CHILDREN'S LEARNING OF FRACTIONS
A COMPARISON STUDY OF
USER-CONTROLLED COMPUTER-BASED LEARNING VS.
NONINTERACTIVE LEARNING ENVIRONMENTS

By

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ABSTRACT

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The purpose of this study was to investigate the effectiveness of the software program Visual Fractions in teaching basic fraction concepts and the effect that student control over the construction of fraction diagrams had on their learning.

The Visual Fractions program provides a diagram and two fractions in numeric form. The diagram consists of a figure divided into partitions with some of the partitions shaded. One fraction represents the shaded parts of the whole and the other represents the unshaded parts. Students can control the total number of partitions and whether each is shaded. Manipulating the diagram changes the value of the fractions.

A Non-interactive (*crippled*) version of the software was designed to eliminate the user-control aspect of the program. Users of this program could click to generate a new fraction, but had no control over the choice of fraction. The computer randomly generated a new fraction and displayed the corresponding diagram each time.

A third treatment, Fraction Flash Cards, was designed to simulate the Noninteractive version of the program, without the computer. The students received Flash Cards containing images of the computer-generated fraction diagrams.

The study consisted of a pilot project during which data collection techniques were tested and revised and the main study. Sixty-four subjects were taken from four intact classes of grade four students. The students were randomly assigned to one of the three Treatment Groups or the Control Group. Three different sets of data were collected: a pretest and postest on fractions, structured interviews, and field notes taken by the researcher during the treatment process.

In Treatment Group One, students used the Interactive Version of Visual Fractions. Here, students could create fractions at their command. There is evidence to suggest that this type of interactive control is a critical factor in learning (Merrill, 1987).

In Treatment Group Two, students used the Noninteractive version of the software. Students could control the rate of observing fractions and fraction diagrams, but not the value of the fraction.

Students in Treatment Group Three used the Flash Cards. Motivation appears to strongly affect one's ability to learn and children appear to be highly motivated to use computers. The purpose of this treatment was to control for any achievement gain that may have been due to the novelty of using computers.

The four Groups were compared using analysis of variance with repeated measures. Significance at the 0.01 level was found for the tests and the interaction. A study of the interaction showed that there was no significant difference between the gains of the Visual Fractions Noninteractive Group, the Flash Card Group, or the Control Group. However the gain achieved by the Visual Fractions Interactive Group was significant.

From this study, it is clear that the Visual Fractions Interactive program which provides students the opportunity to construct fraction diagrams with immediate feedback, is an effective method of teaching fractions.

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Chapter One

Introduction

The Problem

I can't do Math - I don't understand fractions. This common complaint is shared by many school children and even some adults. Too often children do not overcome these initial difficulties and, with confidence eroded, continue to experience further difficulty and frustration in their study of mathematics.

A common finding in much of the research indicates that these difficulties are primarily conceptual (Hector & Frandsen, 1985). Children do not seem to have an adequate understanding of basic fractions upon which to base their further study of fraction equivalence and operations. Elementary students tend to operate on a mechanical or procedural level when working with fractions rather than a conceptual one (Peck & Jencks 1981). They prefer to apply often-inappropriate rules in a rote fashion without trying to interpret the meaning of their answers, rather than trying to find reasonable responses through estimation skills which represent genuine understanding (Behr, Wachsmuth & Post, 1985).

Researchers have been studying the problem of children's learning of fractions for many years. The focus of debate has continually shifted between an emphasis on conceptual and procedural knowledge. Currently, researchers are interested in trying to improve children's understanding of fractions, but also in the relationship between these two types of learning (Hiebert & Lefevre, 1986).

There are several suggestions in the literature concerning the issue of how to develop greater understanding of fractions in elementary school children. At what age should children be formally introduced to fractions? When should children begin their study of fraction equivalence and operations? Should concrete manipulative aids be used and if so, should they be available for longer periods of time and at the upper elementary grades as well as primary? How much time is needed to develop an adequate concept of fraction? A complete understanding of fraction involves several subconcepts (i.e., ratio, decimal, etc.). When should each of these subconcepts be introduced? Should certain subconcepts such as decimals be emphasized over others? If more time needs to be spent studying basic fractions concepts, from where will that extra time come? From which area of the mathematics curriculum (or other subjects) will this learning time be taken? Other relevant issues concern teaching methods, student attitudes, learning environment and available resources. These are just a few of the issues with which the literature on fractions learning is continually concerned.

Concrete Manipulative Aids

The literature on the problem of children's learning of fractions provides many recommendations for helping children to improve their understanding. For example, earlier introduction of basic fraction concepts (i.e., grade three instead of grade four), more time spent on basic concepts before moving on to the study of operations and equivalence, more emphasis on understanding and estimation skills rather than memorization of procedural rules, and a greater use of concrete manipulative aids, especially in the upper elementary grades where these materials are not normally found.

Computer Assisted Instruction

Microcomputers may be an effective classroom tool for teaching fraction concepts. Even at the kindergarten level, many children are using computers as a regular part of their weekly, if not daily routine. Exactly what role micros play in the classroom varies from school to school and from classroom to classroom. Computers are typically used in the schools for word processing, administration, computer assisted instruction, and for computer programming. Word Processing and C.A.I. are the most common ways that micros are currently used in the classroom.

The literature on the use of microcomputers in the teaching of mathematics includes discussion of issues such as what are the best uses of the computer (i.e., drill and practice programs, C.A.I., or programming?), are all C.A.I. programs educationally sound?, are some applications better than others?

Visual Fractions† - A User-Controlled Environment

Although there is great potential for computer assisted instruction in the teaching of mathematics skills and concepts, not all software programs are equally effective in utilizing this powerful potential. What is needed are software programs which utilize computer time uniquely and effectively by providing learning environments which cannot be duplicated with other classroom tools. Visual Fractions is a Macintosh computer software program which allows students to manipulate fraction diagrams and symbols with speed and accuracy. Fractions are created instantaneously at the child's command to represent parts of a whole, equivalence and operations. The most basic part of the program involves only one type of fraction diagram - a vertical bar diagram and the corresponding written fraction symbols. The user is able to change the size of the shaded portion of the fraction diagram to graphically display fractions ranging in size from halves to twentieths. As the size of the fraction is changed, the program supplies the correct mathematical symbol. The child is able to see the association between the fraction represented by the diagram and the symbol. This association takes on meaning because the child is the one determining the size of the fraction parts. The symbol form of each fraction is displayed in large numbers beside the graphic representation to help establish the link between concept and form. Because of the speed and accuracy of the computer, many fractions can be displayed in a very short time and still

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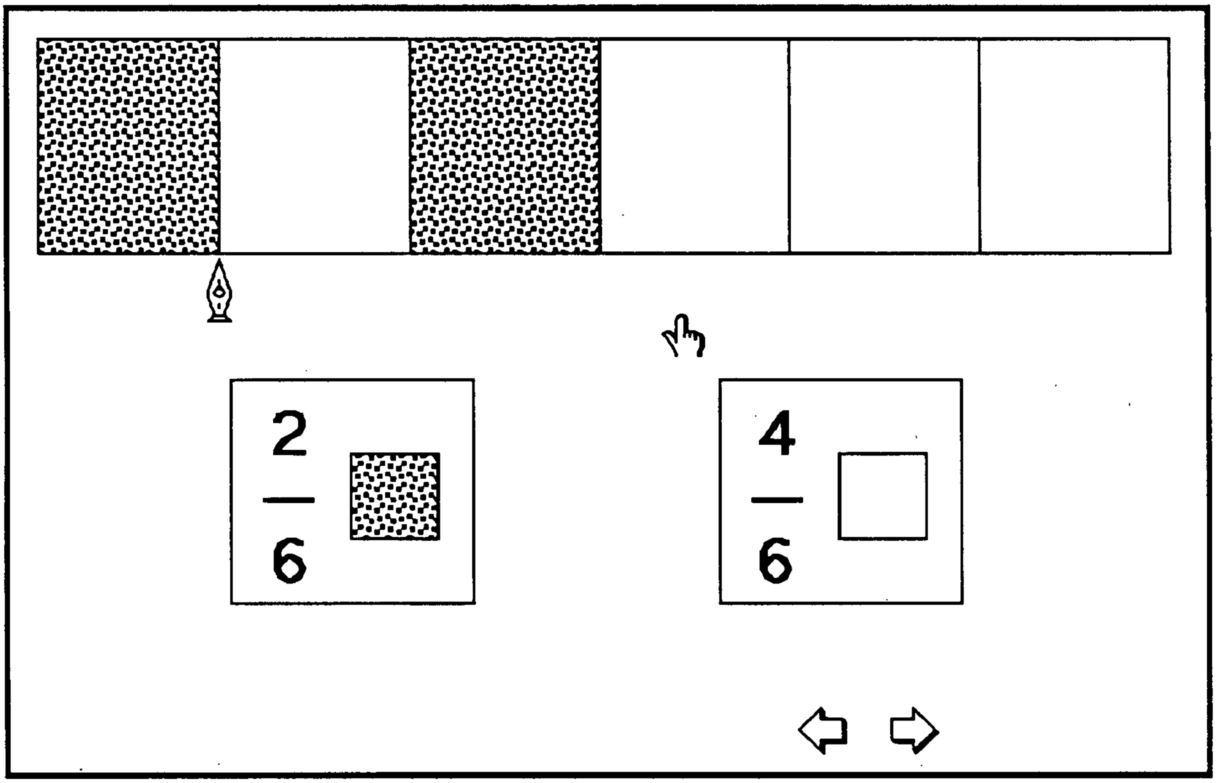
† The Visual Fractions program was developed by Dr. M. Westrom and the Computers in Education Research Group at the University of British Columbia.

retain the interest of the user. Constant repetition of a mathematically-sound visual aid is the principal feature in this unique approach to learning fractions. Also, the concept of equal sized parts should become apparent to the child because he cannot create unequal size pieces regardless of how he may attempt to do so. The computer will always divide the diagram into the nearest equal-sized portions requested.

Figure 1.1 - Example Screen - Visual Fractions



Although not specifically manipulative in the traditional sense, Visual Fractions is interactive and user-directed. It provides a fascinating and motivating experience for students in a child-controlled, structured-learning environment. Although the computer actually draws the fractions, it is the child who *creates* his own knowledge by determining the fraction the computer will make. Because of the intrinsically interesting nature of the program and because children are normally highly motivated to use computers for learning, learning should take place incidentally and without effort.

Noninteractive Learning Environments

In order to determine whether Visual Fractions is a valid use of computer time and student time, the researcher attempted to duplicate the essential elements of the learning environment created by the program in another setting. The researcher designed several sets of fraction Flash Cards (see appendix D) which simulate the type of information displayed by the fraction diagrams in the computer program. The students in the Flash Card Group had an equal amount of time and teacher assistance to explore the cards as the students in the Computer Group had with the Visual Fractions program. With the cards, students could *create* fractions by lifting the flaps on the cards to reveal either a fraction diagram or the corresponding written symbol. These cards were designed to be very similar to the display in the Visual Fractions program, except for two critical differences. First, the child in the Flash Card Group is not really in control of his own learning. In the Visual Fractions treatment, there is a real sense of control over the learning situation; children feel as though they are making the fractions through their own efforts (i.e., sliding the mouse button back and forth on the diagram). This is not quite the same as merely lifting flaps to reveal ready-made fraction diagrams. With the Visual Fractions program, the fraction diagrams do not exist until the child creates them. There is evidence to suggest that this type of interactive control is a critical factor in learning (Merrill, 1987). Also, because motivation appears to strongly affect one's ability to learn and children appear to be highly motivated to use computers, educators should consider utilizing this motivational factor by encouraging greater use of computers to teach concepts currently taught by more traditional methods.

In order to ensure that any positive results gained from the Visual Fractions treatment are not due to the novelty of using computers, the research design includes a third Group which will use the computer to view the same flash cards used by the Flash Card Group. The only difference will be that the cards will be presented randomly on the computer screen. The user may control the rate at which the cards are presented but has no other control over the learning environment. This type of experience is similar to the type of minimal interactive learning involved in turning the pages of a text book or flipping through a stack of fraction flash cards.

Expectations of the Study

It was expected that as a result of using the program for as little as one sixty-minute session, the students in the Visual Fractions Group would have a better understanding of fractions than those in either of the other two experimental Groups. Each Group was given an equivalent amount of time to explore the learning aids. Superior performance was anticipated both from the results of the posttest and from several in-depth interviews with students in each Group. Students in the Visual Fractions Group were expected to improve substantially in their ability to recognize the correct written symbols for fractions and also be able to correctly shade in the required number of fraction parts when provided with the symbol notation. It was assumed that this ability will represent an improved understanding of the concept of fraction defined as part to whole relationship of equal sized parts.

Problem Statement

Does learner-control over the construction of fraction diagrams improve learning?

Set in a larger context, there is the ability of the computer to provide opportunities for learning experiences for students. Visual Fractions is one of these. Is it an effective one? If so, what are it's effective characteristics?

Purpose of the Study

The purpose of the study was to examine whether a software program designed to allow learner-control over the construction of fraction diagrams program is effective in helping children develop a better understanding of fractions. In particular, this study compared the use of computers with a more traditional classroom aid, fraction flash cards. The study examined whether attitude toward computers had any effect on the ability to benefit from the program. This was accomplished by the inclusion of a Third Treatment Group which had an opportunity to view fraction flash cards on a computer screen but without control over the construction of the fraction diagrams.

Need for the Study

Microcomputers are currently not being utilized to their full potential in the school system. Computers still represent a reasonably costly investment and availability is usually limited to between fifteen and thirty machines per school. In order to derive the greatest return on investment, micros should be used to perform functions that cannot be done with other resources. This study examines the use of a microcomputer in a unique way. No other classroom tool can instantly *create* fractions at the child's will in the way Visual Fractions can. This study will examine this unique approach to using computers compared with a more typical use of computers, a drill program. The study compared the use of the computer drill program to traditional flash card materials.

Children often do not have a good conceptual understanding of fractions. Researchers have been studying this problem for many years. Several suggestions have been proposed, but the problem still persists. This study examined a relatively new tool, the microcomputer and determine whether it might be used to help children better understand the concept of fraction.

Limitations of the Study

The following are limitations to this study:

This study used a convenience sample chosen from one elementary school and used two intact classes. There was the possibility of non-representation due to the socio-economic status of the school or the ability level of the two classes. There was also the danger of contamination due to student's discussing the treatment with their classmates in a different Group. However, because of the nature of the learning situation (i.e., it would be difficult to learn simply from a verbal description of the treatment) and the short length of the study, this contamination was unlikely to occur.

The exposure to the program is limited to one one-hour session. It may be that this time limit is not sufficient and student's would have benefitted from a longer exposure. It was assumed in this study that students answered truthfully on the tests and in the interviews and that they performed to the best of their ability during the treatments.

Chapter Two

Review of the literature

The Development of Fraction Concepts

The study of fractions has always been a struggle for children. The research identifies a number of possible reasons for this difficulty. In particular, fractions are considered to be conceptually difficult and involve complex definitions

The Difficulty is Primarily Conceptual

A common finding in much of the research suggests that the difficulties children have with fractions are primarily conceptual. *They go through the motions of operations on fractions but have not been exposed to the kinds of experiences that could provide them with necessary understandings* (Hector & Frandsen, 1981). It is indicated in the literature that there are significant problems in learning and applying concepts related to rational numbers (Behr, Wachsmuth, Post & Lesh 1984). Hope and Owens (1987) refer to fractions as an inherently difficult abstraction which early civilizations went to great lengths to avoid. Hart (1981) states that children are not confident in their use of fractions. She says that whenever possible they will apply whole number concepts to operations on fractions, preferring a remainder type answer to one which states a fraction.

As a result of twenty in-depth interviews with grade six children, Peck and Jencks (1981) report that about 55% of the students were unable to demonstrate that they possessed a meaningful concept of fraction. 35% of the students appeared to have a correct concept of fraction but were unable to extend this knowledge to work with operations and equivalence. The authors state that fewer than 10% of all student's interviewed had acquired an adequate conceptual base to guide them in their study of fractions. The twenty students interviewed are reported to be typical of hundreds of similar interviews the authors have conducted with grade six, seven and nine students.

Definition of Fractions is Complex

One of the reasons that children may have difficulty with fractions is that it is difficult to define what is meant by fraction. The notion of fraction is not an obvious or easy concept possibly because there are many different interpretations. Early work by Piaget defines fractions as a part-whole concept but deals only with continuous quantity tasks such as length and area. Seven subconcepts of a fraction are identified in his work. These subconcepts are summarized by Hiebert and Tonnessen (1978) as the realization that:

1. The whole is subdivisible.
2. A fraction implies a determinable number of parts.
3. The subdivision must be exhaustive.
4. There is a fixed relation between the number of parts and the number of divisions.
5. The parts have a nesting or hierarchical character.
6. The whole is conserved under subdivision.
7. All parts must be equal.

According to Piaget, children's understanding of fraction as a part-whole relationship develops sequentially through several stages beginning with an understanding of division into two equal parts, then into fourths and finally division into thirds, fifths and sixths.

The problem of defining what is meant by fraction is further complicated when one considers the difference between discrete and continuous objects. Hiebert and Tonnessen (1978) argue that Piaget's subconcepts are applicable only for continuous quantity representations and children do not necessarily perceive sets as a whole which must be divided into parts. They state that children's strategies used to solve discrete quantity (set/subset) problems are extremely different than the strategies they employ in continuous quantity tasks. Hunting (1983) suggests that children may be able to transfer knowledge about discrete quantities to continuous problem tasks but probably not the reverse. He recommends that children receive instruction on fractions in both continuous and discrete fraction quantities.

Thomas Kieren (1976) has done extensive work in the area of rational number interpretation. He has identified six subconstructs which he claims are necessary for a fully functional fractional number construct. They are part-whole, decimal, ratio, quotient, measure and operator. Kieren states that instruction with rational numbers is typically based upon a computational construct. Rational numbers, he says, are viewed as objects of computational manipulation and extensions of whole number computations. According to Kieren, little effort has been made to develop in the child a broader construct of rational numbers which would include a variety of experiences in all six subconstruct areas.

Hope and Owens (1987) are in agreement with Kieren. They state that each of these subconcepts is important and a child must have experience in all areas in order to have a fully functional concept of fraction. While it is important for educators to recognize the various interpretations of fraction and address each, many studies are limited to the notion of fraction defined as a part to whole relationship. In order to find out where children have gone wrong in developing a concept of fraction, it is appropriate to start by trying to understand what children know about the most basic concept of fraction. It is generally accepted in the literature that there exists a hierarchy in the development of fraction concept. Novillis (1976) for example, has utilized pictorial models to represent this hierarchy ranging from a part to whole model which she calls region, to a part of a set model and finally to the most difficult model, the number line. Part to whole relationships typically receive the most attention in the formative years of schooling. According to Novillis, this time does not appear to be well-spent or adequate enough to help children develop a good base line on which to build a more complete concept of fraction.

Kieren (1976) has further developed his perspective on the definition of fractions as a part to whole comparison. He refers to this dividing of a whole into equal parts as partitioning. Kieren claims that within this subconcept, understanding is developmental in nature and depends upon the maturation of the child through various age-related stages. Pothier and Sawada (1983) further developed Kieren's theory to identify five levels in the progression of understanding of the partitioning process. They claim that *each level is distinguished by certain characteristics, procedural behaviors, and partitioning capabilities*. The five successive stages are mastering:

1. Level 1: Sharing (notion of 1/2).
2. Level 2: Algorithmic halving (denominators with powers of two).
3. Level 3: Evenness (fractions with even denominators).
4. Level 4: Oddness (fractions with odd denominators).
5. Level 5: Composition (fractions with composite denominators).

Children are not concerned about equality of parts in a fraction until level three. They are able to divide a diagram into eight or even sixteen pieces at level two but do not concern themselves with whether the pieces represent a *fair share*. It takes considerably more time and experience for children to attain level 3 reasoning than is often available in the school system. Research by Peck and Jencks (1981) confirms this theory. When asked to draw sketches of one third, one half, one fourth and other simple fractions, typical grade six students drew sketches which were only vaguely related to the meaning of the fraction symbols. Peck and Jencks report that even at the grade six level, most students interviewed were unaware that the partitions they made had to be the same size. Children struggle particularly with the notion of odd numbered fractions (i.e., thirds and fifths) especially when asked to illustrate these fractions using circular diagrams. Typically children will divide a circle in half and then divide one side into half again to illustrate one third. They do not appear to be concerned with the notion of equal sized pieces even when their partitions are grossly uneven. This concept of equality should be firmly acquired by the child before any further instruction is given on more complex fraction concepts (Peck & Jencks, 1981).

Children's Construction of Mathematical Ideas

Even if one can arrive at a satisfactory definition of what is meant by fraction, it is another task altogether to find out what children understand about rational numbers. Researchers are continually trying to understand how children construct basic mathematical ideas. Hunting (1983) writes:

> *Since basic fraction concepts are the seedbed for many important mathematical ideas--including notions of equivalence, inverse, decimals, probability, ratio, and proportion--the mental mechanisms that support a knowledge of fractions needs to be understood* (p. 182).

How do we know for example, when a student understands a concept such as one half or two thirds? Lesh, Behr and Post (1987) propose an answer to this question based upon their work from three different National Science Foundation funded projects. These researchers have developed five distinctive types of representation that occur in mathematics learning. They are:

1. Experienced-based scripts (i.e., real world events).
2. Manipulative models.
3. Pictures or diagrams.
4. Spoken languages.
5. Written languages.

According to the Lesh et al. (1987), translations among and within these five representations are critical to genuine understanding of mathematical concepts. Thus, if a child really knows what we mean when we say one half, he or she will be able to translate this knowledge in a variety of different ways as illustrated by the five different representational systems. He or she should be able to recognize the idea in a variety of representational systems, flexibly manipulate the idea within each representational system and translate the idea from one system to another. Lesh argues that it is in trying to translate an inadequately developed concept from one representational system to another that students begin to have serious difficulty.

Another point of view expressed in the literature suggests that children come to school with well-defined concepts including an understanding of fractions (Pothier and Sawada, 1983). Virtually every child they say, understands what it means to divide a candy bar in half. However, this type of understanding may simply be limited to a notion of *my share* as opposed to any meaningful insight into the idea of equal parts of a whole. In other words, just because a child understands that an object may be divided into parts of two, three or perhaps even four, does not necessarily mean that this is sufficient knowledge for the acquisition of any meaningful concept of fraction. Pothier and Sawada argue that this type of knowledge is merely rote learning as a result of sharing activities and does not involve an understanding of half in a number sense. They state that this is often indicated by the child's liberal use of the term half in expressions like *break in half in four pieces* and *split in half in three pieces*.

The Connection Between Concepts and Procedures

Even if children arrive at school with some rudimentary ideas about fractions, they are often unable to connect this knowledge with the new information they receive about fraction symbols and operations. According to a study by Peck, Jencks, and Chatterley (1980), students have no idea how to relate fractions symbols with their previous experiences and thus have no base for reasoning in problem situations. Hiebert (1984) states that one possible reason for this failure to link understanding with symbols may be that many children fail to establish meaning for the symbols they are taught. *Mathematics becomes, for* *many students, an exercise in following the right rules* (p. 501). Students appear to follow rules indiscriminately. Even when confronted with the unreasonableness of their answers, children will insist upon *rule* rather than *reason*. According to Hiebert,

> *Children see little or no connection between the understandings they possess about the number system and it's properties and the rules of form they have memorized for operating on symbols (p. 504).*

Peck and Jencks (1981) report that when asked to solve fraction problems, children typically search for rules that often have no meaning for them. They seem to believe that *any rule will do* and even when children misapply previously learned rules, they are often unable to tell that they have done so. Further Peck and Jencks state, these children are *dependent on external influences to determine the validity of their answers* (p. 345). The most likely reason for this is that children are generally poor at estimation skills. They do not like to engage in activities which require practice in estimation. A child seems more secure in a situation which has a right and a wrong answer. According to a study by Behr, Wachsmuth & Post (1985), children who scored low on rational number order and equivalence problems showed uncertain or inaccurate use (if any) of the estimation process. Additionally, students whose rational number concepts are weak will often confuse their knowledge of whole number operations with that of fractions (Behr, Wachsmuth, Post & Lesh, 1984). Estimation skills are however, according to Behr et al., *fundamental to the development of a viable concept of rational number* (p. 128).

The difference between conceptual (development of meanings) and procedural (application of rules) knowledge has been the focus of much debate in mathematics education for many years and continues to this present day. Hiebert and Lefevre (1986) summarize this debate as one which has constantly alternated over the years between one focus and the other. In 1895, for example, Dewey argued for improved understanding. In 1922, Thorndike emphasized the learning of skills in order to maximize retention. Brownwell turned the debate in 1935 back to an argument for increased understanding. In 1960, Bruner reinforced this position until Gagne refocused the argument in 1977 on skill learning.

Currently the debate over these two issues has taken on a different perspective. No longer are the two issues considered in isolation of each other. Instead researchers are interested in the connection between these two types of learning.

The Learning Environment is Critical

One way to help children make the connection between concepts and procedures is to ensure that their learning occurs in an environment most conducive to the assimilation of knowledge.An aspect of learning environments that has been researched and discussed extensively in the literature is the role of physical models.

The Role of Physical Models

According to Lavetelli (1970), the curriculum should apply Piagetian techniques to the learning environment which would encourage students to be active participants in the process of learning. Concrete materials (especially for young children) she says, are an excellent way to allow for this participation. She states that the teacher should guide learning but not force children to parrot the right answers. Being given the right answers does not convince a child. They need to be convinced by their own actions. Being able to construct one's own knowledge, says Lavatelli, may be a key factor in learning and retention.

Simply using concrete materials to teach fractions however, is not the complete solution to the problem. A major focus of the Rational Number Project, a multiuniversity research project funded by the National Science Foundation and conducted over a five year period from 1979 to 1983, was the role of concrete manipulative materials or physical models in facilitating the development of a rational number concept. One of the many findings of the project was that even after extensive instruction on the order and equivalence of fractions, a significant number of fourth grade students demonstrated a substantial lack of understanding when asked to apply fraction knowledge to new situations. (Behr, Wachsmuth, Post and Lesh, 1984). The recommendations of these researchers include introducing fractions in the third grade with a heavy emphasis on unit fractions and plenty of repetitive *hands on* experiences.

The emphasis on teaching the meaning of fractions using manipulative aids in a variety of learning experiences has been clearly established in the literature (Carpenter, Coburn, Reys,& Wilson, 1976). Hiebert (1984) points out however, that the concern over teaching understanding using physical models is not as new a perspective as the literature often suggests. He cites Van Engen's work (1949) as an example of someone whose objective was to help children link mathematical symbols with the concrete objects or events that they represent. Indeed, researchers have been calling attention to the need for a more active participation of the child in his own learning for many years and yet the problem with fractions still persists. Gunderson and Gunderson (1957) noted that children showed good understanding of fractions when using manipulative materials. Children they argued, can benefit from systematic instruction in the meaning and use of fractions rather than memorizing rules and generalizations. Thirty three years however, have passed since these recommendations were made and in spite of the continued use of concrete manipulative materials, children still have problems with fractions. The problem appears to be more complex than an initial examination of the issue reveals (Chaffe-Stengel & Noddings, 1982).

Instructional Time must be Effective

In addition to providing children with concrete manipulative aids, teachers must be prepared to allow students enough time to use the materials effectively and in an appropriate learning environment. Carpenter et al. (1976) recommend a greater focus on initial conceptual work prior to any formal work with algorithms. They point out that merely increasing instruction time will not guarantee better achievement results. Instruction time, they say, should be well organized in order to teach the concept. Further, when faced with an already over-crowded curriculum, most teachers do not welcome the recommendation to simply *spend more time teaching concepts* (Johanson 1988). Additionally, says Johanson, computers are still relatively expensive and their proper use must be justified. Hope and Owens (1987) warn us that children will not develop meaning in vacuo. *A child*, they say, *can develop meaning only through carefully structured experiences in a context or setting used to represent a chosen subconstruct* (p. 29). The purpose of these physical settings is to provide an environment where children can manipulate objects in order to eventually establish a link to a symbolic setting. According to Hope and Owens, these learning environments must be carefully structured in order to establish the meaning of some mathematical concept. They emphasize the importance of understanding the language used in the physical setting. It is unfortunately often true that the development of understanding is inhibited not because of a missing conceptual link, but rather a misunderstanding of the language used to develop the concept. The role that language plays in developing concepts has received a great deal of attention in the literature. For example, Lesh, Behr & Post (1987) report that most fourth through eight grade students have a seriously deficient understanding about the models and languages needed to represent, describe and illustrate mathematical ideas. In attempting to understand children's concepts of fractions, researchers must try to develop methods to ensure that they and the child are using the same language when talking about mathematical ideas. For example, does the child understand terms such as divide, numerator, cut up into equal pieces, shade in the fraction part, etc.?

The Role of Attitude in Student's Learning

In addition to providing an appropriate learning environment, educators must concern themselves with the role of the student's attitude toward that environment.

Connections between Attitude and Achievement

There is evidence in the literature to suggest that a student's attitude toward a learning experience may affect his ability to learn. Extensive reviews have been done of research on attitude towards mathematics (Halyadyna, Shaughnessy and Shaughnessy, 1983). Much of this research has been dedicated to finding a positive relationship between attitude towards and achievement in, mathematics. (Chapman, 1984; Corbett, 1984). The link has been clearly established; anxiety does seem to have an effect upon achievement and clearly many young people identify themselves as suffering from this phobia.

Halaydyna et al. (1983), provide a good definition of what is commonly meant by *attitude towards mathematics*. They define it as a general emotional disposition toward the school subject of mathematics. They go on to suggest that a positive attitude toward mathematics is valued for several reasons, one of which is that attitude is often positively related to achievement.

Math Anxiety

Having accepted the connection between attitude and achievement, several researchers have tried to find ways to reduce anxiety and improve achievement. Mathematics educators have continually been plagued with the question of how one can foster understanding of and thereby improve achievement in school mathematics. The term *Math Anxiety* is not a new one; an abundance of study has been devoted to this one aspect of attitude toward mathematics. The literature gives several different interpretations to the notion of Math Anxiety. It is often referred to as a state of being anxious about mathematics achievement and ability. Words like fear, panic, dislike, hate, and *mathophobia* are used to describe this phenomena (Marion, 1984; Papert, 1980).

In another article on mathematics achievement and attitude productivity, Tsai and Walberg (1983), report that mathematics attitude is influenced by home conditions and achievement.

They suggest that causality may be reciprocal: the more one learns, the higher the attitude, and the higher the attitude, the greater one learns. In conclusion, they state that improving attitude and encouraging greater learning are both important and long-term results.

The indication in the literature is that the use of physical models to facilitate understanding is appropriate but not necessarily effective unless combined with a carefully structured learning environment where language barriers are not a problem and student attitudes toward the learning experience are positive. The learning environment must also be constructed to teach a specific concept in an appropriate time frame. Microcomputers have been shown to be an effective classroom tool requiring significantly less instruction time over conventional classroom techniques (Kulik, Kulik & Bangert-Downs,1985).

The Role of Microcomputers in Education

Just as attitude towards mathematics affects achievement, student attitudes toward computers are important to success in computer related programs (Loyd & Gressard, 1984).

Student Attitudes towards Microcomputers

In a study designed to measure attitude towards computers, these authors reported that students on the whole had a fairly positive attitude toward computers. Dalton and Hannafin (1985), report that children believe computer-made learning is more pleasant than conventional instructional strategies due to the infinite patience and non-judgemental nature of the computer. The students were reported to have developed positive attitudes toward the computer which provides consistent feedback and never showed signs of frustration or anger.

In another study on student attitudes toward computers, evaluated over a two year period, the researchers found that student interaction with computers did not result in less interaction with teachers and classmates inspite of the claims of some critics of C.A.I. (Griswold, 1984). The researcher recommends the use of computers and C.A.I. to encourage students to *recognize their role and gain confidence in controlling the course of their own learning*. The reason for this recommendation is that student's perceptions of their responsibility for success and their academic self-confidence were consistently related to C.A.I. after controlling for the effects of gender, minority status and achievement.

The Role of Intellectual Models

The work of Seymour Papert (1980) lends some further insight into the notion of learning environments, microcomputers and children controlling the course of their own learning. He proposes a theory of learning based upon the idea that each of us develops a collection of models which remain with us throughout our lives and to which we relate all new knowledge. In his case, the gearbox of an automobile served as a model which he appropriated at a very young age and to which he later related his ideas about mathematics. Thus, for Papert, mathematics was easy but only because of the conceptual model of gears to which he related it. In fact, Papert contends that *anything is easy if you can assimilate it to your collection of models* (p vii). The idea of gears is an excellent model to represent mathematical ideas because a child can relate to the mathematical knowledge as well as the *body knowledge* notion of turning gears. He says that the child can in a sense become a *gear* as he tries to go through the motions of understanding how they work. Papert advises educators and researchers to look for ways to create conditions under which intellectual models will take root. We need to realize however, that the notion of intellectual models is a personal matter and each person collects their own set of models as they mature. What serves as an ideal model for one child may do nothing for another child if he cannot relate to it. Papert reminds us that the gears model worked for him because he fell in love with gears.

In order to understand how educators can facilitate this development of intellectual models, it is appropriate to consider how Papert's notion of intellectual models can be related to the idea of structured learning environments. Papert argues that the computer is the ideal medium currently available to provide these learning experiences for children. He argues that computers can in a sense become *all things to all people* because of their power to simulate countless functions in countless ways. Computers can serve as the tool to create *gear box* models or microworlds which will give meaning to concepts as they are presented to or more accurately stated, developed by the child.

It is important not to confuse Papert's notion of microworlds with what is traditionally meant by structured learning environments. Structured learning environments are designed to teach a specific concept. Microworlds are learning environments designed to provide the information and tools necessary for a child to discover knowledge about a particular topic. They are environments which are in part provided for and in part created by the child where only issues of importance to him are relevant. Currently, the onus is on the classroom teacher to facilitate the creation of a microworld for the student. The result may be the same as in a structured learning environment, but in a microworld, the child is constructing his own knowledge and thus learning should be more meaningful and retention greater.

Papert's ideas are strongly linked to Jean Piaget who is known as the author of learning without curriculum. Papert (1980) refers to *Piagetian Learning* as learning which occurs without formal instruction. He deviates from Piaget's theory however, in that Papert believes that although children build their own knowledge, they need rich tools to do so. When knowledge is slow to develop, Piaget would attribute this to the complexity of the concept being learned. Papert on the other hand, believes that an impoverished learning environment is responsible for the slower intellectual development in the child. Thus, as long as the learning environment is rich in tools for building knowledge, the child should be left to develop his own learning. This is not the same thing as a carefully structured learning environment designed to teach a child a specific concept.

Learner Control

For the past decade, a theory of instructional design has been proposed by a group of people under the name of Component Design Theory which suggests that learning, resulting from instruction is most efficient and effective if a proper combination of information presentation forms are used to their greatest advantage. (Merrill, 1987). Embedded in this theory is the promotion of learner control as a mechanism for adapting to individual differences in learning style. Because of increased technology and availability, a wide variety of instructional techniques are now possible. According to Merrill (1987), instructional design theory has not kept pace with the increased capabilities of computer hardware and software. As an extension of the original instructional design theory, Merrill proposes a *New Component Design Theory* which extends the original theory to take advantage of the increased capabilities of computers particularly in the areas of intervention and presentation.

Traditionally, most instructional software was based upon a structured or tutorial type model, also known by the term *Branched Program Instructional Model* (Reigeluth, 1983). In this type of learning situation, the student is presented with some information (usually a screen of text and/or graphics), asked a question, provided with an answer and possibly remedial material depending upon his response, and then the cycle is repeated. Merrill (1987) argues that this type of learning environment is best in two circumstances: to act as a secondary reinforcement to a primary, more experiential environment and to correct or help students overcome misconceptions or misunderstandings after having explored some experiential environment. However, he says that the computer is much more than just just a tutor and should not be limited to this type of function. One of the most effective instructional uses of the computer, he argues, is that which allows the student to interact directly with the subject matter. According to Merrill (1987), an experiential model of instruction is much different from a tutorial model because the student is able to interact directly and have control over some type of experiential representation of subject matter. He refers to an experiential representation as a controllable microworld which allows the student to explore and discover the relationships involved. He warns us, however, that exploration is only one type of transaction and may not be sufficient to enable the student to learn the necessary procedures or to understand all of the relationships included in the experiential simulation. He suggests that students may learn more from a program which includes elements of demonstration, explanation and prediction as well as components of exploration. The student, he argues, may learn more from some form of structured transactions than from open ended learner controlled exploration.

Researchers are not all in agreement with the idea that learner control presents the best learning environment. A study by Belland et al. (1985) into the nature of motivation and learning indicates that self-directed learning environments may not necessarily be the best situation for all people. They argue that computer software programs which do not provide any external prompting may not be effective learning environments for low achievers who can not keep themselves on task. Students who are not self motivated for example, require some external control to help them learn. Software developers, they say, need to address this question when designing educational programs for all types of learners.

Rowland and Stuessy (1988) concur with this finding. They argue that although one of the greatest aspects of C.A.I. is it's ability to provide individualized instruction, it is possible that not everyone is able to benefit from this interactive nature of some computer programs. Their concern is that some learners are predisposed to a particular cognitive style and are not easily adapted to new modes of instruction. In order to test their theory, Rowland and Stuessy conducted a study to determine what effect the mode of C.A.I (tutorial versus simulation (which involves learner control)) has on achievement and understanding of concept relationships. They were also interested in determining whether cognitive learning styles could be purposely matched to the type of C.A.I in order to increase performance. Cognitive learning styles were defined as holist or serialists and were determined from a self-administered study preference questionnaire. The results of their study indicate that cognitive style does interact with mode of C.A.I. to influence student achievement and that learners are more effective when matched to the appropriate mode of C.A.I.

Microcomputers and Mathematics Learning

Microcomputers are already beginning to have an effect on mathematics education. The National Council of Teachers of Mathematics (NCTM) considered this impact at the 1984 conference. A significant statement resulting from that conference was that the major influence of technology on mathematics education is its potential to shift the focus of instruction from an emphasis on manipulative skills to an emphasis on developing concepts, relationships, structures and problem-solving skills (NCTM. report, 1981). Because of the introduction of computers and calculators, the Council argues in their report that it is no longer the most important goal of school mathematics that students be proficient in computational skills. The Council calls for a change in focus to take advantage of emerging technology.

Not all studies of microcomputers and achievement report positive results. The results of a study (Morris, 1983) undertaken to determine how micro-computer activities integrate into regular instruction and how such activities affect student achievement and attitude indicate mixed results. The design of the study included a pretest, posttest and both an experimental (one intact class) and a Control Group (a second intact class). It was determined by an achievement pre-test analysis using an independent t-test, that the two classes were close enough that it was unlikely they came from populations that were different. The study involved a four week unit on co-ordinate geometry where one class received instruction as well as an opportunity to spend time at the computer playing several games which required the use of co-ordinate and directional movement concepts. The second class received only instruction and no computer time. At the end of the four week unit, both classes took an achievement test on the content of the unit. Students were also asked to respond to an attitude scale which related to computers and their use in school. The achievement post-test analysis proved to be significant at the .02 level. The authors concluded that the addition of computers to regular instruction may have significantly improved achievement. Their research design does not however, take into account that any extra reinforcement, computer or otherwise, may have accounted for the improvement. It may simply be that the Experimental Group had lots more opportunity to learn the concepts. The results on the attitude survey were not encouraging. Students were, according to the author, *frustrated by the* [apparent] *limitations of computers, attributing to computers the ability to think and control the world.* This study raises several interesting concerns and makes recommendations for future studies. For example, they ask *What specific aspects or characteristics of computers affect the learning of mathematics?* (Morris, 1983).

In another study of using microcomputers to reinforce arithmetic skills, the results are also not conclusive. Carrier, Post, & Heck (1985) claim that although their study provides some support for the claim that microcomputers enhance achievement in school mathematics, further study is warranted especially in areas other than simply reinforcement of arithmetic skills.

The majority of the literature on computer uses in education however, is generally favorable and indicates that computers may provide an effective means of improving performance in a variety of subjects including mathematics (Henderson, Landesman, & Kachuck, 1985; Griswold, 1984). According to Burns and Bozeman (1981), many studies on the implementation of computer based instruction point to a significant enhancement of learning in mathematics. Jolicoeur and Berger (1988) report in their review of the literature that C.A.I. (computer assisted instruction) is an effective means for improving academic skill, requiring significantly less time than traditional classroom methods. Studies by Kulik et al. (1985) support this position.

Although documented in the literature that the computer may be an effective classroom tool, it is nonetheless true that not all software programs currently available are equally effective (Jolicoeur and Berger, 1988). Some are successful and utilize the computer efficiently and others do not. The development of instructional software has come a long way in the past twenty years. Originally students were not able to interact with the computer to tailor the learning situation to meet their own needs. The potential of the computer was not being realized. Most software programs available were of a linear, drill and practice nature. Today, however, most instructional programs allow much greater use of the interactive capabilities of the microcomputer (Brandon, 1988). Reviews of the literature on the effects of currently available instructional software have generally been positive. Studies typically report improved grades, reduced instruction time and positive attitudes towards computers (Brandon, 1988). Other research indicates that C.A.I. improves academic skills in significantly less time than traditional instructional methods (Kulik et al., 1985).

There are some negative findings in the research concerning the effectiveness of C.A.I.. (Clark, 1985) argues that much of the research on the effects of instructional software contains serious design flaws. Curricular content and method of instruction for example, have not been properly controlled in many studies. He states that instructional software is no more effective than traditional instruction when method and content are appropriately controlled. (Clark 1983). In fact he says, *Media are mere vehicles that deliver instruction but do not influence student achievement any more than the truck that delivers groceries causes changes in our nutrition* (p. 445). According to Mandell and Mandell (1989), critics of computer use in education believe that the current interest in computers is only temporary and will eventually wane just as did the interest in other forms of media as educational tools (i.e., television, programmed-learning books and movies). Advocates of computers in education on the other hand, argue that there is a fundamental difference between computers and other forms of media. Computers allow for active participation and interaction with the computer. Further, unlike television and movies, computers have become pervasive in the work force.

The majority of the literature tends to report positive results in currently available software. Joliceur and Berger (1988) developed a study which analyzed fractions and spelling software programs to determine if they were an effective teaching media. Results from these studies indicated that students who used the fractions software program learned significantly more than students who did not. Some programs were clearly superior to others in the study.

The Design of Educational Software is Important

In designing educational software, a programmer should consider several pedagogical issues. Ease of use of the program, for example, is an important criterion. Software developers must be careful not to design programs which will waste the student's time in learning how to operate the program. Leron (1985) thoughtfully warns us that simply *engaging in mathematical activities does not ensure that children will learn the mathematical concepts involved.* Jolicoeur and Berger (1988) suggest that information processing methods are important cognitive concepts to keep in mind when developing educational software. They say that to *produce optimal learning conditions, educational software must provide students with basic conceptual tools that they can use to build and store more complex concepts* (p 13-19). In addition, they argue, any exercises must encourage students to process information semantically. They state that when not provided with conceptual tools, students merely memorize new information and thus retention can be expected to decline fairly rapidly. What is needed then, are software programs which utilize computer time uniquely and effectively by providing learning environments which cannot be duplicated with other classroom tools.

Summary

Children struggle greatly with the concept of rational number. The difficulty children have with fractions is primarily conceptual. The literature identifies many different interpretations of fraction. Children often hold incomplete ideas possibly because there are so many different definitions. There appears to be a developmental hierarchy in how children develop concepts including those related to fractions. It is important that children begin to understand the concept of fraction by understanding the relationship of equal parts of a whole. Greater emphasis must be placed on the idea that fraction parts must be equal in size. Children struggle with this concept particularly as it relates to fractions with odd numbered denominators and circular diagrams (e.g., illustrating one third of a circle).

The recommendation in the literature is that educators increase their emphasis on teaching meaning and understanding of all the different interpretations of fractions. Children should be taught to analyze the reasonableness of their answers using estimation skills and to rely less on the indiscriminate use of procedural rules. An increased amount of time spent on the topic, earlier introduction, structured learning environments and greater use of concrete manipulative aids are all suggested as possible solutions to the problem children have understanding fraction concepts. Educators should be aware of the role of language when discussing mathematical ideas with children.

In addition to traditional concrete manipulative materials, educators should consider using the microcomputer as a classroom tool. The current literature indicates that the microcomputer has been successfully used in many areas of the curriculum including mathematics instruction. Teachers should choose software programs that are pedagogically sound and take into consideration the individual needs of their students. Some students need more prompting and external motivation than some software programs provide. Also, issues such as ease of use, time on task, student attitude toward the subject and the learning environment and availability of equipment must be taken into consideration.

A possible learning tool for teaching fractions which deserves further exploration is the microcomputer. Ideally, software programs should be designed to teach specific fraction concepts in a time-efficient, structured learning environment. Such a learning environment is intrinsically motivating and utilizes the power of the microcomputer in a unique way.

In other words, this environment relies on the unique abilities of the microcomputer to implement it and cannot be duplicated by other means. It is the hypothesis of this researcher that Visual Fractions is such a program.

Chapter Three
Methodology

The purpose of the study was to determine whether Visual Fractions, a software program designed to teach fraction concepts on the Macintosh computer, is an effective method of instruction. In particular, the issue of learner control over the learning environment was examined. The study was designed to answer three questions:

1. Do children who have control over their learning, learn more?

2. Is Visual Fractions an effective means of teaching basic fraction concepts?

3. Does Visual Fractions help children move toward an understanding of fractions based upon a concept of equal parts of a whole?

This study addressed several issues related to the use of computers for instructional purposes. Because of the findings of several prior research projects, it was assumed that computers are an appropriate method of instruction in mathematics education. A review of the literature also revealed that children typically have a rule-based understanding of fractions and generally do not develop sound basic fraction concepts before being required to perform operations and equivalence tasks. The software program, Visual Fractions was therefore specifically modified for the purpose of this study to provide a learning environment designed to teach basic fraction concepts. This chapter contains a discussion of the sample, pilot study, research design (instrumentation and treatments) and data analysis techniques.

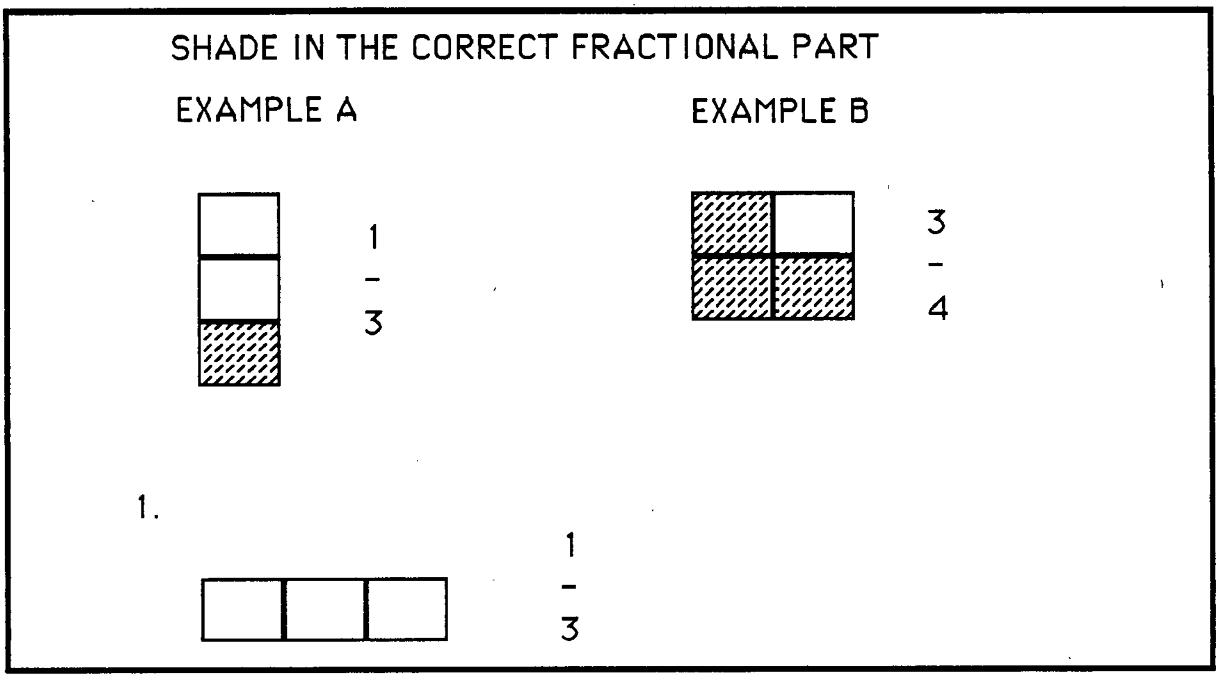
Sampling

The study used a convenience sampling of four classes of grade four students taken from two schools in Burnaby, British Columbia, (a suburb of Vancouver). They are referred to as School 1 and School 2. There were four groups in the study. Two groups were involved using computers, the Visual Fractions Interactive Group and The Visual Fractions Noninteractive Group. The other two groups, Flash Card Group and the Control Group,did not use computers. The computer groups were comprised of students from School 1 and the non-computer two groups were from School 2 All students were randomly assigned to the four Groups.

The Pilot Study

The pilot study occurred at School 1. Three grade three students and four grade four students participated in the pilot study. These students represented a wide range of ability levels. All of the children had worked with Macintosh computers prior to the treatment and were comfortable with the mouse pointing device. The original pretest (appendix E) consisted of 4 pages (a total of 12 questions) arranged in progressive difficulty. Shading questions comprised the first two pages and the last two pages contained questions requiring numeric answers. Two examples were provided for each page of questions. The test was piloted on several children (4 intact classes of grade 3 and 4 children) as well as the 7 students in the pilot study. It was determined from the results of these tests that even grade three students with little understanding or formal instruction in fractions could score relatively high on the test simply by copying the pattern presented in the example questions. For example, on page one of the pretest, the student is asked to shade in the diagrams to illustrate the requested fraction (see figure 3.1) Students simply studied the example and transferred what they saw there to the questions asked. Several students explained this technique to the researcher claiming it was how they answered *lots of worksheets* which they didn't understand.

Figure 3.1 Example of Page One-Pilot Study Pretest



It was decided therefore that this test was an inappropriate measure of understanding of fraction concepts and the pretest was redesigned to address these problems. The subsequent test used during the study was focussed more accurately on the examination of fractions and less on worksheet answering skills. Because the original pretest/posttest was used for the pilot study, it is difficult to determine whether these students showed any improved understanding of fractions. There were however, some important discoveries about the program itself and how children reacted to it.

The three grade three students in the pilot study were extremely enthusiastic and tended to work in a cooperative manner with each other rather than as individual learners. Two were of average ability level but appeared to have no prior knowledge of fractions or at least no formal fraction language to express their ideas when asked prior to the session. The only instruction given to the students at the beginning of the sessions was a directive in the form of a challenge to discover how the program worked. The students willingly shared their discoveries and helped each other to understand how the program worked. They quickly fell into a pattern of challenging each other to *make* different fractions on the computer. Because they did not know the terms *one third* or *two fifths* to describe what they were seeing on the computer screen, they simply used terms like *one out of three* and *two out of five*. They spent a lot of time working with the program button which hides the numbers from view. They would call out a fraction to each other, try to make it on the screen and then uncover the numbers to see if they were correct. These girls had no trouble working with fractions with denominators of twenty and could easily *make* any fraction you asked them to after only thirty minutes of playing with the program.

The third girl tended to work more on her own and was somewhat slower than the other two in understanding how the program worked. She had a problem staying on task with almost any of her required school work, however, her attitude toward school had changed quite considerably after working with the software program. According to this child's aide, she had become extremely enthusiastic about fractions and had designated herself the class expert and tutor! This child could not answer any of the questions on the pretest, but made remarkable improvement on the posttest. In light of this child's normally apathetic approach to her school work, they were amazed at this transformation.

The four grade four students who participated in the study were of varying ability ranges. They were introduced to the computer program in the library of the school where there was only one computer available, thus there was no possibility of peer interaction. The first student, a girl of high ability level, was very quick to understand how the program worked and concentrated very intently for over an hour. She asked no questions and did not make any effort to communicate with us during her entire session. After the hour was over, we had to stop her and ask her to rewrite the test for us. It was later determined that she was an

ESL student and quite shy. Her score improved slightly on the posttest, but as it was quite high to begin with, there wasn't a lot of room for improvement. This student was quite fascinated with the two-way rectangle part of the program and spent much of her time making patterns and seeing relationships between the numbers and the pictures. She wasn't as interested in the button which hid the numbers from sight as the grade three girls were. Although she did not communicate her thoughts, it seemed as though she was trying to really understand exactly how the program worked and why she had to decrease the numerator before she could increase the denominator. She may have benefitted from a more sophisticated version of this program.

The second grade four student was a boy was of average ability and promptly informed us of this fact when asked if he enjoyed mathematics. He did not like the challenge instructions issued to him to simply *play with the computer and try to figure out how the program works*. He really struggled with the lack of directions and in complete contrast to the first grade four student, appeared very frustrated and continually asked questions. After several attempts at trying to understand the program he informed us that he was finished. At this point he was shown how to hide the numeric symbols in order to try and guess which fraction he was creating. From this point on he was able to work much more independently and appeared to enjoy the program very much. He had to be stopped after approximately forty-five minutes because of lunch hour.

The third grade four student was of lower ability level in most subjects including mathematics. She performed extremely well on the posttest to the amazement of her teachers. Extensive interviews with this student lead the researcher to believe that she did not really understand fractions but had somehow learned the system and was able to respond from memory rather than from real understanding.

The fourth student in grade four was exposed to the software program in the computer lab along with the three grade three students. This student was not highly motivated and did not appear to enjoy the program at all. He had trouble with the challenge to find out how the program worked. He seemed to be the type of student whose learning was not self-directed at all and needed constant teacher intervention to keep him on task. He asked to be allowed to return to his class after 30 minutes. Perhaps this type of student would have benefitted more from the program if he was given some assistance in the basic operation of the software or if he had more experience with self-directed learning environments. He did not seem to want to explore on his own and consequently appeared to learn very little

Research Design

The research design adopted was a difference study using a two-factor (treatment and test), quasi-experimental design. Factor A consisted of four Treatment Groups. and Factor B had repeated measures, pretest and posttest.

The Research Design Model

The following model (figure 3.2) illustrates the design of the study:

Figure 3.2 Model of Research Design

GROUP	N	PRETEST	POSTTEST
A1 VISUAL FRACTIONS INTERACTIVE	1 22	B1	B2
A2 VISUAL FRACTIONS NON-INTERACTIVE	1 22	B1	B2
A3 FRACTION FLASH CARDS	1 . . 9	B1	B2
A4 CONTROL	1 . . . 11	B1	B2

Each of the three Treatment Groups was required to write a pretest. Three days after the pretest, each Group was exposed to a treatment session of approximately 20 to 55 minutes in duration. These three Groups then rewrote the same test immediately following the treatment. The Control Group wrote the tests in the morning and rewrote the same test in the afternoon of the same day, but received no treatment. The four Groups in the study were:

1. Visual Fractions Interactive Computer Group.

2. Visual Fractions Noninteractive Computer Group.
3. Fraction Flash Card Group.
4. Control Group.

In addition to this quantitative data, there was a qualitative component to this study, in the form of student interviews. Interviews were conducted prior to the pretest and after the posttest.

Description of Data Collection

Three sets of data were collected: the pretest/posttest of fraction knowledge, student interviews and observations of the treatment process.

The Pretest/Posttest

The purpose of the pretest and posttest (Appendix C) was to make an objective measure of student's understanding of fractions. This was done to determine whether exposure to the program, Visual Fractions, was more effective in developing a good concept of fractions than a similar amount of time spent working with a noninteractive version of the same learning environment. The original pretest/posttest designed for this study was piloted with over 100 grade 3 and 4 students. It was discovered that because of the relatively easy questions at the beginning of the test and the two example questions given for each set of questions, students with little or no understanding of fractions were able to score very well. As a consequence the pretest was redesigned to test fractions rather than worksheet competence. Examples were not provided and the student had to read and answer all of the questions independently using their knowledge of fractions. There were 17 questions on the pretest. The questions were not arranged in any particular order according to type or degree of difficulty. Some questions required shading, others required the student to write numeric symbol answers. Rectangles, squares and circle diagrams were used throughout the test. The same test was administered as the posttest. The maximum possible score on the test was

68. Each answer was rated a nominal scale score between zero and four based on how closely the response resembled a correct understanding of fractions. Answers were given a score of zero if no attempt was made to answer the question. A score of one was given if some effort (mar) was made but there was no obvious relationship between the answer and either the numerator or denominator of the fraction.or if the attempt to shade appeared to be completely arbitrary. A score of two was assigned if the answer corresponded in some way with either the numerator or denominator, but not both or if the shading represented some discernable reasoning process, though not accurate. The answer did not have to be correct as long as there was some apparent purpose for, or relationship between, the numerator or denominator given by the child. For example, question 1 on the test asks the child *What fraction is illustrated by the shaded portion of the diagram ?* (a rectangle divided into fifteen parts with 7 shaded in). If a child answered 7/12, they would have scored 2 points. The numerator 7 corresponds correctly, but the denominator is not correct and there is no apparent logical reason for it. A score of three was assigned if there was a relationship between the answer and the correct numerator and denominator or if the drawing was roughly correct but not as accurate as it could have been. For example, question 16 shows a rectangle with 1/3 shaded .Students who answered 1/2 would have scored three points because there is one shaded part and two unshaded parts.Both the numerator and denominator in the child's answer has a logical purpose, though not correct. A score of four was assigned only to correct answers (numerator and denominator) and reasonably accurate efforts at shading. There were three basic types of questions included in the test. These three types of questions are randomly dispersed throughout the test.

Four types of questions were used:

1. The first type of question was designed to test fraction concept and estimation skill. Students typically work with halves, thirds, fourths and fifths when first learning about fractions. Students with a good understanding of basic fraction concepts

should be able to transfer knowledge learned about these smaller numbers to any problem with a reasonably larger denominator (e.g. fifteenths and twentieths). Some of the questions required shading, others required answers to shaded-in diagrams. Questions in this group were: 1, 8, 9, 10, 14. Figure 3.3 shows an example of a shading problem which tested understanding of a *large* denominator problem (question 14):

Figure 3.3 Example Shading Question With Large Denominator

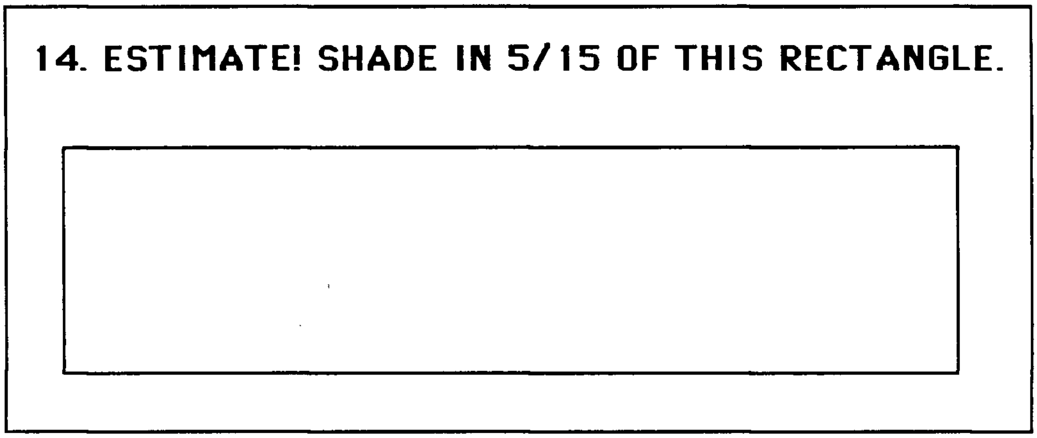
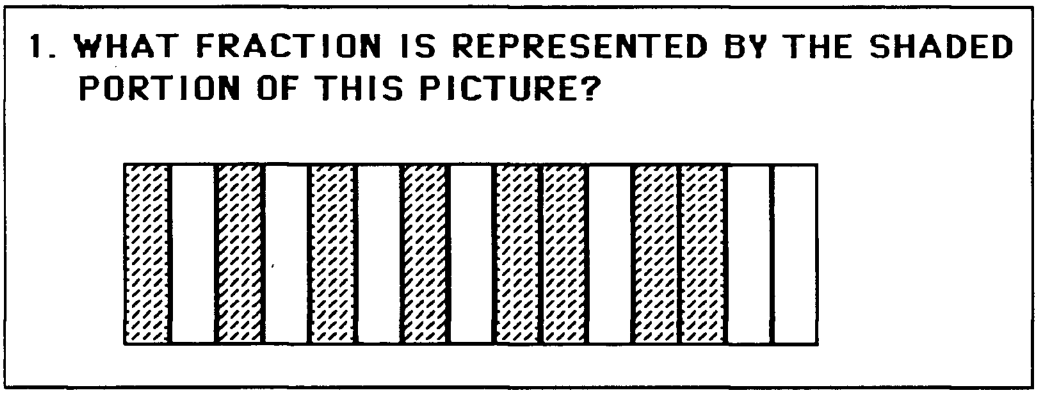


Figure 3.4 shows an example of a question with a large denominator which requires the student to name the fraction part shaded (question 1):

Figure 3.4 Example Naming Question with Large Denominator



2. The second type of question was designed to test the idea that fraction parts must be of equal size. Again, some of the questions required shading and others required answers to shaded-in diagrams. Questions in this group were: 3, 4, 5, 11, 12, 13a, 13b, 15. Figure 3.5 illustrates a typical example of a shading problem of the second type (question 3):

Figure 3.5 Example Partitioning Question

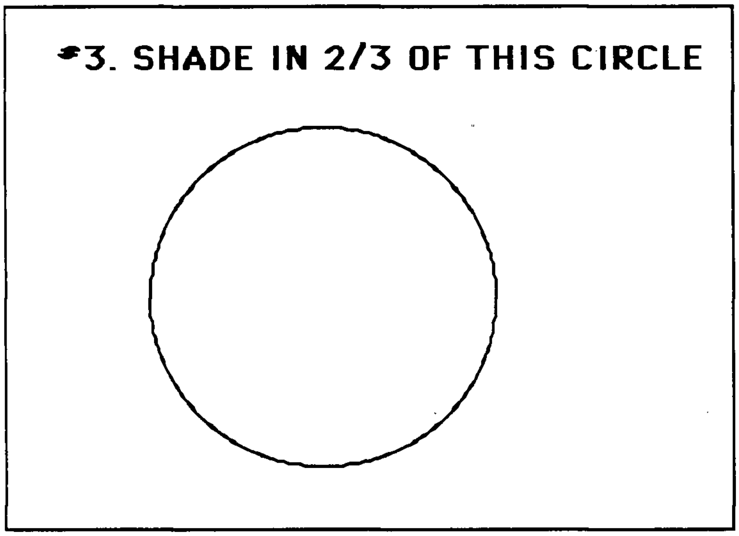
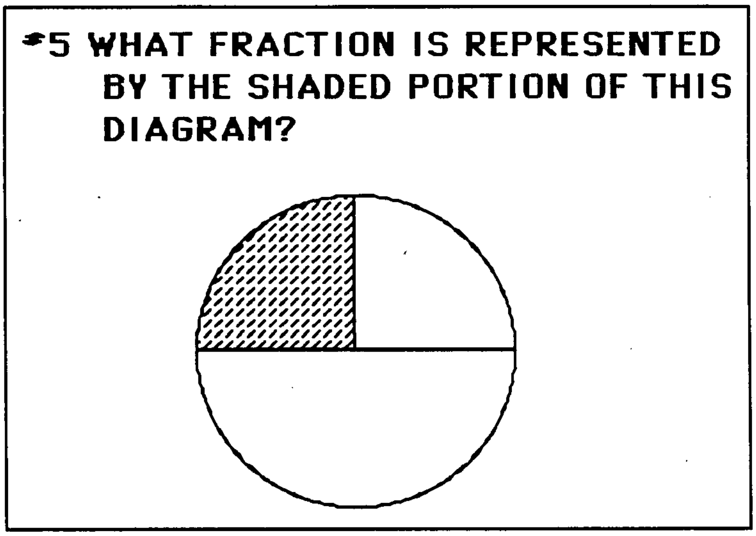


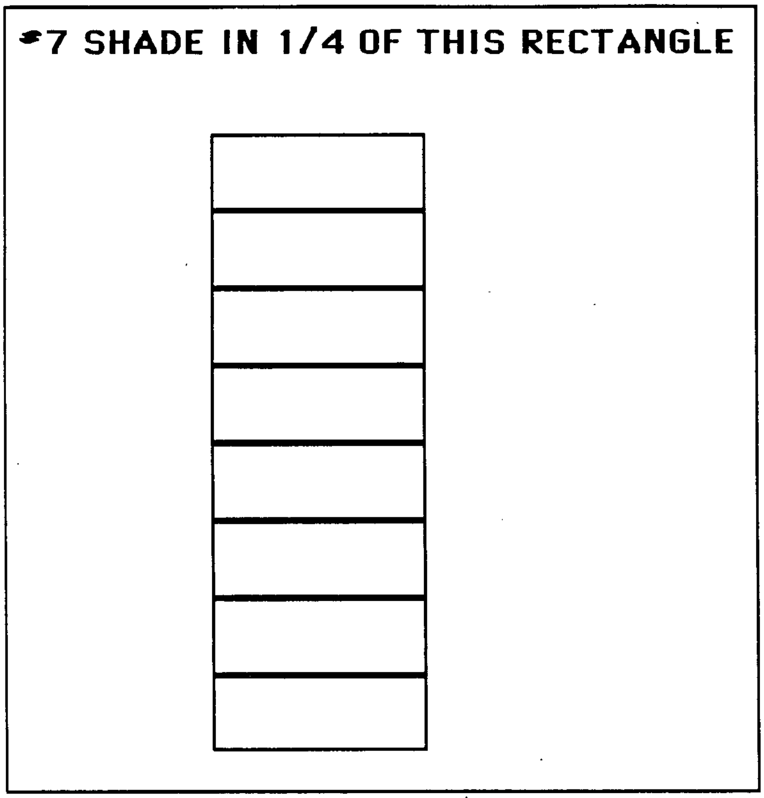
Figure 3.6 (question 5) illustrates the same type of question except that here the student was required to determine what fraction was represented by the shaded portion. (Before the treatment, most students thought that this diagram illustrated the fraction 1/3).

Figure 3.6 Example Naming Question With Unequal Partitions



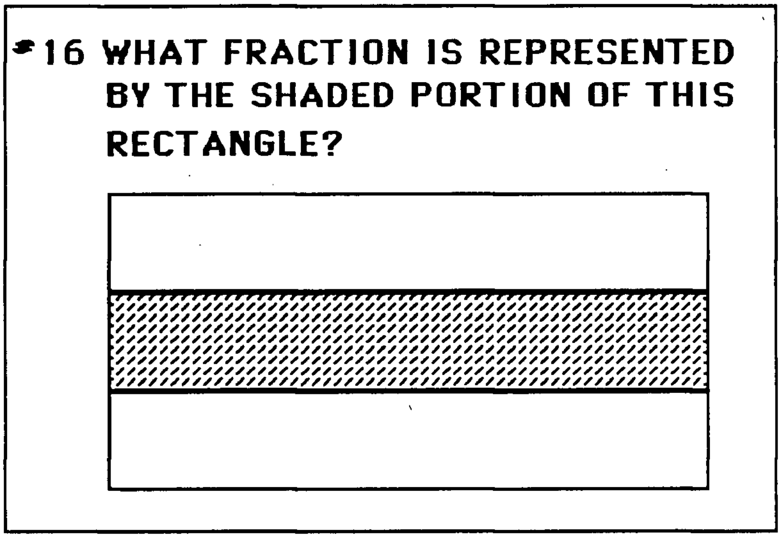
3. The third type of question was designed to test mastery of basic fraction concepts. These questions were in a sense equivalency tasks and go beyond basic understanding, however, in the context of the test and with the diagrams provided, students without formal instruction in equivalent fractions still should have been able to figure out the answer to these two problems. Questions in this group were: 6 and 7. Figure 3.7 (question 6) illustrates an example of this type of problem.

Figure 3.7 Example Shading Question with Equivalent Fractions



4. The fourth type of problem included on the test involved the ability to recognize very commonly used fractions, 1/3 and 1/4. Students were required to recognize the shaded-in portion and write the correct numeric symbol. These questions were included to see whether students who had little or no understanding of fractions (demonstrated by an inability to correctly answer any of the other questions on the test) could still answer these two questions correctly simply from rote memory. They are the most commonly used fraction illustrations and it is possible that students are able to correctly answer these questions without any understanding of what a fraction is. Figure 3.8 (question 16) illustrates an example of this type of question.

Figure 3.8 Example Naming Question with Equal Partitions



Each question was constructed with a specific purpose and diagnostic intent. See Appendix C for illustrations of each question. The intent of each question is described below:

Question 1: This question was included to determine whether students could handle a fraction with a large denominator. The shading is purposely scattered throughout the diagram in a arbitrary manner.

Question 2: This question was included to see whether students can write a fraction symbol correctly for a question they should have seen many times. It is likely that even students with little or no understanding of fractions will have been able to answer this question.

Question 3: This question required the student to shade in 2/3 of a circle. It was a difficult question and was included to see whether the children understood that fraction parts must be of equal size AND were able to act on that knowledge by dividing the diagram correctly.

Question 4: This question was deliberately designed to be confusing. At first glance it looks like one half of the diagram is shaded, which in fact it is. However, the line separating the right hand side makes it seem like the answer could be 2/3. The student who understands that fraction parts must be the same size should have answered 1/2 or 2/4, not 2/3.

Question 5: This question required students to realize fraction parts must be equal and answer 1/4, not 1/3. This is a common textbook problem.

Question 6: This question is difficult for children who do not have a sound concept of fraction and especially a knowledge of what the numerator and denominator represent. The student had to realize that although the question asked for 1/4, they were to shade in 2/8 of the diagram. This was a visual problem, not an equivalency task because the students were provided with a diagram as well as a numeric symbol. The fraction diagram was divided with a broken rather than a solid line to help students realize that the lines were not absolute.

Question 7: This was the probably the most difficult question on the test. It required the student to ignore the solid lines dividing the diagram into fifths and instead to shade in 3/4. This question required a solid understanding of basic fraction concepts in order to answer it correctly.

Questions 8, 9, 10, and 14: These questions were designed to see whether students could estimate large numbers of parts and shade in the correct response.

Questions 11 — 13b: These questions all tested the fraction concept of equal parts of a whole.

Questions 15 and 16: These two questions are designed to see whether the student with a good concept of fraction will answer 1/4 to the top one and 1/3 to the bottom one. They were deliberately placed together to make the student think at first glance that both should be 1/3.

Student Interviews

The purpose of the student interviews was to see how much the students knew about fractions and to see whether there were any differences between the three Groups' responses after the treatment. It is often difficult to determine why a student answers a question in a particular way and the interviews were designed partly as test of the pretest/posttest, to determine whether it was an accurate evaluation of student's knowledge of fractions. Five students were randomly selected from each Treatment Group and interviewed twice. The first set of interviews occured directly after the pretest and prior to the treatment. The same students were interviewed again after the treatment. Students were asked to explain what they were doing as they worked. Interviews were audio recorded. The procedure was the same for both interviews. Students were asked the following seven questions during the interview (Appendix A).

Question 1: Students were asked to demonstrate an ability to take a piece of paper (8 1/2 by 11 inches) and divide it into five sevenths.

Question 2: Students were asked whether they thought fraction parts all had to be the same size. This question was repeated at the end of the interview to see whether the interview process caused the student to change his thinking or if he retained his original answer throughout the interview process. Students were questioned to make sure that they understood the directions of the test and in particular that they knew what part of the fraction the shaded area of the diagram represented. They were asked to identity the shaded and unshaded fraction of the same diagram. The commonly used fractions 1/3 and 2/3 was used for this question to see whether students might have been able to answer this from memory without real understanding. It was assumed that students who could correctly answer this question but no others on the interview, were answering from memory rather than from an understanding of fractions.

Question 3: Students were asked to explain what a fraction was to someone from outer space who had never heard the word before.

Question 4: Students were asked which was bigger, 8/12 of a chocolate bar or one whole chocolate bar? This question was included to see whether students thought 8/12 was larger than one whole. It was also included to see whether they understood the difference between 8 out of 12 chocolate bars and 8/12 of *one* chocolate bar. Students who answered that 8/12 was larger had the question repeated to them changing the word *a* chocolate bar to *one* chocolate bar.

Question 5: Students were asked to determine from both a diagram and a numeric representation, which was bigger, 2/3 or 4/6. The pictures were arranged to make the visual clue most obvious. The purpose was to see whether students could operate from a visual expression of fractions and not just numeric.

Question 6: This question asked students to identify which of two diagrams represented 4/10 when in fact both did. The purpose was to see whether students could identify the bottom picture as 4/10, recognizing that fraction parts must be of equal size or whether they would ignore the size of the fraction parts and answer 1/7.

Question 7: The final question in the interview was also designed to test the student's understanding of fraction size. At first glance it appeared to be 1/3, however the student who clearly understands fraction parts must be the same size should have realized that the correct answer was 1/5.

Observations of the Treatment

In addition to the qualitative data provided by the student interviews, the research design for this study included a report and analysis of the treatment process. Data was collected in the form of notes by the researcher during the treatment sessions for each Group. General impressions, noise level, peer interactions, student comments, frustrations, time on task, etc were noted and reported for each Group. A summary of this data is reported in Chapter four.

The Treatment

There were four Groups, three Treatment Groups and one Control Group.Parental consent forms (see appendix B) were taken to the school by the researcher one week prior to the experiment. The teacher was responsible for the distribution and collection of the signed consent forms. Parents were given three possibilities: allowing their child to participate in the all phases of the experiment, allowing their child to participate in the experiment, but not be interviewed or not allowing their child to participate at all. Two parents did not want their child to participate at all and twelve allowed them to participate, but not be interviewed.

Visual Fractions Interactive Group

Students selected for the first computer treatment were given an opportunity to explore the interactive version of the Visual Fractions software program.There were only four screen on the entire program. The first screen was simply a title screen with an arrow button indicating that the user should click on the arrow to proceed to the second screen. The next three screens displayed different types of fraction diagrams - a horizontal bar diagram , a two-way rectangle diagram (a square) and a circular diagram. Each diagram was on a separate screen which also displayed the corresponding written fraction symbols. The user was able to change the size of the shaded portion of the fraction diagram to graphically display fractions ranging in size from halves to twentieths. As the size of the fraction was changed, the program supplied the correct mathematical symbol. The child was able to see the association between the fraction represented by the diagram and the symbol. The symbol form of each fraction was displayed in large numbers beside the graphic representation to help establish the link between concept and form. The student had the option of clicking on the numeric symbol and hiding it from view. Because of the speed and accuracy of the computer, many fractions could be displayed in a very short time The user could move between the four screens simply by clicking on the arrow button located at the bottom of each screen. . The

Students in this Group were allowed one session of up to an hour in duration.Students were taken out of their regular classroom schedule in two groups to use the computers. Each student worked individually on a Macintosh computer. The researcher explained that the child's task was to discover how the program worked. The researcher remained in the room during the students time on the computers and answered general questions about the operation of the computer, resolved hardware and network problems, explained how to operate the mouse, etc. No instructions were provided to the students on how to operate the program. The researcher did not tell the students how the program worked, but provided positive feedback and reinforcement to students as they began to understand how the program worked. The researcher encouraged students who were having difficulty getting started. The researcher also made notes on interesting student comments, questions and behaviors during the computer session.

Visual Fractions Noninteractive Group

In order to ensure that any positive results gained from the Visual Fractions treatment were not due to the novelty of using computers, the research design included a noninteractive version of the program which used the computer to view the same flash cards used by the Flash Card Group. The only difference was that the fraction diagrams were displayed randomly on the computer screen. The user was able to control the rate at which the cards were presented but had no other control over the learning environment.

The procedure for this Group was the same as the Interactive Group. Instructions were the same and this Group was provided with the same opportunity to spend as long as they wanted up to one hour using the software program.

Flash Card Group

Students selected for the Flash Card Group had an opportunity to explore a more traditional method of instruction on fractions. Students in this Group were provided with a set of fraction flash cards designed by the researcher designed to simulate the noninteractive version of the Visual Fractions program. See Appendix D for an example of the Flash Cards.With the cards, students were able to create fractions by lifting the flaps on the cards to reveal either a fraction diagram or the corresponding written symbol. These cards were designed to be very similar to the display in the Visual Fractions program, except for two critical differences. First, the child in the Flash Card Group was not really in control of his own learning. In the Visual Fractions treatment, there was a real sense of control over the learning situation; children felt as though they were making the fractions through their own efforts (i.e., by sliding the mouse button back and forth on the diagram)

This Group had the same amount of time to examine the materials and figure out how they worked. Students were taken out of their regular classroom schedule in groups of three or four. The researcher remained in the room during the session and observed the activity of the children. Questions were answered about the cards and what to do with them in a vague and general way, usually by responding with another question, i.e., *What do you think you could do with the cards?* The researcher made notes on student behavior, comments and group interactions during the sessions. The researcher administered the treatment to two groups of four students and one group of three students. These students were randomly selected from the two grade four classes at School 2. All students received the treatment on the same day.

Control Group

The Control Group did not receive any treatment. They wrote the test in the morning with the students selected to be in the Flash Card Group and did not know ahead of time which Group they were in. They rewrote the same test at the end of the afternoon on the same day.

Data Analysis

The data from the pretest and posttest was coded on a five point scale from zero to four. Responses were assigned a score based upon how closely they approximated a correct understanding of the question. For example, if the student made no attempt to answer the question, they scored zero. If they made some effort but there was no apparent logic to their response, they score one. If there was a logical (though not necessarily correct) reason for the denominator or the numerator, they scored two. If there was a logical reason for both the numerator and denominator, they scored three. If the answer was correct, they scored four. These scores were analyzed using BMVDP for a repeated measures ANOVA. Data collected from the interview process was summarized and overall generalizations, individual and Group responses were reported. The researcher's observations of the treatment process are also reported. The qualitative data was analyzed by the researcher.

Chapter Four
Results

In this chapter, an analysis of the findings from both the quantitative and the qualitative part of the study are presented. First is a report of the three treatment sessions including student performance, attitudes, etc. The second part focuses on the results of the quantitative data. A repeated measures ANOVA was used with one treatment factor (the four Groups) and one repeated measures factor (the pretest and posttest). The final part of the chapter presents a report of the student interviews including general findings, Group similarities and individual reports where relevant.

Treatment Session Report

The following is a general report of the procedures of each of the Groups in the study.

Visual Fractions Interactive Treatment

The children were exposed to the treatment in two groups. The computer lab was operating efficiently for the first treatment and there were sufficient computers for each student to work by themselves. For the second group, there were some hardware problems and this caused some interference with the session. These students were limited to approximately 50 minutes rather than the full hour exposure. The following comments are with regard to the general condition of the treatment sessions and apply to both sessions.

Most of the students were generally enthusiastic and cooperative. They were simply issued the challenge to *discover how the program worked.* The children seemed confused at first and not certain how to go about the assigned task. Many of them called for clarification and some stubbornly demanded more information. The researcher kept communication to an absolute minimum. The only comments made were designed to encourage the students to keep trying, to play with, and to explore the program.

When students thought they knew what the program was about, they were keen to share it with the researcher and each other. The researcher offered positive praise for the discovery and encouraged any student-initiated peer interaction. Most of the students did not want to leave the lab when the hour was up. There was however, a small group of students from one class were working on a dinosaur project who wanted to go back and finish it. These students said that normally they would have preferred to stay and work on computer, but the dinosaur art project was more appealing. All of the students in both sessions expressed a desire to use the program again.

Visual Fractions Noninteractive Treatment

These students were divided into two different groups for treatment. One of these groups experienced similar hardware problems as one of the Interactive Groups, however, the problem did not interfere with the learning process. Students in the Noninteractive Visual Fractions Treatment had a very different reaction pattern to the treatment sessions. They were very keen at first and understood right away what was expected of them. After approximately five minutes, most of them felt they understood what the program was about and how to operate it. After approximately fifteen minutes, however, they became bored and terminated the program voluntarily. Only four out of the twenty two students who used this version of the program expressed a desire to use it again.

The following problems affected both the Interactive and the Noninteractive Groups:

There were some problems with hardware malfunctions in the Macintosh lab. These problems did not, however, appear to affect the learning process and were dealt with promptly.

One potentially very serious problem in the treatment process may have been the interference from a dinosaur project which the students in one class were working on. It seemed that this unstructured learning activity was not the norm for these students and they clearly did not want to miss out on this unique opportunity to play with plasticine and chat with their friends in an informal session back in the classroom. This probably interfered with their ability to concentrate on and benefit fully from the treatment. However, as students from this class were divided randomly, half into each of the two computer treatments, this problem would have affected both Groups and not just one. Thus the results, for the two different computer methods would not have been contaminated in favour of one Group or the other because of this random placement of students equally into both Groups.

Flash Card Treatment

Students in this Group were divided into three groups and of three or four students each. The groups were exposed to the treatment in a small office containing one round table on which all the flash cards were placed. The students in all three groups seemed to be extremely keen, highly motivated and very cooperative. Only one student tried to examine the cards independently and he was very quickly conscripted into the cooperative activities of his peers. These students were given the same directive as the students in the computer sessions: to examine the materials and try to figure out how they worked or what they could be used for. Most of the students expressed a desire to have a set of the cards in their classroom and could think of several activities associated with them. The students in all three groups sorted the cards by denominator and ranked them from highest to lowest. At no point did the researcher ever suggest that there was a *right* way to use the cards. Some of the students recognized equivalent fractions and started to categorize them according to equivalency. None of the students wanted to leave when their time was up.

The following comments pertain to both the Flash Card and the Control Group:

Although the results do not indicate greater improvement, the students in the Flash Card Group had several advantages, some of which were external and unavoidable and others which were intrinsically part of the research design. None of the students in any the three flash card groups wanted to leave when the session time was up and all said they really enjoyed playing with the cards. It may be that the small group situation and the privilege of being chosen over their classmates may have contributed to the intense interest in the learning situation. The other Treatment Groups did not enjoy this same advantage because everyone in the class (with the exception of those who did not have parental permission) participated in the study. Also, in the Flash Card Group, the novelty of the cards (they were bright, colourful, laminated cards with velcro flaps) may also have influenced the student's behaviour. This high degree of concentration and enjoyment may have been the result of these unforeseen advantages.

Because of the smaller sample size, these students were exposed to the treatment in small groups of three of four. It was clear from the researcher's point of view that these students were more on task and concentrating much more intently that the students in either of the other two treatments. It is possible that the presence of the researcher in a small room, sitting at one round table with a handful of students may have been responsible for focusing their attention on the task at hand. Additionally, these students knew that they were chosen (randomly) to participate in the treatment whereas in the other Groups, everyone who returned their permission slip was permitted to participate. This exclusivity may also have contributed to their greater enthusiasm. The small group may also have been a contributing factor in the peer interaction which was clearly very intent in this Treatment Group. In fact, only one student tried to examine the cards independently and was eventually drawn into the group activities of his classmates. Finally, this class had just completed a unit on fractions and had a greater initial understanding of fractions in general prior to the treatment .This instructional advantage was clearly evident from the pretest results and it may have been responsible for the absence of significant improvement from the pretest to the posttest in spite of the evident enthusiasm and concentration on the treatment (i.e., *a ceiling effect*).

Quantitative Data Collection

The following is a general report of the quantitative data collection, consisting of a statistical report of the pretest and posttest results and a summary rejecting the Null Hypothesis.

Pretest/Posttest

The pretest/posttest was administered to all 64 subjects in the study. There were four Groups in the study, three treatment Groups (two Computer Treatment and one Flash Card Group), and the fourth Group which served as a control and did not receive any treatment. The same test was used as both the pretest and the posttest. The total possible score on the test was 64. The results of the tests were analyzed using BMDP2V for a two-factor (four Groups by two Tests) analysis of variance with repeated measures (pretest and posttest) on the second factor.

Table 4.1 presents a summary table of statistical significance for each of the two factors in the study and their interaction. Factor A represents the main effect *Groups* of which there were four. The ANOVA indicates that when considered by themselves, there is no significant difference between any of the four Groups in the study. Factor B represents the main effect *Tests* of which there were two, the pretest and the posttest. The same test was administered as the pretest and the posttest. The ANOVA indicates that there is a significant difference between the scores on the pretest and the posttest at the .01 level

Table 4.1 Comparison of the Four Groups on the Two Tests

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A (Groups)	875.56	3	291.85	1.58	0.2045
Error	1109.93	60	185.16		
B (Tests)	346.97	1	364.97	8.75	0.0044
AB (Interaction)	482.24	3	160.74	4.05	0.0109
Error	2378.75	60	39.64		

Table 4.2 shows a comparison of the means scores for each Group. Figure 4.1 displays the difference between the mean scores for each Group. The maximum possible score on the test was 68. The greatest difference between pretest and posttest means occurs with Group 1, the Interactive Visual Fractions Group. Group 1 had an almost 10 point (15%) increase in mean scores between the two tests. The results indicate that the Control Group had an almost zero (43.8 - 43.9) increase in overall mean scores. The difference in mean scores between the pretest and posttest for Group 2 and 3 were both less than three points. The pretest score for Group 3 was much higher than the other three Groups. Their posttest score however, was very similar to the posttest score for Group 1, indicating an increase of less than two points. Group 3 (the Flash Card Group) and Group 4 (the Control Group) were comprised of a random selection of students from two different classes, one of which had just completed an intensive introductory unit on fractions. The other two Groups did not have a similar experience that year. It is probable that this additional instructional time accounted for the higher pretest score for Group 3.

Table 4.2 Group Mean Scores and Standard Deviations

Group	Pretest	SD	Posttest	SD	Means
A1	41.20	12.40.	51.00	9.48	46.10
A2	41.00	11.39	43.50	8.96	42.30
A3	49.30	7.95	51.00	7.59	50.10
A4	43.80	11.79	44.00	12.62	43.90
Col Means	42.70		47.20	45.00	

Note:

A1 = Visual Fractions Interactive Group
A2 = Visual Fractions Noninteractive Group
A3 = Flash Card Group
A4 = Control Group

The graph of the overall means (figure 4.1), suggests that the significant factor B (tests) can be explained by the difference between pretest and posttest scores for Group 1 (A1). This difference between the pretest and posttest scores does not seem to hold for the other Groups because the lines appear to be parallel for these Groups within chance. The graph suggests that a special study of the interaction effects should be made.

Figure 4.1

Comparison of Means

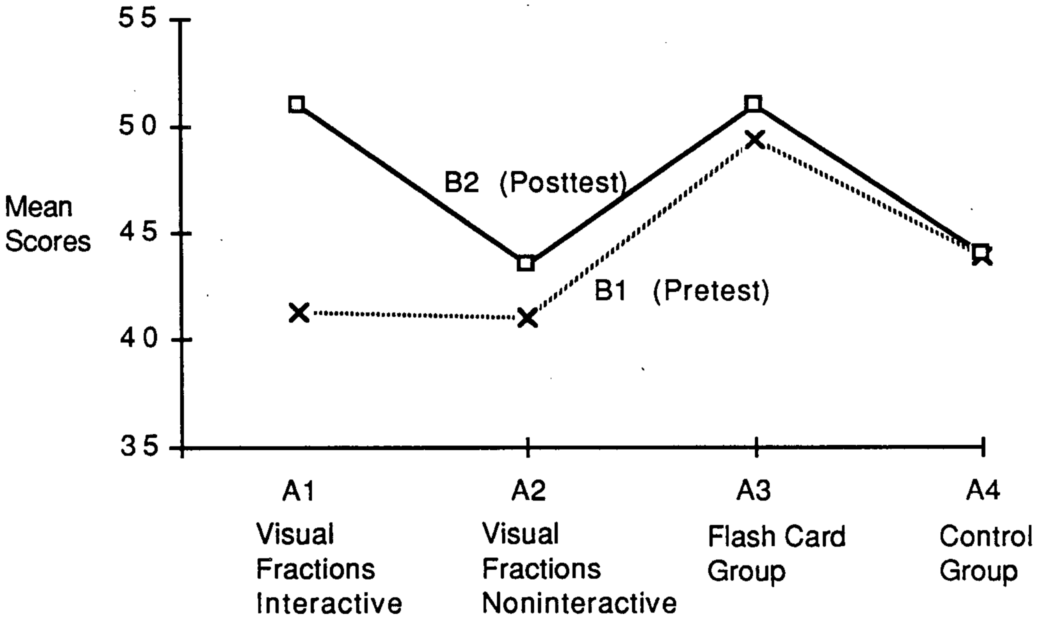


Table 4.3 Calculation of Interaction Effects

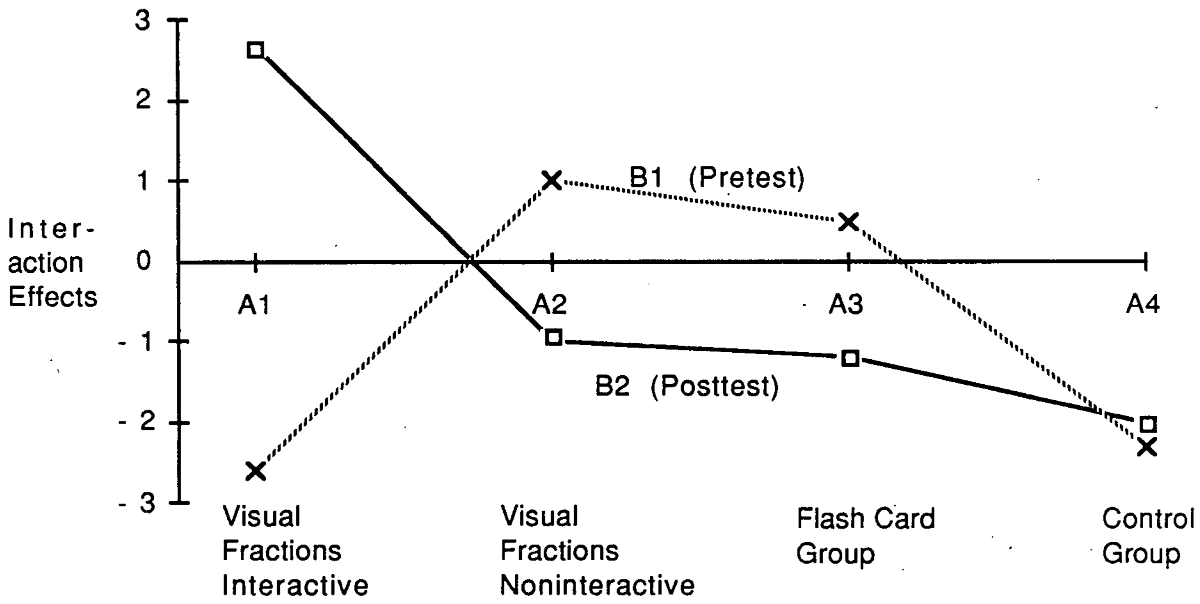
Inter-action	cell value	-	row mean	-	col mean	+	grand mean	=	effect
A1B1	41.2	-	46.1	-	42.7	+	45.0	=	- 2.6
A1B2	51.0	-	46.1	-	47.2	+	45.0	=	+ 2.7
A2B1	41.0	-	42.3	-	42.7	+	45.0	=	+ 1.0
A2B2	43.5	-	42.3	-	47.2	+	45.0	=	+ 0.5
A3B1	49.3	-	50.1	-	42.7	+	45.00	=	- 1.2
A3B2	44.0	-	43.9	-	47.2	+	45.00	=	- 2.3
A4B1	43.8	-	43.9	-	42.7	+	45.00	=	- 2.3
A4B2	44.0	-	43.9	-	47.2	+	45.00	=	- 2.0

Figure 4.2 depicts a comparison of the interaction effects (BA). When the interaction effects are graphed, the results show that for the Group 1, the posttest performance was relatively

superior to the pretest performance. This superiority in performance on the posttest relative to the pretest, however, does not hold for the other three Groups. The graph shows that the AB effects lie close to zero for the other three Groups and therefore the same results as for the first group cannot be substantiated.Although the interaction effect, AB, is not significant at the .01 level, the probability of error is so close to 0.01 ($p = 0.0109$), that it is reasonable to assume there an interaction effect is with Group 1, the Visual Fractions Interactive Group.

Figure 4.2

Interaction Effects



Summary:

There was no difference found between the groups. The significant difference between the pretest and the posttest could not be interpreted because of the significant interaction. An examination of the interaction effects revealed a significant difference in the mean score for treatment one, the Visual Fractions Interactive Group. The other groups did not change significantly between the pretest and the posttest.

Student Interviews

Approximately 12 parents chose the option of allowing their child to participate in the study but not to be interviewed. Many of the children in the class were eager to participate in the interview process and requested to be chosen. Five children from each Treatment Group were randomly selected for interviews from those who had been given parental permission. Children were taken out of their regular classroom routine and interviewed privately for approximately 10 to 15 minutes, in the morning prior to the computer treatment and immediately after writing the pretest. Each child was interviewed a second time as soon as possible after writing the posttest. No one had to wait longer than three hours to be interviewed after the posttest.

Computer treatment Groups. Because of random assignment of children into Treatment Groups, six of the children interviewed at School 1 were assigned to the Noninteractive Group and only four were in the Visual Fractions Interactive Group. Interviews were conducted at a table in the corridor due to limited space available in the school. There was considerable interference due to noise and other distractions, however this did not appear to affect any of the childrens' ability to concentrate on the questions. Most children did not appear to mind missing their classroom activity to participate in the interview, and appeared to try their best at answering the questions. However one boy was not particularly interested in trying to answer the questions and during the second interview he complained that he was missing *sharing time*. It is possible that he knew more than he was willing to reveal.

Control and Flash Card Groups. At School 2, none of the children in the Control Group were interviewed. Four children from one class were interviewed and one from the other class. Again, the uneven representation from the one class was due to random selection of students. A private office was provided for the purpose of interviewing students and all the students appeared to answer the questions to the best of their ability. The data from one student in the Noninteractive Group and one student from the Flash Card Group was not usable as second interviews could not be obtained for these two students because they were absent during the scheduled time for the second interview.

The interview process did not appear to significantly affect the childrens' learning. At the beginning of the interview, every child was asked whether they thought fraction parts had to be of equal size. This question was repeated at the end of the interview. In every case, the children did not change their original answer to this question during either the first or second interview. Students who did change their mind, did so between the end of the first interview and prior to the second interview. It is therefore reasonable to conclude that students did not change their mind as a result of the interview process.

Question One: The task was to show 5/7 of a piece of paper by tearing it.

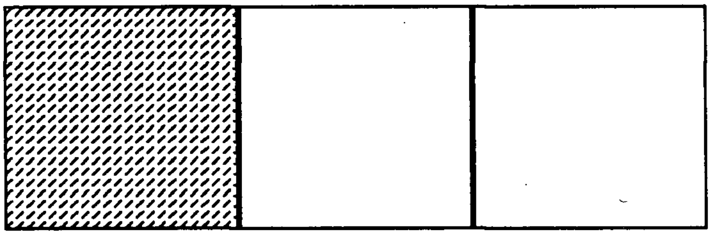
This question appeared to be quite difficult for most students in grade four. Many were not able to attempt and most did not try to measure or make their pieces at all similar in size prior to the treatment. After the treatment sessions, a few students did make some effort to measure, but in most cases this was not very successful.

Visual Fractions Interactive. Students in the Visual Fractions Interactive Group responded to this question with a large variance in ability. Before the treatment, two of the students interviewed would not even attempt the question. The other two were able to divide the paper into seven pieces and give back five of them, but made no attempt to make the pieces even roughly the same size. No one made any effort to measure their work at all. After the treatment, during the second interview, every student at least attempted the question with varying degrees of success. This time two of the students tried to measure very carefully in order to have pieces of the same size. Three students were still unable to answer the question correctly, but it was apparent that they at least understood what to do even though they could not do it. This task appears to be very difficult for students of this age.

Noninteractive. Students in the Noninteractive Group had a greater initial variance in their response, however, each student retained their original answer from the first to the second interview. Three of the five children in this group appeared to understand what was being asked of them. Of these three, no one tried to measure the pieces of paper or make the fraction parts in any way similar, let alone the same size. Of the remaining two children, one student would not even try the problem. The other student ended up with eight unequal pieces of which she kept three and returned five. A significant finding with this question is that for all five children in this group, their initial approach to the question and final answer remained almost identical from the first to the second interview. The treatment did not appear to affect their ability to answer this particular question at all.

Flash Cards. Students in the Flash Card Group were similar to the Noninteractive Group in that they had a large variance in their ability to answer this question and no one changed their original response. One boy measured very carefully and correctly answered the question. Three others were able to divide the paper into seven pieces, and give back five of them, but none of them made any attempt to measure or make the pieces equal. The fifth student did not use the entire piece of paper. He divided part of it into five unequal pieces and when asked what the left over piece was, he called it *two blanks*.

Question 2: Look at the diagram. One part is shaded and two are not. All the parts in this diagram are the same size.



a) Do fraction parts always have to be the same size? (yes)

b) What fraction is represented by the shaded part of this diagram? (1/3)

c) What fraction is represented by the unshaded part? (2/3).

This set of questions was included to see whether students understood the notion of equal parts of a whole and also to see whether they understood the language used during the interview process (i.e., shaded, unshaded, represented etc.) The simple fraction 1/3 which is commonly used in textbooks was chosen to establish whether the child understood the language used. It was discovered that several children did not. They spoke in terms of *one two* (meaning 1 on the top and 2 on the bottom) rather than *one third.* Also, it was apparent that some children could recognize the fraction 1/3 from rote memory with very little understanding of fractions. They could for example correctly respond *one third* but not know which number corresponded with the numerator and which the denominator. (Note: the terms *numerator and denominator* were not actually used; instead students were asked, for example, what the bottom number represented in 1/3.)

Visual Fractions Interactive Group. All four students in the Visual Fractions Interactive Computer Group responded *no* in the initial interview to part A of this question. After the treatment all four students had changed their minds and answered *yes, fraction* *parts do all have to be of the same size* at the beginning of the second interview. Every student was able to respond 1/3 and 2/3 to part B and C.

Visual Fractions Noninteractive Group. Initially, all five students in this Group responded *no* and only one student changed her mind during the second interview. She said that she changed her mind because the computer had always drawn the fraction parts the same size. Four of these students were able to recognize 1/3 and 2/3. One student did not have the verbal language but requested to write his answer. He wrote 1/2 and 2/1 and explained these responses as 1 on the top and 2 parts on the bottom and vice versa. He did not change his response after the treatment. The other student who could not recognize these fractions initially was able to recognize both after the treatment.

The Flash Card Group. Four of the children in this Group did not think fraction parts had to be the same size and one believed they did. After the treatment, no one changed their mind. Everyone in this Group was able to answer 1/3 and 2/3 during both interviews, except for one student who responded *two into one* and *one into two*. He used these terms interchangeably and didn't seem to know which should correspond with the numerator and which the denominator. He made no improvement by the second interview.

Question 3: Describe what a fraction is to someone from outer space who had never heard the word before.

This question resulted in a large variance in student responses. Some students were not able to answer at all, others gave a school context-based or text book type answer, and a few described real life partitioning activities.

Visual Fractions Interactive Group. There were two different types of response to this question. Two of the children (both from the same class) responded in a context-based manner (e.g., *A fraction is parts of a block and some are shaded*). The other two children were able to talk about fractions in a more abstract manner (e.g. *A fraction is a part of something, for example a chocolate bar, that you want to divide into parts and share*). All of these children answered this question in a very similar manner before and after the treatment. One child added the notion of equal-sized parts to his definition of fraction, thus verbalizing a more sophisticated understanding of fraction after the treatment.

Visual Fractions Noninteractive Group. Of the five children in this Group, one was unable to give any sort of answer, two answered from the context of a school-based task (i.e., a math question involving shapes, shading in, numbers, etc.) and two were able to answer in a more general context (i.e., parts of something). After the treatment, the student who could not answer was able to give a reasonable definition (i.e., pieces that you divide up) and another student shifted from a context-based idea to a more general understanding of fractions (i.e., splitting something into Groups). Thus, two of the five children in this Group were able to verbalize a more sophisticated understanding of fractions after the treatment.

Flash Card Group. This Group was similar to the other two Groups in that two of the children referred to fractions as part of something and the other three responded in the context of a mathematical formula (i.e., fractions are numbers with a line in between them and the top number means the shaded part and the bottom is the total parts.) The only change in this Group's answers from one interview to the next was in one boy who couldn't provide a definition during the first interview and was able by the second interview to say *Fractions are Math....*

Question Four: Which is bigger, 8/12 of a chocolate bar or one whole Chocolate Bar?

It seemed from the answers to this question that some children do not distinguish between fraction problems on discrete vs. continuous objects. When children responded 8/12, the question was clearly repeated emphasizing 8/12 of *a* chocolate bar. For example, one girl appeared to be working from a continuous object concept of fractions. Because of this she believed that 8/12 was bigger that one whole. She saw 8/12 as 8 out of 12 chocolate bars even though the question was carefully restated as 8/12 of *one* chocolate bar. Her understanding of this question had not changed by the second interview. It is probably because of this type of thinking orientation that she answered *one seventh* to the question which asked her to name the fraction in question 6. The correct answer was 4/10 but she could only see 7 separate boxes. She did not see that the *big* box was in fact 4 out of 10 parts of one box.

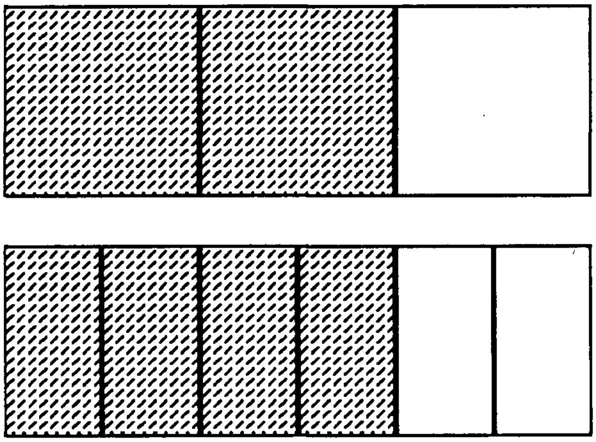
Another interesting observation about children's responses to this question was the tendency to define 8/12 in terms of being similar in size to 1/2 or 1/4, fractions with which they were apparently more familiar.

Visual Fractions Interactive Group. At first only two of the four children in this Group believed that one whole was larger. One student was not sure and the other thought 8/12 was larger. After the treatment, all four children responded that one whole was bigger, and three of them could clearly explain why 8/12 was only part of one whole.

Visual Fractions Noninteractive Group. In this Group, three children believed one whole was larger than 8/12 and two thought 8/12 was larger. After the treatment, only one child changed his mind and thought that perhaps one whole was larger after all.

Flash Cards. In this Group all five children thought that one whole was larger. Three of these were able to say they thought so because one whole was the whole thing and 8/12 was just part. One child referred to 8/12 as being *like 1/4.* No one changed their mind between the two interviews.

Question Five: Which diagram represents a bigger fraction or are they both the same size? (The top diagram illustrates 3 equal parts with two shaded and the bottom illustrates 6 equal *parts with 4 shaded. The area of the two diagrams is the same and they are positioned to make it visually obvious that the shaded portion is the same in both cases.)*



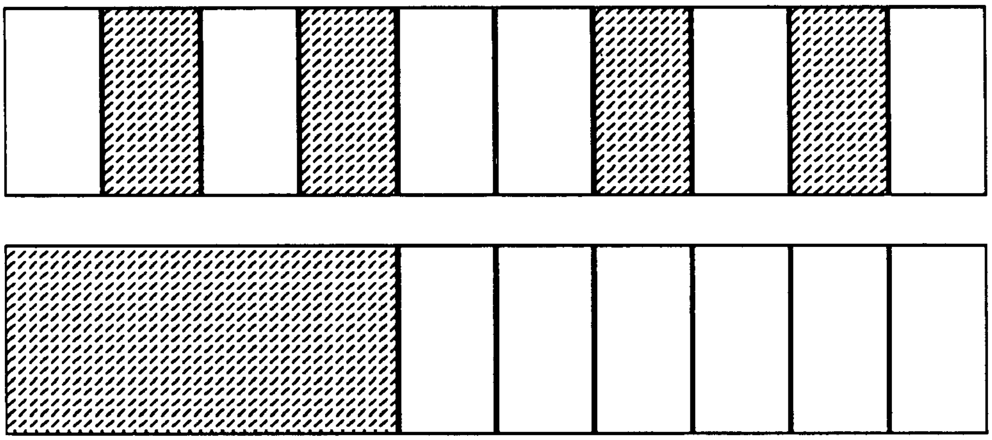
All of the children who believed one fraction was larger than the other chose 4/6 because it *had more pieces*. Even when asked to ignore the numbers and just concentrate on the diagram, many of these children still insisted that the diagram which had four shaded pieces was larger. These children were not operating from a concept of fraction as a representation of area. Clearly fractions were associated here only with numbers and the larger the numbers, the bigger the fraction.

Visual Fractions Interactive Group. Two of the children in this Group responded in the first interview that 4/6 was bigger (because there were more pieces) and the other two thought they were both the same (one because it was obvious from looking at the picture and the other because of a mathematical calculation. After the treatment, all four children realized that 2/3 of the diagram was the same as 4/6. The two who changed their mind both responded that they could *remove the line* dividing the diagram into sixths and make it just thirds. Both children seemed pleased to have discovered the similarity between 2/3 and 4/6 by themselves.

Visual Fractions Noninteractive Group. Of the five children in this Group, only one reported in the first interview that the fractions were the same size. The others all believed that more parts meant bigger fraction. After the treatment, only one student changed her mind and decided that by looking at just the picture you could see they were the same size. The student who originally believed that the fractions were the same size started to change his mind and say that 2/3 was bigger, but after reconsidering, he confirmed that they were really the same.

Flash Card Group. Two of the children in this Group responded that 2/3 was larger because the numbers were smaller. They believed that the smaller the number, the larger the value. One child explained this was because his teacher told them 2 was larger than four thousandths. These children had recently completed a unit on place value and decimals and were probably confusing the procedural rules which they had learned. Two of the children in this Group responded that 2/3 was the same as 4/6 and one thought 4/6 was larger because the numbers were bigger.

Question Six: Which of these figures represents 4/10? (The two diagrams are again placed to demonstrate that the area is the same.)



The general response to this question during the first interview was that the top picture represented 4/10 and the bottom diagram showed 1/7. It was not possible to see any lines dividing the shaded section into fourths, thus children tended to treat this as one large piece rather than four separate ones.

Visual Fractions Interactive Group. This Group understood this question better before treatment than the other two Groups. Three of the children were able to identify both pieces as 4/10 in the first interview and only one said that the top was 4/10 and the bottom was 1/7. This student changed her mind by the second interview and all the students in this Group then realized that both fractions represented 4/10 of the diagram.

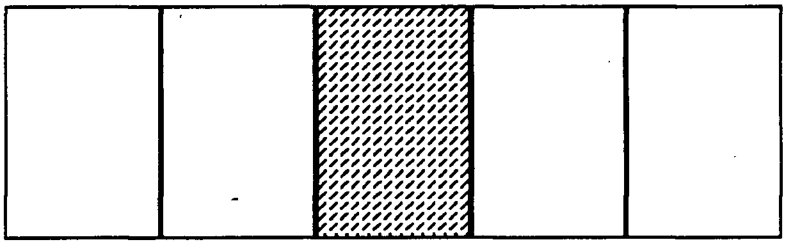
Visual Fractions Noninteractive Group. Only one student in this Group answered the question correctly during the first interview. The others all believed that the top picture represented 4/10 and the bottom was either 1/6 or 1/7. After the treatment, only one student changed her mind and realized both fractions were actually 4/10. The other students all retained their original answer, except for one girl who said the bottom fraction was actually 1/7 and not 1/6 as she had originally thought.

Flash Card Group. Four of the children in this Group were able to give the correct response to this question during the first interview. Only one child thought that the top was 4/10 and the bottom was *1 into 6*. During the second interview this child decided that the bottom was actually *1 into 7*.

Question 7: look at the Diagram.

a) What fraction part is represented by the shaded part of this diagram? (1/5)

b) Do fraction parts all have to be of the same size?



These questions were designed to test whether children who believed fraction parts all had to be equal, could in fact apply that knowledge to correctly answer 1/5. The diagram is intended to look like the more commonly seen fraction 1/3 which was used to ask the same questions at the beginning of the interview. The question: *Do fraction parts have to be equal?* was repeated to see whether any of the students changed their minds as a result of the interview process. In no case did any student answer one way at the beginning of an interview and change their mind by the end. Any changes in response always occurred from the negative to the positive and from the first to the second interview.

Visual Fractions Interactive Group. During the first interview, all but one of the students in this Group believed that fraction parts did not have to be of the same size. One boy thought maybe they did, but wouldn't say for sure. By the second interview, all four promptly answered yes to this question. In the first interview, two children answered 1/3 to this question, one answered 1/3 or maybe 1/5 (he wasn't sure) and only one student gave the correct answer as 1/5. After the treatment, three students answered 1/5 and only one was not sure. She knew it wasn't 1/3 because the pieces weren't the same size, however she was unable to determine what the correct answer should be.

Visual Fractions Noninteractive Group. All but one student in this Group believed fraction parts do not have to be the same size and no one in this Group changed their mind after the treatment. In the first interview, four children in this Group answered 1/3 and one student responded *one two* (meaning one as the numerator and two as the denominator). Even after the treatment, none of these children changed their minds. Not one of them was able to give the correct response of 1/5.

Flash Card Group. All but one student in this Group thought fraction parts did not have to be the same size and none of these students changed their mind by the second interview. The one student who knew they had to be equal realized this before the treatment and did not change his mind by the second interview. Two students responded 1/5 and two responded 1/3 during the first interview. None of these students altered their answers by the second interview. The fifth student responded *one into two* during the first interview and *one into three* during the second interview.

Summary Report of Interview Findings

The interview results are compared below according to similarities and differences between groups.

Similarities Between Groups

The following statements appeared to be true for most of the children interviewed regardless of membership in treatment Group.

Children's understanding of fractions appears to be rule or procedure-based. For a great many children, statements such as *my teacher told us*, or *the books show it like this*, or *a fraction is school stuff like math* reflect the idea that their level of understanding is procedural-based rather than conceptual (i.e., they have memorized rules without pocessing any intuitive sense of what a fraction is). Consider for example, one student's definition of fraction:

> *A fraction is ..... well say you had four boxes and two of them were shaded and two of them weren't, you would have to put a line and the two shaded ones would go on top and the unshaded ones on the bottom, my teacher said.*

This procedural approach did not change after the treatment sessions. Obviously it would take considerably more experience for children to develop a more independent or concept-based learning approach. Again, this same student's posttest response to the question of whether fraction parts all had to be the same size illustrates this point well: *Yes they do because when they* [the computer] *drew the fractions, they were all the same size.*

A related issue had to do with whether children operated from a context-based framework or whether they were able to transfer information learned in one situation (i.e., the computer screen) to another (i.e., the posttest). It seemed clear from the interviews and from talking in-depth to several students after the posttest that this transfer was often not made even though it was clear that a conceptual shift in understanding had occurred. This was especially apparent for the question about whether fraction parts had to be of equal size.

The treatment did not seem to affect the student's ability to define a fraction. Students did not change their original answer to this question during the second interview. Many of them defined fractions with a very limited, context-based definition (e.g., *its numbers with a line between them. The top number is the shaded part and the bottom number is the number of parts*," etc. Very few students answered this question in a more general fashion (e.g., *a fraction is a part of something or equal parts of a whole*).

Differences Between Groups

In addition to these general findings, some specific observations can be drawn from each of the three Treatment Groups. Because of the small sample size of students interviewed in each Group (n= 4 or 5), it is somewhat dangerous to make generalizations. However, there appeared to be differences between the Visual Fractions Interactive Group and the other two Groups on almost every interview question. Some of these differences were rather dramatic and are reported here.

The biggest difference that occurred was in response to question 2a and question 8. Both of these asked the question *Do fraction parts always have to be the same size?* All four students in the Interactive Group responded no to this question during the first interview and all four changed their mind and said yes during the second interview. In contrast, the students in the Noninteractive Group all responded *no* at first, but only one student changed her mind and said *yes, they did have to be always the same size.* Similarly, all four students in the Flash Card Group said no at first and no one changed their mind by the second interview.There was a clear difference between the Interactive Group and the other two Treatment Groups in their response to this question.

Another difference between the Treatment Groups occurred in response to question 7 which required the students to correctly identify the shaded portion of a rectangle as 1/5 (It was divided into three uneven parts and could be interpreted as thirds to the undiscerning eye). Before the treatment, only one student in the Visual Fractions Interactive Group was able to correctly answer this question and even he wasn't totally convinced of his answer. After the treatment, every child in this Group correctly identified the fraction as 1/5. In contrast, none of the students in the Noninteractive Group were able to answer this question after the treatment. Additionally, the Flash Card Group showed no improvement between interviews. Two were able to answer correctly prior to treatment and no one else was able to answer 1/5 afterward. Again this shows a clear difference between the Interactive treatment and other two Groups.

All of the students found the task of dividing a piece of paper into 5/7 difficult. The differences between Groups was more subtle in this case. However, because of the difficulty of the question and the nature of the change (from no measuring to some attempt at measuring), the difference is worth noting. Before the treatment, none of the students in the Interactive Fraction Group made any effort to measure the paper in order to make their divisions equal and two students would not even attempt the problem. After the treatment, everyone attempted the problem and two students measured very carefully. There was also great variance in ability to answer this problem in the other two Groups; however in contrast to the Interactive Group , none of the students in the other two Groups altered their original effort at solving this task. The only student who tried to measure the paper did so during the first interview as well.

An analysis of the findings from both the quantitative and qualitative data revealed that Visual Fractions is an effective method of teaching basic fraction concepts. A discussion of the findings is presented in Chapter Five.

Chapter Five
Summary, Conclusions and Recommendations

The purpose of this study was to determine whether learner-control over a computer learning environment was more effective in teaching basic fraction concepts than a noninteractive version of the same program. The two computer methods of instruction were compared to a Flash Card treatment, designed to control for the influence of computers on learning by simulating the noninteractive computer environment. Data was obtained by means of a pretest/posttest. In addition, students interviews were conducted and results reported. The final source of data is in the form of a qualitative report of student's interaction with the learning environment. This chapter presents a summary report of the findings of the Pilot Study and the research results as well as conclusions and recommendations for further study.

The Pilot Study

The results from the Pilot Study indicated that the original pretest/posttest did not accurately report childrens' understanding of fractions. Students were able to answer the questions simply by looking at the examples and using them to fill in the blanks in a similar manner. Apparently this technique of answering worksheets correctly without really understanding the concepts involved occurs alarmingly often in schools. According to more than twenty children in five different schools who wrote the original test, this copying technique was something which they employed regularly because it seemed to work for them. Unfortunately, the learning which seemed to be occuring had to do with *worksheet wiseness* as opposed to increased conceptual understanding. This finding has broad implications for the way in which we teach all subjects, not just mathematics. Teachers need to consider very carefully what they want students to learn when designing worksheets.

Discussion of Research Results

The sample was taken from the grade four students in four different classes at two elementary schools in Burnaby, British Columbia (a suburb of Vancouver). The sample may not be representative of a random sampling of grade four students and therefore any conclusions that the researcher makes cannot be generalized to the population of grade four students.

The treatment process was limited to one one-hour session. Except for the Group which used the noninteractive version of Visual Fractions, it is probable that many of the students would have benefited from a second exposure. These students were clearly bored after fifteen minutes and it is unlikely that they would have learned anything more from a continued or repeated exposure. Students seemed to work better in smaller groups. The students in the pilot study were observed in groups of four and seemed to be more on task more of the time than the students in the larger Group. Individual instruction may not be as beneficial because the peer interaction appeared to be an important part of the learning process.

Some of the students in both computer groups were not concentrating as intently as they could have been because of a desire to return to their class and work on a dinosaur project. It is the opinion of the researcher that this activity represented an unusual opportunity for student's to interact in a informal way with one another and was highly desirable. They therefore did not spend as much time at the computer as they otherwise might have done This situation could have posed a serious threat to the study, however, because the same situation occurred with one half the students in each computer Group, the problem was the same for both treatments. Further, it only affected approximately five or six out of twenty two students in each Treatment Group

There were several issues concerning the design of the flash cards which may have given the Flash Card Group an advantage over the other Treatment Groups which the researcher did not consider prior to the treatment. For example, the cards were all colour coded by denominators and the students quickly picked up on this fact and began to sort the cards into groups by colour. They also liked to find cards with equivalent fractions and sort them into groups. This ability to look at several cards at once was unique to this Treatment Group and the students clearly enjoyed comparing and sorting the cards. This type of activity was not possible in either of the two other Groups because only one fraction at a time could be seen on the screen. Thus, the intention of duplicating without a computer, the Noninteractive version of the Visual Fractions Program, was not accomplished. Although the design of the Flash Cards was very similar to the Noninteractive software program, they provided two more elements which clearly resulted in a greater enthusiasm and higher level of concentration. It seems therefore that this result lends support to the theory that learner control is is a critical factor in learning. The students who used the Flash Cards had a greater degree of control over their learning environment than they were intended to have and the result was a greater enthusiasm and desire to learn. Without the presence of the possible *ceiling effect,* this Group may well have significantly improved their score from pretest to posttest.

Conclusions - The Value of Visual Fractions

The Visual Fractions Group scored significantly better than the Control Group and this makes it clear that the testing and passage of time were not responsible for their gain. Additionally their gain scores were significantly superior to the Flash Card and Non-interactive Groups indicating that it was not the activity with fractions nor the opportunity to use the computer that caused their high gain. The interactive nature and opportunity for control offered to the students appear to be the main factors contributing to their success.

The results from this study indicated that the interactive version of Visual Fractions is an effective means of teaching basic fraction concepts. The version of the software program used in the study was specifically adapted to test one aspect of learner control. A more sophisticated version of the program exists which allows students to test themselves by selecting a drill button. The program then asks the students to make various fractions and prompts for incorrect efforts. This added feature would probably improve the intrinsic appeal of the program and permit a greater retention of interest and longer time on task, possibly resulting in a greater improvement of understanding of fraction concepts. Additionally, versions of the program exist which allow students to make comparisons between two fractions, (equivalence) and to perform operations tasks.

Recommendations for Future Research

It is possible that students in both the Interactive Visual Fractions Group and the Flash Card Group may have benefitted from a second treatment session. Also, based on the interactions of the students in both the pilot study and the Flash Card Group, smaller Groups may be preferable and result in greater improvement in understanding.

Fraction concepts are difficult to define and even more difficult to measure. Any test designed to evaluate a child's understanding of fractions is bound to be subjective and therefore possibly not an accurate assessment. In-depth interviews appear to reveal more about the concepts children hold than tests do because the researcher has an opportunity to probe and find out why they answered the way they did. A future study of this topic might include more emphasis on the interview process and less on the testing aspect.

In addition to experimenting with smaller groups and longer sessions, it would be interesting to expand the study to include the more sophisticated version of Visual Fractions.

Students could then be tested to see whether their understanding of equivalent fractions and operations (addition, subtraction, etc.) could be improved by exposure to this software.

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Appendix A - Interview Procedure

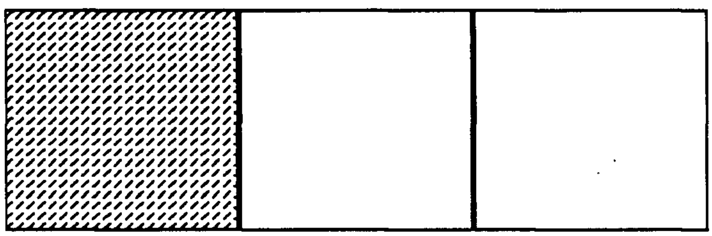
There are 7 questions in this interview. The time approximate for each interview is 10 -15 minutes. Each student should be interviewed privately. Student responses are to be audiotaped. In addition, the researcher is to take notes on such activity as facial expression, student mannerism, etc.... that may help to clarify student responses. The student should be greeted and have the procedure explained briefly. He should be reminded that the purpose is for a university research project on how children learn, that it is not a test and the outcome will not affect his grades in any way. The child should be encouraged to relax and give the best answers he can to each question.

Procedure

1) Ask the student to take a piece of paper, tear it and give back 5/7 of the paper.

2) Ask:

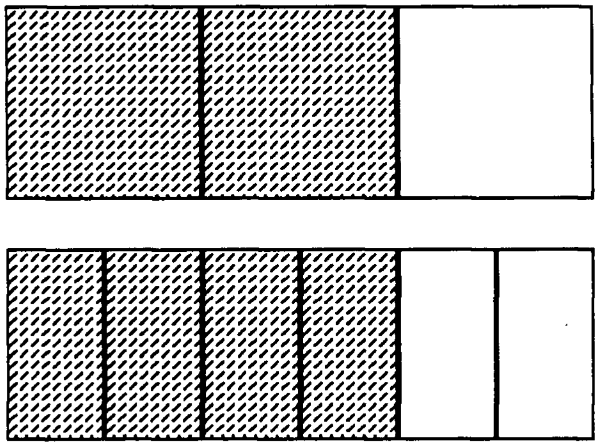
   1) Do fraction parts all have to be the same size?
   2) What fraction is represented by the shaded portion of this diagram?
   3) What fraction is represented by the unshaded portion?



3) Pretend that I am from outer space and have never heard the word fraction before. Can you describe what a fraction is for me?

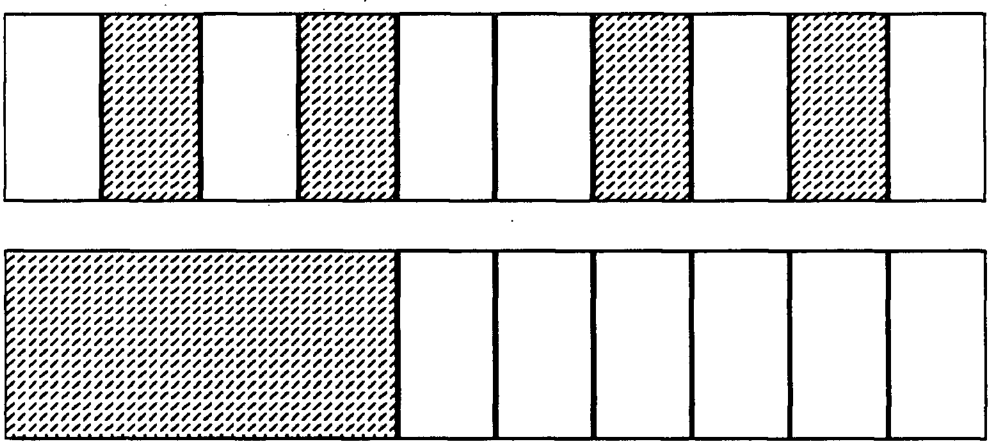
4) Ask: Which is bigger, 8/12 of a chocolate bar or one whole chocolate bar? (show the question written on a piece of paper).

5) Ask: Which diagram represents a bigger fraction?



6) Ask: Which of these pictures represents 4/10?

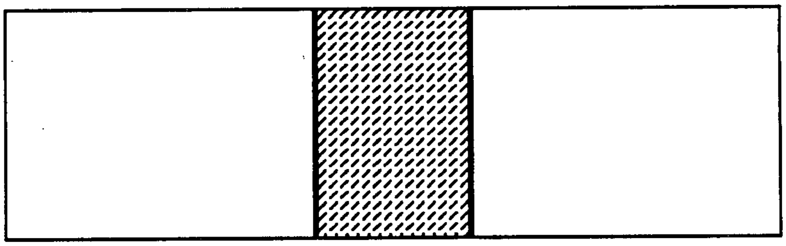
What fraction does the other picture represent?



7) Look at this diagram. Ask:

1) Do fraction parts all have to be the same size?

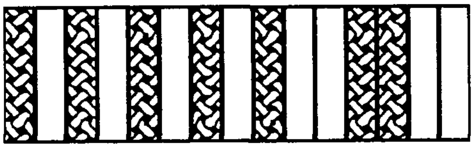
2) What fraction is represented by the shaded portion of this diagram?



APPENDIX C

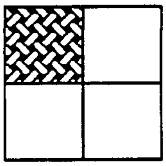
NAME: ____________ GRADE: ________ AGE: __________
BIRTHDAY: ________

1. What fraction is represented by the SHADED portion of this picture?



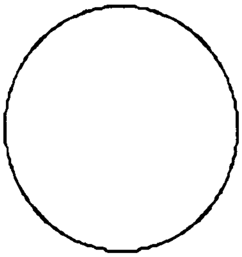
ANSWER: ____________

2. What fraction of this square is SHADED?

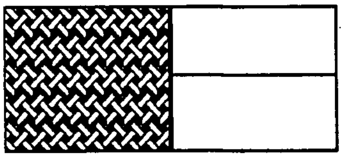


ANSWER: ________

3. SHADE in 2/3 of this circle.

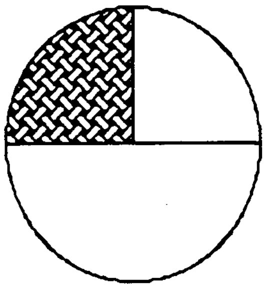


4. What fraction is represented by the UNSHADED portion of this picture?



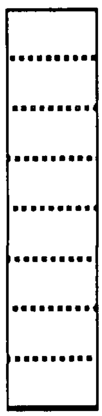
ANSWER: ________

5. What fraction is represented by the SHADED in portion of this circle?

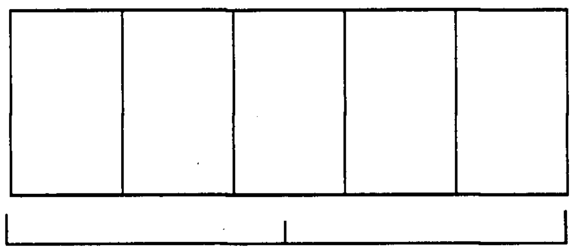


ANSWER: ____________

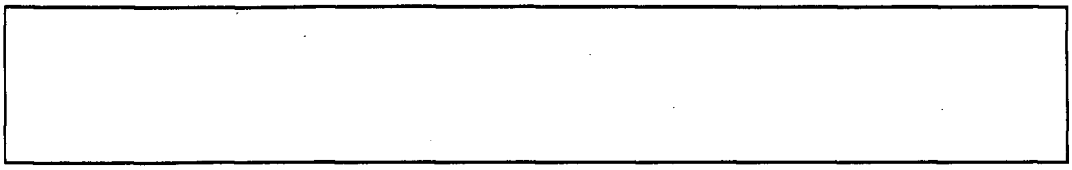
6. SHADE in 1/4 of this rectangle.



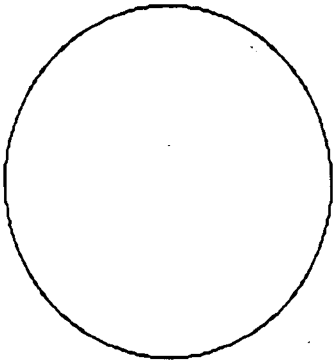
7. SHADE in 3/4 of this rectangle.



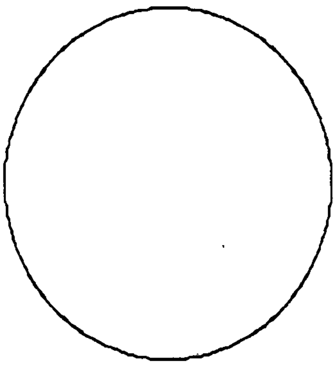
8. ESTIMATE! SHADE in 7/20 of this diagram.



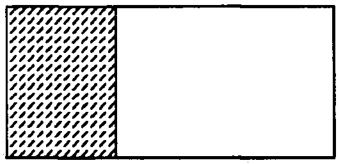
9. ESTIMATE! SHADE in 7/12 of this circle.



10. Can you SHADE in a DIFFERENT 7/12 of the same circle?

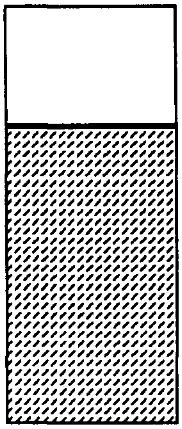


11. What fraction is represented by the SHADED portion of this diagram?



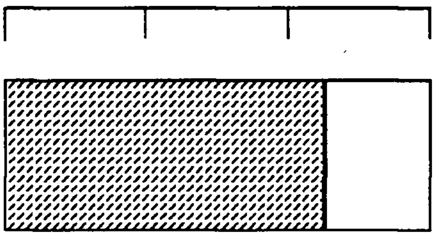
ANSWER: ____________________

12. What fraction of this rectangle is SHADED?



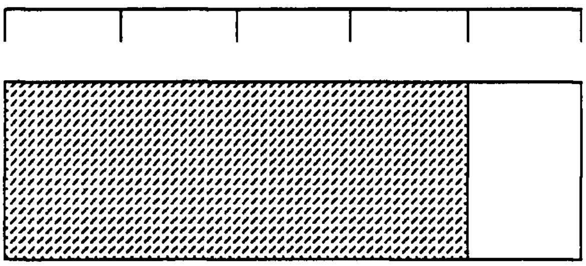
ANSWER: ____________________

13. What fraction of this rectangle is SHADED?



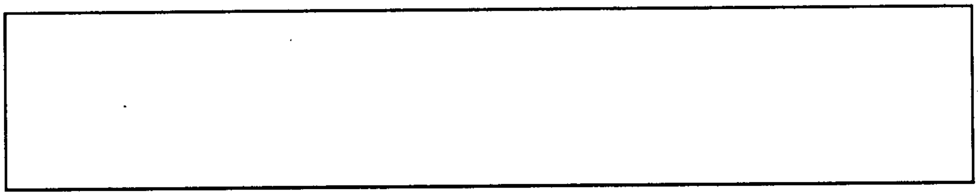
ANSWER: ________________

13. What fraction is represented by the UNSHADED portion of this diagram?

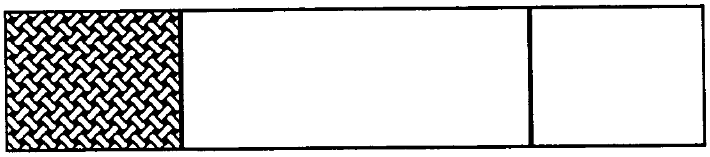


ANSWER: ____________

14. ESTIMATE! SHADE in 5/15 of this rectangle.

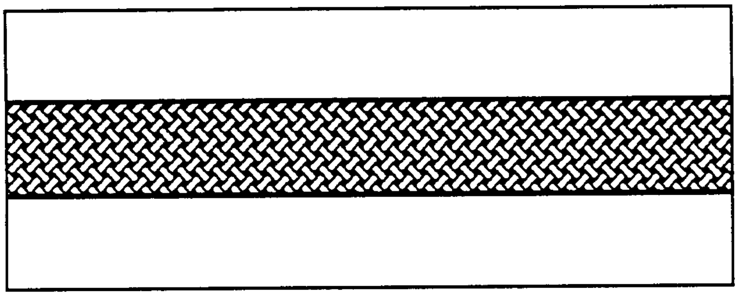


15. What fraction is represented by the shaded portion of this diagram?



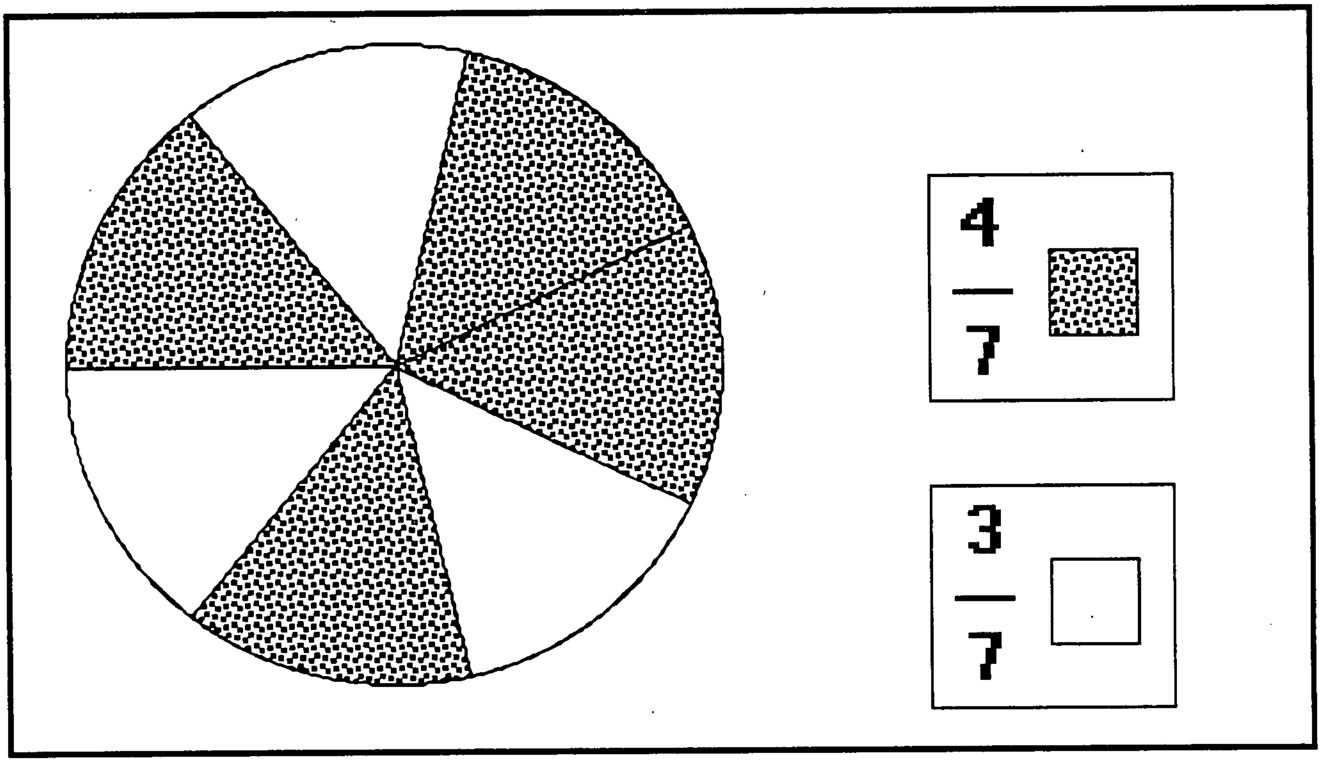
ANSWER: ______________

16. What fraction is represented by the SHADED portion of this diagram?



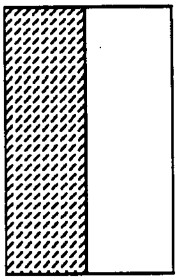
ANSWER: ______________

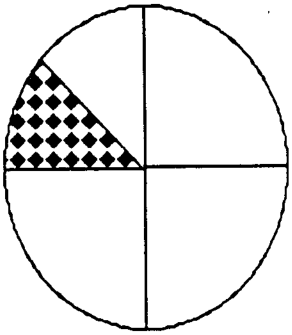
Appendix D - Sample Flash Card



APPENDIX E - PILOT STUDY PRETEST / POSTEST

EXPLORING FRACTIONS





NAME ______________________________

GRADE ______________________________

BIRTHDAY

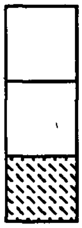
YEAR 19 _____

MONTH _______

DATE _______

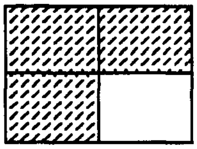
Shade in the correct fractional part.

Example A

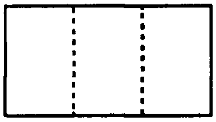


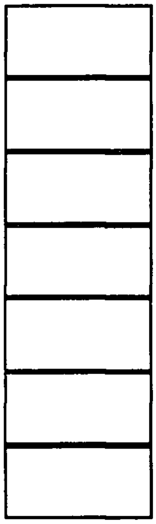
$\frac{1}{3}$

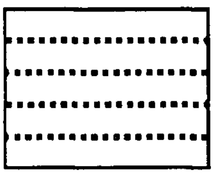
Example B

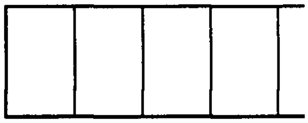


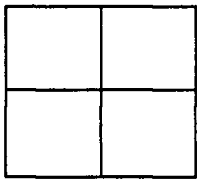
$\frac{3}{4}$

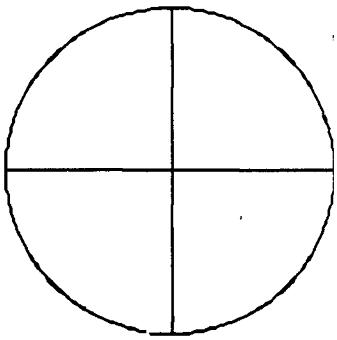
1.  $\frac{2}{3}$

2.  $\frac{2}{7}$

3.  $\frac{2}{5}$

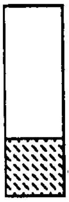
4. 

5. 

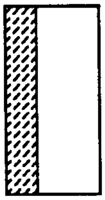
6. 

Shade in the correct fractional part.

Example A

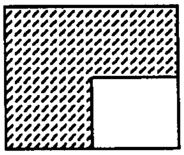


$\frac{1}{3}$ or

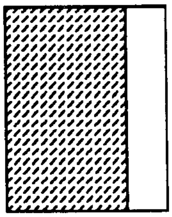


$\frac{1}{3}$

Example B



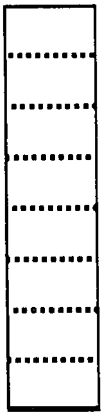
$\frac{3}{4}$ or



$\frac{3}{4}$

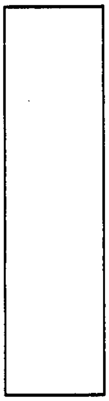
Estimate your answers.

1.



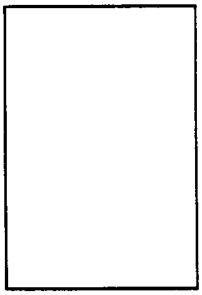
$\frac{1}{4}$

2.



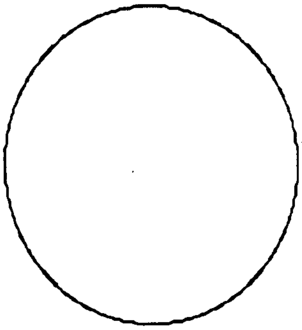
$\frac{3}{6}$

3.



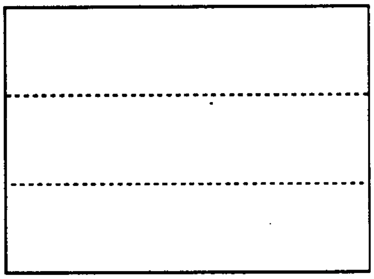
$\frac{1}{2}$

4.



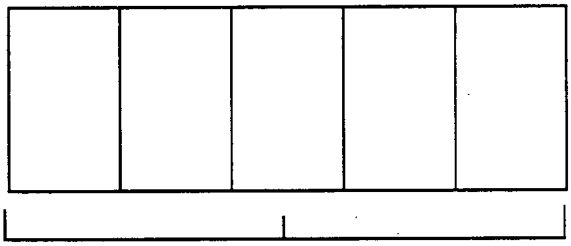
$\frac{2}{3}$

5.



$\frac{1}{3}$

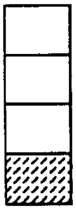
6.



$\frac{3}{4}$

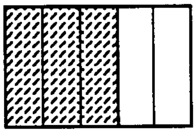
WRITE THE CORRECT SYMBOL FOR EACH FRACTION.

Example A



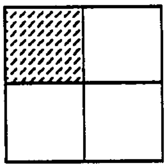
$\frac{1}{4}$

Example B

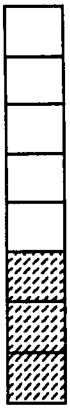


$\frac{3}{5}$

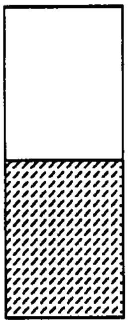
1.



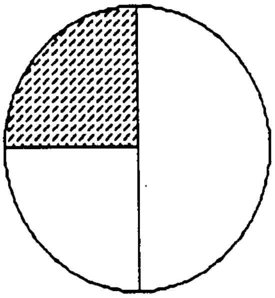
2.



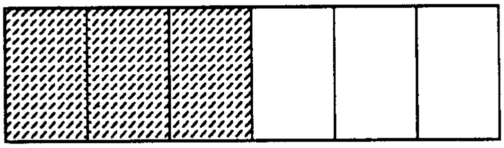
3.



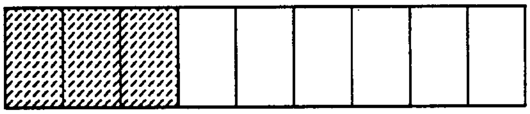
4.



5.

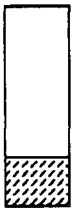


6.



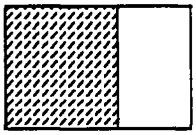
WRITE THE CORRECT SYMBOL FOR EACH FRACTION.

Example A



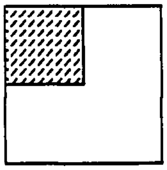
$\frac{1}{4}$

Example B

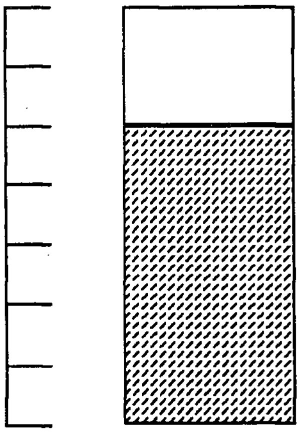


$\frac{3}{5}$

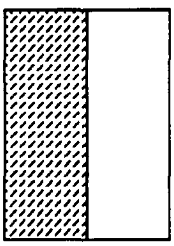
1.



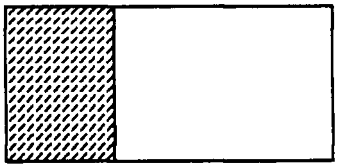
2.



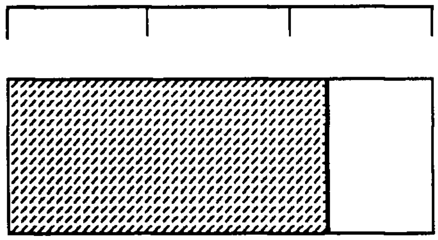
3.



4.



5.



6.