A TEACHER'S PERSONAL JOURNEY: MY EMERGENT UNDERSTANDING ABOUT HONOURING COMPLEXITIES IN A MATH CLASSROOM

by

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ABSTRACT

New insights from complexity theory inform us of a universe with embedded systems of unpredictability and non-linearity. Amidst the chaos sits the potential where individual agents are able to coalesce to a self-organizing and self-adapted collective. Many systems such as a flock of birds, an ant colony, and the stock market have been identified as representing complex phenomena, but more await to be explored, and among those is teaching and learning.

Complexity is embedded throughout the educational system - in our students, our classrooms, teaching practices, learning process, mathematics and in the interrelationships between all theses. To treat education as a dynamic and adaptive system, educators need to walk a fine balance of top-down and bottom-up governance, of randomness and structure, and of diversity and redundancy. When these are embraced, I believe that each learning experience and each classroom setting will be a unique system unlike any other; each will emerge in an unpredictable course defined by students and teachers as experience unfold within the system. In this study, I will attempt to do just this and to explore the challenges, struggles, conflicts and successes of this environment.

I am capturing a journey of living and learning, an experience that is not the result of any "particular methods, texts or activities", but from a genuine "understanding of human existence and cognition" (Davis, Sumara and Kieren, 1996, p 166).

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DEDICATION

This is dedicated to all who believe, that collectively, they can make a difference on the evolution of our educational system.

CHAPTER ONE: A NEEDED SHIFT IN PERCEPTION

1.1 Linear systems

Tic-tock, tic-tock, tic-tock; ring, ring, ring; click-clack, click-clack, click-clack…these are the repetitive, rhythmically patterned sounds of clocks, telephones and typewriters. Inventions provide more than a lifestyle convenience; these innovations deeply support a social, economical, political, technological, scientific, and academic structure that attend to the same line of thinking. That is, a linear way of thinking that reasons everything in our world as predictable, reducible, and constructible. Predictable in the sense that events follow a predetermined path with little variation, allowing predictions to be made based on observation of past results. This perspective also argues that events are reducible to discrete components. As such, to understand the overall effect of any event, one only needs a systematic examination of the elements at the sub-level that give rise to results seen at the main level. Finally, events are constructible in that they follow a causal-effect relationship, allowing processes to be recreated and ends to be replicated unproblematically.

Mechanistic approaches and scientific ideologies had made a deep imprint on society for several centuries. One does not need to look too far to realize that we are surrounded by a deterministic and linear philosophy that impacts the way we make sense of our world, as can be found in Newton's Law of Motion, the Periodic Table of Elements and categorization of species. It is undeniable that these positions have contributed to successful headways in the scientific field; in particular, it supports the use of scientific methods as a credible way to understand our world as a container of guiding principles. The prominence of linear thought has been pervasive and its presence is evident in varied realms of human reasoning, social organizations, and academic research, just to name a few. From the way we symbolize learning as an input-output process, the way we organize ourselves in hierarchal structures organized by class, socio-economic condition, family status, education level, occupation, and place of residence to the quest of scientific research as a Truth-driven discipline, we are consumed, in belief and action, with the idea of a linearly operating world. In the following, I will turn this focus to the field of education.

1.2 Linearity in education

Grade levels, subject-centered curricula, and achievement testing are words that educators and teachers use to communicate a methodological approach to learning. The same plan is revealed in the activity at various levels of the system, starting at the top with the provincial learning outcomes from the ministry and district-wide learning initiatives from school boards, down to the bottom with specific learning goals from individual schools and classroom teachers. The purpose of schooling is then for students to get through a predefined program of identified learning outcomes as governed by age and subject appropriate materials, one that clearly articulates what students should learn at each grade level across all curricular dimensions. The clarity and preciseness of our K-12 learning program also manifest in our need to conduct student assessment as a way to compare student attainment against system-wide learning objectives. In a sense, education is seen as a huge assembly line whereby students go through 13 years of lock-stepped programs with built-in checkpoints that signify the completion of one phase and the beginning of the next.

1.2.1 Linear system in learning

Our perceptions of what knowledge is and what learning looks like resonate with a similar societal obsession with linearity and hierarchy. Many of us view knowledge as external, objective truths, something that we spend our lifetime searching for and to be understood. We examine our surroundings in crave for knowledge that we assume stays the same through time, place and people. The consistency behind the predictability and certainty of these events gives us all the more comfort and confidence that there is a reality, a Truth, out there waiting to be discovered. Learning, then, is the route to these higher understandings. To learn, in this framework, is to grasp, to digest, to internalize, to absorb, and to master particular understandings that remain invariant under any circumstances. We also define learning as something to be taken in steps, as the learner moves through stages of transition from an acquisition of basic facts to the attainment of expert knowledge.

1.2.2 Linear system in teaching

Planning is a big part of a teacher's job, as in course planning, curriculum planning, unit plans, lesson plans, school plans, seating plans, and so on. A lot of time, effort and resources are being put into planning curriculum content and classroom structures prior to students entering the school building. We plan because we strongly believe that specific learning outcomes can be occasioned, that learning follows a logical sequence of topics, and that structures and rules are needed in order for learning to take place. As such, we plan out exactly how we teach and we communicate this plan to our students so that they are aware of what is to be learned as well. In addition, we also make our plans transparent to school administrators and parents so that the ends of our learning objectives are shared with the greater community. While it is pedagogically sound to plan, a deeper meaning behind all this is that teachers inherently trust they can control what can be learned in a class and that their pupils can be expected to meet a common set of prescribed learning goals at the end of the lesson or the course.

1.2.3 Linear system in educational research

Educational research, like all other academic research, adheres to a set of rigorously defined and rule bounded practices often aligned with the scientific community. Although it is slowly retreating, there is still a huge keenness and compliance towards conducting research using quantitative methods. Many in the academia and the general public consider the application of statistical data and graphs as a more accurate frame of educational activity. Still, many more believe that research results that can speak to unfaltering and consistent patterns across context and time differences are the only ones that are worthy of our efforts for future investigative work, in relation to a point I raised earlier on about an external, objective Truth.

1.3 Argument for change

A general conception of mathematics is that it is a linear and objective discipline. Many people, including teachers, believe that the impartiality of mathematics pushes aside any gray area that we might have. And when uncertainty does arise, we attribute it to the fallibility of the human mind to comprehend such higher understandings. I cannot recall how many times I have been told, both as a student and as a teacher, that I could only have one answer for each math question.

As a child, I learned math through memorizing steps and doing repetitive exercises. And now as an educator, many teachers including myself are tempted to follow the same structure in presenting our lesson plan. Rarely do we stray away from our well-trodden path - going over homework questions for the first part of the class, followed by an introduction of the day's lesson by explaining and doing examples on the board, and concluding the class with students working individually on their homework assignments. Not only is each lesson carried out with this generic format of clarifying homework problems, taking notes, and assigning math questions, but we also teach with the same examples and problems. Why do math teachers keep using the same lesson plans year after year? The examples in our class notes often introduce a very particular algorithm leading to a solution such that students can replicate the same steps in their own assignments. Why do math teachers believe that replication is more important than authentic reasoning? Most of us still teach "at" the students using a white board or the overhead. Why do math teachers believe that indoctrination, rather than social constructivism, is a pedagogically sound way to teach math?

Our obsession with linearity and repetition do not enhance, but rather restrict what we try to achieve with our learners. To our peril we ignore many of our students who express their frustration with math in terms such as: "I cannot do math because I cannot remember the steps"; "I get confused easily because there are so many steps to follow"; "Math is boring and repetitive; why do we do the same type of questions 30 times"; "I don't really understand the question, but I know I can get the answer if I follow the steps". How can we continue to think and teach linearly if these overemphases on steps and formulas are the very things that our students struggle with? We have come to a time where we need to stop ignoring the blatant complaints from our students and start thinking about what learning, teaching, classrooms, students, school staffs, and the wider community all mean to our responsibilities as educators (Ollerton, 1999).

1.4 A new way of thinking – Definition of complexity theory

As I look back upon defining moments of my teaching career, I realize that they were all moments that I could not have planned for and that my students were the main instigators in precipitating these events. These moments can be as simple as a student going up to the board and sharing with the class a completely different way of solving a question, where we are treated to a magical moment of seeing a new way of doing and conceptualizing a math problem. It is times like this where I quietly say to myself, "Why didn't I think of this before?" These unexpected additions to the lesson are far richer and deeper than what I can accomplish on my own effort. On these occasions, I feel like I am part of the learning process. The student goes up to the board and explains, while I sit down and enjoy learning from them. These are defining moments for me because the physical, intellectual and emotional boundaries between teacher and students melt away and we become a collective and cohesive group of learners.

I realize later that what I have been describing resembles a new sensibility that can completely change my understanding of, with, and within the world. It is complexity theory.

CHAPTER TWO: LITERATURE REVIEW

2.1 A history into educational theories

Different times bring with them different sets of beliefs, philosophies and action plans. Upon a closer look at the history of our educational theories, one notices a microcosmic stability during defined periods while at the same time realizes that progressive changes occur across long stretches of time. Education, therefore, has gone and is still going through continuous and steady growth as it responds to new knowledge coming from research initiatives, socio-cultural views, and spontaneous partnerships with other disciplines and enterprises.

Hein (1998) defines education as a consortium of knowledge, learning, and teaching. At this breadth, he defines four positions of educational theories: didactic, expository education; stimulus-response learning; discovery learning; and constructivism.

Didactic, expository educational theory was introduced in the mid-sixties, with avid support from Gagné. This position frames knowledge as external, objective truths. Learning requires that learners follow a sequence of step-wise mastery, starting from the most simple and basic to the more complicated. As such, teachers "subscribing to this view represent the 'true' content in an orderly fashion and divide materials into small enough units so that it can be learned" (Hein, 1998, p 26). These perspectives are embedded in many aspects of our educational system. For example, our school curriculum is one big, methodical plan that details learning objects students need to master in a specified time span. Knowledge is associated with grade and age levels, the assessment of which aims to track student performance according to some established (and sometimes seemingly arbitrary) standards.

Further back in the mid-thirties, behavioral and scientific psychologists such as Watson and Thorndike made huge contributions towards stimulus-response learning (Hein, 1998; Tomkins, 1986a). This position is similar to the previous in that they both believe learning and teaching are incremental, thus the necessity to break down our educational objectives into smaller parts and steps. However, stimulus-response learning sees the attainment of knowledge as a constructible experience as opposed to an intake of objective truths. This belief can be seen amongst teachers as far back as the early 1920s, whose interest included educating "the whole child" and attempted to present a curriculum that emphasized skill-building subjects such as manual training, domestic science, agriculture and health and physical education (Tomkins, 1986b).

Discovery learning rejects a mechanical, one-size fits all approach to learning. Although it holds a realist position about knowledge, it eschews past orthodoxies and allows individuals the freedom to choose their own path of reasoning and meaning making around observable ideas. This approach places less responsibility on teachers to instigate learning; rather, it puts faith on each student as he or she engages in an authentic, active process of minds-on, hands-on learning whereby new understandings add to and reshape past knowledge. For the last few decades, this view has been taken up favorably by educators who aim to present opportunities for individual learners to "discover the truth by finding out for herself or to learn through doing" (Hein, 1998, p 31).

The last stance, constructivism, is the most recent initiative in the educational field. It neither proposes knowledge as an infallible truth nor does it define a pre-planned, logical passage for the learning and teaching process. In a constructivist classroom, "learners use both their hands and minds, to interact with the world, to manipulate it, to reach conclusions, experiment, and increase their understanding; that is their ability to make generalizations about the phenomena with which they engage" (Hein, 1998, p 35).

Through all, views on knowledge, learning and teaching practices are deeply impacted by the linearity of a didactic, expository approach to education. Whether it is in the hierarchy of students, teachers, administrators, school districts, and ministry, or in the procedures of our classes, curriculum content, and course pathways, we are bounded and in many ways trapped within our own rules. Although in recent years, teachers and educational researchers are making progress by tapping into new and different opportunities promised by discovery and constructive learning theories, barriers continue to hinder the educational field from successfully taking up these ideologies as a school or district-wide initiative.

2.2 A history into linearity and predictability

It would be fruitful to examine the pervasiveness of linear thoughts within education in light of other things that may have been an influential force. The single most powerful contribution from the last century came from the practice of studying units and variables that give rise to whole systems. Scientists and academic researchers hoped that all observable and non-observable events in the world could be explained by studying the puzzle literally one piece at a time and that the processes behind such phenomena are mechanical and cumulative. A Cartesian-Newtonian model steered human endeavors towards a "world of order, linear sequence and mechanistic prediction" (Crowell, 1989, p 60).

From the seventeenth to the nineteenth century, people benefited from a simplistic way of understanding the world which erupted into an explosion of machines and technology. It brought us "the telephone and the radio, the automobile and the airplane, the phonograph and the moving pictures, the turbine and the Diesel engine, and the modern hydroelectric power plant" (Weaver, 1948, p 536). Weaver (1948) considers these as "simplistic" systems because a thorough knowledge of the parts can predictably lead to understanding of the whole.

These values were endorsed by education during the early 20th century in support of a more systematic, efficient and effective way of formal schooling that consisted of routines, schools of conduct, policies, daybooks, seating plans, staff meetings, authorized textbooks, deductive reading, external examinations, fundamentals, rigorous discipline, a prescribed curriculum of traditional subjects and learning out of a book (Sutherland, 2003). Society's fixation at the time on seeing education as a simple system led to the introduction of scientific progressivism, where standardized intelligence and aptitude tests were used to classify students into appropriate school programs (Thomson, 2000). Along with this was the creation of the platoon system which employed the use of subject-specific teachers, most of whom "conveyed to children, to colleagues, and to parents the sense that there was in each subject only one right way of doing things" (Sutherland, 2003, p 337).

2.3 A history into the emergence of complexity theory

For the past few decades, there has been growing concern about the insufficiencies of a reductionist perspective. There are phenomena in the world that simply cannot be understood by reducing them to discrete parts, such examples as the flight of a flock of birds, the design of the Koch pattern in snowflakes, the fractal pattern in a fern, the formation of ant colonies and the sustainability of a city or culture. Together, they form a group of phenomena described best by an emergent field called complexity theory (Fullan, 1999; Gleick, 1987; Gribbin, 2004; Johnson, 2001; Russell, Murphy, and Peacocke, 1995; Smith, 1998; Waldrop, 1992; Weaver, 1948). Complex systems arise from the amalgamation and interaction among individual agents. Each agent operates within localized rules and exhibits micro-behavior without restriction or order from higher levels. When certain environmental condition enables the coming together of different agents, it will create macro-behavior that emerges neither from an amplification of individual behavior nor order from a higher-level leader, but from the interaction between and among different agents. In contrast to a simplistic system, one cannot predict the overall effect of a complex system by calculating the sum of its parts. Fullan (1999) defines complexity theory as a new science that claims "the link between cause and effect is difficult to trace, that change unfolds in non-linear ways, that paradoxes and contradictions abound and that creative solutions arise out of interaction under conditions of uncertainty, diversity and instability" (p 4).

It is difficult to capture the full effects of complex phenomena as they are under continual influences of unpredictability and non-linearity coming from within, lower and higher levels of the system. Nevertheless, those who have studied complexity theory in depth have identified conditions of rich, random interconnections, iterations, fluctuations, and bottom-up engagement as having the potential to embrace the sensibilities of a complex phenomenon as a self-organizing and self-adaptive emergent system (Axelrod, 1999; Reason, 1999).

2.4 Complexity theory in the educational context

The fields of biology, neurology, economics, meteorology and psychology have all begun to use complexity theory as an alternative way to look at complex phenomena (Davis and Sumara, 1997). In education, changes in learning theories indicate that a similar trend is emerging. We had progressed from behaviorism in the early 20^{th} century, to cognitivism in mid century, and constructivism in the late 20^{th} century. Now, education is starting to pay close attention to complexity theory; this change may even hold the key to give us a more comprehensive way of thinking about education other than the four defined categories from Hein's educational theories.

Education researchers and teachers are starting to make connection between complexity theory and learning (Crowell, 1989; Davis and Simmt, 2003; Davis and Sumara, 1997; Davis, Sumara & Kieren, 1996; Davis, Sumara, Luce-Kapler, 2000; Sumara and Davis; 1997). They argue that classrooms are complex systems, with qualities of self-adaptation and self-organization. Each student is different from but shares certain similarities with his/her classmates and teachers. Collectively, the learners, teachers, curriculum content and the classroom environment participate in a creative dance of knowledge building, with each co-implicating the other. This holistic collaboration results in a deep, meaningful, and socially-constructed learning occasion. In honoring this sensibility, the teacher is never certain about, nor is he/she in control of, the learning experiences that would result for each individual. The lesson plan cannot predestine, nor can it accurately predict, specific learning goals for the day. Crowell (1989) believes models from cooperative learning, complex instruction, and whole language learning are reflective of this new paradigm.

Davis and Simmt (2003) identify five key conditions that teachers can respect to sustain their classrooms as complex systems. The five conditions are: 1) internal diversity, 2) redundancy, 3) decentralized control, 4) organized randomness, and 5) neighbor interactions. These qualities are reiterated by many theorists in this field as the main attributes of complex systems (Gleick, 1987; Johnson, 2001; Waldrop, 1992).

Prolific researchers on complexity theory, such as James Gleick, Steven Johnson, and Mitchell Waldrop, have all elaborated on these features. They all recognize that individual agents in these systems have different skills, strengths, weaknesses, interests and behaviors, which serve as a source of variations that in turn sparks excitement. On the other hand, individual agents are alike to one another in that they share similar responsibilities and goals; this common bond is an important force that acknowledges and binds the communal effort coming from all agents in the collective. In addition, complex phenomena arise in the absence of a higher controller, which complexivists believe could do more harm than good by putting an end to an otherwise spontaneous collaboration. Although control does not come in the form of a leader, blueprint, plan, intention, or predefined order, agents are nevertheless bounded by localized rules of interaction. This intricate balance between control and randomness unfolds in a creative space that renders the possibility for individual agents to self-organize and to self-adapt in an ever-changing environment. Finally, rich interaction amongst agents is crucial to the generation of strange attractors, turning a chaotic system into one that consists of definable yet indeterminate patterns of behavior.

2.4.1 Complexity theory in Mathematics education

Math educators are in their early stages of understanding the theory, application, and benefits of thinking about themselves and their work as complex systems (Davis, Sumara & Luce-Kapler, 2000; Fullan, 1999). Both Chazan (2000) and Pixten (1997) abandon "formal mathematics" in favor of one that is conceptualized with a complexity theory sensibility, believing passionately that mathematics learning should be dynamic and authentic. They both criticize Western schools for putting too much on an Euclidean emphasis in mathematics; the static aspect of points and lines cloud a new geometry that can "mirror a universe that is rough, not rounded, scabrous, not smooth. It is a geometry of the pitted, pocked, and broken up, the twisted, tangled, and intertwined" (Gleick, 1987, p 94).

The way in which math content is presented in textbooks and classroom instructions seems to un-proportionately magnify the orderliness of mathematics. By and large, math textbooks encourage a repetitive practice of pen and pencil exercises, equating development of the mind and knowledge acquisition to a linear training regime. Traditional teachers place more importance on rote learning, memorization, and regurgitation of formulas and steps than on student understanding and pattern creation (Doll, 1989). It may be true that this view is still very much the norm in today's classrooms, but more math teachers are now reaching out for something that can give themselves and their students a livelier and more interactive connection with mathematics.

Doll (1989), an education researcher, believes that teachers' insights into complexity theory would enable their "students to develop their own appreciation of the irregular but beautiful patterns inherent in both mathematics and nature" (p 70).

2.4.2 Complexity theory as a mathematical learning tool

Although theoretical arguments of complexity theory are extensive and growing in the education field, few researchers look into the potentials, pitfalls and practicality of applying this theory in a classroom. Resnick (1996) is one of the few who explores the application of complexity theory in education. He invites students to participate in a computer program called "StarLogo" that can be used as a conceptual tool to understand complex phenomena. The students look at systems of traffic jams, termites and ant cemeteries. From the project, students appreciate how randomness creates order as individual agents in these systems self-adapt and self-organize into emergent events.

Similarly, research that integrates complexity theory into mathematics education seems to be equally narrow. Among the most prominent writers in this area are Brent Davis, Thomas Kieren, Elaine Simmt, and Dennis Sumara. Throughout several of their articles, they illustrate evidence of student work that exemplify depth, diversity and originality when teachers embrace the sensibilities of complexity theory. In one study, students look into ways of expressing a fraction that is more than one-fourth and less than three-fourths (Davis, Sumara, and Kieren, 1996). The varieties in their solution push each student to "recreate, re-interpret and re-negotiate" what they know, and ultimately changing their relationship with the world (Davis, Sumara, and Kieren, 1996, p 158). In another study, students illustrate their understanding of 3 x –4 as ideas of groupings, moving on number lines, repeated addition, and previously learned rules (Davis and Simmt, 2003). More recently, Davis and Simmt (2005) worked with a cohort of 24 teachers to examine how their mathematical knowledge can impact upon daily pedagogical decisions and practices. The teachers explore their own and each other's understanding of multiplication. In the process of doing so, they immerse in a learning experience of complex dynamics, where "new interpretations were tossed into the mix, each addition compelled and elaborated redefinition" (Davis and Simmt, 2005, p 10). Doll (1989) is another researcher who is making contribution to this topic. He, in partner with an elementary school teacher, creates complex math questions for their students. The problems are "hard, interesting, and humorous", and learning is "progressive, constructive and interactive", reinforcing their goal to "reorder the curriculum and instructional methods to promote more dancing and less marching" (Doll, 1989, p 66-67).

Researchers and teachers who have had the chance to honor the sensibilities of a complex system to embrace the non-linearity, unpredictability, and surprising turns of a collective knowledge construction recognize the benefits of this as a powerful learning tool that can deepen and enrich the classroom experience.

2.4.3 Complexity theory as a teaching tool

To educators who are counting on a magic formula that can tell them how to recreate learning occasions as those described above, they will be disappointed to know that there is none. Complexity theory provides us with an alternative way to understand complex world phenomena; however, it makes no promises that learning occasions will always unfold into all of its complexity; neither does it attempt to describe in detail any teaching methods that should be supported in a dynamic classroom environment. Despite these uncertainties, a few patterns can still be identified as characteristics of complex phenomena operating at the creative space between simplistic and chaotic systems – the edge of chaos.

Imposing top-down rules, as in a simple system, results in behaviors that are mundane and repetitive. On the other hand, a chaotic system has "no direction, unclear responsibilities and random communication" (Fullan, 1999, p 24). At the edge of chaos, it has neither "too much structure that it creates gridlock" nor "too little structure that it creates chaos" (Brown and Eisenhardt, 1998, p 14). Teachers are consumed with unlimited possibilities and are inspired to work collaboratively with their students. A complexity theory perspective puts a classroom at a dynamically innovative space where teaching is refined on-the-go and where learning arises from the co-creation of teacher and student input. In a very profound way, this sensibility allows educators to become aware of the type of teaching and learning experience that could be provoked from honoring the complexities embedded within a classroom.

2.5 My response to the literature

This overview of significant findings in the literature confirms a preliminary interest among researchers to view learning as a complex system. They all speak to the potential of increasing the breadth and depth of student knowledge when educators honor learning, teaching and classrooms as complex phenomena. In all cases, knowledge is emergent not prescribed; learning is chaotic not linear; conversations are encouraged not restricted; ideas are critically examined not recapitulated; and last but not least, at the center of the learning circle are student groups not teachers. This hub of interest indicates that we might be on the verge of exploring as a profession a new educational theory as we step into the era of complexity. My study attempts to fill a void in a research community that looks more at the theoretical underpinning rather than the practical application of complexity theory, that explores more at how the theory is utilized in elementary mathematics than in secondary mathematics, and that focuses more on immediate rather than long term effects of embracing mathematics teaching and learning as complex phenomena.

2.6 My research questions

I wish to examine the potential of recognizing classrooms, and in my particular instance mathematics classrooms, as complex systems, and consequently look at the type of pedagogy and learning that would unfold when a teacher has an intention of teaching high school mathematics with a complexity theory sensibility. In this study, my use of complexity theory draws upon the features outlined by Davis and Simmt (2003): internal diversity, redundancy, decentralized control, organized randomness, and neighbor interaction. I will explore the following research questions:

How does a teacher's practice change when teaching with an awareness of complexity theory?

- How do changes in a teacher's beliefs about mathematics and learning unfold when attempting to support the classroom as a complex system?
- What kinds of pedagogical decisions and actions are made in an effort to support the classroom, and in particular the mathematics classroom, as a complex system?

- How do changes in classroom dynamics and student learning experiences inform a teacher's beliefs about teaching mathematics with a sensibility from complexity theory?

CHAPTER THREE: METHOD

3.1 A qualitative action research approach

My Master's thesis study draws upon a qualitative research methodology (Appendix A). Action research is perhaps the best approach that can paint the complexities of this study's context; it is also one that recognizes the research process itself as a complex phenomenon. Thus, this project not only studies complex systems as they emerge from the edge of chaos, it also utilizes this sensibility in a very practical way in order to appreciate the "complex choreography of experiences that constitutes our lived action research practices" (Sumara and Davis, 1997, p 420). Educational researchers are beginning to posit action research as an interpretive framework that holds the key to integrate the space of teaching, learning and research into one, going beyond the formal structures and component analysis that are usually associated with academic research. They recognize this as a research position that opens up "layers of interpretation and reinterpretation" (Radford, 2005, p 10). Although it has its limitation to generalize results; nevertheless, it shifts the research focus to one that conceptualizes "difference, context, processes through time, multi-factor causality and the specificity of situations" (Haggis, 2005, p 10). Increasingly, researchers are making convincing arguments on the use of action research as a methodological approach to study complex phenomena (Phelps and Hase, 2002; Sumara and Davis, 1997). Among many inquiry frames of action research, my study works with a notion of action research as a form of living and meditative practice that comes from understanding "one's self and one's relations to particular communities" (Carson and Sumara, 1997, p xviii).

3.2 Description of site

The research site is situated at a Surrey district public high school in the lower mainland of British Columbia, Canada. The school enrolls approximately 1400 grade 8 to 12 students with 80 staff members.

The front entrance has a stone plaque engraved with our school name; it leads into a staff parking lot and 2 flag poles before one reaches the building. Upon entering, immediately to the right side is the main office and to the left is the library. Straight ahead is a huge open den area where students gather to socialize and to have lunch, it is surrounded on three sides by a cafeteria, a theatre and 2 gyms. On the left hand side of the den are the business, computer, shop and math classes, with science classrooms directly above them. The right side of the den consists of cooking and sewing classes; right above on the second floor are the English, language, and social studies classes. There is a large grass field located at the back, along with five portables lined up off to the right of the school.

The school day starts at 8:30 am in the morning and ends at 2:40 pm in the afternoon, scheduled with four 80-min classes and a 45-min lunch period. The school is on a semester schedule where four courses are offered in the first 5-month semester and the other four courses in the following semester. With the exception of a few teachers, most have their own classrooms and are assigned a specialized teaching area. Students, on the other hand, have to rotate between four different classrooms in any particular day.

Each subject has its own syllabus of learning objectives, which are usually defined early on at the beginning of the course and specified within the Ministry's Integrated Resource Package. Like many other schools on the first day, teachers lay out the specific topics intended to be covered in their courses and address academic / behavioral expectations. Academic assessment is usually within the realm of student achievement across class work, homework assignments, group projects, participation, quizzes and chapter tests. At the end of each semester, students are expected to write a final exam or to complete a final project to demonstrate what they have learned.

In British Columbia, every high school student is obligated to take Ministry required courses, and based on individual interests, they also have their own choices of elective subjects. The course selection process is communicated clearly to students each year so that they have the necessary credits to graduate.

3.3 Description of myself

I am entering into the fifth year of my profession as a high school math teacher. Aside from a few short-term teaching assignments, I have stayed with my current school since the beginning of my career. Although we are located at a less privileged neighborhood and are forced to deal with many challenges of an inner city school, my commitment and desire have not dissipated as a result. In contrast, I am blessed at the opportunity to make a difference to this group of students and, more than ever before, I realize how important education is, because for some of them it is their only way out of poverty, drugs and starvation. I take what I do to heart, and it is evident that there are still many things that I could improve upon as a beginning teacher. Three years ago, I made a decision to go to graduate school to learn more about educational theories and teaching practices. Now, three year later, I am bringing what I learned back into my own classroom.

3.4 Selection of participants for the study

Participants are selected from a second-semester math 8 class in this school. Approximately one month into the course, a counsellor helped me introduce the research initiatives to this particular group of students. They were given information on research procedures and purposes, and were also informed on issues of ethical concerns and confidentiality. Both students and parents were given a week to read over the content and to raise any questions that they may have had about the research before signing and returning the consent forms (Appendix B and Appendix C). No student was excluded from being invited to participate in this research; selection was based purely on voluntary responses from students and their parents. From a class of 28, nine students, with parental consent, agreed to participate in this study.

At our school, we do not offer streaming of different math programs at the grade 8 level. Classes consist of students with a wide range of abilities. In addition, computer-generated class schedules further randomize the spread of students with varying levels of math competency, thus this particular math 8 class has students from across the ability spectrum. As well, it has a diverse cultural mix, ranging from Caucasians to East Indians and Asians. The same can be said for the participant group, itself forming a representative group of students with different strengths, capabilities, and backgrounds.

3.5 Context of research

Each classroom is a unique space filled with unique individuals, and each learning opportunity is a creative occasion filled with special circumstances. It is inevitable then that my research is highly contextualized, imbued within a classroom environment of students, teachers, relationships, and learning objects that is one of its kind.

I do not attempt, nor have the capability, to repeat what other researchers in similar classrooms have done in their attempts to draw upon complexity theory. Nor is it my objective to present my own research in a way that others might replicate it in detail. I only wish to study and to learn from my own unique context, one that is continually growing and reshaping itself through time. However, I hope that a vivid account of my experiences will open doors for other educators who share the same ideal and passion as I, and will enable an increased awareness and understanding of complexity theory as a way of thinking about teaching and learning in our school system.

In honoring my classroom as a complex system, it is important to note that I am not studying any one particular aspect of classroom practice, but the full gamut of learning and teaching in its immediate environment. As such, my study attributes the "ends" to the unfolding occurrence of many types of interaction.

3.5.1 Teacher / student roles

In this research, my students played an active role in the learning process. They were not subjects that things were done unto, but were involved in a complex, participatory collaboration with each other, as well as with the content, the classroom environment and myself. Therefore, this is more than a study that simply looks at the effects of complexity theory on student learning; it attends to the ways in which students interact and on how learning communities emerge when mathematics teaching and learning are approached with a complexity sensibility. Their behaviors, interactions and contributions formed an integral action-piece of my research, impacting both processes and ends every step of the way. In a sense, my students were co-researchers of this study because they had as much impact on the unfolding of our classroom experience as I did.

My role as a teacher-researcher similarly took on numerous overlapping responsibilities. I was at once a teacher, a researcher, and a learner. I was hardly an outsider making objective observations. Rather, I acted in the moment as I responded to my students, content and the classroom setting. My multiple involvements influenced everything around me, which in turn modified the decisions and actions that I made during the study.

This action research project is a collaborative effort between my students and myself as a teacher-researcher. To fully understand the complexities within teaching and learning mathematics, both areas are scrutinized as processes and ends. Moreover, the relationship that developed between me and my students was an important motivator that sustained the unfolding of complex phenomena in our classroom.

3.5.2 The classroom environment

The first thing that comes to mind about classroom environment is the layout of chairs, tables, and whiteboards. However, to fully honor the complexities of a classroom, what comes to the fore should be an interplay between the people, experience and emotions that inhabit this physical space. My research sees a math class as something different from the static, unchangeable backdrop against which everything else happens. The classroom, in its own right, is another action-piece of my study. The setup of my classroom, including seating arrangement and classroom expectations, impacts the level and intensity of student interaction, the results of which could possibly reshape the learning environment in a variety of ways.

3.5.3 The learning environment

From the beginning of the 5-month semester, I taught my Math 8 class with a sensibility to embrace this classroom as a complex system. To respect a complexity view of teaching and learning that unfold under spontaneous and nonlinear occurrences, I was not able to describe my intervention in detail before the research started. Nevertheless, my lessons were inspired by Davis and Simmt's (2003) five conditions of complexity theory. The following captures some of the characteristic features of my lessons:

1) Internal diversity - I recognize the possibility of and encourage variations among student contributions; each student brings with him/her the potential to provide unique and different insights to the same problem.
2) Redundancy - I create a common language/experience/belief that all students can relate to or a common project/objective toward which we can work. A successful collective enables everyone in the class to see a part of themselves in others; the sense of belonging that comes from the ability to relate to one other and our work is a powerful driving force in a learning community.
3) Decentralized control - I am an active part of the learner's circle, but not in the form of an authoritarian or a disciplinarian. I resist from imposing rules, restrictions and limits on my students. The student's own sense of self, power and agency in shaping a collective learning experience is prominent. I provide

students with a level of ownership and freedom to engage themselves in a liberating and emergent learning occasion.

4) Organized randomness - At the same time that students are open to creativity and imagination, I also provide guidance in the form of liberating constraints such that students are not let loose on a confused, directionless, and chaotic learning path. Assignments and class activities contain a balance of direction and freedom.
5) Neighbor interactions - I encourage interaction/discussion/communication among students, and between students and myself. Both students and teachers have important roles in shaping a collective learning pathway. More importantly, interaction between ideas is valued.

The Math 8 Prescribed Learning Outcome (Ministry directive) covers a wide range of topics from basic numeration to patterns and relations, measurement, probability and statistics. The curriculum provides a suitable and appropriate place for students to revisit many of the concepts that they have learned in elementary schools, but to also push and restructure the boundaries of what they know to create an even fuller, deeper, and richer understanding. This course lends itself as a stepping-stone for students to understand the construction of mathematics and the learning of mathematics as complex systems, such as providing students with opportunities to re-explore and to reshape their knowledge in light of new understandings that unfold intentionally and spontaneously in a class.

3.6 Sources of data and Collection methods

The multi-faceted, richly connected and complex unraveling of this research context demanded a similarly dense collection of data sources. They captured the ways in which variables interacted and manifested, all at once, in highly contextualized circumstances. This research utilized a combination of qualitative research methods, such as whole class observation, student reflection, and teacher journaling, to embrace complexities in a classroom. Data collection began about a month into the math 8 course and lasted for about 3 ½ months until the end of the semester.

3.6.1 Whole class observation

Whole class observation seized the varying breadths and depths of interaction and learning that occurred in a classroom context. It opened my perception to the complex emergence that arose between partners and among large groupings; it also allowed me to apprehend the dynamics that progressed throughout human relationship and knowledge formation.

By paying close attention to classroom occurrences and discussions, I wanted to understand how rich, learning occasions unfold from student interactions. I tried to take notice of things that might help bring out and sustain a collective learning environment. Moreover, whole class observation was a holistic approach that expanded my awareness to many other things that I could not anticipate before engaging in the study.

Early on in the research project, I constructed a sensitizing frame to focus my observations. This included looking at small and large group interactions, the interplay of chaos and order, and the delicate balance between redundancy and diversity in affecting complex student interaction and learning. These aspects formed the structure for my observation, though I was prepared to embrace new happenings and was determined not to be confined just to these (Appendix D).

The presence of multiple complex systems and the unpredictability of their emergence added to the difficulty of my research methodology. I decided to use various recording methods, like video recording and written notes, to collect data on classroom interaction. These allowed me to capture, in part, things as they transpired.

Although physical interaction is a main focus of complexity theory, I defined "interaction" in the classroom as participation and engagement that come from student involvement and class discussion of mathematical ideas. Therefore, interaction was anticipated to occur at a more verbal than physical level where student knowledge and understanding were shared and discussed. This type of learning environment had little implication on the video recording of six of my lessons. Video observations concentrated on parts of rather than the whole class where student movements could be controlled readily and taping could be constricted to the participants. Whenever possible, the camera focused only on the teacher and the participants; avoiding chances of non-participants to be captured on screen. Although audio sounds from non-participants were often recorded on tape, these clips were edited out and were not used for this study.

3.6.2 Student reflection

The impact of learning, as a complex phenomenon, on individual students was another area that I wanted to collect data on, amongst other things that emanated from this class. The students themselves were a critical part of this complex system that I was calling my math classroom; they each made unique contributions to the class and influenced the type of learning that unfolded. At the same time, each of them was also uniquely transformed by the episodes that developed in this type of environment. The key point is that every student continually reconstructed his/her learning and personal space to adapt to changes within a classroom. Each student's experience was non-imitable; each was a phenomenon of its own and enriched my knowledge of complexity theory in my daily work.

Throughout the study, students were given opportunities to express their feeling and understanding of learning mathematics in my class. Every student was asked to participate in a self-reflection of their own experience, which I believed was an important learning process that all students would benefit from. At the start of the study, they were given an initial survey that asked about their perceptions and beliefs about mathematics learning. Students gave feedback to 5 questions:

1) Describe what a typical day was like for you in previous math classes.

2) What are your expectations for your math teachers? Describe 3 of the most important job responsibilities that you hope your math teacher can fulfill.

3) Describe 3 things you can do to contribute to or to enhance learning mathematics in your class.

4) Describe 3 teaching methods or activities that you find helpful when you learn math.

5) Describe what you believe an ideal math class should look like.

As the study progressed, students wrote 3 journal entries, as part of a self-reflexive practice to bring forth insights. The following are the topics addressed in the three entries:

1) Describe your views on the class activities we do in class? Do they benefit or impede on your learning of mathematics?

2) What role do you play in how your classmates learn mathematics? What role do your classmates play in how you learn mathematics? (Appendix E)

3) Consider all the different types of learning tasks that we do, such as group activities, class discussions, note-taking, and textbook assignments, which one(s) has/have been beneficial to you and why? Which one(s) has/have not been beneficial and why?

Finally, the nine participants completed a final survey at the end of the study in an attempt to compile a more comprehensive summary on the afterthoughts of this 3 ½ -month research study. Each participant was given one week to give written responses about this personal learning experience. Below is a sample of the survey:

1) Learning of mathematics:
    - Compare the learning experiences that you have had in this class to previous math classes you have taken. What are the similarities? What are the differences?
    - Tell me some of the most memorable experience that you have in this class. What makes them memorable?

- Describe to me some of most intriguing mathematical concepts that you have learned in this class.
- How has your understanding of mathematics developed as a result of your participation?
- How has your understanding of mathematics developed as a result of your classmates' contributions? Describe how important the role your classmates play in your learning of mathematics. Are you learning more or less because of your friends?

2) <u>Relationships:</u>

- Describe the relationship that you have developed with your peers in this class.
- Describe the kind of interaction/relationship you would like to have with your teacher and how do I compare to that?

3) <u>Final thoughts:</u>

- Describe any successes you have encountered during this course that you are proud of.
- Describe any difficulties you have encountered during this course that inhibited you from learning math. How did you overcome these difficulties?
- With the amount of group learning we do in this class, did you have to change or adjust yourself to accommodate learning math in this class? How?

- What are your final thoughts on group learning, how do you feel about learning while you interact with your classmates and learning as a class?
- As a class, do you think we have established ourselves as a cohesive team?

In addition to student journals and surveys, photocopies of student work were used to capture the learning process that came from my effort to embrace various complexities within my classroom.

3.6.3 A teacher's self-reflection

My observation on classroom learning emergences and the students' feedback on this experience undoubtedly provided valuable insights on how I understood learning from a complexity science perspective. Nevertheless, it would be ignorant to take myself out of this interconnected web of relationships. I, after all, represented a crucial part of this learning circle. This study began with my endeavor to treat my students, classroom, learning and mathematics as complex phenomena. My behaviors and the way I planned my classroom activities reflected a similar sensitivity as those of complexity theorists. Throughout the study, these beliefs and actions would shift and reshape as I responded to new events and reacted to the effect that these episodes placed upon myself and my students.

I kept a journal during the course of this study and used it to recount my inner thoughts and observations. In addition, it was used as an opportunity for critical analysis, where I raised questions and navigated an appropriate course of action when problems arose. Journal writing started approximately a week prior to the beginning of the course;

entries were entered every working day so that immediate occurrences could be addressed and dealt with in a timely manner (Appendix F). I also carried out monthly writings to communicate critical moments of my research and to contemplate future plans for my project. Taken together, my journal reflected on various topics of personal beliefs, teaching pedagogy, teacher involvement, classroom dynamics, math curriculum and educational policy as they related to complexity theory. Other emergent themes were also discussed in the journal entries as I saw fit.

The content revealed in these daily journals was a major source of information for my research, and the basis on which I was able to reexamine substantive elements embedded in the learning and teaching discourse of my secondary math classroom.

3.7 Methods of data analysis

Hard data, including written notes from classroom observations, video recordings of selected lessons, copies of student work, student surveys, and both students and teacher journals, were collected. Analysis of these data was done consistently throughout the entire research and thesis writing processes.

During the research, student data was mainly used to explore my students' learning experience and the extent of their mathematical understanding in this new learning context. I focused my analysis on, but not limited to, describing critical and emergent themes that unfolded in both the cognitive and affective domains. Information from the teacher journal was paramount to the data collection, and from which I explored the potential outcome of thinking about classrooms and learning as complex systems, as they impacted upon students and myself. In these reflective journals, I unveiled my own assumptions, beliefs, struggles, internal conflicts and enlightenments that came from embracing teaching and learning as complex phenomena. Moreover, these thoughts, combined with personal experiences and student/class data, informed myself of both unconscious and conscious aspects of this endeavor as I explored the many overlapping complex systems embedded in our schools.

Following the research, I went through an initial reading of all collected data and identified important points from the teacher journal and student survey. Photocopies of student work and clips from video recordings were used as a secondary source in my data analysis; they were only selectively examined to support a rather rich collection of data gathered from journals and student reflections. I started by writing out the critical points on cue cards, where they were re-read for the second time. The cue cards were put up on a wall so that I could easily manipulate and synthesize my understanding from different perspectives. I tried to group these points according to different themes and topics; but at the same time, I stayed away from reducing the ideas down to several main headings. To me, the data analysis procedure provided a valuable opportunity for me to embrace and to understand the complexities of this study at a deeper level. I approached my analysis of data observation with a mindset to learn more from it, as opposed to a way to make things more simple and reduced. The cue cards were read and reexamined many times before critical themes started to emerge. These themes were not identified prior to the data analysis process; they came about as a surprising turn of events upon close data examination (Appendix G). These significant themes will be addressed in the results section of my thesis.

3.8 Embedded systems of my research study

This study interweaves many complex systems: learning, teaching, math education, students, teachers, classrooms and action research. All these aspects mutually affect and are affected by each other's presence. The rich interconnections embedded in the phenomenon of schooling suggest that it is defeating to have a detailed and fined research procedure. All I needed to do was to watch and embrace the emergence of various complexities that would inevitably unfold, and my sincere attempt to understand them would inform the course of my action research.

CHAPTER FOUR: MY JOURNEY ON BREAKING DOWN LINEARITY

4.1 A teacher's story

This is a story of my journey. It is a journey that started many years ago the first time I set foot in a classroom as a kindergarten student and then 19 years later as a teacher. This is the path that I have walked for some twenty odd years of my life. I am all too familiar with the smell of textbooks, the feel of whiteboards, the sound from snapping together binder rings, the heat from turning on the overhead projector, and the sight of tables and chairs organized in rows. For the last five months, I had retraced these steps but living through them with a different perspective. I suddenly found my classroom to be a very unfamiliar place and saw things that I had never noticed before. This is the story that I want to tell; it is a story on how the road I had walked so many times in the past has now become a place where each step is taken with uncertainty and excitement.

4.1.1 My journey through complex layers of a classroom

My understanding of complexity theory began with coming to terms with my beliefs and experiences; every piece of knowledge not only added to, but reshaped, my architectural picture of what this sensibility meant to me. This process had been none other than complex, to truly live in these moments would require looking at each level and simultaneously all levels of my classroom, through time, as complex systems. In order to understand the bigger picture, it was important for me to first embrace the phenomenological forces that are apparent in what I do as a teacher, as well as being inclusive towards more intricate interactions within teaching like the student body and the broader dynamics of which, such as the curriculum and cultural contexts. This way, complexities in education are not looked upon as separate entities, but that actions, thoughts, and feelings are embedded within and across multiple systems. Coincidently, complexity theory recognizes the need to focus on the emergence of phenomena as they unfold before us in order to holistically understand the interstitial relationships with smaller and within larger complex systems. So, as I was studying the effects of honoring complexities in my classroom, it also provided me with the means to approach the overlapping layers of my research. In the following, I will share my understanding of complexity theory in the manner that was revealed to me: i) a generic snapshot of my teaching beliefs, ii) my authentic experience on issues of learning tasks and pedagogy, and iii) the sustaining relationships within the collective community of my class.

4.2 My generic snapshot

My initial contact with complexity theory came primarily from a graduate course that I had taken; ideas from literature and class discussions gave me a different perspective about classroom dynamics. These understandings shaped my beliefs about what needed to be changed as I undertook my journey to embrace my classroom as a complex system. These beliefs were what I called my "generic snapshot", intended to mean that these viewpoints were broad in scope and specific in time. They reflected only my humble thoughts at the beginning, but nevertheless were the basis that grounded all other things that would unfold later on in my research. My transformation was evident from the beginning of the course as I contemplated where I stood on issues of lesson planning, learning tasks and student interaction.

4.2.1 Lesson planning

I owed much of my understanding of complexity theory to the five conditions that Davis and Simmt (2003) have identified as precursors to bring about emergence in a dynamic system; they have been instrumental to my beliefs about its significance in the educational field. To support these beliefs, I had to change many of the practices that I had adopted throughout my teaching career. The first place that I turned to was my lesson plan. In the past, I had used lesson plan templates that were given to me by the university, school and colleagues. Although they all varied in design and style, most of them stayed within confined categories such as the lesson topic, learning objective, assigned tasks for both teachers and students, and the allotted time for each activity.

My first challenge was to reconstruct the lesson plan in a way that would support teaching, learning, and the classroom as complex systems; it required taking out, adding on, and redefining the various categories of the daily plan that I had grown accustomed to. More specifically, I had to incorporate ideologies from writers of complexity theory into my lesson plan, one that would enable me to think about and support the emergence of mathematics learning. On my revised template, I decided to keep the lesson topic as the header, but created six different categories for each learning task: i) teacher activity, ii) student activity, iii) learning concepts, iv) internal diversity & redundancy, v) organized randomness & decentralized control, and vi) neighbor interaction (Appendix H).

The categories of teacher and student activities were the same as I had before; it was where I "outlined" the instructions and rules of the learning task. I renamed the category of learning objectives to learning concepts, to encompass a more creative space that could invite every event as a possibility of occasioning valuable learning. I further subcategorized this heading into expected and unexpected learning concepts, emphasizing the need to be attentive to what could be learned, both intentionally and unintentionally, in a lesson. Although it was still important for me to list specific learning goals for the day, the freedom to consider other learning possibilities was equally important, if not more powerful. To think about expected and unexpected learning outcomes meant that I needed to be a resilient practitioner, in the sense that I would be more prepared to take hold of and capitalize on unexpected learning occasions should they arise. To a great extent, this small change in the lesson plan allowed me to anticipate chaos and to work with unanticipated randomness.

The category of internal diversity and redundancy reflected my need to be a considerate practitioner; I was acutely aware of the differences and similarities within the student body, across areas of academic achievement, content knowledge, learning style, personality, communication style, social network, and family background. All of these made up a unique matrix that was characteristic of my own classroom. For the category of organized randomness and decentralized control, I examined whether learning tasks had the mix of freedom and control to embrace mathematics learning as an unfolding process, whereby students could creatively sort through a muddle of chaos and order. To a large degree, the new lesson plan template enabled me to come to terms about working in and with a delicate balance between randomness and control, which was a difficult thing for me to grapple with. Finally, for the last and probably the most critical component of complexity theory, neighbor interaction, I summarized the type and the level of classroom interaction that could be brought about from the learning task.

4.2.2 Learning tasks

At the same time that literature readings have changed the way I organized my lesson plan, they have also shaped my beliefs around how I designed my learning tasks. From the beginning, I was determined to sustain the notion of learning as a complex system and to go beyond the defined nature of constructive and discovery learning. I also wanted to go beyond our sometimes superficial satisfaction with group projects and activities, such as the assumption that students learn better when they participate in group work. Although physical interaction is crucial, I believe it would take much more than student groupings to result in deep, meaningful learning. More so than physical connection, my contention was to create an intellectual space "where student knowledge would be emergent and the learning path would be chaotic as ideas are shared, discussed and pondered upon" (Journal Entry, Mar 22, 2006).

To cultivate my intellectual space, I started to design learning tasks that could encompass Davis and Simmt's five conditions. First and foremost, I made sure my activities were rich enough to support a wide variety of answers. I believed that these possibilities would open up the chaotic interaction of ideas necessary for learning any concept, which is often filtered out of our provincial curriculum and lessons. Both teachers and teaching resources unknowingly impose harsh constraints on thinking and learning when they oversimplify the learning objects by repackaging them in ways that would make them distilled and disconnected from the learning being, that is, the student. I aimed to bring back the complexity of mathematics learning by giving my students the opportunity to work through the mess, and asking them to give meaning to the redundancy and diversity in their answers. Through the use of group work and class discussion, I expected student interaction to be extensive and dynamic as they worked through activities that were free yet focused in their learning goals, bringing seemingly unrelated ideas into consideration. I believed it would be important to maintain learning activities at critical points, where students were able to understand what was expected of them while not letting randomness of the task lead into endless confusion and frustration. I believed this is the space in which emergent learning could be achieved through students' cooperative efforts to find higher order within loosely confined possibilities.

4.2.3 Student interaction

From the beginning, the topic of student interaction surfaced many of my conflicted values between teaching beliefs and teaching practices, which I had to confront and to resolve in my research. Taking up a new set of beliefs was an easy feat, but to stay consistent with that in my classroom context proved to be a huge challenge. In this section, I will only discuss the former while my struggle with the latter part will be explored upon later in the paper.

I knew as far as wanting to promote my classroom as a collective learning circle that was organized from the bottom-up. In this type of learning community, students would make huge impact on their own and each other's learning experience. They would also see themselves less as passive learners, but more as active participants who could orchestrate learning events in a classroom. The most difficult part was to bring these occasions up in a non-threatening or non-forceful demeanor that could result in a student-initiated and student-sustained learning space.

One area that I particularly wanted to change was the feelings of isolation and egocentrism when students work on a textbook assignment. I believed that assigning 5 to 6 students to become “helpers” during seatwork would encourage a substantiated level of interaction and communication in class, making learning a less individual and mundane process. I also saw the helper method acting as a negative feedback network in my classroom, especially when students could view their peers as somebody whom they could turn to and consult with whenever they encountered a problem.

4.2.4 My thoughts on the supportive structure

The understanding that I got from reading the works of people who had engaged in extensive research on complexity theory and from talking with professors and colleagues on this topic laid out the groundwork of my action research. I took the theoretical arguments from these sources and transformed them into my own beliefs about what complexity meant to my classroom, my students, myself as an educator, and the learning of mathematics. And from which, I made fundamental, but mostly rhetorical, changes to my lesson plan and in the way I organized learning tasks and the student collective. As I made my way back and trampled through a past journey, I carried with me these newfound perspectives and felt a rush of peculiarity as I looked upon a familiar place with a very different sensibility.

4.3 My authentic experience

The next development of my research still centered on myself as a teacher, but explored more of what I did instead of what I believed in. In this, the pedagogy was a key component to my understanding of pupil learning and the classroom as complex systems. My pedagogy led me down paths that I had never taken, and the consequences of which opened my eyes to things that I had never seen before. My pedagogy put my beliefs into actions, giving rise to my own authentic experience where I explored the feasibility, practicality and effectiveness of applying complexity theory in an educational context. During this process, I learned six ideas that deepened my appreciation for seeing and carrying out what I do with a complexity awareness. Each of these will be discussed in detail below.

4.3.1 Chaos in the learning tasks – roll along with variations and mistakes

March 21, 2006 – a lesson on ratios. I drew 2 diagrams on the board, one with 3 stars, 6 circles, and the other had 3 stars, 6 circles and 4 squares. I asked students to compare the objects in each diagram and to represent their proportion in as many ways as they could. Students immediately gave me different types of answers and I proceeded to write them down on the board (Table 1). This is what we had:

★●●★● ●●★●	Diagram #1:		
	3 to 6	$\frac{1}{2}$ to 1	$\frac{3}{6}$
	3:6	1 to 2	6 to 12
	12 to 24	24 to 48	
★●★●■ ■●■●★ ●■●	Diagram #2:		
	3:6:4	3 to 6 to 4	6 to 12 to 8
	12 to 24 to 10	24 to 46 to 32	9:18:12
	$\frac{1}{2}:1:\frac{2}{3}$	$3\frac{6}{4}$	

Table 1. Unfolding of chaos in a ratio activity.

One answer stood out among the rest, it was "$3\frac{6}{4}$" in the second diagram. When I asked the student to elaborate on the answer, it became apparent to me that she had used her prior knowledge of mixed fractions to come up with what she thought would be an appropriate representation for a proportion of three objects.

> This is the chaos. It led us to discuss how we could use fractions to continue to represent 4 or 5 or even 6 objects. Eventually, most students saw that we could only use fractions to represent a ratio of two things, whereas other methods of using "to" or ":" would still work with multiple objects. This is significant! While I used to concentrate on teaching the right content, I now realize that it's just as important to discuss the wrong content. Sometimes chaos is needed such that learning can evolve into richer understanding.
> (Journal entry, Mar 21, 2006)

My question for the class provided the first breaking point for jumping into an enlightened space for learning mathematics. The question provided many angles of exploration, with possibilities of looking at different types of situation, such as the proportion between two and three objects, and at different ways of writing out ratios. I introduced chaos in the lesson and my students responded by thinking about the different varieties and relations within a simple mathematical concept. The activity was important in the sense that it broke down a stereotypically narrow and constricted view of mathematics, particularly the belief that there is only one right answer for every question. To me, the lesson took an unexpected turn and added another degree of disorder when a student tried to write out a ratio for three objects using mixed fractions. I did not anticipate this, but from a teaching perspective, it was better than what I could have imagined. That student's answer consumed our conversation for the remainder of the class and we reacted with wonder and uncertainties. Our learning space was stretched further because it prompted us to explore possibilities that we didn't know existed.

May 23, 2006 – 3-dimensional fractals. On the other hand, I had also been in situations where chaos seemed to arise effortlessly without anyone's attempt and simply required the willingness to roll along with our mistakes. In one of our geometry lessons, my students made 3-dimensional fractals in a repeatedly cubical staircase figure. The task was meticulous, entailing precise measurement, cutting at the right places, and folding in the correct direction. When I walked around to assist my students, I noticed that some of them had made mistakes along the way.

> The "old me" would have told these students to restart the entire project. This time, instead of having them correct their mistakes, I decided to let my students run with their mishaps. I looked at what they had and we talked about ways to continue cutting and folding into other interesting fractal shapes (Figure 1). I encouraged them to just go along with it. At the end, every student was able to make a 3-dimensional fractal that was used later to explore various geometry concepts like surface area and volume.
> (Journal Entry, May 23, 2006)

Chaos meant many things for me; it could be the variations among student responses, a confused concept or an alternative conception; it could be something that was purposely introduced by a teacher or spontaneously combusted by students. The chaos we had in our activities took us off into side tangents but they were paths worthwhile exploring. I learned how to "maintain" learning tasks that were focused yet flexible enough so that my students were free to embark on different tangents of understanding while not losing sight of the main course of action. As a class, what we learned was so much more exciting and fulfilling because chaos gave us more things to learn from. At the end, we constructed a more colorful picture about proportion and fractals.

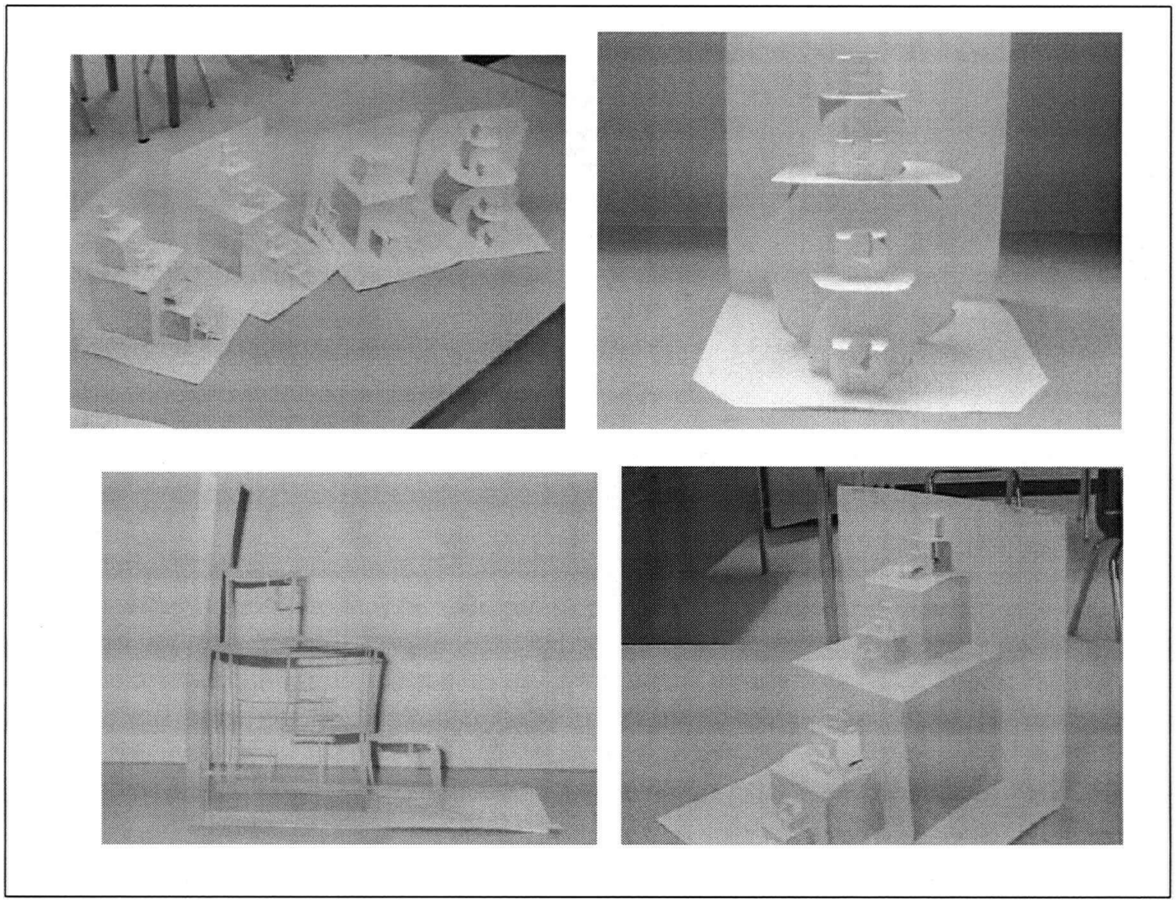


Figure 1. The chaos of 3-dimensional fractals.

4.3.2 Learning tasks at the edge of chaos

April 10, 2006 – a lesson on percents. I prepared a worksheet with 3 diagrams on it and asked students to find as many ways as they could to represent the shaded parts in each. They were given a sufficient amount of time to work in groups of 3's before being asked to share their answers with the class. By now, students were comfortable and receptive about working at the edge of chaos. We immersed in a learning experience that consisted of breadth and depth, making connections from a simple diagram to diverse concepts of decimals, percents, fractions, place value, ratios, scientific notation and exponents (Table 2). More importantly, it was a collective learning experience. We could not have reached this level of knowledge from any single individual or group; it had to be brought about with the effort of the whole class. The following is what we came up with in our collective understanding:

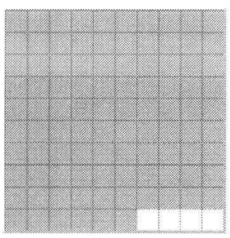


Diagram #1:

0.96, 96%, $\frac{96}{100}$, $\frac{48}{60}$, 96:100, $\frac{24}{25}$, 9 tens and 6 ones,
96×10^{-2}, 96 to 100; $\frac{960}{1000}$, $\frac{192}{200}$, $9.79799^2 : 100$,

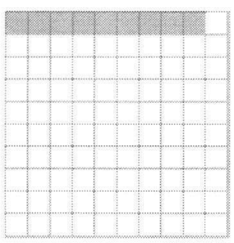


Diagram #2:

0.09, 9%, $\frac{9}{100}$, 0 tens and 9 ones, 9:100, 9 to 100,
9×10^{-2}, $\frac{18}{200}$, $\frac{90}{1000}$, $\frac{4.5}{50}$, 18:200, $3^2 : 100$, $\frac{0.900}{10}$,
$\frac{0.09}{1}$, **0.9**

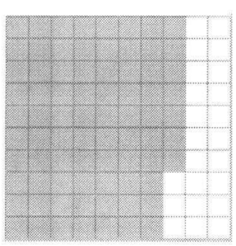


Diagram #3:

0.77, 77%, $\frac{77}{100}$, 7 tens and 7ones, 77 to 100, 77:100,
7.7×10^{-1}, $\frac{154}{200}$, $\frac{38.5}{50}$, $\frac{770}{1000}$

Table 2. Creating order from chaos.

I was impressed with what we created, but was this all there was to learning with a complexity sensibility? No! Our emergent learning did not stop here. Beside the second diagram where 9 boxes out of 100 were shaded, two students wrote down 0.9 on the board. During our class discussion, someone pointed out and explained, by making references to the other two diagrams, how the 0.9 should have been written as 0.09 instead.

> Students are capable of synthesizing and reasoning each other's alternative conceptions. It is an example of learning that comes from the engagement of the lower masses. I didn't have to point out the mistakes; the students corrected themselves and made sure their neighbors are learning the right concepts…sustaining a system where failures and mistakes are fixed firsthand at the lower level.
> (Journal entry, April 11, 2006)

April 6, 2006 – a lesson on square roots. During one of our class discussions, we had an informal talk on square roots. The following is an excerpt of what unfolded in our conversations:

Teacher: Who can give me examples of square roots?

Class:

$\sqrt{16} = 2^2 = 4$ (A student went on to explain that square root of 16 is 4 because 4 times 4 is 16, so is 2^2 times 2^2.)

$\sqrt{9} = 3$

$\sqrt{36} = 6$

$\sqrt{81} = 9$

$\sqrt{4} = 2$

$\sqrt{1} = 1$

$\sqrt{2500} = 250$

$\sqrt{484} = 22$

Teacher: Let's explore one of these answers a little more. Take $\sqrt{4}$ for instance, besides 2, is there another number that I can multiply by itself to get positive 4?

Class:

$\sqrt{4} = -2$

$\sqrt{9} = \pm 3$

Teacher: We have generated an extensive list of examples. Can we think of any other possibilities?

Class:

$\sqrt{11.0889} = 3.83$

$\sqrt{100} = 10$

$\sqrt{49} = 7$

$\sqrt{1{,}000{,}000} = \pm 1000$

$\sqrt{6{,}250{,}000} = 2520$

$\sqrt{3080.25} = 55.5$

$\sqrt{\frac{1}{9}} = \frac{1}{3}$

$\sqrt{930.25} = 30.5$

(Students took out their calculators and commented that you could also take square root of a decimal and fractions)

Teacher: When I take square root of a positive number, it gives me a positive and a negative answer. What do you think happens when I take square root of a negative number?

Class:

$\sqrt{-16}$ is -4.

But $-4 \cdot -4$ is not -16, maybe it's +4.

$+4 \cdot +4$ doesn't give you -16 either.

I don't think anything works; you can't multiply anything together to get -16. When you multiply two positive numbers together, you get a positive answer. When you multiply two negative numbers together, you still get a positive answer.

From our class discussion, students conceptualized their own understanding from listening to each other's input. We went from the most basic and common understanding of square roots, that of a number multiplying by itself to get the radicand, to its relationship to power, perfect squares, non-perfect squares, principal and negative square roots, and imaginary numbers. I simply prompted the class to look beyond the obvious. My students brought up the chaos of mathematics, and it was also them who collectively turned it back to the edge of chaos by making sense of what the mess was telling them. They revealed, one at a time, the layers of what was thought to be a simple math topic, only to realize that it was anything but simple. The examples we came up with as a class reflected the diversity and chaos that could be occasioned by such a task. Our knowledge and understanding were deepened and our experience was more meaningful when we were given the chance to explore the limitations and possibilities of square roots. Nevertheless, the bigger picture could only be appreciated when students engaged themselves in the contributions of each other's work.

Order unfolded itself in various ways, such as from students rationalizing through each other's alternative conceptions to an open dialogue that brought forth the collective understanding of a mathematical concept. The step from chaos to order was critical because it brought about higher learning and (temporary) closure to a class activity. Moreover, the space at the edge of chaos was closely connected to and codependent on the integration of chaos and order; I believed the former provided a rich environment that had the potential to build a more meaningful learning experience. Supporting a meaningful mathematics learning experience also meant seeing the light among chaos; the anarchy from working with diversities gave us a chance to look at many different pieces of a loosely defined puzzle and trying to put them together gave us a fuller and more robust picture of the mess that we were hoping to understand.

4.3.3 Pedagogy at the edge of chaos

In addition to the learning tasks, having a different set of teaching practices also transformed my experience in the classroom. Just as my students had to embrace the complexities within mathematics learning, I needed to live up to my own research as that, which included throwing myself into chaos of teaching.

To accomplish that, it was important for me to find my own way through teaching and learning as complex systems, but to experience its full effect I had to simultaneously immerse myself in various types of learning perspectives aside from those of complexity science. Inadvertently, the chaos from working with different kinds of learning activities and teaching philosophies allowed me to distinguish the similarities and differences among them, making it more obvious the application and potential of honoring complexities in a classroom. A variety of complex and complicated activities were explored, including those that were considered to be simple tasks such as writing notes and doing book assignments. The concept of "simple, complicated and complex" was first introduced by Weaver (1948). He characterizes simple systems as being linear and stable. Complicated systems, which Weaver termed as disorganized complexity, consist of many variables that behave in random ways. Finally, he equates organized complexity as the middle ground between simple and erratic behaviors.

Of all that we did in the classroom, I found the similarities and differences between complicated and complex learning tasks to be the most interesting to explore.

To illustrate this point, I will describe two different "exponent" activities that we did on separate occasions (Table 3 and Figure 2). This is the question for the first activity:

Little Richie is joyously looking forward to turning 13 on August 1, 2006. On his birthday, his parents give him 3 options on what he can choose for his present. For option one, Richie will get $500,000 on August 1. For option 2, his parents will deposit $2 in a bank account on his birthday and will continue to double the amount each day until the end of August. Richie will get whatever the amount is on the last day of the month. For option 3, his parents will deposit $30,000 in a bank account on his birthday and will continue to deposit the same amount daily until the end of August. Again, Richie will get the accumulated amount at the end of the month. Which option should he choose? (adapted from Oswego City School District Regents Exam Prep Center, 2006)

Table 3. An exponent activity.

In the other activity, I asked student pairs to write a cheque payable to their favourite cartoon/movie character for the biggest amount that they could fit on the handout. The following is a sample of what the students were asked to fill out:

	April 5, 2006
Pay to: ______________________________	$ ______________________
0000010 1265 7516878212	Signature ____________________

Figure 2. Sample from an exponent activity.

Although both activities introduced the topic of exponents, they unfolded in very different dynamics and learning outcomes. In the first activity, student groups felt challenged to find the best option. Most groups went about the question by listing out the amounts day by day until the end of the month. There was healthy discussion and interaction between group members as they talked about ways to approach the question and on how they could logically present their solutions on a piece of paper. However, class dynamic seemed to dissipate quickly once students have figured out the answer, and most were disengaged from our class discussion afterwards. The question in the first activity involved an end-knowledge of finding the best option for Richie, and did not do much to push students into other boundaries of exploration or understanding. As a result, only a few students made the connection of this task to the concept of exponents.

In comparison, the second activity was, from a mathematical point of view, much easier to do. Student groups ran wild with their ideas to find ways to write down the biggest number they could fit on the handout. I gave each group 4 cheque templates and they were encouraged to try out different possibilities. Students were eager to share each other's work because no two answers were the same. All the answers varied in the use of digits, place value, and math notations. As these varieties unfolded on the board, they continued to instigate deeper understanding and prompted one another to explore the use of exponents and scientific notation (Table 4). Class dynamic was sustained for a longer period of time and the energy level increased steadily into our class discussion.

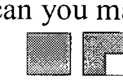
Answer for the first activity:	Answer for the second activity:
Option 1: \$500,000	$999999999999999999999^{999}$
Option 2: \$2,147,483,648	$\overline{9}$
Option 3: \$930,000	$9\textit{google}$
	9^{∞}
	$99999999999999999\overline{9}$
	9999999999999999999
	250,000,000
	280000000
	10,000,000
	10,000,000,000
	90,000,000,000,000,000
	$100000000000000000000000\overline{0}$
	$10{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}00\overline{0}$
	∞
	$\infty \cdot 2^{10}$
	1000000000^{999}

Table 4. Results from the exponent activity.

Honoring learning activities with a complexity sensibility provided a uniquely and rarely explored learning space than what is often attempted in a typical math class. Surely, the use of word problems in the first task might already be a stretch for teachers who do not see the value of implementing different teaching strategies other than giving out notes on the overhead projector. For teachers who are comfortable with the use of group projects, like myself, I might be opening another can of worms by raising the differences inherent within various class activities. In my experience, teaching with a complexity awareness went beyond a mere effort of using cooperative learning and group work; the way in which I designed and phrased the questions greatly impacted the type, degree, and grasp of mathematical understanding. From a comparison of the two tasks, I was impressed at how much fuller and deeper class engagement was when my question had no end in sight, and where collective knowledge was limited only by the students' imagination.

4.3.4 Simplicity in chaos

The rendering of complexities within learning arose from very little effort; it was not uncommon to see that easy questions could generate intricate answers. Throughout the study, I found that the uncomplicated activities were often the ones that manifested in complex learning occasions. These questions were straightforward to understand, easy to decipher, and relatively effortless to engage in. When learning tasks were manageable, every student in the class was invited to join in the intellectual unfolding of math knowledge. Each student was welcomed to have his/her opinions heard and each idea was precious to the collective knowledge of the class. It was precisely this high level of engagement and interaction among students that brought about the complex emergence of any learning tasks, and all of these started from a small and simple inquiry. In the following, I will provide a couple of examples (Figure 3 and Figure 4).

Example 1: A question on fractions: How many ways can you make 1 ¾? Try adding 2 or 3 or 4 or…fractions together to create 1 ¾. 

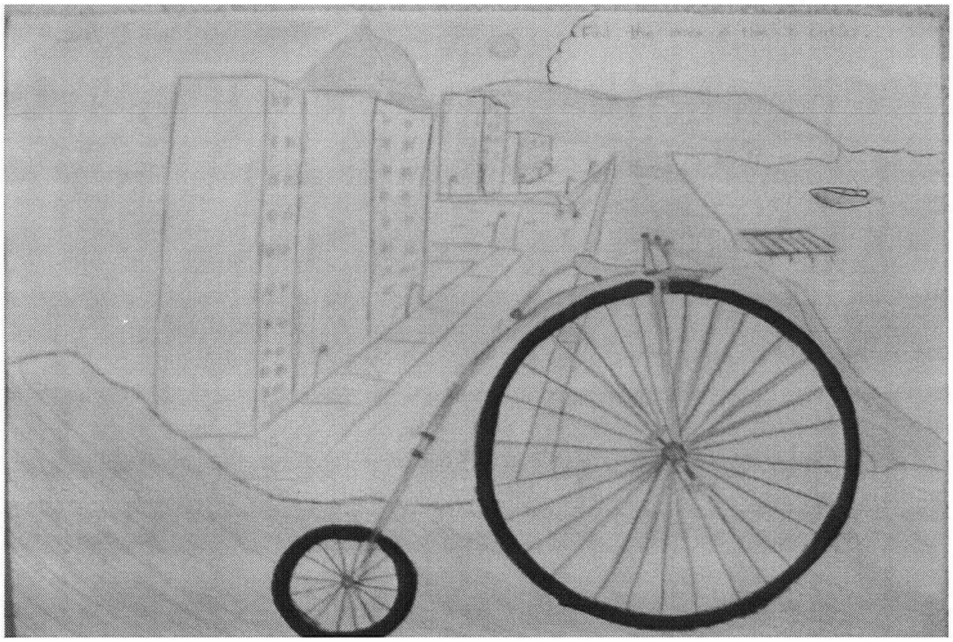
Answers:

$$\frac{16}{16}+\frac{3}{4}$$
$$1+\frac{3}{4}$$
$$\frac{5}{4}+\frac{2}{4}$$
$$1\frac{1}{2}+\frac{1}{4}$$
$$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$$
$$\frac{2}{8}+\frac{2}{8}+\frac{2}{8}+\frac{1}{4}+\frac{3}{12}+\frac{6}{24}+\frac{12}{48}+\frac{24}{96}$$
$$\frac{7}{8}+\frac{7}{8}$$
$$\frac{100}{100}+\frac{75}{100}$$
$$\frac{1000000}{1000000}+\frac{250000}{1000000}+\frac{250000}{1000000}+\frac{250000}{1000000}$$
$$\frac{4}{4}+\frac{3}{4}$$
$$\frac{1}{2}+\frac{1}{2}+\frac{3}{4}$$

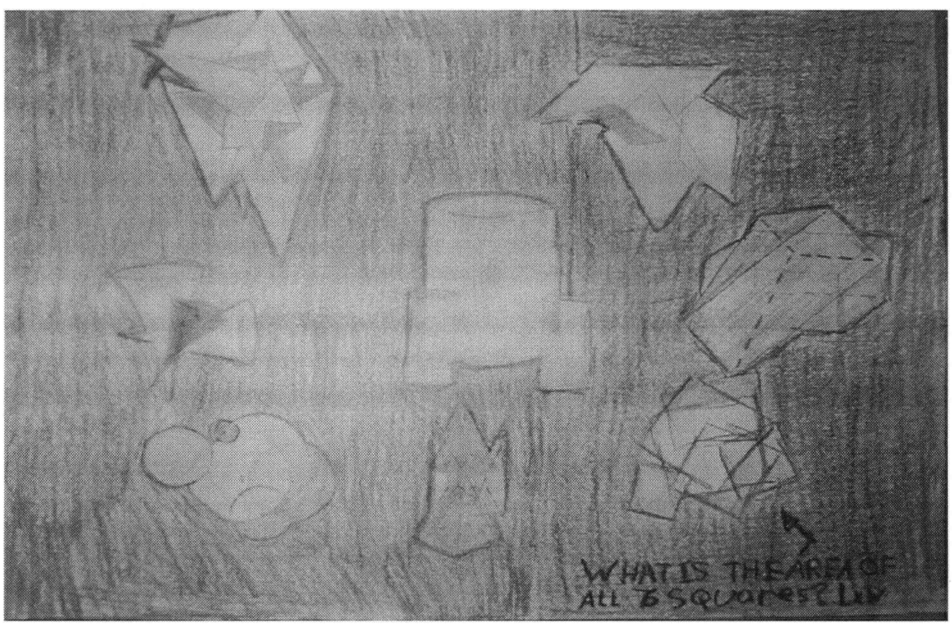
Figure 3. Complex answers from a simple question.

Example 2: An assignment on geometry: Create a stamp design using the theme of geometry. Then, using the finished diagram, I would like you to make up one question that is related to the area of one part of your design.

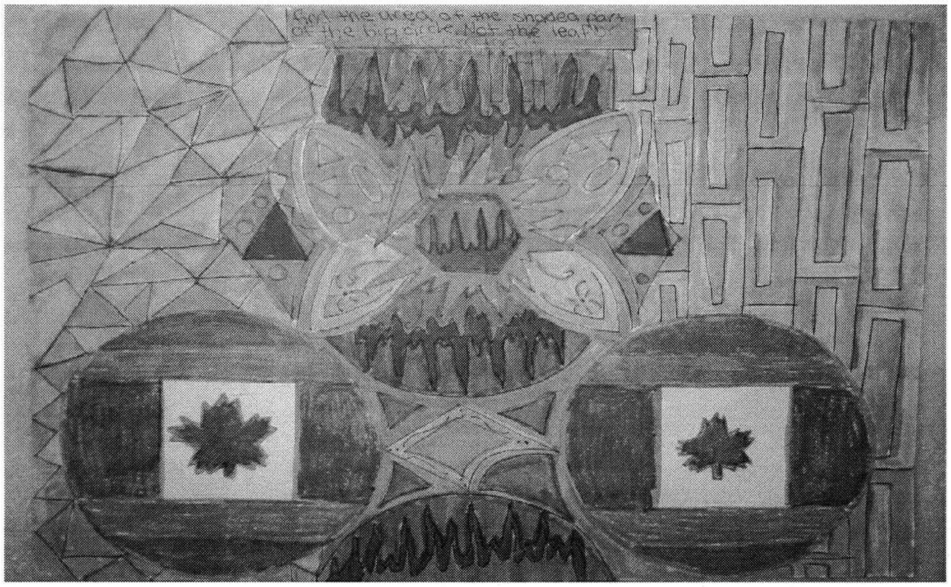
Products:



Question: Find the area of the two circles.



Question: What is the area of all 6 squares (in the bottom right diagram)?



Question: Find the area of the shaded part of the big circle (not the leaf).

Figure 4. Student work on the stamp diagrams.

4.3.5 Taking time to evolve

For the past few years, I was one of those teachers who were mindful of and, to a point, followed faithfully their lesson plan. I saw it as my responsibility to carry out the "plan" in an often tightly packed lesson, and an inability to do so meant either the lesson was not well thought out or I failed to do my job properly. As a result, I did not like it when unexpected events interrupted my lesson, like the inconveniences from a fire alarm or an assembly. These lessons often felt wasted and unproductive.

April 27, 2006 – Sierpinski Triangle. Today, my students had the opportunity to draw a Sierpinski triangle and we used it as a focal point for investigating various mathematical patterns embedded within different levels of the triangle. First, I explained the rules for the drawing, and then asked students to repeatedly connect up the midpoints of all sides in the outer three triangles (Figure 5). I allotted 10 to 15 minutes for the task.

Just as I was about to stop the activity, two students from student council dropped by my class unannounced and requested to take up 10 minutes of my time to complete a school survey.

> It is strange; this unexpected event threw my schedule off balance, but enabled me to go along with the mess. As my students thoroughly enjoyed the task and wanted to keep on doing it, I let them roll with the drawing...the added time allowed my students to explore concepts that would not have been made if things went as planned. Although it was an individual task, it quickly turned into a class activity because they wanted to see what each other was able to do. The students initiated their own conversations and drove each other's learning to a level that I didn't intend at first.
> (Journal Entry, April 27 and 28, 2006)

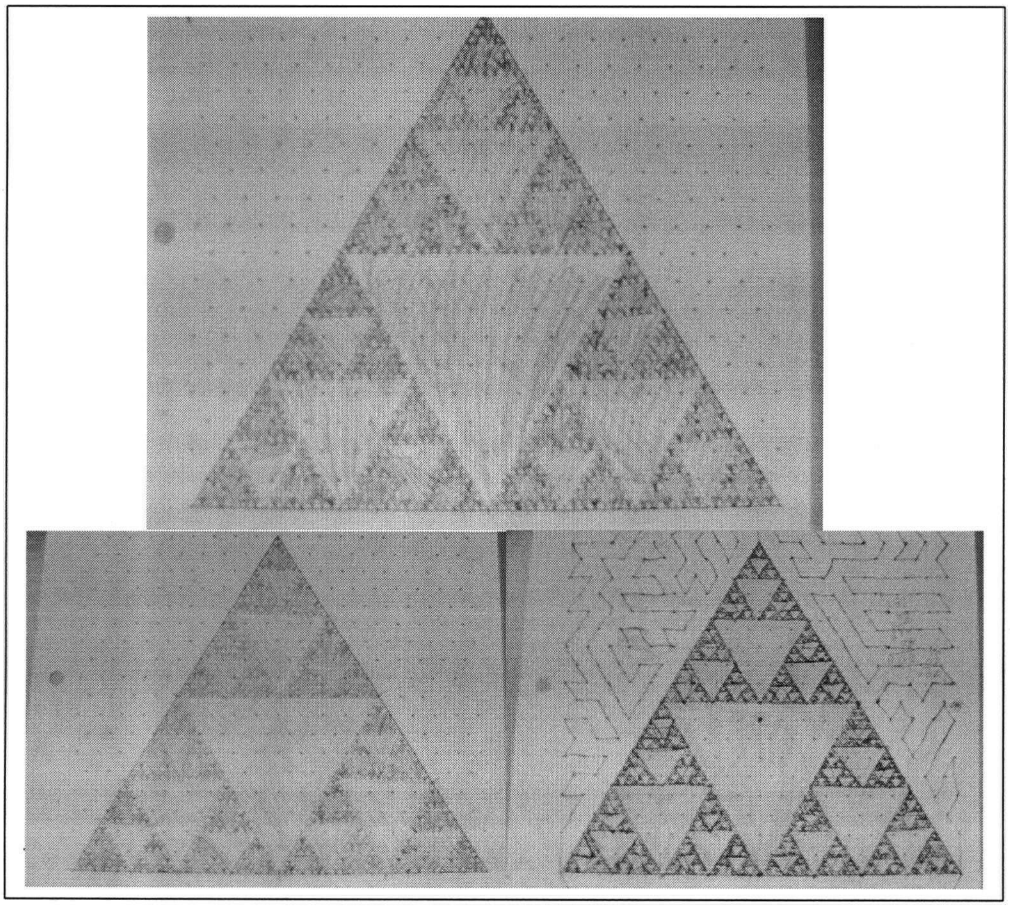


Figure 5. Student work on Sierpinski Triangle.

Our class discussion on the Sierpinski Triangle also had more substance and depth because the students had enough time to play with different ideas. The following is a summary of what was shared by the class (Figure 6):

"all triangles, except the original, point down"
"for each triangle that you draw out, there are three new spaces around it"
"an equilateral triangle has three equal sides and if you make a smaller triangle inside of it, you'll get 3 more triangles equal to the size of the triangle you drew"
"they are all equilateral triangle"
"take the area of the triangle and divide it by four = area of the new triangle"
"the length of the triangle divided by two = length of the new triangle"
"if you keep on drawing, you would eventually cover the entire triangle"
"triangles get smaller and smaller"
"each time, you draw three times as many triangles as the one before"

Figure 6. Dissemination of class discussion on Sierpinski Triangle.

Besides the Sierspinski Triangle assignment, there were other instances where I observed students putting continuous effort into constructing new knowledge by being open to their peers' suggestions, even after they had already done extensive work within their groups. Often, what we shared during our class discussions were more varied and thoughtful than the ideas they were able to come up with either on their own or in groups. I believed more than anything, class discussions provided the extra time that students needed to reflect on what they understood by listening to other people's input on the same topic.

Complex emergence within learning cannot happen at an instance and cannot be rushed; its beauty blossoms when it is given time to ripen. From the many group activities and class discussions that my students and I had engaged in, I experienced the dynamic of complex learning like a bell-shaped curve. Whatever the topic was, student ideas and engagement were sparse at the beginning. The atmosphere took a quick turn when diversities started to appear in their answers or when a student contributed something unusual. It sent a rush of contagious energy through the classroom and instigated students to engage in a cooperative and interactive learning experience. This dynamic was sustained until eventually the engagement level tapered off. In all, mathematics learning unfolded similarly to the development of a story, as it went through phases of rising action, climax and the resolving state. The most important thing that I had learned was that complexities within learning take time to unfold; it is really a test of a teacher's patience to be able to see and allow good things to happen.

4.3.6 Changing shapes of the student body

Throughout the study, I have had to make some tough decisions around student grouping that could sustain a vibrant learning space. At the end, I experimented with different numbers of grouping, such as pair groups or focus groups of 3s, 4s, and 5s, and with different ways of choosing the groups, from students organizing their own teams to assigning them randomly or putting them in groups consisted of varied student academic abilities. It did not matter much what the student group was like; interaction was slow at the beginning when students were still getting to know their neighbors. As they became more comfortable with one another, their behaviors and their flow of conversations got more electric. However, similar to the dynamic of class activities, I also observed this energy diminishing over time. It became apparent to me that it did not matter what I tried, the crucial point is that I needed to keep on changing the shapes of the student groups to prevent it from stagnating. Diversity was important to the unfolding of higher learning; "I wanted to keep a level of excitement in my classroom, forcing my students to bump into the edge of chaos, even if it meant putting students with whom they do not normally associate with" (Journal Entry, March 1, 2006). The exposure to changes and uncertainty could spark new ways of looking at an activity, forcing new ideas and perspectives to surface. I kept on changing the way I organized student groups so that each student had the opportunity to work with different types of people and be exposed to different ways of doing things.

4.3.7 Thoughts on my authentic experience

My authentic experience was a long and slow process of honoring the sensibilities of complexity science within my classroom. I learned to embrace emergences at the edge of chaos, which included sustaining order and randomness within class activities, and keeping intact the similarities and diversities of the student body. It was therefore more than a snapshot of my beliefs because it encompassed a 5-month journey of my own experience, as a teacher / researcher / learner, in a mathematics classroom. My beliefs acted as a supportive structure, out of which my pedagogy unfolded. My experience, as with my pedagogy, was understood differently at different times. In this paper, I could only capture the metamorphosis of my action and understanding as I went through trials of never-ending changes from the stance of a teacher who embraced her learning environment as a complex system. What I understood about teaching practices and student learning became my authentic experience; by this, it was a collection of genuine revelation into my personal endurances of seeing my work with a different awareness and sensibility.

4.4 Changing relationships

While I had focused on transforming the delivery of my pedagogy and in the way I conducted my learning activities, it would be hard not to notice a comparable change in the relationships among students, and between the students and myself. Aside from teaching beliefs, teaching practices and class activities, relationship was probably one of the most important deciding factors for determining the kind of collective learning experiences we would have. Coincidently, of all the complex systems found in a classroom, the one that embraced the student-teacher collective, along with the relationships established within it, was the most difficult for me to bring about. In the following, I will discuss the rapport my students had in this class and my effort as a teacher to bring myself into that learning collective.

4.4.1 Students as agents of a learning community

What I believed and did in a classroom had bearing on how effectively I was able to support the teaching and learning of mathematics as complex systems; however, as I found out later on in my research, it was as much my story as it was a testament to how well my students coped with this sensibility. They had to accept a new way of learning that went beyond individual achievement, in order to see the potential of gaining higher knowledge as a group. From the beginning of the course, I had provided numerous opportunities, through group activities and class discussions, for my students to work together. Although great strides were made in the context of mathematics learning, I did not see the same kind of cohesiveness and dynamicity among student relationships. Students were not opposed to the idea of being put into groups; but deep down, they still held a very conventional view of what encompassed their notion of learning. The following summarizes how my students responded to what they believed would contribute to mathematics learning:

> "Do the work."
> "Listen to the teacher when she is explaining the work."
> "Worksheets. Do your work, study for tests."
> "People working quietly."
> "Teacher teaching the day's lesson and the class paying attention to the teacher, then doing homework for the last 15 minutes or so."
> (Student survey, March 20, 2006)

Many students perceived the classroom as a teacher-centered space, and behaved fittingly as the role of passive students who just sat back and tried hard to take in the lesson. They already had in mind a firm opinion about what counted as learning, from the type of student assignments that were assessed to the kind of behavior that was expected of them, as told by their parents, teachers and peers. Their construct of being this type of a student helped to further a top-down learning environment where they were more concerned about themselves than the other 27 people who were around them. To ask them to behave otherwise would raise doubts and suspicion around the only thing that they were accustomed to - being compared and competitive with each other.

As such, for my students to behave in ways that would respect their peers and classroom as complex systems would mean to abandon a lifetime of values and to accept something unfamiliar. I found that true collaborative learning was not as easy as putting students into groups. I could not be discrete about it but had to make explicit about the complexities that I would like my students to experience in a collaborative learning environment. At the same time, this had to be done carefully so that it was not perceived as another attempt to perpetuate the teachers' authoritative need to indoctrinate yet a new kind of teaching philosophy.

4.4.2 Collective learning that starts with the individual

What does it take to develop a more collective notion of learning in a classroom? I was not forceful in my manner and did not try to shape my students to behave in a certain way, nor did I know what it should look like. My objective was to provide the chance for my students to discuss, to listen, and to learn from one another and throughout the process, allow them to experience the benefits of what they could achieve as a collective.

I decided not to beat around the bush and simply let my students find their own way around this new sensibility. About four months into the course, I asked students a couple of pertinent questions regarding the goals they wanted to accomplish in this class: i) why are we here, ii) what is preventing us from reaching our goals, and iii) what we should do differently in order to achieve these goals (Garmston and Wellman, 1995). Although these questions seemed to be late in coming, I felt I had made an attempt for my class to become more aware of why they were there. For each student, his/her goals might be different, his/her resolution might vary, but they all added an irreplaceable piece to the wellbeing of the class. Their behaviors and attitudes were important to the success of achieving a learning space that was accommodating to both group and individual needs.

We discussed these issues in an open forum, and each student's opinion was heard. The open dialogue gave voice to everyone's needs and concerns, and in doing so, it created a space where each student was not only well aware of but was also respectful of each other's goals. Although these goals might be different for each person, it nevertheless instigated a more cohesive, more collective learning space. This is precisely the type of sensibility that was needed to sustain the dynamic relationships in a classroom.

What I found most interesting was that collective learning did not demand every student to behave and to act in the same way. In fact, it was the opposite of that which I tried to create. Every individual needed to come to terms with his/her abilities and strengths, and to be in tune with what and how he/she would like to contribute in the class. It took me almost five months to understand this point. For students to respect classroom as a complex system, they were to honor the objectives of the class by working within their own capabilities. This way, they were not identical beings having to compromise their individually nuanced perspectives to my needs or the needs of their peers, but they were unique individuals who were committed to work together to serve personal and collective needs.

> I want my students to live in a close-knit classroom community. If they were to become productive, contributing members in our learning circle, I would need to let them work together as a class. Students need to know that each of them plays a role in constructing mathematics learning, and that each affects and is influenced by the overall efforts of the entire class. I believe that when the common objective is clear, each person will behave within his/her ability to add a part of him/herself to the overall learning environment.
> (Journal Entry, May 18, 2006)

4.4.3 My role in a learning community

In addition to the relationships developed within the student body, my relation with them was just as important to this learning space. Throughout the course, I had asked myself many times where I belonged in all this. I knew I wanted to be a teacher who could be part of a dynamic, emergent learning environment without exerting complete control on her students. The means of getting to this end was another part of my journey; it was a process of trial and error every bit as complex as other parts of this study.

I had to be true to the teaching philosophies I held then and now. My past experience as a teacher, including what I learned from my teacher education program, had been a big influence on my daily teaching practices and in the way I interacted with my students. On top of this, my students had preconceived notions about what they expected of me. They saw the responsibility of a teacher as someone who transmits knowledge in a traditional, rule-bounded classroom. According to my students, my job was to:

> "explain everything clearly and to be ready to answer questions"
> "help us when we need it"
> "know what he/she is talking about"
> "describe the work carefully and make sure we understand the work"
> "mark tests correctly; enter marks correctly"
> "explain the concept in a way that I understand"
> "teach us an easy method for working out equations"
> (Student survey, March 20, 2006)

As a result, there was an invisibly large distance between my students and me. From the beginning, my distinctive role, as defined by a conventional view of education, separated my physical and intellectual presence from my students' learning circle. According to that, I would arguably stand at the front of the class and give a talk on what I know, rather than working with the students and thinking along with them at their level. Embracing classrooms and relationships as complex systems meant breaking down the normative construct of an authoritative, hierarchical teacher-student relationship, as viewed by both students and the teaching community.

To completely change the students' views so that they understood my line of thinking was beyond the feasibility of this research, but I was more "in control" of what I could do as an educator, reassessing where I stood within this sensibility and closing the communication gap between me and the students. The dividing line that separates teachers, the knower, from students, the learners, draws a territorial partition between the two sides, with teachers standing at the head, metaphorical and literally, in front of the class while students stay seated behind their desks. There is nothing wrong with this arrangement if teachers like to view learners in a top-down learning hierarchy. My relationship with the students was not changed overnight but progressed gradually as I struggled to find my "self" within the complex systems of a classroom. Unlike embracing the complexity within mathematics learning, which could be accomplished in a relatively short time, changes being made to the traditionally constructed role of teachers had to progress in smaller steps. It involved many other factors, such as giving students the time to embrace this sensibility, and gaining the acceptance and trust of one another.

It took me about four months to let down my guard and be open about giving greater "ownership" to the students. The difficult part was redefining my teaching role, which included reinterpreting issues of control and my level of involvement in the learning circle. I had to respect my classroom with a completely new and different perspective, one that would allow me to experience all levels of complexities in the learning environment. This study forced me to observe my classroom, my students and myself closely for the first time in my teaching career. I was finally making the effort to get to know the people I was teaching to/with, something that I had never done before. I started to realize the diversities embedded within in my classroom, and felt that it was wrong for me to suppress these differences by exercising too much control on my students. In the past, I had a false perception that my students were identical bodies who could think, act, work, and learn in the same way. Once I understood this was not the case, it was a liberating learning experience. It enabled me to become a more attentive teacher. I was more willing to listen and to communicate with my students because I wanted to hear the individual voices, which can sometimes get lost in a large group.

My genuine desire to understand each student as a unique agent helped me to see and to value students as separate entities having different capabilities and needs. Our working relationship developed to a point where I was welcomed to join in their learning circle. I felt as though I melted into the background. I no longer saw myself as the teacher standing at the front of the class; I saw myself through the eyes of my students.

> The most efficient way for me to get down to the students' level is to join them on their activities; I'm not talking about just being with them intellectually but to literally join in on what they do. By joining them, I no longer control the pace of the activity from the outside but was forced to go along with what my students could handle. I knew exactly what my students were doing and could anticipate the problems they would encounter in their work, both of which would have been difficult if I were not a participant in their learning circle.
> (Journal Entry, May 22, 2006)

A few times I sat down with my students and we worked on the class activities together. When I joined in their learning environment, there was less temptation on my part to seize control because I was living in their space. Only feelings of powerlessness would make someone want to have top-down control in the first place. In a way, I did not have to exert control because I had it. In fact, every single person in the class had massive control over his/her personal learning space and had influence over the more global environment. I felt "in control" because I established myself as an interlinking member of the class, living in the students' moment. The biggest change in my relationship with them came when I found myself being in tune with, instead of reacting externally to, my students' emotions and feelings.

4.4.4 My thoughts on relationships

The relationships amongst the students themselves and between my students and me grew stronger towards the end of the course. In the last few weeks of classes, we were a cohesive group. Each of us was willing to step outside of our private zone and take a more proactive role in the well-being of the class. Most of all, my/our journey of embracing complexities became more than a personal feat, it was a team expedition as my students and I went through the voyage of seeing and experiencing our classrooms and each other as a collective.

4.5 Breaking down linearity

It was not until this study that I started questioning our entrenched ideologies with linearity, policy and organization in the educational system. Although I encounter them daily, they represent so much of my own schooling experiences and societal expectations that I am blinded by their constraints. Complexity theory made me skeptical about what we have been doing in the past and opened up my eyes to new ways of seeing. Understanding and honoring my classrooms as complex systems that are part of and give rise to other complexities urged me to look at everything else differently. I explored my own beliefs about lesson planning, learning tasks and student interactions, and then challenging these beliefs by making changes to my pedagogy and redefining my working relationship with the students.

CHAPTER FIVE: MY EMERGENT LEARNING ON COMPLEXITY THEORY

5.1 Emergent learning

My beliefs on complexity theory had evolved from those I held at the beginning of this research, my understanding of which had also deepened after 5 months of experiences and partnerships in a classroom. As such, I have now with me more than just a snapshot or experience, but a panoramic portrait that includes all that and what will become of it in years to come in my teaching career. The chaos of embracing complexities across spectrums of education, like students, learning, teaching, and relationships, were described in the previous chapter. In this chapter, I will turn to what I had learned as a result of living through "the mess" and will recount my perspective on matters of mathematics learning, the classroom, and their implications on our educational system. This is the part of my story where I found order at the edge of chaos as I searched for my emergent understanding of complexity theory. The following is a dissemination of my reflective thoughts on this journey, and this was a journey like no other that could be experienced or told the same way I did.

5.2 Reframing mathematics

> While doing math questions resembles a linear workout, mathematics is not. I think too much focus has been put into the "calculation" and not enough into the "understanding" of mathematics as a discipline. Math as a discipline is much more chaotic, more debatable, and more uncertain than how it is usually projected. (Journal Entry, June 2, 2006)

Ideas from complexity theory challenged me to question what could be done locally in a classroom to work around the linearity of our math curriculum and educational structure. Like all other math teachers out there, I have a responsibility to teach the prescribed content as laid out in the Integrated Resource Package. For practical reasons, it perhaps makes more sense to organize this information by categorizing it into discrete headings or chapters. Regardless of this, our lessons hardly need to mimic the same arrangement.

To honor the complexities within mathematics, I was empowered with a sensibility, and maybe to an extent a teaching tool, to reframe my understanding of this discipline in a different light, one that enabled students to work with multiple topics at a time and to touch upon areas that were not required to be covered in the course. For example, while Euclidian geometry is a common topic in elementary and secondary math, I extended that space by asking students to think about other different kinds of shapes that they came across in life, like man-made or non man-made shapes, and even things that do not have shapes. We expanded the limits of points and lines in a triangle to explore the self-organizing nature of a Sierpinski triangle; similarly, we surpassed the confines of a triangular prism to the prospect of constructing a tetrahedron. Class discussion, such as debating whether it is correct to have a value over 100%, was another hotspot for us to enter the ambiguous places that most teachers do not dare tread.

It is powerful to think of mathematics as complex in nature. It breaks down many past perceptions about the rigidity of mathematics and opens doors for new ways of learning and understanding math. I do not have a fixed syllabus for how the course should be run or preference for a list of topics that should be covered in Math 8, but in some rather broad guidelines, it is a combination of having the right kinds of activities, questions, group interaction, and vibes that can support the complexities of this subject.

5.3 Reframing the learning of mathematics

Since math is a complex phenomenon, learning mathematics needs to encompass this quality so that the field is studied in its true essence and beauty. Traditional games and projects are fun for students but they merely resonate the same learning objectives as writing class notes and doing exercises from the textbooks. From what I had done in this class, I found that working with a complexity sensibility met certain learning goals that other methods could barely touch on, specifically, the goals of making mathematical connections and instilling learning as a life-long process.

When the complexities of mathematics learning were embraced, students were introduced to different ways of thinking and were encouraged to make connections with diverse concepts. The nature of their connection was broadly conceived, anything from past knowledge of a related math topic to real life experiences. During one of our "π" discussions in class, one student was excited to find out that Pi day, Mar 14, falls on his birthday, while other students made a connection from π as an irrational number to the variation of decimal places they saw on different calculators. Each student differed in the type of connection he/she made with a particular learning task. However, the nature of their association was less important than the fact that they were making linkages across prior and present understanding. What came of this type of interaction was my welcoming of "noise"; it was the kind of noise that indicated an avid sharing of ideas and reflected a convoluted and complex process of learning in action. In addition, I found that my class notes were shorter than before but nonetheless contained as much if not more substance because many concepts were brought up in our prior group activities and class conversations. These emergences resulted in a more spontaneous unfolding of learning opportunities where students' many "what ifs…" were taking them to places beyond the usual constraints of a provincial curriculum.

As I became an active part of this learning circle, I was invited to make connections to my students' ideas. It implied listening and understanding their frame of mind so that I could respond appropriately to their comments. I was also making connections between student ideas so that emergences of higher learning could be occasioned. These demanded that I be alert at all times, an awareness of all my five senses to be able to think and learn on my feet. Inadvertently, this turned me into an even more responsive teacher because I was attuned to the people and things within this complex system. This was very strange to me because in the past I was too focused on what I had to say and do as a teacher that I never dared to or even seemed to consider the possibility that I might invade this space. Now that I had, I feel completely alive and awake in my new being.

When students made new connections amongst various math concepts, their understanding was undoubtedly deeper and more meaningful. I believed the heightened emotions around these circumstances could also serve as stepping stones on the path of lifelong learning.

> Although this phrase has been used extensively in the educational community, no one has clearly defined what this means or what it takes to be lifelong learners. I'm beginning to think that treating classrooms as complex system is a start. Instilling life-long learners should not be seen as an individualistic accomplishment; it should be seen as a community of learners with the interest of striding higher grounds together. The success of this lies not only in his/her own learning but in everyone else's learning.
> (Journal Entry, May 5, 2006)

Educators might have been mistaken to think of lifelong learning as something which to imbue individuals, and as something that can be taught like a skill. In face of such ambiguities, it is probably why few people have attempted to define the phrase, let alone try to explain what this means to teachers in the practice setting.

Complexity theory allowed me to think about learning not as a moment in time, but as a process unfolding through time. It reframed my definition of mathematics learning and it allowed me and my students to explore different possibilities and to make connections the best way we knew how for the moment, knowing that some of these mathematical understandings would be refuted in the future while others would be reinforced along the way. At times, my students held misconceptions that they were not aware of. At other instances, they brought up math concepts that they would not normally learn for another 2 years. In my own opinion, this sensibility stretches the boundaries of mathematical knowledge in that learning is not contained to certain age, grade level or course. In addition, it also sets up a viable and sustainable atmosphere for students to understand lifelong learning as a group and collective effort; they will continue to learn from and learn with one another. I believe providing the opportunity for any student to build collegiality with their peers, friends, teachers, and later in life with their colleagues, will be the main driving force for higher learning.

5.4 Mathematics education for the future

Even with all the benefits that came from viewing learning and learning activities as complex phenomena, I had hardly banished doing things I had done in the past. Whether the task was simple, complicated, or complex, I found that they served very different learning outcomes despite any apparent redundancies. For simple activities such as listening to lectures, taking down notes and doing textbook exercises, students learned basic fundamental knowledge and skills - the "calculating" of mathematics. In complicated activities like word problems, students developed reasoning skills – the "thinking" of mathematics. These challenging questions; however, did not necessarily tailor to low student abilities. Class discussions, if they could be brought about at all, were often flat and only served to recapitulate student answers. Last but not least, complex activities were the most difficult to define because it was not what the students did but how they were done that became the essence of their learning. I found that these activities were straight forward, such as having class discussions on an open ended concept. They brought out the differences, similarities and variations of mathematics and encouraged students to make connections among them, all of these ushered a combined effort from students and teacher. Therefore, these tasks held the potential to higher, deeper, and more meaningful emergences – the "living in and of mathematics".

> The linearity in teaching notes produces a culture of learning that emphasizes the need to repeat, where the criteria for success is in the students' ability to copy and to reproduce in an exact manner. The nonlinearity of group activities and class discussions on the other hand instills a culture where spontaneity, individuality, and differences are important to the cooperative learning of the class.
> (Journal Entry, March 31, 2006)

I am not in a place to refute any of these paradigms; rather, I see the benefits in all and I hold in high regards when teachers can bring about all types of learning activities in their classrooms. It is also important that none of these exists in the absence of others. Each serves different purposes of learning, and each is an important linkage in a network of an enriched learning space.

5.5 Reframing the math classroom

It is unfortunate that educators spend an overwhelmingly large amount of time on curriculum and pedagogy that they neglect the attention given to classroom composition, which foremost should be the first thing a teacher considers. After all, how could teachers make decisions on content and teaching practices without consideration for the students?

Yet, there has been little conversation, in the teaching community, around students and classrooms as complex systems, which I believed contributed to the clash often seen between theory and practice. For example, complexity theory recognizes the heterogeneity of student abilities as an important factor for higher learning to emerge. History can tell us that schools have tried to introduce integration and inclusion as ways to overturn educational philosophies like student segregation and streaming. However, teachers' lack of understanding about diversity adds to the frustration they often feel when students of varied abilities are integrated into a single classroom. They see academic and behavioral variance as an impediment to their responsibility for making sure that every student attains an acceptable standard of education. It also transposes, on a larger scale, to other conflicts that teachers sometimes deal with on matters of classroom management and student engagement. My experience told me that complexity theory reveals the barest essence of a classroom, and understanding it gives a greater compatibility between what classrooms are actually like and how we would like them to be.

I thought honoring complexity in a classroom would mean losing control as a teacher; in contrast, it helped me turn my class into a more robust and nourishing place. Certainly, letting go of my power as an authoritative teacher was critical, but what was gained on the other hand was my understanding of how control was redistributed elsewhere in the classroom. Therefore, it was less about losing control and more about finding a different balance of power distribution that could reflect the true spirit of my class. In this study, I progressed through phases of personal conflicts and struggles on the issue of classroom management. At the beginning, I held my authority. Then, the class spiraled into an erratic state when I attempted to abandon top-down rules. After many attempts, we finally reached at a more powerful place where students had more say on what went on in our classroom. To get to this point, I enlisted "student mediators" each class to deal with management issues; they made decisions on giving out warnings, punishments and rewards. Prior to this, students had discussed the kind of classroom they would like to learn in. We also talked about various implementation they could use, such as holding up signs that read "stay on task, pay attention, listen up", giving out warnings by putting up green, yellow, and red colored magnets on the board, and handing out small prizes to recognize exemplary student behavior. It was about "allowing people, who are closest to the problem, to participate in arriving at a satisfactory resolution. Students needed to realize the huge impact they have on their own learning and on what the class learns at the end of the day" (Journal Entry, May 25, 2006). Using ideas from complexity theory to resolve issues of classroom management meant that students had to identify common goals and decide what they could do to accomplish them. This was a form of classroom management that started from the bottom of the typical learning hierarchy. This kind of control felt powerful because students took a more active role and initiative for their own learning and behavior.

Control redistribution was one way for me to understand classroom as a decentralized system; but still, I could not explain the dynamics behind students coming together as a group. My emergent learning on that issue came from an unlikely source, a tetrahedron project. The thought came to me while I stood back and watched as my students worked individually folding triangular prisms, then putting them into a bigger structure as a group, and finally as a class, constructing a gigantic tetrahedron (Figure 7).

> Embedded in this project is the collective undertaking of the whole class and the individual effort from every single student. The tetrahedron means so much more. It resembles my ideal learning space. For me, each student is like a triangle pyramid that comes in different colours and qualities. But if they all have a common goal, they will create something big and beautiful. The key is that everyone has to participate cooperatively within the group. When one fails, it will cause the whole structure to fall down. Each student needs to be able to support him/herself, to support another student, and thereby to also support learning of the collective group.
> (Journal Entry, June 8, 2006)

Figure 7. Tetrahedron project.

For a long time, I could not grasp the immensity of "neighbor interactions" nor comprehend the role that each student plays in this complex system. I went as far back as studying other systems like ant or bee colonies; however, human beings are too unique on their own to be comparable to anything as such. Unlike ants and bees, we are too disparate in our abilities, desires, and goals. Watching the tetrahedron coming together helped me put these confusions into perspective. The tetrahedron epitomized my contention of supporting a classroom as complex system. Students may work on the same task, but each understands it in a uniquely personal way and supports the class in a different manner. Similar to the tetrahedron, some pyramids are at the bottom while others are at the top. But no matter where it is, each is equally salient to the overall structure. Within the confines of each student, he/she is capable of deciding and determining what they can do in class to help each other to learn. To some students, it would be very little. To others, it would be a little more. For some, their actions are overt while others are more covert. As a teacher, I had to embrace the special talents of each student to foster the collective learning environment of a classroom. For example, I turned away from assigning students to become helpers and mediators to soliciting student volunteers to take up those responsibilities because only they were in a position to know whether they could handle the job.

> "I am the one who everyone turns to for the answer if no one else knows the solution. I may not have the highest mark but I see myself as the smartest in the class."
> "The role I play is that I sometimes help people who need help. Another role I play is I do my work and don't bother people."
> "I am the one who asked for help."
> "The role that I play in this class is helping everyone as much as I can when help is needed. I try to be a good mediator as well."
> (Student Survey, May 19, 2006)

Having an awareness for complexity theory emancipated a lively space for me to rethink my teaching responsibilities and my classes; it also opened up the possibility of bringing about learning that was deep and meaningful. My emergent understanding about these could nonetheless not happen if I did not attend to the system from the bottom up, through the eyes of my students. When students felt obligated to be part of a learning collective and worked hard at building relationships with their friends, the environment became one that was robust and positive. So at the same time that I looked at global changes in a classroom, I could not afford to disregard the potential that were embedded within individual efforts and group relationships.

Lastly, complexity theory gave me a new awareness for dealing with an inclusive learning environment, something that has been spoken widely about by educational theorists but remains a difficult topic for teachers to make sense of in their classrooms. From the various types of teaching practices that I tried, it was interesting to see that each perspective tended towards different learning goals and embraced classrooms as different paradigms. I found that a teacher-led, top-down approach to learning, as in teacher lectures and board note taking, disadvantaged those students who are verbal learners, academically weak, or have learning disabilities; while challenging word problems can divide the class along lines of academic ability such that only high-achieving students would benefit from doing the tasks. These perspectives, when taken alone, could be a disservice to a large proportion of the class, ignoring student needs and suppressing their curiosity to learn.

> Education is not about weeding out the weakest or survival of the fittest. Education is about including the underachiever, overachiever, the gifted, the learning disabled and the averaged. It is not about working with the best, but how to work with the varieties that are given to us.
> (Journal Entry, June 5, 2006)

It would be difficult to find any teachers disagreeing to this, although it maybe that few actually walk the talk. I believe the discrepancy between how teachers think and act is largely due to the fact that most of us are still working with a traditional view of schooling and are confined by the linearity across classroom structure and curriculum.

Understanding my classroom as a complex system changed a lot of my decisions around lesson planning, learning activities, and student interaction. In my class, I embraced these undertakings to manifest in a mutual yet dynamic unity. What I did not know until I had a conversation later on with a colleague was that this understanding helped to celebrate the varieties and diversities of my students. Not only did I think more about student collaboration, but in practice I brought the class together while maintaining individual integrity of each student. For example, I tried to design my class activities in a way that would reach a whole spectrum of student capabilities, that each student would work within the confines and possibilities of the tasks to find them both manageable yet challenging at the same time. This was something my colleague described as a "low floor, high ceiling" phenomenon. When I embraced my classroom as a complex system, each student, especially the academically weaker students, was valued as a key contributor to the class, no matter how small or big that contribution might be. I was gratified at how this sensibility allowed me to bridge the gap between theory and practice by welcoming the inclusive effort of every student in a collaborative classroom.

5.6 Reshaping value loaded jargon

It became evident to me that a lot of our current beliefs, practices, and setups needed to be changed, across the board, from students to societies, if classrooms and education were to be understood as complex phenomena. Throughout the study, I felt that there were interferences that deterred me from taking in the full effect of this rendering. These were the blockages that had supported the foundation of our educational system as an hierarchical institution. We are on the verge, and in desperate need, of an educational revolution. I believe many of these so-called traditions need to be torn down and rebuilt from the bottom up. It will be a major renovation and will shake the basis of our foundation that we have depended upon for so many years. Some changes are global, which will unquestionably take longer and more effort to adjust. However, there are also local changes that teachers can implement right here, right now in their classrooms. A few of these I have already mentioned, such as revamping lesson plans, pedagogy, learning activities, student groupings, and teaching philosophies. I believe the other most powerful thing that we can change is in our use of value-laden educational jargon.

It was almost effortless to bring out the complexities of mathematics learning, but for me, to envision classrooms as complex systems was an arduous task. Teachers, including myself, have grown accustomed to educational jargon that impedes the functionality of working in a complex system. Each day, I am bombarded with value-laden words that remind me about working in a linearly structured environment. The word "classroom management" appropriates our use of other auxiliary words and phrases like rules, consequences, gaining attention, and feeling in control. All of these imply a very commanding demeanor on the part of the teacher and because we hear them every so often that we unknowingly live in those roles. Working under this constraint, I had a hard time learning how to let go of control but being aware of the detriments of a particular jargon was a start. I was no longer using the phrase classroom management; in fact, I considered what I did as creating "classroom harmony".

Student achievement, teacher performance, teacher accountability, school goals, district initiatives, etc…are similarly phrases that we use regularly to distinguish the various components within our educational system. This compartmentalization separates these various levels to an extent where it creates unnecessary distance between the objectives of each party. If these varying levels were seen as embedded systems, there would be much more communication and agreement between what is being achieved at the student level and those initiatives at the district office level.

Jargon that teachers use in education is commonly loaded with strong values and meanings. I believe that introducing new vocabularies can help support the complexity of what we do and who we are.

5.7 The chaos that kept me going

There is no ending to this journey, neither was there a definable starting point. As I continue to travel forward, I step backwards for guidance and look sideways for inspiration. The journey I have described thus far was merely one critical incident out of the many that I had and will experience in my life journey. I am certain that while I may take rest stops along the way, I will not stay permanent at one place unless I stop looking for chaos. Chaos will keep me rolling onward and visiting unexpected places. In this study, my determination to engage with questions and confusions turned out to be a rewarding experience, and one which will continually reshape what I will learn in the future. The following is a list of what I considered as the chaotic points in my study:

- How does this type of teaching and learning affect the students? How does it affect my lesson planning?
- What are the key components needed to bring about the complexity of learning?
- What is the role of the teacher in a complex system?

- How does this sensibility affect my classroom as a community of self-organizing learners?
- What are evidences of emergent learning?
- What is the chaos that my students and I experience in this system?
- Are there any order / pattern that might emerge from seeing classrooms as complex systems?
- What is too much control, such that it constricts rich learning possibilities? What is too little control, such that it impedes meaningful learning from taking place?
- How does this sensibility compare with collaborative learning, constructivism, and group work?

These questions kept me intrigued for five months and working through them, I was pulled back and forth between order and chaos. Many times, the strenuous pressure of sorting out the mess was too much to bear. At the beginning, the study went well as I experimented with different learning activities and various ways for my class to be enlightened with the emergence of mathematics learning. My activities got better at bringing about these occasions and I got better at recognizing and capitalizing on these moments when they came up. Then about three months into the course, I was at a complete stalemate. No ideas came in; no thoughts went out; no hint as to where the next direction would be. I had no clue about what I could do to improve on what I had. I was literally searching for the chaos that might have slipped through the cracks. Then there it was; I suddenly realized that I had never even explored the possibility of my classroom and the relationships within it being a complex system. This turned out to be more challenging than honoring the complexity within mathematics learning. It took a long time and a tremendous amount of courage to come face to face with aspects of decentralized control and self-organization. I felt out of place at the beginning, especially when I did not understand the implication of losing control. When I let go of top-down control, I felt as though I was not doing my job as a teacher. My students were also at a lost with the freedom of having voices in my class, and I immediately wanted to step in with my control measures. The comfort of being in order was all too tempting, but I knew I would re-walk an old path if I chose to turn back too far. I learned that order is not a choice that I make alone, but an emergence that comes along when we, being the students and I, could work ourselves out of the mess.

5.8 My thoughts on emergent learning

These two chapters are written to reflect the surprising paths of complexity theory. I started from the bottom, looking at embedded beliefs, pedagogy, and relationships. The chaos of meddling through various snapshots and experiences led to my own emergent learning of complexity theory, which I called my panoramic portrait. Reflecting on the journey so far, I have a deeper appreciation for the discipline of mathematics and regret for all of its exquisiteness that teachers do not do enough to engage in with their students. I also value the potential of seeing learning in this sensibility, one that can effectively address how students make mathematical connections and develop life-long learning skills. Embracing complexity within classrooms resulted in a powerful rendering where every student was respected as an important and indispensable member of the learning collective. Despite of these successful emergences, they came at the price of struggling through innumerable challenges and confronting various restraints that stood in the way. However, as my students could attest to, it was a joy to work in this space.

> "I found that participating in class helped me understand math, if I participate more all my mistakes would be caught and I can be corrected."
> "I think that we did establish ourselves as a cohesive team because we all helped each other through it all. We all seemed to contribute to the progress of the class."
> "Having classmates and friends around helps a lot because we can discuss different ways to solve problems and correct one another if needed."
> "It felt good to hear other peoples' opinions and seeing them for myself."
> (Student survey, June 30, 2006)

5.9 The journey…to be continued

Now that I have lived through the liberation of an emergent learning community, what is next? Well, a new course will start and it will be an embedded system with very different complexities. Will I learn different things from it? Assuredly. I look forward to a group of new students and I wonder what they will teach me this time. As with my story…it is to be continued.

CHAPTER SIX: DISCUSSION

6.1 Research limitations

The traditional notion of rigor in research is impossible to achieve in this project, where the "object" of study is on the interrelationships and the collective unfolding among students, teachers, classroom dynamics, and mathematics learning. I did not have a definite set of research procedures, nor did I pinpoint down in precise detail the particular factors that my study tries to address. All of these unfolded and were defined as my study progressed. They, along with many other uncertainties that might arise within the natural course of development of a classroom complexity, place realistic limitations on my ability to layout the specifics of moral-ethical and research related issues.

6.1.1 Blurring of teacher-researcher responsibilities

It would be difficult to clarify my role in this research project. From an academic standpoint, I was a researcher with a mission to understand pedagogical and epistemological changes that came from embracing the complexity of a classroom. If I were to comply with a scientific model of research methodology, my responsibility would be to generate a critical analysis of the components that impact upon these changes. I would also need to keep an objective stance during the study so to ensure the integrity, objectivity and accuracy of the research data.

However, early on in the research, concepts from complexity theory informed me that the separation alluded to above is nothing more than a convenient and artificial discernment that tries to predefine the role of a researcher. My overlapping identities at the school, that of a researcher, teacher, learner, and colleague, formed the true and authentic me. All of these roles had a huge impact on what I did, how I behaved, and what I understood about my action research project. These identities grew with me and my new self kept on changing with each experience. The contention between people and context has been expressed by Sumara and Davis (1997), who believe that the supposed boundaries between researchers and research situations are blurred into a complex web of relationships, and these could only be honored and sustained by the sensitivities of an action research project. The appropriateness and benefits of using action research are further elaborated by Kemmis and McTaggart (2000); they believe that research practitioners should "understand themselves and their practices as formed through the life-world processes of cultural reproduction, social integration and socialization-individuation and that their efforts to change their practices necessarily involved changing the substance of these processes" (p 589). Throughout the study, my understanding was unceasingly shaped by the transformation occurring within varied systems of teaching, learning, and research practices. The blurring of multiple roles did not present as big a struggle as I thought it would be at first; I felt strongly about what I had to do to honour complexity in the classroom and simply carried out what I believed in without worrying the specific identity I was supposed to assume at any point in time.

6.1.2 Student collaboration in the research process

As I came to terms with my new medley of identities, I felt as though my students were also stepping up to take on other roles in this study aside from that of a learner. The rich learning occasions described in the results section came about because my students saw to it that they could be a co-teacher in this class, a mentor to their friends, and a co-researcher to the study project.

During class discussions, students introduced new concepts and novel ways of connecting those ideas together. The students, collectively, created their own narrative space that put various mathematical topics like decimals, percentages, fractions, place values, scientific notation, power and ratios and proportions into one picture. Through the use of the "helper" initiative, they developed a genuine concern for each other's learning. The mathematically inclined students were eager to provide support for those who were less capable; likewise, students who found math challenging welcomed the help they were getting from their friends.

From my perspective, I enjoyed the collaboration with my students. This study could not have gone the way it did if I did not pay attention to them. They provided key information on the complex emergence of learning and showed me what the possibilities and limits were. They also informed me about the type of group dynamic that could be sustained when honouring complexity in the classroom, which led to our attempts to try out the helper method and various types of group work. The road to reaching these understandings was conflicted and difficult, but my students contributed valuable suggestions so that we could actively co-create a self-motivated and self-sustaining learning circle. In his paper, Collins (2004) vividly captures this position. He writes that "in a classroom that promotes authentic participation, interactions among students and relationships throughout a stratified school structure are important...knowledge is shared, interactions are encouraged and the teacher, who is in a traditional position of authority, learns from students and actively seeks to share decision-making about various aspects of the classroom community" (p 348).

6.1.3 Unpredictability of moral and ethical issues

The academic regulation of university research requires pre-approval of study procedures that do not in breach moral and ethical violation on the rights of the participants. I respect this process and its objective to protect all participants from harm. Nevertheless, as in the case of an action research, there are many surprising events that are impossible to anticipate until they occur. The limitation to predict places greater trust and good judgment on the part of educational researchers to make sound decisions as they arise, in order to shield their participants from emotional distress.

Earlier, I touched upon the discrepancies between intentions and how events actually unfold. In one, my role as a teacher-researcher expanded to include other identities, such as a learner and colleague, that might be considered a conflict of interest in a traditional scientific research model. The blurring of the participants' roles into that of a teacher and co-researcher was another area that emerged from this classroom context, and again something that I could not have predicted before the start of the research. Flinders (1992) believes that

> all research is context-bound, and…the circumstances encountered in a given study will always interact with various ethical frameworks in unpredictable ways. Researchers must learn to 'read' ethical concerns as they emerge, anticipate

> relevant considerations, and recognize alternative points of view. In qualitative research, these skills are not marginal; they are at the heart of what we do. (p 114)

My research continually tested the boundaries of an ever-changing context; as a result, I consistently reaffirmed my responsibility as a researcher to ensure that new identities and contexts did not jeopardize my students' physical and emotional safety.

6.2 The use of action research to my awareness of emergence in the classroom

The data collection practices employed in this action research were vital to the revelation of many emergences in my classroom. The use of student surveys, video recordings, copies of student work and teacher journals combined to paint the complex mosaics that can arise in a mathematics classroom. These practices encouraged my students to self-reflect on their part in building the complexities of a learning environment. More importantly, it also forced me to participate in a similarly self-reflexive activity where I could make sense of the complexity within learning and teaching emergences as I contemplated on the events and challenges along the way. For me, writing journal entries was a welcomed release to the intense atmosphere of my study context. Reflection and journals are non-linear learning approaches that

> intermingle documentation of ideas and experiences from the past, present and 'imagined' futures. Journals need not present a logical, sequential argument but can emerge from experience, interactions and complex thought processes. There is no notion of 'right' or 'wrong' in the experiences documented by learners, and

> variation, individualization and localised experience and knowledge are embraced...Reflection becomes a tool not only for documenting and researching learning, but enhancing and stimulating it. (Phelps, 2004, p 257)

Complex phenomena are all around us, but are often dismissed. We are too busy with our daily lives that we rarely have the chance to question our being and the situations we find ourselves in, never mind seeking any type of explanation for them. Day after day, we act without question; we do without stopping. Writing journals was therefore a powerful medium for my students and me to temporarily detach from our busy schedule so that we could start inquiring into the value of viewing our world as a complex system and what we needed to do to embrace this sensibility.

6.3 My sensitivity towards students as agents of complex systems

My research looks primarily at the influences of embracing complexity - the way I perceived my students, how I honoured complexity in the classroom, the way I discerned my pedagogy, and how I comprehended the purpose of mathematics learning. In the following, I will touch upon each of these aspects in light of literature findings. The first of which will be my new sensitivity towards students as agents in a complex system.

6.3.1 Student autonomy

In retrospect, I focused too much on bringing the class together to a collective circle in neglect of the individual agents that made up this group. At the beginning of the research, I tried many approaches to group work, but none that significantly enhanced my understanding of complex systems. Although complexity theory calls forth the importance of agent interaction, it blinded me from seeing the energies embedded within the autonomy and individuality of each student in my class.

Every student in my class has a unique personality and attitude. Each brings with him/her a complex aggregate of likes, dislikes, behaviors, viewpoints, strengths, weaknesses and life goals. To utilize these skills and desires to the maximum, I needed to let 'free will' guide the way for my students. In this study, they each had opportunities to discover for himself/herself the path they would lay down in the learning process, and during which, they also respected the space of their classmates and appreciated the commonality of their overall goals in this class. In the student survey, they identified what type of learner they were, spanning anywhere from the quiet-working type to the class-clown, distracter, and the helper type. Respecting individuality in a democratic atmosphere is a strong promoter of a self-organizing, self-adaptive environment (Blackman, 2006). It did not matter to me how they categorized themselves, I was more concerned about whether or not my students were able to find a positive role that they could identity themselves with. No matter what the choice was, I knew that only the best would come when each student's strength was embraced.

6.3.2 Student collective

This unique blend of personalities determined the level of engagement that each student was willing to put in, the type of interaction each liked to have with other students and these ultimately influenced the richness of complexity that unfolded in a group dynamic. It took me many thoughtful reflections and journal entries to come to this realization. A collaborative learning environment is not something that I can create, but rather a mutual understanding that emerges from within and between me and my students.

From the students' perspective, they had to recognize the benefits of working together. They needed to see for themselves that communicating their knowledge, however big or small, added an important piece of information to the collective understanding of the class. Moreover, they had an obligation to use their unique strengths to contribute to the overall good of the learning community.

From my perspective, I needed to understand my classroom by embracing the complexities of each student, giving each of them the freedom to shine through his/her individuality. This called upon my ability to shed the guard of a traditional teacher and be inclusive towards student autonomy in creating their own collective learning space. Other factors such as a teacher's encouragement and respect for student choice and decision-making are brought up by those who have had firsthand experience with creating a collaborative learning environment (Clarke and Collins, 2007).

6.4 Reflection on my teaching practice

With my students' help, I have learned many things that will have an ongoing impact on my teaching practices. Most of all, I have a newfound respect for the teaching profession and the way I fit into that role. Now, teaching to me represents opportunities for my class to search for new meanings and to co-create connections. It means putting our heads together where each person contributes in unique ways to explore, to compose and to consume in a stimulating and visionary learning community. This is a far cry from a prevailing conception of education which presupposes teaching to be a transaction between information dissemination and intake, separating learner from knower, and both from a presumably objective body of knowledge.

6.4.1 Lesson "preparation"

Since the start of the study, I tried out various pedagogical approaches that would give me the freedom to live in a creative, collaborative space. The first thing that I changed was the constricted and limited nature of lesson plans. I still believe that a certain degree of planning is necessary; however, I felt that the setup needed to be changed drastically if I wanted to embrace complexities within a classroom.

I did not write out "lesson plans" as prescriptions. In place of such lesson plans, I spent time exploring lesson possibilities, including all the trajectories that my lesson could potentially take. The question of "What am I teaching?" is quickly replaced with "How could we learn it?" I invested more time thinking about various questions and learning activities that could instigate rich learning occasions; at the same time, I also wrote down possible mathematical concepts and connections that could emerge during the lesson. Beairsto (2001) is repulsed by the standardization of lesson planning; he urges that "the notion of 'lesson planning,' which either ignores or presupposes learner responses, is replaced by 'lesson preparation,' which provides a clear framework of intents but assumes that actual classroom activities must be adapted or even initiated on the fly according to student responses, both individual and collective" (p 6). Lesson preparations are now an important part of my job, the ability to anticipate means that I am better able to grasp surprising learning circumstances when they come up and to capitalize on my students' originality.

6.4.2 Learning "possibilities"

Up until the time of this study, I was comfortable with my routine of going over homework questions, introducing the lessons and giving out workbook assignments. However, I was willing to take risks by going outside my comfort zone, and in return, I had an adventure of a lifetime. One area that my research explored was the type of learning activities that could give my students an enriched learning experience other than what could be offered through copying notes from the overhead or doing repetitive exercises from the textbooks.

I started off by trying out activities described in past literature, particularly those that were deemed to be noteworthy exemplars provided by Davis and Sumara, who had both done extensive work on complexity theory and mathematics education. One of those activities asks students to think of as many ways as they could to make a fraction of 1 ¾. Another involved generating various representations to describe a diagram of shaded parts. These learning tasks gave me invaluable insights about some of the features that reside within a complex learning environment. These activities were easy yet difficult, consisted of redundant concepts serving as common ground, but could also amount to infinite diversities depending on what direction and how far the students wanted to go.

As I got more comfortable, I started to create my own learning tasks that were suitable to the topic and audience of my context. The cheque writing activity, the class discussion on square roots, a paper cutting project on 3-dimensional fractals and student drawings of geometric figures were such attempts. Soon after, I was able to pick up key threads that make certain activities more successful than others. First, the leading question in these tasks needed to be simple to comprehend; students at any mathematical level should be able to make a reasonable attempt at the activity. In particular, the weaker students who would normally be discouraged from participating in any sort of group work seemed motivated to put effort into these tasks because they were within their grasp of understanding. Second, solutions of these tasks were not deduced down to one answer but were induced to many possibilities. The varieties that resulted from it led to further critical discussion and knowledge (re)construction within the class community. When complexity is embraced, it opens up interpretative possibilities that the whole learning collective can actively engage in (Bowsfield et al., 2004).

6.4.3. Student assessment

Student assessment was an issue of conflict that I came across in the study. I could not come up with an adequate answer then and nor do I have anything conclusive now. The discrepancy came in the face of trying to fit new initiatives into an old system of standardization. In many ways, we are still operating in an educational framework where students are moved along pre-defined standards from one grade level to the next, and engrained in the centre of this cultural norm is a regime of testing practices. For a long time, assessments have been and are still widely used as reliable indicators for academic performance. At the end of each year, results from various student assessments are used for determining promotion to the next grade level.

I do not know how to assess for student knowledge in a classroom that honours complexity, especially when learning is often brought about in an interactive, rather than individual, fashion. It would be difficult to assess what students have learned, as it varies for each individual, and how well they understood the content, as they each make connections in significantly different ways.

Should assessment be viewed holistically to encompass a group effort? If so, what would assessment look like so that it reflects more of a collaborative rather than individual attempt? And before we go too far into this argument, should we first consider if assessment has a place in a complex educational framework? Perhaps what is suggested by Clark and Collins (2007) will provide a humble start for educators who are struggling with this dilemma. They propose that evaluation should be re-coined to reflect the changes that happened to the "individuals in the setting" as well as the "quality of the collective interactions" (p 24). In another word, evaluation needs to stay away from norms-based testing, where "the purpose of student assessment is to compare students with each other" (Reigeluth, 2004, p 15).

6.5 My newfound perspective on the learning paradigm

As a teacher, this action research altered many previous beliefs that I held about teaching. My students are now an integral body that enlivens a creative and self-organizing learning collective. My view on the purposes of 'lesson planning' has extended to include anticipation of all possible learning occasions, which has also transformed the way I design my learning activities. As a result of all these, I have a new perspective for the learning paradigm.

6.5.1 Enabling network interactions

In Davis and Simmt's paper, they identify neighbor interaction to be an important element of complex phenomenon. However, I think the phrase "network interaction' captures a more realistic dynamic that occur in the embedded systems of education. In a classroom, communication in the form of written, oral, auditory and body language spans between partners, among groups, and within the class in feedforward and feedback loops. From my study, I found that there was not one form of communication that predominated over the others. On the contrary, from the many class activities that I tried, I found that interactions digressed through all forms and shapes. Learning tasks were relatively dense in content that they could carry through many forms of communication without losing the students' natural drive and interests, but seemed to pick up energy as they progressed through neighbor, small group and class interaction.

6.5.2 Enabling interaction of ideas

In the past, educators put a lot of effort into studying the best types of classroom structures and student setups that could enhance learning outcomes. Advice such as arranging tables in horseshoe shapes or putting students in groups of three's when they are working on projects are, to an extent, false idealizations that assume higher learning can be brought about from a particular arrangement of furniture or student groupings. I agree that thoughtful planning is needed around a classroom's physical space, but I believe more effort should be spent on establishing a supportive network that could encourage interaction of mathematical ideas. Group work, class projects and sitting in pairs are simply not enough to support the emergence of complexity within a classroom environment.

From what I observed, precursors to complexities of learning came from two things. One, learning tasks need to have enough substance to sustain conversation, saturated with endless opportunities to re-define the known and to explore the unknown. For example, our class discussion on the possibility of having "over 100%" covered these prospects. Two, ideas from learning tasks need to be openly shared in a class. This includes celebrating authentic understanding and original ways of making sense of phenomena. As individual work intermeshes with other ideas, critical analysis and knowledge reconstruction in the form of an open group discussion will inevitably push both individual and class understanding to new heights. What is noteworthy here is that even more important than student interaction, ideas must be deliberately bumped up against one another, molded, reshaped, and stretched beyond their usual boundaries (Bowsfield et al, 2004). These resonate with my conception of a classroom where complexities in "learning reverberate throughout the room and bounce back altered, changed and sometimes amplified" (Clarke and Collins, 2007, p 16). In my classes, students had ample opportunities to discuss viewpoints and thoughts in small groups and class discussions, which very often were places that provided valuable learning experiences.

6.5.3 Enabling learning beyond the lesson plan

Learning in the context of a complex environment also means welcoming occasions that might take learning beyond what is stated on the lesson plan. This by no means includes getting side tracked or going off on a topic that has no relation to the learning outcomes. It is about allowing students to draw upon past and present resources coupled with the imaginative future to generate learning experiences that are fuller and far more descriptive than anything that could be deciphered or gleaned from a curriculum guide or lesson plan.

In mathematics learning, awareness about learning as a complex phenomenon engenders the possibility for any classroom collective to dig deeper and wider than the intended learning outcomes. It encourages students to relate to prior experiences, to make sense of present knowledge and to be prepared to redefine these understanding in future interpretations. From the class discussions we had, students always attempted to link a specific idea to other mathematical concepts and relationships. In another example, my intention to introduce a particular 3-dimensional fractal ended instead in many variations of artistic structures, which deepened the students' appreciation for non-Euclidean designs. Through all, a teacher has very little control over the course of action because it is largely determined by the enthusiasm and capability of the student group. Regardless of the extent by which a classroom community is able to "address concepts and issues that reach far beyond the objectives outlined in their unit plans", enabling student interaction "removes the teacher from the position of authority, and facilities the productive interaction, rather than conformity, of ideas" (Bowsfield, 2004, p 17). This perspective supports a shift in thinking about lesson planning less of a plan and more as preparation for possible learning pathways.

6.5.4 Enabling learning that harmonizes different perspectives

The education field is a myriad of different, sometimes even contradicting, values. Beginning with learning styles, we have at least visual, auditory, and tactile learners. There is a more extensive list as one looks into learning theories. Gardner's (1993) Multiple Intelligences Theory reasons that the learning process is a combination of varying degrees of visual/spatial, verbal/linguistic, logical/mathematical, bodily/kinesthetic, musical/rhythmic, interpersonal and intrapersonal intelligences. Miller's (1956) Information Processing Theory views knowledge as information "chunks" that are cycled between operations and testing. Rogers (1969) believes that only a learner's will and self-initiation can result in meaningful learning, forming the basis of Experiential Learning. Yet, in the Theory of Genetic Epistemology, Piaget (1969) defines learning as a systematic adaptation of cognitive developments. Bruner (1966), on the other hand, believes that learning takes place when schemas and mental models are constructed from the learners' current and past experiences. As if all these are not confusing enough, teachers are simultaneously exposed to different teaching approaches such as student-led, teacher-directed, and peer group mediation, with an array of activities like seatwork, group work, concept mapping, debates, projects, plays, drawings, experiments and simulations.

This is similar to stepping up to a buffet. In education, choices are vast and endless. However, it is not in our interest to start eliminating things down to a one size fits all approach. As chaotic as it seems, I think the aforementioned ideas, and many more than I could list here, are from their particular perspectives believed to represent best practices. As a teacher, I am obligated to strike a balance that can encompass a healthy variety of these ideologies that are consistent with my teaching philosophy and the needs of my student collective.

The key is to embrace varieties in the choices so that together there is consistency and coherency; this is precisely what I am able to walk away with from studying the complexities of my own classroom. Within the mess, I crafted my own ideology about learning and teaching that supports my sensibility of complex phenomena, and it is embroidered with my appreciation for various learning styles, teaching methods, classroom activities, and student interaction. Any of these approaches on their own might not honour learning as a complex phenomenon, but integrating them together will surely lead to deeper and more meaningful learning collaborations.

6.5.5 Enabling life-long learning

What would you say is the real purpose of education? Many would agree that education "provide learners with the means of associating ideas with other events of their lives by pointing to various aspects of the world in a deliberate attempt to foster different habits of perception/interpretation" (Davis, Sumara and Luce-Kapler, 2000, p 26). In math education, we want students to "put themselves in position of authors of ideas" when they "talk about mathematics, reasoning and mathematical arguments" (Lampert, 1990, p 34). Learning, then, is a personal, analytical, subjective, value-laden, and sensorial experience with no clearly defined beginning and ending. For educators, teaching is about "effecting transformations, and where the effects of teaching linger. Like all of life's events, one's experiences with a teacher are incorporated into one's evolving frame in complex, unruly, unpredictable ways" (Davis, Sumara and Luce-Kapler, 2000, p 44 and 178).

Hardly anyone would argue that the sole purpose of schooling is for students to memorize facts from the textbooks. But no matter what intentions teachers have, the truth rings in that most classrooms do not stray far from activities like copying, memorization and regurgitation. Teachers are experts when it comes to encouraging short-term memory retention, asking that students retain the content long enough until chapter tests and final exams. We envision, with high hopes, that students will learn for life, but often we lack the pragmatic approach to what we can do as educators to instill the habit of life-long learning.

In this action research project, I cultivated a newfound awareness for who I am, who my students are, and the ever-evolving relationships each of us have with our environment and one another. It is a bodily and spiritual restorative healing that puts my whole being in close connection within the webs of our ecological sphere. At the heart of this is my sensitivity and appreciation for thinking about learning as a complex phenomenon. My humble understanding on this topic is already having a big impact on my view of learning; for the first time in my life, and hopefully for my students as well, we acknowledge an open invitation to the continual co-creation and co-construction of knowledge because learning, living, and being are life-long processes.

Complexity theory provides the theoretical ground, which undergirds our understanding, and the practical approaches, which guides our action, to learning. I no longer see life-long learning as something too abstract to comprehend. Though it has often been pushed to the periphery of educational concerns and considered to be too difficult to accomplish; now, it becomes clearer and even simpler to undertake. All it takes is a more insightful perspective on how we come to associate ourselves with our environment, both living and non-living. A change in attitude about the interrelationships of complex world dynamics will generate similar changes to our view of learning. Self-initiative to participate in ongoing knowledge construction comes at one's realization that we are embedded in an universe that is "rich enough to produce not just a few complex forms, but complexity in such quantity that our entire planet is just one insignificant part of it" (Stewart, 2001, p 214).

Every moment, we are drawn into, and are part of, numerous accounts of transformation across the physical, emotional, and spiritual world. These changes purport that we exist in a complex upbringing that unfortunately do not always honor boundaries of people, objects, places, information and time. Life-long learning is therefore a survival skill that merits human necessity to learn and relearn about ourselves in face of constant changes in the environment.

6.6 Framing the research process in a different light

My participation in this study has been an emergent experience of a complex system at work. I intercepted an overly ordered classroom with questions, skepticism and wonder; these interruptions put learning, teaching, and classroom in a dynamic state as they unraveled between orderly structures and chaotic rumblings. Action research provided a critical method for me to study complex phenomena while at the same time that I engaged my own soul searching in a complex unfolding. The "action" piece and self-reflexive nature of this study helped me tremendously in understanding what it means to engage in the many complexities of myself as a human being.

6.6.1 Research at the edge of chaos

For me, this research speaks to how pertinent it is for educators to interrogate norms and daily practices. Critical interrogation invigorates reexamination of beliefs and actions that we often take for granted, and too much of that is already crowding the space of education. High demands and heavy workload leave teachers with no time and energy to do anything viewed as optimal, little do they know that good teachers prioritize consistent reflection and learning at top of the list.

For the past three years, I have been taking graduate courses in addition to my full-time teaching obligations. I admit that balancing my schedule to make room for attending classes, reading literature, and doing assignments has been difficult; nevertheless, what I have already gained is insurmountably more than the sacrifices that I have had to make. Our weekly classes are powerful congregations where ideas, thoughts, and concerns are shared amongst a diverse group of educators who teach different age groups and subject areas at various schools and universities. Professional development at the district and provincial level is another place where perspectives converge and diverge in different directions. Recently, I attended a conference on *Me to We: Educating Creative, Compassionate and Committed Young Leaders*. I was moved to tears by stories of young people around the world making remarkable breakthroughs in children's education. The work of these people is reminiscent of ideas from complexity theory; they showed me just how contagious a project can be when individuals come together and work passionately and tirelessly on a common cause. Complexity theory makes us see that we live in a global community; problems halfway around the world hit close to home and they become our problems. However, it also reminds us that much can be done in our immediate surroundings. Everyday at work, I am exposed to plentiful opportunities to take part in intense conversations and learning occasions with my students and colleagues (Appendix I).

The world is ready for us; many structural networks are already in place so that teachers can reflect on past practices and learn to grow with new understandings about our world. It is unfortunate that many of us do not explore these alternatives because we are easily contented with what we have and accept what is before us. However, I want to take one step back, preceding any type of reflection and learning is the teacher's willingness to step towards the edge of chaos. I was challenged to step out of my comfort zone and to throw myself into the vulnerabilities of unpredictability, of change and non-linearity. Throughout the study, I was inspired to find new meanings to my work by identifying faults and loopholes in the system, and from then on, included other vested individuals in the problem-solving process to find new attractors in our dynamical world.

6.6.2 Time as a one way variable

Time has a special place in my research; it emerges as an attractor across many contexts. Especially in cases of complex phenomena, I came to the realization that time needs to be treated as a one-way variable (Haggis, 2005; Sumara & Davis, 1997). That is, complex systems proceed in a self-propelling motion and they need to be looked upon as unfolding events. One can study the past as they certainly influence and add meaning to present conditions; nevertheless, they do not and cannot accurately predict what could happen at any other time or in any other context. We cannot repeat history, and history cannot repeat itself.

As I am writing this paper, what I understand now is different from what I understood at the time of my action research project. Similarly, what I know now about complexity theory will be altered and reshaped in the future. It is important to note that we ponder on past events, at times when we reflect, and we think laterally about alternatives, at times when we struggle, but these occur in a bigger picture where we are always moving forward in time.

Many aspects of my research were sensitive to time. On a smaller scale, class activities and classroom dynamics incessantly evolved as my students and I co-constructed our "realities" of complex learning occasions. We did not repeat the same thing until it was perfected; rather, we strived for near-perfection because we were moving forward. On a bigger scale, the opportunity to embrace complexity in my classes spilled over to other realms of my realities. Since the study, this understanding has helped me come to terms with the relationships that I have with my colleagues and the parents/partners of my school community. It has also kept me abreast of the kind of leadership I need to sustain as a department head so that other teachers can step up and make a difference on what goes on around the school. I have been working hard to establish a "true dialogue in which people engage with each other, not to be in control but to provoke and be provoked, to learn and contribute to the learning of others, to change their own minds as well as the minds of others" (Stacey, 1996, p 280).

6.7 Difficulties of encompassing a complexity viewpoint in education

I hope that schools, including universities, will move to a position that is informed by complexity theory. What we are doing now shields us from a reality that is filled with unpredictability and chaos. We are blinded by how complex our classrooms, schools, learning, teaching and relationships are, to a point where do not even know how things really behave because we have built our lives around rules, boundaries, procedures and structures. I am afraid that the more comfortable we are with linearity and control, the further we will be from a diverse, electrifying, and integrated community.

The time has come for a reform in education, and this urgency calls forth our immediate action. At present, our "educational system is reactionary, oriented to past values and past technologies...it is a dying and outdated system founded on fragmented and classified data" (McLuhan, 1995, p 249). An educational reform is a complex system in itself, its success begs for everyone's commitment, ranging from students to teachers, administrators, university professors, parents, and the wider community. This action research is a small but a significant part of my contribution to a desperate change on learning and teaching in the 21st century.

The road ahead is bumpy, throwing us unexpected sharp curves when we least expect it. Any type of reform will be arduous; it is important that we do not give up and revert to old ways of doing things. Just as my own research process, there were challenges after challenges, but my determination and perseverance to stay in the journey landed me at a better place than where I started.

Reigeluth (2004) identifies two components that are needed to result in productive changes in the educational system. The first thing is that there "must be sufficient impetus for transformation" (Reigeluth, 2004, p 12). Teachers need to be educated about the significance of our work as contextualized within complex systems, and inevitably how an understanding of this can improve our teaching practices and adopt a more meaningful view of the learning paradigm. This has to come from local actions instead of being an imperative imposed from the top down (David-Gnahoui, 2001). The second is that there "must be sufficient enablers of transformation" (Reigeluth, 2004, p 12). I am proud of what I have done in my own classroom, but I also realize how minute the achievement is. To feel the effect, ideas from complexity theory have to ripple out to other classes, schools, districts, societies and our global village. A successful reform starts from the collective support of a diverse group of individuals.

6.7.1 Need for a new language

Besides the two points mentioned earlier, I also believe that the use of language is something that education reformers have to address. Language speaks volumes to the way we communicate, read, write, understand, and make sense of our world. I feel that there is a need to develop new vocabularies to explain what I was trying to do in my own research. There were times during my study where I felt trapped. I was not trapped by ideas of complexity but was suppressed by having to use an old language to describe a dynamically different phenomenon.

It was not until later in the research that I realized how value-laden our every-day language is towards conforming to linearity. In any given day, we use many words that subtly remind us of this boundary – such as rule of thumb, time line, standard of living, right and wrong (Davis and Sumara, 2005).

Rorty advocates that "changes to habits of thinking demand changes to habits of speaking. New language must be developed if established conceptions are to be interrupted…attentions are thus prompted toward the usually-not-noticed aspects of language and other interpretation practices that support and constrain meanings and perceptions" (Davis and Sumara, 2005, p 306 and 307). In my study, I contested my own use of language that was salient within an Euclidean world and developed new phrases that could describe defining features of complex systems. I stopped using phrases such as "lesson planning" and "classroom management" to make room for new vocabularies like "lesson preparation" and "classroom harmony". Though this is a small change, it unleashed the freedom of thought and the freedom of action to fully live in a complex environment. More importantly, it allowed me to freely embrace the proliferation of new ideas that emerged from the power of a new language.

6.8 Future questions

There are still many unanswered questions about the thinking of classrooms and learning as complex phenomena. More work is needed to take us beyond the theories and into the action around teaching practices and the learning process. What are the continuing challenges and struggles of teaching with this sensibility? What are some common features that characterize the complexity of learning and learning tasks? Little is also known about the type of relationship we need to keep with one another in order to sustain a classroom environment of distributed leadership and control. How do we transform the hierarchal structures of our organizations to encompass a more lateral rather than vertical development? What needs to happen so that students and teachers recognize the pivotal role they each play in the success of the collective? These questions have great implications for the existing relationships we have with our students and colleagues, and on the way we work together as a team towards an integrated and respectful community.

Future questions also have to address various dimensions of our educational system. At the school level, we need to study how a complexity perspective invests in a deeper understanding of mathematics and classroom dynamics that would result in more fruitful purposes of teaching and learning. On a holistic scale, we need to study the means in which we involve the whole community to embrace in complexity thinking because only that can give us the impetus for changing our current view of education.

As we look towards future research to provide new directions, it is important that we do not look at them as external truths. Truths about complex phenomena are sensitive to time, context and people. We should frame future studies as our continual effort to participate in a collective practice of story sharing. In a complex system like education, there are no objective truths, only constructed truths as understood from the embedded relationships amongst people, places and knowledge.

6.9 Direction for practicing teachers

I hope that this action research project can be an inspiration for other educators. More importantly, I want my story to be another critical piece that can instigate and encourage practicing teachers to question and to reflect on their current view of education. As we take this new understanding back to our classrooms, I would like to leave my colleagues with some direction.

Ideas on pedagogy:

- The intent of lesson preparation is to embrace all potential learning opportunities.

Ideas on learning tasks:

- Learning activities
  - need to be manageable.
  - need to be intellectually challenging to all types of learners.
  - can instigate the space for further discussion and concept re-exploration.

Ideas on learning:

- Student interaction and class collaboration allow ideas to be reshaped and molded in light of new understanding.
- In honoring a complexity sensibility, each student is encouraged to make their own connection with the learning object. As well, everyone is invited to engage in the collective unfolding of learning experiences.
- Ideas from complexity theory provide a workable framework for fostering life-long learning skills.

Ideas on research:

- Research cannot exist on its own; it interplays with learning, teaching, classroom, curriculum content, students, teachers and the relationship embedded within them.

Ideas on education:

- New language and new vocabularies need to be established to describe this new sensibility.
- Adopting a stance that is informed by complexity theory has to start from the individual, which demands his enthusiasm and awareness for who he is within the

embedded systems of our universe, and across all systems of our educational paradigm.

6.10 Conclusion – My own embracing

Before I began this research, I thought that I would come to some sort of closure about teaching with a complexity sensibility. I thought that I would have something conclusive to say about my struggles and on what I have learned. Far from this, I did not bring to closure any particular ideas, but I have opened up ideas which will continue to carry with me as I continue my journey.

To the limit of what I am able to describe in this paper, I am only giving readers "my" reality of a certain experience in time, but it is a reality that will keep on reshaping as my learning community lives through and learns from being at the edge of chaos.

Complexity theory believes that a small shift in perspective can lead to astounding results. This is what I experienced in my action research; embracing views of complexity opened up a world that I never knew existed and gave me an invaluable learning experience on teaching, learning, and relationships. For my concluding thoughts, I want to share with you a few defining insights that I had during the research.

> "Classroom that embraces the sensibilities of complexity theory needs a lot of revamping and adjustments, especially in changing students' beliefs about what comprises as rich learning experience."
> (Teacher journals, June 6, 2006)

> "Education embeds within overlapping layers of complex systems. In all, each is a complex system that is closely connected with one another, they cannot be analyzed as separate entities without realizing that each affects and is being affected by all the other complex systems within education."
> (Teacher journals, May 17, 2006)

"Now, math for me is about making connections. It is about learning with other people in the group. It is about bringing up prior understanding and constructing new knowledge with one another. It is about appreciating and seeing mathematics in different lights. I'm welcoming variations, even when variations are going in different trajectories. Instead of molding individual students in my classes; now, I'm trying hard to build a community of learners."
(Teacher journals, May 3, 2006)

"My students see how neat it is to learn from each other, to discuss ideas, and to see what others are thinking."
(Teacher journals, June 6, 2006)

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to submit copies of their written work and to agree to being videotaped for no more than 15 selected lessons of the school term. In addition, participants will be invited to share their thoughts and experiences about this course in a 1-hour videotaped group discussion. Participants are not anticipated to commit to more than 1 hour outside of scheduled class time to participate in this project. All data collection procedures will be done without harm to your physical and emotional well-being and the teacher-researcher will be mindful about carrying out good teaching practices.

Ethical Concerns:
All lessons in this Math 8 class will be taught in the same way to all students; you will all be given equal opportunities to benefit from learning mathematics in a community-like classroom environment. The only difference is in the participant's willingness to submit copies of written work and to be within range of the video camera. Participation in this study is voluntary and you may decide to withdraw from it at any time. Regardless of your decision to participate in or to withdraw from this study, it will not affect the amount of time and the quality of instruction that you will receive; it will most certainly not affect your grades. Data from student work, group discussions and video recordings will only be collected and used from those who have given both student and parental informed consent to participate in the study. For non-participants, student work such as class assignments, concept maps, journals, concept maps and portfolios will not be used in this study. As well, non-participants will not be captured on video during the taping sessions of selected lessons.

Confidentiality:
All data used for this study will be safely stored in a locked cabinet, and computer files will be protected by a password. Participants and their parents will have access to their own data should they feel a need to review and/or to refuse the use of any personal information. Data will not be analyzed in depth until the math course has ended and when final marks have been submitted. Research data will be used, shared, and presented among members of this project team, educators and graduate students. This is important as we believe this study can be used to improve teaching practices in the education community. Results from this project will lead to a published document; however, data will be kept confidential where individual students and the participating school will not be identified. In addition, student responses on group discussions and written work will be identified using pseudonyms. All written and video documents used in the study will be kept locked up for a period of 5 years, after which time they will be destroyed.

----Please detach consent form and return this to the school counsellor----

STUDENT INFORMED CONSENT FORM

THE POTENTIAL OF USING COMPLEXITY THEORY IN MATHEMATICS LEARNING: EXPLORING THE INFLUENCES OF A PRACTICAL FRAMEWORK ON LEARNING AND PEDAGOGY

Your participation in the study is entirely voluntary and you may withdraw from the study at any time without jeopardy to your class standing.

Your signature below indicates that you have received a copy of this consent form for your records.

Please put a check mark beside your indicated decision:

☐ I consent (agree) to participate in this study.
☐ I do not consent (do not agree) to participate in this study.

______________________________ __________________
Student Signature Date

____
Printed Name of the Student signing above

to submit copies of their written work and to agree to being videotaped for no more than 15 selected lessons of the school term. In addition, participants will be invited to share their thoughts and experiences about this course in a 1-hour videotaped group discussion. Participants are not anticipated to commit to more than 1 hour outside of scheduled class time to participate in this project. All data collection procedures will be done without harm to students' physical and emotional well-being and the teacher-researcher will be mindful about carrying out good teaching practices.

Ethical Concerns:
All lessons in this Math 8 class will be taught in the same way to all students; they will all be given equal opportunities to benefit from learning mathematics in a community-like classroom environment. The only difference is in the participant's willingness to submit copies of written work and to be within range of the video camera. Participation in this study is voluntary and your child may decide to withdraw from it at any time. Regardless of your decision to let your child participate in or to withdraw from this study, it will not affect the amount of time and the quality of instruction that your child will receive; it will most certainly not affect your child's grades. Data from student work, group discussions and video recordings will only be collected and used from those who have given both student and parental informed consent to participate in the study. For non-participants, student work such as class assignments, concept maps, journals, concept maps and portfolios will not be used in this study. As well, non-participants will not be captured on video during the taping sessions of selected lessons.

Confidentiality:
All data used for this study will be safely stored in a locked cabinet, and computer files will be protected by a password. Participants and their parents will have access to their own data should they feel a need to review and/or to refuse the use of any personal information. Data will not be analyzed in depth until the math course has ended and when final marks have been submitted. Research data will be used, shared, and presented among members of this project team, educators and graduate students. This is important as we believe this study can be used to improve teaching practices in the education community. Results from this project will lead to a published document; however, data will be kept confidential where individual students and the participating school will not be identified. In addition, student responses on group discussions and written work will be identified using pseudonyms. All written and video documents used in the study will be kept locked up for a period of 5 years, after which time they will be destroyed.

----**Please detach consent form and return this to the school counsellor**----

PARENT INFORMED CONSENT FORM

THE POTENTIAL OF USING COMPLEXITY THEORY IN MATHEMATICS LEARNING: EXPLORING THE INFLUENCES OF A PRACTICAL FRAMEWORK ON LEARNING AND PEDAGOGY

Your child's participation in the study is entirely voluntary and you may withdraw your child from the study at any time without jeopardy to his/her class standing.

Your signature below indicates that you have received a copy of this consent form for your records.

Please put a check mark beside your indicated decision:

☐ I consent my child to participate in this study.
☐ I do not consent my child to participate in this study.

____
Printed name of the student

____________________________________ ____________________
Parent or Guardian Signature Date

____
Printed Name of the Parent or Guardian signing above

APPENDIX D: Sample notes from teacher observation

April 13, 2006

Observation on complicated math problems (vs complex activities)
"If the total price, including 7% GST, of a shirt is $85.60. What is the original price of the shirt before GST is added?

- the activity is complicated to do, accomplishing it would require less of bringing up prior knowledge but more of reasoning and application
- more discussion among members
- more confusion and frustration among students
- the students are more inclined to check their answers with me to confirm whether they have the right answer
- students gave me all kinds of weird solutions, like dividing and multiplying the percentages.
- Higher academic students like to use formulas and equations, giving me a very sophisticated solution; while lower academic students use the calculator and relied on methods of guessing and testing
- It was the lower academic students who were able to give me the answer.
- One of the more capable student said, "Who could you get it when I didn't?"
- It was good that some students realize the importance of learning from each other, and not be stigmatized by an overall perception that someone getting C's and F's are not capable to do math.
- Class discussion was difficult to conduct because their solutions were very different and many were wrong. Yet, to avoid turning the discussion into a teacher-led lecture, I had to bite my tongue until someone gave me the right answer

APPENDIX E: Samples of student responses to surveys

Student data – Journal writing May 19, 2006

1) **What role do you play in how your classmates learn mathematics?**

Student 1: I am the one who everyone turns to for the answer if no one else knows the solution. I may not have the highest mark but I see myself as the smartest in the class.

Student 2: The role I play is that I sometimes help people who need help. Another role I play is I do my work and don't bother people.

Student 3: I have no important role except help my two unsmart friends and no names will be told.

Student 4: In math class, I play the role of a distraction.

Student 5: I am the one who asked for help. My role is to help when needed and not disrupt the class.

Student 6: The role that I play in this class is helping everyone as much as I can when help is needed. I try to be a good mediator as well.

2) **What role do your classmates play in how you learn mathematics?**

Student 1: Looking to me for answers.

Student 2: The role of my classmates are helpers. They help me when I need help!

Student 3: They give answers, explain questions and just be there for you.

Student 4: I don't know.

Student 5: no comment written

Student 6: My classmates play the role of "helper" when I need help too!

APPENDIX F: Samples of teacher's journal entries

Journal Entry #15 March 2, 2006
Topic: Making connections in class discussion

I'm starting to enjoy the class sharing sessions with my students. I get excited about the varieties in their ideas. I'm trying to go beyond individual understanding to see the benefits of group understanding. Group understandings; such as concepts that are shared as a class, are often more in numbers, in breadth and in depth. Two brains are better than one.

But I'm also finding that group sharing is only the beginning. Although it is important to put student ideas on the board, learning that results from it is still pretty static. It is merely a convenient way to see all the ideas in one place. I'm trying to go beyond that by making connections to the ideas on the board. I want group understanding to reach a higher level, a collective understanding.

As a teacher, someone who is endowed with more content knowledge than my student, it is easy for me to point out the connections among ideas on the board. During the class discussion, I've been able to introduce so many other ideas from the answers that my students have shared with me. For example, when I asked students to write down examples of fractions, I was able to make connection to equivalent fractions, mixed fractions, proper fractions, improper fractions and so forth. I can't help but to make connections myself. I am a part of this classroom, and it is becoming natural for me to respond to my students, instead of the content. But I also have to give ownership to my students to make their own connections and to create their own understanding from comparing and contrasting their neighbors' ideas against their own.

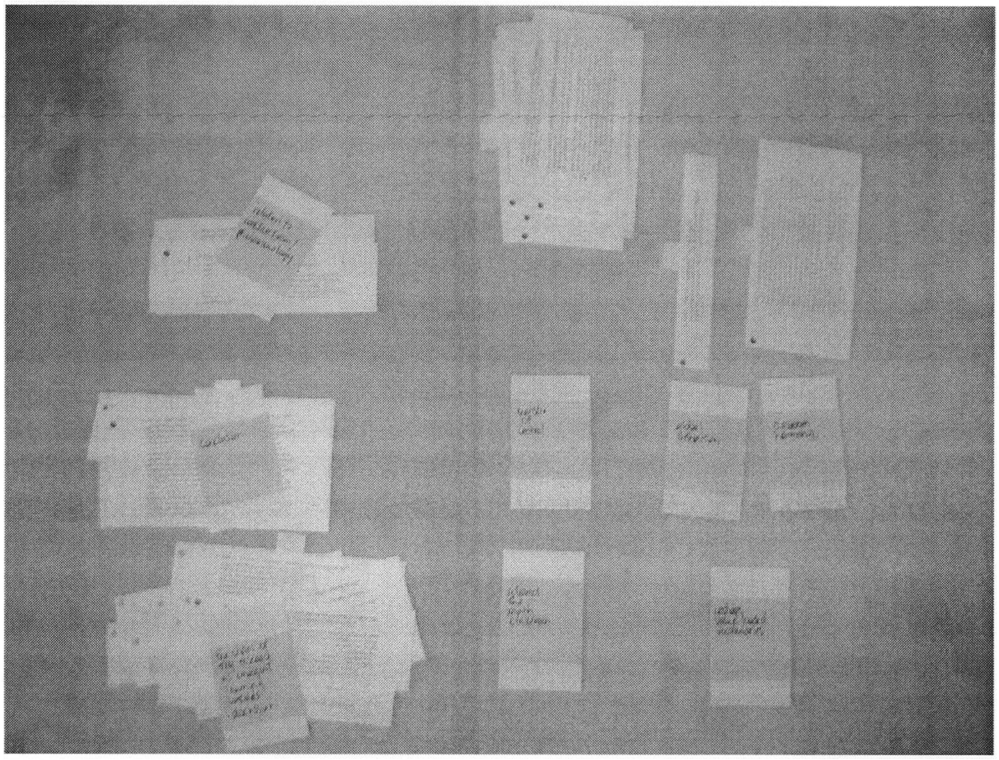
Journal Entry # 16 March 3, 2006
Reflection: I'm learning with the student

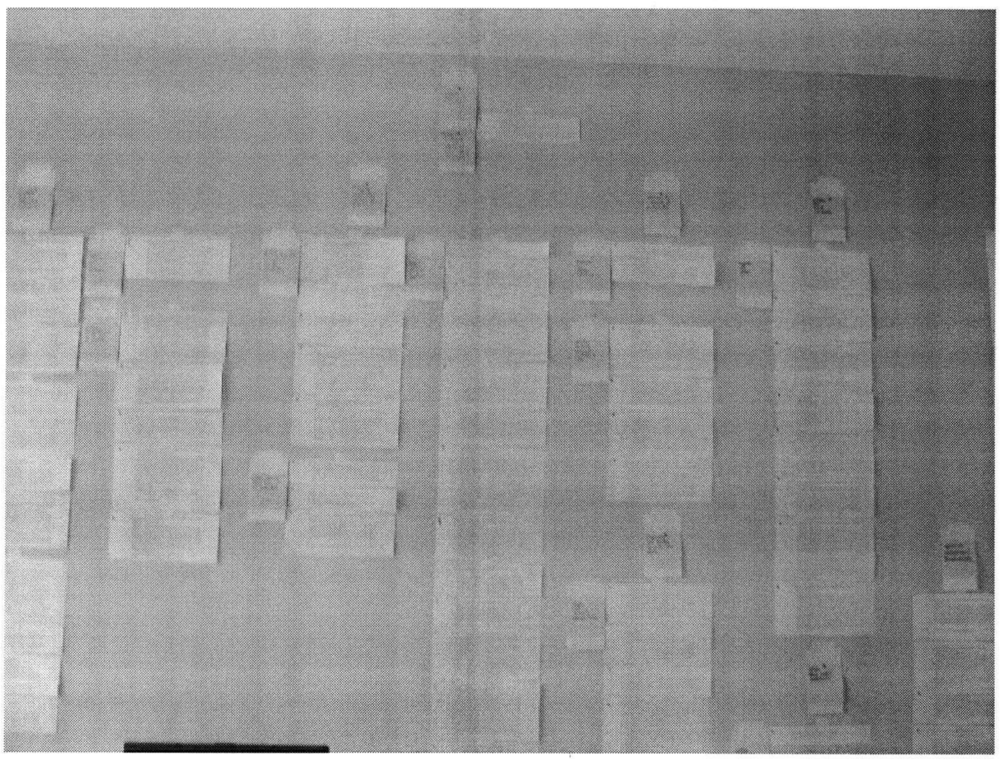
How do I put myself into the "learning system"? How do I become a part of this complex system? Well, it is not something that I can enforce. Rather, it is one of those spontaneous occasions that turn out to be both teachable and learning moments for me as a teacher.

Today's lesson was on dividing fractions. I put students into their focus groups and asked them to illustrate their solutions to "separate 6 into 2 equal parts", "separate 1 ¾ into 2 parts", "separate 6 into ¼ parts", and "separate 1 ¾ into ½ parts". Most groups had no trouble coming up with a correct explanation for the first two questions. For the third problem, some students interpreted it as 6 / 4. I persisted in not giving out the answer right away. Because I had anticipated that my students would have trouble with fractions, I already have in mind a method that I can use to facilitate, but not to teach, the understanding. I started with drawing out 6 circles on the board, and we looked at how we could divide 6 circles into 3's to get 2, then into 2's to get 3, and finally into 1's to get 6. I asked students to look at the patterns on the board. From the trends, students were able to tell me that when they divide 6 circles into smaller parts, the answer would be bigger. I asked someone to come up to the board to show me how they could divide 6 into ½ . The student separated each circle into halves, and from there, we all saw that it generated 12 sections. Using the same reasoning, we proceeded to dividing the circles into 1/3, and finally into ¼.

The knowledge was by no means enlightening. However, I was struck by how together and unison that piece of knowledge came about. It was something we did together as a class, going from a state of confusion and together we progressed to a state of understanding. I also didn't intend the solution to come out the way it did. Originally, I had another explanation in mind. For me, separating 6 into ¼ parts means that 6 represents a quarter, and we needed to add enough circles to the diagram until we can see 6 circles being ¼ of something bigger. However, this is my explanation, I'm so glad that I did not let my preconceptions interfered with those of my students. I was willing to be part of their "learning circle", willing to learn from them. It turns out that they taught me a different and an easier way to think about the problem.

APPENDIX G: Data analysis – dissemination of critical themes





APPENDIX H: Sample of a lesson plan

Course: Math 8
Lesson Topic: numeration
Lesson Concept: to learn the concepts of scientific notation

Teacher activities:	**Student activities:**	**Possible learning concepts**
• Teacher will go through homework difficulties on the board	1) review homework questions	• Expected: students will get an opportunity to clarify their misunderstanding. • Unexpected: students might be able to make new connections through the explanation
Internal Diversities/ Redundancy	**Organized Randomness / Decentralized Control**	**Neighbor Interaction**
• Redundancy: students have established an adequate level of understanding to make sense of the review session. • Diversity: differences in student understanding will determine the easiness with which student can follow the review	• Control: students are listening to explanations on the boards • Randomness: student contributions during the review session will determine the path of the discussion and level of understanding	• Possibility of student contribution

Teacher activities:	**Student activities:**	**Possible learning concepts**
• Teacher will facilitate student discussion on the application and understanding of scientific notation	2) In focus groups, students will try to fill out the biggest number they can possibly think of on a blank cheque. Then, also fill out the smallest positive number they can think of on a blank cheque. - class discussion – to make connections to scientific notation	• Expected: to apply rules of scientific notation. • Unexpected: to make connections to the patterns that appear in scientific notation
Internal Diversities/ Redundancy	**Organized Randomness / Decentralized Control**	**Neighbor Interaction**
• Redundancy: students will have preliminary understanding in word problems and powers • Diversity: students' comfort level with scientific notation will determine the difficulty and variations they can come up with for the problem	• Control: students will work in focus groups to find different ways to figure out the application of powers and scientific notation . • Randomness: variations will occur during student discussion. They will benefit from listening to each other's approach to the problem.	• Focus group • Class discussion

Teacher activities:	Student activities:	Possible learning concepts
• Teacher will clarify and explain the concepts in more detail	3) student notes – on the rules of scientific notation	• Expected: students' understanding on exponents will be reinforced during the note taking session
Internal Diversities/ Redundancy	**Organized Randomness / Decentralized Control**	**Neighbor Interaction**
• Redundancy: students have a basic level of understanding on scientific notation from the focus group and class discussion activity • Diversity: students' prior knowledge in scientific notation will determine the connections they make with the concepts introduced	• Control: students will take notes from the overhead • Randomness: student might ask questions or raise points that are significant to the development of their understanding and logic	

Teacher activities:	**Student activities:**	**Possible learning concepts**
• Teacher will facilitate student while they are working on the assignment	4) student assignment – worksheet on scientific notation	• Expected: students will reinforce their understanding on scientific notation
Internal Diversities/ Redundancy	**Organized Randomness / Decentralized Control**	**Neighbor Interaction**
• Redundancy: students will use their knowledge from the class discussion and notes to complete the assignment • Diversity: differences in student understanding on scientific notation will determine the easiness in which they find their homework assignment	• Control: students will work on the assignment • Randomness: students are encouraged to consult with the helper when they do not understand the concepts	• Interaction with helper

APPENDIX I: Outside triggers

Conversation with colleagues June, 2006

Once a teacher told me that grade 8's do not know what they want, that I have to be tough and throw in the rules. Grade 8's are often described as having low self-discipline, rowdy, loud, talkative, unfocused...

Maybe this is why students are so well behaved when they get to grade 12, because they have been accustomed to the rules and disciplines and realize the consequences from a lack of it. However, we are also creating a culture of very studious but independent learners. Senior students rarely get a chance to mingle, talk and learn from one another, at least not in the same interactive and cohesive manner that they are allowed in elementary schools. This interaction is even less when they get to universities and colleges, creating a very isolated society.

Conversation with colleagues February 13 – March 10, 2006

- Embracing teaching as a complex phenomenon can sometimes be difficult. Most teachers agree with the benefits behind complexity theory; however, the difficulty in the delivery of it often deters teachers from doing so.
    - A feeling that the teacher is not in control
    - It's difficult to recognize teachable moments when they come up
    - A tendency for teachers to be too quick to give out the answers
    - A fear that students will not give you the answers that you are looking for
    - A fear that because you're letting students to construct their own meaning, that they would not be able to handle the task
    - It's difficult to control students when they participate in group work
    - The implementation of activities that honor complexity theory is difficult
    - Time consuming to come up with activities with elements of complexity theory
    - The teacher needs to be knowledgeable in the subject matter
    - Have to deal with students who cannot get along with one another
    - Have to deal with students who cannot stay on task because they are working with close friends

- Class activities such as class discussion and group work provide a way for students to learn other than taking notes. For grade 8 students, this variation can also be a good tactic for "classroom management" because they get a chance to release their energies in a 1 hour and 20 minutes lesson. These activities can keep them engaged longer.

- "glass ceiling?" – activities should be implemented such that each student in the class is being challenged. Each student, no matter what his or her academic level is, should be able to do the activity but still feel that task at hand is challenging.

- We want to create "Math Bliss"
    - Students need to feel that they are making contribution to the class and that they are accomplishing a personal goal
    - Students want to learn but at the same thing need to have the feeling that they can handle the material

- When complex systems are honored, as a teacher you will have the gratification that you have enabled your students to learn meaningfully. A teacher is not there to lecture, he/she should provide students with learning opportunities that they cannot get from the textbook.