In presenting this thesis in partial fulfillment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed with my written permission.

Department of Chemical and Biological Engineering
The University of British Columbia
2216 Main Mall
Vancouver, BC
Canada, V6T 1Z4

Date: August 2000
Abstract

Fault detection and isolation (FDI) has become a crucial issue for industrial process monitoring in order to increase availability, reliability, and production safety. Model-based FDI methods rely on the mathematical model and input-output data of a process to perform detection. The local approach is a new model-based FDI method which aims to detect slight changes of parametric properties of a system. This thesis mainly addresses to the application of FDI using the local approach.

Robustness with respect to model uncertainties is an important issue for the local approach. A new algorithm was proposed to recalculate threshold based on the original threshold and covariance matrix of the estimated parameters in order to reduce false alarms due to the estimation error of process parameters. A similar algorithm was also provided to recalculate threshold to reduce fault alarms due to regular parameter fluctuations. As fault detection algorithms are often applied to closed-loop data, closed-loop fault detection was also investigated. Two methods were proposed to deal with the relevance between system input and output data in closed-loop detection: the dimension reduction method and the indirect detection method. The dimension reduction method uses a linear transformation to reduce the dimension of the normalized residual so that the covariance matrix of the revised normalized residual has full rank. The indirect detection method uses the closed-loop model to calculate the primary residual and the normalized residual. By detecting the changes of the closed-loop parameters, the method also detects the changes of the open-loop parameters. Simulation results show that both of these methods can detect changes of every single parameters of a system. Industrial data from a cross-direction (CD) control system in a paper machine was also used to assess the applicability of the local approach. By dividing the CD databox into small sections, the sensitivity of the detection algorithm was improved and the algorithm successfully detected abrupt faults of a single actuator. However, incipient faults of a single actuator can not be detected due to noise and inaccuracy of the process model.
# Table of Contents

Abstract .............................................................. ii
List of Tables ....................................................... vi
List of Figures ....................................................... vii
Acknowledgements .................................................. ix

1 Introduction ...................................................... 1
  1.1 Background .................................................. 1
    1.1.1 Basic concepts of fault detection ....................... 1
    1.1.2 General structures of fault detection and isolation algorithms .... 2
    1.1.3 Existing methods for fault detection and isolation .............. 4
    1.1.4 The local approach ................................... 6
    1.1.5 Robustness of fault detection and isolation ................. 6
  1.2 Outline and contributions of the thesis ..................... 7

2 Principle of the local approach ................................ 9
  2.1 Residual generation of the local approach ................. 9
  2.2 Fault detection .......................................... 12
  2.3 Fault isolation .......................................... 13
    2.3.1 Sensitivity test .................................... 13
    2.3.2 Min-max test ....................................... 14
  2.4 General procedure of FDI using the local approach ....... 15

3 Robustness of FDI using local approach ..................... 17
  3.1 Introduction ............................................. 17
  3.2 Problem formulation .................................... 18
  3.3 Recalculation of the threshold .......................... 19
    3.3.1 Distribution of the normalized residual using the estimated parameter .... 20
    3.3.2 Evaluation of the normalized residual ................... 21
    3.3.3 Deduction of the threshold .......................... 23
    3.3.4 Optimization of the threshold ....................... 24
    3.3.5 Discussion of the results .......................... 25
    3.3.6 Extension of the result to the case of parameter fluctuations ...... 25
3.4 Simulation results ................................................................. 26
  3.4.1 System specification ......................................................... 27
  3.4.2 Simulation of robustness with respect to model uncertainties .... 27
  3.4.3 Simulation of robustness with respect to parameter fluctuations .. 32
3.5 Experiment results ............................................................. 36
  3.5.1 Identification of the system parameters................................. 37
  3.5.2 Training of the FDI system .............................................. 38
  3.5.3 Test of the false alarm rate ............................................ 38
  3.5.4 Test of the detection power ............................................ 39
3.6 Conclusions ........................................................................... 40
4 Closed-loop fault detection using local approach ....................... 41
  4.1 Introduction ........................................................................ 41
  4.2 Discussion of a simple example ............................................ 42
  4.3 Closed-loop fault detection using the dimension reduction method ... 47
  4.4 Closed-loop fault detection using the indirect detection method .... 51
  4.5 Simulation results .............................................................. 54
    4.5.1 Closed-loop detection using the dimension reduction method .... 55
    4.5.2 Closed-loop detection using the indirect detection method ....... 60
  4.6 Experiment results ............................................................. 64
  4.7 Conclusions ........................................................................ 66
5 Application of fault detection in paper-making processes ........... 68
  5.1 Background ......................................................................... 68
  5.2 Problem formulation ............................................................ 69
    5.2.1 System model .............................................................. 70
    5.2.2 Modeling of actuator faults ........................................... 70
  5.3 Actuator fault detection and isolation ..................................... 72
    5.3.1 Machine-direction and cross-direction data separation .......... 72
    5.3.2 Mapping of databox to actuators .................................... 72
    5.3.3 General algorithms for actuator fault detection and isolation ... 73
    5.3.4 Actuator fault detection by dividing the system output into sections ... 75
    5.3.5 Actuator fault detection by decoupling the system .............. 76
5.4 Simulation results
5.4.1 Process specification
5.4.2 Simulation with real process model and generated data
5.4.3 Simulation with real process data
5.4.4 Discussion of simulation results
5.5 Conclusions

6 Conclusions and future work
6.1 Conclusions
6.2 Future work

References
List of Tables

3.1 False alarm rate in a fault-free case .................................................. 28
3.2 False alarm rates for the robustness scheme and the upgrading scheme .... 31
3.3 False alarm rate in a fault-free case with parameter fluctuations ............ 33
3.4 False alarm rates for the upgrading scheme and the robustness scheme in a fault-free case with parameter fluctuations ................................. 35
Table of Figures

1.1 System representation................................................................. 3
1.2 Simplified block representation of the system................................. 3
1.3 Conceptual structure of FDI using analytical redundancy.................. 4
3.1 Chi-square value of the normalized residual in a fault-free case........ 28
3.2 Chi-square value of the normalized residual in a faulty case (a change occurred at the parameter $a$).......................... 29
3.3 Chi-square value of the normalized residual in a faulty case (a change occurred at the parameter $b$).......................... 30
3.4 Chi-square value for the robustness scheme and for the upgrading scheme in a fault-free case............................................. 31
3.5 Chi-square value of the normalized residual in a fault-free case with parameter fluctuations in parameter $b$ .......................... 32
3.6 Chi-square value of the normalized residual in a faulty case with parameter fluctuations (a change occurred in the parameter $a$).......................... 34
3.7 Chi-square value of the normalized residual in a faulty case with parameter fluctuations (a step change occurred in the parameter $b$).......................... 34
3.8 Chi-square value for the upgrading scheme and the robustness scheme in a fault-free case with parameter fluctuation.......................... 35
3.9 Process diagram of the tank level and temperature control system........ 36
3.10 Test of the false alarm rate in a fault-free case using the revised threshold.......................... 38
3.11 Test of the detection power using the revised threshold.................... 39
4.1 Detection of the change in $a_1$ using the dimension reduction method...... 57
4.2 Detection of the change in $a_2$ using the dimension reduction method...... 58
4.3 Detection of the change in $b_1$ using the dimension reduction method...... 58
4.4 Detection of the change in $b_2$ using the dimension reduction method...... 59
4.5 Detection failure with the dimension reduction method..................... 60
4.6 Detection of the change in $a_2$ using the indirect detection method....... 62
4.7 Detection failure using the indirect detection method..................... 64
4.8 Detection of the change of tank level using the dimension reduction method..............66
5.1 Schematic of a typical Fourdrinier paper machine..............................................69
5.2 Process interaction matrix....................................................................................78
5.3 Detection of a jammed actuator at the 25th actuator using the decoupling method......79
5.4 Detection of a jammed actuator at the 25th actuator using the general method..........80
5.5 Detection of a jammed actuator at the 25th actuator using sectioning method..........81
5.6 Detection of an abrupt fault at the 25th actuator using the sectioning method..........82
5.7 Detection of an incipient fault at the 25th actuator using the sectioning method..........82
5.8 Detection of a jammed actuator at the 25th actuator using the sectioning method........84
5.9 Detection of jammed actuators in section 3 using the sectioning method...............84
5.10 Detection of an incipient fault at the 25th actuator using the sectioning method.........86
5.11 Detection of incipient faults at the 23rd – 27th actuators using the sectioning method....86
5.12 Detection of an abrupt fault at the 25th actuator using the sectioning method...........87
5.13 Prediction of the system output using the system inputs and process model.............88
Acknowledgements

I would like to thank my research supervisor Dr. K Ezra Kwok for giving me the opportunity to work under his guidance. I benefited a lot from his valuable advice and I am also grateful for his support in the past two years.

I also thank Dr. Biao Huang for being my co-supervisor in this research, especially for his support and guidance during my research in the Department of Chemical and Materials Engineering at the University of Alberta. Thanks to Professor Ping Li of Zhejiang University for being my advisor in my study in China. Without his recommendation, I would not have the opportunity to study at UBC under Professor K. Ezra Kwok.

Special thanks go to the following people: Dr. Greg Steward for providing industrial data for this project, Mr. Norman Woo for his assistance in my thesis writing, Mr. Michael Chong Ping, Mr. Stevo Mijanovi, Xin Huang, Sheng Wan and my friends at the Pulp and Paper Center, Department of Chemical and Biological Engineering at UBC and control group of the Department of Chemical and Materials Engineering at the University of Alberta for their friendship during my studies. They made my study and research an enjoyable experience.

And last, I would like to pay special tribute to my parents, sisters and brother for all they have done for me. I can do nothing without their love and support and also want to dedicate this thesis to them.
Chapter 1

Introduction

1.1 Background

With the increasing productivity requirements and performance specifications, chemical processes and automatic control systems are becoming more and more complicated. Such conditions increase the possibility of system failures, which can be characterized by unpredictable changes in the system dynamics. Consequently, there is a growing demand for fault tolerance for the purpose of availability, reliability and production safety while maintaining high performance. Fault tolerance can be achieved not only by providing hardware redundancy and improving the reliability of individual functional units in the system, but also by implementing plant health monitoring systems and online fault detection. Fault detection and isolation (FDI) have drawn growing attention and become a crucial issue for industrial system monitoring in the past several decades. It has been widely investigated both in theory and application.

1.1.1 Basic concepts of fault detection and isolation

A fault can be understood as unpermitted deviation of at least one characteristic property of a system (Isermann, 1994). Usually, three kinds of faults are considered: sensor fault (instrument fault), actuator fault, and component fault. Fault detection (FD) is a decision on whether faults exist in a system. Fault isolation (FI) is a decision on the presence of a faulty mode among a number of possible modes. With respect to the different units where faults can occur in a system, three categories of fault detection exist: instrument fault detection (IFD), actuator fault detection (AFD) and component fault detection (CFD) (Frank 1990).

The nature of possible failure situations may be classified as abrupt (sudden) faults, which are typically modeled as step-like changes and incipient (slowly developing) faults, which are represented by drift-type changes. With respect to the nature of faults, application of fault detection typically can be divided into two broad categories:

(1) Quick detection of abrupt faults of sensors, actuators or other components of a control system;
Early detection of incipient changes in the dynamics of a system and its interpretation and diagnosis (Zhang, 1998)

In abrupt-type fault detection, it is crucial that the algorithm is able to detect the changes quickly to avoid disastrous consequences. In such cases, early detection and isolation are the key objectives. Early detection of incipient changes is also referred to as condition-based maintenance. It is extremely important in the industry, but has not been investigated extensively. The key idea is to replace regular systematic inspection of a system by condition-based inspections (i.e. inspections based on continuous monitoring of the system) in order to prevent possible malfunction or damage from happening. A solution to this condition-based maintenance may be the early detection of incipient changes of the parameters of a system without any artificial excitation. One of the main difficulties in the detection of incipient faults is the compensating effect of feedback control, which tends to diminish the effect of small incipient faults on the control performance.

A novel philosophy has emerged and is increasingly discussed. It is based on the use of analytical redundancy instead of physical redundancy. This implies that the information redundancy contained in the dynamic relations between the system input and measured output is exploited for fault detection and isolation. It is called the model-based fault detection and isolation. Another FD method in use is knowledge-based method. The knowledge-based method differs from the model-based method due to use of qualitative models instead of quantitative models to perform detection. It may be an alternative way to model-based fault detection or complement it (Frank, 1990). These two types of methods can even be combined together into one scheme. This thesis mainly discusses model-based fault detection and isolation.

1.1.2 General structures of fault detection and isolation algorithms

The process and fault model are very important in the performance of FDI algorithms. Figure 1.1 shows a typical dynamic system with input $u$ and output $y$. It consists of an actuator, a sensor and plant components. Because system and measurement noise, modeling errors are inevitable in any system and their effect can lead to false alarms, they must be taken in account in the model. Figure 1.2 is a simplified block diagram of the dynamic system.
It has been widely acknowledged that the FDI problem can be split into two steps:

1. Residual generation. Residuals are functions of the system input $u$ and output $y$, and nominal system model. They are accentuated by the faults and can be regarded as the index of faults in the system.

2. Residual evaluation. Residuals are evaluated to make decisions on the presence of faults and to decide the time, location, type, size, and source of the faults.
Figure 1.3 Conceptual structure of FDI using analytical redundancy (Frank 1990).

Figure 1.3 illustrates the conceptual structure of model-based FDI algorithms. In the residual generation stage, a validation of the nominal relationship is performed with the system input $u$, output $y$ and nominal model. If a fault happens, the nominal relationship will not be satisfied and a residual occurs. The residual is then used to form appropriate decision functions. They are evaluated in the fault decision logic in order to monitor both the time of occurrence and location of the fault.

1.1.3 Existing methods of fault detection and isolation

Various methods of fault detection and isolation have been developed in the past several decades, as can be seen from survey papers by Willsky, (1976); Gertler, (1988); Basseville, (1988); Frank, (1990); Isermann, (1993). Among them are the parity space approach, the detection filter, the fault observer, and parameter estimation techniques. The main idea of each of these approaches will be introduced in the following passage. The comparison of each of these techniques is discussed in Basseville and Nikiforov (1993), Basseville (1997) and Nikiforov, et al (1996).
1. **Parity space approach.** The parity equation based method was first developed by Potter and Suman (1997), Desai and Ray (1981), Chow and Wilsky (1984) and Lou et al (1986). It was further developed by Patton and Chen (1991), Gertler, Hofling and Pfeufer (1994). The parity space approach relies on a check of the parity of the mathematical equations of the system (analytical redundancy relation) by using the actual system input and output. A fault alarm is generated once preset thresholds are surpassed. There are two types of analytical redundancy relationships: direct redundancy which are the relationships among instantaneous redundant sensor outputs (algebraic relations) and temporal redundancy which are dynamic relationships between system inputs and system outputs.

2. **Dedicated observer approach.** Dedicated observer method was developed by Clark et al. (1975), Willsky (1976), Clark (1978a, b), Frank and Keller (1980), Frank (1978b, 1988), Mehra and Peshon (1971) and Willsky (1976). The basic idea of the observer approach is to reconstruct the outputs of the system from the measurements or subsets of the measurements with the aid of observers or Kalman filters using the estimation error or innovation, respectively, as a residual for the detection and isolation of the faults.

3. **Fault filter approach.** The fault detection filter (or fault sensitive filter) is a full-order state estimator with a special choice of feedback gain matrix. It was first proposed by Beard (1971) and Jones (1973) and then developed by Wilbers and Speyer (1989). By proper choice of feedback gain matrix, the residual due to a particular fault is constrained to a single direction or plane in the residual space independent of the model of the fault. A fault is detected when one or more of the residual projections along the known fault direction or in the known fault plane are sufficiently large.

4. **Parameter identification approach.** The use of parameter estimation methods for fault detection of dynamical systems was demonstrated by Hohmann (1977), Baskiotis et al. (1976), Geiger (1982), Filbert and Metzger (1982). The development of parameter estimation methods for fault detection was then summarized by Isermann (1984). This is an alternative approach to the above described methods based on state estimation. It makes use of the facts that faults of a dynamical system are reflected in the physical parameters, as for example, friction, mass viscosity, resistance, capacitance, inductance, etc. The idea of the parameter identification approach is to detect the faults through estimation of the parameter of the mathematical model (Isermann, 1984).
1.1.4 The Local approach

For additive faults, parity check and observer-based detection methods are quite effective. However, monitoring innovations or observer errors is not sufficient for detection of non-additive faults and additive faults in nonlinear systems. In such cases, a local test relying on efficient scores has proven to be useful. A local approach based on system identification and local test has been developed which can be used to detect both incipient faults and abrupt faults. The main idea of this approach is to transform fault detection problems concerning a parameterized process into the universal problem of monitoring the mean of a Gaussian vector. Under the framework of this approach, it can also be used in model validation. Benveniste, Basseville and Moustakides (1987) first proposed the basic idea of the local approach in change detection and model validation. Zhang (1993), Basseville (1993) and Benveniste (1997) have done notable work in this area. They have built up the framework of this fairly general methodology and generalized this approach to a wide arrange of system models. Delyon and Benveniste (1997) discussed the relationship between identification and local test. Zhang (1998) applied the local approach to a fairly wide type of nonlinear systems and proposed a combined input-output and local approach. He also combined the observer-based fault detection method and the local approach together and applied them into monitoring nonlinear dynamical systems (1998). There have been a lot of applications of the local approach in the literature. B. Huang (1999) applied it to the problem of process and control loop performance monitoring. Wahnon and Berman (1990) implemented it the problem of tracking time-varying parameters in linear systems in adaptive signal processing and adaptive control. O’Reilly (1998) applied local approach to the problem of sensor decalibration. This thesis is mainly dedicated to application issues of the local approach: the robustness of the local approach, closed-loop detection and implementation of the local approach in actuator fault detection in paper-making CD control systems.

1.1.5 Robustness of fault detection and isolation

All model-based fault detect and isolation methods rely on the mathematical model of the monitored system to perform detection and diagnosis. If the system model is accurate, fault detection may be very straightforward. However, noise and model uncertainties are inevitable in a practical system. The consequence is that even in a fault free case, the nominal relationship
between the system input and output is not satisfied. It may lead to erroneous decision on the presence of faults in the system. In order to improve reliability and performance of fault detection algorithms, it is of great importance to consider the effects of noise and model uncertainties on the detection algorithms, i.e. robustness of FDI algorithms. Robustness means the ability of the algorithms to detect changes correctly in the presence of model uncertainties, disturbance and noise (Basseville, 1998; Gertler, 1988).

Many efforts have gone into increasing robustness of fault detection algorithms. Investigations on robustness of FDI fall into two types. The first type concerns the residual generation stage. The purpose is to find residuals that are sensitive to faults and insensitive to noise and model uncertainties. The second type concerns the residual evaluation stage. It involves either the selection or calculation of a threshold to accommodate the effect of noise and model uncertainties. Adaptive threshold selection and fuzzy decision logic have been used in the evaluation of the residuals (Basseville, 1998).

Though the robustness problem of FDI has been investigated widely with the parity check and observer-based methods, little attention has been concerned with the robustness of FDI using the local approach. Part of this thesis is about the robustness of FDI using the local approach. As a number of methods have been proposed in the stage of residual generation to improve the insensitivity of the residual to model uncertainties, the robustness of the detection algorithms can also be improved by appropriate selection of thresholds to reduce the false alarm rate. From the application point of view, the selection of thresholds is as important as the generation of the residual.

1.2 Outline and Contributions of the Thesis

In chapter 2, the principle of fault detection and isolation using the local approach is introduced. It includes basic concepts used in the local approach, regular assumptions for the local approach to work, and a general procedure of fault detection and isolation using the local approach.

Chapter 3 considers the robustness of fault detection using the local approach. Since noise and model uncertainties are unavoidable in control systems, it is meaningful to consider their affects on fault detection algorithms. In this chapter, a new algorithm was proposed to revise the threshold to reduce the false alarm rate to the required level while keeping the sensitivity of
fault detection algorithms toward faults. The result was also extended to the situation of regular system parameter fluctuations. A similar algorithm was provided to calculate the threshold to be used in the residual evaluation. With the revised threshold, the detection algorithm will not generate any alarm to acceptable parameter changes.

Chapter 4 addresses the problem of closed-loop fault detection. As most industrial systems are operated in closed-loop, it is important to apply detection algorithms to closed-loop data. Problems that will arise in closed-loop detection are analyzed and two methods were proposed: dimension reduction method and indirect detection method. Simulation studies in this chapter show the effectiveness of the proposed methods.

In chapter 5, the local approach is applied to the problem of actuator fault detection in paper-making cross-direction (CD) control systems. CD control systems generally have many actuators. It is helpful to detect and isolate faults for these actuators to maintain CD control performance and reduce maintenance cost.

Conclusions and future directions will be provided in chapter 6.
Chapter 2

Principle of the Local Approach

The local approach is a statistical FDI method that aims at detecting slight changes in the parameters of a parametric system. The key idea is to transform FDI problems into the general problem of monitoring the mean value of a Gaussian random vector. The terms fault and change will be used interchangeably in the following.

It has been widely acknowledged that the FDI algorithms can be split into two steps: residual generation and residual evaluation. For the local approach, the main purpose of residual generation is to calculate a primary residual and a normalized residual. A primary residual is a vector-valued function of the measurements and the parameters of the monitored system. It has zero mean in the case of no fault and non-zero mean otherwise. Such a function can be regarded as the index of faults in the system. A normalized residual is a vector-valued function built on the primary residual. Under the assumption of small changes, it is Gaussian distributed with a known covariance matrix. In the residual evaluation stage of the local approach, a statistical test is applied to the normalized residual to make a decision on the presence or absence of faults and to be able to further isolate the faults.

To illustrate the procedure of the local approach, the following state-space model is used to describe the monitored system

\[
\begin{align*}
x_{k+1} &= f(\theta, x_k, u_k) \\
y_k &= h(\theta, x_k, u_k) + e_k
\end{align*}
\] (2.1)

where \(x_k, u_k, y_k\) are vectors of state variables, system inputs and outputs at time \(k\) respectively; \(\theta\) is the vector of the system parameters that remain to be monitored; \(e_k\) is noise which could be white or colored.

2.1 Residual generation of the local approach

A primary residual for monitoring a system parameterized by \(\theta\) is a function of the system parameter vector \(\theta\), the system input vector \(u_k\) and system output vector \(y_k\). More precisely, it is a function of the nominal value of the parameter vector \(\theta_0\) and an auxiliary process \(z_k\).
driven by the input $u_k$ and output $y_k$. In general, $z_k$ is generated by a dynamic system of the form

$$
\xi_{k+1} = \varphi(\theta, \xi_k, u_k, y_k) \\
z_k = \theta(\theta, \xi_k, u_k, y_k)
$$

(2.2)

where $\varphi, \theta$ are two appropriate functions. Through this auxiliary process, $z_k$ summarizes in some sense all the observations up to the time instant $k$. The auxiliary vector may be simply composed of a finite sample of observed output and the innovation. A simple example is given for the following first order discrete system

$$
y_k = a u_{k-1} + e_k
$$

The auxiliary vector can be generated through the dynamic system

$$
\xi_{k+1} = u_k \\
z_k = \xi_k
$$

Primary residuals associated with parametric models for the purpose of FDI should satisfy the following requirements.

**Definition 2.1: (Primary residual)** A vector-valued function $H(\theta, z_k)$ is called a valid primary residual if it is differentiable in $\theta$ and

$$
E_\theta H(\theta_0, z_k) = 0, \text{ if } \theta = \theta_0 \tag{2.3}
$$

$$
E_\theta H(\theta_0, z_k) \neq 0, \text{ if } \theta \neq \theta_0 \tag{2.4}
$$

where $\theta_0$ is the nominal value of the system parameter $\theta$, $E_\theta$ means the expectation when the actual system parameter value is $\theta$. In eqns. 2.3 and 2.4, $z_k$ is a function of $\theta$.

A typical example of such a primary residual is the gradient of the least square (LS) prediction error with respect to $\theta$. It is assumed that the sensitivity matrix

$$
M(\theta_0) = E_{\theta_0} \frac{\partial}{\partial \theta} H(\theta, z_k) \bigg|_{\theta=\theta_0}
$$

(2.5)

exists and has full column rank (Zhang, 1998a).

General statistical approaches to residual evaluation require the knowledge of the statistical properties of the FDI residuals. However, even for a simple linear model, the distribution of the primary residual $H(\theta, z_k)$ is unknown. The solution of the local approach to circumvent this difficulty is to calculate a normalized residual with the primary residual $H(\theta, z_k)$ and to make
the assumption that the change in the parameters is small. It assumes that parameters before any change are given by $\theta_0$ and after any change are given by

$$\theta_0 + \frac{\lambda}{N}$$

where $\lambda$ is an unknown, but constant, vector and $N$ is the data sample size. With the small change assumption, the change detection problem can be formulated as the following statistical test:

- **Nominal system:** $H_0 : \theta = \theta_0$  \hspace{1cm} (2.6)
- **Faulty system:** $H_1 : \theta = \theta_0 + \frac{\lambda}{N}$  \hspace{1cm} (2.7)

For large $N$, hypothesis $H_1$ implies the smaller deviations in $\theta$. The small change assumption is reasonable in the sense that with more data available, the detectability will be increased, (i.e. smaller changes can be detected with a larger sample size).

With valid primary residuals a normalized residual can be calculated as the average of the primary residual at $N$ sample time:

**Definition 2.2: (Normalized residual)** Given a primary residual $H(\theta, z_k)$ and $N$-size sample of $z_k$, for $k = 1, 2, \ldots, N$, the normalized residual is defined as

$$\zeta_N(\theta_0) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} H(\theta_0, z_k)$$  \hspace{1cm} (2.8)

For the design of the FDI algorithm, it is also assumed that the covariance matrix

$$\Sigma(\theta_0) = \lim_{N \to \infty} E_{\theta_0} \zeta_N(\theta_0) \zeta_N(\theta_0)^T$$  \hspace{1cm} (2.9)

exists and is positive definite (Zhang, 1998a).

Under the assumption that the changes in the parameters are small, the normalized residual turns out to be asymptotically Gaussian distributed. This property makes it possible to evaluate the normalized residual in a statistical way. The following central limit theorem holds for a large class of dynamic systems, as shown in Basseville and Nikiforov, (1993) and Basseville, (1998).

**Theorem 2.1** (Zhang, 1998) If the normalized residual $\zeta_N(\theta_0)$ defined in eqn. 2.8 is built from a primary residual satisfying assumption 2.3 and 2.4, then it is a asymptotically Gaussian distributed vector under both hypotheses of eqn. 2.6 and 2.7, that is when $N \to \infty$.\]
Chapter 2: Principle of the local approach

\( \zeta_N(\theta_0) \sim N(0, \Sigma(\theta_0)) \) under \( H_0 \) \hfill (2.10)  
\( \zeta_N(\theta_0) \sim N(M(\theta_0)\lambda, \Sigma(\theta_0)) \) under \( H_1 \) \hfill (2.11)

"~" means follow and the matrix \( M(\theta_0) \) and \( \Sigma(\theta_0) \) are defined in eqn. 2.5 and 2.9. This notation will also be used in the following chapters.

Theorem 2.1 shows that under both \( H_0 \) and \( H_1 \), the normalized residual \( \zeta_N(\theta_0) \) is Gaussian distributed and the covariance matrices are the same. Therefore, one can detect changes in the system parameters \( \theta_0 \) by monitoring the mean value of \( \zeta_N(\theta_0) \). With theorem 2.1, the problem of detecting a change is transformed into a statistical problem of detecting whether the mean of a Gaussian distributed vector is zero or not.

### 2.2 Fault detection

With the results in section 2.1, detection of small changes in the parameter \( \theta \) is asymptotically equivalent to the detection of changes in the mean of a Gaussian vector \( \zeta_N(\theta_0) \). It can be shown that for deciding between \( H_0 \) and \( H_1 \), the optimum test statistics is the generalized likelihood ratio (GLR) (Zhang, 1998a):

\[
\chi^2_{\text{global}} = 2 \ln \frac{\max_{\lambda} p_{\lambda}(\zeta_N)}{p_0(\zeta_N)} = \zeta_N^T \Sigma^{-1} M (M^T \Sigma^{-1} M)^{-1} M^T \Sigma^{-1} \zeta_N
\]

where

\[ \Sigma = \Sigma(\theta_0), \quad \zeta_N = \zeta_N(\theta_0), \quad M = M_0 \]
\[ p_{\lambda}(\zeta_N) \] is the density function of the Gaussian vector \( \zeta_N \) under \( H_1 \), \( p_0(\zeta_N) \) is the density function of \( \zeta_N \) under \( H_0 \). Under both hypotheses, \( \chi^2_{\text{global}} \) is asymptotically a chi-square distributed variable, with a number of degrees of freedom equal to the dimension of \( \theta \). It is central under \( H_0 \) and has the non-centrality under \( H_1 \) equal to \( \gamma = \lambda^T M^T \Sigma^{-1} M \lambda \).

When the sensitivity matrix \( M \) is a square matrix, the test can be simplified to \( \chi^2_{\text{global}} = \zeta_N^T \Sigma^{-1} \zeta_N \). A threshold \( \chi^2_\alpha \) can be found from a standard chi-square table according to the false alarm rate \( \alpha \). If \( \chi^2_{\text{global}} \) is found to be larger than the threshold \( \chi^2_\alpha \), a change in the parameters is detected.
2.3 Fault isolation

In the implementation of the FDI algorithm, FD is performed as frequently as possible. FI is performed only after the detection of a fault. The task of FI is to decide which components of \( \lambda \) are nonzero. Without loss of generality, assume that \( \lambda_a \) consists of the first component of \( \lambda \). Letting \( \lambda_b \) be the complementary vector of \( \lambda_a \) in \( \lambda \), i.e.

\[
\lambda = \begin{bmatrix} \lambda_a \\ \lambda_b \end{bmatrix}
\]

with \( n_a = \dim(\lambda_a) \), \( n_b = \dim(\lambda_b) \). FI is to decide between two hypotheses:

\[ H_0 : \lambda_a = 0 \]
\[ H_1 : \lambda_a \neq 0. \]

The value of \( \lambda_b \) is unknown and there are two ways to deal with this problem: assume \( \lambda_b \) to be zero or equal to the least favorable value for making the decision between \( H_0 \) and \( H_1 \). These two solutions result in the sensitivity test and min-max test.

2.3.1 Sensitivity test:

In the sensitivity test, \( \lambda_b \) is assumed to be zero and FI is a decision between two hypotheses

\[
H_0 : \lambda = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]
\[
H_1 : \lambda = \begin{bmatrix} \lambda_a \\ 0 \end{bmatrix}
\]

Partition \( M = \begin{bmatrix} M_a \\ M_b \end{bmatrix} \) so that \( M\lambda = M_a \lambda_a + M_b \lambda_b \). It is shown that the sensitivity test for monitoring \( \lambda_a \) is the GLR test between \( H_0 \) and \( H_1 \), where \( \lambda_a \neq 0 \) (Basseville, 1998)

\[
\bar{\chi}_a^2 = 2 \ln \frac{\max_{\lambda_a} P_{\lambda_a,0}(\zeta_N)}{P_{0,0}(\zeta_N)} = \bar{\zeta}_a F_{aa}^{-1} \bar{\zeta}_a
\]

(2.13)

where

\[
\bar{\zeta}_a = M_a^T \Sigma^{-1} \zeta_N ;
\]

\[ P_{\lambda_a,0}(\zeta) \] stands for the density function of \( \zeta_N \sim N(M_a \lambda_a + M_b \lambda_b, \Sigma) \);

\[ F_{aa} = M_a^T \Sigma^{-1} M_a \] is the covariance matrix of \( \bar{\zeta}_a \).

Under both hypotheses, \( \bar{\chi}_a^2 \) is distributed as a chi-square random variable with \( n_a \) degrees of freedom. This chi-square distribution is central under \( H_0 \) and non-central under \( H_1 \). A threshold \( \bar{\chi}_a^2 \) can be found from a standard chi-square table according to the false alarm rate.
Chapter 2: Principle of the local approach

By comparing $\chi^2_a$ with the threshold one can make a decision on whether $\lambda_a$ is zero or not.

2.3.2 Min-max test:

For the min-max test, the parameters in $\lambda_a$ are considered as nuisances replaced by its least favorable value, namely the value which minimizes the noncentrality parameter $\gamma = \lambda^T M^T \Sigma^{-1} M \lambda$. This is equivalent to the GLR method (Basseville 1998):

$$\chi^2_a = 2 \ln \frac{\max_{\lambda_a, \lambda_b} P_{\lambda_a, \lambda_b} (\zeta_N)}{P_{\lambda_a, 0} (\zeta_N)} \quad (2.14)$$

Letting

$$F = M^T \Sigma^{-1} M$$

and partition it as

$$F = \begin{bmatrix} F_{aa} & F_{ab} \\ F_{ba} & F_{bb} \end{bmatrix} = \begin{bmatrix} M_a^T \Sigma^{-1} M_a & M_a^T \Sigma^{-1} M_b \\ M_b^T \Sigma^{-1} M_a & M_b^T \Sigma^{-1} M_b \end{bmatrix}$$

It has been shown that

$$\chi^2_a = \zeta^*_a F_a^{-1} \zeta_a \quad (2.15)$$

where

$$\zeta^*_a = \bar{\zeta}_a - F_{ab} F_{bb}^{-1} \bar{\zeta}_b, \quad \bar{\zeta}_a = M_a^T \Sigma^{-1} \zeta_N, \quad \bar{\zeta}_b = M_b^T \Sigma^{-1} \zeta_N$$

$$F_a = F_{aa} - F_{ab} F_{bb}^{-1} F_{ba}$$

is the covariance matrix of $\zeta^*_a$.

Under both hypotheses $\chi^2_a$ is a chi-square distributed variable with $n_a$ degrees of freedom. This chi-square distribution is central under $H_0$ and non-central under $H_1$. A threshold value can be found from a standard chi-square table according to the false alarm rate $\alpha$ and one can make a decision on whether $\lambda_a$ is zero or not by comparing $\chi^2_a$ with the threshold value.

2.4 General procedure of FDI using the local approach

As a model-based FDI method, the local approach relies on the nominal model of the monitored system to perform detection. Therefore, the first step in the application of the local approach is to determine a parametric model that can best describe the dynamics of the system and to estimate the value of the system parameters under nominal situations. The primary residual can simply be calculated as the gradient of the estimation error with respect to the
Chapter 2: Principle of the local approach

system parameters. Since a sensitivity matrix and a covariance matrix are used in the residual evaluation, they must be estimated before the local approach can be applied to detect faults. Generally, the implementation of the local approach can be split into two steps: training and detecting.

The purpose of training is to estimate the sensitivity matrix and the covariance matrix. To make the local approach work properly, the sensitivity matrix and the covariance matrix must be estimated under a nominal situation. Given a primary residual $H(\theta_0, z_k)$ and $N$-size training data $z_k$ for $k = 1, 2, \cdots, N$, the sensitivity matrix can be simply estimated using the following average equation:

$$ M(\theta_0) = \frac{1}{N} \sum_{k=1}^{N} \left[ \frac{\partial}{\partial \theta} H(z_k, \theta) \right]_{\theta = \theta_0} $$

(2.16)

Estimation of the sensitivity matrix can simply use the following formula:

$$ \Sigma(\theta_0) = \frac{1}{N} \sum_{i=1}^{N} H(z_k, \theta_0) H^T(z_k, \theta_0) $$

$$ + \sum_{i=1}^{J} \frac{1}{N-i} \sum_{k=1}^{N-i} \left( H(z_k, \theta_0) H^T(z_{k+i}, \theta_0) + H(z_{k+i}, \theta_0) H^T(z_k, \theta_0) \right) $$

(2.17)

with properly selected numbers $J$. In practice, one can gradually increase the value of $J$ until the $H_2$-norm of the change of $\Sigma(\theta_0)$ is smaller than some preset threshold (Huang, 1998).

One alternative way to estimate $\Sigma(\theta_0)$ is proposed by Devauchelle-Gach (1991). According to their suggestion, the data are divided into $L$ segments of length $M$ and the sensitivity matrix can be estimated as

$$ \Sigma(\theta_0) = \frac{1}{L} \sum_{l=0}^{L-1} D_{M,l} D_{M,l}^T $$

(2.18)

where

$$ D_{M,l} = \frac{1}{\sqrt{M}} \sum_{k=1}^{M} H(z_{k+lm}, \theta_0) $$

(2.19)

$L M = N$ is the total length of the sample. Obviously, $\Sigma(\theta_0)$ is generically positive definite for $L$ large enough, $M$ must be taken large enough so that $D_{M,l}$ behaves approximately as a Gaussian random variable (Zhang, 1994).

With the sensitivity matrix and the covariance matrix, FDI can be performed according to
the following procedure:
1. Collect the input and output data in order to calculate the auxiliary variable and the primary residual;
2. Calculate the normalized residual with the primary residual;
3. Perform chi-square test on the normalized residual and generate an alarm when the chi-square value exceeds a pre-designed threshold;
4. Perform fault isolation if a fault has been detected in step 3.
Chapter 3

Robustness of fault detection using the local approach

3.1 Introduction

The task of FDI is to provide prompt information to supervision processes about the occurrence of unexpected changes in a system. The terms change and fault will be used interchangeably in the following. With a nominal model, the model-based fault detection and isolation algorithm calculate residuals which are signals with zero mean in fault-free cases and non-zero in other cases. However, noise and uncertainties are unavoidable in any system. It is of great importance to consider their effects on the detection algorithm (i.e. the robustness of the FDI algorithm). In order to improve the robustness of the FDI algorithm, the approach is to reduce the effect of noise and uncertainties to the detection algorithm, while keeping the sensitivity of the algorithm toward faults. Different kinds of fault detection approaches have different ways to generate residuals and thus have different ways to deal with noise and model uncertainties.

The local approach is a powerful tool for incipient change detection and condition-based maintenance. It is designed to detect slight changes in parameters of a system so as to prevent a possible malfunction or damage. As a model-based fault detection and isolation method, the local approach has the same robustness problem with respect to noise and model uncertainties. Especially since it aims at detecting slight changes in parameters of a system, the issue of robustness should be examined very carefully. As a supervisory tool, the FDI algorithm should not provide any alarm when there is no fault in the system or if changes in parameters are within some acceptable range. Too many false alarms will greatly reduce the applicability of the algorithm.

It is important to consider the impact of noise on the local approach residuals. Even in a fault-free case, the normalized residual is an asymptotically Gaussian vector with zero mean. Due to noise in the system, the normalized residual is not exactly zero. An alarm is triggered when the chi-square test of the normalized residual exceeds some threshold. However, the local approach does not consider the effect of model uncertainties on the method. Since the local
approach is mostly used to detect changes in the parameters of a system, it is natural to think of parameter uncertainties. The meaning of robustness due to parameter uncertainties may be illustrated in the following two cases: 1) Actual value of system parameters can never be known. Instead, an estimated value obtained through identification is used. Due to noise, estimation error is unavoidable. When estimated parameters are used to calculate the primary and normalized residual, they may cause more false alarms. 2) There are regular fluctuations in system parameters due to changes in the operating condition. If the parameters fluctuate within an acceptable range, no alarm should be triggered.

There are two approaches to improve the robustness of FDI with respect to noise and model uncertainties: (1) find a residual which is sensitive to faults and insensitive to noise and model uncertainties and (2) revise the threshold to accommodate the effect of noise and model uncertainties. In this chapter 3, the later approach is considered.

3.2 Problem formulation

The effect of model uncertainties on the robustness of the local approach will be analyzed, followed by the effect of parameter fluctuations.

In fault detection, a parametric system characterized by the following state-space model is considered:

\[
\begin{align*}
x_{k+1} &= f(\theta, x_k, u_k) \\
y_k &= h(\theta, x_k, u_k)
\end{align*}
\] (3.1)

where \( u_k, y_k, x_k \) are vectors of the system input, output and state variable at time \( k \); \( \theta \) is the vector of the system parameters. In order to obtain the primary residual, a vector of auxiliary variables \( z_k \) is calculated using the following system:

\[
\begin{align*}
\xi_{k+1} &= \varphi(\theta, \xi_k, u_k, y_k) \\
z_k &= \vartheta(\theta, \xi_k, u_k, y_k)
\end{align*}
\] (3.2)

The primary residual \( H(\theta_0, z_k) \) and normalized residual \( \zeta_N(\theta_0) \) can be calculated with auxiliary variable \( z_k \) and the nominal value of system parameter \( \theta_0 \). The normalized residual is a Gaussian vector with zero mean in fault-cases and non-zero mean in other cases. With pre-estimated sensitivity matrix \( M \) and covariance matrix \( \Sigma \) (Definitions of the sensitivity matrix and the covariance matrix can be found in chapter 2), a chi-square test is performed on \( \zeta_N(\theta_0) \)
Chapter 3: Robustness of fault detection using the local approach

to detect whether the mean of the normalized residual is zero or not. That is to compare the chi-square value of the normalized residual

\[ t = \xi_N^T (\theta_0) \Sigma^{-1} M (M^T \Sigma^{-1} M)^{-1} M^T \Sigma^{-1} \xi_N (\theta_0) \]  

with a threshold \( \chi^2 \) obtained from a standard chi-square table according to the pre-assigned false alarm rate \( \alpha \) and the dimension of the normalized residual. A fault is detected when \( t \) is larger than \( \chi^2 \).

In practice, the nominal parameters \( \theta_0 \) of a system can never be known. Instead the estimated parameters \( \hat{\theta}_0 \) obtained through system identification are used. Usually \( \hat{\theta}_0 \) is not equal to \( \theta_0 \) because of noise and disturbance around the system. The primary residual \( H(\hat{\theta}_0, z_k) \) and normalized residual \( \xi_N(\hat{\theta}_0) \) calculated with the estimated parameters \( \hat{\theta}_0 \) have non-zero mean even in a fault-free case. Regarding the chi-square test of \( \xi_N(\hat{\theta}_0) \)

\[ \tilde{t} = \xi_N^T (\hat{\theta}_0) \Sigma^{-1} M (M^T \Sigma^{-1} M)^{-1} M^T \Sigma^{-1} \xi_N (\hat{\theta}_0) \]  

\( \tilde{t} \) is a non-central chi-square distributed random variable and the possibility of \( \tilde{t} \) to exceed \( \chi^2 \) will be larger than \( \alpha \). A new threshold value must be found to reduce the false alarm rate and thus improve robustness of the detection algorithm. However, the larger the threshold is, the less sensitive the detection algorithm is to faults. In order to minimize the loss of detection performance, the threshold should be set as small as possible.

The robustness problem can be summarized as follows:

1. Use the original scheme of the local approach and the estimated parameter \( \hat{\theta}_0 \) to calculate the primary residual and the normalized residual and do a chi-square test. In order to keep the false alarm rate small, a new threshold \( \hat{\chi}_\alpha \) must be found such that

\[ P(\tilde{t} > \hat{\chi}_\alpha \mid H_0) \leq \alpha \]  

where \( \tilde{t} \) is calculated using eqn. 3.4 and \( H_0 \) means the fault-free case.

2. In order to keep the sensitivity of the detection algorithm, the threshold \( \hat{\chi}_\alpha \) should be set as small as possible.

3.3 Recalculation of the threshold

The algorithm used to calculate the threshold will be given in the following.
3.3.1 Distribution of the normalized residual using the estimated parameter

As mentioned in section 3.1, the primary residual and the normalized residual, calculated with estimated parameters, have non-zero mean even in the fault free case. This is because of the inherent differences between the estimated parameters and the nominal parameters. The first step is to find the distribution of the normalized residual with the estimated parameters. With the primary residual \( H(θ₀, z_k) \), the normalized residual is calculated as follows:

\[
ζₙ(θ₀) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} H(θ₀, z_k)
\]  \hspace{1cm} (3.6)

With the estimated parameter \( \hat{θ}_₀ \), the normalized primary residual becomes \( ζₙ(\hat{θ}_₀) \). Using a first order Taylor series expansion, \( ζₙ(\hat{θ}_₀) \) can be approximated as:

\[
ζₙ(\hat{θ}_₀) \approx ζₙ(θ₀) + \frac{1}{\sqrt{N}} \left( \sum_{k=1}^{N} \frac{∂}{∂θ} H(θ, z_k) \right|_{θ=θ₀})(\hat{θ}_₀ - θ₀)
\]  \hspace{1cm} (3.7)

By the law of large numbers, when \( N \) approaches infinity

\[
\frac{1}{N} \left( \sum_{k=1}^{N} \frac{∂}{∂θ} H(θ, z_k) \right|_{θ=θ₀} \rightarrow M
\]  \hspace{1cm} (3.8)

where \( M \) is the sensitivity matrix defined in (2.5). Therefore, the approximation of the normalized residual calculated with the estimated parameters \( \hat{θ}_₀ \) can be written as

\[
ζₙ(\hat{θ}_₀) \approx ζₙ(θ₀) + M(\hat{θ}_₀ - θ₀)\sqrt{N}
\]  \hspace{1cm} (3.9)

As mention in chapter 2, under the assumption of small changes, that is to distinguish between two hypotheses \( H_0: θ = θ₀ \) and \( H_1: θ = θ₀ + λ/\sqrt{N} \), \( ζₙ(θ₀) \) follows:

\[
ζₙ(θ₀) \sim N(0, Σ) \hspace{1cm} \text{under } H_0
\]  \hspace{1cm} (3.10)

\[
ζₙ(θ₀) \sim N(-Mλ, Σ) \hspace{1cm} \text{under } H_1
\]  \hspace{1cm} (3.11)

where \( Σ \) is the covariance matrix. The definition of \( Σ \) can be found in chapter 2.

Combining eqn. 3.9 and 3.10, the approximate distribution of the normalized residual under nominal situation is obtained:

\[
ζₙ(\hat{θ}_₀) \sim N(M(\hat{θ}_₀ - θ₀)\sqrt{N}, Σ) \hspace{1cm} \text{under } H_0
\]  \hspace{1cm} (3.12)

Combining eqn. 3.9 and 3.11, the approximate distribution of the normalized residual under faulty situation is obtained:
\[ \zeta_n(\hat{\theta}_0) \sim N(M(\hat{\theta}_0 - \theta_0)\sqrt{N} - M\lambda, \Sigma) \quad \text{under } H_1 \] (3.13)

For simplicity, the following denotation is used:
\[ a = (\hat{\theta}_0 - \theta_0)\sqrt{N} \] (3.14)

The distribution of the normalized residual calculated \( \zeta_n(\hat{\theta}_0) \) with the estimated parameter \( \hat{\theta}_0 \) can then be summarized as
\[ \zeta_n(\hat{\theta}_0) \sim N(Ma, \Sigma) \quad \text{under } H_0 \] (3.15)
\[ \zeta_n(\hat{\theta}_0) \sim N(Ma - M\lambda, \Sigma) \quad \text{under } H_1 \] (3.16)

Comparing eqn. 3.12 and 3.13 with 3.10 and 3.11, one can conclude that the estimated error \( (\hat{\theta}_0 - \theta_0) \) does not affect the covariance matrix of the normalized residual. It only contributes a constant bias \( Ma \) to the mean of the normalized residual under both \( H_0 \) and \( H_1 \).

### 3.3.2 Evaluation of the normalized residual

In the residual evaluation stage, a chi-square test is performed on the normalized residual \( \zeta_n(\theta_0) \) to detect whether the mean value of \( \zeta_n(\theta_0) \) is zero or not. To compute generalized likelihood ratio (GLR) between \( H_1 \) and \( H_0 \):
\[
\hat{t} = 2\ln \frac{\max_x p_x(\zeta_n(\theta_0))}{p_0(\zeta_n(\theta_0))} = \zeta_n(\theta_0)^T \Sigma^{-1} M^T \Sigma^{-1} \zeta_n(\theta_0)
\] (3.17)

where \( I = M^T \Sigma^{-1} M \), \( p_x(x) \) is the density function of Gaussian distribution under \( H_1 \) and \( p_0(x) \) is the density function of Gaussian distribution under \( H_0 \). When \( M \) is a square matrix
\[
\hat{t} = \zeta_n(\theta_0)^T \Sigma^{-1} \zeta_n(\theta_0)
\] (3.18)

The normalized residual calculated with the estimated parameters follows:
\[ \zeta_n(\hat{\theta}_0) \sim N(Ma, \Sigma) \quad \text{under } H_0 \]
\[ \zeta_n(\hat{\theta}_0) \sim N(Ma - M\lambda, \Sigma) \quad \text{under } H_1 \]

Then the generalized likelihood ratio (GLR) between \( H_1 \) and \( H_0 \) is
\[
\hat{t} = 2\ln \frac{\max_x p_x(\zeta)}{p_0(\zeta)} = (\zeta_n(\hat{\theta}_0) - Ma)^T \Sigma^{-1} M^T \Sigma^{-1} (\zeta_n(\hat{\theta}_0) - Ma)
\] (3.19)

It is assumed that \( M \) is square matrix, thus
\[ \hat{t} = (\zeta_N(\hat{\theta}_0) - Ma)\Sigma^{-1}(\zeta_N(\hat{\theta}_0) - Ma) \]

If the estimation error \( \theta_0 - \hat{\theta}_0 \) is known, \( a \) can be calculated using eqn. 3.13. The normalized residual \( \zeta_N(\theta_0) \) can be estimated as \( \zeta_N(\hat{\theta}_0) - Ma \). This is called the upgrading scheme (it will be discussed in the simulation studies). The key idea is to estimate the constant bias \( Ma \) with training data. In detecting faults, one can calculate the normalized residual \( \zeta_N(\hat{\theta}_0) \) with estimated parameters \( \hat{\theta}_0 \) and use an estimated bias to upgrade the normalized residual as \( \zeta_N(\hat{\theta}_0) - Ma \). A chi-square test can be performed on \( \zeta_N(\hat{\theta}_0) - Ma \) and the original threshold can then be used without increasing the false alarm rate. However, the real value of the estimation error \( \theta_0 - \hat{\theta}_0 \) is unknown. Although it can be estimated through identification, the estimated parameter error is not accurate. Therefore, it is better to use the original scheme to perform a chi-square test on \( \zeta_N(\hat{\theta}_0) \) as follows:

\[ \tilde{t} = \zeta_N^T(\hat{\theta}_0)\Sigma^{-1}\zeta_N(\hat{\theta}_0) \]  (3.21)

\( \tilde{t} \) is a distributed non-central chi-square distributed random variable in a fault-free case. The probability of \( \tilde{t} \) to exceed \( \chi_a \) in a fault-free case (false alarm rate) can be larger than the pre-assigned value \( \alpha \). As mentioned in section 3.1, the robustness of FDI using the local approach is dependent on making the false alarm rate no larger than \( \alpha \). Therefore, the threshold \( \chi_a \) needs to be revised as \( \hat{\chi}_a \) so that the criterion

\[ P(\tilde{t} > \hat{\chi}_a | H_0) \leq \alpha \]

is satisfied.

If the probability of \( \tilde{t} \) being less than \( \hat{\chi}_a \) is larger than the probability of \( t \) being less than \( \chi_a \)

\[ P(\hat{t} \leq \hat{\chi}_a) \geq P(t \leq \chi_a) \quad \text{under } H_0 \]

then the false alarm rate with the revised threshold \( \hat{\chi}_a \)

\[ P(\hat{t} > \hat{\chi}_a | H_0) = 1 - P(\hat{t} \leq \hat{\chi}_a | H_0) \leq 1 - P(t \leq \chi_a | H_0) = \alpha \]

Moreover, if \( \hat{t} \leq \hat{\chi}_a \) is always true when \( t \leq \chi_a \), then \( P(\hat{t} \leq \hat{\chi}_a) \geq P(t \leq \chi_a) \). Therefore, if a threshold \( \hat{\chi}_a \) can be found that \( \hat{t} \leq \hat{\chi}_a \) is true when \( t \leq \chi_a \), the probability of \( \tilde{t} \) being less than \( \hat{\chi}_a \) is larger than the probability of \( t \) being less than \( \chi_a \), the false alarm rate of the chi-
square on $\zeta_n(\hat{\theta}_0)$ with the revised threshold will be less than $\alpha$. This is also equivalent to finding a threshold $\hat{\chi}_a$ that when

$$(\zeta_n(\hat{\theta}_0) - Ma)^T \Sigma^{-1}(\zeta_n(\hat{\theta}_0) - Ma) < \chi_a$$

the following inequality is true:

$$\zeta_n^T(\hat{\theta}_0) \Sigma^{-1} \zeta_n(\hat{\theta}_0) \leq \hat{\chi}_a$$

Therefore, the robustness problem is transformed into a problem of finding a threshold $\hat{\chi}_a$ which can guarantee that $i \leq \hat{\chi}_a$ holds when $t \leq \chi_a$.

### 3.3.3 Deduction of the new threshold

Since

$$(\zeta_n(\hat{\theta}_0) - Ma)^T \Sigma^{-1}(\zeta_n(\hat{\theta}_0) - Ma) < \chi_a$$

$$\Leftrightarrow \zeta_n^T(\hat{\theta}_0) \Sigma^{-1} \zeta_n(\hat{\theta}_0) < \chi_a + 2 \zeta_n^T(\hat{\theta}_0) \Sigma^{-1} Ma - a^T M^T \Sigma^{-1} Ma$$

where $\Leftrightarrow$ means equivalent to. It is easy to prove that for any $n$-dimension vectors $X$ and $Y$

$$2X^T Y \leq \frac{1}{k} X^T X + k Y^T Y$$

where $k$ is any positive real number.

Therefore, for positive number $k$

$$2\zeta_n^T(\hat{\theta}_0) Ma \leq \frac{1}{k} \zeta_n^T(\hat{\theta}_0) \Sigma^{-1} \zeta_n(\hat{\theta}_0) + ka^T M^T \Sigma^{-1} Ma$$

Substituting eqn. 3.23 into 3.22, then

$$(\zeta_n(\hat{\theta}_0) - Ma)^T \Sigma^{-1}(\zeta_n(\hat{\theta}_0) - Ma) < \chi_a$$

$$\Leftrightarrow \zeta_n^T(\hat{\theta}_0) \Sigma^{-1} \zeta_n(\hat{\theta}_0) < \chi_a + 2 \zeta_n^T(\hat{\theta}_0) \Sigma^{-1} Ma - a^T M^T \Sigma^{-1} Ma$$

$$\Rightarrow \zeta_n^T(\hat{\theta}_0) \Sigma^{-1} \zeta_n(\hat{\theta}_0) < \chi_a + \frac{1}{k} \zeta_n^T(\hat{\theta}_0) \Sigma^{-1} \zeta_n(\hat{\theta}_0)$$

$$+ ka^T M^T \Sigma^{-1} Ma - a^T M^T \Sigma^{-1} Ma$$

$$\Leftrightarrow (1 - \frac{1}{k}) \zeta_n^T(\hat{\theta}_0) \Sigma^{-1} \zeta_n(\hat{\theta}_0) < \chi_a + (k - 1)a^T M^T \Sigma^{-1} Ma$$

Therefore, when $k > 1$
If the threshold \( \hat{\chi}_a \) is set to be

\[
\hat{\chi}_a = \frac{k}{k-1} \chi_a + ka^T M^T \Sigma^{-1} Ma
\]  

(3.24)

then

\[
\zeta_N^T (\hat{\theta}_o) \Sigma^{-1} \zeta_N (\hat{\theta}_o) \leq \hat{\chi}_a
\]

is always true when \( (\zeta_N (\hat{\theta}_o) - Ma)^T \Sigma^{-1} (\zeta_N (\hat{\theta}_o) - Ma) < \chi_a \) is true.

Therefore, if \( \hat{\chi}_a \) determined by eqn. 3.24 is set to be the new threshold, the false alarm rate of the chi-square test on \( \zeta_N (\hat{\theta}_o) \) will be less than \( \alpha \).

### 3.3.4 Optimization of the threshold

In order to make the threshold value as small as possible and to maintain the detection performance, \( \hat{\chi}_a \) is minimized with respect to \( k \). Let

\[
\chi_\beta = a^T M^T \Sigma^{-1} Ma
\]  

(3.25)

Then

\[
\hat{\chi}_a^{opt} = \min_{k} \left( \frac{k}{k-1} \chi_a + ka^T M^T \Sigma^{-1} Ma \right)
\]

\[
= \min_{k} \left( \frac{k}{k-1} \chi_a + k \chi_\beta \right) \quad (3.26)
\]

\[
= (\sqrt{\chi_a} + \sqrt{\chi_\beta})^2
\]

where \( \chi_\beta \) is defined in eqn. 3.22. \( \hat{\chi}_a \) reaches its minimum value \( \hat{\chi}_a^{opt} \) when \( k = 1 + \sqrt{\chi_a / \chi_\beta} \).

In eqn. 3.26, \( \chi_\beta \) remains to be determined. Usually the value of nominal parameter \( \hat{\theta}_o \) is obtained through system identification and it is assumed that:

\[
\hat{\theta}_o \sim N(\theta_o, \Sigma_i)
\]

then

\[
a = (\hat{\theta}_o - \theta_o) \sqrt{N} \sim N(0, N \Sigma_i)
\]

In practice, \( \chi_\beta \) can be calculated as

\[
\chi_\beta = E(a^T M^T \Sigma^{-1} Ma)
\]  

(3.27)
To be more precise, this $\chi_\beta$ is not the same as that defined in equ. 3.25.

### 3.3.5 Discussion of the results

Noise and model uncertainties are two main factors leading to false alarms. The revised threshold $\tilde{\chi}_a$ can be rewritten as follows:

$$\tilde{\chi}_a^{opt} = (\sqrt{\chi_a} + \sqrt{\chi_\beta})^2 = \chi_a + \chi_\beta + 2\sqrt{\chi_a \chi_\beta}$$

In eqn. 3.28, there are three terms. The first term $\chi_a$ is related to system noise. If there is no noise in the system, the non-zero normalized residual will indicate a fault. The threshold here is set to be zero. Because of the noise, the normalized residual is a Gaussian random vector with zero mean. A threshold $\chi_a$ other than zero is set so that a fault alarm is triggered only when chi-square value of the normalized residual goes beyond the threshold. When there are model uncertainties, the normalized residual has a non-zero mean even in fault-free cases. A term $\chi_\beta$ is introduced to increase the threshold to reduce false alarms due to model uncertainties. While the noise and model uncertainties can interact with each other, the third term $2\sqrt{\chi_a \chi_\beta}$ appears in eqn. 3.28 which takes in consideration the interaction between noise and model uncertainties.

A natural requirement for the revised threshold is that when there is no uncertainty in the nominal parameter, (i.e. $\hat{\theta}_0 = \theta_0$), the threshold does not need to be changed and the revised threshold should be the same as the original one. Through eqn. 3.13 and 3.25 it can be shown that when $\hat{\theta}_0 = \theta_0$, $\chi_\beta = 0$ and $\tilde{\chi}_a^{opt} = \chi_a$.

### 3.3.6 Extension of the results to the case of parameter fluctuations

Secondly, consider the robustness of the local approach with respect to regular fluctuations of system parameters. Suppose that $A$ is the regular range of the parameter fluctuations. The algorithm to revise the threshold is provided as follows:

At any one moment, suppose that the real parameter value is $\theta$ and it is in the range of regular fluctuation. The normalized residual follows:

$$\zeta_n(\theta_0) \sim N(Ma, \Sigma)$$

where

\[25\]
Chapter 3: Robustness of fault detection using the local approach

\[ a = \sqrt{N(\theta - \theta_o)} \]

The chi-square value of \( \zeta_N(\theta_o) \) is

\[ t = \zeta_N(\theta_o)\Sigma^{-1}\zeta_N(\theta_o) \]

Since the mean of \( \zeta_N(\theta_o) \) is not zero, then \( t \) is a non-central chi-square distributed random variable. If the original threshold is used this will lead to increased false alarms. In order to reduce the false alarm rate to the pre-assigned value \( \alpha \), a new threshold should be found such that the false alarm rate

\[ P(t > \chi_a | H_o) \leq \alpha . \]

Let

\[ \tilde{t} = (\zeta_N(\theta_o) - Ma)^T \Sigma^{-1} M I^{-1} M^T \Sigma^{-1} (\zeta_N(\theta_o) - Ma) \]

Since

\[ \zeta_N(\theta_o) - Ma \sim N(0, \Sigma) \]

\( \tilde{t} \) is a central chi-square random variable with the degree of freedom equal to the dimension of the normalized residual. Therefore \( P(\tilde{t} > \chi_a | H_o) = \alpha \). If a new threshold can be found that when \( t \leq \chi_a \) is always true when \( \tilde{t} < \chi_a \), then

\[ P(t > \tilde{t} | H_o) > P(t < \chi_a | H_o) \]

The false alarm rate with the revised threshold \( \hat{\chi}_a \)

\[ P(t > \hat{\chi}_a | H_o) = 1 - P(t \leq \hat{\chi}_a | H_o) \leq 1 - P(\tilde{t} \leq \chi_a | H_o) = \alpha \]

Using the result in section 3.3 and 3.4, the revised threshold can be calculated as

\[ \hat{\chi}_a = (\sqrt{\chi_a} + \sqrt{\chi_\beta})^2 \] (3.30)

where \( \chi_a \) is the original threshold. \( \chi_\beta \) can be calculated as

\[ \chi_\beta = aM^T\Sigma^{-1}Ma \]

In order to make the false alarm rate to be less than \( \alpha \) for all \( \theta \in A \), \( \chi_\beta \) should be computed as

\[ \chi_\beta = \max(N(\theta - \theta_o)^T M^T\Sigma^{-1} M(\theta - \theta_o)), \theta \in A \] (3.31)

3.4 Simulation results

The effectiveness of the proposed algorithms of calculating a new threshold in section 3.3 will be shown by the simulation of a single-input-single-output (SISO) system.
3.4.1 System specification

In the simulation, a first-order SISO system is used:

\[ A(z^{-1})y(t) = B(z^{-1})u(t-1) + e(t) \]  

where \( u(t) \) and \( y(t) \) are the input and output of the system and \( e(t) \) is a white noise.

\[ A(z^{-1}) = 1 + az^{-1}, \quad B(z^{-1}) = b \]

Nominal values of the system parameters \( a, b \) are:

\[ a_0 = -0.6, \quad b_0 = 0.4 \]

The primary residual is calculated as

\[ H(u(t), y(t), a, b) = \begin{bmatrix} -y(t-1) \\ u(t-1) \end{bmatrix} = \begin{bmatrix} y(t) - ay(t-1) - bu(t-1) \end{bmatrix} \]  

(3.33)

3.4.2 Simulation of robustness with respect to model uncertainties

First the parameters \( a, b \) were estimated using the Least-Square (LS) method and the estimated parameters \( \hat{a}_0, \hat{b}_0 \) were found to be

\[ \hat{a}_0 = -0.5899, \quad \hat{b}_0 = 0.4691 \]

The system was excited with a white noise signal with a variance of 1 for 2000 time units. With the input and output data, the primary residual was calculated using eqn. 3.33. With the primary residual, the sensitivity matrix \( M \) and covariance matrix \( \Sigma \) were estimated using eqn. 2.16 and 2.17:

\[ \Sigma = \begin{bmatrix} 1.6173 & 0.0776 \\ 0.0776 & 0.2867 \end{bmatrix}, \quad M = \begin{bmatrix} 1.6360 & -0.0119 \\ -0.0119 & 0.3317 \end{bmatrix} \]

In the estimation of \( M \) and \( \Sigma \), the estimated parameter \( \hat{a}_0, \hat{b}_0 \) were used instead of the nominal parameters \( a_0, b_0 \). The false alarm ratio \( \alpha \) was set to be 0.01 and the original threshold was obtained from the standard chi-square table as \( \chi^2_\alpha = 9.21 \). With the covariance matrix of the estimated parameters \( \hat{a}_0 \) and \( \hat{b}_0 \), the sensitivity matrix \( M \) and the covariance matrix \( \Sigma \), the new threshold could be calculated using eqn. 3.26. The threshold value was found to be \( \hat{\chi}^2_\alpha = 13.57 \).

In order to show the improvement of the new threshold, it was tested under both fault-free
Chapter 3: Robustness of fault detection using the local approach

cases and faulty cases.

(a) Test of the false alarm rate using the revised threshold in a fault free case

The process model 3.32 was excited with a white noise signal with a variance of 1 to 10000 time units. A chi-square value was calculated at every 10 time units in order to reduce the computation load. Every chi-square value was based on the past 100 time units (i.e. a moving window of 100 time units was used). A plot of chi-square value versus time units is shown in Fig. 3.1.

![Chi-square value plot](image)

Figure 3.1 Chi-square value of the normalized residual in a fault-free case

<table>
<thead>
<tr>
<th>Number of chi-square value exceeding threshold</th>
<th>False alarm rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original threshold</td>
<td>23</td>
</tr>
<tr>
<td>Revised threshold</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 3.1 False alarm rate in a fault-free case

Fig. 3.1 and Table 3.1 indicate that when the original threshold was used there were many more false alarms than necessary and the false alarm rate exceeded the expected rate of 1%. When the revised threshold value was used, there were only 7 false alarms. The false alarm rate was 0.7% close to the expected rate of 1%. The simulation results show that the revised threshold is suitable since it reduced the false alarm rate to around the preset value of 1%. If the false rate was well below the expected value of 1%, it may indicate that the revised threshold was too high.
Chapter 3: Robustness of fault detection using the local approach

(b) Test of the detection power the using revised threshold in faulty cases

Missing detection rate measures the probability of chi-square value of the normalized residual going under the threshold when a fault happens. It is also referred to as detection power. One can not obtain a low missing detection rate and a low false alarm rate simultaneously. By increasing the threshold, a low false alarm rate can be obtained. But the missing detection rate will also increase. Therefore, the detection power can be used as another criterion to evaluate a FD algorithm.

The following simulations were done to test the detection power of the revised threshold. For each of the simulations, the process model was excited with a white noise signal with a variance of 1 to 5000 time units. A chi-square value was calculated at every 10 time units. Every chi-square value was based on the past 100 time units. In the first simulation, the parameter $a$ was step changed from $-0.6$ to $-0.8$ at the $2500^{th}$ time unit and the plot of chi-square value versus time units is shown in Fig. 3.2. In the second simulation, the parameter $b$ was step changed from 0.4 to 1.3 at $2500^{th}$ time unit and the plot of chi-square value versus time units is shown in Fig. 3.3.

![Figure 3.2 Chi-square value of the normalized residual in a faulty case](image)

(a change occurred at the parameter $a$)
Fig. 3.2 and 3.3 show that when faults occurred, the effect of the revised threshold toward the detection power was quite small. The detection algorithm could detect a change in the system parameters $a$ and $b$ effectively.

The tests of false alarm rate and detection power indicate that the revised threshold is quite effective. It reduces the false alarm rate below the expected value, while maintaining the detection power.

(c) Comparison of the upgrading scheme and the robustness scheme

As discussed in section 3.3.2, upgrading the normalized residual is a method to deal with constant bias due to parameter uncertainties. The idea is to estimate the constant bias of normalized residual with training data, while simultaneously estimating the sensitivity matrix and covariance matrix. The result is used to upgrade the normalized residual in the detection of faults. Upgrading the normalized residual is called the upgrading scheme while the method of revising the threshold to reduce the false alarm rate is called robustness scheme.
The following simulation was done as a comparison of these two schemes. The process model was excited with a white noise signal with a variance of 1 to 5000 time units. A chi-square value was calculated at every 10 time units. Every chi-square value was based on the past 100 time units.

![Test of false alarm rate for the robustness scheme](image1)

![Test of false alarm rate for the upgrading scheme](image2)

Figure 3.4 Chi-square value for the robustness scheme and for the upgrading scheme in a fault-free case

<table>
<thead>
<tr>
<th></th>
<th>Number of chi-square value exceeding threshold</th>
<th>False alarm rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robustness scheme</td>
<td>Original threshold 34</td>
<td>3.4%</td>
</tr>
<tr>
<td></td>
<td>Revised threshold 8</td>
<td>0.8%</td>
</tr>
<tr>
<td>Upgrading scheme</td>
<td>Original threshold 28</td>
<td>2.8%</td>
</tr>
<tr>
<td></td>
<td>Revised threshold 8</td>
<td>0.8%</td>
</tr>
</tbody>
</table>

Table 3.2 False alarm rates for the robustness scheme and the upgrading scheme

With the upgraded normalized residual and the original threshold, the false alarm rate decreased from 3.4\% to 2.8\%. However it was still well above the preset value of 1\%. The revised threshold reduced the false alarm rate to 0.8\% close to the expected value of 1\%. One can conclude that the upgrading scheme is not as effective as the robustness scheme in dealing with parameter uncertainties.
3.4.3 Simulation with respect to parameter fluctuations

In this simulation, the model parameter is assumed to have a regular fluctuation in $b$:

$$b = 0.4 + 0.1\sin(0.01t)$$

The original threshold was obtained from standard chi-square table as $\chi_a = 9.21$. Using eqn. 3.30 and 3.31, the revised threshold was calculated as $\hat{\chi}_a = 12.90$. In order to show the improvement of the new threshold, a test was conducted under both a fault-free case and a faulty case.

(a) Test of the false alarm rate using the revised threshold in a fault-free case

The process model was excited with a white noise signal with a variance of 1 to 10000 time units. A chi-square value was calculated at every 10 time units. Every chi-square value was based on the past 100 time units. A plot of chi-square value versus time units is shown in Fig. 3.5.

![Figure 3.5 Chi-square value of the normalized residual in a fault-free case with parameter fluctuations at $b$](image-url)
Chapter 3: Robustness of fault detection using the local approach

Table 3.3 False alarm rate in a fault-free case with parameter fluctuations

<table>
<thead>
<tr>
<th></th>
<th>Number of chi-square value exceeding threshold</th>
<th>False alarm rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original threshold</td>
<td>23</td>
<td>2.3%</td>
</tr>
<tr>
<td>Revised threshold</td>
<td>7</td>
<td>0.7%</td>
</tr>
</tbody>
</table>

Fig. 3.5 and Table 3.3 show that when the original threshold was used, the false alarm rate of 2.3% was much higher than the expected value of 1%. The revised threshold value decreased the false alarm rate to 0.7% which is close to the expected value of 1%.

(b) Test of the detection power using the revised threshold in faulty cases

The two following simulations were done to test the detection power of the revised threshold. For each of the simulations, the process model was excited with a white noise signal with a variance of 1 to 5000 time units. A chi-square value was calculated at every 10 time units. Every chi-square value was based on the past 100 time units. In the first simulation, the parameter \( a \) was step changed from \(-0.6\) to \(-0.8\) at the 2500\(^{th}\) time unit and the plot of chi-square value versus time units is shown in Fig. 3.6. In the second simulation, the parameter \( b \) was step changed from around 0.4 to 1.3 at 2500\(^{th}\) time unit and the plot of chi-square value versus time units is shown in Fig. 3.7.

Fig. 3.6 and 3.7 show that when a fault occurred, the effect of the revised threshold toward the detection power was quite small. The local approach was still able to effectively detected changes in the system parameters \( a \) and \( b \).
Figure 3.6 Chi-square value of the normalized residual in a faulty case with parameter fluctuations (a change occurred at the parameter \( a \))

Figure 3.7 Chi-square value of the normalized residual in a faulty case with parameter fluctuations (a step change occurred at the parameter \( b \))
(c) Comparison of the upgrading scheming and the robustness scheme

The following simulation was done to compare the robustness scheme and upgrading scheme in the case of parameter fluctuations. The process model was excited with a white noise signal with a variance of 1 to 5000 time units. For each scheme, a chi-square value was calculated at every 10 time units. Every chi-square value was based on the past 100 time units. The plot of chi-square value versus time units was shown in Fig. 3.8.

![Test of false alarm rate for the robustness scheme](image1)

![Test of false alarm rate for the upgrading scheme](image2)

Figure 3.8 Chi-square value for the upgrading scheme and the robustness scheme in a fault-free case with parameter fluctuations

Table 3.4 False alarm rates for the upgrading scheme and the robustness scheme in a fault-free case with parameter fluctuation

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Original threshold</th>
<th>Revised threshold</th>
<th>False alarm rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robustness</td>
<td>20</td>
<td>7</td>
<td>2.0%</td>
</tr>
<tr>
<td>Revised threshold</td>
<td>0.7%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Upgrading</td>
<td>20</td>
<td>6</td>
<td>2.0%</td>
</tr>
<tr>
<td>Revised threshold</td>
<td>0.6%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
With the upgraded normalized residual and original threshold, the false alarm rate was 2.0%, which was well above the preset value of 1%. The revised threshold reduced the false alarm rate to 0.7% which was close to the expected value of 1%. One can conclude that the upgrading scheme was not as effective as the robustness scheme in dealing with parameter uncertainties.

### 3.5 Experiment results

In order to assess the applicability of the algorithm in a real world unit, a pilot-scale tank level and temperature control system was used to generate data for the algorithm. Fig. 3.9 shows the schematic of the process.

![Figure 3.9 Process diagram of the tank level and temperature control system](image)

In this experiment, only the temperature process was considered. The energy balance of the process is as follows:

\[
C_p A_h \frac{dT}{dt} = C(pF_{in} T_{in} - pF_{out} T) + F_{\text{steam}} H
\]

where

\[C:\text{Heat capacity of water;}\]
Chapter 3: Robustness of fault detection using the local approach

$A$: Area of the tank;

$h$: Water level of the tank;

$\rho$: Water density;

$F_{in}$: Inflow of cold water;

$F_{out}$: Outflow of warm water;

$F_{steam}^\prime F_{sw}$: Flow of steam, determined by the opening of steam valve;

$H$: Steam enthalpy.

At steady state, $F_{in}$ and $F_{out}$ are constant and are determined by the water level in the tank $h$. $C, A$ and $\rho$ were assumed to be constant. Thus the dynamics of the temperature process were only determined by $h$. In the experiment, the tank level $h$ was regulated by a PI controller. Changes in the tank level $h$ were used to simulate faults in the system.

### 3.5.1 Identification of model parameters

Model-based FDI methods rely on the nominal model of the process to perform fault detection. The system model used was

$$A(z^{-1})y(t) = B(z^{-1})u(t) + e(t) \quad (3.35)$$

The process was excited with a PRBS signal to identify the parameters. The sampling interval was 4 seconds. The estimated parameters were:

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} = 1 - 0.5192 z^{-1} - 0.1671 z^{-2} - 0.1392 z^{-3}$$

$$B(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} = 0.0185 z^{-2} - 0.0689 z^{-3} + 0.0110 z^{-4}$$

The process model (3.35) can be rewritten as

$$y(k) = \varphi^T(k)\theta$$

where

$$\varphi(k) = [-y(k-1) - y(k-2) - y(k-3) u(k-1) u(k-2) u(k-3)]^T$$

$$\theta = [a_1 \ a_2 \ a_3 \ b_1 \ b_2 \ b_3]^T$$

The primary residual was calculated as follows:

$$H(\theta_0, y(k), \varphi(k)) = \varphi(k)(y(k) - \varphi^T(k)\theta_0)$$

where $\theta_0$ are the nominal values of the system parameters.
3.5.2 Training of the FDI system

The process was excited with a pseudo random binary signal (PRBS) at nominal state for 5000 sampling times. With the input-output data, the sensitivity matrix $M$ and the covariance matrix $\Sigma$ were estimated using eqn. 2.16 and 2.17. The false alarm ratio was set to be $\alpha = 0.01$. The original threshold value was obtained from the standard chi-square table as $\chi^2_a = 16.81$. With $M$ and $\Sigma$, the revised threshold was calculated as $\chi^2_a = 22.46$ using eqn. 3.27 and 3.28.

3.5.3 Test of the false alarm rate

Under nominal state (the nominal tank level is 14cm), the process was excited with a PRBS signal for 2000 sampling times. The length of the moving window was set to be 100. A chi-square value was calculated for every 5 sampling times. The plot of chi-square value is shown versus sampling time in the Fig. 3.10.

![Figure 3.10 Test of the false alarm rate in a fault-free case using the revised threshold](image)

When the original threshold was implemented, there were 21 false alarms and the false alarm rate was 5.25%. When the revised threshold value was implemented, there were 9 false alarms and the false alarm rate is 2.25%. Although the revised threshold greatly decreased the false alarm rate, it was still not large enough to reduce the false alarm rate to the preset value of 1%. The reason is that only the effect of high frequency noise was considered in both training and detection. In the parameter identification and the training stage, low frequency disturbance was not taken into account. Therefore, the detection algorithm can not distinguish the effects of low frequency disturbance and those of faults. Reducing the effect of low frequency
disturbances toward the detection algorithm may be a future research topic.

3.5.4 Test of the detection power

The process was excited with a PRBS signal for 2000 sampling times. The length of the moving window was set to be 100 time units. A chi-square value was calculated for every 5 sampling times. The tank level of the system changed from 14cm to 11cm at the 800th sampling time. The plot of chi-square value versus sampling time is shown in the Fig. 3.11.

![Figure 3.11 Test of the detection power using the revised threshold](image)

Fig. 3.11 shows that when a fault occurred, the effect of the revised threshold toward the detection power was quite small. The revised threshold has decreased the fault alarm rate while maintaining the ability to detect a real fault.

3.6 Conclusions

In this chapter, the robustness of the local approach, with respect to model uncertainties, was investigated. As for the local approach, an algorithm was proposed to recalculate the threshold value to reduce the false alarm rate to the preset value. A similar algorithm was also proposed to calculate the threshold to accommodate the regular parameter fluctuations.
Simulation results show that the revised threshold can reduce the false alarm rate to a value close to the pre-assigned one and maintain the ability of detecting fault in case of both parameter uncertainties and parameter fluctuations. The robustness scheme was also compared with the upgrading scheme through simulations and it was concluded that the robustness scheme is more effectiveness than the upgrading scheme in reducing the false alarm rate.

The proposed algorithms were also applied to a pilot-scale tank level and temperature control system. Though, the revised threshold was focused to be able to reduce the false alarm rate, it was not large enough to decrease the false alarm rate to the preset value. This probably was due to the low frequency disturbances in the system. A future research topic may be to improve the robustness of the local approach with respect to low frequency disturbances. Possible solutions could either be to further increase the threshold value, or to take in account the low frequency disturbances in the process model.
Chapter 4

Closed-loop fault detection using the local approach

4.1 Introduction

Fault detection and isolation (FDI) have become an important subject in the process control community with the increasing complexity of industrial systems. One of the applications of FDI is condition-based maintenance. The decision for such maintenance can be achieved by early detection of incipient changes in the parameters of a system with respect to their nominal value without any artificial excitation. By doing so, one can prevent possible malfunction or damage before it occurs. A statistical approach to this early detection task, the local approach, has been developed by Basseville (1998) and Zhang et al. (1998). It consists in the design of chi-square tests built on a function of the nominal model parameter and system inputs and outputs.

The local approach is closely related to parameter estimation based detection methods. In parameter estimation based fault detection methods, system parameters are estimated with on-line input-output data and compared with the nominal value. If significant changes occur, a decision on the presence of faults in the system is made. However there is inevitable estimation error in the estimated parameters due to noise and disturbance around the system. Evaluating the significance of the changes with respect to estimation error remains a problem. In fact, fault detection and isolation algorithms are only concerned with changes in the system parameters, not the exact value of the parameters. Thus an alternative way to detect changes in system parameters is to check whether the current system inputs and outputs are still in agreement with the nominal model, i.e. a signal-to-model distance is considered instead (Zhang, et. at. 1998). The local approach is a method based on the above idea and it can be developed in line with system identification.

A lot of research has been done on the local approach both theoretically and experimentally. The effectiveness and reliability of this approach has been demonstrated by its applications in the monitoring of some critical processes such as nuclear power plants, gas turbines, catalytic converter, etc (Huang, 1999). However, few studies have been conducted on closed-loop detection using the local approach. Since most of the control systems work under closed-loop
conditions, it is meaningful to investigate how the local approach can be applied to closed-loop monitoring using closed-loop data.

The fundamental problem in closed-loop identification is the correlation between system inputs and outputs through a feedback controller. The local approach is a detection method naturally related to parameter identification. Therefore, when closed-loop data are used for fault detection, one can anticipate similar problems associated with closed-loop identification; that is insufficient excitation exists in system inputs and outputs. Terms of the primary residual and the normalized residual may be linearly related. From the information point of view, some of the information of the primary residual calculated using closed-loop data is redundant and the information contained in the primary residual is not rich enough to reflect all types of changes in the system parameters. With respect to the computation of the detection algorithm, it is usually assumed that the sensitivity matrix has full rank and the covariance matrix is positive definite. However, this may not be true in closed-loop detection. A chi-square test cannot be performed on the normalized residual if the covariance matrix is not positive definite.

In this chapter the problem of closed-loop detection will be discussed. The original scheme of the local approach will be revised to make it useful for closed-loop detection. Closed-loop identification has received considerable attention in the literature. Several methods have been proposed and analyzed (Forssell, 1999). Some ideas from existing results in closed-loop identification will be applied in the problem of closed-loop detection.

### 4.2 Discussion of a simple example

A simple example is used in the following to outline the approaches for closed-loop detection. A second order process given by the following ARX model is considered:

\[ A(z^{-1})y(k) = B(z^{-1})u(k) + e(k) \]  

(4.1)

where \( u(k) \) and \( y(k) \) are the system input and output respectively at time instance \( k \), \( e(k) \) is a white noise sequence, and the polynomials \( A(z^{-1}) \) and \( B(z^{-1}) \) are defined as

\[ A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} \]

\[ B(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} \]

System 4.1 can be rewritten as

\[ y(k) = \varphi^T(k)\theta + e(k) \]  

(4.2)
where

\[ \varphi(k) = \begin{bmatrix} -y(k-1) & -y(k-2) & u(k-1) & u(k-2) \end{bmatrix}^T \]

\[ \theta = \begin{bmatrix} a_1 & a_2 & b_1 & b_2 \end{bmatrix}^T \]

For simplicity, the following notations are used:

\[ y_k = y(k), \quad \varphi_k = \varphi(k), \quad u_k = u(k), \quad e_k = e(k) \]

The primary residual is calculated as

\[ H(\theta_0, y_k, \varphi_k) = \varphi_k (y_k - \varphi_k^T \theta_0) \]  \hfill (4.3)

Substituting 4.2 into eqn. 4.3, the primary residual becomes

\[ H(\theta_0, y_k, \varphi_k) = \varphi_k \varphi_k^T (\theta - \theta_0) + \varphi_k e_k \]  \hfill (4.4)

Then the normalized residual can be calculated as

\[ \zeta_N(\theta_0) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} H(\theta_0, y_k, \varphi_k) \]

\[ = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \varphi_k \varphi_k^T (\theta - \theta_0) + \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \varphi_k e_k \]  \hfill (4.5)

In the local approach, the assumption of small change is made that

\[ \theta - \theta_0 = \frac{\lambda}{\sqrt{N}} \]  \hfill (4.6)

where \( \theta \) is the system parameter after a fault, \( \lambda \) is considered as an unknown, but fixed vector.

Under the assumption of small change, the normalized residual becomes

\[ \zeta_N(\theta_0) = \frac{1}{N} \sum_{k=1}^{N} (\varphi_k \varphi_k^T) \lambda + \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \varphi_k e_k \]  \hfill (4.7)

Since \( \varphi_k, e_k \) are uncorrelated and \( e_k \) is zero mean, \( \varphi_k e_k \) is also zero mean. By the Central Limit Theorem, the second term in eqn. 4.7 converges to some Gaussian distribution with zero mean when \( N \) approaches infinity. Also by the Central Limit Theorem when \( N \) approaches infinity

\[ \frac{1}{N} \sum_{k=1}^{N} \varphi_k \varphi_k^T \xrightarrow{\text{cov}(\varphi_k)} = M \]

where \( M \) denotes the covariance matrix of \( \varphi_k \).

To summarize if the assumption of small changes is taken, the normalized residual is asymptotically Gaussian distributed:
Chapter 4: Closed-loop fault detection using the local approach

\[ \xi_N(\theta_0) \sim N(M\lambda, \Sigma) \] when \( N \) approaches infinity.

where \( \Sigma \) is the covariance matrix of the second term in equation 4.7. A chi-square test can be performed on \( \xi_N(\theta_0) \) to detect whether \( \lambda \) is zero or not. A non-zero \( \lambda \) indicates a fault.

When the system 4.1 is well excited, both \( M \) and \( \Sigma \) will be of full rank. It also means that all types of changes in the system parameter vector \( \theta \) can be reflected in the system input and output, and thus in the mean value of the normalized residual \( \xi_N(\theta_0) \).

In a closed-loop case, \( M \) and \( \Sigma \) may not be of full rank because of the correlation between the system inputs and outputs. Suppose that a proportional controller with gain \( K_c \) is used to control the system 4.1 and the setpoint \( y_r \) is zero, then

\[ u_k = K_c(y_r - y_k) = -K_c y_k \] 

Therefore,

\[ \varphi_k = \begin{bmatrix} -y_{k-1} & -y_{k-2} & -K_c y_{k-1} & -K_c y_{k-2} \end{bmatrix}^T = P\overline{\varphi}_k \] 

where

\[ \overline{\varphi}_k = \begin{bmatrix} -y_{k-1} \\ -y_{k-2} \end{bmatrix} \quad P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ K_c & 0 \\ 0 & K_c \end{bmatrix} \] 

Then the primary residual becomes

\[ H(\theta_0, y_k, \varphi_k) = \varphi_k (y_k - \varphi_k^T \theta_0) = P\overline{\varphi}_k (y_k - \overline{\varphi}_k^T P^T \theta_0) \] 

Letting

\[ h = \begin{bmatrix} 0 & -K_c & 0 & 1 \end{bmatrix} \]

then

\[ hP = \begin{bmatrix} 0 & 0 \end{bmatrix} \]

and

\[ hH(\theta_0, y_k, \varphi_k) = hP\overline{\varphi}_k (y_k - \overline{\varphi}_k^T P^T \theta_0) = 0 \] 

Therefore, the terms of \( H(\theta_0, y_k, \varphi_k) \) are linearly related.

In the case of closed-loop detection, the sensitivity matrix and the covariance matrix can be written as
Chapter 4: Closed-loop fault detection using the local approach

\[ M = \text{cov}(\varphi_k) = E(\varphi_k \varphi_k^T) = E(P \varphi_k \varphi_k^T P^T) \]
\[ = PE(\varphi_k \varphi_k^T)P^T = \text{cov}(\varphi_k)P^T \quad (4.13) \]

\[ \Sigma = \text{cov}(\varphi_k e_k) = E(\varphi_k e_k \varphi_k^T) = E(P \varphi_k e_k \varphi_k^T P^T) \]
\[ = PE(\varphi_k e_k \varphi_k^T)P^T = \text{cov}(\varphi e)P^T \quad (4.14) \]

\( \varphi_k \) and \( \varphi_k e_k \) are random vectors with length 2. Therefore, \( \text{rank}(\text{cov}(\varphi_k)) \leq 2 \) and \( \text{rank}(\text{cov}(\varphi_k e_k)) \leq 2 \). Since \( \text{rank}(P) = 2 \), then

\[ \text{rank}(M) = \min[\text{rank}(P),\text{rank}(\text{cov}(\varphi_k))] = \text{rank}(\text{cov}(\varphi_k)) \leq 2 \]
\[ \text{rank}(\Sigma) = \min[\text{rank}(P),\text{rank}(\text{cov}(\varphi_k e_k))] = \text{rank}(\text{cov}(\varphi_k e_k)) \leq 2 \]

From equations 4.10 to 4.12, it can be concluded that in the case of closed-loop detection:

- Terms of the primary residual calculated with open-loop model are linearly related. This means that some terms in the primary residual can be calculated as the linear combination of the rest of the terms. They can be eliminated from the primary residual without loss of information about faults.

- The sensitivity matrix \( M \) is not full rank. Therefore, for some non-zero \( \lambda \), \( M \lambda \) can be zero. In such a case, the change in the system parameters \( \lambda \) does not cause the change in the mean value of the normalized residual. A missed detection will occur.

- The covariance matrix \( \Sigma \) may not be of full rank, which means that it is impossible to take the inversion of \( \Sigma \) to perform a chi-square test on the normalized residual and the detection algorithm can not be continued.

In the case of closed-loop detection, insufficient excitation or correlation between system inputs and outputs makes it difficult to detect all types of changes in the system parameters. However, the primary residual and the normalized residual still contain useful information. The local approach may be revised to use such information to detect these changes in the system parameters that can be reflected in the closed-loop data. In the following, the ideas of closed-loop detection will be illustrated using the above example.

With closed-loop data, the normalized residual can be calculated as
Chapter 4: Closed-loop fault detection using the local approach

\[ \zeta_N(\theta_0) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \varphi_k \varphi_k^T (\theta - \theta_0) + \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \varphi_k e_k \]

\[ = \frac{1}{N} \sum_{k=1}^{N} P \varphi_k \varphi_k^T P^T \lambda + \frac{1}{\sqrt{N}} \sum_{k=1}^{N} P \varphi_k e_k \]

\[ = P \left( \frac{1}{N} \sum_{k=1}^{N} \varphi_k \varphi_k^T \right) P^T \lambda + P \left( \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \varphi_k e_k \right) \]  

(4.15)

where \( P \) is defined in eqn. 4.10. Letting

\[ D = \begin{bmatrix} 1 & 0 & K_c^{-1} & 0 \\ 0 & 1 & 0 & K_c^{-1} \end{bmatrix} \]  

(4.16)

then

\[ DP = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

where \( I_2 \) means an unit matrix with number of columns equal to 2. With the following notation:

\[ \bar{\zeta}_N(\theta_0) = D \zeta_N(\theta_0), \quad \bar{\theta}_0 = P^T \theta_0, \quad \bar{\theta} = P^T \theta, \quad \bar{\lambda} = P^T \lambda \]

and then

\[ \bar{\zeta}_N(\theta_0) = DP \left( \frac{1}{N} \sum_{k=1}^{N} \varphi_k \varphi_k^T \right) P^T \lambda + DP \left( \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \varphi_k e_k \right) \]

\[ = \left( \frac{1}{N} \sum_{k=1}^{N} \varphi_k \varphi_k^T \right) \bar{\lambda} + \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \varphi_k e_k \]  

(4.18)

By the Central Limit Theorem, the second term in eqn. 4.7 converges to some Gaussian distribution with zero mean when \( N \) approaches infinity. Also by the Central Limit Theorem, when \( N \) approaches infinity

\[ \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \varphi_k \varphi_k^T \rightarrow Cov(\varphi_k) = \bar{M} \]  

(4.19)

Suppose \( \bar{\Sigma} \) is the covariance matrix of the second term in eqn. 4.19. Therefore, when \( N \) approaches infinity,

\[ \bar{\zeta}_N(\theta_0) \rightarrow N(\bar{M} \bar{\lambda}, \bar{\Sigma}) \]  

(4.20)

In can be proven that in a closed-loop case, \( \bar{M} \) and \( \bar{\Sigma} \) are of full rank. This means that the revised normalized residual \( \bar{\zeta}_N(\theta_0) \) can be used to detect whether \( \bar{\lambda} \) is zero or not, (i.e. the
actual change in $\theta$).

In fact the closed-loop model is

$$A(z^{-1})y(k) = -B(z^{-1})K_e y(k) + e(k)$$

(4.21)

$$[A(z^{-1}) + B(z^{-1})K_c]y(k) = e(k)$$

(4.22)

Since

$$A(z^{-1}) + B(z^{-1})K_c = 1 + (a_1 + K_c b_1)z^{-1} + (a_2 + K_c b_2)z^{-2}$$

(4.23)

The closed-loop parameters are

$$[a_1 + K_c b_1, a_2 + K_c b_2]^T = P^T \theta = \bar{\theta}$$

(4.24)

Therefore, $\bar{\theta}$ is actually the closed-loop parameter vector of the system.

From the above discussion, one can conclude that the local approach can be applied for closed-loop detection in either of the following two ways:

- Use an open-loop system model to calculate the primary residual and the normalized residual. Reduce the dimension of the normalized residual to eliminate information redundancy. The regular local approach can be used to detect changes in the open-loop parameters.

- Using the closed-loop model instead of the open-loop model to calculate the primary residual and the normalized residual and then perform detection on the closed-loop parameters. Since the changes in the closed-loop parameters indicate changes in the open-loop parameters, changes in the open-loop parameters can be detected indirectly.

Based on the above ideas, two closed-loop detection methods are proposed: a dimension reduction method and an indirect detection method. They will be discussed separately in the following.

4.3 Closed-loop fault detection using the dimension reduction method

First, closed-loop detection using the dimension reduction method is discussed. The basic idea of the dimension reduction method is to reduce the dimension of the normalized residual to eliminate information redundancy and reduce the dimension of the covariance matrix in order for it to have full rank. A parametric system given by the following state space model is considered
where \( x_k, u_k, y_k \), are vectors of state variables, system inputs and outputs at time \( k \) respectively; \( \theta \) is the vector of the system parameters of length \( n \); \( e_k \) is a white or colored noise. Using the method given in chapter 2, the primary residual and normalized residual can be calculated using the open-loop model 4.25. Under the assumption of small changes, that is to distinguish between two hypotheses \( H_0 : \theta = \theta_0 \) and \( H_1 : \theta = \theta_0 + \lambda \sqrt{N} \), the normalized residual \( \zeta_N(\theta_0) \) follows:

\[
\zeta_N(\theta_0) \sim N(0, \Sigma) \quad \text{under } H_0
\]

\[
\zeta_N(\theta_0) \sim N(-M\lambda, \Sigma) \quad \text{under } H_1
\]

The generalized likelihood ratio (GLR) between \( H_1 \) and \( H_0 \) is

\[
t = 2 \ln \frac{\max \{ p_\gamma(\zeta) \} }{p_0(\zeta)} = \zeta_N^T(\theta_0)\Sigma^{-1}MF^{-1}M^T\Sigma^{-1}\zeta_N(\theta_0)
\]

where \( F = M^T\Sigma^{-1}M \). \( t \) is a chi-square distributed variable with a number of degrees of freedom equal to the dimension of \( \theta \). This chi-square distribution is central under \( H_0 \), and has non-centrality under \( H_1 \).

Usually the system is operated in closed-loop conditions. When insufficient excitation exists in the closed-loop system, \( M \) and \( \Sigma \) will not be of full rank. Therefore, both \( \Sigma \) and \( F \) will be invertible. The \( t \) value, though, in eqn. 4.28 cannot be calculated.

Generally, the primary residual and the normalized residual have the same dimension as that of the system parameter vector \( \theta \). Therefore, \( M \) and \( \Sigma \) are \( n \times n \) matrices. Assuming that \( M \) and \( \Sigma \) have the same rank \( r \) and using Singular Value Decomposition (SVD) on \( M \) and \( \Sigma \)

\[
M = U_1 S_1 V_1^T, \quad \Sigma = U_2 S_2 V_2^T
\]

where

\[
U_1, V_1, U_2 \text{ and } V_2 \text{ are all unitary matrices;}
\]

\[
S_1 = \begin{bmatrix}
\alpha_1 & 0 & 0 & 0 \\
0 & \ddots & \ddots & \vdots \\
0 & \cdots & \alpha_r & 0 \\
0 & \cdots & 0 & 0
\end{bmatrix}, \quad S_2 = \begin{bmatrix}
\beta_1 & 0 & 0 & 0 \\
0 & \ddots & \ddots & \vdots \\
0 & \cdots & \beta_r & 0 \\
0 & \cdots & 0 & 0
\end{bmatrix}
\]
Chapter 4: Closed-loop fault detection using the local approach

Letting $\bar{S}_1$ and $\bar{\Sigma}$ be the sub-matrices with ranks of $r$

$$\bar{S}_1 = \begin{bmatrix} \alpha_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \alpha_r \end{bmatrix} \quad \bar{\Sigma} = \begin{bmatrix} \beta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \beta_r \end{bmatrix}$$

Since $\Sigma$ is a symmetric and semi-positive matrix, $U_2 = V_2$ and $\beta_1, \ldots, \beta_r$ will be all positive numbers. Therefore, $\bar{\Sigma}$ is a positive definite matrix. Defining

$P = [I_r \ 0]_{r \times r}$

where $I_r$ means the unit matrix with a rank of $r$. therefore,

$$S_1 = P^T \bar{S}_1 P, \ S_2 = P^T \bar{\Sigma} P$$

Then

$$\Sigma = U_2 P^T \bar{S}_2 P U_2^T, \ M = U_1 P^T \bar{S}_1 P V_1^T$$

The distribution of the normalized residual can be rewritten as

$$\xi_N(\theta_0) \sim N(0, \ U_2 P^T \bar{S}_2 P U_2^T) \quad \text{under } H_0$$

$$\xi_N(\theta_0) \sim N(-U_1 P^T \bar{S}_1 P V_1^T \lambda, \ U_2 P^T \bar{S}_2 P U_2^T) \quad \text{under } H_1$$

Using a linear transformation of the normalized residual

$$\tilde{\xi}_N(\theta_0) = T \xi_N(\theta_0)$$

where

$$T = PU_2^T$$

$\tilde{\xi}_N(\theta_0)$ is a $r \times 1$ Gaussian distributed random vector and

$$\text{cov}(\tilde{\xi}_N(\theta_0)) = \text{cov}(PU_2^T \xi_N(\theta_0)) = PU_2^T \text{cov}(PU_2^T \xi_N(\theta_0)) P^T U_2$$

$$= PU_2^T \Sigma P^T U_2 = PU_2^T U_2 P^T \Sigma U_2 P^T$$

Since $U_2$ is a unitary matrix

$$U_2 U_2^T = I_n, \ U_2^T U_2 = I_n$$

and $PU_2^T U_2 P^T = PI_r P^T = I_r$

Therefore,

$$\text{cov}(\tilde{\xi}_N(\theta_0)) = PU_2^T U_1 P^T \Sigma U_2 P^T = I_r \Sigma I_r = \Sigma$$

Letting

$$\bar{M} = PU_2^T U_1 P^T \bar{S}_1, \ \bar{\lambda} = PV_1^T \lambda$$

49
Then
\[
E[\xi_N(\theta_0)] = E[PU_2^T \xi_N(\theta_0)] = PU_2^TE[\xi_N(\theta_0)] = 0 \quad \text{under } H_0
\]
\[
E[\xi_N(\theta_0)] = E[PU_2^T \xi_N(\theta_0)] = PU_2^T(-M\lambda) = -PU_2^TU_1P^T\Sigma_1P\Sigma_1^T\lambda = -M\lambda
\quad \text{under } H_1
\]

The distribution of the revised normalized residual is
\[
\xi_N(\theta_0) \sim N(0, \Sigma) \quad \text{under } H_0 \tag{4.36}
\]
\[
\xi_N(\theta_0) \sim N(-M\lambda, \Sigma) \quad \text{under } H_1 \tag{4.37}
\]

\(M\) and \(\Sigma\) are full rank matrices. Using a linear transformation, the dimensions of the normalized residual are reduced so that the covariance of it is positive definite. A chi-square test then can be applied to the revised normalized residual to detect whether \(\lambda\) is zero or not, i.e. (to detect the change in \(\bar{\theta}\)).

\[
\bar{t} = 2\ln \frac{\max_{\xi} \frac{-P_\xi(\xi)}{P_\xi(\xi)}}{P_\xi(\xi)} = \xi_N^T(\theta_0)\Sigma^{-1}M\bar{F}^{-1}M^T\Sigma^{-1}\xi_N(\theta_0) = \xi_N^T(\theta_0)\Sigma^{-1}\xi_N(\theta_0) \tag{4.38}
\]

where \(\bar{F} = \bar{M}^T\bar{I}^{-1}\bar{M}\). The \(\bar{t}\) value is asymptotically chi-square distributed, with a number of degrees of freedom equal to the dimension of \(\bar{\theta}\). It is central under \(H_0\), and non-central under \(H_1\). One can detect changes in \(\bar{\theta}\) by comparing \(\bar{t}\) with a threshold \(\bar{t}_\alpha\) that can be obtained from standard chi-square table according to the false alarm rate \(\alpha\) and dimension of the reduced parameter \(\bar{\theta}\). Since the reduced parameters \(\bar{\theta} = PV_1^T\theta\) are the linear combination of the original system parameter vector \(\theta\), changes in \(\bar{\theta}\) also indicate the changes in the original system parameter vector \(\theta\). Therefore, changes in \(\theta\) can be detected indirectly.

As a summary, the underlining philosophy of the dimension reduction method for closed-loop detection is that the local approach can be applied to a closed-loop system by using the open-loop model and the system inputs and outputs to calculate the primary and the normalized residual. During the training of the FDI system, the rank of the sensitivity matrix and the covariance matrix should be checked. If they are not full, the dimensions of the normalized residual and the system parameters should be reduced accordingly. Otherwise, nothing needs to be done.
4.4 Closed-loop fault detection using the indirect detection method

It is mentioned in section 4.2 that when the open-loop model and the closed-loop data are used to calculate the primary residual and to perform detection, only those changes that affect the closed-loop dynamics can be detected. Since the structure of the controller is fixed and the closed-loop model is only determined by the open-loop model, changes in the closed-loop parameters indicate changes in open-loop parameters. Thus changes in the open-loop parameters can be detected indirectly. If the structure of the controller is known, the closed-loop model instead of the open-loop model can be used to calculate the primary residual. The nominal value of the closed-loop parameters can also be calculated with the known controller and open-loop system parameters.

Consider a single-input and single-output system that is defined by the following ARX model:

\[ A(z^{-1}) y(k) = B(z^{-1}) u(k) + e(k) \]  \hspace{1cm} (4.39)

where \( u(k) \), \( y(k) \) are the system input and output, \( e(k) \) is a white noise sequence, and

\[ A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n} \]
\[ B(z^{-1}) = b_1 z^{-1} + \cdots + b_n z^{-n} \]

The input signal is determined by:

\[ u(k) = G_c(z^{-1})(r(k) - y(k)) \]  \hspace{1cm} (4.40)

where \( G_c(z^{-1}) \) is a linear controller and \( r(k) \) is the setpoint. It is assumed that the setpoint of the system is zero. Then

\[ y(k) = -G_c(z^{-1})u(k) \]  \hspace{1cm} (4.41)

Suppose that

\[ G_c(z^{-1}) = \frac{D(z^{-1})}{C(z^{-1})} \]  \hspace{1cm} (4.42)

then the system closed-loop model can be rewritten as

\[ y(k) = \frac{D(z^{-1})}{A(z^{-1})C(z^{-1}) + B(z^{-1})D(z^{-1})} e(k) \]  \hspace{1cm} (4.43)

Letting

\[ E(z^{-1}) = A(z^{-1})C(z^{-1}) + B(z^{-1})D(z^{-1}) \]  \hspace{1cm} (4.44)
\[ v(k) = D(z^{-1})e(k) \]

The closed-loop system becomes

\[ E(z^{-1})y(k) = v(k) \quad (4.45) \]

Suppose

\[ E(z^{-1}) = 1 + e_1 z^{-1} + \cdots + e_n z^{-n} \]

\[ \bar{\theta} = [e_1, e_2, \ldots, e_n]^T \]

With the closed-loop model 4.45, the system output \( y(k) \) can be used to detect changes in the closed-loop parameter \( \bar{\theta} \) and thus the changes in the open-loop parameters.

Before calculating the primary residual and performing fault detection with the closed-loop model 4.45, one must deal with the problem that \( v(k) \) is not a white noise. A colored noise will lead to biased estimation in system identification, as it will make the normalized residual non-central even in fault free cases. The following two cases will discuss how the primary residual can be handled.

**Case 1:** \( D(z^{-1}) \) does not have unstable poles.

In this case, \( D(z^{-1}) \) can be used to filter \( y(k) \). The closed-loop system is

\[ E(z^{-1}) \frac{y(k)}{D(z^{-1})} = e(k) \quad (4.46) \]

Letting

\[ y_f(k) = \frac{y(k)}{D(z^{-1})} \]

System (4.46) becomes

\[ E(z^{-1})y_f(k) = e(k) \quad (4.47) \]

Rewriting eqn. 4.18 as

\[ y_f(k) = \phi_f^T(k)\bar{\theta} + e(k) \quad (4.48) \]

where

\[ \phi_f(k) = \begin{bmatrix} -y_f(k-1) & -y_f(k-2) & \cdots & -y_f(k-n_e) \end{bmatrix}^T \]

Then the primary residual can be calculated as

\[ H(y_f(k), \bar{\theta}_0, \phi_f(k)) = (y_f(k) - \phi_f^T(k)\bar{\theta}_0)\phi_f(k) \quad (4.49) \]

52
where $\bar{\theta}_0$ is the value of $\bar{\theta}$ in the nominal situation. Substituting eqn. 4.48 into eqn. 4.49

$$H(y_f(k), \bar{\theta}_0, \varphi_f(k)) = \varphi_f(k)\varphi_f^T(k)(\bar{\theta} - \bar{\theta}_0) + \varphi_f(k)e(k)$$

(4.50)

When $\bar{\theta} = \bar{\theta}_0$

$$E[H(y_f(k), \bar{\theta}_0, \varphi_f(k))] = E[e(k)\varphi_f(k)] = 0$$

and when $\hat{\theta} = \hat{\theta}_0$

$$E[H(y_f(k), \bar{\theta}_0, \varphi_f(k))] = E[\varphi_f(k)\varphi_f^T(k)(\bar{\theta} - \bar{\theta}_0) + e(k)\varphi_f(k)]$$

$$= E[\varphi_f(k)\varphi_f^T(k)(\bar{\theta} - \bar{\theta}_0)] + E[e(k)\varphi_f(k)]$$

$$= \text{cov}(\varphi_f(k))(\bar{\theta} - \bar{\theta}_0) \neq 0$$

Therefore, $H(y_f(k), \bar{\theta}_0, \varphi_f(k))$ defined in eqn. 4.49 meets the requirement of being a valid primary residual.

**Case 2:** $D(z^{-1})$ has unstable poles.

$D(z^{-1})$ is the denominator of the transfer function of the controller. It has an unstable pole in most cases because of the integration action in the controller. Therefore, it can not be used to filter $y(k)$. Otherwise $y_f(k)$ will become a non-stationary signal. If the system 4.45 is rewritten as

$$y(k) = \varphi^T(k)\bar{\theta} + v(k)$$

(4.51)

where

$$\varphi(k) = [-y(k-1) \ - y(k-2) \ \cdots \ - y(k-n_e)]^T$$

The primary residual can be calculated as

$$H(y(k), \bar{\theta}_0, \varphi(k)) = (y(k) - \varphi^T(k)\bar{\theta}_0)\varphi(k)$$

(4.52)

$H(y(k), \bar{\theta}_0, \varphi(k))$ may have non-zero mean even in fault free cases. To illustrate it, suppose that a PI controller is used in the system. The PI controller can be written as

$$G_c(z^{-1}) = K_c(1 + \frac{Ti}{1-z^{-1}}) = \frac{Kc(1+Ti-z^{-1})}{1-z^{-1}}$$

(4.53)

Then $D(z^{-1}) = 1-z^{-1}$. When $\bar{\theta} = \bar{\theta}_0$,

$$H(y(k), \bar{\theta}_0, \varphi(k)) = (y(k) - \varphi^T(k)\bar{\theta}_0)\varphi(k) = v(k)\varphi(k)$$

$$= [e(k) - e(k-1)]\varphi(k) = e(k)\varphi(k) - e(k-1)\varphi(k)$$

(4.54)
Since $e(k)$ is a white noise and $y(k)$ is determined by the ARX model 4.39, it is easy to prove that

$$E[e(m)y(n)] = \begin{cases} 0 & \text{when } m > n \\ \sigma_e & \text{when } m = n \end{cases}$$

therefore,

$$E[e(k)\varphi(k))] = E(e(k)[-y(k-1) - y(k-2) \cdots - y(k-n_e)])^T$$

$$= \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}^T$$

$$E[e(k)\varphi(k))] = E(e(k-1)[-y(k-1) - y(k-2) \cdots - y(k-n_e)])^T)$$

$$= \begin{bmatrix} \sigma_e^2 & 0 & \cdots & 0 \end{bmatrix}^T$$

Therefore,

$$E[H(y(k),\bar{\theta}_0,\varphi(k))] = E[e(k)\varphi(k)] - E[e(k-1)\varphi(k)]$$

$$= \begin{bmatrix} \sigma_e^2 & 0 & \cdots & 0 \end{bmatrix}^T$$

There is a bias in the primary residual defined in 4.52. Bearing in mind that the only requirement for a primary residual is zero mean in a nominal case, one can simply subtract the bias from $H(y(k),\bar{\theta}_0,\varphi(k))$ to fulfill the requirement of being a valid primary residual. Since the bias is only determined by the covariance of noise, one can estimate the bias during the training of the FDI system as

$$h = \sum_{k=i}^{N} H(y(k),\bar{\theta}_0,\varphi(k))$$

where $N$ is the number of data to be used in training. In the detection stage, one can calculate the primary residual and subtract the bias $h$ from it. The new primary will be

$$\bar{H}(y(k),\bar{\theta}_0,\varphi(k)) = H(y(k),\bar{\theta}_0,\varphi(k)) - h$$

(4.56)

can be used to detect the change in the closed-loop system parameter $\bar{\theta}$.

After a suitable primary residual is found, the rest is straightforward. The normalized residual is calculated with the primary residual and a chi-square test can be applied to the normalized residual to detect changes in the closed-loop parameter. The limitation of the indirect detection method is that the controller structure must be known. Otherwise the closed-loop model can not be obtained.
Chapter 4: Closed-loop fault detection using the local approach

4.5 Simulation results

To illustrate the result presented in this chapter, a system given by the following second-order ARX model is considered

\[ A(z^{-1})y(t) = B(z^{-1})u(t) + e(t) \]  

(4.57)

where \( u(t), y(t) \) are the system input and output, \( e(t) \) is a white noise with a variance of 1.

\[
A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} = 1 - 1.1 z^{-1} + 0.3 z^{-2};
\]

\[
B(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} = 0.7 z^{-1} - 0.5 z^{-2}.
\]

The system is operated in closed-loop with the PI controller

\[ G_c(z^{-1}) = \frac{C(z^{-1})}{D(z^{-1})} \]

(4.58)

and a setpoint of zero.

4.5.1 Closed-loop fault detection using the dimension reduction method

The system model (4.57) can be rewritten as

\[ y(k) = \phi^T(k)\theta \]

(4.59)

where

\[
\phi(k) = \begin{bmatrix} -y(k-1) & -y(k-2) & u(k-1) & u(k-2) \end{bmatrix}^T
\]

\[
\theta = [a_1, a_2, b_1, b_2]^T
\]

The nominal value of the open-loop parameters are

\[
\theta_0 = [-1.1, 0.3, 0.7, -0.5]^T
\]

As for the dimension reduction method, the primary residual was calculated using the open-loop model as follows:

\[ H(\theta_0, y_k, \phi_k) = \phi_k(y_k - \phi_k^T \theta_0) \]  

(4.60)

In the training of the FDI system, the process was operated in closed-loop condition and the setpoint was set to be zero. With the closed-loop input and output data of 10000 time units, the primary residual was calculated using eqn. 4.60.

In this case, the sensitivity matrix can be estimated as
\[ M(\theta_0) = \frac{1}{N} \sum_{k=1}^{N} \left[ \frac{\partial}{\partial \theta} H(\theta, y_k, \varphi_k) \right]_{\theta = \theta_0} \]
\[ = \frac{1}{N} \sum_{k=1}^{N} \varphi_k \varphi_k^T \]  

(4.61)

\[ M = \]
\[
\begin{bmatrix}
2.9360 & 2.2981 & 0.4405 & 0.0831 \\
2.2981 & 2.9359 & 0.6066 & 0.4406 \\
0.4405 & 0.6066 & 1.0252 & 0.9978 \\
0.0831 & 0.4406 & 0.9978 & 1.0252 \\
\end{bmatrix}
\]

\[ \Sigma = \]
\[
\begin{bmatrix}
3.1036 & 2.5033 & 0.6325 & 0.2621 \\
2.5033 & 2.9727 & 0.6904 & 0.4870 \\
0.6325 & 0.6904 & 1.0060 & 0.9486 \\
0.2621 & 0.4870 & 0.9486 & 0.9448 \\
\end{bmatrix}
\]

The rank of \( M \) and \( \Sigma \) was 3. Since \( \Sigma \) is not of full rank, the normalized residual need to be revised to reduce the dimension of \( \Sigma \). Using Single Variable Decomposition on \( \Sigma \)
\[ \Sigma = USU^T \]

where
\[ U = \]
\[
\begin{bmatrix}
0.6868 & -0.2741 & 0.6585 & 0.1397 \\
0.6785 & -0.0817 & -0.7267 & -0.0698 \\
0.2159 & 0.6538 & 0.1952 & -0.6984 \\
0.1464 & 0.7005 & -0.0092 & 0.6984 \\
\end{bmatrix}
\]

\[ S = \]
\[
\begin{bmatrix}
5.8313 & 0 & 0 & 0 \\
0 & 1.6709 & 0 & 0 \\
0 & 0 & 0.5250 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The transform matrix was calculated as
\[ T = \]
\[
\begin{bmatrix}
0.6868 & 0.6785 & 0.2159 & 0.1464 \\
-0.2741 & -0.0817 & 0.6538 & 0.7005 \\
0.6585 & -0.7267 & 0.1952 & -0.0092 \\
\end{bmatrix}
\]

In detection, the primary residual \( H(\theta_0, y_k, \varphi_k) \) was calculated using eqn. 4.60. The
normalized residual $\zeta_N(\theta_0)$ was calculated based on the past 100 time units (a moving window of 100 time units was used) using eqn. 2.18. It was calculated for every 10th time unit to reduce computation load. Since the covariance matrix of $\zeta_N(\theta_0)$ was not full rank, it was revised with $T$ using eqn. 3.32 as

$$\bar{\zeta}_N(\theta_0) = T\zeta_N(\theta_0).$$

In the following, the dimension reduction method was applied to detect the change in every single parameter of the open-loop model 4.59. For each situation, the system was operated in a closed-loop condition with the setpoint of zero for 10000 time units. One of the four open-loop parameters $a_1$, $a_2$, $b_1$ or $b_2$ was changed at the 5001st unit. The chi-square value was calculated for every 10th unit. Figure 4.1 to 4.4 shows the chi-squares versus time unit.

![Figure 4.1 Detection of the change in $a_1$ using the dimension reduction method](image-url)
Chapter 4: Closed-loop fault detection using the local approach

Figure 4.2 Detection of the change in $a_2$ using the dimension reduction method

Figure 4.3 Detection of the change in $b_1$ using the dimension reduction method
Fig. 4.1 to 4.4 indicate that a sudden jump of the chi-square value indicates a change in the system parameters. The proposed algorithm can successfully detect the change in every single parameter of the open-loop model.

It is mentioned in section 4.2 that if changes in the open-loop parameters do not affect the closed-loop dynamics, they can not be detected. That is to say, if the change in closed loop parameter $\lambda$ happens to be $M\lambda = 0$, it will not be reflected by the mean value of the normalized residual. One example is that $\lambda = [0.6306 \ 3.5164 \ 0.2048 \ -3.4922]$. In this case, $M\lambda = 0$. The closed loop parameters $\theta$ change from $[-1.1 \ 0.3 \ 0.7 \ -0.5]$ to $[0.4694 \ 3.8164 \ 0.9048 \ -3.7922]$. The process was operated in closed-loop condition for 10000 time units. A chi-square value was calculated for every 10 time units. Fig 4.3 shows the chi-square value versus time unit.
Fig. 4.5 shows no signification changes in the chi-square value, which means that the change in the open-loop parameters was not detected even though the change in $\theta$ at the 5001st unit was significant. Fortunately, the probability for such a change to happen in a real system is very low.

4.5.2 Closed-loop fault detection using the indirect detection method

The closed-loop model of the process defined in eqn. 4.57 with the PI controller in eqn. 4.58...
can be calculated as

\[ E(z^{-1})y(k) = D(z^{-1})e(k) \]  \hspace{1cm} (4.62)

where

\[ E(z^{-1}) = A(z^{-1})C(z^{-1}) + B(z^{-1})D(z^{-1}) = 1 + e_1z^{-1} + e_2z^{-2} + e_3z^{-3} \]  \hspace{1cm} (4.63)

Under nominal situation,

\[ A(z^{-1}) = 1 + a_1z^{-1} + a_2z^{-2} = 1 - 1.1z^{-1} + 0.3z^{-2} \]
\[ B(z^{-1}) = b_1z^{-1} + b_2z^{-2} = 0.7z^{-1} - 0.5z^{-2} \]
\[ C(z^{-1}) = 0.2 - 0.1z^{-1}, \quad D(z^{-1}) = 1 - z^{-1} \]
then

\[ E(z^{-1}) = A(z^{-1})C(z^{-1}) + B(z^{-1})D(z^{-1}) = 1 - 1.96z^{-1} + 1.23z^{-2} - 0.25z^{-3} \]

The model in eqn. 4.63 can be rewritten as

\[ y(k) = \varphi^T(k)\bar{\theta} + v(k) \]  \hspace{1cm} (4.64)

where

\[ \varphi(k) = [-y(k-1) - y(k-2) - y(k-3)] \]
\[ \bar{\theta} = [e_1, e_2, e_3]^T \quad v(k) = D(z^{-1})e(k) \]

The nominal value of the closed-loop parameter \( \bar{\theta} \) is

\[ \bar{\theta} = [-1.96 \ 1.23 \ -0.25]^T \]

The primary residual is calculated based on the closed-loop model 4.64 as

\[ H(y(k),\varphi(k),\bar{\theta}_0) = y(k) - \varphi^T(k)\bar{\theta}_0 \]  \hspace{1cm} (4.65)

Since \( D(z^{-1}) = 1 - z^{-1} \) is improper. There is constant bias in the mean value of the primary residual and the normalized residual. Before the indirect method can be applied to detect changes in the closed-loop parameters, the bias of the primary residual has to be estimated together with the sensitivity matrix and the covariance matrix in the training of the FDI system. The process was operated in closed-loop condition for 10000 time units. With the input and output data, the bias of the primary residual was estimated using eqn. 4.55 as

\[ h = [1.0034 \ 0.0011 \ 0.0019] \]

Then the primary residual can be revised as

\[ \bar{H}(y(k),\varphi(k),\bar{\theta}_0) = H(y(k),\varphi(k),\bar{\theta}_0) - h \]  \hspace{1cm} (4.66)
The sensitivity matrix can be estimated as

\[ M(\theta_0) = \frac{1}{N} \sum_{k=1}^{N} \left[ \frac{\partial}{\partial \theta} \widehat{H}(\theta, y_k, \varphi_k) \right]_{\theta=\theta_0} \]

(4.67)

With the revised primary residual, the covariance matrix can be estimated using eqn. 2.18 and 2.19.

In the following simulations, the indirect detection method was applied to detect changes in every single parameter of the process. For each situation, the system was operated in closed-loop condition with the setpoint of zero for 10000 time units. One of the four open-loop parameters \(a_1, a_2, b_1\) or \(b_2\) was changed at the 5001\(^{st}\) time unit. The chi-square value was calculated for every 10\(^{th}\) time unit to reduce the computation load. Figure 4.6 to 4.9 show the chi-square value versus time unit.

![Figure 4.6 Detection of the change in \(a_2\) using the indirect detection method](image-url)
Fig. 4.6 shows that the change in the open-loop parameter \( a_2 \) was successfully detected. The indirect detection method was also used to detect the change of \( a_1, b_1 \) or \( b_2 \). Simulation results show the effectiveness of the indirect detection method in detecting the change of each of the open-loop parameters. For each of these simulations, the plots of chi-square value, system input, system output versus time unit are similar to those plots obtained in the simulation using the dimension reduction method. Therefore, they were omitted to avoid repetition.

In certain situations, the open-loop parameter changes while the closed-loop remains unchanged. An example is given as following:

The open-loop parameters \( \theta \) changed from \([-1.1 \ 0.3 \ 0.7 \ -0.5]\) to \([-2.1 \ 0.8 \ 5.7 \ -5.5]\). Therefore, the closed-loop model after the change was calculated as

\[
E(z^{-1}) = A(z^{-1})C(z^{-1}) + B(z^{-1})D(z^{-1}) = 1 - 1.96z^{-1} + 1.23z^{-2} - 0.25z^{-3}
\]

where

\[
A(z^{-1}) = 1 - 2.1z^{-1} + 0.8z^{-2}, \quad B(z^{-1}) = 5.7z^{-1} - 0.8z^{-2}
\]

The closed-loop parameter did not change. The process was operated under closed-loop condition with setpoint set to be zero for 10000 time units. The chi-square value was calculated for every 10 time units. This can be seen in Fig. 4.7 of chi-square versus time unit.
Changes in the open-loop parameters were not detected even though they were significant. A detection failure occurs when the change in open-loop parameters does not affect the closed-loop dynamics.

4.6 Experiment results

The proposed algorithms have been applied to the pilot-scale tank level and temperature
control system as described in chapter 3. In the experiment, PI controllers were used to control both the water level and the temperature in the tank. A change in the water level was used to simulate a fault in the system. The input (steam valve) and output (temperature) data of the temperature process were used to detect changes in the tank level.

The open-loop process model was obtained through system identification as described in chapter 3. The process was operated under closed-loop condition for 1000 sampling times in order to train the FDI system. With the training data, the sensitivity matrix $M$ and the covariance matrix $\Sigma$ were estimated using eqn. 2.16 and 2.17. Though $\Sigma$ was of full rank, its condition number was more than $10^6$. Using this $\Sigma$ may bring calculation error to the chi-square value. Using Singular Value Decomposition, the dimension of covariance matrix was reduced to 5. The false alarm ratio was set to be $\alpha = 0.01$ and the threshold value was obtained from standard chi-square table as $\chi^2_\alpha = 16.81$ Using the algorithm provided in chapter 3, the threshold was revised as $\hat{\chi}_\alpha = 17.47$.

In the detection stage, the process was operated in closed-loop for 2500 sampling times and the set-point of the tank level was changed from 14 cm to 11 cm at the sampling time of 800. Using a moving window with a length of 100 sampling times, the normalized residual and chi-square value were calculated for every 10 sampling times. The chi-square value versus sampling time in Fig. 4.8.
Figure 4.8 Detection of the change of the tank level using the dimension reduction method

Figure 4.8 shows a sudden jump of chi-square value after the change in the tank level, which indicates that the change was detected by the algorithm. A remaining problem is that the chi-square value returned below the threshold after a period of time. This may be caused by the processing of the data. The trend of the input-output data was subtracted from them before they were used to calculate the normalized residual to reduce the affect of low-frequent disturbance to the detection algorithm. This also caused the loss of information of the change due to changes in the system parameters. Future research work may need to be done in order to find methods to distinguish the effect of low-frequency disturbances from those of parameter changes.

4.7 Conclusions

In this chapter, closed-loop detection was investigated. When the closed-loop data are used to calculate the primary and the normalized residual, the relevance between the input and output causes a non-full rank covariance matrix. The dimension reduction method used a linear
transformation to reduce the dimension of the normalized residual so that the covariance matrix of the revised normalized residual will be of full rank.

With the closed-loop data, fault detection algorithms can only detect those parameter changes that can affect the closed-loop dynamics. The indirect detection method used the closed-loop model instead of the open-loop model to calculate the primary residual and normalized residual to perform detection. By detecting the changes of the closed-loop parameters, the method also detected the changes of the open-loop parameters.

Simulation studies showed that both the dimension reduction method and indirect detection method detected the change of every single parameter of a second-order linear process operated under the closed-loop condition. The implementation of the proposed algorithm on the tank level and temperature control system proved to be effective. From the simulations, the dimension reduction method and indirect method were found to be equivalent in performance.
Chapter 5

Application of fault detection in paper-making systems

5.1 Background

Figure 5.1 shows a typical Fourdrinier paper machine. In manufacturing the paper, a mixture of fibres and water is pumped into the headbox. The headbox is designed to keep the fibers and water evenly distributed. The pulp is then delivered onto a moving wire known as a web or wire. Water is drained out of the mixture by means of gravity, pressing and drying. The resulting fibres are formed into a paper sheet. After passing through a series of rolls called the calender, the paper sheet is wound up onto a reel.

In the paper-making systems, there are three important properties: (1) basis weight, which is defined as total sheet weight per unit area, (2) moisture, which refers to the percentage weight of the water in the total mass of the paper sheet, and (3) caliper, which is the thickness of the paper sheet. At the end of the paper machine, these properties are measured through a scanner mounted on an O-frame surrounding the moving sheet. The scanner continuously traverses back and forth in the cross machine direction.

Variations in paper properties are considered to be composed of two parts: the machine direction (MD) part and the cross direction (CD) part. Machine direction means the direction of the movement of the paper sheet and cross direction means the direction perpendicular to machine direction. In order to obtain high-quality paper product and high product rate, automation control systems have been designed to control each of the above three properties. Efforts on improving the control of paper machine began in the early 1960s when computers became available for industrial system control. They were primarily dedicated to the control of MD properties. With the development of measurement devices and actuators, CD control has rapidly gained prominence since the 1980's (Dumont, 1995; Kwok, 1997). The adjustment of CD properties is done through a series of actuators mounted across the machine. The weight actuators are mainly the slice screws at the head box. For consistency profiling head boxes, the weight actuators are a series of valves that control the local consistency across the head box. The control of moisture profile across the sheet is carried out by steam boxes and steam
showers installed at the dry section of the paper machine (Jahangir, 1997).

Figure 5.1 Schematic of a typical Fourdrinier paper machine

With the growing demand for fault tolerance, FDI has become an important issue for system control community. In CD control systems of papermaking systems, there are many actuators. Performance of a CD control system not only depends on the control algorithm, but also on the reliability of the actuators. Therefore, it is helpful to on-line monitor the system using system input and output data and detect the actuator faults. By detecting the actuator faults and reporting them to the supervision system, one can repair the faulty actuators, improve the availability of the system and maintain the overall control performance.

The local approach is a useful tool for both incipient change detection and abrupt fault detection. In this chapter, it will be applied to the problem of actuator fault detection for CD control systems of paper-making systems.

5.2 Problem formulation

This chapter addresses the application of local approach on actuator fault detection in CD control systems. Since system and fault models are crucial to the performance of model-based FD algorithms, they will be specified in the following:
5.2.1 System model

An open-loop CD system can be described by the following ARX model:

\[ Ay(t) = Bu(t - d) + e(t) \]  \quad (5.1)

where

- \( B \) is called the interaction matrix which contains the coupling information, \( d \) is the time delay;
- \( A \) is the dynamic matrix of the system;
- \( u \) and \( y \) are vectors of the system inputs and outputs, \( e(t) \) is a vector of white noise sequences.

In order to simplify the problem, it is assumed that the dynamics for all outputs are the same. Under the above assumption, the dynamic matrix can be written as follows:

\[
A = \begin{bmatrix} 1 - az^{-1} & 0 & \cdots & 0 \\
0 & 1 - az^{-1} & \ddots & \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & 1 - az^{-1} \end{bmatrix}
\]  \quad (5.2)

Therefore, system 5.1 can be rewritten as

\[ y(t) + ay(t - 1) = Bu(t - d) + e(t) \]  \quad (5.3)

The purpose is to detect the faults of actuators with system input \( u \) and output \( y \) and identify the faults, i.e. to find the faulty actuators.

5.2.2 Modeling of actuator faults

For simplicity, it is assumed that actuator faults will not affect the dynamic matrix \( A \). The faults only cause the changes of interaction matrix \( B \). Therefore, the actuator faults can be detected by detecting changes of elements of the interaction matrix \( B \).

Because of its high dimension, it is very difficult to detect the change of one single element of the interaction matrix \( B \). Since the fault with an actuator will cause the changes in all of the elements in the column corresponding to this actuator, it is then assumed that the fault affect the elements at the same rate. This means that when a fault happens with the \( p^{th} \) actuator, the interaction matrix \( B \) becomes:
Chapter 5: Application of fault detection in paper-making systems

\[
\bar{B} = \begin{bmatrix}
    b_{11} & b_{12} & \cdots & rb_{1p} & \cdots & b_{1n} \\
    b_{21} & b_{22} & \cdots & rb_{2p} & \cdots & b_{2n} \\
    \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
    b_{m1} & b_{m2} & \cdots & rb_{mp} & \cdots & b_{mn}
\end{bmatrix} = BF
\]

where

\[
F = \begin{bmatrix}
    I_{(p-1)\times(p-1)} & 0 & 0 \\
    0 & r & 0 \\
    0 & 0 & I_{(n-p)\times(n-p)}
\end{bmatrix}
\]

Therefore, the system model 5.3 can be rewritten as

\[y(t) + ay(t-1) = BFu(t-d) + e(t)\]  \hspace{1cm} (5.4)

Since

\[
Fu(t-d) = \begin{bmatrix}
    I_{(p-1)\times(p-1)} & 0 & 0 \\
    0 & r & 0 \\
    0 & 0 & I_{(n-p)\times(n-p)}
\end{bmatrix}
\begin{bmatrix}
    u_1(t-d) \\
    \vdots \\
    u_n(t-d)
\end{bmatrix}
\]

\[= \begin{bmatrix}
    u_1(t-d) \\
    \vdots \\
    u_n(t-d)
\end{bmatrix} + \begin{bmatrix}
    1 \\
    \vdots \\
    1
\end{bmatrix}
\]

Let

\[
U(t-d) = \begin{bmatrix}
    u_1(t-d) \\
    \vdots \\
    0 \\
    u_n(t-d)
\end{bmatrix}
\]

\[f = [1 \cdots r \cdots 1]^T\]

System (5.4) becomes:

\[y(t) + ay(t-1) = BU(t-d)f + e(t)\]  \hspace{1cm} (5.5)

where \(f\) is a \(n\times1\) vector. Under nominal conditions, all elements of \(f\) are 1, i.e. \(f^0 = [1 \cdots 1]^T\). \(f\) can be called the index of fault since changes of its elements directly indicate faults of the correspondent actuators. Then the problem of detecting actuator faults is transformed to the problem of detecting changes of \(f\) with the system input \(u\) and output \(y\).
The advantage of the above method to model actuator faults is that it is much simpler to detect changes of a vector $f$ than a matrix $B$.

5.3 Actuator fault detection and isolation

In this part, algorithms for actuator fault detection in CD control systems will be proposed. Generally the input data of the CD system are setpoints for CD actuators and output data are the system measurements. Data collected by the scanner are referred as raw data and contain MD and CD variations. MD and CD variations are regulated by different controllers. For detection of faults of the CD actuators, CD variation should be separated from MD variation.

5.3.1 Machine-direction and Cross-direction data separation

There are two methods to separating MD and CD information: Exponential Multi-Scan Trending (EXPO) and Estimation and Identification of Moisture Content (EIMC) algorithm. Exponential multi-scan trending is very simple and widely used in the paper industry (Li, 1998). The filter is described by

$$
\hat{y}_n(t) = (1 - \beta)\hat{y}_n(t-1) + \beta y_n(t)
$$

where

- $\hat{y}_n(t)$ is the estimated CD value at CD position $n$
- $y_n(t)$ is the present measured deviation from the scan average at CD position $n$
- $\beta$ is the weight on the measurement, $\beta \in [0,1]$

The trended data are taken as the CD profile of the sheet, and the average of each scan is considered as the MD value.

5.3.2 Mapping of databox to actuators

Paper-making systems are high-dimension systems. Therefore, computation of primary residuals and evaluation of normalized residuals will involve calculation of high dimension matrices. As on-line monitoring tools, fault detection algorithms must be simple enough to meet the computation and capability limitation of existing control systems. Therefore, it is meaningful to reduce the dimensions of system output and model (especially the interaction...
matrix) to avoid unnecessary computation.

Let \( m \) and \( n \) represent the dimensions of system output \( y \) and system input \( u \) respectively. Usually, \( m \) is much larger than \( n \). System output \( y \) is related to input \( u \) through interaction matrix \( G \). So when \( m \) is larger than \( n \), \( G \) is not of full rank and elements of \( y \) will be linearly related. Suppose that \( m \) is a multiple of \( n \) and \( p = m/n \).

Averaging \( y \) for each \( p \) data points:

\[
\bar{y}_i = \frac{1}{p} \sum_{k=1}^{p} y_{(i-1)p+k}
\]

and it is equivalent to take linear transformation of system output:

\[
\bar{y} = Gy
\]

where

\[
G = \frac{1}{P} \begin{bmatrix}
 1 & \cdots & 1 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
 0 & \cdots & 0 & 1 & \cdots & 1 & \cdots & 0 & \cdots & 0 \\
 \vdots & & \ddots & \vdots & & \ddots & \vdots & & \ddots & \vdots \\
 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 1 & \cdots & 1
\end{bmatrix}
\]

Since

\[
Gy(t) + aGy(t-1) = GBU(t - d)f + e(t)
\]

then

\[
\bar{y}(t) + a\bar{y}(t-1) = \bar{B}U(t - d)f + e(t)
\]

(5.7)

where \( \bar{B} = GB \).

With linear transform \( G \), the dimension of system output \( y \) is decreased from \( m \) to \( n \) and the dimensions of the interaction matrix \( B \) from \( m \times n \) to \( n \times n \). This will greatly reduce the computation in calculation of primary residuals and evaluation of normalized residuals. Since \( \bar{y} \) are the linear transformation of \( \bar{y} \), they contains all the information about actuator faults. For simplicity, \( y \) and \( G \) will be used to represent the system output and interaction matrix after averaging.
5.3.3 General algorithms for actuator fault detection and isolation

For fault detection and isolation algorithms, the very first step is to find a suitable primary residual which is sensitive to the faults. Since $A$ is assumed to be unaffected by actuator faults, let

$$y_s(t) = y(t) + ay(t-1) \quad (5.8)$$

then the system (5.3) can be rewritten as a static system;

$$y_s(t) = BU(t-d)f + e(t) \quad (5.9)$$

The primary residual can be calculated as

$$H(y_s(t), U(t-d)) = U(t-d)B^TY_s(t) - BU(t-d)] \quad (5.10)$$

Substituting eqn. 5.9 into eqn. 5.10, obtain

$$H(y_s(t), U(t-d)) = U(t-d)B^TBU(t-d)(f - f^0) + U(t-d)B^Te(t)$$

Under nominal situation, $f = f^0$

$$H(y_s(t), U(t-d)) = U(t-d)B^Te(t)$$

Since $e(t)$ is the vector of white noise sequence, it is easy to prove that:

$$E[H(y_s(t), U(t-d))] = E[U(t-d)BTe(t)] = 0$$

When $f \neq f^0$

$$E(H(y_s(t), U(t-d))) = E[U(t-d)B^TBU(t-d)](f - f^0) + E[U(t-d)B^Te(t)]$$

It can be shown that

$$E[U(t-d)B^TBU(t-d)] \neq 0.$$ 

Therefore,

$$E(H(y_s(t), U(t-d))) \neq 0 \text{ when } f \neq f^0.$$ 

The primary residual defined in eqn. 5.10 fits the requirement that it should have zero mean in fault-free cases and non-zero mean in other cases. Normalized residual can be calculated as follows:

$$\zeta_N = \sum_{k=1}^{N} H(y_s(k), U(k-d)) \quad (5.11)$$
where $N$ is the length of moving window. $\zeta_N$ is a $n \times 1$ vector. The fault of any actuator will cause the change of mean value of $\zeta_N$ and a chi-square test can be applied on $\zeta_N$ to detect whether the mean value of it is zero or not.

Fault isolation always follows fault detection. In this case, it is designed to find out the faulty actuator. There are two fault isolation methods: the sensitivity test and the min-max test. Since the sensitivity test is easier to apply, it will be used in this chapter to isolate the faulty actuator. Detailed information about the sensitivity test and the min-max test can be found in chapter 2.

### 5.3.4 Actuator fault detection by dividing the system output into sections

In the algorithms proposed above, the primary residual and the normalized residual have the same dimension as that of actuators. With model uncertainty and noise present in the system model, the change of one actuator may not have an enough effect on the system output. This may lead to a fault being not detected by the algorithm. In fact, the interaction matrix is a sparse matrix, one output is only determined by the move of several actuators and the move of one actuator will only effect several system outputs. Therefore, one can divide the system output into several regions and detect faults of those actuators that have an effect on the outputs. Since the dimension of the output within a section is much less than that of the total output, one can anticipate that by appropriately dividing the system output into sections the detection algorithms can be more sensitive to the fault of a single actuator.

After averaging, the system inputs and outputs have the same dimension $n$. Suppose the system outputs are divided into $l$ sections and let $p = n/l$. With the following denotation

$$y^t = \begin{bmatrix} y_{(i-1)i+1} & \cdots & y_{il} \end{bmatrix}^T$$

$$U^t = \begin{bmatrix} u_{(i-1)i+1} & \cdots & u_{il} \end{bmatrix}^T$$

$$u^t = \begin{bmatrix} u_{(i-1)i+1} & \cdots & u_{il} \end{bmatrix}^T$$

system 5.3 can be rewritten as

$$y^t(t) + ay^t(t-1) = G^tU(t)f + e^t(t), \text{ for } i = 1, \cdots, l$$

(5.12)
Therefore, the whole system can be divided into smaller systems. For each of those systems, one can detect the faults of those actuators that have effect on the outputs in the particular section. Primary residuals can be calculated as

$$H^i(t) = (G^i U^i)^T [y^i(t) + ay^i(t-1) - G^i U(t)f] \quad \text{for } i = 1, \ldots, l$$

(5.13)

and the normalized residual

$$\zeta^i_N = \sum_{k=1}^{N} H^i(k)$$

(5.14)

### 5.3.5 Actuator fault detection by decoupling the system

For most of the CD control systems interaction matrix $B$ is usually not full rank or has high condition number. Calculation of the inverse of $B$ is not possible or will bring large calculation error. In some cases, $B$ may be full rank. In such cases, one can decouple the MIMO into $n$ SISO systems.

Taking the pseudo-inverse of $B$:

$$D = (B^T B)^{-1} B^T$$

and multiple the both side of eqn. 5.9 by $D$, the system model becomes:

$$D\hat{y}(t) = DBU(t-d)f + De(t)$$

(5.15)

Letting

$$\tilde{y}(t) = D\hat{y}(t), v(t) = De(t)$$

(5.16)

then

$$\tilde{y}(t) = U(t-d)f + v(t)$$

Since $U(t-d)$ is a diagonal matrix,

\[
\begin{bmatrix}
\tilde{y}_1(t) \\
\vdots \\
\tilde{y}_n(t)
\end{bmatrix} =
\begin{bmatrix}
u_1(t-d) & 0 & f_1 \\
\vdots & \ddots & \vdots \\
0 & u_n(t-d) & f_n
\end{bmatrix}
+ \begin{bmatrix}
v_1(t) \\
\vdots \\
v_n(t)
\end{bmatrix}
\]  

(5.17)

The system is decoupled into $n$ SISO systems:

$$\tilde{y}_i(t) = u_i(t-d)f_i + v_i(t) \quad \text{for } i = 1, \ldots, n$$

(5.18)

Therefore, one can detect the fault of each of the actuator respectively though monitoring the SISO systems separately. The primary residual for each of the SISO system can be calculated
as

\[ H_i(\bar{y}_i(t), u_i(t-d), f_i) = [\bar{y}_i(t) - u_i(t-d)f_i]u_i(t-d) \]  

(5.19)

5.4 Simulation results

The best way to test the applicability of proposed fault detection algorithms is to intentionally disable one or several actuators in a CD control system and see whether the algorithms can detect the faults or not. However, it is not feasible to do this, as it may deteriorate control performance. The alternative way is to collect data from a CD control system and simulate actuator faults by modifying the input and output data. If the proposed algorithms can successfully detect the simulated faults, one can anticipate that it will also detect real actuator faults in an actual CD control system.

The simulation studies were carried out in two stages. First, a real system model was used to generate data that contained the information of a single actuator fault. The proposed types of algorithms were applied to detect the fault from the generated data and the performance of these algorithms was compared. Second, real system data were used to simulate actuator faults and the algorithm with the best performance was applied to detect this fault.

5.4.1 System specification

The data used in the simulation studies was collected from the moisture profile CD control system with a steam box application in a Canadian paper mill producing newsprint grade paper. Steam boxes are quite slow systems as the dynamics of the systems depend on the thermal momentum of the paper machine. In the moisture CD control system, there are 50 actuators and 1500 measurements for each scan. The system model can be described as follows:

\[ \Delta y(t) = B\Delta u(t-d) + a\Delta y(t-1) \]  

(5.20)

where \( y(t) \) is the CD profile, \( u(t) \) is the vector of the setpoints for the actuators. \( a = 0.9432 \), \( d = 2 \). Figure 5.2 shows the three-dimensional view of interaction matrix \( B \) and gain of the 25th actuator to system outputs:
The move of one actuator will cause a change of about 10 CD measurements around the actuator. Although $B$ has full column rank, its condition number is very larger: $\text{cond}(B) = 4.0705 \times 10^4$.

5.4.2 Simulation with real model and generated data

In this chapter, three actuator fault detection algorithms were proposed based on a theoretical model of the system. The three algorithms are defined as the decoupling method, general method and sectioning method. Before these algorithms are applied to real industrial data, they will be compared by detecting faults in generated data. In the following simulation study, data were generated with eqn. 5.20 and the parameters are real system parameter.

After a fault occurs with an actuator, the position of the actuator will either stop at the position of last scan, slowly drift to zero, or suddenly drop to zero. They can be called jammed fault, incipient fault and abrupt fault. The proposed algorithms will be applied to detect these three types of faults. The system was excited for 2000 time units by a vector of white noise with covariance of 0.01. A chi-square value was calculated for every 25th unit. The chi-square value was based on the past 100 units (i.e. a moving window of 100 units was used). At the
1001st unit, an artificial fault was introduced at the 25th actuator by fixing the position of the actuator. This is similar to a jammed actuator. Figure 5.3 to 5.7 show the results of fault detection using different algorithms.

Case 1: Actuator detection using the decoupling method

![Diagram of actuator position over time](image1)

![Chi-square value over time](image2)

Figure 5.3 Detection of a jammed actuator at the 25th actuator using the decoupling method

Though the position of the 25th actuator was fixed to a constant value, figure 5.3 shows no obvious change in the chi-square of the normalized residual of the SISO system containing the 25th actuator. This indicates that the fault was not detected. The explanation is the high condition number of interaction matrix $B$. Numerical calculation of the inverse of $B$ matrix brings large calculation error. When the $B$ matrix is used to decouple the system, the calculation error will lead to a large calculation error in the system outputs defined in eqn. 5.17. This error will overcome the change caused by actuator faults and makes the faults undetectable.
Case 2: Actuator fault detection using the general method

After the 2500\textsuperscript{th} time unit, there are more chi-square values exceeding the threshold than before, which may be explained as the presence of fault in the system. But the increase of the chi-square values is not significant. The explanation is as follows: the primary residuals are calculated as the multiple of estimation error and the derivative of estimation error with respect to system parameter. In the entire detection method, faulty index $f$ was regarded as the vector of $n$ parameters of the system. The primary residual and the normalized residual have the same dimension as system outputs. Therefore, normalized residual will not be sensitive the fault of a single actuator. To solve this problem one can count the rate of chi-square value exceeding threshold instead of single chi-square to make decision on the presence of fault. That is actually equivalent to increasing the length of the moving window.

Figure 5.4 Detection of a jammed actuator at the 25\textsuperscript{th} actuator using the general method
Case 3: Actuator fault detection using the sectioning method

Figure 5.5 Detection of a jammed actuator at the 25th actuator using the sectioning method

Significant change of the chi-square value in figure 5.5 indicates an actuator fault in region 5. The algorithm successfully detected the fault with a single actuator. For the other two types of actuator faults, a fault causes more drastic change of the actuator position. Therefore, it is easier for the algorithm to detect them. Figure 5.6 and 5.7 showed the results of detection.
Figure 5.6 Detection of an abrupt fault at the 25th actuator using the sectioning method

Figure 5.7 Detection of an incipient fault at the 25th actuator using the sectioning method
5.4.3 Simulation with real system data

In the following simulation studies, closed-loop data obtained from a CD control system was used to assess the performance of the proposed algorithms. As mentioned before, manually disabling the actuators will deteriorate the control performance. The alternative way to simulate the actuator faults is to modify the output data. Suppose that $u(t)$ is the vector of setpoints for the actuators and $\hat{u}(t)$ is the actual position of the actuators. In the fault free case, $u(t)$ should be equal to $\hat{u}(t)$. Actuator faults will case a difference between $u(t)$ and $\hat{u}(t)$ and further lead to the change of system output. Letting $u_e(t) = u(t) - \hat{u}(t)$

With the system model 5.3, the changes of the system output due to actuator faults can be calculated as

$$y_e(t) = ay_e(t) + Bu_e(t-d)$$  \hspace{1cm} (5.21)

Then the actual measurements should be:

$$\hat{y}(t) = y_e(t) + y(t)$$  \hspace{1cm} (5.22)

In practice, only the faulty output $\hat{y}(t)$ and the control signal $u(t)$ can be obtained. They will be used to detect the simulated actuator faults.

Since the sectioning method was proven to help increase the sensitivity of the detection algorithm, it was uses it in the following simulations. A set of raw data of 400 scans was obtained from a CD control system. There were 50 actuators and 1500 outputs. The outputs were averaged for every 30 points. The CD component was extracted from the raw data using EXPO algorithm mentioned in section 3.1. Figure 5.2 shows that the move of a single actuator will affect 10 outputs around it. Therefore, it is reasonable to divide the actuators into 5 sections and each section contains 10 actuators. For each of the sections, the sensitivity matrix and covariance matrix were estimated with the extracted CD component and input data. A threshold was also calculated using the algorithms proposed in chapter 3. Because of the limited number of data, the moving window of 8 scans was used.

As mentioned in 4.1, there are three types of actuator faults based on the position of actuator after the actuator faults. The most common one is the jammed fault, where the position of the actuator is fixed to that in the last scan. Firstly, the sectioning method was used to detect this
type of fault. The position of the 25th actuator was fixed from the 200th scan. A chi-square value was calculated for every scan. Figure 5.8 shows the result of chi-square test in the section 3.

Figure 5.8 Detection of a jammed actuator at the 25th actuator using the sectioning method
Chapter 5: Application of fault detection in paper-making systems

Time, scan

Figure 5.9 Detection of jammed actuators in section 3 using the sectioning method

There is no significant change in the chi-square value of the normalized residual of the 3\textsuperscript{rd} section. The actuator fault was not detected. In another test, all the actuators in a section 3 (21\textsuperscript{st} - 30\textsuperscript{th} actuators) was fixed at 200\textsuperscript{th} scan. Fig. 5.8 shows no significant change of chi-square value and the faults were not detected.

In some cases when a fault occurred with an actuator, the position of the actuator gradually drops to zero after the fault. This is to simulate a minor fault attending the process for a period of time. Since there is a larger change in the position of the actuator than the first type of fault, one can anticipate that it will be easier to detect such a fault. Fig. 5.9 and 5.10 shows the chi-square value of the normalized residual of section 3.
Figure 5.10 Detection of an incipient fault at the 25\textsuperscript{th} actuator using the sectioning method

Figure 5.11 Detection of incipient faults at the 23\textsuperscript{rd} – 27\textsuperscript{th} actuators using the sectioning method

Figure 5.10 shows that the algorithm can not detect the fault of one single actuator (the 25\textsuperscript{th} actuator). When the number of faulty actuator increased to 5, there were some chi-square
values exceeding the threshold, which indicate faults in section 3. However, the peak of chi-square values around 210 scan was not significant enough to distinguish it from possible peaks caused by disturbance.

For the third type of fault, the position of the actuator will suddenly drop to zero after the failure. Figure 5.16 shows the chi-square value of the normalized residual of section 3.

![Graph showing position of the 25th actuator over time](image)

Figure 5.12 Detection of an abrupt fault at the 25th actuator using the sectioning method

The algorithm successfully detected the fault of a single actuator with the presence of model uncertainty and disturbance.

### 5.4.3 Discussion of the simulation results

With generated data, the proposed algorithm can detect all types of faults at one single actuator. However, the algorithm can only detect abrupt fault at an actuator. It failed to detect slowly developing faults. The explanation is the noise and model uncertainties. Figure 5.12 shows the prediction of one of the system output using the given system model and system
inputs.

![Actual value of the 25th CD output](image1)

![Prediction of the 25th CD output](image2)

Figure 5.13 Prediction of the system output using the system inputs and system model

Actual value of the 25th CD output shows that it is very noisy. The difference between the actual value and prediction of the 25th CD output shows that the system model is very poor. The model inaccuracy in the system model and the noise in the system output make it difficult to detect slight faults of the actuators.

### 5.5 Conclusions

In this chapter, algorithms based on the local approach were proposed to detect actuator faults in CD control systems.

A CD control system is usually of high dimension and interaction matrix has ill-conditioned. Simulation studies shows that by divide the system output into sections, the sensitivity of the detection algorithm was improved. When the algorithm was applied to simulated data, different kinds of faults of a single actuator could be detected.

Real process data was obtained from a CD control system to assess the proposed algorithms. In this case, only abrupt failure of an actuator can be detected. An explanation could be that interaction matrix $B$ is usually unaccurate and the output data are very noisy. The effect of an incipient fault of an actuator was concealed by the noise and the model uncertainty. If the system noise overwhems the information about the fault, it is impossible to detect the fault. Therefore, the improvement in the detection of the fault lies in denoising the output data.
Chapter 6

Conclusions and Future work

6.1 Conclusions

This thesis addresses the application of fault detection and isolation using the local approach. As model uncertainties are unavoidable in practice system and they lead to false alarms in fault detection, the robustness of the local approach, with respect to model uncertainties, was investigated. An algorithm was proposed to recalculate the threshold value to reduce the false alarm rate to the preset value. A similar algorithm was also proposed to calculate the threshold to accommodate the regular parameter fluctuations. Simulation results show that the revised threshold can reduce the false alarm rate to a value close to the pre-assigned value and maintain the ability of detecting fault in case of both parameter uncertainties and parameter fluctuations. The robustness scheme was also compared with the upgrading scheme through simulations and it was concluded that the robustness scheme is more effective than the upgrading scheme in reducing the false alarm rate. The proposed algorithms were also applied to a pilot-scale tank level and temperature control system. Though, the revised threshold was focused to be able to reduce the false alarm rate, it was not large enough to decrease the false alarm rate to the preset value. This probably was due to the low frequency disturbances in the system.

Since most fault detection algorithms are applied to closed-loop system, closed-loop detection using the local approach was investigated. When the closed-loop data are used to calculate the primary and the normalized residual, the correlation between the input and output causes a non-full rank covariance matrix. The dimension reduction method used a linear transformation to reduce the dimension of the normalized residual so that the covariance matrix of the revised the normalized residual will be of full rank. With the closed-loop data, fault detection algorithms can only detect those parameter changes that can affect the closed-loop dynamics. The indirect detection method used the closed-loop model instead of the open-loop model to calculate the primary residual and normalized residual to perform detection. By detecting the changes of the closed-loop parameters, the method also detected the changes of
the open-loop parameters. Simulation studies showed that both the dimension reduction method and indirect detection method detected the change of every single parameter of a second-order linear process operated under the closed-loop condition. The implementation of the proposed algorithm on the tank level and temperature control system proved to be effective. From the simulations, the dimension reduction method and indirect method were found to be equivalent in performance.

Finally, real process data obtained from a CD control system was used to assess the applicability of the local approach. A CD control process is usually of high dimension and interaction matrix is ill-conditioned. Simulation studies shows that by dividing the system output into sections, the sensitivity of the detection algorithm was improved. When the sectioning method was applied to simulated data, different types of faults of a single actuator could be detected. The proposed algorithm was also implemented to real process data. In this case, abrupt fault of one single an actuator was detected. However, freezing faults and incipient faults could not be detected because of the noise and the inaccuracy of the process model.

6.2 Future work

A future research topic in robustness of the local approach may be to improve the robustness of the local approach with respect to low frequency disturbances. Possible solutions could either be to further increase the threshold value, or to take in account the low frequency disturbances in the process model.

In CD control systems in paper-making process, interaction matrix $B$ is usually inaccurate and the output data are very noisy. The effect of an incipient fault of an actuator was concealed if the system noise overwhems the information about the fault. Therefore, the improvement lies in denoising of the process data.
References


Frank, P. M., “Fault Diagnosis in Dynamic Systems Using Analytical and Knowledge-based

Frisk, E. and Nielsen, L., “Robust residual generation for diagnosis including a reference model for residual behavior,” 14th World Congress of IFAC, 1999.


Patton, R.J. and M. Hou, "On Sensitivity of Robust Fault Detection Observers," 14th World Congress of IFAC, Beijing, P.R. China 1999.


Van Den Hof, P. M. J. and Schrama, R. J. P., "An indirect method for transfer function
Wahnon, E. and Berman, N., “Tracking algorithm designed by the local asymptotic approach,”
Zhang, Q. And Basseville, M., “Monitoring nonlinear dynamical systems: a combined
observer-based and local approach,” Proceedings of the 37th IEEE Conference on
Zhang, Q., M. Basseville and A. Benveniste, “Fault detection and isolation in nonlinear
Zhang, Q, “Using nonlinear black-box models in fault detection,” Proceeding of the 35th
conference on Decision and Control, Kobe, Japan, Dec. 1996.
Zhang, Q., Basseville, M. and Benveniste, A., “Early warning of slight changes in systems,”