CHILDREN'S CONCEPTS ABOUT THE SLOPE OF A LINE GRAPH

By

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We accept this thesis as conforming

to the required standard

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ABSTRACT

This study is concerned with how children interpret the slope of a line graph. Today with the vast accumulations of data which are available from computers, people are being faced with an ever increasing amount of pictorial representation of this data. Therefore it is of the utmost importance that children understand pictorial representation. Yet in spite of the popularity of graphs as tools of communication, studies show that many adults experience difficulty in reading information presented in a graphical form.

The slope of the graph was chosen for this investigation because it is in this aspect of graphing (as shown by the results of the 1981 B.C. Assessment) that children in British Columbia seem to have the greatest difficulty when they reach Grade 8. The study dealt with positive, negative, zero and infinite slopes, combinations of these slopes, curvilinear graphs and qualitative graphs, that is, graphs that have no numerical data shown on the axes.

The researcher chose to use a structured individual interview as a means of collecting data about how the students interpreted the slope of a line graph. Graphs used in the interviews dealt with temperature, height, weight and distance. Twenty-two students were chosen for this study.

The students were found to have problems mainly with graphs dealing with distance related to time. This problem may be due to the fact that many students read only one axis and when interpreting
distance seem to include direction as an added dimension of the graph. Infinite slope graphs were misinterpreted by every student, which may be due to the fact that they ignore the time axis. In general students used two methods of interpreting graphs. In some cases they observed the direction of the graph from left to right, that is, whether the slope went up or down from left to right. In other cases they examined the end points on the graph and drew their conclusions from them. The choice of method varied with the contextual material shown on the graph, which may be due to the children's concept of the parameter in the physical world and whether they see the parameter as being able to increase and decrease over time.

From the study the investigator feels that more discussion of graphing by teachers and students is needed if the misconceptions are to be cleared up. Discussion of the parameters of both axes by teachers might help clear up the misconceptions students have about distance travelled over a period of time when this is expressed as a graph. There would be less chance of a graph being read as a map if the relationships between the two axes were demonstrated to students. Teachers also need to be aware of both methods used by students in interpreting graphs.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>LIST OF TABLES.</th>
<th>vi</th>
</tr>
</thead>
<tbody>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>1. THE PROBLEM</td>
<td>1</td>
</tr>
<tr>
<td>Statement of the Problem</td>
<td>5</td>
</tr>
<tr>
<td>Method of Study</td>
<td>7</td>
</tr>
<tr>
<td>Educational Significance of the Study</td>
<td>7</td>
</tr>
<tr>
<td>Limitations of the Study</td>
<td>8</td>
</tr>
<tr>
<td>2. REVIEW OF THE RELATED LITERATURE.</td>
<td>10</td>
</tr>
<tr>
<td>Literature of general Relevance to Graphing</td>
<td>10</td>
</tr>
<tr>
<td>in the Schools</td>
<td></td>
</tr>
<tr>
<td>Research Relevant to Children's Ability to Read Graphs</td>
<td>20</td>
</tr>
<tr>
<td>Research Relevant to the Understanding of Graphs</td>
<td>26</td>
</tr>
<tr>
<td>Summary</td>
<td>30</td>
</tr>
<tr>
<td>3. METHODS AND PROCEDURES.</td>
<td>32</td>
</tr>
<tr>
<td>The subjects</td>
<td>33</td>
</tr>
<tr>
<td>Rationale for Using the Structured Interview Method.</td>
<td>34</td>
</tr>
<tr>
<td>The Development of the Instrument</td>
<td>35</td>
</tr>
<tr>
<td>The Interview Procedure</td>
<td>36</td>
</tr>
<tr>
<td>Method of Analysis</td>
<td>37</td>
</tr>
<tr>
<td>4. THE RESULTS OF THE INVESTIGATION.</td>
<td>39</td>
</tr>
<tr>
<td>Children's interpretation of graphs with a positive or a negative slope.</td>
<td>39</td>
</tr>
<tr>
<td>Children's interpretation of graphs with a zero slope.</td>
<td>44</td>
</tr>
<tr>
<td>Children's interpretation of graphs with an infinite slope</td>
<td>48</td>
</tr>
<tr>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Children's interpretation of two graphs which have different slopes</td>
<td>54</td>
</tr>
<tr>
<td>Children's interpretation of the slope of curvilinear graphs</td>
<td>59</td>
</tr>
<tr>
<td>Children's interpretation of individual graphs each of which has a combination of positive, negative and zero slopes</td>
<td>64</td>
</tr>
<tr>
<td>Qualitative Graphs</td>
<td>69</td>
</tr>
<tr>
<td>Discussion of Results</td>
<td>81</td>
</tr>
<tr>
<td>5. SUMMARY AND CONCLUSIONS</td>
<td>84</td>
</tr>
<tr>
<td>The interpretation of graphs with different contexts</td>
<td>85</td>
</tr>
<tr>
<td>Questions arising from the presentation of different slopes</td>
<td>91</td>
</tr>
<tr>
<td>Qualitative Graphs</td>
<td>92</td>
</tr>
<tr>
<td>Implications for Instruction</td>
<td>93</td>
</tr>
<tr>
<td>Recommendations for Further Research</td>
<td>94</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>96</td>
</tr>
<tr>
<td>APPENDIX A: The Instrument</td>
<td>99</td>
</tr>
<tr>
<td>APPENDIX B: Interviewing Checklist</td>
<td>138</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. CHILDREN'S INTERPRETATION OF GRAPHS WITH A POSITIVE OR NEGATIVE SLOPE.</td>
<td>74</td>
</tr>
<tr>
<td>II. CHILDREN'S INTERPRETATION OF GRAPHS WITH A ZERO SLOPE.</td>
<td>75</td>
</tr>
<tr>
<td>III. CHILDREN'S INTERPRETATION OF GRAPHS WITH AN INFINITE SLOPE</td>
<td>76</td>
</tr>
<tr>
<td>IV. CHILDREN'S INTERPRETATION OF GRAPHS WHICH HAVE DIFFERENT SLOPES.</td>
<td>77</td>
</tr>
<tr>
<td>V. CHILDREN'S INTERPRETATION OF THE SLOPE OF CURVILINEAR GRAPHS.</td>
<td>78</td>
</tr>
<tr>
<td>VI. CHILDREN'S INTERPRETATION OF INDIVIDUAL GRAPHS EACH OF WHICH HAS A COMBINATION OF POSITIVE, NEGATIVE AND ZERO SLOPES</td>
<td>79</td>
</tr>
<tr>
<td>VII. CHILDREN'S INTERPRETATION OF GRAPHS SHOWING NO NUMERICAL DATA.</td>
<td>80</td>
</tr>
</tbody>
</table>
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Chapter 1

THE PROBLEM

The interpretation and use of graphs is an aspect of mathematics which has appeared in the school curriculum for many years. While it is true that at the present time a variety of ways of displaying data is included under the general heading "graphs", traditionally the most common kind of graph that has received great attention in schools, especially secondary schools, has been that which employs a rectangular coordinate system.

The use of coordinates is commonly ascribed to Rene Descartes (1596-1650), although it seems that the notion of a coordinate system played little, if any, part in his work. It is known that a rectangular coordinate system was used by Hipparchus (c 161-126 B.C.) to locate places on the Earth's surface, using a 'longitude', the distance from east to west along the Mediterranean, which was the 'length' of the then known world, and his place of writing, Rhodes, as a meridian. It was probably Professor Chrystal in his Algebra: An Elementary Textbook for the Higher Classes of Secondary Schools and Colleges published in 1886 who first placed the study of graphs in the school curriculum in this country (Kerslake, 1981, p.120).

Kerslake meant by "this country", Britain, and probably Professor Chrystal's book would also have been carried to the United States by teachers of the day, thus influencing that country's curriculum as well.

The understanding of graphs may be of little importance in the daily lives of many individuals, but this does not lessen the dependence of society as a whole on such knowledge. Today people are confronted with various pictorial representations of data and with the vast accumulation of data which will be available in the new computer
age, people will be faced with ever increasing amounts of pictorial information. Therefore it is of the utmost importance that children understand pictorial representations. In spite of the fact that graphs are a popular, useful tool of communication, studies show that many adults experience difficulty in reading information presented in graphical form (Vernon, 1950; Meserve and Sobel, 1972). In a report to the Ministry of Education of the Province of British Columbia, Robitaille (1981) stated that the results of the British Columbia Mathematics Assessment showed a very weak result on item B/19. The graph shows the speed of three cars, A, B and C.

![Graph showing the speed of three cars, A, B, and C.](image)

Which car is travelling fastest?

In order to answer this question students must have some understanding of the relationship between rate and slope. Most respondents chose the graph with the longest line segment as the one which represented the greatest rate. Only 17% of the students gave the correct answer. Analyses of achievement test scores in the Concepts in
Secondary Mathematics and Science study (1979), shows that although very little difficulty was experienced by pupils with elementary items on block graphs, and the use of rectangular coordinates to plot points, there was difficulty with items dealing with the understanding of continuity (Kerslake, 1981 p.122). Another area of difficulty shown by the results of this test was with the distance-time graph (Question 7).

In question 7 the students were told that the graph represented a journey John made to a disco which was 3 miles away from home. They were also told that John walked in one direction and rode the bus in the other direction. The questions asked were:

1. At what time did he get to the disco?
2. What do you think he was doing between 7.00 and 8.45?
The difficulties experienced by students in answering the question did not show during the written test but only when the subjects were interviewed. Kerslake (1981, p.128) states:

However the earlier interviews had suggested that several pupils found difficulty with travel graphs, even though they appeared to be able to give the correct answer. It was clear that several of those interviewed had incorrect perceptual interpretations of the graph. Some thought of the graph as a journey that was up and downhill, or directional on the ground and found it difficult to deal with the abstract of 'distance from an origin', in this case the boy's home.

There is some evidence that those children who are particularly strong visualisers, found extra difficulty with this question (Question 7) and with some other items in the test where a graph can be visually misleading (Kerslake, p.128).

One area of recent educational interest in cognitive psychology has been the role that intuitive beliefs play in the development of formal concepts. A number of investigations have suggested the importance of these early beliefs as they evolve into mature concepts (Ausubel, 1968; Driver and Easley, 1978; Erickson, 1980; Hobbs and Erickson, 1980). It has been suggested that knowledge of these beliefs could benefit the educational community by raising the awareness of teachers and curriculum developers with respect to some of the many perspectives that students bring to the classroom.

Accordingly, this study focuses on children's interpretation of the slope of a line graph in which time is recorded on one axis and temperature, height, weight or distance is recorded on the other axis; that is, the focus is on the misunderstanding of the relationship between slope and rate of change.
Statement of the Problem

The purpose of this study therefore is to collect evidence from Grade 7 students about some of their conceptions concerning the slope of a line graph that has time on one axis and temperature, height, weight or distance on the other axis. The graphs used may show a gain, a loss or both a gain and a loss in temperature, height, weight or distance during a given time interval.

The slope of the graph was chosen for this investigation because it is in this aspect of graphing (as shown by the results of the 1981 B.C. Assessment) that children in British Columbia seem to have the greatest difficulty when they reach Grade 8.

The investigation concerns the ways in which students respond to questions that ask them what they can tell about the real-world changes depicted by a "line graph" simply by considering the slope of the graph. This investigation analysed student responses to questions concerning the interpretation of the data:

1a When shown a graph with a constant positive slope, that is a graph where the parameter in question is increasing with time, or showing a gain in temperature, weight, height or distance.

1b When shown a graph with a constant negative slope, that is a graph where the parameter in question is decreasing with time, or showing a loss in temperature, weight or distance.

1c When shown a graph with a zero slope, that is a graph where the parameter in question is neither increasing or decreasing with
time, or showing no change in temperature, height, weight or distance.

1d When shown a graph with an infinite slope, that is, a graph where the parameter in question increases or decreases instantaneously.

2a When shown two graphs where the slope on one of the graphs is greater than on the other, that is, two graphs where the parameters in question both increased or both decreased but at different rates, one showing a greater rate of change in temperature, growth, weight or distance travelled in a given time than the other.

2b When shown two graphs where the slope on one of the graphs is less than on the other, that is, two graphs where the parameters in question both increased or both decreased but at different rates, one showing a smaller rate of change in temperature, growth, weight or distance travelled in a given time than the other.

3a When shown a graph with a combination of positive, negative and zero slopes, that is, a graph where a parameter increases, remains constant and decreases with time, showing changes in temperature, height, weight or distance travelled.

4a When shown a graph with a positive, but non-constant slope, that is, a graph where the parameter in question increases in a curve, showing that there has been an increase in the rate of change in temperature, height, weight or distance travelled in a given time.

4b When shown a graph with a negative but non-constant slope, that is, a graph where the parameter in question decreases in a curve, showing that there has been a decrease in the rate of change in temperature, height, weight or distance travelled.
Method of Study

The researcher has chosen to use a structured individual interview as a means of collecting data about how the students interprets the slope of a line graph. The researcher is attempting to obtain responses children give concerning the slope when they are shown a line graph and from the responses to identify the difficulties. Also the researcher is attempting to identify whether contextual changes of temperature, height, weight and distance have any influence on those responses. A set of graphs and associated questions was prepared embracing all of the variations of slope of a line graph as listed above (1a-4b). The data were collected by means of structured interviews and the study is descriptive and exploratory in nature. It is hoped that the discussion of the findings of this study will serve to generate more specific hypotheses for future research.

The 22 subjects chosen for this study were a 'sample of convenience' where each student was randomly selected from a group of consenting students.

Educational Significance of the Study

Obtaining knowledge of the intuitive ideas which children have regarding the slope of a line graph would be of great assistance to the educational community. By determining the nature of these intuitive ideas, it may be possible to create special instructional strategies which will promote the optimal development of desired concepts.
In particular, the findings of this study may aid curriculum developers to produce specific materials which emphasize special strategies for dealing with certain misconceptions. Perhaps less emphasis on the teaching of coordinates would be in order, with more emphasis being placed on such things as the parameters being represented on the graph, and on the rate of change shown by a graph. Teachers would benefit from the knowledge of these intuitive ideas by having a better understanding of where children's difficulties lie when presenting graphs. From such knowledge teachers would know where difficulties are likely to occur and thus be ready to correct probable misconceptions during the instruction period.

Limitations of the study

In this study it is assumed that the structured individual interview is a good one for ascertaining the child's intuitive ideas about the slope of a line graph. However, there are limitations as to the validity of this technique especially with reference to determining the correct interpretation of the child's responses. One method which might be employed to validate the final belief summaries would be to have the data analysed by other researchers and then to cross-check their conclusions with the present conclusions.

The small sample size also imposes a restriction on the generalizability of the results. It must be recognized that the findings of the present study concerning the conceptions that children have regarding the slope of a line graph may only be typical of the subjects interviewed. External validity can be increased only if
similar research with a larger, more representative group of children is conducted in the future.

In this study it is assumed that the subjects have similar instructional and experiential backgrounds in graphing. Again however it must be recognized that important differences may exist between subjects in terms of both instruction in school and exposure to graphs in other settings. Only a detailed background research study would reveal the degree of sample homogeneity in this regard.
Chapter 2
REVIEW OF THE RELATED LITERATURE

The literature review for this study has been divided into three categories.

1. Literature which discusses the need for the teaching of graphing in schools and lists the various skills necessary for graphing mathematical concepts.

2. Literature which discusses the ability of children to read graphs, that is, the ability of a child to read numerical data from the graph, such as quantities, and the ability of a child to make use of such things as keys.

3. Literature which discusses the ability of a child to interpret graphs, that is, to relate information given on the graph to the real world changes that are being portrayed.

Literature of general Relevance to Graphing in the Schools.

This section deals with those aspects of graphing that researchers deem to be the most important and essential elements. In 1977 the National Council of Supervisors of Mathematics included reading, interpreting and constructing tables, charts, and graphs in its definition of ten basic skills.

Graphing is not just the concern of the mathematics teacher. From the literature it can be seen that graphing is used by science, social studies and reading teachers as well. Edward Fry (1981) lists several
reasons for graphical literacy including:

1. Graphs quickly communicate a concept, often better than words.
2. Graphs pack a high density of information into a small area.
3. The ability to read graphs is becoming increasingly important because they are being more widely used in newspapers, magazines, textbooks and on television.
4. Computers are being programmed to draw graphs in order to simplify massive amounts of statistics of complex mathematical data.
5. Offset printing now so widely used, makes the reproduction of graphs easier than former printing processes.

Students today can look forward to an increasing use of graphical presentation on computer terminals, in print and in the classroom (Fry, 1981, p.386).

The need for better graphical literacy has been proclaimed for decades. Edwin Eagles in 1942 noted that newspapers showed a definite trend towards an increased use of graphical methods. He complained that the construction and the use of statistical graphs was at the time inadequately treated in mathematical textbooks (Eagles, p.127). In 1939 the Progressive Education Association in their publication Mathematics in General Education recognized the importance of graphical methods (Eagles, 1942). Why then after all these years is the problem still being discussed? Is it that there is not enough known about the best method by which to teach graphing? Before discussion can take place about the best method of teaching graphing more needs to be known about the way that children look at and interpret graphs.
In the search of the literature there were no studies found that were solely concerned with the understanding of graphs. The understanding of graphs seems to have been examined only as a small part of a much larger study in which children's understanding of many mathematical concepts are discussed.

Before one can look at the ways in which children interpret graphs one needs some understanding of what skills are necessary for graphing. One article which looks at these skills is "Graphing as a Communication Skill" by James V. Bruni and Helene Silverman (1975).

Two skills that they deemed necessary are:

1. The ability to sort objects.
2. The ability to match objects.

Another article that lists skills that were considered to be necessary for graphing is "Primary Mathematics a Further Report for the Mathematical Association. 1970." These skills are:

1. The ability to understand relationships,
   a. One-to-one.
   b. Many-to-one.
   c. One-to-many.
   d. Many-to-many.
   e. Comparisons, that is, greater than, less than, more than and other similar comparisons.
   f. Abstract relationships, that is, 1/2 of, twice as many, and other similar relationships.
2. The ability to make deductions after drawing a graph.
3. The ability to make scale drawings. (p.25).

Other articles dealing with graphing skills, discuss skills needed to draw inferences and to make predictions:

1. Organizing observed data.
2. Identifying a pattern in the data.
3. Based on the pattern, predicting events not yet observed.
From the three articles discussed above it can be seen that there is a wide range of skills that are deemed necessary for the reading and drawing of graphs.

In the methodological literature reviewed there seems to be general agreement as to the order in which one should teach the different types of graphs; this order is:

a. Picture graphs.
b. Bar graphs.
c. Sector or circle graphs.
d. Line graphs.

Whether this is the best order for the child is yet to be determined. More may be known concerning the best order for learning to read graphs. K.C. Thomas (1933) found that children could interpret picture graphs most easily; with circle, two dimensional graphs and line graphs following in that order. Culbertson and Powers (1959) stated, "Bar graphs were easier to read because they were clearly connected to the horizontal axis with each abscissa value to be read. Line graphs on the other hand are not clearly connected to the horizontal axis and therefore it is more difficult to pick out points on a line" (p.107). Both of these authors agree that the line graph is the most difficult to read. Perhaps the order is not the same one that should be employed to learn to draw graphs.

James Wesson in his article, "Graphs and Charts, An Important Topic for the Middle Grades", listed some very specific objectives for the teaching of graphs in the science classroom. These objectives, although desirable for the science teacher should also be the goals of
the mathematics teacher if graphing is to become a useful tool to the student. These objectives were:

1. The student should be able to collect data in an organized and useful form.
2. Students should be able to construct a variety of types of graphs and charts in a neat readable fashion.
3. The student should be able to employ reasonable sampling techniques including reasonable estimates of sample size and intuitive understanding of simple satisfaction.
4. The student should be able to group data into useable forms and select scales for a graph or chart which will convey information effectively.
5. A student should develop an understanding of the approximate nature of the information obtained through sampling and other measurements and should develop an appreciation for these techniques as valid problem solving tools.
6. The student should make informal interpolations and extrapolations when interpreting graphs (Wesson, 1980, p. 593).

Although these objectives may appear to be beyond the ability of the average elementary school child, a start has to be made somewhere and therefore these objectives should be considered and adopted where possible. The elementary school child could begin collecting and organizing data into a usable form but perhaps in not so sophisticated a way as an older student, yet by doing so he comes to realize that some organization is necessary in the area of graphing.

The age of the students to be studied must be given careful consideration because in order to understand graphing concepts there are several prerequisite skills needed, as shown by Wesson (1980) and McGalliard and Cooney (1979). It is with these skills and concepts that this study is concerned and it is hoped that an insight will be gained into how children think about graphs and how they learn the necessary skills and concepts.

In order that the researcher might have some knowledge as to what the student has already studied in the area of graphing, three
textbooks were examined. The textbooks chosen were those most commonly used in British Columbian schools; Investigating School Mathematics, Heath Elementary Mathematics, and Project Mathematics.

The research found that each textbook seems to have a different approach to graphing in mathematics and each emphasises a different aspect of graphing. The Investigating School Mathematics (I.S.M.) textbook series seems to be predominantly concerned with Cartesian coordinates. In Book 3 (Grade 3) of I.S.M. the child is introduced to a coordinate grid and is able to give a number pair for the location of an object on the grid. Book 4 (I.S.M.) again emphasises coordinates and simple data collecting. In this book the student meets the graphing of a function and the reading and interpreting of negative numbers. Books 5 and 6 (I.S.M.) review most of the work on coordinates and introduce the graphing of negative numbers and the graphing of number sentences.

The first two levels of the Heath series emphasise bar graphs. These are all vertical bar graphs and are concerned with data collected by the children. This data is displayed in a bar graph using one-to-one correspondence, but towards the end of the second book, scale is introduced with a bar graph in which one square represents five items. In Book 3 the child is asked to graph addition and multiplication sentences, still using the bar graph method taught in the previous books. Line graphs are introduced at this level, followed by making predictions. It is not until Book 5 that graphing of ordered pairs and graphing of a function is introduced. Circle graphs are introduced in Book 6. One type of graph met in Heath
Mathematics which is not met in any of the others is the 'broken line' graph which is also found in Book 6. The 'broken line' graph plots data and joins it together by a broken line because the data being plotted is not of the type that would normally be plotted as a true line graph as it is not of a continuous nature.

Project Mathematics has a completely different approach from the other two textbooks in that far more of the graphing is concerned with data collected by the children and the approach is more "experimental" in nature. This series begins with graphs which use pictures. This is in contrast with Heath where picture graphs are not seen until the final book, and with Investigating School Mathematics where they are not discussed formally at all. After the picture graph Project Mathematics uses circles and squares to represent objects in a one-to-one correspondence which is a similar approach to that of Heath. In Project Mathematics the children move from making graphs, to graphs which show relationships. Some of the relationships discussed are the 2 and 3 "times tables". In Book 3 the advantages of graphing are discussed by comparing first two sets of information and then three or more sets (p.8). Project Mathematics is the only series that deals with this aspect of graphing. In Book 4 scale is introduced and so is the graphing of coordinates. It is at this level that circle graphs are seen for the first time. Graphing of linear relationships does not appear in the Project Mathematics series until Book 6.

Of the three series, the Heath covers the greatest number of different types of graphs. Project Mathematics emphasises the
collecting of data, and makes the area of graphing more meaningful to a child. Investigating School Mathematics places most of the emphasis on the plotting of coordinates while neglecting the areas of bar graphs and circle graphs. None of these textbooks seems to emphasize what a graph really is, that is, a geometrical way of displaying numerical relationship between two or more quantities.

It can be seen then from this discussion that what a child knows about graphing may depend to a large extent on the textbook being used in the school which he or she attends. Also, as the various textbooks introduce topics at different times it was thought that this study should be undertaken with students who would, as far as possible, have completed the first 6 years of instruction and thus have covered most aspects of graphing introduced in the elementary schools. Yet graphing is not taught in the mathematics classrooms only and some children might have information about graphing acquired from other studies in school. Also, some experience with graphs may enter a child's everyday life through the various media with which one is surrounded and interpretive skills may be obtained in this way. Therefore what a child knows about graphing depends on many factors; the textbooks used, the prerequisite skills acquired, and the developmental level of conceptual understanding that the child has reached.

A study arising from the results of the C.S.M.S. (Concepts in Secondary Mathematics and Science, 1979) test on graphs was undertaken by Booth. This work looks at the results of the C.S.M.S test on graphing and then draws conclusions as to why children found certain questions difficult. Booth looks at how mathematics and science
lessons actually treat graphs, and he makes an assessment of the
discrepancies between the practice and the ideal aims of teaching the
topic. Booth hoped that this study would help to explain why children
lack these kinds of understanding which were shown by the results of
the test.

One test item showed that the students were merely plotting on
consecutive ordinates the value of each successive value in a table
containing relational data. This way they were constructing a
"Histogram" type graph, instead of plotting the number pairs repre­
senting the relationship in question as required.

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<td>1.95</td>
<td>- -</td>
<td>1</td>
</tr>
<tr>
<td>2.13</td>
<td>- -</td>
<td></td>
</tr>
</tbody>
</table>

(Booth, 1981, p.2)

Booth states that this tendency to plot, on consecutive ordinates,
successive values in a table containing relational data, may be due to
the student's preference for the direct pictorial method of
representation. This may, in fact, hinder them from coming to terms
with the more abstract relational representation (Booth, 1981, p.3).
Booth feels that the different treatment of graphs by mathematics and
science teachers adds to the confusion that the child might have in
concern with certain types of graphs. The mathematics teacher seems
to be concerned with graphs in terms of discussing $y$ as a function of $x$, while the science teacher is more concerned with drawing a graph of the results. This situation is made even more confusing by the tendency for mathematics and science teachers to use different notation and terminology.

Booth suggests that

If we are to overcome this problem we must focus on mathematical graphs as an "ideal" which may or may not be attained by physical data, and by using the "real" obtained data as the basis for fruitful discussion concerning its observed departures from the hypothesised model, may hopefully help both to clarify children's understanding of the nature of scientific and mathematical graphs, and to assist them towards a new and unified view of their essential interrelatedness (Booth, 1981, p.5).

From this section of literature studied it can be seen that the teaching of graphing is thought to be extremely important and has been considered to be so since 1939 (Progressive Education Association). As technology advances, more and more graphs are used to display data and therefore the ability to read graphs become increasingly important. In order that students may develop the ability to read graphs it has been necessary for researchers to understand what skills were necessary for this task. Several studies list these skills; such studies were undertaken by James V. Bruni and Helene Silverman (1975), and James Wesson (1980). A study of the textbooks used in British Columbian schools gives information as to what students in British Columbia may be expected to know about graphs, thus helping the researcher in this study to decide on appropriate subjects and content of the test to be given. The study by Booth (1981) discusses the confusion that a child may have due to the different treatment given to graphs by teachers of Science and the teachers of Mathematics.
This confusion must also be taken into consideration by the researcher when designing the test to be given, in that all graphs must be described in the same terminology. The child's confusion, if indeed there is any, must also be considered in the light of Booth's study when the researcher is drawing conclusions.

Research Relevant to Children's Ability to Read Graphs.

The major research on the reading of graphs was done between 1926 and 1959. Samuel Weintraub (1967) in his paper, "Reading Graphs, Charts and Diagrams" (a review of research done on the reading of graphs) states that early studies dealt with the ability of children to read graphs and which types of graphs children found easiest to read. The most relevant of these studies seems to be the one undertaken by Katheryne Thomas (1933), as it deals with elementary school children. In this study children from grades 4-7, a total of 355, were tested to investigate their ability to read graphs. The graphs considered were circle graphs, multi-picture graphs and line graphs. Questions asked about each graph included:

1. The reading of rank: What country produces the most wool?
2. The use of keys: How many acres are there in an average farm in Indiana?
3. Interpretation of the significance of the graph, for example, a multipicture graph intended to check the interpretation of symbols and meaning, used the picture of a ship to represent 50,000,000 bushels of wheat exported. The question read:
   a. Does the graph tell you that Canada has more boats than the U.S.?
   b. Does the graph tell you which country raised the most wheat?
   c. Does the graph tell you which country exported the most wheat? (K.C. Thomas, 1933, p.493).

The results of this study showed that the questions involving the use of graphs to gain information concerning rank were the easiest for all grades to answer. The most difficult graph to read rank from, for
all grades, was the line graph. To discover whether the children could interpret the key they were asked to read quantities from the graphs. Inaccuracy in the arithmetical processes made it difficult to determine how many pupils understood how to interpret the key. Thomas found that children could interpret pictorial graphs most easily, with circle, two-dimensional graphs, and line graphs following in that order. Thomas stated that the study was not conclusive at the Grade 4 level because these children had trouble with both reading and arithmetic, and therefore the test does not show to what extent Grade 4 children could use graphs.

Another study on the reading of graphs, done by Eells (1926) came to the conclusion that circle graphs were easier to read than bar graphs. The subjects in this study were psychology students at Whitman College. There were 93 first year students and 97 second year students. The subjects were given 15 circles divided into various sectors and asked to mark the percentages represented by the sectors. After three days, graphs using the same data but in the form of bar graphs were given to the students to read. It was found that they read the circle graphs more rapidly and with more accuracy than the bar graphs. The reason for this result may be due to the fact that the sample in the Eells study were college students. There are many concepts involved in the reading of circle graphs that might make the task more difficult for elementary school children; for example, a knowledge of percentages is necessary for this type of graph.
A later study on the reading of graphs was done by two agricultural scientists, Culbertson and Powers, in 1959. This study was made to determine how much certain factors of graph design contributed to the ease of the comprehension of graphs. The subjects were again young adults, that is, college students and high school students. These researchers felt that although there were many different ways of presenting data in a graphical form to the non-technical reader, there was very little research evidence to show which presentation is the most effective. As a result most choices of presentation are based on tradition or advice given in books (Culbertson and Powers, 1959).

In the Culbertson and Powers study 26 graphs were designed, all based on the same data. Each graph was presented with a series of four to seven multiple choice answers. Comparisons were made between the identification of elements, the different types of labels, keys and pictorial symbols. Various methods of presentation of quantities were given such as placing the numbers on the graph and use of a grid with numbers on the axes. Discrete bar graphs were compared with continuous line graphs to see which were the easiest to read. "Segmented" arrangements for presenting constituent parts of a total, some elements on this graph not originating at zero, were compared with "grouped" arrangements for presenting constituent parts of a total, with all the elements on this graph originating at zero. Also two ways of presenting parts of a whole were compared, that is, a pie chart and a segmented bar for presenting percentages.

The results of the study showed that the bar graphs were easier
to read than the line graphs, a result substantiated by K.C. Thomas (1933). Culbertson and Powers also concluded that:

1. Figures or numbers on the graph are easier to read than grid lines. (p.104)
2. Labels on the graph are easier to read than are keys. (p.101)
3. There was no difference between word labels or pictorial symbols but pictorial symbols were easier to read than keys. (p.104)
4. Students favour vertical bars to horizontal bars. (p.105)
5. Students favour the "grouped" arrangements rather than the "segmented" arrangements. (p.105).

Culbertson and Powers felt that "bar graphs were easier to read because they were clearly connected to the horizontal axis with each abscissa value to be read. Line graphs on the other hand are not clearly connected to the horizontal axis and therefore it is more difficult to pick out points on a line" (p.107).

A recent study undertaken in the United States is "The First Mathematics Assessment of the National Assessment of Educational Progress". The Mathematics Assessment was done in 1972-73 as part of a cycle of learning assessment areas covering various aspects of the school curriculum. Students in various age groups were tested, namely age 9, age 13, age 17 and age 26-35. The tests were given to a scientifically selected sample which took into account such variables as size and type of community, race and geographical region.

In this first assessment, the area of graphing was included under the section "algebra". In this test only 17 year-olds were tested.
"It was found that 17 year-olds were able to graph linear equations with about the same degree of success that they had with other non-trivial algebra exercises. However, exercises dealing with slope and intercept and equations of circles were among the most difficult exercises administered" (Carpenter, Coburn, Reys and Wilson, 1978, p.62). About 20% of those tested could find the coordinates of specific points of a line and correctly graphed the line. Identifying the equation of a horizontal line was of equal difficulty (21% correct responses). Finding the slope and y-intercept of a line was somewhat more difficult (16 and 12% correct responses), and finding the equation of a line given its x and y coordinates of two of its points was more difficult still (5% correct). (Carpenter, Coburn, Reys and Wilson, 1978 p.62).

Graphing was also included in the area of consumer mathematics in this First National Assessment and the results showed that, "direct interpretations from graphs were accompanied by a much higher percent of correct responses than exercises requiring not only interpretation but judgement and decisions" (Carpenter, Coburn, Reys and Wilson, 1978 p.113).

The British Columbia Ministry of Education also conducted a mathematics assessment in 1981. At the Grade 4 level there were 6 items dealing with graphs, including the reading and interpreting of bar, line and picto-graphs. The two items receiving the highest success rate (80%) required the students merely to read graphs. On those items requiring that the student make use of information from the graph to obtain an answer rather than simply reading numerical
data from the graph the results were only satisfactory (68% was the provincial mean) (Robitaille, 1981, p.118).

At the Grade 8 level there were again 6 items dealing with graphs, items on construction and interpretation of bar, line and circle graphs. One item (B/19) gave a very weak result (17% of students answered correctly).

**B/19**  The graph shows the speeds of three cars, A, B, and C. Which car is travelling fastest?

In order to answer this question students must have some understanding of the relationship between rate and slope. Most respondents chose the graph with the longest line segment as the one which represented the greatest rate. The results of the question B/19 are as follows:

<table>
<thead>
<tr>
<th>Option</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>17†</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>42</td>
</tr>
<tr>
<td>Not enough information given</td>
<td>37</td>
</tr>
<tr>
<td>I don't know</td>
<td>2</td>
</tr>
</tbody>
</table>

(† = correct answer)
To sum up, therefore, these studies show that in all cases the main area of difficulty is with line graphs. This group of studies gives an insight into the type of questions that should be asked regarding the reading of graphs; for example, questions need to be asked about the relationship between rate and slope for line graphs.

Research Relevant to the Understanding of Graphs.

The studies in this group of literature differ from those above in that they were designed to test understanding rather than rote recall skills or techniques. They are in some way a follow-up to the assessment studies carried out by researchers in the United States, in that they took the form of an assessment but went one step further and included some individual interviews to further probe student's understanding.

One of these studies was carried out in Britain in a project called "Concepts in Secondary Mathematics and Science", (C.S.M.S.). This was a longitudinal study (1974-1979), in which 10,000 students were tested in various aspects of the mathematics curriculum. As in the First National Assessment not all the students were tested on all the topics. There were thirteen topics in all, including one, "Graphs for 13-15 year-olds".

The graphing section in the C.S.M.S. study was divided into three levels of difficulty.

Level 1 Plotting points (integer, coordinates, or halves).
Interpretation of block graphs.
Recognition that a straight line is constant.
Interpretation of a scattergram.
Level 2  Simple interpolation from a graph.
Recognition of the connection between rate of growth and gradient.
Use of scale shown on a graph and an awareness of the effect of changing the scale.
Interpretation of simple travel graphs.

Level 3  The relationship between a graph and an algebraic equation for example \( y=2x, \ y=2, \) and \( x+y=2. \) (Hart, 1981, p.134.)
The children were described to be at level 1 of understanding if they correctly answered 2/3 of a particular group of items. The results were:

<table>
<thead>
<tr>
<th>Facility Range (Percentage)</th>
<th>Pass Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>78.5 - 97.6</td>
</tr>
<tr>
<td>Level 2</td>
<td>58.2 - 70.9</td>
</tr>
<tr>
<td>Level 3</td>
<td>12.8 - 26.5</td>
</tr>
</tbody>
</table>


98.7 per cent of the children at any level also succeeded on all lower levels. Level 0 was included for those who succeeded at less than 5/7 of the items in Level 1 and who thus were unable to make any coherent attempt at the questions.

The percentage of children who achieved each level for the three
Year groups in 1976 was:

<table>
<thead>
<tr>
<th>Levels by year group (Percentage)</th>
<th>13 yrs</th>
<th>14 yrs</th>
<th>15 yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 0</td>
<td>6.5</td>
<td>9.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Level 1</td>
<td>33.5</td>
<td>32.5</td>
<td>22.5</td>
</tr>
<tr>
<td>Level 2</td>
<td>55.0</td>
<td>48.5</td>
<td>57.0</td>
</tr>
<tr>
<td>Level 3</td>
<td>5.5</td>
<td>10.0</td>
<td>16.5</td>
</tr>
</tbody>
</table>


Very little difficulty was experienced by pupils with elementary items on block graphs, and the use of rectangular coordinates to plot points, when the numbers involved were integers. One of the questions asked of students in this C.S.M.S. study was to pick out rates of travel and arrival times. They managed to answer these questions quite well.

However the earlier interviews had suggested that several pupils found difficulty with travel graphs, even though they appeared to be able to give the correct answer. It was clear that several of those interviewed had incorrect perceptual interpretations of the graph. Some thought of the graph as a journey that was up and down hill, or a directional journey on the ground, and found it difficult to deal with the abstract notion of 'distance from an origin'. (Kerslake, 1981, p.128).

It should be remembered that while the main purpose of a graph must be to illuminate numerical data by using a visual form, there are some occasions when a graph can be visually misleading. An example of this has been seen in the distance/time graph which looks to many children, like a journey in a vertical plane. (Kerslake, 1981, p.135).

The C.S.M.S. study shows that although students may demonstrate a knowledge of learned rules in dealing with graphs they do not always
demonstrate a full understanding of the concepts. This was shown by the introduction of questions which were in an unfamiliar form. Areas of difficulty were:

1. The use of coordinates when curved lines were used.
2. The idea of continuity.
3. The relationship between rate and slope (especially in terms of journeys).

A second mathematics assessment was carried out in the United States in 1977-78. The results of this assessment, in the area of graphing, are discussed in an article by Barbara J. Bestgen in the December 1980 issue of *The Arithmetic Teacher*. Bestgen states;

1. When asked to search for information that required direct reading of a graph or table, performance for all age groups was high (p.26). This means that most subjects were able to take information from graphs where there was a one-to-one correspondence.

2. When asked to perform problem-solving tasks that required more than one bit of information within the graph or table to be processed or required the use of this information with computation, performance declined (Bestgen, 1980, p.26).

For example in a picture graph using a picture to represent 10 students only 30% of 9 year-olds answered correctly.

Bestgen also stated that when given a choice of four graphs and questioned as to which graph represented the data correctly only 29% of 9 year-olds answered correctly. "The most frequent response for
this exercise indicates that although students had some understanding of how bar graphs should appear, they seem to neglect or not understand the importance of the labelling axes" (p.27).

To sum up, therefore, the C.S.M.S. study, from which most previous knowledge of children's understanding of graphs can be found, concluded that although children demonstrate a knowledge of learned rules in dealing with graphs they do not always demonstrate a full understanding of the concept. One area of difficulty mentioned by this study was the relationship between rate and slope (especially in terms of journeys). It is with this in mind that the present researcher has decided to investigate children's understanding of the slope of a line graph, to see if some light can be shed on the reasons for the difficulties which occur in this area. The United States assessment (1977-78) also stated that there were problems with line graph and although students were able to read information directly from a graph, when asked to perform problem-solving tasks using information from the graph, performance declined. This lower performance in the problem solving area may be due to lack of understanding.

Summary.

From all the information gathered in the literature it seems that the choice of subjects should be made in the upper intermediate grades because Thomas found that fourth graders had trouble with the reading and computation necessary and, also, by the time the students have reached the upper intermediate levels they have covered those aspects of the curriculum necessary for the type of questions that will be asked.
Many studies have been conducted dealing with the reading of graphs; the two main ones dealt with in this study were the one by Thomas (1933) in which children were studied to see which graphs they found easier to read, and the other by Culbertson and Powers (1959) where the authors wished to investigate which form of graph would be best to use for displaying data by determining which was easiest to read. Both of these studies concluded that the line graph was the most difficult to read. Other studies discussed in this chapter deal with skills felt necessary for both the reading and drawing of graphs. It is with this knowledge of the skills deemed necessary, that the researcher in this study was able to judge, whether students had the prerequisite skills to answer the questions in this investigation. The study of the textbooks used in British Columbia schools helped in the selection of subjects as it was necessary that the subjects had been exposed to line graphs sometime during their school life. The Booth study (1981) explained how some of the difficulties children experience with graphs may be due to the different treatments given them by Mathematics and Science teachers.

The area that seemed to give the most problem seemed to be line graphs, especially those graphs dealing with rate and slope. This was illustrated in the British Columbia Assessment (1980), the United States Assessment (1977-78) and the C.S.M.S. study (1980).
From previous studies it was found that the main area of difficulty for most upper intermediate students was the line graph. From the British Columbia Mathematic Assessment (1981) it was seen that with respect to line graphs, it was the interpretation of the slope which caused most of the difficulty. It was with these results in mind that the investigator decided to test children's understanding of positive, negative, zero and infinite slopes. In order to do this the graphs were divided into categories as shown below:

1. Individual graphs each with either a positive, a negative, a zero or an infinite slope.
2. Pairs of graphs each with a different slope so that a comparison can be made of the rates of change.
3. Individual graphs each with increases and decreases in a curve showing a change in rate of increase or decrease.
4. Individual graphs each with a combination of positive, negative, zero and infinite slopes.
5. Qualitative graphs, that is, graphs where the numerical data has been omitted.

In all five areas questions were asked dealing with temperature, height, weight and distance. Once the categories were decided upon, graphs were drawn and questions were compiled to cover those aspects of the graph which were to be tested.

This study used a structured interview as a method of data collection. The questions were first piloted with students from the
Grade 6 level. The data from the pilot study were collected and analysed in order to refine both the questions and the interview techniques to be used in the final interviews.

One of the major goals of the study was to acquire detailed information about the students' understanding of the slope of a line graph. The individual structured interview for the collection of data is very time-consuming and therefore only a relatively small sample, twenty-two Grade 7 students, was used.

This chapter is divided into five areas of discussion:
1. The subjects
2. The rationale for using the clinical interview
3. The development of the instrument
4. The interview procedure
5. The method of analysis.

The subjects

Twenty-two children from Grade 7 classes were used and selected at random from one school in the lower mainland of British Columbia. Grade 7 students were used because it was felt that they would have nearly completed their elementary school education and would have been exposed to some of the aspects of graphing discussed in the previous chapter. Also, it was the Grade 8 students who, when tested by the British Columbia Mathematics Assessment, were seen to have difficulty with the types of graphs being investigated in this study. By testing the Grade 7 students it was felt that it might be possible to find what understanding of the slope of a line graph these students have,
and if it is this understanding which seems to be the cause of the

The school selected was chosen for convenience and was located in
a predominantly middle-class residential area. No attempt was made to
determine the subject's achievement level or the parental occupation
of these children, since it was recognized that, due to its small
size, the sample was inappropriate for generalizing to a broader
population. Further studies in this area are required to confirm any
findings of the study.

_Rationale for Using the Structured Interview Method_

A great deal of early research in child psychology by Piaget was
conducted using the clinical interview technique. In these clinical
interviews both the interviewer and the subject became actively
involved in a discussion which was relevant to the investigation.
Often, the child's answers determined the course of questioning.
Hence, the entire format of these interviews could be altered by his
or her own interest. It was felt that this technique provided the
greatest flexibility by permitting children to expand upon many of
their natural beliefs and interests.

This present study made use of a modification of Piaget's
clinical interview technique. In an attempt to control the uniformity
of the questions to all subjects, a set of graphs was drawn up and a
corresponding set of interview questions. However, individualized
probing questions, such as 'Why do you think that is true?' or 'Why
did you think that journey was longer?' were used in an attempt to
determine a deeper understanding of the child's response. It was felt that this method was justifiable because the data collected in this way can give an in-depth study into student's understanding of the slope of a line graph. This technique allows the student to give his or her interpretation of the slope of a line graph.

**The Development of the Instrument.**

The graphs used in the interview were developed to include the five categories listed at the beginning of this chapter. In all five areas questions were asked dealing with temperature, height, weight and distance.

Some of the questions asked were taken from graphs used in the C.S.M.S. study. For example, Graph 9, which shows a journey John made one evening, is a graph taken from the C.S.M.S. study and the story line changed to make it more appropriate to the students being tested. Graphs 11a,b,c,d were also taken from the C.S.M.S study. The graphs describe an elevator travelling from the ground floor to the fifth floor of a building, taking approximately 5 seconds to travel from one floor to the next, stopping at the first, second and fifth floors with each stop taking approximately 10 seconds. The students were asked which graph (a,b,c or d) illustrated the journey made by the elevator. Graphs 17a,b,c too were taken from the C.S.M.S. study. The three graphs illustrate distance over time and the student is asked which graph truly illustrates a journey.
Other graphs used in this study were developed by the author. There were 34 graphs in all. Once the set of questions was compiled they were piloted with Grade 6 students. As a result of analysis of the data of the pilot study, other questions came to mind concerning the students' understanding of a line graph and some of the existing questions were changed in order that they might become more probing. Also, some of the graphs were changed; for example, Graph 13 was changed so that the y axis showed temperatures below zero, in order that questions might be asked as to why the water temperature did not drop below zero. During the pilot study it was found that if the student was required to answer questions pertaining to all 34 graphs the time involved became too long; because of this the graphs were divided into two groups. The first group included Graphs 1-17; these individual graphs had either a positive, a negative, a zero or an infinite slope. The second group contained graphs 18-34; these graphs compared gradients or slopes on the two graphs or showed changing rates of growth, temperature or weight increase or decrease, or varying speeds of travel. With this division each part of the interview took approximately 30 minutes and students were asked to complete only one part, either Graphs 1-17 or Graphs 18-34. (The graphs can be found in Appendix 1)

The Interview Procedure.

The data for this study were all collected during a three day period in the month of May 1984. Students were interviewed during school hours in a room located within the school, each interview
taking approximately 30 minutes. At the beginning of each interview the subject was told that he/she would be asked questions pertaining to graphs, and that whether the answer given was correct or not was not important, but that what was important was the reason for giving that answer. In addition the students were asked if a tape-recording could be made of the conversation.

Each interview was audio tape-recorded so that data could be later transcribed. This served to add further validity to the study by allowing others to verify each child's interpretations as summarized by the investigator. A check list on each subject was also kept in order to note such things as: 'Does the student use his/her fingers to study the numbers along the axis?' or 'Is the student making calculations to acquire the answers? A copy of this checklist can be found in Appendix 2.

Method of Analysis.

Because this study has been to identify children's interpretations of the slope of a line graph, the investigation should show: whether the student can provide a real world interpretation when shown a graph with a positive, a negative, a zero or an infinite slope; whether the student, when shown a graph with two different negative or two different positive slopes, can tell which slope increases or decreases most rapidly; and whether the student can identify and describe increases or decreases in the rate of change.

A preliminary list of responses were made after the pilot study, every response being listed. These were reviewed when the first
analysis of the tape-recordings of the interviews were completed with
the Grade 7 students. The tapes of the interviews were later listened
to carefully and trends in responses or interpretations were then
categorized. A table was drawn up of the responses and then the tapes
were listened to again and the number of incidents for each
identifiable belief were noted. Later, examples of each belief
pattern were transcribed.

As a result of this analysis a "Conceptual Inventory" has been
generated for each of the categories of graphs listed. The inventory
illustrates the many different beliefs which children possess about
each of the questions.
Chapter 4

THE RESULTS OF THE INVESTIGATION

In this chapter the results of the investigation will be discussed as a 'conceptual inventory' of the way children interpret the slope of a line graph. To generate a conceptual inventory, the subjects were asked questions about graphs which were divided into five general areas, and it is under these headings that the results will be discussed.

1. Individual graphs with a positive, a negative, a zero, or an infinite slope.

2. Two graphs with two different slopes so that a comparison can be made of the slopes.

3. Individual curvilinear graphs showing changes in the rate of increase or decrease.

4. Individual graphs with a combination of positive, negative, and zero slopes.

5. Qualitative graphs, that is, graphs where the numerical data has been omitted.

The graphs presented data dealing with changes in temperature, height, weight or distance during a specified time period.

Children's interpretation of graphs with a positive or a negative slope.

In order to investigate children's interpretation of graphs which illustrate a positive or negative slope, eleven children were shown eight graphs, four of which illustrated a positive slope and four a negative slope. Some of the graphs were presented individually while
in other cases both a positive slope graph and a negative slope graph were presented at the same time. This type of presentation was chosen to facilitate the type of questions being asked.

In general, the results indicate that when asked questions about the graphs which illustrated positive or negative slopes, the subjects were able to state that there was a rise or fall in temperature, height or weight as shown by the graph. When the graph dealt with distance, however, the subjects became confused and quite often read the graph as a two-dimensional diagram illustrating direction travelled rather than distance travelled in a given time.

The only variation that occurred in the response to graphs dealing with temperature, height, or weight was the way in which the subjects described how they derived their answers. Some of the subjects obtained answers by looking at the line on the graph and noting in which direction it had been drawn, that is, had it been drawn going up from left to right or going down from left to right. The other subjects looked at the end points of the line and noted the numerical data that each point represented and made a comparison of the numerical values. For example, when shown a graph where the temperature decreases with time (graph 2) the following reply was given:

I: What is happening to the temperature on this day?
S: It is going down.
I: What do you mean going down?
S: It's getting colder.
I: How do you know it's getting colder?
S: The line is going down. (Pointing with his finger and showing the direction in which the graph is going down.)
When the same graph was shown to another subject the reply given was as follows:

I: What happened to the temperature on this day?
S: It's going down.
I: How do you know it is going down?
S: First it was a seven and it went down to zero at this end of the line. (Pointing to the end of the graph.)

These two forms of reasoning to obtain answers were found to be rather evenly divided when discussion concerned temperature, that is six subjects looked at the line and noted the direction of the slope, and five subjects looked at the end points and made a comparison.

When these same eleven children were questioned about graphs illustrating positive and negative slopes that dealt with height (graphs 3 and 4), the comparison of the end points was much preferred (N=9), over the observation of the direction of the slope drawn (N=2). When the same eleven children were questioned about graphs illustrating positive or negative slopes that dealt with weight (graphs 5 and 6) the reverse was true, with the observation of the direction of the slope of the line being favored (N=8) over the comparison of the end points (N=3). While questions about the temperature graphs dealt with single graphs showing either a positive or negative slope, both the height and weight questions referred to two distinct graphs one with a positive slope and the other illustrating a negative slope. In each case the subject was asked to choose which slope
showed an increase and which showed a decrease. There seems to be no contextual material in these graphs which would account for the preference shown in deriving the answers. On the contrary it would seem that the height graphs, which present data related to the altitude of an airplane during take off and landing, would lead the subjects to look at the slope and direction lines because the lines are closely related to the mental image of a flight path. In these graphs (5 and 6) the increase or decrease in height over time is similar to the actual flight path taken by the airplane, yet the subjects seemed to prefer the comparison of end points when explaining how they derived their answers.

When the subjects were shown a graph in which the distance decreased with time (graph 7 and 8) three methods of determining answers were apparent. Two of these methods were similar to those seen in the discussion of temperature, height and weight; that is, the subjects realized that there was a decrease in distance either by comparison of the end points of the graph (N=7) or by noting the direction of the slope of the line (N=2). The third method used by students to derive answers, in regard to graphs illustrating positive and negative slopes dealing with distance, is one in which the graph is seen as a map. A "map" is seen in this discussion as a representation, on a flat surface, of an area with direction and distance travelled represented. The students seem to ignore the time axis and thus they misread the data. With graph 7 and 8 the subject was asked to say which graph, 7 or 8, showed a journey to school and which showed a journey home from school. Two subjects believed that the distance
travelled on the graph may be in either direction depending upon which axis you wish 'home' to be represented. For example the following reply was given by one of the subjects who looked at the lines and the direction in which they went from left to right:

I: Can you tell from the graphs which shows a journey to school and which shows a journey home from school?

S: This one shows the journey to school (pointing to graph 8).

I: So graph 8 shows the journey to school and graph 7 shows the journey home. How do you know that?

S: Because in this one (pointing to graph 7) it is going to zero and you are counting down while in this one you are counting up from zero.

The following reply was given by a subject who looked at the end points and made a comparison before deriving his answer:

I: Which graph shows the journey to school and which shows the journey home from school?

S: O.K. This one must be from school.

I: Graph 7 is home from school. Where is home?

S: Home is there --- at 0 and school is 600 metres away.

I: How do you know that?

S: The graph says that the distance from home and it starts at 0 and moves slowly to 600 metres.

The rest of the subjects, (N=2) when asked which graph represented the journey to school picked graph 8. When questioned about their choice, they stated that they chose graph 8 because the line went up.
When it was pointed out that the line of graph 7 also went up but to the right and not to the left as in graph 8 the subject stated, "Oh! I can't really tell. It could be either." When questioned about where home was represented on the graphs the subject said that it could be at (0,0) or at (0,600).

From the results of the interviews dealing with graphs showing positive and negative slopes, it can be seen that most children do realize that temperature, height and weight increase when the slope is positive and decrease when the slope is negative. When dealing with graphs illustrating distance travelled over time, a third category of interpretation emerges where the subject looks at the graph as a form of map and apparently ignores the time axis.

Children's interpretations of graphs with a zero slope.

In order to investigate children's interpretations of graphs which illustrate a zero slope, the same eleven students were shown individual graphs each of which contained a positive, a negative, and a zero slope. Each graph showed a horizontal line segment to illustrate a zero increase or decrease in temperature, height, weight or distance travelled over time.

When asked questions about graphs with a zero slope the subjects realized that there was a zero increase or decrease in those graphs which dealt with temperature, height or weight (N=11). This was not true with graphs showing distance travelled during a given time. A variety of different beliefs as to what a zero slope was meant to illustrate was manifest in this case.
An example of an interview using a graph showing weight (graph 14) is shown as follows:

I: What happened to the boy's weight?
S: At 10 he had a growth spurt until he reached 15.
I: What happened when he was 15?
S: He stayed the same weight.
I: How do you know that?
S: Because after 15 he is going along the same line.

When the eleven subjects were shown a graph in which the distance travelled in a given time was zero their responses varied. These variations can be categorized as five different responses or interpretations.

1. There was no change in the distance travelled because there was a horizontal segment in the graph (N=3).
2. The person travelling took another means of transportation at the point where the graph became a horizontal line (N=3).
3. The person travelling changed direction (N=3). These subjects are reading the graph as a map.
4. The person travelling was going up hill and the hill levelled off at the horizontal segment on the graph (N=1). This subject is reading the graph as a topographical profile.
5. The line changes direction to indicate a stop at either end of the horizontal segment (N=1).

Examples of interviews illustrating these five belief categories are as follows:
Response Category 1. There was no change in distance because there was a horizontal line, that is a zero slope (graph 9).

I: When was John at the dance?
S: From A to B.
I: How do you know that?
S: Because he stayed in the same place.
I: How do you know he stayed in the same place?
S: Because it's a straight line.

Response Category 2. The person travelling took another means of transportation.

The subject reasoned that in graph 9 John walked from 0 to A because he started out walking, then he took the bus between A and B because it is further and finally he walked from B home.

Response Category 3. The person travelling changed direction (graph 10).

In this interview the subject was unsure of what was happening but finally decided that when the graph showed a horizontal segment John changed his direction of travel, that is, instead of travelling north-east he now began to travel directly east.

Response Category 4. The person travelling was going up a hill and the hill levelled off at the horizontal line (graph 10).

I: What was John doing between the third and fourth hours?
S: He was walking on the flat. The rest of the time he was walking up hill.
I: How do you know that?
S: Because here the line goes straight.
Response Category 5. The line changes direction to indicate a stop at either end of the horizontal line (graph 10).

I: What was John doing between the third and fourth hours?
S: I think at the third hour he stopped.
I: Why do you think he stopped?
S: Because the line joins. He stopped at the third hour and at the fourth hour again.
I: What was he doing between the third and the fourth hours?
S: I guess he was still cycling.
I: Why do you think there is that 'jinks' in the line? (Pointing to the beginning of the zero slope)
S: To show he stopped.

There are five alternative interpretations of a zero slope with graphs illustrating distance travelled during a period of time. Some of the varying responses may arise because the same subjects (N=3) often associate distance with maps and they seem to be reading the graph as a map. One subject visualized the graph as a topographical profile of a hill and the person climbed the hill walked along the top and then down the other side, while three subjects viewed the zero slope as a time when the traveller used another mode of transportation. These variations did not appear when the graphs dealt with changes in temperature, height and weight during a period of time. Response 3 and 4 can be combined to give one response category because they are both regarding the graph as a two dimensional diagram one being in the horizontal plane, that is, response 3, and the other being in the vertical plane, that is response 4.
Children's interpretations of graphs with an infinite slope.

In order to investigate children's interpretations of graphs which illustrated an infinite slope these same students (N=11) were shown graphs which contained infinite slopes alongside graphs which had positive or negative slopes. They were asked questions to see if they realized the difference in a positive or negative slope and an infinite slope when interpreting these graphs.

When the subjects were given graphs showing temperature there seemed to be no realization on the part of the subject that the infinite slope implies that the temperature rose instantaneously, an impossibility in the physical world. One of the subjects seemed to realize that the temperature rose quickly, but there was no indication that he realized how quickly. The graphs used for this inquiry were graphs 15a-c which represent data showing the changes in temperature which occur when milk is taken out of the refrigerator for three meals and replaced after each meal. Some (N=6) of the subjects equated the graph with a map of the milk being carried in and out of the refrigerator and yet all the while talking about the milk becoming warmer or cooler. An example of one of these interviews follows:

I: Which graph shows the milk being taken in and out of the refrigerator?
S: I think it is this one (pointing to graph B).
I: You think it is B.
S: Oh time in hours - No I think it is C. Here is where it is taken out and carried to the table (following the infinite slope with
finger). The milk doesn't take long to warm up. It warms up quickly.

It's out on the table (showing the horizontal line with finger) for about 45 minutes for breakfast.

It cools down quite quickly, then in the refrigerator for a couple of hours and taken out again at noon ——

It warms up quickly at noon —— it's often warm at noon.

I: What is happening in graph B?

S: The milk is warming up slowly — In C it warms up fast and in B it warms up slowly.

The above interview also shows that this subject's attention seemed to be drawn towards the time aspect of the graph rather than the temperature and the subject was concerned with the length of the meals and the time of the meals. Another subject also showed an attraction towards the time aspect of the graph but in this case was more worried about the interval between meals than about the times of the meals. This subject thought that the time between meals in some of the graphs was not sufficient to cool the milk down. Perhaps concentration on only one axis of the graph is characteristic of children of this age.

When the infinite slope graph was shown representing data concerning height it was again misinterpreted. The graphs used to represent height with an infinite slope were Graphs 11A-D which represented an elevator travelling from floor to floor. Graph 11A represented the elevator moving from floor to floor shown with an infinite slope, while the stops on the 1st and 2nd floors were represented by a zero slope. Graph 11B showed one infinite slope
representing the elevator's journey from the 1st floor to the 5th floor. Graph 11C represented the elevator moving from floor to floor with a positive slope, with the stops at the 1st and 2nd floors represented by a zero slope. Graph 11D represented the elevator going from the 1st floor to the 5th floor with a positive slope.

Not one of the subjects realized that the segment of graph showing infinite slope (Graph A) implies that the elevator rose in an instant, or in zero seconds. Most subjects realized that the horizontal line was a stop (N=8) although when discussing zero slope in the previous test all subjects indicated that a zero slope meant no change in height.

An example of an interview which illustrates these points is as follows:

I: Can you tell which graph shows an elevator that stops at the 1st, 2nd and 5th floor?

S: I think it is A.

I: Why do you think it is A?

S: It starts at 5 - it takes 5 seconds for each floor - it goes up in 5 seconds - the line goes to the right for 10 seconds — goes up for 5 seconds - to the right for 10 seconds and then up to the 5th floor.

I: What do you mean the line goes to the right for 10 seconds?

S: It went from 10 to 15 seconds.

I: What does that mean?

S: That means the elevator stopped.

I: Why couldn't it be C?
S: Because the elevator goes up for 5 seconds, stops -- oh!
I: Could it be both?
S: No, it can't -- because this graph (Graph C) goes to the right so it takes 15 seconds. It takes 15 seconds between the 1st and 2nd floors.
I: So it takes 15 seconds between each floor in graph C?
S: No, only between the 1st and 2nd floors.

Those subjects (N=4) who did choose the correct graph seemed to have done so by reading the time taken to travel from one floor to the other and the time while the elevator was stopped. They calculated that five floors would take 25 seconds, and two stops would take a further 20 seconds, then they made their choice of graph based on the end points of the graphs.

It is interesting that one of the subjects thought that only if the line went to a point that had integral coordinates labelled on the axis would it indicate a stop. This type of response may be due to the way in which graphing has been taught. Teachers have put a greater emphasis on the plotting of points than on the recording of data and the reading of graphs. This subject obviously associated points on the graph with the identified coordinates labelled on the axis. The subject also associated the point on the graph with happenings or changes in conditions in the physical world.

In order to test the children's interpretation about an infinite slope in the area of weight graph 16 was used. This graph showed the changes in weight when apples were placed on a scales one at a time.
When questioned about the graph it was found that the children had two interpretations. The first was that the weight went up immediately (N=4). The second interpretation was that it did not matter whether the line went up vertically as long as it shows the correct change in time (N=7). An example of an interview illustrating this second interpretation is as follows:

I; Which of these graphs shows the apples being weighed?
S OK --- C --- because 6x3 --- would take 18 seconds.
I: Why couldn't it be B?
S: It's taking more than 18 seconds therefore the assistant must be weighing more apples.
I: Why couldn't it be graph A? It goes up in 18 seconds.
S: It goes straight up.
I: Why must it take 18 seconds?
S: 6 apples at 3 seconds each is 18 seconds.

It seems from the above interview that the subject is not looking at the slope of the graph but making arithmetical calculations and fitting these calculations to the end points of the graph. This seems to be a similar concept to the one seen when the graphs with positive and negative slopes were examined. Some of the subjects focused their attentions on the end points of the graph in order to answer the questions put to them.

When the infinite slope was presented to the subjects in the form of a distance travelled graph (graph 17), it was found that only one of the subjects realized that the vertical line indicated the distance travelled in zero seconds. Most of the subjects (N=8) took
the line to indicate direction either 'up' or 'north'. An example of one of these interviews is as follows:

I: What happened in graph 17?
S: They travelled this way for half an hour.
I: Which way is this way?
S: East.
I: Then what happened?
S: They travelled north for 20 km, then east for an hour and a half, then north for 40 km.

Here again it appears that the subjects are reading graphs dealing with distance travelled as a map. One interesting point to note was that the subject mixed their reading of the journey in that they interpreted part of the journey as a distance measured in kilometres, while in other sections it was a time interval during which travel took place. For example in the interview illustrated above, the subject stated, "They travelled north for 20 km, then east for an hour and a half, then north for 40 km."

Additionally, the reading of points on the graph rather than the observation of the graph as an entirety is evident. The total length of the journey shown on the graph was 40 km yet the subject quite happily stated that the traveller had travelled 20 km (reading the distance axis) plus whatever distance was covered in an hour and a half (reading the time axis) plus 40 km more (reading from the distance axis).

It can be seen that graphs which represented data as an infinite slope gave rise to more misinterpretation than the other three slopes
so far discussed, that is, the positive, negative and zero slopes. Not one of the subjects interpreted the segments of the graphs showing infinite slopes as illustrating instantaneous increases.

Children's interpretations of two graphs which have different slopes.

In order to test children's interpretations of two graphs which have different slopes, eleven subjects, the second group of students, were shown two graphs drawn on similar axes and asked to compare the slopes. Each pair of graphs dealt with time on one axis and temperature, height, weight or distance travelled on the other. Both graphs in each pair had a positive slope or both had a negative slope.

The results of the interview dealing with these graphs were similar to those where individual graphs showing positive and negative slopes were presented. That is in the area of temperature, height and weight the main variations were in the method by which the subjects derived their answer. Some subjects obtained answers by looking at the segments on the graph and noting in which direction it had been drawn, that is, had it been drawn going up from left to right or going down from left to right. The other subjects looked at the end points of the segment and noted the numerical data that each point represented and made a comparison. When shown a pair of graphs where distance travelled was plotted over time and each graph had a different but positive slope or each graph had a different but negative slope, some subjects (N=4) again read the graphs as maps. Once each of the graphs had been interpreted individually as a map, answers were based on these assumptions. The other subjects (N=7) used the
technique of reading the points on the graph and making a comparison based on their reading of the points

When shown a pair of graphs illustrating the temperature of water under two different conditions (graph 19) all the subjects were able to distinguish which graph represented water getting hotter more quickly. The difference in responses was in the means by which the subjects derived their answers. One group compared the temperature of the water at the end points (N=9). An example of this response is as follows:
The graph being used in this interview is graph 19 which illustrates water left in a white can for three hours and water left in a black can for three hours.
I: Which graph shows the white can?
S: Graph B.
I: How do you know that?
S: Because it only goes up to 25° or 30° in about 3 hours, while graph A goes up to 40° in 3 hours.
The other group of subject (N=2) made a comparison of the time it took to reach a set temperature. An example of one of these interviews is as follows:
I: Which graph shows the white can?
S: Graph A is the black can because it didn't take as long to get up to 40°.
This second method of comparison seems more complex but perhaps it may be just another way of expressing the same comparison made by the previous set of subjects. In this method of comparison it may be that
the subject places a greater emphasis on the time axis. It is interesting that in this second interview not only did the subject make a comparison of the time taken to reach a certain temperature, but when asked which was the white can proceeded to tell which was the black can.

In order to investigate the subjects' interpretation of graphs illustrating changes in height the subjects were shown graphs which compared the height gained by two airplanes over a period of time (Graph 21). Some of the subjects (N=5) reasoned that the airplane rose from the ground more quickly because the line was steeper, for example:

I: Which was the fighter plane?
S: A.
I: Why did you pick A?
S: It's lighter so it can climb more steeply and line A is steeper than line B.

A graph showing the height of airplanes might be easier to read, as the visual imagery of what is happening in the physical world is very similar to what is shown on the graph. A number of subjects (N=4) still relied on the numbers on the axis to supply them with the answer, as they read the end points of the graph and then made numerical comparisons, for example:

I: Which was the fighter plane?
S: A.
I: Why A?
S: Because fighter planes are lighter and can climb more easily, and A got to 600 metres while B went to 300 metres which is not as high.
Two of the subjects confused increase in altitude with distance travelled, that is they read the graph as a map, for example:
I: Which is the fighter plane?
S: A is the fighter.
I: Why A?
S: Because A travels this way (pointing with finger to the right) faster than B.
I: Why is it going faster?
S: Because B is going a shorter distance because it is heavier.

A similar response was seen when the subjects were shown graphs with different but constant rates of weight gain, or different but constant rates of weight loss (Graph 20). Again, all the subjects were able to choose the graph which illustrated the most rapid gain or loss in weight, the difference in response again being the way in which they explained how their answers were derived. Some of the subjects (N=2) looked at both lines to determine their answers, for example:
I: Which rat is gaining weight the faster?
S: Rat A.
I: Why did you pick rat A.
S: Because they start at the same point and rat A goes up more steep.

Other subjects (N=8) read the end points of both lines and made a comparison of the weight, that is 18 and 10 kg. A third group of subjects (N=1) compared the two end points that is the end point of A and the end point of B, and concluded that A was heavier because the end point of the graph was higher.

In order to test the subjects interpretation of graphs illustrating distance travelled in a given time, the subjects were
shown two graphs with constant but positive slopes but with the slope of one of the graphs greater than the slope of the other (Graphs 9). The subjects were equally divided in their interpretations concerning the distance travelled and the gradient of the slope on the graphs. About half (N=6) of the subjects stated either that the longer the line segment the more time was taken to cover the distance or that the less steep the line segment the more time was taken to cover the distance. For example:

I: Which way did John travel by bus and which way did he walk?
S: He walked to the dance (0 to A).
I: How do you know that?
S: Because the line is longer showing he took longer to travel there than he did to come home.

The other half of the subjects (N=5) read the graph as a map and regarded the length of the line to represent the distance covered. That is the longer the line the further the distance travelled, for example:

I: Which way did John walk and which way did he take the bus?
S: He walked at point B.
I: Just at point B?
S: No, he walked from point B to 3.
   He took the bus from 0 to B.
I: How do you know that?
S: Because he took the bus one way and walked the other.
I: But I didn't say which way he walked or which way he took the bus, so how do you know that?
S: The line is longer.
I: Which line is longer?
S: This one (pointing to line B-3).
I: What does that tell us?
S: No one would want to walk that far.
I: What happened at point A?
S: He just went by there.
I: The bus turned the corner or something?
S: Yes, that's right.

From the results of the interviews in which subjects were shown these pairs of graphs, it can be seen that most of the children realize that temperature, height, and weight increase more rapidly when the slope is steeper. When presented with pairs of graphs each dealing with distance travelled over time, the subjects often confuse the graphs with maps, which show direction and distance travelled on a two-dimensional surface rather than a comparison between two variables on a graph, distance and time.

Children's interpretation of the slope of curvilinear graphs.

In order to investigate children's interpretation of graphs which exhibit increases or decreases curvilinearly, the subjects (N=11) were shown graphs in which increases or decreases indicated variable rates of change in temperature, height, weight and distance travelled, over a specified time.

The results of the tests in this area show a great reliance by some of the subjects on the reading of points on the graphs. Some of the subjects also seemed to confuse the greatest value of a graph with the greatest rate of change. Again there seems to be more misinter-
pretation with the distance travelled over time graphs than with the temperature, weight and height graphs.

With the graphs dealing with temperature, some of the subjects (N=3) realized that the greater the slope the greater the increase or decrease in temperature (Graph 26), for example:
I: When was there the greatest increase in temperature?
S: Between 6 and 7.
I: Why between 6 and 7?
S: Because that is where the line is most angled.
I: Angled? Angled from where?
S: The line is most angled up.

Those subjects who did not realize that the greater the slope the greater the increase in temperature in some cases confused the greatest value on the graph with the greatest rate of change. A number of subjects (N=5) seemed to think that the greatest increase was found where the highest temperature was shown or at the peak of the graph, for example:
I: When was there the biggest increase in temperature?
S: Between the 8th and the 10th hour.
I: Why did you pick that time?
S: Because it has the highest temperature.

In order to investigate the subjects' interpretation of graphs representing data on height, the subjects (N=11) were shown a graph which compared the growth of plants over a period of time (Graph 25). The subjects were asked which plant grew the most and which grew the
least. All the subjects were able to choose the graph which illustrated the most or least growth, but again the methods by which answers were derived varied. Some subjects (N=7) made the comparison of growth by referring to the slope of the line, while others (N=4) made their comparison of the growth of the plant by looking at the end points of the graphs and deciding which had the greatest change in height coordinates. This method of determining answers is similar to the results observed when the subjects were tested with graphs having positive and negative slopes.

Three of the subjects referred to the coordinates on the graphs before making a decision as to where the greatest increase occurred. This type of interpretation seems to indicate a reliance on using the points of the graph rather than looking at the slope on the graph. An example of this follows:

I: When was there the biggest increase in temperature?
S: Between the 7th and 11th hours.
I: Why did you choose that time?
S: Because it goes from 7°C to 9°C and that is the biggest increase.

When shown a graph where the weight increased with the curve (Graph 24) again most of the subject (N=6) realized that the steeper the slope the greater the increase in weight, for example:
I: In which month or months did the baby gain most rapidly?
S: Between 4th and 5th months.
I: Why did you pick those months?
S: Well it goes up so much — it's sort of hard to say — it's kind of angled up more.
It was seen that, when dealing with other graphs showing changes in
weight, some of the subjects (N=2) still relied on coordinates and
points on the axis to make a decision as to where the greatest
increase in weight occurred, for example:
I: When did the baby gain most rapidly?
S: Between the 6th and 8th months.
I: Why do you say that?
S: Because it went from 25 to 32.
I: And that is the biggest increase?
S: Yes, because that's where the biggest difference is.
Two subjects expressed the steepness of the slope as "the greatest
curve" for example:
I: When did the baby gain weight most rapidly?
S: Between the 6th and 10th months.
I: Why do you pick those months?
S: Well — there the line is kind of curved?
I: Is it not curved here too?
S: Yes — but it's more curved here.
One subject had difficulty defining increase and chose the top of the
graph as the point of greatest increase because that was where the
weight was greatest, for example:
I: When did the baby gain weight most rapidly?
S: At 35 kg.
I: Why do you pick 35 kg?
S: Because this is where the baby is the heaviest.
    When shown a graph where the distance increased curvilinearly
(graph 29) most subjects (N=9) regarded the graph as a map. The graph represents a journey a man took to and from the same location. During the walk the man's speed varied. Subjects read the time axis as a directional axis. An example of one of the interviews which illustrates this is as follows:

I: Can you describe the walk the man took?

S: He started somewhere — he walked towards something — and then he walked straight — then towards something — then straight — then towards — he probably got there — then he walked down a little bit and went straight and then down a little bit and then he got there.

I: Where did he end up?

S: I don't know — a park?

Eight subjects choose to ignore the time axis even though the interviewer repeated orally that the graph showed the distance walked in a given time. The gradients of the graph were regarded as diversions which did not follow city blocks, for example:

I: Can you describe the walk the man took?

S: He walked this way (pointing at the gradient of the graph with finger).

I: What do you mean by this way?

S: Well he didn't go down the street — perhaps he cut across the park.

I: Well, can you go on?

S: He got to the far side of the park — and then he went this way

I: Which way is that?

S: Well kind of back but not back the way he came.
I: Where is he going to end up?
S: I don't know! Down the block I suppose.

One subject regarded the walk as a climb a man took up a hill. The subject saw the graph as a two-dimensional picture of a hill with the man following the contours. Here is a transcript of a portion of that interview:

I: Can you describe the walk that the man took?
S: He climbed up a hill.
I: So he is walking up a hill?
S: Yes -- then he walks some more, and he gets to the top!
I: What does he do when he gets to the top?
S: He comes back down again.
I: When he has finished walking where is he?
S: At the bottom of the hill.
I: Is he on the same side of the hill as he started with?
S: No -- he is on the other side of the hill.

It can be seen from the results that the subjects seem to confuse the greatest value of a graph with the greatest rate of change. There again seems to be misinterpretation with distance travelled in a given time graphs as they are read as maps or diagrams.

Children's interpretation of individual graphs each of which has a combination of positive, negative and zero slopes.

In order to investigate children's interpretation of graphs which have a combination of positive, negative and zero slopes the students (N=11) were shown graphs 26-29. The subject interpreted the graphs which illustrated temperature and weight in the usual manner, but
height was confused with distance travelled and the graph describing
distance travelled over time was again interpreted as a map by some of
the subjects.

Of the eleven children tested all were able to interpret the
temperature graph in the usual manner and realized the significance of
a positive, negative or zero slope. The subjects were shown a graph
which illustrated the body temperature of a person who was sick. The
graph therefore illustrated changes in body temperature. An example of
one of these interviews is as follows:

I: Can you describe the changes in body temperature of this person?
S: Well, he or she began being normal.
I: What do you mean "normal"?
S: Well what you're supposed to be.
    He stayed that way for a while.
    Then his or her temperature went down.
I: What do you mean "went down"?
S: Well they got colder — they went outside or something.
I: Yes.
S: Then the temperature stayed that way for a while.
    Then here (pointing to the increase in the graph) his or her
temperature started to go back up.
    He or she must have come in.
    Then it stayed the same or normal.

When the subjects (N=11) were shown a combination of positive,
negative and zero slopes on a graph dealing with height, there seemed
to be confusion between height and distance. Six of the subjects
confused height with distance travelled, for example:

I: Describe the journey of the plane in terms of height from the ground.
S: Height!
I: Tell me what is happening to the plane.
S: Goes up — straight up — then it goes — sort of bends — then it goes up — bends to the right — goes straight up — then goes straight a bit — then goes down.
I: What happens when it "goes straight a bit"? Where is it going?
S: It's going to the right!

This example again shows the graph being read like a map. Four of the subjects realized that the height increased and decreased with the slope and that there was no change in height when there was a zero slope, for example:

I: Can you describe the journey of the plane in terms of height from the ground?
S: It changes a lot.
I: What is happening to the plane?
S: It increases a lot.
I: Where does it start?
S: It starts at zero and takes off increasing to 6000 metres.
I: What happens between 0 and 6000 metres?
S: It increases then levels a little bit — for about half an hour — then it goes up again, stays level for about half an hour — goes up stays level at about 6000 meters for about an hour then comes down.
I: Does the plane go anywhere?
S: Yes up.
I: I mean does it go any distance?
S: I suppose so but the graph doesn't say.

When the subjects (N=11) were shown a graph, dealing with weight, containing a combination of positive, negative and zero slopes, they seemed to interpret it in the same manner as they did when dealing with a positive, or a negative or a zero slope in isolation. In graph 28 where a person's weight is illustrated there seemed to be two approaches to the problem. One group (N=6) of the subjects looked at the slope of the line to determine their answers, while the other group of subjects (N=5) determined their answers by referring to the numbers on the axis.

An example of a response from a subject who looked at the slope of the line follows:
I: What happens to the person's weight during the 6 years shown on the graph?
S: It began to fall -- go down -- perhaps he was ill or he was on a diet.
   Then it stayed down for a while along here (pointing to zero slope)
   Then it went up.
I: How did it go up?
S: (Following with finger) it went up a little, then stayed the same, then went up a little then stayed the same, then up a little and stayed the same.
An example of a response from a subject who determined their answers by referring to the numbers on the axis follows:

I: It went down - how?
S: It fell from 60 to 45.
I: Then what happened?
S: It stayed at 45 then it went up and levelled off and reached 50 after 4 years.

After 5 years it was 55 and after 6 years it was back to 60.

When the subjects (N=11) were shown a graph, dealing with distance travelled, containing a combination of positive, negative and zero slopes, similar results to those obtained with other distance graphs were observed. Some subjects (N=2) interpreted the graph correctly but most of the subjects (N=7) read the graph as a map.

There were two subjects who interpreted the graph as a two-dimensional topographical drawing of a hill and a man climbing the hill on his walk. This is different from a 'map' interpretation as a 'map' interpretation seems to be in the horizontal plane while the topographical drawing seems to be in the vertical plane. Both interpretations are similar in that they are visual images of a journey except they differ in that they are in different planes. This is illustrated by the following extract from the transcript:

I: Can you describe the walk that the man took?
S: The first hour he climbed up.
I: So he is walking up a hill?
S: Yes -- then he walks some more, then he climbs up again, he keeps
walking, then he walks down and walks a bit then he walks down again.

I: Where does he end up?
S: At the bottom of the hill.
I: So he ends up on the other side of the hill?
S: Yes.

It can be seen from the results that the subjects interpreted the graphs which illustrated temperature and weight correctly, but height was confused with distance travelled while the graph describing distance travelled over time was interpreted as a map by seven subjects and as a topographical profile by two subjects.

Qualitative Graphs.

To investigate children's interpretation of qualitative graphs the children (N=11) were shown graphs where the numerical data had been omitted (Graphs 30-33). This was done to see if they could still interpret the graphs and whether the omission of the numerical data would make any difference to their interpretation or method of deriving answers.

There did not seem to be any difference between the subjects' interpretation of the qualitative graphs and their interpretation of the other graphs. One example of an interview referring to Graph 30 is as follows:

I: This graph shows a journey. It shows distance from home during a period of time. Can you describe in terms of distance from home?
S: There are no numbers?
I: That's right.
S: Well he starts at the bottom.
I: The bottom of what?
S: The bottom of the hill.
I: Then what happened?
S: He climbed up the hill a way and then the hill levelled off.
I: How do you know it levelled off?
S: Because it shows it here (pointing to the zero slope on the graph)
Then he climbed again and walked along on the level. Then up a little way more to the top. He walked along the top and down the other side.

This was the same student who interpreted Graph 27 as a journey up and down hill.

Four subjects found it difficult to interpret the zero slope, while 3 subjects interpreted them in the usual manner, that is, as no change in the variable of temperature, height, weight or distance. What was noticeable was that 2 subjects who had relied on numbers on the axis to guide them to the correct solution, tended to put in arbitrary numbers to help them with their interpretation, for example:
S: The journey started about 60 km from home.
I: How do you know that?
S: 1, 2, 3, 4, 5, 6 - (count in the squares on the grid along the Y axis).
I: O.K. Then what happened?
S: He went about 10 km then rested for a while then went about 15 km before resting again then went 20 km before resting.
Then went home.
When shown graph 33 illustrating the temperature of melting ice, most of the subjects (N=9) stated that the temperature of the water rose, levelled out, then rose again. Some (N=2) subjects described the rising and falling of the temperature by inserting arbitrary numbers.

I: What happened to the temperature?
S: It went up about 45° then stayed the same then went up about 10 more degrees.

I: How do you know that?
S: Because it goes up 1, 2, 3, 4 and a bit sq. then stays the same for about 2 and a bit hours then up again (following the graph with his finger).

When the subjects (N=11) were shown Graph 31, illustrating the height of a kite over a period of time, there were four interpretations. The first interpretation describes the kite as lying on the ground before rising and remaining in the sky for some length of time and then coming down again. There were five subjects with this interpretation. Four subjects described the kite as moving on the ground for a while, then going up and moving along because the owner is walking or running, and then coming down. The same two subjects as in the previous test put in arbitrary numbers to describe the time and height of the kite while one subject had the kite go straight up into the sky ignoring the zero slope at the beginning of the graph.

When the subjects (N=11) were shown graph 32, which represents data showing weight gained and lost during a period of time, there were three interpretations. Most of the subjects (N=8) interpreted
the graph in the usual manner, while two of the subjects interpreted the graph as a depiction of how much the scales were depressed. They interpreted a negative slope as an increase in weight and a positive slope as a decrease in weight. Again these same two subjects put in arbitrary weight and time values in order to describe the graph.

When the subjects (N=11) were shown graph 30 which represents data showing distance travelled during a period of time, most subjects (N=4) interpreted the horizontal lines as diversions along the route that was to be travelled. One subject interpreted the graph as a walk down a hill which levelled off in places. Three of the subjects read the graph as having rest places where there were horizontal lines.

Three of the subjects had a problem in that they were confused as to which end of the line or slope they should begin to read. That is, 3 subjects said that home was wherever the graph began and when asked where exactly, pointed to the start of the graph on the Y axis. One subject chose to start at the X axis and read up the graph towards the Y axis. Four of the subjects found it difficult to read the zero slope while one subject confused distance with elevation.

These results show that when these children are shown graphs where the numerical data has been omitted they tend to have the same interpretations and methods of deriving answers as they do with the quantitative graphs. Those subjects who relied heavily on the numerical data tended to put in arbitrary numbers to aid them with their interpretation.
Summary tables of interpretation.

Each section discussed in this chapter was taken and the interpretation of the graphs in that section tabulated. The tables show the N values, that is how many subjects interpreted the graph in that way. In each area 11 subjects were tested. A subject may have more than one interpretation; for example he/she may realize that a positive slope on a graph shows an increase and he/she may derive that interpretation by observing the direction of the line. This is why there may be more than 11 subjects accounted for in the interpretations. The 0 shown in the tables means that although this response was applicable to the area being tested none of the subjects responded in that way. The / shows that this response was not applicable in the area being tested.
TABLE I

Children's interpretations of graphs with a positive or negative slope.

1. The positive slope on a graph shows a rise in the parameter over a period of time.
2. The negative slope on a graph shows a decrease in the parameter over a period of time.
3. The student used the direction of the line to interpret the graph.
4. The student used the end points of the line to interpret the graph.
5. The slope may show an increase or a decrease in the parameter over a period of time, depending on at which end of the line you start.

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<th>Height</th>
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TABLE II

Children's interpretations of graphs with a zero slope.

1. A graph which shows a zero slope illustrates that there was a zero increase or decrease in the parameter over a given time.
2. The zero slope means that the person travelling used some other means of transportation.
3. The zero slope means that the person travelling changed direction.
4. The zero slope means that the person travelling went up a hill, the hill then levelled out at the zero slope.
   (Response 3 and 4 can be combined to give one response category because they are both regarding the graph as a two dimensional diagram one being in the horizontal plane, that is, response 3, and the other being in the vertical plane, that is, response 4.)
5. The line changes direction to indicate a stop at either end of the horizontal line.

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TABLE III

Children's interpretations of graphs with an infinite slope.

1. The parameter illustrated increased instantaneously.
2. The parameter illustrated increased very quickly.
3. The parameter illustrated increased during the time shown on the axis, that is, the number shown at the point where the vertical line leaves the axis.
4. The student chose the correct answer by calculating the time the parameter should have taken to increase according to the story line accompanying the graph.
5. The infinite slope indicates the distance travelled 'north' (Map response).

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<td>Response No. 5</td>
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TABLE IV

Children's interpretations of graphs which have different slopes.

1. The steeper the slope of the line on the graph the greater the increase in the parameter in a given time.
2. The steeper the slope of the line on the graph the greater the decrease in the parameter in a given time.
3. The student used the end points of the lines in order to make a comparison.
4. The parameter increased more on a given time because the end point is higher.
5. The line is longer therefore there was more distance travelled.

N Values

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</table>
Children's interpretations of the slope of curvilinear graphs.

1. The greater the slope the greater the increase or decrease in the parameter over a given time.
2. The greatest increase in the parameter is to be found at the maximum.
3. The greatest decrease is to be found at the minimum.
4. The student compared points on the graph to make a decision.
5. A graph showing height was confused with distance travelled.
6. The student thought of the lines on the graph as indicating direction.
7. The curve shows a hill which a person climbs up and down.

(Response categories 6 and 7 can be combined because they are both regarding the graph as a two dimensional diagram one being in the horizontal plane, that is, response 6 and the other being in the vertical plane, that is, response 7.)

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<td>Response No. 7</td>
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TABLE VI

Children's interpretations of individual graphs each of which has a combination of positive, negative and zero slopes.

1. The positive slope shows an increase, the negative slope shows a decrease and the zero slope shows there was no change in the parameter over a given time.

2. When presented with a graph showing varying rates of change in height they seem to confuse this with distance travelled.

3. The graph was read as a map, that is, the student interpreted the lines on the graph as if they showed direction.

4. The slope of the line on the graph was seen as a hill.

(Response 3 and 4 can be combined to give one interpretation category because they are both regarding the graph as a two dimensional diagram, one being in the horizontal plane, that is response 3 and the other being in the vertical plane, that is, response 4.)

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TABLE VII

Children's interpretations of graphs showing no numerical data.

The beliefs in this category were similar to those shown in other areas tested. The subjects used two methods of interpretation.
1. The graph was interpreted by observing the slopes.
2. The graph was interpreted by using arbitrary numbers.

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Discussion of Results

It was seen during the interviews that the subjects used two methods in finding answers to the questions asked concerning the positive and negative slopes on a line graph. One method was to view the graph as a whole and note whether the line was going up or down when read from left to right. The other method was to examine the end points, note the numerical data that each point represented and make a comparison. When dealing with height the subjects favored the first of these two methods, and when dealing with weight they favored the second. Graphs dealing with positive and negative slopes and distance travelled seemed to give some difficulty as the diagrams were sometimes read as maps rather than as graphs; in other words the time axis was ignored and the lines on the graph took on a directional aspect.

When pupils were confronted with graphs having a zero slope, difficulty again arose with the distance travelled graphs and again they were read either as maps, or as topographical profiles of what was happening. The interpretation of distance travelled and the zero slope was one of great confusion. When shown graphs with infinite slopes the subjects were confused no matter what the non-time parameter happened to be; invariably they failed to realize that the infinite slope expressed a change of rate in an instant. Some of the subjects did realize that the rate of change was rapid because they were using the comparison method but they did not realize that the change was instantaneous.

When the subjects were asked to compare the slopes on two graphs both with positive slopes or negative slopes, we saw similar results
to those of the previous tests where the subjects had been shown individual graphs with positive or negative slopes. The subjects used the same methods of interpretation of the graphs before making their comparisons. They also expressed the same misconceptions when reading distance graphs and when confronted with infinite slopes.

When the rates of change themselves were not constant and the graph was curvilinear, one of the problems found was that a few of the subjects did not understand the concept of increase and chose the maximum point on the graph as the time of greatest increase and the lows or minimum point on the graph as the time of greatest decrease. Again with the distance graphs it was observed that many of the subjects interpreted them as maps rather than as graphs. A positive slope was interpreted as a distance travelled in a north easterly direction.

When shown a graph which had a combination of positive, negative and zero slopes the results were again as expected after the previous testing.

The qualitative graphs, too, showed similar results; some subjects who had relied on numbers on the axis to guide them to the correct solution, on other graphs, tended to put in arbitrary numbers on the qualitative graphs.

In summary, there were several areas where the researcher observed variations in children's concepts of the slope of a line graph.

1. The subjects used one of two methods to read graphs. The choice of method depending on the non-time parameter of the graph.
2. The subjects often regarded those graphs which dealt with distance as maps, choosing to ignore the time axis and reading the slope of the line as directional.

3. Most subjects had problems with the infinite slope in all kinds of graphs.

4. Some subjects had problems with the concepts of greatest increase and greatest decrease, choosing the maximum or minimum of the graph rather than section of the graph which showed the greatest slope.
Chapter 5
SUMMARY AND CONCLUSIONS

This chapter, deals with a summary of the study that has been undertaken, discusses the results of the study and makes suggestions for further research.

In chapter 1 the study documented the growing need for children to be able to interpret graphs correctly. It was stated that today, more and more numerical data are accessible through computers, and more is being displayed in graphical form. Yet in spite of the fact that graphs are a popular, useful tool of communication, studies show that many adults experience difficulty in reading information presented in graphical form (Vernon, 1950; Meserve and Sobel, 1972).

From the literature review it was found that what a child knew about graphs depends to a large extent on the textbook being used in the school which he or she attends, as different textbooks emphasize different aspects of graphing. The literature which discusses which graphs were easier or more difficult to read concludes that the line graph is the most difficult. It was found from the study of the literature that children are able to read information directly from a line graph, but difficulties are apparent in the areas of continuity and the relationship between rate and slope (especially in terms of journeys). As a result of the previous studies the investigator decided to test children's understanding of the slope of a line graph. The data collected and analysed can be discussed with reference to the following research questions:
1. Do children's concepts of graphs change when the context of the graph changes?

2. Do children's concepts of graphs change with the different types of slopes?

The interpretation of graphs with different contexts

The question as to whether children's concepts change when the context of the graph changes, based on the evidence of this study, at first seems to be negative. There seems to be a reliance, by all subjects in all context areas, on one axis. When discussing temperature the subjects said 'it rose' or 'it went up' and in no instance did they mention time. The time axis was used in as far as the graph was read from left to right. When discussing height, weight or distance the subjects again did not use the time axis, except when time seemed more important than the other parameter being shown on the graph and then it was used instead. For example, with Graph 11A-D where the subject was asked to pick out the graph which showed an elevator going from the first floor to the fifth floor with stops at the first and second floors, four of the subjects answered this question by reading the time taken to travel from one floor to the other and the time while the elevator was stopped. They calculated that five floors would take twenty-five seconds, and two stops would take a further twenty seconds, then they made their choice of graph based on all of this information. It seems that the subjects relied just on one axis of the graph. Sometimes this might be the axis showing the independent variable, that is temperature, height, weight
or distance, but in other instances it might be the time axis, which ever seemed to the the most relevant to the subject being addressed.

This reliance on one axis may be due to the fact that they read numerical data from each axis in isolation. Consider the subjects' responses to Graph 17, where some of the subjects read from both the distance axis and the time axis, but did not relate these axes. For example, on Graph 17 the total length of the journey shown was 40 km, yet the subjects stated that the traveller had travelled 20 km (reading the distance axis) plus whatever distance was covered in an hour and a half (reading the time axis) plus 40 km more (reading the distance axis). This seems to indicate that children do not relate the data given on one axis to the date given on the other axis.

It is with this reliance on one axis, or perhaps the reading of numerical data in isolation, that the similarity of the interpretation of graphs with different contexts ends. The data show that, in every case distance graphs were interpreted differently from all others. The distance graphs, whether they were given with a positive, a negative, a zero or an infinite slope were interpreted by some subjects as a 'map' having direction. In the case of distance graphs it is evident that the subjects were relying upon only one axis of the graph. Evidence of reliance upon one axis was also found with the other graphs but it was not as prevalent. It becomes more obvious in the distance graphs because students add the dimension of direction to the graph. For example, Graph 10 shows a cycle trip made by John from Town A to Town B 50 km away; this graph was interpreted by some subjects to be a journey of 30 km in a north-easterly direction, then
a distance in an easterly direction (the zero slope) then a
continuation of the journey in a northerly direction.

When subjects interpreted temperature, weight or height graphs
showing a zero slope they were able to give the correct answers not
because the subjects were reading the time axis in relationship to the
other parameters, but rather because the subjects realized that the
other parameters were remaining constant; they realized that the
parameters of temperature, weight and height were not changing.
However, in the case of distance graphs some of the subjects read the
zero slope with reference to the time axis, ignoring the distance
axis, and the time axis was given direction. In these cases the
distance given to these zero slopes was the distance covered in the
time shown (see the above example, Graph 17). Again it can be seen
that children do not seem to see graphs as representations of
relationships between two parameters, but rather as lines from which
data can be read from either of the axes depending on which seems the
most logical.

Whenever the slope was infinite, if the independent variable was
temperature, weight or height, the subjects appeared not to realize
that the implication is that the indicated change occurs instan-
taneously; the most likely cause of their failure to realize this
seems to be that they were concentrating only on the independent
variable and were ignoring the passage of time completely. When the
infinite slope appeared on a distance graph, they imposed the
additional dimension of direction on the graph, just as they did with
distance graphs with other kinds of slope.
One important finding of this study was that subjects choose to derive their answers in different ways. It seems that some subjects choose to look at the direction of the line to interpret the graph while others use the coordinates points on the line to interpret the graph. For example, when subjects were shown Graph 1, showing the rise in temperature during the day, some subjects stated that the temperature rose because the line went up from left to right, while other subjects stated that the temperature rose because the line went from 0°C to 7°C.

The choice of method varies with the contextual material being displayed. When dealing with temperature the choice of methods were evenly distributed, that is, about half of the subjects used the directional method of interpretation while the rest looked at the numbers on the axes. This even distribution of methods may be due to the two types of conceptual imagery that the children are exposed to in their everyday lives. Temperature is associated with thermometers, and the scales on fluid-containing thermometers are almost invariably oriented so that in use the thermometer is placed vertically; it is because of this that any change in temperature becomes associated with movement in a vertical direction. Children also see temperature given in numerical form in the newspaper, on television, in textbooks, and on new digital thermometers and therefore are equally familiar with the numerical concept of temperature. So it is quite understandable that either of the methods would be used depending on the subjects' experience with temperature.

In response to graphs dealing with height, students favoured the use of end points of the line to interpret the graph. This choice
seems contrary to the choice being made due to the conceptual imagery of what is happening in the physical world. One of the graphs dealing with height presented data related to the altitude of an airplane during take-off and landing, and it would seem that this would lead the subjects to look at the slope and direction of the lines because the lines are closely related to the mental image of a flight path. Yet subjects seem to prefer to use the end points of the graph as a means of interpretation. This choice of end points as a means of interpreting a graph may be due to the way in which the ideas of height are always expressed in numerical terms, for example, children are usually asked questions such as — 'how tall?', 'how high?', 'how many metres?' all of which require numerical answers and perhaps their concept of height is a point some distance off the ground or surface rather than the distance between that point and the ground or surface. This would account for their preference for the method of looking at the end points of the graph because they see height as a point.

In response to the graphs dealing with weight students favoured the directional method as a means of interpreting their graphs. This again is difficult to explain because children are asked similar questions about weight that they are about height. Perhaps it is because the weight of a person can fluctuate, whereas the height of an adult remains virtually the same, that there is this difference in methods of interpreting graphs. The interpretation of weight graphs may be linked to the widespread obsession in North American society, displayed on television and in print, with increases in body weight of adults, rather than with specific weights expressed numerically.
only that but typically, most children have experience of being weighed on a set of 'scales' that displays a needle that moves successively over a sequence of increasing digits, or displays a 'window' in which appears, in succession, a sequence of increasing digits. In either case, movement is involved.

This choice of the directional method or the coordinate method was not apparent when the students were shown distance graphs. The only two types of interpretation were shown with respect to distance graphs with those graphs which showed a combination of slopes. For example, where a graph showed a journey away from zero and returning to zero three hours later most of the subjects interpreted it as a journey from A to B or as a climb up and down a hill. The first interpretation arises because the subjects incorporate the dimension of coordinate direction into the graph and do not realize that the distance travelled is directional only in the sense that it is distance from home. The second group of subjects, who may be strong visualizers, incorporate not only coordinate direction to the graph but elevation as well. It can be seen from the discussion that with respect to temperature some children are comfortable with the directional method while others are comfortable with the coordinate method of interpretation. Concerning weight and height, perhaps it can be said that if a parameter is believed by the child usually to be constant he/she favours looking at the end points but where the parameters are believed to be variable the child favours the directional approach. The anomaly concerning the parameter of distance, may be associated with a child's belief that distance in the physical world always has direction, in other words, that distance is
a vector rather than a scalar measure.

These conjectures about the possible types of reasoning used by children when interpreting simple line graphs should be the subject of a more in depth study preferably over an extended period of time.

Questions arising from the presentation of different slopes

Whether the graphs were presented singly or in pairs seemed to make no difference to a subject's response. The subjects seem to read the graphs singly and when given a pair of graphs to interpret they compare the single graphs. Presenting graphs in pairs did not seem to change their method of interpretation.

The same thing appeared to be true when graphs containing a variety of slopes were presented. The subjects read each segment of the graph individually except in the case of distance graphs in which case for some subjects the combination of slopes presented the added dimension of elevation and the journey was interpreted as a climb up and down a hill. The dimension of elevation was not incorporated when the graph had a single slope, but only when different slopes were present on the same graph.

One difficulty which arose with the curvilinear graphs was that it was found that some subjects did not understand the concept of rate of change as applied to the parameters being shown on the graphs. They interpreted the points of greatest rate of increase of the parameter in question to be at the maximum value of the parameter, and the point of greatest rate of decrease to be at the minimum value of the parameter; this may not be entirely a problem of interpretation of the graph but may be partly one of misinterpretation of vocabulary.
Perhaps the choice of a maximum or minimum is a misinterpretation of the graph in that the subjects feel that the rate of increase or rate of decrease are indicated by specific points on the graph rather than by the slope of the set of points joining two specific points. Perhaps because the subjects are referring only to one axis the only possibility for an answer is a point on the graph. Also to locate the point that shows greatest increase the subject must see a relationship between both axes. In order to answer the question of whether the misinterpretation of the concepts of increase and decrease is one of vocabulary or one of the inability of the subject to see a relationship between both axes, a further in depth study would be necessary.

From the above discussion it can be seen that whether graphs are presented singly or in pairs, they are always read singly. Also, when graphs contain a variety of slopes each slope is read individually. In the case of distance graphs with positive, negative and zero slopes their direction with respect to vertical and horizontal caused the subject to add the dimension of elevation. The interpretation of rate of increase and rate of decrease may have been misinterpreted because of the subject's inability to see a relationship between both axes.

Qualitative graphs

There seemed to be a few difference between the results of the investigation with qualitative graphs and those concerned with quantitative graphs. Four subjects did have difficulty with the zero slope questions whereas they did not experience this difficulty when dealing with quantitative graphs; this suggests that these four pupils rely on the numbers on the axes to guide their interpretation.
It was observed also that two of the subjects put in arbitrary numbers to help them with their interpretations; however, it was interesting to note that these numbers were not put in on the time axis and no relationship between the time axis and the other parameters was drawn.

Another interesting observation concerning interpretation of qualitative graphs is the fact that in Graph 32, dealing with the weighing of sugar, two subjects interpreted the graph as a visual depiction of how far the scales were depressed; they interpreted a negative slope as an increase in weight. If these two subjects had had numerical data on the axes this interpretation might not have happened as they might have been reading the data at various points on the graph. These two subjects did not equate a negative slope with a decrease in the parameter so they resorted to visual imagery for their interpretation.

From the observation of subject responses with qualitative graphs it can be seen that there is a heavy reliance on numerical data by most subjects. When this numerical data is removed there are more incidents of misinterpretation of graphs.

**Implications for Instruction**

The investigator believes that more discussion of graphing by teachers and students is needed if the misconceptions displayed by students in this study are to be avoided. In particular discussion of the interrelationship between the parameters of both axes might help clear up the misconceptions students have about distance-time graphs. One way of doing this would be for the teacher to explain the relationships between the two axes by changing the scale on one of
them and demonstrating to the students the effect that this has on the slope of the graph. There would be less chance of a graph being read as a map if these relationships were demonstrated to the students and if a discussion of the parameters shown on the axes occurred before each graph was interpreted.

Children also need to discuss the concept of increase and decrease and what their relationship is to time. Teachers need to be aware of the two methods employed by students in interpreting graphs and need to demonstrate both these methods when teaching the interpretation graphs; that is, pupils need to look at the graph as a whole noting the direction of the slope but they need also to examine the actual value of the end points. It should be pointed out to students that there are points on the graph other than the end points and that they should look at these points on a graph when making comparisons.

**Recommendations for Further Research**

This study was exploratory in nature and therefore the results cannot be generalized in any way. What it has done is to show specific problems which need further investigation. One of these problems is certainly associated with graphs which illustrate distance travelled over time.

Questions that might be asked in this area are:

1. Why are these graphs being read as maps?
   a. Is it because the subject ignores the time axis?
   b. Why are they confusing the graph with a map and adding the extra dimension of direction? Is it because they
believe that distance and travel always have direction in the physical world?

2. Why do some of the subjects visualize the distance-time graphs as a topographical diagram?

Other questions which might be asked about children's interpretations of the slope of a line graph are:

1. Are those subjects who have good visual imagery less likely or more likely to read a line graph correctly?

2. Do subjects who misinterpret any graphs showing infinite slope no matter what the non-time parameter, do so because they chose to ignore the time axis?
   a. If this is true, why do they ignore it?
   b. Is it a lack of understanding of relationships?
   c. If it is a lack of understanding of relationships, is it because of maturation factor?

3. Why do some subjects seem to interpret a graph solely from the direction of the slope, while others seem to rely solely on the end points? With any individual is this a constant conceptual trend or as seems possible from this study, does it change with the context of the graph?
BIBLIOGRAPHY


APPENDIX A

THE INSTRUMENT
The graphs shown in this appendix are not the actual graphs used in the interview. These graphs have been reduced so that they can be displayed on the page along with the questions asked during the interview. When interviewed the subjects were shown full page graphs and the questions were read from a different text, to the subjects by the interviewer.
Graph 1.

This graph shows the changes in temperature over a seven hour time period on January 14th, 1984.

1. Can you tell from the graph what is happening to the temperature on that day?
Graph 2.

This graph shows the changes in temperature over a seven hour time period on February 3rd, 1984.

1. Can you tell from the graph what happened to the temperature on that day?
Graphs 3 and 4.

These graphs show the height of an airplane from the ground during a five minute period.

1. Can you tell from the graph which shows an airplane landing and which shows an airplane taking off?

2. Which airplane shows an increase in height and which shows a decrease in height?
GRAPH 4
Graphs 5 and 6

One of these graphs shows the weight of a growing child over a period of time, while the other shows the weight of a woman on a diet over a period of time.

1. Can you tell from the graphs which shows the growing child and which shows the woman on a diet?
GRAPH 6

TIME IN MONTHS

WEIGHT IN KILOGRAMS

10, 20, 30, 40, 50, 60, 70
Graphs 7 and 8.

These graphs show the distance from home over a given time period.

1. Can you tell from the graphs which graph shows the journey to school and which shows the journey home?
Graph 9.

This is a graph of a journey John made one evening. He left home to go to a dance in Vancouver 3 kilometres away. He walked one way and takes the bus the other way.

1. Can you tell from the graph the time John spent at the dance?
2. Can you tell me which part of the graph represents the bus ride and which represents the time John spent walking?
3. Can you explain why you chose that section of the graph?
Graph 10

This graph shows a cycle trip John made from town A to town B.

1. What do you think John was doing between the third and fourth hour of his trip?
Graphs 11a, b, c, d.

An elevator can travel from the ground floor to the fifth floor of a building, taking approximately 5 seconds to travel from one floor to the next. Each stop taking approximately ten seconds.

1. Which of the following graphs represents the journey of the elevator when it starts from the ground floor, 'g', and stops at the 1st, 2nd, and 5th floors only?
Graph 12

This graph shows the temperature of water as it is heated to boiling point.

1. How do we know from the graph when boiling point was reached?

2. At what temperature did the water boil?
Graph 13.

This graph shows the changes in temperature of a cup of warm water that has been put in the freezer over a period of twenty-five minutes.

1. How can you tell from the graph when the water starts to freeze?
2. After how many minutes did the water start to freeze?
3. How can you tell when the temperature of the water stopped decreasing?
Graph 14.

This graph shows a boy's weight from the age of ten to sixteen years.

1. Can you tell from the graph what happened to his weight during this time?
2. When did he stop gaining weight?
3. For how many years according to the graph was his weight the same?
Milk is taken from the refrigerator for breakfast, lunch, and supper, and then replaced after each meal. Each time it is taken out it gets warmer and when it is replaced in the refrigerator it becomes cool again.
1. Which of the three graphs tells what happens to the milk during the day?

2. Why did you not choose this graph? (Interviewer points to one of the graphs not chosen.)
These graphs show what might happen when a shop assistant weighs 6 apples placing them on the scales one at a time, until all six apples are on the scale. She places an apple on the scale every 3 seconds.
1. Which graph seems most likely to show a true picture?
2. Why?
3. Why couldn't it be one of the others?
Graph 17a, b, c

These graphs show what distance might be travelled over a period of hours.
1. Which of the graphs seems most likely to show a true picture of a journey?

2. Why did you choose that graph?

3. Why did you not choose one of the other graphs?
Graph 18

This graph shows a journey made by two cars. It shows the distance they travelled over a certain time period.

1. Can you tell from the graph which car travelled the faster?
2. Can you tell from the graph which car travelled the further?
In a science experiment two containers were taken, one white and the other black. Each was filled with equal amounts of water and placed in the sun. The temperature of the water in each container was recorded over 3 hours and a graph of each drawn. It was found that the water in the black container became warmer more rapidly than the water in the white container.
1. Can you tell from the graphs which represents the temperature of water in the black container and which represents the temperature of water in the white container?
Graph 20

This graph shows how two rats increased in weight when they were fed different foods.

1. Can you tell which rat gained weight at the faster rate?
Graph 21

This graph shows the take off of a transport airplane and a fighter airplane.

1. From the graph can you tell which is the fighter plane?
This graph shows the change in temperature during the day.

1. Between what hours was there the biggest increase in temperature?

2. Between what hours was there the biggest decrease in temperature?
Graph 23

This graph shows the change in temperature of water after it has been poured from a kettle into a cup and allowed to cool.

1. Between what times does the water lose most of its heat?
Graph 24

This graph shows a baby's weight gain in the first year of its life.

1. In what months did this baby gain most rapidly?
2. In what months did this baby gain least rapidly?
Graph 25

This graph shows how Janet's plant grew each week.

1. Did the plant grow at the same rate all the time?
2. Is there a period when the plant did not grow at all?
3. During which week did the plant grow the fastest?
Graph 26.

This graph shows a person's body temperature.

1. Can you explain what is happening to the person's temperature as shown in this graph?
Graph 27

This graph shows the height at which an airplane flies over a period of seven hours.

1. Can you describe the flight in terms of the height of the plane?
Graph 28

This graph shows a person's weight over a period of 6 years.

1. Can you tell from the graph what happened to his weight during this time?
Graph 29.

This graph shows a walk that a man took one afternoon.

1. Can you describe the walk in regard to the distance he was from home?
This is a graph which shows a journey. It shows the distance from home during a period of time.

1. Can you describe this journey in terms of distance from home from the graph?
Graph 31.

This is a graph showing what happened to a kite one afternoon. It shows the height of the kite during a period of time.

1. Can you tell from the graph what happened to the kite?
Graph 32.

This graph shows what happens to the weight of sugar as it is being weighed.

1. Can you describe what is happening to the weight of sugar on the scales from the graph?
Graph 33

This graph shows the temperature of a block of ice which is left in a cup to thaw. The temperature of the resulting water is plotted on the graph.

1. From the graph can you tell what is happening to the temperature of the water?
CHECKLIST FOR INTERVIEWING.

Does the student use his/her fingers to study the numbers?
   a) along the side axis?
   b) along the bottom axis?

Does the student run his/her fingers from points on the graph to numbers on the axis?

Is the student making calculations to acquire the answers?

Graph positions (Placing graph for student)
   Graph 3 to the left of Graph 4
   Graph 4 to the left of Graph 3
   Graph 3 above Graph 4
   Graph 4 above Graph 3

Is the student indicating answers by pointing to parts of the Graph?

Does the student make gestures with his/her hand?
   a) to indicate horizontal
   b) to indicate vertical
   c) to indicate slope

#10. Did not really understand - 'horizontal'.
CHECKLIST FOR INTERVIEWING.

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**Subject No.** 5  
**Graphs No.** 18-33

**GRAPH No.**

Does the student use his/her fingers to study the numbers?
   a) along the side axis?  
   b) along the bottom axis?

Does the student run his/her fingers from points on the graph to numbers on the axis?

Is the student making calculations to acquire the answers?

Graph positions (Placing graph for student)
   Graph 3 to the left of Graph 4  
   Graph 4 to the left of Graph 3  
   Graph 3 above Graph 4  
   Graph 4 above Graph 3

Is the student indicating answers by pointing to parts of the Graph?

Does the student make gestures with his/her hand?
   a) to indicate horizontal  
   b) to indicate vertical  
   c) to indicate slope

#21 Showed slope increase with hands