DESIGN OF DECOUPLING CONTROL AND TIME-DELAY COMPENSATION FOR A CFSTR

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Abstract

This thesis is concerned with the design of a decoupling compensator and a time-delay compensator for a nonisothermal continuous flow stirred tank reactor (CFSTR).

An expression for the analysis of interaction of the two-variable CFSTR was theoretically derived by using the relative gain method (RGM). For the purpose of improving the stability of the decoupling control system, undercompensation for a decoupled CFSTR system was suggested and the robustness test of such undercompensation decoupler to the modelling error was studied. On the other hand, the proposed time-delay compensation method, unlike conventional Smith's scheme, can rely on the basic property of gain-invariant time-delay. The stability of this time-delay compensation method is not affected by the CFSTR control system time-variant time-delay, while its compensation structure has the same features as the Smith compensator.

The design of a decoupler and that of a time-delay compensator are independent of each other. All compensation structures are physically realizable.

The theoretical results are supported by simulation. Simulation results for a CFSTR demonstrate that the undercompensation decoupling control can tolerate a relatively wide modelling error and reduce the sensitivity of the CFSTR process to parameter variations and unwanted disturbances. Also, simulation results show that the proposed time-delay compensator can provide an improvement over the conventional Smith compensator.

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Chapter 1

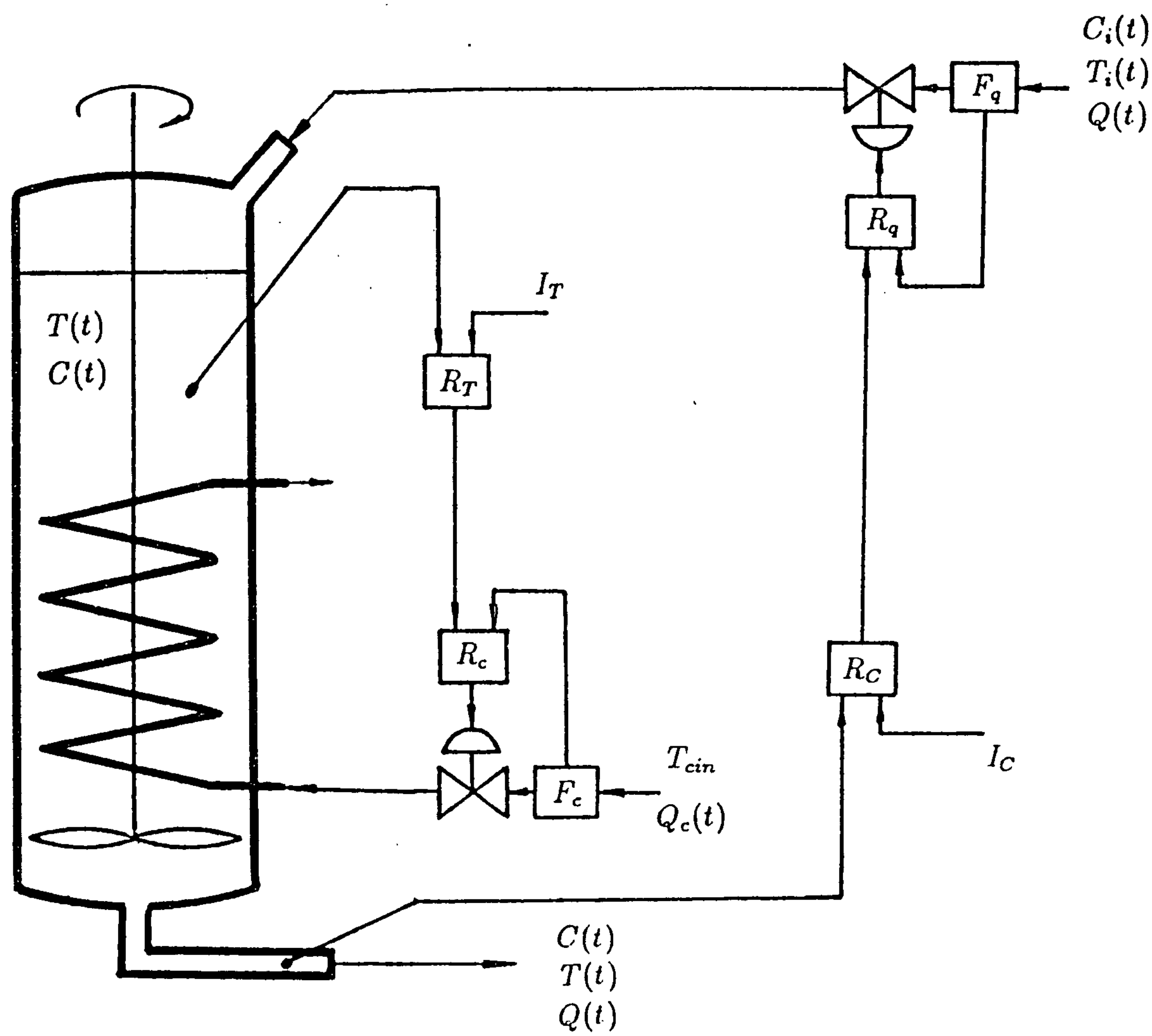
Introduction

1.1 Motivation of the Present Work

A control system is required on a continuous-flow stirred tank reactor (CFSTR) with irreversible, exothermic reactions to ensure that it operates under steady-state conditions. Figure 1.1 illustrates a typical CFSTR control system. When the effluent concentration is not controlled the regulation of the reactor temperature is a single dimensional problem, while the regulation of both reactor temperature and effluent concentration is a multi-dimensional one. The conventional control approach for a single variable linear system without time-delay; using standard type controllers and parameter tuning by rules of thumb and experience from similar processes, works quite well in many cases. However, for a CFSTR process with

- interactive behaviour
- time-delay behaviour
- nonlinear behaviour

it is frequently quite difficult and time consuming to find the appropriate structure and the correct parameters for the controllers if good control performance is required.
The main difficulty in the design of a multivariable control configuration is that individual controllers cannot behave optimally because of control loop interaction. In fact, the controllers in multiloops tuned by classical control techniques cannot overcome internal



- C: effluent concentration of reactant A
- C_i: inlet concentration of reactant A
- F_c: coolant flow rate measuring device
- F_q: reactant flow rate measuring device
- I_C: concentration set point
- I_T: temperature set point
- Q: volumetric flow rate
- Q_c: coolant flow rate
- R_c: coolant flow rate controller
- R_C: concentration controller
- R_q: reactant flow rate controller
- R_T: temperature controller
- t: time
- T: temperature in a reactor
- T_{cin}: inlet temperature of coolant
- T_i: inlet temperature of reactant A

Figure 1.1: Conventional control loops of a CFSTR.

disturbances arising from this interaction behaviour. In Figure 1.1, the reactor temperature is controlled by the flow of coolant while the effluent concentration is controlled by the inlet flow rate. The distinctive feature of the dynamics of a CFSTR is characterized by nonlinear interaction behaviour between the temperature control loop and the concentration control loop and time-delay behaviour because of the measuring delay of the feedback variable.

From the standpoint of process control, the CFSTR poses a considerable challenge to a designer owing to this sort of behaviour. In recent years, there has been an extensive interest in adaptive control systems that automatically adjust the controller settings to compensate for unanticipated changes in the process. Such an adaptive control system requires an on-line digital computer to do some complicated computation. With the evolution of digital control computers, much better designs can be produced without any consideration for hardware realizability. This, in part, has spurred research and development to evolve advanced control strategies for process systems. In spite of the flexibility offered by the general structure of digital computers, most process industrial loops are still controlled by conventional controllers (Mendoza-Bustos, 1990). In fact, in many practical applications, the advanced control system for a small scale subclass process is not always feasible due to the high cost of a computer system or sophistication not accessible to nonexperts. The large number of conventional controllers used routinely for process control may be regarded as an experimental evidence of their usefulness. The reason for their extensive use may lie in the fact that a trained operator can quickly master the controller's behaviour. This is why classical control theory is still going strong.

In most chemical processes, control schemes should be kept as simple as possible, even at the expense of some performance. Simpler controllers tend to be easier to adjust by trial and error, easier for the operations and maintenance personnel to understand, and less sensitive to process parameter changes. For the same reasons, three questions about the CFSTR process control should be answered. They are:

- What is an easy way of avoiding the problem of the interaction between the temperature control loop and the concentration control loop and ensuring the stability of a decoupling system if modelling error occurs?
- Can a time-delay compensator be designed to achieve robust adaptation?
- What can be used as a physically realizable decoupling compensator and time-delay compensator?

This thesis intends to focus on the measurement of interaction, design of a decoupling compensator and the analysis of modelling error. It will also present two physically realizable models for both decoupling compensation and time-delay compensation, which are simple to understand and implement, while possessing a sound fundamental basis.
The basic assumption behind this approach is that the success of a model in engineering has always depended on the valid use of approximations and assumptions to reduce the complexity of the real world to simple and manageable mathematical abstractions; and CFSTR process control is no exception in this respect.
Therefore, the message of the present work is that applicable and simple methods should be sought in an effort to develop a suitable CFSTR controlled model.

1.2 Objectives of the Present Work

1.2.1 Interaction Analysis and Decoupling

Basic control studies of a CFSTR are usually based on mass and energy balance equations, which are coupled and nonlinear. Generally speaking, interactive multivariable systems should be decoupled in order to avoid difficulties in control. However, two problems may arise. Firstly, an ideal decoupling design is by no means a panacea. In fact, adding a decoupler requires more components, more attention, and tends to be less reliable. Secondly, even if a decoupling design is necessary, the process deviations from the decoupled model may lead to unstable control. Therefore, every effort for improving system performance should be made to keep the CFSTR control system as simple as possible.

One area which is still poorly understood is the source of interaction of a CFSTR. To effectively design a CFSTR process control system, the designer must have a basic understanding of (1) interaction analysis of a multivariable system, (2) the relevant factors that affect degree of interaction and (3) the relationship between the decoupling design and degree of interaction.

Therefore, one of studies presented here is an attempt to determine the relationship between the degree of interaction and the process parameters, and to design a decoupling compensator for a CFSTR process with strong interaction.

1.2.2 Time-delay Compensation

Another troublesome area encountered in CFSTR process control is the handling of measurement characterized by time-delay. The control of time-delay processes is usually carried out using a conventional Smith compensator (Smith, 1959). This compensator is sometimes adequate for successful control. But, in fact, the Smith compensator suffers from two shortcomings. Firstly, its robustness is not very good and is sensitive to the deviation from the mathematical model. Secondly, the compensator is physically irrealizable.

For most processes, a "reasonable time-delay compensator" with some "good" values for the model parameters is employed for control purposes. The mismatch between the mathematical model and the true process can lead to serious stability problems for CFSTR process control, especially when measurement feedback delay is uncertain. Thus, another objective of this thesis is the design of a simple and tractable robust strategy for the time-delay compensation that takes care of model uncertainty. This is of paramount importance for the design of a good and efficient control system for a CFSTR process.

In order to fulfill the above objectives, classical control theory, based on the Laplace transform as its main analytical tool, will be considered as a very effective method for system analysis in CFSTR process control.

1.3 Summary

This thesis is organized in the following manner.

- The motivations and objectives of this research are briefly described in Chapter 1.
- A literature review about interaction analysis, decoupling in general and the time-delay compensator is given in Chapter 2.
- Mathematical models for the CFSTR which include a linearized interaction model and a time-delay model are described in Chapter 3.
- In Chapter 4, the relative gain method (RGM) for interaction analysis is introduced and a study on interaction of the CFSTR is given.
- Chapter 5 deals with a decoupling design and contains some results from the simulation.
- Chapter 6 is concerned with the design of a physically realizable time-delay compensator.
- The conclusions and suggestions are presented in Chapter 7.
- The appendices contain the Taylor expansion for a two-variable system, the Laplace transform pairs, the derivation of a closed-loop transfer function with a time-delay, the property of the relative gain for 2×2 system, the transfer of dimensionless variables, the solution of a standard second-order system, the derivation of a physically realizable time-delay compensation model, and the simulation data.

Chapter 2

Literature Review

2.1 Introduction

This thesis is concerned with the design of a two-variable CFSTR process control system using physically realizable control algorithms. Two passes will be made through the literature. In the first pass, attention will be paid to the theory of both multivariable control and time-delay control, or, more precisely, the development of a decoupled control system and a time-delay compensation system. During the second pass, a brief review of CFSTR process control will be presented. The former problem is a problem of control theory, and the latter falls under the heading of applications of control theory.

2.2 Literature on Interaction Analysis and Decoupling Control

There have been many studies on multivariable process control systems. For reviews, the reader may refer to Lloyd (1973), Fossard (1977), Tung and Edgar (1982), Tzafestas (1984), Sinha (1984), Vidyasagar and Kimura (1986), Marino et al. (1987), O'Reilly (1987) and Shen and Lee (1988). The present review on multivariable process control is confined to the measure of the interaction and to decoupling theories.

The earliest study of both interaction analysis and the decoupling of designs seems to have been by Boksenbom and Hood (1949). They introduced the matrix analysis method in the analysis of multivariable control systems and proposed the notion of non-interactive control, namely decoupled control. In spite of many studies on interaction analysis, no successful study of the measure of interaction appeared until Bristol (1966) introduced the relative gain method (RGM), popularized by Shinskey (1979). They defined a decoupling sensitivity. This sensitivity indicates how much error in a decoupler gain can be tolerated by a decoupled control system. An important feature of the RGM is that it is independent of the controller design, and less information about the control theory is required. Thus, a designer does not have to carry out detailed control system designs for processes.

The output feedback control problem for weakly coupled linear systems has been studied by Petkovski and Rakic (1979) using a series expansion approach. Basically, their study is an effective method for analyzing a weakly coupled linear system. Manousiouthakis et al. (1986) extended the RGM to cases in which more than SISO (single-input single-output) controllers were considered and they developed the application of the RGM to the multivariable system. They call their approach " the block RGM ". In another study, Yu and Luyben (1986) described a method for determining the structure, variable pairing, and tuning of multiloop SISO controllers in a multivariable-process environment by using a negative RGM. The basic idea of Yu's method was to produce a stable, workable and simple SISO system. This idea today is still valid for most control system designs.

Most research in multivariable control has been concerned with the decoupling of interactive loops using specially designed networks, with emphasis on the servo problem of decoupling the loops for changes in set point. Among the approaches to the decoupled control problem, three schemes have been recognized to be in a common framework:

1. The diagonal matrix method proposed and developed by Kavanagh (1958). The idea was to design a controller which produced an overall diagonal transfer function

matrix. If such a controller could be found, then the problem of multivariable control system design could be reduced to a number of single loop designs which could be carried out by the well-established classical control methods.

2. The state variable method employed by Falb (1967) and Gilbert (1969). This method was given a significant boost in the early 1970's by Wonham (1970), Francis (1975) and Wonham (1979). They showed that many of the standard problems of multivariable system design could be solved by this means in an abstract state-space setting.

3. Multivariable adaptive control algorithms, which appeared approximately 15 years ago, are still based on rudimentary theory. There have been a number of schools of study on such multivariable control theory. The research efforts have been vigorous. Several papers, for example Wolovich and Falb (1976), Elliot and Wolovich (1984), McDermott and Mellichamp (1984), Dickmann and Sivan (1985), Narendra (1986) and Chien et al. (1987), have been published.

   The original idea of the interactor matrix was proposed by Wolovich and Falb (1976). The interactor matrix is a canonical model which can ensure the use of the minimum order of predictors for multivariable systems with time-delay. McDermott and Mellichamp (1984) studied a decoupling pole-placement self-tuning controller for MIMO processes with open-unstable behaviour. This approach is based on the concept of state. Some helpful discussions on stability robustness have been made by Dickmann and Sivan (1985). They arrived at the conclusion that the decoupling structure can improve system robustness. Chien (1987) discussed a new algorithm for a self-tuning controller with time-delay compensation (STC-TDC) for multivariable decoupling control problems. This approach employed multiple single-input/single-output self-tuning controllers but with a classical decoupling scheme incorporated. Simulation studies utilizing two distillation column models showed that the controller could provide good control performance.

In general, the state variable method and adaptive control algorithms belong in the modern control category.

2.3 Literature on Time-delay Compensation Control

Time-delay is recognized as the most difficult dynamic element naturally occurring in processes (Shinskey; 1988). It is well known for the delay-free case that the use of negative feedback not only modifies system dynamics but also makes the system performance less sensitive to changes in process parameters. An ideal time-delay compensator was described by Smith (1957, 1958 and 1959). Smith proposed a compensation technique to eliminate the delay term in the closed-loop characteristic equation, which is known world-wide as the Smith Predictor. However, at an early date, Buckley (1964) pointed out that if the process deviates from the model, then Smith's time-delay compensator can lead to unstable or at least poorer control than is generally achievable with a standard proportional plus integral controller. Again, Palmor (1980) noticed and explained in different ways that performance improvements by the Smith method can be very sensitive to model error, which means its robustness is very poor.

Over the last 10 years, there has been a dramatic change in the design of time-delay compensators. Vogel and Edgar (1980) used a digital control method for time-delay compensation. This method, which is based on the digital control process, can improve the robustness of a time-delay system. Also, Vogel and Edgar (1980) developed the SISO adaptive time-delay compensator using Dahlin's control algorithm in the Smith predictor structure. Lee and Lu (1984) proposed coefficient assignments which belong to the state-space feedback compensator. A modified Smith predictor was reported by De (1985) which is, in theory, physically realizable. Chandra et al. (1985) provided an ideal adaptive control method for time-delay compensation. Agamennoni et al. (1987) concentrated on an adaptive control scheme for a single-input single-output process with delays by using the Smith method with a dynamic filter to improve the dynamic performance of the control system.

More recently, a methodology for the identification of multivariable processes was developed by Shanmugathasan and Johnston (1988) that can achieve a higher level of controllability by considering a generalized multidelay compensator (GMDC). A simple heat recovery network provided a practical example of the application of Shanmugathasan's method. This method yielded a consistently better closed-loop response than existing compensators.

Annraoi and Ruth (1989) designed a new modified Smith predictor for unstable processes with time-delay, but the problem of physical realizability has not been discussed.

Liu (1989) presented a state-space method for multivariable decoupling with simultaneous time-delay compensation. The importance of the state vector is that, in the case of a deterministic system free of all unpredictable random effects, all future states are completely determined by an initial state and inputs to the system.

All these works, no doubt, are important contributions to the input-output decoupling problem and process time-delay compensation. However, the design of advanced control algorithms usually requires the measurement of system states. In many practical applications, this is not feasible due to either the high cost of states measurement or the inaccessibility for measurement of some of the system states. On the other hand, if the application of a digital computer is not considered, hardly any modern control algorithms are possible in industrial environments.

2.4 Literature on the CFSTR Process Control

A study of the CFSTR process control was reported by Nakanishi and Nanbara (1981), who used a feedforward/feedback control system for both reactor temperature control and effluent concentration control. They also considered decoupling design in the feedforward loop and used a time-domain's multivariable Smith predictor in the feedback loop. Simulations confirmed that the dynamic characteristics of CFSTR with time-delays in the control variables could be improved.

Mukesh and Cooper (1983) gave a brief review of the CFSTR control and used a partial simulation technique to study the dynamic behaviour of a CFSTR. Their study also involved the development of software for the simulation and control of a CFSTR system using a digital computer.

Bartusiak et al. (1986) studied a nonlinear control structure for a CFSTR, and pointed out that a nonlinear controller could provide a better servo and regulatory response relative to linear temperature controllers tuned at different temperatures within the range of operating conditions. In fact, this method indicates that to some extent control system has good robustness.

Nakanishi and Ohtani (1986) pointed out that the traditional procedure [Foster and Stevens, 1967; Bruns and Bailey, 1977; Ray, (1982)] based on a linear, time-invariant, delay-free model of the reactor dynamics cannot be justified for a practically useful control system design of a nonisothermal CFSTR with time-delay. They studied the effects of time-delay, interaction and nonlinearity involved in the mass balance and heat balance equations of CFSTR dynamics. For the purpose of improving the control performance of the decoupling control system, a feedback control system with a Smith compensator was designed for an incompletely decoupled CFSTR. They gave little information on robustness.

Kantor (1988) also studied a finite-state nonlinear observer and a nonlinear state feedback controller for an exothermic stirred-tank reactor operated in continuous mode. Simulation results showed that good performance of CFSTR process control could be obtained, but the effects of modelling error were not studied.

Another control scheme using internal model control (IMC) was studied by Calvet and Arkun (1988). They applied IMC theory to a model of a CFSTR with a single first-order exothermic reaction. This CFSTR simulation example illustrated the power of the nonlinear system design with an IMC structure for disturbance rejection and set-point tracking. Throughout their paper, nonlinear dynamic models were assumed to be available; robustness considerations, with regard to model errors, were not addressed.

More detailed studies of CFSTR process control have been the subject of extensive discussions [See, e.g. Douglas (1972), Seborg and Edgar (1981), Stephanopoulos (1984), and Cinar et al. (1986)]. Nowadays, more effort is being invested in the design of adaptive control techniques with improved robustness properties, and some successful applications have already been reported, as for example, in temperature control systems for chemical reactors (Amhren, 1977 and MacGregor et al., 1984). Seborg et al. (1986) reported several applications of adaptive control in the chemical process control field.

A review by Schnelle and Richards (1986) gave a comprehensive list of references, including difficult problems of industrial reactor control.

From the above review, some conclusions can be drawn regarding the CFSTR process with decoupling as well as time-delay behaviour:

1. Theoretically, advanced multivariable adaptive control techniques for the CFSTR process have been made possible, using digital computer-based systems, though they are still being developed. Practically, many chemical processes involve complex reactions or transport operations that almost defy modelling of the adaptive structure. On the other hand, in process control, a universal complaint is the inability to measure key process variables, such as reaction rate. Therefore, although significant advances have been made in hardware design, the control algorithms used in conventional controllers are still not dying, in spite of all the advances made in control theory.

2. From a practical point of view, the most spectacular developments in recent years have been in robustness analysis. Information on physically realizable as well as robust control algorithms for decoupling design and time-delay compensation of the CFSTR process is somewhat scarce; there are only a few published articles.

3. Although all of the above approaches have the potential for better performance, some of tuning methods are usually not easy, causing difficulties in practice. It is fair to say that modern control algorithms may be used, but they usually require a great deal of effort by very skilled personnel and the support of an on-line computer system. As mentioned above, the major recent change in the process control field is the appearance and not very rapid acceptance by the user of direct digital control systems based on microprocessors. Therefore, physically realizable control algorithms will remain an exciting and practically important area of research for many years to come.

Based on the above evidence, a good robust, physically realizable decoupling control structure as well as a time-delay compensator for the CFSTR process will constitute the main content of the present work.

Chapter 3

Mass Balance and Energy Balance for a CFSTR

3.1 Basic Mathematical Equations Describing a CFSTR

The continuous-flow stirred tank reactor consists of a well-stirred tank into which there is a steady flow of reacting material, and from which the reacted material passes continuously. Deriving a reasonable mathematical model is the most important part of the entire analysis and control of such a CFSTR. The two basic mathematical equations required to describe CFSTR performance are a macroscopic mass balance and an energy balance.

3.1.1 Mass Balance Equation

Since the CFSTR contents are completely uniform with perfect mixing, a mass balance for the rate of change in the mass of reactant A within the reactor can be expressed as

$$\underbrace{V[\frac{dC(t)}{dt}]\Big|_{net}}_{(1)} = \underbrace{Q(t)C_i(t)}_{(2)} - \underbrace{Q(t)C(t)}_{(3)} + \underbrace{V[\frac{dC(t)}{dt}]\Big|_{reaction}}_{(4)} \tag{3.1}$$

where

V=reactor volume

$C(t)$=concentration of reactant A in reactor

$Q(t)$=volumetric flow rate

$C_i(t)$=inlet concentration of reactant A

t=time

The respective terms are as follows:

(1) net rate of change in the mass of reactant A within the reactor,

(2) rate of increase in the mass of A due to its presence in the influent,

(3) rate of decrease in the mass of A due to removal in the effluent,

(4) rate of decrease or increase in the mass of A due to the reaction of A in the reactor.

The last term on the right-hand side of Equation 3.1 will be assigned a negative value if it is assumed that the reaction of A within the reactor results in a decrease in the quantity of A. If the reaction of A within the reactor results in an increase in the quantity of A, a positive value should be assigned to this term.

3.1.2 Energy Balance Equation

In an energy balance over a volume element of a chemical reactor, kinetic and potential terms may usually be neglected relative to the heat of reaction and other heat transfer terms. Assume no density changes and that specific heat does not change with composition. So, the energy balance for the fluid includes energy lost to a cooling coil and heat release by an exothermic chemical reaction. It is

$$\underbrace{V\rho_f c_p \frac{dT(t)}{dt}}_{(1)} = \underbrace{Q(t)\rho_f c_p T_i(t) - Q(t)\rho_f c_p T(t)}_{(2)} + \underbrace{(-\Delta H)V\,[\frac{dC(t)}{dt}]\Big|_{reaction}}_{(3)} \underbrace{\pm h[T(t)]}_{(4)} \quad (3.2)$$

where

ρ_f=fluid density of the reacting mixture

c_p=specific heat of the reacting mixture

$T(t)$=temperature in the reactor

$T_i(t)$=inlet temperature

ΔH=heat of reaction (by thermodynamic convention, $\Delta H < 0$ for exothermic reactions, so that a negative sign is attached to the heat generation term.)

The respective terms are as follows:

(1) accumulation of total energy,

(2) the heat removed from the system through the difference in temperature between inlet and outlet streams,

(3) heat generated by reaction,

(4) $h[T(t)]$ represents external heat addition or removal from the reactor.

Heat Removal Driving Force: An energy balance for the heat transfer fluid gives

$$h[T(t)] = Q_c(t)\rho_c c_c[T_{cout}(t) - T_{cin}(t)] \tag{3.3}$$

and the heat transfer rate between the heat transfer fluid and the reactor is

$$h[T(t)] = A_K U[T(t) - \overline{T}_c(t)] = A_K U[T(t) - \frac{T_{cin}(t) + T_{cout}(t)}{2}] \tag{3.4}$$

where

A_K=heat exchange surface

U=overall heat transfer coefficient

$\overline{T}_c(t)$=average coolant temperature

$Q_c(t)$=coolant flow rate

ρ_c=fluid density of coolant

c_c=specific heat of fluid of coolant

$T_{cin}(t)$=inlet temperature of coolant

$T_{cout}(t)$=outlet temperature of coolant

Eliminating $T_{cout}(t)$ from Equation 3.3 and Equation 3.4 gives

$$h[T(t)] = A_K U[T(t) - \frac{T_{cin}(t) + \frac{h[T(t)]}{Q_c(t)\rho_c c_c} + T_{cin}(t)}{2}] \tag{3.5}$$

Rearranging, we get

$$h[T(t)] = \frac{2A_K U \rho_c c_c Q_c(t)[T(t) - T_{cin}(t)]}{A_K U + 2\rho_c c_c Q_c(t)} \tag{3.6}$$

Substituting Equation 3.6 into Equation 3.2, we have

$$\begin{aligned} V\rho_f c_p \frac{dT(t)}{dt} &= Q(t)\rho_f c_p[T_i(t) - T(t)] \\ &\quad +(-\Delta H)V\left[\frac{dC(t)}{dt}\right]\Big|_{reaction} \\ &\quad -\frac{2A_K U \rho_c c_c Q_c(t)[T(t) - T_{cin}(t)]}{A_K U + 2\rho_c c_c Q_c(t)} \end{aligned} \tag{3.7}$$

Reaction Rates: As is well known, chemical reactions may be classified in one of the following ways:

(1) on the basis of the number of molecules that must react to form the reaction product,

(2) on a kinetic basis by reaction order, or reaction mechanism.

In control of the CFSTR, the latter classification is needed to describe the kinetics of the reaction process and to model the dynamic characteristics of the system. The relationship among rate of reaction (r), concentration of reactant (C), and reaction order (n) can be simply given by the expression

$$r = -\left.\frac{dC(t)}{dt}\right|_{reaction} = KC^n(t) \quad \begin{cases} n = 0 \\ n = 1 \\ n = 2 \\ \vdots \end{cases} \tag{3.8}$$

where K is the reaction-rate constant which is a function of temperature. Arrhenius proposed that the effect of temperature on the reaction-rate constant in a chemical reaction may be described by Equation 3.9:

$$K = A_r e^{[-\frac{E}{RT(t)}]} \tag{3.9}$$

where A_r is the frequency factor, E is the activation energy of the reaction, R is the ideal gas constant, and T is the absolute temperature of the reacting mixture.

Now, substituting Equation 3.9 into Equation 3.8, and then substituting Equation 3.8 into both Equation 3.1 and Equation 3.7, we have

$$V\frac{dC(t)}{dt} = Q(t)[C_i(t) - C(t)] - VA_rC^n(t)e^{[-\frac{E}{RT(t)}]} \tag{3.10}$$

$$\begin{aligned} V\rho_f c_p \frac{dT(t)}{dt} &= Q(t)\rho_f c_p[T_i(t) - T(t)] \\ &\quad +(-\Delta H)VA_rC^n(t)e^{[-\frac{E}{RT(t)}]} \\ &\quad -\frac{2A_K U\rho_c c_c Q_c(t)[T(t) - T_{cin}(t)]}{A_K U + 2\rho_c c_c Q_c(t)} \end{aligned} \tag{3.11}$$

or

$$\frac{dC(t)}{dt} = \frac{Q(t)}{V}[C_i(t) - C(t)] - A_rC^n(t)e^{[-\frac{E}{RT(t)}]} \tag{3.12}$$

$$\begin{aligned} \frac{dT(t)}{dt} &= \frac{Q(t)}{V}[T_i(t) - T(t)] \\ &\quad +\frac{(-\Delta H)A_rC^n(t)e^{[-\frac{E}{RT(t)}]}}{\rho_f c_p} \\ &\quad -\frac{2A_K U\rho_c c_c Q_c(t)[T(t) - T_{cin}(t)]}{V\rho_f c_p[A_K U + 2\rho_c c_c Q_c(t)]} \end{aligned} \tag{3.13}$$

3.2 Transfer Function Representation of the CFSTR Response

Controller design is not based on specific physical or chemical behavior, but on a set of Laplace transformed differential equations called transfer functions. In fact, transfer functions can only be used to characterize the input-output relationships of linear systems. It is a well-known fact that many relationships among chemical processes are not linear. In fact, a careful study of chemical systems reveals that even so-called "linear systems" are really linear only in a limited operating ranges. For this system mass balance Equation 3.12 and energy balance Equation 3.13 are nonlinear due to the reaction rate term (Equation 3.8). In general, in solving a new problem, a simplified model should be built so that a general feeling can be got for the solution. A more complete mathematical model may then be built and used for a more complete analysis. Local linearization appears to be reasonable since most chemical processes are operated at a constant steady-state condition for extended periods of time. Disturbances and changes from normal operating conditions will occur, but they usually have a low amplitude.

This section presents a linearization technique applicable to the nonlinear equations of a CFSTR. In order to obtain a linear mathematical model for Equations 3.12 and 3.13, the following assumptions are made :

(1) the variables deviate only slightly from the normal steady-state operating conditions;

(2) all initial conditions are zero;

(3) the output variables (or controlled variables) are $C(t)$ and $T(t)$, the input variables are $Q(t)$ and $Q_c(t)$, and the disturbance variables are $C_i(t)$ and $T_i(t)$;

(4) the inlet heat transfer fluid temperature $T_{cin}(t)$ has been controlled, i.e., $T_{cin}(t) = T_{cin}$=constant.

If the normal steady-state operating condition of the CFSTR corresponds to C_0, Q_0, T_0, and Q_{c0}, and steady-state values of the disturbance variables are defined as C_{i0} and T_{i0}, then Equation 3.12 and Equation 3.13, which are quadratic functions respectively, may be expanded into a Taylor series about these points (see Appendix A) and the higher-order terms may be neglected. The linear mathematical model of nonlinear Equation 3.12 in the neighborhood of the normal operating condition is then given by

$$
\begin{aligned}
\frac{d\Delta C(t)}{dt} &= \frac{\Delta Q(t)}{V}[C_{i0} - C_0] + \frac{Q_0}{V}[\Delta C_i(t) - \Delta C(t)] - \Delta K C_0^n - K_0 n C_0^{n-1}\Delta C(t) \\
&= \frac{C_{i0} - C_0}{V}\Delta Q(t) \\
&\quad -[\frac{Q_0}{V} + K_0 n C_0^{n-1}]\Delta C(t) \\
&\quad -\frac{C_0^n E K_0}{R T_0^2}\Delta T(t) \\
&\quad +\frac{Q_0}{V}\Delta C_i(t)
\end{aligned} \tag{3.14}
$$

where

$$K_0 = A_r e^{-\frac{E}{RT_0}}$$

$$\Delta K = \frac{E K_0}{R T_0^2}\Delta T(t)$$

For Equation 3.13, similarly

$$
\begin{aligned}
\frac{d\Delta T(t)}{dt} &= \frac{\Delta Q(t)}{V}[T_{i0} - T_0] + \frac{Q_0}{V}[\Delta T_i(t) - \Delta T(t)] \\
&\quad + \frac{(-\Delta H)[\Delta K C_0^n + K_0 n C_0^{n-1}\Delta C(t)]}{\rho_f c_p} \\
&\quad - \frac{2A_K U \rho_c c_c}{V \rho_f c_p}[\frac{A_K U \Delta Q_c(t)(T_0 - T_{cin})}{(A_K U + 2\rho_c c_c Q_{c0})^2} + \frac{Q_{c0}\Delta T(t)}{A_K U + 2\rho_c c_c Q_{c0}}] \\
&= \frac{T_{i0} - T_0}{V}\Delta Q(t) \\
&\quad + \frac{(-\Delta H) K_0 n C_0^{n-1}}{\rho_f c_p}\Delta C(t) \\
&\quad + [\frac{(-\Delta H) C_0^n E K_0}{\rho_f c_p R T_0^2} - \frac{2 A_K U \rho_c c_c Q_{c0}}{V \rho_f c_p (A_K U + 2\rho_c c_c Q_{c0})} - \frac{Q_0}{V}]\Delta T(t) \\
&\quad - [\frac{2 A_K^2 U^2 \rho_c c_c (T_0 - T_{cin})}{V \rho_f c_p (A_K U + 2\rho_c c_c Q_{c0})^2}]\Delta Q_c(t) \\
&\quad + \frac{Q_0}{V}\Delta T_i(t)
\end{aligned} \tag{3.15}
$$

For simplicity, let

$$\alpha_0 = \frac{C_{i0} - C_0}{V} \tag{3.16}$$

$$\alpha_1 = \frac{Q_0}{V} + K_0 n C_0^{n-1} \tag{3.17}$$

$$\alpha_2 = \frac{C_0^n K_0 E}{R T_0^2} \tag{3.18}$$

$$\alpha_3 = \frac{Q_0}{V} \tag{3.19}$$

$$\beta_0 = \frac{T_0 - T_{i0}}{V} \tag{3.20}$$

$$\beta_1 = \frac{|\Delta H| C_0^n E K_0}{\rho_f c_p R T_0^2} + \frac{2 A_K U \rho_c c_c Q_{c0}}{V \rho_f c_p (A_K U + 2\rho_c c_c Q_{c0})} + \frac{Q_0}{V} \tag{3.21}$$

$$\beta_2 = \frac{|\Delta H| K_0 n C_0^{n-1}}{\rho_f c_p} \tag{3.22}$$

$$\beta_3 = \frac{2 A_K^2 U^2 \rho_c c_c (T_0 - T_{cin})}{V \rho_f c_p (A_K U + 2\rho_c c_c Q_{c0})^2} \tag{3.23}$$

Thus, Equation 3.14 and Equation 3.15 may be rewritten as

$$\frac{d\Delta C(t)}{dt} + \alpha_1 \Delta C(t) = \alpha_0 \Delta Q(t) - \alpha_2 \Delta T(t) + \alpha_3 \Delta C_i(t) \tag{3.24}$$

$$\frac{d\Delta T(t)}{dt} + \beta_1 \Delta T(t) = -\beta_0 \Delta Q(t) - \beta_2 \Delta C(t) - \beta_3 \Delta Q_c(t) + \alpha_3 \Delta T_i(t) \tag{3.25}$$

Note here that the Laplace transform of a increment function will be defined by $L[\Delta f(t)] = F(s)$.

Then, taking Laplace transform of each term in both Equation 3.24 and Equation 3.25, we obtain

$$(s + \alpha_1) C(s) = \alpha_0 Q(s) - \alpha_2 T(s) + \alpha_3 C_i(s) \tag{3.26}$$

$$(s + \beta_1) T(s) = -\beta_0 Q(s) - \beta_2 C(s) - \beta_3 Q_c(s) + \alpha_3 T_i(s) \tag{3.27}$$

To simplify the mathematical expressions of the system equations, it is advantageous to use matrix notation. For theoretical work, the notational simplicity gained by matrix operations is most convenient and is, in fact, essential for the analysis and synthesis of a multivariable system. Therefore, Equations 3.26 and 3.27 can be described in matrix form by

$$\begin{bmatrix} (s+\alpha_1) & \alpha_2 \\ \beta_2 & (s+\beta_1) \end{bmatrix} \begin{bmatrix} C(s) \\ T(s) \end{bmatrix} = \begin{bmatrix} \alpha_0 & 0 \\ -\beta_0 & -\beta_3 \end{bmatrix} \begin{bmatrix} Q(s) \\ Q_c(s) \end{bmatrix} + \begin{bmatrix} \alpha_3 & 0 \\ 0 & \alpha_3 \end{bmatrix} \begin{bmatrix} C_i(s) \\ T_i(s) \end{bmatrix} \tag{3.28}$$

Then, by premultiplying by the inverse of the matrix in Equation 3.28, we obtain

$$\begin{aligned} \begin{bmatrix} C(s) \\ T(s) \end{bmatrix} &= \frac{\begin{bmatrix} s+\beta_1 & -\alpha_2 \\ -\beta_2 & s+\alpha_1 \end{bmatrix}}{P(s)} \left\{ \begin{bmatrix} \alpha_0 & 0 \\ -\beta_0 & -\beta_3 \end{bmatrix} \begin{bmatrix} Q(s) \\ Q_c(s) \end{bmatrix} + \begin{bmatrix} \alpha_3 & 0 \\ 0 & \alpha_3 \end{bmatrix} \begin{bmatrix} C_i(s) \\ T_i(s) \end{bmatrix} \right\} \\ &= \frac{1}{P(s)} \left\{ \begin{bmatrix} \alpha_0(s+\beta_1)+\alpha_2\beta_0 & \alpha_2\beta_3 \\ -\alpha_0\beta_2-\beta_0(s+\alpha_1) & -\beta_3(s+\alpha_1) \end{bmatrix} \begin{bmatrix} Q(s) \\ Q_c(s) \end{bmatrix} \right. \\ &\quad \left. + \begin{bmatrix} \alpha_3(s+\beta_1) & -\alpha_2\alpha_3 \\ -\alpha_3\beta_2 & \alpha_3(s+\alpha_1) \end{bmatrix} \begin{bmatrix} C_i(s) \\ T_i(s) \end{bmatrix} \right\} \end{aligned} \tag{3.29}$$

where

$$P(s) = s^2 + (\alpha_1+\beta_1)s + \alpha_1\beta_1 - \alpha_2\beta_2 \tag{3.30}$$

$P(s)$ is the open-loop characteristic equation of the CFSTR system. According to Routh's stability criterion, all the coefficients in the characteristic equation must be positive. So,

$$\alpha_1\beta_1 > \alpha_2\beta_2 \tag{3.31}$$

Figure 3.2 shows the open-loop model for the CFSTR in block diagram form. Figure 3.3 indicates the block diagram of the closed-loop control of the CFSTR. Also, Equation 3.29 can be expressed as

$$\begin{bmatrix} C(s) \\ T(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} Q(s) \\ Q_c(s) \end{bmatrix} + \begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix} \begin{bmatrix} C_i(s) \\ T_i(s) \end{bmatrix} \tag{3.32}$$

where

$$G_{11}(s) = \frac{\alpha_0(s+\beta_1)+\alpha_2\beta_0}{P(s)} \tag{3.33}$$

$$G_{12}(s) = \frac{\alpha_2\beta_3}{P(s)} \tag{3.34}$$

$$G_{21}(s) = -\frac{\alpha_0\beta_2+\beta_0(s+\alpha_1)}{P(s)} \tag{3.35}$$

$$G_{22}(s) = -\frac{\beta_3(s+\alpha_1)}{P(s)} \tag{3.36}$$

$$D_{11}(s) = \frac{\alpha_3(s+\beta_1)}{P(s)} \tag{3.37}$$

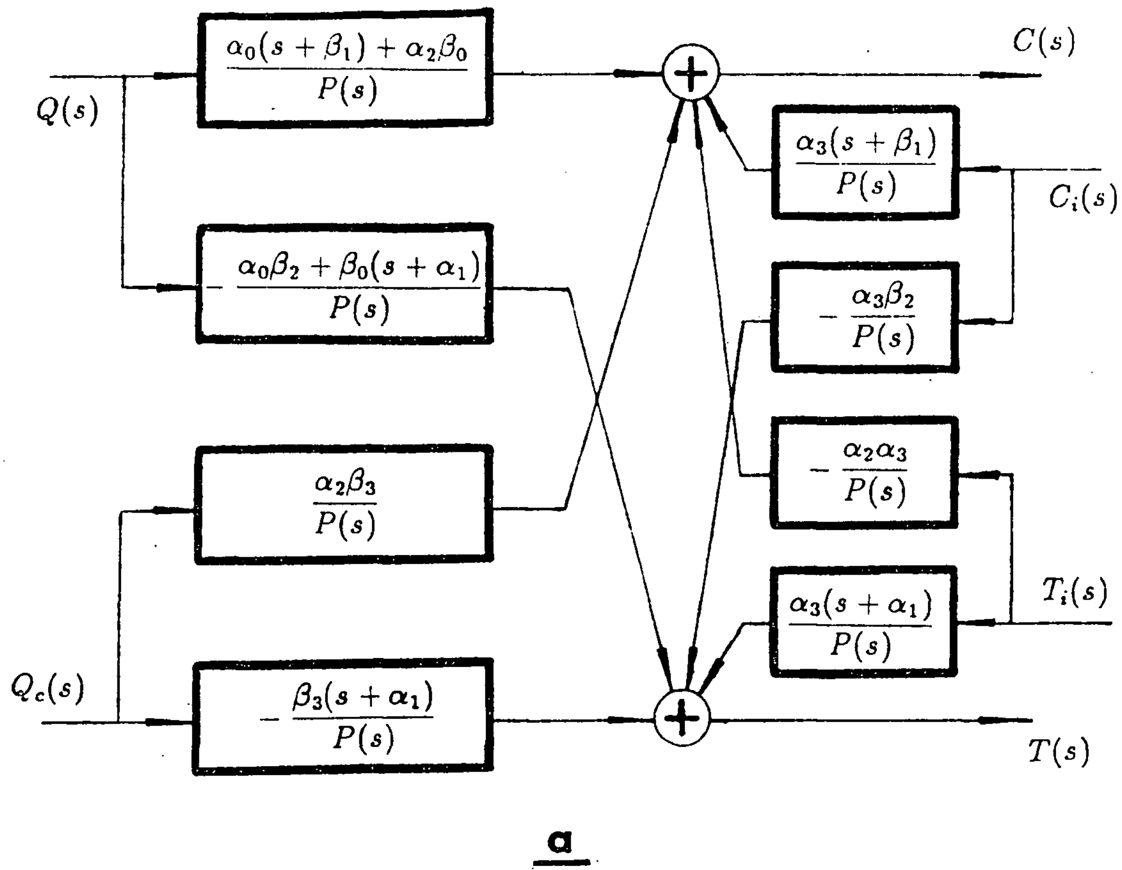
$$D_{12}(s) = -\frac{\alpha_2\alpha_3}{P(s)} \tag{3.38}$$

$$D_{21}(s) = -\frac{\alpha_3\beta_2}{P(s)} \tag{3.39}$$

$$D_{22}(s) = \frac{\alpha_3(s+\alpha_1)}{P(s)} \tag{3.40}$$

3.3 Time-delay Behaviour of the CFSTR Control System

Figure 1.1 illustrates a CFSTR in which the contents are mechanically agitated. The essential feature is the assumption of complete uniformity of concentration and temperature throughout the reactor. So, the CFSTR represents the extreme case of back mixing or longitudinal dispersion. More specifically, the vessel will have a characteristic throughput time $\bar{t}$ and there will be a characteristic time for mixing, t_{mix}. If the process time-delay is



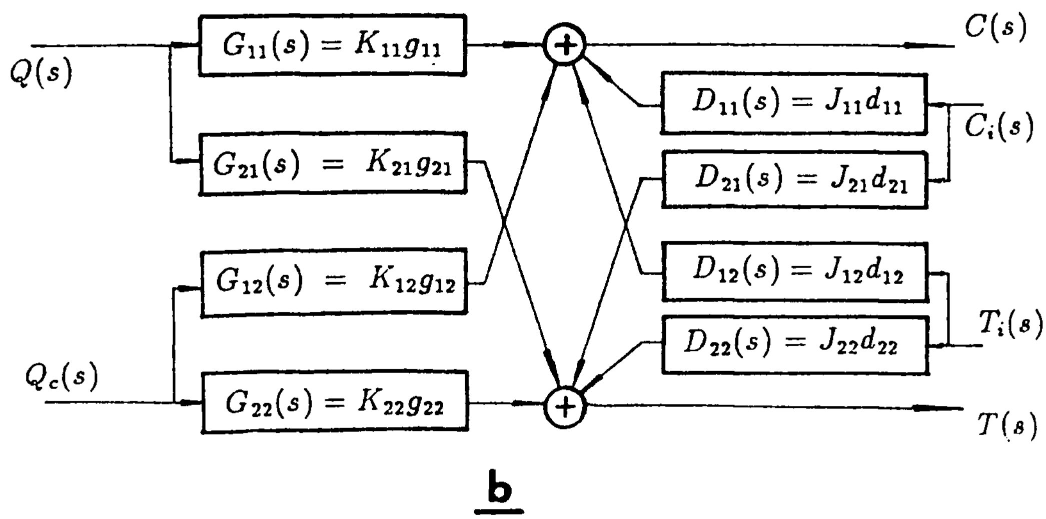


Figure 3.2: (a) Block diagram representing Equation 3.29. (b) A simple representation of (a). G_{ij} and D_{ij} are transfer functions of each channel; K_{ij} and J_{ij} are the steady-state gains of each channel; g_{ij} and d_{ij} are the dynamic gains of each channel.

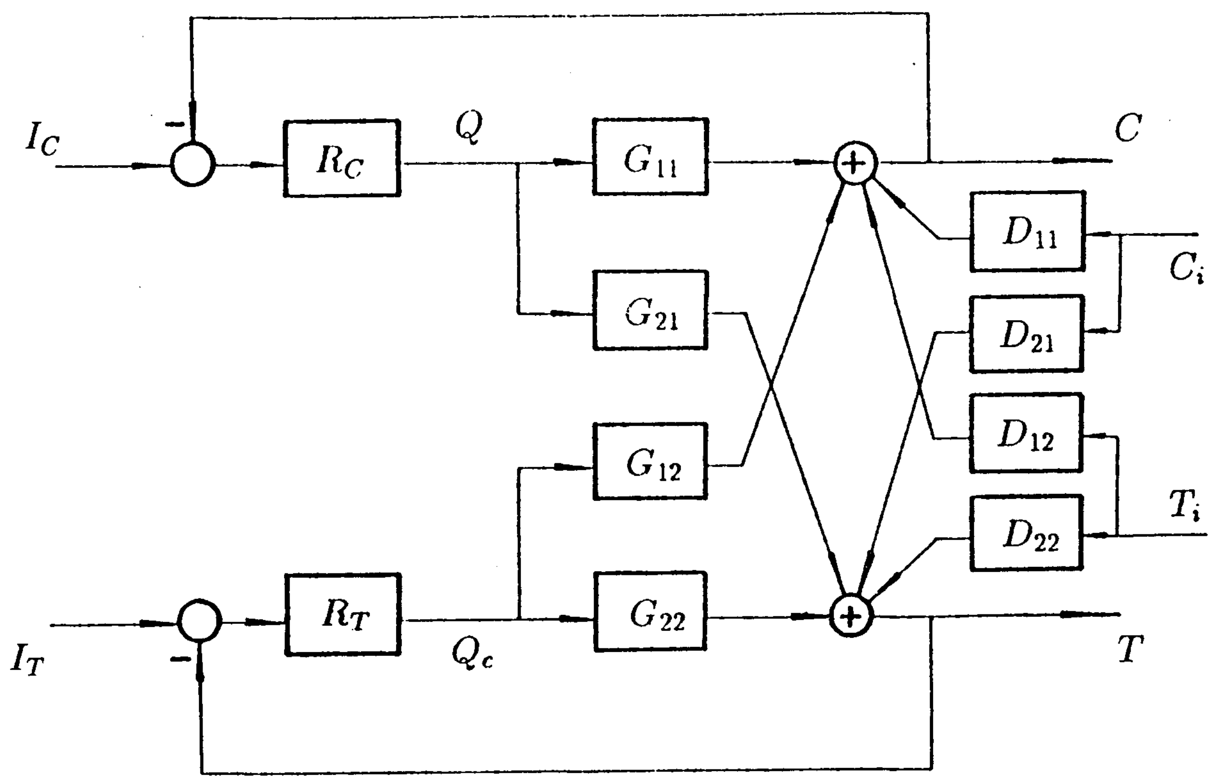


Figure 3.3: Closed-loop control system for the CFSTR, I_C is the concentration set point and I_T is the temperature set point.

considered, the mass balance and energy balance are described as distributed parameter equations, which is beyond the scope of this study. So, consider the case in which t_{mix} is much smaller than $\bar{t}$, thus CFSTR then has perfectly mixed characteristics. On the one hand, the mixing time t_{mix} can be assumed to be small for an ideal CFSTR, but, on the other hand, the time-delay behaviour for a closed-loop control system can still occur because the time-delay of a measuring device is unavoidable even if t_{mix} be zero, that is to say, a sensor's response is also a function of time. Therefore, from the point of view of process control theory, the feedback delay also implies that the CFSTR control system has time-delay behaviour.

Time-delay is defined as the time interval between the initiation of an action and the first observation of a result. It is caused by transportation of material from the point of manipulation to the point of detection. The concentration control loop will contain a time-delay, since the ions or molecules which are sensed by the measuring device must be transported to that point by a flowing stream. Like concentration control, the temperature control loop also has time-delay because heat is transferred both by convection and by conduction, and it is impossible to transport heat from the wall of the vessel to the temperature sensor in zero time.

Time-delay can be measured and expressed in Laplace transform form shown in Appendix B. There is no attenuation or filtering for time-delay behaviour. Since time-delay does not change the magnitude or form of the signal, its gain is unity, and may be left out of any gain-product calculation.

The feedback process containing time-delay produces no immediately observable effect; hence control action of the CFSTR is unavoidably delayed. For this reason, consider that the time-delay behaviour occurs in the feedback channel. Thus, the familiar mass control loop and energy control loop must next be modified to include the time-delay. This modification is shown in Figure 3.4 by the transport lag elements in both feedback loops. Finally, the closed-loop transfer function whose block diagram is shown in Figure 3.4 is expressed as:

$$\begin{bmatrix} C(s) \\ T(s) \end{bmatrix} = \frac{\begin{bmatrix} G_{c11}(s) & G_{c12}(s) \\ G_{c21}(s) & G_{c22}(s) \end{bmatrix} \begin{bmatrix} I_C(s) \\ I_T(s) \end{bmatrix} + \begin{bmatrix} D_{c11}(s) & D_{c12}(s) \\ D_{c21}(s) & D_{c22}(s) \end{bmatrix} \begin{bmatrix} C_i(s) \\ T_i(s) \end{bmatrix}}{P_c(s)} \tag{3.41}$$

where

$$G_{c11}(s) = [1 + e^{-\tau_T s} R_T(s) G_{22}(s)] R_C(s) G_{11}(s) - e^{-\tau_T s} R_C(s) R_T(s) G_{12}(s) G_{21}(s) \tag{3.42}$$

$$G_{c12}(s) = [1 + e^{-\tau_T s} R_T(s) G_{22}(s)] R_T(s) G_{12}(s) - e^{-\tau_T s} R_T^2(s) G_{12}(s) G_{22}(s) \tag{3.43}$$

$$G_{c21}(s) = [1 + e^{-\tau_C s} R_C(s) G_{11}(s)] R_C(s) G_{21}(s) - e^{-\tau_C s} R_C^2(s) G_{11}(s) G_{21}(s) \tag{3.44}$$

$$G_{c22}(s) = [1 + e^{-\tau_C s} R_C(s) G_{11}(s)] R_T(s) G_{22}(s) - e^{-\tau_C s} R_C(s) R_T(s) G_{12}(s) G_{21}(s) \tag{3.45}$$

$$P_c(s) = [1+e^{-\tau_C s} R_C(s) G_{11}(s)][1+e^{-\tau_T s} R_T(s) G_{22}(s)] - e^{-(\tau_C+\tau_T)s} R_C(s) R_T(s) G_{12}(s) G_{21}(s) \tag{3.46}$$

τ_C is the time-delay of concentration feedback,

τ_T is the time-delay of temperature feedback,

$R_C(s)$ is the transfer function of effluent concentration controller,

$R_T(s)$ is the transfer function of reactor temperature controller,

$I_C(s)$ is the concentration set point,

$I_T(s)$ is the temperature set point.

Details for Equations 3.42, 3.43, 3.44, 3.45 and 3.46 are provided in Appendix C.

Clearly, if the decoupling design and time-delay compensation are not considered, Equation 3.41 will result in a complex control algorithm.

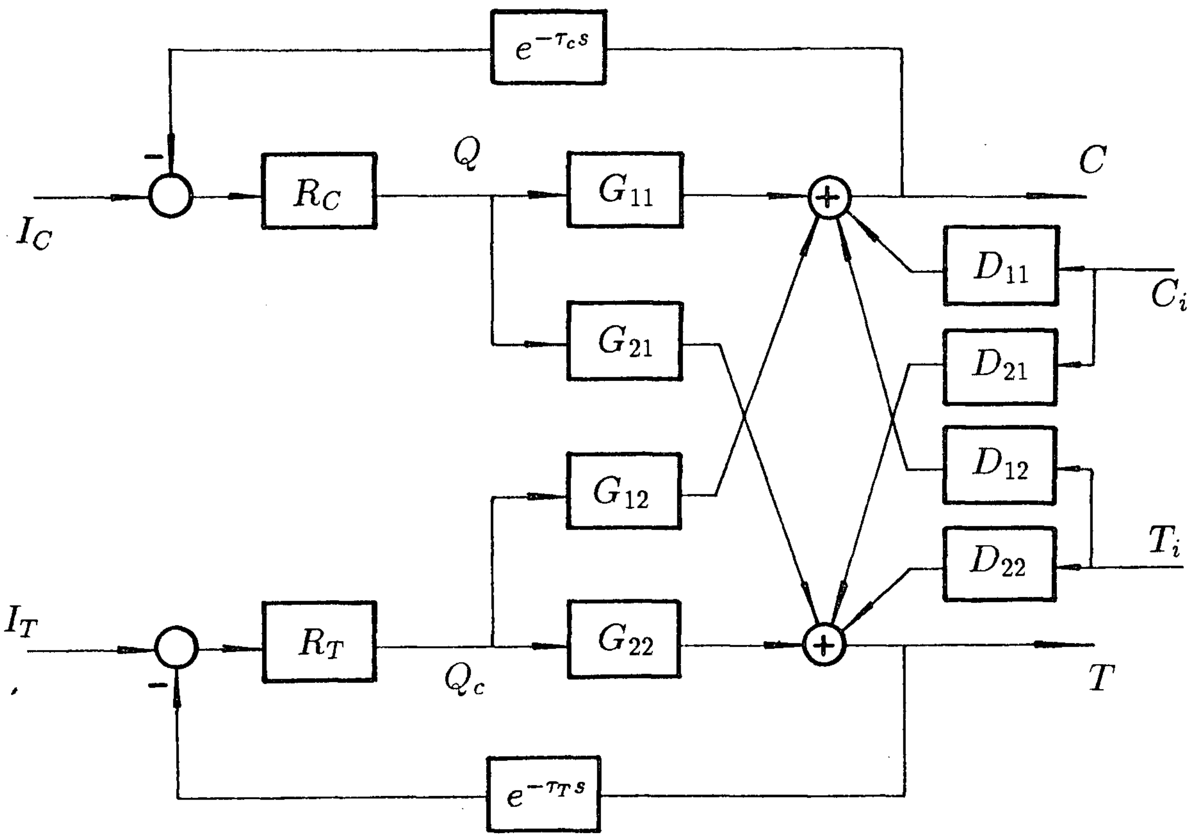


Figure 3.4: Closed-loop control system for the CFSTR with time-delay

Chapter 4

Determination of Interaction Degree of Two Control Loops

4.1 Interaction Behaviour of a CFSTR

It is important for a CFSTR control system designer to be aware of the effects of the parasitic modes of the system even though they are not explicitly modelled. A related effect occurs when simplified models are used to design controllers for complex systems in which several variables are to be controlled. When a change in one loop's manipulated variable causes a change in some other loop's controlled variable, the control loops are said to be coupling. If, in addition to the coupling from the first loop to the second loop, there is coupling from the second loop back to the first loop, then interaction exists.

In the CFSTR of Figure 1.1, the reactor temperature $T(t)$ and effluent concentration $C(t)$ are used as the controlled variables while the cooling water flow rate $Q_c(t)$ and stream flow rate $Q(t)$ are manipulated variables to regulate $T(t)$ and $C(t)$, respectively. Equations 3.26 and 3.27 form the basic model of a CFSTR process control system. At first glance, Equation 3.26 seems to be uncoupled from the heat exchanger system, but the temperature variable in Equation 3.27 is a function of concentration $C(t)$, coolant flow rate $Q_c(t)$, and initial temperature $T_i(t)$. Therefore the input variables $Q_c(t)$ and $T_i(t)$ for the reactor temperature subsystem appear in the mass balance Equation 3.26. Stephanopoulos (1984) has described interaction for a CFSTR in dynamic operation. The concentration feedback control loop can compensate for changes which are caused by variations in either inlet concentration $C_i(t)$ or the desired effluent concentration $C(t)$, or both of them. The controller R_C in the feedback control will regulate for these changes by manipulating the feed flow rate. However, this change in the feed rate also disturbs the reactor temperature. The temperature feedback control loop attempts to compensate for the change in temperature by varying the coolant flow rate, which in turn effects the effluent concentration. On the other hand, attempts to compensate for changes in feed temperature or the desired set point of reactor temperature, may also causes the effluent concentration to vary. Then the concentration loop attempts to compensate for the change in effluent concentration by varying the feed rate, which in turn disturbs the reactor temperature. This interaction can cause oscillations and even instability.

4.2 Bristol Method

4.2.1 Introduction

The control loops of a CFSTR control system can not be considered separately because of the existence of coupling. Thus setting the controller's parameters to produce good control always becomes a difficult problem. Interaction analysis can help provide answers to the following questions:

(1) Can the degree of interaction be determined analytically?

(2) Is there any possibility that the interaction can be neglected? or, can a CFSTR be designed to be easily controllable ?

(3) What is an ideal or simplified decoupling control design?

(4) What is the effect if the decoupling model is in error?

4.2.2 Definition of the Relative Gain

By for the most important, practical, and widely used interaction analysis technique is the relative gain array (RGA) proposed by Bristol (1966) who offered an attractive means of avoiding complex analysis of a multivariable system. The chief advantages of the RGA approach are that it is easy to use and only requires a crude process model, namely, the process gains which can be determined from steady-state information. Before taking up the subject of the RGA analysis for the CFSTR system, it is necessary to review some definitions of the RGA.

Bristol defined a set of open-loop gain coefficients F_{ij} and closed-loop gain coefficients S_{ij} for a multivariable system, where subscript i refers to the controlled variable and subscript j to the manipulated variable. Now, consider a 2×2 system (see Figure 4.5), the definitions of F_{ij} are as follows:

$$F_{11} = \left.\frac{\partial Y_1}{\partial M_1}\right|_{M_2=constant} \tag{4.47}$$

$$F_{12} = \left.\frac{\partial Y_1}{\partial M_2}\right|_{M_1=constant} \tag{4.48}$$

$$F_{21} = \left.\frac{\partial Y_2}{\partial M_1}\right|_{M_2=constant} \tag{4.49}$$

$$F_{22} = \left.\frac{\partial Y_2}{\partial M_2}\right|_{M_1=constant} \tag{4.50}$$

where M_j is a manipulated variable and Y_i is a controlled variable.

The definition of S_{ij} is the open-loop gain evaluated with all other controlled variables constant (see Figure 4.6). Expressions are as follows:

$$S_{11} = \left.\frac{\partial Y_1}{\partial M_1}\right|_{Y_2=constant} \tag{4.51}$$

$$S_{12} = \left.\frac{\partial Y_1}{\partial M_2}\right|_{Y_2=constant} \tag{4.52}$$

$$S_{21} = \left.\frac{\partial Y_2}{\partial M_1}\right|_{Y_1=constant} \tag{4.53}$$

$$S_{22} = \left.\frac{\partial Y_2}{\partial M_2}\right|_{Y_1=constant} \tag{4.54}$$

The relative gain for the assumed pairing is defined as the ratio:

$$\lambda_{11} = \frac{F_{11}}{S_{11}} \tag{4.55}$$

$$\lambda_{12} = \frac{F_{12}}{S_{12}} \tag{4.56}$$

$$\lambda_{21} = \frac{F_{21}}{S_{21}} \tag{4.57}$$

$$\lambda_{22} = \frac{F_{22}}{S_{22}} \tag{4.58}$$

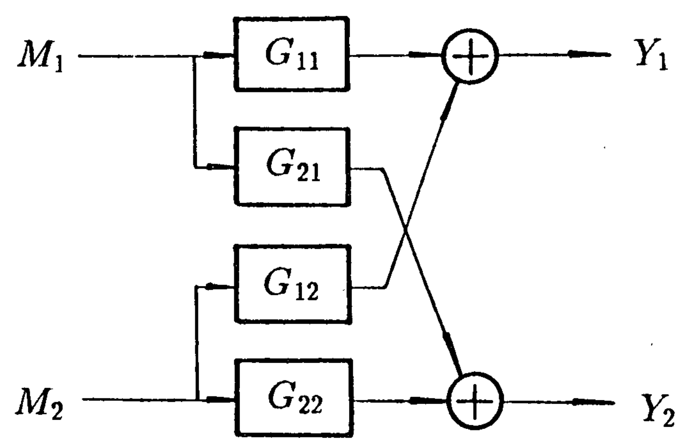


Figure 4.5: Open-loop of a 2×2 system, G_{ij} is the transfer function of each channel.

So, the relative gain is often written as

$$
\begin{aligned}
\lambda_{ij} &= \frac{\left.\frac{\partial Y_i}{\partial M_j}\right|_{M=constant}}{\left.\frac{\partial Y_i}{\partial M_j}\right|_{Y=constant}} \\
&= \frac{F_{ij}}{S_{ij}}
\end{aligned}
\tag{4.59}
$$

λ_{ij} in this case is the measure of interaction of four channels in a 2×2 system. Arrange the four relative gains into a matrix form, which is known as the relative gain array (RGA).

$$
\begin{array}{cc}
 & \begin{array}{cc} M_1 & M_2 \end{array} \\
\begin{array}{c} Y_1 \\ Y_2 \end{array} & \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}
\end{array}
\tag{4.60}
$$

One property of the relative gain array is the relative gains in each column and row add up to unity (See Appendix D), that is

$$\lambda_{11} + \lambda_{12} = 1$$

$$\lambda_{21} + \lambda_{22} = 1$$

$$\lambda_{11} + \lambda_{21} = 1$$

$$\lambda_{12} + \lambda_{22} = 1$$

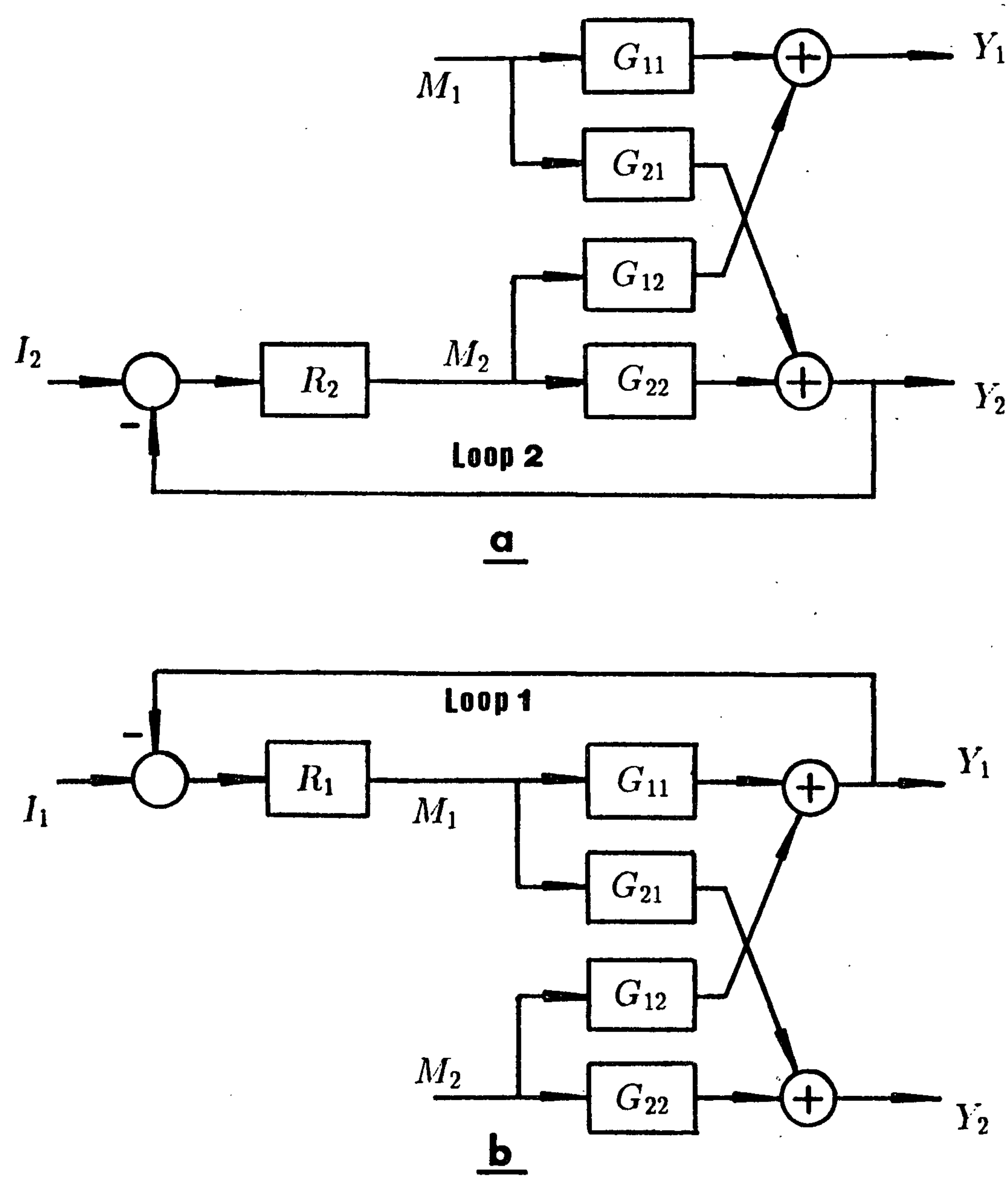


Figure 4.6: Determination of closed-loop gain for a 2×2 system. R_1 and R_2 are controllers. I_1 and I_2 are set-points. (a) Determination of S_{11} and S_{12}; (b) Determination of S_{21} and S_{22}.

4.2.3 Interpretation of the Relative Gain Value

The relative gain λ_{11} represents all the information about the interaction in a 2×2 interacting process. If λ_{11} is known, the other three relative gains can be determined. This is an important property of the relative gain matrix for a 2×2 system (Details are provided in Appendix D),

λ_{11} can take on any value.

- If $\lambda_{11} < 0$, then M_2 cause a strong negative effect on Y_1. In this case, the interaction effect is very dangerous.
- If $\lambda_{11} = 0$, then Y_1 does not respond to M_1 and M_1 should not be used to control Y_1.
- If $\lambda_{11} = 0.5$, then interaction between the two loops is the same.
- If $\lambda_{11} = 1$, then a 2×2 system has two noninteracting control loops, i.e. either loop does not affect the other loop.
- If $\lambda_{11} \gg 1$, then both variables cannot be controlled at the same time.

In order to understand the value of λ_{11} as a measure of interaction in a 2×2 system, Shinskey (1979 and 1988) presented several figures for different values of λ_{11} which illustrate the change of the system's dynamic characteristics in open and closed loop step response due to an interaction effect. These figures are duplicated as Figure 4.7, Figure 4.8, Figure 4.9, and Figure 4.10. Experience has shown that if λ_{11} falls between 0.7 and 1.5 (McAvoy, 1983), then the channel $M_1 \rightarrow Y_1$ (or $M_2 \rightarrow Y_2$) is influenced only slightly by other channels, that is to say, the interaction can be neglected.

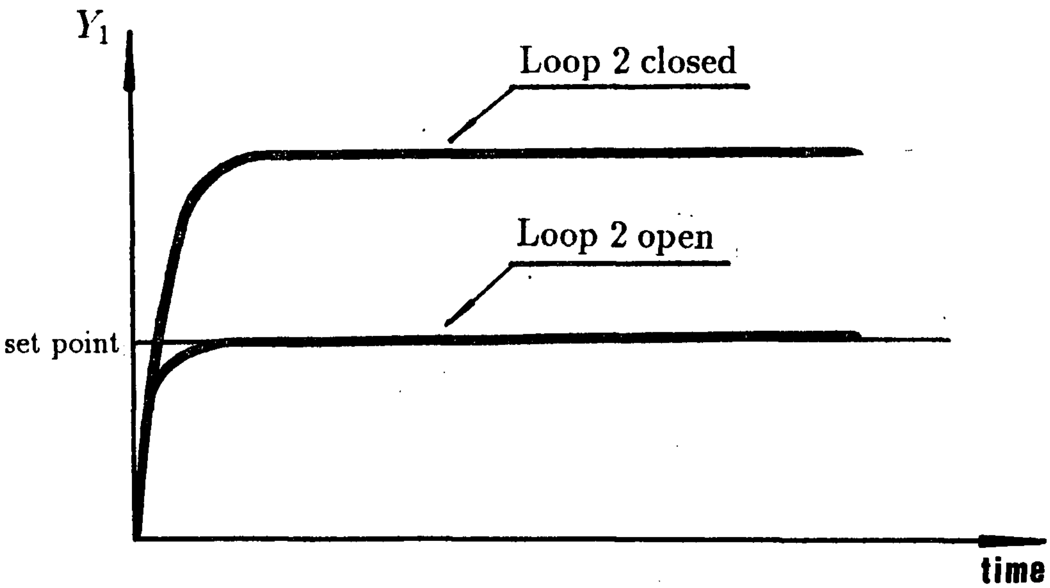


Figure 4.7: Step response of loop 1 open for a 2×2 system with λ_{11}=0.5.

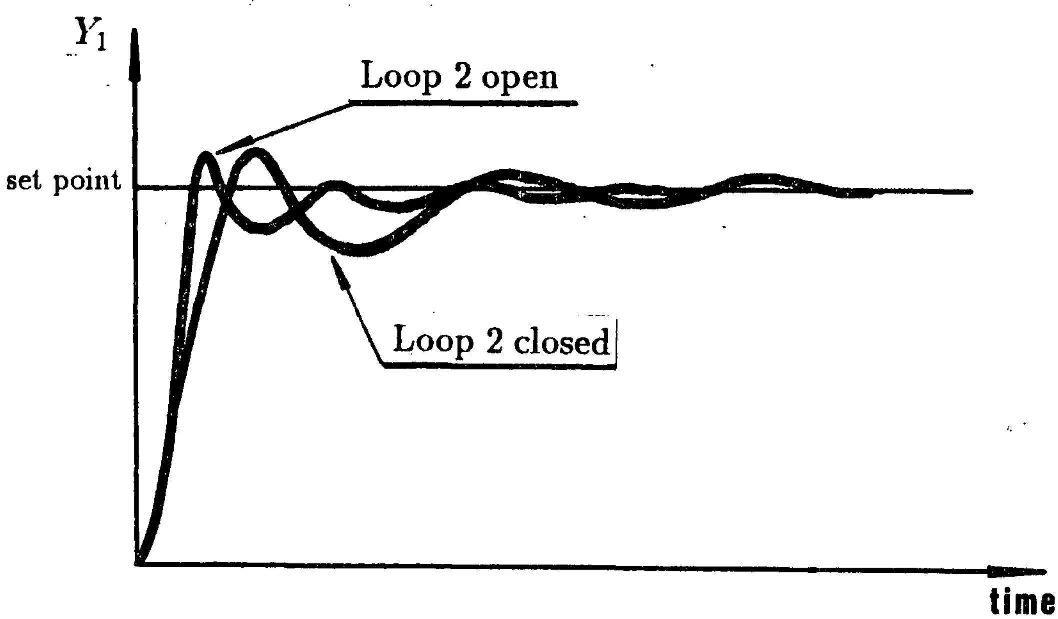


Figure 4.8: Step response of loop 1 closed for a 2×2 system with λ_{11}=0.5.

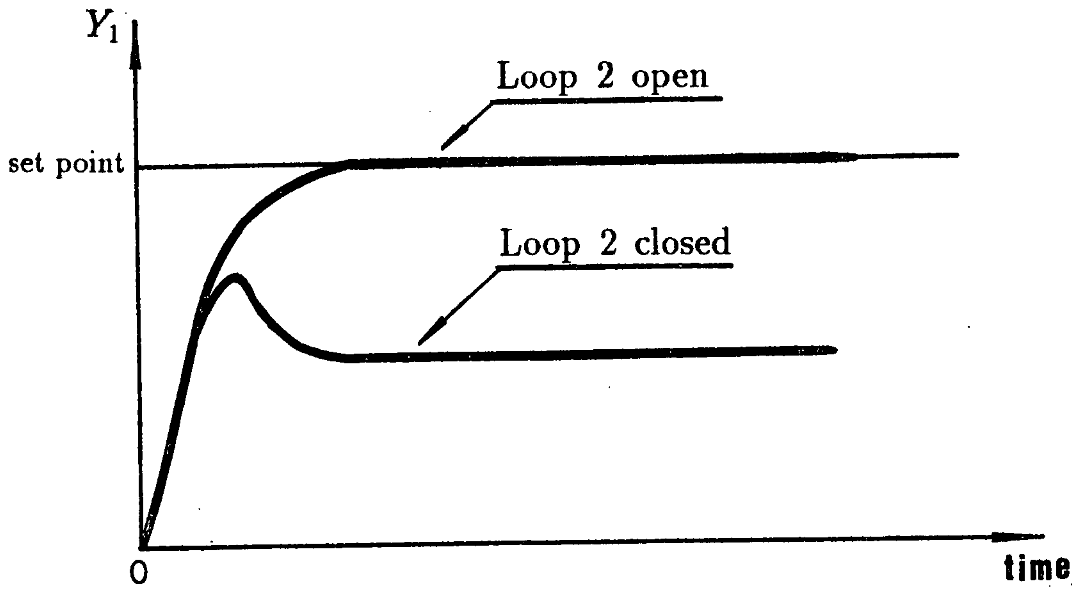


Figure 4.9: Step response of loop 1 open for a 2×2 system with λ_{11}=2.0.

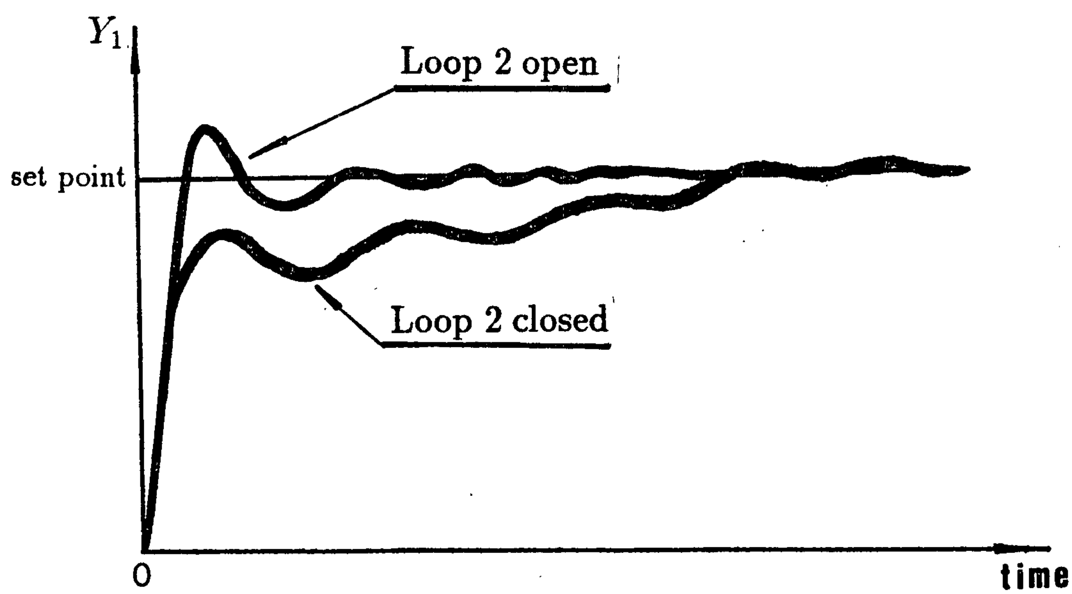


Figure 4.10: Step response of loop 1 closed for a 2×2 system with λ_{11}=2.0.

4.3 Determination of the RGA for the CFSTR

4.3.1 Determination of the Open-loop Gain Coefficient of the CFSTR

The open-loop gain coefficient is nothing else but the steady-state gain K_{ij} of the channel (a manipulated variable $\rightarrow$ a controlled variable) when only this channel is under operation and other channels are open. So,

for $Q(t) \rightarrow C(t)$

$$F_{11} = \left.\frac{\partial C}{\partial Q}\right|_{Q_c=constant} = \frac{\alpha_0\beta_1 + \alpha_2\beta_0}{\alpha_1\beta_1 - \alpha_2\beta_2} \tag{4.61}$$

for $Q_c(t) \rightarrow C(t)$

$$F_{12} = \left.\frac{\partial C}{\partial Q_c}\right|_{Q=constant} = \frac{\alpha_2\beta_3}{\alpha_1\beta_1 - \alpha_2\beta_2} \tag{4.62}$$

for $Q(t) \rightarrow T(t)$

$$F_{21} = \left.\frac{\partial T}{\partial Q}\right|_{Q_c=constant} = -\frac{\alpha_0\beta_2 + \alpha_1\beta_0}{\alpha_1\beta_1 - \alpha_2\beta_2} \tag{4.63}$$

for $Q_c(t) \rightarrow T(t)$

$$F_{22} = \left.\frac{\partial T}{\partial Q_c}\right|_{Q=constant} = -\frac{\alpha_1\beta_3}{\alpha_1\beta_1 - \alpha_2\beta_2} \tag{4.64}$$

By the principle of superposition, the system output is a sum of all input effects. So, writing Equations 4.61, 4.62, 4.63, and 4.64 in matrix form gives

$$\begin{bmatrix} \partial C \\ \partial T \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} \partial Q \\ \partial Q_c \end{bmatrix} \tag{4.65}$$

4.3.2 Determination of the Closed-loop Gain Coefficient of the CFSTR

From the definitions given in Equations 4.51, 4.52, 4.53, and 4.54, the method for determining the closed-loop gain coefficients is easier said than done. For most processes, to measure a gain in one channel while the other channel outputs always remain constant is out of the question. The study reported in this section is an attempt to determine the closed-loop gain coefficients from the open-loop gain coefficients.

From Figure 4.11, consider a change in the manipulated variable $Q(t)$ which is the compound result of effects from changes in the controlled variables $C(t)$ and $T(t)$, then

$$\partial Q = L_{11}\partial C + L_{12}\partial T \tag{4.66}$$

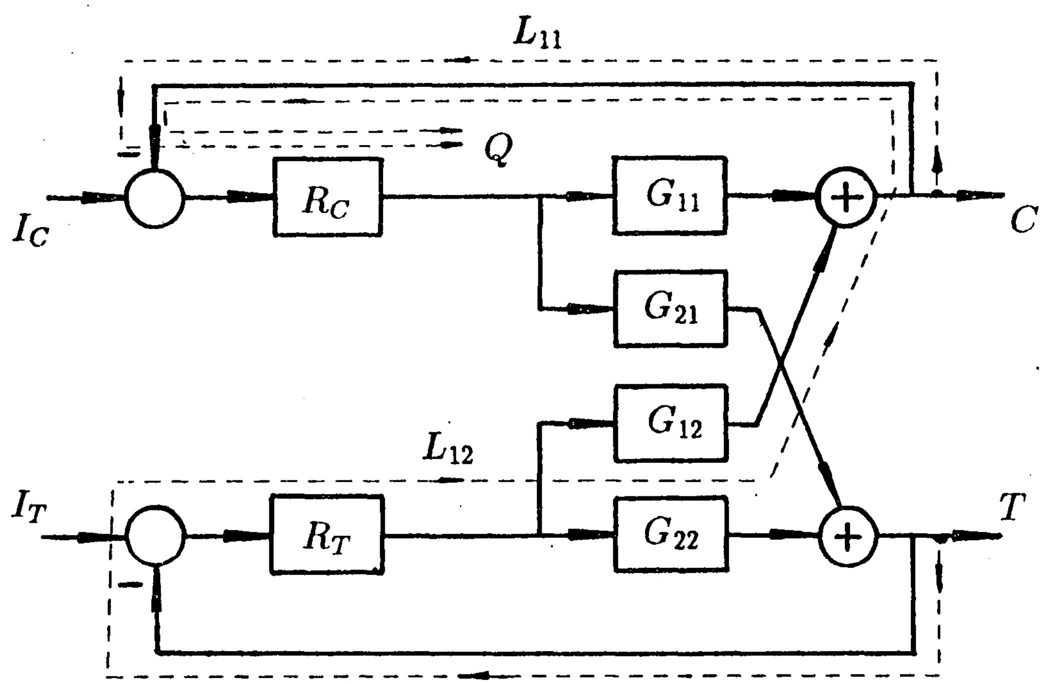


Figure 4.11: Determination of the manipulated variable $Q(t)$ from controlled variables $C(t)$ and $T(t)$.

Similarly, $Q_c(t)$ is also the compound result of changes to $C(t)$ and $T(t)$ (See Figure 4.12). Then

$$\partial Q_c = L_{21}\partial C + L_{22}\partial T \tag{4.67}$$

where L_{11}, L_{12}, L_{21} and L_{22} are assumed channel gains. The ∂ quantities refer to the value of the increment.

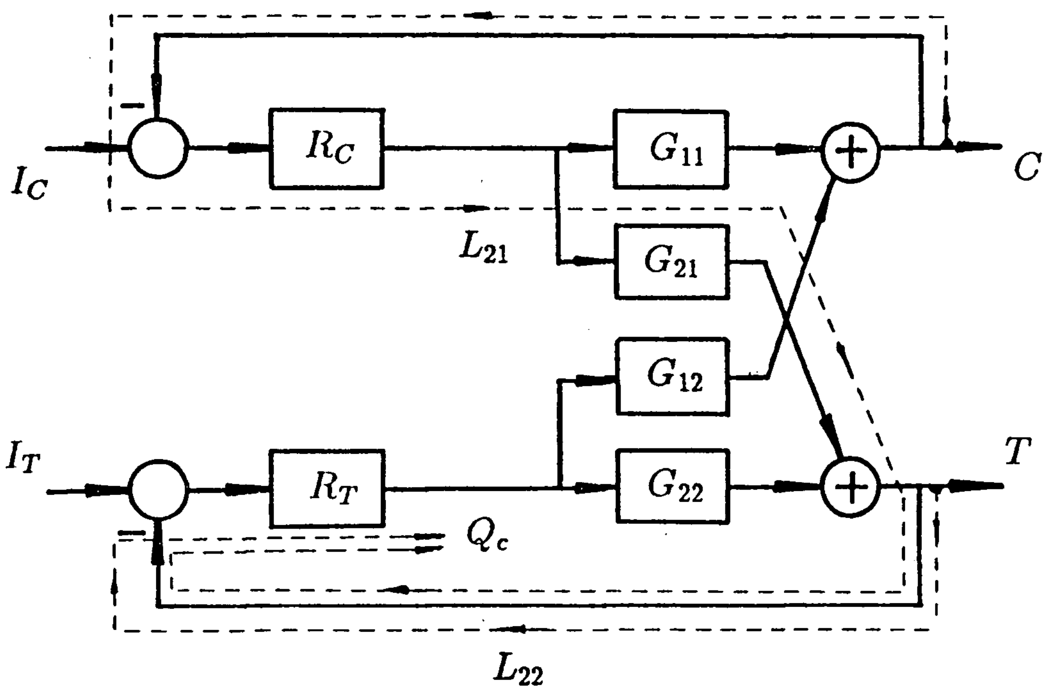


Figure 4.12: Determination of the manipulated variable $Q_c(t)$ from controlled variables $C(t)$ and $T(t)$.

If $\partial T = 0$, we have

$$\left.\frac{\partial C}{\partial Q}\right|_{T=constant} = \frac{1}{L_{11}} \tag{4.68}$$

$$\left.\frac{\partial C}{\partial Q_c}\right|_{T=constant} = \frac{1}{L_{21}} \tag{4.69}$$

If $\partial C = 0$, we have

$$\left.\frac{\partial T}{\partial Q}\right|_{C=constant} = \frac{1}{L_{12}} \tag{4.70}$$

$$\left.\frac{\partial T}{\partial Q_c}\right|_{C=constant} = \frac{1}{L_{22}} \tag{4.71}$$

Equations 4.68, 4.69, 4.70, and 4.71 are just the definitions of the closed-loop gain coefficients. By the principle of superposition, the matrix form of Equations 4.68, 4.69, 4.70 and 4.71 is as follows:

$$\begin{bmatrix} \partial Q \\ \partial Q_c \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} \partial C \\ \partial T \end{bmatrix} \tag{4.72}$$

Eliminating ∂Q and ∂Q_c from Equation 4.65 and 4.72 gives

$$\begin{bmatrix} \partial C \\ \partial T \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} \partial C \\ \partial T \end{bmatrix} \tag{4.73}$$

So

$$\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{4.74}$$

Solving for L_{11}, L_{12}, L_{21}, and L_{22} from Equation 4.74

$$L_{11} = \frac{F_{22}}{F_{11}F_{22} - F_{12}F_{21}} \tag{4.75}$$

$$L_{12} = -\frac{F_{12}}{F_{11}F_{22} - F_{12}F_{21}} \tag{4.76}$$

$$L_{21} = -\frac{F_{21}}{F_{11}F_{22} - F_{12}F_{21}} \tag{4.77}$$

$$L_{22} = \frac{F_{11}}{F_{11}F_{22} - F_{12}F_{21}} \tag{4.78}$$

Therefore, the closed-loop gain coefficients are

$$S_{11} = \frac{1}{L_{11}} = \frac{F_{11}F_{22} - F_{12}F_{21}}{F_{22}} \tag{4.79}$$

$$S_{12} = \frac{1}{L_{21}} = -\frac{F_{11}F_{22} - F_{12}F_{21}}{F_{21}} \tag{4.80}$$

$$S_{21} = \frac{1}{L_{12}} = -\frac{F_{11}F_{22} - F_{12}F_{21}}{F_{12}} \tag{4.81}$$

$$S_{22} = \frac{1}{L_{22}} = \frac{F_{11}F_{22} - F_{12}F_{21}}{F_{11}} \tag{4.82}$$

Then, λ_{11} which is defined as the relative gain of channel $Q(t) \rightarrow C(t)$ is found to be:

$$\lambda_{11} = \frac{F_{11}}{S_{11}} = \frac{F_{11}F_{22}}{F_{11}F_{22} - F_{12}F_{21}} \tag{4.83}$$

Now, substituting Equations 4.61, 4.62, 4.63, and 4.64 into Equation 4.83, we get

$$\begin{aligned}
\lambda_{11} &= \frac{-(\alpha_0\beta_1 + \alpha_2\beta_0)\alpha_1\beta_3}{[-(\alpha_0\beta_1 + \alpha_2\beta_0)\alpha_1\beta_3] + [\alpha_2\beta_3(\alpha_0\beta_2 + \alpha_1\beta_0)]} \\
&= \frac{\alpha_1(\alpha_0\beta_1 + \alpha_2\beta_0)}{\alpha_0(\alpha_1\beta_1 - \alpha_2\beta_2)} \\
&= \frac{\alpha_0\alpha_1\beta_1 + \alpha_1\alpha_2\beta_0}{\alpha_0\alpha_1\beta_1 - \alpha_0\alpha_2\beta_2}
\end{aligned} \tag{4.84}$$

Now, a simple yet important expression for the analysis of interaction of CFSTR has been derived as Equation 4.84. It has been shown how that the relative gain value λ_{11} depends on the process parameters.

At first glance, the relative gain value λ_{11} is seen to be greater than or equal to one because all parameters (α_i and β_i) are greater than zero and $\alpha_1\beta_1$ and $\alpha_2\beta_2$ are subject to the inequality (Equation 3.31) constraint.

An interaction analysis is presented in which the relative gain value λ_{11} is a function of both the system design parameters and the process parameters which influence to a large degree the interaction between the two control loops. The next section will illustrate two examples which indicate that the relative gain of a CFSTR responds to the system design parameters and to the process parameters.

4.4 Illustrative Examples

Example 1: As an example of a nonisothermal CFSTR, consider the design given by Douglas (1965, 1972). The values of the design parameters are given in Table 4.1 (page 50). Despite the fact that this does not correspond to a case where there is an optimum noninteracting design for a two-variable control system, it does provided a set of classical parameters for a general study.

According to Table 4.1 and Equations 3.16, 3.17, 3.18, 3.19, 3.21, and 3.22, the parameters α_0, α_1, α_2, β_0, β_1, and β_2 can be found as

$$\alpha_0 = \frac{0.0065 - 15.31 \times 10^{-5}}{1000} = 6.347 \times 10^{-6}$$

$$\alpha_1 = \frac{10}{1000} + 0.4145 = 0.4245$$

$$\alpha_2 = \frac{15.31 \times 10^{-5} \times 0.415 \times 28000}{1.987 \times 460.91^2} = 4.215 \times 10^{-6}$$

$$\beta_0 = \frac{460.91 - 350}{1000} = 0.1109$$

$$\beta_1 = \frac{27000 \times 15.31 \times 10^{-5} \times 28000 \times 0.415}{1.987 \times 460.91^2} + \frac{2 \times 10 \times 5}{1000(10 + 2 \times 5)} + \frac{10}{1000} = 0.129$$

$$\beta_2 = 27000 \times 0.415 = 11205$$

then

$$\begin{aligned}\lambda_{11} &= \frac{\alpha_1(\alpha_0\beta_1 + \alpha_2\beta_0)}{\alpha_0(\alpha_1\beta_1 - \alpha_2\beta_2)} \\ &= \frac{0.4245(6.347 \times 10^{-6} \times 0.129 + 4.215 \times 10^{-6} \times 0.1109)}{6.347 \times 10^{-6}(0.4245 \times 0.129 - 4.215 \times 10^{-6} \times 11205)} \\ &= 11.3\end{aligned}$$

Evidently, the calculation of λ_{11} is not difficult. In this example, λ_{11} is much greater than one. Therefore, when the designer is confronted with the control of both the effluent concentration and the reactor temperature, he or she should introduce a design of decoupling control.

Example 2: Another example is quoted from Nakanishi and Ohtani (1986). The design specifications for steady-state operation of their CFSTR are given in Table 4.2. In this case, the average coolant temperature is given as $301^o K$, while the inlet coolant temperature was not given. Assuming the average coolant temperature $T_{cin} = 20^o C = 293.15^o K$, the relative gain may be calculated.

$$\alpha_0 = \frac{5 - 1.07}{2 \times 10^{-3}} = 1965$$

$$\alpha_1 = \frac{10^{-5}}{2 \times 10^{-3}} + 0.186 = 0.0236$$

$$\alpha_2 = \frac{1.07 \times 0.0186 \times 9.41 \times 10^4}{8.314 \times 336.1^2} = 0.00199$$

$$\beta_0 = \frac{336.1 - 301}{2 \times 10^{-3}} = 17550$$

$$\begin{aligned}\beta_1 &= \frac{4.18 \times 10^4 \times 1.07 \times 9.41 \times 10^4 \times 0.0186}{10^3 \times 4.18 \times 8.314 \times 336.1^2} \\ &\quad + \frac{2 \times 5.67 \times 10^3 \times 1.78 \times 10^{-7}}{2 \times 10^3(5.67 \times 10^{-3} + 2 \times 10^3 \times 4.18 \times 1.78 \times 10^{-7})} \\ &\quad + \frac{10^{-5}}{2 \times 10^{-3}} \\ &= 0.0202\end{aligned}$$

$$\beta_2 = \frac{4.18 \times 10^4 \times 0.0186}{1000 \times 4.18} = 0.1864$$

then

$$\lambda_{11} = \frac{\alpha_1(\alpha_0\beta_1 + \alpha_2\beta_0)}{\alpha_0(\alpha_1\beta_1 - \alpha_2\beta_2)}$$

$$
\begin{aligned}
&= \frac{0.0236(1965 \times 0.0202 + 0.00199 \times 17550)}{1965(0.0236 \times 0.0202 - 0.00199 \times 0.1864)} \\
&= 8.5
\end{aligned}
$$

Relatively speaking, although λ_{11} in this case is smaller than the one in the previous example, its value is still greater than 1.5. For the decoupling compensation design (see Chapter 5), we will find that the smaller λ_{11}, the better the compensation performance.

4.5 A Few Comments on Interaction Analysis

The purpose for deriving Equation 4.84 was to obtain an exposition and overview of the interaction analysis from the process designer's point of view; that is to say, Equation 4.84 gives an expression with which the interaction in the CFSTR control process can be calculated. It is a simple algebraic operation for a process designer to find the degree of interaction by substituting all the system parameters into Equation 4.84. If the relative gain value is greater than 1.5, it means the interaction between the temperature control loop and concentration control loop cannot be neglected. Thus decoupling will be necessary. On the other hand, it is important to realize that Equation 4.83 and Equation 4.84 are merely two different ways of expressing precisely the same relations, one using open-loop gains, the other the system parameters. Equation 4.83 has practical significance because the relative gain can be determined directly from measurements of all the open-loop gains and a designer needn't have any knowledge of the CFSTR process parameters.

Except for certain applications where any interaction cannot be tolerated, it is desirable that the degree of interaction be sufficiently small, or the relative gains (λ_{11} and λ_{22}) of the main channels should tend to one. For a CFSTR process, a desirable relative gain value must fall within the range from 0.7 to 1.5. In fact, as was mentioned previously, the relative gain value for a CFSTR is always greater than or equal to one for all system parameters. System parameters and design parameters in general are constrained by process demand. Therefore, a desired relative gain value and the needed process design parameters sometimes conflict with each other. In other words, there is only limited possibility that while designing a CFSTR process, a designer can pay attention to reducing the interactive control behaviour by changing the design parameters within the limits of the design objectives. Therefore, it should be emphasized here that the system design, which can deal with the reduction of interaction in the control of a CFSTR, rather than with the decoupling design, depends on the process properties with respect to the process design requirements.

Derivation of the relative gain Equation 4.84 represents a first step in the study of interaction analysis of a CFSTR. Decoupling conditions and decoupling stability will be studied in the next chapter.

Parameter	Description	Nominal value	Unit
V	Volume of reactor	1000	cm^3
T_i	Inlet temperature of feed	350	K
T_{cin}	Inlet temperature of coolant	340	K
A_r	Frequency factor	7.86×10^{12}	s^{-1}
n	Reaction order	1	
E	Activation energy	28000	cal/mol
$-\Delta H$	Heat of reaction	27000	cal/mol
R	Gas constant	1.987	$cal/(mol \cdot K)$
$A_K U$	Heat transfer conductance	10	$cal/(s \cdot K)$
ρ_c	Fluid density of coolant	1.0	kg/cm^3
c_c	Specific heat of fluid of coolant	1.0	$cal/(kg \cdot K)$
ρ_f	Fluid density of feed	1.0	kg/cm^3
c_p	Specific heat of fluid of feed	1.0	$cal/(kg \cdot K)$
C_{i0}	Steady-state inlet concentration of feed	0.0065	mol/dm^3
C_0	Steady-state effluent concentration of feed	15.31×10^{-5}	mol/dm^3
T_0	Steady-state temperature in reactor	460.91	K
Q_0	Steady-state feed flow rate	10	cm^3/s
Q_{c0}	Steady-state coolant flow rate	5	cm^3/s

Table 4.1: Steady-state operation condition of a CFSTR, copied from Douglas (1965).

Parameter	Description	Nominal value	Unit
V	Volume of reactor	2×10^{-3}	m^3
T_i	Inlet temperature of feed	301	K
T_{cin}	Inlet temperature of coolant	293.15	K
A_r	Frequency factor	7.86×10^{12}	s^{-1}
n	Reaction order	1	
E	Activation energy	9.41×10^4	$kJ/kmol$
$-\Delta H$	Heat of reaction	4.18×10^4	$kJ/kmol$
R	Gas constant	8.314	$kJ/(kmol \cdot K)$
$A_K U$	Heat transfer conductance	5.67×10^{-3}	$kJ/(s \cdot K)$
ρ_c	Fluid density of coolant	1×10^3	kg/m^3
c_c	Specific heat of fluid of coolant	4.18	$kJ/(kg \cdot K)$
ρ_f	Fluid density of feed	1×10^3	kg/m^3
c_p	Specific heat of fluid of feed	4.18	$kJ/(kg \cdot K)$
C_{i0}	Steady-state inlet concentration of feed	5	$kmol/m^3$
C_0	Steady-state effluent concentration of feed	1.07	$kmol/m^3$
T_0	Steady-state temperature in reactor	336.1	K
Q_0	Steady-state feed flow rate	1×10^{-5}	m^3/s
Q_{c0}	Steady-state coolant flow rate	1.78×10^{-7}	m^3/s

Table 4.2: Steady-state operation condition of a CFSTR, copied from Nakanishi and Ohtani (1986).

Chapter 5

Decoupling Design for the CFSTR

5.1 Introduction

The aim of the decoupling design is to find a compensation network for overcoming the interaction naturally existing in the CFSTR process. If perfect decoupling is achieved, a change in set point for one variable will only effect the controlled variable associated with that set point, and all other controlled variables will be unaffected. The theoretical problems associated with the decoupling design are usually solved by the diagonal matrix method. On the other hand, there are two kinds of schemes for the decoupling configuration. One is the ideal decoupling design in which the decoupled system is just the original system without coupling channels. Another is the simplified decoupling design. Generally speaking, the advantage of simplified decoupling is that the decouplers are always physically realizable. In this chapter, the ideal decoupling design will be discussed briefly because it has been tried with some chemical process simulations and has proven to be very sensitive to modelling errors (Weischedel and McAvoy, 1980; and McAvoy, 1981.).

In this chapter attention will be paid to both simplified decoupling design and the modelling error analysis by applying the relative gain method.

5.2 Ideal Decoupling Design

The fundamental problem in designing multivariable feedback controllers lies in the interactions which occur between the various input and output variables. If a system had no coupling between variables and the number of control variables equalled the number of outputs to be controlled, then the system in the transform domain would have a diagonal open-loop transfer function. Ideal decoupling design permits a decoupled process to behave as if the original interaction channels were not present, i.e., the response of each control loop is independent of all other control loops. Figure 5.13 shows a block diagram for the ideal decoupling of a CFSTR control system.

The open-loop transfer matrix relating $[\ C(s)\ \ T(s)\ ]^{-1}$ and $[\ M_C(s)\ \ M_T(s)\ ]^{-1}$ is

$$\begin{aligned}\begin{bmatrix} C(s) \\ T(s) \end{bmatrix} &= \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} N_{11}(s) & N_{12}(s) \\ N_{21}(s) & N_{22}(s) \end{bmatrix} \begin{bmatrix} M_C(s) \\ M_T(s) \end{bmatrix} \\ &= \begin{bmatrix} G_{11}(s)N_{11}(s) + G_{12}(s)N_{21}(s) & G_{11}(s)N_{12}(s) + G_{12}(s)N_{22}(s) \\ G_{21}(s)N_{11}(s) + G_{22}(s)N_{21}(s) & G_{21}(s)N_{12}(s) + G_{22}(s)N_{22}(s) \end{bmatrix} \begin{bmatrix} M_C(s) \\ M_T(s) \end{bmatrix}\end{aligned} \tag{5.85}$$

where $N_{11}(s)$, $N_{12}(s)$, $N_{21}(s)$ and $N_{22}(s)$ are decoupling compensators; $M_C(s)$ and $M_T(s)$ are output variables of the controller $R_C(s)$ and the controller $R_T(s)$, respectively.

For ideal noninteraction, define

$$G_{11}(s)N_{11}(s) + G_{12}(s)N_{21}(s) = G_{11}(s) \tag{5.86}$$

$$G_{21}(s)N_{11}(s) + G_{22}(s)N_{21}(s) = 0 \tag{5.87}$$

$$G_{11}(s)N_{12}(s) + G_{12}(s)N_{22}(s) = 0 \tag{5.88}$$

$$G_{21}(s)N_{12}(s) + G_{22}(s)N_{22}(s) = G_{22}(s) \tag{5.89}$$

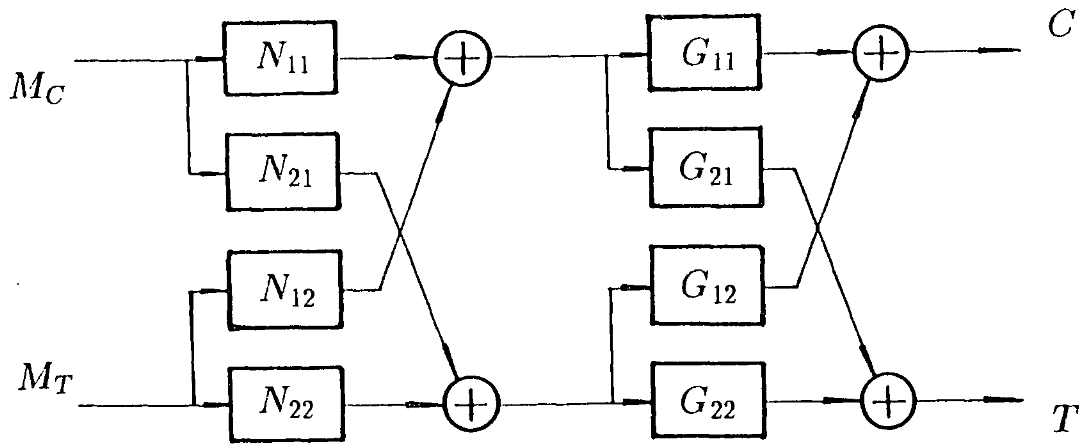


Figure 5.13: A block diagram for ideal decoupling system of the CFSTR. $N_{11}(s)$, $N_{12}(s)$, $N_{21}(s)$ and $N_{22}(s)$ are decoupling compensators.

Equation 5.86 and Equation 5.87 can be rewritten in matrix form as

$$\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} N_{11}(s) \\ N_{21}(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) \\ 0 \end{bmatrix} \tag{5.90}$$

then

$$\begin{bmatrix} N_{11}(s) \\ N_{21}(s) \end{bmatrix} = \frac{\begin{bmatrix} G_{22}(s) & -G_{12}(s) \\ -G_{21}(s) & G_{11}(s) \end{bmatrix} \begin{bmatrix} G_{11}(s) \\ 0 \end{bmatrix}}{G_{11}(s)G_{22}(s) - G_{12}(s)G_{21}(s)} = \frac{\begin{bmatrix} G_{11}(s)G_{22}(s) \\ -G_{11}(s)G_{21}(s) \end{bmatrix}}{G_{11}(s)G_{22}(s) - G_{12}(s)G_{21}(s)} \tag{5.91}$$

For Equation 5.88 and Equation 5.89, similarly,

$$\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} N_{12}(s) \\ N_{22}(s) \end{bmatrix} = \begin{bmatrix} 0 \\ G_{22}(s) \end{bmatrix} \tag{5.92}$$

then

$$\begin{bmatrix} N_{12}(s) \\ N_{22}(s) \end{bmatrix} = \frac{\begin{bmatrix} G_{22}(s) & -G_{12}(s) \\ -G_{21}(s) & G_{11}(s) \end{bmatrix} \begin{bmatrix} 0 \\ G_{22}(s) \end{bmatrix}}{G_{11}(s)G_{22}(s) - G_{12}(s)G_{21}(s)} = \frac{\begin{bmatrix} -G_{12}(s)G_{22}(s) \\ G_{11}(s)G_{22}(s) \end{bmatrix}}{G_{11}(s)G_{22}(s) - G_{12}(s)G_{21}(s)} \tag{5.93}$$

Therefore

$$N_{11}(s) = N_{22}(s) = \frac{G_{11}(s)G_{22}(s)}{G_{11}(s)G_{22}(s) - G_{12}(s)G_{21}(s)} \tag{5.94}$$

$$N_{12}(s) = -\frac{G_{12}(s)G_{22}(s)}{G_{11}(s)G_{22}(s) - G_{12}(s)G_{21}(s)} \tag{5.95}$$

$$N_{21}(s) = -\frac{G_{11}(s)G_{21}(s)}{G_{11}(s)G_{22}(s) - G_{12}(s)G_{21}(s)} \tag{5.96}$$

Obviously, not only are four compensators needed for ideal decoupling, but also their structures are not simple, and they may be very difficult to design. In many cases, they are not physically realizable if the model order is high. For this reason, another basic approach to algorithmic decoupling design will be discussed in the next section, namely simplified decoupling.

5.3 Simplified Decoupling Design

If both Equation 5.87 and Equation 5.88 can be satisfied, the transfer function matrix in Equation 5.85 can still be a diagonal matrix. Thus, the system is uncoupled since the controller's output variable $M_C(s)$ has no effect on the controlled variable $T(s)$ and the other controller's output variable $M_T(s)$ has no effect on the controlled variable $C(s)$, either. Therefore, the process will be decoupled. The conditions for simplified decoupling are as follows:

$$G_{21}(s)N_{11}(s) + G_{22}(s)N_{21}(s) = 0$$

$$G_{11}(s)N_{12}(s) + G_{12}(s)N_{22}(s) = 0$$

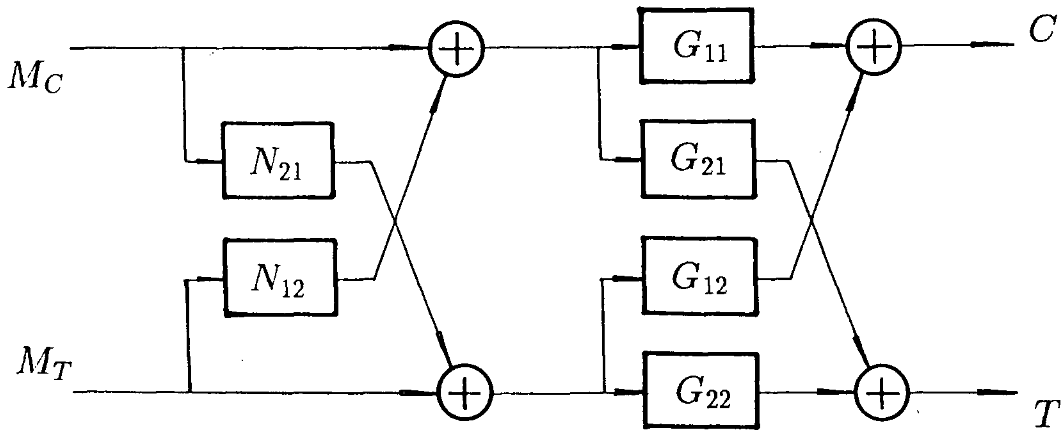


Figure 5.14: A block diagram for a simplified decoupling system of a CFSTR

Now, letting

$$N_{11}(s) = N_{22}(s) = 1 \tag{5.97}$$

and by defining $N_{12}(s)$ and $N_{21}(s)$ as

$$N_{12}(s) = -\frac{G_{12}(s)}{G_{11}(s)} \tag{5.98}$$

$$N_{21}(s) = -\frac{G_{21}(s)}{G_{22}(s)} \tag{5.99}$$

only two decoupling compensators are needed. Figure 5.14 shows the block diagram for a simplified decoupling system of a CFSTR.

Substitute Equations 3.33 and 3.34 into Equations 5.98 and substitute Equations 3.35 and 3.36 into Equation 5.99, and rearrange to obtain

$$N_{12}(s) = -\frac{\alpha_2\beta_3}{\alpha_0(s+\beta_1)+\alpha_2\beta_0} \tag{5.100}$$

$$N_{21}(s) = -\frac{\alpha_0\beta_2+\beta_0(s+\alpha_1)}{\beta_3(s+\alpha_1)} \tag{5.101}$$

the two steady-state gains of $N_{12}(s)$ and $N_{21}(s)$ are identified as k_{12} and k_{21}, then

$$k_{12} = -\left. N_{12}(s)\right|_{s=0} = -\frac{F_{12}}{F_{11}} = -\frac{\alpha_2\beta_3}{\alpha_0\beta_1 + \alpha_2\beta_0} \tag{5.102}$$

$$k_{21} = -\left. N_{21}(s)\right|_{s=0} = -\frac{F_{21}}{F_{22}} = -\frac{\alpha_0\beta_2 + \alpha_1\beta_0}{\alpha_1\beta_3} \tag{5.103}$$

For the concentration main channel $M_C(s) \rightarrow C(s)$, we have

$$\begin{aligned} G_{11}(s) + G_{12}(s)N_{21}(s) &= \frac{\alpha_0(s+\beta_1) + \alpha_2\beta_0}{P(s)} - [\frac{\alpha_2\beta_3}{P(s)}][\frac{\alpha_0\beta_2 + \beta_0(s+\alpha_1)}{\beta_3(s+\alpha_1)}] \\ &= \frac{(s+\alpha_1)[\alpha_0(s+\beta_1)+\alpha_2\beta_0] - \alpha_2[\alpha_0\beta_2 + \beta_0(s+\alpha_1)]}{P(s)[(s+\alpha_1)]} \\ &= \frac{\alpha_0[s^2 + (\alpha_1+\beta_1)s + \alpha_1\beta_1 - \alpha_2\beta_2]}{P(s)[(s+\alpha_1)]} \\ &= \frac{\alpha_0}{s+\alpha_1} \end{aligned} \tag{5.104}$$

and for temperature main channel $M_T(s) \rightarrow T(s)$, we have

$$\begin{aligned} G_{21}(s)N_{12}(s) + G_{22}(s) &= -[\frac{\alpha_0\beta_2 + \beta_0(s+\alpha_1)}{P(s)}][\frac{\alpha_2\beta_3}{\alpha_0(s+\beta_1)+\alpha_2\beta_0}] + \frac{\beta_3(s+\alpha_1)}{P(s)} \\ &= \frac{-\alpha_2\beta_3[\alpha_0\beta_2 + \beta_0(s+\alpha_1)] + \beta_3(s+\alpha_1)[\alpha_0(s+\beta_1)+\alpha_2\beta_0]}{P(s)[\alpha_0(s+\beta_1)+\alpha_2\beta_0]} \\ &= \frac{\alpha_0\beta_3[s^2 + (\alpha_1+\beta_1)s + \alpha_1\beta_1 - \alpha_2\beta_2]}{P(s)[\alpha_0(s+\beta_1)+\alpha_2\beta_0]} \\ &= \frac{\alpha_0\beta_3}{\alpha_0(s+\beta_1)+\alpha_2\beta_0} \end{aligned} \tag{5.105}$$

From Equations 5.100, 5.101, 5.102 and 5.103 just derived, a block diagram of a CFSTR can be drawn, as shown in Figure 5.15(a). Simplification of this block diagram results in Figure 5.15(b).

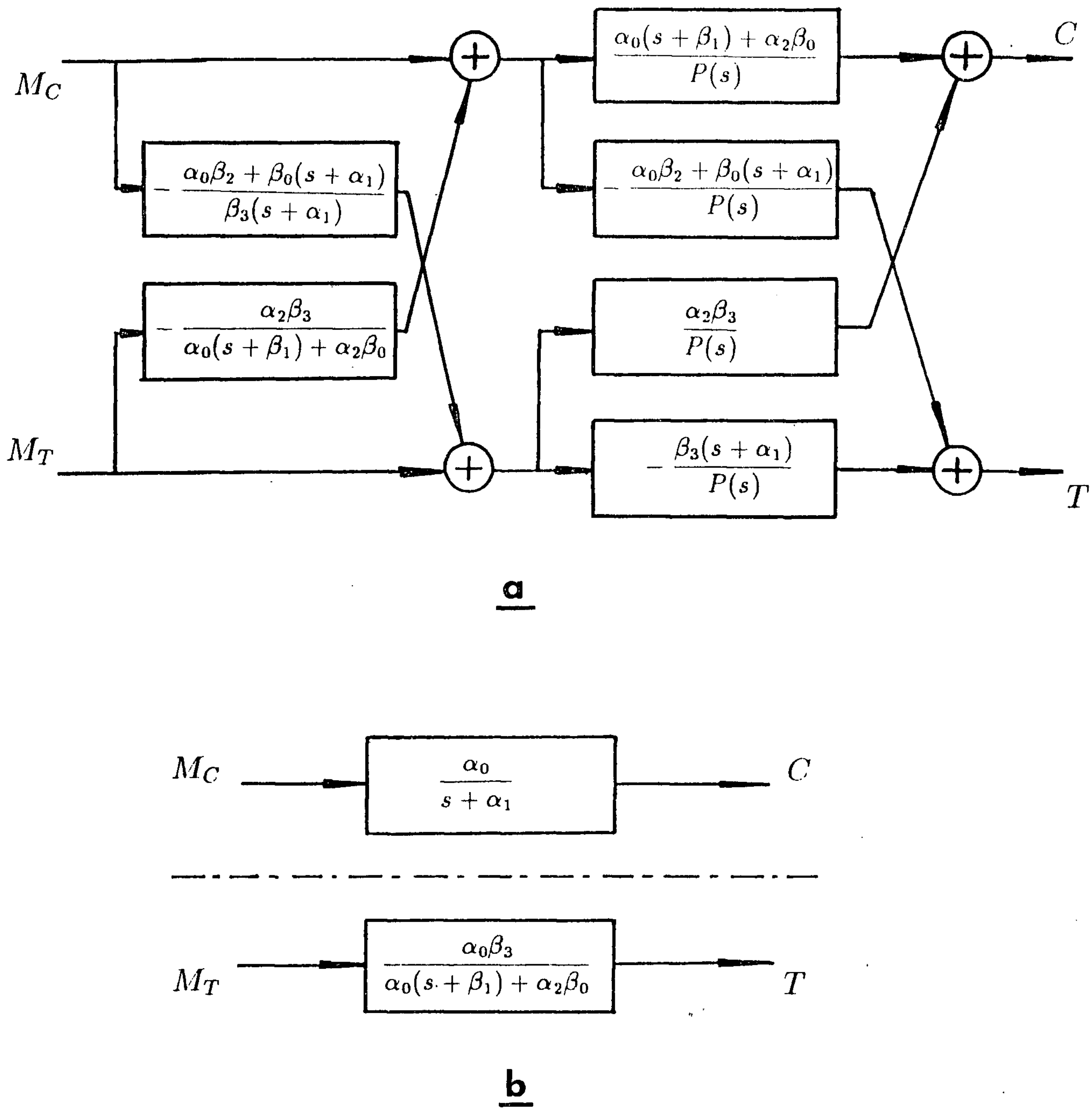


Figure 5.15: (a) A block diagram of a decoupled CFSTR system. (b) A CFSTR process with superficial noninteractive behaviour.

5.4 Analysis of Modelling Error by the RGM

As described above, classical decoupling design requires that the dynamics of the CFSTR process be known, either in terms of differential equations or transfer functions. However, in many cases, detailed dynamic studies are not feasible, or they may not be worthwhile because of uncertainty regarding the proper form of the objective function to be used in designing the decoupling network or controller action. For this reason, improving the robustness of the decoupling system will play an important part in a CFSTR process control design. It should be pointed out that the relative gain method is still applicable to the analysis of the decoupling system design. Now, consider the open-loop gain coefficients F_{ijd} of the decoupled CFSTR system, as shown in Figure 5.16.

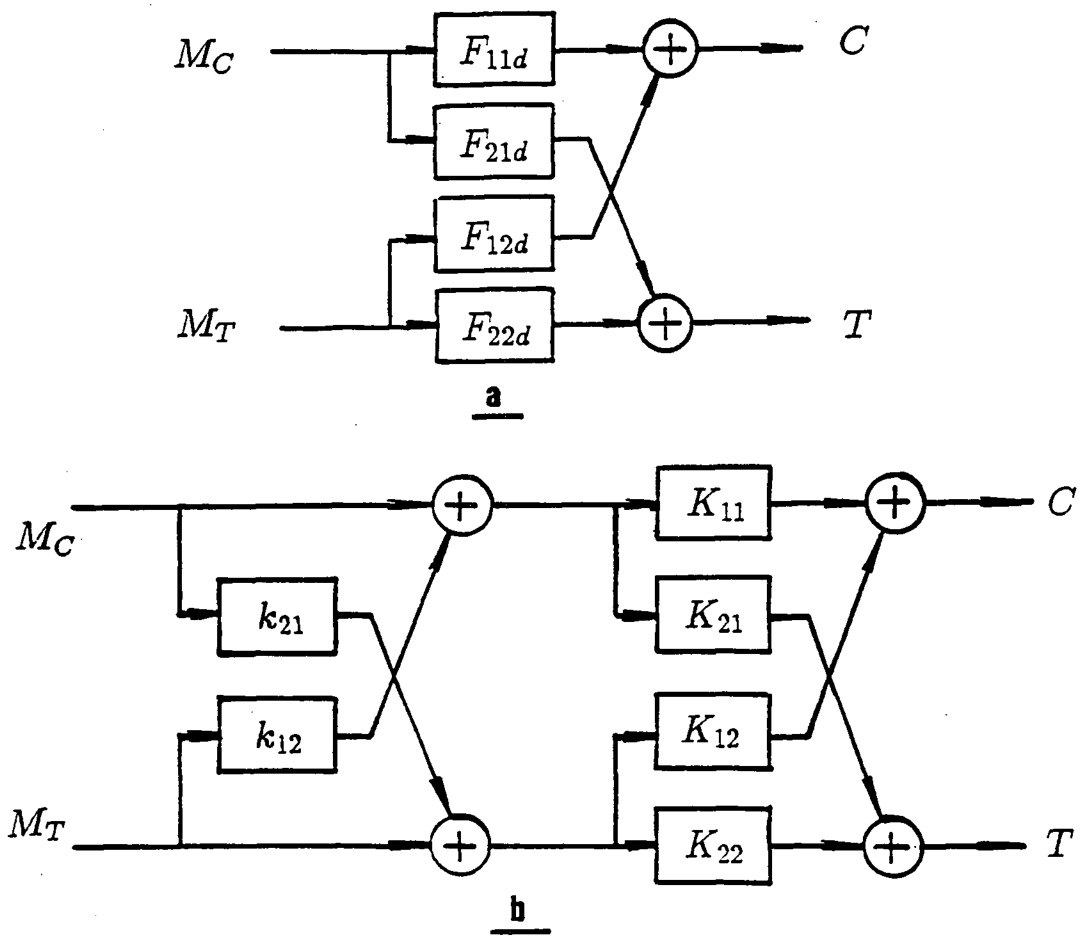


Figure 5.16: (a) The open-loop gain coefficients F_{ijd} of the decoupled CFSTR system. (b) The steady-state gain of each element in the open-loop decoupled CFSTR system.

for $M_C(t) \rightarrow C(t)$

$$F_{11d} = K_{11} + k_{21}K_{12} = F_{11} + k_{21}F_{12} \tag{5.106}$$

for $M_C(t) \rightarrow T(t)$

$$F_{12d} = K_{21} + k_{21}K_{22} = F_{21} + k_{21}F_{22} \tag{5.107}$$

for $M_T(t) \rightarrow C(t)$

$$F_{21d} = K_{12} + k_{12}K_{11} = F_{12} + k_{12}F_{11} \tag{5.108}$$

for $M_T(t) \rightarrow T(t)$

$$F_{22d} = K_{22} + k_{12}K_{21} = F_{22} + k_{12}F_{21} \tag{5.109}$$

According to Equation 4.83, the relative gain of the decoupled system of the CFSTR, which is identified as λ_{11d}, can be inferred as follows:

$$\begin{aligned} \lambda_{11d} &= \frac{F_{11d}F_{22d}}{F_{11d}F_{22d} - F_{12d}F_{21d}} \\ &= \frac{1}{1 - \frac{F_{12d}F_{21d}}{F_{11d}F_{22d}}} \\ &= \frac{1}{1 - \frac{(F_{21}+k_{21}F_{22})(F_{12}+k_{12}F_{11})}{(F_{11}+k_{21}F_{12})(F_{22}+k_{12}F_{21})}} \end{aligned} \tag{5.110}$$

Obviously, if

$$k_{12} = -\frac{F_{12}}{F_{11}} \tag{5.111}$$

$$k_{21} = -\frac{F_{21}}{F_{22}} \tag{5.112}$$

then $\lambda_{11d} = 1$; this is a perfect noninteracting system.

Consider the "worst case" possibility, which can drive the system out of control, that is to say, $\lambda_{11d} \rightarrow \infty$. Clearly, the condition which λ_{11d} tends to infinity is when the denominator of Equation 5.110 tends to zero, or

$$\frac{(F_{21} + k_{21}F_{22})(F_{12} + k_{12}F_{11})}{(F_{11} + k_{21}F_{12})(F_{22} + k_{12}F_{21})} \rightarrow 1 \tag{5.113}$$

From Equation 5.111 and Equation 5.112, if

$$k_{12}k_{21} = 1 \tag{5.114}$$

then substituting above Equation 5.114 into the denominator of Equation 5.110, we get

$$\begin{aligned}
1 - \frac{(F_{21} + k_{21}F_{22})(F_{12} + k_{12}F_{11})}{(F_{11} + k_{21}F_{12})(F_{22} + k_{12}F_{21})} &= 1 - \frac{F_{12}F_{21} + k_{12}F_{11}F_{21} + k_{21}F_{12}F_{22} + k_{12}k_{21}F_{11}F_{22}}{k_{12}k_{21}F_{12}F_{21} + k_{12}F_{11}F_{21} + k_{21}F_{12}F_{22} + F_{11}F_{22}} \\
&= 1 - \frac{F_{12}F_{21} + k_{12}F_{11}F_{21} + k_{21}F_{12}F_{22} + F_{11}F_{22}}{F_{12}F_{21} + k_{12}F_{11}F_{21} + k_{21}F_{12}F_{22} + F_{11}F_{22}} \\
&= 1 - 1 \\
&= 0
\end{aligned} \tag{5.115}$$

So, the relative gain $\lambda_{11d} \rightarrow \infty$. It should be noted that although, in theory, the interactive behaviour of a CFSTR can be compensated for by the diagonal matrix method, in practice, the modelling error can still make the decoupled system deviate from the optimal state so that the CFSTR process goes out of the control. Therefore, it is necessary to improve the decoupling design in order to make the system performance more stability.

5.5 Error, System Stability, and Robustness

The aim of stability analysis is to find bounds on the decoupling modelling error that leads to divergence of the CFSTR process. Let e_1 and e_2 be two compensation factors for decoupling elements N_{12} and N_{21}, respectively. The modelling compensation factors represent the errors associated with the model mismatch between the interactive behaviour of the CFSTR and the decoupler. Rewriting Equation 5.111 and Equation 5.112 as

$$k_{12} = -e_1\frac{F_{12}}{F_{11}} \tag{5.116}$$

$$k_{21} = -e_2\frac{F_{21}}{F_{22}} \tag{5.117}$$

If $e_1 = e_2 = 1$, this is the condition for perfect decoupling.

If $k_{12}k_{21} = 1$, then

$$e_1 e_2 = \frac{F_{11}F_{22}}{F_{12}F_{21}} \tag{5.118}$$

and the CFSTR system is out of control because $\lambda_{11d} \to \infty$. This is the condition which indicates that both the effluent concentration variable and the reactor temperature variable cannot be controlled at the same time. As mentioned in Chapter 4, the original relative gain value of the CFSTR process is

$$\lambda_{11} = \frac{F_{11}F_{22}}{F_{11}F_{22} - F_{12}F_{21}} = \frac{1}{1 - \frac{F_{12}F_{21}}{F_{11}F_{22}}} \tag{5.119}$$

Substituting Equation 5.118 into Equation 3.119, we obtain

$$\lambda_{11} = \frac{1}{1 - \frac{1}{e_1 e_2}} \tag{5.120}$$

Therefore

$$e_1 e_2 = \frac{\lambda_{11}}{\lambda_{11} - 1} \tag{5.121}$$

Equation 5.121 indicates that under the unstable condition, the modelling compensation factors depend on the original relative gain value. As is well known, a model of a realistic CFSTR process is seldom completely known and, if known, it is seldom linear. Local linearization, as described in Chapter 3, forms the basis for most of the currently applied control theory; but unfortunately, it allows good performance only for small departures of the operating variables from their nominal trajectories. In most cases, the main reason for the control problems associated with an unstable process is uncertainty which can lead to modelling error. Uncertainty in a CFSTR process model may have three origins. (1) There are always parameters in the linear model which are only known approximately. (2) The parameters in the linear model may vary due to nonlinearities or changes in the operating conditions.

(3) Outside disturbances can effect the process parameters.

Therefore, from the viewpoint of control engineering, the decoupling design should consider that the error in the original relative gain value, λ_{11} can vary within the limits of the objective conditions. Now, consider two different cases: overcompensation and undercompensation. To simplify the algebra and avoid complicated computations, let the two compensation factors be the same, i.e., $e_1 = e_2 = e$. Thus, for overcompensation, the compensation factor e is greater than one; and for undercompensation, the compensation factor e is less than one.

5.5.1 Overcompensation of Interaction

In this case, substituting Equation 5.116 and Equation 5.117 into Equation 5.110, then the relative gain λ_{11d} can written

$$\begin{aligned}\lambda_{11d} &= 1/[1 - \frac{F_{12}F_{21}(1-e)^2}{(F_{11} - \frac{F_{12}F_{21}}{F_{22}}e)(F_{22} - \frac{F_{12}F_{21}}{F_{11}}e)}] \\ &= 1/[1 - \frac{F_{11}F_{12}F_{21}F_{22}(1-e)^2}{(F_{11}F_{22} - F_{12}F_{21}e)^2}] \\ &= 1/[1 - \frac{F_{11}F_{22}(1-e)^2}{F_{12}F_{21}(\frac{F_{11}F_{22}}{F_{12}F_{21}} - e)^2}] \qquad (5.122)\end{aligned}$$

From Equation 5.119, we have

$$\frac{F_{12}F_{21}}{F_{11}F_{22}} = 1 - \frac{1}{\lambda_{11}} \qquad (5.123)$$

Now, substituting Equation 5.123 into Equation 5.122, we get

$$\lambda_{11d} = 1/[1 - \frac{(1-e)^2}{(1 - \frac{1}{\lambda_{11}})(\frac{1}{1-\frac{1}{\lambda_{11}}} - e)^2}] \qquad (5.124)$$

Clearly, the relative gain of the decoupled system is a function of both the original relative gain of the CFSTR process and the modelling compensation factor. Considering that under the overcompensation e is greater than one, the family of curves λ_{11d} obtained from Equation 5.124, with various values of both λ_{11} and e is shown in Figure 5.17 and

Figure 5.18 (See Table H.3, H.4, and H.5). It is important to note that once the theoretical relative gain λ_{11}, which is a unique value in design, is determined from Equation 4.84, the relative gain λ_{11d} of the decoupled system is only a function of the compensation factor e because e, in practice, can be regulated. As shown in Figure 5.18, the overcompensated system, with e around 1.10, will be unstable when the original relative value of λ_{11} is greater than about 5.65. If, as an Example 1 in Chapter 4, the original relative gain λ_{11} of the CFSTR is around 11.3, the compensation factor e must be less than 1.03. When the compensation factor e is less than 1.02, relatively speaking, the overcompensated system has good performance if the original value of λ_{11} is less than 16.95.

From the foregoing analysis, it can be seem that if the original relative gain λ_{11} is much greater one, the overcompensation factor must be smaller for good compensation performance. In other words, if the overcompensation factor is too large, it is difficult to obtain good decoupling. When λ_{11} is near the unstable boundary condition the decoupled CFSTR process can experience a nonlinear, divergent change. In short, the overcompensation of an interacting system which has a large λ_{11} may lead to unstable or poorer control than when this system is controlled without a decoupler.

5.5.2 Undercompensation of Interaction

Equation 5.124 is also true for the analysis of undercompensation. In this case, assume that the undercompensation factor falls between 0.90 and 0.99 (See Figure 5.17). With changes in the undercompensation factor, the relative gain of the decoupled system, with a known original relative gain, can deviate from the desirable value as well. The deviation from the desirable value, however, is basically proportional to the original relative gain value over a wide range, and the rate of deviation is not very sensitive to the compensation factor e. The important point is that there are not any nonlinear divergent phenomena. Consider Example 1 in Chapter 4, the original relative gain value is about 11.3, the biggest deviation value for the decoupled CFSTR process, when the undercompensation factor is 0.9, is about 1.39 (See Table H.3). Therefore, the undercompensated system, which can tolerate a relatively wide undercompensation range, performs better than with overcompensation under the same original relative gain value.

5.5.3 Stability Analysis

In the above studies, it was found that although the absolute deviation of the compensation factors from unity in the two cases are the same, the effect of the value of the relative gain of the decoupled CFSTR system are quite different. Overcompensation will probably lead to instability, while undercompensation can hardly be made unstable. The reason is that in Equation 5.121, if e_1e_2 is greater than one, the value of the right-side of this equation probably equals the product of e_1 and e_2 since λ_{11} always is greater than one. On the contrary, if the product of e_1 and e_2 is less than one, then Equation 5.121 is never satisfied. Therefore, a significant improvement in the decoupled control system robustness can be obtained if the decouplers, N_{12} and N_{21}, always work under a condition of undercompensation. It is conceivable that if the product of e_1 and e_2 is less than one, that Equation 5.121 becomes

$$e_1e_2 < 1 \;\neq\; \frac{\lambda_{11}}{\lambda_{11}-1} \geq 1 \qquad\qquad \lambda_{11} \geq 1 \tag{5.125}$$

Equation 5.125 essentially gives a bound on the decoupling stability condition. This condition is simple and physically realizable.

5.5.4 An Illustrative Example

Assuming that each parameter in the CFSTR model process has a increment or a decrement, there will be thousands of combinations possible. It is therefore difficult to simulate all these combinations. As an example of the robustness test, one case will be studied in which it is assumed that the main channel of the concentration control loop, in practice, may be disturbed independently from outside so that the open-loop gain deviates from the theoretical value F_{11}. Now, consider the practical open-loop gain $\hat{F}_{11}$, which is given by

$$\hat{F}_{11} = dF_{11} \tag{5.126}$$

where d, which should be greater than zero, is the deviation factor.

The practical relative gain becomes

$$\hat{\lambda}_{11} = \frac{1}{1 - \frac{F_{12}F_{21}}{dF_{11}F_{22}}} \tag{5.127}$$

Substituting Equation 5.119 into Equation 5.127, we get

$$\begin{aligned} \hat{\lambda}_{11} &= \frac{1}{1 - \frac{1}{d}(1 - \frac{1}{\lambda_{11}})} \\ &= \frac{d\lambda_{11}}{d\lambda_{11} - \lambda_{11} + 1} \end{aligned} \tag{5.128}$$

or

$$d = \frac{(1 - \lambda_{11})\hat{\lambda}_{11}}{(1 - \hat{\lambda}_{11})\lambda_{11}} \tag{5.129}$$

The practical relative gain of the decoupled system becomes

$$\hat{\lambda}_{11d} = 1/[1 - \frac{(F_{21} + k_{21}F_{22})(F_{12} + k_{12}dF_{11})}{(dF_{11} + k_{21}F_{12})(F_{22} + k_{12}F_{21})}] \tag{5.130}$$

Substituting Equations 5.116, 5.117 and 5.129 into 5.130, we have

$$\hat{\lambda}_{11d} = 1/[1 - \frac{F_{12}F_{21}(1-e)(1-ed)}{(dF_{11} - \frac{F_{12}F_{21}}{F_{22}}e)(F_{22} - \frac{F_{12}F_{21}}{F_{11}}e)}]$$

$$
\begin{aligned}
&= 1/[1 - \frac{F_{11}F_{22}(1-e)(1-ed)}{F_{12}F_{21}(\frac{dF_{11}F_{22}}{F_{12}F_{21}} - e)(\frac{F_{11}F_{22}}{F_{12}F_{21}} - e)}] \\
&= 1/[1 - \frac{(1-e)[1 - e\frac{(1-\lambda_{11})\hat{\lambda}_{11}}{(1-\hat{\lambda}_{11})\lambda_{11}}]}{(1-\frac{1}{\lambda_{11}})(\frac{1}{1-\frac{1}{\hat{\lambda}_{11}}} - e)(\frac{1}{1-\frac{1}{\lambda_{11}}} - e)}] \qquad (5.131)
\end{aligned}
$$

In Equation 5.131, the theoretical relative gain λ_{11} is 11.3, and it is assumed that the practical relative gain can be changed between 5 and 25. Figure 5.19 and Figure 5.20 show a comparison between overcompensation and undercompensation. It is found that the $\hat{\lambda}_{11}$ is very sensitive to the changes in the overcompensation factor. If $\hat{\lambda}_{11}$ changes because of a $\pm 20\%$ change in λ_{11}, the overcompensation factor must fall between 1 and 1.03 in order to keep $\hat{\lambda}_{11d} \leq 1.4$ (See Table H.6), while the undercompensation factor can vary between 0.90 and 1 for equivalent control stability. Therefore, undercompensation has good robustness. The primary purpose of using undercompensated decouplers is to reduce the sensitivity of the CFSTR process to parameter variations and unwanted disturbances. For the decoupling design, the undercompensation factors must be selected very carefully so that the CFSTR system can operate under undercompensated conditions. Figure 5.21 shows a block diagram of an undercompensated decoupled CFSTR system. The result of this compensator design is that a 2×2 CFSTR system has effectively become two separable single control loops. However, if time-delay behaviour occurs in each feedback channel, an unstable CFSTR system is still probable. Time-delay compensation for a CFSTR process will be studied in the next chapter.

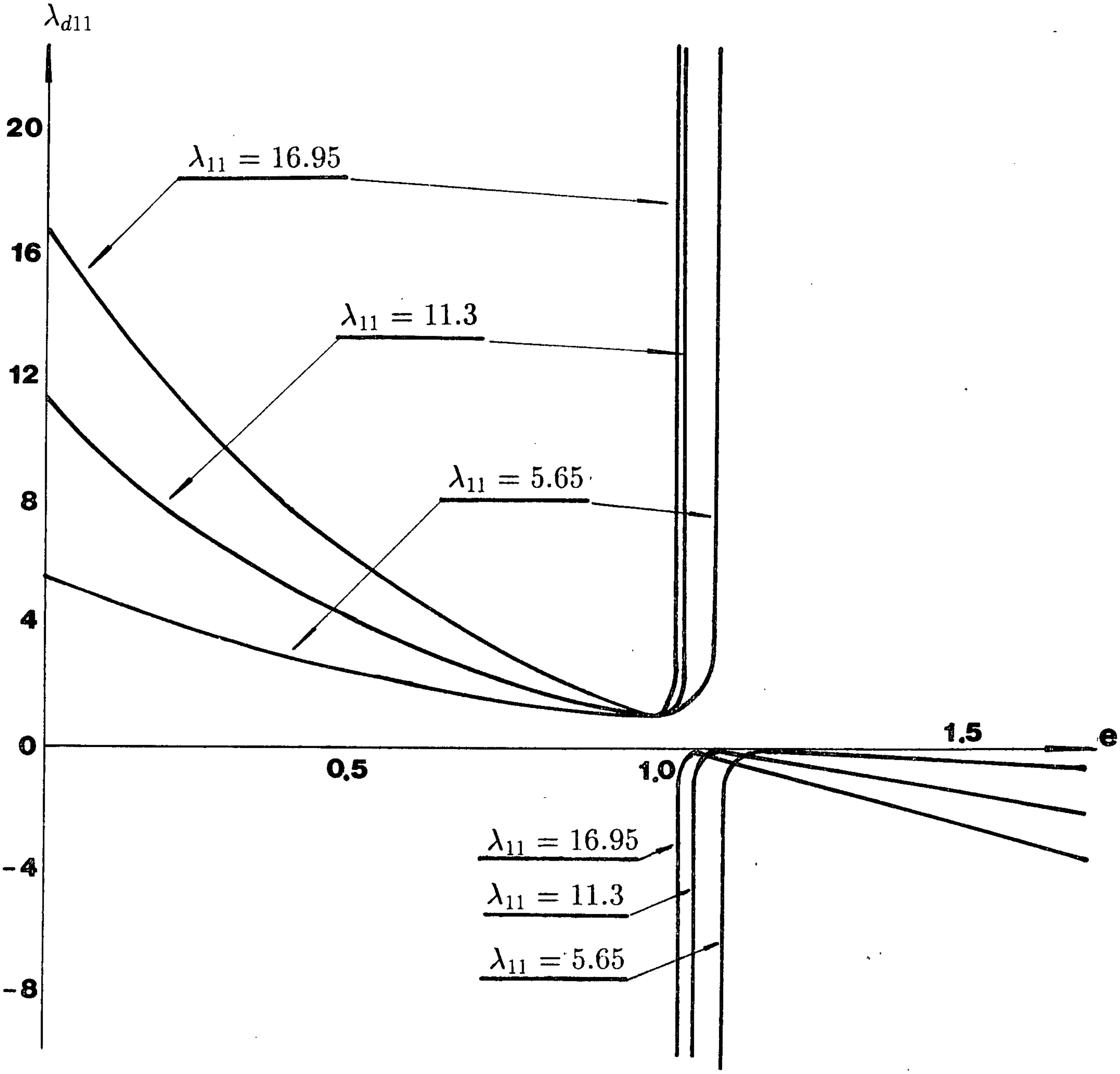


Figure 5.17: The relative gain λ_{11d} versus the compensation factor e

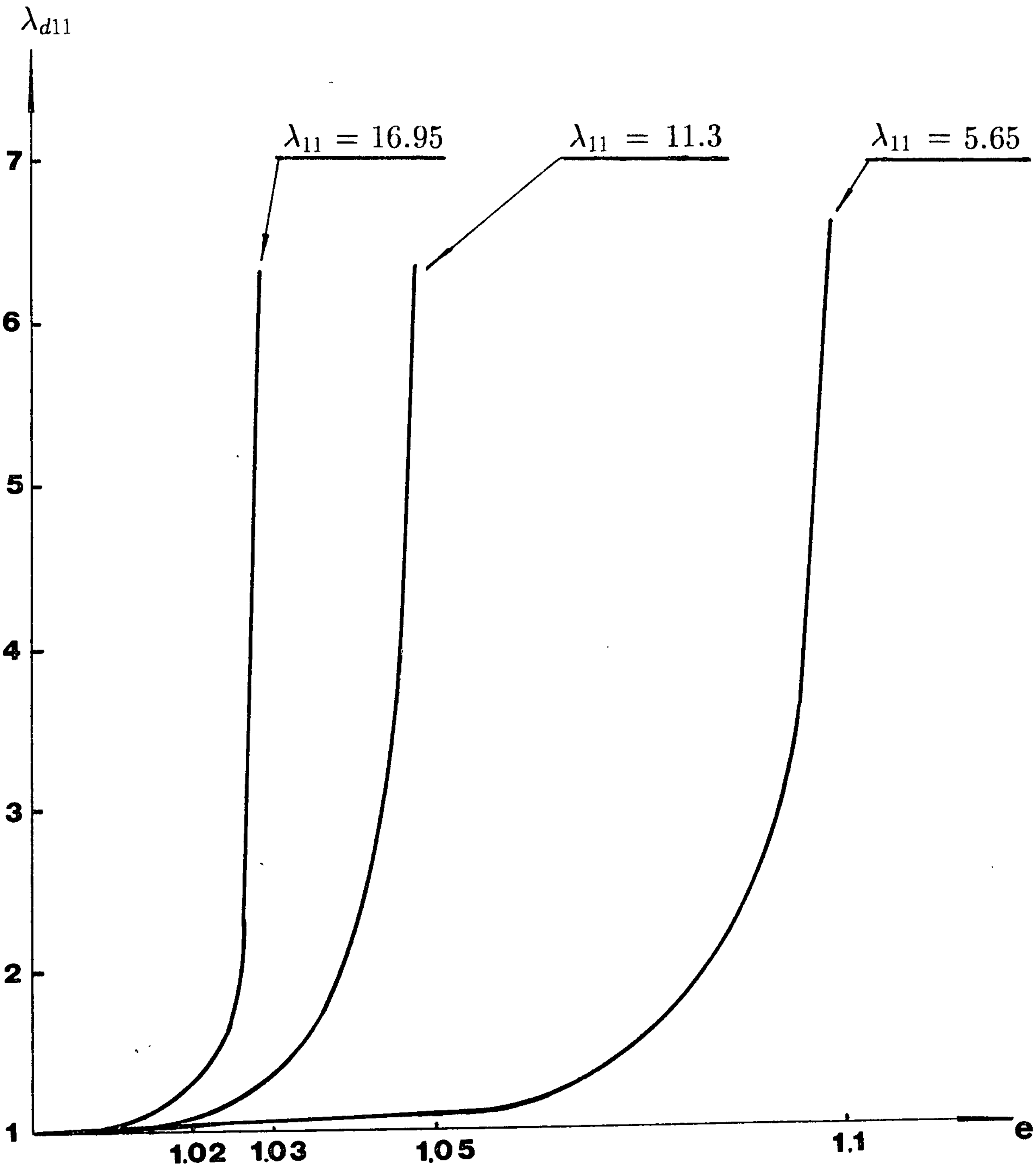


Figure 5.18: The relative gain λ_{11d} versus the overcompensation factor e

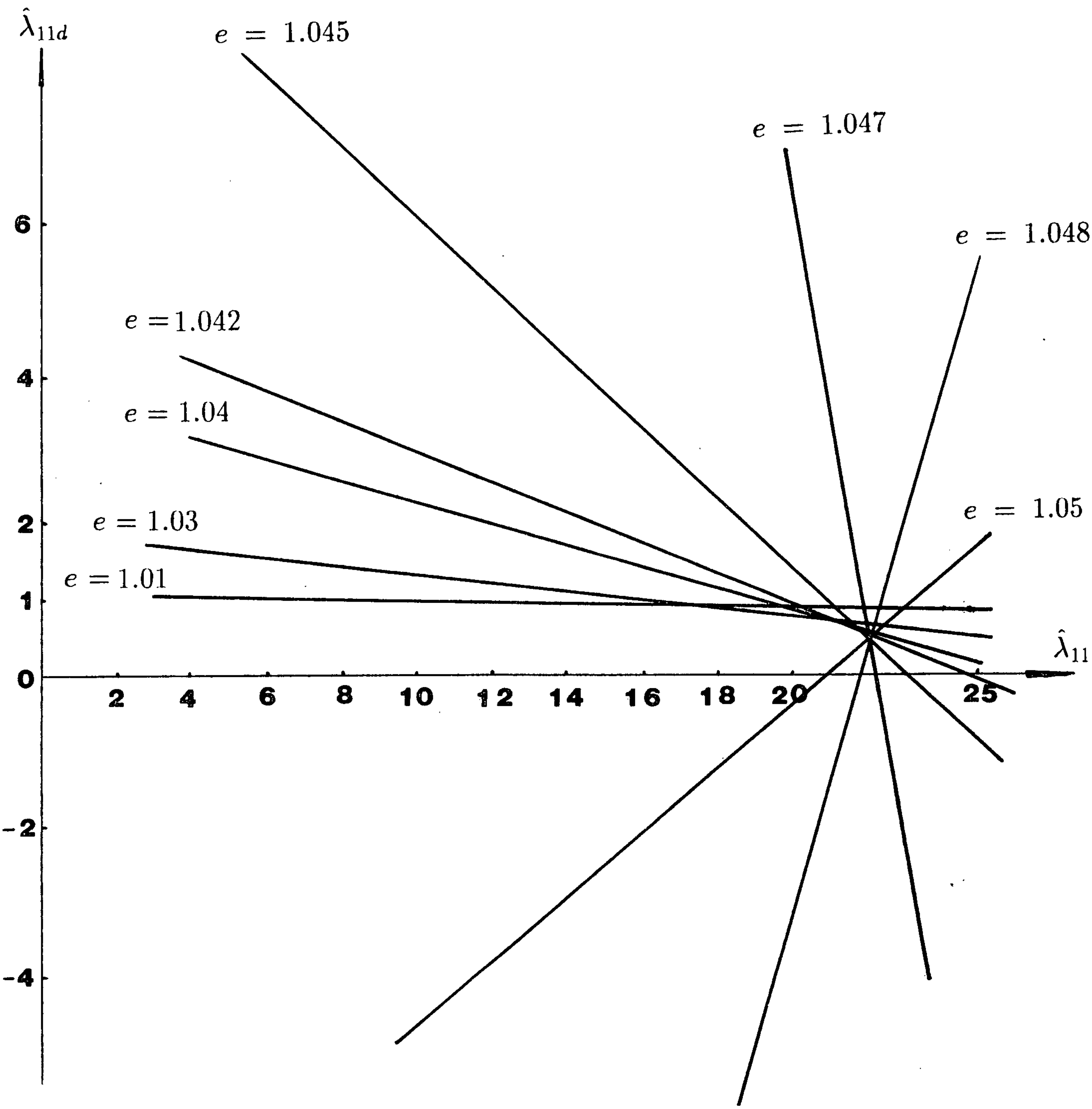


Figure 5.19: $\hat{\lambda}_{11d}$ versus $\hat{\lambda}_{11}$ with $e > 1$.

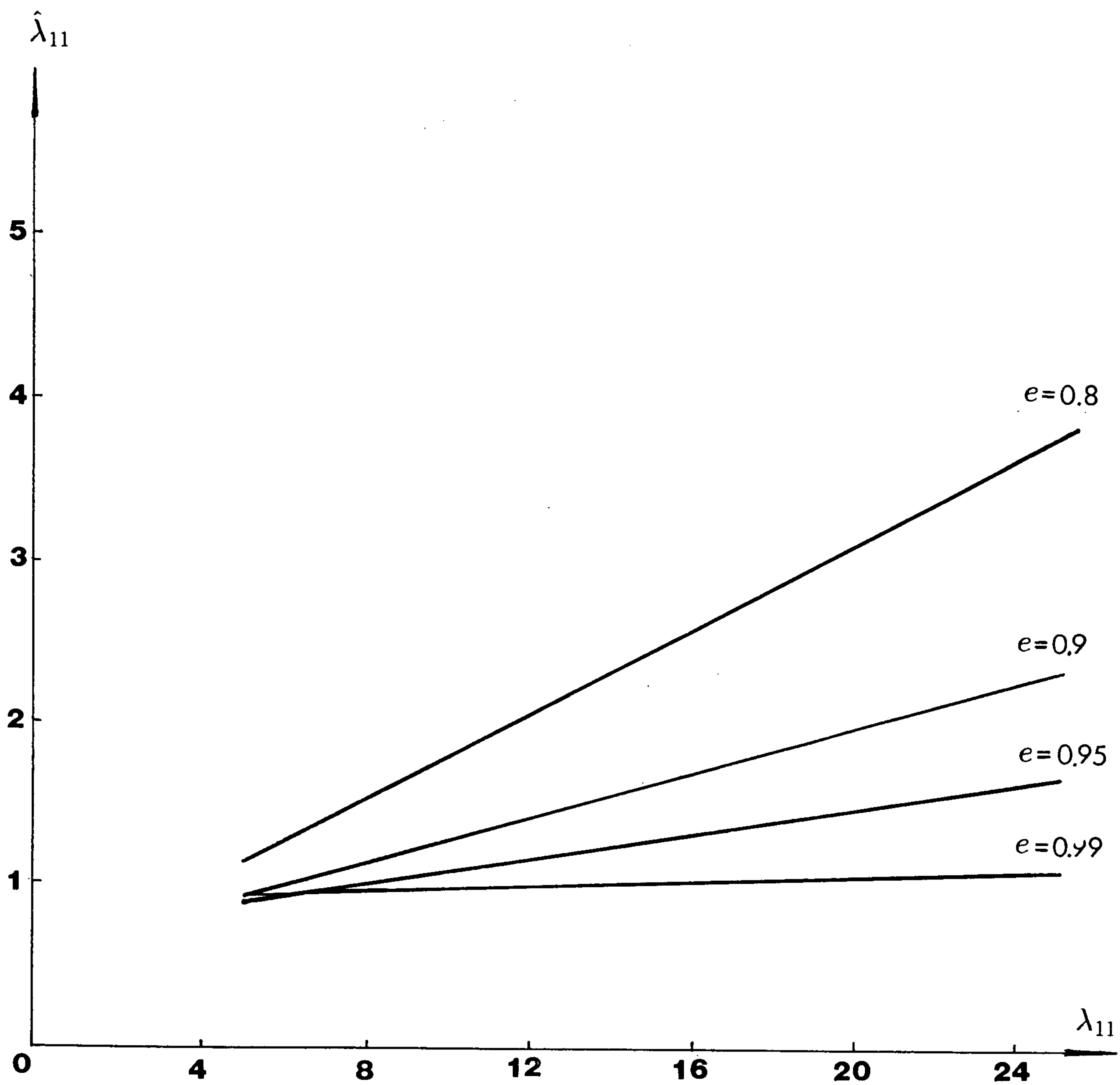


Figure 5.20: $\hat{\lambda}_{11d}$ versus $\hat{\lambda}_{11}$ with $e < 1$.

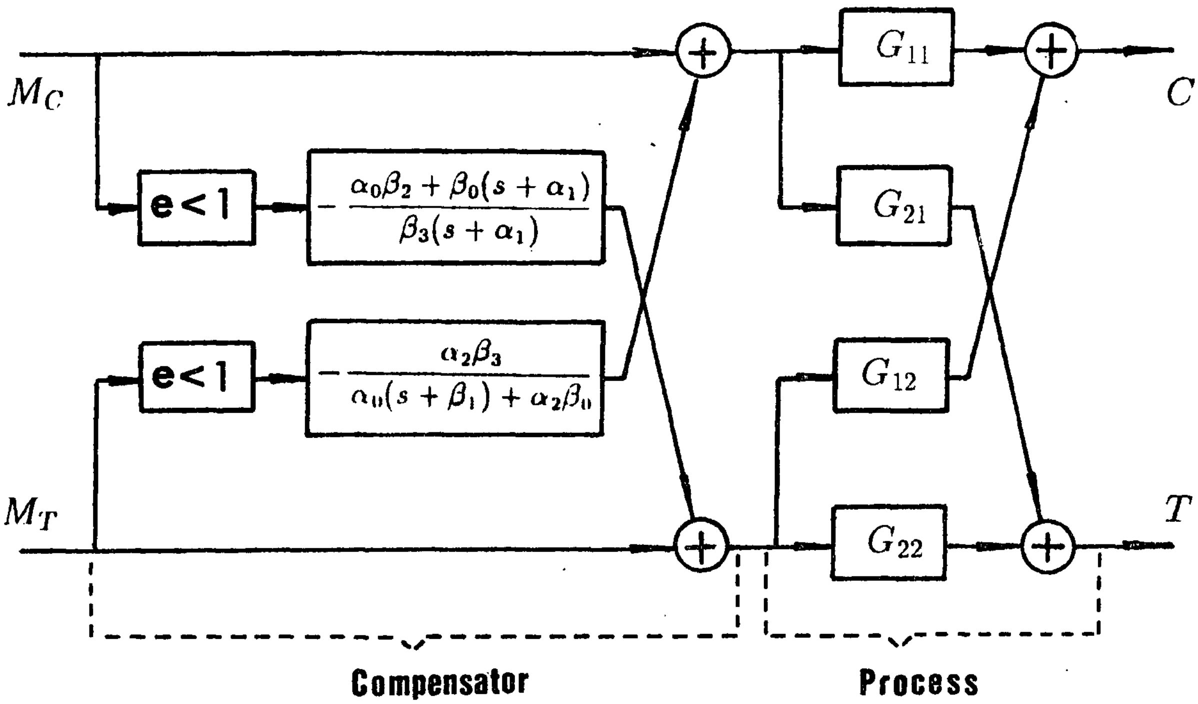


Figure 5.21: A block diagram of an undercompensated, decoupled CFSTR system

Chapter 6

Design of Time-delay Compensation

6.1 Introduction

By the careful design of a decoupling system, a two-variable CFSTR process has been separated into two single-variable control loops. As pointed out in Section 2.2, for the delay-free case, the use of negative feedback can make the system performance less sensitive to changes in process parameters. However, when the CFSTR process exhibits a time-delay in the control loop, this process is still not a simple one, that is to say, the time-delay can effect system performance and it can even lead to instability, while the conventional controllers cannot minimize the delay effect at all. Therefore, it is necessary to study time-delay compensation for the stable operation of a CFSTR. The following sections will be concerned with discussion of the CFSTR process with and without a time-delay, an ideal and nonideal Smith compensator, and a simple and physically realizable time-delay compensator. As mentioned in Chapter 2, problems involved with the sensitivity analysis of the Smith compensator have been reported in the last 20 years. So, the effect of a nonideal Smith compensator, i.e., the modelling error, will be presented briefly, while this chapter will concentrate on studying a simple and physically realizable time-delay compensator.

In general, the concentration control loop of the CFSTR has a bigger time-delay than the temperature control loop has. In the light of this, the time-delay compensation for the concentration control is considered in detail and the result for the temperature control it can be deduced easily by analogy.

6.2 Control of the Concentration Process without a Time-delay

In analyzing practical processes, it is often desirable to change the units of a variable or to normalize a given variable. The results in terms of normalized dimensionless variables are useful because they can be applied directly to different systems having similar mathematical equations. Appendix E provides an outline of the derivation of the dimensionless variables for concentration control.

Figure 6.22 shows the block diagram of the concentration control system without time-delay. For simplicity and insight, the controller K_C is considered to have proportional control action. The open-loop transfer function between the manipulated $Q(s)$ and controlled variables $C(s)$ is

$$\frac{C(s)}{Q(s)} = G_{11}(s) = \frac{\hat{\alpha}_0}{s + \hat{\alpha}_1} = \frac{\frac{\hat{\alpha}_0}{\hat{\alpha}_1}}{\frac{1}{\hat{\alpha}_1}s + 1} \tag{6.132}$$

where $\frac{\hat{\alpha}_0}{\hat{\alpha}_1}$ is the dimensionless open-loop gain, and $\frac{1}{\hat{\alpha}_1}$ is the time-constant of the system.

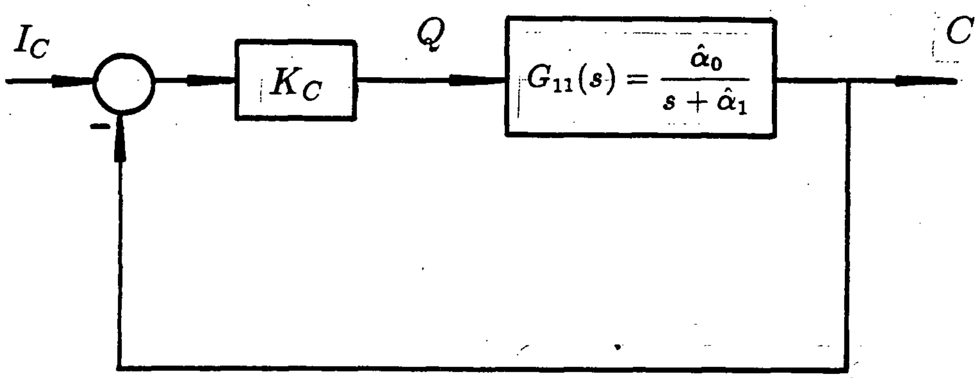


Figure 6.22: Concentration control system without time-delay

The closed-loop transfer function between the set point $I_C(s)$ and the controlled variable $C(s)$ is

$$\frac{C(s)}{I_C(s)} = \frac{K_C G_{11}(s)}{1 + K_C G_{11}(s)} = \frac{K_C \frac{\hat{\alpha}_0}{s+\hat{\alpha}_1}}{1 + K_C \frac{\hat{\alpha}_0}{s+\hat{\alpha}_1}} = \frac{K_C \hat{\alpha}_0}{s + (\hat{\alpha}_1 + K_C \hat{\alpha}_0)} \tag{6.133}$$

where K_C is the amplifier gain or proportional sensitivity.

Unit-step Response of the System: Since the Laplace transform of the unit-step function is $\frac{1}{s}$, substituting $I_C(s) = \frac{1}{s}$ into Equation 6.133, we obtain

$$\begin{aligned} C(s) &= \frac{1}{s}[\frac{K_C \hat{\alpha}_0}{s + (\hat{\alpha}_1 + K_C \hat{\alpha}_0)}] \\ &= \frac{K_C \hat{\alpha}_0}{\hat{\alpha}_1 + K_C \hat{\alpha}_0}[\frac{1}{s} - \frac{1}{s + (\hat{\alpha}_1 + K_C \hat{\alpha}_0)}] \end{aligned} \tag{6.134}$$

Consider Example 1 in Chapter 4, where $\hat{\alpha}_0 = 0.41456$ and $\hat{\alpha}_1 = 0.4245$ (See Appendix E) and taking the inverse Laplace transform of Equation 6.134, we obtain

$$\begin{aligned} C(t) &= \frac{K_C \hat{\alpha}_0}{\hat{\alpha}_1 + K_C \hat{\alpha}_0}[1 - e^{-(\hat{\alpha}_1 + K_C \hat{\alpha}_0)t}] \\ &= \frac{0.41456 K_C}{0.4245 + 0.41456 K_C}[1 - e^{-(0.4245 + 0.41456 K_C)t}] \end{aligned} \tag{6.135}$$

Equation 6.135 states that initially the output variable $C(t)$ is zero and finally it becomes $\frac{0.41456K_C}{0.4245+0.41456K_C}$ or unity if $0.41456K_C \gg 0.4245$. As seen from Equation 6.135, the steady-state is reached mathematically only after an infinite time. In practice, however, a reasonable estimate of the response time is the length of time the response curve needs to reach the 4% line of the final value, or four time constants. Note that a discrepancy between a set-point and a practical value is always of occurrence with proportional control. By regulating K_C, the system response can be improved. The concentration control system without time-delay is always stable for all values of K_C. Figure 6.23 shows that the bigger K_C, the better the response curve.

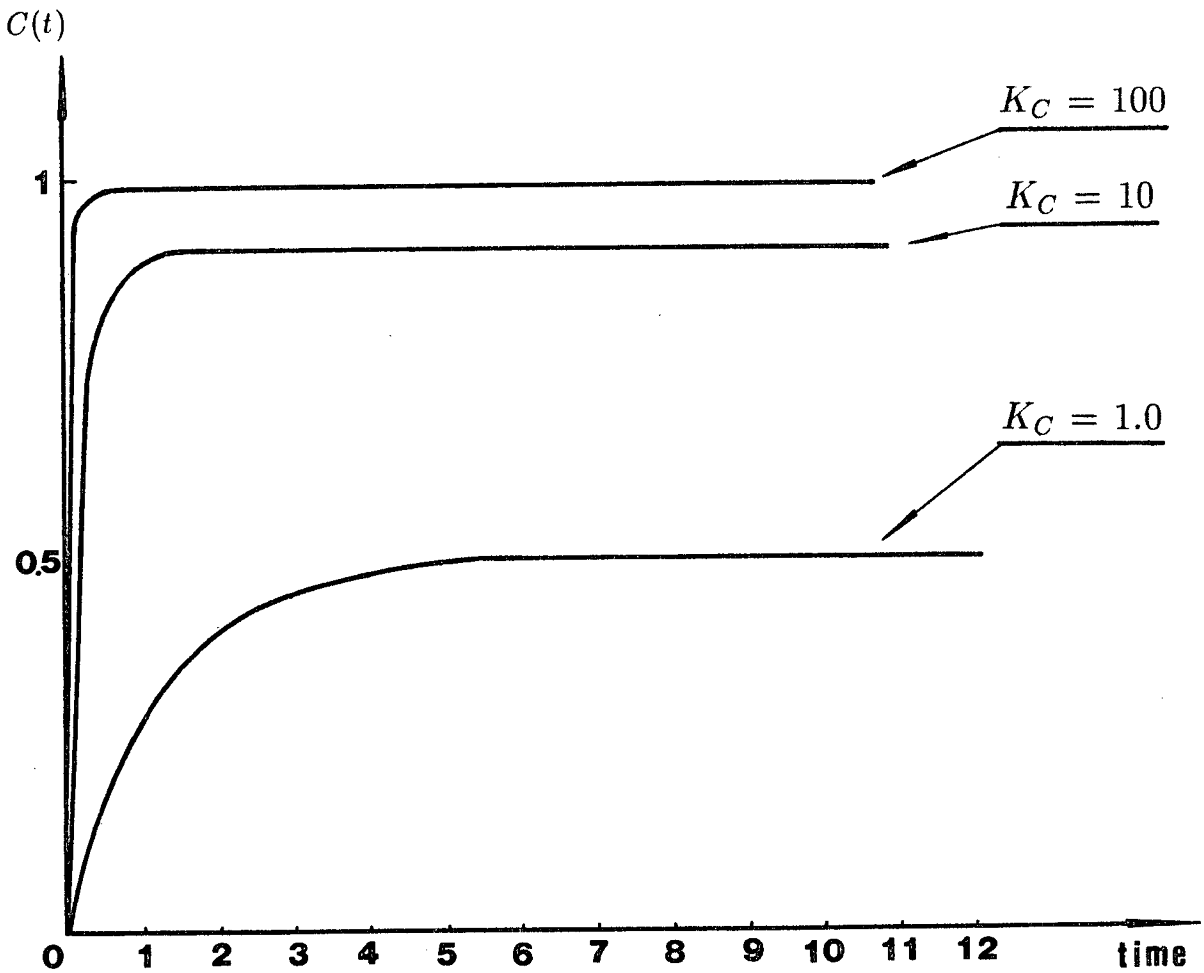


Figure 6.23: Unit-step response curves of the concentration control system without time-delay

6.3 Control of the Concentration with a Time-delay

Figure 6.24 illustrates a practical concentration control system with a measurement time-delay. The closed-loop transfer function is

$$\begin{aligned}\frac{C(s)}{I_C(s)} &= \frac{K_C G_{11}(s)}{1+K_C e^{-\tau_C s} G_{11}(s)} \\ &= \frac{K_C \frac{\hat{\alpha}_0}{s+\hat{\alpha}_1}}{1+K_C \frac{\hat{\alpha}_0}{s+\hat{\alpha}_1} e^{-\tau_C s}} \\ &= \frac{K_C \hat{\alpha}_0}{s+\hat{\alpha}_1+K_C \hat{\alpha}_0 e^{-\tau_C s}} \end{aligned} \tag{6.136}$$

Usually, $e^{-\tau_C s}$ can be approximated by

$$\begin{aligned} e^{-\tau_C s} &= \frac{e^{-\frac{1}{2}\tau_c s}}{e^{+\frac{1}{2}\tau_c s}} \\ &\approx \frac{1-\frac{\tau_C}{2}s}{1+\frac{\tau_C}{2}s} \\ &= \frac{2-\tau_C s}{2+\tau_C s} \end{aligned} \tag{6.137}$$

Equation 6.137 is the *Padé* approximation which is reasonablely accurate for many purposes.

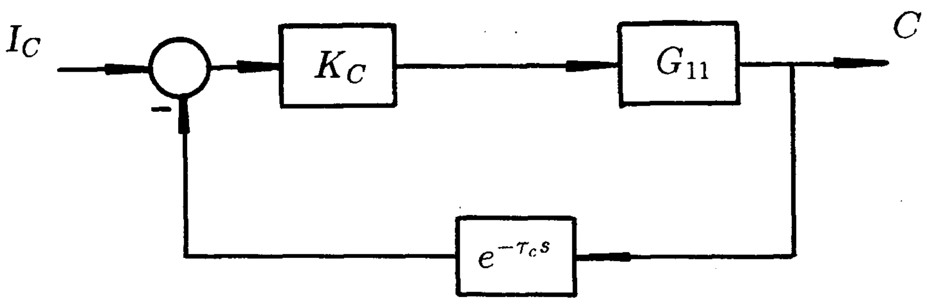


Figure 6.24: Control of the concentration process with measuring time-delay

Substituting Equation 6.137 into Equation 6.136, we get

$$\begin{aligned}\frac{C(s)}{I_C(s)} &= \frac{K_C\hat{\alpha}_0}{s+\hat{\alpha}_1+K_C\hat{\alpha}_0(\frac{2-\tau_C s}{2+\tau_C s})} \\ &= \frac{K_C\hat{\alpha}_0(2+\tau_C s)}{\tau_C s^2+(2+\hat{\alpha}_1\tau_C-K_C\hat{\alpha}_0\tau_C)s+2(\hat{\alpha}_1+K_C\hat{\alpha}_0)}\end{aligned} \tag{6.138}$$

The characteristic equation of the closed-loop system can be obtained by setting the denominator of Equation 6.138 equal to zero. As is well-known, the stability of a system is independent of the input excitation, and the characteristic equation determines system stability. Obviously, the coefficients in the characteristic equation of the closed-loop system may be less than zero when K_C is sufficiently large. For stability, all coefficients in the characteristic equation of second-order systems must be positive. Therefore, although the amplifier gain K_C of a first-order system can be set at a high value in the absence of time-delay, it cannot be set too high if time-delay is present. According to the Routh criterion, we have

$$2+\hat{\alpha}_1\tau_C-K_C\hat{\alpha}_0\tau_C>0 \tag{6.139}$$

or

$$K_C<\frac{2+\hat{\alpha}_1\tau_C}{\hat{\alpha}_0\tau_C} \tag{6.140}$$

For the system considered here, the value of the amplifier gain K_C must be less than $\frac{2+\hat{\alpha}_1\tau_C}{\hat{\alpha}_0\tau_C}$ for stable operation. Nevertheless, the smaller K_C, the poorer is the response of the system.

Equation 6.138 is a second-order system. Its standard analytical solutions are provided in Appendix F. The dynamic behavior of second-order systems can then be described in terms of two parameters ξ and ω_n.

By defining the undamped natural frequency ω_n and the damping factor ξ as

$$\omega_n^2=\frac{2(\hat{\alpha}_1+K_C\hat{\alpha}_0)}{\tau_C} \tag{6.141}$$

and

$$\xi = \frac{2 + \hat{\alpha}_1\tau_C - K_C\hat{\alpha}_0\tau_C}{2\tau_C}\frac{1}{\omega_n} = (\frac{2 + \hat{\alpha}_1\tau_C - K_C\hat{\alpha}_0\tau_C}{2\tau_C})\sqrt{\frac{\tau_C}{2(\hat{\alpha}_1 + K_C\hat{\alpha}_0)}} \tag{6.142}$$

Equation 6.138 can be rewritten as

$$\begin{aligned}\frac{C(s)}{I_C(s)} &= \frac{\frac{K_C\hat{\alpha}_0}{\tau_C}(2+\tau_C s)}{s^2 + \frac{2+\hat{\alpha}_1\tau_C - K_C\hat{\alpha}_0\tau_c}{\tau_c}s + \frac{2(\hat{\alpha}_1+K_C\hat{\alpha}_0)}{\tau_c}} \\ &= (\frac{2K_C\hat{\alpha}_0}{\tau_C\omega_n^2})[\frac{\omega_n^2(1+\frac{\tau_c}{2}s)}{s^2 + 2\xi\omega_n s + \omega_n^2}] \end{aligned} \tag{6.143}$$

The parameters ω_n and ξ are very important for characterizing a system's response. Note from Equation 6.143 that ω_n is the radian frequency of oscillation when $\xi = 0$. As ξ increases in value from 0, the oscillation decays and becomes more damped. When $\xi \geq 1$, oscillation does not occur. For a standard second-order system which is shown in the square brackets of Equation 6.143, as $t \to \infty$, its steady-state value under a unit-step input tends to one. Now, the steady-state value of the concentration control system depends on the closed-loop gain $\frac{2K_C\hat{\alpha}_0}{\tau_c\omega_n^2}$. Notice that the ultimate response, after $t \to \infty$, never reaches the desired set point. There is always a discrepancy called offset which is equal to

$$\text{offset} = (\text{set point}) - (\text{ultimate value of the response}) \tag{6.144}$$

The final value theorem (See Appendix B) provides a convenient way to find the steady-state performance of a system, thus

$$\begin{aligned}\text{offset} &= 1 - \lim_{t\to\infty} C(t)|_{closed-loop} \\ &= 1 - \lim_{s\to 0} sC(s)|_{closed-loop} \\ &= 1 - \lim_{s\to 0} sI_C(s)[\frac{K_C G_{11}(s)}{1 + K_C e^{-\tau_C s}G_{11}(s)}] \\ &= 1 - \lim_{s\to 0}\frac{K_C\frac{\hat{\alpha}_0}{\hat{\alpha}_1+s}}{1+K_C e^{-\tau_C s}\frac{\hat{\alpha}_0}{\hat{\alpha}_1+s}} \\ &= 1 - \frac{K_C\hat{\alpha}_0}{\hat{\alpha}_1 + K_C\hat{\alpha}_0}\end{aligned} \tag{6.145}$$

So

$$\begin{aligned}
\text{offset}|_{Min.} &= 1-\left(\frac{K_C\hat{\alpha}_0}{\hat{\alpha}_1+K_C\hat{\alpha}_0}\bigg|_{K_C=Max.}\right)\\
&= 1-\left(\frac{K_C\hat{\alpha}_0}{\hat{\alpha}_1+K_C\hat{\alpha}_0}\bigg|_{K_C=\frac{2}{\hat{\alpha}_0\tau_c}+\frac{\hat{\alpha}_1}{\hat{\alpha}_0}}\right)\\
&= 1-\frac{\frac{2}{\tau_c}+\hat{\alpha}_1}{2\hat{\alpha}_1+\frac{2}{\tau_c}}\\
&= \frac{\tau_C\hat{\alpha}_1}{2(\tau_C\hat{\alpha}_1+1)}\\
&= \begin{cases} 0 & \text{if } \tau_C=0\\ 0.5 & \text{if } \tau_C\hat{\alpha}_1\gg 1\end{cases}
\end{aligned} \tag{6.146}$$

Clearly, the bigger the time-delay, the bigger the offset, but the maximum offset tends to a limit of 0.5. The simulation results are shown in Figure 6.25 and Figure 6.26.

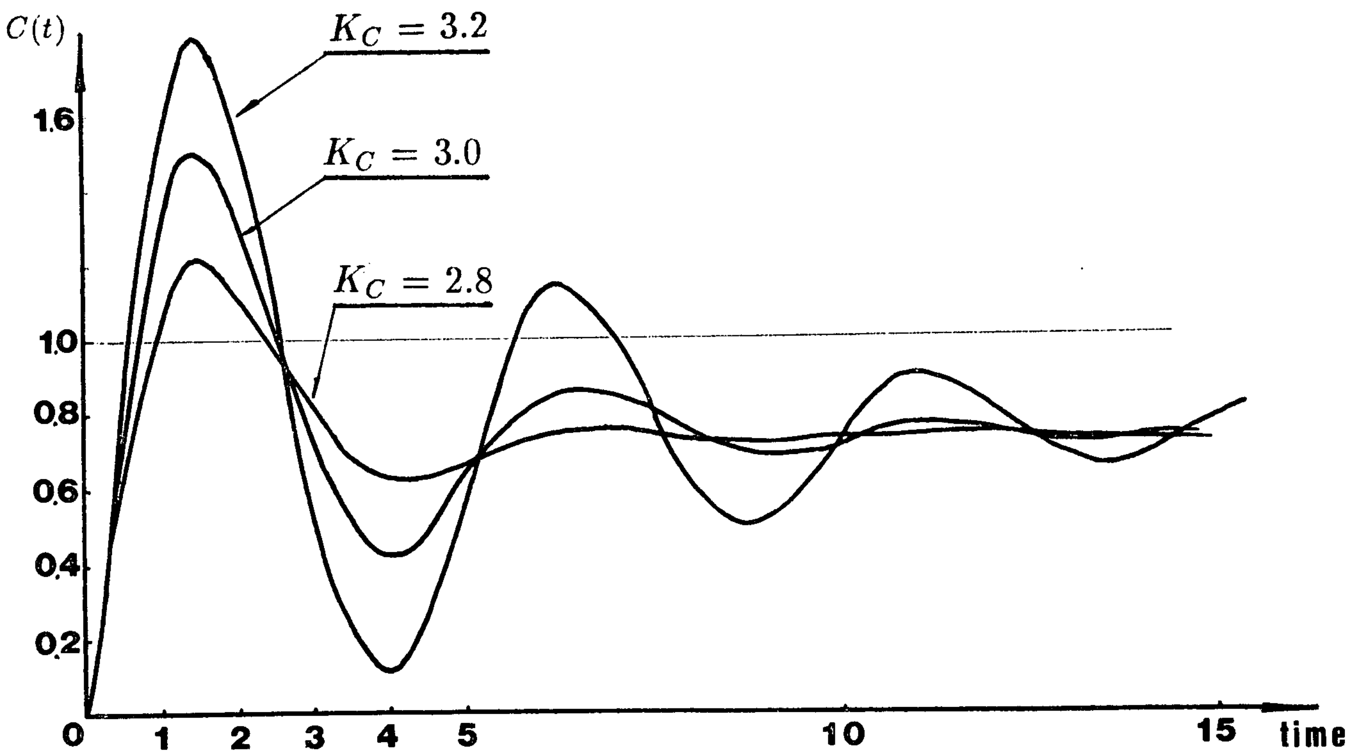
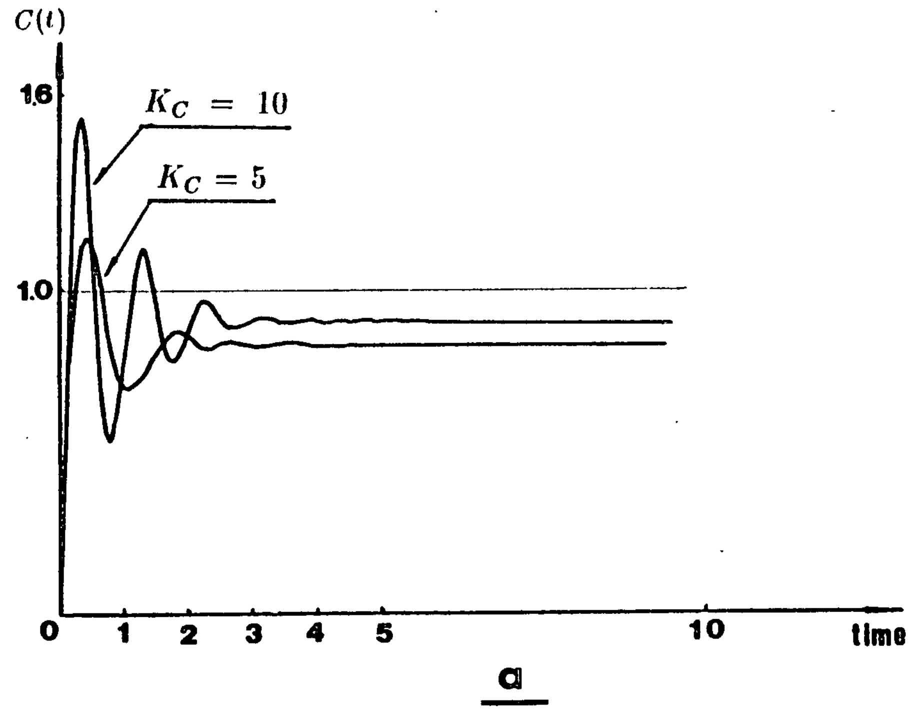


Figure 6.25: Unit-step response curves of the concentration control system with measuring time-delay. $\tau_C = 2$ sec.



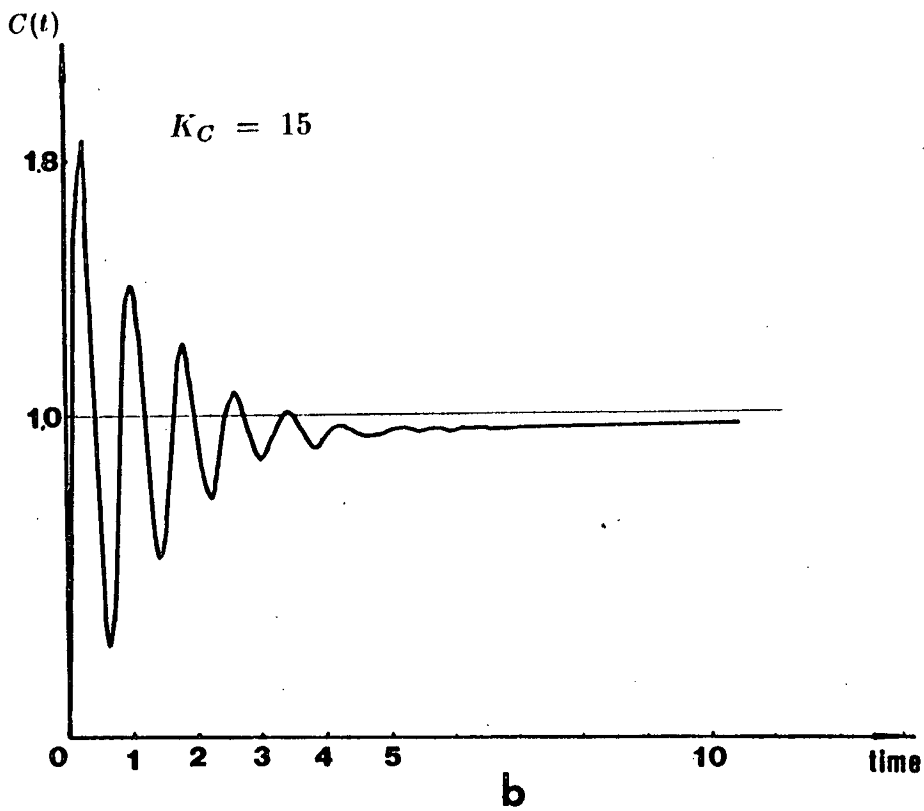


Figure 6.26: Unit-step response curves of the concentration control system with measuring time-delay. $\tau_C = 0.2$ sec. (a) the amplifier gains are 5 and 10. (b) the amplifier gain is 15.

6.4 Control of the Concentration Process with a Smith Compensator

Figure 6.27 shows a block diagram of the concentration control loop with a Smith compensator (Smith, 1957). The basic idea of the Smith scheme is very simple, namely let the closed-loop characteristic equation of a time-delay system which contains an intentional time-delay model be equal to a new characteristic equation without the time-delay factor. From Figure 6.27, the transfer function of the closed-loop is as follows

$$\frac{C(s)}{I_C(s)} = \frac{K_C G_{11}(s)}{1 + K_C[G_s(s)H_s(s) + e^{-\tau_C s}G_{11}(s)]} \tag{6.147}$$

where $G_s(s)H_s(s)$ is an intentional model.

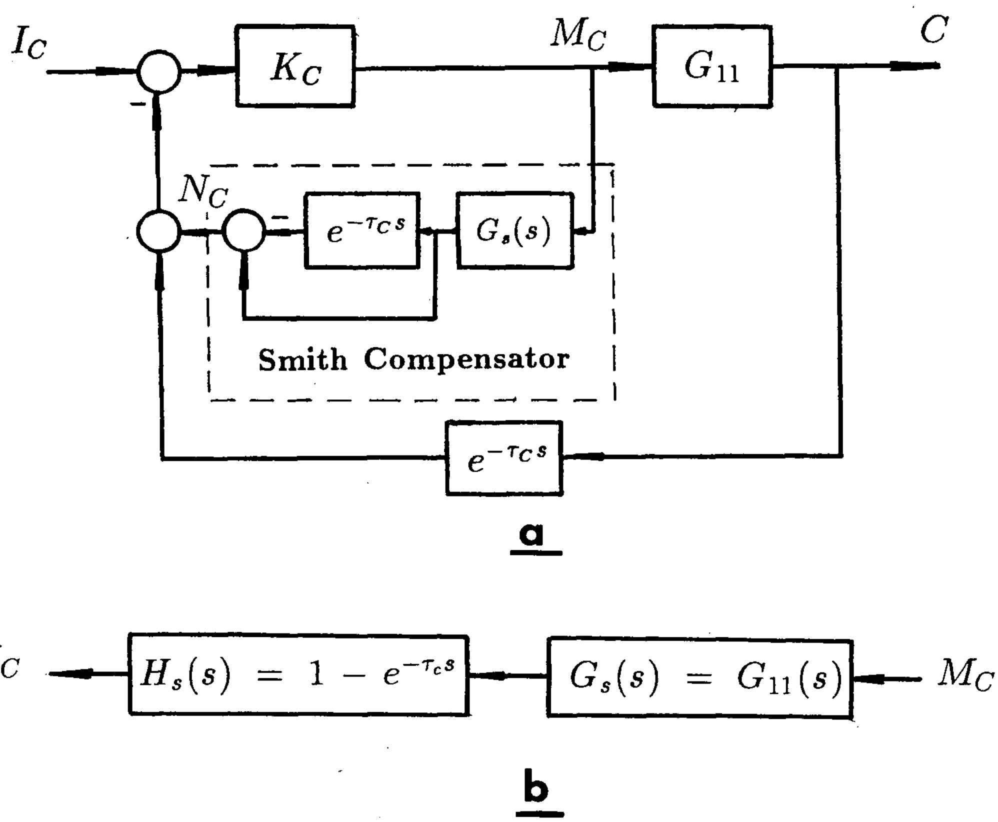


Figure 6.27: (a) Control of the concentration process with the Smith compensator. (b) A block diagram of the Smith compensator.

For the Smith compensator, substituting $H_s(s) = 1 - e^{-\tau_C s}$ and $G_s(s) = G_{11}(s)$ into Equation 6.147, we have

$$\begin{aligned} \frac{C(s)}{I_C(s)} &= \frac{K_C G_{11}(s)}{1 + K_C[G_{11}(s)(1 - e^{-\tau_C s}) + e^{-\tau_C s} G_{11}(s)]} \\ &= \frac{K_C G_{11}(s)}{1 + K_C G_{11}(s)} \end{aligned} \tag{6.148}$$

So, Equation 6.148 is the same as Equation 6.133, which is for the delay-free case. However, it is universally accepted that the block $H_s(s)$ is physically irrealizable because of the transcendental function $e^{-\tau_C s}$. On the other hand, if the real transfer function deviates from the theoretical model, then the characteristic equation of the closed-loop becomes

$$1 + K_C[G_{11}(s)(1 - e^{-\tau_C s} s) + e^{-\hat{\tau}_C s} \hat{G}_{11}(s)] = 0 \tag{6.149}$$

where $e^{-\hat{\tau}_C s} \hat{G}_{11}(s)$ is the real process. Many researchers have pointed out that the performance of the Smith scheme is very sensitive to the accuracy with which actual process time-delay is identified. For extensive discussions of the modelling error of Smith's scheme, see Buckley (1964), Marshall (1979), Palmor (1980) and Stephanopoulos (1984).

6.5 A Physically Realizable Time-delay Compensator

One of the most important qualitative properties of a control system is its stability. One feature of the ideal Smith's scheme is that the time-delay factor $e^{-\tau_c s}$ can be eliminated in the system characteristic equation by subtraction, i.e., if the Smith model and process control model are in exact agreement, the term in square brackets of Equation 6.149 becomes $G_{11}(s)$. However, in practical situations, the time-delay of a feedback analyzer is time-variant. So, an improperly designed control system can lead to system instability, mistrust by operators and maintenance headaches. In order to improve the stability of the time-delay compensator, a proposed compensation scheme is presented in Figure

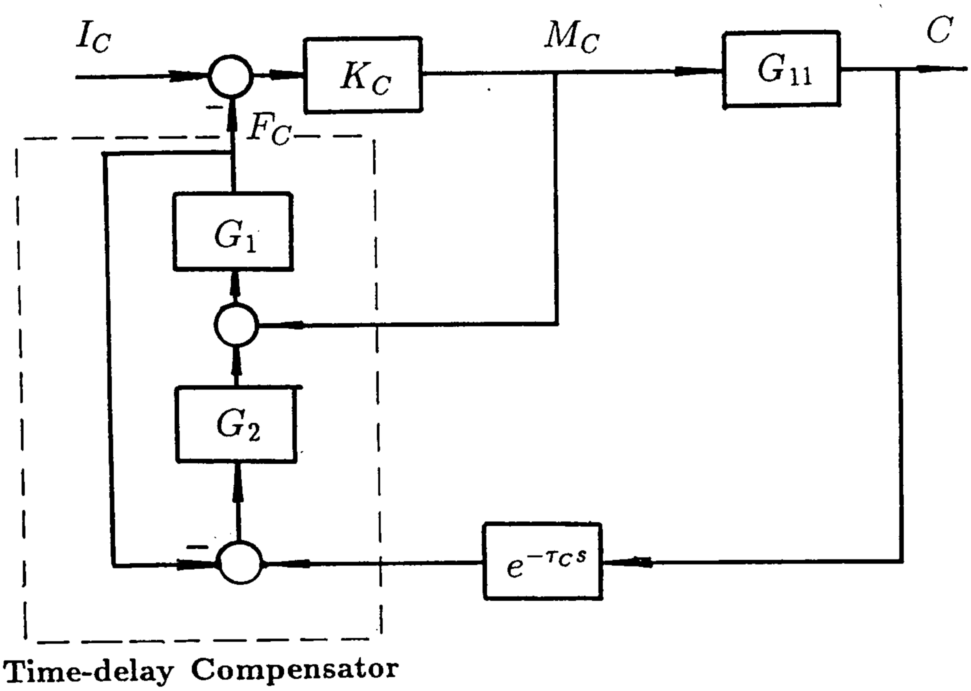


Figure 6.28: A physically realizable time-delay compensator

6.28 to eliminate the effect of time-delay through division. The transfer function of Figure 6.28 between $C(s)$ and $I_C(s)$ is

$$\begin{aligned}\frac{C(s)}{I_C(s)} &= \frac{K_C G_{11}(s)[1+G_1(s)G_2(s)]}{[1+G_1(s)G_2(s)]+K_C G_1(s)[1+G_2(s)e^{-\tau_C s}G_{11}(s)]} \\ &= \frac{K_C G_{11}(s)}{1+K_C G_1(s)\frac{1+G_2(s)e^{-\tau_C s}G_{11}(s)}{1+G_1(s)G_2(s)}}\end{aligned} \tag{6.150}$$

The detailed derivation of Equation 6.150 is shown in Appendix G.

As mentioned before. the magnitude of $e^{-\tau_c s}$ is always unity. So, if

$$|G_2(s)e^{-\tau_c s}G_{11}(s)| \ll 1 \tag{6.151}$$

$$|G_1(s)G_2(s)| \ll 1 \tag{6.152}$$

then

$$\frac{C(s)}{I_C(s)} \approx \frac{K_C G_{11}(s)}{1+K_C G_1(s)} \tag{6.153}$$

When $G_1(s) = G_{11}(s)$, Equation 6.153 is the same as Equation 6.148 which is for the perfect Smith compensator.

6.5.1 Stability Analysis

For $G_{11}(s) = \frac{\hat{\alpha}_0}{\hat{\alpha}_1+s}$, let $G_1(s) = K_1$ and $G_2(s) = K_2$, then Equation 6.150 becomes

$$\begin{aligned}
\frac{C(s)}{I_C(s)} &= \frac{K_C\frac{\hat{\alpha}_0}{\hat{\alpha}_1+s}}{1 + K_C K_1 \frac{1+K_2(\frac{2-\tau_c s}{2+\tau_c s})(\frac{\hat{\alpha}_0}{\hat{\alpha}_1+s})}{1+K_1K_2}} \\
&= \frac{(1+K_1K_2)K_C\frac{\hat{\alpha}_0}{\hat{\alpha}_1+s}}{1+K_1K_2+K_CK_1[1+K_2(\frac{2-\tau_c s}{2+\tau_c s})(\frac{\hat{\alpha}_0}{\hat{\alpha}_1+s})]} \\
&= \frac{(1+K_1K_2)K_C\hat{\alpha}_0(2+\tau_c s)}{a_2 s^2 + a_1 s + a_0}
\end{aligned} \tag{6.154}$$

where

$$a_0 = 2[(1+K_1K_2+K_1K_C)\hat{\alpha}_1 + K_1K_2K_C\hat{\alpha}_0] \tag{6.155}$$

$$a_1 = (1+K_1K_2+K_1K_C)(2+\tau_C\hat{\alpha}_1) - K_1K_2K_C\tau_C\hat{\alpha}_0 \tag{6.156}$$

$$a_2 = \tau_C(1+K_1K_2+K_1K_C) \tag{6.157}$$

For stability,

$$(1+K_1K_2+K_1K_C)(2+\tau_C\hat{\alpha}_1) - K_1K_2K_C\tau_C\hat{\alpha}_0 > 0 \tag{6.158}$$

or

$$(1+K_1K_2)(2+\tau_C\hat{\alpha}_1) > K_1K_C(K_2\tau_C\hat{\alpha}_0 - 2 - \tau_C\hat{\alpha}_1) \tag{6.159}$$

Obviously, if $K_2\tau_C\hat{\alpha}_0 - 2 - \tau_C\hat{\alpha}_1 > 0$, then

$$K_C < \frac{(1+K_1K_2)(2+\tau_C\hat{\alpha}_1)}{K_1(K_2\tau_C\hat{\alpha}_0 - 2 - \tau_C\hat{\alpha}_1)} \tag{6.160}$$

if $K_2\tau_C\hat{\alpha}_0 - 2 - \tau_C\hat{\alpha}_1 < 0$, then

$$K_C > \frac{(1+K_1K_2)(2+\tau_C\hat{\alpha}_1)}{K_1(K_2\tau_C\hat{\alpha}_0 - 2 - \tau_C\hat{\alpha}_1)} \tag{6.161}$$

Notice that the right-side of Equation 6.161 is less then zero. Therefore, the value of the amplifier gain K_C, which is always greater than zero, is not constrained at all, that is to say, for all values of K_C, the concentration control system is stable.

On the other hand, in order to satisfy Equations 6.151, 6.152 and 6.161, the conditions are

$$K_2\tau_C\hat{\alpha}_0 - 2 - \tau_C\hat{\alpha}_1 < 0 \tag{6.162}$$

$$K_2 \ll \frac{1}{|G_{11}(s)|} \tag{6.163}$$

$$K_2 \ll \frac{1}{|G_1(s)|} \tag{6.164}$$

For chemical processes, the system frequency ω, in general, is very low. So, Equation 6.163 becomes

$$K_2 \ll \frac{1}{\sqrt{\frac{\hat{\alpha}_0^2}{\omega^2+\hat{\alpha}_1^2}}} \approx \frac{\hat{\alpha}_1}{\hat{\alpha}_0} \tag{6.165}$$

In practice, assuming $K_1 = \frac{\hat{\alpha}_0}{\hat{\alpha}_1} = 0.976$ (see Appendix E) and K_2 is less than one tenth of the magnitude of $G_{11}(s)$, we get

$$K_2 = 0.1\frac{\hat{\alpha}_1}{\hat{\alpha}_0} = 0.1\frac{0.4245}{0.41456} \approx 0.1 \tag{6.166}$$

Thus, Equations 6.151, 6.152 and 6.161 are satisfied. The transfer function of the concentration control system becomes

$$\frac{C(s)}{I_C(s)} = \frac{\frac{K_C\hat{\alpha}_0}{\hat{\alpha}_1+s}}{1+K_CK_1} = \frac{K_C\hat{\alpha}_0}{(1+K_CK_1)(\hat{\alpha}_1+s)} \tag{6.167}$$

For a unit-step input, we get

$$C(s) = \frac{1}{s}\frac{K_C\hat{\alpha}_0}{(1+K_CK_1)(\hat{\alpha}_1+s)} \tag{6.168}$$

The time-domain solution of Equation 6.168 is

$$C(t) = \frac{K_C\hat{\alpha}_0}{(1+K_CK_1)\hat{\alpha}_1}(1-e^{-\hat{\alpha}_1 t}) \tag{6.169}$$

and

$$
\begin{aligned}
\text{offset} &= 1 - \lim_{s\to 0} sC(s)|_{closed\ loop} \\
&= 1 - \lim_{s\to 0} sI_C(s)[\frac{K_C G_{11}(s)}{1 + K_C G_1(s)\frac{1+K_2 e^{-\tau_C s} G_{11}(s)}{1+K_2 G_1(s)}}] \\
&= 1 - \frac{K_C \frac{\hat{\alpha}_0}{\hat{\alpha}_1}}{1 + K_C K_1 \frac{1+K_2\frac{\hat{\alpha}_0}{\hat{\alpha}_1}}{1+K_2 K_1}} \\
&= 1 - \frac{K_C \frac{\hat{\alpha}_0}{\hat{\alpha}_1}}{1 + K_C \frac{\hat{\alpha}_0}{\hat{\alpha}_1} \frac{1+K_2\frac{\hat{\alpha}_0}{\hat{\alpha}_1}}{1+K_2\frac{\hat{\alpha}_0}{\hat{\alpha}_1}}} \\
&= 1 - \left.\frac{K_C \frac{\hat{\alpha}_0}{\hat{\alpha}_1}}{1 + K_C \frac{\hat{\alpha}_0}{\hat{\alpha}_1}}\right|_{K_C\to\infty} \\
&= 0
\end{aligned}
\tag{6.170}
$$

Figure 6.29 shows the unit-step response of Equation 6.153.

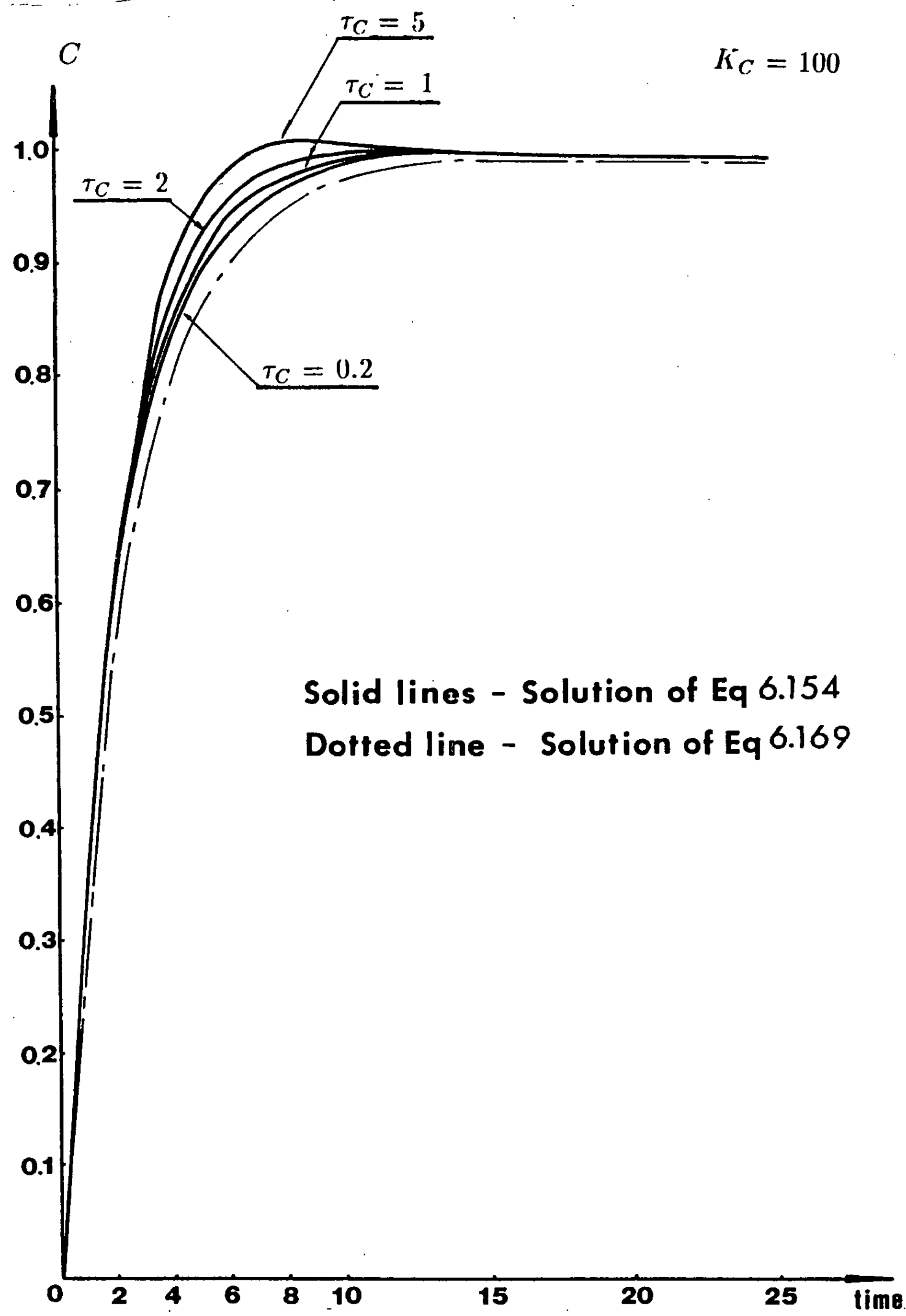


Figure 6.29: The unit-step response of the concentration control loop with a physically realizable time-delay compensator. $K_C = 100$, $K_1 = 0.976$ and $K_2 = 0.1$.

6.5.2 A Few Comments on the Control Mechanism

1. The time-delay compensation of Figure 6.7 possesses two advantages. The first of these is that of avoiding the solution of the transcendental function $e^{-\tau_C s}$ which is physically irrealizable, and the second is the inherent stability of the compensation structure which can withstand a time-variant time-delay.

2. The present method virtually retains the benefits of feedforward control. The output of the controller is through both the process model $G_{11}(s)$ and the feedforward model $G_1(s)$ without a time-delay factor. The dynamic response error of the system can be reduced by comparing the input $I_C(s)$ and the output of feedforward model $G_1(s)$ (see Figure 6.28).

3. Basically, the measuring delay can be reduced, but it can not be totally eliminated by $G_2(s)$. When the system reaches a steady-state, the difference between the process output and the feedforward model output can improve the steady-state response of the system, i.e., the offset will tend to zero.

4. In practice, $G_1(s)$ may be a process model $G_{11}(s)$ or any other compensator, such as a lead compensator or a lag compensator or a lag-lead compensator. Also, $G_2(s)$ may have any structure, but the magnitude of $G_2(s)$ must be small in order to satisfy Equations 6.151, 6.152 and 6.162.

Chapter 7

Conclusions and Suggestions

7.1 Conclusions

1. A measure of the interaction of the two-variable CFSTR control system has been derived and is given in Equation 4.84, which can provide information on interaction for a process designer before setting up a two-variable CFSTR system. Once the process parameters and design parameters are known, in theory, the relative gain value of the interaction can be calculated easily from Equation 4.84. If the relative gain value is greater than 1.5, then a decoupling design should be considered. If the relative gain value is less than 1.5 and tends to 1, then the two-variable CFSTR control system can be regarded as two single control loops, that is to say, the interaction between the concentration loop and the temperature loop can be neglected.

2. For the simplified decoupling design of a CFSTR process, the modelling error can probably cause an unstable behaviour. Nevertheless, if the simplified decoupled CFSTR system can work with undercompensation, the control system gives good stability. It is worth stressing that the product of two compensation factors (e_1 and e_2) must be less than one for the CFSTR system not to have nonlinear divergence. In a practical design, the compensation factors (e_1 and e_2) can be considered as two proportional amplifiers which are physically realizable.

3. Generally speaking, if the time-delay value is greater than one tenth of the system

time-constant, time-delay compensation is necessary. As in Example 1 of Chapter 4, when a time-delay (τ_C) is 0.2 (sec), which is less than one tenth of the system time-constant ($\frac{1}{\hat{\alpha}_1} = 2.36$ sec), Figure 6.26 shows that the concentration loop may work under the high-gain amplification, but rather grudgingly. In order to compensate a big time-variant time-delay, the compensation scheme of Figure 6.28 is proposed which can rely on the basic property of gain-invariant time-delay. In other words, the compensation structure only depends on the magnitude of both $G_{11}(s)$ and $G_1(s)$, no matter how big the control system time-delay is. Stability analysis indicates that if $K_2 < \frac{2-\tau_c\hat{\alpha}_1}{\tau_c\hat{\alpha}_0}$ (See Equation 6.161), for all values of K_C the concentration control system is stable. Besides the compensation structure of Figure 6.28 is easily physically realizable, and it has the same features as the Smith compensator when $G_1(s) = G_{11}(s)$.

4. The scheme of Figure 6.28 hold true for the temperature control loop time-delay compensation. The design procedures for the temperature loop are analogous to those for the concentration loop time-delay compensation.

5. Figure 7.30 shows an overall decoupling compensator and time-delay compensator for a two-variable CFSTR control system, which has thus been made to react like two separate delay-free single control loops. All compensation structures are physically realizable.

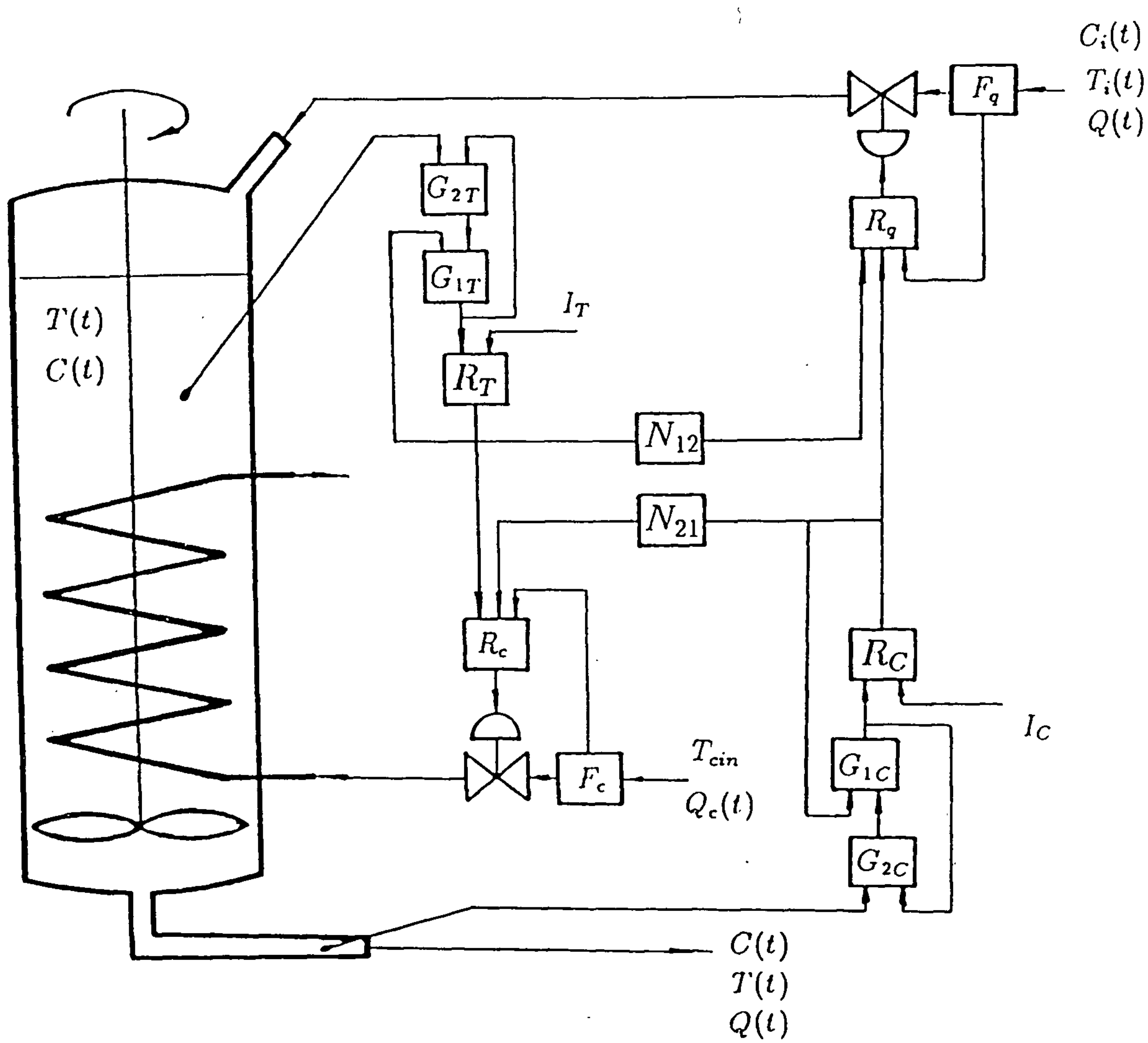
7.2 Suggestions

This study of a two-variable CFSTR system is relatively abstract and some questions still remain unanswered. To reach wide acceptance for practical use, further research needs to be carried out covering:

1. Equation 4.84 gives a theoretical analysis of the interaction whereas the practical significant interaction analysis should rely on Equation 4.83 which comes directly from the definition of the interaction. Therefore, the relative gain from measuring the response of a real CFSTR system should be obtained for comparison with a theoretical value.

2. As mentioned in Section 6.5.2., both $G_1(s)$ and $G_2(s)$ models can have different kinds of structures. Which structures is better for the system's dynamic performance? This should be studied further.

3. The strongly nonlinear process compensation and disturbance rejection should receive further attention to assure that the CFSTR system can be operated over a wider range.



C:	effluent concentration of reactant A	Q:	volumetric flow rate
C_i:	inlet concentration of reactant A	Q_c:	coolant flow rate
F_c:	coolant flow rate measuring device	R_c:	coolant flow rate controller
F_q:	reactant flow rate measuring device	R_C:	concentration controller
G_{1C}:	concentration process compensator	R_q:	reactant flow rate controller
G_{1T}:	temperature process compensator	R_T:	temperature controller
G_{2C}:	concentration time-delay compensator	t:	time
G_{2T}:	temperature time-delay compensator	T:	temperature in a reactor
I_T:	temperature set point	T_{cin}:	inlet temperature of coolant
I_C:	concentration set point	T_i:	inlet temperature of reactant A
N_{12}, N_{21}:	decoupling compensator		

Figure 7.30: An overall control system of the CFSTR with decoupling compensation and time-delay compensation.

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Notation

Roman

		Reference or Equation
a_i	Coefficient	6.155~ 6.157
A_K	Heat exchange surface	3.4
A_r	Frequency factor	3.9
C	Effluent concentration of reactant A	3.1
c_c	Specific heat of coolant	3.3
C_i	Inlet concentration of reactant A	3.1
C_{i0}	Steady-state inlet concentration	3.14
C_0	Steady-state concentration operating condition	3.14
c_p	Specific heat of reacting mixture	3.2
d	Deviation factor	5.126
d_{ij}	Dynamic gains of disturbance channel	Fig. 3.2
D_{cij}	Closed-loop transfer function of disturbance channel	3.41, Appendix C
D_{ij}	Transfer function of disturbance channel	3.32, 3.37~ 3.40
e	Compensation factor	pp.63, 5.122
e_i	Compensation factor	5.116, 5.117
$e^{-\tau_C s}$	Laplace transform of concentration feedback time-delay	Fig. 3.4, Appendix B
$e^{-\tau_T s}$	Laplace transform of temperature feedback time-delay	Fig. 3.4, Appendix B
E	Activation energy	3.9
F_c	Coolant flow rate measuring device	Fig. 1.1
F_C	Feedback parameter	Fig. 6.28, Appendix G

$\hat{F}_{11}$	Uncertain open-loop gain	5.126
F_{ij}	Open-loop gain coefficient between M_i and Y_1	4.47~ 4.50
F_{ijd}	Open-loop gain coefficient of decoupled system	5.106~ 5.109
F_q	Reactant flow rate measuring device	Fig. 1.1
$F(s)$	Laplace transform of $f(t)$ or $\Delta f(t)$	3.26, 3.27, Appendix B
g_{ij}	Dynamic gain of a 2×2 system	Fig. 3.2
G_1	Process compensator	Fig. 6.28, 6.150
G_{1C}	Concentration process compensator	Fig. 7.30
G_{1T}	Temperature process compensator	Fig. 7.30
G_2	Time-delay compensator	Fig. 6.28, 6.150
G_{2C}	Concentration time-delay compensator	Fig. 7.30
G_{2T}	Temperature time-delay compensator	Fig. 7.30
G_{cij}	Process closed-loop transfer function	3.14, Appendix C
G_{ij}	Process open-loop transfer function	3.32~ 3.36
G_s	Smith compensation of process	6.147, Fig. 6.27
$h[T(t)]$	Heat addition or removal from a reactor	3.2
H_s	Smith compensation of time-delay	6.147, Fig. 6.27
I_i	Set point or input variable	Fig. 4.6
I_C	Concentration set point	Fig. 1.1, 3.41
I_T	Temperature set point	Fig. 1.1, 3.41
J_{ij}	Steady-state gain of disturbance channel	Fig. 3.2
k_{ij}	Gain of compensator	5.102, 5.103, Fig. 5.16
K	Reaction-rate constant	3.8
K_{ij}	Steady-state gain of a 2×2 system	Fig. 3.2, Fig. 5.16
K_1	Gain of process compensator	pp.85, 6.154
K_2	Gain of time-delay compensator	pp.85, 6.154

K_C	Amplifier gain	Fig. 6.22
$L[\_]$	Notation of Laplace transform	Appendix B
L_{11}	Channel gain from C to Q	Fig. 4.11, 4.66
L_{12}	Channel gain from T to Q	Fig. 4.11, 4.66
L_{21}	Channel gain from C to Q_c	Fig. 4.12, 4.67
L_{22}	Channel gain from T to Q_c	Fig. 4.12, 4.67
M_C	Manipulated variable of concentration	Fig. 5.13
M_i	Manipulated variable	Fig. 4.5
M_T	Manipulated variable of temperature	Fig. 5.13~5.15
n	Reaction order	3.8
N_C	Output of Smith compensator	Fig. 6.27
N_{ij}	Decoupling compensator	5.85, Fig. 5.13
p_i	Coefficient	Appendix F
$P(s)$	Open-loop characteristic equation of a CFSTR	3.30
$P_c(s)$	Closed-loop characteristic equation of a CFSTR	3.46, Appendix C
Q	Volumetric flow rate	3.1
Q_c	Coolant flow rate	3.3
Q_{c0}	Coolant flow rate steady-state operating condition	3.15
Q_0	Reactant flow rate steady-state operating condition	3.14
r	Rate of reaction	3.8
R	Gas constant	3.9
R_i	Controller	Fig. 4.6
R_c	Coolant flow rate controller	Fig. 1.1
R_C	Concentration controller	Fig. 1.1, pp.28
R_q	Reactant flow rate controller	Fig. 1.1
R_T	Temperature controller	Fig. 1.1, pp.28

s	Complex variable of Laplace transform	Appendix B
S_{ij}	Closed-loop gain coefficient	4.51$\sim$ 4.54
t	Time	3.1
$\overline{T}_c$	Average coolant temperature	3.4
T	Temperature in a reactor	3.2
T_i	Inlet temperature of reactant A	3.2
T_{cin}	Inlet temperature of coolant	3.3
T_{cout}	Outlet temperature of coolant	3.3
T_{i0}	Steady-state inlet temperature	3.15
T_0	Steady-state temperature operating condition	3.14
U	Overall heat transfer coefficient	3.4
V	Reactor volume	3.1
Y	Output variable	Appendix F
Y_j	Output variable	Fig. 4.5

Greek

$\hat{\alpha}_0$	Transformed coefficient	6.132, Appendix E
$\hat{\alpha}_1$	Transformed coefficient	6.132, Appendix E
α_i	Coefficient	3.16$\sim$3.19
β_i	Coefficient	3.20$\sim$3.23
∂	Increment	pp.43
ρ_c	Fluid density of coolant	3.3
ρ_f	Fluid density of reacting mixture	3.2
Δ	Increment	pp.22, pp.23, Appendix A
ΔH	Heat of reaction	3.2
τ_C	Concentration feedback time-delay	pp.28, Appendix C

τ_T	Temperature feedback time-delay	pp.28, Appendix C
$\hat{\lambda}_{11}$	Uncertain relative gain	5.127
$\hat{\lambda}_{11d}$	Uncertain relative gain of a decoupled system	5.130
λ_{ij}	Relative gain between M_i and Y_j	4.55∼ 4.58
λ_{ijd}	Relative gain of decoupled system	5.110
ξ	Damping factor	6.142, Appendix F
ω	System frequency	pp.86, 6.165
ω_d	Damped natural frequency	Appendix F
ω_n	Undamped natural frequency	6.141, Appendix F

Appendix A

The Taylor Series Expansion for a System with Two Dependent Variables

Consider a nonlinear system whose output y is a function of two dependent inputs x_1 and x_2, so that

$$y = f(x_1, x_2) \tag{A.171}$$

In order to obtain a linear approximation to this nonlinear system, we may expand Equation A.171 into a Taylor series about the normal operating point y_0, x_{1_0}, x_{2_0}. Then Equation A.171 becomes

$$\begin{aligned} y &= f(x_1, x_2) \\ &= f(x_{1_0}, x_{2_0}) + \left[\left. \frac{\partial f}{\partial x_1} \right|_{\substack{x_1 = x_{1_0} \\ x_2 = x_{2_0}}} \cdot (x_1 - x_{1_0}) \right. \\ &\quad \left. + \left. \frac{\partial f}{\partial x_2} \right|_{\substack{x_1 = x_{1_0} \\ x_2 = x_{2_0}}} \cdot (x_2 - x_{2_0}) \right] \\ &\quad + \text{higher-order terms} \end{aligned} \tag{A.172}$$

Near the normal operating point, the higher-order terms may be neglected. The linear mathematical model of this nonlinear system in the neighborhood of the normal operating condition is then given by

$$
\begin{aligned}
y \quad = \quad & f(x_1, x_2) \approx f(x_{1_0}; x_{2_0}) \\
& + \left.\frac{\partial f}{\partial x_1}\right|_{\substack{x_1 = x_{1_0} \\ x_2 = x_{2_0}}} \cdot (x_1 - x_{1_0}) \\
& + \left.\frac{\partial f}{\partial x_2}\right|_{\substack{x_1 = x_{1_0} \\ x_2 = x_{2_0}}} \cdot (x_2 - x_{2_0}) \qquad \text{(A.173)}
\end{aligned}
$$

Lettting

$$K_1 = \left.\frac{\partial f}{\partial x_1}\right|_{\substack{x_1 = x_{1_0} \\ x_2 = x_{2_0}}} \qquad \text{(A.174)}$$

$$K_2 = \left.\frac{\partial f}{\partial x_2}\right|_{\substack{x_1 = x_{1_0} \\ x_2 = x_{2_0}}} \qquad \text{(A.175)}$$

$$\Delta x_1 = x_1 - x_{1_0} \qquad \text{(A.176)}$$

$$\Delta x_2 = x_2 - x_{2_0} \qquad \text{(A.177)}$$

$$\Delta y = y - y_0 \qquad \text{(A.178)}$$

So

$$\Delta y \approx K_1 \Delta x_1 + K_2 \Delta x_2 \qquad \text{(A.179)}$$

Appendix B

Laplace Transformation

B.1 Delay Function

In order to obtain the Laplace transform of the delay function $f(t-\tau)$, $f(t)$ is assumed to be zero for $t < 0$ or $f(t-\tau) = 0$ for $t < \tau$. Then, for $0 < t < \tau$, we have

$$\begin{aligned} F(s) &= \int_0^\infty f(t-\tau)e^{-s(t-\tau)}dt \\ &= e^{\tau s}\int_0^\infty f(t-\tau)e^{-st}dt \end{aligned} \tag{B.180}$$

Thus,

$$L[f(t-\tau)] = \int_0^\infty f(t-\tau)e^{-st}dt = e^{-\tau s}F(s) \tag{B.181}$$

This last equation states that the time-delay of function $f(t)$ by τ corresponds to the multiplication of the $F(s)$ by $e^{-\tau s}$.

B.2 Final Value Theorem

If $f(t)$ and $df(t)/dt$ are Laplace transformable, if $\lim_{t\to\infty} f(t)$ exits, then let s approach zero in the equation for the Laplace transform of the derivative of $f(t)$, or

$$\lim_{s\to 0}\int_0^\infty \left[\frac{d}{dt}f(t)\right]e^{-st}dt = \lim_{s\to 0}[sF(s) - f(0)] \tag{B.182}$$

Since $\lim_{s\to 0} e^{-st} = 1$, we obtain

$$\int_0^\infty \left[\frac{d}{dt}f(t)\right]dt = f(\infty) - f(0) = \lim_{s\to 0} sF(s) - f(0) \tag{B.183}$$

So,

$$f(\infty) = \lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s) \tag{B.184}$$

Appendix C

Derivation of the Closed-loop Transfer Function for the CFSTR with Time-delay

For Figure 3.4, we have

$$\begin{aligned}
\begin{bmatrix} C(s) \\ T(s) \end{bmatrix} &= \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} Q(s) \\ Q_c(s) \end{bmatrix} + \begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix} \begin{bmatrix} C_i(s) \\ T_i(s) \end{bmatrix} \\
&= \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} R_C(s) & 0 \\ 0 & R_T(s) \end{bmatrix} \begin{bmatrix} I_C(s) - e^{-\tau_C s} C(s) \\ I_T(s) - e^{-\tau_T s} T(s) \end{bmatrix} \\
&\quad + \begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix} \begin{bmatrix} C_i(s) \\ T_i(s) \end{bmatrix} \\
&= \begin{bmatrix} R_C(s)G_{11}(s) & R_T(s)G_{12}(s) \\ R_C(s)G_{21}(s) & R_T(s)G_{22}(s) \end{bmatrix} \left\{ \begin{bmatrix} I_C(s) \\ I_T(s) \end{bmatrix} - \begin{bmatrix} e^{-\tau_C s} & 0 \\ 0 & e^{-\tau_T s} \end{bmatrix} \begin{bmatrix} C(s) \\ T(s) \end{bmatrix} \right\} \\
&\quad + \begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix} \begin{bmatrix} C_i(s) \\ T_i(s) \end{bmatrix} \qquad \text{(C.185)}
\end{aligned}$$

For simplicity, letting

$$[G][R] = \begin{bmatrix} R_C(s)G_{11}(s) & R_T(s)G_{12}(s) \\ R_C(s)G_{21}(s) & R_T(s)G_{22}(s) \end{bmatrix} \qquad \text{(C.186)}$$

$$[I] = \begin{bmatrix} I_C(s) \\ I_T(s) \end{bmatrix} \qquad \text{(C.187)}$$

$$[E] = \begin{bmatrix} e^{-\tau_C s} & 0 \\ 0 & e^{-\tau_T s} \end{bmatrix} \quad (C.188)$$

$$[Y] = \begin{bmatrix} C(s) \\ T(s) \end{bmatrix} \quad (C.189)$$

$$[D] = \begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix} \quad (C.190)$$

$$[Y_i] = \begin{bmatrix} C_i(s) \\ T_i(s) \end{bmatrix} \quad (C.191)$$

$$[U] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (C.192)$$

Then, Equation C-185 can be rewritten as

$$[Y] = [G][R]\{[I] - [E][Y]\} + [D][Y_i] \quad (C.193)$$

or

$$[Y] = ([U] + [G][R][E])^{-1}([G][R][I] + [D][Y_i]) \quad (C.194)$$

Expanding Equation C-194, we get

$$\begin{bmatrix} C(s) \\ T(s) \end{bmatrix} = \frac{\begin{bmatrix} G_{c11}(s) & G_{c12}(s) \\ G_{c21}(s) & G_{c22}(s) \end{bmatrix} \begin{bmatrix} I_C(s) \\ I_T(s) \end{bmatrix} + \begin{bmatrix} D_{c11}(s) & D_{c12}(s) \\ D_{c21}(s) & D_{c22}(s) \end{bmatrix} \begin{bmatrix} C_i(s) \\ T_i(s) \end{bmatrix}}{P_c(s)} \quad (C.195)$$

where

$$G_{c11}(s) = [1 + e^{-\tau_T s} R_T(s) G_{22}(s)] R_C(s) G_{11}(s) - e^{-\tau_T s} R_C(s) R_T(s) G_{12}(s) G_{21}(s) \quad (C.196)$$

$$G_{c12}(s) = [1 + e^{-\tau_T s} R_T(s) G_{22}(s)] R_T(s) G_{12}(s) - e^{-\tau_T s} R_T^2(s) G_{12}(s) G_{22}(s) \quad (C.197)$$

$$G_{c21}(s) = [1 + e^{-\tau_C s} R_C(s)G_{11}(s)]R_C(s)G_{21}(s) - e^{-\tau_C s} R_C^2(s)G_{11}(s)G_{21}(s) \quad \text{(C.198)}$$

$$G_{c22}(s) = [1 + e^{-\tau_C s} R_C(s)G_{11}(s)]R_T(s)G_{22}(s) - e^{-\tau_C s} R_C(s)R_T(s)G_{12}(s)G_{21}(s) \quad \text{(C.199)}$$

$$D_{c11} = [1 + e^{-\tau_T s} R_T(s)G_{22}(s)]D_{11}(s) - e^{-\tau_T s} R_T(s)G_{12}(s)D_{21}(s) \quad \text{(C.200)}$$

$$D_{c12} = [1 + e^{-\tau_T s} R_T(s)G_{22}(s)]D_{12}(s) - e^{-\tau_T s} R_T(s)G_{12}(s)D_{22}(s) \quad \text{(C.201)}$$

$$D_{c21} = [1 + e^{-\tau_C s} R_C(s)G_{11}(s)]D_{21}(s) - e^{-\tau_C s} R_C(s)G_{21}(s)D_{11}(s) \quad \text{(C.202)}$$

$$D_{c22} = [1 + e^{-\tau_C s} R_C(s)G_{11}(s)]D_{22}(s) - e^{-\tau_C s} R_C(s)G_{21}(s)D_{12}(s) \quad \text{(C.203)}$$

$$P_c(s) = [1 + e^{-\tau_C s} R_C(s)G_{11}(s)][1 + e^{-\tau_T s} R_T(s)G_{22}(s)] - e^{-(\tau_C + \tau_T)s} R_C(s)R_T(s)G_{12}(s)G_{21}(s) \quad \text{(C.204)}$$

Appendix D

An Important Property of the Relative Gain for a 2×2 system

Bristol (1966) pointed out that the rows and columns of the RGA sum to 1.0. Therefore, for a 2×2 system, we have

$$\lambda_{11} + \lambda_{12} = 1$$

$$\lambda_{21} + \lambda_{22} = 1$$

$$\lambda_{11} + \lambda_{21} = 1$$

$$\lambda_{12} + \lambda_{22} = 1$$

thus

$$\lambda_{11} = \lambda_{22}$$

$$\lambda_{12} = \lambda_{21} = 1 - \lambda_{11}$$

So, the relative gain array (RGA) becomes

$$\begin{array}{cc} & \begin{array}{cc} M_1 & M_2 \end{array} \\ \begin{array}{c} Y_1 \\ Y_2 \end{array} & \left[\begin{array}{cc} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{array}\right] \end{array} \qquad \begin{array}{cc} & \begin{array}{cc} M_1 & M_2 \end{array} \\ \begin{array}{c} Y_1 \\ Y_2 \end{array} & \left[\begin{array}{cc} \lambda_{11} & 1 - \lambda_{11} \\ 1 - \lambda_{11} & \lambda_{11} \end{array}\right] \end{array} \tag{D.205}$$

Needless to say, the main channels ($M_1 \rightarrow Y_1$ and $M_2 \rightarrow Y_2$) have the same relative gain.

Appendix E

Dimensionless Variable Transformation

Consider

$$\frac{C(s)}{M_C(s)} = \frac{\alpha_0}{s + \alpha_1} \tag{E.206}$$

the differential equation form of Equation E-206 is

$$\frac{d\Delta C(t)}{dt} + \alpha_1 \Delta C(t) = \alpha_0 \Delta M_C(t) \tag{E.207}$$

where $C(\_)$ is the output variable and $M_C(\_)$ is the manipulated variable (See Equation 5.104), which are not the same dimension.

Letting

$$\hat{C}(t) = \frac{\Delta C(t)}{C_{max}} \tag{E.208}$$

$$\hat{M}_C(t) = \frac{\Delta M_C(t)}{M_{Cmax}} \tag{E.209}$$

where C_{max} and M_{Cmax} are the assumed maximum value of the output variable and the manipulated variable, respectively.

Dividing Equation E-207 by C_{max}, we get

$$\frac{d[\frac{\Delta C(t)}{C_{max}}]}{dt} + \alpha_1 \frac{\Delta C(t)}{C_{max}} = \frac{\alpha_0}{C_{max}} \Delta M_C(t) = \frac{\alpha_0 M_{Cmax}}{C_{max}} \hat{M}_C(t) \tag{E.210}$$

or

$$\frac{d\hat{C}(t)}{dt} + \hat{\alpha}_1 \hat{C}(t) = \hat{\alpha}_0 \hat{M}_C(t) \tag{E.211}$$

where $\hat{\alpha}_0 = \frac{\alpha_0 M_{Cmax}}{C_{max}}$ and $\hat{\alpha}_1 = \alpha_1$.

So,

$$\left.\frac{\hat{C}(s)}{\hat{M}_C(s)}\right|_{dimensionless} = \frac{\hat{\alpha}_0}{s + \hat{\alpha}_1} \tag{E.212}$$

For the convenience of expression (see Equation 6.132), we still write Equation E.212 as

$$\left.\frac{C(s)}{M_C(s)}\right|_{dimensionless} = \frac{\hat{\alpha}_0}{s+\hat{\alpha}_1}$$

Assuming $C_{max} = 1.3$ and $M_{Cmax} = 1.3$, then

$$\hat{\alpha_0} = \frac{\alpha_0 M_{Cmax}}{C_{max}} = \frac{6.347 \times 10^{-6} \times 1.3 \times 10}{1.3 \times 15.31 \times 10^{-5}} \approx 0.41456 \tag{E.213}$$

$$\hat{\alpha_1} = \alpha_1 = 0.4245 \tag{E.214}$$

Appendix F

The Standard Solution of a Second-order System

Consider a second-order system as

$$\frac{Y(s)}{I(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{F.215}$$

For a unit-step input, $Y(s)$ can be written

$$Y(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} \tag{F.216}$$

If $0 < \xi < 1$, then

$$Y(t) = 1 - e^{-\xi\omega_n t}[cos(\omega_d t) + \frac{\xi}{\sqrt{1-\xi^2}} sin(\omega_d t)] \qquad t \geq 0 \tag{F.217}$$

where $\omega_d = \omega_n\sqrt{1-\xi^2}$.

If $\xi = 1$, then

$$Y(t) = 1 - e^{-\omega_n t}(1 + \omega_n t) \qquad t \geq 0 \tag{F.218}$$

If $\xi > 1$, then

$$Y(t) = 1 + \frac{\omega_n}{2\sqrt{\xi^2 - 1}}(\frac{e^{-p_1 t}}{p_1} - \frac{e^{-p_2 t}}{p_2}) \qquad t \geq 0 \tag{F.219}$$

where $p_1 = (\xi + \sqrt{\xi^2 - 1})\omega_n$ and $p_2 = (\xi - \sqrt{\xi^2 - 1})\omega_n$.

Appendix G

Derivation of the Transfer Function for a Time-delay Compensation System

For the system block diagram shown in Figure 6.28, the $M_C(s)$ and $I_C(s)$ are related as follows:

$$M_C(s) = K_C(s)[I_C(s) - F_C(s)] \tag{G.220}$$

and

$$F_C(s) = G_1(s)\{G_2(s)[e^{-\tau_C s}C(s) - F_C(s)] + M_C(s)\} \tag{G.221}$$

Solving $F_C(s)$ from above equation gives

$$F_C(s) = \frac{[M_C(s) + G_2(s)e^{-\tau_C s}C(s)]G_1(s)}{1 + G_1(s)G_2(s)} \tag{G.222}$$

So

$$M_C(s) = K_C(s)\{I_C(s) - \frac{[M_C(s) + G_2(s)e^{-\tau_C s}C(s)]G_1(s)}{1 + G_1(s)G_2(s)}\} \tag{G.223}$$

or

$$M_C(s) = \frac{K_C(s)\{I_C(s)[1 + G_1(s)G_2(s)] - G_1(s)G_2(s)e^{-\tau_C s}C(s)\}}{1 + G_1(s)G_2(s) + G_1(s)K_C(s)} \tag{G.224}$$

and then

$$\begin{aligned} C(s) &= G_{11}(s)M_C(s) \\ &= \frac{K_C(s)G_{11}(s)\{I_C(s)[1 + G_1(s)G_2(s)] - G_1(s)G_2(s)e^{-\tau_C s}C(s)\}}{1 + G_1(s)G_2(s) + G_1(s)K_C(s)} \end{aligned} \tag{G.225}$$

Finally, the transfer function of the closed-loop system is

$$\begin{aligned} \frac{C(s)}{I_C(s)} &= \frac{K_C(s)G_{11}(s)[1 + G_1(s)G_2(s)]}{1 + G_1(s)G_2(s) + K_C(s)G_1(s) + K_C(s)G_1(s)G_2(s)G_{11}(s)e^{-\tau_C s}} \\ &= \frac{K_C(s)G_{11}(s)}{1 + K_C(s)G_1(s)\frac{1+G_2(s)e^{-\tau_C s}G_{11}(s)}{1+G_1(s)G_2(s)}} \end{aligned} \tag{G.226}$$

Appendix H Simulation Data

No.	e	λ_{11d}	No.	e	λ_{11d}	No.	e	λ_{11d}
1	0.0000	11.30	21	1.0463	11.34	41	1.0483	-13.28
2	0.1000	9.42	22	1.0464	12.40	42	1.0484	-11.88
3	0.2000	7.84	23	1.0465	13.69	43	1.0485	-10.73
4	0.3000	6.49	24	1.0466	15.03	44	1.0486	-9.78
5	0.4000	5.34	25	1.0467	17.36	45	1.0487	-8.98
6	0.5000	4.33	26	1.0468	20.08	46	1.0488	-8.30
7	0.6000	3.45	27	1.0469	23.86	47	1.0489	-7.70
8	0.7000	2.68	28	1.0470	29.42	48	1.0490	-7.19
9	0.8000	1.99	29	1.0471	38.48	49	1.0500	-4.22
10	0.9000	1.39	30	1.0472	55.80	50	1.0600	-0.53
11	1.0000	1.00	31	1.0473	102.17	51	1.1000	-0.0008
12	1.0200	1.08	32	1.0474	629.73	52	1.2000	-0.32
13	1.0300	1.28	33	1.0475	-149.74	53	1.3000	-0.72
14	1.0400	2.17	34	1.0476	-66.61	54	1.4000	-1.09
15	1.0420	2.76	35	1.0477	-42.71	55	1.5000	-1.45
16	1.0440	4.06	36	1.0478	-31.36	56	1.6000	-1.78
17	1.0450	5.52	37	1.0479	-24.74	57	1.7000	-2.09
18	1.0460	9.05	38	1.0480	-20.39	58	1.8000	-2.37
19	1.0461	9.69	39	1.0481	-17.32	59	1.9000	-2.64
20	1.0462	10.45	40	1.0482	-15.04	60	2.0000	-2.89

Table H.3: The relative gain λ_{11d} versus the compensation factor e, $\lambda_{11} = 11.3$

No.	e	λ_{11d}	No.	e	λ_{11d}	No.	e	λ_{11d}
1	0.0000	5.65	21	1.1020	91.50	41	1.1400	-0.31
2	0.1000	4.80	22	1.1021	138.18	42	1.1500	-0.18
3	0.2000	4.08	23	1.1022	283.51	43	1.1600	-0.11
4	0.3000	3.46	24	1.1023	-5096.36	44	1.1700	-0.06
5	0.4000	2.93	25	1.1024	-254.25	45	1.1800	-0.03
6	0.5000	2.46	26	1.1025	-130.14	46	1.1900	-0.01
7	0.6000	2.06	27	1.1026	-87.35	47	1.2000	-0.005
8	0.7000	1.70	28	1.1027	-65.68	48	1.3000	-0.07
9	0.8000	1.39	29	1.1028	-52.58	49	1.4000	-0.21
10	0.9000	1.14	30	1.1029	-43.81	50	1.5000	-0.36
11	1.0000	1.00	31	1.1030	-37.53	51	1.6000	-0.51
12	1.0600	1.22	32	1.1040	-15.24	52	1.7000	-0.65
13	1.0800	1.74	33	1.1050	-9.43	53	1.8000	-0.79
14	1.0900	2.70	34	1.1060	-6.76	54	1.9000	-0.91
15	1.0920	3.12	35	1.1070	-5.22	55	2.0000	-1.03
16	1.0940	3.74	36	1.1080	-4.23	56		
17	1.0960	4.76	37	1.1090	-3.53	57		
18	1.0980	6.74	38	1.1100	-3.01	58		
19	1.1000	12.18	39	1.1200	-1.07	59		
20	1.1010	21.20	40	1.1300	-0.54	60		

Table H.4: The relative gain λ_{11d} versus the compensation factor e, $\lambda_{11} = 0.5 \times 11.3 = 5.65$

No.	e	λ_{11d}	No.	e	λ_{11d}	No.	e	λ_{11d}
1	0.0000	16.95	21	1.0307	46.23	41	1.3000	-1.43
2	0.1000	14.04	22	1.0308	110.38	42	1.4000	-2.02
3	0.2000	11.61	23	1.0309	-272.38	43	1.5000	-2.57
4	0.3000	9.54	24	1.0310	-60.43	44	1.6000	-3.08
5	0.4000	7.76	25	1.0320	-6.45	45	1.7000	-3.55
6	0.5000	6.21	26	1.0330	-3.20	46	1.8000	-3.98
7	0.6000	4.86	27	1.0340	-2.03	47	1.9000	-4.39
8	0.7000	3.66	28	1.0350	-1.43	48	2.0000	-4.77
9	0.8000	2.60	29	1.0360	-1.07	49		
10	0.9000	1.67	30	1.0370	-0.83	50		
11	1.0000	1.00	31	1.0380	-0.66	51		
12	1.0200	1.30	32	1.0390	-0.53	52		
13	1.0220	1.45	33	1.0400	-0.43	53		
14	1.0240	1.69	34	1.0500	-0.07	54		
15	1.0260	2.14	35	1.0600	-0.002	55		
16	1.0280	3.25	36	1.0700	-0.01	56		
17	1.0300	9.49	37	1.0800	-0.05	57		
18	1.0302	12.17	38	1.0900	-0.09	58		
19	1.0304	17.12	39	1.1000	-0.15	59		
20	1.0306	29.37	40	1.2000	-0.79	60		

Table H.5: The relative gain λ_{11d} versus the compensation factor e, $\lambda_{11} = 1.5 \times 11.3 = 16.95$

$\hat{\lambda}_{11}$	$\hat{\lambda}_{11d}(e=1.045)$	$\hat{\lambda}_{11d}(e=1.042)$	$\hat{\lambda}_{11d}(e=1.04)$	$\hat{\lambda}_{11d}(e=1.03)$	$\hat{\lambda}_{11d}(e=1.01)$
5	8.6	4.1	3.1	1.6	1.08
6	8.1	3.8	2.9	1.6	1.07
7	7.6	3.6	2.8	1.5	1.06
8	7.2	3.4	2.6	1.5	1.05
9	6.7	3.2	2.5	1.4	1.04
10	6.2	3.0	2.4	1.4	1.03
11	5.7	2.8	2.2	1.3	1.02
12	5.3	2.6	2.1	1.2	1.01
13	4.8	2.4	1.9	1.2	0.99
14	4.3	2.2	1.8	1.1	0.98
15	3.9	2.0	1.6	1.1	0.97
16	3.4	1.8	1.5	1.0	0.96
17	2.9	1.6	1.3	0.9	0.95
18	2.5	1.4	1.2	0.9	0.94
19	1.9	1.2	1.0	0.8	0.93
20	1.5	1.0	0.9	0.8	0.92
21	1.0	0.8	0.7	0.7	0.91
22	0.5	0.6	0.6	0.7	0.89
23	0.1	0.4	0.4	0.6	0.88
24	-0.4	0.2	0.3	0.6	0.87
25	-0.8	-0.04	0.1	0.5	0.86

Table H.6: $\hat{\lambda}_{11d}$ versus $\hat{\lambda}_{11}$ with $e > 1$

$\hat{\lambda}_{11}$	$\hat{\lambda}_{11d}(e=1.1)$	$\hat{\lambda}_{11d}(e=1.07)$	$\hat{\lambda}_{11d}(e=1.05)$	$\hat{\lambda}_{11d}(e=1.048)$	$\hat{\lambda}_{11d}(e=1.047)$
5	0.016	-0.41	-6.38	-30.2	51.7
6	0.013	-0.37	-6.40	-28.4	48.7
7	0.011	-0.33	-5.97	-26.6	45.8
8	0.008	-0.29	-5.55	-24.8	42.8
9	0.005	-0.25	-5.12	-23.0	39.8
10	0.003	-0.21	-4.69	-21.0	36.8
11	0.000	-0.17	-4.27	-19.4	33.8
12	-0.003	-0.13	-3.84	-17.6	30.8
13	-0.005	-0.09	-3.41	-15.8	27.8
14	-0.008	-0.05	-2.99	-14.1	24.8
15	-0.011	-0.01	-2.56	-12.3	21.8
16	-0.013	0.03	-2.13	-10.5	18.8
17	-0.016	0.07	-1.71	-8.6	15.8
18	-0.019	0.11	-1.28	-6.8	12.8
19	-0.021	0.15	-0.85	-5.1	9.8
20	-0.024	0.19	-0.43	-3.3	6.8
21	-0.027	0.23	0.00	-1.5	3.8
22	-0.029	0.26	0.43	0.3	0.8
23	-0.032	0.30	0.85	2.09	-2.2
24	-0.035	0.34	1.28	3.88	-5.2
25	-0.037	0.38	1.71	5.69	-8.2

Table H.7: $\hat{\lambda}_{11d}$ versus $\hat{\lambda}_{11}$ with $e > 1$

$\hat{\lambda}_{11}$	$\hat{\lambda}_{11d}(e = 0.8)$	$\hat{\lambda}_{11d}(e = 0.9)$	$\hat{\lambda}_{11d}(e = 0.95)$	$\hat{\lambda}_{11d}(e = 0.99)$
5	1.2	0.9	0.91	0.95
6	1.3	1.0	0.94	0.96
7	1.4	1.0	0.98	0.97
8	1.6	1.2	1.02	0.98
9	1.7	1.2	1.06	0.99
10	1.8	1.3	1.09	0.99
11	1.9	1.4	1.13	1.01
12	2.1	1.4	1.17	1.02
13	2.2	1.5	1.21	1.03
14	2.3	1.6	1.25	1.03
15	2.5	1.6	1.28	1.04
16	2.6	1.7	1.32	1.05
17	2.7	1.8	1.36	1.06
18	2.9	1.9	1.39	1.07
19	3.0	1.9	1.44	1.08
20	3.1	2.0	1.47	1.09
21	3.2	2.1	1.51	1.09
22	3.7	2.1	1.55	1.11
23	3.5	2.2	1.58	1.12
24	3.6	2.3	1.62	1.13
25	3.8	2.3	1.66	1.17

Table H.8: $\hat{\lambda}_{11d}$ versus $\hat{\lambda}_{11}$ with $e < 1$

t	$C(t)_{K_C=1}$	$C(t)_{K_C=10}$	$C(t)_{K_C=100}$	t	$C(t)_{K_C=1}$	$C(t)_{K_C=10}$	$C(t)_{K_C=100}$
0.0	0.000	0.000	0.000	4.0	0.477		
0.1	0.040	0.333	0.975	4.2	0.480		
0.2	0.076	0.543	0.989	4.4	0.482		
0.4	0.141	0.761	0.990	4.6	0.484		
0.6	0.195	0.849	0.990	4.8	0.485		
0.8	0.242	0.884	0.990	5.0	0.487	0.907	0.990
1.0	0.281	0.898	0.990	6.0	0.491	0.907	0.990
1.2	0.314	0.903		7.0	0.493	0.907	0.990
1.4	0.341	0.906		8.0	0.493	0.907	0.990
1.6	0.365	0.907		9.0	0.494	0.907	0.990
1.8	0.385	0.907		10.0	0.494	0.907	0.990
2.0	0.402	0.907	0.990	11.0	0.494	0.907	0.990
2.2	0.416			12.0	0.494	0.907	0.990
2.4	0.428						
2.6	0.438						
2.8	0.447						
3.0	0.454	0.907	0.990				
3.2	0.460						
3.4	0.466						
3.6	0.470						
3.8	0.474						
4.0	0.477	0.907	0.990				

Table H.9: Unit-step response of the concentration control system without time-delay

t	$C(t)_{K_C=2.8}$	$C(t)_{K_C=3}$	$C(t)_{K_C=3.2}$	t	$C(t)_{K_C=2.8}$	$C(t)_{K_C=3}$	$C(t)_{K_C=3.2}$
0.0	0.00	0.00	0.00	6.0	0.73	0.82	1.13
0.2	0.28	0.32	0.35	7.0	0.76	0.85	1.02
0.4	0.53	0.62	0.71	8.0	0.75	0.75	0.62
0.6	0.74	0.89	1.04	9.0	0.73	0.70	0.52
0.8	0.92	1.11	1.33	10.0	0.73	0.72	0.75
1.0	1.05	1.28	1.56	11.0	0.73	0.76	0.92
1.2	1.13	1.40	1.71	12.0	0.73	0.76	0.83
1.4	1.18	1.46	1.79	13.0	0.73	0.75	0.68
1.6	1.20	1.46	1.80	14.0	0.73	0.74	0.67
1.8	1.18	1.43	1.74	15.0	0.73	0.74	0.77
2.0	1.14	1.35	1.61	16.0	0.73	0.75	0.82
2.2	1.09	1.25	1.44	17.0	0.73	0.75	0.77
2.4	1.02	1.13	1.23	18.0	0.73	0.74	0.72
2.6	0.95	1.00	1.01	19.0	0.73	0.74	0.73
2.8	0.89	0.88	0.79	20.0	0.73	0.74	0.77
3.0	0.82	0.76	0.58				
4.0	0.63	0.46	0.11				
5.0	0.65	0.61	0.60				

Table H.10: Unit-step response of the concentration control system with measuring time-delay.

t	$C(t)_{K_C=5}$	$C(t)_{K_C=10}$	$C(t)_{K_C=15}$	t	$C(t)_{K_C=5}$	$C(t)_{K_C=10}$	$C(t)_{K_C=15}$
0.0	0.00	0.00	0.00	2.4	0.82	0.94	0.89
0.1			0.82	2.5			1.01
0.2	0.59	1.12	1.58	2.6	0.82	0.89	1.06
0.3			1.85	2.7			1.04
0.4	1.06	1.54	1.55	2.8	0.82	0.88	0.96
0.5			0.94	2.9			0.88
0.6	1.14	1.01	0.42	3.0		0.91	0.84
0.7			0.28	3.1			0.87
0.8	0.98	0.57	0.54	3.2		0.93	0.93
0.9			0.99	3.3			0.98
1.0	0.79	0.72	1.34	3.4		0.91	1.00
1.1			1.40	3.5			0.98
1.2	0.72	1.04	1.18	3.6		0.90	0.94
1.3			0.86	3.7			0.90
1.4	0.76	1.07	0.63	3.8		0.90	0.89
1.5			0.61	3.9			0.91
1.6	0.83	0.89	0.78	4.0		0.91	0.94
1.7			1.02	4.1			0.97
1.8	0.86	0.80	1.17	4.2		0.91	0.97
1.9			1.16	4.3			0.95
2.0	0.86	0.88	1.02	4.4		0.91	0.93
2.1			0.86	4.5			0.91
2.2	0.84	0.96	0.76	4.6		0.90	0.91
2.3			0.78	4.7			0.92

Table H.11: Unit-step response of the concentration control system with measuring time-delay.

t	$C(t)_{\tau_c=0.2}$	$C(t)_{\tau_c=1}$	$C(t)_{\tau_c=2}$	$C(t)_{\tau_c=5}$	$C(t)_{Eq.6.169}$
0	0.00	0.00	0.00	0.00	0.00
1	0.37	0.38	0.38	0.38	0.34
2	0.60	0.61	0.62	0.63	0.56
3	0.75	0.76	0.77	0.79	0.70
4	0.84	0.85	0.86	0.89	0.80
5	0.89	0.90	0.92	0.95	0.86
6	0.93	0.94	0.94	0.98	0.90
7	0.95	0.96	0.97	0.99	0.93
8	0.97	0.97	0.98	1.00	0.94
9	0.98	0.98	0.98	1.00	0.96
10	0.98	0.98	0.99	1.00	0.96
11	0.98	0.99	0.99	1.00	0.97
12	0.99	0.99	0.99	1.00	0.97
13	0.99	0.99	0.99	1.00	0.97
14	0.99	0.99	0.99	0.99	0.97
15	0.99	0.99	0.99	0.99	0.98
16	0.99	0.99	0.99	0.99	0.98
17	0.99	0.99	0.99	0.99	0.98
18	0.99	0.99	0.99	0.99	0.98
19	0.99	0.99	0.99	0.99	0.98
20	0.99	0.99	0.99	0.99	0.98

Table H.12: The unit-step response of the concentration control loop with a physically realizable time-delay compensator. $K_C = 100$, $K_1 = 0.976$ and $K_2 = 0.1$.