Dimensional Similitude and the Hydrodynamics of Three-Phase Fluidized Beds

By

Michael Safoniuk
B. Eng, McGill University, 1989
M. Eng., McGill University, 1992

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

In THE FACULTY OF GRADUATE STUDIES (Department of Chemical and Bio-Resource Engineering)

We accept this thesis as conforming to the required standard

Department of Chemical and Bio-Resource Engineering
The University of British Columbia
Vancouver, B.C., Canada
July 1999
© Michael Safoniuk, 1999
In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Chemical and Bio-Resource Engineering

The University of British Columbia
Vancouver, Canada

Date July 20, 1999
Abstract

It is proposed that scaling of three-phase fluidized bed hydrodynamics can be carried out based on geometric similarity and matching of a set of five dimensionless groups: (i) the M-group, \[ M = \frac{g \cdot \Delta \rho \cdot \mu_L^4}{\left(\rho_L^2 - \sigma^2\right)} \]; (ii) an Eötvös number, \[ E_o = \frac{g \cdot \Delta \rho \cdot d_p^2}{\sigma} \]; (iii) the liquid Reynolds number, \[ Re_L = \frac{\rho_L \cdot d_p \cdot U_L}{\mu_L} \]; (iv) a density ratio, \[ \beta_d = \frac{\rho_p}{\rho_L} \]; and (v) a superficial velocity ratio, \[ \beta_u = \frac{U_g}{U_L} \]. These were varied in an experimental study where four dimensionless hydrodynamic parameters were measured: (i) gas hold-up, \( \varepsilon_g \); (ii) bed expansion ratio, \( \beta_e \); (iii) the ratio of mean bubble diameter to particle diameter, \( \frac{d_b}{d_p} \); and (iv) the ratio of mean bubble rise velocity to gas superficial velocity, \( \frac{U_b}{U_g} \). This approach was validated experimentally by matching the dimensionless operating conditions from a kerosene-nitrogen-ceramic three-phase system with those in an aqueous magnesium sulphate solution-air-aluminum particle fluidized bed. There was good agreement between the gas hold-ups and bed expansion ratios in the two systems.

A pilot-plant scale cold-flow co-current upwards-flowing three-phase fluidized bed column of inside diameter 292 mm was built and operated using three different liquids (tap water, an aqueous 44 mass % glycerol solution, and an aqueous 60 mass % glycerol solution), air, and cylindrical aluminum particles of diameter 4 mm and length 10 mm. The fluids and solids were carefully selected to result in dimensionless group values in the range of those of an industrial hydroprocessor. Specially built conductivity probes and pressure transducers were used to measure the hydrodynamic properties for different gas and liquid superficial velocities. Special attention was required to provide for drift and calibration when recording and analyzing data from the conductivity probes.
Gas hold-ups were in the range of 5 to 20% by volume and were correlated as a function of liquid-phase Reynolds number and superficial velocity ratio. The gas hold-ups were a strong function of the velocity ratio with little influence of the liquid-phase Reynolds number. Bed expansion, in the range of 0-200%, was similarly correlated. Although bed expansion was dependent on the velocity ratio, the liquid Reynolds number had a much more significant influence. Bubble rise velocities, pierced chord lengths, and frequencies were also determined for a range of operating conditions.

A new approach for estimating gas hold-up based on the gas-perturbed liquid model, using calculated values of interstitial liquid velocity, was also developed. This approach gave favorable agreement with gas hold-up measurements based on pressure drops.
# Table of Contents

Abstract ii  
Table of Contents iv  
List of Tables viii  
List of Figures x  
Acknowledgements xviii  

Chapter 1 – Introduction 1  
1.1 Advantages and disadvantages of fluidized bed technology 7  
1.2 Bitumen processing 8  
1.3 Scaling: The journey from laboratory unit to industrial application 10  
1.4 Some definitions of fluidization phenomena 12  
1.5 Research objectives 15  

Chapter 2 – Basis for Dimensional Analysis and its Application 16  
2.1 Existing information for hydrodynamic predictions 17  
2.1.1 Effect of surface tension 17  
2.1.2 Effect of liquid viscosity 24  
2.1.3 Effect of particle size 24  
2.1.4 Summary of influence of variables 24  
2.2. Empirical correlations 26  
2.3 Models 28  
2.3.1 Richardson-Zaki model for liquid-solid systems 28  
2.3.2 The bubble wake model 29  
2.3.3 The Drift Flux model 33
2.4 Dimensional similitude approach and existing tools

2.4.1 The Buckingham Pi theorem

2.4.2 Dimensional similitude for two-phase systems

2.4.3 Dimensional similitude for three-phase fluidized beds

2.5 Present application of dimensional similitude

2.5.1 Learning from the past

2.5.2 Forming the groups

2.6 Selection of the operating conditions based on dimensionless operating groups

Chapter 3 – Experimental Equipment

3.1 292-mm diameter column

3.2 83-mm diameter column

Chapter 4 – Measurement Techniques

4.1 General discussion of phase hold-up measurements

4.2 General discussion of bed expansion measurements

4.3 General discussion of bubble property measurements

4.4 Pressure gradients

4.4.1 Bed expansion

4.4.2 Phase hold-ups

4.5 Conductivity probes

4.5.1 Theory

4.5.2 Discrimination criteria

4.5.3 Probe design
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5.4 Data acquisition</td>
<td>82</td>
</tr>
<tr>
<td>4.5.5 Signal filtering</td>
<td>85</td>
</tr>
<tr>
<td>4.5.6 Data analysis program</td>
<td>92</td>
</tr>
<tr>
<td>4.5.7 Probe response time</td>
<td>94</td>
</tr>
<tr>
<td>4.6 Borescope</td>
<td>100</td>
</tr>
<tr>
<td>4.7 Measurement of physical properties</td>
<td>102</td>
</tr>
<tr>
<td>4.7.1 Viscometer</td>
<td>102</td>
</tr>
<tr>
<td>4.7.2 Surface tensiometer</td>
<td>104</td>
</tr>
<tr>
<td>4.8 Qualitative analysis of three-phase fluidized bed</td>
<td>104</td>
</tr>
<tr>
<td>Chapter 5 – Validation Experiments of Dimensionless Groups</td>
<td>108</td>
</tr>
<tr>
<td>Chapter 6 – Bed Expansion Measurements</td>
<td>114</td>
</tr>
<tr>
<td>6.1 Experiments with tap water</td>
<td>117</td>
</tr>
<tr>
<td>6.2 Experiments with 44 mass % aqueous glycerol solution</td>
<td>119</td>
</tr>
<tr>
<td>6.3 Experiments with concentrated aqueous glycerol solution</td>
<td>129</td>
</tr>
<tr>
<td>6.4 New approach to use bed expansion data to estimate gas hold-up</td>
<td>132</td>
</tr>
<tr>
<td>Chapter 7 – Gas Hold-up Measurements</td>
<td>140</td>
</tr>
<tr>
<td>7.1 Experiments with tap water</td>
<td>140</td>
</tr>
<tr>
<td>7.2 Experiments with 44 mass % aqueous glycerol solution</td>
<td>143</td>
</tr>
<tr>
<td>7.3 Experiments with concentrated aqueous glycerol solution</td>
<td>145</td>
</tr>
<tr>
<td>7.4 Radial profiles of local gas hold-ups</td>
<td>147</td>
</tr>
<tr>
<td>Chapter 8 – Bubble Rise Velocities and Pierced Chord Lengths</td>
<td>157</td>
</tr>
<tr>
<td>8.1 Tap water system</td>
<td>157</td>
</tr>
<tr>
<td>8.2 Aqueous 44 mass % glycerol system results</td>
<td>167</td>
</tr>
</tbody>
</table>
8.3 Use of bubble characteristics to estimate $\varepsilon_g$ and probe sampling area 175

Chapter 9 – Conclusions and Recommendations 180

9.1 Achievement of objectives 180

9.2 Additional conclusions and accomplishments 183

9.3 Recommendations for future work 183

Nomenclature 185

References 188

Appendix A: Design Calculations and Schematics for 0.292-m column 201

A.1 Number of holes required for sparger and distributor plate 201

A.2 Design drawings for miscellaneous components 203

Appendix B: Conversion of Pressure Drop Readings to Gas Hold-up 209

Appendix C: Software Written for this Project 212

C.1 Data acquisition program (Test6.BAS) 212

C.2 FFT filtering software (Filter.FOR) 220

C.3 Data analysis program used on FFT-smoothed data (Cal_S98.BAS) 229

Appendix D: Conversion of Local Gas Hold-up to Average Axial Gas Hold-up 238

Appendix E: Radial Gas Hold-up Profiles 241
List of Tables

Table 1.1: Definitions of some three-phase fluidization phenomena 13

Table 2.1: Literature correlations of gas hold-up and bed expansion 18
Table 2.2: Literature correlations for bubble rise velocity and diameter 22
Table 2.3: Summary of qualitative influence of operating variables 25
Table 2.4: Physical properties of fluids and solids used in industrially operated 0.91-m column and UBC unit 46
Table 2.5: Aqueous solutions and their physical properties 48
Table 2.6: Ratios of values of dimensionless groups for systems reported in the literature to values in industrial hydrocracker 52
Table 2.7: Potential liquids for matching dimensionless operating parameters 53
Table 2.8: Potential solids for matching dimensionless operating parameters 53

Table 6.1: Physical properties of liquids used in 292-mm column 115
Table 6.2: Fitted coefficients for equation (6.2) pertaining to liquid component of bed expansion data 118
Table 6.3: Correlations coefficients $B_1$ and $B_2$ for equation (6.3) for various $Re_L$ for water system 120
Table 6.4: Correlations coefficients $B_1$ and $B_2$ for equation (6.3) for various $Re_L$ for aqueous 44 mass % glycerol system 127
Table 6.5: Correlations coefficients $B_1$ and $B_2$ for equation (6.3) for various $Re_L$ for aqueous 60 mass % glycerol system 133

Table 6.6: Comparison of gas hold-ups from pressure measurements and from use of interstitial velocity method for aqueous 44 mass % glycerol system 139

Table 7.1: Values of “a” and “b” for equation (7.1) as fitted in Figures 7.1 to 7.3 141
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Modes of gas-liquid-solid fluidization [from Fan, 1989]</td>
<td>2</td>
</tr>
<tr>
<td>1.2</td>
<td>Schematic diagram of flow patterns [from Zhang, 1996]</td>
<td>6</td>
</tr>
<tr>
<td>1.3</td>
<td>Schematic representation of typical co-current gas-liquid-solid fluidized bed</td>
<td>14</td>
</tr>
<tr>
<td>2.1</td>
<td>Schematic representation of classical bubble wake model</td>
<td>31</td>
</tr>
<tr>
<td>2.2</td>
<td>Schematic representation of a three-phase fluidized bed with (a) gas liquid flowing, and (b) after gas and liquid flow have been terminated</td>
<td>34</td>
</tr>
<tr>
<td>2.3</td>
<td>M-group values for various concentrations of aqueous MgSO₄</td>
<td>49</td>
</tr>
<tr>
<td>3.1</td>
<td>Schematic of 292-mm diameter column and (a) its auxiliary equipment and (b) overhead view of storage tank</td>
<td>55</td>
</tr>
<tr>
<td>3.2</td>
<td>Bottom view of gas sparger ring used in pre-mix area of 292-mm column</td>
<td>57</td>
</tr>
<tr>
<td>3.3</td>
<td>Assembled pre-mix area showing sparger and distributor plate</td>
<td>57</td>
</tr>
<tr>
<td>3.4</td>
<td>Design diagram of distributor plate grid</td>
<td>58</td>
</tr>
<tr>
<td>3.5</td>
<td>Schematic representation of secondary 83-mm diameter column</td>
<td>61</td>
</tr>
<tr>
<td>4.1</td>
<td>Idealized pressure profile in a fluidized bed</td>
<td>67</td>
</tr>
<tr>
<td>4.2</td>
<td>Typical axial pressure profile in the column as measured by pressure transducers</td>
<td>71</td>
</tr>
<tr>
<td>4.3</td>
<td>Schematic representation of set-up of multi-tube barometers</td>
<td>73</td>
</tr>
</tbody>
</table>
Figure 4.4: Idealized conductivity probe signals for a bubble passing and piercing a two-tipped probe 75

Figure 4.5: Typical pierced chord length distribution 79

Figure 4.6: Design schematic of two-tip conductivity probe used in this research 81

Figure 4.7: Diagram depicting connection of two probe tips to Wheatstone bridges 81

Figure 4.8: Flowchart used for data acquisition software 83

Figure 4.9: Conductivity probe signal from lower tip before (top) and after (bottom) processing through Fast Fourier Transform subroutine 86

Figure 4.10: Logic flowsheet for FFT filtering program (Filter.For) 89

Figure 4.11: Effects of cut-off frequency on baseline calculations 91

Figure 4.12: Small segment of FFT-modified conductivity probe signal from lower tip 93

Figure 4.13: Conductivity probe signal from lower tip: (a) before processing; (b) after processing once; and (c) after processing twice 95

Figure 4.14: Frequency count for curves shown in Figure 4.12 96

Figure 4.15: Example of signal processed using Filter.For 97

Figure 4.16: Frequency count for curves shown in Figure 4.15 97

Figure 4.17: Close-up of calibration signal for conductivity probe 99

Figure 4.18: Schematic diagram of rigid borescope 101

Figure 4.19: Typical image taken inside the bed using the borescope 103

Figure 4.20: Typical image taken at the wall of the column from the outside 103

Figure 4.21: Bubble frequency in the freeboard 105
Figure 5.1: Bed expansion versus \( \text{Re}_g \) for various liquid-phase Reynolds numbers

Figure 5.2: Apparent gas holdup in freeboard region versus \( \text{Re}_g \) for various liquid Reynolds numbers

Figure 5.3: Apparent gas hold-up in freeboard region and ebullated bed versus \( \text{Re}_g \) for various liquid Reynolds numbers in the 0.91-m diameter unit

Figure 6.1: Temperature dependence of viscosity of the concentrated glycerol solution

Figure 6.2: Bed expansion versus \( \text{Re}_L \) with no gas flow for tap water

Figure 6.3: Bed expansion versus \( \beta_u \) for tap water with \( \text{Re}_L = 356 \)

Figure 6.4: Bed expansion versus \( \beta_u \) for tap water with \( \text{Re}_L = 430 \)

Figure 6.5: Bed expansion versus \( \beta_u \) for tap water with \( \text{Re}_L = 536 \)

Figure 6.6: Bed expansion versus \( \beta_u \) for tap water with \( \text{Re}_L = 646 \)

Figure 6.7: Bed expansion versus \( \beta_u \) for tap water with \( \text{Re}_L = 716 \)

Figure 6.8: Correlation for \( B_1 \) as a function of \( \text{Re}_L \) for tap water

Figure 6.9: Correlation for \( B_2 \) as a function of \( \text{Re}_L \) for tap water

Figure 6.10: Bed expansion versus \( \text{Re}_L \) with no gas flow for aqueous 44 mass % glycerol solution

Figure 6.11: Bed expansion versus \( \beta_u \) for aqueous 44 mass % glycerol solution with \( \text{Re}_L = 73.1 \)

Figure 6.12: Bed expansion versus \( \beta_u \) for aqueous 44 mass % glycerol solution with \( \text{Re}_L = 105 \)
Figure 6.13: Bed expansion versus $\beta_u$ for aqueous 44 mass % glycerol solution with $Re_L = 127$

Figure 6.14: Bed expansion versus $\beta_u$ for aqueous 44 mass % glycerol solution with $Re_L = 156$

Figure 6.15: Correlation for $B_1$ as a function of $Re_L$ for aqueous 44 mass % glycerol solution

Figure 6.16: Correlation for $B_2$ as a function of $Re_L$ for aqueous 44 mass % glycerol solution

Figure 6.17: Bed expansion versus $Re_L$ with no gas flow for aqueous 60 mass % glycerol

Figure 6.18: Bed expansion versus $\beta_u$ for aqueous 60 mass % glycerol solution with $Re_L = 31.7$

Figure 6.19: Bed expansion versus $\beta_u$ for aqueous 60 mass % glycerol solution with $Re_L = 41.6$

Figure 6.20: Bed expansion versus $\beta_u$ for aqueous 60 mass % glycerol solution with $Re_L = 50.0$

Figure 6.21: Correlation for $B_1$ as a function of $Re_L$ for aqueous 60 mass % glycerol solution

Figure 6.22: Correlation for $B_2$ as a function of $Re_L$ for aqueous 60 mass % glycerol solution

Figure 6.23: Parity plot comparing measured bed expansions to values predicted by correlations presented in equations (6.6), (6.9) and (6.12)

Figure 6.24: Correlated $A_1$ and $P$ for equation (6.2) for various values of $M$
Figure 6.25: Bed height versus liquid interstitial velocity

Figure 6.26: Comparison of gas hold-ups measured with $U_L^*$-method and pressure drop method for aqueous 44 mass % glycerol solution

Figure 7.1: Average gas hold-up versus $Re_g$ for various $Re_L$ for tap water

Figure 7.2: Average gas hold-up versus $Re_g$ for various $Re_L$ for aqueous 44 mass % glycerol solution

Figure 7.3: Average gas hold-up versus $Re_g$ for various $Re_L$ for aqueous 60 mass % glycerol solution

Figure 7.4: Parity plot of measured gas hold-up values versus predictions from Equations (7.2) through (7.4)

Figure 7.5: Fitted factor “a” from equation (7.1) versus M-group

Figure 7.6: Fitted factor “b” from equation (7.1) versus M-group

Figure 7.7: Radial profiles of local gas hold-up at four different heights for aqueous 44 mass % glycerol solution

Figure 7.8: Radial profiles of local gas hold-up for tap water with $Re_g = 430$ and $Re_L = 170$

Figure 7.9: Radial profiles of local gas hold-up for tap water with $Re_g = 430$ and $Re_L = 470$

Figure 7.10: Radial profiles of local gas hold-up for tap water with $Re_g = 536$ and $Re_L = 356$

Figure 7.11: Radial profiles of local gas hold-up for tap water with $Re_g = 716$ and $Re_L = 170$
Figure 8.1: Ratio of bubble rise velocity to liquid superficial velocity versus $Re_g$ for tap water system

Figure 8.2: Bubble rise velocity versus $Re_g$ for various $Re_L$ in tap water

Figure 8.3: Histograms of (a) bubble rise velocity and (b) pierced chord length for tap water with $Re_g = 233$ and $Re_L = 536$

Figure 8.4: Ratio of pierced chord length to particle diameter versus $Re_g$ for tap water system

Figure 8.5: Pierced chord lengths in tap water measured by lower tip of probe versus those measured by upper tip

Figure 8.6: Frequency of bubble passing vertically across probe tips versus $Re_g$

Figure 8.7: Percentage of bubbles passing probe completely vertically versus $Re_g$

Figure 8.8: Radial profiles of frequency of vertically rising bubbles, mean pierced chord length and mean bubble rise velocity for tap water

Figure 8.9: Ratio of bubble rise velocity to liquid superficial velocity versus $Re_g$ in aqueous 44 mass % glycerol solution

Figure 8.10: Ratio of pierced chord length to particle diameter versus $Re_g$ for aqueous 44 mass % glycerol solution

Figure 8.11: Histograms of (a) bubble rise velocity and (b) pierced chord length for aqueous 44 mass % glycerol solution with $Re_g = 104$ and $Re_L = 131$

Figure 8.12: Frequency and percentage of vertically rising bubbles versus $Re_g$ for aqueous 44 mass % glycerol solution
Figure 8.13: Average gas hold-up measured by conductivity probe versus gas
hold-up measured by pressure drops in aqueous 44 mass %
glycerol solution 174

Figure 8.14: Estimated gas hold-up based on equation (8.4) versus correlated
values from Figures 7.1 and 7.2 177

Figure 8.15: Estimated $U_g$ from equation (8.9) versus actual value for tap water
and aqueous 44 mass % glycerol solutions 179

Figure A.1: Schematic of (a) side view of the distributor, and (b) how the
distributor was attached to the column 203

Figure A.2: Design diagram for pre-mix area shown in Figure 3.1 204

Figure A.3: Side view of sparger 205

Figure A.4: Design drawing of column section immediately above distributor
$(z = 0$ to $0.61\text{ m})$ showing ports 206

Figure A.5: Design drawing of second column section above the distributor
$(z = 0.61$ to $1.22\text{ m})$ 206

Figure A.6: Design cross-sectional drawing of section immediately above
distributor (side view shown in Figure A.5) 207

Figure A.7: Design cross-sectional drawing of expanded section at top of column 208

Figure B.1: Schematic representation of two tubes in barometer set connected
to fluidized bed 209

Figure D.1: Local gas hold-up data presented in Figure 7.10 for $z = 0.051\text{ m}$ 238
Figure E.1: Radial profiles of local gas hold-up for aqueous 44 mass % glycerol solution with $Re_g = 233$ and $Re_L = 430$

Figure E.2: Radial profiles of local gas hold-up for aqueous 44 mass % glycerol solution with $Re_g = 356$ and $Re_L = 430$

Figure E.3: Radial profiles of local gas hold-up for aqueous 44 mass % glycerol solution with $Re_g = 170$ and $Re_L = 536$

Figure E.4: Radial profiles of local gas hold-up for aqueous 44 mass % glycerol solution with $Re_g = 233$ and $Re_L = 536$

Figure E.5: Radial profiles of local gas hold-up for aqueous 44 mass % glycerol solution with $Re_g = 470$ and $Re_L = 536$

Figure E.6: Radial profiles of local gas hold-up for aqueous 44 mass % glycerol solution with $Re_g = 170$ and $Re_L = 646$

Figure E.7: Radial profiles of local gas hold-up for aqueous 44 mass % glycerol solution with $Re_g = 233$ and $Re_L = 646$

Figure E.8: Radial profiles of local gas hold-up for aqueous 44 mass % glycerol solution with $Re_g = 356$ and $Re_L = 646$

Figure E.9: Radial profiles of local gas hold-up for aqueous 44 mass % glycerol solution with $Re_g = 470$ and $Re_L = 646$

Figure E.10: Radial profiles of local gas hold-up for aqueous 44 mass % glycerol solution with $Re_g = 233$ and $Re_L = 716$

Figure E.11: Radial profiles of local gas hold-up for aqueous 44 mass % glycerol solution with $Re_g = 356$ and $Re_L = 716$
Acknowledgements

A work of this nature is never a solo effort. It requires the help and collaboration of many people and I would like to thank those people at this time.

The first group I would like to thank is Syncrude Canada Ltd. for believing in this project and for their generosity in sponsoring the work. I would also like to especially thank Maja Veljkovic, Larry Hackman, and Craig McKnight for their encouragement, advice, and technical support.

The second group of people are those who work at the department. Thanks to Alex, Chris, Doug, Helsa, Horace, Lori, Peter, Qi, Robert, and Shelagh, for all their fantastic help, super suggestions, moral support, and friendship. I would also like to thank all the professors in general within the department who were always available for help and advice. I would especially like to thank Norm Epstein, Sheldon Duff, and Colin Oloman.

I would also like to thank Jesse Zhu for his belief in me, bringing me in from Montreal to work on this project, and for leading me through the initial learning curve involved in this project. Kok-Seng Lim also deserves a lot of thanks for the great help and moral support he provided. He also often spent some late nights with me working on the rig and I thank him for all his help. Arturo Macchi has also been a fantastic help and great sounding board for many of the ideas that are presented here.

Many friends made the transition to Vancouver easy and enjoyable and I must thank them for their encouragement and help. The Petroff family took me in and I can never repay their kindness for making my move painless and in treating me like family. I would also like to thank Linda, Steve, Anna, Irene, Margaret, Junping, Xiatao, Joe, Allan, Denis, Ian, Abba, and all the other past and present students that made the time at UBC pass quickly and enjoyably.

I especially thank my mentor, John Grace, who has been an exceptional project supervisor. He has continued to support, encourage and allow me the room within this research project to learn and grow. I appreciate his patience and his incredible ability to always make time for his students regardless of his busy schedule.

Last but certainly not least I would like to thank my family for continuing to support and encourage me. Thanks to my mom and dad, along with Anna, Peter, Andrew, and Elena for all their encouragement. I would also like to thank the newest additions to my family, Ingrid, Maureen, Michael, Melissa, and David for welcoming me into their family. Finally I would like to thank my wife, Margaret, who gave me the energy, desire, and love to finish this thesis. Certainly I could not have done it without her.
Chapter 1 - Introduction

Three-phase fluidized beds have been used in a wide range of applications including hydrotreating and conversion of heavy petroleum and synthetic crude, coal liquefaction, methanol production, sand filter cleaning, electrolytic tinning, conversion of glucose to ethanol, aerobic waste water treatment, and various hydrogenation and oxidation reactions (Fan, 1989; Wild and Poncin, 1996). There have been extensive studies into many aspects of three-phase fluidization and some excellent reviews since 1980 (Epstein, 1981; Wild et al., 1984; Darton, 1985; Muroyama and Fan, 1985; Fan, 1989; Nacef et al., 1992; Wild and Poncin, 1996; Kim and Kang, 1997). However, three-phase systems are complex and there remain many unanswered questions (Barkat and Diboun, 1995; Tarmy and Coulaloglou, 1992).

Within the field of three-phase systems there are several configurations and contacting modes. Three-phase fluidization requires three distinct phases. The present work is limited to gas-liquid-solid systems. Fluidized beds are further classified by the direction of flow: cocurrent upflow, cocurrent downflow, countercurrent, and liquid batch with gas upflow. A schematic showing these contacting modes, produced by Fan (1989), appears in Figure 1.1. The present work is limited to cocurrent upflow of the gas and liquid, the most widely used type of three-phase flow (Wild and Poncin, 1996).

When designing three-phase fluidized beds for industrial use, the kinetics, heat transfer, and mass transfer must all be considered. For a typical catalytic reaction, there are eight key steps to be considered (Smith, 1981):
<table>
<thead>
<tr>
<th>Mode Designation</th>
<th>E-I-a-1</th>
<th>E-I-a-2</th>
<th>E-I-b</th>
<th>E-I-a-1</th>
<th>E-I-a-2</th>
<th>E-II-b</th>
<th>E-III-a</th>
<th>E-III-b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schematic Diagram</td>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
<td><img src="image5" alt="Diagram" /></td>
<td><img src="image6" alt="Diagram" /></td>
<td><img src="image7" alt="Diagram" /></td>
<td><img src="image8" alt="Diagram" /></td>
</tr>
<tr>
<td>Continuous Flow</td>
<td>Liquid</td>
<td>Gas</td>
<td>Liquid</td>
<td>Gas</td>
<td>Liquid</td>
<td>Gas</td>
<td>Liquid</td>
<td>Gas</td>
</tr>
<tr>
<td>References (Chapters)</td>
<td>1, 2, 5, 7, 10, 11, A</td>
<td>1, 4, 6, 10, 11, A</td>
<td>1, 2, 11</td>
<td>1, 5, 6, 7, 8</td>
<td>5, 9, 11</td>
<td>1, 5, 7, 9, 11</td>
<td>1, 4, 7, 11</td>
<td>1, 5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode Designation</th>
<th>T-I-a-1</th>
<th>T-I-a-2</th>
<th>T-I-b</th>
<th>T-II-a</th>
<th>T-II-b</th>
<th>T-III-a</th>
<th>T-III-b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schematic Diagram</td>
<td><img src="image9" alt="Diagram" /></td>
<td><img src="image10" alt="Diagram" /></td>
<td><img src="image11" alt="Diagram" /></td>
<td><img src="image12" alt="Diagram" /></td>
<td><img src="image13" alt="Diagram" /></td>
<td><img src="image14" alt="Diagram" /></td>
<td><img src="image15" alt="Diagram" /></td>
</tr>
<tr>
<td>Continuous Flow</td>
<td>Liquid</td>
<td>Gas</td>
<td>Liquid</td>
<td>Gas</td>
<td>Liquid</td>
<td>Gas</td>
<td>Liquid</td>
</tr>
<tr>
<td>References (Chapters)</td>
<td>1, 6, 7, 8, 11</td>
<td>1, 4, 10, 11, A</td>
<td>1, 6</td>
<td>1, 9, 11</td>
<td>1, 9</td>
<td>1, 6, 11</td>
<td>1, 6, 9</td>
</tr>
</tbody>
</table>

(S) Independent Charge of Solids from Fluid  (S) Independent Discharge of Solids from Fluid  (+S) Charge or Discharge of Solids with Fluid  
A = Appendix A

Figure 1.1: Modes of gas-liquid-solid fluidization [reprinted from Fan, 1989]
1) Gas-liquid interface mass transfer,
2) Liquid-solid interface mass transfer,
3) Internal diffusion to the catalyst surface,
4) Adsorption of reactants at the catalyst surface,
5) Reaction on the catalyst surface,
6) Desorption of adsorbed products,
7) Transport of products from catalyst interior sites to outer surface,
8) Transport of product from catalyst surface to bulk fluid.

Depending on the system, any of these can be the rate-limiting step. To completely understand a fluidized bed, each of these aspects must be studied. The first of these steps is commonly the rate-controlling one, and it is clear that it depends strongly on bed hydrodynamics. For this work, only the bed hydrodynamics were studied: specifically the gas hold-up, bed expansion, average bubble size and bubble rise velocity. For a given reactor volume and fluid flow rates, a cloud of small, spherical, slow-moving bubbles will have a greater opportunity to allow mass transfer than a few large, quick-moving slugs. A better understanding of hydrodynamics would also be helpful in understanding axial and lateral dispersion, factors which are also important in multiphase reactors.

The work presented here deals with the hydrodynamics within a cocurrent upflowing gas-liquid-solid fluidized bed. The major motive is to gain a better understanding of bed behaviour for industrial applications such as the production of synthetic crude oil from bitumen and to explore a new approach for using small-scale cold-flow experimental units to predict behaviour under industrial conditions.
Traditional engineering research has focused on "big" breakthroughs resulting in new processes or dramatic improvements of existing technologies. Without doubt, this is a worthwhile goal; however, recent efforts have been directed more towards fundamental understanding and small but steady improvements to maximize process efficiency. The work in this thesis is an attempt to accomplish exactly that. By proposing and validating a new method of simulating industrial processes, some fundamental engineering information has been gathered to help gain a better understanding of the process being simulated.

Fan (1989) identified three flow regimes in which three-phase fluidized beds can operate: bubbling, slugging, and transport. Within the bubbling regime, there are two sub-categories: the dispersed bubble and coalesced bubble regimes. The separation between regimes is often qualitative and not well defined. Zhang (1996) and Zhang et al. (1996) identified seven distinct flow regimes for gas-liquid-solid co-current fluidized beds and identified a number of quantitative methods for determining transitions:

1) Dispersed bubble flow: Usually corresponds to high liquid velocities and low gas velocities. Results in small bubbles of relatively uniform size. Little bubble coalescence despite high bubble frequency.

2) Discrete bubble flow: Usually occurs at low liquid and gas velocities. It is similar to the previous regime with respect to small bubble size and uniform size. However, the bubble frequency is lower.

3) Coalesced bubble flow: Usually found at low liquid velocities and intermediate gas velocities. The bubbles are larger and show a much wider size distribution due to increased bubble coalescence.
4) Slug flow: Not often seen in industrial applications, this regime is characterized by large bullet shaped bubbles with a diameter approaching that of the column and lengths that exceed the column diameter. Some smaller bubbles are also observed, especially in the wakes of the slugs.

5) Churn flow: Churn flow is similar to the previous regime, but much more chaotic and frothy. As gas flow is increased, an increase in downward liquid flow near the wall is usually observed. Note that while Darton (1985) also identified a Churn-turbulence regime, he defined it as the transition between bubbling and slug flow, and his definition, based upon two-phase fluidized systems, should probably be classified as a coalescing bubble regime.

6) Bridging flow: A transitional regime between churn flow and annular flow where liquid and solids effectively form “bridges” across the reactor which are continuously broken and re-formed.

7) Annular flow: At extremely high gas velocities, a continuous gas phase appears in the core of the column.

A schematic of these flow regimes reprinted from Zhang (1996) is shown in Figure 1.2. Under Fan’s previous classification of regimes (Fan, 1989), Zhang’s dispersed bubble flow, discrete bubble flow, and coalesced bubble flow can be grouped under the heading “bubbling regime”, while churn flow, bridging flow, and annular flow can all be classified as belonging to the transport regime.

The bubble sizes and shapes in the bubbling, slugging, and transport regimes differ considerably from one another. As mentioned above, slug flow has bullet shaped bubbles with a cross-sectional dimension almost equal to the reactor column. In
Figure 1.2: Schematic diagram of flow patterns [reprinted from Zhang, 1996]
the transport regime, bubbles are practically non-existent as the gas forms a continuous phase as it conveys liquid droplets and solid particles through the fluidized bed. In the bubbling regime, under dispersed bubble flow, the bubbles tend to be spherical, small and relatively uniform in size. For coalesced flow, however, the bubbles tend to be larger, with a wider size distribution. Spherical-cap or spheroidal bubbles are also commonly found, and these can have significant wakes that also affect the reactor performance (Matsuura and Fan, 1984). Wakes are responsible often for increased particle mixing and are the reason some beds contract initially when the gas flow is increased (Epstein and Nicks, 1976).

Another important aspect of the flow regimes is the particle action. In a transport system, many particles are entrained and removed from the system; hence particles need to be replenished continuously. In a bubbling bed, entrainment of particles is much less significant, although some caution must still be taken to prevent emptying of the column over time.

Although most classifications of the fluidization regime are related to liquid and gas superficial velocities, it is important to also consider particle size, shape, and density (Muroyama and Fan, 1985)

1.1 Advantages and disadvantages of fluidized bed technology

There are several advantages of fluidized beds relative to fixed bed processes (Le Page et al., 1992; Beaton et al., 1986):

a) The ability to maintain a uniform temperature. This is especially important if one wishes to limit possible competing reactions.
b) Significantly lower pressure drops which reduce pumping costs.

c) Catalysts may be withdrawn, reactivated, and added to fluidized beds continuously without affecting the hydrodynamic performance of the reactor. This also allows the catalyst activity to be controlled.

d) Bed plugging and channeling are minimized due to movement of the solids.

e) Lower investments for the same feed and product specifications.

f) New improved catalysts can replace older catalysts with minimal effort.

There are, however, also some disadvantages to fluidized beds:

a) Catalyst attrition due to particle motion.

b) Special care must be made to minimize carryover of particles.

c) Reactor size must be larger than for fixed beds due to bed expansion.

d) Catalyst-fluid contact per unit volume is reduced due to bed expansion.

1.2 Bitumen processing

At the United Nations' third international conference on heavy crude and tar sands, held in 1985, it was estimated that about 70% of the world's oil supply consists of heavy crude and bitumen. Canada has about 82% of the world's bitumen in its oil sands deposits (Parkinson, 1995), found primarily in Northern Alberta between Fort McMurray, Peace River and Lloydminster. Deposits include the Athabasca, Peace River, Cold Lake, Wabasca, and Buffalo Head Hills deposits. Some estimates place the production potential of these five deposits at 1.7 billion barrels, approximately five times the conventional oil reserves of Saudi Arabia and enough to meet Canada's energy demands for the next 475 years (Syncrude, 1993). Current oil sands projects produce over 40% of
Canada's yearly oil needs (Financial Post, 1997). However, bitumen is considerably more difficult to extract than conventional crude oil. Heavy oils also have higher percentages of sulfur and metals, resulting in processing difficulties.

Oil sands are composed of sand, bitumen, mineral rich clays and water. The composition of the oil sands mined by Syncrude at Mildred Lake averages 10.5% bitumen, and 4% water (Syncrude, 1993). To separate the bitumen from the other components, a hot water separation process is usually used (Bott, 1993). In some cases, when mining is difficult, this separation is done in situ. The oil sands are mixed with hot water and caustic soda, and aerated. This results in a layer of froth, containing most of the bitumen, floating on water. The solid clays and sand tend to settle to the bottom. The bitumen froth is then de-aerated and naphtha added as diluent. This mixture is sent to a series of settlers, centrifuges and a naphtha-recovery unit where the bitumen is recovered. Typically, over 91% of the bitumen from the oil sands is recovered in this manner (Syncrude, 1993).

In its natural state, bitumen has a viscosity greater than 10 Pa·s presenting transport problems. To convert it to a usable product requires upgrading. There are several methods which can accomplish this goal: H-Oil, LC Fining, Coking, HYCON, RDS, Resid Fining, Gulf HDS, etc. (Le Page et al., 1992). These processes all break the long-chain bitumen molecules into lighter fractions at high pressures and temperatures. There are two major routes to accomplish this: hydrogen addition, and carbon rejection. In hydrogen addition, as in LC-Finers, hydrogen gas is mixed with the bitumen in the presence of catalysts. The long-chain bitumen molecules are then broken as hydrogen
breaks some carbon-to-carbon bonds. In carbon rejection, as in Cokers, elevated temperatures cause long hydrocarbon molecules to thermally crack forming coke.

The present project is strongly related to the three-phase fluidized bed LC Finer (Le Page et al., 1992). Typical industrial LC-Finer reactors are of the order of 4.5 m in diameter, over 30 m high and have volumes in excess of 250 m$^3$ (Le Page et al., 1992). The catalysts used in the LC-Finer are porous cylindrical ceramic particles with catalytic metals deposited on their surface. From the LC-Finer, the lighter fractions are sent to a hydrotreater where sulfur and nitrogen are removed to form synthetic crude oil. Typical synthetic crude oils contain no residue, while conventional crude oils contain as much as 8% residue (Syncrude, 1993).

In the early 1980s a barrel of sweet crude oil sold for about $37 US. By November, 1998, the price was down to $12.12 (US dollars) per barrel. Typical production costs are approximately $12 Canadian per barrel (Syncrude News Release, 1998), compared to over $20 Canadian per barrel in 1997. Based upon a prediction of continued Asian-market troubles, some experts are forecasting even lower oil prices (Canadian Press, 1998). As of April 28, 1997, more than a dozen companies had committed to investing over 25 billion dollars into various oil sands projects by the year 2020 (Financial Post, 1997). Clearly, in order to stay profitable, oil production costs must be minimized.

1.3 Scaling: The journey from laboratory unit to industrial application

Although there has been extensive effort already on the hydrodynamics of three-phase fluidized beds, a major problem that continues to limit their industrial application
involves difficulties in scaling-up the results from small laboratory units to larger industrially significant units. It is common for results found with small scale test units to be unachievable when the unit size is increased. This problem is due in part to the materials used in the bulk of the previous work: air, water and spherical glass beads, while industrial applications mostly involve non-spherical particles at high temperatures and pressures. These temperatures and pressures result in fluid properties considerably different from air and water (Tarmy et al., 1984a,b; Jiang et al., 1992; Luo et al., 1997a,b).

This problem is not unique to three-phase systems and has also plagued two-phase fluidization. The first major documented case of a severe scale-up problem was in a Fischer-Tropsch synthesis plant in Brownsville, Texas in the early 1950s (Squires et al., 1985). The conversion achieved in the industrial unit was only about 50% of that achieved in the laboratory reactors. Such a large reduction in conversion would cripple most processes.

This thesis presents a novel approach designed to prevent scale-up surprises and to make experimental results from small laboratory units more pertinent to industrially relevant reactors. In particular, the approach focuses on ensuring effective scaling of bed hydrodynamics. For two-phase systems, scale-up problems are almost always attributable to bed hydrodynamics. Since bed hydrodynamics control the degree of fluids-solids contact, a good understanding of the hydrodynamics is essential to eliminate significant scale-up effects. For two-phase systems, much work has been carried out to solve this problem. Glicksman et al. (1994) presented a comprehensive review outlining the development and application of scaling laws for two-phase fluidized bed systems.
Horio et al. (1986) also presented an approach for scaling up two-phase systems based on manipulating and non-dimensionalizing the governing equations. However, this approach has not been applied previously to three-phase systems.

In the current work, dimensional similitude and the Buckingham Pi Theorem are used in Chapter 2 to form a set of parameters for achieving dynamic similarity. This approach is especially important for systems that are mass-transfer controlled and hence controlled by bed hydrodynamics. The experimental equipment is outlined in Chapter 3. The measurement techniques and methodology are given in Chapter 4. Experiments are then presented to validate the approach in Chapter 5. Bed expansion measurements for various conditions are examined in Chapter 6. Gas hold-ups and bubble properties are investigated in Chapters 7 and 8, respectively.

1.4 Some definitions of fluidization phenomena

Some of the characteristics to be studied in this work are defined in Table 1.1. Figure 1.3 is a schematic of a typical co-current gas-liquid-solid fluidized bed operating in the dispersed or coalescing bubble flow regimes. In Figure 1.3 some fundamental regions of the bed are identified.
Table 1.1: Definitions of some three-phase fluidization phenomena

<table>
<thead>
<tr>
<th>Term</th>
<th>Symbol</th>
<th>Physical Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bed expansion</td>
<td>$\beta_{be}$</td>
<td>Measures the extent of fluidization of the bed, [ \beta_{be} = \frac{H_e - H_o}{H_o} ]</td>
</tr>
<tr>
<td>Gas hold-up</td>
<td>$\varepsilon_g$</td>
<td>Measures the fractional volume occupied by the gas, [ \varepsilon_g = \frac{\text{volume of gas}}{\text{total bed volume}} ]</td>
</tr>
<tr>
<td>Liquid hold-up</td>
<td>$\varepsilon_L$</td>
<td>Measures the fractional volume occupied by the liquid, [ \varepsilon_L = \frac{\text{volume of liquid}}{\text{total bed volume}} ]</td>
</tr>
<tr>
<td>Solids hold-up</td>
<td>$\varepsilon_p$</td>
<td>Measures the fractional volume occupied by the solids, [ \varepsilon_p = \frac{\text{volume of solids}}{\text{total bed volume}} ] Note that: $\varepsilon_g + \varepsilon_L + \varepsilon_p = 1$</td>
</tr>
<tr>
<td>Porosity</td>
<td>$\varepsilon$</td>
<td>Measures the volume occupied by both the liquid and the gas [ \varepsilon = \varepsilon_g + \varepsilon_L = 1 - \varepsilon_p ]</td>
</tr>
<tr>
<td>Bubble rise velocity</td>
<td>$U_b$</td>
<td>The actual velocity of a bubble as it rises through the bed. Note that $\varepsilon_g \approx U_g/U_b$</td>
</tr>
<tr>
<td>Minimum fluidization velocity</td>
<td>$U_{mf}$</td>
<td>The minimum superficial velocity at which the bed becomes fluidized. For the operating conditions in this work, the particles are fluidized primarily by the liquid.</td>
</tr>
</tbody>
</table>
Figure 1.3: Schematic representation of a typical co-current gas-liquid-solid fluidized bed
1.5 Research Objectives

The following goals provided the impetus for the present work:

1) To establish dynamic and geometric similarity criteria with which to scale up three-phase data from cold-flow units to industrial conditions and to validate the approach experimentally;

2) To measure gas hold-up under conditions pertinent to those used in industry based on the scaled series of parameters for a co-current three-phase fluidized bed;

3) To measure bed expansion under conditions pertinent to those used in industry based on the scaled series of parameters for a co-current three-phase fluidized bed;

4) To measure bubble characteristics, in particular bubble rise velocity and bubble diameter, under conditions pertinent to those used in industry based on the scaled series of parameters for a co-current three-phase fluidized bed.
Chapter 2 - Basis for Dimensional Analysis and its Application

To design a three-phase fluidized bed chemical reactor many different aspects must be predicted and quantified. Most often, to achieve desired reactor goals, chemical reaction kinetic rates and stoichiometry dictate operating temperatures and pressures. The fluids are mandated by the reaction one wishes to achieve, while the fluid properties are directly related to the specified temperature and pressure. Availability and cost of catalysts frequently establish the particle shape and size range. Care must also be taken to provide adequate heat transfer within the reactor to meet the overall requirements. Given these operating parameters, one must also ensure that global reaction rates, which, as discussed above, may depend on mass transfer rates, are sufficient to meet the goals. To do so requires fundamental knowledge of how the operating conditions affect the hydrodynamics. For the given fluid and solids properties, the operating gas and liquid superficial velocities must then be set and the reactor size determined based upon the expected bed expansion and hold-ups.

There are essentially three ways of addressing this last item: laboratory studies, pilot-plant studies, and mock-up studies (Euzen et al., 1993). In a laboratory study, the fluidized bed hydrodynamics, or any other aspect of the process (reaction kinetics for example), may be studied for various differing fluid and solids under idealized conditions. This is definitely the least expensive method and can generate significant amounts of data relatively quickly. In a pilot-plant study, the work is focused more closely on the expected operating conditions of the full-scale reactor. This may include operating the unit for extended periods of time and varying process parameters such as operating conditions and feed impurities. Even at this level of process-directed research,
not all scale-up issues are addressed adequately (Euzen et al., 1993). In mock-up studies, experiments are conducted in a unit that is typically about 1/10 the linear size of the expected final design. This is a very expensive process and can delay final reactor design by several years (Euzen et al., 1993) due to the time it takes to commission and run such a unit. Often the implementation of a process can go through all three methods since scale-up problems, as discussed above, have led to caution within industry.

2.1 Existing information for hydrodynamic predictions

There is a significant amount of detailed research data available in the literature based on experiments conducted in small-scale columns. Some of the more widely known correlations for gas hold-up and bed porosity, directly related to bed expansion, together with the authors who presented the work and some information on the systems studied, are listed in Table 2.1. Correlations for bubble rise velocity and bubble diameter are summarized in Table 2.2.

2.1.1 Effect of surface tension

In recognizing that the results from air-water-glass bead systems do not accurately predict the hold-up of industrial units, several authors have focused on obtaining higher gas hold-ups in their laboratory columns by lowering the surface tension through the use of surfactants (e.g. Kelkar et al., 1983; Fan et al., 1987; Gorowara and Fan, 1990). The results show that as the surface tension is lowered with surfactants, measured gas hold-ups increase as the bubbles become smaller and more rigid, thereby lowering the bubble rise velocity and the tendency to coalesce. A problem associated with the use of
Table 2.1: Literature correlations of gas hold-up and bed expansion in three-phase fluidized beds

<table>
<thead>
<tr>
<th>Authors</th>
<th>Correlation</th>
<th>Gas/Liquid/Solids or Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baker et al. (1977) cited in El-Temtamy and Epstein (1978)</td>
<td>$\varepsilon = \left[ \frac{U_L - U_g k}{U_t (1 - \varepsilon_g - k \varepsilon_g)} \right]^{1/n} \left[ 1 - \varepsilon_g (l + k) \right] + \varepsilon_g (l + k)$</td>
<td>Based on wake model</td>
</tr>
<tr>
<td>(k = 1.617 \left( \frac{U_L}{U_g} \right)^{0.610} \sigma^{-0.654} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Begovich and Watson (1978)</td>
<td>$\varepsilon = \left( 0.048 \right) U_g^{0.720} D_p^{0.168} D_e^{-0.125}$</td>
<td>169 data points from Kim et al., 1975; Bhatia and Epstein, 1974; Michelsen and Ostergaard, 1970; Efremov and Vakhrushev, 1970; Ostergaard and Michelsen, 1968</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon = 1.057 U_g^{0.271} U_L^{0.041} (\rho_p - \rho_L)^{0.316} D_p^{-0.268} U_t^{0.035} D_e^{-0.033}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>cited by Wild and Poncin (1996) as best for $\varepsilon$ (along with Han et al., 1990)</td>
<td></td>
</tr>
<tr>
<td>Bhatia and Epstein (1974)</td>
<td>$\varepsilon = \left[ \frac{U_L - U_g k (1 - x)}{U_t (1 - \varepsilon_g - k \varepsilon_g)} \right]^{1/n} \left[ 1 - \varepsilon_g (l + k - kx) \right] + \varepsilon_g (l + k - kx)$</td>
<td>Generalized wake model</td>
</tr>
<tr>
<td></td>
<td>$k = \left( 0.61 + \frac{0.037}{\varepsilon_g + 0.013} \right) e^3$</td>
<td></td>
</tr>
<tr>
<td>Bloxom et al. (1975) cited in Wild and Poncin (1996)</td>
<td>$\varepsilon_g = 0.159 \left( \frac{U_g (\rho_p - \rho_L)}{g U_L \sigma} \right)^{0.150}$</td>
<td>Suggested as best correlation of an &quot;unreliable&quot; lot by Wild and Poncin (1996)</td>
</tr>
<tr>
<td>Chern et al. (1984)</td>
<td>For dispersed bubble regime:</td>
<td>Air/water/glass spheres &amp; PVC cylinders</td>
</tr>
<tr>
<td></td>
<td>$\frac{U_g}{\varepsilon_g} = \frac{U_g}{1 - \varepsilon_p} + \frac{U_L}{1 - \varepsilon_p} + 10.16 + 14.88 \sqrt{\frac{U_g}{1 - \varepsilon_p}}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>For coalesced bubble regime:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_g = 0.098 U_L^{-0.98} U_g^{0.70}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>For slug flow regime:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{U_g}{\varepsilon_g} = 1.783 \left( \frac{U_g}{1 - \varepsilon_p} + \frac{U_L}{1 - \varepsilon_p} \right) + 0.35 \sqrt{g D_e}$</td>
<td></td>
</tr>
<tr>
<td>Costa et al. (1986)</td>
<td>$\varepsilon = \frac{3 f_D (1 - \alpha) (U_L - U_g)^2}{4 f_D [p_p - (1 - \alpha) \rho_L]} \frac{1}{n_c}$</td>
<td>Water &amp; aqueous solutions of CMC/air, He, CO$_2$, CH$_4$/glass, aluminum and benzoic acid covered with a paint film</td>
</tr>
<tr>
<td></td>
<td>for $Re_p &lt; 10^3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_D = 24/Re_p \left( 1 + 0.15 Re_p^{-0.687} \right)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>for $10^3 &lt; Re_p &lt; 10^5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_D = 0.44$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$n_c = 5.7 - 8 U_g$</td>
<td></td>
</tr>
<tr>
<td>Dakshinamurty et al. (1971)</td>
<td>( \alpha = \frac{3.464 \times 10^{-9} U_L^{-0.66} (\phi_d)^{0.30} \rho_p^{0.30}}{1 + 1.74 (\frac{U_L}{U_L + U_g})^{0.34} H_0^{0.43} (\rho_L - \rho_g)^{0.08} \mu_L^{0.08} D_0^{0.23}} )</td>
<td>Air/water/glass beads, iron shot, sand or Rockwool shot</td>
</tr>
<tr>
<td>Dakshinamurty et al. (1972)</td>
<td>( \varepsilon = 2.12 (\frac{U_L}{U_t})^{0.41} (\mu_L U_t / \sigma)^{0.08} ) for Re ( t &lt; 500 )</td>
<td>Air/water/glass beads &amp; lead shot</td>
</tr>
<tr>
<td></td>
<td>( \varepsilon = 2.85 (\frac{U_L}{U_t})^{0.8} (\mu_L U_t / \sigma)^{0.08} ) for Re ( t &gt; 500 )</td>
<td>Air/water/glass beads, kerosene/glass beads, iron shot or Rockwool shot</td>
</tr>
<tr>
<td>Darton and Harrison (1975)</td>
<td>( \varepsilon = \left[ \frac{U_L - U_g k}{U_t (1 - \varepsilon_g - k \varepsilon_g)} \right]^{1/n} ) ( [1 - \varepsilon_g (1 + k)] + \varepsilon_g (1 + k) ) ( k = 1.4 \left( \frac{U_L}{U_g} \right)^{0.33} - 1 )</td>
<td>Based on wake model</td>
</tr>
<tr>
<td>El-Temtamy (1974) cited in El-Temtamy and Epstein (1978)</td>
<td>( \varepsilon = \left[ \frac{U_L - U_g k}{U_t (1 - \varepsilon_g - k \varepsilon_g)} \right]^{1/n} ) ( [1 - \varepsilon_g (1 + k)] + \varepsilon_g (1 + k) ) ( k = 0.462 \left( \frac{U_L}{U_g} \right) d_p^{-0.33} )</td>
<td></td>
</tr>
<tr>
<td>Fan et al. (1987)</td>
<td>( \varepsilon = 1.837 F_{g, dh}^{0.315} \rho_{g, dh}^{-0.909} M_0^{0.02} C^{0.145} (1 + 34.09 d_p^{-0.346}) ) ( ) where ( C ) is the term for the number of carbon atoms in the alcohol molecule</td>
<td>Air/water-alcohol solutions/glass beads</td>
</tr>
<tr>
<td></td>
<td>( ) Surfactants added</td>
<td></td>
</tr>
<tr>
<td>Gorowara and Fan (1990)</td>
<td>Low foaming solutions similar to water: ( \varepsilon_g = 0.132 F_{g}^{0.285} R_{CL}^{0.0892} )</td>
<td>Air/water-alcohol solutions/glass beads</td>
</tr>
<tr>
<td></td>
<td>Medium foaming: ( \varepsilon_g = 0.132 F_{g}^{0.3117} R_{CL}^{0.1166} )</td>
<td>Surfactants added</td>
</tr>
<tr>
<td></td>
<td>Middle and high gas hold-up regions: ( \varepsilon_g = 0.8140 F_{g}^{0.3997} R_{CL}^{0.0977} )</td>
<td></td>
</tr>
<tr>
<td>Han et al. (1990)</td>
<td>For initial expansion: ( \varepsilon = (\frac{U_L}{U_t})^{1/n} (1 + 0.123 F_{g}^{0.347} W_{em}^{0.037}) )</td>
<td>Air/water or aqueous glycerol solution/glass beads</td>
</tr>
<tr>
<td></td>
<td>For initial contraction: ( \varepsilon = (\frac{U_L}{U_t})^{1/n} \left[ 0.359 F_{g}^{0.552} W_{em}^{0.124} + \exp \left( \frac{U_L}{U_t} \right)^{0.305} \right] F_{g}^{0.25} )</td>
<td>combined with over 5000 points from the literature</td>
</tr>
<tr>
<td></td>
<td>( \varepsilon_L = (\frac{U_L}{U_t})^{1/n} (1 - 0.374 F_{g}^{0.178} W_{em}^{0.173}) )</td>
<td>cited by Wild and Poncin (1996) as the best for ( \varepsilon_L ) and ( \varepsilon )</td>
</tr>
<tr>
<td>Hassamien, et al. (1984) cited in Elenkov et al. (1986) &amp; Nacef (1991)</td>
<td>( \varepsilon = 0.205 U_L^{0.388} U_g^{0.061} (\rho_s - \rho_L)^{0.231} d_p^{0.305} )</td>
<td></td>
</tr>
<tr>
<td>Author(s) (Year)</td>
<td>Equation</td>
<td>Notes</td>
</tr>
<tr>
<td>-----------------</td>
<td>----------</td>
<td>-------</td>
</tr>
<tr>
<td>Hu et al. (1986)</td>
<td>$e_g = 0.0645 U_g^{0.842} U_L^{-0.095} \mu_L^{-0.419}$</td>
<td>Air/water/glass beads</td>
</tr>
<tr>
<td>Jean and Fan (1986)</td>
<td>$\frac{d_p}{D_L} = \left( \frac{\rho_p}{\rho_p - \rho_L} \right)^{1.6} \left( \frac{U_L}{U_g} \right)^{-0.896}$</td>
<td>Air/water/glass, alumina or lead spherical or near-spherical particles</td>
</tr>
<tr>
<td>Kato et al. (1981)</td>
<td>$e_L = \left[ 1 - 9.7 \left( \frac{350 + Re_L^{1.1}}{0.092} \right)^{0.5} \right]^{0.02} \left( \frac{U_L}{U_g} \right)^{-0.896}$</td>
<td>Air/water or aqueous solutions of CMC/glass beads</td>
</tr>
<tr>
<td>Kato et al. (1985)</td>
<td>$\frac{1}{f_2} = 0.1(1 + 4.43K_{0.165}^{0.9}) Re_L^{0.9}$</td>
<td>$f_2$ is a function of $Re$, and is plotted in the article</td>
</tr>
<tr>
<td>Kelkar et al. (1983)</td>
<td>$e_g = 0.3 \left[ \left( 1 + 0.01 Re_L^{0.62} \right) \left( 1 + 1.1y^{1.5} \right) \right]^{0.19}$</td>
<td>Air/water-alcohol solutions/glass beads Surfactants added</td>
</tr>
<tr>
<td>Kim et al. (1975)</td>
<td>$e = 1.40 Fr_L^{0.170} We_L^{0.878}$</td>
<td>Air/water or aqueous acetone or aqueous sugar or aqueous CMC/glass beads or gravel</td>
</tr>
<tr>
<td>Lee and de Lasa (1986)</td>
<td>$e = 1 - 0.631 \exp(-0.418 U_L - 0.004 U_g)$</td>
<td>Air/water/glass beads</td>
</tr>
<tr>
<td>Nacef et al. (1992)</td>
<td>Based upon definition of slip-velocity: $e_g = e \left( \frac{U_g - U_{GL}}{U_g + U_{GL}} \right)$, $e_L = e \left( \frac{U_L + U_{GL}}{U_g + U_L} \right)$</td>
<td>Data compiled for 15 different research systems to correlate $z_1$ and $z_2$ (dependent on distributor and column diameter)</td>
</tr>
<tr>
<td>Peterson et al. (1987)</td>
<td>For 152-mm diameter column: $e = 0.427 \left[ 1 + (46.9 U_L)^{4.91} \right]^{0.08}$ for 76-mm diameter column: $e = 0.422 \left[ 1 + (29.2 U_L)^{4.91} \right]^{0.05}$ Hydrogen/organic solvent mixture or water/glass cylinders</td>
<td></td>
</tr>
<tr>
<td>Saberian-Broudjenni et al. (1987)</td>
<td>$e = \left( \frac{14 \phi_s}{1 + \phi_s} \right)^{1/3} \left[ \frac{U_L}{U_{mfo}} \right]^{0.27} \left[ 1 + 0.070 \left( \frac{\rho_L U_p d_p}{\mu_L} \right)^{0.34} \right]$</td>
<td>$N_2$, He or CO$_2$/cyclohexane, kerosene, gas oil, tetrachloroethylene, or water/2 types of glass beads and 2 types of cylindrical catalyst particles</td>
</tr>
<tr>
<td>Source</td>
<td>Equation</td>
<td>Description</td>
</tr>
<tr>
<td>------------------------</td>
<td>--------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Sinha et al. (1984)</td>
<td>$\varepsilon = 0.125 \text{Re}_L^{1.1} \left( \frac{U_L}{\sigma L} \right)^{-0.40} \left( \frac{U_g}{\sigma L} \right)^{0.431} - 0.201 \text{Fr}_g^{0.167} - 1.714 \text{Re}_g^{-0.33} \text{Fr}_g^{0.205}$</td>
<td>Air/aqueous $t$-pentanol solution/cylindrical particles</td>
</tr>
</tbody>
</table>
| Song et al. (1989)     | $\varepsilon = 7.62 U_L^{0.204} U_g^{0.139} (\rho_p - \rho L)^{-0.252} (d_p \phi)^{0.175} \mu_L^{0.000}$ | For dispersed large bubble regime: $\varepsilon_g = 1.81 \text{Fr}_g^{0.221} \text{Re}_L^{-0.431} \text{Mo}^{0.020}$  
For transition regime: $\varepsilon_g = 0.654 \text{Fr}_g^{0.385} \text{Re}_L^{-0.051} \text{Mo}^{0.020}$  
For dispersed small bubble regime $\varepsilon_g = 2.61 \text{Fr}_g^{0.210} \text{Re}_L^{-0.372} \text{Mo}^{0.020}$  
Air/water/cylindrical particles |
| Song et al. (1989)     | For coalesced bubble regime: $\varepsilon_g = 0.342 \text{Fr}_g^{0.073} \text{Re}_L^{-0.192}$  
For dispersed bubble regime: $\varepsilon_g = 0.280 \text{Fr}_g^{0.156} \text{Re}_L^{-0.0873}$ | nitrogen/n-heptane/cylindrical extrudates                                   |
| Soung (1978)           | $\varepsilon = \frac{(\text{Ho})}{(\text{He})_{U_g=0}} = 1.0 - 1.22 \left( \frac{U_L}{U_t} \right)^{1.20}$ For $U_L/U_t \geq 0.25$  
$\varepsilon = \frac{(\text{Ho})}{(\text{He})_{U_g=0}} = 1.0 - 4.50 \left( \frac{U_L}{U_t} \right)^{2.15}$ For $U_L/U_t \leq 0.25$ | Air/aqueous sucrose solution/Ca-alginate gels                               |
| Sun et al. (1988)      | $\varepsilon = 2.1 \times 10^{-3} \text{Fr}^{0.41} \text{Bo}^{0.80} \text{Ga}^{0.10}$ | Air/aqueous $t$-pentanol solution/cylindrical particles                     |
| Tavakoli (1989)        | $\varepsilon_L = 0.033 U_g^{0.68} d_p^{0.07} \mu_L^{-0.02}$ | Air/water/cylindrical particles                                             |
| Yu and Kim (1988)      | $\varepsilon = 3.697 d_p^{0.309} U_L^{-0.022} U_g^{0.701}$ | Air/aqueous sucrose solution/Ca-alginate gels                               |
| Zaida et al. (1990)    | $\varepsilon_p = 0.03 \text{Fr}^{0.23} \text{Fr}_L^{-0.24} \text{Mo}^{-0.07}$ | Air/water/glass beads                                                      |
| Zheng et al. (1988)    | $\varepsilon_p = \frac{z_1}{z_2} \text{Fr}^{22} \text{Ar}^{23} \text{Bo}^{24} \left( 1 + \frac{U_L}{U_t} \right)^{25} \left( 1 - \varepsilon_p \right)^{26}$ | Air/water/glass beads                                                      |

For "homogeneous bubbling":  
$Z_1 = 0.281$  
$Z_2 = 0.243$  
$Z_3 = -0.0159$  
$Z_4 = 0.135$  
$Z_5 = -0.150$  
$Z_6 = 0.104$  
$0.00611 \leq \text{Fr} \leq 0.134$  
$109 \leq \text{Ar} \leq 39600$  
$3.26 \leq \text{Bo} \leq 136$  
$0 \leq 1/\beta_u \leq 4.20$  

For "transition":  
$Z_1 = 36.5$  
$Z_2 = 0.229$  
$Z_3 = -0.411$  
$Z_4 = 0.756$  
$Z_5 = -0.152$  
$Z_6 = 0.320$  
$0.0414 \leq \text{Fr} \leq 0.698$  
$109 \leq \text{Ar} \leq 39600$  
$3.25 \leq \text{Bo} \leq 135$  
$0 \leq 1/\beta_u \leq 1.62$  

For "turbulent bubbling":  
$Z_1 = 42.2$  
$Z_2 = 0.203$  
$Z_3 = -0.428$  
$Z_4 = 0.760$  
$Z_5 = -0.160$  
$Z_6 = 1.77$  
$0.113 \leq \text{Fr} \leq 10.3$  
$109 \leq \text{Ar} \leq 39600$  
$3.25 \leq \text{Bo} \leq 136$  
$0 \leq 1/\beta_u \leq 0.977$
Table 2.2: Literature correlations for bubble rise velocity and diameter in three-phase fluidized beds

<table>
<thead>
<tr>
<th>Authors</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deshpande et al. (1992)</td>
<td>$U_{br} = z_1 d_p^{22} U_L^{23} U_g^{24} \mu_L^{25} \sigma^{26}$</td>
</tr>
<tr>
<td></td>
<td>For coalesced regime &amp; 0.0025 ≤ $d_p$ ≤ 0.005 m:</td>
</tr>
<tr>
<td></td>
<td>$Z1 = 0.0180$, $Z2 = -0.7544$, $Z3 = -0.4219$, $Z4 = 0.0569$, $Z5 = 0.025$, $Z6 = 0.179$</td>
</tr>
<tr>
<td></td>
<td>For dispersed bubble &amp; 0.0025 ≤ $d_p$ ≤ 0.005 m:</td>
</tr>
<tr>
<td></td>
<td>$Z1 = 0.1204$, $Z2 = -0.4111$, $Z3 = -0.8027$, $Z4 = 0.6621$, $Z5 = 0.025$, $Z6 = 0.179$</td>
</tr>
<tr>
<td></td>
<td>For $U_L &lt; 0.095$ m/s &amp; 0.001 ≤ $d_p$ ≤ 0.0025 m:</td>
</tr>
<tr>
<td></td>
<td>$Z1 = 0.0008$, $Z2 = -0.7544$, $Z3 = -0.9584$, $Z4 = -0.7860$, $Z5 = 0.025$, $Z6 = 0.179$</td>
</tr>
<tr>
<td></td>
<td>For 0.095 &lt; $U_L &lt; 0.117$ m/s &amp; 0.001 ≤ $d_p$ ≤ 0.0025 m:</td>
</tr>
<tr>
<td></td>
<td>$Z1 = 0.0100$, $Z2 = -0.7544$, $Z3 = -0.5558$, $Z4 = 0.7714$, $Z5 = 0.025$, $Z6 = 0.179$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Authors</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deshpande et al. (1992)</td>
<td>$U_{br} = z_1 d_p^{22} U_L^{23} U_g^{24} \mu_L^{25} \sigma^{26}$</td>
</tr>
<tr>
<td></td>
<td>For coalesced regime:</td>
</tr>
<tr>
<td></td>
<td>$Z1 = 0.0051$, $Z2 = -0.7544$, $Z3 = -0.4219$, $Z4 = 0.0569$, $Z5 = 0.025$, $Z6 = 0.5$</td>
</tr>
<tr>
<td></td>
<td>For dispersed bubble:</td>
</tr>
<tr>
<td></td>
<td>$Z1 = 0.1041$, $Z2 = -0.4111$, $Z3 = -0.8027$, $Z4 = 0.6621$, $Z5 = 0.025$, $Z6 = 0.179$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Authors</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deshpande et al. (1992)</td>
<td>$U_{br} = z_1 d_p^{22} U_L^{23} U_g^{24} \mu_L^{25} \sigma^{26}$</td>
</tr>
<tr>
<td></td>
<td>For coalesced regime &amp; 0.015 &lt; $U_g$ ≤ 0.046 m/s:</td>
</tr>
<tr>
<td></td>
<td>$Z1 = 0.0026$, $Z2 = -0.7544$, $Z3 = -0.4219$, $Z4 = 0.0569$, $Z5 = 0.025$, $Z6 = 0.5$</td>
</tr>
<tr>
<td></td>
<td>For coalesced regime &amp; 0.046 &lt; $U_g$ ≤ 0.061 m/s:</td>
</tr>
<tr>
<td></td>
<td>$Z1 = 0.0051$, $Z2 = -0.7544$, $Z3 = -0.4219$, $Z4 = 0.0569$, $Z5 = 0.025$, $Z6 = 0.5$</td>
</tr>
<tr>
<td></td>
<td>For dispersed bubble:</td>
</tr>
<tr>
<td></td>
<td>$Z1 = 0.1041$, $Z2 = -0.4111$, $Z3 = -0.8027$, $Z4 = 0.6621$, $Z5 = 0.025$, $Z6 = 0.5$</td>
</tr>
</tbody>
</table>

| Fan and Tsuchiya (1990)  | $U_b \left( \frac{p_L}{\sigma} \right)^{1/4} = \left( \frac{Mo^{-1/4}}{K_b d'_{e}} \right)^{-f} + \left( \frac{2c + d'_{e}}{d'_{e}} \right)^{-f/2} -^{1/2}$ |
|                          | where $d'_{e} = d_{e} \left( \frac{g\rho_L}{\sigma} \right)^{1/2}$ |
|                          | in Luo et al. (1995), $c = 1.4$, $f=1.6$, and $K_b = 12$                  |

<table>
<thead>
<tr>
<th>Authors</th>
<th>Gas/Liquid/Solids or approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deshpande et al. (1992)</td>
<td>Air/water/glass beads</td>
</tr>
<tr>
<td>Deshpande et al. (1992)</td>
<td>Nitrogen/water/cylindrical particles</td>
</tr>
<tr>
<td>Deshpande et al. (1992)</td>
<td>Helium/kerosene/cylindrical particles</td>
</tr>
<tr>
<td>Fan and Tsuchiya (1990)</td>
<td>High pressure nitrogen/Paratherm NF heat transfer fluid/glass beads</td>
</tr>
</tbody>
</table>
surfactants is the tendency for foaming to occur. In addition, surfactants lead to surface tension gradients around bubbles that are difficult to characterize and replicate.

Several correlations have been presented for the gas hold-up under these conditions (Kelkar et al., 1983; Fan et al., 1987; Gorowara and Fan, 1990). Two of the correlations include a term representing the number of carbon atoms on the straight chain of the surfactant used. These correlations appear to be of little value for industrial systems.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Equation</th>
<th>Medium/Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Khayat and Mahmood (1988)</td>
<td>[ \sqrt{\frac{\rho_g (\rho_L - \rho_g)}{\rho_L U_b}} = 77.05 Re_{g,2}^{0.201} Re_{L,2}^{0.106} Re_{inf}^{-0.054} ]</td>
<td>Air/water or aqueous acetone or aqueous sugar or carboxymethyl cellulose (CMC)/glass beads or gravel</td>
</tr>
<tr>
<td>Kim et al. (1977)</td>
<td>[ d_b = 13.4 U_L^{0.052} U_g^{0.248} \gamma^{-0.008} \sigma^{0.034} \gamma = K \gamma^{n-1} ] [ U_b = 83.1 U_L^{0.065} U_g^{0.339} \gamma^{-0.025} \sigma^{0.179} ]</td>
<td>Air/water or aqueous acetone or aqueous sugar or carboxymethyl cellulose (CMC)/glass beads or gravel</td>
</tr>
<tr>
<td>Lee and de Lasa (1986)</td>
<td>For ( U_g = 0.01214 \text{ m/s} ): ( U_g &lt; 15.6 \text{ mm} \quad U_b = (0.197) L_b^{0.342} ) ( U_g &lt; 15.6 \text{ mm} \quad U_b = (0.06754) L_b^{0.732} ) ( U_g &lt; 0.01896 \text{ cm/s} ): ( U_g &lt; 10.1 \text{ mm} \quad U_b = (0.2067) L_b^{0.293} ) ( U_g &lt; 10.1 \text{ mm} \quad U_b = (0.1043) L_b^{0.589} )</td>
<td>Air/water/glass beads</td>
</tr>
<tr>
<td>Matsuura and Fan (1984)</td>
<td>[ U_b = 3.4 \sqrt{g d_b} ]</td>
<td>Air/water/glass beads</td>
</tr>
<tr>
<td>Meernick and Yuen (1988)</td>
<td>[ d_b = d_p [1.544 (\sigma/(\Delta \rho d_p^2 g)) + 0.143] ]</td>
<td>Nitrogen/variety organic solvents or water or octanol/glass (mostly spheres but some cylinders)</td>
</tr>
<tr>
<td>Razumov et al. (1973)</td>
<td>[ \epsilon_g = 0.578 - 3.198 U_L - 0.538 U_g ] [ \epsilon_L = 0.422 + \frac{0.135 U_L}{d_p^{0.562}} - 1.82 U_g ]</td>
<td>Air/water/glass beads or sand</td>
</tr>
<tr>
<td>Rigby et al. (1970)</td>
<td>[ U_b - (U_g + U_L) ((1-\epsilon)/\epsilon)^2 = 32.5 (0.57 d_b)^{1.53} ] [ U_b - (U_g + U_L) ((1-\epsilon_L)/\epsilon_L)^2 = 27.7 (0.57 d_b)^{1.54} ]</td>
<td>Air/water/glass beads or sand</td>
</tr>
<tr>
<td>Yu and Kim (1988)</td>
<td>[ L_b = 4.295 \times 10^{-3} d_p^{-0.129} U_L^{-0.069} U_g^{0.124} ]</td>
<td>Air/water/glass beads</td>
</tr>
</tbody>
</table>
2.1.2 Effect of liquid viscosity

Although the viscosity of a typical industrial hydrocarbon unit can differ by an order of magnitude from that of water, there has been little work on the effect of viscosity. Kim et al. (1975) found that liquid hold-up increased with increasing viscosity, especially for smaller particles (1 mm as opposed to 6 mm glass spheres). On the other hand, Kim et al. (1977) showed slight increases in both $d_b$ and $U_b$ with increasing viscosity, resulting in lower gas hold-ups. In contrast, Begovich and Watson (1978) cited the work of Bloxom et al. (1975) where the gas hold-up was unaffected by liquid viscosity while the solids hold-up decreased with increasing $\mu_L$.

2.1.3 Effect of particle size

Several researchers (e.g. Dakshinamurty et al., 1971; Tarmy et al., 1984b; Kim et al., 1987; Deshpande et al., 1992) have reported that the bubble size decreases and the gas hold-up increases with increasing particle diameter.

2.1.4 Summary of influence of variables

A qualitative summary concerning operating variables and their effect on the hydrodynamics is shown in Table 2.3. Although bubble rise velocity, $U_b$, is not included in the table, the consensus is that it is directly related to the bubble diameter, $d_b$ (Kim et al., 1977; Clift et al., 1978; Matsuura and Fan, 1984; Fan, 1989), and, therefore, the qualitative influence of the variables on it is the same as for $d_b$.

For the present work, it is assumed that no significant local gradients in physical properties exist and that the bulk values are representative of the entire liquid. However
some previous work by Saberian-Broudjenni et al. (1987) suggests that for two different liquids with similar bulk fluid properties, different flow regimes may be observed under the same operating conditions due to higher foamability of one of the liquids. The questions of foamability, the relationship between bulk and local physical properties, and their influence on bubble coalescence are being examined in another doctoral thesis project currently in progress at UBC.

Table 2.3: Summary of Qualitative Influence of Operating Variables
(The phenomenon of initial bed contraction is not considered in this table)

<table>
<thead>
<tr>
<th>Variable</th>
<th>( \epsilon_g )</th>
<th>( \rho_{be} )</th>
<th>( d_b )</th>
<th>Remarks/cited research</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \uparrow U_g )</td>
<td>( \uparrow )</td>
<td>( \uparrow )</td>
<td>( \uparrow )</td>
<td>As gas flow increases, bubble diameter tends to increase due to greater opportunity to coalesce. ( \epsilon_g ) increases despite this due to the greater gas flow (Wild and Poncin, 1996; Saberian-Broudjenni et al., 1984; most correlations in Table 2.1).</td>
</tr>
<tr>
<td>( \uparrow U_L )</td>
<td>( \approx )</td>
<td>( \uparrow )</td>
<td>slight ( \downarrow )</td>
<td>As liquid flow increases, bubbles have less time to coalesce as they pass through the system. Therefore ( d_b ) is reduced. The influence on ( \epsilon_g ) is less well understood. The correlations in Table 2.1 predict both decreases and increases under similar circumstances.</td>
</tr>
<tr>
<td>( \uparrow \rho_p )</td>
<td>?</td>
<td>( \downarrow )</td>
<td>( \uparrow )</td>
<td>Intuitively one expects little influence of particle density. However as ( \rho_p ) increases, the bed expansion must decrease for given operating conditions as the gravitational forces on a particle increase. Since the bed is more tightly packed, coalescence increases, bubble size increases slightly and ( \epsilon_g ) decreases. Wild and Poncin (1996) predict decreases, as do some correlations (Costa et al., 1986; Hassanien et al., 1984), while others (Bloxom et al., 1975; Jean and Fan, 1986) predict increases.</td>
</tr>
<tr>
<td>( \uparrow \rho_L )</td>
<td>( \downarrow )</td>
<td>( \uparrow )</td>
<td>?</td>
<td>A decrease in gas hold-up, together with increased bed expansion with increasing liquid density, was reported by several groups (e.g. Bloxom et al., 1975; Hassanien</td>
</tr>
</tbody>
</table>
et al., 1984; Wild and Poncin, 1996). It should be noted that Kato et al., 1985, predict an increase in $e_g$ with increasing $p_L$.

| $\uparrow \rho_s$ | $\equiv$ | $?$ | $?$ | Saberian-Broudjenni et al. (1984) showed little or no effect of $\rho_s$. However, several people have indicated significant effects of pressure (Tarmy et al., 1984a,b; Jiang et al., 1992). |
| $\uparrow \sigma$ | $\downarrow$ | $?$ | $\uparrow$ | As $\sigma$ increase, the bubbles become larger and hence $e_g$ decreases. (Kelkar et al., 1983; Fan et al., 1987; Gorowara and Fan, 1990) |
| $\uparrow \mu_L$ | slight $\downarrow$ | $\uparrow$ | slight $\uparrow$ | As liquid viscosity increases, the bed expansion increases due to increased drag. The maximum stable bubble size also increases. This may result in larger bubbles rising faster, and hence lower $e_g$ (Kim et al., 1977; Han et al., 1990; Bloxom et al., 1975). |
| $\uparrow d_p$ | slight $\downarrow$ | $\downarrow$ | $?$ | Kim et al. (1987) reported a small decrease in gas hold-up with increasing particle size. As particle diameter increases, the gravitational forces on a particle increase slightly relative to the drag forces; hence a decrease in bed expansion is observed (Wild and Poncin, 1996). |
| Particle shape | No effect | $?$ | $?$ | Sinha et al. (1986) |

| $\uparrow D_c$ | $\downarrow$ | $?$ | $?$ | Should have no effect for proper scale-up provided that $D_c >> d_p$ and $d_b$. However, some researchers (e.g. Hu et al., 1986; Begovich and Watson, 1978) have found that the gas hold-up decreases with increasing column diameter. |

### 2.2 Empirical correlations

While it seems from Table 2.1 that there has been extensive work on cocurrent upward three-phase fluidized bed hydrodynamics, most of the studies have been performed on air-water-glass bead systems. A good example of this is the correlation for
bed porosity presented by Han et al. (1990). While this correlation incorporates over 5000 data points (from Han et al., 1990, among others) over 90% of the data come from air-water-glass bead systems. Such systems are unlikely to provide a good representation of the hydrodynamics for non-aqueous industrial systems. Tarmy et al. (1984a,b) noted this problem and reported that, for a two-phase bubble column, increasing pressure resulted in significant increases in gas hold-up. The gas hold-ups in air-water systems at the same gas superficial velocities were significantly lower (at a superficial velocity of 0.05 m/s, of the order of 10% for air-water compared to 20% at 345 kPa, 30% at 520 kPa, and 40% in a pilot plant containing an organic liquid). Correlations tend to disagree with one another, even for simple air-water-glass bead systems (Nacef et al., 1992).

There are further difficulties with the correlations. Many fail to predict the initial behaviour (either contraction or expansion) of three-phase fluidized beds when gas is first introduced. There are some correlations that do take into account the two different types of system, but these require advanced knowledge whether the bed will initially contract or expand (e.g. see Han et al., 1990; Kim et al., 1975).

Another concern with the correlations is that almost all of the data have been collected in small-diameter columns, often 80 mm or less. Industrial units, on the other hand, are often of the order of several meters in diameter. Since capital investment is usually approximately proportional to the diameter to the power of 0.7, it is economically expedient to build large reactors (Euzen et al., 1993). As a result some correlations incorporate a term for column diameter, \( D_c \) (for example Han et al., 1990, Chern et al., 1984, among others). While such dependence most likely applies for smaller units where wall effects can be significant, it is unlikely to be applicable for industrial columns.
Many correlations are specific to the flow regime in which the bed is to be operated. Hence to give accurate predictions of reactor performance, one would have to know the flow regime during the design. While an estimate can be made as to the bed’s operating flow regime, there is uncertainty and this makes design even more difficult.

While empirical correlations provide useful information in estimating conditions and represent a wealth of information, they do not provide an effective means for predicting bed performance with any degree of confidence (Wild and Poncin, 1996). A database was established by Han et al. (1990) and extended by Nacef (1991). It now contains over 7000 data points, but more than half of these data are from air-water-glass bead systems at atmospheric temperature and pressure in small columns (Wild and Poncin, 1996). To create a correlation that could more closely predict bed behaviour under various conditions, it is vital that considerably more data be obtained for systems that differ significantly from these idealized systems.

2.3 Models

2.3.1 Richardson-Zaki correlation for liquid-solid systems

Apart from completely empirical correlations, there has also been some significant work combining experimental data with theory to provide models for predicting bed behaviour. The Richardson and Zaki correlation (Richardson and Zaki, 1954) establishes, for two-phase liquid-solid fluidized beds, that the bed porosity, ε, (in this case equal to the liquid holdup, εL, since there is no gas) is related to the liquid superficial velocity, the terminal settling velocity of a single particle, and an index n:

\[
\frac{U_L}{U_t} = \epsilon^n. \tag{2.1}
\]
A successful correlation for three-phase fluidized beds should have a two-phase liquid-solid relationship, such as the Richardson-Zaki equation (2.1), as its limit as the gas velocity approaches zero. There are various empirical correlations available in the literature to estimate values of the Richardson-Zaki index, n, and the particle terminal settling velocity, Ut.

2.3.2 Bubble wake model

The bubble wake approach is based on the study of the behaviour when gas is first introduced into a liquid-fluidized bed. First observed in the 1960’s, the phenomenon of initial bed contraction upon introduction of gas is attributed to bubble wakes (Rigby and Capes, 1970; Epstein, 1976; Epstein and Nicks, 1976). Bubbles tend to rise through a fluidized bed quicker than the liquid mean velocity. If significant wakes occur (more likely with non-spherical bubbles) containing few if any solids, a portion of the liquid is conveyed behind the bubble effectively by-passing the bed and not participating in fluidizing the bed; the remaining liquid then slows down. On the other hand, bubbles decrease the volume available for the liquid and solid and hence increase the interstitial liquid velocity. When the effect of bubble wake activity is greater than that of the increase in interstitial velocity due to the volume occupied by the bubbles, initial contraction is observed. If the effect of the increased interstitial velocity is always greater than the wake transport effects, then no contraction is observed, i.e. the bed expands continuously.

Bhatia et al. (1972) and Armstrong et al. (1976) carried out experiments supporting this mechanism by comparing non-wettable particles with wettable ones under
similar conditions. Teflon-coated wettable particles did not show any initial contraction, unlike their non-wettable counterparts. The authors attributed this to significantly less energy required to overcome the work of adhesion for non-wettable particles as compared to wettable particles. Tsutsumi et al. (1991) observed small bubbles adhering to non-wettable particles. Epstein (1976) suggested a criterion to predict initial contraction. Wild and Poncin (1996) suggest that a rule of thumb has been established which predicts initial contraction only for particles smaller than about 2.5 mm in diameter. However, they also note that this is not a firm rule as exceptions have been observed (e.g. see Han et al., 1990).

The bubble wake model divides the fluidized bed into three regions: gas bubbles, bubble wakes (containing solids and liquid), and a liquid-solid fluidized bed (containing solids and liquid) outside the bubbles and wakes. A schematic is shown in Figure 2.1.

The ratio of the volume occupied by the wakes to that occupied by the bubbles, $k$, is defined as

$$k = \frac{\varepsilon_w}{\varepsilon_g}$$

(2.2)

The ratio of solids hold-up in the wake region to that in the liquid-solid fluidized bed, $x$, is defined as

$$x = \frac{\varepsilon_{p,w}}{\varepsilon_{p,f}}$$

(2.3)

One can then make material balances on the solids and the liquid.

For the solids, some particles are entrained in the bubble wake and some in the liquid-solid fluidized region. The latter have a net flow directed towards the base of the
Figure 2.1: Schematic representation of classical bubble wake model

Since all particles are assumed to remain in the bed, the two volumetric flowrates must be equal in magnitude. Hence, for a constant cross-sectional area,

\[ V_g \varepsilon_{p,w} \varepsilon_w + V_{SF} \varepsilon_{p,f} (1 - \varepsilon_g - \varepsilon_w) = 0 \]  

Substituting equation (2.3) into (2.4) and rearranging yields

\[ V_{SF} = -\frac{V_g \varepsilon_w}{1 - \varepsilon_g - \varepsilon_w} \]  

Similarly, for the liquid, the following material balance must hold,

\[ V_g \varepsilon_w (1 - \varepsilon_{p,w}) + V_{LF} \varepsilon_{L,f} (1 - \varepsilon_g - \varepsilon_w) = U_L \]

Upon rearrangement,

\[ V_{LF} = \frac{U_L - V_g \varepsilon_w (1 - \varepsilon_{p,w})}{(1 - \varepsilon_g - \varepsilon_w) \varepsilon_{L,f}} \]  

The Richardson-Zaki equation, e.g. (2.1), can be applied to the liquid-solid region to relate the relative velocity between liquid and solids, \( V_0 \), to the bed porosity, i.e.
\[ V_f = \frac{U_f}{\varepsilon_{LF}} = V_{LF} - V_{SF} = U_t e^{u_f} \] (2.8)

Substituting equations (2.5) and (2.7) into (2.8) and rearranging for \( \varepsilon_{LF} \), yields

\[ \varepsilon_{LF} = \left[ \frac{U_L - V_g \varepsilon_w (1-x)}{U_t (1-\varepsilon_g - \varepsilon_w)} \right]^{1/n} \left[ \frac{U_L - U_g k(1-x)}{U_t (1-\varepsilon_g (1+k))} \right]^{1/n} \] (2.9)

Next, if one makes a volumetric balance of the liquid in the column, based upon constant cross-sectional area, the liquid hold-up, \( \varepsilon_L \), can be calculated as

\[ \varepsilon_L = \varepsilon_{LF} (1 - \varepsilon_g - \varepsilon_w) + \varepsilon_w (1 - \varepsilon_{pw}) \] (2.10)

And the overall porosity is

\[ \varepsilon = \varepsilon_g + \varepsilon_L = \varepsilon_{LF} (1 - \varepsilon_g - \varepsilon_w) + \varepsilon_w (1 - \varepsilon_{pw}) + \varepsilon_g \] (2.11)

If equations (2.2) and (2.3) are used to replace \( \varepsilon_w \) and \( \varepsilon_{pw} \), and equation (2.9) to replace \( \varepsilon_{LF} \), then equation (2.11) becomes the classical generalized wake model shown in Table 2.1:

\[ \varepsilon = \left[ \frac{U_L - U_g k(1-x)}{U_t (1-\varepsilon_g - k\varepsilon_g)} \right]^{1/n} [1 - \varepsilon_g (1+k-kx)] + \varepsilon_g (1+k-kx) \] (2.12)

This model succeeds in explaining observed bed behaviour and is an excellent means for estimating bed hydrodynamic properties. The major drawback, however, is that \( k \) and \( x \) have to be determined experimentally. Several researchers (Baker et al., 1977; Bhatia and Epstein, 1974; Chern et al., 1984; Darton and Harrison, 1975; El-Temtamy, 1974, Jin et al., 1993) have correlated the values of \( x \) and \( k \). In most cases, researchers have assumed that the wakes are free of solids (\( x=0 \)) and therefore only a correlation for \( k \) is necessary. Unfortunately, these models have the same sort of problems as the purely empirical correlations; they have been mostly tested for ideal air-water-glass bead systems and there are discrepancies among the results reported by
there is one additional constraint on the use of this model: the gas hold-up, $e_g$, must be known or estimated to obtain the liquid hold-up or bed porosity. It is also interesting to note that the wake structure behind small bubbles, the size expected in most industrial applications, is almost non-existent (Jiang et al., 1992).

2.3.3. Drift flux model

Another model used to explain bed hydrodynamics is the drift flux model. Based upon earlier work in two-phase systems (Wallis, 1969), this concept was reported by Chen and Fan (1990) to be valid for three-phase fluidized beds operated in the dispersed bubble regime. The basis of the model is that the bubble rise velocity in a three-phase fluidized bed consists of the sum of the solids-free superficial velocity, and the bubble drift velocity defined by Chen and Fan (1990). The authors also validated the model with a series of bed collapse tests, involving simultaneously stopping the gas and liquid flow. As the gas and liquid flows stopped, four regions develop in the fluidized bed as shown schematically in Figure 2.2: liquid-solid packed bed; liquid-solid sedimentation region; gas-liquid-solid three-phase region; and gas-liquid freeboard region. As the bed settles, the interfaces between regions also migrate as shown by the arrows in Figure 2.2(b). The model assumes that the boundaries, or interfaces, between regions form immediately upon cessation of the gas and liquid flows. By making mass balances across boundaries, Chen and Fan (1990) develop an expression for the bubble rise velocity, $U_b$:

$$U_b = \frac{U_g}{e_g} = \frac{U_g + U_L}{1 - e_p} + U_{bd}$$

(2.13)
Figure 2.2: Schematic representation of a three-phase fluidized bed with (a) gas and liquid flowing, and (b) after gas and liquid flows have been terminated.
where $U_{bd}$ is defined as the bubble drift velocity. Experimental data were used to obtain a correlation for $U_{bd}$. As mentioned above, the model is further limited by the assumption that the bed is operating in the dispersed bubble regime. This is, however, an acceptable restriction for many industrial applications. Many hydrocracking processes have had significantly higher gas hold-ups than expected from predictions based upon the ideal air-water-glass bead systems (Tarmy et al., 1984b). Some of this difference is likely attributable to smaller-than-expected bubbles suggesting that these industrial beds likely operated in the dispersed bubble flow regime.

A slightly different drift flux model based upon a slip velocity, was developed by Saberian-Broudjenni et al. (1984). These researchers defined a slip velocity, $U_{GL}$, as

$$U_{GL} = U_g - \frac{\varepsilon_g}{\varepsilon} (U_g + U_L)$$

(2.14)

It should be noted that this slip velocity is equivalent to the drift flux; equations (2.13) and (2.14) are identical if $U_{GL} = U_{bd} \varepsilon_g$. The drift flux must be correlated experimentally. However, Saberian-Broudjenni et al. found that while the drift flux was a function of gas velocity and liquid density, it was not significantly affected by liquid velocity, particle diameter, particle density, or liquid viscosity. They developed a correlation for the drift flux in a column of diameter 52 mm, but found that it was invalid for a different column. However they suggested that for a given column diameter and distributor, the drift flux could be correlated by an equation of the form,

$$U_{GL} = a U_e^b$$

(2.15)
An appealing aspect of this model is its relatively minor dependence on liquid properties. This gives it the potential to accurately predict gas hold-up if data can be obtained for cold-flow fluidized beds of similar scale.

Some researchers, as detailed in Wild and Poncin (1996), have shown that the correlation for drift flux is, as suggested by Saberian-Broudjeni et al. (1984), dependant on the bed distributor. Therefore, the drift flux can be used as a measure of the effectiveness of the distributor system used.

The multitude of empirical and model-based correlations can provide estimates of bed hydrodynamics. Unfortunately, due to many of the issues raised above, they do not reliably predict bed behaviour under industrial conditions (Tarmy et al., 1984). In an attempt to solve this problem, a new approach is proposed here based on the use of dimensional similitude.

### 2.4. Dimensional similitude approach and existing tools

Dimensional similitude is an approach that reduces the number and complexity of experimental variables for a given physical phenomenon, effectively compacting the problem (White, 1986). If the phenomenon one wishes to study depends upon \( N \) dimensional variables, this method serves to reduce the problem to a study of \( m \) dimensionless parameters, where

\[
m = N - j
\]  

(2.16)

with \( j \) being the number of fundamental dimensions. In the case of fluid dynamics, the fundamental dimensions are mass, length, time, and sometimes, temperature (White, 1986). When temperature effects are minor, only three fundamental dimensions are
required (mass, length, and time). This procedure results in an enormous economy of time and effort in many cases and also often allows researchers to study phenomena that are difficult to study in their natural environment (e.g. molten lava flow). More important in the current context is the ability to provide scaling laws.

There are several means of achieving dimensional similitude. One can non-dimensionalize the governing equations and boundary conditions, establish force balances, or use the Buckingham Pi Theorem. The advantage of the Buckingham Pi Theorem approach is that it is quick and relatively easy to produce a set of dimensionless groups. Its main disadvantages, compared to using the governing equations as a basis, are that it does not provide a means to determine whether the initial list of N variables is complete, and the physical significance of the dimensionless groups is not as clear.

2.4.1 The Buckingham Pi theorem

The basis for the present work is the Buckingham Pi theorem (Buckingham, 1914). As shown in equation (2.16), the theorem reduces the number of parameters in a problem from N dimensional variables to m dimensionless ones. The approach requires a sound understanding of the variables that influence the phenomena to be studied.

The first step is to identify all variables that are expected to significantly affect the properties to be studied, e.g. the hydrodynamics for our three-phase fluidization problem. Common sense and experience play important roles in cataloguing the important variables. Identification of all the variables that may significantly influence the bed hydrodynamics is crucial. Failing to include a key variable will almost certainly lead to misleading and confusing results. Inclusion of a variable that ultimately does not
significantly influence the hydrodynamics is not as serious, for while it creates additional superfluous experiments resulting in unnecessary time loss, the extra group should ultimately be shown experimentally to be insignificant. Once all the independent variables have been selected, the number of dimensionless groups can be determined from equation (2.16). It is essential that all the dimensionless groups formed be independent of each other (not a combination of any subset of the other groups). The chosen set of groups can differ significantly from researcher to researcher. Mindful of the advantage of using groups that are familiar to others, it is generally advisable to use standard dimensionless groups (e.g. Reynolds number, Froude number, Eötvös number, Prandtl number, etc) where possible.

Once the dimensionless groups have been selected and geometric similarity has been achieved, experimental results can be used to describe the phenomena studied for all systems with the same values of the dimensionless groups. Hence it is possible to predict behaviour in systems considerably different from the one studied. In essence, equality of dimensionless groups assures that the dependent dimensionless parameters are identical at the same relative locations while geometric similarity makes the dimensionless boundary conditions equivalent. Hence the results are no longer specific to the unit on which the research was done, but rather to all dynamically similar units. This allows for scale-up if the appropriate independent variables have been chosen correctly. If the dimensionless groups can be properly matched, it may even allow scale-up from experiments at ambient temperature and pressure to high temperature, high-pressure applications.
2.4.2 Dimensional similitude for two-phase systems

The dimensional analysis approach has been used successfully for two-phase gas-solids fluidized beds by a number of researchers: e.g. Glicksman (1984); Fitzgerald et al. (1984); Horio et al. (1986); Glicksman (1988); Chang and Louge (1992). For a comprehensive review, see Glicksman et al., 1994.

Glicksman (1984) used the equations of motion, assuming incompressible fluids (although he reported that the scaling rules did not depend upon this assumption) and omitting interparticle (e.g. van der Waals) forces:

For the gas: \( \rho_g e_g \left[ \frac{\partial U_g}{\partial t} + U_g \cdot \nabla U_g \right] + \dot{i} \rho_g g e_g + \nabla p + \beta (U_g - U_p) = 0 \) (2.17)

For the solid: \( \rho_p (1 - e_g) \left[ \frac{\partial U_p}{\partial t} + U_p \cdot \nabla U_p \right] + i \rho_p g (1 - e_p) - \beta (U_g - U_p) = 0 \) (2.18)

where \( \dot{i} \) is the unit vector in the vertical direction. By non-dimensionalizing these equations and manipulating the results, Glicksman obtained a series of six dimensionless groups that could be used to scale two-phase fluidized beds:

\[
\frac{\beta d_p}{\rho_p U_g}, \quad \frac{g d_p}{U_g^2}, \quad \frac{\Delta z}{d_p}, \quad \frac{D_c}{d_p}, \quad \frac{\rho_g}{\rho_p}, \quad \frac{\Delta P}{\rho_p U_g^2}
\] (2.19)

Furthermore, he eliminated the sixth term dealing with pressure, stating that the absolute pressure would not change sufficiently to cause thermodynamic changes in the gas. Apart from these dimensionless groups, the reactor geometry and particle size distribution were required to be maintained geometrically similar. The term \( \rho_g/\rho_p \) was replaced by various forms of the Reynolds number depending upon whether viscous or inertial terms were prevalent.
Horio et al. (1986) developed another set of scale-up dimensionless groups using the governing equations of bubble and interstitial gas dynamics. These parameters were later shown (Glicksman, 1988) to be a subset of the terms developed earlier.

2.4.3 Dimensional similitude for three-phase fluidized beds

The only previous attempt to use dimensional similitude in three-phase beds was by Deshpande (1992). However, there was no experimental verification of the process and no attempts were made to simulate any industrial process experimentally. Deshpande proposed six dimensionless groups that could be used to quantify phase hold-ups based upon earlier correlations and models:

\[
\frac{U_l}{U_g}, \frac{\rho_l}{\rho_p}, \frac{D_e}{d_p}, \frac{\rho_l U_l d_p}{\mu_l} = Re, \quad \frac{U_l^2}{g d_p} = Fr, \quad \frac{U_l^2 d_p \rho_l}{\sigma} = We
\]  

(2.20)

2.5 Present application of dimensional similitude

In the present work, a systematic approach has been taken to apply the Buckingham Pi Theorem to three-phase fluidized beds.

2.5.1 Learning from the past

The first step is to identify the dependant variables to be studied. In this case, these are the gas hold-up, \( \varepsilon_g \), and the bed expansion, while a secondary goal was to measure bubble diameters, \( d_b \), and rise velocities, \( U_b \). As mentioned above, the next step, identifying all important independent variables, is crucial to forming a proper set of dimensionless groups. A search of the previous work on three-phase fluidized beds, as summarized in Table 2.3, indicates that several important variables must be included.
To begin with, consider the gas hold-up correlation of Bloxom et al. (1975) which is the one recommended by Wild and Poncin (1996) and probably the best of an unreliable lot. This is

$$\varepsilon_g = 0.159 \left( \frac{U_g^5 (\rho_p - \rho_L)}{g U_L \sigma} \right)^{0.150} \tag{2.21}$$

Six variables are identified as being important for gas hold-up: gas superficial velocity, $U_g$, solids density, $\rho_p$, liquid density, $\rho_L$, superficial liquid velocity, $U_L$, interfacial gas-liquid surface tension, $\sigma$, and acceleration due to gravity, $g$.

Another important set of correlations is that of Han et al. (1990) which incorporates over 5000 data from various sources:

(when initial expansion occurs)

$$\varepsilon = (U_l/U_t)^{1/n} \left( 1 + 0.123 \text{ Fr}_{g}^{0.347} \text{ We}_{m}^{0.037} \right) \tag{2.22}$$

(when initial contraction occurs)

$$\varepsilon = \left( \frac{U_L}{U_t} \right)^{1/n} \left[ 0.359 \text{ Fr}_{g}^{0.352} \text{ We}_{m}^{0.124} + \exp \left( - \left( \frac{U_L}{U_t} \right)^{0.305} \text{ Fr}_{g}^{0.5} \right) \right] \tag{2.23}$$

These two equations indicate that the overall bed voidage, and hence the bed expansion, are directly related to seven variables: liquid superficial velocity, $U_l$, gas superficial velocity, $U_g$, particle diameter, $d_p$, acceleration due to gravity, $g$, liquid density, $\rho_L$, gas-liquid surface tension, $\sigma$, and column diameter, $D_c$. Although the particle terminal velocity appears in both correlations, it is itself a function of $g$, the densities of the solid and the liquid, and the liquid viscosity (Clift et al., 1978) and is therefore not an independent variable.

Another important correlation is that of Begovich and Watson (1978),
\[ \varepsilon = 0.371 \ U_L^{0.271} \ U_g^{0.041} (\rho_p - \rho_L)^{-0.316} \ d_p^{-0.268} \ \mu_L^{-0.055} \ D_c^{-0.033} \] (2.24)

which provides for a dependence of porosity on the liquid viscosity.

A correlation by Meernick and Yuen (1988b),

\[ d_b = d_p \left[ 1.544 \left( \frac{\sigma}{(\Delta \rho \ d_p^2 \ g)} \right) + 0.143 \right] \] (2.25)

shows a strong dependence of bubble diameter on mean particle diameter, \( d_p \), and a buoyancy term, \( g \Delta \rho \), where \( \Delta \rho \) is the density difference between the liquid and gas phases as well as the surface tension and the acceleration due to gravity.

Kim et al. (1977) provide empirical correlations for bubble diameter and bubble rise velocity and show the following dimensional relationship between bubble rise velocity and bubble diameter:

\[ U_b = 18.0 \ d_b^{0.989} \] (2.26)

This equation indicates that the bubble rise velocity does not add any new variables, and this is consistent with the findings of other researchers (e.g. Matsuura and Fan, 1984; Deshpande et al., 1992)

### 2.5.2 Forming the groups

Nine variables have been identified which have or may have a significant influence on the bed hydrodynamics (\( U_L \), \( U_g \), \( \mu_L \), \( \sigma \), \( d_p \), \( \rho_L \), \( \rho_p \), \( \Delta \rho g \), \( D_c \)). Note that, except for being incorporated into the buoyancy term, \( g \Delta \rho \), the gas density is excluded from the list of significant variables. Under some circumstances \( \rho_g \) may be important to bed hydrodynamics, especially when gas densities are high due to elevated pressures or when considering flow regimes at high velocities (Tarmy et al., 1984a,b; Luo et al., 1997a,b). However, in most cases \( \rho_g \ll \rho_L \) and \( \rho_g \ll \rho_p \), so that the gas density is of
secondary importance (Sabierian-Broudjenni, 1984), especially in the low velocity regimes (Deckwer et al., 1980). It is also excluded from common correlations for gas hold-up as summarized by Ramachandran and Chaudhari (1983) and Fan (1989).

Although the column diameter was found to influence the hydrodynamics in several of the studies in the literature, it is not considered a significant variable providing that $D_c \gg d_p$ and $D_c \gg d_b$. Its omission from the list of variables takes into account the flow regime in which the experiments in this research were expected to operate. It is expected (Tarmy et al., 1984a,b) that industrial hydrocarbon units operate with smaller bubbles in the dispersed bubble regime. In this flow regime, bubble sizes are relatively small and coalescence is minimized. Therefore, once the column diameter is large enough to avoid significant wall effects, there should be little further influence of column diameter on the hydrodynamics. This would likely not be the case in other flow regimes (for example, slug flow).

Begovich and Watson (1978) give

$$\varepsilon_g = 0.048 U_g^{0.720} d_p^{0.168} D_c^{-0.125}$$

This equation predicts a substantial decrease in gas hold-up with increasing column diameter for fixed $U_g$ and $U_L$. In fact, larger industrial units (operating with much smaller bubbles) most often result in much higher gas hold-ups (Tarmy et al., 1984b).

Hence for the conditions of interest here, a list of eight significant independent variables is established:

$$U_L, U_g, \mu_L, \sigma, d_p, \rho_L, \rho_p, \Delta \rho_g$$

There are also four hydrodynamic dependent variables that are to be studied:

$$\varepsilon_g, \beta_{be}, d_b, U_b$$
Three fundamental dimensions, mass, length, and time, are needed to express all of the variables, assuming negligible temperature variations. From equation (2.16) it is clear that five independent dimensionless groups can be formed according to the Buckingham Pi Theorem to express the variables listed in (2.28). Mindful of the advantage of groups that are familiar to most engineers, we choose

\[ M = \frac{g \Delta \rho \mu_l^4}{\rho_l^2 \sigma^3}, \quad E_o = \frac{g \Delta \rho d_p^2}{\sigma}, \quad Re_L = \frac{\rho_L d_p U_L}{\mu_L}, \quad \beta_d = \frac{\rho_p}{\rho_L}, \quad \text{and} \quad \beta_u = \frac{U_g}{U_L} \quad (2.30) \]

These five dimensionless groups (M-group, Eötvös number, Reynolds number, density ratio, and velocity ratio) are designated as the \textit{dimensionless operating groups}. The dependent variables can be expressed in dimensionless form as

\[ \varepsilon_g, \quad \beta_{be}, \quad \frac{d_b}{d_p}, \quad \frac{U_b}{U_g} \quad (2.31) \]

Since gas hold-up and the bed expansion ratio are already dimensionless, they constitute valid dimensionless variables. These four dimensionless groups are termed the \textit{dependent hydrodynamic dimensionless groups}.

By assuring geometric similarity and by carefully selecting the appropriate physical properties and operating variables such that the five dimensionless operating groups match those for an industrial unit, dynamic similarity can be achieved. In other words, the dependent hydrodynamic dimensionless groups should be the same in two independent systems if the five dimensionless operating groups are matched, even if the individual dimensional variables (fluid and solid properties) differ considerably between the two systems. Recall that we require also that significant wall effects be avoided, i.e. \( d_p \ll D_c \) and \( d_b \ll D_c \). Regardless of this consideration, particle geometric similarity and dimensionless size distribution must be maintained.
It is also important to note that it is the dimensionless hydrodynamics that will be similar in the two systems. For example, the bubble diameters in two such matched systems may be unequal, but, the ratio of the bubble diameter to the particle diameter should be the same for the two cases if appropriate matching has been achieved.

2.6 Selection of operating conditions based on dimensionless operating groups

A significant problem in any scaling study is to obtain accurate industrial data with which to compare the laboratory results. When industrial data are available, their accuracy is commonly limited due to the difficulties in obtaining measurements in large high-temperature, high-pressure units. In the present work, bed expansion and gas hold-up data, gathered in an industrially operated (Amoco) large cold-flow 0.91-m diameter column were made available by the project sponsor. Kerosene, hydrogen, and ceramic catalyst particles were used as the liquid, gas, and solids respectively in this industrially operated cold-flow column. The physical properties of the fluids and solids were based, for the most part, on values provided by the researchers responsible for operating the unit. However, where properties were not available, it was necessary to estimate some properties from data in the literature for kerosene-nitrogen systems. The five dimensionless operating groups for this system were then calculated. The values for the significant variables and the dimensionless parameters are listed in Table 2.4.

As shown above, there are five dimensionless operating groups that must be matched for the two units to be dynamically similar. Due to its ready availability, compressed air was selected as the gas to be used in the UBC unit. The next step was to determine either the liquid or the solid. Since the M-group relies primarily on the liquid
Table 2.4: Physical properties of fluids and solids used in industrially operated 0.91-m diameter cold flow three-phase fluidized bed and in the UBC unit and resulting dimensionless groups

<table>
<thead>
<tr>
<th>Property</th>
<th>0.91-m diameter column</th>
<th>UBC column</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column diameter, $D_c$</td>
<td>0.914 m</td>
<td>0.083 m</td>
</tr>
<tr>
<td>Liquid density, $\rho_L$</td>
<td>800 kg/m³</td>
<td>1183 kg/m³</td>
</tr>
<tr>
<td>Liquid viscosity, $\mu_L$</td>
<td>$1.5 \times 10^{-3}$ Pa·s</td>
<td>$3.6 \times 10^{-3}$ Pa·s</td>
</tr>
<tr>
<td>Liquid velocity, $U_L$</td>
<td>0 to 0.019 m/s</td>
<td>0.007 to 0.031 m/s</td>
</tr>
<tr>
<td>Particle density, $\rho_p$</td>
<td>1.68 kg/m³</td>
<td>2.68 kg/m³</td>
</tr>
<tr>
<td>Particle diameter, $d_p$</td>
<td>0.8 mm</td>
<td>1.0 mm</td>
</tr>
<tr>
<td>Particle length, $L_p$</td>
<td>2.4 mm</td>
<td>2.6 mm</td>
</tr>
<tr>
<td>Gas density, $\rho_g$</td>
<td>2.72 kg/m³</td>
<td>1.2 kg/m³</td>
</tr>
<tr>
<td>Gas velocity, $U_g$</td>
<td>0 to 0.09 m/s</td>
<td>0 to 0.122 m/s</td>
</tr>
<tr>
<td>Gas-liquid surface tension, $\sigma$</td>
<td>$2.5 \times 10^{-2}$ kg/s²</td>
<td>$7.7 \times 10^{-2}$ kg/s²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dimensionless groups</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>M-group</td>
<td>$M = \frac{g \Delta \rho \mu_L^4}{\rho_L^2 \sigma^3}$</td>
<td>$4.0 \times 10^{-9}$</td>
</tr>
<tr>
<td>Eötvös No.</td>
<td>$E_0 = \frac{g \Delta \rho d_p^2}{\sigma}$</td>
<td>0.20</td>
</tr>
<tr>
<td>Reynolds No.</td>
<td>$Re_L = \frac{\rho_L d_p U_L}{\mu_L}$</td>
<td>0 to 9.9</td>
</tr>
<tr>
<td>Density ratio</td>
<td>$\beta_d = \frac{\rho_p}{\rho_L}$</td>
<td>2.1</td>
</tr>
<tr>
<td>Velocity ratio</td>
<td>$\beta_u = \frac{U_g}{U_L}$</td>
<td>0 to ∞</td>
</tr>
</tbody>
</table>
properties, it was used to select an appropriate liquid.

Table 2.5 lists the physical properties of some liquid systems. The list contains only liquids that passed an initial screening to eliminate those liquids that did not result in a similar value of the $M$-group. Because of its 4$^{\text{th}}$ power dependence on $\mu_L$ and 3$^{\text{rd}}$ power dependence on $\sigma$, the $M$-group varies considerably with concentration for solutions. A plot of the $M$-group value versus concentration is shown in Figure 2.3 for aqueous magnesium sulphate solutions. By comparing the $M$-group value of the system to be simulated, one can determine the appropriate liquid concentration required for dynamic similitude such that

$$[M]_{\text{Experimental System}} = [M]_{\text{Industrial Unit}} \quad (2.32)$$

The next step is to ensure that particles can be found for this system to maintain geometric and dynamic similarity. To do so, one looks at the value of the Eötvös number divided by the particle diameter squared for the necessary solution concentration. Since

$$[E_0]_{\text{Experimental System}} = \left[ \frac{g\Delta\rho d_p^2}{\sigma} \right]_{\text{Exp.Sys.}} = [E_0]_{\text{Ind Unit}} \quad (2.33)$$

one can determine the required particle diameter for a given solution by re-arranging equation (2.33) to give

$$[d_p]_{\text{Exp.Sys.}} = \sqrt{\frac{[E_0]_{\text{Ind Unit}}}{[E_0]_{\text{Exp.Sys.}}}} \left[ \frac{\sigma}{g\Delta\rho} \right]_{\text{Exp.Sys.}} \quad (2.34)$$
Table 2.5: Aqueous solutions and their physical properties  
(Perry and Green, 1984; Weast, 1985)

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Density (kg/m³)</th>
<th>Surface Tension (kg/s²)</th>
<th>Viscosity (kg/ms)</th>
<th>Eötvös no./dp²</th>
<th>M-Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acetic Acid 1%</td>
<td>999.6</td>
<td>0.068</td>
<td>0.00101</td>
<td>1.44 x 10⁵</td>
<td>3.25 x 10⁻¹¹</td>
</tr>
<tr>
<td>Acetic Acid 5%</td>
<td>1005.2</td>
<td>0.0601</td>
<td>0.001103</td>
<td>1.64 x 10⁵</td>
<td>6.65 x 10⁻¹¹</td>
</tr>
<tr>
<td>Acetic Acid 10%</td>
<td>1012.1</td>
<td>0.0546</td>
<td>0.001208</td>
<td>1.82 x 10⁵</td>
<td>1.27 x 10⁻¹⁰</td>
</tr>
<tr>
<td>Acetic Acid 50%</td>
<td>1056.2</td>
<td>0.0384</td>
<td>0.002154</td>
<td>2.70 x 10⁵</td>
<td>3.53 x 10⁻⁹</td>
</tr>
<tr>
<td>Acetic Acid 70%</td>
<td>1067.3</td>
<td>0.0343</td>
<td>0.002624</td>
<td>3.05 x 10⁵</td>
<td>1.08 x 10⁻⁸</td>
</tr>
<tr>
<td>Ethyl Alc 34%</td>
<td>948.5</td>
<td>0.03324</td>
<td>0.002762</td>
<td>2.80 x 10⁵</td>
<td>1.64 x 10⁻⁸</td>
</tr>
<tr>
<td>Ethyl Alc 60%</td>
<td>891.1</td>
<td>0.02756</td>
<td>0.002542</td>
<td>3.17 x 10⁵</td>
<td>2.20 x 10⁻⁸</td>
</tr>
<tr>
<td>Ethyl Alc 72%</td>
<td>862.9</td>
<td>0.02628</td>
<td>0.002144</td>
<td>3.22 x 10⁵</td>
<td>1.32 x 10⁻⁸</td>
</tr>
<tr>
<td>Ethyl Alc 96%</td>
<td>801.3</td>
<td>0.02304</td>
<td>0.001339</td>
<td>3.41 x 10⁵</td>
<td>3.22 x 10⁻⁸</td>
</tr>
<tr>
<td>Glycerol 10%</td>
<td>1021.5</td>
<td>0.0729</td>
<td>0.001288</td>
<td>1.37 x 10⁵</td>
<td>6.82 x 10⁻¹¹</td>
</tr>
<tr>
<td>Glycerol 20%</td>
<td>1045.9</td>
<td>0.0724</td>
<td>0.001734</td>
<td>1.42 x 10⁵</td>
<td>2.23 x 10⁻¹⁰</td>
</tr>
<tr>
<td>Glycerol 30%</td>
<td>1071.7</td>
<td>0.072</td>
<td>0.002453</td>
<td>1.46 x 10⁵</td>
<td>8.88 x 10⁻¹⁰</td>
</tr>
<tr>
<td>Glycerol 50%</td>
<td>1125.4</td>
<td>0.07</td>
<td>0.0060275</td>
<td>1.58 x 10⁵</td>
<td>3.35 x 10⁻⁸</td>
</tr>
<tr>
<td>Glycerol 85%</td>
<td>1221.9</td>
<td>0.066</td>
<td>0.0999</td>
<td>1.81 x 10⁵</td>
<td>2.78 x 10⁻³</td>
</tr>
<tr>
<td>Glycerol 100%</td>
<td>1261.1</td>
<td>0.0634</td>
<td>1.487</td>
<td>1.95 x 10⁵</td>
<td>1.49 x 10⁻²</td>
</tr>
<tr>
<td>n-propanol 1%</td>
<td>996</td>
<td>0.0493</td>
<td>0.001054</td>
<td>1.98 x 10⁵</td>
<td>1.01 x 10⁻¹⁰</td>
</tr>
<tr>
<td>n-propanol 50%</td>
<td>906.6</td>
<td>0.02434</td>
<td>0.0037805</td>
<td>3.65 x 10⁵</td>
<td>1.53 x 10⁻⁷</td>
</tr>
<tr>
<td>n-propanol 80%</td>
<td>834.1</td>
<td>0.02366</td>
<td>0.003072</td>
<td>3.45 x 10⁵</td>
<td>7.91 x 10⁻⁸</td>
</tr>
<tr>
<td>n-propanol 90%</td>
<td>809.55</td>
<td>0.02341</td>
<td>0.002607</td>
<td>3.39 x 10⁵</td>
<td>4.36 x 10⁻⁸</td>
</tr>
<tr>
<td>Sucrose 10%</td>
<td>1059</td>
<td>0.0725</td>
<td>0.00188</td>
<td>1.43 x 10⁵</td>
<td>3.04 x 10⁻¹⁰</td>
</tr>
<tr>
<td>Sucrose 20%</td>
<td>1118</td>
<td>0.073</td>
<td>0.001957</td>
<td>1.50 x 10⁵</td>
<td>3.31 x 10⁻¹⁰</td>
</tr>
<tr>
<td>Sucrose 40%</td>
<td>1235</td>
<td>0.0741</td>
<td>0.00621</td>
<td>1.63 x 10⁵</td>
<td>2.90 x 10⁻⁸</td>
</tr>
<tr>
<td>Sucrose 55%</td>
<td>1323</td>
<td>0.0757</td>
<td>0.02828</td>
<td>1.71 x 10⁵</td>
<td>1.09 x 10⁻⁵</td>
</tr>
<tr>
<td>NaOH 5.66%</td>
<td>1061.1</td>
<td>0.07585</td>
<td>0.001383</td>
<td>1.37 x 10⁵</td>
<td>7.75 x 10⁻¹¹</td>
</tr>
<tr>
<td>NaOH 16.66%</td>
<td>1182.2</td>
<td>0.08305</td>
<td>0.003235</td>
<td>1.40 x 10⁵</td>
<td>1.59 x 10⁻⁹</td>
</tr>
<tr>
<td>NaOH 30.56%</td>
<td>1333.6</td>
<td>0.09605</td>
<td>0.01442</td>
<td>1.36 x 10⁵</td>
<td>3.59 x 10⁻⁷</td>
</tr>
<tr>
<td>NaOH 35.90%</td>
<td>1389.1</td>
<td>0.10105</td>
<td>0.0250135</td>
<td>1.35 x 10⁵</td>
<td>2.68 x 10⁻⁶</td>
</tr>
<tr>
<td>MgSO₄ 1.19%</td>
<td>1010.3</td>
<td>0.07301</td>
<td>0.001063</td>
<td>1.36 x 10⁵</td>
<td>3.19 x 10⁻¹¹</td>
</tr>
<tr>
<td>MgSO₄ 5.68%</td>
<td>1056.8</td>
<td>0.07378</td>
<td>0.0013814</td>
<td>1.40 x 10⁵</td>
<td>8.42 x 10⁻¹¹</td>
</tr>
<tr>
<td>MgSO₄ 10.75%</td>
<td>1111.8</td>
<td>0.07485</td>
<td>0.001928</td>
<td>1.46 x 10⁵</td>
<td>2.91 x 10⁻¹⁰</td>
</tr>
<tr>
<td>MgSO₄ 19.41%</td>
<td>1212.6</td>
<td>0.07735</td>
<td>0.0039071</td>
<td>1.54 x 10⁵</td>
<td>4.07 x 10⁻⁹</td>
</tr>
<tr>
<td>MgSO₄ 24.53%</td>
<td>1277</td>
<td>0.07925</td>
<td>0.0068997</td>
<td>1.58 x 10⁵</td>
<td>3.50 x 10⁻⁸</td>
</tr>
</tbody>
</table>
Figure 2.3: M-group values for various concentrations of aqueous magnesium sulfate solutions (using data of Weast, 1985)
This determines the required particle diameter. In some cases, this results in diameters too small or too large for practical purposes. Such liquids can be eliminated from further consideration.

The density ratio, $\beta_d$, is also compared:

$$\left[ \frac{\rho_p}{\rho_L} \right]_{\text{Exp.Sys.}} = \left[ \frac{\rho_d}{\rho_L} \right]_{\text{Ind.Sys}} \left[ \frac{\rho_L}{\rho_d} \right]_{\text{Exp.Sys.}}$$  \hspace{1cm} (2.35)

This gives the necessary solids density. Again, if the results are impractical, the corresponding liquid is eliminated.

If suitable combinations of solids and liquid can be found to match $M$, $E_o$, and $\beta_d$ the required liquid superficial velocity for dynamic similarity is then calculated from the Reynolds numbers, i.e.

$$\left[ U_L \right]_{\text{Exp.Sys.}} = \left[ \frac{\mu_L}{\rho_L d_p} \right]_{\text{Exp.Sys.}}$$  \hspace{1cm} (2.36)

This step could also eliminate solids-liquid combinations if the calculated velocities are impractical.

The final step in matching the two systems is to determine the required experimental gas velocities. Matching the value of $\beta_u$ for the industrial unit with that for the experimental system leads to

$$\left[ U_g \right]_{\text{Exp.Sys.}} = \left[ \frac{\mu_L}{\rho_L d_p} \right]_{\text{Exp.Sys.}}$$  \hspace{1cm} (2.37)

For the case of the 0.91-m diameter industrially operated cold-flow unit, the matching resulted in several possibilities. Ultimately, as shown in Table 2.4, aqueous magnesium sulfate (commonly known as Epsom salts for use in bath water) and aluminum particles were selected as the liquid and the solids. Although this did not result
in a perfect match, considering the uncertainty in the industrial unit's exact operating properties, the enormous range of M values found in practice and experimental error in general, the match was considered to be sufficiently close.

The ultimate goal was to simulate an industrial unit to gain further understanding of co-current three-phase fluidization. An identical matching procedure was followed for Syncrude's LC-Finer hydrocracker, the main priority of this research. The properties of this system were estimated from several internal Syncrude reports. Due to the confidential nature of the process, the individual properties are not listed here. However, Table 2.6 compares the ratios of operating dimensionless groups in the present work and in various works from the literature to the corresponding groups for the industrial hydrocracking unit. Clearly the previous work has not operated under conditions dynamically similar to those of the hydrocracker being simulated in the present work. It should be noted, of course, that the studies cited were not attempting to simulate the hydrocracker, and, as such, it is not surprising that the values for the dimensionless operating groups did not agree. The main difficulty is in matching the M-group values.

In matching the industrial hydrocracker's dimensionless operating groups, several choices were available as outlined in Tables 2.7 and 2.8. Since the solid that would be easiest to use would be aluminum it was chosen. This required pairing it with an aqueous glycerol solution. Since geometric similarity of the particles must be maintained, it was then necessary to use cylindrical particles of diameter of 4.0 mm and length 10.0 mm. Particles of this size were produced by cutting aluminum wire to precise lengths of 10 mm as described in the next chapter.
Table 2.6: Ratios of values of dimensionless groups for co-current three-phase systems reported in the literature compared with values in industrial (Syncrude) hydrocracker

<table>
<thead>
<tr>
<th>Researcher</th>
<th>System</th>
<th>Ratio of M</th>
<th>Ratio of Eo</th>
<th>Ratio of Re</th>
<th>Ratio of βd</th>
<th>Ratio of βd</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Present Research</strong></td>
<td>44 mass % aqueous glycerol-air</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Typical Water-Air System</td>
<td>water-air</td>
<td>0.0055</td>
<td>0.056</td>
<td>0.67</td>
<td>0.49 - 2.9</td>
<td>Can be set to 1.0</td>
</tr>
<tr>
<td>Tarmy et al. 1984</td>
<td>coal liquefaction unit</td>
<td>0.051</td>
<td>0.0072</td>
<td>0.0040 - 0.0058</td>
<td>1.0</td>
<td>3.2 - 19.9</td>
</tr>
<tr>
<td></td>
<td>coal liquefaction unit</td>
<td>0.052</td>
<td>0.28</td>
<td>0.018 - 0.027</td>
<td>1.2</td>
<td>3.4 - 4.8</td>
</tr>
<tr>
<td>Han et al. 1990</td>
<td>Glycerol-air</td>
<td>2.7 x 10^2 - 2.7 x 10^4</td>
<td>0.38 - 5.0</td>
<td>0.0054 - 0.50</td>
<td>0.82 - 0.86</td>
<td>0 - 14.3</td>
</tr>
<tr>
<td>Kang and Kim 1988</td>
<td>Kerosene-air</td>
<td>0.10</td>
<td>0.33 - 7.4</td>
<td>0.28 - 6.2</td>
<td>1.3</td>
<td>0.083 - 4.8</td>
</tr>
<tr>
<td></td>
<td>mineral oil - air</td>
<td>8.7 x 10^2</td>
<td>0.92 - 6.4</td>
<td>0.077 - 0.57</td>
<td>1.2</td>
<td>1.0 - 2.9</td>
</tr>
<tr>
<td>Deshpande et al., 1992</td>
<td>Kerosene-He</td>
<td>0.13</td>
<td>0.76</td>
<td>0 - 0.8</td>
<td>0.85</td>
<td>wide range (0 -&gt; oo)</td>
</tr>
<tr>
<td>Saberian-Broudjenni et al. 1987</td>
<td>kerosene - N_2/He/CO_2</td>
<td>0.080</td>
<td>0.23 - 1.2</td>
<td>Not available</td>
<td>Not available</td>
<td>Not available</td>
</tr>
<tr>
<td></td>
<td>cyclohexane-N_2/He/CO_2</td>
<td>0.043</td>
<td>0.21 - 1.1</td>
<td>Not available</td>
<td>Not available</td>
<td>Not available</td>
</tr>
<tr>
<td></td>
<td>gas oil - N_2/He/CO_2</td>
<td>1.7 x 10^2</td>
<td>0.22 - 1.2</td>
<td>Not available</td>
<td>Not available</td>
<td>Not available</td>
</tr>
<tr>
<td></td>
<td>C_2Cl_4 - N_2/He/CO_2</td>
<td>0.0080</td>
<td>0.37 - 2.0</td>
<td>Not available</td>
<td>Not available</td>
<td>Not available</td>
</tr>
<tr>
<td></td>
<td>water - air</td>
<td>0.0019</td>
<td>0.10 - 0.56</td>
<td>Not available</td>
<td>Not available</td>
<td>Not available</td>
</tr>
</tbody>
</table>
### Table 2.7: Potential liquids for matching dimensionless operating parameters

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Required $\rho_s$ (kg/m$^3$)</th>
<th>Required cylinder diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acetic acid-water solution</td>
<td>2600</td>
<td>2.8</td>
</tr>
<tr>
<td>Ethanol-water solution</td>
<td>2100</td>
<td>2.8</td>
</tr>
<tr>
<td>Glycerol-water solution</td>
<td>2700</td>
<td>4.0</td>
</tr>
<tr>
<td>n-propanol-water solution</td>
<td>2400</td>
<td>3.5</td>
</tr>
<tr>
<td>Sucrose-water solution</td>
<td>2900</td>
<td>4.0</td>
</tr>
<tr>
<td>NaOH-water solution</td>
<td>2900</td>
<td>4.3</td>
</tr>
<tr>
<td>MgSO$_4$-water solution</td>
<td>3000</td>
<td>4.1</td>
</tr>
</tbody>
</table>

### Table 2.8: Potential solids for matching dimensionless operating parameters (based upon requirements from Table 2.7)

<table>
<thead>
<tr>
<th>Solid material</th>
<th>$\rho_s$ (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teflon</td>
<td>2200</td>
</tr>
<tr>
<td>Silicon nitride</td>
<td>2400</td>
</tr>
<tr>
<td>MgO ceramic porous rod</td>
<td>2500</td>
</tr>
<tr>
<td>TriBoron silicide</td>
<td>2500</td>
</tr>
<tr>
<td>Magnesium pyrophosphate</td>
<td>2560</td>
</tr>
<tr>
<td>Silica</td>
<td>2650</td>
</tr>
<tr>
<td>Aluminum alloys</td>
<td>2700-2800</td>
</tr>
<tr>
<td>Alumina ceramic rods</td>
<td>2800</td>
</tr>
<tr>
<td>Beryllium oxide</td>
<td>3000</td>
</tr>
<tr>
<td>Aluminum nitride</td>
<td>3100</td>
</tr>
</tbody>
</table>
Chapter 3 - Experimental Equipment

Two experimental columns were used during the course of this work. A 292-mm diameter column, was designed and commissioned expressly for this project. The other unit, featuring a 83-mm diameter column, was designed and used primarily for a previous thesis project (Zhang, 1996).

3.1 292-mm diameter column

The primary tool for this research was a 292-mm diameter column. A schematic diagram is shown in Figure 3.1(a) with an overhead view of the storage tank in Figure 3.1(b). The column was constructed from acrylic (Plexiglas) and had a possible ebullated bed height of 1.8 meters. The use of acrylic permitted observation of the interior and allowed bed expansion to be measured by determining the interface between the ebullated bed and freeboard regions.

Liquid and air entered the column at the bottom through a pre-mixing area below the distributor grid. The pre-mix area and distributor were not geometrically similar to those in the industrial hydrocracker due to practical considerations including the proprietary nature of the latter. Although procedures for designing distribution systems for two-phase systems are well documented, there is little information for three-phase systems. To design the system for the present work, a two-tiered approach was used. First a sparger ring was designed to distribute the gas into the liquid in the section denoted as “pre-mix area” in Figure 3.1(a). This region was also filled with ceramic and plastic Raschig rings, primarily to increase the pressure drop and to enhance the distribution of liquid and gas. The gas sparger designed for this research is shown
Figure 3.1: Schematic of 292-mm diameter column and (a) its auxiliary equipment and (b) an overhead view of the storage tank
schematically in Figure 3.2 in a bottom view. A photograph of the pre-mix area without the Raschig rings appears in Figure 3.3. The holes in the sparger face downward to increase the mixing time.

The building compressor was used to supply air to the column. Towards the end of the project, an oil filter was installed on the air line to remove traces of contaminants. No significant differences in fluid properties or experimental results were noted prior to and following installation of the filter. The air passed through a 76-mm ID pipe into the sparger and into the column. The flow of air was monitored and controlled by a flow meter and valve (Hedland valve). At the top of the column, air was vented to the atmosphere.

The second stage of the distribution system was a perforated plate, denoted as "distributor" in Figure 3.1. A schematic of the distributor plate is shown in Figure 3.4. This plate was designed to evenly distribute the liquid within the column. Calculations for the number of required holes for both the distributor plate and the gas sparger are given in Appendix A. To prevent particles from falling into the distributor holes and plugging the system, a screen with square holes approximately 3 mm in length, was placed on top of the grid.

Tests were conducted during commissioning of the system to determine the effectiveness of the distributor system. To facilitate observation, no particles were loaded in the column during the first set of tests. It was suspected that if the particles had any effect, it would be to improve the distribution. The first tests used a 0.25 m depth of stagnant water in the column and introduced gas through the sparger system. The resulting air distribution, as observed from the top of the column confirmed that the gas
Figure 3.2: Bottom view of gas sparger ring used in pre-mix area of 292-mm column

Figure 3.3: Assembled pre-mix area showing sparger and distributor plate. Raschig rings are placed in this area during normal operations.
Figure 3.4: Design diagram of distributor plate grid

sparger did effectively distribute the air under those conditions. To test the liquid distribution, particles were loaded into the column to a settled bed height of 0.15 m and liquid was passed through the system (with no gas flow). The resulting bed interface was smooth, indicating that the velocity profile was likely to be evenly distributed. These
tests indicated that the design method was appropriate.

Three different solutions were used as the liquid: water, an aqueous glycerol solution containing 44 mass % glycerol, and a higher concentration of glycerol in water (approximately 60 mass % glycerol). The liquids were mixed and stored in a large covered plastic storage tank. Superficial velocities up to 0.16 m/s could be obtained with the aid of a 5 Horsepower Goulds Centrifugal Pump (model 53BFIIJ5B2). From the storage tank the liquid passed through a large ball valve, 76-mm diameter PVC piping, and a flow straightener. The pressure drop across an orifice meter was used to determine the liquid flow rate after calibration. A safety valve was used in a by-pass loop to bypass the column in the event that the distributor system became plugged. Unlike the gas, the liquid was recovered at the top of the column and returned to the storage vessel. Periodic checking of the solution was carried out by withdrawing samples from the storage tank, and water was added to the tank when needed to maintain the proper concentration, i.e. to replace water lost by evaporation. An aluminum partition in the storage vessel minimized entrainment of air from the recycle stream back into the column.

The fluidization column is divided into three main sections: distributor and pre-mix region, ebullated bed region, and expanded section. Schematics of these sections are provided in Appendix A. Rubber O-rings were used between the column sections to provide good seals. Care was taken to align all holes on each set of flanges to allow the sections to be interchanged or to remove some sections entirely. The column inner diameter was, as noted above, 292 mm with a wall thickness of 6.4 mm. Along the height of the column, various 6.3-mm NPT and 12.8-mm NPT female ports were added which could be used to insert various probes or transducers. When not in use each of
these could be plugged. These ports were located at intervals of 102 mm, beginning 51 mm above the distributor plate. Although only one port was available at each height, the sections could be rotated to change the angular position of the ports. Since the column was expected to operate in the dispersed bubble regime, it was expected that radial and angular gradients would be small. In the upper section of the column (above a height of 1.27 m), the spacing between probe ports increased to 152 mm. The top of the column was of larger cross-section, 0.62 m in diameter section, (expanding over 0.305 m at an angle of 62°) to reduce the superficial gas and liquid velocities, and hence the drag on any entrained particles. A particle screen (square holes with lengths of approximately 3 mm) was used to prevent particles from being entrained into the liquid recycle stream. The latter returned to the storage vessel through a flexible pipe of 0.15 m inner diameter.

3.2 83-mm diameter column

An existing 83-mm diameter column, of height 2 m, was used for validation experiments. This column, also constructed from acrylic, was designed and commissioned by Zhang (1996) and was modified for this research. The modifications allowed the liquid to be recycled. A schematic is given in Figure 3.5. The fluid flow was controlled using two sets of rotameters. Compressor air was used as the gas and was vented to the atmosphere after passing through the column. As for the larger column, the liquid properties were important and, hence, the liquid was continuously recycled. The gas and the liquid entered the column at the base in a pre-mix chamber packed with 25-mm Raschig rings. The mixed solutions then passed through a perforated distributor plate, similar to the one used in the 292-mm diameter column.
Figure 3.5: Schematic representation of the 83-mm diameter column modified from Zhang (1996)
Chapter 4 - Measurement Techniques

Good measurement techniques are essential to successful use of dimensional similitude for scale-up. There are many different methods currently available to measure bed hydrodynamics.

4.1 General discussion of phase hold-up measurements

To determine hold-ups, the main methods are pressure drop measurements, conductivity probes, photography, a quick-shut-down technique, sampling probes, tracer techniques, optical fiber probes, and high or low frequency pulse probes. The methods used for the present research consist of pressure drop measurements and the use of an electrical conductivity probe. These are discussed in detail in sections 4.4 and 4.5, respectively.

The pressure drop can be related directly to the density of the individual phases, as follows

\[
\frac{\Delta P}{g \Delta z} = \rho_g \epsilon_g + \rho_L \epsilon_L + \rho_p \epsilon_p
\] (4.1)

The solids hold-up can be calculated based upon the observed bed expansion and the known solids loading of the bed:

\[
\epsilon_p = \frac{W_s}{\rho_p A H_e}
\] (4.2)

And since the only phases present in the reactor are the gas, liquid and particles,

\[
\epsilon_g + \epsilon_L + \epsilon_p = 1
\] (4.3)

Based upon equations (4.1), (4.2) and (4.3), gas hold-ups can be estimated. However,
equation (4.2) assumes a constant solids hold-up throughout the expanded bed, which may not be a valid assumption, and that the interface between the freeboard and ebullated bed regions can be clearly discerned, which is often difficult. For industrial systems, the solids inventory is rarely known accurately.

Photography is more common for systems designed to be “two-dimensional” (rectangular in cross-section with a small thickness) (Yates and Simons, 1994; Wild and Poncin, 1996). The wall effects that occur due to the thin thickness can retard and deform bubbles (Grace and Baeyens, 1986). Lacking geometric similarity, the results from such a system are therefore unlikely to be similar to those of an industrial unit. Photography does have the advantage of being able to measure bubble velocities simultaneously with bubble diameters and gas hold-up. An attempt to use a borescope in connection with visual photography is discussed below.

A quick-shut-down technique was used by Jin and Zhang (1990) to determine over-all phase hold-ups in a three-phase fluidized bed. There are two steps for this technique. First, the gas flow-rate is terminated impulsively while the bed is being fluidized and the fluidized bed height is then measured as a function of time. Second, the gas and liquid feed valves to the system are closed simultaneously and instantaneously and the bed height is again measured as a function of time. The gas hold-up and liquid hold-up are determined from the measurements of bed height. This technique is best suited for a flow regime in which there are significant bubble wakes.

A simpler version of this technique involves only one shut-down test. Liquid and gas flow are shut-off simultaneously and time is allowed for all the gas to escape the system. The over-all gas hold-up can then be estimated by measuring the volume of gas
that was in the system (the difference in height between the exit port and the liquid level multiplied by the cross-sectional area) and dividing by the overall reactor volume. There are some difficulties with this simplification, however. Small bubbles may be trapped within the settled solids, giving results that are somewhat too low. Refluidization of the solids with the liquid only to remove these trapped bubbles yields more accurate results (Saberian-Broudjenni et al., 1984). In addition, this method lumps the freeboard gas hold-up with the ebullated bed freeboard. One way of avoiding this problem is to minimize the freeboard volume. A further complication is that the gas hold-up includes gas that is in the plenum chamber pre-mix area, which, in some cases, may be very significant. The liquid hold-up can then be determined by subtracting the measured gas hold-up and the known solids hold-up (from the solids loading) from unity. This technique was not used in the current project.

Sampling probes are quite intrusive and not widely used. Kato et al. (1985) used a shutter system to isolate sections of their column so that the gas, liquid and solids content could be determined in those sections. While this may be possible for small diameter columns, it becomes markedly more difficult in columns of larger diameter since the shutter must close virtually instantaneously to yield practical results. As well, large particles can jam shutter mechanisms preventing them from closing. This method was not used due to its very intrusive nature and the relatively large size of column needed for this project.

Tracer techniques can yield very accurate measurements, but are often expensive. Gas or liquid tracer is injected into the feed streams. It is clearly important that the tracers be non-reactive with the system. The most common liquid tracer is a salt solution
in combination with a conductivity probe (Wild and Poncin, 1996), while helium is a
common gas-phase tracer. Other common tracers are radioactive, requiring special
precautions. Either a point probe or testing the outlet streams in combination with a pulse
or step-change injection of tracer can yield residence time distributions (RTDs). These
distributions can be used to evaluate phase mean residence times and hence hold-ups,
since the phase hold-up is equal to the mean residence time multiplied by the flow rate
divided by the phase volume. An added advantage of this method is that it can provide
information with regard to the mixing characteristics within the column. Tracer response
tests were not attempted in this project, but will be studied in a subsequent study.

Optical fiber probes have often been used successfully in two-phase gas-solid
fluidized beds (de Lasa et al., 1984; Goure, 1992). While some fibers transport light to
the measuring volume, other fibers carry reflected light to the photocells. These probes
need to be calibrated and cannot work effectively with translucent or opaque fluids. Due
to difficulties with translucent and opaque fluids, this method was not pursued in this
work.

High or low frequency pulse probes provide yet another method to calculate phase
hold-ups (Yates and Simon, 1994; Wild and Poncin, 1996). These are usually the most
expensive methods. Some typical units involve γ-rays, x-rays (Chaouki et al., 1997), and
ultra-sound (Stolojanu and Prakash, 1997; Uchida et al., 1988). The pulses pass through
the column and detectors are used to determine such features as the scatter, the
absorption, reflection, or transit times. In the case of γ-rays, the attenuation is
proportional to the mass absorption coefficient, ψ, which is related to the ratio of atomic
number to atomic mass of the material (Lim, 1994; Daly et al., 1995). The intensity of
the gamma-rays after scattering, $\varphi$, to the initial intensity, $\varphi_0$, is related to the individual phases as follows,

$$-\ln\left(\frac{\varphi}{\varphi_0}\right) = \psi_g \rho_g e_g + \psi_L \rho_L e_L + \psi_p \rho_p e_p$$  \hspace{1cm} (4.4)

If both gamma-ray attenuation and pressure drops are measured, equations (4.1), (4.3) and (4.4) can be used to calculate phase hold-ups. This eliminates any concerns regarding accuracy of bed inventory and expanded bed heights. However, as noted by Lim (1994), in practice, with the exception of hydrogen, most elements have comparable absorption coefficients due to similar atomic number to mass ratios. Therefore, unless hydrogen is used, equation (4.4) is reduced to a redundant expression of equation (4.1).

Nuclear Magnetic Resonance (NMR) is another technique that could be used to map the regions of a fluidized bed and determine phase hold-ups. However this is a very expensive technique, of the order of $500,000 - $800,000 US for a complete unit appropriate for the size of the primary column used in this research (Hammer, 1996). In addition, the large aluminum particles used in this research would interfere with the magnetic field and make imaging almost impossible.

4.2 General discussion of bed expansion measurements

To determine the bed expansion, the two most common methods are pressure profiles, and visual observation of the bed-freeboard interface. In the former, pressure readings are taken along the height of the column. Because of the presence of heavier, or denser, particles in the ebullated bed, the pressure gradient should be greater in this region than in the freeboard. A typical pressure profile is shown in Figure 4.1. From
Figure 4.1: Idealized pressure profile in a fluidized bed.
such a plot, the expanded bed height can be estimated. This method was not used in the present work for the reason cited in section 4.4.1.

Visual observations require that the interface between the ebullated bed and the freeboard be discernable. If this is the case, then the height at which the interface is noted, effectively the expanded bed height, can be measured directly. This is the primary method used in this project.

Neutron radiographic means were used by Chiba et al. (1996) to determine bed expansion. The bed is bombarded at different levels with neutrons and the bed height is determined by noting the difference in neutron absorption. This method was not used in the current work.

4.3 General discussion of bubble property measurements

For determining bubble velocities and bubble diameters the most common measurement techniques are photographic (either using visible light or x-rays), conductivity probes, and optical probes. As discussed above, visual photography can be used to take pictures of bubbles in two-dimensional columns. However, two dimensional columns are generally not representative of three-dimensional columns. Alternatively, one could take photographs of bubbles at the wall of three-dimensional columns, but this leads to distortion and bias due to wall effects. This last approach is not suitable for the coalescing regime where larger bubbles, and the bulk of the gas, are more likely to be found towards the center of the column. X-rays (Yates and Cheesman, 1992; Yates and Simons, 1994) and laser Doppler imaging (Chen and Fan, 1992; Chen et al., 1994; Mudde et al., 1992) have been used to map the interior of the column. These methods
can be expensive and are constrained by the materials used within the column. However, the method could provide an effective non-intrusive technique to obtain hydrodynamic measurements, but it is limited to columns of relatively small diameter.

Conductivity probes are also in wide use to determine bubble properties (e.g. Buchholz et al., 1981; Burgess et al., 1981; Clements and Schnelle, 1963; Khang and Fitzgerald, 1975; Lamb et al., 1960; Matsuura and Fan, 1984; Park et al., 1969; Prausnitz and Wilhelm, 1956; Rigby et al., 1970; Uribe-Salas et al., 1994). Although they are intrusive and can effect hydrodynamics (Rowe and Masson, 1981), conductivity probes are relatively inexpensive and give quick responses. This method works on the principle that the conductivity of the liquid and the gas differ considerably. Local values of the gas hold-up and bubble properties can be measured using this technique. This method has been used extensively in the present research. Capacitance probes have also been employed extensively in previous work (e.g. Geldart and Kelsey, 1972; Werther et al., 1993; Wittman et al., 1981; Gunn and Al-Doori, 1985).

Optical probes have also been used to determine bubble characteristics (Lee et al., 1986; Meernick and Yuen, 1988a). Composed of a bundle of optical fibers, with alternating layers of light-emitting fibers and light-receiving fibers in parallel, optical probes are quite similar to conductivity probes in that they are intrusive but economical. As bubbles or particles move upwards in the column, they pass a light-emitting fiber and then a receiver fiber. By measuring the difference in time between arrival of bubbles at different probes, one can determine bubble velocities. Optical probes too require a set of discrimination criteria. For the purposes of this work, they were not considered due to their more fragile nature and the murkiness of the liquid solutions used.
4.4 Pressure gradients

As mentioned above, pressure measurements are commonly employed to estimate phase hold-ups and bed expansion. Attempts were made in the present work to use this method for both objectives.

4.4.1 Bed expansion

Attempts to use pressure profiles to determine bed expansion were unsuccessful. Pressure profiles obtained typically resembled that shown in Figure 4.2, as opposed to the idealized, and expected, curve shown in Figure 4.1. There is no clear indication of a pressure gradient change between the expanded bed and the freeboard zones. Therefore, this method was not suitable for measuring the bed height. However, the slope does indicate that it was likely that a significant gradient in solids concentration existed in the bed. Wild and Poncin (1996) noted that the pressure profile approach was not likely to be effective for systems using small or light particles due to particle entrainment in the freeboard. They add that this is especially true at high gas flow rates.

4.4.2 Phase hold-ups

Pressure drop measurements were used successfully to obtain gas hold-ups. Equations (4.1), (4.2) and (4.3) were solved for the three phase hold-ups at various heights along the column. Two different methods were used to obtain the pressure drops. For the 83-mm column, a differential pressure transducer (Omega PX750-150DI) was
Figure 4.2: Typical axial pressure profile in the column as measured by the pressure transducers. Solution is aqueous glycerol solution (44 mass %) $U_L = 0.127$ m/s, $U_g = 0.12$ m/s, $W=25.5$ kg.
used. The two ends of the differential pressure transducer were attached to 6.4 mm NPT ports located along the column wall at different heights (z = 0.10 m, 0.41 m, 0.71 m, 1.02 m, 1.32 m, 1.63 m, and 1.93 m) and the pressure drop measured.

For the 292-mm diameter column a series of pressure barometers had been set-up before the pressure transducer was obtained and was therefore used. There were some difficulties in setting up the system. A fluid which was insoluble and unreactive with the aqueous glycerol was required. Various oils might have been employed, but that would necessitate inverting the position of the barometer. While that was possible, it would make it difficult to maintain the barometer fluid in the tubes when the column was drained or empty. While mercury would work, the pressure drops were sufficiently low that it would be difficult to measure them accurately with that fluid. A compromise was selected and two sets of multi-tube barometers were constructed, as shown schematically in Figure 4.3. The bottom of each barometer tube was connected to a port along the column with flexible hosing. The tops of each of the barometers were connected together, pressurized with compressed air, then sealed. The barometers were attached to 6.4 mm NPT ports along the column height with Quick-Connect fittings. When connected together, the two halves of these fittings allow complete flow. When the column is to be drained, the fitting can be quickly uncoupled and the fluid flow is completely restricted. Details of the calculations to convert the readings from barometer readings to pressure drops are given in Appendix B.
Figure 4.3: Schematic representation of set-up of one set of multi-tube barometers. Drawing not to scale.
4.5 Conductivity probes

As mentioned above, conductivity probes utilize local differences in conductivity to obtain information on gas hold-up and bubble characteristics. The technique is applicable only to systems where the gas and liquid have substantially different conductivities. For example, the difference in conductivity between tap water and air is ample.

4.5.1 Theory

Figure 4.4 depicts how the method works. To obtain bubble velocities two conductive tips are needed, while for gas hold-up only one tip is necessary. To reduce the intrusive nature of the probe, the probe should be as thin as possible. However, it must also be sufficiently sturdy to resist deformation by the particles and the fluids. As a bubble progresses upwards along the column, it passes across both probe tips in succession. As shown in Figure 4.4, it pierces the lower tip at \( t_1 \), then continues upwards and pierces the upper tip at \( t_2 \). As it rises further, the bottom of the bubble then passes the lower tip at time \( t_3 \). Finally, at \( t_4 \), the bubble bottom completely passes the upper probe. In such an idealized situation, the signal from the two probes can be measured. As shown in Figure 4.4(a), as the bubble is pierced by the lower probe tip, the signal drops towards zero since the conductivity of the gas is negligible compared to that of the liquid. Once the bubble disengages from the lower probe tip, the signal from that tip returns to the liquid conductivity value. Similarly in Figure 4.4(b), the signal drops as the bubble reaches and is pierced by the upper tip. The time at which the upper probe tip is pierced, \( t_2 \), is necessarily greater than the time at which the lower tip pierces the bubble, \( t_1 \).
(a) isolated signal from lower tip

(b) isolated signal from upper tip

Figure 4.4: Idealized conductivity probe signals for a bubble passing and piercing a two-tipped probe.
Similarly $t_4$ must be greater than $t_i$. Since the distance between the two probes, $s$, is known, the bubble velocity can be calculated from the difference in piercing time, $t_2 - t_1$, i.e. 

$$U_b = \frac{s}{t_2 - t_1} \quad (4.5)$$

while the pierced chord length, $L_b$, is given by 

$$L_b = U_b(t_3 - t_1) \quad (4.6)$$

The average bubble diameter can be estimated from the pierced bubble chord. Since the bubble can be pierced along any vertical chord and most bubbles are non-spherical, care is needed to relate the average pierced chord length to the actual average bubble size. This is especially true for non-spherical bubbles such as spherical-caps and slugs. The pierced chord lengths can be compared to the probability distribution for the appropriate shape and the true average bubble diameter can be determined (Clark and Turton, 1988; Han and Kim, 1993; Lim and Agarwal, 1990; Matsuura and Fan, 1984; Meernick and Yuen, 1988a; Tsutsui and Miyauchi, 1980; Turton and Clark, 1989). The gas hold-up is measured by dividing the sum of all the intervals when the signal is below the baseline (i.e. where the tip is enclosed by bubbles) by the total sampling time.

Since not all bubbles travel vertically up the column, the possibility exists that two distinct bubbles can pierce the set of probe tips in quick succession in such a way that the signals from the two tips resemble the idealized signals in Figure 4.4. However, this would clearly not yield a valid measurement for the bubble rise velocity. In some cases, a bubble may also become momentarily "stuck" at the probe tip, and this would result in overestimating the bubble chord length and underestimating the bubble velocity. A further complication is the possibility of swarms of attached bubbles to pass the probe. Since the probe response time would not allow the probe to register the swarm as a
distinct collection of bubbles, the signal would most likely register as one large extended bubble. Obviously these sorts of problems can lead to misrepresentative results, and therefore a set of discrimination criteria is required.

4.5.2 Discrimination criteria

To discern "true" bubbles travelling vertically upward from anomalies such as independent bubbles travelling at angles, or touching clusters of bubbles, a set of discrimination criteria was established, based upon previous work by Matsuura and Fan (1984). When the signals from two tips are compared, as in Figure 4.4(b), a bubble is considered to be a "true" vertically-rising bubble if the following three conditions are met:

1) \( t_1 < t_2 \)  \hspace{1cm} (4.7)
2) \( t_3 < t_4 \)  \hspace{1cm} (4.8)
3) \( 0.9 < \frac{(t_3 - t_1)}{0.5[(t_3 - t_1) + (t_4 - t_2)]} < 1.1 \)  \hspace{1cm} (4.9)

With this set of criteria, Matsuura and Fan (1984) found that for a bed operating in the dispersed regime typically 50% of the possible bubbles were rejected. For the coalesced regime, the rejection proportion increased to 65%, while for slug flow, this ratio was as high as 80%.

This set of discrimination criteria eliminates bubbles travelling obliquely or downwards. It does not, however, eliminate clusters of bubbles being taken as one large bubble. Although such a cluster would not affect the accuracy of the gas hold-up and bubble rise velocity measurements, it would falsely increase the average bubble chord length. To avoid this problem, a simple criterion was added to the above three criteria
proposed by Matsuura and Pan (1984). If a measured pierced bubble chord length is greater than 5 times the average bubble chord length then it is not considered when calculating bubble properties (although it still is counted when calculating gas hold-up). In other words, the bubble is rejected if

\[ 4) \quad L_b > 5 \bar{L}_b \]  

The multiple of five used in this criterion was based upon initial experiments. A typical pierced bubble chord length graph is given in Figure 4.5. As expected for a dispersed spherical bubble regime, there is a relatively narrow distribution of pierced bubble size. However, clearly there are also some “bubbles” that seem to be have much larger pierced lengths (in this case, there is one with an apparent pierced length of approximately 68 mm). These are likely to be bubble swarms or bubbles stuck at the tip for a brief period of time. In practice one would expect that real bubbles of that size would appear from time to time at the column wall and would result in large bubble bursts at the top of the column. Neither of these was observed. The bubble motion observed at the column walls did indicate swirling bubbles and bubble swarms that could likely lead to large pierced bubble chord lengths. In Figure 4.5, the factor of five would allow all pierced chord lengths up to 35.6-mm to be included, but the apparent 68-mm-long bubble would be excluded.

4.5.3 Probe design

Several 3.2-mm diameter probes were built and tested, but they did not survive within the column very long. Most broke or bent whenever the probe’s radial position needed to be changed. There also seemed to be no difference in their measuring
Figure 4.5: Typical pierced chord length distribution. Results are from system with aqueous glycerol solution (44 mass %), $U_L = 0.135 \text{ m/s}, U_g = 0.070 \text{ m/s}$, with probe at a height of 0.864 m and $W_s = 25.5 \text{ kg}$. Average $L_b$ is 7.11 mm.
capabilities whether the tips of the probe were at an angle or not. The final probe main
diameter of 8 mm was selected because it was sufficiently robust to withstand the
environment within the column. It was also the same diameter as the borescope so that
only one set of port fittings was required.

A schematic of the probe design is shown in Figure 4.6. The probe design was
based on previous work at UBC with conductivity probes (Zhang, 1996). The essential
components are two lengths of 1.0-mm diameter wire and the stainless steel shaft through
which they run. The wires are coated with Teflon to isolate them electrically from one
another and from the probe shaft. The very tips of the wires are clean and uncovered to
allow sensing of the conductivity surrounding the tips. The probe is tapered, reducing in
diameter from 8 to 3.2 mm near the tip, making the probe less intrusive in the immediate
area being measured. Recall that the particles themselves had a larger diameter, 4 mm.
Solder was used to connect the larger shaft with the smaller cross-sectional area to ensure
a good electrical connection. Epoxy was used at the tip of the probe to continue isolating
the wires from the shaft and to maintain the position of the wires relative to one another
and to the probe shaft.

The two sensors, or wires, in the conductivity probe are attached to two
Wheatstone bridges as shown in Figure 4.7, as in previous work at UBC (Zhang, 1996).
The two bridges are balanced to provide a signal as close as reasonably possible to zero
when there is negligible conduction between the wire tips and the probe shaft (as when a
bubble occupies the region). A further constraint is that the signal must be less than 5.0
volts when there is high conduction between the tips and the shaft (as when liquid
surrounds the region). The signal from the Wheatstone bridges were collected and stored
Figure 4.6: Design schematic of two-tip conductivity probe used in this research

Figure 4.7: Diagram depicting connection of two probe tips to Wheatstone bridges.
using a Keithley Industries DAS-1202 data acquisition board attached to a 100-MHz Pentium computer.

The energy sources for the two bridges were two separate 9-volt DC batteries. This presented unforeseen problems in the signals. Much of the previous work (e.g. Clements and Schnelle, 1963; Khang and Fitzgerald, 1975) in the field used AC-sources, rather than DC-batteries, and used platinum or platinized wires to reduce polarization. The previous work at UBC showed no need for this and the results in a water-air system worked reasonably well with a non-platinized, DC system.

4.5.4 Data acquisition

As mentioned above, the output from the two Wheatstone bridges was sent to a DAS-1202 data acquisition card. This card allows sampling at up to 100,000 samples per second and can accommodate up to 8 differential-ended analog inputs. Only signals between ±5 V can be measured. Typically, the board has an accuracy of 0.01% of the signal obtained. The board is plugged directly into a single ISA expansion slot in a Pentium computer. The data acquisition card was accompanied by a series of card drivers, QuickBasic subroutines, utility files, example files, and other miscellaneous files.

The data acquisition program was written in QuickBasic to match the supplied call subroutines already available in QuickBasic. An outline of flowsheet used to create the data acquisition software is shown in Figure 4.8. The program (test6) is listed in Appendix C. The board was set up to allow high speed DMA transfer, allowing the board to transfer data to the PC-memory without using the CPU so as to achieve the highest possible data transfer and sampling rates. Under DMA control, the data
Figure 4.8: Flowchart used for data acquisition software
acquisition card can continue to acquire data while transferring other data to the PC's random-access memory (RAM) simultaneously. Ideally the data should be transferred immediately to the hard drive, but the time required to access the hard drive was large enough to conflict with the data gathering process. By using DMA and the computer's RAM, the total sampling time was extended to the maximum allowable under the disk operating system (DOS) control (640 kilobytes). This total time could have been increased if a Windows-based system, such as Visual Basic, was used since that would allow practically full use of the computer's RAM, including high-memory (upwards of 16 megabytes, or 25 times greater than the amount used in this research). However, that required additional expertise in Visual Basic and the purchase of special call routines and drivers for the Windows-based system. For each sampling point, the program gathered ten separate packets of signals from the probe tips, each packet having a sampling time of 5.5 s and a sampling rate of 2500 Hz for each tip. The size of each file, or packet, from test6 was of the order of 320 kilobytes, or 3.2 megabytes per sampling point. An increase in sampling time through the use of a Windows-based system would also increase the required practical size of the hard drive.

The use of DMA creates a memory stack to which the data are temporarily written. For the program used for this research, under DOS limitations, a buffer of 15,000 data points is created (7500 from the upper tip of the conductivity probe and 7500 from the lower tip). The data from the probe tips are continuously stored sequentially in this stack. When the end of the stack is reached, the pointer is automatically reset to the beginning of the stack and the data storage continues uninterrupted overwriting the previously stored data. As shown in Figure 4.8, the acquisition program continuously
monitors the status of the data stack pointer. When the pointer is in the first half of the stack, the program transfers the information contained in the second half to a temporary array in the computer’s RAM. Similarly, when the pointer is in the second, or lower, half of the stack, data from the first half are stored in the RAM. Once the required data are obtained, the data stored in the temporary arrays are transferred to the hard drive. As mentioned above, a typical size of such a file is 320 kilobytes. The program then resets and seeks another batch of data. This loop is repeated ten times to allow for sufficient repeats at a given sampling point. A typical section of data obtained from the acquisition program for one batch of conductivity probe data for one tip is shown as the upper plot in Figure 4.9. These data are for the freeboard region and clearly show the effect of bubbles passing the conductivity probe. For the ebullated bed region, the highly conductive aluminum particles, created large upwards spikes resembling the inverse of the valleys caused by the bubbles, shown in Figure 4.9.

4.5.5 Signal filtering

In the present research, significant signal shifting was observed as shown in Figure 4.9. The upper graph in Figure 4.9 shows the raw signal data for measurements in the freeboard region. The gray line superimposed on this plot represents the baseline. Clearly, the baseline oscillates, unlike the flat and stable idealization shown in Figure 4.4. However, the baseline oscillation is at a very low frequency compared to the fluctuations.

The baseline oscillation can be attributed to several factors. The first has been discussed by previous researchers dealing with conductivity probes who found it necessary to use platinized wires and AC-power sources to prevent significant shifting
Figure 4.9: Conductivity probe signal from the lower probe tip before (top) and after (bottom) processing through Fast Fourier Transform subroutine. Liquid is aqueous glycerol. $U_g = 0.109 \text{ m/s}$, $U_l = 0.12 \text{ m/s}$, $\rho_l = 1105 \text{ kg/m}^3$, $W_s = 25.47 \text{ kg}$, $H_e = 0.89 \text{ m}$, $H_{settled} = 0.27 \text{ m}$. Probe is positioned at $H = 1.42 \text{ m}$, $r = 0.60 \text{ m}$. 
Platinizing wires and using AC current are important when one is measuring absolute conductivity (e.g. when measuring concentrations of slurries or when the bubbles are significantly smaller than the probe sampling area) as an indication of local concentration. For the present work, differential conductivity measurements were sufficient to discern between the gas and the liquid. As mentioned above, similar non-platinized, DC current conductivity probes had been used successfully in previous work (Zhang, 1996) in three-phase fluidized beds in this research group. Hence, the difficulties encountered were not anticipated.

The present research has also identified other factors contributing to signal fluctuation. Specifically, the calibration tests showed that when the probe was inserted into an area where there are particles, the baseline shift was much larger than in the freeboard where there were very few particles. The calibration tests also showed that baseline oscillations were higher for a flowing liquid than for a stagnant liquid. Initially it was thought that the oscillation might be due to flakes of aluminum, formed as a result of the particles rubbing against each other. This explanation was, however, discarded when similar oscillations were observed with clean solutions. It has also been suggested that small bubbles in the vicinity of the probe tip that are not pierced by the tip could lead to oscillation. There is no conclusive evidence to determine whether this is possible. Certainly no noticeable change in oscillation was noted when the probe was modified to reduce the distance between its shaft and tip. Based upon the calibration tests, the oscillation is most likely due to a combination of charging which occurs due to the non-platinized wire coupled with the DC-source and the motion of highly conductive particles in the tip area.
To correct for this baseline oscillation, a program was written to remove the low-frequency fluctuations and to flatten the baseline through the use of Fast Fourier Transforms (FFT’s). Initially, features in a commercial plotting program, Origin, were used to obtain the baseline but this required a long user-involved process. To speed data-processing, a filtering program, Filter.for, was written in Fortran. This program is listed in Appendix C, while the logic flowsheet for the program is shown in Figure 4.10. Raw data, obtained from Test6, are processed by Filter.for. The second step of the program involves adding “zeros” to the end of the data packet being processed. This is required due to the nature of Fast Fourier Transforms which requires $2^n$ data points for calculations (Ramirez, 1985). In this case, $2^{14}$, or 16384, data points were used. Since the raw data in a file consisted of only 13750 units for each tip, an additional 2634 data were added to the end of the raw signal. These data were only present to make the calculations possible and did not significantly influence the processing. The value of these “zeros” was set as the average signal value for the sampling period to be processed. In addition, to minimize the effect of the bubble-valleys in the signal, especially the larger ones, any signal that was significantly below the average value was excluded in calculating the low-frequency baseline and were replaced by the average value as well. After these two processing steps, the data were transformed from the time domain to the frequency domain using the subroutines provided in “Numerical Recipes in Fortran: The Art of Scientific Computing” (Press et al., 1992). To obtain the baseline data, only the low-frequency components were used. The higher frequency signals were attributed to system noise and the presence of bubbles.

For the purposes of this research, a baseline cut-off value was used. This cut-off
calculate average signal value

\[ \downarrow \]
add "zeros" to data stream

\[ \downarrow \]
eliminate any bubble "valleys"

\[ \downarrow \]
transform the data into the frequency domain

\[ \downarrow \]
eliminate all high frequencies components
to obtain the signal baseline

\[ \downarrow \]
re-invert the baseline data to the real domain

\[ \downarrow \]
calculate baseline average value

\[ \downarrow \]
normalize re-inverted baseline data with new average

\[ \downarrow \]
subtract processed baseline from original data

\textbf{Figure 4.10: Logic flowsheet for FFT filtering program (Filter.for)}
value was based upon qualitative analysis of previous signals, as seen in Figure 4.11. While sufficient low-frequency components were required to define the baseline, it was essential to remove the mid-range, and higher, frequency components. Including too high frequencies would cause the calculated baseline to be influenced by passing bubbles. Clearly if all frequencies were used, the calculated baseline would be exactly the same as the raw data. Figure 4.11 shows the effects of increasing the range of frequency components used in the baseline calculations. As the cut-off becomes higher, allowing higher frequencies to be included in the FFT calculations, the calculated baseline resembles the raw data more closely. In Figure 4.11 (c) the calculated baseline is obviously greatly influenced by the peaks and valleys in the raw signal data. Both Figure 4.11 (b) and Figure 4.11 (a) show significantly less influence of the bubble motion on the baseline calculations. Based upon these observations, it was determined that the first 25 frequency-components would be satisfactory to determine the baseline while minimizing the influence of passing bubbles. The importance of setting an appropriate cut-off value for the baseline calculations is shown in the next section.

Once the raw data were converted and the high frequency components were discarded (by setting their amplitudes to zero), the data were re-inverted to return to the time domain using subroutines provided by Press et al. (1992). In the case of inversion, the subroutine does not normalize the data and, therefore, normalization had to be written into the program. The normalized calculated baseline was then subtracted from the original data to yield the smoothed data. The smoothed data were then used to determine the gas hold-up and bubble properties.
Figure 4.11: Effects of Cut-off frequency on baseline calculations. Raw signal data are plotted in thinner lighter line showing peaks and valleys attributable to solids and bubble motion. The thicker darker smoother line in each case represents the calculated baseline based upon the FFT smoothing program (Filter.For). In case (a) only the first 25 frequency components of the signal were used to calculate the baseline. For (b) the first 50 were used, and for (c) the first 100 components.
4.5.6 Data analysis program

A third program was written to analyze the smoothed data. This program, written in QuickBasic, is called Cal_S98.Bas and is given in Appendix C. It uses two loops to calculate the required hydrodynamic values. In the first loop, gas hold-up is determined. In the second a search is made for possible bubbles based upon the three criteria set out above, i.e. equations (4.7), (4.8) and (4.9). The fourth criterion, equation (4.10), is applied once all possible bubbles have been identified.

A section of some filtered data is shown in Figure 4.12. In contrast to the idealized schematic in Figure 4.4, the valley due to bubble motion in Figure 4.12 is considerably sharper, and there is no evident valley bottom where the signal remains for a brief period of time. Due to the sharpness of the valley walls, it is clearly important to set a threshold limit whereby one can determine whether the signal indicates the presence of gas as opposed to liquid at the probe tip. If the threshold is set too low, then some of the bubble signal will be missed. In the case of Figure 4.12, if the threshold were set to – 0.5, for instance, then some small bubbles would be missed, resulting in erroneously low calculated gas hold-ups and high average bubble sizes. However, it is also important to set a threshold that will not allow signal noise to be included as bubbles (resulting in erroneously high gas hold-ups). With a threshold set very close to the baseline, any signal oscillation has a significantly detrimental effect on measurement accuracy. An oscillation “upwards” (or to a higher voltage) would create a situation whereby smaller bubbles are missed and bubble “valleys” in the signal would be intersected at lower points in their valleys, resulting in calculated gas hold-ups which are too low. A “downwards” oscillation (to a lower voltage) would actually push much of the liquid and
Figure 4.12: Small segment of FFT-modified conductivity probe signal from lower tip. Liquid is aqueous glycerol. $U_g = 0.109 \text{ m/s}$, $U_l = 0.12 \text{ m/s}$, $\rho_l = 1105 \text{ kg/m}^3$, $W_s = 25.47 \text{ kg}$, $H_e = 0.89 \text{ m}$, $H_{settled} = 0.27 \text{ m}$. Probe is positioned at $H = 1.42 \text{ m}$, $r = 0.60 \text{ m}$. 
noise portions of the signal into an area where the analysis program would consider it as indicating the presence of the gas and over-estimate the gas hold-up. Clearly, therefore, it is necessary to set the threshold as close as practical to the baseline and to eliminate baseline oscillations.

To determine an appropriate value for the threshold, experiments were conducted with no gas flow in a liquid-fluidized bed using the 292-mm column and an aqueous glycerol solution (44 mass % glycerol). The raw signal obtained was processed through the FFT-filtering program to remove the baseline oscillation. Examples showing the difference between the raw and the smoothed data are shown in Figures 4.13 and 4.15. Figure 4.13 shows data processed in the early stages of the research using the commercial program, while Figure 4.15 shows data processed more recently using the specially written software. The only fluctuations in the signal can be attributed to noise and these are eliminated from consideration if the threshold is set to −0.1. Hence, all smoothed signals with a value below −0.1 are considered to represent air bubbles at the probe tip. The frequency counts of the raw signals are shown in Figures 4.14 and 4.16. Clearly, at first the data show a relatively dispersed range of signal values. Upon smoothing, however, the data show much less dispersion, with no signal noise at −0.1 or lower values.

4.5.7 Probe response time

A series of calibration experiments was carried out in an attempt to quantify the response time. The probe tip was inserted in three different solutions, swirled around and removed quickly to the air. The three liquids used were deionized water, tap water, and
Figure 4.13: Conductivity probe signal from lower tip: (a) before processing; (b) after being processed with FFT analysis once; and (c) after being processed through a Fast Fourier Transform subroutine twice. Liquid is aqueous glycerol. \( U_g = 0 \text{ m/s}, U_l = 0.14 \text{ m/s}, \rho_l = 1105 \text{ kg/m}^3, W_s = 25.47 \text{ kg}, H_e = 0.77 \text{ m}, H_{settled} = 0.27 \text{ m}. \) Probe is positioned at \( z = 0.76 \text{ m}, r = 0.04 \text{ m}. \)
**Figure 4.14**: Frequency count for curves shown in Figure 4.12. Count of original conductivity probe signal from lower tip: (a) before processing; (b) after being modified with Fast Fourier Transform analysis once; and (c) after being modified twice with FFT analysis. Liquid is aqueous glycerol. \( U_g = 0 \text{ m/s}, U_1 = 0.14 \text{ m/s}, \rho_1 = 1105 \text{ kg/m}^3, W_s = 25.47 \text{ kg}, H_e = 0.77 \text{ m}, H_{settled} = 0.27 \text{ m}. \) Probe is positioned at \( z = 0.76 \text{ m}, r = 0.04 \text{ m}. \)
Figure 4.15: Example of signal processed using FilterFor. The original data are shown in (a) together with the calculated baseline. The smoothed data (where the baseline has been subtracted from the original data) are shown in (b).

Figure 4.16: Frequency count of signals shown in Figure 4.15.
an aqueous glycerol solution (44 mass % glycerol). There was no significant dependence of response times or signal shape on the liquid used. Particles were present in the beaker during all three sets of tests. An example of the signal obtained in an aqueous glycerol solution test is shown in Figure 4.17.

When the probe is inserted or removed from the liquid, as in the case of a bubble passing the probe tip, four distinct regions can be identified in the probe signal. The first is the period of time in which the probe is totally immersed in the liquid, including the entire signal to the left of line AB and to the right of line CD. The second region is the time during which the probe tip is shedding the liquid, including any thin layers that remain. This region is marked AB in Figure 4.17. The third region is the time during which the probe tip is surrounded by gas (or a bubble in the case of the experimental work). This is the region of a low signal between lines AB and CD in Figure 4.17. The fourth region corresponds to the time during which the probe tip is being wetted again and becoming surrounded by liquid, shown as CD. In all the test cases, the re-wetting of the tip occurred more quickly than the drying or shedding of liquid, i.e. the magnitude of the slope of CD is greater than that of AB, with the ratio of shedding to wetting rates varying from 1.1 to 3.3.

As discussed above, a threshold value (-0.1 volts) was set to determine whether or not a signal was in the gas or liquid phase. The response time is related to the probe's ability to measure a bubble. The probe must contact the bubble for a minimum period of time to be able to positively identify the bubble. The response time of the system can be determined from the calibration tests and includes both the probe's individual response time and the response time of the data acquisition system. The response time can be
Figure 4.17: Close-up of calibration signal for conductivity probe. The probe tip was immersed in an aqueous glycerol solution (44 mass % glycerol). At periodic intervals the probe was quickly taken out and exposed to air.
estimated from the difference between the time it takes line AB to intersect the threshold value and the time it takes line CD to go from the threshold to the baseline. If shedding and wetting occurred at the same pace, the probe's response time would be nearly instantaneous. However, in the test cases, due to quicker wetting, the average response time was of the order of 4.0 ms with the majority of the test cases showing response times better than 3.0 ms. This is comparable to the probe response time of 5 ms found by Burgess et al. (1981) for their probe. In the case of Figure 4.17, the response time was of the order of 3.0 ms. The size of bubble that can be detected by the probe depends on the bubble velocity. For bubbles travelling at 1 m/s, the average size the probe should be able to detect, according to the calibration tests, is of the order of 4.0 mm, while for slower bubbles travelling at 0.5 m/s, the average detectable bubble size is about 2.0 mm. Since the expected bubble size in the aqueous glycerol solution was expected to be about 4.0 mm, the response rate could interfere with the bubble measurements considerably leading to some under-prediction of gas hold-up. As shown below in Chapters 8 and 9, this was not a serious problem.

4.6 Borescope

An Olympus rigid borescope (model F-0800-028-000-55) was used in a series of initial experiments to attempt to film bubbles within the experimental column. A schematic diagram of a borescope is shown in Figure 4.18. The borescope is relatively intrusive as it has a diameter of 8 mm, and care must be taken in its use since the series of lens within the borescope are quite fragile and subject to breakage, even under mild conditions. A photograph taken with the borescope with the tip inserted into the fluidized
Figure 4.18: Schematic diagram of rigid borescope used in this research. Diameter is 8 mm and working length is 280 mm. The direction of view is direct and the field of view is 55° [reprinted from Lorenz, 1990].
bed is shown in Figure 4.19. Although a halogen light source is used, the picture quality is poor due to the lighting and murkiness of the solution. A picture taken with the borescope from outside the column appears in Figure 4.20. This photograph shows bubbles near the wall to be of the order of 4.0 mm in diameter as expected based upon the estimated value of the bubbles in the industrial unit and the scaling factor due to dimensionless group matching. Although the borescope can be a useful tool, it was not often used. The main problems were the poor lighting, low F-stop needs of the lens and low depth-of-field, which resulted in images of poor quality.

4.7 Measurement of physical properties

It was important to monitor and maintain the physical properties of the solutions to preserve the matching of the dimensionless groups. Since water (vapor pressure of 17.5 mm-Hg at 20°C, Weast, 1985) could evaporate much quicker than glycerol (vapor pressure of 1 mm-Hg at 125.5°C, Weast, 1985), it was important to add water periodically to the aqueous glycerol solutions. This was especially true during the hotter summer months and when doing prolonged runs with heating occurring due to the pump used to recirculate the liquid. Similarly, care had to be taken with the magnesium sulfate solutions to prevent crystallization as water evaporated.

4.7.1 Viscometer

Three separate viscometers were used. A Falling Ball Viscometer (size 3 from Gilmont Instruments) and a Canon-Manning Semi-Micro Viscometer, size 100 (No. B598) were used for quick tests. The falling ball viscometer was accurate but tended to
Figure 4.19: Typical image taken inside the bed using the borescope. Poor image quality is due to limited lighting and cloudy solution. Shutter speed = 1/250 s
(W= 25.47 kg, U₁ = 0.130 m/s, U₉ = 0.12 m/s, z = 0.56 m, r/R = 0.62)

Figure 4.20: Typical image taken at the wall of the column from outside.
Shutter Speed = 1/250 s
(W= 25.47 kg, U₁ = 0.130 m/s, U₉ = 0.12 m/s, z = 0.36 m, r/R = 1.0)
get plugged if any solid impurities were in the liquid being sampled. For analysis of
temperature effects a larger, computerized rotational Haake VT500 viscometer (relates
torque to viscosity) was used (with sensor tip NV).

4.7.2 Surface tensiometer

To analyze the surface tension, a ring-type Interfacial Tensiometer was used
(Cenco model No. 70545). After calibration, this device gave direct measurements of
surface tension in N/m.

4.8 Qualitative analysis of three-phase fluidized bed

Most of the data presented in this thesis were gathered in the 292-mm column
(described in section 3.1 and shown schematically in Figure 3.1). Apart from the
quantitative analysis in the following chapters, some qualitative observations were made.
The most striking feature of the fluidized bed under conditions dynamically similar to
those in an industrial hydrotreater is the flow pattern. Considering that the bed is
expected to operate in the dispersed bubble regime, one might expect a gently-bubbling
stream with bubbles rising vertically over the entire cross-section of the bed, and a clear
interface between the ebullated bed and the freeboard. Instead, the flow within the
column causes turbulent upward and downward swirl patterns that appear and disappear
quite quickly (in agreement with observations by Jiang et al., 1992). As a result bubbles
do not continuously rise vertically, but are swirling; a small number (less than 1%) can
even go in the opposite direction for brief periods of time. The particles are similarly
affected. Figure 4.21 depicts the bubble frequency as a function of angle (based upon the
Figure 4.21: Bubble frequency in various directions in the freeboard of a system dynamically similar to that of an industrial hydrotreater. A conductivity probe was used to make the measurements and was at a horizontal position 1.27 m above the distributor grid on the axis. Conditions for these data are $H_e = 1.0$ m, $U_g = 7.0$ cm/s, $U_L = 13.4$ cm/s, $\beta_s = 0.52$, $\beta_d = 2.37$, $M = 3.13 \times 10^7$, $Eo = 2.75$, and $Re_L = 61.1$. 
discrimination criteria established in section 4.5.2 and given in equations (4.7) to (4.10)).

Clearly, although the majority of bubbles have a mostly vertical direction, there are a significant number of bubbles rising obliquely. The temporary swirling flow patterns are likely influenced by the end effects caused by the liquid recycle exit and entry ports (see Figure 3.1(a)).

The interface between the ebullated bed and the freeboard was quite clear at low gas velocities, but became increasingly agitated and difficult to discern with increasing gas velocities. This finding is consistent with the observations of Wild and Poncin (1996), indicating increased particle entrainment into the freeboard with increasing gas flow rates.

Zhang et al. (1997) presented a series of flow regime maps for air-water-glass bead three-phase fluidized beds. Their results predict that the operating conditions used in the present work should result in either dispersed bubble flow or slug flow (at the higher gas flow rates). However, the flow regime map used for this prediction is based on work with 4.5-mm diameter glass beads. The aluminum particles used in the majority of the present work have an equivalent diameter of 6.4 mm. The flow regime maps presented by Zhang et al. (1997) suggest that as particle size increases, at a given liquid superficial velocity, the transition between dispersed bubble flow and slug flow is delayed to higher gas superficial velocities. This in turn suggests that the column in the present work operated primarily in the dispersed bubble regime. This is consistent with qualitative observations of bubble size and shape at the walls of the column which indicated little change in bubble shape or size for all three liquid solutions used in this work (tap water, aqueous 44 mass % glycerol solution, and aqueous 60 mass % glycerol
solution), and all operating conditions

Another feature common to many fluidized beds is to find particles moving mostly downwards near the wall. Since the 292-mm column was made of acrylic, particle motion near the wall could be observed. It was found that the particles did indeed tend to travel mostly in the downward direction near the wall. However, for brief periods, the particles near the wall temporarily moved upwards.

Dead zones were observed at the base of the column, directly above the distributor. These zones were indicated clearly by particles that did not move. In most cases, the particles became stacked lengthwise vertically. In some cases the dead zones were quite large, e.g. several particles lengths in height and about a dozen particle diameters in width. The dead zones were most evident at high liquid flow rates and low gas flow rates.

A final general observation concerns the liquid level at the top of the column. As the liquid flowrate was increased, the level increased as the particle screen became plugged with particles, leading ultimately to overflow. Increasing the gas flowrate also increased the level, while also causing the upper surface to become much more turbulent.
Chapter 5 - Validation Experiments of Dimensionless Groups

To test the validity of the dimensionless operating groups formed in Chapter 2, equation (2.30), a series of experiments was conducted using the 83-mm diameter column described in section 3.2 and depicted in Figure 3.5. As mentioned in section 2.6, one of the most difficult aspects of validating a scale-up technique is the lack of accurate industrial measurements with which to compare laboratory results. In the case of this research, data were obtained for an industrial cold model involving kerosene-nitrogen-ceramic particle fluidized bed in a column of diameter 0.91 m with physical properties given in Table 2.4. The liquid was an aqueous solution of 20 mass % magnesium sulphate, while the gas was air. Solid aluminum cylindrical particles with the correct length-to-diameter ratio (2.4 mm length and 1 mm diameter) were used.

The $d_p/D_c$ ratio between the two units could not be maintained as the volume of particles and column size would have been excessive. Relaxing the requirement for the $d_p/D_c$ ratios to be matched is only possible if significant wall effects are avoided. Both systems were expected to operate in the dispersed bubble regime. The expected bubble size was unknown since there were no measurements available from the industrial cold model unit. However, estimates using the empirical equation of Meernick and Yuen (1988b) predicted a bubble size of about 10 mm for the magnesium sulphate solution. Observation of bubbles near the walls of our column indicated that the actual bubble size was somewhat smaller, no larger than 5 mm in diameter. With the UBC column diameter 34 times larger than the maximum particle length (2.4 mm) and 16 times larger than the observed bubble size (5 mm), it was expected that wall effects would be small enough to be acceptable. Geometric similarity between the two systems was also impractical as, in
the case of the 0.91 m diameter industrial unit, there was a 229-mm-o.d. downcomer in
the center, while minimizing wall effects made it impossible to incorporate a central tube
in our column. In addition, the distributor system was not geometrically scaled, as noted
above.

A differential pressure transducer (Omega PX750-150DI) was connected to ports
along the column at 0.10 m, 0.41 m, 0.71 m, 1.02 m, 1.32 m, 1.62 m, and 1.93 m above
the distributor. The pressure differences between each position were measured for
periods of 60 s at a sampling rate of 50 Hz. The data acquisition program used for this
purpose was a simpler version of the program used for the conductivity probe (see
Section 4.5.4 and Appendix C.1 for details). Due to the much lower sampling rate (50 Hz
as compared to 2500 Hz), sufficient data could be acquired without using DMA data
transfer and looping, discussed in section 4.2.4. Differential pressure transducers were
also used in the 0.91-m diameter column measurements.

Figure 5.1 shows the bed expansion for the two systems versus $Re_g$ (the product of $Re_L$
and $\beta_U$) for various values of $Re_L$. It should be cautioned that while the data is plotted as
a function of $Re_g$ and $Re_L$, the only two parameters within those dimensionless groups
that were varied significantly for each liquid were the gas and liquid superficial
velocities. This is true for most of the data presented in this thesis. However, this should
not create any difficulty in interpreting results since the principal of dynamic similarity
suggests that it is the global dimensionless group rather than the values of the individual
parameters which is important. The filled-in data points represent the data from the UBC
column, while the open data points depict those from the 0.91-m diameter unit. The error
bars indicate the fluctuation of the bed interface during the UBC experiments. The
Figure 5.1: Bed expansion versus $Re_g$ for various liquid Reynolds numbers. Filled-in points represent UBC data while the 0.91-m diameter unit data are represented by open points. Physical properties are listed in Table 2.4.

average of the minimum and maximum heights was used to determine the bed expansion.

No estimate of uncertainty is available for the 0.91-m diameter data.

Since the UBC experimental column was built from acrylic, the axial position of the interface between the three-phase ebullated bed section and the two-phase (gas-
liquid) freeboard area could be observed from outside the column. The bed expansion is defined as the expanded bed height minus the settled bed height divided by the settled bed height. Although the operating conditions covered in the UBC experiments cover a wider range, some comparison can still be made. The data in both cases agree with previously reported trends (Lee, 1986). In particular, the bed expansion shows a strong dependence on the liquid flow rate, and hence on the liquid Reynolds number, and a much weaker dependence on the gas flow rate, and therefore on Re_g. Although the exact operating conditions were not matched for this set of experiments, the similarity in trends is encouraging, as is the relative placement of the six lines in order of increasing Re_L.

Figure 5.2 plots the freeboard gas hold-up versus Re_g for various values of Re_L. The pressure drop was taken in the freeboard immediately above the three-phase bed surface in both the 0.91-m diameter column and in the UBC unit. Measurements were taken as close as possible above the interface so that coalescence in the freeboard would be minimized. Most of the UBC data agree quite well with those for the 0.91-m diameter column. Previously reported trends (Fan, 1989; Nigam and Schumpe, 1996) are repeated in that the gas hold-up is a strong function of gas velocity, and hence of Re_g, while there is no significant dependence on the liquid flow rate, i.e. on Re_L.

The ebullated bed and freeboard gas hold-ups measured in the industrial unit are plotted in Figure 5.3. In the UBC 83-mm column, there were significant difficulties in reading the gas hold-up in the ebullated bed accurately. Most likely these arose due to gradients in solids concentration making the use of pressure drop measurements difficult and preventing accurate readings. The 0.91-m diameter data under these dynamic conditions indicate that there was little difference between the gas hold-ups in the
ebullated bed and the freeboard region immediately above the bed. However, in deriving these results, the industrial analysis included a number of unspecified correction factors. The pressure measurements were corrected to account for what the experimenters perceived to be errors arising from unbalanced DP gas purge rates, traces of liquid in DP lines, and drift in DP cell electronics.

Figure 5.2: Apparent gas holdup in freeboard region versus Re₉ for various liquid Reynolds numbers. Filled-in points represent UBC data while the 0.91 m diameter unit data are represented by open points. Physical properties are listed in Table 2.4.
Figure 5.3: Apparent gas holdup in freeboard region and ebullated bed versus $Re_g$ for various liquid Reynolds numbers for the 0.91 m diameter unit. Data for the freeboard region are represented by unfilled points. The ebullated bed measurements are symbolized by data points containing plus signs in the center (+) and are connected by solid lines. Physical properties are listed in Table 2.4.
Chapter 6 - Bed Expansion Measurements

A series of experiments was conducted in the 292-mm column described in Chapter 3 with three solutions: tap water, a 44 mass % aqueous glycerol solution, and a more concentrated (approximately 60 mass %) glycerol-water solution. The pertinent properties of the solutions, as measured by the techniques discussed in Chapter 4, are given in Table 6.1. The most important variable to control is the viscosity. Viscosity is particularly sensitive to solution temperature, and careful control of the cooling water flowrate through the cooling coil had to be maintained. This is especially true since the ambient temperature changed on a daily basis and differed significantly between summer and winter months. Figure 6.1 shows the dependence of viscosity on temperature. A 0.5 x 10\(^{-3}\) Pa-s change in viscosity around 11.5 x 10\(^{3}\) Pa-s results in a change in the Reynolds number of only 4.3%. However, for the M-group, the viscosity is raised to the fourth power so that the sensitivity is much greater. Obviously it was vital to maintain a constant fluid temperature during an experimental run.

Normal operation occurred within a 1°C range to minimize viscosity variance. The solution temperature usually rose with time, primarily due to heating caused by the pump. For higher liquid viscosities, viscous dissipation increased, and the solution became hotter more quickly. For the concentrated glycerol solution (60 mass %), ice cubes were periodically packed around the pump casing to reduce the increase in temperature. In practice, this allowed continuous operation of the unit for two to three hours before a break was required to allow the pump and the liquid to cool down to within one degree of the set temperature. Maintaining the temperature was particularly necessary for the bed expansion measurements due to the sensitivity of bed expansion to
Table 6.1: Physical properties of liquids used in 292-mm column

<table>
<thead>
<tr>
<th>Liquid Solution</th>
<th>Temperature (°C)*</th>
<th>μ (Pa·s)</th>
<th>σ (kg/s²)</th>
<th>ρ_L (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tap water</td>
<td>12 ± 0.2</td>
<td>1.2 x 10⁻³</td>
<td>72 x 10⁻³</td>
<td>999</td>
</tr>
<tr>
<td>44 mass % (by weight) aqueous glycerol</td>
<td>21 ± 0.4</td>
<td>4.5 x 10⁻³</td>
<td>67.4 x 10⁻³</td>
<td>1097</td>
</tr>
<tr>
<td>60 mass % (by weight) glycerol solution</td>
<td>24 ± 0.5</td>
<td>11.5 x 10⁻³</td>
<td>66.8 x 10⁻³</td>
<td>1150</td>
</tr>
</tbody>
</table>

Increased operating temperature results from increased viscous dissipation at higher liquid viscosities.

Figure 6.1: Temperature dependence of viscosity of the concentrated glycerol solution.
the liquid viscosity.

For each set of runs, measurements were taken at various times. The results include scatter due to uncertainty in the flow rate measurements, as well as temperature fluctuations. Since the liquid flow was always sufficient to fluidize the solids without the presence of gas (i.e. at $\beta_u=0$) and since increasing the gas flow always caused increased expansion, the bed expansion (BE) data for given values of $M$, $E_o$, and $\beta_u$, were fitted by an equation of the form

$$BE = \text{Liquid term} + \text{Gas term} \quad (6.1)$$

The liquid term was a function of the liquid Reynolds number while the gas term showed a significant dependence on $\beta_u$, and on liquid Reynolds number. The liquid term was of the form

$$BE(\text{Liquid Term}) = A_1 \cdot |Re_L - Re_{Lmf}|^P \quad (6.2)$$

where $Re_{Lmf}$ is the liquid Reynolds number based on the liquid superficial velocity for minimum fluidization velocity and $A_1$ and $P$ are empirical coefficients. The gas term can be correlated by

$$BE(\text{Gas Term}) = B_1 (\beta_u)^{B_2} \quad (6.3)$$

These correlations were fitted using the non-linear curve fitting routine available in a commercial software package (Origin version 5.0 by Microcal). The fitting sessions used by the subroutines are based upon the Levenberg-Marquardt algorithm, described as the most widely used algorithm in non-linear least squares fitting (Origin manual, 1997). The error bars on all correlated coefficients, gas hold-ups, and bubble properties presented in Chapters 6 to 8 are based on standard errors calculated by the program and related to the sample population standard deviation divided by the square root of the
number of data in the sample population. Error bars for bed expansion measurements reflect the fluctuations in bed level.

6.1 Experiments with tap water

For the water experiments, three dimensionless groups were constant: $M = 5.94 \times 10^{11}$, $E_o = 2.24$, and $\beta_d = 2.70$. Figure 6.2 shows the data for the two-phase fluidized bed case where $\beta_u = 0$. The coefficients for equation 6.2 for this data are given in Table 6.2. For this case $Re_{\text{lmf}}$ is 240, corresponding to $U_{\text{l}_{\text{mf}}} = 7.2 \text{ cm/s}$. Most three-phase correlations for $U_{\text{l}_{\text{mf}}}$ include a term for gas flow rate and cannot be used to predict the minimum liquid flow required when the gas flow is equal to zero (Begovich and Watson, 1978; Costa et al., 1986; Song et al., 1989; Yang et al., 1993). To obtain a prediction for $U_{\text{l}_{\text{mf}}}$ for the liquid-solids system, one turns to the numerous two-phase correlations available. Zhang (1996) summarized correlations of Ermakova et al. (1970), Begovich (1978) Saxena and Vogel (1977), Babu et al. (1978), Grace (1982), and Chitester et al. (1984) of the form

$$Re_{\text{lmf}}(\beta_u = 0) = \sqrt{a^2 + bA_{RL}} - a \quad (6.4)$$

where

$$A_{RL} = \frac{d^3 \rho_L (\rho_P - \rho_L) g}{\mu^2} \quad (6.5)$$

The $U_{\text{l}_{\text{mf}}}$ for the tap water/glass bead system is predicted, by these correlations, to be within the range of 5.6 to 7.7 cm/s. This matches well with the value of 7.2 cm/s calculated above from Figure 6.2.

The bed expansions in the water-air-aluminum system for various liquid Reynolds
Table 6.2: Fitted coefficients for equation (6.2) pertaining to liquid component of bed expansion data.

<table>
<thead>
<tr>
<th></th>
<th>Re\textsubscript{Lmf}</th>
<th>A\textsubscript{i}</th>
<th>P</th>
<th>Interpolated U\textsubscript{Lmf} (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tap water</td>
<td>240 ± 39</td>
<td>0.00705 ± 0.01005</td>
<td>1.55 ± 0.21</td>
<td>7.2</td>
</tr>
<tr>
<td>44 mass % aqueous glycerol solution</td>
<td>39.1 ± 2.7</td>
<td>0.109 ± 0.032</td>
<td>1.47 ± .06</td>
<td>4.0</td>
</tr>
<tr>
<td>60 mass % aqueous glycerol solution</td>
<td>11.6 ± 1.1</td>
<td>1.01 ± 0.04</td>
<td>1.3*</td>
<td>2.9</td>
</tr>
</tbody>
</table>

This value was set at 1.3 from average range of observed values (1.1–1.6)

Figure 6.2: Bed expansion versus liquid Reynolds number with no gas flow for tap water and aluminum particles in 292-mm column. Physical properties of the liquid are given in Table 6.1. M= 5.94 x 10\textsuperscript{-11}, Eo = 2.24, and β\textsubscript{d} = 2.70.
numbers are given in Figures 6.3 to 6.7 as a function of the velocity ratio, \( \beta_u \). For these conditions, the correlation of Begovich and Watson (1978) predicts bed expansions from \(-7.9\%\) to \(24.5\%\), significantly lower than observed in the present work. The correlation does, however, correctly predict the qualitative trends observed in the present work of increased bed expansion with increased liquid viscosity and increased gas and liquid superficial velocities.

Correlations of the form of equation (6.3) were next fitted to the data. Table 6.3 shows the coefficients \( B_1 \) and \( B_2 \) obtained by fitting. Both \( B_1 \) and \( B_2 \) are influenced by the liquid Reynolds number. The influence of Reynolds number on \( B_1 \) is shown in Figure 6.8 while the influence of Reynolds number on \( B_2 \) is plotted in Figure 6.9. In both cases, the coefficient increases rapidly for \( \text{Re}_L > 600 \). The resulting equation for the tap water system (\( M=5.94 \times 10^{11} \), \( \text{Eo} = 2.24 \), \( \beta_d = 2.70 \), \( 430 \leq \text{Re}_L \leq 716 \)) is

\[
100 \beta_{be} = 0.00705 (\text{Re}_L - 240)^{1.55} + B_1 (\beta_u)^{B_1},
\tag{6.6}
\]

where

\[
B_1 = 16.6 + 6.03 \times 10^{17} (\text{Re}_L)^{6.38}
\tag{6.7}
\]

and

\[
B_2 = 0.299 + 0.0023 \text{Re}_L - 9.35 \times 10^6 \text{Re}_L^2 + 1.05 \times 10^8 \text{Re}_L^3.
\tag{6.8}
\]

### 6.2 Experiments with 44 mass % aqueous glycerol solution

For these experiments, a somewhat more viscous solution was used as seen from Table 6.1. While the liquid Reynolds number (\( \text{Re}_L \)) and velocity ratio (\( \beta_u \)) were varied, the other three dimensionless operating groups were kept constant at \( M = 1.20 \times 10^8 \), \( \text{Eo} = 2.55 \), and \( \beta_d = 2.46 \). As in the tap water case, the bed expansion at zero gas flow rate
Table 6.3: Correlation coefficients $B_1$ and $B_2$ for equation (6.3) for various liquid Reynolds number values for the water-air-aluminum system.

<table>
<thead>
<tr>
<th>$Re_L$</th>
<th>$B_1$</th>
<th>$B_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>356</td>
<td>$14.6 \pm 0.6$</td>
<td>$0.398 \pm 0.113$</td>
</tr>
<tr>
<td>430</td>
<td>$20.5 \pm 2.0$</td>
<td>$0.476 \pm 0.217$</td>
</tr>
<tr>
<td>536</td>
<td>$38.2 \pm 2.4$</td>
<td>$0.435 \pm 0.100$</td>
</tr>
<tr>
<td>646</td>
<td>$61.1 \pm 2.0$</td>
<td>$0.709 \pm 0.047$</td>
</tr>
<tr>
<td>716</td>
<td>$113.9 \pm 11.1$</td>
<td>$0.988 \pm 0.103$</td>
</tr>
</tbody>
</table>

Figure 6.3: Bed expansion versus velocity ratio for tap water-air-aluminum three-phase system in 292-mm column. Liquid Reynolds number = 356. Physical properties of the liquid are given in Table 6.1. $M= 5.94 \times 10^{11}$, $Bo = 2.24$, $\beta_d = 2.70$. 
Figure 6.4: Bed expansion versus velocity ratio for tap water-air-aluminum three-phase system in 292-mm column. Liquid Reynolds number = 430. Physical properties of the liquid are given in Table 6.1. \( M = 5.94 \times 10^{11}, E_0 = 2.24, \beta_d = 2.70 \).

\[ \chi^2 = 23.5 \]
\[ a = 23.9 \]
\[ B_1 = 20.5 \pm 2.0 \]
\[ B_2 = 0.476 \pm 0.217 \]

Figure 6.5: Bed expansion versus velocity ratio for tap water-air-aluminum three-phase system in 292-mm column. Liquid Reynolds number = 536. Physical properties of the liquid are given in Table 6.1. \( M = 5.94 \times 10^{11}, E_0 = 2.24, \beta_d = 2.70 \).

\[ \chi^2 = 19.6 \]
\[ a = 47.5 \]
\[ B_1 = 38.2 \pm 2.4 \]
\[ B_2 = 0.435 \pm 0.100 \]
Figure 6.6: Bed expansion versus velocity ratio for tap water-air-aluminum three-phase systems in 292-mm column. Liquid Reynolds number = 646. Physical properties of the liquid are given in Table 6.1. M= $5.94 \times 10^{11}$, Eo = 2.24, and $\beta_d = 2.70$.

Figure 6.7: Bed expansion versus velocity ratio for tap water-air-aluminum three-phase systems in 292-mm column. Liquid Reynolds number = 716. Physical properties of the liquid are given in Table 6.1. M= $5.94 \times 10^{11}$, Eo = 2.24, and $\beta_d = 2.70$. 
Figure 6.8: Correlation for $B_1$ as a function of liquid Reynolds number for air-water-aluminum three-phase system. $M = 5.94 \times 10^{11}$, $E_0 = 2.24$, and $\beta_d = 2.70$.

\[
\chi^2 = 40.2 \\
y = a + bx^2 \\
a = 16.6 \pm 5.6 \\
b = 6.03 \times 10^{-17} \pm 5.78 \times 10^{-16} \\
c = 6.38 \pm 1.45
\]

Figure 6.9: Correlation for $B_2$ as a function of liquid Reynolds number for air-water-aluminum three-phase system. $M = 5.94 \times 10^{11}$, $E_0 = 2.24$, and $\beta_d = 2.70$.

\[
Y = Z_1 + Z_2 x + Z_3 x^2 + Z_4 x^3 \\
Z_1 = 0.299 \pm 2.607 \\
Z_2 = 0.0023 \pm 0.0152 \\
Z_3 = -9.35 \times 10^6 \pm 0.00003 \\
Z_4 = 1.05 \times 10^{-6} \pm 1.75 \times 10^{-8}
\]
was first plotted versus $\text{Re}_L$, leading to the graph shown in Figure 6.10. These data were fitted to equation (6.2). The results, shown in Table 6.2, indicate a minimum liquid-solid fluidization value, $\text{Re}_{Lmf}$, of 39.1, corresponding to a superficial velocity of 4.0 cm/s. For comparison, the correlations summarized by Zhang (1996) predict a range of 4.2 to 6.1 cm/s, in reasonable agreement with the experimental value.

The bed expansions for the aqueous 44 mass % glycerol-air-aluminum system for various liquid Reynolds numbers as a function of the velocity ratio, $\beta_u$, are given in Figures 6.11 to 6.14. The trends observed in this series of experiments resemble those for the water-air-aluminum system. The correlation of Begovich and Watson (1978) predicts bed expansions from -2.2 % to 38.3%, again significantly lower than observed in the present work. However, the qualitative trends observed in the present work are again correctly predicted by the correlation (increased bed expansion with increased liquid viscosity and increased gas and liquid superficial velocities).

Correlations of the form shown in equation (6.3) were again fitted to the data. Table 6.4 shows the coefficients $B_1$ and $B_2$ obtained by fitting. In both cases, as shown in Figures 6.15 and 6.16, the liquid Reynolds number influences the fitted values. For the aqueous 44 mass % glycerol solution ($M=1.20 \times 10^{-8}$, $Eo = 2.55$, $\beta_d = 2.46$, $73 \leq \text{Re}_L \leq 156$), the bed expansion is correlated as

$$100\beta_{be} = 0.109(\text{Re}_L - 39.1)^{0.47} + B_1(\beta_u)^{B_2}$$

(6.9)

where

$$B_1 = 16.0 + 1.29 \times 10^{-5} (\text{Re}_L)^{3.10}$$

(6.10)

and

$$B_2 = 0.311 + 2.55 \times 10^{-10} \text{Re}_L^{4.24}$$

(6.11)
Figure 6.10: Bed expansion versus liquid Reynolds number with no gas flow for aqueous 44 mass % glycerol-aluminum three-phase system in 292-mm column. Physical properties of the liquid are given in Table 6.1. $M = 1.2 \times 10^8$, $E_0 = 2.55$, and $\beta_d = 2.46$.

Figure 6.11: Bed expansion versus velocity ratio for aqueous 44 mass % glycerol-air-aluminum three-phase system in 292-mm column. Liquid Reynolds number = 73.1. Physical properties of the liquid are given in Table 6.1. $M = 1.2 \times 10^8$, $E_0 = 2.55$, and $\beta_d = 2.46$. 
Figure 6.12: Bed expansion versus velocity ratio for aqueous 44 mass % glycerol - air - aluminum three-phase system in 292-mm column. Liquid Reynolds number = 105. Physical properties of the liquid are given in Table 6.1.

\[ \text{M} = 1.2 \times 10^8, \text{Eo} = 2.55, \text{and } \beta_d = 2.46. \]

Figure 6.13: Bed expansion versus velocity ratio for aqueous 44 mass % glycerol - air - aluminum three-phase system in 292-mm column. Liquid Reynolds number = 127. Physical properties of the liquid are given in Table 6.1.

\[ \text{M} = 1.2 \times 10^8, \text{Eo} = 2.55, \text{and } \beta_d = 2.46. \]
Figure 6.14: Bed expansion versus velocity ratio for aqueous 44 mass % glycerol-air-aluminum three-phase system in 292-mm column. Liquid Reynolds number = 156. Physical properties of the liquid are given in Table 6.1. $M = 1.2 \times 10^8$, $Eo = 2.55$, and $\beta_d = 2.46$.

Table 6.4: Coefficients $B_1$ and $B_2$ for equation (6.3) for various liquid Reynolds number values for the aqueous 44 mass % glycerol-air-aluminum system.

<table>
<thead>
<tr>
<th>$Re_L$</th>
<th>$B_1$</th>
<th>$B_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>73.1</td>
<td>$23.6 \pm 1.3$</td>
<td>$0.327 \pm 0.120$</td>
</tr>
<tr>
<td>105</td>
<td>$41.3 \pm 0.8$</td>
<td>$0.420 \pm 0.048$</td>
</tr>
<tr>
<td>127</td>
<td>$58.7 \pm 1.1$</td>
<td>$0.515 \pm 0.036$</td>
</tr>
<tr>
<td>156</td>
<td>$98.8 \pm 4.5$</td>
<td>$0.825 \pm 0.064$</td>
</tr>
</tbody>
</table>
Figure 6.15: Correlated $B_1$ factors as a function of liquid Reynolds number for aqueous 44 mass % glycerol-air-aluminum system. $M = 1.2 \times 10^8$, $E_o = 2.55$, and $\beta_d = 2.46$.

$\chi^2 = 1.38$
$y = a + b \times x^c$
$a = 16.0 \pm 2.4$
$b = 1.29 \times 10^5 \pm 1.46 \times 10^5$
$c = 3.10 \pm 0.22$

Figure 6.16: Correlated $B_2$ factors as a function of liquid Reynolds number for aqueous 44 mass % glycerol-air-aluminum system. $M = 1.2 \times 10^8$, $E_o = 2.55$, and $\beta_d = 2.46$.

$\chi^2 = 0.00025$
$y = a + b \times x^c$
$a = 0.311 \pm 0.022$
$b = 2.55 \times 10^{-10} \pm 6.99 \times 10^{-10}$
$c = 4.24 \pm 0.54$
6.3 Experiments with concentrated aqueous glycerol solution

The liquid properties for the highest viscosity solution (aqueous 60 mass % glycerol solution) are given in Table 6.1. While the Reynolds number and the velocity ratio were varied, the other three dimensionless operating groups were kept constant with \( M = 5.00 \times 10^7 \), \( Eo = 2.70 \), and \( \beta_d = 2.35 \). As in the previous cases, the bed expansion at zero gas flow rate was first plotted versus the liquid Reynolds number to obtain the graph shown in Figure 6.17, and these data were then fitted to equation (6.2). It should be noted that with only six data points (two data are superimposed at both \( Re_L = 31.7 \) and \( Re_L = 41.6 \)), it is difficult to obtain precise information when matching the curves. Reasonable fits for values of the exponent \( P \) in equation (6.2) were from 1.1 to 1.6. To estimate the minimum liquid-solid fluidization velocity for the liquid-solid case, \( P \) was fixed at 1.3. The results, shown in Table 6.2, indicate a minimum liquid-solid fluidization value, \( Re_{Lmf} \), of 11.6, corresponding to a superficial velocity of 2.9 cm/s. For comparison, the correlations summarized by Zhang (1996) predict a range of 2.7 to 4.4 cm/s, in good agreement.

The bed expansions for the concentrated aqueous glycerol-air-aluminum system for various liquid Reynolds numbers as a function of the velocity ratio, \( \beta_u \), are given in Figures 6.18 to 6.20. The trends observed in this series of experiments are similar to those already seen and are correctly predicted by the correlation of Begovich and Watson (1978). The correlation also succeeds in predicting the observed trend of increasing bed expansion with increasing solution viscosity. However, the correlation again predicts expansions much lower than those observed, in the range of 3.5 to 54.5%.
Figure 6.17: Bed expansion versus liquid Reynolds number with no gas flow for aqueous 60 mass % glycerol-aluminum three-phase system in the 292-mm column. Physical properties of the liquid are given in Table 6.1. \( M = 5.0 \times 10^7 \), \( E_0 = 2.70 \), and \( \beta_d = 2.35 \).

Figure 6.18: Bed expansion versus velocity ratio for aqueous 60 mass % glycerol-air-aluminum three-phase system in 292-mm column. Liquid Reynolds number = 31.7. Physical properties of the liquid are given in Table 6.1. \( M = 5.0 \times 10^7 \), \( E_0 = 2.70 \), and \( \beta_d = 2.35 \).
Figure 6.19: Bed expansion versus velocity ratio for aqueous 60 mass % glycerol-air-aluminum three-phase system in 292-mm column. Liquid Reynolds number = 41.6. Physical properties of the liquid are given in Table 6.1. $M = 5.0 \times 10^7$, $Eo = 2.70$, and $\beta_d = 2.35$.

Figure 6.20: Bed expansion versus velocity ratio for aqueous 60 mass % glycerol-air-aluminum three-phase system in 292-mm column. Liquid Reynolds number = 50.0. Physical properties of the liquid are given in Table 6.1. $M = 5.0 \times 10^7$, $Eo = 2.70$, and $\beta_d = 2.35$. 

\[ \chi^2 = 12.0 \]
\[ a = 84.4 \]
\[ B_1 = 31.5 \pm 0.8 \]
\[ B_2 = 0.201 \pm 0.061 \]
Correlations of the form found in equation (6.3) were again fitted to the data. Table 6.5 gives the coefficients $B_1$ and $B_2$ obtained by fitting. As shown in Figures 6.21 and 6.22, both $B_1$ and $B_2$ vary with $Re_L$. It should be noted that for this more viscous liquid, data were available for only three liquid Reynolds numbers. For the concentrated glycerol solution ($M=5.00 \times 10^{-7}$, $E_0 = 2.70$, $\beta_d = 2.35$, $31 \leq Re_L \leq 50$), the data were correlated as

$$100\beta_{be} = 1.01(Re_L - 11.6)^{1.3} + B_1(\beta_u)^{B_2}$$  \hspace{1cm} (6.12)

where

$$B_1 = -49.3 + 1.93 Re_L$$  \hspace{1cm} (6.13)

and

$$B_2 = 0.213 + 1.45 \times 10^{13} \exp (-Re_L)$$  \hspace{1cm} (6.14)

A comparison of the measured data for all three solutions with the predictions from the correlations given by equations (6.6), (6.9) and (6.12) is shown in Figure 6.23 in a parity plot. It is seen that there is excellent agreement.

$A_1$ and $P$ from equation (6.2) for the three solutions tested are plotted versus the $M$-group in Figure 6.24. Since only three data points are available, caution must be exerted when attempting to conclude any relationship.

### 6.4 New approach to use bed expansion data to estimate gas hold-up

Based upon the bed expansion experiments, a new method for estimating gas hold-up is proposed. The method is based upon the gas-perturbed liquid model presented by Zhang et al. (1995). This model considers the sole effect of the gas to be occupation of volume within the column, thereby decreasing the volume available for the liquid, thus
Table 6.5: Coefficients $B_1$ and $B_2$ for equation (6.3) for various liquid Reynolds number values for the aqueous 60 mass % glycerol-air-aluminum system.

<table>
<thead>
<tr>
<th>$Re_L$</th>
<th>$B_1$</th>
<th>$B_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>31.7</td>
<td>$11.7 \pm 0.6$</td>
<td>$0.464 \pm 0.117$</td>
</tr>
<tr>
<td>41.6</td>
<td>$31.5 \pm 0.8$</td>
<td>$0.200 \pm 0.061$</td>
</tr>
<tr>
<td>50.0</td>
<td>$47.0 \pm 3.1$</td>
<td>$0.225 \pm 0.111$</td>
</tr>
</tbody>
</table>

Figure 6.21: Correlated $B_1$ factors as a function of liquid Reynolds number for aqueous 60 mass % glycerol-air-aluminum three-phase system. $M = 5.0 \times 10^7$, $Eo = 2.70$, and $\beta_d = 2.35$. 

$Y = A + B \times X$

$A = 49.3 \pm 1.8$

$B = 1.93 \pm 0.04$

$r^2 = 0.99976$ SD = 0.547
Figure 6.22: Correlated $B_2$ factors as a function of liquid Reynolds number for aqueous 60 mass % glycerol-air-aluminum system. $M = 5.0 \times 10^7$, $E_0 = 2.70$, and $\beta_d = 2.35$.

\[ y = y_0 + A \cdot e^x \]
\[ y_0 = 0.213 \]
\[ A = 1.45 \times 10^{13} \]

Figure 6.23: Parity plot comparing measured bed expansions to values predicted by correlations presented in equations (6.6), (6.9) and (6.12)
Figure 6.24: Correlated $A_1$ and $P$ in equation (6.2) for various values of $M$. 
increasing its interstitial velocity. Therefore, for conditions similar to those used in this work, an increase in gas flow rate results in an increase of bed height since the increased gas flow occupies space and hence increases the interstitial liquid velocity. In other cases, under conditions not covered by the current research, in particular with particles smaller than 2.5 mm, bed contraction has been observed upon introduction of gas (e.g. see Epstein and Nicks, 1976; Wild et al, 1984). Since the interstitial velocity increases, the drag on the particles increases and the bed expands further. The method presented below relates the bed expansion directly, and solely, to the interstitial liquid velocity.

Extension of the model to relate bed expansion to gas hold-up requires the following steps:

1) measure the mass of particles in the column, $W_s$;

2) measure the bed height in the liquid-solid two-phase system (i.e. with no gas addition) for various liquid superficial velocities;

3) convert the superficial velocities to interstitial velocities based upon the bed height and the solids loading, using equation (4.2). to determine $\varepsilon_p$:

\[
is \quad \text{superficial velocity} : U_L = \frac{Q_L}{A_c} \quad (6.15) \\
\quad \text{and} \quad \text{interstitial velocity} : U_L^* = \frac{U_L}{\varepsilon_L} \text{ or } \frac{U_L}{(1-\varepsilon_p)} \quad (6.16)
\]

These three steps then yield a curve of bed height versus interstitial velocity for a liquid-solids system. An example of such a figure is given in Figure 6.25 for the intermediate viscosity solution.

When the bed is fluidized in the presence of gas,

\[
\varepsilon_g + \varepsilon_L + \varepsilon_p = 1 \quad (4.3)
\]
Figure 6.25: Bed height versus interstitial velocities. Liquid is aqueous 44 mass % glycerol solution. 4 mm x 10 mm aluminum cylinders used as particles. Temperature of the solution is approximately 20°C.

\[ Y = A + B \times X \]

\[ A = -36.6 \pm 4.8 \]

\[ B = 7.49 \pm 0.27 \]

\[ r^2 = 0.99622 \]
so that equation (6.16) must be rewritten as

$$U_L^* = \frac{U_L}{(1 - \varepsilon_p - \varepsilon_g)}$$

(6.17)

The liquid superficial velocity can be calculated from equation (6.15), and the over-all solids hold-up from equation (4.2). One can then use the measured bed height and the plot of interstitial liquid velocity versus bed height to calculate the required interstitial velocity, $U_L^*$. Equation (6.17) can then be solved for the sole remaining unknown, the gas hold-up, $\varepsilon_g$.

As an example, if the initial liquid superficial velocity (with no gas flow) is 0.126 m/s, an expanded bed height of 0.997 m is observed, and the equivalent interstitial velocity is 0.179 m/s from equation (6.16). When gas is added at 0.07 m/s, the bed expands to 1.22 m. From Figure 6.25, we find that an interstitial liquid velocity of 0.212 m/s is required to attain that bed expansion. One then solves equation (6.17) to obtain a gas hold-up of 16.5% in good agreement with the value of 17.5% measured from pressure drop experiments.

It is important to note that this method is limited to situations in which the gas flowrate is not expected to have any significant effects other than on the liquid interstitial velocity. When the particles are relatively light or small or gas velocities are considerably higher, this will not be the case and this method will not be applicable, since the gas could then fluidize the particles directly.

This method was applied to the aqueous 44 mass % glycerol solution-air-aluminum system. Good agreement between predictions and experimental values were obtained as shown in Table 6.6 and in Figure 6.26. The error bars in the figure reflect the fluctuation of the height of the ebullated bed-freeboard interface.
Table 6.6: Comparison of gas hold-up values from pressure measurements and from use of interstitial velocity method for aqueous 44 mass % glycerol-air-aluminum system.

<table>
<thead>
<tr>
<th>$U_g$ (cm/s)</th>
<th>$U_L$ (cm/s)</th>
<th>Average $\varepsilon_g$ from interstitial velocity method</th>
<th>Error margin based on bed level fluctuation ±</th>
<th>Average $\varepsilon_g$ from Pressure drop</th>
<th>Average $\varepsilon_p$ from Pressure drop</th>
<th>Average $\varepsilon_L$ from Pressure drop</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>7.9</td>
<td>0.116</td>
<td>0.0260</td>
<td>0.147</td>
<td>0.375</td>
<td>0.479</td>
</tr>
<tr>
<td>7.0</td>
<td>7.9</td>
<td>0.138</td>
<td>0.0422</td>
<td>0.172</td>
<td>0.363</td>
<td>0.465</td>
</tr>
<tr>
<td>10.7</td>
<td>7.9</td>
<td>0.160</td>
<td>0.0630</td>
<td>0.183</td>
<td>0.352</td>
<td>0.465</td>
</tr>
<tr>
<td>14.1</td>
<td>7.9</td>
<td>0.170</td>
<td>0.0718</td>
<td>0.209</td>
<td>0.347</td>
<td>0.444</td>
</tr>
<tr>
<td>5.1</td>
<td>10.5</td>
<td>0.141</td>
<td>0.0131</td>
<td>0.134</td>
<td>0.290</td>
<td>0.576</td>
</tr>
<tr>
<td>7.0</td>
<td>10.5</td>
<td>0.162</td>
<td>0.0339</td>
<td>0.157</td>
<td>0.281</td>
<td>0.562</td>
</tr>
<tr>
<td>10.7</td>
<td>10.5</td>
<td>0.206</td>
<td>0.0313</td>
<td>0.178</td>
<td>0.264</td>
<td>0.558</td>
</tr>
<tr>
<td>14.1</td>
<td>10.5</td>
<td>0.221</td>
<td>0.0538</td>
<td>0.205</td>
<td>0.258</td>
<td>0.537</td>
</tr>
<tr>
<td>5.1</td>
<td>12.6</td>
<td>0.135</td>
<td>0.0229</td>
<td>0.165</td>
<td>0.252</td>
<td>0.582</td>
</tr>
<tr>
<td>7.0</td>
<td>12.6</td>
<td>0.165</td>
<td>0.0280</td>
<td>0.175</td>
<td>0.242</td>
<td>0.583</td>
</tr>
<tr>
<td>10.7</td>
<td>12.6</td>
<td>0.193</td>
<td>0.0483</td>
<td>0.222</td>
<td>0.232</td>
<td>0.546</td>
</tr>
</tbody>
</table>

Figure 6.26: Comparison of gas hold-ups measured with $U_L$-method and pressure drop method for aqueous 44 mass % glycerol solution-air-aluminum system.
Chapter 7 - Gas Hold-Up Measurements

Experiments were conducted to measure gas hold-up under various conditions in the 0.292 m column. Three different liquids were used: tap water, an aqueous 44 mass % glycerol solution, and an aqueous concentrated glycerol solution. The physical properties of these liquids were given in Table 6.1. In all cases, air and aluminum cylindrical particles (4 mm in diameter and 10 mm in length) were used as the gas and the solids, respectively. The experimental equipment and procedures are outlined in Chapters 3 and 4. Pressure drop readings were used to calculate gas hold-ups for both aqueous glycerol solutions, but not for tap water due to large fluctuations, which were of the same order of magnitude as the pressure difference expected, in the latter case. Conductivity probes were used to calculate the gas hold-up in both tap water and the aqueous 44 mass % glycerol solution. The conductivity probe did not work properly in the concentrated glycerol solution.

7.1 Experiments with tap water

The gas hold-ups measured in the water system are shown in Figure 7.1. Measurements were obtained using the conductivity probe shown in Figure 4.6 and discussed in section 4.2. The probe has the advantage of not requiring knowledge of solids or liquid hold-up to determine gas hold-up. However, since the conductivity probe yields local measurements, the data had to be integrated over the column cross-section to yield average gas hold-ups at each measurement height. A sample calculation is given in Appendix D. This method ignores areas of the column in which no local measurements were made, thereby potentially introducing errors for sections where there are significant
Table 7.1: Values of “a” and “b” for equation (7.1) as fitted in Figures 7.1, 7.2, and 7.3.

<table>
<thead>
<tr>
<th>Liquid used in system</th>
<th>a-factor</th>
<th>Error</th>
<th>b-factor</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tap water</td>
<td>0.0139</td>
<td>±0.0028</td>
<td>0.426</td>
<td>±0.035</td>
</tr>
<tr>
<td>Aqueous 44 mass % glycerol</td>
<td>0.0359</td>
<td>±0.0045</td>
<td>0.346</td>
<td>±0.028</td>
</tr>
<tr>
<td>Aqueous concentrated glycerol</td>
<td>0.0486</td>
<td>±0.0060</td>
<td>0.322</td>
<td>±0.035</td>
</tr>
</tbody>
</table>

Figure 7.1: Average gas hold-up versus modified gas Reynolds number \((Re_g = Re_L * \beta_u)\) for various liquid Reynolds number values. Hold-ups were determined by integrating conductivity probe local measurements from the 0.292-m column with tap water and air as the fluids. \(M = 5.46 \times 10^{-11}\), \(\varepsilon_o = 2.18\), \(\beta_d = 2.75\), \(W_s = 53.4\) kg.
radial gradients. However, in general, the method provides an effective means for measuring the average gas hold-up.

From Figure 7.1, it is evident that the gas hold-up depends strongly on the modified gas Reynolds number. This trend, which was observed with all three solutions used, is correctly predicted by the correlation of Bloxom et al. (1975). Clearly in Figure 7.1, there is also a slight increase in measured hold-up with decreasing liquid Reynolds number. The correlation of Bloxom et al. (1975) does predict such a relationship. However, results for the other liquid systems, presented below, do not conform to such a prediction and show no discernible dependence of gas hold-up on liquid superficial velocity. For tap water as the liquid and the range of operating conditions covered, the correlation of Bloxom et al. (1975) predicts gas hold-ups in the range of 0.069 to 0.160, slightly lower than the measured values. There is a good agreement between the results and predictions at the lowest values of the product (henceforth referred to as modified gas Reynolds number) of liquid Reynolds numbers, ReL, and velocity ratio, βu. The results are lower than predicted at higher ReL·βu values. However, the results are within the 40% deviation expected with this correlation (Wild and Poncin, 1996).

The results were fitted to a power-law equation passing through the origin (zero gas hold-up at zero gas flow) with no dependence on liquid Reynolds number giving

\[
\varepsilon_g = a \text{Re}_g^b
\]  

(7.1)

The fitted values of a and b are given in Table 7.1, and the resulting correlation is compared with the experimental data in Figure 7.1. For the tap water system (M=5.94 x 10^{-11}, Eo = 2.24, βd = 2.70, 430 ≤ ReL ≤ 716), the data were correlated as

\[
\varepsilon_g = 0.0139 \text{Re}_g^{0.426}
\]  

(7.2)
7.2 Experiments with 44 mass % aqueous glycerol solution

Figure 7.2 shows measured gas hold-ups with an aqueous 44 mass % glycerol solution as the liquid. For this set of experiments, a bank of barometers was used to obtain pressure drop measurements, as discussed in section 4.1. As in the water system, the hold-up is directly related to the gas Reynolds number. Unlike the situation with water, there seems to be little or no consistent effect of liquid Reynolds number.

For the aqueous 44 mass % glycerol solution, the correlation of Bloxom et al. (1975) predicts gas hold-ups in the range of 0.075 to 0.172. As in the case of the tap water experiments, the predictions are slightly lower than the measured values but well within the 40% deviation possible with this correlation (Wild and Poncin, 1996).

The data were again fitted to equation (7.1) and the fitted constants appear in Table 7.1 and Figure 7.2. The data was correlated (for the range of conditions, i.e. \( M=1.20 \times 10^{-8}, E_o = 2.55, \beta_d = 2.46, 73 \leq \text{Re}_L \leq 156 \)) as

\[
\varepsilon_g = 0.0359 \text{Re}_g^{0.346}
\]  

(7.3)

The open-faced data points in Figure 7.2 are from a set of early experiments in which the gas hold-up was measured in the freeboard region immediately above the three-phase fluidized bed region. These results were checked using a conductivity probe inserted just below the interface and the measurements agreed quite well, suggesting that, for the conditions studied in this work, the gas hold-up immediately above the fluidized bed region is similar to the bed gas hold-up. The filled-in data points shown in Figure 7.2 also indicate this. These points represent results from pressure measurements across as much of the fluidized bed as possible (minimizing any effects of gradients in solid concentration) and they fit earlier data quite well. Note that these two series of
Figure 7.2: Average gas hold-up, determined from pressure drops, versus modified gas Reynolds number for various liquid Reynolds numbers. Experiments conducted in a 0.292-m column with an aqueous 44 mass % glycerol solution and air as the fluids.

\[ y = a \times x^b \]
\[ \chi^2 = 0.00013 \]
\[ a = 0.0359 \pm 0.0045 \]
\[ b = 0.346 \pm 0.028 \]

\( M = 1.20 \times 10^8 \), \( Eo = 2.55 \), \( \beta_d = 2.46 \), \( W_s = 53.4 \text{ kg} \).
experiments were conducted for two separate batches of aqueous 44 mass % glycerol solutions, more than two years apart. The earlier data (open-faced data points) were taken with no air line filter, while the later data (filled-in data points) were taken after the air filter had been installed. The data give a measure of the reproducibility of the data and indicate that the addition of the oil filter did not have a significant influence on the results.

7.3 Experiments with concentrated aqueous glycerol solution

Figure 7.3 plots the average gas hold-up in the fluidized bed for the most concentrated glycerol solution. The hold-ups were determined, as for the other glycerol solution, from pressure drop measurements. The results again show no significant dependence of gas hold-up on liquid Reynolds number.

For the aqueous 60 mass % glycerol solution, the correlation of Bloxom et al. (1975) predicts gas hold-ups in the range of 0.075 to 0.172. As in the previous cases, the predictions are slightly lower than the measured values, but well within the 40% deviation possible with this correlation (Wild and Poncin, 1996).

Fitted constants for equation (7.1) are listed in Table 7.1 and in Figure 7.3. The data were correlated (for the range of conditions covered, i.e. \(M=5.00 \times 10^7\), \(Eo = 2.70\), \(\beta_d = 2.35\), \(31 \leq \text{Re}_L \leq 50\)) by

\[ \varepsilon_g = 0.0486 \text{Re}_g^{0.322} \]  

(7.4)

The open-faced points in Figure 7.3 represent data from pressure drops within the fluidized bed, while the filled-in data points are for the region in the freeboard immediately above the three-phase fluidized bed surface. As in the earlier case, there
\[ y = a \times x^b \]

\[ \chi^2 = 0.00004 \]

\[ a = 0.0486 \pm 0.0060 \]

\[ b = 0.322 \pm 0.035 \]

Figure 7.3: Average gas hold-up, determined from pressure drops, versus modified gas Reynolds number for various liquid Reynolds number values. Experiments conducted in 0.292-m column with a concentrated (60 mass %) aqueous glycerol solution and air as the fluids. Fluid properties are listed in Table 6.1.

\[ M = 5.01 \times 10^7, \ E_o = 2.70, \ \beta_d = 2.35, \ W_a = 53.4 \text{ kg}. \]
was little difference between the gas hold-ups in these two regions.

A parity plot is presented in Figure 7.4 which compares measured gas hold-ups for all three solutions used with predictions from equations (7.2) through (7.4). It is seen that there is good agreement.

Figures 7.5 and 7.6 plot the fitted factors “a” and “b”, respectively, from equation (7.1) versus the M-group for the liquid systems used. These factors are plotted as functions of the M-group rather than the Eötvös number or the density ratio, since these last two groups were each only varied over a range of about 20%. Some caution must clearly be exercised when interpreting these results given the small number of data points (three). Further work is required to substantiate the relationship. Certainly it seems that the factors have opposite dependence on the M-group. From Figure 7.5, has

\[ a = 0.106 + 0.00898 \log_{10} M \] (7.5)

while, from Figure 7.6,

\[ b = 0.143 - 0.0269 \log_{10} M \] (7.6)

7.4 Radial profiles of local gas hold-ups

Figure 7.7 shows local gas hold-up measurements in the 0.292 m diameter column with the conductivity probe for the aqueous 44 mass % glycerol solution. This series of experiments was prompted by earlier work in which the conductivity probe was inserted at a height of 0.051 m above the distributor plate and registered very small and often negligible gas hold-ups, with respect to greater heights. Initially, the reason for this was thought to be that all bubbles were very small, smaller than the conductivity probe could
Figure 7.4: Parity plot of measured gas hold-up values versus predictions from equations (7.2) through (7.4).
Figure 7.5: Fitted factor “a” from equation (7.1) versus M-group.

\[ a = 0.106 + 0.00898 \times \log M \]

\[ r^2 = 0.999 \]

Figure 7.6: Fitted factor “b” from equation (7.1) versus M-group.

\[ b = 0.143 - 0.0269 \times \log M \]

\[ r^2 = 0.975 \]
Hold-ups were determined by conductivity probe with a 44 mass % glycerol solution and air as the fluids. Fluid properties are listed in Table 6.1.

\( \text{Re}_L = 58.5, \text{Re}_g = 49.6, M = 1.20 \times 10^8, \text{Eo} = 2.55, \beta_d = 2.46, W_s = 53.4 \text{ kg}. \)
measure, in this region. As shown by Figure 7.7, the problem is primarily maldistribution of air at the base. At the lowest measurement height of 0.051 m, gas bubbles are found solely near the outside of the column. This maldistribution corrects itself, however, as one moves upwards. The non-uniformity almost completely disappears within the lowest 0.356 m of the bed, and is completely eliminated by $z=0.559$ m. It should be recognized that due to the geometric scaling factor of the particles, this height of correction is equivalent to only 0.089 m (or 0.140 m for complete elimination of gradients) in the industrial unit being simulated. Following this series of experiments, the packing in the pre-mix (or “windbox”) region of the column, shown in Figure 3.3, was replaced with packing that resulted in a significantly higher pressure drop in an effort to improve the flow distribution. In retrospect, it is conceded that the design of the distributor system could have been better to eliminate the observed maldistribution.

Figures 7.8 to 7.11 represent local gas hold-up profiles at different heights for various liquid and modified gas Reynolds numbers. For this series of figures, the liquid was water. Despite the higher pressure drop packing, maldistribution still occurred above the distributor. Clearly, despite the efforts to improve the distributor system, as discussed earlier in section 3.1, maldistribution continued, though at a lower level. The probable reason can be seen from Figure 3.3, where one notices that the liquid is introduced by 76 mm piping into the center of the column with considerable momentum. This momentum makes it more difficult for the gas to penetrate into the central area of the column. However, the good news is that in all fifteen cases examined (only five are presented in this chapter, with the other results being similar and presented in Appendix E), the fluidized bed self-corrected the maldistribution completely within 0.559 m of the
The effect of varying the superficial gas velocity can be determined by comparing Figures 7.8 through 7.11. Comparison of Figures 7.8 and 7.9 indicates that as the gas Reynolds number increases significantly while the liquid Reynolds number is nearly constant, more gas appears in the central region of the column. However, as shown in Figure 7.10, this is true only for relatively low liquid Reynolds numbers. At higher liquid velocities, and hence Re_L, the gas still has difficulty penetrating the inner core of the column at low heights. As seen in Figure 7.11, at the lowest measurement height (z=0.051 m), for systems where the gas velocity is considerably lower than the liquid velocity, i.e. Re_L ≫ Re_g, the radial profile is similar to those seen in Figures 7.8 and 7.10.
Figure 7.8: Local gas hold-up at various radial positions and four different heights. Hold-ups were determined by conductivity probe in 0.292-m column with tap water and air as the fluids.

\[ \text{Re}_L = 430, \ \text{Re}_g = 170, M = 5.46 \times 10^{11}, E_o = 2.18, \beta_d = 2.75, W_g = 53.4 \text{ kg}. \]
Figure 7.9: Local gas hold-up at various radial positions and four different heights. Hold-ups were determined by conductivity probe in 0.292-m column with tap water and air as the fluids. 

\[ \text{Re}_L = 430, \text{Re}_g = 470, \text{M} = 5.46 \times 10^{11}, \text{Eo} = 2.18, \beta_d = 2.75, W_s = 53.4 \text{ kg}. \]
Figure 7.10: Local gas hold-up at various radial positions and four different heights. Hold-ups were determined by conductivity probe in 0.292-m column with tap water and air as the fluids.

Re_L = 536, Re_g = 356, M = 5.46 x 10^{-11}, Bo = 2.18, \beta_d = 2.75, W_s = 53.4 kg.
Figure 7.11: Local gas hold-up at various radial positions and four different heights. Hold-ups were determined by conductivity probe in 0.292-m column with tap water and air as the fluids.

Re_L = 716, Re_g = 170, M = 5.46 x 10^{-11}, \text{Eo} = 2.18, \beta_d = 2.75, W_s = 53.4 \text{ kg.}
Chapter 8 - Bubble Rise Velocities and Pierced Chord Lengths

The conductivity probe assembly discussed in section 4.5 was also used to detect bubble characteristics for the systems using tap water and the aqueous 44 mass % glycerol solution. The discrimination criteria discussed in section 4.5.2 were used to eliminate bubbles not travelling in a vertical direction from consideration. All work discussed in this chapter was carried out with cylindrical aluminum particles (4 mm in diameter and 10 mm in length) and air as the solids and gas, respectively, in the 0.292-m column.

The correlation of Yu and Kim (1988) predicts ranges of $\frac{U_b}{U_L}$ of 4.1 to 5.4 and 7.0 to 8.9 for tap water and aqueous 44 mass % glycerol solutions, respectively, in reasonable agreement with the measurements in the present work. The correlations for bubble size predict a considerably wider range of values. The correlation of Meernick and Yuen (1988b) predicts a bubble size of 2.0 mm for all three liquid solutions used, while that of Kim et al. (1977) predicts a range of 53 to 70 mm, for tap water, and 52 to 69 mm, for the aqueous 44 mass % glycerol solution.

8.1 Tap-water system

Figure 8.1 shows a plot of the ratio of bubble rise velocity to liquid superficial velocity for the tap water system for various modified gas Reynolds numbers. Each point in Figure 8.1 represents, on average, 5250 individual data points measured over time spans of approximately 600 s. The plotted error bars represent standard errors which are equal to the standard deviation divided by the square root of the number of bubbles detected. The data represented by the filled-in upside down triangles in Figure 8.1 are for
Figure 8.1: Ratio of bubble rise velocity to liquid superficial velocity, $\beta_b$, versus modified gas Reynolds number, $Re_g$, for various liquid Reynolds number values. Liquid for this series of experiments was tap water. The filled-in and open points are for heights of 0.864 m and 0.559 m above the distributor plate, respectively. The expanded bed heights, except for $Re_L = 430$, exceeded 0.864 m. For $Re_L = 430$, the expanded height was between 0.749 m and 0.800 m. Each plotted point represents an average of 5250 bubbles and a sampling time of 600 s.
bubbles in the freeboard region, while all other data are for the ebullated bed. Since the bubbles in the freeboard would be unimpeded by particles, they are expected to rise more quickly than bubbles of similar volume in the ebullated bed. In addition, bubbles in the freeboard are more likely to coalesce and become larger, resulting in larger buoyancy effects.

Although there is considerable scatter, the bubbles in the ebullated bed seem to have similar trends at the two heights of the measuring probe. In both cases, a significant increase in velocity is measured with increasing modified gas Reynolds number. There is some increase in bubble velocity with increasing depth. There is also some evidence, although not conclusive, that the ratio of bubble rise velocity to liquid velocity decreases as the liquid velocity increases (i.e. as the liquid Reynolds number increases) (contrary to what was reported by Lee et al., 1990). However, as shown in Figure 8.2, the actual dimensional bubble rise velocity increases with increased liquid velocity. Figure 8.3(a) shows a typical histogram of the distribution of measured bubble rise velocities.

Figure 8.4 plots $\beta_L$, the ratio of bubble pierced chord length, $L_b$, to the cylinder diameter, $d_p$, for various liquid and modified gas Reynolds numbers. As in Figure 8.1, the filled-in data points represent values measured at 0.864 m above the gas-liquid distributor, while the open points are for measurements at 0.559 m. The filled-in upside down triangles are again for the freeboard region. The plot suggests that there is no significant effect of liquid velocity on the $L_b/d_p$ ratio. However, for the range studied, the bubble size is seen to increase slightly with increased gas flow (or modified gas Reynolds number). These findings are in good agreement with those of Kwon et al. (1994). The dependence on gas velocity is considerably greater in the freeboard region. The chord
Figure 8.2: Bubble rise velocity versus modified gas Reynolds number, $Re_g$, for various liquid Reynolds number values in tap water 0.864 m above the distributor plate. The expanded bed heights for all the experiments exceeded 0.864 m. Each data point represents an average of 5250 bubbles and a sampling time of 600 s.
Figure 8.3: Histograms of occurrences of (a) bubble rise velocity and (b) pierced chord length for a liquid Reynolds number value of 536 and a gas Reynolds number equal to 233 for 5685 vertically rising bubbles over a period of 605 s. The average bubble rise velocity and average bubble pierced chord length were 0.923 m/s and 5.47 mm, respectively. Liquid for this series of experiments was tap water.
Figure 8.4: Ratio of pierced chord length, $L_b$, to particle diameter, $d_p$, versus modified gas Reynolds number, $Re_g$, for various liquid Reynolds number values. Liquid for this series of experiments was tap water. The filled-in and open points are for heights of 0.864 m and 0.559 m above the distributor plate respectively. The expanded bed heights, except for $Re_L = 430$, exceeded 0.864 m. For $Re_L = 430$, the expanded height was between 0.749 m and 0.800 m. Each plotted point represents an average of 5250 bubbles and a sampling time of 600 s.
lengths in Figure 8.4 are average values measured by the two probe tips. The measured lengths from the two tips are plotted against each other in Figure 8.5. The 45° line plotted in Figure 8.4 represents the equality between the two tips. Clearly the measurements at the two tips agree quite well. Figure 8.3(b) shows a typical histogram of the distribution of pierced chord lengths.

Figure 8.2 indicates that the bubble rise velocity increases with increasing gas Reynolds number while Figure 8.4 indicates that the bubble size does not increase as strongly. This could suggest that the gas hold-up decreases with increased gas flow but this is contrary to both common sense and to previous trends as seen in Figures 7.1, 7.2 and 7.3. The explanation lies in the frequency of bubbles that the probe measures as seen in Figure 8.6. As the gas flow increases, the frequency of bubbles increases as well. Figure 8.6 also shows that the frequency of bubbles rising vertically increases with increased liquid Reynolds number. Note also that a higher frequency of bubbles moving vertically is detected at a z = 0.864 m compared to that at 0.559 m. This can be attributed primarily to the axial solids concentration gradient. As observed in connection with the pressure drop experiments (see Section 4.4 and Figure 4.2), there seemed to be a gradient of solids along the height of the bed, with fewer particles at the top of the ebullated bed than at the bottom. This would result in fewer bubbles being deflected by particles with increasing height, and hence more bubbles travelling vertically. Figure 8.7 shows the percentage of the total number of bubbles detected by the lower tip (as indicated by valleys in the signal) that met the discrimination criteria and were considered to be bubbles rising completely vertically. There does not seem to be a significant effect of gas
Figure 8.5: Pierced chord lengths in tap water measured by the conductivity probe’s lower tip versus those measured by the upper tip. Each plotted data point represents an average of 5250 bubbles and a sampling time of 600 s.
Figure 8.6: Frequency of bubbles passing vertically across the probe tips versus modified gas Reynolds number, $Re_g$, for various liquid Reynolds number values in tap water. The filled-in and open points are for heights of 0.864 m and 0.559 m above the distributor plate, respectively. The expanded bed heights, except for $Re_L = 430$, exceeded 0.864 m. For $Re_L = 430$, the expanded height was between 0.749 m and 0.800 m. Each plotted point represents an average of 5250 bubbles and a sampling time of 600 s.
Figure 8.7: Percentage of bubbles which pass the probe tip vertically versus modified gas Reynolds number, $Re_g$, for various liquid Reynolds number values in tap water. The filled-in and open points are for heights of 0.864 m and 0.559 m above the distributor plate, respectively. The expanded bed heights, except for $Re_L = 430$, exceeded 0.864 m. For $Re_L = 430$, the expanded height was between 0.749 m and 0.800 m. Each plotted point represents an average of 5250 bubbles and a sampling time of 600 s.
velocity (i.e. of \( Re_g \)) on this parameter. However, more bubbles travel vertically at higher liquid velocities (i.e. at higher \( Re_L \)).

Plots of typical bubble characteristics versus radial position are given in Figure 8.8 for \( z = 0.559 \) m with cross-sectional averages for this height shown as dashed lines. The results are generally quite consistent across the radius. The lower than average values observed in the region of \( r/R = -0.4 \) to \(-0.2 \) seem to be associated with the proximity of the exit port for the recycle stream.

**8.2 Aqueous 44 mass % glycerol system results**

Figure 8.9 plots the ratio of bubble rise velocity to liquid velocity for various liquid and modified gas Reynolds numbers. As in Figure 8.1, the \( \frac{U_b}{U_L} \) ratio increases with increasing gas Reynolds numbers. There is relatively little difference between the bubble velocity ratios for \( Re_L = 110 \) and 131 at \( z = 0.864 \) m. However, there does seem to be a significant difference between these results and those at \( z = 0.559 \) m and \( Re_L = 70 \). In comparison to the tap water results in Figure 8.1, the bubble velocity ratio in the aqueous glycerol system is approximately 40% higher at equivalent gas superficial velocities. Figure 8.11(a) shows a typical histogram of the distribution of bubble rise velocities in the aqueous glycerol system.

This increase in bubble rise velocity is most likely due to larger bubbles in the aqueous glycerol solution, as shown in Figure 8.10. The ratio of pierced chord length to particle diameter, \( \beta_L \), exhibits a stronger dependence on gas flow rate in the aqueous glycerol solution than shown in Figure 8.4 for the tap water system. The large difference in bubble rise velocities in Figure 8.9 between \( Re_L = 70 \) and the higher \( Re_L \) (110 and 131)
Figure 8.8: Frequency of vertically rising bubbles, mean pierced chord length, and mean bubble rise velocity versus radial position for tap water as the liquid at 0.559 m above the distributor. The dashed lines represent the cross-sectional average values. \( \text{Re}_g = 233 \) and \( \text{Re}_L = 646 \). Each plotted data point represents an average of 546 bubbles and a sampling time of 55 s.
Figure 8.9: Ratio of bubble rise velocity to liquid superficial velocity, $\beta_b$, versus modified gas Reynolds number, $Re_g$, for various liquid Reynolds numbers. Liquid for this series of experiments was an aqueous 44 mass % glycerol solution. The filled-in and open points are for heights of 0.864 m and 0.559 m above the distributor plate respectively. The expanded bed heights exceeded 0.864 m. Each plotted point represents an average of 2022 bubbles and a sampling time of 275 s.
Figure 8.10: Ratio of pierced chord length, $L_b$, to particle diameter, $d_p$, versus modified gas Reynolds number, $Re_g$, for various liquid Reynolds numbers. Liquid for this series of experiments was an aqueous 44 mass % glycerol solution. The filled-in and open points are for heights of 0.864 m and 0.559 m above the distributor plate respectively. The expanded bed heights exceeded 0.864 m. Each plotted point represents an average of 2022 bubbles and a sampling time of 275 s.
is consistent with the measurements in Figure 8.10. The pierced chord lengths are seen to be significantly higher for the lower liquid Reynolds number system, leading to larger buoyancy effects and higher bubble rise velocities. A typical histogram for one of the cases where there were large measured bubbles is shown in Figure 8.11(b). This shows that the results do not reflect a few large bubbles skewing the results but rather genuine measurement of larger bubbles than expected.

Figure 8.12 shows the frequency of vertically rising bubbles, and the fraction of total detected bubbles which met the discrimination criteria for various liquid modified gas Reynolds numbers. These results can be compared to those for tap water as the liquid shown in Figures 8.6 and 8.7. For higher Re_L, the frequency trends are similar to those in Figure 8.6. More bubbles met the discrimination criteria as the liquid Reynolds numbers increase, with a stronger dependence on liquid Reynolds number than for tap water. The actual frequencies are similar in magnitude as well. However, the percentage of total bubbles detected by the lower tip which met the discrimination criteria is slightly lower for the aqueous glycerol system.

The average gas hold-ups, obtained from the conductivity probe measurements (as discussed in Section 4.5), for the aqueous 44 mass % glycerol solution and for various liquid and modified gas Reynolds numbers, are shown in Figure 8.13. For comparison purposes, the correlation for gas hold-up determined from the pressure drop data shown earlier in Figure 7.2 for the same conditions is plotted as a solid line. Clearly, except for one point, there is good agreement between the two methods of measurement.
Figure 8.11: Histograms of frequency of (a) bubble rise velocity and (b) measured pierced chord lengths for $Re_g = 104$ and $Re_L = 131$. The average measured pierced chord length was 6.47 mm with an average bubble rise velocity of 1.12 m/s. Liquid for this series of experiments was an aqueous 44 mass % glycerol solution. A total of 2900 vertically rising bubbles were measured over a span of 275 s.
Figure 8.12: Frequency of vertically-rising bubbles, and fraction of bubbles which are rising vertically versus modified gas Reynolds number, $Re_g$, for various liquid Reynolds number values with aqueous 44 mass % glycerol solution as the liquid. The filled-in and open points are for heights of 0.864 m and 0.559 m above the distributor plate respectively. The expanded bed heights exceeded 0.864 m. Each plotted point represents an average of 2022 bubbles and a sampling time of 275 s.
Figure 8.13: Average gas hold-up measured with conductivity probe versus modified gas Reynolds number, $Re_g$, for various liquid Reynolds numbers. The solid curve is from the pressure drop measurements shown in Figure 8.2. Liquid for this series of experiments was an aqueous 44 mass % glycerol solution. The measurements were taken 0.864 m above the distributor plate. The expanded bed heights exceeded 0.864 m. Each plotted point represents an average of 2022 bubbles and a sampling time of 275 s.
8.3 Use of bubble characteristics to estimate \( \varepsilon_g \) and probe sampling area

The measured bubble characteristics can be used to verify the gas hold-up and to estimate the effective sampling area of the probe. A volumetric balance can be made on the gas over an infinitely thin cross-section of the column as follows,

\[
Q_g = U_g A_c = \sum_{i=1}^{n} U_{bi} A_i
\]

(8.1)

where \( Q_g \) is the volumetric gas flow, \( U_g \) is the superficial gas velocity, \( A_c \) is the cross-sectional area of the column, \( U_{bi} \) is the \( i \)th bubble rise velocity, \( A_i \) is the cross section of the \( i \)th bubble at the given cross-sectional area, and \( n \) is the total number of bubbles over a given cross-section. If, as a first approximation, we assume that \( U_{bi} \) and \( A_i \) are uncorrelated, then the average bubble rise velocity, \( U_b \), can be substituted into equation (8.1),

\[
U_g A_c = U_b \sum_{i=1}^{n} A_i
\]

(8.2)

The gas hold-up at a given cross-section, is equal to the area, i.e.

\[
\varepsilon_g = \frac{\sum_{i=1}^{n} A_i}{A_c}
\]

(8.3)

Rearranging equation (8.2) and substituting into equation (8.3) yields, upon simplification, the equation presented by Østergaard (1965),

\[
\varepsilon_g = \frac{U_g}{U_b}
\]

(8.4)

Equation (8.4) can be used to relate the measured bubble velocity to the gas hold-up. Some caution should be used in applying this principle. The bubble velocity used in equation (8.4) should be the mean directional bubble velocity to account for relatively
large numbers of bubbles not travelling directly upwards (as many as 80% of the bubbles in the system). The bubble velocities calculated in this chapter are based solely on bubbles which are travelling directly upwards. In addition, in practice, $U_{bi}$ is expected to be correlated with $A_i$ as larger bubbles travel faster. As can be seen in Figure 8.14, the result is that, as expected, the gas hold-up estimated with this method is significantly lower than that measured by pressure drop calculations.

The total gas flow rate for the cross-section can also be related to the bubble frequency and average bubble size, i.e.

$$Q_g = U_g A_c = F_b V_b$$  \hspace{1cm} (8.5)

where the average volume of a bubble is $V_b$ and the total frequency of the bubbles over the entire cross-section is $F_b$. If the gas velocity and bubble size are uniform across the cross-sectional area, the total frequency of bubbles can be related to the local frequency, $f_b$, as measured by the conductivity probe as follows,

$$\frac{F_b}{A_c} = \frac{f_b}{A_s}$$  \hspace{1cm} (8.6)

where $A_s$ is the effective sampling area of the probe. Note also that

$$A_s = \frac{\pi d_s^2}{4}$$  \hspace{1cm} (8.7)

$$V_b = \frac{\pi d_b^3}{6}$$  \hspace{1cm} (8.8)

where $d_s$ is the effective sampling diameter of the probe, and $d_b$ is the bubble diameter.

Substituting equations (8.6) through (8.8) into equation (8.5) and rearranging yields
Figure 8.14: Estimated gas hold-up based on equation (8.4) versus correlated values of gas hold-up from Figures 7.1 and 7.2. Liquids were tap water (150 < $Re_g$ < 500, 430 < $Re_L$ < 720) in (a) and an aqueous 44 mass % glycerol solution (40 < $Re_g$ < 140, 55 < $Re_L$ < 125) in (b). Estimates were based on average bubble velocities 0.864 m above the distributor plate. Straight lines denote parity.
Equation (8.9) can be used to determine the effective probe sampling area. Figure 8.15 shows the results of such a calculation. Fitting the data obtained from conductivity probe measurements, the effective sampling area diameter of the probe in tap water is found to be approximately 6 mm. In the aqueous 44 mass % glycerol solution, the effective sampling diameter increases to 8 mm. It should be noted that these effective sampling diameters are calculated under the additional assumption that the local bubble frequencies measured by the probe are accurate and that no bubbles are diverted (and hence missed) due to the intrusive nature of the probe. This is not likely to be completely true and so the effective sampling diameter is most likely somewhat smaller. Certainly an increase in sampling area for a less conductive material (the aqueous glycerol solution) is not expected. It is likely that the difference between the two systems is attributable to a greater portion of bubbles being missed by the probe for the more viscous solution.
Figure 8.15: Estimated gas superficial velocity from equation (8.9) versus measured gas superficial velocity. Liquids were tap water (150 <Re_g< 500, 430<Re_L<720) in (a) and an aqueous 44 mass % glycerol solution (40 <Re_g< 140, 55<Re_L<125) in (b). Estimates were based on average pierced chord lengths and bubble frequency 0.864 m above the distributor plate. The effective probe sampling diameter was set to 6 mm in (a) and 8 mm in (b). Straight lines denote parity.
Chapter 9 - Conclusions and Recommendations

9.1 Achievement of objectives

1) Dynamic and geometric similarity criteria were established to scale up three-phase hydrodynamic data from cold-flow units to industrial conditions:

- A review of previous work in three-phase fluidized beds identified the significant operating parameters which affect the bed hydrodynamics.
- A set of five dimensionless operating groups (\( M = \frac{g\Delta \rho \mu_L^4}{\rho_L^2 \sigma^3} \), \( E_o = \frac{g\Delta \rho d_p^2}{\sigma} \),
  \[ Re_L = \frac{\rho_L d_p U_L}{\mu_L}, \beta_d = \frac{\rho_p}{\rho_L}, \text{ and } \beta_a = \frac{U_g}{U_L} \]) was established.
- Particle geometric similarity is also required to be maintained.
- For the dispersed bubble conditions of interest, the gas density was not considered except insofar as it influences the buoyancy in the E\ötvös number and M-group. The ratio of column diameter to particle dimensions was also deemed to be of secondary importance so long as it is greater than about 10.
- The dimensionless approach was substantially validated by comparing results from an industrial unit with kerosene as the liquid with a dynamically similar unit involving an aqueous MgSO_4 solution.

2) Gas hold-up was measured under conditions pertinent to those used in industry based on the established set of scaled parameters:
- The industrial conditions of a hydrocracker were matched under a wide range of operating conditions using aqueous glycerol solutions of different liquid properties and cylindrical aluminum particles simulating catalysts.
• Measured gas hold-ups were typically of order 5-20%. Correlations for gas hold-up obtained under these conditions were developed of the form $\varepsilon_g = a \text{Re}_g^b$. The results are limited to the range of conditions studied.

i) For tap water as the liquid$^1$ the hold-up was correlated as $\varepsilon_g = 0.0139 \text{Re}_g^{0.426}$.

ii) For a 44 mass % glycerol aqueous solution$^2$, the hold-up was correlated as $\varepsilon_g = 0.0359 \text{Re}_g^{0.346}$.

iii) For a 60 mass % glycerol aqueous solution$^3$, the hold-up was correlated as $\varepsilon_g = 0.0486 \text{Re}_g^{0.322}$.

3) Bed expansion was measured under conditions pertinent to those used in industry based on the established set of scaled parameters:

• The industrial conditions of a hydrocracker were again matched. As for gas hold-up, the correlations for bed expansion are limited to the range of operating conditions used in this research.

• The results were correlated as $100 \beta_{be} = A_1(\text{Re}_L - \text{Re}_{Lmf})^p + B_1(\beta_u)^{B_1}$. 

i) For tap water$^1$: $100 \beta_{be} = 0.00705(\text{Re}_L - 240)^{1.55} + B_1(\beta_u)^{B_1}$

where $B_1 = 16.6 + 6.03 \times 10^{-17} (\text{Re}_L)^{6.38}$ and

$B_2 = 0.299 + 0.0023 \text{Re}_L - 9.35 \times 10^{-6} \text{Re}_L^2 + 1.05 \times 10^{-8} \text{Re}_L^3$.

ii) For a 44 mass % glycerol aqueous solution$^2$:

$100 \beta_{be} = 0.109(\text{Re}_L - 39.1)^{4.47} + B_1(\beta_u)^{B_1}$ where $B_1 = 16.0 + 1.29 \times 10^{-5} (\text{Re}_L)^{3.10}$ and $B_2 = 0.311 + 2.55 \times 10^{-10} \text{Re}_L^{4.24}$.

---

$^1$ For tap water: $M = 5.94 \times 10^{-11}$, $E_o = 2.24$, $\beta_d = 2.70$, $430 \leq \text{Re}_L \leq 716$

$^2$ For aqueous 44 mass % glycerol solution: $M = 1.20 \times 10^{-8}$, $E_o = 2.55$, $\beta_d = 2.46$, $73 \leq \text{Re}_L \leq 156$
iii) For a 60 mass % glycerol aqueous solution:

\[ 100 \beta_{se} = 1.01 \left( \text{Re}_L - 11.6 \right)^{1.3} + B_1 \left( \text{Re}_L \right)^{B_2} \]

where \( B_1 = -49.3 + 1.93 \text{Re}_L \) and \( B_2 = 0.213 + 1.45 \times 10^{13} \exp \left( -\text{Re}_L \right) \).

4) Bubble properties were measured under conditions pertinent to those used in industry based on the established set of scaled parameters:

- The industrial conditions of a hydrocracker were matched as for gas-hold-up.
- Only 20-30 % of the bubbles detected by the conductivity probe traveled vertically upwards (as determined by the criteria discussed in section 4.5.2). As many as 1-2 % of the bubbles traveled downwards.
- In general, \( U_g/U_L \) increased with \( \text{Re}_g \).
  i) For tap water, \( U_g/U_L \) varied between 4 and 9.
  ii) For a 44 mass % glycerol aqueous solution, \( U_g/U_L \) varied between 7 and 14.
- In general, \( L_b/d_p \) also increased with \( \text{Re}_g \).
  i) For tap water, \( L_b/d_p \) varied between 1 and 2.
  ii) For a 44 mass % glycerol aqueous solution, \( L_b/d_p \) varied between 1.25 and 2.6.
- The bubble frequency tended to increase slightly with \( \text{Re}_g \) and significantly more with \( \text{Re}_L \).
  i) For tap water, bubble frequency varied between 4 and 14 Hz.
  ii) For a 44 mass % aqueous glycerol solution, bubble frequency varied between 4 and 10 Hz.

---

\(^3\text{For aqueous 60 mass % glycerol solution: } M = 5.00 \times 10^{-7}, E_0 = 2.70, \beta_d = 2.35, 31 \leq \text{Re}_L \leq 50\)
9.2 Additional conclusions and accomplishments

- A new method for estimating gas hold-up based upon the interstitial liquid velocity is presented, together with experimental validation.

- Despite careful attention to the design of the gas and liquid distributor at the base of the column, maldistribution of the phases occurred near the base of the experimental ebullated bed. However, the maldistribution was self-correcting and was no longer significant at a height of 0.36 m or more above the distributor.

- A pilot scale cold-flow co-current upwards-flowing three-phase fluidized bed column was successfully designed, fabricated, commissioned and operated.

- Conductivity probes were built and used to determine gas hold-ups and bubble characteristics, after resolving a number of issues with respect to signal analysis.

- Several computer programs were written to gather data, with fast Fourier transforms to remove baseline fluctuations and to extract gas hold-up and bubble characteristics from the data.

9.3 Recommendations for future work

➢ Further validation of the dimensionless approach is required. This would include significantly varying the Eötvös number, M-group, and the density ratio, $\beta_d$.

➢ Experiments are required to examine the effect of gas bubble entrainment in the liquid recycle stream and to determine if this is sufficient to increase the hold-up in the cold-model to a level closer to that of the industrial unit.
A method for properly designing an effective and simple distributor system for three-phase fluidized beds, similar to those available for two-phase systems, is required. This could be a combination of a different distributor system and improved column design (for instance tapered corners at the inside base of the column).

To reduce exit-effects, a more efficient and non-intrusive method of withdrawing and recycling liquid is necessary.

To better determine whether there are significant gradients of hydrodynamics over the height of the column, other than end/entrance effects, the over-all column height should be increased.

Further work should be conducted using particles of different shapes and sizes.
Nomenclature

a correlation factor for the slip velocity used by Saberian-Broudjenni et al. (1984), -
A area, m²
A₁ factor used to correlate bed expansion liquid term in equation (6.2), -
Ar Archimedes number, \( d^3 \frac{g(\rho_s-\rho_l)\rho_l}{\mu_l^2} \), -
Ar_L Archimedes number based upon volume-equivalent diameter, \( d_e^3 \frac{g(\rho_s-\rho_l)\rho_l}{\mu_l^2} \), -
b correlation factor for the slip velocity used by Saberian-Broudjenni et al. (1984), -
B₁, B₂ factors used in correlating bed expansion data in equation (6.3), -
BE bed expansion percentage, \( (H_e-H_o)/H_o \times 100 \), -
Bo Bond number, \( g \frac{d^2 \rho_c}{\sigma} \), -
c Fan and Tsuchiya (1990) bubble rise correlation factor, -
C number of carbon atoms in alcohol molecule, -
d diameter, m
dₑ spherical volume-equivalent diameter of a bubble, m
dₑ dimensionless volume-equivalent diameter of bubble, \( d_e (g \rho_l / \sigma)^{1/2} \), -
dₜ hydraulic column diameter, used by Fan et al. (1987), defined as difference between inner and outer column diameters, m
dₚ actual particle diameter, m
Dₑ column diameter, m
E₀ Eötvös number, \( g \Delta \rho \frac{d_p^2}{\sigma} \), -
f Fan and Tsuchiya (1990) bubble rise correlation coefficient, -
f_b bubble frequency, s⁻¹
f²p friction factor for individual particle, -
f₂ Kato et al. (1981) correlation coefficient, -
Fr Froude number, i.e. ratio of inertial to gravitational forces, \( U^2/\rho g \), -
Fr_g,dh gas Froude number used by Fan et al. (1987), \( U_d^2/(d_b \ g) \), -
Fr_l,dh liquid Froude number, \( U_l^2/(d_b \ g) \), -
g acceleration due to gravity, 9.81 m/s²
Ga Galileo number, \( g \frac{D_c^3}{\rho_L^2} \), -
Hₑ expanded bed height, m
H₀ static or settled bed height, m
j number of fundamental physical dimensions, -
k ratio of volume of wake region to that of the bubble region, -
K dimensionless term used by Kato et al. (1981), \( \rho_l U_f^4/(g \sigma) \), -
K_b Fan and Tsuchiya (1990) bubble rise correlation coefficient, -
L length, m
Lₑ bubble pierced chord length, m
m number of dimensionless groups that can be formed, -
M M-group, \( g \frac{\mu_L^4}{(\rho_L \sigma^3)} \), -
n Richardson and Zaki index, -
nₑ porosity exponent, -
N number of dimensional variables, -
P factor used to correlate liquid term of bed expansion data in equation (6.2), -
ΔP pressure drop, Pa
Q volumetric flow rate, m$^3$/s
Re$^b$ modified bubble Reynolds number, $\rho_g \, d_b \, U_b / \mu_g$, -
Re$^g$ modified gas Reynolds number, $Re_L \beta \, d_p \, U_g / \mu_g$, -
Re$^{g2}$ modified gas Reynolds number, $\rho_g \, d_p \, U_g / (\mu_g \varepsilon_p)$, -
Re$^L$ liquid Reynolds number, $\rho_L \, d_p \, U_L / \mu_L$, -
Re$^{L2}$ modified liquid Reynolds number, $\rho_L \, d_p \, U_L / (\mu_g \varepsilon_p)$, -
Re$_{Lmf}$ minimum fluidization Reynolds number based on particle diameter,
$\rho_L \, d_p \, U_{Lmf} / \mu_L$, -
Re$^{"Lmf}$ minimum fluidization Reynolds number based on equivalent diameter,
$\rho_L \, d_p \, U_{Lmf} / \mu_L$, -
Re$_t$ terminal particle Reynolds number, $(1-\alpha) \rho_L \, d_p \, U_t / \mu_L$, -
s distance between two probe tips, mm
t time, s
U superficial velocity, m/s
U$^*$ interfacial velocity, m/s
Ub bubble rise velocity, m/s
Ue extrapolated terminal velocity as the voidage approaches unity for the liquid-solid fluidized bed, m/s
U$_{GL}$ slip velocity of gas, $U_g - \frac{\varepsilon_g}{\varepsilon} (U_g + U_L)$, m/s
U$_i$ extrapolated superficial liquid velocity in liquid-solid fluidized bed as bed voidage approaches 1, m/s
U$_{Lmf}$ minimum liquid fluidization velocity in a three-phase fluidized bed, m/s
U$_{mf0}$ minimum fluidization velocity in corresponding liquid-solid fluidized bed, m/s
U$SF$ velocity of solids in fluidized bed according to bubble wake model, m/s
V interfacial velocity, m/s
Ws particle inventory, kg
We$_m$ modified Weber number, ratio of inertial to surface tension forces, $U_L^3 \rho_L D_c / \sigma$, -
x ratio of solids hold-up in the wake region to that in the liquid-solid fluidized bed region, -
x$_c$ factor used to correlate liquid term of bed expansion data, must be equal to Re$_{mf0}$, -
y dimensionless parameter used by Jean and Fan (1986) defined in Table 2.1, -
z height above distributor, m
$\Delta z$ height interval, m
Z1,Z2,Z3,Z4,Z5,Z6 factors used by present author and by Zheng et al. (1988) in various correlations, -

Greek Letters

$\alpha$ void fraction filled by gas, $\varepsilon_g / \varepsilon$, -
$\beta$ coefficient of drag force used by Glicksman (1984), kg/m$^3$/s
$\beta_0$ ratio of bubble rise velocity to superficial liquid velocity, $U_b / U_L$, -
$\beta_{be}$ Bed expansion ratio $=(H_e-H_0)/H_0$, -
$\beta_d$ ratio of densities, $\rho_g / \rho_L$, -
\( \beta_L \) ratio of pierced chord length to particle diameter, \( L_p/d_p \), -
\( \beta_s \) ratio of superficial velocities, \( U_g/U_L \), -
\( \chi^2 \) chi-square used for statistical tests, -
\( \Delta \rho \) density difference = \( \rho_L - \rho_g \), \( \text{kg/m}^3 \)
\( \varepsilon \) hold-up (equal to the bed porosity, \( = \varepsilon_s + \varepsilon_L \) if no subscript present), -
\( \phi \) geometric shape factor of particles used by Costa et al. (1986), -
\( \phi_s \) sphericity, -
\( \gamma \) generalized liquid viscosity constant used by Kim et al. (1977), \( \text{mN s}^n/\text{m}^2 \)
\( \phi/\phi_0 \) ratio of intensity of gamma-rays after scattering to original intensity, -
\( \mu \) viscosity, \( \text{Pa} \cdot \text{s} \)
\( \rho \) density, \( \text{kg/m}^3 \)
\( \sigma \) surface tension, \( \text{kg/s}^2 \)
\( \psi \) mass absorption coefficient, -

**Subscripts**

- \( a \) available
- \( c \) column
- \( b \) bubble
- \( f \) in liquid-solid fluidized region according to bubble wake model
- \( g \) gas
- \( i \) \( i^{th} \) value
- \( l,L \) liquid
- \( o \) initial
- \( p \) particle
- \( t \) terminal velocity
- \( w \) wake
References


Hammer, B.E., Department of Radiology, University of Minnisota, personal e-mail, May 16,1996.


Appendix A - Design Calculations and Schematics for 0.292 m Column

A.1 Calculation of number of holes required for sparger and distributor plate

Assume:
- \( h_{settled \ bed} = 0.91 \) m
- \( \varepsilon_{settled} = 0.5 \)
- \( h_{expanded \ bed} = 1.83 \) m

Known:
- \( \rho_{solid} = 2700 \) kg/m\(^3\)

Estimate:
- \( \rho_{liquid + gas} = 1200 \) kg/m\(^3\)

Step 1) Determine pressure drop required across distributor

For 2-phase (gas-solid) fluidized beds (Kunii and Levenspiel, 1969)

\[
\Delta P_{distributor} = \max (0.1 \Delta P_{bed}, 35 \text{-cm H}_2\text{O}) \quad \text{(A.1)}
\]

Expected pressure drop across fluidized bed,

\[
\Delta P_{bed} = \Delta P_{solids} + \Delta P_{gas+liquid} \quad \text{(A.2)}
\]
\[
\Delta P_{solids} = (h_{settled \ bed})(\varepsilon_{settled})(\rho_{solids}) \quad \text{(A.3)}
\]
\[
\Delta P_{gas+liquid} = (h_{settled \ bed})(1-\varepsilon_{settled})(\rho_{liquid + gas}) + (h_{expanded \ bed} - h_{settled \ bed})(\rho_{liquid + gas}) \quad \text{(A.4)}
\]

Substituting the values listed earlier into equations (A.1), (A.2), and (A.3)

\[
\Delta P_{bed} = 2.8 \times 10^4 \text{ Pa}
\]
\[
0.1 \Delta P_{bed} = 2.8 \times 10^3 \text{ Pa}
\]

Compare 0.1 \( \Delta P_{bed} \) and 35-cm H\(_2\)O and take maximum

\[
\therefore \Delta P_{distributor} = 35 \text{-cm H}_2\text{O} = 3.4 \times 10^3 \text{ Pa}
\]

Step 2) Determine size and number of holes required according to 2-phase equation

\( U_o = \) superficial fluid velocity
\( N_o = \) number of orifices
\( D_o = \) diameter of orifices
\( C_d = \) drag coefficient
\( D_c = \) column diameter
\( \rho_o = \) fluid density
\[ d_{or}^2 N_{or} = U_o \frac{d_c^2}{C_d} \sqrt{\frac{\rho_o}{2\Delta P_{dist}}} \]  \hspace{1cm} (A.5)

a) For Gas

\[ \rho_g = 1.2 \text{ kg/m}^3 \]
\[ U_g = 0.15 \text{ m/s} \]
\[ \mu_g = 1.8 \times 10^{-4} \text{ kg/m/s} \]
\[ D_c = 292 \text{ mm} \]

For Reynolds number, \( Re_g = 2923 \), from Kunii and Levenspiel (1969)
\[ C'_{d} = 0.6 \]

Substituting these values into equation A1, we obtain, for the gas,
\[ N_o d_o^2 = 2.83 \times 10^{-4} \text{ m}^2 \]

If we define \( d_o = 3.2 \text{-mm} \) \( \Rightarrow \) \( N_o = 28 \)

Therefore, for the gas sparger, we need approximately 28 holes with diameters of 3.2-mm

If we define \( d_o = 6.4 \text{ mm} \) \( \Rightarrow \) \( N_o = 7 \)

Therefore seven 6.4-mm holes are required to distribute the gas well with the distributor plate.

b) For Liquid

\[ \rho_l = 1100 \text{ kg/m}^3 \]
\[ U_l = 0.15 \text{ m/s} \]
\[ \mu_l = 5.0 \times 10^{-3} \text{ kg/m/s} \]
\[ D_c = 292 \text{ mm} \]

For Reynolds number: \( Re_l = 9.64 \times 10^3 \), from Kunii and Levenspiel (1969)
\[ C'_{d} = 0.6 \]

Substituting these values into equation A1, we obtain, for the gas,
\[ N_o d_o^2 = 8.96 \times 10^{-3} \text{ m}^2 \]

For \( d_o = 6.4 \text{ mm} \), this gives \( N_o = 219 \)

Therefore we need 219 holes of 6.4 mm diameter for the liquid.
Adding the holes required for the liquid and the gas yields a total requirement of 226 holes of diameter 6.4 mm.

A.2 Design drawings for miscellaneous components of the 292-mm diameter column

Figure A.1: Schematic drawing showing (a) side-view of the distributor, and (b) how the distributor was attached to the column.
Figure A.2: Design diagram for pre-mix area shown in Figure 3.1. Liquid enters through the 76-mm hole at the base, with air entering the sparger (not shown) through the side port (of inside diameter 25 mm).
Figure A.3: Design side view of sparger ring. Bottom view is shown in Figure 3.2.
Figure A.4: Design drawing of column section immediately above distributor (z = 0 to 0.61 m) showing ports.

Figure A.5: Design drawing of second column section above the distributor (z = 0.61 to 1.22 m)
Figure A.6: Design cross-sectional drawing of original section immediately above distributor (side-view shown in Figure A.5). 12.7 mm NPT ports are not shown in this cross-sectional view.
Figure A.7: Design cross-sectional drawing of expanded section at top of column
Appendix B - Conversion of Pressure Drop Readings to Gas Hold-up

Figure B.1: Schematic representation of two tubes in barometer set connected to fluidized bed

A pressure balance is made on each of the tubes in Figure B.1. For tube (1), the pressure on the left-hand side of the barometer tube (LHS) must equal that on the right-hand side of the barometer (RHS) in the same connected liquid. The LHS includes all the weight of the gas-solid-liquid mixture above the port + the amount of liquid in the tube from the port to the RHS of the barometer. The "ground point" shown in Figure B.1 is an arbitrary and imaginary point which does not influence these calculations. The RHS
includes the weight of the liquid in that arm of the barometer, in addition to a term, $P^*$, to account for the pressurization of the barometer bank.

$$[(L_1^* \rho_L) + (L_2^* - L_1^* + L_{extra})(\epsilon_p \rho_p + \epsilon_L \rho_L + \epsilon_g \rho_g)]g = [L_1 \rho_L]g + P^* \quad (B.1)$$

For the second barometer, (2) in Figure B.1, a similar balance yields

$$[(L_2^* \rho_L) + (L_{extra})(\epsilon_p \rho_p + \epsilon_L \rho_L + \epsilon_g \rho_g)]g = [L_2 \rho_L]g + P^* \quad (B.2)$$

If we then subtract equation (B.1) from equation (B.2), the following equation is obtained

$$[L_2^* - L_1^*] \rho_L - (L_2^* - L_1^*)(\epsilon_p \rho_p + \epsilon_L \rho_L + \epsilon_g \rho_g) = (L_2 - L_1) \rho_L \quad (B.3)$$

$[L_2^* - L_1^*]$ is simply the distance between the two pressure ports, while $(L_2 - L_1)$ is the difference in height of fluid in the two barometer tubes. The densities of each of the components can be measured separately. Since there are three phases present in the column,

$$\epsilon_g + \epsilon_L + \epsilon_p = 1 \quad (B.4)$$

This leaves two equations (B.3 and B.4) with three unknowns $(\epsilon_g, \epsilon_L, \epsilon_p)$.

(a) Freeboard region calculations:

In the freeboard region, $\epsilon_p = 0$ since negligible solids are present. In addition, the gas density is significantly lower than the liquid density and can be held to be effectively zero, for the purposes of these calculations. Hence equation (B.3) is reduced to

$$[L_2^* - L_1^*] \rho_L - [L_2^* - L_1^*] (\epsilon_L \rho_L) = (L_2 - L_1) \rho_L \quad (B.5)$$

or

$$(1 - \epsilon_L) = \frac{(L_2 - L_1)}{[L_2^* - L_1^*]} \quad (B.6)$$
(b) Ebullated bed calculations:

Equations (B.3) and (B.4) are still valid. The gas density is assumed to be negligible with respect to that of the liquid and solids and, hence, equation (B.3) can be reduced to

\[
[L_2^* - L_1^*]\rho_L - (L_2^* - L_1^*)(\epsilon_p\rho_p + \epsilon_L\rho_L) = (L_2 - L_1)\rho_L
\]

(B.7)

To solve this equation one of the phase hold-ups must be determined independently. The most frequently used methods involves estimating the solids hold-up, \(\epsilon_p\), from the loading of particles in the reactor, \(W_s\), and the measured expanded bed height, \(H_e\), as per equation (4.2) shown earlier:

\[
\epsilon_p = \frac{W_s}{\rho_p AH_e}
\]

(B.8)

With this assumption, equation (B.7) can be solved to yield the liquid hold-up, \(\epsilon_L\). Subsequently, equation (B.4) can be used to give the gas hold-up.
Appendix C - Software Written for this Project

C.1 Data acquisition program (Test6.Bas)

'****************************************************************************
' test6.BAS                  DAS-1600/1400/1200
'
' - To run from the QuickBASIC Environment (up to Ver 4.5) you must load
'  the appropriate quick library using the command line switch /L,
'  as follows
'
'    QB /L D1600Q45 QBEXAMP1    (using version 4.5)
'
'  (we are using version 4.5)
'****************************************************************************

' This file includes all function DECLARATION supported by the driver.
' $INCLUDE: 'c:\dAS1600\basic\Q4IFACE.BI'

'Dimension integer array to receive A/D data. Note that, for reliable
'operation, this array should dimensioned twice a big as needed.

'Statement of subroutine

    DIM BUFFA(30000) AS INTEGER  ' destination buffer for acquired data
    DIM loop1(7500) AS SINGLE    ' local array used to store 1st set of 7500 data points
    DIM loop2(7500) AS SINGLE    ' local array used to store 2nd set of 7500 data points
    DIM loop3(7500) AS SINGLE    ' local array used to store 3rd set of 7500 data points

' Variable used by driver functions.
    DIM NumOfBoards AS INTEGER   ' number of boards used (1)
    DIM DERR AS INTEGER         ' Error flag
    DIM STARTINDEX AS INTEGER   ' Actual Index where data starts
    DIM DEVHANDLE AS LONG       ' Device Handle
    DIM ADHANDLE AS LONG        ' A/D Frame Handle
    DIM STATUS AS INTEGER       ' Status indicator during DMA (0=idle,
                                '      1=active, 2=data overrun)
    DIM count AS LONG           ' Holds DMA transfer count
    DIM addr AS LONG            ' Holds address needed by KSetDMABuf
    DIM TFP AS LONG             ' Total number of sample points
    DIM FF AS STRING            ' Saved data file name
    DIM CR AS LONG              ' clock rate = 10 MHz/F
    DIM D1 AS SINGLE            ' real value of data from channel 1
    DIM D2 AS SINGLE            ' real value of data from channel 2
DIM a AS STRING ' temporary string variable for y/n answers
DIM I AS INTEGER ' variable used to hold the buffer addresses
DIM iflag AS INTEGER ' used to determine whether which half of the
' buffer should be read next (0=2nd, 1=1st)
DIM iwrite AS INTEGER ' holds the corrected buffer address
DIM T AS INTEGER ' duration of one data acquisition loop (in seconds)
DIM F AS INTEGER ' frequency of data acquisition (in Hz)

CLS
COLOR 10, 8
LOCATE 1, 3:
PRINT "TEST5.BAS" DAS 1202"
PRINT
PRINT "This program is used to acquire data,"
PRINT
PRINT "Please be aware that:"
PRINT
PRINT "1. Two channels, #0 and #1 will be scanned"
PRINT "2. Sampling frequency is 2500 Hz"
PRINT "3. Total sampling time is 9 s."
PRINT "4. The raw signals are stored in a binary file "
PRINT "in c:\data\test\result.dat"

PRINT
COLOR 7, 8

"Name the results file.

INPUT "Input a file name for the results (automatic with '.BIN'):"; FF$
5 IF LEN(FF$) > 7 THEN
BEEP: INPUT "File name should have less than 8 characters. Type it again. ", FF$
PRINT
GOTO 5
END IF
ON ERROR GOTO ErrorHandler
OPEN "c:\data\test" + FF$ + ".BIN" FOR INPUT AS #2

CLOSE #2
COLOR 13, 8
BEEP: PRINT FF$; : COLOR 14, 8: PRINT " has been used. Enter another file name."

COLOR 12, 8: INPUT "If you want to overwrite it TYPE y, if not TYPE file name again"; a$
PRINT
COLOR 7, 8
IF a$ = "Y" OR a$ = "y" THEN
GOTO ok
ELSE
FF$ = a$
END IF
GOTO 5
ok:
CLOSE #2

' Sampling time (T) * Sampling frequency (f) should <=7500
T = 3
F = 2500
1020 CLS

'------------------------------------------------------------------------------------------------------------------------
' STEP 1: This step is mandatory; it initializes the internal data tables according to the information contained in the configuration file 'DAS1600.CFG. Specify another filename if you are using a different configuration file. (ie. DAS1400.CFG, DAS1200.CFG)
'
    a$ = "c:\data\test\1202.CFG" + CHR$(0)
    DERR = DAS1600DEVOPEN%(SSEGADD(a$), NumOfBoards)
    IF DERR <> 0 THEN BEEP: PRINT "ERROR "; HEX$(DERR); " OCCURRED DURING '.DEVOPEN'": STOP

'------------------------------------------------------------------------------------------------------------------------
' STEP 2: This step is mandatory; it establishes communication with the driver through the Device Handle.
'
    DERR = DAS1600GETDEVHANDLE%(0, DEVHANDLE)
    IF (DERR <> 0) THEN BEEP: PRINT "ERROR "; HEX$(DERR); " OCCURRED DURING '.GETDEVHANDLE'": STOP

'------------------------------------------------------------------------------------------------------------------------
' STEP 3: To perform any A/D operations, you must first get a Handle to an A/D Frame (Data tables inside the driver pertaining to A/D operations).
'
    DERR = KGetADFrame%(DEVHANDLE, ADHANDLE)
    IF (DERR <> 0) THEN BEEP: PRINT "ERROR "; HEX$(DERR); " OCCURRED DURING 'KGETADFRAME'": STOP
STEP 4: Before specifying the destination buffer for the acquired data, you must first call KMAKEDMABuf% to determine a suitable buffer address for the DMA controller to use.

TFP = INT(T * F * 2)
LOCATE 15, 1
' IF TFP > 15000 OR TFP = 0 THEN COLOR 11, 12: BEEP: PRINT "WARNING!!! Buffa is too large OR =0. Reduce T or F. T*F<=7500 BUT <> 0": COLOR 7, 8: GOTO 1010
DERR = KMAKEDMABuf%(TFP, BUFA(), addr, STARTINDEX)
IF DERR <> 0 THEN PRINT "Make Dma Buffer Error": STOP

STEP 5: Assign the data buffer address to the A/D Frame and specify the number of A/D samples.

DAS16000ERR = KSetDMABuf%(ADHANDLE, addr, TFP)
IF DAS16000ERR <> 0 THEN BEEP: PRINT "ERROR "; HEX$(DAS1600ERR); " OCCURRED DURING 'KSetDMABuf'": STOP

STEP 6: Choose the Start and Stop channels and overall gain code to use during acquisition.

DERR = KSetStartStopChn%(ADHANDLE, 0, 1)
IF DERR <> 0 THEN BEEP: PRINT "ERROR "; HEX$(DERR); " OCCURRED DURING 'KSetStartStopChn'": STOP

STEP 7: This example program uses the internal (by default) conversion clock source; the following call specifies the divisor to the Clock Source (1MHz or 10MHz)

CR = 10000000 / INT(F)
DERR = KSetClkRate%(ADHANDLE, CR)
IF DERR <> 0 THEN BEEP: PRINT "ERROR "; HEX$(DERR); " OCCURRED DURING 'KSetClkRate'": STOP

STEP 8: Specify Continuous mode.

DERR = KSetContRun(ADHANDLE)
IF DERR <> 0 THEN BEEP: PRINT "ERROR "; HEX$(DERR); " OCCURRED DURING 'KSetContRun'": STOP
COLOR 17, 11

LOCATE 15, 1: PRINT "Press a key to START A/D Acquisition..."
COLOR 7, 8
DO
LOOP WHILE INKEY$ = ""

STEP 9: Start data acquisition according to the setup performed above.

CLS
LOCATE 10, 3
COLOR 16, 11
PRINT "The computer is acquiring data"
PRINT ""
PRINT "DO NOT TOUCH THE KEYBOARD"
PRINT ""
PRINT "Time="; T; " sec", "Frequency="; F; "Hz."
COLOR 7, 8

FF$ = "c:\data\test\test.DAT"
OPEN FF$ FOR OUTPUT AS #1

' iflag is set initially to 0 then after the first 7500 samples are taken
' it is set to 1 so that we know not to take the 2nd half data when there is
' nothing there yet!
iflag = 0

BEEP: PRINT "starting .... !"
contr1 = 0

DERR = KDMAStart%(ADHANDLE)
IF DERR <> 0 THEN BEEP: PRINT "ERROR "; HEX$(DERR); " OCCURRED
DURING 'KDMA'": STOP

STEP 10: Monitor the status and sample transfer count until done

' start the loop ........

77 contr1 = contr1 + 1
100 DERR = KDMAStatus%(ADHANDLE, STATUS, count)
IF DERR <> 0 THEN BEEP: PRINT "ERROR "; HEX$(DERR); " OCCURRED
   DURING 'KDMAStatus'": GOTO 300
'   LOCATE 13, 1: PRINT "COUNT : "; count

IF INKEYS <> "" GOTO 300

IF count < 7500 AND iflag = 1 THEN

   iflag = 0 ' change the flag: the next section to be written will
      ' be the 1st half

   FOR I% = 3750 TO 7499 ' do-loop to write 2nd half!
      D1 = BUFFA(2 * I% + 1 + STARTINDEX) / 16
      D1 = SGN(D1) * (ABS(D1) - 2048) / 4096 * 10
      D2 = BUFFA(2 * I% + 2 + STARTINDEX) / 16
      D2 = SGN(D2) * (ABS(D2) - 2048) / 4096 * 10
      IF (contr1 = 2) THEN
         loop1(I%) = D1
      ELSEIF (contr1 = 3) THEN
         loop2(I%) = D1
      ELSEIF (contr1 = 4) THEN
         loop3(I%) = D1 'this is the final part of a 3x loop
         iflag = 2 'set flag to terminate process
      END IF
   NEXT I%

   IF iflag = 2 THEN GOTO 333 'once requested data is stored in local
      'arrays, terminate the acquisition

ELSEIF count >= 7500 AND iflag = 0 THEN

   iflag = 1 ' change the flag: the next section to be written will
      ' be the 1st half

   FOR I% = 0 TO 3749 ' do-loop to write 1st half!
      D1 = BUFFA(2 * I% + 1 + STARTINDEX) / 16
      D1 = SGN(D1) * (ABS(D1) - 2048) / 4096 * 10
      D2 = BUFFA(2 * I% + 2 + STARTINDEX) / 16
      D2 = SGN(D2) * (ABS(D2) - 2048) / 4096 * 10
      IF (contr1 = 1) THEN
         loop1(I%) = D1
      ELSEIF (contr1 = 2) THEN
         loop2(I%) = D1
      ELSEIF (contr1 = 3) THEN
         loop3(I%) = D1
      END IF
   END IF
NEXT I%
GOTO 77
END IF

GOTO 100

' STEP 11: Stop DMA operation in case user interrupted or an error occurred.
' This step is not required upon normal termination of DMA; but it can't hurt!

300 DERR = KDMAStrp%(ADHANDLE, STATUS, count)
IF DERR <> 0 THEN BEEP: PRINT "ERROR "; HEX$(DERR); " OCCURRED
   DURING 'KDMAStrp'": STOP

' STEP 12: Save the data just acquired...

333 FOR contr2 = 1 TO 3
    FOR contr3 = 1 TO 7500
        IF (contr2 = 1) THEN
            PRINT #1, loop1(contr3)
        ELSEIF (contr2 = 2) THEN
            PRINT #1, loop2(contr3)
        ELSEIF (contr2 = 3) THEN
            PRINT #1, loop3(contr3)
        END IF
    NEXT contr3
NEXT contr2
CLOSE #1

' STEP 13: Free a frame and return it to the pool of available frames
DERR = KFreeFrame%(ADHANDLE)
IF DERR <> 0 THEN BEEP: PRINT "ERROR "; HEX$(DERR); " OCCURRED
   DURING 'KDMAFreeRun'": STOP

CLOSE #1
LOCATE 13, 1: PRINT "Data acquisition completed. . . "
   BEEP: BEEP: BEEP
LOCATE , , 1
SCREEN 0
WIDTH 80

1100 END

ErrorHandler:
    RESUME ok
C.2 FFT filtering software (Filter.For)

PROGRAM FILTER

C
C this program is used to use FFT's to filter the raw
C data in D to C:\data\lc\
C
character*64, filenm, filenm2
character*11, file1, tempo, filetp
INTEGER isign, n, samp, cutoff, ifile, i, j, k
REAL data1(16384), orig1(16384), data2(16384), orig2(16384)
real base1(16384), base2(16384)
REAL thresh1, thresh2, ave1, ave1b, ave2, ave2b

print *, ' Results are stored in c:\data\apr98\FILTER\'
print *, ' The file group should be in c:\data\apr98\CONVERT\'
print *, ' please enter the FIRST 2 CHARACTERS of the SOURCE ? '
filenm = "d:"
print *, ' '
read '(A)', filetp

if (len_trim(filetp) .gt. 2) then
   print *, ' ** only first 2 character please **'
goto 111
endif
filel="12345"
filel(1:2) = filetp

do 100, i = 1, 2
   if (i .eq. 1) then
      filel(3:3) = '0'
   elseif (i .eq. 2) then
      filel(3:3) = '1'
   endif

100 do 200, j = 1, 10
   if (j .eq. 1) then
      filel(4:4) = '0'
   elseif (j .eq. 2) then
      filel(4:4) = '1'
   elseif (j .eq. 3) then
filel(4:4) = '2'
elseif (j .eq. 4) then
  filel(4:4) = '3'
elseif (j .eq. 5) then
  filel(4:4) = '4'
elseif (j .eq. 6) then
  filel(4:4) = '5'
elseif (j .eq. 7) then
  filel(4:4) = '6'
elseif (j .eq. 8) then
  filel(4:4) = '7'
elseif (j .eq. 9) then
  filel(4:4) = '8'
elseif (j .eq. 10) then
  filel(4:4) = '9'
endif

do 300, k =1, 10
C
C  this next small subsection is used to stop processing (non-existant
C  files ahead ...)
C
if ((i .eq. 2) .and. (j .eq. 3) .and. (k .gt. 7)) then
  goto 300
endif

if (k .eq. 1) then
  if ((j .eq. 1) .and. (i .eq. 1)) then
    goto 300
  endif
  filel(5:5) = '0'
elseif (k .eq. 2) then
  filel(5:5) = '1'
elseif (k .eq. 3) then
  filel(5:5) = '2'
elseif (k .eq. 4) then
  filel(5:5) = '3'
elseif (k .eq. 5) then
  filel(5:5) = '4'
elseif (k .eq. 6) then
  filel(5:5) = '5'
elseif (k .eq. 7) then
  filel(5:5) = '6'
elseif (k .eq. 8) then
elseif (k .eq. 9) then
file1(5:5) = '8'
elseif (k .eq. 10) then
file1(5:5) = '9'
endif

temp = 5
filenm (4:4+temp) = file1
filenm (4+temp:10+temp) = "_x.dat"
C PRINT *,''

filenm2 = "c:\data\lcl"
print *,'
filenm2 (12:12+temp) = file1
filenm2 (12+temp:18+temp) = "_x.cnv"

n = 16384
samp = 13750
cutoff = 25

C have to account for files already processed
C if (i .eq. 1) then
C goto 100
C endif
C
C if ((j .eq. 1) .and. (k .le. 9)) then
C goto 300
C endif
C

do 177, ifile=1, 10

if (ifile .eq. 1) then
  filenm(5+temp:5+temp) = "1"
  filenm2(13+temp:13+temp) = "1"
elseif (ifile .eq. 2) then
  filenm(5+temp:5+temp) = "2"
  filenm2(13+temp:13+temp) = "2"
elseif (ifile .eq. 3) then
  filenm(5+temp:5+temp) = "3"
  filenm2(13+temp:13+temp) = "3"
elseif (ifile .eq. 4) then
  filenm(5+temp:5+temp) = "4"
  filenm2(13+temp:13+temp) = "4"
elseif (ifile .eq. 5) then
  filenm(5+temp:5+temp) = "5"
if ilem2(13+temp:13+temp) = "5"
elseif (if ile . eq. 6) then
  filem(5+temp:5+temp) = "6"
  filem2(13+temp:13+temp) = "6"
elseif (if ile . eq. 7) then
  filem(5+temp:5+temp) = "7"
  filem2(13+temp:13+temp) = "7"
elseif (if ile . eq. 8) then
  filem(5+temp:5+temp) = "8"
  filem2(13+temp:13+temp) = "8"
elseif (if ile . eq. 9) then
  filem(5+temp:5+temp) = "9"
  filem2(13+temp:13+temp) = "9"
elseif (if ile . eq. 10) then
  filem(5+temp:5+temp) = "0"
  filem2(13+temp:13+temp) = "0"
endif

C print *, filem, filem2
C read '(A)', tempo
C remove the next line to resume to normal ....
C goto 177

ave1 = 0.0
ave2 = 0.0
open (1, file = filenm, status='old')
print *, filenm

do 10 iloop =1,samp
  read (1,*) data1(iloop), data2(iloop)
  ave1 = ave1 + data1(iloop)
  ave2 = ave2 + data2(iloop)
  orig1(iloop) = data1(iloop)
  orig2(iloop) = data2(iloop)
10 continue

close (1)

ave1 = ave1/samp
ave2 = ave2/samp

C add "zeros" to end of data stream to bring total up to 8192 and eliminate
C wrap-around error! These additional data are not used for any other
C purpose and need not be recorded after the application. Originally these
C "zeros" were actually equal to 0 but there seemed to be significant
C 'wrap-around' problems so I decided to continue by adding the average signal
C value, hopefully thereby reducing any error introduction.
C
do 20 iloop=samp+l, n
   data1(iloop) = ave1
   data2(iloop) = ave2
20  continue

C C 'isign' is used by the FFT subroutines to determine whether a transform
C (isign = 1) or an inverse transform (isign = -1) is performed. Remember
C that no normalization occurs for inverses.
C    isign = 1
C
C now, we need to eliminate the bubbles from frequency analysis so they
C do not falsify the low frequency signal
C
    thresh1 = 0.85 * ave1
    thresh2 = 0.85 * ave2
    do 40 iloop =l,n
       if (data1(iloop) .lt. thresh1) then
          data1(iloop) = ave1
       endif
       if (data2(iloop) .lt. thresh2) then
          data2(iloop) = ave2
       endif
40  continue

C C now use the REALFT subroutine to transform the data into the
C frequency domain !
C
    call realft(data1,n,isign)
call realft(data2,n,isign)
    isign = -1
do 60 iloop = 1, n
   if (iloop .gt. cutoff) then
      C this will eliminate the high (noise) and mid-level (bubbles) frequency components
      C
      data1(iloop) = 0.0
      data2(iloop) = 0.0
   endif
60 continue

call realft(data1,n,isign)
call realft(data2,n,isign)

ave1b = 0.0
ave2b = 0.0

do 70 iloop = 1, n
   ave1b = ave1b + data1(iloop)
   ave2b = ave2b + data2(iloop)
70 continue

ave1b = ave1b/n
ave2b = ave2b/n

do 80 iloop = 1, n
   data1(iloop) = data1(iloop)*ave1/ave1b
   base1(iloop) = data1(iloop)
   data2(iloop) = data2(iloop)*ave2/ave2b
   base2(iloop) = data2(iloop)
80 continue

open (2, file = filenm2, status='new')
write (2, *) "filename = ", filenm1
write (2, *) "number of samples = ", samp
write (2, *) "cutoff for FFT inversion = ", cutoff

do 140 iloop = 1, samp
   data1(iloop) = orig1(iloop) - data1(iloop)
   data2(iloop) = orig2(iloop) - data2(iloop)
   write (2, *) data1(iloop), data2(iloop)
140 continue

close (2)
SUBROUTINE four1(data,nn,isign)

INTEGER isign,nn
REAL data(2*nn)
INTEGER i,i,istep,j,m,mmax,n
REAL tempi,tempr
DOUBLE PRECISION theta,wi,wpi,wpr,wr,wtemp
n=2*nn
j=1

do 11 i=1,n,2
  if(j.gt.i)then
    tempr=data(j)
    tempi=data(j+1)
    data(j)=data(i)
    data(j+1)=data(i+1)
    data(i)=tempr
    data(i+1)=tempi
  endif
  m=n/2
1   if ((m.ge.2).and.(j.gt.m)) then
     j=j-m
     m=m/2
     goto 1
  endif
j=j+m
11 continue
mmax=2
2 if (n.gt.mmax) then
  istep=2*mmax
  theta=6.28318530717959d0/(isign*mmax)
  wpr=-2.d0*sin(0.5d0*theta)**2
  wpi=sin(theta)
  wr=1.d0
  wi=0.d0
  do 13 m=1,mmax,2
    do 12 i=m,n,istep
      j=i+mmax
      temp=sngl(w)*data(j)-sngl(wi)*data(j+l)
      tempi=sngl(wr)*data(j+1)+sngl(wi)*data(j)
      data(j)=data(i)-temp
      data(j+1)=data(i+1)-tempi
      data(i)=data(i)+temp
      data(i+1)=data(i+1)+tempi
    continue
    wtemp=wr
    wr=wr*wpr-wi*wpi+wr
    wi=wi*wpr+wtemp*wpi+wi
  continue
  mmax=istep
  goto 2
endif
return
END

C (C) Copr. 1986-92 Numerical Recipes Software -)+M!&l.)5+

SUBROUTINE realft(data,n,isign)
INTEGER isign,n
REAL data(n)
C USES fourl
INTEGER i,i1,i2,i3,i4,n2p3
REAL c1,c2,h1i,h1r,h2i,h2r,wi,wrs
DOUBLE PRECISION theta,wi,wpi,wpr,wr,wtemp
theta=3.141592653589793d0/dble(n/2)
c1=0.5

if (isign.eq.1) then
  c2=-0.5
  call fourl(data,n/2,+1)
else
  c2=0.5
  theta=-theta
endif
wpr= -2.0d0*sin(0.5d0*theta)**2
wpi = sin(theta)
wr = 1.0d0+wpr
wi = wpi
n2p3 = n+3
do 11 i=2,n/4
  i1 = 2*i-1
  i2 = i1+1
  i3 = n2p3-i2
  i4 = i3+1
  wrs = sngl(wr)
  wis = sngl(wi)
  hlr = c1*(data(i1)+data(i3))
  hli = c1*(data(i2)-data(i4))
  h2r = -c2*(data(i2)+data(i4))
  h2i = c2*(data(i1)-data(i3))
data(i1) = hlr+wrs*h2r-wis*h2i
data(i2) = hli+wrs*h2i+wis*h2r
data(i3) = hlr-wrs*h2r+wis*h2i
data(i4) = -hli+wrs*h2i+wis*h2r
wtemp = wr
wr = wr*wpr-wi*wpi+wr
wi = wi*wpr+wtemp*wpi+wi
11  continue
if (isign.eq.1) then
  hlr = data(1)
data(1) = hlr+data(2)
data(2) = hlr-data(2)
else
  hlr = data(1)
data(1) = c1*(hlr+data(2))
data(2) = c1*(hlr-data(2))
call fourl(data,n/2,-1)
endif
return
END
C.3 Data analyzing program used on FFT-smoothed data (Cal_S98.Bas)

REM

***************************************************************************
**
REM * This program takes the FFT-processed conductivity probe signal *
REM * and calculates bubble velocities, and bubble chord lengths. *
REM *
REM * Results are put in a file with the same name and the suffix *
REM * '.BUB' for bubble data and '.GHU' for gas hold up *
REM *
REM * copywritten by Mike Safoniuk, 1996 *
REM *
REM***************************************************************************
**

REM

DIM itime1(2000)
DIM itime2(2000)
DIM itau1(2000)
DIM itau2(2000)
DIM bubvel(2000)
DIM chord(2000)
DIM chord2(2000)
DIM r1gas(10), r2gas(10)
DIM iloop, jloop, kloop, subloop AS INTEGER

rthresh = -.1
total = 0!
uave = 0
lave = 0

ON ERROR GOTO ErrorHandler

1 INPUT "First -2- letters of ??*.cnv file series .....", a$  
IF LEN(a$) > 2 THEN
   PRINT "-ONLY- the first 2 letters please ..."
bad$ = "c:\mike\data\" + a$ + "_err.log"
OPEN bad$ FOR OUTPUT AS #13

REM FOR iloop = 1 TO 2
FOR iloop = 1 TO 1
  IF iloop = 1 THEN
    i$ = "0"
  ELSEIF iloop = 2 THEN
    i$ = "1"
  END IF

REM FOR jloop = 1 TO 10
FOR j loop = 1 TO 8
  IF jloop = 1 THEN
    j$ = "0"
  ELSEIF jloop = 2 THEN
    j$ = "1"
  ELSEIF jloop = 3 THEN
    j$ = "2"
  ELSEIF jloop = 4 THEN
    GOTO 8003
  ELSEIF jloop = 5 THEN
    j$ = "3"
  ELSEIF jloop = 6 THEN
    j$ = "4"
  ELSEIF jloop = 7 THEN
    j$ = "5"
  ELSEIF jloop = 8 THEN
    j$ = "6"
  ELSEIF jloop = 9 THEN
    j$ = "7"
  ELSEIF jloop = 10 THEN
    j$ = "8"
  END IF

FOR kloop = 1 TO 10
  IF kloop = 1 THEN
    IF iloop = 1 AND jloop = 1 THEN
      GOTO 8001
    END IF
  END IF

GOTO 1
END IF
231

```
k$ = "0"
ELSEIF kloop = 2 THEN
    k$ = "1"
ELSEIF kloop = 3 THEN
    k$ = "2"
ELSEIF kloop = 4 THEN
    k$ = "3"
ELSEIF kloop = 5 THEN
    k$ = "4"
ELSEIF kloop = 6 THEN
    k$ = "5"
ELSEIF kloop = 7 THEN
    k$ = "6"
ELSEIF kloop = 8 THEN
    k$ = "7"
ELSEIF kloop = 9 THEN
    k$ = "8"
ELSEIF kloop = 10 THEN
    k$ = "9"
END IF

f$ = "C:\data\lc" + a$ + i$ + j$ + k$ + ".bub"
REM PRINT f$
OPEN f$ FOR OUTPUT AS #3
WRITE #3, "bubble velocity, tip 1 length, average length"

FOR subloop = 1 TO 10
    IF subloop = 1 THEN
        sub$ = "_0"
    ELSEIF subloop = 2 THEN
        sub$ = "_1"
    ELSEIF subloop = 3 THEN
        sub$ = "_2"
    ELSEIF subloop = 4 THEN
        sub$ = "_3"
    ELSEIF subloop = 5 THEN
        sub$ = "_4"
    ELSEIF subloop = 6 THEN
        sub$ = "_5"
    ELSEIF subloop = 7 THEN
        sub$ = "_6"
    ELSEIF subloop = 8 THEN
        sub$ = "_7"
    ELSEIF subloop = 9 THEN
        sub$ = "_8"
    ELSEIF subloop = 10 THEN
```

```
sub$ = "_9"
END IF

f$ = "c:\data\lc" + a$ + i$ + j$ + k$ + sub$ + ".cnv"

REM REM ** temp for debugging ** vvvvvvvv
REM PRINT f$
REM GOTO 8000
REM REM ** temp for debugging ** ~~~~~~
ibad = 0
OPEN f$ FOR INPUT AS #1
IF ibad = 1 THEN
   CLOSE #1
   CLOSE #3
   bad$ = a$ + i$ + j$ + k$
   WRITE #13, "Error = ", ERR, "filename => ", bad$
   ibad = 0
   GOTO 8001
END IF

REM
REM ibub1 & ibub2 are used to determine the number of bubbles recorded
REM iflag1 and iflag2 are used to determine whether or not we are currently
REM recording a bubble (0 = no, 1 = yes)
REM rgas= gas hold-up based on tip #1
REM
ibub1 = 0
ibub2 = 0
iflag1 = 0
iflag2 = 0
rgas = 0
rgas2 = 0

REM time to get rid of the first three lines in the data file!

INPUT #1, b$
INPUT #1, b$
INPUT #1, b$

FOR icount = 1 TO 13750

INPUT #1, sig1, sig2

IF sig1 < rthresh THEN
\[ r_{gas} = r_{gas} + 1 \]

IF \(iflag1 = 1\) THEN

REM This means that this bubble is already being recorded

\[ itau1(ibub1) = itau1(ibub1) + 1 \]

ELSE

REM this means that this is the beginning of the presence of a bubble
REM \(itimel\) and \(itime2\) record the starting time of the bubble presence
REM for tips 1 and 2 respectively.

\[ ibub1 = ibub1 + 1 \]
\[ iflag1 = 1 \]
\[ itau1(ibub1) = 0 \]
\[ itime1(ibub1) = icount \]

END IF

ELSE

IF \(sig1 \geq r_{thresh}\) AND \(iflag1 = 1\) THEN

REM this means that the bubble has left the tip area

\[ iflag1 = 0 \]
\[ itau1(ibub1) = itau1(ibub1) + 1 \]

REM this offsets the fact that the first segment we miss
REM approximately 1/2 a time count and so we add approximately 1/2
REM to the end

END IF

ENDIF

IF \(sig2 < r_{thresh}\) THEN

\[ r_{gas2} = r_{gas2} + 1 \]
\[ iflag2 = 1 \]

REM This means that this bubble is already being recorded

\[ itau2(ibub2) = itau2(ibub2) + 1 \]

ELSE

REM this means that this is the beginning of the presence of a bubble
REM \(itimel1\) and \(itime2\) record the starting time of the bubble presence
REM for tips 1 and 2 respectively.

\[ ibub2 = ibub2 + 1 \]
iflag2 = 1
itau2(ibub2) = 0
itime2(ibub2) = icount
ENDIF
ELSE
   IF sig2 >= rthresh AND iflag2 = 1 THEN
   REM this means that the bubble has left the tip area
   REM
      iflag2 = 0
   REM
      itau2(ibub2) = itau2(ibub2) + 1
   REM
   REM this offsets the fact that the first segment we miss
   REM approximately 1/2 a time count and so we add approximately 1/2
   REM to the end
   REM
      END IF
   END IF
   NEXT icount
CLOSE #1
rlgas(subloop) = rgas / 13750
r2gas(subloop) = rgas2 / 13750

REM Now to calculate bubble velocities and pierced chord lengths
REM
REM
REM s = distance between the two probe tips in meters
REM igood = # of 'good' bubbles
REM uave = average bubble rise velocity
REM lave = average bubble chord length
REM
s = .0011
igood = 0
i1 = 0
i2 = 0
REM
REM i1, and i2 are counters that allow us to cycle through all
REM the signal valleys to identify pairs as valid bubbles
REM

10  i1 = i1 + 1
    IF i1 > ibub1 THEN GOTO 999

20  i2 = i2 + 1
    IF i2 > ibub2 THEN GOTO 999

REM
REM let's only look at any t2's that are greater than t1 since any
REM bubbles at tip 2 that start earlier than t1 are either travelling
REM downward or have not passed through both tips.
REM
    IF itime2(i2) <= itime1(i1) THEN GOTO 20
REM
REM now we next check to ensure that the 2nd tip's signal drop
REM starts BEFORE the end of the 1st tip's signal return to baseline
REM
    IF itime2(i2) > itime1(i1) + itau1(i1) THEN
        i2 = i2 - 1
        GOTO 10
    END IF
REM
REM now we also want to ensure that the signal for the 2nd tip
REM doesn't both start and end during the drop in the 1st tip's
REM signal
REM
    IF itime2(i2) + itau2(i2) < itime1(i1) + itau1(i1) THEN GOTO 20
REM
REM  Now the final check (as per Matsuura & Fan, 1984)
REM
    rchk = 2 * itau1(i1) / (itau1(i1) + itau2(i2))
    IF rchk < .9 OR rchk > 1.1 THEN
        i2 = i2 - 1
        GOTO 10
    END IF
REM
REM After allllll these checks, -finally- a good bubble!
REM
    igood = igood + 1
REM    total = total + 1
bubvel(igood) = s / (itime2(i2) - itime1(i1)) * 2500  
chord(igood) = itaul(i1) / 2500 * bubvel(igood)  
chord2(igood) = (itaul(i1) + itau2(i2)) / 2 / 2500 * bubvel(igood)  
uave = uave + bubvel(igood)  
lave = lave + chord(igood)  
lave2 = lave2 + chord2(igood)  
GOTO 10  

WRITE #3, igood  
FOR icon = 1 TO igood  
   WRITE #3, bubvel(icon), chord(icon), chord2(icon)  
NEXT icon  

7777 IF total = 0 THEN  
   WRITE #3, "NO bubbles found .... sorry"  
ELSE  
   WRITE #3, "average bubble velocity = ", uave / total  
   WRITE #3, "average tip 1 length = ", lave / total  
   WRITE #3, "average averaged length = ", lave2 / total  
   total = 0!  
END IF  

REM no longer need this line ....  CLOSE  

8000 NEXT subloop  

CLOSE #3  
f$ = "C:\data\le\" + a$ + i$ + j$ + k$ + ".ghu"  
OPEN f$ FOR OUTPUT AS #2  

f$ = a$ + i$ + j$ + k$ + ".cnv"  
WRITE #2, "file series =", f$  
WRITE #2, "threshold = ", rthresh  
r1ave = 0!  
r2ave = 0!  
FOR icount = 1 TO 10  
   WRITE #2, "gas hold-up = ", r1gas(icount), r2gas(icount)  
   r1ave = r1ave + r1gas(icount) / 10  
   r2ave = r2ave + r2gas(icount) / 10  
NEXT icount  
WRITE #2, "-----------------------------------------------"
WRITE #2, "AVERAGE = ", rlave, r2ave

rlsdev = 0!
r2sdev = 0!
FOR icount = 1 TO 10
   rlsdev = rlsdev + (rlgas(icount) - rlave)^2
   r2sdev = r2sdev + (r2gas(icount) - r2ave)^2
NEXT icount
rlsdev = (rlsdev/9)^.5
r2sdev = (r2sdev/9)^.5
WRITE #2, "STAN DEV (n-1) = ", rlsdev, r2sdev
CLOSE #2

REM PRINT " "
REM INPUT delay$
800
8001 NEXT kloop
   PRINT f$
   PRINT " "
8002 NEXT jloop
8003 NEXT iloop

CLOSE #13
END

' error handling routine handles only Bad File Name
' aborts on any other error

ErrorHandler:
IF ERR = 53 THEN
   ' get another file name
   bad$ = a$ + i$ + j$ + k$ + ".cnv"
   PRINT bad$, "not found."
ELSE
   ' some other error, so print message and abort
   PRINT "Unreccoverable error--"; ERR
END IF
ibad = 1
RESUME NEXT
Appendix D - Integration of Local Gas Hold-Up to Average Radial Gas Hold-Ups

The basis for the approach applies integration of the results based upon a circular cross-section and the use of the trapezoidal rule for approximate integration (Gear, 1978).

As an example, consider some actual data from Figure 7.10:

![Graph](image)

**Figure D.1**: Local gas hold-up data presented in Figure 7.10 for $z=0.051$ m

The left-hand side of the graph in Figure D.1 including all data with $-1<r/R<0$ (denoted by “LHS”) is considered separately from the right side (RHS), i.e. $0<r/R<1$. An average gas hold-up is calculated for each side separately and the over-all average axial gas hold-up is then equal to the average hold-up of the two halves.
It is important to note that the calculated averages are only valid for the area of the column for which measurements were taken. In this case, this means that we can only measure the average gas hold-up on the LHS from -0.86 to 0 since no measurements were taken from -1 to -0.86. Similarly for the RHS the average gas hold-up can only be determined for the area between 0.91 and 0 since no measurements were taken between 0.9134 and 1. When severe gradients occur (as in the data presented here) this can lead to misleading results since the "ends" are essentially ignored.

For the LHS, the average gas hold-up is given by

$$\bar{\varepsilon}_g \pi r_i^2 = \sum_{i=1}^{n-1} \left[ \left( \pi r_i^2 - \pi r_{i+1}^2 \right) \left( \frac{\varepsilon_{g_i} + \varepsilon_{g_{i+1}}}{2} \right) \right]$$ (D.1)

where \( n = \) number of points on the LHS. In this case we have 6 actual measured points and the mid-point, for a total of \( n=7 \). The latter part of this equation, which averages adjacent gas hold-ups, is an application of the trapezoidal rule. For the purposes of these calculations on the LHS, the \( r/R = 0 \) point, is considered to have the same gas hold-up as the first data point on the RHS (the point marked by \( \bullet \) in Figure D.1). Equation D.1 can be rewritten as

$$\bar{\varepsilon}_g \left( \frac{r_i}{R} \right)^2 = \sum_{i=1}^{n-1} \left[ \left( \frac{r_i}{R} \right)^2 - \left( \frac{r_{i+1}}{R} \right)^2 \right] \left( \frac{\varepsilon_{g_i} + \varepsilon_{g_{i+1}}}{2} \right)$$ (D.2)

Equation D.2 can then be applied to the LHS of Figure D.1 to obtain an average gas hold-up for that side. The sequence in which the data points are used always follows from the wall region \( (r/R \rightarrow 1) \) to the center of the column \( (r/R=0) \).

For the RHS, equation D.2 is also used, except that
- The zero point is assumed to now have the same value as the first data point on the LHS of the graph.

- The term on left hand side of equation D.2 is \( \bar{e}_g \left( \frac{r_L}{R} \right)^2 \) instead of \( \bar{e}_g \left( \frac{r_i}{R} \right)^2 \) to reflect that this average is for the area limited by data point \( L \).

The final step in the process is a simple averaging of the average values obtained for the two sides.
Appendix E - Radial gas hold-up profiles

The graphs in this appendix are for the air-tap water-aluminum three-phase system. Hold-ups were determined by conductivity probe in the 0.292-m column with cylindrical aluminum particles of 4 mm diameter and 10 mm length as the solids. M = 5.46 x 10^{-11}, Eo = 2.18, \( \beta_d = 2.75 \), \( W_s = 53.4 \) kg.

![Diagram of local gas hold-up at various radial positions and four different heights for \( \text{Re}_L = 430 \) and \( \text{Re}_g = 233 \).]

**Figure E.1:** Local gas hold-up at various radial positions and four different heights for \( \text{Re}_L = 430 \) and \( \text{Re}_g = 233 \).
Figure E.2: Local gas hold-up at various radial positions and four different heights for $Re_L = 430$ and $Re_g = 356$. 
Figure E.3: Local gas hold-up at various radial positions and four different heights for $Re_L = 536$ and $Re_g = 170$. 
Figure E.4: Local gas hold-up at various radial positions and four different heights for $Re_L = 536$ and $Re_g = 233$. 
Figure E.5: Local gas hold-up at various radial positions and four different heights for $Re_L = 536$ and $Re_g = 470$. 
Figure E.6: Local gas hold-up at various radial positions and four different heights for $Re_L = 646$ and $Re_g = 170$. 
Figure E.7: Local gas hold-up at various radial positions and four different heights for $\text{Re}_L = 646$ and $\text{Re}_g = 233$. 
Figure E.8: Local gas hold-up at various radial positions and four different heights for $Re_L = 646$ and $Re_g = 356$. 
Figure E.9: Local gas hold-up at various radial positions and four different heights for \( Re_L = 646 \) and \( Re_g = 470 \).
Figure E.10: Local gas hold-up at various radial positions and four different heights for $Re_L = 716$ and $Re_g = 233$. 
Figure E.11: Local gas hold-up at various radial positions and four different heights for $Re_L = 716$ and $Re_g = 356$. 