# MATHEMATICS ANXIETY AND THE FIRST YEAR UNIVERSITY INTRODUCTORY CALCULUS COURSE

by

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ABSTRACT

**Mathematics anxiety and the first-year university introductory calculus course**

Mathematics departments experience large dropout rates among first-year students. Attempts have been made to remedy this attrition by focusing on curriculum reform and teaching-learning techniques. Less effort has gone into exploring the differences in values, beliefs, expectations and experiences of teachers and students in a first year calculus course.

The purpose of this study was to identify these differences in an effort to determine the circumstances under which teaching and learning takes place in the classroom. Identification of possible mismatches may provide a beneficial insight towards improving the pedagogy of mathematics education in the first year calculus classroom.

To that extent, I conducted open-ended interviews and questionnaires with five faculty members and with five students. The research was carried out at the mathematics department of a Research University in Eastern Canada.

Interpretative analysis of the data focused on three spheres of interest: beliefs about:

- the nature of mathematics,
- the pedagogy of mathematics education, and
- the aims of mathematics education and post-secondary university education.

It was found that differing perspectives for the first two spheres contribute to mathematics anxiety among first year students. To address mathematics anxiety within the first-year introductory calculus course, the study suggests that there is a need to (1) develop a social constructivist theory of mathematics anxiety, (2) develop within the

professional practice of post-secondary mathematics education an awareness of the role of communication, and (3) develop within post-secondary educational institutions an awareness of the benefit of nurturing research among instructors into their individual teaching practices.

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**Acknowledgement**

To my children Haley and Tynan:

"I tried to thank you along the way, but

If I didn't, let me thank you now."

(Leonard Cohen, 1992)

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# Chapter I

## Introduction to the Study

### The Problem

There is a mismatch between the beliefs, values, expectations, and experiences of students and their teachers in the first-year introductory calculus course. Witness for instance the experience of a first-year calculus student in his description of his instructor:

> It's a sorry excuse for a professor, if the best way they can teach is reading from the book. I can do that. The bum off the street can do that if I grab him, as long as he can read or write. The professor should have some insight and some reason for why we are learning this. Describe it in a different way or better way than the book is describing it. Show us another angle. Show us another view so that way we can have more understanding...

Compare this with the way an instructor of a first-year calculus course describes the commitment he brings to his teaching:

> I try to mostly...be extremely energetic. I like to bounce around a lot. I try to bring some humor to the classroom. And I try to bring a lot of rigor and precision. I try to speak very carefully.
>
> ...
>
> And yet I try to be emotional. I, I don't try to hide my emotional, eh, connection to mathematics, my sense of beauty about the subject. I try to get that kind of stuff into my calculus classes. I try to get my own enthusiasm in whenever there is an opportunity.

The student and instructor were not related to each other, but it raises the question if the experience for students is in general as the student describes, or is teaching in general as the instructor describes it? What was the student's instructor really like? Did the student's instructor perceive his or her own teaching style as 'emotional.' What was the quoted instructor's style as perceived by students? Was the instructor reading from the book?

Descriptive studies of mathematics anxiety suggest that students are motivated through satisfaction in involvement with successful work, but tend to blame their dissatisfaction on the teacher. Students seem to appreciate teachers who provide an environment for a structured, logical progression in learning, as well as sufficient explanation, encouragement and friendliness. Math anxious students tend to avoid challenge, presumably to avoid the anxiety involved in dealing with failure. In mathematics, perhaps because it is considered to be hard and difficult and only for smart people, students tend to internalize their experiences into self-concept more than in other subject matters. (Middleton and Spanias,1999) Negative experience can translate into math anxiety and manifest itself through high attrition rates in courses, despite the students' beliefs in the importance of such courses. A negative experience for first-year calculus students can have serious effects for the students, the mathematical community, and society at large. The students may foster dislike for mathematics in their future careers and personal lives, thus creating second-order generational effects. The mathematical community will be faced with a challenge of student attrition, declining enrollment, loss of jobs, and loss of status in society at large. And society at large continues with stereotypes and misconceptions about mathematics and mathematicians which will further the bad reputation that mathematics faces in society. (Zaslavsky, 1994)

**Background to the Problem**

Math anxiety is the phenomenon in which an individual perceives mathematics as difficult and his or her personal ability to do mathematics as poor, resulting in the avoidance of mathematics, if possible.(Middleton and Spanias, 1999) Negative attitudes towards mathematics stem from myths and misconceptions. (Zaslavsky, 1994) These misconceptions tend to grow as students progress into and through high school. Despite the fact that students consider mathematics important, the number of students choosing mathematics courses is steadily

declining according to the Conference Board of Mathematical Sciences (CBMS) 1995 survey. Educational studies suggest that decline in positive attitudes towards mathematics is in part a function of lack of teacher supportiveness and experiences in the classroom environment. Transitions such as progressing from elementary to junior high at grade seven and having to deal with new rules for determining success in doing mathematics accelerate the formations of negative attitudes. (Middleton and Spanias, 1999)

Although the aforementioned references relate primarily to the K-12 education system, it could be argued that similar issues are relevant in the post-secondary education system, in particular in the first-year introductory calculus course. For instance, the perspectives on teaching for the scientist at the college level are quite different from those of a high school teacher. Not only are there differences in what it means to know something (the scientific reductionist perspective of the university in contrast with the educational holistic perspective of high school), there are important differences between high school students and college students in the constrains on their behavior and what they expect of instructors. The discourse structures of schools and scientific communities are fundamentally different in that the scientific community communicates by engaging in persuasive and challenging discourse, while school communication is more egalitarian oriented. (Richmond, 1996; Gerofsky, 1996) Richmond argues that the above statement although derived within the context of natural science, applies across the disciplines: "What is central is the difference in cultures between university and school educators." (p. 214)

Additionally there are fundamental conflicts between the expectations of mathematics instructors at the college level and the issues considered important by mathematics education researchers. Mathematics faculties want (a) access to resources, (b) descriptions of up-to-date subject matter, and (c) text materials in a "ready-to-wear" format. (Schoenfeld, 1991) From the community of mathematicians, there have been (1) no calls for epistemological change or

inquiry, (2) no calls for learning research, and (3) no perceptions that fundamental assumptions regarding teaching and learning need to be questioned. Schoenfeld describes the perspective of the mathematical community as follows:

> We know how to teach, and can do it well if we devote the time and energy to it. (It is a shame we're as busy as we are and can't spend as much time grading assignments, etc.) What we need is for people to give us the resources that make things easier—means of keeping current and good classroom materials, ready for use.

Instructors perceive the problems facing them to be external to their teaching practice. Declining enrollments, and high level failure rates in first-year calculus courses have resulted in diverse efforts at calculus reform, focusing on curriculum issues such as mathematical considerations and technological possibilities. (Ferrini-Mundy and Graham, 1991) In these reform efforts less attention has been focused on communication in the classroom.

There is also the similar problem of attrition at the introductory calculus course level as there is when students enter junior high school level at grade seven. Garfunkel and Young (1998) document a trend of enrollment decrease in calculus courses at the undergraduate level. They warn that: "We have to take a long look at what is actually happening. Where and how is mathematics being taught at our nations' colleges? ... This is not an issue of left or right, of reform or status quo. This is an issue of survival. We are losing students; we are losing faculty. We need to understand why and, if possible, find ways to reverse these trends ... The future of the profession and the next generation of mathematicians depend upon it.'"

Framing the pragmatic issues facing the mathematics community at the university level through the above argument, makes it possible to entertain the question of math anxiety being a fundamental factor to be faced by mathematics instructors in the first-year university course. Negative attitudes, misconceptions, and failure to communicate across the culture differences will become issues to be addressed if math anxiety is a fundamental factor. Understanding the

nature and extent of differences in beliefs, values, expectations, and experiences become essential. The consequences of a mismatch are misconception, predetermined notions, and negative attitudes resulting in finding math hard and irrelevant, inducing anxiety, and failure, thus repeating the process that Middleton and Spanias (1999) describe for the grade seven student.

**The Study Proposal**

Little attention has been paid to the circumstances under which meaningful discourse in the calculus course at the first-year level is to take place. No matter what form this discourse will take, conversation will play an important role. Lack of awareness of the differences in personal history and context can hinder communication across the differences. Prior experiences of inadequacy, especially when they become self-perpetuating, as in the case of math anxiety, can be serious barriers to successful communication. (Burbulus and Rice, 1991)

Recognizing the importance of the values and beliefs for the educational endeavor, the goal of this study is to inquire into the nature of the differences between the conceptions, values, beliefs, experiences, and expectations of instructors and students. What are some of the spheres of influence in the educational process where these differences manifest themselves? In this study I will focus on the "what", the "why", and the "how" for the participants. The what focuses on the sphere of mathematics: the perspective that brings the participants together in the classroom. The why refers to why the participants are in the classroom, the curriculum goals and objectives. The how describes the pedagogical sphere: expectations and social experiences that each of the participants brings into the classroom.

The research question that guided this study can be narrowed down to:

> What are the differences in values, beliefs, expectations, and experiences of students and teachers in an introductory first-year calculus class in the

areas of mathematics, purpose of mathematics education and post-secondary education, and pedagogy of mathematics education?

## The Study

To obtain research data, I interviewed five first-year mathematics students and five mathematics faculty members. I used semi-structured interviews to elicit openness. The advantages of this type of descriptive, qualitative study using semi-structured interviews are the opportunities to obtain rich sources of information, which is not possible in more rigid interviewing or with collection of data through paper-and-pencil questionnaires. The limitations of this study are the difficulties in generalizing and summarizing a small sampling. I agree with Deborah Tannen (1990) in *You Just Don't Understand* that there is a lot of framing going on in the interview conversation and in the data analysis. It is hoped that this is more than offset by the quality of the data obtained in a manner that allowed participants to go deeper into issues that concerned them. This also enabled interviewees to have some control over the information brought forward and to have some direction over the outcome of this study. (Fontana and Frey, 1994)

## The Study Purpose

The purpose of this study was to qualitatively analyze information for evidence of differences, and to generalize on the attitudes of the students and instructors in the mathematics classroom. I read my transcripts through numerous times, each time filtering out what I considered non-essential. This combing effect resulted in condensed interviews that were then explored for information related to the three spheres of the nature of mathematics, aims of mathematics education and post-secondary education, and pedagogical concerns. Similarities within groups and differences between groups were subsequently catalogued.

## Chapter Organization

Chapter One presents an introduction to the study with a brief introduction to the problem, some background situating of the importance of the problem, and an overview of the study into the problem. Chapter Two focuses on a review of the literature in math anxiety, beliefs about mathematics, pedagogy and post-secondary education. Chapter Three provides further information about the research methodology, and the instruments used to collect and analyze data, and the context within which this data was collected. Chapter Four is organized around the analysis of the data. Representative samples of data analysis are presented together with overall findings. Chapter Five concludes this study with a summary of the findings of this study, the implications of these findings on classroom practices, and suggestions to improve on those practices.

# Chapter II

## Review of the Literature

This literature review has been organized in five sections. The first section "Math Anxiety-Avoidance" explores the concept of mathematics anxiety. The second part focuses on a review of motivation as it pertains to mathematical beliefs, decision making and mathematics achievement. This section is followed by a look at beliefs, in particular, beliefs and attitudes of students and instructors at the university level. The next part examines the mathematics classroom culture and the role of communication in avoiding mathematics anxiety. The chapter finishes with a summary of the literature review as it guides this study into the differences in values, beliefs, expectations and experiences between students and instructors.

### Math Anxiety-Avoidance

Values, beliefs, perspectives, and experiences about mathematics, mathematical education and purpose of education in general, are all factors that affect learning outcomes. (see, for instance, Nickson, 1992) The importance of these affective domain factors on the outcomes of educational goals is considered important enough by the National Council of Teachers of Mathematics (NCTM, 1989) to list, as its top two educational goals in their K-12 standards, that the students learn to value mathematics, and that they become confident in their ability to do mathematics. (p.5)

An important factor in the affective domain is mathematics anxiety. Mathematics anxiety came to academic attention in the mid-1970's when feminist studies in inequity and the need for reform of the mathematics curriculum converged. Quantitative and qualitative research studies focused on gender, race, and ethnicity factors in mathematics anxiety. (Fennema and Sherman, 1976; Tobias, 1993) The accepted term "math anxiety," as used in the research

literature, and the less common terms "mathophobia" and "math panic" should not be confused with the pathological or psychological use of the words anxiety, phobia, or panic. Because the term "anxiety" could be taken in a self-defeating way to refer to some innate disability or debilitating affliction which should be treated clinically or through therapy, Hilton (1981) and Segal (1978) recommended the use of "math avoidance". The latter term refers more to behavior than emotion.

Some researchers view the nature of mathematics anxiety as merely a lack of confidence in the ability to learn mathematics. (Fennema and Sherman, 1976; Reyes, 1984) Using the phrase "mathophobia", Lazarus (1974) describes the anxiety as "an irrational and impeditive dread of mathematics." Wood (1988) defines math anxiety as "the general lack of comfort that someone might experience when required to perform mathematically." Williams (1988) refers to math anxiety as "both an emotional and cognitive dread of mathematics." "A general fear of contact with mathematics, including classes, homework, and tests" is how Hembree (1990) describes this distress. Bessant (1995) suggests that mathematics anxiety is used as an euphemism for debilitating test stress, low self-confidence, fear of failure, and negative attitude toward mathematics learning. Middleton and Spanias (1999) describe math anxiety as the phenomenon in which individuals perceive mathematics as difficult and their ability to do mathematics as poor, resulting in the avoidance of mathematics, if possible. In this study, which is prompted by a concern about the attrition rates in first-year calculus courses, the term math anxiety is used with a preference for Middleton's and Spanias' description, and with an emphasis on the avoidance perspective as proposed by Hilton and Segal.

Zaslavsky's (1994) descriptive feminist study focuses on the experiences of the underprivileged students in elementary and secondary education. She identifies as factors influencing math-avoiding students: inadequate schools, poor teaching, inappropriate mathematics programs, and stereotypes about who can and who should do mathematics. New

elements in students' lives, such as enrollment in college, may cause negative feelings to "come out of the closet." Zaslavsky recommends that colleges offer the remediation that these people should have received in earlier school years as well as helping individuals overcome their math anxiety. (p.19) Teachers are seen as a key factor: "good teachers make math interesting and enjoyable, and encourage the students to do well." (p.115) Lazarus (1974) claims that it is particularly at the college and university level that mathematics anxiety comes forth. It is here that many people are brought into forcible contact with relatively advanced mathematics. Mathematics anxiety has its roots in earlier education, but may remain unnoticed because students are able to pass by just memorizing formulas and algorithms. This tactic becomes unworkable at the advanced level when students are faced with discontinuity in their coping mechanism. (Robert and Schwarzenberger, 1991)

Fear of mathematics is the result and not the cause of negative math experiences. (Tobias, 1993) Feelings are at the heart of the problem. Tobias observes that not just women and minorities are being disserved by mathematics education. Both sexes are beginning to reassess their mathematical potential. (see also Reyes, 1984) Tobias claims that research studies confirm the hypothesis that perceived incompetence is often the result of common myths about mathematics. She notes that apart from general intelligence, which is probably equally distributed among males and females, the most important elements in determining success for learning mathematics are motivation, temperament, attitude, and interest. Buxton (1991) indicates that the most important elements influencing mathematics anxiety are attitudes towards this subject, relevance of the curriculum, the relation between mathematics and a person's moral worth (the "you are right—you are wrong" dichotomy), the role of symbols, and psychological issues of authority and power.

Mathematical anxiety affects both the extent to which students avoid mathematical activity and their achievement in these mathematical activities. Schoenfeld (1989) claims a

strong negative correlation between mathematics anxiety and mathematical performance, but that there is no clear cause-and-effect relationship between mathematics anxiety and achievement. His findings echo the general trend that there is a negative correlation between mathematics anxiety and achievement. (see for instance Reyes, 1984; Hembree, 1990; Middleton and Spanias, 1999) In general, analyses of the associations between mathematics anxiety and attitudinal variables indicate a strong relationship. However, these analyses are complicated by inconsistencies in the meaning and usage of terms such as beliefs, attitudes, emotions, values and anxieties.

Inconsistencies in research findings may be attributed to an inadequate theoretical foundation for the affective concepts. (Bessant, 1995) Mathematics anxiety is seen as having its roots in a much larger, vague, and elusive construct of anxiety. (Hembree, 1990) Bessant reports on a complex interaction process between levels of mathematics anxiety, forms of mathematics anxiety, attitudinal factors towards mathematics anxiety, and learning processes. Schiefele and Csikszentmihalyi (1995), noting that research has investigated the relation between cognitive and motivational predictors and affective outcome measures such as self-esteem, satisfaction, attitude and anxiety, believe that there is a neglect of indicators that measure subjective experiences of students being engaged in mathematics in natural settings. The researchers examined how quality of experience, a multi-dimensional construct, is as an outcome measure related to interest, achievement motivation and mathematical ability. Quality of experience (including the dimensions of potency, affect, concentration, intrinsic motivation, self-esteem, importance and perceived skill) is reported to be significantly related to interest in mathematics and, to a lesser extent, to achievement motivation. Interest, through its connection with intrinsic motivation, is considered crucial for achieving success in school.

Descriptive studies show that, in addition to the prevailing myths about mathematics in society, teacher interaction plays an important role in mathematics anxiety. Teachers are

perceived as critical elements: good teachers make mathematics interesting and enjoyable, and encourage students to do well. "It is not only the *content* but also the *classroom climate* that made many of us anxious about math." (Tobias, p.39) Newstead (1998) reports i n a study involving nine to eleven-year-old students that pupils who were exposed to a traditional teaching approach reported more mathematics anxiety than those who were exposed to an alternative approach. Newstead's research confirmed that in classroom approaches that allow students to construct their own strategies for problem solving and to discuss these with peers, students responded with less mathematics anxiety.

In this exploratory study the focus will be on a construct of mathematics avoidance, as it may surface when students are faced with discontinuities in their coping methods, with an emphasis on the role of the classroom climate. Although mathematics avoidance is the motivating concern of this study, the characteristics of the student learning experience in the first year calculus course is the underlying theme in this research.

**Motivation**

To encourage students to do well is not as easy for teachers as it may seem. Steen (1991), in his foreword to Buxton's *Math Panic,* notes that "Even simple strategies that teachers take for granted can contribute in unintended ways to emotions that block rational thought. Asking questions, offering praise, enlisting parents—all generally accepted as good teaching practices—can in some cases provoke an emotional revolt against authority that erases any hope that the mathematics that follows will be engaged or understood." (p.x)

"Motivations are reasons individuals have for behaving in a given manner in a given situation. They exist as part of one's goal structures, one's beliefs about what is important, and they determine whether or not one will engage in a given pursuit." (Ames, 1992) Research in the interplay between motivation, behavior, beliefs, perceptions of competence, ability, and

achievements indicate that these relationships are not of a simple linear cause-and-effect nature and are not necessarily consistent from study to study, or from one motivational theory orientation to another. This situation becomes further complicated by the fact that it is difficult to separate research on anxiety towards mathematics from research on beliefs about mathematics. At times, beliefs and attitudes are interchanged with motivations. In addition, teachers' and students' practices are not always consistent with their professed beliefs and values. (Schoenfeld, 1989; Raymond, 1997)

Ames and Ames (1984) distinguish between two types of motivation in education: motivation as a quantitative variable, and motivation as a qualitative variable. The quantitative form of motivation is concerned with issues such as working harder, persisting longer, spending more time on a task, and/or performing better. The qualitative form values specific types of information, outcomes and cognitive factors (ability-evaluative, task mastery, and moral responsibility orientation). Middleton and Spanias (1999) point out that there are two further distinct types of academic motivation: intrinsic and extrinsic motivation. Students who are intrinsically motivated find rewards in math activity because:

They have a drive or desire to engage in learning for "it's own sake".

1. They engage out of enjoyment.
2. Learning is important to them with respect to their self-image.

Their motivation centers on learning goals with a focus on:

1. Understanding.
2. Mastery of math concepts.

Furthermore, intrinsically motivated students exhibit pedagogically desirable behaviors such as increased time on task, persistence in the face of failure, greater creativity and risk taking, deeper and more efficient performance and learning strategies. Intrinsic motivation is related to students' perception of competence.

Students who are extrinsically motivated engage in mathematics activities because:

1. They desire favorable judgment from teachers, parents or peers (rewards), or
2. They desire to avoid negative judgments from the same of their competence (punishments).

Their motivation centers on performance goals (or ego goals) with a focus on:

1. Obtaining good grades and approval, or
2. Avoiding bad grades and disapproval.

Middleton and Spanias (1999) in their review of research in the area of motivation for mathematics achievement, describe mathematics anxiety as a motivational attitude that does not fit well within existing theoretical orientations on motivation. (p.77)

Exploring mathematics anxiety within theoretical motivational orientations provides teachers with the potential to adjust their classroom practices to motivate their students. The extent to which students and instructors are aware of the intricate role played by intrinsic and extrinsic motivation may facilitate through classroom practices desired attitudes towards mathematical learning. This study will explore student and instructor practices for evidence of motivational awareness.

**Student Motivation.**

Mathematics anxious students tend to derive satisfaction from a task when they are involved in successful work and they tend to blame their dissatisfaction on the teachers. (Middleton and Spanias, 1991) Quilter and Harper (1988) observe that "whereas research tends to focus principally upon the cognitive/conceptual/intellectual area in diagnosing pupil difficulties, the learners themselves stress the overriding importance of the learning environment (teacher attitude and competence, and 'relevance') and its influence upon motivation." (p. 127) The students seem to appreciate teachers who provide a structured logical progression for

students' work as well as sufficient explanation, encouragement, and friendliness. When teachers emphasize understanding of mathematical concepts and provide facilitative classroom environments, students tend to be more receptive and less anxious with regard to mathematical activities than when teachers stress rote activities and are perceived to be authoritarian. Students who have good experiences in mathematics tend to be less math-anxious and less inhibited in pursuing mathematics-related careers than students who have bad experiences. In mathematics, perhaps because it is viewed as a difficult and important subject, students tend to internalize their experiences into their self-concept more than in other subject areas. (Buxton, 1991; Middleton and Spanias, 1999) Although descriptive studies (Tobias, 1993; Buxton, 1991;Zaslavsky, 1994) show great similarities of bad mathematics experiences, students differ in how they could achieve satisfaction. Some find the challenge satisfying, while others stress the understanding behind the problems. Some just want to be able to do the problem successfully; others are concerned with the grades earned through their work effort.

Middleton and Spanias (1999) point out that it is likely that students must feel comfortable with mathematics, must be challenged to achieve, and must be expected to succeed before the development of intrinsic motivation can begin. Declines in positive attitudes toward mathematics can be explained in part as functions of lack of teacher support and experiences in the classroom. They state that "motivational patterns are learned and that students generally learn to dislike mathematics, and that this dislike becomes an integral part of their mathematical self-concept." (p. 67) Regardless of the theoretical motivational orientation, Middleton and Spanias observe that there are some consistencies, all influential on mathematics anxiety. These are:

1. Students' perceptions of success in mathematics are highly influential in forming their motivational attitudes.

1.1. The effort a student is willing to expend on a task is determined by the expectation that participation in the task will result in successful outcomes, mediated by either value attached to the task or the extrinsic reward associated with success.

1.2. Students should be encouraged to attribute their success to a combination of ability and effort, and their failures to either insufficient effort, or to confusion or reliance on inappropriate strategies.

1.3. Students must not be given cause to believe that their failures are due to lack of ability for fear of exacerbating their feelings of learned helplessness.

2. Motivations toward mathematics are developed early, are highly stable over time, and are influenced greatly by teacher action and attitudes.

2.1. Consolidated attitudes from middle school predict the courses taken and the mathematics achievement in high school and college.

2.2. Preponderance of students' recollections of bad experiences explains, in part, why students' liking of mathematics tends to decrease and why enrollment in higher-level mathematics courses has declined. Students try to avoid the anxiety resulting from involvement in mathematics tasks.

2.3. Students tend to attribute their feelings about mathematics to their identification with influential teachers, or to their reactions to bad experiences for which they blame teachers.

2.4. Students are expected to dislike mathematics and are not provided direction or support when they fulfill this expectation.

3. Providing opportunities for students to develop intrinsic motivation in mathematics is generally superior to providing extrinsic incentives for achievement. Students must understand that the mathematics instruction they receive is useful, both in immediate terms

and in preparing them to learn more in the fields of mathematics and in areas in which mathematics can be applied.

4. Inequities exist in the ways some groups of students in mathematics classes have been taught to view mathematics.
5. Achievement motivation in mathematics, though stable, can be affected through careful instruction design.
   5.1. A supportive, authoritative teacher serving as a model and as a friend gives students the confidence and feelings of self-worth necessary to be comfortable in mathematics.
   5.2. Teachers who are more attuned to bettering their students' motivational beliefs are better able to adjust their classroom practice to motivate their students.
   5.3. Classroom practice can be positively reinvented so that the culture of the classroom can become conducive for learning and enjoying mathematics.

To decide if an academic activity comes to be regarded as intrinsically motivating, students will evaluate the stimulation it provides and the personal control the activity affords. If arousal and control requirements are consistently met, students may include the activity among their interests. (Middleton, Littlefield, and Lehrer, 1992)

In my exploration of the circumstances under which teaching and learning takes place in the first year calculus course I will look for evidence of motivational practices, either as professed by the instructor, or as experienced by the student.

**Instructor Motivation.**

Most motivational research has been done with students' attributions. Teacher beliefs, attributions, or personal constructs of what makes mathematics intrinsically motivating tend to parallel and reinforce those of their students. (Middleton, 1995) Teachers may unwittingly

undermine their students' achievement motivation by reinforcing failure attributions or failing to meet the needs of students' personal constructs.

Ames and Ames (1984) suggest that teachers process information about their own behavior and the performance of their students in the context of a value orientation that assigns a level of importance to various goals related to teaching. According to this "value-belief" framework, teachers are thought to select and pursue a goal because attainment of the particular goal implies something desirable about themselves, such as being competent discussion leaders, concerned about the welfare of their students, or interested in helping students achieve a certain level of excellence. This value-belief framework assumes a more general value orientation that teachers are responsible for students' learning and welfare. Such a value for responsibility involves three key beliefs: teaching is an important work activity, teachers act intentionally to produce positive student outcomes, and students' success is feasible in the context of situational constraints. Table 1 presents the systems of qualitative teacher motivation.

*Table 1: Systems of Teacher Motivation according to Ames and Ames*

| Cognitive Factors | Systems | | |
|---|---|---|---|
| | Ability-Evaluative | Task mastery | Moral responsibility |
| Goal | Teacher self-enhancement | Student task accomplishment | Welfare of others |
| Salient information | Teacher vs. student and student vs. student | Individual student goals and past performance | Moral standard of helpfulness applied to teacher and student behavior |
| Attributional focus | Student and situational factors: Ego-enhancing/protective | Teacher and student effort: Problem focused | Teacher effort: Non-defensive and self-effacing |
| Self-evaluational and strategy focus | Act to demonstrate high or avoid demonstrating low ability | Act to maximize student competence | Act to help students. |

Ames and Ames report that teachers' values and goals affect their attributions, related perceptions, and general strategy beliefs. Differing motivational states are derived from

particular constructions of social reality. Each construction involves different values and goals, perceptions, attributions and strategy beliefs. Both environmental and individual difference factors affect the particular construction of social reality. Ames and Ames conclude that "...classroom goal structure affects both student and teacher perceptions in quite similar ways. On the other hand, individual difference factors in both students and teachers may in some manner attenuate these effects. The interaction of teacher and student factors with classroom structural variables is an area in need of further study." (p. 553).

Ernest (1991) proposes a five-category beliefs system based on a broader construction of social reality. Ernest five beliefs systems, his educational ideologies, reflect social and cultural orientations and include the influence of social constructivism in mathematics education. His cognitive belief factors are organized in two sets, the primary and secondary belief elements. Ernest's five ideologies are the Industrial Trainer, the Technological Pragmatist, the Old Humanist, the Progressive Educator, and the Public Educator; each ideology representing a particular political and social perspective. An overview of the values and beliefs of Ernest's schema of Five Educational ideologies is provided in Table 2.

*Table 2. Five educational ideologies according to Ernest*

| *Social Group* | *Industrial Trainer* | *Technological Pragmatist* | *Old Humanist* | *Progressive Educator* | *Public Educator* |
|---|---|---|---|---|---|
| Political Ideology | Radical Right, 'New Right' | Meritocratic, conservative | conservative/ liberal | liberal | Democratic socialist |
| View of Mathematics | Set of Truths, and Rules | Unquestioned body of useful knowledge | Body of structured pure knowledge | Process view: Personalized mathematics | Social constructivism |
| Moral Values | Authoritarian 'Victorian' values, Choice, Effort, Self-help, Work, Moral Weakness, Us-good, Them-bad | Utilitarian, Pragmatism, Expediency, 'wealth creation', Technological development | 'Blind'Justice, Objectivity, Rule-centred Structure, Hierarchy, Paternalistic, 'Classical' view | Person-centred, Caring, Empathy, Human values, Nurturing, Maternalistic, 'Romantic' view | Social Justice, Liberty, Equality, Fraternity, Social awareness, Engagement and Citizenship |

*Table continued*

*Table 2 continued*

| *Social Group* | *Industrial Trainer* | *Technological Pragmatist* | *Old Humanist* | *Progressive Educator* | *Public Educator* |
|---|---|---|---|---|---|
| Theory of Society | Rigid Hierarchy, Market-place | Meritocratic Hierarchy | Elitist, Class stratified | Soft Hierarchy, Welfare state | Inequitable hierarchy needing reform |
| Theory of the Child | Elementary School Tradition: Child 'fallen angel' and 'empty vessel' | Child 'empty vessel' and 'blunt tool' Future worker or manager | Dilute elementary school view, Character Building, Culture tames | Child-centred, Progressive view, Child: 'growing flower' and 'innocent savage' | Social Conditions view: 'clay moulded by environment' and 'sleeping giant' |
| Theory of Ability | Fixed and inherited Realized by effort | Inherited ability | Inherited cast of mind | Varies, but needs cherishing | Cultural product: Not fixed |
| Mathematical Aims | Back-to-Basics: numeracy and social training in obedience | Useful maths to appropriate level and Certification (industry-centred) | Transmit body of mathematical knowledge (Maths-centred) | Creativity, Self-realization through mathematics (child-centred) | Critical awareness and democratic citizenship via mathematics |
| Theory of Learning | Hard work, effort, practice, rote | Skill acquisition, practical experience | Understanding and application | Activity, Play, Exploration | Questioning, Decision making, Negotiation |
| Theory of Teaching Mathematics | Authoritarian Transmission, Drill, no "frills" | Skill instructor, Motivate through work relevance | Explain, Motivate, Pass on structure | Facilitate personal exploration, Prevent failure | Discussion, Conflict Questioning of content and pedagogy |
| Theory of Resources | Chalk and Talk only Anti-calculator | Hands-on and microcomputers | Visual aids to motivate | Rich environment to explore | Socially relevant, Authentic |
| Theory of Assessment in Maths | External testing of simple basics | Avoid cheating, External tests and certification, Skill profiling | External examinations based on hierarchy | Teacher led internal assessment, Avoid failure | Various modes. Use of social issues and content |
| Theory of Social Diversity | Differentiated schooling by Class Crypto-racist, Monoculturist | Vary curriculum by future occupations | Vary curriculum by ability only (maths neutral) | Humanize neutral maths for all: use local culture | Accommodation of social and cultural diversity a necessity. |

*Note.* From " The Philosophy of Mathematics Education," by P. Ernest, 1991, Hampshire, UK: The Falmer Press.

To explore "the interaction of teacher and student factors with classroom structural variables..." which Ames and Ames (1984) refer to, I will use Ernest's categorizations of ideologies extensively within the research question of this study. Further references in this research to Ernest's five ideologies will be supported with quotes from *The Philosophy of Mathematics Education.* (Ernest, 1991)

**Beliefs**

Mathematics anxiety, through perceived incompetence, is often the result of common myths about mathematics. Recurring common myths found in descriptive studies are:

- Mathematics ability is inherent. Only very few can do mathematics. Mathematics is hard. Only a genius or a "math-brain" can understand it.
- Mathematical insight comes instantly if it comes at all; mathematics must be done fast.
- Mathematics is a male domain.
- Mathematics is mainly arithmetic, working with numbers. If you are not good in arithmetic you can't learn higher level mathematics, such as calculus.
- Mathematics involves a lot of memorization of facts, rules, formulas, and procedures. You must follow procedures set down by the teacher and the textbook.
- Every problem has just one right answer, and it must be exact.
- You must never use hands-on materials to help solve a problem.
- You must work on mathematics alone. Working with others is cheating.
- You must keep at it until you have solved the problem.
- Mathematical language is unrelated to ordinary everyday language.
- Mathematics is rigid, uncreative, cut-and-dried, and complete. It does not involve imagination, discovery, or invention. There is nothing new in mathematics.
- Mathematics is exact, logical, and certain. Intuition has no place.

- Mathematics is abstract. It is unrelated to history or culture.
- Mathematics is a mystical science accessible to few.
- Mathematics is a discipline in which judgments on both personal intellect and personal worth will be made.
- Mathematics is value-free. It is the same for everyone all over the world.

(Tobias, 1993; Buxton, 1991; Zaslavsky, 1994)

Students, teachers and society have varying beliefs about mathematics. Some beliefs will encourage students to learn. Others will actually hinder interest in and the understanding of the subject. Kloosterman and Stage (1992) point out that through defining the aims of belief research it is possible to determine which beliefs need to be studied. An assumption behind this research is that certain beliefs result in high motivation on the part of the students to learn mathematics, whereas other beliefs diminish this motivation. In addition, it is assumed that higher motivation results in better achievements. (Fennema, Wolleat, Pedro, and Becker, 1981) Reyes' 1984 overview study of affective variables and mathematics education points out that the affective domain plays an important role in students' decision making. Of the four affective variables discussed by Reyes (confidence in learning mathematics, mathematics anxiety, attributions of success and failure in mathematics, and perceived usefulness of mathematics), perceived usefulness of mathematics is ranked higher by students in their course selection decisions. High achievers view mathematics as more useful than low achievers. Reyes observes that perceived usefulness may be the easiest to change, and that teachers are in a good position to bring about such change. Reyes also reports a strong correlation between confidence in learning mathematics (an affective variable) and mathematical achievement. Confidence in learning mathematics is also a strong predictor of mathematics course selection. Research studies show a consistent negative correlation between mathematics anxiety and mathematical achievement.

Beliefs about mathematics can be subdivided into groupings about beliefs in the nature of mathematics (knowledge and activity), and beliefs in teaching and learning mathematics (pedagogy). In examining the research in beliefs and behaviors, it is important to distinguish between *deep* (or *principled*) and *surface* (or *pragmatic*) beliefs, and to differentiate between corresponding practices such as *pervasive* versus *superficial* practices. (Ruthven and Coe, 1994; Raymond, 1997) Thompson (1992) points at the difficulty to distinguish between beliefs and knowledge, and recommend the following distinctions: beliefs can be held with varying degrees of conviction; beliefs are not consensual; beliefs do not meet the criteria of evaluating and judging validity. Thompson also draws attention to the different belief systems (the organization of an individual's beliefs): primary and derivative beliefs, central and peripheral, and the clustering of beliefs.

**Student Beliefs.**

Motivation related beliefs identified by Kloosterman and Stage (1992) appropriate for the college level are:

1. Beliefs as a learner of mathematics:
   1.1. I can solve time consuming mathematics problems.
   1.2. Understanding concepts is important in mathematics.
2. Beliefs about the discipline of mathematics:
   2.1. There are word problems that can not be solved with simple step-by-step procedures.
   2.2. Word problems are important in mathematics.
   2.3. Effort can increase mathematical ability.

Schoenfeld (1989) reports the following findings about beliefs, values and perspectives from a questionnaire to 230 students, all in an academic, college-bound track program in three highly regarded high schools.

1. Beliefs about the role of the learner in mathematics:
   - It is work and not good luck that accounts for good grades.
   - Students place more emphasis on work than on inherent talent.
   - Students firmly believe in native ability, particularly in mathematics.
   - If students do poorly, it is considered their own fault.
   - Teachers' attitudes towards their students are not considered to be a factor in grading.
2. Beliefs about the discipline of mathematics:
   - Mathematics is considered to be an objective, and objectively graded, discipline that can be mastered.
3. Beliefs about teaching and learning mathematics:
   - Students are expected to master the subject matter, by memorization, in bite-size bits.
   - Students stress memorization.
   - Doing mathematics requires lots of practice in following rules.
   - Students are exposed to the "rhetoric of mathematical understanding": the consistent claims by their teachers that studying mathematics will help them to think mathematically.
   - Most questions are pointed, quick to solve, and aim more at evoking quick recall than at stimulating deep thought.

The students in Schoenfeld's 1989 study are a highly motivated college-bound population with the following characteristics:

- The students work at the mathematics in large part because they want to do well in the course that is required in their programs.
- They study mathematics for its intrinsically valuable reasons (to think more clearly in general) rather than for extrinsic reasons (avoid looking dumb, or to impress teachers).

- Students' overall academic performance, their expected mathematical performance, and their sense of their own mathematical ability all correlate strongly with each other.
- Students who think less of their mathematical ability tend more to attribute their mathematical success to luck and their failures to lack of ability.
- Students who think themselves good at mathematics attribute their success to their abilities.
- The better the student is, the less likely he or she is to believe that mathematics is mostly memorizing, that success depends on memorizing, or that problems get worked from the top down in step-by-step procedures.
- Those who are good at mathematics also tend to find it interesting.
- The better students perceive themselves as working harder than most.
- Motivation to do well did correlate with general academic performance, mathematical performance, and perceived ability, but not as significantly.

Rodd (1993) hypothesized that the fallibilist-absolutist dimension of beliefs in the nature of mathematics, and the investigative-didactic dimension of beliefs about how mathematics is learned would strongly correlate. The fallibilist pole would correlate with the investigative pole, and the absolutist with the didactic. Her results, however, obtained from small interviews do not support this hypothesis.

Ruthven and Coe (1994) found support for Rodd's observation that there are no simple systematic relationships between students' beliefs about the nature of mathematical knowledge and activity, and about the teaching and learning of mathematics. Their results obtained from a structured questionnaire point to the emergence of principled and pragmatic beliefs dimensions of students about mathematics, enabling them to explain the inconsistencies between the stated beliefs of students about proofs and their demonstrated practices regarding proofs. This distinction in beliefs is reflected in Schoenfeld's (1989) observation that "What counts in

problem-solving situations is the students' behavior, and that behavior seems to be driven much more by students' experiences than by their professional beliefs. If that is so, the advances of mathematics education ... have been largely in our acquiring a more enlightened goal structure, and in having the students pick up the rhetoric—but not the substance—related to those goals." (p. 349)

Ernest proposes a mathematical model of educational ideologies with two levels of beliefs: (1) the primary level comprising the deeper elements of the ideology, and (2) the secondary level, made up of the derived elements pertaining to education. Table 3 summarizes the elements for both the primary and secondary levels.

*Table 3: Belief elements of Ernest's ideologies*

| Primary Elements | | Secondary Elements | |
|---|---|---|---|
| 1 | Epistemology | 7[a] | Aims of Mathematics Education |
| 2[a] | Philosophy of Mathematics | 8[a] | Theory of School Mathematical Knowledge |
| 3[a] | Set of Moral Values | 9[a] | Theory of Learning Mathematics |
| 4[a] | Theory of the Child | 10[a] | Theory of Teaching Mathematics |
| 5[a] | Theory of Society | 11[a] | Theory of Assessment of Mathematics Learning |
| 6[a] | Educational Aims | 12[a] | Theory of resources for Mathematics Education |
| | | 13[a] | Theory of Mathematical Ability |
| | | 14[a] | Theory of Social Diversity in Mathematics Education |

*Note:* From "The Philosophy of Mathematics education." By P. Ernest, 1991, p.134. Hampshire, UK: The Falmer Press.
[a] Listed in further detail in Table 2.

For the purpose of this study, several of Ernest's belief elements have been used to explore for beliefs of students and instructors.

**Instructors' Beliefs.**

The literature review about mathematics anxiety and student motivation indicates the important role teachers plays in students' mathematical involvement, achievement, and beliefs and perspectives on success and failure. Fennema, Peterson, Carpenter, and Lubinsky, stated in

1990 that little is known about teachers attributions and beliefs and about the influence of those beliefs on learning. Research into university teachers' emotional domain of beliefs, attitudes and perspectives about mathematics, and about teaching first-year university students is even scarcer. Schoenfeld (1991) comments that there needs to be a serious change in current instructional practices because the epistemological and pedagogical stances of most mathematics faculty have some serious negative consequences. He argues that mathematicians, the teachers at the college and university level, assume that they have little need for a substantial portion of the current research in mathematics education at the college level. Robert and Schwarzenberger (1991) note that at the post-secondary level mathematical notions are presented in an unimaginative lecturing method. They question whether the fundamental premise is if teachers believe that the purpose of learning vast quantities of abstract concepts is part of a wider scientific, critical and even creative form of advanced thinking, or whether the premise is to merely be able to reproduce learned materials and mechanical skills. In terms of psychological and cognitive characteristics of university students, Robert and Schwarzenberger hypothesize that university teachers expect that students should be able to have an enhanced capacity to reflect on their own activity and that students should be able to distinguish between mathematical knowledge and meta-mathematical knowledge (the correctness, the relevance, or elegance of a piece of mathematics). In addition, students should carry a substantial quantity of mathematical knowledge, experience of mathematical strategies, working methods, and be able to communicate them with teachers or other students. For most students the conflicts between their own past experiences, present teacher expectations, and their first year learning experiences endanger the transition to advanced mathematical thinking. Robert and Schwarzenberger state that it is possible to change students' views, but that given the rigidity of institutional structures, a similar teacher change would be more difficult. University mathematics instructors are primarily recipients of Ph.D's with no training in pedagogical issues.

Dossey (1992) comments that "most professional mathematicians think little about the nature of their subject as they work within it." (p.42) For the working mathematician this may not be a problem, but for mathematics education this can create a problem for students. The teachers' beliefs about the nature of mathematics may have a great deal to do with the way in which mathematics is characterized in classroom teaching. Messages communicated to students about mathematics and its nature may affect the way students grow to view mathematics and its role in society. Discontinuities in views presented to students may cause discontinuities in learning processes. Dossey identifies five mathematics conceptions, two with an external (Platonic) view, and three with an internal (Aristotelian) perspective. The first two ideas differ from each other in their view of mathematics as a static structure, but agree on a mastery perspective of the given factual oriented curriculum. The other three conceptions focus on mathematics as a personally constructed or internal set of knowledge. They differ in the models and processes assumed in the construct forming (see also Thompson, 1992, Nickson, 1992).

Raymond (1997) uses a five-category scale to classify teachers' beliefs about the nature of mathematics based upon work by Ernest. (1989) This scale differs from Dossey's perspective in that it tries to distinguish more between views of mathematics and views of learning mathematics. One pole of the scale is represented by the category "Traditional", characterized by a view of mathematics as an unrelated collection of facts, rules and skills, and that this collection creates a mathematics which is fixed, predictable, absolute, certain and applicable. At the other pole is situated "Nontraditional" which views mathematics as dynamic, problem driven, and continually expanding—making a mathematics possible which can be surprising, relative, doubtful, and aesthetic.

Raymond (1997) developed a five-category scale to differentiate between teachers' beliefs about teaching mathematics. At one end of the scale is the view of the teacher in the role of dispenser of mathematical knowledge in a cognitive science approach to the study of

mathematics from a Platonic perspective. At the other end of the scale is the teacher's role as promoter and guide to knowledge sharing within a structure that values process over product with emphasis on the student's autonomy. Raymond developed in a similar way a five-scale classification of teaching practices.

Teachers in higher education expect their students to acquire specific skills and knowledge. They also wish to broaden and deepen their students' experience of the field and the meaning of that personal experience. They aim to enable the students to engage in practice in the informed way that is characteristic of competent practitioners (Dall'Alba,1993) The meaning of the course content must be developed concurrently with the students' understanding of the perspective of the field. There are three different ways in which teachers see the content of their courses: (1) as a body of knowledge and skills; (2) as concepts and principles to which knowledge and skills are linked; and (3) as experiences of a field of study and practice. The third way will meet the aim of higher education as envisioned by Dall'Alba. The meaning that experiences have differs for students from the meaning of the experiences for teachers. The focus of enriching students' experiences of what it means to engage in the field as a competent practitioner must be maintained throughout the course. If such a focus is not sustained, students will revert back to learning new content in less experienced ways. In many instances, the enrichment of the students' experiences will involve major changes for them. (Tall, 1991)

Ernest (1991) identifies five different mathematics educational ideologies focusing on the political ideologies of the social interests groups within the educational processes. His five categories are the industrial trainer, the technological pragmatist, the old humanist, the progressive educator, and the public educator. (see Table 2 for a detailed categorization) Both Raymond's beliefs scales and Ernest's beliefs' categorization have been used for the purpose of this exploratory investigation.

## Classroom Culture and Communication

Nickson (1992), in stressing the role of the mathematical classroom culture, writes that "by focusing on culture we can learn more about how the 'invisible' components in the teaching and learning situation can contribute to or detract from the quality of the mathematical learning that takes place. An exploration of such issues as the influence of differing perceptions of mathematics as a subject, of teacher beliefs and actions, and of pupil perspectives may help clarify how some of these components contribute to the cultural context of the mathematics classroom." (p.102) To respond to the hidden meanings and beliefs that all participants bring to the classroom, they must first of all be identified. She concludes that: "there will be greater variations in the cultures of mathematics classrooms. With this increased breadth of possibilities, however, comes increased potential for lack of consensus, in particular for mismatches between teachers' views and goals and those of pupils. This can result in what we might call productive classroom cultures and nonproductive classroom cultures." (p.111)

Mathematical enculturation, as described by Bishop (1988), explains how students' values and beliefs about mathematics are shaped by their experiences in the social setting of the classroom. Bishop notes that "Mathematical communication is not just about words and language. Neither is it just a skill to be learnt on a training course. Particularly for a Mathematical *enculturator*, it needs to involve values, perceptions, and beliefs of a personal nature, the communication of which is conveyed strongly by behaviour towards the Mathematical cultural group itself and towards the people with whom one is communicating." (p.167)

Nodding (1996) observes that "there seems to be something about the subject or the way that it is taught that attracts a significant number of young people with underdeveloped social skills. ... But when the group is examined from a social perspective, many talented young people may question whether they *want* to be part of it. The contemporary emphasis on cooperative

learning, open communication, and the social aspects of mathematics may be powerful in overcoming such social barriers, but some thoughtful minority educators have expressed doubts about these strategies. ... the result may be continued exclusion. ... Indeed, if the common perception of the "math crowd" is accurate, *mathematical* communication may aggravate inequalities, and both insiders and outsiders may opt for exclusion." (p. 611) Pirie (1997) in *Encyclopedia of Language and Education* stresses the importance of the use of language in the classroom. She describes how traditionally the research emphasis on talk was on the teacher to be clear in the delivery of the course content. The talk emphasis has since shifted from teacher talk, to teacher and student talk, and to teacher and student interaction, i.e. talking and listening. Pirie comments that "There is little doubt that certain kinds of talk in the mathematics classroom can engender more effective mathematical understanding. What we must now explore and define are the mechanisms that, within the complex interactions that form the classroom environment, provide such effective discourse... The teacher's role lies in educating students in the processes of conjecturing, explaining and justifying their ideas, and classroom talk has a major part to play in such education." (p. 235-236) Just as classroom talk can engender mathematical understanding, it can also obstruct the cultural process of learning

Gerofsky (1996) found discourse features in first-year introductory calculus courses to be the "hard sell" pitch of salespeople and questions whether this approach will reach or engage first-year students. Gerofsky stereotypes the first-year introductory calculus courses as follows:

1. Often disliked by faculty and students.
2. Usually are prerequisites for other non-mathematics undergraduate programs as well as for degrees in mathematics.
3. Instructors face large classes.
4. Students are not interested in mathematics
5. Students want high-grade averages for academic standing.

6. Both students and teacher perceive the course as a "weeder" course in which a certain percentage will fail.

She found that there is a fake dialogue style of rhetoric questioning present in the lecture discourse. Fake dialogue presumes an audience that cannot respond, either because they are not present, or because they are incapable of responding, or because they are socially constrained from responding. The pedagogical question then becomes should students be treated as if they cannot or must not respond? Students know that the "hard sell" in society represents a triumph of persuasion over criticism, reasoned thought and a person's "better judgment". The lecture discourse form conveys an attitude of condescension and paternalism.

Sfard (1996) makes a distinction in the forms of practice in mathematics education through the use of metaphors: the "Acquisition Metaphor" and the "Participation Metaphor." Sfard notices that the acquisition metaphor is likely to be more prominent in older texts while the participation metaphor takes the lead in more prominent studies. A schematic comparison between the acquisition and participation metaphor is given in Table 4.

*Table 4: A comparison between the acquisition and participation metaphor*

| Acquisition Metaphor | | Participation Metaphor |
|---|---|---|
| Individual enrichment | **Goal of learning** | Community building |
| Acquisition of something | **Learning** | Becoming a participant |
| Recipient (consumer), (re-) construction | **Student** | Expert participant preserver of practice/discourse |
| Property, possession, commodity (individual, public) | **Knowledge concept** | Aspect of practice/discourse/activity |
| Having possessing | **Knowing** | Belonging, participating, communicating |

Sfard comments that it is beneficial to consider reality to be constructed from a variety of metaphors. Ernest (1991) models of the industrial trainer, the technological pragmatist and the

old humanist can be considered to coincide with the acquisition metaphor, while the progressive educator and the public educator represent the participation metaphor.

Not all classroom participants speak the same language, especially in a mathematics class. For those learning the subject, the mathematics language can become a "foreign language", a "dead language", a "nonsense language" or an "abstract language". The result can be a distancing from and anxiety about mathematics. (Usiskin,1996) To overcome "a distancing from and anxiety about mathematics" mathematical communication must pay attention to the differences across which meaning has to be communicated. It may well be that one reason for math anxiety to surface in first year calculus courses is that we fail to communicate across the difference. (Burbulus and Rice, 1991)

**Summary**

Fear of mathematics is the result and not the cause of negative experiences with mathematics. Students can develop mathematics anxiety early on in their education, but learn to cope with it through memorization. Math anxiety will surface when a student is faced with experiences that interrupt their coping mechanism. Students consider teachers a key factor: good teachers make mathematics interesting and enjoyable, and encourage the students to do well Quality of experience is reported to be significantly related to interest in mathematics and crucial for achieving success in school.

Motivations and beliefs of students and teachers play an important role in the context of the classroom setting. Teachers who wish their students to become participants in their field of practice need to recognize that success will depend on their ability to motivate the students in this endeavor. The culture of the classroom is the cumulative effect of what teachers and students bring to it in terms of knowledge, beliefs and values and how these affect the social interactions within the context. It is not only the *content* but also the *classroom climate* that made many of us

anxious about mathematics. The mathematical community needs to have a good look at what it is that they communicate to a reluctant audience. The ability to communicate across the differences in values, beliefs, attitudes and experiences in the mathematical classroom will be a contributing factor to how much math anxiety-math avoidance will surface in the first-year calculus course.

# Chapter III

# Methods

The purpose of this study was to investigate the differences in values, beliefs, attitudes, expectations, and experiences between instructors and students in a first-year introductory calculus course with regard to mathematics, post-secondary education and teaching and learning. In keeping with this aim, research methods were used that would elicit belief statements from students and instructors in a first year calculus course. A description of the participants in this study and the information collection method, as well as explanations of the instruments used in the data gathering and compilation procedures, are presented in this chapter.

## Participants

The study was carried out at a research university in Eastern Canada. I interviewed five first-year calculus students and five mathematics faculty members, all instructors of first-year calculus courses. Within the mathematics department, first-year calculus is taught in approximately ten sections typically ranging in size from 60 to 120 students. All sections follow a common curriculum. Course evaluation consists of a combination of assignments, midterm examinations, and one final common exam. Assignments vary from section to section. Most sections consign ten homework assignments cumulatively worth approximately ten percent. A section can have one or two midterm examinations, worth approximately forty percent of the total grade for the course. Sometimes these midterm examinations are given in common with other sections; sometimes they are given independently. The final exam, worth approximately fifty percent, is common to all section. The first-year calculus course consists of two sequential one-term courses, each worth one-half credit. No special calculus course for mathematics majors

is taught at the first-year level. A science faculty requirement stipulates that, in order to graduate, all science students must have one mathematics credit, with at least half of this credit in calculus. The department uses part-time staff to teach some of the calculus sections. These sessional instructors were not included in this study.

First-year calculus students were selected from a random ordering of the registrar's list of first-term calculus students. The first five students that responded positively to a request to participate in this study were to be interviewed. Students were initially contacted by e-mail in late October and early November. Two attempts, each involving ten randomly selected students, resulted in only one student volunteering to participate. Because of approaching final exams, no further attempts were made that term. Given the low response rate to my original e-mail requests, I reasoned that the less-than-personal approach of an e-mail message did not generate any interest among the students to participate in this study. I also assumed that because e-mail does not often require an immediate decision, the decision-making process was postponed. With the passing of time, my request was eventually ignored. Additionally, students became pre-occupied with approaching exams.

I decided to repeat the student selection process in early January; and since I had already one student volunteer from the previous term, I needed only four more. This time students were contacted by telephone. The first six calls resulted in four additional participants. After an initial meeting, however, one of these students decided not to participate. One additional phone call produced one more participant.

Due to the small department size and limited specialization, I did not select instructors randomly. Department members were informed by internal mail of the intent to interview a sampling of five instructors, representative of the department's demographics. I asked faculty

members to nominate by e-mail five departmental instructors considered demographically representative. The results were tallied, and the top ranked members were then approached. All selected faculty members agreed to take part in the study. All were regular instructors of the first-year calculus course. However, because of course load, and other circumstances, two faculty members were not teaching first-year calculus that particular term. No attempt was made to assure that student participants attended the same course sections as being taught by the instructor participants

**Data Collection**

The initial student volunteer was interviewed before the Christmas holidays. The other four students were interviewed in January and February following their first-term calculus course. The five instructors were interviewed in November and December during the first term. Procedures for data collection consisted of audiotaped interviews during which time the subjects were asked a number of pre-determined questions as well as several unstructured ones. Both forms of questions provided participants with an opportunity for open-ended treatment of issues raised.

To supplement the open-ended questions, interviewees were given questionnaires in the form of "cue cards." (see following section) The total length of the interviews ranged from three to five hours. Participants were interviewed over two to three sessions in my office, except for one instructor who was interviewed in the instructor's office. The interviews were open-ended and semi-structured to elicit openness and provide qualitative information.

Participants were provided with aliases for this project. Audiotapes and questionnaire results were stored in a locked file cabinet for additional confidentiality. Furthermore, I

transcribed all the audiotapes myself. The printed transcripts were also kept locked up, along with a back-up copy on diskette. As an extra measure of security, the electronic copies on diskette and hard drive were access protected.

**Data Instruments**

Qualitative studies provide rich information, something not possible in rigid interview settings, or with paper-and-pencil questionnaires. Characteristics essential to a successful interview are according to Fontana and Frey (1994): rapport, commitment to understanding, inter-personal format, free form modes of communication, and genuine interest and concern for the views of the interviewees. I aimed to develop interpersonal relationships to receive understanding rather than just explanations. My conversational style was open-ended and unstructured, in the sense of not restricting dialogue to any pre-determined script. The interviews were somewhat structured however, because this study was guided by the research question's three spheres of interests: beliefs about the nature of mathematics, beliefs about the aim of post-secondary education, and beliefs about teaching and learning mathematics. The interviews were unstructured or open-ended in that deviation from a pre-selected list of questions was expected.

Unstructured interviews allow for a conversation within a naturalistic setting. Having some sections of the interview structured and other parts unstructured permit for mini open-ended interviews to explore for further meaning (Denzin, 1989). The structured questions were loosely grouped together to allow a focus on four themes for conversation. Table 5 provides an overview of these themes. These four themes have been chosen to cover the three main spheres of interest that frame this study: beliefs about the nature of mathematics, beliefs about the

pedagogy of mathematics education and beliefs about the purpose of mathematics education and post secondary education.

*Table 5: Four themes of structured interview questions.*

| Instructors | Students |
|---|---|
| Perceptions about mathematics | Perceptions about mathematics |
| Experiences about the calculus course | Experiences about the calculus course |
| Perceptions about students | Perceptions about instructors |
| History of instructor: | History of student: |
| -university | -university |
| -educational | -educational |
| -social | -social |

To explore the interview data for beliefs relevant to the three main spheres of interest, I have used the beliefs framework developed by Ernest (1991). (see Table 2 and Table 3 in chapter two) The four specific themes of Table 5 allow for coverage through qualitative probing of Ernest' inter-related and interdependent elements of educational beliefs. Each instructor participant was asked the same set of structured questions in each theme. A similar procedure was used for student participants. To illustrate, Table 6 presents a list of structured questions in the "experiences about the calculus course" area of interest for faculty.

*Table 6: Structured questions for the "experiences about the calculus course" theme*

| |
|---|
| Are you satisfied with your teaching situation? |
| Can you identify the values that you hold as a teaching professional? |
| Are these values at risk or are they an integral part of your teaching practice? |
| Imagine what you might like the situation to be so that it is in keeping with your values? |
| To what extent are you working the way you wish? |
| What do you need to change to improve the situation? |

An unstructured conversation around these general questions can provide information about any of Ernest' elements of beliefs. Questions were developed with the aim of collecting information about the belief elements identified by Ernest. As Ernest developed his ideologies along social and political classifications, questions were included about the political and social realm. The set of questions and cue cards were tested on a volunteer instructor for participant reaction. Structured interview questions were considered appropriate in opening up interview themes to be explored further with unstructured questions. One set of cue cards, dealing specifically with scientific paradigms, was dropped from the interview as being not of relevance to the purpose of this study. The complete set of structured questions for faculty is provided in Table A1, Appendix A.

By giving participants questionnaires in the form of cue cards, hereafter called the educational ideology instrument, some multiple-choice information was obtained regarding Ernest' (1991, pp.138-139) descriptions of his primary and secondary belief elements. (see Table 2 in chapter two) Ernest categorized the belief elements for five educational interest groups: the industrial trainer, the technological pragmatists, the old humanist, the progressive educator, and the public educator. Raymond (1997), using Ernest's (1989) categories of teaching, learning and the nature of mathematics, developed a similar categorization to make comparisons of beliefs about the nature of mathematics, mathematics teaching, mathematics learning, and practice consistent. Raymond labels her categories traditional, primarily traditional, an even mix of traditional and nontraditional, primarily nontraditional, and nontraditional. Both Raymond's and Ernest's categories have been used for data compilation.

The educational ideology instrument consists of a set of cue cards, the "educational ideology cards", used to explore the respondents' educational ideologies in relation to the five

social interest categories identified by Ernest. Cue cards in this set are essentially a multiple-choice questionnaire with every question presented on a separate 5 x 8 file-card. Cue cards were presented to the participants one at a time for consideration. Participants were asked to identify those statements that agreed with their beliefs, or that they reacted neutral to, were unaware of, or disliked. An example of an educational ideology cue card dealing with the philosophy of mathematics (one of the elements of the model of educational ideologies from Table 2) is outlined in Figure 1. Included in Figure 1 are Ernest's corresponding social groups of educational utility. Raymond's scale, which I have used for the compilation of qualitative information from the interview's data, is also incorporated in Figure 1.

*Figure 1: Example of multiple-choice educational ideology cue card and associated categorizations l Ernest and Raymond*

Cue card:

| **I present mathematics in my teaching as:** | | | | | |
|---|---|---|---|---|---|
| **View of Mathematics** | Set of Truths, and Rules | Unquestioned body of useful knowledge | Body of structured pure knowledge | Process view: Personalized maths | Social constructivism |

Ernest's educational utility categories and Raymond's beliefs categories

| | | | | | |
|---|---|---|---|---|---|
| **Ernest's Social Interest Group** | Industrial Trainer | Technological Pragmatist | Old Humanist | Progressive Edu | Public educator |
| **Raymond's 5-level scale** | Traditional | Primarily traditional | An even mix of traditional and nontraditional | Primarily nontraditional | Nontraditional |

A full listing of these cue cards is provided in Appendix B.

Administration of the cue card instrument was spread out over the entire interview duration; the timing dictated by interview circumstances. When educational terms were used that interviewees were not familiar with, the tendency was not to explain the term if the participants were instructors, and to guide students in their sense making of the terminology.

## Data Compilation

Results from the cue cards provide a quick insight into beliefs about the elements of mathematical education. The information obtained from the cue cards can be used to create a profile of values and beliefs of mathematical education of students and of instructors.

Wolcott (1990) observed that the critical task for qualitative studies is "not to accumulate all the data you can, but to 'can' (i.e., get rid of) most of the data you accumulate." (p. 35) Wolcott suggests the use of display, through tables, graphs, charts, diagrams, etc. to facilitate management, analysis, and presentation of data. "Display formats provide alternatives for two of our most critical tasks, data reduction and data analysis. ...They invite us to sort and categorize data, to explore what goes with what, and to contemplate how seemingly discrete data may be linked in previously unrecognized ways." (p. 63) I started to sort through the data by first identifying data with sets of beliefs, values, perceptions and experiences that I considered of relevance for this study. Table 7 presents the belief instrument used in this primary sorting process.

*Table 7: Unstructured Interview Beliefs Instrument.*
*Sets of participant's beliefs relevant to research framework*

| Beliefs, perceptions |
|---|
| 1. Beliefs about the nature of mathematics |
| 2. Perspectives on teaching practices |
| 3. Beliefs about the learning of mathematics |
| 4. Beliefs about the teaching of mathematics |
| 5. Beliefs about the aims of post secondary education |
| 6. Beliefs about the aims of mathematics education |

The sets of beliefs from Table 7 and Ernest's elements of beliefs from Table 3 are related to the three main spheres of interest of this study. This relationship is not one-to-one or unique and varies from participant to participant. For instance, information concerning the beliefs about the aims of mathematical education, quite conceivably, may provide information about the nature of mathematics, post secondary education and the pedagogy of mathematics education. Table 8 provides an overview of a mapping between the two sets of beliefs and the areas of interest of this study.

*Table 8: Mappings of belief sets onto areas of research interest.*

| | Ernest's Elements of Beliefs (Table 3) | Unstructured Interview Beliefs Instrument (Table 7) |
|---|---|---|
| Beliefs about Mathematics | 2,7,12, 13 | 1,6 |
| Beliefs about Post Secondary Education | 4,7,11,13,14 | 5,6, |
| Beliefs about the Pedagogy of Mathematics Education. | 3,4,8,9,10,11,12,13 | 2,3,4,6 |

Using Wolcott's data sorting suggestions, I read the transcripts numerous times, each time filtering out what I considered non-essential, and selecting materials in accordance to their affinities with the belief sets of Table 6. After this primary combing of all the data, the resulting condensed interview groupings were then further explored for analysis. At this stage, I made use of criteria, hereafter also called instruments, developed by Raymond (1997) based on work done by Ernest. (1989) Raymond's criteria for beliefs categorization cover the beliefs about the nature of mathematics, and beliefs about the pedagogy of mathematics. The beliefs about the nature of mathematics is covered with one instrument. For the beliefs about the pedagogy of mathematics education I have used three instruments developed by Raymond: The criteria for categorization of beliefs about mathematics teaching, of perceptions about the instructor's teaching practice,

and of beliefs about learning mathematics. Table 9 provides an example for the "Criteria for the Categorization of Teachers' Beliefs about the Nature of Mathematics" instrument. Similar tables for belief sets 1-4 of Table 7 are listed in Appendix C.

Table 9: *Criteria for the Categorization of Teachers' Beliefs About the Nature of Mathematics*

| | |
|---|---|
| Traditional | Mathematics is an unrelated collection of facts, rules and skills.<br>Mathematics is fixed, predictable, absolute, certain, and applicable. |
| Primarily traditional | Mathematics is primarily an unrelated collection of facts, rules and skills.<br>Mathematics is primarily fixed, predictable, absolute, certain, and applicable. |
| Even mix of traditional and nontraditional | Mathematics is a static bit unified body of knowledge with interconnecting structures.<br>Mathematics is equally both fixed and dynamic, both predictable and surprising, both a and relative, both doubtful and certain, and both applicable and aesthetic. |
| Primarily nontraditional | Mathematics is primarily a static but unified body of knowledge.<br>Mathematics involves problem solving.<br>Mathematics is primarily surprising, relative, doubtful, and aesthetic. |
| Nontraditional | Mathematics is dynamic, problem driven, and continually expanding.<br>Mathematics can be surprising, relative, doubtful, and aesthetic. |

Note: From "Inconsistency Between a Teacher's Beliefs and Practice" by A.M. Raymond, 1997, *Journal for Research in Mathematics Education, 28,* (5), 550-576.

The participant's interview material sorted under the heading of "Teachers' Beliefs about the Nature of Mathematics" was then scrutinized for descriptors matching the criteria of Table 9, in the process developing a profile of the beliefs of the individual.

The last step in the analysis was a comparison of the preceding profile with elements of Ernest's (1991) classification of educational interests groups. The information obtained through the cue card instruments and the unstructured interview beliefs instruments were then compiled using Table 8 as a guide.

Although Table 9 describes the beliefs of teachers about the nature of mathematics, a similar table can be constructed with minor changes for students. Similar processes using Ernest's descriptors have been used for the beliefs about the aims of post secondary education and the beliefs about the aims of mathematics education, the belief sets 5 and 6 from Table 7.

**Study Limitation**

The limitations of this study are the difficulties in generalizing and summarizing a small sampling. I am aware that in any communication much depends on how things are done and said. Deborah Tannen (1990) in *You Just Don't Understand* refers to the meta-messages behind each message as the framing that is going on in conversation. I admit that some framing in interviews and subsequent data analysis takes place. It is hoped that this is more than offset by the quality of the data obtained through the semi-structured conversation format which allows participants to go deeper into issues that concern them. This also enabled interviewees to have some control over the information brought forward and to have some direction over the outcome of this study. (Fontana and Frey, 1994)

It should be kept in mind that generalization of this exploratory study is limited in that it explores practice at one particular mathematics department. The fact that instructors were interviewed during the end of the term may influence their response. The use of instruments based on Ernest's ideologies, with a focus on social orientation, render the study atypical. However, the same particulars that may influence this study will also affect classroom interaction in a similar manner. If particulars of time, place and attitudes color this study, so will it color the classroom discourse. In this study I have assumed that all participants, students and instructors, are committed to teaching and learning. At least that's how I have tried to answer for

myself why they all gave so generously of their time in what I felt was for all a genuine attempt at conversation and (re-)searching. This commitment infuses the information obtained through the interviews with an authenticity and validity which warrants the probing for trends in beliefs, values expectations, and experiences.

# Chapter IV

# Findings

Chapter four focuses on reporting some of the data analysis. Observations about the beliefs and perception of students and instructors are organized in three main sections in accordance to he spheres of interest of this study: beliefs about the nature of mathematics, beliefs about the pedagogy of mathematics education and beliefs about the aims of mathematics education and post-secondary education. The second topic, "beliefs about pedagogy of mathematics", explores the following three pedagogical elements: beliefs about teaching mathematics, perspectives on the practice of teaching mathematics, and beliefs about learning mathematics. Within each section, beliefs are first explored for students, followed by an examination of the same set of beliefs for instructors. Findings have been developed using instruments such as the cue cards or Raymond's criteria for categorizations of beliefs. In exploring the aims of education, Ernest' and Raymond's terminology have been used interchangeably, i.e. the term "progressive educator" has been equated with the term "primarily nontraditional", although there is actually no corresponding instrument among Raymond's instruments.

## Beliefs about the nature of mathematics

### Description and categorization of students' beliefs about the nature of mathematics.

Students were asked how mathematics had been presented at high school and university. Through the use of educational ideology cards, they were also asked to identify their preferred way of being taught mathematics. ("I would like to see mathematics presented as:") For most students, high school mathematics experiences paralleled the industrial trainer or the technological pragmatist perspective, or to use Raymond's classification, the traditional or

primarily traditional perspective. In general, students' university calculus class experiences are the same as their high school ones. Robert's high school experience, on the other hand shows a much larger variation, ranging from primarily traditional to primarily nontraditional, which probably reflects the role of student-teacher interaction that Robert experienced in high school. Robert refers to his teachers as "friends" and "equals". Four students identify with a preference for nontraditional or primarily nontraditional perspectives on the nature of mathematics, or in Ernest'(1991) nomenclature, respectively the public educator's and progressive educator's perspective. (see first column Table 10) Two student, Mona and John, mention the traditional perspective. John, the only student in this study who came to university via community college, is the sole student who exclusively chooses the traditional perspective of the industrial trainer. Students' cue card responses for their beliefs about the nature of mathematics are presented in Table 10.

*Table 10: Students' beliefs about the nature of mathematics. Results obtained from cue card responses*

| Students | Experienced high school mathematics as: | Experienced university mathematics as: | Prefer to see mathematics as: |
|---|---|---|---|
| Mona | Traditional<br>Primarily traditional | Traditional<br>Primarily traditional<br>Even mix | Traditional<br>Primarily nontraditional |
| Cindy | Primarily traditional | Primarily traditional | Primarily nontraditional |
| John | Traditional | Traditional<br>Primarily traditional | Traditional |
| Robert | Traditional<br>Primarily traditional<br>Primarily nontraditional<br>Nontraditional | Traditional<br>Primarily traditional | Primarily traditional<br>Primarily nontraditional<br>Nontraditional |
| Noreen | Traditional | Traditional | Primarily nontraditional |

I have chosen Robert' interview to illustrate the analysis of interview data for descriptors about the nature of mathematics. Robert, who is enrolled in computer science, took enriched

mathematics in high school which he describes as interactive, strict, demanding, personal , and formal. He compares mathematics with other subject matters such as physics and history:

> It [physics] was a more relaxed course. Math, we had so much to cover in so little time. It was ... you make jokes for the first five minutes, and then bang get ready with math. And it was ... we worked on it, worked on it, worked on it. But physics, it was a little, physics is a little easier. Especially to understand. It's more concepts than math is. Math sometimes is just number crunching. But physics, you actually get to see what is going on.
> ...
> Physics is always more of a journey if I can use that word.
> ...
> Eh, in history ... you have to, in order to write anything, you have to have your personal opinion ingrained in it. But math, it's hard to do that. Right? Any kind of formula is a formula. It is static, and it does not change. ... because of this we have that. That kind of thing. As per mathematical theorem we know this. We are able to create this and that.

One of Robert's favorite high school teachers was his mathematics teacher who involved students through a project oriented curriculum in problem solving: "... he called a couple of us up, who knew a little bit about computers and about math. And we, we started a group, eh, that combined the two. ... one of the coolest things was that he learned with us ... To tell you the truth I don't really remember what he taught ... but he taught me how to think about math ... the way to deal with calculus." Robert seems to look for commitment from instructors to connect with the subject matter and with the students. Besides relating to the rational and objective nature of mathematical knowing, Robert connects in an affective way with the subject:

> ...math is a always been a very precise thing for me. And I have never sought of emotions as precise and defined or anything like that. But that does not mean you can't be intrigued about math. But music stirs emotions. I suppose math does too...

Robert enjoys nontraditional mathematics activities, but has problems reconciling the static impression about the mathematical factual knowledge base with the dynamic perspectives of knowledge acquisition. Robert's process orientation echoes Ernest' description of the progressive educator, which is:

> The philosophy of mathematics is absolutist, viewing mathematical truth as absolute and certain. But it is progressive absolutism, because great value is attached to the role of the individual in coming to know this truth. Humankind is seen to be progressing, and drawing nearer to the perfect truths of mathematics. On the basis of the connected values mathematics is perceived in humanistic and personal terms, and mathematics as a language, its creative and human side, and subjective knowledge are valued and emphasized. But this is coupled with absolutism. Thus the view of mathematics is progressive absolutist, the absolutism coloured by the humanistic, connected values. (Ernest, 1991, p. 182)

Robert's beliefs about the nature of mathematics are primarily nontraditional.

Other students' interviews have been analyzed in the same fashion. Most students have a traditional or primarily traditional perspective on the nature of mathematics. Robert's primarily nontraditional perspective is the sole variant. Students' responses on the beliefs about the nature of mathematics from interview analyses are summarized in Table 11.

*Table 11: Students' beliefs about the nature of mathematics. Results from interview analysis.*

| Students | Interview analysis result |
|---|---|
| Mona | Traditional |
| Cindy | Traditional |
| John | Primarily traditional |
| Robert | Primarily nontraditional |
| Noreen | Traditional |

**Description and categorization of instructors' beliefs about the nature of mathematics.**

All five instructors chose, as nature of mathematics, "a body of structured pure knowledge" on the educational ideology cue card. Of the five instructors three opted for additional descriptor of "an unquestioned body of useful knowledge." The instructors' responses to the educational ideology cue card on the nature of mathematics point to the image of the old humanist and the technological pragmatist. Instructors' cue card responses are presented in Table 12. According to the cue card responses, instructors perceive their beliefs about the nature of mathematics to be composed of the primarily traditional and the even mix perspectives.

*Table 12: Cue card results and interview analysis results for instructors' beliefs about the nature of mathematics*

| Instructor | Cue card result<br>Presents mathematics in teaching as: | Interview analysis |
|---|---|---|
| Robin | Even mix of traditional and nontraditional | Even mix of traditional and nontraditional |
| Lesley | Primarily traditional<br>Even mix of traditional and nontraditional | Even mix of traditional and nontraditional |
| Chris | Even mix of traditional and nontraditional | Primarily traditional |
| Stacey | Primarily traditional<br>Even mix of traditional and nontraditional | Primarily traditional |
| Alex | Primarily traditional<br>Even mix of traditional and nontraditional | Primarily traditional |

Robin's interview has been used to demonstrate the analysis process. Robin does mathematics "because it esthetically appeals." Mathematicians' activities are located "somewhere in between those of the people in arts and those of the people in science." There is "more kinship with the philosophers than...with the scientists." Robin describes mathematics as:

...it became obvious that there were rules to the game, and you just played the game within those rules. And I really liked that as a student. And the thing that appealed to me about the mathematics was, that it was a surer form of knowledge than what we were getting in physics and chemistry classes. ... I had the teacher that I mentioned in grade ten emphasizing that what was so beautiful about this stuff that he was teaching was how sure it was... And that was a, that was a worship for me that year. To be listening to that guy and realize "Yes there where some things that were really, that were really true." ... I considered there were some things that were more sure than God's word. And what was allegedly God's word and so on. And eh, and this, and this was, you see, very exiting for me about mathematics. That's why I wanted, I kept wanting to do it. Because of the certainty, the beauty of the stuff. ... I learned in a very young, young age, to debate the merits of eh, of mathematical ideas.

Mathematics is a model of truth and beauty:

I think it is been kind of a treasury of the, of the human race, this, this thing that has, has been accumulated. Which has great value. Which is there as a model for truth and beauty. Which should be stored away and valued for its own sake. Even though only a tiny part of it has been useful.

For Robin mathematics is, however, more than a rational process:

It is interesting you know, I, I was expressing this concern about the denial of the spiritual and the emotional life, but because of rationality, the rational movement. But if, if you take mathematics correctly positioned in that content, in that enterprise, and you, and you realize that the mathematics involved people who go around talking about a proof "out of the book", jeez, it is not much more spiritual than that. To be, to be contemplating some mathematical ideas as being "out of the book". I mean, to even, to even have that expression make sense, suggests to

me that the person is an extremely spiritual and emotional person. It is just that maybe, they are maybe, they are not organizing their thoughts in Christian-Judaic lines.

Robin represents Ernest' old humanist profile:

> The old humanist considers pure knowledge to be worthwhile in its own right. In particular, the mathematical old humanists regard mathematics as intrinsically valuable, a central element of culture. Mathematics is a supreme achievement of human kind, 'queen of the sciences', a perfect, crystalline body of absolute truth. It is the product of an elite, a small band of genius. Within mathematics rigour, logical proof, structure, abstraction, simplicity, elegance are valued. Based on these values the aim for mathematics education is the communication of mathematics for its own sake." (Ernest, 1991, p.168)

Robin's perspective on the nature of mathematics is an even mix between a traditional and nontraditional perspective. Instructors' beliefs about the nature of mathematics, obtained in similar fashion to Robin's, are primarily traditional or an even mix between traditional and nontraditional perspective. Table 12, previously mentioned in relation to instructors' cue card results, also summarizes the interview results for instructors.

## Beliefs about the pedagogy of mathematics education

To address beliefs related to the pedagogy of mathematics education, three belief elements have been explored. As mentioned earlier, the elements are beliefs about the teaching of mathematics, the perspectives on teaching practices, and beliefs about the learning of mathematics. (see Table 7 in chapter three) Each of these belief sets will be addressed in a separate section. The first three sections will deal with the beliefs of students. Instructors' beliefs are presented in the next three sections.

### Description and categorization of students' beliefs about teaching mathematics.

In this section, students' beliefs, preferences, values and experiences about the teaching of mathematics, are explored. I have used Noreen's interview to demonstrate the analysis of the interview for beliefs' descriptors about teaching mathematics. Noreen, studied calculus in high school, and is interested in a career in physiotherapy. She likes "discussions a lot more than just

listening ... people more interacting between each other" She observes: "... I enjoy math ... I find it an interesting course. But I like to, I like to find a course that you know spends more exploring than studying what is true." Noreen needs to see the relevance of mathematics:

> Why is it important? What does it have to do with being a doctor? What does it have to do with being a computer engineer? Why is it there? It is obvious important, but nobody has taken the time to tell us why."

Teaching should encourage her:

> ...to be able to be open minded. And not being taught this is the way it should be. Just being taught: "Well what do you think?" Allowing students to improvise on their own opinions, and taking other opinions and sort of bring it all into one.

An instructor should be "one that sort of speaks both languages ...the language of the subject ... technical terms, big words ... Also explain it in terms that...the students understand. ...one who listens to the students...don't just walk in, give a lecture, and leave." Noreen expresses a preference for the primarily nontraditional approach to teaching mathematics. Ernest (1991) comments on the theory of mathematics teaching in the progressive educator's ideology that it:

> Consists of encouragement, facilitation, and the arrangement of carefully structured environments and situations for exploration. Ideally, it will involve the use of teacher or school constructed mathematics curriculum, offering a 'circus' of different mathematical activities around the classroom, and employing multi-disciplinary projects. The role of the teacher is seen to be that of manager of the learning environment and learning resources, facilitator of learning, with non-intrusive guidance and shielding from conflict, threat and sources of negative feelings. (Ernest, 1991, p. 192)

All students, except for John, prefer the primarily nontraditional mathematics teaching method. John's description of his preferred approach to being taught mathematics resembles a primarily traditional perspective. This may reflect John's experience in Community College were his math courses were taught in a small group setting. (approximately 10 students) According to John these courses followed an industrial pragmatist ideology. Table 13 presents cue card results and interview analysis results for the students' beliefs about teaching mathematics.

*Table 13: Cue card results and interview analysis results for students' beliefs about mathematics teaching*

| Student | Cue card result | Interview analysis |
|---|---|---|
| Mona | Primarily traditional<br>Even mix of traditional and nontraditional | Primarily nontraditional |
| Cindy | Even mix of traditional and nontraditional<br>Nontraditional | Primarily nontraditional |
| John | Even mix of traditional and nontraditional | Primarily traditional |
| Robert | Primarily traditional<br>Even mix of traditional and nontraditional<br>Primarily nontraditional<br>nontraditional | Primarily nontraditional |
| Noreen | Nontraditional | Primarily nontraditional |

**Description and categorization of students' perspectives on teaching practices.**

In this section students' experiences as they pertain to teaching practices in their first year calculus course are explored. A collage of student responses is presented first. The interview impressions of the students' perspectives about instructor's teaching practices are summarized in Table 14 for each student.

Cindy finds the calculus course cut and dry "this is the way that it is, this is where it is derived from, now use it." There is not much opportunity for discussion, "they never give you the option to question it." She feels inhibited about asking questions, "I just attribute it to my lack of knowledge rather than it actually being a valid question" Class is described as "you sit down and [the instructor] come in and write the four questions on the board. And [the instructor] will do them and you copy them down. And that's about it." Students "mostly listened" or "just point out things that [the instructor] did wrong."

For Mona calculus was taught in an atmosphere of "your standard lecture, where you have somebody teach you different things … you have to absorb it and hope for the best. And hope that you can do the best with the knowledge and information you have been given. … You weren't encouraged to debate … you are taught a certain way of thinking and you are expected to follow that way of thinking … just go to class and take notes and go write tests and that is it." Cindy found assessment processes discouraging because "I get things like a zero out of six when I have half the question right" and "I didn't even get credit for having the right idea." She is not very motivated by evaluation methods, "If I am gonna work hard and get no recognition for it, then I did not want to continue it."

Cindy found the instructor encouraging and discouraging at the same time: "[The instructor] is encouraging because [the instructor's] like: "Oh, this is easy, this is easy" … But then it is discouraging, because…you know it is encouraging until you run into a problem … And [the instructor] is still saying, it's easy." She finds the instructor "rushed … wants to get the work done … and it is new to us … flying around the board … just likes get the problem done."

Robert comments that instructors should earn respect because people tend "not to listen to people they don't respect. And if their professor is up there just reading off notes, then anyone can do that … anyone can just copy them on the board and read them." Robert notices that copying notes is particular to the calculus course, a distinction from his physics, computer science and discrete mathematics course. I ask Robert if there is disrespect for the students from the instructor:

> Well, not disrespect, just … irrelevant. They just stand there and read notes. They don't disrespect us. They don't respect us. They are just there.
>
> …
>
> It was not interesting. I could read out of my text book for that. Sit home in my bed with my cup of coffee and do the same thing. Know more. There was no reason for me to be there. It has to keep it interesting.

Robert finds it important for instructors:

> … to describe why they like it so much. I mean, you're not going to be a professor and hate the subject you are in. At least I hope you don't, but...

Noreen describes a calculus course as straight lecturing without much student interaction "they just sort of sit there. And they take up what the teacher says. ... Always by the book ... if you had a question you ask ... if nobody had a problem, we continue on with another example. Another section."

John, critical of the large impersonal lecture format, prefers a smaller class size with more personal contacts:

> I guess you can describe it sort of like ... the bank machine and the you know, the personal teller. Like a bank usually if it is a small branch, after a while, you get to know some of the people there. It's just, like a nicer atmosphere to, to, sort of if you walk into the bank, they say: "oh, hello, how is it going?" ... Whereas outside, it is just a cold impersonal beep, beep, beep: the bank machine. So I think, I think it helps maybe stimulate learning a little bit more, sort of a more friendly atmosphere.
>
> ...
>
> Instead of you know..."Hello number One Three Four Zero. What is your problem?" ...Stuff like, "this, this, this and this. You understand? Yes? Ok. Next."

John finds it difficult to ask questions because of the "embarrassment, intimidation factor."

Overall students' experiences of instructors' teaching practices reflecting traditional and primarily traditional teaching practices, are summarized in Table 14.

*Table 14: Unstructured interview results of students' perspectives on instructor's teaching practice*

| Student | Instructors teaching practices are like: |
|---|---|
| Mona | Traditional |
| Cindy | Primarily traditional |
| John | Primarily traditional |
| Robert | Traditional |
| Noreen | Traditional<br>Primarily nontraditional |

**Description and categorization of students' beliefs about learning mathematics.**

At some point during the interview with Cindy we were talking about her summer reading, the novel *Summer Sisters*. She mentions how, in the book, the main characters grow apart:

> Eh, I guess sometimes like maybe sometimes people they are not really who you think they are. Like you think they are the same as you in every way. But you know that's only because you've been presented with the same questions and when you ask the question differently, you realize that you're not the same after all. ....it creates tension. Just because you and your questioning how ... They are going their way and you are going your way. And your ideas can't meet in the middle.

I asked Cindy if the notion of questioning differently is something that could be applied to mathematics and she comments:

> Maybe there is a way of looking at it, you know mathematically. But then maybe ... logically that is ... even possible. Like come up with ... an example type answer. Like ... how can this be applied ... Solve this problem. ... word it. Put the problem into words. And don't use any kind of numbers.
> ...
> You know, I, I know it can happen with some word processes ... you probably don't even realize that you are using math. But you ... are. But you don't really know it, because you are not using any numbers. Cause you think math, numbers right? So maybe you just present a problem with words that you have to figure out logically. And then present it, like with numbers that you figure out mathematically. That would be pretty neat.
> ...
> ...some thing that does not require a mathematical answer just ... a general answer, which is not so right or wrong.

Cindy experiences calculus as difficult and "very isolated, very unfriendly." Feelings are important to Cindy and she likes to see more room for individual perspectives in the learning of mathematics:

> ...you have always, you know you have always been brought up to believe that, and a lot of people have been brought up to believe that it is very difficult. And just, I think with the beliefs, when you just use your thoughts on calculus, it is very society oriented, but your feelings are individual. So you can look at it two ways. Like your individual perspective and then the perspective that you have been brought up to have.
> ...
> Like ... three classes of math a week, make ... at least half of one ... about the actual subject. Not so much like the application of it and the evaluation of it. But ... what the subject actually is and how it effects everything.
> ...
> ... actually raise some questions about which people had opinions. There are feelings behind those opinions.

When Cindy refers to "the actual subject" she is referring to the non-mathematical, non-textbook aspects of calculus; the social and cultural embeddedness. She likes opportunities for debate as "hearing other people's ideas will probably help me to fully realize what I want to say." Cindy describes herself as hands-on with a need to create a personal affective understanding of mathematics. She wants to involve herself with math by "practical experience ... apply it ...

think about it ... understand where it is coming from ...being able to visualize it ... hands-on or anything like that." She could enjoy a historical insight in "how the person actually came up with it ... what where they thinking ... break it down into ... components ... how they all came together to mean one thing." In her calculus course, through her own initiative, Cindy became involved in working with other people on assignments. She describes how the original aim of coming up with the right answer changed:

> It started out with getting the right answer. But as we got more into depth in the course, it was more helping each other and making sure that we did actually understand it. It wasn't so much the right answer anymore, it was the concept behind it. If we actually knew how to do this or not.

She likes to learn through project work, "assign one really hard problem, and have like groups of four and have them all work together ... that would bring people closer together and they would interact with each other." Cindy would then "enjoy the subject more. Which means I try harder." She loves creativity and variety which she does not find in calculus because "the answer is already there ... we already have our formulas and our theories." If there was more room for creativity she would "look at it with more enjoyment ... want to learn more ... be curious about things ... not afraid to question things." Cindy feels that she is doing well in calculus but:

> Even though I am doing well here, it's, I don't get the satisfaction of knowing that I enjoy it. And for some of my courses, I do have the satisfaction of knowing that actually I know the material.
> ...
> Some of them I know what I am doing. I apply it, and I understand it. And math I don't have that feeling that I, I know it.

Cindy's preferred mode of learning mathematics resembles that of the nontraditional public educator and the primarily traditional progressive educator. Ernest' (1991) describes the public educator's philosophy of mathematics as social constructivism:

> ...this entails a view of mathematical knowledge as corrigible and quasi-empirical; the dissolution of strong subject boundaries; and the admission of social values and a socio-historical view of the subject, with mathematics seen as culture-bound and value-laden. (Ernest, 1991, p. 197)
> ...
> the theory of mathematics learning is that of the social construction of meaning ...knowledge and meaning are internalized 'social constructions' resulting from social interactions, the negotiation of meaning and engagement in 'activity'. (Ernest, 1991, p. 208)

Ernest' description of the theory of learning mathematics in the progressive educator's ideology states that:

> This involves the students' active responses to environment, autonomous inquiry by the child, seeking out relationships and creating artefacts and knowledge. Learning involves investigation, discovery, play, discussion, and cooperative work. The environment in which learning takes place must be rich and challenging, but must be secure, fostering self confidence, positive attitudes and good feelings. Thus the learning of mathematics is first and foremost active, with the child learning through play, activity, investigations, projects, discussions, exploration and discovery. (Ernest, 1991, p.192)

Students' interview results vary from primarily traditional to nontraditional. Table 15 presents cue card results and interview analysis results for the students' beliefs about learning mathematics.

*Table 15: Cue card results and interview analysis results for students' beliefs about learning mathematics*

| Student | Cue card result | Interview analysis |
|---|---|---|
| Mona | Traditional<br>Primarily traditional<br>Even mix of traditional and nontraditional<br>Primarily nontraditional<br>Nontraditional | Primarily traditional |
| Cindy | Primarily traditional<br>Even mix of traditional and nontraditional<br>Nontraditional | Primarily nontraditional<br>Nontraditional |
| John | Even mix of traditional and nontraditional | Primarily traditional |
| Robert | Traditional<br>Primarily traditional<br>Even mix of traditional and nontraditional<br>Primarily nontraditional<br>Nontraditional | Nontraditional |
| Noreen | Primarily nontraditional | Primarily nontraditional |

**Description and categorization of instructors' beliefs about teaching mathematics.**

To demonstrate the evaluation of the interview for teacher's beliefs about teaching mathematics, I have used Chris' interview. Chris believes that in mathematics teaching there are "too many students who get no individual treatment at all … a lot of them should not be in the same room as a lot of the others." Chris expects from students "clear thinking, literate, being able to write good sentences, and with a substantial background." Chris admits that " I am not good with the weak ones … I would not have the patience." However, "a lot of the middle ones would come in my top favorite ones to teach." Chris spends a lot of time encouraging students to "use a very specific language in a syntactically acceptable way. This instructor's teaching style is self-described as "non threatening:"

> I try to mix hard things and easy things within any one class. So that there is something for everybody. Bearing in mind that I got everybody. I try to make sure that the students have things to do while I am teaching some of the time. For example if I show them some process, and if I do two examples, then I make them do two, for example. And I will wait while they do it. I, I don't want to feel that I am talking solidly for fifty minutes. Because that's hard on everybody. They like to feel that they achieved something. Obviously they have to be little things. Or to me it is obvious they have to be little things.

Chris describes an admired mentor's teaching style as characterized by "the writing on the board was always clear and large …always enthusiastic …he didn't try to achieve too much." Chris learned that "it was possible to be clear … the fact that they are only students does not mean that the subject can't be interesting … even when teaching hundreds of students simultaneously, to give the impression that we are talking to just one or two of them."

> I think I am probably better with a large group than with the small groups. I come across to a large group perhaps as nicer than I am. … Because in the large group, I can always say to myself: "Oh, you know, some of the class really knows what I am talking about." So I can tune out some of the others.
> …
> And because I can be patient with a lot of them, even the ones that I have little time for don't see that side of me.

Chris does not use technology in the classroom because " I am not good with micro-computers … I don't have time or whatever." Teaching mathematics should:

> encourage some sort of logical thinking. It doesn't always, I mean, mostly it does not achieve that. It, it, it points out how extremely difficult it is for students to follow more then a couple of logical steps.
> …

think we try to make the students aware of both aspects. This sort of idealized, pure, beautiful kind, and the useful, technologically necessary kind.

...

I talk a lot and mostly the students just listen. They, well sometimes talk back. I think they always feel that they could talk back. But with the large group they recognize that this isn't always practical. That everybody asks every question then no progress will be made. And it puts extra pressure on me to try to be very clear at the beginning, so that people don't need to ask too many questions.

This instructor works with "setting identifiable goals at the beginning ... and then reach them." To get students interested in learning "we could tone down some of the more obscure theoretical things ... bring in more real world problems ... break up the work in manageable pieces ... and produce a time frame that individual students have a hope of completing." It is important for students to "have people behind it all that they can see and work with and meet and talk to and recognize as being simultaneously interested in them." Chris' believes that "there isn't time for a personal exploration" of mathematics in teaching. The instructor observes that:

I like some sort of depth of historical knowledge. I like to feel that this, the weight of history that has accumulated all these things, that we aim to transmit ... A lot of the modern theories would downplay those at the expense of more self discovery and so on, which I think is a very inefficient way. That students don't have time to discover everything over again. Both are important. But there is a substantial amount of transmissible skill and knowledge.

...

Uhm, a lot of modern educators would say that I talk too much, and should allow more space in class time for students to do things. But I find that if I am transmitting a language then they need to hear the language spoken. And they need to hear it spoken reasonably accurately. Rather than hearing it spoken very badly.

Chris presents a math-centered approach to teaching mathematics, and exhibits the characteristics of the old humanist: an even mix of traditional and nontraditional perspectives.

Overall instructors' responses vary from primarily traditional to the even mix perspective, to the primarily nontraditional belief, with a larger emphasis on the primarily traditional perspective. Instructors' unstructured interview beliefs about teaching mathematics are summarized in Table 16 together with cue card responses.

*Table 16: Results cue card and interview analysis of instructors' beliefs about teaching mathematics*

| Instructor | Cue card result (Theory of teaching mathematics) | Interview analysis |
|---|---|---|
| Robin | Primarily traditional<br>Even mix of traditional and nontraditional | Even mix of traditional and nontraditional |
| Lesley | Even mix of traditional and nontraditional | Primarily traditional |

*Table continued*

*Table 16 continued*

| Instructor | Cue card result (Theory of teaching mathematics) | Interview analysis |
|---|---|---|
| Chris | Traditional<br>Primarily traditional<br>Even mix of traditional and nontraditional | Even mix of traditional and nontraditional |
| Stacey | Even mix of traditional and nontraditional<br>Primarily nontraditional | Primarily traditional |
| Alex | Primarily traditional | Primarily nontraditional |

**Description and categorization of instructors' perspectives on teaching practices.**

In exploring teaching practices it is necessary to keep in mind that teaching takes place in a departmental environment with common traditional assessment procedures, common curriculum, and lecture oriented content delivery. Instructors may thus be structurally restricted in their practices of teaching mathematics. Lesley's interview is used to illustrate the instructors' perspectives on teaching practices.

Instructors are confronted with "reluctant learners" says Lesley. "Any kind of learning is not cool ...[students] aren't as well equipped." Students are learning "almost too late ... with great effort, great frustration often ... what for some reason ...[was] never learned in high school." Lesley comments that "we live in a certain society ... that does not value learning as much as other societies." The instructor believes that "there is a certain social responsibility in mathematics." Lesley questions the good of pure mathematics and suggests a practice which advocates:

> ... that in science in general, and in mathematics, we have to think through everything, question everything, question all the dogmas, including perhaps our own. Uh, I think that is an important role... And I think that I try to instill in my students a certain feeling of not taking anything for granted. And I hope that, once they go out in the real world that they continue to think along those lines.

Asked if there is a social responsibility from the instructor to the student, the answer is:

> Eh, social of course can be interpreted in different ways. Uh, being considerate and kind to them. Social obligation like if they have problems, that I help them out. That I show flexibility. That is one level. On the other hand, on a different level, considering them as social beings, and teaching them mathematics in such a way that they can function in society. That is a different social obligation. I feel a social obligation towards everybody. Including the students. Like in a wider framework. Uhm, social obligation also means to be fair. Not to give unfair disadvantage

... advantages to pushy students who know the ropes. So, social obligation sometimes means being tough and denying sometimes, which at first side looks like, oh, I am being un-social, antisocial.

Lesley rejects the idea of "putting artificial social content into the curriculum," a concern extended to problems in textbooks:

So I would want problems in there, that are just as valid in thirty years time, than they are now. So some of the word problems ... are of that kind. Like the lighthouse, the famous lighthouse problem, the ship problem, and most of them are of that kind. No problem at all! Pick that book up in thirty years time, uhm, it is neutral. .... It sounds a little bit paradoxical because on the one hand I seem to portray a social conscience here. At the other hand I am strongly against putting social context in the textbook. But I do believe that that's the way it has to be done.

Asked if the instructor is happy with present teaching circumstances, Lesley comments "if we accept the present organization as a framework, then I aim fairly satisfied ... there are things one would like changed ... for instance, smaller classes." There are "adverse circumstances ... stick to a strict schedule ... having to teach out of a textbook ... If we have ten sections we have to work together." Advantages are that "if I agree with my colleagues on a common framework, then all I have to do just to get through the course is stick to the framework ... I have to do less thinking about these matters and can concentrate on other things." To the question, what is required of a mathematics instructor, the instructor responds:

Well, first of all, a mathematics teacher has to be entirely comfortable with mathematics. So, a mathematics teacher , in any level, should have a full mathematics degree. And that, that's the absolute requirement. And any teacher who is in front of a class to teach mathematics, should have a mathematics degree. And secondary of course is a pedagogical education. But I would say only secondary. Because as I mentioned already it is important that [the teachers ]are themselves. Doesn't teach according to a certain theory, or a certain dogma sometimes that [he or she] learned. But teaches mathematics, which [he or she] knows intimately, because ... he or she has a degree in mathematics. ... They have to be very comfortable with the subject. They have to like it. And that's the most important thing. And then the pedagogical instruction perhaps. Some hints as to how to deal with the psychology of, well, thirteen-year-olds. That's a great part. But not with: "How to teach mathematics to those thirteen year olds." But how to deal with them as people, what goes through them, and so on.

Lesley believes that learning is "un-cool" in society and "work towards better teacher education" will bring about a change because "part of the problem is that many teachers are scared of mathematics." Lesley is referring to teachers in the elementary and secondary school system. Lesley observes that " I often learn the stuff along with the students. So, I am more facilitator. ... Sometimes ... I am only a coach. And the most effective teaching is when I do just

a little bit of gentle guiding. That's the ideal thing, where the student is, say an apprentice. And I am the, well I shouldn't say master, but the same idea."

I do what feels right. I do a lot of things instinctively. And of course, for my own experience, I had good teachers, I had bad teachers. I had first rate teachers, I had lousy teachers. And that of course I have over the years, over the decades incorporated. So I have a feel for what I think is good learning, good teaching, and what is not good learning and good teaching. But I can not formulate a theory.

This instructor responds to a question about preferred teaching styles with "well, one word: blackboard. I believe in the blackboard."

I like lecturing. I tried other things too. But I always come back to lecturing. Uhm, even in problem solving, it turns out I do lecturing. So I have to mention it. That is part of my teaching style.

Lesley represents through the apprenticeship model the tradition of mathematical stewardship. Traits of an even mix of traditional and nontraditional perspectives are displayed. This instructor believes mathematics and mathematics education to be neutral. However when teaching is seen as a social activity, it is a bureaucratic or administrative social activity practiced from a primarily traditional perspective, and disconnected from culturally embedded mathematics. Lesley's teaching practice is characterized by the technological pragmatist, the primarily traditional perspective.

Results from the instructors' interviews reveal that the nature of all instructors' teaching practices is primarily traditional. Table 17 summarizes these results together with instructors' responses to the theory of teaching mathematics cue card.

*Table 17: Results cue card and interview analysis of instructors' beliefs about instructor's teaching practice*

| Instructor | Cue card result (Theory of teaching mathematics) | Interview analysis |
|---|---|---|
| Robin | Primarily traditional<br>Even mix of traditional and nontraditional | Primarily traditional |
| Lesley | Even mix of traditional and nontraditional | Primarily traditional |
| Chris | Traditional<br>Primarily traditional<br>Even mix of traditional and nontraditional | Primarily traditional |

*Table continued*

*Table 17 continued*

| Instructor | Cue card result (Theory of teaching mathematics) | Interview analysis |
|---|---|---|
| Stacey | Even mix of traditional and nontraditional<br>Primarily nontraditional | Primarily traditional |
| Alex | Primarily traditional | Primarily traditional |

**Description and categorization of instructors' beliefs about learning mathematics.**

Instructor Alex considers "discussions … demonstrating … interactions between the students and professors … and among the students" a learning process. But it is considered "slightly less" prevalent in mathematics classes than science classes. To learn, students need "to know the reasons behind it … the facts behind it …the definitions … the general theories …the key is you have to understand the theory well." Students need to be "independent … responsible … keen." Learning goes hand in hand with "curiosity … and … probably a degree of frustration." Students are "interested in new stuff." Students' attitudes will change if they know that what they are learning is "helpful … useful …applicable." Learning can improve if instructors "could help them in another way then in just the traditional teaching way. … inspire the students in the sense we … let them discover the results for themselves." Making mistakes " is the way of learning … as long as you are learning, you always make mistakes." Alex describes learning as practice in asking questions, but acknowledges that in math class "you don't have too much debate …because for mathematics …most math theories … you don't have to debate." Alex shows characteristics of learning through exploring, developing understanding through active participation, and supporting a student centered learning environment. Alex leans towards the primarily nontraditional perspective in learning mathematics.

Instructors' responses, in this section, vary from primarily traditional to the even mix perspective, to the primarily nontraditional belief, with a larger emphasis on the primarily traditional perspective. Results of instructors' unstructured interview beliefs about learning mathematics are summarized in Table 18.

*Table 18: Results cue card and interview analysis of instructors' beliefs about learning mathematics*

| Instructor | Cue card result (Theory of teaching mathematics) | Interview analysis |
|---|---|---|
| Robin | Traditional<br>Primarily traditional<br>Even mix of traditional and nontraditional<br>Primarily nontraditional<br>Nontraditional | Primarily nontraditional |
| Lesley | Even mix of traditional and nontraditional | Even mix of traditional and nontraditional |
| Chris | Traditional<br>Primarily traditional<br>Even mix of traditional and nontraditional<br>Primarily nontraditional<br>Nontraditional | Even mix of traditional and nontraditional |
| Stacey | Primarily traditional<br>Even mix of traditional and nontraditional<br>Primarily nontraditional | Primarily traditional |
| Alex | Even mix of traditional and nontraditional | Primarily nontraditional |

## Beliefs about the aim of mathematics education and post-secondary education

### Students' perspectives on mathematics education and post-secondary education.

Student John believes that universities "encourage some people to think … if you are predisposed to sort of thinking." John wants to experience "some of the more social events" as well as "the learning that goes along with it." For John, mathematics education forms:

> the foundation of a lot of other courses. And like just in basic live you need common math skills to be able to get by Like you know, eh your basic adding and subtracting and sort of like you know, like doing your taxes and, and that stuff, So it's really handy to have as a tool.
> …

Yeah, yeah, because there is stuff that can't be, that can't be computed without Calculus. Certain things like physics or chemistry or, just especially like in electronics and stuff like that.

Post-secondary education, according to John:

should be to help you sort of to do what you want to do in life. And, and I guess, there is the research aspect, and stuff like that. ... But, I think, I think it should be something that helps you eh, to learn, to get together the skills necessary to do what you want to do. ... Most of it, I think it should be sort of like to help you with knowledge and requirement what your goals it might further your career or whatever you like to do.

The aim of mathematics education is described as:

organize the information into a form that you can use ... sort of like, analytically and be able to ... see certain patterns and sort of repetitions and things ... you get this good ability to recognize things sort of on an abstract basis

Besides offering industry-centered descriptions, John sees the value of post-secondary education and mathematics education, just for the sake of knowledge. However, John's views seem to be rooted in the technological pragmatist ideology. John presents a primarily traditional perspective. Ernest (1991) describes the aim of mathematics education according to the technological pragmatist's ideology as:

> The aim of this group for the teaching of mathematics are utilitarian; students should be taught mathematics at the appropriate level to prepare them for the demand of adult employment. This aim has three subsidiary components: (1) to equip students with the mathematical knowledge and skills needed in employment, (2) to certify students' mathematical attainments to aid selection for employment, and (3) to further technology by thorough technological training, such as in computer awareness and information technology skills. (Ernest, 1991, p. 162)

Beliefs about the purpose of mathematics education and post-secondary education have been obtained for all students in a similar fashion. Results of students' unstructured interview beliefs about the purpose of mathematics education and post-secondary education are summarized in Table 19 together with their cue card responses. Cue card responses show a similar pattern with the exception of Noreen who opts for a primarily nontraditional aim on her cue car response.

*Table 19: Results cue card and interview analysis of students' beliefs about mathematics education and post-secondary education*

| Instructor | Cue card result (Mathematics Aims) | Interview analysis |
|---|---|---|
| Mona | Primarily traditional | Primarily traditional |

*Table continued*

*Table 19 continued*

| Instructor | Cue card result (Mathematics Aims) | Interview analysis |
|---|---|---|
| Cindy | Even mix of traditional and nontraditional<br>Primarily nontraditional | Primarily traditional |
| John | Primarily traditional<br>Even mix of traditional and nontraditional | Even mix of traditional and nontraditional |
| Robert | Primarily traditional<br>Even mix of traditional and nontraditional | Even mix of traditional and nontraditional |
| Noreen | primarily nontraditional | Primarily traditional |

**Instructors' perspectives on mathematics education and post-secondary education.**

Instructor Stacey describes one goal of university education the promotion of:

> Learning about different things, you know learning about that that matters, and so on.
> ...
> In some ways you want to be a little different. You want to bring all these ideas, exploring new areas. Thinking for yourself, you know. Just finding out...

Because "in most cases high school councilors did not know a thing about career, not really ...university should be ... more clued in to that kind of assistance for students." Stacey believes that mathematics is taught because "it is the alphabet of the language of science. ...whenever you try to analyze a bit of data ... you can not, you have to have, it's like why do we teach the alphabet before we teach spelling? Well, cause if you don't have the alphabet, you can't spell. That's it. That's why we teach math." When asked about the purpose of post-secondary education, the instructor responded with "I wouldn't wanna venture there." But Stacey does leave a suggestion that, whereas in the past " money wasn't a big thing," nowadays "money is everything, possessions, consumerism ... all driving factors." Stacey's educational purposes are career oriented. Stacey resembles the technological pragmatist, and fits the primarily traditional approach.

Instructors' belief sets about the purpose of mathematics and university education vary from primarily traditional through the even mix perspective to the primarily nontraditional perspective. There is a tendency to lean towards the primarily traditional perspective. Instructors' unstructured interview results for beliefs about the purpose of mathematics education and post-secondary education are summarized in Table 20.

*Table 20: Results cue card and interview analysis of instructors' beliefs about mathematics education and post-secondary education*

| Instructor | Cue card result (Mathematical aims) | Interview analysis |
|---|---|---|
| Robin | Even mix of traditional and nontraditional<br>Primarily nontraditional | Primarily traditional<br>Even mix of traditional and nontraditional |
| Lesley | Primarily traditional<br>Even mix of traditional and nontraditional<br>Primarily nontraditional | Even mix of traditional and nontraditional<br>Primarily nontraditional |
| Chris | Even mix of traditional and nontraditional<br>Primarily nontraditional | Even mix of traditional and nontraditional |
| Stacey | Primarily traditional<br>Even mix of traditional and nontraditional | Primarily traditional |
| Alex | Primarily nontraditional | Primarily traditional |

# Chapter V

## Summary, Implications and Suggestions

The purpose of this study was to investigate the differences in beliefs between instructors and students in a first-year introductory calculus course with regard to the nature of mathematics, the pedagogy of mathematics education and the purpose of mathematics education and post-secondary education. Beliefs about the main spheres of interest have been reported in chapter four. Beliefs of student participants and instructor participants are further analyzed and compared for differences and similarities in the first section of this chapter, the summary section. Results will be explored in the same order as the analysis in chapter four. The "Implications" section interprets the observations of the previous section in relation to the construct of "mathematics avoidance." Suggestions pertaining to the outcomes conclude this chapter.

## Summary

### Beliefs about the nature of mathematics.

According to Table 10 and 11, most students have experienced a traditional or primarily traditional perspective on the nature of mathematics. Robert's primarily nontraditional perspective is the sole variant. The results of Table 11 are impressions from students' interviews. Cue card results show agreement between the nature of mathematics as it was presented to the students in high school and as it was presented to the students in university. (see Table 10) However, students show a preference (4 out of 5 students) for a perspective on the nature of mathematics that is primarily nontraditional. There is a distinct difference between what students are experiencing as the nature of mathematics, and what the students prefer as the nature of mathematics. The preference is for a process view of mathematics in which "great value is attached to the role of the individual on coming to know this truth." (Ernest, 1991, p.182) In the

process the students experience mathematics as primarily aesthetic, doubtful, relative, and surprising. In contrast, students experience mathematics as a body of pure knowledge, primarily applicable, certain, absolute, predictable and to be unquestioningly accepted.

All instructors, according to the cue card results (see Table 12), present the nature of mathematics in their teaching as an even mix of the traditional and primarily traditional perspectives, the old humanist ideology according to Ernest. Three out of 5 also mention the primarily traditional perspective. This corresponds well with the interview impressions which categorize 3 out of 5 instructors as having a primarily traditional perspective, and 2 out of 5 as having an even mix of traditional and nontraditional perspectives. A comparison of students' preferred view of the nature of mathematics, of instructors' view of mathematics, and of students' experienced view of mathematics is presented in Table 21. Students' preferences regarding the nature of mathematics are not in congruence with their experiences and with the perspectives transmitted by instructors.

*Table 21: Nature of mathematics according to students and instructors*

| Nature of mathematics presented, as perceived by students | Nature of mathematics as presented by instructors | | Nature of mathematics as preferred by students |
|---|---|---|---|
| Traditional | Primarily traditional | Even mix of traditional and nontraditional | Primarily nontraditional |
| Set of Truths and Rules | Unquestioned body of useful knowledge | Body of structured pure knowledge | Process view: Personalized math |
| An unrelated collection of facts, rules, and skills. | Primarily an unrelated collection of facts, rules, and skills. | A static but unified body of knowledge with interconnecting structures | Primarily a static but unified body of knowledge |
| Fixed | Primarily fixed | Equal mix of fixed and dynamic | Involves problem solving |
| Predictable | Primarily predictable | Equal mix of predictable and surprising | Primarily surprising |

*Table continued*

*Table 21 continued*

| Nature of mathematics presented, as perceived by students | Nature of mathematics as presented by instructors | | Nature of mathematics as preferred by students |
|---|---|---|---|
| Traditional | Primarily traditional | Even mix of traditional and nontraditional | Primarily nontraditional |
| Absolute | Primarily absolute | Both absolute and relative | Primarily relative |
| Certain | Primarily certain | Both doubtful and certain | Primarily doubtful |
| Applicable | Primarily applicable | Both applicable and aesthetic | Primarily aesthetic |

Instructors, in their beliefs about the nature of mathematics, convey an ideology with the main features: an unquestioning acceptance of existing structures and models, an action oriented world-view, and the treatment of scientific and technological progress as the means to social development. Epistemologically, they present the view of pure knowledge representing an absolutist perspective to be accepted unquestioningly, mixed with the view of purity of value free logic and reasoning. The pure knowledge perspective is tempered with an appreciation for applied knowledge. Applied knowledge is multiplistic, and the choice between many equally valid methods depends on the skills and knowledge of the professional experts who decide on pragmatic grounds between different approaches. There is a tendency to see applied mathematics as inferior to pure mathematics, "the earthly shadow of an eternal, celestial body of truth." (Ernest, 1991) The set of moral values in this perspective consists partly of utility, expedience, pragmatism, self or group interests, and the impartial application of the rules of justice for all, without concern for individual human issues and concerns. Scientific and technological progress is valued because these elements serve social development.

The educational aims in the above perspective is a mix of the need to equip students with the knowledge and skills needed for employment, and to produce liberally educated individuals, while recognizing that the system is elitist and that only a few will manage to achieve these objectives.

However, the progressive educator's perspective, preferred by students, corresponds more with relating, nurturing, comforting and protecting. Instead of a rigid presentation of rules, structure, logic, objectivity, and static form, there is a concern with human relationships, a connectedness with the human dimensions of situations: creativity, feelings, subjectivity, expression, and dynamic growth. The educational goal is the self-development and personal fulfillment of each individual. The aims are purist because they concern the development of the students for development sake, as something of intrinsic value. The view of mathematics in this perspective is progressive absolutist: absolutism colored by humanistic, connected values. The public educator's perspective reflects a social constructivist perspective. Mathematics is fallible and corrigible, culture-bound and value-laden. The aim of mathematics education, according to this philosophy, is to empower individuals to be confident solvers and posers of mathematical problems embedded in social context. Mathematics education serves a democratic purpose. Instructors socially profess a set of values which differs from the set of values they profess to bring into the calculus classroom.

Regarding the calculus classroom culture there seems to be:

1. A difference between students' experienced beliefs about the nature of mathematics and students' preferred beliefs about the nature of mathematics.
2. A difference between instructors' beliefs about the nature of mathematics and students' beliefs about the nature of mathematics.

**Beliefs about the pedagogy of mathematics education.**

According to interview results from Table 13, most students (4 out of 5) have a primarily nontraditional perspective on the beliefs about teaching mathematics. John's primarily traditional perspective is the sole variant. The results of Table 13 are impressions from the students' interviews. Cue card results included in Table 13, show greater diversity in answers, although

the range of answers does include the interview impressions. Students seem to recognize, besides a more personal process of exploration, the value of primarily traditional product oriented skill acquisition.

Interview results for students' beliefs about learning mathematics show less consistency. Three out of 5 students associate with a nontraditional or primarily nontraditional perspective, while 2 out of 5 students associate with a primarily traditional perspective on learning mathematics. (see Table 15) Cue cards again show a great variety of choices per participant. Students seem to be willing to acknowledge the validity of different learning perspectives when presented with options. Yet when expressing personal perspectives in conversation their preferences become more focused and emphasize a singular approach.

Interview results show solidarity in students' perspectives on instructors' teaching practices. According to Table 14 all students perceive their instructors' teaching practices as traditional (3 out of 5) or primarily traditional (2 out of 5). Noreen, in addition to describing her instructor as traditional also depicts the teacher as having primarily nontraditional perspectives.

There seems to be a divide between students' results for teaching mathematics and students' results for instructors' perceived teaching practices. A comparison of the two perspectives is presented in Table 22.

*Table 22: A comparison between students' perspective on teaching mathematics and students' perception of their instructors' teaching practice*

| Perspective on teaching mathematics | Perspective on instructors' teaching practice |
|---|---|
| Non-intrusive guidance | Authoritarian role of teacher |
| Exploration, investigation, discovery, play, discussion and project work | Strict discipline, hard work, effort, self-discipline. |
| Teacher facilitates and guides | Teacher dispenses knowledge as a stream of facts |

*Table continued*

*Table22 continued*

| Perspective on teaching mathematics | Perspective on instructors' teaching practice |
|---|---|
| Little lecturing | Rigid lesson plans |
| Valuing of process more than product | Right answers are more important than the process |
| Valuing of understanding over memorization | Valuing of memorization over understanding |
| Learning through problem solving, project work | Learning through standard questions |
| Textbook just a resource | Instruction out of the textbook |
| Student directed discourse | Students engage in individual practice tasks |
| Environment encourages active learners | Environment encourages passive learners |
| Self confidence oriented | Mastery oriented |
| Self expression focus | Template focus |

Observations of students' beliefs about learning mathematics, Table 15, show greater variation. Students' interview results vary from primarily traditional to nontraditional.

At the classroom level we observe an environment in which students are faced with two inconsistencies: (1) the nature of mathematics as presented in comparison to the nature of mathematics as preferred and (2) the nature of instructors' teaching practice as compared to the nature of students' preferred teaching practice. Students face an intellectual perplexity, in addition to an instructional perplexity. Students have to reconcile what is taught and how it is taught with their own belief systems.

Interview results for instructors' beliefs about teaching mathematics, according to Table 16, show 2 out of 5 instructors described as having an even mix perspective, 2 out of 5 described as primarily traditional, and one described as primarily nontraditional. Instructors' beliefs about the teaching of mathematics can be presented as an even mix of traditional and nontraditional perspectives with some preference towards the primarily traditional perspective. Instructors' cue card responses reflect a similar profile, an even mix perspective with a tendency to lean towards the primarily traditional perspective.

From interview impressions with regard to the beliefs about the learning of mathematics instructors beliefs can be described as representing an even mix of traditional and nontraditional perspectives with a slight leaning towards the primarily nontraditional perspectives. Cue cards results from the instructors show a much larger variation with two instructors recognizing all modes of learning as important. Cue card results may present "theoretical" beliefs while interview results may better represent "practiced" beliefs.

Teachers' perspectives on their own teaching practice, obtained through interview analysis, reveal a practice solidly rooted in the primarily traditional perspective. (see Table 17) This analysis seems to support the students' perceptions about instructors' practices.

**Beliefs about the purpose of mathematics education and post-secondary education.**

Results reported in Table 19 and 20 present agreement between students and instructors on the purpose of mathematics education and post-secondary education. In terms of Ernest's (1991) educational ideology, the purpose reflects the perspectives of the technological pragmatist and the old humanist. Thus, instructors and students seem to agree on the importance of education for careers in science, technology, and industry. In addition, they value the knowledge and cultural tradition of a hierarchical, stratified society.

## Implications

This study has explored differences in beliefs about the nature of mathematics, the pedagogy of mathematics education and the purpose of mathematics education and post-secondary education. Having observed what seem to be significant differences in the beliefs about the nature of mathematics and the pedagogy of mathematics education, it raises the question as to whether these differences encourage the inquisitive mind or become an impediment to the learning of mathematics. In my deliberations in this section I will argue that

these differences may indeed form an obstacle to the learning of mathematics and contribute to mathematics avoidance at the post-secondary mathematics education level.

If mathematics is not culture free and not value neutral, as in Ernest's (1991) progressive educator's and public educator's ideology, then it raises the question of what mathematics education's role ought to be in society? Sfard (1996) comments that the Participation Metaphor has a potential to lead to a new, more democratic practice of learning and teaching mathematics but warns that "in the final account, it is up to those who translate ideas into practice ... whether the introduction of the new metaphor will, indeed, lead to a democratization of learning and to the improvement of learner's conditions." (p. 408) Learning mathematics in the participation metaphor is perceived as a process of becoming a member of the mathematical community. This entails, above all, the ability to communicate the language of the community and to act according to its particular norms. These metaphors have consequences for the practices of the participants in the calculus classroom. For students, the challenge is twofold: (1) to learn the right language for communication, and (2) to meet the criteria, norms, and standards of the practitioners in the field of mathematics.

Usiskin (1996) comments that learning dead, nonsense and abstract languages is more difficult than learning a native or a living second language. By presenting mathematics in the acquisition metaphor, instructors imply a language that is dead (lacking currency), nonsense (neither context nor currency) and abstract (no rationales for ideas). Students, operating in the acquisition metaphor, work with mathematics as a native language, or a living second language. The implications for communication are that students might not find any common ground in language for access to the content of mathematics. Where the student seeks debate and involvement, the student is confronted with the high-pitch sales discourse of the calculus instructor. (Gerofsky, 1996)

Dall'Alba (1993) stresses the importance for instructors, to focus on the engagement of students in the field of practice in higher education courses, in addition to having expectations that students acquire specific skills and knowledge. Course content and meaning as a professional practitioner must be developed concurrently. For students to become members of the mathematical community, students need engagement and involvement with this community. Learning mathematics involves a qualitative congruence between the learners' values and values of the institute of learning as reflected in the classroom. While students seek engagement and involvement through the participation metaphor, students find instead an intellectual life that is focused on the acquisition of skills and techniques. The purpose of the first-year calculus course is not directed towards engaging students in matters of intellectual substance related to the professional practice of mathematics. Not only do students perceive a communication conflict in their attempt at access to the community of learning, students also perceive a conflict in their attempt at access to the community of practitioners through a teaching and learning process which does not value involvement and engagement. Thus, students stand twice rebuffed. Students' intent and commitment to learn are perceived as not being met at the classroom level by the instructors. The primary source for students to commit to the subject matter and their own learning is perceived as missing. An implication of this is that student retention is in jeopardy. Not only do students withdraw from calculus courses, but students will also feel shunned by the post-secondary mathematics community. Math avoidance is not only possible but also plausible.

## Suggestions

The suggestions in this section are centered around the hypothesis that tensions between the beliefs of the participants in the classroom activities are natural and should be embraced in the educational endeavor. In my deliberations I will not address so-called external, structural or organizational "solutions", such as streaming, class-size, faculty-student ratios, resources, etc.,

but will focus on the situational characteristics of the classroom culture and the role of communication.

In his "Social Constructivism as a Philosophy of Mathematics" Ernest (1998) proposes a philosophical grounding of *Public Educator Ideology* and breaks with the traditional neutral free vision of mathematics. Traditionally, mathematics is regarded as value neutral, and traditionally, mathematicians have carried that message in their lecturing. Ernest writes that "The argument for accepting that conversation has a special role to play in epistemology is that language and discourse play an essential role in the genesis, acquisition, communication, formulation, and justification of mathematical knowledge. ... Without conversation and its feedback mechanisms, the individual appropriation of collective knowledge cannot be conducted or validated. ... to reduce conversation to a string of sentences is to sacrifice the structure within speech, essential for proofs, as well as its intrinsically social and interactive aspect." (pp.166-167) Accepting mathematics as a social activity makes all mathematical activity open to a diversity of perspectives. Participants in teaching and learning should not only develop an awareness of the diversity of perspectives, but should consider the exploration of these diversities and the tensions between them as essential in their educational endeavors. The development of a social constructivist theory of mathematics anxiety, in conjunction with further research into mathematics anxiety at the post-secondary level, would benefit this educational endeavor in a pro-active manner. This research in the diversity and tensions could be a vehicle for students and faculty engagement in process oriented mathematics.

In the education process, there is a need to focus more on the formation and development of particularly communicative relations devoted to inquiry and understanding than on specific predetermined learning outcomes. Arguing from a perspective that difference implies sameness, Burbulus and Rice (1991) suggest a framework in which difference and sameness are in constant interaction with one and another. A central feature of this focus on communicative relations is to

regard differences as providing educational opportunities, not as intimidating barriers. Attempting dialogue across differences and persisting in the attempts even when it becomes difficult, not only allows us to broaden our horizons as travelers in world of diverse beliefs, values, and actions, but also to develop communicative virtues such as tolerance, patience and willingness to listen.

The authors suggest that the nature of these dispositions is that they are acquired in relation to communicative partners and improved by practice. The issue of conversational practices, situated inside the practice of education, raises the issue of practice in general. To develop an awareness of the personal and mathematical culture of the participants, classroom practice will have to involve a research into these domains. Faculty's natural preference for research positions the instructors favorably to incorporate in their teaching a research orientation. The challenge will be for institutional organizations such as mathematics departments, departments of education, and offices of instructional development to foster this educational research orientation among faculty.

In summary, I identify: (1) within the mathematics education research community, the benefit of developing a social constructivist theory of mathematics anxiety, (2) within the professional practice of post-secondary mathematics education, the benefit of developing an awareness of the role of communication, and (3) within post-secondary instructional institutions, a benefit of nurturing research among instructors into their individual teaching practices.

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## Appendix A

Table A1: The structured interview questions—faculty

| *History of instructor: society* |
| --- |
| Who is your role model in society and why? |
| Who is your favorite politician, alive or dead? Why? |
| What would you say is your political ideology? |
| What is your ideal model of society? |
| Use three words to describe present day society. Elaborate. Describe present day society. |
| What would you like to see changed in present day society? |
| What would your definition, description be of a community, or concept of community? |
| Would you describe yourself as an individualistically centered or socially centered person? |
| Would you consider the department to be a community? Elaborate! |
| What comes to mind if you hear the word community of learners? |
| Where do you think one finds community of learners? |
| Do you know the mission statement of your university? |
| Do you feel kinship to the university? |
| Are you active in communities? What kinds? And why? |
| What do you do for recreation? |
| What do you read? Name three favorite books. Elaborate why they are your favorite books. |
| What is your view on technology? |
| What is your perspective on technological progress? |
| What do you think about consumerism? |

*(table continues)*

Table A1. *(continued)*

| *Perceptions about students* |
|---|
| Use three words to describe the present young generation. |
| Please elaborate on your choice. |
| Are first year students well equipped for their task? |
| How responsible are students? |
| How well prepared are students? |
| Describe first year mathematics students in general. |
| Describe with three key words a good mathematics student. |
| Describe your favorite student. |
| What would you tell your students about studying math, about a career in math? |
| What would you tell your students about the role of mathematics in society? |
| What would you like to see changed in the make-up of first year students? |
| Do you feel a social obligation toward your students? |
| Describe what you see as an appropriate model of interaction between you and first-year students in your courses. |
| *History of instructor: university* |
| Use three words to describe colleagues in the humanities. Elaborate. |
| Use three words to describe colleagues in science. Elaborate. |
| Use three words to describe yourself as an academic. Elaborate. |
| What is your perspective on the discipline of mathematics ? (cue-cards will be used for faculty to choose from, see appendix C) |
| Use three words to describe your most revered mentor. |
| Describe his/her teaching style |
| What did you learn from him/her? |
| Use three words to describe the essentials of a good mathematician. Elaborate. |

*(table continues)*

Table A1 *(continued)*

| *History of instructor: eductational* |
|---|
| Where did you receive your academic training? |
| How long ago was this approximately? (50's, 60's, 70's, etc.) |
| Are there any specific social consequences with regard to your research or teaching in particular. |
| *Perceptions about mathematics* |
| What kind of research do you engage in? |
| What kind of applications are there for this research? |
| Are there any humanitarian aspects to your research? |
| Is there any social consequence from your research? |
| How important is research for/in your career? |
| How much professional effort (percentage) goes towards research? |
| *Experiences about the calculus course* |
| Use three words to describe your teaching style. |
| How would you characterize your teaching style? (Subject will be given cue cards to choose from. See Cue Cards Appendix D) |
| How would you describe your moral values? |
| What are the aims of mathematical education? |
| What learning theories do you value? |
| What teaching methods do you practice? |
| What is the purpose of post-secondary education? |
| Is mathematics important for social reform? |
| What would you like to see changed in present day teaching practices? |
| Are you satisfied with your teaching situation? |
| Can you identify the values that you hold as a teaching professional? |

*(table continues)*

Table A1. *(continued)*

| *Experiences about the calculus course* |
| --- |
| Are these values at risk or are they an integral part of your teaching practice? |
| Imagine what you might like the situation to be so that it is in keeping with your values? |
| Being a mathematician, what is it like to be? |
| Being a faculty member, what is it like to be? |
| To what extend are you working the way you wish? |
| What do you need to change to improve the situation? |

## Appendix B

**Cue Cards used to explore educational ideologies:**

| **I would characterize the political values I represent as:** | | | | | |
|---|---|---|---|---|---|
| **Political Ideology** | Radical right, 'New Right' | Meritocratic, conservative | conservative/liberal | liberal | Democratic socialist |

| **I present mathematics in my teaching as:** | | | | | |
|---|---|---|---|---|---|
| **View of Mathematics** | Set of Truths, and Rules | Unquestioned body of useful knowledge | Body of structured pure knowledge | Process view: Personalized maths | Social constructivism |

| **In terms of educational ideologies I would characterize the moral values I espouse to be:** | | | | | |
|---|---|---|---|---|---|
| **Moral Values** | Authoritarian 'Victorian' values, Choice, Effort, Self-help, Work, Moral Weakness, Us-good, Them-bad | Utilitarian, Pragmatism, Expediency, 'Wealth creation', Technological development | 'Blind' Justice, Objectivity, Rule-centred Structure, Hierarchy, Paternalistic 'Classical' view | Person-centred, Caring, Empathy, Human values, Nurturing, Maternalistic, 'Romantic' view | Social Justice, Liberty, Equality, Fraternity, Social awareness, Engagement and Citizenship |

| **My educational views are formatted by the following perspective on society:** | | | | | |
|---|---|---|---|---|---|
| **Theory of Society** | Rigid Hierarchy, Market-place | Meritocratic Hierarchy | Elitist, Class stratified | Soft Hierarchy, Welfare state | Inequitable hierarchy needing reform |

| In teaching I work with a model of the learner, analogue to the following theory of development: | | | | | |
|---|---|---|---|---|---|
| **Theory of the Child** | Elementary School Tradition: Child 'fallen angel' and 'empty vessel' | Child 'empty vessel' and 'blunt tool' Future worker or manager | Dilute Elementary School view, Character building, Culture tames | Child-centred, Progressive view, Child: 'growing flower' and 'innocent savage' | Social Conditions view: 'clay moulded by environment' and 'sleeping giant' |

| My perceptions of a student's ability are: | | | | | |
|---|---|---|---|---|---|
| **Theory of Ability** | Fixed and inherited Realized by effort | Inherited ability | Inherited cast of mind | Varies, but needs cherishing | Cultural product: Not fixed |

| The mathematical aims in my teaching are: | | | | | |
|---|---|---|---|---|---|
| **Mathematical aims** | 'Back-to-Basics': numeracy and social training in obedience | Useful maths to appropriate level and Certification (industry-centred) | Transmit body of mathematical knowledge (Maths-centred) | Creativity, Self-realization through mathematics (Student-centred) | Critical awareness and democratic citizenship via mathematics |

| My theory of learning is closely matched by: | | | | | |
|---|---|---|---|---|---|
| **Theory of Learning** | Hard work, effort, practice, rote | Skill acquisition, practical experience | Understanding and application | Activity, Play, Exploration | Questioning, Decision making, Negotiation |

| My theory of teaching mathematics can be summarized by: | | | | | |
|---|---|---|---|---|---|
| **Theory of Teaching Mathematics** | Authoritarian Transmission, Drill, no 'frills' | Skill instructor, Motivate through work-relevance | Explain, Motivate, Pass on structure | Facilitate personal exploration, Prevent failure | Discussion, Conflict Questioning of content and pedagogy |

| My perspective on the role of resources can be described as: | | | | | |
|---|---|---|---|---|---|
| **Theory of Resources** | Chalk and Talk Only, Anti-calculator | Hands-on and Micro computers | Visual aids to motivate | Rich environment to explore | Socially relevant, Authentic |

| Evaluation and assessment should be guided by: | | | | | |
|---|---|---|---|---|---|
| **Theory of Assessment in Maths** | External testing of simple basics | Avoid cheating, External tests and certification, Skill profiling | External examinations based on hierarchy | Teacher led internal assessment, Avoid failure | Various modes. Use of social issues and content |

| My perceptions of the social role of mathematical education can be characterized by the following : | | | | | |
|---|---|---|---|---|---|
| **Theory of Social Diversity** | Differentiated schooling by Class Crypto-racist, Monoculturist | Vary curriculum by future occupations | Vary curriculum by ability only (maths neutral) | Humanize neutral maths for all: Use local culture | Accommodation of social and cultural diversity a necessity. |

## Appendix C

Table C1 lists the criteria used to establish the categories of beliefs about mathematics learning for teachers. A similar table can be used to analyze student responses.

Table C1
*Criteria for the Categorization of Teachers Beliefs About learning Mathematics.*

Traditional

Students passively receive knowledge from the teacher.
Students learn mathematics by working individually.
Students engage in repeated practice for mastery of skills.
There is only one way to learn mathematics.
Memorization and mastery of algorithms signify learning.
Students learn mathematics solely from the textbook and worksheets.
Many students are just not able to learn mathematics.
Students' learning of mathematics depends solely on the teacher.

Primarily traditional

Students primarily engage in practice for mastery and skills.
Memorization and mastery of algorithms provide primary evidence of learning.
The teacher is more responsible for learning than the student.
Mathematics is learned primarily from the textbook and worksheets.
Students work individually except perhaps to work on homework.
Students are primarily passive learners, raising questions on occasion.

Even mix of traditional and nontraditional

Students should learn mathematics through both problem solving and textbook work
Students should both understand and master skills and algorithms.
Students should do equal amount of individual and group work.
Most students can learn mathematics.
There is more than one way to learn mathematics.
Learning mathematics is equally the responsibility of students and teachers.
Trying hard is likely to aid mathematics learning as is being naturally good.
Repeated practice is as likely to help in the learning of mathematics as is having insights as a result of explorations.

Primarily nontraditional

Students primarily learn mathematics through problem-solving tasks.
Students primarily learn mathematics from working with other students.
Learning is evidenced more through ability to explain understanding than through expert memorization and performance of algorithms.
Students are more responsible for their own learning than the teacher.
Students learn mathematics primarily as active learners.

Nontraditional

The students' role is that of an autonomous explorer.
Students learn mathematics only through problem-solving activities.
Students learn mathematics without textbook or paper-and-pencil activities.
Students learn mathematics through cooperative group interactions.
Students are active mathematics learners.
All students can learn mathematics.
Each student learns mathematics in his or her own way.

*(table continued)*

Note: From "Inconsistency Between a Teacher's Beliefs and Practice" by A.M. Raymond, 1997, *Journal for Research in Mathematics Education, 28,* (5), 550-576.

Criteria to analyze instructors' beliefs about teaching mathematics are presented in Table C2. Student responses can be explored in a similar fashion.

Table C2
*Criteria for the Categorization of Teachers Beliefs About teaching Mathematics.*

Traditional

The teacher's role is to lecture and to dispense mathematical knowledge.
The teacher's role is to assign individual seatwork.
The teacher seeks "right answers" and is not concerned with explanation.
The teacher approaches mathematical topics individually, a day at a time.
The teacher emphasizes mastery and memorization of skills and facts.
The teacher instructs solely from the textbook.
Lessons are planned and implemented explicitly without deviation.
The teacher assesses students solely through standard quizzes and exams.
Lessons and activities follow the same pattern daily.

Primarily traditional

The teacher primarily dispenses knowledge.
The teacher primarily values right answers over process.
The teacher emphasizes memorization over understanding.
The teacher primarily (but not exclusively) teaches from the textbook.
The teacher includes a limited number of opportunities for problem solving.

Even mix of traditional and nontraditional

The teacher includes a variety of mathematical tasks in lessons.
The teacher equally values product and process.
The teacher equally emphasizes memorization and understanding.
The teacher spends equal time as a dispenser of knowledge and as a facilitator.
Lesson plans are followed explicitly at times and flexibly at others.
The teacher has students work in groups and individually in equal amounts.
The teacher uses textbook and problem solving activities equally.
The teacher helps students both enjoy mathematics and see it as useful.

Primarily nontraditional

The teacher primarily facilitates and guides, with little lecturing.
The teacher values process somewhat more than product.
The teacher emphasizes understanding over memorization.
The teacher makes problem solving an integral part of class.
The teacher uses the textbook in a limited way.

Nontraditional

The teacher's role is to guide learning and pose challenging questions.
The teacher's role is to promote knowledge sharing.
The teacher clearly values process over product.
The teacher does not follow the textbook when teaching.
The teacher provides only problem-solving, manipulative-driven activities.
The teacher does not plan explicit, inflexible lessons.
The teacher has students work in cooperative groups all the times.
The teacher promotes students' autonomy.
The teacher helps students to like and value mathematics.

Note: From "Inconsistency Between a Teacher's Beliefs and Practice" by A.M. Raymond, 1997, *Journal for Research in Mathematics Education, 28,* (5), 550-576.

Criteria to analyze instructors' teaching practices are presented in Table C3.

Table C3
*Criteria for the Categorization of Teachers' Mathematics Teaching Practices.*

Traditional

The teacher instructs solely from the textbook.
The teacher follows lesson plans rigidly.
The teacher approaches mathematics topics in isolation.
The teacher approaches mathematics instruction in the same pattern daily.
The teacher has students engage only in individual paper-and-pencil tasks.
The teacher creates an environment in which students are passive learners.
The teacher poses questions in search of specific, predetermined responses.
The teacher allows no student-to-student interactions.
The teacher evaluates students solely via exams seeking "right answers."

Primarily traditional

The teacher instructs primarily from the textbook with occasional diversions from the text.
The teacher creates an environment in which students are passive learners, occasionally calling on them to play a more active role.
The teacher primarily evaluates students through standardized quizzes and exams, occasionally using other means.
The teacher primarily encourages teacher-directed discourse, only occasionally allowing for student-directed interactions.

Even mix of traditional and nontraditional

The teacher teaches equally from textbook and problem-solving activities.
The teacher creates a learning environment that at times allows students to be passive learners and at times active explorers.
The teacher evaluates students' learning equally through standard quizzes and exams and alternative means, such as observations and writing.
The teacher encourages teacher-directed and student-directed discourse.

Primarily nontraditional

The teacher primarily engages students in problem-solving tasks.
The teacher primarily presents an environment in which students are to be active learners, occasionally having them play a more passive role.
The teacher primarily evaluates students using means beyond standard exams.
The teacher encourages mostly student-directed discourse.

Nontraditional

The teacher solely provides problem-solving tasks.
The teacher selects tasks based on students' interests and experiences.
The teacher selects tasks that stimulate students to make connections.
The teacher selects tasks that promote communication about mathematics.
The teacher creates an environment that reflects respect for students' ideas and structures the time necessary to grapple with ideas and problems.
The teacher poses questions that engage and challenge students' thinking.
The teacher has students clarify and justify their ideas orally and in writing.
The teacher has students work cooperatively, encouraging communication.
The teacher observes and listens to students to assess learning.

Note: From "Inconsistency Between a Teacher's Beliefs and Practice" by A.M. Raymond, 1997, *Journal for Research in Mathematics Education, 28,* (5), 550-576.