A COMPARISON OF THREE APPROACHES TO PROBLEM SOLVING IN GRADES SIX AND SEVEN

by

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ABSTRACT

This study was undertaken to compare the effectiveness of two problem-solving strategies at the grade six and seven levels. Both strategies were designed to aid students in associating the verbal statement of a problem with its corresponding mathematical equation. One approach, the Translation Method, stressed literal, carefully structured translation of word problems, while the second, the Inductive Method, encouraged students to create their own problems, using mathematical equations given by the teacher. A control group practiced word problems without any instructional guidance.

Forty-eight students from the sixth and seventh grades of a private elementary school in Vancouver, British Columbia were combined and assigned to the three treatment groups on the basis of their performance on a pretest in translation. For a period of four school days, all subjects used materials prepared by the investigator.

Two criterion measures were used. Posttest One was composed of traditional word problems requiring only one mathematical operation for the correct solution. Posttest Two was constructed with novel or challenging word problems requiring more than one operation for the correct solution. Each test contained eight items and was designed for one forty-minute period. Scores of the tests were analyzed using multivariate analysis of variance for the two dependent measures. The three factors considered were Treatment, Sex, and Grade, and a simple main

effects analysis was employed to examine male-female differences within each treatment level.

Statistical comparisons among the three groups offered no evidence of superiority for one approach over another. In addition, no interaction was found between treatment and sex. Boys were found to be significantly superior to girls in performance on the posttests. Further analysis indicated that Posttest One scores for the Translation Group students differed significantly between boys and girls, with the girls' performance particularly weak for this measure.

Subjective observation revealed differences in attitude. Students found the Translation Method burdensome. Students in the Inductive Group enjoyed that approach, and students in the Control Group seemed interested in the practice sequence of word problems.

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Chapter 1

THE PROBLEM

Problem solving in mathematics in its most advanced form represents the application of mathematical concepts, formulas, and logic within all aspects of life. In fact, the goal of mathematical education has often been viewed as enabling students to use mathematical tools to solve the problems and challenges that they face outside the classroom. Mathematicians of the Cambridge Conference on School Mathematics (Goals for School Mathematics, 1963) urged curriculum developers to devote more time and energy to the creation of problem sequences, particularly those which introduce new mathematical ideas.

The kind of problem solving which takes place in the schools, however, tends to be more of an extension of arithmetical examples into the domain of word problems. Yet even in this context, the "simple" word problems enable students to gain experience with the process of problem solving and to deduce principles which they will later need in coping with more complex problems. Wilson (1967), in his content taxonomy for mathematics, divides the cognitive skills of mathematics into the four hierarchical domains of computation, comprehension, applications, and analysis. He places the solution of routine problems in the domain of applications, and the solution of nonroutine problems in the domain of analysis. Thus, problem solving demands the highest order cognitive skills that students may develop. These skills are described by Cohen and Johnson (1967) as "observing, exploring, decision-making, organizing, recognizing, remembering, supplementing, regrouping, isolating, combining, diagramming,

guessing, classifying, formulating, generalizing, verifying, and applying" (p. 261).

It is hardly surprising that mathematical word problems at all levels frustrate many students. Current learning assessment projects, discussed below, show that weakness in problem solving, particularly solving problems of a more complex nature than the one-step problems using whole numbers, is a widespread characteristic of junior high school and high school students.

Learning Assessment Projects

One such project, the National Assessment of Educational Progress (NAEP), (Carpenter, Coburn, Reys, and Wilson, 1975) was conducted in 1972-73 with 90 000 individuals participating at four age levels: students aged 9, 13, and 17, and young adults between 26 and 35 years of age. The abilities tested ranged from recall through analysis or problem solving in a variety of content areas. One example was:

> A sports car owner says he gets 22 miles per gallon of gasoline. How many miles could he go on seven gallons of gasoline?

It was found that for this problem, 89 percent of the 17-year-olds and 90 percent of the adults obtained the correct solution.

However, an example which was more complex, such as the following, did not produce such encouraging results:

> Weathermen estimate that the amount of water in nine inches of snow is the same as one inch of rainfall. A certain Arctic island has an annual snowfall of 1.63 inches. Its annual snowfall is the same as an annual rainfall of how many inches?

In this case the correct answer was found by 31 percent of the 13-year-olds, 53 percent of the 17-year-olds, and 58 percent of the adults. When the individuals who attempted to use division but did not obtain the correct solution are included, only about half of the 13-year-olds could analyze the problem sufficiently to determine the appropriate operation, and about three-fourths of the 17-year-olds and adults could do so. These results were similar to those found on other exercises. All problems were read to students by a tape recording in order to minimize the effect of poor reading ability.

Another project, the British Columbia Mathematics Assessment project (Robitaille and Sherrill, 1977), was undertaken in 1977 and over 100 000 students from grades 4, 8 and 12 were tested, using the three domains of computation, comprehension, and applications for a number of different content areas. The results for problem solving in grade 12 were found to be "disappointing," indicating that many students are unable to apply the computational skills they have learned to certain types of problems, especially in geometry and measurement. Although certain difficulties were again experienced with more complex, multi-step problems, results for the grade 8 students concerning problem solving were satisfactory overall.

For both of these assessments, the researchers summarize their findings by urging that greater emphasis be placed on problem solving. The NAEP organizers report, "As a whole, these age groups need to develop more problem-solving skills" (Carpenter et al, p. 470). Robitaille and Sherrill (1977) of the British Columbia Mathematics Assessment recommend: "Teachers and teacher educators need to stress the overriding importance of problem solving in mathematics, and

their students need to learn strategies to use in attempting to solve problems in mathematics" (p. 33).

Problem-Solving Strategies

Henderson and Pingry (1970) suggest, "Unless students study the process of solving problems as an end in itself, there is scant likelihood that they will learn the generalizations which will enable them to transfer their ability to solve problems to new problems as they arise" (p. 233). Viewed from this perspective it would seem natural that at least a moderate amount of time should be devoted to teaching students some useful method for solving problems. Yet Stilwell (1967) found that a relatively small amount of class time was devoted to discussing a problem-solving method for general use: less than three percent of all problem-solving time!

Teachers seem to avoid teaching a general strategy which could apply to all kinds of verbal problems and instead rely on a collection of "problem-type" strategies, each pertaining to one particular type of problem only. This method of attack has been criticized for being the least transferable for students of all popular techniques. Students who are taught rate problems, or interest problems, for example, are limited to solving only those types of problems (Spitler, 1976).

One of the most widely known general strategies was developed by Polya (1957). In How to Solve It, Polya suggested a step-by-step solution, basically heuristic in nature, which involves reading to understand the problem and planning for a solution using the following questions as a guideline:

1. What is the unknown?

2. What are the given facts?

3. What condition relates the unknown to the data given?

Finding a plan for the solution involves the use of heuristics such as analogies, solving part of the condition, and other strategies, with examining the obtained solution as a final step. This process has become known as the wanted-given approach.

Difficulties in attempting to use Polya's checklist to analyze the problem-solving procedures of 56 eighth grade students of above average ability were reported by Kilpatrick (1967):

> "...Whatever merits Polya's list has for teaching problem solving, it is of limited usefulness, as it stands, for characterizing the behavior of these subjects. Many of the categories were unoccupied; subjects seemingly did not exhibit behavior even remotely resembling actions suggested by the heuristic questions" (p. 44).

Polya's strategies are of unchallenged value in the construction of a general problem-solving model. Yet, to a large extent, this model is intended for the solution of nonroutine and challenging problems, and it may not be as directly useful for students who are having difficulty with simpler routine problems.

An alternate problem-solving strategy was suggested by Maurice Dahmus (1970) in a paper presented to the Central Association of Science and Mathematics Teachers at a convention in 1970. This method is directed towards the lower 90 percent of mathematics classes who seem to lack a viable approach to solving word problems. Students are asked to record each phrase of a problem in mathematical terms without first reading through the entire problem. The series of resulting

symbols and equations are then combined by substitution methods until only the final equation or system of equations remains to be solved. Dahmus characterizes his method as "DPPC":

D-direct:	proceed directly from the order of the problem's statement,
P-piecemeal:	translate piece by piece as you read,
P-pure:	no operations done before the translation is complete,
C-complete:	all facts, ideas, and questions used for the solution to be mathematically recorded.

This approach has become known as the translation method.

Dahmus' method of literal translation is supported by a large body of research which links verbal skills to problem-solving abilities. Studies of the language factor in mathematics have been concerned with the possibility that verbal skills have as much influence as--or even more than--computational ability on problem solving (Martin, 1964; Balow, 1964; Knifong & Holton, 1976, among others). Structural analysis of word problems to determine those components which contribute the most to errors have indicated that linguistic factors occupy positions of primary importance in the determination of the difficulty of the problem (Jerman & Rees, 1972; Segalla, 1973; Cook, 1973, among others). Training to increase mathematical vocabulary is one example of a language-based program which has helped students to significantly increase their problem-solving abilities (Dresher, 1834; Johnson, 1944; and Vanderlinde, 1964).

Indeed, most of the pages of a booklet prepared by the International Reading Association entitled "Teaching Reading and Mathematics" (Earle, 1976) are devoted to the steps necessary for students to perceive symbols and attach literal meanings. These steps are seen to be just as crucial to the problem-solving process as is the learning of new vocabulary to a student in literature. Cohen and Johnson (1967) expressed the following sentiments in this regard:

> "The ability to translate accurately from the written (or spoken) description of the physical situation to an appropriate mathematical sentence enables a person to cope with a large number of problems in mathematics in an orderly, logical manner. The ability to translate a given situation into mathematical symbolism is considered to be the 'most useful tool in problem solving'" (p. 262).

Yet few studies have examined the potential of using a language-based strategy to approach the solution of word problems.

The Dahmus translation method, while providing this direction for problem solving, is quite a rigorous strategy which minimizes the role of sudden insight or inspiration and the type of heuristics recommended by Polya. According to Kilpatrick (1967) and others, however, the assumption that problem solving in reality occurs in well defined, sequential stages is one which should be avoided. Burch (1953) conducted a study of formal analysis or the rigid sequence of questions or steps to be followed before computational work is begun. He found that training elementary school children in formal analysis methods for solving problems was troublesome and confusing for them. Whether the Dahmus translation method is too rigid or too similar to formal analysis is open to question.

An attempt to maintain flexibility and creativity on the part of students, while at the same time bridging the gap which exists between the verbal problem

and its corresponding mathematical statement, is found in a strategy suggested by Spitler (1976). As one of many suggested techniques for improving problem solving, she proposes the technique of having students create their own problems from a given mathematical equation. Students are taught to associate certain situational possibilities and different meanings for mathematical operations with a simple equation such as $10 = N \div 5$. This association of real world context with symbolic equations should lead to deeper understanding of verbal problems, thus improving problem-solving abilities.

The advantage of this approach, which is inductive rather than deductive, is that students who were directed in creative and divergent thinking patterns have shown increased abilities to accomplish divergent-type tasks. This was demonstrated to be something of a mixed advantage by Richards and Bolton (1971). They found that when a sample of students was tested on "mechanical" arithmetic tasks, children taught by discovery methods were significantly lower in performance than students taught by traditional or combined traditional-discovery methods. Yet on a test of divergent-thinking ability, the discovery and balanced methods were superior to the traditional group.

The practical usefulness of a creative, inductive method for problem solving, which entails training of students without the use of specific word problems has not been fully investigated. However, Suydam and Weaver (1977), in their summary of research on problem solving, report that creative or divergent thinking is a successful strategy.

The available research relating to the testing of different instructional strategies has tended instead to focus on comparisons with some variant of Polya's

method against another deductive method. Wilson (1967), Jerman (1973), and Post and Brennan (1976), for example, contrasted some variation of the wanted-given method with some other strategy, obtaining differing results. One study, conducted by Bassler, Beers, and Richardson (1972), contrasted Dahmus' translation approach with the wanted-given strategy. No differences were found between two groups of ninth-graders on a solution criterion. No further comparisons could be found using the translation method, and no experimental studies have been conducted with Spitler's inductive approach.

It remains to be seen whether an inductive, creative method would be effective in the domain of novel problem situations. The question of whether such a method would also improve skills in solving simple word problems should be studied as well. These questions can also be considered for a deductive, formal translation strategy. Will literal translation prove useful in the solution of one-step word problems? If so, will it also help students who face complicated problem-solving tasks? The questions indicate that a comparison of these two translation-oriented strategies should be conducted using both a simple problem-solving criterion and a higher-level, more challenging measure of problem-solving skills.

Sex Differences

The study of Bassler, Beers, and Richardson (1972) did not consider sex differences because it was conducted in an all girls' school. Other researchers who have been able to use larger samples of male-female subjects have usually relied on randomization to remove any possible sex differences and, in general, Suydam and Weaver (1977) concluded from research that sex is not an important

factor in problem solving.

However, the report from the British Columbia Mathematics Assessment (Robitaille and Sherrill, 1977) shows that males outperformed females on all the problem-solving objectives, although most differences were small. This is paralleled by the findings from the NAEP (Carpenter et al, 1975), in which an analysis by sex of word problem results indicated that males generally did better than females at all ages.

With these recent findings taken into consideration, it seems that the question of sex differences is sufficiently relevant to problem solving to be used as one of the factors in an analysis of the problem-solving strategies. In addition, it was felt that if some interaction between treatment and sex could be found, this would be of particular interest.

Definition of Terms

For the purposes of this study, the following definitions of terms are outlined for reference:

The Translation Method refers to the strategy proposed by Dahmus which requires literal translation from the verbal statement of a problem to an appropriate mathematical equation.

The Inductive Method is a program to aid students' problem solving by having them create their own word problems from a given mathematical equation.

Problem-type Strategies refers to the collection of different methods for solving different kinds of word problems which is currently used by teachers, i.e., one mode of attack for rate problems, one for work problems, and so forth.

One-step Problems are word problems which require only one mathematical operation for the correct solution.

Multi-step Problems are word problems which require more than one mathematical operation for the correct solution.

Statement of the Problem

The purpose of this study is to compare the relative effectiveness of two methods for teaching problem solving, using a control group who practice word problems with no instruction as to method. Both strategies are designed to aid students in associating the verbal statement of a problem with its mathematical counterpart. One is a deductive, literal translation method, while the other is an inductive process which encourages creative thinking.

At the same time, this study will evaluate the differential effect of the two strategies on male and female students, with the purpose of determining whether there is an interaction between problem-solving method and sex.

Two criterion measures will be used. One represents traditional one-step word problems, and the other contains novel or more challenging multi-step problems.

Students at the grade 6 and grade 7 levels were selected for the study since it was felt that they would not have been instructed previously in any problem-solving method.

Treatment-Related Research Questions

Does learning a particular strategy for problem solving improve the performance of students more than providing an unstructured practice sequence without

instructional guidance? That is, will the groups learning the Translation Method or the Inductive Method do better than the control group on the simple or the complex posttest or both?

Will one particular strategy for problem solving prove superior to the other on either or both of the posttests?

Does an interaction exist between any of the treatment methods and the sex of the students? That is, will boys improve more under one treatment while girls improve more under another method?

Sex-Related Research Questions

Will grades six and seven boys perform better than girls, as research seems to indicate, on a standard problem-solving test? Will they prove superior on a non-standard problem-solving test as well? If the boys prove to be better at problem-solving than the girls, is this finding consistent through all treatments or will any one treatment help to equalize the performance of the sexes?

Grade-Related Questions

If there are significant differences between the achievement of the grade six and grade seven students, are these differences consistent across both sexes and all treatments? Will they be demonstrated in both posttests?

Chapter 2

REVIEW OF RELATED LITERATURE

Research relevant to a comparison between problem-solving strategies falls into three general categories. First, research is reported which serves as a general background to the need for linguistic approaches to mathematical problem solving. Next literature relevant to the Translation Method and then the Inductive Method is discussed. Finally, problem-solving comparisons with relevance to the present study are examined. In all categories, literature of general relevance precedes inspection of specifically related research. After literature in these three categories is examined, it will be necessary to examine some research dealing with sex differences. This research will be dealt with only briefly.

Research of General Relevance to Translation-oriented Strategies

Problem-solving ability has been correlated with verbal skills for several years. Martin (1964) found that the partial correlation between reading comprehension and problem-solving abilities, with computational ability partialed out, was higher than the partial correlation between computational ability and problem solving with reading comprehension partialed out. Balow (1964), on the other hand, found that computational ability is more influential to successful problem solving than is reading ability, but he suggests that general reading ability may have a greater effect on problem solving than the effect that he found in his study.

Harvin and Gilchrist (1970) concluded from their investigations that there exists a positive relationship between problem-solving ability and reading ability. This relationship is not great enough to conclude that the second is a predictor of the first, but it is significant enough to suggest that arithmetic teachers should also teach those reading skills which are peculiar to the nature of mathematics.

Structural analyses. Manheim (1961) states that "the word problem remains a generator of fear and frustration for many students... If we ask our students, or ourselves, why such a problem is more difficult than a non-word problem, we are apt to find the difficulty attributed to 'the non-mathematical nature of the problem'" (p. 234).

Attempts to ascertain exactly where the difficulties lie in mathematical word problems have inspired structural analyses of problems in the last decade. Studies have been attempted to isolate those variables, whether linguistic or computational, which contribute most heavily to the errors found among students at different age levels in solving algebra word problems.

One study was conducted by Cook (1973), who described 26 variables to which difficulty might be attributable. Some of these were drawn in turn from other studies (Krushinski, 1973, and Suppes, Jerman and Brian, 1968) and they were added to Cook's independently formulated variables. Cook analyzed the variables to determine which accounted for the most variance from the correct solutions to algebra word problems. The following results were reported, with variables listed in order of relative importance as steps along the regression line:

1. length of words in the problem statement;

2. a "translation" variable--signifying the number of unknowns used in solution of other unknowns;

3. recall of formulas;

4. number of digits in the quotient of divisions;

5. number of steps required to isolate the unknown once the equation is found; and

6. the number of operations necessary to solve the problem.

The findings in this study suggest the significance of different linguistic factors and skills in the solution of word problems. Although the subjects were college students, the problems chosen are representative of the type of word problems such as distance problems, age problems, encountered by secondary school students.

Jerman (1972, 1973, 1974) has been a most prominent figure in the area of structural analyses. One of his most comprehensive studies (Jerman and Mirman, 1974) involved 340 students from grades four to nine, and compared the results for the upper elementary grades of four through six with the results for the junior secondary levels of seven through nine. Seventy-three linguistic variables were isolated for this study, and they were combined with computational variables from previous studies. In the analysis of the data from the lower grades, four through six, Jerman found that three computational variables entered the regression equation. Two of these variables were in the first two steps, accounting for 54 percent of observed variance from the correct solutions. Yet these two variables were no longer significant in the analysis of grades seven through nine. In the higher

grades, a linguistic variable occupied the first step in the regression equation. Two computational variables followed in order of importance, but no other computational variables entered the regression equation.

Jerman concluded that computational variables accounted for most of the variance from the correct solutions for grades four through six, but that linguistic variables had become more significant by grades seven through nine. In a study of college-level students, no computational variables entered in the first twelve steps, thus effectively disappearing as determiners of the difficulty of word problems. Thus, a developmental trend for linguistic variables appears to exist, while computational variables progressively decrease in importance.

Other studies have noted that the sequencing of the problems has a greater significant effect on their difficulty than other computational-type variables (Suppes, Loftus and Jerman, 1969; Rosenthal and Resnick, 1971). This means that students find a problem much more difficult if it is not similar in type to the problems which preceded it.

The relevance of these studies in the development of problem-solving approaches is strong. For example, it seems quite reasonable to assume that the problem-type strategy used by many teachers is based on the concept of sequencing. Students who are taught to handle one specific type of problem will feel quite comfortable with the presentations found in most secondary school textbooks where examples of that type of problem are all grouped together. While the problem-type strategy effectively eliminates the sequencing variable as an obstacle in the classroom, it loses most of its usefulness when students face the problems in unfamiliar contexts.

Language-based strategies. The growing realization of the importance of linguistic skills to verbal problem solving has generated attempts to raise problem-solving ability by improving such skills as reading and vocabulary. Henney (1969), for example, compared a large group of fourth graders who were given lessons in reading verbal problems with a second group who studied and solved verbal problems in any way they chose. Results showed that although both groups improved significantly, there was no significant difference between the two groups. Other studies on the effect of teaching reading skills in mathematics classes (Lyon, 1975; Parler, 1975) also failed to find significant improvement in mathematical achievement or in problem solving.

Yet investigations into the benefit of training in vocabulary for increased mathematical achievement have been quite productive. After pupils were given specific training in mathematical vocabulary, gains in problem-solving ability were found by Dresher (1934), Johnson (1944), Lyda and Duncan (1967) (although this study has been shown to be poorly designed (Kane, 1967)) and Vanderlinde (1964). These studies seem to indicate that mathematics teachers have been too casual about introducing new symbols and terms to their classes. Students appear to improve as problem solvers when these terms are clarified and more time is spent on their mathematical vocabulary.

Literature Relevant to the Translation Method

The general success of mathematical vocabulary training, and the increasing recognition of the language factor as a major cause of difficulty in solving word problems, indicate that a translation approach is in order. General translation

strategies have been developed (Maffei, 1973, Taschow, 1969, and Earp, 1970), but Dahmus (1970) has developed the most rigorous and highly detailed method.

The Dahmus Translation Method is aimed at the lower 90 percent of mathematics students, who are intimidated by long verbal presentations of mathematical information and have no tools for dealing with such problems on a step-by-step basis. For example, students are not permitted to first read through a problem, but are to translate it one phrase at a time. All translations and related information are set down under the caption of Translation, and only then may the student begin to substitute and solve the resulting equation(s).

Dahmus claims that he has used this method often with a great deal of success at all age levels. Specific studies which have contrasted his method, or strategies similar to the Translation Method, with other problem-solving approaches, will be reviewed later under "Comparisons of Problem-Solving Strategies."

The careful structure of this method drew a protest from Boersig (1970). Her criticism was levelled at two aspects of the Translation Method: First, she notes that implied relationships and recalling formulas form a large part of finding the solution to word problems. Dahmus makes no provision for this. Then, in a reaction to the literal and highly structured nature of the strategy, she asserts:

> "A student should not be cheated out of learning problem-solving processes by just teaching him to translate according to verbal word patterns. Rather, he should be given the opportunity to wrestle with a problem and have the satisfaction of resolving the conflict" (p. 643).

Wilson and Becker (1970), in a general discussion of problem solving, echo the same sentiments:

> "We feel that students come to equate problem solving with "answer-getting" and, in doing so, miss the real heart of mathematics. Solving a mathematics problem should involve the systematic application of one's knowledge to the novel situation, lead to the complete understanding of the problem and its solution, and in turn, increase one's knowledge through learning new information (the solution), enhancing problem-solving skills (the process), and discovery of new relationships" (p. 293).

Literature Relevant to the Inductive Method

Another approach has been taken by some educators who emphasize the benefits of creative or divergent thinking in the solution of verbal problems. One such attempt is the Inductive Method, suggested by Spitler (1976). Students are encouraged to generate their own word problems in response to equations given by the teacher, with the goal of enabling students to establish a link between the verbal statement and the mathematical equivalent inductively.

Research supporting this kind of creative thinking is difficult to categorize because studies have used the terms "discovery" and "inductive" approaches interchangeably with "creative" or "divergent" approaches. The literature in question has dealt with every aspect of non-traditional teaching. However, it may loosely be characterized as dealing with learning which allows the student to abstract principles from information or experience with a minimum of guidance. (This is usually referred to as inductive or discovery learning.) When the student is encouraged to generate the experiences from which to draw generalizations, the terms "creative" or "divergent" thinking are often used. Literature discussing any of these approaches was considered relevant to the Inductive Method, regardless of which particular term was used.

Studies of discovery learning. The relationship between mathematical ability in general and discovery teaching methods has been studied increasingly since the 1960's emphasis on "new math," but consistent results have been difficult to find. One extensive study by Worthen (1968) involved 538 fifth-and-sixth-grade pupils. The investigator concluded that expository methods were superior to discovery methods on tests of initial learning, but discovery was significantly superior for concept transfer, transfer of heuristics, and retention.

However, a later study (Worthen and Collins, 1971) criticized certain statistical methods used by Worthen (1968). Reanalysis of the data with proper statistical procedures yielded no significant differences between treatments on any transfer or retention test. Thus the earlier conclusions by Worthen could not be supported.

A study which is often cited was conducted by Richards and Bolton (1971), who studied 265 children in their last year at three junior schools. The subjects were matched, while the three schools used different mathematical instruction techniques. One used discovery methods, another used traditional methods, and the third balanced the two approaches. They found that divergent thinking is a minor factor in the determination of general mathematical ability, with general ability being the most important determiner. (Few proponents of creative thinking approaches would question the fact that general ability, as well as verbal ability, are the factors most highly correlated with mathematical ability.) A second part of their study found that students at a school which emphasized discovery teaching and divergent thinking were inferior on tests of mechanical performance in mathematics, but were superior to students of a traditional school on tests of divergent thinking.

Olander and Robertson (1973) used 374 fourth-grade pupils for a comparison of discovery and expository learning on tests of computation, concepts, applications, and attitudes. They found that students under the expository treatment were significantly better in computation on both a posttest and a retention test. However, students in the discovery group better retained their ability to apply mathematical knowledge and showed significant improvement in attitudes. Since solving word problems is considered to be an "application" skill, this study indirectly supports the possibility that an inductive teaching approach may be helpful in problem solving although it may not be strongly correlated with general mathematical ability or performance on tasks of low cognitive complexity.

Studies of divergent thinking. Few studies have specifically related divergent or creative thinking to the process of solving mathematical word problems. Some have attempted to determine whether it is possible to increase divergent thinking in and of itself by means of special treatments. Dirkes' (1974) study is an example of this type of research.

Dirkes administered a fourteen-day divergent thinking program in problem solving to fifty-two geometry students and found that they showed significant gains in creative productivity of verbal content when compared to a control group. However, she did not test the students for problem-solving ability either before or after the treatment.

Maxwell (1974) categorized 105 students as either divergent or convergent thinkers in a clinical study to better understand problem-solving processes. Her classification was based on a six-problem test which she constructed, using three

divergent-type items and three convergent-type items. Forty-nine of the students were then observed as they solved one particular problem, described their methods, and reworked the problem on a second trial. Maxwell found that divergent thinkers used fewer generalizations with more trial and error, and they took more time with the second trial. These results, however, are based on only one actual problem-solving opportunity. In addition, Maxwell does not draw conclusions about the first attempt to solve the problem, but rather concentrates on the students' ability to dissect their methods and rework the problem.

Literature of specific relevance to an inductive approach. Wallace (1968) provides one of the few examples of studies which examined the relationship of different factors to the ability of students to solve mathematical word problems using the discovery method. He administered a battery of standardized tests to 548 freshmen at a college in Pennsylvania, and employed regression analysis of the data to draw the following conclusions:

1. The greater the student's mathematical ability, the greater his ability to solve mathematical problems by the discovery method;

2. The student's ability to solve mathematical problems by the discovery method was dependent to some extent upon his verbal ability;

3. There was a substantial relationship between a student's mathematical achievement and his ability to discover the solution to a mathematical problem; and

4. Female students displayed a slightly greater ability to solve mathematical problems by the discovery method than did male students.

However, it appears that no comparison with a control group had been made. Also, the first three conclusions do not seem to establish new relationships which had not already been found to be true of general problem-solving ability.

Dodson (1970) charted the characteristics of successful problem solvers and found that high scores on divergent thinking were directly related to success with problem solving. In general, although more research supporting these conclusions would be desirable, many mathematics educators would agree with Dirkes (1974) when she assumes that divergent thinking precedes convergent reasoning and evaluation in the problem-solving process. As Manheim (1972) suggests:

> "One of the big lessons of 'modern' mathematics is that the creation of new mathematics often is inductive rather than deductive. Thus we often try to abstract certain commonalities from a few cases and then generalize to a very large set... . But students should be encouraged to imagine, to postulate, to 'give it a try'. For trying is the essence of induction" (pp. 235-36).

Of particular relevance to the Inductive Method, where students write their own problems, was a study by Keil (1964). The purpose of the study was to determine whether students who wrote and solved their own problems in mathematics would prove superior to students who solved textbook problems. Data were obtained from test scores of 226 sixth-grade students in eight classrooms of eight schools in a midwestern state. All classes were given two standardized mathematics tests and one test of mental ability. Students were classified by sex, as well as by three levels of intelligence and two levels of socio-economic status. Four experimental and four control classrooms followed their regular textbook program for four days of the week. One day each week was devoted to the investigator's materials. The experimental group wrote and solved problems about a given situation while the control group solved textbook type problems about the same situation.

At the end of sixteen weeks, students were given two standardized mathematics tests. An analysis of covariance indicated that the experimental group

scored higher than the control group on both tests. Further analysis showed that for one test, differences were significant only for low socio-economic pupils. On the other criterion test, subjects in the experimental groups in each of the following categories: boys, girls, high intelligence, average intelligence, and low socio-economic, outperformed their counterparts in control groups. Kiel concluded that, in general, pupils who wrote and solved problems of their own were superior in arithmetic problem-solving ability to pupils who had the usual experiences in problem solving.

Comparisons of Problem-Solving Strategies

Research of general relevance dealing with problem solving has been plentiful, to say the least. Suydam (1967) conducted an impressive review of all published research on elementary school mathematics from 1900-1965, and found that problem solving was the most widely researched topic, with 84 of a total of 799 reports. At the same time, she found conflicting results and a generally low quality of research design and reporting to be prevalent. This was especially true of experimental studies. Studies contrasting different problem-solving approaches are typical, and problem-solving studies of this nature were used by Suydam as examples of research yielding inconsistent conclusions.

Some of the investigations into the relative effectiveness of problem-solving strategies are presented, despite the fact that they do not deal with either the Translation Method or the Inductive Method. Research of specific relevance to a comparison of these two methods is discussed later. It should be noted that no comparison was ever drawn between the Inductive Method and any other method, and that experiments using the Translation Method are rare.

Studies of the effectiveness of formal strategies. Research has indicated that there is some question as to the usefulness of imposing a structure, and especially a highly rigid structure, on problem solving. The classic study of this kind was conducted by Burch (1953) with 305 elementary school children who were trained to solve arithmetic problems using formal analysis (i.e., having students answer a specific sequence of questions before beginning computational work). The children attained higher scores on tests which did not require formal analysis than on one which did. Moreover, fifty-one students who were later interviewed indicated that they never used formal analysis unless required to do so, and many became confused when they attempted to do problems in this way.

Similarly, Kinsella (1951) compared the effects of a step method against no formal teaching at all using as a criterion the students' success in selecting the correct process for the solution of a problem. His findings showed that success was not dependent on prior success with any certain step or combination of steps, and that these in fact might lower the level of performance on the solution of the whole problem. This again supports the hypothesis that answering any specific set of questions may not produce superior results.

More recently, Post (1968) examined the question of structure in general as an aid to problem solving in mathematics. He assembled a list from research of the mental operations underlying the problem-solving process, and referred to this as problem-solving "structure." He divided ten grade 7 classes into experimental and control groups, and gave the experimental groups three days of instruction in methodology with a six-week reinforcement period, while the control group solved identical problems and then proceeded with other work. He

concluded from the lack of significant differences between groups that exposure to the structure of the problem-solving process does not enhance problem solving. Burch's investigations of formal analysis seem more relevant to the question of rigidly-imposed techniques than does Post's analysis of the benefits of a generally defined "structure." Yet the conclusion remains as to the doubtful benefit of imposing restrictive guidelines.

It seems that a problem-solving strategy should be a guideline which is specific enough to be of some use, and yet not too highly restrictive for students. Kilpatrick (1969) discusses the need for "finding methods and devices that would improve problem solving without putting the child in the kind of straightjacket provided by formal analysis and other prescriptive techniques" (pp. 529-30).

Research of general relevance to problem-solving strategies. Most of the comparisons which have been drawn between different methods have used some variation of Polya's method as one of the treatments. Wilson (1967), for example, compared a version of the wanted-given method as suggested by Polya with an action sequence popularly used in the elementary schools. In the action sequence, the student looks for operations suggested by the sequence of actions in a problem (thus actions dealing with "joining" would signify addition, and "separation" actions would signify subtraction). Results favored the wanted-given approach when the two methods were contrasted with a control group.

One of the more recent studies was conducted by Post and Brennan (1976), who compared formal instruction in Polya's strategy with an informal presentation of general heuristics for problem solving and found no significant differences between

the two. However, the study did not take teacher effect into consideration, and it used no control group.

A modified wanted-given approach was compared with a general problem-solving program in grade 5 by Jerman (1973) and, again, no differences were found in either the posttest or on a follow-up test between the two experimental groups and a control. The wanted-given group did use correct procedures on the posttest more often, but conclusions may be validated by the presence of group differences which were not controlled.

One study which seems more relevant to the current study was conducted by Gawronski (1972). Its purpose was to ascertain the existence of deductive or inductive learning styles. This study differs from the general comparison of strategies because the two types of instruction were administered to all subjects, with content matter differing slightly from one group to the next. Thus not only were deductive and inductive learning programs compared, but the hypothesis under investigation was that different students would benefit from each program.

Three hundred eighty-one eighth-grade subjects were stratified by sex and mathematical ability. Students who lacked prerequisite skills or who had had previous experience with the content matter were eliminated. The remaining 298 were randomly assigned to classification programs which used programmed texts to present two concepts, one deductively, and the other inductively. Students who scored above the median on the posttest following the deductive concept and below the median on the inductive concept posttest were classified as deductive learners, and students who scored below the median on the deductive posttest and above it on the inductive posttest were considered inductive learners. Using this method, 32 deductive and 22

inductive learners were found.

These 54 subjects were then given two additional programs of instruction, one inductive and the other deductive, and posttests were again administered at the close of each program. While it was expected that inductive learners would score higher than deductive learners on the inductive program posttest, with the reverse holding true for the deductive program posttest, this was not the case.

Although this study is of particular interest because it involves a deductive-inductive comparison, all posttests used by Gawronski used items which required low cognitive ability. This means that aside from the fact that no significant results were reported, the relevance to the current study is limited by the fact that Gawronski did not consider mathematical word problems in her investigation.

Problem-solving comparisons of specific relevance. No comparison similar to that undertaken by Gawronski (1972) could be found in the field of problem solving. However, one study by Shoecraft (1971) contrasted three translation approaches to problem solving, using twelve grade 7 mathematics classes and ten grade 9 mathematics classes. One group, considered the control group, was taught algebra word problems by direct translation (they were called the low imagery or "LO" group). Another group was taught by translation with accompanying materials for illustration (the high imagery: materials, or "HIM" group). The third group learned translation methods with drawings preceding the translation (the high imagery: drawings, of "HID" group). All groups were taught number, coin, and age problems for eight days, and work and mixture problems for four days.

HIM was found most effective for low achievers, but otherwise students in the LO group generally performed comparably to HIM and better than HID.

Shoecraft concluded that the popular assumption that materials or drawings, in and of themselves, enhance problem-solving achievement is unjustified. This study does provide some support for a translation method, but it is difficult to generalize from it to the study at hand because no group was taught word problems or practiced them without benefit of translation. Thus LO (direct translation) may be better than HIM or HID, but it does not necessarily follow that LO would be superior to having students simply practice the problems without instructional guidance.

The only study which used Dahmus' Translation Method in a comparison with another, non-translational strategy, was one conducted by Bassler, Beers and Richardson (1972). Here it was compared with a variation of Polya's method.

The experiment was based on a relatively small sample of 48 ninth-grade students in a parochial school for girls, with the sample divided into three ability levels. The instruction was given by videotape presentation to eliminate the factor of teacher effect. Seven forty-minute periods of instruction included teaching the solution of linear equations as well as the particular strategy for problem solving. A posttest of ten problems was given following the treatment, and an unannounced retention test followed four weeks later. Two criteria were used to score the test: the first was a solution criterion and the second was an "equation" criterion in which students were scored on their ability to find a single equation which would solve the problem.

Results from the tests were quite low, with means on the solution criterion ranging from 25 percent to 75 percent, and equations-criterion means ranging from two percent to 65 percent for the six cells. There were no significant differences on the solution criterion scores, but the Polya method students did

significantly better on the equations criterion. Both groups improved significantly on the retention tests.

The researchers point out that the results should be viewed with some caution in light of the generally low scores. They also note that the instructional medium may not have been as effective as a teacher-based instruction, where open questioning methods in earlier grades favored the Polya method, which might account for some of the differences.

One more reservation about the study should be recorded, since the style and design of Bassler et al (1972) is most closely aligned with that of the present study. No control group was used by them for comparison purposes. Therefore, it is not known whether either of the methods used was in fact superior to non-instructional methods.

Bidwell (1972) points to other limitations in his abstract of the study. He questions the appropriateness of using an equations criterion, when a careful inspection of the Translation Method will show that students are not being trained to direct their efforts toward producing a single equation. Rather, they are taught to use many variables in many minor equations. He also criticized the time length for the tests which, although not explicitly stated in the study, seems to be one forty-minute class period. Ten problems are too many in this short time sequence and do not allow for effective problem solving. In Bidwell's opinion, this would invalidate the research results.

Of all the research to date in this area, however, this comparison serves as the most useful precedent. A follow-up study to measure the different effects of two translation-oriented strategies, one deductive and formal, the other inductive

and creative, seems in order. Any such study would use teachers as the means of instruction, and would use a lower grade level to eliminate the problems cited by the researchers. It would also have to consider sex differences, which were not relevant to the Bassler, Beers and Richardson study because their subjects were all female.

Literature Relevant to Sex Differences

As indicated in Chapter 1, the learning assessment projects have found that sex differences, although small, do exist. Robitaille and Sherrill (1977) conclude from the British Columbia Mathematics Assessment project, for example, that female students are more competent at lower level cognitive tasks, while male students obtain higher scores in such high level cognitive tasks as problem solving.

Yet Suydam and Weaver (1975) conclude from their review of research that sex differences do not appear to exist in the ability to solve problems. This conclusion does not seem to be justified upon inspection of related studies and other general reviews of sex differences in mathematics education.

One such review of mathematics learning and the sexes was undertaken by Fennema (1974), and it appears from this review that there are conflicting results from studies which have used sex as a factor in their analyses. In the thirty-six studies which are reported, the total of significant differences which were found depended largely upon the age of subjects used. Of the thirteen investigations conducted with pre-school age children and early elementary levels, nine studies found no significant differences between the sexes. Of the remaining four, three studies found significant differences favoring the girls, and one favored the boys, thus indicating that there are no consistent differences in the early years.

The question of sex differences becomes more confused at higher grade levels, as is evident from the twenty studies reviewed in grades 4 through 9. These studies are listed, together with their results, in Table 2.1. This table appeared as Table 3 in the review by Fennema (1974). From this tabulation it may be concluded that no significant differences consistently appear in studies at these age levels, but where significant differences do exist, girls tend to perform better in tests of mathematics computation and boys tend to perform better in tests of mathematical reasoning.

The National Longitudinal Study of Mathematical Ability (NLSMA) provides more insight into the sex differences. Data from one group, the X-population were collected as the students progressed from grade 4 to grade 8 (Carry and Weaver, 1969; Carry, 1970). Data from a Y-population were collected for four years as the group progressed from grade 7 to grade 10. Tables 2.2 and 2.3 summarize the total number of tests for each population, with all grades for the population represented in the totals, together with the number of significant differences found between the performance of girls and boys at each level of cognitive skills.

It seems evident from these tables that boys outperformed girls at all levels of cognitive complexity, with results being particularly strong at the ninth and tenth grades (although no breakdown by grade is shown in these tables). Also, boys appear to do better than girls at the higher cognitive levels, as noted previously.

Several complicating factors appear to exist at the high school level. Lower ability boys tend to drop out more often than low-ability girls, so that the high school boys who are sampled may be of higher ability than the girls. Also,

TABLE 2.1

SEX DIFFERENCES IN MATHEMATICS ACHIEVEMENT
UPPER ELEMENTARY AND HIGH SCHOOL YEARS

Author	Grade	Dependent Variable(s)	Results
D'Augustine (1966)	5,6,7	Geometry and topology	No significant differences found.
Jarvis (1964)	6	Standardized achievement test	Boys tended to excel in reasoning at all IQ levels. Girls performed better in fundamentals in 3/4 IQ levels.
Unkel (1966)	1-9	Discrepancies between actual achievement as measured by standardized achievement test and anticipated achievement as determined by CA, MA & grade placement; 3 SES groups used as independent variables	Arithmetic reasoning: no significant differences in discrepancy scores between girls and boys. Arithmetic totals: no significant differences between girls and boys on total, yet at grades 6,7, 8, girls have significantly higher discrepancy scores with convergence again at grade 9.
Cleveland & Bosworth (1967)	6	Standardized achievement test	No significant differences found. "Virtually no differences between the sexes in any aspect of arithmetic achievement."
Zahn (1966)	8	Arithmetic achievement and reasoning (standardized test)	On 5 out of 32 subtests boys performed significantly better than girls; on 0 out of 32 subtests girls performed significantly better than boys.
Parsley, et al (1963)	2-8	Standardized achievement test	No significant differences found.
Parsley, et al (1964)	4-8	Standardized achievement test	Boys with IQ of 125+ outperformed girls with similar IQs on arithmetic reasoning. Girls with IQs of 75-124 outperformed boys with similar IQs on arithmetic fundamentals. The overall differences appear to be nonsignificant.

TABLE 2.1 - continued

Author	Grade	Dependent Variable(s)	Results
Singhal & Crago (1971)	5-16 years K-11 grades	Wide range achievement test (Level 1)	Before instruction girls had higher grade-equivalent scores in arithmetic as a total group and at most grade levels. After six weeks (approx.) of instruction boys made gains significantly higher than girls in grades 3, 4, and 9. The differences in the total gains for boys and girls were nonsignificant.
Olander & Ehmer (1971)	4,5,6	Mathematics vocabulary of children's contemporary mathematics tests	Girls were significantly better at all 3 grades
Sowder (1971)	4-7	Discovery of patterns	No significant differences.
Wozencraft (1963)	6	Standardized achievement test	No significant differences found in arithmetic reasoning. Girls performed significantly better on arithmetic computation. On arithmetic average girls in middle IQ range performed significantly better.
Overholt (1965)	4	Standardized achievement test and conservation-of-substance test	When scores were adjusted for differences in intelligence, boys scored significantly higher than girls on total score, understanding of concepts, and problem-solving ability. No significant differences in ability to conserve were found.
Alexander 1962	7	Arithmetic reasoning test	No significant differences found in ability to solve verbal problems.
Muscio (1962)	6	Quantitative understanding	Boys performed significantly better than girls.
Sheehan (1968)	9	Ability to learn to solve algebra problems	Ambivalent results.

TABLE 2.1 - continued

Author	Grade	Dependent Variable(s)	Results
Carry (1970); Carry & Weaver (1969)	Longitudinal 7-10	Computation, comprehension, application, analysis in algebra, geometry, and number systems	In 38 out of 75 tests boys performed significantly better than girls. In 16 out of 75 tests girls performed significantly better than boys.
Kilpatrick & McLeod (1971); McLeod & Kilpatrick (1969, 1971)	Longitudinal 7-10	Computation, comprehension application, analysis in algebra, geometry, and number systems	In 25 out of 54 tests boys performed significantly better than girls. In 10 out of 54 tests girls performed significantly better than boys.
McGuire (1961)	Junior High	Standardized achievement test	No significant differences found.
Gainer (1962)	6-12 years	Standardized achievement test	No significant differences found.
Hilton and Berglund	Longitudinal (5,7,9,11)	Standardized achievement test	No significant differences at grade 5. At grades 7, 9, and 11, college-bound boys scored significantly higher than college-bound girls. At grade 11, non-college-bound boys scored significantly higher than non-college-bound girls.

TABLE 2.2

SEX DIFFERENCES IN ACHIEVEMENT : GRADES 4-8
NLSMA: X-POPULATION*

	Total Tests	Significance Found Favoring: Boys	Girls
Computation	31	6	14
Comprehension	34	22	2
Application	3	3	0
Analysis	7	7	0
All tests	75	38	16

*Figures from Carry (1970) and Carry and Weaver (1969).

TABLE 2.3

SEX DIFFERENCES IN ACHIEVEMENT : GRADES 7-10
NLSMA: Y-POPULATION*

	Total Tests	Significance Found Favoring: Boys	Girls
Computation	18	5	7
Comprehension	14	6	1
Application	5	3	0
Analysis	14	11	2
All tests	51	25	10

*From Kilpatrick and McLeod (1971) and McLeod and Kilpatrick (1969, 1971).

for whatever reason, girls do not choose mathematics electives as often as boys, and it is therefore possible that a larger percentage of bright girls than bright boys do not continue in math. This would result in the sample of girls from mathematics classes being of correspondingly lower ability than the boys.

On the basis of the research done at the high school level, no conclusions could be drawn.

The sex differences favoring boys at the junior high school and upper elementary levels are supported by some of the studies discussed in the preceding pages, although not all used sex as a factor. For example, Kilpatrick (1967), in his study at the grade 8 level, found that girls said "I don't know" as a response more often than boys, and they used equations less often than boys. Maxwell (1974) found that boys were better problem solvers than girls in tenth grade.

On the other hand, Post (1968) found that sex was not a factor in his grade 7 study. Other studies either did not take the factor of sex into consideration or examined male subjects separately from female subjects (Gawronski, 1972).

However, it does seem that the indications from research are sufficiently strong to justify using sex as a factor in the analysis of performance of students in sixth and seventh grades. Specifically of interest is the possibility that of the two teaching strategies being compared, one will serve to benefit one sex more than the other. This is suggested by Wallace (1968), who found that female students displayed a slightly greater ability to solve mathematical problems by the discovery method than did male students.

Summary of Review of Related Literature

Although many aspects of problem-solving research defy generalizations, some patterns do emerge from the literature. There exists a strong correlation between linguistic ability and problem-solving ability, paralleling and perhaps even exceeding the correlation between computational ability and problem-solving ability. The need to aid students in the language aspect of mathematical word problems was evident from the prominence of linguistic factors in the determination of the difficulty of word problems, particularly at the grade 7 level and beyond.

Assorted attempts to teach students methods to improve problem-solving skills, were often successful, although not consistently. The effectiveness of carefully structured problem-solving techniques was questioned by a number of studies. Yet studies which tried to improve problem-solving by divergent or discovery techniques were rare, and no conclusions could be drawn from studies that do exist. Although the ability to use divergent thinking is considered a minor factor in terms of general mathematical ability, it was related to successful problem solving.

Comparisons of problem-solving techniques generally used some variation of Polya's method, and tended to be inconclusive or inconsistent. No contrasts could be found which compared detailed, deductive strategies such as the Translation Method with creative, inductive strategies such as the Inductive Method.

Sex differences in favor of boys do appear in the research, although not consistently. In general, boys seem superior at higher-level cognitive tasks and

girls at lower-level cognitive tasks when differences are found in the upper elementary and lower secondary grades. The sex differences are sufficiently in evidence to justify using sex as a factor in a study of problem-solving performance.

Chapter 3

DESIGN AND PROCEDURE

This study was undertaken after a search of literature revealed that no experimental situation had as yet been constructed which would contrast certain translation-based strategies with a noninstructional practice sequence. These strategies, the Translation Method and the Inductive Method, are discussed after a description of the sample involved and the tests used in the study.

The actual experimental procedure and the statistical procedures anticipated for the later analysis of data are then detailed. Results of the study are examined in subsequent chapters.

Sample

Grades six and seven were chosen as the level for the treatments, since it was felt that these grades were just beginning to experience a higher concentration of word problems in their texts and curricula and had not yet evolved a consistent strategy for dealing with these problems.

The study was performed with students in a private Jewish elementary school in Vancouver, British Columbia. This meant that the subjects did not constitute a random sample of the population at large, and significant results of an experiment done with them would indicate that more extensive research should be done before final conclusions are drawn. Two small classes of sixth graders with a total of 26 students between them and one grade seven class consisting of 24 students were collapsed into one sample group of 50 mixed sixth and

seventh graders for the interim of the treatment. This was done both to enlarge the sample and to eliminate variance which would have resulted from class differences. The justification for combining these grades was that careful scrutiny of the word problems in textbooks being used by both levels revealed little change in the complexity of grade 7 problems as compared with grade 6 problems. (As a precaution, grade was used as a factor in the analysis of the results of the study.)

The original sample, containing 16 students in Group A and 17 in each of the other two groups, was further reduced by two students who missed more than one treatment period. Thus, by the completion of the experiment, the number of subjects was 15 in Group A, 16 in Group B and 17 in Group C, for a total sample size of 48 subjects.

DEVELOPMENT OF MEASURING INSTRUMENTS

Word Problem Pilot Test

A word problem test was constructed to be used as a pilot test. It was originally planned to measure the problem-solving abilities of the students from the sample for later use as a pretest or covariate for the final statistical analysis. Then, when the results were tabulated, it served as a gauge to determine the optimum length of the subsequent posttests as well as to set the level of difficulty for future tests.

The pilot test consisted of six word problems which approximated the type of problems found in the grade six and seven textbooks (Investigating School Mathematics, Eicholz, O'Daffer and Fleenor, 1973). All items were short and uncomplicated, only whole number operations were needed for the solutions, and the computation involved was minimal. The operations required were two division calculations, two subtraction, one addition, and one multiplication. The test was administered in one forty-minute period but many students were finished after 30 minutes.

Answers for each item were scored on a scale of 0, 1, 2, with 2 as the maximum for the correct operation with no computational errors. One point was given for the correct operation with computational errors, and no points were allowed for an answer which did not use the correct operation. All work and responses were recorded on the test pages so that the paper score could be ascertained from the scratch work shown. A copy of the Word Problem Pilot Test may be found in Appendix A, together with scores obtained by students.

Because the test proved to be of low reliability (as discussed in Chapter 4), it was not utilized in the final analysis of results.

Translation Pretest

Because one of the treatments involved the ability to translate directly from an English statement of the problem to the mathematically equivalent symbols, it was felt that all three treatment groups should be more or less equivalent in their translation skills when the treatments began. The translation pretest was thus constructed to ascertain the students' translation ability for use when assigning students to groups.

No standardized test existed for this purpose, so the test was independently constructed by the researcher with English phrases such as "16 kilometres less than the distance." The students were instructed to indicate the proper mathematical phrase using symbols, numbers and letters (in this case, D - 16). The 30 items consisted of six addition-only examples, seven subtraction-only items, four multiplication-only items, seven division-only items, two items requiring only equality, one addition with equality, one subtraction with equality, one requiring addition with multiplication, and one multiplication with division. Correct translation was scored as a maximum possible of two points for an item. If the operation was correctly translated but other mistakes were made, or if only one of two required operations was correct, partial credit of one point was given (for example, "the sum of Jack's score and 25" written as N + 25 = N). Appendix B contains the translation pretest, which was designed for one 40-minute period.

Posttest One

Posttest One consisted of eight word problems of varying degrees of difficulty, but involving only one operation for the solution. Problems were drawn from a variety of sources, including Investigating School Mathematics, Books 6 & 7, (Eicholz et al, 1973), the 1973 edition of the Canadian Test of Basic Skills, and MP-1 Problem Solving (Spitler, 1976), and some were constructed by the investigator. Items required only whole number operations, with a minimum of difficulty in the computation necessary for reaching the solution. The test contained two addition items, two subtraction items, two multiplication items, and two

division items, one of which called for a remainder. The test was administered in one forty-minute period.

All work was to be shown on the test paper, and on this basis, partial credit was given. Each item was worth three points. One point was deducted if only one of the following mistakes was found: (1) dollar signs and decimal point omitted, (2) remainder not interpreted properly, or (3) a simple careless or computational error. If a student used the figures incorrectly or in an improper order, or had two of the above errors, then two points were deducted from the item, with one point allowed for identification of the proper operation. No credit was given if the student did not choose the correct operation.

The first posttest was designed to represent the traditional word problem test taken by students as a standard exercise in the application of whole number operations to the field of problem situations. Typical problems such as rate, distance, money and so forth all appear in the test, which is given in full in Appendix C.

Posttest Two

Posttest Two was constructed by the experimenter to be on a much more challenging level. The eight items in this test all required a multi-step computational process, and although only whole number operations were required, the solution of each item involved the use of two operations. The problems were drawn from the "think" problems found in Books 6 and 7 of Investigating School Mathematics (Eicholz et al, 1973), as well as from sources used for the first posttest.

The operations required for the test items were as follows:

Item 1: addition and multiplication

Item 2: subtraction and division

Item 3: division and multiplication

Item 4: addition and division

Item 5: multiplication and subtraction

Item 6: addition and division

Item 7: division and multiplication

Item 8: subtraction and multiplication

When the system of scoring was developed, it was apparent immediately that it would not be feasible to mark the papers allowing different degrees of partial credit, since the errors varied so widely from item to item as well as from student to student that it was impossible to delineate in any consistent manner how much work merited a certain amount of credit. It was therefore decided to give partial credit only for the same careless errors listed for Posttest One, with no credit for conceptual errors of any kind. Each item was given a total value of four points, and one point was deducted for each of the clerical errors previously noted (i.e., (1) omitting dollar signs and decimal points, (2) not using remainders properly, or (3) for a simple careless or computational error).

The test was administered in one forty-minute period, with all work shown on the face of the test paper. Ample room was left for these calculations and all test papers were carefully examined to discriminate between computational and conceptual errors. The second posttest, with directions for its administration, can be found in Appendix D.

DESCRIPTION OF INSTRUCTIONAL TREATMENTS

Treatment A : Translation Method

This method, developed by Maurice Dahmus (1970), is a structured treatment of word problems based on the assumption that the primary difficulty with problem solving lies in describing English problems in accurate mathematical terms. To meet this difficulty, the Dahmus method trains students to literally translate, phrase by phrase, from the statement of the problem into mathematical symbols. As a first step in the treatment, then, detailed instruction is necessary in the translation of such phrases as "increased by," "the difference between,", etc.

The actual translation strategy can best be illustrated using a simple example on a grade six level:

Jack has 15 more pennies than Betty. If Betty has 48, how many does Jack have?

A. Translate: Jack / has / 15 more pennies than Betty.

J = 15 + B

If Betty / has / 48, / how many / does Jack have?

B = 48 ? = J

B. Relationships or formulas suggested by key words in the problem.

(This example needs no outside formulas.)

C. Solution - usually by direct substitution into statement of relationship

i.e., J = 15 + B

? = 15 + 48 and solve.

Using the Dahmus Method, the student's exercise book should look something like this:

Translate	Solve
$J = 15 + B$	$J = 15 + B$
$B = 48$	$? = 15 + 48$
$? = J$	$? = 63$

The teaching outline for the four days of translation lessons was as follows:

Lesson 1 : Familiarized students with common language equivalents of mathematical terms (decreased by, a total of, etc.).

Lesson 2 : Introduced solving word problems by the translation method.

Lesson 3 : Extended the method to include cases requiring outside formulas or relationships not specifically stated in the problem.

Lesson 4 : Reviewed method; general practice.

The complete set of lesson plans, worksheets, and other materials for Treatment A can be found in Appendix E.

Treatment B : Inductive Method

This method was suggested by Gail Spitler (1976). The assumption was that students lack some vital link which they need to make the connection between word phrases and symbols, and that it would be possible to reduce this lack by having students "create" word problems to match suggested equations. Students were encouraged to use as many different contexts and equivalent word phrases for the operation as possible.

The equation used as an example here is identical to the derived equation in the example for the Dahmus Method:

$$N = 48 + 15$$

Teacher: "What kind of problem situation could this mathematical statement be describing? Can you write at least three word or story problems which are as different as possible, which would fit the same equation?"

Examples of Possible Answers:

1. If a man drove from his home a distance of 48 km in the morning and 15 km more in the afternoon to reach a second city, how far apart are the two cities?

2. Jane is 15. If she is 48 years younger than her grandmother, how old is her grandmother?

3. If Mrs. Johnson has only $15 left after buying a coat which cost $48, how much money did she have originally?

- -

(Students who have difficulty generating problems at first might look through different story problems in the textbook for context ideas.) In the beginning lessons, students compared answers and each exercise such as the one above was to be followed by a summary list of the different possible solutions or contexts which the students have used in their examples; here they might be categorized as

1. distance
2. age
3. money

and so forth.

Later lessons emphasized that equations may correspond to different meanings of the same operation, and that word problems can vary accordingly. For example, in the problem above, the first sample answer corresponds to $N = 48 + 15$ as a simple example of the sum of two quantities. However, in the third sample answer, the appropriate equation is probably better described as $N - 48 = 15$, or a word problem using the addition operation as the inverse of subtraction. Variety in this area was also to be encouraged.

The teaching outline for the four days of inductive lessons was as follows:

Lesson 1: Expressed the same mathematical relationship in very different English terms.

Lesson 2: Introduced the concept of creating word problems.

Lesson 3: Emphasized different meanings of operations.

Lesson 4: General practice.

The complete set of lesson plans for Treatment B can be found in Appendix F. No worksheets were necessary since the work was generated from a series of equations which were written on the board by the teacher.

Treatment C : Control Method

This method was designed to duplicate as closely as possible a noninstructional sequence of problem solving. The emphasis was on having students solve as many problems as were being done in the first, experimental method, without any guidance as to a general strategy for the solution. When students requested help, they were given the proper equation for the solution with as few words of explanation as possible.

Generally the purpose of this method was to determine whether a concentrated exposure of word problems is all that is really necessary to raise the achievement levels of students in word problems. No lesson plans were required, therefore, and only worksheets were distributed to this group. These worksheets are presented in Appendix G.

DEVELOPMENT OF INSTRUCTIONAL MATERIALS

"How to Teach Word Problems" (Dahmus, 1970) provided the outline for the instruction necessary for Treatment A, and MP-1 Problem Solving (Spitler, 1976) was used as a source for the general introduction of the lesson plans for Treatment B.

All lesson plans, most worksheets, and visual aids were created by the experimenter to correspond to the instructional aims of the treatment methods. Puzzles and teasers included in the worksheets for Group C, and the "Line Code" worksheet for Group A's introductory lesson, were drawn from Mathimagination (Marcy, 1973).

DESIGN

The study basically follows the outline set by Campbell and Stanley (1963) of the "true experimental design" for the posttest-only control group design since the pretest was not used in the final analysis. Here R refers only to the random

assignment to groups rather than to general randomization, since the sample was not randomly selected from the population. In order to present a general scheme of the study, the following figure outlines the experimental design:

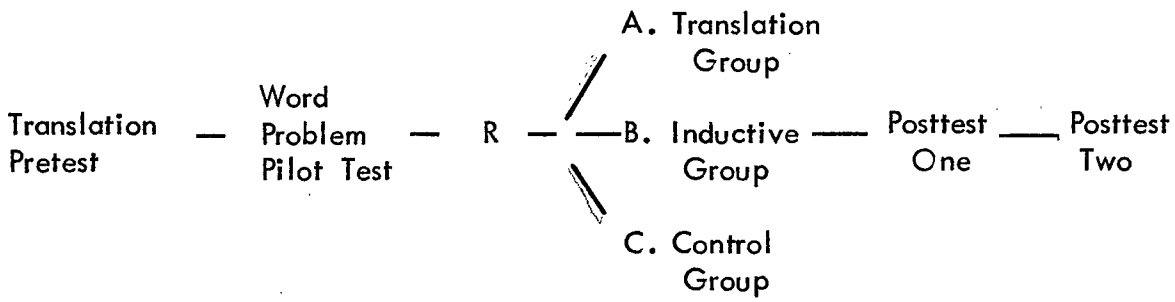


Figure 3.1 Outline of Study

CONTROLS

Problem-Solving Background

Since the same mathematics teacher had taught each of the three original classes during the year, the classes were assumed to have had similar mathematical experience in problem solving, and none of them had been given any concentrated exposure in this field. The teacher, however, did have a personal preference for teaching problem solving by translating, although he had never taught translation in any prolonged fashion. It was felt that if all three treatment groups were equated on translation skills before the treatments began, any problem-solving superiority that might be later discovered in the translation group could be assumed to be a result of the translation treatment and would not be attributable to having a higher proportion of strong translators in the group.

Teachers

The teacher referred to above was extremely receptive to the inductive strategy as well as the translation method, and agreed to teach both Treatment A and Treatment B. This effectively eliminated the possibility of variance between different teachers. During the many consultations before and during the actual teaching week, he was fully cooperative, and after observing his lessons on the first and third days of the treatment week, the researcher concluded that he was carefully following the lesson plans for each method.

Meanwhile the control group was supervised by a teacher who had little mathematical background and ordinarily taught the students an entirely different subject. They therefore did not press him for explanations of the word problems which they did independently of any instruction and, in this way, the control group did simulate as closely as possible the conditions of pure practice without a specific teaching strategy.

Treatments

In order to ensure that all treatment groups practiced an equal number of problems of equal difficulty, the lesson plans and worksheets were coordinated in the following way: a set number of problems were planned for each lesson of Treatment A, and the identical problems were used by the control group in their practice worksheets. Since it was anticipated that students of Treatment C would finish the problems in less time, working independently, than students of Treatment A who were using these problems to learn a particular strategy, the worksheets for Treatment C contained puzzles and unrelated games for students who finished earlier.

Treatment B did not require any word problems because this method involves students creating problems with equations given by the teacher. For this reason, keeping the difficulty of word problems on a par with the other groups was accomplished by using equations which were derived from word problems being used by those groups.

ASSIGNMENT TO GROUPS

In order to insure that the treatment groups were equivalent, three things were taken into consideration when the fifty subjects were randomly assigned. First, it was preferable to have an equal number of sixth and seventh graders in each group. Secondly, for reasons cited previously, it was felt that the groups should be roughly matched on their translation skills when the treatments began. Lastly, the three students for whom English was not a native language were randomly assigned to three separate groups.

Thus, on the basis of performance on the translation pretest, students from grade seven were divided into three general levels: superior translators, average translators, and poor translators. Students in the highest level were randomly assigned to Groups A, B, or C. The second level students were then similarly assigned to the groups, and finally the third level. This process was then repeated with students from grade six.

It should be noted here that subjects were not matched with respect to gender, although it became apparent later that this would have been appropriate. As detailed in Chapter 2, however, the significance of sex in relation to problem

solving had not been sufficiently clarified, especially at these particular grade levels, to justify random assignment by gender as well as by grade and translation skills, and such an assignment seemed at the time to be unnecessarily contrived. Later, an analysis of variance of results of the translation pretest seemed to support the method chosen for the random assignment to groups. (The reader is referred to Chapter 4 for this analysis.)

After the assignment, Group A contained 16 students, and Groups B and C had 17 students in each. At the completion of the experiment, however, the final sample sizes for treatment groups A, B, and C were 15, 16 and 17 respectively.

PROCEDURE

The Word Problem Pilot Test and the Translation Pretest were administered during the regularly scheduled mathematics lessons of the original three classes (two sixth and one seventh grade class), with the translation skills being tested two weeks before the treatments began, and the pilot test given on the last day before the treatments. Students were informed of their assignment to groups on that same day, in their official classes. At this time they were told that they were taking part in a special attempt to improve their problem-solving abilities in preparation for the upcoming administration of the Canadian Test of Basic Skills.

The problem-solving lessons involved four consecutive school days, with each lesson lasting forty minutes. On these days, the groups were alternately taught one after the other, with the control group meeting one period later, as

summarized in Figure 3.2.

	Day 1	Day 2	Day 3	Day 4
Period 1	Group A	Group B	Group A	Group B
Period 2	Group B	Group A	Group B	Group A
Period 3	Group C	Group C	Group C	Group C

Figure 3.2 Treatment Sequence

After the four days of lessons, students returned to their original classes and took the first posttest on the day immediately following the treatments. This meant that they did not take their posttest in treatment groups, but instead took it within their sixth or seventh grade mathematics classes. All students took the test under as similar conditions as possible, with the same mathematics teacher supervising.

The second posttest was administered on the third day following the first posttest. This test was also given to students in their original classes, but the second posttest was taken simultaneously by all three classes, with identical test instructions and time limitations provided for each of the three teachers involved as supervisors. Figure 3.3 describes the time element involved in the study.

	Week 1					Week 2					Week 3					Week 4				
	M	T	W	Th	F	M	T	W	Th	F	M	T	W	Th	F	M	T	W	Th	F
Group A			P							P	Treatment				T			T		
			R							I	A				E			E		
			E							L					S			S		
Group B			T							O	Treatment				T			T		
			E							T	B									
			S												O			T		
Group C			T							T	Treatment				N			W		
										E	C				E			O		
										S										
										T										

Figure 3.3 Procedure

STATISTICAL ANALYSIS

General statistical procedures can be divided into two categories: the analysis of the measuring instruments themselves for overall reliability, and the analysis of the results of the two posttests. All calculations were performed at the University of British Columbia Computing Centre.

Data

Test papers and scores were obtained from the 48 subjects for each of the following tests:

1. Word Problem Pilot Test
2. Translation Pretest
3. Posttest One
4. Posttest Two

In addition, work and exercises were obtained from each group according to their lessons. Thus, students from Group A handed in each day's worksheet for four days, as did students from Group C. Student-generated problems were

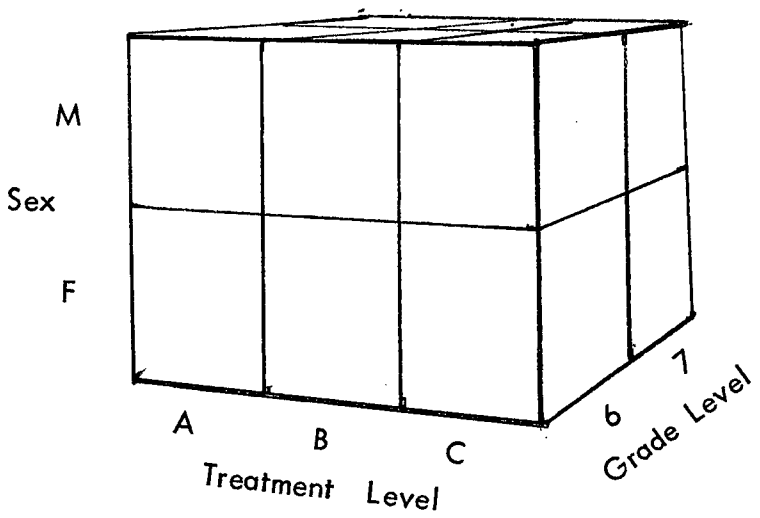
collected as the result of their instruction for each of the four days from Group B.

Analysis of Tests

The Word Problem Pilot Test and the Translation Pretest, as well as the two posttests, were analyzed by the LERTAP item analysis (Nelson, 1974) program to determine the reliability of the tests. An analysis of variance was also performed on the results of the translation pretest in order to determine whether the subjects differed significantly by grade and/or sex, which would justify the assignment of students to groups.

Analysis of Posttest Results

Since more than one dependent variable was involved, a multivariate analysis of variance was performed on results of the posttests by the computer program MULTIVAR (Multivariance, 1978). Three factors were being tested for significance: sex, grade, and treatment effect. These factors can be pictured as a completely randomized, 2x2x3 fully crossed factorial design, utilizing two posttests as the dependent measures:



Differences with regard to Sex, Grade, and Treatment are regarded as fixed rather than random effects, and therefore the following multivariate linear model for fixed effects was considered appropriate: (The model should be viewed as two-dimensional in consideration of the fact that two dependent measures were used.)

$$Y_{ijkm} = \mu + \alpha_i + \beta_j + \lambda_k + \alpha\beta_{ij} + \beta\lambda_{jk} + \alpha\lambda_{ik} + \alpha\beta\lambda_{ijk} + \varepsilon_{ijkm}$$

where

Y_{ijkm} represents the score for a particular subject on the dependent measures:

μ represents the general mean, a constant for all the subjects,

α_i represents the effect of the ith level of sex, constant for all subjects in that population,

β_j represents the effect of the jth level of grade, constant for all subjects in that population,

λ_k represents the treatment effect of level k, constant for all subjects in that population,

with the terms $\alpha\beta_{ij}$, $\beta\lambda_{jk}$, $\alpha\lambda_{ik}$ and $\alpha\beta\lambda_{ijk}$ representing the interaction between the corresponding effects, and

ε_{ijkm} represents the error for each individual, or that part of the dependent measures which cannot be accounted for by the sum of the main and interaction effects.

The use of a covariate for this analysis was considered and rejected, since scores for the Canadian Test of Basic Skills were not available for every one of

the subjects, and too many subjects would have had to be dropped from the study. Using the Translation Pretest results was not feasible because the correlation between translation skills and problem solving would mean that some of the posttest results would be drawn away as part of the covariate (the correlation between the translation pretest and the first posttest was 0.396, and the correlation between the pretest and the second posttest was 0.626). Missing data analyses were also not considered appropriate when the data is all missing from the same measure.

Since each group did not have the same number of subjects, the order of entry of the main effects has been shown to influence the results of the analysis. In this situation, the need for the "unchangeable" or fixed factors of Sex and Grade to take priority over experimental factors which may be manipulated such as treatment level was recognized by researchers (Overall and Spiegel, 1969). Thus Sex and Grade factors were given an "a priori" type of analysis over the treatment factor. However, a reordering of the two unchangeable main effects was necessary to confirm that any significance in either of these two factors, Sex and Grade, was not due solely to the order of testing for results. Consequently a second run of the multivariate analysis of variance was made with the factor of Grade entered before the Sex factor.

If the multivariate F-ratios were found to be significant at the 0.05 level, a closer look at the univariate F-ratios would then be appropriate to find which posttest was contributing to the significance, or whether both were. In the event that the Treatment factor, or any interaction with treatment levels, were found to be significant, further analysis for multiple comparisons between groups on this

factor could then be conducted.

If, however, the effect of Treatment was not statistically significant while one of the descriptive factors of Sex or Grade was significant, further analysis would take the form of altering the representations of the linear model on a univariate level in order to examine whether significance on either Grade or Sex differed across treatment levels. Any nonsignificant factor of Grade, for example, could be collapsed and a further statistical investigation would use a simple main effects analysis to examine the following sources of variance: treatment differences, sex differences in the first treatment group, in the second treatment, and in the third treatment.

The new model for the analysis, using only the relevant two factors and treating each dependent variable separately, would be as follows:

Simple Main Effects Model

$$Y_{ijk} = \mu + \alpha_i + \beta_{j(i)} + \varepsilon_{ijk}$$

where

Y_{ijk} represents the score for a particular subject,

μ represents the mean, a constant for all subjects,

α_i represents the effect associated with the ith level of treatment, constant for all subjects in that population,

$\beta_{j(i)}$ represents the separate, additive effects for the j level of a fixed factor (male-female, for example) within each treatment level i, constant for all subjects in that population, and

ε_{ijk} represents the error for each individual, or that part of the dependent

measures which cannot be accounted for by the sum of μ, α_i, and $\beta_{j(i)}$.

This simple main effects model is drawn from Marascuilo and Levin (1970), and is suitable for situations where no interaction effect between a fixed effect such as Sex and a manipulated effect such as Treatment is found, but when it is desirable to examine one of the factors as a function of the other.

Statistical Assumptions

Use of the multivariate analysis of variance rests on two assumptions: the assumptions of normality of distribution of scores, and of homogeneity of variances of the treatment populations with respect to each posttest variance and the covariance of the two posttests.

The assumption of normality posed no particular difficulty because inspection of the data indicated no great departure from normality, and in any case, the multivariate analysis of variance is fairly robust with respect to violations of normality.

The assumption of homogeneity of within-cell variance, however, is not as evident from the data and is complicated by the fact that cell sizes are not equal. (The test is known to be robust with respect to violations of homogeneity of variance with equal cell sizes.) Although the multivariate analysis "can survive a certain amount of heterogeneity among the dispersions of the groups," (Hope, 1968, p. 29), statisticians advise that a preliminary test of homogeneity be conducted and the results reported before the multivariate analysis of variance is undertaken.

This test was therefore performed, using the SPSS computer program (Kita, 1977), and the results are reported in the following chapter.

Chapter 4

RESULTS OF THE STUDY

Discussion of test results will be preceded by a report on the overall reliability of all tests, and a discussion of the random assignment to groups. The outcome of the various research hypotheses are then described, with an accompanying description of the validity of the assumption of homogeneity of variance.

Analysis of Test Reliabilities

Word Problem Pilot Test. The pilot test consisted of six items, and the Hoyt estimate of reliability was determined to be 0.53, with a standard error of measurement of 1.08. A check of the biserial correlations indicated that the correlation between each item and students' performance on the test as a whole was at a fairly high level. This meant that the students who obtained a correct answer to any particular problem generally performed well on the entire test. A summary of the item reliability results can be found in Appendix H, Table H.1.

Since the reliability of this test was so low as to render it useless as a covariate, no further reference to it will be made in the course of this report. Its sole function in the study was to clarify that future posttests should contain at least eight problems, and that these problems could be considerably more difficult than was at first anticipated.

Translation Pretest. The Hoyt reliability for the Translation Pretest proved to be 0.87, and the standard error of measurement was 4.01 (with a maximum

possible score of 60). Biserial correlations of the thirty items all discriminated in the proper direction, and are listed in Table H.2 of Appendix H.

The test was judged to be sufficiently reliable for use in the assignment to groups.

Posttest One. Both the first and the second posttests consisted of eight items. Posttest One had an estimate of reliability of 0.72, with a standard error of measurement of 2.00 (and a total point spread of 24). The correlation of the performance on each item with overall test score, as described in Appendix H, Table H.3 was considered acceptable.

Posttest Two. The Hoyt estimate of reliability for the second posttest was slightly higher than that of the first, at the 0.76 level. The standard error of measurement was 3.73, but the maximum possible score on the posttest was 32. All items were judged to have acceptable biserial correlations. For a list of item reliabilities, the reader is referred to Table H.4 of Appendix H.

Although it would have been desirable to have obtained higher reliability statistics for both posttests, this was difficult to accomplish when each test consisted of only eight items. Practical considerations limited the testing to one forty-minute period per test, and it was felt that students could not handle more problems in that amount of time. However, the posttests were considered sufficiently reliable to proceed with the statistical analysis of their results.

Analysis of Variance for Pretest

In order to determine whether the groups were generally matched on translation skills, an analysis of variance was performed on results of the pretest.

Since no significant differences were found among treatment groups, it was concluded that no group had an initial advantage or handicap with respect to translation abilities. When results of the pretest were analyzed by Grade and Sex, Grade was found to be a significant factor while Sex did not figure significantly in the outcome. Table 4.1 contains the analysis of variance results for the Translation Pretest, which seem to justify the assignment to groups by Grade and not by Sex, as well as supporting the assumption that all groups began with roughly matched translation skills.

RESULT OF RESEARCH HYPOTHESES

The statistical hypotheses to be verified will be presented in the same general order as the research questions which were posed in Chapter 1. Results of all treatment-related hypotheses will be discussed prior to the inspection of sex differences, and therefore the Treatment-by-Sex interaction is considered before the main effect of Sex.

Individual scores for all subjects on the three relevant tests are listed in Appendix I, while summary data can be found in Table 4.2.

However, the description of results is incomplete without recording the outcome of testing for homogeneity of variance, since it is one of the assumptions upon which the multivariate analysis of variance rests.

Homogeneity of Variance

Because the test for homogeneity of variance was complicated by the presence of different factors in the study as well as the existence of more than one

TABLE 4.1

ANALYSIS OF VARIANCE FOR THE PRETEST

Source	df	Mean Square	F-ratio	Probability less than
Treatment	2	17.8953	0.1438	0.8666
Sex	1	172.9292	1.3895	0.2462
Grade	1	1084.1038	8.7112	0.0056*
Sex x Grade	1	19.8547	0.1595	0.6920
Sex x Treatment	2	8.7353	0.0702	0.9324
Grade x Treatment	2	71.5320	0.5748	0.5680
Sex x Grade x Treatment	2	42.5331	0.3418	0.7128
Error	36	124.4500		

* $P < 0.01$

TABLE 4.2

SUMMARY DATA FOR THREE TESTS

	Translation Pretest	Posttest One	Posttest Two
Number of observations	49	49	49
Maximum possible score	60	24	32
Mean	42.9	20.5	19.7
Mean as percent	72%	85%	62%
Standard deviation	11.2	4.0	8.1
Highest score	59	24	32
Lowest score	14	6	0

dependent variable, the factor of Grade, later found to be non-significant, was collapsed, and only six cells of the remaining 2 x 3 factorial design were tested. The analysis was done separately for each dependent variable using both Cochran's C formula and the Bartlett-Box F formula, with the results shown in Table 4.3.

It can thus be seen that for Posttest One, homogeneity of variance cannot be assumed. Posttest Two produced somewhat more homogeneous results. Since there exist few alternatives to the multivariate analysis of variance for direct analysis of posttest results, the analysis proceeded and the reader is advised to view the results with caution because of the partial violation of the assumption of homogeneity of variance.

Hypothesis One : Treatment Differences

The first statistical hypothesis to be tested was:

Hypothesis One: There will be no significant differences among treatment groups A, B, and C as measured by either or both of the posttests.

Results of the multivariate analysis of variance with the two dependent variables did not justify rejecting this hypothesis. The differences among the means of the three treatment groups, which are outlined in Table 4.4, did not prove to be significant at the accepted α level of 0.05. Multivariate F-ratios, formed from each of the possible sources of variation, are given in Table 4.5.

The fact that the multivariate F for the treatment hypothesis was not significant did not merit further investigations of the univariate analysis of variance for each posttest.

TABLE 4.3

HOMOGENEITY OF VARIANCE

	Probability for: Cochran's C	Bartlett-Box F
Posttest One	0.025*	0.001**
Posttest Two	0.129	0.237

*$p < 0.05$
**$p < 0.01$

TABLE 4.4

MEANS OF TREATMENT GROUPS

		Posttest One	Posttest Two
Treatment Group	A	19.93	17.53
	B	20.62	20.44
	C	20.82	21.24

TABLE 4.5

RESULTS OF MULTIVARIATE ANALYSIS OF VARIANCE

Source	Multivariate F-ratio		Probability Less Than	
Treatment	0.3536		0.8407	
Sex	5.6590	(5.5786)**	0.0075*	(0.0079)*
Grade	1.8392	(1.9196)	0.1740	(0.1618)
Sex x Grade	0.4279		0.8700	
Sex x Treatment	1.3982		0.2436	
Grade x Treatment	0.3299		0.8570	
Sex x Grade x Treatment	0.5326		0.7122	

*$p < 0.01$

**Figures in parentheses represent results when Grade was entered before Sex in the order of analysis.

Hypothesis Two : Treatment-Sex Interaction

Another statistical hypothesis involving treatment to be tested was:

Hypothesis Two: There exists no interaction between Treatment and Sex.

The differential means for boys and girls of each treatment group are given in Table 4.6.

The differences found among the means were not statistically significant. This means that there was no significance attached to being in one particular cell as opposed to another cell in the six cells represented in the "Sex x Treatment" source of variation. (The reader is again referred to Table 4.5.) It was concluded that the difference between the problem-solving achievement on the part of the males as compared to the females in one treatment group was not significantly different from the same comparison in another treatment group. Since a non-significant multivariate F does not permit analysis of individual results from the two posttests, no further investigations of the interaction effect were conducted.

Hypothesis Three : Sex Differences

Shifting the focus from treatment-related questions to sex-related questions, the next statistical hypothesis to be considered was:

Hypothesis Three: The mean of all male students' performance on either or both of the posttests, regardless of Treatment or Grade, will be no different from the mean test score for female students.

The means of male students on the two posttests, together with the two means for female students, are given in Table 4.7.

TABLE 4.6

MEANS OF TREATMENT-BY-SEX LEVELS

	POSTTEST ONE			POSTTEST TWO		
	Treatment Group			Treatment Group		
	A	B	C	A	B	C
M	22.75	21.67	21.73	19	23.44	23
F	16.71	19.29	19.17	15.86	16.57	18

TABLE 4.7

MEANS OF MALE STUDENTS COMPARED TO FEMALE STUDENTS

	POSTTEST ONE	POSTTEST TWO
M	22.0	22.0
F	18.35	16.75

Wide differences between the two sexes are evident from Table 4.7, with boys consistently claiming the higher mean. A statistical analysis of these results, as reported under "Sex" in Table 4.5, shows that there was indeed a significant difference on the factor of Sex, with $p < 0.01$. (The F-ratios when the factor of Grade was allowed to enter the analysis before the Sex factor, as explained in Chapter 3, "Statistical Procedures," are entered in the table in parentheses.) The change in F-ratios because of the order of entry was negligible. Because the results proved significant, the null hypothesis was rejected and it was concluded that Sex is an important factor in the comparison of subjects' achievement.

When a multivariate F shows evidence of statistical significance, consideration of the univariate F's is the next step to determine which of the posttests is contributing to the significance, or whether it is caused by both tests. This further inspection of the results for Sex corresponds to the next research hypothesis.

Hypotheses Four and Five : Sex Differences Within Tests

Hypothesis Four and Hypothesis Five were the following:

Hypothesis Four: For Posttest One, no differences in achievement between male and female students will be found.

Hypothesis Five: For Posttest Two, no differences in achievement between male and female students will be found.

With the multivariate F-ratio of 5.659 exceeding the critical value of 5.29 at the 0.01 level, the separate univariate F-ratios may now be examined.

They are listed in Table 4.8.

Results reported in this table are based on the analysis with Sex preceding Grade in the order of entry, since reversing the order produced basically similar results. The table shows that significant differences exist within both posttests, with the probability for Posttest One results found to be less than 0.005, and the findings for Posttest Two significant at the 0.05 level.

Significant results found in the univariate F-ratios for both posttests permitted investigations within treatment levels of the male-female differences. However, further analyses using the standard model of main effects with interaction effects were not possible, since sex differences within treatment levels requires a special partitioning of main effects. Therefore an appropriate model, the simple main effects model, was utilized to examine the next statistical hypotheses. (This procedure is explained in full in Chapter 3, "Statistical Procedures.")

Hypotheses Six, Seven, and Eight: Sex Differences Within Treatment Levels Using the Simple Main Effects Model

Hypothesis Six: Male students will not differ in performance from female students in posttest results for Treatment A.

Hypothesis Seven: Male students will not differ in performance from female students in posttest results for Treatment B.

Hypothesis Eight: Male students will not differ in performance from female students in posttest results form Treatment C.

These hypotheses were tested to determine whether any of the treatment levels showed results with no significant male-female difference. Lack of significant differences within any one treatment level would suggest that in some

TABLE 4.8

RESULTS OF UNIVARIATE ANALYSIS OF VARIANCE BY SEX DIFFERENCES

Variable	Hypothesis Mean Square	Error Mean Square	F-ratio #	Probability less than
Test 1	155.4292	13.5866	11.4399	0.0018*
Test 2	321.5652	66.7130	4.8201	0.0347**

#with degrees of freedom of 1, 36

*$p < 0.005$

**$p < 0.05$

way, treatment tends to bring the achievement level of the boys and girls closer together.

The result of examining each treatment for a significant male-female difference is summarized in Table 4.9.

While the male students consistently outperformed the female students as previously described in Table 4.6, the results of Table 4.9 indicate that in only one treatment, Treatment A, did the comparison of the sexes show a significant difference. Thus only Hypothesis Six was rejected, but the null hypotheses for the other treatments, Hypotheses Seven and Eight, were not rejected. In Treatment A the probability that the boys would show superior achievement was found to be less than 0.01.

It should be noted here, however, that the general level of significance was not applicable to the simple main effects model. As recommended by Kirk (1968), the alpha level of 0.05 is divided by three because there are three treatment levels to correspond to the partitioning of main effects, and the critical level used for this analysis was 0.0167.

Further analysis within this treatment level became appropriate when the multivariate F was shown to be significant and, therefore, the results of the two posttests were individually examined to determine whether either or both would show similarly significant differences. Table 4.10 reports the results of the univariate F analysis for each of the posttests.

TABLE 4.9

COMPARISON OF MALE-FEMALE DIFFERENCES FOR EACH TREATMENT

		Multivariate F-ratio	Probability less than
Treatment Group	A	5.3991	0.0083*
	B	1.4905	0.2373
	C	1.0437	0.3614

*p < 0.01

TABLE 4.10

UNIVARIATE F-RATIO FOR TREATMENT A SEX DIFFERENCES

Variable	Hypothesis Mean Square	Error Mean Square	F-ratio #	Probability less than
Posttest One	136.0048	14.1279	9.6267	0.0035*
Posttest Two	36.8762	63.4951	0.5808	0.4503

#with degrees of freedom 1, 42
*p < 0.005

As indicated by this table, results were not consistent across both posttests. While the differences between the performances of boys as compared to girls in Posttest One were found to have a probability of less than 0.005, Posttest Two results were not significantly different.

Hypothesis Nine : Grade Differences

The research hypothesis dealing with grade differences was:

Hypothesis Nine: No differences will be found between the problem-solving performance of Grade Six students as compared to the posttest results for the Grade Seven students.

As demonstrated in Table 4.5, the multivariate F-ratio for the variance attributable to Grade was 1.8392 (or 1.9196 if Grade is considered before Sex). This falls short of the critical F value of 3.28, with degrees of freedom of 2 and 35 at the 0.05 level of significance. Therefore the null hypothesis of no difference between grades was not rejected, and the factor of Grade was not considered in further analysis.

STUDENT REACTION TO TREATMENTS

F-ratios and probability statistics cannot fully describe the outcome of this study. To completely evaluate the effectiveness of the teaching strategies, the reactions of students to these methods should be examined.

Treatment A : Translation Method

Students generally did not enjoy using this strategy as it was presented to them. Several times in class, they complained about having to do problems in one particular way when the answer was obvious to them without this method. The clearest example of their reluctance to translate as a method of solving a problem is shown by their reaction during the first posttest when urged by the teacher to solve the problems using the method that they had been taught. Many students from Group A seemed quite upset by this instruction and strongly expressed their preference for doing it "their own way."

The students did, however, understand what the method involved, and for the most part, they translated the majority of the problems in Posttest One. For Posttest Two they were not instructed to solve the problems in any special way, and consequently very little translation is in evidence on the papers of the Group A students.

Treatment B : Inductive Method

The inductive method of creating word problems from equations produced extremely positive reactions from students in Group B and the teacher, for this reason, enjoyed teaching it. This was evident from observation of the class, where students competed enthusiastically to create the funniestoorsstrangest word problems and vied with each other to read their problems aloud. The class was lively but fully cooperative and generally the stories did correspond to the given equations. Appendix F contains several examples of the students' work in response to the equations.

Students in this treatment were encouraged to use a variety of different approaches to the same mathematical equation. Apparently, they found the instructions dealing with different meanings of operations difficult to apply in creating their problems. Stories collected from the group at the end of the final two lessons do not show a great variety of mathematical meaning. It seemed that practically all of the efforts of the class were directed toward inventing unusual contexts for the problems, to the possible detriment of some of the less exciting purposes of this method.

Control Group

Group C was reported to have enjoyed doing independent work with problem solving, and students were quite receptive to the worksheets being handed out on each day.

Chapter 5

SUMMARY, CONCLUSIONS, AND IMPLICATIONS

The purposes of this study and the procedures involved are summarized prior to a discussion of conclusions which were drawn from the results described in Chapter 4. After some of the limitations of the study are noted, implications for the classroom and suggestions for future research are presented.

Summary

This study was undertaken to compare the effectiveness of two problem-solving strategies at the grade six and grade seven levels. Both strategies were designed to aid students in associating the verbal statement of a problem with its corresponding mathematical equation. One approach, the Translation Method, stressed literal, carefully structured translation of word problems, while the second, the Inductive Method, encouraged students to create their own problems, using mathematical equations given by the teacher. A control group was formed to ascertain whether sixth- and seventh-grade students would benefit as much from solving a sequence of word problems as they might from learning either of the two problem-solving approaches presented. This group practiced word problems without any instructional guidance for the duration of the treatment.

A review of literature indicated that boys were often superior to girls on measures of high cognitive skills such as problem solving. Therefore, the differential effect of the problem-solving approaches on male and female students

was examined, with the purpose of determining whether any interaction existed between learning strategy and sex.

Grades six and seven were chosen for the study because it was found that these students had not had previous instruction in problem solving strategies, and they were beginning to experience difficulty in this area. Possible differences between performance of grade six and grade seven students were considered in the final statistical analysis.

In order to determine whether superiority of one treatment would be evident through all levels of problem-solving ability, two criterion measures were used. Posttest One was composed of traditional word problems requiring only one mathematical operation for the correct solution. Posttest Two was constructed with novel or challenging word problems requiring more than one operation for the correct solution. Each test contained eight items and was designed for one forty-minute period.

For the purposes of the study, the sixth and seventh grades of a parochial elementary school in Vancouver were combined, thus enlarging the sample size. The fifty students were then assigned to three treatment groups on the basis of performance on a pretest in translation. The groups were later found not to be different on translation ability.

For a period of four school days, Group A was taught the Translation Method and Group B learned the Inductive Method using materials and worksheets created by the investigator. During this time, Group C practiced word problems without instruction. After the four days, Posttest One and later Posttest Two were administered to all students.

Scores of the tests were analyzed using multivariate analysis of variance for the two dependent measures. The three factors considered in the analysis were Treatment, Sex, and Grade. This resulted in a 3x2x2 factorial design. A simple main effects model was employed to examine the male-female differences within each treatment level.

Results

General results of the statistical analyses will be described in the order that they were first presented in Chapter 1, with a discussion of these results to follow.

Treatment-related research questions. Statistical comparisons among the groups offered no evidence of superiority for any one method over another. This means that no significant increase in problem-solving skill was found between students in the strategy-oriented Groups A and B as compared to the control group, Group C, and that neither the Inductive Method nor the Translation Method could be judged as superior to the other. In addition, no interaction was found between type of treatment and sex of the student.

Non-statistical comparisons, however, revealed differences in attitude among the groups. Students in Group B learning the Inductive Method were enthusiastic about creating their own problems. Students in the control group also appeared to enjoy their method of working independently on sequences of word problems. Students in Group A seemed uncomfortable with the restrictions imposed by the Translation Method. It became apparent from examination of their test papers that many students of this group did not translate properly, and quite a few

Group A pupils did not use the translation strategy altogether.

Sex-related research questions. Sex differences proved to be significant, with boys outperforming girls on each of the criterion measures. Further male-female comparisons were conducted within each of the treatment levels. Although the performance of the boys was consistently superior to that of the girls, these differences were found to be significant only for Posttest One scores of the students in Treatment A.

Grade-related research questions. No differences were found between performance of the grade seven students as compared with the grade six pupils' performance on either of the two word problem posttests. One finding of some interest which was not directly related to the questions which dealt with problem solving was discovered in the analysis of the Translation Pretest. Here Grade did appear as a significant factor, while Sex and Treatment were not found to be important in determining test scores. (Treatment groups were compared to insure that initially there were no significant differences among groups on translation skills.)

Conclusions and Discussion

Although no significant differences among treatments appeared in the statistical analysis of posttest results, there were strong indications that the Translation Method was not a viable strategy for students at a grade six or seven level. While students in the two other treatment groups seemed to enjoy their experiences with problem solving, pupils learning the Translation Method needed prompting to solve problems by translating. They seemed to feel that the strategy

was more of a burden than an aid to problem solving.

Examination of boys' performance as compared to girls' achievement supported this impression as well. In all three treatment groups, the scores of boys were consistently higher than the scores of girls, but significant differences appeared only in the Translation group. Here the differences on Posttest One scores were not caused by especially high scores on the part of the boys, but were attributable to particularly weak performance on the part of the girls. The significant differences between male and female achievement disappeared in Posttest Two results. Yet again, this was not caused by higher scores of the girls, but was rather caused by a marked decrease in scores of Group A boys as compared to boys of the other two treatments. It is reasonable to speculate that not only were the girls of Group A unaided by the Translation Method, but that boys of this group may have actually been hampered in their performance on a novel and challenging posttest by the method's formal structure.

Students of Group B who practiced problem solving using the Inductive Method did not fully understand the concept of varying meanings for mathematical operations in the word problems which they created. Yet they did seem to benefit from the treatment as far as attitude was concerned, and thus, the advantages of this method may be in a reduction of the tension with which students often approach word problems. While no objective evidence was obtained to support the Inductive Method over the Translation Method, the subjective response to the Inductive Method by both the teacher involved and the investigator was clearly positive.

The most unexpected conclusion which was drawn from this study regarded the benefits of a noninstructional sequence of practice word problems. Students of Group C who simply solved problems for four days were found to have enjoyed the experience. This may have been because they were motivated to complete the specified problems by the desire to proceed to the brain teasers and puzzles included in each day's worksheet. In addition, students in Group C scored as well as the other groups on the posttests. Concentrated practice in solving problems may be all that is really necessary to enhance problem-solving abilities for most students at the grade six or seven level.

It should be stressed that all findings are relevant to the sixth and seventh grades only and should not be generalized to higher grade levels. As Jarman and Mirman (1974) indicate, a large portion of the difficulty with word problems experienced by elementary school students is caused by the computational work involved, not by linguistic factors. After grade seven, however, linguistic considerations assume a much more important role as determiners of the complexity of word problems. It is then, perhaps, that students may better appreciate a translation strategy such as the one suggested by Dahmus.

The same may be true of any carefully structured problem-solving strategy. Sixth and seventh graders apparently do not perceive word problems as extremely complex statements, requiring the aid of formal strategies. The difficulties which they encounter in the area of problem solving are greater than the trouble that they find with computational work, but this is to be expected of problems which demand higher cognitive skills than computation. Yet it cannot be compared to

the uncertainty and fear with which many students in grades nine or ten react to complicated algebra problems. Therefore, rigorous strategies such as the Translation Method which confine and hinder the performance of grade six and seven students may prove quite helpful to students who are at a more advanced level.

In connection with the development of linguistic skills, it is interesting to note the difference between results of the word problem posttests and results of the Translation Pretest. While problem-solving activity was not affected by the difference between sixth-graders' ability as compared to seventh-graders', this was not the case with the translation skills. On the Translation Pretest, grade level was found to be the only significant consideration in the determination of test scores of the three factors examined. This finding indicates that the ability to solve problems does not vary from grade six to grade seven, and by possible extension from any one grade to the next, but translation skills do seem to improve as students mature.

No specific conclusions may be drawn in this respect, but one explanation may be offered. It is possible that translation ability is more of an acquired and learned skill than is problem-solving ability, and as such, the age of the student would more directly affect the results of a translation test than a problem-solving test. If this is true, then the research indications that translation ability is correlated with problem-solving ability suggest that improving problem-solving ability by teaching translation skills may still be a viable solution, despite the findings of this study.

Limitations

Several limitations of the study will be dealt with before implications for the classroom may be examined. The major limitation of this study was the small sample size involved. The 48 subjects were divided into levels of sex, grade, and treatment, and the 12 resulting cells averaged only four students per cell. In one case, a cell contained only two members. The assumption of homogeneity of variance within cells was consequently not supported for Posttest One. Since the final statistical analysis was based on this assumption, results should be viewed with caution.

The sample in turn was drawn from a Jewish parochial school, whose students may not be typical of the population as a whole. This should be considered before attempts are made to generalize from this study to all sixth- and seventh-grade pupils.

Lastly, the short duration of the treatment period should be noted. Students who are learning a strategy for solving word problems should ideally be exposed to the strategy over a long period of time. In this way, the method used would seem more of a tool to enable them to solve word problems, and less like a short-term topic to be studied as a separate entity and then perhaps forgotten.

Implications for the Classroom

The present study found no evidence of superiority for either of the problem-solving approaches when compared with the practice-only method on measures of achievement in word problem tests. This seems to indicate that there is little benefit to be derived from training grade six or seven students in literal translation

methods which are as restrictive as the Dahmus Translation Method. On the other hand, students may benefit from exposure to more creative approaches such as the Inductive Method in order to improve their attitude toward this difficult topic. Students of this age may not feel a direct need for any particular strategy for problem solving, particularly rigorous ones, and concentrated practice in solving problems may be the most practical suggestion for teachers. Other studies which have attempted to instruct students in problem-solving approaches without discovering any strong benefits support the possibility that solving problems may not be aided by using only one specific strategy. Teachers should also be aware that girls have more trouble with word problems than boys, and perhaps extra time should be devoted to their needs in this area.

Suggestions for Further Research

The findings of the present study suggest numerous research possibilities. Investigations of a translation method can be undertaken at a grade nine level or for older students, using a much larger sample of subjects, and hopefully over longer periods of time. The strategy need not necessarily be the one developed by Dahmus, which is quite formal in nature and may inhibit students' achievement rather than improve it. The benefit of such a method should be examined, particularly for students who are experiencing great difficulty with word problems and who have been found to be weak in translation skills.

The potential of the Inductive Method, which was explored only to a limited extent in this study, should be investigated from a number of aspects. Students of different ages should be exposed to this approach in order to determine whether it

can be of specific use to students who are having difficulty with word problems in algebra. It would also be interesting to find out whether students who learn this approach improve on measures of creative thinking and on measures of attitude. Subjects might be exposed to both a literal translation approach and a method such as the Inductive Method to determine whether the two approaches might be combined to produce an effective translation-oriented training process.

All such studies should include control groups in order to measure the relative effectiveness of a program of practicing word problems against the types of strategies being suggested.

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APPENDIX A

Word Problem Pilot Test and Results

APPENDIX A

NAME_____

GRADE_____

Word Problem Pilot Test

1. The average life span of man, 69 years, is three times the average life span of a horse. What is the average life span of a horse?

2. Dave now weighs 55 kilograms. He gained 6 kg during the past year. How much did he weigh at the beginning of the year?

3. The average distance from the earth to the sun is about 150,000,000 km. The average distance from the earth to the moon is about 384,000 km. How much farther is it from the earth to the sun than from the earth to the moon?

4. A hockey team has played all of its scheduled games. Altogether the team won 27 games, lost 15, and tied 8. How many games did the team play?

5. A wave as high as 35 metres has been reported. If each floor of a building is 5 metres tall, the wave would be as tall as a building with how many floors?

6. A plane flew at an average speed of 342 kilometres per hour 7 hours. How far did it fly?

Table A.1

SCORES OF STUDENTS ON WORD PROBLEM PILOT TEST

SUBJECT *	SCORE**	SUBJECT *	SCORE**
1	9	25	8
2	12	26	11
3	11	27	7
4	12	28	12
5	8	29	12
6	12	30	12
7	12	31	11
8	12	32	12
9	12	33	11
10	10	34	10
11	10	35	4
12	10	36	8
13	11	37	12
14	7	38	11
15	12	39	10
16	12	40	10
17	11	41	12
18	12	42	11
19	12	43	11
20	10	44	12
21	12	45	12
22	12	46	12
23	11	47	12
24	11	48	12

*of a total of 48 students who took the test
**with a maximum possible score of 12

APPENDIX B

Translation Pretest

APPENDIX B

Name: _____

Grade: _____

Translation Pretest

Translate the following phrases to mathematics. You will need to use letters instead of words, but the exact letter you choose is not important. Your answers should use symbols, numbers and letters.

EXAMPLE: the length multiplied by 5 L x 5

_____ 1. some pennies decreased by 8

_____ 2. 25 more than a number

_____ 3. the quotient of 4 and 32

_____ 4. 16 kilometres less than the distance

_____ 5. three times as old as Sue

_____ 6. the total cost of some oranges at 20¢ each

_____ 7. the temperature fell 4 degrees

_____ 8. the number of marbles increased by 75

_____ 9. kilometres per litre if Jack travelled 100 kilometres on N litres of gas

_____ 10. 19 divided by some number

_____ 11. the sum of Jack's score and 25

_____ 12. twice the side's length

_____ 13. 75 metres further than Victoria

_____ 14. all the cookies separated into three parts

_____ 15. Karen and Ann together have 10

_____ 16. one half the cost of a sandwich

Name_____

_____ 17. what remains of your money after spending $6.00

_____ 18. the difference between my age and yours

_____ 19. how many fives in the number

_____ 20. Bob has the same amount as Rob

_____ 21. his age five years ago

_____ 22. the product of side "a" and side "b"

_____ 23. 100 added to the product of side "a" and side "b"

_____ 24. Jack's age exceeds Robin's age by 4

_____ 25. 15 cm shorter than Jackie

_____ 26. lots of eggs: how many dozen?

_____ 27. the total length of four sides of a square

_____ 28. his car went as far as the horse

_____ 29. four inches longer than the first kind

_____ 30. triple some number divided by 10

APPENDIX C

Posttest One

APPENDIX C

Name_____

Grade_____

WORD PROBLEMS - POSTTEST ONE

DIRECTIONS: For each of the following problems, SHOW ALL WORK.

Write your final answer in the box provided.

1. The sugar maple tree sometimes grows to 125 metres tall. This is 5 times as tall as a fully grown dogwood tree. About how many metres tall is a fully grown dogwood tree?

1.

2. The regular flight from Vancouver to Toronto is $271. and the fare from Toronto to Montreal is $49. If Mr. Jackson wants to fly on the regular fare from Vancouver to Montreal with a stop in Toronto, how much will he pay for his total trip by plane?

2.

3. A delivery truck with a cargo weight of 1807 kg leaves on its delivery route. At the first delivery stop a package weighing 39 kg is unloaded. How much does the truck's cargo weigh when it leaves the first stop?

3.

Name_____

4. If your heart beats 72 times a minute, how many times does it beat in an hour?

4.

5. A fireman's ladder leaning against a building reaches a window of the building which is 18 metres high. If the roof is 23 metres above this window, how tall is the building?

5.

6. 75 students plan a two-day trip to Victoria. If each student must pay $7.00 to cover the cost of the trip, what are the total expenses for the class trip?

6.

7. A bus leaves the main bus depot for the town of Lawrence and continues on from there to the next stop at Cedarhurst, which is 50 km from the main depot. If Lawrence is 37 km from the depot, how far is it from Cedarhurst to Lawrence?

7.

8. Michael earns $88 per month at a part-time job. If he is interested in photography equipment which costs $550, how long will it take him to earn enough money to buy the equipment?

8.

APPENDIX D

Posttest Two

APPENDIX D

Teacher's Directions for Word Problem Posttest Two

The following are directions to be given to your students before they begin Word Problem Test #2:

Please read the problems carefully and do the best you can. You may find that these problems are harder than the ones you had on the first test, but remember that there will be heavy partial credit given if you have found a correct equation or the right process even if you couldn't finish the problem. For this reason, be careful to show all your work. If you need more room, you should use the back of the test paper.

To the teacher: In order to insure that each class takes the test under similar conditions, please note the following:

1. Except for clarifying typographical errors or simple reading uncertainty, do not give students additional aid or suggestions. (If students ask the meaning of the phrase "approximate age" in the "Mercury years" question, you may explain that any form of the answer is acceptable (- the actual answer does not require the fractional remainder, but this point is hard to express without revealing that the problem requires division)).)
2. Please have the students begin work as soon as possible, and do not let them take any extra time beyond the class period.
3. If students seem too anxious, remind them that the test is quite hard and that this fact will be taken into consideration.

Name _____

Grade _____ Group _____

Word Problem - Posttest Two

DIRECTIONS: Show ALL work for the following problems, since partial credit will be given. Correct solutions should be written in the box provided.

1. The sum of two numbers is 19. The product of the same two numbers is 60. What are the numbers?

1

2. A hamburger and a bag of potato chips cost 50¢. Two hamburgers and a bag of potato chips cost 85¢. What is the cost of a bag of potato chips?

2

3. Since it takes the planet Mercury 88 of our days to travel once around the sun, a "year" on Mercury is 88 of our days. If a person were 10 years old, give his approximate age in "Mercury years." (Use 365 days for an Earth year.)

3

Name_____

4. On Jim's paper route, there are three times as many weekly customers as Saturday-only customers. If his total number of customers is 72, how many are weekly customers?

4

5. Mr. and Mrs. Wengen decide to take a boat cruise with their young daughter. The regular adult fare for this cruise is $387 per person, but children may travel at a special fare. If the Wengen family pays a total of $1028 for the cruise, what is the special fare for children?

5

6. Jackie has some dresses. If she had twice as many dresses, she would have 6 more than Jean. Jean has 10 dresses. How many does Jackie have?

6

Name_____

7. The perimeter (total length of all the sides) of a square is 36. What is the area of the square?

7

8. A train leaves Vancouver for Calgary, traveling at 80 kilometres per hour. At the same time, another train also leaves Vancouver for Calgary, traveling at 72 kilometres per hour. After the trains have traveled for 5 hours, how far apart are the two trains?

8

APPENDIX E

Treatment A: Translation Method

TRANSLATION LESSON 1 - TRANSLATING PHRASES

Objectives: A. Students will recognize the phrases corresponding to particular operations (increased by → +), and

B. Students will condense long phrases into shorthand symbols (the length of a slide → S).

1. Introduction

Distribute "Code line" Worksheet (Worksheet #1) to students: allow them approximately 10-15 minutes to complete.

2. Procedure - classwork

A. Write on board, allowing plenty of space for each category:

+ - x ÷

"What answer did you get for the code? Which problems gave you the most trouble? Let's go down the list and write down the key words from each in the right column (indicate board)."

With the students, place each phrase in the appropriate column.

(List should look like Table 1 until line of asterisks.)

B. Take suggestions from class as to other possibilities for phrases which translate as +, -, x, ÷. (More are listed under the Table E.1 line of asterisks: those underlined are slightly more difficult than the others and should be emphasized. Otherwise the list should be copied exactly as is from Table E.1) Have students copy this table into their exercise books.

Shortening Long Phrases

Take out translation exercises and distribute to class.

"Now let's see if we can get all of these problems by translating them into mathematics as closely as we can."

(As each problem is reviewed, train students to condense the longer phrases such as "the length of a slide" into the shorter symbol "s". Try to de-emphasize the use of N for every case, since the use of many variable symbols in one expression, such as "Jack has as many marbles as Ben," lends itself more to representative symbols such as J = B.)

*At this point, add an EQUALS column (see below) to board for students to copy.

3. Seatwork

The following examples provide more practice in the translation of phrases. Keep encouraging the students to ignore - in the actual translating - the unnecessary words and explanations.

1. the number of pears decreased by 10 —— km (P - 10)
2. the distance from home to school is about 5 miles —— km. (D = 5)
3. After collecting money for the class trip, Jane found that she had $7.50. (J = 7.50)
4. There are only 25 arithmetic books in the classroom. (B = 25)
5. An astronaut can jump twice as high as the average person. (A = 2xP)

* *

EQUALS
"is"
as much as
as far as

TABLE E.1 : TRANSLATIONS

+	-	×	÷
GENERAL TERMS			
SUM	DIFFERENCE	PRODUCT	QUOTIENT
EXAMPLES			
8 more than: 8 +	7 less than: - 7	9 times: 9 x	36 divided by: 36 ÷
16 increased by: 16 +	740 reduced by: 740 -	9 multiplied by: 9 x	
76 added to: 78 +			
* * * * * * * *	* * * * * * * *	* * * * * * * *	* * * * * * * *
75 further than: 75 +	decreased by 8: - 8	twice the length: 2 x L	separated into 3: ÷ 3
25 taller than: 25 +	15 cm shorter: - 15	10 groups of 3: 10 x 3	how many dozen: ÷ 12
TYPES OF QUESTIONS ASKED			
What is the total?	What is the number remaining?	What is the total?	How many per ...?
How many altogether?	How many are left?	How many altogether?	How much for each?
How many in all?	How many more are needed?	How many in all?	How much for one ...?

TRANSLATION LESSON 2 - THE TRANSLATION STRATEGY

Objectives: Students will translate word problems directly from the English statement to the mathematical equation and solve them.

1. Introduction

Write on board: Fermez la fenêtre, s'il vous plaît.

Translate: Fermez = Close fenêtre = window

la = the s'il vous plaît = please

"Some of you think that math is also a foreign language. In a way, it is. Many words in English, like "added to" or "less than" need a special translation into math symbols. For other words we may use symbols such as a letter in the translation. Now we are going to learn a way to solve word problems by translating them almost straight from English. Let's take a simple example now. You may guess the answer before we finish, but remember the important thing in this example is how we do it."

On board: Paul is about 10 years older than Jane. If Jane is 15 years old, how old is Paul?

(This should be written straight across the board, with the translating of phrases being done directly underneath each appropriate phrase or operation description.)

SEE NEXT PAGE

2. Procedure - Classwork

Underline each phrase as it is being translated, so that students appreciate that the process is an orderly, phrase-by-phrase translation. Students may volunteer the answers for each step.

A. Paul is about 10 years older than Jane.

P = 10 + J

If Jane is 15 years old, how old is Paul?

J = 15 ? = P

"Now that we have translated the problem, what mathematical statements do we have?"

Write on board: TRANSLATED - P = 10 + J

J = 15

? = P

B. "The next thing we have to do is solve this problem. What do we want to find? What is the best way to solve this mathematical problem?" (Guide students toward the idea of solving by substitution of what is known, J = 15, into an equation that contains the unknown.)

Write on board: SOLVE - P = 10 + J

? = 10 + 15

? = 25 or Paul is 25 years old.

(Students should have TRANSLATED section and SOLVE section copied into their exercise books next to or under the word problem statement.)

"Now reread the problem, substituting the answer, and see if it makes sense. This is how we check the problem."

C. Distribute worksheet #2 for lesson 2 to the class.

"Let's do one example together before you try the others on your own." Write the first problem from the worksheet on the board, again leaving room between phrases and translating with the students in a phrase-by-phrase way.

The Great Pyramid of Egypt was originally 144 metres tall.

P = 144

The Great Pyramid was as tall as a building of how many stories

P = ? x S

if you use 4 metres per storey?

4 = S

(All translation and solving can be done in the space provided on the students' worksheets.)

TRANSLATED -	SOLVE -
P = 144	P = ? x S
P = ? x S	144 = ? x 4
4 = S	? = 144/4

The number of stories needed is 36.

"Don't forget to reread this problem with your answer to check it after you've finished."

3. Seatwork

Allow students time to do problems 2 and 3 of their worksheet on their own, and then review these. Do not start problem 4 unless there is enough time to insure that this can be reviewed too.

TRANSLATION LESSON 3 - MORE DIFFICULT PROBLEMS IN TRANSLATING

Objectives: Students should extend their ability to solve by translation to a variety of more difficult problems, such as problems involving formulas derived from the problem statement.

1. Introduction

Write on board:

25 jibbers grabbled at the wocky. If another 11 jibbers gribbled at the wocky, how many jibbers grabbled and gribbled altogether?

"Can you translate this problem and solve it the same way as yesterday's problems? What are the key words here?"

Students should arrive at the following setup:

TRANSLATED -		SOLVE -	
$J = 25$		$J + 11 = ?$	
$J + 11 = ?$		$25 + 11 = ?$	

Total jibbers were 36.

2. Procedure - Classwork

"Today we need to look at some kinds of problems which are a little harder to figure out than even the jibber problem. At your seats, try this next problem by yourselves first."

A. Dave now weighs 82 kg. He gained 9 kg during the past year. How much did he weigh at the beginning of the year?

Allow the students some time, and then guide them to see the following points:

1. As they read the problem initially, they should observe that the element of time is what is in the question: Before vs. After
2. All statements should be made in terms of WHEN, rather than in terms of WHO. Their setup should look like this:

TRANSLATED:	SOLVE:
N (now) = 82	B + 9 = N
B (before) + 9 = N	? + 9 = 82
? = B	? = 82 - 9

He weighed 73 kg at the beginning.

B. "There are also other types of examples where you will notice that there is a question of time involved, and you should pay careful attention to the way that they should be translated. Try this one for example." Sharon can read about 30 pages per hour. How long will it take her to read a book with 150 pages?

(Quite a few students should be able to guess the answer without really translating each phrase. If they volunteer the correct solution, have them come up to board and demonstrate their translation. If they have trouble, point out the following:)

1. Certain problems involve miles per hour, or km per hour, or, as in this example, pages per hour. This means that the more hours that are involved, the more miles travelled, pages read, etc.
2. A good way to handle this kind of "formula", using this problem as an example, is this:

Sharon can read about 30 pages per hour.

P (pages) = 30 per h

How long will it take her to read a book with 150 pages?

? = h P = 150

TRANSLATED: P = 30 per h.
? = h
P = 150

SOLVE: P = 30 x h
150 = 30 x ?
150 / 30 = ?

It will take her 5 hours.

3. Seatwork: Worksheet #3

Assign as many problems from the second worksheet, #3, as you feel can be finished and checked in class. Since the next lesson consists of all practice problems, any problems not covered in this lesson may be done in the next. Similarly, if there is extra time, problems from the next lesson may be borrowed.

TRANSLATION : LESSON 4 - PRACTICE AND REVIEW

Objectives: Students do enough word problems of different types to be thoroughly comfortable with the translation method.

1. Classwork - procedure

In this lesson, students should be given Worksheet #4 and Worksheet #4, cont'd., and asked to work the problems quietly, at their own pace, as a review for a test to be given the next day. Problems should be reviewed with class as a group toward the end of the period. Note that problems 6 and 7 in particular are word problems which involve two operations and students may need a little extra time for reviewing these.

WORKSHEET #1 GROUP A
(For Lesson 1)

TEACHER'S COPY WITH SOLUTIONS

C
L
O
I
D
N
E
E

DIRECTIONS:

FIRST, work any problem below and find the answer in the coded line at the bottom of the page.

SECOND, each time the answer appears in the code, you must write the letter of the problem above it.

KEEP DOING PROBLEMS UNTIL YOU HAVE DECODED THE LINE

D = THE SUM OF 12 AND 25 = 37
C = THE PRODUCT OF 9 AND 8 = 72
H = THE DIFFERENCE OF 18 AND 15 = 3
W = THE QUOTIENT OF 39 AND 39 = 1
M = 8 MORE THAN THE PRODUCT OF 5 AND 4 = 28
N = 9 TIMES THE DIFFERENCE OF 12 AND 10 = 18
E = 36 DIVIDED BY THE SUM OF 5 AND 1 = 6
L = 3 INCREASED BY THE QUOTIENT OF 10 AND 5 = 5
T = TWICE THE ANSWER WHEN 6 IS DIVIDED BY 3 = 4
O = 7 LESS THAN HALF OF 18 = 2
G = 21 REDUCED BY THE DIFFERENCE OF 9 AND 1 = 13
I = 4 GREATER THAN THE PRODUCT OF 3 AND 7 = 25
A = 20 DECREASED BY THE PRODUCT OF 3 AND 4 = 8
S = 6 MULTIPLIED BY THE DIFFERENCE OF 12 AND 10 = 12

****

A TIMELY TALE

****

A * W A T C H D O G * I S * A D O G *

8 1--8--4--72--3--37--2--13 25--12 8 37--2--13

T H A T * C A N * T E L L * T I M E *

4--3--8--4 72--8--18 4--6--4--5 4--25--28--6

WORKSHEET #2 - GROUP A

NAME _____

1. The Great Pyramid of Egypt was originally 144 metres tall. The Great Pyramid was as tall as a building of how many stories, if you use 4 metres per storey?

TRANSLATED SOLVE

2. Grandpa is 75 years old, and Jane is 20 years old. How much older than Jane is Grandpa?

TRANSLATED SOLVE

---

3. Generally, a person's arm is about three times as long as his hand. Mike's arm is 81 centimetres long. How long is his hand?

TRANSLATED SOLVE

4. 325 students are going on a field trip to see an experimental forest. The students are travelling by bus and each bus will hold 42 students. How many buses are required to carry all the students?

TRANSLATED SOLVE

WORKSHEET #3 - GROUP A

NAME _____

1. A teacher puts the desks in her room into rows. There are six rows. If there are 42 desks, how many desks are there in each row?

TRANSLATED SOLVE

2. The temperature dropped 20°C in one hour. At the beginning of the hour the temperature was 40°C. What was the temperature at the end of the hour?

TRANSLATED SOLVE

---

3. The average number of persons per square km in North America is 12. The area of North America is 24 400 900 square km. About what is the population of North America?

TRANSLATED SOLVE

4. Mr. Regan drives from his home at 8:00 in the morning and arrives at 11:00 A.M. at a business convention which is being held in a hotel. If the hotel is 170 km from Mr. Regan's home, how many km per hour does he average?

TRANSLATED SOLVE

WORKSHEET #4 - GROUP A

NAME _____

1. Mr. Jones bought a stove for $279 and a TV set for $349. How much more did he pay for the TV set than the stove?

TRANSLATED SOLVE

2. There are 100 bags of peas in a box. The grocer bought five boxes. How many bags of peas did he buy?

TRANSLATED SOLVE

---

3. The B.C. Lions scored two touchdowns and one field goal. They also scored an extra point after each touchdown. What was the Lions' total score? (A touchdown scores 6 points and a field goal is worth 3 points.)

TRANSLATED SOLVE

4. A noted baseball pitcher knows that his fastball is four times as fast as his screwball. His fastball travels at 120 km per hour. How fast does his screwball travel?

TRANSLATED SOLVE

WORKSHEET # 4 - GROUP A - cont'd.

5. A carpenter is fitting a door into a frame. The door is 1.7 metres wide. The door frame allows for a door 1.5 metres wide. By how much should the carpenter decrease the width of the door?

TRANSLATED SOLVE

6. Season tickets to all 32 basketball games cost $145. Individual tickets cost $5 each. How much do you save if you buy a season ticket?

TRANSLATED SOLVE

---

7. Two boys each had 60 cookies. One of the boys ate half of his cookies the first day and one third of the remaining cookies the next day. The other boy ate one third of his cookies the first day, and half of the remaining cookies on the next day. Who had the most cookies left?

TRANSLATED SOLVE

APPENDIX F

Treatment B: Inductive Method

INDUCTIVE LESSON 1 - SAYING IT DIFFERENTLY

Objectives: A. Students recognize that any mathematical relationship can be expressed in several different ways (Jack has twice as many = Jill has half as much)

B. The same mathematical relationship may exist in any number of different situations.

1. Introduction

Place two containers on the desk. (They may be milk containers, beakers, flour sacks, etc.) One should be visibly twice the size of the other.

"In your exercise books, write a sentence using numbers to describe this situation."

(Some students will write that one is twice the other, or holds twice as much as the second, or that one is half the other, etc.)

Have the students compare their answers, and elicit the general conclusion that (on board):

There are many different ways of describing the same mathematical relationship.

2. Procedure - classwork

A. As one more example, have one tall student and one shorter one come to the front of the room. Measure the difference in their heights, without explaining verbally what is being done, and write the number only on the board (10 cm, for example).

"Can you write a description of this situation in 3 ways?"
(A taller than B by 10 cm; B 10 cm shorter than A; If B were 10 cm taller, he/she would be as tall as A)

Have students compare answers to emphasize the variety possible.

Practice examples

On board, write the following three phrases. Students are asked to describe the same situation in at least two other ways.

1. the length of a caterpillar is half the length of a butterfly.

   (possible answers include: the butterfly is twice as long as the caterpillar; the length of the butterfly is double the length of the caterpillar; etc. Encourage creative answers.)

2. The train travels 80 kilometres per hour.

   (80 km for each hour; 80 km an hour; 80 km every hour; etc.)

3. There were 62 left.

   (62 remaining; there were 62 more earlier; the difference was 62; etc.)

B. Different possible contexts

"Let's stop now and take a look at that last example. We can now see that there are a few different ways of saying that there were 62 left without changing the basic mathematical relationship, but what about the real life situation that it is describing? There are 62 of what left? Can you give an example of a situation in which there might be 62 of something left?" Or: "If this is the answer to some word problem, what do you think the problem was?"

(To help students, write on board:)

After _____, there were 62 _____ remaining.

"How could you fill in these blanks?" (Students should be able to think of any number of situations to volunteer.)

(Possible answers: after the earthquake destroyed 1,000 buildings, there were 62 buildings left; how many were there originally?

or - after 10 students failed, 62 graduated; what was the class size?)

Practice examples

If there is enough time, have students do the following two or three examples to emphasize the fact that context possibilities are practically endless. Have students compare answers.

1. The _____ is 5_____more than the _____.

   (The new cigarette is 5 cm more than (or longer than) the old one, etc.)

2. Jack wants 2 _____, at _____ each.

   (Two suits at 125 dollars each, etc.)

3. The _____ is twice as _____ as the _____.

   (The distance from A to B is twice as far as the distance from B to C....)

INDUCTIVE LESSON 2 - CREATING WORD PROBLEMS

Objectives: Students write their own word problems to correspond to a given equation.

1. Introduction

Write on the board: $N = 10 + 15$

"Does this problem look a little easy for you? This part should be easy, but there is more to it than you can see right now. This describes a mathematical relationship in some situation like the ones we tried to think of yesterday. Can you think of a word problem or story problem that could correspond to this equation?"

(Give students some time to think of possibilities. If they do not understand what is expected, offer the following example:

There are 10 students working on a math problem. Fifteen more are doing an English project. How many students are there in the class?)

"Now write at least two word problems for this equation on the paper provided. Try to make them as different as possible."

2. Classwork - procedure

Please follow the following 4 steps for all the examples to come.

1. Have at least 4 or 5 students read one of their problems to the class.
2. After each problem is read, have the class decide whether it fits the equation. (Note that this will get a bit more complex in the next lesson.)

3. List the different contexts used in each problem on the side board. Do not erase this list; it can grow cumulatively through all the different examples (in this lesson) as an aid to the students in creating problems and to inspire further creativity. Students should be encouraged to add new situations with each example.

4. Without listing them on the board, verbally note the different possible descriptors of operations (such as 15 more than; a gain of 15; 15 miles further; etc.). Encourage students to vary these as much as possible.

- -

Write the following equation on the board; 144 = N x 4

"Write at least two word problems for this equation on the paper provided. Try to make them as different as possible."

FOLLOW THE 4 STEPS AS LISTED ABOVE. (Possible answer: If a field is planted with 144 tomato seedlings, with 4 rows in the field, how many seedlings are in each row? Or: 144 cookies in 4 packages, how many per package? etc.)

3. Seatwork

The following are really practice equations to be done as a group. After students write their problems for the first example, follow the four steps listed above, and then proceed to the next.

1. 75 - 20 = N
2. 81 = 3 x N
3. 325 ÷ 42 = N (Constantly encourage variety)

INDUCTIVE LESSON 3 - MEANINGS OF OPERATIONS

Objective: Students broaden the range of their created word problems by recognizing the different meanings of operations.

1. Introduction

Write on board: 25 + 11 = N

A. "Can anyone change this equation around without changing the value of N?"

(Elicit: 11 + 25 = N, or N = 11 + 25, or N = 25 + 11)

WRITE THIS (OR THESE) EQUATIONS UNDER THE FIRST, AND WRITE ABOVE THIS GROUP: ADDITION FORMS

B. "Can anyone write another equation, also using the same numbers and with N staying the same, that will look like a SUBTRACTION problem?"

(Elicit: 25 = N - 11 or 11 = N - 25)

WRITE THESE EQUATIONS ON THE OTHER SIDE OF THE BOARD, AND ABOVE THEM LABEL: SUBTRACTION FORMS

Draw a line under the equations of both addition and subtraction sides in order to continue the same process but with a different original equation.

Write on board: (on the addition side of the board) N + 5 = 8

"Can you write this equation in other addition forms? Can you write a subtraction form?" WRITE PROPER VARIATIONS ON APPROPRIATE SIDES.

(Answers: N = 8 - 5; 5 + N = 8)

CONCLUDE ON BOARD: Any addition equation can also be written as an equivalent subtraction equation.

2. Procedure - classwork

A. RETURN TO ORIGINAL PROBLEM: 25 + 11 = N

"In your paper provided, write a word problem for this equation which uses addition ideas." (If students seem confused; Have 25 pennies, found another 11, now have ? or other "sum" problems.)

Elicit examples from students, and write one typical word problem on board under the addition side of the equation.

B. "Now can you write a word problem using the subtraction form of the same equation?" (Example: (for 25 = N - 11) Had some money, lost 11¢, have 25¢ left, how much originally?)

After students have written their examples in their papers, elicit examples and write one on board.

CONCLUDE ON BOARD: We can write many different types of word problems for the same equation if we think of the equation as an addition problem AND as a subtraction problem.

Multiplication and division equivalence

Write on board: 150 = 30 x N

Follow the same procedure as above to elicit multiplication-division equivalence; i.e.,

	1. Multiplication forms	2. Division forms
ELICIT THESE EQUATIONS AND SAMPLE WORD PROBLEMS: WRITE THEM ON BOARD.	150 = 30 x N 150 = N x 30 (Coat costs $150; pay $30 per month for how months?)	N = 150 ÷ 30 (150 pencils to be divided among the students in a class; class has 30 students)

3. Seatwork

"Write the following equations in two possible forms just as we have done, and then write a word problem to fit each of the different forms. Try to make your problems as different and as interesting as possible."

*Note: some students may find it a bit difficult to write problems that correspond to different meanings of the same operation. Do not belabor the point: multiplication-division differences are especially hard to grasp.

1. $N + 9 = 82$
2. $42 \div 6 = N$
3. $24\ 400\ 900 \times 12 = N$
4. $N = 170 \div 3$

INDUCTIVE LESSON 4 - PRACTICE AND REVIEW

Objective: Students write word problems to correspond to a variety of different equations as practice.

1. Classwork - procedure

The following equations may be listed on the board as a group, with students working at least 2 word problems for every example at their own pace. Examples need not be shared and compared until 10 minutes before the end of the period, unless some students request an example or further explanation during their work on a difficult equation. Otherwise the examples should be gone over as a group when students seem to be finished.

EQUATIONS: with some suggested possibilities.

1. $N = 349 - 279$

2. $N = 100 \times 5$

3. $N = 2 \times 6 + 1 \times 3 + 2$

   (Total spent = two belts, one tie, and (belts @ 6.00 ea., ties @ 3.00 ea.,) lost two dollars)

4. $120 = 4 \times N$

5. $1.5 = 1.7 - N$

6. $A = 5 \times 32$ (Bus travels distance A for 5 hours @ 32 km per hour; If desired destination is only 145 km away, how much extra distance is covered by bus?)

   $N = A - 145$

7. I. $A = 60 \div 2$ II. $A = 60 - (60 \div 3)$

$B = A - (A \div 3)$ $B = A \div 2$

$N = IB - IIB$ OR $N = IIB - IB$?

(Suggest that I and II are two different possibilities, and ask which is better: for further hints add that A and B are like step 1 and step 2.)

Sample Word Problems Created by Group B Students

EQUATION: 24 400 900 X 12 = N

Sample Stories:

1. There are 24 400 900 people living in Canada. Twelve times this amount of people live in the whole world. How many people live in the world?

2. Some books were published. 24 400 900 people bought books, each person bought 12 books. How many books were bought?

EQUATION: N = 170 ÷ 3

Sample Stories:

1. 170 scars were made by "Too Tough" Blumsky. He only hit three people. How many scars did he give each person?

2. There were 170 beer cans in a liquor store. Three men came and bought them all out. Each man bought the same as the other two. How many beer cans did each man buy?

APPENDIX G

Treatment C: Control Method

GENERAL DIRECTIONS FOR TEACHER : GROUP C

1. Please record absentees on a daily basis.
2. Each day students are to receive a worksheet with word problems, and they are to complete as many of the problems as they can during the class period, on an independent basis. All worksheets should be turned in at the end of the period, whether finished or not. (Students should be reassured at the outset that they will not be penalized in any way for examples not completed; that these are practice problems to help raise their scores in word problem solving.)
3. Please distribute paper to the class as well and have students use this instead of their exercise notebooks. All work should be collected at the end of each class period.
4. During the last 10 minutes of class, please write the correct solutions to the problems on the board (these will be found in the teacher's copy of the worksheet for the day). <u>If students request further explanation,</u> the accompanying equation can then be written on the board. Otherwise equations need not be written. Verbally, the teacher should not say more than "If you got the correct solution, then you probably did the problem the right way. Otherwise, check your equation..." when questioned at the end of class.
5. If students request help during the class, they should first be reminded that all answers will be given at the end of class. If they insist, the teacher can tell them the proper equation to use on an individual basis, <u>without any further explanations</u>.

Name_____ Group C

Grade_____

Worksheet #1

DIRECTIONS: Please do as many of the following practice problems as you can on the separate paper provided. Answers will be given at the end of class.

1. Paul is about 10 years older than Jane. If Jane is 15 years old, how old is Paul?

2. The Great Pyramid of Egypt was originally 144 metres tall. The Great Pyramid was as tall as a building of how many stories, if you use 4 metres per storey?

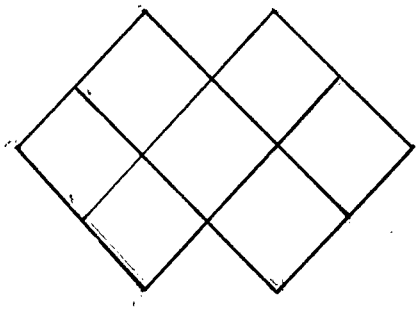
3. Grandpa is 75 years old, and Jane is 20 years old. How much older than Jane is Grandpa?

4. Generally, a person's arm is about three times as long as his hand. Mike's arm is 81 centimetres long. How long is his hand?

5. 325 students are going on a field trip to see an experimental forest. The students are travelling by bus and each bus will hold 42 students. How many buses are required to carry all the students?

6. 25 jibbers grabbled at the wocky. If another 11 jibbers gribbled at the wocky, how many jibbers gribbled and grabbled altogether?

IF YOU FINISH EARLY, YOU MAY TRY THIS PROBLEM: Try to put the numbers 1 to 7 in the seven diamonds below so that the numbers in each row of three diamonds add up to 10. Each number can be used only once.



Name _____ Group C

Grade _____

Worksheet #2

1. Dave now weighs 82 kg. He gained 9 kg during the past year. How much did he weigh at the beginning of the year?

2. Sharon can read about 30 pages per hour. How long will it take her to read a book with 150 pages?

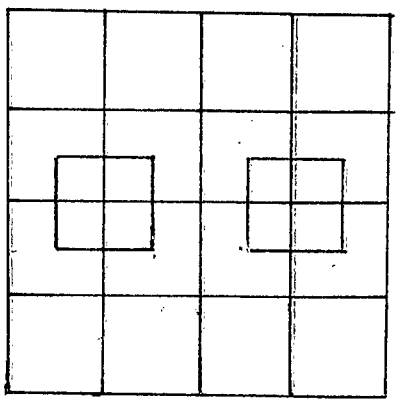
3. A teacher puts the desks in her room into rows. There are six rows. If there are 42 desks, how many desks are there in each row?

4. The temperature dropped 20°C in one hour. At the beginning of the hour, the temperature was 40°C. What was the temperature at the end of the hour?

5. The average number of persons per square km in North America is 12. The area of North America is 24 400 900 square km. About what is the population of North America?

6. Mr. Regan drives from his home at 8:00 in the morning and arrives at 11:00 A.M. at a business convention which is being held in a hotel. If the hotel is 170 km from Mr. Regan's home, how many km per hour does he average?

IF YOU FINISH EARLY, YOU MAY TRY THIS PROBLEM:
How many complete squares can you count in this drawing?



Name_____ Group C

Grade_____

Worksheet #3

1. Mr. Regan drives from his home at 8:00 in the morning and arrives at 11:00 A.M. at a business convention which is being held in a hotel. If the hotel is 170 km from Mr. Regan's home, how many km per hour does he average? (NOTE: If you completed this problem on the previous worksheet, continue to the next problem.)

2. Mr. Jones bought a stove for $279 and a TV set for $349. How much more did he pay for the TV set than the stove?

3. There are 100 bags of peas in a box. The grocer bought 5 boxes. How many bags of peas did he buy?

4. The B.C. Lions scored two touchdowns and one field goal. They also scored an extra point after each touchdown. What was the Lions' total score? (A touchdown scores 6 points and a field goal is worth 3 points.)

5. A noted baseball pitcher knows that his fastball is four times as fast as his screwball. His fastball travels at 120 km per hour. How fast does his screball travel?

IF YOU FINISH EARLY, YOU MAY TRY THIS PROBLEM:
In the following addition problem, the letters A, B, and C stand for three different digits. Figure out which digit each letter stands for.

$$\begin{array}{r} A\,A \\ +\;\;B\,B \\ \hline C\,B\,C \end{array}$$

Name_____ Group C

Grade_____

Worksheet #4

1. Jackie had 52 gerbils. All but 19 of them did not live past the first week. How many gerbils did she have left by the second week?

2. A carpenter is fitting a door into a frame. The door is 1.7 metres wide. The door frame allows for a door 1.5 metres wide. By how much should the carpenter decrease the width of the door?

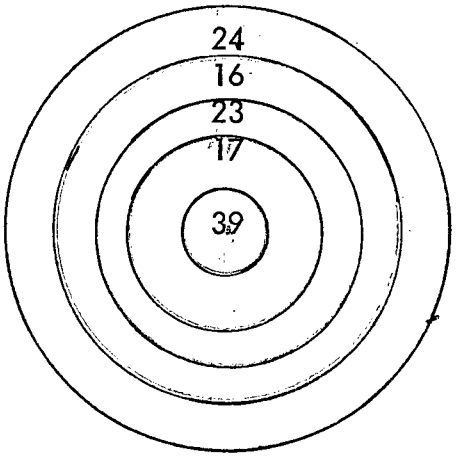
3. Season tickets to all 32 basketball games cost $145. Individual tickets cost $5 each. How much do you save if you buy a season ticket?

4. Two boys each had 60 cookies. One of the boys ate half of his cookies the first day and one third of the remaining cookies the next day. The other boy ate one third of his cookies the first day, and half of the remaining cookies on the next day. Who had the most cookies left?

IF YOU FINISH EARLY, YOU MAY TRY THE FOLLOWING PROBLEMS:

1. Ten pink socks and ten purple socks are all mixed up in a drawer. The twenty socks are exactly alike except for color. The room is pitch dark. How many socks do you need to take out to be sure you have a matching pair?

2. Shown below is a rather strangely numbered target. One day a marksman was practicing. He took six shots and made a score of exactly 100. Where did his shots land?



APPENDIX H

Item and Test Statistics

Table H.1

ITEM RELIABILITY STATISTICS FOR WORD PROBLEM PILOT TEST

Item	Point Value	Frequency*	Frequency as Percent	Biserial Coefficient of Correlation	Means on Total Test**
1	0[a]	4	8.3	-0.89	8.0
	1[b]	4	8.3	-0.73	8.5
	2[c]	40	83.3	0.98	11.3
2	0	2	4.2	-1.04	7.0
	1	0	0.0	0.0	0.0
	2	46	95.8	0.87	11.0
3	0	4	8.3	-1.21	7.0
	1	13	27.1	-0.16	10.5
	2	31	64.6	0.64	11.4
4	0	3	6.3	-0.73	8.3
	1	1	2.1	-0.19	10.0
	2	44	91.7	0.61	11.0
5	0	2	4.2	-0.90	7.5
	1	1	2.1	-0.68	8.0
	2	45	93.8	0.84	11.0
6	0	3	6.3	-1.33	6.3
	1	3	6.3	-0.04	10.7
	2	42	87.5	0.79	11.1

*of a total of 48 students who took the test

**with a maximum possible score of 12

[a]represents scoring of incorrect solution with no partial credit given

[b]represents scoring when partial credit was given

[c]represents scoring of fully correct solution

Table H.2

ITEM RELIABILITY STATISTICS FOR THE TRANSLATION PRETEST

Item	Point Value	Frequency*	Frequency as Percent	Biserial Coefficient of Correlation	Means on Total Test**
1	0[a]	13	26.5	-0.87	31.0
	1[b]	0	0.0	0.0	0.0
	2[c]	36	73.5	0.87	47.3
2	0	5	10.2	-0.55	32.2
	1	2	4.1	-0.23	37.5
	2	42	85.7	0.51	44.5
3	0	13	26.5	-0.76	32.5
	1	0	0.0	0.0	0.0
	2	36	73.5	0.76	46.7
4	0	17	34.7	-0.61	35.7
	1	2	4.1	0.19	47.5
	2	30	61.2	0.55	46.7
5	0	1	2.0	-0.22	37.0
	1	1	2.0	0.11	46.0
	2	47	95.9	0.05	43.0
6	0	11	22.4	-0.66	33.2
	1	5	10.2	0.05	44.0
	2	33	67.3	0.52	46.0
7	0	5	10.2	-0.60	31.4
	1	1	2.0	-0.11	40.0
	2	43	87.8	0.53	44.4
8	0	6	12.2	-0.69	30.3
	1	0	0.0	0.0	0.0
	2	43	87.8	0.66	44.7
9	0	29	59.2	-0.46	39.6
	1	2	4.1	0.02	43.5
	2	18	36.7	0.46	48.2
10	0	9	18.4	-0.39	36.7
	1	1	2.0	-0.71	24.0
	2	39	79.6	0.48	44.9
11	0	6	12.2	-0.67	30.7
	1	3	6.1	-0.41	34.0
	2	40	81.6	0.69	45.5
12	0	4	8.2	-0.22	38.5
	1	0	0.0	0.0	0.0
	2	45	91.8	0.20	43.3
13	0	6	12.2	-0.53	33.2
	1	2	4.1	0.07	44.5
	2	41	83.7	0.41	44.3

Table H.2 - cont'd.

Item	Point Value	Frequency*	Frequency as Percent	Biserial Coefficient of Correlation	Means on Total Test**
14	0	10	20.4	-0.70	32.2
	1	2	4.1	-0.17	39.0
	2	37	75.5	0.67	46.1
15	0	17	34.7	-0.57	36.2
	1	6	12.2	-0.11	41.0
	2	26	53.1	0.58	47.8
16	0	18	36.7	-0.52	37.0
	1	0	0.0	0.0	0.0
	2	31	63.3	0.52	46.4
17	0	18	36.7	-0.31	39.4
	1	0	0.0	0.0	0.0
	2	31	63.3	0.31	45.0
18	0	7	14.3	-0.97	26.0
	1	11	22.4	-0.02	42.6
	2	31	63.3	0.59	46.9
19	0	16	32.7	-0.78	33.3
	1	1	2.0	0.04	44.0
	2	32	65.3	0.76	47.7
20	0	10	20.4	-0.72	31.8
	1	8	16.3	-0.18	39.9
	2	31	63.3	0.66	47.3
21	0	10	20.4	-0.83	30.1
	1	1	2.0	-0.48	30.0
	2	38	77.6	0.86	46.7
22	0	15	30.6	-0.73	33.7
	1	4	8.2	0.06	44.3
	2	30	61.2	0.64	47.4
23	0	1	2.0	-0.01	16.0
	1	18	36.7	-0.58	36.3
	2	30	61.2	0.70	47.8
24	0	34	69.4	-0.33	41.1
	1	2	4.1	-0.17	39.0
	2	13	26.5	0.40	48.5
25	0	11	22.4	-0.62	33.7
	1	6	12.2	-0.03	42.3
	2	32	65.3	0.52	46.2
26	0	19	38.8	-0.49	37.6
	1	5	10.2	0.13	45.4
	2	25	51.0	0.41	46.5

Table H.2 - cont'd.

Item	Point Value	Frequency*	Frequency as Percent	Biserial Coefficient of Correlation	Means on Total Test**
27	0	16	32.7	-0.75	33.8
	1	5	10.2	-0.02	42.6
	2	28	57.1	0.76	48.3
28	0	17	34.7	-0.75	34.1
	1	1	2.0	0.15	47.0
	2	31	63.3	0.72	47.7
29	0	17	34.7	-0.68	34.9
	1	2	4.1	-0.29	36.0
	2	30	61.2	0.72	48.0
30	0	7	14.3	-0.76	29.6
	1	9	18.4	-0.40	36.4
	2	33	67.3	0.77	47.6

*of a total of 49 students who took the test
**with a maximum possible score of 60
[a]represents scoring of incorrect solution with no partial credit given
[b]represents scoring when partial credit was given
[c]represents scoring of fully correct solution

Table H.3

ITEM RELIABILITY STATISTICS FOR POSTTEST ONE

Item	Point Value	Frequency*	Frequency as Percent	Biserial Coefficient of Correlation	Means on Total Test**
1	0[a]	8	16.3	-1.03	14.3
	1[b]	1	2.0	0.06	21.0
	2[b]	3	6.1	-0.10	19.7
	3[c]	37	75.5	0.83	21.9
2	0	0	0.0	0.0	0.0
	1	1	2.0	-1.50	6.0
	2	1	2.0	-0.15	19.0
	3	47	95.9	0.77	20.8
3	0	3	6.1	-0.86	13.7
	1	0	0.0	0.0	0.0
	2	3	6.1	-0.35	17.7
	3	43	87.8	0.70	21.1
4	0	1	2.0	-1.29	8.0
	1	1	2.0	-0.36	17.0
	2	5	10.2	-0.33	18.2
	3	42	85.7	0.60	21.1
5	0	4	8.2	-1.31	10.8
	1	0	0.0	0.0	0.0
	2	1	2.0	0.16	22.0
	3	44	89.8	1.02	21.3
6	0	0	0.0	0.0	0.0
	1	1	2.0	-1.50	6.0
	2	8	16.3	-0.55	17.1
	3	40	81.6	0.78	21.5
7	0	16	32.7	-0.90	16.5
	1	0	0.0	0.0	0.0
	2	0	0.0	0.0	0.0
	3	33	67.3	0.90	22.4
8	0	4	8.2	-1.01	13.0
	1	7	14.3	-0.32	18.4
	2	22	44.9	0.02	20.6
	3	16	32.7	0.60	23.1

*of a total of 49 students who took the test

**with a maximum possible score of 24

[a]represents scoring of incorrect solution with no partial credit given

[b]represents scoring when partial credit was given

[c]represents scoring of fully correct solution.

Table H.4

ITEM RELIABILITY STATISTICS FOR POSTTEST TWO

Item	Point Value	Frequency*	Frequency as Percent	Biserial Coefficient of Correlation	Means on Total Test**
1	0[a]	13	26.5	-0.97	10.2
	3[b]	0	0.0	0.0	0.0
	4[c]	36	73.5	0.97	23.2
2	0	10	20.4	-0.91	9.6
	3	1	2.0	-0.04	19.0
	4	38	77.6	0.87	22.4
3	0	23	46.9	-0.78	14.4
	3	1	2.0	-0.04	19.0
	4	25	51.0	0.79	24.6
4	0	43	87.9	-0.73	18.4
	3	0	0.0	0.0	0.0
	4	6	12.2	0.75	29.7
5	0	5	10.2	-1.02	5.6
	3	1	2.0	-0.09	18.0
	4	43	87.8	0.88	21.4
6	0	9	18.4	-0.90	9.3
	3	0	0.0	0.0	0.0
	4	40	81.6	0.89	22.1
7	0	33	67.3	-0.83	16.2
	3	0	0.0	0.0	0.0
	4	16	32.7	0.83	27.1
8	0	12	24.5	-0.80	11.6
	3	6	12.2	0.39	24.8
	4	31	63.3	0.46	21.9

*of a total of 49 students who took the test
**with a maximum possible score of 32
[a]represents scoring of incorrect solution with no partial credit given
[b]represents scoring when partial credit was given
[c]represents scoring of fully correct solution

APPENDIX I

Summary of Data

APPENDIX I

Table I.1

SUMMARY DATA FOR THREE TESTS

Subject	Sex	Grade	Treatment Group	Score on Translation Pretest	Posttest One	Posttest Two
1	M	6	A	30	21	0
2	F	6	A	44	19	20
3	M	7	A	57	23	27
4	F	6	A	47	6	4
5	M	6	A	46	24	19
6	M	6	A	36	24	24
7	F	7	A	40	11	4
8	M	7	A	59	22	24
9	M	7	A	39	24	16
10	M	6	A	39	24	23
11	F	7	A	54	21	23
12	F	7	A	49	19	28
13	F	6	A	22	18	12
14	F	7	A	52	23	20
15	M	7	A	45	20	16
16	M	6	B	53	20	24
17	F	7	B	45	20	28
18	M	7	B	57	24	31
19	F	6	B	34	17	12
20	M	7	B	47	23	20
21	M	7	B	51	24	28
22	F	6	B	56	22	28
23	M	6	B	50	22	24
24	F	7	B	23	18	0
25	M	6	B	20	21	20
26	F	6	B	24	20	4
27	M	6	B	42	23	28
28	F	6	B	37	17	16
29	F	7	B	57	21	28
30	M	7	B	32	17	16
31	M	6	B	44	21	20
32	M	7	C	47	23	20
33	M	6	C	43	24	28
34	F	6	C	16	8	8
35	M	6	C	40	15	18
36	F	6	C	52	24	16

SUMMARY DATA FOR THREE TESTS - continued

Subject	Sex	Grade	Treatment Group	Score on Translation Pretest	Score on Posttest One	Score on Posttest Two
37	M	6	C	32	19	12
38	M	7	C	49	23	32
39	F	6	C	45	16	20
40	M	7	C	58	23	32
41	M	7	C	49	23	27
42	F	7	C	54	24	28
43	M	7	C	42	23	20
44	M	6	C	44	24	20
45	F	7	C	48	24	16
46	M	6	C	44	24	24
47	M	7	C	49	18	20
48	F	6	C	14	19	20