# THE ROLE OF A NEW TEXTBOOK IN THE TEACHING AND LEARNING OF ALGEBRA 9 FOR ENGLISH AS SECOND LANGUAGE STUDENTS

By

Esther Tello

B.Ed The National University of Cordoba

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF ARTS

In

THE FACULTY OF GRADUATE STUDIES

Department of Curriculum Studies

We accept this thesis as conforming

to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

June 2004

# Library Authorization

In presenting this thesis in partial fulfillment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Esther Tello — 29 June 2004

Name of Author *(please print)* — Date (dd/mm/yyyy)

Title of Thesis: The Role of a New Textbook in the Teaching and Learning of Algebra for English as Second Language Students

Degree: MA — Year: 2004

Department of Curriculum Studies
The University of British Columbia
Vancouver, BC Canada

# ABSTRACT

The purpose of this study is to explore the role a new textbook plays in facilitating the teaching and learning of Algebra 9 for English as a Second Language students. The research constitutes a case study based on fifteen ESL students in a mathematics grade 9 class.

Data was collected using a qualitative research methodology. Relevant class events and student interview outcomes that contributed to answering the research questions constituted the core of the raw data. Data included a demographic questionnaire, pre and post-tests, a teacher professional journal, and post-lesson interviews with students.

The information obtained was analyzed and synthesized through the constant comparative method. The patterns that emerged from the analysis allowed me to categorize them into three fundamental areas of significance: adaptations that aimed to increase text reading comprehension, adaptations that aimed to increase the understanding of algebraic processes, and assignment of textbook exercises that promoted accurate algebraic procedural skills. These three main categories illustrate the way the textbook was used and modified by me in the classroom. The results revealed that the textbook facilitated more the development of algebraic procedural skills than the conceptual understanding and mathematical language acquisition.

This research has implications for mathematics instructors in adapting textbook algebraic activities for ESL students. In addition, this study offers educators suggestions for assessing and adapting algebraic textbook activities to maximize the teaching of

Mathematics 9 for all students. Moreover, the results of this study suggest editors activities to enhance the quality of a textbook to facilitate the learning outcomes expected from ESL students who read and study their textbooks. The results of this case study warrant further research into the comprehensive approach that textbooks take in consideration of ESL students.

# TABLE OF CONTENTS

# LIST OF TABLES

# ACKNOWLEDGEMENTS

Many people made this research possible. Without their contributions, this study would have not taken place.

I would like to thank my thesis advisor, Dr. Lyndon Martin, for taking the risk to guide me through a process of critical and independent thinking during the stages of this study. Thanks also for his willingness of sharing his expertise with me. His advice and academic insights contributed to shape this study.

Thanks to Dr. Cynthia Nicol who sat with me to provide me with critical feedback. She contributed to enhance this study and her help proved invaluable. Her expertise improved the quality of this research.

I would also like to express my sincere appreciation to Dr. Samson Nashon for his valuable suggestions regarding this study. His contributions facilitated the construction of the research as a whole.

Thanks to my students for contributing to the research assessment process. Without their willingness to share part of their time and learning with me, this study would not have been possible.

I owe special thanks to my family for their patient and emotional support. They allowed me to spend the quality time I needed to reflect on this study and its implications.

# CHAPTER ONE

# INTRODUCTION

This research was a case study in which a new Mathematics 9 textbook was used for the first time in an ESL (English as a Second Language) class. The goal of this study was to explore the role of the new textbook in the teaching and learning of Mathematics 9 for ESL students.

At the school where I teach, there are two levels of mathematics ESL classes: ESL Mathematics Level One in which students learn Mathematics 8, and ESL Mathematics Level Two in which students learn Mathematics 9. The goal of these ESL mathematics courses is to prepare students with English as a second language to succeed in mainstream mathematics 10 or higher-grade courses.

Students are placed in either of these classes according to their linguistic abilities and mathematical skills even if their mathematical skills are higher than grade 9. To be placed in ESL Mathematics Level Two, students must have acquired an intermediate level of English language and have mastered a minimum of grade 8 mathematical skills.

I teach ESL mathematics classes as well as regular mathematics courses. In ESL mathematics courses, I implement a language-content approach where students are first introduced to the language features they need to know to be able to learn the necessary mathematical concepts later. In this study, the teacher, myself, was the researcher and the participants were my ESL mathematics 9 students.

## Significance of the Problem

In my four years of teaching mathematics to ESL students, I have identified a number of learning patterns that I believe deserve the attention of educational

researchers. ESL students cannot always learn Mathematics using the activities suggested in mathematics textbooks. Sometimes, reading comprehension problems arise as a result of students' poorly developed vocabulary, and they are unable to understand the textbook. At other times, they understand the meaning of words but struggle with the comprehension of mathematical concepts and skills.

In this study, I have selected to examine the topic of algebra which is chapter two in the textbook. From my personal experience, algebra is one of the most complex themes for ESL students to learn and for mainstream students as well. The topic of algebra at the secondary school level is loaded with complex sentences and problem solving exercises. Once students overcome their language barrier and understand what they read, algebraic symbolic sense and conceptual understanding are the next challenges for students to overcome, challenges that not every student can meet. In other words, ESL students need to be introduced not just to the new English vocabulary but also to the new mathematical terminology so they can communicate with their teacher, among themselves and interact with the textbook.

At my school, a mathematics textbook, Minds on Math 9 (1996), was used in mainstream mathematics classes for the first time this year, and I wanted to know how appropriate the textbook was for ESL students and what changes I needed to make to it as I used it. In my experience, ESL students are better integrated into mainstream Mathematics courses when they have been familiarized previously with textbooks that have the same structure and the same approach to the learning of Mathematics in the textbooks they will use in mainstream courses. The mathematics department head teacher of the school decided to replace the old mathematics 9 textbook because she did

not find this book aligned with the latest mathematics standards and after assessing two different textbooks she decided to use Minds on Math 9 (1996). She thought this textbook had a sufficient number of exercises to practice and the word problems had an appropriate level of difficulty.

Until this year, I have based most of my instruction on the old Mathematics 9 (1987) textbook, but I have had to spend a considerable amount of time adapting the learning activities suggested in the textbook to suit ESL students' learning needs. As a consequence of adapting textbook activities and spending a considerable amount of time implementing them, I have been able to cover barely fifty per cent of the Mathematics 9 textbook before students are transferred to Mathematics 10 the following year.

Nelson (1962) encourages teachers to evaluate curricular resources on the basis of their classroom experience to identify textbooks' weaknesses and strengths to be able to professionally select the best curricular resource for each situation. It is my experience that an effective educational resource to teach Mathematics 9 for ESL students would be one that introduces new mathematical vocabulary and concepts by bridging students' previous mathematical literacy and abilities with current mathematical 9 concepts and skills. In addition, I would like to be able to teach a minimum of 70% of the grade 9 curriculum by the end of the course because students will be promoted to mainstream Mathematics 10 the year after.

The purpose of this study was to determine to what extent and in what ways chapter two of algebra in a new Mathematics 9 textbook facilitates the teaching and learning of algebraic concepts and skills in an ESL class. Chapter two introduces the following mathematics contents:

The Unit: Chapter Two - Algebraic Operations and Equations

Table of Contents



Figure 1.1

I taught the above sections in the same order the textbook suggests because I considered that the sequence of the contents the textbook offers gradually introduces and connects the main algebraic concepts taught trough this chapter.

It is hoped that this study will provide ESL mathematics teachers as well as mainstream teachers with some of the support they need to determine which educational resource help their students to learn the expected mathematics concepts and skills and what teaching strategies can reinforce the learning of algebra 9 for ESL students.

# CHAPTER TWO

# LITERATURE REVIEW

This chapter contains a selective integrative review of the role of mathematics textbooks in the classroom, the teaching of algebra for understanding, and the instruction of mathematics with a content-based approach for ESL students.

## The Importance of Textbooks for All Students

The method of teaching and learning mathematics changes as the needs of society and the needs of individuals evolve. An increased language diversity in classrooms (NCTM, News Bulletin, V39, I2, 2002) and future algebra demands in post secondary entry level exams (Huntley, 2000) invite researchers to examine textbooks as the main curricular resource in order to determine their effectiveness in student being able to learn mathematics (Kang & Kilpatrick, 1992).

Most mathematics educators are experiencing a world that is increasingly interconnected (Kimball in NCTM News Bulletin, 1990, p. 604; Tate, 1997). Moreover, Silver and Kilpatrick (1994) and the current president of the NCTM Johnny Lott reported that most scholars predict an increasing diversity in classrooms regarding ethnicity and language in North America and in most countries around the world (NCTM, 2002, I1). An increase of diversity in the classroom challenges educational practitioners to provide all students with the same learning opportunities regardless of their backgrounds and languages (D' Ambrosio, 1994; NCTM, 2000).

To build an equal environment in mathematics education, educational practitioners should place high expectations on all students regardless of their specific characteristics or background (D'Ambrosio, 1994).

Educators should consider selecting curricular materials that will bring students closer to the standards expected in mathematics, "especially from students with language barriers who are very often victims of low expectations" (NCTM, P and S for M, 2000, p.1). Mathematical standards describe the mathematical contents and processes that students should learn and constitute a guideline for educators to strive for the improvement of mathematics (NCTM, 2000, p. 11). There is a need for a better integration of ESL mathematics students into the mainstream, and Stiff (2001) encouraged educational practitioners to include teaching strategies and resources that include underserved students such as ESL students across the US and Canada (NCTM, 2001).

In Mathematics, algebra contains specific vocabulary with variations in structure. For ESL students, algebra is sometimes one of the most difficult subjects to learn. Nevertheless, algebra is an important topic to master. Huntley (2000) and Musik (2002) noted that algebra is one of the most relevant themes in the secondary mathematics curricula because it is one of the most important foundations for higher mathematical and scientific studies.

Huntley encouraged educators to research new approaches to teaching algebra to better enable their students to meet future educational demands in mathematics. For example, in 2005, some post secondary educational institutions in North America will have higher expectations of students on entry-level exams such as the SAT, especially in advanced algebra and trigonometry (NCTM, News Bulletin, I2, 2002).

The teaching of algebraic concepts depends primarily on the main educational resources used and on the way educational professionals implement them. The

adaptations implemented often depend on the teacher's assessment of the textbook and the teacher's experience and knowledge of the subject.

Since textbooks are the primary resource used to teach the mathematics curriculum, it is important that educational practitioners examine them carefully and closely to determine what content is emphasized and which processes are suggested (Kang & Kilpatrick, 1992; Chandler, 1995; Pizzini, 1995; NCTM: P and S for SM, 2000).

Sometimes students fail to understand the mathematics in their textbooks because the algebraic concepts are not introduced properly. For example, textbooks should introduce new ideas based on old ones (Bruner 1966; Hativa, 1985). In addition, the sequence, timing, and the complexity of mathematical tasks should be appropriate to the developmental stage of students. According to Hativa (1985) a good sequence of topics reflects a shift, moving from the familiar to the unfamiliar, from the easy to the difficult, and from the concrete to the abstract.

## Teaching Algebra for Understanding

In high school, the teaching of algebra is framed into three main contemporary approaches that provide teachers with guidelines to facilitate the teaching of algebra: conceptual understanding, procedural knowledge, and problem solving. (Musik, 2000)

Conceptual understanding constitutes the most contemporary approach to the teaching of algebra as opposed to the procedural approach, which was the main core of traditional algebraic curricula.

The Principles and Standards for School Mathematics (2000) and the Mathematics Framework for the 2003 National Assessment of Educational

Progress (2002) emphasized the importance of teaching mathematics for understanding, promoting the use of symbolic algebra to express relationships, the application of properties, and the interpretation of the results of calculations.

Procedural knowledge involves the mastery of symbolic manipulation from words to algebraic equations and the understanding of the use of mathematical properties to calculate the results (Murphy, 2002). Procedural knowledge involves the ability to select and to apply appropriate algorithms to achieve a numerical result.

The balance between conceptual and procedural knowledge is a challenging idea. (Huntley, et. el., 2000) Those who are in favor of symbolic-calculation argue that mastery of symbolic manipulation is essential for problem solving and mathematical learning. Proponents of a curricular change for less emphasis on symbolic-calculation argue, "students with moderate symbol-manipulation are able to achieve high levels of mathematical representation for problem situations and on tasks that require interpretation of calculated results" (Huntley, 2000, p. 357).

Whatever pedagogical approach educators implement, NCTM (2000) invited teachers to take a moderate approach to balance procedural knowledge and conceptual understanding and to select educational resources accordingly. NCTM (2000) claimed that developing a concept for the properties of numbers can also help students solve problems and understand the operations to number systems that are new to them.

Steinberg (1990) claimed that, often, students lack understanding in basic algebraic concepts such as equivalence. He encouraged editors to include more exercises on equivalent equations in textbooks and to emphasize that equivalent equations refers to those equations where the left side of the equation has the same value as the right side.

Steinberg found that many students who are able to solve equations and to apply the properties of equivalence had a lack of understanding of algebraic concepts. He suggested educators not only teach equivalence as part of the procedural methodology, but also that they teach the concept of equivalence per-se.

Steinberg developed an instrument to assess conceptual understanding of equivalent equations based on twenty-one pairs of equations where students were asked to decide whether the equations were equivalent or not and to justify their answers. The list of exercises provides educators with guidelines to teach students about conceptual understanding in their classrooms and Steinberg encouraged educators to use it to supplement textbook activities.

Other researchers recommend the use of heuristic approaches in teaching algebraic concepts, to increase conceptual understanding in this area (Bransford, Brown, & Cocking 1999, cited in NCTM P and S for S M, 2000; Hunthley, et. el., 2000; Piaget, 1973). Educators and textbooks offer different options in approaching similar concepts, exercises and problems, facilitate the development of student content knowledge and understanding. However, it is important that educators require pupils to reflect on their answers and verify their strategies to determine if they are suitable in each case.

Similarly, Pirie and Kieren (1994), Pirie and Martin (1997), and Kang and Kilpatrick (1992) emphasized the importance of mathematical understanding through processes that move students to higher levels of mathematical abstraction as opposed to transitory didactic approaches that may help students reach a temporary answer intuitively but prevent them from acquiring the symbolic, stable representation of the problem and as a consequence, preparing them for further mathematical knowledge.

Kilpatrick (1992) defined didactic transposition of knowledge as a device used to communicate abstract knowledge in an organized, connected, and instructional way. In textbooks, knowledge has to be presented and written down. Students have to comprehend the main concepts in order to incorporate them into their own knowledge for future application and retainment. A didactic transposition may have different forms such as a simple narration, a rigorous numerical formula or the use of a mnemonic system. However, Kilpatrick (1992) warned educators to be "epistemologically vigilant about some pathological mathematical didactic transpositions and aware of their educational limitations" (p.3). He claimed that some didactic approaches may promote very different meanings from that of the original. Some didactic transpositions may detract from conceptual thinking and tend to create in students transitory understanding that cannot be transferred to further mathematical abstraction. Textbooks sometimes communicate knowledge in a format that can be discarded after mastering the skills.

For example, in algebra, Pirie and Martin (1997) observed that often, linear equations and the concept of equivalence are taught with pictures of a balance and weights. They found that this didactic transposition, as defined by Kilpatrick (1992), provides a poor strategy to promote conceptual understanding of linear equations because the physical representation of a balance and weights may mislead students to relate the principle of equivalence with a center of gravity, positive numbers with physical weights or objects, and negative numbers with meaningless pictorial representations of weights. In other words, to understand the concept of linear equations, it is important that students are not mislead into a process of weighing objects after objects orderly. Students need to comprehend linear equations as a symbolic "whole" to relate letters to unknown values

rather than to physical objects. Relating linear equations to the physical world limits the scope of possibilities for solving them and the level of abstraction students can reach.

Regardless of the contents textbooks emphasize, conceptual understanding should be one of the main goals in the teaching of algebra (Huntley, et. el, 2000; Musick, 2002; NCTM, 2000). Huntley et. el (2000) compared the effects of the curriculum designed to implement the NCTM "standards" to that of more conventional curricula. He found that if students are frequently asked to formulate mathematical models and to interpret results of algebraic calculations, they develop greater understanding of the steps and skills in those processes (2000, p. 354). He has also found that students who calculate algebraic equations in the context of meaningful problem settings develop skills in using those situations as guides to their formal algebra.

To promote understanding, educators should facilitate the development of algebraic concepts through a process of cognitive inquiry. Costa classified the cognitive level of questions in three groups: input, processing, and output questions. Input questions require students to recall information. Processing questions ask students to relate the recalled data to concrete situations in which students have to sequence, compare, contrast, and classify processing questions. Output questions require students to apply the data in new ways: to hypothesize, to speculate, to generalize, to create, and to evaluate (1985, cited in Pizzini, 1995). Costa encouraged educators to expose students to a wide variety of cognitive questions to develop their transition from concrete algebraic thinking to a more abstract understanding.

## Word Problems in Algebra

Word problems are an important component of algebra, and it seems that students have more difficulties in this type of activity than in the others. Stockdale (1991) investigated some of the difficulty elementary students have in solving word problems. She suggests that it is important to determine the degree of difficulty that word problems pose to students. There are important elements to assess the level of difficulty of a problem such as, the amount of extra information (Nescher, 1976, cited in Stockdale, 1991) and the number of multiple operations required to solve a problem (Loftus & Suppes, cited in Stockdale, 1991).

Similarly, Hutchinson (1986) researched cognitive theory and algebraic problem solving strategies and suggested that to represent an algebraic problem first, there is a translation from the statement of a problem (words) to an internal coherent representation enabling students to select what is relevant. Secondly, there is a logical internal integration of the algebraic features in the mind so mathematical operations can be followed to find the solution. She defined this process as "schema". A schema represents a prototypical abstraction of a concept that has been constructed from previous experiences (Hutchinson, 1986).

It is important to develop a good schema in students because schemata guide new incoming information to those clusters in the mind where problems are solved. She added, "effective instruction in algebra problem solving would be expected to induce individuals to construct relevant schemata based on mathematical structure which would guide problem representation" (Hutchinson, 1986, p. 26). Besides, "it can serve as a retrieval mechanism during recall"(p.14). She emphasized the importance of having

students participate in class because they are the ones who have to generate the schema processes themselves. Furthermore, it is important to assign exercises that promote the development of the same mathematical skills and concepts without boring students.

Some researchers are concerned about promoting reasoning and fostering symbol sense in students to raise their understanding in mathematics. (Driscoll, 1999; NCTM, 2000) Driscoll claims that classroom questions should foster algebraic habits in the minds of students. Based on three basic algebraic- thinking habits of the mind: doing-undoing or the process of working backwards, building rules to represent functions organizing input and output information, and abstracting from computation or thinking about computations independently of a particular number used, Driscoll suggests assessing student understanding by asking specific questions about conceptual understanding (1999, p.14).

He suggested some questions that will help teachers assess and support the development of symbol sense. Questions such as, "how can I describe the steps without using specific inputs?" "Is there information here that lets me predict what is going to happen?" "What process reverses the one I am using?" "Which steps am I doing over and over again?" "Is there a rule or relationship here?" "How can I write this expression in terms of things I care about?"

Regardless of the method of pedagogy and textbook approaches, the transition from arithmetic to algebra seems to be a challenging one. Several studies have been completed on algebraic thinking. For instance, Driscoll's (1999) research seemed to indicate that regardless of the language proficiency of students, students need some extra time to work with variables to fully grasp the concept of a variable. Until grade eight,

students do more concrete mathematical operations. In algebra grades eight and nine, students are required to think more conceptually than operationally and the idea of substituting a number for a letter and its meaning "can provoke a sense of real loss" (Murphy, 1996).

## The Language of Algebra

The language of algebra is a complex theme, often overlooked by educators and textbooks (Lochhead & Mestre, 1988). Research emphasizes two main factors that influence the understanding of algebraic language: the syntax and the structure of the language.

Lochhead and Mestre (1988)) and Murphy (1996)) conducted research into the learning of algebra and conclude that many students appear to have difficulties translating written language to symbolic algebraic symbols even when they possess good reading and mathematical skills. They both found patterns where the students showed a strong tendency to perform from left -to-right- word- order when they translate written language to algebraic expressions when in fact, it is processed right to left. For instance, when students have to translate an English sentence into mathematical language such as, "Four more than five times a number equals one less than six times that number". A frequent answer found is:"$5n + 4 = 1 - 6n$", rather than $5n + 4 = 6n - 1$" (p. 131). In addition, many students tend to confuse variables with labels. Lochhead and Mestre added that the symbols "S" and "P" are often interpreted as labels for "students" and "professors" and that students interpret $6S = P$ to mean "six students for every one professor" (p. 130).

To fully prepare students for higher- level mathematics, Lochhead and Mestre suggested that students be exposed to practical algebraic drills for as long as they require to master the translation of words into symbolic algebra first, before moving ahead to higher algebraic skills such as the application of algebra to real- life situations.

Similarly Hutchinson (1986) and Wilson (1993) found that the presence of syntactic relational propositions in algebraic problems such as, " Mary is twice as old as Betty was two years ago. Mary is forty years old. How old is Betty?" increases the conflict between students' previous schemata and problem representation. Hutchinson noted that first, pupils have to translate each proposition into symbolic language and then, integrate them into a unified whole which specifies that mathematical relationships among the propositions. From a syntactical point of view, the subject and verb of the above sentence are familiar linguistic parts of a sentence. The predicate however, may involve an unfamiliar syntax for students that cannot be understood until the reader reads the last subordinate clause of the predicate. In other words, most pupils will mentally and directly process the meaning of the subject, verb, and predicate as they read from left to right. However, due to the syntax of algebra, students have to cognitively process the subject and verb directly from left to right, stop and often, they have to cognitively process the whole predicate as an entity and later, synthesize the subject, verb and predicate together to construct the idea of equivalence demonstrated by natural language (Hutchinson, 1986).

# The Challenge of Mathematics Textbook Reading Comprehension for ESL Students

A considerable number of studies have been completed concerning the difficulty students have in decoding textbooks. Hubbard (1990) and Ghosh (1987) findings showed that mathematics textbooks are heavily loaded with linguistic prepositions, unfamiliar mathematical terms and a highly sequential structure where each concept is developed from earlier ones. MacGregor (1990) and Clarkson (1992) have also investigated the effect that some metalinguistic components have on the learning of mathematics. Their research considered students who had limited English proficiency and they concluded that language is not only a means to communicate, but it is also used as an organizer of knowledge, as a tool of reasoning; therefore, a low level of English proficiency may limit student achievement in algebra, although it is not necessarily a barrier to success.

Marshall (1992) and Jones (1982) further noted that students with limited language proficiency will sometimes fail on exercises because they misinterpret words rather than performing incorrect mathematical operations. Jones (1982) suggested that reading comprehension skills in mathematics refer to the ability of students not only to know the meaning of words but also to understand the different mathematical relationships of mathematical terms. He cited the research of Nesher and Teubal (1975) who worked with the mathematical definitions of the words "more" and "less" for ESL students. Jones concluded that students who learn mathematics through a second language are at an immediate disadvantage because they often confuse terms. Sometimes, ESL students confuse the phrase "five is more than three" that describes the

relationship 5>3, with the phrase "five more than three" which involves the relationship 5 + 3.

## Reading Comprehension as a Cognitive Process

In their recent research on text comprehension in multimedia environments for ESL level II students, Chun and Plass (1997) concluded that the combination of pictures and text play an important role in vocabulary acquisition and text comprehension. Their approach views reading comprehension as a cognitive process rather than a collection of skills or acquired knowledge. The authors emphasized processes such as recognizing words, connecting previous content/world background knowledge with new experiences and contents, and analyzing the syntactic and semantic structure of sentences as the main components of a larger metasystem of comprehension (1997). By analyzing the several components that influence the comprehension of a text, Chun and Plass mentioned the study of Laufer and Sim (1985) that divides reading into four areas mentioned in order of decreasing importance: a) knowledge of vocabulary, subject matter, discourse markers, and syntactic structure.

Chun and Plass mentioned the research of several other authors (Cohen, 1987; Hanley, Herron, and Cole, 1995; Leow, 1995; Oller, 1996; Omaggio, 1979;and Seculas, Herron, and Tomasello, 1992) to emphasize that "in facilitating L2 reading comprehension, the use of sound, pictures, and animated pictures or video in addition to text, have played an important role in vocabulary acquisition and in overall text comprehension, and are unquestioned components of instructional material for language learning."

For Chun and Plass (1997) reading comprehension as a process involves, among other factors, the construction of a mental representation of the text's linguistic surface structure, the construction of a prepositional representation of the semantics of the text, and eventually the construction of a mental model of the subject matter described in the text with the help of existing cognitive schemata (p. 5).

Chun and Plass (1997) maintain that the difference between learning from a text and learning from pictures is that text information is processed sequentially word by word. In addition, text reading comprehension requires an indirect transformation between the symbolic representation of the text and the analog mental model. Pictures on the other hand, convey information by means of a visual-spatial structure that is processed simultaneously and can be directly mapped onto the mental schemata. Finally, Chun and Plass concluded that visual information can aid in text comprehension in three different functions: (a) in selecting information, (b) in organizing the selected information into a coherent structure of propositions using cognitive schemata, and (c) in integrating these propositions into the mental model (1997).

## Teaching Mathematics to ESL Students with a Content Based Approach

There seems to be some controversy in the literature about the best approach to teaching mathematics to students with English barriers. Spanos (2000), for instance, has worked on the acquisition of mathematical skills in ESL students and conducted two case studies in a public school in Virginia. Spanos used a content-based approach to teach mathematics and concluded that "the primary benefit seemed to be related more to classroom management, study skills, and critical thinking than to language or content

learning per se" (p. 24). His research seemed to indicate that most mathematical content can be learned despite language barriers if students are engaged in the right activity. Similarly, the authors of a cognitive analysis of American and Chinese students' mathematical performance concluded that, "Asian children have higher mathematical achievement than American children because, among other factors, their language is linked with the number system, and this fact may account for their advantage for mathematics learning" (NCTM, JRME Monograph #7, p. 28).

Dawe (1983) explored the ability of bilingual students to reason deductively in mathematics. The data was collected through a set of tests that measured deductive reasoning, reading comprehension, and logical connectives in mathematics. He concluded that bilingual students have more cognitive flexibility of thought and a more diversified structure of intelligence that facilitates divergent thinking and the ability to look for different possible solutions to exercises.

Since mathematics is dominated by deductive reasoning, these children have an advantage over monolinguals. However, students with low language competency in the first and second language show a poor knowledge of logical connectives and a tendency to switch languages more frequently. Dawe (1983) advocated a heuristic approach in mathematics and suggests promoting different modes of thought such as to reason by analogy, induction and deduction, particularly if students use English textbooks, and to use verbal-logical and visual-spatial means toward problem solving in mathematics.

Borgen (1998) also worked with ESL students and her research focused on the relationship between teachers' use of the English language and students' understanding of mathematics. She concluded that many elements can affect student understanding of

mathematical concepts. Borgen emphasized that how fast a teacher speaks, affects understanding. In addition, when the teacher speaks quickly, ESL students have more difficulties understanding. She also found that vocabulary was another element that sometimes hinders understanding. When teachers use commonly known words in mathematical contexts, students tend to misinterpret the new mathematical meaning. Borgen's results also showed that syntax, or the order of words put together in a sentence, also affects understanding. ESL students are trained to look for the most important words in a paragraph to extract the main idea. In algebra, every word is important and has relevant meaning.

Similarly, Ghosh (1987) suggested that the transition from a second language math classes to mainstream should occur smoothly by using very simple language. Ghosh's s suggestion raises the question of how many new language features can be introduced in each mathematical course. In fact, and according to research done in second language courses, the amount of vocabulary introduced in each curriculum should not exceed 1475 new words (Rivers, 1975, cited in Groves, 1995). Pedagogical approaches and layouts in textbooks are crucial for ESL mathematics teachers because they determine the amount of work and time dedicated to the planning and to delivery of each mathematical concept.

In conclusion, Silver and Kilpatrick (1994) identified classroom-based research as an important trend in mathematics and encourage scholars to explore how and what students learn from instruction. Special attention should be given to students who have language barriers because recommended textbooks may not always offer the variety of learning approaches these students need to learn mathematics (NCTM, 2000). It is hoped

that this research will contribute some guidelines to adapt textbook activities to facilitate the learning of algebra for ESL and non-ESL students. The successful and mainstreaming of ESL students may reduce stress on the educational system and relieve students of undue hardship. An early and successful incorporation of ESL students in mainstream could save resources to educational systems. In addition, students may be able to join mainstream mathematical courses sooner, and feel better as they will not be removed from mainstream classes.

# CHAPTER 3

# METHODOLOGY

## Introduction

The purpose of this study was to determine the role of a new textbook will play in the teaching and learning of mathematics 9 for ESL students. I used, for the first time, a new mathematics textbook as the main educational resource in a class of fifteen ESL students. The research and instruction was conducted simultaneously by myself at a Vancouver public school during the month of May and June 2002. I chose the second chapter of the textbook because, in my experience, algebra is one of the most complex themes for ESL students to learn because it is filled with linguistic abstractions and word problem solving exercises.

This chapter describes how the study was conducted. I explain the characteristics of the students that participated in this study and it offers a review of the methodology and methods chosen. I describe how the data was collected and documented as well as the methods used to analyze the information.

## Introduction to the Students

At my school, ESL students are grouped in levels one, two and three according to their English language level. Students at level one have reading comprehension and writing skills equivalent to regular grade one and two levels. Students at level two or intermediate levels have reading comprehension and writing skills equivalent to regular grades three and four. Students at level three have reading comprehension and writing skills equivalent to regular grades five, six, and seven.

Students at level two are placed in ESL mathematics 9 classes; however, it is not always possible to have a homogeneous level two group in class. In this study, most students in the classroom are at level two and few at levels one and three.

For the purposes of this study, ESL students academically at level two and three, in the last transition period prior to being mainstreamed following year were the main focus of this research.

From a development point of view, the students' age ranged from 14 to 18 years old, although most students were 15 and 16 years old. There were 9 females and 6 males.

From an academic point of view, fifty-seven per cent of the students took mathematics eight last year, twenty-two percent mathematics eleven, and twenty one percent mathematics seven. Regarding the grade means obtained by the students in the last Mathematics course they took, fifty-seven percent of students scored seventy percent, thirty-six of the students scored sixty percent, and seven percent of the students failed. Seventy percent of students expressed a desire to go onto higher education after high school.

From a social point of view, most students seemed to have been able to establish new friendships in Vancouver and have no plans to return to their countries of origin. Fifty per cent of the students came to Canada two years ago and the rest has been in Vancouver for a shorter period of time.

From a language point of view, most students speak their first language at home and with their friends. For most of them, school is the only place where they are exposed to the English language. In class, most students think and process information in their first language; they spend most of their time translating information. Fifty per cent of

students understand 70-100% of what is said in class, however, 63 % of them understand 50 to 69 % or less of what they read in the math textbook.

## General Methods of Research

The qualitative nature of this research allowed me to use methods that portray and capture specific learning outcomes and textbook adaptations and insights. Wiersma (2000) emphasized the benefits of the inductive nature of qualitative research. He noted that the study of specific situations will eventually allow the researcher to arrive at general conclusions and apply the findings to similar situations (p.12). Moreover, the inductive approach research facilitates the incorporation of any unexpected and relevant variables as they emerge, thereby, allowing the researcher to gain insight, open to various behavior participants may exhibit (Palys, 1997).

This study constituted a case study based on a population of fifteen ESL students in a mathematics nine class. Lancy (1993) argued that a case study is the best method for studying innovation or making evaluations, and Anderson (1998) added that case studies are concerned with the cause and effect aspects of certain events (p.152-153). For the purpose of this research a case study approach was taken where a new textbook was used and adapted to determine to extent to which it facilitates the teaching and learning of algebra 9.

## Specific Methods of Research

To address the research questions, I used a critical incident technique (Tripp, 1993) as the framework to collect data and, to analyze this data, I drew on the constant comparative method.

*The Critical Incident Technique*: this strategy is a valuable method in observing and interpreting class events to illuminate what is required to produce positive instructional changes. Schon (1987) mentioned that occasionally, routines in class produce unexpected pleasant or unpleasant surprises that make educators reflect on the events and as a result, shape their future actions (p.26). Similarly, Tripp (1993) believed that critical incidents in class provide the reason to improve the practice. He claimed that critical incidents are an excellent means building a good framework for teaching and allowing educators to focus on their own concerns (p.22). He said, "incidents happen, but critical incidents are produced by the way we look at a situation: a critical incident is an interpretation of the significance of an event" (p. 8). Tripp added, "...the point is that incidents only become critical because someone sees them as such. This is really another aspect of awareness..."(p.27).

To collect and interpret incidents, Tripp suggested that there are two stages. The first includes the production of an incident: "a phenomenon is observed and noted, which produces a description of what happened" (p.25). The second stage includes the categorization of the critical incident: the incident is seen as an example of a category in a wider context. He emphasized that professional judgment be based upon scholarly analysis of ones' ideas of the meaning of the incidents and that how much one learns from an incident depends on ones' interpretation of it.

In terms of internal validity, Tripp (1993) added that there is always a personal value component in the interpretation of incidents. Nevertheless, the incident has to be understood within the situation in which it occurred "to ensure that our reflection and

evaluation are grounded in actuality", in other words, in its context (p.31). There are other aspects to consider as well.

Palys (1997) mentioned that it is important to consider external validity and defined it as the extent to which the results of a study can be generalized (p. 257). In terms of external validity, Schumacher and McMillan (1993) added that it can be increased if the findings are contrasted to the results of prior research (p. 395). For instance, the results of similar case studies can offer a good source of comparison to determine the extent to which the latest results can be generalized.

*The Constant Comparative Method*: Mertens (1998) mentioned that the constant comparative method of analysis as it is sometimes called, constitutes an excellent analytical framework that provides guidelines to a non-linear interpretation of the data. As Strauss and Corbin (1990) mentioned, there are three main steps in which the researcher moves back and forth with the data. If the researcher is interested only in the analysis of a particular theme, the final step can be omitted. However, the third or final step is necessary if the researcher wants to create a theory. In this research, I was not intending to generate a theory, but to use the method to produce a framework of descriptive and explanatory categories to help characterize underlying patterns and themes observed in the data.

The first step called "open coding" involves the breaking down of the data by comparisons that require similarities and differences in labeling phenomena. To categorize phenomena, Strauss and Corbin (1990) recommended asking questions such as, Who? When? Where? What? How? How Much? And Why? concerning the phenomenon that occurs.

The second step Strauss and Corbin (1990) mentioned is called "axial coding" and it is the part of the analytical process where the researcher brings the parts together to make connections between categories. They say that the researcher continues to ask questions, but this time, the questions are related to the relationships between categories.

To increase internal validity in the constant comparative method, Mertens (1998) claimed that researchers have to establish a high correspondence between what participants mean and what the researcher portrays (p. 181). Mertens (1998) and Guba and Lincoln (1989) suggested the use of multiplicity of sources to provide evidence to increase the credibility of the results and triangulate of the data to strengthen the interpretations and conclusions of a study (cited in Mertens, p, 354, 1988).

Similarly, Guba and Lincoln (1989) recommended higher quality of results by increasing the transferability or external validity (p. 183, cited in Mertens, 1998). They explain, "in order for readers to generalize the results to other situations, the researcher need to provide sufficient details to enable the reader to make his own judgment. Any change in methodology is expected to be documented to increase the reliability of the methodology"(p. 183).

In this research and in view of expected communication barriers between the students and myself, I selected assessment tools that allowed me to gather data in a variety of ways. Post- lesson interviews of students, a teacher professional journal, demographic questionnaires and pre-post tests offered a description of student outcomes, textbook adaptations and class contextuality. Each data collection method generated information from different perspectives and ultimately, facilitated the answering of the research questions.

Post-Lesson Student Interviews.

Individual semi-structured interviews are a good source of data in qualitative research (Mertens, 1998). They allow the researcher to have more intimate conversations with the participants and they provide insight for the interviewer and interviewee (p.321). The researcher may have a list of open questions that as the study evolves, the questions become more structured and formal. Kvale (1996) noted that semi-structured questions allow researchers the latitude to ask new questions in order to clarify or extend the inquiry plus add notes to the student answers.

Teacher Professional Journal

Arhar, Holly and Kasten (2001) claimed that a teacher professional journal represents a comprehensive, unstructured detail of what the researcher is experiencing (p.18). It constitutes a living document used as a tool for scholarly reflection, professional development, the collection of data, interpretation, analysis, and synthesis.

To support the credibility of the findings and increase the reader autonomy to make his own judgement, Schumacher and McMillan (1993) suggested, among other strategies, to record precise, almost literal, and detailed descriptions of people and situations (p.387). They also recommended writing excerpts with low-inference descriptors as opposed to abstract language. Teacher journals should include entries that portray the circumstances of a critical incident. However, the richness of spontaneous entries may be limited by time constraints; consequently, restricted to point form notes that may not offer sufficient information.

Demographic Questionnaires

Many authors agree that questionnaires are a good method to collect data. For instance, Schumacher and McMillan (1993) and Palys (1997) claimed that questionnaires have the advantage of ensuring anonymity; the questions can be written for specific purposes, and that can have standardized questions. To increase internal validity, Schumacher and McMillan recommended listing specific objectives and then writing questions to address them. They also mentioned that it is also important to establish beforehand how each piece of information will be used. There are many forms used in constructing statements. Any decisions should be based on the advantages, uses, and limitations of each study. For instance, closed questionnaires are best for obtaining demographic information (p.243) and have the advantage of being easily categorized easily. The Likert-scale questionnaire is the most widely used. To answer the questions, the questionnaire offers a series of gradations, levels and values that describe various degrees of something. The participants choose places on the scale that best reflect their beliefs. Schumacher and McMillan (1993) noted that it is important to include a neutral choice, otherwise, participants may be forced to either make a choice that is incorrect or may not respond at all.

Pre and Post-Tests

Pre and post-tests can be useful tools in gathering data if the elements of validity and reliability are taken seriously. Schumacher and McMillan (1993) explained that validity is " a situation-specific concept: validity is assessed depending on the purpose, population, and environmental characteristics in which measurement takes place" (p. 223). They emphasized that the quality of questions is an important factor in a test in

order to produce accurate findings and increase internal validity. Achievement tests are used to measure what has been learned (p.235). They suggested examining the items of the test and making a professional judgment about the relationship between what was taught and what is being tested. Another important element to consider when administering a test is the gradient level of the questions, making sure it matches the ability of the students.

Regarding reliability, Schumacher and McMillan (1993) claimed that reliability minimizes the influence of external variables unrelated to a study. Consequently, if external factors are not taken into consideration, student scores may develop an imperfect assessment of their knowledge of the subject. For instance, to increase reliability, it is important that the test has the appropriate reading and language level, and that the same person administers the test.

Schumacher and McMillan (1993) maintained that when a researcher needs to administer a pre and post-test to assess a change of skills or behaviors across time, a reliability coefficient of equivalence and stability should be established (p. 229). It involves administering to the same group of participants, one form of a test at one time and a second form at a later day. Schumacher and McMillan (1993) said that "this is the most stringent type of validity and it is especially useful for studies involving gain-scores or improvement"(p.229).

According to Schumacher and McMillan (1993) the pretest-posttest design has potential external variables that can affect the results other than the ones studied. A possible external variable is history (p.305). Post-test results can be affected by the maturity of the students or by other external variables that took place between the pre and

post-test. Schumacher and McMillan suggested using this design along with other assessment tools to minimize the threats of uncontrolled variables. In addition, they mentioned using reliable instruments and short pretest-posttest time intervals, and when possible, using other designs that will control some of these threats (p.307). They suggested using the nonequivalent pretest-posttest control design in which the researcher uses the entire class, gives a pretest, administers the treatment condition to the group, and gives the posttest to the same group (p.317).

## Data Collection

### Introduction

The resources utilized in this study were: post lesson student interviews, a teacher professional journal, demographic questionnaires, and pre and post-tests. In this section, I describe how I adapted the data gathering tools to fit the needs of this case study.

#### Post-Lesson Student Interviews

The objective of the individual post-lesson interviews was to collect information about student comprehension with regard to algebraic concepts and skills. For the purpose of the interviews, six ESL students, academically at level two and three, were the focus of the information gathering.

Mertens (1998) maintained that semi-structure interviews provides flexibility to interact with the interviewee to gather quality answers and at the same time, they provide some structure that guide the process of inquiry to the research questions.

While some interview questions were taken and adapted from the assessment section of the teacher resource manual, others, were open-ended questions in which student anecdotal feedback about their algebraic concepts, procedural understanding, and

language difficulties were collected. All the interviews used the same written questions concerning each section of the textbook.

Most interviews were scheduled to occur after school or at lunchtime. Few interviews, however, took place unexpectedly in the classroom while the rest of the students worked on the exercises assigned for practice. After introducing two or three new algebraic concepts, one student was called after class to be interviewed about the algebraic content studied that day. It was interesting to note that students who were interviewed showed more signs of confidence, independence, and co-operation in class than the students who were not interviewed.

I had to modify the original interview sample in order to maximize the results of the interviews. For instance, I modified the length of the interviews, and in the beginning, to make each interview more manageable, I restricted it to twenty minutes. Later on, I realized that in most cases, twenty minutes was not enough time to gather relevant answers so, I decided to extend the time of the interview to at least thirty minutes. Students needed more time to be able to think critically and to relay their views. Kvale (1996) maintained that conducting interviews is generally not time-consuming; he claimed that analyzing the interviews on the other hand, requires a considerable amount of time. Unlike Kvale's assertion, I found that interviewing students required as much time as analyzing the transcripts. It was a time consuming activity but the in depth assessment made it possible for me to relate how the textbook activities affected the extent of student comprehension.

Despite the fact that most questions were structured, in the beginning, I felt disoriented about the questions I had to ask students to find out what their learning

difficulty involved. I did not know how extensively I had to question students to perceive whether or not they had understood the expected concepts and skills. After the first three interviews, I realized that I needed to ask more open ended and unstructured questions to identify individual learning problems. Unlike what Mertens (1998) said about beginning an interview with open questions and then structuring them to produce a progressive interview, it was easier for me to start with structured questions and then, to ask open questions based on each particular case (p. 321). The first set of questions in each interview guided me to a deeper inquiry about student understanding.

If I detected a students' lack of understanding about something, I narrowed my questions down until I was able to determine whether that student had not understood something because of language problem, or an instruction deficiency, or lack of basic mathematical skills. At the end of the interviews, I realized that the more experience I had in assessing student learning processes, the more meaningful questions I was able to pose, and the more accurate and relevant responses I was able to garner. My improved interviewing skills allowed me to gather relevant information in a shorter period of time.

Another modification that I had to make after the first three interviews was to change the method I was using to collect the data. In the beginning and following the suggestions of Schumacher and McMillan (1993), I used a tape recorder so that I could listen to what students had said later on. Unfortunately, students were not able to express their ideas clearly and I was not able to fully understand what they meant when I listened to the tapes. So, my questions and the student responses were recorded with pen on paper because I discovered that the students were better able to express themselves in writing in

Mathematics than in conversation. This methodology allowed me to gather more written data from students and then look at this information retrospectively for further analysis.

To maintain anonymity in the transcripts, the names of the students interviewed were replaced by number codes, but their transcripts have not been modified. Syntax and grammar mistakes were kept as they were in the original scripts since they did not appear to impede understanding. Citations from the teacher professional journal have been shortened to maintain brief quotations and focus. Three dots, (...), have been incorporated in the transcripts to indicate that a portion of the transcripts was omitted. In addition, each transcript was identified by a code to indicate the location of the transcript in the raw data source. The accuracy of the transcript interpretations was done to the best of my abilities.

Teacher Professional Journal

The objective of the teacher professional journal was to collect information about student participation and inquiries in class, and the interaction between myself, and the new textbook.

Entries in the teacher professional journal took place before and after class time. I highlighted the learning and teaching events that surprised me or challenged me as a teacher. I also documented the most relevant discoveries that have emerged from class activities and those that have changed the way I instruct mathematics and the way I assess educational resources today. Critical incidents such as student difficulty answering questions in class and student inquiry about the same exercises in the practice section occurred spontaneously as I was instructing algebra.

In narrative form, I illustrated the adaptations implemented and the rationale behind them. I updated the professional journal each time I planned a new lesson, evaluated the old one or when spontaneous thoughts and/or insights came to mind.

Demographic Questionnaires

The objective of the demographic questionnaires was to gather data to better contextualize the information collected in the interviews, pre and post-tests, and class observations. The data provided by the demographic questionnaires offered relevant information about the students' language proficiency, as perceived by them, along with the other already gathered, and their viewpoints on social immersion in the North American culture. This information was utilized to determine what problem contexts could have been foreign for most students and to determine the language activities planned for class and homework assignments.

I administered semi-open questionnaires to all the students in the beginning of the study. Some questions followed the likert-type questions others, the semantic differential-type questions (Palys, 1997). Most questions had a range of categories to choose from and students had to indicate the degree of agreement with each one. I was in the classroom to clarify any misunderstanding the students had with regard to the questions.

Prior to the elaboration of the questions, I defined three objectives: a) to find out the structure of the students' families and school activities after school, b) to find out the students' language proficiency in informal and formal settings, and c) to assess their attitudes towards Mathematics and future educational and professional goals.

The demographic questionnaires posed twenty-two questions based on the three

objectives mentioned above. Questions such as "How long have you been in Canada?" "What do you do after school?" and "How many members of your family are living with you?" helped me to frame the students' personal situations and degree of integration into the North American culture. My first concern was the emotional status of students. It is well known that sometimes, students are sent to other countries without any relatives and experience emotional problems that interfere with the learning process. My second concern was to learn about student interests and activities to be able to assign the most relevant math problems and adapt the introductions of math concepts in a way that the learning of math was framed into meaningful contexts for ESL students.

The second objective was to assess student English language proficiency. Questions such as "In what language do you watch T.V?" and "In what language do you think when you do your math?" gave me an idea of how immersed students are in English so I can determine what additional resources to bring to class.

The third objective addressed student attitudes toward Mathematics. Questions such as "How much do you understand the English written in your math textbook?" and "What do you want to do after high school?" "Do you have a math tutor or any other extra help in math?" provided information about the degree of understanding perceived by students and how relevant Mathematics was for them.

Although I tried to be as clear as possible, some questions needed clarification. To find out the social and emotional situation of students, one of the questions geared to that objective asked them to say whether they had friends. However, questions such as "How many close friends do you have?" was not clear to them. Some students asked, "here or in my first country?" others asked, "what does a close friend mean? So, I realized

that I should have asked, "How many close friends do you have in Vancouver?" to be more specific.

Pre and Post-Tests

The objective of the pre and post-tests was first, to confirm to what extent the suggested standardized evaluation reflected what students learned by comparing it with the mathematical assessment carried out during the interviews including my classroom observations. Second, I wanted to assess to what extent students had learned solely as a consequence of my instruction and the use of the textbook as opposed to the algebraic concepts learned prior to my classes.

I assigned a pretest to the entire class in the beginning of chapter two and an identical one at the end. After professionally assessing it, I used the standardized test that the textbook recommended.

Prior to the instruction of algebra, I assigned a test to students using seventeen questions that the teacher resource manual suggested. Students were asked to write down all the steps of their approaches. They could try different strategies and not to worry about mistakes because the pre-test was not for marks.

In addition, I asked students to comment on the nature of their difficulties. Basically, I asked them to group their comments into two categories: language and/or mathematical difficulty. As a result, most students indicated whether they had language difficulties or lacked the necessary mathematics skills when they were unable to solve an exercise or problem.

Students were not permitted to use calculators and dictionaries neither on the pre-test nor on the post-test. Calculators are not needed when the mathematical operations

are basic. Dictionaries were not used because students were expected to know everyday English and the mathematical terminology written on the test before hand. This required that as a teacher, I use no new word or mathematical concepts on the test for these ESL students. Consequently, the algebraic concepts and skills were introduced and reviewed in advance to promote mathematical communication among the students, the teacher and the textbook.

## Data Analysis

I began the analysis of the data as soon as the first interview was completed. Interviews and journal transcripts that described the same learning difficulty were piled up on one corner of my working table. When an interview showed a new learning difficulty its transcript was placed on a different corner of my table and piled up with the new ones that described the same pattern. By looking at differences among the piles I was able to group transcripts in different low-level categories.

For example, students' transcripts 2.5 and 22.4 showed the students inability to apply algebraic concepts to real-life situations. Therefore, they were grouped in the same pile. However, student 2.2 and teacher journal transcript 1.9 showed a lack of comprehension in final numerical values of some exercises and thus were grouped in a different pile. By looking at the commonalities between these two low-level categories, I was able to determine that in both piles, the transcripts showed a poor conceptual understanding regarding the meaning of variables and the algebraic equations. As a result, a new higher-level category emerged between the two piles of transcripts. I gave the new category the name of "Conceptual Understanding of the Variables."

Even though most low-level categories emerged from patterns found in student interviews and teacher journal, few ones emerged solely as a result of my professional observations. Such was the case of my teacher journal transcript 1.8 that describes my concern regarding the lack of an introduction of properties of equivalence. This constitutes an example where a low-level category emerged as a result of my professional observation even though patterns were not found. However, I considered the absence of the introduction of these properties an important obstacle for students to reach higher algebraic thinking. As a result, I included this finding to the low-level category called "Addition of Prototypical Examples" and implemented additional activities to supplement the textbook's.

During the process of data collection and analysis, I was constantly looking for similarities and differences among the raw data to be able to identify patterns to label the low-level categories. As a result, some low- level categories were relocated under a new or different higher-level category. For example, in the beginning of the study, I placed the interview transcripts and journal entries that described the difficulty students had understanding the language of algebra under the high-level category called "Text Reading Comprehension." I placed these transcripts under the last category because the data in the transcripts were related to poor language comprehension. As a consequence of revisiting the raw data constantly, I realized that although the transcripts showed a language comprehension problem, the problem was not directly related to an ESL problem but was to the learning of algebra. As a result, I decided to include this finding to a much greater category that involves the understanding of algebraic processes. I

considered the translation of algebraic sentences into symbolic algebraic language the first step of the learning of algebraic processes.

# CHAPTER 4

## RESULTS

### Introduction

The interpretation and analysis of the results illustrate how the textbook was utilized. While the first and second category of the results illustrate how the textbook activities were adapted to meet the language and algebraic learning needs of ESL students, the third category describes how some textbook activities were assigned directly because they did not require adaptations. I found patterns in the raw data that allowed me to group the findings into different categories. Based on the raw data collected, three main categories emerged. They are:

1) Adaptations that Aim to Increase Textbook Reading Comprehension.
2) Adaptations that Aim to Increase Understanding of Algebraic Processes.
3) Assignment of Textbook Exercises that Promote Accurate Algebraic Procedural Skills.

In the first category, "Adaptations that Aim to Increase Textbook Reading Comprehension", what became evident was the necessity for:

1) Adaptations that Aim to Increase Textbook Reading Comprehension:
    a. Decrease the Amount of New Vocabulary in Problem Statements.
    b. Omission of Problems With Foreign Context.
    c. Add Relevant Visual Aids.

These three elements constituted the main changes I implemented to facilitate the comprehension of the textbook's vocabulary and problematic contexts.

In the second category "Adaptations that Aim to Increase the Understanding of Algebraic Processes", four elements appeared:

1) Adaptations that Aim to Increase Understanding of Algebraic Processes:

   a. Addition of Language Exercises to Increase Understanding of Algebraic Syntax.

   b. Addition of Prototypical Examples.

   c. Addition of Conceptual Questions.

These elements illustrate how I complemented the textbook activities to promote algebraic reasoning in ESL students. The adaptations were implemented to encompass algebraic processes that varied from understanding concrete procedural skills to more abstract algebraic conceptual interpretations for all grade nine students. In the last element "Addition of Conceptual Questions", two sub divisions became apparent:

2) d. Addition of Conceptual Questions:

   - Addition of Questions that Require Interpretations of Final Numerical Values.
   - Addition of Questions that Require Application of Algebraic Concepts to New Situations.

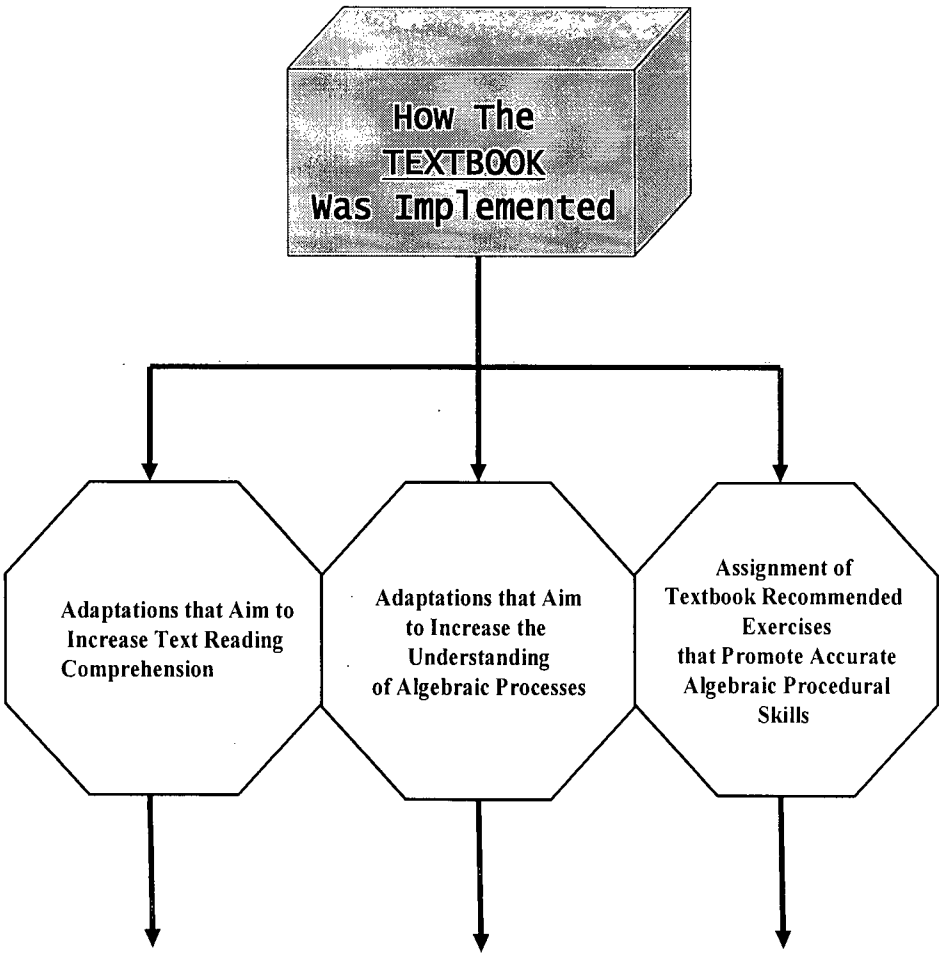
In the third category called, "Implementation of Textbook Exercises that Promote Accurate Algebraic Procedural Skills", two elements arose:

3) Assignment of Textbook Exercises that Promote Accurate Algebraic Procedural Skills:

   a. Assignment of Textbook Recommended Exercises to Solve First Degree Linear Equations

   b. Assignment of Textbook Recommended Exercises to Solve Inequalities.

These two elements illustrate how the textbook recommended exercises facilitated, without adaptations, an effective use of procedural skills. For a better understanding of the transcripts mentioned in this chapter, please refer to figure 1.2 on the next page.




- Reduction of Vocabulary in Problem Statements
- Omission of Problems with Foreign Context
- Addition of Relevant Visual Aids

- Addition of Language Exercises Aim to Increase Understanding of Algebraic Language
- Addition of Prototypical Examples
- Addition of Conceptual Questions re:

  a) Addition of Questions that Promote Interpretation of Final Numerical Values

  b) Addition of Questions that Promote Applications of Algebraic Concepts to New Situations

- Assignment of Textbook Recommended Exercises to Solve First Degree Linear Equations
- Assignment of Textbook Recommended Exercises to Solve Inequalities

Figure 1.2

## Description of the Categories

### Adaptations that Aim to Increase Text Reading Comprehension

There are several components that decrease the reading comprehension of a text such as an excessive amount of new vocabulary for the audience and foreign context for the reader (Chun & Plass, 1997, p. 61). For ESL mathematics teachers these two components are particularly relevant because usually, mathematics textbooks lack language aid activities that include ESL students; consequently, students may fail because they do not comprehend the text rather than the mathematical concept itself.

The reading comprehension difficulties that have been identified in the transcripts of the teacher and students in this category show how the two areas sometimes converge into one. However, their differences are believed to be relevant enough to require individual discussion.

*a) Reduction of Vocabulary in Problem Statements*

Vocabulary is the range of words that form the basis of expression. The processing of words requires a lower-level processing capacity (Chun & Plass, 1997). Most ESL students at this level are able to comprehend intuitively the content of a sentence even when they do not know the meaning of every word. However, when the number of unknown words in a sentence is high and students do not recognize most of them, it is difficult for them to construct the necessary mental representation or conceptual content that is conveyed by the sentence.

It is important to notice that, since the maximum amount of new vocabulary introduced in each class should not exceed more than fifteen words per class (Rivers, 1975, cited in Groves, 1995), it is expected that ESL students be able to comprehend the

textbook after the introduction of the new vocabulary in the beginning of each class; otherwise, the textbook might not be considered by the teacher and/or ESL students to be a convenient educational resource for teaching or learning the subject matter. Rivers' findings are consistent with my experience in class. It seems that fifteen new words to learn in mathematics classes is an adequate task to assign to ESL students.

The students and myself identified the need to decrease and / or clarify the number of new words in the statements of some problems and in the introductory examples of the textbook.

The following transcript is part of my entry into the *teacher professional journal* and illustrates how I had to reduce the vocabulary in the introductory activities by limiting the number of unknown words, introducing the mathematical terminology beforehand, and changing the inquiry approach that the textbook recommends for an expository one.

> *"To introduce the concept of a variable, the textbook has two group activities in which students have to look at three big squares that are divided into smaller squares, and at a rectangular flower bed. Students then have to answer six questions in each activity in which they have to make a table, to look for patterns, to find a formula that can be used to determine further patterns, and to draw a graph. For instance, the second activity that the Minds on Math 9, p. 113 offers, which is as long as the first one, shows a rectangular flower bed that has a length of 5m and reads as follows:*
>
> *1. Calculate the perimeter of the flower bed for a width of 2 m, and for a width of 3.5 m.*
> *2. Calculate the perimeters for four other widths. Record the results in a table.*
> *3. a) Suppose you know the width of a flower bed with length 5m. How would you find the perimeter?*
> *b) Let w metres represent the width of the flower bed. Write an expression for its perimeter.*
> *c) Write this expression in another way.*
> *d) What kind of number is w?*
> *4. Suppose the width is 2.4 m. Use your expression to determine the*

*perimeter.*

5. *Suppose the perimeter is 13 m. Use your expression to determine the width.*
6. *a) Use your table to draw a graph of perimeter against width. Should you connect the points in the graph? Explain.*
   *b) How would you use the graph to determine the perimeter if you know the width?*
   *c) How could you use it to determine the width if you know the perimeter?*

*'In planning the second activity of section 2.1mentioned above, the concepts of a variable, I decided to reduce the level of reading difficulty and the teaching approach suggested by the textbook. Instead of having a group discussion about finding and extending patterns, I decided to introduce the concept of a pattern and a variable in an expository way. It has been my experience that ESL students at intermediate level do not have sufficient English skills to follow long instructions or to interact mathematically in groups if they have not been previously introduced to the vocabulary and, this would have required more than thirty minutes, preparing students for the introductory activities, time that would better be spent on the practice or closing moments of the lesson where I wrap up mathematical concepts and terminology...The instructions were too long and wordy for ESL students at this level and I had to introduce the terms I was going to refer to during the teaching of this section. Terminology such as algebraic expression, ..., and algebraic equation was introduced and listed on the sideboard in a vocabulary list...After the introduction of the main examples, I queried students about their understanding. They seemed to comprehend the examples I had shown them. They answered most of my questions properly without asking for clarification or further examples. So, I decided to move on and assign some exercises for practice.*"(1.1)

The next transcript was taken from the *teacher professional journal* and illustrates how I omitted a problem because it has an excessive amount of new vocabulary for ESL students.

Problem 7 in <u>*Minds on Math 9*</u>, page 152 reads as follows:
'Adrian and Jasmine live near a mountain road. There is a viewpoint on the road, higher up the mountain. The two students cycled up the viewpoint and back. The total traveling time was 3 h. Going up to the viewpoint, they averaged 5 km/h, but returning from the viewpoint they averaged 25 km/h. How far is it the viewpoint?"
*"After introducing section 2.7, <u>Solving Problems in Different Ways</u>, for practice, I chose exercises that encouraged different approaches and left*

*out some others (exercise 7, p.152,) because they had too much new vocabulary.* (1.13)

The two transcripts mentioned below were taken from the *student interviews* and seemed to indicate that both students were unable to rephrase what the question asked; consequently, their answers lacked understanding due to unknown vocabulary.

Student 2.3 was unable to explain what the question asked. The question asks students to explain the meaning of 8000 and 9n on the right side of the equation and not to solve the equation as the student responded. It is important to note at this point, that this student has had previous knowledge of algebra as stated in his demographic questionnaire. If that is the case, and since he knew that ultimately he had to solve for "c", he was probably guessing what the question asked and consequently and due to the lack of comprehension, was unable to rephrase the question.

The problem was taken from the textbook, page 142, and reads as follows:

*The cost, C dollars, of producing a school yearbook is given by the formula C = 8000 + 9n, where n is the number of yearbooks printed.*

*a) What does each term on the right side of the formula represent?*

*b) The yearbook committee has a budget of $10000. To determine the number of yearbooks that can be produced for $10000, substitute 10000 for C and solve the equation for n.*

During the interviews the following dialog took place:

*Teacher: [referring to question (a)] "Do you understand what question (a) says?"*

*Student 2.3: "Yes. It is asking what the value of n is, but I don't know why n is there."*

*Teacher: "n is the number of yearbooks issued. Do you know what C means?"*

*Student 2.3: " Is the total cost at the books"*

*Teacher: "Do you know what 8000, n, and 9 mean?"*

*Student 2.3 :" 8000 means the number of year books that has just been printed right now; n and 9, I don't know" (Student 2.3, section 2.5, problem 2)*

Similarly, in answering question b student 2.4 was able to apply all the right procedural steps and even to find the value of n but, he was unable to explain what his final answer, n = 2222.22, meant.

*Student 2.4:* $10000 = 8000 + 9n$
$-9n = -2000$
$n = 2000 \div 9$
$n = 222.22$
*Teacher:* *(referring to question b) "O.K., What does n mean? Could you answer in a complete sentence?"*
*Student 2.4: (after a long pause)"n means" (a long pause)*
*Teacher:* *"Do you understand what the questions says?"*
*Student 2.4: "No, I don't understand much. I think it is not right, because the n means the number of the yearbooks printed" (Student 2.4-section 2.5-problem 2b)*

The following transcript was taken from the student interviews and refers to problem number 8, extracted from the textbook, page 146. It illustrates a student's lack of comprehension concerning a question of a problem. I had introduced the most important vocabulary in the beginning of the class; nevertheless, student 2.6 was unable to rephrase the question.

The problem gives information about the prices of long distance calls and gives the formula: C = 4.88 + 1.22 (n - 3) to find the cost a long distance telephone call where n stands for minutes. The first question reads as follows,

*a) To determine how long you could talk for $10, substitute 10 for C and solve the equation for n."*
*Teacher:* *"What exercises do you have to translate more than two Words to be able to understand the problem statement?"*
*Student 2.8: "In section 2.5, p. 146 8(a) determine, substitute, equation this word are hard."(Student 2.8 - textbook assessment-question 3)*

Student 2.8 said explicitly that he did not understand the meaning of the three most important words in the question; consequently, he was unable to understand what he had to do.

*b) Omission of Problems with Foreign Contexts*

Context is a phrase or sentence in which the words just before and after a certain word help make clear what the word means (Collins English Dictionary, 1987). Cognitively, when students read, the incoming words form a meaningful structure in their minds that are integrated into a previously established mental model or "squema". According to Hutchinson (1986) squemas are internal cluster of previous learning experiences in the mind where new information is integrated to generate meaning. These "squemas" are culturally constructed and based on formal and informal life learning experiences (Chun & Plass, 1997, p. 65).

ESL students have different cultural life learning experiences compared to non ESL students in North America and consequently, some of the situational contexts of mathematical problems may be totally foreign to them; in other words, the context of the problems cannot be immediately incorporated into their "schemas". As a consequence, foreign problem contexts may delay the comprehension of the mathematical concepts intended to be learned.

In the new textbook, some problem statements and introductory examples have been identified by the teacher as "probably unfamiliar" to ESL students and consequently, a barrier to the comprehension of the most important information given in the problem statements. My personal criteria used to consider whether a context was foreign or not was based on the information gathered from the students' demographic questionnaires, my previous experience teaching ESL students, and my personal world/background knowledge. My pedagogical approach to problems with probable unfamiliar contexts was either, to simplify the context if the problem promoted

important algebraic skills, or to omit the problem if the textbook offers other problems with simpler contexts that require similar algorithms to solve it.

The following transcript was extracted from the *teacher professional journal* and shows how I shortened a wordy and probable unfamiliar context to all ESL students by selecting the most relevant information.

The problem said:

> *'Every October, Canine Visions Canada sponsors a national Walk-a-dog-a-thon. The money raised provides blind and visually-impaired Canadians with a free 26-day dog guide handling course. Last year, Ashok, Krishnam and Lisa Crosbie took part in the walk-a-thon. Lisa twisted her ankle during the walk and had to drop out. Ashok completed the walk. Ashok walked 6 km farther than Lisa. Together they walked a total distance of 14km. How far did Lisa walk?'*

My comment was:

> *" Section 2.8 focuses on how to solve problems algebraically...I would have liked to omit the second example because the contextualized problem was too wordy and the situational context, though interesting and informative, was unfamiliar to these students. I am not sure if most students have seen, experienced or read about a walk-a-thon, a walk-a-dog-a-thon, sponsors that support associations, and a dog guide handling course. Depending on the political, economic, and social situation of the countries where students originated, the context of the statements mentioned in this problem may not exist in their minds....but I realized that the example introduces important algebraic concepts, so I decided to simplify the statement ...*
> *After simplifying the context the statement read:*
> *Ashok and Lisa went for a walk. Ashok walked 6km farther than Lisa. Together they walked a total distance of 14km. How far did Lisa walk? (1.3)*
> *After changing the context, I explained the steps and most students seemed to understand them. The majority of the students, answered my questions correctly.*
> *The expected steps read as follow,*
> *Ashok:* $x + 6$
> *Lisa:* $x$
> $(x + 6) + x = 14$
> $2x = 14 - 6$
> $x = 8 \div 2$

*$x = 4$*
*Lisa walked 4 km."*

During the *student interviews*, I assigned students a similar problem with a simple context and similar steps compared with the one mentioned above. The problems call for a translation from sentences into algebraic terms, a creation of an equation, and a solution to the problem. Most of students were able to solve it without difficulty. The following transcript was taken from the student interviews and reads as follows:

> *The mass of a cat and a dog is 21 kg. The dog's mass is 2.5 times the mass of the cat.*
> *a) Write an equation for the statement mentioned above.*
> *b) Find the mass of each animal.*

Student 2.8 said:

*Student 2.8:* *The mass of a cat and dog = 21 kg*
*The dog's mass = 2.5x (x)*
*cat (x)*
*dog 2.5(x)*
*Teacher:* *What is your equation?*
*Student 2.8:* $x + 2.5\ (x) = 21$
*Teacher:* *What do you have to do to find the mass of each animal?*
*Student 2.8:* $x + 2.5\ (x) = 21$
$3.5x = 21 \div 3.5$
$x = 6$
*cat mass is 6 kg dog's mass is $2.5 \times 6 = 15$ kg."*

Similarly, student 2.6 wrote:

*Student 2.6:* *"$x + 2.5\,x = 21$*
$x + 2.5\,x = 21$ $\quad$ $21 - 6 = 15$
$3.5\,x = 21$
$x = 6$
*The cat weight 6 kg. And dog weight 15."*

*Analogously, student 2.4 wrote,*

*Student 2.4:* *"$2.5\,x + x = 21$*
$2.5 + x = 21$ $\quad$ $2.5 \times 6 = 15$
$3.5x = 21$
$x = 6$
*The cat's mass is 6 kg. The dog's mass is 15 kg."*

After explaining what "write an equation" meant, a fourth student answered:

*Student 2.2:* *"21 = 2.5x + x*
*21 = 2.5x + x*
*21 = 3.5x* *6 × 2.5 = 15*
*6 = x*
*cat' mass is 6*
*dog's mass is 15."*

As we can see here, the above students were all able to complete successfully all the algebraic steps required to solve the word problem.

Other times, problems with probable unfamiliar contexts to ESL students were directly omitted. For instance, the following transcript was from the *teacher journal* and illustrates that a problem was omitted because the words "apple orchard and Macintosh and Delicious varieties of apples" could have been probable foreign context for ESL students.

Problem 19 on page 169 reads as follows:

> *An apple orchard is selling baskets of Macintosh and Delicious apples. The orchard has 8 times as many baskets of Macintosh apples as Delicious apples. The orchard has a total of 153 baskets of apples. How many baskets of each type are there?*
> *"The expected steps are,*
> *Delicious: x*
> *Macintosh: 8x*
> *x + 8x = 153*
> *9 x = 153*
> *x = 153 ÷ 9*
> *x = 17*
> *8x = 17 × 8*
> *8x = 136*
> *There are 17 Delicious apples and 136 Macintosh apples.*

In my journal I wrote:

> *"The chapter review offers a sufficient number of exercises to review all the sections; however, I had to omit some exercises such as number 17 on page 169 because I was afraid that the "apple orchard and the Macintosh*

*and Delicious varieties of apples" mentioned several times in the problem could be a foreign context for most ESL students. As far as I know, due to physical and geographic differences, one of the main differences among countries is the variety of names given to vegetables and fruits.*
*Once the context had been simplified, ESL students showed no difficulty solving them."*

The above comment illustrates my personal criteria to decide when a context represents a foreign context for ESL students. I am an experience teacher and am constantly exposed to multicultural environments. My world/background experience and professional knowledge and experience helped me to decide whether a context of a problem could be a foreign one or not for most ESL students. However, to decide whether a context is foreign or not for ESL students represents a challenge for me since I feel that I am always assuming what could be "foreign" for most students.

The following transcript was taken from the *teacher journal* and illustrates how I omitted another probable unfamiliar problem for ESL students. The problem refers to the Celsius and Fahranheit scales.

The problem in Minds on Math 9, page 117 reads as follows:

*Canada started using the Celsius scale for temperatures in the 1970s. Suppose a tourist from the United States wants to convert a Celsius temperature to Fahrenheit. Let C represent a temperature reading in degrees Celsius. Let F represent the equivalent reading in degrees Fahrenheit.*

*a) A rule of thumb for converting Celsius to Fahrenheit is "to double and add 30". Write a corresponding formula for F.*
*b) The exact formula for converting Celsius temperatures to Fahrenheit is $F=1.8C+32$. Choose some values of C. Determine how closely the rule of thumb gives the correct Fahrenheit temperatures.*

My journal noted:

*"In section 2.1, page 117, exercise 11 was omitted because of the foreign context for ESL students? The problem talks about two different scales to measure temperature, Fahrenheit and Celsius, and as far as I know, the Fahrenheit scale is used mostly in North America and the Celsius scale is*

> *used internationally in the scientific arena." The formula is a highly useful formula for students in any grade science course but, I would need sometime to explain the whole context of the scales and their applications and unfortunately, I do not count with it." I would assign instead a similar problem with a more familiar context for these students."*

*c) Addition of Relevant Visual Aids*

Visual aids are visual information such as pictures that add information to a text and facilitate the comprehension of it. I considered the lack of relevant visual aids to help students understand the meaning of the vocabulary and/or the algebraic concepts a weakness of the textbook. Due to the lack of visual aids, I selected the basic textbook vocabulary and drew, to the best of my abilities, the meaning of mathematical words and ideas on the chalkboard and overhead. The disadvantage of not having visual aids printed in the textbook is that students may forget to draw the drawings in their binders and consequently, forget the meanings later. In addition, they would have trouble understanding mathematical concepts.

. The following transcript was taken from the *teacher journal* and illustrates my effort to find visual aids in the textbook to explain the vocabulary;

> *"To communicate mathematical concepts, I always look for visuals to show to students and the textbook is the first resource I examine. In this particular textbook, I did not find many pictures in the textbook that I could have used as visual aids to introduce the new vocabulary. In fact, I was not able to find a relationship between the pictures and the mathematical contents, nor were the pictures useful to explain the context of the exercises. In my opinion, most of the pictures were attractive and colourful, but added little to the learning of the mathematical concepts."* (teacher journal, p.11)

The following transcripts were taken from the *student interviews* and reveal that the textbook images did not facilitate the learning of algebraic vocabulary or concepts. During the student interviews, students were asked what they thought about

most textbook pictures, and whether the majority of the illustrations helped them to understand mathematics. The following transcripts reveal students' disappointments with the textbook illustration:

> *Student 2.6: "No, makes the book look better. Pictures are nice to see, but don't help."*

Another student responded:

> *Student 2.3: "No sometimes the confeusd me It is not related to the math".*

Student 2.8 said:

> *Student 2.8: "Pictures don't help me"*
> *Teacher: "What kinds of pictures would you like to see?"*
> *Student 2.8: "Vocabulary pictures, a question explain by the pictures"*

Student 2.2 added:

> *Student 2.2 was looking at page 112 and 113 that introduce the concept of a variable. There are six colourful pictures, two pencils, two tables, a flowerbed and a square showing patterns. The following comment refers to these pages:*
> *Student 2.2: P. 112. Pencils not help. The other picture not help."*

Although some students admitted that the pictures looked good, they did not find them useful and even found some pictures confusing. These comments seem to indicate that ESL students rely strongly on visuals to understand mathematical concepts.

Adaptations that Aim to Increase the Understanding of Algebraic Processes

Algebraic processes include operations that range from the manipulation of symbolic algebraic language and the understanding of procedural skills, to the development of higher conceptual understanding of algebraic reasoning. (Driscoll, 1999)

Raw data seemed to indicate that the textbook activities needed some adaptations to maximize the development of algebraic processes for ESL students. I have

implemented learning activities that aimed to facilitate the translation of words into algebraic symbolic language, the learning of the properties of equivalence, and the development of reflective, interpretative answers.

In this study, the understanding of algebraic process included four components that played an important role in the way the textbook was used and how I adapted it in my classroom: "Addition of Language Exercises to Increase Understanding of Algebraic Syntax" "Addition of Prototypical Examples" "Introduction to Properties of Equivalence," and "Addition of Conceptual Questions."

*a) Addition of Language Exercises that Aim to Increase Understanding of Algebraic Syntax*

The syntax of some algebraic sentences may constitute unusual structures for beginner algebra students. Some words in the predicate of algebraic sentences are considered entities or direct objects (a group of words that cannot be separated to comprehend the conceptual meaning) that pose a precise meaning (Wilson, 1993; Hutchinson, 1986). Knowing the meaning of each word is not enough to comprehend the conceptual content involved in a sentence. For instance, most ESL students probably know the meaning of "larger", "times", "smaller", "more", "than, and "twice". However, the sentence, "Three times the smaller number is 14 more than twice the larger number" has a predicate in which a transitive verb leads to a direct object that has an unusual comparison "14 more than twice the larger number."

I have considered the lack of understanding of the syntax of algebra an obstacle to the translation of algebraic sentences into symbolic algebraic equations. It is important to note that this problem not only exists for ESL students, but also for non-ESL students as

well. I promoted the translation of sentences into symbolic equations by reviewing homework every class, providing additional examples, and allocating additional time to the teaching of the language of Algebra.

The following transcript was taken from the *teacher professional journal* and illustrates the ways in which students struggle with algebraic phrases. Students struggled with the comprehension of phrases such a, "a number plus two" and "four times a number decreased by five" and consequently, they were unable to write symbolic equations and solve the problems.

> *"Before introducing the next section, 2.8, I reviewed orally the vocabulary learned so far in this chapter and the homework I had assigned. I asked for volunteers to pick a word from the vocabulary list and explain using any medium, written, oral visual, numerical or through visual dramatic antics, the meaning of the mathematical terms. Half of the class raised their hands and explained all the mathematical terms of the list except 'there are three times as many nickels as dimes', 'three-quarters full' and 'half-full'. To review the last two terms, I used drawings to show the meaning of fractions and left the drawings beside the words on the list. However, I had to spend more time to explain the first term. To facilitate the comprehension of algebraic sentences, I introduced additional language exercises on the chalkboard. I assigned phrases such as, a number plus 2 (x+2), 4 times a number (4x), 8 divided by 3 times a number (8 ÷ 3x), and the 4 times a number decreased by 5(4x-5) to students to be translated into equations"* (teacher journal, p. 20 l)

The following transcript was taken from the *teacher professional journal* and illustrates the difficulty of students comprehending algebraic sentences despite the language learning activity I improvised during the last class.

> *" Despite how well-organized section 2.8, <u>Solving Problems Using Equations</u>, is and how smoothly the introduction of this lesson went, some ESL students found the vocabulary challenging, especially those ones with lower language skill. Statements such as, 'one number is nine times the other' or 'three times the smaller number is 14 more than twice the larger number' are challenging phrases for ESL students to understand. The predicate is too complex for them. Usually, ESL students work with simple, compound and complex sentence structures where the direct and*

> *indirect objects constitute simple ideas that do not require much elaboration"* (teacher journal, p. 22)

The following transcript was taken from the *teacher professional journal* and illustrates how students struggle with algebraic language. Since students have not yet overcome this learning problem, I decided to break the sentences down into their grammatical structures, subject, verb and predicate, and explain their meaning individually.

> *"Section 2.9 introduces inequalities and the meaning of symbols such as ≥,>, ∠, and ≤. As always, I began by reviewing the most difficult exercises I had assigned for homework previously. I was frustrated when I saw that some students still had difficulty understanding the algebraic meaning of 'The dog is three times as heavy as the cat' or 'The burger cost 40 cents more than the fries', or 'Three times the smaller number is 14 more than twice the larger number.' So, I reviewed the meanings of these algebraic sentences again as a class activity by giving examples of the same kind, until I saw the majority of the students answered the examples correctly."* (teacher journal, p. 24)

Most students demonstrated difficulty understanding the meaning of algebraic sentences. The following transcript was taken from the *student interviews* and illustrates a lack of comprehension of algebraic sentences.

The problem in Minds on Math 9 reads as follows:

> *"A pile of nickels and dimes has a value of $4.50. There are three times as many nickels as dimes. How many nickels and how many dimes are there?"*
>
> *Teacher: (After I explained the vocabulary for this section)'What problems did you find more difficult to understand and why?'*
> *Student 2.8: 'Exercises 5, p.152, because of the English three times as many nickel as dimes."*

That day, I had explained the meaning of algebraic sentences such as " there are three times as many nickels as dimes." Nevertheless, some students kept struggling with the meaning of some algebraic sentences.

*b) Addition of Prototypical Examples*

The lack of or poor development of common highly representative examples of the algebraic concepts promoted later in the practice section, was identified by myself and the ESL students as an obstacle to the development of procedural understanding.

In the textbook, chapter two has nine sections. Adaptations were implemented to three sections: 2.1, The Concept of a Variable, 2.6, Simplifying Equations Before Solving, and 2.8, Solving Problems Using Equations. The lack of important mathematical concepts and an insufficient number of steps to show students how an exercise can be solved, represented the most common textbook activities. These were modified to introduce algebraic concepts to ESL students. As a result of a poor introduction, I sometimes omitted exercises or explained to students individually the missing concepts or steps as they asked for help. Other times, I added exercises and /or steps to improve comprehension.

I omitted exercises in the practice part of section 2.1. The introduction of section 2.1 required many adaptations for ESL students and the time allocated for the preparation of this lesson limited the algebraic skills taught. The following transcript was taken from the *teacher professional journal* and illustrates the omission of some questions concerning graphing in the practice section due to the poor development of the introductory example.

The transcript refers to the following problem, taken from Minds on Math 9, page 116 and reads as follows:

> *A rectangular flower bed has a length of 5 m.*
> *a) Calculate the area of the flower bed if its width is 2 m, and if its width is 2m and if it is 3.5 m.*

*b) Calculate the areas for four other widths. Record the results in a table.*
*c) Let w metres represent the width of a flower bed and so on.....*
*g) Use your table to graph area against width.*
*h) Compare your results with the results of Activity 2 on page 113. What similarities and differences can you find?*
*i) Suppose you were to graph area against length for the rectangles in these exercises. What do you think the graph would look like?""*

In my journal, I wrote:

> *"In the practice section 2.1,the concept of a variable, p.116, as I was selecting some exercises for practice, I had to omit a couple of them such as h) and i) where students had to transfer and plot some values from a table of patterns to a graph because I had not explained how to do it earlier. I made the decision to omit these introductory activities because they were poorly developed in the introductory section and were not clearly related to the concept of a variable (p. 114). (teacher journal1.5, p. 8)*

Analogously, the introduction of section 2.6, Simplifying Equations Before Solving, does not talk about the concept of equivalence. Nevertheless, in the practice section, there is one exercise that requires identifying equivalent equations from a pool of equations.

The following transcript was taken from the *teacher professional journal* and shows how, as a result of a lack of an introduction to the concept of equivalence, students asked for help more frequently than usual in the practice section. Consequently, I explained the concept individually and missed those students who never asked for clarification.

Problem 7, page 146, reads as follows:

> *For each equation, decide which of the other two equations is equivalent to it. Explain your choice.*
>
> *7a)*

$$\frac{3x}{2}+\frac{9}{5}=12$$

$$3x + 9 = 120$$
$$15x + 18 = 120$$

7b) $5=\frac{9}{4}-\frac{r}{3}$

$$60 = 27 - 4r$$
$$5 = 27 - 4r$$

In my journal I noted that:

> *"While I was assigning the exercises for practice, I did not realize that I had not explained the concept of equivalent equations at the beginning of the class. The introductory examples in the textbook do not explicitly explain what 'equivalent equations' means but show how to solve equations isolating variables. There is an exercise in the practice section (7, p.146) that asks students to identify equivalent equations from a pool of algebraic equations but some students were unable to identify equivalent equations. Consequently, I had to explain to students as they asked for help." (teacher journal 1.6, p.15)*

The following transcripts were taken from the *student interviews* and reveal the difficulties some students had with the concept of "equivalence" as a consequence of the lack of introducing the idea of equivalent equations in the introductory examples of section 2.6 of the textbook. Student answers refer to the following context-free exercise that calls for symbolic manipulation and interpretation of the algebraic expressions:

> *"The expressions 6 (4x + 9) and 24x + 54 are equivalent. How would you explain it to a friend on the phone?" (Interviews-exercise 2.2)*
> *Teacher: "How would you explain that these expressions are equivalent?"*
> *Student 2.2: "I don't know"*
> *Teacher: "What does equivalent mean?"*
> *Student 2.2: "I don't know"*
> *Teacher: "Can you write the same statement but in an algebraic equation, for example, in Algebra we cannot write "and". We have to substitute "and" for a mathematical symbol. What symbol would you write instead of "and"?"*
> *Student 2.2: " No, I can explain in Chinese. But can't in English"*

*Teacher:* *"O.K, do not explain it, could you write the algebraic expression?"*
*Student 2.2:* *"No"*

*Another student replied:*
*Teacher:* *"Why are these expressions equivalent?"*
*Student 2.4:* *"Equivalent means equal?"*
*Teacher:* *" Yes, could you write the algebraic equation?"*
*Student 2.4:* *"What do I do with "and"?*
*Teacher:* *" If you have to change what you have in the statement into an expression, what would the translation of "and" in this case be?"*
*Student 2.4:* *"Oh, 6 (4x + 9) = 24x + 54"*

*Another student answered:*
*Teacher:* *"Could you translate the statement into an algebraic expression?"*
*Student 2.5:* *"I don't know"*
*Teacher:* *"Do you know what equivalent means?"*
*Student 2.5:* *"Yes"*
*Teacher:* *"With what mathematical symbol would you replace the word "and". Could you give me an example?"*
*Student 2.5:* *"I don't know. Oh, maybe with an equal sign. 6(4x + 9) = 24x + 54"*

Similarly, the textbook does not introduce comprehensively the properties of equivalence. Properties can be defined as the relationships between two or more changes. For instance, the subtractive property of equality justifies that "if c + 6 = 15, then c + 6 - 6 = 15 - 6". To isolate variables, students may need to apply the additive, divisive, multiplicative, subtractive, reflexive, transitive, and/or symmetric properties. I considered the absence of these basics an obstacle to understanding the procedural steps and consequently, in the future, to the application of these properties to other exercises. Data from the *teacher professional journal* indicates that it is an area of concern for me.

The following transcript was taken from the *teacher professional journal* and illustrates my concern about the lack of introduction of the properties of equivalence. To supplement the textbook, I implemented a class activity that involved the additive,

divisive, multiplicative, subtractive, reflexive, transitive, and symmetric properties as algorithms that can be applied to a wide variety of mathematical exercises.

> *"Chapter two has nine sections and from section three to nine, students have to isolate variables to solve equations. In these seven sections there are prescriptive instructions that tell students what number to add, to divide, to multiply and/or to subtract from both sides of the equation, but the sections do not explain why and how the properties work. Consequently, students may not know if they can use the same steps in other exercises or if the steps involved can be applied only to the exercises they are doing. So, I decided to explain the mathematical properties of equality in section 2.5. I showed students how the inverse subtraction, addition, multiplication and division do not change the value of an algebraic equation and can be used to isolate a variable. The introductory examples have a clear procedural explanation but fail to explain why students have to apply the same fundamental operations on both sides of the equation. Therefore, I spent sometime showing how the value of an algebraic expression is independent when the same numerical value is added, subtracted, multiplied and/or divided on both sides of the equation." (teacher journal, 1.8, p. 12)*

The textbook offers a very prescriptive and clear method of solving equations that most students can understand easily. However, I am concerned about the lack of introduction of mathematical properties. Prescriptive recipes that do not help students reach higher mathematical understanding.

Similarly, I added steps to some introductory examples in section 2.8. The following transcript was taken from the *teacher professional journal* and illustrates how section 2.8, Solving Problems Using Equations, poorly introduces the manipulation of algebraic terms involved in problem solving. The translation from sentences into algebraic terms is not clearly explained. In addition, the creation of an equation from the algebraic terms would require further information to be easily understood. As a result, and in order to translate sentences into algebraic symbols and create the algebraic terms, I

added a table to the introductory example and incorporated additional prototypical examples to facilitate the development of problem solving strategies.

The journal entry read:

*"After simplifying the context, the introductory example 2 of section 2.8, page 156 read as follows,*
*'Ashok and Lisa went for a walk. Ashok walked 6 km farther than Lisa. Together they walked a total distance of 14 km. How far did Lisa walk?*
*Solution*
*Let x kilometers represent the distance Lisa walked.*
*Then, the distance Ashok walked is (x + 6) kilometers.*

*I added the following table and explanations:*

| *Names* | *Kilometres Walked* |
|---|---|
| *Lisa* | *X* |
| *Ashok* | *X + 6* |
| *Total distance* | *14 km"* |

*Book continues with procedural steps:*
*Since the total distance they walked is 14 km.*
$X + (x + 6) = 14$
$2x + 6 = 14$
$2x + 6 - 6 = 14 - 6$
$2x = 8$
$2x \div 2 = 8 \div 2$
$x = 4$
*Lisa walked 4 km.*
*Check: Ashok walked 4 km + 6 km = 10 km.*
*The total distance was 4 km + 10 km = 14 km.*
*The solution is correct.*
*Similarly, section 2.8 shows two examples to explain how to solve problems algebraically without explaining how to get the algebraic terms from the information given in the problem. In my experience, this section is the most challenging one in this chapter because students have to translate words into algebraic equations and usually, they have trouble understanding accurately the meaning of the sentences. Consequently, I added two more introductory examples in which I showed students first, how to translate words into algebraic terms and how to transcribe the data onto tables to understand the meaning of the symbolic algebraic language, and how to create the equation they need to solve the problem."*

The above journal entry illustrates how added a table to organize the translation of sentences into algebraic terms and later into an algebraic equation.

The journal entry continues as follows:

> *"Analogously, I incorporated additional prototypical examples to section 2.1 and 2.8. Some sections have a deficient number of introductory examples to facilitate learning (2.1, 2.8). Section 2.1 called the, <u>The Concept of a Variable</u>, has just two group activities with an inquiry approach to introduce the concept of a variable and no additional examples to explain through expository instruction.*
> *The first introductory example added was taken from the practice section and reads as follows:*
> *"'An airplane travels eight times as fast as a car. The difference in their speeds is 420 km/h. How fast is each vehicle traveling?'*
> *To determine the algebraic terms that would identify the speed of the airplane and car, the following table was drawn on the chalkboard,*

| | |
|---|---|
| *Car* | *X* |
| *Airplane* | *8x* |
| | |

> $8x - x = 420$
> *First, to find out the speed of the car, x was isolated,*
> $7x = 420$
> $x = 420 \div 7$
> $x = 60$ *The speed of the car was 60 km/h.*
> $7 \times 60 = 480$ *km/h* *The speed of the airplane was 480 km/h.*
> *The second example incorporated reads as follows:*
> *Mary is three years older than Ann, and the sum of their ages is 35. How old is Ann?*
> *To introduce this problem, the same approach was taken,*
> *Ann* $x$
> *Mary* $3x$
> $3x + x = 35$
> $4x = 35$
> $x = 35 \div 4$
> $x = 8.75$ *Ann is 8.75 years old.*
> $3 \times 8.75 = 26.25$ *Mar is 26.25 years old."*
> *(teacher journal, table, 1.7-p.25)*

I chose the above examples because they show all the algebraic steps required to solve problems. First, there is a translation of algebraic sentences into symbolic algebra. Second, the algebraic terms are organized into algebraic equations, the variable is isolated, and the final numerical value interpreted. In my experience, students can

understand the algebraic processes easily if the prototypical examples show all steps required to reach the final answer.

*c) Addition of Conceptual Questions*

Conceptual understanding plays an important role in developing algebraic reasoning. Understanding the algebraic process involves the understanding of why what has been done works and also involves the development of algebraic thinking. To promote understanding and to help students get ready to acquire higher algebraic concepts, textbooks should offer activities that require students to justify why answers are correct or incorrect, to establish relationships between word problems and equations, to apply what was learned to new situations, and to give meanings to variables in equations (Driscoll, 1999). It is important to mention that the lack of conceptual questions may delay the development of algebraic concepts to ESL and non-ESL students respectively.

Differences found in the data allowed me to divide this element in two factors: "Addition of Questions that Promote Interpretation of Final Numerical Values" and "Addition of Questions the Promote the Application of Algebraic Concepts to New Situations."

*c1) Addition of Questions that Promote Interpretation of Final Numerical Values:*

Questions that engage students to reflect and synthesize are important elements that form the algebraic foundation of knowledge in this subject. I considered the students' inability to understand the final numerical values an obstacle to the development of conceptual thinking.

The following transcripts were taken from the *teacher professional journal* and indicate that a considerable number of exercises lacking in inquiry, regarding what variables stand for, the meaning of the algebraic terms in formulas, and the stating of the meaning of the answers.

In my journal I noted:

> *"After teaching section four where students had to solve simple equations, I noticed that some students did not understand what their answers meant and I did not know what the problem was. So, I took a closer look at the exercises assigned for practice in section four and in the previous sections as well, and I noticed that sections two, three and four have many exercises that promote procedural understanding and few ones that promote conceptual understanding. For instance, section two called, Representing Variables and Expressions, offers thirty five sets of exercises for practice but just seven questions are about conceptual questions where students are asked to explain or to think about the answers or the meaning of variables in formulas. Similarly, section three called, Combining Like Terms, has fourteen sets of exercises for practice where students can develop procedural knowledge but just one exercise asks students about the advantages of different approaches in solving one particular exercise. In the same way, section four poses thirteen sets of exercises; however, just five exercises have conceptual inquiries."* (teacher journal 1.9, p. 8)

> *"Section 2.7 introduces four different approaches to solving problems. The systematic trial, the use of tables, the use of algebraic equations, and the method of reasoning out the solution are explained and encouraged in this section...Since most problems do not challenge student understanding of the basic concepts, I decided to increase the number of conceptual questions in each exercise to promote better understanding. That day at night, while I was writing my journal, I realized that I had learned something new again. Mathematical oral and written communication in class is very important to understanding mathematical concepts. Based on students' assessments, I realized that I have gradually increased the number of activities that promote communication in order to facilitate understanding of mathematical concepts. Proper communication takes place only after a series of mental processes take place such as, sequencing steps, understanding concepts, organizing thoughts, and bringing words coherently together. The challenge is to find activities that promote mathematical communication for ESL students that can be utilized in a limited amount of time so that the development and learning of further mathematical skills and concepts can take place as well."* (teacher journal 1.12, p.17)

The above transcripts show my concern about the insufficient number of exercises that call for conceptual understanding and interpretation of the steps taken to solve an exercise.

The following transcript was taken from the *student interviews* and illustrates a student's inability to explain the meaning of the final numerical value. Even though he was able to apply all the steps correctly to solve the equation, and find the final numerical value, he did not have full understanding of what the solution meant.

The following transcript refers to problem 8 of the textbook, page 146 and shows the comments I wrote on the side of the student's note and the student's answers:

> *"...I asked student 2.2 to explain to me the following exercise. The problem had the formula C = 4.88 + 1.22 (n - 3), where C is the cost in dollars, n is the time in minutes, and n is greater than or equal to 3 and refers to the first three minutes of a call that costs $ 4.88 and additional minutes for $1.22. He had to determine how long he could talk for $10.*
>
> *Student 2.2:*
> $$10 = 4.88 + 1.22\,(n-3)$$
> $$10 = 4.88 + 1.22n - 3.66$$
> $$10 + 3.66 - 4.88 = 1.22n$$
> $$9.88 = 1.22\,n$$
> $$8 = n$$
>
> *All the steps seemed to be correct but, I wanted to know how much of what he had done was understood.*
>
> *Teacher: "I see that you got the right answer,", "What does n = 8 mean"?*
>
> *Student 2.2: "One minute for $ 8."*
>
> *Then, I realized that even though he knew the mechanical steps, he did not understand the meaning of n. At this point, I wondered if there was a language problem or a lack of conceptual understanding so, I asked him to write down what the questions said,*
>
> *Student 2.2: "A little bit, if you have $10, so how many minutes you van talk"*
>
> *Realizing that there was not a language problem, I continued my inquiry about conceptual understanding.*
>
> *Teacher: "Now think again, does n = 8 mean that you can talk one minute for $8'?*
>
> *Student 2.2: "Yes, does the right answer"*
>
> *Teacher: "Now", I proceeded, "What do you have to find out"?*
>
> *Student 2.2: "The number of C"*

*Teacher: "What does C mean?"*
*Student 2.2: "How many minutes for $10?"*
*Teacher: "O.K", "What does n mean?"*
*Student 2.2: "How many dollars for a minute?"*
*Teacher: "So, does it mean minutes or dollars?"*
*Student 2.2: "Minute".*

*Based on the students' assessment that day, I decided to increase the emphasis on the meaning of variables in formulas and the meaning of the final numeric values. From that day, and regardless of which problems I asked students to solve, I asked them to answer in complete sentences explaining what the final numerical value meant and explain the meaning of each algebraic term in each formula. In addition, I began to assign fewer exercises that promoted mechanical skills and increased the number of exercises that promoted problem-solving skills." (teacher journal 1.11, p.15, 2.6)*

*c2) Addition of Questions that Promote the Application of Algebraic Concepts to New Situations:*

Application type questions that ask students to use the algebraic concepts learned in real life situations, stimulate and clarify further thinking and promote understanding (Driscoll, 1999). I have found that an insufficient number of textbook exercises that promote the application of algebraic concepts constitute a hindrance to algebraic reasoning.

Data from the *teacher professional journal* illustrate my concerns about the lack of applicative questions in most sections of chapter two.

*"Section 2.6 explains how to simplify equations when they have like variables on both sides of the equation and how to solve equations involving fractions as coefficients....In the practice section, I found just one exercise that asks students to substitute the variable with the values given. The rest of the exercises promoted procedural understanding. So far, students were developing good mechanical skills according to my expectations; in other words, they could identify like terms, simplify equations and isolate variables correctly; however, the concept of a variable and its application to real life situations or its implications in formulas were not fully grasped. My goal has always been to teach for understanding. My next challenge was to implement activities that avoid mechanical solutions and promote understanding. Since I spent a*

*considerable amount of time explaining concepts and reviewing vocabulary in class, I decided to increase the amount of homework and to assign more conceptual questions." (teacher journal 1.10, p. 14)*

The following transcripts were taken from the *student interviews* and show students inability to apply the algebraic concepts to real-life situations and personal notes that I made to myself to remember my thoughts at the time of these interviews.

*" ...I wanted to determine if students understood the concept of a variable so, I asked a student to solve the following problem. I gave him a word bank of known vocabulary to help him make sentences."*

The written question offered was:

*In a game, the total points earned by a hockey team can be described using the expression 2w + t. What does each variable represent?*

*Word Bank*

| |
|---|
| Mathematically tie total score win written tournament written equation |

Student 2.5 wrote:

*Student 2.5: (without hesitation he wrote), "w = win and t = tie",*
*Teacher: "Very good student 2.5!"*

My notes read:

*"His representation of the variables coincided with their meanings. Consequently, I thought that he understood the concept of the variables. Nevertheless, I wanted to assess his understanding further, so I asked him to describe a situation in which the algebraic expression "3c-14" might be used. Again, I gave him a word bank to help him put sentences together.*

*Word Bank*

| |
|---|
| Items store dollars on sell discount total price books |

*One expected answer would be:*
*In a bookstore, there are some items on sell: if you buy three books with the same price, you have a discount of $14.00."*

Student 2.5 wrote:

> *Student 2.5: (after deliberating for at least a couple of minutes and wondering whether he should say yes or no), "yes, in a candy store, a boy bought three candies each cost 14cents, when he was paying, the owner of the stores told him that if he buy 3 candies or more, he can get one free"*

Analyzing this example, I discovered that, even though the student knew the vocabulary written in the word bank, he was unable to apply the formula to a real life situation.

I encountered a similar situation when I asked student 2.4 the same question. Student 2.4 interview read as follow:

> *Teacher: "Could you describe a situation in which the algebraic equation C = 3c-14 might be applied"?*
> *Student 2.4: (after thinking for a while), "Yes, C is the total number of books left in the library and c is the number of books the person return and -14 is another person borrow 14 books"*

My notes read:

> *"I discovered that she was unable to apply the algebraic expression to a new situation and that this was not a language problem but rather a conceptual one. Even though a number of students were able to identify and relate some variables to their contexts in class, they did not fully comprehend the idea of a variable, or at least be able to apply it to real life situations." (interviews 1.9, p. 8)*

<u>Assignment of Textbook Exercises that Promote Accurate Algebraic Procedural Skills</u>

This category illustrates textbook activities that I did not need to modify and promoted an appropriate level of procedural algebraic skills. Procedural skills involve the ability to arrange an ordered sequence of steps to accomplish a certain goal. Data collected in the student interviews and in the post written test, reveal that the textbook facilitated the learning of a high level of procedural skills. Raw data seem to indicate that

students gained high procedural skills in two different areas: "Assignment of Textbook Recommended Exercises to Solve First Degree Linear Equations" and "Assignment of Textbook Recommended Exercises to Solve Inequalities."

*a) Assignment of Textbook Recommended Exercises to Solve First-Degree Linear Equations*

The textbook offered a sufficient number of exercises that facilitated the development of the procedural skills used to isolate variables and solve linear equations. The following transcripts were taken from the *student post- tests* and show a high level of procedural skills mastery.

Exercise 6, p.12, reads as follow,

*Solve.*

$$4x-3=\frac{7}{3}-\frac{4x}{3}$$

*Student 2.10:*

$$4x-3=\frac{7}{3}-\frac{4x}{3}$$

$$4x+\frac{4x}{3}=3+\frac{7}{3}$$

$$\frac{12x+4x}{3}=\frac{9+7}{3}$$

$$\frac{16x}{3}=\frac{16x}{3}$$

*x=1* *x equals 1*

*Student 2.6:*

$$4x-3=\frac{7}{3}-\frac{4x}{3}$$

$$4x+\frac{4x}{3}-3=\frac{7}{3}$$

$$5\frac{1}{3}x=3+\frac{7}{3}$$

$$5\frac{1}{3}x=5\frac{1}{3}$$

$$x=1$$

*Student 2.2:*

$$4x-3=\frac{7}{3}-\frac{4x}{3}$$

$$12x - 9 = 7 - 4x$$

$$16x = 16$$

$$x=1$$

Students were able to collect and isolate variables confidently using different approaches to solve an algebraic equation. While student 2.10 showed all the steps required to combine like variables on one side of the equation and numbers on the other side, student 2.10 showed a more advance skill that involves a short cut in which the a common factor is utilized to get ride of the denominator and solve the exercise.

*b) Assignment of Textbook Recommended Exercises to Solve Inequalities*

The textbook exercises offer a wide variety of problems that promote the development of procedural skills to solve first-degree inequalities with one variable, to display the solutions on a number line, and to demonstrate multiple-step equations by maintaining equality at each step.

The following transcripts were taken from the *student interviews* and illustrate students' developed skills in solving inequalities.

The exercise reads as follows:

*"1) Solve.*
*a)* $-4(7+2x)\geq 3x + 5$
*b)* $-4(7+2x)=3x+5$

*2) How are the solutions alike? How are they Different?*

Student 2.8 wrote:

*Student 2.8:*

*1)*

*a)*

$$-4(7+2x) \geq 3x+5$$
$$-28-8x \geq 3x+5$$
$$-8x-3x \geq 5+28$$
$$-11x \geq 33$$
$$x \leq 33 \div -11$$
$$x \leq -3$$

*b)*

$$-4(7+2x) = 3x+5$$
$$-28-2x = 3x+5$$
$$-8x-3x = 5+28$$
$$-11x = 33 \div 11$$
$$x = -3$$

*2)*

The interview transcript read:

*Teacher:* *How the solutions are alike?*
*Student 2.8:* *because it's same answer in - sign and ≥/= this are similer.*
*Teacher:* *O.K, how are they different?*
*Student 2.8:* *if we have =sign we don't have to chang it other side but ≥ sign chang to the -≥ this side big what # is big that side it change.*
*Teacher:* *O.K, so what is the difference between these two answers, could you give me examples, : $\underline{x \leq -3}$ and $x = -3$*
*Student 2.8:* *in x equal to -3, example, -3 = -3, just one value.*

Similarly, another student wrote:

*Student 2.6:*

*1)*

*a)*

$$-4(7+2x) \geq 3x+5$$
$$-28-8x \geq 3x+5$$
$$-28 \geq 11x+5$$
$$-33 \geq 11x$$
$$-3 \geq x$$

*b)*

$$-4\,(7 + 2x) = 3x + 5$$
$$-28x - 8x = 3x + 5$$
$$-28 = 11x + 5$$
$$-33 = 11x$$
$$-3 = x$$

The interview transcript read:

*2) Teacher:* *How are they alike?*
*Student 2.6:* *They both might be equal to -3, but for the second one is one possibility, but for first one there is possibility that it's bigger than -3.*
*(interviews - section 2.9)*

Students were able to solve inequalities and interpret the solutions. In addition, they were able to realize that when an inequality is multiplied or divided by a negative number, they must reverse the inequality symbol to keep the inequality true.

In the next chapter, I interpret the results discussed above and will draw the final conclusions concerning this research. I will make some recommendations which will shed light on the use of mathematics textbooks to mathematics instructors, school administrators, editors and may demonstrate the need for further research.

# CHAPTER 5

# CONCLUSION

The focus of this research was to determine the role of a new mathematics nine textbook in the teaching and learning of algebra in an ESL class and how I needed to adapt it. I utilized the textbook as the main educational resource for the instruction of algebra nine and adapted certain textbook activities to bridge the linguistic and mathematical level of the students.

The findings of this study seem to indicate that the textbook focused more on the teaching and learning of algebraic procedural skills than the development of conceptual understanding and mathematical language acquisition.

This study helped me improve my classroom expertise and increase my personal awareness about assessing educational resources. I am more aware of the different approaches textbooks have towards the learning of mathematics and therefore, I can plan my lessons taking into consideration the adaptations needed to maximize the teaching of mathematics for understanding.

Moreover, this research helped me assess the extent to which my students understood algebraic concepts while increasing my ability to teach algebra. I became more alert in both, the critical incidents in class and the fact that ESL students of level two and three English could understand and succeed in algebra. I improved my assessment skills. This allowed me to understand the implications of student questions and answers which aided me to determine the extent of student conceptual understanding.

The contributions of this study have implications for math educators, editors, and scholars. To math educators, this study provides suggestions on how to assess mathematics textbooks. In addition, it provides examples of algebraic language activities to increase the acquisition of algebraic language for ESL and non-ESL students.

To editors, this study provides specific suggestions to improve the quality of the pictures and consequently the comprehension of the mathematical language for ESL students. This study also provides examples of conceptual questions that may increase conceptual understanding for all students.

In the scholarly community, this case study constitutes an investigation of the approach that a textbook takes in the teaching of mathematics in consideration to ESL students.

In this chapter, I interpret and analyze the data previously synthesized in chapter 4. I discuss the extent to which the new textbook facilitated the learning of algebra and the implications that this holds for educators. Finally, I suggest implications for teaching and further research in this field.

## Student Understanding of Algebraic Processes

### The Language of Algebra and the New Textbook:

According to the results found in this study, most students had difficulty with the language of Algebra, especially, translating words into algebraic symbols. ESL students seemed to indicate that the textbook activities poorly facilitated the manipulation of symbolic algebra to express relationships in algebraic terms. Such was the case with student 2.8 who, despite an introduction to algebraic sentences in the beginning of the class, was unable to comprehend what "three times as many nickels as dimes" meant.

The difficulty students had in translating words into symbolic algebraic language found in this study, seemed to coincide with what was found by Lochhead and Mestre (1998). The authors emphasize that most beginners in linear algebra struggle with the algebraic translation of problem statements. Lochead and Mestre suggest that students should be exposed for as long as they need to the practice of algebraic skills because the beginning of Algebra constitutes the most important building block to higher algebraic understanding.

The Introduction of Prototypical Examples and the New Textbook:

Since the ability of students to communicate is limited, direct instruction and prototypical introductory examples are important components in the instruction of mathematics for ESL students. This study indicates that a poor introduction of algebraic concepts and skills decreases the understanding and ability of students to solve problems in the practice section. For example, the journal transcript 1.6 and the comments of students 2.2, and 2.4 who were unable to identify equivalent equations from a pool of equations any that were equivalent. The journal transcript suggests that the textbook does not introduce the concept of equivalence in the beginning of the section.

A poorly developed introduction of the fundamental algebraic concepts and skills affect all students of algebra. This finding seems to coincide with that of Steinberg's (1990). He found that students often lack the understanding the concept of equivalence and suggests that instructors not only teach it as part of the procedural methodology but also as a conceptual study of equivalent equations.

Similarly, in order to promote understanding of procedural skills, the journal transcript 1.8 illustrates how I added the properties of equivalence to the introductory part of some sections in order to counteract the prescriptive instructions of the textbook.

*Conceptual Understanding and the New Textbook:*

According to the findings of this study, the promotion of conceptual understanding constitutes another area that needs improvement. Data from this study seem to indicate that the new textbook illustrates how to multiply, divide, subtract or add in both sides of each algebraic equation without introducing the mathematical principle or theory behind it. As a result, I explained the properties of equivalence and the steps to solve algebraic equations in general as algorithms to be used later in higher mathematical learning. For example, students 2.2, 2.5, 2.4, 2.6 and journal transcript 1.10. These transcripts illustrate the difficulties students had with stating the meaning of variables, interpreting the final numerical value, and applying algebraic concepts to new situations.

To promote conceptual understanding, I had to implement a significant number of adaptations. I supplemented the textbook activities by adding conceptual questions to the textbook exercises. Such was the case of the teacher journal transcript 1.12 where I decided to increase the number of conceptual questions. The questions ranged from asking students to explain final values in complete sentences and explaining the meaning of the variables in formulas to apply the same algebraic concepts to new real-life situations.

An increase of the number of conceptual questions in the textbook will hopefully encourage students to reflect on their answers and increase algebraic reasoning. As stated in the Principles and Standards for School Mathematics 2000, "algebraic problems can

incorporate concepts and skills successfully and provide opportunities to reflect on the answers if they are carefully designed"(p. 335). For ESL students and according to the study, problems that introduce familiar contexts and short statements but challenge them to reflect on their answers, constitute adequate exercises to stimulate students without taking over the process of thinking for them.

## Procedural Skills and the New Textbook

Algebraic procedural skills were highly promoted by the textbook. Most students were able to learn how to expand expressions applying the distributive law, to combine like terms, to isolate variables, to simplify equations when they contain fractions and to solve inequalities as stated in student transcripts 2.6, 2.10, 2.2, and 2.8.

## The Language Proficiency and the Comprehension Level of the New Textbook

This study seems to indicate that ESL students cannot comprehend word problems if they have a high number of unknown words. Such was the case with students 2.3 and 2.4 when they were unable to answer, understand or rephrase the question of a problem even though, they had some idea of how to solve the exercises because they had studied algebra before in their first language.

When ESL students encounter problems that have known vocabulary and familiar contexts, they seemed to have no problem comprehending the information given in a problem and can solve it algebraically. For instance, students 2.7 and 2.2 successfully solved a problem which had a short statement and a familiar context. Students were able to manipulate algebraic symbols, apply the properties of equivalence to find the final value of the variable, and interpret the final answer.

Regarding visual aids, most students indicated disappointment when they were asked to comment on the pictures of the textbook. Student comments seem to indicate that they often looked for textbook illustrations to help comprehend words and concepts mentioned in the problems. Regardless, the pictures did not seem to help them much. Such was the statement of student 2.6, "no, makes the book look better. Pictures are nice to see, but don't help" and student 2.3, "no, sometimes they confused me. It is not related to the math." I considered the number of pictures sufficient to illustrate concepts or words problems. Most of them are colorful and make the textbook look more appealing to read. However, pictures that are more relevant to the vocabulary utilized in the word problems and to the main mathematical concepts introduced in each chapter would help ESL students to decode the mathematical language and concepts introduced.

Linguistically, most students were expected to be at level two English language proficiency. Consequently, all the textbook adaptations were geared to level two students. Even though I used bilingual techniques to support ESL students, this study seems to indicate that ESL students at level one struggled with the learning of mathematics. That was the case illustrated in the journal transcript p.22 where I observed that students with lower language skills find the language of algebra challenging. This finding seems to coincide with the ones found by MacGregor (1990) and Clarkson (1992). MacGregor (1990) and Clarkson (1992) claim that students with low-level language proficiency may be limited in their achievement in algebra because they misinterpret words and may not be able to decode the information given. Similarly, journal transcript p. 22 indicates that students in level one English language proficiency

struggle with the comprehension of sentences and are unable to understand what the problems mean. As a result, they are unable to learn algebra.

Finally, the results seem to indicate that the new textbook needed a considerable number of language adaptations to teach Algebra to ESL students. For example, journal transcripts 1.1, 1.13, and 1.3 illustrate how often I reduced the amount of vocabulary, modified, or contextualized the statements of problems to facilitate understanding.

Some textbook activities were supplemented with vocabulary worksheets using visual aids. Other activities were supplemented with language exercises to introduce the syntax of algebraic terminology. In addition, handouts with simplified problem statements were effective adaptations that allowed students to overcome reading comprehension problems and focus on the algebraic skills and concepts promoted by the textbook.

Based on the findings of this research a number of recommendations to textbook writers can be made. To improve the quality of comprehension and to facilitate ESL student understanding, it is recommended that current pictures in the textbook be substituted with visual aids that are related to the algebraic concepts taught and to the vocabulary utilized in the word problems. The number, colour and size of the pictures seemed to be attractive to students but did not provide the support students were looking for. Also, that the textbook should increase the number of language enhancing activities through the assignment of questions that promote mathematical communication and interpretation of final values, and that word problems have short, contextualized statements. The Principles and Standards for School Mathematics (2000) claims, " mathematical concepts, processes and ideas can successfully be introduced in

contextualized problems if they are carefully sequenced, designed, and engaging. However, they do not have to be long and they have to provide opportunities to reflect on the answers"(p. 335).

A New Textbook as the Main Source of Algebraic Instruction for ESL Students: Implications for Educators

An adequate educational resource can facilitate the learning and teaching of mathematics for ESL students and the instructor, reduce the level of stress on both students and teachers, and increase the content covered in a school year. The new textbook utilized in this study was a valuable educational resource for the learning of algebraic procedural skills. Nevertheless, the textbook requires additional activities to maximize the development of conceptual understanding and algebraic language acquisition in ESL students.

One of the main strengths of the textbook is the variety of exercises available and the number of problems that promote algebraic procedural skills as stated in journal transcripts 1.9 and 2.3. In addition, despite the adaptations implemented that tend to add deeper understanding to the entire chapter, the sequence of algebraic skills introduced through the chapter was adequate, and facilitated a gradual understanding of procedural skills.

Nonetheless, this study indicates that the textbook poses certain conceptual algebraic constraints. An increase of conceptual type questions and activities that promote mathematical interpretation of variables, final numerical values and the application of concepts can optimize the teaching and learning of algebraic concepts.

Based on the findings of this study and the literature review, I will use the new textbook as the main educational resource to teach mathematics 9 along with supplementary worksheets to reinforce algebraic language acquisition and conceptual understanding for ESL students.

## Implications for Further Research

I believe that Mathematics textbooks can challenge ESL students to reach their potential without jeopardizing a positive attitude in learning. How many words per class a student is capable of learning, what level of mathematical abstraction they are able to reach, and how many mathematical concepts they can grasp in a school year, influence the performance of students concerning contemporary mathematical standards and constitute factors that deserve further research in this field.

It would be interesting to determine the extent to which educators could cover the standard mathematics nine curriculum in order to best facilitate ESL students. Perhaps, if instructors use a comprehensive textbook and are able to cover a sufficient number of mathematical themes prior to being mainstreamed, ESL mathematics courses could be registered and counted as official creditation toward high school graduation. More importantly, ESL students could join the mainstream courses quickly and finish high school sooner.

This study attempted to advance a number of potentially effective strategies to assess mathematics textbooks, to adapt textbook activities for ESL classes, to assess student outcomes, and to disseminate the knowledge I gained in my pedagogical research. Moreover, in order to bring about satisfactory results in this subject, instructors need the support of editors. Textbooks that suit curricular demands and take into consideration

every student in the classroom, would constitute an important asset to any educator and play a major role in raising the potential and standards of our students in Mathematics.

# REFERENCES


Alexander, et. al. (1996). Minds on Math 9. Canada: Addison-Wesley Publishers Limited.

Alexander, et. al. (1987). Mathematics 9. Canada: Addison-Wesley Publishers Limited.

Arhar, J., Holly, M., Kasten, W. (2001) Action research for teachers. New Jersey: Prentice-Hall, Inc.

Borgen, K. (1998). Teacher use of language which helps or hinders understanding of mathematics by second language learners. M.A thesis. B.C.: UBC Library.

Bruner, J. (1966). Toward a theory of instruction. Cambridge, MA: Harvard University Press.

Chandler, D. (1995). A comparison between mathematics textbook content and a statewide mathematics proficiency test. School Science & Mathematics 95 (3), 118-123.

Clarkson, P. & Galbraith, P. (1992). Bilingualism and mathematics learning: another perspective. Journal for Research in Mathematics Education 23 (1), 34-44.

Collins Cobuid English Dictionary, 1987. First Edition. London: Harper Collins Publisher.

Chun, D., Plass, J. (1997). Research on text comprehension in multimedia environments. Journal for Language Learning & Technology. 1 (1), 60-81.

D' Ambrosio, U. & D' Ambrosio, B. (1994). An international perspective on research through the JRME. Journal for Research in Mathematics Education 25 (6), 685-696.

Dawe, L. (1983). Bilingualism and mathematical reasoning in English as a second language. Educational Studies in Mathematics 14 (4), 325-353.

Driscoll, M. (1999). Fostering algebraic thinking. NH: Educational Development Center, Inc.

Ghosh, S., Giri, S. (1987) Understanding secondary mathematics: analysis of linguistic difficulties vis-a-vis errors. International Journal Mathematics Education Science Technology 18 (4), 573-579.

Groves, F. (1995). Science vocabulary load of selected secondary science textbooks. School Science and Mathematics 95 (5), 231-235.

Hativa, N. (1985). A study of the organization and clarity of mathematics lessons. International Journal Mathematics Education Science Technology 16 (1), 89-99.

Hubbard, R. (1990). Teaching mathematics reading and study skills. International Journal Mathematics Education Science Technology 21 (2), 265-269.

Huntley, A., Rasmussen, C., Villarubi, R, Sangtong, J. & Fey, J. (2000). Effects of standards -based mathematics education: a study of the core-plus mathematics project algebra and functions strand. Journal for Research in Mathematics Education 31 (3), 328-361.

Hutchinson, N. (1986). Instruction of Representation and Solution in Algebraic Problem Solving With Learning Disabled Adolescents. PhD thesis. Simon Fraser University.

Instructional-Design Theories. (2003). Module 5: Principles for Teaching Principles. Indiana Education. Web site: http://www.indiana. edu/~idtheory/methods/module_5_4.html.

Jones, L., P., (1982). Learning mathematics in a second language: a problem with more and less. Educational Studies in Mathematics 13 (3), 269-287.

Kang, W. & Kilpatrick, J. (1992). Didactic transposition in mathematics textbooks. For the Learning of Mathematics 12 (1), 2-7.

Kelly, A & Lesh, R. (2000). Handbook of research design in mathematics and science education. NJ: Lawrence Erlbaum Associates, Inc.

Kvale, S. (1996). Interviews. An introduction to qualitative research interviewing. California: SAGE Publications, Inc.

Lancy, D. (1993). Qualitative research in education. An introduction to the major traditions. New York: Longman Publishing Group.

Lochhead, K & Mestre, J. (1988). From words to algebra: mending misconceptions. The Ideas of Algebra, K-12. National Council of Teachers of Mathematics. Reston: The National Council of Teachers of Mathematics, Inc.

MacGregor, M., Price, E. (1999). An exploration of aspects of language proficiency and algebra learning. Journal for Research in Mathematics Education 30 (4), 449-467.

Marshall, S., Gilmour, M., Lewis, D. (1992). Lexical knowledge and mathematics education in science and technology in Papua New Guinea. International Journal Mathematics Education Science Technology 23 (2), 211-218.

Mertens, D., (1998). Research methods in education and psychology. California: Sage Publications, Inc.

Murphy, M. (1996). Using Calculators to Teach Algebra: From Numbers to Operations to Structure. MA Thesis. The University of British Columbia.

Musick, Mark, et. el. (2002). Mathematics Framework for the 2003 National Assessment of Education Progress. Washington: National Assessment Governing Board. www.nagb.org/pubs/math_framework/toc.html.

National Council of Teachers of Mathematics. News Bulletin. (2002). Volume 39, Issue 1.

National Council of Teachers of Mathematics. News Bulletin. (2002). Volume 39, Issue 2.

National Council of Teachers of Mathematics. News Bulletin. (2001). Volume 38, Issue 5.

National Council of Teachers of Mathematics. News Bulletin. (2000). Volume 39, Issue 1

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: Author.

National Council of Teachers of Mathematics. (1995). Journal for Research in Mathematics Education. Monograph number 7. A cognitive analysis of U.S. and Chinese students' mathematical performance on tasks involving computation, simple problem solving, and complex problem solving. Reston, VA: Author.

National Council of Teachers of Mathematics (1990). News Bulletin. Volume 83, p. 604

Nelson, L. (1962). Relation of textbook difficulty to mathematics achievement in junior high school. Thesis: Doctor of Philosophy. University of Minnesota.

Palys, T. (1997). Research decisions. Quantitative and qualitative perspectives. Ontario: Harcourt Canada Ltd.

Paiget, Jean (1973). To understand is to invent: the future of education. New York: Grossman's Publishing Co.

Pirie, S. & Martin, L. (1997). The equation, the whole equation and nothing but the equation! One approach to the teaching of linear equations. Educational Studies in Mathematics 34, 159-181.

Pirie, S. & Kieren, T. (1994). Beyond Metaphor: Formalising in mathematical understanding within constructivist environments. For the Learning of Mathematics 14 (1), 39-43.

Pizzini, E., Shepardson, D., Abell, S. (1995). The questioning level of select middle school science textbooks. School Science and Mathematics 92 (2), 74-79.

Schon, D. (1987). Educating the reflective practitioner. California: Jossey-Bass Inc.
Silver, E., Kilpatrick, J. (1994). E pluribus unum: challenges of diversity in the future of mathematics education research. Journal for Research in Mathematics Education 25 (6), 734-754.

Spanos, G. (2000). ESL math and science for high school students: two case studies. Third National Research Symposium in Limited Proficient Student Issue: Focus on Middle and High School Issues (2000). Paper on line: http:/learnweb.Harvard.edu/alps/thinking/docs/article2.html.2000

Steinberg, R., Sleeman, D., Ktorza, D. (1990). Algebra students' knowledge of equivalence of equations. Journal for Research in Mathematics Education 22 (2), 112-121.

Stockdale, Susan. (1991). A study of the frequency of selected cue words in elementary textbooks word problems. School Science & Mathematics 91 (1), 15-21.

Tate, W. (1997). Race-ethnicity, ses, gender, and language proficiency trends in mathematics achievement: un update. Journal for Research in Mathematics Education 28 (6), 652-679.

Tripp, D. (1993). Critical incidents in teaching. NY: Routledge.

Wiersma, W. (2000). Research methods in education. An introduction. MA: Allyn & Bacon.

Wilson, P. (1993). Research ideas for the classroom. High school mathematics. NY: Macmillan Publishing Company.

Date : _____
Number: _____

## Post-Lesson Student Inteviews: Sample

**Objective**: to assess the extent of students' understanding of mathematical concepts and skills in each section of algebra.

### Section 2.1

**Students Learning Outcomes:** students will learn to describe the meaning of a variable.

**Assessment:**

1) In a game, the total points earned by a hockey team can be described using the expression 2w +t.
   a) Rephrase the statement

Word Bank

| | | | |
|---|---|---|---|
| Mathematically | tie | total score | |
| tournament | win | written | equation |

**Rate:**

_____

_____

   b) What does each variable represent?

_____

_____

_____

**Rate:** _____

2) Describe a situation in which the expression 3c-14 might be used.

_____

_____

_____

**Rate:** _____

### Section 2.2

**Students Learning Outcomes:** students will learn to work with algebra tiles to simplify and evaluate expressions and to extend that knowledge to symbolic representations.

**Assessment:**

1) Use algebra tiles to show why 3(x-2) can be represented by 3x - 6. Explain or draw your work.

_____

_____

_____

**Rate:** _____

- What does the statement tell you?

Word Bank

| |
|---|
| Variable the same as with equal |
| Equivalent to equal value |

_____

_____

**Rate:** _____

2) The expressions 6(4x + 9) and 24x + 54 are equivalent. How would you explain it to a friend on the phone?

Word Bank
Multiply times plus
By add same value

_____

_____

_____

**Rate:** _____

- Identify the variables

_____

___

**Rate:** _____

**Section 2.3**

**Students Learning Outcomes:** students will learn to identify like terms and combine them to simplify expressions.

**Assessment:**

1) Which of the following are like terms?

16x, 7y, -7n, 8z, -12x, -15x, -15y, -3z, n%2

_____

_____

**Rate:** _____

2) Give an example of two unlike terms.

_____

**Rate:** _____

3) Tell why 10a - 3b is in simplest form.

_____

_____

**Rate:** _____

**Section 2.4**

**Students Learning Outcomes:** students will learn to solve equations using algebra tiles and to record the corresponding algebraic statement.

**Assessment:**

1) Solve:

$5y + 9 = -5 - 2y$

**Rate:** _____

a) What are you been asked to do?

Word Bank
Value replace isolate
Find evaluate variable

_____

_____

_____

**Rate:** _____

b) What is your first step?

_____

_____

**Rate:** _____

c) How can you know that your solution is correct?

_____

_____

**Rate:** _____

d) Show me another way to solve the equation.

_____

_____

_____

**Rate:** _____

## Section 2.5

**Students Learning Outcomes:** students will learn to solve equations algebraically.
**Assessment:**

1) Write the steps for solving the equation:
$4y = -18 + 2y$

_____
_____
_____
_____

**Rate:** _____

2) Describe how solving $C = 28.50 + 0.15d$ for d is like solving $-x = 16 + 3x$ and how it is different.

_____
_____
_____
_____

**Rate:**_____

3) Describe a method for checking the solution of an equation, and explain why it makes sense.

_____
_____
_____

**Rate:** _____

4) Use $p = (l + w)$
Find P when l=5cm and w=3cm

## Section 2.6

**Students Learning Outcomes:** students will learn to solve equations that require simplification first.
**Assessment:**
1) Three students solved $x\%3 - 3 = 3\%4 + 1$

**Rate:** _____

5) What other method would you use to solve the same equation?

_____
_____
_____

**Rate:** _____

## Section 2.7

**Students Learning Outcomes**: students will learn to select from various problem-solving strategies, including systematic trial, making a table, using an equation, and reasoning.

**Assessment:**

1) a) Find four different ways to solve the following problem. Select the method you think is most appropriate. Explain your choice.

A parking meter contains $36.85 in dimes and quarters. There are 223 coins in total. How many quarters are there? (Final answer, 97 quarters)

_____
_____
_____
_____
_____
_____
_____

**Rate:** _____

b) Rewrite the statement in a different way.

Word Bank

| |
|---|
| Amount number coins of 25cents coins of 10 cents bills algebra total has equation maximum |

_____
_____
_____
_____
_____

c) What information are you given that will help?

_____
_____
_____

**Rate:** _____

d) What method would you use?

_____
_____
_____

**Rate:** _____

e) How could you check that you have the correct answer?

_____

_____

_____

**Rate:** _____

f) Identify the most important information given in the following problem:

Suppose it took Martha 5 h to travel 91 km. She cycled for part of the way and ran the other part. Her cycling speed is 21 km/h and her running speed is 7 km/h. Do you think she spent more time running or cycling?

_____

_____

_____

_____

**Rate:** _____

g) Create a similar problem like the one above.

_____

_____

_____

_____

**Rate:** _____

## Section 2.8

**Students Learning Outcomes:** students will learn to describe the advantages and disadvantages of solving a problem using an equation.

**Assessment:**

1) Find three numbers whose product is 2431.

_____

_____

_____

_____

**Rate:** _____

2) The mass of a cat and a dog is 21 kg. The dog's mass is 2.5 times the mass of the cat. a) Find the mass of each animal.

_____
_____
_____
_____

**Rate:** _____

b) Write an equation for the statement mentioned above.

_____
_____
____

**Rate:** _____

3) A piece of rope is 75m long and is cut into two pieces. One piece is five times the length of the other.

a) Draw the most important information given in the statement.

**Rate:** _____

b) How long is each piece?

_____
_____
_____
_____

**Rate:** _____

c) What other method would you use to solve the problem mentioned above?

_____
_____
_____
_____

**Rate:** _____

d) Create a similar problem to the mentioned above.

_____
_____
_____
_____

**Rate:** _____

## Section 2.9

**Students Learning Outcomes:** students will learn to solve inequality, interpret and graph the solution.

**Assessment:**

1) Solve. How are the solutions alike? How are they different?

Word Bank
Values only many one
Part have can

_____
_____
_____
_____

**Rate:** _____

2) How is solving an equation like solving an inequality?

Word Bank
Steps reverse negative
Sign number divide by
Same whenever multiply
Add subtract

_____
_____
_____
_____

**Rate:** _____

## Demographic Questionnaire

**Objective:** to provide information about the classroom context. The information gathered will be organized in categories for further analysis.

- Please complete:

Age: _____ Gender (please circle): F / M

1) How long have you been in Canada?

_____

3) In what country were you born?

_____

4) How many close friends do you have?

_____

5) What is your first language?

_____

6) Please indicate how much you understand the spoken language mentioned above

100 - 70% 50 - 69% Less than 50%

7) What is you second language?

_____

Please indicate how much you understand when you read in that language (please circle)

100 - 70% 50 - 69% Less than 50%

8) In what language do you think when you do your mathematics?

_____

9) How many members of your family are living with you? (Please circle)

1 2 3 4 5 6 7 8 9 10

10) Does your mother or father live with you? (Please circle)

Both Mother Father Neither

11) What do you do most days after school? (Please circle)

Talk to friends Watch T.V Study Play Games Read

Others _____

12) In what language do you do the activity/ies mentioned above?

_____

13) What language do you speak at home?

_____

14) Do you do math homework at home? (Please circle) Yes / No

14) How often? (Please circle)

Once a week Twice a week Three times a week or more

15) For how long? (Please circle)

15 min or less 30 min 45 min 60 min or more

16) Do you have a math tutor or any other extra help in math? (Please circle)

Yes / No

17) How much do you understand the English spoken in your mathematics classroom?

100 - 70% 50 - 69% Less than 50%

18) How much do you understand the English written in your mathematics textbook?

100 - 70% 50 - 69% Less than 50%

19) Do you like mathematics?

10 9 8 7 6 5 4 3 2 1 0

20) What was you average in the last math course you took? (Please circle)

0 - 45% 46 - 59% 60 - 69% 70 - 100%

21) What was the last mathematics course you took equivalent to the Canadian educational system? (Please circle)

Grade 1 2 3 4 5 6 7 8 9 10 11 12

22) How much do you like Canada? (Please circle)

10 9 8 7 6 5 4 3 2 1 0

23) What do you want to do after high school? (Please circle)

Work In what area

_____

Study In what field

_____

At what level: College University Others

_____

24) Are you going back to your first country in the next 5 years? (Please circle)

Yes / No