MATHEMATICAL LEARNING AND CHILDREN'S LITERATURE

by

DONNA JENNER

B.A. (Specialized Honours), York University, 1993

B.Ed., York University, 1993

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ARTS

in

THE FACULTY OF GRADUATE STUDIES

Department of Curriculum Studies

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

October 1999

ABSTRACT

This study investigated one teacher's use of children's literature in her mathematics program during the first term of Grade Two. The study was guided by two questions: What is it about the shared reading situation that creates a fertile context that helps individual children make mathematical connections? How does quality children's literature capture their imaginations in a way that enhances the possibility for them to think mathematically in deeper, more reflective ways?

A qualitative research methodology was employed in this investigation. The collected data consisted of videotapes and a handwritten, personal journal. The videotapes documented classroom literature sessions and the journal recorded the teacher's observations, thought, and conjectures. Field notes and photographs were also kept to accurately document and describe students' work on chalkboards.

The teacher videotaped the reading and discussion of many different books with her Grade To class, but this thesis focuses on a detailed analysis of just one book, Selina and the Bear Paw Quilt by Barbara Smucker, within the context of the previous readings. The shared reading context and the book provoked he students to engage in mathematical discussion as well as stimulating deep, reflective mathematical thinking.

Using an interpretive inquiry approach, initiated by one student's mathematical comment on an illustration in this book, the thesis seeks to uncover the ways in which children's literature promotes the growth of mathematical thinking and conceptual understanding in young children. The power, of well-written and illustrated literature for children, to enhance mathematical understanding, has been a long-held personal conviction of this teacher.

TABLE OF CONTENTS

LIST OF FIGURES

ACKNOWLEDGMENTS

I am grateful to my supervisor, Dr. Susan Pirie for her continual enthusiasm for this project and for helping me to step out of my comfort zone to take some risks in writing this Story. To Dr. Ann Anderson, thank you for your willingness to listen and encourage me along each step of the way. Thank you, Marlene Asselin for your feedback at the proposal stage and for offering your help if I needed assistance.

I thank the parents and students for their willingness to participate in this project. In addition, I would like to thank my school administrator Rhoda Lank for her continual interest and encouragement. I would also like to offer grateful thanks to all my friends who have asked continually about this work, given me feedback, and provided prayer support.

To my husband, Jim, thank you for always believing that I could finish this work and for your constant love and support along the way. To my daughter, Chris and Candace, this wouldn't be done without all the ways you have helped out. To my family, this thesis is dedicated to you.

The Seed

When I began to think about children's literature and the kind of mathematical learning that comes out of using it with children, I deliberated for a time about whether it was appropriate to center research on something I treasured. How could I preserve the beauty of a story and the story experience while developing my thesis? What would be appropriate ways to collect data? What, in fact, would be appropriate data?

Children's literature and mathematical thinking and learning are the two things that I cherish about teaching. So it seemed natural to bring together my two loves as my research focus. I have a background in quantitative research and, initially, thought this would be the research model I would use to examine my topic. Yet, the more I thought about the richness of literature and the shared reading context, the less willing I became to subject these experiences to the restrictive rigour of clinical number crunching. How could numbers tell the 'whole story'? I began to contemplate how I would conduct qualitative research and the form this type of research would take. Around this same time, I was introduced to the narrative form of writing in a graduate course. I was struck by how the writing captured all the intricate layers of the research experiences and I determined that I, too, must find a form of writing that would fully capture the reading experience. More than anything, my desire is to preserve The Story – the story about my students' experiences with books, linked to the story about their personal mathematical learning and thinking and, as an aside, the story of my own professional growth.

In essence, what follows is only part of the story of a much larger Story that involves my students and myself as we proceed on our journey together through learning. But it is also a story about our own personal journeys. The children's journeys center on the growth of their mathematical understanding and thinking. My journey is centered on the playing out and solidifying of my beliefs about children's literature and what happens to children's learning and understanding of mathematics in the midst of experiencing and talking about literary tales.

This is a journey of growth for my students and myself. The data collection method[1] and analysis[2] allows for both to be revealed. By examining and offering a multi-layered interpretation of one shared reading episode, I attempt to conduct an interpretive inquiry[3] to better understand my own teaching and the growth of my students' mathematical understanding. As in all interpretive work, my reading of the episode is but "one voice among many".[4]

For me, this is a conscious attempt to change my practice. Although this is documented in my journal, it is only relevant in this report as it impinges on the main thesis of the effect of shared reading of literature on children's growth of mathematical understanding.

What follows is not a typical thesis format. To preserve the integrity of a story, I have attempted to write a narrative – the story of an unfolding story. Initially, I tried to write a separate review of the literature chapter but it did not blend with the rest of my writing. The chapter appeared to stand alone as an add-in. Separate sections supporting my methodology and methods of analysis also interrupted the flow of the learning tale in much the same way as a 'commercial break' interrupts the flow of a television drama.

The story is a story of growth and, staying with this metaphor has enabled me to fashion a structure for the thesis that remains true to my intent to fertilize children's mathematical understanding through working with literature. The background of my own broad reading, the earlier work and thinking of other researchers, even my formal and informal research 'methods', together form the soil in which understanding – the children's and mine – has grown.

The research methods used and the review of related and background research, therefore, have been woven into the story through references and endnotes. The analysis of the data is set up in a fashion that I hope will make its reading more understandable to the reader: portions of video transcripts are interwoven sequentially with my interpretation of the children's comments and actions, sections that address the children's mathematizing, supporting research, and comments about past experiences with children, as well as, from time to time, reflections about my own practice and professional growth. The innermost story – that of the classroom events – can be glimpsed by skipping all but the portions of the text in italics. This multiple narrative documents one chapter in the learning life of myself and my students: one shared reading experience of one book out of a whole year.

How then to read this thesis? The choice lies with the reader, but I offer three suggestions:

1. Read only the italics for the story about the reading and sharing of Selina and the Bear Paw Quilt.
2. Read the italics and the text, for the story I tell of mathematical understanding.
3. Read the italics, text, and endnotes if you want to follow up on the background and justification through the literature for what I am saying.

Germination of an Idea

Everyone settles on the carpet in the story area while I take my place on the storyteller's chair. My favourite time of the school day is about to begin. Expectant faces eagerly wait to see what today's selection will be – which picture book will we explore together. I love children's books and sharing the adventure with my students is a genuine pleasure. This is a time of the day when we all get to take a break from the hustle and bustle of classroom life to merely relax, settle comfortably on the carpet, and be transported beyond the immediate to new realms and ideas. When the book cover is revealed, we all draw closer together, tightening the circle and forming an intimate gathering. I sense the connectedness between the children and myself, the rising comfort level as some youngsters move closer to peer over my arm at the page. There is something about the coming together on the carpet to hear a story that charges the group with expectancy. Beginning to read, I hear the children's expressions of enjoyment, delight, and willingness to explore themes and ideas that emanate from the story. By the end of the story, we are all refreshed from our literary journey just as one returns from a vacation, renewed and invigorated by one's trip to a faraway place.

Two years ago, I introduced children's books into my mathematics programme. In some ways, it was a little selfish. This would give me a way to blend my two greatest loves in teaching: children's books and teaching mathematics. Beyond my own intrinsic reasons, it struck me that books were hooking my students into a learning mode in a powerful way. At story time, the children rarely needed reminders to focus their attention and listen when viewing and listening to an excellent picture book. A well-chosen book acts like a magnet, drawing them in closer to me and to each other. Like a well-defined magnetic field, the group clusters tightly around the storyteller's chair. Two years ago, I was already using books to launch us into exploring science and theme topics. I was comfortable with reading books to my class and engaging them in discussions about ideas and information that emerged from specific books. But I was in charge of the discussion and my discomfort rose if the talk strayed too far from my purpose for reading the book. That was wasting our valuable class time! How would I maintain control of the discussion when we talked about mathematical ideas surfacing in books? Would I understand what a student was trying to explain to me? Was my own mathematical knowledge broad and deep enough to grasp their images for different mathematical concepts?

I started attending any workshop that offered ideas about using children's literature in mathematics. Bit by bit, I devised a starting point for myself. I would choose a book that I believed dealt with a certain mathematical concept, then would read and carefully pose mentally scripted questions to the children. Following this format, I would be sure that the children got out of the story what I intended. A course director once told me that it was impossible to plan the perfect lesson because children won't allow it to be perfect. The first shared reading session confirmed this prediction. To my great consternation, the children continually went off on tangents rather than sticking with responding to my 'thought-provoking' questions. I was so busy trying to redirect the discussion and focus them on specific mathematical ideas that the whole session became frustrating.

As part of an assignment for a graduate course I was taking at UBC, I decided to videotape a shared reading session during a mathematics lesson. Sure enough, there I was frantically trying to direct the flow of talk like a traffic officer during rush hour traffic. The children rushed on blurting out their ideas as something surfaced for them, at times ignoring my posed questions. What emerged for me after viewing this tape was how strongly my students' thinking was being primed by the story and how my directive stance and interjections were shutting down the group discussion.[5] Could I loosen up my hold on the discussion and allow the children to lead the discussion?

It was at this juncture that I began paying closer attention to the learning objectives in the Mathematics I.R.P. (Integrated Resource Package).[6] I was also becoming more familiar with the NCTM Standards.[7] Both these documents emphasize the importance of children learning to "communicate mathematically" and to "communicate their mathematical ideas" to others.[8] It suddenly dawned on me that children talking about mathematics and their own understanding of concepts were critical objectives in these documents. Not only was it alright for children to 'think out loud' and give voice to their thinking but it should be encouraged. What a relief, now I could relax a bit and let children take the lead in talking about mathematical themes in stories. The ownership of our talk surrounding stories should belong to the children. The spotlight should be on specific children and not on me. I, the teacher, should fade into the background to become the object they could bounce ideas off.

It became a little easier to give up control of the discussion but I still needed to learn how to actively listen to my students' mathematics. In my rush to be reflective and helpful, I tended to speak too soon, cutting children off in mid-sentence and in mid-thought. Only brave students would say "no, that's not it" when I was off base in reflecting back what I believed they were thinking. More often, children would merely nod their heads in reluctant consent to my erroneous assumptions. They gave up trying to make me hear and understand their ideas. If I did realize that I was off-track, it was often too late, further thinking was no longer possible. I had turned off the power switch and nothing could regenerate that particular train of thought for that child. I needed to become quieter, more patient and willing to allow children's ideas to flow and ebb. Mathematical talk is hard but given sufficient time to pursue the twists and turns in their thinking, they would be able to tell me about their ideas with greater clarity and insight.

And now, three years later, I find myself entering into story time, trusting that most children desire to learn and share new ideas. I see my students as capable of talking about their thinking and about what they know and understand with a minimum of help from me. My students and I have much to teach each other when we are willing and open to learn. Something almost magical occurs when we read and discuss stories together. That 'something' is what I would like to explore in my research. What is it about the shared reading situation that creates a fertile context that helps individual children make mathematical connections? How does quality children's literature capture their imaginations in a way that enhances the possibility for them to think mathematically in deeper, more reflective ways?

Growth

The two video cameras are in position on either side of our classroom story corner. One is focused on me and the book, while the other is on the group of children clustered around my chair. Four previous shared reading sessions have been taped so the children are familiar with the equipment and set-up and are comfortable (and excited) about being videotaped.

During the previous mathematics literature experiences, I had shared books where mathematical concepts were either explicit or were used in obvious ways by the characters. For example, the story theme in 10 For Dinner[9] centered on the experiences of 10 guests at a birthday party. In The Greedy Triangle the children were encouraged to identify various two-dimensional shapes in the illustrations of different objects and buildings. Counting by five's and number patterns was the main focus in Arctic Fives Arrive while the total amount of money the peddler would make from selling all his caps was the central discussion point in Caps for Sale. After sharing all these books, the children invariably commented that these were great books because they "make you learn about math". With the exception of The Greedy Triangle, these books were identified as mathematical books since each text incorporated the use of number or arithmetic into the story. But arithmetic is only one small section in the broader spectrum of mathematics.

Today I have chosen Selina and the Bear Paw Quilt for several reasons. I wonder what my students' response will be to a book where the mathematics is not obvious in the story. The mathematics in Selina and the Bear Paw Quilt is not explicit and I wonder if the children will recognize the book as a "math book" as they had previously.[10] (In fact, after sharing this text with the children, not one child comments that this is a mathematical book. Yet, for close to sixty minutes, they think in deep mathematical ways).

The format of this book is quite unusual. Each page features a specially designed quilt pattern that is used simply to frame the illustration that accompanies the text. This is a strong historical fiction book that features a young Mennonite girl and her family's experiences during the Civil War and their subsequent journey to religious freedom in Upper Canada. The exquisitely detailed pictures are closely tied to and support the accompanying text. The author's use of rich language and descriptive writing along with engaging illustrations marks this book as quality literature.[11] The book makes implicit use of pattern in both the quilt designs and the various pieces of cloth. But the theme of pattern is extended to the story theme as well. Quilt patterns are an integral part of the Mennonite family's lives. They keep memories of the old alive in their new life and weave a continuous link between the past and the present.

There are fifteen different quilt designs featured in Selina and the Bear Paw Quilt. The patterns in each quilt design are formed from tessellating geometric shapes. It is important to note that what is provoked by the story is not what I expected and so I was not totally prepared for their ideas. I thought the children might focus on patterns but instead the children's comments center around ideas about transformational geometry.

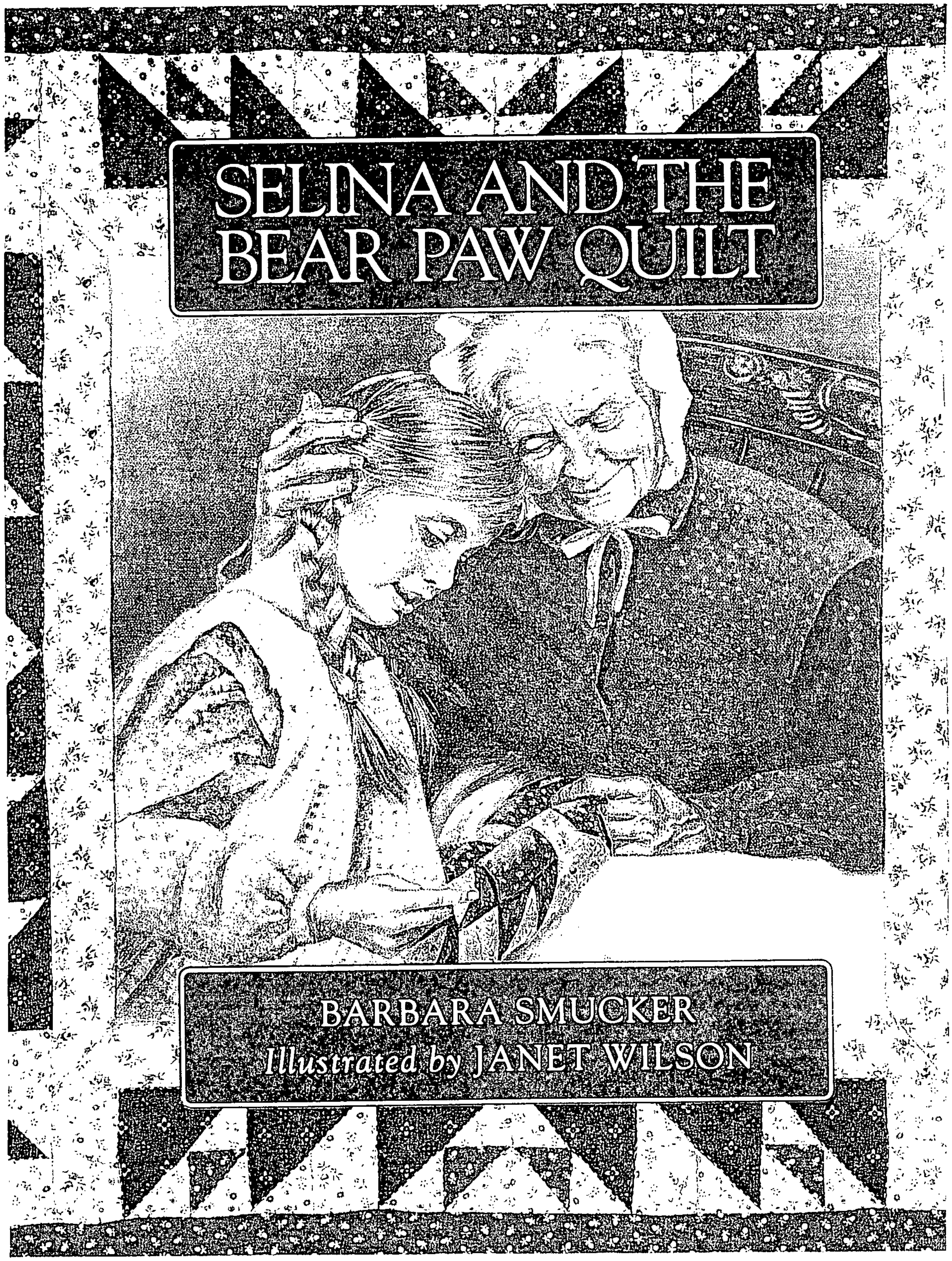


Figure 1: Front Cover of Book

Our journey through intricately detailed quilt patterns and the accompanying story begins with a discussion about the cover of the book.

Discussing the cover illustration and title of a book is a pre-reading strategy that I frequently use with my Grade Two students to set the stage for a story, to help them enter a text with greater appreciation and understanding.

My question, "what do you see?" prompts the children to describe colours and to speculate about the story theme.

In other shared reading sessions, noticing and describing colour patterns is frequently the children's initial reactions to a story's illustrations.

As the pages turn and more quilt patterns are revealed, the discussion shifts to noticing and describing the fabric pieces as "flowery" and as squares and diamonds, other attributes beside colour.

In discussing other books, I have noticed that my students often respond superficially to illustrations, noting surface details such as colour patterns before going deeper to focus on specific details and mathematical ideas.

In this book, the quilt designs that frame the pictures are not specific to the story. The patterns in the borders are not referred to in the text but the significance of each piece of cloth is illuminated in the story, linking the cloth to a special memory or person. Nonetheless, I notice that the children focus on the borders rather than on the pictures within the quilt frame. I have no idea whether this is the author's intention.

This is a departure from the other four books mentioned above. For example, in Arctic Fives Arrive, groupings of animals figure prominently on each page, displaying and reinforcing the skill of counting by five's. As well, in Caps For Sale, the focus of the story is to count the monkeys and the caps and the illustrations encourage the children to do so.

The central characters of the classroom story I am telling here will be Kevin and Jacob, two boys in the first term of Grade Two. I have chosen to focus on Kevin and Jacob as a way of using the micro-view to inform the macro-view. I shall try to elucidate how their mathematical thinking is revealed and provoked through the reading of this one particular book. By focusing on Kevin and Jacob, I try to understand the ways that children's literature impacts the mathematical learning and understanding of children in a more general sense. Other students and other books help to give the story substance and provide a context to my intent to reveal the hidden power of literature in mathematical learning by both student and teacher.[12] The central idea of language will be revisited again and again.

We move through the first seven pages of the Mennonite tale. Within these pages, we are introduced to Selina, a young Mennonite girl, and a simple way of life filled with family events centered on activities such as quilting. Our journey has begun with superficial reactions to the story but now the students lead the way, unprompted by me, as they begin to see mathematics in this non-mathematical book. Delving deeper into the book the children begin to focus on certain shapes that make up the quilt pattern. Talking about the squares and triangles that make up this quilt leads the children into a discussion of personal images of shapes and how each shape might move in space.

Central to the dialogue, is how Kevin and Jacob manipulate the shapes mentally, and talk about the shapes' motions and how each movement will affect the resulting shape and pattern. It is clear that the illustrations and discourse bring specific ideas to the surface for other children and sharpen their focus on certain geometric concepts. Their growth, however, is not the focus for this thesis, though as a teacher I must remain alive to everyone's learning.

In the following document, I have added copies of book pages and my own drawings to illustrate the picture that the children are referring to and the manipulations suggested by the students' hand and body movements.[13] Such physical actions play an important role in helping my students express their thinking. Burgoon's (1994) work substantiates the importance of nonverbal signals such as these during interpersonal communications. Nonverbal actions play a significant and often, dominant role in communicating meaning in person-to-person interchanges. When words fail them, my students will often perform physical acts such as making hand and body gestures or manipulating objects to make themselves understood. In this session this was particularly evident.[14]

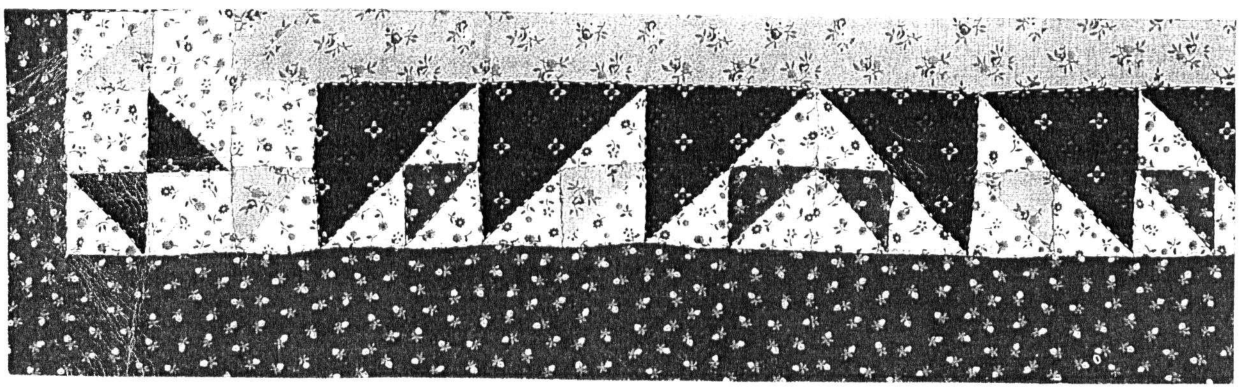
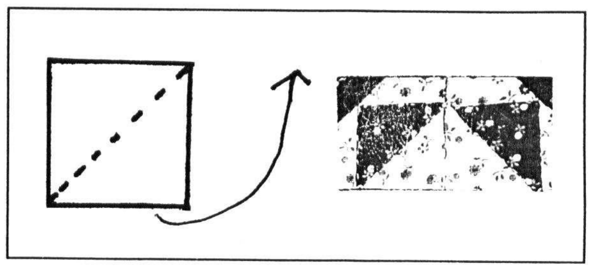


Figure 2: Portion of the Illustration on page 7

Kevin: I think they cutted the square so it's

like that.

(pointing to the two red triangles)



The quilt is a fixed, sewn together pattern. Yet, Kevin sees the quilt as being made up of moveable shapes – as of course it originally was. This quilt notion causes Kevin to begin thinking about how shapes can be created cut up. The intricate quilt patterns provoke geometrical thinking and for Kevin, the triangles are now held in place by stitches but were previously turned so that different and new shapes were formed. In this case, he sees the triangles resulting from cutting up a square.

Previously, when I shared The Greedy Triangle with my students, Kevin repeatedly noted how each square or diamond in the pictures could be cut diagonally or "in half" to form two triangles. When Kevin begins to talk about triangles and squares, he is entering into the discussion from his own entry point. Anderson and Anderson[15] highlighted the highly personal nature of children's literature while working with their daughter, Terri. They conclude that:

> Because the child initiates the mathematical event, it is usually at an appropriate level of difficulty since the child tends to deal with ideas with which she (he) is comfortable.

Kevin is beginning to tell the group and myself about a mathematical idea that he is comfortable talking about. Triangles can be made from a square.

Allowing children to enter into the discussion using their own level of mathematical understanding,[16] is one aspect of literature that continues to excite me. My journal is filled with accounts of how surprised I have been by how a book allows children to enter the discussion from many levels of complexity. For example, when reading Sea Squares with a previous class, it became evident that the book allowed children to talk about the story and illustrations using a diverse range of mathematical knowledge. After reading "2 white gulls, with 2 eyes each, have 4 bright eyes to watch the beach", some youngsters counted each seagull then commented on the total number of seagulls on the page. One child talked about colour patterns she saw emerging in the book. Others explained how they knew there would be 4 eyes because 2 plus 2 is 4 while some children noted "2 seagulls with 2 eyes each is 2 times 2". The same text and picture but four different entry points and explanations of the mathematics children saw as personally relevant. In fact, after explaining to this group of children how the picture also showed us how squared numbers work, a few of them began to talk about the multiplication sentences and related squared numbers they saw on subsequent pages.

It isn't very likely that Grade Two children would begin a discussion of squared numbers of their own volition. But it is significant that some children used my description of the illustration and explanation of squared numbers to identify and talk about squared numbers in a sophisticated manner. It is important to note, however, that when I shared this same story with the next class of children, they ignored my explanation of squared numbers. Explaining the relationship between multiplication and squared numbers did not redirect these children at all. The second group continued to follow their own trains of thought and observations. Unlike my previous class, these children did not seem to be ready to learn about squared numbers. Vygotsky[17] would argue that my explanation did not fit these children's way of thinking about numbers. They were not ready to be challenged by this new verbal information. He believed that children could accommodate new information through interactions that they were cognitively prepared to receive. My explanation was possibly understandable to the first group discussed because they were cognitively ready to fit the new information into their current knowledge.

Fitting new knowledge into an existing framework is an aspect of knowledge building that is discussed extensively in the literature.[18] Piaget coined the term assimilation to describe how a child deals with new information. Children assimilate new information when a new idea can be fitted into an existing knowledge framework.[19] The image of a partially constructed jigsaw puzzle illustrates the notion of a framework of ideas. Each new puzzle piece, like the information about squared numbers, is only useful and contributes to conceptual understanding if it can be fitted into the collection of pieces already assembled.[20] When the new piece is recognized as part of the puzzle then its inclusion helps to make sense of the entire picture. The first group of children mentioned above seemed willing and capable of placing this new piece of knowledge into their number framework. On the other hand, the second group listened politely but did not see the information about squared numbers as fitting into their number puzzle. They brushed aside my explanation and continued to discuss the story using other mathematical concepts. I concentrated on listening to them.

As classrooms have become more inclusive, I have also become more sensitive to the needs of mainstreamed[21] and English as Second Language students[22] in my class. I am acutely aware that continual efforts must be made to draw these children into learning situations. As I have watched videotaped literature sessions, I have been heartened by how my special learners are engrossed in the story. Their sharing often involves literal interpretations and simple mathematical concepts but it is shared freely with enthusiasm and pride. The inclusive nature of a story is affirming and celebrates <u>all</u> my learners' diverse levels of mathematical knowledge. Whether one can count and talk about the total number of seagulls on a page or comprehend square numbers, a book invites children to enter comfortably and joyfully into a mathematical experience.

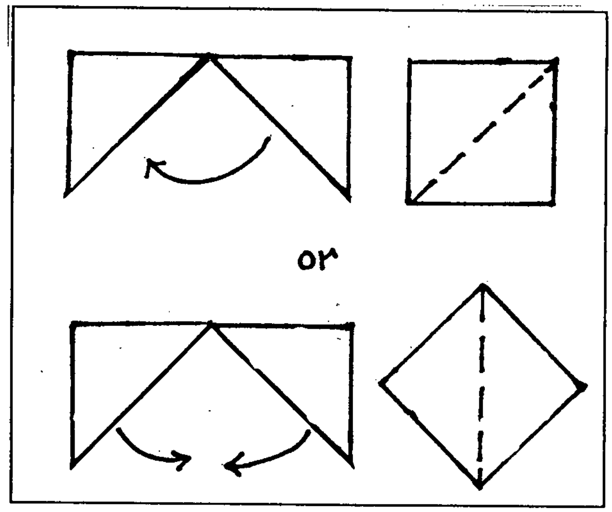
(Back to the discussion of Selina and the Bear Paw Quilt and the illustration on page 7).

Me: *If we put those two back together, what shape would we have?*

Kevin: *Square.*

Jacob: *Or a diamond.*

Me: *(silent personal question) Does he see these as different?*



The question I ask is meant to clarify whether Kevin knows two triangles can also reform a square. It is one thing to know that a square can be cut into two triangles but it is another to reverse the actions to create a square from an arrangement of two triangles. I also want to ensure that I have heard Kevin correctly. My journal reflects my struggles to appreciate a child's current knowledge in a busy, chatty group setting. Part of my journey has been the growing awareness that I needed to slow down and give my full attention to what a child was telling me. Along with this awareness has come a sense that I needed to ask children questions that would clarify for myself (as clearly as possible) the level of current knowledge. von Glasersfeld[23] argues that knowledge resides in the mind of the child, constructed on the basis of what he knows based on his own experience. My task, then, is to continually make an attempt to glimpse a child's interior world of knowledge. I want to be respectful of children's knowledge when questioning their statements but I also want to try and glimpse a youngster's mathematical reality.

I want to understand my students' knowledge but I also want to clearly understand what has been stated. Initially, my questions were very directive and had a didactic feel to them. However, with practice my questions have become more thoughtful and supportive of children's learning.

The question asked of Kevin is also meant to direct the group's attention to Kevin's observations of triangles and squares. Whitin[24] contends that "sharing their own personal connections to the stories enriches the group's understanding of mathematical ideas". The group does benefit when children talk about their personal observations of a story but some children need help to focus and listen to another child's ideas. My Grade Two class includes English as Second Language students, some attention disordered children, and an autistic child. They need continual refocusing and cues to follow the action and to listen actively to another child,[25] but for all these children story time is an effective learning experience too. Pointing to the quilt pattern under discussion and asking Kevin to re-state his knowledge of triangles and squares, is done in the hopes that more passive, inattentive students will hear Kevin's explanation. For some children, a child's voice (and informal language) may be more understandable than more formal information I might give them about squares and triangles.[26]

Pirie and Kieren's[27] model of the growth of understanding has established how children develop images for a concept and can use the images they have constructed to perform mathematical actions. Kevin is visualizing the triangles as flat, two-dimensional shapes that can be pushed apart and back together to form a square. I come to this realization based on his comments and the way he holds his fingers up to the book and demonstrates how the shapes could shift to form a square. He is comfortable with moving a familiar shape on a flat plane. Previous experiences with pattern blocks in Kindergarten and Grade One would have likely established his understanding of how triangles can be turned so that the two edges meet and form a square and for Kevin this is a clear image for triangles. He no longer needs to manipulate materials such as pattern blocks to determine how a square is formed from two triangles. Kevin can now use mental images to visualize such an action.

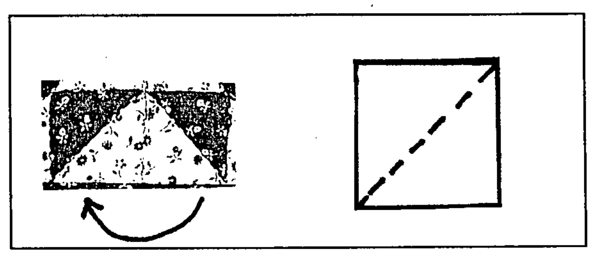
Kevin used the terms squares and diamonds interchangeably when talking about objects in The Greedy Triangle book's illustrations but many young children define a shape as a diamond using different criteria. What does Jacob understand by the words? As seen in the above excerpt, some children accept that a diamond is also a square and use the labels loosely. This became evident when Jacob responds to Kevin that the resulting shape could be a square "or a diamond". For other children, a shape's orientation together with sides and angles determines the shape. Children attach personal meaning to shapes based on their perception of each shape.[28] For some children, a square consists of two horizontal sides and two vertical sides while a diamond has all four sides that are on an angle to the horizontal. Unlike these children, Kevin and Jacob reveal from the fluidity of previous and subsequent comments that they each have a meaningful image for a square/diamond. Kevin's image allows him to re-present and talk about the shape in flexible ways – a square can be a diamond and a diamond can be a square.

In the scenario recorded above, Kevin is able to connect what he already knows about the relationship between a square and triangles to the current task of constructing a square from two triangles. Thus, demonstrating the strength of his knowledge of geometric shapes. Conceptual knowledge is actively constructed when children relate new experiences to what is already known.[29] During an earlier pilot study, I was amazed by how one of my low-achieving students was able to build on his real world knowledge of money while we read Caps for Sale.

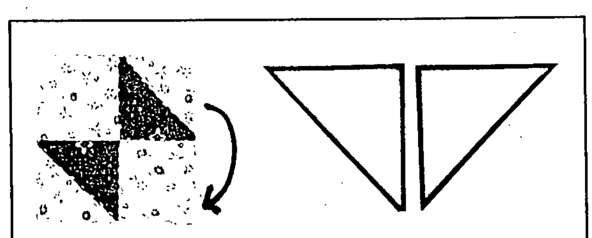
Neil struggled all year to master rudimentary money concepts and by spring of Grade Two he still did not seem to have a clear grasp of coin values or how to make equivalent groups using different coins. Yet in this shared reading situation, Neil was able to use his knowledge of what athletic caps cost in order to explore money calculations. Neil's entry point was knowing that 50 cents is "pretty cheap" to pay for a cap that would cost him thirty dollars or more to buy in an athletic shop. Using his knowledge that "50 cents and 50 cents is a dollar", Neil was eventually able to calculate (with support from me) that the peddler would make eight dollars by selling all 16 caps. Neil's understanding of money was initially very limited. Nevertheless, Neil was able to use this knowledge to not only enter the discussion, but also as a basis to deepen and broaden his understanding of money. Common knowledge and mathematical knowledge come together through literature. [30]

Continuing the discussion about the illustration of page 7 of Selina and the Bear Paw Quilt, Jacob compares two different configurations of triangles in the quilt border pattern.

Jacob: If you were to squeeze these ones back together that'd make a square.



but these ones wouldn't 'cause it hasn't both sides.



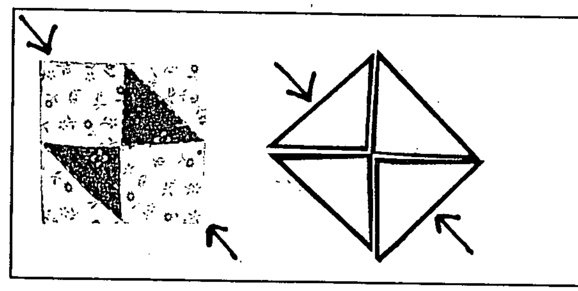
The "squeeze" action suggested by Jacob accompanies a demonstration involving his thumb and finger. Holding his thumb and finger up to the illustration, Jacob brings his thumb and finger together in a pinching or squeezing together action thus indicating how he sees the two triangles merging into another shape.

Jacob concludes that only one set of triangles can be made to form a square. The other is missing "sides" or pieces and can't become a square. It is clear to him that one of the triangles in the first box can easily be turned to create a square. However, a similar action on the triangles in the second box will create a larger triangle composed of two smaller triangles not a square.[31] *Jacob is scanning the two designs and making comparisons. He notices that turning one triangle in the first box will create the two sides (seen as diagonals) needed to fill in the empty space and form a square. Turning one triangle in the second box will not form a square because "it hasn't both sides" – the resulting figure will not have the one long diagonal needed to complete a square.*

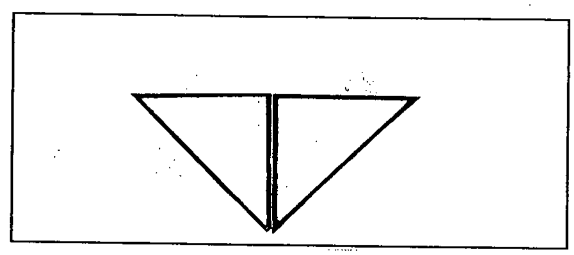
It is interesting to note that Jacob is able to mentally perform the same action on two different triangles while at the same time, monitoring the results of these actions. Wheatley and Cobb[32] also found some Grade Two children who were able to mentally rotate shapes to anticipate how a shape would be created by certain rotations. They argue that children like Jacob construct an image for a design that relates the component shapes to each other and to the resulting shape. If we follow Wheatley and Cobb's argument, Jacob formed an image for the relationship between the two triangles and each of the resulting shapes. It is this image that guided Jacob to mentally position each triangle and 'imagine' how the resulting shapes would differ. His mental manipulations, however, seem to preclude actually moving one shape to a totally different position. His focus is rotation not translation.

Returning to Selina and the Bear Paw Quilt, the dialogue continues with a discussion of whether or not the group agrees with Jacob's image of the transformed triangles.

Me: *(to the whole group of students) Do you agree?*



Kevin: *No... if you put two more triangles in there it'd make a square.*



Jacob: *Yeah, if you put two more triangles in there but if you just squished it back together. It couldn't make a square.*

As previously, Jacob uses his fingers to describe "squishing". Holding his finger and thumb up to the book, Jacob demonstrates how squishing or rotating [33] each triangle towards the other will not result in a square.

Kevin just sees a way to make a square. The two triangles can still create a square but only if the extra spaces are filled in with two more triangles. Jacob agrees that filling in the empty spaces with the triangles would help to form a square but merely turning one triangle would not be sufficient to make a square shape.

When talking about the second configuration of triangles, it appears that Jacob and Kevin possess different images of what shapes will be formed by shifting one triangle's position. Jacob sees a larger triangle emerging while Kevin continues to envision a square. He would need to fill it out with two triangles but it is still possible to make a square. The image of a square seems to be a strong one for Kevin. It is also possible that Kevin is trying to be helpful and wants to assist Jacob in finding a way to make a square from this configuration of triangles as they did with the first arrangement.

Jacob makes a statement that seems to be interpreted by Kevin as a problem situation that needs to be resolved. How could he make a square and solve this question? Kevin comes up with a solution but not within the boundaries set out by Jacob. The problem needs to be solved, not by adding in more triangles, but by turning the one triangle. Jacob insists that it is not possible without the addition of other triangles. A problem situation is posed by Jacob, not in the usual question format, but based on his observations of the quilt design. Other writers have also documented how children's literature supports problem posing.[34] Stories often have supporting details that learners can change in different ways to create a multitude of related problems.[35] Brown and Walter[36] saw the value of such playfulness, for they state that "we understand something best in the context of changing it". Building on this perspective, I have observed my students muse out loud "I wonder what would happen if….", often with amusement. This is what Brown and Walter saw as valuable, playing with ideas and posing new questions and problem situations. During a recent shared reading of <u>Arctic Fives Arrive</u>, one of my students, Jason suggested that perhaps there would be more room on the iceberg for other animals if the walruses would leave. In this instance, Jason takes the story's description of five walruses on the iceberg and poses a situation that other children begin to ponder – how many other animals could replace the walruses on the iceberg? The story's text and Jason's observation that each huge beast was taking up a lot of space sparked a discussion as the children pondered this new problem situation. Notions of area and volume were at stake here but not, I suspect, in the mind of the author.

(Returning to the previous dialogue and Jacob's comments about what shapes would emerge from "squishing" the different triangles together) *My question, "do you agree?" is posed to the group for two reasons. My initial reaction to Jacob's assertion that the second configuration of triangles could not be turned to form a square is to mentally disagree. I'm not positive but I figure there must be a way to manipulate the triangles to form a square. I am not sure how - not yet. I subtly ask the children if someone has figured out a strategy, one that will also help me to think through a way to make a square.*

I am stalling for time, enough time for me to think carefully about what Jacob and Kevin are saying about the shapes. But I also want to prod other children to begin thinking about this situation.

It has mildly irritated me in the past how some children would shift to the fringes when other children began talking about their thinking. It appeared to me that these children felt like they were off the hook in a cognitive sense. As long as others were talking about their thinking, it wasn't necessary to actively join the dialogue. At one point in my journey, this type of scenario caused me to seriously question whether reading and talking about children's literature really was a valid classroom practice. My journal reflects the depth of my questioning:

> It bothers me that a core (of students) only are participating. Are they different children than in other settings? Are other children participating mentally while not saying their thinking out loud to me or others? Do all experiences fit all children or is it not okay to give only a small percentage

a great experience?"

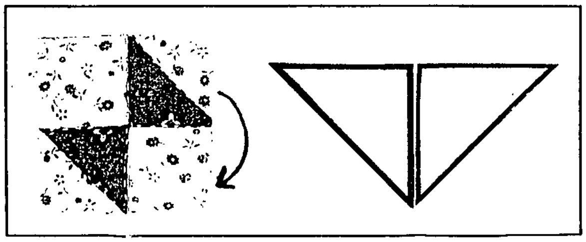
(excerpt from my journal dated June 10, 1998).[37]

Was it legitimate to continue doing something that didn't seem to hook all the children at the same time? I was reluctant to abandon something that seemed to give a small segment of my class such a rich mathematical experience. June 10, 1998 marks my turning point. I realized that each time we shared a book, a nucleus of children would connect strongly with the story ideas. And, yes, each time they were often different children. When I reflected about my teaching in general, it struck me that I was constantly trying different approaches in the hopes that I could meet the diverse learning styles and needs of my students. The question of whether or not to continue using children's literature was resolved. Knowing that my classroom offered students diverse opportunities for learning made it reasonable for me to continue a practice that was so rich and rewarding for some children.

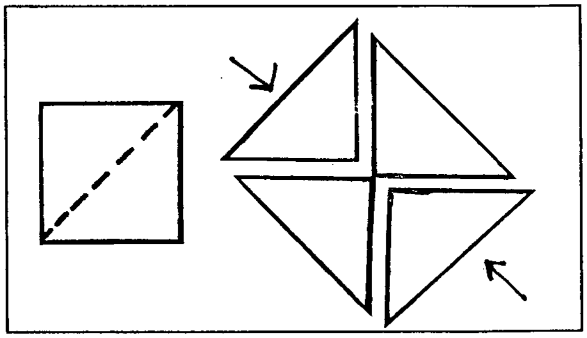
What wasn't acceptable was to allow children to remain passive and uninvolved in a learning situation. I pose questions like "do you agree?", "why or why not?" to try and jolt children out of their passivity. I want them to join the rest of the group, if not verbally at least cognitively. My question 'do you agree' in the current scenario has a two-fold purpose. Firstly, to help clarify my thinking and secondly, to stimulate my students' thinking.

Continuing with our discussion of the illustration on page 7 of Selina and the Bear Paw Quilt:

Me: *So if I just moved this one, I just rotated it back . . . . .*

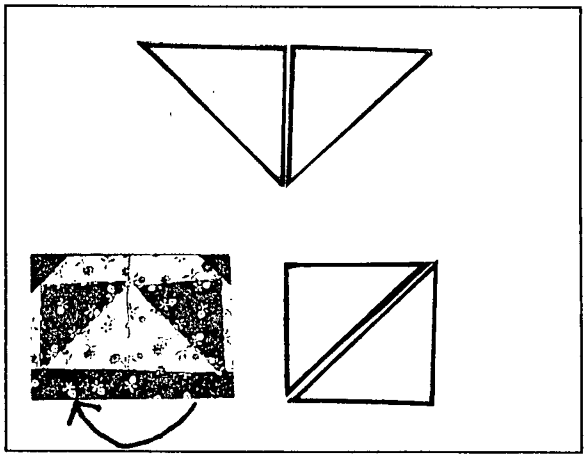


Kevin: *(interrupting) Or if you cutted up a square you could move it in like that.*



Me: *(continuing) If you only rotated this one would you have a square?*

Jacob: *You wouldn't. If you didn't fill it out with anything, squish it together like the red one it wouldn't make a square but the red one would.*



Kevin continues to maintain that cutting up squares and filling out the empty spaces can make a square. He dismisses my statement in favour of a "filling in" strategy. Why rotate a triangle when you can cut up a square and place the newly fashioned triangles in the opening? Is his thinking influenced by the context of quilting: cutting up shapes and fitting them *together? Jacob seems to use the term "squish" as a way to describe a single two-dimensional movement. He seems to be telling us about a way to bring the two closest sides of each triangle together.*

My questions in the transcript above are an attempt to understand Kevin and Jacob's images as well as to prod the boys to examine the reasonableness of their images and mental transformations.[38] In responding to my questions, Jacob and Kevin challenge each other's thinking while debating and defending their images. It has been well documented how children's literature creates a mental stage where student thinking can be challenged and reexamined.[39] On such a stage, Jacob and Kevin are each asked to individually look again at his own image and then verbally explain and justify the reasonableness of his thinking.

I want to understand both Kevin and Jacob's geometrical thinking. In particular, I want clearly to understand what actions each boy is mentally performing and how the resulting shapes would differ. Many writers have documented how children's books encourage a free exchange of ideas where thinking can be probed and discussed in an environment marked by intellectual and emotional safety.[40] I want to understand what Kevin and Jacob are telling me but their loose and casual statements are often only partial explanations. This is frequently the case when my students and I share literature together. Grade Two children are limited in the range and depth of language available to them when communicating with others. During the reading of The Greedy Triangle, Kevin tried on several occasions to explain his thinking and faltered. He obviously had an idea he wanted to share about the pictures of shapes and the story but after long periods of waiting time Kevin still could not find the words to express his thinking. In other shared reading times, all some children needed was extra time in order to express their impressions of a story. I am reminded of Vygotsky's[41] notion that thoughts are not merely expressed by words but come into existence through them. A dynamic connection then, exists between thought and word. Words and the expression of those words can clarify and bring form to a child's thinking. All some children require is extra time to find the necessary words to tell about his or her thinking. It is important to note however, that the absence of the language to express one's thoughts does not immediately mean that a child also lacks a coherent way of thinking about mathematical concepts. It may simply reveal that he does not have the necessary language to express his mathematical thoughts. Pirie [42] cautions teachers to separate a child's use of language from indications of lack of understanding. We cannot assume that because children cannot talk coherently about a mathematical task (concrete or mental) they have performed that the youngsters lack mathematical understanding. Grade Two children often struggle to find a way to talk about their impressions and thinking. Kevin may have had a coherent notion of the mathematics he saw in the Greedy Triangle but did not have the language (informal or formal) to tell us about his impressions.

The group situation may also have contributed to Kevin's difficulties in talking about his thinking. While I want as many children as possible talking about their ideas, the enthusiastic responses are sometimes overstimulating and distracting. I have sometimes found myself at a loss about what I was going to share with a child. The idea that seemed so salient a moment ago suddenly vanished in the midst of swirling conversation. I continually remind the children to wait until someone has finished speaking before sharing their ideas but even then, the waiting seems to affect their responses. For example, in a shared reading session with last year's class, Judith lost her idea while waiting for some children to finish talking about the animals in Dinner at the Panda Palace. Judith had patiently allowed other children to talk. When it was her turn, a look of amazement crossed her face and she said "I forgot!" My reaction at the time was tinged with sadness and made me realize the shortcomings of whole group discussions. Having to wait too long causes some young children's thinking to slowly deteriorate or move to follow other discussion paths, leaving no accessible trace of their unexpressed ideas. I would need to continually find ways to strike a balance between fully exploring one child's ideas while addressing the needs of all children to share their thinking. This will continue to be part of my journey for I realize that such a balance will be difficult to find when working with large groups of children. Perhaps the closest I can come to a balance is to ensure that our discussions allow different children to have their 'moment in the sun'.

In this current scenario, I ask the questions about the transformed shape because I want to focus both boys' attention on the rotation of the one triangle. Jacob insists that it is impossible to form a square by merely turning the triangle. He does not seem to be thrown off when I talk about rotating back the triangle. He seems to equate the word rotating with his term "squishing". An atmosphere of acceptance has grown as my students and I have read and talked about books. The children know that I will value their contributions to the discussion. My students realize that I am more interested in what they are thinking and understand than the words chosen to describe personal knowledge. My students also assume that I will understand what they tell me.[43] Vygotsky's writing notes a similar assumption existing in the shared social world that evolves as an adult and child engage in the negotiation of mathematical meanings. He too, acknowledged that an understanding exists that what the child says will be understood by an adult. The term taken-as-shared is used by researchers[44] to describe how participants interact as if they interpret a math topic in the same way. Jacob may not use the same words as me but we seem to have a shared understanding that we are talking about the same transformation. I use the word 'seem' deliberately because there is no way I can know for certain what Jacob is thinking, only his words give me a clue.

Acknowledging children's informal language does not mean that I also accept incorrect descriptions and terminology. For example, when we were reading The Greedy Triangle, one of my English as Second Language students, Gerard, described a picture that was obviously a rectangle, as a triangle. As we interacted together, we arrived at an understanding of how we both defined a rectangle. We shared an understanding of a rectangle but differed in how we labeled the shape. When I realized Gerard was saying triangle instead of rectangle, I corrected him. This happened several times before Gerard started to use the correct vocabulary with his description of rectangles. Gerard's understanding and description of the shape were correct but not his terminology. I want to value children's language and ways of knowing but I have a responsibility to model accurate mathematical language as well. There is a difference between Pirie's quasi-mathematical language[45] and incorrect mathematical language.

Kevin continues to insist "filling in" with more triangles can create a square. Jacob and Kevin have different beliefs about how to make a square and are beginning to talk past each other. The boys are heading towards an impasse that they won't be able to resolve.

Cobb, Yackel, and Wood[46] use the term incommensurability to describe such a situation. They argue that the teacher plays a critical role at this point. As a representative of the mathematical community, the teacher needs to be both a facilitator and a role model. In this scenario, I seem to be legitimizing Jacob's explanation while asking Kevin to look again at his argument. In a sense, I act as the group's facilitator by constraining Kevin's explanation so that we can all focus on Jacob's strategy.

A tension exists in group work between children trying to build personal understanding of concepts and trying to understand others' explanations. Cobb, Yackel, and Wood[47] use the term circularity to describe this interactive process of learning. Children's learning is influenced by each other's activity in the group while they themselves are influenced by their own interpretations of the activity. My role in the group is to help Kevin and Jacob negotiate a productive collaborative relationship so that the group process does not hinder either student's mathematical learning.

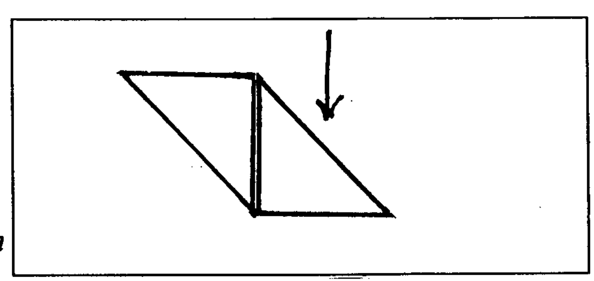
It is often difficult in a group setting to focus the group's attention on all the mathematical ideas raised by children. Part of my role is to select the most salient idea at the moment to pursue either one on one with a child or as a whole group discussion. Part of my own growth has centered on resolving this dichotomy of group discussion and individual participation. When is it appropriate and necessary to stay with one child's thinking when it means suspending, for a time, the spontaneous sharing of other children? In this situation, it is evident that Jacob is performing an intricate transformation that warrants further examination and discussion. There have been other times when the spontaneous sharing of some children has overshadowed what could have been a rich group investigation and discussion. For example, in a previous year, while reading Arctic Fives Arrive, one child made a comment about the size of an animal and the size of the iceberg. The incident happened quickly and the moment passed as other children commented on other aspects of the picture and story. A fleeting thought went through my mind that this was an interesting comment. I had a momentary struggle over whether to pursue this comment with the child and in the end decided we could maybe revisit the book at another time and explore the concept of area and even ratio embedded in the picture. This was one of the first shared reading sessions that was taped and I felt very awkward and nervous. I was not yet comfortable in front of the camera. It is likely that my unease came into play at this point. I may have eliminated this potential item for discussion based on my own level of discomfort. While reviewing the videotape, I realized what a rare opportunity this could have been to talk about area in a relaxed, supportive atmosphere. Revisiting the text at another time would not likely have the same impact on my students' understanding as perhaps staying with this child's comment and exploring the relationship between an iceberg's size and the animals. Even when one is aware and watching for opportunities to explore new ideas there is always the possibility that opportunities for such probing will be missed. I was interested when Jason raised the same issue this year and, this time, allowed the discussion to flourish.

Back to today and the same illustration on page 7. *My questions are meant to focus the group on Jacob's strategy of rotating a triangle as well as the bigger question of what shape will emerge.*

Me: *(personal silent thinking) Can we always make a square?*

Kevin: *Oh, it wouldn't make a square (Jacob agrees). It would make a diamond because it's kind of on its side. When you put one right there it would make a diamond on its side. If you left those out and those sides go in.*

Me: *(personal silent thinking) Kevin has changed his thinking about the emerging shape. Rather than turning the triangle inward, Kevin seems to be mentally making a slide*



with the triangle. Is he visualizing the transformed shape as a kind of *diamond that is resting on its side? But Kevin distinguishes this kind of diamond from his previous description of a diamond as a square turned on its corner. For Kevin, a square can also be a diamond but apparently now, not all diamonds can be squares. I am confused by what Kevin is telling me about how a square/diamond can be created by turning* a *triangle. I need more information from Kevin to help me imagine what he visualizes will happen to the triangles. From what he is saying, I am unable to see how a diamond is being formed.*

When talking about the second configuration of triangles, it appears that Jacob and Kevin have different images of what shapes will be formed by shifting one triangle's position. Jacob sees a larger triangle emerging while Kevin at first envisions a square then a diamond. When Jacob insists that "squishing" the triangles together will not make a square, Kevin switches to calling it a diamond.

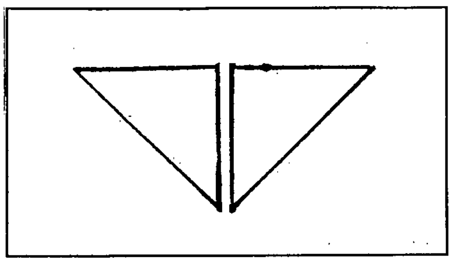
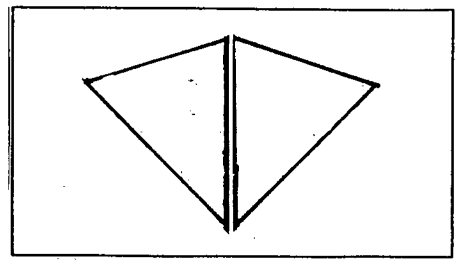
Me: I'm not sure I'm following what you're saying, tell me a little more.

Kevin: It would make a diamond if you make it a little more along the side.

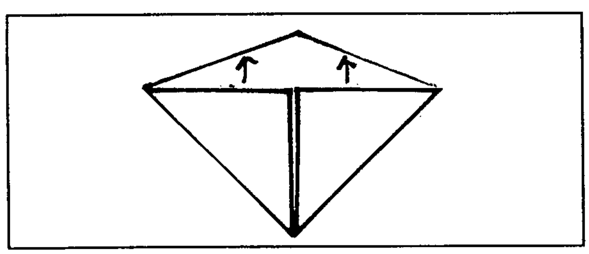
Me: (personal silent thinking) I'm baffled! I'm very confused about how he can possibly see the transformation resulting in a diamond shape. He seems to have changed his interpretation of what transformation has taken place. Is he changing his thinking from a slide action back to a rotation?

At the time, I was confused by what Kevin was saying and how he saw the shapes merging into new squares or diamonds. It was only after watching the videotape several times that I began to understand what Kevin was saying about the movement of the triangles.[48] This was a major mathematical moment for Kevin. He was trying to express his idea of a diamond being more than just a square on it's side.

In Kevin's thinking, rotating one triangle does not make one longer continuous side of a larger triangle.[49]

NOT  BUT 

Kevin defines the first shape as a diamond because it has four sides and is "kind of on its' side" and I assumed from Kevin's actions and description that he had the image of a 'square on a slant'. In fact, he was seeing the resulting shape as having four sides, two of which form an enhanced diamond by pushing them both upward a bit.



The two individual triangles that make up the larger triangle retain their respective sides. By pushing those two sides upwards a bit more, the two sides can be made more closely to resemble the shape that Kevin defines as a diamond. The image of a diamond being a square resting on a corner was a strong one at the time for <u>me</u>, because my students repeatedly commented on how rotating a square turned the shapes into diamonds in <u>The Greedy Triangle</u>. As well, Jacob a few minutes earlier had made the observation in this current book that you can tell a diamond is a square by turning your head over to one side and then the diamond looks like a square, concluding "a diamond is just a square standing on its corner". Pirie and Kieren[50] argue that learner images that are incomplete or erroneous hinder learning and the student needs to "fold back" to the level of Image Making to develop a broader range of images. In this instance, my dominant image of squares and diamonds was incomplete and Kevin's conclusion that a diamond will be formed is erroneous! At the heart of his difficulty is the fact that Kevin visualizes the movement of the triangle as resulting in a four-sided figure.

Since I am confused by this switch in Kevin's thinking, I need more information to inform my understanding.

It has taken me some time to be able to say to a child, "I don't understand what you are trying to tell me". Somehow I felt it was just too embarrassing to admit that I couldn't fathom what a student was telling me. To counter my past unease, I would make an assumption about what I heard and would continue the dialogue without checking my comprehension by reflecting back what I had heard or by asking for more information. But it was a mistake to assume I knew what a child was telling me and the consequences have been both amusing and sad. Watching myself on initial videos has made me shake my head in disbelief and embarrassment. How could I be so thick as to not catch what a particular child was trying to say? Why would I have made such a comment to him or her when it was clear that he or she was telling me something quite different? It was like watching two parallel paths of conversation that forever stayed separate and never intersected at any point. Now I try not to assume understanding and attempt to check out what I am hearing by questioning and reflection. I am still surprised, however, by how off-track my comprehension can be and how poorly I can listen. For example, while talking about the groups of animals on an iceberg in

Arctic Fives Arrive, Jacob speculates that "20 more groups would be higher than the sky". What I "hear" is 20 more animals not groups. My question, "20 more animals would be how many?" reflects the fact that I have not accurately heard what this child said to me and that my dominant images at this point are arithmetical and associated with the repeating pattern of adding more animals. What could have been an interesting problem-solving situation is down-played to a simpler one of calculation. What is also interesting in this exchange between Jacob and myself, is that he didn't correct me. Did Jacob hear what I said or was he confused by my question? When Jacob responded to my question, he did so by giving the new total as including the number of additional animals rather than the groups of animals. Someone in the group whispered this number to Jacob, so it is also possible that he merely 'parroted' the answer and was diverted from thinking about the problem situation he had posed to the group.

The previous discussion about groups versus animals could also have centered on proportion. Another opportunity for a discussion of proportionality came up later in the year when we were reading and talking about Counting on Frank. Jack made a comment in the middle of the story that related to the concept of proportion but was completely missed by me. The conversation and story proceeded and it was only later when I reflected on the shared reading experience that it dawned on me how significant that child's comment had been. We could have engaged in a valuable discussion about the number of "Franks" it would take to fill a whole house.

Towers[51] concludes from her work that it is difficult for teachers, even those who take the time to study their practice for "listening", to really attend to listening to students in classroom situations that involve groups of children. She advocates that teachers adopt a different approach to listening, one where listening for understanding is the objective. By questioning and reflecting back what I think I have heard, it is my hope that I too can listen more effectively for an understanding of my students' thinking.

I am having a difficult time understanding Kevin because I do not share his image. Listening effectively is only part of the story when trying to understand children's thinking.

This situation has occurred with other children in a group discussion as well. When I began using children's books to talk about mathematical ideas, I was unaware that I had limited images for certain concepts.[52] Several years ago, I had videotaped my class of Grade Two and Grade Three children as we talked about how much money the peddler in Caps for Sale would make if he sold all his caps. At one point, a Grade Three student was explaining his multiplication strategy to the group by first using the blackboard and then using his words. When I reflected back to Arin what I had heard, it was in a very different form from what he had told me. While analyzing the episode, I was puzzled by what I had said and why Arin so reluctantly agreed that my response was correct. As I pondered this instance and my own understanding of multiplication, it struck me that I really only had one image for multiplication. Arin had a very different image from mine. When Arin put his strategy on the blackboard he was using that image. It was Arin's work on the board not his words that convinced me that we did not share the same image. My understanding of the process of multiplication would not have been described by the diagram Arin drew on the board. For whatever reason, Arin did not disagree with my explanation. It is possible that my image conflicted with Arin's and he was left feeling confused. I was dismayed that Arin, a very confident student, could not tell me 'no, that's not it' if he disagreed with my interpretation of his multiplication strategy.

> *Me: If we take this triangle and if we flipped it right over so that edge was with that edge am I understanding you to say that would be a diamond then?*

Kevin is silent at this point, possibly because I have introduced the term "flipped" but more likely, because he is unsure how to respond to my question. At any rate, Kevin is unable to explain his thinking. His ability to explain verbally has become limited. I believe he needs a way to supplement his language and express bodily what he cannot communicate with language. So I decide a concrete material might help him demonstrate the triangle's movements. I bring out Power Blocks (a manipulative marketed by Math Their Way) to supplement the illustrations and to help Kevin explain how the triangles will move. The Power Block triangles are larger shapes than other materials in the classroom and so, I believe these shapes will be easier for Kevin to turn and will allow the group to follow what he is doing.

Me: Can you hold it like it is in the book?

I restate to Kevin my understanding of how he sees a diamond being formed but in reflecting back to Kevin, I substitute the word "flipped" for rotation. This is a term that is related to three-dimensional not two-dimensional motion. "Flipped" implies rotation "out of the page" and both the flat pictures and the idea of the quilt itself (flat pieces that one can shift in position but one does not usually turn the material over) tend to inhibit three-dimensional thinking in the children. The illustrations of quilt patterns play an important role in the children's musings about geometric concepts. It is the quilt pictures that spark and support my students' mathematical thinking and discussion. Along my journey, I have shared a variety of books with my students, and my journal records the children's reactions and mathematical comments to different books. In particular, the entry documenting the children's experience with <u>Sea Sums</u> cements the importance of illustrations when selecting books:

I am convinced now the best books are those that have strong images and story line. Sea Sums, while lacking a plot, has vivid pictures full of details that strengthens the text and supports their thinking.

(excerpt from my journal dated June 8, 1998).

The richly detailed illustrations of the quilt patterns in Selina and the Bear Paw Quilt are highly engaging for my students. Over and over, I am asked to turn again to this page or that page as the children check out recurring details, shapes, or patterns. Of all the children's literature we have shared this year, this book appeared to elicit the most spontaneous mathematical comments from a large number of children.

On page 7 of the book, the arrangement of triangles in the Bear Paw quilt pattern give Kevin and Jacob a type of semi-concrete stimulus to mentally manipulate and use to describe how triangles can form squares or diamonds or larger triangles. Harsh[53] describes how listening to and looking at books that deal with specific concepts is a semiconcrete experience. The illustrations support and guide thinking and explanations by providing a constant stimulus. For example, the notion of quilting may possibly provoke the mental cutting and fitting activity. The children continually refer to the picture and patterns as they mentally manipulate the triangles. However, I believe Kevin needs another means to help him express how the arrangement of triangles is transformed into a diamond.

It is important to note that it was very unusual to engage in such an activity in the midst of a story. Like other proponents of children's literature in math programmes, I too, want to preserve the integrity and enjoyment of a story.[54]

I hold the book vertically with the picture facing towards the class. Kevin is able to copy the position of the first triangle after scrutinizing the illustration. He knows that the quilt *pattern involves two triangles but has some difficulty positioning the second triangle in relation to the first. After looking several times at the picture, Kevin (with some help from me) holds the plastic triangle in position against the first triangle. Because I am holding the book vertically in the air, this makes the task more difficult for Kevin. To match the illustration Kevin has to also position the triangles vertically in the air. It may have been easier for Kevin to place the triangles in position on a flat surface but I want the camera to capture the action. I also want the group to see Kevin's demonstration.*

Actually, it is difficult to see on the video if Kevin has the triangles put together the same as the book since he holds them up out of the view of the camera! All this happens quickly, too fast for the camera to be adjusted so that Kevin and the triangles come into view.[55]

Jacob: It would be a triangle not a diamond.

In this instance, no prompting is needed by me for Jacob to react to Kevin's ideas. We can't focus on both boys' ideas at the same time, so I make a decision to postpone talking about Jacob's assertion that the shape is a triangle. Instead, I direct the group to focus on Kevin's argument that a diamond is being formed and ask Jacob to hold his comments until later.

As mentioned earlier, there is always a danger, in such a request that the child will lose their train of thought. This was the case when I was reading and discussing <u>Panda Palace</u> with my class last year. We were talking about various mental calculation strategies different children were using to calculate the number of animals in each group that asked to be seated in the restaurant. I had just asked the group a question that involved using a related multiplication strategy. When I responded to Bart's raised hand, he began to talk about another mathematical idea so I told him "let's finish this strategy first". By the time we finished and I asked Bart to share his comments, he had forgotten. My journal entry for March 26, 1998 reflects my dilemma about children sharing:

> "... while spontaneously saying their thinking/ideas is best, kids waiting too long forget what they are thinking".

This is a dilemma, one that is difficult to avoid in large group discussions.

(We look at the illustration on page 7 and watch Kevin demonstrate with the Power Block triangles).

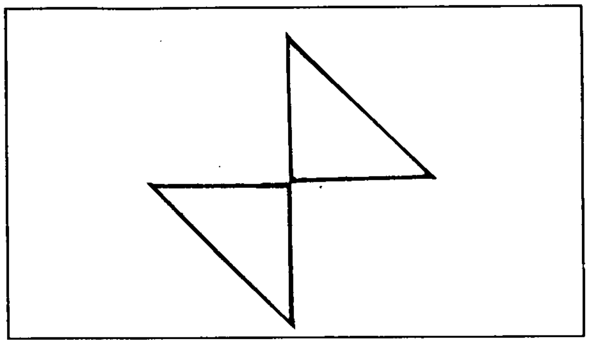
> *Me: How would you put them together so that it would make a diamond?*

Kevin holds the triangles in a similar way to the book but is unsure how to make a diamond. It has been a bit of a struggle for Kevin to replicate the position of each plastic triangle and, consequently, he has a difficult time rotating each piece to form a diamond. He appears to want to alternately turn each triangle and as a result, has a difficult time making a diamond shape. He seems to be turning the shapes at random, searching for how the two triangles will merge into a diamond. It is not clear whether he is using a mental image to guide his actions. Even if he is trying to track a mental transformation, this will be a very difficult cognitive task to carry out.

Wheatley and Cobb[56] analyzed young children's attempts to construct such spatial arrangements to determine the relationship between their use of imagery and spatial reasoning. They conclude that by noting the position of only one shape in a pattern at a time, some children experience difficulty constructing an image that relates shapes to each other. Kevin may also be experiencing a more physical difficulty. It could be that two hands are not enough to hold one triangle stationary while making a turn with the other.

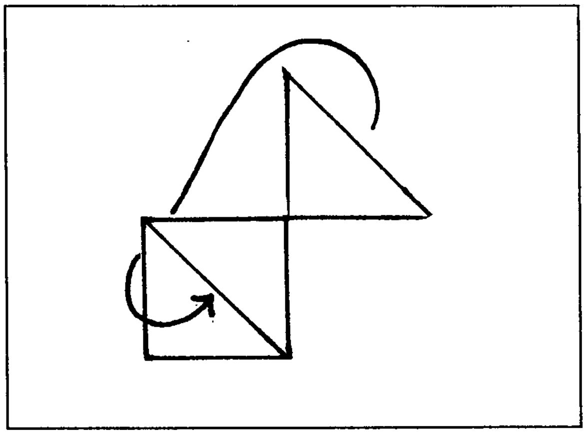
Me: *We started off with a design that looked like that...*

(When Kevin hesitates and has trouble shifting the triangles, I intervene).



Me: *So if we flipped it we'd have a diamond or if we turned it we'd have a square.*

(I move one triangle three dimensionally behind the other, turning it as I go until the two long edges coincide).



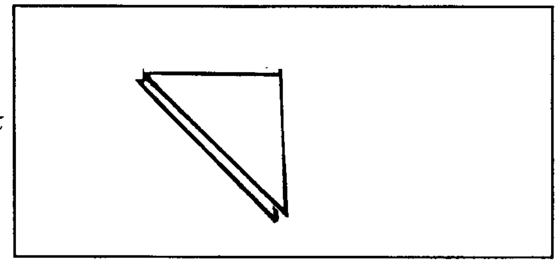
In my eagerness to help, I may have intervened too soon. With more time shifting the triangles, it is possible that Kevin would have figured out how the shapes could form "a diamond" (and in fact seen that it was really a larger triangle). I also introduce the word 'flipped' which is not part of any of the children's thinking. Up until now, they have ignored my insertion but now I put the term into action and make a transformation that is impossible for a flat shape restricted to two-dimensional motion.

As mentioned previously, when I began experimenting with children's literature as part of my mathematics' programme, I was continually uneasy about allowing children to respond spontaneously to the book. What if I couldn't picture in my mind what a child was seeing in the story? To combat my own discomfort, I would redirect the discussion, ensuring that what we talked about fit with my impressions. That way, I was on safe ground and couldn't end up looking unintelligent and wouldn't lose control of the discussion! Yet, as this scenario points out, my image of the pattern and how a diamond can be formed is quite different from Kevin's images. The differing images of teachers and students and the impact on student learning has been explored by Pirie, Martin, and Kieren.[57] In this instance, my image of the pattern and how the triangles could move to form a diamond are grounded in how I visualize a solid moving through space. As I rotate the triangle for Kevin, I move the shape in a manner similar to the way a solid will turn rather than how a flat shape will move on a two-dimensional surface. It is clear that my image of how the shape can be transformed into a diamond is quite different from Kevin's. Wheatley and Cobb[58] conclude from their analysis of Grade Two children's spatial constructions that children give their own meaning to spatial tasks in ways that are often incompatible with an adult's conception of a particular task.

I see from the video that my demonstration with the triangles takes place very quickly, so fast that I now believe it was impossible for Kevin to follow or understand what I was showing him. It was also too fast for me to see the difference in our images. This realization underscores the power of video analysis and why I chose videotaping as the means to collect my data.[59] What is impossible to discern while in the midst of a learning situation becomes more salient with repeated viewings of the taped scene. I have the luxury of slowing the action and reflecting on what took place and why. It is through this process that I realized the significance of what happened and how different were Kevin and my images. The possibility that we could have different images didn't really occur to me until I began analyzing the videotape. Consequently, I have been moved along on my own journey. I deepened my own image of what geometry might mean to children. This experience also underscored what pictures and a story might invoke in children.

Now that Kevin has completed his demonstration (and my own), the spotlight is focused on Jacob.

Jacob: I meant if you just folded them back they'd make one triangle.



Jacob uses the Power Block triangles and demonstrates the triangle's movement. The folding back action results in the two triangles superimposing and so, is seen as one triangle. Not a larger triangle but one with the same dimensions as the original triangles. He is now using a three-dimensional flip action and has moved away from talking about turning (or squishing) a triangle. He seems to have changed his mental image.

The task is simple in Jacob's thinking. You just take one triangle and fold it back onto the other like one would with paper and you have one triangle. From Jacob's "squishing" actions and verbal description I infer that Jacob initially envisioned the transformed shape as a larger triangle. Rotating one triangle in two dimensions, as Jacob described, will make a triangle double the size of one smaller triangle. Now he describes folding one triangle on top of the other. One small triangle will be the result. It is possible that my use of the term flipped redirected Jacob's thinking. I think the flat two-dimensional image conjured up by a quilt and the sudden introduction of the freedom of a three-dimensional movement of solid objects is a factor here. Previously, Jacob demonstrated an ability to compare two different configurations of triangles and imagine the differing results of rotating a triangle. Now making such a comparison reveals to me that he has some understanding of sides and angles. He notices before performing the action, that the two triangles are congruent, their sides and angles are the same. He knows, then, that one triangle will fit neatly on top of the other and the sides will line up.

I seem to have a different image in my mind about how the triangles will move and the resulting shape. Although it is difficult to recollect now, I think that I imagined a square emerging from either turning a triangle two times or from performing two flips. Jacob's explanation came as a bit of a surprise but was an interesting observation on his part that the triangles' sides and angles were the same.

(Continuing with our discussion of Jacob's explanation of "folding back" the triangles).

> *Me: So you'd lay one on top of the other. There's a special name for shapes that you can do that with. We say they are symmetrical.*

I reflect back to Jacob my understanding of his demonstration. I also use his explanation as a springboard to explain symmetry. Jacob's description of folding back one triangle to make one identical triangle gives me a teachable moment. Children's mathematical learning involves both a process of active individual construction[60] and a process of acculturation into the mathematical practices of the wider society.[61] As a representative of the mathematical community, I play an active role in influencing the mathematical aspects of knowledge that my students construct. I recognize that learning situations need to be open and rich enough so that children can build their own understanding of math concepts. Yet, I also have a responsibility to provide my students with ways of knowing that are compatible with those of a wider society. Jacob gives me an opening to talk about symmetry in an informal fashion.

Steffe and Tzur[62] state that we help children construct and modify their knowledge through interactions while at the same time we impart our conceptual knowledge to them. However, a balance needs to be struck between 'telling' and supporting children's efforts to build personal knowledge. Without the kind of 'discovery' activities that literature provides, Jacob would not have an arena for playing out his knowledge. I wouldn't know what he already knew about symmetry or when to insert such knowledge into our discussion. It is like a dance where sometimes a student leads and I follow with specific knowledge. But during this dance I may also lead with information and my students will follow my cognitive steps. Without some knowledge structure in place, my words would have been empty and meaningless to Jacob. In Davis' (1993) words: "telling might give them some words, but it would not help them to build up the metaphoric mental imagery that is the basis for true understanding". I could not 'tell' children all that is needed to build true mathematical understanding. They need active experiences such as shared reading to learn and develop conceptual understanding.[63]

But how would I balance 'telling' about aspects of math in a story with letting children spontaneously respond to the story?

My journal entry after sharing Remainder of One last year reflects this internal struggle:

> I struggled with how quiet I should be. I know that I struggled with when to highlight things in the book and when to stay quiet. I don't want to direct the kids to focus on 1 or 2 specific things that would make them think that's all I want them to notice in the book. But don't I have a role to play as teacher too – what about the teachable moment? This is a setting when I could be teaching something too. Where is the balance? Between teaching? Focusing attention on certain things/highlighting specific math? Showing a connection or how this is linked to____? I am quite puzzled and feel the internal conflict in the midst of reading the story.
>
> (excerpt from my journal dated June 4, 1998)

Leaving the mathematics, I turn the page and read on. Upon returning home, Selina's father informs the family that the war is coming and that Mennonite people like themselves *were now believed to be the enemy. They must flee to Upper Canada. Selina is sad to leave her relatives, especially her grandmother. But Selina's grandmother gives her a Bear's Paw quilt to take along, to keep Selina's memories of her family alive and strong. The family boards a train and is soon on their way across the Niagara River into Upper Canada. The children relax into the tale, noting many aspects of the story and illustrations. The discussion weaves back and forth through the book as the children compare the sequence of quilt designs presented with each text page. The array of quilt patterns on the back of the book prompts the youngsters to speculate about which creation will come next and most importantly, are they right?*

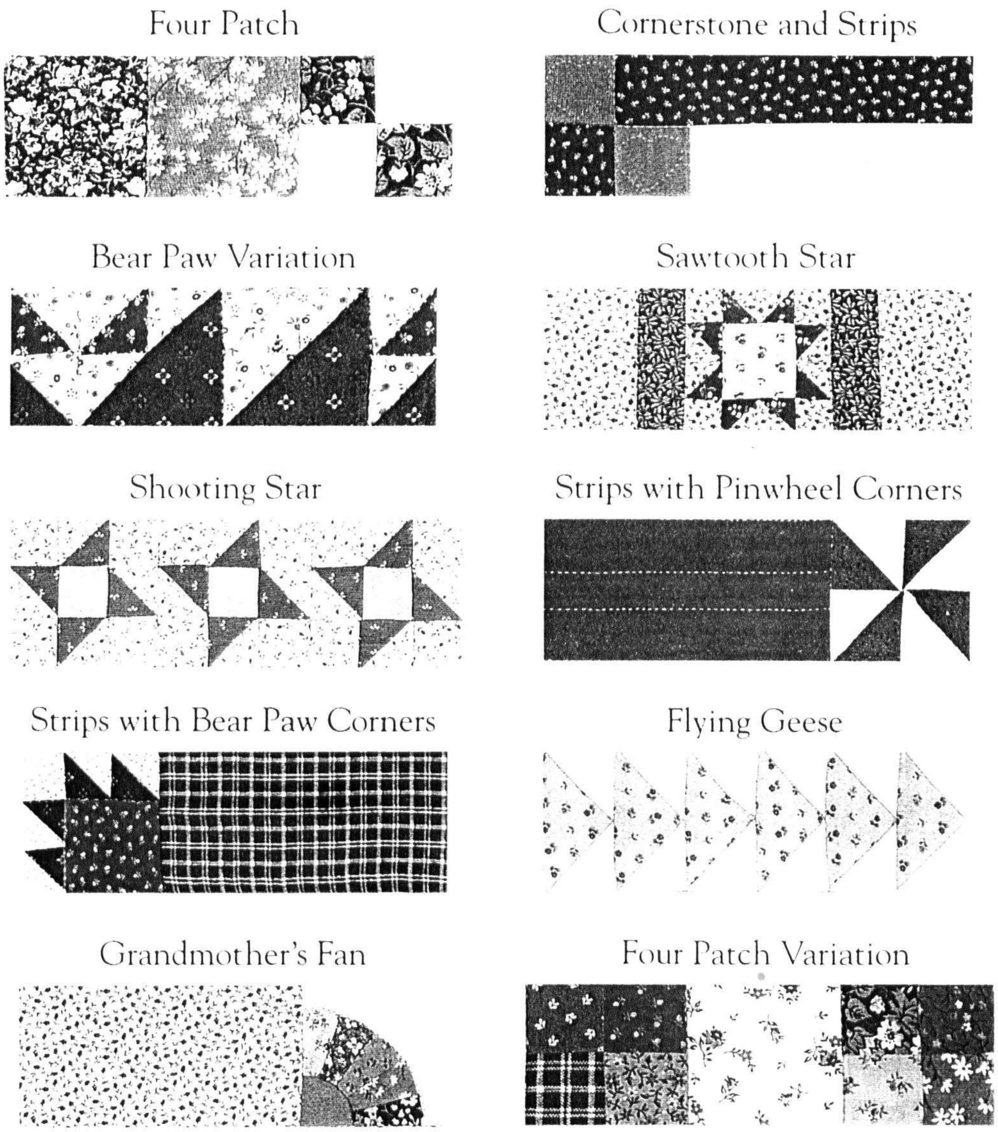


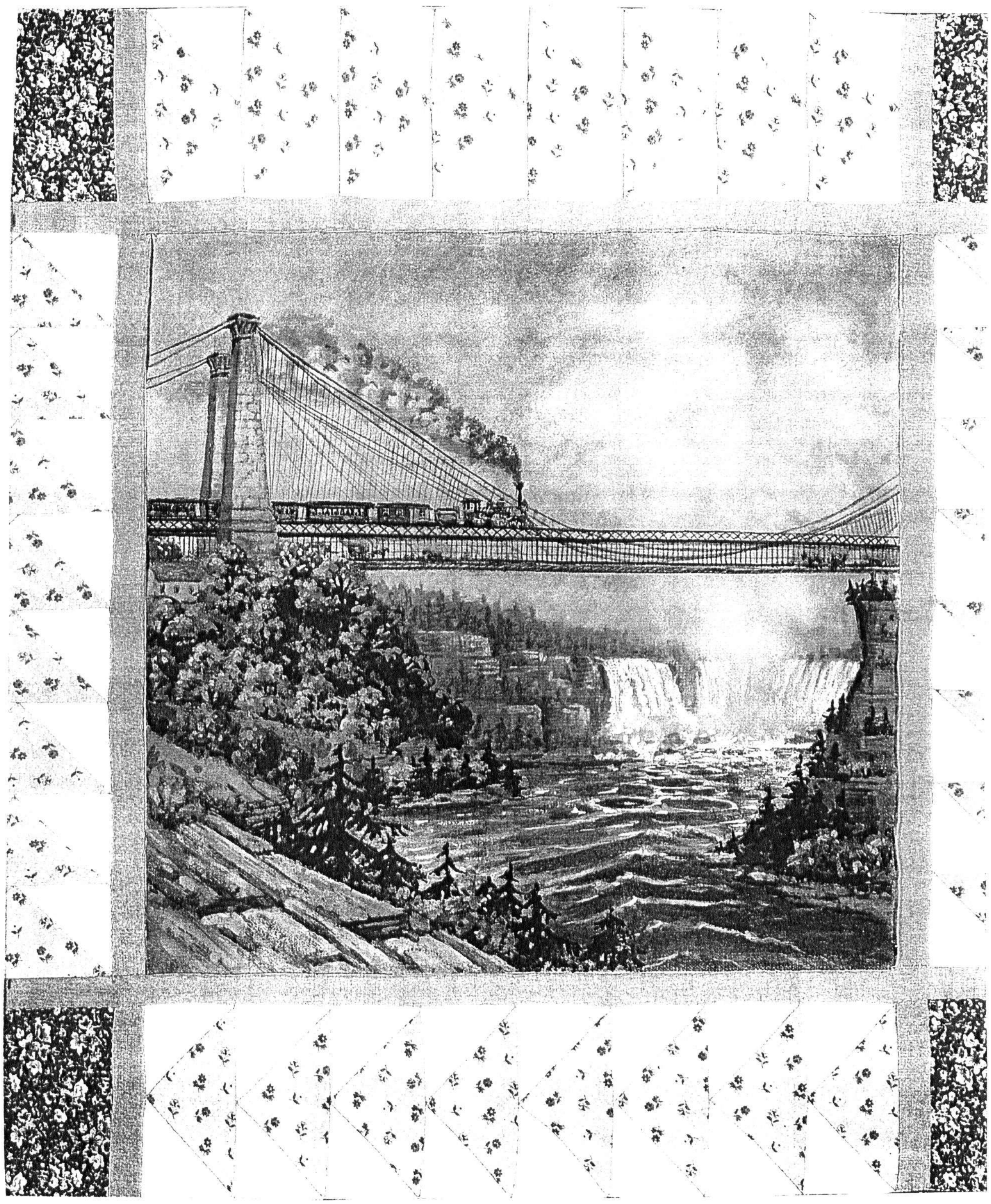
Figure 3: Back Cover of Book

The back cover of the book (see attached illustration) shows in colourful detail each quilt design in the order they appear in the text. Each one of the quilt borders was especially created by a quilter who incorporated old methods and patterns into the original designs. The illustrator then used these quilt patterns to frame the illustrations, page by page. The comparing activity extends to specific pieces of fabric incorporated into the quilt borders. The children identify pieces of fabric as also being part of quilts seen earlier in the book. This comparison activity is noteworthy, as it seems to set the stage for the discussion that follows.

The children and I flip backwards and forwards through the book looking at, comparing, and discussing aspects of the quilt patterns found on various pages. They are interested in the fabric and how the pieces change or are repeated in subsequent quilt designs. The shape and details on each fabric piece are of particular interest to the group. The children's comments center on how different fabric pieces have been incorporated into each preceding quilt design. Initially, the group debates whether or not the Bear Paw design is being created as the book proceeds and each page reveals an altered design. Now, the group decides that not one but many quilt patterns are being featured in the text. A consensus emerges, the back cover will show them the order that the quilt designs will be found in the book.

We stop reading the story text and go back to discussing the illustrations on pages 21 and 22 and the "Flying Geese" pattern.

Figure 4: Illustration on page 21 including the Flying Geese Pattern



Kevin is engrossed in finding a way to make current quilt designs match earlier designs. He is trying to relate what he sees in the "Flying Geese" pattern with one aspect of the "Four Patch and Hourglass" design.

Kevin continues to use hand and body motions to help him explain how the various triangles can be turned to become 'like' the configuration of triangles on pages 17 and 18 in the "Four Patch and Hourglass" design (see Figure 3). But the comparing and physical manipulations also extend to Kevin's earlier explorations and attempts to make a square from two triangles.

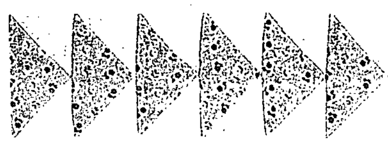


Figure 5: Small Frame of Flying Geese Pattern on page 22.

Kevin: Know these one up here? Just turn that around, that one around, and would make kind of like (voice trails off).

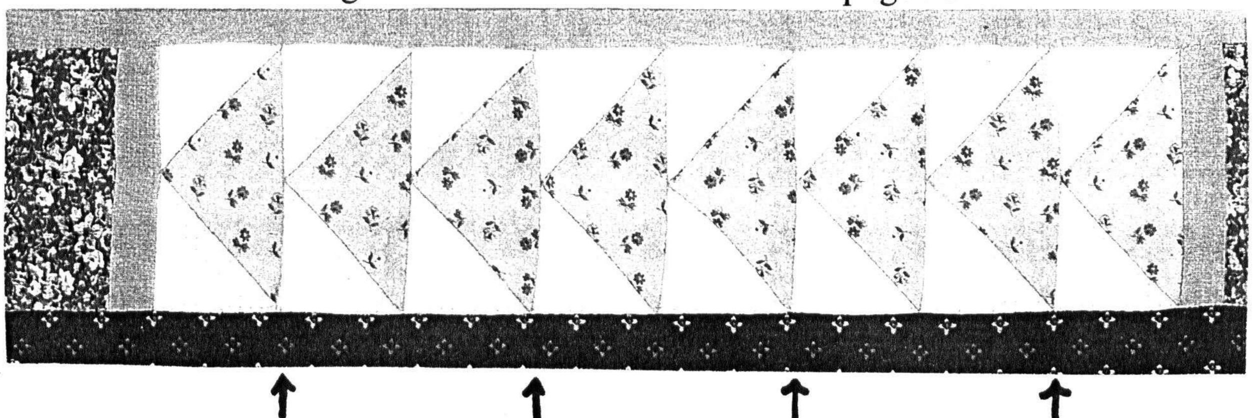
Kevin points to the small, framed quilt design that is placed above the story text on page 22. This box contains six triangles arranged in a row. Kevin states that turning every second triangle will make a different arrangement of triangles. He tries to explain what the new design will look like.

This discussion seems to prime Kevin's thinking about the array of triangles at the top of the page. Other writers have documented how children's literature and the groups' comments stimulate mathematical thinking in group members.[64]

Kevin seems to be imagining how the string of triangles can be altered to become similar to something he has seen before – the "hourglass" arrangement of triangles in the Four Patch and Hourglass design. He is pointing to every second triangle in the pattern and describing orally and with hand actions how turning every second triangle will create a row of three hourglasses.

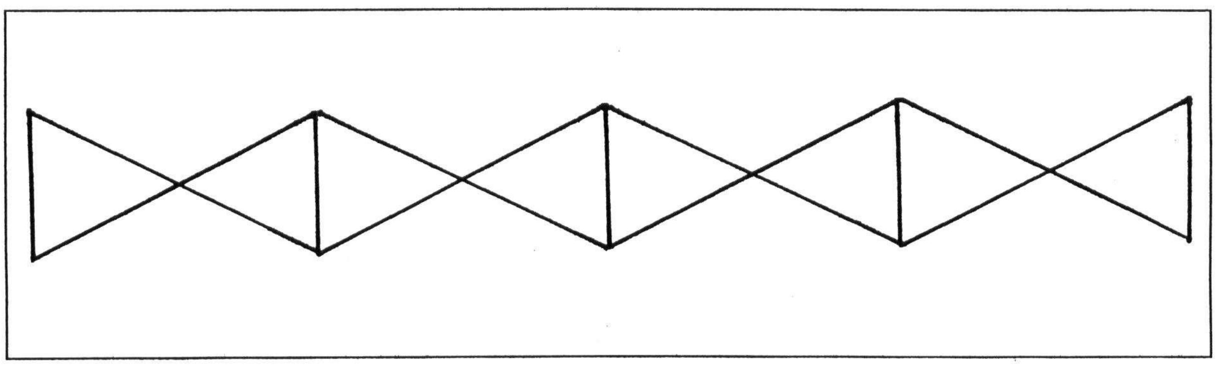
When I remain quiet and do not respond to his previous comments about making hourglasses out of the row of triangles, Kevin points to a row of 8 larger triangles positioned at the bottom of the quilt frame. Pointing with his finger, Kevin explains how turning every second triangle (starting from the right side of the row) would create four 'hourglass' arrangements of triangles.

Figure 6: Portion of Illustration on page 21



Kevin: same with this...If you turned that one around and that one around and that one around and that one around and that one around it would be like skipping.

At first, it wasn't clear to me which triangles Kevin was referring to since his body shielded the camera. Even though I couldn't see each of the triangles Kevin pointed to, he ended the discussion by pointing to the last triangle on the left. Coupled with his comment that "it would be like skipping", I infer that Kevin is mentally turning every second triangle (beginning on the right side). By turning the first, third, fifth, and seventh triangles, four "hourglasses" will emerge.



Kevin: If you turned it around it would be as the page before. Go back to the page before this. You know how I was thinking about that the last time.

(Kevin waves his arm and finger to the left as if to indicate an earlier page).

Come back to this page.

(Kevin points to the Bear Paw design on page 7).

Me: (silent personal thinking) I really don't know what "thinking" Kevin is referring to or which observation is he talking about. Is Kevin referring to the triangles in the "Four Patch and Hourglass" pattern?

We look at the "Four Patch and Hourglass" design and the "Flying Geese" pattern for awhile before Kevin explains again, that no, he doesn't mean the "Four Patch and Hourglass" design but one seen previously. I cannot follow his thinking but remain silent, waiting for Kevin to explain further.

As mentioned previously, I am learning to not leap ahead of the children in my efforts to be helpful. Kevin needs this time to sort out his own thinking about the shapes and how each configuration compares with arrangements of triangles seen on previous pages.

In fact, Kevin is quiet for a long period of time. He seems to be using his current mental image of an "hourglass" as a means to make a comparison to the configuration of triangles he now recalls are on page 7, in the "Bear Paw" design.

Me: (silent personal thinking) But how does Kevin see an "arrow" arrangement of triangles as being related to our previous discussion and manipulations of triangles?

Kevin: If you turned it around it would be the same?

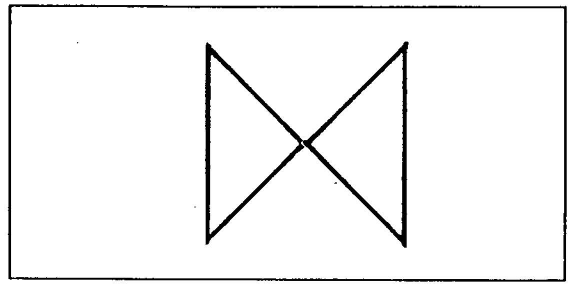
Me: *How would you turn them?*

(Kevin places one hand by the first triangle in the row and makes a motion with his left cupped hand, as if he was turning a door knob).

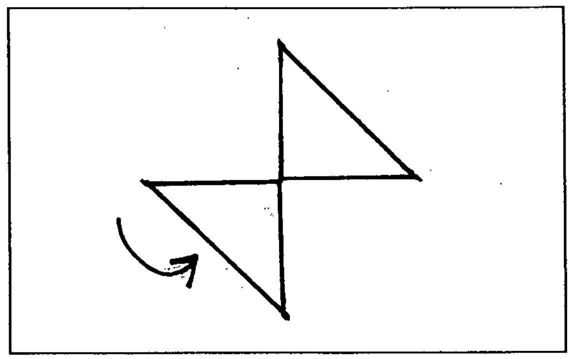
Kevin falls back to using hand gestures to help him state verbally how he mentally visualizes the triangles turning and becoming "the same" as the triangles in the "Bear Paw" quilt. As mentioned previously, Kevin relies on physical gestures and motions, when words fail him, to explain his thinking.

After watching the tape repeatedly, I now understand why I couldn't follow Kevin's explanation. In the midst of our discussion, I believe Kevin is still discussing how he mentally turns one triangle to form an "hourglass". Since this is my understanding, I am unable to recognize how Kevin is relating this configuration to the Bear Paw design. Now, I realize that the "turning" action Kevin is describing is directed at the "hourglass" image. Kevin has taken his thinking one step further than I initially realized. The image Kevin holds in his mind and later turns, it that of an "hourglass", not a triangle. In going that one step more, Kevin mentally turns the hourglass and realizes that the arrangement "is the same" as the configuration previously discussed in the Bear Paw quilt. Since I cannot "see" Kevin's mental image, it takes repeated viewing of the videotape to realize that it is an "hourglass" not a triangle that is being turned in Kevin's mind.

Kevin: Like this (Kevin uses the Power Block triangles to copy the arrangement of triangles in the book).



Kevin: If you turn them around and they'd be like this.



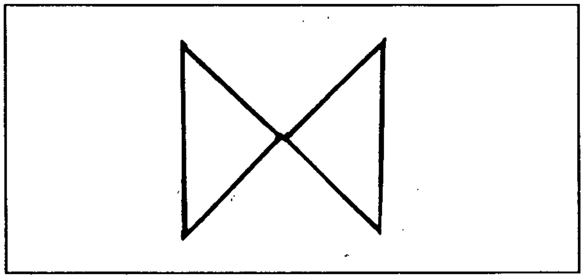
Kevin begins his explanation by modeling how the triangles are arranged in the book on page 21. When he turns the one triangle, he creates an "hourglass" design. This development causes Kevin to hesitate in his explanation – something doesn't seem to look right!

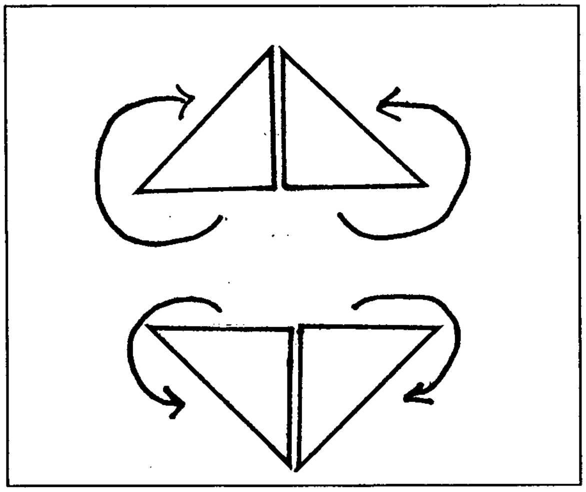
The Power Block triangles appear to be more of a hindrance than a help in this situation. Unlike previously when the triangles were used in Kevin's explanation, the concrete triangles seem to divert Kevin from his mental images and transformations. Relying solely on his own images allowed Kevin to mentally shift and turn the triangles with a fluidity and sophistication not possible with stiff, plastic triangles. In fact, the concrete materials may interrupt his mathematical thinking. Kevin makes an assumption that he must take a mental step backwards and begin the demonstration by first copying the position of the triangles as they are in the "Flying Geese" design. Kevin's next step is to turn one triangle two times so that an "hourglass" is formed. Suddenly, Kevin is unsure how to proceed. When Kevin relied solely on his mental images, the initial image of an hourglass was effortlessly turned and recognized by Kevin as being the same configuration as in the Bear Paw quilt.

Kevin: If you put it like this...

Kevin: Go back to the other page.

(Kevin looks at the Four Patch and Hourglass quilt and copies the hourglass).





Kevin: Trying to make the square like I made last time.

Kevin looks at the row of triangles in the "Flying Geese" pattern and positions the triangles to make an "hourglass". He then begins turning a triangle upward to form a larger triangle then downwards to again form another larger triangle. Kevin continues to repeat these same actions for some time, forming and reforming larger triangles with each upward and downward turn. Eventually, Kevin admits that he is having difficulty making a square as previously accomplished when we discussed the "Bear Paw" quilt.

It appears that Kevin is trying to bring together all the bits of information he has gleaned about triangles. In particular, Kevin is attempting to use his new understanding about how shapes can be formed by turning triangles in various ways. It isn't a surprise that Kevin has difficulty recalling how two triangles were transformed into a square. In this previous discussion and demonstration, I moved one triangle in a nonmathematical fashion. As mentioned before, the demonstration was quick and involved some impossible motions for a two-dimensional shape. It is not surprising then, that Kevin now experiences difficulty figuring out how the triangles can become a square. What is noteworthy, however, is that Kevin is trying to consolidate all this previous information to help him in this current task of making a square out of two triangles.

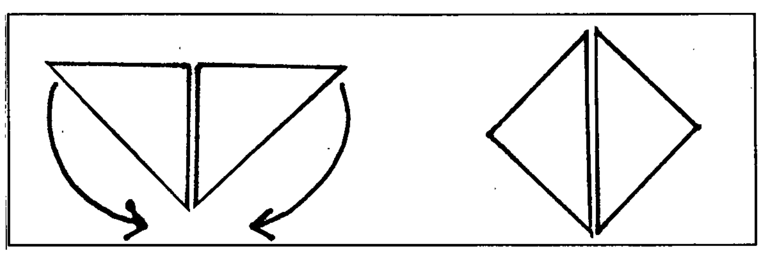
(Continuing with the dialogue).

Jacob: You need more.

Jacob informs Kevin that the only way to make a square from an "hourglass" arrangement is to fill in the two empty spaces with two more triangles.

Jacob has adopted one of Kevin's initial strategies for making a square! This `filling-in' approach was previously discussed as a way to make a square out of two triangles. It is interesting that Jacob has fallen back to this strategy. It could be as well, that Jacob is merely reminding Kevin of this previous strategy in an effort to be helpful.

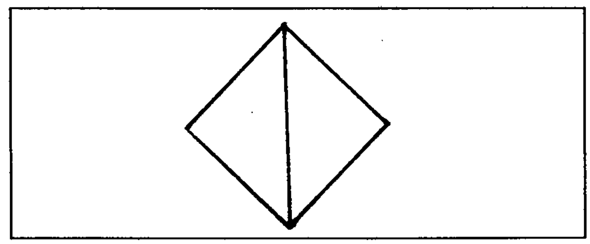
Alan: Just have to go like this.



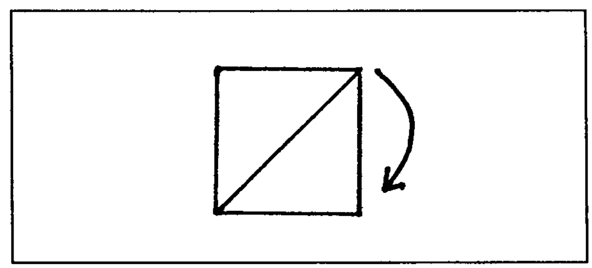
Alan leans forward and shows how the triangles are each rotated downwards to form a square. Kevin is showing the group a large triangle formed by holding both smaller triangles together (Kevin repeats this actions for several minutes). Alan grasps each of Kevin's hands and turns each small triangle downward. In this way, Alan swings the two long edges of the *triangles around until they meet and form a "diamond". Alan then rotates the shape to reveal a square.*

Alan has not been a vocal participant in this scenario but it is evident that he has been thinking geometrically while we read and discussed the story. Alan is now ready to use what he knows about how shapes can be transformed and shows Kevin how a diamond/square is made by turning both triangles.

Kevin: It would make a diamond



Jacob: And it would make a square.



Kevin uses the Power Block triangles himself to repeatedly perform the actions Alan used to create a square. Kevin continually rotates the shape to convince himself that the newly-formed shape really is a diamond. Or, as added by Jacob, also a square.

Earlier discussions centered on how a square is also a diamond. Kevin, Jacob, and Alan come back to this loose definition of square/diamond and demonstrate with the plastic triangles that a diamond becomes a square and vice versa by rotating the shape. After all, for these boys, a diamond is just a square resting on a corner!

The problem has been solved. A square can be made by first turning two triangles into an "hourglass" then by turning each triangle downward twice (first into a larger triangle) until a *diamond/square is formed. Now we can continue enjoying the tale and Selina's journey to Upper Canada!*

It is difficult for Selina to adjust to a new country and home but her grandmother's "Bear Paw" quilt brought her comfort and warm memories of the home and family she had left behind. When her cousins show Selina that their quilt also includes pieces of Grandmother's wedding dress, Selina's heart becomes lighter. The story concludes as Selina realizes that there is peace in her heart. The memory of her grandmother will always be with her, woven into the delicate design of her "Bear Paw" quilt.

Blossoming

This Story began with the seed of an idea – bringing together my love of children's literature and my fascination with how children think and learn mathematically. Was it possible to ensure that the lens of research wouldn't destroy the richness of the story setting? I passionately wanted to preserve the beauty of literature while documenting the intricate layers of my students' responses and mathematical thinking and understanding. The seed, then, encompasses the desire to preserve The Story – a story that reveals within its pages my student's experiences with books linked with their personal mathematical learning and thinking and as a subtext, the story of my own professional growth.

When I began thinking about the shape my research would take, I struggled with my dual roles as researcher and classroom teacher. What form would my teaching take if I was also the researcher? All teachers need to be researchers in their own classrooms but usually in informal ways.[65] Whether as educator or researcher, I am in a privileged position and can capitalize on the close personal relationships that develop with children. It is through such interpersonal relationships that children's math activity can be analyzed. As a researcher, I can tease out recurrent features of the math activity that is building a blueprint of students' math knowledge. It is this knowledge that informs my teaching and my research and guides my interactions with different children. Teaching, researching and personal learning are inextricably interwoven, but as a result of the creation of this Story, I am a little closer to understanding my role as a teacher as well as researcher. As argued by Cobb and Steffe[66], teaching and researching go together. I am learning to trust my instinct about what is needed when reading and talking about a story, to either remain quiet or interject with information.

The seed began to germinate as I turned my teaching gaze back over the last few years. I reflected on why I felt and thought so passionately about children's books and my students' mathematical experiences with each tale. It was then that I realized my passion was being fed by what was happening when we read and talked about books. Something powerful was happening every time we shared a story together. As the children talked about the math they saw in stories, I was struck by the realization that there was something unusual and exciting about this learning experience. In my musings about the past few years, I pondered how I, too have been caught up and altered by the same learning experience. It is out of this type of reflection that I began to question my practice: how did I know for sure that my students' learning was being impacted by reading and talking about books? My sense remained that something special was happening when we read and discuss books together. It was this "something" that became the focus of my research. Others have written about this is general terms. I wanted to try to look in fine detail at a specific classroom incident to capture the essence of the experience.[67]

Out of the seed of an idea sprouted two questions. Firstly, what is it about the shared reading situation that creates a fertile context that helps individual children make mathematical connections? Secondly, how does quality children's literature capture their imaginations in a way that enhances the possibility for them to think mathematically in deeper, more reflective ways?

Videotaping was selected as a viable means to put our shared reading sessions under a research "lens". Videotaping gave me the luxury of time to listen again and again to the children's comments while we read and talked about Selina and the Bear Paw Quilt together. I was able to listen to and reflect on what the children's comments and gestures revealed about their mathematical understanding and thinking. At the same time, I was able to examine my own teaching practice, and set the chosen incident against my analysis of other videotaped shared readings.

The research questions form a landscape. By looking at the fine detail of this landscape, the story of Kevin and Jacob in the shared reading context, I attempt to illuminate the questions. I did not expect to find discrete results (i.e. it is numbers 1, 2, and 3 that make shared reading valuable). Rather, through the minute examination of an event occurring during shared reading (which is typical of what goes on when I share books with children) I am illustrating for the reader the kind of exciting mathematical understanding that can take place and explaining why it is the shared reading that helped it to happen. I am not claiming that this is the only way it can happen, just suggesting the value of shared reading.

Out of research activity, grew four main stems of understanding about children's literature and children's learning and understanding of mathematics. I will address these stems of understanding by connecting them to each of the research questions.

What is it about the shared reading situation that creates a fertile context that helps individual children make mathematical connections? The first stem of understanding is linked to the highly personal learning experience that literature gives children.[68] Personal responses to literature make it possible for a teacher to help students scrutinize their responses, thus, deepening their understanding. Kevin and Jacob entered the experience from their own entry points and levels of mathematical understanding. The inclusive nature of a story spans the diverse learning needs of a classroom by providing a bridge that gives all learners an opportunity to strengthen and build on their knowledge.[69] The story context seems to give children a safe environment to work out their understanding of mathematical concepts. In this setting, children merely chat about their conceptual knowledge as opposed to having to perform or demonstrate their understanding. During the sharing of Selina and the Bear Paw Quilt, Kevin and Jacob were able to explore their understanding of aspects of geometry, consolidate their personal knowledge, and use that new understanding to manipulate shapes. The second stem of understanding relates to the ways a child's voice and informal language can be celebrated and maximized in this learning experience. Kevin and Jacob were able to use their own words such as "squishing" to talk about their current understanding of geometrical concepts and to create new connections between aspects of their personal knowledge. As well, Kevin and Jacob gave voice to transformational ideas that were no part of their formal curriculum work. During the shared reading of Selina and the Bear Paw Quilt, Kevin and Jacob deepened their understanding of geometry.

How does quality children's literature capture their imaginations in a way that enhances the possibility for them to think mathematically in deeper, more reflective ways? The third stem of understanding involves all the ways that the illustrations and text engage children's imaginations, evoking mental visualization and enabling mental manipulations of such things as shapes. The quilt patterns framing the illustrations in Selina and the Bear Paw Quilt were powerful mental stimuli eliciting geometric images. Aspects of the quilt patterns stimulated and supported Kevin and Jacob's mental manipulation of triangles as they shifted and transformed the triangles into squares, diamonds, and 'hourglass' shapes. The fourth stem of understanding is related to the ways children's literature and the shared reading context stimulate and, in particular, offer an opportunity to sustain mathematical thinking. While responding to the story, Kevin and Jacob engaged in problem solving and (to a lesser extent) problem posing. These boys challenged their peers' ideas while debating the reasonableness of their own and others' thoughts. The discussion evoked by the text and illustrations challenged Kevin and Jacob to explain and justify their thinking. While ideas were being probed and examined, Kevin and Jacob's thinking was also clarified. While sharing Selina and the Bear Paw Quilt the text, illustrations, and comments made by other children and the teacher stimulated mathematical thinking in the whole group.

The lens of scrutiny also revealed an additional stem of understanding – with respect to my own teaching role and practice. I am one partner in the dance of learning. My performance is a balance between respecting the child's need to construct personal knowledge and performing my role as representative of the mathematical community. I provide the nourishment and elements for growth but my students do the growing – of their own mathematical knowledge.

Growth of a practice and a learning experience has begun and in time, fruit may form. As we continue to share books and our thinking together, it is my personal desire that my students' and my own learning will continue to grow into the solid fruit of greater mathematical knowledge.

I have written Chapter One. I have offered my interpretation of one shared reading session. But even a "good interpretation, is not definitive and final".[70] Over time I, and others, will continue to probe the effects of literature on learning and will add to the Story. In particular, we need to explore much further the incidental, unintended mathematical understanding that can grow from reading <u>everyday</u> stories together, which has been the focus of this study.

The Soil

[1] Videotaping and a handwritten personal journal.

[2] Micro analysis of language, pictures, and actions.

[3] Jardine, D. W. (1998) describes interpretive inquiry as way to uncover "a truth, an understanding" of the incidents of our lives. The inquiry begins with an "incident", when "something addresses us" (Godamer, 1989), when one is struck by something that seems already familiar but which is not fully understandable. What follows are my attempts to understand in fresh and new ways what previously was believed to be already understood. I intuitively understood my students' mathematical understanding was enriched by the literature experience. Now, my journey is to understand more fully how and why a child's book sparks and supports mathematical learning and to effectively communicate that understanding to potential readers. My journey takes a form that is eloquently described by Jardine:

> "Producing a "reliable" interpretive reading of this instance requires living with this instance for a period of time in order to learn its ways: turning it over and over, telling and retelling it, finding traces of it over and over again ... scouring the references colleagues suggest, searching my own lived-experiences for analogues, testing and retesting different ways of speaking and writing about it to see if these different ways help engage and address possible readers of the work to follow."

[4] Jardine, D.W. (1998) is clear that "interpretive inquiry does not wish to literally and univocally say what (an) instance is. Rather, it wishes to playfully explore what understandings this instance makes possible". In this current study, I scrutinize the shared reading event and offer the reader possible but not literal, interpretations of the mathematical significance of my students' explanations or physical actions. The interpretations also surfaced through indepth discussions with Susan Pirie (my thesis supervisor) that occurred frequently during my analysis of the videotaped literature session. The importance of such discussions is underscored by Jardine in his writing:

> "None of us necessarily knows all by ourselves the full contours of the story each of us is living out. That is why dialogue and conversation figure so predominantly in interpretive work."

[5] (In press). Jenner, D. & Anderson, A. (1999). Math Through Literature, The Story of Neil. Teaching Children Mathematics. Reston: NCTM.

[6] Ministry of Education of British Columbia (1995) mandated the implementation of the Mathematics Integrated Resource Package (I.R.P.) in Kindergarten to Grade 12 classrooms.

[7] National Council of Teachers of Mathematics (1991). The U.S.A. produced three documents one of which summarizes research on mathematical teaching and learning (Research discussed was conducted prior to 1991. A new document of standards has been written to address research completed after 1991). Key areas of focus are identified and are included as

'standards', markers against which educators can examine their own teaching practices and assess the mathematical learning of their students. The B.C. Mathematics I.R.P. (Integrated Resource Package) was written with the NCTM Standards as a guiding document.

[8] National Council of Teachers of Mathematics (1991).

[9] See Appendix 1 for a listing of children's books and their respective authors.

[10] I have kept a detailed log of chosen books including why specific books were chosen to share with the children at certain points in the research.

[11] Gailey, S. (1993) outlines six criteria to assess whether or not a book can be considered quality literature:

1. Interesting format that engages children
2. Appropriate size of type
3. Text and illustrations that interact with each other
4. Mathematically correct
5. Logical, sequential organization of content
6. Age-appropriate vocabulary

[12] Jardine, D.W. (1998) contends that the interpretive inquiry of specific incidents involves viewing an instance "as a text which must be read and reread for the possibilities of understanding that it evokes". As I examined the current shared reading session, other literature sessions and the contributions of various students were recalled and viewed with a new lens of understanding. The process of trying to understand the current shared reading experience evoked earlier instances of children's mathematizing and their responses to specific books. With the help of the current literature episode, past sessions could be viewed with new understanding. Discussions of earlier sessions became woven into this Story as a way of illuminating my new understanding as well as underscoring the power of literature in encouraging the growth of children's mathematical understanding.

[13] The diagrams within the boxes have been inserted into the text to provide the reader with illustrations of my understanding of what each boy is saying about how the triangles move.

[14] Tracking children's hand and body gestures as well as how they manipulated the plastic triangles played a critical role in helping me to understand the children's mathematizing. This underscores the value of videotaping to collect data. Without the images showing me what the children were doing in the midst of their explanations, my understanding of their mathematical thinking would have been limited. The videotapes allowed me to revisit the episodes numerous times to listen and watch their accompanying actions over and over again. In some cases, only then, did I begin to understand what each boy was trying to explain to the group.

[15] Anderson, A. & Anderson, J. (1995) conducted a case study of their daughter, Terri to examine the shared reading situation. They document Terri's spontaneous reactions to the books and her mathematical thinking and understanding while reading and talking about selected children's literature.

[16] Lewis, B., Long, R., and Mackay, M. (1993) also document how one story, So Many Cats, encouraged Grade One children to enter the discussion from many different entry points. The authors note that "all the students were able to contribute to the dialogue, even though they were at different developmental levels, because a `correct' answer was not expected."

[17] Vygotsky (1934) coined the term "zone of proximal development" to describe the optimal learning situation. While interacting with adults, a child's everyday conceptual thinking comes into contact with an adult's formal knowledge. When the child's understanding is stretched by new information but is still comprehensible, the interaction has occurred within a child's zone of proximal development. If the gap between the new idea and the child's knowledge is so large that it cannot be bridged, then the learning situation is outside the child's zone of proximal development.

For further elaboration on Vygotsky's ideas, see Berk, L. & Winsler, A. (1995); Smith, L., Dockerill, J., and Tomlinson, P. (1997).

[18] See Baroody and Ginsburg (1990) who contend that children are interested in 'moderately novel' information and will make the effort to assimilate such information because it makes some sense and is, therefore, important to them.
Also, Davis (1992) who argues that conceptual understanding develops when a child can fit a new idea into his/her larger framework of previously assembled ideas.

[19]see various writings of Piaget (eg.1967) for a broader discussion of assimilation.

[20] von Glasersfeld (1987).

[21] Winzer (1993) defines mainstreaming as "both a philosophy and a process. It is the physical, intellectual, social, and emotional integration of exceptional children and youth into the regular educational milieu". Children who are exceptional are individuals who differ from the norm in some way.

Crealock and Bachor (1995) extend the definition of mainstreaming: "Mainstreaming is a philosophy that involves acceptance, attitudes and values that assume all students can learn and act in an appropriate manner".

Ministry of Education in B.C. incorporates both definitions and in advocating mainstreaming for exceptional children also promotes the concept of normalization (philosophical belief that all exceptional individuals should be provided with an education as close to normal as possible) and the concept of least restrictive environment. It is mandated that exceptional children will receive a normal education in the least restrictive environment.

[22] Law, B. and Eckes, M. (1990) define English as a Second Language as a term ESL "referring to those students whose first language is other than English, and whose proficiency is not high enough to perform equally with their English-speaking peers." The acquisition of a second language parallels how children learn a first language. Firstly, children learn a language by listening to the language being spoken around them. Secondly, by speaking the language and later, by reading and writing the language. Language exposure needs to be meaningful and can

be maximized through cooperative learning situations where communication is natural and rich whole language experiences exist. They advocate the use of visuals such as pictures and picture books and related discussions to promote language acquisition and comprehension.

[23] von Glasersfeld's (1987) view of learning blends sociocultural and radical constuctivist theories. Learning is a private process which cannot be witnessed by anyone else; all one has to go on is the visible results of those mental operations, the writing, speaking or other behaviour of students. Even though children construct their own knowledge, their constructions are constrained by society and interactions with others. "Knowing" is shaped by the child's experience, but also by the influence of social interactions.

[24] Whitin, D. (1992) contends that literature experiences are times when children can spontaneously respond to a story's ideas. The shared reading experience is open-ended and unstructured allowing children to make personal statements about the math in a story. It is these personal statements that allow children in a group to hear multiple interpretations of story themes. Hearing about how other children interpret the math in a story enriches the group's mathematical understanding.

[25] Winzer, M. (1993) suggests that teachers of students with learning disabilities refocus children continually in learning situations using verbal cues such as "listen carefully" or "this is important".

Osborne, S., Kosiewicz, M., Crumley, E., & Lee, C. (1987) identify distractibility as a particular difficulty for special learners (i.e. children who are impulsive, have short attention spans, or difficulty concentrating). The authors suggest that teachers use focusing techniques to help such children pay better attention during learning situations.

Bootzin, R. and Acocella, J. (1988) contend that autism's most distinguishing feature is self-absorption. Some autistic children respond to shaping and modeling techniques to develop appropriate behaviors such as better listening and attending skills. The authors advocate placing autistic children in group situations where they can learn by observing others. Group situations provide a way for teachers to model listening and attending behaviours as well as give autistic children a way to practice these skills.

Law, B. and Eckes, M. (1990) emphasize the importance of promoting listening as a specific language skill with English as Second Language students. They urge teachers to use a range of strategies to strengthen listening skills such as: focusing children's attention in physical ways, using linguistic cues like "look or watch" to direct student's attention.

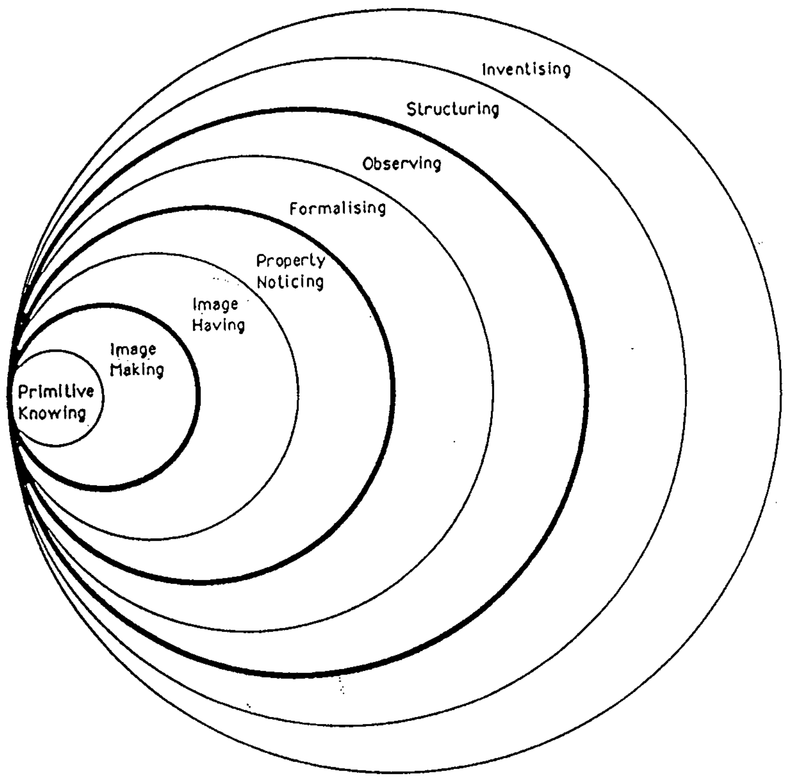
[26] Pirie, S. (1998) uses the term quasi-mathematical language to describe language used by pupils that has, for them, a mathematical significance not always evident to their teacher. The author contends that the use of quasi-mathematical language can often enhance mathematical understanding by helping students form a language-linked image that has personal relevance for learners.

Pimm, D. (1987) also views mathematics as a language children need to learn and use as a way to talk about mathematical ideas. He contends that children's informal languages are attempts to describe mathematical ideas in metamorphical terms. The term metaphor is used by Pimm to describe how children use informal language as a means to experience and understand mathematical ideas within the framework of their own language.

[27] The Pirie-Kieren (1994) model of the growth of mathematical understanding emphasizes that children develop understanding (and therefore, a multiplicity of images and an ability to move flexibly between images) in a nebulous, non-linear, and fluctuating fashion (see attached diagram). The model indicates how a student's understanding for a concept can be built from varying starting points. I am only concerned in this thesis with the inner layers. Initially, the actions (actual or mental) of a learner during a learning experience are attempts to get some idea of the concept. The authors call this stage Image Making. If the learner Has some Images for a concept she does not need to rely on actions but can carry and use the ideas she has constructed.. However, at the Image Having stage, images may be incomplete, inappropriate or insufficient. Learner images that are limited or inaccurate hinder learning and the student needs to "fold back" to the level of Image Making to develop a broader range of images. The Property Noticing phase of the Pirie-Kieren model is useful to consider how conceptual understanding unfolds with children. Manipulating or combining aspects of their images allows children to develop flexible, useful conceptual knowledge. The term "don't need boundaries" describes a child's progression in developing understanding. Sfard (1991, 1994) describes this progression in understanding as reification. She argues that reification, the movement from an operational (process) to a structural mode of thinking, is a difficult but necessary transition to develop understanding of abstract concepts. When a mathematical object comes into existence, mathematical understanding is deepened. Learners are now able to work with ideas without the need to mentally or physically refer back to specific images (hence they "don't need" the knowledge contained within the boundary). They can now operate at a symbolic level without reference to basic concepts.

See Pirie, S. & Kieren, T. (1994a and 1994b) for more details about the Pirie-Kieren model.

Figure 7: Pirie-Kieren (1994a) Model of the Growth of Mathematical Understanding



[28] Wheatley and Cobb (1990) come to this conclusion based on their analysis of Grade Two children's spatial constructions.

[29] Baroody and Ginsburg (1990)

[30] Griffiths, R. and Clyne, M. (1991) contend that literature has the potential to move children from using informal language and recording to formal mathematical language and recording. They argue that this movement will not occur spontaneously. Literature experiences assist this movement by providing opportunities for children to have: "varied experiences of the concept or topic, opportunities to share ideas, demonstrations from other children and adults of ways of dealing with the concept or topic, challenges to their thinking and to stimulate thought, opportunities to reflect on and refine their ideas."

Edwards, D. & Mercer, N. (1987) define common knowledge as that knowledge that comes to be shared between people. The authors contend that when two people communicate, there is a real possibility that by pooling their experiences they achieve a new level of understanding beyond that which either had before.

[31] Jacob's finger actions, that of holding his fingers up to the book and performing the 'squishing' movements, plus his verbal explanations were used to describe the appearance of the two resulting shapes.

[32] Wheatley, G. and Cobb, P. (1990) examined Grade Two students working with geometric shapes to explore the images, re-presentations and transformations that children engage in when acting on geometric shapes.

[33] Jacob appears to use the word "squishing" to describe a rotation.

[34] Ohanian, S. (1989) contends that reading and discussing books encourages divergent thinking. Stories provide a context for children to make up story-related questions for investigation. She also suggests that the questions raised in a text encourage children to pose similar or related questions for others to solve.

Griffiths, R. and Clyne, M. (1991) view books as a powerful means of communicating ideas and concepts, of raising issues and posing problems. Stories pose problems directly or implicitly involve children intrinsically in solving problems. The authors contend that as children work through the problems posed, they become involved in "creating and solving new problems".

[35] Whitin, D. and Wilde, S. (1992) suggest that children's literature supports the art of problem posing. The original story and supporting details become the basis for related problems.

[36] Brown , S. and Walter, M. (1983) write extensively on the topic of problem posing. The more opportunities children have to alter a particular problem, the greater their understanding of the underlying mathematical concepts. Playing with problem variables produces the irony that "we understand something best in the context of changing it"(page 123).

[37] Writing in a journal is part of my methodology for this research. I needed a way to record my reflections (summary of observations, feelings, conjectures, etc.). A journal took the place of structured field notes since what I have recorded are reflections written after each literature session. However, field notes and photographs were also kept to accurately document and describe students' work created on the chalkboard to illustrate their thinking and strategies. The journal entries record my journey and professional growth as I document my questioning and cognitive and emotional struggles surrounding how I was using books in my mathematics programme.

[38] Pirie, S. & Kieren, T. (1992) contend that a constructivist environment is created by a teacher who acts on the belief that all mathematical knowledge is personally constructed and organized. The authors discuss a classroom episode where a routine task triggers different, yet compatible understandings that contribute to the growth of understanding within the class. In a constructivist environment, the assumption of different understandings and different levels of understanding leads a teacher to allow for and seek the different mathematical understandings shown by students. The authors argue that teachers must prompt students to justify what they say or do, thus revealing their thinking and logic. Questioning and prompting students, then, exposes different levels of understanding.

[39] Ohanian, S. (1989) advocates using children's literature as the means to generate open discussion and honest criticism of ideas.

Griffiths, R. and Clyne, M. (1991) contend that "story telling is a universal and age-old form of communication". Books involve children in "making and checking hypotheses and explaining and justifying solutions".

[40] Ohanian, S. (1989) discusses how children's literature encourages "genuine give and take in the classroom". The spark for rich mathematical discussions comes from books as children respond to stories in personal ways. Since their responses are personal, the children are in control of the discussion, of the direction the discussion takes, of the knowledge that is being built in the midst of talking about math ideas.

Lewis, B., Long, R., and Mackay, M.(1993) discuss how books can engage children in communicating mathematical ideas. The authors document how books provide a natural, meaningful context for encouraging students to talk about mathematics. When sharing books, the authors note that teachers provoke mathematical thinking through questioning or by just allowing time and opportunity for students to respond spontaneously to the ideas in the story.

[41] Vygotsky, L.(1934) elaborates on the relationship that exists between the development of thought and language. Thought that is not expressed stays immature and eventually dies away. Vygotsky states that language is necessary to the formation of thoughts as well as to express already formed thoughts.

[42] Pirie, S. (1997).

[43] Simmt, E. (1998) describes her parent-child mathematics programme. She notes that in each parent-child partnership discussion played a major role in the mathematical thinking and learning that took place. The parent's thinking stimulates the child's and in turn, the adult's thinking is stimulated by the child. Simmt also notes that the parent-child interactions enhance their personal and shared understanding of mathematics and that possibly the close family bond allows an assumption of understanding.

[44] Vygotsky, L. (1934) uses the term intersubjectivity to describe the shared social world where an adult and child engage in the negotiation of meanings. In this shared world, an understanding exists that what the child says will be understood by the adult.

Cobb, P., Yackel, E., and Wood, T. (1992) argue against the notion that math learning is just a matter of acquiring an accurate internal representation of an environment. The authors propose instead a metaphor of mathematics as an evolving social practice that is constituted by the constructive activities of individuals. Cobb, Yackel, and Wood argue that students actively construct their mathematical ways of knowing as the teacher initiates them into the taken-as-shared mathematical practices of the wider society. Teachers and students together interpret and influence each other's learning.

Voight, J. (1994) proposes an interactionist perspective on mathematical learning that is compatible with the constructivist perspective. Mathematical meaning emerges from between individuals, as the product of their social interactions. However, mathematical meaning must be negotiated between teacher and student. Voight uses the term "taken-to-be-shared" in a similar way to Cobb, Yackel, and Wood. Taken-to-be-shared describes how participants can interact as if they interpret the math topic in the same way even though one can never be sure that the two persons are thinking identically. Therefore, Voight contends that students don't learn math, they learn to negotiate math meaning with an expert, the teacher.

[45] Pirie, 1998.

[46] Cobb, P., Yackel, E., and Wood, T. (1992).

[47] Cobb, P., Yackel, E., and Wood, T. (1992) argue that learning is both a constructive and an interactive process. Thus learning is both an individual and a social process. In discussing their socioconstructivist view of learning, they argue that learning is constrained by the individual's current math understanding as well as by the group process. A tension exists in group work between children trying to build personal understanding of concepts and trying to understand others' explanations. The learner must learn how to negotiate a productive collaborative relationship for learning math. The authors use the term circularity to describe this interactive process of learning.

[48] I analyzed the video data by viewing the tapes numerous times. Each time, I would take notes about what I thought was happening with the children's thinking and mathematizing. It was after watching this particular segment many times that I began to understand what Kevin was saying about the triangles. Initially, I thought he was explaining a slide motion that would result in a diamond shape. After watching his hands with great care and listening to him, it became apparent that this was not Kevin's image. As my next class interaction reveals, I did not see all this at the time.

[49] The diagrams represent my understanding of what the child is describing and imaging based on their physical actions and comments.

[50] Pirie, S. and Kieren, T. (1994a) model of mathematical understanding, as described above.

[51] Towers, J. (1996) examined classroom discussion and the role of the teacher in "listening for understanding". Towers concludes that in knowing what they expect to hear, teachers listen for and credit students with an understanding that these same students many not have. She adopts the phrase "listening for understanding" from Davis (1996) to remind teachers that listening is an aid to understanding and that adopting a different approach to listening may promote greater understanding.

[52] Pirie, S. and Kieren, T. (1994a) note that "for any math topic there will always be a multitude of images formed". Individuals will continually form different images from the work they are doing. For any one math concept, the possibility exists that many different images can be formed (e.g. division: sharing and grouping).

[53] Harsh, A. (1987) contends that using books to teach mathematics fits the developmental learning sequence required by young children (i.e. Teaching Learning Sequence, Kennedy, 1984). Children first need concrete hands-on experiences preceded by activities at the semiconcrete-pictorial and the abstract-symbolic level. Harsh argues that children need experience with manipulatives (concrete level) but listening to and looking at books that deal with these same concepts give children a semiconcrete experience.

[54] Griffiths, R. and Clyne, M. (1991) address the concern expressed by many that using literature to explore mathematical ideas is "disemboweling literature for the sake of mathematics". The authors go on to state that: "it is true that teachers who attach to a story trivial or boring mathematical exercises will make children antagonistic to both the story and the mathematics...However, teachers who are sensitive both to the text and to the needs and interests of the children will assist children to reach a deeper understanding and enjoyment of both the text and the mathematical ideas inherent in the story".

Whitin, D. and Wilde, S. (1992) advise teachers to remember that first and foremost, books with a math dimension are good literature and need to be used appropriately with children. They suggest that teachers "don't destroy the magic of a story by interrupting it with mathematical questions as you read it aloud. Each book is a unique literary experience and should be enjoyed for its own sake. The first step... is an uninterrupted reading with time for spontaneous, unstructured personal response". In my class, the "spontaneous response" usually interrupts the reading!

[55] Pirie, S. (1996) comments on the claim made by others that video-recording is a way to capture everything that is taking place in the classroom. She contends instead that "who we are, where we place the cameras, even the type of microphone that we use governs which data we will gather and which we will lose". It is inevitable that the camera will not catch everything that is happening in the classroom.

In this current study, two cameras were used in all taping sessions. One focused on the book being shared with the children and the other panning the whole group of children. The first camera documented each page as the children talked about the story. This same camera also recorded my body language and facial expressions during the literature sessions. The second camera recorded all the children's actions and comments during the reading. Yet, even with this careful set-up of cameras I missed recording how Kevin was holding the triangles.

[56] Wheatley, G. and Cobb, P. (1990).

[57] Pirie, S., Martin, L., and Kieren, T. (1994) discuss the differing images held by students and their teacher and how these different images interact during the learning of specific math concepts. The authors use the term "cycles of responsibility" to describe how teacher and student images impinge on the growth of understanding (i.e. teacher to student). If the teacher holds limited images, this will have a significant impact on the images constructed by her pupils and on their ability to understand concepts. The authors indicate that a multiplicity of images and an ability to move flexibly between those images enables students to more easily grasp new concepts. Therefore, if a teacher has only one image her students' progress will be hindered.

[58] Wheatley, G. and Cobb, P. (1990).

[59] Pirie, S. (1996) discusses how videotaping allows classroom episodes to be preserved in their entirety to be examined and re-examined in the future. Videotape buys the researcher time, granting him/her the leisure to reflect on classroom events.

[60] von Glasersfeld, E. (1987).

[61] Yackel, E. and Cobb, P. (1996).

[62] Steffe, L. and Tzur, R. (1994) emphasize that social interaction is a primary means of engendering learning and building children's mathematical knowledge. They contend that learning is directly related to the individual's capability to change his or her conceptual structures in response to perturbation.

[63] Davis, R. (1992) and Davis, R. (1993).

[64] Lo, J., Wheatley, G., & Smith, A. (1994) investigated student discussion in a group setting. They conclude that listening to other students' explanations prompts and stimulates the mathematical thinking of individuals. Their observations of "Brad" in the study led the authors to conclude that class discussion can also provide opportunities for individual students to connect and integrate their mathematical knowledge.

Whitin, D. (1992) provides illustrations and scenarios that support his contention that children's literature supports mathematical communication.

Anderson, A. & Anderson, J. (1995) conclude that sharing a book with a child seems to naturally involve talking, a natural scaffold to prompt and develop mathematical thinking.

[65] Cobb, P. and Steffe, L. (1983).

[66] Cobb, P. and Steffe, L. (1983).

[67] My intention is to use a micro-view to inform the macro. This thesis is an attempt to examine in fine detail the shared reading context to see if such a scrutiny will shed some light on the broader questions about children's mathematical understanding in the story context.

[68] Rosenblatt, L. (1938) states that "the reading of any work of literature is, of necessity, an individual and unique occurrence involving the mind and emotions of some particular reader". An interaction occurs between what the reader brings to the literature session (knowledge, previous experiences, images) and the knowledge engendered by the book. The author contends that readers project onto the story, elements from their own experience, that may or may not be relevant to the story. These projections may be something only vaguely suggested by the story. The author contends that literature provides the reader (and hearer) with a medium to explore his/her thoughts.

[69] This is not to suggest that other learning tasks cannot or are not also inclusive and allow for personal entry points. I am suggesting instead that this is one way that I have been examining in this research.

[70] Jardine, D.W. (1998).

Bibliography:

Anderson, A. & Anderson, J. (1995). Learning Mathematics Through Children's Literature: A Case Study. *Canadian Journal of Research in Early Childhood Education,* 4, 2, 1-9.

Baroody, A. Ginsburg, H. (1990). Children's Mathematical Learning: A Cognitive View. In *Journal for Research in Mathematics Education, Monograph Number 4.* Reston, VA: NCTM. 51-64.

Berk, L. & Winsler, A. (1995). *Scaffolding Children's Learning: Vygotsky and Early Childhood Education.* Washington: National Association for the Education of Young Children.

Bootzin, R. and Acocella, J. (1988). *Abnormal Psychology.* New York: Random, 454 – 461.

Brown, S.I. & Walter, M.I. (1993). *Problem Posing: Reflections and Applications.* Hillsdale, N.J: Lawrence Erlbaum.

Burgoon, J.K. (1994). Nonverbal Signals. In M. Knapp and G. Miller (Eds.). *Handbook of Interpersonal Communication.* London: Sage, 229 –285.

Cobb, P. & Steffe, L., (1983), The Constructivist Researcher as Teacher and Model Builder, *Journal for Research in Mathematics Education,* 14, 2, 83-94.

Cobb, P., Yackel, E., & Wood, T., (1992), Interaction and Learning Mathematics Classroom Situations, *Educational Studies in Mathematics,* 23, 1, 99-122.

Crealock, C. & Bachor, D. (1995). *Instructional Strategies for Students With Special Needs.* Scarborough: Allyn & Bacon.

Davis, R. (1992). Understanding "understanding. *Journal of Mathematical Behavior.* 11, 225-241.

Davis, R. (1993). The Theoretical Foundations of Writing in Mathematics Classes. *Journal of Mathematical Behavior.* 11, 225-241.

Edwards, D. & Mercer, N. (1987). *Common Knowledge – The Development of Understanding in the Classroom.* London: Methuen.

Gailey, S. (1993). The Mathematics-Children's Literature Connection. *Arithmetic Teacher,* 258-261.

Gadamer (1989). *Truth and Method,* New York: Crossroad, 299.

Griffiths, R. & Clyne, M. (1991). Once Upon a Time ... *Australian Mathematics Teacher*, 47, 1, 10-13.

Harsh, A. (1987). Teach Mathematics With Children's Literature. *Young Children*. 42, 6, 24-29.

Jardine, D. W. (1998). *To Dwell with a Boundless Heart.* New York: Peter Lang.

Kennedy, L. (1984). *Guiding Children's Learning of Mathematics* (4th ed.). Belmont: Wadsworth.

Kieren, T. (1990). Understanding for Teaching for Understanding. *The Alberta Journal of Educational Research.* 36, 3, 191-201.

Law, B. & Eckes, M. (1990). *The More Than Just Surviving! Handbook.* Winnipeg: Peguis.

Lewis, B.A., Long, R., Mackay, M. (1993). Fostering Communication in Mathematics Using Children's Literature. *Arithmetic Teacher*, 40, 8, 470-473.

Lo, J., Wheatley, G., Smith, A. (1994). The Participation, Beliefs, and Development of Arithmetic Meaning of a Third-Grade Student in Mathematics Class Discussions. *Journal for Research in Mathematics Education,* 25, 1, 30-49.

Ministry of Education of British Columbia (1995). *Mathematics Integrated Resource Package.*

National Council of Teachers of Mathematics (1991). *Professional Standards for Teaching Mathematics.* Reston, VA: NCTM.

Ohanian, S. (1989). 'Readin' 'Rithmetic – Using Children's Literature to Teach Math. *Learning*, October, 32-35.

Osborne, S., Kosiewicz, M., Crumley, E., & Lee, C. (1987). Distractible Students Use Self-Monitoring. *Teaching Exceptional Children.* Winter, 66 – 69.

Piaget, J. (1973). *To Understand is to Invent.* New York: Grossman.

Pimm, D. (1987). *Speaking Mathematically.* London: Routledge & Kegan Paul

Pirie, S. (1988), Understanding: Instrumental, Relational, Intuitive, Constructed, Formalised ...? How Can We Know? *For the Learning of Mathematics,*8, 3, 2-6.

Pirie, S. (1996). Classroom video recording: When, why, and how does it offer a valuable data source for qualitative research? Paper presented at the 18th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (Panama City, FL.). ERIC ED 401 128, 17 pages.

Pirie, S. (1997). The Use of Talk in Mathematics.. In B. Davies and D. Corson (Eds.) *Encyclopedia of Language and Education.* Dordredht: Kluwer Academic Publishers, 229-238.

Pirie, S. (1998). Crossing the Gulf Between Thought and Symbol: Language as (slippery) Steppingstones. In M. Bussi, A. Sierpinska, and H. Steinbring (Eds.) *Language and Communication in the Mathematics Classroom.* Reston: NCTM. 7-29.

Pirie, S. & Kieren, T. (1992). Creating Constructivist Environments and Constructing Creative Mathematics. *Educational Studies in Mathematics,* 23, 505-528.

Pirie, S. & Kieren, T. (1994a). Growth in Mathematical Understanding: How Can We Characterise it and How can we Represent it? *Educational Studies in Mathematics,* 26, 165-190.

Pirie, S. & Kieren, T. (1994b). Beyond Metaphor: Formalising in Mathematical Understanding within Constructivist Environments. *For the Learning of Mathematics.* 14, 1, 39-43.

Pirie, S.E.B., Martin, L. & Kieren, T.E. (1994). *Mathematical images for fractions: help or hindrance?* Proceedings of the 18th International Conference on the Psychology of Mathematics Education (Lisbon), 3, 247–254.

Rosenblatt, L. (1938). *Literature as Exploration.* New York: Appleton-Century-Crofts.

Sfard, A. (1991). On the Dual Nature of Mathematical Conceptions: Reflections on Processes and Objects as Different Sides of the Same Coin. *Educational Studies in Mathematics,* 22, 1-36.

Sfard, A. (1994). Reification as the Birth of Metaphor. *For the Learning of Mathematics,* 14, 1, 44-54.

Simmt, E. (1998). Reflections on an Extracurricular Parent-Child Mathematics Program. *delta-K,* 35, 1, 56-60.

Smith, L., Dockerill, J., & Tomlinson, P. (1997). *Piaget, Vygotsky, and Beyond: future issues for developmental psychology and education.* London: Routledge.

Steffe, L.P. & Tzur, R. (1994). Interaction and Children's Mathematics. In P. Ernest (ED.) *Constructing Mathematical Knowledge: Epistemology and Mathematical Education.* London: Falmer Press, 8-32.

Towers, J.M. (1996). False Impressions: Listening for Understanding in the Mathematics Classroom. Proceedings of the 18th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Panama City, FL., October, 1996.

Voight, J. (1994), Negotiation of Mathematical Meaning and Learning Mathematics, *Educational Studies in Mathematics*, 26, 275-298.

von Glasersfeld, E.(1987). Learning as a Constructive Activity. In C. Janvier (Ed.) *Problems of Representation in the Teaching and Learning of Mathematics*, Hillsdale, NJ: Lawrence Erlbaum, 3-17.

Vygotsky, L. (1934) *Thought and Language*. A. Kozulin (Ed.). Cambridge: MIT Press.

Wheatley, G. & Cobb, P. (1990). Analysis of Young Children's Spatial Constructions. In L. Steffe and T. Wood (Eds.) *Transforming Children's Mathematics Education.* Hillsdale: Lawrence Erlbaum, 161- 173.

Winzer, M. (1993). *Children With Exceptionalities.* Scarborough: Prentice Hall.

Whitin, D.J. (1992). Explore Mathematics Through Children's Literature. *School Library Journal*, 38, 24-28.

Whitin, D.J. & Wilde, S. (1992). *Read Any Good Math Lately?* Portsmouth: Reid.

Yackel, E. & Cobb, P. (1996), "Sociomathematical Norms, Argumentation, and Autonomy in Mathematics", *Journal for Research in Mathematics*, 27, 4, 458 – 477.

Appendix 1

Bibliography of Children's Literature

Bogart, J. (1989). 10 for Dinner. Toronto: Scholastic.

Burns, M. (1994). The Greedy Triangle. New York: Scholastic.

Calmenson, S. (1991). Dinner at the Panda Palace. New York: Harper Collins

Clement, R. (1991). Counting on Frank. Milwaukee: Gareth Stevens

Friedman, A. (1994). King's Commissioners. New York: Scholastic.

Hulme, J. (1991). Sea Squares. New York: Hyperion.

Hulme, J. (1996). Sea Sums. New York: Hyperion.

Pinczes, E. (1996). Arctics Fives Arrive. New York: Scholastic.

Pinczes, E. (1995). A Remainder of One. New York: Scholastic.

Slobodkina, E. (1968). Caps For Sale. Harper & Row

Smucker, B. (1995). Selina and the Bear Paw Quilt. Toronto: Stoddart.