AN INVESTIGATION OF THE EFFECTS OF CONVERGENT/DIVERGENT TEACHING METHODS ON THE MATHEMATICAL PROBLEM-SOLVING ABILITIES OF GRADE TEN STUDENTS

by

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ABSTRACT

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It was the purpose of this study to investigate the effects of convergent/divergent teaching methods on student performance on two mathematical problem solving tasks (routine/non-routine problems). A concurrent purpose was to investigate the interaction between the convergent/divergent teaching methods and the thinking style (either convergent or divergent) of the learner.

Four grade ten classes were randomly selected from the eleven academic mathematics classes in the secondary school involved in the study. Due to subject absenteeism a total of sixty-six subjects were used for the analyses. Each subject was given the Watson-Glaser Test of Critical Thinking (Form YM) and the Torrance test of Thinking Creatively with Words (Booklet A) to determine their level on the independent measures of convergent and divergent thinking, respectively. Each subject was taught by one teacher using one method for approximately two hours. The content of these lessons involved the Fibonacci Sequence and Pascal's Triangle. At the end of treatment, each subject received a test on the dependent measures (routine/non-routine problems). Trained observers were used to ensure consistency of teaching method. Analysis of covariance using the regression model was performed with convergent/divergent thinking styles as the covariates.

There was no significant difference between convergent teaching methods and divergent teaching methods ($p \leq 0.05$).

Convergent thinkers scored significantly higher than did divergent thinkers on both dependent measures. However, as convergent thinking is far more highly correlated with intelligence than is divergent thinking, this result may have been confounded by intelligence. Therefore, in further studies in this area, the variance in problem solving due to intelligence should be partialled out.

Only one of eight interaction effects was significant ($p \leq 0.05$). This suggested that non-divergent thinkers did better with convergent (as opposed to divergent) teaching methods and that non-convergent thinkers did better with divergent (as opposed to convergent) teaching methods. The lack of other significant interactions indicated that intelligence may have been a confounding effect in this study.

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CHAPTER I

THE PROBLEM

Many teachers of mathematics are concerned with finding better teaching methods, in particular better methods of teaching problem solving. Suggested methods have ranged from drilling students on particular classes of problems, to programmed learning, where a student is lead by small steps to the desired conclusions, and to discovery learning, where examples are presented in such numbers that the student discovers a correct method of solution.

The difference in these methods is a major philosophical issue in education: the difference between instruction and teaching. The distinction is essentially one of directive versus discovery teaching. It is generally assumed that to ensure the best transfer of training, the method should correspond with the intended product. Kersh (1958) aptly states:

> If meaningful learning is the key concept, it should make no difference whether learning occurs with or without direction, so long as the learner becomes cognizant of the essential relationships. However, some learning procedures may be superior to others simply because they are more likely to cause the learner to become cognizant of the relationships. (p. 282)

In the case of problem solving it is essential that the learner develop methods or processes for attacking problems. The multiplicity of possible problems precludes teaching problem solving from a directive approach. Gagné

(1966) points out:

> Problem solving, when considered as a form of learning, requires discovery, since the learner is expected to generate a novel combination of previously learned principles. (p. 150)

The emphasis in on the learner combining his learned skills to become a problem solver. The problem for the teacher is how to effectively facilitate these combinations of previous information so that the learner will become an efficient problem solver. "There is no convincing evidence that one can learn to be a discoverer, in a general sense; but the question remains an open one." (Gagné, 1966, p. 150).

Many studies have been done to try to determine what factors affect the teaching of problem solving and, hence, help to create better problem solvers. Memory (Gagné, 1960) and IQ (Kline, 1960) of the learner, personality of the teacher (Torrance, 1962; McNary, 1967), motivational level of the learner (Kersh, 1958; Brown, 1975) and cognitive style (Merrifield, et. al., 1960, 1967) seem to be some of the factors which influence the teaching of problem solving in mathematics.

One factor that has been investigated is the relationship of creativity to problem solving. Many studies have been done involving creativity and problem solving (Clark, 1967; Behr, 1970; Maier, 1970; Ruse, et. al., 1976). These studies show that creativity has a positive relationship to problem solving. This lends support to Gagné's previously stated position.

However, one critical factor as yet uninvestigated is the interaction between creativity and teaching method: specifically, between the thinking style of the learner and the teaching method of the teacher. Perhaps one type of thinker is a better problem solver, no matter in what teaching situation he/she is involved. Perhaps one type of teaching method tends to create better problem solvers, regardless of initial differences in thinking style. Perhaps it is the interaction between thinking style and teaching method which is the important factor in producing better problem solvers. It is to this last premise that this research is addressed.

Background of the Study

In teaching mathematics, one endeavours to transmit both facts and processes. The teacher hopes that the student, through the use of these tools, will be able to recognize and solve problems which occur in his daily life which are mathematical in nature. This expectation of mathematics teachers does not simply mean the ability of students to achieve correct answers for specialized mathematical textbook problems. It rather purports that " ... the real aim of learning mathematics lies in the ability to apply its methods to new situations." (Avital and Shettleworth, 1968, p. 3). Curriculum guides and various study groups (CEEB, 1959, p. 2; CCSM, 1963, p. 7) list problem solving as one of the major outcomes of the mathematics curriculum. As Lucas (1974) states:

> The basis of mathematics is problem solving; therefore, if the cause of mathematical education is to be served, effective means of teaching problem solving must be clarified. (p. 45)

Before effective teaching of problem solving can occur, it is necessary to determine what problem solving in mathematics entails. Scandura (1968) states that:

> ... most meaningful learning including problem solving, may involve the recombination of previously learned rules into new patterns ... In effect, that problem solving be viewed simply as a form of transfer. (p. 9)

This transfer may just involve synthesis of relevant data or it may involve the ability to take the pertinent data and test it against many models (either previously existing or being created to meet the need) which will offer possible solutions for the given problem. If transfer is a necessary component of problem solving, as Guilford (1965) suggests, then thinking styles which enhance transfer should have a positive effect on problem-solving ability. Therefore, students who possess certain characteristics - divergent thinking, flexibility, synthesis or convergent thinking - (Guilford, 1965) should be better problem solvers than those who do not possess these characteristics or a combination thereof.

Merrifield, Christensen, Guilford and Frick (1960) showed that divergent production of semantic transformation (hereafter termed originality) had a high correlation with (1) the ability to think of attributes of a desired goal and (2) the ability to deduce logically sufficient antecedents. Problem solving can be thought of as, first, an awareness that there is a problem; secondly, a thinking of many possible solutions; and, thirdly, a decision as to the plausibility and effectiveness of these solutions (Feldhusen and Treffinger, 1977). It would seem that a good problem solver exhibits both abilities as described by Merrifield, et. al. (1960). Therefore originality would appear to be one charactistic of the good problem solver. Perhaps by promoting originality in the classroom, problem solving abilities will improve. If this is the case, one should try to create an atmosphere in mathematics classrooms whereby more students will be encouraged to think divergently as well as to display divergent thinking processes.

Producing better problem solvers is one of the major goals of school learning (Ausubel, 1963; Dirkes, 1975). However, the methods to best achieve this goal are somewhat elusive. Skemp (1971) suggests that the manner of presentation of mathematics should be fitted to the mode of thinking of the learner. While Skemp was making reference to Piagetian levels of operation, it is also possible to apply this notion to other models such as Guilford's (1957,1960) Model of the Intellect. This could best be exemplified through the convergent/divergent thinking aspect of the Operations axis in Guilford's (1965) model (See Figure 2.1).

Gagné (1960) proposed that one way to test Guilford's (1957) Model of the Intellect is to experimentally show that people who have scores on the Contents axis - figural, symbolic and semantic - learn better if content is presented in these modes. However, as problem solving is a process, it seems more appropriate to use the Operations axis as the source of experimental manipulation. Taylor (1965) suggests that:

> ... teacher training programs could focus in turn on various kinds of thinking in students as well as on subject matter; and we are advocating that both of these kinds of learning should happen simultaneously in the classroom. (p. 261)

These theoretical considerations suggest a need to devise a way to test whether one method of teaching is more effective in creating divergent thinkers. However, there is little guarantee that training teachers in divergent thinking processes will produce divergent thinking in their students. As Hutchinson (1965) pointedly remarks:

> ... we are faced with the challenge of how to train teachers so that their students will display primarily divergent thinking and remain at this thinking level without hurrying on through convergent to evaluative thinking. (p. 263)

Before looking for the means of training teachers, one first needs to ensure that teaching will indeed influence student thinking. McNary (1967) indicates that certain teacher characteristics did effect both convergent and divergent thinking of gifted elementary students. Convergent areas were most effected by teachers who were submissive, dependent, alert, cheerful, and seemed to have a natural warmth and liking of people; while divergent areas were most effected by teachers who were presistent, energetic, emotionally mature, friendly, and without fixed methods for gaining social approval. The Sutherlin Program (1964) indicates that creativity is nurtured in a student by matching him with a teacher who is creative. The two studies indicate that teachers can influence students' patterns of thought. It is further necessary to determine the relationship between teaching methods and student thinking styles.

Based on the literature, the following were assumed to be true for the present study: (1) increasing the mathematical problem-solving ability of students is one of the most important tasks set before mathematics teachers and (2) teaching method and thinking style of the learner are factors that may affect problem-solving skills in the mathematics classroom.

Given these two assumptions, the present study was designed to investigate two specific teaching methods and their two corresponding thinking styles.

Statement of the Problem

The present study has been designed to investigate the relationship between convergent/divergent teaching methods and student performance on a problem solving task. The relationship between these two teaching methods and the students' thinking style (either convergent or divergent) has also been investigated.

Research Questions

Some studies (Dahmus, 1970; Campbell, 1964) suggest that a directed approach to learning enhances problem-solving abilities. Other studies (Torrance, 1962; Flanders, 1965; Clark, 1967) claim that an open-ended approach to learning has a greater influence on the development of problem-solving abilities. Taylor (1965) and Prouse (1967) suggest that both approaches are necessary. To clarify these conflicting research findings, the following research question was asked:

1. For which teaching style (convergent/divergent) do subjects score significantly more correct answers on the dependent measures?

The teaching styles - convergent and divergent - have been defined to reflect consonance with convergent production and divergent production as defined by Guilford (1960, 1965). Convergent teaching emphasizes correctness of response, validity of inference from given choices and suggestions made by the teacher and correctness of interpretations from a limited group of possible alternatives. It is teacher-centered and the teacher asks leading questions which will lead the students to the desired response. Divergent teaching emphasizes the thinking process involved in the problem-solving process, making hypotheses, utilizing problem-solving tools and recombining the various hypotheses to form new hypotheses. It is a student-centered approach and the teacher acts as a catalyst, providing open-ended questions, listing student suggestions and rephrasing the question to stimulate further thought.

Maier and Janzen (1969) suggest that creative people are " ... superior problem solvers in that they are more likely to find correct answers to difficult problems."(p. 100). This would seem to indicate that divergent thinkers (ones who display a high degree of fluency, originality, and elaboration) would be better problem solvers than convergent thinkers (who have a good facility for transforming and redefining problems and for making valid inferences and choosing the best alternative). However, as Treffinger, Renzulli and Feldhusen (1971) point out:

> If creativity is viewed as a complex kind of human problem-solving (in which case perhaps the term "creative problem solving" would be preferrable), divergent thinking may be a necessary, although not sufficient, component. (p. 108)

This basically implies that a non-divergent thinker is not necessarily a convergent thinker and, conversely, that a non-convergent thinker is not necessarily a divergent thinker. As these thinking types may have some overlap and as both may be necessary components of the good problem solver, the following question is posed:

2. For which thinking style (convergent/ divergent) do subjects score significantly more correct answers on the dependent measures?

While many studies have dealt singularly with either thinking style or teaching method, few have tried to look at the correlation between the two. One such study by Vos (1976) showed that low ability students performed better when " ... emphasis on presenting a problem and reviewing past knowledge that may be helpful for the problem situation followed by specific instruction in the concept ..." (p. 274) was used as the teaching procedure. Vos further showed that high ability students performed better when the teaching procedure involved only instruction and the problem-solving task, rather than these two procedures combined with review of past knowledge which might be helpful to solving the problem. While the Vos study shows an interaction effect between ability and methodology, both methods incorporated an advanced- and post-organizer which used together certainly confounded the effects (Ausubel and Fitzgerald, 1962).

Skemp (1971) suggests that teachers should fit their teaching to the learner's mathematical schemas and that the manner of presentation should be consonant with the learner's mode of thinking. It was this suggestion which prompted the following research question:

3. Does a statistically significant interaction exist between teaching method and student thinking style (convergent/divergent) in terms of students' ability to solve problems on the dependent measures?

It was hoped that the comparison of divergent production and convergent production in both the process of teaching and the process of thinking would provide some insight into the partitioning of the variance in problem-solving abilities. It was further hoped that by allowing these two operations to range over the Contents and Products axes (Guilford, 1960,1965) that significant differences would result at the $p \leq 0.05$ level.

CHAPTER II

REVIEW OF THE PERTINENT LITERATURE

The review of the literature is divided into three major areas of investigation: (1) Convergent/Divergent Thinking, (2) Problem Solving in Mathematics and (3) Interaction between Problem Solving and Teaching Methods in Mathematics. A summary section is provided to focus on the interrelationships among these areas and to depict how the research and literature have formed a basis for the present study.

Convergent/Divergent Thinking

The whole concept of convergent and divergent thinking has its basis in Guilford's (1957,1960,1965) Model of the Intellect. This model contains three major axes: (1) Operations, (2) Contents, and (3) Products. Each of these axes contains categories as illustrated in Figure 2.1.

These axes and categories are defined in Guilford and Merrifield (1960). Operations are intellectual types of processes of "things that the organixm does with the raw materials of information." (p. 5). The Operations axis has five categories: cognition (recognition of information), memory (retention of information), divergent production (generating a variety of output from the same source), convergent production (generating unique or best outcomes from the given information), and evaluation. The Contents axis is defined to be general varieties of information. It contains four categories: figural (images), symbolic (signs

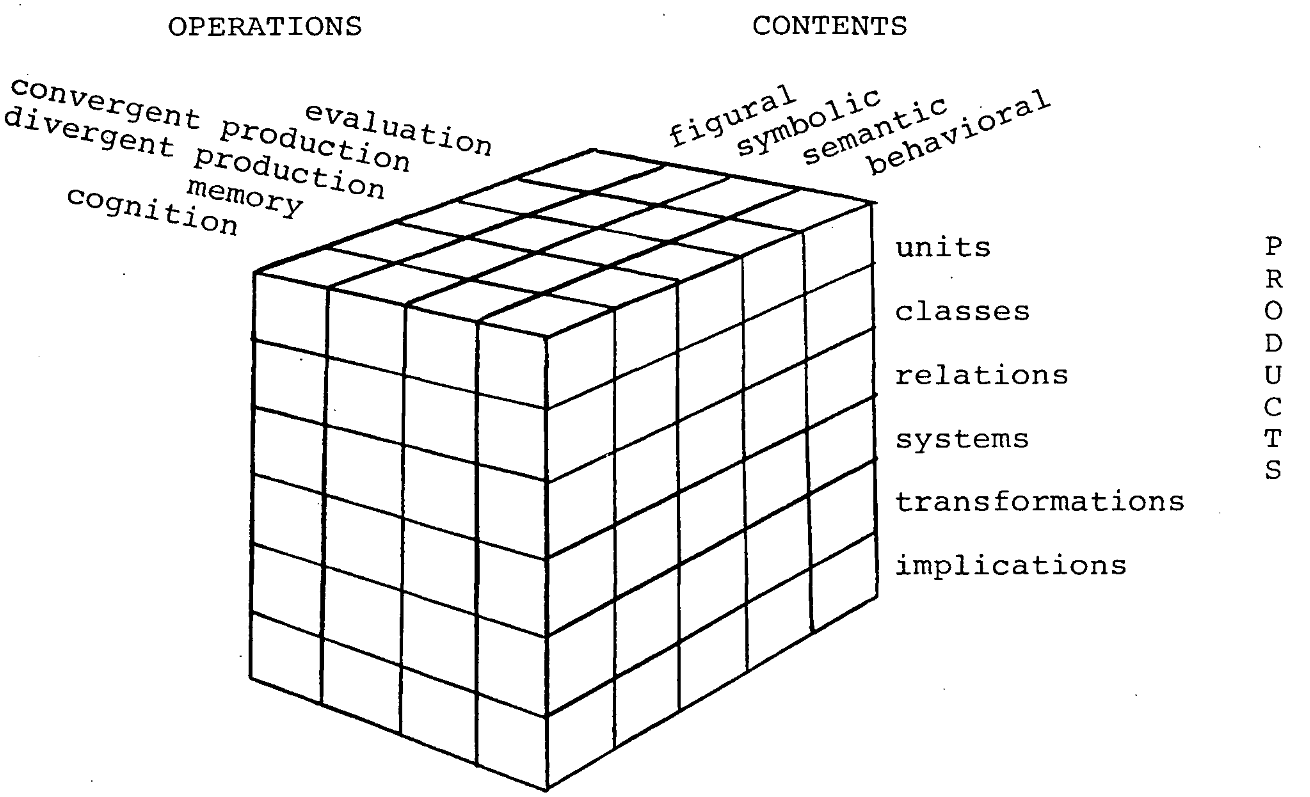

FIGURE 2.1

GUILFORD'S MODEL OF THE INTELLECT

and words), semantic (verbal) and behavioral (non-verbal). The Products axis is defined as "results from the organism's processing of information."(ibid., p. 5). It contains six categories: units (segrated items), classes (items grouped by a common property), relations (recognized connections between units), systems (structured aggregates of information), transformation (changes in production) and implications (predictions, extrapolations of information).

It is the Operations axis wherein the abilities involved in learning will be found. "The concept of thinking, itself, is to be applied to the three operation categories of divergent production, convergent production, and evaluation." (Guilford and Merrifield, 1960, p. 15). Furthermore, creative thinking "clearly points toward the category of divergent production" (ibid., p. 11) which includes the factors of fluency, flexibility, originality, and elaboration. While Guilford and Merrifield (1960) first try to equate creative thinking and divergent production, they later limit their statement by commenting that some aspects of creative thinking may involve some convergent production and even evaluation. However, convergent production has more to do with deduction which "implies the drawing of conclusions or the making of inferences ..." (ibid., p. 10).

Guilford's (1957, 1960, 1965) Model of the Intellect has been used to relate convergent/divergent thinking to problem solving in the studies by Merrifield, Guilford, Christensen and Frick (1960,1967). Their findings suggested which attributes of thinking were accounted for by particular blocks within the model. The ability to classify objects of semantic classes and the ability to find different relationships were accounted for by divergent production of semantic units as well as convergent production of semantic classes and implications. Other abilities involved in the process of problem solving (thinking rapidly of attributes, thinking of alternative outcomes, thinking of attributes of a desired goal and deducing logically sufficient antecedents) were accounted for by allowing the Product axis to range over its possibilities for divergent production x semantic content (Refer to Figure 2.1).

Behr (1970) used Guilford's (1965) Model of the Intellect in presenting verbal and figural teaching methods of modular arithmetic. Cognition of semantic relations and both teaching methods had a significant interaction ($p \leq 0.05$) with tests of knowledge of structure. While these results do not directly relate to the thesis of this study, they do show that some aspects of problem solving in mathematics are affected by teaching. Behr (1970) further suggests:

> If the factors that suggest ability to succeed at "higher order" learning situations can be identified, information from tests which measure these factors might be helpful in predicting a student's chance of being successful at various educational levels. (p. 39)

As stated earlier, it is the premise of this study that convergent/divergent thinking (and teaching) are factors in the process of improving problem-solving skills. While Merrifield, et. al. have used all three axes of Guilford's (1965) Model of the Intellect for their investigations, this study is confined to one axis: the Operations axis. This study explores the relationship between convergent/divergent teaching methods (Refer to Teaching Methods, this chapter and Definition of Terms section in Chapter III) and convergent/divergent thought processes in students (Refer to Definition of Terms section in Chapter III). It was felt that not restircting these two operations to any particular category of the Contents and Products axes would provide a first step towards developing characteristic operations involved in the process of problem solving in mathematics.

In a study involving male undergraduate students, sixteen of whom had high scores on the Remote Associates Test and another sixteen who had low scores, Klein and Kellner (1967) found that "highly creative" students scored significantly higher than "low creative" students in finding a pattern in a probability choice problem. The investigators pointed out that the "high creative" took more time to shift choice of solution than did the "low creative". This means that once the optimal solution had been reached, the "high creative" were more likely to maintain that pattern than were the "low creative" who would often switch to less optimal choices. Ideational fluency appears to be a critical variable in differentiating between creative and non-creative students.

In a study investigating the relationship between verbal response heirarchies and problem solving, Staats (1957) reported a positive correlation between fluency of verbal response (fluency of verbal response is one of the measures of divergent thinking as defined by Guilford (1965)) and problem-solving time. This finding, together with those of Merrifield, Guilford, Christensen and Frick (1960, 1967) and Behr (1970), has led to the choice of the Torrance test of Thinking Creatively with Words (Booklet A) (hereafter referred to as the Torrance test) as a measure of divergent thinking for the present study. The Torrance test provides reliable measures of fluency, flexibility, and originality of verbal problem solving. The Torrance tests started at the elementary level and an entire battery of tests developed through the first year college level. This progression of test development seemed to provide a stronger base for testing junior high subjects than would the Guilford battery which started at the adult level and developed downward into the high school area.

Prouse (1967) developed divergent and convergent thinking tests in mathematics and found:

> Correlation coefficients between scores on the divergent-thinking items and creativity test composite scores (one of Guilford's) ranged from 0.10 to 0.64, while correlation

> coefficients between scores on the convergent-thinking items and creativity scores ranged from 0.01 to 0.23. This would seem to corroborate Guilford's assertion that the more prominent creative abilities appear to be concentrated in the divergent-thinking category. (p. 879)

While the content of the tests in the Prouse study were inappropriate for the age level involved in the present study, the correlations show indicate that divergent thinking may be used as a measure of creativity, and therefore reinforces the choice of the use of the Torrance test.

In a comprehensive review of ability and creativity in mathematics, Aiken (1973) discusses both general and mathematical measures of creativity. He suggests that as IQ tends to be a catch-all for most significant differences attributed to the creativity factor, choosing measures which have low correlations with IQ will help in eliminating this problem. This suggests that the differences occurring in studies of convergent/divergent thinking in mathematical problem solving could be attributed to IQ differences, rather than creativity factors. Again, this is a justification for using the Torrance test as the most appropriate measure of divergent thinking. Studies involving correlations between the Torrance test and IQ show correlations ranging anywhere from 0.16 to 0.32.

Of the available tests of convergent thinking, the Watson-Glaser Test of Critical Thinking (Form YM) (hereafter referred to as the Watson-Glaser test) was chosen for two primary reasons: (1) its constructs of Critical Thinking (inference, recognition of assumptions, deduction, interpretation and evaluation of arguments) most closely resemble Guilford's (1960) convergent production category, and (2) it has the lowest correlation (0.55 - 0.73) with IQ (other tests of convergent thinking had correlations of 0.80 or better).

Problem Solving in Mathematics

Kilpatrick (1969) reviewed problem solving and creative behaviours in mathematics ans summarized the various findings by stating: "It appears that creativity, though it may be related to certain facets of problem solving, bears no simple relationship to problem-solving performance." (p. 168). Torrance (1966) suggests that perhaps it is the interaction between creativity and learning modes which bears a more direct relationship to problem solving than just measures of creativity alone.

What is meant by a problem when one is discussing problem solving is often misunderstood. Duckner (1945) suggests a problem is a circumstance occurring when someone has a goal but does not know how to obtain this goal. Thus many of the so called "problems" in mathematics are really not a problem as the student has readily available algorithms which he knows apply to this particular "problem" situation.

When agreement has been reached as to what are problems, it would be nice if one could predict which students would a good problem solver. Kilpatrick (1969) found that:

> Subjects who attempted to set up equations (they had not yet had an algebra course) were significantly superior to the others on measures of quantitative ability, mathematics achievement, word fluency, general reasoning, logical reasoning, and a reflective conceptual tempo. (p. 165)

These subjects obtained higher scores on the problem-solving measures employed and thus students who were better problem solvers displayed superiority on both divergent (word fluency) and convergent (logical reasoning) measures. The question is, "Which one of these two factors is more significant as a predictor of good problem solvers?".

Scandura (1968) attempted to define problem solving and its characteristics when he first pointed out that problem solving "may involve the recombination of previously learned rules into new patterns ... simply a form of transfer." (p.9). He then continued by saying that problem solving is "more than selection and integration of previously learned rules." (p. 13). In solving a problem a student first needs to define the problem and then test trial solutions. The development of trial solutions is one aspect of divergent thinking, while the selection of the most appropriate approach is part of the convergent thinking process (Covington, 1968). It would seem that perhaps both factors are needed by the good problem solver.

It is interesting to note than many investigators (Gagné, 1959; Merrifield, Guilford, Christensen, and Frick, 1960,1967; Polya, 1965; Taba, 1965; Troutman, et. al., 1967) agree on the general phases of the problem-solving process: preparation, analysis, production, verification, and reapplication (Merrifield, Guilford, Christensen, and Frick, 1960, p. 2). With this kind of general consensus on the process of problem solving, it seems imperative to begin investigation of the factors which contribute to creating better problem solvers in mathematics and to incorporate these factors into the educational process.

Interaction between Problem Solving and Teaching Methods in Mathematics

The two teaching methods used in the present study are defined in Chapter III. However, it seems appropriate to describe these two methods (convergent/divergent) prior to presenting the related literature.

The convergent teaching method utilizes teacher-formed alternatives, stepwise questioning and an emphasis on correct solutions in mathematical problem-solving situations. The teacher presents alternative methods for attacking the problem and asks the students to choose the one they feel is the best. The emphasis in on the student making consistent inferences and interpretations of the alternatives the teacher is presenting. The answer to the problem, not the method of solution, is emphasized.

The divergent teaching method utilizes open-ended questioning on the part of the teacher with an emphasis on "We have an interesting problem. What means do we have available to possibly solve this problem?" attitude. The teacher questions the students so that the students make suggestions how to proceed. These suggestions are listed and then tried. The emphasis is on the student utilizing previous information regarding mathematical problems, recombining this information to form new hypotheses about the solution of the given problem. The method of solution, not the answer to the problem, is emphasized.

Taba (1965) suggested that the ability to think is not an automatic by-product of studying another's disciplined thought processes. She further pointed out that reflective thinking is not dependent on the volume of facts presented. Freudenthal (1972) reinforced this viepoint when he stressed the need to allow mathematics to be reinvented. The implication for teaching mathematics seems clear: if one wants to create good problem solvers, one must allow the students to apply mathematics to the world around them. One of the ways to do this seems to be to stimulate divergent thinking patterns (Hutchinson, 1965).

Lucas (1974) attempted to do this through heuristic teaching of thirty university students enrolled in two calculus classes. In the experimental group, problems were discussed to reinforce the learning of problem-solving strategies, while in the control group problems were discussed mainly to reinforce mathematical concepts. Lucas found significant differences at the $p \leq 0.05$ level using a chi-square analysis on problem-solving performance scores between two groups in favour of the experimental group. However, one should view the significance of these findings with caution due to the low n involved. Like Lucas, Gurova (1969) found that by making fifth- and sixth-graders aware of the processes they used in actually solving problems, their problem-solving abilities improved.

It is interesting to note that recent studies involving aptitude-treatment-interaction have shown no significant differences for either the treatment effect or the aptitude-treatment-interaction (ATI) effect. Post and Brennan (1976) used 47 pairs of subjects in comparing a General Heuristic Problem-Solving-Procedure and "normal" instruction in geometry. While they found no significant differences for the treatment or the interaction between treatment and ability level, they did suggest that "... identification of 'typical' problem-solving behaviors within well-defined topical domains and subsequent attention to the development and maintenance of those behaviors." (p. 64) may be one way to approach future research in the field of problem solving in mathematics. The present study has attempted to look at convergent/divergent teaching methods and their subsequent effect on the problem-solving behaviours on routine/non-routine problems regarding sequences.

Kantowski (1977) in a study using eight high ability ninth-grade algebra students investigated the use of heuristics in problem solving in mathematics. The use of heuristics is similar to the divergent teaching method in that both emphasize the process involved in solving problems as opposed to the product (or solution) of the problem. She suggested that "The effects of heuristic instruction versus expository instruction should be investigated with the use of heuristics as the dependent variable." (p. 175). Eastman and Behr (1977) attempted to do something similar to this, although they still used problem solving as the dependent variable. Their ATI study involved 208 ninth-grade algebra students using logical inference. The figural-inductive group was taught using figural-inductive programmed material and the symbolic-deductive group was taught using symbolic-deductive material. The subjects had ninety minutes to study the programmed material and were tested both one day later and two weeks later. None of the problems used in the tests were defined to be unfamiliar (non-routine). No significant differences were found. However, the authors presented questions regarding: "What aptitudes are appropriate for predicting differential achievement in mathematics learning?" (p. 381). It is the premise of the present study that perhaps convergent/divergent, while close to deductive/inductive, are more accurate descriptors of the actual differential thought processes of students and may more accurately describe the basic differences found in the teaching process of mathematical problem solving.

It is interesting that the Eastman and Behr (1977) study used teaching material which corresponded to the thinking style of the learner. This idea is incorporated in Skemp's (1971) suggestion that the task of the mathematics teacher is three-fold:

> He must fit the mathematical material to the state of development of the learners' mathematical schema; he must also fit his manner of presentation to the modes of thinking (intuitive and concrete reasoning or intuitive, concrete reasoning and also formal thinking) of which his pupils are capable; and finally he must be gradually increasing their analytic abilities to the stage at which they no longer depend on him to pre-digest the material for them. (p. 67)

These statements suggest that the mode of teaching should correspond to the student's thinking style and infers (from the final task) that divergent teaching may be one way to help establish this student independence of thinking.

McNary (1967) attempted to relate teacher characteristics to the degree of change displayed by gifted elementary students in both the convergent and divergent areas of thinking. While these characteristics are related to the personality of the teacher, perhaps they give a clue to methodological considerations as well. She found that a teacher who was dependent and stood by society's standards effected the convergent style of thinking. Perhaps such a teacher would teach in accordance with this personality and tend to foster dependency and choices of the best acceptable outcome when teaching problem solving. Similarly, McNary found that teachers without inflexible patterns for obtaining social approval influenced the divergent style of thinking. Perhaps this teacher would also teach in accordance with this personality and be flexible in accepting many different suggestions, solutions and methods from students in a problem-solving situation.

Torrance (1962) also supported the concept that teaching method may make a difference. In a study on under- and over-achievement of fifth-graders, he found that with a low creative teacher, highly creative children were underachievers, while low creative children tended to overachieve. He also found that with a highly creative teacher, both types of children seemed to overachieve. This seems to suggest that teaching method, as inferred by characteristics of the teacher, may influence performance in a classroom.

The Sutherlin Program (1964) used an idea inventory for teaching the creative. Their study involved students from grades seven through twelve and their concern was to find ways to nurture creativity. They established five teaching principles which they found encouraged creative thinking: (1) treating the students with respect, (2) treating imaginative ideas with respect, (3) placing value on pupil ideas, (4) allowing students to explore learning situations without always being evaluated, and (5) tying evaluation with cause and consequence. Principles (2), (3), and (4) are certainly part of the basis of the divergent teaching method.

Summary

There is considerable research suggesting that consistency in teaching methods and student thinking styles is important in order to produce better problem solvers in mathematics (Sutherlin Program, 1964; Taylor, 1965; Behr, 1970; Skemp, 1971). Skemp (1971) suggested that mathematics teachers need to plan ways of teaching which take into account not only the student's previous mathematical experience but also their level of thinking (p. 114). This seems to indicate that there will be an interaction between the way in which something is taught and the thinking style of the learner.

Taylor (1965) suggested focusing on the student, not only what he learns, but how he learns. He further suggested the use of the factors in the Operations axis of Guilford's (1960) Model of the Intellect as the means of this investigation. This leads directly to the area of thinking and hence the areas of convergent/divergent production.

From Torrance's (1962) findings it would seem that it is particularly important for the divergent thinker to be matched with a divergent teaching method. However, McNary (1967) implies that gifted children need both convergent and divergent types of teachers. Taylor offers a solution:

> By using appropriate classification systems, one could start logging the responses of the teacher and students to find out what thinking and learning processes in students are evoked by various behaviors and teaching methods of the teacher. (p. 258)

If, as the research indicates, there is some kind of interaction between teaching method and thinking style of the student, then there is a definite need to acquaint teachers with divergent methods of teaching and to consciously promote divergent thinking patterns and methods of solution to problems in the mathematics classroom.

CHAPTER III

DESIGN OF THE STUDY

The third chapter is partitioned into the following sections: (1) Definition of Terms, including actual examples of convergent/divergent thinking and teaching; (2) Sample, who was involved, how and why they were chosen; (3) Teachers, who they were and how they were trained; (4) Procedures, what actually happened during the study; and (5) Statistical Hypotheses, nine one-tailed hypotheses, each group of three relating to one of the three research questions as outlined in Chapter I.

Definition of Terms

Convergent Thinker

Prior to treatment, the Watson-Glaser Critical Thinking Appraisal (Form YM) was administered to all subjects. This test involves five sections - inference, recognition of assumptions, deduction, interpretation, and evaluation of arguments. These variables are consistent with Eberle's (1965) description of convergent thinking activities which involve redefining the problem, transforming the problem, and recognition of the best or conventional solution to a given problem. While individual subtest reliabilities are low (0.53 - 0.74) on the Watson-Glaser test, reliability of the entire test at the grade ten level is 0.86. Consequently, only total scores were considered for this study.

Convergent thinkers were defined to be those subjects who scored one standard deviation or more above the mean on the Watson-Glaser test. The normative data did not include the time of testing at the grade ten level; therefore, it was felt that it was better to be conservative and use the grade eleven mean and standard deviation as the subjects in the present study were in their last quarter of their grade ten year. The grade eleven mean was 64.4 (compared with 61.7 at the grade ten level) and the standard deviation was 11.0 (the same as the grade ten level). Any student who had a raw score of 76 or better on the Watson-Glaser test was defined to be a convergent thinker.

A non-convergent thinker was defined to be a subject who scored one or more standard deviations below the mean on the Watson-Glaser test. Subjects with a raw score of 53 or less were defined as non-convergent thinkers. A non-convergent thinker should not be equated with a divergent thinker. A non-convergent thinker may be thought of as a person who does not readily redefine or transform problems and who is unable to make the best choice of several alternatives. This does not imply that he/she is able to be original or generate novel solutions as would a divergent thinker.

The following is a hypothetical example of how a convergent thinker might solve a problem. The student is given the sequence of numbers 1, 2, 6, 15, ... and is asked to find the next number. A convergent thinker would probably writes down the sequence, mentally note that adding the numbers did not yield a constant sum; however, the difference between the numbers gave a sequence of 1, 4, 9, ... which appears to be the set of perfect squares; let's see, the last one is three squared, so the next must be four squared which is 16, so the next number is 31 (15 plus 16), right? Good, now I'm finished.

Divergent Thinker

To continue the example given above, the divergent thinker might solve the problem in a manner similar to the following. He might write down the sequence, but above it he would probably write the term numbers because he likes to think of individual things as part of a whole:

1	2	3	4	5
1	2	6	15	?

He might then note that the first is the same as the first term, the second is the same as the second term, you double 3 to get 6, you triple 4 then add 3 to get the fourth term, maybe you quadruple 5 then add 4 (or maybe 6) to get the next term. What else? Let's see, square 1 to get 1, square 2 and subtract 2 to get 2, square 3 and subtract 3 to get 6, square 4 and subtract (oops) 1 (oh, that's OK) for the next just square, then square subtract 2, square subtract 3, square subtract 1. Oh, the difference between terms gives 1, 4, 9, ... so that would make the next difference 16 and the next number 31. Mmm, wonder what else: take the first multiply by 2 to get the second, multiply second by 3 to get third, multiply third by 3 subtract 3 to get fourth, the pattern could just keep repeating.[1]

Prior to treatment, all subjects were given the Torrance test of Thinking Creatively with Words (Booklet A). This test involves seven activities: the first three deal with asking questions, guessing causes and guessing consequences regarding an elfin type creature looking at his reflection; the fourth involves product improvement of a stuffed toy elephant, unusual uses of cardboard boxes is the fifth activity, while unusual questions regarding cardboard boxes is the sixth; a just suppose question involving strings attached to clouds is the final activity. Each activity is scored for fluency (the actual number of responses on an item), flexibility (changes in types of response on an item), and originality (as compared to the responses of others in the norming group).

The total score index of all three categories provides a more stable index of the creative energy which a subject has available and/or is willing to use and the reliabilities are higher for the total score than for the separate scores. Total means and total standard deviations were not available in the norming data; consequently, the means for the three separate scores (fluency = 94.6, flexibility = 40.2, and originality = 45.2) were added to give a composite mean of

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1 It should be noted that the author does not fit the definition of a divergent thinker, but this thinking is designed to simulate divergent thinking.

180. While standard deviations are not additive, if a positive correlation exists between subtests (as is the case in the Torrance test), then adding the individual standard deviations would give a sum which would be greater than the actual total standard deviation (as the covariance would be subtracted from this sum). Therefore, to give a conservative estimate of the total test standard deviation, the standard deviations (fluency = 32.5, flexibility = 9.0, and originality = 23.2) were added to give an s = 64.7 (approximately 65).

A divergent thinker was defined to be a subject who scored one or more standard deviations above the mean on the Torrance test. Using the above data, this meant that any student who had a raw score of 245 or more on the Torrance test was defined to be a divergent thinker.

A non-divergent thinker was defined as a subject who had a total raw score of 115 or less on the Torrance test.

Convergent Teaching Method

The convergent teaching method was based on Eberle's (1965) convergent thinking activities (transformation, redefinition, and the ability to pick the best choice of several alternatives). As these had no mathematical base, the author and one of the committee members devised the following list of expected teacher behaviours:

(1) The teacher makes suggestions as to how to solve the problems.

(2) Questions are leading and aiming towards correct conclusions and answer.

(3) Emphasis is on being correct, making valid inferences from possible choices, making correct interpretations from limited possible alternatives as presented by the teacher.

(4) If the class is slow, the teacher does not rephrase the question, but rather tells, through example or many little leading questions, how to get the desired answer.

(5) The tone of the class is teacher directed.

The content for this method was taken from Jacobs' (1970) Mathematics: A Human Endeavor (See Appendix A for the Convergent Teaching Lessons).

The following is a hypothetical example of how a convergent teacher might approach teaching the 1, 2, 6, 15, ... sequence as presented earlier. The teacher would write the sequence on the board and then ask for guesses of the next number (accepting only 31 as an answer) then asking for the next number (again accepting only 56). The teacher would ask if anyone knew how to get these answers (if no guesses were forthcoming), "What about the operation of subtraction? What is the difference between the first and second terms? the second and third? the third and fourth? Do you see a pattern in these numbers? Recall the operation of squaring numbers. (This should be a big enough hint, but if not...) Let's look at numbers whose squares we know: 1^2, 2^2, 3^2, ... Therefore, when confronted with sequences of this kind the best way to try to find the pattern is to look for differences between terms."

Divergent Teaching Method

The divergent teacher teaching the same sequence might approach doing so in the following manner. "Here is a sequence of numbers. How do you think this sequence of numbers was arrived at? (As students answer, the teacher is accepting and writes the suggestions on the board.) (if the class is slow ...) Does everyone know what a sequence is? What does the word itself mean? Do you know of any other sequences in math? Could the number line be thought of as a sequence? How did we develop that? Can you apply that to our present problem? Again, does anyone have a suggestion as to how I arrived at this particular sequence of numbers?"

The divergent teaching method was based on Eberle's (1965) divergent thinking activities (fluency, flexibility, originality, and elaboration). The author and one of the committee members devised the following list of expected teacher behaviours:

(1) The teacher's questions are open-ended, aiming toward getting more suggestions from the students.

(2) Students are to make the suggestions as to how to solve the given problems.

(3) Emphasis is on how the thinking is being done, making hypotheses as to how to solve problems, utilizing previous information and tools for solving problems, recombining these to get new hypotheses. The answer is nice, but the method is the most interesting facet of the solution.

(4) If the class is slow, the teacher rephrases the question, or asks the class to tell what they think that the question means, or the teacher asks a different question and then comes back to the original question or defines a term which the class may not know and then asks the original or a rephrased question.

(5) The tone of the class is student-directed. The teacher is to act as a catalyst and a resource person, and also as a lister of student suggestions for possible solutions.

The content for this lesson was taken from Jacobs' (1970) Mathematics: A Human Endeavor (See Appendix B for Divergent Teaching Lessons).

Routine Problems

Thirty-four questions were constructed by the author to reflect the content taught in the convergent and divergent lessons. The questions include ten "true-false-sometimes" questions based on the Fibonacci Sequence and Pascal's Triangle; ten multiple choice questions based on the same two areas; seven questions involving division and squaring of Fibonacci numbers; and seven questions involving Pascal's Triangle, powers of eleven, and Chinese symbols. These questions were routine as they directly tested the content as given in the two lessons. While the routine questions were predominantly convergent type questions, a few divergent ones were included (See Appendix C).

Non-Routine Problems

Thirty-four questions were constructed by the author to reflect content similar (i.e. sequence-based) to that in the convergent and divergent lessons. These questions, again, were mainly convergent in nature, but included a question on magic squares which was divergent and required a grading system similar to the one used by Prouse (1967) for fluency, flexibility, and originality (See Key for Divergent Question in Non-Routine Problems, Appendix D). These questions were non-routine as they did not include any content taught in the convergent and divergent lessons. The first ten questions were "true-false-sometimes" problems based on four sequences (positive integers, odd whole numbers, even whole numbers, and square whole numbers). The next ten questions were multiple choice questions based on the same four sequences. The next five questions involved a coded sequence. The next two questions involved a magic square (this included the special divergent question mentioned above). The last seven questions dealt with triangular, square, and pentagonal numbers (See Appendix D).

Sample

Subjects for this study consisted of students from four grade ten classes randomly selected from the eleven academic mathematics classes taught at one secondary school. Vos (1976) indicated that "mathematical maturity was a definite factor in problem-solving ability." (p. 274). He also indicated that students in a second year algebra course were superior on problem-solving tasks as compared to non-academic and first year algebra students. As the grade ten students selected for this study were in the final quarter of their second year of algebra, it was hoped that their mathematical maturation level would be high enough to cope with the problem-solving tasks presented to them in the study. At the grade ten level, normative data were available for both standardized tests used to measure convergent or divergent thinking.

Each class of subjects was randomly assigned to teaching method (either convergent or divergent) and to teacher (either A or B) by the toss of a coin.

The final sample included all students who were in attendance for the pre-tests (Watson-Glaser and Torrance), the treatment, and the post-test which contained both the routine and non-routine problems.

Group 1 - This group consisted of 15 subjects and was taught convergently by Teacher B.

Group 2 - This group consisted of 18 subjects and was taught convergently by Teacher A.

Group 3 - This group consisted of 16 subjects and was taught divergently by Teacher A.

Group 4 - This group consisted of 17 subjects and was taught divergently by Teacher B.

Teachers

The teachers involved in the present study were both members of the Mathematics Education Department at the University of British Columbia. Both had over five years teaching experience at the secondary school level. Both of these teachers were known to use both convergent and divergent teaching techniques in some of their classes, and this was the major reason for choosing them.

Two weeks prior to treatment, both teachers were given the convergent and divergent lessons (See Appendices A and B). They were also given the list of expected teacher behaviours

(Refer to Definition of Terms, this chapter). They were instructed to become familar with both lessons and to note their similarities and differences in style.

In a study done by Taylor (1965), teachers were instructed for an hour each day for a week prior to beginning instruction of students. This training was to ensure that the teachers would treat the students as thinkers during instruction, rather than treating them as learners as was considered the conventional manner. Due to conflicting schedules, one three-and-one-half hour training session was held four days prior to treatment in the present study. During the training, emphasis was given to the discrepancies between the questioning techniques of the two methods. In the convergent method a vague general question could be broken down into "little leading questions", guiding students to the desired conclusions. In the divergent teaching method, the teacher was not at liberty to give hints, but rather rephrased the question. Throughout the convergent lesson, emphasis was placed on correct answers and accurate conclusions drawn from the teacher's suggestions. The emphasis in the divergent lesson was on the procedures used to answer questions with the solution coming from the students. The divergent lesson also placed emphasis on patterns and relationships rather than correct conclusions.

Each teacher taught one group on one day for a period of 100 minutes (the first two morning periods of 50 minutes each). The next day each teacher taught a different group for a period of 100 minutes. Each teacher taught the convergent method one day and the divergent method on the other day (or vice versa; see Table 3.1).

Procedures

A Latin Square Design was selected to eliminate differential effects due to time and teacher differences. Each teacher taught one class per day using either the convergent or divergent method. Each class was assigned to one and only one treatment. The treatment period lasted for the first two morning periods (of approximately 50 minutes duration each). During the third period, the tests on the two dependent measures (routine and non-routine problems) were administered.

TABLE 3.1

DESIGN

DAY	TEACHER A	TEACHER B
1	C*	D**
2	D	C

*C means convergent teaching method
**D means divergent teaching method

A toss of a coin determined which teacher was designated as Teacher A and which was Teacher B. Classes were randomly assigned by the toss of a coin to teaching method and teacher. One half of each class first received the routine and then the non-routine problems, while the other half of each class received the same problems in reverse order.

The experimental groups were tested once on the dependent measures, therefore a short term stability estimate of these measures was required. Hoyt Estimates of Reliability were calculated to provide an index of short term stability of the dependent measures administered during the experiment.

There were thirty-four questions administered for each dependent measure. It was hoped that at least twenty of these would have sufficiently high correlation with the total dependent measure as to be retained in the analysis. The overall Hoyt Estimate of Reliability was low (0.58) for the routine problems (Group 1 = 0.53, Group 2 = 0.82, Group 3 = 0.53, and Group 4 = 0.85). An item analysis was performed using the programme LERTAP (Larry R. Nelson, April, 1974) available at the Computing Centre at the University of British Columbia. Those items with a negative or zero correlation were omitted (items 1, 3, 8, 10, 16, 19, and 22). Subsequent reliabilities were obtained as follows: Group 1 = 0.60, Group 2 = 0.82, Group 3 = 0.60, and Group 4 = 0.71. This gave an overall Hoyt Estimate of Reliability for the routine problems of 0.74, thus raising the overall reliability to acceptable standards. Further analysis of the routine problems was done with the above omissions being made.

On the non-routine problems, no item omissions were necessary and the Hoyt Estimates of Reliability were as follows: Group 1 = 0.85, Group 2 = 0.85, Group 3 = 0.84, and Group 4 = 0.93. This gave an overall Hoyt Estimate of Reliability for the non-routine problems of 0.88.

There were two observers involved in the study. They were the author and a fellow graduate student in mathematics education at the same university as the teachers who taught the lessons in the study. Both observers participated in the teacher training session. Their purpose was to observe each lesson to ensure that both teachers taught the same content, as well as to ensure that the method for that particular lesson was adhered to. This was strictly an extra precautionary measure (See Table 4.5). Due to unavoidable delay on the second day, Group 2 was only observed for the last twenty minutes. However, during this time Teacher A was teaching convergently and completing the content as outlined in the lesson.

It was hoped that by having a short treatment period that differences due to modality (convergent/divergent teaching methods) would appear as the major contributor to the variance of any differences between groups. Spache (1976) suggests:

> We can readily show that differences in short term learning, as a single lesson of a certain type, seem to be related to modality. (p. 70)

While the treatment was of short duration (approximately two hours), it was felt that if significant results could be obtained, these would probably be more due to treatment than to previous mathematical experience. Gagné, Mayor, Garstens and Paradise (1962) found this to be the case when students were learning addition of integers for the first time. Fattu, Mech and Kapos (1954) also found that a two hour treatment did significantly affect the scores of problem solvers on a set of gear-train problems. It was felt that as the present study is an exploratory study, the two hour treatment might be sufficient to show significant results, particularly as the post-test was administered immediately after treatment.

The post-tests (dependent measures were administered immediately after the treatment as there is ample evidence (Postman, 1963,1964; Petersen and Petersen, 1959) to indicate that the manner in which people store material changes with time. That is to say, the longer the period of delay, the more likely the subject is to return to his/her habitual thought patterns. If the dependent measures were allowed to be delayed, one might expect the established patterns of coding to take precedence in the manner to which the subject would respond to the test. Postman (1964) found that this coding process could be expected to interfere with the experimental method if the testing period is delayed beyond the critical period (12 - 14 hours) and might confound the results.

Statistical Hypotheses

The following statistical hypotheses were tested to answer the research questions as posed in Chapter I of this study:

(1a) Subjects convergently taught will score significantly more correct answers on routine problems than subjects divergently taught.

(1b) Subjects divergently taught will score significantly more correct answers on non-routine problems than subjects convergently taught.

(1c) Subjects divergently taught will score significantly more correct answers on the total problem set than subjects convergently taught.

The above questions look at the teaching method as related to question type and propose that the teaching style which is consonant with the questions will more significantly effect scores.

The next set of questions indicate that a student's thinking style will effect his ability to solve problems whose type is similar to his thought processes:

(2a) Convergent thinkers will score significantly more correct answers on routine problems than will divergent thinkers.

(2b) Divergent thinkers will score significantly more correct answers on non-routine problems than will convergent thinkers.

(2c) Divergent thinkers will score significantly more correct answers on the total problem set than will convergent thinkers.

The final set of hypotheses have been posed to analyze the interaction between teaching method and thinking style and its possible effect on problem solving:

(3a) Convergent thinkers taught convergently will score significantly more correct answers on routine problems than divergent thinkers taught divergently.

(3b) Divergent thinkers taught divergently will score significantly more correct answers on non-routine problems than convergent thinkers taught convergently.

(3c) Divergent thinkers taught divergently will score significantly more correct answers on the total problem set than convergent thinkers taught convergently.

A linear regression model was used for the statistical analysis. An analysis of variance was first used in the regression as subjects had been randomly assigned to groups and groups were randomly assigned to treatment. However, when checking for the possible confounding effects (as mentioned earlier in this chapter), the author also investigated whether there had been a difference between groups on the mean scores of the Watson-Glaser test (convergent) and/or the Torrance test (divergent). There were significant differences found between Group 3 and Group 1 and between Group 3 and Group 4 on the Torrance test (See Table 4.7). For consistency with theoretical considerations (i.e. thinking style to be covaried with the dependent measures) a further analysis of covariance was made using both the Watson-Glaser test and the Torrance test as covariates. The analysis of covariance corrected the fact that there had not been an opportunity to block the cells, as entire mathematics classes were being used. It should be further noted that the analysis of covariance is a more sensitive analysis than the original analysis of variance. The direction of any significant results will be determined by using a t-test.

CHAPTER IV

ANALYSES AND RESULTS

Since, for studies like the present one, there is great concern about the results being explained, not by the statistical hypotheses, but by characteristics inherent in the design of the study, the present chapter is divided into two major parts: (1) Confounding Effects and (2) Results of the Study. First the data will be analyzed to ensure that no significant effects were evident on variables which might tend to confound the results of the study. Second, the data will be analyzed with respect to the statistical hypotheses as presented at the end of Chapter III.

Confounding Effects

There were four major areas of concern, regarding the design of the study, which might have confounded the results. The first was that one teacher might be better, regardless of the method used (Teacher Effect). The second was that subjects might perform better on one day of treatment than on the other day of treatment, regardless of the teacher of the method used (Time Effect). Thirdly, there was concern that the lesson content might not be completed and/or that a teacher might change from a divergent to a convergent teaching method (or vice versa) (Observer's Checklist). Lastly there was some concern that some subjects might not be responding to the lesson according to the directions of the teacher (Student's Notes).

Teacher Effects

One of the concerns arising from the design of this study was the use of two different teachers. While the design was balanced for teachers, there was still the concern that a teacher might do better on one method than on the other method. It was therefore necessary to determine that there were no significant effects due to the use of two different people as instructors. To ensure this, the means and standard deviations were calculated for Teacher A's and Teacher B's classes (these data are presented in Table 4.1). An *F* - ratio was performed at the one percent level of convidence, and no significant differences were found on either the routine ($F(33,31) = 1.19$), the non-routine ($F(31,33) = 1.20$), or the total problem set ($F(31,33) = 1.20$), which meant that pooled sums could be used for the *t*-test to determine whether or not there were significant teacher effects. The results of the *t*-tests are presented in Table 4.2. All of these values showed no significant difference at the $p \leq 0.01$ level. Since using different teachers seemed to have no effect on the problem-solving abilities of the subjects, further analysis of the data can be made.

Time Effects

Another primary concern was that there could be differential results due to treatment occurring on two different days for subjects. While there were no obvious factors,

TABLE 4.1

MEANS AND STANDARD DEVIATIONS FOR
TEACHER EFFECTS ON DEPENDENT MEASURES

	Teacher A*			Teacher B**		
	Routine	Non-Routine	Total	Routine	Non-Routine	Total
Mean	12.29	33.42	45.00	11.91	29.91	41.82
Standard Deviation	4.41	7.72	10.90	3.70	9.28	11.63

*N = 34

**N = 32

TABLE 4.2

RESULTS OF *t*-TESTS FOR
TEACHER EFFECTS ON DEPENDENT MEASURES

Problem Set	*t*-value*
Routine	0.380
Non-Routine	1.563
Total	1.300

* d.f. = 64

Critical *t*-value ($p \leq 0.01$) = 2.390

there may have been some unforseen difference in learning which occurred because a subject was taught on the first day of treatment as opposed to the second day (or vice versa).

The means and standard deviations were calculated for Day 1 and Day 2 of treatment (these data are presented in Table 4.3). An *F*-ratio was performed at the $p \leq 0.01$ level of confidence to assure homogeneity of variance. This was necessary to enable the use of pooled sums for the *t*-test of the difference between means on Day 1 and Day 2 subjects. No significant differences were found on either the routine ($F(35,31) = 1.35$), the non-routine($F(35,31) = 1.24$) or the total problem set ($F(35,31) = 1.47$). As no significant differences were found, the appropriate *t*-tests were performed to determine whether or not the day difference significantly effected the results. The results of the *t*-tests for significant differences between the means are given in Table 4.4. All of these values showed no significant difference at the $p \leq 0.01$ level. The treatment occurring on two consecutive days had no effect on the problem-solving achievement of the subjects; these results indicate that further analysis of the data can be made.

Observer's Checklist

The observer checklist was used to ensure that relatively the same content was taught in all lessons and that the correct method (either convergent or divergent) was being used to present this content. The observers were the author and a

TABLE 4.3

MEANS AND STANDARD DEVIATIONS FOR
DAY EFFECTS ON DEPENDENT MEASURES

	DAY 1*			DAY 2**		
	Routine	Non-Routine	Total	Routine	Non-Routine	Total
Mean	12.23	31.61	43.84	12.00	31.62	43.69
Standard Deviation	3.41	7.65	8.98	4.60	9.49	13.18

*N = 31

**N = 35

TABLE 4.4

RESULTS OF *t*-TESTS FOR
DAY EFFECTS ON DEPENDENT MEASURES

Problem Set	*t*-value*
Routine	0.010
Non-Routine	0.849
Total	0.053

* d.f. = 64

Critical *t*-value ($p \leq 0.01$) = 2.390

fellow graduate student in mathematics education, both of whom had taught for more than three years in secondary mathematics classrooms and both of whom were in attendance during the training session of the teachers on the convergent and divergent teaching methods. The results of the checklist for all classes are presented in Table 4.5.

The first column of the checklist represents the content taught in the two lessons as presented to each student. While it would appear that only one class of the convergently taught subjects completed the entire content, this was really not the case. Due to unavoidable dalay, one of the observers could not attend the Group 2 lesson until the last twenty minutes. However, immediately after the testing, the author and Teacher A carefully went over both lessons and discussed what had happened in the Group 2 class. The bracketed twos (2) in Table 4.5 indicate what happened in the class according to Teacher A and were subsequently verified from the students' notes during the lesson.

In both classes taught convergently (Groups 1 and 2) all of the content was covered for both lessons. The reviews in both classes were teacher-directed. During the lessons the most frequently used technique was that of "little leading questions" (See Convergent Teaching Method, Chapter III) to obtain the desired content and algorithms from the students. The second most frequently used teaching technique was that of giving possible solutions to the students and

TABLE 4.5

OBSERVER'S CHECKLIST

CONTENT	Gave Choices for Answers	Little Leading Questions	Teacher Directed	Open Ended Ques.	Student Suggest List	How to Solve
	FIBONACCI CHECKLIST					
1. Number Trick		1 (2)	1 (2)	3	3 4	
2. How Number Trick Works	(2) 3 4	1 (2) 4	1	3 4	3 4	1 3 4
3. Change Machine	1	1 (2)	1 (2)	3	3 4	3 4
4. Dittoed Sheets		1	1 (2)	3 4		
5. Even/Odd Numbers	(2)	1 (2)	(2)	3	3 4	3 4
6. Multiples of Numbers	1 (2)	1	(2)		4	3 4
7. Sum of 1st N Numbers	1 (2) 3	1 (2)	1	3 4		
8. Review			1 (2)			3 4
	PASCAL CHECKLIST					
1. First 3 Rows	1 (2)		1 (2)	3 4	3 4	3 4
2. Next Row	1	(2) 3	1 (2)	3 4	3 4	3 4
3. Row Sums	(2)	1 (2)	1 (2)	4	3 4	3 4
4. "Right" Column Sums		1	1 (2)	4	4	
5. Sums of Shapes	2	1 2	1 2	3		
6. Right 3rd Column Sums		1 2	1 2	4	4	
7. Fibonacci Pascal Diagonal	1 2	1 2 4	1 2	4	4	1 2
8. Review			1 2	4		

1 and 2 Convergently taught 3 and 4 Divergently taught

having them decide which choice would be the best for the problems as presented in the content of the lesson. There were only two content items where the teachers in both convergent classes used techniques from the divergent method portion of the checklist. These occurred in the discussion of triangular-shaped- and diamond-shaped-sums in Pascal's Triangle portion of the lesson when attempting to determine these sums from some other entry in Pascal's Triangle. The divergent technique used was to discuss how a solution was arrived at emphasizing the similarity to other patterns in Pascal's Triangle. However, the solution was teacher-directed rather than student-suggested and "little leading questions" were given prior to the discussion of patterns. Consequently, while part of a divergent teaching method was used, it was employed from a convergent viewpoint. Since this departure in method could only effect the data by making significant results harder to obtain, it was not considered a major problem.

In both classes taught divergently (Groups 3 and 4) all of the content for the Fibonacci lesson was completed. However, in Pascal's Triangle lesson, Group 4 did not complete the sums of shapes of triangles nor diamond shapes and Group 3 did not complete the last three sections ("Right" version of Pascal's Triangle, 3rd column sums and adding the diagonal upwards on Pascal's Triangle to obtain the Fibonacci sequence). Group 3 also did not receive a review of Pascal's Triangle lesson. Failure to complete the content was due mainly to the fact that in both divergent classes the lessons started slowly. However, as students became involved in the problem-solving process, they made many suggestions over and above the proposed content in the Fibonacci Sequence (See discussion in Suggestions for Future Research, Chapter V).

The majority of the divergent teaching involved the teacher asking open-ended questions ("How might we do this?", "Does anyone else have an idea?", "How do you think I got that answer?", "Does that relate to anything we've done earlier?") and listing solutions and suggestions as proposed by the students (See Student's Notes, Groups 3 and 4, this Chapter).

There were three instances in each divergent class where the teacher used a convergent technique. These occurred when open-ended questions had been tried and no results were obtained and there was a long pause (anywhere from 25 to 75 seconds of silence) where the students were obviously puzzled as to which direction to proceed. In both classes the situation first occurred when the students were asked to decide how the teacher had been able to obtain anyone's number sums from only asking for the seventh number. The hints given in both classes suggested a choice of using algebra to assist in solving the problem and then in both classes $\underline{x}$ was chosen as the first unknown and then discussion ensued as to just what the $\underline{x}$ was to represent, from this point both classes proceeded covering more content than anticipated or planned for in the lesson.

In Group 3 (divergently taught) use of a convergent technique next occurred when the teacher suggested skipping numbers in the sequence to try and find the answer for the sum of the first n numbers in the Fibonacci Sequence and occurred again when the students found difficulty in finding the fourth row of Pascal's Triangle (they had many suggestions - 1 2 3 2 1; 2 3 3 2; 1 1 3 1 1) and the teacher suggested a choice of 1 and 4 for the first two numbers at which point the students completed the row and then began to diverge into the elevens times tables prior to going back and developing more rows in Pascal's Triangle.

In Group 4 (divergently taught), the teacher asked a "little leading question" (convergent method) in aiding the students in finding out how the number trick worked in the Fibonacci lesson. He asked, "What does the final sum have to do with the seventh number?" He did this only after asking several open-ended questions such as: "How do we generalize this?", "Can you think of a way to get the final sum?". He also used a "little leading question" when trying to get at the relationship between Pascal's Triangle and the Fibonacci Sequence along the diagonals by commenting, "Oh, I see a goody! Look at adding in the rows of the right triangle version." This led to the students suggesting adding upwards and downwards on the diagonals to both the right and the left as well as columns and L-shaped patterns.

While it was unfortunate that some discrepancies from the divergent teaching occurred, they were done in such a way that they acted as a catalyst for students to bring forth more ideas regarding possible solutions (ideational fluency and flexibility) rather than converging on one solution. It was felt that these discrepancies did not significantly effect the overall divergent teaching method.

Student's Notes

At the beginning of each lesson all subjects were given four pages of blank computer print-out paper. They were instructed to place their class block on the first page (and their name, if they wished to do so). The purpose of this was two-fold: to facilitate pattern searches and involvement in the actual lessons and to provide a check for the author to ensure that subjects were actively involved in the lesson and that the content of the lesson was being adequately perceived by the subjects. These sheets were collected by the observers at the end of Pascal's Triangle lesson and prior to testing on the dependent measures so that any algorithms derived in the lesson (either by the class or by the individual subject) were unavailable during the testing session.

In Group 1 (convergently taught), all subjects handed in completed notes on both the Fibonacci Sequence and Pascal's Triangle. Each subject's notes consisted of the following information:

1) Ten numbers in a column created by adding the first two to get the third, second and third added to get the fourth and so on until the tenth number plus the final sum. (See Appendix E for examples of student's actual notes).

2) Ten algebraic expressions obtained in a similar manner to the above beginning with a and b.

3) A table of cheque values, possible pay offs and count the number of ways to pay off from the change machine with values from $0.00 to $5.00.

4) A listing of the Fibonacci Sequence and their sums using subscripted notation.

5) The first four rows of Pascal's Triangle.

6) The sum of these first four rows together with the powers of two in side by side columns.

7) A "right" triangle version of Pascal's Traingle. This had an arrow on the third column plus upward diagonal sums showing the relation to the Fibonacci Sequence.

8) A regular version of Pascal's Triangle with both triangle shapes and diamond shapes outlined with arrows pointing to the sums which were entries lower down in Pascal's Triangle.

There was very little evidence of extra guesses or doodles on these notes and, except for page placement of the sketches and a few papers which did not have the diamond or else the triangle sums marked, there was almost identical notes taken by each subject. This would indicate that there was a high level of involvement with the lessons and that subjects were following the directions given by the teacher very explicitly.

In Group 2 (convergently taught), thirteen of the eighteen subjects had a complete set of notes on both lesson areas; two subjects completed only Pascal's Triangle portion of the lesson; one subject handed in a still blank computer print-out sheet.

Each subject's notes consisted of the same information as outlined for Group 1 with the following additions or deletions:

1) Same.

2) Same, except instead of a and b, x and y were used.

3) Same, except a sketch of the change machine was included.

4) No subscripted notation, instead comments about odds and evens and divisibility rules for the Fibonacci Sequence.

5) Same.

6) Same.

7) Same, plus the fact that the sum in the columns can be obtained by going one over and one down from the last number in the column which was being added in the sum.

8) Same.

Again, there was no evidence of extra guesses or doodles on these notes. Some students also included notes regarding the third column adjacent pair sums being related to square numbers (4, 9, 16, 25, ...). The consistency among subject's notes was high.

All sixteen subjects in Group 3 (divergently taught) completed the notes for both lessons on the computer print-out sheets provided. One student engaged in some doodling making a mushroom and outlining some of his conjectures and conclusions. The note-taking of this group varied far more than that of the previous two groups. Many students made extra conjectures which were not listed on the board by the teacher, however, most of these were fallacious in nature.

In general each subject's notes consisted of information similar to that of the previous groups; however, there were some notable exceptions due to the divergent teaching method as indicated below:

1) Same.

2) A list of ideas of how the sum might be obtained as per (1) plus many other patterns or notions:
   a) take the 7th number, add zero to the end of it and then add the 7th number to that number.
   b) multiply the 7th number by eleven.
   c) try all pairs
   d) prime number pairs and then generalize
   e) use a variable to test guess
   f) let x be 7th (no) answer (no) first number
   g) 1st plus second equals third
   h) 10 - x is the other number
   i) use another variable a

   At this point students came up with (2) as did the two previous groups, however without assistance, the teacher just acted as a recorder. Letters used were x and a.

3) Most omitted change machine entirely from their notes, those who put it in just listed 1, 2, 3, 5 and noted that it was the same pattern as before. The teacher was a recorder of student guesses on the board and 8 did come up in the class session as the number of ways to make change for $5.00.

4) Here the dittoed sheets were used exclusively and the following conjectures and relations were noted by the students in their notes:
   a) to get number you would add the two before
   b) to get the 3rd add the 1st and 2nd
   c) in number of terms column, adding 2 consecutive numbers gives you the odd numbers
   d) every 5th number is divisible by 5
   e) every 4th number is divisible by 3
   f) every 6th number is divisible by 8

g) every 7th number is divisible by 13
h) if numbers are a, b, c then c - a = b
i) a + b - d = -b
j) b - c = -a

5) Same, except additional guesses regarding the 4th row occurred: 131, 232, 12321, 233, 2332, 11311, and finally 14641.

6) Did not occur.

7) Only the right triangle version of Pascal's Triangle occurred.

8) Using both versions of Pascal's Triangle, conjectures were made about summing:
a) top number of triangle should be 1.
b) every number in the 7th row is divisible by 7 except the ones.
c) third row you add one more
d) sum of the first 6 rows equals the 7th row
e) add diagonally down go one to the left diagonally and will get the sum
f) add pairs adjacent and go down one number to get the sum (in the right version)
g) add the right angles works (this is the same as the triangle pattern in the convergent groups)
adding squares works

Far more ideas (between 10 and 12) were generated during the divergent lessons (Groups 3 and 4) than in the convergent lesson which used only ideas posed by the teacher and did not have student suggestions occurring. In Group 3 most of the content of both lessons was completed, even if not being explicitly done by the teacher. The student involvement in the lesson was high and there were many student-student interactions taking place, particularly during the times when sections (2), (4), and (8) of the student's notes (as listed above) were occurring. Even though not requested to do so, most students did take rather thorough notes of the lists that the teacher was recording on the board.

Four of the seventeen subjects in Group 4 (divergently taught) completed notes on the Fibonacci Sequence only. Of these four subjects, there was one who obviously played tic-tac-toe with himself during the latter part of the lesson. The other thirteen subjects were fairly consistent in their note-taking. Group 4 subjects did not display as many extra conjectures as did the subjects in Group 3, but they did tend to record conjectures that arose in class and were recorded by the teacher. In general, each subject's notes consisted of the following information:

1) The same.

2) Conjectures were first made as to how the 7th number related to the total sum:
   a) first and last numbers same then the middle number is the first two added together.
   b) number is the first two added together
   c) 7th number times 3rd equals the sum (vetoed by a vote of the class)
   d) 7th times ten then add itself
   e) eleven times the 7th number

   And then after deciding to use an "open symbol" the algebraic sums were derived as in Group 3.

3) The coin machine problem was done using amount, ways and number of ways, although the order of these was different for different students.

4) The dittoed sheet was used, no subscripted notation was recorded by students; the following facts were noted:
   a) odd-odd-even
   b) every 1st term is a multiple of one
   c) every 3rd term is even
   d) every 3rd term is divisible by 2
   e) every 4th term is divisible by 3
   f) every 5th term is divisible by 5
   g) every 6th term is divisible by 4
   h) every 6th term is divisible by 8
   i) every 7th term is divisible by 13
   j) every 8th by 21

5) Same.

6) Same, except student-suggested and not teacher-directed.

7) Same, in addition the students also discovered sums in downward diagonals if you work from the edge of the right triangle version of Pascal's Triangle then the sum is the number just below the last number you add along the diagonal.

8) Same, students found the triangles and diamonds, where the diamonds in the right version gave the odd numbers as sums if you started from the top; the triangle shape was disguised as the L-shape (as in Group 3). Students tried to put columns of zeroes and negatives on the left side of the right triangle version, but things began to get too far afield for the majority of the class and time ran out.

In examining each group of student's notes (as presented by 1 - 8 for Groups 1 - 4 above) it is apparent that all groups completed approximately the same materials. It would also seem that the convergent classes (Groups 1 and 2) were taught in the same manner as their notes were consistent. It appears that the divergent classes (Groups 3 and 4) were taught in a similar manner, because their notes were relatively consistent, but not to the high degree of the two convergent classes. There were, however, some notable differences between the notes of the two convergent classes and the two divergent classes. These differences are particularly evident in items (2), (4), and (8) of the subject's notes (as listed above), where the divergently taught subjects listed choices and the convergently taught subjects listed only the choices given to them by the teacher involved. Based on the observations and the student's notes, it appears that the subjects were taught (and perceived the teaching) in the manner to which they were assigned.

In this study there was a possibility that a teacher could have changed methods during the lesson. This was controlled for by having an observer with a checklist watching the method employed for each content item. The analysis of the checklists revealed that both the convergent and divergent lessons were appropriately taught. The content of the students' notes indicated that lessons had essentially covered the same material. The respective style of students' notes was also consistent with the teaching method (convergent or divergent). The nature of these notes revealed that the divergent lessons were taught very differently from the convergent lessons (as they were supposed to be). There was also the possibility that one teacher would do better than the other. Analyses of the dependent measures showed no significant differences ($p \leq 0.01$) between teachers. There was the possibility that on one day subjects would do better than on the other day. Analyses of the dependent measures showed no significant differences ($p \leq 0.01$) between days. On the basis of these findings it would appear that the teachers adhered to the method and content of the lessons on both days, and that the teaching method (convergent or divergent) was clearly distinguishable.

Results of the Study

The regression analysis as used in this study is based on techniques described by Kerlinger and Pedhazur (1973).

Regression analysis was chosen as it enabled the use of both continuous and dichotomous variables as independent measures. This was particularly important in this study as the Watson-Glaser and the Torrance tests represented continuous variables, while the teaching method (convergent/divergent) and the instructor (Teacher A/Teacher B) represented dichotomous variables.

A stepwise regression was chosen so that variables which may have been good predictors in the early stages of the regression could be removed if they were no longer useful in the regression equation. The stepwise regression analysis was performed using Stepwise Regression (BMD O2R) (Jason Halm, 1974) as adapted from BMD (UCLA) (Dixon, 1970) which is available at the Computing Centre of the University of British Columbia.

Both the Watson-Glaser and Torrance tests were used as covariates in the regression (See discussion at the end of Chapter III). Since the correlation between the two tests was shown to be 0.04, the two tests were treated as two variables independent of each other. Analysis of covariance was used since it was not possible to block the convergent/divergent subjects into classes on an equal basis. Using the scores on the Watson-Glaser test (convergent and the Torrance test (divergent) as covariates allowed all the groups to be equated regarding these two variables.

Analysis of covariance was also used to improve the sensitivity of the analysis (Kerlinger and Pedhazur, 1973,

p. 266). The assumptions underlying the use of analysis of covariance were met as academic mathematics students had been randomly assigned to their particular classes within the school (within the constraints of the timetable by the school administration) and the classes used were randomly selected from the academic mathematics classes and were randomly assigned to treatment. It was further assumed that within each treatment group the residuals were independently and normally distributed with a mean of zero and homogeneous variance. A linear regression of the dependent measures on the independent measures was assumed with homogeneous regression coefficients. The treatment did not have an effect on the covariates (Watson-Glaser test and Torrance test), as these data were collected prior to treatment. If the regression coefficients are heterogeneous, the *F*-test performed in the regression analysis would be conservative (for further discussion, see Meyers, 1973, p. 327).

The means and standard deviations for each group on both the Watson-Glaser and Torrance tests is given in Table 4.6. A *t*-test using pooled sums was employed to test for significant differences between the means of each group. While there were no significant differences on the Watson-Glaser test between any of the groups, there were significant differences between Group 3 (divergently taught) and Group 1 (convergently taught) and between Group 3 and Group 4 (divergently taught) on the Torrance test with the mean of Group 3 being significantly

TABLE 4.6

MEANS AND STANDARD DEVIATIONS FOR
WATSON-GLASER AND TORRANCE TESTS

	Group 1	Group 2	Group 3	Group 4
<u>Watson-Glaser</u>				
N	15	18	16	17
Mean	63.87	63.17	59.06	61.59
Standard Deviation	11.31	9.45	9.31	8.66
<u>Torrance</u>				
N	15	18	16	17
Mean	171.13	202.11	231.25	174.71
Standard Deviation	52.37	74.47	56.31	64.28

TABLE 4.7

GROUP COMPARISON OF MEANS ON
WATSON-GLASER AND TORRANCE TESTS

	t-value
Watson-Glaser	
Group 1 vs Group 2	0.189
Group 1 vs Group 3	1.252
Group 1 vs Group 4	0.624
Group 2 vs Group 3	1.235
Group 2 vs Group 4	0.499
Group 3 vs Group 4	0.783
Torrance	
Group 1 vs Group 2	1.314
Group 1 vs Group 3	2.970**
Group 1 vs Group 4	0.166
Group 2 vs Group 3	1.236
Group 2 vs Group 4	1.129
Group 3 vs Group 4	2.599**

** Significant at the $p \leq 0.01$ level.

higher than either the mean of Group 1 or that of Group 4 (See Table 4.7). This finding suggests that there were differences between groups on the measure of divergent thinking. As was mentioned earlier (See discussion at the end of Chapter III) it was originally planned to use an analysis of variance in the regression analysis. However, the differences between groups on the Torrance test made it necessary to use the Torrance test results as a covariate. While significant differences were not noted between groups on the Watson-Glaser test, it was felt, for the reasons stated in the earlier discussion (See end of Chapter III) that the Watson-Glaser test would also be used as a covariate.

As this study is looking for relationships among teaching method, thinking style and problem solving, another advantage of using the regression analysis is the multiple *R* obtained. "R^2 indicates the portion of the total variance of the dependent variable that the independent variable accounts for..." (Kerlinger and Pedhazur, 1973, p. 98). Thus, within the analysis, information should be gained regarding the contributions of both teaching method and thinking style to the total variance on both routine and non-routine problems.

The model for the regression analysis used assumed that each of the dependent measures (routine, non-routine and total problem set) were a linear combination of the following: the covariates (Watson-Glaser/Torrance), the dichotomous variables (method - convergent/divergent and instructor - Teacher A/

Teacher B), the interactions (Method x Instructor, Watson x Method, Watson x Instructor, Torrance x Method, Torrance x Instructor, Watson x Method x Instructor, and Torrance x Method x Instructor), and the error terms.

Means and standard deviations are provided for both the independent and dependent variables in Table 4.8. Table 4.9 contains the correlation matrix for the independent and dependent variables.

Since the subjects in the present study were in the last quarter of grade 10 and there was no indication when in the grade 10 year the normative data were collected for the Watson-Glaser test, the grade 11 normative data were used. Though there is not a significant difference between the grade 10 and grade 11 normative data for the Watson-Glaser test (Grade 11: $\overline{X} = 64.4$, $s = 11.0$; Grade 10: $\overline{X} = 61.9$, $s = 11.0$) it was concluded that the more conservative approach was appropriate. Since there is no significant difference between the two means, the interpretation of the results of the present study is not affected.

The means and standard deviations on the Torrance test were likewise conservatively chosen (Fluency: $\overline{X} = 94.6$, $s = 32.5$; Flexibility: $\overline{X} = 40.2$, $s = 9.0$; Originality: $\overline{X} = 45.2$, $s = 23.2$) giving a total mean of 180.0 and a total standard deviation* of approximately 65. The mean and standard deviation found

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* See discussion, Divergent Thinker, Chapter III.

TABLE 4.8

MEANS AND STANDARD DEVIATIONS
OF ALL VARIABLES

VARIABLE	MEAN	STANDARD DEVIATION
Dependent Measures		
(3) Routine Problems	12.11	4.12
(4) Non-Routine Problems	31.65	8.78
(5) Total Problem Set	43.76	11.49
Independent Measures		
Covariates		
(1) Watson-Glaser	61.89	9.92
(2) Torrance	195.12	67.73

TABLE 4.9

CORRELATION MATRIX FOR
DEPENDENT AND INDEPENDENT VARIABLES*

		2	3	4	5	6	7
(1)	Watson-Glaser (convergent)	.04	.44	.41	.47	-.16	-.08
(2)	Torrance (divergent)		-.14	.06	-.01	.10	.32
(3)	Routine Problems			.53	.76	-.25	.05
(4)	Non-Routine Problems				.95	-.09	.19
(5)	Total Problem Set					-.16	.16
(6)	Method						-.06
(7)	Instructor						

* N = 66

in this study ($\bar{X}$ = 195.12 and s = 67.74) were not significantly different ($p \leq 0.01$) from this conservative estimate from the Torrance normative data, and should therefore not affect the results of the study.

While an analysis of covariance became necessary, the analysis of variance had already been performed. Therefore, both analyses are presented in the study for completeness. The results of the regression analyses of variance are presented in Tables 4.10-4.12. The results of the regression analyses of covariance are presented in Tables 4.13-4.15. These tables will be discussed in the subsequent analysis of the statistical hypotheses.

<u>Statistical Hypothesis 1a</u>

> Subjects convergently taught will score significantly more correct answers on routine problems than subjects divergently taught.

When using the analysis of variance regression, the contribution made to the variance of the routine problems by the convergent teaching method was significant at the $p \leq 0.05$ level of significance. While the analysis indicated that there were significant differences, a further post hoc analysis was done to determine the directionality. The post hoc analysis revealed that convergent teaching was superior to divergent teaching on routine problems, which supported the hypothesis. However, after adjusting for differences due to convergent/divergent thinking within classes, no significant effect for method was found ($p \leq 0.05$). In light of this further analysis, the hupothesis was not supported. (Refer to Tables 4.10 and 4.13).

Statistical Hypothesis 1b

> Subjects divergently taught will score significantly more correct answers on non-routine problems than subjects convergently taught.

The hypothesis was not supported ($p \leq 0.05$) by either the analysis of variance or covariance within the regression analysis (Refer to Tables 4.11 and 4.14).

Statistical Hypothesis 1c

> Subjects divergently taught will score significantly more correct answers on the total problem set than subjects convergently taught.

This hypothesis was also unsupported by either the analysis of variance or covariance within the regression analysis ($p \leq 0.05$) (Refer to Tables 4.12 and 4.15).

Statistical Hypothesis 2a

> Convergent thinkers will score significantly more correct answers on routine problems than will divergent thinkers.

Using the regression analysis of covariance this hypothesis was supported at the $p \leq 0.01$ level. The Watson-Glaser test accounted for approximately 19 percent of the total variance of the routine problems. This was the highest single variable contributor to the variance of the dependent measure. The post hoc analysis verified this result. (Refer to Table 4.13).

Statistical Hypothesis 2b

> Divergent thinkers will score significantly more correct answers on non-routine problems than will convergent thinkers.

This hypothesis was not supported in the analysis of covariance. However, again, the Watson-Glaser was the single highest contributor to the variance on the non-routine problems as well as on the routine; accounting for approximately 17 percent of the total variance attributed to the non-routine problems. This is a statistically significant result. A further post hoc analysis confirmed that convergent thinkers did better than did divergent thinkers on the non-routine problems ($p \leq 0.05$) (Refer to Table 4.14).

Statistical Hypothesis 2c

> Divergent thinkers will score significantly more correct answers on the total problem set than will convergent thinkers.

The analysis of covariance indicated that this was not the case. Precisely the opposite occurred with the Watson-Glaser contributing approximately 22 percent of the total variance attributed to the total problem set. This finding was again verified by post hoc analysis and found to be significant at the p 0.01 level in favour of convergent thinkers over divergent thinkers (Refer to Table 4.15).

Statistical Hypotheses 3

a) Convergent thinkers taught convergently will score significantly more correct answers on routine problems than divergent thinkers taught divergently.

b) Divergent thinkers taught divergently will score significantly more correct answers on non-routine problems than convergent thinkers taught convergently.

c) Divergent thinkers taught divergently will score significantly more correct answers on the total problem set than convergent thinkers taught convergently.

As the contributions made to the variance by the interaction variables (Watson-Glaser x Method and Torrance x Method) ranged from a high of one percent to a low of three hundredths of a percent, it was felt that separate analyses were not indicated.

Hypothesis 3 was not supported by the analysis of variance or covariance (Refer to the interaction portion of Tables 4.10-4.15). There were other interesting relationships with the covariance regression analysis. However, as these did not bear on the statistical hypotheses of this study, these will be discussed in Chapter V (See Findings - 3).

TABLE 4.10

RESULTS OF THE REGRESSION ANALYSIS WITH ROUTINE PROBLEMS AS THE DEPENDENT VARIABLE (N = 66)

DEPENDENT VARIABLE	SOURCE OF VARIATION	F-VALUE TO ENTER/REMOVE	F-OBS.[2]	R WITH DEPENDENT VARIABLE	Δ RSQ[3]
Routine Problems	Instructor	0.1443	0.1574	0.4740	0.0022
	Method	4.0672	4.3281	0.2505	0.0605*
	Method x Ins.	0.0812	0.0858	0.2529	0.0012
	Torrance x Ins.	7.7365	7.5401	0.4115	0.1054**
	Watson x Ins.	0.3240	0.3219	0.4169	0.0045
	Torrance x Met.	1.0855	1.0659	0.4344	0.0149
	Watson x Met.	0.9998	0.9801	0.4500	0.0137
	Wat x Met x Ins	0.8932	0.8799	0.4634	0.0123
	Tor x Met x Ins	0.1731	0.1769	0.4660	0.0024

[1] This represents a step-wise regression using analysis of variance.

[2] This is the *F*-value used to calculate statistical significance.

[3] The change in the square of the multiple correlation represents the proportion of the variance in the dependent variable accounted for by the independent variable.

TABLE 4.11

RESULTS OF THE REGRESSION[1] ANALYSIS WITH NON-ROUTINE PROBLEMS AS THE DEPENDENT VARIABLE (N = 66)

DEPENDENT VARIABLE	SOURCE OF VARIATION	F-VALUE TO ENTER/REMOVE	F-OBS.[2]	R WITH DEPENDENT VARIABLE	Δ RSQ[3]
Non-Routine Problems	Instructor	2.3273	2.4035	0.1873	0.0351
	Method	0.3941	0.4109	0.2027	0.0060
	Method x Ins.	0.0003	0.0000	0.2027	0.0000
	Torrance x Ins.	5.9204	5.8068	0.3549	0.0848
	Watson x Ins.	1.6036	1.5613	0.3856	0.0228
	Torrance x Met.	0.0562	0.0548	0.3905	0.0008
	Watson x Met.	0.2091	0.2054	0.3895	0.0030
	Tor x Met x Ins	1.4190	1.4106	0.4160	0.0206
	Wat x Met x Ins	0.6223	0.6231	0.4268	0.0091

[1] This represents a step-wise regression using analysis of variance.

[2] This is the <u>F</u>-value used to calculate statistical significance.

[3] The change in the square of the multiple correlation represents the proportion of the variance in the dependent variable accounted for by the independent variable.

TABLE 4.12

RESULTS OF THE REGRESSION[1] ANALYSIS WITH TOTAL PROBLEM SET AS THE DEPENDENT VARIABLE (N = 66)

DEPENDENT VARIABLE	SOURCE OF VARIATION	F-VALUE TO ENTER/REMOVE	F-OBS.[2]	R WITH DEPENDENT VARIABLE	ΔRSQ[3]
Total Problem Set	Instructor	1.6846	1.8112	0.1601	0.0256
	Method	1.4355	1.5353	0.2176	0.0217
	Method x Ins.	0.0077	0.0071	0.2179	0.0001
	Torrance x Ins.	8.3630	8.1223	0.4029	0.1148**
	Watson x Ins.	1.4208	1.3725	0.4263	0.0194
	Torrance x Met.	0.0424	0.0425	0.4346	0.0006
	Watson x Met.	0.4791	0.4670	0.4339	0.0066
	Tor x Met x Ins	1.3067	1.2877	0.4550	0.0182
	Wat x Met x Ins	0.0989	0.0991	0.4566	0.0014

1 This represents a step-wise regression using analysis of variance.

2 This is the F-value used to calculate statistical significance.

3 The change in the square of the multiple correlation represents the proportion of the variance in the dependent variable accounted for by the independent variable.

TABLE 4.13

RESULTS OF THE REGRESSION[1] ANALYSIS WITH ROUTINE PROBLEMS AS THE DEPENDENT VARIABLE (N = 66)

DEPENDENT VARIABLE	SOURCE OF VARIATION	F-VALUE TO ENTER/REMOVE	F-OBS.[2]	R WITH DEPENDENT VARIABLE	Δ RSQ[3]
Routine Problems	Watson	15.0702	15.6111	0.4366	0.1906**
	Torrance	1.9898	2.0312	0.4641	0.0248
	Instructor	1.5516	1.5726	0.4843	0.0192
	Method	1.8782	1.8756	0.5073	0.0229
	Met x Ins	0.4156	0.4177	0.5124	0.0051
	Tor x Ins	2.7132	2.6357	0.5431	0.0324
	Wat x Ins	1.1501	1.1221	0.5556	0.0137
	Tor x Met	0.9554	0.9337	0.5657	0.0114
	Wat x Met	0.0321	0.0328	0.5661	0.0004
	Tor x Met x Ins	1.6896	1.6627	0.5837	0.0203
	Wat x Met x Ins	0.0003	0.0000	0.5837	0.0000

[1] This represents a step-wise regression using analysis of covariance with the scores of the Watson and Torrance as the covariates.

[2] This is the F-value used to calculate statistical significance.

[3] The change in the square of the multiple correlation represents the proportion of the variance in the dependent variable accounted for by the independent variable.

TABLE 4.14

RESULTS OF THE REGRESSION[1] ANALYSIS WITH NON-ROUTINE PROBLEMS AS THE DEPENDENT VARIABLE (N = 66)

DEPENDENT VARIABLE	SOURCE OF VARIATION	F-VALUE TO ENTER/REMOVE	F-OBS.[2]	R WITH DEPENDENT VARIABLE	ΔRSQ[3]
Non-Routine Problems	Watson	13.0781	14.5827	0.4119	0.1697**
	Torrance	0.1318	0.1461	0.4140	0.0017
	Instructor	3.7359	4.0474	0.4674	0.0471*
	Method	0.0017	0.0000	0.4675	0.0000
	Met x Ins	0.0296	0.0344	0.4679	0.0004
	Tor x Ins	1.6175	1.7444	0.5219	0.0203
	Wat x Ins	2.6161	2.8530	0.5021	0.0332
	Tor x Met	0.1154	0.1289	0.5233	0.0015
	Wat x Met	0.0665	0.0773	0.5241	0.0009
	Tor x Met x Ins	5.1597	5.3450	0.5804	0.0622*
	Wat x Met x Ins	2.9805	2.9836	0.6096	0.0347

1 This represents a step-wise regression using analysis of covariance with the scores of the Watson and Torrance as the covariates.

2 This is the $\underline{F}$-value used to calculate statistical significance.

3 The change in the square of the multiple correlation represents the proportion of the variance in the dependent variable accounted for by the independent variable.

TABLE 4.15

RESULTS OF THE REGRESSION[1] ANALYSIS WITH TOTAL PROBLEM SET AS THE DEPENDENT VARIABLE (N = 66)

DEPENDENT VARIABLE	SOURCE OF VARIATION	F-VALUE TO ENTER/REMOVE	F-OBS.[2]	R WITH DEPENDENT VARIABLE	Δ RSQ[3]
Total Problem Set	Watson	18.2674	20.5661	0.4712	0.2220**
	Torrance	0.0490	0.0556	0.4719	0.0006
	Instructor	3.9365	4.2985	0.5187	0.0464*
	Method	0.2793	0.3057	0.5219	0.0033
	Met x Ins	0.1363	0.1482	0.5235	0.0016
	Tor x Ins	3.2208	3.4832	0.5582	0.0376
	Wat x Ins	2.3088	2.4457	0.5814	0.0264
	Tor x Met	0.0050	0.0093	0.5816	0.0001
	Wat x Met	0.0222	0.0278	0.5816	0.0003
	Tor x Met x Ins	5.3210	5.4102	0.6298	0.0584*
	Wat x Met x Ins	1.8930	1.8900	0.6458	0.0204

[1] This represents the step-wise regression using analysis of covariance with the scores of the Watson and Torrance as the covariates.

[2] This is the *F*-value used to calculate statistical significance.

[3] The change in the square of the multiple correlation represents the proportion of the variance in the dependent variable accounted for by the independent variable.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The purpose of this study was to investigate the effect of two teaching methods (convergent/divergent on student performance on two problem-solving tasks (routine/non-routine problems). Would one method be better for both kinds of problems? Would one method be better on one type and the other method be better on the other type? The study also investigated the relationships between the convergent/divergent teaching methods and the student's thinking style (convergent/divergent). Would students taught by a method similar to their thinking style be better than those who were taught by a method dissimilar to their thinking style? Would there be an interaction among thinking style, teaching method, and problem type?

Summary of the Study

Four grade ten classes were randomly selected from the eleven academic mathematics classes in a secondary school involved in the study. Due to subject absenteeism for either the pre-tests and/or the treatment, a total of sixty-six subjects were involved in the study. Each subject was given the Watson-Glaser Test of Critical Thinking (Form YM) and the Torrance test of Thinking Creatively with Words (Booklet A) to determine their level on the two independent measures of convergent and divergent thinking. A Latin-design was used to assign classes to treatment (convergent/

divergent teaching) and instructor (Teacher A/Teacher B). Each subject was taught by one teacher using one method for approximately two hours. The content for the lessons involved the Fibonácci Sequence and Pascal's Triangle and was taken from Jacobs' (1970) Mathematics: A Human Endeavor. At the end of the treatment each subject received a test on the dependent measures (routine/non-routine problems). Trained observers were used to ensure consistency of teaching method. Analysis of covariance using the regression model was performed with convergent/divergent thinking styles as the covariates.

Findings

1. Convergent teaching was superior to divergent teaching on routine problems only. When the variance due to thinking style was removed, this finding was no longer significant. The superiority of the convergent teaching method was in fact due to differences between subjects on the convergent and divergent thinking measures. This finding questions studies that suggest that directed teaching is superior to non-directed teaching for short term effects (Dahmus, 1970; Campbell, 1964). This result may, however, be a socio-cultural phenomena in that current teaching practices tend to be largely convergent and directive in nature.

2. Convergent thinkers were found to score significantly more correct answers on all the dependent measures than did divergent thinkers. While there is some evidence that

convergent thinking is highly correlated with intelligence (Furst, 1950), the Watson-Glaser test was only moderately correlated with intelligence (0.55-0.73) (See convergent/divergent thinking, Chapter II, for further discussion). The correlation of the dependent measures used in this study with intelligence is unknown. Therefore it is unknown to what extent intelligence may have influenced scores on the dependent measures.

3. Only one out of the eight interaction effects tested was statistically significant. This interaction was only found for routine problems and suggested that non-divergent thinkers did better with convergent teaching as opposed to divergent teaching, while non-convergent thinkers did better with divergent teaching as opposed to convergent teaching. Unfortunately, the n was so limited as to make a generalization of this finding questionable. It is interesting to note that a similar finding was not found when the definition of groups was divergent as opposed to non-convergent, which supports the division of these two groups (See definition of non-convergent in Chapter III). Only this one finding supports the theoretical hypothesis of this study that an interaction effect would be found between teaching method and thinking style. The fact that none of the other findings support the theoretical hypothesis gives further indication that intelligence may have been a confounding factor, i.e. those who scored low on the Watson-

Glaser and/or Torrance tests were of lower intelligence compared to those who had high scores on these measures.

4. When the significant differences for method disappeared on the routine problems due to changing from an analysis of variance to an analysis of covariance within the regression model, a significant difference for the Watson-Glaser (convergent thinking) appeared. This would seem to indicate that it is the thinking style rather than the convergent teaching method which has the greater effect on problem solving. It should be noted that this significant effect for convergent thinking style was found for all the dependent measures.

5. The analysis of variance showed a significant effect for the Torrance x Instructor interaction. Upon further analysis, using covariance, this effect disappeared entirely on the routine problems and two other significant effects were found (Instructor and Torrance x Method x Instructor) in both the non-routine problems and the total problem set.

The teacher effect must be qualified by the fact that the Torrance x Method x Instructor effect also occurred. When the post hoc analyses of the comparison between group means were done (as presented in Table 4.7) one of the groups taught divergently by Teacher A (Group 3) scored significantly ($p \leq 0.01$) differently from the two groups taught by Teacher B (Groups 1 and 4). This may have influenced the results and this fact has a limiting effect on the generalizability of this study.

6. In analyzing the correlation matrix, there was a positive correlation between the Watson-Glaser test and the routine problems (0.4366) and a positive correlation between the Watson-Glaser test and the non-routine problems (0.4119). This would indicate the the convergent portions of both dependent measures was adequate. However, there was a slight negative correlation between the Torrance test and the routine problems (-0.1404) and a slight positive correlation between the Torrance test and the non-routine problems (0.0576). This seems to indicate that some revision is needed in the divergent portions on both dependent measures. The correlation of the Watson-Glaser test with both problem sets may have influenced the results of the study in favour of the convergent thinkers, which may have accounted for the lack of significant results for the divergent thinkers.

Implications of the Study

The results of this study suggest that in a short term situation, academic grade ten students will likely benefit most from being taught in a convergent manner. Students who are convergent thinkers will benefit most from this type of instruction.

The results of this study also indicate that convergent thinkers, in a short term situation, do better than divergent thinkers on a mathematical problem-solving task. However, this result may be confounded by intelligence.

Limitations of the Study

1. The study involved students from academic grade ten mathematics classes and therefore generalizations should be limited to groups from a similar population.

2. Generalizability of the findings is limited by the fact that one teacher seemed to get better results than the other teacher involved and that teacher also interacted with the Torrance test and divergent teaching method.

3. The results of the study should only be generalized for short term effects.

4. Work is needed on the dependent measures to increase their correlation with the Torrance test and thus bring their measures of divergent problem-solving abilities in line with their measures of convergent problem-solving abilities.

5. It should be noted that due to the time constraints of this study, convergent thinking and convergent teaching may have been given unfair advantage. Convergent lessons completed more of the specified material and had the advantage of reviewing with the students the important points of the lesson.

Recommendations for Further Research

If the quality of mathematics teaching is to improve, it is essential that research involving problem solving be implemented in such a way as to maintain the complexity of the problem-solving process. This will ultimately involve further investigation and understanding of the trait-treatment interaction of thinking style and teaching method as a minimal starting point.

Further research is needed to specify and clarify the critical variables involved in both thinking style and teaching method. While the results of the current study suggest that a convergent thinking style and a convergent teaching method facilitate problem solving, it is important to investigate possible confounding variables.

It is recommended that any further research in the area of convergent/divergent thinking style as related to mathematical problem solving be designed to partial out the variance attributable to intelligence. This is particularly important in the light of the significant interaction between non-divergent and non-convergent thinkers and the convergent/divergent teaching methods.

It is recommended that any further research utilizing lessons similar to those used in the study incorporate some of the content from the Fibonacci Sequence as found by the divergently taught subjects (See Observer's Checklist, Groups 3 and 4, Chapter IV).

An increase in the time factor would probably improve the effectiveness of the divergent teaching method. It might also show that there are differential effects over time between teaching methods and thinking styles.

As Taylor (1965) suggests, perhaps both convergent and divergent teaching methods need to be used in conjunction with one another to truly improve the quality of problem solving in students. One further variation of this study might be to use three teaching methods: convergent, divergent, and a combination method of the two over a long period of time and study the effects that each of these methods have on the problem-solving abilities of students in mathematics.

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APPENDIX A

CONVERGENT TEACHING LESSON

FIBONACCI NUMBERS

I. Introduction
Observer introduce the teacher by name, as well as self. Observer take seat at back. Observer is there to note content covered and consistency with convergent teaching.

II. Lesson
1. Pass out a piece of paper to each student.
2. Have each student write the numbers 1-10 in a column along the left hand margin.
3. Have each student choose two numbers between 1 and 10.
4. Have them write their first choice by the (1) in their column and the second choice by the (2).
5. The third number is the sum of the first and second. The fourth is the sum of the second and third.
6. Have students fill in all ten numbers in a similar way.
7. Have students find the sum of the ten numbers.
8. Steps 3-7 should be done by teacher at the board along with the students to ensure that they understand the procedure. Teacher should determine the sum of his own ten numbers by multiplying the seventh number by 11. Then should check with a few students for their 7th number.
9. Teacher tells the class the sum of the numbers for the students selected. Then asks another student for his seventh number and tells the class the sum.
10. "Notice, I am asking everyone for their seventh number before telling them the sum. You know that numbers may be related to each other through the arithmetic operations (Check to be sure that students know these are addition, subtraction, multiplication and division). Which of these operations relates the total sum to the seventh number?"
11. "We have found that this works for many cases. However, in mathematics we often want to find out if something works for any numbers chosen. We can use algebra to represent what we have done for the general case of any two numbers and then deduce whether this number trick does indeed work no matter what numbers we choose."
12. Let $\underline{a}$ represent the first number and $\underline{b}$ represent the second, then our ten numbers look like:

(1)	a
(2)	b
(3)	$a + b$
(4)	$a + 2b$
(5)	$2a + 3b$
(6)	$3a + 5b$
(7)	$5a + 8b$
(8)	$8a + 13b$
(9)	$13a + 21b$
(10)	$21a + 34b$
Sum	$55a + 88b$

(a) looking at the sum, is there any common factor between the two terms?

(b) recalling the distributive property $ab + ac = a(b+c)$
$11(5a) + 11(8b) = 55a + 88b$

(c) Will our trick work for any two numbers?

13. Consider the coefficients (the numbers in front)of the a's (Agreement: when no number appears in frong of a or b we agree the number is one.)
Can you best describe the pattern of coefficients as a product? a sum? or a difference?
What reasoning led you to this choice?
How do the doefficients of the a's bets relate to the coefficients fo the corresponding b's?(Start at the third or fourth term). Would you say that they are one ahead? One behind? the same?
What led you to believe your conclusion?

14. "A certain machine makes change for any whole number of dollars. However, it will only pay out in one dollar or two dollar bills. The change in bills may come out in any order, though. For instance, if change is wanted for three dollars, it may come out in any one of the following 3 ways: \$1 followed by another \$1 followed by another \$1; OR \$1 followed by a \$2; OR a \$2 followed by a \$1. If we agree to denote a \$1 bill by A and \$2 bill by B, we get AAA, AB, BA as the three possible ways of making change for three dollars.

15. On the back of your piece of paper, make the following headings:

Amount of Change to be paid	Possible ways of paying out	Numbers of ways of paying out

16. Next try a \$5 bill to change, what are the possible ways of paying out? What are the number of ways?

17. Suppose we had no money. The possible ways to pay out are? How many ways is this?

18. Fill in the table up to and including \$5 to be changed.

19. Look at the numbers in the third column. Do they look familar? Recall the numbers which were in front of the a's and b's in our trick problem. How did we get those numbers? Do you think that we get these numbers in the same way? What would you predict the number of different ways the machine could make change for \$10 would be?

20. Pass out dittoed sheets on Fibonacci Sequence. There are many kinds of things we might be able to do with this sequence. You may note how it is related to our number trick.

21. What about even and odd numbers in the sequence? Could you describe a pattern to tell someone else which terms are even and which terms are odd?

22. What about multiples of certain numbers?
2(evens)?
3?
(Work out together at the board)
Notice that multiples of 4 do not occur every 5th term.
Do multiples of 5 occur every 6th term.
Notice that multiples of the Fibonacci numbers do occur in a particular pattern.

23. Could we also sum terms (demonstrate what is meant by this)
What if we wanted to add the first ___ terms?
Should we try to figure out the general case first?
Or should we take specific cases and look for a pattern?
How should we choose these specific cases? Choose anywhere?
Start at the beginning? Start at the end? What gave you a hint as to shich cases to choose? (Recall the change machine).

24. Review what has been learned about Fibonacci Sequences.
Emphasize correct choices and conclusions.

(The following is an outline of notes as given to the teachers to use during the lesson on the Fibonacci Sequence.)

1.*
1) 5
2) 4
3) 9
4) 13
5) 22
6) 35
7) 57
8) 92
9) 149
10) 241

Sum 627

Trick
(11 x 57 = 627 = sum)

11 x 7th # = sum - do some

how is the sum related to the 7th number?

numbers can be related by by operations
add, sub., mult., div.

Now does it work for all numbers? Can we use algebra to represent the numbers? How?

---
* These numbers (1 - 8) refer to the numbers on the Observer's Checklist and Student's Notes, see Chapter IV, Table 4.5.

2.

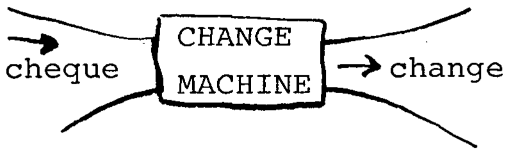
1)	a	+	
2)			b
3)	a	+	b
4)	a	+	2b
5)	2a	+	3b
6)	3a	+	5b
7)	5a	+	8b
8)	8a	+	13b
9)	13a	+	21b
10)	21a	+	34b
Sum	55a	+	88b

Common Factor
Distributive
Pattern of coefficients
need a, b 10?

Are coefficients sums, products, differences

What pattern relates <u>a</u>'s and <u>b</u>'s

3. Change machine for cheques



A = $1 bill

B = $2 bill

order counts as a separate way of doing change.

	Amount of cheque to be paid	Possible ways of Paying out	Number of Ways of Paying out
Do third	0	---	1
	1	A	1
	2	AA B	2
do first	3	AAA BA AB	3
	4	AAAA AAB BAA ABA BB	5
do second	5	AAAAA AAAB AABA ABAA BAAA BBA BAB ABB	8

Fourth - relate to coefficients in two (2.)
Fifth - Predict for $10

4. Pass out dittoed sheets of Fibonacci Sequence.

   Explain. Familar?

5. Even and odd numbers - pattern

   (even is every 3rd)

6. Multiples of numbers

	2	3	4	5	8
every	3rd	4th	6th	5th	6th

   Then look at ONLY multiples of the Fibonacci numbers.

7. Sum of Fibonacci Numbers

		Sum of first n	n
F_1	1		
F_2	1	1	1
F_3	2	2	2
F_4	3	4	3
F_5	5	7	4
F_6	8	12	5
F_7	13	20	6
F_8	21		

Sum of first n = $F_{n+2} - 1$

THE FIBONACCI SEQUENCE

NAME OF TERM	NUMBER OF TERM	TERM
F_1	1	1
F_2	2	1
F_3	3	2
F_4	4	3
F_5	5	5
F_6	6	8
F_7	7	13
F_8	8	21
F_9	9	34
F_{10}	10	55
F_{11}	11	89
F_{12}	12	144
F_{13}	13	233
F_{14}	14	377
F_{15}	15	610
F_{16}	16	987
F_{17}	17	1,597
F_{18}	18	2,584
F_{19}	19	4,181
F_{20}	20	6,765

PASCAL'S TRIANGLE

1. Watch this sequence closely. It builds in a different way than the Fibonacci Sequence did.

   1. 1 1
   2. 1 2 1
   3. 1 3 3 1

2. What things do you notice about the rows so far? How do the numbers between the rows relate to the row number?

3. Do you think that the next row will be 14641? 14541? or 14441?

4. The next row is 1 4 6 4 1 . Did you notice that the ones remain the same? Notice that the numbers in the row above are not directly above the numbers in the row below. Do you see a relationship between any 2 numbers in the previous row and a number in the row below?

5. On the Fibonacci Sequence we found some very interesting patterns, particularly when we looked at sums. Let us consider a row as a minature sequence (subsequence) and add across the rows and look for a pattern.

   Sum of numbers in the nth row is 2^n.
   What does the shape of our sequence suggest? (triangle) Is there a portion missing? How might we use what we have just found about the sums of numbers in rows to help us find the answer?

   Get the top 1 in the triangle, goes with 0 row and $2^0 = 1$.

6. What if we changed the shape of the "triangle" slightly to form a right triangle? Have we changed the rows?

7. Now we can look at what instead of rows? To find the sum of the first ___ numbers in any column, go over 1 and down 1 from where you ended in that column to find the answer.

8. Consider the triangle shape itself (use outer row) Go back to the original form of the triangle. Could we look at the sums in any other shape? Now consider a diamond shape (use outer edge).

9. Go to the right triangle version. Consider the third column. What if instead of adding up all the numbers in the column to a certain point, what if you added up any 2 adjoining numbers and looked at the sequence formed?

10. Which of the following does the question mean?
You add up 1 and 3, then add up 6 and 10, then 15 and 21?
or You add up 1 and 3, then 3 and 6, then 6 and 10?

11. The sequence formed is 4, 9, 16, ...
What do you notice about these particular numbers?
Would 2^2, 3^2, 4^2, ... be an equivalent sequence?
Would 2, 3, 4, ... be an equivalent sequence?

12. Do you see any way in which this sequence and the Fibonacci sequence are related?

13. We've looked at rows and columns, what is left to look at for a pattern?
Should we look at the upward or downward diagonals?

14. Review the relationships found in Pascal's Triangle. Again emphasis is on answers and conclusions.

(The following is an outline of notes as given to the teachers to use during the lesson on Pascal's Triangle.)

1.*

```
     1   1
   1   2   1
 1   3   3   1
```

Next row: 14641?
14541?
14441?

Relationship between row above and one below.

Let's see if we can find any relationships.

2.

	Row	Sums	
1)	1 1	2	2^1
2)	1 2 1	4	2^2
3)	1 3 3 1	8	2^3

Get row sums

Work to 0 row

---

* These numbers (1-8) refer to the numbers on the Observer's Checklist and Student's Notes, see Chapter IV, Table 4.5.

3.

0)	1							
1)	1	1						
2)	1	2	1					
3)	1	3	3	1				
4)	1	4	6	4	1			
5)	1	5	10	10	5	1		
6)	1	6	15	20	15	6	1	
7)	1	7	21	35	35	21	7	1

Look at the column sums of first so many

one over and one down from last number added.

4.

```
          1
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
1   5  10  10   5   1
                6   1
                  7   1
```

Sums of little triangles

Sums of diamonds

5.

				Sums
1				
1	1			
1	2	1		1
1	3	3	1	4
1	4	6	4	9
1	5	10	10	16
1	6	15	20	25

Sums of 3rd column adjacent pairs

Interpretation of question
1+3, 6+10, 15+21
OR 1+3, 3+6, 6+10 (yes)

Sum sequence 4, 9, 16...

is 2^2, 3^2, 4^2 or 2, 3, 4

6.

```
   1   1
1        2
1  1        3
1  2  1        5
1  3  3  1        8
1  4  6  4  1        13
1  5  10 10 5  1
1
```

Sums of diagonals upwards
(Get Fibonacci Sequence)

7. Review: Answers found in lesson.

APPENDIX B

DIVERGENT TEACHING LESSON

FIBONACCI NUMBERS

1 - 9 Identical with convergent lesson.

10. How did I know the sum? Yes, I found out about the seventh number, but how does that relate to the sum? How do numbers relate to one another any way? (list suggestions on the board)
Do you think that this trick will work for any two numbers?
What suggestions do you have for finding out?
(Make list of suggestions on the board)
(Have students come up with some form of representation which is general for any 2 numbers)
What do you notice about the numbers that you see in the general representation?
Do they appear to be special in any way?
How are they related to each other?
(List suggestions).

13. This content should come out of the above discussion.

14. A certain machine makes change for any whole number of dollars. However, it will pay out in $1 or $2 bills only. The change in bills may come out in any order and we want to consider the order that the bills come out in to be a different way of making change even though you may end up with the exact same number of bills - e.g. getting $2 first and the $1 is different than getting $1 and then a $2. If we want to find out how many different ways of cashing a cheque of any particular amount, how can we do it?
(list suggestions on board)
Which should we try first? Why are you making that choice?

15-18 The content of the convergent lesson should be brought out, but through participation and suggestions and should not particularly be teacher led.

20. What properties do you know about numbers?
(List suggestions froj students such as odd, even, multiples, more than, less than,... <u>must</u> be student suggested). Pass out dittoed sheets of Fibonacci Sequence. What kinds of things can we do with

these numbers? What ways of combining numbers do you know of? What kinds of patterns might you expect? Let's make a list of suggestions which we might make to combine these numbers and which might give a pattern?

21. Are there any patterns which are obvious in the sequence? Hope that odd/even will come out of this discussion as well as multiples which are Fibonacci numbers.

22. (As per 21)

23. Are there any things that we have done previously that might suggest something to do with the Fibonacci Sequence? (Hope that : $F_1 + F_2 + \dots + F_n = F_{n+2} - 1$ comes out of this).

24. Review what has been going on in class regarding Fibonacci Sequence. Emphasis on the ways we found for solving problems, relationships, combining one or more previously learned facts to make a new conjecture, using old tools and applying to new situations, etc.

(The following is an outline of notes as given to the teachers to use during the lesson on the Fibonacci Sequence).

1.*

1) 5
2) 4
3) 9
4) 13
5) 22
6) 35
7) 57
8) 92
9) 149
10) 241

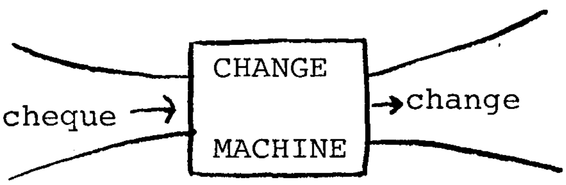
Sum 627

Trick (11 x 7th = sum) DO NOT TELL

Relationships between numbers (list suggestions)

Ways to find out if trick works for any two (list suggestions)

2.



$2 then $1 is different way to make change than $1 then $2.

Find out how many ways to make change for any particular cheque (Suggestions? list these.)

---

* These numbers (1-8) refer to the numbers on the Observer's Checklist and Student's Notes, see Chapter IV, Table 4.5.

3. Pass out dittoed sheets of Fibonacci Sequence:
   a) What properties do you know about numbers?
      (List suggestions like even/odd, greater than, less than, equal to, multiples, divisible by, constant sum, etc.)
   b) How do these relate to the sequence?
      Look for patterns.
4. Get content of multiples of Fibonacci

F_1	F_2	F_3	F_4	F_5	F_6
1	1	2	3	5	8

   Divisible by 2, repeating every 3rd

   Divisible by 3 presents a pattern of every 4th
5. How much is the sum of the 1st <u>n</u> Fibonacci Numbers?
   How can we find out? (list suggestions)

   $F_1 + F_2 + \ldots F_n = \quad F_{n+2} - 1$
6. Review - How have we found relationships?
   How have we solved problems?
   What techniques did we use?

PASCAL'S TRIANGLE

1. Identical to convergent lesson.

2. How do you think this sequence is formed?
   List suggestions on board.
   Just how can we test these hypotheses?
   What will the next row be given your hypothesis or rule?
   (list these beside the suggestions)

3. Suppose I choose the next row to be 1 4 6 4 1? Which of these rules could I use?

4. Did all of the hypotheses work? Did more than one of the hypotheses work? What would we do if none of our hypotheses worked and somebody said that 1 4 6 4 1 was the correct next row of the sequence that they were developing. Using one or more hypotheses, now predict the next row (again list beside suggestions)
   (If time here to expand - maybe look at some of the other hypotheses and wee what things come out of them.)

5. On the Fibonacci sequence we found some very interesting patterns in the sequence. What might we look for in this sequence?
   (List many suggestions, explore each briefly, utilize the mos productive if time is short).

6. Identical with convergent lesson.

7. What might we now be interested in looking at?
   (List these new suggestions on board, explore).

8. Go back to the original form of the triangle. Could we look at the sums of any other shape? What other shapes do you know? Where should we start?
   (Lists and conjectures and conclusions on the board).

9. If we want to look at pairs of adjacent numbers, how would we add them (Discussion)
   How could we go about solving this?
   Make a guess (hypothesis)(list on board) Let's see if they work.
   Try some of these guesses and test workability.

10. Discuss sequence formed.

10-12 Omit, should get this content with discussion if not, that's acceptable.

13. Which triangle should we try in looking for patterns? What made you choose that one? Are there any patterns we might have missed in our exploration so far? (list on board)

14. Review our approach to solving the problems presented so far. Emphasis on procedures, lools used, how we made hypotheses, etc.

(The following is an outline of notes as given to the teachers to use during the lesson on Pascal's Triangle.)

1.*		
1.	1 1	Suggestions for next row? List What rules do you need to get next row? (list) If 1 4 6 4 1 is next row, which row works? Row sum (may ask question) Add zero row (use this content only if student suggested) List suggestions
2.	1 2 1	
3.	1 3 3 1 Next Row/Rule Guesses	
2.	Right triangle as listed in convergent lesson	a) list suggestions of what might now look at. b) Pairs of adjacent numbers (vertical) How to add? List Suggestions. c) Using 3rd column, what pattern?

3. Relationship between Pascal's Triangle and Fibonacci Sequence? How to find - list suggestions - try.

4. Review - Relationships, conjectures.

---

* These numbers are assoicated with the Observer's Checklist (See Chapter IV, Table 4.5) where 1 goes with 1 and 2; 2 goes with 4,5 and 6; 3 goes with 7; and 4 goes with 8.

APPENDIX C

ROUTINE PROBLEMS

NAME: ______________________ BLOCK: __________

REFERENCE
FIBONACCI SEQUENCE

1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1,598

PASCAL'S TRIANGLE

```
       "Equilateral"                        "Right"
             1                              1
           1   1                            1  1
         1   2   1                          1  2  1
       1   3   3   1                        1  3  3  1
     1   4   6   4   1                      1  4  6  4  1
   1   5  10  10   5   1                    1  5 10 10  5  1
 1   6  15  20  15   6   1                  1  6 15 20 15  6  1
```

****

I. For each of the following questions, circle the ONE answer which BEST fits:
- T if you think the statement is ALWAYS true.
- ST if you think the statement is SOMETIMES true.
- F if you think the statement is ALWAYS false.

1. The fifth term of the Fibonacci sequence is 5. T ST F

2. The sum of two Fibonacci numbers gives another Fibonacci number. T ST F

3. The sum of the numbers in any row of Pascal's Triangle after row 1 is just the row number multiplied by two. T ST F

4. Numbers in the fifth column of the "right" version of Pascal's Triangle are multiples of five. T ST F

5. One way to get the sixth Fibonacci number is to add one to the sum of the first four terms. T ST F

6. The sum of two numbers in the same row of Pascal's Triangle will give a number in the next row of the triangle. T ST F

7. If you know that a certain number is a Fibonacci number, then you know that it is also a number in Pascal's Triangle. T ST F

8. In the Fibonacci Sequence every third term is a multiple of 2. In general, every (n+1)th term is a multiple of n: e.g. if n = 3, then every fourth term is a multiple of 3. T ST F

9. If you add a number in the first column of the "right" version of Pascal's Triangle to a number in the second column the result will be a number in the second column. T ST F

10. If you know that a particular row of Pascal's Triangle contains ten numbers, then you know that you are in the eleventh row. T ST F

****

II. For each of the following problems, choose the BEST answer and place the letter of your choice in the column at the right.

1. The number of terms in the Fibonacci Sequence is infinite, yet only two Fibonacci numbers are squares. They are _____
(a) 1 and 8 (b) 8 and 144 (c) 1 and 144 1. ___

2. Both 4 and 8 divide every _____ term of the Fibonacci Sequence.
(a) sixth (b) fifteenth (c) twenty-first 2. ___

3. If you take the difference between adjacent Fibonacci numbers the answer is _____
(a) a Fibonacci number (b) the Fibonacci number just before the adjacent pair (c) the Fibonacci number just after the adjacent pair. 3. ___

4. When you add the first nine Fibonacci numbers the sum is _____
(a) an even number (b) one less than the eleventh Fibonacci number (c) one more than the eleventh Fibonacci number. 4. ___

5. If you know that a certain even number was less than 10 and was also a Fibonacci number and a Pascal's Triangle number, then the number would be _____
(a) 2 (b) 8 (c) 2 and 8 5. ___

6. To get the row above in the "equilateral"version of Pascal's Triangle you should _____
(a) Take the sum of two adjacent numbers and write it in-between and above these numbers.
(b) Take the product of two adjacent numbers and write it in-between and above these numbers.
(c) Take the difference of two adjacent numbers and write it in-between and above these numbers. 6. ___

7. In the "right" version of Pascal's Triangle the sum of the first 5 numbers in any column may be obtained by _____
(a) going over one and down one from the 5th number.
(b) going over one and up one from the 5th number.
(c) going over one and down one from the 1st number. 7. ___

8. In row 3 of Pascal's Triangle, the third number is _____
(a) 3 (b) 1 (c) 6 8. ___

9. Starting at the second row, the numbers in Pascal's Triangle _____ from left to right.
(a) increase (b) increase then decrease
(c) decrease 9. ___

10. We have found a pattern relating to the "right" version of Pascal's Triangle and the Fibonacci numbers by summing along _____
(a) columns (b) diagonals (c) rows 10. ___

****

III. When you divide the numbers in the Fibonacci Sequence by 4 some interesting results occur when you look at the remainders after division.

e.g.

$$\begin{array}{r} 1 \\ 4\overline{)\,5} \\ \underline{4} \\ 1 \end{array}$$

1 remainder after division

The first four remainders are just the terms themselves.

1. Complete the following table of remainders by dividing 4 into each number of the Fibonacci sequence and writing the remainders.

Number	1	1	2	3	5	8	13	21	34	55
Remainder	1	1	2	3	1					

2. List the complete pattern that repeats. _______________

3. If you were to "re-invent" the Fibonacci Sequence, what two numbers would you start with?

_____ _____

****

IV. A. The squares of the first eight Fibonacci numbers are:

$(F_1)^2$	$(F_2)^2$	$(F_3)^2$	$(F_4)^2$	$(F_5)^2$	$(F_6)^2$	$(F_7)^2$	$(F_8)^2$
1	1	4	9	25	64	169	441

By taking adjacent pairs of this square sequence e.g. 1 1, 1 4, 4 9, ... the following sequences are formed:

Sequence A	2	5	13	34	89	233	610
Sequence B	0	3	5	16	39	105	272

1. How is Sequence A formed from the square sequence? _________

2. How is Sequence B formed from the square sequence? _________

3. Choose either Sequence A or Sequence B and predict the eighth term ______________________________

4. Explain why you made that prediction, be as explicit as possible. ______________________________

____

B. 1. Complete the following table of powers of 11.

$11^1 =$ _____
$11^2 =$ _____
$11^3 =$ _____

2. How do these powers of 11 relate to Pascal's Triangle?

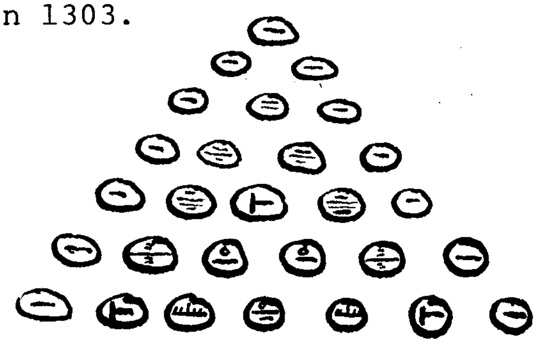
____

3. Does your statement in #2 hold true for 11^2 and 11^3?

____

****

V. This diagram was taken from a Chinese Mathematics book written in 1303.



1. What does this diagram represent? ______________________
2. What is the Chinese symbol for 10? ______________________
3. What is the Chinese symbol for 15? ______________________
4. Use this diagram to make a Chinese symbol for 25. ______________________
5. Use this diagram to make a Chinese symbol for 30. ______________________

APPENDIX D

NON-ROUTINE PROBLEMS

NAME: ____________________ BLOCK: ________

REFERENCES

Number of Term (n)	1	2	3	4	5	6	...
SEQUENCE 1	1,	2,	3,	4,	5,	6,	...
SEQUENCE 2	1,	3,	5,	7,	9,	11,	...
SEQUENCE 3	2,	4,	6,	8,	10,	12,	...
SEQUENCE 4	1,	4,	9,	16,	25,	36,	...

****

I. Each of the following questions refer to the sequences above. For each of the following questions circle the ONE answer which BEST fits:

T if you think the statement is ALWAYS true.
ST if you think the statement is SOMETIMES true.
F if you think the statement is ALWAYS false.

1. The numbers in SEQUENCE 2 are prime numbers. T ST F

2. If you have SEQUENCE 2 and want to get SEQUENCE 3 then you just add one to each term in SEQUENCE 2 T ST F

3. If you add adjacent pairs from SEQUENCE 1 then you get all of SEQUENCE 2. T ST F

4. If you have SEQUENCE 3 and want to get SEQUENCE 4 then you just square each term in SEQUENCE 3 and subtract one. T ST F

5. SEQUENCE 4 is a sequence of perfect squares. T ST F

6. No number in SEQUENCE 2 is divisible by 2. T ST F

7. The terms in SEQUENCE 4 are double the corresponding terms in SEQUENCE 1. T ST F

8. If you add adjacent pairs of numbers in SEQUENCE 3 you get the numbers in SEQUENCE 2. T ST F

9. SEQUENCE 3 consists entirely of even numbers. T ST F

10. If you have SEQUENCE 1 and want to get SEQUENCE 2 then just double each term in SEQUENCE 1, add two and the result is SEQUENCE 2. T ST F

****

II. Each of the follwoing questions refers to Sequence 1 - 4 above. For each of the following problems, choose the <u>BEST</u> choice and place the letter of your choice in the column at the right.

1. The general term (nth term) of SEQUENCE 1 is _____
   (a) 2n-1 (b) n (c) 2n 1. _____

2. The general term (nth term) of SEQUENCE 2 is _____
   (a) 2n-1 (b) 2n (c) n^2 2. _____

3. The general term (nth term) of SEQUENCE 3 is _____
   (a) 2n-1 (b) n^2 (c) 2n 3. _____

4. The general term (nth term) of SEQUENCE 4 is _____
   (a) 2n (b) n^2 (c) 2n-1 4. _____

5. To get SEQUENCE 4 from SEQUENCE 3
   (a) subtract 1 and then square each term in Seq. 3
   (b) divide by 2 and then square each term in Seq. 3
   (c) square and then subtract 1 from each term in Seq. 3 5. _____

6. If you add the first 3 terms of SEQUENCE 2, you get as a sum
   (a) the third term of SEQUENCE 4
   (b) the fifth term of SEQUENCE 2
   (c) both (a) and (b) above. 6. _____

7. If you add the first 5 terms of SEQUENCE 2, you get as a sum
   (a) the fifth term of SEQUENCE 4
   (b) the seventh term of SEQUENCE 2
   (c) both (a) and (b) above. 7. _____

8. Sequences which have the same first term are:
   (a) 1 and 2 (b) 1, 2, 3 (c) 1, 2, 4 8. _____

9. To get SEQUENCE 1 from both SEQUENCE 2 and SEQUENCE 3 you
   (a) add the terms of SEQUENCES 2 and 3
   (b) choose terms alternately from SEQUENCES 2 and 3, beginning with SEQUENCE 2
   (c) choose terms alternately from SEQUENCES 2 and 3, beginning with SEQUENCE 3 9. _____

10. To get SEQUENCE 1 from SEQUENCE 4 you
   (a) take the square root of each term in Seq. 4
   (b) take the positive square root of each term in Seq. 4
   (c) halve each term in Seq. 4 10. _____

****

III. The following code was found in the bottom of a mathematician's trunk:

There was a note which said  = eleven.

1. What does this sequence represent? ________________
2. What is the code symbol for 9? ________________
3. What is the code symbol for 5? ________________
4. Use this code to make a symbol for 24. ____________
5. Use this code to make a symbol for 30. ____________

IV. The following is a 4 by 4 magic square. If you take particular sets of 4 numbers you will find that they all give the same magic sum.

16	3	2	13
9	6	7	12
5	10	11	8
4	15	14	1

What is the magic sum? ________

Look for patterns that give you the magic sum. List as many DIFFERENT patterns as you can in words (not using numbers) describing where the pattern starts and ends and how you follow it.*

________________________ ________________________

________________________ ________________________

________________________ ________________________

________________________ ________________________

________________________ ________________________

* This is the divergent problem whose key follows the non-routine problems

V. A TRIANGULAR number is a number which can be represented with dots in the shape of a triangle.

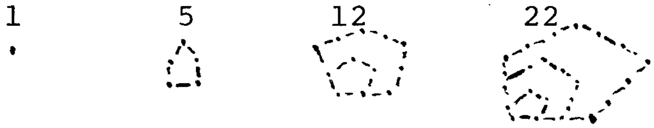
1 3 6 10

A SQUARE number is a number which can be represented with dots in the shape of a square.

1 4 9 16

A PENTAGONAL number is a number which can be represented with dots in the shape of a pentagon.

1 5 12 22



Consider the following chart:

Number of term (n)	1	2	3	4	5	6	...
Triangular number	1	3	6	10			
Square number	1	4	9	16			
Pentagonal number	1	5	12	22			

1. Describe how you would get the next two triangular numbers using the pattern from the first four:

   ____

2. Describe how you would get the next two square numbers using the pattern from the first four:

   ____

3. Describe how you would get the next two pentagonal numbers using the pattern from the first four:

   ____

4. What relationship do you see between the triangular numbers and the square numbers?

   (a) Describe in words: ______________________________

   ____

5. What relationship do you see between the pentagonal numbers and the other two (i.e. triangular and square)

   (a) Describe in words: ______________________________

   ____

6. In the space below using dots:

   (a) Answer 4a (b) Answer 5a

KEY FOR DIVERGENT QUESTION
IN NON-ROUTINE PROBLEMS

	FLEXIBILITY	FLUENCY	ORIGINALITY
Row 1	1	1	0
Row 2	0	1	0
Row 3	0	1	0
Row 4	0	1	0
Column 1	0	1	0
Column 2	0	1	0
Column 3	0	1	0
Column 4	0	1	0
Diagonal Top to Bottom	0	1	0
Diagonal Bottom to Top	0	1	0
Bottom right 4	1	1	1
Bottom left 4	0	1	1
Top right 4	0	1	1
Top left 4	0	1	1
Centre 4	0	1	1
Column 2 - 1st 2 and Column 3 - 2nd 2 (and the reverse)	1	1	1
Column 1 - 1st 2 and Column 4 - 2nd 2 (and the reverse)	0	1	1
Z-shape (3, 7, 10, 14)	1	1	1
Every 2nd number	1	1	1
4 corners	1	1	1
Row 1 - 1st 2 and Row 4 - last 2	1	1	1
TOTAL POSSIBLE	7	21	11

* See Magic Square p. 114

APPENDIX E

EXAMPLE OF STUDENT'S NOTES

Relations

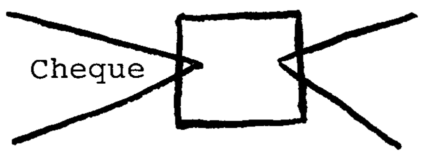
1. 4	1. Add the two before to get the third
2. 8	2. Every 3rd no. is third
3. 12	4. adding 2 consecutive nos. get consecutive odd nos. 2 consecutive odd get even
4. 20	
5. 32	5. every 5th number is divisible by 5
6. 52	6. term column: every 3rd is divisible by 2
7. 84	7. (number of term) every 4th no. by 3
8.136	8. every 6th is divisible by 8
9.220	9. every 7th no. (term) is divisible by 13
10.356	
924	

---

1. one more than each other in sequence

---

1. x	
2. a	
3. a + x	
4. 2a + x	
5. 3a + 2x	1$ - 1
6. 5a + 3x	2$ 1 - 2
	2 - 1
7. 8a + 5x	
8. 13a + 8x	3$ 3 - 1
9. 21a +13x	2 - 1 - 1
	1 - 1 - 2
10. 34a +21x	
11(8a+5x)	



every 3rd div by 2

4th	3
5th	5
6	8
7	13

each term is one more than
the one before
add two consecutive gives
odd (no. of term)
skip one
subtract
term column 3 nos. in a row
c - a = b
c - b = a
do in threes 3's
4's
5's

1
1   1 + 1 + 2 = 4
2

16-4 = 12 = 13-1

3
5   3 + 5 + 8 = 16
8

1
1
2
3 = 2 more than 8
5 = 3 more than 13
8 = 5 more than 21
13 = 8 more than 34
21
34
55 = 13 more than 55

1. 1 1
2. 1 2 1
3. 1 3 3 1
4. 1 4 6 4 1
5. 1 5 10 10 5 1
6. 1 6 15 20 15 6 1

2 4 8 16 32
2 2^2 2^3 multiply by 2

next 131   232 (add one to each)
- up and down
233 - x

2332

1 1 3 1 1 put nos. down

14641 - 121 x 1211

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

7 21 35 35 21 7 1

1 2 4 8 16 32 64