From Mathematics Learner to Mathematics Teacher: Preservice Teachers' Growth of Understanding of Teaching and Learning Mathematics

by

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Abstract

This study was designed to determine whether using the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding as a model enhanced the growth of preservice teachers' understanding of teaching and learning mathematics. The study also investigated the efficacy of using the theory as a framework with which to analyze that growth.

The study charts the growth of understanding of four preservice, secondary, mathematics teachers during the integrated portion of their teacher education program during which they were introduced to the Pirie-Kieren theory and encouraged to consider it during their reflections on their learning. This portion of the program concentrated on mathematics teaching and learning and was designed to help them develop an understanding and practice of teaching and learning which reflects the present-day conceptions of mathematics and mathematics education as dynamic processes. The Pirie-Kieren Dynamical theory was modeled for the preservice teachers because of its alignment with these views.

Video data on the four preservice teachers was collected during the integrated portion of their teacher education program and during their practicum experience. Analysis of this data resulted in a portrait of each of the four individuals. These portraits were then used as the data for the analysis of developing understanding of teaching and learning mathematics. The definitions of the terms used in the Pirie-Kieren theory to describe the different levels of understanding of mathematics were modified, retaining the integrity of the

original, to determine whether the theory could be used as an analytic tool to discuss the growth of understanding of teaching and learning mathematics. A modified model that reflected these definitions was also developed.

The analysis indicated that the modified definitions provided an effective structure for discussing the growth of understanding of teaching and learning mathematics, both for the preservice teachers and the researcher. It also indicated that, as with the learning of mathematics, the developing understanding of the activity of teaching and of what it means to learn mathematics is an individual experience and is based on one's own background, or one's own Primitive Knowing. This development is a dynamic process which involves Folding Back to previously held Images to examine them in light of newly acquired concepts.

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Chapter 1

Introduction to the Study

There is an abundance of literature on the beliefs of teachers about teaching and learning and also on the beliefs that preservice teachers hold regarding the nature of mathematics and mathematical learning. This literature seems to indicate that often teachers' beliefs about teaching and the nature of learning are determined before they enter a teacher education program and that the effect of the program is minimal in changing these beliefs. One possibility cited for this lack of change is that the length and intensity of the teacher education programs are not sufficient to influence the understanding of the prospective teachers regarding the process of teaching and learning. Beliefs, it is felt, often are not founded on fact and therefore need proof and time in order to effectively be re-defined.

1.1. Rationale for the study

Being an effective teacher involves having the "ability to transform subject content through a range of techniques, suitable examples, analogies, etc., to make the content more assessable and memorable to the learner" (Smith, 2001; p. 117) or to be able to translate one's own knowledge into classroom curricular events that make that knowledge accessible to students, using the students' interests and motivations to learn a particular topic (Carter, 1990). A central tenet of teaching is "an understanding of how to recognize, evaluate, and implement activities with pupils' learning in mind" (Carter, 1990; p. 295). However, people tend to become teachers in subject or content areas in which they did well and therefore often do not understand the problems that others might experience in the learning of the subject matter. An intermediary step might be to integrate theory of understanding as well as time to practice and discuss these theories in teacher education programs. However, in teacher education, and thus in teaching, there is a perpetual struggle between integration of theory and practice. While theory cannot capture the practice/enactment of the situation, "it is in the explication of why particular roles might be productive that theorizing can be beneficial" (Roth & Tobin, 2002; p. 168). And, while awareness of different educational theories and research may help develop different forms of praxis that apply to different teaching situations, the "significance [is] not just understanding that much of teaching occurs as *habitus* that generates practices but also of understanding the critical need to connect theories to the experience of teaching" (Roth & Tobin, 2002; p. 296).

Presenting preservice teachers with one theory that gives them a way of thinking about students' developing understanding of mathematics, and by exposing them to situations in which they think about and discuss student learning using that theory as their lens may help them come to a better understanding of ways in which students develop understanding of specific topics and this in turn may help them consider different ways in which to present material so as to make it more meaningful to their students.

1.2. Teacher education

Learning to teach is a process that begins possibly before one starts grade school and which continues throughout one's years as a student in the classroom (Lortie, 1975; Britzman, 1991; Smith, 2001; Roth & Tobin, 2002). However, to officially become a teacher in North America, or to be certified as a teacher, one must complete a teacher education program, which, in most situations, will involve at least one university degree. When an individual reaches this point in his/her development as a teacher, "[l]earning to teach should ... be seen as a process of experiential learning and the student teachers' experiences should be taken as starting points for learning" (Koster, Korthagen & Wubbels, 1998; p. 77). Thus, the teacher education program and the classroom experience both during and after the teacher education program should provide experiences that will help further develop one's ideas and concepts about teaching, using one's personal background as the starting point (Roth & Tobin, 2002; Smith, 2001; Feiman-Nemsor, 1990). A teacher education program that takes into consideration the backgrounds of the preservice teachers as well as presenting them with further ideas regarding the nature of teaching and learning would seem a good program within which a prospective teacher is made to feel secure in his/her own understanding while, at the same time, allowing for the learning of new ideas and the development of new understandings.

1.2.1. Attitudes and beliefs

The usefulness of teacher education programs in changing the attitudes and beliefs that one holds about teaching, about the nature of the subject matter being taught and about the manner in which subject matter is learned has been questioned (Kagan, 1992; Smith, 1999). Research seems to indicate that most teacher education programs have had little effect on prospective teachers' images of what it means to teach and to learn (Feiman-Nemser, 1990; Pajares, 1992; Cooney & Shealy, 1997). Therefore, whether or not a teacher education program can help prospective teachers develop a philosophy or image of teaching that is different from that which they have already formed before entering the program and if this new image can be rendered strong enough to maintain when they begin teaching is an important point for consideration (Cooney & Shealy, 1997; Feiman-Nemsor, 1990). In order to determine the possibility of a program having the ability to change a prospective teacher's perspective on teaching and learning, one must therefore consider possible reasons that the teacher education programs are not sufficient to change these perspectives and understandings.

Since beliefs are formed over time, in order to change them, it is likely that pedagogical practices that support changing them can only be constructed by engaging prospective teachers, over time, in experiences that incorporate the desired changes. Also, since many attitudes and beliefs are non-rational and intuitive, some of them may be changed only by dealing with them directly and experientially rather than by attempting to influence them through rational, analytic means. To address this issue, prospective teachers must be exposed to innovative teaching styles throughout their preservice education (Wubbels, Korthagan & Broekman, 1997). A program that considers these findings, a program that presents preservice teachers with a theory at the beginning of their teacher education program and which exposes them to situations in which they study student understanding through the lens of that theory, as well as one which encourages them to use the language embedded in the theory in discussion of their own and students' understanding would seem an appropriate way in which to consider changes that might occur in the understanding and beliefs about teaching and learning of prospective teachers as they complete their teacher education program.

1.2.2. The use of theories

Learning environment research indicates that each theoretical framework and/or theory involves a set of constructs and provides only one window through which to view learning and the environment. For example, activity theory indicates that we are co-creators of our learning and have the power to act based on the subject, object and tools available in a community guided by rules and regulations (Roth & Tobin, 2002). Thus, while each theory provides its own perspective on a situation, we must be active in the construction of our own understanding of a situation. By being presented with another point of view, being presented with a theory that is used and discussed in practice, preservice teachers may be helped to view teaching and learning from a different perspective.

Having a new perspective on and a new way of viewing teaching and learning may also help preservice teachers develop the essentials of pedagogical content knowledge, knowledge which is considered necessary in order to effectively teach content knowledge. Pedagogical content knowledge goes beyond knowing subject content knowledge by providing ways of presenting that knowledge so as to make it comprehensible to others (Shulman, 1986). Prestage and Perks (2001) refer to this as a distinction between "*learner-knowledge* in mathematics [and] *teacher-knowledge* in mathematics, the former is the knowledge needed to pass examinations, to find the solutions to mathematical problems; the latter is the knowledge needed to plan for others to come to learn the mathematics" (p. 102) (italics original). In order to design appropriate classroom activities, analysis of topics through reflection "to reconsider their own personal understandings of mathematics" (Prestage & Perks, 2001; p. 108) is necessary to develop the *teacher-knowledge* with which to inform teaching that will challenge and meet the needs of the students.

Thus, teaching mathematics for understanding requires knowledge of mathematics, knowledge of student learning, and knowledge of mathematics pedagogy. This implies the need to have both conceptual knowledge, the "rich relationship, linking new ideas to ideas that are already understood" (Stump, 2001; p. 210) and procedural knowledge, the "formal language and symbol systems, as well as algebraic algorithms and rules" (Stump, 2001; p. 210) of the content. An understanding of the manner in which students develop understanding of mathematical content would seem essential in a teacher education program for prospective secondary mathematics teachers.

1.2.3. The use of imagery in teacher education

Schön (1988) believes that, in order to develop an understanding of how students come to understand mathematics, preservice teachers' perceptions about teaching and learning must be sought and that the preservice teachers must be helped in finding meaningful metaphors in which to frame their work so that they can reflect on their own understanding in relation to that of their students. Smith (1999) indicates that imagery is a possible means of helping prospective teachers reconstruct their pre-formed world images of the meaning of teaching, and the use of metaphors and/or analogies, such as that of comparing learning to teach to learning to play a musical instrument, can be used during teacher education to help preservice teachers realize that the *apprenticeship of observation* as described by Lortie (1975) is inadequate preparation for learning to teach. Theory and first hand experience are necessary if one is to develop competency as a teacher (Smith, 1999).

In considering the use of theory, one must realize that theory does not tell one how activities will be manifested or how one's students will act, but rather that it provides a framework for considering different approaches they might take (Kieren, 1997). Awareness of different theories and research results may help preservice teachers consider different forms of praxis, and this knowledge may also help them realize why the different practices lead to different productivities (Kreber, 2002; Roth & Tobin, 2002).

1.2.4. Requisites of teacher education

Undergraduates often view learning as "acquisition of knowledge, memorizing, utilization and/or use of knowledge [as opposed to] abstraction of meaning ... and an interpretive process aimed at understanding reality" (Hattie & Marsh, 1996; p. 531). However, in order to make sense of how students think and learn, a teacher must be attentive to the ways in which his/her own learning is the same as, or is different from the way in which the students think and learn (Schön, 1987). A teacher needs to be able to think as a student would think and to build on the images that the students might have, not only on his/her own images (Wubbels, *et al.*, 1997). Thus, a requisite of teacher education programs should be to help prospective teachers, if necessary, re-adjust their image of teaching and learning by having them reflect on the manner in which they were taught, what they think teaching should entail, and how they think students learn (Wubbels *et al.*, 1997). The program should help prospective teachers reflect on their own learning experiences, realizing that students have different preferences for learning, different learning styles, and that several possible explanations can be given for a specific situation (Kreber, 2002). An effective teacher education program needs to help prepare teachers to teach a curriculum that may have different characteristics from that which they themselves experienced and to teach students who have different learning styles and needs than they themselves have (Wubbels *et al.*, 1997). The focus of teacher education should, therefore, not be on how one teaches, but rather should be on how students learn (Davis, Sumara, & Luce-Kapler, 2000).

During their teacher education program, prospective teachers need to learn theory, need to observe others teaching in order to become familiar with the task of teaching and the structure and nature of the learning process, as well as have the opportunity to practice what they have thus learned (Roth & Tobin, 2002). While this involves further observation, and observation, it has been indicated, is not sufficient in learning to teach, it should be noted that the observations made by students as preservice teachers will be very different from those experienced by them as students in a classroom attempting to learn the subject content (Pajares, 1992). The new observations, informed by theory and discussion, should help the preservice teachers notice how different practices affect different students and lead to difference outcomes. Thus, the teacher education program should provide prospective teachers with an opportunity to observe the teaching process and should help them learn what and how to attend to the particulars of practice. The observations should be followed by discussion of what is observed in light of theories learned (Kreber, 2002; Roth & Tobin, 2002) and these discussions must then be followed by practice so that the prospective teachers not only learn about, but can also experience what they have learned to see how the theory applies *in situ*. Theory and practice are therefore inseparable (Schulman, 2002).

1.3. Images of mathematics education

In the realm of mathematics education, Wubbels, Korthagan and Broekman (1997) indicate that emphasis has shifted from a mechanistic, structured approach in which mathematics is viewed as a set of algorithms with rules for applying them in an organized deductive system, to a more holistic view whereby individuals construct knowledge and principles from concrete situations. The Platonic view of mathematics as 'out there' and 'linear' has been challenged by mathematics educators who view the subject as dynamic, and who view the learning of mathematics as a to-and-fro process of interrelating ideas whereby individuals construct their own images of what it means. Thus, the learning of mathematics is often seen as unpredictable (Pirie & Kieren, 1994a; Cooney & Shealy, 1997).

Mathematical learning does not begin and end in the classroom but is a dynamic whole with the environment (Davis *et al.*, 2000). Small group discussions are seen as a means by which to develop informal approaches which can later be developed into more standardized, formalistic forms if necessary (Wubbels *et al*, 1997; NCTM Principles and Standards, 2000). However, "many teachers' traditional experiences with and orientations to mathematical pedagogy hinder their ability to conceive and enact a kind of practice that centers on mathematical understanding and reasoning and that situates skills in context" (Ball, 1993 p. 162 cited in Stump, 2001; p. 208). Thus, as preservice mathematics teachers may have been taught by mathematics teachers who, even if they did not explicitly state it, thought of mathematics as linear and cumulative, the images and attitudes the preservice teachers hold toward the teaching and learning of mathematics will have been influenced by that approach. This, in turn, will affect their ability to enact a new form of curriculum unless it is specifically addressed in their teacher education program.

1.3.1. Requisites of mathematics education

Most students who enter a secondary mathematics teacher education program do so because they were good at mathematics (Roth & Tobin, 2002). As they enter the teacher education program, their images of what it means to teach and to learn mathematics are based on their own ability to have succeeded in the system and they may not be aware of different perspectives and approaches and of the different understandings that students may develop. To teach the variety of students that they will be faced with, and to apply the more interactive, progressive manner of teaching implied by the new philosophies toward the teaching and learning of mathematics will require that materials be prepared differently from those from which they themselves learned. Preservice teachers must be made aware of and must have experiences with the mathematical knowledge that is required for a particular application, and of the different learning styles that their students may exhibit (Wubbels *et al.*, 1997).

Although a teacher education program may be designed to help preservice teachers develop an understanding of the need to look at mathematics teaching and learning differently, there is no guarantee that this will happen or that the prospective teachers will be able to carry the new ideas into practice (Hart, 2002). In order to do so, they need the opportunity to practice their newly developing understandings about teaching and learning in a supportive atmosphere. This is supposed to be the purpose of the practicum experience. However, the schools in which preservice teachers are placed often work in the traditional manner and use textbooks that present mathematics in the traditional manner. As these are constraints within which preservice teachers must work, and which govern the daily teaching practice of their in-school experience, their ability to incorporate into practice the newly developing theories, which they have been learning in the university classroom, is questionable (Wubbels *et al.*, 1997).

A theory base which will help preservice mathematics teachers overcome the over-emphasis on conforming to existing practices at the cost of overlooking new developments and insights gleaned through research is therefore needed in teacher education programs (Koetsier & Wubbels, 1995). The program must be such that it will render strong enough and rich enough the experiences of the preservice teacher so that the new images presented and the progressive attitudes developed can be incorporated into his/her daily practice.

Modeling is a particularly good method by which to teach abstract behaviors (Bandura, 1997; Hart, 2002), and thus, modeling of a theory during the teacher education program may be a good means by which to encourage preservice mathematics teachers to integrate new methods into their practice.

1.4. Purpose of this study

In light of the above research, it would appear that, if a teacher education program for prospective secondary mathematics teachers is to influence their images of mathematics and mathematics teaching, and if this program is to create images that are strong enough to be integrated into practice, the program must not only <u>teach</u> the theory associated with the image that it wishes to develop, but must also <u>model</u> it and provide the opportunity for the preservice teachers to practice their developing understanding in a supportive environment. A program concentrating on the manner in which learning develops and which places this into the context of the preservice teachers' daily classroom life seems to be needed if prospective teachers are to have the opportunity to construct new understandings and to develop new techniques. Prospective teachers need, not only to learn about a theory, but they must also see its application as a teaching technique in order to develop a better understanding of the activities of teaching and learning mathematics. Seeing the theory in action would allow for the construction of new knowledge *en route* and *in situ* and possibly lead to a greater predisposition to consider student learning differently. Also, by considering their own learning and understanding in light of the theory studied, the prospective teachers would be able to develop a better understanding of the process that they themselves had to go through to come to their understanding of the subject matter. This, in turn, could inform their understanding of students' developing understanding which could then lead to a new form of praxis (Schön, 1987).

The purpose of this study is to address this issue of influencing prospective teachers' understandings of the processes of teaching and learning mathematics. In order to address this issue, it was felt that the presentation of a theory for the learning of mathematics to preservice secondary mathematics teachers, describing, demonstrating and discussing the theory with them, and having them consider their own understanding and learning through the lens of the theory would be an appropriate means by which to consider the usefulness of the theory in influencing their understanding of student understanding.

Many different theories of learning exist, and each theory is useful for its own purposes. Also, many theories of learning exist which are specifically related to the learning of mathematics. For example, there is Skemp's (1976) categorization of mathematical learning as relational vs. instrumental, the van Hiele's (1986) five leveled model, and the age and topic specific two-tiered model of Herscovics & Bergerson (1988), each of which can be used to describe students' mathematical understanding in an appropriate manner. There is also the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding, and it is this theory that I chose as most appropriate for this study. This theory is discussed in detail in the next chapter, but here I state that it was chosen specifically because it is about <u>growth</u> of understanding which is what this research investigates, not what can be done at any one time, but rather, the changes that occur as one develops understanding of a particular topic.

Since many prospective mathematics teachers hold a more traditional view of mathematical teaching and understanding than that implied by the Pirie-Kieren theory, this theory, it was felt, would problematize their beliefs more than the other theories would. This theory, it was hoped, would possibly give them a new way of viewing the teaching and learning of mathematics as well as a vocabulary with which to discuss it. The main, crucial feature, however, for its choice as the theoretical basis for this research was the fact that it is the only complete theory that pertains to the growth of understanding. That is, it looks not at the static existence of understanding by a learner at any one moment, but at the dynamic process of acquiring understanding in any learning situation.

Specifically, then, this study considered the questions:

- Does using the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding (the Pirie-Kieren theory) as a model enhance the development of preservice teachers' understanding of the teaching and learning of mathematics?
- Does the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding offer an illustrative framework with which to analyze the growth of preservice teachers developing understanding of teaching and learning mathematics?

In order to answer these questions, it was necessary first to consider how prospective teachers indicated their understanding of what it means to teach and how students learn. It was then important to consider whether statements they made about their understanding of teaching and learning, and about their understanding of the theory of how students develop an understanding of mathematics, that are in line with the theory, carried over to classroom practice during their practicum. That is, were they able to integrate what they learned about teaching and learning into classroom practice?

1.5. The organization of the chapters

This chapter has presented an overview of the need for this research, indicating some of the shortcomings in teacher education programs and areas that have been identified as needing to be enhanced and has identified the focus of this research.

Chapter 2 presents in detail, the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding, the theory that was used as a model for the preservice secondary mathematics teachers in this study and which was modified for use in analyzing the data.

Chapter 3 offers a review of the literature related to teacher education with some consideration to mathematics teacher education.

Chapter 4 presents a review of the literature related to the method of data collection (video) and the first stage of analysis (case-study and portraiture).

Chapter 5 presents the methodology used for the study from the collection of the data, the writing of the portraits, through to the development of a method of analysis of the portraits using a modified version of the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding.

Chapter 6 presents the portraits of four preservice teachers who were involved in an intensive integrated program designed to help them develop a better understanding of how mathematical understanding is developed.

Chapter 7 forms the second portion of the analysis by considering the growth of the understanding of teaching and learning mathematics of the preservice teachers. The Pirie-Kieren theory was modified to use as a tool of analysis for this chapter.

Chapter 8 provides a summary of the findings of the study based on the analysis of the portraits of the four individual preservice teachers, answers the research questions, and makes suggestion for further research.

Chapter 2

The Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding

The Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding (the Pirie-Kieren theory) is a theory in which the growth of mathematical understanding is viewed as "a whole, dynamic, leveled but non-linear, transcendentally recursive process" (Pirie & Kieren, 1991b; p. 1). The theory is in agreement with the biologically based, self-referencing systems as described by Maturana & Varela and by Tomm and the work of Vitale and Margeneau, who discuss the recursiveness involved in the learning process, whereby recursiveness implies self-defined levels that are different from the previous and in which the new knowing "frees one from the actions of the prior knowing" (Pirie & Kieren, 1989; p. 8). Thus, the Pirie-Kieren theory also reflects the more dynamic view of mathematics teaching and learning as described by the National Council for the Teaching of Mathematics, and in considering the learning of mathematics as a non-linear, dynamic to-and-fro flow of ideas, it goes further than Sierpinska's (1990) model of understanding as a series of overcoming obstacles by realizing that these obstacles often force one to fold back so as to revisit and re-image previous understandings. The Pirie-Kieren Theory is thus in accordance with the latest thinking on the nature of mathematics and the way it can best be learned.

2.1. Background to the Pirie-Kieren theory

Having its foundation in constructivism, the Pirie-Kieren theory had its conception in the research of two mathematics education theorists and practitioners, Dr. Susan Pirie and Dr. Thomas Kieren. Although working in different countries and involved in different areas of mathematics education research at the time, they were brought together in 1988 by a shared belief that learning is a dynamical process and that one never completely understands, but is always in the process of developing understanding (Pirie & Kieren, 1991a). Pirie and Kieren have extended the constructivist view of understanding to incorporate an enactivist perspective, whereby understanding is individual but is influenced by external processes (Kieren, 1992; Kieren, Reid & Pirie, 1995). As new research has developed and expanded different aspects of the theory, it has been adapted and modified to increase its power as a research tool (Pirie & Kieren, 1994a; Kieren *et al*, 1995). Thus, the Pirie-Kieren theory has been in the process of developing for almost twenty years, a process of growth and development that can be paralleled with and is consistent with the belief that Pirie and Kieren have regarding the growth of mathematical understanding.

For the purposes of this study, it is important to note that the Pirie-Kieren theory is a theory for, not a theory of, the growth of mathematical understanding. Thus, it provides one way to consider the growth of mathematical understanding and provides one framework within which to discuss the process of learning (Kieren, 1992). Also, being a theory about growth of understanding, it is not used to determine what is understood at this time, as in assessment, but rather, it is used to describe the process by which a student continually reorganizes knowledge into a more meaningful whole. For the purpose of this study this is an important aspect in that the theory can be used to describe the growth of understanding of any mathematical concept and at any level. The theory provides a language with which to discuss these differing understandings and does not put a hierarchical value on that understanding.

In this study, preservice teachers were presented with the theory and had it modeled for them in their university classes. In order to use the theory as an analytic tool, the terms which describe the levels of understanding were carefully re-defined to fit this new context while staying consistent with the original definitions relating to growth of mathematical understanding.

2.2. A model for the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding

In order to discuss developing understanding using their theory, Pirie and Kieren developed a two dimensional representation, or a model of their theory (see Figure 1). As can be seen, the model consists of a set of eight nested circles which implies eight potential layers, or levels in the development of mathematical understanding (Pirie and Kieren, 1989). It must be stressed that this model is not the theory, but that it can be used to discuss and map a particular person's understanding of a particular topic at a particular time.

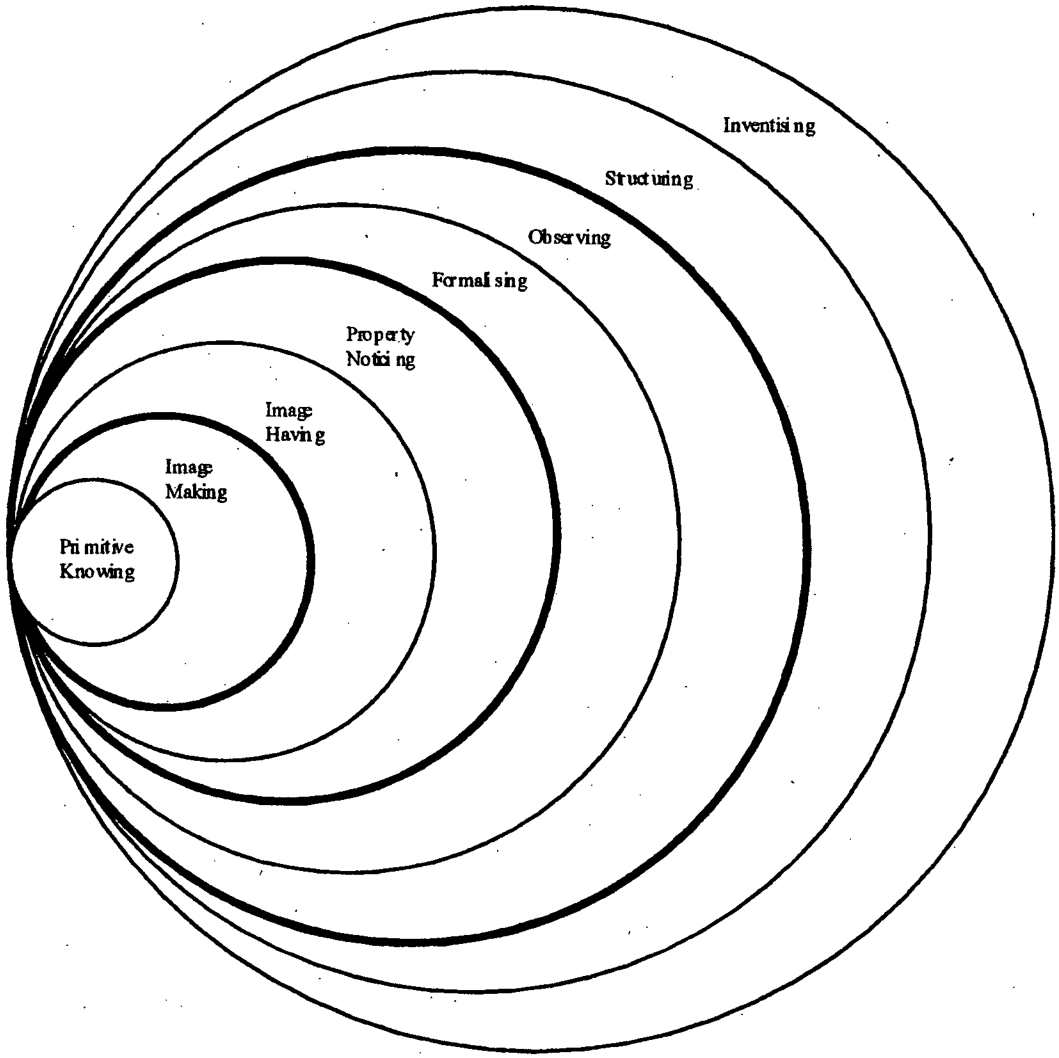

Figure1: A Model for the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding

2.2.1. Mappings

A mapping on the Pirie-Kieren model is a method whereby an individual's developing understanding, as it is observed, can be traced on the Pirie-Kieren model (see Figure 2 as an example). A mapping diagram of a student's developing understanding is achieved by placing a student's observed understanding at the appropriate layer on the Pirie-Kieren model and following it through, using solid lines to indicate connected understanding and broken lines to indicate disconnected understanding. Connected understanding implies that the student has made the necessary transitions from one layer to the next in developing understanding while disconnected understanding implies that these connections have not been made. Disconnected understanding inhibits an

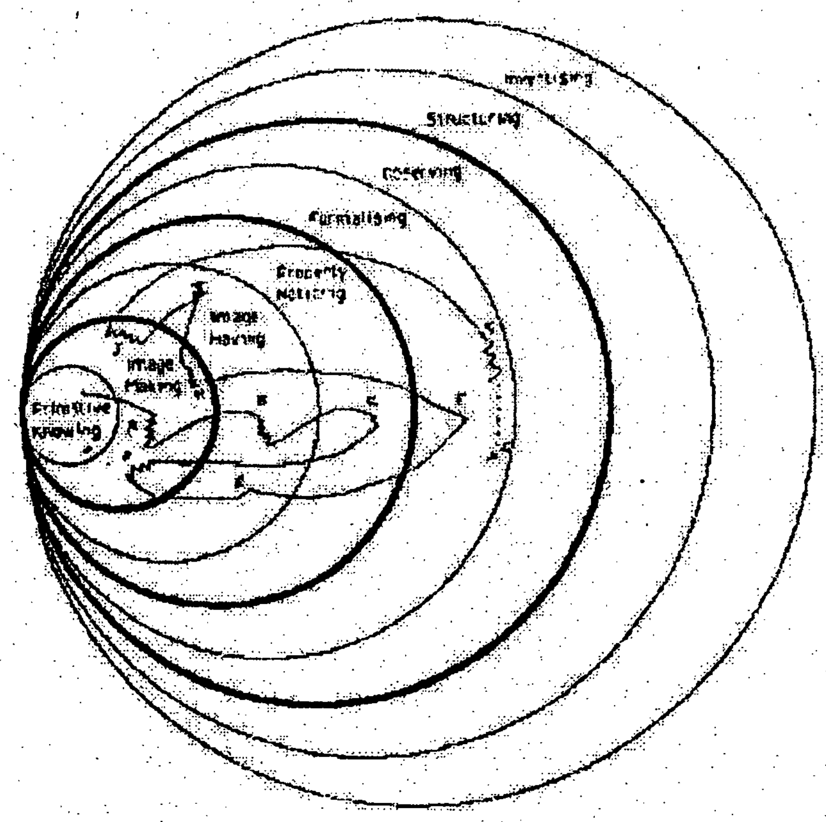

Figure 2: Sample mapping of student understanding using the Pirie-Kieren model

individual from constructing further knowledge in a meaningful way (Pirie & Kieren, 1994a) and may result from the teacher trying to force the student into an outer layer of understanding before he/she has had a chance to <u>work at</u> the inner level to develop his/her understanding. The mapping of a student's developing understanding is a useful process that can help the teacher determine what understanding the student has and can also be useful in helping determine the source of some of the student's misconceptions which lead to errors.

In this study, the model has been modified to suit a new context, maintaining the integrity of the original, and will be used in Chapter VII to trace the developing understanding of teaching and learning mathematics of four preservice mathematics teachers, Sophia, Lance, Ellie and Wayne, as they progress through their year of teacher education in the Secondary Mathematics Integrated Program (SMIP) at the University of British Columbia.

2.2.2. Transcendental recursiveness

Recursion is an overriding aspect of the Pirie-Kieren theory and is used throughout the theory in describing the complex phenomenon in which each layer of understanding occurs as a whole process that is structurally similar to, but not reducible to, any previous state (Kieren & Pirie, 1991). "The metaphor of *recursion* is used to highlight the fact that the dynamical understanding of a person involves states which differ in character but are self-similar" (Pirie, Martin & Kieren, 1996; p. 147) (italics original).

Like Varela, Thompson and Rosch (1991), Pirie and Kieren believe that individuals are self-referencing, autopoietic beings in whom knowing occurs through actions that are bound in previous actions. Recursion "forms a basis for looking at the process of developing understanding in which an individual's current understanding acts together with previous understanding and integrates them in the sense that they are called into current knowing actions" (Pirie *et al.*, 1996; p. 147).

The transcendental recursiveness of developing understanding is featured in the eight layers of the Pirie-Kieren model, firstly, in that they are divided into four sub-sections by what are referred to as Don't Need Boundaries. These boundaries are indicated by the darker circles in figure 1 and represent a 'crossing over' in understanding so that the individual no longer needs to work with the previous layer of understanding but can work at the new layer. The two layers of each of the four subsections are analogous to those of the other layers. Thus, transcendental recursion occurs in that the relationship between Primitive Knowing and Image Making is similar to the relationship between Image Having and Property Noticing, between Formalising and Observing, and between Structuring and Inventising where the outer layer involves working on the inner.

The Don't Need Boundaries also indicate the transcendentally recursive nature of the Pirie-Kieren theory in that there is a similar relationship between layers as one crosses over the boundaries from Image Making to Image Having, from Property Noticing to Formalising, and from Observing to Structuring. In each case, the crossing over frees the individual from the need to work with specific examples and enables him/her to consider a more generalized situation. Thus, once an individual has an Image, he/she no longer needs to work at Image Making; once he/she has Formalised a concept, the need to work with specifics no longer exists; and once a Structure is determined, actions can be independently carried out at a meta-mathematical level (Kieren, 1992; Pirie & Kieren, 1992a).

A further example of the transcendental recursiveness in the development of understanding as described by the Pirie-Kieren theory is that, within this developing understanding, at any point, there can be a redefinition of the levels. What was Formalised understanding of one concept may become the Primitive Knowing for another, or the Image Having if the same topic is considered at a later time. For example, in the study of rational expressions in algebra, the Formalised knowledge that a student has about working with fractions in arithmetic forms the basic Images upon which he/she must build the structure for working with the rational expressions. He/she must Fold Back to this knowledge and use it to construct new Images of the meaning of the operations involved when the fractions are no longer numeric in nature.

2.3. The layers of understanding in the Pirie-Kieren theory

In this section, the form that learning takes at each layer of understanding as represented by the circles of the model will be discussed.

Beginning from an individual's Primitive Knowing, the next three layers of understanding as represented on the Pirie-Kieren model demonstrate informal modes of action which are local and context dependent, while the second three are formal modes of mathematical activity, making them less context dependent and more abstract (Kieren, 1992). If the development of understanding is not disconnected, the continual back and forth flow through these layers of understanding can lead to the final stage, Inventising, at which new mathematical concepts can be developed (Kieren *et al*, 1995).

2.3.1. Layer 1: Primitive Knowing

The innermost circle of the Pirie-Kieren model is referred to as Primitive Knowing. Primitive Knowing is the knowledge that a student brings to the situation, except the knowledge he/she has about the concept that is to be developed (Pirie & Kieren, 1991a). Somewhat analogous to Polanyi's (1983) tacit knowing, it is the background knowledge that a student comes with and from which he/she must work. Primitive, in this context, does not imply a lack of sophistication but simply a starting point, the point at which the student begins the discussion and, as such, it depends on the background and maturity of the individual (Pirie & Kieren, 1991b, 1992). The Primitive Knowing of a six year old, for example, is considerably different from that of a sixteen year old simply because of the life experiences of each both in and out of the classroom. Although a teacher can never be certain of the Primitive Knowing of a student, it is that which he/she has to consider when beginning a mathematical discussion. It is the Primitive Knowing of the student that will inform his/her understanding of the teacher's presentation of a topic and/or his/her understanding of the terminology that is implicit in that topic.

2.3.2. Layer 2: Image Making

The second layer of understanding in the Pirie-Kieren theory is that of Image Making, an important and active period in the developing of understanding. At this level, the student begins to build on his/her knowledge of the topic under discussion, working with this knowledge in an attempt to formulate Images that will, at least temporarily, help determine the meaning of the activities. The Images need not be physical models, and the action need not be physical. Either can be enacted entirely in the mind, depending on the situation and the maturity of the student. For example, in working with fractions, a young student might cut a 'pie' or a 'pizza' in half, then quarters, etc. using a diagram, whereas an older student will likely be able to call upon the abstract Image of 'half' as being two equal parts, not having to use physical models. A teacher is released from the need to continuously individualize instruction by allowing the students the opportunity, or task-space, to engage in Image Making activities as they see fit (Kieren *et al*, 1996). By working at this level, possibly forming different Images depending upon what Primitive Knowing is being recalled until he/she begins to internalize the Images, a student is able to develop connected understanding so that he/she is able to cross into the next level with new knowledge rooted in existing knowledge.

Pirie & Martin (2002) suggest Guided Image Making as a means of motivating and encouraging weaker students in their developing understanding. For Guided Image Making to be effective, it is imperative that the teacher reviews with the students the Primitive Knowing that he/she wishes them to use, then leads the class in Image Making by use of a question that is not immediately solvable. In this process, the teacher constantly encourages the students in their work so that he/she, "almost from the beginning [has the students] strategy hunting deliberately using their mathematical knowledge" (Pirie & Martin, 2002; p.

16). The students will thus develop an understanding based upon thinking about a problem, not in trying to remember a process. Guided Image Making, then, leads the students to a way of thinking, not to a formalized process.

2.3.3. Layer 3: Image Having

The move from the Image Making to the Image Having layer is an important step in a student's developing mathematical understanding. This is the step at which he/she crosses the first Don't Need Boundary. It is often considered as the "Ah ha" crossing in that the student, for the first time, may internalize the understanding of the concept and feel that he/she really understands it. He/she will no longer have to work on the Images of the concept, but will have an Image or mental object with which to work. This crossover frees him/her from the need for particular actions and provides an understanding of the concept that is often metaphoric (Pirie & Kieren, 1994a). The student may think that the equation is the balance, or that three-quarters is three out of four pieces. It must be stressed that, although the student has now formed an Image of the concept, this Image is not complete and may even be incorrect (Pirie & Kieren, 1994a). Incomplete or incorrect Images may create problems later on, obstacles which may interfere with the student's developing understanding of a topic. What is meant by an incomplete Image is, for example, concerning fractions, a student might think of 'half' as being one of two equal parts of a whole, but not considering it as a separation of a number of objects into two equal sized groups. That is, he/she could have the Image of 'half' as an object being cut into two equal parts, but not that three people out of six people form one half of the group.

An incorrect Image of 'half' could be that it implies cutting an object into two parts without realizing that the parts have to be equal.

Preservice mathematics teachers may be unaware of the incomplete and incorrect Images that students might have. Presenting them with the Pirie-Kieren theory early in their teacher education program as well as modeling it throughout the program, it was hoped, would make them more aware of students' understandings and possible misunderstandings and that this would help them in thinking about this in a more active manner which, in turn, would help them be able to think about and plan more appropriate and/or involving teaching and learning activities.

A student working <u>at</u> the Image Having layer of developing understanding has different Images about a topic as well as Images about different topics and is likely to hold these as separate, often unrelated entities. He/she, at some point, may question this, and when this happens, may begin to see relationships among them. He/she has passed into the next layer of understanding, Property Noticing, as represented on the Pirie-Kieren model.

2.3.4. Layer 4: Property Noticing

As mentioned earlier, Property Noticing is somewhat akin to Image Making but the individual is now working at a higher level of sophistication for his/her level of maturity and understanding of the concept under consideration. He/she begins to bring together discrete Images, previously seen as disconnected, and examines them for specific properties. Understanding becomes a simile rather than a metaphor (Pirie & Kieren, 1994a): The equation <u>is</u>

like a balance, or ¾ is like taking three people from four people. At this level, the student's thinking is more abstract and he/she needs to work with his/her different Images, seeing the properties for him/herself until he/she is able to get what might be referred to as 'a feel' for the topics. It is important that the teacher does not try to force him/her to too quickly cross the next Don't Need Boundary into the Formalising level. However, many preservice teachers, like many practicing teachers, because of their background experiences and their own learning situations, are not aware of the need for students to notice relationships themselves, and may have a tendency to lead students too quickly into a Formalised mathematical 'understanding'. Since he/she was likely 'good' at mathematics, he/she may not have had to spend much time at the Property Noticing level. That is, already having connected and Formalised understanding, he/she may not recall the process through which he/she had to go to develop that understandin. Similarly, preservice teachers (and many practicing teachers) may not realize the difference between students developing mathematical understanding and their ability to perform mathematical algorithms, surmising that since the students can do the questions assigned they must understand the mathematics. However, often the student may simply be applying an algorithm and has memorized a number of applications but does not see the relationship among them.

2.3.5. Layer 5: Formalising

The fifth layer of the Pirie-Kieren theory, the layer at which formulas or algorithms are often developed, is officially referred to as the Formalising level.

At this level generalization occurs and the student comes to realize that a method works 'for all' cases in that category. By abstracting, the student begins to use more formal mathematical activities and can work with concepts rather than needing specific instances. To illustrate, in an interesting example cited by Pirie and Kieren (1994a), a student had been working with paper folding at the Image Making level of fractions and had noticed that $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}$, etc. (Image Having). He then noticed that the multiplication did not have to be by twos but could be by threes, yielding $\frac{1}{2}=\frac{3}{6}=\frac{9}{18}$, etc. (Property Noticing). The student then happily stated: "I'll bet it even works for sevenths!" He had abstracted from having to work with specific numbers as, to him, sevenths were the epitome of all possible fractions. For him, 'even sevenths' was a 'for all' statement and he Formalised his understanding of equivalent fractions.

If allowed to properly develop, as in this case, Formalisations become abstractions or meta-analyses of one's own ideas on which one must work to create a pattern (Pirie & Kieren, 1992a). Formalisations do not have to involve formulas, or even be expressed symbolically. However, they are usually expressed verbally or in some external manner before they are internalized.

By presenting preservice teachers with this aspect of the learning process, it was hoped that they would be able to realize the importance of allowing a student to develop his/her own understandings. It was hoped that by making them aware of this aspect of learning mathematics, the preservice teachers, in their planning and teaching, would see the need to allow time for students to work with their Images and Formalizations, that is, allow time for Image Making and Property Noticing, in order to develop their own understandings rather than leading them directly into a Formalisation or trying to force their own Images onto the students.

2.3.6. Layer 6: Observing

The sixth level of developing understanding in the Pirie-Kieren theory involves an individual's conscious effort to further his/her understanding. Already having a Formalised understanding of different aspects of mathematics, he/she actively searches for a pattern or relationship among them, searching for commonalities so as to see 'the big picture'. This may be the level that separates mathematical learning from that of other areas (Pirie, 2001; personal communication). At this level, the individual wants to work at the mathematics him/herself to see the patterns that exist and the relationships among topics.

In the development of the Pirie-Kieren theory, and in the Mapping of individuals' developing understanding of mathematical concepts, the upper levels (six, seven and eight), have not been observed and documented as often as the lower levels. It is at these levels of mathematical understanding, however, that teachers have to work as they think about and prepare lessons so that they will be meaningful to students and so that students can develop their own Images and Formalisations. Thus, it is important that preservice teachers are aware of these levels of thinking so that they can understand the abstractions that they are trying to teach their students.

2.3.7. Level 7: Structuring

When an individual is able to take his/her previous understandings of concepts and establish a relationship among them, he/she becomes more confident about the concepts and is able to verify or justify a process or proof. In working at the Observing level an individual may try to re-form his/her own knowledge into an over-arching theorem. Structuring occurs when he/she is able to do so and he/she can see a pattern and formulate an encompassing explanation or an abstraction of these Observings (Pirie & Kieren, 1992a). An example of a Structuring relationship is that a student, thinking about the linear, quadratic and cubic functions studied as separate entities realizes that the patterns can be generalized to 'polynomial function'. The generalized relationship then releases him/her from having to think of separate patterns because he/she is able to tie existing knowledge to some other area or aspect of knowing.

Once an individual has abstracted a concept, he/she is able to see the general relationship and consider special cases under that Structure, thus being freed from the need to classify each example individually. Although Structuring involves abstraction, it must be pointed out that it can occur at any age or level of mathematical development. It is the individual's understanding and level of mathematical development that determines his/her level on the model, not the abstractness of the concept.

The Structuring aspect of learning mathematics may, again, be a level at which the teacher tries to force understanding by presenting it as <u>fact</u> to the student rather than by letting him/her think about the relationships involved and determining for him/herself that the Structure exists. In the polynomial function example, for instance, the teacher might give the students the definition without allowing time for the students to see how the previously learned functions qualify. For this study, it was hoped that if preservice teacher were aware of this, and if they gave thought to it, they would be able to use their own, new understanding of student understanding in order to develop lessons that would allow students to determine the existing Structures for themselves.

2.3.8. Level 8: Inventising

While working at the Structuring level, if an individual asks him/herself: "What would happen if I changed a part of this structure?" he/she, on that topic, has reached the Inventising or highest level of mathematical understanding "where, with full structural understanding of a piece of mathematics, he/she can deliberately ask questions that break out of this structure" (Pirie & Kieren, 1991b; p. 171). The term, Inventising was coined specifically for this theory, so as not to be confused with the term inventing since students may invent at any level of understanding (Kieren, 1992; Pirie & Kieren, 1991a, 1992).

While Inventising, an individual is capable of "developing, without giving up the previously understood knowledge, a completely new way of looking at and building from phenomena developed in the previous structure" (Kieren, 1992; p. 215). He/she is able to create new questions and observations that have the potential of becoming a new topic or area of study. Inventising can involve complex concepts (What might be implied by the fifth or sixth dimension?) or it can involve quite simple mathematics (What happens if you take 9 away from 7?) depending on the age and mathematical development of the individual.

The examples given above for the different levels of the Pirie-Kieren theory indicate that at any level of mathematical development, an individual can experience different layers of understanding. It is the individual's knowledge of and manner of working on or with the information that determines his/her level of developing understanding with respect to the Pirie-Kieren model, not his/her age level.

2.4. Features of the Pirie-Kieren theory

The Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding must be considered as a whole interactive, dynamical unit. The features of it are intrinsically intertwined and yet discrete. Some reference has been made to this in previous sections. In this section, the focus is on the four features of the Pirie-Kieren theory that represent its dynamic nature: Folding Back, Interventions, Don't Need Boundaries, and Complementarities.

2.4.1. Feature 1: Folding Back

An important aspect of the Pirie-Kieren theory is its emphasis on the non-linearity of coming to a better understanding. That is, it stresses the importance of the need to move to-and-fro through the layers of understanding. While working at a particular layer, one may have to work for some time, or one may pass through to the next level very quickly. At other times, when one faces a challenge that is not immediately solvable, one may have to move inwards to a previous level of understanding in order to reconstruct knowledge or to extend current inadequate understanding (Pirie & Kieren, 1991a). This is referred to as Folding Back. On Folding Back, an individual is "able to extend their [sic] current inadequate and incomplete understanding by reflecting on and then reorganizing his/her earlier constructs for the concept in question, or even to generate and create new images, should their [sic] existing constructs be insufficient to solve the problem" (Pirie, Martin & Kieren, 1996; p. 148).

Martin (1999) "explored the act of folding back to examine the actions that a learner engages in when folding back, and to consider the interactions that facilitate, occasion and constitute this process" (p, 74). Highlighting the complexity of the phenomenon as observed in action, he states:

> Folding back involves more than mere borrowing from a lower level construct on a higher level or to objectify an experience. Instead it is concerned with the process and act of re-visiting, re-constructing and re-shaping these experiences, accepting that one may never have a full understanding and thus growth involves a constant shifting from formal to informal, formulated to unformulated understanding. The notion of *folding back* recognizes that growing understanding is not defined in terms of a move from the concrete to the abstract, but that one enfolds the other and that the two cannot be split (pp. 57-58) (italics original).

Martin (1999) identified three over-arching aspects as discrete but interrelated aspects of folding back. These three aspects are Source, Form and Type (see Table 1). The Source of folding back may be 1. material (manipulatives), 2. an individual (self, peer or teacher), and 3. it can be intentional or non-intentional. Folding Back can take one of four Forms: 1. to collect data from an inner level, 2. to work at an inner level, 3. to move out of a topic and work there, or it can 4. cause a discontinuity (Martin, 1999). Table 1 below summarizes these findings.

1. "SOURCE"
Encompassing four main categories:
Invocative Teacher intervention
Invocative Peer Intervention
Invocative Material Intervention
Self Invoked
Each of these interventions can be divided into two subcategories:
Intentional
Unintentional
An Intentional Intervention can be further divided into two subcategories:
Explicit
Unfocused

2. "FORM"
Encompassing four main categories:
Working at an inner layer using existing understanding
Collecting at an inner layer
Moving out of the topic and working there
Causing a discontinuity

3. "OUTCOME"
Encompassing four main categories:
Uses extended understanding to work on overcoming an obstacle-i.e. effective
Cannot use extended understanding to work on solving problem-i.e. ineffective
The first of these uses also divided into two further subcategories:
Returns to outer layer with external prompt
Returns to outer layer without external prompt

A special case: 'Not taken as invocative' as an outcome

Table 1: The framework for describing folding back (adapted from Pirie & Martin, 2000; p. 143)

Generally, Folding Back tends to be to a layer that precedes a Don't Need Boundary. At such a level, one is able to re-construct Images, re-consider properties, or re-observe relationships. That is, once a student has formed an Image, he/she proceeds to do mathematics using that Image. If it is a correct Image, then, for the time being the student is likely to be happy with his/her work and will likely 'get the right answers'. If the Image is incorrect, it is likely that this will be evidenced in that the student will get incorrect answers. If the Image is incomplete, he/she is likely to get contradictory and inconsistently correct and incorrect solutions. In such cases, he/she would likely be prompted to stop, Fold Back, and re-consider the Images held so as to reconstruct them and form new Images that would apply to the situation at hand. The new Images being developed would be informed by new understanding and the returned-to level would be 'thicker' with the fold encompassing or bringing back information which could be used to reconstruct or modify the previous understanding while at the same time further extending the outer level of understanding (Pirie & Kieren, 1991a, 1992a). The information encompassed in the 'fold' would provide the necessary data for altering or enlarging the existing Image. Subtle, but significant changes can thus enlarge the previous understanding so as to support more formal understanding so that the new level transcends but is compatible with the previous level (Kieren & Pirie, 1991b). However, if a student has Disconnected Understanding, and if pure memory fails, then he/she may not be able to Fold Back to re-evaluate and re-form the Images that are associated with the concept. The student will not be able to think back to his/her previous understanding to make the necessary connections and will be forced into memorizing a new or different method, or may become discouraged and want to give up.

While it is helpful to identify and classify the various aspects of Folding Back so as to have a language with which to discuss these, it is also important to consider how these acts affect the teaching and learning of mathematics. The act of Folding Back does not necessarily lead to growth of mathematical understanding. While the phenomenon of Folding Back is an integral part of developing mathematical understanding, it is possible that an individual does not return to the outer layer of understanding without help or that he/she may not see the usefulness of the inner layer of working (Martin & Pirie, 1998). In such a case, Folding Back is considered ineffective. Martin (1999) postulates that it is the form of Folding Back and the related Intervention that are significant if understanding is to occur. The form of Folding Back needs to be appropriate to the needs of the individual, and the individual must be aware of what it is he/she needs (to collect, to move outside the topic, etc.).

Martin (1999) indicates that it is teacher Intervention, intentional or unintentional, that plays the greatest role in Invoking students to Fold Back. He warns, however, that there is real danger if the teacher attempts to give too much guidance and that the language that the teacher uses is important in determining the effectiveness of Folding Back. Explicit teacher Interventions are more directive than unfocussed Interventions and leave little room for exploration. Martin (1999) highlights the similarity between the sense of Discontinuity often created by these explicit Interventions which the student may not take as Invocative and Jaworski's (1994) 'didactic tension' created when the teacher's specific aims interfere with the student's own constructions. He suggests that this may occur because, when using an explicit Intervention the teacher makes an assumption regarding the student's understanding. The Intervention then directs the student to the teacher's perceived need, which does not allow the student time for reflection on the limitations of his/her own thinking. Explicit Interventions may be effective, however, if they follow an unfocused Intervention which has already focused the student on his/her limitation. Generally, intentional, unfocused Interventions appear to be most effective in that, being less directive, they provide 'thinking space', allowing the student to explore his/her own Primitive Knowing or Images more freely (Martin 2000).

When a teacher is present, he/she is usually viewed as the source of authority by a student and his/her Intervention is given credence over peer Interventions. However, Martin (1999) identified peer Interventions as having the potentiality to play as great a role in another student's Folding Back and he observed that peer Interventions occurred most often when the teacher had made the classroom a collaborative work environment. However, a collaborative work environment did not guarantee that the Interventions would be effective since the peer intervener is in the process of developing his/her own understanding of the concept so that any Intervention on his/her part is likely to also involve self-invoked Folding Back to seek assistance in his/her own thinking. To be effective or to develop a shared understanding, the Intervention needs to encourage exploration and negotiation in light of the intervener's uncertainty, as well as willingness and the ability on the part of his/her partner(s) to participate in the exploration. If the perceived need by the peer intervener is not seen as a need by the partner(s), then the Intervention would be ineffective.

The ability to self-invoke Folding Back is a useful dynamic in the developing understanding of a student as this implies a natural tendency to reflective thinking. However, it is unclear whether Folding Back can be <u>taught</u> to students. As with many practiced activities, Folding Back could become part of an individual's repertoire of alternative actions if he/she was encouraged to do it often enough through any form of Invocation. Although most textbooks are linearly formatted, Martin (1999) suggests that they could be written so as to promote Folding Back as a response to obstacles and that this would promote growth of understanding on the part of the students.

For this study, the concept of Folding Back was considered to be important in that many preservice teachers do not realize the significance of this process. Preservice teachers (and many practicing teachers) often do not understand the importance of the process and the need for students to re-create Images as being a major component of the process of developing understanding. Many feel that students should be taught correctly and linearly so there will be no stumbling blocks along the path of developing understanding. However, if this were possible, then the teaching of mathematics would be a simple process, and all students would learn mathematics in the same manner. Thus, it was hoped, exposing the preservice teachers to this aspect of student learning would enable them to realize that students have to Fold Back to previous understandings and Images many times and that they have to re-work Images so that they can develop a meaningful understanding of a mathematical concept. It was hoped that this would help the preservice teachers realize that student understanding is always in a state of development, and that at times they develop incomplete and incorrect understandings, not as the result of bad teaching, but as a necessary aspect of learning.

2.4.2. Feature 2: Interventions

Interventions are means by which an individual's thinking is stimulated by an action, either internal or external, to re-evaluate his/her present working. Pirie and Kieren (1991b) classified Interventions as Provocative, Invocative or Validating. A Provocative Intervention is an Intervention that prompts a student to consider an outer or more sophisticated level of understanding; an Invocative Intervention prompts him/her to see the need to Fold Back; a Validating Intervention establishes the student within a level, encouraging him/her to some form of expression (Kieren & Pirie, 1992). While the teacher often plays an important role in Interventions, in the context of students' developing understanding it is the student's <u>response to</u> or <u>action on</u> the Intervention that determines its nature (Pirie & Kieren, 1994b). Thus, although a teacher may pose a question that is intended to be a Provocative Intervention, for example, it may actually Invoke the student to Fold Back in order to re-establish some concept before being able to elaborate on it (Kieren *et al.,* 1995). Interventions may thus take a form different from that intended, and Towers (1998a), with an interest in determining the role of the teacher in supporting and encouraging student learning, "began to investigate the complex web of interactions that interweave to support the growth of understanding in the classroom" (p. 41).

2.4.2.1. Themes of Interventions

Working within an enactivist framework, Towers (1998a) believed that "understanding can be interpreted as being dependent on, but not determined by, the actions of the teacher" (p. 43) and was interested in teacher talk and teacher questioning strategies, but also in the interactions and Interventions between participants in the learning process, specifically in how teacher Interventions occasion student growth of understanding. Rather than considering when a teacher should Intervene in student learning, Towers (1998a, 1999) recognized the need to ask questions about how teachers Intervene. Initially applying the three categories of Intervention (Invocative, Provocative and Validating), and recognizing the need for "a space for discussion of understanding and cognition which recognizes the interdependence of all the participants" (Towers, 1999; p. 162), Towers (1998a) found that the three categories did not adequately describe the complexity of the observed actions in the moment. Purposefully choosing active verbs to describe Interventions in action, Towers (1998a) identified fifteen Strategies involved in Interventions:

1. Checking: The teacher determines if students are paying attention and/or confirms their understanding.
2. Showing and Telling: The teacher presents material, often giving new material and not checking for understanding.
3. Leading: The teacher directs the students, through a series of guided questions, toward a specific answer or procedure.
4. Reinforcing: The teacher repeats for the purpose of confirming the statements of a concept.
5. Inviting: The teacher presents open-ended questions which encourage alternative interpretations.
6. Clue-giving: The teacher purposefully directs the student to a line of thinking.
7. Managing: The teacher engages in administrative or behavioral tasks.
8. Enculturating: By alluding to the larger mathematical community, the teacher introduces the student to conventional notations and language.

9. Blocking: The teacher prevents a student from following a particular path.

10. Modeling: The teacher explicitly models a process or thought pattern.

11. Praising: The teacher commends an individual or the class as a whole.

12. Shepherding: The teacher subtly nudges or coaxes the student on a particular path.

13. Rug-pulling: The teacher deliberately destabilizes the student's thinking.

14. Retreating: The teacher deliberately withdraws, leaving the student to think about his/her work.

15. Anticipating: Anticipating the student's thinking, the teacher actively attempts to remove obstacles before they are encountered.

2.4.2.2. Teaching styles

Realizing that the teacher does not cause learning to happen, but provides occasions for it to happen, Towers (1998a) considered turning points in the student's understanding – those instances where he/she extended to an outer layer or Folded Back to an inner one. Drawing together student Mappings of developing understanding with teacher Interventions, she then attended to the teacher Interventions that occasioned cognitive shifts, defining occasioning as "a situation in which growth of understanding is allowed for, not caused" (Towers, 1998a; p. 229). Using these observations, she identified three major Teaching Styles, later referred to as Intervention Styles (Towers, 2001) which differ from Intervention Strategies in that a strategy usually involves only a brief interaction while a style implies a general pattern of teaching. The identified Teaching Styles are: 1. Showing and Telling, 2. Leading, and 3. Shepherding. A teacher typically displays one dominant Style, while calling on a repertoire of Strategies. Certain Intervention Strategies are typical of particular Teaching Styles. Table 2 below summarizes these classifications. Consistent with Towers' (1998a) observations that Showing and Telling and Leading are the most common Teaching Styles, Truxaw, DeFranco and McGivney-Burelle (2002) observed that discourse within the classroom tends to be univocal, rather than dialogic, implying these two styles are dominant in the teaching setting.

Showing and Telling	**Leading**	**Shepherding**
Reinforcing	Clue-giving	Clue-giving
Enculturating	Blocking	Inviting
Modelling	Anticipating	Retreating

Table 2: Towers' intervention styles and related teaching strategies

Referring to Smith (1996), Towers states: "[T]eaching by telling allows teachers to build and maintain a sense of efficacy by defining a manageable content base and by providing clear descriptions of how they should present the content" (Towers, 1998b; p. 29). Such a Style places the teacher-as-authority, and breaking away from this may be cause for discomfort. Towers therefore (1998a) describes a "dilemma of trying not to tell" (p. 225) as problematic for teachers, and identifies the reform movement as being partially responsible as it has encouraged the active participation of students in their own development of mathematical understanding. In elaborating on this, she notes that enactivism provides an environment that may make teachers more comfortable in assuming a less directive role. By realizing that the growth of student understanding is, as stated earlier, dependent on but not determined by the teacher, she believes that teachers will be able to assume a more Shepherding Style of teaching which she feels is more conducive to students developing understanding than are Leading or Showing and Telling.

Many preservice teachers, having experienced 'teaching by telling', a style of teaching which is consistent with Towers' Showing and Telling and some aspects of Leading, tend to re-enact this in their own teaching. For this study, it was hoped that if preservice teachers were aware of different forms of questioning as well as their relationship to the different forms of teacher Interventions it would encourage them to reflect on their own ideas and Styles of teaching and how these would impact student understanding.

2.4.2.3. Towers' observations on the use of Interventions

Towers (1998a) suggests that there exists a kind of continuum of Intervention streams where the focus of the viewer shifts from teacher to student as Style of teaching moves along the continuum and that, at the same time, the interactivity of students and teacher also shifts. Expectation of response to questioning shifts from rhetorical, or low level questioning where the student's response involves guessing what the teacher is thinking, to questioning where there is little doubt as to the answer, to questions which provide the student with the opportunity to develop ideas through interaction (Towers, 1998b, 2000). In this light, Towers (1998a) noticed that some students experienced jumps in understanding. Prior to this, it was believed that a student moved through each layer of understanding as outlined by the Pirie-Kieren theory, although the length of stay could be very short. Towers noticed that jumps seemed to occur with inviting Interventions for brighter students, and at these times they were able to demonstrate a strong understanding of one aspect of understanding while developing a broader knowledge base.

Introducing preservice teachers to the different aspects of teacher Interventions and the effect they have on student learning, as outlined by Towers, it was hoped would help them in dealing with their own dilemmas. Thus, instead of using Clue Giving, Anticipating or Blocking Interventions to ease the situation for the student, it was hoped that they would realize that they could leave the student to struggle with the concept so as to develop his/her own understanding. This would return "control of the learning to the students and offer them the space to reach conclusions without direct intervention from the teacher" (Towers, 1998a; p. 217-218). To make preservice teachers aware of these aspects of teaching, it was felt, would help them in focusing on their own methods and Teaching Styles and of other possibilities that exist.

2.4.3. Feature 3: Don't Need Boundaries

As previously mentioned, Don't Need Boundaries are indicated on the Pirie-Kieren model by the darker rings (see Figure 1) which separate the model into four subsections and which represent increasingly abstract modes of thinking. Crossing over a Don't Need Boundary is an important step in a student's developing understanding in that it frees him/her from the constraints of less formal and less sophisticated action of concentrating on specific cases and allows for more general and abstract thought (Kieren & Pirie, 1991; Pirie &

Kieren, 1991a). The previous layers are embedded within the new layer and can be accessed if and when needed, but the individual working at the new level does not need to constantly reference them. With connected understanding, he/she can, however, Fold Back to the previous levels when confronted with a problem where existing practices do not provide a solution.

2.4.3.1. The importance of the crossover from Property Noticing to Formalising

As mentioned earlier, Formalising is the first level at which the student works at an abstract level of mathematical thinking. Much of the original research involving the Pirie-Kieren theory provides examples of students working at the inner layers of the model, from Primitive Knowing through to Formalising. Less has been written about working at the outer layers even though students of all ages can work at them. One reason that less is known about working at the outer levels may be that teachers often perceive a need to move students beyond the Don't Need Boundaries as quickly as possible so that they are able to work at the Formalising level. At this level, students are able to do their mathematics assignments, often with speed and accuracy. However, if prematurely Formalised, the student may be able to act but not understand (Kieren & Pirie, 1991). Polanyi (1983) would say this is the difference between konnen (knowing how) and wissen (knowing what) and that understanding is a process, grounded within an individual which co-emerges within an environment. In agreement, Pirie and Kieren (1994b) indicate that for true understanding to take place, a student's informal understanding must be sufficiently developed before an action by the teacher can successfully Provoke understanding at the Formalising level. The student must be able to apply the abstract meaning to the symbols from within his/her own Image-based system if this understanding is to take place. Being dropped into the Formalising level by being given the abstractions and/or formulas before he/she has had time to Property Notice affects the individual's mechanism of growing structural understanding and this in turn may hinder his/her further development.

For this study, it was considered possible that preservice teachers may need to work at their own understanding of how students develop an understanding of mathematics as they had already reached or passed the Formalising level of understanding in a learning environment that worked for them – that is, in an education system in which they were able to learn. However, since they had been good at mathematics the preservice teachers may have experienced Towers' jumps in understanding, and may not realize, unless it is specifically drawn to their attention, that most students will need to spend time at the Image Making and Property Noticing levels and that they will form incomplete and incorrect understandings of concepts. The tendency for the preservice teachers to want to push students into the Formalising level may be a strong, albeit an inappropriate action for a student struggling with the concept. By exposing preservice teachers to the tenets of the Pirie-Kieren theory, it was hoped that they would become aware of the need for students to develop their own understanding in order to make meaning of the mathematical concept being considered.

2.4.4. Feature 4: Complementarities

Within each of the six inner layers of the Pirie-Kieren Theory for the Growth of Mathematical Understanding, there exist two mathematical activities (see Figure 3). Referred to as Complementarities, these activities consist of an appropriate Mathematical Action and the appropriate Mathematical Expression which reflects upon and justifies the action (Kieren, 1992).

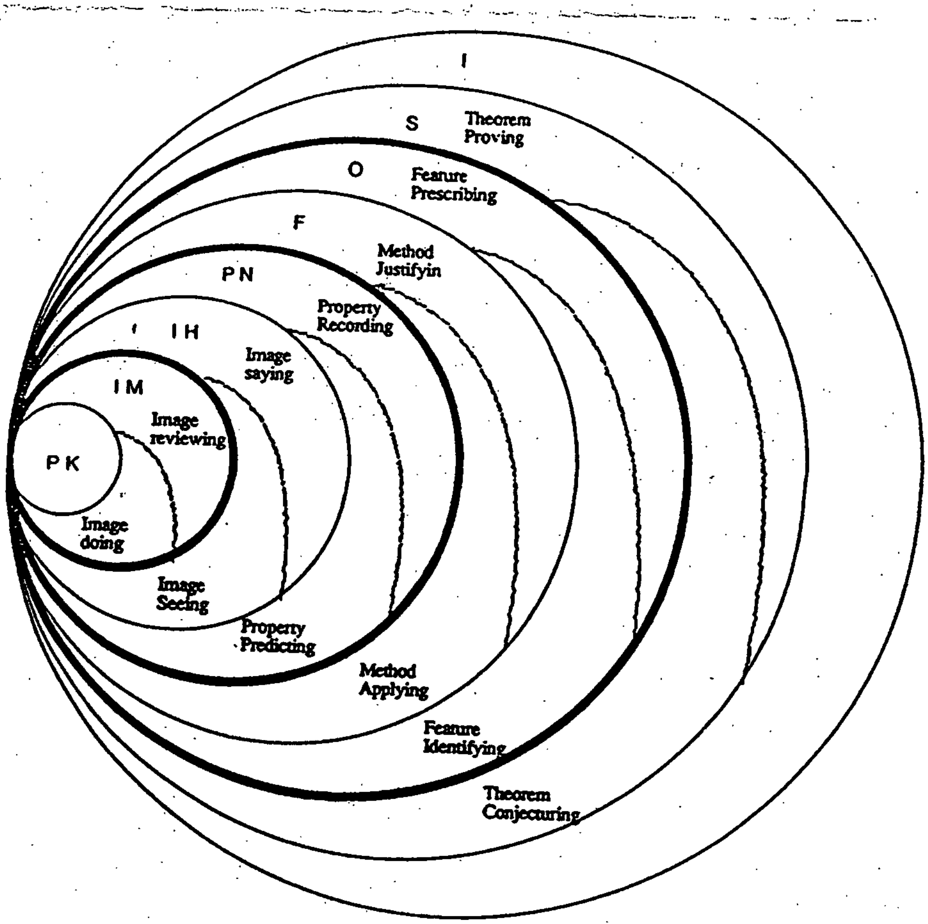

Figure 3: The Pirie-Kieren model incorporating the Complementarities

Data indicates that the <u>expression</u> of the <u>action</u> is an important and necessary process in the developing understanding of a student. An individual can engage in an action such as Image Making, but it is the expressing of that action that allows the individual to reconsider and consolidate understanding so he/she can progress to the next layer (Kieren, Mason, Davis & Pirie, 1993).

Acting can be a mental or a physical act, depending on the mathematical maturity of the individual. Expressing need not be verbal but it is often the verbalization that facilitates abstraction or provides the occasion for an individual to perceive the need to Fold Back and correct or enlarge his/her understanding (Kieren & Pirie, 1991). Thus, Expressing at an inner level is important in the student's construction of understanding at an outer level as it provides an orienting action facilitating abstraction (Kieren & Pirie, 1991) and affects his/her own acceptance of comments made by others as being appropriate to his/her personal understanding (Pirie & Kieren, 1994a). Through Expressing in a more conventional or appropriate manner, and by refining his/her expression, the student comes to internalize the meaning of the mathematical object or action.

Lastly, while it is essential that one engages in Acting activities before Expressing, there is also a need to reflect before expressing. Reflecting, therefore is a component of the Acting activity, and considers how previous understanding was constructed. Expressing articulates what was involved in the actions (Pirie & Kieren, 1994b). Within a level of developing understanding, there is a constant to-and-fro movement between Acting and Expressing (Kieren *et al*, 1993; Pirie & Kieren, 1994b). Only through some form of Expressing can an individual progress to the next level, and only through some form of externalization (visible expressing) can an observer infer what construction of knowledge has taken place. While for the teacher an Action may exist to create an object, for the student the Actions are often central with the objects created and Expressing of the results indicates his/her understanding of the Action.

Prospective teachers have often not given sufficient thought to how students come to an understanding of a mathematical concept and may think that the student understands the concept because he/she can complete assignments. Through discussing with the preservice teachers in this study how students come to make meaning of mathematics through Acting and Expressing, it was hoped that they would begin to think more about the meaning making involved in learning and that they would think about their own learning and that this would be used to inform their understanding of how their students learn mathematics and of how they can design lessons that allow for the students to develop their own understanding. By reflecting on their own learning, it was hoped that they would think about possible incomplete and/or incorrect Images that students might develop along the way.

2.5. Extensions and elaborations

In a manner similar to the growth of mathematical understanding as described by the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding, affiliated researchers (e.g. Towers, Martin, Davis, Covey, etc.) have been extending and elaborating the theory, in effect, Folding Back to re-address, re-define and broaden the understandings of the different layers and features within it as well as trying new applications. Since 1998 a series of publications have dealt with the elaborations and extensions as developments have co-emerged.

2.5.1. Elaborating the enactivist nature of the theory

In their initial research and Mappings, much of the discussion of Pirie and Kieren entailed students working at the inner levels of understanding. In 1998 they made a conscious effort to examine further the more formal layers of developing understanding. Specifically, they considered the character of the Formalising level and this led to increased discussion into the enactivist nature of the theory. Understanding as knowing in action (defined as prolonged actions, which produce a range of responses not predicted or suggested by the teacher) became prominent in and central to the discussion (Kieren & Pirie, 1998). Observation of students <u>in action</u>, always an integral part of the theory as a means of understanding understanding, became even more prominent as a way to gain insights as to how others might Provoke a student to more sophisticated levels of thinking.

The simile of set forming and set ordering emerged as a useful one through which to discuss observer understanding of learners' emerging understanding from Images to Formalisings (Kieren, 2001). Understanding in action involves "continual generation and collection as well as working with and expressing" (Kieren, 2001; p. 227). Formalising requires a method, and a conscious ability to justify it "supported by actions which are like one is ordering or has ordered a set in some way and works with that ordering" (Kieren, 2001: p. 224). Thus, in the process of coming to Formalised understanding, one must have an Image or

Images and then be able to delineate properties of them to fit criteria in an orderly and sequential fashion, and set forming can be seen as a way of deliberately forming a collection that obeys the rules for inclusion into a set. Formalisings can be achieved through pathways other than producing formulas and applying them and can be exhibited without the use of formalized methods (Kieren & Pirie, 1998).

Within the preservice teacher education program, one must consider the preservice teachers' Images and possibly Formalisings of the meaning of teaching and learning mathematics as these will affect the manner in which they view teaching and learning the subject. It is possible that through too early developing Formalisings of the meaning of teaching and learning, the preservice teachers will not be flexible enough to reflect upon the process that their students must enact in their developing understanding. For this study, therefore, it was considered important to have the prospective teachers consider these processes and discuss their own understanding through the lens of this Pirie-Kieren theory so that they would have a foundation on which to build their own processes and understandings.

2.5.2. Alternative forms of Mapping

Mapping on the Pirie-Kieren model is often limited to a single set of eight nested circles (refer back to Figure 1). This, however, is a simplistic view of the model. Pirie and Kieren indicate throughout their research that coming to a better understanding is a dynamic process of back and forth activity. In developing understanding, an individual may have to Fold Back to collect and/or Fold Back to a different topic and work there. Indicating the complexity of this developing understanding, Pirie & Martin (2000) offer a multifaceted representation of the process of developing understanding which involves several interconnected models to indicate how a student's thoughts weave in and out of the necessary topics in a dynamic, enactive process of developing understanding (see Figure 4). This Mapping clearly indicates the complexity of developing understanding as well as the ability of the theory to capture some of that complexity. This is precisely what was needed for my research.

As a "pragmatic move resulting from the difficulty [faced] in an attempt to represent a students understanding over a period of several months" (Towers, 1999; p.164), Towers adapted the Pirie-Kieren nested circle Mapping to a more linear model (see Figure 5). This Mapping allowed her to observe patterns, and the connection between the student action and the teacher Intervention that may not have been so evident using the traditional model. While this appeared problematic for some individuals who felt that the integrity of the theory was lost in this linearity, a member of the audience at a presentation (Borgen, 2001) pointed out that the concept of nesting could be maintained by 'fanning' Towers' mapping.

Borgen and Manu (2002) introduced the concept of 'clustered confusions' in their mapping of a student's misunderstanding of a concept by using a 'broken circle' (see Figure 6) to indicate a grouping of semi-related but confused ideas which the student was not able to clarify and which resulted from disconnected understanding.

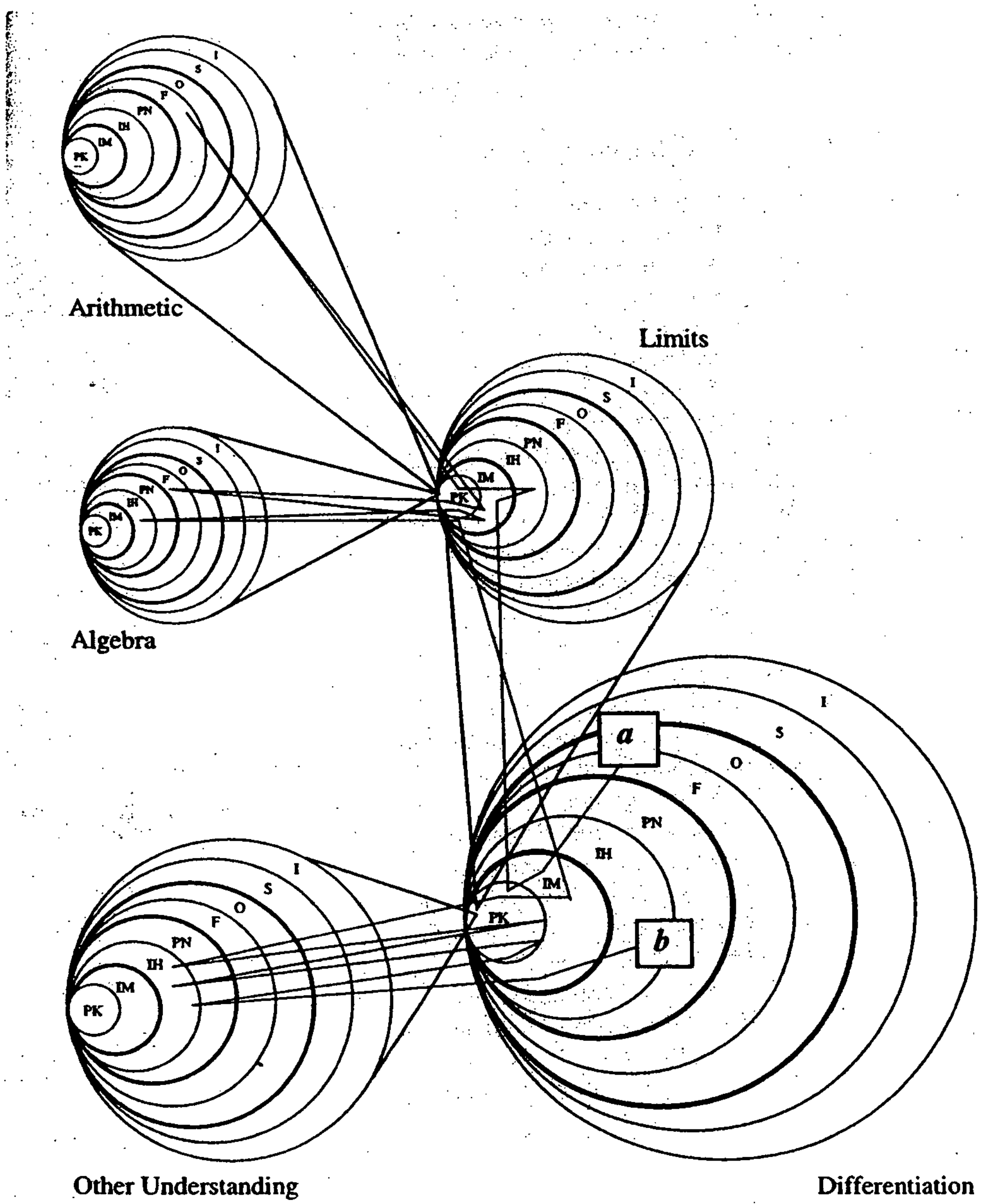

Figure 3. Ann's path of growth of understanding of differentiation.

Figure 4: Pirie and Martin's Complexity Drawing (Pirie & Martin, 2000)

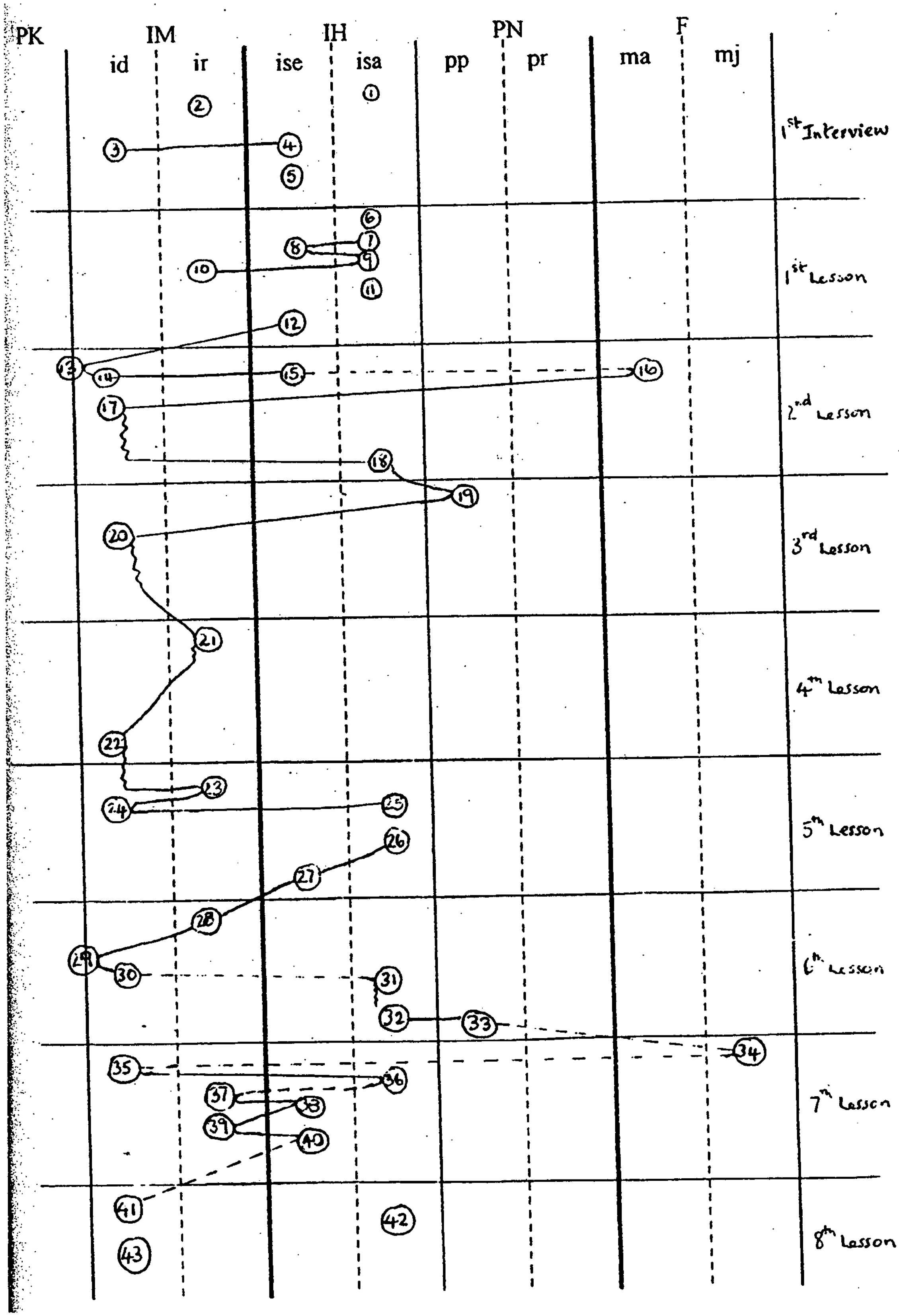

Figure 5: Towers' Linear Model (Towers, 1998)

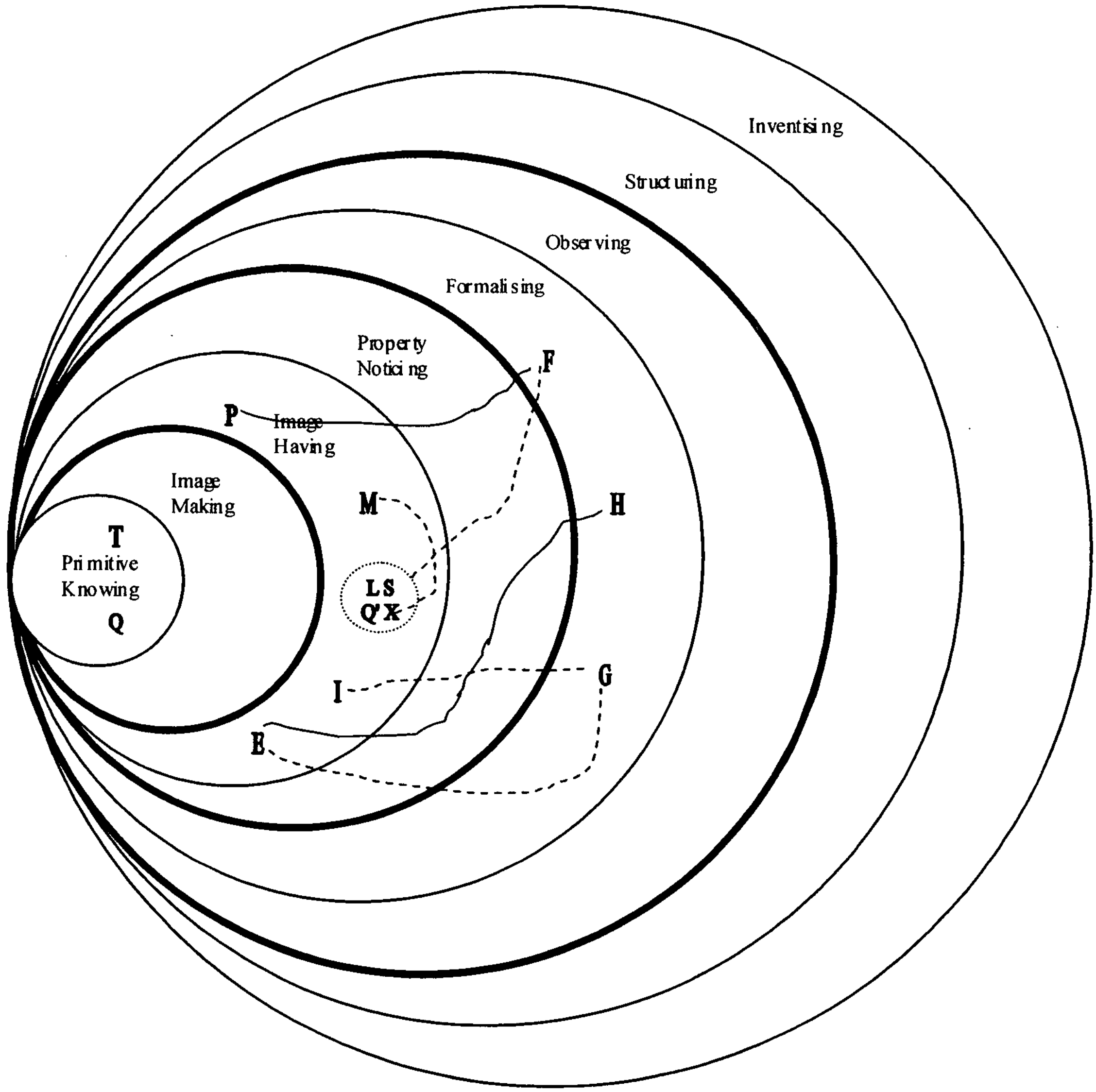

Figure 6: Borgen and Manu's Clustered Confusions Mapping

2.5.3. Collective understanding and shared understanding

While the Pirie-Kieren theory allows for seeing different Images formed by different students and through this to make sense of their deliberations and the emerging understandings, little research has been done in applying it to group situations. Pirie and Martin (2000) discuss the idea of understanding as shared during interactivity but note that the understanding changes on an individual level.

In tracing the growing understanding of two students working together on a problem, Towers and Davis (2002) found that a teacher Intervention could prompt two students to Fold Back simultaneously, creating 'coupled activity' or 'overlapping understanding' developing into "a tightly entwined structural dynamic ... in which it becomes clear that each student is complicit in the unfolding of the other" (p. 326). Although this does not indicate specifically that there is collective understanding, it does indicate that growth of understanding can be a collective action and that the collective action can "simultaneously occasion differing understanding (which in turn prompts further collective action as they struggle to understand each other and the prompts from the teacher) and converge on shared understanding" (Towers and Davis, 2002; p. 331). While this is not specifically detailing collective understanding, consistent within the enactivist environment on which the Pirie-Kieren theory is based (an intertwining of the activities of teacher, student, and environment), Towers and Davis (2002) conclude: "The collective structural dynamic of the two students, the teacher, and the mathematical content presents the possibility that the mathematical understanding is shared" (p. 332).

Martin (2001), drawing on the theory of enactivism and Davis' (1996) definition of listening, drew the metaphor of teaching and learning as listening and sharing. He states that the Pirie-Kieren theory "provides a way to consider understanding which recognizes the interdependence of all the participants in an environment" (Martin, 2001; p. 246). In considering understanding not as a state to be achieved but as an ongoing phenomenon whereby individual understanding is entwined with that of the environment and the other participants, he has used this as a basis to explore the collective understanding of a class, using teacher listening as a means of exploring images and misconceptions within a group dynamic.

2.5.4. Prospective teachers' growth of mathematical understanding

In teaching mathematics, a teacher must call upon not only his/her own understanding of mathematics, but also on his/her understanding of teaching strategies, of the way students learn, and his/her own personal philosophy of mathematics. Berenson, Cavey, Clark, and Staley (2000, 2001, 2002, etc.) have used the Pirie-Kieren theory as a conceptual framework within which to study the growing <u>mathematical</u> understanding of prospective teachers. Preliminary results of a longitudinal study indicate that Folding Back is essential in developing this understanding and that conversations with others are useful Interventions to occasion it. As prospective teachers have few teaching strategies developed or understood, Cavey, Berenson, Clark and Staley, (2001) suggest that providing them with the opportunity to work together on a lesson plan facilitated their mathematical understanding so that they were better able to incorporate different Images of the task at hand and more clearly see the connections between college mathematics and school mathematics.

After several preliminary studies (Berenson and Cavey, 2000; Berenson, Clark and Staley, 2001; Clark, 2001), Cavey (2002) observed that in planning a lesson, a teacher must Fold Back to his/her own understanding of the topic to be presented, collecting mathematically precise information from different Primitive Knowings that are necessary to the lesson. Each Folding Back 'thickens' and informs the understanding, affecting how he/she thinks about the topic and how he/she prepares the lesson to present it to students as effectively as possible. Cavey and Berenson (2005) further demonstrated 'lesson plan study' as useful in helping preservice teachers grow in their understanding of a particular mathematical concept. By working together, they noticed that it is often the saying or verbalization of their Images that Provoked growth in their understanding. "Image saying is noted as a catalyst for growth because of its potential for creating self-awareness in understanding for the learner" (Cavey and Berenson, 2005; p. 186).

Thus, many aspects of the Pirie-Kieren theory have recently been developed to expand and elaborate its usefulness, both as a theoretical and a practical tool, for studying the growth of mathematical understanding, and have provided a languaging by which to discuss the concepts and processes deeply embedded within the theory. Pirie & Martin (2002) indicate that the theory can be used to help explain incomprehensions and sources of problems as students relate new topics to previously learned material. Martin's (1999, 2000, 2001, etc.) work helps to explain how students can overcome problems by appropriate Interventions and Folding Back. In this light, Cavey (2002) suggests that further use of the Pirie-Kieren theory in the area of teacher training would be appropriate in helping prospective teachers consider the teacher-learner situations as they interact with the mathematics and the students. By recognizing that each activity co-emerges in the environment to become a part of the growing awareness of the individual or group, as teachers they may be able to use concepts from the theory to help inform their teaching and to promote growth of mathematical understanding.

2.6. Modification of the Pirie-Kieren theory to a new context

In discussing the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding, it is apparent that it has:

> evolved into a theory that can be used by teachers or researchers as a tool for listening and observing in the context of mathematical activity .... It is a theoretical thinking tool for a person who is observing mathematical understanding and who might be interacting with students who are engaging in understanding activities (Pirie and Martin, 2000; p. 129).

While the above presentations indicates the usefulness of the Pirie-Kieren theory in discussing the developing understanding of an individual on a specific topic in mathematics, for this study, it must be pointed out, that the theory was used in two distinct, interconnected ways. Firstly, it was presented to the SMIP students as a <u>theory</u> which they could use to consider the growth of mathematical understanding of their students. They were, in addition, encouraged to use the theory as a theoretical framework to think about their own growth of understanding. Secondly, it was used by the researcher, in a modified form, to Map the growth of understanding of teaching and the growth of understanding of learning of the SMIP students. One challenge of this study, then, was to develop definitions that would strictly adhere to the principles of the theory but which could be applied to this new context. Below I present the modified definitions that will be applied in the Chapter 7 analysis of the growth of understanding of teaching and learning of the preservice teachers. These definitions were verified through consultation with Dr. Susan Pirie as to their integrity.

2.6.1. The modified definitions

PRIMITIVE KNOWING (TEACHING & LEARNING): As with the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding, Primitive Knowing is the information the individual brings to the situation except that which is related to the topic. It includes his/her background experiences that affect his/her outlook on life.

IMAGE MAKING (TEACHING): A preservice teacher is involved in Image Making of what it means to teach mathematics when he/she is trying to make meaning out of a lecture, a video or a presentation by considering what teaching involves. When a preservice teacher observes a peer presentation and tries to determine if that activity could be useful in teaching, he/she is engaged in Image Making.

> *The preservice teacher observes another's lesson in which a student was asked to explain his incorrect solution and says: "You know, I never would have thought of having the students explain their solution when they did it*

incorrectly, and I'm thinking, maybe you can teach by taking what they did and building on it."

IMAGE MAKING (LEARNING): Image Making regarding what it means to learn mathematics occurs when the preservice teacher considers activities and thinks what students might learn from them.

The preservice teacher notices that students MIGHT have different styles of learning. He/she may have observed a presentation by another preservice teacher and realized in the discussion afterwards he/she has made different Images of what the manipulatives were for from those that some others made. He/she says. "Oh, I got something quite different out of the activity than Mary did. I was trying to write it all out in x's and y's, but she just laid out the tiles to make a rectangle and said 'Ta-da!' "

IMAGE HAVING (TEACHING): When a preservice teacher, through demonstration, indicates that he/she believes a certain technique is involved in the process of teaching mathematics, or when he/she states: "Teaching involves … ." or "A part of teaching is…", he/she has crossed the Don't Need Boundary into Image Having of what is the activity of teaching. He/she has determined for him/herself that this particular activity is part of the definition of teaching.

The preservice teacher states: "A teacher should provide the background information so that students will know what they have to know." Or he/she indicates: "Teaching involves knowing where your students have made mistakes and fixing that."

IMAGE HAVING (LEARNING): Image Having of what it means to learn mathematics occurs when the preservice teacher indicates that a particular activity can help students develop understanding. He/she is able to state: "Learning involves being able to ..."

The preservice teacher states: "I think that getting a good mark on a test indicates that a student has learned this unit." Or he/she indicates: "You can tell that learning is taking place if the student can use the right words."

PROPERTY NOTICING (TEACHING): For Property Noticing to occur about what it means to teach mathematics, the preservice teacher will consider his/her own Images of teaching mathematics and will examine them to try to reconcile them into a meaningful understanding of the process. This activity may be quite subtle and he/she may not explicitly state his/her beliefs, but may reason through observed activities in trying to rationalize how they can be defined as teaching.

The preservice teacher states that he/she believes that teaching is telling the students what you want them to know, and realizes that he/she also thinks that the teacher should provide opportunities for the student to discover for him/herself. He/she looks at the conflicting Images and tries to determine how they can be accommodated into a meaningful definition of teaching. Or, he/she might say: "You know, it is really interesting all these different ways people present their material, and I never would have thought of some of them. I'll have to think if they will work for me."

PROPERTY NOTICING (LEARNING): Property Noticing of a developing understanding of what it means to learn mathematics occurs when the individual realizes that he/she has different ideas of what it means to learn and he/she tries to reconcile these different ideas so that he/she can explain what he/she thinks learning means.

The preservice teacher says: "Johnny seems to have learned the work because he got a 90% on his test. But, Paul also seems to know what he's doing because he explains it in class - but he didn't do well on the test. Maybe it's not the math that is the problem. Maybe he just can't write tests. "

FORMALISING (TEACHING): After a preservice teacher has thought about his/her Images (Property Noticing), he/she may come up with a summary statement of the various Images. Formalising occurs when he/she defines an activity as teaching mathematics.

The preservice teacher states: "Teaching is just a matter of leading students to the right conclusion through guided questioning." Or, the preservice teacher presents several examples and each time leads the students to the solution with leading questions.

FORMALISING (LEARNING): After considering his/her various Images of learning (Property Noticing) a preservice teacher may reach the conclusion that a certain activity indicates that the corresponding mathematics has been learned and that it has been demonstrated in a particular manner. Formalising occurs when he/she offers a definition (visual or verbal) of leaning.

A preservice teacher may state: "A student has learned a concept when he can apply a formula correctly in a problem."

OBSERVING (TEACHING & LEARNING): At the Observing layer of understanding of what it means to teach mathematics, there is an interface with what it means to learn mathematics. The preservice teacher makes a connection between teaching and learning and analyzes how different teaching methods/approaches affect the learning process of the student. Thus, at this level, the preservice teacher's understanding of what it means to teach mathematics becomes aligned with what it means to learn mathematics. In the process of learning to teach, this is an important step, and after having arrived at this level, it is likely the preservice teacher will Fold Back to previous understandings and re-examine them. Observing is being undertaken when the preservice teacher consciously works to align his/her definitions of teaching and learning. It is the corresponding layer to Property Noticing, working on Formalisings as opposed to Images, and also to Image Having where he/she is working on Primitive Knowings and new ideas come together to form new Images.

> *The preservice teacher has Formalised that teaching is leading the students in the right direction and that learning is demonstrated by getting good marks on a test. He/she states: "I showed the students the best way to do these questions and look, they all did well on the test following my method. If I continue with this method of teaching my students will do well."*

STRUCTURING (TEACHING & LEARNING): Structuring of the developing understanding of what it means to teach and learn mathematics occurs when the preservice teacher uses his/her Observings to make a decision about what he/she believes is the essential activity of teaching mathematics based on his/her acceptance of what this implies and how it relates to his/her understanding of what it means to learn mathematics. He/she has passed through the next Don't Need Boundary and has given meaning to the activities that he/she believes describe the process of teaching mathematics based on his/her perception of the implications this has on the learning of mathematics. When an individual answers the question for him/herself: "What does it mean to learn mathematics?" and has reconciled this with his/her answer to: "What does it mean to teach mathematics?" or answers the question: "How can one teach mathematics effectively?" he/she has Structured his/her understanding of teaching and learning.

> *The preservice teacher may conclude: "It is the teacher's job to be sure that a student has had enough practice so that he/she can apply mathematics to any problem" (i.e. uses a Formalisation that learning depends on practice, and teaching is therefore providing practice, Observing the necessary connection between teaching and learning).*

INVENTISING (TEACHING & LEARNING): Inventising of what it means to teach and learn mathematics involves a preservice teacher in thinking 'outside the box'. He/she examines other alternatives to those observed and attempts to determine of a method of teaching mathematics whereby the structure creates a different learning environment for the learning of mathematics by the students.

The preservice teacher, after considering various forms that define teaching (Observing), reaches a conclusion about the basic structure that, in his/her mind unites teaching and learning, then problematizes it and wonders: "What if I didn't tell them if they were right or wrong? What if I just left them to check for themselves? Would that be teaching? Would they be learning?"

Within these definitions, it must be pointed out that, at any level, the preservice teacher may have only partial, or even incorrect Images of teaching and/or learning as is the case with the Pirie-Kieren theory regarding the learning of mathematics and that there is recursiveness implied in this discussion.

2.6.2. The modified Pirie-Kieren model

The Pirie-Kieren model for the Dynamical Theory for the Growth of Mathematical Understanding consists of eight nested circles to indicate the embeddedness of the growth of mathematical understanding (see Figure 1). In discussing and analyzing the growth of the understanding of teaching and learning of mathematics of the preservice teachers in this study, it was found that at the Observing level, there was a union of the two understandings as defined above. That is, when an individual reached the Observing level of understanding of teaching and learning he/she realized that there was a connection between the process of teaching of mathematics and the process of learning of mathematics. The model, therefore, had to be adjusted to take this into consideration. To reflect this, the modified model represents the teaching and learning as two separate sets of embedded circles for the first five levels of developing understanding and the next three levels 'umbrella' them. The modified model is presented below in Figure 7 and will be used is Chapter 7 to Map the growth of understanding of teaching and learning mathematics for four preservice teachers.

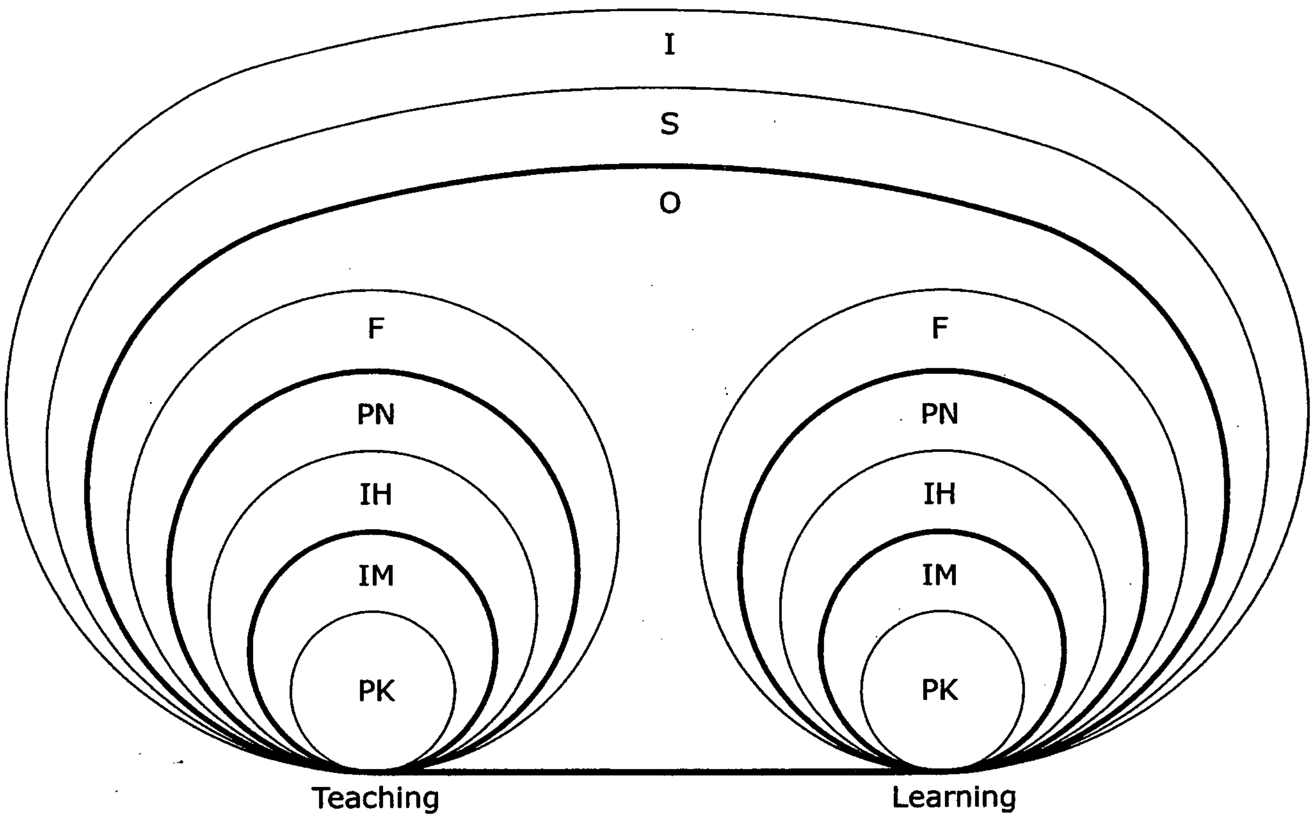

Figure 7: Borgen's Modified Dual Model for the Mapping of the Growth of Understanding of Teaching and Learning.

Chapter 3

Review of Literature - Teacher Education

3.1. A context for discussion of preservice teacher education

Over the years, teacher education has undergone a multitude of changes. Licensing and training/education of teachers in North America have always involved both a theoretical and a practical component, although not always with the same emphasis (Shulman, 1998), and have generally involved a component which provided opportunities for classroom observations and practice teaching (Feiman-Nemser, 1990; Shulman, 1998). As philosophies shifted, emphasis shifted from the education of teachers to the training of teachers (Britzman, 1991; Shulman, 2002) with the assumption being that theory learned in isolation would lead to informed practice. However, often the theoretical methods learned in isolation were difficult to apply in context (Hiebert, Gallimore & Stiger, 2002). An interest in education by cognitive scientists, who saw student experience (and thus the student-teacher experience) as a source of knowledge, brought new developments in teacher education. Emphasis on the importance of prior experience led to a redefining of "[t]he recurrent challenge of all professional learning [in] negotiating the inescapable tension between theory and practice" (Shulman, 1998; p. 517). Piaget's developmental stages created a context for learning through experience (Brown, Cooney & Jones, 1990) and practical experience in the classroom came to be viewed as an apprenticeship for experimenting with different techniques, as well as a means by which to verify and develop new theory. Emphasis within education shifted from "preparation of citizens, the enhancement of individual abilities and talents, [and] the intellectual attainment in academic subjects" (Corrigan & Haberman, 1990; p. 197) to the teaching of basic skills as prerequisite to all other goals. The effect on teacher education was to "narrow the broad knowledge base formerly required of teachers to emphasize their preparation with know-how in teaching extremely narrowly defined basic skills" (Corrigan & Haberman, 1990; p. 197). More recently, teaching has come to be thought of in a more holistic manner (Brown *et al*, 1990) and as a continual process whereby all those involved in the process learn from each other (Shulman, 1998). 'Being in' is part of the process of 'becoming' and teaching can be seen as "an epistemology wherein the knowledge that is teaching is only evidenced in the practice that is teaching" (Roth & Tobin, 2002; p. 21). This encourages one to objectify experiences, reflect upon them and thus engage in activities which allow for the construction of self as 'teacher' (Britzman, 1991). From this perspective:

> [L]earning to teach is not a mere matter of applying decontextualized skills or of mirroring predetermined images; it is a time when one's past, present and future are set in dynamic tension. Learning to teach - like teaching itself - is always the process of becoming: a time of formation and transformation, of scrutiny into what one is doing, and who one can become (Britzman, 1991; p. 8).

Thus, in the process of learning to teach it is assumed that research into teacher education programs has played an important role as it is through the teacher education process that beliefs and opinions about teaching and learning may be reconfigured from those that are learned through one's own experiences in a classroom as a student.

While there have been general changes in the philosophies surrounding teacher education, there have also been changes specific to subject content areas. In the area of mathematics, specifically, the shifts in philosophy of education over the past century, possibly beginning with Dewey's progressive education movement and its emphasis on understanding curriculum as a connected enterprise, have had a profound influence on the philosophy and activities that are considered mathematics. There was a structural change in the teaching of mathematics in the 1960's, manifested through *the new mathematics* which emphasized the axioms and structure of the subject. This placed emphasis on connected understanding as well as on producing more mathematically sophisticated students (Brown *et al.*, 1990). This resulted in an increased emphasis on the need for teachers to know more mathematics. However, a *competency-based teacher education*, rooted in behaviourism, with an emphasis on accountability soon followed this movement. Less emphasis was placed on the need for subject content knowledge and more on the need to learn and teach the basics of the content (Corrigan & Haberman, 1990). Recently, schools have been more inclusive of students, and students of all abilities and skills remain in and are required to complete higher level mathematics courses than before. There has been a movement to a problem solving and application approach as "an effort to learn and appropriately invoke heuristics to deal with not only what is already known ... but also what is unknown" (Brown *et al*, 1990; p. 641). Problem solving and application approaches again create a need for deeper mathematical understanding on the part of the teacher. However, the understanding that is needed may not be the theoretical knowledge base that allows them to obtain a degree in mathematics. Research has indicated that teachers need to better understand the manner in which students learn as many of the students they face will have differing learning styles than their own. It has become important, then, that preservice teacher education programs include some theories of learning in their curriculum, with theories specifically related to subject content considered useful.

Thus, during the entire latter part of the twentieth century, with the shift toward *mathematics for all*, there has been an emphasis on the "concomitant necessity to redefine mathematical content in terms of the capacities of good workers ... [and] the historically resilient desire for basic mathematical literacy now tends to be paired with the need to develop sound reasoning skills" (Davis, 2001; p. 18). These changes in view as to the nature of mathematics have involved a change from seeing mathematics as a set of theorems and postulates to be learned, to seeing mathematics as a human endeavor in which communication skills are an essential component so that students will be better able to involve themselves in the technological changes and complexities of the twenty-first century (Cooney & Shealy, 1997). Ours has become "a culture that is obsessed with and utterly reliant upon mathematics" (Davis, 2001; p. 19). Mathematics has become an integral part of our lives, defining the way we look at the world, often being thought of as much a humanity as a science. This has created a need for a different form of teacher education.

From the above discussion, it is obvious that there have been many changes in the philosophy of teaching and in the philosophy of teacher education over the past century. Freiberg and Waxman (1990) claim, however, that there has actually been little change in teacher education. The resistance to change by teachers and administrators, and the popularization of education that has led to an erosion of the authority by teacher educators in determining the requirements for entrance and exit of programs, they claim, is partially the cause of this.

While there may have been few overall changes in teacher education at the time of Freiberg and Waxman's writing, there have been some specific changes in the area of mathematics and mathematics teaching as well as mathematics teacher education that have resulted from recent changes in views of what mathematics is and what it means to understand mathematics. Also, recent studies on the scholarship of education may be having an influence on university education, which in turn may be influencing teacher education and thus teaching. These studies, aimed at considering the relationship of *teaching* as scholarly activity compared to *research* as scholarly activity, have resulted in changes in the consideration of what is defined as scholarly activity and of the teaching done at the university level. This may be creating opportunities for changes in teacher education.

3.2. A discussion about the scholarship of teaching and teacher education

Teacher education and teaching in schools are obviously interrelated. Recent research on the scholarship of teaching, with an emphasis on teaching at the university level where, in the past, it was considered sufficient that one know one's subject matter to be able to teach it, now indicates that it is important that teachers at any level have the skills to teach. "The process of teaching the subject matter of a discipline forces academics to clarify the big picture into which their specific research specialization fits" (Marsh & Hattie, 2002; p. 604). Scholarship of teaching emphasizes that it is important to be aware of the constructions of knowledge on the part of students. Imparting that knowledge is insufficient, and it is seen as important to develop strategies that reward deep, not surface, learning. Existing research, however, indicates that there appears to be little relationship between research as scholarly activity and scholarship of teaching, except to a minor degree, in faculties of education (Hattie & Marsh, 1996; Cunsolo, Elrick, Middleton & Roy, 1996; Seijts, Taylor & Latham, 1998; Kreber, 2002). As a result, much of the preparation of teachers is done by people trained in research methodology rather than by people trained in pedagogy (Cunsolo *et al.*, 1996). Thus, as students at the university move from their subject area specialty into a teacher education program they come with subject content knowledge, but not necessarily with pedagogical content knowledge.

Study of the scholarship of teaching reconfirms the effects of Lortie's (1975) *apprenticeship of observation* in which he indicates that, as students in a class, people are actually identifying what teaching means to them. They are learning to teach in the manner in which they were taught. As a result, the majority of undergraduates view learning as "acquisition of knowledge, memorizing, utilization and/or use of knowledge" (Hattie & Marsh, 1996; p. 531) as this is how they were taught. Students entering teacher education programs often tend to view learning as a reproduction of what is stated as opposed to the more creative "abstraction of meaning, and learning as an interpretive process aimed at understanding reality" (Hattie & Marsh, 1996; p. 531).

Scholarship implies integration, application and transmission of knowledge, and, it would seem, if preservice teachers became more aware of the manner in which learning takes place, the manner in which students come to understand subject content material, they may approach teaching in a more enlightened manner. Thus, although scholarship of teaching is usually considered a university activity promoting an institutional environment supportive of teaching and learning (Kreber, 2002), the concepts may be applied to teacher education by providing a language and understanding that permit ways of viewing practice and of promoting critical reflection of practice. There is a seen need to share teaching as a discipline, going to the literature to learn about the successes and failures of others, grounding one's own theory in the work of others if one wishes to improve teaching (Shulman, 2000) since, although the opposite is not true, scholarship of teaching research has found that increased research does improve teaching (Hattie & Marsh, 1996).

This study, then, which exposes preservice mathematics teachers to theory for understanding – particularly, theory for developing understanding of mathematics - is a step which fits into this thinking of understanding teaching as a scholarly activity. Therefore, regardless of the substantive content area that teachers are called upon to teach, all teachers should have a working knowledge of pedagogical principles and practices which can be brought to bear in the classroom.

3.3. What is teaching?

Viewing teaching and learning from the point of view of complex theory, Davis *et al*, (2000) indicate that these processes do not begin and end in the classroom but are a dynamic whole with the environment, with learning dependent upon, but not determined by, teaching. The process of learning, although individually experienced, is socially constructed and determined by cultural circumstances (Davis *et al.*, 2000). Clarke and Erickson (in press) define teaching as "the professional practice of engaging learners in the construction of knowledge directly related to a particular area of study" (p. 22).

To be a teacher involves the internalization of the teacher's role and the teacher's professional identity. This assumes knowledge and ability, as well as the essential norms, values and attitudes of the role (de Ponte & Brunheira, 2001). Teaching involves helping the student see the relationship between experience and the classroom (Ball, 1997). In keeping with the more holistic view of teaching previously mentioned, in order to teach effectively, not only must one be familiar with the subject matter, have an understanding of the learning process, and have knowledge of the students and the culture of teaching, one must also have a sense of the *complexity* of how they fit together (Ball &

McDiarmid, 1990; Britzman, 1991; Davis *et al.*, 2000). Thus, teaching is a complex process in which:

> The teacher's *task* is "to assist the learners in the continuous process of understanding previous experiences and knowledge in terms of new events and circumstances (Davis *et al*, 2000; p. 91 (italics added).

> The *goal* of teaching is to assist students in the development of intellectual resources that enable them to *participate in*, not merely *know about*, the major domains of human thought and enquiry" (Ball & McDiarmid, 1990; p. 438) (italics added).

> "The *art* of teaching is to maintain the difficult balance between richness of detail and narrowness of focus ... *effective teaching* is more a matter of listening than telling – that is, of attending and responding to the sense that learners are making" (Davis et al, 2000; p. 10) (italics added).

> [T]eaching, *fully understood*, is an extraordinary process of creating understanding and knowledge" (Shulman, 2000; p.6) and is "a very time-consuming activity in that it requires sound knowledge of one's discipline as well as a good understanding of how to help students grow within, and perhaps beyond, the discipline" (Kreber, 2002; p. 9) (italics added).

The focus of teaching should therefore not be on how one teaches, but rather on how students learn (Davis *et al.*, 2000). The Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding is a theory about the developing understanding of mathematics. Knowledge of this theory, it would seem, provides an appropriate place to start a dialogue in the education of prospective mathematics teachers.

Teaching should involve the possibility of 'spaces opening in the moment' and the need and ability to respond and react to them. Plans for teaching must be developed so as to expect surprises and to choose from different possibilities in order to help students notice what they know (Davis *et al*, 2000). Teachers "must be able to work with content for students in its growing, unfinished state, they must be able to do something perverse: work backward from mature and compressed understanding of the content to its constituent elements" (Ball, 2000; p. 245). Put succinctly, Clarke and Erickson (in press) indicate that there can be little distinction between teaching and learning: teaching involves continuous learning, with listening and learning on the part of the teacher as an integral part of the dynamic.

3.3.1. The role of theory in teaching

As mentioned in Section 1.3, there occurs in teacher education, and thus in teaching, a perpetual struggle between mediation of theory and practice. Learning environment research indicates that each theoretical framework and/or theory involves a set of constructs and provides only one window through which to view the learning and the environment. Activity theory indicates that we are co-creators of our learning and have the power to act based on subject, object and tools available in a community guided by rules and regulations (Roth & Tobin, 2002). Thus, awareness of different educational theories and research may help develop different forms of praxis that apply to different teaching situations. The "significance [is] not just understanding that much of teaching occurs as *habitus* that generates practices but also of understanding the critical need to connect theories to the experience of teaching" (Roth & Tobin, 2002; p. 296). While theory cannot capture the practice/enactment of the situation, "it is in the explication of why particular roles might be productive that theorizing can be beneficial" (Roth & Tobin, 2002; p. 168).

Having a theoretical framework with which to discuss their understanding of teaching and learning, therefore, would be useful to preservice teachers. The Pirie-Kieren theory gives preservice teachers, not only a theory upon which to build their understanding of student understanding, but also a language with which to discuss and consider student, as well as their own, learning. A teacher must continually question what he/she teaches and how he/she teaches it. It is essential that teachers understand the value of theories as <u>models</u> of understanding which can then be adapted to specific content. "Using theory to inform practice then cannot, and should not, occur in the form of a direct application of a recipe to a given problem. It rather implies a decision-making process on the part of the teacher" (Kerber, 2002; p. 11). The Pirie-Kieren theory provides a forum that can Invoke preservice teachers to question their own understanding of mathematics and the manner in which they believe students learn mathematics.

While indicating that research may not be the best foundation on which to build an essential professional knowledge base, Kieren (1997) notes that research should be used for its exploratory power and that ideas drawn from theories will not tell one how students will use material but rather provides a framework for considering different approaches students might take. By listening *to* the student, the teacher is able to inform his/her practice through developing a deeper understanding of the student's understanding. Theory may help in this exploration. “Because teaching is a highly cognitive task, access requires sharing otherwise covert thought processes by reading the works of others, actively seeking feedback, and discussing teaching decisions with other” (Seijts *et al*, 1998; p.160). Teaching itself thus becomes a means of theorizing, and separation of theory and practice is impossible (Shulman, 2000).

3.3.2. The role of reflection in teaching

Reflection on action provides an important method by which teachers can monitor their knowledge in action, using teaching acts as objects of reflection, while applying theory (Roth & Tobin, 2002). Mature reflection on teaching shares many of the same features as the scientific method in that it involves questioning, evidence, interpretation based on that evidence, critical appraisal, and conclusions drawn from this. Through thinking about teaching and discussing it with colleagues, teachers can improve learning by forcing the consideration of strategies that make the content knowledge more accessible to the learner (Cunsolo, *et al*, 1996). Clarke and Erickson (in press) argue that teaching becomes routine without enquiry and reflection and that practice becomes less professional when teachers are not inquisitive about how students learn. Thus, along with Schön (1988) they agree that how students learn - their actions and words - must be the cornerstone on which to build activities to help students make sense of the world. Seijts, Taylor and Laytham (1998) suggest that research indicates that observation of others teaching and reviewing video-tapes of oneself teaching could help in identifying concerns and create opportunities to critically analyze alternative models of teaching.

In this study, preservice teachers observed themselves and other preservice teachers during presentations and were encouraged to think about the teaching and learning that took place. As well, they observed videos of teachers in regular classrooms and discussed these as they proceeded through their teacher education classes. While they were *in situ* during their practicum and had a chance to observe students developing understanding, and when they returned to their university classes, they discussed these activities in terms of the teaching and learning that took place. In this manner, they were encouraged to discuss their understanding of teaching and learning and the Pirie-Kieren theory provided them with a format and a vocabulary with which to do so meaningfully.

3.3.3. The role of knowledge in teaching

Teaching invokes many kinds of knowledge such as knowledge of the subject matter, knowledge of pedagogy, pedagogical content knowledge, and knowledge of how students learn. As well, there is a practical knowledge involved in teaching, some of which may only be learned through experience, observation and/or through discussion/reflection.

3.3.3.1. The role of subject content knowledge in teaching

For instruction to be meaningful to a student, it must be based on what he/she knows and the teacher must base instruction on the cognitive connections he/she assumes exist and can be further developed (Cooney & Shealy, 1997). Based on the tenets of the Pirie-Kieren theory, this implies that the teacher must try to access the student's Primitive Knowing and his/her Images. To do this, the importance of subject content knowledge on the part of the teacher cannot be downplayed (Ball, 2000; Davis *et al*, 2000; Thompson, 1992; Ball & McDiarmid, 1990). "Subject matter, or content knowledge includes knowledge of facts, concepts, and procedures that define a given field and understanding of how those facts fit together. It also includes knowledge about knowledge – where it comes from, how it grows, how truth is established" (Feiman-Nemser, 1990; p. 221). As well, subject content knowledge includes understanding about the subject, "the relative validity and centrality of different ideas or perspectives, the major disagreements within the field ... how claims are justified and validated, and what doing and engaging in the discourse of the field entails" (Ball & McDiarmid, 1990; p. 440).

A well-developed understanding of subject matter on the part of the teacher, and a deep conceptual understanding of a topic allow the teacher to respond to all aspects of the pupils' needs more readily so that he/she can teach the content for understanding (Prestage and Perks, 2001; Davis *et al*, 2000; Thompson, 1992). "Knowing content is also crucial to being inventive in creating worthwhile opportunities for learning that take learners' experiences, interests and needs into account" (Ball, 2000; p. 242) since learners learn in different ways. A teacher with inaccurate and/or narrowly conceived understandings will not be able to respond to the sense the students are making of the content, and this lack of understanding will be passed on to his/her students through failure to challenge their misconceptions which will perpetuate a state of not knowing (Carter, 1990). Thus, the teacher must know the subject content and be able to approach it from different perspectives in order to make it meaningful to the student. He/she must be capable of seeing him/herself as an authority in evaluating materials and practice while being flexible enough to change those beliefs in the face of conflicting evidence (Cooney & Shealy, 1997). The teacher, at the very least, must know the subject content well enough to be able to determine if the student's <u>expression</u> of it indicates understanding. "Both the ability to create flexible tasks and the capacity to follow student leads clearly depends on fairly broad and flexible understanding of the task at hand" (Davis *et al*, 2000; p. 143). Thus, the ability to respond to the sense that the learner is making requires subject content knowledge. Since "meaningfulness of all knowledge, regardless of its level of abstraction, derives from one's own experience with the world" (Davis *et al.*, 2000; p. 26), knowledge of the students here and now is important in knowing how to approach the teaching of a topic to make it understandable to students. According to the Pirie-Kieren theory, this implies a need to access the students' Primitive Knowing.

3.3.3.2. The role of mathematical content knowledge in the teaching of mathematics

Disciplinary knowledge impacts the organization and instruction of a subject as well as the teacher's ability to re-present it to students (Carter, 1990). Greater knowledge of mathematical content leads to more conceptual teaching strategies in mathematics, the ability to relate it to outside activities and the ability to engage students in problem solving activities (Steinburg, Haymore & Marks (1985) cited in Carter (1990)). An important component of mathematical content knowledge is understanding possible representational systems that students may have for a topic as well as the potential misconceptions they may have (Shulman, 1986). Simple subject content knowledge is likely to be insufficient to be an effective teacher of mathematics (Ball, 2000). Alternative images of that knowledge are necessary so that one is able to understand the mathematical concepts from the different perspectives the students might have, as well as the ability to be able to relate it to activities that are involving for students. Based on the Pirie-Kieren theory, this implies that the teacher must have a vast repertoire of Images on which to draw if he/she is to understand student representations. What is important, then, in teachers' mathematical knowledge, is not just what they know, but also how they know it and how they represent it.

3.3.3.3. The role of pedagogical content knowledge

A central tenet of teaching is "an understanding of how to recognize, evaluate, and implement activities with pupils' learning in mind" (Carter, 1990; p. 295). However, since people tend to become teachers in subject or content areas in which they did well, not all teachers know the content in a way that they can use it to help students. Being an effective teacher involves having the "ability to transform subject content through a range of techniques, suitable examples, analogies, etc. to make the content more assessable and meaningful to the learner" (Smith, 2001; p. 117) or to be able to translate one's own knowledge into classroom curricular events that make that knowledge accessible to students, using their interests and motivations to learn a particular topic (Carter, 1990). This is pedagogical content knowledge, knowledge which goes beyond knowing subject content knowledge, to knowing ways of presenting it to make it comprehensible to others (Shulman, 1986).

> Teachers draw on their own wits, observations, intuitions, and articulations of what it is that they do and how they do it to guide their practice. Often idiosyncratic, rarely documented, and always complicated, this knowledge is the essence of their teaching and bears the imprint of an authentic rendering of the complexities associated with a highly social and inherently situated practice (Clarke & Erickson, in press; p. 27).

> When teachers develop expertise, they not only mediate theoretical knowledge about education with their knowledge derived from personal teaching experience, they also develop increasingly better ways of helping students understand the subject matter. When expertise in the discipline is effectively combined with knowledge of how to teach, the latter being derived from both educational theory as well as experience, we witness the construction of *pedagogical content knowledge* (Kerber, 2002; p. 15).

Pedagogical content knowledge thus implies the blending of content knowledge and knowledge of pedagogy and involves knowing and applying "useful ways to conceptualize and represent commonly taught topics in a given subject, plus an understanding of what makes learning those topics difficult or easy for students of different ages and backgrounds" (Feman-Nemsor, 1990; p. 221-222). Within the context of the Pirie-Kieren theory, pedagogical content knowledge implies being aware of the different Images students might hold, and the different levels of understanding at which they might be working.

Pedagogical content knowledge thus involves formal and informal knowledge constructed about teaching by combining knowledge of the discipline with knowledge of teaching, declarative knowledge (which can be gained through reading books and articles about teaching and theory), along with procedural knowledge about teaching (which can be gained through experience) and implicit knowledge of self-regulation (Shulman, 1986). It can be considered as knowing both the how and the why of the content or subject matter - knowing the structure of and knowing the potential sources of error within the topics. This can only be achieved through being able to recognize the knowledge in its basic, tactic dimensions (Shulman, 1986), through being able to understand the content from different perspectives and being able to discuss it in different manners and at different levels.

3.3.3.4. The role of pedagogical content knowledge in mathematics teaching

Prestage and Perks (2001) refer to the distinction between "*learner-knowledge* in mathematics [and] *teacher-knowledge* in mathematics, the former is the knowledge needed to pass examinations, to find the solutions to mathematical problems; the latter is the knowledge needed to plan for others to come to learn the mathematics" (p. 102) (italics original). In order to design appropriate classroom activities, analysis of topics through reflection "to reconsider their own personal understandings of mathematics" (Prestage & Perks, 2001; p. 108) is necessary for an individual to develop the *teacher-knowledge* with which to inform teaching that will challenge and meet the needs of the students. Thus, teaching mathematics for understanding requires knowledge of mathematics, knowledge of student learning and knowledge of mathematics pedagogy. This implies the need to have both conceptual knowledge, the "rich relationship, linking new ideas to ideas that are already understood" (Stump, 2001, p. 210) and procedural knowledge, the "formal language and symbol systems, as well as algebraic algorithms and rules" (Stump, 2001; p. 210) of the content. "Pedagogical content knowledge highlights the interplay of mathematics and pedagogy in teaching. Rooted in content knowledge, it comprises more than understanding the content oneself" (Ball, 2000; p. 245-246). It is important that teachers know something about how students come to know the mathematical content, what representation and Images students might have, as well as being able to identify the teacher's role in developing this. The Pirie-Kieren theory, being a theory for the learning of mathematics provides a good base upon which to build an understanding of possible Images and representations that students might have as well as considering where they might experience difficulties or obstacles in their developing understanding.

3.3.3.5. The role of practical knowledge in teaching

While Prestage and Perks (2001) agree with Shulman and others that pedagogical content knowledge is important, they indicate that even it is insufficient. Experience, personal theories and opinions, and ways of interacting are also important aspects to teaching. These affect one's actions in a situation and one's ability to reflect-in-action. Only when combined with practical knowledge, "the knowledge teachers have of classroom situations and the practical dilemmas they face in carrying out purposeful action in these settings" (Carter, 1990; p. 299) or with "the kinds of knowledge that practitioners generate through active participation and reflection on their own practices" (Heibert *et al*, 2002; p. 4) does pedagogical content knowledge help inform the teacher as to how students learn. Informed by one's practical knowledge, knowledge of self, and one's values and intentions can enhance teaching possibilities. This knowledge, it seems would best be developed through reflection on one's own learning, on the process of developing understanding, and on one's beliefs about the subject matter.

3.3.4. The role of attitudes and beliefs in teaching

It is commonly believed that the manner in which a curriculum is implemented is dependent on one's knowledge and beliefs about the nature of the subject, about teaching and about how students learn (Pajares, 1992; Thompson, 1992; Cooney & Shealy, 1997). "What teachers and students believe about [the subject matter] is influential to how they understand themselves, others, knowledge, and future possibilities" (Davis *et al*, 2000; p. 234). Pajares (1992) suggests that not only do beliefs influence how tasks are defined and organized but also that they predict behaviour to a greater extent than does knowledge. Hattie and Marsh (1996) point out that there are "two teaching conceptions: teaching that involves preparing concepts so that the student can learn without too much interpretation, and teaching that aims to involve the students in interpretive and structuring work" (p. 530). These, they say, coincide with two configurations of teaching; the instructional method and the student-centered method. If knowledge is believed to be objective and as involving creation or discovery, "it would seem consistent to think that it requires the transmission and absorption through a separately conceptualized teaching process" (Marsh & Hattie, 2002; p. 629). But if knowledge is viewed as a product of communication and negotiation, then "the relationship between teaching and learning becomes an intimate one" (Marsh & Hattie, 2002; p. 629).

However, beliefs are difficult to determine. What one says one believes is often more a reflection of what one thinks one should believe than of one's true beliefs (Davis *et al*, 2000). Also, "[t]here is no reason to believe that one's beliefs are consistent and logically arranged, as if considerable thought had gone into the networking of beliefs. Some beliefs are held in isolation from one another ... [others] can exist in separate clusters, segregated and protected by one another" (Brown *et al*, 1990; p. 652). If beliefs are held peripherally or in isolation, they may contradict other beliefs held in the same manner, leading to apparently contradictory incorporation of activities (Cooney & Shealy, 1997).

Reflection on experience is considered paramount to learning, particularly in determining what one truly believes about teaching and learning. This reflection can occur only when the individual is able to "step out of the stream of direct experience, to re-present a chunk of it, and to look at it *as though* it were direct experience, while remaining aware of the fact that it is not" (von Glasersfeld, 1991, p. 47, cited in Cooney and Shealy, 1997; p. 100) (italics original). This reflection can be used to help one determine what one truly believes about the nature of the subject and the nature of student learning and thus impacts the manner in which one implements the curriculum.

3.3.4.1. The importance of changing attitudes and belief about the nature of mathematics

Recent changes in the way we view and enact mathematical activity has led to changes in what we consider mathematical proof and mathematical understanding (Cooney & Shealy, 1997). The naïve idea that mathematics is hierarchical, that mathematical learning involves the learning of *a priori* knowledge in an axiomatic, deductive system with irrefutable solutions and that performance indicates that learning has taken place has been challenged (Davis & Hersh, 1986; Brown, *et al*, 1990; Cooney & Shealy, 1997). These changes in perspective have required more than simple changes in questioning and ways of problem posing in mathematics, but necessitate changes in one's beliefs of the nature of mathematics and to challenging one's philosophic views of the nature of the subject and its assessment. This shift in perspective in mathematics has led to a widening view of the nature of mathematics and has created uncertainty in a field previously considered certain, and to a questioning of basic beliefs and epistemological perspectives (Franke, Fennema & Carpenter, 1997). Testing for understanding, formerly considered easy in mathematics because of the structure and organization of the subject, has become more complex and problematic (Cooney & Shealy, 1997; Brown *et al*, 1990).

The paradigmatic shift in thinking about the nature of mathematics forces one to believe that students <u>have</u> mathematical understanding which they can further develop without direct instruction. It forces the tenet that the teacher can learn from listening to the students and that this listening can and should be used to inform instructional decisions. The teacher, with the students, develops an understanding of mathematics negotiated through interaction in a cultural setting (Cobb, Jaworski & Presmeg, 1996). However, the inevitable resistance to change by some teachers who hold absolutist views about mathematics and mathematics teaching means that they are unlikely to participate in such activity (Cooney & Shealy, 1997; Hart, 2002). Thus, while there are fundamental changes in the philosophy of what mathematics is, and this affects the education of students and teachers, there are pockets of resistance in the field.

In order to actuate change in the teaching of mathematics, "mathematics teachers must change the way they learn before they can change the way they teach" (Pereira, 2000; pg. 205). This will need to be a requisite of teacher education programs, and it can possibly be approached through presenting preservice teachers with alternative views about the nature and meaning of mathematics, presenting them with ways of thinking about and presenting material that make them question their own understandings, and by allowing them to practice the new approaches in a safe environment.

3.4. What is effective mathematics teaching?

"Effective mathematics teaching requires understanding what students know and need to learn and then challenging them and supporting them to learn it well' (NCTM, 2000; p. 16). For mathematics to be meaningful and understandable to the student, the teacher must be able to merge these various aspects of teaching and learning together into a meaningful whole that creates a more understandable unit for both. Effective mathematics teaching goes well beyond knowing the subject matter to being able to draw on a broad background of mathematical knowledge, having some understanding about how students learn it, and having the ability to reflect upon what mathematics means to the student as well as to oneself (NCTM, 2000). It involves knowing different representations that students may hold and being able to see the relationship between these representations and the students' understanding of them. In many cases preservice teachers must learn these representations and relationships during their teacher education programs as their previous learning, based on their own ability to do mathematics with relative ease, may not have involved the concept of different understandings.

3.5. What is learning to teach?

Learning to teach may be thought of as "a process of learning to understand, develop, and use oneself effectively. The teacher's own personal development is a central part of teacher preparation" (Feiman-Nemsor, 1990; p. 225) and forms the basis for his/her personal development as a teacher.

Students entering professional programs other than teaching must define their position in that profession, but students entering the field of education are entering, albeit at a different level, the same scene (the classroom) that they have previously experienced (Pajares, 1992). They have already formed an image of teaching based on their own experiences in the classroom and those experiences and Images will serve as the filter through which their teacher education is processed (Carter, 1990). Thus, grounded in one's personal experiences, one's own student experiences and fashioned through one's university experience, the transformation from student to teacher will involve knowledge of self, knowledge of how students learn, knowledge of subject matter, and knowledge of the system into which they fit (Davis *et al*, 2000).

However, learning is a social interaction and learning to teach must therefore also be thought of as a social action, intertwined in a socio-cultural setting in which individuals work together to create shared meanings. Experience is an important aspect in learning to teach (Smith, 2002). During the process of learning to teach, there is a need for observation of others to become familiar with the task as well as the structure and nature of the learning process (Roth & Tobin, 2002). This observation of the classroom is from a different perspective than that which the individual has previously experienced, and it is through the process of seeing and experiencing theories in practice, theories that had been presented in abstract, that they will become internalized (Smith, 2002). It is important that preservice teachers, in the process of learning to teach, become active inquirers into their practices (Clarke & Erickson, in press) so that they will develop into inquiring teachers who are capable of supporting their students' developing understanding.

3.5.1. The effect of prior schooling on beliefs and Images of teaching

"Learning to teach, like teaching itself, is a time when desires are rehearsed, refashioned and refused. The construction of the real, the necessary, and the imaginary are constantly shifting as student teachers set about to accentuate the identities of their teaching selves in contexts that are overpopulated with the identities and discursive practices of others" (Britzman, 1991; p. 220). The ability to experiment and to try something new, however, is affected by the fact that in the process of learning to teach an individual is constrained by various forces including the political context in which he/she works (Britzman, 1991), the textbook used (Stump, 2001), the school climate (Roth & Tobin, 2002), one's own ideas and experiences (Lortie, 1975; Britzman, 1991) as well as one's conflicting position as both a student and a teacher (Roth & Tobin, 2002).

As mentioned, Lortie (1975) refers to the years spent as a student in school as the *apprenticeship of observation* and suggests that these years may be more influential than any teacher-training program. Feiman-Nemser (1990) concurs: "Many people believe that teacher education is a weak intervention incapable of overcoming the powerful influence of teachers' own personal schooling or the impact of experience on the job" (p. 229).

As students, prospective teachers "draw from their subjective experiences constructed from actually being there" (Britzman, 1991; p. 3) in developing their understanding of what teaching means. The *habitus* that they have developed through their own experience - the practices, perceptions and expectations that they have - form the catapult for what and how they perceive the experience of teaching (Roth & Tobin, 2002). Thus, in developing an Image of what it means to teach or to learn, prospective teachers are greatly influenced by their own experiences in the classroom (Lortie, 1975; Britzman, 1991; Pajares, 1992; Smith 2001; Roth & Tobin, 2002). However, these Images are often superficial and may impose stereotypes on what teaching and learning mean (Britzman, 1991). This may impact the view that the individual has of the content knowledge which, in turn, will affect his/her future practice (Pajares, 1992).

Many preservice teachers have a naïve idealism in which they assume that they can be taught the right way to teach while others believe that they already know it, being unrealistically optimistically biased in favor of the belief that they themselves hold the attributes most important for successful teaching (Shealy, 1993; Cooney & Shealy, 1997). Still others hold views that are incomplete and possibly dysfunctional. For example, they may perceive the teacher as 'friend' or 'guide' to students (Carter, 1990). Other prospective teachers enter the teacher education program with a view of teaching as a replication of the *learner knowledge* they hold (Prestage and Perks, 2001) but with the conviction that they will avoid the teaching styles that they found unhelpful or inappropriate when they were students, not realizing that for different learners, those styles may be helpful. As Britzman's (1991) Jack August stated: "[There is a] desire to do something creative, something other than the traditional lecture model so endemic both to the high school classes observed recently and to his own educational biography" (p. 128). These preservice teachers are convinced that they will teach differently than they were taught (Britzman, 1991; Davis *et al*, 2000; Hart, 2002). Thus, one's past experiences affect one's views, and it is essential that teacher education programs address these issues.

3.5.2. Effects of previous mathematical experiences on beliefs and Images of teaching mathematics

In the areas of mathematics and mathematics education, the NCTM Principles and Standards (2000) indicate that students' understanding of mathematics and their confidence and willingness to do it are reflections of their own classroom experiences. Those experiences are reflections of their teachers' attitudes and beliefs about teaching and learning and "many teachers' traditional experiences with and orientations to mathematical pedagogy hinder their ability to conceive and enact a kind of practice that centers on mathematical understanding and reasoning and that situates skill in context" (Ball, 1993; p. 162 cited in Stump, 2001; p. 208). In considering the education of prospective high school mathematics teachers, this impacts the experiences and practice of their preservice observations and student teaching experiences.

Thus, in mathematics education, with the changing perspective on the nature of mathematics and mathematical understanding, there is a great need to challenge pre-established absolutist beliefs and practices early in the education program so that preservice teachers will be more likely to accept the uncertainties of the subject, uncertainties which they possibly did not experience in their own learning. The willingness to accept uncertainty must be made strong enough so as to be held, even when confronted by others who hold absolutist views on the nature of mathematics (Cooney & Shealy, 1997). In effect, then, presenting preservice mathematics teachers, at the beginning of their teacher education program, with a theory such as the Pirie-Kieren theory which encompasses the changing understandings of the nature of mathematics and the manner in which students grow in understanding may have an impact on the preservice teachers' understanding of teaching and learning mathematics.

3.5.3. The effect of teacher education programs on beliefs and Images of teaching

Changing beliefs about teaching may be a difficult task since, as mentioned previously, prospective teachers are entering a scene that they have already experienced and, "(f)or insiders, changing conceptions is taxing and potentially threatening. These students have commitments to prior beliefs, and efforts to accommodate new information and adjust existing beliefs can be nearly impossible" (Pajares, 1992; p. 323). Thus, preservice teachers often experience internal conflicts between how they have been taught and how they think teaching should be enacted (Britzman, 1991), between their own beliefs and values and the norms of the traditional setting in which they find themselves (Hart, 2002), as well as between what they are presented with at the university and the reality of the classroom in which they are expected to enact the curriculum in its entirety (Smith, 2002). Even if beliefs are challenged and changed during a preservice program, and prospective teachers indicate a change of perspective on teaching, the strength of their prior beliefs and the pressures of the situation may be such that these beliefs are not enacted or they may dissolve when the individual is confronted with a problematic situation in practice. They may take much longer to move from intellectual belief to practical action. Thus, the question remains as to whether teacher education programs can help prospective teachers develop a philosophy or vision of teacher and teaching and of learner and learning that is different from that which they hold when they enter the program and which is strong enough for them to maintain when they begin teaching (Cooney & Shealy, 1997). The teacher education program must therefore address this issue, as well as the issue of changing perspectives. It must provide the preservice teacher with a view of teaching and learning that is strong enough so that he/she can see the benefits of it and put it into practice. To do so, he/she would have to be able to, not only learn theory, but see it in practice, and have an opportunity to experiment with new perspectives him/herself so as to develop and understanding of the working of the theory.

3.5.4. The effects of the practicum experience on beliefs and Images of teaching

Da Ponte and Brunheira (2001) consider the student teaching experience as a highly valued activity in the process of learning to teach. They believe that diversity of observations or of objects of observation during the student teaching experience provides a means whereby preservice teachers can be helped to become aware of the complexity that is teaching. Field experience should provide prospective teachers with the opportunity to put their theories into practice (Hart, 2002) and while many schools seem to expect them to want to try new ideas (Smith, 1996) other school organizations may resist their attempts at pedagogical experimentation (Britzman, 1991). Thus, preservice teachers are learning to teach by working within the given constraints and a political situation in which interests, values and contexts are used to maintain the status quo, and the orientation toward power and identity may make it difficult for them to develop a critical voice in order to participate in the negotiation of change. The school, its culture and its climate, have a great influence on the experience of and views expressed by preservice teachers (Roth & Tobin, 2002).

During preservice field experience, while sponsor teachers often encourage preservice teachers to try a variety of activities, they do not generally suggest any alternative approaches, but rather focus on ""the Ex's", i.e. explanation, examples and exercises" (Smith, 2001; p.27). Often the sponsor teacher will advise the use of the same method that he/she would use. The student part of the student-teacher necessarily responds to the authoritative/supervisoratory aspects of the program, limiting his/her attempts at trying new teaching techniques. In order to cover the content matter and to meet the expectation and approval of the supervisory teacher, the preservice teacher often finds him/herself slipping back into the manner in which he/she was taught (Britzman, 1991; Davis *et al*, 2000) or the manner that the sponsor teacher advises (Smith, 2001) as the expectations of the in-school sponsor teacher may have a greater influence on the preservice teacher's enactment of the curriculum than the theories and practices learned in the university environment of the teacher education program.

Generally, then, teacher education programs make it difficult for preservice teachers to change in that, in the process of teacher education, there is a pattern of observing and being observed, of evaluating and being evaluated

(Roth & Tobin, 2002), a structure which dictates that a prospective teacher is part student and part teacher (Britzman, 1991). The authoritative/evaluative position of the inservice teacher or university supervisor during preservice practice creates a tension between the theoretical view held by, and the practice enacted by the preservice teacher. As a result, he/she may struggle between realizing his/her own ideologies and subjectivities and the demands placed on him/her by practicing teachers in the existing institutional settings. Caught between the official expectations (spoken and unspoken), the expectations of supervisory personnel, and his/her own desires, interests, expectations and philosophical views on teaching and learning, "[t]he student teacher is confronted with [the dilemma] between tradition and change - ... confronted not only with the traditions associated with those of past teachers and those of past and present classroom lives, but with the personal desire to carve out one's own style, and make a difference in the education of students" (Britzman, 1991; p. 19).

3.6. What can teacher education programs do?

The above discussion concerning what is meant by teaching and by learning to teach and what factors impact these, leads one, of necessity, to consider what teacher education programs can actually do to support and promote changing attitudes, beliefs and practices regarding these. Some researchers believe that early field experience can positively influence the teaching attitude and performance of preservice teacher whereas others believe that it "can foster bad habits and narrow vision" (Freiberg and Waxman,1990; p. 626). Thus, one must consider the role of the preservice education program and how it should be applied if it is to be effective in preparing teachers to teach in the present day classroom.

Prior to field experience, preservice teachers need to be adequately prepared in both content and method (da Ponte & Brunheira, 2001). However, there is, at present, a perceived gap between what is taught at the university in teacher education courses and what is required as a teacher (Prestage & Perks, 2001; Ball, 2000; Smith, 1996). Smith (2001) states specifically: "There is not a close match between the teaching required of the student teachers and the activities that had been suggested in college" (p. 37) and Roth and Tobin (2002) indicate: "What they hear in university courses is generally declarative and procedural knowledge about teaching that has a timeless character … . In their teaching, on the other hand, they have to cope with situations without having time out to reflect on the next move" (p. 5). Thus, the preservice teacher, in his/her practicum, faces two conflicts. The one being that there is a disconnection between the theory he/she learns at the university and the enactment of curriculum that is expected in the school setting, and the other being the disconnection between theory about teaching practice and actions and the reality of the time commitments and response times in the actual classroom.

Teacher education courses should help preservice teachers develop pedagogical content knowledge as well as help them develop alternative teaching approaches (Ball, 2000). To do so, the program should help them know about the pupils (cultural influences, economic influences, etc.) and should help them become aware of key educational influences and school expectations. This can be achieved by focusing on both conceptual and procedural knowledge, which allows prospective teachers to think about students' knowledge in different ways and to expand their own views of a topic while considering areas of possible difficulty and misunderstanding (Stump, 2001). "The overarching problem across many examples is that the prevalent conceptualization and organization of teachers' learning tends to fragment practice and leave to individual teachers the challenge of integrating subject matter knowledge and pedagogy in the context of their work" (Ball, 2000; p. 243). This issue must be addressed through re-organization of content and pedagogical knowledge, combining the two so that preservice teachers learn the how as well as the what to teach within their subject specialty. Unfortunately, teacher education often aids in the conditioning and socialization of the past when methods of instruction are taught without the context of subject matter (Britzman, 1991). The perceived gap between university preparation courses and classroom experience could be decreased if there was a re-examining of what content knowledge is needed by the teacher (Ball, 2000).

In order to bring out and develop both a deep understanding of the pedagogical issues and the subject matter understanding to help understand ways that students come to understand, Roth and Tobin (2002) point out the importance of what they refer to as 'cogenerative dialogue', dialogue which uses current understanding to describe, identify and articulate pedagogical instances and/or problems students might have been experiencing, as a means of framing options for change which reflect on occurrences and problems students might have been experiencing with another. They suggest that through dialoguing with others in the field, knowledge increases in both subject matter and pedagogical matters. Cogenerative dialogue, they feel, should be an integral part of preservice teacher programs because of the deeper structural understanding of the subject and of teaching that is thus generated.

Attention in teacher education programs must be given to how knowledge is constructed as well as to focused-reflection-on/self-regulated-learning-about teaching. "The act of reflecting on beliefs and behaviors allows teachers to make connections between thoughts and actions and to recognize, expose and confront contradictions and inconsistencies" (Hart, 2002; p. 6). "Transforming practice, then, is hinged to the exercise of uncovering core assumptions and webs of belief about what knowledge is (an object? an action?), what learning is (acquisition? transformation?), what schools do (inform? enculturate?)" (Davis *et al*, 2000; p. 41). A requisite of teacher education programs should be to challenge pre-existing beliefs and ideas and to develop practitioners who learn about their students and their teaching through reflective practice involving collective dialogue. The SMIP program was designed so as to accomplish this task.

3.7. Mathematics teacher education programs

Most secondary preservice mathematics teachers choose the area because they were 'good' at mathematics. However, much of the content knowledge that is taught in secondary mathematics is not revisited at the university level (Ball, 2000). This means that secondary mathematics preservice teachers must call upon their own high school experiences when deciding how to teach the content. Lerman (1999) suggests that preservice courses do not provoke students to confront their preconceived notions of teaching mathematics. As a result, many teachers of mathematics enter the profession applying loose fragments of knowledge gleaned from their own learning experiences as to what the art of teaching and the meaning of mathematics are (Ball & McDiarmid, 1990) and do not have at hand even the requisite subject content knowledge to be able to predict student behavior (Ball, 2000).

> In many cases ... education students' understanding of school mathematics lacks the depth, vastness, and thoroughness required to teach well. Understanding coursework, taken as a series of isolated independent courses does little to help students see the connectedness of mathematics within the discipline or outside of it. Students' mathematical knowledge for an undergraduate degree in mathematics is not the same as the mathematical knowledge needed for teaching mathematics for understanding (Nicol, 2002; p. 30).

Mathematics teacher education must provide preservice teachers with the opportunity to develop and practice different representational forms and expose them to nontraditional situations to broaden their perceptions on mathematical understanding (Stump, 2001) if we expect them to practice enlightened teaching methods of the subject matter. Lack of exposure to a rich meaningful knowledge of how students solve mathematics problems may lead to an inability to make coherent connections among different aspects of pedagogical knowledge (Carter, 1990). Mathematics teacher education must help guide prospective teachers in finding a method whereby they can learn/discover alternative teaching approaches and means of understanding student understanding.

Using student work as a means of analyzing and interpreting what the student knows and is learning can help preservice teachers work on learning the content so that they can present it flexibly and meaningfully to students (Ball, 2000). Change in practice is limited for preservice teachers who learn mathematical content differently than they learn their mathematical teaching methods (Hart, 2002). Using content knowledge as the basis for learning pedagogical methods is another way in which teacher education courses are able to support preservice teachers in their understanding of how students learn as well as to help them develop new methodologies (Hart, 2002). Therefore, an integrated approach to learning to teach, an approach which encompasses theory and methodology and which helps develop pedagogical content knowledge, would be a useful approach to teacher education.

Thus, in order for the teacher education program to be meaningful for prospective mathematics teachers, "just as teaching mathematics needs fluid and connected knowledge of mathematics (teacher-knowledge) so too mathematics educators need an articulated, fluid and connected understanding of teaching mathematics education – the teacher knowledge of mathematics education" (Prestage and Perks, 2001). That is, in order to help prospective teachers of mathematics develop in their role of becoming effective mathematics teachers, the teacher-educator must be an effective teacher of teaching how to teach mathematics, being cognizant of subject content knowledge, pedagogical content knowledge and 'pedagogical *teaching* content knowledge', that knowledge which enables him/her to help the prospective teacher become aware of the need for, and to develop the ability to develop pedagogical content knowledge. Only then will the divide between theory and practice be balanced.

3.8. Concluding remarks on teacher education

The meaning of teaching and learning has undergone many changes over the years. The constructivist and enactivist perspectives of recent times have greatly influenced our understanding of how students come to a greater understanding of subject content matter. This is particularly true in the area of mathematics learning which, once thought to be linear and cumulative, is now perceived as a dynamic growth of building upon previous knowledge which involves the need to revisit and rebuild structures to make them more meaningful to the present situation. This has led to a reform movement in mathematics teaching and a need to re-evaluate teacher education programs.

Many of the present preservice teachers have been exposed to this more enlightened/reform perspective, but many have not. If, as Lortie (1975) indicates, beliefs about teaching are developed through years of *apprenticeship of observation* that occur as a student in a classroom, then we may not be on the road to a more enlightened teaching force. Also, as the most recent mathematical experiences of most preservice teachers has been the lecture methods of their university mathematics courses, this may have had an effect on their ideas of teaching and learning so that they will have to be re-initiated into the more progressive methods. Thus, there is a need in the mathematics teacher education program to consider applying new concepts and theories and to present preservice mathematics teachers with theories and applications that address this issue of re-view of the nature of mathematics and of learning mathematics. Preservice teachers need to be exposed to and experience the dynamic of the two changed perceptions if they are to enact them in their future classrooms.

Chapter 4

Review of the literature: Video data and Portraiture

The purpose of this study was to determine whether using the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding as a model enhanced the development of preservice teachers' understanding of the meaning of teaching and learning mathematics. Video was used as the main source of data collection for the first phase of the study. This data was used to paint the portraits of four preservice teachers, Sophia, Lance, Ellie and Wayne, based on their actions and discussions as they progressed through their preservice teacher education program. The portraits, and the data from which they were drawn, were then used to analyze the growth of understanding of teaching and learning mathematics of the four preservice teachers using the modified version of the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding which was specifically modified for this purpose.

In this chapter, I present a review of the literature related to the method of data collection, video, and for the initial stage of analysis, the drawing of the portraits.

4.1. A context for video as a source of data collection

Originally used in ethnographic studies in anthropology, video recording as a means of data collection has been used in quantitative studies for the purpose of counting incidents that were difficult to enumerate in live observation (Bottoroff, 1994). Its use in qualitative studies has increased greatly in the past few decades, particularly in the sphere of educational research, as an aid to understanding classroom culture, interactions and the implication of these (Pirie, 1996b). Video data has also been used in educational research in demonstrating specific teaching methods, as a means of observing and reflecting on one's own practices to improve teaching, and in analyzing student behaviour as a means of determining understanding (Wood, Cobb &Yackel, 1991; Maher, Martino & Davis, 1994).

4.1.1. Collecting video data

While the presence of a video camera might seem intrusive in a classroom situation, establishing a good rapport with the participants and explaining the reason for which the data is being collected, with assurances of the confidentiality of the data will help alleviate problems of behaviour distortion (Marland, 1984). Having the camera present for a period of time before the actual data collection begins may also help the participants forget about its presence or at least be less distracted by it (Bottoroff, 1994; Cudmore & Pirie, 1996). As well, people have become more used to being videoed (Erickson, 1992) and thus tend to be only temporarily affected by the presence of the camera (Mercer, 1995) with the intrusive effect decreasing the longer the video camera is present (Towers, 1998).

While video is a rich source of data collection, there are obvious limitations placed on the quality of the data collected based on the limitations of the recording devices, the lighting, or location of the camera (Marland, 1984). These can obscure the sound and/or the features of the individuals being videoed and in a group situation, it is often difficult to place the camera so that one is able to see the faces of all those involved. The number of and placement of the camera(s) during data collection as well as the quality of the audio must therefore be considered.

Wood, Cobb and Yackel (1991) and Maher, Martino and Davis (1996) are among those who have used several cameras in their video data collection. Combining this data with field notes, students' work, and meetings with teachers, they felt, allowed them to make better selections of particular incidents for further study. Cudmore & Pirie (1996) preferred to use a fixed camera at the back of the room as they felt that this was less intrusive to the classroom action. Using one or two fixed cameras on a tripod Pirie and Kieren (1989, 1991, etc.), in developing their theory of developing mathematical understanding used in this study, focused on small groups of students working on a specific problem for a short time, occasionally drawing these students out of class and recording them while they worked on a specific mathematical task, while at other times recording them in class. In extending the theory developed by Pirie and Kieren, Towers (1998) also focused on a small group of students, videoing them during their regular mathematics classes for the duration of a unit of work and, to obtain more specific data, videoing them working on specific mathematical tasks outside the regular classroom.

Goldman-Segal (1998) chose an interactive method of videoing, tucking her camera under her arm and mingling with the students involved in her research. She and her camera thus became an integral part of the classroom during the year in which she obtained her video data.

Other researchers have found that the zoom capabilities of video data collection allowed for closer observation of individuals without infringing on the individual's personal space (Bottoroff, 1994). This method has been useful in classroom observations in that it allows for focusing on different groups of students at different times without disrupting the class by moving the camera.

In considering the quality of the audio aspect of videoing, simply letting the camera pick up the sounds of the classroom in process is a suitable means by which to view the class in action. An external microphone can be used if one wishes to record the conversations of a small, selected group of students. Cudmore and Pirie (1996) chose small clip-on microphones in order to minimize the background noise. Since these microphones were fairly inconspicuous, students tended to forget that they were there, thus lessening their impact on the collection of data. Towers (1998) used a more conspicuous microphone in her study on teacher interventions. This allowed her to record the 'teacher talk' and the general classroom questions while minimizing the distraction of other students' talk, thus allowing her to focus on the students being studied. It did, however, have a negative effect on one of her subjects who withdrew from the study because of self-consciousness.

Video recorded data is selective in that a conscious decision is made by someone (usually the videographer) with regards to when, what and on whom to focus, and it is limited by what can be picked up through the camera lens

(Bottoroff, 1994; Pirie, 1996b) and the possible bias of the interpreter. "Who we are, where we place the cameras, even the type of microphone we use governs which data we will gather" (Pirie, 1996; p. 553). Even the angle of a shot and the point at which one begins and ends a taping affect the data collected (Goldman-Segal, 1993). Video data is thus data collected in the eye of the beholder, the videographer, and this places limits on the actions that are recorded (Bottoroff, 1994) creating a bias, albeit usually an unconscious one, by the subjective nature of this choice of focus (Pirie, 1996b).

During a video session some members of the group may move out of range of the camera, limiting the interpretive aspect of facial expressions and other physical sign that are important to the data. Also, if an individual who is not part of the research enters the field of focus, unless the researcher can obtain his/her permission to use the data, this portion of the video will have to be eliminated.

Other limitations are implied in video research when the camera operator is not the primary researcher. If the videographer is not the primary researcher, he/she may not shift the emphasis of focus to the specific aspects of the situation that are most significant to the research question and this can affect the richness of the data collected (Goldman-Segal, 1993). Data collected by video recording, therefore, although offering more insights into occurrences than do many other forms of data collection is still incomplete and not without problems.

4.1.2. Advantages of video data

One of the main advantages of using video data in qualitative research is that it allows for re-viewing the data and re-examining actions in light of past and future actions (Mercer, 1995; Pirie, 1996) which reduces the tendency to make premature interpretations of events (Erickson, 1992). As well, video allows one to both hear and see the individuals being studied, thus providing more detailed information than field notes and anecdotal responses. Many of the signs that we respond to are subtle, and not obvious to us *in situ*. Body language and facial expressions often reveal information that cannot be captured by means other than video, the physical actions revealing more than the spoken word, often indicating that the spoken word should not be taken 'at face value' (Pirie, 1996b; Bottoroff, 1994). Video is a means by which one can help determine the 'do/say problem': What a person does may in contradiction to what he/she says (Ruhleder and Jordan, 1997). That is, when asked specifically about a problem or a situation, an individual will often state what he/she thinks is the expected or desired response but actions may reveal something quite different (Davis, Sumara & Luce-Kapler, 2000). With video, one is able to re-view the actions to determine if they agree with the statements and to compare the two at a later date.

Video also frees one from "the limits of sequential occurrence of events in real time" (Erickson, 1992; p. 209). It allows for re-observations of interactions, and enables the viewer to see actions that he/she did not originally observe. Instantaneous and brief reactions of a teacher to a student, missed in a 'live' observation, might be picked up in the re-viewing of a video. With video, one is able to focus on a specific hand movement that, in passing, may not have seemed important but on closer scrutiny may reveal the individual's attitude or feelings about the situation. "What video tapes can do is give us the facility through which to re-visit the aspect of the classroom recorded, granting us greater leisure to reflect on classroom events and pursue the answers we seek" (Pirie, 1996b; p. 553). In the re-view, one may see and hear things that were not originally seen or heard but which may have occasioned an action or reaction on the part of an individual. The re-view allows for the separation of the 'smaller picture' from the 'bigger picture' and vise versa, and for concentrating on those aspects that are important to the research at the time. With video, one is able to consciously pay close attention to specific occurrences or activities in order to make sense of the situation based on both the initial understanding as well as what has been gleaned from further, closer, observation. By considering an incident from different perspectives and re-examining the original interpretation, the complexity of the interactions may become more apparent and help in understanding earlier actions (Bottoroff, 1994), putting them into a new context, and providing the opportunity to obtain confirming or alternative interpretations of the results drawn from the original data (Bottoroff, 1994; Goldman-Segal, 1993). Video allows for the placing of the incident into context both visually and orally (Pirie, 1996b).

Re-examining video data allows one to observe the timing and sequence of events and behaviours in order to develop rich descriptions and accurate records of the situation without having to prematurely exclude behaviours or make interpretive judgments. "[N]o longer constrained by the sequential occurrences of events in real time, instances of similar recorded events at different points in time can be compared and contrasted easily" (Bottoroff, 1994; p. 246). By comparing later actions to earlier ones, one can determine if individuals are responding significantly differently in similar situations or if the response is simply a different manifestation of the same behaviour, brought on by different stimuli but not showing a change in attitude. "[B]ecause the behavior is analyzed in context, the likelihood of being able to identify possible antecedents and consequences of behavioral or interactive patterns is enhanced" (Bottoroff, 1994; p. 258). This is essential to any research looking at change.

With video data, this refinement of interpretation is based on the actions that take place, not just the recollection of the action that took place (Ruhleder & Jordon, 1997). Together with the verbal, the visual can provide a more complete, albeit, a more complex picture of the situation or individual being studied than either can provide individually (Pirie & Kieren 1991). Thus, a major advantage of video data over field notes or audio data is that it allows one to actively listen to and view the data after the fact any number of times (Pirie & Kieren, 1991). Observing video data frees one from the tendency to specifically notice frequently occurring events and allows one to observe for less common, but possibly revealing incidents (Erickson, 1992).

4.1.3. A process for analyzing video data

Video data is an exceptionally rich source of data and there are techniques which allow for the analysis of the data in an unbiased, orderly manner. The process of analyzing video data is likely to involve viewing the data several times, doing a 'trace' every three to five minutes. A' trace' is a statement of occurrences made, without bias, for that period of time. By doing a trace, the researcher becomes familiar with the data and may be able to temporarily eliminate some that do not appear to relate to the research question and he/she can also flag incidents that appear significant (Pirie, 1996).

Major concerns in the interpretation of video data are that the amount of information collected can distract from the research question (Mercer, 1991) and that the plethora of raw data collected renders the analysis of video data time consuming and difficult (Goldman-Segal, 1993). Simply stated, video data cannot be scanned quickly as can field notes and/or transcripts. The researcher may be required to watch hours of video that is of little or no relevance to the study (Pirie, 1996). He/she is faced with the difficult task of determining what to look for in the data. This, naturally, is guided by the research question and is facilitated by the fact that one can re-view the data in its original form (Cudmore & Pirie, 1996). However, in the beginning, anything may be relevant (Pirie, 1996b).

In non-video research, due to an immediate decision that has to be made by the researcher *in situ*, much information is omitted from the data because the researcher has to make an immediate decision with regards to its relevance. This is not the case with video data. The videographer could possibly suggest the elimination of some of the videoed data through the use of field notes, indicating its irrelevance at the time, but this contravenes the purpose of using video as a means of data collection. Interpretation of video data thus involves an interactive analysis with multiple viewings and layerings of data to obtain nuances and subtleties of the situation, which may not be observed at the time of videoing. Eliminating parts of the data before re-viewing it could eliminate important information (Goldman-Segal, 1993).

Interpreting video data is much more difficult than interpreting text (Pirie, 1996b; Goldman-Segal, 1993) and a major concern is that it produces an overabundance of data (Hammersley & Atkinson, 1983). Due to the profusion of data that becomes available, organizing it requires a systematic approach to analysis. While various computer programs such as VPrism, MacSHAPA®, VideoNoter®, NVivo, Nud*ist and Transana are available that can assist in analysis, and some of them can interface with video and audio editing programs such as Apple's Final Cut Pro and Sonic Foundry's Acid link (Thorn, 2002) this does not eliminate the hours of observation required.

Video-based interaction analysis (Ruhleder & Jordon, 1997) is a process which allows for in-depth multidisciplinary analysis of video data. In video-based interaction analysis, the "analysts allow the categories to emerge out of a deepening understanding of the taped participants' interactions ... [and] emerging patterns of interactions are checked against other sequences of tape, and against other forms of ethnographic observations including field notes, interviews, transcripts, documentary materials, etc." (p. 6) to neutralize bias while generating a set of categories for exploration. Following a process similar to Flanagan's (1954) critical incident technique which allows one to look at "observable human activity that is sufficiently complete in itself to permit inferences and predictions to be made" (p. 327), and Glaser and Strauss's (1967) constant comparative method which allows for "generating and plausibly suggesting (but not provisionally testing) many categories, properties and hypotheses" (p. 104), video-based interaction analysis provides one means to study video data.

4.1.4. Validity in video research

Eisner (1991) suggests that when one is describing or interpreting complex acts such as those that occur in the process of teaching and learning in a classroom, one is not actually trying to present things as they are, but is seeking to interpret them as seen. Such subjectivity leads many to question the validity of qualitative research. Since "[v]ideotape records as a constant reference point, a point of departure, permit grounded, well considered conclusions during interpretation of original field data" (Schaeffer, 1995; p. 256), the subjective viewing and interpretation of the data by different persons may help establish validity of the data by confirming the original interpretation. On the other hand, a second observer might view the data differently, repudiating the initial interpretation. Alternative concerns can be discussed and initial interpretations can be repudiated or a deeper, thicker interpretation can result. "[H]aving to articulate and argue for my perception helps to crystallize them into either useful or inappropriate descriptions" (Pirie, 1996b; p. 556).

While thick descriptions (Geertz,1973) can be used "to address the validity in ethnographic descriptions of cultures [whereby it is] the role of the ethnographer to build thick enough descriptions of events, such as the closing of the eyelids, so that the reader of the description can come close to understanding what the gesture might have meant for the person whose action is being described" (Goldman-Segal, 1993; p. 262) interpreting video data is affected by and determined by the interpreter "in much the same way that a writer of fiction determines the telling of the story" (Goldman-Segal, 1993; p. 262). An individual's description of occurrences and movements is subjective and the descriptions he/she provides may not instill the same image to another person. Actions, described in a transcript, lose tone and demeanor and once an interpretation of an action has been written, the action is less likely to be considered differently.

4.2. Considering case study and portraiture

4.2.1. Case study

Case studies are a useful means of developing knowledge of action and reflection on process and provide a particularly effective method for studying educational programs (Stake, 1995). According to Snow and Anderson (1991), case studies

> are relative holistic analyses of systems of actions that are bounded socially, spatially, and temporally; they are multi-perspectival, and polyphonic; they tend toward triangulation; they allow for the observation of behavior over time and thus facilitate the processual analysis of social life; and they have an open-ended, emergent quality (p. 152).

A case study can transform an individual experience into a group discussion so that practice in action becomes a focus for reflective practice, helping focus attention on specific professional actions (Shulman, 1998). This can provide a language and a procedure by which to generalize from a particular instance for the development of possible actions in other situations (Roth & Tobin, 2002). A case study involves circumscribing units of analysis, contextualizing the observations, and adding rich details where necessary. It is in "the study of the particularity and complexity of a single case, coming to understand its activity within important circumstances ... [emphasizing] episodes of nuance, the sequentiality of happenings in context, the wholeness of the individual" (Stake, 1995; xi-xii) that gives strength to case study. A case is chosen, then, not so much for its generalizability, but "to maximize what we learn" (Stake, 1995; p. 4). In terms of generalizability it might be better to think in psychological terms rather than mathematical probability (Donmoyer, 1990) and to use the results to "expand and enrich the repertoire of social constructions available to the practitioners and others" (p. 182). Thus, while a case may be chosen for its typicality, it may also be chosen because it appears intrinsically interesting.

4.2.2. Portraiture

Portraiture is a method of qualitative research which combines art and science so as to blur

> the boundaries of aesthetics and empiricism in an effort to capture the complexity, dynamics, and subtlety of human experience and organizational life. Portraitists seek to record and interpret the

> perspectives and the experience of the people they are studying, documenting their voices and their visions. ... The relationship between the two is rich with meaning and resonance and becomes the arena for navigating the empirical, aesthetic, and ethical dimensions of authentic and compelling narrative (Lawrence-Lightfoot and Hoffman Davis, 1997; p. xv).

Portraiture is used in an attempt to capture the essence of the situation, creating what Lawrence-Lightfoot and Hoffmann Davis (1997) refer to as "life drawings" (p. 4).

Research involving portraiture has been conducted in teacher education programs (Broyles, Kulawiec & Fryling, 1988; Kozleski, 1999) and within this genre has combined the impressions of the researcher with the hard, scientific approach, attempting to emphasize the goodness of the situation rather than its deficiencies (Lawrence-Lightfoot and Hoffmann Davis, 1997). "With its focus on narrative, with its use of metaphor and symbol, [portraiture] seeks to seduce the readers into thinking more deeply about issues" (Lawrence-Lightfoot & Hoffmann Davis, 1997; p 10).

4.2.3. Choosing case study and portraiture

In qualitative research, the uniqueness of the individual is important (Stake, 1995). The aim of qualitative research is to "discover essences and then to reveal those essences with sufficient context, yet not become too mired trying to include everything" (Wolcott, 1990; p. 35). In writing case studies, it is important to emphasize the person within the context of time and place. Case study is a suitable approach to take to observe and discuss ongoing situations.

Researchers have classified portraiture as a case study approach (Merriam, 1988; Yin, 1999) and have indicated that, by use of this methodology, there is a blurring of the line between the researcher and the researched. The subject matter is more about rare events than commonly occurring events (van Mannen, 1988) in that the tale is the tale of an individual's journey rather than a replicable finding. English (2000) argues that the portraitist cannot find the truth of the situation, and that he/she uses his/her position to control the information that is included. Marble (1997), however, indicates that portraiture allows for multiple ways of understanding the data and Lawrence-Lightfoot and Hoffmann Davis (1997) suggest that the portraitist's approach is an attempt to explain the situation from an insider point of view.

Stories and narratives are useful means of providing vicarious experiences (Stake, 1995). As the anthropologist, Clifford Geertz (1973) stated, a fundamental aspect of any cultural description is the researcher's imagination, and the depiction will bring us to better understand the un-experienced situation. The depiction must pay close attention to the reality of the social and human experiences, paying attention to all aspects of interaction. In this type of research, then, the pictures drawn of the individuals represent interpretations based on observation. By providing a thick description as outlined by Geertz (1977), of the individuals in action, a portrait becomes embedded into context, with the interpretation of actions based on understandings gleaned from observations over time and in a number of situations so that the portrait painted is a detailed and layered understanding of the portrayed.

4.3. Writing of a portrait

Kozleski (1999) relates portraiture to photography. Accordingly, when using portraiture, like when taking a photograph, the research would

> result in a snapshot of their work within a specific timeframe [much like a photograph]. The aperture of the lens, the angle that the photographer shoots from, the time of day, and the available natural lighting are some of the variables that shape the final image. In the end, any photograph likely omits far more than it reveals but it remains an artifact of what was (p. 4).

Portraiture, like collecting video data, is thus restricted to the viewing and interpretation that takes place and the results can be considered as that which has occurred under the research eye of the observer. Individuals' subjective realities vary according to the context and the events of their experiences (Lincoln & Guba, 1985) and thus, in order to obtain a larger view of the landscape, one might consider using a number of portraits drawn from the same surrounding.

"The portraitist comes to the field with an intellectual framework and set of guiding questions. The framework is usually the result of a review of relevant literature, prior experience in similar settings, and a general knowledge of the field of inquiry." (Lawrence-Lightfoot and Hoffmann Davis, 1997; p. 185). According to the anthropologist Clifford Geertz (1973), the author will, of necessity, present his/her sense of the situation which means that a fundamental aspect of the description is the researcher's imagination. Thus, in order to present an authentic portrait, the portraitist must <u>listen for a story</u>. The author must evaluate and juggle different kinds of data to produce coherence through themes which emerge from the data so as to represent the order that exists in what, to the outsider appears as disarray.

Interpretation is an important feature of art. The artist is the one who draws attention to particular aspects of the portrayed, but it is the viewer who does the interpretation. So it is with portraiture. The portraitist is involved in a process of evaluating and judging the data in order to determine which are the significant features of the themes that emerge and the ones to which he/she wants to draws attention. However, it is the reader who interprets them and puts them into a context that he/she understands. An intention of portraiture, then, is to capture, from an outsider's view, the insider's understanding of the situation, to capture the essence of the situation, so that it makes meaning to the outsider but allows him/her to draw his/her own conclusions about the veracity of the work.

Specifically, portraiture "makes the researcher's biases and experiences explicit, in essence becoming a lens through which the researcher processes and analyzes data collected throughout the study" (Hackmann, 2002, p. 52) keeping in mind the five essential features of voice, relationship, context, emergent themes and aesthetic whole (Lawrence-Lightfoot and Hoffmann Davis, 1997). Voice is that personal input through which actions are interpreted and described through the eye of the portraitist. Relationships of trust must be developed so that the portrayed allows the portraitist to see his/her inner feelings and context must be considered so that the resulting themes can paint a picture which results in genuine representation of the portrayed.

4.3.1. Rigor in portraiture

Tradition dictates that researcher bias should be eliminated or at least limited in research. More recently, especially in qualitative research, it has been acknowledged that the researcher will impact the interpretation of data (Van Maanen, 1988; Marshall and Rossman, 1999) and specifically that portraiture "makes the researcher's biases and experiences explicit, in essence becoming a lens through which the researcher processes and analyzes data collected throughout the study" (Hackmann, 2002, p. 52).

The argument that rigorous data analysis is not a part of portraiture is countered by Lawrence-Lightfoot (1986) who indicates that in portraiture the researcher must use a number of different pieces of data collection and, using these, must try to find points of convergence, a form of triangulation. Themes may be revealed through the rituals and normal activities of individuals (Lawrence-Lightfoot and Hoffmann Davis, 1997).

4.3.2. Trust in portraiture

As is necessary when collecting video data, in portraiture there is a need for a strong bond of trust and relationship between the researcher and the researched, and for the researcher's presence to be accepted as non-interfering and non-judgmental (Lawrence-Lightfoot & Hoffmann Davis, 1997). The "[p]ortraitist must always be mindful of the seriousness of their [sic] work on site and the ease with which their [sic] actions can unintentionally be injurious" (p. 167-168) while maintaining "the attitude of respectful learners, insiders with the opportunity to perceive themselves as experts and teachers" (Lawrence-Lightfoot and Hoffmann Davis, 1997; p.167). It is imperative, then, that in research involving portraiture, rapport and trust exist between the researcher and the portrayed.

4.3.3. Authentification in portraiture

"It is the actors who are most vulnerable to whatever distortion they may find in the image reflected in the portraitist's mirror" (Lawrence-Lightfoot and Hoffmann Davis, 1997; p. 172). In writing, the portraitist "must guard the relationships that are established throughout the writing of the final portraits" (Lawrence-Lightfoot and Hoffmann Davis, 1997; p. 173). The rapport between portraitist and portrayed "is imprinted on the portrayal, embodied in the carefulness with which descriptions of individuals are shaped and presented" (Lawrence-Lightfoot and Hoffmann Davis, 1997; p. 176). Since the portrait drawn must reflect the fact of the situation, the complexity of the situation must be maintained, as well as the authenticity of the individual's experiences (Marshall & Rossman, 1989). Thus, in organizing data, complexity must be retained and each recorded incident must embody the essence of the individual. The portrayal must be authentic. This authentification, it is suggested, can be achieved through allowing the portrayed an opportunity for reciprocity, allowing him/her the opportunity to review his/her portrait and to comment on its integrity, putting constraints around factual error (Lawrence-Lightfoot and Hoffmann Davis, 1997).

4.4. Concluding remarks on video data collection, case study and portraiture

Portraiture as a means of presenting data offers many advantages when one wishes to depict situations that are open to interpretation. Just as art depicts the situation as seen by the artist but is interpreted by the beholder, portraits depict the individual as seen by the portraitist, but it is the reader who interprets them. In this study, the development of understanding of teaching and learning of the four preservice teachers, Sophia, Lance, Ellie and Wayne is presented through the lens of the researcher's eyes and this development is thus open to the interpretation of the reader.

Chapter 5

Methodology - Outline of Study

5.1. The setting

As indicated in Chapter 1, Wubbels, Korthagan and Broekman (1997) believe that prospective teachers need to be exposed to innovative teaching styles throughout their teacher education program if change in their practice is to occur. Smith (1999) states that the use of imagery in teacher education programs may be useful in helping preservice teachers redefine their images and attitudes about what it means to teach. And Schön (1987) believes that images are important in creating knowledge that can inform new practice. Piecemeal modifications to existing programs are not seen to bring about the necessary change in prospective teachers' beliefs and there is a need to improve the context in which they are placed if change is to occur (Ross, 1990). In order for a program to be successful, a more inclusive approach is needed. In their university classroom, preservice teachers need to be presented with an Image of teaching and learning, an Image that is applicable to the actions and activities of that classroom, and which incorporates their peer discussions and activities so that they can see how that approach to teaching can be manifested in day-to-day high school classroom activities.

The teacher education program at the University of British Columbia provided an opportunity to test these ideas and the Pirie-Kieren Dynamical

Theory for the Growth of Mathematical Understanding provided a suitable model to present to the preservice secondary mathematics teachers involved.

Students in the Secondary Mathematics Integrated Program (SMIP) were enrolled in an intensive one-year program in which their Methods, Principles of Teaching and Communications courses were integrated and taught by one instructor. This provided the opportunity to present and practice with them, an Image of the manner in which students learn mathematics, an Image that incorporates the present understanding of the teaching and learning of the subject as "a whole, dynamic, leveled but non-linear, transcendentally recursive process" (Pirie & Kieren, 1991a; p.1). This Image of learning mathematics is implicit in the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding (the Pirie-Kieren theory) and it was thus considered a good Image to present to prospective teachers regarding the teaching and learning of mathematics. As this situation provided an excellent opportunity to study preservice teachers' developing understanding of teaching and learning mathematics, a Request for Ethical Review was completed and permission to carry out the study was granted (See Appendix A).

At the beginning of their teacher education program, prospective secondary mathematics teachers were presented with the Pirie-Kieren theory as a way to consider the manner in which school students come to understand mathematics in order to determine if knowledge of and modeling of the theory would enhance their understanding of the process of teaching and learning mathematics. The theory was revisited, specifically, twice during the preservice in-class sessions during the fall term of the teacher education program as well as being used throughout the term as a lens through which to discuss the teaching and learning that was taking place by the prospective secondary mathematics teachers, by school students observed in videos and by the high school students in the preservice teachers' short practicum experience.

The purpose of the study was to examine the growth of understanding of teaching and learning of preservice secondary mathematics teachers, considering this growth through the lens of the modified Pirie-Kieren theory, as they progressed through their intensive one-year teacher education program. The specific questions that it addressed were stated in Chapter 1.

5.2. Framework under which the study was be carried out

The Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding is a theory developed by Dr. Thomas Kieren and Dr. Susan Pirie to discuss the manner in which students develop in their understanding of a mathematical concept. As discussed in Chapter 2, the theory is a theory <u>for</u> the <u>growth</u> of mathematical understanding, not a theory <u>of</u> mathematical understanding. As such, it is not a theory that can be used in assessment, but rather, it can be used to discuss the development of meaning making by an individual student about an individual concept. Following is a description of the manner in which the theory was used in the SMIP program. The proposed course outline is provided in Appendix B.

5.2.1. Presentation of the Pirie-Kieren theory

For the purposes of this study, the Pirie-Kieren theory was presented as a model to describe and discuss the manner in which students develop mathematical understanding. As such, it provided a language with which preservice teachers could discuss their own understanding of the development of mathematical understanding as well as providing them with a way of thinking about the development of mathematical understanding. Specifically, the theory was used in the following manner:

1. Prospective teachers enter a teacher education program with beliefs and Images of what it means to teach and of the manner in which students learn. During the first two days of the SMIP program, the prospective secondary mathematics teachers were presented, by Dr. Susan Pirie, with the Pirie-Kieren theory as a possible alternative method to think about and discuss the manner in which students come to a greater understanding of mathematics. During this presentation, the students were provided with the model of the theory, with examples of work depicting the different levels, with videos of students working so that they could discuss their working with reference to the theory, as well as being given opportunities to work at mathematical activities and discuss their own understanding of the working with reference to the theory. As such, they were given a short, intensive course on the theory.
2. Change in behaviour is not random, but conditions can be designed so as to assist obtaining the desired result. Modeling of desired behaviours is

one method that appears to have a positive effect (Bandura, 1997; Hart, 2002). However, what is being modeled is often not apparent to those to whom it is being demonstrated and it is often necessary to specifically address the modeling so that the benefits can be more noticeable (Hanney, Smeltzer Erb & Ross, 2001). Thus, the Pirie-Kieren theory was modeled in the teaching of the SMIP program so that the preservice teachers would be better able to see how it could be used as a means of considering student understanding. Preservice teachers were asked to consider their own mathematical learning experiences and to discuss their impact on their learning opportunities. Regular references to and re-visitations to aspects of the theory in classroom discussions were made. For example, the preservice teachers observed videos of students working on mathematical tasks and were then given the opportunity to discuss the understanding that they observed. Twice during the term, guest speakers were invited to discuss and reinforce the tenets of the theory. Dr. Jo Towers presented some of her findings to the class in mid-October, and Dr. Susan Pirie returned in late-November to further discuss aspects of the theory. These presentations were used to reinforce the preservice teachers' understanding of the embedded ideas.

3. In order to teach a concept effectively, one must consider what background knowledge is required and what knowledge it is assumed that the students have about the topic. (Schön, 1987; Wubbels *et al,* 1997). Through the lens of the Pirie-Kieren theory, one could express this as

specifically accessing and assessing the Primitive Knowing of the students, and in so doing, possibly Folding Back to one's own learning experience. During the program, preservice teachers were therefore encouraged to Fold Back to their own learning experiences and understandings and to reflect upon their assumptions about student understanding while planning for and discussing their lessons for presentation in the SMIP class and during their practicum. That is, they were asked to consider the learning of mathematics as a recursive process by returning to inner layers of their own understanding to inform their understanding of outer layers to help in planning meaningful lessons. Specifically, they were given mathematical tasks to work on and were asked to discuss their own stages or levels of understanding with reference to the theory. Thus, as practice is considered important in learning new behaviours (Wubbels *et al,* 1997), the preservice teachers were encouraged to use the Pirie-Kieren theory and the language which it provided in their university classroom thinking and discussions and during their preservice practicum.

The Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding thus provided both a theoretical model for the prospective teachers to use in thinking about and discussing the teaching and learning of mathematics as well as a practical model around which to plan lessons and to discuss the processes.

5.2.2. Data collection

Video was the main form of data for this study, but the data also included questionnaires, field notes, e-mail messages and class assignments. Data was collected throughout the fall term of the SMIP program as well as throughout the practicum experience of the four students who were selected for further study.

On the first day of their teacher education program, prospective candidates for this study received an explanation of the purpose of the study. It was explained that those who did not wish to participate would not be videoed and that their data would not be used in the study. Demographic information was obtained, and all participants, including the instructor were given the chance to choose a pseudonym. Other than Dr. Susan Pirie and Dr. Jo Towers, people who appear in the discussion and who did not provide pseudonyms were given fictitious names. The placement schools of the four preservice teachers followed in the practicum have also been given fictitious names.

5.2.3. Initial questionnaire

Preservice teachers' past experiences form the basic knowledge upon which they draw in thinking about teaching and learning (Pajares, 1992; Kreber, 2002; Roth & Tobin, 2002). In the language of the Pirie-Kieren theory, this forms their Primitive Knowing, their Images, and even, possibly, some Formalisings of what it means to teach and to learn mathematics.

As the instructor of the SMIP program wanted to obtain an initial view of the prospective secondary mathematics teachers' attitudes and beliefs about teaching and learning the subject, and as it was important that I have an unbiased pre-view of their beliefs and attitudes, the SMIP students were asked to complete an initial questionnaire designed to obtain this information. The first portion was a survey (See Appendix C: Images Questionnaire) consisting of thirty statements which were rated on a four-point Likkert-like scale to indicate how strongly the prospective teachers agree or disagree with the statements. Modeled on a survey used by Hart (2002) to determine preservice elementary teachers' attitudes and beliefs about mathematics teaching and learning, the questions were modified to suit the purposes of this study. The SMIP students were asked to complete this questionnaire as quickly as possible so as to obtain their 'gut' reactions or unbiased views with regard to the statements. They were then given ample time to complete and elaborate on eight sentences (See Appendix C: Views Questionnaire) concerning their views about teaching in general and how teaching other subjects is the same as or different from teaching mathematics.

5.2.4. Outline of video data collection

While the questionnaires were designed to obtain an initial overview of the stated images and beliefs held by the prospective teachers, there was need to obtain their lived beliefs and understandings. What is important to people will be revealed in their day-to-day conversations and interactions and this can be determined by watching and listening to them with as few pre-conceptions as possible (Richer, 1975). Thus, to best capture the honest reactions of the SMIP students in their natural environment and to determine their true beliefs about teaching and learning mathematics, videoing their day-to-day discussions and interactions as unobtrusively as possible was an appropriate means of data collection. Video data allows one to put the stated and enacted understandings of the people into the context of present, past and future actions and statements and enables one to review any relevant incidents any number of times in as authentic of a setting as possible, confirming or altering an interpretation in light of later data (Pirie & Kieren, 1991a).

For the above reasons, video was chosen as a means of data collection for this study. A single, semi-fixed video camera was placed on a tripod at the back of the room. It was small and light enough so that it could be moved around the periphery of the room to get a better angle when videoing small groups. In this manner, I was able to video the class as a whole, an individual presenter or a selected small group. The microphone on the camera was of high quality and was used as audio for large groups. For small group discussions a small directed microphone which cut out background noise was used.

Of the twenty-two students in the SMIP class, nineteen initially agreed to participate by signing the Informed Concept Forms (See Appendix E). Of the three remaining students, one indicated that she did not think that she 'fit' the criterion as she was from another country and was recertifying as a teacher so that she could teach in Canada. She became a willing participant and signed the form when I explained that I thought she might add an interesting perspective to the discussions as she already had classroom experience. One week into the study, one of the other two indicated that the camera did not affect him as much as he had thought it would, and therefore he would like to be an integral part of the study. When he realized that he was the only one not being videoed, the remaining student approached me, indicating that he was a 'nervous' person but that he did not mind being videotaped providing he was not one of the people chosen for in-depth study. He signed a form to this effect (See Appendix F). I was therefore able to video all of the students in the SMIP program in any combination, an important issue as, at the beginning of the research, I did not know who would be followed during the practicum situation.

Thus, while it may be argued that the presence of a video camera is likely to have an effect on individual's reactions (Cudmore & Pirie, 1996), in this case, the presence of the camera over time minimized its effect on the class (Marland, 1984). Also, the intrusion of the video camera was also decreased in that, as videographer, I became an observant participant (Towers, 1998a), capturing not only the SMIP students, but myself on video as I was interacting with them and the instructor, participating in activities and helping with presentations. As one of the students indicated: "It's just like having another person here, watching but not talking."

5.2.5. General data collection

As mentioned, Dr. Susan Pirie presented and discussed the tenets of the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding during the first two days of the SMIP program. Following the initial presentation of the Pirie-Kieren theory, the next three weeks of classes involved the preservice teachers in discussions of their understandings and beliefs about mathematics as a subject, about teaching in general, and specifically about the teaching and learning of mathematics. All discussions were videotaped. The camera was at times focused on the class as a whole, on small groups, or on individuals, which ever seemed appropriate at the time for the purposes of obtaining the best data. As it was not known at this time which students would be followed during their practicum, an effort was made to obtain data on as many, and as equally as possible, all SMIP students and to have them working and discussing in different combinations so as to obtain a comprehensive overview of the actions and beliefs of all of them and of their interactions with different groupings. This was made possible since, as mentioned, all the preservice teachers had agreed to be videoed

A second set of video data was obtained during the second and third weeks of classes when the SMIP students were videoed in self-selected groups of two to three outside class time. These video sessions were arranged at a time convenient for the students, and during the sessions they discussed their understanding of the nature of the teaching and learning of mathematics, and specifically their understanding of the Pirie-Kieren theory as it was presented to them in this context. The purpose of these discussions was to obtain, in a more relaxed atmosphere, the preservice teachers' stated beliefs and understandings and their opinions and understandings of the Pirie-Kieren theory and to determine if there was a connection between the two.

Throughout the Fall term, there were many informal meetings with the SMIP students – some simple conversations in the hallways walking to class, some organized over coffee, and others at planned social events. While these were not videotaped, any informative statements made by students were recorded as soon after the event as possible in the form of field notes.

5.2.6. Practicum data collection

"A student cannot at first understand what he [sic] needs to learn, and can learn it only by beginning to do what he [sic] does not yet understand" (Schön, 1987; p. 93). For the purpose of this study, it was felt that it was appropriate, if not essential, to follow students during their practicum as it is in doing that one often reveals one's true understanding and beliefs. By observing the students in *in situ*, a more realistic or honest view of their actual beliefs about teaching and learning mathematics would be obtained.

It was physically impossible to follow all twenty-one of the SMIP students who agreed to be a part of the study during the practicum. A sample of four students was thus selected for more in-depth study. The selection, although somewhat random, was also based on convenience. They would each be observed several times during the practicum, thus students who were placed outside the Lower Mainland were eliminated. Based on their initial questionnaires and the observations during the fall term, students with somewhat different profiles and stated beliefs were then selected. That is, in considering possible selections, I looked at their stated beliefs and experiences and selected some students who indicated that they held quite traditional views of the teaching and learning of mathematics and some who seemed more progressive. I also considered their background experiences and chose some who had 'outside' experience and others who had followed through the standard educational system. As a result, it was possible to follow two male and two female SMIP students who were placed in different socioeconomic areas of the city. The gender balance and different placement areas, it was felt, would allow for a comprehensive view of the SMIP students' practicum experience. Necessary permission from the School District, the principals of the schools involved, as well as the sponsor teachers was obtained (See Appendices G, H, and I respectively). A minimum of four classes taught by each of the four selected SMIP preservice teachers was videoed.

5.2.7. Reflection videos

Since "[t]he act of reflecting on beliefs and behaviors allows teachers to make connections between their thoughts and actions and to recognize, expose, and confront contradictions and inconsistencies" (Hart, 2002; p. 6), it was important to have the preservice teachers reflect on their own teaching. They were therefore involved in two further video sessions. The first of these took place as soon as possible after the class that was videoed. The sponsor teacher was considered important in these discussions because he/she has a significant impact on the attitudes and teaching behaviours of the student teachers and how they enact the curriculum (Koster *et al*, 1998). Therefore, the sponsor teacher was invited to attend these sessions and be part of the discussion.

The second post-classroom video session was a *retrospective viewing* which occurred after the practicum experience was completed. A *retrospective viewing* involved the SMIP student observing and commenting on one of the videos of him/herself teaching. *Retrospective viewings* differed from video-stimulated recall sessions (Bloom, 1953; Marland, 1984) as the intention was not to try to determine what the individual was thinking at the time, but rather, what he/she observed *in retrospect*, and to discuss this in light of other experiences. In this case, the SMIP students were to discuss their observations through the lens of the Pirie-Kieren theory. These sessions allowed the preservice teacher to consider his/her interactions with the students in the class and further consider how he/she enacted the teaching process and to discuss if these were in agreement with his/her stated beliefs about mathematics teaching and learning.

5.2.8. Debriefing

As was planned at the beginning of the program, a session was arranged in which the students in the SMIP program could meet as a whole after their practicum. As such an opportunity was not provided in the teacher education program, it was arranged outside class time, one week after their practicum ended. Twenty of the twenty-two SMIP students were able to attend. This session was videoed, and while they did share their experiences, it was difficult to obtain specific information on any one student. The SMIP students tended to use it more as a social activity, building on and maintaining their sense of unity and support.

5.2.9. Field notes and class assignments

As mentioned earlier, video was the major source of data collection. However, field notes were also used and class assignments were collected throughout the study. Recall that, as videographer, I was interacting with the class, and therefore, the field notes were not detailed, but were used to draw attention to specific actions or activities that I thought might be important and which took place out of the focus of the camera. I also used field notes to record interactions that occurred out of class and which I thought might have impact on the data collected during the study. For example, after the social evenings and coffee sessions that I attended, I would often make brief notes concerning students' statements. Individual preservice teachers were contacted when I felt that further clarification was needed and email messages within the SMIP list-serve through which we communicated on a regular basis, especially during the practicum were also used. All notes were dated and identified as to where and how the information was obtained so that it could be fitted into the correct place in data analysis.

5.2.10. Discussion of the collection of the data

As outlined above, the collection of data for this study provided a comprehensive collection of the preservice teachers' activities and as such revealed much about their changing understanding about teaching and learning mathematics. By collecting data in this manner, it was possible to capture the experiences of the prospective teachers in various forms and to be able to isolate those activities which depicted their Images, attitudes and beliefs and their developing understandings of the teaching and learning of mathematics. This provided the necessary information to answer the research questions.

5.3. An outline of the data analysis

Data analysis involving videotapes is a time-consuming process, and this study was no exception. While, during the collection of the data, I reviewed videos on a random basis, I left the detailed reviews until after all data was collected. I chose to do this as I would then have an overview of the entire development and would be viewing the data with greater knowledge of the actions in context. By the time I had collected all the data, I had more than two hundred forty hours of video.

I began by viewing the tapes chronologically to re-familiarize myself with the initial data. Since at this point, I knew which preservice teachers I had followed through the practicum, as I viewed the videos I kept that in mind. After viewing approximately forty hours of videotapes, I randomly selected other videotapes from the fall term and viewed approximately forty more hours of video. At that point, I felt that I was ready to proceed with the formal analysis as I was immersed in the data.

Because of the data I had obtained and the information I hoped to extract from it, I chose not to use any of the available programs designed to assist in video analysis as I felt that they constricted my intentions and purposes. Instead, I set up my computer beside my video monitor and engaged in *post-field-note-taking*. As in a 'trace', as described earlier, I noted the time and activity at regular intervals. However, my *post-field-notes* were more directed to the interactions that took place, in that I typed notes related to my observation, making special note of when any of the four individuals chosen for further observation were involved. This viewing was done chronologically and I was able to identify a number of videos that seemed unrelated to my purpose as well as identify many that seemed specifically significant. Although I use the term 'chronologically' here, I do not mean that I did not go back and re-view previous videos in light of new information. That is, if, for example, on video #93 I witnessed an action that reminded me of a previous incident, I would go through my notes, locate the video that it reminded me of, and re-view that video to determine if there was in fact a connection or a contradiction to the previous observation. In this manner, I was constantly Folding Back to my previous interpretation, refining and redefining them as necessary.

During the viewing and *post-field-note* taking, I would rewind and review videos so that I could obtain verbatim, statements of preservice teachers which seemed relevant at the time as well as making note of physical reactions and to re-observe actions that seemed to be instances of understanding of teaching and learning. Since the validity or trustworthiness of video data can be confirmed and/or denied through sharing interpretations with others, a sample of my video data was shared with Dr. Susan Pirie for discussion and confirmation or repudiation of interpretation.

After having viewed and re-viewed all the in-class sessions, I viewed and re-viewed the videos of the four preservice teachers in their practicum settings. These I viewed chronologically, one student at a time, again noting verbatim, the interactions that appeared significant and making notes as to their interactions with the class and sponsor teacher. By the time I began to write my portraits, I had approximately three hundred and twenty-five pages of notes.

Analysis thus began with the initial impressions made during the observations in class, and continued with the viewings and re-viewings of the videos, and with making of *post-field-notes.* Since I had used video as the primary means of data collection for the first part of the study, I had been able to observe the case in two ways – first during the actual proceedings, and then in hindsight, observing the videos later. This enabled me to put actions in the context of past and future occurrences. Throughout the study, I was neither teacher nor learner, but a participant in both, as well as being an outsider, observing the situation from afar so that as I collected data and analyzed it I was able to live through the experiences of the preservice teachers both in reality and vicariously.

In the viewing of the video data, I had to observe for themes that developed in the understanding of teaching and learning mathematics for the preservice teachers and to observe how they portrayed this understanding in their presentations to students in class, both at the university and in their practicum. As I wrote my observations of the videos and chose quotes to use to portray the individual's understanding, I had to review previous data to determine more explicitly what had transpired and the context in which the quotes were made. Was this a 'one-timer' or did it truly capture the individual's understanding?

5.4. The drawing of the portraits

A portrait must provide readers with "good raw material for their own generalizing" (Stake, 1995, p. 102). In this case, this meant that I had to portray each of four individuals, Sophia, Lance, Ellie and Wayne, as they developed their understanding of the meaning of teaching and learning mathematics, as I perceived it through the lens of the modified Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding. By observing the preservice teachers in a number of different situations, keeping the modified theory in mind, but leaving the interpretation of actions open and ongoing, I was able to obtain an extensive overview of each individual under consideration and his/her reactions to different situations.

The video data and resulting notes along with all other data gathered through the fall term formed the 'paint' with which I would portray each of the four preservice teachers. A portrait can lead one to assume the sitter to be of calm and intelligent disposition, although this is not stated in any one feature of the picture, but is surmised from the whole portrait. As such, my intention was not to simply describe the preservice teachers as I observed them, but to truly 'paint a portrait' of each. The portrait would, of course, be my interpretation of the individual, just as was, for example is Whistler's portrait of his mother. And, just as a viewer interprets Whistler's portrait to form his/her own image of the lady, the reader would interpret my portraits to form their own images of the preservice teachers.

Thus, as the portraitist in this case, the portraits would be my interpretation of each individual after having listened to them and empathized with them, and their experiences would be interpreted through my own experiences and understandings as a teacher and researcher. As Marshall and Rossman (1989) suggest, this part of the analysis was challenging in that I had to try to maintain the authenticity of each individual's experiences and understandings while I was interpreting them through my own. Not only was it important to organize and classify the incidents that I was selecting, it was essential that the complexity of each be maintained and that each incident presented in the portrait embodied the essence of the individual's understanding. In construction of the portrait, I had to search for patterns that developed in each person's profile and choose quotes that authentically represented their development. At the same time I had to watch for "the deviant voice", incidents that seem to contradict the norm (Lawrence-Lightfoot & Hofmann Davis, 1997) for that individual and determine how they fit into the whole. Even the manner in which the preservice teachers expressed themselves when describing their classes was significant in determining their understanding of teaching and learning and the behaviour they expected of their students as these provided their metaphors for understanding and learning. By finding threads of connection between themes, I was able to present each person as an individual who reacted to the environment in a specific manner and who interpreted outcomes in a particular manner.

Having had experience as a preservice teacher (albeit many years previous), as a teacher, as a sponsor teacher and as an instructor of preservice teachers, and having done an extensive review of the relevant literature of teacher education and teachers' attitudes and beliefs, I was able to consider the data collected in this study from the perspective of the relevant literature, my personal experiences and general knowledge of the area, all of which were part of my "anticipatory schema" (Lawrence-Lightfoot and Hoffmann Davis, 1997; p.

186). From the perspective of the Pirie-Kieren theory which was used in this study, these formed part of my Primitive Knowing. Thus, I entered the field of research with a framework and plan, while expecting that both my intellectual agenda and my methods of obtaining data may have to be adapted to the situation. Also, as I chose to portray more than one preservice teacher, I would be allowing my interpretation of their actions to be seen through different lenses which, in essence, would help authenticate the portraits.

In painting the portrait of each individual, consideration had to be given to his/her positioning in the whole. The depiction of the situation had to pay close attention to the reality of the social and human experiences, and to all aspects of interaction. Thus, by initially choosing to video not only the individuals who would be followed through during their practicum, but rather by videoing the entire SMIP class, I was considering not only "the voices and experiences of the range of actors of focal concern but also the perspectives and actions of other relevant groups of actors and the interaction among them" (Snow & Anderson, 1991; p. 154). By observing the preservice teachers in the larger class, and not pin-pointing specific students from the beginning, I was able to gain a more thorough understanding of the individuals' attitudes and beliefs without interference of guidelines that limited their expression.

In portraiture, an important aspect is trust. In this study, there were several layers of trust demonstrated, and evidence of willingness to share experiences. Through the building of relationships with the subjects, it was possible to learn more about their conceptual and emotional understandings of students' ways of learning – empathy, trust – so as to understand the other (the preservice teacher) and to see him/her in light of my own experiences. The first evidence of trust was that all SMIP students allowed me to video them and were not restricted in their actions in class while being videoed. At several points, they even 'played the part' of the instructor and myself, knowing that all would be taken in good stead. The instructor and I were also invited to the many social gatherings organized by the SMIP students.

For those who would be followed into the student teaching classroom, the bond of trust needed to be strong enough for the them to feel at ease in allowing me to video them during their teaching sessions, a time when they would be practicing newly learned techniques and when errors might occur and mistakes might be made. It was thus important that they not perceive me as judgmental as this would have diminished the likelihood of sharing of negative experiences (Lawrence-Lightfoot and Hoffmann Davis, 1997).

The trust and willingness of the four individuals to have me in their classes was demonstrated by the fact that they took the initiative to ask their sponsor teachers to allow me to video their classes and that each one arranged for me to meet with the sponsor teachers and the principals of the school in which they were placed to obtain the necessary approval. Each of the preservice teachers indicated a willingness to share negative experiences with me. For example, one of the preservice teachers, Wayne, on one occasion, indicated that his students "were apprehensive" and that it might be best that I not be there the next day. He said that his interactions with them at that precise time were not as "healthy" as they should be and he needed time for their relationship to become smooth again. It was essential that I honor his interpretation of the situation so as not to lose his trust or to have my presence interfere with the natural behaviour of the students, even though the observation might have provided some useful data to determine what it was that made Wayne feel this way. Similarly, when Lance indicated that his class was uneasy due to the fact that one girl "injured herself", it was important that I not ask too many questions about the situation and that I wait until he felt that the incident was under control and he wanted to share the experience with me, which he did later in confidence. Ellie discussed her concerns over her sponsor teacher thinking her presentations were too theoretical and Sophia chatted about how pleased she was with her own approach to teaching the solving of equations and her unease about the teaching that she had witnessed, knowing that I would not reveal her statements to her sponsor teachers. These examples indicate that there was a bond of trust developed in which we could share experiences -- the positive, the negative and those of concern.

After considering the data collected in this study and the essence of portraiture, I wrote my portraits. Each portrait begins with a statement made by the individual, a statement which depicts his/her understanding of the process of teaching and learning at a particular stage in his/her development. The individual's personal history, his/her own Primitive Knowing, if you like, is provided for background so as to create a holism in each portrait which allows for the analysis of each within the context of the case, taking into consideration the viewer's own observations and experiences, that is, my own Primitive Knowings and Images. This is in agreement with recent acknowledgements that, especially in qualitative research, the researcher's bias can, and likely should impact the interpretation of data (van Maanen, 1988; Marshall and Rossman, 1999).

Portraitists often present individuals as omnipotent and as if they can see things that others cannot. The portraits presented here, however, are written using a collage of the individual's own words extracted from class discussions or written assignments. They therefore capture the essence of the individual in a style that represents his/her mode of expression in real, day-to-day activities. Since the portrait is written in the words of the portrayed, he/she should appear to the reader as a real person, not as an individual who is 'bigger than life' or as one who has been 'airbrushed'. He/she should appear as an actual person who is having day-to-day experiences.

In drawing the portraits of the four preservice teachers, I drew on my interpretation of the situation, but I wanted to ascertain that the portrayed felt that I had captured their images. To verify this, I gave each participant an opportunity for reciprocity. After the initial study, while writing up the data, I kept in e-mail contact with and met with the participants on several occasions, both individually and in groups. We discussed, not only the study, but also our present lives and we shared information and ideas on teaching and learning. After the writing of the portraits, I gave each of the portrayed the opportunity to review his/her own portrait and to comment on its integrity, thus allowing them the opportunity to authenticate my Image and to note if there were factual errors. Putting constraints around factual errors gave them the impression that the portrait was indeed a finished product, even if at times there were aspects of the portrait that they would have preferred not be made public (Lawrence-Lightfoot and Hofmann Davis, 1997). I wanted them to be able to say "Yes, that is me." even if it was not how they would like to portray themselves. All four accepted the portrait as an authentic Image, and as a result, no changes were necessary.

The final portraits of Sophia, Lance, Ellie and Wayne are presented in Chapter 6 as data for the next stage of analysis, the discussion of their growth of understanding of teaching and learning mathematics, which is presented in Chapter 7. The analysis of the growth of understanding of teaching and learning mathematics of each individual is presented in the same order as the portraits are presented so that the reader is easily able to follow one individual through, or may read the portraits as a unit so as to feel the interaction of the individuals and obtain a sense of the SMIP class.

5.5. Analysis of the portraits

Analysis of the portraits, like analysis of the video data, began long before sitting down to do the analysis. It began before the construction of the portraits or the collection of the video data, activities during which I was actively involved in Image Making. Since the Pirie-Kieren theory was being used in this study in several ways, a thorough understanding of it was essential, and thus, developing this understanding was the beginning of the analysis. After having immersed myself in the theory, the next stage, involving it, was to re-define the terms of the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding to fit the new context of growth of understanding of teaching and learning mathematics. In order to do this, I had to consider the meaning of mathematical understanding and its similarities and/or differences to the meaning of understanding understanding. That is, I had to think about the relationship between what it means to learn something oneself and what it means to understand the process of developing understanding. This, then, had to be bifurcated to the two separate areas of developing understanding of the meaning of teaching and developing understanding of the meaning of learning.

Beginning at the beginning, so to speak, it was fairly easy to define Primitive Knowing as this is the information one brings with them, except that information directly related to the topic at hand. However, since everyone has had experiences with learning, it was important to think in terms of developing an understanding of the learning process, or developing an understanding of how one learns, and to keep this distinct from learning (or simply developing understanding). The process of developing further definitions was somewhat more complex.

Developing understanding of mathematics is quite different from developing an understanding of <u>how</u> one learns mathematics, and quite different from developing an understanding of how one <u>teaches</u> mathematics. Thus, in developing the definitions for the different levels of understanding, it was first necessary to determine if indeed these levels did exist and if so, to think of exemplars for them. As a practicing teacher, I had to Fold Back to my own learning of how to teach and my own learning of how I had developed an understanding of how students learn. Using this and the Pirie-Kieren theory as my lens, the process began with examining my own understanding of the original definitions and re-formatting them to fit the new contexts. Indeed, it appeared, in so doing that the different levels existed. However, after several trial definitions and thinking of examples that fit, I realized that at the outer levels of developing an understanding of teaching and learning, the two became amalgamated so that at the Observing level, Structuring level and Inventising level one had to be able to rationalize the relationship between teaching and learning. In order to accommodate this, a dual model following the tenets of the Pirie-Kieren model was designed (See Figure 7 in Chapter 2) and which can be used to map the growth of understanding of both teaching and learning mathematics.

During the process of developing definitions and thinking about exemplars that fit the new context of developing understanding, I was in regular contact with Dr. Susan Pirie to ensure that the integrity of the original definitions was maintained.

Once I thought that the definitions were completed and exemplars determined, it was time to analyze the portraits. It became apparent that I had over-simplified some of the definitions and that some activities that I would have defined as Formalisings were really Property Noticings, Recording. The Formalising levels for both growth of understanding of teaching and learning were redefined, as was the Observing level. New exemplars were determined for the Inventising level. Thus, throughout the process of developing definitions, my thinking about them was following a path of the to-and-fro movement of a mapping on the Pirie-Kieren model. I was using my own understanding to develop Images of what I thought was involved at that level. I then used these definitions to analyze the developing understanding of teaching and learning of the SMIP students, and when I met with an obstacle, I had to Fold Back and re-examine my Images, reconciling them (Property Noticing) into a more meaningful understanding. This, in my mind, re-confirmed that the model could be used to discuss the growth of understanding in the new context.

Once the definitions were complete, the final analysis of the portraits was begun. Again, the process was not linear. I first had to determine which statements and actions of the preservice teachers seemed to indicate a particular level of understanding according to my new definitions. Classifying the incidents onto the appropriate level took great concentration, and it soon became apparent that I wanted, too quickly, to assume that the preservice teacher had developed beyond the level that he/she had. I <u>wanted</u> him/her to show development since the aim of the study had been to determine growth of understanding, and the purpose of the program had been to give them a foundation upon which to build understanding. However, I was trying to force onto the individual's profile, an understanding that was not there. Thus, often if he/she was at the Reviewing level of Image Making, or if an individual made an emphatic statement or repeated a statement, I would classify it as Formalising. I then had to go back and re-classify when I realized that he/she had not had engaged in Property Noticing. I had to be reminded that I was tracking the growth of what actually happened, not what I hoped had happened: "Tracking his growth is about what actually happened. If I plant a seed in the ground and take infinite care to water it, warm it, protect it, etc. and a mole comes along and digs it up (as they do here), then however good my intentions, however little it was my fault, I cannot say the seed grew to be a plant" (Susan Pirie, personal communication, 2006). I also had to be reminded that, at this stage of their development, most preservice teachers would likely be involved, as much, if not more, at the Image Making level than at the Property Noticing level and that this was good because it meant they were open to new ideas. I therefore had to 'back off' and be less emotionally involved. The distinction had to be made between making a statement (Property Noticing - Recording) and Formalising, when the individual actually enacted and discussed the understanding. Thus, I had to reconsider and re-classify several incidents, realizing that enactment was necessary before the preservice teacher could be said to have crossed a Don't Need Boundary.

Another dilemma in classification occurred when an individual stated that he/she believed one thing but enacted another. At face value, the statement would be taken as an Image, but the later action enacted a different Image. Thus, the statement was likely an Image Making - Reviewing statement and had not been encompassed into the individual's repertoire of Images. In these situations, it was only after considering the later Images and actings that I was able to properly classify the incident. As a result, in the process of identifying levels on the theory it was necessary to identify incidents, classify them at the time, and then, possibly, re-classify once they were placed into context.

The final decision with respect to the analysis was on how to best display the developing understanding as a form of Mapping so that the movement between levels was apparent. The modified, duel model would be used as a summary of the results, but it could not be used to indicate all the connections of approximately ninety incidents for each preservice teacher. After considering several different models and trying a number of methods, I chose a vertical display of the incidents, charting the individual's growth through the levels of the modified Pirie-Kieren theory using numbers provided in the text of the analysis. In this way, it was possible to visualize the growth of understanding of teaching and learning at the same time, and to see the connections between them. This form of display provided a good visual representation of the developing understanding of the individual. The results of this analysis are presented in Chapter 7.

Chapter 6

Presentation of the Portraits

6.1. Introduction

In this chapter I present the portraits of the four preservice teachers, Sophia, Lance, Ellie and Wayne, whom I followed through their practicum to observe their growth of understanding of teaching and learning mathematics. The purpose of these detailed portraits is to give the reader a 'picture' of the four students as they passed through the SMIP course. It is hoped that by forming such a picture, the reader will be in a better position to understand the data and follow and agree with the analysis performed in Chapter 7. For the analysis, it is essential that the students be seen as individuals, as real people in a real situation and I felt that merely quoting snippets from the video data would not have supplied enough information and background for the reader to be convinced of the analysis. The analysis of the individuals will be presented in the same order as the portraits. Thus, the reader can follow one individual by reading a section of Chapter 6 followed by the corresponding section of Chapter 7. However, reading the four portraits first will enhance the understanding of the group dynamics of the SMIP program and will better situate each individual in the group. As mentioned, the portraits have been written as much as possible in the words of the individual. In each portrait, *italics* print was used to give background information or when I interpreted the individual's reaction. **Bold** print was used when the individual emphasized a word or phrase.

6.2. A portrait of Sophia

It is the first day back in class after the short practicum. Preservice teachers are excitedly talking about their experiences: The experience was enjoyable. But, it was also found to be a real 'eye-opener'. They would have to work on questioning, pacing, being authoritative. Sponsor teachers were great, students were great, things were happening. And suddenly:

> Sophia: It was horrible. I attended six classes and was **bored** out of my **skull** in **every one**.
> Lorne (instructor): What do you mean?
> Sophia: It was really – REALLY – boring – nothing - teacher was trying to – mayhem – *nothing* – ***like nothing happened.***
> Trinity (student): What do you mean, nothing happened?
> Sophia: Nothing! Nobody learned a thing!
> Val: How could nothing happen? No instruction? Like turn to chapter 2?
> Sophia: "You're grade 8 – doing fractions since grade 4 --- so you know how to do this." So he gave them an assignment – kids don't work – some **amazing** behaviour so he just walked away – ***not one spot of math for the whole time*** –

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Originally from a small city in the interior of the province, immediately after high school, Sophia entered the Faculty of Science to study engineering. After only two years, she realized that this was not what she wanted to do so went to Ghana for six months on a cultural studies program. Returning to university, she switched focus and finished a degree in mathematics. After that, she spent two years in Peru as a volunteer working in a rural community development project where she had a chance to teach English and practice her favorite activity, mountaineering. "I'm an idealistic girl and I want to make the world a better place through education. To tell you the truth, I'd rather be teaching outdoor education than mathematics".

However, the enthusiasm she displays for mathematics seems to belie this statement.

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"I don't think a teacher can understand the confusion or difficulties in a student's head by evaluating written work - or heaven forbid a multiple choice exam - one needs to see a student work through the problem, hear their 'umm's' to know where they are having logic problems etc. Also by encouraging **understanding** of problem solving, and not just procedures. There are a few students for whom mathematics is fabulous --- just comes naturally and placing a textbook in their hand would be more than sufficient. But for others --- probably the majority --- they need to be taught more clearly --- more precisely --- and it's the teacher's responsibility to open up their horizons with her passion and energy."

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A sense of comradery develops in the SMIP class and Sophia is quickly identified as someone who not only has good ideas, but also as someone who is not afraid to present provocative ideas. Discussions in which Sophia is involved are always lively and she freely expresses her thoughts while, at the same time, being willing to listen to others. To her, human nature is complex, and so is learning mathematics, so when Evan, a fellow student, suggests that learning mathematics is a linear process, she jumps in:

"Mathematics has a **natural learning progression**. --- But it's from **tangent** to **tangent**, not book chapter to book chapter. Sure, there is stability in mathematics. Truths you learn today will be true tomorrow, but your understanding is always expanding and changing and going back. A sense of continuity is important, and math can't be presented as a bunch of little units outlined in the textbook. I think it's really important that a teacher understands **where** the students are coming from - like their Primitive Knowing, wasn't it? - and that she listens to the kids in order to understand what they are saying. Each learns differently and --- you can't just teach them all the same. I really want to learn how to reach all kinds of learners --- to find ways to challenge the top of the class without leaving the bottom behind --- as well as vice versa. Teaching, I think, is like a guided tour - fabulous --- like mountaineering. The guide knows general directions and knows the terrain and overall picture so --- so --- well, she can detour depending on the needs of the tourists and then the tour is different each time depending on the tourists' interests and abilities to deal with the terrain. Yeah, I think that that's what teaching is like. I like that – mountaineering. That's a good simile. Fabulous. And anyway, learning --- I think it has to be a continuous process and the teacher must be a continual learner of – well, of both mathematics and of how to teach it. I mean --- you even have to look at it differently depending where you're teaching. I remember that when I was teaching in Peru, the students had lots of problems with the concepts of how to enquire about quantity in English. It was completely **foreign** to them to use different expressions for, I mean like time --- how long --- or quantity --- how many/how much --- or maybe distance --- how far --- 'cuz in Spanish all would be expressed using the word 'Cuanto.' And so, if I were trying to define the nature of mathematics in Spanish, I would simply say that Mathematics is what happens whenever you try to answer the question 'Cuanto?' ... To me this is the basic nature of mathematics, and many of the other ways we define mathematics are simply descriptors of this basic nature. I would compare it to my basic nature as a human being. There are many ways to describe me – tall or short, old or young, hairy or bald, introverted or extroverted, etc. – but these are **only** descriptors of my qualities and do **not** themselves sum up **my** nature of being human."

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Sophia is impressed by a video the class observed. Alwyn, the teacher of a low-level mathematics class introduces algebra using boxes rather than x's or some other variable. He begins his lesson with a fairly complex question, not the usual kind where the student can just look at it and tell the answer.

"You know, that makes sense --- to do it that way --- you know, what the teacher did there. The boxes really show that something goes there. It was **fabulous** the way the kids just dug in and got going on the problems. Understanding is important and maybe --- juuuust maybe --- those boxes might help students make sense out of it – they can maybe see that something goes **into** that box --- the answer --- it's not just some abstract entity --- it actually **fits** in there. Fabulous! I think I might --- **yeah, I will definitely** try that in my teaching. I think it was good."

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When the preservice teachers are given a problem to work on, they are asked to work in groups and to think about and discuss how they are solving the problem.

"Hey, I just Formalised there! Wow! I never thought of that before. Fabulous! I didn't realize that **even we** went through these steps in the theory when **we** worked on a problem. I think it is neat, just fabulous, to discuss the levels that we are working at - use the terminology and discuss. --- seeing how we go through problem solving and discussing it with peers --- will --- will --- well --- will help understanding what our kids will go through and how to push them to get beyond the level at which they are working. ... I think that **labeling** always serves a purpose and what I like about the Pirie-Kieren theory is that you can look at a problem and you can say that I went to Structuring or whatever and you could see what level you progressed to whereas you just said that was a good problem - that's good, but it doesn't define the limitations of the problem or the benefits of the problem so I think that if we discuss the problems in terms of this and how we can push the students to conceptualize further or less or what level will most of the students get to and it puts a nice exactness to the problem that you can then put on other problems. Oh --- this is how I can make my teaching better -- it's more of a retro-scale of your teaching the problem of how your teaching goes --- that was a nice experience, a good problem but you don't really --- but if you don't use the label, define it more discretely --- I think it is more nebulous about how good your teaching is – how good their learning is".

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In discussing homework, many of Sophia's classmates indicate that they think students are given too much. Students should be able to decide how much they need to do, but Sophia does not totally agree.

"Students don't learn just by being **shown** how to do a question or two. They do have to **practice**. They don't have to **do** like 20 questions, maybe, but they --- well, to understand, they --- **practice** gives kids the chance to break through a barrier so that the work can become --- more automatic --- at a higher level. They might be able to **do** the questions by following an example – but being able to do is only a part of understanding. Sometimes, *maybe* the doing can lead to the understanding --- Like going through the knowledge boundary. By doing, you can maybe suddenly come to understand --- like move **up** a level in math, that's supposed to be like --- its like spelling ---- not supposed to have to **think** about every step --- you should --- you have to just do as a functional process … if you have to think of each one as a cognitive process there is just too much and they get lost ".

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Each student in the SMIP program is required to make several presentations to the class. Sophia's lessons are always interactive and involve social contexts to which high school students can identify and with which they can engage: linears are introduced using mountaineering, a class to which she brings her rope and compass, and for which she fabricates an enticing story; in a lesson on areas and perimeters, Harry Potter is 'chained to a building'; and, manipulatives are a standard part of her presentations.

"I think it is **really** important that students feel **engaged** with their work. I think they have to have **fun** with it --- and they have to understand. Sometimes it's hard for them, but the teacher has to encourage --- tell them they are doing fabulously --- she has to have them make connections some way or other – and it's not really too hard for the kids to get the connections if they come up with the connections. The teacher should be helping them maneuver through math. Like when Trinity gave her problem on probability --- often not a really interesting thing --- but she made it about AIDS with **real** numbers and got us to **guess** at the answer **before** we started. Boy, was **I** off --- but then she --- well, it was just **fabulous** the way she made us do all the work - and figure it out --- ourselves. We'll remember **that**. Fabulous!"

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A positive attitude and the use of encouragement will go a long way in helping students learn, from Sophia's perspective.

"Not that it will make everything perfect --- it's okay to make mistakes. You can learn from them if you don't get all gloomy --- But, the teacher must give students some challenging work to do. When I was in school, well ---- My own grade 7 teacher as really strong – encouraged us to learn more. But, in grade 8 and 9, it was **boring – so boring** because those teachers **didn't** push. Or maybe it was the curriculum, I don't know, but I remember ---- **Finally** --- in grade 10 we **learned something new** --- some trigonometry --- and it was 'Like --- **Wahoo! Fabulous! Finally** we learned something new. Praise be to God!'

"All students should be given the opportunity to do the best they can. And, I think they **all** can do some things well. They just need to be encouraged -- develop their own skills -- and think about what they already know. A major goal of mathematics education should be to get students to be **independent** thinkers and problem **solvers**. More hands on --- Manipulatives --- give the physical and work with them until they can see where it comes from --- the standard, rigorous form - maybe all don't need it --- the rigorous Formalisations --- unless they are going into science or math, but I think you are doing a great disservice to your kids if you don't prepare them to the level that they can get out there and that they are able --- I mean, they'll be good problem solvers and they will likely be able to solve the problem --- but I think you have to give them the **tools** to be able to **read** the language that everybody else is writing in and to be able to express their thoughts in those ways. Not all need to be able to do the abstract stuff --- the typical standard form. For them, it is the brain impulses that are important --- but give the weaker student a chance to **physically** figure things out while --- make it less boring as kids who are strong --- they might be able to pick up other ideas in the hands-on approach. You know, sort of like we did when we worked with the Algetile – we figured out some stuff we hadn't thought of before --- maybe the better kids would too.

"Really, you shouldn't judge students by their mathematical ability, but rather --- well, you should assume that they can do it - and let them know that it is **their** responsibility to do so."

Sophia is not so naïve as to think that all students will reach the same level, and admits that not all students will love and understand mathematics.

"There are some who just want to learn the process and get out of there - but even these should be encouraged to understand and maybe they will come to a better feeling about the subject. All students should be given the opportunity to develop their own way of thinking. The teacher should be one with the students – seeing how they are learning it --- and she really has to encourage them."

With this positive attitude and 'fabulous' outlook, Sophia entered her short practicum.

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Sophia's experience during her short practicum is encapsulated at the beginning of her portrait.

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"The attitude of the school stinks! Nobody should be made to feel that they can't do good --- the teacher needs --- the students are **fabulous** and they **need to know that**. How can they learn if the teacher – if they're not expected to? It sucks! It was **horrible** the way students got placed in some classes. They were put in an Essentials class just because there was no room in the Principles class there was this one girl – in Essentials – and she was a fabulous little thing – from Guatemala – talked to her in Spanish and she really knew all the math ---

could do all sorts of things --- her parents, engineers --- don't speak much English and don't know – scared to go and push. It was terrible!"

Sophia was not impressed with the laissez faire attitude of many of the teachers she met, nor with the school's policy of assigning second language students to lower level mathematics classes based on their lack of English skills or simply because the other classes were full. The experience made Sophia think about how she will teach while on her "long practicum" at John's High.

"My Father once said – after he retired and had time to think --- that if schools focused on teaching kids how ***to think***, boy, education would be amazing! That fairly sums up the academic side of my goals as a teacher. But --- one really has to think about the social aspect also. I hope, well --- that my classroom will represent --- a microcosm of the world I'd like to live in --- where everyone is respected and valued. School is a place to learn about the world. People of all sorts are a part of society, and so students with exceptionalities should be included in classes. I disagree with you, Wayne --- People have to live in society with everyone and so they may as well learn it in the classroom --- social benefits outweigh the negative – potential negative implications on learning. Yes, you have to consider the majority, and yes, people do spend the majority of their time with people of similar interests --- but you have to consider the **minority,** not just the majority, at **all** times, and respect people for their differences. Just because it --- well benefits the majority doesn't mean you can ignore the minority - for example, people with fetal alcohol syndrome --- they aren't going to find a group of people with similar disabilities to interact with ---

they have to be part of the society --- we have to augment them in the regular classroom. But, then, also, sometimes we don't address the problems of the gifted. The education system, however, often focuses too much on the lower end and doesn't do enough for the gifted because they will manage anyway. I think we have to think about this --- equity --- challenging --- spend time on making questions that are diverse enough so that people in the lower end won't feel right out of it but people at the upper end are challenged but if you could theoretically come up with extension questions where people at the upper end could be challenged, there is no reason why you couldn't have people of completely varied abilities in one classroom. Equity - and a part of that is technology. --- It is a **fundamental** for the information and communication age we live in and there are **so many** fabulous ways it can be used in a math class. Just imagine how --- if used properly --- it can help enhance and deepen the students' understanding of certain mathematical concepts. For many, just giving them experience of **using** technology with ease is useful --- helps create 'a zone of equality' --- help limit the inequality between the haves and the have nots".

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After various presentations in class where the SMIP students have been presenting on different topics, they talk about the methods they used and ones that they think might be used in the classroom to help students understand.

"Peer people, peers explaining, as long as the peer understands is so much more effective than the teacher explaining so that I would probably do it in groups and then just like --- can you see a faster way to do it? - just ask questions --- and somebody is going to come up with a more efficient way --- even if its not your way ... like I'll bet there are four or five people who come up with a more efficient way and then get them to teach the people nearest them and then you'll have everyone progressing at a closer to uniform rate than if you let everyone work on their own and you have someone progressing and helping everyone else get done."

After some work with algetiles, Sophia is concerned the switching from one representation to another might be confusing for students.

"Whatever method the students are using, they need time to develop. They need to notice this, this and this, and they notice in so many different ways, 'cuz it's all new to them and so the teacher needs to be aware of all **sorts** of different possible connections. She shouldn't outline everything too much --- you can't show the --- you can't lead them through **everything** because then the students will not be able to **develop** the understanding that will be necessary when new topics are introduced. They'll just have the teacher's understanding and when they try --- well, they have to call upon their **own** previous understanding."

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Looking at the world as a whole, and taking a pragmatic perspective, Sophia thinks that tests are useful but the design of them needs to be considered:

"Tests are a part of life and kids will have to take tests of different forms throughout their lives. Not all will be paper and pencil --- answer this question and that --- but they **will** be tested --- one way or another --- so why not prepare them when they are working in a --- well --- safe environment? **But** --- most tests given in schools don't **really** evaluate a student's understanding. Open-ended questions --- ones with no right answer. --- then you have to rely on how well the student can argue and you can better determine the process they go through --- **that's** what's important. And, during assessment, it is important to consider the kinds of mistakes students make: Some are just careless, and these should be pointed out --- but if they don't affect the understanding, students should not be penalized too greatly for them. But, if something is something --- **completely wrong** --- well, the teacher needs to be concerned and try to determine what epistemological obstacle is there and help them fix it. Multiple choice tests. Well, they shouldn't be completely discarded --- but to be useful, you really have to know the **kinds** of mistakes the kids will make and why and go over all this with them otherwise they will be useless. Don't want them to do what Lance did --- just substitute back in."

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John's High is not designated as an inner-city school in which case it would receive extra funding, but it has many of the characteristics of such a school. There is a large number of Special Needs students, some in special classes, but integrated into regular classrooms for some of their courses. There are also many students who are not designated as special needs, but who need extra help. Many are first or second generation immigrants and poverty is a problem. In some families the children have to work to help make ends meet.

However, there are also families in which there is a stay-at-home mom, there are relatively few families on welfare, and there is a relatively low proportion of single parent families. Because of cultural traditions and financial necessities, parental involvement in school activities is low. While academic performance at the school is poor, the school prides itself on its athletics. This is a concern for some teachers and the school has recently introduced a mini-school program for the more able students. There is, however, a significantly large and vocal group of teachers who express the opinion: "With what we've got to work with, what can you expect?"

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A few days into her long practicum, one of Sophia's sponsor teachers, the one she had previously complained about, has put her in charge of the class. Although no longer meeting regularly at the university, the preservice teachers keep in email contact with each other. Sophia expresses dismay at the situation, again, but delights with the way her class was responding.

> Well, i thought instead of being the last ones to write this time, I'd start it off. So hope everyone's practica are going spendidly Hey – not fabulously!). I've had the eventful time as always. one of my sponsor teachers had to rush back to England for a family emergency, and so there's been a different TOC every day now. and the other, well.... Let me just say that i was once again ranting to someone about the teaching i'm witnessing. ... but my own lessons today went great. Lorne (FA) came to my second one. I did that Cubey-Doobie-Doo thingy we learned, (looking for patterns in the cube structure, translating it into equations) and the kids WERE ENGAGED! Although lots of them gave up quickly, but some were really problem solving. good emotion in their eyes. and **hardly anyone** threw the blocks at each other. and lots of them built guns and tanks ... got to love it. ... it was lovely. *(email communiqué)*

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Sophia is pleased that she will be able to introduce algebra to her class. She models the unit on the lesson that she observed in the Methods class as she thought she would. She doesn't copy Alwyn's lesson, but rather adapts it to suit her personality and her students – a relatively small class of 17 boys and 8 girls which has been described as 'rowdy'. She begins the first class of the unit by writing on the chalkboard: $\square + \square 5 = \square + \square + \square + 12$.

"What number do you think goes in the box?" *The students don't know.* "Guess – guess any number." *Someone guesses 52.* "Fabulous, now, plug it in and work it out. ... Does that work? ... No? ... How do you know? ... Okay, then what do we need a bigger or a smaller number? ... Smaller? How do you know? Okay. What would you choose now? ... Why?"

Students are not afraid to answer. They seem to feel comfortable giving their opinion because even if it is wrong, Sophia doesn't say it's wrong. She lets them work with their number until they see that it 'doesn't work'.

Sophia gives them another question on the chalkboard and they work on it in groups, discussing the process and their results. She hands out a worksheet. Within minutes, the classroom buzzes with sound. Students are working in small groups. Hands are shooting up faster than Sophia can get around to them. Her identification badge, hooked on the waistband of her slacks, she makes no attempt to hurry, acknowledging by eye contact or a little wave that she will come as soon as she can. Meanwhile:

"Could you try working it out? Get help from someone?" *It is hard to believe that Sophia has taught this class for only three days.*

After a few examples, Sophia tries to introduce variables, but many students resist. She does not force them to switch. She lets them work with the boxes because they feel comfortable with them and they seem to understand what they are doing. The next day she hands out another worksheet.

"It was too hard to do all those boxes with the computer, so I just used the letter B. But, if you want -- when you write it in your book, if you want a box, you can use it". *Over the next few days, most students do change from using the boxes to using variables in their equations, many using the 'B' that Sophia used as her first example. They continue to refer to it as 'the box' and talk about the answer as 'what fits in it'.*

"You know, learning mathematics is a process of thinking --- not a step by step process --- you can't just follow rules, you have to think --- and verify and interpret --- always checking and justifying. Be sure to substitute your answer back in to see if it works," *Sophia says to the class.*

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There is respect in Sophia's classroom – respect <u>*and*</u> *laughter.*

"Today you can choose who you want to work with. But, **think**. Don't necessarily choose your best buddy. This is for marks and you will want to have someone in the group who knows how to do the work." *The class laughs.*

In the after class discussion, Sophia states:

"Students need to be reminded that everything they do is important and marks are always important to students, so this is a good way of reminding them – you know, when I told then it was for marks --- that everything they do is important and will affect their future. They --- they will always be marked --- its not graded – like outside school, I mean --- so I think they should know it is always important to do your best."

Sophia expresses her respect for her students in many ways. She spends very little time explaining and doing questions – she trusts their ability to learn. At the front of the class, she listens as students explain their methods. They come and do the work on the board and then discuss it.

"It doesn't matter if it isn't done correctly" *she tells the class.* "Errors simply provide an opportunity for learning to take place - a chance to develop new understandings and create better/more accurate images. You can't learn without making mistakes. No one is perfect".

Even when she is reprimanding someone for inappropriate behaviour, Sophia shows respect for the student. Often it is just silence - the reprimand - silence with a smile – a little lop-sided – accompanied by a slight tilt of the head and a raised eyebrow. Or sometimes, it is just a reminder:

> Sophia: Chan, did you just throw that eraser?
> Chan: Yes, ma'am.
> Sophia: Do we throw erasers in this class?
> Chan: No, ma' – well, yes, ma'am I did but I shoudn't 'a. Sorry.

However, at times, discretions must be brought to the fore: "Jason, you're busted. You're so busted ....." *Sophia retains her ever-present smile, but her sparkling eyes speak volumes as she walks over to Jason and puts out her hand* *so that he will give her his Discman. There aren't many rules in Sophia's class, but Jason has been caught breaking one of them and he is called on it. Since the students helped in drawing up the few rules that exist, Jason knows he is in the wrong and he does not feel demeaned or put down. Nor do any of the other students make fun of him. Not saying anything more about the indiscretion, Sophia's eyes and tone of voice tell Jason that she expects better from him as she reminds him that he will have a detention, a fairly strict form of discipline for her to give. But, even on this Sophia puts a positive spin:* "You can get extra help" *she says*, "and if anyone else wants help, well, I'll be here visiting with Jason and you are welcome to join us."

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'Received wisdom' in North America often regards teaching as a 'stand and deliver' process whereby each class begins with a five minute review followed by ten minutes to go over students' problems with their homework. Ten to twenty minutes are then spent on presenting new work, interspersed with time for questioning and for students to try progressively harder questions.

On the surface, then, it seems that Sophia seldom teaches. She may put a question or two on the chalkboard. She then walks about the class, answering questions with questions.

"The students can do the work. They are capable. No doubt they will have problems, but that is when you have to ask them what their problem is and in explaining it, they sometimes figure it out themselves. Or, they should talk to their friends and discuss. They just have to think about what they know and they learn so much from each other – Peer teaching is fabulous. It saves my voice!"

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Sophia continues to move about the class, returning to a group of four boys at the back for the fourth time. Two of them are quite 'good' at mathematics, she has indicated, but the other two have 'low skills'. This time they have stopped working.

> Sophia: What are you doing?
> Chan: Nothing. We can't do this question.
> Sophia: Try using different numbers – easier ones.
> Ali: Oh, yeah!
> ***Learning was taking place!***

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"Each class is a surprise" *says Sophia.* "It never goes as expected and that is what keeps me going - the excitement of developing deeper understanding, of questioning and probing for more knowledge."

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After viewing herself on videotape, Sophia reflects on her teaching and recalls various incidents through the practicum.

"They really seem to have --- really seem to have gotten the equations stuff. I liked the conceptualization I saw when the kids were doing it. Instead of x being some unknown weight --- It was **fabulous**. Well, they seemed to see the box as some number and they got the clue that it had to be the same each time. I thought the box thing was great to have them work with numbers because their other teacher had them use calculators so much I found that their mental numeric abilities were so – so **very poor** --- so I thought with this - with lots of calculations as well, it helps them. There's a lot --- and just the concept that this box is not a weight --- it's a **number** and it's **always the same** and I think it's fabulous the way --- it really took off --- like it was a puzzle and something to find. People understand something hidden – puzzles – but with x it's algebra and a lot of kids have this mental thing even before they start 'cuz they've heard about it. So the boxes --- they didn't even realize, well some of them did --- sort of like your situation analysis you told us about instead of calling it problem solving. Anyway, when we made the transition, they --- even the weak ones --- they will have a good **mental** visualization of what x means and that sort of thing --- build their Images, sort of and they won't just forget it or try to follow rules. I think it was a fabulous way to try the -- to teach equations.

"There are some --- well, they just don't want to be there, so you have to make it like they want to come --- Jade, well, he sort of knows what's going on, but Steven --- well --- so a lot of the stuff is trying to let them have fun in a social context and hopefully there is some math going on at the same time – if they work individually its just chaos --- at least this way – group work --- they may pick up something from their neighbors and get the concept but really a lot of it is their own Image in this setting. Peer teaching is great and they can learn so much. I remember at one time, there was a **hysterical** discussion with a group of girls --- the ones at the front --- remember them? Anyway, about '$2x$' when 'x' was '6'. Was that two, 2 times 6 or 6 times 6 or something else? --- like they got 60 out of it so I guess they thought it was 6 times 10 --- and there was one student going

'It's this way.' and another student saying 'No it isn't'. There was something about the --- the **energy** they were putting in to defend their argument --- **that** was fabulous math going on right there. Then there was Serb who kept explaining things to Ron who is very low level math --- just like --- I don't know --- the **care** that I saw -- just the good mathematical learning. That was good --- just with a few questions a lot of teaching goes on without my having to lose my voice. I don't have to talk all the time. I think they really **learn** it this way and I don't have to lose my voice. I really saw some great teaching going on between the kids and I think maybe it worked best when I let them pick their own groups because of the increased --- fraternity that was in the groups. I thought that went really well"

About punishment: "Well, I learned long ago as a camp counsellor, that sometimes it is best to get angry before you really are angry. Punishment --- has to have different levels for different students. I don't consider myself a disciplinarian, but I expect the kids --- students --- to behave. A few rules are good, but I never really had a problem. Sometimes, I think it is just keeping the kids occupied and engaged. If they have something interesting to do and they can do it, I think they are all just fabulous. They'll do it."

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The next year, Sophia took a job in a remote northern area of the province.

6.3. A portrait of Lance

It is the end of the first class Lance taught for Mr. Forsyth. The class went fairly smoothly but Lance is concerned that he really doesn't know how difficult he should be making the assignments.

Lance: Something tells me if I just do what is in the book, I'm not trying very hard. I have to look for other things to do ---

Mr. Forsyth: First of all, **THIS** is the curriculum. *(Picking up the textbook in two hands he lays it firmly down on Lance's worksheet.)* Textbook writer wrote the textbook based on the curriculum. They spent a lot of time --- **Everything** you need is here. If you go beyond that --- first of all you could be out of your riding --- research has been done into **this** *(tapping the textbook with his right pointer finger)* and **this** is what you teach.

Lance: Oh. Okay.

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Lance is a quiet, sincere young man who has lived in the city all his life. He is the youngest of three sons of a University professor and a stay-at-home mom. As was expected of him, he went to University immediately after high school, and, after almost "failing out", finished a degree in Commerce, studying part-time and trying his hand at a few jobs – delivery boy, bartender, magazine distribution manager. None of these proved satisfying, except, possibly when he had to train people for new responsibilities, so he decided to follow in his father's footsteps and become a teacher. He did not find that his own teachers had been an inspiration for him, and he was hoping that there was a way to make mathematics classes a little more interesting than the ones he had had.

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"I've given it a lot of thought and I feel this is a good decision, and the right one for me. I have to say that I am beginning the program a bit concerned about my own understanding of high school mathematics, but since mathematics is ordered and logical, I guess I will be okay. Sure --- the ideas are still there, but I'm --- uh --- just not sure if I remember all the ... umm --- you know, the words. See, I believe that learning mathematics requires higher-level thinking than many other subjects. For example, social studies you can memorize it. But in mathematics everything is either right or wrong, and can be proven true or false through a series of steps. Learning mathematics requires practice, focus and patience --- and I think it is the teacher's job to guide and educate my students, providing -- well --- thoughtful, precise, you know --- detailed lessons. Except for the incredibly gifted who'll understand each concept immediately, mathematics may seem strange and foreign to kids until there is a sudden insight. As a teacher of mathematics, I'll have to put myself in their --- the students --- shoes and try to remember how I learned the concepts. It's much easier to teach a concept if I remember what it was like for myself to learn the same concepts."

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At the end of the second day of the SMIP program, the preservice teachers were left with a proof to do, based on a hand- on activity they had done. Although Lance was hesitant about his own understanding of high school mathematics, he was the only student who came to class the next day ready to demonstrate his 'proof' of the theorem. True to his admitted weakness of the language of mathematics, a weakness that persists throughout the course, Lance talks about "six-gons" and "squished squares" instead of hexagons and trapezoids. He is not nervous in front of the class, and although he makes a few *mistakes which are corrected by his fellow classmates, this does not bother him and he discusses his presentation.*

"You know now I think about it, I reckon my lesson was OK --- a bit jumpy and not very organized maybe 'cuz I hadn't really thought about how the others would follow my presentation and I wonder if, --- Because --- you know, mathematics comes easy to us students in SMIP, if we will be able to anticipate the problems students will have in school. I don't know, I'd say I was a learn by doing kind of guy who likes to figure things out by myself. When I had to train some people at work, I planned my, you know --- planned my lessons on **my** understanding and the manner in which I learned to do the work. Learning mathematics is a bit like construction of a building --- I read that somewhere and I agree. It said that a building must have a strong foundation, but if you don't fill it, it's useless – and if the foundation is not strong, it will collapse. So that's like math, and the teacher should direct the student's learning --- create a strong structure so ... you know ... more learning can take place."

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Lance's honesty and sincerity make him a person who is easy to talk to, and everyone in the class seems to like him. His female classmates tease him that all the high school girls will fall in love with him because of his classic good looks and his athletic body. He self-consciously smiles this aside.

"That's not so important. You have to consider **what** people do. Think about Armie. Here she is, all the way from Iran where she taught, and here she has to re-certify and she's doing it. That takes real strength, courage. I'm impressed by that and I hope --- well, I know I can learn a lot from her. You know, she's from a different country, but you know, mathematics is everywhere – the multiplication of rabbits is a series, shells are spirals – you can't talk about anything without mathematics. Even linguistics is structured mathematically and they don't have the math to do the structuring – it just happened naturally – maybe the math we do is just trying to make sense of what we see. But how do you explain that to students? You know, they should know that it is important, that they really have to use it – so maybe word problems are good, but maybe what that teacher in the video --- you know, the one Lorne showed us where he kept telling them that they **must** be smart if they could learn this stuff 'cuz it was hard. Maybe telling them that they are smart is a substitute for relevance. --- I'll have to think about that 'cuz, everything we know and do in mathematics is based on what we already know. You can't go ahead unless you already have a place to start and maybe if you tell the kids – let them know you think they can do it, that that will, you know, make them think they can. --- And then that makes me wonder --- Is math created as a product of our environment, as a way to relate? If we grew up on the moon, with no gravity, would that affect it? If an alien race came to earth, would they have math and, you know, would it be the same as ours? Could we have gotten to this point without math?"

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Lance watches carefully as other students present their lessons and when they are given videos to watch of teachers in classrooms.

"Actually, talking in these groups in SMIP about what everybody did in school and seeing all these different teaching techniques that are effective, I realize how much I don't know – because – I keep analyzing – but I don't think I ever had a teacher in high school that ever, you know --- that used a questioning style or that --- well, I rarely remember working in groups as a project – they actually discouraged it because they saw that as a way for kids to cause trouble --- just do your work and they made sure the desks were far enough apart --- so those are a lot of the things I'm learning – keeping kids involved, getting them to learn without being told – working together, hearing information from all sides – you just naturally absorb. But, you know, sometimes I can't figure out what it is that you're trying to teach - Was it factoring or multiplying that that you were teaching, Trinity? It really wasn't very clear in my mind.

"Really, when I entered the program I thought I would have liked it if I had just been given a teaching certificate and went out to teach so's I could learn on the job. 'Cuz I pictured the program being more a telling kind of thing and I never really liked that. Still, in this first month of the teacher ed program, I can see how much I'm actually --- you know --- learning. I mean learning techniques and tricks of how to get through to students, things I'd never previously even thought about. The teaching I got was the lecture-type and --- but that's the way it --- well -- it's amazing how many different teaching styles there really are, you know --- and how much better they seem to work than what I suffered! I had to try to figure it out myself and we are learning to help students figure it out for themselves so they can remember – comes from your own head rather than someone telling you and that's what I always wanted but never --- and it is good to see that happen so much because I didn't know how to do it --- I' m sure if I'd gone out to teach I'd have been just like my teachers."

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For one of his presentations to the SMIP class, Lance begins the lesson with a game/puzzle:

"Pick a number. ... Double it. ... Add 9. ... Add the original number. ... Divide by 3. ... Add 4. Okay, subtract the original number. ... What's your answer, Erin? No! Let me guess." *His hand goes up to his head, he thinks for a few seconds, then:* "The answer is 7." *The preservice teachers are intrigued. Lance goes on to explain how/why he knew the answer would be 7. He had made this question up himself and he demonstrates the algebraic reasoning to the class, purposefully making errors. His classmates correct him.* "Just making sure you are paying attention. I think that this might be a good idea in teaching --- make mistakes to see if the kids are paying attention. Something I never thought of before. What do you guys think?"

As is often the case, Lance has again started a discussion. Later, when discussing the presentation: "You know, I just told you guys how it worked – and I know this is not the best way for students to learn. We talked about questioning and all that. You know, I guess I'm just used to --- I guess I need to work on this – not telling, but trying to get the class to figure it out for themselves. It's really hard to break habits. This is what I'm used to, but it's not the best."

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"I suppose I'd also describe myself as a visual learner. I really enjoyed geometry. I could look at a geometry question and see a ten-step proof, but algebra was more abstract and I had to start it – write things down you know --- before the steps would fall into place. That's the way my mind works. That's probably why I'm learning a lot about teaching by watching you guys present – you know, the mini-lessons Lorne has us do --- where we have to try something new --- those fifteen-minute lessons on something and we talk about it. I think I'll probably try to incorporate some of those activities in my lessons. You know, like I'm eager to use what Peter – that arithmetic series – I'm a visual - and I never thought to relate areas to addition of sequences. This will be good to teach with. I like getting things others have done. These ideas – I can use them - so they're good. There were a lot of good ideas in those demos."

Lance seems able to find something good to say about almost anything he has to critique. After presenting a teach-by-telling lesson in which Armie simply presented material to students in a lecture format, she criticizes herself, but Lance cuts in with: "You're teaching in a second language, but you are so knowledgeable. Don't be so hard on yourself". *And when another student, Mer presented a class in which he faced the board for the entire explanation, and during which it was hard to hear what he was saying, Lance commented:* "You were so calm. I could learn from that – not putting me under pressure."

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Lance gives some thought to student behaviour, and as is often the case, his thoughts jump from topic to topic. "You know, maybe we have to think about inappropriate behaviour by the kids. You know, it might be caused by something outside the class – like the student might have personal problems or learning problems. How will a kid feel if the teacher cuts him off or did a question differently from the way that the kid suggested? You know, Evan followed the way we suggested that he do the question and later told us that that wasn't what he intended. I felt really actuated when you did that. It made me feel, you know, like you cared how I thought of the question and how I would do it. But, what if he hadn't? How would I have felt, and how would a kid who thought he knew what he was doing didn't have the teacher, you know, accept it? Would he think he was wrong or what? And then, what about assessment? Do you assess where the kids are or how far they have come? It may not be the student's fault that he/she doesn't know everything – it may be the result of previous, poor teaching – or even my own."

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Lance confided: "I really felt sorry for Wayne that he was thinking of leaving the program - you know, when he missed class the other day. He said he didn't really know why, but I think it's, you know, 'cuz he really hasn't had any work experience. I had a few jobs before - you know to make money - but that was all they were. They didn't mean much, but -- and I hope this doesn't sound too corny -- now I feel committed. Yeah, I think teaching takes commitment. And, it is more beneficial to society than my other jobs. That does sound corny, but that's how I feel and if --- well, hopefully Wayne will remain. For myself -- even if I don't stay, I've learned a lot. Team dynamics, relating to people, understanding where they come from. – you know, even these are enough for Wayne to remain if he learns these. That's what I told him, anyway.

"In school, you know, maybe it's the focus on grades that's the problem. I know I was, you know --- stifled by being "told" what to learn and by worrying about marks. I worried about marks and I wasn't really learning - I could maybe **do** the work, but I wasn't understanding. I think its good the way this course, you know, is a pass-fail course 'cuz I'm not focused on getting that percentage --- not like --- questions like this will be on the test and I maybe focus on the wrong thing and don't see the big picture. When I was studying math, it was, you know, learn how to do it and it didn't really matter. But here, because of the structure - not focused on that **score** I'm learning a lot more. Its important for kids to "look at this and this" and try to make the connections – try to digest and so to go through a process ... I guess that it makes sense, I guess, I don't know, but maybe 'cuz that's the way I learned – not memorizing like I was supposed to. That's the way I **really** learned, and maybe that's the way it should happen – not focusing on marks in school, but on discussion, getting kids to think about what they do."

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"Teaching involves a lot of organization and you have to be able to express ideas in different ways. I don't want to dominate and control my class but to teach I will, you know, have to express my knowledge. I want to find out from all you guys the **best way** to do things 'cuz I don't want my students to get the wrong idea. If you learn something wrong, it will be hard to get back on track. You know --- I taught myself how to subtract in elementary school by adding on.

You know, 10 subtract 7 is 8, 9, 10, I would think, so --- three. But then I had to learn **real** subtraction 'cuz that wasn't the right way and now I just draw the numbers in my head --- the line and all, you know --- and do **real** subtraction. Learning takes place in the head and if you figure it out yourself it will make sense and you will remember it --- but if your learned it wrong, you'll have to re-learn it and that's hard --- I don't want my kids to have to re-learn things 'cuz they learned them wrong."

Worry may be a way to describe Lance, worry and concern over fairness. Lance is always a part of every conversation, often through posing questions that stimulate the discussion. "What does it mean to know? - does it mean knowing more or knowing better. Are they trying to put too much into the high school curriculum? What is the best configuration of desks in a class? If students are seated together, won't they talk too much and not do their mathematics? You know, the best teaching may be done by giving a naturally occurring example."

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"My short practicum went pretty good. I watched a lot of classes that were, you know, **right out of my league** --- band --- textiles --- wow --- but they had to have different techniques and that was neat to see. Then, there was a great example in a mathematics class where the teacher used graphics calculators to demonstrate slope and y-intercept. This really worked great, and you know, I'll probably use this with my own classes."

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Lance gives thought to what it means to teach mathematics after his short practicum.

"I read, you know --- in elementary school there are specialists for French and physical education, so why not mathematics? It's important, but by grade 3, 70% of the boys say they hate mathematics, and ---- well, I don't want to stereotype, but most elementary teachers fear mathematics. It isn't giving the students a fair shake – specialists for PE and French but not math." *Lorne interjects, and says that he doesn't think he understands mathematics well enough to teach elementary school. Other students concur. This surprises Lance.* "That really scares me, but, you know, as Elisa says, that may true for most of us. We are beyond that. We do all that math stuff that they do but automatically. Hey, does that mean we've crossed a Don't Need Boundary? Neat! --- but --- doesn't that mean we should be able to - what was the word? You know, go back and retrieve it? I'm not sure if I could now that you mention it."

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As time goes on Lance continues to use incorrect terminology – perfect polygon, for example, instead of regular polygon – but he is not afraid to speak up, and his understanding of the topics is developing as he discusses them with the class. He remains open to learning other methods, but he is comfortable with his own.

"I liked working with algebra tiles the **second** time. I was able to think more about how they worked. I thought about the problems the students might have by making up the problems for myself: You know, like I made up one where there weren't enough *x*'s to take away and I tried to figure out how I would do it. Then, when Elisa demonstrated her method I **saw** that it works, but I **understood** mine. Then I realized with factoring using algebra tiles, I saw that if it doesn't make a rectangle, you can't factor it. You know, these are good, but I wonder if you would show them to the students first and then teach decomposition or if I found them interesting because I already knew decomposition. What do you think about this – which way does the learning go? Don't you think it will take too much time if the --- the kids all have to figure it out by themselves? I don't want to just give rules, but there is a lot of work to cover. There must be some way to work this out --- you know, some way so they can learn it without you telling them everything, but not so's they have to do it all themselves?"

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Lance has been developing different approaches to teaching during the SMIP program. At the beginning of the year, when he did his presentation on the proof of the theorem about the shapes, he was serious and matter of fact. Now he and Sophia dress the part of Lome and myself and act out the question for their class presentation as Frank reads it. Lance goes to the chalkboard and draws a representational diagram. He practices various questioning techniques to elicit information from his fellow classmates.

"Do you think the diagram should be more real? Like I made the banks crooked, not straight lines and then, you know, I wasn't very accurate with my angle. In a real class do you think it should be sort of accurate? You know, looking more like 40 degrees. I think that that is what I would really do so's not to lead them astray."

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As the fall term nears its end, and thus, also, the close contact with his SMIP cohorts, Lance wants to find out as much as he can. When a visiting professor, Dr. Jo Towers, presents to the class, he is one of the few students who remains behind and talks to her. He is impressed with how openly she criticizes her own teaching. He wonders if she would teach it differently, now.

"I'm often not sure what to do. You know, even Dr. Towers made mistakes. She didn't see that the one girl had the right answer at the beginning. So, was this a good learning experience for that girl? And then people talk about testing as a good learning tool. Do they mean the actual test or the preparing for it? On multiple-choice tests --- like I said earlier, I used to just play with the answers, plugging them in, to see which fit the question. This was not a learning experience! So, what would make a good learning activity? A good activity should make the students **want** to play with it and they sure don't, you know --- want to play with tests. Doing something with the class like Sophia and Frank and I did would be fun if you could get another teacher to help and they could then do the question. Or, you know, remember Sophia's question about the --- I can't remember, but it had something to do with Harry Potter --- But, you know, for a lot of activities --- I --- it would be hard to make the leap from the activity to the abstract. I'm not sure how I could get the kids to do that. They'd have the picture --- uh --- Image --- but I don't know if they could switch. And, you know --- what --- how much time do you need? What if your lesson takes too much time? Not enough time? How can you make sure those open-ended questions will get to what you want them to --- you know, will the kids --- will they learn what you want them to? How do you grade them? Can you ever give anyone a 5 out of 5? Isn't there always room for improvement? And who is to say that this student's approach is better than that one's? It might just be that the one said it --- I guess looked at it more like you did and so you might give a better grade. Is that fair? And, then, if a kid --- you know, student --- does switch from physical manipulations to drawing, have they gone over some Boundary – or are they just Property Noticing? How do you know if the student made a lucky guess or if he really understood the process? These things really bother me."

During the last week of classes, before the long practicum, Lance was unusually quiet.

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Eric's High is in a well-to-do area of the city. It is known to be relatively traditional in its attitude, and staff and students alike are proud of their academic performance. About 80% of the students go on to post-secondary education. Mathematics is considered to be an important subject, and the mathematics office is adorned with awards and trophies - tribute to the good performance by the students on the mathematics contests from the University of Waterloo. Eric's High is also proud of its performance in other areas, and have many athletic and fine arts awards displayed in the front hall to prove this. Thus, at this school, *there seems to be emphasis on a well-rounded education. Students are expected to do their best in whatever they are doing. Teachers are friendly and cooperative and seem happy with the administration. Because of its reputation Eric's High is a sought after school by teachers to transfer to.*

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Mr. Forsyth announces to the class that Lance, who had been observing for several days, will be teaching them for the next 2 ½ months. He moves to the side of the room and Lance walks to the front of the class. Although this grade 8 class had obviously not been previously told that Lance would be teaching them, they accepted it without question as they did Lance's announcement that I would be videoing the class – him, not them. There are 30 students in the class, 15 boys and 15 girls, mostly of Asian descent. The lack of reaction by the class to Lance's taking over and to the videoing is almost eerie, but Lance is relaxed as he talks about rules and about learning mathematics.

"Respect -- Mutual respect is what I expect. I expect your attention when I am talking and when someone else is talking you should pay attention to them. To learn new mathematics, you always have to rely on what you already know so you shouldn't be afraid to ask questions. To let you know about myself, I've worked in other areas and I have found that it is always important to ask if you don't understand. But after working at other places, I found I don't want to, you know, sit at a desk. I want to teach, so maybe we should start."

Lance's lesson begins with a review, section by section, of the formulas from the work covered by Mr. Forsyth the previous day. He asks the students if *there were any problems, and having a student read the first question, he uses a diagram and some algebraic notation, writing the information on the chalkboard. He acknowledges that this is a very difficult question (It is a question that asks about maximizing perimeter of a rectangle with a fixed area.) and has students try to identify the important information given. Many indicate that they do not really know where to begin.*

"Okay, what do you know about areas? --- Right they are always square units. Now, if this is a rectangle, what do you know about it? ... That's right, there are all right angles. ... What's that? ... Yes, the sides **can** be the same length. What do you call a rectangle where the sides are the same length? ... Right – a square. ... So, suppose the area was 10,000 square meters. What would the dimensions be, knowing it is a square? ... Now, what might they be if it isn't a square?"

Lance makes good use of diagrams as he has different students describe 'their' rectangle. One girl, Grace, talks about making a 'less square rectangle', and Lance, knowing what she means (after all, this is the way he talks about them), follows through, drawing first a rectangle where the length is about twice the width, then makes other rectangles that are 'skinnier and skinnier', as Grace describes them. Lance is unconcerned about the terminology. Grace seems to know what she is talking about. The students conclude: 'The less square the rectangle is, the greater its perimeter will be.'

"Good going! Right! Right on. The skinnier the rectangle, the greater the perimeter. Now this is a problem that takes some time. I only expected that you could think about it and maybe come up with the idea that the square has the least perimeter. But, I guess --- when it works out --- maybe you can think that if you maximize either the base or the height --- that you will get --- you know --- the perimeter will be biggest. Good work. So, let's go on to the next section. Circles. What can you tell me about circles?"

The class reviews the terms associated with circles: radius, circumference, pi.

"Pi is the ratio of circumference to diameter. At the end of the period I will be giving you an activity that will help you understand this. But we have to get more work done today, so we'll take a look at the next section. Composite figures. Does anybody know what I mean by a composite figure? ... Right, Grace. They are made up of different shapes. Now, they can be divided into distinct regions --- do you know what I mean by distinct regions? --- Yes? --- Okay, and the areas of these regions can be added together to find the total area. Let's do one together."

Lance draws an L-shaped figure on the board, divides it into two rectangles, finds their areas and adds them together. The class, not really engaged with the question, but listening attentively and copying the work in their notebooks, agrees that this would be the area of the composite figure. Lance assigns a few questions, and, working individually, the students begin the assignment.

By raising his hand to get their attention, Lance 're-groups' the class to discuss their work. They discover that they have not all divided the figures up in *the same manner. Using this as a lead, Lance chooses one question that he has noticed has been done three different ways. Drawing the diagram on the board, he asks three students to come up, one at a time to describe their different methods of finding the area.*

"Now, what do you notice about these answers? Are they different? No. --- You're right. It's the same figure so it doesn't matter how you divide it up. It has the same area irregardless. Now, turn to page 228. See the activity there. That's what you're going to do. Here's a photocopy – one for each person – I want you to do this 'cuz it's really important to know what you are doing --- I want it to make sense. You can't do more math unless you understand --- know what you're doing. This activity will teach you about pi."

For the last fifteen minutes of class, students work individually or in pairs on the activity and Lance walks about the class. The activity, the students find, is simple enough, and there is little questioning. After class, Lance expresses concern to Mr. Forsyth over the depth to which he should go in teaching and if he should be bringing in outside activities. It is clear (see the opening statement) what Mr. Forsyth believes.

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Lance's next lesson is based on the next two sections in the textbook.

"Be sure to draw diagrams, and remember, you can work together. You don't have to work alone." *But, it seems that the students are used to working alone, and most continue to do so.* "Remember, you can do **more** than I assign if you want to. It never hurts to do extra work. John --- I can see that you aren't working --- and don't throw that!" *John is surprised that Lance saw his movements as he appears to be facing the other direction. Lance seems to have 'eyes in the back of his head' and the students are now aware that they will likely be caught if they try to act up.*

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Lance phoned me a few nights later. Concern in his voice, he says:

"I don't think it would be a good idea if you came tomorrow. I don't want to talk too much about it now, but there has been a problem with a student - a personal problem, and she did harm to herself. I'm afraid that if you're there with the camera the class might react – and you know --- when she comes back, we should probably wait a day or two to see if she's okay."

Lance's concern over his student is sincere. The privacy of the girl involved and the welfare of all the class are foremost in his mind. Over the next while, he does keep in touch with me via the telephone, and with the whole SMIP group by taking an active part in the SMIP email list-serve. He lets down his natural reserve, and reveals a sense of humor that had not originally been obvious.

> For some reason, they [the students] don't share my enthusiasm in solving quadratic equations so I've gotten into the routine of telling silly, innocent, one line jokes: What did the tide say when it rolled in? --- Nothing, just waved. I'm starting to run out of ones that aren't too risqué. Anyone have some to share? ... And, no, Sophia – none of the girls has asked me to marry them --- yet. *(email communiqué)*

And later:

> While teaching grade 8's composite areas, it is not recommended to use the example of a cylinder with a semi-circle attached to it, especially if you are no great artist and your cylinder is slightly curved ... Giggles spread

> across the class and I had no idea why until I looked back ... It's hard not to laugh yourself. *(email communiqué)*

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Lance lets me know that he thinks it would be okay if I retuned to his class. When I do, he is teaching the 'stem and leaf plot' and the 'box and whisker plot' as outlined in the text. After doing a few questions involving stem and leaf plots, he gives the class a worksheet on which he has outlined the method for drawing a box and whisker plot. After class, he discusses his reasoning and reactions.

"Rather than teaching it directly, I like to give them instructions on paper. They still like to work by themselves, but --- you know, for this activity I wanted them to work together. But they got confused. They couldn't even seem to follow the instructions. I tried to let Zao explain 'cuz she's pretty good, but they were noisy. I had to raise my hand to quiet them down several times. I ended up having to explain it myself. As I did, I had to use my arms to demonstrate the different proportions, holding them out to represent the whole amount, bringing them in to indicate the center half, etcetera. But it just gets frustrating with the pace I have to go in getting through the work. These kids don't really have a very good understanding of what's going on. They really just learned mean, median and mode, you know – only yesterday! --- and now they have to use it here to do this stuff. It seems that they don't ever have time to think and absorb what they need to learn. --- But they just need time to make sense of the whole thing."

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After Lance observed the video of himself teaching, he spoke freely about how he felt about the experience and his reaction to the teaching he had observed at Eric's High. And, even though he had displayed a presence in the class that let the students know that he was the authority, (a presence that kept him distanced just enough from his students so that, as his fellow SMIPPERs had feared would happen, none of the students 'fell head over heals in love with him') he indicated that he was not totally sure of his position as a teacher.

"I encouraged the students to ask questions and I'm pretty sure I was able to answer them, but I don't think I really got to do a lot that was differentish. I think I'm better at using the right words, now, but I --- you know --- I was often really tired. I was working a night job during the practicum, and some nights I didn't get enough sleep. Sometimes I just didn't connect with what the kids said, and I just went on. Like I really missed it when – who was it – oh well, it doesn't matter who it was, but she asked about stem and leaf plots that were in the hundreds. I just passed over it. I guess, partly 'cuz it wasn't in the book and I hadn't thought of it – and partly 'cuz Mr. Forsyth told me that everything I needed was in the book – and 'cuz he said I shouldn't teach anything else 'cuz I might --- can't remember his word, but you know – somebody might complain. I felt that working that way really limited me. You know, put all sort of constraints on me.

"I think I got down the questioning a bit and didn't always just tell them what to do, but I had trouble – couldn't really get them working on activities that would, you know, get them to figure it out themselves. I didn't see any of my sponsor teachers do anything differentish. They just sort of stuck to the book. But when I see people do good things --- different --- and I like it, I can use it. I need to **see** somebody do these different things when they are teaching so I can see how they work --- like we did in SMIP. But it, you know, didn't really happen at Eric's High.

"Teaching is really different from other jobs. You're just thrown in the fire and away you go – not like taking training and then being watched or working with someone. You know --- it's really --- once you're there, you're there. I'd like to watch more differences and then try them out – different teaching styles, that is – I guess I learned that being a teacher --- knowing how to teach is a big thing. You have to --- well, it's more than knowing your subject matter --- even more than understanding it --- there's a difference between a mathematician and a math teacher. The mathematician loves math but the math teacher first loves teaching – am I being corny? But math is – teaching math, well, you have to know the math and know what the kids will think like in the P-K theory. It makes sense. You can't just jump in at any old point. You know, you have to have all the knowledge --- and be able to Fold Back – can't Property Notice unless you know some properties and -- Most teachers I watched just showed, you know --- here is how it works now do it --- forcing them through the Property Noticing to Formalised.

"You know – I watched one class where they were working on standard deviation and the teacher just --- you know, **said**: 'You don't really have to figure it out. You have a computer that will work it out.' That was so frustrating and I ---

well, that's what happened and --- well, you know I wasn't impressed. But, what does teaching mean?

"I think maybe it should involve some leading of students in the right direction, but also, listening. What do the kids say? I know there was one girl who said something and I sort of ignored her when she asked. I guess that happens sometimes, but if it isn't really related, how much time --- if it's a tangent – how much time do you give? I always felt that I couldn't spend time 'cuz Mr. Forsyth said to teach what was in the book. But what if the kids were interested in something else? And if they have a wrong idea, I think you need to discuss that so they can understand what is wrong --- not just tell them: 'This is the right way.' I know I sometimes just droned in front of the class, but I really --- well, sometimes I wasn't really engaged, but I think I would like to get more activities that would involve the class. And there were students who shouldn't be in that class --- didn't have the prior knowledge they needed – two of them – you know, when we were doing the circle – knew circle, but not radius or pi or diameter - and couldn't do much for them."

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The following year, Lance took a job teaching mathematics in a relatively new, small private school.

6.4. A portrait of Ellie

It was the beginning of the third week of the fall term in the Teacher Education program. The preservice teachers had been given some student work to grade. The mathematics question was open-ended, asking students to explain and justify their choice of which player they would choose for a bowling team given only the scores of two boys from six games. In the discussion that followed:

> Ellie: I **never** would have thought of giving a question like that! --- but if I --- It's still bothering me - but a good question but I'm not sure if I'm right – I would have **serious reservations** giving this --- what is expected? --- looking for clear justification --- How would I mark it? maybe I have lower expectations --- I would **never** give this kind of question.

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Ellie entered the Secondary Mathematics Integrated Program (SMIP) of Teacher Education at UBC with a background in computing science and mathematics. She had moved to British Columbia from Manitoba to take a Master's degree in Computing Science. However, when she realized that the work she did as a contract computer programmer did not satisfy her, she decided to not finish her thesis but to become a teacher instead. She indicated that she liked sports and coached volleyball at a local, private girls' high school and that she thought of hiking as a good way to keep in shape. During the summer she did the "Grouse Grind" regularly, usually finishing it in an hour. Her classmates were impressed, as this local mountain trail is considered quite challenging. Ellie initially indicated that she believed it is a teacher's role to stimulate students *about the subject being taught, and that mathematics teaching requires some different skills than teaching most other areas.*

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"When I think of math, even though it can be applied practically, often it seems more abstract. The concepts being taught are of an analytical nature and usually deal with questions that have only one answer. Of course, physics, chem --- and other sciences also involve the use and understanding of formulas so perhaps math is not so different from them. The concepts are more abstract and require **true** understanding (not just opinions) ... and you need to get the kids to relax and get down to understanding of not just the concepts but the process behind each one ... there is more onus put on math teachers to really understand the subject if we are going to teach it well than there is on other teachers. I like math because it is objective and logical, but this --- sort of --- abstractness --- scares many people and that's the very reason I love it – **most questions have just one answer!** I don't like just opinion kinds of things.

"I think students have to feel good about what they are doing if they are going to do well. The teacher will have to be --- sort of, a modeler of clay. The teacher must know how to work the clay --- the kids --- be able to figure out what they are thinking -- in order to get good results. She needs to get across to kids that doing the work may be interesting --- important --- and the mindset of thinking this way is important because they need to practice in order to learn. Many kids lose confidence because of a bad math teacher and you can't get it back if you're not **totally** a good math student. So it is important to follow proper procedures and to practice, too. The teacher is responsible for this, and she really has to know her math. To follow up on my clay thing --- You can't really do a delicate model without a competent potter regardless of how good the clay is."

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"Some people are intimidated by the logic of mathematics and think that the order and system of mathematics stifle creativity, but I don't see it that way. It's not a limitation, but rather it's a power of the subject so that with patience --- mathematics can be explained to everyone. But, too often by the time they reach high school, most students have **emotions** about mathematics. Some students are good *(she uses finger quotes around this word)* at mathematics while others are not and it's the teacher's job to guide these students through obstacles and processes, building competence in them. Helping them take the baby steps they need in order to learn."

Ellie listens as the other SMIP students discuss their ideas about mathematics. She's a good listener, and likes to summarize her ideas on the topics under discussion and to clarify what it is the others have been discussing. She is not shy about expressing her ideas, or sometimes what she thought were a lack of ideas.

"Mathematics has always just **been there** for me. I never ever **thought** that people thought differently about it --- sort of, the mathematical objects we've been talking about. I **really** never realized that students might think of these things differently than I do. **Honestly**, **I never** thought about that. But, Dr. Pirie's discussion of Primitive Knowing really made me realize how little we as teachers can assume about our students ---- Like she said, we can know what they were taught but not --- sort of what they learned or understood or how they understood it. I guess it seems sort of --- obvious --- but having heard it, it makes me think. I don't want to sound too naïve, but I had never fully grasped the fact that just because a student cannot **do** what the teacher asks, doesn't imply that he understands --- or the opposite --- sort of, he might be able to --- not be able to understand, but he might be able to do it. But --- I think, sort of, that the doing is important. So ---without the skills --- I really don't think a student can --- succeed in mathematics unless he can do it – I really don't see how he can understand and not be able to do."

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The SMIP students are required to do a number of presentations. After each presentation, they spend time critiquing each other's performance. First, the presenter relates his/her impression of how the presentation went, and then the class members add their ideas. They are always positive, and point out what their fellow classmate did that impressed them. After one of her presentations, Ellie pipes up:

"But **critique** me! You know it wasn't all good. I want to learn --- to improve. If all you tell me is what went good, it won't help. I won't know what needs improving." *Val reminds her: "Ellie,* ***you*** *said that it is the teacher's responsibility to make every student feel special and that she – or he – shouldn't play favorites --- must find something good to say about every student's work. Remember?* ***You*** *told us that you should always find something good to say –*

even if it is just 'Great, Mary, you remembered your book today.' I thought that was funny, but now as I listen, I realize you were right. You said that this comment may be the first baby step to helping Mary set goals that she can achieve. I think that when we give positive comments, it's like that – and anyway, Lorne says we have to be positive (the class laughs). Anyway, I remember you saying that and it helped me change some of my ideas. And I think that it might be a way to improve without, like, thinking of the bad – but if we think about the positive and build on that, well ... And it was ***your*** *comment about Mary that really got me thinking this way." Classmates nod. Ellie says nothing for a moment. She blushes and it is clear that she is moved.*

"I hadn't really thought of it the way Val put it. Sort of --- well, I like to find good things about others, but in myself, I look at what I didn't like. I never thought of taking what went good and building on it and that, well, then the bad --- sort of there's not room for it and it disappears. I really believe that every student can achieve at some level and it may just be in the way the teacher works with them. For some, the 'big picture' of mathematics might be scary. They need those baby steps – and all sorts of little positives to learn the process and build confidence, but others learn the other way and for some students, so maybe for them – the not big picture ones - you shouldn't start with generalizations, but show them step by step first. But, for sure, generalize at the end. Sort of, at the end of each step or section, the teacher should summarize to bring to focus what has happened --- hopefully what has been learned --- providing the correct math terms. --- Summarizing is always important because then at least they have it down. If they write it correctly, they'll have it if they need it. The teacher should be sure they have it."

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It bothers Ellie that she has never really thought about: 'What is mathematics?' before and she enjoys the class discussions around this definition. Numeracy, number sense, representation, use of algorithms, problem solving and checking and interpreting answers are essential parts of mathematics, the class agrees.

"Like I said, I never really would have thought of that kind of question." *Ellie is responding to discussions on the open-ended question to which she had had a fairly negative reaction.*

"Problems may – work, though. 'Cuz you want an answer, and with a problem, you can, sort of, look for a formula that works --- or a pattern. Yeah, a pattern. Usually you can classify a question. Categorization is a part of mathematics – **real** mathematics, that is. High school mathematics is just a stepping-stone on to real mathematics. It's a mandatory program that moves to a higher level of abstraction than previously considered that allows for exploration of new and separate topics, more refined problem solving, a broad base preparation for life. If it requires mathematics - in high school - you should be able to determine the procedure. But real mathematics goes **way** beyond that. It **cannot** be limited. So in school, problem solving is likely a good way to teach some things – can show application. But, numeracy doesn't mean that one always gets the answer – some days you just don't, but you still have numeracy

– and justification may be as important as proof - justification means giving good reasons for your work and is a critical part of mathematical thinking. This is part of being able to break things down so that you can understand them -- part of the logical process and being able to properly apply an algorithm."

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"You know, assessment is different from testing. Assessment may involve reasoning and problem-solving activities while testing is usually just finding an answer. Problems that make the student think outside the box may really be helping them develop mathematically. But, if the student doesn't get it, well, I don't know. It might sort of be a dead end." *Ellie hesitates, then continues, looking serious:* "These dead ends ----- though ---- they may not be **useless** because they can help the teacher figure out the student's thought processes. But, you know, there just isn't going to be time to do that. The teacher is expected to give a grade and time is needed to test. But, what if the test indicates that the students haven't learned something? There isn't going to be time to re-teach and re-test. --- but --- It seems unreasonable to go on if the students don't understand. Learning math is a work in progress and knowing how each student learns and what came before is important --- for us --- us teachers --- and helping them figure out where they went wrong."

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As the SMIP classes proceed, the preservice teachers are given more mathematics problems to do. Some are open-ended, and others follow fairly *regular patterns. As they work through the problems together, questioning takes on a whole new perspective for Ellie.*

"I used to always use closed questions: 'What is the next number?' or 'What should I write now?' But, now I'm seeing ways of asking that get at understanding – like when sometimes you ask me about what I'm thinking. And, because I had to explain how I did a question and then **why** I did it that way, I'm learning a lot of mathematics. I never really thought about this before. And, well --- hadn't really realized that other students might, sort of, take a different approach. I always thought everybody would do the question sort of the same. But now I am sort of thinking about what I'm thinking --- you know what I mean. But here in SMIP, these open problems, we can just explore. So, maybe open questions are okay here. We are talking about thinking and understanding. We don't have a specific place to go to with these questions – but in high school we have a curriculum to cover. Teaching is different from conversation. We have to get it all done – the curriculum. And that makes a difference."

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It is Ellie's turn to do a presentation to her SMIP cohorts. She has chosen to do a lesson from the shape and space section – two-dimensional to three-dimensional geometry.

"First of all, I'll step back and review the stuff you'll need to do this 'cuz you haven't worked with this material for a while." *Ellie gives the class a summary of the background material she would cover with her students then gives them a question to work on. She has Tom explain how he did the question,* *but Wayne jumps in saying that he did it differently.* "Just a minute, Wayne. Let's let Tom finish." *Tom finishes his explanation.* "Okay, Wayne, would you like to come up now and show us how you did the question?" *Wayne does so.* "Now, what happened here? --- Are these both right? Why don't you talk about that with your table-mates?" *After a couple minutes there is consensus that they are both correct.* "Right, Tom looked at the question as if it were a rectangle, but Wayne recognized right off that it was a square. No problem because a square is a rectangle. Both ways work."

For Ellie's next question, the class must find the area of a rectangle on which she has drawn some lines, dividing it into various triangles. She has them cut the rectangle into the triangles and put these back together in a different configuration. The area appears to be different. "How can this be?" *she asks, and without waiting for a response, she suggests:* "Look at the slopes of the parts. That should tell you something. Now, finish it for homework and we'll discuss it next day."

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Rules in the classroom are a concern for all SMIP students, and Ellie has given some thought to this because of her coaching experiences.

"I think that we really need rules. You have to have some control. With coaching it has been important that I have rules, but if one of the better players breaks them, it is harder to keep them – like when one of my best players missed a practice. But when we go student teaching, what will we do? The regular classroom teacher will have his rules and what if they're different? Will we have to ---? Then, there are the school rules that have to be kept – school and even broader, like if students are caught drinking or something like that. You can't ignore them. These have to be handled by other people. There's a lot of things like that that you have to know, but how do you learn them all?"

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Again Ellie presents to her peers.

"There are 1000 finalists in a contest and every second one is removed. This continues until only one remains and this person is the winner. Where would you stand if you wanted to win? Who's in? Who's out?" *Before giving the class a chance to work on the question, Ellie hints:* "Try this --- try using a smaller number to see what happens, and look for a pattern." *And Ellie starts the process, using only 10 people. Together with Ellie, the SMIP students work through this simple example.*

"This question is good, I think. It makes you have to try to determine some approach and using a smaller number is often a good way to work on a problem. It was modeled --- well, I modified the question from the original because it had been too morbid for a high school class – everyone died except the last person. Anyway, so okay, so let's try it now with 50 people. Watch for a pattern and let's see if we can find a formula that works." *As Ellie begins to write her solution on the board David proposes a different approach. Ellie does not heed his suggestion, but goes on to lead the class to a tentative algorithm. They must verify it for homework.*

Later, commenting on Song's and Armie's presentations:

"I think this would take too long – you know, like when Song let Niki tell him how she would do it and it was **way** more complicated than it had to be. If this was a real class we wouldn't have time for all that exploring. But, I liked the way Armie was so organized and wrote out the reasons, wrapping things up smoothly and making us talk about why it worked in her presentation. And she took Wayne's method 'cuz, I guess, it --- well, it wasn't too difficult. When we teach --- well, to follow a student's path might not be good if it isn't the way we want them to do it. It's best to have it smooth. Like in my presentation, I didn't let David tell us --- well, I wasn't sure where that would go so I didn't want to let him try. So I sort of --- I told you how I wanted you to do it. There's a difference between teaching and working on a problem. In working on a problem, I go different routes and try different methods to see how to find a solution. I look for a pattern -- a formula that will help me get to the final answer -- but the route is not straight. But the one I teach should be."

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Ellie thought that her short practicum went well and she hadn't been as nervous as she thought she might be.

"I felt a bit self-conscious when I dressed up for the Halloween Dance and none of the other teachers did. They didn't really get into it. Anyway, the students weren't as bad as I had expected. But, it was really frustrating that, sort of, some kids just **don't get it** – even simple things got stuck with one kid."

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In the days that follow the short practicum, the SMIP class further discusses understanding and teacher/student interaction.

"But I'm not sure about having students explain their work. There was this girl – quiet and shy, but she got the right answer this time, so I asked her to explain how she did the question. **Immediately** she withdrew – started mumbling something and she assumed she'd done it wrong. So I said 'no, no – you got it right. I just want to know how you got it' --- because, you, know, kids are never asked unless they sort of got it wrong – that's what they think, anyways and I felt really bad for her. I think I may have confused her more and now she'll never get it!"

Ellie has opened her mind to new ways of thinking about topics. She asks for help from her fellow SMIPPERs.

"How would you teach linears to grade nines? I need some different ideas so that when the kids come up with ideas --- ways that are different from mine, I might be able to know if they're right. Sort of --- what **Image** would they have if they thought of it that way and would that Image make it easier or harder for them to learn later on? I really don't like the idea --- the Image that kids get when you talk about negative numbers using the idea of bad as negative and good as positive. It may help them work with signed numbers but I don't want them to think of them as bad." *Thus, when fellow SMIPPERs discuss different ideas of explaining signed number -- left/right, up/down, sandbags in air balloons, red and black markers - Ellie writes down their ideas and tries to work with them.*

"I have to know if I'm comfortable with that way – see if it works for me – 'cuz if I can't make sense out of it I sure won't be able to explain it to my students! And, if it is a manipulative – like the markers --- well, when should you go to the --- the --- to the formal --- the algebraic representation. I really don't know if I can use the manipulatives. I mean, I can figure out how they work 'cuz I already **know** the stuff, but can the kids really, sort of, learn using them and then switch to the **real** math?"

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The students in the SMIP class must present a problem as a group. During the presentation by Lance, Sophia and Frank, Ellie notices that David tried to present a different method to find the solution than the one they were using as he had during her presentation. She also notes that once again, he was ignored.

"How did that make you feel, David – to be ignored again – like I did to you? Did it affect the way of thinking? I wonder, in class, how many times we sort of --- stifle our students --- make them think that their way is wrong or they aren't as good as we are because we ignore their way of thinking. I think that maybe this would affect their future work. They might have had a really good idea and it might have been really creative but they wouldn't develop it because we cut them off. Did you feel that, David? "

For her group presentation, Ellie works with Elisa and Evan (E^3, they call themselves). They present a question that deals with the probability of two *people in the class having the same birthday. As soon as the question is read, Ellie draws a chart to help the class visualize what is happening.*

"I would use Excel in a real class", *she says as she has the SMIP class explain what each cell of her chart represents, making them reword it if she is not sure that they really have the correct understanding.*

"I can't believe how much I had forgotten about probability but this question really made me think about it. --- using a simpler case --- that's what helped me understand and I think it might be a good idea to suggest this to students when they are stuck. Making things simpler makes it easier to --- well --- relate to. And kids --- well, you guys --- can come up with some really neat ideas. I can't believe it --- all the ideas and different ways people think about stuff that I sort of took for granted the way I did it. **And** it's amazing how much math I've forgotten but how much each of these exercises forces me to think more about it and not only am I remembering it, I'm getting all sort of new insights. Sort of, when I used to just look for the formula, I am now – 'cuz we really have to think about it – I'm seeing why that works and where it all comes from and what else you might do. And --- this makes you think about how to teach it so the kids can get it – so it makes sense – so they can see the pattern you can. Sort of, at least for me that's the way it works and I hadn't realized that before. And I think if I think of it beforehand, I can make better assignments and can think about what to test and different things to test and how to mark those so that I am finding out what the kids know. I'm testing for a 'thing' not just giving them questions to do that are like the questions they always do from the book. Sometimes you need to mark for different things. Sometimes English may even be important and the kids should know that. And if one goes beyond your expectations – well, you can't penalize the others and give him a higher mark if their work was what you had expected. Like in the open-ended. How do you know what is worth a 5/5 'cuz it doesn't have a **right** answer?"

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Ellie is very comfortable with technology: "... but I think many teachers are afraid of it. Well, not **of** the **actual** machine – but simply that they don't know how to really use it as a learning device. Kids have to realize that it - any technological device - is a tool, not the thing that gives the answer. They have to know that, but if the teacher just uses the calc, for example, to get an answer, that's what the kids will do."

Ellie takes an active part in helping her fellow SMIPPERs use the computer and the calculator for various class activities. Her assistance is appreciated as she explains different ways of using the functions on the calculator and different ways to program them. When someone questions her approach and wonders about a different one, Ellie replies:

"I don't know about that. You know, I'll have to think about it. What if a student asked me something like that and I couldn't answer? Do you think it's okay if the teacher doesn't know and has to tell the student she has to think about that? And what would the student think – I'm dumb 'cuz I can't tell him the answer? But there are a lot of things that can be done differently but if you haven't thought of it you might not just right there be able to think of what --- of how that method would follow out – sort of like you have your way and it works and you haven't even thought of that."

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"You know what is interesting! I just realized that just 'cuz a kid says something or asks something that seems on a tangent, she might not --- well, she might just be really interested and have thought of something beyond what you were going to teach or thought she'd get." *Visiting professor, Dr. Jo Towers, has just presented some of the work she had done with the Pirie-Kieren theory to the class. In the video she showed them, one girl had not been able to follow what Dr. Towers had been doing and, on her own went beyond the scope of what had been intended.* "Students may do this --- sort of --- might see the question differently or might already know what you want to teach and see things --- other things --- that they can do and it might be hard, as the teacher to see that – what they are really doing – they're not doing what you want and so you might not see what they're doing. I'm really going to have to watch --- I'm always so sure that the way I'm doing it is the way to – is the **right** way. Haven't really thought about how I would act in different situations – like when students come up with other ways and they lead in a different direction. But, high school kids – well, I'm not sure that they will do much of that 'cuz I think that they just want to get it and get it done and they don't really think about it – sort of these 'motivational' questions we do. Yeah, they're okay for us 'cuz we want to do that --- we want to --- we like doing that and seeing what else we can do, but high school kids just want to get it done. And, the special education teacher that I worked with at Windor doesn't even like word problems! She says that the words interfere with the student's ability to do math so some of these motivational things we are looking at would just get in the way. The kids wouldn't be learning the stuff they need to learn."

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Although, as she indicated, Ellie had found it frustrating during the short practicum that some of the students 'just didn't get it', she is looking forward to her long practicum at Windor High. She had made a good connection with her sponsor teacher, Mr. Smeel, and he is very supportive.

"I think it will be okay – scary – but okay."

Windor High is considered a fairly 'average' school. Nothing about it stands out as being really great or very bad. There is a mixed population – very multi-ethnic, upper-lower to lower-middle income families, mostly intact. Ellie has heard that the students are known to be very polite and helpful. She can't really comment on this, but hadn't had any negative experiences during her short practicum. She is ready to start teaching.

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> My first day at Windor went well. I actually didn't teach because all the math classes are writing their midterm exams today and tomorrow. I start tomorrow with an IT class and I can't wait! It's very scary though when you really start to think about managing **everything** about the class, not just teaching a lesson here or there. Sort of, you have to keep track of everything and attendance and who's late and what time and all that. Good luck, all. *(email communiqué)*

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It is Ellie' first day of teaching a grade 9 mathematics class. She looks elegant in her white blouse, black skirt and silver necklace and earrings – except *that her right hand is in a bulky temporary cast and seems heavy and uncomfortable. She had broken the thumb a few weeks earlier during a sporting activity and is waiting to see if she needs surgery. Fortunately, she is left-handed, so that although she has to hold her hand in a vertical position to keep down the swelling, and it is a bit awkward, it does not really interfere with her ability to write and teach. She chooses the overhead projector as her primary means of writing as this way she can prepare some of the activities and questions ahead of time and can rest her elbow on the stand, making it a bit easier to keep her hand vertical.* "I have observed them, before," *she told me before class,* "so they know me, but this is the first time I will teach them. Mr. Smeel has told them I will be their teacher for a while."

Ellie smiles as she addresses the class for the first time as their teacher. She reassures them that her hand is not a problem.

"After all, I'm left handed, so this is just extra. I'll find out next week what's happening with it. Don't worry. It'll be okay." *Although she appears energetic and enthusiastic, to someone who knows her, the dark circles under her eyes and the strain in her voice, belie this. She is tired, the thumb taking a greater toll on her than she would like to admit.*

Ellie writes a question on the overhead. The class is to do the question in their notebook, but they question her immediately about how to do it.

"Chan? Yes, you will need to use the Laws of Exponents. You don't remember how they work? How many of you don't remember how they work? Oh, a lot of you. Okay, we'll just quickly run through them."

Using fairly simple examples, Ellie reviews with the class when to add, subtract or multiply exponents, then gives them another question to do.

"So, Ali, what did you get? Yes, that's what I got. Okay --- yes, Seti?" *Ellie seems to already know most of the students' names.* "Okay, Seti, so that's different than what we got. Can you explain how you got that?" *Seti explains and Ellie say,* "Okay. I see. You took a shortcut. **But**, unfortunately **that** shortcut only works for certain cases – special types of questions. It's really specific. So you have to use the general way here. It's not really a good idea to use the shortcuts 'cuz mostly they are for special times. I'll always use the general ways and I think you should too. Shortcuts --- well, they save time if you use them right, but usually you should use the long way. Won't make as many mistakes."

Ellie continues with her lesson. She has cut out a number of rectangles, all of which have sides in the ratio of 9:7. She uses these to explain ratio and to work on questions about perimeter. The noise level increases as students discuss their work, and although the noise seems to bother her, Ellie wants the students to work in groups. She next gives them a question about trees and their shadows.

"The ratio of the height of one tree to its shadow will be the same for each tree and shadow, so if it is 4:3 here *(pointing to one tree)* and this tree is 8, the shadow will be 6. Do you see how I got that? Now --- can you get other ratios for these two? Each question can be done in more than one way. Oh, yes – remember to show all your work 'cuz that's the only way I can tell what you're thinking. I can't see in your brain, you know, so you have to show me what you did." *When Ben complains about algebra, saying that he will never use it, Ellie replies, laughingly,* "You'll never be finished with algebra until you breathe your last breath. It's **that** important."

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Having taught her first full mathematics lesson, Ellie takes time out to think about it and discuss it with her sponsor teacher, Mr. Smeel.

"I didn't get all the answers I wanted from the students, so I didn't get to ask all the neat questions I had prepared," *she worries. Mr. Smeel, tells her that he thinks she went into too much theory and that's why the kids didn't give her the answers. "The kids just want to know how to do it," he says.*

"Well, I don't know. I wanted – was trying to get them to figure it out themselves. We learned about this theory in class --- the Pirie-Kieren theory --- and how if kids figure it out for themselves, they will remember it better --- this thing about then they can Fold Back to it if they forget 'cuz they'll have it in their heads, but if you just tell them --- I think that was Formalising --- they won't know where it came from and they won't really remember it. There's this bit about Images and Folding Back to them and how the Images are important 'cuz that's what they need to Fold Back to so they can understand a lot of what they do. Does that make sense to you? I was just trying to get them to understand by trying to do the questions themselves."

Mr. Smeel listened and nodded. "Okay", he says, "so that will be something you have to work on. You're right – Yeah, that idea of Folding Back is *good, but they need the formulas. Maybe you'll have to summarize more, but I like that idea of Folding Back. There's something to that."*

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Ellie has been away from school for a few classes. She has had surgery on her thumb, and her new cast is even larger than the old one. Mr. Smeel had given the class a test during her absence and she returns them to the students, going over some of the questions that she identified as problematic.

"This is a good learning experience - going over the test like this. If you don't go over the questions that they got wrong, how can they learn?"

Mr. Smeel had introduced polynomials to the class while Ellie was away. She reviews the concept by writing a number of examples on the overhead projector and asking the students if they are polynomials. "Why? Why not?" *She is very relaxed, and encourages everyone to try. She rewards them with a* "Brilliant, brilliant!" *when they give a reasonable argument. The students seem to have a fairly good working knowledge of what a polynomial is, and can explain it in their own words. Ellie gives them the formal definition she has prepared. She then displays an overhead with four sections. In each section a different algebraic expression is displayed.*

"Now, take time to think. Don't spoil this for others by yelling out the answer. Now, which one of these is not like the others?" *she asks them, copying a game from the children's TV program, Sesame Street. The students recognize this, and Ellie replies:* "Yes, I did that on purpose. And I'll tell you a story about Sesame Street after you're finished this. Okay, Lisa, which one of these is not like the others?" *Lisa's answer is not what Ellie expected and Hark loudly disagrees with Lisa. He calls out a different answer. His answer is not the one that Ellie expected either. Ellie asks Lisa to explain why she chose the one she did.*

"Oh, but that doesn't explain which **isn't** a polynomial. Sorry, I should have specified that. I want the one that is **not** a polynomial – not like the others because it isn't a polynomial. So --- can you pick out the one that is not like the other – not a polynomial? ... Riiiight! Now can you explain why each of the others **is** a polynomial?"

After the explanations, the students remind Ellie that she was going to tell them a Sesame Street story. "Well, I was once on Sesame Street – me and my grandfather. We had a short bit where we were gardening. It was a **long** time ago, but I still have it on tape. If you're good, I'll show it to you one day. But, now, here is one last question to do."

The bell rings, but the students stay until they have completed the question. Ellie is pleased that they were so involved.

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Again, Ellie is able to discuss the class with her sponsor teacher when the students leave. It was clear that he still felt that she was too theoretical in her presentation and that she spent too much time worrying about terminology. He is also concerned about her interaction with students. "Amy was off topic and Art was a bit of a loudmouth – answering all the questions. Are they taking advantage of you and can you stop this?" he wonders.

"Well, Art is really quite intelligent. He asks good questions and just has --- test phobia? --- so I want to give him a chance to say things so that he knows what --- that he can do it. And Amy ---- well ---- she's vulnerable. Have to treat her with kid gloves. I need to let her be a bit sarcastic. She could **hate** me in a second. But I know we read the same books, so we have made a sort of connection. Need to let her feel a bit of success – not picked on – she's a bit of a project for me. Anyway, did you notice where I **really** missed it? I was so focused on the idea of polynomials that I **completely** missed the chance to have the students explain **their** choices on the Sesame Street thing. I could have let them and that would have maybe tangented onto something and helped them learn --- expand their knowledge. I really goofed on that one."

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When Clare, one of the better students in the class gives a theoretical explanation of a solution, Ellie repeats it in simpler terms, and when Roz gives a general solution, Ellie provides more detail. She patiently listens to the students' questions and asks others if they can provide answers, always telling them that they have done a good job. By her behaviour, Ellie is modeling her beliefs that students should support one another. She treats them like they are adults – like they can do the work and that they do know how to behave. Thus, she visibly pales when Anna, a student who was presenting her solution to the class, tells Ryan, who suggested a different way to do the problem: "You're wrong and that isn't the way to do the question."

"But --- but --- but, Anna --- Ryan was just trying to figure out how to do the question." *"Well," says Anna, "that's the way teachers talk." And she finishes her explanation. Ellie takes a few minutes to politely state her expectations, ending with:* "I want you to treat others like you want them to treat you." *It is obvious **she** doesn't want to be identified as a teacher who 'talks that way' to the class.*

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But, Ellie is finding some parts of teaching very challenging:

"... and I'm not sure that I'm helping them. I really think they needed more time multiplying before we went into factoring, but Mr. Smeel said we had to go on. They seem to see, start with two binomials and multiply them --- but they can't seem to reverse it – decomposition, Mr. Smeel calls it. I'd sort of like to show the – try factors of the first and last terms --- inspection, I think its called --- 'cuz they know when they multiply, but ... Trying to rack my brain for an analogy but I'm somehow not giving them the right Images. Gees, these poor kids! I'm screwing up their factoring for the rest of their lives – thought I was prepared. I'm not quite believing yet, what Mr. Smeel said – that some of them will never get it. Just need to find a way for them to look at it – and of course, time. Sort of, maybe more with Algetiles even though about a third of the students didn't really get it with them because they just wanted to be **told** how to do the questions. Some of them liked it, but I only did one class 'cuz Mr. Smeel thought it was a waste of time."

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Before class, Ellie reminds me:

"This is my last day of really teaching the grade 9 class – only a review period and a test remain until the end of my practicum." *In the middle of simplifying a rational expression Karl announces that he doesn't really believe that Ellie was on Sesame Street. Ellie, with a grin, tells them to finish the question* "and then I **will** show you my 1 minute and 15 seconds of fame. **I** haven't forgotten what I promised." *She holds up the tape of her and her grandfather on Sesame Street and tells the students:* "You can't criticize my clothes. I had no say in them. My mother picked them out. And you can't say anything about my grandfather. He's very special to me even though he is no longer alive."

Ellie plays the video for the class. Although she was only 7 years old at the time of the taping, the students can clearly identify her. They watch quietly for the 1 minute 15 seconds, then with comments of "That was neat!", "How much did they pay you?", "Were you scared?", "How long did it take to do the filming?", they settled back to work on their assignment.

"I really liked the way you all went back to work," *Ellie says at the end of class.* "It shows that you are responsible. And, remember. I promised something and I did it. You should do that too – keep your promises. You know, I'm going to miss you."

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"Wow. Watching yourself on video can be really revealing. I wasn't **as** bad as I thought I might have been – and that's really something for me to say 'cuz

I'm usually pretty hard on myself. But it really reminded me of some of the things that I believe about teaching and some of the things I didn't like. Sort of, well, I never really had my own class. Mr. Smeel usually stayed there. It was good to have him there to help. Mr. Smeel --- well, he's a good teacher but --- It was like – remember the time we talked about the P-K theory? Well, he sort of agreed with it and talked about it, but --- but mostly he just wants to tell the kids what they should do. Well, maybe he's just set in his ways. He only has two years until retirement, and maybe I was overly sensitive about this, but I really think that I focused on the right thing in class – getting the definitions straight and such. I was starting to get out of the stand and deliver things and he didn't like it so much. I liked my interaction with the class and I thought that I had a fair number of them interested in what we were talking about. I usually looked forward to next class. I've already thought about some changes I'll make ... adding more group activity.

"And, boy – It's really **hard** to listen to all the kids have to say. Sort of, you hear one and you miss another and you can't be on top all the time. Like the time Dr. Towers missed that the one girl had the right answer from the start – sometimes you just miss what happens or what someone says either because of noise or they word it so it doesn't sound right but they didn't mean it that way and you don't know what they mean. You can sort of see more when you mark papers – but who has time to look at all the errors! Wow. Marking can take **forever** if you want to do it so that you see what problems the kids have. Maybe it's better to give fewer questions --- I don't know.

'Oh! And remember that day Clare --- no, it wasn't Clare, it was Anna --- said 'That's the way teachers act'? I couldn't believe it. But, I sort of saw that in some of the stuff I observed – not quite so blatant, but --- Anyway, I think that they realized after I expected that they wanted to be treated nicely and wanted to be listened to, and so they should do the same. We're all in this together and if they help each other, they will learn more. If someone does something wrong, it doesn't mean that they're stupid, just that they didn't know how to do the question and we should all try to help them learn, or figure out why they did it wrong. Supportive behaviour is important, and listening to what others have to say, well it's important."

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Ellie spent her first year teaching mathematics and computing science at a private Catholic school in a large city in the Lower Mainland.

6.5. A portrait of Wayne

Wayne has been teaching the mathematics 11 class for a week. He has been trying to develop their terminology.

Tom (student): The name you gave that doesn't make sense. You called one plus three, and one plus three plus five and that row a 'sum'.
Wayne: Right. This column is the sum of consecutive odd numbers starting at one.
Tom: But I learned in elementary school that the sum is the answer when you add - like what we have in the next column, four, nine - those are the sums of the other column.
Wayne: No. One plus three and one plus three plus five and those are sums, and those sums are four and nine and so on. Just like if you have two times three, that is a product and six is the product. Three and two are factors of six. It's what we call them.
Tom: But ---
Chien (student): Maybe look at it like this. There is an equality, so what he is saying is that since they are equal - column one and two - so maybe we can call them the same thing. That's the way I'm looking at it, anyway. That's the only way I can see it.

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Wayne is a sincere young man, seemingly quiet and shy. He blushes when spoken to and prefers to sit near the edge of the class. He likes hiking, philosophy and chess, and has completed his bachelor's degree with a philosophy-mathematics double major. He had several summer jobs but they were nothing special. He originally thought of doing graduate studies in philosophy but decided that he should have some life experiences first so only recently decided to take the teacher education program. He's not sure that he really wants to be a teacher, but he wants the experience of going through the program. He has had some personal considerations recently that make it hard for him to concentrate on his work, but he feels that he is getting his life sorted out *and that he is intelligent enough to be able to catch up on his own on any work he misses because of distractions.*

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"If it weren't for social pressures and parental pressures – all inadvertent and non-explicit I would never have taken a math degree – I enjoy logic and problem solving and ---- philosophy – all are found in math or can be brought out in math but none of them are math – and they – logic – well, I don't need to go into it – I was good at math and I was encouraged – my dad had some sort of double standard – externally he said that I was free to do what I wanted but in actuality he wanted me to go into the maths and sciences and I was usually at the top of my grade – came from a school with really high pressure to achieve. I go back and people I don't recognize know me and my name and assume I have done a science degree and wonder what I have – and all the time my big thing was philosophy and the motivation – I enjoyed the discipline – to have something that would actually help people --- I wanted to know: Are world statements objectively true or not and if so, what are they? What ought people to be doing? And so I embarked on that and it is partly why I took a degree in philosophy – I wanted to help and I have a **huge** ambitious goal --- take hours to explain – in a nutshell I want to axiomatize parts of analytic philosophy so that the structure is boiled down to certain axioms or principles that are most basic. That's my real goal. I don't know if it's possible, but that's what I want to pursue and I'd like to develop that --- I could go on about the philosophy side of things and I would like to develop that in logic and epistemology – epistemology, largely. I'm interested in metaphysics and I might tie some of that in with ethics and social political philosophy and philosophy of law and I'd enter it into a computer and people could enter their data, their own axioms and value judgments and it would churn out consequences of their beliefs."

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In an interview involving himself and Lance, Wayne reveals his Image of teaching:

"The role of the teacher is to guide discussion of the subject matter and let students know why content is structured in a certain way. For example, the teacher should let the students know why the quadratic formula is correct. The teacher should **guide** the discussion in the direction of the **most effective** solution. Mathematics is a creative endeavor but most students think that it is a subject that is learned rotely. But, it is best if students really understand mathematics rather than have rote understanding. Philosophically speaking, I believe that students should learn mathematics by themselves, even though it is often necessary for the teacher to show them how to do it. Guided growth might be the best way to describe mathematical understanding. The teacher has to guide the students in the direction of the most effective solution as far as has been discovered so far. Provide guidance in their growth. Teaching is a matter of guided discovery. If lucky, a student will experience intuition and sudden insight in mathematics. This is what it is about – but I don't know how to teach insight. ... If math is taught like other creative disciplines --- as all disciplines are --- else how were they created? --- it's easier to remember. The good mathematics teacher must learn to present mathematics as creative and meaningful. That is, not a matter of rules but of well justified decision. ... Students need to get an understanding of why mathematical problems are approached in a particular way, and this is done by the teacher who guides them in **that** direction --- the direction of the most efficient solutions. That being said, certainly, there are particular **skills** that have to be taught. Calculator use, for example. This, strictly speaking is not mathematical at all – it is technological. But it is a useful skill to learn how to use a calculator well so I suppose math is an adequate place to teach the skill. It is important to distinguish learning this skill from learning math, though, among other reasons to try to curb use of calculators in place of math by students."

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"I always liked studying mathematics but got frustrated with it when truths became uncertain because, well, mathematics is objective – right or wrong – and although mathematics proofs can be discussed philosophically, logically, it should be prescriptive. Math is itself, not empirical --- and there is one question they ask: 'Is math more fundamental than philosophy?' Why does this world – **they** say world, but mean universe – exist instead of any other possible world and one explanation is because this is the most mathematically parsimonious – or mathematically most beautiful or something which --- a lot of people scoff at that but I think that the motivations for making these claims --- well there is an assumption in analytic philosophy that logic is necessarily true --- and since people believe that math is largely logic, there is a tendency to believe that a mathematical universe is going to be the universe that --- I think it's **bogus** - there is a crossover --- but math is a distinct body of knowledge, really different from every empirical science, every social science --- and until September third my life – well, until today, my life has been consumed with the abstract thinking and in some sense decontectualized thinking and not exclusively --- and this really is a shift and I felt it for the first two weeks --- it is a shift in perspective a different way of being in the world. I've made a lot of beginnings and endings lately and I have to try to distinguish --- self-analyze --- what causes what."

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Although Wayne revealed some of his ideas in the intimacy of an interview involving himself and Lance, and another one where he was the only participant, after a few weeks in class, he remained on the quiet side. When he spoke, he tended to make generalizations at the end of lively group discussions.

"Vocabulary is important in math. Mathematicians have taken the vocabulary and chunked it together so they won't make mistakes. It all depends on grammar, rhetoric and logic. The Socratic method is a part of the structure. **We all** have a value laden idea that the view from the top – the more mathematics we know – the better we understand but this is an erroneous idea."

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The SMIP class is given an assignment in which they are asked to define, in two pages each, teaching and mathematics.

"It is dangerous to ask a philosopher to define teaching – or to define anything at all, in fact – in just two pages. If a philosopher has defined something to his own satisfaction, he has invariably written a volume. ... A philosopher would define teaching in terms of outcomes ... for an educator --- it is more useful to define it in terms of methods. But, are we concerned with what teaching **actually is** or what teaching **should be**? ... Similarly with mathematics: Simply, mathematics is the study of relationships between quantifiable entities. ... Secondary mathematics is ... the integration of **mathematics proper** with many fields related to math ... logic, problem solving, interpreting the real world meaning of mathematical statements and understanding the significance of creating and applying algorithms ... and its influence on and by technology, society, culture and religion. We have to refine the question. I think the world places too much emphasis on the logic of math --- Whether the math is human made and corresponds to something internal – because there is **no** dispute that we make --- custom systems that are mathematical. The question is whether or not they correspond to external reality. I don't see why a mathematician should be given that --- power to make distinctions --- he has no insight into philosophical questions. He does math."

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In discussions involving class organization and ability grouping, Wayne has some opinions:

"Based on my own experiences, I think that maybe there should be --- well, homogeneous groupings, maybe, because then advanced thinkers can work together – but they might --- if in heterogeneous, get the others motivated to emulate them. It's hard to say. Maybe it depends on the teacher. I would want to teach so that it can be generalized. I remember one of my teachers who taught us – had us prove one little thing then another and another and pretty soon we had proven the whole binomial theorem --- led us through to the **best way** and we **didn't even know** what we had done until we were done. There's a lot of cumulation in math and if a student missed something elementary then he would be lost. There's not much emphasis put on understanding at the elementary level because elementary students aren't capable of abstract thinking. Like, what do kids who do second year calculus at thirteen **really** understand? But I think a lot of kids can like math at the elementary level but then they lose the excitement of it when they get stuck with the boring -- well, dull high school math. I agree with Niki --- high school math should be more understanding – understanding of the **facts** that they have to learn in elementary school but don't have the ability to understand. In high school they should be able to see the system. It should be about **proving**. Problem solving and modeling can be useful outside math -- and numeracy --- to a degree. I'd like to push proving as one of the big things in math but that won't ride. People don't like that. --- I wish we discussed more psychology in class. Like, cognitive psychology – how students learn. We are discussing this and that and getting consensus sometimes, but how about some research? We learn what **we** think but if there is **research** that **shows** something, I would like to know it. If someone has **proven** something about what we are talking about, I'd like to know that. If there are biological stages of learning that aren't in dispute, I want to know that – not just our generally accepted personal biases. Like, I think there are some kids who will **never** understand, but I want to know if that's been proven. We did the P-K theory the first day and that was good. Well, I sort of thought that already, but I want to know other ways of thinking that others have **proven**. Have they ever **proven** that theory?"

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The class discusses the organization of high school mathematics courses. Sophia points out that each course seems to have one 'odd ball' topic and that it is usually at the end of the course. She doesn't think this is a good idea because then the students will think it unimportant. Wayne agrees:

"I'd put it at the beginning and then try to relate it to other units. But, there is a problem if you don't follow the textbook because the problems might relate to previous stuff. You'd have to look really carefully over the questions before you assign them to be sure the students knew everything they needed. I think open-ended questions are good. Problems that have no real solution – like choose your favorite number then try to think of the number that is least like it and explain why that is so."

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It is time for each SMIP student to do his/her first class presentation. Wayne observes and is quiet. Even though the class discusses each presentation, Wayne seldom makes comments. When it is his turn to do a presentation he stands at the front until the class is quiet.

"You are a grade eight class and will be working in groups. You know how to do squares, take square roots and you can use formulas. Now, here are three squares for each group. Find their areas. It's easy. Just count the little squares. Then, discuss the relationships between the areas of the three." *He draws a table on the whiteboard.*

"Okay, now give me your numbers" *and he writes the squared values for* a, b *and* c *for each set he has handed out in the appropriate column.* "Now, what is the sum of a^2 and b^2?" *Wayne hands each group a triangle. Four triangles have right angles and two do not.* "Fit the squares to the sides of the triangles and tell me what you notice." *The students notice that the three squares fit on the sides of the triangle.* "But, do you see that for some of the triangles, $a^2 + b^2 = c^2$? That this only happened for the right-angled triangles?" *The class notices, but some are confused because the squares fit on the non-right triangle. Tom asks about this.*

"But, the sum of the numbers doesn't work, so they don't fit the pattern. You can see that. So they don't work," *says Wayne.*

The next week when the class watches the video of this presentation, Wayne looks on intently.

"I think I had good pace," *he says* "and that I asked lots of questions. I made them think."

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It is Wayne's turn to present a mini-problem to class. He hands out pennies to each group.

"Use the pennies to form an equilateral triangle with three rows of pennies. How many moves will it take to reverse the triangle so the vertex is pointing down instead of up? Two? Good. Now what if you have six rows? And keep trying until you can generalize."

After a few minutes of work, Wayne asks the class if they can generalize. Lance thinks it will require $n-1$ *moves and Val agrees. She summarizes, drawing a table on the board, then demonstrates her moves for the last row.*

"How do you know that is the fewest moves? Are you **sure** it can't be done in fewer moves? Did anyone else do it differently?"

Erin points out that it doesn't work for five rows "because there is no hexagon." Suddenly, Lance asks if anyone knows why Fibonacci numbers are just out there.

"That's not really relevant here. We're not discussing Fibonacci. Erin, do you have any more to say about the hexagons?" *Erin goes to the board and explains the difference between figures that have an even number of rows and those that have an odd number of rows.*

In the discussion that follows the presentation, Wayne indicates that he is pleased with it.

"I think it went pretty well. I tried it without using notes and I felt that I wasn't as constricted as my last presentation where I tried to follow my outline. I think my questioning was good, and I showed the class that you can prove some things but other times you only conjecture but that those conjectures aren't always true."

****

During the last few weeks before the short practicum, Wayne is more involved in the class discussions and comments on the presentations of other people. After Lance's presentation in which students were to determine the score of a cricket match when all he gave them seemed like unrelated clues, Wayne commented:

"It was sort of interesting the way you made it Elizabeth and Charles. But, what would happen if the score of the cricket match was zero?" *Trinity explained the consequences.* "Oh," *said Wayne, unconvinced,* "Okay, I guess."

When Niki wondered about the best way to mark angles Wayne stated: "When I wrote my provincial exam it was acceptable to use numbers for angles." *And, when David let some of the class lead him on a tangent, Wayne said:* "Do you think that tangent was useful, or was it just using up time --- too much time for what it did?" *When Song got a suggestion from one of the students as to how to do a question and followed through, later showing his own method that was shorter, Wayne stated:* "That was good, Song. You showed us the **best** way to do it, the **better** method – that the other took too long so we would know that there is a reason we do it that way. And, you were able to catch your own mistakes."

Later: "It was also good when Stephanie did a problem that involved Alvin and the Chipmunks. She wanted the class to realize that the logic and organization are more important than the final answer. And I think, because it's difficult to think of real world implications so this was a way to relate - dealing with something the students might know about. And she dealt with the need to go backward, **and** she got through the discussion **quickly** by **guiding** us through the process she wanted. I think it's important to do the definitions, too. Too often we gloss over them and the students don't really understand. They have to know what things really mean --- definitions. Students have to know that it is **us** who decide what things are – what they really mean. **We're** the ones who decide to define things certain ways because that's useful for us to make sense of things. You have to use this to help the students - what was it? ------ Make Images, but how can you make sure students have the same Images – the Images **you** want them to have? I guess just by making sure you define things properly."

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Wayne described his short practicum:

"My sponsor teacher had an accident just prior to the short practicum, so the department head of Prince School took me on. Other teachers wanted a student teacher so that it would decrease their workload - and they argued about who would get the UBC credit. So in the end I had three different ST's, as different as day and night. One even spat grape seeds on the staff room floor. It went okay with the students, though. I broke up a fight. It was easy. I just walked over to them and they quit."

****

Following the short practicum, the SMIP students often discussed what they have learned about student understanding and their own understanding about the mathematics they are required to teach. Most like acronyms, but only after they have properly taught the theory.

"BEDMAS is okay to use if the students know **why** we use it. It isn't arbitrary. We do what is most powerful first. Exponents are an extension of multiplication so we do it before multiplication --- and extension of addition, so multiply before adding --- there is some sort of intuitive appeal to the conventions that we use. And so, I like to start off my lessons with an exercise that requires **true** understanding rather than facility and at the same time I take attendance then walk around and get a sense of what they understand, then I know where to start my lesson and what Images I have to correct. Like with grade nine --- I gave x+5=7 and asked them to write out what they did to solve it. Some had an idea but many didn't have a clue."

****

The SMIP students must present problems in groups of three. After a presentation during which Sophia, Lance and Frank act out a skit and then solve the problem on the board, Lance thought that a drawing he did might confuse students because he had used 'squiggly, not straight lines' to represent the river banks.

"It's good to show the students what we, as scientists or mathematicians **really** do. **We** would actually draw it wavy because it isn't really straight so we should do that for the students so they learn. And with the calculator --- **I specifically** tell them to keep **all** the digits until the end. I think it is important that they know this. That this will give the correct answer rather than rounding off."

****

It is Wayne, Win and Jane's turn to present to the class. They do a lesson on surface area and volume of pyramids and prisms. Wayne is responsible for developing the concept of the relationship between their volumes. He has made cardboard models and filled them with clay for the demonstration.

"Okay, here I have a prism and a pyramid. They have the same base. See - I'll show you. The bottoms fit together perfectly. And, when I stand them, you can see that they have the same height. Now, how do you think they --- their volumes are related? Go ahead. Discuss it with your tablemates."

Wayne lets the class discuss and makes no comments. Then:

"Okay, everyone. Eyes up. See, here I have taken the clay out. Now I'll pack the clay from the pyramid into the prism. Do you think it will fit? ... Oh, there'll be room left over. Let's just see. See here. It comes to the mark I made. I marked the one-third height and the clay comes up to that. The volume of the pyramid is one-third the volume of the prism. I'll show you that another way. Here. Let me weigh the clay from the prism. Now I'll weigh the clay from the pyramid. See, it weighs one-third the prism's. So, again, you can see that it works."

After the presentation, Wayne discusses his performance.

"I think it is important for the students to have input. They should be able to express their ideas articulately. And they need to see different ways of doing the question. I showed them by volume **and** weight that the relationship was one-third. You have to **show** them the relationships so they will understand them and learn them. I think I did a good job of this because I spent a lot of time thinking of different ways to show it. People often use water or rice or something, but I think that by using the clay and weighing it I would have had them have a better Image."

****

When the SMIP class discusses testing, Wayne also has ideas. He stresses the importance of people using the correct terminology:

"I think testing is a useful tool when it comes to teaching. If there is a test, it forces student to pull things together. And you have to make tests so that they use them all --- all the things they've learned, not in isolation, but all in one question. That way you can find out what they really know not if they can just do each thing in isolation. And making them think - like the question we looked at where the students had to choose a bowler just given their scores on each game. Oh, yes - and we have to be careful about the way we describe the tests. Some people were discussing this as qualitative. The distinction on tests is not subjective/qualitative. Those two aren't even paired. Qualitative is paired with quantitative and subjective with objective. There's a big difference. Anyway, in grading that, I didn't like using the original rubric we had because I don't think it looked at the right things. We have to look at **how** the student **describes** why he made a specific decision. Teacher bias is always there and I think **we** would likely go for consistency than over-all average if we were to choose, so our bias would be to a student who thought that way. But, really, it really doesn't matter if the student makes a final decision because what we **should** be interested in is **how** he argues --- understands the trade-offs. This isn't really math. It's more statistics. It is really important to use correct terminology because it is like logic. If you state math correctly, use proper grammar, you can apply logic."

****

The SMIP class has observed a video of students making cubes of different dimensions from snap-blocks and painting the blocks different colors depending on the number of faces exposed to try to determine a pattern. They are to discuss the working with reference to the Pirie-Kieren theory. Again Wayne expresses concern over vocabulary:

"Really, we **can't** say no sides exposed. We have to say zero sides exposed, one side exposed, and so on. It's like logic. We have to word it correctly."

Working with David, Sophia, Trinity and Evan, Wayne indicates concern over the process used by the teacher in the video:

"We need to discuss the Images, what prior knowledge they need in order to see the relationships, and think about how we would do the question. What Images do you think the teacher in the video had? And then, I guess, figure out how the students interpreted that.

"You know, it's **inefficient** if you have to spend too much time on Images. Every time you use a new apparatus to help explain something you have to form a new Image of it. I think --- Really, if you want the students to learn **efficiently**, you should think it through beforehand and think of the best way to present it so you can **guide** them through the Images you want them to have. Having them have different Images and having to Fold Back and modify them is just **too** time consuming. So you would want to break it up, **lead** them through questioning. Like, my next step would be to have them realize that they just have to count them for one side and then multiply by six --- that might be the next step in their cognitive develop --- and then after that I'd want them to notice that there is always a square on each side and then after that --- Do you follow?"

When Sophia and Trinity expressed concern that this might be too structured, Wayne explains:

"No, no --- it doesn't have to be formal. I'm just **totally** trying to bring it together so that they can see it. I basically want them to figure out those things themselves, then put them together, probably without an end and probably with only words something like 'something times two times the same thing minus two times six' or whatever. Ideally, every student would do this for himself, but that's not likely to happen. There are students who aren't likely to get it. But if the teacher walks around the class and asks pointed questions, that will help the students develop on their own in the right direction. Students have different abilities and I'm thinking that maybe --- well, what are the arguments for inclusion of all?"

Wayne has a way of segwaying topics.

"Anyway, I'm not sure if I'm totally into inclusion of people with less ability. In general, in society, people don't spend time with --- in general society, people spend the majority of their time with people with similar interests and --- and I'm not really drawing the line between people with special needs and the rest, I'm drawing it halfway between. I want a **standard route** for different types of learning objectives, but students who come to me with a different assignment, students who are self-motivated - could come to me with a semi-related aspect they want to develop and I would have to look at it to see if it conforms to the IRP and I would have to determine a different way of looking at it. I don't know, but I think that the classes need to be different."

The rest of his group does not agree with Wayne, and he withdraws from the conversation.

****

A few days later there is a discussion about the use of technology, specifically graphing calculators:

"We think that they --- the students --- that if they do it by hand rather than on a calculator, that they will understand better because we are forcing them through smaller steps --- the object being that **they** are **doing** something, experiencing something and if they go through those steps they will learn. I'm not opposed to the graphing calculator. I think that we are afraid, though, that if we use them they might supplant the students' understanding and we just have to be conscious and be sure that they understand the mathematics and that we assess separately and that they can use technology that they will use for the rest of their lives. There is a real bias on the part of students to use anything the teacher tells them, so if it is a calculator the teacher gives them --- they will use the calculator the way the teacher does. It is sort of like in a problem, they will try to use any information in it even if it is irrelevant because they think they have to use everything the teacher gives them."

Wayne is noticeably quiet again the last week of classes before the long practicum.

****

Prince School is located in a fairly affluent area of the city. It offers a few special programs for students such as an outdoors education program and a program for the gifted, learning disabled. As with most local schools, many of the students are English as a second language and it is multi-ethnic in population. Prince School, however, is one that does not have a specific pre-dominant culture. Nor does it have a reputation as either a "good" school or a "bad" school. Students are generally content as are most teachers, many of whom have been there for a number of years and are nearing retirement. Parents are fairly involved and generally expect that their children will go to university. It is here that Wayne will complete his long practicum.

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Wayne has been teaching a mathematics 11 class for a week. Although they were not forewarned that I would be videoing Wayne, the students accept my presence without comment. Wayne gives them a question involving patterning and wants the class to work in pairs, discussing what the pattern is and how they might describe it: 1→1, 1+3 → 4, 1+3+5 → 9.

"It's so that you can build up your terminology so that you can explain in words. It's not easy. It's not intuitive --- won't roll right off your head, but you can do it. Can anybody tell me the next term? Raise your hand. Right."

He writes: 1+3+5+7 → 16. Wayne then makes a third column to the left of the two he has on the board and numbers the rows, 1, 2, 3, 4, etc.

"Now, we want to name these columns. We want to describe them carefully in words." *There is some discussion. The ensuing discussion is represented in the opening statement. After Chien's statement, Tom still appears unconvinced and mumbles something under his breath as Wayne continues with the class:*

"I'll give you time to digest. This isn't easy, you know, but it is what you will **have** to do as you describe your conjectures."

After several attempts, the class has determined a formula and Wayne moves on to last day's homework. They review three questions with Wayne articulating definitions for them:

"I want to be sure that you understand the terminology. It's very important. And remember, you should **never** use special cases. For example, if it says triangle, don't make it equilateral or right or anything. Be sure you make it general.

"You know, we are having to take up a lot of questions here. You could've come for help yesterday, but didn't. So, now we have to go on. Take out your notebooks. You'll be taking notes. Now, everyone pick a number. --- Yes, it can be negative, but you probably don't want too big of a number. Now, double it. Add eight. Divide by two. Subtract the original. Multiply by five. Now, I bet everyone got twenty --- well, most. What was that, Allan? --- You think as long as it is smaller than some number, the answer will be twenty? --- That's your conjecture. Good. Amanda? You say it worked for one thousand nine hundred ninety four? Okay. What we are going to do is deductive reasoning. I'll **show** you how that question works after you take notes."

Wayne gives notes on the meaning of deductive reasoning. He then goes to the board, and using n for his number, he generalizes the oral question he had given the students:

"See, here I have **proven** for you that regardless of what number - whatever number you started with, the answer will always be 20. I have proven it for you."

As the class continues, it is apparent that the students have a reasonably good grasp of basic mathematics, and that they are a well disciplined group. When given time to work, they work - usually individually, but sometimes in pairs. They think about what is being asked. Thus, when Wayne gives a question that involves two odd numbers:

> Angie: If you say two odd numbers, they could both be $2x+1$.
> Wayne: No. It says **any** two odd numbers. And, so, no, we wouldn't really have to prove it for if they were both the same. We would have the second number as $2x-1$ or maybe $2x+3$ because it says **any** two odd numbers.
> Nathan: But, sir, if it is **any** two odd numbers and one was $2x+1$, the other would **have** to be $2y+1$.
> Wayne: No --- no. We aren't looking for a counter-example, we are trying to prove that something is true, so I don't think so. We don't need that.

The class ends.

****

The class is on 'logic sentences'. They have briefly discussed these last day. Wayne has a number of examples of logic sentences and their classifications written on the overhead:

"Now, here are some examples of each of these. What I want you to do is look for connecting words, use letters for statements, and to translate these into logic. Think of how you would describe each of the words --- the ones we are naming each sentence. Like contra-positive, bi-conditional --- what do they mean? Talk it over with a friend and see if you can describe them - make a definition."

Students work together and discuss the questions, arguing with each other as they find the solutions. They laugh at some answers, and then discuss why they were wrong. When it is apparent that several students are done, Wayne states:

"Here's a question for you if you're finished. Use the numbers 0-8, a different number for each letter, and no carrying, so that xyz+abc=pqr. Do it on scrap." *After a short time, Siu says it won't work: "I'm terrible at math. But, if you can't carry ..."' and she explains explicitly why the question won't work.*

"Great, Siu. That was good. Now, let's get back to conditional statements. Here are the definitions you should have come up with." *And Wayne gives the students his definitions.*

Jan: Can a 'when' statement be a conditional? - like 'When I am sick, then I feel bad.' Can that be a conditional?

Wayne: No. You **cannot** write logic that way. You have to use if and then. You might interpret it as logic, but to be logic you have to have the premise and conclusion. Now, it is **very** important to think of the truth-value of each type of statement as it relates to the original.

Like the contra-positive truth-value is always the same as the truth-value of the original.

Nathan: Yeah. Two negatives make a positive so it makes sense.

Wayne: No, this is a little different.

Nathan: But the principle is the same. Double negatives make a positive.

Wayne: It's **not** the same. There is a difference and this is logic. You have to think of the truth value, not the double negatives. --- Now, a bi-conditional statement's a combination of a statement and its conditional.

Nathan: Isn't it a conditional and its converse?

Wayne: Yes, we combine a statement and its conditional to an if and only if statement. Can anyone give me an example?

Siu: A polygon is a triangle if and only if it has three sides. Is that correct?

Wayne *(not responding to her question):* Now, write three statements and make all the others, matching their truth-values. The originals don't have to be true. When you are done, raise your hand and I'll come and give you your homework so you can have a head start.

****

The class is beginning geometry. Wayne gives them a few opening questions which he has written on the overhead:

"These questions are fairly simple --- they are review – to get you thinking in the right direction."

Wayne takes attendance and works at the computer at the front of the class while the students do the questions.

"Okay, how did you do this one? Let's take a look. --- That's right. This is the perpendicular bisector ..." *Wayne reviews each question with the class, correcting their terminology and writing the solutions on the overhead.*

"Okay. I have prepared a book for you. Take it out and work through it with a partner. You can follow it. It is very organized. You'll be having a quiz on this stuff --- aiming to get to theorem eleven. If everyone is working hard and doesn't get that far, I'll move back to an appropriate place."

The geometry booklet he has given them is designed as guided discovery to basic circle theorems. Rob suggests that a ray is a chord, the longest chord of the circle and Aaron asks if a diameter is the perpendicular bisector of each chord.

"Yes, Rob, that's right. That's a good conjecture, Aaron. Why don't you think about it?"

Even though Wayne has suggested discussing with a partner, there is little class-talk. He walks around the class and responds to students who raise their hands.

"Okay, now, everyone take out some scrap paper 'cuz it's time for the quiz." *The quiz consists of drawing diagrams that represent the different theorems that were presented in the geometry book.*

****

The next time I plan to attend Wayne's class, he asks me not to come.

"To be honest with you, some of them have been very apprehensive lately and it's been affecting the mood of the class. I've been trying hard to calm them down and so I'm not sure if having you come in and observe with your video camera will be the best for the solution. I need to think about it, I guess. I've decided that they are writing a test on Monday, so that day wouldn't be useful to you, anyway. Maybe I'll have a good sense after Monday about how having you in Wednesday might affect things."

****

The class was not ready the next week and I am thus unable to observe again until after the Spring break. By this time, the class is working on coordinate geometry and the students are using the slope, midpoint and distance formulae. Wayne asks Joe to do his question on the board. Tom indicates that he did it differently. Wayne looks at his work:

"That was coincidental. That won't usually work. You have some of the wrong ideas because you weren't here last day."

Next, students are asked to draw lines on graph paper.

"If you have the equation, y equals something *(writes* $y = 2x - 3$*)*, then this number here (*points to the* 2*)* is very special because it tells us something about the line. --- Right, the slope, m. ... No, I don't know why they use m. Now, on the graph paper, draw a line with slope 2. Then one with slope of $-\frac{1}{2}$. Then another set with slopes -3 and $\frac{1}{3}$ and think about how they are related."

Students come to the front and draw their lines on the overhead and Wayne discusses the meaning of slope. Students conclude that when the slopes of two lines are negative reciprocals, then the lines are perpendicular. Nathan asks: "Didn't we do something like that with them? Over one, up one, then over one, up four or something?" Wayne responds:

"Yes, but that's not really the same thing. You have a right idea --- the rise and the run to get the next point --- but it's a bit off here. Now, here's something you **should** get into you notes. If the multiplication of two slopes is negative one, then the lines with those slopes are perpendicular. See, $2 \times -\frac{1}{2} = -1$ and

$-3 \times \frac{1}{3} = -1$. I guarantee you - promise you - that if you do a whole bunch, that if you multiply them together and their products will be negative one, that those two slopes will be perpendicular. What kind of reasoning are we using to figure this out? Della, do you know? --- Deductive. No, not deductive. What is the other one? -- Inductive --- where you do a whole bunch of examples and get the same result each time and so conclude it is always going to happen. Here we did two examples and both times the lines were perpendicular so we are going to conclude -- generalize that the lines are always perpendicular when the products are negative one. I'm going to tell you that it is always true but you did some inductive reasoning ...."

****

After the practicum, Wayne is very open to discussing his ideas and the classes he taught.

"I think my classes went pretty well. I got the students to work together fairly well. As for the P-K theory, to be honest with you, I think my --- my --- my **Images** for a lot of the components of the model was formed before I – well partly through university. So I don't actually see its any diff – it's given me a language but I don't see it as new to the way I thought about teaching.

"I never had things explained well to me in school - like what subtends means. So I tried to explain it well, but I wanted students to grapple with ideas. A lot of math isn't taught well -- I guess in terms of the P-K you'd say they jump over Don't Need Boundaries. But during the practicum I worked hard with people who aren't succeeding. The standard deviation shrank. One girl who wasn't doing well, after I talked to her, she really started working hard. She got 53 for second term! Her words for algebra were terrible. She'd say 'two times x' when she meant 'x squared'. But she could explain what she meant. It was her symbolization was way off and manipulations. Going through that and building confidence helped.

"I have a bit of latitude with both sponsor teachers - more with Don than Al --- and I used it within the political constraints. If I brought up cognitive psychology, Don just sort of smiled and dismissed it. It's Al's philosophy that most students and most teachers just want the teacher to provide ways to get the answer. Well, you read him on the list serve. He comes up with these new theorems that he wants to brag about to other teachers. Al - well, he was the teacher and I am his apprentice - well, more his appendage so if there is any problem, he dealt with it - well, wants to --- but Don - no real problems in his class because he has a more strict discipline bent than I do. Some kids don't even know anyone in their class. He doesn't relate to them. And Al doesn't like the Math Power book because he says it brings up irrelevant things. I felt frustrated in his class. I wanted to teach so that it's all hypothetical.

"There is a real disconnect between what we learn at the U and what Don and Al practice --- but otherwise they fit nicely. They are traditional and in their classrooms, there are those who 'get' it and those who struggle and I think those that get it are the ones who can fill in the gaps themselves.

"When I was in high school, I saw how terrible my teachers were - most of them - and I succeeded, not because I was bright --- although I was --- but ---

that's not all, it was because whenever I was presented with something I --- I spent my time thinking about it - developed a **complete** understanding. And I think I taught fairly well because I did the same thing before I taught - I thought about it, thought about what was needed and what it really meant and then taught it so it was the **easiest** route.

"A lot of the things I thought about relate to the Pirie-Kieren theory - different terminology, but same idea. A lot of theory comes from an unconscious, intuitive --- internalized level and these ideas, some are in the Pirie-Kieren theory --- were there **long** before I had the language. It's difficult for me to separate theory from practice because I do a lot of those things without thinking about it. There are some, and I say this with confidence, not to say I don't learn a lot from reflecting on the theory, but I sort of think the structure is there - the framework is already there for me. I tutored a kid who was failing in math and in six weeks I got him to pass --- asked simplified questions and **provided** definitions. Same with a friend who couldn't do math eleven until I taught him math eight and nine. A good teacher has to have patience, willingness, desire - drive, even - to understand why students are doing what they are doing and then try to find ways to get them on the right track on their own by giving them prompts that will lead them in the right direction.

"I liked student teaching for a while then I hated it because I was putting in lots of time and energy and was getting neutral responses from my sponsor teachers. Even the kids hated me because I was making them think - making them work, moved them if they talked too much. Some were defensive and accusatory - the class you usually observed - and said that I'm not teaching well. So I'm getting these kinds of things and not a lot of positive feedback even though what I'm doing **is** good for these kids. I don't want to say 'you guys are full of crap and this is good for you so get to work'- don't want to be too heavy handed but need to be firm and sympathetic.

"They're learning, though, because I can see that the mean scores are going up - some kids marks - the poorer ones - have come up, and a few of the higher ones have come down because they used to be able just to memorize and now they have to think - and the standard deviation is getting smaller. I think I have made good progress with the students. I've made a difference. But it's frustrating when some of them don't get it **even though I've** broken it down **very** well because they just don't understand simple things like how to multiply by 2/3. Can do 1/3 but not 2/3. Seems like to teach them I would have to start with adding and subtracting. Then I could **lead** them **straight** through. I never realized how much students didn't understand from their previous work, previous years.

"Every time you see me with my arms crossed and looking stern, that's because I'm frustrated with the teaching they didn't get before - the education that **didn't** take place. It's frustrating to see the kids that can't learn because they don't know the previous work. If I was teaching a grade eight class, I would go back and teach all the stuff that they **should** have learned in elementary school. Probably take only about three weeks to teach them that - basic numeracy - just lead them through. Really, I'm **very** pleased with the way my student teaching went though the students were feeling pressured and frustrated. It's not my fault.

Here and there there might have been things that contributed to it but I blame that more on the way they have been taught than what I'm doing. Over all, I think I did a good job of teaching under constraints."

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Wayne has just observed the video of the class on 'logic sentences' which was discussed earlier:

"You know, I don't really have class policies --- haven't thought of them very heavily. I don't want kids to volunteer without raising their hands, though. I've had to change seating plans a few times. Sam, my FA, said he always changed his every few weeks so I tried to do that. He said he did it to give them experience working with others. I did it because the students weren't working. I don't like to punish. I try to keep management by drawing on students.

"To tell you the truth, what I did in that class wasn't my idea. I was going to give them the definitions and then give them examples, but Sam, called the night before with --- I'm being facetious --- but, with **his** plan. He suggested that I turn **my** lesson plan on its head --- give them the examples first and then give them the definitions --- they'd have exposure, he said, and I could fill in the gaps later --- so I sort of changed. I didn't know how it would go. I guess it worked okay. I guess I covered all the same things in the end, but in that format. Anyway, if it was **my** class, I'd do it differently. Takes too much time this way – had to do it twice."

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The following year, Wayne took a position in the city as a teacher-on-call.

Chapter 7

Analysis of the Portraits

7.1. Introduction

Once the portraits of the four preservice teachers in this study had been drawn and authenticated, the next stage of analysis involved considering their developing understandings of teaching and learning mathematics based on the modified definitions of the Pirie-Kieren theory provided in Chapter 2. To remind the reader of the meanings of the levels, I summarize:

- Image Making is thinking about Primitive Knowing.
- Image Having is summarizing an activity and no longer having to reference the thinking that led to it.
- Property Noticing occurs when an individual thinks about his/her Images and tries to reconcile them.
- Formalising occurs when he/she summarizes and enacts the Images in a coherent manner.
- Observing requires looking at Formalisations of both teaching and learning to determine if there is a 'fit' between them so that the teaching, as defined, leads to learning, as defined.
- Structuring is a summarization of what is the integration of teaching and learning.
- Inventising occurs when the individual considers other structural alternatives that might define teaching and learning.

These modified definitions are used here to discuss the growth of understanding of teaching and learning of each of the preservice teachers, Sophia, Lance, Ellie and Wayne, in the same order as their portraits appear in Chapter 6.

Each analysis begins with a discussion, using the modified Pirie-Kieren theory definitions as the lens, of the developing understanding of the preservice teacher. Evidence for the classifications of the action with respect to the levels of developing understanding of teaching and learning is provided, in quotes in italics, from his/her portrait. I could have referred the reader back to the portrait using an identified notation. However, I did not want to compromise the integrity of the portrait. This section is designed so that it can be read without referencing these quotes should the reader so desire. As this is a study into growth of understanding of teaching and learning, to maintain the authenticity of the preservice teachers' growth, throughout the analysis I have retained each individual's wording rather than interpret it into technical/theoretical language.

Throughout the analysis discussion, incidents that indicate a positioning on the theory are numbered in bold print. After the analysis I present two visual renditions of the results, one with the incidents numbered as in the analysis, charting the growth of understanding of teaching and learning for each of the preservice teachers, and the second on the modified dual model which very clearly provides a summary of each individual's path of growing understanding. Blue items relate to teaching, and red items to the learning of mathematics. Each of these mappings is followed by a discussion of the individual's growth.

Throughout this chapter I use the following abbreviations concerning the levels of the Pirie-Kieren theory: PK for Primitive Knowing, IM for Image Making, IH for Image Having, PN for Property Noticing, F for Formalising, O for Observing, S for Structuring, and I for Inventising.

7.2. Analysis of Sophia's portrait

7.2.1. Classifying Sophia's levels

Based on her experiences volunteering in other countries and on the fact that she had a number of relatives who were teachers, Sophia entered the teacher education program with a fairly well defined Image of teaching as making the world a better place **(1)**. *"I'm an idealistic girl and I want to make the world a better place through education."* Teaching, to Sophia, involved understanding students' confusions by listening to their discussions (IH) **(2)** *"I don't think a teacher can understand the confusion or difficulties by evaluating written work ... needs to see a student work through the problem, hear their 'umm's' ...* and it should lead to understanding *...encouraging understanding of problem solving, and not just procedures."* (IH) **(3)**. For most students, to develop this understanding, she thought, required clear instruction (IH) **(4)**. *"...the majority - they need to be taught more clearly..."* In this vein, she believed that it was the teacher's responsibility to create opportunities for students to learn (IH) **(5)**. *"... and it's the teacher's responsibility to open up their horizons..."*

After the presentation of the Pirie-Kieren theory, during discussions on the nature of mathematics and mathematical learning, Sophia was involved in Image

Making **(6)**, thinking about how students learn and she indicated that she believed that learning, or developing mathematical understanding, was progressive - not linearly, but tangentially (IH) **(7)**. *"Mathematics has a **natural learning progression**. But it's from **tangent** to **tangent**, not book chapter to book chapter ... math can't be presented as a bunch of little units outlined in the textbook."*

Sophia Folded Back to her teaching in Peru (PK) **(8)** and related mathematics teaching to mountaineering (IM) **(9)**, indicating that to her teaching involved knowing the students' backgrounds (IH) **(10)** *"... important that a teacher understands **where** the students are coming from ..."* and knowing different ways to approach a topic so as to make it meaningful to the particular students (IH) **(11)**. *"The guide knows the general directions and knows the terrain ... can detour depending on the needs of the tourists ... you even have to look at it differently depending where you're teaching ... "* Thus, teaching must vary for different learners (IH) **(12)**. *"Each learns differently and --- you can't just teach them all the same."* Thinking about the meaning of learning (IM) **(13)**, Sophia stated that that teaching involves continual learning (IH) **(14)**. *"... learning --- I think it has to be a continuous process and the teacher must be a continual learner of - well, of both mathematics and how to teach it."*

A video of a teacher, Alwyn, presenting an introductory lesson on solving equations served as an Image Making Intervention **(15)** for Sophia and she formed Images of teaching as allowing students to explore **(16)**, challenging them and helping them develop understanding or conceptualization of the concept

(17). *"It was fabulous the way the kids just dug in ... Understanding is important ... they can maybe see that something goes* ***into*** *that box ... it actually* ***fits*** *...* ***I will definitely*** *try that."*

While discussing her own levels of understanding with reference to the Pirie-Kieren theory Sophia was involved in Image Making **(18)** with respect to learning. *"I didn't realize that* ***even we*** *went through these steps in the theory when* ***we*** *worked on a problem."* She was involved in Property Noticing while discussing how she worked through problems and thought about her own understanding **(19)**. *"... seeing how we go through problem solving and discussing it with peers ..."* She realized that this type of activity would help a teacher understand students' problems (IM) **(20)** *"... will help us understand what our kids will go through ..."* and thus improve teaching because the teacher would have a more clearly defined situation (IH) **(21)**. She also re-iterated that learning involves conceptualization (IH) **(22)**. *"... define the limitations of the problem or the benefits of the problem ... we can push the kids to conceptualize further ... and it puts a nice exactness to the problem ... its more a retro-scale of your teaching."*

In discussing homework, Sophia returned to the discussions of the Pirie-Kieren theory and actively engaged in Property Noticing **(23)**, discussing the Images of how students develop understanding through working on assignments. She concluded that practice is an essential part of learning (IH) **(24)**, *"Students don't learn just by being* ***shown*** *how to do a question or two. They have to have practice ...* ***practice*** *gives the kids a chance to break through a barrier."* and that being able to do the work efficiently was a part of understanding (IH) **(25)**. Thus the two, doing and understanding, were both a part of learning for Sophia. *"Being able to do is only a part of understanding. Sometimes maybe the doing can lead to understanding."*

Class presentations and the discussions that followed afforded Sophia time to practice some of her Images of teaching and learning. She revisited her Images of teaching as actively involving students and providing engaging activities (IH) **(26)**, *"... it is really important that students feel **engaged** with their work ... Like when Trinity gave her problem on probability ... made it about AIDS with **real** numbers..."* and these were enacted as well in her own lessons (PN) **(27)**. Learning, she indicated, should be an enjoyable activity (IH) **(28)**, *"I think they have to have **fun** with it."* and teaching involves encouraging students (IH) **(29)** *"... but the teacher has to encourage --- tell them they are doing fabulously..."* and helping them make connections (IH) **(30)**. *"...she has to have them make connections."* She indicated that students learn best by developing their own connections (IH) **(31)**, *"It's not too hard for the kids to get the connections if they come up with the connections ... it was **fabulous** the way she made us do all the work"* possibly with the teacher as guide (IH) **(32)**. *"Teacher should be helping them maneuver through math."* Learning, according to Sophia involved understanding (IH) **(33)** and likely occurred best if teaching provided real-life situations (IH) **(34)**. Reflecting on her own experiences (IM) **(35)**, she thought about the interplay among these different Images and realized that they might work together to encourage student understanding. She made a pre-formalising statement that teaching involves preparing students to the best level they can achieve by forming them into independent thinkers **(36)**.

Folding Back to her own learning experiences (IM) **(37)**, Sophia recalled some of them as being boring until finally something she learned something new. She revealed that she thought of good teaching as encouraging/challenging students to do their best (IH) **(38)**, *"When I was in school ... my own grade seven teacher was really strong - encouraged us to learn more..."* and indicated that she thought of teaching as giving students an opportunity to learn (IH) **(39)**. *"[T]he teacher must give the students some challenging work to do ..."* Thinking about student learning she realized that students learn differently (IH) **(40)**. *"All students should be given the opportunity to do the best they can ... They just need to be encouraged -- develop their own skills -- and think about what they already know."* She re-stated her pre-formalising notion that a fundamental purpose of teaching mathematics is to develop independent thinking and problem solving abilities in students (PN) **(41)**. *"A major goal of mathematics education should be to get students to be* ***independent*** *thinkers and problem* ***solvers.****"*

Discussions on the use of manipulatives as a teaching tool and on the amount of rigor that is needed in the classroom (PN) **(42)**, *"Manipulatives --- give the physical and work with them until they see where it comes from" "the standard, rigorous form - maybe all don't need it ... Not all need to be able to do the abstract stuff."* led Sophia to indicate that teaching provides students with the necessary language and tools to be able to function to the best of their ability (IH) **(43)**. *"You have to provide them with the* ***tools*** *to be able to* ***read*** *the language* *that everyone else is writing and to be able to express their thoughts in those ways."* Sophia then noticed (IM) **(44)**, that they (the SMIP students) learned differently when using the manipulatives than they had when using their formalized understanding and realized that providing students with these different forms of teaching might create similar learning experiences for them (IH) **(45)**, particularly for the more able. *"... they might be able to 'pick up' other ideas in the hands on approach. You know, sort of like we did when we worked with the Algetiles .... Maybe the better kids would too."* She then considered her different Images of learning (PN) **(46)** as involving different styles and as being able to take place on different levels, and determined that it was the teacher's responsibility to let the students know they are responsible for their own learning. Thus, she indicated, teaching works toward creating independent thinkers (PN) **(47)**. *"... assume that they can do it - and let them know that it is* ***their*** *responsibility to do so."* She further discussed the idea of teaching as encouraging understanding/thinking (IH) **(48)** by providing the opportunities for students to learn to their best ability (IH) **(49)**. *"... even these should be encouraged to understand and maybe they will come to a better feeling about the subject ... all students should be given the opportunity to develop their own way of thinking."* Again, she indicated that it is the teacher's responsibility to understand how students learn (IH) **(50)** and that encouragement is an important aspect in this process (IH) **(51)**. *"The teacher should be one with the students - seeing how they are learning it --- and she really has to encourage them."*

After her short practicum, Sophia revealed that she had witnessed what she thought of as poor - or non-teaching (IH) **(52)**. She was shocked at the activities that she saw in the school that passed as teaching and learning situations *"Nobody should be made to feel that they can't do good. ... How can they learn if the teacher - if they're not expected to?"* and, informed by her own understanding of the processes of teaching which were partially based on her family experiences **(53)**, *"My Father once said - after he retired and had time to think --- that if schools focused on teaching kids how* ***to think****, boy, education would be amazing!"* she considered the positioning of the school in the larger society. *"... school is a place to learn about the world. People of all sorts are part of society ..."* She further discussed the design of the education system and the effect this has on different learners *"... often focuses too much on the lower end and doesn't do enough for the gifted ... equity --- challenging."* and considered what equity meant in the school situation (PN) **(54)**. She concluded that teaching should provide a safe learning environment and respect for all learners (IH) **(55)** *"... a microcosm of the world I'd like to live in ... you have to consider the* ***minority****, not just the majority at all times..."* as well as providing equal learning opportunities for them all (IH) **(56)**. *"... questions that are diverse enough so that people at the lower end won't feel right out of it but people at the upper end are challenged."*

Discussions on technology prompted Sophia to further Property Notice **(57)** and she re-confirmed her belief that teaching involves providing learning opportunities (IH) **(58)** and she indicated that it is the teacher's responsibility to use available technology to help students develop understanding (IH) **(59)**. *"Equity ... is fundamental for the information and communication age ... can help enhance and deepen the students' understanding of certain mathematical concepts..."*

Presentations and the discussions that followed them revealed Sophia's Image of teaching **(60)** and learning **(61)** as occurring through peer tutoring. *"... peers explaining, as long as the peer understands is so much more effective than teacher explaining."* Sophia believed that teaching involved trusting that students can learn (IH) **(62)** and again revealed the Image that teaching involves providing the opportunity for this to occur (IH) **(63)**. *"I'll bet there are four or five people who come up with a more efficient way and then get them to teach the people nearest them ... progressing and helping everyone else."* She realized that students need time to develop understanding (IH) **(64)**, which requires the ability to see relationships and connections (IH) **(65)** by using previous understandings (IH) **(66)**. *"Whatever method ... they need time to develop. They need to notice 'this and this',... be able to* ***develop*** *the understanding that will be necessary ... They have to call on their own previous understanding."* Sophia revealed Images of teaching as being aware of different approaches students might take **(67)** and of allowing students to develop their own understanding **(68)**. *"... the teacher needs to be aware of all* ***sorts*** *of different possible connections ... can't lead them through everything because then the students will not be able to develop the understanding ... They'll just have the teacher's understanding."*

Discussions on the use of tests, on assessment vs. testing, and on considering the types of mistakes that are made by students revealed that Sophia held an Image of teaching as preparing students for life **(69)**, *"Tests are a part of life and kids will have to take tests of different forms throughout their life."* and that it involves understanding students' thoughts **(70)**. *"... rely on how well the students argue and you can better determine the process they go through."* She further revealed that teaching involves knowing what mistakes students might make (IH) **(71)**, *"... it's important to consider the kinds of mistakes the students made..."* and in what situations the teacher should intervene (IH) **(72)** since teaching involves helping students understand the sources of their errors (IH) **(73)**. "... *if they don't affect their understanding, students should not be penalized ... But, if something --- **completely wrong** the teacher needs to be concerned and try to determine what epistemological obstacles is there and help them fix it."*

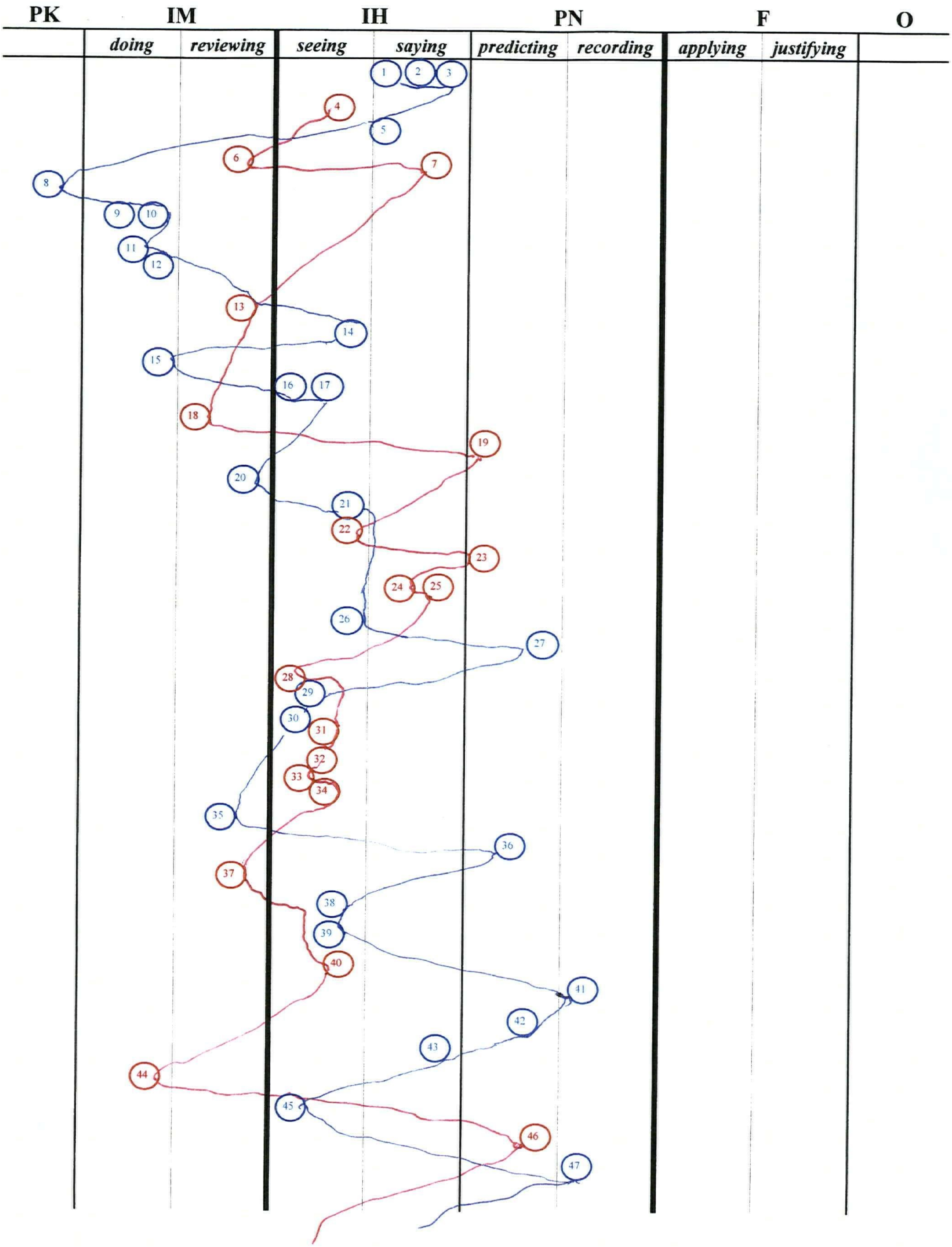
During her long practicum, Sophia actively practiced many of her Images of teaching, working at the Formalising level with respect to creating independent thinkers and problem solvers **(74)**. She placed emphasis on having students actively involved in their learning and thought about how they reacted. *"... the kids WERE ENGAGED! Although lots of them gave up quickly, but some were real problem solving."* Most of Sophia's classes had the students actively involved in working in groups and explaining to each other (PN) **(75)**, *"How do you know? What would you choose now? ... Could you try working it out? Get help from someone?"* thus demonstrating her belief that students are able to learn (IH) **(76)** and that they can help each other do so (IH) **(77)**. "*They should talk to their friends and discuss.*" Sophia also demonstrated her Image of learning as requiring time (IH) **(87)** and developing one's own understanding (IH) **(79)** by giving them the time and permission to work with their Images. "*... if you want a box, you can use it ... learning mathematics is a process of thinking --- not a step by step process --- you can't just follow rules, you have to think --- and verify and interpret.*" She re-stated her Image that mistakes are a legitimate avenue to learning (IH) **(80)**. "*... errors simply provide an opportunity for learning to take place - a chance to develop new understandings and create better/more accurate images.*" Her Image of teaching as respecting students was evident in the manner in which she approached discipline **(81)**. "*Jason, you're busted. You're so busted ....*" *Sophia retains her ever-present smile, but her sparkling eyes speak volumes ...*"

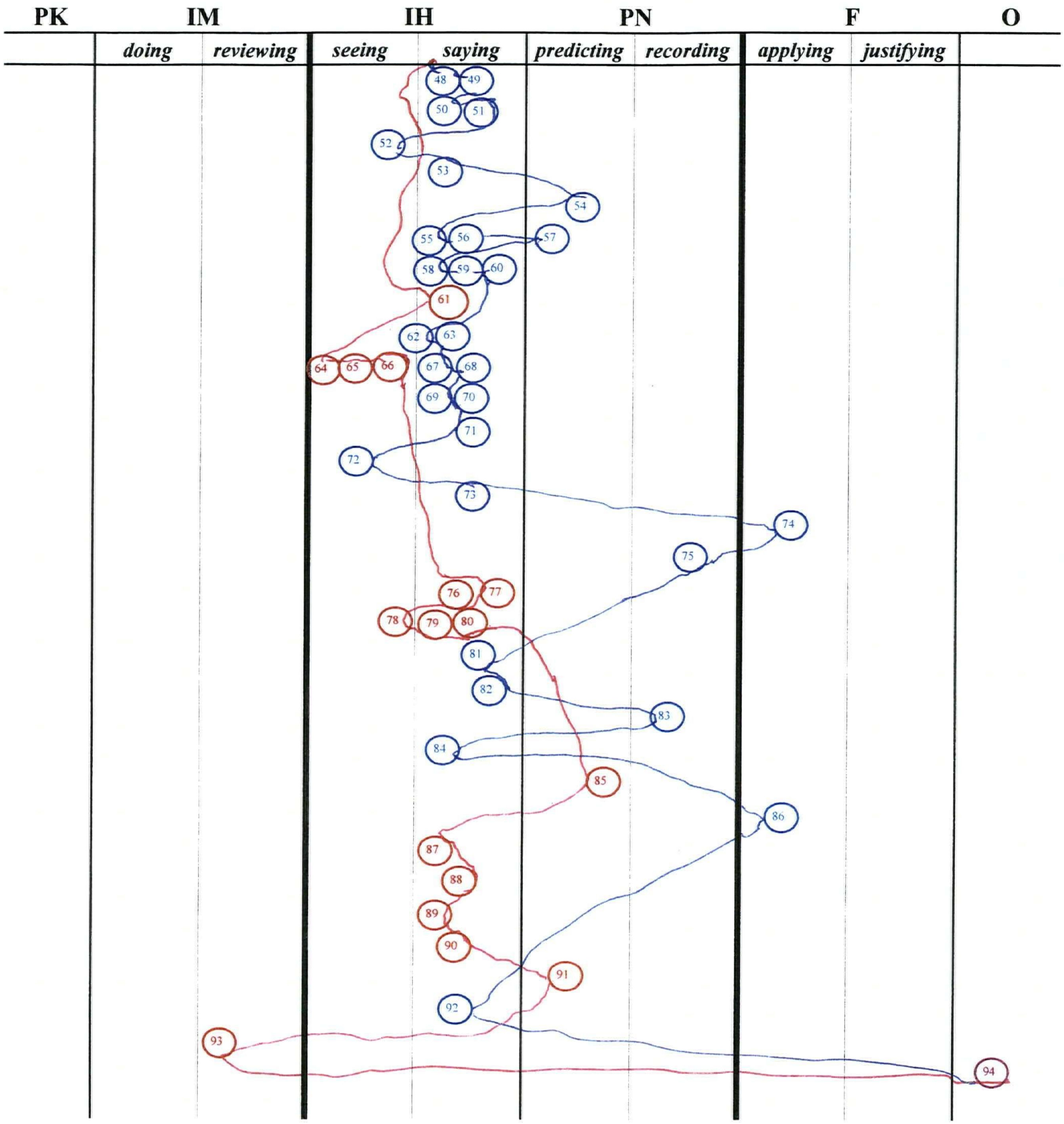
During the practicum, Sophia revealed an Image of teaching as providing hints or leads, an Image which she had not previously stated **(82)**, "*Try using different numbers - easier ones.*" and she reminded the students of their future, implementing and discussing (PN) **(83)** the concept of teaching as preparing for real life. "*I told them it was for marks - that everything they do is important and will affect their future. Students need to be reminded that everything they do is important ... and affects their future.*" She also expressed the Image of teaching as helping students determine their own understanding **(84)**. "*No doubt they will have problems, but that is when you have to ask them what their problem is and in explaining it, they sometimes figure it out themselves.*"

Viewing herself on video prompted Sophia to think about (PN) **(85)** the learning that had taken place in her classes. She was impressed with the conceptualization the students had been able to achieve, *"I liked the conceptualization I saw ... they seemed to see the box as some number ... I found that their mental numeric abilities were so - so* ***very poor*** *- so I thought this - with lots of calculations as well, it helps them"*, thus enacting (F) **(65)** the understanding of teaching as creating independent thinkers. She also indicated that she thought that there was a certain amount of emotion involved in learning (IH) **(87)** *"... it's algebra and a lot of kids have this mental thing even before they start. There are some --- well, they just don't want to be there, so you have to make it like they want to come..."* and that she thought that Images were important (IH) **(88)**. *"... they will have a good* ***mental*** *visualization of what x means..."*

Social interaction was seen as an important part of learning by Sophia (IH) **(89)** *"... so a lot of the stuff is trying to let them have fun in a social context..."* and thus peer interventions were useful in learning (IH) **(90).** *"...group work -- they may pick up something from their neighbors."* In discussing this, Sophia thought about these images (PN) **(91)** and considered that teaching could take the different forms of peer-teaching or teacher-teaching (IH) **(92)**, *"That was good --- just with a few questions a lot of teaching goes on without my having to lose my voice... I think they really* ***learn*** *it this way ... I really saw some great teaching going on..."* and that learning could take place without specific teacher interventions (IM) **(93)**. *"There was something about the* ***energy*** *they were* *putting in to defend their argument -* ***that*** *was fabulous math going on right there..."* Realizing that the teacher provides the setting, Sophia placed the student at the center of learning and had moved to the level of Observing **(94)** with the enactment of her beliefs relating teaching and learning. Thus, according to Sophia, learning occurred when the teacher provided an interesting opportunity for engagement at the appropriate level. *"... just keeping the kids occupied and engaged. If they have something interesting to do and they can do it, I think they are all just fabulous. They'll do it. ... I think they really* ***learn*** *it this way and I don't have to lose my voice. ... I really saw some great teaching going on between the kids and I think maybe it worked best when I let them pick their own groups because of the increased --- fraternity that was in the groups. I thought that went really well..."*

7.2.2. Charting Sophia's growth

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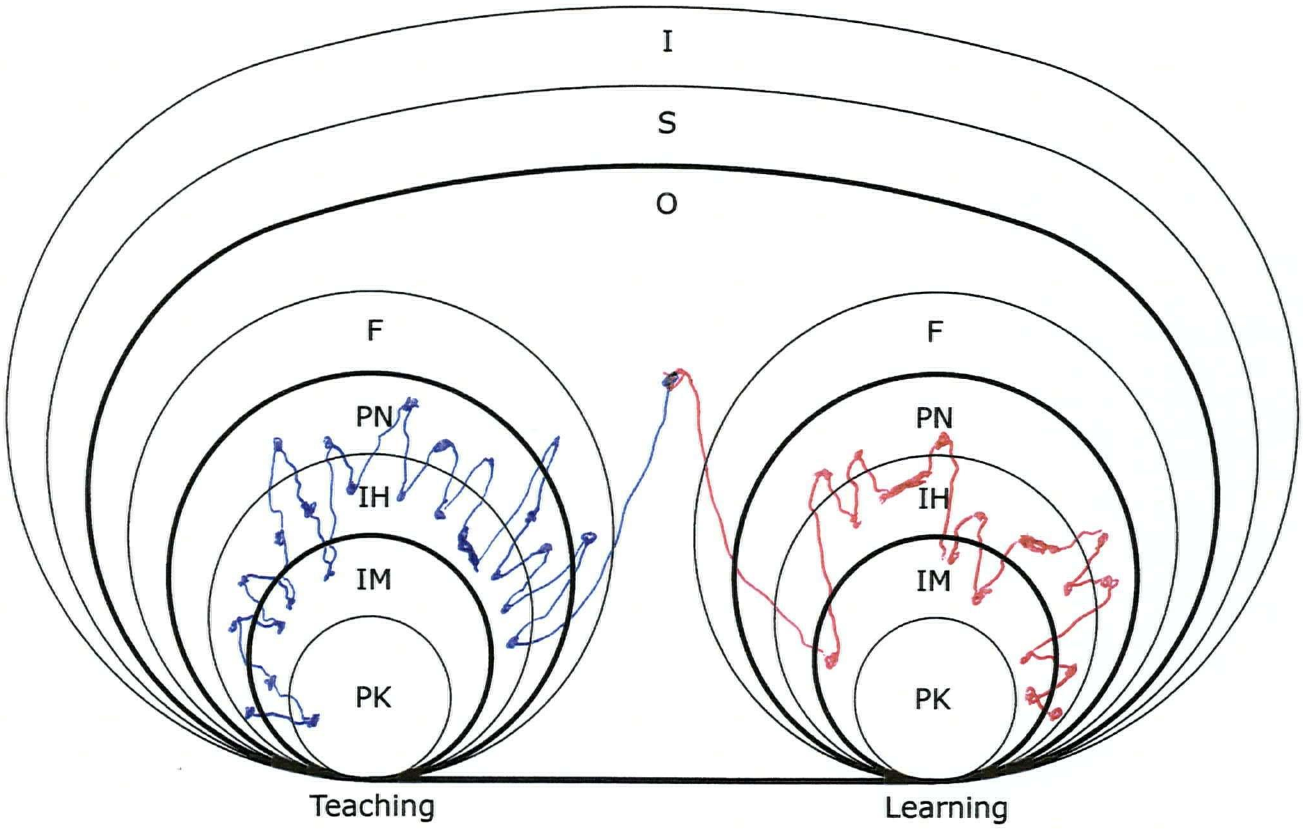


Figure 8: The Mapping of Sophia's Growth of Understanding of Teaching and Learning

7.2.3. A discussion of Sophia's growth

From Sophia's portrait, one could see a preservice teacher who was actively involved in learning about teaching and learning. Based on her own experiences, she had some pre-formed Images of what was meant by teaching and learning mathematics. These Images were, unknowingly, somewhat aligned with the Pirie-Kieren theory and other modern ideas on the teaching and learning of mathematics. Even so, Sophia was open to new ideas.

Various activities in the program provided catalysts for Sophia to think differently about teaching and learning and provided Image Making and Property Noticing opportunities. Of significance was the viewing of the video of Alwyn teaching, using a method different from that which Sophia visualized. Using this method, he wanted the students to conceptualize the meaning of solving equations. This Provoked Sophia to think about how this method would affect the students' learning and she incorporated it into her repertoire of teaching methods. A second significant activity for her was when the SMIP students worked through a problem while thinking about their own level of understanding. This prompted her to think about how her own thinking and understanding of a situation could impact her teaching of the concept and thus also impact the learning of the students.

An overview of Sophia's developing understanding of both teaching and learning reveals that she used many in-class activities as Image Making and Property Noticing experiences and that she used her own experiences to inform her understanding of a situation. In in-class discussions, she revealed many of the Images she held of teaching and learning. Often, an Image of teaching was followed immediately by an Image of learning or vice versa, indicating that to Sophia, the two were intertwined.

In reviewing Sophia's developing understanding of teaching prior to her practicum, four themes emerge: 1. Teaching involves understanding students and their thinking; 2. Teaching involves providing opportunities for students; 3. Teaching requires knowledge of the subject matter; and 4. Teaching is an act that involves student learning. The learning aspect also revealed four themes: 1. Learning is an active process; 2. Learning is an individual process; 3. Learning takes time; and 4. Learning involves seeing relationships. These Images are in agreement with progressive views of mathematics teaching and reflect the tenets of the Pirie-Kieren theory regarding the manner in which mathematical understanding develops.

In her short practicum, Sophia witnessed what she considered some very poor teaching practices and this Provoked her to consider what teaching was.

During her long practicum, Sophia was free to enact her Images of teaching and learning mathematics and on occasion worked at the Formalising level. She engaged students in activities, challenging them and providing opportunities for them to learn. She had students work together, explaining their work to each other, and she let them carry on their discussions, using their own terminology, trusting that they would be able to work through to an understanding.

In the post-practicum video review and discussion, Sophia was able to recall specific teaching incidents and methods and relate them to their possible effects on learning. She concentrated on how learning was enacted and, using these Images, she thought about the act of teaching (PN). It may be that her previous background placed her at an advantage because within her family she was used to discussing educational issues, but it also appears that the exposure to the Pirie-Kieren theory gave her 'food for thought' and prompted her to think in greater detail about what is involved in the processes of teaching and learning.

Sophia was able to consider her Images of both teaching and learning, act on them, and discuss the interactions after the fact. In summary, Sophia left the teacher education program with some Formalising and even thought about the interplay between the teaching and learning aspects, entering the Observing level of understanding. She had entered the SMIP program with some Images of teaching and learning mathematics. She left it with many more Images on which she was actively working.

7.3. Analysis of Lance's portrait

7.3.1. Classifying Lance's levels

Lance entered the teacher education program unsure of his Images of teaching and learning but, realizing that his own teachers had been uninspiring (IH) **(1)**, he hoped to make mathematics classes more interesting than they had.

Since he viewed mathematics as logical and ordered, Lance's initial Images of learning mathematics included the idea that it required high-level thinking **(2)** *"... learning mathematics requires higher level thinking than many other subjects..."* practice, focus and patience (IH) **(3)**. *"Learning mathematics requires patience, focus and practice."* As well, he thought that learning mathematics would be difficult for some students but that for others understanding could occur automatically (IH) **(4)**. *"... mathematics may seem strange and foreign to kids until there is a sudden insight."* Teaching mathematics therefore would involve guiding students in a linear manner (IH) **(5)**. *"... can be proven right or wrong through a series of steps ... it is the teacher's job to guide and educate my students..."* using detailed plans (IH) **(6)** *"... thoughtful, precise, you know, detailed lessons..."* and the need to think about one's own learning experiences (IH) **(7)**. *"I'll have to put myself into their --- the student's shoes and try to remember how I learned the concepts..."*

After doing a short presentation to the class, Lance engaged in some discussions regarding the development of understanding of mathematics (IM) **(8)**. Concerned that the SMIP group, as able mathematics students might not be able to anticipate problems, he thought about the way he learned **(9)** and how he prepared when he had to train people on the job **(10)**. *"I was a learn by doing kind of guy who likes to figure things out for myself ... planned my lessons on* ***my*** *understanding and the manner in which I learned to do the work."* This reminded Lance of a metaphor that he had read about, and to which he could relate. Learning mathematics was compared to building construction (IM) **(11)** which led to the Image that a solid foundation on which to construct more knowledge is needed in mathematics. *"Learning mathematics is a bit like constructing a building ...a building must have a strong foundation, but if you don't fill it, it's useless - and if the foundation is not strong, it will collapse. So that's like math."* Based on this thinking about learning, Lance thought of teaching as providing the foundational knowledge (IH) **(12)** and then directing the learning (IH) **(13)**. *"Teaching mathematics involved directing student's learning - create a strong structure so ... you know ... more learning can take place"*

Lance gave thought to personal interactions, *"Think about Armie. Here she is all the way from Iran..."* and this prompted him to discuss a number of natural occurrences of mathematics. "*The multiplication of rabbits is a series, shells are spirals,... Even linguistics is structured mathematically..."* He then questioned the basic nature of the subject (IM) **(14)**. *"...is math created as a product of our environment, as a way to relate? If we grew up on the moon with no gravity, would that affect it ... Could we have gotten to this point without math?"*

Watching the same video of Alwyn that had provoked Sophia's thinking also prompted Lance's thinking, but in a different manner. Lance concentrated on the teaching technique used (telling students that they were intelligent) (IM) **(15)** and he considered how this might affect student learning. *"... he kept telling them that they* ***must*** *be smart if they could learn this stuff 'cuz it was hard. Maybe telling them that they are smart is a substitute for relevance..."* Later, he re-iterated that learning mathematics requires foundational knowledge (IH) **(16)** and further understanding is built on what is already known (IH) **(17)**. *"... everything we know and do in mathematics is based on what we already know . You can't go ahead unless you already have a place to start."*

Listening to other SMIP students' discussions about their own learning experiences and watching their presentations to the class prompted Lance to Fold Back to his own learning experiences (IM) **(18)**. *"I realize how much I don't know because - I keep analyzing - but I don't think I ever had a teacher in high school that ever, you know --- that used a questioning style or that --- well, I rarely remember working in groups as a project - they actually discouraged it because they saw that as a way for kids to cause trouble --- just do your work and make sure the desks were far enough apart."* Using the discussions of the SMIP class related to keeping students involved and getting them to develop their own understanding, as a basis for his thoughts, Lance re-examined his understanding of teaching (IM) **(19)**. "... *talking in these groups in SMIP about what everybody did in school and seeing all these different teaching techniques that are effective ... I'm learning - keeping kids involved, getting them to learn without being told - working together, hearing information from all sides - you just* *naturally absorb."* Lance acknowledged these as possibilities (IM) **(20)** for his repertoire of what it means to teach.

Lance moved out to work at Property Noticing **(21)** and, he realized that based on his own classroom experiences, he thought of teacher as lecturer (IH) **(22)** and that he probably would have taught that way had he not heard what the other SMIP students had to say. *"I'm sure if I'd gone out to teach I'd have been just like my teachers."* He realized that his developing Image (IM) **(23)** in the SMIP class was of teaching by helping students see the relationships for themselves. *"I had to try to figure it out myself and we are learning to help students figure it out for themselves so they can remember - comes from your own head rather than someone telling you and that's what I always wanted."* However, in his own presentation, he showed the students the solution to his question, reverting to his firmly held Image of teaching as telling (IH) **(24)**. He was, however, quick to pick up on this in his discussion of the lesson and he thought about his own teaching methods and those that he had observed and related these to student learning, realizing that he had not incorporated the Images he had stated (PN) **(25)**. *"You know, I just told you guys how it worked and I know this is not the best way for students to learn. We talked about questioning and all that. You know, I guess I'm just used to --- I guess I need to work on this - not telling, but trying to get the class to figure it out for themselves, It's really hard to break habits."*

By Folding Back to his own experiences and realizing that he was a visual learner, Lance also realized that he was learning to teach by observing others' presentations (IM) **(26)**. *"I suppose I'd also describe myself as a visual learner. ... That's probably why I'm learning a lot about teaching by watching you guys present."* He was concerned about student-teacher interaction and how a teacher's actions could affect student learning (IM) **(27)**. *"How will a kid feel if the teacher cuts him off or did a question differently from the way the kid suggested? ... and how would the kid who thought he knew what he was doing didn't have the teacher, you know, accept it. Would he think he was wrong or what?"* After some mulling through his thoughts, Lance decided that learning is based on teaching (IH) **(28)**. *"It may not be the student's fault that he/she doesn't know everything - it may be the result of previous, poor teaching."*

In a general discussion, Lance revealed his Image of teaching as requiring commitment and as important to society (IH) **(29)**. *"I think teaching takes commitment. And, it's more beneficial to society than my other jobs."* Again, he Folded Back to his own learning experiences (IM) **(30)**, *"In school ... I was, you know, stifled by being 'told' what to learn"* and to discussions related to the Pirie-Kieren theory, *"Like Dr. Pirie said, I could maybe* ***do*** *the work, but I wasn't understanding"* and, thinking about how grading had affected his learning, refined his idea of teaching to consider it as possibly helping students learn to think (IM) **(31)**, *"... and maybe that's the way it should happen - not focusing on marks in school, but on discussion, getting the kids to think about what they do..."* He clarified his Image of learning as making connections (IH) **(32)**. *"Its important to get kids to 'look at this and this' and try to make the connections - try to digest and so to go through a process".*

The imminent short practicum brought out three more of Lance's Images of teaching: 1. Teaching requires organization **(33)**. *"Teaching involves a lot of organization..."* 2. Teaching involves telling **(34)**. *"I will, you know, have to express my knowledge."* and 3. There is a best method to use in teaching (IM) **(35)**. *"I want to find out from all you guys the* ***best way*** *to do things..."* The impending practicum also brought out two, somewhat conflicting, Images of learning: 1. There is a right way to learn **(36)**. *"I taught myself how to subtract in elementary school by adding on ... But then I had to learn* ***real*** *subtraction 'cuz that wasn't the right way ... If you learn something wrong, it will be hard to get back on track ... you'll have to re-learn it and that's hard."* and 2. Learning is as an individual mental activity **(37)**. *"Learning takes place in the head and if you figure it out yourself it will make sense and you will remember it."*

During the short practicum, Lance was actively involved in Image Making, attending a number of classes and thinking about the teaching that took place **(38)**. *"I watched a lot of classes that were, you know,* ***right out of my*** *league --- band --- textiles --- wow --- but they had to have different techniques ... a great example in a mathematics class where the teacher used graphics calculators ... This really worked, and you know, I'll probably use this with my own classes."*

Discussions about elementary school mathematics teaching provided another Image Making experience for Lance **(39)**, and he questioned what his understanding meant if he could not Fold Back to the elementary level. *"We are beyond that... but doesn't that mean we should be able to ... retrieve it?"* Working with manipulatives also provided an Image Making experience for Lance

(40), *"I liked working with the algebra tiles the second time. I was able to think more about how they worked."* and led him to the Image of learning/understanding as figuring out for oneself **(41)**. *"I tried to figure out how I would do it. Then, when Elisa demonstrated her method, I **saw** that it works, but I **understood** mine."*

Lance questioned the learning of mathematics as being from abstract to concrete or vice versa (PN) **(42)** and he concluded that teaching by allowing students to figure it out themselves is too time consuming (IH) **(43)**. *"It will take too much time if the --- the kids all have to figure it out themselves."* Working at Property Noticing **(44)**, he realized that teaching should not be telling and there must be some way to teaching that involves some telling and some figuring it out. *"I don't want to just give them rules ... some way so they can learn it without you telling them everything, but not so's they have to do it all themselves?"*

Lance's involvement in a group presentation allowed him to practice a new Image of teaching, the Image of the teacher being actively involved **(45)** and the discussion following it was used as an Image Making experience **(46)** as he questioned his own presentation. *"Do you think the diagram should be more real? ... In a real class do you think it should be more real?"* He concluded that teaching involves leading students in the right direction (IH) **(47)**. *"I think that that is what I would really do so's not to lead them astray."*

The presentation by Dr. Towers caused Lance to once again wonder about the effect of a teacher's reaction on a student (IM) **(48)**. *"So was that a good learning experience for that girl?"* He was also concerned about the effects of testing on students (IM) **(49)**. *"And then people talk about testing as a good learning tool. Do they mean the actual test or the preparing?"* Lance concentrated on his own experiences (IM) **(50)**, *"...like I said earlier, I used to just play with the answers, plugging them in, to see which fit the question. This was not a learning experience! So, what would make a good learning activity?"* and, although maintaining his previous Images of teaching (IH) **(51)**, he concluded that learning should involve active participation (IH) **(52)**. *"A good activity should make the students **want** to play with it."* He also thought that learning should lead to Formalising (IH) **(53)** *"For a lot of activities, --- I ---- it would be hard to make the leap from the activity to the abstract"* and that students would have a difficult time reaching that stage (IH) **(54)**. *"I'm not sure how I could get the kids to do that - they'd have the **picture** --- uh --- Image, but I don't know if they could switch."* Lance's Image of teaching further included that it involved time-constraints (IH) **(55)** *"What if your lesson takes too much time? Not enough time?"* and making sure that the students arrived at the correct conclusion (IH) **(56)**. *"How can you make sure those open-ended questions will get to what you want them to --- you know, will the kids --- will they learn what you want them to?"*

At the beginning of the long practicum, Lance indicated that he believed (IH) that teaching **(57)** and learning **(58)** involved the participants having respect for each other *"Respect --- mutual respect is what I expect"* and he re-iterated his Image that learning mathematics involves building on previous understandings (IH) **(59)**. *"To learn mathematics, you always have to rely on what you already know."* His initial class modeled his Image of teaching as ordered and logical (he reviewed section by section) **(60)** and that teaching involved determining a student's understanding and building on that (IH) **(61)**. *"Okay, what do you know about areas? ... What do you know about that?"* Lance followed students' leads, including their terminology, *"Good going, Right, the skinnier the rectangle, the greater the perimeter."* modeling his Image that teaching involves allowing students to learn (IH) **(62)** and that learning occurs through building on one's previous understanding (IH) **(63)**. "*You can't do more math unless you understand --- know what you're doing."* He also modeled his Image that learning can take place in different ways **(64)** by allowing several students to explain their method of doing the same question and his belief that learning requires active participation **(65)** by providing students with an activity to do at the end of the class. *"This activity will teach you about pi."*

When Lance tried to use an after class discussion with his sponsor teacher as a time to review and to consider different aspects of teaching, the sponsor teacher clearly indicated that he expected Lance to follow the textbook. *"**Everything** you need is here."* Lance was therefore encouraged to continue with his structured approach to teaching (IH) **(66)** rather than to try something new. He did, however, put into practice one of his new Images, that of learning as involving active participation, by encouraging group activity (IH) **(67)**. *"... you can work together. You don't have to work alone."* When there was a problem, he practiced the Image of teaching as caring for the well being of the students (IH) **(68)**. *"I don't want to talk too much about it but there has been a problem with a student - a personal problem, and she did some harm to herself. I'm afraid if* *you're there with the camera the class might react."* He also revealed a new Image, that learning should be fun, by bringing jokes into the class (IH) **(69)**. *"I've gotten into the routine of telling silly, innocent, one line jokes."*

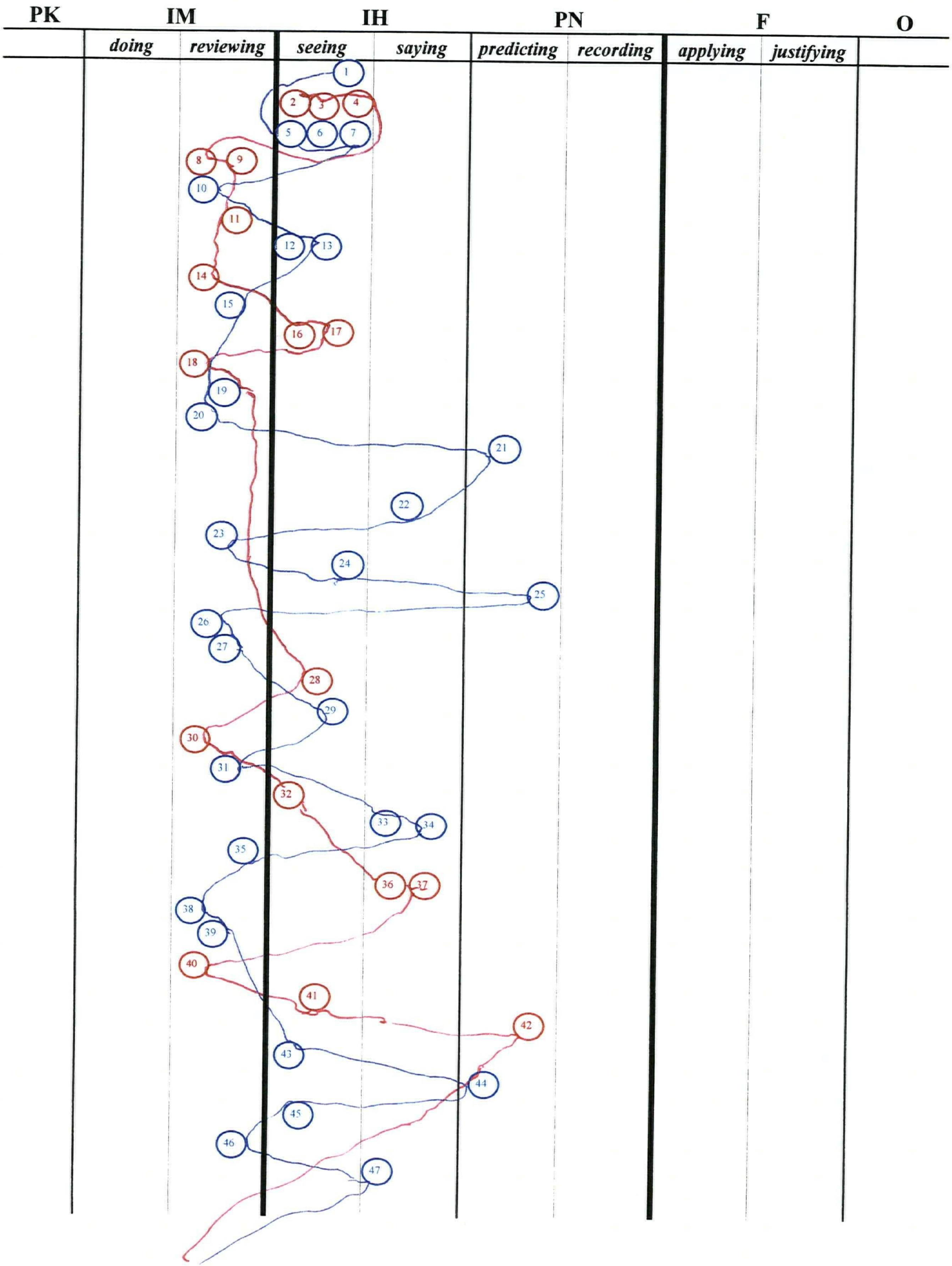
In an after class reflection on his teaching Lance thought about the activities and he began to think that ideally students learn by developing their own understanding (IM) **(70)**, *"Rather than teach it directly..."* and that other students might help them learn (IM) **(71)**. *"I tried to let Zao explain because she's pretty good..."* However, his actions and instructions indicated that he had not given up the Image of teaching as telling (IH) **(72)**, *"I like to give them instructions on paper"* and that he did not entirely trust that students would be able to develop their own understanding (IH) **(73)**. *"I ended up having to explain it myself."*

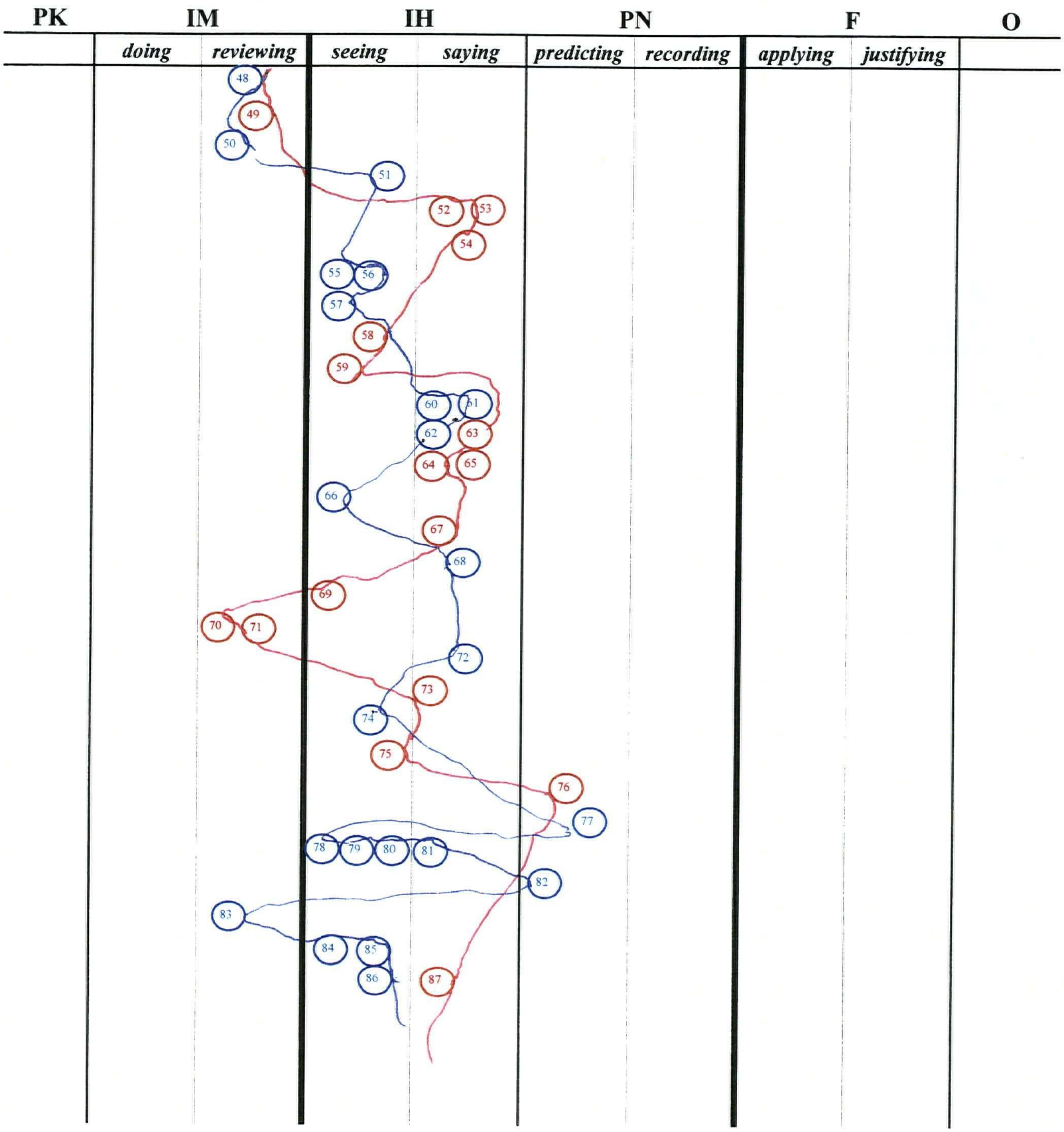
The Images of teaching as time-constrained **(74)** and of learning as requiring time and practice **(75)** created frustrations for Lance, as he noticed the difficulties these different expectations raised for the students (PN) **(76)**. *"But it gets frustrating with the pace I have to go .... they don't even have time to think and absorb what they need to learn. --- But they need time to make sense of the whole thing."*

Viewing the video of himself teaching did not Invoke Lance to think about that particular lesson but the discussion that followed revealed many of his conflicting Images. Frustration over not seeing anything innovative in the school (PN) **(77)**, *"I didn't see any of my sponsor teacher do anything differentish."* and feeling stifled by his sponsor teacher *"I felt that working like that really limited me. You know, put all sorts of constraints on me.... I always felt that I couldn't spend* *time 'cuz Mr. Forsyth said to teach what was in the book."* left Lance feeling unfulfilled and again raised the problem of time-constraints in teaching (IH) **(78)**. *"I just passed over it. ... partly because Mr. Forsyth ... said I shouldn't teach anything else..."* Thus, while he had worked at Property Noticing with respect to teaching he was unable to reconcile his new Images with his old. He then revealed a new Image of questioning as being important in both teaching **(79)** *"I got down the questioning a bit and didn't always just tell them what to do"* and in learning. *"I encouraged the student to ask questions"*

Lance concluded his discussion with an Image of teaching as being fundamentally different from other professions **(80)**. *"Teaching is really different from other jobs. You're just thrown in the fire and away you go."* He also thought that there is not enough observation of other teachers involved in the teacher education program, confirming his earlier Image that he was learning how to teach by observation **(81)**. *"I'd like to watch more differences and then try them out."* Thinking about his experiences and the Pirie-Kieren theory, Lance realized that teaching requires subject content knowledge, pedagogical knowledge and pedagogical content knowledge (PN) **(82)**. *"I learned that being a teacher --- knowing how to teach ... well, it's more than knowing your subject matter --- even more than understanding it --- teaching math, well, you have to know the math and know what the kids will think like in the P-K theory. It makes sense. You can't just jump in at any old point. You know, you have to have all the knowledge -- and be able to Fold Back - can't Property Notice unless you know some properties."* Asking himself: "What does teaching mean?" (IM) **(83)** Lance concluded that teaching involved leading students in the right direction (IH) **(84)**, *"I think it should maybe involve leading of students in the right direction"* listening to student's questions and input (IH) **(85)**, *"... but also, listening"* and discussing student understanding and misunderstanding (IH) **(86)**. *"And if they have a wrong idea, I think you need to discuss that so they can understand what is wrong."* To Lance, this later Image was an important aspect of how students learn as was having the requisite prior knowledge on which to build understanding (IH) **(87)**.

7.3.2. Charting Lance's growth

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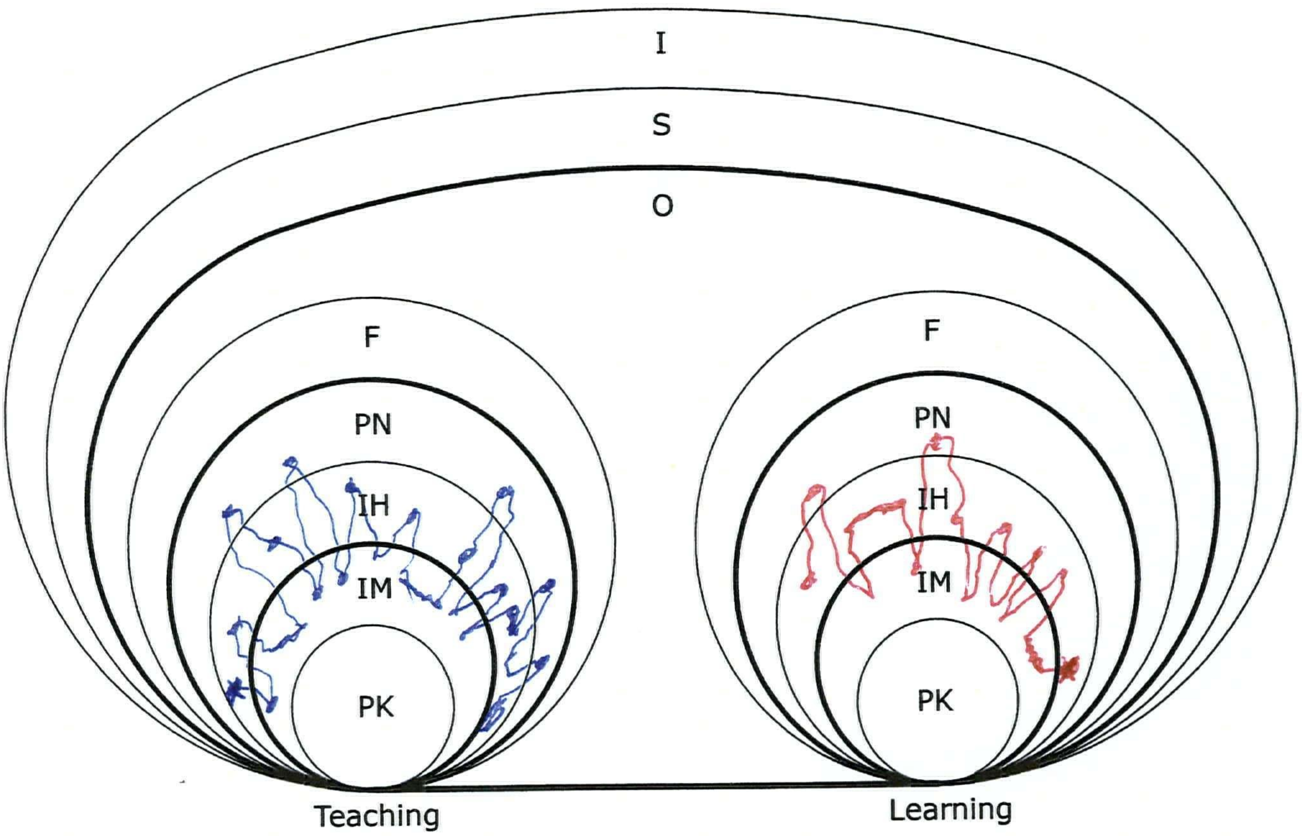


Figure 9: The Mapping of Lance's Growth of Understanding of Teaching and Learning

7.3.3. A discussion of Lance's growth

Lance's portrait revealed a young man who had made a decision about what he wants to do with his life. He had decided to be a teacher and he entered the program questioning what that really meant. The over-riding themes of his understandings about teaching and learning at the beginning of the program would be of teaching as being a very organized and well-planned activity in which knowledge is presented to learners, and of learning as requiring individual effort on the part of the learner to make meaning of the presentation. These images were based on his experiences with lecture-type teaching and his own need to think through concepts to make sense of them.

During the teacher education program Lance could be described as the thinker and the questioner. Like Sophia, he used classroom activities as catalysts to think about teaching and learning and in this vein, he spent a fair bit of time working at the Image Making level and some time at the Property Noticing level of developing understanding. He was open to the ideas of others and often seemed to make connections or to question his own understanding based on some unidentified prompt.

His past experiences appeared to have affected his developing understanding of teaching and learning in that his Image of teacher as lecturer seemed firmly embedded. This was possibly due to the fact that this was the type of teaching he had experienced and it was probably the type of teaching his father, as a University professor, described.

An important event for Lance was when he realized that he had enacted his traditional Image of teaching-by-telling when he told the SMIP class how he had done a question rather than getting their input and letting them figure it out themselves. From this point, he consciously tried to enact some of the new Images into his activities. However, while he worked at Property Noticing, he was unable to reconcile his old Images with the new and tended to fall back to his comfort zone, so that, although he realized that possibly the new Images would create a better teaching and learning environment, he was not to able to entirely break from the old method.

An intervention by his sponsor teacher early in his practicum placed Lance in a compromised position and possibly further limited his development. The sponsor teacher indicated that Lance should follow the text as curriculum. Thus, since Lance's newly developing Images were tentative, mostly unpracticed and contrary to his traditional Images, many still being at the Image Making (Reviewing) level, he was not secure enough to put them into practice and he followed the text as guide. In his post-practicum discussion, he indicated that the directive from his sponsor teacher did affect his implementation of new ideas as he did not feel that he was able to go on the tangential leads of the students or that he could explore areas of interest. However, it should be mentioned that he did incorporate some of his newer Images of learning by involving students in activities and trying to build on their understanding, working at Property Noticing, both levels.

Lance seemed concerned about student-teacher interaction and how it might affect learning and mentioned it on several occasions. For example, the same video of Alwyn that had prompted Sophia to thinking about conceptualization of concepts, prompted Lance to think about different ways of teaching and learning and he wondered if a teaching technique could affect learning. The presentation by Dr. Towers also prompted him to think about this relationship.

In the post-practicum discussion, Lance indicated that not seeing any innovative teaching practices at the school, and being told to follow the textbook had disempowered him and therefore he was not as creative as he had hoped. During this discussion, he revealed a new Image of the importance of questioning in teaching, and he worked at the Property Noticing level while discussing the complexity of the process and enactment of teaching. He realized that there are many different types of knowledge that are required of a teacher. He left the program still uncertain about the meaning of teaching and learning but with many new Images with which to work, and an understanding that teaching and learning could be enacted differently than the manner in which he had experienced them.

7.4. Analysis of Ellie's portrait

7.4.1. Classifying Ellie's levels

Ellie entered the teacher education program with an Image of teaching mathematics as teaching that required true understanding **(1)** as opposed to teaching other subjects where opinions could suffice. *"The concepts are more abstract and require **true** understanding (not just opinions) ... there is more of an onus put on math teachers to really understand the subject."* This Image was based on her understanding of mathematics as an objective subject that was abstract, has a process and a correct answer. *"When I think of math ... it seems more abstract... it is objective and logical ...**most questions have just one answer...**"* She also felt that most people had an emotional reaction to mathematics and that to learn it, one had to have a positive attitude (IH) **(2)**. *"I think that students have to feel good about what they are doing."* Within this Image, the teacher was responsible for developing (modeling) the student mathematically **(3)** and this implied that he/she had to understand his/her students **(4)** *"... the teacher must know how to work the clay -- the kids..."* and that practice was important to learning (IH) **(5)**. *"... because they need to practice in order to learn..."* This Image encompassed the Image that poor teaching could lead to lack of understanding and a negative predisposition to learning mathematics (emotion is involved in learning) (IH) **(6)** and that the onus was on the teacher to ensure that correct learning took place (IH) **(7)**. *"You can't really do a delicate model without a competent potter regardless of how good the clay is."*

After the presentation of the Pirie-Kieren theory and during discussions that took place in the first few weeks of the SMIP class, Ellie elaborated on some of her Images, reconfirming her Image that the ability to learn mathematics is tied to emotions **(8)**. "*... most students have **emotions** about mathematics...*" She believed that mathematics can be learned by everyone (IH) **(9)** providing the teacher explains properly, and patiently guides students through their difficulties (IH) **(10)**. *"It's the teacher's job to guide the students through obstacles and processes, building competence in them..."* The ensuing discussions prompted Ellie to Fold Back (IM) **(11)** to her own learning experiences *"Math has always just **been there** for me..."* and, talking about the Pirie-Kieren theory, she thought of what it might imply regarding her understanding of teaching mathematics (IM) **(12)**. Thinking about the tenets of the theory Ellie, for the first time, realized that students would learn mathematics differently than she did (IM) **(13)**. *"I **really** never realized that students might think of these things differently than I do."* From this discussion, she began to realize that in order to effectively teach, a teacher needed to understand something about the students' backgrounds (IH) **(14)**. "*But, Dr. Pirie's discussion of Primitive Knowing really made me realize how little we as teachers can assume about our students...*" The discussions on the theory also prompted Ellie to explore the difference between being able to do mathematics and being able to understand mathematics (IM) **(15)**. "*... but I had never fully grasped the fact that just because a student can **do** what the teacher asks, doesn't imply that he understands --- or the opposite...*" She concluded that being able to do was an essential element of understanding or learning (IH) **(16)**. *"I really don't think a student can --- succeed in mathematics unless he can do it."*

Ellie's own presentation to the SMIP class elicited positive comments from other students. Using these comments, Ellie Folded Back to her Image of how she thought she had learned mathematics, revealing the Image that learning implies that one needs to know what errors had been made **(17)**. *"But **critique** me! You know it wasn't all good. I want to learn --- to improve. If all you tell me is what went good, it won't help."* Prompted by the comments of another student, *"Ellie, **you** said that it is the teacher's responsibility to make every student feel special ... you must find something good to say about every student's work."* Ellie began to think about her own learning and how she related to other people (IM) **(18)**. *"... well, I like to find good things about others but in myself, I look at what I didn't like."* As a result, she incorporated into her Image of teaching the possibility that teaching involves building on what is known/good **(19)**. *"I never thought of taking what went good and building on it..."* Ellie re-iterated that all students can learn mathematics (IH) **(20)** and that it is the teacher's responsibility to ensure that this happens (IH) **(21)**. *"... every student can achieve at some level and it may just be in the way the teacher works with them."* She identified two types of learners; those who like to see the 'big picture' and those who should be guided through 'baby-steps' (IH) **(22)**. These therefore implied the need for different styles of teaching to accommodate them. *"... so maybe for them - the not big picture ones - you shouldn't start with generalization, but show them step by step first..."* For each group, however, teaching should involve summarization by the teacher (IH) **(23)** *"But for sure, generalize at the end. Sort of at the end of each step or section"* to ensure that students have the proper languaging (IH) **(24)**. *"... providing the correct math terms..."* and it is the teacher who is responsible to ensure that the learning takes place (IH) **(25)**. *"The teacher should be sure they have it."*

Discussions around "What is mathematics?" and engaging in problem solving activities prompted Ellie to some active thinking about what is involved in learning mathematics. These discussions also prompted her to think of the role of problem solving in teaching and she allowed that teaching can be accomplished using these activities (IM) **(26)** *"Problems may work, though ... problem solving is likely a good way to teach some things..."* but she held firmly to the Image of learning as applying the correct algorithm **(27)**. *"... you should be able to determine the procedure..."* She thought about critical thinking as being a part of mathematical thinking, and therefore sometimes the answer was not as important as the ability to justify one's methods (IM) **(28)**. *"... justification means giving good reasons for your work and it is a critical part of mathematical thinking."* However, she retuned to the Image that correct application of an algorithm is the aim of the logical process of learning **(29)**. *"This is part of being able to break things down so that you can understand them --- part of the logical process and being able to properly apply an algorithm."*

In discussing testing and assessment, Ellie noticed that students' problems may create opportunities for the teacher to learn about student thinking (IM) **(30)**. *"Those dead ends --- though --- they may not be* ***useless*** *because they* *can help the teacher figure out the student's thought processes..."* However, considering an aim of teaching as completing the curriculum (IH) **(31)**, she realized that there was a conflict between teaching time (teaching is time constrained) and the fact that students need time to learn (IH) **(32)**. *"There isn't going to be time to re-teach and re-test. --- but --- it seems unreasonable to go on if the students don't understand."* This conflict, for Ellie, created a dilemma. *"Learning is a work in progress and knowing how each student learns and what came before is important, helping them figure out where they went wrong."*

Thinking about her own positioning (level) on the Pirie-Kieren model while being engaged in problem solving activities provoked Ellie to more Image Making **(33)** and the Image of teaching as involving good questioning emerged **(34)**. *"But, now I'm seeing ways of asking that get at understanding..."* As well, Ellie re-iterated the Image of the possibility of students using different methods **(35)** *"... hadn't really realized that other students might, sort of, take a different approach."* However, as she worked at her own understanding, thinking about the uses of problem solving in teaching (PN) **(36)**, she could not rationalize the use of open-ended problems as a teaching technique. She concluded that teaching required completing the curriculum (IH) **(37)**. *"Here in SMIP we can explore ... but in the high school we have a curriculum to cover ... We have to get it done – the curriculum."*

When Ellie presented her own lesson to the SMIP class, she revealed an Image of teaching as ensuring that students have the correct background knowledge **(38)**. *"First of all, I'll step back and review the stuff you'll need to do* *this 'cuz you haven't worked with this material for a while."* She indicated that teaching should allow for student input **(39)** and again she noted that she realized that students learn differently (IH) **(40)**. A new Image emerged when Ellie presented another question: teaching can involve providing leads or hints as to how students might approach the problem **(41)**. *"How can this be? ... Look at the slopes of the parts. That should tell you something."* Again, she indicated that practice was important to learning (IH) **(42)**. *"Now finish it for homework and we'll discuss it next day."*

Ellie's Image of teaching included the fact that the teacher should have control in the classroom **(43)**. *"... we really need rules. You have to have some control ..."* This prompted her to think about the complexity of the school environment and the rules that are involved (IM) **(44)**. *"There are a lot of things like that that you have to know, but how do you learn them all?"*

During some final presentations before the short practicum Ellie re-confirmed the Image of teaching as providing hints **(45)**. *"Try this --- try a smaller number to see what happens, and look for a pattern ... It makes you have to try to determine some approach and using a smaller numbers is often a good way to work on a problem."* She indicated that learning involved looking for a pattern or formula (IH) **(46)**. *"Watch for a pattern and lets see if we can find a formula."* Assigning homework reconfirmed the Image of learning as requiring practice (IH) **(47)**, and while discussing presentations Ellie revealed her Image of teaching as time-constrained **(48)**. *"... if this was a real class we wouldn't have time for all that exploring..."* Thus, although learning might involve trying different approaches (IH) **(49)**, *"In working on a problem, I go different routes and try different methods..."* teaching should be an organized presentation providing reasons for each step (IH) **(50)**. *"I like the way Armie was so organized and wrote out all the reasons"*. Ellie concluded that teaching should be linear (IH) **(51)**. *"The one I teach should be straight."*

After the short practicum, Ellie referred back to it several times, indicating that it had been an Image Making experience **(52)** and it provided her with some conflicting Images of teaching. She questioned the value of having students explain their work because of the effect it might have on their understanding (IM) **(53)**. *"I'm not sure about having students explain their work ... because, you know, kids are never asked unless they sort of got it wrong..."* As well, she revealed the Image of teaching as showing concern for students **(54)**, *"I felt really bad for her."* and the Image of learning as being irreversible **(55)**. *"I think I may have confused her more and now she'll never get it."*

Ellie actively involved in Image Making **(56)** when she asked the other SMIP students questions about how they would teach certain topics, *"How would you teach linears to grade nines...?"* and she worked at Property Noticing **(57)** when she considered the Images that students might develop through different presentations. *"What Image would they have if they thought of it that way?"* Her Image that learning is tied to emotions re-emerged when she worried if students would form negative Images **(58)**. *"I really don't like the idea – the Image that kids get when you talk about negative numbers as being bad."* Ellie thought about the fact that the teacher needed to understand the method being used (IM)

(59), *"I have to know if I'm comfortable with that way – see if it works for me – 'cuz if I can't make sense out of it I sure won't be able to explain it to my students!"* and the Image of learning as knowing the abstract **(60)**. *"I can use the manipulatives. I mean, I can figure out how they work 'cuz I already* ***know*** *the stuff ... the* ***real*** *math."* She realized that this was important to her.

Further presentations and discussions about them revealed that Ellie continued to feel that teachers should be concerned about students' feelings (IH) **(61)** *"How did that make you feel, David?"* and, she wondered how teacher interaction might affect student learning, expressing concern that the teacher might stifle the students' initiative (IM) **(62)**. "*They might have had a good idea and it might have been really creative but they but they wouldn't develop it because we cut them off"?* Thus, she indicated that the teacher had an important role in learning, or that he/she was responsible for learning (IH) **(63)**.

Preparing for a presentation and discussing it with the class prompted Ellie to think more about learning. She thought about how she had developed understanding (IM) **(64)** and decided that her method of using simpler examples would be good for the students (IH) **(65)**. *"I can't believe how much I had forgotten ... but this question really made me think about it. --- using a simpler case --- that's what helped me understand and I think that it might be a good idea to suggest this to the students."* She discussed the processes she was using and those she saw others using and thought about how many different ways of presenting material there are (PN) **(66)**. *"...these exercises force me to think about it and not only am I remembering it, I'm getting all sorts of new insights ...*

come up with some really neat ideas ... all the ideas and different ways people think about stuff ... this makes you think about how to teach it so the kids can get it..." She then indicated that teaching was helping students see the pattern (IH) **(67)**. *"... so they can see the pattern you can."*

Being comfortable working with technology, when the SMIP class used computers and calculators in class, Ellie revealed that she felt that it was the teacher's role to model proper usage of these devices (IH) **(68)**. *"... but if the teacher just uses the calc, for example, to get the answer, that's what the kids will do."* By helping fellow SMIP students with their problems, she also revealed the Image of teacher as helper **(69)**. She questioned what the consequences would be if the teacher didn't have all answers at hand (IM) **(70)**, *"What if the student asked me something like that and I couldn't answer? ... And what would the student think?"* but, since she realized that students might think of things differently, she also realized the impossibility of having all answers at hand. *"But there are a lot of things that can be done differently ... and you haven't even thought of that."*

The presentation by Dr. Jo Towers shortly before the long practicum was an Image Making experience for Ellie **(71)**. After the presentation, she discussed her reactions, and again realized that learning can take different routes (IH) **(72)**. "*Students may do this --- sort of --- might see the question differently..."* She realized that students can take different viewpoints and develop different understanding than the teacher had intended (IH) **(73)**. *"I just realized that just 'cuz a kid says something or asks something that seems on a tangent, she might* *not --- well, she might just be really interested and have thought of something beyond what you were going to teach."* She realized that the teacher cannot be aware of all that happens in the classroom or in the students' minds (IH) **(74)**. *"... they're not doing what you want and so you might not see what they're doing."*

Observations of Ellie during her practicum indicated that she practiced many of her stated Images. During her lessons, she was always friendly, polite and positive, smiling and re-assuring students (IH) **(75)**. *"Don't worry. It'll be okay. ...I want you to treat others like you want them to treat you."* She modeled her Image of teaching as providing the necessary background when she realized that a number of students had forgotten some necessary information and she reviewed it for them (IH) **(76)**. *"How many of you don't remember how they work? Oh, a lot of you. Okay, we'll just quickly run through them."* Similarly, she modeled her Image of involving students by asking them for the answer **(77)** *"So, Ali, what did you get"?* as well as by having them explain their results **(78)**. *"Can you explain how you got that?"*

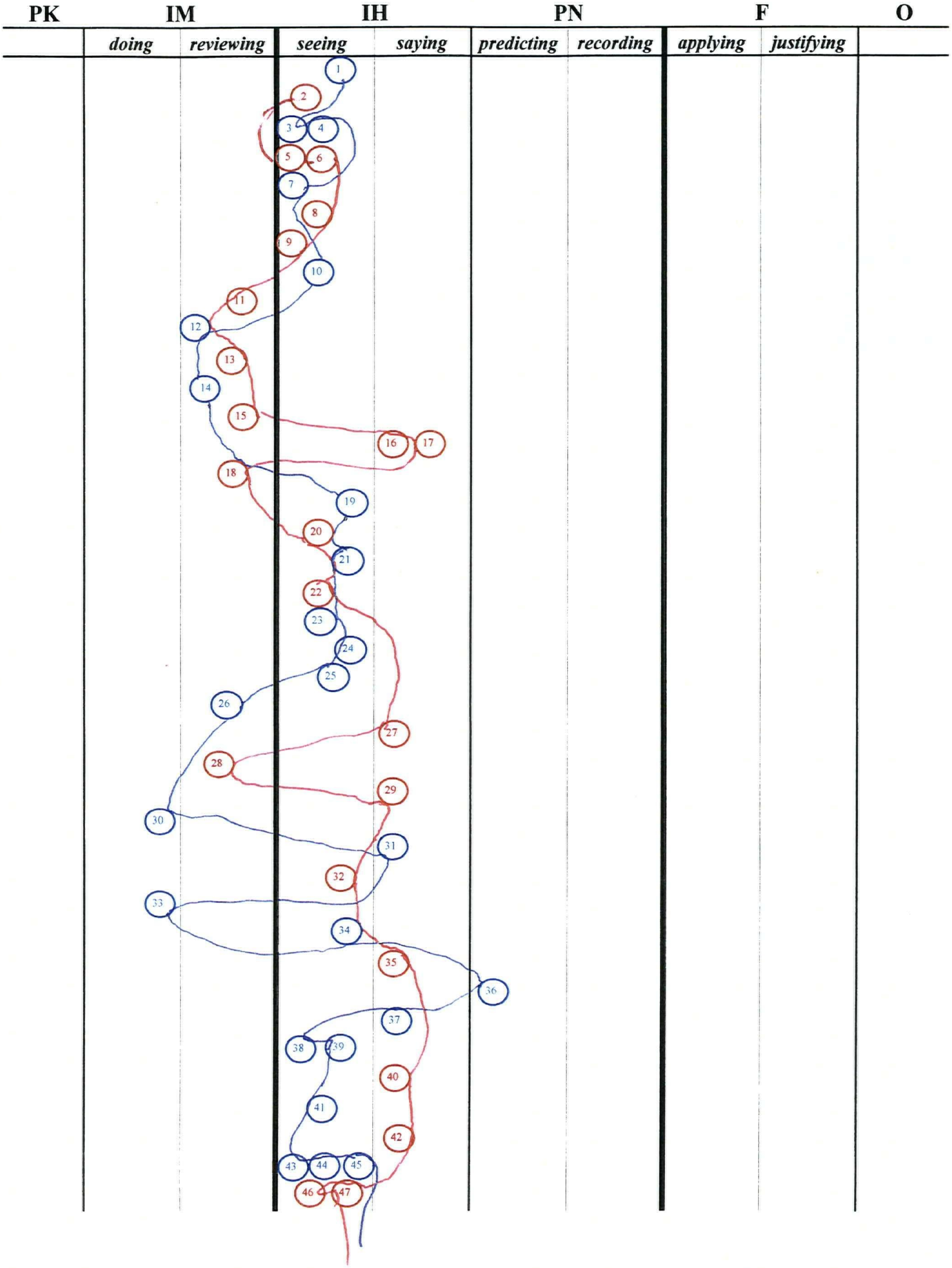
In discussions with her sponsor teacher, Mr. Smeel, who thought that perhaps she was too theoretical, Ellie indicated that she thought students learn by figuring out the process themselves (IH) **(79)**. *"I wanted - was trying to get them to figure it out themselves."* As she explained some aspects of the Pirie-Kieren theory to Mr. Smeel, she worked at the Property Noticing level, bringing together her ideas of teaching and learning to try to explain why she was teaching in a specific manner. She explained that she wanted to use tenets of the theory as a model to help students develop understanding rather than just giving them formulae (PN) **(80)**. *"If kids figure it out for themselves, they will remember it better --- this thing about then they can Fold Back to it if they forget 'cuz they'll have it in their heads, but if you just tell them --- I think that was Formalising --- ... and how the Images are important 'cuz that's what they need to Fold Back to..."*

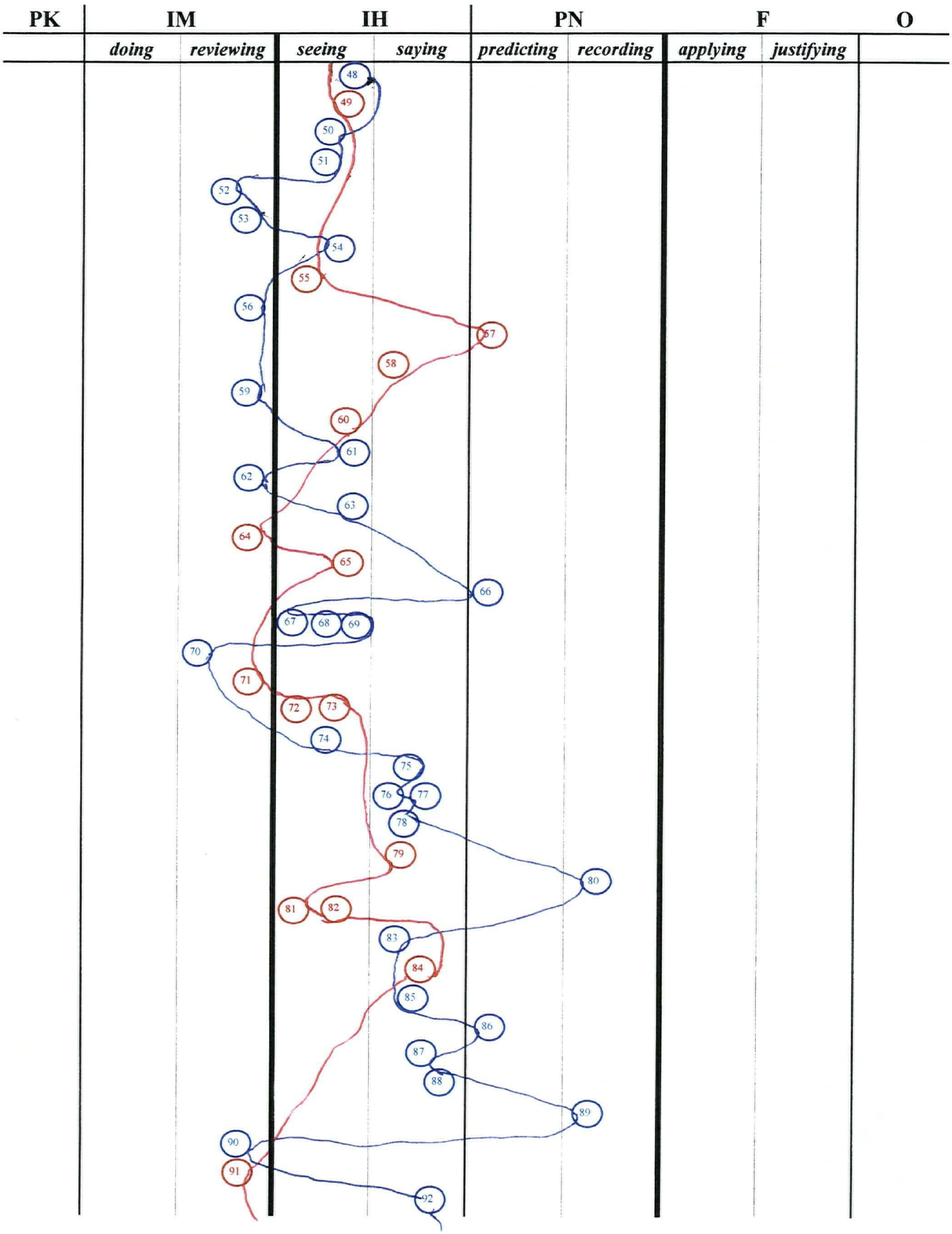
Ellie modeled her Image of learning as knowing what you did wrong (IH) **(81)**. *"If you don't go over the questions that you got wrong, how can you learn?"* The Image of learning as occurring in a collaborative atmosphere **(82)** and that she felt it was important to understand students' backgrounds **(83)** were clear when she explained her responses to students to her sponsor teacher. *"Amy --- well --- she's vulnerable. ... need to let her feel a bit of success - not picked on."* She also indicated an understanding of different styles of learning (IH) **(84)**. *"Art is really quite intelligent. He asks good questions and just has --- test phobia? --- so I want to give him a chance to say things so he knows what --- that he can do it."* The time constraints on teaching (IH) **(85)** were frustrating for Ellie, *"I really think they needed more time multiplying before we went into factoring ... and of course, time."* and she was also frustrated by the fact that her sponsor teacher directed the manner in which she was to teach, not allowing for another method that she thought could lead to better understanding (PN) **(86)**. *"I'd like to show the - try factors of the first and last term..."* Again she indicated that she believed that it was the teacher's responsibility to ensure correct learning occurred (IH) **(87)**. *"I'm somehow not giving them the right images."*

When Ellie reviewed the video of herself teaching, she revealed that she felt constrained by the authority of her sponsor teacher and the need to accommodate his wants within the classroom (IH) **(88)**. *"I never really had my own class. Mr. Smeel usually stayed there."* She discussed how her style was different from that of Mr. Smeel's and how she thought that maybe her method would help students understand better. *"... mostly he just wants to tell the kids what to do. ... but I really think that I focused on the right things in class - getting the definitions straight and such. I was starting to get out of the stand and deliver things and he didn't like it so much. I liked my interaction with the class and I thought I had a fair number of them interested in what we were talking about."* She also gave thought to changes she might make in the future (PN) **(89)**. *"I've already thought about some changes I'll make ... adding more group activities."*

Ellie Folded Back to the presentation by Dr. Towers and realized the complexity of teaching (IM) **(90)** *"... it's really **hard** to listen to all the kids have to say ... sometimes you just miss what happens..."* and that students might <u>understand</u> differently than they can explain (IM) **(91)**. *"Or they word it so it doesn't sound right but they didn't mean it that way..."* Finally, Ellie reiterated her Image of teaching as providing a safe environment in which respect plays a major role (IH) **(92)**. *"We're all in this together and if we help each other, they'll learn more. If someone does something wrong, it doesn't mean that they're stupid, just that they didn't know how to do the question and we should all try to help them learn."*

7.4.2. Charting Ellie's growth

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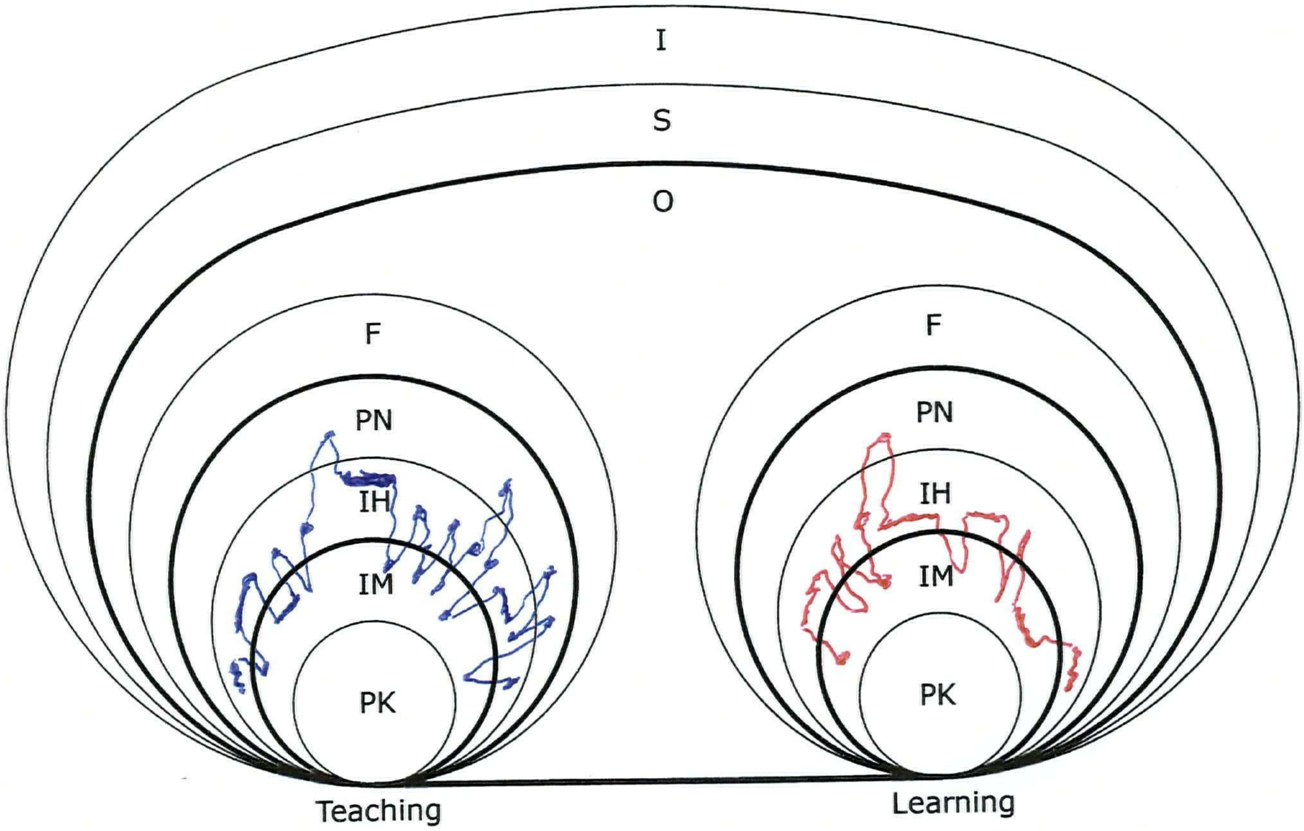


Figure 10: The Mapping of Ellie's Growth of Understanding of Teaching and Learning

7.4.3. A discussion of Ellie's growth

Ellie's portrait revealed a sensitive young woman who, at the beginning of the teacher education program, held a fairly traditional view of teaching and learning of mathematics, and that the learning aspect involved an emotional commitment. Other than the fact that she had a computing science background, she revealed little over the term about her background that would account for her Images.

The presentation of the Pirie-Kieren theory at the beginning of the teacher education program was a catalyst for Ellie's thinking about teaching and learning mathematics and she referred back to it several times, clearly indicating that the presentation had been an Image Making experience for her. She had never realized that people learned differently than she had, nor had she thought about student-teacher interactions and the need for the teacher to be aware of students' backgrounds. In her later teaching, she indicated clearly that she had incorporated this understanding into her Image of teaching.

The presentation by Dr. Towers during the term also had an influence on Ellie's understanding. Having considered earlier the effect of the teacher not knowing all answers immediately, after this presentation she realized that, because students learn and think differently, it is possible that the teacher might not have thought about that particular method and that he/she might miss some of the things that students do. Also, the teacher might not understand the student's understanding or way of thinking. An over-riding image for Ellie was that the teacher is responsible for ensuring that learning takes place, and that this often could be achieved through summing up or generalizing for the students.

Throughout the program, Ellie often worked at the acting stages of the Pirie-Kieren theory (Image Making, Property Noticing) with respect to teaching, and she formed many new Images. She used her own learning experiences in the SMIP class as a springboard for thinking about student learning, and as she grappled with the Images she began to think about teaching and learning at a higher level than before. She began thinking about both in a new, more informed manner so that by the time she left for her long practicum, she had gone through several phases of developing understanding. She still grappled with the idea of using open ended questions (they didn't have a direction) and seemed to think that they were inappropriate for most high school situations but she had accepted the idea that students would learn differently than she had, and that it was important that a teacher realize this in planning lessons so that the differences could be accommodated.

During her long practicum, Ellie modeled many of her stated Images of teaching and learning. She had accepted the tenets of the Pirie-Kieren theory as an explanation for the manner in which learning takes place and demonstrated this by explaining it to her sponsor teacher so that he would understand why she taught in a particular manner.

A new, strong Image of teaching as caring for students and providing a safe environment emerged during the practicum and seemed to be the reason Ellie provided opportunities for different styles of learning.

In her practicum, she did not enact her Image that it was the teacher's responsibility to summarize so that students would "have it", indicating that, possibly she began to realize that students are responsible for their own learning. In the post-practicum discussion, she verbalized her enacted Image of teaching as providing a safe environment and she noticed that different teaching techniques (her method of factoring, for example, compared to Mr. Smeel's) would likely have an effect on learning, both Saying and Recording at the Property Noticing level.

It is evident that during the teacher education program Ellie made new Images of teaching and learning. Her initial Images of teaching, which aligned with the traditional view of teaching-as-telling, had been redefined to include a more student-centered Image in which each student was an individual who would learn in his/her own manner. Ellie, in attempting to put these Images into practice during her practicum, did so with some success and came to a realization of the complexity of teaching and learning.

7.5. Analysis of Wayne's portrait

7.5.1. Classifying Wayne's levels

Wayne began the teacher education program with a view of himself as a philosopher rather than as a teacher or a mathematician. *"If it weren't for social and parental pressures ... I would never have taken a degree in math ... and all the time my big thing was philosophy..."* His Image of teaching was that of the teacher as a guide who knows the best route and that he/she guides students along that route to understanding **(1)**. *"The teacher should **guide** the discussion in the direction of the **most effective** solution..."* According to Wayne, it is the teacher's job to tell students why we (teachers) do things as we do **(2)**. *"For example, the teacher should let the students know why the quadratic function is correct."* He stated that, while he thought of mathematics as creative, and that learning mathematics should involve true understanding (IH) **(3)**, others thought of mathematics and mathematical learning as rote memorization (IH) **(4)**. *"Mathematics is a creative endeavor but most students think of it as a subject that is learned rotely".*

Referring to his view as a philosopher, Wayne revealed contradictory images of learning: It was best if students learned by themselves (IH) **(5)**, *"...it is best if students really understand mathematics rather than have rote understanding. Philosophically speaking, I believe that students should learn mathematics by themselves..."* and learning usually involved the need for the teacher to show students how to do it (IH) **(6)**. *"... even though it is often necessary for the teacher to show them how to do it."* Wayne again described understanding as guided growth **(7)**, *"Guided growth might be the best way to describe mathematical understanding."* and after briefly mulling through ideas on philosophy and his own background (IM) **(8)**, he again verbalized his understanding of teaching as guided discovery (IH) **(9)**. *"The teacher has to guide the students in the direction of the most effective solution ... Provide guidance in their growth. Teaching is a matter of guided discovery."*

Wayne indicated that learning mathematics involved intuition and insight (IH) **(10)**, *"If lucky, a student will experience intuition and sudden insight ... This is what it is about."* but since he did not know how to teach insight, he thought the teacher should teach mathematics as creative and meaningful (IH) **(11)**. *"... the good mathematics teacher must learn to present mathematics as creative and meaningful."* According to Wayne, understanding implied that students could see why a particular approach was the best one (IH) **(12)**, "*Students need to get an understanding of why mathematics questions are approached in a particular way...*" and teaching involved letting the students know (telling them) the structure of mathematics **(13)** "*let students know why content is structured in a certain way*" and guiding them along a particular path (IH) **(14)**. "... *teacher who guides them in **that** direction - the direction of the most efficient solutions.*"

Using his Images of mathematics as an objective subject, *"... mathematics is objective - right or wrong - ... it should be prescriptive ..."* and of himself as a philosopher, *"... until September third my life - well until today, my life has been consumed with abstract thinking..."* Wayne revealed that he thought teaching mathematics would be described in terms of outcomes by a philosopher (IH)

(15), and in terms of methodology by a teacher (IH) **(16)**. *"A philosopher would describe teaching in terms of outcomes ... for an educator --- it is more useful to define it in terms of methods."* He further indicated that he thought of vocabulary as essential to mathematical understanding (learning) so that mistakes would not be made (IH) **(17)**, implying that teaching it (vocabulary) would be important (IH) **(18)**. *"Vocabulary is important in math. Mathematicians have taken the vocabulary and chunked it together so they won't make mistakes."*

Discussions regarding inclusion Invoked Wayne to Fold Back to his own learning experiences, *"Based on my own experiences..."* and served as an Image Making experience **(19)**. Having listened to others, he thought about his own experience and indicated that learning would occur best among like-minded or like-ability thinkers **(20)** but conceded that for less able students, role models (more able students) might be helpful **(21)**. *"I think that maybe there should be --- well, homogeneous groupings, maybe, because then advanced thinkers can work together - but they might --- if in heterogeneous, get others motivated to emulate them..."* Continuing working with his past experiences, Wayne revealed the Image that teaching involves presenting material in a systematic manner (IH) **(22)**, leading students to the final conclusion at which time they would understand what had been taught (IH) **(23)**. *"I remember one teacher who taught us-- had us prove one little thing then another and pretty soon we had proven the whole binomial theorem --- led us through the* ***best way*** *and we* ***didn't even know*** *what we had done until we were done."*

Wayne indicated that he thought that learning mathematics was cumulative **(24)** and that a student would not be able continue if he/she was missing some information (IH) **(25)**. "*There's a lot of cumulation in math and if a student missed something elementary then he would be lost.*" Within this Image he held the Image that learning was enacted differently in elementary schools than in high schools **(26)**. Elementary students, he thought, were not capable of higher-level thinking and they are/should simply be taught facts which then must be proven in high school. "*... elementary students aren't capable of abstract thinking . ... high school math should be more understanding - understanding of the* ***facts*** *that they have to learn in elementary school but don't have the ability to understand.*"

Wayne indicated that proof and problem solving are important aspects of mathematics and that proof should be emphasized in teaching (IH) **(27)** so that student can see the system (IH) **(28)**. "*In high school they should be able to see the system. It should be about* ***proving.***" He also thought that theories and research could possibly explain biologically how students learn (there is a specific path to learning) (IH) **(29)** and that some students simply cannot learn (IH) **(30)**. "*... if there are biological stages in learning that aren't in dispute, I want to know that. ... Like I think there are some kids who will* ***never*** *understand ...*" Referring to the Pirie-Kieren theory, he indicated that he believed that he already thought along those lines and wondered if there were other explanations (IM) **(31)**. "*We did the P-K theory the first day and that was good. Well, I sort of* *thought that already, but I want to know other ways of thinking that others have* ***proven*** *..."*

During an in-class discussion on textbooks, Wayne implied that teaching should relate different topics for students (IH) **(32)**, *"... then try to relate it to other units ..."* and that teaching is often textbook bound (IH) **(33)**. *"But, there is a problem if you don't follow the textbook ... You'll have to look really carefully over the questions before you assign them to be sure the students knew everything they needed."* At the same time, he thought that teaching should be creative and that open-ended questions are a means by which to achieve this (IH) **(34)**. *"I think open-ended questions are good. Problems that have no real solution."*

In his first presentation in the SMIP class (Pythagorean Theorem), Wayne modeled the Image that having the background information is important for learning, and that once it was learned, it would be accessible **(35)**. *"You know how to do squares, take square roots and you can use formulas ..."* By providing several hands-on examples, Wayne implied that he thought that active participation was important for learning (IH) **(36)** and by directing the class to a generalization, he enacted his Image that teaching should lead the class along the correct or desired route to the right solution **(37)**.

An Image of teaching-as-telling **(38)** and of learning as having the facts **(39)** emerged when the SMIP students questioned Wayne about the values of the numbers in his presentation. He simply told them they did not fit the pattern so did not work. *"But the sum of the numbers doesn't work, so they don't fit the pattern. You can see that. So they don't work."*

Wayne's second presentation also allowed for student involvement (IH) **(40)** *"Use the pennies to form an equilateral triangle with three rows of pennies. How many moves will it take to reverse the triangle?"* and re-emphasizing the Image of guiding students to generalizations **(41)**. *"Now what if you have six rows? And keep on trying until you can generalize."* Within this Image, an Image of keeping students on the right track (not following student leads) appeared **(42)** when some unknown catalyst prompted Lance to ask a question about Fibonacci numbers. *"That's not really relevant here. We're not discussing Fibonacci."*

As the term progressed, Wayne became more involved in commenting on presentations, and indicated that time constraints were a concern in teaching (IH) **(43)** and this was a consideration in the reason for guiding students along the best route **(44)**, the Image to which he repeatedly returned. *"Do you think that tangent was useful, or was it just using up time --- too much time for what it did ... That was good, Song. You showed us the* ***best*** *way to do it, the* ***better*** *method - that the other one took too long. ...* ***and*** *she got through the discussions* ***quickly*** *by* ***guiding*** *us through the process she wanted."* He indicated that he thought correct definitions were very important to learning (IH) **(45)**. *"I think it's important to do the definitions, too.... They have to know what things really mean --- definitions ... but how can you make sure students have the same Images - the Images* ***you*** *want them to have? I guess just by making sure you define things properly."*

Following the short practicum, by describing the reasoning behind BEDMAS to his peers, Wayne reiterated his image that teaching involves letting students know why a specific process is used/works (IH) **(46)**. *"It isn't arbitrary. We do what is most powerful first. ..."* He indicated that he believed teaching involves knowing what understanding students have (IH) **(47)**, *"I like to start my lessons with an exercise that requires* ***true*** *understanding rather than facility ... and get a sense of what they understand ..."* and then using this understanding to correct their incorrect Images (IH) **(48)**. *"... then I know ... what Images I have to correct."*

Wayne's Image of self as philosopher, took a turn and he viewed self as scientist/mathematician whose job as a teacher is to enculturate students into the field (IH) **(49)**, *"It's good to show students what we, as scientists or mathematicians* ***really*** *do."* and that the teaching often involves telling the students what you want them to do **(50)**. *"****I specifically*** *tell them to keep* ***all*** *the digits until the end."*

In a group presentation with Win and Jane, Wayne displayed an Image of teaching as giving students time to discuss **(51)**, *"Now, how do you think they --- their volumes are related? Go ahead. Discuss it with your tablemates."* and he later explained that being able to express oneself clearly is an important part of learning (IH) **(52)**. *"I think it is important to have student input. They should be able to express themselves accurately."* He again modeled his Image of teaching as showing/telling the students what you want them to learn **(53)** by demonstrating to the class the volume activity. *"See, here I have taken the clay out. Now I'll pack the clay from the pyramid into the prism. ... The volume of the pyramid is one-third the volume of the prism. ... You have to* ***show*** *them the* *relationships so they will understand them and learn them."* He then revealed an Image of teaching as showing results in more than one way **(54)** *"I'll show you that another way. Here. Let me weigh the clay ..."* and that teaching involves pre-thinking to determine alternative ways of showing students what you want them to learn (IH) **(55)**. *"I think I did a good job because I spent a lot of time thinking of different ways to show it."*

In a class discussion on testing and assessment, Wayne stated that he thought that learning was best demonstrated by being able to amalgamate various pieces of information into an understandable whole (IH) **(56)**. *"And you have to make tests so that they use them all --- all the things they have learned, not in isolation, but all in one question. That way you can find out what they really know not just if they can just do each thing in isolation."* He also indicaed that teaching involved making students think (IH) **(57)** *"And making them think."* and that it involved bias (IH) **(58)**. *"Teacher bias is always there."*

A video presentation which the SMIP students were to use as a basis for discussing the learning that they thought took place with reference to the Pirie-Kieren theory led to Wayne to re-emphasize his Image of the importance of vocabulary in learning **(59)** *"Really, we* ***can't*** *say no sides exposed. We have to say zero sides exposed."* And, while using the terminology from the theory, *"We need to discuss the Images, what prior knowledge they need ..."* he reiterated that teaching involves thinking through what you want the students to learn and then leading them there (IH) **(60)**. Also, there is a best way/path to do so (IH) **(61)**. *"Really, if you want the students to learn efficiently, you should think it* *through beforehand and think of the best way to present it so you can* ***guide*** *them through the Images you want them to have."* Allowing students to develop their own understanding, he said, was too time-consuming (IH) **(62)**. *"You know, its* ***inefficient*** *if you have to spend too much time on Images. ... Having them have different Images and having to Fold Back and modify them is just* ***too*** *time consuming."* Again he stated that there was a best method to teach a concept (IH) **(63)**. *"... you should think it through beforehand and think of the best way to present it ..."* Teaching, therefore, would involve breaking the subject content down to smaller parts and guiding the students through questioning to the desired result (IH) **(64)**. *"So you would want them to break it up, lead them through questioning."* Learning, according to Wayne, resulted when one was guided along the right path (IH) **(65)**. *"I basically want them to figure out those things themselves ... Ideally every student would do this for himself, but that's not likely to happen. ... But if the teacher walks around the class and asks pointed questions, that will help the students develop on their own in the right direction..."*

Wayne referred back to a previous discussion on inclusion and again revealed the Images that learning takes place best among like-minded people **(66)** *"I'm not sure if I'm totally into inclusion of people with less ability.... in general society, people spend the majority of their time with people with similar interests."* and he indicated that there are learners who are self-motivated and those who are not **(67)**. The non-self-motivated need the teacher's guided direction and the motivated ones are able to try other approaches (IH) **(68)**. *"I* *want a standard route for different types of learning objectives ... students who are self-motivated - could come to me with a semi-related aspect they want to develop and I would have to look at it to see if it conforms to the IRP ... I think the classes need to be different."*

Shortly before the long practicum, Wayne revealed the Image that learning requires 'doing' **(69)** and that teaching therefore involves showing the step-by-step procedures **(70)**. *"... that if they do it by hand rather than calculator, that they will understand better because we are forcing them through smaller steps ... and if they go through those steps, they will learn..."* He left for the practicum with an Image of students as learners who would follow the teacher's lead **(71)**. *"There is a real bias on the part of the students to use anything the teacher tells them."*

All but one of Wayne's observed classes in his long practicum followed a pattern that modeled his Images of teacher as guide **(72)** and students as followers and/or accepters of teacher statements **(73)**. He guided the class to the discovery of a formula. *"Can anyone tell me the next term? ... Now we need to name these columns."* He provided them with a guided discovery geometry booklet he had made (IH) **(74)**, *"Okay. I have prepared a book for you. Take it out and work through It with a partner. You can follow it. It is very organized."* and, using only two examples, he provided them with the conclusion that when two lines are perpendicular, the product of their slopes is negative one (IH) **(75)**. *"I guarantee you - promise you - that if you do a whole bunch, that if you multiply* *them together and their products will be negative one, that those slopes will be perpendicular."*

True to his stated Image, terminology was introduced and stressed as important on several occasions during the practicum **(76)**. *"It's so you can build up your terminology so that you can explain in words. ... I want to be sure that you understand the terminology. It's very important. ... No, you cannot write logic that way. You have to use if and then."*

In a lesson on logic, Wayne enacted the Image of providing the opportunity for students to construct their own understanding **(77)**. *"What I want you to do is look for connecting words, use letters for statements, and to translate these into logic. Think of how you would describe each of the words -- the ones we are naming each sentence. Like contra-positive, bi-conditional --- what do they mean? Talk it over with a friend and see if you can describe them - make a definition."* Also, he revealed a new Image that allowed for teaching as giving the students a break from their work **(78)** by providing a diversion - an unrelated question that all students should be able to do. *"Now, everyone, pick a number. ... Now double it. Add eight. Divide by two. Subtract the original. Multiply by five."* The Image of teaching as 'showing students how' or 'telling them how' was modeled when he provided them with a formal explanation to this question rather than have one of the students demonstrate his/her strategy (IH) **(79)**. *"I'll show you how the question works after you have taken notes."*

In the post-practicum discussions and video observation, Wayne referred to the discussions about the Pirie-Kieren theory, and articulated that he thought that his Images of learning aligned with it **(80)**. *"As for the P-K theory, to be honest with you, I think my --- my --- my **Images** for a lot of the components of the model was formed before … its given me a language but I don't see it as new to the way I thought about teaching. … A lot of theory comes from an unconscious, intuitive --- internalized level … were there **long** before I had the language."* He also Folded Back to his own learning experiences and indicated that his Image of teaching was based on the poor teaching he had experienced and his desire to improve on it **(81)**. *"I never had things explained well to me in school … So I tried to explain it well. … A lot of math isn't taught well … When I was in high school, I saw how terrible my teachers were!"*

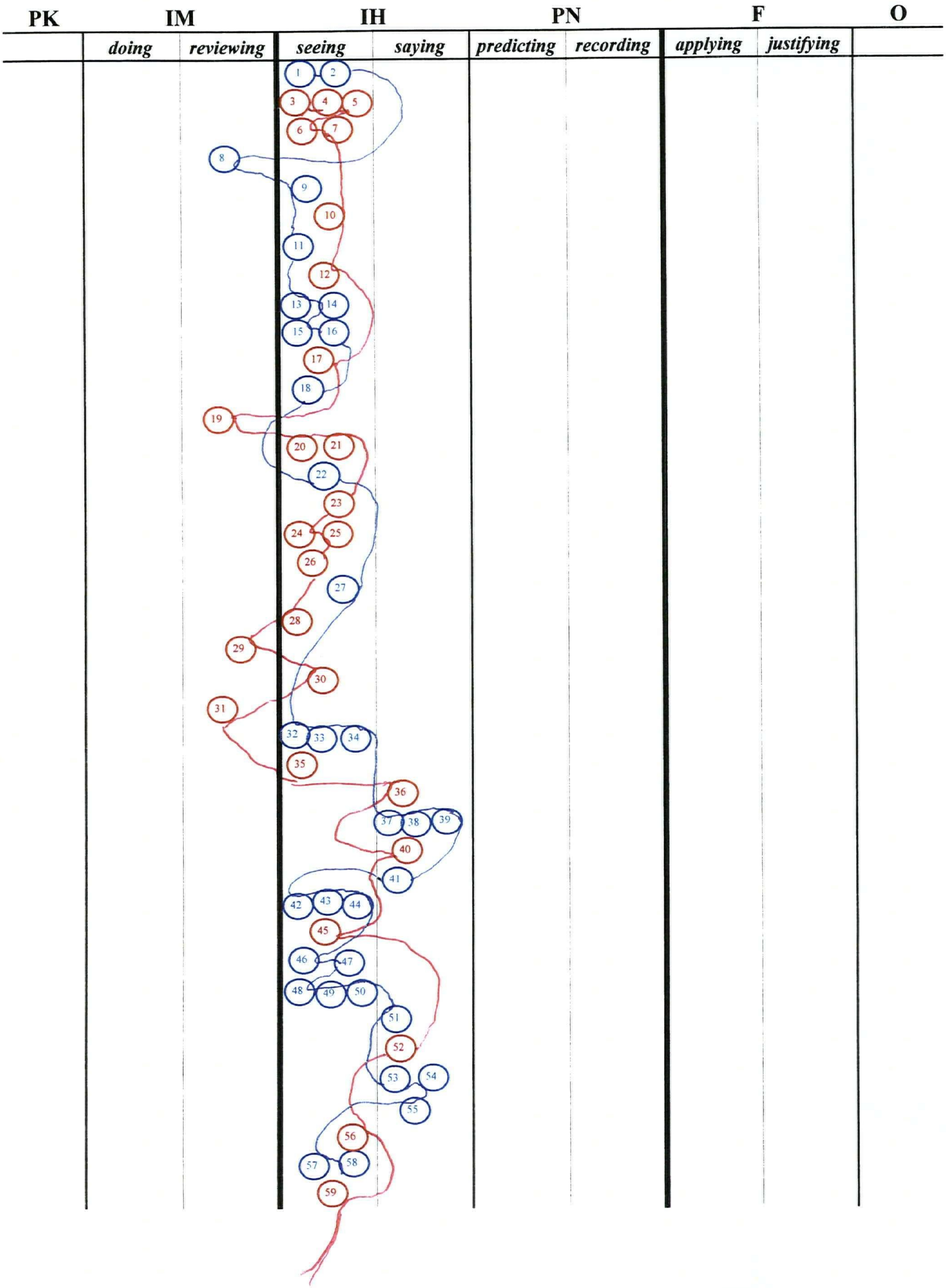
Wayne further identified Images of teaching as requiring complete understanding **(82)**, *"I spent my time thinking about it - developed a **complete** understanding."* and again, the need to pre-think the concept so that it can be presented in the simplest, most direct form **(83)**. *"And I think I taught fairly well because I did the same thing before I taught - I thought about it, thought about what was needed and what it really meant and then taught it so it was the **easiest** route."* Thus, he concluded that teaching was providing the best route (IH) **(84)**.

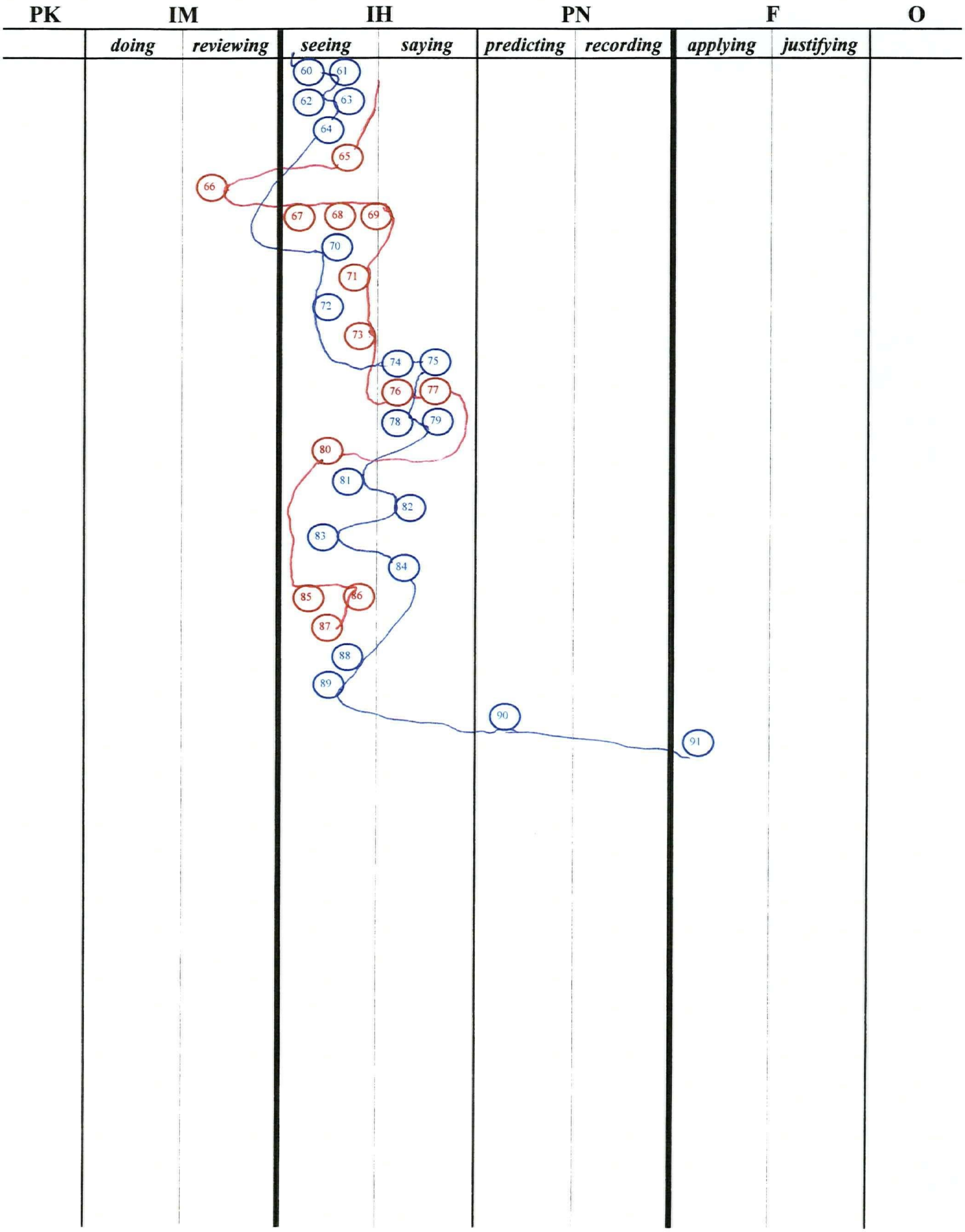
Wayne again re-iterated the image that learning involves knowing correct terminology **(85)**, *"I tutored a kid who was failing in math and in six weeks I got him to pass-- asked simplified questions and **provided** definitions …"* and he revealed a new Image that learning is measured by improved grades **(86)**. *"… they're learning, though, because I can see that the mean scores are going up."*

He also restated the Image that learning was based on previous understanding **(87)**. *"... couldn't do math eleven until I taught him math eight and nine ... It's frustrating to see the kids that can't learn because they don't know the previous work ..."* and in further discussion he revealed the Image of teaching as being hard work and not very rewarding **(88)**. *"I liked student teaching for a while and then I hated it because I was putting in lots of time and energy and was getting neutral responses from my sponsor teachers. Even the kids hated me."*

The final Images of teaching and learning that Wayne revealed were Invoked through watching the video of himself teaching the lesson on logic sentences. He revealed that the lesson plan had not been his idea, and re-confirmed his Image of teaching-as-telling **(89)**, *"To tell you the truth, what I did in that class wasn't my idea. I was going to give them the definitions and then give some examples."* and that having students develop their own understanding was too time consuming (PN) **(90)**, concluding that his method was best (F) **(91)**. *"I guess it worked okay. I guess I covered all the same things in the end, but in that format. Anyway, if it as* ***my*** *class, I'd do it differently. Takes too much time this way - had to do it twice."*

7.5.2. Charting Wayne's growth

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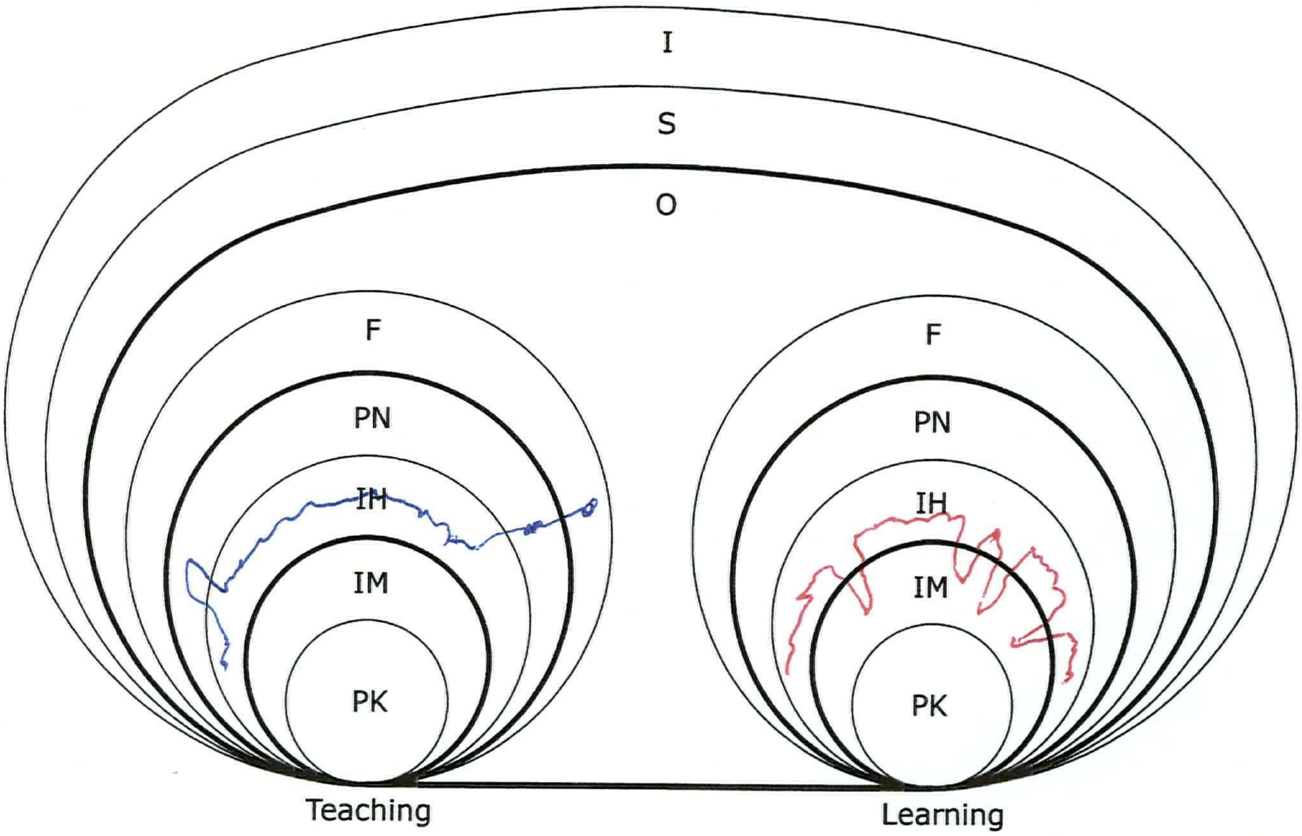

Figure 11: The Mapping of Wayne's Growth of Understanding of Teaching and Learning

7.5.3. A discussion of Wayne's growth

Wayne's portrait revealed an insecure young man who viewed himself as a capable student, mathematically, and as a philosopher. He seemed to believe that he could analyze most situations and generalize from them. A review of his portrait indicates that he tended to summarize discussions in generalities rather than attempting to use the discussions to obtain new ideas.

In considering Wayne's growth of understanding, it is clear that most of his time was spent at the Image Having level. He repeatedly stated his Images and did not consider alternative ideas. He clustered Images of teaching somewhat separate from those of learning.

Wayne entered the teacher education program with a firm Image of teaching as guided discovery, and without considering other alternatives he made a firm statement to this effect early in the program. This remained the over-riding theme of his Image of teaching mathematics. He had two recurring, related Images of teaching and learning mathematics: 1. There is a best method to teach any topic and the teacher should determine this by pre-planning and thinking through the topic completely. The implication for learning here is that students would learn the topic in that manner, that all students would learn the same, and that if learned properly, there was no need to Fold Back to previous understandings. 2. Correct vocabulary is essential for learning. Thus, the implication for teaching was that the teacher should begin by supplying this vocabulary.

Wayne's over-riding traditional view of teaching-as-telling, is occasionally broken by statements that indicate that <u>he</u> (as opposed to others) has the Image of teaching as being creative and student centered, but his practice belies this. In his presentations and his long practicum, he enacted his Images of teaching as guided learning or guided discovery and as showing/telling students 'the best way'.

During the practicum, a new Image of teaching which included providing interesting, unrelated side-bits as a distraction or to lighten the focus appeared. Even in this, Wayne enacted his Image of teaching by telling by providing the students with the solution rather than letting them figure it out.

Wayne maintained (repeatedly stated) his belief in the importance of students knowing and using proper terminology and he attempted to model this by providing them with definitions. It is important to note, however, on several occasions students pointed out that he had used incorrect terminology or notation and that Wayne never acknowledged this. Thus, he did not truly enact this Image.

In his post-practicum discussion, as well as at other times, Wayne indicated that he thought his ideas about teaching and learning mathematics were aligned with thinking about learning as described by the Pirie-Kieren theory and that he thought that he taught in a manner that allowed students to develop their own understanding. However, again, his practice indicated otherwise and he specifically stated that he thought it would take too much time to allow students to develop their own understandings. He clearly indicated that the one lesson in which he had allowed the students to develop their own ideas was done to appease his faculty advisor, thus indicating that he did not <u>have</u> the this specific Image. That is, there was a 'disconnect' between his stated Image and his enacted Image. While talking about the presentation, he Formalised that 'his' method, the Image he had come to the program with, was the right way to teach.

Even when he was confronted by problems (neutral responses from his sponsor teacher and “even the kids hated” him), Wayne did not question his methods, but assumed his Images were right. He “talked the talk”, but he did not “walk the talk” - not even with respect to his own Image of the importance of proper vocabulary. Thus, he began the program with an Image of teaching as guiding the students along the right route to understanding and he left the program with this as a Formalisation. He had been taught and had learned in this manner, and therefore he decided it would work ‘for all’.

Chapter 8

Conclusion

8.1. Introduction

This chapter draws together the results of the findings of this study. The study set out to explore the usefulness of using the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding as a model in a teacher education program for prospective secondary mathematics teachers. It also set out to determine if the same theory could be modified to serve as a tool for analysis for growth of understanding of teaching and learning. It, in fact, did much more. Since the theory was used as a model for the preservice mathematics teachers during their teacher education program as well as being modified to use as a means of analyzing their growth of understanding of both the areas of teaching and the learning of mathematics, the implications of the findings of the study are important to both the extension of the theory and to teacher education.

In the next section I bring together the experiences of the four preservice teachers and the relevance of the theory to their individual understandings of teaching and learning of mathematics. This is followed by a discussion of some of the general influences of the theory during the program. I then discuss the implications this study has for the Pirie-Kieren theory and the implications it has for teacher education programs. The final sections of this chapter provide suggestions of areas for future research.

8.2. Main findings related to growth based on use of the Pirie-Kieren theory

There is evidence in the data of this study to suggest that using the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding had an influence on the preservice teachers' understanding of learning to teach mathematics as well as their understanding of the process of learning mathematics, albeit in different ways for each individual. When they entered the program, each preservice teacher had an Image of teaching and learning based on his/her own experiences. Most of them had been 'good' at mathematics and therefore some of them had never thought of how other students had to struggle with the concepts or that they might misinterpret meanings or have only partial understanding. Discussion of the theory helped them realize these possibilities.

8.2.1. Sophia

Sophia came from an academic background in which several family members were involved in the teaching profession, and they apparently discussed teaching and learning as a regular part of their everyday lives. This, along with the fact that she had some teaching experiences volunteering in Peru, influenced her initial Images of teaching and learning and informed her changing Images as she saw other aspects of the practice of teaching and learning emerge in the teacher education program. At the beginning of the program, she was aware of differences in students' learnings, and the struggles that they had in developing understanding. She was also aware that teachers need to understand those struggles. Introduction of the Pirie-Kieren theory provided her with a language to discuss her understanding of student understanding and provoked her to think more about this and about how teaching would influence it.

Sophia was greatly influenced by what she saw as good teaching (Alwyn), and this prompted her to think about the type of learning that was taking place. She used the tenets of the Pirie-Kieren theory as a way to focus on this. The process of thinking about her own understanding based on the levels of understanding described in the Pirie-Kieren theory as she worked through a mathematical problem in class also influenced her understanding of the relationship between teaching and learning. She realized that, even the preservice teachers went through the stages as outlined in the theory when they were working on a new problem. The experience during her short practicum in which she observed what she considered non-teaching also seemed to provoke her to think about a more active and involving teaching and learning process.

Sophia was able to effectively practice her Images of teaching and learning mathematics during her practicum. Reasons for this include the fact that her sponsor teachers did not tend to interfere with or intervene in her teaching. Specifically, they were seldom present so that Sophia was free to enact her ideas, working at the Property Noticing (Recording) level. As a result, Sophia was able to engage her students in what she referred to as 'peer tutoring'. This activity incorporated Martin's (1999) peer interventions and helped encourage a collaborative environment. Her teaching style reflected Towers' (1998) inviting and praising styles, styles that encourage the development of understanding.

Sophia consciously thought about and discussed the learning that she had seen taking place and reflected on what prompted that learning, reaching the Formalising level in her understanding. The Pirie-Kieren theory, she indicated, provided her with an effective way to think about the relationship between teaching and learning mathematics and provided a language through which to discuss it.

While Sophia's Images of teaching and learning mathematics did not change greatly as she progressed through the teacher education program, they were refined and clarified as she actively worked at developing her understanding of the relationship between the two. Her discussions after viewing herself on video and thinking of the learning that took place were such that she was beginning to work at the Observing level. She was thus able to move to the outer levels of understanding of teaching and learning mathematics as she progressed through the teacher education program. She consciously used the Pirie-Kieren theory as a model for this development.

8.2.2. Lance

Lance's family background and experiences as a student had noticeable influences on his initial Images of teaching and learning, as did his experience working at other jobs in which he had to train people. With a university professor as a father, there were high expectations that Lance would achieve well, and he had attended a private boys' high school that concentrated on preparing students for further education. His personal experience of having to develop his own understanding of mathematics after having been given the information by his teachers informed him that students do need to do this, and his experience training people on the job informed him that he had to think about how he had learned a process if he was to teach it to someone else.

During the SMIP program Lance saw that there were ways of presenting material that differed from that which he had experienced (lecture type teaching) and that these other methods, such as allowing for student involvement, could possibly help students develop better Images or develop them more readily. Lance tried to incorporate these ideas into his repertoire of Images of teaching and learning but did not get beyond the level of Image Making with many of them. The Pirie-Kieren theory provided him with a way to think about learning and provoked him to realize that learning takes different forms. Considering this and watching other SMIP students further provoked him to consider (Image Making - Reviewing) different Images of teaching. This was most evident when he realized that he had 'taught' in the traditional method when he told the SMIP students how to prove his presentation rather than let them develop the method themselves.

Lance was greatly influenced by the presentation of Dr. Towers when she displayed her own shortcomings in a teaching situation and this provoked him to consider the relationship between teaching and learning, but he was reticent to act on this. He held it at the Image Making (Reviewing) level.

Lance's practicum experience is evidence of the influence a sponsor teacher can have on a student teacher who is still developing new Images of teaching. Lance's sponsor held a traditional view of teaching from the text and indicated that Lance should do so also. In his post-practicum discussion Lance indicated that, because of the constraints put on him by the directive that the textbook was the curriculum, he did not go beyond it or try different approaches. However, on observing Lance's interaction with the students, although he maintained quite a traditional stance, it was apparent that he did, in fact, put many of his newly developing Images of teaching and learning into practice (Property Noticing - Recording), trying to involve students in activities and trying to have them discuss among themselves and explain their thinking.

Although he did not say so specifically with respect to his practicum teaching, comments made earlier in his preservice education seem to indicate that, as Cavey (2002) suggested is useful, Lance Folded Back to his own learning in planning lessons, thinking about how he had learned. His own learning experiences were probably also influential in his use of a more traditional teaching method as this is what his experiences implied was teaching. Thus, it appears, that while Mr. Forsyth had tried to 'drop Lance into Formalising' of teaching in the traditional manner, Lance's developing Images were strong enough to allow him to try them out.

Lance's post-practicum discussion revealed that he had come to an understanding of the complexity of the knowledge base that is needed for effective teaching, and that learning is a process that requires being able to Fold Back to previous understandings. Lance had incorporated the Image of learning as aligned to the Pirie-Kieren theory into his repertoire of learning. He realized that many teachers did not teach this way and that this, therefore, may be why many students were not learning well.

8.2.3. Ellie

Ellie revealed little about her personal background that could be said to have been influential in shaping the Images she held of teaching and learning mathematics at the beginning of her teacher education program. Fundamental to her Images, though, was that she saw mathematics as ordered and logical, that it had always been easy for her, and that learning mathematics involved a certain emotional commitment. Presentation of the Pirie-Kieren theory and discussions about other preservice teachers' experiences were enlightening to Ellie and made her consider the learning of mathematics differently than she previously had. She had assumed that all people learned mathematics in the same manner as she had, and that learning mathematics was a linear process. While she had previously thought that a teacher could simply teach a topic in mathematics, discussing the Pirie-Kieren theory helped her realize how little teachers know about students and how important it is that the teacher has an understanding of both the personal and the mathematical backgrounds of his/her students if he/she is to teach effectively.

Working through problems in the SMIP program after having had the Pirie-Kieren theory explained as a model prompted Ellie to think about her own learning and how, as a teacher, she would have to be aware of other approaches than that which she would typically take. She also realized that there are different 'learnings' that can take place in any situation (students will not necessarily learn what the teacher planned) and that some of them may not even be 'correct'.

Ellie tended to model Towers' (1998) clue-giving and modeling teaching styles in her teaching, but it is significant to note that she did usually leave time for the students to work through their own understandings. Thus, while she was still not totally confident that students were capable of taking control of their own learning, she had accepted the tenet of the Pirie-Kieren theory that students develop their own understanding based on their own experiences. Of significance is the fact that Ellie explained the theory to her sponsor teacher as justification for teaching in the manner that she did. She had accepted the theory as a way of discussing developing understanding and her willingness to share this with an experienced teacher indicates that she was confident that it was an acceptable explanation.

8.2.4. Wayne

Wayne's experiences as a capable mathematics student in an enriched mathematics program and as a philosophy major had a significant influence on his initial Images of teaching and learning mathematics. He felt that most teachers of mathematics did not teach the subject properly and that the role of a teacher was to guide students along the right/most effective path to learning. That is, as Shealy (1994), and Cooney and Shealy (1997) suggest of some preservice teachers, Wayne already <u>knew</u> the right way to teach and he thought that students would learn mathematics correctly if they were guided along this path. Wayne's background in philosophy led him to believe that the world could be 'axiomatized' and thus terminology was essential to the development of understanding so that one could express oneself precisely. These Images did not change as he progressed through the SMIP program.

When discussions involved the Pirie-Kieren theory, Wayne indicated that he thought that his Images of learning were aligned with it, and that presentation of the theory merely provided him with terminology. Contradictorily, he clearly stated that to have students learn in the manner described by the theory was too time-consuming, demonstrating that he did not accept the way of learning posited by the theory. Wayne clearly had not accepted the tenet that students need to develop their own understanding and he only gave lip-service to the theory, an action which agreed with the concept expressed by Davis, Sumara and Luce-Kapler (2000) who indicated that people often say what they think others want to hear, but that their actions indicate otherwise.

Generally, Wayne assumed that he was practicing Guided Image Making (Pirie & Martin, 2000). However, he took the expression at face value and missed the key element - that of providing a question that was not immediately solvable and letting the students work at it. Instead, he led (guided) the students along his pre-determined path. He forced a structure on the students' learning and did not allow for individual development of understanding.

During his practicum, Wayne enacted his stated Images of teaching in all but one observed lesson. In this lesson, he allowed the students to <u>work on</u> their understanding, but he later indicated that the format of that lesson was not his idea, but that of his faculty advisor. He stated that he thought that this method had not been as effective or efficient as his own method (giving the students the definitions) would have been. Wayne had applied a technique of teaching with which he did not believe and, in thinking about it, Formalised his original Image. Wayne' s initial, pre-SMIP Image of teaching was so firmly embedded in his nature that nothing anyone did or said impelled him to think otherwise.

8.2.5. General findings with respect to the Pirie-Kieren theory

Learning about the Pirie-Kieren theory at the beginning of the teacher education program, and seeing examples of how students work at mathematics as well as practicing some activities the first two days of class in which they had to consider that own working in light of the theory, provided the SMIP students with a foundation upon which to base their learning about teaching mathematics as well as providing them with a means of considering how students learn mathematics. It also provided them with a common language to discuss their understanding. Observations of videos of students working on mathematical problems and discussing their developing understanding based on the theory gave the preservice teachers an opportunity to consider student learning from a different perspective than many had previously thought about it, and consideration of their own understanding based on the Pirie-Kieren theory proved to be an enlightening experience for many of them and helped them focus on how students develop understanding.

Obviously, all preservice teachers entered the program with Images of teaching and learning. These they indicated either through direct statement, or, as often as not, through actions. If there was consistency between the two, this tended to confirm the Image. If there was not, one had to question which was the held Image and rely on further observations. These sometimes revealed that the statement or action was at the Image Making level and the preservice teacher had to work at it until he/she was able to incorporate it into his/her repertoire.

Throughout the teacher education program, some of the Images that the preservice teachers formed were modified and clarified, and new Images emerged. Moves to both inner and outer layers of understanding reveal that the preservice teachers were involved in clarifying or refining their understanding. The mappings clearly indicate that each preservice teacher went through a different process in developing his/her understanding of teaching and learning mathematics.

An overview of the activities in the program and of the analysis of the developing understandings of the four preservice teachers reveals that some instances served as catalysts for changing Images for one particular student while some affected more than one, albeit in different manners. For example, the video of Alwyn teaching was an inspiration to try a new form of teaching for Sophia while it invoked Lance to think about substituting encouragement for relevance. The initial presentation informed Ellie that there were different paths to learning and Dr. Towers' presentation influenced both Ellie's and Lance's understandings. Most significant, however, was the over-riding effect of the Pirie-Kieren theory in the preservice teachers' discussions. At some point, each referred back to it and used it to explain some of their understandings, even Wayne who did not truly apply the tenets.

Consistent with Seijts, Taylor and Laytham's (1998) findings, observing others and themselves on video helped the preservice teachers self-reflect. Throughout the program, watching others teach and discussing the activity helped the preservice teachers create new Images and refine old. Viewing him/herself teaching provided each of them with an opportunity to think about and discuss what he/she thought had happened during that lesson and often provoked him/her to think about other lessons. The theory provided the preservice teachers with a way of thinking about learning and a language with which to discuss it. It prompted all but Wayne to 'think outside the traditional view' of teaching and learning mathematics and provided a common language with which to discuss it.

8.3. Implications for the Pirie-Kieren theory

This study has provided an important extension to the Pirie-Kieren theory. Previously, the theory has been used only to consider and map the growth of mathematical understanding of a particular person on a particular topic in mathematics. This study has added a new dimension to the theory. By adjusting the definitions of the levels of understanding to fit the new context, being careful to keep them within the integrity of the original definitions, this study has shown that the theory can be adapted to discuss and map the growth of understanding in areas other than mathematics, specifically, in the areas of growth of understanding the teaching and learning of mathematics. This is significant in that mathematics has its own integral structure and the theory was created with that structure in mind and to account for it.

A second implication of the research to the advancement of the theory relates to the model. The process of redefining and applying the modified definitions so that the theory could be used in the new contexts indicated that at the Observing level of understanding of teaching and learning a duality emerged and the two different developing understandings became melded into one. That is, the structural wholeness of understanding of the two only emerged when the developing understanding of learning merged with the developing understanding of teaching. It was therefore necessary to develop a modified model which maintained the embeddedness of the levels of understanding but which incorporated this duality.

8.4. Implications for teacher education

The presentation of the Pirie-Kieren Dynamical Theory for the Growth of Mathematical Understanding had an affect on the Images of teaching and learning mathematics for the preservice teachers in this study. While each came to the teacher education program with pre-formed Images of the meanings of teaching and learning based on their own experiences, most were open to new ideas. Providing them with a visual Image (the Pirie-Kieren model) of a way to think about student learning, and discussing the Image using examples of student work at the beginning of their program, gave them a focus for discussion and a language with which to discuss their understanding of the teaching and learning of mathematics. This language was an important aspect of their understanding as it meant that they could discuss learning from the same perspective. In essence, to borrow a term from the Pirie-Kieren theory, they had the same Image with which to work.

Using the theory as a model in the preservice classroom, and having the preservice teachers discuss their own and other's work, helped build their understanding of the theory and of the manner in which students develop mathematical understanding. For example, Sophia, whose understanding of teaching and learning mathematics aligned with the present day understanding that learning mathematics is a dynamic process, was impressed with the way in which the theory described developing understanding and used it as a means of thinking about her own understanding. The theory gave Lance a focus, and encouraged him to think more deeply about student understanding, as well as giving him confidence to practice some of his newly developing Images. Ellie was able to use the theory to explain why she taught as she did. And, although there was no noticeable growth in Wayne's understanding from the perspective of the theory, he did use the terminology to discuss learning.

Since the preservice teachers in this study benefited from having a program-wide theory from which to work, an implication for teacher education programs is that presenting preservice teacher with a theory, and having them model it during their program is a useful means of helping them develop into more reflective practitioners, not only because they have a theory base, but also because they have a common language with which to discuss their ideas. The Pirie-Kieren theory, because of its precise language was found to be a useful theory with which to work.

Another implication for teacher education relates to the fact that the preservice teachers in this study were involved in a special program in which they took three courses together from the same instructor. Being an integrated unit for a large portion of their courses allowed them time to develop their understandings about teaching and learning mathematics as a team and the theory provided them with a language to do so. Application of the theory by a number of different instructors, if these courses were taught separately, might have been more difficult to enact and might not have had the same positive results. On the other hand, it would have allowed the preservice teachers an opportunity to observe the implementation from different perspectives, which might also have had positive results. If the theory had been applied in only one context, say their Methods course, it may not have been as powerful. Further research would be needed to determine this.

8.5. Other suggestions for future research

This study has explored the growth of understanding of teaching and learning of mathematics of four preservice teachers in an integrated program in which they were presented with and had modeled for them, a theory on the learning of mathematics which, as quoted earlier views the learning of mathematics as "a whole, dynamic, leveled but non-linear, transcendentally recursive process" (Pirie & Kieren, 1991a; p. 1). The study has shown that using such a theory is beneficial to the development of the preservice teachers' understanding of teaching and learning mathematics. It has also extended the use of the Pirie-Kieren theory as a means of discussing student growth of mathematical understanding to a new, albeit related area - the growth of preservice teachers' understanding of teaching and learning mathematics. There are several areas that can be identified for further research as a result of this study.

This study focused on the patterns of growth in the preservice teachers' developing understanding of teaching and learning. It did not focus in detail on the specific changes that took place or what incidents prompted those changes. Within the data, there remain opportunities to do so. Further, with the proliferation of original data, there remain opportunities for many secondary researchers to study different aspects of teacher education. For example, the data relating to the preservice teachers working on a non-conventional mathematical problem while thinking of their own positioning on the Pirie-Kieren model could be used to research preservice teachers' understanding of their own learning. Lance's Folding Back to his own learning and to the manner in which he trained people on the job could be used as a starting point to study how personal experiences might affect later teaching, as could Wayne's experiences with like-minded learners. Thus, a major implication for further research is in the sharing of the data to enquire into different aspects of preservice teacher education.

From the data it seems that the sponsor teacher had an effect on the manner in which the SMIP students were able to teach and enact their own ideas. More research into this aspect of a teacher education program is suggested.

Another area for further research identified as a result of this study is with regard to the long-term effects of this teacher education program. While there is evidence to show that the preservice teachers responded to the Pirie-Kieren theory during their teacher education program, follow up research is needed to determine the manner in which, as teachers, they enact their understanding in their own classroom. Long-term results are, after all, the aim of any good teaching, including the teaching of preservice courses.

This research was intended as an investigation into the efficacy of using the Pirie-Kieren theory as an analytic tool by both preservice teachers and the researcher. Having shown its worth, it has provided the basis for use of the theory, and the model developed for these purposes, in deeper more focused study of developing understanding of teaching and learning mathematics.

The final area of research opportunity opened by this study is to determine if the Pirie-Kieren theory can be extended to discuss developing understanding in other areas of study. It has been demonstrated that, while it was developed to describe developing understanding of mathematics, it was successfully extended and revised, keeping within the tenets of the theory, to the related areas of understanding teaching and learning of mathematics. However, further research is needed to determine if it can be applied to less closely related subject areas.

8.6. Conclusion

Learning mathematics is a complex process. Learning to teach mathematics effectively is an even more complex process. As Davis (2000) suggests, a teacher education program should focus on how students learn, not on how one teaches. This study, which focused on four preservice teachers in a teacher education program designed to help them think about how students develop mathematical understanding, indicates that such a program - a program that focuses on how students learn - is possible and is beneficial in the development of effective mathematics teachers.

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Appendix A

Ethics Permission

The University of British Columbia
Office of Research Services and Administration
Behavioural Research Ethics Board

18 July 2002

Notice of Ethical Review

PRINCIPAL INVESTIGATOR	DEPARTMENT	NUMBER
Pirie, S.E.B.	Curriculum Studies	B02-0378

INSTITUTION(S) WHERE RESEARCH WILL BE CARRIED OUT
UBC Campus

CO-INVESTIGATORS:
Borgen, Katharine, Curriculum Studies

SPONSORING AGENCIES

TITLE:
An Analysis of Change in Preservice Teachers' Understanding of Teaching Secondary Mathematics Using the Pirie-Kieren Model

The Committee has reviewed the protocol for your proposed study, and has withheld issuing a *Certificate of Approval* until the following conditions have been satisfied or information provided:

Please highlight or underline changes to consent form(s) or letter(s) and submit only one copy. Provide other requested information in a letter or memo. Do not re-submit the Request for Ethical Review form.

Appendix B

Course Outline

Tentative Schedule of Topics:

We will be covering the following topics this term. The timing is subject to change.

Sept. 3-6 Week 1	Learning and Understanding Mathematics. We will address the questions how do students learn Mathematics, what are the conditions for learning Mathematics and others related to students experience of learning mathematics.
Sept. 9-13 Week 2	Definitions and Purposes of Teaching and Teachers. We ask and discuss different definitions of teaching and develop some goals of teaching in general.
Sept. 16-20 Week 3	Mathematics as a Content Area. Will discuss the nature of Mathematics as it relates to the secondary curriculum. The IRPs will be discussed. Issues that will be discussed are objectives for mathematical learning and understanding.
Sept. 23-27 Week 4	Issues and Ideas in Contemporary Mathematics Education. The ideas of coherence throughout the curriculum, equity and material from the professional standards will be discussed. Lesson planning will be discussed in terms of what one should be thinking of when planning lessons and units. We will also talk about whole year planning with an emphasis on connections within the whole curriculum. In addition, we will discuss interdisciplinary work with mathematics, history of mathematics, gender and cultural issues. Finally, we will discuss interdisciplinary topics, especially between math and physics.
Sept. 30-Oct.4 Week 5	Questioning. We will be exploring the importance of using questioning as part of presenting mathematical material to students. We will look at different kinds of questions and questioning techniques. You will be given the chance to practice these as well as critique the questioning seen in the videos.
Oct. 7-11 Week 6	Presentation of Activities and Mathematical Material. Practice using a clear speaking voice with appropriate volume will be given in this section of the course. We will also go over the use of visual representations of equations, functions, and graphs and their use in communicating key mathematical ideas.
Oct. 14-18 Week 7	Continuation of Presentation. In addition, we will language and vocabulary and how these topics relate to mathematics education. Here we will look at reading of mathematical texts and material. Here we will be presenting sample mini-lessons and discussing attributes of these presentations.

Classroom Management. The importance of setting up a classroom environment that is conducive to learning will be discussed as well as some of the factors that influence classroom environment. In addition, we will talk about specific issues that may arise during your practicum and discuss your questions and concerns about the field experience.

Oct. 21-Nov 1

EDUC 319. Short Practicum. October 25 is the BCAMT Conference.

Nov. 4-8
Week 8

Post 319 Discussions.

Mathematics Lessons. We will look at various kinds of Mathematics lessons and discuss the strengths and weaknesses of these lessons. We will look at a wide variety of lessons including, teacher led (lecture style), groups and individual explorations, projects, and games. We will revisit all of the ideas previously discussed in the class up to this point. Technology will be addressed at this time. In addition, homework and considerations related to the assigning of homework and review of previously assigned homework will be addressed.

Nov. 11-15
Week 9

Continue our discussion of lessons and lesson planning. We will look at enrichment activities and recreational mathematics.

Cooperative learning. We will discuss reasons for group work and situations in which group work is important. We will also analyze implementations of cooperative learning.

Nov. 18-22
Week 10

Assessment and Evaluation. We will discuss the importance of knowing what your students understand and what they do not understand. Techniques of assessing student understanding during a lesson will be discussed and analyzed. Also, the importance of using tests as a learning tool will be treated in addition to the usual use as an evaluation tool.

Nov. 25-29
Week 11

Continue with assessment and evaluation. Practice making test items and designing unit tests.

Practical examples, summary, and applications to your upcoming practicum.

Assignments:

There will be a written assignment for each of the topics through week 9. This will be a short (2-3 page) paper on your views and ideas about the topic we have been covering. In addition, there will be a longer (5-8 pages) paper on teaching secondary mathematics due at the end of classes. The problem statement will be given to you at the end of the 4th week. I recommend that you keep a journal of this class to use as material for the final paper.

In addition, there will be in class presentations of thoughts on reading excerpts or mini lessons.

Appendix C

Images Questionnaire

Pseudonym: ____________________________

Please circle the statement that best describes your beliefs.

1. Some people are good at mathematics and some aren't.

   True More true than false More false than true False

2. A good math teacher can explain the relationships among the various topics studied.

   True More true than false More false than true False

3. You cannot do mathematics unless you understand it.

   True More true than false More false than true False

4. Good math teachers show you the exact way to answer the math question you will be tested on.

   True More true than false More false than true False

5. In mathematics something is either right or it is wrong.

   True More true than false More false than true False

6. Students learn math best by working independently.

   True More true than false More false than true False

7. One learns very little about teaching secondary mathematics by sitting in on the class of a poor mathematics teacher.

   True More true than false More false than true False

8. Good mathematics teachers show students lots of different ways to look at the same question.

   True More true than false More false than true False

9. If students has been taught mathematics properly there is no need to return to previous topics.

   True More true than false More false than true False

10. In order for students to understand math, it is important that they do all their homework.

    True More true than false More false than true False

11. To teach mathematics effectively, a teacher must know what the students have already been taught.

    True More true than false More false than true False

12. Males are better at math than females.

    True More true than false More false than true False

13. Students need to have a clear understanding of one mathematical concept before they can understand the next.

    True More true than false More false than true False

14. To solve most math problems you have to be taught the correct procedure.

    True More true than false More false than true False

15. Getting the right answer in mathematics is not always important.

    True More true than false More false than true False

16. Discussion is a good way to learn mathematics.

    True More true than false More false than true False

17. The best way to do well in math is to memorize all the formulas.

    True More true than false More false than true False

18. It is important that a teacher has lesson plans and sticks to them.

    True More true than false More false than true False

19. Some ethnic groups are better at math than others.

    True More true than false More false than true False

20. To be good at math you must be able to solve problems quickly.

    True More true than false More false than true False

21. One can learn a lot about teaching mathematics by sitting in on a class presented by a good mathematics teacher.

    True More true than false More false than true False

22. Everything important about mathematics is already known by mathematicians.

    True More true than false More false than true False

23. Work with concrete materials should rarely be necessary in the secondary mathematics class.

    True More true than false More false than true False

24. Test results determine if a student understands math.

    True More true than false More false than true False

25. Students will understand mathematics if it is properly presented.

    True More true than false More false than true False

26. Getting the right answer is more important in math than being able to explain what you have done.

    True More true than false More false than true False

27. Student will get things wrong regardless of how good your teaching is.

    True More true than false More false than true False

27. Math problems can be done correctly in only one way.

    True More true than false More false than true False

28. I know what it takes to be a good math teacher.

    True More true than false More false than true False

29. In mathematics you can be creative and discover things for yourself.

True	More true than false	More false than true	False

30. I am confident about my understanding of high school math.

True	More true than false	More false than true	False

Appendix D

Views Questionnaire

Pseudonym: ______________________

Please complete the following sentences and elaborate on your statements.

1. The role of the teacher in the classroom is
2. Teaching secondary mathematics is/is not different from teaching other secondary subjects because
3. Learning to be a secondary mathematics teacher is/is not different from learning to be a teacher of other secondary subject matter because
4. In working with students to whom I will be teaching mathematics, my main goals will be
5. In order to reach these goals I will need to
6. A teacher can best help a student understand secondary mathematics by
7. Teaching and learning mathematics are connected because
8. During this teacher education program I hope to learn

Appendix F

Letter of Permission

Having been introduced to the study "Breaking New Ground in Teacher Education: Using multi-media as a tool to see whether theory can be profitably transplanted from one field to another" by Katharine Borgen, I agree to be videoed along with the other students in the class. However, I do not agree to be one of the people who will be studied in detail.

Appendix G

Permission from the Vancouver School Board

VSB Vancouver School Board
School District No. 39 (Vancouver)
DISTRICT LEARNING SERVICES
1580 West Broadway
Vancouver, B.C. V6J 5K8
Telephone: (604) 713-5000
Fax: (604) 713-5244

Katharine Borgen

September 27, 2002

Dear Katharine,

Thank you for your research proposal called *An Analysis of Change in Preservice Teachers' Understanding of Teaching Secondary Mathematics Using the Pirie-Kieren Model.* Please accept this letter as approval for you to approach Vancouver Secondary schools that plan to have preservice teachers working with students.

The VSB/UBC Research Committee understands that the purpose of your work is to support new teachers in reflecting on their practice particularly when teaching mathematics. Committee members look forward to receiving a copy of your final report.

Thank you for bringing forward your request. I wish you the best of luck as you proceed with your work.

Sincerely,

Valerie Overgaard, Associate Superintendent
Learning Services