the attitude and AChievement of tenth grade GENERAL MATHEMATICS STUDENTS AS EFFECTED BY THE USE OF DESK CALCULATORS
by

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#### Abstract

The purpose of the study was to determine whether the use of a desk calculator in a grade ten general mathematics class would produce a positive change in a pupil's attitude toward mathematics, to see if achievement in quantitative thinking (problem solving) would be increased, and to see if there was a correlation between any such changes.

Two teachers, each having two classes of grade, ten general mathematics, taught an experimental and a control class. The experimental classes were instructed in the use of desk calculators, and encouraged to use the calculators to arrive at solutions to a prearranged program in mathematics. The control grouns were taught in a similar manner, but did not have access to desk calculators. The classes were pre-tested and post-tested for attitude and achievement.

A Mathematics Attitude Test was constructed which yields (1) a "Preference for Calculation" score, (2) a "Preference for Quantitative Thinking" (Problem Solving) score, and (3) a composite "Mathematics Attitude" score. The test comprised sixteen three-part situations consisting of (A), a calculation situation, (B), a quantitative thinking (problem solving) situation, and a distracter, (C), a


non-mathematical activity. The punils were asked to state a like and a dislike among the three offered. On item analysis using biserial $r$, all but one of the items proved satisfactory.

When using the Spearman-Brown orophecy formula for estimating reliability, the reliability was found to be sufficient for differentiating between means of groups, but not sufficient for differentiation of individual differences.

Analysis of Covariance was used to compare the groups for a significant positive change in attitude, and also for a significant improvement in achievement. The correlation of a suspected positive change in attitude and improvement in achievement was checked for significance. At the 0.05 level, there was no significant change in attitude and no significant change in achievement. At the 0.08 level, however, there was a significant improvement in achievement. There was no significant correlation of a positive change in attitude and improvement in achievement. The conclusion drawn was that the use of a desk calculator makes no significant change in attitude, but perhaps might change achievement with a carefully planned program. No correlation of a positive change in attitude and achievement was found.

The attitude test constructed may, with modifications, be of use to other experimenters.

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## Chapter 1

## BACKGROUND

One of the froblems of the teacher is how to deal with the low achiever in mathematics. That low ability is the chief determiner for low achievement is not necessarily the case. Aiken in his review of research on attitudes toward mathematics points out "mathematical ability may be a less important determiner of the achievement of students having extreme attitudes toward mathematics than those having more moderate attitudes.". Studies have shown that the low achiever generally has a negative attitude while the good achiever has a positive attitude toward mathematics. ${ }^{2}$

Dutton has listed several areas that contribute to the negative attitudes toward arithmetic some being that arithmetic "takes too long" and was "boring or stale" with "too much memorization." ${ }^{3}$

The acquiring of negative attitudes covers a diverse field. Some of the contributing factors toward the acquiring of a negative attitude are a slow learning rate, parental attitude, course content, and the teacher's approach. 4 The classroom teachers cannot hope to eliminate all the contributing factors but they might be able to modify the existing ones.

The pupil's self concept in relation to mathematics is partly develoned by his exneriences in working with mathematics. If he fails in a task many times then self confidence may be reduced and hostility may be increased. According to Aiken "The teacler must nrovide for success experiences in the learning." ${ }^{5}$

Alnert et al lead us to conclude low achievement generates a negative attitude and a negative attitude generates low achievement. ${ }^{6}$ To change an existing negative attitude to more positive one might be one way of improving achievement. There are indications that such a change can be brought about. ${ }^{7,8}$

However, the attitude revealed by the pupils is not a unique attitude but a multidimensional attitude. ${ }^{9}$ To measure what attitudes are present and what attitudes can be changed and how they can be changed is a comnlex problem.

Dutton has given us indications of the attitude dimensions or facets. ${ }^{10}$ If a nupil were to dislike arithmetic because he finds it slow and tedious then any situation requiring arithmetic could nave a negative attitude transferred to it.

Perhaps by the removal of the necessity of doing tedious calculations the situation that formerly would have negative connotations could have a positive connotation. Perhaps the situation in itself might be interesting and rewarding which previously could not be because of the
negative attitude toward calculations. Several such programs have been attempted with varying results. $11,12,13$ In order to reduce the work involved in solving questions posed, desk:calculators have been used in home and industry. The manufacturers of desk calculators have expressed their optimism about the machines use in school. Newspapers have carried opinions from observations but do not validate the results. Headines such as "Calculator Takes Sting Out of Math" have generated questions about the use of calculators in a mathematical program. ${ }^{14}$

PURPOSE OF THE STUDY

In this chapter various sources have been cited which make claims about the effects of a calculator based program on the attitude and achievement of the low achiever. From these claims about improved attitude and achievement it would seem that, a calculator based program might be expected to produce several outcomes for the low achiever:

1. A positive change in general attitude toward mathematics.
2. A general improvement in problem solving achievement.
3. A positive correlation between a positive change in attitude and improvement in problem solving achievement.

A proposal for a study was put forth which would compare the attitudes and achievement of four classes
of grade 10 general mathematics students. Two classes using desk calculators and two control classes not using the calculators were to be used. Two teaciers would be involved each having one control and one exnerimental class. The classes would be taught as similarily as was feasible. The attitudes and achievements would be measured by selected tests before and after the experimental period. The measures would be statistically analysed for significant changes and correlations.*

## DEFINITION OF TERMS

(a) quantitative thinking: the assimilation of supplied data together with the application of basic operations of arithmetic to arrive at a solution to a question to which the student has not been told what steps or procedures are to be taken.
(b) general mathematics: a program designed by the British Columbia Department of Education to be given to trose students who are not on an academic-technical nrogram. It is normally given to those whose academic achievement is such that they would not be able to compete successfully with others more able.
(c) desk calculator: a machine such as the Olivetti Underwood Divisi-Suma 24 which is a ten key machine

[^0]performing addition, subtraction, multiplication, and division with printed output.

HYPOTHESES

Three hypotheses were formulated.

1. There will be a positive change in general attitude toward mathematics for the grade ten General Mathematics students using desk calculators.
2. The achievement in doing quantitative thinking
will show greater improvement for those using desk calculators than for those not using desk calculators.
3. Where there is a change in attitude toward quantitative thinking there will be a change in the same direction in the achievement in doing quantitative thinking.

## Chapter 2

REVIEW OF THE LITERATURE

## REVIEW BY ATKEN

In Aiken's (1969) review of the research concerning attitudes toward mathematics he discusses several methods of measuring attitudes even though some maintain that there are no valid measures. 15 Aiken notes that "observation is superficially the most objective measure but ...teacher observation (is found) to be inadequate."l6 A second method is a questionaire where students are asked to state true or false to a statement and the responses are tabulated to indicate an overall attitude or, in anotrer examnle, a preference to a subject field. ${ }^{17}$

The Thurstone and Likert attitude scaling techniques were found to be popular, but Guttman's scalogram analysis was infrequently used. A study by Nealeigh used picture preferences to measure attitudes and achievement proneness. 18 Picture preference was found to be discriminating at the third grade.

Physical indicators of anxiety such as electrical skin resistance, breathing characteristics, blood pressure and heart beat rate were found by Aiken to be used by a few experimenters. 19

When the attitudes were measured it was noted "that very definite attitudes toward arithmetic may be formed as early as the third grade. ${ }^{20}$ It was further noted that Junior High School was the period when a large percentage of prospective teachers formed their attitude. 21.

Aiken summarizes the section on measuring attitudes by saying:

> ...attitudes are probably not very stable in the early grades. In addition, the oreciseness with which pupils can express their attitudes varies with level of maturity. Finally it is clear that attitudes toward different asnects of arithmetic and mathematics are measured by "general attitude" instruments administered at different grade levels. Attitude toward materials to be learned by rote, such as the multiplication table, is not the same variable as attitude toward word problems and algebraic symbols. Of more importance than the exact frequency of attitudes at different grade levels, however, are the causes and effects of these attitudes.

It was noted that Alpert et al (1963) made an analysis of the relationships among attitude, expectation and performance, they viewed the level of exnectation and performance as a kind of self perpetuating cycle affecting the child's self concept. 23 Another study by Brown and Abell found the "correlation of attitude and achievement was higher for arithmetic than for spelling, reading or language." 24 Durrance found significant correlation between performance in mathematics and measures of attitude and anxiety. 25 It was noted by Cech that achievers had a more positive attitude toward mathematics than underachievers. 26 A study by Jackson maintains that it is only at
"the extremes - highly positive or highly negative - that attitude affects achievement in any significant way."27

Aiken summarizes his review of the relationships
among attitude, exnectation, and performance by saying:
Collectively, the findings of studies relating personality variable to mathematics attitude and achievement indicate that individuals with more positive attitudes and higher achievement tend to have better personal and social adjustment than those with negative attitudes and low achievement. These results must be kept in persnective, however. The correlations are relatively low, and it is a truism that correlation does not imply causation. Personalsocial adjustment, attitudes, and achievement not only interact with each other, but they are the effects of other home, school, and community variables.

In his review Aiken has also noted that "in order to change a pupil's attitude toward mathematics his perception of himself in relation to mathematics materials must be changed." ${ }^{29}$ That pupils who constantly fail lonse self confidence, develop dislike and hostility was noted by Lerch and others. ${ }^{30}$ The pupil must be provided with situations in which he can experience success. Low success experiences can be brought about is up to many people: the teacher, the text writer, the home and others.

At least one investigator (Tulock) recommended that
"games, contests and other audio-visual aids be used to heighten interest in mathematics." ${ }^{31}$ While Natkin suggests for the pupil "to associate mathematics with something pleasant may alter his attitude or anxiety toward the subject." ${ }^{32}$

Aiken exnressed himself by saying "the concept of a general attitude toward mathematics should be summemented with the attitudes toward more specific asnects of mathematics, for examrle, problem solving and routine drill.133

## ATTITUDE

The problem of the rupils' negative attitudes toward mathematics is not a new one. The solution to the problem has not yet been found, and as such, a study by Poffenberger and Norton has helped to show how negative attitudes are formed. They felt the nresent lack of interest in mathematics is largely a cultural phenomenon and that the attitudes of some children a re conditioned by the family.

They state that:
Attitudes are develoned in the home in some cases before the child begins school. In the first and second grades he is affected not only by his teacter and his readiness to deal with numbers, but also by the attitude of his parents toward the subject matter. He carries into his high school mathematics classes attitudes that are long in building and difficult to change. Certainly it is logical to exnect that the student wo goes into a class with the thought "Here is another lousy math class", is severly handicapned. 34

A predetermined attitude is difficult to overcome; as they relate:

Students with an initially negative attitude toward mathematics may go into the classroom with a mental attitude set against the subject which may be maintained even when positive identification with the teacler is made. It is therefore of considerable importance that parents and teaclers in the early grades make every effort to give positive experiences with arithmetic. ${ }^{35}$

In order to determine what the attitudes of pupils toward arithmetic were, Dutton has develoned an attitude scale. ${ }^{36}$ Dutton's scale consists of twenty-two questions which are to be checked if the person agrees with them. It is an attempt to objectively measure the attitudes of a ruril or teacher. From the administration of his scale Dutton has concluded that pupils dislike arithmetic because of the following:

1. Lack of understanding--confused by thoughtproblems without practical applications.
2. Arithmetic was hard--made poor grades--irregular attendance.
3. Poor teachers--punishment--frightening experiences. 4. Not sure of self--insecure--mental blocks--fear of failure.
4. Time factors--not enough time--takes too long-pressure.
5. Arithmetic was boring, stale--slow learning, behind.
6. Too much drill--memorization. ${ }^{37}$

It seems from the responses received by Dutton, then, that if some of the reasons for disliking arithmetic were removed or reduced, improvement in achievement could result. For again, Dutton states:

How pupils feel toward arithmetic is imnortant. Liking arithmetic has a nronounced effect unon the amount of work attempted, the effort expended, and the learning that is acquired. 38

He also concludes that:
Apparently lasting attitudes toward arithmetic are developed at each grade $3 g$ vel. Grades V and VII were pronounced most crucial.

Some contradiction of conclusions occurs for a study
by Stright concludes that a consistently better score is made by girls in general studies, while boys have a slight etge in arithmetic, geography, and science. The Stright study found that girls liked arithmetic better than boys. It made no attempt to correlate arithmetic achievement and attitude toward arithmetic. 40

## ATTITUDE A $\because$ D ACHIEVEMENT

It has been hypothesized that attitudes are formed early and that they play a role in the general achievement of the student. 41 A study by Fedon set out to determine some of the attitudes held toward arithmetic. Fedon then used the scaled attitudes toward a mathematical program to evaluate that program. In his summary he states:

The scales discussed in this paper are an attemnt to cast light on some aspects of the arithmetic curriculum that are liked and disliked. Attitudes nlay an important part in the success of the arithmetic orogram. If we feel that they are a valid criteria for evaluating the effectiveness of our program, then the application of this scale will provide better onnortunities to study children's reactions as they exnerience arithmetic in daily life. 42

To relate attitude and achievement, Stenhens used a histogram to compare the attitudes of an accelerated and a remedial class. The histogram indicated that the accelerated
class did have a more positive attitude toward arit?metic than the remedial class. ${ }^{43}$

Another study by Lyda and Morse reaches the conclusions:

1. When meaningful methods of teaching arithmetic are used, changes in attitudes toward arithmetic take place. Negative attitudes become positive, and the intensity of positive attitudes becomes enhanced.
2. Associated with meaningful methods of teaching arithmetic and changes in attitude are significant gains in arithmetic achievement, that is, in arithmetical computation and reasoning. 44

This study, then, indicates that methods change attitudes. However, the effect attitude has on achievement is open to question as Aiken and Dreger conclude that "...the hypotriesis of significant contribution of mathematics attitudes to prediction of achievement is borne out for females, but not males."45

## CALCULATORS, ATtitude And AChtevement

A study by Fehr, McMeen, and Sobel, using hand operated computing machines at the grade five level made the hypothesis:

Pupils who use comnuting maci:ines to learn arithmetic will gain significantly in paper and pencil com-utations, and in arithmetic reasoning over those who do not use the computing machines. ${ }^{46}$

This hypothesis was substantiated by their exneriment. It
was concluded by the experimentors that:

1. Machine-taught students gain more in reasoning ability;
2. Machine-taught students gain more in commutation ability;
3. Machine-taught students learn more also, because they understand machine computation as well as ordinary arithmetic;
4. The interest of students and teachers in arithmetic is heightened by the use of machines;
5. The machines fit into our present culture. They give additional learning. 47

The conclusion reached by Fehr, McMeen, and Sobel was not that reached by Durrance in his experiment, for he concludes that "...there is no proof that the use of the calculator will significantly enable a student to achieve in arithmetic. ${ }^{48}$

A study was performed by Cech using desk calculators with ninth grade general mathematics students to test for improvement in (1) their attitude toward mathematics (2) their paper and pencil computational skills (3) the use of calculators to obtain computational results. The PYOll ProMath Composite test developed by SMSG was used and as measured by this test showed no significant differences in attitude. This test does not, however, differentiate among attitude toward computation, toward quantitative thinking and toward mathematics in general.

The Stanford Diagnostic Arithmetic Test, Test 2, Parts $A, B$ and $C$ was used to measure the computational skills. The results showed no significant differences in the skills of the experimental and the control groups. The Stanford test did not test for imnrovement in achievement in more complex quantitative thinking.

The experiment did show significant improvement of the experimental g roup in computing using calculators as opposed to the group computing without them. 49

Various grade levels have been used to test the effect of calculators or similar machines on the achievement of the pupils. The studies by Fehr et al, Durrance, and Cech give quite dissimilar results. It is supnosed trat attitude affects learning, but not much has been shown that substantiates this claim for mathematics. Little research has been done at the grade ten level to see if attitudes can be changed. It seems that attitudes are formed early, but it has not been determined that they can be changed at the later stage.

The attitudes of children toward arithmetic are formed early in the home. Children's attitudes can sometimes be moderated by a positive identification with the teacher and positive experiences with arithmetic. According to Dutton, the amount of work attempted, effort expended, and learning acquired is dependent upon the attitude toward arithmetic. The relationshin of attitude and achievement is open to question, for Stephens indicated that the relationship has a positive correlation, while Aiken and Dreger found that the relationshin was a predictor for girls but not for boys.

Studies using calculators are not consistent in their general conclusions. Fehr et al found that there was a
significant improvement in arithmetic comnutation and reasoning, while Durrance found no significant change in achievement.

## Chapter 3

## DESIGN OF THE STUDY

## PURPOSE

The purpose of the study was to determine whether the use of a desk calculator would produce a positive change in a pupil's attitude toward mathematics, to see if achievement in quantitative thinking would be increased, and to see if there was a correlation between any such changes.

## PROCEDURE

Two teachers, including the experimenter, and four classes of grade ten general mathematics pupils, were involved in the experiment. Each teacher had two classes, one class using desk calculators, the other using no calculators.

All classes involved were grouped by the school adininistrators at the start of the school year according to the pupils' programs, the electives chosen and classes available. No steps were made to randomize the pupils. Each class was pre-tested with the Dutton attitude test, the experimenter's own mathematics attitude test and the Iowa achievement test. The calculators were situated in one room to be used by the experimental group. Tise control
group was taught in a different room to prevent contamination due to the presence of the calculators.

The teachers met before and during the experiment to discuss the lessons to be presented. Each class was kept as similar as was feasible with different teachers and classes. While the experimental group was being taught to use the calculators, the control groun was taught to find and eliminate errors in calculating.

During the first lessons the control groun was taught some ways of renembering numbers, copying numbers correctly and adding certain number combinations. Grouns of numbers were supplied which the pupils were to copy and add. Some exercises were timed and checked for accurracy. In the second lesson pupils were asked to copy and add numbers with three digits in columns and check. The pupils were grouped in pairs in which one read and the others copied, then added. The third lesson consisted of exercises for quick addition and subtraction.

In the experimental groups the pupils were grouped in pairs with one macinine. They were instructed in the function and operation of the machine. While one was using the machine, the other would mentally add and verify some of the results. The ways of remembering and conying numbers were explained. The two remaining lessons were similar with both pupils alternately operating the machine. The operations on the machine were gradually broadened in scone.

The lessons of the experimental and the control groups were as similar as was feasible. For examole, one lesson dealt with the weekly earnings of a number of employees. The control group was expected to do twenty computations while the experimental group because of the speedier calculations was expected to do thirty computations. The type of question was not varied, only the number of questions. The standard types of questions and exercises were utilized. The questions dealt with payroll computations, commission earnings, simple interest and installment buying. The calculations involved the basic operations either singly or in combination.*

There were sixty pupils in the experimental grouns, and fifty-seven in the control groups. The exneriment ran for seven weeks during February, March and Anril of 1968. All three tests were again administered to all pupils at the completion of the experimental program.

## HYPOTHESIS

Three hypotheses were formulated.

1. There will be a positive change in general attitude toward mathematics for the grade ten General Mathematics students using desk calculators:
2. The achievement in doing quantitative thinking will show greater improvement for those using calculators than for those not using calculators.
*For further details please see the anpendix.
3. Where there is a change in attitude toward quantitative thinking there will be a chauge in the same direction in the achievement in doing quantitative thinking.

## TEST SELECTION

## Arithmetic Attitude

A test to determine a pupil's attitude toward arithmetic has been constructed by Dutton. 50 Dutton had college students write statements conceining their likes and dislikes toward arithmetic. From these statements, some forty-five statements were retained. These were screened further and twenty-two statements were retained.

The statements on examination were found to refer to arithmetical operations or procedures which could elicit various feelings. The feelings mentioned annear to be both positive or negative to varying degrees. The statements could be considered as having face validity for an indication of a person's attitude toward arithmetic.

By using methods suggested by Thurstone and Chave, a scale of value for each of the twenty-two statements was established. 51 These scaled values range from 1 to 10.5 . The pupils were asked to check the statements that they agreed with. They could check as many or as few as they wished. The scores for each statement checked were tabulated and totalled, and an average response score was arrived at. That is, if five statements were checked whose
values totalled thirty, then the average resmonse score would be six.

The reliability of the test using the average response score method on a test-retest is renorted to be $0.94 .^{52}$

Quantitative Thinking Achievement

Selection. In selecting a test to measure the achievement attained in quantitative thinking, a test was sought which covered material apnropriate for the pupils involved with the general mathematics course at th:e grade 10 level, a test which could be administered within the one hour period alloted for this course, a test which could be administered and scored easily, as well as a test that had suitable standardization and statistical background.

The test chosen was "The Iowa Test of Educational Development Test 4: Ability to do Quantitative Thinking: Form $Y$ - $3 S^{\prime \prime}$. The questions are concerned with the ability of the student to use basic arithmetic ideas and nrinciples to arrive at solutions to the questions nosed. The questions involve mainly finance and measurement situations. There is no attempt, however, to measure the ability to do abstract thinking or to identify students who have special mathematical ability. The test was designed for use with grades nine to twelve. There was a class version consisting of thirtythree multiple-choice questions that could be administered
within the sixty minute period. The test did not require laborious calculations by the students, but did require an analysis of the question followed by a selection of the correct arithmetical operations. The test did require that a student have a good arithmetical sense as the distracters in the multiple choices were such that the student must know what is correct before he could, excent by chance, select the correct response. 53

For example, one of the questions was:
In a short cut method of multiplying $4087 \times 198$, one first multiplies $4087 \times 200$. What product must then be subtracted from this result in order to obtain the correct answer?

1) $13 \times 2$
2) $4087 \times 2$
3) $4087 \times 13$
4) Not given
5) $198 \times 2$

The student could know that 4100-13=4087 and 200-2 $=198$ but not know what to do with them. They then could be distracted to the wrong response.

The test could be machine scored readily by having the student place his responses on a mark sense card. The test has been widely administered. It is rated as good to excellent by the Mental Measurements Yearbook. 54

Reliability. The reliability is said to de about
0.91 for pupils of the same grade. 55

No test was found by the experimenter that would measure the attitude toward calculating and also toward quantitative thinking. It was then determined trat such a test should be constructed.

## TEST CONSTRUCTION

A test was sought which would differentiate from those who like simple calculations but do not like questions involving planning, evaluating or other mental caracities, as well as a test which would differentiate those who would prefer the latter but not the former. Three areas were selected which might allow this type of differentiating.

Definition of Terms

Calculation: (A): the performance of a specified operation or operations on a given set of numbers.

Quantitative Thinking: ( $B$ ): the assimilation of supplied data together with the application of basic operations of arithmetic to arrive at a solution to a question to which the student has not been told what stens or procedures are to be taken.
Non-mathematical Activity: (C): an activity
either physical or mental not involving mathematics.

A grade ten geometry class was asked to think of one desirable activity and one undesirable activity in mathematics and also outside mathematics. The author then supplied some others of his own. Some of the suggested or supnlied items were then placed in the following categories; Calculation (A), Quantitative Thinking (R), and Non-mathematical Activity (C) and nresented as a group of three to the nupils, who were asked to nut a check for the one that they liked the most and a cross for the one they liked the least. An example was given as follows:

1. (a) Add up a column of 20 three digit numbers
(b) Solve the equation $3 x+2=20$ (c) Wash the breakfast dishes

| $A$ | $B$ | $C$ |
| :---: | :---: | :---: |
| $V$ |  | $X$ | There were in the original test some thirty situations, each with a statement for each category. The original test was labelled Type $I$, and administered to two classes of General Mathematics 9. Since the students were General Mathematics students they were assumed to be as near as possible to the experimental classes in attitude and achievement.

The responses of the tiwo General Mathematics 9 classes were then tabulated for each of tiae categories in the thirty situations. From an insnection of the resnonses, certain statements were revised to be more selective. One of the original statements was "Watch television". This
statement was changed to "Watch Batman on television". The original had received many like but no dislike resnonses; thus it was revised so that it elicited both positive and negative responses.

The revised thirty item test was labelled Tyne II, and administered to another two classes of grade nine general mathematics students.

A tabulation was made of the responses from this tabulation, many statements were rejected or reassembled in different combinations. The final form consisted of sixteen situations, each containing a statement from category $A, B$ and $C$. This final revision was administered to a grade nine algebra class that was homogeneously grouped according to past marks in mathematics. Only those students with C+ or better were placed in this class. The resultant scores were compared with the teacher's expectations of student attitude and found to indicate a strong preference for calculating followed closely bs a preference for problem solving and a low preference for non-mathematical activities.

## Scoring

A score for each category was obtained by allotting two marks for each positive response in that category (a check mark), one mark for each blank in that category, and no marks for a negative response (a cross) in that category. Hence, there is a possible high score of
thirty-two, and a low score of zero. Each of the three categories, $A, B$, and $C$, received serarate scores.

Scores were than available for an attitude or preference toward calculation (A), quantitative thinking ( $B$ ), and $t$ e indicated non-mathematical activities (C). A mathematics attitude score was derived by adding tine $A$ and B scores.

Item Analysis
An item analysis was performed on the test using methods suggested by Garrett ${ }^{56}$ whereby the top and bottom twenty-seven percent of the scores were compared. A biserial 'r' was computed for each item in each category and with one exception all items were found to be above the suggested 0.20 minimum. The exception was number twelve of the problem solving category. A comolete analysis may be found in the appendix.

Reliability of Attitude Test
A test for reliability using tle split-half method was performed. The sixteen items were split on an odd-even basis. A response of a coeck was scored one, and a blank or a cross scored zero. Tlie odd-even scores were tabulated on this basis for each of the three categories.

The correlation for the category $A$ odd-even was found to be 0.4782 . The correlation for the category $B$ odd-even was 0.5433 , and for the category $C$ odd-even was
0.7001. When using the Spearman-Brown nronhecy formula for estimating reliability from snlit halves, ${ }^{57}$ figures of 0.6470 for $A, 0.7048$ for $B$, and 0.8236 for $C$ were obtained. The reliability figures are above the minimum of 0.60 which Garrett ${ }^{58}$ considers necessary for differentiating between means of groups, but not sufficient for the differentiating of individual differences.

## STATISTICAL DESIGN

The Mathematics Attitude Test yielded two subscores designated "Attitude toward Calculation" (A) and "Attitude toward Quantitative Thinking" (B). Category (C) was used as a distractor. The score for "Attitude toward Mathematics" was derived from the sum of these two subscores $(A+B)$.

Data for the individuals were obtained from the scores on the Mathematics Attitude Test and were designated $A_{1}, B_{1}$ and $A_{1}+B_{1}$ for the nre-test and $A_{2}, B_{2}$ and $A_{2}+B_{2}$ for the post-test. The Dutton Attitude Scale yielded scores designated $D_{1}$ and $D_{2}$ for the pre-test and nost-test, while the Lowa Achievement Test scores we:e designated respectively $P_{1}$ and $P_{2}$ for the pre-test and post-test.

A score was obtained from the Mathematics Attitude Test for change in attitude toward Quantitative Thinking by finding the difference between the post-test and pre-test scores of Attitude toward Quantitative Thinking ( $B_{2}-B_{1}$ ).

A score for change in achievement was found by taking the difference between the post-test and pre-test
scores of the Iowa Achievement Test, and was designated $P_{2}-P_{1}$.

Analysis of Covariance was used to evaluate the scores from Dutton's Test ( $D_{1}$ and $D_{2}$ ), the Mathematics Attitude Test $\left(A_{1}+B_{1}\right.$ and $\left.A_{2}+B_{2}\right)$, and the Iowa Achievement Test on Quantitative Thinking ( $P_{1}$ and $P_{2}$ ). The levels of significance were found from a table by Dixon and Massey. 59

The correlation of the scores from Change in Achievement $\left(P_{2}-P_{1}\right)$ and Change in Attitude toward Quantitative Thinking ( $B_{2}-B_{1}$ ) were calculated for the experimental and the control groups. These correlations were then checked for significance with the respective number of degrees of freedom.

## Chapter 4

ANALYSIS OF TIE RESULTS

## FIRST HYPOTHESIS

The hypothesis that there would be a positive change in general attitude toward mathematics for students using calculators was tested using analysis of covariance.

The number of observations made was fifty-seven for the control group and sixty-one for the experimental group for a total of 118 observations. Dutton's Attitude Scale yielded results as follows:

CONTROL
Mean S. D.
Pre-test $5.376 \quad 1.247$
Post-test
5.326
. 1.127

EXPERIMENTAL Mean S. D. 5.5131 .413 5.562 1.373

The F-ratio was found to be 0.89 which, with 115 degrees of freedom, is not significant.

The results of the Mathematics Attitude Test using trie subscore "Attitude toward Mathematics" ( $A+B$ ) were as follows:

|  | CONTRCL |  | EXPERTMENTAL |  |
| :--- | :--- | :--- | :--- | :--- |
|  | Mean | S.D. | Mean | S.D. |
| Pre-test | 33.70 | 6.902 | 32.20 | 6.623 |
| Post-test | 31.98 | 7.308 | 30.98 | 6.828 |

The F-ratio for the "Attitude toward Mathematics" subscore was found to be 0.00 which with 125 degrees of freedom is not significant. The conclusion must be that there was no significant change in attitude.

## SECOND HYPOTHESIS

The hypothesis that the achievement in doing quantitative thinking would show greater improvement for those using calculators than those not using calculators was tested using the Iowa Achievement Test results with analysis of covariance. The results of the test were as follows:

|  | CONTROL |  | EXPERIMENTAL |  |
| :--- | :--- | :--- | :--- | :--- |
|  | Mean | S.D. | Mean | S.D. |
| Pre-test | 11.82 | 4.445 | 13.08 | 3.625 |
| Post-test | 12.74 | 4.249 | 14.51 | 3.404 |

A F-ratio of 3.33 with 115 degrees of freedom was not significant at the 0.05 level. It is, however, significant at the 0.08 level. The conclusion must be that there is no significant change in the achievement in doing quantitative thinking by using a calculator-based program.

## THIRD HYPOTHESIS

The raw scores were handled by a computer and it was asked to establish the necessary calculations. Since the computer was pre-programmed without prior knowledge of any
results, the third hyrothesis was tested along with the first two.

The hypothesis that where there is a positive change in attitude towards quantitative thinking there will be a change in the achievement in doing quantitative thinking was tested by their correlation. The correlation between change in attitude toward quantitative thinking as measured by the Mathematics Attitude Test and the achievement in doing quantitative thinking as measured by the Iowa Achievement Test was found to be for the experimental groups $\mathbf{- 0 . 0 4 9 2 ,}$ and for the control groups, 0.2105. Neither of the correlation figures is significant at the 0.05 level. The conclusion must then be that there is no correlation of change in attitude toward quantitative thinking and change in achievement in doing quantitative thinking.

## Chapter 5

SUMMARY AND CONCLUSIONS

Research into attitude formation indicates that there is a positive correlation of attitude and ac!ievement in mathematics. Some of the attitudes toward mathematics are formed in junior high school. It was thought that some of the reasons for disliking mathematics might be decreased if a desk calculator could be used to work the tedious calculations.

The purpose of the study was to determine whether the use of a desk calculator would change a pupil's attitude toward mathematics, to see if achievement in quantitative thinking would be increased, and to see if there was a correlation between any such changes.

Four classes of grade 10 General Mathematics pupils were used with two teachers. Each teacher taught one control and one experimental class using desk calculators. The classes were pre-tested and post-tested using Dutton's attitude scale, the Iowa Achievement Test and the experimenter's own Mathematics Atti.tude Test.

For the nrogram used, the grade level tested, and the tests used to measure possible changes in attitude toward mathematics, increases in achievement in quantitative thinking and correlation between any such changes, there
is no evidence for a positive correlation of change in attitude and achievement while the increased achievement in doing quantitative thinking using desk calculators is not significant at the 0.05 level it is significant at the 0.08 level.

## LIMITATIONS

The study was carried out with general mathematics students at the grade ten level. No randomization was done as the groups were not nre-selected for the program as they had been placed by the administration in their respective classes. Often the placement is done in conjunction with some specific program being predominant in a class. The program predominance was not taken into account. It was assumed that some equivalence of attitude and achievement could be accomodated for in the test results by analysis of covariance.

Poffenberger's study suggested that the attitudes of pupils are established early. It is only after several positive mathematical experiences have been encountered that a change of any degree could occur. Because attitudes are formed early and several positive exneriences must be encountered before a chainge is made could mean that any experience would have to be long-lasting and of considerable strength to shift attitude at the grade ten level. The general mathematics students have not met with a great deal
of success in mathematics and hence may have different attitudes from students in regular mathematics classes.

The two groups were taught in different rooms, and the environment may have had some influence on their attitude. The control groups were taught in a relatively new class room using conventional desks. The experimental groups were taught in an older building using tables and chairs. Going to the older building necessitated a trin outside, frequently in the rain. The physical factors could have had a negative effect on the attitude of the pupils taking part in the experimental group.

Some room changes were necessary in order to conduct the study. Pupils frequently do not like changes in their established patterns. They like a particular seat in a particular row, and any changes threaten their feeling of security. Changes in seats and rooms may again have had a slight negative influence on attitudes and achievement. Other factors that may have affected the results of the study include the noise generated by the calculators when operating. The noise may have been distracting to the pupils. The program selected may not have been the best possible for any interaction between pupil, teacher, and calculator. The duration of the study was seven weeks, which may not have been long enough to remove a possible Hawthorne effect or to overcome any deep seated negative feelings toward mathematics.

It has been argued that while the experimental group was learning to operate the machines, the control group could have been doing some irrelevant exercises. It was thought by the experimenter that the procedure used was fair to both groups in that the experimental groups would not have an edge over the control groups in reading, remembering, and copying numbers. Reading, remembering, and conying were thought to be separate from machine operation influence. With two teachers there may have been a bias for one teacher for a particular class. It was hoped by the experimenter that teacher bias was kept to a minimum. An attitude test is open to suspicion in that it is not known if the pupil can relate what his feelings are toward a subject. He may put down what he thinks he is expected to put down, rather than what he actually feels. We may not have measured what we should have measured.

A test which uses multiple choice responses for selection of the correct response may be quite different in results from one which is open-ended. Hence the Iowa achievement test shows only what can be done on that test, not what a pupil might do on an open-ended test question such as he would find in a real life problem situation. The concepts involved in problem solving or in quantitative thinking are not clearly understood. Perhaps what was measured was not that which was influenced or changed. The organization of a pupil's thinking was not
tested, and it is this organization, perhaps, which is more aptly shaped by the calculator, rather than what was tested. A study was done with grade ten general mathematics pupils at a school whose socio-economic level is one of lower middle class, stable, home situations. It has strong European cultural background. Tine cultural, social and economic influences present make it difficult to generalize to other grades, types of classes, cultural, social or economic situations.

## IMPLICATTONS FOR FURTHER RESEARCH

The study was with grade ten general mathematics pupils whose attitudes are usually firmly entrenched. The study was of a relatively short duration and, as such, it may not have been of sufficient length to overcome the inertia of the deep-seated attitudes. A study involving younger pupils for a longer period of time might yield significant changes.

There are now on the market newer, quieter machines which may not have the distracting noise level of the machines used. A study using electronic calculators might prove of significant worth.

The results of particular students were not analysed. What kind of attitude or achievement is changed? Would the program have been the same for pupils with an initially higher level of achievement, but poor attitude?

During the operation of the calculator, the nupils had to know what numbers and what operations were to be used. However, some would use a trial and error method in solving the question. If multiplication did not work, then they would try division. Perhaps it might be advisable to teach organization of the data by means of a flow chart. That is to say, that students would be required to show what numbers and what operations were to be performed before the calculations were done by the machine. The flow chart procedure might lead to a better insight into how to solve a problem.

The role of the calculator in facilitating the pupil to achieve better is not clearly understood. Whether the additional experience in working with the aid of the calculator to do more work in a given time, whether the required organization of the situation before the calculations are performed or whether some other factors are enabling the pupil to greater achievement are questions yet to be investigated.

The teacher needs information about a punil's attitude in order to conduct remedial procedures. The mathematics attitude test constructed for the study seems to be a valid and reliable test. With some modifications, the test could be useful to others.
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## APPENDIX A

## SAMPLE PROBLEMS

Some of the questions asked were similar to the following. A. Payroll Computation

Name Hours Rate Gross Pay Income Tax Pension Net per hour Pay

|  |  | 45 |
| :--- | :--- | :--- |
| b | 42 | 3.25 |
| c | 40 | 2.00 |
| d | 40 | 2.75 |
| e | 43 | 2.75 |
| f | 36 | 2.50 |
| g | 32 | 2.50 |

TOTALS
B. Commission

1. Calculate the commission received by the agent in each of the following:
Sales
a) 1560.00
Rate of Commission
$15 \%$
b) 750.00
22\%
Commission
2. Calculate the rate of commission in each of the following:
Commission
Rate
a) 4000.00
b) $70,000.00$
340.00
+200.00
3. Calculate the amount of sales in each of the following:

Commission
a) 320.00
b) 460.00

Sales
C. Simple Interest

1. Calculate the interest in each of the following:
Principal
a) 450.00
b) 1250.00
Rate
$5 \%$
$3 \frac{1}{2} \%$
Time
2 yr.
3立 yr .
2. Find the rate of interest in each of the following:
a) $\begin{aligned} & \text { Principal } \\ & \$ 500.00\end{aligned}$
b) 1200.00
Interest
30.00
90.00
Rate
Time
1

yr. | $1 \frac{1}{2} \mathrm{yr} \mathrm{yr}$. |
| :--- |

3. Calculate the principal that must be invested at $6 \%$ to yield $\$ 60.00$ interest in 8 months.
4. Calculate the time in each of the following:
Principal
Interest
Rate
Time
a) $\$ 750.00 \quad 40.00$
b) $350.00 \quad 19.25$
$5 \frac{1}{2} \%$
D. Installment Buying
5. Tom Barrow borrowed $\$ 100$. He promised to repay the loan in 10 installments of $\$ 10$ each at $1 \frac{1}{2} \%$ monthly on the unpaid balance. Make a table showing:

Payments on principal
Balance due Interest charges Each monthly payment Total interest charges Total of repayments and interest

APPENDIX B

DUTTON'S ATTITUDE SCALE

1. I think about arithmetic problems outside of school and like to work them out.
2. I don't feel sure of myself in arithmetic.
3. I enjoy seeing how rapidly and accurately I can work arithmetic problems.
4. I like arithmetic, but I like other subjects just as well.
5. I like arithmetic because it is practical.
6. I don't think arithmetic is fun, but I want to do well in it.
7. I am not enthusiastic about arithmetic, but I have no real dislike for it either.
8. Arithmetic is as important as any other subject.
9. Arithmetic is something you have to do even though it is not enjoyable.
10. Sometimes I enjoy the challenge presented by an arithmetic problem.
11. I have always been afraid of arithmetic.
12. I would like to spend more time in school working arithmetic.
13. I detest arithmetic and avoid using it at all times.
14. I enjoy doing problems when I know how to work them well.
15. I avoid arithmetic because I am not very good with figures.
16. Arithmetic thrills me, and I like it better than any other subject.
17. I never get tired of working with numbers.
18. I am afraid of doing work problems.
19. Arithmetic is very interesting.
20. I have never liked arithmetic.
21. I think arithmetic is the most enjoyable subject I have taken.
22. I can't see much value in arithmetic.

## APPENDIX C

## MATHEMATICS ATTITUDE TEST

NAME $\qquad$
BLOCK $\qquad$
DATE
ATTITUDE TEST FOR MATHEMATICS
You will be presented with 16 situations in which you have three choices (A), (B), (C). From these three choices you are asked to select the one you like the least and the one you like the most.

Put a check ( $\sqrt{ }$ ) in the square for the one you like the best. Put a cross ( $X$ ) in the square for the one you like the least.

## EXAMPLE

1. a. Add up a column of 20 three digit numbers.
b. Solve the equation $3 x+2=20$.
c. Wash the breakfast dishes.


SCORES


1. a. Add up a series of fractions.
b. Find the number when four more than the number is three less than three times the number.
c. Watch Batman on TV.
2. a. Add up a column of 15 numbers.
b. Determine the number of nine inch tiles needed to cover this floor.
c. Learn to waltz.
3. a. Multiply a series of fractions.
b. Find the length of a rectangle when the perimeter and width are known.
c. Paint a picture of some colorful stones.
4. a. Multiply five, two digit numbers.
b. Two men together can do a piece of work in six days. One man working alone can do the job in ten days. How long would it take the second man himself?
c. Type out this page.
5. a. Find the square of 3.2.
b. You paid $\$ 60$ for a radio at a sale which advertised that all prices were reduced by one third. What was the original price?
c. Dust the living room.
6. a. Work some subtraction questions involving decimals.
b. Mary is ten and her aunt is forty. When was Mary one-seventh as old as her aunt?
c. Give an oral report in Social Studies.

| $A$ | $B$ | $C$ |
| :--- | :--- | :--- |
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7. a. Find the square root of 169 .
b. There are 120 apple and peach trees in an orchard. There are $2 / 3$ as many peach trees as apple trees. How many trees of each are there?
C. Run laps during P. E.
8. a. Find out what percent one number is of another.
b. The head of a fish is 10 inches long, the tail is as long as the head plus one half of the body; the body is as long as the head and the tail together. How long is the fish?
c. Rake up the leaves on the lawn.
9. a. Find out what percentage one number is of another.
b. A baseball team won 25 more games than it lost. If it won $3 / 5$ of its games, how many did it play?
c. Make a drawing of a coffee table that you are going to have built.
10. a. Round off seventy numbers to the nearest hundred.
b. A toy train consists of an engine, a dining car and a caboose. The whole train is 26 inches long. The relation of the lengths of the dining car and the engine is $(3,2)$ and the relation of the lengths of the caboose and the dining car is (1,2). What is the length of each section?
c. Mow the lawn.
11. a. Work out five division questions involving numbers of seven digits.
b. A policeman chases a thief. The policeman takes two steps while the thief takes three. But each sten the noliceman takes covers as much distance as two of the thief's stens. How many steps will the policeman take before he catches the thief?
c. Design a desk for your homework.

12. a. Determine the weight of 990 bags weighing seventy-five pounds each.
b. Determine the cost of making your own tent.
c. Listen to an oral report by a classmate about weeds.
13. a. Determine the volume of a fish tank.
b. On a remote island there are two tribes, The Tee's and the L's. The Tee's always tell the truth. The L's always lie. A stranger to the island comes to a groun of three natives who all look alike. He asks the first what tribe he belongs to but can't make out the answer so he asks the second native what he said and gets the reply "He said he was a Tee." The third says the second was a lier. To what tribe did the third man belong?
c. See a stage play.
14. a. Calculate the price of an article if it is to be reduced by $1 / 3$.
b. You have planned to take a boat trip for 20 people, two were unable to go. You each had to pay an additional three dollars. How much was the boat rental?
c. Write a letter of thanks for a birthday gift.
15. a. Compute the weight of 1000 yards of wire at ten pouncls for each 1000 feet.
b. Solve an algebraic equation.
c. Go for a car ride with your grandparents.
16. a. Find the common divisor of four numbers.
b. Graph an algebraic equation.
c. Sing in a mixed choir.


## APPENDIX D

IOWA TEST OF QUANTITATIVE THINKING

## ABILITY TO DO

## QUANTITATIVE THINKING

DIRECTIONS: In working these problems, pay no attention to the suggested answers until after you have got your own answer. If your answer agrees with one of the suggested answers, mark the corresponding box on the answer sheet. If your answer does not agree with any of the answers given, mark the last box in the corresponding row on the answer sheet. There are many problems to which the correct answer is not given. Do not rework a problem simply because your answer is not among those suggested. Instead mark the fifth box ("Not given" or "None of these") and go on to the next problem. The three sample problems have been marked correctly on the answer sheet. Notice how they are marked, and mark the remaining problems similarly.

Samples:
0 . What is the cost of 3 pounds of butter at $50 \phi$ per pound?

1) $\$ 1.15$
2) $\$ 1.65$
3) $\$ 1.35$
4) Not given.
5) $\$ 1.50$
0. Mrs. Smith, in paying for $60 \phi$ worth of groceries, gave the clerk a dollar bill. How much change should she receive?
1) 254
2) $60 \phi$
3) $30 \phi$
4) Not given.
5) $50 \phi$
0. If one gallon of paint will cover 200 square feet, approximately how many gallons of paint will be needed to cover 380 square feet?
1) I gallon
2) 4 gallons
3) 2 gallons
4) 5 gallons.
5) 3 gallons

NOTE: Do your figuring on scratch paper. Make no marks on these pages!

1. A three-pound beef roast contains five ounces of bone. The weight of the bone is what part of the total weight?
1) $3 / 80$
2) $15 / 16$
3) $5 / 48$
4) Not given.
5) $5 / 19$
2. The ingredients for making 24 chocolate marshmallow balls, as given in a particular recipe, are:

2 squares (2 oz.) unsweetened chocolate
l cup evaporated milk
1/2 cup granulated sugar
12 marshmallows, halved
1 cup finely chopped walnuts

How many ounces of chocolate would be needed in order to. 57 make 3 dozen marshmallow balls?

1) $21 / 4$
2) $31 / 2$
3) $21 / 2$
4) Not given.
5) $31 / 3$
3. In a short-cut method of multiplying $4087 \times 198$, one first multiplies $4087 \times 200$. What product must then be subtracted from this result in order to obtain the correct answer?
1) $13 \times 2$
2) $4087 \times 2$
3) $4087 \times 13$
4) Not given.
5) 198 x 2
4. How many pencils selling at 2 for $5 \neq$ can be bought for $60 \phi$ ?
1) 30
2) 6
3) 15
4) Not given.
5) 12
5. The map of a summer camp was drawn according to the scale 1 inch $=1 / 8$ mile. What is the length of the camp ground in miles if the corresponding distance on the map measures 10 inches?
1) .8 Mile
2) 80 miles
3) 1.25 miles
4) Not given.
5) 8.2 miles

NOTE: If you do not recognize quickly how a problem should be worked, skip it and go on to the next. You may come back to the harder problems later if time permits.
6. What is the next term in the series $20,6,1.8, .54, .162$ ? 1) . 0162 2) . 0486 3) . 0540 4).0810 53 Not given.
7. The following quotation is from an automobile club pamphlet.
"If you are in an accident while your car is traveling under 40 miles an hour, there is one change in 44 that someone will be killed. If an accident occurs while your car is traveling over 40 miles an hour, there is one chance in 19 that someone will be killed."

Which of the following statements best interprets the meaning of this quotation?

1) A car traveling faster than 40 miles an hour will kill 25 more people than will a car traveling under 40 miles an hour.
2) If all people drove faster than 40 miles an hour, one in 19 would be killed.
3) If all people drove at 30 miles an hour, nobody would be killed.
4) If people drive more than 40 miles an hour, the danger of fatal accidents increases.
5) Out of 1,000 people traveling by car, 19 will be killed by fast drivers, and 44 by slow drivers.
8. Exactly how many hundreds are contained in the number
1) 63.517
2) 517
3) 635.17
4) 63,500
5) Not given.
9. To make bookshelves, a board 8 feet 9 inches long was cut into three equal sections. How long was each section?
1) 2 feet, 6 inches 2) 2 feet, 11 inches
2) 3 feet, 5 inches 4) 3 feet, 10 inches 5) Not given.
10. An agent received commissions at the rate of $2.5 \%$ on a sale of $2,500,3 \%$ on a sale of $3,000,3.2 \%$ on a sale of $\$ 3,200,3.5 \%$ on a sale of $\$ 3,500$, and so on. According to this pattern, how much money would his commission amount to on a sale of $\$ 4,000$ ?
1) $\$ 16$
2) $\$ 40$
3) $\$ 160$ 4) $\$ 400$
4) Not given.
11. A family bought a two-family house, planning to rent one apartment and live rent-free in the other. They were not interested in making any further profit. Yearly expenses on the house were estimated at $\$ 140$ for taxes; $\$ 340$ for insurance, upkeep and depreciation; and $\$ 200$ for other expenses. What monthly rent should the family charge the tenant?
1) $\$ 29$
2) $\$ 55$
3) $\$ 57$ 4) $\$ 64$
4) $\$ 68$.

Problems 12 and 13 are based on the circle graph below, representing the budget plan of a family.

12. Wrich of these conclusions is supported by this graph?

1) The family spends too much for food.
2) The family is wealthy.
3) The family has a very low income.
4) The family is extravagant.
5) None of the conclusions given above can be drawn.
13. On the basis of this budget plan, how much would be alloted for rent if the family's income were $\$ 4,000$ ?
1) $\$ 400$
2) $\$ 480$
3) $\$ 1,000$
4) $\$ 1,600$
5) Not given.
14. 

| Height in <br> Inches | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | Over <br> 60 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| No. of <br> Students | 2 | 1 | 0 | 2 | 3 | 1 | 5 | 12 | 8 | 6 |

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The table above gives, to the nearest inch, the heights of the students in a certain class. The table shows that of the 40 students in the class, 2 students are 52 inches tall, 1 student is 53 inches tall, etc.

Exactly one-eighth of the members of the class are the same height (to the nearest inch). How tall are the students in that group?

1) 55 inches
2) 56 inches
3) 58 inches
4) 60
inches
5) Not given.
15. It has been recommended that a person watching television sit at least 6 feet away from the set if the screen measures 10 inches across, and 7 inches farther away for each additional inch of screen. What is the minimum distance satisfying this requirement for a 20 -inch screen? 1) 11 feet, 8 inches 2) 11 feet, 10 inches 3) 12 feet 4) 17 feet, 8 inches 5) Not given.
16. A housewife purchased linens from a wholesaler for resale from her home. She invested $\$ 104.00$ in merchandise, and made sales amounting to $\$ 79.50$. Her inventory then showed that she still had merchandise on hand for which she had paid $\$ 42.00$. What was her gross profit, to date? 1) $\$ 17.50$ 2) $\$ 24.50$ 3) $\$ 37.50$ 4) $\$ 66.50$ 5) Not given.
17. Many fire insurance policies operate under an $80 \%$ clause. This clause provides that the owner of a house insured for $80 \%$ of its value will receive the complete cost of damages (up to the face value of the pollcy) in case of fire while the owner of property insured for less than $80 \%$ of the value will receive only 60/80 of damage costs resulting from the fire. A house valued at $\$ 15,000$ was insured for $\$ 7,500$. Damages caused by fire cost $\$ 2,000$. How much should the company pay the owner?
1) $\$ 1,000$
2) $\$ 1,500$
3) $\$ 1,600$ 4) $\$ 2,000$
4) Not given.

## Problems 18 to 20_are_dased

FEDERAL BUDGET:
RECEIPTS AND EXPENDITURES (In Billions of Dollars)

18. What is the total of the estimated receipts for the years 1952 and 1953 combined?

1) 32 billions
2) 70 billions
3) 132 billions
4) 155 billions 5) 287 billions.
19. For which one of the following years were the receipts approximately the same as the expenditures?
1) 1941
2) 1944
3) 1949
4) 1952
5) None of these.
20. Assuming the population of the United States to be 150 miliion, what is the estimated per capita tax for 1953? $\begin{array}{lllll}\text { 1) } \$ 470 & \text { 2) } \$ 570 & 3) \\ \$ 1\end{array} \$ 700$ 4) $\$ 1,050 \quad$ 5) $\$ 1,330$.
21. People with checking accounts in Bank A pay the bank $10 \phi$ for every check issued but nothing for deposits made, while those dealing with Bank 13 pay $5 \notin$ for each check issued and $5 \phi$ for each deposit made. What kind of depositor will save money dy dealing with Bank B?
1) All depositors
2) Those who issue checks more frequently than they make deposits.
3) Those who make deposits more frequently than they issue checks.
4) Those who make deposits just as frequently as they issue checks.
5) There is no difference, since in the long run each person deposits as much as he withdraws.
22. In playing a game, John makes a score of -35 and Henry makes a score of +65 . What is the difference in their
scores?
1) 30
2) 35
3) 90
4) 100
5) Not given.
23. During World War II, about 9륭 of girl high school graduates were recruited for the Army Cadet Nurse Corps. This is equivalent to saying that the Nurse Corps recruited which of the following?
1) 9 out of 15 girl graduates.
2) 9 out of 50 girl graduates.
3) 10 out of 95 girl graduates.
4) 19 out of 100 girl graduates.
5) 19 out of 200 girl graduates.
24. Mr. Noll accepted a temporary job for a week. At the end of the week he was paid \$60. He was told that he had completed $5 / 6$ of the work, and could return the next week to complete the job at the same rate of pay. How much would he be paid to complete the job?
1) $\$ 6$
2) $\$ 10$
3) \$12
$\$ 50$
4) Not given.
25. The annual premium for an insurance policy is figured at the rate of $54 ¢$ per $\$ 100$. For premiums paid for 3-year periods in advance, the 3 -year premium is set at $21 / 2$ times the l-year premium. What would be the average yearly rate on a 3 -year premium paid in advance? $\begin{array}{lll}\text { 1) } 36 \$ \text { per } \$ 100 & \text { 2) } 45 \phi \text { per } \$ 100 & 3) \\ 44 \\ 4\end{array} \mathbf{4}$ per $\$ 100$
4) $64 \phi$ per $\$ 100$ 5) It would depend upon the face value of the policy.
26. The hundreds digit of a three-digit number is $h$, the tens digit is $t$, and the units digit is $u$. Which of the. following expressions represents this number?
1) $h x t x u$ 2) $h+t+u$ 3) $100 h+10 t+u$ 4) (100h)(10t)(u) 5) None of these.


To determine the distance between points $\underline{A}$ and $\underline{B}$ on opposite sides of a pond, line BC is laid off perpendicular to the line of sight between $\underline{A}$ and $B$, as shown in the diagram above. The distances BC and CA are then measured. What is the distance between $A$ and $B$ in yards? ( Do not try to estimate the distance from the figure, since it is not drawn to scale.)

1) 18 2) 20 3) 21 4) 23 5) Not given.
28. One commonly used method of allowing for annual
depreciation in value consists of deducting the same per cent each year of the value at the beginning of that year. According to this method, what is the value at the end of two years of a car costing $\$ 2,000$ new if the rate of depreciation is $25 \%$ per year?
1) $\$ 1,000$
2) $\$ 1,125$
3) $\$ 1,500$
4) 帑1,250
5) Not given.
29. The energy requirement of adults $20-60$ years old for 1 hour of slight exercise is . 4 calorie per pound of body weight. The energy requirement for adults $60-70$ years old is $10 \%$ less. What is the corresponding calorie requirement per pound of body weight for these older persons?
1) . 096
2) . 06
3) . 03
4). 36
4) Not given.
30. An automobile dealer determines the trade-in value of a certain make and model of car by means of the followirg table.

| $Y=$Number of Years <br> Since Purchased | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: |
| $T=$Trade-in Value <br> in Dollars | 2,000 | 1,700 | 1,400 | $1,100\}$ |

Which of the following formulas expresses the relationship between trade-in value ( $T$ ) and the age of the car ( $Y$ )?

1) $\mathrm{T}=2,000-150 \mathrm{Y}$
2) $T=2,500-500 Y$
3) $T=1,100+600 Y$
4) $T=2,300-300 Y$
5) None of these.


What is the area in square feet of the city lot shown in the diagram above?

1) 4,800
2) 7,600
3) 8,000
4) 8,125
5) Not given.

Let $A=$ the cash price of an article of merchandise.
$I=$ the installment price, ie., the total cost when purchasing on the installment plan.
$D=$ the down payment.
$\mathrm{n}=$ the number of monthly payments.
Then $r$, the approximate rate of interest paid when purchasing on the installment plan, is given by the formula:

$$
r=\frac{24(I-A)}{(A-D)(n+I)}
$$

32. Which of the following expressions represents the amount of each payment?
1) $\frac{I-D}{n}$
2) $\frac{A-D}{n}$
3) $\frac{A}{n}$
4) $\frac{I}{n}$
5) None of these.
33. A television set sells for $\$ 300$ cash, or $\$ 50$ down and $\$ 40$ per month for 7 months. What is the approximate rate of interest paid by an individual purchasing this set on the installment plan?
1) $7 \%$
2) $10 \%$
3) $17 \%$
4) $26 \%$
5) $36 \%$.

## APPENDIX E

ITEM ANALYSIS OF MATHEMATICS ATTITUDE TEST

## A - Attitude Toward Calculation

| Top 27\% |  |  |  | Bottom 27\% |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Question Number | Number of Responses | Percent of Responses | $\mathrm{r}_{\text {bis }}$ | Percent of Responses | Number of Responses |
| 1 | 22 | 69 | 49 | 22 | 7 |
| 2 | 24 | 75 | 48 | 25 | 8 |
| 3 | 14 | 44 | 53 | 6 | 2 |
| 4 | 18 | 56 | 34 | 22 | 7 |
| 5 | 8 | 25 | 50 | - | 0 |
| 6 | 23 | 72 | 37 | 37 | 12 |
| 7 | 15 | 47 | 45 | 10 | 3 |
| 8 | 26 | 81 | 71 | 10 | 3 |
| 9 | 15 | 47 | 45 | 10 | 3 |
| 10 | 20 | 62 | 37 | 28 | 9 |
| 11 | 10 | 32 | 30 | 10 | 3 |
| 12 | 20 | 62 | 33 | 28 | 9 |
| 13 | 12 | 37 | 61 | - | 0 |
| 14 | 20 | 62 | 64 | 6 | 2 |
| 15 | 21 | 65 | 29 | 37 | 12 |
| 16 | 28 | 87 | 63 | 22 | 7 |

B - Attitude Toward Quantitative Thinking

Top $27 \%$

| Question <br> Number | Number of <br> Responses | Percent of <br> Responses | $\mathbf{r}_{\text {bis }}$ | Percent of <br> Responses | Number of <br> Responses |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 15 | 46 | $66^{+}$ | - | 0 |
| 2 | 15 | 46 | 45 | 9 | 3 |
| 3 | 15 | 46 | 36 | 16 | 5 |
| 4 | 16 | 50 | $68^{+}$ | - | 0 |
| 5 | 28 | 88 | 63 | 25 | 8 |
| 6 | 20 | 63 | 68 | 3 | 1 |
| 7 | 22 | 69 | 26 | 44 | 14 |
| 8 | 15 | 46 | 66 | - | 0 |
| 9 | 12 | 37 | 50 | 3 | 1 |
| 10 | 14 | 44 | 65 | - | 0 |
| 11 | 18 | 56 | 60 | 6 | 2 |
| 12 | 12 | 37.5 | $09 *$ | 31 | 10 |
| 13 | 15 | 46 | 64 | 3 | 1 |
| 14 | 17 | 53 | 49 | 13 | 4 |
| 15 | 8 | 25 | 35 | 6 | 2 |
| 16 | 11 | 34 | 34 | 9 | 3 |

+ the value is greater than the indicated number.
* the value is too low to be accentable.

C - Attitude Toward Non Mathematical Activity

| Top 27\% |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Question <br> Number | Number of <br> Responses | Percent of <br> Responses | $\mathrm{r}_{\text {bis }}$ | Percent of <br> Responses | Number of <br> Responses |
| 1 | 20 | 77 | 63 | 15 | 4 |
| 2 | 18 | 69 | 72 | 4 | 1 |
| 3 | 22 | 85 | 72 | 12 | 3 |
| 4 | 18 | 69 | 65 | 8 | 2 |
| 5 | 21 | 81 | $82^{+}$ | - | 0 |
| 6 | 5 | 19 | 34 | 4 | 1 |
| 7 | 12 | 46 | $66^{+}$ | - | 0 |
| 8 | 19 | 73 | $79^{+}$ | - | 0 |
| 9 | 22 | 85 | 50 | 38 | 10 |
| 10 | 17 | 65 | 62 | 8 | 2 |
| 11 | 23 | 88 | 68 | 19 | 5 |
| 12 | 6 | 23 | 39 | 4 | 1 |
| 13 | 26 | 100 | $77^{+}$ | 31 | 8 |
| 14 | 21 | 81 | $82^{+}$ | - | 0 |
| 15 | 21 | 81 | $79^{+}$ | 4 | 1 |
| 16 | 19 | 73 | $79^{+}$ | - | 0 |

[^1]
[^0]:    * A more detailed outline is to be found in Chapter 3 pages 26-27.

[^1]:    + the value is greater than the indicated number.

