COMPUTERS IN TRIGONOMETRY:
A USEFUL TOOL AS A TEXTBOOK SUPPLEMENT?

By

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We accept this thesis as conforming
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ABSTRACT

The purpose of this study was to compare the effectiveness of utilizing microcomputers with traditional, textbook-based classroom instruction for teaching the graphing of trigonometric functions in grade 12 algebra.

The study was conducted over six teaching days in two phases. In phase one, the Microcomputer First group was given a classroom demonstration using a Macintosh microcomputer and the software program "Master Grapher". The students in this group then completed exercises in a microcomputer laboratory utilizing the same software program, working either individually or in pairs.

The Textbook First group was given a traditional lesson and an assignment from a textbook.

A researcher-developed test to assess comprehension of graphing trigonometric functions was administered to both groups on the third day.

In phase two of the study, the treatments were reversed. A second posttest was administered on the third day of phase two. This permitted the comparison of the effectiveness of the order of the treatments: microcomputer followed by textbook instruction with textbook followed by microcomputer instruction.

Posttest 1 and 2 scores were analyzed using an analysis of covariance, with an achievement score comprised of Algebra 12 test scores received prior to the experiment as the covariate. As well, Posttest 2 scores were analyzed using two covariates, the Algebra 12 test scores and Posttest 1 results, to determine the effect of the treatments.

On posttest 1, the adjusted mean score of students in the Textbook First group was significantly higher (p < .05) than the adjusted mean score of the students in the Microcomputer First group. The analysis of
covariance using two covariates on posttest 2 indicated no statistically significant difference between the adjusted mean score of students in the Textbook First group who had just completed the microcomputer treatment, and the adjusted mean score of students in the Microcomputer First group who had just completed the textbook treatment.

On posttest 2, the analysis of covariance using one covariate showed that the difference between the adjusted mean scores of the two groups was not statistically significant indicating that the order of treatment was not related to achievement. However, effect sizes of .86 for posttest 1 and .22 for posttest 2 indicated that in both phases, students completing the textbook treatment scored higher on the achievement tests than students completing the microcomputer treatment.

When the data for both posttests were arranged according to gender, time of the day the class was taken, and whether the course was being repeated, no statistically significant relationships were detected.

In conclusion: (a) traditional, textbook-based classroom teaching produced higher achievement scores on a test of graphing trigonometric relations than teaching which used a microcomputer and the software program "Master Grapher"; (b) student achievement was not affected by the order in which they were given the two treatments; and (c) there was no relationship between the treatment and factors of gender, time of day, and repetition of the course.
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Chapter 1
THE PROBLEM

This chapter discusses the background of the problem and its significance in the field of education as well as how the problem was originally formulated. The limitations of the problem are explained, the research hypothesis and related questions are outlined, and the terms to be used in the study are defined.

Background of the Problem

Microcomputers have infiltrated many aspects of society, and the school system is no exception. A few years ago, it was considered a novelty for a school to have one microcomputer. Today, it is not uncommon for a school to have a classroom filled with microcomputers (Bork, 1984; Eisenrauch, 1985; Becker, 1987). This increase in the number of computers in the school system is as a result of societal demands, provincial curriculum changes, and a desire for educational growth on the part of teachers (Forman, 1981; Parry, Thordilksen, Biery, & Macfarlane, 1986; Collis, 1988).

Microcomputers have a huge capacity for storing and retrieving information as well as for carrying out complex calculations quickly and accurately. As a result, microcomputers have been welcomed into the business world as a useful tool. Parents, aware of this new technological aspect of the business place, want their children to be exposed to microcomputers early and often. Consequently, many of the original microcomputers present in schools were funded by parent groups donating countless hours and/or money to their school's 'computer fund' to ensure
that their children would have the opportunity to become a microcomputer user. (Burns & Bozeman, 1981; Bracey, 1982; Eisenrauch, 1985).

School boards, under constant pressure to answer to the parents’ concerns, have begun to allocate funds towards microcomputers in their schools by way of inservice for teachers as well as the purchase of hardware and software to support the curriculum (Savage, 1989). It is generally perceived by the public as being positive to have microcomputer-equipped schools with computer science programs in place. Responding quickly to this pressure, funds have been allocated and spent, even though very little research has gone into proving or disproving the effectiveness of microcomputers in the classroom.

The Government of British Columbia has also responded to the public and its perception of the needs of today's students by channeling funds to technology. Those funds have grown from a yearly allotment of $6.83 million through the Fund for Excellence in 1986-87 to an annual budget of $16 million in 1988-89 (Savage, 1989). This $9.17 million, or 57% increase, in funding over the past three years was also as a result of the report of the Royal Commission on Education (Sullivan, 1988) which recommended greater use of technology, including microcomputers, in schools.

For its part, the Ministry of Education has also requested and supported the implementation of various curriculum revisions over the past few years which include statements regarding the use of microcomputers in the curriculum. Specifically, in the revised mathematics curriculum for British Columbia, it is stated that computers should be used wherever possible to "formulate mathematical models and as a tool in problem solving" (Mathematics Curriculum Guide, 1986, p. viii).
The National Council of Teachers of Mathematics (NCTM) also endorsed microcomputers. At their 1985 conference it was recommended that "all students and teachers will have access to calculators and computers for the study of mathematics in the classroom and at home....and that all students will experience appropriate application of computers in the study of each school discipline" (Corbitt, 1985, p. 244).

It is therefore apparent that microcomputers are being supported by many facets of society. Pressure is being placed squarely on the shoulders of the schools, and ultimately the teachers, to implement their use. As Olson stated:

Some call the effects of microcomputers on schools a revolution. Revolution may seem a strong word to describe the advent of 'educational computing'. It isn't. Nothing before has so stirred schools into action. (1988, p.1)

Teachers, meanwhile, are continually attempting to improve their teaching strategies and to grow professionally. Many see the microcomputer as a useful tool which may enable them to become more effective in their teaching. They are aware of the support being offered from the provincial government as well as from their local associations and they are excited about the 'computer revolution'. However, it seems that in education there is an ongoing "revolution" of one form or another, and with constant pressure from society to improve the educational system, it is easy to understand why educators are tempted to grasp onto innovations. Now, the microcomputer 'revolution' is gaining momentum... or is it? Is the microcomputer only a passing fad or gimic... another quick but ineffective solution to a complex problem? (Gleason, 1981; Eisenrauch, 1985; Hill, Manzo, Liberman, York, Nichols, & Morgan, 1988) Is it being...
promoted as the next tool to save the day for education when in reality it will only end up covered in cobwebs and unused in classrooms? Considering the relatively high cost of a microcomputer as compared to the relatively low price of a textbook ... is it worth it? Are the benefits that can be gained through microcomputer applications significant enough to warrant their integration into the curriculum? These, then, were questions that underlay this study.

Motivation for the Study

At Burnaby Central Secondary School, as in many other schools, there was a laboratory equipped with 15 computers. The researcher had assumed that these microcomputers were heavily used and that, as a classroom teacher, it would be difficult to schedule a time to bring a class into the lab even if it was known what lesson was to be taught once there. However, upon passing the lab several times in a day the researcher noticed that, in reality, it was seldom used. After pondering this revelation, the researcher decided that someone should be making use of this room full of very expensive microcomputers. Furthermore, as a member of the mathematics department, if the researcher was fearful of using the lab, imagine how many other teachers on staff felt. (It is always assumed that mathematics teachers KNOW about microcomputers!) Someone had to start making some progress in the use of this laboratory. The researcher decided to make that move and take a class to the Macintosh lab.

Once that decision was made, the problem then became what to do with a class of students once in the lab. This question was answered during the MathTech conference held in Burnaby, British Columbia in late February, 1989. Impressed with the software on graphing equations that was being
presented, the researcher organized a unit on graphing trigonometric functions for grade 12 students. This topic was chosen because it is a particularly tedious concept to teach with blackboard and chalk, and it is difficult for the students to grasp, especially when all points must be hand plotted. The researcher believed that the microcomputer could be used to great advantage in doing all the necessary calculations freeing the students to concentrate on understanding the overall concepts.

Thus, as a result of the financial support from the provincial government and the school board in that they provided the microcomputers, the pressure from the curriculum outlines stating that the microcomputer must be used, a desire to be a more effective teacher and a curiosity from within... a study was born.

Statement of the Problem

The purpose of this study was twofold. Firstly, it was intended to compare the relative effectiveness of the microcomputer as a teaching tool for graphing trigonometric functions with a traditional textbook approach. Secondly, it was intended to compare the relative effectiveness of having students do work on the microcomputer first followed by work from the textbook with having students do work from the textbook first followed by work on the microcomputer.

At the same time, this study evaluated the differential effects of the two strategies on male and female students, as well as determining whether there was an interaction between the factors of treatment and sex; the treatment effect on the morning class versus the afternoon class, and if there was an interaction between the factors of treatment and time of day; and the relationship between repeating the course and student achievement
based upon the treatments, and if there was an interaction between the factors of treatment and number of times the course had been taken.

Two posttests were given to evaluate progress, and initial differences of the groups were adjusted by using each student's Algebra 12 achievement score as a covariate. Students of grade 12 level were selected for the study because the researcher had easy access to these students, there were four classes (one in each of the four time slots offered at Burnaby Central), and the researcher was very familiar with the curriculum for this grade level. It should also be noted that very little research has been done at this grade level. Becker's (1987) study of 2331 American high schools revealed that little use was made of microcomputers in mathematics at higher grade levels. Therefore, it was felt that a contribution could be made by doing a study at this level.

The Research Hypothesis and Related Questions

The research and related literature on the use of microcomputers for assisting instruction is not conclusive. However, as will be detailed in Chapter 2, the majority of the research looks upon the use of the microcomputer as favourable for enhancing learning. The following hypothesis was developed based upon that information:

There will be a significant difference between the adjusted mean posttest scores on a test of graphing trigonometric functions of the students following the microcomputer treatment and of the students following the textbook treatment.

It was thought that students using a microcomputer to graph trigonometric functions would score higher on a test of achievement than students using only a textbook because a microcomputer can plot many
graphs very quickly, allowing students the freedom to observe the shifts that occur, rather than tediously plotting a few graphs to observe these same shifts.

In addition, the following questions will also be examined:

1. Will there be a significant difference in the adjusted mean posttest scores of the students who have completed the microcomputer treatment first followed by the textbook treatment compared to the students who have completed the textbook treatment first followed by the microcomputer treatment?

2. Will there be a significant difference in the adjusted mean posttest scores of males and females on a trigonometric graphing test? Will males respond differently to the microcomputer treatment and the textbook treatment than females?

3. Will there be a significant difference in the adjusted mean posttest scores of those students taking the course in the morning compared to those students taking the course in the afternoon? Will there be an interaction between treatment and time of day the course is taken?

4. Will there be a significant difference in the adjusted mean posttest scores of those students repeating the course compared to those students taking the course for the first time? Will there be an interaction between treatment and number of times the student has taken the course?

Limitations of the Study

This research was confined to examining students' ability to interpret, graph, and determine the equations of trigonometric functions.
The study was carried out in an urban high school in British Columbia. The sample was restricted to 78 students in grade twelve. This sample included all but seven of the students registered in Algebra 12 in the second half of the academic school year 1988/89. The length of this study was only six school days due to the specific nature of the material being presented and its place in the overall Algebra 12 curriculum outline.

The Macintosh computer was chosen for this study as it was the microcomputer in place in this school. The microcomputer study of trigonometric relations was limited by the use of the software program 'Master Grapher' (see definition of terms). This program was selected for the study as a result of its availability for use by the researcher and for its suitability for this topic.

DEFINITION OF TERMS

The following terms will be defined for the purposes of this study.

**MicroComputer First Group (MCF)**

This is the group of students who received two days of instruction involving the microcomputer followed by two days of instruction from the textbook.

**Textbook First Group (TBF)**

This is the group of students who received two days of instruction from the textbook followed by two days of instruction involving the microcomputer.
**Microcomputer Treatment**

This treatment consisted of two lessons on graphing trigonometric functions involving the use of the Macintosh microcomputer and the software program "Master Grapher".

**Textbook Treatment**

This treatment consisted of two lessons on graphing trigonometric functions using the textbook and chalkboard. This would be considered traditional mathematics instruction.

**Microcomputer Demonstration**

The teacher used the program "Master Grapher" on a Macintosh microcomputer to display to the students the graphs of various trigonometric functions. This demonstration was done using a Kodak Liquid Crystal Display (LCD) unit and an overhead projector to project the image from the microcomputer screen onto a classroom screen.

**Trigonometric Functions**

These are functions involving the sine, cosine, and tangent along with their reciprocal functions of cosecant, secant, and cotangent. The students studied both the equations and the graphs of these trigonometric functions.

"**Master Grapher**"

This is a software program produced by the Addison-Wesley Company and written by Franklin Demana, Bert Waits, and David New of Ohio State University. It enables the user to generate the graphs of many different relations (not just trigonometric relations).
**Algebra 12**

This is the course as outlined by the Ministry of Education of the Province of British Columbia (1978).

**Individual practice**

This term is used to define the time given to students to work on assigned questions as well as to devise questions of their own while sitting at a microcomputer, or while working from a textbook on an assignment at their desk.

**Posttests 1 and 2**

There were two parallel forms of the posttest, A and B, and two sittings, 1 and 2. At the first sitting (Posttest 1), form A was administered to students taking Algebra 12 in the morning, and form B was administered to students taking Algebra 12 in the afternoon. These are referred to as Posttest 1A and 1B respectively. At the second sitting (Posttest 2), form B was administered to students taking Algebra 12 in the morning, and form A was administered to students taking the course in the afternoon.

**Summary**

The microcomputer has been introduced into schools and is currently being supported at all levels of the school system. A considerable amount of money has been channelled into funding the use of the microcomputer as an educational tool (Becker, 1984; Savage, 1989). Time and more research will determine whether these funds are generating an enhanced learning environment for students of British Columbia. This study was designed to
contribute to the information available regarding the effectiveness of microcomputers for improving learning by students in the mathematics classroom.

Specifically, this study compared the results of students who were exposed to textbook instruction with students who had microcomputer-based instruction. The researcher also probed to see whether the order of the presentation of the two types of instruction influences students' learning. Is it better to use the textbook first and then supplement its use with microcomputer practice, or is it more effective to observe information and interact with the microcomputer first, and then complete work from the textbook?
Chapter 2
REVIEW OF THE LITERATURE

This chapter includes an introduction followed by a description of the early research (1970s) relating to the use of the computer in the classroom, the recent research (1980s) on microcomputer use, a summary of the research reviewed, and the implications of the review on this study.

The literature has been grouped according to whether it was written in the 1970s or the 1980s to indicate the shift from the use of mainframe computers to microcomputers around 1980 (Eisenrauch, 1985; Kearsley, Hunter, & Streibel, 1983; Chambers & Sprecher, 1980). Although the literature is often not specific, it can be assumed that most of the early research, that of the 1970s, was conducted on mainframe computers whereas most of the recent research, that of the 1980s, was conducted on microcomputers. Although this study is only concerned with the use of microcomputers, the results obtained through research using mainframe computers provides helpful background information.

Additionally, this study is mainly concerned with the use of the microcomputer as a tool, although aspects of the tutor mode are also present (Taylor, 1980). This combined usage of the microcomputer is commonly called Computer-Assisted Instruction (CAI), Computer-Assisted Learning (CAL), Computer-Based Education (CBE), and a variety of other terms which all generally mean that students respond to work given to them on the microcomputer or use the microcomputer to generate information.
INTRODUCTION

The introduction includes a statement about how the literature review was conducted, a discussion of various educators' opinions on the use of the microcomputer in the classroom, and a restatement of the research problem as it relates to those opinions.

Conducting the search

A few of the studies discussed in this review were located by a computer search of Educational Resources Information Center (ERIC) during the summer of 1988. The descriptors of "mathematics education", and "secondary education", and "microcomputers" identified 12 journal articles from 1983 to 1988 indicating that there is a paucity of research in this area. The bibliographies of these articles were used to locate related studies on the use of the microcomputer in the mathematics classroom. Additionally, a manual search of periodical indexes, Dissertations Abstracts International, and newspapers and magazines was conducted in the libraries of the University of British Columbia, Simon Fraser University, and San Diego State University. An attempt to limit the literature used in this review to only those studies that dealt with secondary mathematics (grades 8 to 12) was abandoned because many of the research studies located involved elementary as well as secondary students, and several involved language arts as well as mathematics.

Educators' Opinions

Since the advent of the microcomputer in the late 1970s, its use in classroom instruction has increased rapidly (Chambers & Sprecher, 1980). It seems apparent, however, that although the hardware has become more
easily accessible to most students, and the software is becoming more readily available and "user friendly" (Bork, 1984; Eisenrauch, 1985), there have not been significant increases in the use of microcomputers in the average mathematics classroom (Olsen, 1988). Certainly, the National Council for Teachers of Mathematics (NCTM) and the Ministry of Education in British Columbia are attempting to bring about change by stressing that students and teachers should have access to microcomputers to alleviate the burden of the computational aspects of mathematics, and to open the doors to more creative endeavours (Corbett, 1985; Mathematics Curriculum Guide, 1986). The B.C. Mathematics Curriculum Guide (1986) states that computers should be used whenever possible to formulate mathematical models, and as a tool in problem solving.

Wilkinson (1984) stresses that the microcomputer has been treated as an "object of instruction" rather than a "tool of instruction". He argues that the use of microcomputers in instruction will not solve all of our educational problems but, because of that, microcomputers should not be rejected. He is also of the belief that microcomputers can relieve us of tedious tasks, thus allowing more time for problem solving, which he thinks should be the goal of education.

Fey (1984) speculates on how the microcomputer will effect algebra, geometry, and calculus but warns educators that they "stagger from one shocking change to another as the victims, not the shapers, of events" (p. 71). Fey is chiding teachers that they should become the architects of their futures, and that they must take the time to plan where they want the microcomputer to take them in education rather than changing for the sake of change with little thought going into the process.
Many educators do believe in the value of the microcomputer as an enhancer of learning in the classroom. Often, however, this opinion seems to be more intuitive than research-based. Educators do not believe that microcomputers will replace teachers but that they will change the role of teachers from that of transmitters of information to the far more "significant role of planning and providing those higher-order learning experiences that cannot be provided by technological devices" (Gleason, 1981, p.18).

Educators believe that microcomputer-based instruction should be regarded as a tool for enhancing teacher effectiveness, and that this can happen in two ways: (a) by improving the general learning environment for the students; and (b) by freeing the teacher from unproductive, time-consuming tasks (Trollip & Alessi, 1988). Trollip and Alessi (1988) argue that the microcomputer can be used to improve the learning environment by teaching difficult material, providing the student with practice, reducing cost, improving motivation, and reducing or eliminating the logistical difficulty of teaching certain topics. Also, wherever a computer can save teacher time, it provides the opportunity of enhancing the quality of the student-teacher interaction.

Collis (1988, p. 2) muses as to whether we are in the midst of a revolution in education, and she quotes Alfred Bork from the University of California who is a leader in the field of microcomputers:

The computer is the most powerful new learning device since the invention of the printing press and the textbook. The computer has the potential to solve most of our current educational problems. Within twenty years the computer will be the major delivery system for
education at all levels and in practically all subject areas, replacing books and lectures.

This appears to be a very powerful statement from a man who has spent most of his life studying computers.

Kelly (1988) is also most dramatic as he looks at mathematics in the 90s when he says that "invention and subsequent refinement of the computer represents a 'discontinuous jump' in the evolution of mathematical methods and techniques." (p. 9) In other words, he believes that educators are being faced with a major new force in the teaching of mathematics... one of such magnitude that it will have a profound effect on the way that mathematics is taught in the future.

The opinions presented by the preceding educators are typical of the opinions expressed throughout much of the literature on this topic (see e.g., Billstein, 1985; Usiskin, 1985; Marsh, 1987; Colorado, 1988; Olson, 1988). Generally, the literature has a positive, optimistic tone toward the use of the microcomputer in the classroom. However, it also carries a message of warning in that the microcomputer can be a very powerful force, and educators should choose their uses of the microcomputer wisely... but, more importantly, they should choose to use it.

Restating the Problem

The focus of this study is on the effectiveness of the microcomputer in teaching grade 12 students to graph trigonometric functions. As this topic is very specific, no other identical studies could be found. However, the general effectiveness of the microcomputer in the classroom has been endorsed by the opinions of many educators, and a review of the research should provide an indication of the results likely to be obtained from this
study. Thus, the remainder of this chapter contains a broad review of the research available on microcomputers through the 1970s and 1980s.

LITERATURE OF THE 1970s

A review of the research from the 1970s indicates a favorable report for the use of microcomputers in the classroom. The findings of studies reported by Ronan (1971), Vinsonhaler and Bass (1972), Jameson, Suppes, and Wells (1974), Edwards, Norton, Taylor, Weiss, and Dusseldorp (1975), Hartley (1977), and Thomas (1979) have supported the statement that CAI plus regular classroom instruction is more effective than traditional classroom instruction alone.

Reviews of research

Vinsonhaler and Bass (1972) did a narrative and box-score summary of the results of 10 major studies of CAI which included both mathematics and language arts. In their review, there were approximately 10,000 subjects of elementary school age involved in over thirty separate studies conducted in California, Michigan, Mississippi, and New York. The studies they chose to include had to meet three major criteria: they used a mode of CAI described as drill and practice; the method for measuring was a standardized test—principally, the Stanford Achievement Test (SAT) for mathematics; and basic experimental/control group design was used. From a review of these 10 studies, they concluded that the results indicated "a substantial advantage for CAI augmentation of traditional classroom instruction, where standardized tests are used as the criteria for educational performance" (p. 29). They reported their box-scores by subtracting the control group means from the experimental group means stating that the result should be zero if
no gains are attributed. This type of reporting does not take into account the initial differences of groups involved in the various studies nor does it account for differences in teacher methodology.

Jameson, Suppes, and Wells (1974) surveyed research on the effectiveness of Traditional Classroom Instruction (TI), Instructional Radio (IR), Instructional TV (ITV), Programmed Instruction (PI), and CAI. Their results were reported narratively throughout, as well as in tabular form for some individual studies cited. They found that at the elementary school level, students exposed to supplementary instruction through CAI performed at a higher level than those students not so exposed. It is interesting to note that at the secondary and college level, the evidence of no difference between CAI taught and traditionally taught students predominated when CAI was used as a replacement for classroom instruction, and that the presence of traditional instruction along with CAI was imperative for improved results. The researchers admitted at the outset of their survey that their results were based upon relatively few studies, original sources remained unchecked in some instances, and some results lacked scientific standards of analysis. Thus, they cautioned that their survey is "inherently spotty in its conclusions" (p. 41).

Edwards, Norton, Taylor, Weiss, and Dusseldorp (1975) used a box-score count combined with narrative to review 12 drill and practice studies, and eight tutorial studies. They concluded that of the nine studies which involved CAI as a supplement, all showed greater student achievement (6 elementary and 3 college level), and the studies which involved CAI as a substitute were not as favorable but most were at least equal to traditional classroom instruction. Only two studies showed mixed or negative results. One of these studies involved the microcomputer in a "problem-solving"
mode in a high school algebra class, and the other involved the computer in a "simulation" mode in a grade 6 economics class. The reviewers did not expand on these results. These reviewers simplified the results of the various studies to "+" and "-" categories thus eliminating all statistical comparisons. They defined a "+" to indicate that the computer group did better than the control group but by doing so, they have hidden the actual amount of the improvement which may very well be minimal, and due to novelty or confounding effects other than the treatment itself.

Hartley (1977) reviewed 153 experimental studies in mathematics comparing CAI; cross-age and peer tutoring; Individual learning; and programmed instruction. Although she found that tutoring was the superior technique for increasing mathematics achievement, CAI was found to be more effective than individual or programmed learning which Hartley called "traditional instruction". She found that CAI raised student achievement by .41 standard deviation or from the 50th percentile to the 66th percentile. She made the point that the CAI classes were much larger than the tutoring classes and that comparison of large scale tutoring with CAI was unavailable, which may have accounted for tutoring being ranked above CAI.

Thomas' (1979) narrative review dealt only with secondary school studies of language arts and mathematics. After a thorough search, he only included those studies which included "hard" data. He found much the same as the previously discussed reviews. He concluded that the studies "overwhelmingly support CAI as a viable instructional alternative" (p. 106). Additionally, he stated that at the secondary level the studies indicated "higher achievement scores whether measured by teacher-made tests, gain scores on local or standardized tests, or on predicted versus actual scores using regression analysis methods" (p.106). He did uncover one study by
Toggenburger and McDaniel in 1977 which reported normal (not improved) achievement growth.

In summary, the findings of the reviews of research from the 1970s were reported mostly in box-score and narrative fashion without reference made to specific gains with the exception of Hartley (1977). This type of reporting tends to indicate trends but it is not helpful in testing for statistical significance as opposed to gains which can be attributed to factors other than the treatment itself.

**Individual Studies**

Ronan (1971) did an interesting study using two classes of algebra/trigonometry students being taught by the same teacher. The study took place over nineteen weeks. In general, he found that the students who had used the computer were significantly higher in mathematics skills than the students who did not use the computer. At varying stages in the study he found: no differences between the groups; significant differences favoring the treatment group; and significant differences favoring the control group. For example, after the treatment involving algebraic review material and radicals in equations; trigonometric functions and complex numbers; and circular functions and their inverses he found no significant differences between the control and the experimental group. However, after treatment involving exponential functions and logarithms, he found the experimental group (using the computer) had attained a significantly higher level of achievement than the control group. Following treatment involving trigonometric identities and formulas, he found the control group attained a significantly higher level of achievement than the experimental group. It
appears, therefore, that the computer is more useful for certain topics than it is for others.

When pre- and posttest scores for each group were compared individually, Ronan (1971) found that both groups showed significant growth in the understanding of mathematical concepts and their ability to perform mathematical problem-solving, but that the control group showed no significant growth in the development of computational skills after 19 weeks whereas the computer group did.

Not all studies have yielded positive results. A study by Robitaille, Sherrill, and Kaufman (1977) reported unfavorable results for CAI. Their sample included 174 grade nine students in six mathematics classes in Vancouver, British Columbia. They used two schools to conduct their study and used three classes in each school. In each school, one class used a computer throughout the study, a second class used the computer for the first third of the study, and the third class (the control) did not use the computer at all. Instrumentation consisted of standardized pre- and posttests to measure students' attitude, and a standardized pretest combined with teacher-developed posttests to assess mathematics achievement. The studies in both schools started in September, but in one school it was completed in December while in the other school it was not completed until May thus giving a shorter term and longer term base for comparison. Their results indicated a significant difference in mathematical abilities but not in favour of the computer group. They attempted to explain why their results were contrary to those found in most studies of this nature, and they concluded that variation in teacher methodology, which they had not attempted to control, may have been the
cause. Therefore, the finding of their study may be better categorized as a study of teaching styles rather than of the computer versus non-computer.

Findings regarding the saving of time

Many of the studies done evaluating CAI reported a substantial saving in the amount of time needed by students to learn materials (Vinsonhaler & Bass, 1972; Jameson, Suppes, & Wells, 1974; Edwards, Norton, Taylor, Weiss, & Dusseldorp, 1975; Thomas, 1979). According to Edwards, Norton, Taylor, Weiss, and Dusseldorp (1975), of nine studies which were carried out on "time saving", all showed that by using the microcomputer students could learn the same amount of material in less time. Vinsonhaler and Bass (1972) found that drill-and-practice CAI produced gains of between one and eight months of learning time in children taught via the computer as compared to those who were taught by traditional methods. However, there was little evidence available regarding retention of the material learned. Thomas (1979) found retention to be equal to that obtained in traditional instruction. Edwards, Norton, Taylor, Weiss, and Dusseldorp (1975) included three studies which dealt with retention, and two of the three studies indicated that those who had learned through CAI retained material less well than non-CAI students.

Findings regarding student attitude

Students' attitudes toward mathematics were said to be improved as a result of using a computer in studies done by Thomas (1979) and by Robitaille, Sherill, and Kaufman (1977). Students enjoyed working with computers and were generally more enthusiastic toward their math classes as a result of the introduction of the computer into their lessons, and
certainly improved attitudes favour the likelihood of increased learning. Robitaille, Sherill, and Kaufman (1977), however, state that this improvement in attitude was obtained only in the shorter term evaluation. They did not find any significant difference in attitude in the longer term study.

Summary of the 1970s

Research of the 1970s can be summarized as follows: the studies reviewed which attempted to evaluate the effectiveness of CAI overwhelmingly support CAI as a viable instructional supplement; the use of CAI enables students to learn the same amount of material in less time than traditional teaching methods; and students appear to have a positive attitude toward the use of the computer. However, the reviews reported were largely narrative in nature and lacked sufficient data to perform statistical tests of significance.

LITERATURE OF THE 1980s

Research studies of the 1980s have supported the findings of the earlier studies but have not added anything substantial to the findings. The general premise that CAI along with traditional instruction is more effective than only traditional teaching methods is supported by many researchers and reviewers of the 1980s (Chambers & Sprecher, 1980; Burns & Bozeman, 1981; Forman, 1981; Gleason, 1981; Bracey, 1982; Ragosta, Holland, & Jamison, 1982; Kearsley, Hunter, & Seidel, 1983; Gallitano, 1984; Parry, Thorkildsen, Biery, & Macfarlane, 1985; Eisenrauch, 1985; Bitter & Cameron, 1986; Colorado, 1988; and Roblyer, 1988). Some researchers, such as Kulik, Bangert, & Williams (1983) and Bangert-Drowns, Kulik, & Kulik
(1985) completed meta-analyses of large numbers of studies in an attempt to draw together the research.

Meta-analysis

In their most recent meta-analysis, Bangert-Drowns, Kulik, and Kulik (1985) reviewed 42 studies conducted with students from grades 7 through 12, all of which had reported quantitative results in language arts and mathematics. They found that in 32 of the 42 studies, students in the CBE class had higher mean achievement scores than control class students, and in 18 of those studies the improvement was reported to be statistically significant. In 16 of those 18 cases, the significant difference favored the CBE class, whereas only two studies favored conventional teaching. The average effect size in the 42 studies was 0.26 (which is a small effect size) and standard error was 0.063. In other words, the average CAI student would perform at the 60th percentile while the average control student would perform at the 50th percentile. Interestingly, the reviewers found that programs of computer enrichment did not add anything substantial to student learning in that they raised examination scores by only 0.07 standard deviations in the typical study. Thus, it appears that the microcomputer is best used as a tool for assisting and managing basic instruction rather than as a tool for enriching course material. The findings of this meta-analysis certainly do not provide 'overwhelming' support for CAI.

Some of the findings of Bangert-Drowns, Kulik, and Kulik (1985) regarding the way in which articles were reported were also interesting. They found that more published studies than unpublished studies reported statistically significant results; statistically significant results were
more frequently found in shorter studies than in longer ones; more recent studies reported statistically significant results than older studies; and the effects tended to be larger when different teachers taught the experimental and control groups than when both groups were taught by the same instructor.

The criteria used for selecting studies for this analysis were strict and it appears that the reviewers were very thorough in their scientific approach to the analysis. However, a meta-analysis is a review of a compilation of studies, each of which contains its own inaccuracies, thus contributing to an overall result perhaps containing many inaccuracies.

**Reviews of Research**

Burns and Bozeman (1981) synthesized the findings in 40 studies in which CAI was used in conjunction with mathematics instruction at the elementary and secondary levels. They, too, arrived at the conclusion that while "no ultimate final answer related to CAI effectiveness or guarantees of success can be presented, the analysis and synthesis of many studies do point to a significant enhancement of learning in instructional environments supplemented by CAI" (p. 35). Of particular interest from their findings was that the achievement of average ability students was not significantly enhanced by supplemental drill/practice CAI. This finding was also reinforced by Fisher (1983), Kulik, Bangert, and Williams (1983), and Eisenrauch (1985) all of whom stated that CAI was significantly more effective at raising achievement scores among low-achieving and high-achieving students than it is for students of average achievement.

Parry, Thorkildsen, Biery, and Macfarlan (1985) did a narrative review of 13 major studies and found that CAI supported instruction was
significantly more effective in 9 of those 13 studies. They added that the use of the microcomputer for instruction was more motivational than traditional teaching in 5 of those 13 studies, whereas in 8 studies it seemed to have no motivational effect.

Two reviewers who reported findings contrary to the norm include Streibel (1986) and Clark (1983). Streibel’s (1986) analysis "revealed that the learner’s personal intellectual agency was decreased rather than increased in the case of computer-based tutorials even though the rhetoric promised the exact opposite" (p. 138). His arguments were based upon an informed opinion rather than quantitative studies and he did make a strong case for his biases. For example, he concluded in his article that "drill and practice courseware programs alter the nature of the sub-skill acquisition, tutorial courseware programs restrict the full range of personal intellectual agency, and computer programming and simulations delegitimize non-technological ways of learning and thinking about problems. Taken together, is this worth the price?" (p. 158). In other words, he found very little to be positive about the use of a computer for educational purposes. He felt that it limits students intellectual curiosity and does not even enable them to learn drill-type questions more effectively. Certainly most of the research studies do not support this statement about drill-and-practice.

Clark (1983), after scrutinizing look the research and reviews, concluded that it was the uncontrollable effects of novelty and instructional method which accounted for the existing evidence for the effects of various media on learning gains. Certainly Bangert-Drowns, Kulik, and Kulik (1985) concur with this in their finding that short term studies generate more statistically significant results than longer studies.
Clark stated that each new medium rallies advocates around it and they make claims of "improved" learning. He said,

Five decades of research suggest that there are no learning benefits to be gained from employing different media in instruction, regardless of their obviously attractive features or advertised superiority. All existing surveys of this research indicate that confounding has contributed to the studies attributing learning benefits to one medium over another and that the great majority of these comparison studies clearly indicate no significant differences. (p. 450)

He concluded that it was the method of instruction that led more directly and powerfully to learning. He concluded that media are delivery methods for instruction and that it was what the teacher did—the teaching—that influenced learning. This tends to be a difficult concept to argue against.

Individual Studies

Ragosta, Holland, and Jamison (1982) carried out a 4-year study of computational skills of students in 4 elementary schools in Los Angeles, California. They concluded that with only 10 minutes per day of math CAI, students made statistically significant gains in their computational skills compared to control students. Additionally, with 20 minutes per day of math CAI, students doubled gains in computational skills. After controlling for initial differences, CAI students performed at the 64th percentile of their within-class control groups at the end of one year, at the 71st percentile by the end of two years, and at the 76th percentile at the end of three years. These results were based upon standarized mathematics tests.
On curriculum specific tests, CAI students performed at the 79th percentile after one year, the 82nd percentile after two years, and the 89th percentile at the end of three years. This study warrants respect because of its length and the procedural methods which are clearly stated in the report.

In Mattson's (1983) study of three elementary schools in the Pasco School District, CAI gains were statistically significantly higher in computation and total mathematics skills than traditional gains. Mattson also discovered that the grade 3 students benefitted most from CAI while the grade 6 students benefitted the least. The finding that younger students achieve greater benefits than older students was supported by other researchers as well (Ragosta, Holland, & Jamison, 1982; Kulik, Bangert, & Williams, 1983). However, this position is disputed by Robyler (1988), who stated that while past reviews found that elementary levels seemed to profit most from use of microcomputer applications, he found highest effects in college and adult populations.

Bishop (1986) described in a three page editorial a study on transformational geometry skills involving 18 fifteen-year-olds of below average mathematics ability. He conducted his study over a six-week period. He found that the students who had used the microcomputer did better than the control group on the specific skill that he was teaching but that anticipated transfer of learning to other areas did not occur. This is an area where very little research seems to have been conducted to date.

Some examples of studies which have produced negative or neutral effects include that of Signer (1982), Hathaway (1983), and Gallitano (1984). Signer (1982) had used a non-equivalent control group pretest-posttest design to obtain a formative and summative evaluation of CAI in two schools—one control and one treatment. The sample was all Algebra II
students enrolled at each school (120 to 150 students per group). The instruments used were both researcher-designed as well as those adapted from existing CAI evaluation studies. Her formative evaluation stated that student attitudes were neutral to CAI at the end of the study and included the recommendation that CAI be used with small groups of students for drill and practice instead of whole class presentation for developmental work.

Signer's (1982) summative evaluation reports a significant difference for algebra achievement favoring the control group. The length of her study is not indicated in the abstract, and thus it is difficult to adequately "label" her study as brief or long in looking at attitudinal results. Her choice of two schools for treatment and control raises questions about the validity and reliability of the results obtained as no mention is made of their similarities or differences.

Hathaway (1983) was the director of a two-year evaluation study of the effectiveness of CAI on reading and mathematics skills conducted in the Portland Public School system. It involved six elementary (grades 5-8) schools. The study included only those students who had experienced at least one semester and 300 minutes of computer time. The results indicated: student learning in the basic skills did not improve; mean achievement gains for students participating in each CAI program were less than normally expected of groups throughout the district and less than those of other similar groups; but that students liked working on the computer and thought it helped them learn. The recommendation was for this program to be terminated as the results did not indicate any reasons for continuing and a modification of the CAI program was not warranted.

Gallitano (1984) conducted a five-month experiment on the effects of a computer-based approach to teaching trigonometry using researcher-
developed software. The study involved two New Jersey high school classes, and each student in the experimental group used a Commodore Pet microcomputer while each member of the control group had use of a TI-30 calculator. The students were pre- and posttested with the Fennema-Sherman Mathematics Attitudes Scales and the NLSMA Ideas and Preferences Inventory. The results indicated an improvement in motivation and attitude toward mathematics by the experimental group. The analysis of scores on a researcher-developed instrument to measure mathematics achievement indicated no significant overall differences between the two groups. However, a question-by-question analysis showed a 10 percent better performance by the computer students. Gallitano made a good point by stating that the computer students performed as well as, if not better than, the non-computer group in spite of the fact that they covered more material. By this, she meant that the computer group had also had to learn Basic programming. Gallitano concluded that the use of computers increased efficiency and has advantages over calculators for teaching trigonometry, and that positive attitude changes occurred in the computer group but no attitude changes occurred in the control group.

Surveys

In 1980-81, a pilot project was conducted in the elementary and secondary schools of British Columbia to determine the most beneficial uses of microcomputers in the schools and to provide information regarding acquisition and development of courseware and integration of CAI materials into the curriculum. The project was funded by the Ministry of Education and coordinated by Joint Educational Management (JEM) research group at the University of Victoria, under the direction of William Tennant (Forman,
The Apple II was chosen as the microcomputer for use, and 100 computers were installed in 12 districts, selected from among 50 that had submitted proposals for inclusion in the project. As a part of the project, in-service training was provided, covering major aspects of using microcomputers as well as acquiring, developing, and using courseware. The results, which were gathered largely through questionnaires to participating teachers, indicated that the microcomputers were used extensively to support the education process, and that most teachers felt such use was valuable and should be continued and expanded. Student achievement was reported as being as good as traditional instruction by 59% of the teachers, and as being better by 33% of the teachers. The project reaffirmed that the microcomputer is at its best when used as an aid to instruction, not as a substitute.

In 1980, Chambers and Sprecher surveyed current CAI trends in the United States as well as throughout the world. They found that the microcomputer was viewed as a positive force in the classroom. They listed the advantages of CAI as the following: it involved the individual actively in the learning process; it permits students to proceed at their own pace; it provides immediate and systematic reinforcement of learning; and students can explore time and space as well as mix chemicals, etc. safely. The disadvantages that they cited included the need for trained personnel, the cost, and the fact that the microcomputer is still in a primitive form; and there is a lack of coordination amongst educators involved in its evolution.
Findings regarding the saving of time

Findings from the 1970s regarding the effectiveness of microcomputers in reducing the amount of time required for students to learn material is supported by all of the major reviews and studies of the 1980s (Chambers & Sprecher, 1980; Gleason, 1981; Ragosta, Holland, & Jamison, 1982; Kulik, Bangert, & Williams, 1983; Bangert-Drowns, Kulik, & Kulik, 1985; Parry, Thorkildsen, Biery, & Macfarlane, 1985; Eisenrauch, 1985; Bitter & Cameron, 1986; Colorado, 1988; and Roblyer, 1988).

In the meta-analysis conducted by Kulik et al. (1983), two studies were identified that reported findings of substantial time savings of students using CAI. Studies conducted with secondary students revealed 39% and 88% savings in student learning time. If no other positive effects were indicated with CAI, reduced learning time offers the potential for cost savings or opportunites for greater student advancement than does traditional instruction (Eisenrauch, 1985). In one study, a cost benefit analysis concluded that the costs of CAI were equivalent to the benefits that might be gained from equal amounts of tutoring (Ragosta, Holland, & Jamison, 1982). It appears that this is an area of research that could have a very significant impact on the desire to integrate the microcomputer into the classroom if studies continue to impress with the conservation of time.

Findings regarding student attitude

The results of studies of attitude and motivation were also similar to those found in the 1970s. The general consensus of the 1980s is that students enjoy working on the microcomputers and that their attitudes toward their work seem to improve as a result (Chambers & Sprecher, 1980; Forman, 1981; Gleason, 1981; Bracey, 1982; Ragosta, Holland, & Jamison,
1982; Hathaway, 1983; Kulik, Bangert, & Williams, 1983; Mattson, 1983; Bangert-Drowns, Kulik, & Kulik, 1985; Bishop, 1986; Parry, Thorkildsen, Biery, & Macfarlane, 1985; Trollip & Alessi, 1988). Robyler (1988) did not concur, however, stating that from his review of the literature of the 1980's there was little evidence to support the "widely-held belief that good attitudes toward computers result in better attitudes toward school work and higher achievement" (p. 88). In the face of so many supportive studies, it is difficult to give credence to Robyler's statement.

Findings regarding gender differences

The research in this area is extremely limited to date. Gallitano (1984) found in her study that overall there was no statistically significant difference between the experimental and control groups in achievement. However, within the experimental group, females scored higher than males.

Robyler (1988) found in his review of studies that while males tended to achieve slightly higher than females, the results were not statistically significant.

Summary of the 1980s

A summary of the literature of the 1980s reads very much like the summary of the 1970s. Researchers generally agree that the use of the microcomputer enhances traditional classroom instruction by improving student achievement. Its use reduces learning time and produces positive attitudes in students. It can be added from the more current research of the 80s that elementary students appear to benefit more from the use of the microcomputer than secondary students, and that low-achieving and high-achieving students tend to benefit more than average-achieving students.
The question regarding the retention of material learned via CAI has still not been answered.

**SUMMARY**

Research has shown that microcomputers are good at providing drill-and-practice, particularly in the field of mathematics. The microcomputer is a tireless and noncomplaining tool which can provide repetitious practice for which human teachers often have too little time. The microcomputer's interactive capabilities enable it to communicate with students on a one-to-one nonthreatening basis providing personalized feedback which deals specifically with the user's response to a given question. This feedback provides immediate and relevant information to the learner and causes the student to become more actively engaged in learning. Most importantly, the advent of features such as graphics, music, and color capabilities make the microcomputer a vivid, exciting tool which can make learning faster and possibly more enjoyable.

**RESEARCH CLOSELY RELATED TO THE PRESENT STUDY**

In Ronan's study in 1971, following the teaching of trigonometric identities and formulas, the control group scored significantly higher than the experimental group. Gallitano in 1984, received slightly higher results (although not statistically significant) from the experimental group during her trigonometry study. However, Ronan's study was conducted on a mainframe computer, and the description of the lesson as being trigonometric identities is quite different from the lesson of this study which is to graph trigonometric functions. Consequently, it is more likely that this study will produce results such as those obtained by Gallitano.
Generally, the research seems to indicate that in this study, the microcomputer group can be expected to do at least as well as the textbook group if not better. However, the studies reviewed involving trigonometry provide conflicting results.

Since the present study involves grade 12 students, the research seems to indicate that they may not benefit as much as students at a lower grade level. Robyler’s (1988) review of CAI research studies in which he concluded that the highest achievement effects were found in college and adult populations rather than at the elementary level casts some doubt on this finding.

Similarly, since these students are mostly of average ability, the effectiveness of the microcomputer is not likely to be as significant as if they were either high or low-achieving students (Burns & Bozeman, 1981; Fisher, 1983; Kulik, Bangert, & Williams, 1983; Eisenrauch, 1985).

The question raised in this study relating to gender differences is similarly confronted with controversy. This appears to be an area where there is indeed a shortage of research. Do males and females respond differently to instruction given on a microcomputer? Perhaps this study will generate an answer.

Is there a “better” order in which to present material to students? Is it more effective for students to struggle with paper and pencil prior to allowing the microcomputer to do calculations and generate graphs? Is it more effective for students to get an idea of the “whole picture” from the microcomputer prior to using paper and pencil?

These are some of the questions left unanswered by the review of the literature. Hopefully this study will provide some insight, and possibly some answers.
The very fact that controversy exists as to the effectiveness of CAI makes this a worthwhile study to complete. The results of this study will add to what is currently an inconclusive body of knowledge.
Chapter 3
METHOD AND PROCEDURE

The sample, teachers, assessment program, treatments, procedures, research design, and statistical analyses, along with the null hypotheses, are described in this chapter.

Sample

This study was conducted at Burnaby Central Secondary School, which is situated in the Lower Mainland of British Columbia, Canada. This is a large, urban high school enrolling 1300 students in grades 8 through 12. These students come from varied socio-economic backgrounds but the majority could best be described as middle-class. Burnaby Central is a semestered school. Therefore, students receive instruction in each of their courses daily for one semester, a period of five months. The class periods at Burnaby Central are one hour and fifteen minutes in length.

It was intended that the sample would consist of all 85 (44 males and 41 females) grade 12 students enrolled in Algebra 12 at the time of the study. However, the results of 7 of those students (3 males and 4 females) were not included in the final sample because they missed either Posttest 1 or Posttest 2. It will be shown later in the chapter that this group of 7 students was fairly typical of the remaining 78 students (41 males and 37 females), and therefore it seems reasonable to assume that the deletion of their scores did not significantly affect the results.

Teachers

There were two teachers involved in the study. They had been teaching together at Burnaby Central for two years at the time of this
study. Both had a thorough knowledge of the Algebra 12 curriculum. They had similar teaching styles and often shared teaching materials and ideas. Prior to the study they spent three to four hours together discussing the instructional procedures that would be followed in order to minimize any contamination that might result from inconsistent presentation of materials.

Instrumentation

Algebra 12 Achievement Score

Since there would possibly be differences in mathematical ability between the classes in this study, it was important to control for these initial differences. A pretest was not justified because the students' knowledge of graphing trigonometric functions would have been minimal, and consequently such an instrument would not have been useful as an indicator of the amount learned as a result of the treatment. The researcher had considered using IQ scores and the student's Algebra 11 letter grades as covariates. However, the IQ scores were missing from a number of personal record cards, and the Algebra 11 grades would have offered limited information in that students had been assigned letter grades rather than numerical grades. The researcher did not feel that this five point scale (A through P) would help distinguish the differences in the students. Therefore, it was decided that a mark calculated from scores earned on the four chapter tests (151 marks) that the students had already completed would be a more reliable and accurate measure of each student's mathematical ability at the time of the study. This mark, therefore, was selected as the covariate in the analysis of the data. A sample of one of these chapter tests is included in Appendix H, along with the coefficient of
reliability for the marking methods of the two teachers involved in this study (Appendix I). The composition of this Algebra 12 achievement score will be detailed later in this chapter as will the determination of the reliability of the marking styles for the two teachers.

**Algebra 12 Trigonometric Graphing Tests (Posttests A and B)**

Since trigonometric graphing is a small part of the Algebra 12 curriculum, the teachers involved felt that a 15 mark test would be adequate to test students' knowledge on this topic. The researcher used previously written provincial exams to develop a pool of 32 possible questions about trigonometric functions. Since the provincial exam questions are written by groups of teachers from throughout the province, and since preparation for provincial exams is an integral part of this course, the researcher felt that these questions would be valid. Also, these posttests were comparable to tests typically given to Algebra 12 students on this topic.

It was decided to construct two forms of the test, one test to be administered to the morning classes, and the second test to be administered to the afternoon classes. In this way, students in the afternoon classes would not have an advantage over the students in the morning classes by being informed of the questions over lunchtime. The researcher selected 10 multiple choice questions for each posttest. The selected questions were comparable in difficulty as judged by the two teachers involved in the study. (See Appendix D for Posttests A and B titled "Algebra 12 Trigonometric Graphing Test"). These questions were computer marked as they were multiple choice. The remaining 5 marks on each test consisted of two two-mark questions and one one-mark question. The teachers involved felt that
there should be some student-generated work as well as multiple choice questions. These three questions were hand-marked by the teachers. A bonus question given on each posttest was not included in the analysis.

Treatments

There were two treatments given. One treatment involved the use of the microcomputer and related worksheets. The other treatment involved the use of a textbook. This second treatment could be referred to as traditional classroom instruction. Each treatment was sufficient to adequately cover the topic of graphing trigonometric functions.

Microcomputer Treatment

The microcomputer treatment used the software program Master Grapher (Waits, Demana, & New, 1988) plus teacher-produced worksheets. This software program generates the graphs of most equations, not just trigonometric equations. The user can define the domain and range desired for the equations entered, and specify that the domain be graphed in radians or standard units of measure.

This treatment consisted of two lessons over two days. On day one, in lesson one (Thursday, May 11), the students were given a classroom demonstration using a Macintosh computer, a Kodak Liquid Crystal Display (LCD) unit, and an overhead projector to project the microcomputer screen onto a classroom presentation screen. The topic was graphing trigonometric functions and the software program was "Master Grapher". During the demonstration, the students were taught the concepts of "amplitude" and "period" and were given a basic idea of how to graph trigonometric functions from observing the graphs on the presentation screen.
screen. The students were instructed to copy the graphs shown on the screen onto graph paper supplied to them so that they would be taking a more active part in the demonstration. As well, they were asked to predict the period and amplitude of the functions prior to the microcomputer generating the graphs. Throughout the demonstration, students were being instructed how to use the "mouse" and the microcomputer because they would be going to the Macintosh lab the following day. (See Appendix A for a complete outline of the lesson).

At the end of day one, the students were given eight questions which were to be completed for homework in preparation for their next day in the Mac lab. These eight questions ranged greatly in degree of difficulty. They included questions as routine as graphing $y = -2 \sin x$ as well as much more difficult equations to graph such as $y = \sin (\sin x)$. Upon checking homework the next day, the teachers found that students had attempted most of the questions; however, their graphing attempts ranged widely in creativity and accuracy.

On day two, in lesson two (Friday, May 12), these students were taken to the Mac lab. Their first assignment was to enter the equations which they had been asked to graph for homework into the microcomputer, and to copy the graphs of any equations which they had done incorrectly or had not been able to do. This tended to be a rather slow process as most of the students had not previously worked on a microcomputer. The fact that they had been able to observe the teacher entering information the day before did help somewhat. Eventually, the students managed to enter all the equations and gain feedback from observing the computer-generated graphs. Not all of the exercises that were included in the work for lesson two in the lab were completed by most students. There appeared to be too much work for
most of them to complete. It appears that if their typing and/or computer skills had been better, they would have been able to complete all of the work given to them (See Appendix B). At the end of the class, students were given an answer key for the work from lesson one and lesson two (See Appendix C). The students were informed that they would be getting a 15 mark test on the following day.

On day 3 (Monday, May 15), the students in the morning class were given Posttest 1A and the students in the afternoon class were given Posttest 1B. All students were given exactly 27 minutes to complete the tests. Questions on the tests were not discussed following the administration of the tests. Students then went on to another section of the chapter (proving trigonometric identities) for the remainder of the period.

**Textbook Treatment**

This treatment also consisted of two lessons over two days in which students were given traditional classroom instruction on graphing trigonometric functions. On day one, in lesson one (Thursday, May 11), the teachers used blackboard and chalk to "hand-plot" the sine and cosine curves and define the terms "amplitude" and "period". Both teachers used section 13.4 of *Algebra and Trigonometry: Structure and Method - Book 2* (Dolciani, Sorgenfrey, Brown, and Kane, 1986, pp. 587-591). An outline of the lesson that was used can be found in Appendix E. Following the lesson, the classes did the oral exercises and were then assigned the odd-numbered exercises on page 591 from #1 to #43. They were given a solution key so that they were able to check their answers as they did their homework (See Appendix G).
On day two, in lesson two (Friday, May 12), the material from lesson one was reviewed, and the students were introduced to the tangent and cotangent graphs as well as touching on the secant and cosecant graphs. Little time was spent on the reciprocal graphs. The students were then assigned the odd-numbered exercises (#1 to #31) on page 467-468 from Algebra and Trigonometry: Structure and Method (new edition) - Book 2 (Dolciani et al., 1984). These questions included more work on graphing basic trigonometric functions. (See Appendix F.) The students were given an answer key to go along with this assignment. (See Appendix G). The students were informed that they would be given a 15 mark test the following day.

On day 3 (Monday, May 15), the students in the morning class were given Posttest 1A and the students in the afternoon class were given Posttest 1B. They were given exactly 27 minutes to complete the test. The questions were not discussed following the test and students proceeded to work on another section of the chapter (proving trigonometric identities) for the remainder of the period.

Treatment Verification

The researcher observed the other teacher on two of the four instructional days and concluded that both teachers were carefully following the lesson plans for each method.

Procedure

This section will outline how the classes were assigned to treatments. In addition, it will give details of the two phases of the study.
Selection Process

Two weeks prior to the beginning of the study, the two teachers tossed a coin to see which of their two classes would be selected to become the Microcomputer First group (MCF) or the Textbook First group (TBF). There were four Algebra 12 classes being offered. Blocks A and B were held in the morning and Blocks C and D in the afternoon. Each of the teachers taught two blocks: Teacher I taught Blocks B and C and Teacher II taught Blocks A and D. The coin toss determined that for Teacher I, Block B would be part of the MCF group while Block C would be part of the TBF group, and for Teacher II, Block D would join the MCF group and Block A would join the TBF group. By randomly assigning one class of each teacher to the microcomputer treatment first and the other to the textbook treatment first, the researcher attempted to control for contamination of the results by teacher methodology. Table 3.1 summarizes the composition of the MCF and TBF groups for the study. The MCF group consisted of a total of 46 students (29 males and 17 females) while the TBF group consisted of a total of 32 students (12 males and 20 females).

Treatment Sequence

This study took place from Thursday, May 11, 1989 until Thursday, May 18, 1989, a total of six school days. There were two phases of the study. Phase one lasted for three days (May 11, 12, and 15) as did phase two (May 16, 17, and 18). Prior to the first phase, two groups were randomly assigned as the MCF group (Blocks B and D) while the remaining two groups became the TBF group (Blocks A and C). In phase two, the treatments were reversed whereby the MCF group completed the textbook work, and the TBF group completed the microcomputer work.
Table 3.1
Composition of Groups

<table>
<thead>
<tr>
<th>Group</th>
<th>Block</th>
<th>Teacher</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sex</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>Mortality</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Study</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>M</td>
</tr>
<tr>
<td>B</td>
<td>I</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>M</td>
<td>D</td>
<td>II</td>
<td>16</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
<td>31</td>
</tr>
<tr>
<td>A</td>
<td>II</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>T</td>
<td>C</td>
<td>I</td>
<td>7</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
<td>13</td>
</tr>
</tbody>
</table>

At the end of phase one of the study, Posttest A was given to the morning classes (called Posttest 1A), and Posttest B was given to the afternoon classes (called Posttest 1B). To ensure that the second set of posttests was not more difficult than the first set, the same tests were administered, but in reverse order by time, on day 6 (called Posttest 2A and Posttest 2B). The posttests were not discussed in class nor were the
students given their results of the first posttests until after the second posttests had been written. Table 3.2 summarizes the treatment sequence.

Table 3.2
Treatment Sequence

<table>
<thead>
<tr>
<th>PHASE 1</th>
<th>PHASE 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intact Classes</td>
<td>Treatment</td>
</tr>
<tr>
<td></td>
<td>Test</td>
</tr>
<tr>
<td>Intact Classes</td>
<td>Treatment</td>
</tr>
<tr>
<td></td>
<td>Test</td>
</tr>
<tr>
<td>Days 1 and 2</td>
<td>Day 3</td>
</tr>
<tr>
<td>Days 4 and 5</td>
<td>Day 6</td>
</tr>
<tr>
<td>Block B</td>
<td>Microcomputer</td>
</tr>
<tr>
<td>Block D</td>
<td>Microcomputer</td>
</tr>
<tr>
<td>Block A</td>
<td>Textbook</td>
</tr>
<tr>
<td>Block C</td>
<td>Textbook</td>
</tr>
<tr>
<td></td>
<td>Posttest 1A</td>
</tr>
<tr>
<td></td>
<td>Posttest 1B</td>
</tr>
<tr>
<td></td>
<td>Textbook</td>
</tr>
<tr>
<td></td>
<td>Posttest 1A</td>
</tr>
<tr>
<td></td>
<td>Posttest 1B</td>
</tr>
<tr>
<td></td>
<td>Microcomputer</td>
</tr>
<tr>
<td></td>
<td>Posttest 1B</td>
</tr>
</tbody>
</table>

**Phase one of the study.** On day 1 (Thursday, May 11) and day 2 (Friday, May 12), Blocks B and D (the MCF Group) were given the microcomputer treatment while Blocks A and C (the TBF Group) were given the textbook treatment as described earlier in this chapter. On day 3 (Monday, May 15), Blocks A and B were administered Posttest 1A (morning test), and Blocks C and D were administered Posttest 1B (afternoon test).
Phase two of the study. Following phase one of the study, the treatments were interchanged for phase two.

On day 4 (Tuesday, May 16) and day 5 (Wednesday, May 17) the students in Block B and D who had completed the microcomputer treatment (the MCF group) were given the textbook treatment. Similarly, the students in Blocks A and C who had completed the textbook treatment (the TBF group) were given the microcomputer treatment on day 4 and 5.

On day 6 (Thursday, May 18), the students in morning Blocks A and B were given Posttest 1B (originally the afternoon test), and the students in afternoon Blocks C and D were given Posttest 1A (originally the morning test). The first posttest was not given back or discussed with any of the classes prior to the administration of the second posttest.

Upon completion of the study, the students were given their test scores for both posttests. The students had been told that their mark for this trigonometric graphing section would be an average of the two posttest scores.

Design

In this section the design for the study will be outlined. As well, threats to internal and external validity will be discussed.

The Counterbalanced Design

A counterbalanced design, as outlined by Campbell and Stanley (1963, pp. 50-52) and Borg and Gall (1983, pp. 703-705) was selected for the study. This design was used since it was decided that all groups should receive both treatments. Borg and Gall suggest that a counterbalanced design is an appropriate design for examining the influence of order of treatments on the
dependent variable. Assignment of individuals to treatment groups was not possible since students were in intact classes. However, the two classes of each teacher were randomly assigned to the treatment groups. Table 3.3 shows a diagram of the research design.

Table 3.3
Outline of Research Design

<table>
<thead>
<tr>
<th>MCF Group</th>
<th>O₀</th>
<th>X₁</th>
<th>O₁</th>
<th>X₂</th>
<th>O₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBF Group</td>
<td>O₀</td>
<td>X₂</td>
<td>O₁</td>
<td>X₁</td>
<td>O₂</td>
</tr>
</tbody>
</table>

The following abbreviations are used in Table 3.3:

- MCF - Microcomputer First group
- TBF - Textbook First group
- O₀ - the covariate (algebra 12 achievement score)
- O₁ - the first posttest
- O₂ - the second posttest
- X₁ - microcomputer treatment
- X₂ - textbook treatment

All students received the same instruction by the end of the study, which is one of the strengths of this type of design. This type of a design was necessary so that students would not be denied the benefits of either one of the instructional methods.
Analysis of Covariance (ANCOVA)

Posttest 1 results were analyzed by ANCOVA using one covariate, the Algebra 12 achievement score of each student. This provides a test of the main effect for treatment. Posttest 2 results were analyzed in two different ways by ANCOVA. Firstly, the results were analyzed using one covariate, the Algebra 12 achievement scores, to test for the effects on achievement of the order of the two treatments. Secondly, the Posttest 2 results were analyzed using two covariates, the Algebra 12 achievement scores and the Posttest 1 results, to test for the main effect of the second treatment on achievement.

Threats to Internal Validity

Campbell and Stanley (1963) assert that the counterbalanced design controls for threats to internal validity caused by history, maturation, testing, instrumentation, regression, selection, and mortality. This would seem to be the case in this study.

History does not appear to be a threat in that an event that might affect scores on the dependent variable would be experienced by both groups. Also, the brief period of six days for this study tends to control for contamination due to historical factors.

Maturation does not appear to be a threat to internal validity since all subjects in this study are about the same age, and because this study is of such a short duration.

Testing is not a factor since there was no pre-test given. Similarly, with respect to instrumentation, there was no change in the type of instrument used, and both sets of instruments were marked at the same time.
With respect to selection bias, as will be shown later in Chapter 3, a t-test revealed that there was no statistically significant difference in the algebra 12 achievement scores of the two groups in this study. The effect of the relatively small difference that did exist in the algebra 12 achievement score means of the two groups was controlled by using those scores as a covariate in the analyses of the data.

With respect to mortality, seven of the 85 students present at the start of the study were not included in the analysis because they missed either Posttest 1 or Posttest 2. Four of these students were female and three were male. Their Algebra 12 achievement scores were as follows: 89.1%, 80.6%, 65.9%, 64.6%, 57.3%, 49.9%, and 42.7%. The mean of these scores is 64.3% which seems to indicate that this group is similar to the remaining 78 students (41 males and 37 females) whose mean score was 66.6%, and therefore it did not seem unreasonable to assume that the deletion of their scores in any way influenced the results. Thus, mortality was not a threat to the internal validity of this study.

With respect to regression toward the mean, the students selected for this study were not selected because of their above or below average abilities and, therefore there was no reason to believe that regression would be a threat.

Threats to External Validity

Campbell and Stanley (1963) suggested that the interaction of testing and the treatment, selection and the treatment, reactive arrangements, and multiple treatment interference are possible threats to external validity posed by the counterbalanced design.
Testing was not likely to be a threat since no pre-test was administered, and the posttests were the same for both groups.

Additionally, there was no reason to think that the treatments would be related to student characteristics specific to this sample. In fact, the students in this sample had a mean score of 66.83% on the 1989 provincial Algebra 12 exam. This mean was very similar to the provincial mean for Algebra 12 of 67.48% (Ministry of Education, 1989).

The possibility of an effect of reactive arrangements as a threat to external validity should be considered when interpreting the results of the study. The students were aware that they were involved in a study, however, there was no evidence to suggest that this affected the results.

The threat of multiple treatment interference is the main problem with this type of research design. Certainly in this study there is a carry-over effect of the information absorbed in phase one onto phase two. However, since both groups have the same opportunity for this carry-over effect, it tends to neutralize itself. As well, this carry-over effect was taken into account when the data on Posttest 2 were analyzed. This was accomplished by using two covariates: the knowledge gained during phase one of the study (Posttest 1), and the initial differences prior to the start of the study (algebra 12 achievement scores). In this way, the interference of the multiple treatment was controlled. Part of the study was to determine if there was a more effective treatment order, and according to Borg and Gall (1983) this is a strong design for conducting such an analysis.

Statistical Analyses

All calculations were performed at the University of British Columbia Computing Centre using the SPSS:X statistical package.
The level of significance accepted for rejecting null hypotheses was .05 which is the most commonly used probability level in education research (Borg & Gall, 1983, p. 373; Ary, Jacobs, & Razavieh, 1979, p. 144).

Data Collected

Test papers and scores were obtained from the 78 subjects for each of the following shown in Table 3.4. Results of individuals can be found in Appendix K.

Table 3.4
Data Collected

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
<th>Date written</th>
<th>Possible score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapter 9a</td>
<td>Conics Test 1</td>
<td>February 21, 1989</td>
<td>32</td>
</tr>
<tr>
<td>Chapter 9a</td>
<td>Conics Test 2</td>
<td>March 3, 1989</td>
<td>39</td>
</tr>
<tr>
<td>Chapter 10a</td>
<td>Logarithms</td>
<td>March 22, 1989</td>
<td>40</td>
</tr>
<tr>
<td>Chapter 11a</td>
<td>Sequences and Series</td>
<td>April 19, 1989</td>
<td>40</td>
</tr>
<tr>
<td>Posttest One</td>
<td>Trigonometry</td>
<td>May 15, 1989</td>
<td>15</td>
</tr>
<tr>
<td>Posttest Two</td>
<td>Trigonometry</td>
<td>May 18, 1989</td>
<td>15</td>
</tr>
</tbody>
</table>

These chapters are from Using Advanced Algebra (Travers, Dalton, Brunner, & Taylor, 1977). The Algebra 12 achievement score (the covariate) is comprised of these four tests.
Reliability of scores from Teacher 1 and II

To determine a coefficient of reliability for the marking styles of the two teachers, the teachers randomly selected the tests of 5 students from each of their two classes following the writing of the Chapter 11 test (Sequences & Series). Both teachers independently marked these 20 test papers. Scores were recorded and the Pearson-Product Correlation was used to find a coefficient of reliability of .995 which indicated that inter-rater reliability was satisfactory. Scores are shown in Appendix I. Therefore, the marks from Chapters 9, 10, and 11 which were summed to produce a composite mark of algebra 12 achievement for each student were thought to be a reliable measure to use as the covariate.

Initial Group Comparison

To determine if there were statistically significant differences between the two groups used in this study, an unpaired t-test was run. The results are summarized in table 3.5.

<table>
<thead>
<tr>
<th>Group</th>
<th>Mean</th>
<th>SD</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCF</td>
<td>64.30</td>
<td>16.01</td>
<td>-1.60</td>
<td>.1154</td>
</tr>
<tr>
<td>TBF</td>
<td>70.03</td>
<td>15.08</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Since the probability of this t-value occurring is greater than .05, the initial differences in these two groups is not considered to be statistically significant. Statistical significance, of course, is related to sample size. The difference in the groups means (MCF = 64%; TBF = 70%) might, however, have been statistically significant had the sample size been larger. Therefore, it was decided that analysis of covariance, rather than analysis of variance, was the appropriate statistical test to use in this case.

T-tests were also run on the Algebra 12 achievement scores to determine if there were any initial differences by gender, time of day of the course, and for the number of times the course had been taken (first time as compared to second time). The results have been summarized in table 3.6.

Males and females were not different on Algebra 12 achievement prior to the study. Similarly, time of day had no significant effect on Algebra 12 achievement.

The mean score of the students who were taking the course for the second time was significantly greater than the mean score of the students taking the course for the first time. In other words, the 12 students who were repeating the course entered this study with a much higher Algebra 12 achievement mean score (81%) than the 66 students who were taking the course for the first time (64%).
Table 3.6

T-tests of Algebra 12 achievement scores of students arranged by sex, time of day, and number of times course taken

<table>
<thead>
<tr>
<th>Factor</th>
<th>Count</th>
<th>Mean</th>
<th>SD</th>
<th>t-value</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>41</td>
<td>66.19</td>
<td>15.73</td>
<td>-.27</td>
<td>.79</td>
</tr>
<tr>
<td>Female</td>
<td>37</td>
<td>67.16</td>
<td>16.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Morning</td>
<td>33</td>
<td>67.79</td>
<td>16.97</td>
<td>.55</td>
<td>.59</td>
</tr>
<tr>
<td>Afternoon</td>
<td>45</td>
<td>65.81</td>
<td>15.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>First time</td>
<td>66</td>
<td>64.03</td>
<td>14.93</td>
<td>-3.71</td>
<td>.0004*</td>
</tr>
<tr>
<td>Second time</td>
<td>12</td>
<td>81.05</td>
<td>12.66</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .05

Analysis of Posttest Results

Analysis of variance is the appropriate statistical technique for treating data collected with the counterbalanced design (Campbell & Stanley, 1963; Borg & Gall, 1983). However, since prior algebra achievement scores were available for each student, it was decided to use an analysis of covariance as a means for adjusting for any initial group differences.

Posttest 1 results were analyzed by an analysis of covariance to test the statistical significance of differences in the performance of the treatment groups. The covariate used was the Algebra 12 achievement
score. In this way, the mean scores obtained on the posttests could be adjusted for the initial differences between the groups. ANCOVA on Posttest 1 was also used to analyze the main effects of sex, time of day, and number of times taking the course, as well as to analyze treatment by sex, treatment by time of day, and treatment by number of times taking the course.

Posttest 2 results were analyzed using two covariates. The Algebra 12 achievement scores were used to account for initial differences and the Posttest 1 results were used to control for knowledge gained in phase one of the study. The purpose of this analysis was to determine the effect of the second phase of the treatment. Posttest 2 results were also analyzed by ANCOVA for the main effects of sex, time of day, and number of times taking the course, as well as for differences resulting from interactions of treatment and gender, treatment and the time of day of the class, and treatment and the number of times the course was taken.

Additionally, the Posttest 2 results were analyzed using one covariate (the Algebra 12 achievement score) to determine whether the order of the treatment affected the student's final results.

Effect Sizes

Effect sizes were calculated for Posttest 1 and 2 using the pooled variance model (Cohen, 1977). In this model the effect size is calculated by subtracting the control-group mean from the experimental-group mean and then dividing the difference by the pooled standard deviation. The complete formula used in this study can be found in Appendix J.

Effect sizes (ES) are independent of sample size. Calculating an effect size helps the researcher determine if a trend is evident in the data being
studied, and if perhaps the results would have been statistically significant had the size of the sample been much larger (Borg & Gall, 1983). For the purposes of this study, a small effect size will be .2, a medium effect size will be .5, and a large effect size will be .8 as suggested by Cohen (1977).

Null Hypotheses Tested

The data gathered from the covariate, Posttest 1, and Posttest 2 were used in a statistical analysis designed to test the following null hypothesis:

Null Hypothesis 1

A. There will be no significant difference between the adjusted mean Posttest 1 scores on the Algebra 12 Trigonometric Graphing Test of the students who received the microcomputer treatment and of the students who received the textbook treatment using Algebra 12 achievement as the covariate.

B. There will be no significant difference between the adjusted mean Posttest 2 scores on the Algebra 12 Trigonometric Graphing Test of the students who received the microcomputer treatment and of the students who received the textbook treatment using Algebra 12 achievement and Posttest 1 as covariates.

Several questions posed as null hypotheses were also tested.

Null Hypothesis 2

There will be no significant difference between the adjusted mean Posttest 2 scores on the Algebra 12 Trigonometric Graphing Test of the
students in the MCF group and of the students in the TBF group using Algebra 12 achievement as a covariate.

**Null Hypothesis 3**

A. There will be no significant difference between the adjusted mean Posttest 1 scores on the Algebra 12 Trigonometric Graphing Test of the students in both treatment groups arranged according to sex, using Algebra 12 achievement as the covariate.

B. There will be no significant difference between the adjusted mean Posttest 2 scores on the Algebra 12 Trigonometric Graphing Test of the students in both treatment groups arranged according to sex, using Algebra 12 achievement and Posttest 1 as covariates.

C. There will be no interaction effect of "Treatment" and "Sex" on the adjusted mean scores of Posttest 1, using Algebra 12 achievement as the covariate.

D. There will be no interaction effect of "Treatment" and "Sex" on the adjusted mean scores of Posttest 2, using Algebra 12 achievement and Posttest 1 as covariates.

**Null Hypothesis 4**

A. There will be no significant difference between the adjusted mean Posttest 1 scores on the Algebra 12 Trigonometric Graphing Test of the students in both treatment groups arranged according to number of times the course has been taken, using Algebra 12 achievement as the covariate.

B. There will be no significant difference between the adjusted mean Posttest 2 scores on the Algebra 12 Trigonometric Graphing Test of the students in both treatment groups arranged according to number of times
the course has been taken, using Algebra 12 achievement and Posttest 1 as
covariates.

C. There will be no interaction effect of “Treatment” and “Number of
Times” on the adjusted mean scores of Posttest 1, using Algebra 12
achievement as the covariate.

D. There will be no interaction effect of “Treatment” and “Number of
Times” on the adjusted mean scores of Posttest 2, using Algebra 12
achievement and Posttest 1 as covariates.

Null Hypothesis 5

A. There will be no significant difference between the adjusted mean
Posttest 1 scores on the Algebra 12 Trigonometric Graphing Test of the
students in both treatment groups arranged according to the time of day the
course is taken, using Algebra 12 achievement as the covariate.

B. There will be no significant difference between the adjusted mean
Posttest 2 scores on the Algebra 12 Trigonometric Graphing Test of the
students in both treatment groups arranged according to the time of day the
course is taken, using Algebra 12 achievement and Posttest 1 as covariates.

C. There will be no interaction effect of “Treatment” and “Time of Day”
on the adjusted mean scores of Posttest 1, using Algebra 12 achievement as
the covariate.

D. There will be no interaction effect of “Treatment” and “Time of Day”
on the adjusted mean scores of Posttest 2, using Algebra 12 achievement
and Posttest 1 as covariates.
SUMMARY

In this chapter the procedures followed in the study and the statistical analyses are discussed. The researcher realized that an analysis of variance (ANOVA) is adequate for analyzing the data in this study as it has been shown that the initial differences of the two groups being studied were not statistically significant. However, more accurate results would be generated by using an analysis of covariance (ANCOVA) so that the small initial differences in the two groups could be taken into account throughout the analyses.
Chapter 4
RESULTS

Following a brief description of the statistical procedure used to analyze the data, and a summary of the data for the group as a whole, this chapter deals with the disposition of the statistical, or null, hypothesis stated in Chapter 3. Additionally, the several questions listed in the preceding chapter and posed as null hypotheses will also be examined. The level of significance for rejecting the null hypothesis was set at .05, which is the most commonly used probability level in education research (Borg & Gall, 1983, p. 373; Ary, Jacobs, & Razavieh, 1979, p. 144). Thus, the probability of rejecting the null hypothesis while it was true (a Type I error) was set at 5%.

The analyses presented in this chapter were performed using the SPSS:X statistical package at the University of British Columbia Computing Center.

Introduction

The statistical procedure, analysis of covariance (ANCOVA), was used to analyze the data collected by the assessment instruments. Algebra 12 achievement scores were used as covariates in all the analyses. Posttest 1 scores were used as covariates in some of the ANCOVA's of Posttest 2 scores. ANCOVA increases the precision of the research analysis by removing the effects of initial differences, and thereby identifying more clearly differences that can be attributed to the experimental treatment (Ary, Jacobs, & Razavieh, 1979). Even though the initial difference between the two groups on the Algebra 12 achievement covariate was not
statistically significant (see Table 3.5), the decision was made to use ANCOVA in order to increase precision.

Overview of Data

Individual scores for all students on both posttests as well as their algebra achievement score are listed in Appendix K. Summary data of those scores can be found in Table 4.1. These results are provided to give an overall picture of the data prior to analyzing it for the effects of the treatments and their various interactions.

Table 4.1
Summary Data of Covariate, Posttest 1, and Posttest 2

<table>
<thead>
<tr>
<th></th>
<th>Covariate</th>
<th>Posttest 1</th>
<th>Posttest 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Observations</td>
<td>78</td>
<td>78</td>
<td>78</td>
</tr>
<tr>
<td>Maximum Possible Score</td>
<td>100</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Mean</td>
<td>66.65</td>
<td>8.45</td>
<td>11.25</td>
</tr>
<tr>
<td>Mean as percent</td>
<td>66.7%</td>
<td>56.3%</td>
<td>75.0%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>15.80</td>
<td>2.62</td>
<td>2.25</td>
</tr>
<tr>
<td>Highest Score</td>
<td>100</td>
<td>13</td>
<td>15</td>
</tr>
<tr>
<td>Lowest Score</td>
<td>32</td>
<td>3.5</td>
<td>5</td>
</tr>
</tbody>
</table>
Disposition of Hypotheses

The statistical tests of the null hypothesis and the related questions, which were posed and tested as null hypotheses, are presented along with interpretations of the meaning of the findings.

Null Hypothesis 1: Treatment Differences:

A. There is no significant difference between the adjusted mean Posttest 1 scores on the Algebra 12 Trigonometric Graphing Test of the students who received the microcomputer treatment and of the students who received the textbook treatment, using Algebra 12 achievement as the covariate.

Data from Posttest 1 were analyzed to test this hypothesis and the results of the analysis are found in tables 4.2 and 4.3.

Table 4.2
Posttest 1: Observed and Adjusted Means
on the Trigonometric Graphing Test
(One Covariate: Algebra 12 achievement)

<table>
<thead>
<tr>
<th>Group</th>
<th>Number</th>
<th>Treatment</th>
<th>Observed Mean</th>
<th>SD</th>
<th>Adjusted Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCF</td>
<td>46</td>
<td>Microcomputer</td>
<td>7.62</td>
<td>2.42</td>
<td>7.90</td>
</tr>
<tr>
<td>TBF</td>
<td>32</td>
<td>Textbook</td>
<td>9.70</td>
<td>2.44</td>
<td>9.42</td>
</tr>
</tbody>
</table>
Table 4.3
Posttest 1: Analysis of Covariance for the Adjusted Mean Scores on the Trigonometric Graphing Test
(One Covariate: Algebra 12 achievement)

<table>
<thead>
<tr>
<th>Sources of Variation</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>1</td>
<td>42.28</td>
<td>42.28</td>
<td>11.81</td>
<td>.001*</td>
</tr>
<tr>
<td>Within</td>
<td>75</td>
<td>268.42</td>
<td>3.58</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .01

Results. The probability of this result occurring by chance is one in one thousand, and the null hypothesis was rejected. In this case, the adjusted mean score of the students in the textbook treatment was significantly higher than the adjusted mean score of the students in the microcomputer treatment on Posttest 1 (F = 11.81; df=1, 75; p < .01). The adjusted mean score of the microcomputer group was 7.9 out of 15 while that of the textbook group was 9.4 out of 15.

An analysis of Posttest 2 was conducted using two covariates: (a) Algebra 12 achievement score, and (b) Posttest 1. The students' algebra 12 achievement score was used to control for initial differences and their
Posttest 1 score was used to control for the learning that had occurred in phase one of the study. The results are summarized in tables 4.4 and 4.5.

**Table 4.4**
Posttest 2: Observed and Adjusted Means on the Trigonometric Graphing Test
(Two Covariates: Algebra 12 achievement and Posttest 1)

<table>
<thead>
<tr>
<th>Group</th>
<th>Number</th>
<th>Treatment</th>
<th>Observed Mean</th>
<th>SD</th>
<th>Adjusted Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCF</td>
<td>46</td>
<td>Textbook</td>
<td>11.04</td>
<td>2.11</td>
<td>11.46</td>
</tr>
<tr>
<td>TBF</td>
<td>32</td>
<td>Microcomputer</td>
<td>11.55</td>
<td>2.45</td>
<td>11.13</td>
</tr>
</tbody>
</table>

**Table 4.5**
Posttest 2: Analysis of Covariance for the Adjusted Mean Scores on the Trigonometric Graphing Test
(Two Covariates: Algebra 12 achievement and Posttest 1)

<table>
<thead>
<tr>
<th>Sources of Variance</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>1</td>
<td>1.63</td>
<td>1.63</td>
<td>.60</td>
<td>.439</td>
</tr>
<tr>
<td>Within</td>
<td>74</td>
<td>199.63</td>
<td>2.70</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Results. In phase two of the study, students' adjusted mean Posttest 2 scores indicated that both the textbook treatment and the microcomputer
treatment were equally effective ($F = .439; \text{df}=1, 74; p > .05$). The null hypothesis could not be rejected because the probability of this result occurring was not less than the alpha level set (.05) at the outset of this study.

**Null Hypothesis 2: Order of Treatment Differences:**

There is no significant difference between the adjusted mean Posttest 2 scores on the Algebra 12 Trigonometric Graphing Test of the students in the MCF group and of the students in the TBF group, using Algebra 12 achievement as the covariate.

Since this hypothesis was testing for the most effective order of treatment, Posttest 2 was analyzed using only one covariate: Algebra 12 achievement score. In this way, the end result will reflect knowledge gained in both phases, and the differences being tested will be the order of the treatments. The results obtained were very similar to those described using two covariates. The data are summarized in table 4.6 and 4.7.

**Table 4.6**

Posttest 2: Observed and Adjusted Means on the Trigonometric Graphing Test arranged according to order of treatment
(One Covariate: Algebra 12 achievement)

<table>
<thead>
<tr>
<th>Group</th>
<th>Number</th>
<th>Observed Mean</th>
<th>SD</th>
<th>Adjusted Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCF</td>
<td>46</td>
<td>11.04</td>
<td>2.11</td>
<td>11.32</td>
</tr>
<tr>
<td>TBF</td>
<td>32</td>
<td>11.55</td>
<td>2.45</td>
<td>11.27</td>
</tr>
</tbody>
</table>
Table 4.7
Posttest 2: Analysis of Covariance for the Adjusted Mean Scores on the Trigonometric Graphing Test
(One covariate: Algebra 12 achievement)

<table>
<thead>
<tr>
<th>Sources of Variation</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>1</td>
<td>.06</td>
<td>.06</td>
<td>.02</td>
<td>.885</td>
</tr>
<tr>
<td>Within</td>
<td>75</td>
<td>207.77</td>
<td>2.77</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Results. The mean scores of the two groups indicates that the MCF Group improved considerably after the textbook treatment. They went from a mean score of 7.6 on Posttest 1 to a mean score of 11.0 on Posttest 2. This is an improvement of 3.4 marks out of a possible 15 marks which is an increase of 23%. That group "caught up" to the TBF Group. The textbook treatment group improved not as dramatically following the microcomputer treatment in that they went from a mean score of 9.7 on Posttest 1 to a mean score of 11.6 on Posttest 2. This is an improvement of 1.9 marks out of a possible 15 marks, which is an increase of 13%. It appears as though the use of the microcomputer may have actually slowed the learning of the TBF Group whereas the textbook work seemed to have accelerated the learning of the MCF Group. However, the difference between the adjusted means of the two groups on Posttest 2 was not statistically significant, and the null hypothesis was not rejected. It does not appear to make any difference whether students are exposed to the microcomputer followed by the textbook, or if they complete the textbook treatment first followed by
treatment on the microcomputer. Both groups of students achieved essentially the same result after having completed both treatments. The results are diagramed in Figure 1.

Effect Size

Effect sizes (standardized mean difference) were calculated from the data for Posttests 1 and 2 (See Appendix J). An ES of .86 for the difference between the mean scores of the MCF Group and the TBF Group on Posttest 1 was large and favoured the TBF Group. Although smaller, an effect size of .22 for the difference between the means of the two groups on Posttest 2 favoured the MCF Group, who received the textbook treatment. In both cases, the mean score of the group receiving the textbook treatment exceeded that of the group receiving the microcomputer treatment.

Null Hypothesis 3: Sex Differences:

A. There will be no significant difference between the adjusted mean Posttest 1 scores on the Algebra 12 Trigonometric Graphing Test of the students in both treatment groups arranged according to sex, using Algebra 12 achievement as the covariate.

B. There will be no significant difference between the adjusted mean Posttest 2 scores on the Algebra 12 Trigonometric Graphing Test of the students in both treatment groups arranged according to sex, using Algebra 12 achievement and Posttest 1 as covariates.

C. There will be no interaction effect of "Treatment" and "Sex" on the adjusted mean scores of Posttest 1, using Algebra 12 achievement as the covariate.
FIGURE 1: Observed and Adjusted Means on Posttest 1 and 2

OBSERVED AND ADJUSTED MEANS ON POSTTEST 1 & 2
(Two Covariates: Algebra 12 achievement and Posttest 1)

OBSERVED AND ADJUSTED MEANS ON POSTTEST 1 & 2
(One Covariate: Algebra 12 achievement)
There will be no interaction effect of "Treatment" and "Sex" on the adjusted mean scores of Posttest 2, using Algebra 12 achievement and Posttest 1 as covariates.

Table 4.8 summarizes the observed and adjusted mean scores for Posttest 1 and 2 arranged according to sex. The results of ANCOVA are summarized in tables 4.9 and 4.10.

Table 4.8
Posttest 1 and 2: Observed and Adjusted Mean Scores
on the Trigonometric Graphing Test arranged according to Sex

<table>
<thead>
<tr>
<th></th>
<th>Observed Scores</th>
<th>Adjusted Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male Mean SD</td>
<td>Female Mean SD</td>
</tr>
<tr>
<td>MCF Group</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>29 17</td>
<td></td>
</tr>
<tr>
<td>Posttest 1</td>
<td>7.52 2.31</td>
<td>7.79 2.65</td>
</tr>
<tr>
<td>Posttest 2a</td>
<td>11.02 2.33</td>
<td>11.09 1.73</td>
</tr>
<tr>
<td>TBF Group</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>12 20</td>
<td></td>
</tr>
<tr>
<td>Posttest 1</td>
<td>9.50 2.56</td>
<td>9.82 2.42</td>
</tr>
<tr>
<td>Posttest 2a</td>
<td>11.37 2.50</td>
<td>11.65 2.48</td>
</tr>
</tbody>
</table>

*Posttest 2 adjusted mean scores calculated using two covariates: Algebra 12 achievement and Posttest 1.*
### Table 4.9

Posttest 1: Analysis of adjusted mean scores arranged according to sex
(One Covariate: Algebra 12 achievement)

<table>
<thead>
<tr>
<th>Sources of Variance</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>1</td>
<td>34.93</td>
<td>34.93</td>
<td>9.58</td>
<td>.003*</td>
</tr>
<tr>
<td>Sex</td>
<td>1</td>
<td>2.24</td>
<td>2.24</td>
<td>.62</td>
<td>.435</td>
</tr>
<tr>
<td>Interaction</td>
<td>1</td>
<td>.09</td>
<td>.09</td>
<td>.02</td>
<td>.876</td>
</tr>
<tr>
<td>Within</td>
<td>73</td>
<td>266.18</td>
<td>3.65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* p < .01

### Table 4.10

Posttest 2: Analysis of adjusted mean scores arranged according to sex
(Two covariates: Algebra 12 achievement and Posttest 1)

<table>
<thead>
<tr>
<th>Source of Variance</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>1</td>
<td>1.94</td>
<td>1.94</td>
<td>.70</td>
<td>.405</td>
</tr>
<tr>
<td>Sex</td>
<td>1</td>
<td>.49</td>
<td>.49</td>
<td>.18</td>
<td>.674</td>
</tr>
<tr>
<td>Interaction</td>
<td>1</td>
<td>.33</td>
<td>.33</td>
<td>.12</td>
<td>.730</td>
</tr>
<tr>
<td>Within</td>
<td>72</td>
<td>198.93</td>
<td>2.76</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Results. F-values of .435 (df=1, 73; p > .05) and .674 (df=1, 72; p > .05) for Posttests 1 and 2 arranged according to sex were not statistically significant. Therefore, the null hypothesis was accepted; that is, sex of the students was not related to scores on Posttests 1 and 2.

Similarly, F-values for interaction of sex with treatment were not statistically significant for either Posttest 1 (F = .876; df = 1, 73; p > .05) or Posttest 2 (F = .730; df = 1, 72; p > .05).

Null Hypothesis 4: Number of Times Taking the Course Differences:

A. There will be no significant difference between the adjusted mean Posttest 1 scores on the Algebra 12 Trigonometric Graphing Test of the students in both treatment groups arranged according to number of times the course has been taken, using Algebra 12 achievement as the covariate.

B. There will be no significant difference between the adjusted mean Posttest 2 scores on the Algebra 12 Trigonometric Graphing Test of the students in both treatment groups arranged according to number of times the course has been taken, using Algebra 12 achievement and Posttest 1 as covariates.

C. There will be no interaction effect of "Treatment" and "Number of Times" on the adjusted mean scores of Posttest 1, using Algebra 12 achievement as the covariate.

D. There will be no interaction effect of "Treatment" and "Number of Times" on the adjusted mean scores of Posttest 2, using Algebra 12 achievement and Posttest 1 as covariates.

The observed and adjusted mean scores for both posttests arranged according to the number of times the student had taken the course can be found in Table 4.11 while the results of the ANCOVA on Posttest 1 are given
in Table 4.12, and the results of ANCOVA on Posttest 2 are given in Table 4.13.

Table 4.11
Posttest 1 and 2: Observed and Adjusted Mean Scores
arranged according to Number of Times Course Taken

<table>
<thead>
<tr>
<th></th>
<th>Observed Mean</th>
<th>Adjusted Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First SD</td>
<td>Second SD</td>
</tr>
<tr>
<td>MCF Group</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>41</td>
<td>5</td>
</tr>
<tr>
<td>Posttest 1</td>
<td>7.45</td>
<td>2.44</td>
</tr>
<tr>
<td>Posttest 2&lt;sup&gt;a&lt;/sup&gt;</td>
<td>10.93</td>
<td>2.16</td>
</tr>
<tr>
<td>TBF Group</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>25</td>
<td>7</td>
</tr>
<tr>
<td>Posttest 1</td>
<td>9.38</td>
<td>2.67</td>
</tr>
<tr>
<td>Posttest 2&lt;sup&gt;a&lt;/sup&gt;</td>
<td>11.19</td>
<td>2.57</td>
</tr>
</tbody>
</table>

<sup>a</sup>Posttest 2 adjusted mean scores were calculated using two covariates: Algebra 12 achievement and Posttest 1.
Table 4.12
Posttest 1: Analysis of adjusted mean scores arranged according to number of times taking the course
(One covariate: Algebra 12 achievement)

<table>
<thead>
<tr>
<th>Sources of Variance</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>1</td>
<td>16.73</td>
<td>16.73</td>
<td>4.59</td>
<td>.035*</td>
</tr>
<tr>
<td>Previous</td>
<td>1</td>
<td>1.10</td>
<td>1.10</td>
<td>.30</td>
<td>.584</td>
</tr>
<tr>
<td>Interaction</td>
<td>1</td>
<td>.81</td>
<td>.81</td>
<td>.22</td>
<td>.638</td>
</tr>
<tr>
<td>Within</td>
<td>73</td>
<td>265.97</td>
<td>3.64</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* p < .05

Table 4.13
Posttest 2: Analysis of adjusted mean scores arranged according to number of times taking the course
(Two Covariates: Algebra 12 achievement and Posttest 1)

<table>
<thead>
<tr>
<th>Sources of Variance</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>1</td>
<td>.25</td>
<td>.25</td>
<td>.09</td>
<td>.762</td>
</tr>
<tr>
<td>Previous</td>
<td>1</td>
<td>2.55</td>
<td>2.55</td>
<td>.93</td>
<td>.338</td>
</tr>
<tr>
<td>Interaction</td>
<td>1</td>
<td>.12</td>
<td>.12</td>
<td>.04</td>
<td>.836</td>
</tr>
<tr>
<td>Within</td>
<td>72</td>
<td>197.08</td>
<td>2.74</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Results. An $F$ of $0.584$ ($1, 73; p > .05$) on Posttest 1 and an $F$ of $0.338$ ($1, 72; p > .05$) on Posttest 2 for the number of times the course was taken were not statistically significant, and thus the null hypotheses were not rejected. Those students who were taking the course a second time were statistically different from the first-time students prior to the start of the study, $t(76) = -3.71, p < .01$. However, this finding indicates that the adjusted mean score of students taking the course a second time was not significantly different than the adjusted mean score of students taking the course for the first time for either treatment.

Similarly, an $F$ of $0.638$ ($1, 73; p > .05$) on Posttest 1 and an $F$ of $0.836$ ($1, 72; p > .05$) suggests that there was no significant interaction effect of treatment and the number of times the student had taken the course.

**Null Hypothesis 5: Time of Day Differences:**

A. There will be no significant difference between the adjusted mean Posttest 1 scores on the Algebra 12 Trigonometric Graphing Test of the students in both treatment groups arranged according to the time of day the course is taken, using Algebra 12 achievement as the covariate.

B. There will be no significant difference between the adjusted mean Posttest 2 scores on the Algebra 12 Trigonometric Graphing Test of the students in both treatment groups arranged according to the time of day the course is taken, using Algebra 12 achievement and Posttest 1 as covariates.

C. There will be no interaction effect of "Treatment" and "Time of Day" on the adjusted mean scores of Posttest 1, using Algebra 12 achievement as the covariate.
There will be no interaction effect of "Treatment" and "Time of Day" on the adjusted mean scores of Posttest 2, using Algebra 12 achievement and Posttest 1 as covariates.

The results of these analyses are included in tables 4.14, 4.15, and 4.16.

Table 4.14
Posttest 1 and 2: Observed and Adjusted Mean Scores
arranged according to Time of Day

<table>
<thead>
<tr>
<th></th>
<th>Observed Mean</th>
<th>Adjusted Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Morning</td>
<td>SD</td>
</tr>
<tr>
<td>MCF Group</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>25</td>
<td>21</td>
</tr>
<tr>
<td>Posttest 1</td>
<td>7.28</td>
<td>2.66</td>
</tr>
<tr>
<td>Posttest 2</td>
<td>11.18</td>
<td>2.30</td>
</tr>
<tr>
<td>TBF Group</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>Posttest 1</td>
<td>10.50</td>
<td>2.20</td>
</tr>
<tr>
<td>Posttest 2</td>
<td>11.75</td>
<td>2.17</td>
</tr>
</tbody>
</table>

Posttest 2 adjusted mean scores were calculated using two covariates: Algebra 12 achievement and Posttest 1.
Table 4.15
Posttest 1: Analysis of adjusted mean scores arranged according to time of day
(One Covariate: Algebra 12 achievement)

<table>
<thead>
<tr>
<th>Source of Variance</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>1</td>
<td>29.05</td>
<td>29.05</td>
<td>8.05</td>
<td>.006*</td>
</tr>
<tr>
<td>Time</td>
<td>1</td>
<td>3.34</td>
<td>3.34</td>
<td>.93</td>
<td>.339</td>
</tr>
<tr>
<td>Interaction</td>
<td>1</td>
<td>.34</td>
<td>.34</td>
<td>.09</td>
<td>.759</td>
</tr>
<tr>
<td>Within</td>
<td>73</td>
<td>263.43</td>
<td>3.61</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p < .01

Table 4.16
Posttest 2: Analysis of adjusted mean scores arranged according to time of day
(Two Covariates: Algebra 12 achievement and Posttest 1)

<table>
<thead>
<tr>
<th>Source of Variance</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>1</td>
<td>5.02</td>
<td>5.02</td>
<td>1.91</td>
<td>.172</td>
</tr>
<tr>
<td>Time</td>
<td>1</td>
<td>1.23</td>
<td>1.23</td>
<td>.47</td>
<td>.497</td>
</tr>
<tr>
<td>Interaction</td>
<td>1</td>
<td>10.18</td>
<td>10.18</td>
<td>3.87</td>
<td>.053</td>
</tr>
<tr>
<td>Within</td>
<td>72</td>
<td>189.44</td>
<td>2.63</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Results.** F-values of .339 (1, 73; p > .05) and .497 (1, 72; p > .05) for Posttests 1 and 2 arranged according to time of day were not statistically significant, and the null hypothesis was not rejected. It was concluded, therefore, that time of day was not a factor in student achievement.

F-values for interaction of time of day with treatment were not statistically significant for either Posttest 1 (F = .759; df = 1, 73; p > .05) or Posttest 2 (F = .053; df = 1, 72; p > .05). Therefore, the null hypothesis was accepted.

**Summary**

Students in the group that received the textbook treatment achieved higher mean scores (although statistically significant in Posttest 1 only) than students who received the microcomputer treatment. This pattern was noted by effect sizes that favoured the textbook treatment on Posttest 1 (.86) and Posttest 2 (.22). However, upon completion of both treatments, the groups achieved almost the same mean score on Posttest 2 indicating that the order of treatment was not significant.

Furthermore, it does not appear that gender was a factor; it does not appear that time of day is related to student performance; and finally, the level of trigonometric graphing achievement of students who are repeating the course was no greater than that of students taking the course for the first time. Similarly, there does not appear to have been an interaction effect of treatment with any of the main effects of sex, time of day, or number of times the course had been taken.
Chapter 5
SUMMARY, CONCLUSIONS, AND IMPLICATIONS FOR FUTURE RESEARCH

The purpose of this study, the procedures involved, and the results are summarized, prior to a discussion of conclusions and their implications for future research.

Summary of the Study

This study compared the use of the microcomputer program “Master Grapher” with the textbook as means of teaching the graphing of trigonometric relations in grade 12 algebra classes. A review of the research suggested that the use of a microcomputer to supplement traditional classroom instruction is likely to improve student achievement (Vinsonhaler & Bass, 1972; Edwards, Norton, Taylor, Weiss, & Dusseldorp, 1975; Hartley, 1977; Burns & Bozeman, 1981; Bracey, 1982; Ragosta, Holland, & Jamieson, 1982; Mattson, 1983; Bangert-Drowns, Kulik, & Kulik 1985). Therefore, it was hypothesized that students receiving the microcomputer treatment would score higher on a test of graphing trigonometric functions than students receiving the textbook treatment. Data were collected and analyzed to test this hypothesis, as well as to investigate the following questions.

1. Will there be a significant difference in the adjusted mean posttest scores of the students who have been treated with the microcomputer first and then the textbook compared to the students who have been treated with the textbook first and then the microcomputer?
2. Will there be a significant difference in the adjusted mean posttest scores of males and females on a trigonometric graphing test? Will males respond differently to the microcomputer treatment and the textbook treatment than females?

3. Will there be a significant difference in the adjusted mean posttest scores of those students taking the course in the morning compared to those students taking the course in the afternoon? Will there be an interaction between treatment and time of day?

4. Will there be a significant difference in the adjusted mean posttest scores of those students repeating the course compared to those students taking the course for the first time? Will there be an interaction between treatment and number of times the course is taken?

The Nature of the Study

The study consisted of two phases conducted over a total of six days. The treatments of each phase lasted three days, and consisted of two one-hour and fifteen minute-periods of treatment followed by one period of testing. There were four Algebra 12 classes and two teachers involved in the study. Each teacher taught two classes; one in the morning, and one in the afternoon. One class of each teacher was randomly selected to receive the microcomputer treatment while the other was given the textbook treatment during phase one.

The microcomputer treatment consisted of a demonstration on a Macintosh microcomputer during period one, followed by a series of exercises using the software program "Master Grapher" during period two. The textbook treatment, considered traditional classroom instruction,
consisted of questions from a typical Algebra 12 textbook during period one, and an in-class assignment taken from another typical Algebra 12 textbook during period two.

Following phase one of the study, both treatment groups were given comparable tests, each consisting of 15 marks.

The treatments were then reversed, and a second set of tests were administered following the two day treatments. In this way, all four classes completed the same program, both the microcomputer and the textbook treatment, by the end of the study.

Data collected by the administration of the Posttest 1 were treated with analysis of covariance, with prior algebra 12 achievement as the covariate. For Posttest 2, an analysis of covariance was carried out using one covariate (Algebra 12 achievement score) to analyze the effect of the order of treatment, and two covariates (Algebra 12 achievement score and Posttest 1 results) to analyze the effect of the treatment alone.

Null hypotheses were formed from the questions posed for the study, and were treated with the analysis of covariance.

Summary of Results

Prior to the investigation, a research hypothesis was formulated to be tested—in the null form—by the analysis of the data collected from Posttest 1. Additionally, several questions were developed and posed in the form of null hypotheses. These questions were to be tested by the analyses of data collected from Posttest 1 and 2.

Null hypotheses were rejected, and differences were considered significant only when the probability of observing such differences was .05 or less due to chance.
Table 5.1 illustrates the disposition of the hypothesis and the questions which were posed as hypotheses in the null form.

**Table 5.1**
Disposition of Null Hypothesis
and Questions Posed as Null Hypotheses

<table>
<thead>
<tr>
<th>Null Hypothesis/Question</th>
<th>Disposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A There is no significant difference between the adjusted mean Posttest 1 scores on</td>
<td>rejected</td>
</tr>
<tr>
<td>the Algebra 12 Trigonometric Graphing Test of the students who received the microcomputer</td>
<td>(p &lt; .01)</td>
</tr>
<tr>
<td>treatment and of the students who received the textbook treatment using Algebra</td>
<td></td>
</tr>
<tr>
<td>12 as the covariate.</td>
<td></td>
</tr>
<tr>
<td>1B There is no significant difference between the adjusted mean Posttest 2 scores on</td>
<td>accepted</td>
</tr>
<tr>
<td>the Algebra 12 Trigonometric Graphing Test of the students who received the</td>
<td></td>
</tr>
<tr>
<td>microcomputer treatment and of the students who received the textbook treatment</td>
<td></td>
</tr>
<tr>
<td>using Algebra 12 achievement and Posttest 1 as covariates.</td>
<td></td>
</tr>
<tr>
<td>2  There is no significant difference between the adjusted mean Posttest 2 scores on</td>
<td>accepted</td>
</tr>
<tr>
<td>the Algebra 12 Trigonometric Graphing Test of the students arranged according to</td>
<td></td>
</tr>
<tr>
<td>order of treatment.</td>
<td></td>
</tr>
<tr>
<td>3 A &amp; B There is no significant difference between the adjusted mean Posttest scores</td>
<td>accepted</td>
</tr>
<tr>
<td>on the Algebra 12 Trigonometric Graphing Test of the students in both treatment</td>
<td>accepted</td>
</tr>
<tr>
<td>groups arranged according to sex.</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.1 (continued)

<table>
<thead>
<tr>
<th>Null Hypothesis/Question</th>
<th>Disposition</th>
<th>Posttest 1</th>
<th>Posttest 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 C &amp; D There is no interaction effect of &quot;Treatment&quot; and &quot;Sex&quot; on the adjusted mean scores of the Posttest.</td>
<td>accepted</td>
<td>accepted</td>
<td></td>
</tr>
<tr>
<td>4 A &amp; B There is no significant difference between the adjusted mean Posttest scores on the Algebra 12 Trigonometric Graphing Test of the students in both treatment groups arranged according to number of times the course has been taken.</td>
<td>accepted</td>
<td>accepted</td>
<td></td>
</tr>
<tr>
<td>4 C &amp; D There is no interaction effect of &quot;Treatment&quot; and &quot;Number of Times&quot; on the adjusted mean scores of the Posttest.</td>
<td>accepted</td>
<td>accepted</td>
<td></td>
</tr>
<tr>
<td>5 A &amp; B There is no significant difference between the adjusted mean Posttest scores on the Algebra 12 Trigonometric Graphing Test of the students in both treatment groups arranged according to time of day the course is taken.</td>
<td>accepted</td>
<td>accepted</td>
<td></td>
</tr>
<tr>
<td>5 C &amp; D There is no interaction effect of &quot;Treatment&quot; and &quot;Time of Day&quot; on the adjusted mean scores of the Posttest.</td>
<td>accepted</td>
<td>accepted</td>
<td></td>
</tr>
</tbody>
</table>

Effect sizes were calculated. An effect size of .86 for Posttest 1 indicated that the students who received the textbook treatment scored...
almost one standard deviation higher on a test of trigonometric graphing than the students who received the microcomputer treatment. An effect size of .22 on Posttest 2 indicated that there was a trend which favoured the use of the textbook over the use of the microcomputer. In each phase of the study, the students using the textbook achieved higher results than the students using the microcomputer.

Conclusions

Although a t-test indicated that the difference between the two groups on algebra 12 achievement was not statistically significant, an analysis of covariance was used in an effort to produce more accurate results since the group that received the microcomputer treatment first had an algebra 12 mean achievement score of 64%, and the textbook group had a mean of 70% entering the study.

After phase one of the study, the microcomputer treatment group scored an actual mean of 7.6 out of 15 or 51%, while the textbook treatment group scored an actual mean of 9.7 out of 15 or 65%. The microcomputer by itself (supplemented with worksheets) was not as effective as the textbook in preparing students for a test on graphing trigonometric functions. This result reinforces the findings of Ronan (1971), Robitaille, Sherrill, and Kaufman (1977), Signer (1982), Hathaway (1983), and Streibel (1986) all of whom found traditional instruction more effective than instruction involving the computer.

Following phase two, the Microcomputer First Group scored an actual mean of 11.0 out of 15 or 73% and the Textbook First Group scored an actual mean of 11.6 out of 15 or 77%. The difference was not statistically significant. Thus, the order of the treatments did not seem to effect
student achievement as both groups achieved comparable levels of proficiency by the end of the study. However, it can be argued that the students who were given the textbook treatment first were actually inhibited by the microcomputer treatment, in that their mean achievement gain was minimal in phase two of the study (from 9.7 to 11.6). In comparison, the students who were given the microcomputer treatment first had a much greater mean achievement gain following the textbook treatment (from 7.6 to 11.0). It can also be argued, however, that the students in the TBF Group were likely closer to their maximum performance by the time that they were exposed to the microcomputer, and thus had limited capacity for gains in achievement.

When the data were arranged according to sex, the time of the day that students took the course, and the number of times students had taken the course, no main effects or interaction effects for these factors were found to be statistically significant. The question of gender was likely the most interesting, and it does not appear that males and females react any differently to either treatment. This result is supported by the research of Robyler (1988) who found that while boys tended to achieve slightly more with microcomputer applications, the differences were not statistically significant.

Implications for Future Research

There are a number of factors that may have contributed to the results obtained in this study and which should be taken into consideration in future studies of this nature.

While the textbooks have been revised and rewritten many times over the past years to continually upgrade their effectiveness, the software that
is currently available is still in its infancy. The software program used in this study ("Master Grapher") was not programmed to ask students basic questions on the topic of graphing trigonometric functions such as "What is the amplitude of this graph?" and "What is the period of this relation?" Although these questions were included on the students' lab assignments, the microcomputer application may have been more effective had the students been forced to respond to questions of this type by the microcomputer program. In future studies, in order to assess the effectiveness of the microcomputer for teaching mathematics, the software program and textbook program must be comparable. To this end, when software is selected for future studies comparing computerized instruction with textbook instruction, it is important to ensure that the content of both teaching instruments is similar.

The posttests that the students were given to assess knowledge of graphing trigonometric functions may have been more textbook-oriented than microcomputer-oriented. An analysis of the posttests indicates that out of 15 marks, 8 marks were on 'visual' or graphic questions and 7 marks were on 'non-visual' or strictly word questions. This half visual, half non-visual set-up was intended to neutralize advantages gained by either treatment group. Perhaps a test should be devised that can be given on the microcomputer to make the testing process more similar to the exercises completed during the treatment program. Although the tests given were made up of standard type questions that a student should be able to answer on this topic, it could be that the knowledge that students were gaining from using the microcomputer was not being tested by these "typical exam questions". Perhaps current testing techniques do not, and should not, apply to microcomputer work. In order for the microcomputer to be judged at its
most effective level, new evaluation techniques may have to be devised. In future studies perhaps consideration should be given to testing students in the medium in which they were taught.

The inexperience of the students on the microcomputer could have influenced the results. Most of the students were not familiar with the Macintosh microcomputer so it took them a while to get comfortable. Valuable learning time was used up while the students acquired the skills of operating the microcomputer. Students know what is expected of them when they are set to work from a textbook. That preconception is not present when it comes to working with the microcomputer. Students were on task throughout both periods of their microcomputer treatment just as they were on task throughout the textbook portion but it is likely that more of their time was devoted to the “textbook” questions than was devoted to the questions asked while they were in the Macintosh laboratory. As a result of their microcomputer inexperience, the actual amount of time on task was likely quite a bit less when students used the microcomputer. All changes take time and the microcomputer is no exception. The literature states that students can learn the same amount of material in less time by using a microcomputer (Vinsonhaler & Bass, 1972; Jameson, Suppes, & Wells, 1974; Edwards, Norton, Taylor, Weiss, & Dusseldorp, 1975; Thomas, 1979; Kulik, Bangert, & Williams, 1983; Bitter & Cameron, 1980; Colorado, 1988; Robyler, 1988), but there must be some initial outlay of time before these benefits can be reaped. The focus of a future study could be to determine how much time on a microcomputer is needed before these time-saving benefits become apparent.

In summary, the following recommendations are made for future studies: (a) In order to test the effectiveness of the microcomputer as
compared to the textbook, the software program selected must be comparable to the textbook. (b) Students should be given time prior to the start of a study to familiarize themselves with the microcomputer so that learning time is not lost as a result of operating inexperience. (c) A posttest should be given on the microcomputer following the microcomputer treatment and on paper following the textbook treatment.


APPENDIX A

LESSON ONE: MICROCOMPUTER TREATMENT
Lesson #1: Microcomputer Treatment

Set grid for horizontal reading of -10 to 10 and vertical reading of -5 to 5. Have students copy demonstration graphs on their own graph paper.

<table>
<thead>
<tr>
<th>Number</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demo #1:</td>
<td>$y = \sin (x)$</td>
</tr>
<tr>
<td>Demo #2:</td>
<td>$y = 3 \sin (x)$</td>
</tr>
<tr>
<td>Demo #3:</td>
<td>$y = \frac{1}{4} \sin (x)$</td>
</tr>
<tr>
<td>Demo #4:</td>
<td>$y = \sin 4(x)$</td>
</tr>
<tr>
<td>Demo #5:</td>
<td>$y = \sin \frac{1}{2}(x)$</td>
</tr>
<tr>
<td>Demo #6:</td>
<td>$y = 2 \sin \frac{1}{3}(x)$</td>
</tr>
<tr>
<td>Demo #7:</td>
<td>$y = \cos (x)$</td>
</tr>
<tr>
<td>Demo #8:</td>
<td>$y = 4 \cos (x)$</td>
</tr>
<tr>
<td>Demo #9:</td>
<td>$y = -\cos (x)$</td>
</tr>
<tr>
<td>Demo #10:</td>
<td>$y = 3 \cos 3(x)$</td>
</tr>
<tr>
<td>Demo #11:</td>
<td>$y = 3 + \sin (x)$</td>
</tr>
<tr>
<td>Demo #12:</td>
<td>$y = \cos(x) - 2$</td>
</tr>
<tr>
<td>Demo #13:</td>
<td>$y = \sin (x - \frac{\pi}{4})$</td>
</tr>
<tr>
<td>Demo #14:</td>
<td>$y = \sin (x + \frac{\pi}{4})$</td>
</tr>
<tr>
<td>Demo #15:</td>
<td>$y = 4 + 2 \sin \frac{1}{3}(x)$</td>
</tr>
<tr>
<td>Demo #16:</td>
<td>$y = -1 - 3 \cos (x)$</td>
</tr>
<tr>
<td>Demo #17: ( y = \tan(x) )</td>
<td></td>
</tr>
<tr>
<td>Demo #18: ( y = 2 \tan(x) )</td>
<td></td>
</tr>
<tr>
<td>Demo #19: ( y = \tan(5x) )</td>
<td></td>
</tr>
<tr>
<td>Demo #20: ( y = \sin(x) ) and ( y = \csc(x) )</td>
<td></td>
</tr>
<tr>
<td>Demo #21: ( y = \cos(x) ) and ( y = \sec(x) )</td>
<td></td>
</tr>
<tr>
<td>Demo #22: ( y = \tan(x) ) and ( y = \cot(x) )</td>
<td></td>
</tr>
</tbody>
</table>

If time permits, show some graphs and have students come up with the equations of those graphs.
Lesson #1: for Microcomputer Treatment

Homework assignment:

Predict the following for tomorrow.

A. Do a sketch of at least one period on the graph paper provided and label at least 4 points on your sketch.
B. State the amplitude and the period of each graph.
C. State the range of each graph.

1) \( y = -2 \sin(x) \)
2) \( y = \frac{3}{4} \cos(x) \)
3) \( y = \tan(x + \frac{\pi}{3}) \)
4) \( y = 3 - \sin 2(x) \)
5) \( y = \sin^2x + \cos^2x \)
6) \( y = \sin(x) - \cos(x) \)
7) \( y = 2 + \frac{1}{3} \cos 4(x) \)
8) \( y = 6 \sin 7(x) \)
APPENDIX B
LESSON TWO: MICROCOMPUTER TREATMENT
TO GET STARTED IN THE MAC LAB:

1. Turn on the Macintosh at the back.
2. Place your "Grapher" disc in the opening at the front of the computer so that the arrows are pointing into the computer.
3. By moving your "mouse", you can slide the arrow on the screen until it is over top of the picture of a disc on your screen. Click on that icon that says "Grapher" so that it is blackened.
4. Then, double click quite quickly on that icon and it will "open up".
5. You will then see some more icons. Move the arrow by shifting your "mouse" until it is on the one titled "Master Grapher" and double click to open it up.
6. On the screen will be the name of the developers of this program. Click the mouse to continue.
7. You will then be given another menu to choose from. Place the arrow by moving the mouse onto "Function Grapher".
8. The screen will look like the graphic on the next page.
9. First click onto the "Change or Remove Function" dot. Once in this program, you can then enter 8 equations to be graphed. Remember to use an * for multiplication and to use brackets. Once you have entered all 8 of your equations, select just one of them to be graphed by moving the arrow and clicking next to that equation. You will also deselect (if it has a black dot) by clicking the mouse next to that equation. Once you have selected one equation, then select "Previous Menu".
10. Once back into the "Function Grapher" menu, change the scale to π/2 by selecting that dot, and change the viewing rectangle by selecting that dot. You will then use the TAB key to change the vertical view from -10 to -5 and from 10 to 5.
11. Then you select "Graph Function" and voila!!!
12. To exit from this graph, select "Return to Prev. Menu".
13. You repeat this process to choose a different function to be graphed and/or to change the functions to be graphed. Note that you can select more than one equation and have them all graphed on the same grid... It just gets a little difficult to see.
14. Once you are finished, then select "Exit Program".
15. At the very top of your screen there will be some headings. Move the arrow up on to the heading titled "Special" and hold down the mouse button and drag down that menu until the arrow is on a command that says "Shut Down". Now release the mouse button and your disc will automatically eject. NOW you can turn off your Mac with the switch at the back.
Lesson #2: Microcomputer Treatment

Name: _______________________________

You will be responsible for handing in as many of the following assignments as you are able to complete during this period. Be sure to accurately draw the graphs of the equations onto the graph paper provided and to label at least four points on each graph. USE THE COMPUTER TO CONFIRM YOUR GRAPHS AND/OR TO HELP YOU GRAPH.

Assignment #1:
Check your homework by entering the 8 questions all at once into the computer and then by graphing them one at a time. Be sure to copy down the correct graphs for each equation if you graphed them incorrectly in your homework.

Assignment #2:
On the same grid, graph the following:

\[ y = \sin(x), \quad y = 5 \sin(x), \quad y = \frac{1}{5} \sin(x) \]

Make a statement as to a general rule for the above.

Rule: ________________________________

Assignment #3:
On the same grid, graph the following:

\[ y = \cos(x), \quad y = \cos(3x), \quad y = \cos(\frac{1}{3}x) \]

Make a statement as to a general rule for the above.

Rule: ________________________________
Assignment #4:
On the same grid, graph the following:
y = 1 + \sin(x), \ y = -1 + \sin(x), \ y + 3 = \sin(x)

Make a statement as to a general rule for the above.
Rule:_________________________________________________________

Assignment #5:
On the same grid, graph the following:
y = \tan(x - 1), \ y = \tan(x + 2)

Make a statement as to a general rule for the above.
Rule:_________________________________________________________

Assignment #6:
Create a graph which has a range of \(-2 \leq y \leq 4\) and a period of \(\pi\)

What is the equation of this graph?________________________________________
Assignment #7:
Create a graph which has a phase shift of $\pi/3$ to the left, a range of $3 \leq y \leq 8$, and a period of $2\pi$.

What is the equation of this graph? ______________________________

Assignment #8:
What is an equation for the following graphs?

Equation: ______________________

Equation: ______________________
Assignment #9:
Graph the following: (Sketch each graph on the graph paper provided)
y = 1 + sec (x)
y = tan²x
y = |3 sin (x)| (Use ABS for the absolute value signs)
y = x sin (x + π/2) (Convert π/2 to approximate radians)
y = \frac{\sin x}{\csc^2 x}

Can you make any predictions from the above graphs?

Assignment #10:
Use your imagination and creativity to devise some interesting trigonometric graphs. Be sure to record your equations and to copy your graphs onto the paper provided.
APPENDIX C

ANSWER KEYS: MICROCOMPUTER LESSONS
ANSWER KEY for Lesson #1: Microcomputer Treatment

***You should have copied the graphs from the overhead monitor during the demonstration so the graphs are not included with this answer key.

Set grid for horizontal reading of -10 to 10 and vertical reading of -5 to 5. Have students copy demonstration graphs on their own graph paper.

<table>
<thead>
<tr>
<th></th>
<th>Amplitude</th>
<th>Period</th>
<th>Label 4 pts. (Range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demo #1: y = sin (x)</td>
<td>1</td>
<td>$2\pi$</td>
<td>$-1 \leq y \leq 1$</td>
</tr>
<tr>
<td>Demo #2: y = 3 sin (x)</td>
<td>3</td>
<td>$2\pi$</td>
<td>$-3 \leq y \leq 3$</td>
</tr>
<tr>
<td>Demo #3: y = 1/4 sin (x)</td>
<td>1/4</td>
<td>$2\pi$</td>
<td>$-1/4 \leq y \leq 1/4$</td>
</tr>
<tr>
<td>Demo #4: y = sin 4(x)</td>
<td>1</td>
<td>$\pi/2$</td>
<td>$-1 \leq y \leq 1$</td>
</tr>
<tr>
<td>Demo #5: y = sin 1/2 (x)</td>
<td>1</td>
<td>$4\pi$</td>
<td>$-1 \leq y \leq 1$</td>
</tr>
<tr>
<td>Demo #6: y = 2 sin 1/3(x)</td>
<td>2</td>
<td>$6\pi$</td>
<td>$-2 \leq y \leq 2$</td>
</tr>
<tr>
<td>Demo #7: y = cos (x)</td>
<td>1</td>
<td>$2\pi$</td>
<td>$-1 \leq y \leq 1$</td>
</tr>
<tr>
<td>Demo #8: y = 4 cos (x)</td>
<td>4</td>
<td>$2\pi$</td>
<td>$-4 \leq y \leq 4$</td>
</tr>
<tr>
<td>Demo #9: y = - cos (x)</td>
<td>1</td>
<td>$2\pi$</td>
<td>$-1 \leq y \leq 1$</td>
</tr>
<tr>
<td>Demo #10: y = 3 cos 3(x)</td>
<td>3</td>
<td>$2\pi/3$</td>
<td>$-3 \leq y \leq 3$</td>
</tr>
<tr>
<td>Demo #11: y = 3 + sin (x)</td>
<td>1</td>
<td>$2\pi$</td>
<td>$2 \leq y \leq 4$</td>
</tr>
<tr>
<td>Demo #12: y = cos(x) - 2</td>
<td>1</td>
<td>$2\pi$</td>
<td>$-3 \leq y \leq -1$</td>
</tr>
<tr>
<td>Demo #13: y = sin (x - $\pi/4$)</td>
<td>1</td>
<td>$2\pi$</td>
<td>$-1 \leq y \leq 1$</td>
</tr>
<tr>
<td>Demo #14: y = sin (x + $\pi/4$)</td>
<td>1</td>
<td>$2\pi$</td>
<td>$-1 \leq y \leq 1$</td>
</tr>
<tr>
<td>Demo #15: y = 4 + 2 sin 1/3(x)</td>
<td>2</td>
<td>$6\pi$</td>
<td>$2 \leq y \leq 6$</td>
</tr>
<tr>
<td>Demo #16: y = -1 - 3 cos (x)</td>
<td>3</td>
<td>$2\pi$</td>
<td>$-4 \leq y \leq 2$</td>
</tr>
<tr>
<td>Demo #17: $y = \tan(x)$</td>
<td>-</td>
<td>$\pi$</td>
<td>reals</td>
</tr>
<tr>
<td>Demo #18: $y = 2 \tan(x)$</td>
<td>-</td>
<td>$\pi$</td>
<td>reals</td>
</tr>
<tr>
<td>Demo #19: $y = \tan(5x)$</td>
<td>-</td>
<td>$\pi/5$</td>
<td>reals</td>
</tr>
<tr>
<td>Demo #20: $y = \sin(x)$ and $y = \csc(x)$</td>
<td>-</td>
<td>$2\pi$</td>
<td>reals</td>
</tr>
<tr>
<td>Demo #21: $y = \cos(x)$ and $y = \sec(x)$</td>
<td>-</td>
<td>$2\pi$</td>
<td>reals</td>
</tr>
<tr>
<td>Demo #22: $y = \tan(x)$ and $y = \cot(x)$</td>
<td>-</td>
<td>$\pi$</td>
<td>reals</td>
</tr>
</tbody>
</table>
Lesson #2: Microcomputer Treatment

You will be responsible for handing in as many of the following assignments as you are able to complete during this period. Be sure to accurately draw the graphs of the equations onto the graph paper provided and to label at least four points on each graph. USE THE COMPUTER TO CONFIRM YOUR GRAPHS AND/OR TO HELP YOU GRAPH.

Assignment #1:
Check your homework by entering the 8 questions all at once into the computer and then by graphing them one at a time. Be sure to copy down the correct graphs for each equation if you graphed them incorrectly in your homework.

1. \( y = -2 \sin(x) \)
   Amplitude = 2  Period = \( 2\pi \)
   Range: \(-2 \leq y \leq 2\)

![Graph of \( y = -2 \sin(x) \)]

2. \( y = \frac{3}{4} \cos(x) \)
   Amplitude = \( \frac{3}{4} \)  Period = \( 2\pi \)
   Range: \(-\frac{3}{4} \leq y \leq \frac{3}{4}\)

![Graph of \( y = \frac{3}{4} \cos(x) \)]

3. \( y = \tan(x + \pi/3) \)
   Period = \( \pi \)
   Range: all real numbers

![Graph of \( y = \tan(x + \pi/3) \)]

4. \( y = 3 - \sin(2x) \)
   Amplitude = 1  Period = \( \pi \)
   Range: \( 2 \leq y \leq 4\)

![Graph of \( y = 3 - \sin(2x) \)]
5. \( y = \sin^2x + \cos^2x \)
   Amplitude = 0
   Range: \( y = 1 \)

6. \( y = \sin(x) - \cos(x) \)
   Amplitude = Period = \( 2\pi \)
   Range:

7. \( Y = 2 + \frac{1}{3} \cos 4(x) \)
   Amplitude = \( \frac{1}{3} \)
   Period = \( \frac{\pi}{2} \)
   Range: \( 5/3 \leq y \leq 7/3 \)

8. \( y = 6 \sin 7(x) \)
   Amplitude = 6
   Period = \( \frac{2\pi}{7} \)
   Range: \( -6 \leq y \leq 6 \)

Assignment #2:
On the same grid, graph the following:
\( y = \sin(x) \), \( y = 5 \sin(x) \), \( y = \frac{1}{5} \sin(x) \)

Make a statement as to a general rule for the above.
Rule: The number in front of the trig function determines the amplitude (or height) of the graph.
Assignment #3:
On the same grid, graph the following:
\[ y = \cos(x), \quad y = \cos 3(x), \quad y = \cos \frac{1}{3}(x) \]

Rule: The number following the trig function but in front of the variable determines the number of times the graph will repeat over a 2\pi period.

Assignment #4:
On the same grid, graph the following:
\[ y = 1 + \sin(x), \quad y = -1 + \sin(x), \quad y + 3 = \sin(x) \]

Rule: A positive number added to the trig function slides the graph up that many units and a negative number added to the trig function slides the graph down.

Assignment #5:
On the same grid, graph the following:
\[ y = \tan(x - 1), \quad y = \tan(x + 2) \]
Rule: A positive number added inside the bracket shifts the graph that many units to the left and a positive number subtracted inside the bracket shifts the graph right.

Assignment #6:
Create a graph which has a range of $-2 \leq y \leq 4$ and a period of $\pi$.

What is the equation of this graph? $y = 1 + 3 \sin(2x)$

Assignment #7:
Create a graph which has a phase shift of $\pi/3$ to the left, a range of $3 \leq y \leq 8$, and a period of $2\pi$.

What is the equation of this graph? $y = 5.5 + 2.5 \sin(x + \pi/3)$
Assignment #8:
What is an equation for the following graphs?

Equation: \( y = 2 \cos 4x \)

Equation: \( y = -2 + \sin (x-\pi/6) \)

Equation: \( y = 1 + 3 \cos (x-\pi/2) \)
Assignment #9:
Graph the following: (Sketch each graph on the graph paper provided)

- $y = 1 + \sec(x)$
- $y = \tan^2x$
- $y = |3 \sin(x)|$
- $y = x \sin(x + \pi/2)$
- $y = \frac{\sin x}{\csc^2x}$

Can you make any predictions from the above graphs?
When absolute value is used, the range will be positive. Similarly, if the function is squared, the range values become positive. Combining variables with trig functions makes the graph difficult to predict as does dividing one trig function by another.
APPENDIX D
ALGEBRA 12 TRIGONOMETRIC GRAPHING TESTS
(POSTTEST A and B)
AND ANSWER KEYS
1. What is the amplitude of the curve shown below?

A. -3  B. 3  C. 6  D. 2π

2. What is the period of the trigonometric function graphed below?

A. 2π/5  B. 6π/5  C. 2π  D. π

3. The amplitude of the curve \( y = \frac{2}{3} \sin 4x \) is:
A. 1/6  B. 2/3  C. 8/3  D. 4

4. What is the period of the curve \( y = 4 \sin \pi x \) ?
A. 2π  B. 4π  C. 2  D. 4

5. Which one of the following functions is represented by the graph below?
A. \( y = 4 \sin 3(x + \pi/6) \)
B. \( y = 4 \sin 3(x - \pi/6) \)
C. \( y = 8 \cos 3(x - \pi/6) \)
D. \( y = 8 \sin 3(x + \pi/6) \)
6. Which of the following is an equation of a sine curve with a maximum value 2, minimum value 0, and period \( \pi/3 \)?

A. \( y = 2 + \sin 3\pi x \)   B. \( y = 1 + \sin 6\pi x \)
C. \( y = 2 \sin 3x \)   D. \( y = 1 + \sin 6x \)

7. What is the range of \( y = 2 \sin 3x - 1 \)?

A. \(-1 \leq y \leq 3\)   B. \(-3 \leq y \leq 1\)
C. \(-1 \leq y \leq 1\)   D. \(-2 \leq y \leq 2\)

8. Which of the following relations has a graph which is the same as the graph of \( y = 3 \cos (x + \pi/2) \)?

A. \( y = -3 \sin x \)   B. \( y = 3 \sin (x - \pi/2) \)
C. \( y = 2 \cos 1/2 x \)   D. \( y = 3 \cos x \)

9. One asymptote of the curve \( y = \tan (x - \pi/4) \) is:

A. \( x = 0 \)   B. \( x = \pi/4 \)   C. \( x = 3\pi/4 \)   D. \( x = 5\pi/4 \)

10. Which one of the following is an equation of the graph below over the interval \( 0 \leq x \leq 3\pi \)?

A. \( y = \tan 2x \)   B. \( y = \tan 1/2 x \)
C. \( y = \cot 1/2 x \)   D. \( y = \cot 2x \)

Part II:

11. On the grid provided accurately graph \( y = 2 \cos 3x \). Label at least 4 points in one period. (2 marks).
12. Write an equation for a sine curve with an amplitude of 3 and a period of $\pi/4$. (one mark)

13. Given the function $y = -2 \sin 1/3 x$ (2 marks)
   a) What is the amplitude?
   b) What is the range?
   c) Sketch its graph on the grid below labeling at least five points accurately.

Bonus: Sketch a graph of at least one period of $y = \frac{\sin 2x}{(\sin x)(\cos x)}$.
Algebra 12 Trigonometric Graphing Test (Posttest IB)

Name: ___________________________  Score ______ / 15

Part I: Multiple Choice (Choose the best answer for the following)

1. What is the amplitude of the curve shown below?
   ![Graph](image)
   A. 4π  B. 2π  C. 6  D. 3

2. What is the period of the trigonometric function graphed below?
   ![Graph](image)
   A. 4π/3  B. 2π/3  C. 3π/4  D. π

3. The amplitude of the curve \( y = -3/5 \sin 2x \) is:
   A. -3/5  B. 3/5  C. 6/5  D. -6/5

4. What is the period of the curve \( y = 2 \cos 3/2x \) ?
   A. 4π/3  B. 3π  C. 3/2  D. 2

5. Which one of the following functions is represented by the graph below?
   ![Graph](image)
   A. \( y = 2 \sin 3(\alpha - 30^\circ) \)
   B. \( y = 2 \sin 3(\alpha + 30^\circ) \)
   C. \( y = 2 \sin 1/3 (\alpha - 30^\circ) \)
   D. \( y = 2 \sin 1/3 (\alpha + 30^\circ) \)
6. A sine function has range values \(-2 \leq y \leq 4\) and a period of \(\pi\). What is one possibility for its equation?
A. \(y = 3 \sin x + 1\)  
B. \(y = 6 \sin x + 1\)  
C. \(y = 3 \sin 2x + 1\)  
D. \(y = 6 \sin 2x + 1\)  
6. ________

7. What is the range of \(y = 3 \sin 2x + 1\)?
A. \(-2 \leq y \leq 4\)  
B. \(-4 \leq y \leq 2\)  
C. \(-1 \leq y \leq 1\)  
D. \(-3 \leq y \leq 3\)  
7. ________

8. Which of the following relations has a graph which is the same as the graph of \(y = 3 \cos (x + \pi/2)\)?
A. \(y = -3 \sin x\)  
B. \(y = 3 \sin (x - \pi/2)\)  
C. \(y = 2 \cos 1/2x\)  
D. \(y = 3 \cos x\)  
8. ________

9. One asymptote of the curve \(y = \tan (x + \pi/4)\) is:
A. \(x = 0\)  
B. \(x = \pi/4\)  
C. \(x = 3\pi/4\)  
D. \(x = -5\pi/4\)  
9. ________

10. Which one of the following is an equation of the graph below over the interval \(0 \leq x \leq 3\pi\)?

![Graph Image]

A. \(y = \tan 2x\)  
B. \(y = \tan 1/2x\)  
C. \(y = \cot 1/2x\)  
D. \(y = \cot 2x\)  
10. ________

Part II:

11. On the grid provided accurately graph \(y = 2 \sin 3x\). Label at least 4 points in one period. (2 marks.)

![Grid Image]
12. Write an equation for a cosine curve with an amplitude of 4 and a period of $\pi/3$. (one mark)

13. Given the function $y = -2 \cos 1/3 x$ (2 marks)
   a) What is the amplitude?
   b) What is the range?
   c) Sketch its graph on the grid below labeling at least five points accurately.

Bonus: Sketch a graph of at least one period of $y = \frac{(\sin x)(\cos x)}{\cos^2 x}$
Algebra 12 Trigonometric Graphing Test (Posttest 1A)

Name: __________________________ Score: _______ 15

Part I: Multiple Choice (Choose the best answer for the following)

1. What is the amplitude of the curve shown below?

A. -3  B. 3  C. 6  D. 2π

2. What is the period of the trigonometric function graphed below?

A. 2π/5  B. 6π/5  C. 2π  D. π

3. The amplitude of the curve $y = \frac{2}{3} \sin 4x$ is:

A. 1/6  B. 2/3  C. 8/3  D. 4

4. What is the period of the curve $y = 4 \sin \pi x$?

A. 2π  B. 4π  C. 2  D. 4

5. Which one of the following functions is represented by the graph below?

A. $y = 4 \sin 3(x + \pi/6)$  B. $y = 4 \sin 3(x - \pi/6)$
C. $y = 8 \cos 3(x - \pi/6)$  D. $y = 8 \sin 3(x + \pi/6)$

B
6. Which of the following is an equation of a sine curve with a maximum value 2, minimum value 0, and period $\pi/3$?
   A. $y = 2 + \sin 3\pi x$  
   B. $y = 1 + \sin 6\pi x$  
   C. $y = 2 \sin 3x$  
   D. $y = 1 + \sin 6x$  
   6. _____

7. What is the range of $y = 2 \sin 3x - 1$?
   A. $-1 \leq y \leq 3$  
   B. $-3 \leq y \leq 1$  
   C. $-1 \leq y \leq 1$  
   D. $-2 \leq y \leq 2$  
   7. _____

8. Which of the following relations has a graph which is the same as the graph of $y = 3 \cos (x + \pi/2)$?
   A. $y = -3 \sin x$  
   B. $y = 3 \sin (x - \pi/2)$  
   C. $y = 2 \cos 1/2 x$  
   D. $y = 3 \cos x$  
   8. _____

9. One asymptote of the curve $y = \tan (x - \pi/4)$ is:
   A. $x = 0$  
   B. $x = \pi/4$  
   C. $x = 3\pi/4$  
   D. $x = 5\pi/4$  
   9. _____

10. Which one of the following is an equation of the graph below over the interval $0 \leq x \leq 3\pi$?
    A. $y = \tan 2x$  
    B. $y = \tan 1/2 x$  
    C. $y = \cot 1/2 x$  
    D. $y = \cot 2x$  
    10. _____

Part II:

11. On the grid provided accurately graph $y = 2 \cos 3x$. Label at least 4 points in one period. (2 marks).
   \[\begin{align*}
   &(-\frac{2\pi}{3}, 2) \quad (0, 2) \quad \frac{\pi}{6}, 1) \quad (-\frac{\pi}{3}, -2) \quad (\frac{2\pi}{3}, 2)
   
   &\frac{\pi}{6} \quad \frac{\pi}{3} \quad \frac{\pi}{2} \quad \frac{2\pi}{3}
   \end{align*}\]
12. Write an equation for a sine curve with an amplitude of 3 and a period of \( \pi/4 \). (one mark)

\[ y = 3 \sin 8x \]

13. Given the function \( y + 1 = -2 \sin \frac{1}{3}x \) (2 marks)

a) What is the amplitude? 2

b) What is the range? \(-3 \leq y \leq 1\)

c) Sketch its graph on the grid below labeling at least five points accurately.

Bonus: Sketch a graph of at least one period of \( y = \frac{\sin 2x}{\sin x \cos x} \)

\[ y = \tan x \]
Algebra 12 Trigonometric Graphing Test (Posttest 1B)

Part I: Multiple Choice (Choose the best answer for the following)

1. What is the amplitude of the curve shown below?

```
\[ A. \ 4\pi \quad B. \ 2\pi \quad C. \ 6 \quad D. \ 3 \]
```

2. What is the period of the trigonometric function graphed below?

```
\[ A. \ 4\pi/3 \quad B. \ 2\pi/3 \quad C. \ 3\pi/4 \quad D. \ \pi \]
```

3. The amplitude of the curve \( y = -3/5 \sin 2x \) is:

```
\[ A. \ -3/5 \quad B. \ 3/5 \quad C. \ 6/5 \quad D. \ -6/5 \]
```

4. What is the period of the curve \( y = 2 \cos 3/2x \)?

```
\[ A. \ 4\pi/3 \quad B. \ 3\pi \quad C. \ 3/2 \quad D. \ 2 \]
```

5. Which one of the following functions is represented by the graph below?

```
A. \( y = 2 \sin 3(\theta - 30^\circ) \)
B. \( y = 2 \sin 3(\theta + 30^\circ) \)
C. \( y = 2 \sin 1/3(\theta - 30^\circ) \)
D. \( y = 2 \sin 1/3(\theta + 30^\circ) \)
```

```
6. A sine function has range values $-2 \leq y \leq 4$ and a period of $\pi$. What is one possibility for its equation?
   A. $y = 3 \sin x + 1$  
   B. $y = 6 \sin x + 1$  
   C. $y = 3 \sin 2x + 1$  
   D. $y = 6 \sin 2x + 1$

7. What is the range of $y = 3 \sin 2x + 1$?
   A. $-2 \leq y \leq 4$  
   B. $-4 \leq y \leq 2$  
   C. $-1 \leq y \leq 1$  
   D. $-3 \leq y \leq 3$

8. Which of the following relations has a graph which is the same as the graph of $y = 3 \cos \left( x + \frac{\pi}{2} \right)$?
   A. $y = -3 \sin x$  
   B. $y = 3 \sin \left( x - \frac{\pi}{2} \right)$  
   C. $y = 2 \cos \frac{1}{2}x$  
   D. $y = 3 \cos x$

9. One asymptote of the curve $y = \tan \left( x + \frac{\pi}{4} \right)$ is:
   A. $x = 0$  
   B. $x = \frac{\pi}{4}$  
   C. $x = \frac{3\pi}{4}$  
   D. $x = -\frac{5\pi}{4}$

10. Which one of the following is an equation of the graph below over the interval $0 \leq x \leq 3\pi$?
    A. $y = \tan 2x$  
    B. $y = \tan \frac{1}{2}x$  
    C. $y = \cot \frac{1}{2}x$  
    D. $y = \cot 2x$

Part II:
11. On the grid provided accurately graph $y = 2 \sin 3x$. Label at least 4 points in one period. (2 marks).

   \[
   \begin{array}{c}
   (-\frac{\pi}{3}, 0) \\
   (0, 0) \\
   (\frac{\pi}{6}, 2) \\
   (\frac{\pi}{3}, 0) \\
   (\frac{\pi}{2}, -2) \\
   (\frac{2\pi}{3}, 0) \\
   (\frac{3\pi}{6}, 2) \\
   \end{array}
   \]
12. Write an equation for a cosine curve with an amplitude of 4 and a period of $\pi/3$. (one mark)

$$Y = 4 \cos 6 \pi x$$

13. Given the function $y - 1 = -2 \cos 1/3 \pi x$ (2 marks)
a) What is the amplitude? 2
b) What is the range? $-1 \leq y \leq 3$
c) Sketch its graph on the grid below labeling at least five points accurately.

![Graph of the function $y - 1 = -2 \cos 1/3 \pi x$]

Bonus: Sketch a graph of at least one period of $y = \frac{\sin x \cos x}{\cos^2 x}$

$$y = \tan x$$
APPENDIX E

LESSON ONE: TEXTBOOK TREATMENT
Lesson One: Textbook Treatment:

13-4 Graphs of the Sine and Cosine

Objective To graph the sine and cosine and related functions.

To graph $y = \sin x$ and $y = \cos x$ we first make a table of values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>$\frac{\pi}{6}$</th>
<th>$\frac{\pi}{3}$</th>
<th>$\frac{\pi}{2}$</th>
<th>$\frac{2\pi}{3}$</th>
<th>$\pi$</th>
<th>$\frac{4\pi}{3}$</th>
<th>$\frac{5\pi}{3}$</th>
<th>$\frac{7\pi}{6}$</th>
<th>$2\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin x$</td>
<td>0.50</td>
<td>0.87</td>
<td>1</td>
<td>0.87</td>
<td>0.50</td>
<td>0</td>
<td>-0.50</td>
<td>-0.87</td>
<td>-1</td>
<td>-0.87</td>
</tr>
<tr>
<td>$\cos x$</td>
<td>1</td>
<td>0.87</td>
<td>0.50</td>
<td>0</td>
<td>-0.50</td>
<td>-0.87</td>
<td>-1</td>
<td>-0.87</td>
<td>-0.50</td>
<td>0</td>
</tr>
</tbody>
</table>

When we plot the ordered pairs $(x, \sin x)$ and $(x, \cos x)$ and join them by smooth curves, we obtain the graphs shown in Figures 9 and 10.

As noted on page 582, $\sin x$ and $\cos x$ each have period $2\pi$. Thus, the graphs of $y = \sin x$ and $y = \cos x$ consist of what we have drawn above repeated over and over, as shown in Figures 11 and 12.

Notice that the graph of the sine function is symmetric with respect to the origin (the sine is an odd function). The graph of the cosine function is symmetric with respect to the y-axis (the cosine is an even function).
Figure 13 shows the graphs of three functions of the form \( y = a \sin x \) \((a > 0)\). In each case the value of the function varies from a minimum value of \(-a\) to a maximum value of \(a\). The number \(a\) is called the amplitude of \(a \sin x\). Thus, the amplitude of \(2 \sin x\) is 2, of \(\sin x\) is 1, and of \(\frac{1}{2} \sin x\) is \(\frac{1}{2}\). In general, we have the following.

If a periodic function has maximum value \(M\) and minimum value \(m\), then its amplitude is \(\frac{M - m}{2}\).

Whereas the \(a\) in \(y = a \sin x\) causes a vertical stretch or compression of the graph of \(y = \sin x\), the \(b\) in \(y = \sin bx\) causes a horizontal stretch or compression. That is, the coefficient \(b\) affects the period.

For example, to find the period of \(\sin 3x\), let \(f(x) = \sin 3x\). Then \(f(x + p) = \sin 3(x + p) = \sin (3x + 3p)\). Therefore \(f(x + p) = f(x)\) if \(\sin (3x + 3p) = \sin 3x\), and this will hold if \(3p = 2\pi\). Thus, \(p = \frac{2\pi}{3}\). The graph of \(y = \sin 3x\) is shown with the graph of \(y = \sin x\) in Figure 14.

In general, the period of \(\sin bx\) \((b > 0)\) is \(\frac{2\pi}{b}\), with a similar result for the cosine. In summary, the functions \(a \sin bx\) and \(a \cos bx\) \((a, b > 0)\) have amplitude \(a\) and period \(\frac{2\pi}{b}\).
Example 1  Graph \( y = 2 \cos \frac{x}{2} \).

Solution  The amplitude is 2 and the period is \( \frac{2\pi}{\frac{x}{2}} \), or 4. The graph is shown below.

Example 2  Each graph below has an equation of the form \( y = c + a \cos bx \) or \( y = c + a \sin bx \). Find an equation of each graph.

Solution  

a. The graph is the same as the one in Example 1 except that it has been shifted 2 units upward.  

\[ y = 2 + 2 \cos \frac{x}{2} \]  

Answer

b. Since the maximum \( M \) is \(-1\) and the minimum \( m \) is \(-2\), we have

\[ a = \frac{M - m}{2} = \frac{-1 - (-2)}{2} = \frac{1}{2} \]  

The graph shows a function with period \( \pi \). Therefore, \( \frac{2\pi}{b} = \pi \) and \( b = 2 \). Finally,

\[ c = \frac{M + m}{2} = \frac{-1 + (-2)}{2} = \frac{-3}{2} \]  

Since at \( x = 0 \) the graph is halfway between the maximum and minimum, it is a sine curve.

\[ y = -\frac{3}{2} + \frac{1}{2} \sin 2x \]  

Answer

Notice the effect of the constant \( c \) in Example 2. If \( c > 0 \), the graph is shifted upward \( c \) units. If \( c < 0 \), the graph is shifted downward \(|c|\) units.

Working from the graph, \( c \) may be found using the formula

\[ c = \frac{M + m}{2} \]
Oral Exercises

State the amplitude and period of each sine or cosine function graphed below.

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

In Exercises 9–12 match each description with the letter of the function that fits the description.

9. The amplitude is 2. 
   a. \( y = 2 + \sin 2\pi x \)
10. The period is \( \pi \). 
    b. \( y = 1 + \sin \pi x \)
11. The minimum value is 0. 
    c. \( y = 2 \cos 5x \)
12. The period is 1. 
    d. \( y = 2 + \sin x \)
    e. \( y = 3 \cos 2x \)

Written Exercises

In Exercises 1-12:
a. State the amplitude of each function.
b. State the maximum and minimum values.
c. State the period.
A 1. \( y = \frac{1}{2} \sin x \)  
2. \( y = 3 \cos x \)  
3. \( y = \sin 3x \)  
4. \( y = \cos 2x \)  
5. \( y = 4 \cos 4x \)  
6. \( y = \sin 2\pi x \)  
7. \( y = 2 \sin \frac{1}{2}x \)  
8. \( y = \frac{1}{2} \cos 2x \)  
9. \( y = \frac{1}{3} \cos \pi x \)  
10. \( y = 2 \sin \frac{1}{2}\pi x \)  
11. \( y = 1 + \cos 2\pi x \)  
12. \( y = 3 + 2 \sin \pi x \)  
13-20. Find an equation of each curve pictured in Oral Exercises 1-8.

Find an equation of a sine curve that satisfies the stated conditions. 
(M = maximum value, \( m \) = minimum value.)
21. \( M = 2, m = -2, \) period \( \pi \)  
22. \( M = 1, m = -1, \) period \( 6\pi \)  
23. \( M = 1, m = -1, \) period \( 1 \)  
24. \( M = 3, m = -3, \) period \( 4 \)  
25. \( M = 5, m = 1, \) period \( 2\pi \)  
26. \( M = 6, m = 0, \) period \( 2 \)  
27-32. Sketch the graph of each function given in Exercises 1-6. Show at least two periods.

B 33-38. Sketch the graph of each function given in Exercises 7-12. Show at least two periods.

In a function of the form \( y = \sin (x - d) \) or \( y = \cos (x - d) \), the number \( d \) determines the phase shift of the graph. In general, for any function \( f \), the graph of \( y = f(x - d) \) is obtained by shifting the graph of \( y = f(x) \) to the right \( d \) units if \( d \) is positive or to the left \( |d| \) units if \( d \) is negative. The graph of \( y = -f(x) \) is obtained by reflecting the graph of \( y = f(x) \) in the \( x \)-axis.

Use the information given in the preceding paragraph to sketch the graph of each of the following equations. Show at least two periods.
39. \( y = \sin \left(x - \frac{\pi}{4}\right) + 1 \)  
40. \( y = -3 \cos 2x \)  
41. \( y = -\sin 2\left(x + \frac{\pi}{3}\right) \)  
C 42. \( y = -\cos (3x - \pi) + 1 \)  
43. \( y = \frac{1}{2} - 2 \sin (4x + \pi) \)  
44. \( y = 4 - 2 \sin (2\pi x - \pi) \)  
45. Use the equation \( \sin 2x = 2 \sin x \cos x \) to show that the graph of \( y = \sin x \cos x \) is a sine curve. Sketch the graph.
46. Use the equation \( \cos 2x = 1 - 2 \sin^2 x \) to show that the graph of \( y = \sin^2 x \) is a cosine curve. Sketch the graph.

Trigonometric Graphs and Identities

APPENDIX F

LESSON TWO: TEXTBOOK TREATMENT
Lesson Two: Textbook Treatment:

13-5 Graphs of the Other Trigonometric Functions

Objective To graph the tangent, cotangent, secant, and cosecant functions.

Before graphing the tangent function, let us list a few of its properties.

1. The period of \( \tan x \) is \( \pi \). We know that \( \sin (x + \pi) = -\sin x \) and \( \cos (x + \pi) = -\cos x \) (see Oral Exercise 11, page 580). Therefore

\[
\tan (x + \pi) = \frac{\sin (x + \pi)}{\cos (x + \pi)} = \frac{-\sin x}{-\cos x} = \tan x.
\]

2. The function \( \tan x \) is odd. Since \( \sin x \) is odd and \( \cos x \) is even, we have:

\[
\tan (-x) = \frac{\sin (-x)}{\cos (-x)} = \frac{-\sin x}{\cos x} = -\tan x
\]

3. As \( x \) increases toward \( \frac{\pi}{2} \), \( \tan x \) increases without bound. Refer to Figure 15. In Exercise 26, page 581, it was shown that for \( 0 < x < \frac{\pi}{2} \), \( \tan x = \frac{AQ}{AP} \). (The letter \( s \) was used instead of \( x \).) Thus, the nearer \( x \) is to \( \frac{\pi}{2} \), the closer \( P \) is to \( B \), and the larger \( AQ \) is. The function \( \tan x \) is undefined at \( x = \frac{\pi}{2} \).

We start the graph of \( y = \tan x \) by using a short table of values and the third property above to obtain Figure 16.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \tan x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{\pi}{6} )</td>
<td>0.52</td>
</tr>
<tr>
<td>( \frac{\pi}{4} )</td>
<td>0.79</td>
</tr>
<tr>
<td>( \frac{\pi}{3} )</td>
<td>1.05</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>1.57</td>
</tr>
</tbody>
</table>

The dashed line is an asymptote of the graph (recall Section 9-5). Next we use the fact that \( \tan x \) is odd and reflect this part of the graph in the origin to obtain Figure 17. Finally, we use the fact that \( \tan x \) has period \( \pi \) to obtain the final result shown in Figure 18.
Lesson Two: Textbook Treatment

Using methods similar to the ones above, we obtain the graphs of the cotangent, secant, and cosecant functions. These are shown below in red along with the graphs of their reciprocal functions.

Oral Exercises

Give the period of each function.

**Example** \( y = \tan 2\pi x \)

**Solution** \( \tan 2\pi(x + p) = \tan (2\pi x + 2\pi p) = \tan 2\pi x \) if \( 2\pi p = \pi \).

\[
\therefore p = \frac{1}{2}.
\]

1. \( y = \tan \frac{1}{2} x \)
2. \( y = \sec 2x \)
3. \( y = \frac{1}{2} \csc x \)
4. \( y = \frac{1}{2} \cot x \)
5. \( y = 2 \cot \pi x \)
6. \( y = 2 \tan \frac{\pi}{2} x \)
7. \( y = -1 + \sec 2x \)
8. \( y = 1 + \csc \frac{1}{2} x \)

Note. Algebra and Trigonometry: Structure and Method - Book 2, (c) 1984

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Written Exercises

A

1. Draw the graph of \( y = \sin x \) for \(- \frac{\pi}{2} \leq x \leq \frac{3\pi}{2}\).

2. Draw the graph of \( y = \cos x \) for \(- \pi \leq x \leq \pi\).

In Exercises 3–8:

a. State the maximum and minimum values of each function.

b. State the period.

c. Sketch the graph. (Include at least two periods of the function.)

3. \( y = 3 \sin x \)

4. \( y = 2 \cos x \)

5. \( y = \cos 2x \)

6. \( y = \cos 4x \)

7. \( y = 5 \sin 2x \)

8. \( y = \sin 3x \)

State an equation for each sine wave or cosine wave.

9. \[ \begin{aligned} \begin{array}{c} y \end{array} \end{aligned} \]

10. \[ \begin{aligned} \begin{array}{c} y \end{array} \end{aligned} \]

11. \[ \begin{aligned} \begin{array}{c} y \end{array} \end{aligned} \]

12. \[ \begin{aligned} \begin{array}{c} y \end{aligned} \]

In Exercises 13–21:

a. State the maximum and minimum values of each function.

b. State the period.

c. Sketch the graph. (Include at least two periods of the function.)

B

13. \( y = \cos \frac{1}{2}x \)

14. \( y = \sin \frac{1}{3}x \)

15. \( y = 4 \sin \frac{1}{2}x \)

16. \( y = \cos \frac{\pi}{10}x \)

17. \( y = \cos \frac{2\pi}{365}x \)

18. \( y = 3 \sin 2\pi x \)

19. \( y = 3 + \sin x \)

20. \( y = -1 + 2 \sin x \)

21. \( y = 2 + \frac{1}{2} \cos 3x \)

State an equation for each sine wave or cosine wave.

22. \[ \begin{aligned} \begin{array}{c} y \end{array} \end{aligned} \]

23. \[ \begin{aligned} \begin{array}{c} y \end{array} \end{aligned} \]
State an equation for each sine wave or cosine wave.

24. \[ y = \sin x \]

25. \[ y = \cos x \]

26. A sine wave varies from \(-5\) to \(5\) with a period of \(365\). What is its equation?

27. A cosine wave varies from \(-32\) to \(32\) with a period of \(52\). What is its equation?

28. A cosine wave has period \(12\). Its maximum and minimum values are \(10\) and \(2\). What is its equation?

29. A sine wave has period \(\frac{1}{10}\). Its maximum and minimum values are \(7\) and \(-3\). What is its equation?

C 30. a. Sketch the graphs of \(y = 2\cos 2x\) and \(y = -2\cos 2x\) on the same set of axes.

b. How are the graphs in part (a) related?

31. Repeat Exercise 30 for \(y = \sin x\) and \(y = -\sin x\).

Self-Test 2

VOCABULARY

<table>
<thead>
<tr>
<th>Reference Angle (p. 458)</th>
<th>Periodic Function (p. 463)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular Functions (p. 462)</td>
<td>Period (p. 463)</td>
</tr>
<tr>
<td>Sinusoidal (p. 463)</td>
<td>Amplitude (p. 464)</td>
</tr>
<tr>
<td>Sine Wave (p. 463)</td>
<td></td>
</tr>
</tbody>
</table>

Evaluate using tables.

1. \(\cos 26°10'\)  
2. \(\sin 1.38°\)  
3. \(\cos 67.4°\)  
(Obj. 12-4, p. 455)

Evaluate using reference angles and tables.

4. \(\sin 120°20'\)  
5. \(\cos 3.65\)  
6. \(\sin 294.8°\)  
(Obj. 12-5, p. 458)

7. Solve \(\sin \theta = 0.7683\) if \(\theta\) is between \(0°\) and \(360°\). Give your answer in decimal degrees.

Sketch the graph of each function for \(0 \leq x \leq 2\pi\).

8. \(y = \cos x\)  
9. \(y = \frac{1}{2}\sin x\)  
(Obj. 12-6, p. 462)

Check your answers with those at the back of the book.

468 CHAPTER 12

APPENDIX G
ANSWER KEYS TO TEXTBOOK LESSONS
1. \( y = \frac{1}{2} \sin x \)
   a) \( \frac{1}{2} \)
   b) \( \frac{1}{2}, -\frac{1}{2} \)
   c) \( 2\pi \)

3. \( y = \sin 3x \)
   a) \( 1 \)
   b) \( 1, -1 \)
   c) \( \frac{2\pi}{3} \)

5. \( y = 4 \cos 4x \)
   a) \( 4 \)
   b) \( 4, -4 \)
   c) \( \frac{\pi}{2} \)

7. \( y = 2 \sin \frac{1}{2} x \)
   a) \( 2 \)
   b) \( 2, -2 \)
   c) \( 4\pi \)

9. \( y = \frac{1}{3} \cos \pi x \)
   a) \( \frac{1}{3} \)
   b) \( \frac{1}{3}, -\frac{1}{3} \)
   c) \( \frac{2}{3} \)

11. \( y = 1 + \cos 2\pi x \)
    a) \( 1 \)
    b) \( 2, 0 \)
    c) \( 1 \)

13. \( y = 2 \sin x \)

19. \( y = 3 + 3 \cos \frac{\pi}{4} x \)

21. \( y = 2 \sin 2x \)

25. \( y = 3 + 2 \sin x \)

27. - 37. see graphs above.

39. \( y = \sin(x - \pi/4) + 1 \)
    Amplitude = 1
    Period = \( 2\pi \)
    Range: \( 0 \leq y \leq 2 \)

41. \( y = -\sin 2(x + \pi/3) \)
    Amplitude = 1
    Period = \( \pi \)
    Range: \( -1 \leq y \leq 1 \)

43. \( y = \frac{1}{2} - 2\sin(4x + \pi) \)
    Amplitude = 2
    Period = \( 2\pi \)
    Range: \(-\frac{3}{2} \leq y \leq \frac{5}{2}\)
Answer Key - Lesson Two: Textbook Treatment (Dolciani pp. 467/468)

Written Exercises, pages 467-468

1. $y = \sin x$

3. a. maximum: 3; minimum: -3 b. $2\pi$
   c. $y = \sin x$

5. a. maximum: 1; minimum: -1 b. $\pi$
   c. $y = \sin x$

7. a. maximum: 5; minimum: -5 b. $\pi$
   c. $y = \sin x$

9. $y = \frac{1}{2} \sin 2x$
11. $y = 4 \sin 2x$
13. a. maximum: 1; minimum: -1 b. $4\pi$
   c. $y = \sin x$

15. a. maximum: 4; minimum: -4 b. $4\pi$
   c. $y = \sin x$

17. a. maximum: 1; minimum: -1 b. 365
   c. $y = \sin x$

19. a. maximum: 4; minimum: 2 b. $2\pi$
   c. $y = \sin x$

21. a. maximum: $2\frac{1}{2}$; minimum: $1\frac{1}{2}$ b. $\frac{2\pi}{3}$
   c. $y = \sin x$

23. $3 \cos \frac{\pi x}{4}$
25. $y = 4 + 2 \cos \frac{\pi x}{5}$
27. $y = 32 \cos \frac{x}{26}$
29. $y = 2 + 5 \sin 20nx$
31. a. $y = \frac{1}{2} \sin \frac{1}{2}x$
   b. $y = -\frac{1}{2} \sin \frac{1}{2}x$
APPENDIX H
SAMPLE OF ALGEBRA 12 CHAPTER TEST
ALGEBRA 12 CHAPTER 9 (CONICS) TEST A

1. MULTIPLE CHOICE. Select the best answer for each question.

1. What are the coordinates of the vertex of the parabola \( x = -2(y - 2)^2 - 5 \)
   A. (5, -2)  B. (2, -5)  C. (-2, 5)  D. (-5, 2)
   
2. Over the set of real numbers, how many solutions does the following system have? (Do not solve)
   \[
   \frac{x^2}{36} + \frac{y^2}{36} = 1
   \]
   \[ y = x^2 - 4 \]
   A. 0  B. 2  C. 3  D. 4
   
3. What are the coordinates of the focus for the parabola \( y = -\frac{1}{16}(x + 4)^2 - 7 \) ?
   A. (-8, -7)  B. (0, -7)  C. (-4, -11)  D. (-4, -3)
   
4. Which of the following is the equation of a circle with center (-3, 2) and radius \( \sqrt{7} \) ?
   A. \( x^2 + y^2 + 6x - 4y + 6 = 0 \)
   B. \( x^2 + y^2 - 6x - 4y + 6 = 0 \)
   C. \( x^2 + y^2 - 6x + 4y + 6 = 0 \)
   D. \( x^2 + y^2 + 6x + 4y + 6 = 0 \)
   
5. What is the equation of the conic graphed below?
   \[
   \frac{x^2}{16} - \frac{y^2}{9} = 1
   \]
   \[
   \frac{y^2}{16} - \frac{x^2}{9} = 1
   \]
   \[
   \frac{x^2}{9} - \frac{y^2}{16} = 1
   \]
   \[
   \frac{y^2}{9} - \frac{x^2}{16} = 1
   \]
   
6. What is the constant of variation equal to if \( X \) varies jointly as \( Y \) and \( Z \) and inversely as \( W \)?
   A. \( \frac{XYZ}{W} \)  B. \( \frac{XYW}{Z} \)  C. \( \frac{XZW}{Y} \)  D. \( \frac{XW}{YZ} \)
7. What is the range of the quadratic relation \( y = \frac{-1}{4}(x - 2)^2 + 2 \)?
   A. \( y: y \geq 2 \)  
   B. \( y: y \leq 2 \)  
   C. \( y: y \geq -2 \)  
   D. \( y: y \leq -2 \)  

8. What are the coordinates of the foci of the conic \( 9x^2 + 4y^2 = 36 \)?
   A. \((0, \pm \sqrt{13})\)  
   B. \((\pm \sqrt{13}, 0)\)  
   C. \((0, \pm \sqrt{5})\)  
   D. \((\pm \sqrt{5}, 0)\)  

9. What is the equation of the parabola with directrix \( x = 7 \) and focus \((3, 2)\)?
   A. \( y = \frac{1}{8} (x - 5)^2 + 2 \)  
   B. \( y = -\frac{1}{8} (x - 5)^2 + 2 \)  
   C. \( x = \frac{1}{8} (y - 2)^2 + 5 \)  
   D. \( x = -\frac{1}{8} (y - 2)^2 + 5 \)  

10. Points \( P(-8, 16) \) and \( Q(6, -8) \) are the endpoints of a line segment \( PQ \). Point \( M \) is the midpoint of segment \( PQ \), and point \( A \) is the midpoint of segment \( MQ \). What are the coordinates of \( A \)?
    A. \((-1, 4)\)  
    B. \((-1/2, 2)\)  
    C. \((1/2, -2)\)  
    D. \((5/2, -2)\)  

11. The graph of \( 9x^2 + 4y^2 - 18x + 24y + 9 = 0 \) is an ellipse whose center has coordinates:
    A. \((1, -3)\)  
    B. \((-1, 3)\)  
    C. \((3, -1)\)  
    D. \((-3, 1)\)  

12. What are the coordinates of the midpoint of the line segment joining the vertex of the parabola \( y = 3(x - 2)^2 - 1 \) and the center of the circle \( (x - 6)^2 + (y - 5)^2 = 16 \)?
    A. \((-2, -3)\)  
    B. \((4, -3)\)  
    C. \((4, 2)\)  
    D. \((2, 3)\)  

13. The graph of an ellipse has \( x \)-intercepts \( \pm 8 \) and \( y \)-intercepts \( \pm 3 \), then the sum of the distances from any point on the ellipse to the foci is:
    A. 6  
    B. 8  
    C. 11  
    D. 16  

14. The graph of the relation \( y = \sqrt{4 - x^2} \) is located in which quadrant(s)?
   A. I only  B. I and II only
   C. I and IV only  D. I, II, III, and IV

15. If \( y \) varies inversely as the square root of \( x \), what is the value of the constant of variation if \( y = 2 \) when \( x = 16 \)?
   A. 512  B. 8  C. 1/2  D. 1/128

FOR THE FOLLOWING SECTIONS CIRCLE YOUR FINAL ANSWERS / SHOW YOUR WORK.

II. Solve the following over the set of complex numbers (2 marks each)

16. \((x^2 - 5)(9x^2 - 1) = 0\)

17. \(4x^2 + y^2 = 13\)
   \(2x - y = 1\)

18. \(x^2 + y^2 = 13\)
   \(x^2 - y = 7\)

19. \(2x + y - 3z = -4\)
   \(4x - 2y + 2z = 12\)
   \(6x + 3y - z = 4\)

III. Solve the following problems (2 marks each)

20. The solve \( V \) of a gas kept at constant temperature varies inversely as the pressure \( P \). If the volume \( V \) is 250 cm\(^3\) when the pressure is 80 kPa, find the volume when the pressure is 25 kPa.
21. The weight of a body is inversely proportional to the square of its distance from the center of the earth. If a man weighs 142 pounds on the earth's surface, what will he weigh 200 miles above the earth (assume radius of earth to be 4000 miles).

22. Given the ordered pairs (5, x) and (15, 4) find x such that:
   a) the ordered pairs vary directly
   b) the ordered pairs vary inversely

IV. Solve the following problems by setting up a system in two variables (3 marks each)
23. When the area of one square is subtracted from three times the area of a smaller square, the result is 11 m\(^2\). The sum of the two areas is 89 m\(^2\). Find the dimensions of each square.

24. A rectangular garden of area 20 m\(^2\) is to be surrounded by a grass border 1.5 m wide. The area of the grass and garden together is 70 m\(^2\). Find the dimensions of the garden.

25. A surface is now 10 m from a source of light. How far from the source would the light have to be in order to receive half as much illumination if the intensity of illumination (Ix) varies inversely as the square of the distance from the source?
ALGEBRA 12 CHAPTER 9 (CONICS) TEST B

NAME:______________________________

1. MULTIPLE CHOICE. Select the best answer for each question.

1. What are the coordinates of the vertex of the parabola \( x = -3(y + 4)^2 + 7 \)
   A. \((-7, 4)\)  
   B. \((-4, 7)\)  
   C. \((4, -7)\)  
   D. \((7, -4)\)

2. Over the set of real numbers, how many solutions does the following system have? (Do not solve)
   \[
   \begin{align*}
   \frac{x^2}{36} + \frac{y^2}{16} &= 1 \\
   \frac{x^2}{25} - \frac{y^2}{9} &= 1
   \end{align*}
   \]
   A. 0  
   B. 2  
   C. 3  
   D. 4

3. What are the coordinates of the focus for the parabola \( y = \frac{1}{8}(x - 2)^2 - 5 \) ?
   A. \((4, -5)\)  
   B. \((2, -7)\)  
   C. \((0, -5)\)  
   D. \((2, -3)\)

4. Which of the following is the equation of a circle with center \((3, -2)\) and radius \(\sqrt{10}\)?
   A. \(x^2 + y^2 + 6x - 4y + 3 = 0\)  
   B. \(x^2 + y^2 - 6x - 4y + 3 = 0\)  
   C. \(x^2 + y^2 - 6x + 4y + 3 = 0\)  
   D. \(x^2 + y^2 + 6x + 4y + 3 = 0\)

5. What is the equation of the conic graphed below?
   A. \(\frac{x^2}{16} - \frac{y^2}{9} = 1\)  
   B. \(\frac{y^2}{16} - \frac{x^2}{9} = 1\)  
   C. \(\frac{x^2}{9} - \frac{y^2}{16} = 1\)  
   D. \(\frac{y^2}{9} - \frac{x^2}{16} = 1\)

6. What is the constant of variation equal to if \(X\) varies jointly as \(Y\) and \(Z\) and inversely as \(W\)?
   A. \(\frac{XYZ}{W}\)  
   B. \(\frac{XYW}{Z}\)  
   C. \(\frac{XZW}{Y}\)  
   D. \(\frac{XW}{YZ}\)
7. What is the domain of the quadratic relation $x = \frac{-1}{4}(y - 1)^2 + 2$?
   A. $\{x : x \geq 2\}$  
   B. $\{x : x \leq 2\}$  
   C. $\{x : x \geq -2\}$  
   D. $\{x : x \leq -2\}$
   7. ____

8. What are the coordinates of the foci of the conic $9x^2 - 4y^2 = 36$?
   A. $(0, \pm \sqrt{13})$
   B. $(\pm \sqrt{13}, 0)$
   C. $(0, \pm \sqrt{5})$
   D. $(\pm \sqrt{5}, 0)$
   8. ____

9. What is the equation of the parabola with directrix $x = -7$ and focus $(-3, 2)$?
   A. $y = 1/8 (x + 5)^2 + 2$
   B. $y = -1/8 (x + 5)^2 + 2$
   C. $x = 1/8 (y - 2)^2 - 5$
   D. $x = -1/8 (y - 2)^2 - 5$
   9. ____

10. Points $P(-8, 16)$ and $Q(6, -8)$ are the endpoints of a line segment $\overline{PQ}$. Point $M$ is the midpoint of segment $\overline{PQ}$, and point $A$ is the midpoint of segment $\overline{MQ}$. What are the coordinates of $A$?
    A. $(-1, 4)$
    B. $(-1/2, 2)$
    C. $(1/2, -2)$
    D. $(5/2, -2)$
    10. ____

11. The graph of $4x^2 + 9y^2 - 24x + 18y + 9 = 0$ is an ellipse whose center has coordinates:
    A. $(1, -3)$
    B. $(-1, 3)$
    C. $(3, -1)$
    D. $(-3, 1)$
    11. ____

12. What are the coordinates of the midpoint of the line segment joining the vertex of the parabola $y = 3(x + 2)^2 + 1$ and the center of the circle $(x - 4)^2 + (y - 3)^2 = 16$?
    A. $(-1, -1)$
    B. $(-3, -1)$
    C. $(1, 2)$
    D. $(1, 1)$
    12. ____

13. The graph of an ellipse has $x$-intercepts $\pm 10$ and $y$-intercepts $\pm 4$, then the sum of the distances from any point on the ellipse to the foci is:
    A. 8
    B. 10
    C. 14
    D. 20
    13. ____
14. The graph of the relation \( x = \sqrt{4 - y^2} \) is located in which quadrant(s)?
   A. I only  B. I and II only  
   C. I and IV only  D. I, II, III, and IV

15. If \( y \) varies inversely as the square root of \( x \), what is the value of the constant of variation if \( y = 2 \) when \( x = 16 \)?
   A. 512  B. 8  C. 1/2  D. 1/128

FOR THE FOLLOWING SECTIONS CIRCLE YOUR FINAL ANSWERS / SHOW YOUR WORK.

II. Solve the following over the set of complex numbers (2 marks each)

16. \((x^2 + 3)(4x^2 - 1) = 0\)

17. \(y = x^2 - 3x + 2\)
   \(x = y + 2\)

18. \(4x^2 + 9y^2 = 36\)
   \(x^2 = 4 - y^2\)

19. \(3a + b + 2c = 5\)
   \(2a - 3b + 4c = -1\)
   \(5a + 2b - 6c = -14\)

(3 marks)

III. Solve the following problems (2 marks each)

20. If two circular disks are connected by a pulley, the number of revolutions made by each disk varies inversely as the disk's circumference. If the smaller disk has a circumference of 20 cm. and revolves at a rate of 350 rpm, how many revolutions/minute would the larger disk of circumference 50 cm. make?
21. The weight of a body is inversely proportional to the square of its
distance from the center of the earth. If a man weighs 142 pounds on the
earth's surface, what will he weigh 200 miles above the earth (assume
radius of earth to be 4000 miles).

22. Given the ordered pairs (7, 12) and (x, 60) find x such that:
   a) the ordered pairs vary directly
   b) the ordered pairs vary inversely

IV. Solve the following problems by setting up a system in two variables
    (3 marks each)
23. The difference of the squares of the digits of a two digit number is 39.
    If the digits are reversed the resulting number is 27 less than the
    original number. What is the original number?

24. A steel reinforcing rod is 24 m. long. It is bent into the shape of a right
    triangle with the hypotenuse 10 m. long. Find the lengths of the legs of
    the triangle.

25. A surface is now 10 m. from a source of light. How far from the source
    would the light have to be in order to receive half as much illumination
    if the intensity of illumination (Ix) varies inversely as the square of
    the distance from the source?
APPENDIX I

COEFFICIENT OF RELIABILITY FOR
MARKING STYLES OF TEACHERS I AND II
## COEFFICIENT OF RELIABILITY

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By using the Pearson-Product Correlation the following coefficient of reliability was calculated:

### Correlation Coefficient: Teacher I and Teacher II

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APPENDIX J

EFFECT SIZES FOR POSTTEST 1 AND 2
Effect Size

\[ Sd = \sqrt{\frac{Sd_t^2(n_t - 1) + Sd_c^2(n_c - 1)}{(n_t + n_c) - k}} \]

\[ ES = \frac{X_t - X_c}{Sd} \]

Effect Size for Posttest1:

\[ Sd = \sqrt{\frac{(2.416)^2 (45) + (2.439)^2 (31)}{(46 + 32) - 2}} \]

\[ = \sqrt{\frac{262.667 + 184.410}{76}} \]

\[ \therefore Sd = 2.425 \]

\[ ES = \frac{(9.703 - 7.620)}{2.425} = 0.8589 \approx 0.86 \]

Effect Size for Posttest2:

\[ Sd = \sqrt{\frac{(2.105)^2 (45) + (2.447)^2 (31)}{(46 + 32) - 2}} \]

\[ = \sqrt{\frac{199.35 + 185.38}{76}} \]

\[ \therefore Sd = 2.249 \]

\[ ES = \frac{(11.547 - 11.043)}{2.249} = 0.2241 \approx 0.22 \]
APPENDIX K

SUMMARY OF DATA
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