PRIMARY CHILDREN'S PERCEPTIONS
OF MATHEMATICAL PROBLEM SOLVING

by

JANICE HEATHER NOVAKOWSKI

B. A. (English Literature) University of British Columbia, 1989
B. Ed. (Elementary) University of British Columbia, 1991
Dipl. in Ed. (Math and Science) University of British Columbia, 1993

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Department of Centre for the Study of Curriculum and Instruction

The University of British Columbia
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Abstract

This study explores primary children's perceptions of mathematical problem-solving. A review of current literature in this area indicated a need to focus on children's thoughts and ideas. The children's perceptions of what a math problem is, their problem-solving process and their stated use and preference for problem-solving strategies were investigated.

For this study, seven grade two students from the researcher's primary classroom were interviewed individually. A set of research questions provided a guide for the researcher during the interviews, but allowed flexibility so that both the subject and the researcher could add questions or information as required. A natural conversational flow was desired. The interviews were transcribed and analyzed. Individual student descriptions as well as comparisons between students in significant areas were prepared.

It was found that the children had varied definitions for what a math problem is. Although some children had difficulty articulating a definition, they demonstrated an understanding through their examples. Most of the children stated that a math problem has words and some math and is like a story. A few children also indicated that a math problem could just involve numbers, depending on the level of difficulty.

All of the children indicated that they had a problem-solving process in place. This process ranged from "try to figure out the answer and write it down" to a multi-step metacognitive process that involved thinking about the problem, trying different ways to solve it and checking your work. Five of the seven children stated that they included a "looking back" step as part of their problem-

solving process.

This group of children could name several problem-solving strategies and indicated that they each used a variety of strategies for different purposes. The children indicated they preferred the strategies of mental calculations, using a calculator, working with number patterns and asking someone for help. When asked what strategies they use, all the children indicated they used drawings and mental math for some problems. When asked why they used particular strategy, the most common response was because, "they help me."

Because the interviews proved viable for this age group and gave the interviewer a wealth of information about the students' thinking, it was suggested that this type of assessment could be applied to classroom use.

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Chapter 1

Introduction

The pendulum in education is constantly in motion between traditional and innovative approaches. One of the content areas that has seen dramatic changes over the years is primary mathematics. Research has supported and energized a change from a focus on arithmetic skills to real-life, active mathematics that encourages thinking, communication and problem-solving. Business and academics both support this change as being necessary for the future of society.

Central to this change in mathematics education is the emphasis on problem-solving. This emphasis is supported by the National Council for Teachers of Mathematics (NCTM, 1989) and is inherent to the mathematics curriculum of the Primary Program of British Columbia. As stated in the Foundation Document (1990), "Problem-solving should permeate the entire program and provide the context in which concepts and skills are developed. Mathematics evolves naturally from problem situations that have meaning to children, and are regularly related to the environment" (p. 281). Goals include using problem-solving approaches and developing and applying strategies to solve a wide variety of problems. Teachers are given a list of strategies to model and encourage children to use in their classrooms.

British Columbia's Primary Program also promotes a child-centred approach which encourages children to be aware of and develop their individual learning styles. Closely linked to this premise is the philosophy that teachers need to encourage children to represent their knowledge and thinking in a variety of ways. For example, the Foundation Document describes and gives examples of the three levels of representation - concrete, transformational and symbolic. These levels are closely tied to the early work of Piaget and his developmental theory. Children's selection of strategies to solve mathematical problems can be categorized according to these levels. For example, at the concrete level, a child might use manipulatives or act out the problem. At the transformational level, a child might draw a picture or record a tally. Strategies at the symbolic level would include writing an equation.

This study aims to explore children's perceptions of the mathematical problem-solving process and the children's use and choice of strategies.

There are a variety of interpretations as to what constitutes a "problem" in mathematics. Based on a review of the current literature, the most common definition of a "problem" is that of Polya's: "To have a problem means: to search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable aim. To solve a problem means to find such action" (cited in Hembree, 1992, p. 248). For primary children, a simple one-step story/word problem may constitute a problem while for older children, a more complex, multi-step or open-ended word problem could be considered a problem. One of the goals of this study is to formulate a definition of what a problem is to primary students, based on their own comments and responses.

Listening to the children's voices is the essence of the study. The children's thoughts and opinions are to be the "data". Researchers have indicated that it is vital to understand children's thinking about mathematics (Confrey, 1981). This corresponds to a child-centred teaching philosophy that is grounded in constructivist theory in which the classroom program is planned for, by, and with the children, based on their needs and abilities. The researcher believes that children are the heart of education and should thus, be meaningful contributors to the knowledge base we are developing about mathematical problem-solving. In their discussion of early childhood research, Swadener and Marsh (1995) confirm this belief by stating that it is "vital to recover and listen to, the voice and perspective of children" (p.174). This belief is also based on the work of Piaget and his supporters such as Margaret Donaldson and Eleanor Duckworth. Vivian Paley is another contemporary researcher whose work focusses on observations and dialogues with young children. The researcher believes that primary children have many worthwhile things to say, and their age should not be an inhibitor to probing their thoughts. Significant portions of the children's interview transcriptions are included in the study so that the reader is able to get a sense of what each child's thinking is like. This study aims to contribute to the growing body of research mainly by including open-ended interview questions and by including the comments and thoughts of the children involved in the study.

Statement of the Problem

The purpose of this study is to explore primary children's perceptions of mathematical problem solving. Through interviews, the children will describe and give examples of what they think mathematical problems are. The children will also describe whether or not they have a problem-solving process in place at this point. Another area of investigation is asking the children what strategies they use to solve mathematical problems. This group of children will also respond to questions and discuss their strategy selection process and their preferences for specific strategies.

While pursuing the problem stated above, the objective is to conduct a study that utilizes the language and perceptions of the children, as shared with the researcher.

Specific Research Questions

1) How do children define mathematical problems?

2) What do children understand about the process of solving problems?

3) What strategies do children say they use to solve mathematical problems?

4) Do children have a preference for specific strategies and why?

Educational Significance of the Study

At a theoretical level, information regarding children's actual perceptions of mathematical problems, the process of problem-solving and the use of strategies, contributes to our understanding of a child's cognitive as well as conceptual development.

At a classroom or practical level, the more teachers understand about the children they teach, the more informed their planning and teaching can be. Teachers would be able to plan activities that build on and extend a child's mathematical knowledge. This constructivist approach to teaching relies on continual assessment and awareness of the children's understandings and developing concepts. This study models a type of assessment that may be used in the classroom. Interviews or conferences with children provide rich information about a child's learning that may not otherwise be discovered. It is hoped this study will reveal worthwhile information obtained by talking and listening to children.

Noteworthy Features of the Study

Although this study begins with a set of interview questions, these are just used as a framework. All the students involved responded to the pre-designated questions but there were many other questions and sub-discussions that came up in some interviews and not in others. Thus, when comparisons are made, they are limited to the common areas.

Another important feature of this study is that the researcher is both teacher and interviewer to the students. This role assignment may concern some readers since there is potential whereby the child in a student role might be questioning, "What does she want me to say?" instead of just sharing his or her thoughts. However, this potential concern is balanced by the accurate and valid data which result from the interviewer knowing the children very well. A distinct element of trust between the researcher and the children involved has developed over time as well as the researcher's ability to sense when a child needs a probing question to further expand his or her ideas. As stated in Anderson, Herr, and Nihlen (1994) the role of the practitioner researcher in qualitative research "gives the researcher a sense of immediacy and a depth of understanding by virtue of his or her position and by the chosen research paradigm" (p. 110). This dual role also provides a more natural setting for the child (Flake, 1995). The researcher firmly believes that her role as teacher strengthens rather than limits the study.

The small sample (n=7), obviously affects generalizability of the results to population. That is, the responses given by this group of students may not correspond to those of another group of grade two students. However, this sample size permits the descriptive analysis which constitutes the core of this research

Chapter 2

Review of the Related Literature

The National Council of Teachers of Mathematics (NCTM, 1989) has identified problem solving as the central focus for mathematics curriculum and emphasizes the importance of the process of problem-solving. In addition, NCTM (1989) standards outline how problem-solving should be incorporated into daily mathematics curriculum. Children need to be encouraged to "share their thinking and approaches with other students and with teachers, and they should learn several ways of representing problems and strategies for solving them" (NCTM, 1989, p. 23). This focus is also present in British Columbia's Primary Program (1990) which suggests that problem solving should "permeate the entire program and provide the context in which concepts and skills are developed" (p. 281). With problem solving central to math programs, children's success at solving problems, along with the strategies they use to solve problems should be central to instruction. Thus, to modify and enhance their math programs, teachers need knowledge of their students' problem solving skills.

Most of the research studies in the area of mathematical problem-solving with primary-aged students involve task-based or clinical interviews using arithmetical equations and simple translation word problems (Carpenter, Moser, & Bebout, 1988; English, 1996; Hembree, 1992; Vakali, 1984). The types of solution strategies are often controlled and the children are rarely asked to articulate their thoughts about how they solve mathematical problems. For the most part, inferences are made by the researchers based on their observations and the written results of the problems the children solved. When children are asked to contribute, it is usually through a "think aloud" method, first used by Soviet educational researchers and introduced to North America by Kilpatrick (Lester, 1983). The "think aloud" method has children describe but not explain what they are doing. Thus, this technique looks at the conceptual understanding and processes used by the children but is limited by the researcher's explanation and interpretation of what is observed and recorded.

Although most of the research done prior to to the 1980's is quantitative in nature, there appears to be a growing body of research that is more descriptive in its approach to viewing how young children solve mathematical problems (Hembree, 1992). Since the current study stresses children's perceptions of problem solving, it seems more appropriate to review more current studies in this vast research area.

Two main bodies of knowledge contributed to this review. The first grouping of studies explores the effects of the teacher or instructional intervention on students' mathematical problem-solving abilities. Most of these studies have found that teachers greatly influence the development of students' problem-solving knowledge and because it is the classroom teacher that is also the researcher for this study, any correlation between student ability and teacher or instruction effect is significant. The second grouping of studies reveals the types of strategies that elementary school-aged children use in clinical interview situations.

Effects of Teacher and Instructional Intervention

Hembree (1992) conducted a meta-analysis of 487 educational studies that looked at characteristics of problem solvers, conditions for harder and easier problems, effects of different instructional methods on problem-solving performance, and effects of classroom-related conditions on problem-solving performance. The studies used ranged from the 1920s to the late 1980s. Hembree defined the purpose of meta-analysis as: "to integrate groups of criterion data" (p. 246).

In the area of effects of instructional methods, Hembree's general findings were that explicit training appears essential with heuristics training being the most successful. In the area of effects of classroom-related conditions, Hembree's general findings were that "better performance seems likely in classes where problems are studied intensively and whose teachers are trained in heuristic methods" (p. 267).

In contrast to Hembree's meta-anaysis, Cobb (1986) looked at one first grader, Melissa, to see how her understanding of basic mathematical concepts related to her methods for solving simple arithmetic questions. Cobb noted that most of the literature in this area is from a constructivist view or an information processing view and that these two points of view promote the importance of conceptual understanding. Cobb's clearly defined purpose was "to investigate Melissa's methods for finding sums, missing addends, and differences, and thus to infer the meanings she established in arithmetical situations" (p. 38).

Cobb had two sessions with Melissa, both of which were videotaped. In between sessions, Cobb inserted a teaching sessions during which he modelled addition and subtraction methods for Melissa.

Cobb drew some general conclusions from this one-shot case study. He felt the results of the teaching session supported his belief that Melissa "had already constructed number as an arithmetical object" (p. 43). Cobb also presumed that "her advances were triggered by perceived limitations in her current methods, not by the demonstration of alternative methods" (p. 44). Comments made by Melissa also led Cobb to conclude that "unfortunate consequences can arise when a child is allowed to rely unduly on manipulatives" (p. 44). Melissa indicated that she got to use "little blocks" when adding or subtracting with big numbers. Cobb implies this use of manipulatives caused Melissa to use primitive solution methods, and therefore implies that the use of manipulatives is a negative approach.

Another study that included changes in students' solution methods due to intervention is Carpenter, Moser and Bebout's (1988) study of the ways first and second grade students translate word problems into arithmetic equations. The purpose of their study was "to investigate children's representation of addition and subtraction word problems with open number sentences" (p. 347).

Carpenter, Moser and Bebout (1988) employed a pre/post-test control group design with twenty-two first graders and fourty-one second graders from a middle-class school in Madison, Wisconsin. The children were randomly assigned to the control group or the experimental group. Each group received two thirty-minute instructional sessions on writing number sentences for word problems. The control group used only standard sentence problems (i.e., a + b = ___) and the experimental group used both standard and open number sentences, (i.e., a + ___ = b). The posttest consisted of twelve word problems covering all possible number sentences for join, separate, combine, compare and equalize problems.

Carpenter et al. found that only the experimental groups wrote open number sentences during the posttest and the second graders were highly successful in using them. The investigators concluded that direct instruction was a critical variable for second graders. The control group members used other counting methods to solve open number sentence-type problems. The investigators concluded that "the semantic structure of word problems directly influences the number sentences that young children write to represent them" (p. 354). Children seem to clearly focus on the action of the problem. First graders had most success with number sentences that directly related to the action of the problem. The investigators concluded that a "more extended period of instruction appears necessary for most first graders to master the open sentence format" (p. 356).

In the introduction to this study, Carpenter et al. state that children invent counting and modelling procedures but rely on convention when using symbols. Carpenter "suggests that instruction may limit the range of symbolic representations that children recognize as acceptable" (p. 347). The authors state that it is widely accepted that the most commonly used number sentences in the primary years are a+b=__ and a-b=__. Thus, Carpenter et al. indicate that children need to become familiar with nonstandard symbolic representations and ways of thinking in order to become successful problem solvers.

Gray (1991) studied children's simple arithmetic problem-solving preferences and also found that direct teacher intervention was necessary in some instances. Gray's first implication for teaching suggests that deductive approaches are not being directly taught. Gray states that "the usual practice in both schools was pen and paper" (p. 559). This background information seems to suggest a bias. If children are not exposed to alternative methods on a regular basis, it is unlikely they will use alternative methods in a research situation.

Gray's second implication for teaching focussed on conceptual understanding of number. This conceptual understanding seemed of utmost importance to Gray as he concluded that "the younger below average child does not receive any feedback from the counting procedure; the process is not being encapsulated into a known concept" (p. 569). Gray felt that his study revealed that counting does not lead to learning facts for the below average child. This has implications for children with learning difficulties and different learning styles. Gray's study shows that alternative solution methods must be directly modelled for these children.

External factors that influence children's problem solving such as curriculum, teacher knowledge and instructional methods have been discussed above. In an era of rapidly expanding use of technology, studies looking at how the use of computers affects children's problem solving are going to be crucial. Another contemporary factor that could possibly have a profound affect on children is television. Hall, Esty and Fisch (1990) looked at how the educational Square One TV program affects children's problem solving ability and strategy use.

The authors described the broad goals of the daily broadcasted, thirty minute show. The three broad goals are to promote a positive attitude toward mathematics, to promote the use of problem solving processes and to present sound mathematical content. Hall et al.'s study looked at the second goal of the program.

Twelve fifth graders were chosen from each of four schools in Corpus Christi, Texas because this city was one of a few that had not started broadcasting the program. All four schools used the same mathematics textbook and curriculum. The four schools were matched in pairs based on test results, racial/ethnic composition and socioeconomic status. One pair had mostly lower SES students while the other pair served mostly middle SES students. Within each pair, one school was designated as the experimental (viewing) and the other was the control (nonviewing). The 48 subjects were also matched as pairs within their matching schools.

The experimental groups watched one Square One TV program each weekday for six weeks. The exposure to the program was sustained, unaided viewing. There was no discussion of the program in class. Teachers did not alter their mathematics instruction in any way. The control groups did not view the program at all.

The pretest and posttest consisted of three problem solving activities (PSAs) given to each student during two 55 minute interviews over two successive days. An attitude interview also occurred during the second day. After solving an activity, the researcher used an interview protocol and questioning to get at the child's thinking. During the interview, no connection was made between Square One TV and the problems. Children's verbal reports and overt behaviours were recorded and analyzed.

Children were scored on two measures. The P-score involved the number and variety of problem solving actions and heuristics used. The M-score looked at the mathematical completeness and sophistication of the solution. Specific findings included that from pretest to posttest, children in the experimental group made significantly greater P-score gains on all three PSA's than the members of the control group ($p<.001$). Hall et al. also found that from pretest to posttest, children in the experimental group made significant greater M-score gains on two of the three PSA's than the members of the control group ($p<.001$). The P-scores and M-scores were significantly correlated, $r=.52$ ($p<.001$). General findings included no significant differences in P-scores and M-scores due to gender, SES or minority/non-minority children.

A study that looked at observed student behaviors as well as external influences, mainly teacher knowledge, was Carpenter, Fennema, Peterson, and Carey's (1988) look at teachers' pedagogical content knowledge of students' problem solving. Because most of the problem-solving research had focussed on how children solve mathematical problems, Carpenter et al. developed a research method that looked directly at teachers.

The focus of Carpenter et al.'s study was teachers' understanding of how children think about mathematics. Research questions addressed distinctions between different problem types, the strategies children use, teachers' predictions of their students' success and strategy use and relationships between student achievement and their teachers' content knowledge. The subjects for the study were 40 first-grade teachers from 27 schools in or near Madison, Wisconsin.

Teachers' knowledge of strategies was measured by showing teachers a videotape twice of three different children solving different problems. The teachers were then asked to describe how these children would solve related problems. Teachers' knowledge of their own students was measured by having teachers predict how six children, randomly selected from each of their classes, would solve six word problems. Students' performance was measured using a number facts test and a problem solving test.

General findings included that teachers were highly successful at writing word problems for number sentences (mean score was 11 out of a possible 12) and performance was also high on the problem difficulty test, except for teachers overestimating how difficult join-change-unknown problems were. Teachers also had difficulty articulating why they made their choices and the differences they saw in the problems. Eleven teachers focussed their assessment of difficulty on how difficult it would be to determine whether to add or subtract. Carpenter et al. also found that teachers identified modelling strategies successfully but had difficulty identifying counting and derived fact strategies. Teachers also had difficulty modifying the latter two strategies to other related problems. Teachers were able to accurately predict a student's successful solution to a problem 75% of the time and a student's strategy choice 50% of the time. Teachers were most successful in predicting a+b=___ and a-b=__problems. Carpenter et al. also found that teachers consistently overestimated the strategy choices of modelling and recall of facts and underestimated the selection of counting strategies.

Based on their findings, Carpenter et al. concluded that most teachers did not have a framework for classifying problems and they could not articulate their distinctions between problems. Carpenter et al. felt that many teachers did not seem to recognize that problems that can be directly modelled are easier than those that cannot. The investigators concluded that most teachers had the basic knowledge to predict their own students' problem-solving performance.

Ford (1994) did a related study with fifth grade teachers and their students. The subjects were interviewed and questioned about their beliefs about mathematical problem solving, what causes strong or weak performance and beliefs about the teaching and learning of problem solving.

Ford's results showed that the teachers believed that problem solving is basically just an application of computational skills. The activities used in the classroom were found to enhance computational ability rather than problem-solving skills. Ford also revealed that the teachers' focus was on right answers and that the use of calculators was strongly discouraged. Ford found that "students' beliefs about mathematical problem solving, are, for the most part, consistent with the beliefs held by the teachers" (p. 314).

Ford's study emphasizes the significant role of the classroom teacher and the mathematics program being used has on how children perceive mathematical problem solving.

Strategy Selection and Use

Gray's (1991) study looked at children's preferences when solving simple arithmetic problems. Gray outlined the differences between the procedural and deductive approaches to problem solving. He speculated that children of different ability levels would have different approaches to solving arithmetic problems. This hypothesis was based on a considerable amount of research that concluded that children with learning difficulties used procedural methods to solve basic arithmetic facts. Gray presented a model of preferential hierarchy including solution strategies and points out regression between deductive and procedural methods. This model is based on Gray's previous work.

Subjects were chosen from twelve classes from two English schools. Six children were chosen from each class, with two being of high ability, two of average ability and two of low ability based on the advice of the classroom teachers. The total sample consisted of seventy-two children varying in age from seven to twelve years. Gray never clearly defines whether he used "problems" or whether he used arithmetic equations. His discussion suggests that equations were used.

The research procedure consisted of task-based interviews followed by numerical and descriptive analysis. Each child was interviewed twice. At each interview the child was asked to solve a group of addition and subtraction numerical problems. The problems at the first interview included facts to ten and the problems at the second interview included facts to twenty. Manipulatives were available. Researchers recorded the child's solution strategy and the child was asked to describe the strategy he or she used.

Findings were very specific and looked at these key areas: known facts, use of alternative strategies, diverging use of strategies and the use of deductive approaches. Specific findings included that 25% of the children knew all of the addition facts to ten and 24% of the children knew all of the subtraction facts to ten. Only 3% of the children had to totally use count-all or take-away methods. The balance of the children used alternative strategies such as using derived facts, counting on or counting back.

When the children were asked what was the best way to get a solution to a problem, 91% of the sample said that knowing the answer (known fact) is the best way. The next best way for almost all of the average and below average students was counting. Only older children (11 and 12 years) were able to state that derived facts were an alternative.

Gray suggests that the child reverts back through a preferential hierarchy when he or she cannot solve a problem with a known fact. Gray stated that the evidence showed that the deductive approach is not used by below average ability children, who tend to move directly to a procedural approach if they can not recall a basic fact. Gray presented two significant implications for teaching practice. The first is to directly teach deductive approaches such as counting on. The second is to focus on conceptual understanding of number for those below average students.

Along with alternative counting and derived fact strategies, there is an abundance of literature that supports the use of manipulative materials in problem solving. Burton (1992) outlined her three key research questions: 1) Will the children select materials which match the story context in the problem when this can be done? 2) What is the effect of matching manipulatives to the problem context? and 3) What strategies will the children use to solve the problems?

Burton developed a set of twelve division problems based on the work of an earlier researcher. The subjects were from five second grade classes in two schools. Of the 117 subjects, 56 were male and 61 were female. An individual interview included time to explore the materials, assurances the study would not affect grades and time to attempt all twelve problems. The interviews took a mean time of 23 minutes. No treatments were involved.

General findings of this study were that 38 out of 117 children never used any of the manipulatives for any of the problems and that when manipulatives could be matched to the content of the problem (i.e., eggs), they were the most frequently chosen manipulative for that problem. Burton also found, unlike other researchers, that partition and measurement problems were of about equal difficulty for these children. She found that grouping manipulatives was the most often used strategy (73 out of 117 children) and that 56 children used direct recall to solve at least one of the problems. Twenty-nine children in the study attempted to use the calculator. Burton indicated that many children tried several strategies and/or manipulatives to reach a solution.

Burton mentioned three main implications for teachers. The first being that second grade children are capable of solving division problems. The second implication was that, contrary to other research "the children in this study did not appear to be distracted by the 'cuteness' of the manipulatives" (p.14) and that some children derived an "instrinsic satisfaction from having realistic materials to use" (p.14). Burton's other teaching implication was that children need to be taught how to use calculators properly and be able to judge the reasonableness of answers from calculators. Burton also provided suggestions for further research in the above three areas.

Carpenter, Fennema, Peterson, and Carey (1988) indicated a significant difficulty when working with young children and problem-solving. They suggested that many students do not consistently use a single strategy so strategy prediction becomes very difficult for teachers. Carpenter et al. presented an interesting hypothesis at the end of their study. They suggested that instructional decisions are based on teachers' knowledge of students' success in attaining correct answers, not strategy selection, thus, student achievement is based on student success levels, not sophistication of strategy use.

English (1996) has a different perspective on the issue of children's strategy selection and what affects it. She analyzed case studies of both low and high-achieving nine year-olds and their construction and analogical transfer of mathematical knowledge. The children were presented with novel problems in both written and hands-on forms. One of her conclusions was that "through problem experience, children acquire not only knowledge about the particular problem domain, but also knowledge about their own strategies as they apply to the problem. Thus they come to realize how a particular strategy works, why it works, and why it is the most appropriate strategy for the problem" (p. 85).

The results of English's study confirm the need for children's exposure to problem solving experiences and practice in a variety of strategies. It is only with experience and practice that children then feel confident in selecting a strategy from their repertoire.

Summary

Many important issues are addressed in the previous review of literature, but there are significant gaps in this area of mathematics research. Although growing in number, more qualitative studies in this area are needed, particularly those that include children's ideas, thoughts and comments. No studies were found that looked directly at children's perceptions of their problem-solving processes and strategy preferences. It is because of this absence in the literature that this study was formed.

Based on the indicated gaps found in the literature in this area, the specific research questions for this study were developed. It was also decided that although pertinent information is collected during task-based interviews, this method has been used frequently in the past ten to fifteen years and a different approach would be taken for this study9. An approach that limited researcher subjectivity and focussed on the actual language of the children would be a unique study. Because the researcher was also the children's classroom teacher, valid and thorough responses to questions and rich, meaningful discussions were anticipated to form the focus of this study.

Chapter 3

Methods and Procedures

The methods and procedures used for this study are described in this chapter. Sections describing the subjects involved and the researcher and her classroom program provide a context for the study. Since data collection involved interviews between the researcher and individual students, a rationale for using interviews, the interview protocol and interview procedures are provided. This chapter concludes with an explanation of the method of analysis for the data collected during this study.

The Subjects

Seven children were selected from the researcher's English class of grades one, two and three. The class is located in a Lower Mainland school district in British Columbia. The school has both English and French Immersion programs. All of the subjects were in grade two at the time of the study. This grouping was selected because of a convenient number that allowed the researcher to use an entire grade grouping rather than having to select only a portion of another grade grouping. There were four female subjects and three male subjects. One of the female subjects was an English as a Second Language student but was able to communicate fluently in English.

Although no specific data were collected regarding the students' socioeconomic or family background, as with most of the school's population, these students were from middle to upper-middle class homes where parents are very well informed, interested and involved in educational matters. All of the children involved were of average to above-average ability in the area of mathematics as determined by the researcher/classroom teacher.

Pseudonyms for the students are used for the reporting of the results to ensure confidentiality.

The Researcher

The researcher is a primary classroom teacher who was working on this study as part of her Master of Arts degree in Curriculum and Instruction. She had been teaching for four years at the time of the study, all years with the same school and school district. Of the seven subjects, three had been in the researcher's class for two years, while the other four were completing their first year with the researcher/teacher.

The Researcher's Rapport with the Subjects

An important feature of this study is that the researcher is both interviewer and teacher to the subjects. As stated in the introduction, the researcher believes this contributes to the strength of the study by enabling the researcher to obtain more accurate and valid data. An essential part of the teacher-student relationship is trust. Trust develops over time. Three of the seven subjects involved had been taught by the researcher for two school years. The other four subjects had been taught by the researcher for one full school year. Over this time, both administrators and parents had commented that the researcher's teaching philosophy encourages teacher-student relationships based on openness and mutual respect. Observers often describe the students in the class as independent, confident risk-takers who care for and support each other. These qualities are encouraged and nurtured by the classroom teacher.

Evidence of the openness of the relationships can be seen in the report cards the students write about the teacher at the end of each year. Sherri stated, “I think you are fare (sic) and gave us challenging math.” Joan felt comfortable giving the teacher constructive criticism, “She is nice. She neads (sic) to cleen (sic) her desk.” Overall, feedback from parents and other adult observers in the class has indicated that there is a very strong rapport between teacher and students.

A strong rapport in the teacher-student relationship will enable the researcher to be acutely aware of the interactions involved during the researcher-subject relationship.

The Classroom Program

The class of grade one, two and three students is organized and planned for to achieve the goals of British Columbia's Primary Program. A child-centred and democratic approach that involves active learning, open-ended experiences and individualized programs is the overall plan for the classroom program. The math program involves a balance of skill development and practice with active explorations with materials and concepts. Textbooks are not used. Whole class oral problem-solving experiences are woven into the day and often integrated with other related subject areas. The children often write their own mathematical problems and enjoy trying to challenge themselves and their peers with these. This experience often causes the children to be more thoughtful about their problems, a phenomenon also experienced by Silverman, Winograd and Strohauer (1992). Written mathematical problems are sometimes solved cooperatively in groups or are individualized for different levels of development among the students. At the beginning of the school year, teacher modelling of problem-solving strategies occurs along with students sharing their successful strategies. The list of strategies used in the interview protocol resulted from a classroom “brainstorm” of different strategies that the children in the class use.

The last thirteen weeks of the math program were taught by a university preservice teacher. Although not realized at the time, the resulting significance of this is suggested in the concluding chapter.

Rationale for Using the Interview Method

Using semi-structured questions that are open-ended in nature allows the researcher to receive the desired information in a manner that also allows the child's thinking to be a focus. As stated by Huinker (1993), “Interviews, like windows, allow us to see students' mathematical understanding and reasoning more clearly” (p.86). Regular probing encourages the child to extend his or her thinking and explain his or her thoughts in different ways. As stated in Schumacher and McMillan (1993), the purpose of the ethnographic interview is “to obtain data of participant meanings-how individuals conceive of their world and how they explain or “make sense” of the important events in their lives” (p. 423). The questions are such that they often take the interview to another related area, which allows for a greater richness in the students' responses. The student's articulated thoughts begin to build on each other and the student is able to develop a clearer understanding of his or her thoughts as well.

The data collected from these interviews were abundant and enabled the researcher to portray each child's thinking in a comprehensive manner. Data from individual interviews were compared and analyzed to form categories and develop patterns.

The Interview Protocol

The researcher followed a series of questions that were formulated based on previous classroom-based action research projects done by the researcher in this area. The questions were meant as a framework for the interview and transgressions were made as deemed necessary by the student or the researcher. A natural conversational flow was desired although returning to the interview schedule involved some unnatural pauses in places.

The intention for these interviews was to hold a conversation with the student. Tasks were not a planned part of the interview, but were to be used if they arose naturally during conversation or if they helped either the student or the researcher with clarification.

The interview began with rapport-building questions such as "What do you like about math at school?" The next set of questions was based on what the child understood about what a problem was and was asked to give some examples of what she or he thought a math problem was. The following set of questions focussed on the child's knowledge of a process for solving math problems and included questions like, "How do you know when you have solved a math problem?" The lengthiest set of questions involved the use of strategies for solving math problems with questions like, "Does the type of strategy you use depend upon the problem? Explain." Time was also given, toward the end of the interview, for the student to make any extra comments or for the researcher to go back and clarify certain points. A list of the scheduled interview questions is included in Appendix A.

The Interview Procedure

The data for this study were collected during a four day period late in June 1995. Informed written parental consent was obtained prior to the interviews. The student's oral consent was obtained at the beginning of the interview. The student and parents were able to select an interview time either before or after school, whatever was most convenient for them. Reminders about the interviews were sent home with the students with their assigned time. The students were interviewed in an empty seminar room, close to their own classroom. At the beginning of the interview, students were informed that they would be asked several questions and that we were going to talk about math problems. They were told to answer the questions as best as they could and to ask for clarification if they did not understand something. Interview times ranged from twenty-five to fifty-five minutes.

One student, whose family is non-English speaking, had her mother remain in the room for comfort purposes. The researcher felt that this did not inhibit the student from responding in any way.

The interviews were audio-taped and video-taped to ensure accurate transcriptions. The audio-tapes were used for the transcriptions and the videotapes were used for clarification of some phrases and to add facial or body gestures to the transcriptions where appropriate. A sample transcription is provided in Appendix B.

Method of Analysis

The objective of the study was to listen to children's perceptions and understandings around mathematical problem-solving. It is inherent in the analysis that the children's language stand for itself with limited interpretation by the researcher. The researcher had a close relationship with all the students involved and was able to add background information or possible interpretations that enhanced the students' responses, but was careful to not distort the children's actual responses.

After the interviews were transcribed, each child's interview was read over carefully. A "snapshot" description of each student involved was prepared. The structured interview questions had been grouped into sections to support the specific research questions which had been formulated based on field work and previous research:

1) How do children define mathematical problems?

2) What do children understand about the process of solving problems?

3) What strategies do children say they use to solve mathematical problems?

4) Do children have a preference for specific strategies and why?

Inductive analysis was used to look at each grouping of questions. Each grouping of questions formed a large category. Within each category, data segments were coded in the transcriptions. A portion of a coded transcription may be found in Appendix C. The patterns and sub-categories that arose as a result of the analysis lead to a descriptive synthesis of the data. This synthesis, divided into categories, along with the students' snapshot descriptions are contained in the following chapter.

Chapter 4

Results

In this chapter, the analysis of the data is presented in descriptive form. A snapshot of each student including some relevant background information is provided along with key responses to questions in the interview. Background information comes from the teacher's classroom-based assessment and report card comments. These comments were included to give the reader a picture of the student involved and pull the reader in to seeing each subject as an individual. The first paragraph of each individual student description is based on classroom assessment and the remaining description is based on the analysis of the child's interview. After these descriptions, comparisons are made between the children and within the categories set out as the specific research questions. The analysis will be provided under the following categories:

1) Definition of a Mathematical Problem
2) The Process of Solving a Mathematical Problem
3) Knowledge of Strategies
4) Preferential Use of Strategies

In the following descriptive analysis, pseudonyms will be used to protect the confidentiality of the children involved.

Individual Student Descriptions

Kyle

Kyle is a male grade two student with strong abilities in the areas of both mathematics and language arts that exceed the expectations for students his age. He is a confident and introspective student who prefers to work independently. Kyle is very proud of often being able to be one of only a few students in the class who is able to solve the most challenging math problems presented.

During his interview, Kyle commented that he liked everything about math. Kyle defined a math problem as something that's about math questions and it has words. Kyle was very clear in the distinction between a math problem and a math question:

> Can you give me an example of a math problem that might be challenging for you?
>
> K: Like 57 divided by 8.
>
> And is that a math problem?
>
> K: It's a math question.
>
> Can you give me an example of a problem?
>
> K:There are 520 fish and along came a big fish and ate 432 .
>
> And what would you do to solve that? You'd write down those numbers?
>
> K: nods yes
>
> And what makes that different than 57 divided by 8?
> (P. A. Interruption)
> How is the problem about the fish and the big fish eating them different from 56 divided by 8? How is one a problem and one's not a problem?
>
> K: Cause one of them have words and its a different type of question.

How is it different?

K: It has words instead of numbers.

According to Kyle, his process for solving math problems involved counting numbers in his head, writing the answer down and having a teacher check it. He felt that sometimes he knew on his own if he had solved the problem correctly. Kyle said he tries to solve the problem in a different way if he can't solve it the first time he tries. Kyle explained that he felt happy when he completes a problem because it is hard work.

Kyle thought at first a strategy was a challenge of some sort and needed the researcher's explanation to help make this clear (see Appendix A). The only strategy that Kyle could think of at first was doing mental calculations. During the interview it was clear that Kyle focussed on this strategy while he did comment that he sometimes did drawings, made tallies for dividing and used equations. Kyle was able to articulate that when dividing, he mentally used the patterns in the multiplication tables to help him solve division questions. Kyle clearly preferred the strategy of "figuring it out in his mind" and commented that this was probably because it was the only strategy he could think of. It is important to note that Kyle is also cognitively able to mentally solve most mathematics presented to him. Kyle was also able to articulate why he might use other strategies:

> You said that sometimes you use drawings and that you use tallies when you do division type problems?
>
> K: There's tallies and I put them in the groups.
>
> And how does that help you?
>
> K: It helps me understand what the problem is and so I can solve it.
>
> And why do you use drawings sometimes?

K: Help me figure out what the problem is.

So if you can't figure it out in your mind it helps to draw a picture?

K: Most of the time.

Would you say that most of the time you just figure it out in your mind?

K: nods yes

This excerpt reveals that Kyle systematically chooses specific strategies for specific purposes.

David

David is a male grade two student who is meeting learning expectations for his age in the area of mathematics and is meeting the minimum expectations in the area of language arts. He has a strong oral vocabulary and enjoys participating in class discussions. David often demonstrates an intuitive understanding of the math problems we solve orally as a class. David is a physically impulsive child who has a hard time sitting in one place for more than a few minutes. For the interview, David held and played with a few pieces of Lego which is a technique used in the classroom to help keep him settled.

David began his interview by stating that he likes math because "you have to think," and that he liked adding and subtracting at school. When asked to explain what a math problem was, David gave an example of a simple translation word problem. He differentiated between problems and equations by explaining that equations were just "normal" math:

Can you explain to me what is a math problem?

D: Like there were 20 frogs and like 15 got away, how many were left?

Okay. What makes that a math problem?

D: Cause that's, you have to take away the frogs.

So if I gave you on a piece of paper, 15-9, would that be a math problem?

D: No.

How come?

D: Cause you don' t have to...that's just normal math, its not a math problem.

As our conversation about math problems continued, David also indicated that problems usually have words and most involve something being taken away. He explained that take aways were harder for him and that's why they were problems. David did concede that problems can involve adding...sometimes.

David seemed to have a clear problem-soving process in place. He stated he thought about the problem in his mind, counted in his head and thought about what it was. His next step was to write down the answer. David felt he either needed to use a calculator or to check with the teacher to see if his response was correct. If he could not solve a problem on his own, David said he usually asked a friend or maybe a teacher.

David gave a unique response to the question, “What is a strategy?” that shows that this word is part of his out-of-school vocabulary as well:

What is a strategy? (pause)

D: Like if you had to go camping and you didn't know how to make a tent and you were like thinking, you were thinking because you didn't know what it was. So you had to think to make one.

Try to figure it out?

D: Yah.

Okay. In math, what do you think a strategy is? When you're solving a math problem, what is a strategy?

D: Like to count backwards or count forwards.

David showed that he had a conceptual understanding of what a strategy was by being able to apply the term confidently in different contexts. David was also the only student to come up with the strategies of counting forwards and backwards. David also indicated that he sometimes used patterns, made drawings, used his fingers and that he sometimes goes and gets little toys or rabbits. David was able to explain that he used certain strategies for problems with specific operations:

> Does the type of strategy you use depend on the problem? Like, do you use different strategies for different types of problems?
>
> D: Yah.
>
> Can you explain that?
>
> D: Like for math (addition), I would use sometimes rabbits and for take aways I could use my head and for groups of, if there's four groups of four, I would put 1, 2, 3, 4, circles and then put four in each one.

As seen in the above partial transcription, David was able to come up with clear responses to the questions and seemed to understand what the researcher was asking of him. He was able to think of specific examples, something many of the other children struggled with.

Colin

Colin is a male grade two student who is easily meeting the learning expectations for children his age both in the areas of mathematics and language arts. Colin is a quiet and dedicated student who has set high standards for himself. He often "pushes" himself to do similar quality work to the older children in the class. Colin has a strong oral vocabulary which enables him to carry on sophisticated conversations with adults.

At the beginning of the interview, Colin explained that he likes math because it is both hard and easy. He stated that he worked on Kumon math at home which is a fact drill-based program. Colin was very clear about what he thought a math problem was:

> What is a math problem?
>
> C: Its something you write and you have to solve it.
>
> What do you mean by solving it?
>
> C: Well, its like a math problem.
>
> Can you think of an example of a math problem?
>
> C: Sort of like a story, a story.
>
> It's kind of like a story ...
>
> C: And you put a bit of math in it.
>
> Okay. Can you think of an example? Can you make one up in your head right now?
>
> C: There were 18 frogs on a log. 7 jumped off. How many were left?
>
> Okay. And what makes that a problem?
>
> C: 'Cause you got to think about it.

Colin was able to come up with an example of a typical translation word problem. His response to the question, "And what makes that a problem?" suggests a higher level of cognitive activity than computing facts.

Colin has a clear problem-solving process in place. He stated he looked at the question, thought about it, considered what it would equal and then wrote down the answer. If he did not know the answer he explained how he used little dot drawings to help him figure out the answer or he asked someone beside him to help him a bit. Colin articulated that you had to check your answer when you were finished. He said he did this in his head, sometimes with a calculator and sometimes he had to ask a teacher.

Colin did not know what a strategy was at first, but after the researcher explained that it was something you use to help you solve something and told him that his idea of drawing litttle dots or asking someone to help him were strategies, he was able to come up with the following strategies on his own: using a calculator, using your fingers and using your head. Colin felt he used his "head" and drawings the most "Cause it helps me a lot." With further questioning, Colin indicated he frequently used tally marks, sometimes used manipulatives, graphs for specific problems and patterns for counting coin combinations. He also said he wrote down number equations when the problem involved big numbers.

An interesting comment Colin made during our conversation about strategies suggests that his initial definition of a math problem may be flexible:

C: Yah, like 7 X 4.

Okay, is that a problem?

C: Sort of, when you can't really think about it.

Colin's comment suggests that a problem does not necessarily have to have words or be a story, rather, it is the "extra" thinking involved. For a grade two student, a multiplication fact such as 7 X 4 might indeed be considered a problem, as few students at this age would have memorized their entire times tables.

Sherri

Sherri is a female grade two student who exceeds the expectations for children her age in the area of mathematics and meets the expectations in the area of language arts. Sherri's family speaks Japanese at home, but she was born in Canada and speaks English fluently. Her accent and English as a Second Language background sometimes affect her spelling and her comprehension of written work, such as written mathematical problems. Sherri has articulated that it is important to her parents that she do very well in math.

Sherri's interview began with a discussion about the kind of math she liked. She stated that she liked doing multiplication facts and math reviews. Sherri also commented that she did Kumon math at home and liked that because she got to time herself. Sherri defined a math problem as "the kind when you write a story and at the end you write a problem." She gave an example of a translation word problem. We further discussed the difference between a problem and a numerical equation:

> Okay, so its a story. What's the difference between, here's a story one and here's one with just numbers, 20-15=. Is this a math problem?
>
> S: No.
>
> How come?
>
> S: Because its kind of not like a problem at the beginning...
>
> What's the difference?
>
> S: There's numbers and in the problems (P.A. interruption) longer.
>
> Which one's easier, the problem or just numbers?
>
> S: Just numbers.
>
> How come?

S: Because I know what numbers I have to use and don't have to use.

Sherri's comment about knowing what numbers to use in an equation makes it easier, implies that the difficulty in a problem is determining what to do. As other children have commented, this requires extra thinking. For Sherri her comprehension of the English language is also likely a factor in this area.

Sherri explained her process for problem solving as first reading the problem and then writing the answer. After she has written the "whole thing" she checks it herself, sometimes using a calculator. If she in unsure of what to do or does not understand the problem she "asks some people for some help."

Sherri replied that she did not know what a strategy was, when asked. The researcher explained that a strategy was what you do to help you solve a math problem and explained to Sherri that her suggestion of using a calculator was a strategy. After this explanation, Sherri was able to come up with the strategies of using her hand, using a calculator and using her head. "Using her head" was the strategy Sherri felt she used most often because she felt she was fast at it. During questioning, Sherri also commented that she used drawings sometimes, counted on her fingers when it was really hard and used facts and equations which she usually did mentally.

Sherri commented that multiplication and big numbers make problems challenging as does a "tricky" way of writing a problem so that it is confusing to know what to do. Sherri said she enjoyed writing her own problems and found solving her peers' problems challenging.

Pam

Pam is a female grade two student who exceeds the expectations for children her age in the area of language arts and meets the expectations in the area of mathematics. Pam is a highly articulate young girl with a flair for the dramatic. She tends to dominate class discussions and does not hesitate to ask questions when she is unclear about something. Pam tends to get somewhat frustrated when she does not understand something and often compares herself to other children in the class. Pam is a perfectionist in many areas.

Pam particularly enjoyed the interview situation and viewed it as an avenue to discuss all sorts of her ideas. Our conversations often went off in many directions and the researcher had to use some leading questions to get Pam focussed back on the discussion we had started. Another observation was that Pam seemed not toclearly listen to the questions being asked and often responded with her own questions and non-related answers. This was the longest interview, approximately fifty-five minutes.

At the beginning of the interview, Pam commented that she liked math because math she knew was easy so then it was fun. She stated that she did Kumon math at home and explained that she had about ninety-two more levels to do in Kumon. Pam also expressed her anxiety about learning division next year and that new math is not fun until you know how to do it.

Pam had a clear and lengthy definition for what a math problem is:

What is a math problem?

> P: A math problem is something, you can figure it out, its something that has to do with numbers, taking away the numbers, adding the numbers, and like dividing the numbers, and you can, you can like mush the numbers all around, take any number you want and then you'll like have a number, umm, a math problem is like something where somebody thinks of a question, they write it down and you try to figure it out using different ways, like a calculator or in your head and stuff like that.

Pam's elements for a math problem included numbers, figuring out, writing it down and using different ways to solve it. When asked to give an example of a math problem, Pam asked whether she should write pluses or another operation. This focus on operations continued throughout the interview as seen in the following example:

What makes it a problem?

P: Oh, I see, okay, never mind, it's something you have to figure out because its a problem, so and you use these signs to figure the problem out, say there were 20 princesses and 18 got taken away, so where's the take away sign? Right there, so you can use that take away sign and go 20 - 18 = like that.

So Pam, what if the problem was there were 18 princesses and half of them got taken away...

P: I think, I think you would use the divided sign, I don't know..maybe.

Pam continually came back to focussing on what operation the problem involved, not the problem itself.

When asked for an example of a math problem, Pam wrote an example of a double-digit addition question and used the procedural method to solve it. She was firmly resolved that this was a math problem. During the entire interview we kept coming back to what a math problem was. I found myself redirecting Pam to continue to try and explain her thinking, for example:

What kinds of problems would you use your fingers?

P: Well sometimes easy math questions I still have to...

Okay Pam, you keep saying math questions, can you tell me the difference between a math question and a math problem?

P: Umm,

'Cause we're trying to talk about problems and I...

P: Umm,

I'm not sure if you mean the same thing.

P: Mmm, a math question is...

If we look at these ones you've done here....which ones are math questions and which ones are math problems?

P: These are questions and this is a problem.

'Kay, what makes these questions?

P: Umm,

And what makes this a problem and how are they different?

P: This one uses words and this one uses numbers.

Does a math problem always have words?

P: Nope, I don't think so, not always.

And again at the end of the interview, the researcher tries to clarify, for herself, Pam's thinking once more:

I'm just trying to get you to think about this and explain what you're thinking.

P: Because I mostly think that, like that one, that math questions all have the same connections, also they use numbers and stuff but so does the time but still, time is usually a math problem so you've asked me that question, is this a math question or a math problem, I would probably say both but most of the time people would answer one of those two.

Do you think a math question and a math problem mean the same thing?

P: nods yes

Yah, that's what you think?

P: nods yes

By the end of the interview, it was clear that Pam had been using the terms math problem and math question interchangeably and did not have distinct definitions for either term. Pam appears to still be developing her understanding of this concept at this time, as this distinction between question and problem was discussed at length several times over the course of the interview.

As Pam's definition of a problem could fit most mathematical tasks, questions about the problem-solving process could apply to many different types of math questions or problems. It is not clear what Pam had in her mind when she described her process but it appears that she was thinking about mathematical word problems. Pam explained what she does when presented with a math problem:

> P: Umm, of course I take out my pencil (laughs) and I think like, is this plus, and if its plus okay then like, is this one of the things I know, well, I let's read through it, say it was, I don't know, plus, well I know how to do plus so I should probably be able to figure it out then

Again, Pam continues to focus on the operation of the problem but she also describes reading over the problem to try and figure out what she has to do, which is an important first step in problem-solving. Pam explained that she knew when she was finished a problem by going over it three or four times herself, "but mostly when you've checked it," meaning that the teacher had checked it. Pam expressed that she often felt tired after solving a math problem but she also felt scared. She explained that she felt scared because:

> P: Because usually I go around saying look at this, look at that, and people usually go "So, so" and I'd be embarrassed and usually that does happen.
>
> Why do you go around talking, showing the other kids? Are you asking for their help or...?
>
> P: Well, I'm just showing them my math, what I just did.

Pam does not seem particularly confident in her problem-solving abilities and seems to need either peer or teacher validation.

As seen in the following excerpt, Pam had a clear, general definition for what a strategy is and she was also able to think of math strategies on her own without any support from the researcher. She also was able to describe why she used the strategies she did.

What is a strategy?

P: A strategy is something, something that you use to help you and you use for math, like, a strategy you use, its somethinng that you use to help you for like, you could use to help you for almost everything.

Okay. What are all the different math strategies you can think of?

P: Mmm, one's a calculator, you could use a calculator, umm, you could use the computer inside your head, sometimes it doesn't work too good, umm, what else, you could like, you could go over the question so that you sort of like so you like understand it but like you think you understand it but you really don't, you could use umm, (grunts) I keep coming back to the calculator.

So that's a strategy?

P: Yah, a calculator.

What strategies do you usually use to solve a math problem?

P: Mmm, sometimes I go over to the people and they help me and sometimes you say that I can use a calculator and sometimes I can look in a book and I know that there's no book but there has to be one in the whole wide world, that tells you how to do a math problem.

Okay.

P: Or, look back at my work and look back at how I did those.

Okay. Why do you think you use these strategies?

P: Mostly to help me so, like I don't get frustrated and angry at myself because I haven't really done it, yah, so a strategy is to help you, umm, do math and well mostly everything.

Pam appears to have a process and a repertoire of strategies in place for solving problems. During specific questioning in part four of the interview (see Appendix A), other strategies that she indicated she used were making drawings for multiplication, using her fingers, using a calculator and equations. Pam commented that she could solve problems using small numbers in her head.

Cathy

Cathy is a female grade two student who meets the expectations for children her age in the areas of mathematics and language arts. Cathy is a quiet, hard-working student who enjoys cooperative group work and is an extremely patient and helpful peer tutor. In the area of math, Cathy often requires repeated instructions or teachings to understand a new procedure or concept but perseveres until she does understand.

Cathy began the interview by explaining that she liked math because it was hard for her and she can think about it in her head. She also commented that she liked working as a group when the class did math. This could be because Cathy tends to lack confidence in the area of mathematics or because she genuinely likes working with others.

Cathy confidently responded to the question, "What is a math problem?" as follows:

> C: A math problem is where you write down something and then you write a number and then you write down something and then you write down another number and then you try and figure it out and draw a picture.

What Cathy seems to be describing is a typical word problem, and these are the type of problems Cathy gave when asked to give examples of math problems.

Cathy commented that math problems were different than math, meaning typical skill-type math and facts. Cathy further added to her definition of what a math problem is by saying, "Math is numbers and math problems you have to write and do the numbers," suggesting that math problems need to have writing. This coincides with the examples of math problems that Cathy suggested.

Cathy's process for solving problems was very basic. She said she tries to figure out the answer and write it down. She added that she also draws a picture and colours it but this does not appear to be a problem-solving strategy in this case, just a decorative way of completing a problem. After this, Cathy said she handed in the work. She made no mention of checking her answer or trying different strategies to solve it. When asked how she knew when she was finished, Cathy replied that she could use a calculator or she does it in her head. If she could not solve a problem, Cathy said she would ask someone.

Cathy admitted she forgot what the word "strategy" meant so the researcher supplied her with a brief definition, similar to that used for the other students who needed it (see Appendix A). Cathy had some difficulty thinking of what strategies she might use until the researcher said that her idea of checking her work with a calculator was a strategy. At this point, Cathy said she could ask an adult, use unifix cubes or use other objects such as erasers or pencils. Cathy explained that she used these strategies because "they help me try and figure out the answer." During specific questioning in part four of the interview, Cathy also indicated she used the following strategies: drawing pictures, using tally marks, using her fingers, using a calculator and using facts or equations. An interesting comment that Cathy made, that is often typical of older children, was that she only used unifix cubes or counters when she was in kindergarten or grade one. The look on Cathy's face suggested that using counters was not a desirable thing to do. The strategies Cathy used definitely seemed to help her with problem-solving and often helped her to clarify the meaning of the problem:

> Can you think of a problem that drawings have helped you solve?
>
> C: Well once I was doing a really hard one and it helped me a lot.
>
> How do you think it helped you?
>
> C: Because it showed me how the picture was and it showed me what the answer was.

At the end of the interview, Cathy commented that she liked writing her own math problems and solving the problems that her peers had written. She explained that some of the students made the problems really hard by using really hard numbers.

Joan

Joan is a female grade two student who meets the expectations for children her age in the areas of mathematics and language arts. Joan is a friendly and outgoing member of the class who enjoys working with others. Joan is often hard on herself and starts crying if she does not understand something. Joan has articulated that she feels pressure to do well at school from her parents. She appreciates one-on-one assistance when she has difficulty and often just needs reassurance that she is on the right track.

During the interview, Joan said she likes math because it was fun. She commented the only other place, besides school, that she did math was Kumon, at a church. Joan commented that Kumon "gets easier when its harder 'cause I get used to it more," which is a similar comment to Pam's about the same issue.

When asked what a math problem was, Joan gave an example of a simple translation word problem, its corresponding equation and the answer.

Joan was unable to explain why her example was a math problem. When the researcher asked her whether 10 - 2= was a math problem, she thought it was. The researcher needed to do redirecting during this converstion to clarify Joan's thinking:

> Would you call this a math problem? 5-2=? Would that be a math problem?
>
> J: nods yes
>
> If I wrote down that problem you said, "There were 90 caterpillars, 10 got eaten?
>
> J: It doesn't matter.
>
> How many were left? Is that a math problem?
>
> J: Yup, wait...(looks at paper)
>
> You don't know?
>
> J: No.
>
> What do you see as the difference between these ones and this one here?
>
> J: Because there's writing in this one and there's numbers.

Even after this discussion, Joan could not articulate what a math problem was. It appeared she thought that both numerical equations and word problems were math problems yet changed her mind during the conversation. The researcher felt that Joan was trying to read the researcher's reactions to her answers and this affected how she responded.

Joan was unable to articulate a process for solving a math problem, other than asking for help, maybe using a calculator and then going on to the next problem. Joan commented that the only way she knew of to find out whether she was finished a problem or not was to have the teacher check it. This corresponds to her classroom behaviour and her lack of confidence in this area.

Joan did not know what the word "strategy" meant and needed an explanation provided by the researcher. Joan had previously mentioned using a calculator and this was given as an example of a strategy. After this conversation, Joan thought that asking for help and using her brain were other strategies she used. The researcher asked Joan to think of what other strategies children in the class might use. Joan thought of using "googol" blocks or little ducks or dinosaurs. During part four of the interview, Joan also indicated that she made drawings, used her fingers for multiplication, used counters, used a calculator and filled out a chart or graph when it was part of the problem. Joan was able to give a specific example of when a problem-solving strategy helped her:

> What kind of problem do you use drawings for?
>
> J: Mmm, circles with ten dots in each one, like 4 circles and ten dots in each one, like that.
>
> If you had to figure out four groups of ten?
>
> J: Uh-hmm, (affirmative)
>
> Why do you think it helps you to draw the circles and dots?
>
> J: Umm, because like let's say its times, I'm not that good at times so I would probably use those to help me get better.

Joan showed use of a strategy for a specific purpose, in this case the operation of multiplication.

At the end of the interview, the researcher and Joan engaged in further dialogue about what constitutes a problem.

> Do you think 2+2 is a problem?
>
> J: Yah.

How come?

J: Because it has numbers and I don't know why but its just a problem.

If you had to call some things math questions and the other things math problems, which ones would you call math questions and which ones would you call math problems?

J: Math questions, I would call the answer, no like, umm, like,umm say

2+2 would be a math question and math problem would be like same thing 2+2 maybe 4+4, that's what they are.

So they're both a question and a problem?

J: Uh-hmm. (affirmative)

Would the one with the words be a question or a problem?

J: A problem, a math problem.

During this conversation, Joan continued to look at the researcher for feedback and was not confident in her responses. It seemed that Joan had not formulated an opinion on this idea and maybe had not considered it before this interview.

Definition of a Mathematical Problem

Two out of the seven children involved could not articulate what a math problem was. Joan admitted she did not know and got frustrated trying to explain her thinking. Joan did give an example of a word problem when asked what a math problem was. David also gave an example of a word problem when asked what a problem was. His only contribution to a definition was that math problems are different than "normal" math but he could not explain why. The researcher engaged in a "teaching" session during the interview which included teacher direction and leading to help David clarify his thinking so this part was not included in the analysis.

Pam was able to give information about what a math problem was but was not confident in her explanations herself. Pam admitted at the end of the interview that she considered a math problem and a math question to be the same thing but she did say that some math problems have words, but some can just be numbers. Pam also indicated that math problems needed figuring out, needed to include a “sign” (operation) and you could use different things like calculators to figure them out. These bits and pieces of information rose throughout the interview, not in response to the question specifically asking her what a math problem was.

Four out of the seven children were able to confidently give a response to the question, “What is a math problem?” Cathy explained that a math problem was a combination of writing and numbers and that a math problem was different than “math”. Sherri stated that a math problem is longer than other math and it is not a problem if it is just numbers. Sherri explained she would write a math problem by writing a story and ending it with a problem. Kyle replied that a math problem is something about math questions and it has words. Colin was able to give a detailed definition. He said a math problem is something you write and you have to solve it and that it is like a story with a bit of math in it. He also added that what makes a problem a problem, is that “you got to think about it.” Later in this interview, Colin also suggested that a multiplication fact, such as 7 x 4 could be a problem, again because you have to think about it, as opposed to knowing the answer right away.

When asked to give an example of a math problem, six of the seven children created simple translation word problems with varying levels of numbers using subtraction. The class had not been studying subtraction at the time. Kyle used three-digit numbers in his problem while Cathy suggested a problem based on the fact 2 - 1. It is interesting to note that all of the children who gave word problems used subtraction as an operation. Reasons for this could be that they found subtraction harder so they consider it more problematic, that subtraction word problems are easier to think of or that the children have been more exposed to these types of problems. It would be pure speculation as to why the children all used subtraction but a further analysis of this is provided by the researcher in the conclusions. Pam gave an example of a double-digit addition question when asked for an example of a problem but brought up other problems later in the interview. She commented that she enjoyed writing and answering problems based on our classroom studies and recalled one problem in particular. Three of the other children also mentioned this problem, which was an open-ended, multi-step problem involving a princess rescuing some princes and their horses. It is interesting to note that this problem was explored during one of only a few math lessons that the researcher taught during the last term of the school year.

Many of the children commented on how they enjoyed writing their own problems and that they like solving the problems that their peers wrote.

The Process of Solving a Mathematical Problem

Polya's (1957) process for solving problems seems to be the process still referred to in current literature. His four steps for solving mathematical problems are:

1. Understanding the problem
2. Devising a plan
3. Carrying out the plan

4. Looking back.

Most of the children involved in this study had a clear process in place, although many used only one or two steps and most lacked the metacognitive aspects suggested by Polya.

Sherri was very concise in her description of what she does to solve a problem. She reads the problem, writes the answer and then checks it herself. By reading the problem, Sherri is presumably "understanding" the problem. She does not verbalize devising or carrying out any sort of plan, rather she just knows the answer. Sherri could very well think about a plan and then carry it out but she has not verbalized these steps. She also included the important step of checking her answer herself or "looking back".

Both Kyle and David thought about the problem in their heads, counted the numbers in their head and then wrote the answer down. Both commented on checking the answer with the teacher and David also mentioned he could check it himself with a calculator. Kyle also mentioned that he could try different ways to solve it if he did not get it the first time. Again, although they do not specifically say that they have a plan and carry it out, their comments suggest that this is what they are doing, particularly Kyle who will try different strategies. Both boys also seem to have articulated the importance of thinking the problem through and looking back.

Colin explained that he looked at the question and thought about it, thought about what it would equal and then wrote down the answer. If he could not figure it out in his head he explained his strategy of drawing little dots. Colin also stressed the importance of checking his answer himself. Pam's process was similar to Colin's in that she talked about spending time thinking about the problem and then deciding on which operation was involved. Pam explained she checked the problem herself three or four times but really needed the teacher to check it.

Cathy and Joan both appear to have less developed processes in place. Cathy stated she tries to figure out the answer and write it down. Joan explained that she asks for help and then goes on to the next problem.

Five of the seven children commented that they needed to check their work when the problem was finished. It is unclear whether this is due to their understanding of the importance of "looking back" or whether they are following a procedure that has been taught to them. Four of the children mentioned that the only way they really knew whether they were finished or not was to have the teacher check their work. Reasons for this are suggested in the conclusions.

Knowledge of Strategies

The children's knowledge of strategies can by divided into two parts: 1) what they think a strategy is and 2) strategies the students could think of without direct questioning.

Four out of the seven children in the study stated that they did not know or that they forgot what the word strategy meant. Although this term was used frequently at the beginning of the year when different strategies were brainstormed and modelled, it had been used considerably less at the end of the year. Kyle thought a strategy was some sort of challenge. David's interesting response about using a strategy to make a tent when you go camping demonstrated his understanding of the term in an applied context. Of all the students, Pam described a strategy clearly:

> P: A strategy is something, something that you use to help you and you use for math, like, a strategy you use, its something that you use to help

you for like, you could use to help you for almost everything.

Pam's explanation is similar to the definition the researcher supplied to the students who needed support in understanding what a strategy was.

Without questioning and examples being given, all the children were able to think of at least three different strategies each. Five of the children thought asking someone was an important strategy and one that was used frequently. The classroom program is set up to support this kind of interaction. SIx of the children thought of using calculators, which are readily available for use in the classroom. Joan and Cathy both suggested using objects such as unifix cubes or little dinosaurs. Five of the children stated that they would use their head, brain or mind. Pam described this as using the computer in her head. David was the only student to suggest counting forwards or backwards as a strategy. Colin was the only student to suggest making a drawing or using your fingers. Pam also described looking back at other problems to see how she had done those to give her ideas. Kyle described the complex manipulation of numbers that he perfomed mentally:

> What are all the different strategies you can think of?
>
> K: If it's divided, I times it. If its plus I can plus of it to make a higher number then take away the amount and then I'll have the answer.
>
> So you always just do that in your head?
>
> K: nods yes

Kyle's sophisticated understanding of number concepts and operations permeated his interview and mental solving of problems is clearly the strategy he uses most often at this point in his development.

Preferential Use of Strategies

Many of the children articulated what strategies they tend to use most often but they also indicated several other strategies that they use. The children were asked what strategies they use most often before a list of possible strategies was provided to them. These results are the strategies the children could recall on their own that they used most often. According to the children's statements, the most preferred strategies were performing calculations mentally, using a calculator, working with number patterns and asking someone for help.

Kyle preferred to use mental math and number patterns. David most often used number patterns, counting forwards and backwards and used little rabbits (counters). Colin stated he usually used his head and made drawings. Sherri preferred to solve problems in her head because she is fastest this way. Pam usually asked someone for help, used a calculator or used other problems to help her. Cathy liked to ask someone for help and use a calculator. Joan preferred to solve problems in her head or use a calculator.

Many of the students replied that they use some strategies only sometimes or for specific problems that require the use of that strategy (i.e., chart or graph). Results from part four of the interview, which focussed on children responding to a variety of questions about specific strategies, as well as those strategies they thought of on their own, are included in the following table that indicates what strategies the students indicated they used :

	Kyle	David	Colin	Sherri	Pam	Cathy	Joan	Total
Drawings	√	√	√	√	√	√	√	7
Tally marks	√		√			√		3
Fingers		√	√	√	√	√	√	6
Manipulatives		√					√	2
Calculators	√	√		√	√	√	√	6
Charts/graphs	√		√				√	3
Patterns	√	√	√					3
Facts/Equations	√	√	√	√	√	√		6
Mental math	√	√	√	√	√	√	√	7
Ask someone		√	√	√	√	√	√	6
Counting (for/back)		√						1
Using other problems					√			1

All of the children indicated that they used drawings for solving some problems and all stated that they solved some problems mentally. Six out of the seven children said they used their fingers sometimes, used a calculator, asked someone for help and used facts or equations. The least common strategies from the interview questions were using tally marks, manipulatives or counters and using charts and graphs.

All of the students used a wide variety of different strategies and could often give examples of the types of problems for which they would use each strategy. The most common response for why they use certain strategies most often is that "they help me." These strategies seem to both help the students understand the problem as well as come to a solution.

Chapter 5

Conclusions

As chapter four attests, the children shared many interesting and informative statements. Presented here are four conclusions this research drew from the findings:

1) Children's perceptions of what a math problem is are varied.
2) The children gave examples of translation word problems involving subtraction when asked to give an example of a math problem.
3) All of the children had a process for solving mathematical problems.
4) All of the children knew of and spoke of using a variety of problem-solving strategies.

These conclusions are discussed in more detail in this chapter and contextualized by referring to classroom incidents and instruction. Following this discussion will be implications of this study and recommendations for future research.

Definition of a Math Problem

The children had varied responses to the question, "What is a math problem?" Some children were unable to clearly articulate a definition though they demonstrated an understanding through their examples. This was particularly true for Joan, who kept changing her mind depending on how the researcher reacted or questioned her. This is also typical of Joan's lack of confidence displayed in the classroom and could have been anticipated for this interview situation. This does not mean that Joan does not necessarily understand what a math problem is, but rather she may have been unable to verbalize her thoughts clearly.

Many of the children limited their definitions to a brief phrase or statement. Some statements made by the students were, that a math problem:
-is different than "normal" math
-have words sometimes
-has just numbers sometimes
-needs figuring out
-needs a sign (operation)
-is a combination of writing and math
-is like writing a story
-needs you to think about it
-could be a fact if you have to think about it.
The idea that a math problem has words and math and is like a story, came through for most of the children. This could be because the children rarely hear the term "math problem" in the classroom, except for when the teacher may ask the students to look on the chart and try and solve the math problem. The math problem in this case would be written out using words. The children are also often asked to write their own math problems which often take the form of stories or simple and multi-step word problems. In this context, the children seem to be echoing what the teacher has presented formally as a "math problem".

It is interesting that, although a math problem has been demonstrated as a story or word problem in the classroom, a few children indicated that a math problem could also just be numbers. Colin in particular was able to articulate that a hard multiplication fact could be a math problem because you had to think of a way to solve it. This suggests that some children view different types of math activities as problems, depending on the difficulty level of the activity and on the child's development and understanding of the concept involved.

Use of Translation Subtraction Problems

When asked to give an example of a math problem, six of the seven children responded with a translation problem involving the operation of subtraction. The children may have found a translation problem to be the easiest to think of in this situation. The classroom context for the last term of the school year was different and the children were exposed to an unbalanced proportion of simple word problems which could also explain why they thought this way.

It was interesting to note that all of the children chose to use the operation of subtraction in their math problem examples. The children had been exposed to all primary math concepts and operations over the school year and had experienced problems with fractions, time, money, geometry and other math concepts as well as the operations of addition, subtraction and multiplication. Some of the class had also been introduced to the operation of division, but not all of the subjects involved with this study. It was the end of the school year when this study occurred and there was no particular emphasis on any mathematical area at that time. All skills were being reviewed and practiced so the researcher anticipated a variety of problems from the students.

Upon further consideration and knowledge of her students, the researcher believes that if the children were asked specifically, they would state that subtraction was indeed the most difficult operation for them. David actually stated this during his interview. This is supported by the work of Vakali (1984) during which she tested ninety-two grade three students. She concluded that "subtraction would appear to be a more complicated mental act than addition" (p. 108). Although the children also had worked with mutlplication, the researcher feels that the children view addition and multiplication similarly based on classroom experience. If the children felt that subtraction was more difficult than other choices, this could contribute to their definition of what a math problem is - something that is hard to do.

Problem-Solving Process

All of the children in this study had a process in place for solving mathematical problems. This process ranged from "try to figure out the answer and write it down" to a metacognitive approach involving thinking about the problem in your head, trying different ways to solve it and then checking your work. None of the children clearly verbalized Polya's stages of devising and carrying out a plan, but the children's descriptions and examples indicated that they were doing so. Kersh and McDonald (1991) suggested a three-step process that combines steps two and three of Polya's model that they feel is more appropriate for the elementary school student. The researcher has also observed that the children in her class naturally combine these two steps.

Five out of the seven children involved in this study included a "looking back" step in their process. This step is not fully developed in the sense that most of the children meant merely checking their answer again, but for primary-aged children to realize this step is important is significant. This result is in contradiction to other researchers' findings, such as Kantowski (1977). Using the "think aloud" method, Kantowski studied eight ninth-graders' problem-solving processes while they solved geometry problems. She concluded that "the use of 'looking back' strategies did not increase as problem-solving ability developed" (p. 169). Kantowski's study involved instructional interludes where this step was practiced but it was not applied in the test situation.

The result that all of the children involved could explain their problem-solving processes can be contributed to two factors. The first is that these children demonstrated strong metacognitive abilities and were verbally capable of explaining themselves. Boekaerts, Seegers and Vermeer (1995) state that "metacognitive aspects are crucial in problem solving. Doing mathematics requires not only knowledge of rules, facts and principles, but also an understanding of when and how to use that knowledge" (p. 242). Most of the students involved in this study seem to be very aware of the process and when to apply their knowledge.

The second factor that could have contributed to the children being able to explain their problem-solving processes is direct instruction in the classroom. The problem-solving process is modelled not only in mathematics but also in scientific explorations and in resolving playground or social conflicts. The process is often worked through step-by-step during whole class meetings or instructional time. The two children who described the most complete processes, Colin and Kyle, had been in this classroom for two years and thus, had plenty of exposure to and experience with the problem-solving process. Although the children's verbal capabilities and direct classroom instruction likely contributed to the children's ability to explain their problem-solving processes, no direct cause and effect relationship can be determined.

Problem-Solving Strategies

This group of children could name several strategies and they stated they used a wide variety of different strategies. The children had been introduced to and encouraged to use different problem-solving strategies over the school year. Many of these strategies were modelled by the teacher or shared by the students. Thus, the class, as well as individual students, developed a repertoire of strategies to draw upon. When asked what strategies the students used most often, the four most common responses were mental math, using a calculator, using number patterns and asking someone for help.

Many of the children are at a stage where they are able to compute many addition, subtraction or multiplication facts or larger equations mentally. In the class, mental math is encouraged during whole group problem-solving sessions where a problem may be posed orally and students are asked to solve it in their minds. Some of the children implied this was a preferred strategy because it meant they were good at math.

The researcher was pleased to see the children chose to use a calculator to support them in problem-solving. Calculators are readily accessible in the classroom and children are encouraged to use them to solve problems. The researcher's beliefs are that the challenge of a problem should be figuring how to solve it and that there are enough other exercises used to practice computational skills. Sometimes a problem may involve multiple steps and the children may write the necessary equation down on paper, solve it on their own and then check their answer with a calculator. A study done by Ford (1994) indicated that a group of grade five teachers felt that "students needed to learn to solve problems on their own without relying on the calculator" (p. 319). The students of these teachers also felt very strongly that "using calculators in problem solving was 'CHEATING'" (p. 319). It is significant that in both this study and in Ford's study that the students' opinions seem to mirror the teacher's.

The researcher was suprised to see that using number patterns was a preferred strategy by some children as this is understood as a symbolic type of strategy and requires quite a strong understanding of number. This strategy was modelled and taught in the classroom. The students seemed to use it specifically for multiplication and division and for counting combinations of coins.

The fourth preferred strategy was asking someone for help. Again, this can be contributed to the classroom environment where children are encouraged to support and assist each other.

Of the nine strategies presented in the interview protocol, all of the children stated that they used mental math or did drawings for some problems. The strategies of using calculators, facts or equations and using one's fingers were used by six out of the seven children. If we look at these five most common strategies with relation to their level of representational knowledge, we find that only one of these strategies is at the concrete level (fingers), one is at the transformational level (drawings) and the remaining three are at the symbolic level (mental math, facts or equations and calculators). Assumptions are often made that most primary children function at a concrete level. This particular group seems to function more toward the symbolic level. Of course, within the group there is a range of abilities.

Although Kyle solves most of his math mentally, for a new operation, such as division, he relies on more transformational-level types of strategies such as drawing and making tally marks. It is also interesting to note that Kyle said he did not use any concrete-type strategies such as using manipulatives or counting on his fingers. This corresponds to what he demonstrates in the classroom.

All of the other students displayed a range of strategies from concrete to symbolic levels. The researcher believes it is important for the students to have a variety of strategies that they feel comfortable using for different purposes.

Implications of the Study

This study points to many relevant issues for classroom teachers. In this study, the researcher's use of interviews proved successful in uncovering children's perceptions and knowledge of problem-solving in mathematics. This then adds support to the use of conversations and conferences not only as a research method but as a classroom-based method of assessment. Conversations or conferences with students as a form of assessment have the potential to provide rich information about students' thinking. Assessment should be a tool that is used to inform classroom practice (NCTM, 1995). To that end, issues found during the analysis of the interviews for this study are shared for classroom teachers' consideration.

Children seem to need repeated use of mathematical terms such as "problem," "process," and "strategy" . Repeated used of the language is necessary until the children use the language naturally and confidently. An example of this was seen in this study when, during the last term of the school year the context for mathematics instruction was different and the children were not regularly exposed to the language they had previously learned. This seemed to affect their ability to use these terms confidently during the interviews. As with any skill or concept a child is acquiring, repetition and use of the language is necessary.

Children's perceptions of what a math problem is are diverse in that a math problem means different things to different children. If it is important to a teacher to have standardized meanings for clarity during class discussions, it is necessary for the teacher to probe and question the students involved to see if everyone is using the term in the same way. Teachers should not assume that a common term means a common understanding or definition.

The children in this study all had some sort of process for solving mathematical problems. Although some were one or two step processes, a few students implied in their interview that they followed all four steps of Polya's problem-solving process. This may be due, at least in part, to their experience with the problem-solving process in the classroom, including direct teaching of different problem-solving processes in different subject areas. This implies that primary children are capable of working through steps in solving problems if exposed to and given experience with this.

It appears that by grade two, these children have a repertoire of strategies for solving mathematical problems in place. Again, it is important to remember that these children had exposure to and practice in a variety or problem-solving situations and strategies. In addition, these children had opportunities to share their strategies with each other and to have teachers directly model and expose them to new strategies to build on their existing knowledge of problem-solving.

Most of the children in this study commented on the need to check their work, and a common strategy was to ask an adult. This points to one area that needs further development and attention, for instance, how might teachers encourage children to be more reflective independently, lessening the need for an adult to check their work for them. As Anderson (1993) states, "we need to create an environment of respect, understanding, and tolerance. We must seek and consider children's opinions and encourage them to talk. We must select and construct many open-ended mathematical tasks and encourage children to assert their autonomy and creativity" (p.105). Obviously, children need to develop confidence in "looking back" and checking their work, if they are to be empowered mathematically, (NCTM, 1989) and as Anderson (1993) suggests, the task as well as the role of the teacher and classroom environment influences self-validation.

Recommendations for Future Research

This study provides many ideas for future research. Because the sample size of this study is small (n=7) generalizability to population would occur with a larger sample size, a variety of grade levels and a cross-section of school sites. Further research could also make comparisons between classrooms, between schools and districts, between private and public schools or between English and French Immersion classes. Because all schools in British Columbia are required to follow the same curriculum, it would also be interesting to compare British Columbia results to results from another province or state that has a math curriculum similar to ours.

Further research could look at specific populations in more detail. For example, similar studies could address issues specifically concerning English as a Second Language students. Although there was only one ESL student involved with this study, her lack of confidence in problem-solving compared to other mathematical areas may be linked to her comprehension of the language used in word problems.

An interesting study into children's perceptions and use of problem-solving strategies would also prove interesting. It seems worthy to ask children the same types of questions about process and strategies used in this study and then have them "think aloud" as they complete some problem-solving tasks. In this way, we would gain insights into the relationship between what children think they do and what they actually do.

Another variation of the study would be to conduct it two to four times over a school year, monitoring instruction and experience in the classroom. This would help determine whether the context of the most recent instructional setting affects the children's perceptions and examples.

Another current issue is the role of metacognition in learning. Garofalo (1987) states that "metacognition refers to the knowledge and control one has of one's cognitive functioning" (p. 22). All the children involved in this study demonstrated an ability to think about their thinking, although the researcher felt that the children had difficulty articulating what they were thinking at times. It would be interesting to further explore young children's metacognitive abilities by investigating children's abilities to verbalize their thinking and the other factors which contribute to metacognitive awareness.

Bibliography

Anderson, A. (1993). Assessment: A means to empower children? In N. Webb and A. Coxford (Eds.) Assessment in the Mathematics Classroom. (pp. 103-110). Reston, Va: The National Council of Teachers of Mathematics.

Anderson, G., Herr, K., & Nihlen, A. (1994). Studying Your Own School: An Educator's Guide to Qualitative Practitioner Research. Thousand Oaks, Ca.: Corwin Press, Inc.

Boekaerts, M., Seegers, G., & Vermeer, H. (1995). Solving math problems: Where and why does the solution process go astray? Educational Studies in Mathematics, 28, 241-262.

Burton, G.M. (1992). Young children's choices of manipulatives and strategies for solving whole number division problems. Focus on Learning Problems in Mathematics, 14, 2, 2-17.

Carpenter, T.P., Fennema, E., Peterson, P.L., & Carey, D.A. (1988). Teachers' pedagogical knowledge of students' problem solving in elementary arithmetic. Journal for Research in Mathematics Education, 19, 5, 385-401.

Carpenter, T.P., Moser, J.M., & Bebout, H.C. (1988). Representation of addition and subtraction word problems. Journal for Research in Mathematics Education, 19, 4, 345-357.

Cobb, P. (1986). Concrete can be abstract: A case study. Educational Studies in Mathematics, 17, 37-48.

Confrey, J. (1981). Concepts, processes and mathematics instruction. For the Learning of Mathematics, 2, 1, 8-12.

English, L. D. (1996). Children's construction of mathematical knowledge in solving novel isomorphic problems in concrete and written form. Journal of Mathematical Behavior, 15, 81-112.

Flake, C. (1995). Teacher as researcher. Phi Delta Kappan, 76, 5, 405-407.

Ford, M. I. (1994). Teachers' beliefs about mathematical problem solving in the elementary school. School Science and Mathematics, 94,6, 314-322.

Garofalo, J. (1987). Metacognition and school mathematics. Arithmetic Teacher, 34, 9, 22-23.

Ginsburg, H. P. (Ed.) (1983). The development of mathematical thinking. New York, N.Y.: Academic Press.

Gray, E.M. (1991). An analysis fo diverging approaches to simple arithmetic: Preference and its consequences. Educational Studies in Mathematics, 22, 6, 551-574.

Hall, E. R., Esty, E. T. & Fisch, S. M. (1990). Television and children's problem-solving behavior: A synopsis of and evaluation of the effects of Square One TV. Journal of Mathematical Behavior, 9, 161-174.

Hembree, R. (1992). Experiments and relational studies in problem solving: A meta-analysis. Journal for Research in Mathematics Education, 23, 3, 242-273.

Huinker, D. A. (1993). Interviews: A window to students' conceptual knowledge of the operations. In N. Webb amd A. Coxford (Eds.) Assessment in the Mathematics Classroom. (pp. 80-86). Reston, Va: The National Council of Teachers of Mathematics.

Kantowski, M. G. (1977). Processes involved in mathematical problem solving. Journal for Research in Mathematics Education, 8, 3, 163-180.

Kersh, M. E. & McDonald, J. (1991). How do I solve thee? Let me count the ways. Arithmetic Teacher, 39, 2, 38-41.

Lester, F. K. (1983). Trends and issues in mathematical problem solving research. In R. Lesh and M. Landau (Eds.) Acquisition of Mathematical Concepts and Processes. (pp. 229-261). Toronto, On.: Academic Press, Inc.

Ministry of Education, (1990). Primary Program Foundation Document. Province of British Columbia: author.

National Council of Teachers of Mathematics, (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, Va.: author.

National Council of Teachers of Mathematics, (1995). Assessment Standards for School Mathematics. Reston, Va.: author.

Polya, G. (1957). How to solve it. New J ersey: Princeton University Press.

Schumaker, S. & McMillan, J. H. (1993). Research in education: A conceptual introduction. New York: Harper Collins College Publishers.

Silverman, F. L., Winograd, K. & Strohauer, D. (1992). Student-generated story problems. Arithmetic Teacher, 39, 8, 6-12.

Swadener, B. & Marsh, M. (1995). Toward a stronger teacher voice in research: A collaborative study of antibias early education. In J. Amos Hatch (Ed.) Qualitative Research in Early Childhood Settings. (pp.173-190). Westport, Con.: Praeger.

Vakali, M. (1984). Children's thinking in arithmetic word problem solving. Journal of Experimental Education, 53, 2, 106-113.

Appendix A

Interview Questions

Part 1 Rapport-building

1. Do you like math? Explain.
2. What do you like about math at school?
3. Do you do math other places than school? Where?

Part 2 Understanding of what a problem is

1. What is a math problem?
2. Please give me an example of some math problems.

Part 3 Knowledge of process for solving problems

1. When you need to solve a math problem, what do you usually do first? Then...?
2. When you have finished solving a math problem, how do you feel?
3. How do you know when you have solved a problem?
4. What do you do when you can not solve a problem?

Part 4 Use of strategies for solving problems

1. What is a strategy?

*(If a student did not know what the word strategy meant, the researcher supplied the following definition: "I think it means something that you do to help you solve a problem or to help you do something.")

2. What are all the different strategies you can think of?

3a) What strategies do you usually use to solve a math problem?

3b) Why do you think you use these strategies?

4. Does the type of strategy you use depend upon the problem? Explain.

5. Do you use ______ to help you solve math problems? Explain. (What kind of problem would you use this strategy for?)

   a) drawings
   b) tallies
   c) fingers
   d) manipulatives/counters
   e) calculator
   f) charts/graphs
   g) patterns
   h) number sentences/equations/facts

6. What kind of problems do you solve mentally, in your mind?

Part 5 Other Comments/Questions

Appendix B

Sample Interview Transcription
(Interview #6-Cathy)

Part 1 Rapport-building

Do you like math?

C: Yah.

What do you like about math?

C: I like that its hard for me and its not too easy.

What do you like about math at school?

C: I like that it's like the numbers are really hard for me and I can think about it in my head instead of using my fingers.

What kinds of things do we do during math that you like?

C: I like working as a group when we do math.

Like games or...?

C: Yah.

Group projects or problems?

C: Yah.

Do you do math other places than school?

C: I sometimes do math at my dad's.

What kind of math do you do?

C: I do adding and subtracting.

Facts and that kind of thing?

C: Yah.

Part 2 Understanding of what a problem is

What is a math problem?

C: A math problem is where you write down something and then you write a number and then you write down something and then you write down another number and then you try and figure it out and draw a picture.

Okay. Can you give me an example of some math problems?

C: Mmm, like once upon a time there was a princess, she had two horses, one ran away, how many were left? And then you try and figure it out.

Okay. Can you give me another example of a math problem?

C: You can do, once there was 3 princes, they each had 5 horses, two ran away and then you try and figure that out.

Okay. Now what makes those problems?

C: It makes them so they're easy for the person or hard.

But how come those are math problems and other things may not be math problems?

C: Well math problems are different than math.

Okay, what do you mean by that, do you want to write that, can you write down what just math is, an example for math.

C: (writing on paper)

What have you written there?

C: Math is something you have to try and guess.

Can you give me an example of something you might do for math?

C: (pause then writing on paper)

Okay, can you read that out for me?

C: 145 + 346

And that's math?

C: Yah.

And how is math different than a math problem?

C: Math is numbers and math problems you have to write and do the numbers.

So you've written here, math is something that you have to try and guess, so a math problem is something you have to... (pause)

C: You have to try and get the guess so it's right or try and get it right.

Okay.

Part 3 Knowledge of process for solving problems

When you need to solve a math problem, what do you usually do first?

C: Well you try and figure out the answer and write it down and do, what you, say I did 4 x 3 and then I'd do the answer and then I'd draw the picture to it.

How do you know to do 4 x 3?

C: 4 x 3 is really simple for me.

But how do you figure out that the problem wants you to do 4 x 3?

C: Well, if I had it in my math problem then I'd do that.

If you wrote it?

C: Yah.

But what if it was one I gave you?

C: Then I would just do it, that one if you gave it to me.

So you, what you do first is you solve it and write it down the answer?

C:Yah.

And then what do you usually do?

C: Then you colour the picture and hand it in.

When you've finished solving a math problem, how do you feel?

C: I feel good about myself when I'm finished it.

How come?

C: I'm glad I've done it so I can do other things.

How do you know when you've solved a math problem?

C: Umm, either you let us use a calculator or we have to do it in our heads.

What do you do when you can't solve a problem?

C: Well we ask someone and if they didn't know we have to ask an older person.

Part 4 Use of strategies for solving problems

Can you tell me what is a strategy?

C: I forgot.

You forgot what that word means?

C: Yup.

When I usually think of the word strategy, I think that it means something that you do to help you solve a problem or to help you do something, so like some people when they play chess, they think of a strategy so that they can win,

C: Yah.

Or when they're playing other games they think of a strategy so that they can win and then when you're solving a math problem, you think of different ways that you can use, you can do different things to help you get the answer.

C: Yah.

What are all the different strategies you can think of?

C: One is I can help myself with a strategy to learn how to do math,

Okay, you mentioned earlier that when you can't solve a problem you could ask a friend, or someone in the class and that's a good strategy or another time you talked about checking your work with a calculator,

C: Yup.

And that's a strategy. What are some other things you could do to help you solve a math problem?

C: Well, you can ask an adult or you can use unifix cubes if you're doing adding or subtracting and you can use erasers, pencils, anything else you can use to add.

What strategies do you usually use to solve a math problem?

C: Well, I usually ask a friend or use a calculator.

Why do you think you use these strategies?

C: Because they help me.

How do they help you?

C: Well they help me try and figure out the answer.

Does the type of strategy you use depend on what the problem is?

C: Yes.

Why?

C: 'Cause you can use a lot of things, if I use a strategy, like with a friend, they can help me a lot try to..

But can you think of a certain type of problem that you wouldn't use a calculator for?

C: If it was like something else that your teacher could give you, then you can probably use something else besides a calculator but if it was math review you couldn't use a calculator.

Do you ever use drawings to help you solve math problems?

C: Yup.

Can you give me a kind of problem that you would use drawings for?

C: If we had to do lots of math problems, then we'd have to draw pictures quickly.

Can you think of a problem that drawings have helped you solve?

C: Well once I was doing a really hard one and it helped me a lot.

How do you think it helped you?

C: Because it showed me how the picture was and it showed me what the answer was.

Okay. Do you ever use tallies?

C: Yup, sometimes.

What kind of problem would you use tallies for?

C: Well if you were using popsicles, you would use that if it was Groundhog Day or if you went to use them because your teacher told you to.

So if I said, use this strategy, then this might be the only time you might use tallies?

C: Yup.

Do you ever use your fingers to help you solve math problems?

C: Sometimes.

What kinds of problems would you use your fingers for?

C: Sometimes for subtraction, times and adding if its really hard.

Do you ever use manipulatives or counters?

C: No.

You talked about using unifix cubes, those are like counters.

C: Yup, but only when I was in grade 1 and kindergarten.

Okay, do you ever use the calculator to help you solve math problems?

C: Once.

What kind of problem was that?

C: It was, I think it was 13 girls and there were and 4 were stuck on each volcano and there were 3 princes, how many trips did each prince have to make?

And a calculator helped you?

C: Yah.

Do you ever use charts or graphs to help you solve math problems?

C: No.

Do you ever use patterns to help you solve math problems?

C: No.

Do you ever use facts or number sentences or equations to help you solve math problems?

C: Sometimes.

Can you think of a problem that you might use them?

C: (shrugs no)

What kind of problems do you solve mentally, in your mind?

C: I solve all the easy ones and the hard ones I can try to think of.

<u>Part 5</u> <u>Other Comments/Questions</u>

What makes a problem challenging for you?

C: Well, I think that I try and make it harder for me, like the first time I did one I made it really easy for me and then I tried to make it harder and harder.

How did you make it harder?

C: Well, I did those 13 princesses ones and that made it really hard for me.

Why was it hard?

C: It was hard for me 'cause I didn't know how to do it and you helped me with it and that helped me a lot.

Okay, can you make up a problem that would be challenging for you?

C: There are 20 princesses on a mountain. The mountain was about to collapse and 4 princes came to help them. How many trips did the princes have to make?

Now what makes that challenging?

C: There's 4 princes and 20 princesses so it would be like hard for me because I'd have to try and work it out but its too hard, I'd have to keep trying and trying.

How do you think you'd figure that one out?

C: Well,

What strategy would you use?

C: I'd have to use a piece of paper and draw a picture.

Okay, do you have anything else you want to tell me about math?

C: No.

Do you like doing math problems?

C: Yah.

Why?

C: Because I make them really hard for myself.

Do you like writing your own?

C: Yah.

Do you like solving the ones that other kids have written?

C: Yah.

How come?

C: Because sometimes they make them really hard for me and for other kids.

How do they make them really hard?

C: Well they sometimes do really hard numbers but they don't think they're hard but they're hard for other kids to do.

Okay, well thanks Cathy.

Appendix C

Example of coded transcription

Part 4 Use of strategies for solving problems

Can you tell me what is a strategy? → ? when was the last time she would have heard this term?
no B: I forgot.
You forgot what that word means?
B: Yup.
When I usually think of the word strategy, I think that it means something
teaching that you do to help you solve a problem or to help you do something, so like some people when they play chess, they think of a strategy so that they can win,
B: Yah.
Or when they're playing other games they think of a strategy so that they
teaching can win and then when you're solving a math problem, you think of different ways that you can use, you can do different things to help you get the answer.
B: Yah.
What are all the different strategies you can think of? ← not clear
B: One is I can help myself with a strategy to learn how to do math,
Okay, you mentioned earlier that when you can't solve a problem you could ask a friend, or someone in the class and that's a good strategy or another time you talked about checking your work with a calculator,
B: Yup.
And that's a strategy. What are some other things you could do to help you solve a math problem?
ask, B: Well, you can ask an adult or you can use unifix cubes if you're doing
manip adding or subtracting and you can use erasers, pencils, anything else you can use to add.
What strategies do you usually use to solve a math problem?
cal. B: Well, I usually ask a friend or use a calculator.
Why do you think you use these strategies?
B: Because they help me. → why she uses strat.
How do they help you?
B: Well they help me try and figure out the answer.
Does the type of strategy you use depend on what the problem is?
B: Yes.
Why?
B: 'Cause you can use a lot of things, if I use a strategy, like with a friend,

they can help me a lot try to..
But can you think of a certain type of problem that you wouldn't use a calculator for?
B: If it was like something else that your teacher could give you, then you can probably use something else besides a calculator but if it was math review you couldn't use a calculator.
Do you ever use drawings to help you solve math problems?
draw B: Yup.
Can you give me a kind of problem that you would use drawings for?
B: If we had to do lots of math problems, then we'd have to draw pictures quickly. ?
Can you think of a problem that drawings have helped you solve?
B: Well once I was doing a really hard one and it helped me a lot. why she used it
How do you think it helped you?
B: Because it showed me how the picture was and it showed me what the answer was. how it was helpful
Okay. Do you ever use tallies?
tal. B: Yup, sometimes.
What kind of problem would you use tallies for? recall of classroom application (graph)
B: Well if you were using popsicles, you would use that if it was Groundhog Day or if you went to use them because your teacher told you to.
So if I said, use this strategy, then this might be the only time you might use tallies?
B: Yup.
Do you ever use your fingers to help you solve math problems?
fing B: Sometimes.
What kinds of problems would you use your fingers for?
B: Sometimes for subtraction, times and adding if its really hard.
Do you ever use manipulatives or counters?
B: No.
You talked about using unifix cubes, those are like counters.
B: Yup, but only when I was in grade 1 and kindergarten. age-related "attitude" already negative?
Okay, do you ever use the calculator to help you solve math problems?
cal. B: Once.
What kind of problem was that?
B: It was, I think it was 13 girls and there were and 4 were stuck on each volcano and there were 3 princes, how many trips did each prince have to make? math lesson I taught ~similar problem
And a calculator helped you?
B: Yah.