DETERMLNATIOM OF GA: EPFECTTVE DIFFUSTVITES IN POROUS BOLIDS, DISPBRSION COEFFTCIETG'S IT PACKED BLD: BDD MOLECULAR DIFIUEIVITY OF BINARY SYGDTE
by
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M.A.Sc., University of British Columbia, 1963

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Of

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December, 1965

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# The University of British Columbia 

 FACULTY OF GRADDUATE STUDIESPROGRAMME OF THE

FINAL ORAL EXAMINATION

## FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

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|  | Ottawa |
| Research Sup | pervisor: D. S. Scott |AND THE LONGITUDINAL DISPERSION COEFFICTENT

Present methods of measurement of effective diffusivities are not generally adaptable to the pellets in a packed bed, for example a catalytic reactor, An unsteady state pulse method has been developed employing simple gas chromatographic rate theory.

The method is generally applicable to pellet sizes down to about 2 mm . : With homogeneous pellets reasonable agreement was obtained on comparison of effective diffusivities measured by a steady state method. For anisotropic solids the unsteady state diffusivity can be quite different from the steady state value due to differences in diffusion path.

Pulse dispersions measured in beds of non porous pellets have revealed a laminar flow regime where the dispersion coefficient is dependent on the square of the velocity. This regime was reported for flow in straight pipes but has not previously been demonstrated in packed beds.

## SECTION II

## DEVELOPMENT OF AN UNSTEADY STATE FLOW METHOD FOR MEASURING BINARY GAS DIFFUSION COEFFICIENTS

Effusion measurements of one gas from a packed bed of known geometry (porosity and tortuosity) into a second flowing gas have been evaluated as a versatile technique for the determination of binary gas diffusion coefficients.

The molecular diffusivities measured ( $\mathbf{(} 10 \%$ ) approached the scatter encountered by other methods $( \pm 5 \%)$ and satisfactory results ( $\pm 3 \%$ ) are envisaged by optimising parameters in the method.

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## ABGTRACT

## BECHTON I

AN EXPERIMETTAL METHOD FOR TYH MEASUREENT OF EFFECTVE GAS DIFPUSIVIFTES IN POROUS PELLETB, AND THE LOGIUUDINAL DISPERSION COEPTICIENT IH PACKED BDS

Prosent methods for measuring effectivo diffusivibies in emali porous particies are not applicable to assemblages of such pellets, for erample, as in catalytic reactors, and recuire special techniques or apparatus, A pulee technique has been developed which can successfully yield a reasonable value of the diffusivity by onalysis of pulse dispersion in tems of simplo chromatographic rate theory. A non-adsorbing pulse gas is nocossery, and kydrogen is nearly incal. Pecause of the high molecular diffusivity of hydrogen ine smallest size of particle which con be testea with this sas is about 2 ma aicherer. The unsteady state pulse effective diffusivity measurcment rinich should be more realistic for catalytic studies was compared with a conventicnel steady state method and good agreement obtained in a spherical isotropic pellet. ( $4, \%$ ); however, as may be expected agreement was poor with anisotropic pellets.

A regime vas found in a study of beris of non porous pellets wiexe the dispersion coefficient is oroportional to the square of the velocity. This regime is reported for pipes but, has not been realized as a separate regime in pecked beds. This dispersion data is compared rith tho limited daca or otner workers although the ranges of axperimental conditions do not overiap.

## SECMTON II

DEVELOPIVENT OF AN UNGEEADY SCATR FLOH METHOD FOR MEASURING BIMGYY GAS DIFTUSION COEFFICIENTS

Effusion of one gas from a packed bed of known geometry into a second flowing gas has been evaluater as a versatile technique for aeternination of binary gas diffusion coerficients having few limitetions of pressure, temp- $\because$. enturc and analysis method. Oplimization of experimental paraneters shouid yiela saiisfactory results $( \pm 3 \%)$.

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INTRODUCTION

## A. THIELE MODULUS

The rate of reaction in a porous solid catalyst can be limited by the rate at which reactants and products can diffuse in and out of the solid. Thiele (1) quantitatively described this effect with a mathematical treatment which is applied to a simple case of an infinite slab in the derivation below. In Fig. I.l a single pore of radius $r$ and length $L$ is shown. A first order gas phase reaction with rate $=k C_{A}$ moles $/(\mathrm{sec})\left(\mathrm{cm}^{2}\right)$ is assumed to be taking place isothermally on the pore walls, and a constant concentration $C_{A O}$ moles $/ \mathrm{cm}^{3}$ is maintained at each face of the slab at the pore mouth. A material balance around the element $d x$ yields,

$$
\begin{equation*}
-D \frac{d C_{A}}{d x} \pi r^{2}-(-D)\left[\frac{d C_{A}}{d x}+\frac{d_{2} C_{A}}{d x^{2}} \delta x\right] \pi r^{2}-k C_{A} 2 \pi r \quad 6 x=0 \tag{1.1}
\end{equation*}
$$

which may be simplified to,

$$
\begin{equation*}
\frac{d_{2} C_{A}}{d x^{2}}=\frac{2 k}{r D} C_{A} \tag{1.2}
\end{equation*}
$$

The boundary conditions, $C_{A}=C_{A O}$ at $x=0$, and $\mathrm{dC}_{A} / d x=0$ when $x=L$, may be applied to the solution of (1.2) to give the concentration in the pore,

$$
\begin{equation*}
C_{A}=C_{A O} \frac{\cosh \left[h\left(1-\frac{x}{i}\right)\right]}{\cosh h} \tag{1.3}
\end{equation*}
$$

where $h=L \sqrt{\frac{2 k}{D r}}$, commonly known as the Thiele modulus.
The rate of reaction is given by the rate of diffusion of $A$ into the pore mouth, which in turn is given by,

$$
\begin{align*}
\text { rate/pore } & =-D\left(\frac{d C_{A}}{d x}\right)_{x=0} \pi r^{2}  \tag{1.4}\\
& =\frac{D C_{A O}}{L} h \tanh (h) \pi r^{2}
\end{align*}
$$



Figure 1.1
Pore liodel For Derivation Of Effectiveness Factor

If the whole pore contains gas at the surface concentration, $\mathrm{C}_{\mathrm{AO}}$, the rate of reaction will be a maximim, given by $k . \mathrm{C}_{\mathrm{AO}}$. $2 \operatorname{TrL}_{\mathrm{L}}$ moles/ sec. The ratio of the rate given by equation (1.4) and this naximum rate is defined as the eifectiveness factor, .

$$
\begin{equation*}
\frac{\text { rate/nore }}{\text { maximum rate }}=E=\frac{r D}{2 k I^{2}} \quad h \tanh h=\frac{\tanh h}{h} \tag{1.5}
\end{equation*}
$$

The effectiveness factor is a function of the Thiele modulus only, and can be used to calculate the rate of reaction when diffusional resistances are controlling.

$$
\begin{equation*}
\text { rate of reaction/pore }=k C_{A O} 2 \pi r L E \tag{1.6}
\end{equation*}
$$

Additional equations can be derived for other orders of reaction, (2) other assumed pore geometries (3), or for cases where the stoichiometry does not allow equimolar counter diffusion to occur (1)(4).

In practical cases it is very difficult to define accurately the pore geometry of a porous solid, and rate constants are more commonly based upon unit mass of catalyst. It is convenient mathenatically to treat the porous solid as a homogeneous medium having an effective diffusivity, rather than attempting to use the true interstitial diffusivity together with the void fraction and suitable assumptions about the pore geometry. B. DIFFUSION MECIIANISMS

There are two basic gas transport processes which occur in porous solids, and which obey Fick's laws of diffusion, namely, molecular or bulk diffusion which occurs through intermolecular collisions, and Knudsen diffusion, which depends only upon wall collisions. In addition, a phenomenon known as "surface diffusion" can take place, but this is not a well understood process. Surface diffusion is believed to result from multilayers of gas molecules condensed to a liquid-like state, which flow from the areas with several layers to those of lower surface concentration. This process results in diffusion rates much larger than those possible by collision mechanisms.

Gases above their critical temperature are less likely to display this phenomenon, because of the reduction in surface adsorption under these conditions.

## Molecular Diffusion in Pores

This mode of diffusion predominates when the ratio of the pore radius to mean free path is greater than about 10. Fick's law, or the one dimensional flux equation for steady-state molecular diffusion in a two component mixture takes the form,

$$
\begin{equation*}
N_{A}=-D_{B} \frac{d C_{A}}{d x}+\left(N_{A}+N_{B}\right) \frac{C_{A}}{P_{m}} \tag{1.7}
\end{equation*}
$$

where the last term accounts for bulk fiors thich may be caused by nonequimolar counter diffusion rates of the two gases with rospect to stationary coordinates. In order to apply the equation to a porous stmuture the flux is taken per total unit area of solid and pore, rather thon unit area of pore only,

$$
\begin{equation*}
N_{A}^{2}=N_{A} \epsilon_{p}=-\left[\frac{D_{B} \epsilon_{2}}{\lambda}\right] \quad \frac{d C_{A}}{\overline{C X}}+\left(N_{A}^{2}+\Pi_{B}{ }^{2}\right) \frac{C_{A}}{\rho_{m}} \tag{1.8}
\end{equation*}
$$

Where $\lambda$ is the "tortuosity" which corrects for the fact that the pore length is greater than the geometric Iength of the structure. The terns thich are grouped with the diffusivity form the definition of an "effective diffusivity" which will be discussed later.

Fick's second law which describes the unsteady state diffusion process is usually expressed as,

$$
\begin{equation*}
\frac{\partial C_{A}}{\partial t}=D_{B} \frac{\partial \Xi C_{A}}{\partial x^{2}} \tag{1.9}
\end{equation*}
$$

In a porous solid, introducing the concept of an eifective diffusivity, this equation should be modified as shown below:

In unit area of a porous infinite slab, a mass balance over the element $8 x$ when no chemical reaction is occurring and alloving for a net bulk flow gives the rate of change of gas content in terms of the effective diffusivity, $\mathrm{D}_{\mathrm{i}}$, as,

$$
\begin{equation*}
\epsilon_{P} \frac{\partial C_{A}}{\partial t} \delta x=+D_{E}\left(\frac{\partial_{2} C_{A}}{\partial x^{2}}\right) \delta x-\frac{\partial\left({ }^{u C_{A}}\right)}{j x} \delta x \tag{1.10}
\end{equation*}
$$

where $u$ is the superficial bulk velocity.
Simplication of 1.10 gives,

$$
\begin{equation*}
\frac{\partial C_{A}}{\partial t}=+\frac{D_{E}}{\epsilon_{P}} \frac{\partial_{2} C_{A}}{\partial x^{2}}-\frac{1}{\epsilon_{\mathrm{P}}} \frac{\partial\left(u C_{A}\right)}{\partial x} \tag{1.11}
\end{equation*}
$$

For equimolal counter diffusion, this reduces to the usual form of Fick's second law, that is, equation (1.9) except that the molecular diffusion coefficient is replaced by the effective diffusion coefficient divided by the porosity.

If equimolar counterdiffusion occurs then equation (1.8) for the.. steady state reduces to

$$
\begin{equation*}
N_{A}{ }^{\perp}=-D_{E} \frac{d C_{A}}{d x} \tag{1.12}
\end{equation*}
$$

The binary molecular diffusion coefficient of a gas is
proportional to the absolute temperature to about the 1.7 power, and inversely proportional to the pressure.

## Knudsen Diffusion

This mechanism predominates when the mean free path of the gas molecules is greater than the pore radius, and because wall collisions contribute primarily to the process in these circumstances, the diffusion coefficient is independent of the presence of other gases. Bulk flow is not distinguishable from diffusion in this case, and so Fick's law in the form of equation (1.12) applies.

- 6 -

The Knudsen diffusion coefficient in cylindrical straight
pores is given by,

$$
\begin{equation*}
D_{K}=2 / 3 r \bar{v} \tag{1.13}
\end{equation*}
$$

Where $\vec{v}$ is the average velocity of the gas molecules, and $r$ the pore radius. In consequence, the value of this coefficient is independent of pressure, and proportional to the square root of the temperature.

## Intermediate or Mixed Diffusion Coefficient

In the intermediate range between molecular and Knudsen diffusion there is a region where both the above diffusion mechanisms occur. The ratio of pore radius to mean free path lies approximately between the following limits in the intermediate zone:

| Knuāsen | Intermediate |  | Miolecular |
| :---: | :---: | :---: | :---: |
| 0.1 | $\frac{r}{\lambda}$ |  |  |

By assumine round capillaries, rigid sphere kinetics and diffuse molecular reflection from the walls, Scott and Dullien (5) derived the following relationship for the flux in a binary gas mixture in the intermediate region.

$$
\begin{equation*}
N_{A}=-\frac{P}{R T} \frac{d y A}{d x}\left[\frac{1}{\frac{I-j y_{A}}{D_{\mathrm{B}}}+\frac{1}{D_{K A}}}\right] \tag{1.24}
\end{equation*}
$$

where $j=1+N_{A} / N_{B}$, and $y_{A}$ is the mole fraction of $A$.
If the term in brackets is considered as the diffusion coefficient, it is obvious that in this region the coefficient is dependent upon the concentration and flux. A diffusion coefficient defined by an equation or the form of (1.12) and measured in this region is not valid for use in the Thiele modulus as defined previously, as the stoichiometry of the chemical reaction imposes a flux ratio which is unlikely to be the same as the flux ratio obtained in an independent non-reactive determination.

An effective diffusion coefficient is defined in equation (1.8)
for molecular diffusion where two factors are used to modify the true or interstitial diffusivity. The porosity, or void fraction, is a fairly easily defined and measured absolute quantity, and in a granular bed may be of the range of 0.3 to 0.5 . However, the tortuosity is a derived quantity, and is therefore usually a less well-defined property, especially in nonuniform pore structures. Although a value of around 1.5 might be expected from simple pore models, it can vary from 1 to 100 when calculated from experimental results. Thus, a typical simple structure may have an effective diffusivity about 4 times less than the interstitial value.

The large range of tortuosity values can be attriouted to tro sources. First, the pores are not necessarily open-ended and so the mass transfer may be only occurring in a limited number of passages. pecond, the pore radius is liable to vary along the length of the pore, and it has been shom (6) (7) that the rate of diffusion is smaller through a pore of varying radius than it is through a cylindrical pore of equivalent volume to surface ratio.

The effective diffusivity can serve as a simple correction to the diffusion mechanism so that the diffusion equation describes the transport behaviour in a uniform porous structure. On the other hand, porous structures can be so haphazard that any of the mechanisms described may occur at the same time in series or in parallel in the same solid. The use of an effective diffusivity in this case amounts to forcing the behaviour observed to fit one of the diffusion equations, and so the result cannot be used to predict the diffusive behaviour under other conditions.

## C. EXPERIMENCAL ESCIMATION OF THE EFFECTIVE DIFFUSIVIIY

## Prediction

The basis for the prediction of the effective diffusivity has been briefly outlined in the previous paragraph, and clearly rests on some physical idealization of the pore structure. Prediction methocis based upon porosity and experimental tortuosity values are often not too satisfactory due to the non-uniform nature of many porous solids. However, a variety of catalyst pellets can be approximated by the "pile or bricks" structure which yields a model consisting of a honeycomb of connected passages. This approach has been described in detail by Wheeler (2), with rules for predicting the effective diffusion coefficient defined by this model.

Other simple pore models include unconnected parailel cylindrical pores (8) (9), and pores with "ink bottle" capacities (10) which are used to explain the hysteresis in certain adsorption-aesorption curves.

Possibly of more general application to the problems involved in catalytic kinetics is the bidisperse pore structure model proposed by Wokao and Smith (11) and Mingle and Smith (12). In the latter paper, a concept of larger macro pores in series with micro pores is used. In the former, three parallel mechanisms are considered; first, diffusion through the macro pores between the basic particles from which the pellet is pressed, second, diffusion in the micro pores of the basic particle, and finally, series diffusion from micropores to macro pores or vice versa. The model does not require empirical constants, or assumptions regarding the mode of diffusion in any of the pores, but porosities ana a pore size frequency distribution function are required in addition to tortuosity values.

In this method, a eylinerical catalyst pellet is fittod into a tube and two test gases of known composition arc passed continuously across the ends. The two exit streams are anolyzed, and from an appropriate solution of the diffusion equation the eifective diffusivity can be computed (13) (5).

This method has also been used to obtain molecular diffusivities (14), because calibration of the porous pellet by a gas pair of known diffusivity allows calculation of the diffusivities of other gas pairs by making the assumption that the tortuosity is independent of the sas system. As a technique for measuring molecular diffusivities it has the advantage that it is a flow method, and so analysis in situ is not required. On the other hand, care must be taken that a narrow pore size distribution exists and that Knudsen diffusion does not occur.

When used as a method for measuring the effective difiusivity in porous pellets one must be sure that the correci dilfusion equation has been used (e.g. eqn. (1.8) or (1.I2)). The method can be applied to the mixed diffusion range if measurements are made at varying total pressures. However, it has the limitation of being tedious if a representative average value is needed, because each pellet must be tested separately, and cracks and fissures have an overwhelming influence on the result. The technique is not convenient for use with other than cylindrical shapes, and thereIore other shapes must be machined to cylinders. If the peliet is not isotropic, this procedure may result in a faulty value of the diffusion coefficient.

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The result obtained by the steady state method in a bidisperse pellet weighs the diffusivity in favour of the larger pores, but in the chemical reaction case most of the conversion takes place in the micro pores. This bias is frequentiy not serious as the micropores are generally short, and so a micropore effectiveness factor of unity is common (11). Hence, the dilfusional resistance to reaction is in the macropores, and the steady state effective diffusivity value may be quite adequate. Chemical Reaction Method

It is obviously possiole to carry out a chemical reaction of known kinetic behaviour at constant conditions using successively smaller sizes of pellet until the reaction rate becomes constant, indicating that an effectiveness factor of unity has been reached. From these effectiveness factors the Thiele modulus, and hence effective diffusion coefficients, can be calculated providing the kinetic behaviour is not complex. This method is not easy to apply experimentally, and is subject to many errors. Unsteady State Methods

A typical procedure for measuring the molecular diffusivity of a gas (Loschmidt method) consists of flushing two cylinders with the test gases, and then bringing them together at time zero'with the lighter gas on top. One or both of the cylinders is removed at a given time, and the total contents analyzed. The diffusion coefficient is then calculated From the solution derived from Fick's secona law (equation (1.9)). It is aifficult to achieve accuracy in this experiment due to the tendency for edaies to be created either when the cylinders are fitted together or by the action of temperature gradients.

For porous pellets the analog of the avove experiment canot be readily applied due to the rapiciity or the diffusion process in goses. For example, if a one cm . diameter pellet of typical pore structure is initially bathed in one gas, and at time zero the surface is flushed with another gas, then $98.8 \%$ of the first gas in the peliet is removea by Uiffusion in 10 seconds if the diffusivity $D_{\mathrm{E}} / \mathrm{E}_{\mathrm{P}}$ is $0.01 \mathrm{~cm}^{2} / \mathrm{sec}$. (Bee Appenaix III for details of this calculation.) Thus, it is obvious that some means to extend the time scale in experiments with small pellets would be very desirable.

Currie (6) has developed a non-flow apparatus of this type which can be used only at normal temperatures and pressures for measuring diffusivities in soils and other granular beds. Only rather complex frequency response techniques, discussed below, are presently available for the measurement of effective diffusion coefficients by transient response nethods.

Frequency Response and Pulse Methods
McHenry and Wilhelm (15) have described a method for measuring the eday diffusivity in packed beds, and this apparatus has been used also by Deissier and Wilhelm (16) to measure both the effective diffusivity and the eddy diffusivity in packed beds. The method is based on frequency response techniques using a concentration sine wave generated in the feed to the bed, with amplituacs and phase angles recorded at the entrance ana exit of a test section.

In the same way, Van Deemter, Zuiderweg and Klinkenberg (17) have applied the delta function (that is, an ideal pulse) to packed beds in the form of gas chromatography columns and ion exchange beds. Hougen (18)
has pointed olit that there is a real dirference bewreen the results obtained by a delta function or by a frequency response method.

In the work of ven Deenter et al the diepersion effects due to rolecular diffusivity, edny diffusivity and a mass transfer coefficient are each round, on the basis of the theory developed, to have a different velocity dependence, which allows separation of the influence of each factor on the delta function. The mass transfer coefficient can be derived in temas of the efrective difrusivity or the porous pellet, and hence, if this quantity can be evaluated, an effective ciffusion coefficleat may be calculated from it. The theory on which this approached is bascd is dealt with more fully in succeaing sections.

## Comparison of Various Mothods

In porous solids there is a basic difference betreen the opplication of difrusion coefficients to the steady state and the unsteady state. 'Mhis difference is the result of the capacitance.effects which manirest thenselves in the unsteady state. In other words, the time of diffusion from a porcus solid condaining dead end pores would be moch greater than the effective diffusivity measured by a stcaiy sifie method would indicate. This efeect is ailoved for in equation (i.11) because inctead of the effective difusivity alone, the effective diffusivity diviaed hy a capacitance term (the porosity) is utilized. Similarly, if adsorption occurs on the surface of the solid then the volume of gas adsorbed must be adaca to the porcus volune or porosity in the divisor. (This last statement regarding adsorption assumes that the adsorption process is effectively at equilibrium and that the isothem is linear, othervise the simple dirfusion equation vould no longer hold.) With the comect dirfusion equation, there shoula be no
hasic difference between effective diffusivity in ar isotropic solid detemined by a steady state or unsteady state method.

If buik diffusion is the transport mechanism there is no difficulty in correctily defining the effective coefficient for either the steady state or unsteady statc methods. However, this is not true when Knudsen difrusion ypeaminated. Consider o simple model of dead end pores of equel leneth in parallel in which Knudsen diffusion is taking place. The total composition of cach pore (after a step change in surface concentration) will vary according to its radius. Initially, the large pores will yield the major flux, but after a time the lower flux in the smaller pores will result in larger concentration gradients, which will eventually result in the flux from the smaller pores equalling or exceeding that from the larger pores. Hence, an unstcady state experiment in the Knudsen regime may yield a diffusivity which varies with time.

An interesting aspect of this latter conclusion arises because the majority of a solid-surface catalyzed chemical reaction occurs in the smaller pores (due to the large surface area), and if these pores are long then they may not be fully effective. The steady state method is insensitive to the resistance which may occur in dead end pores, while the unsteady state method is potentially capable of allowing for this resistance. The unsteady state method will give a dirfusivity which is some average value of all pore resistances, and the ability of this value to describe the rate or a diffusion limited chemical reaction may depend upon the weighting by the experimental procedure or the experimenter. For example, most unsteady state methods involve an initial period before readings are taken in order to allow the application to the data of simple solutions of the diffusion equation applicable at longer times. Thus, in this case, the
ijefusivity obtainei from such experiments may be expecied io be weighted in favour of the small pores if Knudsen difinusion predominates. D. OBJECRIVAS OF THE PRESERT DORK

On the basis of the foregoing comparison of methods for obtainine a value of the effective dirfusivity, it is apparent that, in most cases, a
 Ell the pores contribute to the diffusional process may well be a better value for use in chemically reacting systems. In many instances, steadystate experiments may also give suitable values, but this cannot be assuned. vithout considerable knowledge of the particular porous structure.

It would be useful to develop a method using a pulse technique, Which would avoid nost of the experimental difficulties of frequencyresponse measurements, while giving the arvantages of an unsteady-state method and which could be applied to a representative sample of pellets without requiring special shaping. It might ve possible to make use of such a technique to follow changes in catalyst diffusional behaviour with a'se. Recent advances in the theory of transport processes in chromatographic columns suggest that it might be possible to interpret pulse dispersion results in such a way as to yiela an effective diffusion coefficient.

The primary objective of the present work was to attempt the development of a pulse method as a means of measuring effective diffusivities oi gases in porous pellets, a technique not previously reported. A secondary objective vas to be the investigation of the use of unsteady state flow methods for measuring the binary diffusion coefficient of gases. The flow methods possess the advantage of allowing analysis outside the apparatus, by any convenient means. Further, the use of a porous bed of unit tortuosity woula also allow such a measurement to give absolute values of the

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citefusion coefficient without any calibration being necessary. Freedom from convective effects would aid in making possible measurements at Widely varying temperatures ana pressures, as does the freedom in choice oi concentration measurement.


## A. DERIVA'PION OF VAN DETMMER EQUAITON

Height Equivalent to a Theortical Plate
The performance of a chromatograph colum is generally measured in terms of a "height equivalent to a theorctical plate" (HETP). In a gas chromatograph column a narrow bana of sample gas is injectea into a stream of carrier gas which passes through the column to a detecting device. The components of the sample have differing retention times in the column depending upon the properties of the gas component and the liquid stationary phase in the column. It is ovvious that a column which results in a broadening of the puise is detriraental to the separation desired, and the height equivalent to a theoretical plate (HETP) which is defined below is a measure of the degree of longitudinal dispersion.

The FEPP is obtained by postulating that the mechanism of pulse oroadening is caused by equilibration of the stationary naterial in a given plate with the mobile gas phase which then passes on to the next plate.

A linear absorption isotherm $W C_{n}=C_{L n}{ }^{\prime}$ (where $C_{n}=$ mobile phase concentration, $C_{L n}=$ stationary phase concentration) is assumed and a material balance around the nth plate (see Figure l.2) with an increment of gas flow dU yields,

$$
\begin{equation*}
d U\left(C_{n-1}-C_{n}\right)=(V+W v) d C_{n} \tag{1.15}
\end{equation*}
$$

from which is obtained,

$$
\begin{equation*}
\frac{d C_{n}}{a U}=\frac{C_{n-1}-C_{n}}{V+W v} \tag{1.16}
\end{equation*}
$$



## Figure I.?

Model For Derivation Of Plate Theory
where $V=$ volume of mobile phase in plate and $v=$ volume of plate stationary phase.

Assume that all the pulse gas is initially in stage 1 yielding an initial gas concentration $C^{\prime}$. Applying $n=1$ to equation (I.16), $C_{n}=C_{1}$, and $\omega_{n-1}=0$,

$$
\begin{equation*}
\frac{-\alpha C_{1}}{C_{1}}=\frac{\frac{\alpha}{} U}{V_{p}} \tag{1.17}
\end{equation*}
$$

where $V_{p}=$ Volume of plate $=(V+W v)$. On integration, (1.17) gives,

$$
\begin{equation*}
C_{2}=K \exp \left(-\frac{U}{V_{p}}\right) \tag{1.18}
\end{equation*}
$$

When $U=0, C_{2}=C^{\prime}$, and therefore $K=C^{\prime}$ in (1.18). Hence,

$$
\begin{equation*}
C_{1}=C^{\prime} \exp \left(-\frac{U}{V_{p}}\right) \tag{1.19}
\end{equation*}
$$

Now applying the above result to equation (1.16) with $n=2$,

$$
\begin{equation*}
\frac{d C_{2}}{d U}+\frac{C_{2}}{V_{p}}=\frac{U^{\prime}}{V_{p}} \exp \left(-\frac{U}{V_{p}}\right) \tag{1.20}
\end{equation*}
$$

Wquation (1.20) can be solved by usc of the integrating factor, $\exp \left(+\frac{U}{V_{\mathrm{p}}}\right)$

$$
\begin{equation*}
C_{2} \exp \left(\frac{U}{V}\right)=\frac{C^{\prime}}{V_{p}} \exp -\frac{U}{V_{p}}+\frac{U}{V_{p}} d U=C^{\prime} \frac{U}{V_{p}}+K \tag{1.21}
\end{equation*}
$$

When $U=0, C_{2}=0$, and so $K=0$, yielding the result,

$$
\begin{equation*}
C_{2}=C^{\prime} \frac{U}{V_{p}} \exp \left(-\frac{U}{V_{p}}\right) \tag{1.22}
\end{equation*}
$$

Hence by continuing this process to the nth stage

$$
\begin{equation*}
c_{n}=\frac{c^{\prime} U^{n-1}}{(n-1)!V_{p}^{n-1}} \quad \exp \left(-\frac{U}{V_{p}}\right) \tag{1.23}
\end{equation*}
$$

This is a Poisson distribution function, and for a large number
of plates this distribution approaches a Gaussian or normal distribution. The mean of the above distribution is $\frac{U}{V_{p}}$ and the variance is $\frac{U}{V_{p}}$ (that is, $\sigma=\sqrt{\frac{U}{V_{p}}}$. so that the $(\text { Mean })^{2} /(\text { standard deviation })^{2}=\left(\frac{U}{V_{p}}\right)^{2} / \frac{U}{V_{p}}=\frac{U}{V_{p}}$ Now $U$ is the total volume of gas which has flowed, and $V_{p}$ is the volume of a theoretical plate, so that when the mean approaches the end of the column the $(\text { mean })^{2} / \sigma^{2}=$ no. of theoretical plates.

By definition, therefore,

$$
\begin{equation*}
\operatorname{HEPP}=I\left(\frac{\sigma}{\text { mean }}\right)^{2} \tag{1.24}
\end{equation*}
$$

where $L$ is the column length.

Measurement of HBPP
For a large number of stages the output can be assumed to de a Gaussian distribution, and the mean and variance may be road direcily from the record of the output at the end of the column by using the properties of the Gaussian distribution shown in Figurel.j. inis representation is not strictly correct, in that the outpu'c is Gaussian with respect to position in the column, while the recorded profile at the end of a column is with respect to time. However, if the time of purge of the pulse is small relative to the time of the mean, the error in readins this time distribution compared to the distance distribution is regligible.


INJECTION

Figure 1.3
Gaussian Distribution Properties

Inasmuch as $90 \%$ or a nomol ajetribution lies betwern he linits, then the time of purge which is approximately $4 \sigma$, must be<< mean to achicve a Gaussian distribulion. If now both sides of this inequality are squored and multiplied by $L$, on rearranging

$$
\frac{16 \mathrm{LJ}^{2}}{\operatorname{mean}^{2}} \ll \mathrm{~L} \text { or } \mathrm{L} . \geqslant \quad 16 \mathrm{HLCP}
$$

Hencc, a column must contain much more than 16 plates to saiisfy an assumption or a Gaussian distribution in the output record.

## Inpui Polse Distribuiion

The derivation of the Imer assumed that all the pulse is in the first stage at the start, horever, it is obvious that if the pulse extended over several stages an effect would be noticed in tho oulput. It has been shown by Van Demter (17) that the erfect of the initial disuribution can be ignored if,

$$
\begin{equation*}
\frac{A_{s}}{{V_{p}}^{n}}<0.5 \tag{1.25}
\end{equation*}
$$

where $A_{s}$ is the volume of gas in the inilial pulsc and $n$ is the number of theoretical plates.

Rate Theory
The theoretical plate modul does not, at iempt to cxplain the rate processes occurring in a chromatograph column, put relies on the fact that the sum of several distribulions tend to approach a normal Geussian distribution, having a mean made up of the sum of the incependent means, and having a variance made up of the sum of the independent veriances (I9).

One oovious mechamism which occurs to cause a pulse to broaden is molecular diffusion in the mobile phase. Longitudinal diffusion in the
stationary phase can generally oc ignored as the stationary phase is discontinuous in a packeu bed, and, in acdition, the diffusion coefficient is small in this phase.
'There is a group of little understood processes which cause a . pulse to disperse due to the Clow pattern in the packed bed. Fortunately, In a deep bed these aberations arc of a otatitucal nature whish tend to result in a Gaussian distribution as obtained for molecular diffusion, so that they can be grouped together in a term described as the eday diffusivity. In the work of Van Deenter et al (17) the eddy diffusivity (below a particle Reynolds number of l) is considered to be caused by the difference in flow paths between particles. These concepts are discussed in the following sections.

A pulse broadening mechanism analogous to the theoretical plate mechanism described earlier can also occur in the chromatograph column. If a resistance exists preventing equilibrium between the mobile and stationary phase, then the degree of pulse broadening caused by the capacitance of the stationary phase is increased due to the fact that although less material enters the stationary phase the time taken to get out again causes the pulse to broaden more than would be the case for the equilibrium situation.

Lapidus and Amundson (2) have derived an expression based on a diffusion model to describe the concentration profile for the conitions Where a pulse gas passes through a packed bed containing a stationary phase with a linear absorption isotherm between the gas and stationary phase. The pulse is not assumed to be in equilibrium with the stationary phase, due to a resistance defined by a mass transfer coefficient, $\propto$. Longitudinal diffusion including molecular and eddy contributions is
characterized by a dispersion coefficient, $D_{L}$, and is assumed to occur in the mobile phase, but not in the stationary phase. The model is shown in Figure 1.4.

A material balance around the element $\delta \mathrm{x}$ yields

$$
\begin{align*}
& F_{1} \frac{\partial C_{1}}{\partial t}=F_{2} D_{L} \frac{\partial C_{2} C_{1}}{\partial x^{2}}-F_{1} U \frac{\partial C_{1}}{\partial x}+\alpha\left(W C_{2}-C_{1}\right)  \tag{1.26}\\
& F_{2} \frac{\partial C_{2}}{\partial t}=\alpha\left(C_{1}-W C_{2}\right) \tag{1.27}
\end{align*}
$$

Where W is the equilibrium constant between the mobile and stationary phases. If the stationary phase is a porous solid $W$ can be replaced by $\frac{1}{\epsilon_{p}}$.


## Figure 1.4

Nathematical Model for the Colurn

For a small pulse injection time, $t_{0}$, and an inivial pulse concentration $C_{0}$, Lapicius and Amundson (20) obtained the follorins solution to the ajove equations,

$$
\begin{gathered}
\frac{\mathrm{C}_{1}}{C}=\frac{x t_{0}}{2 t^{2} \sqrt{H D_{L} t}} \exp \left(-\frac{(x-u t)^{2}}{4 D_{L} t}-\frac{\alpha t}{F_{2}}\right)+\int_{F\left(t^{1}\right) d t^{1}}^{2 t^{2} \sqrt{\pi D_{L} t^{2}}} \exp \left(-\frac{\left(x-u t^{1}\right)^{2}}{4 D_{L} t^{1}}\right) X
\end{gathered}
$$

where

$$
F(t)^{1}=\left(\frac{2 V t^{1}}{F_{1} F_{2}\left(t-t^{2}\right)}\right)^{1 / 2} \exp \left(\frac{\alpha V}{F_{2}}\left(t-t^{1}\right)-\frac{\alpha t^{1}}{F_{1}}\right) I_{1}\left(\sqrt{\frac{2 V_{1}^{1}\left(1-t^{2}\right)}{F_{1} P_{2}}}\right)
$$

where $t$ is time, $t_{0}$, time of initial pulse with concentration $C_{0}, x$ is distance along the column, and $I_{1}$ is the hyperbolic Bessel function. It has been shown by Van Deemter et al (17) that the above solution can be reduced to a Gaussian distribution under certain conaitions. Trese conditions are that the height of a transfer unit $\frac{T H}{\alpha} \ll L$, the height of the bed, and the longituainal mixing stage $\frac{2 D_{L}}{u} \ll L$. Essentially, these requirements state that the column must contain a large number of theoretical plates, in which case the concentration profile reduces to,

$$
\frac{C_{1}}{C_{0}}=\frac{\beta_{t_{0}}}{\sqrt{2 \pi\left(\sigma_{2}^{2}+\sigma_{2}^{2}\right)}} \exp \left(\frac{L / u-\beta t}{2\left(\sigma_{2}^{2}+\sigma_{2}^{2}\right)}\right)
$$

where

$$
\begin{equation*}
\frac{I}{\bar{\beta}}=I+\frac{F_{2} W}{F_{I}}, \sigma_{I}^{2}=\frac{2 D_{L} L}{u^{3}} \text { and } \sigma_{2}^{2}=\frac{2 \beta^{2} F_{2}^{2} L}{F_{I} W^{2}} \tag{1.30}
\end{equation*}
$$

This is a Gaussian distribution with mean $L / u$ or $B t$ ana variance $\sigma_{1}{ }^{2}+\sigma_{2}{ }^{2}$. As mentionea above, the variance of a Gaussian distribution is composed of the sum of the individual variances, so equating the ratio $\frac{\sigma^{2}}{m e a n^{2}}$ for the above solution yields the following which can be combined With the Himp derivation of equation (1.24).

$$
\begin{align*}
\frac{\sigma_{2}^{2}+\sigma_{2}^{2}}{(I / u)^{2}} & =\frac{2 D_{1} L}{u^{3}}\left(\frac{u^{2}}{L^{2}}\right)+\left(\frac{1}{1+\frac{F_{2}}{F_{I}}}\right)^{2} \frac{2 F_{2}^{2} L}{\alpha F_{1} V^{2} u}\left(\frac{u^{2}}{L^{2}}\right)=\frac{\sigma^{2}}{\operatorname{mean}^{2}} \quad \text { (1.3I) } \\
& =\frac{2 D_{L}}{u}+\left(\frac{1}{1+\frac{W F_{I}}{F_{2}}}\right)^{2} \frac{2 F_{I u}}{\alpha}=\frac{\sigma^{2}}{\operatorname{mean}^{2}} \times L=\operatorname{HrCP} \quad \text { (1.32) } \tag{1.32}
\end{align*}
$$

The diffusivity $D_{L}$ in equation (1.32) refers to any axial mixing
 dispersion coefficient, including the eddy dirfusivity. It was pointed out by Van Deemter that in the laminar region the eddy diffusivity in a packed bed is probably created by the difference in flow patierns in the bed. A perfectly uniform bed thus conceivably has no eddy term.

The moiecular dirfusivity corrected for the path lengthening in a packed ved by a tortuosity factor, and the eddy diffusivity $D_{\mathrm{L}}{ }^{*}$ are comnonly assumed to be additive, so that the dispersion coefficient $D_{i}$ is given by, $D_{L}=\frac{D_{B}}{. \lambda}+D_{L}^{*}$
where $D_{L^{*}}$ depends on the axial dispersion caused by the flow patterns. This assumption is discussed by Klinkenberg and Sjenitzer (19) and they concluded that this approach is justifiable if the theory adequately describes the results. The abundant work on gas chromatography appears to lena support to the assumption of adativity of coefficients. At high flow rates, the molecular dispersion becomes negligible compared to the turbulent dispersion, so that the overall aispersion is the same as the flow dispersion and can be called the ediy diffusivity.

At low flow rates, e.g. particle Reynolds numbers 1 , the eddy diffusivity can be representea according to Van Deemter et al by the expression $D_{L}{ }^{*}=u d_{p}$. Thus equation (1.32), after introducing the
concept of aditive coerficients stated in (1.33), takes the form,

$$
\begin{equation*}
\operatorname{HETP}=2 \gamma_{p}+\frac{2 D_{B}}{\lambda u}+\left[\frac{1}{1+\frac{V_{1}}{F_{2}}}\right]^{2} \frac{2 F_{1 u}}{\alpha} \tag{1.34}
\end{equation*}
$$

The quantity $\gamma$ is reported to decrease with larger diameter pellets, having a value of about 8 for 200 mesh, and practically zero for 30 mesh, partleles.

Kass Transfer Coefficient and Efective Diffusivity
Dirfusional resistance to mass transfer from the mobile phase to the intcrior of the pellets (stationary phase) is made up of two parts, the first being due to resistance in the mobile phase and the second to the resistance within the pellets. The solution of Lapidus and Arundsen (2) used by Van Deemter et al (17) (equation 1.28) treats the resistance in terms of a mass.transfer coefficient.

Van Deenter et al treated the two resistances as separate mass transfer coefficients which could be combined by the resistances-inseries ruic. (A mass transfer coefficient is really a conductance rather than a resistance hence the reciprocals are the adaitive property.)

$$
\begin{equation*}
\frac{I}{\alpha}=\frac{I}{\alpha_{1}}+\frac{W}{\alpha_{2}}=\frac{1}{k_{2} A_{p}}+\frac{W}{k_{2} A_{p}} \tag{1.35}
\end{equation*}
$$

Where $\alpha_{l}$ is the mobile phase coefficient/unit vol. of bed, $\alpha_{2}$ is the stationary phase coefficient and $\alpha$ is the overall coefficient with $k i$ and $k_{a}$ being the corresponding surface mass transfer coefficients. $W$ is the partition coefficient, winich is necessary in gas chromatography because the diffusion in the stationary phase occurs in a liquid and the liquidphase concentration oradients are expressed in terms of equivalent equilibrium gas phase concentrations in order to make equation (1.35)
consistent. In diffusion in porous solids, the effective diffusivity is derined on the basis of the interstitial gas concentrations and so the partition coefficiont becomes a quantity relating interstitial concentrations to stationary phase concentrations, that is $\frac{1}{\epsilon_{p}}$, where $\boldsymbol{\epsilon}_{p}$ is the pellet porosity.

## Bxtemal Mass transfer Coerficient

Basei on the work of Ergun (21) Van Deenter et al suggested the use of the following correlation for the mass transfer coefficient in the nobile phase,

$$
\begin{equation*}
\mathrm{k}_{2}=\frac{25}{6} \frac{D_{\mathrm{b}}}{\mathrm{~F}_{2}} A_{\mathrm{p}} \mathrm{~cm} . / \mathrm{sec} . \tag{1.36}
\end{equation*}
$$

Where $k_{1}$ is the mass transfer coefficient per unit area and $A_{p}$ is the surface area per unit volume of bed,

$$
\begin{equation*}
\alpha_{1}=A_{p} H_{2} \sec ^{-1} \tag{1.37}
\end{equation*}
$$

In a bod of spherical particles of dianeter $d_{p}$, the surface area per unit volune, $A_{p}$, is given by the following, if the bed porosity is $\mathrm{F}_{1}$,

$$
\begin{equation*}
A_{p}=6\left(1-F_{1}\right) / a_{p} \tag{1.38}
\end{equation*}
$$

Internal Mass Pransfer Coefficient
In this work it is dosired to obtain the effective diffusivity in the porous pellet, and so it is necessary to find a relationship between the mass transfer coefficient, $\alpha_{2}=k_{2} A_{p}$, and the effective diffusivity. Such an expression was given by Van Deemter et al without derivation, but it can be obtained in a pellet of radius $R$ as shown below. Let

$$
\begin{equation*}
D_{\mathrm{E}}\left(\frac{\partial_{u}}{\partial r}\right)_{r=\mathrm{R}}=\mathrm{K}_{2}\left(\mathrm{C}_{\mathrm{S}}-\mathrm{C}_{\text {avg }}\right) \tag{1.39}
\end{equation*}
$$

where $C_{s}$ and $C_{\text {avg }}$ are the surface and average concentrations in the pellet respectively.

Crank: (22) (page 233) has obtained solutions of the diffusion equation for a spherical pellet of radius $R$ which give the concentration $\hat{O}_{\mathrm{A}}$ at any radius $r$ and time $t$ when the surface concentration changes stepwise from 0 to $C_{S}$,

$$
C_{A}=C_{s}+\frac{2 R C_{s}}{\pi r} \sum_{n=1}^{\infty} \frac{(-1)^{n}}{n} \sin \left(\frac{n \prod_{x}}{R}\right) \exp \left(-\frac{D n^{2} \pi^{2} R^{2}}{R^{2}}\right)_{(1.40)}
$$

Similarly the average concentration is given by:

$$
\begin{equation*}
c_{a v g}=c_{s}-\frac{6 c_{s}}{\pi^{2}} \sum_{n=1}^{\infty} \frac{1}{n^{2}} \quad \exp \left(-\frac{D_{d i} n^{2} \pi^{2} t}{R^{2}}\right) \tag{1.41}
\end{equation*}
$$

If $t$ is large then only the first terms of the series solutions need be considered. This amounts to suggesting that $C_{S}$ approaches $C_{\text {avg }}$ and in view of the rapidity of gas diffusion as demonstrated in the example given in the introduction, this assumption would appear to be reasonable.

From (1.4.0),

$$
\begin{equation*}
\frac{C_{A}}{C_{S}}-1=\frac{C_{A}-C_{S}}{C_{S}}=\frac{-2 R}{\pi r} \text { is in }\left(\frac{\pi r}{R}\right) \exp \left(-\frac{D_{E} \Pi^{2} R_{t}}{R^{2}}\right) \tag{1.42}
\end{equation*}
$$

From (1.41),

$$
\begin{equation*}
\frac{C_{a v g}}{C_{s}}-1=\frac{C_{a v g}-C_{s}}{C_{s}}=-\frac{6}{\pi^{2}} \exp \left(-\frac{D_{E} \Pi^{a_{t}}}{R^{2}}\right) \tag{1.43}
\end{equation*}
$$

Divide (1.42 by (1.43)

$$
\begin{equation*}
\frac{C_{A}-C_{S}}{C_{a v g}-C_{S}}=\frac{2 R \pi}{6 r} \quad \sin \frac{\pi r}{R}=\frac{C_{S}-C_{A}}{C_{S}-C_{a v g}} \tag{1.44}
\end{equation*}
$$

Let $\frac{C_{S}-C}{\Delta r}=\left(\frac{\partial C}{\partial r}\right)_{r=R}$, thensubstitute in equation (1.39)

$$
D_{\mathrm{E}} \frac{\mathrm{R} \pi}{3 r} \sin \frac{\pi r}{R} \quad \frac{C_{E}-C_{a v g}}{\Delta r}=k_{2}\left(C_{S}-C_{a v g}\right)
$$

Taking the limit as $\Delta r \rightarrow 0$

$$
\begin{equation*}
\operatorname{limit}\left[-\frac{D E \pi^{2}}{3 \mathrm{k}_{2}}\left[\frac{\sin -\frac{\pi r}{R}}{\frac{\pi r}{R}}\right] \frac{1}{\Delta r}=1\right] \tag{1.45}
\end{equation*}
$$

or $\frac{2 \prod{ }^{2} \mathrm{DE}_{\mathrm{E}}}{\mathrm{k}_{2} \mathrm{~d}_{\mathrm{p}}}=1$ whers $\mathrm{d}_{\mathrm{p}}=2 \mathrm{R}$
therefore $k_{2}=\frac{2}{3} \pi^{2} \quad \frac{D_{E}}{d_{p}}$
The mass transfer per unit volume of ded is obtained by multiplying $k_{2}$ by the surface area per unit volume $A_{p}$ given by equation (1.36).

The two mass transfer coefficients could be combined using equation (1.35). However, at this stage, the conditions of the present work dirfers from that of Van Demter et al. If $k_{l}$ is large conpared to $k_{2}$, then when the inverse is sumned in equation (1.35), $k_{2}$ may be icnored. Trne expression for $k$ suggestē by Van Deenter et al applies only to the laminar filow region, but in the turbulent region the vailue will be Greater rather than less, and so if reason can be found to neglect, $k_{j}$ in the laminer region, it need not be considered in evaluating mass transfer in the turbulent flow region.

If we assume the effective diffusivity in the porous pellet is I/5 of the molecular diffusivity as suggested in the introduction, and assume a bed porosity of 0.4 , then the ratio of $k_{1} / k_{2}$ from equations (1.36) and (1.45) is around 28. 'Inus, at most, the resistance outside the pellet makes a $3 \%$ contribution anu can be ignored.

The derivation for $k_{2}$ was made on the assumption of a step change in the surface concentration, but in this work a Gaussian curve is expected to describe the suriace concentration. It would be desirable to have a derivation applicable to other surface functions, or at least to the Gaussian function. An attempt to obtain an alternate expression for $k_{2}$ using a ramp surface function could not be made to reduce to the expression of (1.46), because the exponential functions woula not cancel
out as is the case in the step yielding equation (1.4 ). Thus, a further aegree of approximation results fiom using (1.46), but it is provable chat an experimental constant other than $2 / 3 \pi^{2}$ can be found which would yield a satisfactory diffusivity Irom a pulse experiment.

## Van Deemter's Equation

 suostituted in equation (1.34). Ignoring $k_{1}$, and combining (1.35) (1.38) and (1.46)

$$
\begin{equation*}
\alpha=2 / 3 \pi^{2} \frac{D_{\mathrm{E}}}{d_{\mathrm{p}}} \quad 6 \frac{\left(I-F_{1}\right)}{d_{p}} \tag{1.47}
\end{equation*}
$$

Substituting (1.47) into (1.34),

$$
\begin{equation*}
F W T P=2 \gamma a_{p}+2 \frac{D_{B}}{\lambda u}+\left[\frac{1}{1+\frac{V F_{2}}{F_{2}}}\right]^{2} \frac{2 F_{2} \dot{a}_{p}^{2} u}{4 \pi^{2} D_{E}\left(I-F_{1}\right)} \tag{1.48}
\end{equation*}
$$

Meiking the substitutions,

$$
F_{I}=\epsilon_{\mathrm{B}}
$$

$$
F_{2}=1-F_{1}=\left(I-\epsilon_{3}\right)
$$

$$
W=\frac{I}{\epsilon_{p}}
$$

$$
\begin{equation*}
H E P=2 \gamma a_{p}+\frac{2 D_{B}}{\lambda u}+\left[\frac{1}{1+\frac{\epsilon_{B}}{E_{p}\left(1-\epsilon_{B}\right)}}\right]^{2} \frac{2 \epsilon_{B} \dot{C}^{2} u}{4 \pi^{2} \frac{1}{D}\left(1-\epsilon_{B}\right)} \tag{1.49}
\end{equation*}
$$

Whis is the equation derived by Van Deerater et al (17) and it may be observed to be of the general form

$$
\begin{equation*}
H E P P=A+B / u+C u \tag{1.50}
\end{equation*}
$$

wherc $A, B$ and $C$ would be constants for a given packed bed and a given sys tem.

A sketch of the behaviour of this equation is shown in Figure 1.5 indicating the physical significance of the constants $A, B$ and $C$.

The magnitude of the A berm, which nay be called the eddy cifeusivity term, depends largely on the valua o: $\gamma$. As poineal out by Von Demter et al, $\begin{aligned} & \text { is } \\ & \text { expectod to be cuite small lor iarge packing sizes, }\end{aligned}$ e. . 30 mesh diameter or larger. Therefore, it is likely that the oday dirfusivity term may not seriously mask the other terms.

At low flow rates, the quantity $B / u$, or molecular diffusivity term, may be expected to dominate, while at high flow rates the eflective diffusivity, or $C u$, term villl predominate.

The effective diffusivity, or Cu term, is of prinary interest and so this term will be considered in more detail. It was mentioned earlier that a lower mass, transfer coefficient or effective diffusivity causes the pulse to broaden, and in the effective diffusivity term a jower effective diffusivity does indeed result in a larger HETP, thich is a measure of the amount of pulse dissipation. Similarly, a non porous pellet will have zero porosity $\left(\epsilon_{p}\right)$, and so the pellet capacity term becomes zero. This implies that for a bea of non porous pellets the following equation should apply:

$$
H E T P=A+B / u
$$

The influence of the bod porosity $\epsilon_{B}$ on the magritude of the effective diffusivity term is very small over the range of porosities comonly found in random pellet packings. On the other hand, the pellet diameter vhich has an exponent of 2 has a strong influence on the vaiue of the effective diffusivity term, and suggests that this term will be much inore easily evaluated for larger peliets. Typical Values of the fffective Diffusivity Term (C)

In table 1.I folloring, some values of the effective diffusivity term are calculated for some typical porous pellet properties and aimensions,

- 31 -
and for a range of possible efective diffusivities. The velocities where ohe molecular diffusion term equals the effective diffusivity term in equation (1.49) (i.e. the manmu shown in Figure 1.5) are also calculateu. for an effective diffusivity of $0.01 \mathrm{~cm}^{2} / \mathrm{sec}$, an assuncu wolecular aiffusion coefricient of $0.2 \mathrm{~cm}^{2} / \mathrm{sec}$, and a tortuosity factor of 1.33 . It can easily be shown that,

$$
u_{\min }\left[\frac{2 D_{B}}{\frac{1}{1+\frac{\epsilon_{\mathrm{B}}}{\epsilon_{\mathrm{p}}\left(1-\epsilon_{B}\right)}}} \frac{\epsilon_{3} d_{\mathrm{o}}^{2}}{2 \mathrm{D}_{\mathrm{L}}\left(1-\epsilon_{\mathrm{B}}\right)}\right]^{1 / 2}=\frac{3}{\mathrm{C}}^{1 / 2}
$$

Because the velocity at the minimum is proportional to $1 / a_{p}$ the particle Reynolds number, $d_{p} u p / \mu$, at each of the minima obtained for different particle sizes will be the same, and has a value of around 6 if a value of $1 / 6$ is taken for the kinematic viscosity.

TABLE 1.1
THE EFFECT OF PELLET DIAMETER AND EFFECTIVE DIFFUSIVI'「Y ON THE EFFECTIVE DIFFUSIVITTY TERM (C) IN EQUATION (1.50) ( $\left.\epsilon_{B}=0.4, E_{p}=0.33\right)$

| $\hat{a}_{p}$ cus. Eff.Dinf. | 1 | . 5 | . 25 | . 1 |
| :---: | :---: | :---: | :---: | :---: |
| . $1 \mathrm{~cm}^{2} / \mathrm{sec}$ | . 0375 | . 00938 | . 00235 | . 000375 |
| . 01 | . 375 | . 0938 | . 0235 | . 00375 |
| . 001 | 3.75 | . 938 | . 235 | . 0375 |
| Velocity at minimum rrom $u_{\min }=\left(\frac{B}{C}\right)^{26} / \mathrm{m} / \mathrm{sec}$ | . 895 | 1.79 | 3.16 | 8.95 |

From the first three lines of calculations in lable l.I, it is Obvious that for high effective ajiffusivities and small pellet diameters a pulse dispersion method may brealk dow because the effective diffusion


Figure 1.5
Typical Plot of IECP Vs. Velocity (Equation 1.50)
term becomes too small with respect to the other terms unleas extremely nigh flow rates can be used. For example, assuming the figures given above, the constant $B=.15$ and $A=d_{p}$ if the quantity $2 \gamma=1$ is used. At very high flow rates, the description of eddy diffusivity suggested by Van Deemter ei al (17) would not be expected to hold. However, if the
 suggested by MoHenry and WiIhelm (15), then the eddy diffusivity $\mathrm{D}_{\mathrm{L}}{ }^{*}$ is Siven by $D_{L} * I / 2 u d_{p}$ for superficial Reynolds numbers from about 10 to 4:00. This expression compares favorably with that suggested by Van Deemter et ai $\left(D_{L}{ }^{*}=\gamma u_{p}\right)$. However, the constant given as $I / 2$ actually varie $\bar{c}$ with Reynolds number from $1 / 1.8$ to $1 / 2.2$ in Mchenry and Wilhelmis experimental work, and this drift would tend to cause errors in determining the effective diffusivity term in equation (1.50) if pellets with high diffusivity and small diameter were useu.

As mentioned earlier, the velocities at the bottom of the table l.I show the location of the minimum in $\operatorname{Figure~} 1.5$, and correspond to a constant particle Reynolas number or about 6 which is well above the flow range considered by Van Demter et al. Nevertheless, it is apparent that if the effective diffusivity term is to be raximised relative to the other terms, higher flow rates and Reynolds numbers must be used. Least Square Error fit of data to Van Deemter Equation

Consiacr an equation or the type

$$
\begin{equation*}
H=A+B / u+C u \tag{1.51a}
\end{equation*}
$$

Given an adequate number of experimental points relating $H$ to $u$ for $a$ given packing bed and gas system, the best values of the constants $A, B$ and $C$ can be determined by a "least squares error" fit.

Mree simuiteneous equations invoiving $A$, $B$ adl $C$ can bo obtaincd in the usual way, that is,
I. sum all whe $n$ cata points:

$$
\begin{equation*}
n A+\sum \sum \frac{Z}{u}+c \sum u=\sum n \tag{1.510}
\end{equation*}
$$

2. Nultiply by the coefficient of 3 :

$$
\begin{equation*}
A \sum \frac{1}{u}+B \sum \frac{1}{u}^{2}+n C=\sum \frac{n}{u} \tag{1.51c}
\end{equation*}
$$

3. Multiply by the coefficient of c :

$$
\begin{equation*}
A \sum u+n B+C \sum u^{2}=\sum H u \tag{a}
\end{equation*}
$$

Eliminatines $A$ and $B$ from (1.51b), (1.5lc), and (1.5la)

Hence,

$$
\begin{align*}
& B=\frac{\sum H-\frac{n \sum\left(\frac{H}{u}\right)}{\sum\left(\frac{1}{u}\right)}-\dot{\left(\sum u-\frac{n^{2}}{\sum\left(\frac{1}{u}\right)}\right)}}{\sum\left(\frac{1}{u}\right)-\frac{n \sum\left(\frac{1}{u}\right)^{2}}{\sum\left(\frac{1}{u}\right)}}  \tag{1}\\
& A=\frac{\sum n-c \sum u-B \sum \frac{1}{u}}{n} \tag{1.51g}
\end{align*}
$$

## B. LONGITUDINAL DISPERSION COEFFICIENTS

There are two models generally used to describe the longitudinal dispersion in packed beds of non porous solids. The dispersed plug flow model superimposes co-ordinates moving at the average stream velocity, $u$, on the appropriate solution of the diffusion equation. Thus, the variation of axial concentration profile $C$ with time $t$ and axial distance $x$ of a quantity $M$ per unit area of gas with diffusivity $D_{\bar{L}}$, initially on a plane
at $x=0$ is given by (22)

$$
\begin{equation*}
C_{A}=\frac{M}{2 \sqrt{\pi^{1} D_{L}^{t}}} \exp \left(\frac{-x^{2}}{2\left(2 D_{L} t\right)}\right) \tag{1.52}
\end{equation*}
$$

which is a Gaussian distribution with mean 0 and variance $2 D_{L}$.
With plug flow at a velocity u this becomes

$$
\begin{equation*}
C_{A}=\frac{M}{\sqrt{2 \pi D_{L}}} \quad \exp \left(-\frac{(I-u t)^{2}}{2\left(2 D_{L} t\right)}\right) \tag{1.53}
\end{equation*}
$$

with mean $L \quad \because$ and variance $2 D_{L} t$.
A second model is the "perfect mixers in series" which can be developed by applying the theoretical plate aerivation described earlier, again yielding a Poisson distribution, with a mean of $\frac{U}{V_{L}}$ and variance $\frac{U}{V_{L}}$, where $U$ is the volume of gas which has passed through, and $V_{L}$ the volume of each mixer. The number of perfect mixers in series is equivalent to the number of stages and is given by $U / V_{L}$ as before.

$$
\text { Equating the ratio } \frac{(\text { mean })^{2}}{\text { variance }}=\text { number of mixers, } N \text {, for yoth }
$$

models,

$$
\begin{equation*}
\frac{\left(\frac{U^{2}}{V_{L}}\right)}{\left(\frac{U}{V_{L}}\right)}=N=\frac{L^{2}}{2 D_{I} t}=\frac{u L}{2 D_{I}} \tag{1.54}
\end{equation*}
$$

If one mixer is assumed to correspond to each layer of particles, then the number of mixers $=L / \alpha_{p}$, and therefore,

$$
\begin{equation*}
D_{L}=I / 2 u d_{p} \tag{1.55}
\end{equation*}
$$

The only work, (apart from a few data points in the laminar
flow regime obtained by Carberry and Bretton (23)) which has been carried out with gases for the purpose of investigating dispersion models in packed beds has been done by McHenry and Wilhelrn (15), using a frequency response technique. They found that over a particle Reynolds number
(based on superficial velocity) range of $10-400$ the above reladionship helu reasonably well.

Other factors which influence the axial dispersion coefficient are buoyancy effects which may be expected when flow rates approach laminar conaitions, and wall effects which Hiby (24) has shown greatly increase the apparent dispersion coefficient.

## Velocity Profile Contribution

Taylor (25) has separated the velocity profile contribution to the dispersion coefficient in pipe flow. Taylor found that four dispersion regimes exist in pipe flow. The first is due to molecular diffusion which predominates at low flow rates. As the velocity increases the parabolic profile contributes to the longituainal dispersion of a pulse, wit the molecular diffusivity is able to largely remove the radial concentration proriles. This yields an eddy diffusivity,

$$
\begin{equation*}
D_{L}^{*}=K \frac{u^{2} R^{2}}{D_{B}} \tag{1.56}
\end{equation*}
$$

where $u$ is the mean velocity, $D_{B}$ the molecular diffusivity, and $R$ is the pipe radius. It may be noted that high molecular diffusivity gases reduce the eddy diffusivity in this region. The constant $K$ is $\frac{1}{48}$ for pipes, but Aris (26) has shown that the constant depends upon the geometry of the system. The range of application of the above regime is describad by furner (27) as,

$$
\begin{equation*}
\frac{7 D B}{R} \ll u<\frac{4 D_{B} I^{2}}{R^{2}} \tag{1.57}
\end{equation*}
$$

Where $L^{\text { }}$ is the length of test section containing most of the pulse. Uithin the above limits the molecular diffusivity contribution is negigible so $D_{L}^{*}=D_{\mathrm{I}}$. Since a Gaussian distribution is assumed we can say that $95 \%$ of the pulse exists in $10 u r$ stendard deviations.

Ir $L^{\prime}$ is defined as $4 \sigma$ then,
$I^{\prime}=4 \sqrt{2 D_{\mathrm{L}}{ }^{t}}$
Substituting for $D_{\text {I }}$ Pron (1.56) and setting $t=\frac{L}{u}$, where $L$ is the length of column (or mean), the upper limit becomes,

$$
u \ll 216^{2} K_{1} \frac{L \cdot D_{B}}{N^{2}}
$$

For pipes $K_{I}=\frac{1}{48}$ giving an upper limit of
$u \ll 10 \frac{L D_{3}}{R^{2}}$
Turner (27) in his derivation obtained $5 L D / R^{2}$ for the R.H.s. of (1.58), apparently because of the omission of a factor of 2 in befining $L$. 'The residence time is introduced if the velocity is replaced by $L$ then,

$$
\begin{equation*}
\frac{L}{t}=u \ll 10 \frac{L D_{B}}{R^{2}} \quad \text { or } t \gg 10 \frac{R^{2}}{D_{B}} \tag{1.58}
\end{equation*}
$$

In other words, a pulse must be allowed to fiow for some finite time arter the injection before the eddy diffusivity is defined by equation (1.56). Ir the dispersion is measured very shortiy after injection (a time less than $R^{2} / 10 D_{B}$ ), then the eady diffusivity is given by some undefined function. This latter function does not include the molecular diffusivity, and so resembles the function for the turbulent regime.

Longitudinal dispersion in the turbulent flow regime in pipes has been dealt with by Taylor by use of the universal velocity profile. This approach yielded

$$
\begin{equation*}
D_{L}^{*}=7.14 \mathrm{Ru} \sqrt{\mathrm{P}} \tag{1.59}
\end{equation*}
$$

where $f$ is the Fanning friction factor.
'The application of the Taylor derivation to packed beds has been somewhat limitcd, although Bischofi and Levenspeil (28) have consilered Gine overail profile in a packeu bed. Inasmuch as the velocity proilile in packed beds approaches plus fiow the contribution of the overall profile to axial dispersion is small.

Saffman (29) has developed a model based on a network of capillaries of length $\boldsymbol{l}$ which are joined in a random manner. Assumptions must be made regarding the length and diameter of the capillaries. Saffman cerived the following expression to cover the transition from laninar to an eddy regime and by assumming a capillary lensth to diameter ratio or 5 the experimental results of Hiby (24) for liquid were fitted

$$
D_{L}=\frac{u L}{6}\left[\operatorname{Loge} \frac{2}{2} \frac{u_{1}}{D_{B}}-\frac{17}{12}-\frac{1}{8} \frac{R^{2}}{\ell^{2}} \frac{u_{L}}{D_{B}}\right]+\frac{D_{B}}{\lambda}+\frac{4}{9} D_{B}+0\left[\frac{D_{B}}{2 u}\right]^{2}(1.60)
$$

The value of the tortuosity $\boldsymbol{\lambda}$ obtained by Saffman from this model approscines 3 ta 4, considerably higher than the tortuosities normally encountered in beds of spheres.

Saffman's model appears to show the most potential at present in jescribing the axial mixing in packed beds, but as in Taylor's work on pipes assumptions made concerning the nature of the flow lead to different solutions. Hence, the basic flow mechanism must be understood berore one can apply the appropriate solution from the model.

III

## APPARATU'

## A. DEVELOPMENT

The Initial work to test the calculation of effective diffusivities by means of the Van Deemter equation was carried out on an apparatus based on a gas chromatograph as shown in Figure 1.6. A cyclopropane puise was injected by a chromatograph sample valve into a helium carrier fas and was detected on a GOW MAC model 9238-D thermal conductivity cell. The flow rate was measured with a soap bubble flow meter at the cell outlet, and the detector output was recorded on a Leeds and Northrup -l to 10 mv recorder. The test section vas mounted in a vertical plane, although initially a set of results were taken with a horizontal bed, but the settiing of the packing resulted in a channel along the top of the bed. The first vertical apparatus suffered from the following defects:

1. The detector vould only operate within a limited gas flow range (about $50 \mathrm{mls} / \mathrm{min}$.$) .$
2. The smail ports in the sample injector restricted the flow of gas.
3. No provision existed for adjusting the recorder chart speed, and at the available chart speed the pulse output was not broad enough to make accurate measurements of standard deviations possible.

## B. DESCRIPTIOIN OF APPARATUS

The apparatus with which the bulk of the results were taken is shown in Figure 2.7. The shortcomings of the earlier apparatus were climinated in this set-up by the following modifications:


Figure 1.6
Apparatus For Exploratory Tests


Figure 1.7

1. A cartosian manos at was fitter on the columa exit to maniain a slight positive pressure in whe colum ( $1 / 2$ to ? anchos of mercury). This pressure made it possibic to direct a sade stream through a capillary $\ddagger 0$ supply the detector at a fixed flow rate. At gas flor rates less than the amount neoder for the detector the manostat supplied additional gas, thus reversing the flow direction between the manifold and the manostat.
2. A sample or pulse injection valve was constructed having large poris as shown in Figure 1.8. This Figure also shows an experimental pulse injection system which was used to test the effect of varying pulse size.
3. A Bausch and Lomb 0. 10, 100, 1000 mv recorder with chart speed adjustments from 0.05 to 20 inches $/ \mathrm{min}$. allowed the pulses to be recorded in such a way that good accuracy could be obtained in measuring the dispersion of the pulse.

The apparatus was set up with the test bed mounted in a vertical plane, anc resting on a manirold block at the discharge end. The side strean for the detector was taken from the manifold, and the main column effluent, gas discharged through the manostat. A port connected to a mercury manoneter indicated the manifold absolute pressure.

Air carrier gas was taken at either $300_{0}^{f} \mathrm{RH}$ from the ouiluing supply or from a cylinder of dry air. The air passed through a regulator set for 22 psig. downstream pressure, and then to a flow meter consisting of a sp. gr. I oil manometer and capillary tube. A series of capillary tubes were calibrated using soap bubble flow meters or a wet test gas meter, so that a wide range of flows could


Figure 1.8
Pulse Injectiors

- be covered. Ho attempt was made to size the capillaries so that they remained in their linear range. From the flow meter, the carrier gas passea through a Moore Constant Differential gas flow control and by-pass loop to the pulse injector. The pulse injector was mounted on a vertical rail so that it sould be adjusted over a six foot range to allow for varying column lengths. Polyethylene tuibing was used to supply the carrier gas, as well as the pulse gas to the injector. The flexibility of the polyethylcne tubing allowed the injector to he adjusted anywhere on the rail without the need of piping alverations.

A microswitch mounted on the injector was either opened or closed at each movement of the injectors. This action operated an event marker on the recorder to indicate the start of each run.

## こ. DETECTORS

## Fyarogen Flame Ionisation Detector

In equation (1.49), it was evident that lover eiffective diffusivicies
increased the magnitude of the $C$ term. To take advantage of this, the hydrogen flame ionisation detector was selected, as it allowed the use of air and hydrocarbons of any convenient molecular weight, as opposed to the need for hydrogen or helium (which have high diffusivities) as one of the fases in thermal conductivity detectors if high precision is desired. In adaition, the hydrogen flame detector is linear over a 5 decade range, and the high sensitivity allows the use of extromely small pulse volumes.

The detector was constructed from the circuit described by Harley, Nels, and Pretorius (30) and is shown in Figure 1.9. A power supply was also constructed to supply the detector, however, the AC filament supply vas found to create excessive noise in the output so the detector tube was powered from a 6 volt accumulator.


Figure 1.9
Hydrogen Flanc Detector

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No output could be obtazned initially from the carcuit as Hescribed, anc on investigation the voltages on the 6 SN 7 tube were found to be outside the range in which a response coula be expected. To correct this problem it was necessary to change the two load resistors from $10 \mathrm{~K} \Omega$ to $100 \mathrm{~K} \Omega$. It is concluded that a misprint has occurred in the original publication. The actual ionisation or combuetion chamber wes constructed To minimize the holdup time of the primary air containing the traces of pulse gas from the manifold. The air entered through the annular space in the glass tubes and joined with the hydrogen before passing through the stainless steel orifice which formed one electrode. Hydrozen was suppliec from a cylinder via a Moore flow controller and a rotameter at a rate of about $150 \mathrm{mls} / \mathrm{min}$. Lower flow rates increased the detector output but in the extreme, the flame became unstable. Air and pulse gas arrived through the capillary at the rate of about $0.7 \mathrm{mls} / \mathrm{sec}$. The volume of the detector air side and supply tubes from the manifold was estimated at 0.2 mls , giving a time lag of about 0.3 seconds.

Initially, the flame oririce was made flush with the metal electrode, but the heat from the flame caused the glassware to crack and so the orifice was nodified by addine about $11 / 2^{\prime \prime}$ of $1 / 8$ inch stainless steel tube. The whole assembly was held on a rubber bung, thus suppiying the insulation for the platinum electrode which was supported by a heavy Fire inserted in the rubber. Shielded cable connected the aetector to the electrical system, and a grounded copper chimney shieldec the flame Irom draughts.

Other modifications to the reference circuit (also shown in Figure 1.9) included a fourth position on the selector switch with a

10 meg resistance to ground, and a coarse and fine zero setting using 20 K and 50 K variable resistors in parallel. The 10 megohm position was usea on all runs.

Fibre glass filters were fitted in the hydrogen tune and the manifold to reciuce the noise in the detector caused by dust. The detcetor still gave occasional characteristic jumps in output, probably caused by aust: in the secondary air, but no attempt was made to correct thie. Inemal Conductivity Detector

A "Gov mac" nociel 9238D tungsten wire themal conductivity detector was used with the recommended conventional auxiliary circuits. A 6v. battery supplied the current for the detector and the event marker on the recorder. The output to the recorder vas fitted to an attenuator having $1,2,5,10,50,100$ and 500 ratios, but only the 1,2 and 5 positions were needed in the pulse apparatus.

The reference side of the detector was supplicd through a neeale valve from the 22 psig air linc, and a small bleed maintained. The capillary supplying the measuring side of the detector from the manifold was sized to give approximately $46 \mathrm{mls} / \mathrm{min}$. of air at $1 / 2^{\prime \prime} \mathrm{Hg}$ gauge manifold pressure.

## EXPERIMENTAL PROCEDURE

## A. OUTLINE OF EXPERIMENTAL INVESTPIGATION

The experimental work was carried out in three parts to (a) test the applicability of Van Deemter's equation with large pellet diameters and higher flow rates; (b) measure the effective diffusivity in some samples of porous pellets using the pulse method, and (c) compare the effective diffusivity obtained with the pulse experiment to those obtained by an independent method. Part (a) was carried out by injecting methane and hydrogen pulses in beds containing non-porous pellets, while (b) was an obvious extension of (a) to porous pellets. The well-tested steady state method was selected to obtain an independent effective diffusivity value. It was convenient, however, to develop a specific solution of the diffusion equation to fit pellets with curved faces. The details of this section of the work are recorded in Appendix 1.
B. NON POROUS PELLETS IN PULSE APPARATUS

A simple gas chromatograph assembly was used for some early exploratory runs with a cyclopropane pulse in an helium or air carrier gas flowing through beds of 2 mm . glass spheres. These results were discarded due to limitations of the apparatus, which included a limited supply of the 2 mm . glass beads necessitating short beds, as well as the defects already listed.

With the development of the more sophisticated apparatus, a series of runs using methane pulses in air was carried out with various bed diameters and lengths packed with three kinds of non porous pellets: 0.208 cm . No. 9 lead shot, 0.568 cm . glass beads, and 1 cm . diameter ceramic beads.

Because the value of the quantity "C" in equation (1.50), HETP $=A+\frac{B}{u}+C u$ was not found to be zero in the exploratory work with non porous pellets, runs 50 to 55 were designed to determine the magnitude of this term, and to investigate ways of minimizing it. lo check the possibility that this effect was caused by a high velocity "by-pass" flow at the wall, run 50 vas made with a 5 cm . diameter column packed with the 0.208 cm . iead shot, and having a maximum particle Reynolds number of 2. Run 50 was cifferent from the other runs in that a higher pressure was used, giving a lower diffusivity. Run 51 was made with a 2.5 cm . column packed with the lead shot to see if particle to tube diameter ratio had much influence on the wall effect. Run 52 duplicated run 50, but used a higher Reynolds number range, and normal column pressure. A run designated 51D was also made on the 2.5 cm . bed, but five doughnut rings were distributed evenly down the column in an attempt to eliminate the wall effect.
fun 53 was made with a 6.27 cm . diameter column packed with the 1 cm . ceranic spheres. The experimental sample injection system using a solenoid valve, which allowed varying pulse sizes, was introduced in this column. Fun 54 was made on a $1 / 4$ " polyethylene tube packed̃ with 3 mra . flass spheres, and run 55 was made with a 1.2 cm . diameter bed packed with the 1 cm . balls to see if a tube/pellet diameter ratio <2 could eliminate the wall effect. This latter case has been designated as a "single pellet" bed. In all the foregoing runs, the test system used was a methane pulse in air as a carrier gas.

Following these tests, runs 56 to 62 with porous pellets were carried out. One of the porous pellets, an activated alumina, gave
abnormally low values for the effective diffusivity. Further investigation, which is summarized in Appendix IV, showed that methane was adsorbed to a significant degree on the activated alumina. The need for a non-adsorbing system resulted in further runs using the non porous pellets being carried out wath a hydrogen pulse, as vell as the methane pulse, in air system. Runs 63 to 66 were carried out with 0.568 cm . glass spheres, both eas systems and two column diameters, including one for a single pellet diameter. Runs 69 to 72 were a similar set of results with the 1 cm . diameter spheres, two column dianeters and two gas systems. Runs 69 and 72 using methane are duplicates of runs 53 and 55 , but covered a wider range of Reynolds number. Table l.II summarizes the values of the variables pertaining to each run number.

TABLE 1.II
SUNMARY OF 'L'HE PELLET AND IUBE 'TO PELLE'I DIAMETER RATIO' COVEREU BY THE EXPERIMENTAL RUNS

| Pulse Gas |  | 1 | 3 | 6 | 12 | 25 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Methane | .208 cm . |  | 54* |  | $\begin{aligned} & 51 \\ & 510 \\ & \hline \end{aligned}$ | 50 52 |
|  | .568 cm . | 64 | 65 |  |  |  |
|  | 1.0 cm . | 72 55 |  | 69 53 |  |  |
| Hydrogen | .567 cm . | 63 | 66 |  |  |  |
|  | 1.0 cm . | 71 |  | 70 |  |  |

*pellet diam. 0.29 cm .
©. POROUS PELLERS IN PULOE APPAPATUO
Three samples of porous spherical pellets were acquired for testing. These included $1 / 8^{\prime \prime}$ and 1/4" diameter HI5l Alcoa activated alumina pellets, and $1 / 2^{\prime \prime}$ diameter Norton Alundum catalyst supports. The physical characteristics of these pellets are sumarized in Appendix III. One of the difficulties experienced in setting experimental conditions was that the activated alumina test pellets could no le adequately dried in the steady state apparatus, as the epoxy resin holding the sample could not stand the necessary drying temperature. In view of this problem, the pulse investigation was attempted on the "wet" pellets, because it was found that the moisture content of the pellets which had been open to the atmosphere was quite stable even though the atmospheric humidity varied from $30 \% \mathrm{RH}$ to $100 \% \mathrm{RH}$. The only problem remaining concerned the true porosity of the wet pellets, but the manufacturcr's literature (31) indicated that the water existed as liquid water, and hence could be assumed to have a density of l. Thus, the porosity could be computed from the dry pellet porosity and the moisture content. The details of these calculations and other confirming experiments with respect to the porosities of the pellets are included in Appendix III.

The pulse technique was first applied using a methane pulse, in run 56 , to the $1 / 4^{\prime \prime}$ diameter H 151 activated alumna pellets in a four foot long single pellet diameter bed. The pellets were in equilibrium with air at room temperature. Unexpectedly high dispersion of the pulse (HETP) caused some doubt about the number of transfer units, so the bed was lengthened for run 57 by adding two bends and two further four foot lengths to create a trombone configuration. A $20 \%$ change in $C$ was found
between the long bed and the short. As shown lator in the "Resulis", the short column was found to contain insufficient transfer urite for a Gaussian uistribution. In run 58, the same bed as that used in run 57 was employed but the pellets were previously cried. At this staje, the possibility of surface adsorption of methane by the alumina was appreciated, and run 59 was conducted at higher flow rates in the hope that the adsorption was a slow process and would not occur to a signiricant extent under thesc conditions.

In run 60, a methane pulse was used in a single pellet diameter trombone bed, which was packed with 1/2" diameter Norton catalyst carrier pellets. In run 61, the use of a hydrogen pulse was tested on the same dry 1/4" diameter $H 151$ activated alumina pellets from runs 58 and 59, while in run 62 the same bed was wetted back to the normal moisture content, and the hydrogen puise applied again.

Run 73 was carried out on a four foot long by $3 / 4^{\prime \prime}$ diameter bed packed with $1 / 8^{\prime \prime}$ Hl $5 l$ activated alumina pellets and using a hydrogen pulse.

## D. INDEPENDENT EFFECTIVE DIFFUSIVITY VEASUREMEN'I

A conventional steady state method was selected for a second determination of effective diffusivity, but the technique was adapted for use with spherical pellets. This modification consisted of mounting the pellets with epoxy resin in a hole in a plate about 0.75 pellet dianeter in thickness. The two spherical caps on each side of the plate were ground off when the resin had dried. The solution for the differential diffusion equation with this geometry is included in Appendix $I$, along with the results and details of this experiment. Only the $1 / 4^{\prime \prime}$ and $1 / 2^{\prime \prime}$ pellets
$\because$ ere testeu. On the basis of the manufacturer's data Knucison diffusion vas expected in the $1 / 4^{\prime \prime}$ alumina pellets, while molecular diffusion was expected in the $1 / \mathbf{2}^{\prime \prime}$ Norton pelleis.

The major problem with this part of the investigation was the moisture content of the pellets. The activated alumina could only be dried in situ, but the epoxy resin would not survive the drying temperature. Since the moisture content of the "wet" pellets remained relatively constant, as mentioned previously, it was decided to test the pellets wet and correct the porosity accordingly.

## E. PREPARATCION OF THE TEST COLUMNS

The packed beds (columns) were constructed from glass tuoing with rubber bungs or tubing in the ends. The dimensions of the beds were generally obtained with a metric rule except for small diameter tubes, where a caliper rule was used. The bed porosities were obtained either by weighing the beds full and empty if the pellet density was known, or by addition of water and weighing. For the single pellet diameter beds, the porosity was calculated by counting the number of pellets in a siven length of bed, and calculating the pellet volume from the mean pellet diameter.

The mean pellet diameter was measured by placing a known number of pellets in line and measuring the overall length.

For the porous pellet beds, the porosity was calculated as for the single pellet beds above, or from the weight of pellets in the bed with the characteristic data of the pellet. All the columns were then mounted in a vertical plane.
. Joints in trombone columns were made with rubber tubing.

## F. OPERATION OF PULITE APPARANU

1. One of the four calibrated flow meter capillaries vas selected and ritteci.
2. The column was assembled, (after taking the necessary data for the porosity calculations), and fitted to the apparatus.
3. The air supply was turned on with the flow capillary bypass open, and the column pressure was set at a convenient level (usually around $0.5^{\prime \prime} \mathrm{Hg}$ ), using the cartesian manostat.
4. The column was tested for leaks with soap solution.
5. The appropriate detector was started up as described below.
6. The appropriate pulse gas was set to flow at a low bleed rate using the cylinder regulator and valves. The gas was bubbled in water at the exit to estimate the flow.
7. A suitable air flow rate was passed through the column using the flow meter and control. The flow meter manometer reading was recorded.
8. The recorder chart was started at any speed (unless previous experiments suggested a specific chart speed), and a pulse injected. When the pulse was produced, the height of the pulse was adjusted on the attenuators (recorder attenuator for $\mathrm{H}_{2}$ flow or attenuator box for thermal conductivity detector) and the width noted. Using the initial pulse, the equipment was adjusted to give a convenient peak height (e.g. 0.75 scale), and a pulse width on the chart of at least 1.5 cm .
9. A series of pulses were injected, each at a different gas flow rate, to give about ten results covering the flow range desired.
10. During the course of each run the room temperature, atmospheric pressure and column pressure were recorded.

Hydrogen Flame Detector

1. The detector was connected to the manifold with the correct capillary.
2. Hydrogen flow was started at around $150 \mathrm{mls} / \mathrm{min}$ (using rotameter) and the flame was ignited.
3. The power supply was turned on and connected to a 6 v battery for filament and event marker.
4. The recorder was turned on and the zero of the recorder and detector adjusted. The selector switch on the detector anplifier was always set at the No. 4 position for all runs.

## Thermal Conductivity Detector

1. The manifold was connected with the correct capillary.
2. The reference air bleed was turned on and adjusted to give a slow positive flow (e.g. by bubbling in water).
3. The filament current was adjusted to 100 ma . after connecting to 6 v supply along with event marker leads.
4. The recorder was set to zero and the detector to zero signal.

## A. NON POROUS PELEERC.

## Treatment of Data for Non Porous Pellets

For each pulse input the primary data consisted of: the llow rate of carrier gas, Q mls $/ \mathrm{sec}$, at $\mathrm{s}^{\prime \prime} \mathrm{P}$, which is actually recoried as a manometer reading and transformed using the calibration charts in Appendix II to a flov rate, the width of the pulse at half the height (WIDiA) taken from the recorder chart and also the "mean" or distance from the pulse injection to the peak of the pulse (cesignated TOTAL). 'These daba points are printed (in cm. units) in columns 8,6 and 7 respectivaly of the tables of results in Appendix II.

In adaition to the above raw data, each table in Appendix II is headed with details of the columns pertinent to the individual run. Mese include a "Run number" which starts at 50 for the sophisticated apparatus, but a run (No. 1) from the preliminary results obtained on the initial simple apparatus is included. The column length (L), diameter ( $\tilde{a}_{t}$ ) and porosity ( $\mathrm{E}_{\mathrm{B}}$ ) are included in the heading along with pellet porosity ( $E_{p}$ ) and diancter ( $d_{p}$ ), and the carrier gas temperature ( $7^{\circ} \mathrm{K}$ ), molecular weight and pressure ( P ). The pulse gas-carrier diffusivity is also printed for the run temperature and pressure.

The values for the molecular diffusivity of the pulsc-carrier gas systems are taken from the following sources:

The diffusivity of hydrogen in air was taken from the experimental results of Currie (6). Currie found a temperature dependence of
diffusivity to the 2.715 power for this system, and this vas used to anterpolate from the experimental results a aiffusivity of $0.755 \mathrm{~cm}^{2} / \mathrm{sec}$ at $298^{\circ} \mathrm{K}$ and 1 a.tmosphere.

The diffusivity of methone in air was calculated from the Firschrelder equation using the force constants tabulated in Birc, itewart and Lightfoot (32). The computation, which is shown in Appendix III, yielded a diffusivity for methane in air of $0.212 \mathrm{~cm}^{2} / \mathrm{sec}$ at $2980^{\circ} \mathrm{K}$ and 1 atmosphere.

Values corresponding to the table headings were fed uirectly to the computer except for the pulse gas-carrier gas diffusivities which were modified to the run temperature and pressure assuming an inverse pressure dependence and a tenperature dependence to the 1.7 power.

Provision was included to read in the carrier gas viscosity, out in the computations shown in Appendix II the viscosity value read in has been over-ruled in the program by a viscosity for air computed from the Sutherland equation (33),

$$
\begin{equation*}
\mu=0.01709\left[\frac{273+3.14}{T+114}\right]\left(\frac{T}{273}\right)^{2 / 3} \tag{1.61}
\end{equation*}
$$

The carrier gas density was calculated assuming the perfect
gas law

$$
\begin{equation*}
P=\frac{M 01 \cdot W t}{22400} \frac{273}{T} \times P \tag{1.62}
\end{equation*}
$$

Finally, the hydraulic diameter was calculated from the following equation,
$\left.h_{D}=4 \frac{\text { Free Volume }}{\text { Wetted Area }}=\frac{d_{T} \epsilon_{B}}{\left(\frac{3}{2} \frac{d r}{d p}\left(1-\epsilon_{B}\right)+1\right.}\right)$
As mentioned earlier the primary data of flow rate at STP, WIDTH ( $=2.360$ ) and TOTAL (mean) are given in columns 8, 6 and 7,
 Huajing of each table, the follorne cajculatin wate are pra. .

In column 1 the interanishal viocdty was calcuiaw. ...o.: tubs
$\therefore$ iameter $d T$, and the flow rate $i$, corrcche for iemprature a air pressure 2.

$$
\begin{equation*}
u=Q \frac{T}{273} \frac{1}{P}\left[\frac{1}{\bar{\pi} \cdot L^{2}}\right] \quad \frac{1}{E_{B}} \tag{1.64}
\end{equation*}
$$

The HEPCP vas calculated as defined by equation (1.26)

$$
\begin{equation*}
\operatorname{HETP}=\frac{L \sigma^{2}}{\operatorname{mean}^{2}}=I \quad\left[\frac{V I D H}{C .36}\right]^{2}\left[\frac{I}{T O T A L}\right]^{2} \tag{1.65}
\end{equation*}
$$

Three Reynolds numbers were calculated for comparing the axial dispersion data with data of other workers and are defined as follors: the particle Reynolds number shown in colum 4 of the table of results in Appendix II is given by $\frac{u d_{p} P}{\mu}$, the superificial Reynolds number shown in column $11, \frac{u \epsilon_{B} a_{p} \rho}{\mu}$, and the hydraulic Reynolds number based on the nyaraulic diameter, u hDP/ $\boldsymbol{N}$, in column 13.

The aispersion coefficient $D_{L}$, was obtained from equation (1.34), which for non porous pellets reduces to,

$$
\begin{equation*}
H E T P=\frac{2 D_{\mathrm{L}}}{u} \tag{1.66}
\end{equation*}
$$

$$
\text { so } D_{L}=\operatorname{HETP} \frac{u}{2}
$$

and this value is printed in column 9 under the heading of "eddy iiffusivity". In fact, it is the sum of the molecular and eddy aiffusivities as given by equation (1.33).

The number of transfer units (NTU), defined by $\frac{u L}{2 D_{L}}$, must be large for equation (1.30) to be satisfied, however, the values calculated and
recorded in column 5 are based on the molecular diffusivity rather than the dispersion coefficient $D_{\mathrm{L}}$. Inspection of the term shows that the NTU is smallest at low velocities, and since low velocities imply the existence of the molecular diffusivity regime, the NTU's based on these diffusivities are an adequate test. The use of "long" beds has generally eliminated the NIU as a limiting criterion in this work.

So make possible comparisons between the eddy diffusivity computed from this work and the correlations and results of other workers, the Peclet and Schmidt numbers were also calculated.

The molecular and so-called "eddy" Peclet numbers are recorded in columns 3 and 10, respectively, and were computed from the following definitions,

$$
\begin{array}{ll}
\text { Molecular Peclet number } & \frac{u d_{p}}{D_{B}} \\
\text { Eddy Peclet number } & \frac{u d_{p}}{D_{L}}
\end{array}
$$

This eddy Peclet number should probably be called the dispersion Peclet number, however, because the eddy diffusivity $\mathrm{D}_{\mathrm{L}}{ }^{*}$ has not been separated from the dispersion coefficient $D_{L}$ in this work the eddy Peclet or dispersion Peclet are interchangeable.

The schmidt number based on the dispersion coefficient is recorded as the inverse Schmidt number in column l2, that is,

$$
\frac{1}{\text { schmidt }}=D_{L} \rho / \mu
$$

At the base of each table the least square error fit of the HETP vs. u data to equation (1.50) is computed and the best values of the constants A, B and C are printed out. The span of certain runs was restricted to the eddy diffusion regime, and the scatter of the data points could cause anomalous values of the $B$, or molecular diffusion, term, which
vas a relatively small quantity in this ranse. To offset thic proolem a second least squares computation is carrica out on the aata to fit the equation,

$$
\begin{equation*}
H E M P^{\prime}=A A+C C u \tag{i.67}
\end{equation*}
$$

wherc HEMP' $=\operatorname{HETP}-\frac{B}{u}$.
Ine value of $B$ is set at $2 \times .75 \times \nu_{B}$, where 0.75 represents the inverse oi the tortuosity $I / \lambda$ in equation (1.49).

From the value of $B$ derived from the three constant equation (1.50), the inverse of the tortuosity has been calculated for each mun. Since $\boldsymbol{\lambda}$ varies from 1 to $\infty$ as discussed in the introduction, then the inverse ranges from 1 to 0 . The usual value expected in a packed bed is about 0.67 to 0.8 . The result printed on the computer sheet (Appendix III) is in the nomenclature oricinated by Van Deemter (17)and so the inverse tortuosity computed from equation 1.49 as $\frac{1}{\lambda}=\frac{2 D_{13}}{B}$ is given under the neading GAMMA.

Similarly, the value of the constant characteristic of the ediy diffusivity which has been designated $\gamma$ is computed from the value of the eddy diffusion term $A$ using equation (1.49), that is $\gamma=A / 2 d_{p}$. Van Deemter et al (17) suggest that $\gamma$ varies from about 8 for 200 mesh particles to about zero, for, say, $1 / 8$ inch particles. The computer has printed the values of $\gamma$ under the heading LAMDA (from Van Deenter et al.) in Appendix III.

## Results for Beds of Non Porous Pellets

some typical curves of the HEPP vs. velocity are shown in Figure 1.10 and 1.11. Figure 2.10 shows the results for run 52 which covered both the molecular and eday diffusion regimes while Figure 1.11 shows the results for runs 51,69 and 70 . The five straight lines shown


Figure 1.10
mestr vs. Velouiry for liun 52


Figure 1.11
HEPP Vs. Velocity For Runs 51, 69 anc 70.
on the plots represent the equation $H E T P=A+C u$, using values of $A$ and C determined from applying equation (1.50) to the data. Runs 69 and 70 were made in a bed with a tube to particle dianeter ratio of 6 containing the 1 cm . spheres, but a methanc pulse was used in run 69 and a hyarogen pulse in run 70. It may be noted that both sets of data have the same intercept, indicating that Van Deemter's definition of the eddy diffusivity given by $D_{L^{*}}=\gamma u d_{p}$ is valid, but a further mechanism which depends on the gas diffusivity and has a velocity exponent of 2 must be added to account for the presence of a "c" term.

Van Deemter et al (17) suggested that the eddy diffusivity term in equation (1.49), $A=2 \gamma d_{p}$, decreased with increase in pellet diameter, due to the decrease of the coefficient $\gamma$. In Figure 1.12 it may be noted that with the larger pellets and generally higher flow rates in this work the trend has been reversed, and " $A$ " increases with pellet diameter. If a straight line is put through the points in Figure 1.12 a slope around unity is obtained, making $\gamma=1 / 2$ corresponding to the value obtained by McHenry and Wilhelm (15) with gases at Reynolds numbers greater than 10. Of the early runs, only run 1 , which was carried out on a 134 cm . bed with a cyclopropane pulse in an air carrier gas stream is included in the data. The results of this run together with additional results from initial tests with methane pulses in beds of non porous pellets (runs 50 to 55) are summarized in Table 1.III. The most significant feature of these results is that over a range of pellet diameters from 0.2 cm . to 1 cm. , with tube to pellet diameter ratios from 1 to 25 , the wall effect or "C," term, which might mask the dispersion effect due to pellet porosity, gave results which varied in value only from 0.04 to 0.08 .

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In Runs 50 and 52 the $\mathbb{C}$ term calculated from tests in the lower Reynolds number range (run 50) is considerably higher than the value ootained in the same column at higher Reynolds numbers (run 52). This suggests that either the wall effect term is not constant or that the exponent of the velocity in the dispersion coefficient is less than 2. The plots of the dispersion coefficient vs. $u$ given in Figure 1.13 would appear to substantiate the latter view.

Comparisons of Runs 51 and 51D demonstrate that artificial mixing devices or wall barriers do not reduce the wall dispersion effect.

No data in the regime in which molecular diffusivity is important were taken in run 51D, so that the comparison is best made using the $C C$ value from run 51D, which was calculated using equation (1.65) as described previously. The value of $B$ found from the results of run $51 D$ represents a molecular diffusivity more than double the normal gas diffusivity (GAMMA $=2.07$ ), demonstrating the failure of equation (1.50) when results in the eddy regime only are used in the least squares evaluation of the three constants A, B and C. Values obtained in run 54 also demonstrate this point.

The large diameter pellets in runs 53 and 55 show a large intercept, or A term, compared to the other runs which show essentially zero intercept. Inasmuch as $A$ is approximately proportional to $d_{p}$, this difference is to be expected. It is rather interesting that a bed with a single pellet diameter (run 55) has essentially the same or less slope (i.e. C value) at high Reynolds numbers as the bed six particles in diameter of run 53. This, as well as other results given in Table l.III, indicate that the dispersion due to the wall effect is not a function of tube diameters.

IABLE 1.III
DISPERSION RESULTS WITH BEDS OF NON POROUS PELLETS

| Run | Pellet <br> Diameter | Column <br> Length | Column <br> Diameter | Column to Pellet Diameter Ratio | A | B | C | AA | CC | Range of Reynolds Number |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.22 | 134.6 | 2.61 | 11.9 | 0.13 | 0.18 | 0.07 |  |  |  |
| 50 | 0.208 | 111.8 | 5.0 | 24 | -0.07 | 0.35 | 0.07 | $\div 0,28$ | -0.150 | 0.5-2.4 |
| 51 | 0.208 | 118.1 | 2.6 | 12.5 | 0.050 | 0.31 | 0.052 | 0.04 | 0.053 | 0.29-31.3 |
| 51 D | 0.208 | 118.1 | 2.6 | 12.5 | -0.27 | 0.87 | 0.071 | -0.037 | 0.064 | 2.6-32.6 |
| 52 | 0.208 | 111.8 | 5.0 | 24 | 0.001 | 0.37 | 0.041 | 0.032 | 0.037 | 0.8-33.0 |
| 53 | 1.03 | 186.3 | 6.27 | 6.1 | 0.68 | 0.36 | 0.071 | 0.72 | 0.064 | 5.0-24.0 |
| 54 | 0.297 | 185.4 | 0.415 | 1.4 | -0.22 | 3.88 | 0.079 | 0.177 | 0.069 | 16.0-79.0 |
| 55 | 1.005 | 121.0 | 1.15 | $-1.1$ | 0.601 | 0.16 | 0.060 | 0.260 | 0.122 | 3.0-48.0 |

TABLE 1.III (Continued)


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The experimental pulse injector shown in Figure 1.8 was used in run 53 on the six particle diameter bed containing 1 cm . spheres. Methane pulses are used and the effect of pulse size (as mcasured by peak height) at a particle Reynolds number of 62.4 is shown in Table $1 . I V$. Over a 13 fold range different pulse sizes resulted in essentially the same HEPP values. It must be pointed out, however, that these values are not incluacd in the Cata for Run 53. At the time when the data were taken a maximum particle Reynolds number of 35 was employed in the hope that Van Deemter's assumption regarding the eddy diffusivity could be extended to a Reynolds number of 35 without serious crror. This limitation was later discarded, and the four points in Table 4 were included with those of fun 53. However, they were found to change seriously the constants of the least square equation (2.50), indicating an inconsistency. Run 60 repeated the conditions of Run 53, but employed the normal pulse injection, and these data were consistent with the results at low flow rates in Run 53, but not with the four points in Table l.IV. It is concluded that the inconsistency was created by the experimental injector at high flow rates because of the failure of the pop valve in the injector to close cleanly. The requirements suggested by Van Deemter to ensure that the feed pulse size does not influence the exit distribution (equation 1.25) were easily satisfied in this work, particularly with a large diameter column such as that used in Run 53.

## TABLE 1.IV

EFFEGC OF PULSE SILE (PEAK HEIGHT) ON HESP at Particle Reynolds number of 62.4 Run 53

PEAK HEICHT

FURTHER DISPERJION RESULS WITH BEDS OF NON POROUS PELLET:

| Run | Pellet Diameter | Column <br> Length | Column Diameter | Column to Pellet Diameter Ratio | Pulse | A | B | Inverse Tortuosity | C | AA | CC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 63 | 0.568 | 421.0 | 0.66 | 1.16 | $\mathrm{H}_{2}$ | 0.21 | 1.88 | 1.25 | 0.019 | 0.51 | -0.021 |
| 64 | 0.568 | 421.0 | 0.66 | 1.16 | $\mathrm{CH}_{4}$ | -0.06 | 0.79 | 1.88 | 0.081 | 0.24 | 0.048 |
| 65 | 0.568 | 119.5 | 2.175 | 3.83 | $\mathrm{CH}_{4}$ | c. 12 | 0.37 | 0.901 | 0.057 | 0.18 | 0.019 |
| 66 | 0.568 | 119.5 | 2.175 | 3.83 | $\mathrm{H}_{2}$ | 0.12 | 0.87 | 0.59 | 0.027 | -0.13 | 0.039 |
| 69 | 1.03 | 186.3 | 6.27 | 6.1 | $\mathrm{CH}_{4}$ | 0.7 C | 0.32 | 0.76 | 0.063 | 0.703 | 0.063 |
| 70 | 1.03 | 186.3 | 6.27 | 6.1 | $\mathrm{H}_{2}$ | 0.68 | 0.86 | 0.57 | 0.023 | 0.54 | 0.030 |
| 71 | 1.005 | 122.0 | 1.15 | 1.1 | $\mathrm{H}_{2}$ | 0.31 | 1.14 | 0.79 | 0.028 | 0.34 | 0.627 |
| 72 | 1.005 | 122.0 | 1.15 | 1.1 | $\mathrm{CH}_{4}$ | 0.64 | 0.17 | 0.41 | 0.06 | 0.59 | 0.062 |

Table I.V (Continued)

| Run | Range of <br> Reynolds <br> Numbers | Number <br> of <br> Points | Remarks |
| :--- | :---: | :---: | :---: |
| 63 | 6 | -35 | 10 |
| 64 | 6 | -33 | 9 |
| 65 | 4 | -28 | 14 |
| 66 | $0.6-125$ | 20 |  |
| 69 | 5 | -180 | 16 |
| 70 | 7 | -130 | 13 |
| 71 | 4 | -183 | 14 |
| 72 | 10 | -181 | 12 |

Table I.V shows the later results with non porous peliets which extend. the range of the earlier data, and allows comparison of the hydrogen pulse technique with methane pulse results. Once again the wall aispersion effects ( $C$ value) for the methane vary only from 0.057 to 0.081 with pellet sizes from 0.56 to 1.0 cms . For the hydrogen pulses, the value of the $C$ term varied from 0.019 to 0.028 in the sane beds. These data confirm the previous conclusions regarding the effects of tuve Ciameter and pellet diameter.

Runs 63 and 64 show high B values (inverse tortuosity), indicating that insufficient data has been obtained in the molecular diffusivity region, and the $A A$ and $C C$ values are probably more meaningful than the A and C terms.

The values of $A A$ and $A$ for all the data are plotted versus pellet diameter in Figure l.12, which indicates, in spite of considerable scatter, the approximately linear dependence of the packed bed eddy aiffusivity on pellet diameter for a wide range of Reynolds numbers, as suggested by Van Deemter. The deviation from linearity could be ascribed to variation of the constant $\gamma$ in Van Deemter's eddy diffusion expression. However, the data from this work aligns itself well with the typical values of $\gamma$ quoted by Van Deenter (17), as shown in Table 1.6, except that the trend is reversed with larger pellets, and $\gamma$ increases with pellet diameter.

## TABLE 工.VI

VALUES OF THE EDDY DIFFUSIVITY TERM CONSTANT; $\quad \gamma=\frac{u d_{p}}{D_{L}}$

|  | Pellet dianeters cms |  |  |  |
| :---: | ---: | ---: | ---: | ---: |
| Van Deemiter (17 | .003 | - | .0074 | 8 |
| " | .015 | - | .025 | 3 |
| " | .035 | - | .083 | $\approx 0$ |
| This work |  | .2 |  | 0.06 |
|  |  | .6 | 0.13 |  |
|  |  | 1.0 |  | 0.37 |



Figure 1.12
Eday Diffision Perm, A, (Eiuation 1.50) Vis. Pell + Diameter
B. LONGIIUDINAL DISPERSION COBFFICIENTS

The data obtained in the beas or non porous pellets were computed as overall dispersion coefficients (that is, eddy plus molecular coefficients) and are compared with the correlations and theories of other vorkers in Figures 1.15, to 1.17. In Figure 1.13 all the data except those from run 1 are plotted as dispersion coefficients vs. the interstitial velocity (u). The data points form smooth curves but the slopes in the turoulent region vary, showing an exponential velocity dependence of 1.5 for the larger 1 cm pellets, increasing to an exponent of 2 for the smaller packing sizes. A' low velocities, the dispersion coefficients approach the value of the molecular diffusivity. In Figures 1.14-1.17, the smoothed data from Figure 1.13 has been used, and is shown as a continuous curve with identifying symbols marking the start and finish of the line.

In Figure 1. $\mathrm{I}^{h}$ the inverse dispersion Peclet number $\frac{\mathrm{DI}_{\mathrm{L}}}{\mathrm{ucp}_{\mathrm{p}}}$ is
Flotted vs. the superficial Reynolds number $u \in_{B} p / \rho$ to compare the results with those of McHenry and Wilhelm (15) obtaincd by the frequency response method in a bed of 0.3 cm diameter spheres. The data from this work are not entirely consistent with McHenry and Wilhelm's, but the lack of acrement is probably due to a difference in the ichmidt number since ifHenry and Wilhelm used a 50\% hydrogen stream while the pulses in this work used only a trece of hydrogen. The Reynolds number above is thus not a complete criterion, particularly in the transition flow regimes as pointed out by Hiby (24). 'The data of Cairns and Prausnitz (3) for Iiqujers are also included in Figure 1.14. Hiby (24) suggested that at low flow rates (approaching the molecular regime) the inverse dispersion Peclet nuaber is better plotted against the molecular Peclet numbers as.. . . shom in Figure 1.15. Un $\begin{gathered}\text { fortunately the nolecular Peclet numbers could not: }\end{gathered}$ be celculatod from wellenry and Wilhelms'puolication without access to


Figure 1.13
Disprarsion Coefficient, $D_{L}$ Vs. Intersinial Veiocity, u.


Figure 2.14
Inverse midy peclet lumber Vs, Supurficial Reynolds Nurber




Figure 1.16

[^0]

Figure 1.17
Empirical Dispersion Coefficient Correlation

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primary data and so a comparison could not be made, but it may be noticed that in Figure 1.25 the resultis from this work are not so scattered as in the previous illustrations. The data for Iiquids publishod by Hiby (2h) are also included in Figure 1.15, but the values are lower than the results from this work. This decrease was to be expected because Hiby took pains to eliminate the high porosity wall section, and thus renove the dispersion due to wall effect. It is also significant that Hiby considered the results of MeHenry and Wilholm to show lower values of the eddy diffusivity or dispersion coefficient than would be expected in a bed with wall effects.

In Figure 1.16, the correlation suggested by bischorf and Levenspiel (28) is examined by plotting the data as inverse dispersion Schmidt number vs. the Reynolas number based on hydraulic diameter. The covergence of the data is no better than in the other plots.

The Saffman model (29) could not be tested because the boundaries of the dispersion regimes in packed beds are not known as they are in the case of dispersion in pipes. Lowever, it would appear that the Saffman molel may have the greatest potential in providing a correiation for eddy diffusion in packed beds.

In the absence of a more logical correlation, an empirical correlation has been developed below, which is an extension of the simpler form proposed by Bischori and Levenspiel (28).

$$
\begin{equation*}
D_{I}=0.75 D_{B}+0.6 u h_{D}+\frac{0.02 u^{2} h_{D} 0.6}{0.75 D_{B}+0.022 u h_{D}} \tag{1.68}
\end{equation*}
$$

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This correlation is plouted in Figure 1.17 as experimental ve. calculated values of axial dispersion ccejficiont, anc although the agreement is not good, the methou is sufficiently accurate to allor a correction to be calculated for the "C" term in equation (1.50), which will correct the value of this te-m in porous pellet tests where any cffects of eddy dispersion are not allow rer in the eduy dirfusion, or "A", term.
C. POROUS PELLEES

Porous Pellet Damples
The properties of the three porous pellet samples tester are sumarizec in Table 1.VII. However, there was originally some question about the pellet properties, the petails of which are discussed in Appendix III. A knowledge of the pellet porosity is essential for this Work, but the manufecturers' data supplied with the pellets seemed to be somewhat inconsistent.

The data on the $1 / 2^{\prime \prime}$ Norton catalyst support pellet were generally satisfactory. However, in the trade literature a $4 \mathrm{j} \%$ porosity was quoted for these pellets. In a private communication, a value of 36-40\% was given, and a simple experimental measurement described in Appendix III found a $36 \%$ porosity. A value of $38 \%$ has thus been accepted as a reasonable average.

With the activated alumina pellets, in addition to the inconsistency of the manufacturer's and supplier's data, the amount of moisture contained in the pellet presented a problen. As discussed earlier, the epoxy resins used to mount the test pellet in the steady state diffusion apparatus

## A. LLr I.VII

## PROPERTIS OF POROUS PLLIEN SA:PLES


could not stand the drying temperature necessary, and so it was decided to make the diffusion tests on the wi pellets. The moisture content of the wet pellets was found to be stable, and not sensitive to atmospheric humidity, remaining between $10-14 \%$ by weitht. In addition, the manuffacturtr's literature (31) suggested that adsorbed water existed in liquid form, so that if the dry pellet porosity could be found, the porosity of the wet pellets could be calculated.

The suppliers quoted a dry pellet porosity of $60-65 \%$, while the manufacturer's literature stated 50\%. The moisture content in equilibrium with $60 \%$ R.H. air was given as $20-24 \%$, but at no time could more than $15 \%$ water actually be found in the pellets. Examination of some of the manufacturer's drying data indicated that after 6 months a $12-15 \%$ moisture content was normal. In order to obtain a better value of the dry pellet porosity, special measurements were carried out. One of the experiments for this purpose described in Appendix III involved putting pellets under vacuum and then flooding them with water. 'This test suggested that the $50 \%$ porosity was correct, and this value was later verified more exactly by placing dry pellets in a chromatograph sample loop, and measuring the resulting reduction in sample volume of the loop. Hydrogen gas was used at the sample gas in the loop. This experiment gave a $50 \%$ porosity for the dry pellets and yielded $28 \%$ and $33 \%$ porosities for wet pellets having a. $12 \%$ moisture content. A porosity of $31 \%$ corresponds to a $50 \%$ dry porosity in a pellet containing le\% by weight water in the liquid state. In addition, the alumina pellets are not homogeneous in that they are apparently manufactured by seeding a colloidal solution. Examination of a slice of pellet on a microscope slide showed pores up to 150 microns in the centre core, compared to a pore diameter of $50 \AA$ in the

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outer shell. These pellets provide an excellent example of an instance in Which the steady state method of measuring diffusivities would give a poor result for use in catalysis work, while the unsteady state method would yield an average diffusivity value which would be more likely to be suitable.

Steady State Apparatus Results (Appendix I)
The effective diffusivity of hydrogen and nitrogen in $1 / 2$ " Norton SA203 spheres was found to be $0.0667 \mathrm{~cm}^{2} / \mathrm{sec}$. at $23^{\circ} \mathrm{C}$ and 760.7 mm . Hg.

The effective diffusion coefficient of hydrogen in $1 / 4^{\prime \prime}$ diameter Alcoa H15l activated alumina pellets containing l2\% by weight of vater vas found to be $0.0067 \mathrm{~cm}^{2} / \mathrm{sec}$. at $26^{\circ} \%$.
reatment of Data for Pulse Apparatus
For the porous pellets, the same measurements and computations are recorded in Appendix II as for the non porous pellets, except that the eddy diffusivity calculations in columns 8, and subsequent columns are omitted. Column 3 contains the inverse velocity rather than tho molecular Peclet number which was used with the non porous pellet results.

Equation 1.50 was fitted to the data, and the quantity $C$ found thereby was corrected, using the differential of the last term of the empirical correlation equation (1.68) to remove the eddy diffusivity contribution as follows,

$$
\begin{equation*}
\text { Correction }=\frac{d H E T P}{d u}=\frac{0.3 D_{B} h D \cdot 66}{\left(0.75 D_{B}+0.2 u^{*} h_{D}\right)^{2}} \tag{1.69}
\end{equation*}
$$

where $u^{*}$ is the mean velocity from all the data points and allows the correction to be made in the middle of the velocity range of interest.

The correction is subtracted from the slope $C$ and the corrected slope $C$ applied in the calculation of the effective diffusivity from equation 1.49 and 1.50 using the form,

$$
D_{E}=\frac{\epsilon_{B}}{2 C_{C O R R}\left(1-\epsilon_{3}\right)}\left[\frac{-83-}{\pi\left(1+\frac{a_{p}}{\epsilon_{p}\left(1-\epsilon_{B}\right)}\right)}\right]^{2}
$$

Porous Pellet Results
In Table 1.VIII, runs 56 to 62 were made with 1/4" Activated Alumina pellets, except run 60 which was made with the $1 / 2^{\prime \prime}$ Norton Catalyst support, and mun 73 in which the $1 / 8^{\prime \prime}$ Activated Alumina pellets were used. All the results were taken in single pellet diameter beds, except run 73 which used a bed having a 7:l diameter ratio.

Runs 56 and 57 differ only in the length of column, while in 58 the same bed was used as in 57, except that the pellets were dried. Run 59 was essentially unsatisfactory, but it shows the results of an attempt to eliminate the adsorption effect with extremely high flow rates. Kun 61 repeated 58 , and 62 repeated 57, except that hydrogen pulses were used. The hydrogen pulse was also used in run 73. Run 60 employed a methane pulse in the single pellet diameter bed packed with the $1 / 2^{\prime \prime}$ Norton Catalyst supports.

The results for porous pellets are summarized in Table l.VIII. It may be noted that there appears to be an end effect in comparing runs 56 and 57. However, under Table l. VIII the values of the criterion for Gaussian dispersion, $\frac{F_{1}}{\mathbf{o c}}$ are given, and for run 56 these are greater than the column length due to the large dispersion caused by the adsorption of the methane pulse. Thus Van Deemter's solution (17) to obtain equation (2.31) would not hold.

The effects of adsorption on a catalyst pellet have been mentioned in section $C$ of the introduction under "Comparison of Methods". The adsorption of methane on dry Aicoa 1/4" Activated Alumina pellets was

DISPEREION RESUL: FOR POROUS PELLETS

| Run | Pellet | Moisture <br> Content <br> wt., \% | Pellet <br> Porosity | Column Length Cm. | Colurn Diameter cm. | Pulse Gas | Pellet <br> Diameter | A | B | C |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 56 | 1/4" activated Alumina | 12 | 0.31 | 129 | 0.66 | $\mathrm{CH}_{4}$ | 0.597 | -0.26 | -0.64 | 1.32 |
| 57 | 1/4" activated <br> Alumina | 12 | 0.31 | 421 | 0.66 | $\mathrm{CH}_{4}$ | 0.597 | -0.22 | 0.58 | 1.61 |
| 58 | 1/4" activated Alumina | 0 | 0.50 | 421 | 0.66 | $\mathrm{CH}_{4}$ | 0.597 | -2.2 | 4.65 | 1.699 |
| 59 | 1/4" activated Alumina | 0 | 0.50 | 421 | 0.66 | $\mathrm{CH}_{4}$ | 0.597 | 68 | -592 | -C. 33 |
| 60 | $\begin{aligned} & \text { I/2" Norton } \\ & \text { Caialyst Support } \end{aligned}$ | - | 0.38 | 420 | 1.6 | $\mathrm{CH}_{4}$ | 1.3 | 0.29 | 0.50 | 0.44 |
| 61 | 1/4:" activated Alumina | 0 | 0.50 | 421 | 0.66 | $\mathrm{H}_{2}$ | 0.397 | 0.64 | 0.87 | 0.22 |
| 62 | 1/4" activated Alumina | 10 | 0.34 | 421 | 0.66 | $\mathrm{H}_{2}$ | 0.597 | 0.38 | 1.5 | 0.237 |
| 73 | 1/8" activated Alumina | 12 | 0.31 | 119.4 | 2.17 | $\mathrm{H}_{2}$ | 0.32 | -0.015 | 1.43 | 0.096 |

Run Number Height of Transfer Unit $\frac{\mathrm{F}_{2} \mathrm{u}}{\alpha}<\mathrm{L}$ at max. velocity

Continued.......

166 cm.
3.38 cm .
20.2 cin.
5.38 cm .

TABLE 1.VIII (Continued)
Dispersion Results for Porous Pellets

|  |  | Diffusivity <br> Assuming |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Run | Slope <br> Correction | Corrected <br> Slope | Diffusivity | Equilibrium <br> Adsorption | Bed <br> Porosity | Reynolds <br> Range |
| 56 | 0.061 | 1.26 | 0.00085 |  | 0.471 | $3-48$ |
| 57 | 0.062 | 1.54 | 0.00069 |  | 0.471 | $2-142$ |
| 58 | 0.061 | 1.63 | 0.0012 | 0.0045 | 0.471 | $2-42$ |
| 59 | 0.019 | 0.31 | -- |  | 0.471 | $47-318$ |
| 60 | 0.12 | 0.32 | 0.0193 |  | 0.522 | $5-33$ |
| 61 | 0.019 | 0.20 | 0.0102 |  | 0.471 | $3-95$ |
| 62 | 0.019 | 0.22 | 0.0056 |  | 0.471 | $7-86$ |
| 73 | 0.017 | 0.079 | 0.0045 |  | 0.39 | $1-19$ |

measured, and the procedure, which involved taking pressure and volume measurements of a gas pellet sample trapped in the leg of a mercury manometer is described in Appendix IV. The results or this experiment, which showed methane adsorbed to the extent of $1.37 \mathrm{mle} / \mathrm{ml}$ of pellct, are utilized in run 58 to calculate the effective diffusivity assuming equilibriwn of the adsorbed gas in the pulse apparatus. If equilibrium had been attained, then the diffusivity calculated with the increased capacity due to adsorption in the pellet taken into account, should be aquivalent to the diffusivity found in run 61 using a hydrogen pulse with ary pellets (after correcting for the different gas system).

In Table l.IX the diffusivities adjusted to those equivalent to hydrogen diffusion are compared for all the runs. The effective diffusivity with a hydrogen pulse in run 61 lies between the two diffusivities calcuilatea from run 58 with the methane pulse, (a) assuming no adsorption and (b) assuming equilibrium adsorption. This result would indicate that the methane probably does not approach equilibrium adsorption closely in the pulse apparatus.

The values of the constants $A$ and $B$ from equation 1.50 presentec in Table l. VIII would appear to represent a breakdown of the theory and/or an inconsistency with the results from the edoy diffusion runs rith non porous pellets, but if the fact that the $C$ terms are extrenely large due to the adsorption of methane in runs 56 to 58 is considered, tilen the $A$ and $B$ terms are negligible, and correction of them has very little influence on the slope, or $C$, term. For the remaining runs, the $C$ terms are smaller but at the same time the values of the $A$ and $B$ terms are within the expected range.

COMPARISON OF EXPERIMENTAL EFFECTIVE DIFFUSION COEFFICIENTS
Diffusivities $\mathrm{cm}^{2} / \mathrm{sec}$ units
Pulse Method Factor to Convert Result as a Assuming

| Run | Pulse <br> Gas | Pulse Method <br> Experimental <br> Result | Factor to Convert <br> to Hydrogen or <br> $\mathrm{H}_{2}-\mathrm{N}_{2}$ Diffusivity | Result as a <br> Hydrogen <br> Diffusion | Assuming <br> Adsorption at <br> Equilibrium | $\lambda_{\text {fron }}^{\text {Column } 5}$ | Steady <br> State |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 56 | $\mathrm{CH}_{4}$ | 0.00085 | $\sqrt{\frac{16}{2}}$ | 0.0024 |  |  | 0.0067 |
| 57 | $\mathrm{CH}_{4}$ | 0.00069 | $\sqrt{\frac{16}{2}}$ | 0.00195 |  | 3.02 | 0.0067 |
| 58. | $\mathrm{CH}_{4}$ | 0.00127 | $\sqrt{\frac{16}{2}}$ | 0.0036 | 0.017 | 2.64 |  |
| 60 | $\mathrm{CH}_{4}$ | 0.0193 | $\frac{0.750}{0.208}$ | 0.0694 |  | 4.14 | 0.067 |
| 61 | $\mathrm{H}_{2}$ | 0.0102 | 1 | 0.0102 | 0.93 |  |  |
| 62 | $\mathrm{H}_{2}$ | 0.0056 | 1 | 0.0056 | 1.05 | 0.0067 |  |
| 73 | $\mathrm{H}_{2}$ | 0.0045 | 1 | 0.0045 | 1.31 | $0.0067^{*}$ |  |

*For 1/4" pellets
Interstitial Diffusivities
$D_{B}$ Nitrogen and Hydrogen $=0.755 \mathrm{~cm}^{2} / \mathrm{sec}$. at $1 \mathrm{a} . \mathrm{m}$. and $296^{\circ} \mathrm{K}$
$\mathrm{D}_{\mathrm{K}}$ Hydrogen in $\begin{gathered}50 \mathrm{~A} \text { pores } \\ \text { at } 296^{\circ} \mathrm{K}\end{gathered}=\frac{2}{3} \times \frac{25}{10^{8}} \times 1.84500 \times \frac{296}{273}=0.019 \mathrm{~cm}^{2} / \mathrm{sec}$.

In order to comparc resultis, Cable I.IX presents the pulse data converted to the equivalent hydrosen diffusivity (or hycrogen-nitrogen for bulk diffusion). If the resulto $i$ th the $1 / 2^{\prime \prime}$ Nortion pellets in run 60 ary examined, it is scen that, the pulse method agieec with th: Unay state value within $4 \%$.

Run 62 should give the same dinfusivity for hydrogen in the $1 / 4 "$ Alcoa activated alumina pellets as the steady state apparatus, ui. the latter result is $20 \%$ higher than tho pulse result. If the difcusivity in the $1 / 8^{\prime \prime}$ pellets could be expected to be the same as that in the $1 / 4^{"}$ size, the steady state result is $52 \%$ higher than the result from run 73. In view of the lack of homogeneity of the alumina pellets these results are not surprising.

The pellet tortuosity values calculated from the true interstitial diffusivities and the pellet porosities shown below Table l.IX also indicate that the Alcoa activated alumina pellets are not homogeneous, as the tortuosities are much lower than would be expected for this type of material. The steady state results should be even more influenced by the macroporous pellet centre or seed because of the removal of part of the microporous shell, and if the tortuosity is calculated from the steady state result an impossible value of 0.88 is obtained. The reason for this anomalous result is because the pore size has been assumed to be 50 A in the calculation of the Knudsen diffusion coefficient, when in fact the centre core has some pores up to 150 microns in diameter as measured under a microscope.

## VI

## DISCUSION

## A. NON POROUS PELLETE

## IIE $\mathbb{C P}$ vs. Velocity Results

The HETP vs. velocity curves shown in Figures 1.10 and 1.11 appear to fit Van Deemter's equation (1.50)well. However, a velocity uependent, or Cu term, was not to be expected with non porous peilets on the basis of Van Deemter's analysis. From Figures 1.10 anā 1.11, as well as from the results in Tables L.III and L.V, the magnitude of this term can be seen to be indcpendent of particle diameter, but inversely proportional to the molccular diffusivity of the gas systom. If the Cu term (rhich Is a velocity dependent axial disporsion effect) in non porous pellets is caused by the higher velocity annulus which resulto from the high packing porosity at the wall, then by analogy with van Deemter'c treatmont, the relative velocity between the flow in the wall annulus and in the packing core could create a term which would be inversely proportional to the molecular diffusivity. This reasoning implies that this auditional dispersive effect for non porous pellets'is caused by a wall ericct.

On the other hand, the above model becomes less satisfactory if single pellet diameter beds are considered, so it would appear that another but similar mechanism occurs in single pellet beds, or that the above physical explanation is questionable.

The intercept, or A term, (which is a dispersion due to the mixing effect of the packing) of equation (1.50) depends on pellet dianeter at Reynolds numbers less than 1, according to Van Deemter et al (17), and a similar relationship for high Reynolds numbers based on the mixind stage model has been obtained by McHenry and Wilhelm. The results from this
work as shom in 'rables l.III and l.V, and in Figure l.12, also show that the intercept $A$ is an approximately linear ( $\pm 50 \%$ ) function or the packing dianeter for diameters from about 0.2 to $1 \mathrm{~cm} .$, and for a wide range of tube: pellet diameter ratios. Axial Dispersion Coefficient

If the same data as above are considered in terms or the dispersion coefficient (i.e. the data for non porous pellets are not fitted to equation (1.50)) as defined by equation (1.66), then it would appear that the wall effect is not the major contribution to the mixing due to packing geometry.

Figure 1.13 shows thet the smaller pellets tend to yield a dispersion coefficient proportional to the square of the velocity which sould correspond to the Cu term in equaition 1.50 , but the larzer pellets show a lower exponent of 1.5 .Hiby (24) obtained the following empirical correlation for liquids,

$$
D_{L}=0.67 D_{B}+\frac{0.65\left(u d_{\mathrm{i}}\right)^{1.5}}{\sqrt[7]{D_{\mathrm{J}}}+} \sqrt{\mathrm{u}_{\mathrm{p}}}
$$

and at low flow rates where $7 \sqrt{D_{B}}>\sqrt{u_{p}}$ this expression has a velocity exponent to the 1.5 power. In the same work ( 24 ), the resuits of other workers with liquids are sumazizen. In general, the axial disperaion coefficient found by other workers is a little larger than that obtained by Hiby, who eliminated the vall effect, but Hiby points out that the data of McHenry and Wilhelm, who worked with gas systems, gives the appearance of having the wall effect removed. This effect may be due to the fact that end corrections were applied to the bed data by McHenry and Wilhelm, because in their work relatively short beds were used (1', 2' and $3^{\prime}$ long), with only the largest being comparable to the bed lengths in the present work.

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In Figure 1.14, it may be seen that the data from this work shows higher dispersion coefficient values than does that of McHenry and Wilhelm (15), so that the data from this work would appear to be consistent with those of Hiby (24).

The use of the hydraulic diameter to describe the system as proposed in Bischoff and Levenspiel's work (28), and as shown in Figure 1.16, does not appear to improve the correlation. The hydraulic diameter would only be expected to account for the wall effect, and if the wall effect is not predominant, as suggested by Hiby, then a major improvement in correlation would not be likely to result.

As mentioned in the "Theory", Saffman's model (29) of a series of interconnected cylindrical capillaries would appear to show the most potential for describing the longitudinal dispersion in a packed bed. Bince the results presented here were generally obtained between particie Reynolds numbers of 1 and 100 (i.e. in the intermediate region between laminar and turbulent flow), then it is quite conceivable that a velocity profile mechanism equivalent to that described by Raylor (25) occurs, resulting in regions where the dispersion coefficient is proportional to the squares of the velocity and inversely proportional to the molecular diffusivity.

The upper limit of the region was found to be, from (1.57) $u \ll 10 L D_{B} / R^{2}$
where $u$ is the gas velocity, $L$ the tube length, $R$ the radius and $D_{B}$ the molecular diffusivity.

Let the capillary length be $K_{1} \dot{d}_{p}$ and radius $K_{2} d_{p}$ in the Saffman model, which should be a reasonable assumption for packings of uniformly sized spheres.

Then,
$u \ll \quad \frac{10 K_{12} p_{p} D_{B}}{K_{2 d p}^{2}}$
or $\quad u \ll K_{3} \frac{D_{B}}{\bar{d}_{p}}$
where $K_{3}=K_{1} / K_{2}$
Thus, the smaller the pellet diameter, $\left(d_{p}\right)$, the larger the right hand side of the above equation.

This model would explain therefore, why the smallest pellets showed a velocity exponent of 2 as compared to 1.5 or 1.7 for the larger pellets. A large molecular diffusivity would also increase the upper limit of the region, and may explain why a maximum is seen in McHenry and Wilhelm's results at a superficial Reynolds number of about 100 in Figure 1.14.

It would appear that at least two mechanisms are operating here; 1.) the velocity dependent dispersion described by equation (1.55) which is caused by the difference in flow paths between adjacent parts of the bed, and which can also be described by the mixing stage theory, and 2.) the effects of velocity profile (equation 1.56) in the individual channels, which yield a velocity exponent of 2 within the flow limits derived by Taylor, given in equation (1.57). Thus, the resultant dispersion coefficient has a velocity exponent between 1 and 2. As pointed out above, a high molecular diffusivity would result in a higher upper limit of significance for the velocity profile range. Nevertheless, McHenry and Wilhelm's results for eddy diffusivity using hydrogen approach a velocity dependence of 1 , possibly because although the molecular diffusivity is high, the magnitude of the contribution to the dispersion due to the velocity profile in the capillaries is smaller with higher diffusivity
gases (equation 1.56), and so the mechanism of equation (1.55) would predominate.

In pipes, when the flow becomes turibulent, the profile contribution changes from the velocity squared dependence of equation (1.56) to a function of velocity and friction factor. In this turbulent region, the dispersion coefficient is independent of the molecular diffusivity and the same independence would be expected in a packed bed.

## Correlation of the Axial Dispersion Coefficient

As may be seen from Figures 1.13 to 1.16 , several attempts were made to obtain a correlation for the dispersion coefficient. In addition to these efforts, dimensional analysis and a least square calculation based on the resulting expression using all the non porous pellet results yielded the following correlation,

$$
\begin{equation*}
\frac{D_{L}}{u h_{D}}=\frac{1}{0.495}\left[\frac{u h D}{D_{B}}\right]^{0.6 .1}\left[\frac{u^{2}}{g h_{D}}\right]^{0.24}\left[\frac{N}{h_{D} u \rho}\right]^{0.41} \tag{1.70}
\end{equation*}
$$

where $\mu$ and $\rho$ are the carrier gas viscosity and density. The above correlation shows an exponent for the hydraulic diameter of nearly unity, and a velocity exponent of 1.67 , which is an average of the values shown in Figure 1.13. Equation (1.70) does not provide a particularly good fit to the data, which is not surprising because the velocity exponent is obviously not constant, a fact clearly evident in Figure 1.13. Of the correlations of the above type, that of Hiby recomended for the transition region and shown in Figure 1.15 seems to be most satisfactory, but due to a dependence on the packing diameter squared, the degree of correlation is less satisfactory than that given by equation (1.70).

Bischoff and Levenspiel (28) suggest the following expression, which does have the virtue of allowing for the experimental fact that the
velocity dependence is 2 for small pellets and approaches 1.5 for larger ones. The expression is based on the Taylor transition regime in which velocity profile effects are significant, but the molecular diffusivity is replaced by a radial diffusivity which includes a velocity dependent term.

$$
D_{L}=\frac{D_{B}}{\lambda}+\left[\frac{K_{1} u^{2} a_{n}^{2}}{\frac{D_{B}}{\lambda}+K_{2}^{2} u a_{p}}\right)
$$

Although better than equation (1.70), a further consiāerable improvement in fit was achieved by reducing the packing diameter exponent from 2 to 1 . However, the equation then becomes dimensionally inconsistent. The correlation finally utilized essentially makes the longitudinal dispersion coefficient a summation of a molecular term, a mixing stage term as suggested from McHenry and Wilhelm's work (15) and a velocity profile term as suggested by Taylor (25) or by Saffman's model (29).

$$
D_{L}=0.75 D_{B}+0.6 u h_{D}+\frac{0.02 u^{2} h_{D} 0.6}{\left(0.75 D_{B}+0.0212 u h_{D}\right)}
$$

The above expression is plotted in Figure 1.17 as experimental vs. calculated results.

## B. POROUS PELIETS

The effect of gas adsorption on the measured diffusivity presents interesting features of significance in any type of unsteady state diffusion measurement. The method used to measure the degree of adsorption, described in Appendix III, has been developed since this work was done and reported as a technique for determining adsorption isotherms for gases on solids (40). If the amount of gas adsorbed from a methane pulse were close to equilibrium, methods of estimating the diffusivity could still be worked out. Unfortunately, the adsorption is not indicated to be at equilibrium on the alumina pellets in this work, but as the adsorption data were derived for large concentrations (1 atm.) of methane, while the pulse apparatus uses trace concentrations in the presence of air, the state of the
equilibrium cannot really be claimed to be conclusively known. Inconsistency of Steady State and Pulse Results for Activated Alumina Pellets

The results of runs 62 and 73 with $1 / 4^{\prime \prime}$ and $1 / 8^{\prime \prime}$ activated alumina illustrates the potentially serious errors possible with non homogeneous pelleted materials in measuring the unidirectional diffusion through a part or all of a pellet, as, for example, in the steady state apparatus, when in the actual reaction diffusion occurs towards the centre and out again. There are, of course, other potential reasons for differences in the results from steady state and pulse methods, which have already been discussed.

The pulse method in this work maintains either bulk equimolar counter diffusion or Knudsen diffusion in the pellet so that equation 1.13 is valid no matter what mechanism occurs. In the case of the alumina pellets, the outer shell has a uniform structure with $50^{\circ} \mathrm{A}$ pores so that Knudsen diffusion occurs, and settles the choice of equation for the steady state apparatus. Thus, the discrepancy between the steady state and pulse apparatus must be caused largely by the macroporous seed which carries a disproportionately large portion of the diffusion flux in the steady state apparatus.

The $1 / 4^{\prime \prime}$ alumina pellets were examined under a microscope and the seed in the centre was seen to be approximately $1 / 8^{\prime \prime}$ across with pores up to 150 microns, as compared to the $50^{\circ} \mathrm{A}$ pore size in the deposited outer layer. The seed in the $1 / 8^{\prime \prime}$ pellets was not visible by eye and it is possible that these pellets either had an extremely small seed or none at all. This would account for the lower diffusivity of the $1 / 8^{\prime \prime}$ pellets compared to the $1 / 4^{\prime \prime}$ ones.

If the interstitial Knudsen diffusivity is calculated for $50^{\circ} \mathrm{A}$ cylindrical pores, the extremely large pores in the seed would account for the tortuosity value of less than unity obtained by the steady state method and given in Table l.IX. Another factor which could account for the difference in diffusivity values from the pulse and steady state apparatus is that the alumina pellets were prone to break down in annular layers. With caps ground off each side in the steady state apparatus, the strata of these layers are exposed and may represent a low resistance diffusion path through the pellet.

## Porosity

One of the critical factors in applying the pulse technique is an accurate knowledge of the pellet porosity. A $1 \%$ change in porosity can result in a $4 \%$ variation in diffusivity. As a check on the manufacturer's data, an experiment was carried out using a gas chromatograph and a 15' by $1 / 2^{\prime \prime}$ diameter empty tube as a dispersing system. Samples oi the $1 / 8^{\prime \prime}$ "wet" alumina pellets were placed in the sample loop of the chromatograph and a hydrogen pulse injected in an air carrier gas. The height of the pulse output compared to the height obtained in the same way from the empty sample loop gave a good measure of the solid volume of the porous pellet. The sample gas of hydrogen had to be diluted with air to keep the detector in the linear range, but it would appear reasonable that if a pulse apparatus was to be utilized, a porosity measuring device of this type would be very useful, so that the porosity of the pellets as tested is neasured.

Non Spherical Pellets
There should be no reason why the effective diffusion coefficient of granular pellets of almost any form could not be measured by applying an
appropriate shape factor, and a surface-to-volume pellet diameter as used for effectiveness factor charts (3). A derivation was attempted to express the mass transfer coefficient for cylinders in terms of an effective diffusivity, (as in equation (1.46)) but no simplified approximation could be made, due to the presence of Bessel functions in the solution. Thus, for shapes other than spheres, a constant based on experiment would seem to be required to relate the mass transfer coefficient and effective diffusivity if the simplified form of equation (1.49) is to be preserved. Methane Pulse

The use of a methane pulse seems to be of little value. The correction to the dispersion measured for a bed of porous pellets which is due to eddy diffusion effects is no higher for hydrogen than the correction term for methane. The desirable amplification of the pellet capacity dispersion term can be achieved by a high velocity, rather than attempting to use a gas of lower molecular diffusivity. The hydrogen flame detector could conceivably have a lower response lag as compared to the thermal lag in a hot wire detector (thermal conductivity), but this does not appear to be a problem in this work.

## Errors

The errors in the result caused by the mathematical manipulations are not readily estimated, however, the effects of inaccuracics in the measured values are considered below. The effective diffusivity is given by,

$$
D_{E}=\left[1+\frac{1}{\epsilon_{\mathrm{p}}\left(1-\epsilon_{B}\right)}\right]^{2} \quad \frac{2 \epsilon_{B}\left(\mathrm{a}_{\mathrm{p}}\right)^{2}}{4 \pi^{2} \mathrm{C}\left(1-\epsilon_{B}\right)}
$$

where $C$ is the term from equation (1.50).

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The overall potential crror may be estimated by adding the effects of individual errors for a given typical set of vaiucs.

In the following table typical variable magnitudes are given alone with the estimated error and the effect on the resultant effective diffusivity.

| TABLE 1. X |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  | POTVITIAL ERRORE |  |
| Variable | Magnitude | Degree of Uncertainty | Percentage error in Effective Diffusivity |
| C | $\begin{aligned} & 0.375 \\ & \mathrm{~cm} / \mathrm{sec} \end{aligned}$ | $\pm 5 \%$ | 5\% |
| ${ }^{\text {d }}$ p | 1.0 | $\pm 2 \%$ | $4 \%$ |
| $\epsilon_{p}$ | 0.33 | $\pm 20 \%$ | 16\% |
| $\epsilon_{B}$ | 0.40 | $\pm 5 \%$ | $\frac{2 c_{p}^{\prime}}{27_{\%}^{\prime}}$ |

It is fairly obvious that more accuracy in the pellet porosity values would radically improve the results, but at the same tine it is extremely improbable that all the errors would be in the same sense and yield the above overall error. It should be mentioned that the above error estimates apply to inaccuracies in mean values obtained from a reasonable sample. For example, although the pellet diameters could show $50 \%$ variation between individual pellets, the mean of 20 to 40 pellets was not found to vary when a grab sample was taken.

## COMTCLUSIONB

1. 'The effective diffusivity of gases in porous pellets can be adequately measured using a hydrogen pulse technique. A $27 \%$ random error is conceivable due to errors in the measured variables; however, this can be halved with better methods of measuring the pellet and bed poroijties. In addition, a probable error exists from the mathematical derivations. This latter error should be a relatively constant percentage, thus lending itself to elimination by calibration.
2. An eddy diffusion mechanism exists in the transition recion between laminar flow and turbulent flow in packed beds such that the axial dispersion coefficient is proportional to the square of the velocity.

## VIII

## RECOMMETDACTOMS

The method for the measurement of the porosity of pellots by injecting a pulse of hydrogen which has been purged from the sangle loop of a chromatograph containing the test pellets, should be developed further and incorporated into the pulsc apparatus. The main problen to overcome is that of minimizing the interparticle volume by packing in as many peliets as possible.

By extending the flow ranges covered in this work, the range of the region where eddy diffusivity is proportional to the square of the velocity may be determined. The results may then be compared with the results obtained in empty pipes by Taylor (25).

DEVELOPMENT OF AN UND DEAUY STATM WIO:. MDMHOD FOR MEASURING BINARY GA: DIPFUSICN COEFFICIENLO

## I

## IWERODUC TON

The bulk, or molecular diffusion coefficient of binary gas mixtures is not readily measured experimentally. One of the oloest techniques is the Loschmidt method which is based on bringing two cylinuers containing the gases (lighter on top) together and measuring concentration variation with time. However, this method is sensitive to convection or thermal eddies.

In Stefan's method the rate of diffusion of a vapour in a vertical glass capillary tube is measured by following the drop in level of a liquid meniscus as evaporation occurs. The open top end of the glass tube is flushed with the second component. This method obviously cannot be used for gases above the critical temperature, and, in practicc, is limited to narrow ranges of temperature and pressure.

The longitudinal dispersion coefficient in a straight tube, Within the limits described in Section I on Taylor's work (25), is a function 0 the molecular diffusivity. Thus by measuring the dispersion in a straight tube by a method similar to that described in Section $I$, the molecular diffusivity may be obtained. Chromatography apparatus can also be used for this type of work. Good results can be obtained, although the apparatus is not simple, and experimental conditions feasibly are limited (35)(36).

The molecular diffusivity at hich temperatures has been measured by Walker and Westenberg (37) by a point source technique in vhich a trace of one gas is fed through a capillary which is mounted in the centre of a tube in which the second gas is flowing. The profile of the trace gas in the bulk stream is measured downstream from the source, and the molecular diffusivity can be calculated using the appropriate solution of the diffusion equation. Very careful experimental technique is required to obtain accurate values by this method, although wide temperature ranges can be covered.

Other methods, such as measurement of diffusion rates through porous barriers, have been employed by numerous workers, but these do not give absolute values, and require calibration, and a correct interpretation of results. In particular, there appears to exist no absolute methods which can be used to give acceptable values of the binary diffusion coefficient over wide ranges of both temperature and pressure, and which will allow some investigation of concentration effects also. The present work is an attempt to develop a measurement technique which will satisfy all these requirements.

An unsteady state flow method similar to the Stefan technique was selected, as offering the possibility of analysis of an effluent stream remote from the diffusion cell by any convenient means and at any necessary conditions. The cell itself could be maintained at any temperature and pressure desired. By varying flowing and cell gas compositions, concentration effects might be studied. However, convection effects in the cell must be absent, and so some form of packing to produce capillary channels would also be a part of the construction.

THEORY
A.

## SIMPLIFIED SOLUTION OF A DIFFUSION EQUATION

It has been shown (6) that for equimolar diffusion in a porous solid Fick's second law of diffusion takes the following form,

$$
\begin{equation*}
\frac{\partial C_{A}}{\partial t}=-\frac{D_{\mathrm{E}}}{\epsilon_{\mathrm{B}}} \frac{\partial_{2} C_{A}}{\partial x^{2}} \tag{2.1}
\end{equation*}
$$

where $D_{E}$ is the effective diffusivity, $\epsilon_{B}$ the porosity, $C$ the concentration and $t$ the time. The absence of significant surface adsorption is also implied by the above equation.

The relationship between the effective diffusivity $D_{E}$ and the true binary diffusion coefficient $D_{B}$ of the free gas $D$ is given by,

$$
\begin{equation*}
D_{E}=\frac{D_{B} E_{B}}{\lambda} \tag{2.2}
\end{equation*}
$$

Where $\boldsymbol{\lambda}$ is the tortuosity with values varying from 1.0 for straight parallel pores to about 100 for a structure containing dead end pores. If $D_{E}$ from equation (2.2) is substituted into (2.1), then for a bed with tortuosity 1.0 the solution of (2.1) would yield the molecular diffusivity $D_{B}$ of the gas.

A simple solution of the diffusion equation (2.1) for the model shown in Figure 2.1 is obtained if the assumption is made that the vessel is initially bathed in a gas concentration $C_{o}$ and then at time zero the plane at $\mathrm{x}=\mathrm{L}$ is maintained at zero concentration.

Mathematically, the boundary conditions are:

$$
\begin{aligned}
& x=0, \frac{\partial C}{\partial x}=0 \\
& C_{A}=C_{0} \text { for all } x \text { when } t \leqq 0 \\
& C_{A}=0 \text { when } x=L \text { for } t>0
\end{aligned}
$$



Figure 2.1
Shull of The Bed For Proposed Diffusion Expurinent

Grank (22) (p.97) has given the oluticn oit equation (2.1) with these


$$
\mathrm{C}_{\mathrm{A}}=\frac{4_{1} \mathrm{C}_{0}}{\pi} \sum_{\dot{n}=0}^{\infty} \frac{\left(-1 i^{n}\right.}{(2 n+L)} \quad \exp -\frac{D^{2}}{B} \frac{\left.\left(\varepsilon^{n}+\right]^{2} \pi\right)^{2}}{4 L^{2}} ; \frac{\left(\varepsilon n \frac{1}{2}\right) \pi x}{(2,3)}
$$

In order to find the flux from the end of the vassel the above solution must be differentiated with respect to $x$, anc the resulting expression solved to give the concentraition gradient at the enc ( $x=L$ ). This gradient may then be applied in conjunction with Fick's first law of aiffusion,

$$
\begin{equation*}
N_{A}=-D_{E} \quad \cdot \frac{d C_{A}}{d x} \tag{2.4}
\end{equation*}
$$

where $N_{A}$ is the flux of gas $A$ in moles/ $(\mathrm{sec})\left(\mathrm{cm}^{2}\right)$ under conditions of equimolar counter diffusion, which must exist in the model of Figure 2.1. The series solution for the concentration gradient given by (2.3) when $x=L$ is,

$$
\begin{equation*}
\left[\frac{d C_{A}}{d x}\right]_{x=L}=\frac{2 C_{0}}{I} \sum_{n=0}^{\infty} \exp \left[-\frac{D E}{B} \frac{(2 n+1)^{2} \pi^{2} t}{4 L^{2}}\right] \tag{2.5}
\end{equation*}
$$

This solution can be simplified by taking into account only times greater than the time when the second term of the series is less than $7 \%$ of the first, or in other words,

$$
\begin{equation*}
\operatorname{Ln} 0.01-\frac{D_{E}}{\epsilon_{B}} \frac{\pi^{2} t}{4 L^{2}}=-\frac{9 D_{E}}{4 \epsilon_{B}} \frac{\pi^{2} t}{L^{2}} \tag{2.6}
\end{equation*}
$$

If a molecular diffusivity of $0.75 \mathrm{~cm}^{2} / \mathrm{sec}$ ( $\mathrm{e} . \mathrm{g}$. hydrogennitrogen) is assumed, and a diffusion path of unit tortuosity and length of 10 cm ., then solving 2.6 gives $t=31$ seconds. Similarly if the gas diffusivity is taken as $0.1 \mathrm{~cm}^{2} / \mathrm{sec}$ then the time before the second term can be ignored becomes 232 sec .

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If now the boundary condition requiring that the end of the bed at $\mathrm{x}=\mathrm{L}$ should be at zero concentration is achieved by sweeping the end rapidly with a second gas, then the concentration of the displaced gas in the exit stream will be proportional to the flux at the end of the bed. If, in addition, sufficient time as calculated above is allowed to elapse before concentrations are recordea so that the second and higher terms become negligible, then the flux equation from (2.5) reduces to the form,

$$
\begin{equation*}
\operatorname{Ln} C_{\text {exit }}=\operatorname{Ln} 2-\frac{D_{E} \Pi^{2} t}{\epsilon_{B} 4 L^{2}} \tag{2.7}
\end{equation*}
$$

Where $Z$ is a constant including the unwanted terms from the material balance, and from equations 2.4 and 2.5.

$$
\begin{equation*}
Z=\frac{A_{B} D_{E}}{Q} \cdot \frac{2 C_{O}}{L} \tag{2.8}
\end{equation*}
$$

$Q$ is the displacing gas flow rate in $\mathrm{mls} / \mathrm{sec}$., and $A_{B}$ the area of bed $a 亡 x=$ L.

A semi-logarithmic plot of exit concentration vs. time for a constant flow rate should yield a straight line with slope $\frac{\mathrm{DE}_{\mathrm{E}}}{\mathrm{B}} \frac{\Pi^{2}}{4 \mathrm{I}^{2}}$. If the bed is packed with parallel tubes the tortuosity should be 1 , and the slope of the plot becomes $\frac{D_{B}}{4} \frac{\Pi^{2}}{L^{2}}$, thus providing a means for measuring the free gas molecular diffusivity without calibration of the apparatus.

## B. MORE RIGOROUS SOLUTION

A problem with the above experimental model arises, in that if a displacing gas flow rate high enough to satisfy the boundary condition that $C_{A}=0$ at $x=L$ is maintained, then by the time analysis is started the concentration is so small that an extremely sensitive analytical method is required. Possibly this very high gas flow rate could be used
anyway, but a second problem due to turbulence caused by the high velocities entering above the packed section could arise and cause eddies in the aiffusion zone.

In order to minimize the displacing gas flow rate, the end zone through which the displacing gas flows must be as small as possible, but too narrow an end zone would result in pressure drops which could cause bulk flow in the diffusion section.

Experimentally, it was not found possible to achieve the ooundary conditions described above, but a solution for the diffusion equation with a well mixed fluid at the end of the diffusion zone (i.e. a finite end zone) has been obtained by Carslaw and Jaeger (38) for heat conduction from a solid. Essentially, the solution expressed in terms of the mass diffusion case discussed above is for the following boundary conditions,

$$
\begin{array}{ll}
\frac{d C_{A}}{d x}=0 & \text { at } x=0 \text { for all } t \\
C A=C_{0} & \text { for all } 2 \text { at } t=0
\end{array}
$$

and a material balance around the well mixed end zone yields,

$$
\begin{equation*}
-D_{\mathbb{B}} A_{B}\left[\frac{\partial C_{A}}{\partial x}\right]_{x=L}-Q C_{A O}=\ell A_{B} \frac{\partial C_{A O}}{\partial t} \tag{2.9}
\end{equation*}
$$

where $A_{B i s}$ the area of the end of the bed, $l$ is the height of the end zone, and $C_{A O}$ is the well-mixed end zone concentration. As before, $Q$ is the gas flow rate, so that the loss of displaced gas from the system is proportional to the concentration in the end zone and also the gas flow rate.

The solution obtained by Carslaw and Jaeger in terms of heat
gives the tomperature $v$ at time $t$ in the region $0<x<I$ with initial uniform temperature $V$, and no heat loss at the plane $x=0$. At $x=L$, contact is assumed with a mass of well stirred fluid $M^{\prime}$ per unit area of contact, and
specific heat $c^{\prime}$ which is cocling by a radiation mechanism at $H$ times its temperature. The initial temperature of the fluid is taken as zero. The latter boundary condition is not compatible with the apparatus proposed for this work but this does not influence the solution at large times which is the region of intercst in this work.

$$
\begin{equation*}
v=2 v \sum_{n=1}^{\infty} \frac{\operatorname{aecs}\left[-k \alpha_{2}{ }^{2} t\right)\left\{h_{1}-k \alpha_{n}{ }^{2}\right\} \cos \left(\alpha_{n} \pi\right)}{\left\{L\left(h-k \alpha_{n}^{2}\right)+\alpha_{n^{2}}(I+k)+h\right\} \cos \left(\alpha_{n} I\right)} \tag{2.10}
\end{equation*}
$$

where $h=H / K^{\prime}, k=\frac{M^{\prime} c^{\prime}}{\rho c}$ and $\alpha_{n}$ are the consecutive roots of

$$
\begin{equation*}
\alpha \tan \alpha L=h-k \alpha^{2} \tag{2.11}
\end{equation*}
$$

In the above equations, $P$ is the density of the bed, with heat capacity $c$ and thermal diffusivity $K^{\prime} \mathrm{cm}^{2} / \mathrm{sec}$. The thermal conductivity is $K$ cals/sec $\mathrm{cm} 2\left({ }^{\circ} \mathrm{K}\right) / \mathrm{cm}$.

The above solution has been transposed to the equivalent diffusion case, and the tabulation which follows may assist in explaining the diffusion parameters.

|  | Heat Transfer | Mass Transfer |
| :---: | :---: | :---: |
| vPc | cais/cm ${ }^{3}$ | $\mathrm{C}_{\mathrm{A}}$ concentration moles/ $\mathrm{cm}^{3}$ |
| $p \mathrm{c}$ | cais/cm ${ }^{3} \mathrm{O}_{\mathrm{K}}$ | 1.0, unless in a porous bed when equals porosity $\boldsymbol{\epsilon}_{\mathrm{B}}$ |
| K | $\mathrm{cm}^{2} / \mathrm{sec}$ |  |
| $K^{\prime}$ | $\mathrm{cals} / \mathrm{sec} \mathrm{cm}^{2}\left({ }^{( } \mathrm{K} / \mathrm{cm}\right)$ | $\mathrm{D}_{\mathrm{E}}$ |
| H v | cals/sec $\mathrm{cm}^{2}$ | $Q C_{A O} / A_{B}$ moles $/ \mathrm{sec} \mathrm{cm}^{2}$, where $Q$ is the gas flow rate, $\mathrm{cm}^{3} / \mathrm{sec}$, and $A_{B}$ the bed area, $\mathrm{cm}^{2}$ |

Other quantities appearing in equation (2.10) when written for the diffusion case are,
$v / V=C / C_{0}$, where $C_{0}$ is the initial concentration in the bed.

$$
\begin{align*}
& h=\frac{Q}{A_{B} D_{E}}  \tag{2.12}\\
& k=\frac{A_{B} \ell}{\epsilon_{B} P} \frac{\rho_{M}}{A_{B}}=\frac{\ell}{\epsilon_{B}} \tag{2.13}
\end{align*}
$$

where $\rho_{\mathrm{M}}$ is the molar density and $l$ is the length of the end zone. Rewriting equation (2.10) and setting $x=I$ yields,

$$
\begin{equation*}
C_{A O}=\frac{2 C_{0} \sum_{n=1}^{\infty}\left(h-k \alpha_{n}^{2}\right) \exp \left[-D_{E}(\alpha)^{2} t\right]}{L\left(h-k \alpha_{n}^{2}\right)^{2}+\varepsilon_{n}^{2}(L+k)+h} \tag{2.14}
\end{equation*}
$$

If the time, $t$, is large then the second term in the series becomes negligible compared to the first, and equation (2.14) becomes,

$$
\begin{equation*}
C_{A O}=\frac{2 C_{0}\left(h-k \alpha_{1}^{2}\right) \exp \left(-\frac{D_{E} \alpha_{1}^{2}}{B}\left(h-k \alpha_{1}^{2}\right)^{2}+\alpha_{1} \alpha^{2}+k\right)+h}{(L+k} \tag{2.15}
\end{equation*}
$$

or

$$
\begin{equation*}
\operatorname{In}\left(C_{A 0}\right)=I_{n}(Z)-\frac{D_{E}}{\overline{\epsilon_{B}}} \alpha_{I}^{2} t \tag{2.16}
\end{equation*}
$$

where $2=\frac{2 C_{0}\left(h-k \alpha_{1}{ }^{2}\right)}{L\left(h-k \alpha_{1}^{2}\right)+\alpha_{1}^{2}(L+k)+h}$
Thus a plot of $\operatorname{In}\left(C_{A O}\right)$ versus $t$ for large times should yield a straight line of slope $-\mathrm{D}_{\mathrm{E}} \boldsymbol{\alpha}_{\mathrm{I}}{ }^{2} / \epsilon_{\mathrm{B}}$. It is also of interest to note that an absolute value of the concentration is not needed. For example, the peak height of a chromatograph is proportional to the concentration at low concentrations, and so the logarithr of peak heights rather than concentrations may be plotted versus time.

Equation (2.16) must be solved simultaneously with the auxiliary equation (2.11) in order to obtain a diffusion coefficient from a set of exit gas concentration versus time data. Examination of the equations shows that an analytical solution is not possible. In order to obtain a trial and error solution the following iterative procedure was applied using the Newton-Raphson method (39).

The first root of equation (2.11) must lie between $\alpha=0$ and $\pi / 2 L$. Selection of an initial value approaching zero could result in a break down of the iterative operation because the second approximation falls outside the zero to $\pi / 2$ I range. A further reason for selecting a root close to $\pi / 2 L$ is apparent, because on substitution of $\alpha=\pi / 2 \mathrm{~L}$ back in (2.16) the simplified solution given by (2.7) is obtained. Because the apparatus was designed to approach the simpler boundary conditions, it is reasonable to assume that $\alpha=\pi / 2 L$ will be close to the actual . root. Also, due to the iterative nature of the solutions to (2.16) and (2.11), the time when the second root can be ignored cannot easily be derived, but as the more rigorous solution approaches the simplified solution it is reasonable to suppose that the time calculated from the simpler solution (2.6) and (2.7) is an adequate criterion.

From the assumed value of $\alpha_{1}=\Pi / 2 L$ and the slope of the semilogarithmic concentration vs. time plot one gets,

$$
\begin{equation*}
\text { slope }=\frac{D_{E}}{\epsilon_{B}} \alpha_{1}^{2} \tag{2.17}
\end{equation*}
$$

and so an initial value of the diffusivity $D_{E} / \epsilon_{B}$ is obtained. Equations (2.12) and 2.13) may then be substituted in equation (2.11), but since the value of $D_{E} / \epsilon_{B}$ is an initial approximation, equation (2.11) is corrected by a term for the resulting error, $\Delta$,

$$
\begin{equation*}
\Delta=h-k \alpha_{I}^{2}+\alpha_{I} \tan \alpha_{1} L \tag{2.18}
\end{equation*}
$$

Differentiating (2.18) with respect to $\alpha_{3}$,

$$
\begin{equation*}
\frac{d \Delta}{d \alpha_{I}}=-2 k \alpha_{I}-\frac{\alpha_{1} I}{\left(\cos \alpha_{I} L\right)^{2}}-\tan \alpha_{I I} \tag{2.19}
\end{equation*}
$$

The second approximation for the first root $\alpha_{1}$ can then be obtained from the first approximation, and equations (2.18) and (2.19).

$$
\begin{equation*}
\alpha_{1}=\alpha_{1} \text { (first approximation) }-\frac{\Delta}{\frac{\mathrm{d} \Delta}{\mathrm{~d} \alpha_{1}}} \tag{2.20}
\end{equation*}
$$

With the second approximation the process can be repcated from equation (2.i7) until a satisfactory result is obtained.

## C. COMPUTATION OF SLOPE OF DECAY CURVE WIYH A RESIDUAL CONCEMCRATION

In cases where the gases are not pure, or where there are dead zones in the apparatus which are not easily purged, a plot of experimental data according to equation (2.7) may yield a curve. A problem was experienced in finding the value of the steady state (or infinite time) concentration which the data should approach with time. This value must be subtracted from the results to yield a straight line. It was round that on a log plot a slight change in the steady state value caused a large change in the slope, and hence uncertainty in the resulting value of the diffusivity.

To eliminate the need for judgment on the part of the experimenter in deciding on a value of the steady state level a least squares solution was prepared for the equation $Y-C=A . e^{-B t}$ where $A, B$ and $C$ are the constants to be determined, $Y$ represents the concentration (or peak height), and $t$ is the time.

There is reason to question the use of an equation of the above form as it tends to weight the solution in favour of data at short times. However, as there is evidence that the so-called steady state value is dependent on the gas flow rate in this apparatus, weighting in favour of shorter times where the steady state value is negligible would seem to be justifiable. The derivation of the expression for evaluating $B$ is given below.

$$
\begin{equation*}
E^{2}=\sum\left[Y_{i}-C-A_{e}^{-B t_{i}}\right]^{2} \tag{2.21}
\end{equation*}
$$

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where $C$ represents the value of $Y$ approached at infinite time and $A$ and $B$ are constants of the systom when equation (2.16) is fitted to (2.21), with $E$ being the error to be minimizea.

Differentiating 2.21 by $A, B$ and $C$ yields.

$$
\begin{align*}
& \frac{d B^{2}}{d A}=\sum 2\left[Y_{i}-C-A e^{-B t_{i}}\right]\left(-e^{-B t_{i}}\right)  \tag{2.22}\\
& \frac{d E^{2}}{d B}=\sum 2\left[Y_{i}-C-A e^{-B t_{i}}\right]\left(-A t_{i} e^{-B t_{i}}\right)  \tag{2.23}\\
& \frac{d B^{2}}{d C}=\sum 2\left[Y_{i}-C-A e^{-B t_{i}}\right](-1) \tag{2.24}
\end{align*}
$$

Betting (2.22), (2.23) and (2.24) equal to zero and eliminating $A$ and $C$ yields the following, where $\Delta$ represent the error resulting from assuming an incorrect value of $B$.

$$
\begin{aligned}
\boldsymbol{A}= & -\sum 1 e^{-B t} \sum \frac{t e^{-B t}}{u} \sum e^{-B t}+\sum Y e^{-B t} \sum t e^{-2 B t}+\sum t e^{-B t} \frac{\sum Y \sum e^{-2 B t}}{n} \\
& -\sum Y e^{-B t} \sum e^{-2 B t}-\sum Y e^{-B t} \frac{\sum t e^{-2 B t}}{n}+\sum Y t e^{-B t}\left(\frac{\left.\sum e^{-B t}\right)^{2}}{n} \quad\right. \text { (2.25) }
\end{aligned}
$$

To apply the Newton-Raphson method (39),

$$
\begin{align*}
& \frac{\partial \Delta}{\partial B}=\frac{\sum Y e^{-B t}}{n}\left[\left(\sum t e^{-B t}\right)^{2}+\sum e^{-B t} \sum t e^{-B t}\right]+\sum t e^{-B t} \sum e^{-B t} \frac{\sum y t e^{-B t}}{n} \\
& -2 \sum Y e^{-B t} \sum t^{2} e^{-2 B t}-\sum t e^{-2 B t} \sum Y t e^{-B t}-\frac{2 \sum Y \sum t e^{-B t} \sum t e^{-2 B t}}{n}+ \\
& \sum Y \sum e^{-2 B t} \sum t e_{e}-B t+2 \sum y t e^{-B t} \sum t e^{-2 B t} \quad+\sum e^{-2 B t} \sum Y t e^{-2 B t} \\
& +\frac{2 \sum y \sum e^{-B t} \sum t^{2} e^{-2 B t}}{n}+\sum y \sum t e^{-2 B t} \frac{\sum t e^{-B t}}{n}-\frac{2 \sum y t e^{-B t} \sum e^{-B t}}{n} \sum t e^{-B t} \\
& -\left(\sum e^{-B t}\right)^{2} \frac{\sum y t^{2} e^{-B t}}{n}  \tag{2.26}\\
& \text { Hence } B_{2} \doteq B_{1}-\frac{\Delta}{d \Delta} \tag{2.27}
\end{align*}
$$

In (2.27), $B_{2}$ represents a better vaiue of $B$ than the previous assumed value, that is, $B_{1}$.

## apparatui

A constant temperature air bath was fitted with the hardvare for a gas chromatograph, ana a vessel containing the bed for the diffusion measurement. The test vessels vere soldered from pieces of brass or copper pipe and were filled to the brim with the packing material. A rubber gasket was used to provide the spacer for the "well mixed end zone" as shown in the sketches in Figure 2.2. 'Wwo entrance flow patterns ware used in the beds, a tangential entry in the 5 cm . dia. vessel, and a direct sweep across the bed in one direction in the 2.5 cm . cell.

A schematic diagram of the apparatus is shown in Figure 2.2. Moore constant differential flow controliers were used to maintain constant gas flov rates, while a soap bubble meter was used for measurement of the effluent stream flows. In order to reduce the hold up of the apparatus due to valves and fittings, the two gas feed systems vere connected to the diffusion cell with $1 / 4^{\prime \prime}$ polyethylene tubing, and switching from one gas to the other was done by disconnecting one tube at the entrance to the constant temperature zone and connecting the second. A bypass valve at the entrance to the constant temperature bath allowed gas to flow directly to the flow meter. The use of $1 / 8^{\prime \prime}$ tubing to connect the test vessel to the chromatograph sample valve provided sufficient resistance to flow to make the bypass valve effective without shut-off valves.

Test gases used in diffusion runs were:

| Nitrogen | Prepurified Matheson Co. | $99.9 \%$ |  |
| :--- | :---: | :---: | :--- |
| Ithane | CP | $"$ | $90 \%$ |
| Hydrogen | Prepurified | $"$ | $99.0 \%$ |
| Butane | CP | $"$ | $99.9 \%$ |
|  |  |  | $99 \%$ |



Figure 2.2
Diffusion Appareius

## - 114 -

The details of the packed beds tested are shown in Table 2.I. The chromatograph colwms were packed with $25 \%$ Nujol on chromasorb. The separation of nitrogen and ethane was accomplished with a $9^{\prime} \times 1 / 4^{\prime \prime}$ diam. column using a helium carrier. Hydrogen and nitrogen were analyzed on the same column but with a hydrogen carrier so that only nitrogen showed as a peak. Butane and nitrogen were analyzed by an $18^{\prime \prime}$ column with heliun carrier gas. Ten psig carrier gas pressure was used in the long columns but the short column neeied only 2 psig.

## IV

## PROCEDURE

A. SELECTION OF THE DISPLACED AND DISPLACING GAS

In the selection of displaced and displacing gas from a gas pair two factors must be considered. The tail normally encountered in gas chromatography peaks tends to mask a following peak, and this efiect may be particularly serious when the colwans are made as short as possible to reduce analysis time. Thus, it vas necessary to make the displaceã gas the first peak to appear on the chromatograph. The second effect to be considered is that the lighter gas should be placed on top, and if the end zone is also at the top of the bed then this latter requirement is contradictory to the first, as the lighter gases usually tend to appear first in the chromatographic trace.

TABLE 2.I
DIFFUSION CELL PROPER PIES

## Parallel Tube Packing

Bed Length, cms.
Bed Diameter, cms.
Length of "End Zone", cms.
Porosity

Properties of
Packing material
10.0
5.0
0.27
0.52
"Kimax" melting point tubes 10 cm . long $x$ 1.2 mm O.D. $\times 0.8 \mathrm{~mm}$ I.D.

Selas ol Microporous synthetic ceramic
average pore size 4.5 Specific surface area $0.577 \mathrm{~m}^{2} / \mathrm{cm}^{3}$
or $1.10 \mathrm{~m}^{2} / \mathrm{cm}^{3}$
by B.E.T.
Ref. (5)

Three gas systens were tested on each jed, hydrogen-nitrogen, ethane-nitrogen and butane-nitrogen. The problems described aoove vere overcome for the first pair by using a hydrogen carrier gas so that the hydrogen peak was lost completely. For ethane-nitrogen, it was hoped that because of the identical molecular weights density effects would not be significant, however, this system does represent a more difficult separation if chromatosraphy is used for analysis. If the bed packing is firmly held then obviously an inverted bed can be readily used also with gas chromatography for the analysis. Butane and nitrogen were readily separated in the analysis, providing butane was used as the displacing gas.

## B. OPERATION OF EQUIPNENT

To start a run the constant temperature air bath was brought up to its control temperature, $\left(95^{\circ} \mathrm{F}\right)$, the carrier gas was put on stream, and a purge of about one $\mathrm{ml} / \mathrm{sec}$. of the displaced gas was passed across the bed (by-pass closed). When the bed had been thoroughly purged, a sample of the purge gas was taken.

After purging, the bypass was opened, and the displacing gas line connected and put on stream. The displacing gas was allowed to purge for about 10 minutes while the flow rate was measured on the soap bubble meter and adjusted to the desired range. The stop watch was started at the same time as the bypass valve was closed. Samples were taken and injected into the chromatograph at convenient times, until the displaced gas peak had become too small to give a satisfactory analysis, or until sufficient results had been obtained. In general, the highest concentration
inclucled in a run was about $25 \%$ by volume of the displaced gas and calibrations of the chromatograph indicated a linear response up to about $40 \%$. Therefore, absolute values of concentrations were not usually used, but rather peak height readings.

At the end of the run the flow rate was checked. If any discrepancy from the initial value was found, the later measurement was utilized because the Moore flow controls were found to drift for the first few minutes after a setting change. No flow measurements were taken during a run as the soap bubbles caused a visible increase in pressure in the system. The room temperature and atmospheric pressure were recorded for each run, and the temperature of the air bath was checked.

## v

RESULTS
A. TREATMLITT OF DATA

The raw data, computer program and computed results are recorded in Appendix $V$ for each run. The value of the dirfusivity recorded is actually the $D_{B} / \lambda$ value which is obtained by this experiment. The diffusivity value is for the temperature of the bed, but is corrected to one atmosphere assuming no pressure drop in the vent lines. The effective diffusivity is computed for the same conditions.

The data for each bed are printed along with the constants and sums for the least mean square line computed from the data. Ten iterations were used for this least square calculation, but 4 or 5 were generally sufficient to obtain four figure accuracy. The number of iterations for the diffusivity calculation was set by a test of the
magnitude of the error, and this number is recorded. Certain data points were rejected as described in the following. These points are recorded, but they were not used by the computer.

The results were calculated by a two-part computer program. A subroutine used the Newton-Raphson (39) iteration described in the "Meory" to compute the least mean square fit of the equation $Y$ - $C=A$ $\exp (-B t)$ to the data of peak heights ( $Y$ ) vs. time, ( $t$ ). Then using the solution of the diffusion equation described in "Theory" (equation 2.16), the main program calculated the diffusivity from the slope of the least squares line with a second Newton-Raphson iteration.

The least squares fit of the equation in the form $Y-C=A$ $\exp (-B t)$ weighs the line in favour of the small time (large $Y$ ) points. 'Thus, if the first or second point was inconsistent with the rest of the results, the computed slope showed this inconsistency in spite of all the other points. From the plot of Log Y vs. t, points which appeareả to be inconsistent when plotted have been discarded before arriving at the values in the following tables.

The residence time of analysis gascs in the chromatograph was extremely shori for the butane-nitrogen system, with the result that the recorder was not able to follow the sharp narrow peaks. The las of the recorder caused the peak heights to be non-linear with composition unless small peak heights were used. Thus, computations for the butane-nitrogen system are based on considerably longer times than the minimum for ccceptable data indicated in the discussion of theory. Other reasons for rejecting data points are discussed where applicable.

In order to compare the data, the tortuosity of the beds as calculated from each data point offers a convenient parameter. This calculation requires a knowledge of the value of the molecular diffusivity
for each gas pair used, and the values in rable $2 . V$ show that available published results are not reliable beyond $\pm 5 \%^{\prime}$. Because or this discrepancy the tortuosity only gives a good indication of the consistency or the method, but its absolute value depends upon the value of molecular dirffusivity selected. 'The tortuosity is shown in the following tables, but in table $2 . V$ the computed value $D_{3} / \lambda$ for each set of gas systems and beds are averaged, and then ratioed with the results for the ethanenitrogen system. These ratios may then be compared with the same ratios of the published experiments and calculated values, and give a comparison less dependent upon experimental error.

## B. PARAILEL TUBE PACKING

The first experiments were carried out on a bed packed with 1.2 mm diameter melting point tubes, thus providing a bed with unit tortuosity parallel to the tube bundle. The details of this bed are given in Table 2.I, while the diffusion results are summarized in Table 2.II and shown graphically in Figures 2.3, 2.4 and 2.5 as plots of the log peak heights vs. time. In some of the runs shown a Millipore Type HA Pilter ( $80 \%$ porosity) was placed over the bed of tubes to prevent eddy currents in the diffusion channels due to the flowing displacing gas. The results shown suggest that such currents are not significant.

An inspection of the tortuosities in Table 2.II shows that the results scatter over $a \pm 9 \%$ range. Murning the bed on its side so that gravity effects became influential increased the diffusivity by, 50\%. The reason for the scatter can be seen in the run with the hydrogen-nitrogen system at a flow rate of $0.563 \mathrm{ml} / \mathrm{sec}$. Three data points had to be ciscarded because the recorder automatic standardization operated and thus

TABLE 2.II
RESUL? FOR PARALLEL RUBE BED



Figure 2.3
Results :ith Parallil 'ube Bed. Hydrogn-ilitrog:n


Figure 2.4
Results With Parallel Tube Bed. Ethane-IVitrogen


Figure 2.5
Results With Parallnl Mube Bed. Butane-IVitrogen

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caused a shift in the peak height proportionality with concentration. Removal of these points caused the least square line slope to change from 0.00446 to 0.00434 with a resulting change in the $D_{3} / \lambda$ value from 1.008 to $0.873 \mathrm{~cm}^{2} / \mathrm{sec}$. The three discarded points are not in error by more than $4 \%$, yet in a total of 15 points these three cause a $3 \%$ variation in the siope, wheh in turn amees a i5y difference in tho aitiusivity.


## C. POROUS SOLID PACKING

The results of the runs using parallel tubes were initially calculated by hand from slopes obtained by graphical means. The sensitivity of the method to slight errors was not appreciated at the time, and errors were tolerated in the iterative calculation as well as those caused by the uncertainty of placing a straight line through a slightly curved set of points to obtain the slope.

Because it was originally felt that these errors could also be due in some measure to eday diffusion within the relatively coarse-pored tubular packing, additional experiments were carried out using fine porous solids as a diffusion medium.

A Selas 01 ceramic filter medium solid rod was fitted tightly into a 2.61 cm . diameter vessel thereby halving the former bed diameter, but the pore diameter was also reduced from 0.8 mm ( 800 microns) to 4.5 microns. The details of this bed are given in Table 2.I, and the results are summarizeủ in 'rable 2.III.

Because of the smaller diameter end zone, the flow pattern was changed from the former tangential inlet arrangement to one having flow in one direction across the chamber.

| RESULIR FOR POROUS SOLID PACKING Bea Temp. $306^{\circ} \mathrm{K}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Displacing Gas | Displaced Gas | $\begin{aligned} & \text { Plow } \\ & \text { Rate } \\ & \mathrm{cm}^{\mathrm{B}} / \mathrm{sec} \end{aligned}$ | Slope $_{\text {sec }}$ | $\begin{aligned} & \frac{D_{B}}{\lambda}=\frac{D E}{\epsilon_{B}} \\ & \mathrm{~cm}^{2} / \mathrm{sec} \end{aligned}$ | Molccular Diffusivity $\mathrm{cin}^{2} / \mathrm{sec}$ | $\lambda$ |
| Hydrogen | Nitrogen | 0.562 | 0.0139 | 0.565 | 0.82 | 1.45 |
|  |  | 0.793 | 0.0159 | 0.596 | 0.82 | 1.37 |
|  |  | 0.928 | 0.0171 | 0.533 | 0.82 | 1.54 |
|  |  | 1.25 | 0.0191 | 0.538 | 0.82 | 1.52 |
|  |  | 1.83 | 0.0245 | 0.651 | 0.82 | 1.26 |
|  | Average |  |  | 0.577 |  | 1.43 |
| Ethane | Nitrogen | ${ }^{\circ} 0.39$ | 0.00450 | 0.114 | 0.151 | 1.34 |
|  |  | 0.82 | 0.00490 | 0.108 | 0.151 | 1.40 |
|  |  | 1.32 | 0.00525 | 0.112 | 0.151 | 1.35 |
|  |  | 1.90 | 0.00523 | 0.108 | 0.151 | 1.39 |
|  | Average |  |  | 0.1105 |  | 1.37 |
| n Butane | Nitrogen | 0.594 | 0.00337 | 0.0747 | 0.099 | 1.32 |
|  |  | 1.14 | 0.00380 | 0.0802 | 0.099 | 1.23 |
|  |  | 2.06 | 0.00400 | 0.0823 | 0.099 | 1.20 |
|  | Average |  |  | . 0791 |  | 1.25 |

An examination of the results in Table 2.III. shows that the tortuosities are fainly consistent, with each gas system showing about a $\pm 5 \%$ scatter from the mean. However, the butane-nitrogen system tortuosities are lower than those obtained from the other gases, indicating that a true diffusivity value higher than that used would be appropriate. In order to avoid the "tail effect" mentioned earlier, nitrogen was made the displaced gas. The fact that butane is almost double the density of nitrogen would probably lead to gravity effects and could cause an apparent increase in the diffusivity. The difference between the average tortuosity or the hydrogen-nitrogen and ethane-nitrogen systems is not significant as it
depends upon the assumed value of the diffusivity. For example, if a value of $0.80 \mathrm{~cm}^{2} / \mathrm{sec}$ is assumed for the hydrogen-nitrogen diffusivity rather than $0.82 \mathrm{~cm}^{2} / \mathrm{sec}$, both systems give an average tortuosity of 1.37 to 1.38 .

The result at high flow rate for the hydrogen containing system is included in the averages. If this result is ignored it would appar that this system is showing about $5 \%$ lower diffusivity relative to the ethane system. The averase pore diameter of the Selas Bed is 4.5 microns, while the mean free path of hydrogen at NTP is 0.18 microns. It is unlikely that the pore size distribution is so narrow that some percentage of the pores are not smaller than, say, 1.8 , at which pore size the resultant of the mixed Knudsen and bulk diffusion rates could be $5: \%$ less than the bulk diffusion alone.

Thus, in spite of the fact that the results look fairly good, use: of the Selas 01 bed is questionable with high difiusivity gases at room temperature. Such a packing also suffers from the need to calibrate the bed to find the tortuosity before it can be used on gases of unknown diffusivity.

## D. SPIERICAL PACKING

The relationship of porosity to tortuosity has been published (6) for beds of spherical particles, and this provides an obvious means of overcoming the need to calibrate a porous solid type of packing to first determine its tortuosity. 'The bed vesscl was the same as that which hela the Delas 01, but it was packed with $42 \mu$ diameter glass spheres. However, the porosity obtained with the spherical packing was considerably less than For the porous soliu, and the resulting reduced bed capacity led to a decay curve that rapidly decreased below the rance of analysis by chromatography. The hydrogen-nitrogen results were most influenced by this effect.

## RESULTS FOR SPHERICAL PACKING

Bed Temp. $306^{\circ} \mathrm{K}$

| Displacing Gas | Displaced Gas | Flow <br> Rate $\mathrm{cm}^{3} / \mathrm{sec}$ | Slope | $\begin{aligned} & \frac{D_{B}}{\lambda}=\frac{D_{E}}{B} \\ & \mathrm{~cm}^{2} / \mathrm{sec} . \end{aligned}$ | $\begin{gathered} \text { Molecular } \\ \text { Diffusivity } \\ \mathrm{cm}^{2} / \mathrm{sec} \end{gathered}$ | $\lambda$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Hydrogen | Nitrogen | 0.456 | 0.0164 | 0.682 | 0.82 | 1.20 |
|  |  | 0.832 | 0.0218 | 0.687 | 0.82 | 1.19 |
|  |  | 1.25 | 0.0242 | 0.661 | 0.82 | 1.24 |
|  | Average |  |  | 0.677 |  |  |
| Ethane | Nitrogen | 0.604 | 0.00524 | 0.117 | 0.151 | 1.29 |
|  |  | 0.919 | 0.00521 | 0.112 | 0.151 | 1.35 |
|  |  | 1.36 | 0.00520 | 0.109 | 0.151 | 1.39 |
|  |  |  | age | 0.113 |  |  |
| Butane | Nitrogen | 0.596 | 0.00364 | 0.0795 | 0.099 | 1.25 |
|  |  | 0.979 | 0.00370 | 0.0784 | 0.099 | 1.26 |
|  |  | 1.24 | 0.00369 | 0.0775 | 0.099 | 1.28 |
|  |  |  | age | 0.0785 |  |  |

The graphical plots of the data in Figures 2.6, 2.7, and 2.8 shows a sharp change of slope at longer times. It is possible that the diffusion flux measured is the resultant of two decay processes, one due to the diffusion from the bed and the other due to diffusion from stagnant portions of the piping. This latter contribution would normally be negligible for a diffusion cell with a sufficiently large capacity. It may be noticed that for the hydrogen data with this bed (Appendix V), the least square computation has shown the decay curve to approach a value higher than the data for larger times. For this reason, the slope and hence the diffusivity (see Table 2.III) is higher for these runs, giving a lower tortuosity.


Figure 2.6
Results With Spherical Packing Bed. Eycrocon-intiocrn


Figure 2.7
Results With Spherical Packing Bed. Ethane-Nitrogen
The lower graph shows the above points after the steady state constant has been subtracted


Figure 2.8
Results With Spherical Packing Bcủ. Dutane-Nitrogen

| PUBLISHED DIFFUSIVITIES REF 40 |  |  |  |  |  | EXPERIMLINTAL RESULTS FROM 'LHIS WORK |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Calculated |  |  | Experimental |  |  | Melting D | Point Ratio | $\begin{aligned} & \text { Sele.s } \\ & \underline{\text { DB }} \end{aligned}$ | 01 Ratio | 42 Micron Spheres |  |
| $\mathrm{Temp}_{\circ}^{\mathrm{K}} .$ | Diff. | Ratio | Temp. ${ }^{\circ} \mathrm{K}$ | Diff. | Ratio |  |  |  |  | $\underline{\mathrm{DB}_{\mathrm{B}}}$ | Ratio |
| $\mathrm{H}_{2}-\mathrm{N}_{2}$ |  |  |  |  |  |  |  |  |  |  |  |
| 273.2 | 0.656 |  | 273.2 | 0.674 |  |  |  |  |  |  |  |
| 288.2 | 0.718 |  | 288.2 | 0.743 |  |  |  |  |  |  |  |
| 293.2 | 0.739 |  | 293.2 | 0.76 |  |  |  |  |  |  |  |
| 306* | 0.790 | 5.27 |  | 0.814 | 5.28 | 0.788 | 2.24 | 0.577 | 2.22 | 0.677 | 5.99 |
| $\begin{aligned} & \mathrm{C}_{2} \mathrm{H}_{6}- \\ & 298.2 \end{aligned}$ | 0.144 |  | 298.2 | 0.148 |  |  |  |  |  |  |  |
| 306* | 0.1498 | 1.0 |  | 0.154 | 1.0 | 0.1505 | 1.0 | 0.1105 | 1.0 | 0.113 | 1.0 |
| $\mathrm{nC}_{4} \mathrm{H}_{2}$ |  |  |  |  |  |  |  |  |  |  |  |
| 298.2 | 0.0986 |  | 298.2 | 0.0908 |  |  |  |  |  |  |  |
| 306* | 0.1025 | 0.685 |  | 0.0944 | 0.613 | 0.084 | 0.567 | 0.0791 | 0.716 | 0.0785 | 0.692 |

*Extrapolated Values

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Again the use of the heavier gas as the displacing gas in the butane-nitrogen experiments may have caused the diffusivity to be relatively somewhat higher than that of the ethane-nitrogen system, similar to the effect apparent in Table 2.III aiso.

## IV

## DISCUOGION

The overall potential error of the method cannot be estimated by the conventional methods due to the iterative nature of the solution. Nevertheless, the extreme sensitivity of the procedure to errors is indicated by the example in the "Results" section (for the hydrogen-nitrogen system in a bed of parallel tubes) where a $3 \%$ change in the slope causes a $15 \%$ change in the diffusivity. An understanding of the potential accuracy of the method may be aided by examining the first root of the auxillary equation,

$$
\tan \alpha L=\frac{h}{\alpha}-\frac{k}{\alpha}=\frac{Q}{A_{B} D_{E} \alpha}-\frac{l \alpha}{\epsilon_{B}}=\frac{Q \lambda}{A_{B} D_{B} \epsilon_{B} \alpha}-\frac{l \alpha}{\epsilon_{B}}
$$

If the right hand side (RHS) of the equation is large, then $\alpha I$ approaches $\pi / 2$ and $\alpha$ becomes independent of the flow rate $(Q)$, bed porosity $\left(\epsilon_{B}\right)$, bed area ( $A_{B}$ ), end zone length ( $\ell$ ) and gas diffusivity ( $D_{B}$ ). Thus in order to reduce the present $10 \%$ scatter of the experiments, it would appear to be necessary to achieve a large value of $\alpha \mathrm{L}$, that is to increase $h / \alpha$, and minimize $k \alpha$.

Both $h$ and $k$ are inversely proportional to the porosity, so if $h / \alpha \gg k \alpha$, a porosity decrease will increase the RHS, however, the reduction of bed capacity which results, decreases the time available for
analysis of efiluent concentrations. It is noticeable that the results from the low porosity 42 micron bed are less scattered.

The term $h$ increases as the bed area decreases, but experimentally, reduction of the area has the same limitations as a decrease of the porosity, except that the benefits do not depend upon the relative magnitude of $h$ and $k$.

The gas diffusivity has the same influence as the bed area, and so high diffusivity gases are most susceptible to error. The parallel tube bed with the hydrogen-nitrogen system would be expected to have most scatter of the experimental results. Unfortunately, there are not enough data points to carry out any form of statistical comparison.

High flow rates of the displacing gas increase the term $Q$ and therefore $h$, but once again experimental factors will restrict the maximum flow rate because of the turbulence, which can enter the bed packing to some extent, thereby increasing the effective diffusivity and making the result flow dependent. Coupled to this is the effect of pressure gradients from Priction losses, or changes in kinetic energy at the entry port, which could cause bulk flow in the bed. Even the parallel tube bed is susceptible to bulk flows as the tubes are not sealed at the blank end.

An increase in the end zone lensth will have the deleterious effect of increasing $k$ and hence decreasing the RHJ. In the experimental apparatus used in this work, $k$ was negligible, so that the end zone depth could probably be doubled without too much influence on the magnitude of $\alpha$. This depth increase might assist in minimizing another potential source of error, in that the solution to the differential equation assumes perfect mixing in the end zone. The use of a deeper end zone would allow larger
scale eddies to increase the mixing, but at the same time the larger eddies should not be able to penetrate too far into the bed. The millipore filter used to discourage eddy penetration does not show any influence on the results, but this is probably to be expected because the added resistance would not amount to more than $0.3 \%$ of the total while the results scatter to $\pm 10 \%$. The millipore filter may help to reduce the penetration of edides into the bed but it would not be expected to stop the bulk flow effects discussed earlier.

Finally, the length of the bed, $L$, may be increased to make $\alpha$ small and hence $h / d$ large. On first inspection this is an obvious improvenent, however, there are limitations. The dead time, before the second term of the series may be dropped, is increased four fold by doubling the bed length. In the case of the $0.1 \mathrm{~cm}^{2} / \mathrm{sec}$ diffusivity gas the dead time was found to be 232 secs for a 10 cm bed (see introduction). In a 20 cm bed a 15 min dead time would be required.

At the same time the effluent gas concentration must be considered. From equation 2.7, the effluent concentration would change only linearly, with bed length. Thus, at the time when the second term represents $1 \%$ of the first, the 20 cm bed after 232 secs would have doubled the concentration of the 20 cm bed after 928 secs. The longer bed thus has the effect of lengthening the time scale, and would allow more gas chromatograph analysis to be carried out before the samples are too dilute, but at the same time would start from a lower concentration.

The use of the three constant equation to fit the curved data would not appear to be responsible for the variations in the results because the data for the butane-nitrogen system as shown in Figure 2.5 for
the tubular bed are not curved, yet the dirfusivities calculated are badly scattered. It may be noticed, hovever, that at a flow rate of $0.903 \mathrm{mls} /$ sec the points at 200 and 300 seconds in Figure 2.5 deviate slightly from the other points, and since the least square equation favours the lower times the scatter may be caused by the small deviations of the first two points.

Nitrogen decay was followed in most of the runs, and so the trace of air in the gas systems appeared as nitrogen, resulting in a curve $Y=A e^{-B t}+C$ instead of $Y=A e^{-B t}$ (where $Y$ and $t$ are the variables), which can be plotted as a straight line. The use of high purity gases might simplify the interpretation of results.

In summary, there are at least three major sources of error which may influence the diffusivity obtained by this nethod: (a) eddies from the end zone penetrating the bed to increase the diffusivity (b) bulk flow in the bed caused by pressure gradients in the end zone, which also act to increase the diffusivity and (c) poor mixing in the end zone causing a lowered diffusivity.

There is some indication of the presence of the last of these errors in the large diameter parallel tube bed (see Table 2.II). Examination of the data in Tables 2.II, 2.III and 2.IV shows that for the parallel tube bed (Table 2.II) there was no significant increase in diffusivity with increasing gas flow rate. The other beds used, which were more isotropic in structure, do tend to show such an increase with flow rate. As the range of flow rates used in all beds was comparable, there appears to be a slight effect of the first two sources of error mentioned in all but the parallel tube bed. The results for this latter bed (see Table 2.II) also indicate that there may be some evidence for poor mixing in the end zone.

## CONCLUSION

The method as used in the present apparatus is satisfectory for measuring gas molecular diffusivities for binary systems within plus or minus 10\%. Analysis of sources of error suggest that by redesigning the apparatus a probable accuracy of $21 / 2 \%$ could be readily achieved.

## VIII

## RECOMMETDATIONS

On the basis of this work, it is apparent that a bed of the following dimensions could minimize the potential sources of error encountered in the present experiments.

Parallel tube packing 1 mm or less $O D$.
Length 20 to 30 cms .
Diameter 5 cms .
End Zone length $0.25-0.5 \mathrm{cms}$.
The sealing off of the tubes and prevention of buik flows through the bed is also advisable. It would be advantageous to be able to invert the bed and also much time could be saved if the displaced gas could be purged through the bed, particularly if the dead time is increased to 15 mins. or half an hour by the larger bed.

Further Study
The advantages of the larger bed should be experimentally verified and the magnitude of the flow effects, like buik flow and turbulence, should be investigated if the larger beds are used to reduce the scatter. The effect of the end zone length on the mixing should also be investigated.

HOI WICLAPDRU
$A, B$ and $C$, Constants in least square cquation.
$A_{B} \quad$ Area of bed, $\mathrm{cm}^{2}$
Ap Specific surfacc area/unit, volune of bed, $\mathrm{cm}^{-1}$
$A_{s}$ Jample or pulse volumc, mls.
$\mathrm{F}_{2} \quad$ Area fraction of mobile phase $=\epsilon_{B}$
$\mathrm{F}_{2} \quad$ Area fraction of stationary phase $=\left(1-\epsilon_{\mathrm{B}_{1}}\right)$
H HETP, cms.
HETP
K
Constants
I Length of bed, cms.
$I_{A} \quad$ Nolar flux, moles/sec of component A per $\mathrm{cm}^{2}$
it $_{A}^{l}$ holar flux per unit geomotrical aroa, moles $/\left(\mathrm{cmin}^{2}\right)^{2}$ sec.

P

Q
dip Column diameter, cmis.
$f$ Franing rriction factor.
$h_{D}$ Hyüraulie diameter, cms.
$h \quad$ Thiele modulus or in section II $h=Q / A_{B} D_{E}$
$\ell \quad$ End zonc-lengtn, cons.
n fumber of theoretical plates, or number of term in aerion folution. Pressure, atm.

Sotal flux, moles/sec., or gas flow rate, mls./isce. or pellet volune in Appendix IV

Gas consant or radius dinension
Feraperabure, ${ }^{\circ} \mathrm{K}$
Volume of gas, mis.
Volum or gas phase in theorctical plate, $\mathrm{cm}^{3}$
Volune of theoreical plase cms ${ }^{3}$
Adsorpiion or partition coefricients.
Corficient of exponential.
Pellet diameter, cans.
$I+\mathrm{N}_{\mathrm{A}} / \mathrm{N}_{\mathrm{B}}$
First order rate constant, sec. ${ }^{-1}$
Mass tronsfer coefficzont in robile phase, cm/sec.
Vess transfer coerficient in stationary phase, cm/sec.

Pore radius, or radius variable in afferential equation, cas.
Time, seconds
Interstitial velocity, cra/sec.
Volume of liquid sidw of theorerical plete, $\mathrm{cm}^{3}$
Average velocity of a gas molecule, cms/sec.
x
Distance in direction of flux or flow, ems.

Mole fraction or peak height.
Mass transfer cocfficient, sec. ${ }^{-1}$ (Jection 1)
$\alpha_{n}$. Consecutive roots of equation (2.11) (ibection2)
$\Delta$
$\epsilon_{\mathrm{p}}$
$\epsilon_{B}$
$\rho_{m}$
$\lambda$
P. Density, Erams/ml.

Viscosity, cps
Eddy diffusivity coefficient.
Tortuosity
Standard deviation.
Error in equality of equation.
Pellet porosity
Bed porosity.
Nolar density, moles/ml.

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DETERMINATION OF TEE EPFECRIVE GAS DIFFUSIVITY IN A POROUS SPHERICAL PELLET BY A STEADY ITATS METHOD

## INTRODUCTION

In order to evaluate the results obtained by the pulse technique an apparatus was constructed to measure the effective gas difiusivity in the test materials by a well established procedure. The steady state method described by Weist (13) was selected. THEORY

Different diffusion resimes, Knudsen and bulk, were anticipated in the two samples which were examined and so two solutions are needed For the dilfusion equation.

## Knudsen Dirfusion

The molar flux $N_{A}$ is given in terms of the Effective Diffusivity $D_{\mathrm{E}}$ and the concentration gradient by Fick's first law,

$$
N_{A}=-D_{E} \frac{d}{d x} \text { at any plane } x \text {, moles } / \mathrm{sec} \mathrm{~cm}^{2}
$$

Referring to Figure A I.I,
The total flux $Q$ is given by $Q_{A}=N_{A}$ (Area of plane) $=N_{A} \Pi\left(R^{2}-x^{2}\right)$ moles $/ \mathrm{sec}$

$$
\begin{gathered}
Q_{A}=D_{E} \pi\left(R^{2}-x^{2}\right) \frac{d C_{A}}{d x} \\
\int \frac{d x}{R 2 x^{2}}=-\int \frac{D_{T} \pi}{Q_{A}} d C_{A} \\
\frac{1}{2 R}\left[\operatorname{Ln} \frac{R+x}{R-x}\right]_{-t}^{+t}=-\frac{D_{E} \pi}{Q_{A}}\left[C_{A}\right]_{1}^{2} \\
-D_{E}=\frac{Q_{A}}{2 \pi R} \frac{1}{C_{A_{2}}-C_{A_{1}}} \operatorname{Ln}\left[\frac{R+t}{R-t}\right]^{2}
\end{gathered}
$$

Since $C_{A}=\rho_{m} y_{A}$ where $y$ is the mole fraction and $\rho_{m}$ the molar density

$$
D_{E}=\frac{Q_{A}}{2 \pi R \rho_{m}} \operatorname{Ln}\left[\frac{R+t}{R-t}\right]^{2} \quad \frac{1}{y_{A_{1}}-y A_{2}} \quad \mathrm{~cm} 2 / \mathrm{sec}
$$

BRASS BLOCK


Figure A I.I
Sample Mounting In steady State Apparatus

Fick's lav is again applied but with a correction for the bulk flow caused by non-equimolar counter diffusion.

$$
\begin{aligned}
& N_{A}=-D_{E} \frac{d C_{A}}{d x}+\left(N_{A}+N_{B}\right) y_{A} \text { moles } / \mathrm{sec} \mathrm{~cm}^{2} \\
& Q_{A}=N_{A}(\text { Area })=-D_{E} \pi\left(R^{2}-x^{2}\right) \frac{d C_{A}}{d x}+\left(Q_{A}+Q_{B}\right) y_{A} \\
& D_{E} \Pi\left(R^{2}-x^{2}\right) \rho_{m} \frac{d y_{A}}{\overline{d x}}=-Q_{A}+Q_{A} y_{A}+Q_{B} y_{A}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{\left(1+\frac{Q B}{Q_{A}}\right)}\left[\operatorname{Ln}\left[y_{A}\left(1+\frac{Q B}{Q A}\right)-1\right]\right]_{y_{A_{2}}}^{y_{A_{2}}}=\frac{Q A}{D+\Pi \rho_{m}}\left[\frac{1}{2 R} \operatorname{Ln}\left(\frac{R+x}{R-x}\right)\right]_{-t}^{+t} \\
& D_{B}=\frac{Q_{A}}{2 \pi R \rho_{m}}\left(1+\frac{Q_{B}}{Q_{A}}\right) \frac{\operatorname{Ln}^{2}\left(\frac{R+t}{R-t}\right)^{2}}{\operatorname{In}\left[\frac{Y_{A_{2}}\left(1+\frac{Q_{B}}{Q_{A}}\right)^{-1}}{Y_{A_{1}}\left(1+\frac{Q_{B}}{Q_{A}}\right)^{-1}}\right]} \quad \mathrm{cm}{ }^{2} / \mathrm{sec}
\end{aligned}
$$

## APPARAITUS

The apparatus shown in Figure A I. 2 was assembled around a pair of brass blocks between which a third block (shown in Figure A I.I) containing the sample pellet was bolted. The brass blocks were constmucted in such a way that two different gases could be flushed through mirror image passages across the two faces of the sample. Streams of hydrogen and nitrogen from their respective cylinders flowed through their respective cylinder pressure regulators and "Moore" constant differential flow controls to the reference sides of a pair of "Gow Mac" NIS model 9220 thermal conductivity cells. From the reference cells the gases could be


Figure A I. 2
Steady State Apparatus
diverted either across the sample faces, or via a by-pass to the measuring side of the cells, and from here the gases were vented to atmosphere through a needle valve and soap bubble flow meter.

Manometer taps in the sample blocks were located opposite the centre of the sample faces and were connected to an inclined oil filled manometer. Polyethylene $1 / 4^{\prime \prime}$ diameter tubing was used for connecting the apparatus, and this allowed flushing of dead end lines by loosening of the fittings.

The test samples were mounted by bathing them in epoxy resin, and then fitting them into the brass block which was drilled with a clearance hole. After the resin had set the faces of the pellet were ground by means of sand paper on a glass plate.

Two "DORION" potentiometers were used to measure the output of the cells which formed part of a conventional bridge circuit. PROCEDURE

Calibration of Thermal Conductivity Cells
These "diffusion-type" cells have the property of being relatively independent of flow rate, and at low concentrations a linear output with concentration can be assumed. In order to calibrate the nitrogen cell a fairly high flow was set through the cell and sample block by-pass. The nitrogen flow rate was measured with the bubble meter and the cell zero adjusted electrically. A flow of hydrogen was set through its system anã measured on the appropriate bubble meter. The polythene tube from the hydrogen was then disconnected and reconnected into a point on the by-pass of the nitrogen system so that the hydrogen now appeared in the measuring
side of the nitrogen cell. The system was allowed to come to equilibrium and the output measured on the potentiometer. The concentration was calculated from the flow rates of the two gases.

Operation
The two gas flows were set to convenient levels and measured While passing through the sample by-pass system. The outputs of the two detectors were set to zero, and then the flows were diverted to pass across the sample faces. The manometer legs were bled and the outlet measuring valves were adjusted to be at maximum opening but maintaining zero pressure difference across the pellet. The system was allowed to come to equilibrium, and then detector outputs were taken at convenient intervals over a period of twenty minutes. The gas streams were set back on the by-pass and the zero drift of the detectors in the course of the experiment recorded along with the flow rates of the gases.

## RESULTS

## Calibration of thermal conductivity detectors

Nitrogen content in hydrogen cell
Nitrogen flow: 25 mls in $72.0,72.2$ seconds $=0.346 \mathrm{mls} / \mathrm{sec}$.
Hydrogen flow: 50 mls in $8.2,8.2,8.6$ seconds $=6.1 \mathrm{mls} / \mathrm{sec}$.
Mole $\%$ nitrogen $=\frac{0.346}{6.1+0.346} \times 100=5.37 \%$
Output of detector 9.56 millivolts or $1.78 \mathrm{mv} / 1 \%$ nitrogen
Hydrogen content in nitrogen cell
Nitrogen flow: 50 mls in $7.5,7.5$ seconds $\quad=6.66 \mathrm{mls} / \mathrm{sec}$.
Hydrogen flow: 25 mls in $68.0,68.5$ seconds $=0.367 \mathrm{mls} / \mathrm{sec}$. Mole $\%$ hydrogen $=\frac{0.367}{6.66+0.367} \times 100=5.22 \%$

Output of detector $11.205 \times 5$ millivolts or $10.72 \mathrm{mv} / 1 \%$ hydrogen

It is of interest to compare the above result with the calibration of $\operatorname{Cox}$ (41)who obtained several points with a similar apparatus and verified the linearity of the response. He obtained a slope of $10.85 \mathrm{mv} / 1 \%$ hydrogen.

Activated Alumina Pellet 1/4" Diameter

## Pellet Characteristics

"Alcoa H15l Activated alumina sphere" having 42 A mean pore diameter.

Diameter of pellet used in test $=0.255^{\prime \prime}, 0.262^{\prime \prime}, 0.262^{\prime \prime}$

$$
\text { Average Dia. } \quad=0.66 \mathrm{cms}
$$

Thickness of mounting plate, i.e. across flats of pellet $=3 / 16^{\prime \prime}=$

$$
0.476 \mathrm{cms}
$$

The mean free path of hydrogen at $0^{\circ} \mathrm{C}$ and 1 atmosphere $=180 \times 10^{-7}$ cms. (Ref. 42 ) $=1800$ A versus 42 A pore size hence Knudsen diffusion will be the predominant mechanism.

The amount of nitrogen which diffused into the hydrogen stream in this experiment was so small that with the lower sensitivity of this detector the output was of the same order as the zero drift during the course of the experiment. For this reason the diffusivity is calculated from the hydrogen flux,

Hydrogen flow rate: 50 mls in $20.5,20.5 \mathrm{sec}$. before test

$$
22.0,22.2 \mathrm{sec} . \text { after test }
$$

Nitrogen flow rate: 50 mls in $18.8,18.8 \mathrm{sec}$. before test

$$
19.0,19.0 \mathrm{sec} . \text { after test }
$$

Room temperature $26^{\circ} \mathrm{C}$
Atmospheric Pressure 755.6 mm Hg .

Analysis of Streams
Hyarogen in nitrogen miv: 1.47, 1.44, 1.42, 1.11, 1.405, 1.405 , 1.405 zero drift ada 0.17 mv yielding 1.575 mv .

Nitrogen in hydrogen mv: 0.07, 0.07, 0.07, 0.095, 0.07, 0.08, 0.08 zero drift add 0.045 mv yielding 0.12 mv

Subscript A refers to hydrogen

$$
\begin{aligned}
& Q_{A}=\frac{50}{19.0} \times \frac{1.575 \rho_{m}}{10.85 \times 100}=.00382 \rho_{\mathrm{m}} \mathrm{moles} / \mathrm{sec} \\
& y_{A_{1}}=1.0-\frac{.125 x .01}{1.78}=0.9993 \text { mole fraction } \\
& \mathrm{yA}_{2}=\frac{1.575}{10.72}=0.147 \%=.00147 \text { mole Iraction } \\
& D_{K}=\frac{Q \Delta}{2 \pi R P_{m}} \quad \ln \binom{2 R+2 t}{2 R-2 t}^{2} \quad \frac{1}{y_{1}-y_{2}} \\
& =\frac{.00382 \rho_{m}}{2\left(\frac{.66}{2}\right) \rho_{m}} \quad \operatorname{In} \cdot\left[\frac{.66+.476}{.66-.476}\right)^{2}-\frac{1}{0.9993-.00147} \\
& =0.0067 \mathrm{~cm}^{2} / \mathrm{sec}
\end{aligned}
$$

Knudsen diffusivity of hydrogen in pellet $=0.0067 \mathrm{~cm}^{2} / \mathrm{sec}$ at $26^{\circ} \mathrm{C}$ "Norton" Catalyst Support 1/2" Diameter (Alundum)

Pellet Characteristics
Maximum diameter of pellet $=0.55^{\prime \prime}$
Minimum diameter of pellet $=0.525^{\prime \prime}$
. Mean diameter $=0.538^{\prime \prime}$ or 1.365 cms .
Thickness of samples plate 0.90 cms .
Pore diameter $90 \%$ in range 2 to 40 microns. Hydrogen has a mean free path around 0.18 microns (Rep. 42) so that bulk diffusion will be the predominant mechanism.

Nitrogen flow rate 50 mls in $28.0,27.0,27.5$ seconds before test 28.1, 28.0 seconds after test

Hydrogen flow rate 50 mls in $27.2,27.1$ seconds before test $29.8,30.0$ seconds after test

Room temperature $23^{\circ} \mathrm{C}$
Atmospheric pressure 760.7 mm Hg

## Analysis of streams

Hydrogen in nitrogen millivolts

Nitrogen in hydrogen millivolts

| $17.495 \times 5$ | $17.44 \times 5$ | 3.91 | 4.09 |
| :--- | :--- | :--- | :--- |
| 17.485 | 17.46 | 3.95 | 4.10 |
| 17.491 | 17.45 | 3.97 | 4.10 |
| 17.46 | 17.44 | 4.00 | 4.10 |
| 17.47 | 17.37 | 4.02 | 4.13 |
| 17.45 | 17.35 | 4.04 | 4.15 |
| 17.43 | 17.39 | 4.07 |  |
| 17.39 | 17.46 | 4.10 |  |
| Average = 17.421 mv |  | 4.10 | add 0.22 |
| Zero drift: add 0.0 |  | $4.32 / 1.78=2.425 \%$ |  |
| $17.421 \times 5 / 10.72=8.11 \%$ |  |  |  |

Subscript A refers to hydrogen

$$
\mathrm{y}_{\mathrm{A}_{2}}=0.081 \quad \mathrm{y}_{\mathrm{A}_{I}}=0.97575
$$

$$
\begin{aligned}
& Q_{A}=\frac{50}{28.05} \times .0811 \rho_{\mathrm{m}}=0.1445 \rho_{\mathrm{m}} \text { moles } / \mathrm{sec} \\
& Q_{B}=\frac{50}{29.9} \times .02425 \rho_{\mathrm{m}}=0.0405 \rho_{\mathrm{m}} \mathrm{moles} / \mathrm{sec}
\end{aligned}
$$

D. Effective $=\frac{Q_{A}}{2 \pi R \rho_{m}}\left(1+\frac{Q B}{Q A}\right)^{\prime}$


$$
=\frac{0.1445 \rho_{\mathrm{m}}}{2 \pi \frac{1.365}{2} \rho_{\mathrm{m}}}\left(1-\frac{0.04405}{0.1445}\right) \quad \frac{\operatorname{In}\left(\frac{1.365+0.90}{1.365-0.90}\right)^{2}}{\operatorname{In}\left(\frac{0.0811(.7195)-1}{0.97575(.7191)-1}\right)}
$$

$$
=0.0667 \mathrm{~cm}^{2} / \mathrm{sec}
$$

Effective buik aiffusivity of hydrogen and nitrogen in the $1 / 2^{\prime \prime}$ Norton catalyst supports was found to be $0.0667 \mathrm{~cm}^{2} / \mathrm{sec}$ at $23^{\circ} \mathrm{C}$ and 760.7 mm Hg pressure.

Bcott and Dullien (5) pointed out that the ratio of fluxes of two gases diffusing at constant pressure in capillaries should be inversely proportional to the ratio of the square root of their molecular weights. In this experiment a ratio of 3.57 was obtained as compared with a value of 3.74 for the square root of the molecular weights. The difference is probably caused by the difficulty in keeping the pressures identical across the pellet.

No absolute pressure measurements were taken in the test cell and so the actual pressure of the measurement may be expected to be slightly higher than the ambient atmospheric pressure. However, care was taken to operate with the valves wide open except to balance the different pressure drops caused by the difference in viscosity of the gases. Results on other equipment at similar flow rates indicate that a $1 / 4$ " tube at flow rates such as used here, the pressure drop is not measurable on a mercury manometer.

## CONCLUSION

The diffusion coefficient for the Knudsen diffusion of hydrogen in a $1 / 4^{\prime \prime}$ dia. Alcoa H 151 activated alumina spheres was found to be $0.0067 \mathrm{~cm}^{2} / \mathrm{sec}$ at $26^{\circ} \mathrm{C}$. The moisture content of the pellet is taken to be $12 \%$ by wt from analysis of similar pellets, however, the actual moisture of the test pellet during the test was not obtainable.

The diffusion coefficient for the bulk diffusion of hydrogen and nitrogen in $1 / 2^{\prime \prime}$ dia. Norton SA 203 Alundum catalyst carrier spheres was found to be $0.0667 \mathrm{~cm}^{2} / \mathrm{sec}$ at $23^{\circ} \mathrm{C}$ and 760.7 mm Hg . No moisture adsorption was found in these pellets.
05/2p/es

05/27/0s

$$
\text { Pase } z
$$

$\approx=$








olffusivity
PELLET
Porosity
0.102000000
$\begin{array}{cl}\text { MYO OIA } & \text { OENSITY } \\ 0.0824221 & 0.0011940\end{array}$
它
 $31 \mathrm{cma}_{0,04}$

2.6100 0.2200
IEMP KELVIN PRESS ATM VISCOSITY
를물


## -137-








[^1]








## 


 $A A=0.5390088 \quad C C=0.0297571$

GAMMA 0.571649

| RUN | Lencim | COLUMN | PELLET | 8ED | PELLET | diffusivity |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | LENGIM CMS | diameter | OIAMETER | POROSITY | POROSITV |  |
| 11 | 122.0000 | 1.150 | 1.0050 | 0.4540000 | 0. | 0.72521957 |


| CARRIER MH | TEMP KELVIN PRESS ATM VISEDSITY | HYO DIA DENSITY |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 29.00000 | 290.00000 | 0.99400 | 0.0179282 | 0.2695177 | 0.0012114 |




IIME LGHRS GIMIM 12.6SEC




| $\begin{aligned} & \text { COLUMN } \\ & \text { NO } \end{aligned}$ | COLUMN LENGTH CHS | $\begin{aligned} & \text { COLUHN } \\ & \text { OIAMETER } \\ & \text { CMS } \end{aligned}$ | PElLET DIAMETER CMS | $\begin{gathered} \text { BED } \\ \text { POROSITY } \end{gathered}$ | $\begin{aligned} & \text { PELLET } \\ & \text { POROSITY } \end{aligned}$ | DIFFUSIVITY |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 56 | 129.6000 | 0.6600 | 0.5970 | 0.4710000 | 0.3100000 | 0.207605682 |


| CARRIER MH TEMP KELVIN PRESS ATM VISCOSITY | HYD DIA | DENSITY |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 29.00000 | 296.00000 | 1.02800 | 0.0182169 | 0.1655945 | 0.0012275 |


| VELOCITY | METP | RECIP VEL SECICM | RE | NTU | WIDTH |  | ef/SEe |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.95941 | 0.58797 | 1.04231 | 3.85938 | 299 | $2.7500^{\text {Cm }}$ | 17.3000 | 0.1600 |  |
| 1.31919 | 1.00155 | 0.75804 | 5.30665 | 411 | 2.3900 | 11.5200 | 0.2200 |  |
| 2.27860 | 2.14265 | 0.43887 | 9.16602 | 111 | 1.7600 | 5.8000 | 0.3800 |  |
| 2.87823 | 2.85963 | 0.34744 | 11.57814 | 898 | 1.5600 | 4.4500 | 0.4800 |  |
| 3.05812 | 3.46638 | 0.32700 | 12.30177 | 454 | 7. 7000 | 19.9500 | 0.5100 |  |
| 4.79705 | 5.35024 | 0.20846 | 19.29689 | 1497 | 5.8500 | 12.2000 | 0.8000 |  |
| 5.75646 | 7.57958 | 0.17372 | 23.15627 | 1796 | 1.1700 | 2.0500 | 0.9600 |  |
| 6.17620 | 7.30867 | 0.16191 | 24.84475 | 1927 | 5.1000 | 9.1000 | 1.0300 | - |
| 7.37546 | 9.87107 | 0.13558 | 29.66898 | 2302 | 4.9500 | 7.6000 | 1.2300 | $\checkmark$ |
| 7.97509 | 10.41804 | 0.12539 | 32.08109 | 2489 | 4.5500 | 6.8000 | 1.3300 |  |
| 8.63469 | 12.80894 | 0.11581 | 34.73441 | 2695 | 4.6000 | 6.2000 | 1.4400 |  |
| 6.75461 | 11.75458 | 0.11423 | 35.21643 | 2732 | 4.3000 | 6.0500 | 1.4600 |  |
| 9.59410 | 12.69980 | 0.10423 | 38.59379 | 2994 | 4.1000 | 5.5500 | 1.6000 |  |
| 10.31365 | 12.82180 | 0.09696 | 41.48832 | 3219 | 3.8600 | 5.2000 | 1.7200 |  |
| 11.99262 | 14.12322 | 0.08338 | 48.24224 | 3743 | 3.3500 | 4.3000 | 2.0000 |  |
| 4 |  | B |  | c |  | SIGMA | $N$ |  |
|  | 26101748 | -0.63 | 14265 | 1.32 | 2167980 | 0.6 | 388615 |  |

EFFECTIVE DIFFUSIVITY $=0.00085020$
CORRECTION OF SLUPE FOR DIFFUSIVITY TERM = 0.0605318 AND NEM SLOPE C $=1.26115$



| $\begin{aligned} & \text { COLUMN } \\ & \text { NO } \end{aligned}$ | COLUMN EENGTH CHS | COLUMN DIAMETER CMS | PELLET DIAMETER CMS | $\begin{aligned} & \text { BED } \\ & \text { POROSITY } \end{aligned}$ | $\begin{aligned} & \text { PELLET } \\ & \text { POROSITY } \end{aligned}$ | DIFFUSIVITY |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 59 | 421.0000 | 0.6600 | 0.5970 | 0.4710000 | 0.5000000 | 0.211745851 |


| CARRIER MH | TEMP KLLVIN PRESS ATM | VISCOSITY | HYD DIA | DENSITY |  |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 29.00000 | 296.00000 | 1.00790 | 0.0182169 | 0.1655945 | 0.0012035 |






EFFECIIVE DIFFUSIVITY $=0.00561640$
CORRECTION OF SLOPE FOR DIFFUSIVITY TERM $=0.0192997$ AND NEN SLOPE $C=0.21859$


EFFECTIVE DIFFUSIVITY $=0.00447061$
CORRECTION OF SLOPE FOR DIFFUSIVITY TERM $=0.0166358$ AND NEW SLOPE $C=0.07911$
TIME ITHRS 2IMIN S5.4SEC

## Manometers Inches Oil

Low Flow (LF)

| 1.8 | 0.338 |
| :--- | :--- |
| 3.65 | 0.674 |
| 7.9 | 1.416 |
| 10.7 | 1.87 |
| 17.5 | 2.82 |
| 23.7 | 3.65 |
| 26.1 | 3.96 |

High Flow (HF)

| .91 | .811 |
| ---: | ---: |
| 1.5 | 1.31 |
| 2.8 | 2.37 |
| 4.1 | 3.21 |
| 5.1 | 3.91 |
| 7.7 | 5.88 |
| 7.0 | 5.38 |
| 10.2 | 6.90 |
| 15.1 | 9.46 |
| 8.1 | 5.85 |
| 14.4 | 9.00 |
| 18.1 | 10.50 |
| 21.1 | 21.70 |
| 24.9 | 12.68 |
| 24.0 | 13.45 |

Very High Flow (VHF)

| 1.65 | 12.9 |
| :--- | :--- |
| 2.35 | 15.9 |
| 5.3 | 26.3 |
| 8.7 | 33.9 |
| 14.4 | 44.0 |
| 20.05 | 52.0 |
| 26.1 | 59.5 |

TABLE A II.I (Continued)

26.75 323
$17.6 \quad 278$
11.85 180
6.35 152
3.77
.95
73.6
45.3


Figure A II. 1
Flow Meter Calibration

## APPENDIX III

TIME OF DIFFUSION OP A GAS FROM A BPHERICAL PELLET WITA A STEP CHANGE IN SURFACE CONCENTRATION

## RATE OF DIFFUSION FROM A SPHERICAL PRLLET

The diffusion of a gas from a spherical pellet with a step change in the surface concentration is given by Crank (Ref. 2, page 86).

The amount of gas diffused at time $t,\left(M_{t}\right)$ as compared to the amount of gas diffused at infinite time ( $M_{\infty}$ ) is given by,

$$
\frac{M_{t}}{M_{\infty}}=1=\frac{6}{\pi^{2}} \sum_{n=1}^{\infty} \frac{1}{n} \exp \left(-\frac{\operatorname{DEn}^{2} \pi^{2} t}{a^{2}}\right)
$$

where a is the pellet radius.
For the second term in the series to be less than $1 \%$ of the first,

$$
\begin{aligned}
& \quad \exp \left(\frac{-D_{E} \Pi^{2}{ }_{t}}{a^{2}}\right)=\frac{100}{4} \exp \left(-\frac{4 D E \pi^{2 t}}{a^{2}}\right) \\
& \operatorname{Ln}(25)=\frac{4 \Pi^{2} D_{E} t}{a^{2}}-\frac{\Pi^{2} D_{E t}}{a^{2}}=\frac{3 \Pi^{2} D E t}{a^{2}}
\end{aligned}
$$

If $D_{E}=0.01 \mathrm{~cm}^{2} / \mathrm{sec}$ and $a=0.5 \mathrm{cms}$

$$
t=\frac{.25 \operatorname{In}(25)}{3 \pi^{2} \times .01} \quad=2.82 \text { seconds }
$$

Thus after ten seconds

$$
\begin{aligned}
\frac{M_{t}}{M_{\infty}}=1-\frac{6}{\pi^{2}} \text { e }\left(-\frac{.01-2 \times 10}{.25}\right) & =.9881 \\
& =98.81 \%
\end{aligned}
$$

Thus in ten seconds $98.8 \%$ of the gas will diffuse out of a 1 cm diameter pellet if the effective gas diffusivity is $0.01 \mathrm{~cm}^{2} / \mathrm{sec}$ MANUFACTURERS DATA ON POROUS PELLETS (INCLUDING CONTRADICTIONS AND VERIFICA:CIONS) Norton Catalyst Supports, 1/2" diameter SA 203 mixture

Information from two sources is summarized below, the first from the manufacturer's general information sheet, and the second from data supplied by the manufacturer in a private communication.

- 185 -

Apparent Porosity
Water absorption
Buik density
Apparent Specific Gravity Packing Density Burface Area Pore Diameter Range

Manufacturer's Data
Private Commication

$$
\begin{aligned}
& 0.41 \\
& 20 \% \\
& 2.1 \text { grams } / \mathrm{ml} \\
& 75-78 \mathrm{lb} / \mathrm{rt}^{3} \\
& \text { less than } 1 \text { neter } 2 / \text { gram }
\end{aligned}
$$

$90 \%$ in $2-40$ microns

Using the bulk density value of 2.1 and the specific sravity of 3.5 , a porosity of 0.382 can be calculated. As a further check on the consistency of the data, if the water adsorbed is assumed to exist as liquid water occupying the pores, then $17 \%$ water indicates a porosity of $0.37 \%$. In an experimental check, a pellet was dried by heating to $500^{\circ} \mathrm{F}$, and then put in a vacuum which was released by vater so that the pellet absorbed as much water as possible. Weighing the pellet vetween each operation showed that the initial water content was negligible, but the evacuation and saturation procedure yielded a water content of $16.8 \%$ which again is an indication of a $37 \%$ porosity if the pellet specific gravity of 3.5 is acceptea.

In conclusion, a value of $38 \%$ porosity has been taken for the pulse apparatus experiments and the information available suggests an error limit of $\pm 1 \%$.

Activated Alurnina Pellets Alcoa H 151 1/4" and 1/8" djaneter
Thro sources of information vere azain used to determine the properties of these pellets, but some of these data were contradictory. Because one of these sources was private communcation in the form of a letter from the supplied, which differed from the manuiactureris data it was concluded that the supplier had not furnished the correct data. This conclusion is justified in the following paragraphs.

- 186 -
ivanufacturer's Data Private Communication

Packing Density
Specific Gravity
Pore Volume
Pore Diameter
Surface Area (BET)
Pore Volume greater than $30 A^{\circ}$
Average Porc Diameter in 90 to
Static Adsomption at 60\% RH

52-55 $1 \mathrm{~b} / \mathrm{It}^{3}$
$51-53 \mathrm{Ib} / \mathrm{ft} 3$
3.1-3.3
$0.3 \mathrm{mls} / \mathrm{cm}$
$50 \mathrm{~A}^{\circ}$
350 meter $2 / \mathrm{gm}$
$0.5-0.55 \mathrm{mls} / \mathrm{gm}$
$40 \mathrm{~A}^{0}$
$390 \mathrm{~meter}^{2} / \mathrm{gm}$
$0.28 \mathrm{mis} / \mathrm{gm}$
$30 A^{\circ}$ region $42 A^{\circ}$

A pore volume of 0.3 ml per gram represents a porosity of $50 \%$ if the specific gravity of the pellets is taken as 3.2 , while a pore volume of 0.5 indicates a porosity of $63 \%$. The problem amounts to deciaing which set or data above are consistent.

Placing the pellets in a vacuum and releasing with water to measure the water absorbed showed a porosity of around 50 to $55 \%$ which is somewhat indetemninate, lying as it does between the data from the two sources. However, for the $1 / 8$ inch activated alumina pellets the test described below was applied.

The test pellets were placed in the sample loop of a gas chromatograph and a hydrogen purge put on the loop. Air carrier gas was put on the column, which consisted of a 20 ft. length of $1 / 2^{\prime \prime}$ plastic hose to cause dispersion and create a Gaussian pulse distribution. The height of the pulse was proportional to the gas in the pulse, and so by noting the aifference in height between the pulse with the pellets in the sample loop, and without, the volume of the solids in the pellets could be determined. The peak heights were calibrated in terms of gas volume by injecting known volumes of hydrogen with a syringe.

The volume of the sample loop vas found to be 4.80 mls while the volume of gas when 47 dried pellets occupied the tube was 4.40 mls . From the mean diameter the overall volume of the pellets was calculated to be 0.81 mls and so the porosity is given by ( $0.81-0.40$ ) $/ 0.81=50.5 \%$.

Similarly for 47 wet ( $12 \%$ ) pellets, porosities of 28.4 and $33 \%$ were obtained.

If the water is assumed to exist as liquid water (31), then the porosity of the wot pellets can be calculated from the moisture content if the dry pellet porosity is known. If a $50 \%$ dry pellet porosity is assumed, then for a $12 \%$ wet pellet the porosity comes out to be $30.8 \%$. Thus, the manufacturer's data appears to give the best agreement with the observations.

Finally, two individual pellets were weighed and the dimensions of three diameters measured on callipers. This allowed the apparent density of the pellets to be calculated, and if the water content is taken into account, the porosity of the dry pellets can be obtained from a knowledge of the true specific gravity.
0.699 cm pellet weighed 0.3140 grams $\quad$ so density $=1.76 \mathrm{gms} / \mathrm{ml}$
0.617 cm pellet weighed 0.2300 grams $\quad$ so density $=1.87 \mathrm{gms} / \mathrm{ml}$

If $12 \%$ water in the pellet is assumed then the densities become 1.57 and 1.67 respectively. In conjunction with the specific gravity the porosities of these two pellets when dried are;

$$
1.57 / 3.2=49 \% \quad 1.67 / 3.2=52 \%
$$

The only anomaly left in the manufacturer's data is the claim of 22 to $25 \%$ static adsorption of moisture in air of $60 \% \mathrm{RH}$. Even with soaking in water only $14 \%$ water was adsorbable. However, in an Alcoa product data

- 188 -
bulletin, ("Activated and Catalytic Aluminas", Feb. I, 1963, Section GB2A, Figure 2, page 8), it is shown that after about six months operation the adsorptive capacity of this material drops to 13 or $14 \%$. The samples used in this work were stored for six months before work was started, and so it is possible that the low moisture contents are to be expected.

The dry pellets were assumed to have a $50 \%$ porosity in the pulse apparatus determinations and the porosity of the wet pellets was taken as $31 \%$ corresponding to a $12 \%$ wet pellet. The $1 / 4^{\prime \prime}$ and $1 / 8^{\prime \prime}$ pellets were assumed to have the same properties.

POROSITY OF PACKED BEDS
Two gencral methods were used to obtain the porosity of the non porous pellet beds
A.) If the density of the pellet packing was known, the bed was weighed before and after filling and the pellet density used to convert the packing weight to a packing volume. The overall volume of the vessel was calculated from the internal dimensions of the bed.

## Example

Run 50: 5 cm . diameter by 111.8 cm . length bed packed with No. 9
lead shot having a density of $10.808 \mathrm{gm} / \mathrm{ml}$.

| Weight of column + bungs + screens $_{n}$ | $=1051$ grams |
| ---: | :--- |
|  | $=35.5 \mathrm{Ib}$. |
|  | $=161028$ grams |

Weight of packing
Volume of packing $\quad 15,051 / 10.808 \quad=1392 \mathrm{ml}$.
Volume of bed $\frac{\pi(5.0)^{2}}{4} \quad 111.8$
Bed Porosity $=\frac{2195-1392}{2195}$
$=25,051$ grams
$=2195 \mathrm{ml}$.
$=36.6 \%$
B.) The alternate method of porosity measurement was to weigh the bed, (a) empty, (b) packed, (c) packed and filled with water, (d) unpacked and filled with water. If the density of water is taken as unity the bed porosity is given by (c-b)/(d-a). Example

Run 54: 1/4" polyethylene tube packed with 2.975 mm glass beads. Column length 184.5 cm and diameter 0.415 cm .


Porosity of bed $=(89.0-72.0) /(70.0-43.0)=63 \%$
The porosities of the beds of porous pellets are treated individually depending on the reliability of the available manufacturer's data.

Norton Catalyst Support 1/2" diameter SA 203 Mixture, RUN 60
The moisture content of these pellets was found to be negligible and so the manufacturer's pellet density was accepted as a value of 2.05 grams/ ml. With the weight of pellets in the bed measured, the porosity of the bed (not including pellet pores) was calculated by method A) above. Activated Alumina Pellets Alcoa $11511 / 4^{\prime \prime}$ diameter, RUNS 56,57,58,59,61,62

The bed used for these runs was a single pellet diameter and the porosity was measured as follows. The average diameter was used to calculate the mean pellet volume and then the number of pellets in a measured length of tube was measured. The volume of the pellets is thus known and the volume of the bed over the measured length can be calculated from the internal dianeter of the vessel.

## Example

Forty-four pellets in line occupy 25.0 cm . makins a mean pellet diameter of 0.568 cm . (Note the pellets were graded so that a small pellet was not adjacent to a large pellet, which would introduce an error into the result.)

In the packed bed 18 pollets occupied 10 cm . The volume of the pellets is thus $18 \pi(0.568)^{3} / 6=1.727 \mathrm{mls}$. and the volume of the vessel is given by $10 \pi(0.66)^{2 / 4}=3.42 \mathrm{mls}$.

The bed porosity is (3.42-1.727)/3.42 $=49.7 \%$ Activated Alumina Pellets, Alcoa H 151 1/8" Diameter, RUN 73

The above method could not be applied to a bed of several particle diameters thick. The moisture content of the pellets was determined to be $12 \%$, and weighing the bed before and after filling showed that 467 grans of the wet pellets packed the bed. Thus the weight of dry pellets was 411 grams and since the density of the dry pellets was given as 3.1 to $3.3 \mathrm{gm} / \mathrm{cm}$. by the manufacturer, the volume of solid can be calculated to be 129 ml . The 119.4 cm . long by 2.175 cm . diameter bed contains a volume of 441 ml . and so the porosity of the bed can be evaluated by method A if the dry pellets are assumed to be 50\% porous.

The porosity is thus (441-129/0.5)/441 = 41\%
As a corollary to the above calculation, the porosity of the dry activated alumina pellets is unlikely to be $65 \%$ as clairned in the supplier.'s literature. 'The assumption of a $65 \%$ pellet porosity yields impossible bed porosities, for example, a value of $54 \%$ is obtained and a $54 \%$ bed porosity is extremely unlikely in a random packed bed of uniform spheres.

Estimation of the Molecular Diffusivity of the Methane Air System Ref (32)


At $298^{\circ} \mathrm{K} \quad \frac{\mathrm{KP}}{\epsilon_{\mathrm{AB}}}=\frac{298}{115.4}=2.54$

From Table B. $2 \Omega=0.990$

$$
\begin{aligned}
D_{A B} & =0.001858 \sqrt{\frac{\left.\left.(293)^{3} \frac{1}{16.04}\right)^{+( } \frac{1}{28.97}\right)}{1(3.7195)^{2} \cdot 990}} \\
& =0.212 \mathrm{~cm}^{2} / \mathrm{sec}
\end{aligned}
$$

## APPENDIX IV

## ADSORPTION OF GASES BY ACTIVATED ALUMLNA PELLETS

## THEORY AND APPARATUS

This experiment was carried out in order to obtain the degree of methane adsorption in dry alumina pellets, adsorption is known to influence the effective diffusivity in a porous pellet (2). The following two assumptions were made, the adsorption isotherm is linear, i.e. moles adsorbed/gm. solid $=W$ (partial press.)
and the presence of other gases does not affect the adsorption isotherm.
The apparatus is shown in Figure A IV.1. The test chamber BC could be evacuated while the burette zone $A B$ was purged with the test gas. The stop cock $A$. was then turned to shut off the purge gas and open the mercury tube to the burette. The amount of gas used in the test could be adjusted by regulating the burette zone vent at $B$ and adjusting the manometer level.

The vacuum in the test chamber could be shut off at $C$, and the burette and test chamber connected at $B$. Thus by knowing the volume of the test chamber and the tube connecting to the burette zero, a series of measurements of the volume and pressure of trapped gas could be made by altering the manometer position. A series of burette readings (volume) and manometer readings (pressure) were taken at corresponding points as well as the atmospheric pressure. The volume of sample solids was also obtained.
'Total gas in the system $=\frac{\mathrm{P}_{0} V_{0}}{R T}$ Moles
A material balance of the trapped gas yields,

$$
\frac{P_{0} V_{Q}}{R T}=\frac{P V}{R T}+W P P_{p} Q
$$



Figure A VI.I
Adsorption Measurement Apparatus

$$
\text { or } V=\frac{P_{0} V_{0}}{P}-W R T P_{p}^{Q}
$$

Where $\rho_{p}$ is the pellet density, and $Q$ the volume of pellets. Hence a plot of volume $V$ against $\frac{1}{\bar{P}}$ should yield a straight line having an intercept WRT $\rho_{p} Q$ which is the volume of gas adsorbed. If the overall volume of the pellets (including pores) is known then the volume of adsorbed gas per unit volume of pellets is easily obtained.

## RESULTS

Table A IV.I shows the manometer and burette readings along with data in terms of volume and inverse pressure for the methane, hydrogen and nitrogen. Two sets of data are recorded for methane, one assumed atmospheric pressure and the other at about $1 / 2$ atmospheres in order to try and approach the concentration in the pulse apparatus. These points are plotted in Figure A IV.2.

Following are the characteristics of the apparatus which had to be known to prepare Table A IV. 1,

$$
\begin{array}{ll}
\text { Volume from zero of burette to stop cock } B & =6.7 \mathrm{ml} . \\
\text { Volume of empty test chamber } & =25.29 \mathrm{ml} . \\
\text { Pellets ALCOA H } 151 \text { activated alumina spheres } 1 / 4 " \text { diameter. } \\
\text { Weight of dry pellets in sample } & =7.68 \text { grams } \\
\text { Volume of solid excluding pores } & =7.65 / 3.2=2.4 \mathrm{ml} . \\
\text { Overall volume of pellet ( } 50 \% \text { porosity) } & =4.8 \mathrm{ml} .
\end{array}
$$

The overall volume of the pellets was also computed from the dimensions of the pellets and the same result was obtained.

Hence volume to be added to the burette reading $=25.29+6.7-2.4$

$$
=29.59 \mathrm{ml} .
$$

Volume of gas $=29.59+$ Burette reading
Pressure in chamber = Atmospheric $\pm$ manometer pressure :difference

The intercept of the hydrogen data is of the order of the accuracy of the experiment, and so it may be concluded that the hydrogen intercept represents zero adsorption. The intercepts were computed by the least squares technique and methane showed 5.22 ml . adsorbed at a half atmosphere and 5.387 ml , at one atmosphere. Since the hydrogen intercept of +0.316 is taken as zero this must be added to the methane result giving $5.22+.31=$ 5.53 ml . in 4.8 ml . of pellet at a half atmosphere and 6.60 ml . in 4.8 ml . pellet at one atmosphere.

Hence the methane adsorbed per unit volume of pellet material is 1.15 ml . at a half atmosphere and $1.375 \mathrm{ml} . / \mathrm{ml}$. pellet at one atmosphere. The results for nitrogen are not particularly of interest but it can be seen from Figure A IV. 2 that nitrogen adsorption is slightly higher than that of hydrogen as may be expected. A least squares computation for the nitrogen data was not carried out.


Figure A IV. 2
Gas Volume vs. Inverse Pressure For Adsorption Measurement

## RESULIS FOR ADSORPTION APPARATUS

| METHANE Low Pressure |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Atm. Pressure Room Temp. |  | $\begin{aligned} & 759.0 \mathrm{~mm} \mathrm{Hg} \\ & 22^{\circ} \mathrm{C} \end{aligned}$ |  |  |
| Burette mls. | Man. cm. | Press. mm Hg | atm. ${ }^{-1}$ | Vol. <br> mls. |
| 0 | - 1.5 | 74.4 | 1.02 | 29.59 |
| 2.3 | -6.2 | 69.7 | 1.09 | 31.89 |
| 4.2 | - 8.2 | 67.7 | 1.12 | 33.79 |
| 6.5 | -12.8 | 63.1 | 1.20 | 36.09 |
| 11.9 | -19.7 | 56.2 | 1.35 | 41.49 |
| 19.1 | -26.8 | 49.1 | 1.55 | 48.69 |
| 25.4 | -31.8 | 44.1 | 1.72 | 54.99 |
| 29.4 | -35.0 | 40.9 | 1.85 | 58.99 |
| 34.01 | -37.5 | 38.4 | 1.98 | 63.60 |
| 49.0 | -45 | 30.9 | 2.45 | 78.59 |

## HYDROGEN <br> 765.2 mm Hg $22{ }^{\circ} \mathrm{C}$

| Burette <br> mls. | Man. <br> cm. | Press. <br> mm Hg | $1 / \mathrm{P}_{-1}$ <br> $\mathrm{~atm}^{2}$ | Vol. <br> mls. |
| :---: | :---: | :---: | :---: | :---: |
| 41.3 | -14.0 | 62.5 | 1.215 | 70.89 |
| 43.0 | -15.4 | 61.1 | 1.242 | 72.59 |
| 36.7 | -9.7 | 66.8 | 1.138 | 66.29 |
| 30.7 | -3.2 | 73.3 | 1.038 | 60.29 |
| 25.3 | +4.3 | 80.8 | .940 | 59.89 |
| 23.9 | +6.4 | 82.9 | .917 | 53.99 |
| 21.6 | +10.7 | 87.2 | .872 | 51.19 |
| 28.4 | 0 | 76.5 | .994 | 57.99 |

$\sum_{1^{1}}{ }^{2}=25.4517$
$\sum \frac{1}{P}=15.33$
$\sum V=477.71$
$\sum \frac{V}{P}=799.7635$
Intercept $=-5.2208 \mathrm{mls}$.

$$
\begin{aligned}
\sum \frac{1}{\bar{P}}^{2} & =8.86418 \\
\sum_{\Sigma} \frac{1}{\bar{P}} & =8.356 \\
\sum V & =487.62 \\
\sum \frac{V}{\bar{P}} & =517.23384 \\
\text { Intercept } & =0.3167 \mathrm{mls} .
\end{aligned}
$$

NITROGEN
Atm. Pressure $\quad 765.2 \mathrm{~nm} \mathrm{Hg}$
Room Temp. $22^{\circ} \mathrm{C}$

| Birette <br> mls. | Man. <br> cm. | Press. <br> mm Hg | $1 / \mathrm{P}$ <br> $\mathrm{atm} .-1$ | Vol. <br> mls. |
| :--- | :---: | :---: | :---: | :---: |
| 42.3 | -17.0 | 59.5 | 1.279 | 71.89 |
| 39.0 | -14.0 | 62.5 | 1.215 | 68.59 |
| 35.2 | -10.5 | 66.0 | 1.152 | 64.79 |
| 31.0 | -6.1 | 70.4 | 1.08 | 60.59 |
| 25.95 | 0 | 76.5 | .994 | 55.54 |
| 23.4 | +3.3 | 79.8 | .953 | 52.99 |
| 20.6 | +7.7 | 84.2 | .901 | 50.19 |

## METHANE High Pressure 749.5 <br> $22.5^{\circ} \mathrm{C}$

| Burette mls. | Wau. cm. | Press. mm Hg . | $\begin{aligned} & 1 / P \\ & \text { atm. }^{-1} \end{aligned}$ | Vol. <br> mls. |
| :---: | :---: | :---: | :---: | :---: |
| 31.0 | -10.6 | 64.3 | 1.185 | 60.6 |
| 33.3 | -12.7 | 62.2 | 1.225 | 62.9 |
| 37.55 | -16.4 | 58.5 | 1.30 | 67.2 |
| 40.8 | -18.9 | 56.0 | 1.354 | 70.4 |
| 26.8 | - 6.5 | 68.4 | 1.11 | 56.4 |
| 23.3 | - 2.3 | 72.6 | 1.01:5 | 52.9 |
| 21.2 | 0 | 74.9 | 1.018 | 50.8 |
| 19.0 | + 3.3 | 78.2 | 0.965 | 48.6 |
| $\sum \frac{1}{\mathrm{p}^{2}}$ | $=10.71984$ |  |  |  |
| $\sum \frac{1}{P}$ | $=9.202$ |  |  |  |
| $\sum \frac{V}{P}$ | $=548.0431$ |  |  |  |
| $\sum \mathrm{V}$ | $=469.8$ |  |  |  |
| n | $=8$ |  |  |  |
| Interecp | $=-6$ | 87 moles |  |  |




1SYT2EB OSYTEBT
64 FORMATIIGX,7HSEGT = E12.5/16X,7HSYEBI $=$,F12.3/16X,7HSY =.
LE12.5/16X,7HSTE日T $=$,E12.5/16X,7HSTE2BT $=$,E12.5/16X,7HSEPBT = 2E12.5./16X,7HST2EBT* .E12.5/16X,7HST2E2G= EE12.5/16X,7HSYTZEB: 3E12.5/16X,7HSYTEBT $=$, E12.5 ।
656 RETURN
66 ENO

NO MESSAGES FOR ABOVE ASSEMBLY
TBME 17MRS OOMIN 39.3SEC

```
PARALIEL TUBE BED
-201-
HYDAOEEN-NITROGEIN
```



```
\begin{tabular}{|c|c|c|}
\hline IIMt SEC. & PEAK & \\
\hline 200.0 & 2925.00 & \\
\hline 300.0 & 2050.00 & 2858CTED P0DNS \\
\hline 460.0 & 1400.00 & 100 4050 \\
\hline 500.0 & 1000.00 & \\
\hline 600.0 & 700.00 & \\
\hline 700.0 & 440.00 & \\
\hline
\end{tabular}
```

```
CCNSTANTS FOR LEAST SAUARES FIT OF DAIA IN Y-C= AOEXPI-BII
```

CCNSTANTS FOR LEAST SAUARES FIT OF DAIA IN Y-C= AOEXPI-BII
A = 6035.51483
A = 6035.51483
S=0.00364182
S=0.00364182
C. 13.98370361
C. 13.98370361
SUMMATIONS FROM LEAST SQUARES CALCULATION
SEGT = n.14035E Ol
SYLUT = 0.27037E 04
SY =0.85550E 04
STEBT = त.44346E O3
STE2BT= 1.12707E 03
SE28F = U.44471E-v0
ST<EBT= 1.20602E O6
ST2E2H= 1.42225E 0S
SYT\&EB= (1.25173F 09
SYTERT: n.17353E 06
UIFFUSIVITY= U.S\$121716
LAMOA = 1.4B761696
EFFECIIVE DIFFUSIVITY= 0.2HOん3293
PUBLISHEO DIFFI'SIVBTYA O.H2ONOOOO
ALPHAS U.IHIZ?977 PUMETR ITFRATIUNSE 21

```

\section*{-202-}

－203－

\section*{HYDRUGEN－NITRUGEN}

BEO DATA
RIJV UAIA


SUPMATBONS FRGM LEAST SUUARES CALCULATIUN
SEBT＝O．185iOEE O1 SYtuT＝U．dblh2E US SY＝O． 24725 EF 04 STEBT \(=\) C．50833C 03 SIE2日fa 0．14210E U3 SEZdT U．6701OE OO ST2FBT＝C． 19846 F 06 ST2t2t：0．342tOE 05 SYT2EHE し．45377t OR SYFPBT＝0．18236t O6 DIFFUSIVIIY： 0.87263739 LAMOA＝0．9346B014 EFFECTIVE DIFRUSIVFTYE O．43377145 PUBLISHED DIFFLSIVIFY：O．H2000000 ALPHA⿷ C．07U60957
hyORUGEN-NITRIGEN

BED OATA
RUV BATA

AED DFAMETER CMS HED LENGTH CMS ENO ZCNE HEIGHT PUROSITY 0.52400
3.03000 10.05000 0.77000


TIME SEC. PEAK HFIliHTS BETEETGU SUNTS
\(1 \mathrm{Cu} .0 \quad 33.5 \mathrm{u} 50.53\).
\(150.0 \quad 10.50\)
200.0 \(1<.00\)
\(250.0 \quad 1.50\)
\(360.0 \quad 5.00\)
\(350.11 \quad 3.50\)
4CG.0 3.00

CUNSTANIS FUR LEAST SAUARES FIT UY DAYA IIN V-C: A*EXPI-BTI
\(\triangle\) : 100.16878
日 = 0.01154304
1.69351013

SUMMATIDNS FROM LEAST SGUARES CALCULATIIIN
SEET = C. 70020K UO
SYEHT = U.1SB72E U2
SY \(=0.840\) OOC C2
SIEBI: \(=11141 t 03\)
STE 28F=0.17835E 02
SE2BT - \(1.14510 t=0\) O
STREBT= C. 21.144 O4.
SI2E23=0.243nyt 04
SYI2EGa 0.283ヶTE O6
SYTEBT \(=0.19974 F 04\)
DIFFUSIVIJYa 0.77602791
LAMDA : 1.05660079
ERHECTIVE OIFHISIVITI= 0.40353536
PUBLISMEU DIFFUSIVFTYa U. B20nOUUO
ALPHAE 0.12234301

\section*{-205.}

\section*{NPTHOGEN-ETHANE}
```

    BED DATA
    RUN IBATA

```


nftrugen－ethaine
bec data Ruiv l／ala

sumations thom lafist sounhes galculailon
SEET \(=0.133 \mathrm{sit} 01\) SYEGT \(=0.114\) ite 03 SY \(=\) C．Sist） 03 STEGT＝0．5日BABL 03 STE2HT＝0．13972t 03 SE2BT \(=0.46742 \mathrm{t}-00\) ST2EGT＝ 0.350 InE OG St2t2s＝c．bis6ltt os SYTくEか＝ \(0.24912 E\) OB SYTEAT \(=0.601+5 \mathrm{~F}\) 05 Diffusivily \(\quad 0.147424 \pi s\)
LAMDA＝ 1.02148467
EFFECIIVE DIFFISSIVITV＝ PUBLISHED OIFFUSIVITY＝ ALPMA 0.14160762
0.07686850
0.15100000
-207-
ngtrogen-ethane


\section*{NYTROGEN-ETHANE}


\section*{nftrugentethane}

\section*{hED DATA}

\section*{RUN IAATA}
\begin{tabular}{lr} 
HED QIAMETEA CMS & 5.03000 \\
SEC LENGTH CMS & 19.05000 \\
END LCNE HEYGHT & \(J .27000\)
\end{tabular}

\section*{FLDW KATE CC/SEC= 3.074, RUIJM TEMMERATURE* 295.000 O ATM.PRESSURE M:A HG, 150.0 JEO TEMPERATURE K= \(309.0 U I O\)} PORUSETY 0.52000

TIME SeC. PEAM meigits
\(\begin{array}{llll}200.0 & 115.00 & 100 & 165\end{array}\)
\(40.0 \quad 69.00\)
\(500.0 \quad 52.00\)
\(500.0 \quad 52.00\)
\(\begin{array}{ll}600.7 & 42.00 \\ 760.13 & 34.00\end{array}\)
\(\begin{array}{ll}860.1 & 29.00\end{array}\)
\(\begin{array}{ll}10.1) & 25.00 \\ 1000.1\end{array}\)
1000.1 22.50
1100.3 20.30

CONSTANTS FOR LEASY SAUARES FIT UF DATA I:I \(Y-G=A \in E X Q-E T I\)
\(A \quad 174.37181\)
\(\begin{array}{ll}\mathrm{C}= & \mathrm{C} .10378764 \\ \mathrm{C}= & 14.94706340\end{array}\)

SUMMATIONS FRCM LEAST SQUARES CALCULATION
SEBT \(\left.=0.17 b^{\prime}\right) 2 t\) DL
SYEUT \(=0.13444 E 03\)
SY \(=0.476\) VOE 03
STEAT = 0.74433E 03
\(S T E Z B F=0.11032803\)
SE2BT \(=0.53637 \mathrm{E} 0\)
ST2E8T=0.40273E 06
ST2L2b=0.63719E 05
SYI2EU= 0.18426t OU
SYTEST: \(0.44269 t 05\)
\(\begin{array}{ll}\text { OIFFUSIVITYE } & 0.14867553 \\ \text { LAMLA } & 1.01563447\end{array}\)
LFFECIIVE OIFFUSIVIYY= 0.07I31129
PUBLISHED DIFFUSIVITYE 0.15100000
ALPHAE U.14884170

\section*{-210-}
mptrugev-ethaive



NITROGEN-BUTANE
GEO DAYA RUN OATA


SUnmations from least souares calculation
SEBT \(=0.35027 E 01\)
SYEBT \(=0.60839 E 03\)
SY = 0.16130E O4
STEBT = \(0.23890 E 04\)
STE2BT: \(0.10196 E 03\)
SE2BT = 0.13159E O1
ST2EBT= 0.20596E O7
ST2E2B- \(0.46022 E\) OO
SYT2EB= 0.21130E O9
SYTEBT= 0.32363E 06
DIFFUSIVITY= 0.07939063
LAMOA = 1.14661468
EFFECTIVE DIFFUSIVITYO 0.04128313
PUBLISHED DIFFUSIVITYO 0.09500000
ALPHAE O.14327731

\section*{-213-}
```

N}TROGEN-GUTANE
BEO DATA RUN DATA

```

```

            TIME SEC. PEAK HEIGHTS
                    300.0 138.00
                        400.0 110.00
                        500.0 92.00
                    600.0 76.00
                    700.0 61.00
                    800.0 51.00
                    900.0 42.00
                    1000.0 34.00
                    1150.0 25.00
    CONSTANTS FOR LEASY SAUARES FIT UF DATA IN Y-G= A\&EXP{-BT}
A = 249.21826
B=0.00202606
SUMMATIONS FROM LEAST SOUARES CALCULATION
SEBT =0.24793E 01
SYEBT = 0.21855E 03
SY =0.62900E 03
STEBT = 0.14174E 04
STE2BTa 0.41079t 03
STE28Ta 0.41079E 03
ST2EET=0.9542OE O6
ST2E2B=0.22771E OG
SYY2EB=0.5793BE 08
SYTEBT= 0.10412t 06
IFFUSIVITYa 0.09057913
AMDA = 1.01568647
EFFECTIVE DIFFUSIVITYa
EFFECTIVE DIFFUSIVITYa 0.04710115
PUBLISHED OIFFUSIVITY= 0.09200000
ALPHAK O.09200000 NUMBER ITERATIONS:
ENO-OF-DATA ENCOUNTERED UN SYSTEA INPUT FILE.
TIME 17HRS O1MIN 2S.3SEC

```
```

POROUS SOLID_BER
MYURUGEN-NITROCEIG

```

```

            TIME SEC. PEAK HFIGHTS
                            150.0 123.00
                            200.0 61.00
                            250.0 34.00
                            300:0 18.00
                            350.0 10.00
                            400.0 6.20
                            450.0---
                            500.0 3.20
    COVSTANTS FOR LEAST SAUAQES FIT UF CATA IN Y-C= ADEXP(-HT)
A = 901.76734
B=
gupmations frum least sluares calculation
SEBT = n.24827E-00
SYEBT = प. 20592E 02
SY = 0.25990E 03
STENT = 0.49247E 02
STE2BT= 0.34536[ 01
SE2\&T=0.20727E-01
STZEBT= 0.10852E 05
ST2E2B= C.57838t 0s
SYIZEE= C.60443E OO
SYTEBT= 0.34520t O4
DIFFUSIVITY\& 0.56476959
LAMOA = 1.45191953
EFFECTIVE DIFFUSIVITY= 0.33321406
PUOLISHEN DIFFUSIVITYZ` \cdots 0.82000000
ALPHA= 0.15623251
NUMGER ITERATIUNS= 21

```

HYOROGEN-NITHOGEV


\section*{\(-216=\)}

\section*{HYDRUGEN-NBTROGEN}
```

    HEO DATA
    RUN DATA

```



\section*{-217-}

\section*{hydrugen-nithcgen}

```

                                    -218-
    MYDRQGEN-NITRUGEN
BED DAIA NUN DATA

```

```

time slc. peak heights
$50.0 \quad 124.00$
$100.0 \quad 37.00$
$150.0 \quad 14.00$
$200.0 \quad 5.00$
$250.0 \quad 2.00$

```

```

SUMMATBONS FHOM LEAST SOUARES CALCULATION
SEBT $=0.41632[-00$ SYEBT =0.40112t 02 SY = 0.18iC00E 03 STEST $=0.29271 E 02$ STE28T= 0.51क51E OI SE2BT $=0.94899 E-01$ ST2EBT= 0.26156L 04 ST2E28=0.30902E 03
SYT2EB= 0.1331 万E 06
SYIEAT= त. 220 CH2E 04
OIFFUSIVITY= 0.650931I7
LAMDA = 1.25973380
EFFECTIVE DIFFUSIVITY= 0.38404939
PUBLISHED DIFFUSIVITY= n.820U0000
ALPHA: 0.1931C470

```
-219-
NHTRLGEN-ETHANE
    BEO OATA RUN DATA
```



```
\begin{tabular}{rrr} 
TIMF StC & PLAK HEIGHTS & \\
250.0 & 212.50 \\
400.00 & 117.00 \\
550.0 & 68.00 \\
700.0 & 43.50 & \\
850.0 & 30.50 & \\
1060.0 & 24.50 & \\
\hline 1150.0 & 20.50 &
\end{tabular}
```

```
CONSIANTS FOR LEAST SAUARES FIT OF DATA IN Y-C= AeEXP{-BTI
```

CONSIANTS FOR LEAST SAUARES FIT OF DATA IN Y-C= AeEXP{-BTI
A = 598.64044
A = 598.64044
B= 0.00448394
B= 0.00448394
SUMMATBONS FROM LEAST SUUARES CALCULATION
SEBT = 0.65974E UC
SYEBT = 0.97459E O2
SY 0.51650E O3
STEGT=0.26179F O3
STEZBT= 0.43494E 02
SC2BT = 0.14366E-00
ST2E\&T= C.128BOE O6
ST2E2B= 0.14699E O5
SYT2EB= 0.11034E OB
SYTEET= 0.30583E O5
OIFFUSIVITY= 0.11308980
LAMOA = 1.33527204
EFFECTIVE DIFFUSIVITY= O.06672298
PUBLISHED DIFFUSIVITYZ O. 15100000
ALPHA= 0.17822310
-220-
NbTRUGEN-ETHANE


TIME SEC. PEAK HEIGHISS. :
200.6 116.0n
300.0 77.50
$400.11 \quad 53.00$
$300.0 \quad 38.30$
$600.0 \quad 29.50$
$700.0 \quad 24.00$
$1200.7-16.00$


SUPMATIUNS FROM LEAST SOUARES CALCULATIUN
SEOT $=0.72750 \mathrm{CDO}$
SYEUT $=0.74499 \mathrm{O}$
SY $=0.35450 \mathrm{O}$ O3
STEBT $=0.30404 E 03$
STE2BT $=0.38910 t 02$
SE23T = 0.2276OE-00
ST2EBT=0.12007L 06
ST2E2B=0.17く30t 05
SYT2EB=0.64333F 07
SVTEBT $=0.20383 \mathrm{t} \mathrm{05}$
DIFFUSIVITVE 0.10786807
LAMOA = 1.39985816
EFFEGTIVE DIFFUSIVITY= 0.06364216
PUBLISHED DIFFUSIVITY: 0.15100000
ALPHA: $0.2116 G 63^{\circ}$ NUMBER ITERATIONS = 11

```
-221-
NBTROGEN-ETHANE
    GED DATH RUN OATA
\begin{tabular}{lll} 
BED DIAMEIER CHS & 2.61000 \\
BED LENGTH CMS & 7.00000 \\
END LONE HEIGHT & 0.27000 \\
PGROSHYY \(0.59 C O U\) &
\end{tabular}
    FLOW RATE CC/SEC= 1.32324
BED LENGTH CMS
PGROSBIY 0.S9COU
IIME SEC. PEAK HEIGHIS
100:0'-- 122.00
200.0 78.50
300.0 33.00
400.0 37.50
500.0 28.50
000.0 23.50
CONSTANTS FOR LEAST SAUARES FII OF DATA IN Y-C= A*EXP(-BT)
A= 177.89097
B=0.00525755
C= 15.67274153
SUPmATIGNS fRUM least squares calculation
SLBT m 0.1384DE 01
SYEBT =0.118135 03
SY =0.34300t 03
STEAT = 0.3014/E O3
STL2BT= 0.81814E 02
SE2dr = 0.53609E 00
ST2EHT= 0.91409t 0S
ST2E2b= 0.165STE OS
SYT2EB=0.44111E 07
SYTEBT= 0.19442F O5
DIFFUSIVITY* 0.11160270
LAMOA * L.35301381
EFFECFIVE UIFFUSIVITY= 0.C6584559
PUBLISHED DIFFUSIVIIY= 0.15100000
ALPHA= 0.21606H73 NUMGER ITERATIONS= 10
```


## NFTRGGEM-ETHANE



## nftrugen-butane



## nbTrogen-butane



## NFTRCGEN-BUTANE




```
HYDRCGEN-NITROGEN
```



```
\begin{tabular}{rr} 
IIME SEC. & PEAK HEIGHTS \\
50.0 & 460.00 \\
100.0 & 201.50 \\
150.0 & 95.00 \\
200.0 & 50.50 \\
250.0 & 28.00 \\
300.0 & 18.00 \\
350.0 & 13.00 \\
400.0 & 11.00 \\
430.0 & 10.00 \\
500.0 & 9.00
\end{tabular}
CONSTANTS FOR LEAST SAUARES FIT UF DATA IN Y-C= AOEXPT-BTI
    A= 1023.41085
    = 0.01643952
    SUMMATIOVS FRCM LEAST SOUARES CALCULATTON
    SEGT . U.78411E 00
    SYEBY = 0.23290t 03
    SY = 0.90700t 03
    STEBT = 0.69449E 02
    STE2BT= 0.14342E 02
    SE2BT=0.23949E-00
    S12E8T= 0.89128E (14
    ST2E2B= 0.10975E 04
SYT2EB= 0.12131E OT
SYTEBT= G.lS885E 05
    DIFFUSIVITY= 0.68188812
    LABOA = 1-205゙34330
    EFFECTIVE DIFFUSIVIIYY 0.26798203
    PUBLISHEO DIFFUSIVIIY: 0.82000000
    ALPHA= 0.15708618
-227-
HYDRUGEN-NITAUGEN


\section*{-228-}

\section*{hyOROGEN-NITRUGEN}
```

    BED DATA
    RUN DATA

```

time sec. pear. heights
\(50.0 \quad 122.25\)
\(100.0 \quad 38.00\)
\(150.0 \quad 15.00\)
200.0 6.80
\(250.0 \quad 4.10\)
300.03 .20

CGNSTANTS FOR LEAST SAUARES FIT OF DATA IIN \(Y\)-G= AEEXP(-BT)
\begin{tabular}{|c|c|}
\hline + & 394.9892, \\
\hline B & 0.02479557 \\
\hline C & 3.44353834 \\
\hline
\end{tabular}

SUMMAIIONS FROM LEASI SQUARES CALCULATIUN
SEBT \(=0.42173[-00\)
SYEBI \(=0.40004502\)
SY \(=0.18915 t 03\)
STEHT = 0.79899t 07
STE26T= 0.52955t 01
SE2HT = 0. 3 h982t-01
ST2EBT= 0.27263E O4
ST2t28=0.31592t 03
SYI2EA= \(0.13589 E\) O6
SYIEAT = 0.22711E OG
nIfFUSIVItVE 0.66116180
LAMOA \(=1.24024105\)
EFFECIIVE Dirfilsivitya 0.2598365H
PUBLISHED DIFFISIVITY= 0.82000000 ALPHYE 0.19394036

\section*{NPTROGEN-ETHANE}
hen data
hun data


SUMMATPONS FROM LEAST SQUARES CALCULATIOA
SEBT \(=0.85416 E 00\)
SYEGT \(=0.79842 E\) 07
SY =0.34490t 03
STEBT = U.24116E 03
STL2BT \(=0.49: 98 t\) 07
SE28T = 0.26231[-00
ST2EBT= 0.97431E US
ST2E2B= 0.11567E U5
SYT2EB= 0.44904E 07
SYTEHT= O.164S8t 05
DIFFUSIVIIY=
0.11735025

LAMDA \(\quad 1.28074738\)
EFFECIIVE DIFFUSIVITY= 0.04611861
PLBLISHEU CIFFUSIVITYE 0.15100000
ALPHAE 0.21198461
-230-

\section*{nithogen-einane}

-231-
Nitrugen-EThane

-232-

\section*{nhtrugf n-bugane}


\section*{NITROGFN-BUTANE}

\begin{tabular}{lll} 
GED DIAMFTER CMS & 2.61000 \\
HED LENGTH CMS & 7.00000 \\
HNE LONE HEIGHI & 0.27000 \\
POROSITY 0.39300 &
\end{tabular}

FLOW RATE CC/SEC= 0.77945 HUOM TFMPFKATURE \(=297.5000\) ATM. PRESSURE MM HG,
BEI TEMPERATURE K= \(\mathbf{3 0 9 . 0 0 0 0}\)

-234-
nbirugen-butane
```


[^0]:    Inverse daly ishniat ifumer Vs. Hyirculic Diadeter Reynolas liurber

[^1]:    GAMMA 0.850906
    LAMDA= 0.331342

