29.1	19334.	0.616	995.6	8.15	5.58	2.71	0.1065		
2.75	36.	29.1	19334.	0.616	995.6	8.15	5.56	2.71	0.1062		
3.25	38.	32.9	19163.	0.621	994.6	7.52	5.26	2.47	0.1111		
3.25	40.	32.9	19163.	0.621	994.6	7.52	5.00	2.47	0.1056		
3.25	41.	32.9	19163.	0.621	994.6	7.52	4.88	2.47	0.1031		
3.75	35.	32.9	19163.	0.621	994.6	7.52	5.71	2.47	0.1206		
3.75	35.	36.7	19004.	0.626	993.4	6.97	5.71	2.26	0.1327		
3.75	52.	36.7	19004.	0.626	993.4	6.97	3.85	2.26	0.0995		
3.75	45.	36.7	19004.	0.626	993.4	6.97	4.44	2.26	0.1032		
3.75	41.	36.7	19004.	0.626	993.4	6.97	4.88	2.26	0.1134		
4.25	38.	40.5	18855.	0.631	992.2	6.49	5.26	2.08	0.1339		
4.25	48.	40.5	18855.	0.631	992.2	6.49	4.17	2.08	0.1062		
4.25	45.	40.5	18855.	0.631	992.2	6.49	4.44	2.08	0.1130		
4.25	49.	40.5	18855.	0.631	992.2	6.49	4.08	2.08	0.1039		
4.25	46.	40.5	18855.	0.631	992.2	6.49	4.35	2.08	0.1107		
4.25	50.	40.5	18855.	0.631	992.2	6.49	4.00	2.08	0.1018		
4.25	39.	40.5	18855.	0.631	992.2	6.49	5.13	2.08	0.1306		
4.75	38.	44.3	18715.	0.636	990.8	6.06	5.26	1.93	0.1460		
4.75	48.	44.3	18715.	0.636	990.8	6.06	4.17	1.93	0.1157		
4.75	51.	44.3	18715.	0.636	990.8	6.06	3.92	1.93	0.1088		
5.25	42.	48.0	18584.	0.641	989.3	5.67	4.76	1.79	0.1435		
5.25	56.	48.0	18584.	0.641	989.3	5.67	3.57	1.79	0.1076		



Table 16

Experimental Data of Block A,  $(T_T - T_B) = 69.0^\circ\text{C}$ 

LIQUID = Water  
 $d_p$  (microns) = 1.011  
 $T_T$  ( $^\circ\text{C}$ ) = 76.3  
 $T_B$  ( $^\circ\text{C}$ ) = 7.3  
 $n = 36$

$P_{os}$	$\tau$ (s)	$T_{PC}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1}\text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\mu/\rho T_{PK}$ ( $\text{m}^2\text{s}^{-1}\text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.25	24.	19.6	24103.	0.601	997.5	10.14	8.33	3.47	0.0996
1.75	25.	24.3	23809.	0.609	996.6	9.05	8.00	3.05	0.1101
1.75	28.	24.3	23809.	0.609	996.6	9.05	7.14	3.05	0.0983
2.25	27.	29.1	23538.	0.616	995.6	8.15	7.41	2.71	0.1162
2.25	29.	29.1	23538.	0.616	995.6	8.15	6.90	2.71	0.1082
2.25	30.	29.1	23538.	0.616	995.6	8.15	6.67	2.71	0.1046
2.25	30.	29.1	23538.	0.616	995.6	8.15	6.67	2.71	0.1046
2.75	31.	33.8	23288.	0.622	994.3	7.39	6.05	2.42	0.1074
2.75	32.	33.8	23288.	0.622	994.3	7.39	6.25	2.42	0.1108
2.75	36.	33.8	23288.	0.622	994.3	7.39	5.58	2.42	0.0989
2.75	33.	33.8	23288.	0.622	994.3	7.39	6.06	2.42	0.1074
2.75	30.	33.8	23288.	0.622	994.3	7.39	6.67	2.42	0.1182
2.75	30.	33.8	23288.	0.622	994.3	7.39	6.67	2.42	0.1182
2.75	35.	33.8	23288.	0.622	994.3	7.39	5.71	2.42	0.1012
3.25	36.	38.4	23057.	0.629	992.9	6.75	5.56	2.18	0.1105
3.25	35.	38.4	23057.	0.629	992.9	6.75	5.71	2.18	0.1135
3.25	36.	38.4	23057.	0.629	992.9	6.75	5.56	2.18	0.1105
3.75	42.	43.0	22843.	0.634	991.3	6.20	4.76	1.98	0.1054
3.75	39.	43.0	22843.	0.634	991.3	6.20	5.13	1.98	0.1136
3.75	37.	43.0	22843.	0.634	991.3	6.20	5.41	1.98	0.1198
3.75	42.	43.0	22843.	0.634	991.3	6.20	4.76	1.98	0.1054
3.75	36.	43.0	22843.	0.634	991.3	6.20	5.56	1.98	0.1231
3.75	42.	43.0	22843.	0.634	991.3	6.20	4.76	1.98	0.1054
3.75	35.	43.0	22843.	0.634	991.3	6.20	5.71	1.98	0.1264
3.75	45.	43.0	22843.	0.634	991.3	6.20	4.44	1.98	0.0983
4.25	41.	47.5	22646.	0.640	989.5	5.72	4.88	1.80	0.1196
4.25	42.	47.5	22646.	0.640	989.5	5.72	4.76	1.80	0.1166
4.25	39.	47.5	22646.	0.640	989.5	5.72	5.13	1.80	0.1257
4.75	45.	52.0	22463.	0.645	987.6	5.30	4.44	1.65	0.1197
4.75	46.	52.0	22463.	0.645	987.6	5.30	4.35	1.65	0.1173
4.75	44.	52.0	22463.	0.645	987.6	5.30	4.55	1.65	0.1227
5.25	50.	56.5	22295.	0.650	985.6	4.94	4.00	1.52	0.1181
5.25	55.	56.5	22295.	0.650	985.6	4.94	3.63	1.52	0.1072
5.75	54.	61.0	22141.	0.655	983.3	4.61	3.70	1.40	0.1190
6.25	59.	65.4	21998.	0.659	981.0	4.33	3.39	1.30	0.1183
6.25	53.	65.4	21998.	0.659	981.0	4.33	3.77	1.30	0.1316

Table 17

Experimental Data of Block A,  $(T_T - T_B) = 78.0^\circ\text{C}$ 

LIQUID = Water       $d_p$  (microns) = 1.011  
 $T_T$  ( $^\circ\text{C}$ ) = 85.6       $T_B$  ( $^\circ\text{C}$ ) = 7.6  
 $n = 34$

$P_{os}$	$\tau$ (s)	$T_{PC}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1}\text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\mu/\rho T_{PK}$ ( $\text{m}^2\text{s}^{-1}\text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.75	27.	27.0	26961.	0.613	996.1	8.54	7.41	2.86	0.0963
1.75	22.	27.0	26961.	0.613	996.1	8.54	9.09	2.86	0.1101
1.75	25.	27.0	26961.	0.613	996.1	8.54	8.00	2.86	0.1039
2.25	27.	32.3	26626.	0.620	994.7	7.62	7.41	2.51	0.1111
2.25	24.	32.3	26626.	0.620	994.7	7.62	8.33	2.51	0.1248
2.25	23.	32.3	26626.	0.620	994.7	7.62	8.70	2.51	0.1304
2.75	35.	37.6	26320.	0.628	993.1	6.85	5.71	2.22	0.0977
2.75	29.	37.6	26320.	0.628	993.1	6.85	6.90	2.22	0.1181
2.75	28.	37.6	26320.	0.628	993.1	6.85	7.14	2.22	0.1222
2.75	30.	37.6	26320.	0.628	993.1	6.85	6.67	2.22	0.1141
3.25	38.	42.8	26040.	0.634	991.3	6.21	5.26	1.98	0.1018
3.25	30.	42.8	26040.	0.634	991.3	6.21	6.67	1.98	0.1291
3.25	35.	42.8	26040.	0.634	991.3	6.21	5.71	1.98	0.1106
3.25	34.	42.8	26040.	0.634	991.3	6.21	5.88	1.98	0.1138
3.25	38.	42.8	26040.	0.634	991.3	6.21	5.26	1.98	0.1018
3.25	35.	42.8	26040.	0.634	991.3	6.21	5.71	1.98	0.1106
3.25	36.	42.8	26040.	0.634	991.3	6.21	5.56	1.98	0.1076
3.75	36.	48.0	25784.	0.641	989.3	5.67	5.56	1.78	0.1208
3.75	40.	48.0	25784.	0.641	989.3	5.67	5.00	1.78	0.1086
3.75	35.	48.0	25784.	0.641	989.3	5.67	5.71	1.78	0.1241
3.75	38.	48.0	25784.	0.641	989.3	5.67	5.26	1.78	0.1143
3.75	41.	48.0	25784.	0.641	989.3	5.67	4.88	1.78	0.1060
4.25	43.	53.2	25551.	0.646	987.1	5.21	4.65	1.62	0.1126
4.25	44.	53.2	25551.	0.646	987.1	5.21	4.55	1.62	0.1102
4.25	40.	53.2	25551.	0.646	987.1	5.21	5.00	1.62	0.1210
4.75	42.	58.2	25338.	0.652	984.7	4.81	4.76	1.47	0.1276
4.75	40.	58.2	25338.	0.652	984.7	4.81	5.00	1.47	0.1340
4.75	50.	58.2	25338.	0.652	984.7	4.81	4.00	1.47	0.1072
4.75	42.	58.2	25338.	0.652	984.7	4.81	4.76	1.47	0.1276
4.75	40.	58.2	25338.	0.652	984.7	4.81	5.00	1.47	0.1340
5.25	48.	63.3	25145.	0.657	982.1	4.46	4.17	1.35	0.1229
5.25	53.	63.3	25145.	0.657	982.1	4.46	3.77	1.35	0.1112
5.75	50.	68.3	24969.	0.661	979.3	4.15	4.00	1.24	0.1291
5.75	59.	68.3	24969.	0.661	979.3	4.15	3.39	1.24	0.1094

Table 18

Experimental Data of Block A,  $(T_T - T_B) = 81.7^\circ\text{C}$ 

LIQUID = Water  $d_p$  (microns) = 1.011  
 $T_T$  ( $^\circ\text{C}$ ) = 88.8  $T_B$  ( $^\circ\text{C}$ ) = 7.1  
 $n = 35$

$P_{os}$	$\tau$ (s)	$T_{PC}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1}\text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{m s}^{-1}$ ) $\times 10^6$	$\nu/\rho T_{PK}$ ( $\text{m}^2\text{s}^{-1}\text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.75	30.	27.4	28264.	0.613	996.0	8.45	6.67	2.82	0.0936
1.75	22.	27.4	28264.	0.613	996.0	8.45	9.09	2.82	0.1139
1.75	20.	27.4	28264.	0.613	996.0	8.45	10.00	2.82	0.1254
1.75	19.	27.4	28264.	0.613	996.0	8.45	10.53	2.82	0.1320
2.25	23.	33.0	27899.	0.621	974.5	7.50	8.70	2.46	0.1266
2.25	27.	33.0	27899.	0.621	994.5	7.50	7.41	2.46	0.1078
2.25	20.	33.0	27899.	0.621	994.5	7.50	10.00	2.46	0.1455
2.25	21.	33.0	27899.	0.621	994.5	7.50	9.52	2.46	0.1395
2.75	39.	38.6	27566.	0.629	972.8	6.73	5.13	2.17	0.0856
2.75	35.	38.6	27566.	0.629	992.8	6.73	5.71	2.17	0.0953
2.75	31.	38.6	27566.	0.629	992.8	6.73	6.45	2.17	0.1077
2.75	35.	38.6	27566.	0.629	992.8	6.73	5.71	2.17	0.0953
2.75	30.	38.6	27566.	0.629	992.8	6.73	6.67	2.17	0.1113
3.25	33.	44.1	27264.	0.636	990.9	6.08	6.06	1.93	0.1149
3.75	41.	49.5	26989.	0.642	988.7	5.53	4.88	1.73	0.1042
3.75	33.	49.5	26989.	0.642	988.7	5.53	6.06	1.73	0.1295
3.75	30.	49.5	26989.	0.642	988.7	5.53	6.67	1.73	0.1425
3.75	37.	49.5	26989.	0.642	988.7	5.53	5.41	1.73	0.1156
3.75	32.	49.5	26989.	0.642	988.7	5.53	6.25	1.73	0.1335
3.75	33.	49.5	26989.	0.642	988.7	5.53	6.06	1.73	0.1295
3.75	37.	49.5	26989.	0.642	988.7	5.53	5.41	1.73	0.1156
3.75	43.	49.5	26989.	0.642	988.7	5.53	4.65	1.73	0.0993
4.25	36.	54.9	26739.	0.648	986.3	5.07	5.56	1.57	0.1328
4.25	41.	54.9	26739.	0.648	986.3	5.07	4.88	1.57	0.1165
4.25	39.	54.9	26739.	0.648	986.3	5.07	5.13	1.57	0.1225
4.25	36.	54.9	26739.	0.648	986.3	5.07	5.56	1.57	0.1328
4.75	42.	60.2	26513.	0.654	983.7	4.67	4.76	1.42	0.1261
4.75	40.	60.2	26513.	0.654	983.7	4.67	5.00	1.42	0.1325
4.75	48.	60.2	26513.	0.654	983.7	4.67	4.17	1.42	0.1105
5.25	58.	65.5	26308.	0.659	980.9	4.32	3.45	1.30	0.1008
5.25	56.	65.5	26308.	0.659	980.9	4.32	3.57	1.30	0.1043
5.25	46.	65.5	26308.	0.659	980.9	4.32	4.35	1.30	0.1271
5.75	57.	70.7	26123.	0.664	977.9	4.02	3.51	1.19	0.1125
5.75	65.	70.7	26123.	0.664	977.9	4.02	3.08	1.19	0.0987
5.75	50.	70.7	26123.	0.664	977.9	4.02	4.00	1.19	0.1282

Table 19  
Experimental Data of Block B,  $(T_T - T_B) = 37.7^\circ\text{C}$

LIQUID = Hexane  $d_p$  (microns) = 1.011  
 $T_T$  ( $^\circ\text{C}$ ) = 50.2  $T_B$  ( $^\circ\text{C}$ ) = 12.5  
 $n = 31$

$P_{os}$	$\tau$ (s)	$T_{PC}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1}\text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\mu/\rho T_{PK}$ ( $\text{m}^2\text{s}^{-1}\text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.25	76.	18.5	12026.	0.132	660.8	3.24	1.97	1.68	0.0972
1.25	88.	18.5	12026.	0.132	660.8	3.24	1.61	1.68	0.0795
1.75	83.	20.9	12124.	0.131	658.6	3.17	1.73	1.64	0.0870
1.75	79.	20.9	12124.	0.131	658.6	3.17	1.85	1.64	0.0931
1.75	88.	20.9	12124.	0.131	658.6	3.17	1.59	1.64	0.0801
1.75	89.	20.9	12124.	0.131	658.6	3.17	1.56	1.64	0.0788
1.75	83.	20.9	12124.	0.131	658.6	3.17	1.73	1.64	0.0870
2.25	89.	23.3	12224.	0.130	656.3	3.10	1.54	1.59	0.0793
2.25	81.	23.3	12224.	0.130	656.3	3.10	1.77	1.59	0.0907
2.25	84.	23.3	12224.	0.130	656.3	3.10	1.63	1.59	0.0861
2.75	85.	25.8	12326.	0.129	654.1	3.03	1.63	1.55	0.0853
2.75	87.	25.8	12326.	0.129	654.1	3.03	1.58	1.55	0.0825
3.25	78.	28.2	12432.	0.128	651.8	2.96	1.82	1.51	0.0972
3.25	87.	28.2	12432.	0.128	651.8	2.96	1.55	1.51	0.0930
3.25	83.	28.2	12432.	0.128	651.8	2.96	1.66	1.51	0.0889
3.75	80.	30.7	12540.	0.126	649.5	2.89	1.73	1.46	0.0944
3.75	87.	30.7	12540.	0.126	649.5	2.89	1.73	1.46	0.0944
3.75	80.	30.7	12540.	0.126	649.5	2.89	1.53	1.46	0.0935
4.25	86.	33.3	12650.	0.125	647.1	2.81	1.53	1.42	0.0854
4.25	97.	33.3	12650.	0.125	647.1	2.81	1.27	1.42	0.0707
4.25	79.	33.3	12650.	0.125	647.1	2.81	1.74	1.42	0.0969
4.25	80.	33.3	12650.	0.125	647.1	2.81	1.71	1.42	0.0951
4.25	94.	33.3	12650.	0.125	647.1	2.81	1.33	1.42	0.0744
4.75	84.	35.8	12764.	0.124	644.8	2.74	1.56	1.37	0.0890
4.75	88.	35.8	12764.	0.124	644.8	2.74	1.45	1.37	0.0828
5.25	83.	38.4	12881.	0.123	642.4	2.66	1.56	1.33	0.0911
5.25	96.	38.4	12881.	0.123	642.4	2.66	1.24	1.33	0.0721
5.25	80.	38.4	12881.	0.123	642.4	2.66	1.65	1.33	0.0964
5.75	85.	40.9	13002.	0.122	640.0	2.59	1.48	1.29	0.0881
5.75	92.	40.9	13002.	0.122	640.0	2.59	1.30	1.29	0.0775
6.25	94.	43.6	13125.	0.121	637.5	2.51	1.22	1.24	0.0746

Table 20

Experimental Data of Block B,  $(T_T - T_B) = 39.5^\circ\text{C}$ LIQUID = Hexane  $d_p$  (microns) = 1.011 $T_T$  ( $^\circ\text{C}$ ) = 52.1  $T_B$  ( $^\circ\text{C}$ ) = 12.6 $n = 34$ 

$P_{os}$	$\tau$ (s)	$T_{PC}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1}\text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\mu/\rho T_{PK}$ ( $\text{m}^2\text{s}^{-1}\text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.25	80.	18.8	12569.	0.132	660.5	3.23	1.83	1.68	0.0970
1.25	78.	18.8	12569.	0.132	660.5	3.23	1.90	1.68	0.0901
1.25	71.	18.8	12569.	0.132	660.5	3.23	2.07	1.68	0.0984
2.25	91.	23.9	12786.	0.129	655.8	3.08	1.49	1.58	0.0736
2.25	82.	23.9	12786.	0.129	655.8	3.08	1.73	1.58	0.0855
2.25	77.	23.9	12786.	0.129	655.8	3.08	1.89	1.58	0.0934
2.25	88.	23.9	12786.	0.129	655.8	3.08	1.57	1.58	0.0773
2.75	83.	26.5	12899.	0.128	653.4	3.01	1.68	1.54	0.0847
2.75	92.	26.5	12899.	0.128	653.4	3.01	1.44	1.54	0.0728
2.75	73.	26.5	12899.	0.128	653.4	3.01	2.01	1.54	0.1014
2.75	83.	26.5	12899.	0.128	653.4	3.01	1.68	1.54	0.0847
3.25	83.	29.1	13014.	0.127	651.0	2.93	1.66	1.49	0.0854
3.25	76.	29.1	13014.	0.127	651.0	2.93	1.88	1.49	0.0968
3.25	90.	29.1	13014.	0.127	651.0	2.93	1.47	1.49	0.0757
3.25	79.	29.1	13014.	0.127	651.0	2.93	1.78	1.49	0.0916
3.75	79.	31.7	13133.	0.126	648.6	2.86	1.75	1.45	0.0924
3.75	85.	31.7	13133.	0.126	648.6	2.86	1.58	1.45	0.0930
3.75	84.	31.7	13133.	0.126	648.6	2.86	1.60	1.45	0.0844
3.75	89.	31.7	13133.	0.126	648.6	2.86	1.47	1.45	0.0774
3.75	80.	31.7	13133.	0.126	648.6	2.86	1.72	1.45	0.0907
4.25	86.	34.3	13255.	0.125	646.1	2.78	1.52	1.40	0.0820
4.25	93.	34.3	13255.	0.125	646.1	2.78	1.35	1.40	0.0726
4.25	76.	34.3	13255.	0.125	646.1	2.78	1.83	1.40	0.0985
4.25	83.	34.3	13255.	0.125	646.1	2.78	1.61	1.40	0.0865
4.25	87.	34.3	13255.	0.125	646.1	2.78	1.50	1.40	0.0805
4.75	86.	37.0	13381.	0.124	643.7	2.71	1.49	1.35	0.0824
4.75	83.	37.0	13381.	0.124	643.7	2.71	1.58	1.35	0.0870
5.25	80.	39.7	13510.	0.123	641.2	2.63	1.64	1.31	0.0926
5.25	90.	39.7	13510.	0.123	641.2	2.63	1.36	1.31	0.0769
5.25	81.	39.7	13510.	0.123	641.2	2.63	1.61	1.31	0.0908
5.25	88.	39.7	13510.	0.123	641.2	2.63	1.41	1.31	0.0777
5.75	89.	42.4	13643.	0.121	638.6	2.55	1.35	1.26	0.0784
5.75	84.	42.4	13643.	0.121	638.6	2.55	1.49	1.26	0.0862
5.75	90.	42.4	13643.	0.121	638.6	2.55	1.33	1.26	0.0770

Table 21

Experimental Data of Block B,  $(T_T - T_B) = 44.9^\circ\text{C}$ LIQUID = Hexane  $d_p$  (microns) = 1.011 $T_T$  ( $^\circ\text{C}$ ) = 57.7  $T_B$  ( $^\circ\text{C}$ ) = 12.8 $n = 29$ 

$P_{os}$	$\tau$ (s)	$T_{PC}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1}\text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\nu/\rho T_{PK}$ ( $\text{m}^2\text{s}^{-1}\text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.25	65.	19.8	14201.	0.131	659.6	3.20	2.40	1.66	0.1021
1.25	81.	19.8	14201.	0.131	659.6	3.20	1.79	1.66	0.0762
1.25	69.	19.8	14201.	0.131	659.6	3.20	2.22	1.66	0.0945
1.75	74.	22.7	14338.	0.130	656.9	3.12	2.01	1.61	0.0871
1.75	65.	22.7	14338.	0.130	656.9	3.12	2.18	1.61	-0.1034
1.75	76.	22.7	14338.	0.130	656.9	3.12	1.93	1.61	0.0840
1.75	67.	22.7	14338.	0.130	656.9	3.12	2.29	1.61	0.0994
1.75	76.	22.7	14338.	0.130	656.9	3.12	1.93	1.61	0.0840
1.75	73.	25.6	14479.	0.129	654.3	3.04	2.02	1.55	0.0897
2.25	77.	25.6	14479.	0.129	654.3	3.04	1.88	1.55	0.0834
2.75	77.	28.5	14625.	0.127	651.6	2.95	1.85	1.50	0.0842
2.75	81.	28.5	14625.	0.127	651.6	2.95	1.72	1.50	0.0784
2.75	70.	28.5	14625.	0.127	651.6	2.95	2.11	1.50	0.0961
2.75	82.	28.5	14625.	0.127	651.6	2.95	1.69	1.50	0.0770
3.25	71.	31.4	14775.	0.126	648.8	2.87	2.04	1.45	0.0953
3.25	73.	31.4	14775.	0.126	648.8	2.87	1.96	1.45	0.0717
3.75	84.	34.4	14929.	0.125	646.1	2.78	1.58	1.40	0.0755
3.75	78.	34.4	14929.	0.125	646.1	2.78	1.76	1.40	0.0842
4.25	71.	37.4	15089.	0.124	643.3	2.69	1.08	1.35	0.0974
4.25	85.	37.4	15089.	0.124	643.3	2.69	1.52	1.35	0.0746
4.25	81.	37.4	15089.	0.124	643.3	2.69	1.63	1.35	0.0603
4.75	89.	40.4	15254.	0.122	640.5	2.61	1.38	1.30	0.0696
4.75	80.	40.4	15254.	0.122	640.5	2.61	1.63	1.30	0.0824
5.25	77.	43.5	15424.	0.121	637.6	2.52	1.69	1.25	0.0879
5.75	81.	46.6	15600.	0.120	634.7	2.43	1.52	1.20	0.0816
5.75	75.	46.6	15600.	0.120	634.7	2.43	1.72	1.20	0.0922
5.75	88.	46.6	15600.	0.120	634.7	2.43	1.32	1.20	0.0710
5.75	82.	46.6	15600.	0.120	634.7	2.43	1.49	1.20	0.0800
6.25	80.	49.7	15783.	0.118	631.8	2.34	1.51	1.14	0.0835

Table 22

Experimental Data of Block C,  $(T_T - T_B) = 34.6^\circ\text{C}$ 

LIQUID = Water  
 $d_p$  (microns) = 0.790  
 $T_T$  ( $^\circ\text{C}$ ) = 42.1  
 $T_B$  ( $^\circ\text{C}$ ) = 7.5  
 $n = 36$

$P_{os}$	$\tau$ (s)	$T_{PC}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1}\text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\nu/\rho$ $T_{PK}$ ( $\text{m}^2\text{s}^{-1}\text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.75	36.	15.9	11792.	0.595	998.1	11.13	5.56	3.86	0.1223
1.75	46.	15.9	11792.	0.595	998.1	11.13	4.35	3.86	0.0957
1.75	41.	15.9	11792.	0.595	998.1	11.13	4.88	3.86	0.1073
2.25	37.	18.2	11717.	0.599	997.8	10.48	5.41	3.60	0.1281
2.25	38.	18.2	11717.	0.599	997.8	10.48	5.26	3.60	0.1245
2.25	49.	18.2	11717.	0.599	997.8	10.48	4.08	3.60	0.0966
2.25	48.	18.2	11717.	0.599	997.8	10.48	4.17	3.60	0.0987
2.25	43.	18.2	11717.	0.599	997.8	10.48	4.65	3.60	0.1101
2.75	42.	20.5	11645.	0.603	997.4	9.90	4.76	3.38	0.1210
2.75	40.	20.5	11645.	0.603	997.4	9.90	5.00	3.38	0.1271
2.75	50.	20.5	11645.	0.603	997.4	9.90	4.00	3.38	0.1017
3.25	41.	22.9	11576.	0.606	996.9	9.37	4.88	3.17	0.1328
3.25	57.	22.9	11576.	0.606	996.9	9.37	3.51	3.17	0.0956
3.25	45.	22.9	11576.	0.606	996.9	9.37	4.44	3.17	0.1209
3.25	50.	22.9	11576.	0.606	996.9	9.37	4.00	3.17	0.1039
3.75	48.	25.2	11509.	0.610	996.5	8.88	4.17	2.99	0.1213
3.75	57.	25.2	11509.	0.610	996.5	8.88	3.51	2.99	0.1021
3.75	51.	25.2	11509.	0.610	996.5	8.88	3.92	2.99	0.1140
3.75	44.	25.2	11509.	0.610	996.5	8.88	4.55	2.99	0.1323
3.75	52.	25.2	11509.	0.610	996.5	8.88	3.85	2.99	0.1120
4.25	67.	27.5	11445.	0.613	996.0	8.44	2.99	2.82	0.0927
4.25	48.	27.5	11445.	0.613	996.0	8.44	4.17	2.82	0.1293
4.25	51.	27.5	11445.	0.613	996.0	8.44	3.92	2.82	0.1215
4.25	57.	27.5	11445.	0.613	996.0	8.44	3.51	2.82	0.1088
4.25	62.	27.5	11445.	0.613	996.0	8.44	3.23	2.82	0.1001
4.25	55.	27.5	11445.	0.613	996.0	8.44	3.64	2.82	0.1128
4.75	68.	29.7	11384.	0.617	995.4	8.03	2.94	2.66	0.0969
4.75	52.	29.7	11384.	0.617	995.4	8.03	3.85	2.66	0.1269
4.75	59.	29.7	11384.	0.617	995.4	8.03	3.39	2.66	0.1118
4.75	50.	29.7	11384.	0.617	995.4	8.03	3.39	2.66	0.1118
5.25	68.	32.0	11324.	0.620	994.8	7.66	2.94	2.52	0.1029
5.25	69.	32.0	11324.	0.620	994.8	7.66	2.90	2.52	0.1015
5.25	57.	32.0	11324.	0.620	994.8	7.66	3.51	2.52	0.1228
5.25	63.	32.0	11324.	0.620	994.8	7.66	3.17	2.52	0.1109
5.25	62.	32.0	11324.	0.620	994.8	7.66	3.23	2.52	0.1130
5.75	59.	34.3	11267.	0.623	994.2	7.32	3.39	2.39	0.1257

Table 23

Experimental Data of Block C,  $(T_T - T_B) = 56.0^\circ\text{C}$ 

LIQUID = Water  
 $T_T (^\circ\text{C}) = 63.6$   
 $T_B (^\circ\text{C}) = 7.6$   
 $n = 36$   
 $d_p$  (microns) = 0.790

$P_{os}$	$\tau$ (s)	$T_{pc}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1}\text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\mu/\rho T_{pk}$ ( $\text{m}^2\text{s}^{-1}\text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.75	29.	21.3	19251.	0.604	997.2	9.71	6.90	3.31	0.1084
2.25	25.	25.2	19066.	0.610	996.5	8.88	8.00	2.99	0.1404
2.25	37.	25.2	19066.	0.610	996.5	8.88	5.41	2.99	0.0950
2.25	25.	25.2	19066.	0.610	996.5	8.88	8.00	2.99	0.1404
2.25	26.	25.2	19066.	0.610	996.5	8.88	7.69	2.99	0.1350
2.25	32.	25.2	19066.	0.610	996.5	8.88	6.25	2.99	0.1097
2.25	38.	25.2	19066.	0.610	996.5	8.88	5.26	2.99	0.0923
2.75	28.	29.0	18892.	0.616	995.6	8.17	7.14	2.72	0.1391
2.75	32.	29.0	18892.	0.616	995.6	8.17	6.25	2.72	0.1218
2.75	35.	29.0	18892.	0.616	995.6	8.17	5.71	2.72	0.1113
3.25	40.	32.7	18730.	0.621	994.6	7.55	4.17	2.48	0.0897
3.25	31.	32.7	18730.	0.621	994.6	7.55	6.45	2.48	0.1387
3.25	35.	32.7	18730.	0.621	994.6	7.55	5.71	2.48	0.1228
3.25	32.	32.7	18730.	0.621	994.6	7.55	6.25	2.48	0.1344
3.25	48.	32.7	18730.	0.621	994.6	7.55	4.17	2.48	0.0897
3.25	39.	32.7	18730.	0.621	994.6	7.55	5.13	2.48	0.1103
3.75	43.	36.4	18577.	0.626	993.5	7.01	4.65	2.28	0.1099
3.75	38.	36.4	18577.	0.626	993.5	7.01	5.26	2.28	0.1243
3.75	56.	36.4	18577.	0.626	993.5	7.01	3.57	2.28	0.0843
3.75	41.	36.4	18577.	0.626	993.5	7.01	4.88	2.28	0.1153
3.75	35.	36.4	18577.	0.626	993.5	7.01	5.71	2.28	0.1349
4.25	57.	40.1	18434.	0.631	992.3	6.53	3.51	2.10	0.0907
4.25	40.	40.1	18434.	0.631	992.3	6.53	5.00	2.10	0.1232
4.25	39.	40.1	18434.	0.631	992.3	6.53	5.13	2.10	0.1325
4.25	41.	40.1	18434.	0.631	992.3	6.53	4.89	2.10	0.1261
4.25	46.	40.1	18434.	0.631	992.3	6.53	4.35	2.10	0.1124
4.75	46.	43.8	18299.	0.635	991.0	6.11	4.35	1.94	0.1223
4.75	42.	43.8	18299.	0.635	991.0	6.11	4.76	1.94	0.1339
4.75	41.	43.8	18299.	0.635	991.0	6.11	4.88	1.94	0.1372
4.75	49.	43.8	18299.	0.635	991.0	6.11	4.08	1.94	0.1147
5.25	62.	47.5	18173.	0.640	989.6	5.73	3.23	1.80	0.0985
5.25	58.	47.5	18173.	0.640	989.6	5.73	3.45	1.80	0.1052
5.25	53.	47.5	18173.	0.640	989.6	5.73	3.77	1.80	0.1150
5.75	61.	51.1	18054.	0.644	988.0	5.39	3.17	1.68	0.1044
5.75	47.	51.1	18054.	0.644	988.0	5.39	4.26	1.68	0.1403
5.75	65.	51.1	18054.	0.644	988.0	5.39	3.08	1.68	0.1015



Table 24

Experimental Data of Block C,  $(T_T - T_B) = 68.9^\circ\text{C}$ 

LIQUID = Water  
 $d_p$  (microns) = 0.790  
 $T_T$  ( $^\circ\text{C}$ ) = 76.5  
 $T_B$  ( $^\circ\text{C}$ ) = 7.6  
 $n = 35$

$P_{os}$	$\tau$ (s)	$T_{PC}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1}\text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\nu/\rho T_{PK}$ ( $\text{m}^2\text{s}^{-1}\text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.75	21.	24.6	23771.	0.609	996.6	8.99	9.52	3.03	0.1322
1.75	36.	24.6	23771.	0.609	996.6	8.99	5.56	3.03	0.0772
1.75	21.	24.6	23771.	0.609	996.6	8.99	5.56	3.03	0.1322
2.25	29.	29.3	23502.	0.616	995.5	8.10	6.90	2.69	0.1091
2.25	21.	29.3	23502.	0.616	995.5	8.10	9.52	2.69	0.1505
2.25	21.	29.3	23502.	0.616	995.5	8.10	9.52	2.69	0.1505
2.75	39.	34.0	23253.	0.623	994.2	7.36	5.13	2.41	0.0916
2.75	23.	34.0	23253.	0.623	994.2	7.36	8.70	2.41	0.1554
2.75	32.	34.0	23253.	0.623	994.2	7.36	6.25	2.41	0.1116
2.75	38.	34.0	23253.	0.623	994.2	7.36	5.26	2.41	0.0939
3.25	42.	38.6	23023.	0.629	992.8	6.72	4.76	2.17	0.0953
3.25	43.	38.6	23023.	0.629	992.8	6.72	4.65	2.17	0.0931
3.25	41.	38.6	23023.	0.629	992.8	6.72	4.88	2.17	0.0977
3.25	36.	38.6	23023.	0.629	992.8	6.72	5.56	2.17	0.1113
3.25	36.	38.6	23023.	0.629	992.8	6.72	5.56	2.17	0.1113
3.75	39.	43.2	22811.	0.635	991.2	6.17	5.13	1.97	0.1143
3.75	39.	43.2	22811.	0.635	991.2	6.17	5.13	1.97	0.1143
3.75	35.	43.2	22811.	0.635	991.2	6.17	5.71	1.97	0.1272
4.25	57.	47.8	22615.	0.640	989.4	5.70	3.51	1.79	0.0965
4.25	43.	47.8	22615.	0.640	989.4	5.70	4.65	1.79	0.1146
4.25	49.	47.8	22615.	0.640	989.4	5.70	4.08	1.79	0.1006
4.25	45.	47.8	22615.	0.640	989.4	5.70	4.44	1.79	0.1095
4.75	36.	52.3	22433.	0.645	987.5	5.28	5.56	1.64	0.1508
4.75	55.	52.3	22433.	0.645	987.5	5.28	3.64	1.64	0.0987
4.75	47.	52.3	22433.	0.645	987.5	5.28	4.26	1.64	0.1155
4.75	43.	52.3	22433.	0.645	987.5	5.28	4.65	1.64	0.1261
5.25	57.	56.7	22266.	0.650	985.4	4.92	3.51	1.51	0.1042
5.25	51.	56.7	22266.	0.650	985.4	4.92	3.92	1.51	0.1164
5.25	42.	56.7	22266.	0.650	985.4	4.92	4.76	1.51	0.1413
5.25	58.	56.7	22266.	0.650	985.4	4.92	3.45	1.51	0.1024
5.75	62.	61.2	22113.	0.655	983.2	4.60	3.23	1.40	0.1044
5.75	51.	61.2	22113.	0.655	983.2	4.60	3.92	1.40	0.1268
5.75	61.	61.2	22113.	0.655	983.2	4.60	3.28	1.40	0.1061
5.75	49.	61.2	22113.	0.655	983.2	4.60	4.08	1.40	0.1319
5.75	56.	61.2	22113.	0.655	983.2	4.60	3.57	1.40	0.1154

Table 25

Experimental Data of Block D,  $(T_T - T_B) = 37.8^\circ\text{C}$ 

LIQUID = Hexane  $d_p$  (microns) = 0.790  
 $T_T$  ( $^\circ\text{C}$ ) = 45.6  $T_B$  ( $^\circ\text{C}$ ) = 7.8  
 $n = 31$

$P_{os}$	$\tau$ (s)	$T_{PC}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1} \text{m}^{-1} \text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1} \text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$u/\rho T_{PK}$ ( $\text{m}^2 \text{s}^{-1} \text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.75	86.	16.2	12161.	0.133	663.0	3.31	1.93	1.72	0.0922
2.25	79.	18.6	12259.	0.132	660.7	3.24	2.12	1.68	0.1032
2.25	86.	18.6	12259.	0.132	660.7	3.24	1.92	1.68	0.0934
2.75	81.	21.1	12364.	0.131	658.4	3.16	2.05	1.63	0.1016
2.75	97.	21.1	12364.	0.131	658.4	3.16	1.64	1.63	0.0813
2.75	88.	21.1	12364.	0.131	658.4	3.16	1.85	1.63	0.0917
2.75	95.	21.1	12364.	0.131	658.4	3.16	1.69	1.63	0.0838
2.75	80.	21.1	12364.	0.131	658.4	3.16	2.08	1.63	0.1031
3.25	102.	23.6	12468.	0.130	656.1	3.09	1.53	1.59	0.0772
3.25	83.	23.6	12468.	0.130	656.1	3.09	1.98	1.59	0.1000
3.25	90.	23.6	12468.	0.130	656.1	3.09	1.79	1.59	0.0704
3.25	98.	23.6	12468.	0.130	656.1	3.09	1.61	1.59	0.0813
3.25	85.	23.6	12468.	0.130	656.1	3.09	1.92	1.59	0.0769
3.75	101.	26.1	12574.	0.129	653.8	3.02	1.54	1.54	0.0791
3.75	86.	26.1	12574.	0.129	653.8	3.02	1.87	1.54	0.0972
3.75	101.	26.1	12574.	0.129	653.8	3.02	1.54	1.54	0.0791
3.75	90.	26.1	12574.	0.129	653.8	3.02	1.78	1.54	0.0915
3.75	93.	26.1	12574.	0.129	653.8	3.02	1.71	1.54	0.0979
4.25	106.	28.6	12682.	0.127	651.4	2.95	1.43	1.50	0.0754
4.25	87.	28.6	12682.	0.127	651.4	2.95	1.84	1.50	0.0969
4.25	97.	28.6	12682.	0.127	651.4	2.95	1.63	1.50	0.0843
4.25	99.	28.6	12682.	0.127	651.4	2.95	1.56	1.50	0.0822
4.75	96.	31.2	12799.	0.126	649.0	2.87	1.61	1.45	0.0864
4.75	98.	31.2	12799.	0.126	649.0	2.87	1.57	1.45	0.0843
4.75	112.	31.2	12799.	0.126	649.0	2.87	1.32	1.45	0.0708
5.25	108.	33.8	12913.	0.125	646.6	2.80	1.36	1.41	0.0748
5.25	99.	33.8	12913.	0.125	646.6	2.80	1.53	1.41	0.0842
5.25	104.	33.8	12913.	0.125	646.6	2.80	1.43	1.41	0.0787
5.25	97.	33.8	12913.	0.125	646.6	2.80	1.57	1.41	0.0864
5.75	100.	36.3	13031.	0.124	644.2	2.72	1.50	1.37	0.0841
5.75	102.	36.3	13031.	0.124	644.2	2.72	1.46	1.37	0.0818

Table 26

Experimental Data of Block D,  $(T_T - T_B) = 54.9^\circ\text{C}$ LIQUID = Hexane  $d_p$  (microns) = 0.790 $T_T$  ( $^\circ\text{C}$ ) = 62.7  $T_B$  ( $^\circ\text{C}$ ) = 7.8 $n = 26$ 

$P_{os}$	$\tau$ (s)	$T_{PC}$ ( $^\circ\text{C}$ )	$dT/dx$ ( $^\circ\text{K m}^{-1}$ )	$k_f$ ( $\text{J s}^{-1}\text{m}^{-1}\text{K}^{-1}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$\mu$ ( $\text{kg m}^{-1}\text{s}^{-1}$ ) $\times 10^4$	$v_{th}$ ( $\text{ms}^{-1}$ ) $\times 10^6$	$\mu/\rho T_{PK}$ ( $\text{m}^2\text{s}^{-1}\text{K}^{-1}$ ) $\times 10^9$	$\alpha$
1.75	65.	19.7	17346.	0.131	659.7	2.67	3.08	1.66	0.0927
2.25	75.	23.1	17548.	0.130	656.5	2.24	2.67	1.60	0.0800
2.25	63.	23.1	17548.	0.130	656.5	2.74	3.17	1.60	0.0978
2.25	58.	23.1	17548.	0.130	656.5	3.02	3.45	1.60	0.1078
2.75	75.	26.7	17763.	0.128	653.2	2.22	2.67	1.53	0.0817
2.75	52.	26.7	17763.	0.128	653.2	3.40	3.85	1.53	0.1250
3.25	69.	30.3	17986.	0.127	649.8	2.43	2.90	1.47	0.0920
3.25	76.	30.3	17986.	0.127	649.8	2.16	2.63	1.47	0.0818
3.25	77.	30.3	17986.	0.127	649.8	2.13	2.60	1.47	0.0807
3.25	57.	30.3	17986.	0.127	649.8	3.04	3.51	1.47	0.1151
3.25	70.	30.3	17986.	0.127	649.8	2.39	2.86	1.47	0.0905
3.25	54.	30.3	17986.	0.127	649.8	3.23	3.70	1.47	0.1223
3.75	58.	33.9	18211.	0.125	646.5	2.96	3.45	1.41	0.1156
3.75	69.	33.9	18211.	0.125	646.5	2.41	2.90	1.41	0.0941
3.75	73.	33.9	18211.	0.125	646.5	2.25	2.74	1.41	0.0879
3.75	67.	33.9	18211.	0.125	646.5	2.50	2.99	1.41	0.0976
3.75	79.	33.9	18211.	0.125	646.5	2.04	2.53	1.41	0.0797
4.25	75.	37.6	18452.	0.123	643.1	2.16	2.67	1.34	0.0870
4.25	79.	37.6	18452.	0.123	643.1	2.02	2.53	1.34	0.0814
4.25	68.	37.6	18452.	0.123	643.1	2.43	2.94	1.34	0.0979
4.25	73.	37.6	18452.	0.123	643.1	2.23	2.74	1.34	0.0898
4.75	75.	41.2	18695.	0.122	639.7	2.13	2.67	1.28	0.0889
4.75	63.	41.2	18695.	0.122	639.7	2.63	3.17	1.28	0.1097
5.25	89.	45.0	18956.	0.120	636.2	1.68	2.25	1.22	0.0727
5.25	73.	45.0	18956.	0.120	636.2	2.17	2.74	1.22	0.0939
5.25	81.	45.0	18956.	0.120	636.2	1.90	2.47	1.22	0.0823

## Appendix III

### Sample and Error Calculations

#### 1. Calculation of $\alpha$ and Error Estimation

This section provides two sample calculations of  $\alpha$ , one for water and one for hexane. The error estimates give the largest errors, not the most probable ones. Two simple rules were employed to calculate errors:

Rule 1: When variables are added or subtracted, add their absolute errors.

Rule 2: When variables are multiplied or divided, add their relative errors.

The absolute errors for both the water and hexane subsections fall into two categories: derived absolute errors and stipulated absolute errors. The former are calculated from the previously mentioned rules and equations. The latter were obtained directly from construction tolerances, calibration results, or measurement errors. The given estimates for coefficients in the equations obtained by the regression programme can be considered as measures of the 95 per cent confidence limit of each coefficient.

## a. Water

The following equations are required:

$$v_{th} = \frac{0.0002}{\tau}$$

Note: since  $v_g$  for water is negligible, Equation [38] reduced to the above.

$$\frac{x}{h} = 1 - \frac{P_{os}}{7.5}$$

$$qh = A (T_T - T_B) + \frac{B}{2} (T_T^2 - T_B^2) + \frac{C}{3} (T_T^3 - T_B^3)$$

For purposes of error estimation only:

$$T_{PC} = \frac{\frac{x}{h} (qh) - (AT_T + \frac{B}{2}T_T^2 + \frac{C}{3}T_T^3)}{A}$$

$$k_f = A + BT_{PC} + C(T_{PC})^2$$

$$\rho = 999.168 - 0.0042607 (T_{PC})^2$$

$$\frac{1}{\mu} = 21.482 ((T_{PC} - 8.435) + (8078.4 + (T_{PC} - 8.435)^2)^{\frac{1}{2}}) - 1200.$$

Note: Reference (24) gives no estimate of error for the coefficients used in the viscosity equation. For purposes of calculation, therefore, all the error was assumed to be in  $T_{PC}$ .

$$\frac{dT}{dx} = - \frac{qh}{(h)(k_f)}$$

$$T_{PK} = T_{PC} + 273.2$$

$$\alpha = - \frac{v_{th}}{\frac{\mu}{\rho T_{PK}} \frac{dT}{dx}}$$

The following values pertain to the first datum point in Table 13, Appendix II.

$$d_p = 1.011 \text{ microns}$$

$$P_{os} = 2.25$$

$$T_T = 57.4^\circ\text{C}$$

$$\tau = 50s$$

$$T_B = 7.2^\circ\text{C}$$

The sample and error calculations for this point are shown in Table 27. The stipulated absolute errors are listed in italics. Comments on footnoted entries are given at the end of the table.

Table 27

## Sample and Error Calculations--Water

Variable	Value	Absolute Error	Relative Error
0.0002	0.0002m	$\pm 1.42 \times 10^{-5} m^*$	$\pm 0.0840$
$\tau$	50s	$\pm 1s$	$\pm 0.0200$
$v_{th}$	$4.00 \times 10^{-6} ms^{-1}$	$\pm 0.416 \times 10^{-6} ms^{-1}$	$\pm 0.1040$
$P_{os}$	2.25 grid spaces	$\pm 0.10 \text{ grid spaces}$	$\pm 0.0445$
7.5	7.5 grid spaces	$\pm 0.10 \text{ grid spaces}$	$\pm 0.0130$
$P_{os}/7.5$	0.300	$\pm 0.0173$	$\pm 0.0575$
$x/h$	0.700	$\pm 0.0173$	$\pm 0.0247$
$T_T$	57.4°C	$\pm 0.5^\circ C$	$\pm 0.0087$
$T_B$	7.2°C	$\pm 2.7^\circ C^{**}$	$\pm 0.3750$
A	$0.567558 Jm^{-1} s^{-1} \circ K^{-1}$	$\pm 0.0019 Jm^{-1} s^{-1} \circ K^{-1}$	$\pm 0.00034$
B	$1.862289 \times 10^{-3} Jm^{-1} s^{-1} \circ K^{-2}$	$\pm 4.6 \times 10^{-5} Jm^{-1} s^{-1} \circ K^{-2}$	$\pm 0.0250$
C	$-7.15006 \times 10^{-6} Jm^{-1} s^{-1} \circ K^{-3}$	$\pm 4.6 \times 10^{-7} Jm^{-1} s^{-1} \circ K^{-3}$	$\pm 0.0640$
$T_T - T_B$	50.2°C	$\pm 3.2^\circ C$	$\pm 0.0630$
$A(T_T - T_B)$	$28.4 Jm^{-1} s^{-1}$	$\pm 1.74 Jm^{-1} s^{-1}$	$\pm 0.0633$

(Table 27 - continued)

Variable	Value	Absolute Error	Relative Error
$T_T^2$	$3294.8^\circ\text{C}^2$	$\pm 57.3^\circ\text{C}^2$	$\pm 0.0174$
$T_B^2$	$51.8^\circ\text{C}^2$	$\pm 38.9^\circ\text{C}^2$	$\pm 0.7500$
$T_T^2 - T_B^2$	$3243.0^\circ\text{C}^2$	$\pm 96.2^\circ\text{C}^2$	$\pm 0.0300$
$\frac{B}{2}(T_T^2 - T_B^2)$	$3.021\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.1670\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0550$
$T_T^3$	$189119^\circ\text{C}^3$	$\pm 4936^\circ\text{C}^3$	$\pm 0.0261$
$T_B^3$	$373^\circ\text{C}^3$	$\pm 420^\circ\text{C}^3$	$\pm 1.125$
$T_T^3 - T_B^3$	$188746^\circ\text{C}^3$	$\pm 5356^\circ\text{C}^3$	$\pm 0.0284$
$\frac{C}{3}(T_T^3 - T_B^3)$	$-0.450\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.042\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0924$
$qh$	$31.0\text{Jm}^{-1}\text{s}^{-1}$	$\pm 1.946\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0628$
$(\frac{x}{h})(qh)$	$21.7\text{Jm}^{-1}\text{s}^{-1}$	$\pm 1.899\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0875$
$AT_T$	$32.6\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.346\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0106$
$\frac{B-2}{2}T_T^2$	$3.069\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.130\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0424$
$\frac{C-3}{3}T_T^3$	$-0.451\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.041\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0901$
$AT_T + \frac{B-2}{2}T_T^2 + \frac{C-3}{3}T_T^3$	$35.22\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.517\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0147$
$(\frac{x}{h})qh - [AT_T + \frac{B-2}{2}T_T^2 + \frac{C-3}{3}T_T^3]$	$-13.5\text{Jm}^{-1}\text{s}^{-1}$	$\pm 2.416\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.1790$
$T_{PC}$	$22.9^\circ\text{C}$	$\pm 4.2^\circ\text{C}^{***}$	$\pm 0.1830$



(Table 27 - continued)

Variable	Value	Absolute Error	Relative Error
$BT_{PC}$	$0.0427 \text{ Jm}^{-1} \text{ s}^{-1} \text{ }^{\circ}\text{K}^{-1}$	$\pm 0.0089 \text{ Jm}^{-1} \text{ s}^{-1} \text{ }^{\circ}\text{K}^{-1}$	$\pm 0.2080$
$CT_{PC}^2$	$-0.0037 \text{ Jm}^{-1} \text{ s}^{-1} \text{ }^{\circ}\text{K}^{-1}$	$\pm 0.0016 \text{ Jm}^{-1} \text{ s}^{-1} \text{ }^{\circ}\text{K}^{-1}$	$\pm 0.4300$
$k_f$	$0.606 \text{ Jm}^{-1} \text{ s}^{-1} \text{ }^{\circ}\text{K}^{-1}$	$\pm 0.0124 \text{ Jm}^{-1} \text{ s}^{-1} \text{ }^{\circ}\text{K}^{-1}$	$\pm 0.0205$
999.168	$999.168 \text{ kgm}^{-3}$	$\pm 0.5 \text{ kgm}^{-3}$	$\pm 0.0005$
-0.0042607	$-0.0042607 \text{ kgm}^{-3} \text{ }^{\circ}\text{C}^{-2}$	$\pm 2.1 \times 10^{-5} \text{ kgm}^{-3} \text{ }^{\circ}\text{C}^{-2}$	$\pm 0.0490$
$-0.0042607 T_{PC}^2$	$2.23 \text{ kgm}^{-3}$	$\pm 0.93 \text{ kgm}^{-3}$	$\pm 0.4150$
$\rho$	$996.9 \text{ kgm}^{-3}$	$\pm 1.43 \text{ kgm}^{-3}$	$\pm 0.0014$
$T_{PC}-8.435$	$14.555^{\circ}\text{C}$	$\pm 4.2^{\circ}\text{C}$	$\pm 0.2890$
$(T_{PC}-8.435)^2$	$211.85^{\circ}\text{C}^2$	$\pm 122.5^{\circ}\text{C}^2$	$\pm 0.5780$
$8078.4+211.85$	$8290.2^{\circ}\text{C}^2$	$\pm 122.5^{\circ}\text{C}^2$	$\pm 0.0148$
$(8290.2)^{1/2}$	$91.05^{\circ}\text{C}$	$\pm 0.674^{\circ}\text{C}$	$\pm 0.0074$
$T_{PC}-8.435+91.05$	$105.61^{\circ}\text{C}$	$\pm 4.874^{\circ}\text{C}$	$\pm 0.0461$
$21.482(105.61)$	$2268.6 \text{ ms kg}^{-1}$	$\pm 104.7 \text{ ms kg}^{-1}$	$\pm 0.0461$
$\frac{1}{\mu}$	$1068.6 \text{ ms kg}^{-1}$	$\pm 104.7 \text{ ms kg}^{-1}$	$\pm 0.0980$
$\mu$	$0.000936 \text{ kgm}^{-1} \text{ s}^{-1}$	$\pm 9.2 \times 10^{-5} \text{ kgm}^{-1} \text{ s}^{-1}$	$\pm 0.0980$
$-\frac{dT}{dx}$	$17073^{\circ}\text{Km}^{-1}$	$\pm 1564^{\circ}\text{Km}^{-1}$	$\pm 0.0916$

(Table 27 - continued)

Variable	Value	Absolute Error	Relative Error
$T_{PK}$	296.1°K	$\pm 4.2^{\circ}\text{K}$	$\pm 0.0142$
$\alpha$	0.0739	$\pm 0.0228$	$\pm 0.3092$

\* Absolute error from the measurement of one-half a grid space,  $\pm .42 \times 10^{-5} m$ .

\*\* Absolute error is a sum of the absolute error in  $T_W$ ,  $\pm 0.2^{\circ}\text{C}$ , and an estimate of the temperature drop in the film resistance in the cooling water channel. From the Dittus-Boelter Equation [41], this drop was considered to be  $2.5^{\circ}\text{C}$ .

\*\*\* Absolute error is a sum of that obtained from the approximation for  $T_{PC}$ ,  $\pm 4.1^{\circ}\text{C}$ , and an estimate of the error due to the iteration itself,  $\pm 0.1^{\circ}\text{C}$ .

## b. Hexane

The basic equations are those listed in the previous subsection, with the following changes:

$$v_{th} = \frac{0.0002}{\tau} - v_g$$

$$v_g = \frac{(\rho_p - \rho) d_p^2 g}{18\mu}$$

$$\rho = 931.5 - 0.928 T_{PK}$$

$$\mu = 0.00117 - (2.9 \times 10^{-6}) T_{PK}$$

The values are the first datum point in Table 26,

$$d_p = 0.790 \text{ microns}$$

$$P_{os} = 1.75$$

$$T_T = 62.7^\circ\text{C}$$

$$\tau = 65\text{s}$$

$$T_B = 7.8^\circ\text{C}$$

The sample and error calculations are in Table 28 with the stipulated absolute errors italicized.

Table 28

## Sample and Error Calculations--Hexane

Variable	Value	Absolute Error	Relative Error
$P_{os}$	1.75	$\pm 0.1$	$\pm 0.0571$
$P_{os}/7.5$	0.233	$\pm 0.0163$	$\pm 0.0701$
$\frac{x}{h}$	0.767	$\pm 0.0163$	$\pm 0.0213$
$T_T$	62.7°C	$\pm 0.5^\circ\text{C}$	$\pm 0.0080$
$T_B$	7.8°C	$\pm 2.7^\circ\text{C}$	$\pm 0.3461$
A	$0.140\text{Jm}^{-1}\text{s}^{-1}\text{C}^{-1}$	$\pm 0.001\text{Jm}^{-1}\text{s}^{-1}\text{C}^{-1}$	$\pm 0.0072$
B	$-4.4 \times 10^{-4}\text{Jm}^{-1}\text{s}^{-1}\text{C}^{-2}$	$\pm 5.0 \times 10^{-6}\text{Jm}^{-1}\text{s}^{-1}\text{C}^{-2}$	$\pm 0.0114$
$T_T - T_B$	54.9°C	$\pm 3.2^\circ\text{C}$	$\pm 0.0583$
$A(T_T - T_B)$	$7.686\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.5034\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0655$
$T_T^2$	$3931.3^\circ\text{C}^2$	$\pm 62.9^\circ\text{C}^2$	$\pm 0.0160$
$T_B^2$	$60.8^\circ\text{C}^2$	$\pm 42.09^\circ\text{C}^2$	$\pm 0.6922$
$T_T^2 - T_B^2$	$3870.5^\circ\text{C}^2$	$\pm 104.99^\circ\text{C}^2$	$\pm 0.0271$
$\frac{B}{2}(T_T^2 - T_B^2)$	$-0.8515\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0328\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0385$
qh	$6.835\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.5362\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0784$

(Table 28 - continued)

Variable	Value	Absolute Error	Relative Error
$\frac{x}{h}(qh)$	$5.242\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.5227\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0997$
$AT_T$	$8.778\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.1334\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0152$
$\frac{B}{2}T_T^2$	$-0.8649\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0237\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.0274$
$\frac{x}{h}(qh) - [AT_T + \frac{B}{2}T_T^2]$	$-2.671\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.6798\text{Jm}^{-1}\text{s}^{-1}$	$\pm 0.2545$
$T_{PC}$	$19.7^\circ\text{C}$	$\pm 5.3^\circ\text{C}$	$\pm 0.2690$
$BT_{PC}$	$-0.00867\text{Jm}^{-1}\text{s}^{-1}\text{K}^{-1}$	$\pm 0.0024\text{Jm}^{-1}\text{s}^{-1}\text{K}^{-1}$	$\pm 0.2804$
$k_f$	$0.131\text{Jm}^{-1}\text{s}^{-1}\text{K}^{-1}$	$\pm 0.0034\text{Jm}^{-1}\text{s}^{-1}\text{K}^{-1}$	$\pm 0.0260$
$T_{PK}$	$292.9^\circ\text{K}$	$\pm 5.3^\circ\text{K}$	$\pm 0.0181$
931.5	$931.5\text{kgm}^{-3}$	$\pm 5.0\text{kgm}^{-3}$	$\pm 0.0054$
-0.928	$-0.928\text{kgm}^{-3}\text{K}^{-1}$	$\pm 0.01\text{kgm}^{-3}\text{K}^{-1}$	$\pm 0.0170$
$-0.928T_{PK}$	$-271.8\text{kgm}^{-3}$	$\pm 9.540\text{kgm}^{-3}$	$\pm 0.0351$
$\rho$	$659.7\text{kgm}^{-3}$	$\pm 14.54\text{kgm}^{-3}$	$\pm 0.0220$
0.00117	$0.00117\text{kgs}^{-1}\text{m}^{-1}$	$\pm 10^{-5}\text{kgs}^{-1}\text{m}^{-1}$	$\pm 0.0085$
$-2.9 \times 10^{-6}$	$-2.9 \times 10^{-6}\text{kgs}^{-1}\text{m}^{-1}\text{K}^{-1}$	$\pm 0.15 \times 10^{-6}\text{kgs}^{-1}\text{m}^{-1}\text{K}^{-1}$	$\pm 0.0052$
$-2.9 \times 10^{-6}T_{PK}$	$8.49 \times 10^{-4}\text{kgs}^{-1}\text{m}^{-1}$	$\pm 1.98 \times 10^{-5}\text{kgs}^{-1}\text{m}^{-1}$	$\pm 0.0233$
$\mu$	$0.000321\text{kgs}^{-1}\text{m}^{-1}$	$\pm 2.98 \times 10^{-5}\text{kgs}^{-1}\text{m}^{-1}$	$\pm 0.0928$

(Table 28 - continued)

Variable	Value	Absolute Error	Relative Error
$\frac{dT}{dx}$	$17346^{\circ}\text{Km}^{-1}$	$\pm 1955^{\circ}\text{Km}^{-1}$	$\pm 0.1127$
$\rho_p$	$1048\text{kgm}^{-3}$	*	*
$d_p$	0.790microns	$\pm 0.001 \text{ microns}$	$\pm 0.0013$
$d_p^2$	$0.624(\text{microns})^2$	$\pm 0.0016(\text{microns})^2$	$\pm 0.0026$
$g$	$9.8\text{ms}^{-2}$	*	*
$\rho_p - \rho$	$388.3\text{kgm}^{-3}$	$\pm 14.54\text{kgm}^{-3}$	$\pm 0.0374$
$v_g$	$0.411 \times 10^{-6}\text{ms}^{-1}$	$\pm 0.0546 \times 10^{-6}\text{ms}^{-1}$	$\pm 0.1328$
$\tau$	65.0s	$\pm 1.0s$	$\pm 0.0154$
$.0002/\tau$	$3.08 \times 10^{-6}\text{ms}^{-1}$	$\pm 0.306 \times 10^{-6}\text{ms}^{-1}$	$\pm 0.0994$
$v_{th}$	$2.67 \times 10^{-6}\text{ms}^{-1}$	$\pm 0.361 \times 10^{-6}\text{ms}^{-1}$	$\pm 0.1352$
$\alpha$	0.0927	$\pm 0.0353$	$\pm 0.3808$

\* No error estimate available.

### c. Observations

These calculations gave conservative estimates for the experimental errors of the  $\alpha$ 's. Note that a large error in  $T_B$  is quickly suppressed so that the film resistance in the cooling water channel can be safely neglected.

## 2. The Upper Velocity Limit of Stokes Law

Many of the calculations performed in this work are based on the particles obeying Stokes Law. This section shows that all measured and calculated velocities are well within the applicable range of this Law.

The Reynolds Number for a particle is defined as:

$$Re = \frac{d_p \rho v_p}{\mu}$$

The upper limit of Stokes Law is reached when  $Re \approx 0.1$ . Thus, the maximum velocity,  $v_m$ , is defined as:

$$v_m = \frac{0.1 \mu}{\rho d_p}$$

The following values of  $v_m$  were found for the conditions used in the previous section:

Table 29  
The Upper Velocity Limit of Stokes Law

Fluid	$d_p$ (microns)	$T_c$ (°C)	$\rho$ (kg m <sup>-3</sup> )	$\mu$ (kg m <sup>-1</sup> s <sup>-1</sup> )	$v_m$ (m s <sup>-1</sup> )
Water	1.011	22.9	996.9	0.000936	0.093
Water	0.790	22.9	996.9	0.000936	0.119
Hexane	1.011	19.7	659.7	0.000321	0.048
Hexane	0.790	19.7	659.7	0.000321	0.062

Since the velocities in this work were of the order of  $10^{-6}$  m s<sup>-1</sup>, it is clear that Stoke's Law applied.

### 3. The Temperature Drop Across Each Disk

Fourier's Law of one-dimensional heat conduction states:

$$q = k_B \frac{\Delta T}{\Delta x}$$

or

$$\Delta T = \frac{qh}{hk_B} \Delta x$$

where  $k_B$  is the thermal conductivity of brass (119 J m<sup>-1</sup> s<sup>-1</sup> °C<sup>-1</sup>--Table 1);  $\Delta x$ , the thickness of the disk face's (1/16 inch--Figures 3 and 5); and  $\Delta T$ , the temperature drop. Table 27 gives a  $qh$  for water of 31.0 J m<sup>-1</sup> s<sup>-1</sup>. Therefore, from the above equations, the axial heat flux and  $\Delta T$  equal 2.95 J s<sup>-1</sup> and  $4 \times 10^{-5}$  °C, respectively. The



temperature drop is negligible.

#### 4. The Temperature Rise of Cooling Water in Passing through the Bottom Disk

The temperature rise,  $\Delta T$ , of the cooling water can be estimated from a simple heat balance:

$$q = m_w C_w \Delta T$$

where  $m_w$  is the mass flux of the water through the cooling channel and  $C_w$  is the specific heat of water,  $4200 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$  (24). The experimental volumetric flow rate of the cooling water and axial heat flux are  $3 \text{ l min}^{-1}$  and  $2.95 \text{ J s}^{-1}$ , respectively. Therefore, the above equation predicts  $\Delta T = 0.014^\circ\text{C}$ , which is negligibly small.

#### 5. The Effect of Temperature Dependent $k_f$ upon $T_{PC}$ and $dT/dx$

The present section estimates the changes which arise in the local fluid temperature,  $T_{PC}$ , and temperature gradient,  $dT/dx$ , when the thermal conductivity of the liquid is assumed to be independent of temperature. If the subscript LIN denotes the case of constant  $k_f$ , then:

$$T_{LIN} = T_T - (T_T - T_B) \frac{x}{h}$$

and

$$-\frac{dT}{dx}_{LIN} = \frac{(T_T - T_B)}{h}$$

The differences between these parameters and the actual values are measures of the effect of the temperature dependent thermal conductivity.

a. Water

From section 1:

$$T_T = 57.4^{\circ}\text{C}$$

$$\frac{x}{h} = 0.700$$

$$T_B = 7.2^{\circ}\text{C}$$

$$- \frac{dT}{dx} = 17073^{\circ}\text{K m}^{-1}$$

Hence:

$$T_{LIN} = 22.3^{\circ}\text{C}$$

$$- \frac{dT}{dx}_{LIN} = 16733^{\circ}\text{K m}^{-1}$$

and the percentage errors are:

$$100 (T_{PC}/T_{LIN} - 1) = 2.7\%$$

$$100 ((dT/dx) / (dT/dx)_{LIN} - 1) = 2.03\%$$

These values are small but not negligible.

b. Hexane

From section 1:

$$T_T = 62.7^{\circ}\text{C}$$

$$\frac{x}{h} = 0.767$$

$$T_B = 7.8^{\circ}\text{C}$$

$$-\frac{dT}{dx} = 17346^{\circ}\text{K m}^{-1}$$

and so

$$T_{\text{LIN}} = 20.6^{\circ}\text{C}$$

$$-\frac{dT}{dx}_{\text{LIN}} = 18300^{\circ}\text{K m}^{-1}$$

The percentage differences are -4.4 per cent and -5.21 per cent, respectively. The minus signs are present because  $k_f$  for hexane falls with temperature whereas  $k_f$  for water rises.

## Appendix IV

### Thermophoretic Force and Velocity for the Large Particle Regime

This section outlines the Epstein solution for the thermophoretic force and velocity of a spherical particle in an attempt to clarify the role of thermal creep in the large particle regime. The discussion will consist of two parts: first, the solution of the Fourier heat conduction equation to obtain the tangential temperature gradient at the particle surface and its use in the Maxwell thermal creep equation; second, the solution of the Navier-Stokes Equation for creeping flow to yield the thermophoretic force and velocity. The Maxwell equation is required because the thermal creep velocity constitutes a boundary condition for the Navier-Stokes Equation.

Figure 25 shows a stationary particle in a fluid that is moving at a velocity (at  $r \rightarrow \infty$ )  $v_f$  in the negative x-direction. The fluid has a constant temperature gradient,  $dT/dx$ , imposed on it. A polar co-ordinate system has its origin at the centre of the particle. At its surface, a tangential temperature gradient,  $dT/ds$ , and thermal creep velocity,  $\underline{v}_s$ , are oriented in the negative  $\theta$  direction.

Maxwell's Equation for the thermal creep velocity is:

$$\underline{v}_s = \frac{3}{4} \frac{\mu}{\rho T_K} \frac{dT}{ds} \quad [\text{IV-1}]$$

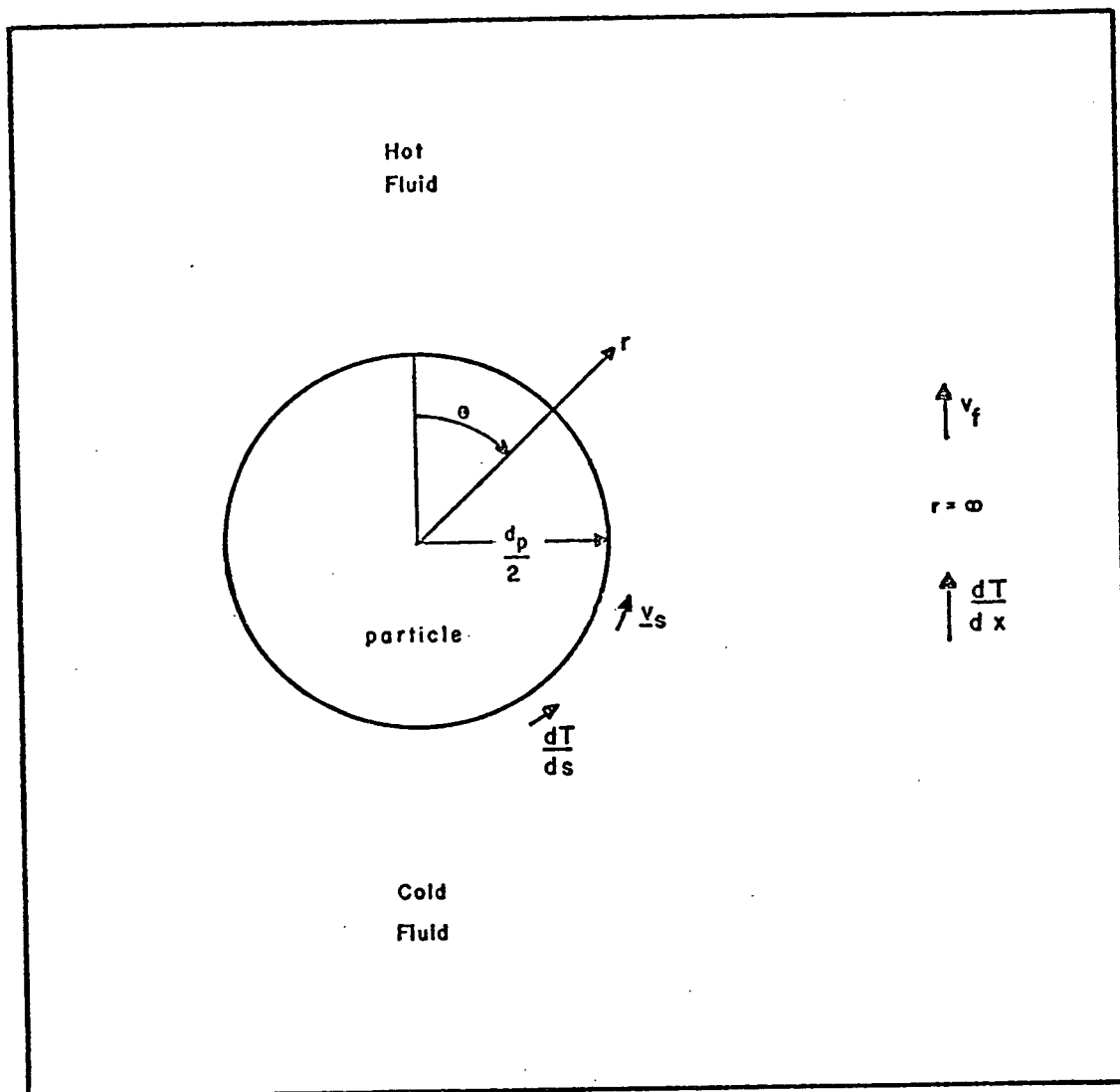


Figure 25

Co-ordinate System and Variables for the Large Particle Regime

Epstein (12) obtained the tangential temperature gradient by solving the Fourier Equation:

$$\Delta^2 T = 0 \quad [\text{IV-2}]$$

subject to the following boundary conditions. First, the temperature and radial heat flux at a point on the particle surface and in the fluid adjacent to that point are equal. Second, the temperature gradient at  $r = \infty$  is unaffected by the presence of the particle, i.e., it is  $dT/dx$ . Epstein's equation for  $dT/ds$  is:

$$\frac{dT}{ds} = \frac{3k_f}{2k_f + k_p} \frac{dT}{dx} \sin(\theta) \quad [\text{IV-3}]$$

Hence Equation [IV-1] can be restated as:

$$\underline{v_s} = \omega \sin(\theta) \quad [\text{IV-4}]$$

where

$$\omega = \frac{9k_f}{8k_f + 4k_p} \frac{\mu}{\rho T_K} \frac{dT}{dx} \quad [\text{IV-5}]$$

Since the particle is stationary, the fluid creeps up the temperature gradient. If the particle were free to move, however, the fluid would force the particle down the gradient. Thermal creep differs, therefore, from hydrodynamic slip in that the fluid itself causes particle movement.

The Navier-Stokes Equation is:

$$E^4 (\psi) = 0 \quad [\text{IV-6}]$$

where the operator, E, is defined as:

$$E^2 = \frac{\partial}{\partial r^2} + \frac{\sin(\theta)}{r^2} \frac{\partial}{\partial \theta} \left( \frac{1}{\sin(\theta)} \frac{\partial}{\partial \theta} \right) \quad [\text{IV-7}]$$

and the stream function,  $\psi$ , is related to the fluid velocity components,  $v_r$  and  $v_\theta$  by:

$$v_r = - \frac{1}{r^2 \sin(\theta)} \frac{\partial \psi}{\partial r} \quad [\text{IV-8}]$$

and

$$v_\theta = \frac{1}{r \sin(\theta)} \frac{\partial \psi}{\partial \theta} \quad [\text{IV-9}]$$

The method of solving Equation [IV-6] follows that outlined by Happel and Brenner (20).

Equation [IV-6] is a fourth order partial differential equation and thus four boundary conditions must be specified. These are:

$$r \rightarrow \infty \quad v_r = v_f \cos(\theta) \quad [\text{IV-10}]$$

$$r \rightarrow \infty \quad v_\theta = -v_f \sin(\theta) \quad [\text{IV-11}]$$

$$r = d_p/2 \quad v_r = 0 \quad [\text{IV-12}]$$

This meets the requirement of no flow through the particle surface.

$$r = d_p/2 \quad v_\theta = 0 \quad [\text{IV-13}]$$

This means no hydrodynamic slip is present. However, the thermal creep velocity appears within one mean free path,  $L$ , of the particle surface. In the case of liquids this is approximately twice the molecular diameter. Since  $L$  is very small, the boundary condition at  $r = d_p/2 + L \approx d_p/2$  is:

$$v_\theta = -v_s = -\omega \sin(\theta) \quad [\text{IV-14}]$$

Using Equations [IV-8] and [IV-9], the boundary conditions can be transformed to:

$$r \rightarrow \infty: \quad \frac{\partial \psi}{\partial \theta} = -v_f r^2 \sin(\theta) \cos(\theta) \quad [\text{IV-15}]$$

$$r \rightarrow \infty \quad \frac{\partial \psi}{\partial r} = -v_f r \sin^2(\theta) \quad [\text{IV-16}]$$

$$r = d_p/2 \quad \frac{\partial \psi}{\partial \theta} = 0 \quad [\text{IV-17}]$$

$$r = d_p/2 \quad \frac{\partial \psi}{\partial r} = -\omega \frac{d_p}{2} \sin^2(\theta) \quad [\text{IV-18}]$$

A general solution of Equation [IV-6] is:

$$\psi = \sin^2(\theta) \left( \frac{A_0}{10} r^4 - \frac{B_0}{2} r + C_0 r^2 + \frac{D_0}{r} \right) \quad [\text{IV-19}]$$

where  $A_0$ ,  $B_0$ ,  $C_0$ , and  $D_0$  are constants determined from the boundary conditions. Differentiating Equation [IV-19] gives:

$$\frac{\partial \psi}{\partial \theta} = \sin(\theta) \cos(\theta) \left( \frac{A_0}{5} r^4 - B_0 r + 2 C_0 r^2 + \frac{2D_0}{r} \right) \quad [\text{IV-20}]$$

and



$$\frac{\partial \psi}{\partial r} = \sin^2(\theta) \left( \frac{2A_0 r^3}{5} - \frac{B_0}{2} + 2C_0 r - \frac{D_0}{r^2} \right) \quad [\text{IV-21}]$$

Substituting Equation [IV-15] in Equation [IV-20] results in:

$$-v_f r^2 = \frac{A_0 r^4}{5} - B_0 r + 2C_0 r^2 + \frac{2D_0}{r} \quad [\text{IV-22}]$$

When  $r \rightarrow \infty$ , there can be no power of  $r > 2$  on the right-hand side of Equation [IV-22]. Therefore,

$$A_0 = 0 \quad [\text{IV-23}]$$

In addition, all terms but  $2C_0 r^2$  can be neglected because  $r^2 \gg r$  or  $1/r$  as  $r \rightarrow \infty$ . Therefore,

$$C_0 = -\frac{v_f}{2} \quad [\text{IV-24}]$$

Substituting Equation [IV-17] in Equation [IV-20] gives:

$$0 = -B_0 \frac{d_p}{2} - v_f \left( \frac{d_p}{2} \right)^2 + \frac{4D_0}{d_p} \quad [\text{IV-25}]$$

and Equation [IV-18] in Equation [IV-21] results in:

$$-\omega = -\frac{B_0}{d_p} - v_f - \frac{8D_0}{(d_p)^3} \quad [\text{IV-26}]$$

Dividing Equation [IV-25] by  $d_p^2/2$  and adding the result to Equation

[IV-26] gives:

$$B_0 = \frac{3 d_p}{4} \left( \frac{2}{3} \omega - v_f \right) \quad [IV-27]$$

The force exerted by the fluid on the particle,  $F_f$ , is:

$$F_f = - 4 \pi \mu B_0 \quad [IV-28]$$

Combining Equation [IV-27] and Equation [IV-28] yields:

$$F_f = 3 \pi \mu d_p v_f - 3 \pi \mu d_p \left( \frac{2}{3} \omega \right) \quad [IV-29]$$

or

$$F_f = F_D - F_{th} \quad [IV-30]$$

When the particle is free to move (rather than held stationary) equilibrium between the drag and the thermophoretic forces exists and  $F_f = 0$ . Hence:

$$v_f = \frac{2}{3} \omega \quad [IV-31]$$

If the fluid is stagnant, the thermophoretic velocity of the particle is:

$$v_{th} = - v_f = - \frac{2}{3} \omega \quad [IV-32]$$

or, utilizing Equation [IV-5]:

$$v_{th} = - \frac{3}{2} \frac{k_f}{2 k_f + k_p} \frac{\mu}{\rho T_K} \frac{dT}{dx} \quad [IV-33]$$

This is Epstein's equation for thermophoretic velocity. The thermophoretic force is then

$$F_{th} = 3 \pi \mu d_p v_{th} \quad [IV-34a]$$

$$= - \frac{9}{2} \frac{k_f}{(2 k_f + k_p)} \frac{\pi \mu^2 d_p}{\rho T_K} \frac{dT}{dx} \quad [IV-34b]$$