A NATIONAL ASSESSMENT OF MATHEMATICS PARTICIPATION: A SURVIVAL ANALYSIS MODEL FOR DESCRIBING STUDENTS' ACADEMIC CAREERS

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Abstract

One of the most striking facts disclosed in national reports is the large number of students who avoid mathematics courses, especially electives. The problem has become a serious public concern because it bears social and individual consequences: (a) a technologically advanced society demands a mathematically literate workforce, yet a large number of students drop out of mathematics; (b) inadequate preparation in mathematics seriously limits future educational and occupational opportunities of individuals.

Although research on school and teacher effects has revealed the effects of school structure and policies and teaching practices on mathematics achievement, researchers have paid little attention to the course of students' academic careers. Even the few existing studies are compromised by serious methodological flaws. Researchers, thus, have not been able to provide policymakers with reliable answers to their basic concerns about mathematics participation. This study tackles these problems, employing the six-wave data from the Longitudinal Study of American Youth (LSAY). The primary purposes of this study are (a) to estimate the probability of students' dropping out of mathematics, conditional on psychological and sociological variables, including sex, socioeconomic status (SES), prior mathematics achievement, prior attitude toward mathematics, prior mathematics anxiety, and prior self-esteem, over a five-year period from grade 8 to 12, (b) to identify conditions that affect the probability, and (c) to determine whether there are critical transition points, and if so, whether certain factors have stronger effects at these points. Survival analysis is used to overcome the difficulties conventional statistical techniques have in modeling probability.

Analyses of mathematics participation indicate that (a) students are most likely to drop out of mathematics in grade 12; (b) males are more likely than females to participate in mathematics in grade 12; (c) the effect of SES decreases over grades; (d) prior attitude toward mathematics is as important as prior mathematics achievement, and their effects are almost constant over grades; (e) the longitudinal effect of prior mathematics achievement or prior attitude toward mathematics depends on students' sex and SES.

Analyses of participation in advanced mathematics show that (a) students are most likely to drop out of advanced mathematics in grade 12; (b) males are more likely than females to participate in advanced mathematics in grade 12, and sex differences are similar across different levels of SES; (c) there is a male advantage in participation in advanced mathematics even when there is a male disadvantage in SES; (d) SES plays a critical role in the early grades, and socioeconomic differences are similar across different levels of mathematics achievement or attitude toward mathematics; (e) prior attitude toward mathematics has the strongest effect in the later grades, whereas the effect of prior mathematics achievement decreases over grades; (f) the effect of prior mathematics achievement varies across different levels of attitude toward mathematics, and vice versa; (g) the longitudinal effect of prior mathematics achievement or prior attitude toward mathematics depends on students' sex and their initial mathematics achievement and attitude toward mathematics.

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To My Parents

Chapter 1

Statement of the Problem

Introduction

There have been a number of public hearings and debates about mathematics education in the United States public schools in recent years. One of the most striking facts disclosed in national reports such as A Nation at Risk: The Imperative for Educational Reform (National Commission on Excellence in Education, 1983) is that a large number of high school students avoid mathematics courses, especially elective courses. Galambos (1980) believes that inadequate preparation in mathematics is perhaps the primary obstacle to the rapid expansion of high technology. Sherman (1982a) found that only 13% of girls and 57% of boys take their fourth-year high school mathematics. Sells (1973) referred to mathematics as the "critical filter"; she reported that 92% of first-year female students entering the University of California at Berkeley had such inadequate mathematics preparation that they would eventually lose 70% of the career choices available to them. "The problem seems to be serious enough to warrant concern and further investigation" (Stefanich & Dedrick, 1985, p. 274).

Mathematics educators face a serious dilemma: a technologically advanced society demands a mathematically literate workforce (National Council of Teachers of Mathematics, 1989), yet a large number of high school students drop out of the study of mathematics (National Commission on Excellence in Education, 1983). Careers that were comfortably free of mathematics in the 1960s are more dependent than ever on mathematics in the 1990s (National Council of Teachers of Mathematics, 1991). Modern society operates based more and more on sophisticated mathematics models, elaborate accounting systems, and computerized data analyses (National Council of Teachers of Mathematics, 1989, 1991). The social demand for a mathematically literate workforce will grow at an unprecedented pace into the twenty-first century (Cohen & Kosler, 1991). Ironically, however, several national surveys, such as the National Assessment of Educational Progress (NAEP), indicate that only half of high school graduates enroll in mathematics courses beyond the 10th grade (Dossey, Lindquist, & Chamber, 1988; National Center for Education Statistics, 1984). The recent International Adult Literacy Survey (IALS) shows that more than 20% of the population in the United States scores low in quantitative literacy (Statistics Canada, 1995). The number of earned graduate mathematics degrees in the United States reached a twenty-year low in 1988 (McGlone, 1988). Thus, despite unemployment problems, there is a job shortage in the highly skilled technical positions (Carnoy, 1995; Gallup, 1984).

"The need for greater awareness and training in science, math and technology is one none of us can ignore" (Cetron, 1984, p. 90). The National Academy of Science emphasized facility with basic mathematics, including algebra, geometry, and trigonometry (Turlington, 1985). Even educators in programs training students in apparently less technical areas have felt the need for increased mathematical sophistication (Martel & Mehallis, 1985). However, many new programs and initiatives in mathematics and science education will not benefit large proportions of females and minorities unless the number of mathematics dropouts is reduced (New York State Education Department, 1986). As a matter of fact, eliminating the behavior of dropping out of mathematics, particularly by female and minority students, is essential for meeting the growing demand for mathematics and science competency (New York State Education Department, 1986).

Purpose of the Study

Research on school and teacher effects has revealed the effects of school structure and policies and teaching practices on mathematics achievement, but researchers have paid little attention to the course of students' academic careers. Lee and Bryk (1988) reviewed studies on school effects, and concluded that "there was little empirical scrutiny of even basic questions, such as who takes what kinds of courses and the subsequent consequences of students' course of study on their academic achievement and future educational and work opportunities" (p. 78). Even the few existing studies are compromised by serious methodological flaws (see Willett & Singer, 1991). As a result, researchers have not been able to provide decision-makers with reliable answers to their basic concerns: how serious is the problem of high school students' dropping out of mathematics in the United States, and what can be done to stop or reduce this problem?

This study will tackle these questions. It will employ data from the Longitudinal Study of American Youth (LSAY), a national six-year panel study of public middle and high school students which focused on mathematics and science education (see Miller & Hoffer, 1994). The LSAY data were collected during 1987 to 1992, covering student social and psychological background, students' schooling experiences, and school organizational and social context variables. The primary purposes of the current study are (a) to estimate the probability of high school students' dropping out of mathematics courses, conditional on psychological and sociological variables, over a five-year period from grade 8 to 12, (b) to identify conditions that substantially affect the probability of students' dropping out of mathematics courses, and (c) to determine whether there are critical transition points, and if so, whether certain factors have stronger effects at these points.

A wide range of factors affect mathematics participation, including psychological factors such as self-esteem and mathematics anxiety; and sociological factors such as socioeconomic status (SES). This work will provide insight into many specific questions. For example, what is the likelihood that a female student will stop taking mathematics courses, given her family background and her level of mathematics achievement? At what grade level is she most likely to stop taking mathematics courses? Are students with a particular SES more likely to stop taking mathematics courses early in high school, or are there some critical transition points in the later years? The LSAY data are particularly well suited to address these questions because they follow students throughout their entire secondary schooling and include many measures of psychological and sociological constructs.

Definition of Terms

The literature uses the term "mathematics avoidance" to describe students' behavior of ceasing to take mathematics courses. However, there has been no theoretically coherent and generally accepted definition of mathematics avoidance. One reason why such a definition has not emerged is the tendency of researchers to consider many psychological concepts self-evident. For example, Gough (1954) suggested that the term "mathemaphobia" has no need of a definition because it is a well-known phenomenon.

This study prefers the term "mathematics dropout", which refers to the event when a student drops out of school, or stays in school but ceases to take mathematics courses. Note that a measure of mathematics dropout can be constructed only by following students throughout their academic careers.

Operationally, this study realizes the complexity in terms of why students drop out of mathematics courses, and does not assume that students cease to take mathematics courses because they dislike mathematics or have low ability in mathematics. This study will, however, examine the relationship of prior achievement, attitude, anxiety, and self-esteem to the likelihood that a student will drop out of mathematics courses. The definitions of educational terms used in this study are presented below:

Socioeconomic status (SES) refers to the relative position of a family or individual on an hierarchical social structure, based on their access to, or control over, wealth, prestige, and power (Mueller & Parcel, 1981). In LSAY, SES is a composite measure of parents' education, occupation, and income, based on an updated version of Duncan's (1961) socioeconomic index (SEI).

Mathematics achievement refers to the amount of mathematical skills and knowledge that an individual knows and possesses (Secada, 1992). Operationally, mathematics achievement is defined as the proportion of cognitive exercises that a student can answer successfully (Anderson, 1981). The achievement tests used in LSAY measure three dimensions of mathematical skills: (a) simple recall and recognition, (b) routine problem-solving, and (c) more complicated problem-solving. Note that researchers often prefer objective scores from standardized tests over subjective grades from mathematics teachers. However, grades may play a role in shaping students' attitude toward mathematics. Because mathematics grades are not available in LSAY, this study assumes that any effect of grades on attitude are captured on the measure of attitude.

Attitude toward mathematics has been used as a general term to include students' feelings and beliefs about self and mathematics (Haladyna, Shaughnessy, & Shaughnessy, 1983; Kulm, 1980; Leder, 1987; Reyes, 1984). It usually includes whether students like mathematics, whether they consider it to be difficult, whether they believe that it is relevant to their life, and whether they feel that they are good at the subject (see Neale, 1969). In LSAY, students' attitude toward mathematics is a composite measure of four attitudinal scales: (a) interest, (b) utility, (c) ability, and (d) anxiety. For some research questions, one must distinguish among these constructs. For example, a student could feel that he or she had low ability in mathematics, but not feel anxious about the subject. Conversely, a student could feel that he or she had relatively high ability in mathematics, but be very anxious about the subject. This distinction may be particularly important when students are in their final years of secondary school, and begin to assess the skill requirements for various courses. Therefore, based on the LSAY attitudinal scales, this study refers to attitude toward mathematics as a composite measure of interest, utility, and ability, and treats anxiety as a separate construct.

Mathematics anxiety refers to "the general lack of comfort that someone might experience when required to perform mathematically" (Wood, 1988, p. 11). In LSAY, mathematics anxiety is an equally weighted composite of two items with which students either agree or disagree on a five-point scale: (a) Doing mathematics often makes me nervous or upset; (b) I often get scared when I open my mathematics book and see a page of problems.

Self-esteem refers to an individual's perception of self in relationship to his or her environment (Shavelson, Hubner, & Stanton, 1976). In LSAY, self-esteem is an equally weighted composite of six items with which students either agree or disagree on a five-point scale: (a) I take a positive attitude toward myself; (b) I feel I am a person of worth, on an equal plane with others; (c) I am able to do things as well as most other people; (d) On the whole, I am satisfied with myself; (e) I wish I could have more respect for myself; (f) All in all, I am inclined to feel that I am a failure.

The methodological approach used in this study relies on "survival analysis". Survival analysis is a comprehensive set of statistical techniques often used in medical research to describe the mortality experience of a population, and, in particular, to examine the factors that determine when and why people acquire and die from certain diseases. Recently, survival analysis has been employed by biostatisticians, demographers, and public health researchers to model human lifetimes (e.g., Chiang, 1984; Cox, 1972; Cox & Oakes, 1984; Kalbfleisch & Prentice, 1980; Miller, 1981; Namboodiri & Suchindran, 1987), by economists and sociologists to model social transitions (e.g., Allison, 1982; Blossfeld, Hamerle, & Mayer, 1989; Mayer & Tuma, 1990; Tuma & Hannan, 1984), and by engineers to model industrial product reliability (e.g., Lawless, 1982). However, survival analysis has rarely been applied to problems in educational research. The aptness of survival analysis in the examination of mathematics participation will be discussed later.

Criteria of Variable Selection

Carroll (1962) encourages researchers to consider variables that reflect five sources of differences in school learning when building models for describing students' academic life — three residing in the individual and two stemming from external conditions:

> Factors in the individual are (1) aptitude — the amount of time needed to learn the task under optimal instructional conditions, (2) ability to understand instruction, and (3) perseverance — the amount of time the learner is willing to engage actively in learning. Factors in external conditions are (4) opportunity — time allowed for learning, and (5) the

> quality of instruction — a measure of the degree to which instruction is presented so that it will not require additional time for mastery beyond that required in view of aptitude. (p. 729)

With respect to Carroll's (1962) model, sex, SES, mathematics achievement, attitude toward mathematics, mathematics anxiety, and self-esteem are considered variables that may affect mathematics participation. Race and ethnicity were not included in LSAY due to political and technical reasons (J. Miller, 1995, personal communication, April 12, 1995). Curriculum tracking was not used in this study because it was measured inadequately and with a substantial proportion of missing data. SES is an individual-level attribute, and is associated with students' aptitude, ability to understand instruction, and perseverance in the learning of mathematics (see Secada, 1992). The measures of mathematics achievement, attitude toward mathematics, mathematics anxiety, and self-esteem are individual-level attributes. These will be associated not only with students' individual aptitude, ability to understand instruction, and perseverance, but also with the opportunity to learn and the quality of instruction in the classroom (see McLeod, 1992; Secada, 1992).

Research Questions

This study estimates probabilities of dropping out of mathematics courses for high school students, and traces these probabilities over a five-year period from grade 8 to 12, based on the LSAY data. It also investigates some educational, psychological, and sociological background variables, and traces how these variables affect mathematics participation of high school students over the five-year period. The main research questions are:

1. What are the probabilities of dropping out of mathematics courses for high school students in each year over the five-year period between grade

8 and 12, without statistical adjustment, and with statistical adjustment for sex, SES, mathematics achievement, attitude toward mathematics, mathematics anxiety, and self-esteem?

2. What are the most important background variables, among sex, SES, mathematics achievement, attitude toward mathematics, mathematics anxiety, and self-esteem, that affect the probabilities of dropping out of mathematics courses for high school students in each year over the five-year period?

3. What are the differences in the patterns of dropping out of mathematics courses among students with differing characteristics?

Methodological Concerns

The lack of preparation in high school mathematics constrains students' future educational and occupational choices. A number of studies have tried, with some success, to explain the reason why high school students drop out of mathematics courses (e.g., Lee & Ware, 1986). However, Willett and Singer (1991) pointed out some serious methodological concerns about studies of student dropout. Three major methodological limitations must be resolved if researchers intend to provide reliable insight into various patterns of student dropout.

Some cross-sectional studies have documented the characteristics of students' dropping out of mathematics courses during the secondary grades (see Kasten & Howe, 1988). However, little is known of the developmental nature of mathematics dropout behaviors. According to Willett and Singer (1991), this lack of knowledge is due mainly to limitations associated with cross-sectional research. Only longitudinal data "permit a more refined and realistic view [of dropout patterns], an ability to track factors [associated with dropout patterns], and increased statistical power as well" (Willett & Singer, 1991, p. 411).

The second limitation speaks to the nature of the research questions of student dropout. Researchers have traditionally asked "whether" students drop out of mathematics courses before the end of high school. Willett and Singer (1991) argued that, rather than asking "whether", researchers should ask "when" students are at the greatest risk of dropping out:

> Although logically intertwined, these two types of questions are also conceptually distinct, the "When?" question being far more general than the "Whether?" In fact, ... by asking when events occur, a researcher learns not only whether these events occur by each of several points in time but much much more. (p. 408)

Researchers' asking "whether" instead of "when" is fairly understandable, given that traditional educational statistics fail to offer appropriate tools to address the "when" question. Willett and Singer (1991) correctly pointed out that familiar statistical techniques such as regression analysis or analysis of variance are not capable of handling censored event times:

> No matter when data collection begins and no matter how long any subsequent follow-up, some study participants do not experience the target event while the researcher watches — some students do not drop out; some children do not leave day care; some teachers do not quit. These people have censored event times. What value of the outcome should they be assigned? Will they experience the event soon after the end of data collection, or will some of them never undergo the transition of interest? (p. 408)

Researchers cannot answer these questions on the basis of traditional educational statistics. Willett and Singer (1991) suggested that survival analysis is an effective, perhaps the best, statistical approach to cope with censored data.

Willett and Singer (1991) also pointed out another limitation of conventional statistical methods which can be overcome in survival analysis: "traditional analytic methods offer few mechanisms for including predictors whose values vary over time or for permitting the effects of predictors to fluctuate over time" (p. 426). As a result, researchers have to use "values corresponding to a single point in time, the average of values over time, or the rate of change in values over time" in their analyses (p. 426). They concluded that "traditional methods force researchers into building static models of dynamic processes; survival methods allow researchers to model dynamic process dynamically" (p. 427).

In line with the methodological suggestions of Willett and Singer (1991), this study employs longitudinal data (from grade 7 to 12) to investigate the "when" question in mathematics dropout. This study also accommodates both "time-invariant variables" such as gender, which have values that are constant over time, and "time-varying variables" such as mathematics achievement and attitude toward mathematics which have values that may fluctuate over time.

Justification for the Study

The research community has devoted considerable effort to understanding the causes of mathematics avoidance (see Howe & Kasten, 1992). The survival analysis model to be developed in this study also allows one to gauge the relative importance of several psychological and sociological variables in mathematics participation. More important, because this study is longitudinal, it is possible to trace how each variable affects mathematics participation over the five-year period from grade 8 to 12 and determine the most important variables that cause high school students to drop out of mathematics courses in each year over the five-year period. Therefore, this study contributes to a better understanding of the developmental nature of mathematics dropout, based on a more reliable statistical model for describing students' academic careers.

Researchers have just begun to examine the actual patterns of course enrollment among high school students (Alexander & Cook, 1982; Lee & Bryk, 1988). This study will compare the patterns of mathematics dropout among students with different psychological and sociological backgrounds. For example, some mathematics educators have the impression that female students with similar family background to their male counterparts are more likely to drop out of mathematics courses. The survival and hazard functions of males and females will be compared to test this conjecture. For another example, this study will assess the effects of SES on the probability of dropping out of mathematics courses, thereby providing insight into an important aspect of educational inequality.

Overall, the probabilities of mathematics dropout will indicate when students are at the greatest risk of dropping out of mathematics courses, and how the developmental pattern of mathematics course-taking varies among students. In an era of declining educational resources, intervention programs have to be implemented at an appropriate time with minimum costs and for maximum benefits. Because this study will demonstrate when a specific group of students is at the greatest risk of dropping out of mathematics courses, it will indicate the most appropriate time for educational intervention. In sum, this type of study can provide decision-makers with theoretical and practical insight into the formation, prevention, and elimination of mathematics dropout.

Faced with overall declining enrollments, increasing costs, and scarcity of funds, educational institutions must strive to become more efficient and effective. Hodgkinson (1983) and Pocock (1983) argued that those institutions whose leadership can react to current information swiftly will fare the best. Indeed, enrollment trends dictate that colleges, especially community colleges, must respond quickly with new program options if they are to survive (Corey, Jaksen, & Pritchard, 1984). Recognizing this, Babcock (1983) emphasized the necessity of realistic planning. Many critical decisions in education are based on enrollment forecasts (Martel & Mehallis, 1985). Given an accurate enrollment forecast, colleges and universities can make plans and decisions that are beneficial both academically and financially (Martel & Mehallis, 1985; Miller & McGill, 1984), and educational funding can also be satisfactorily planned by politicians (Martel & Mehallis, 1985). Greenfield (1979) believes that, with accurate enrollment estimates, negative trends may be forestalled and minimized through appropriate actions. The survival analysis model to be established in this study can be used to accurately forecast participation in education. Decision-makers will be able to rely on information generated through the model to make more rational educational policies.

Finally, as mentioned earlier, considerable methodological difficulties have plagued research on mathematics participation. Traditional educational statistics fail to provide appropriate techniques to estimate conditional probabilities, especially in the presence of censored cases. Willett and Singer (1991) introduced some basic principles of survival analysis. There has been little effort in establishing a systematic statistical model for describing students' academic careers. The methodological significance of this study is that it demonstrates the power of survival analysis to educational research.

Similar survival models can then be used to forecast the likelihood of dropping out of academic courses for high school students, and differentiate among variables that seem to be salient regarding the likelihood of dropping out.

Limitations of the Study

The research literature has exposed pronounced differences in mathematics participation among racial and ethnic groups. For example, Ayabe (1982) found that Asian students surpass those of other races in mathematics participation at every grade level. Leap, et al. (1982) claimed that mathematics dropout can be described as a crisis among American Indian students. Non-participation in academic programs is also alarming among Black (Anick, Carpenter, & Smith, 1981) and Hispanic students (Carter & Segura, 1979; Moore & Smith, 1985; Pallas & Alexander, 1983; Rendon & Triana, 1989). However, because data on race and ethnicity were not included in LSAY, racial and ethnic differences in mathematics participation cannot be examined.

Course enrollment is related to curriculum tracking which is often regarded as an organizational device to guide students' choices of high school courses. For example, students in college-preparatory tracks have greater access to advanced courses (Gamoran, 1987), and pursue significantly more rigorous high school programs (Massachusetts State Department of Education, 1986). These students learn more mathematics even after their different levels of intelligence are taken into account (Kulik & Kulik, 1984). Lee and Bryk (1988) suggested that curriculum tracking is one of the critical determinants of students' enrollment in advanced mathematics courses. As mentioned earlier, the LSAY data do not allow for an examination of the effects of curriculum tracking. However, this is not a serious limitation because many high schools have well-defined curriculum tracks, and the important distinction is among students' course-taking patterns (Garet & DeLany, 1988).

As the first step into a systematic description of students' academic careers and an accurate estimation of participation in mathematics courses, this study mainly concerns the effects of individual characteristics on mathematics dropout. All variables used in this study describe characteristics of students not their teachers or schools. The effects of school policies and practices on mathematics participation are not examined in this study. Neither are the roles of teachers and parents in mathematics dropout of high school students.

An examination of the effects of student, teacher, and school-level variables on dropping out of mathematics over a period of time would require a multilevel survival model. Such models have been described theoretically (Goldstein, 1996), but the software for estimating them is still in the early stages of development. The use of modeling that combines the strength of survival analysis and multilevel regression is, therefore, considered beyond the scope of this study. The final chapter, however, will make suggestions about how this work could be extended when the software can be more readily adopted to this problem.

Organization of the Study

The remainder of this study is organized into four chapters. Chapter 2 reviews the research literature, establishing an educational, psychological, and sociological context for understanding mathematics dropout. Chapter 3 depicts the methodology for this study. Chapter 4 describes statistical analyses and reports results. Chapter 5 summarizes the research findings, discusses their implications for educational policy, and provides recommendations for further investigation. The bibliography and appendices follow Chapter 5.

Chapter 2

Review of the Literature

The purpose of this review is to form a context in which mathematics dropout can be understood educationally, psychologically, and sociologically. There are a number of ways to present research findings on mathematics participation. This review will examine the role in the study of mathematics dropout of the factors selected in the previous chapter. It is divided into four sections. The first presents the theoretical background on mathematics dropout. The second discusses the effects of several psychological and sociological factors on mathematics participation. The third depicts sex differences in mathematics participation. The chapter concludes with a summary of important conclusions from the review as they pertain to the research questions in this study.

Theoretical Perspectives of Mathematics Dropout

Mathematics dropout is one of the new research areas in mathematics education. The investigation of mathematics dropout began in the early 1970s. Researchers investigated mathematics dropout because many students, even gifted students, had inadequate preparation in high school mathematics. The phenomenon is complex, and attempts have been made to understand mathematics dropout from educational, psychological, and sociological perspectives.

The Educational Perspective

As mentioned earlier, Sells (1973) documented that 92% of first-year female students who entered the University of California at Berkeley had inadequate mathematics preparation. Consequently, most students would have to spend additional time in remedial mathematics programs. The more serious consequence was that female undergraduate students would eventually lose 70% of the career choices available to them. From this investigation arose the term "critical filter" (Sells, 1973, 1976, 1978, 1980), because students who have an inadequate background in mathematics are effectively screened out of occupations and professions that require competence in quantitative skills (e.g., Donady & Tobias, 1979; Pearson, 1980; Sherman, 1982a).

Many educators reason that educational policies and regulations explain the increasing impact of the critical filter. The National Commission on Excellence in Education recommended stiffer state and local graduation requirements, particularly in mathematics, science, and computer programming (Gallup, 1984). Forty-two states have already followed this recommendation (Medrich, et al., 1992). The Board of Regents Action Plan to Improve Elementary and Secondary Educational Results in New York State, for example, included enhanced mathematics and science requirements for all students (New York State Education Department, 1986). But other educators fear that more stringent requirements for graduation will make it even more difficult for students to graduate from high school, resulting in a higher rate of school dropout (McDill, Natriello, & Pallas, 1985, 1986). They argue that the most important thing to stop mathematics dropout is to improve the quality of mathematics education.

Many efforts have been directed at high school students to convince them that their mathematics coursework in high school may well determine the career opportunities that will be open to them (Peng & Jaffe, 1979). This represents one perspective on mathematics dropout; that is, students themselves are responsible for dropping out of mathematics. Dropout rates are high because students are near-sighted and unable to recognize the importance of mathematics training in their future careers. Another perspective emphasizes the role of mathematics teachers in mathematics participation. For example, Casserly (1979) found that the changes in recruitment and teaching strategies significantly increase female participation in advanced mathematics courses.

Mathematics, as a discipline, may encompass levels of abstraction that go beyond the cognitive maturities of many students. Conscientious students often find themselves unable to master assigned materials, resulting in frustration, anxiety, and withdrawal (National Commission on Excellence in Education, 1983; Schimizzi, 1985). This situation becomes worse if mathematics teachers do not have appropriate expectations for their students. The sharp reduction in the number of students who choose mathematics elective courses may not be due to the lack of qualification of mathematics teachers, but to the perception on the part of students that some mathematics teachers have unrealistic expectations for their students' performance (National Commission on Excellence in Education, 1983). Furthermore, some mathematics teachers are perceived by their students as unapproachable or lacking effectiveness in teaching mathematics (National Commission on Excellence in Education, 1983). Stefanich and Dedrick (1985) point out that many mathematics and science educators lack even the basic knowledge of behavioral psychology, especially how principles of reinforcement and punishment affect learning. Therefore, students' mathematics dropout is considered a result of repeated exposure to poor teaching practices in mathematics education (New York State Education Department, 1986).

The Psychological Perspective

Researchers have used Seligman's (1975) theory of "learned helplessness" to account for students' dropping out of mathematics.

Seligman found that when animals learn that outcomes are unpredictable and beyond their control, they tend to give up further efforts needed to understand the situation. He called this phenomenon "learned helplessness", and showed that its symptoms include deficits in motivation, understanding, and emotional affect. The phenomenon was later attributed to humans who find themselves incapable of controlling their destinies. The theory has been applied to explain mathematics dropout (e.g., Parente & Chisholm, 1980; Weiner, 1972). The repeated frustration in the learning of mathematics creates and strengthens students' perception of being helpless when working with numbers and shapes. This learned perception causes students to drop out of mathematics as a way to avoid further unpleasant experiences in mathematics.

The theory of "fear of success" has been offered to explain female mathematics dropout. This theory is based on Horner's (1968) concept of motive to avoid success which was initially proposed to understand sex differences in achievement motivation. Horner suggested that success in an environment of competitive achievement is more congruent with the male role than the female role in the Western society, which stereotypes females to be anxious about their success in academic achievement and produces a negative effect on their academic performance. He further hypothesized that females who have successfully engaged in competitive activities in the past are most vulnerable to fear of success, because they are frequently in a dilemma in which high achievement orientation is associated with some negative consequences of being academically successful, especially "when the tasks involved are generally considered masculine, such as tasks of mathematical, logical, spatial, etc., ability" (p. 24). Horner believed that

> There are two potential sources for the negative consequences of success, i.e., loss of one's sense of femininity and self-esteem regardless of whether anyone finds out about the success or not, and/or social rejection because of success. (p. 16-17)

Ishiyama and Chabassol (1985) found that fear of academic success tends to decrease over age, but females generally have a greater fear of success than males. Smithers and Collings (1977) reported that female 6th formers in England who took "A-level" science courses tended to regard themselves as less attractive to boys than girls who did non-science "A-level" examinations. Consequently, bright female students manifested fear of success, believing that their academic success would socially disadvantage them.

Simkin's (1979) study of Australian high school students showed that males tended to value academic success most highly, whereas females tended to value popularity with peers most highly. He contended that fear of success accounted for the fact that only 24% of female 11th graders indicated that they intended to pursue a career in mathematics- or science-related areas, compared with 53% of male 11th graders. Sherman (1982a) also found that girls who took four years of mathematics courses in the United States demonstrated more anxiety about academic success than those who took fewer mathematics courses. Leder (1982) concluded that

> It appears that girls who perform well in mathematics are more likely to be high in FS [fear of success] and yet that, for some, high FS tends to be incompatible with continued high performance in mathematics. Possibly some of the girls high in FS resolve their conflict situation by either opting out of intensive mathematics studies or by lowering their performance and thus no longer continuing to be conspicuously successful. (p. 133)

These findings highlight the need for counteracting the still-prevalent, stereotyped beliefs that certain careers are more appropriate for one sex than the other (Leder, 1982).

The Sociological Perspective

Educational sociologists have hypothesized that certain social conditions encourage mathematics dropout. For example, spending less for more is one characteristic of consumerism. Similarly, many students want to enroll in easy subjects to invest less time for better grades, and in practical subjects to immediately enjoy their benefit after graduation. Consumerism is, to a large extent, responsible for the tremendous increase in vocational-technical programs (Martel & Mehallis, 1985) and the serious enrollment decline in mathematics programs in community colleges (Smith, 1984). Mathematics dropout is regarded as a direct product of the popular trend toward consumerism in society (Guzzardi, 1979; Naisbitt, 1982). Galambos (1980) believes that students have avoided highly technical fields, particularly mathematics, in order to stay in the consumeristic mainstream.

The majority of studies have focused on sorting out socioeconomic determinants of educational performance, stemming from the work of Blau and Duncan (1967). These studies formed the theoretical framework of "social distribution of educational attainment". Five large-scale studies were particularly influential in the 1970s (Alexander, Cook, & McDill, 1978; Hauser, Sewell, & Alwin, 1976; Heyns, 1974; Jencks & Brown, 1975; Rosenbaum, 1980). All of them adopted the "Wisconsin model" of educational stratification, which examines the relationship among students' socioeconomic background, intervening social-psychological variables, and subsequent educational and occupational attainment. These studies portrayed high school curriculum as being divided into college preparatory and non-college tracks.

Three of the five studies examined academic participation. Jencks and Brown (1975) found that students in the college track completed about a third of a year more schooling than students with similar socioeconomic background in the non-college track, whereas Hauser, et al. (1976) indicated that students in the college track completed nearly two-thirds of a year more schooling than comparable students in the non-college track. The difference in their estimates may be due to the fact that Jencks and Brown (1975) included a control for pre-high-school educational aspirations. Rosenbaum (1980) examined the influence of the curriculum track on college attendance, and reported that students in the college track had a 25% higher probability of attending college in the year immediately after high school than those in the non-college track.

Recent reviews of educational stratification continue to conclude that "variation in students' experiences in different groups and tracks contribute to inequality in cognitive outcomes" (Gamoran, 1996, p. 63). In other words, students in high-status ability groups and curriculum tracks gain more knowledge and skills, because instructional materials and teaching methods vary systematically, favoring those in high-status tracks and working against those in low-status tracks (Oakes, Gamoran, & Page, 1992). Cross-national socialization research also shows that stratification between schools has significant effects on students' achievement growth (see Gamoran, 1996).

Lee and Bryk (1988) conducted a study on social stratification of mathematics achievement, using the High School and Beyond (HS&B) data to examine the academic experience of high school students. They were particularly interested in differences between Catholic schools and public schools in curriculum tracking and enrollment in academic courses. Students are more likely to be assigned to rather than choose the academic track in Catholic schools than in public schools. The placement in the academic track is more closely linked to aspirations for college graduation in Catholic schools than in public schools. Catholic school students take more academic courses, and their course-enrollment patterns are less dependent on their family background or prior academic achievement. Lee and Bryk (1988) concluded that "track placement and course of study are the major mediating factors that link students' background (social class, minority status, and academic background) with academic achievement" (p. 78).

Specifically, Lee and Bryk (1988) reported that, in the academic track, students take 3.58 years of mathematics in Catholic schools in comparison to 3.20 years in public schools. Sophomore mathematics achievement, Catholic sector effect, academic background, academic remedial program, and social class, in this order, determine students' enrollment in advanced mathematics courses. In the general track, students take 2.59 years of mathematics in Catholic schools in comparison to 1.54 years in public schools. Sophomore mathematics achievement, Catholic sector effect, academic background, social class, and academic remedial program, in this order, determine students' enrollment in advanced mathematics courses. In the vocational track, students take 2.23 years of mathematics in Catholic schools in comparison to 1.22 years in public schools. Sophomore mathematics achievement, Catholic sector effect, and academic background, in this order, determine students' enrollment in advanced mathematics courses. Note that minority status have no appreciable effects on students' enrollment in advanced mathematics courses after taking into account their academic tracks.

Factors Affecting Mathematics Participation

The research literature described above suggests that there are four sets of factors that affect whether a child will drop out of mathematics. The first set pertains to their home environment, and includes factors such as parental education, family income, and family structure. The second set pertains to psychological factors, and includes children's self-esteem, their anxiety about mathematics and their attitude toward the subject. The third set of factors concerns children's ability, which includes their general cognitive ability, as well as their specific abilities in mathematics-related skills. The fourth set of factors concerns school-related factors, such as the quality of curriculum and instruction, and the organizational features of the school. These four sets of factors are of course related, and interact with one another from the time a child enters kindergarten. The intent of this study is to determine which sets of factors most strongly affect children's dropping out of mathematics at different grades during their secondary schooling. This study is also concerned with whether the sets of factors that are most predictive of dropping out of mathematics differ between males and females.

Kasten and Howe (1988) made a useful distinction between two groups of students who are learning substantively less mathematics than they should. The first group consists of the "typical or usual" potential school dropouts and underachievers. These students have lower than average achievement scores in most of their subjects, and are more likely to drop out of school altogether. The second group is termed "nominal mathematics students". They stay in high school and may even attend college, but their mathematics preparation does not adequately allow them maximum educational and occupational choices. Kasten and Howe (1988) believe that these two groups of students probably account for a significant amount of the reason why national assessment scores in mathematics have not shown much improvement over time.

Approximately 20% of students who enroll in the United States public schools become part of the first group, and two-thirds of these students come from families below the poverty line (Kasten & Howe, 1988). Among those students who complete high school, another 20 to 25% cannot perform at a satisfactory level in mathematics (Dossey, Lindquist, & Chamber, 1988). Unlike potential dropouts, these nominal students seldom cause mathematics teachers serious concerns in school (Kasten & Howe, 1988). They do not manifest behavior problems and are not viewed as potential problems for society. However, these students are not likely to continue their study of mathematics beyond basic requirements in high school, nor are they likely to consider a career that uses mathematics. They are at risk of underachieving economically because their level of understanding and competency in mathematics is substantially below their capacity. Also, many of them will not be able to apply mathematics when needed in their daily life.

With this distinction in mind, this study attempts to discover whether particular sets of factors play a more important role for typical mathematics dropouts than for nominal mathematics students. This can be achieved through examining the effects of the predictive factors on the likelihood of dropping out of mathematics generally, and on the likelihood of dropping out of advanced mathematics.

This section of the review discusses issues pertaining to the measurement of each of these sets of factors. It assumes that within each set it is essential to have three or four good measures that represent that set, and that with respect to explaining dropout rates in mathematics, the contribution at the margin of additional variables within that set is minimal. Thus the task of this section is to identify the most important factors within each set.

The Effects of Home Factors

Kasten and Howe (1988) contended that certain family characteristics contribute to typical mathematics dropouts. Typical mathematics dropouts often come from families with patterns of dropouts and cyclical poverty. Family problems associated with alcohol, drugs, and child abuse encourage typical students to drop out. Typical students also drop out of mathematics because of low parental expectations of success. Living in a single-parent home, having parents with less than high school education, and coming from families where English is not the primary language put typical students at risk of dropping out. Kasten and Howe (1988) also argued that the most important family characteristic that causes nominal mathematics students to have inadequate mathematics preparation in high school is low parental expectations for experience and success in mathematics.

All the above characteristics are associated with a powerful indicator of family environment: parental socioeconomic status (SES). Researchers have been using SES as a basic sociological factor in educational studies. Early theories proposed that poor socialization of working-class students or the academic deficiencies they brought to school would lead them to academic failure and discipline problems (Cloward & Ohlin, 1960; Merton, 1968). Cohen (1955) illustrated that many students from working-class backgrounds, when confronted with the middle-class values inherent in the school, would become frustrated and fail. Many sociological theories are based on the Wisconsin model of educational stratification discussed earlier. For instance, the theory of "economic deprivation" grows out of the educational and socioeconomic disadvantages of single-parent families (Herzog & Sudia, 1973). The status attainment model usually relies on a family's socioeconomic structure to locate individual positions in the educational and occupational sphere (Sewell, Haller, & Ohlendorf, 1970).

SES is a consistent predictor of students' dropping out of high school (National Center for Education Statistics, 1987). School dropouts are disproportionate in low SES families (Ekstrom, Goertz, Pollack, & Rock, 1986; Bachman, Green, & Wirtanen, 1971; Rumberger, 1983). Dropouts in every curriculum track are more likely to come from low SES households (Natriello, Pallas, & Alexander, 1989). On the other hand, Rosenbaum (1980) found that students whose SES is 1 standard deviation (SD) above the average had a probability of entering the college track about 12% higher than students with average SES. Students from more affluent and better-educated families are more likely to pursue more rigorous coursework in mathematics and science (Gamoran, 1987) and attend college (Bachman, O'Malley, & Johnson, 1978; Rehberg & Rosenthal, 1978; Sewell & Shah, 1967). SES of female students is predictive of their dropping out of mathematics (Brush, 1980). SES has been used as an explanation of the lack of academic achievement and participation for all minorities. Ethnic differences in academic achievement are usually strong, but the effects of ethnicity tend to diminish as long as SES is controlled (Hill, 1979; Rumberger, 1983).

Armstrong (1980) reported that components of SES, such as parent education and parent occupation, are significantly correlated with mathematics course-taking. Schools where students are from middle-class backgrounds offer more chances for enriched and rigorous academic programs (Willms, 1986). However, students from working-class homes receive relatively less academic attention and support from schools than their counterparts from middle-class homes (Bowles & Gintis, 1976; Grubb, 1984;

Grubb & Lazerson, 1975). Students' SES also affects curriculum tracking. Lee and Bryk (1988) reported that track placement is more strongly correlated with social class in public schools than in Catholic schools. The SES composition of a school was also found to affect the availability of track positions (Jones, Vanfossen, & Spade, 1985).

One popular explanation of educational differences along social-class lines is that parents in high SES families are more likely than those in low SES families to be involved in the educational affairs of their children (Fehrmann, Keith, & Reimers, 1987; Lareau, 1987; Stevenson & Baker, 1987). These researchers suggested that parents' cooperation with teachers and schools improves their children's educational opportunities as well as their academic achievement.

There are a number of other home factors that may explain in part why students do not persist in mathematics. Some researchers suggest that the role of parents is important in mathematics participation (e.g., Armstrong & Kahl, 1979). For example, parental encouragement has been found to affect whether students take elective mathematics courses (Armstrong, 1980; Kaczala, 1980; Smith, 1980). The New York State Education Department (1986) concluded that students' mathematics dropout is largely a result of the lack of parental support in the study of mathematics. Furthermore, parents' attitude toward mathematics appears to transfer to offspring (Aiken, 1976; Lazarus, 1974). Fathers seem to play a more important role in their children's mathematics participation (Carlsmith, 1964; Ernest, 1976). The negative attitude of students toward mathematics which later causes them to drop out of mathematics is attributable to the influence of father's rather than mother's attitude toward school (Aiken, 1976). Chisholm (1980) also found that father's characteristics are significant predictors of mathematics dropout.

The Effects of Psychological Factors

Affective factors such as anxiety, confidence, and self-efficacy have emerged as salient predictors of mathematics achievement and participation (see Eccles & Jacobs, 1986; Hackett & Betz, 1989; Kloosterman, 1988). Kasten and Howe (1988) described typical mathematics dropouts as having low self-esteem, low persistence, low personal expectations, and low interest in school activities. Typical mathematics dropouts also manifest discipline problems in school. Kasten and Howe (1988) also thought that the most important psychological factors that affect nominal mathematics students were mathematics anxiety, persistence on difficult tasks, and personal expectations in mathematics. These psychological characteristics can be summarized or measured through three factors: attitude toward mathematics, mathematics anxiety, and belief about self (see Aiken, 1970a, 1976; McLeod, 1992).

Attitude toward mathematics. Tobias (1980) considered mathematics dropout a natural consequence of negative attitudes toward mathematics that develop as a result of student's early troublesome experiences with mathematics. Negative experience in mathematics classes, negative parental attitude toward mathematics, and societal stereotyping of mathematics cause many students to develop negative feelings about mathematics, which in turn cause mathematics anxiety and mathematics dropout (Taylor & Brooks, 1986). Research has provided empirical evidence for the hypothesis that negative attitude toward mathematics leads to mathematics dropout (e.g., Aiken, 1970a; Kaczala, 1980; Smith, 1980). In general, positive feelings about mathematics and one's ability in mathematics show a strong relationship with active mathematics course-taking (Armstrong, 1980). The number of years that students enroll in mathematics courses correlates with their attitude toward mathematics (Minnesota State Department of Education, 1976). Research has demonstrated that attitude toward mathematics is the most important factor that determines the election of high school mathematics (e.g., Armstrong & Kahl, 1979; Pedro, Wolleat, Fennema, & Becker, 1981). Similarly, attitude toward mathematics has been shown as one of the best predictors of mathematics persistence (Gemmill, Bustoz, & Montiel, 1982).

Attitude toward mathematics as a general descriptor of the affective domain may not be effective in explaining mathematics participation because many students who are in favor of mathematics also fail to elect mathematics courses. Researchers therefore begin to examine more specific components of attitude toward mathematics. For example, Brush (1980) found that the perceived ability in mathematics, largely a matter of students' self-perception as either able or unable, is a consistent predictor of the terminal level of mathematics coursework. Moreover, students' perception of the future usefulness of mathematics has been found to affect the elective course-taking in mathematics (Kaczala, 1980; Smith, 1980). Chisholm (1980) reported that perception of the usefulness of mathematics is a significant predictor of mathematics dropout. Armstrong (1980) asked students to identify their reasons for taking mathematics courses. Students in both grades 8 and 12 indicated that the usefulness of mathematics is the most important factor in determining whether or not to take mathematics courses, followed by confidence in mathematics and enjoyment of mathematics.

Many female students do not recognize the usefulness of mathematics to their life plans and have negative mathematical self-concepts, which interferes with the amount of time spent in studying mathematics (Hilton & Berglund, 1971). Haven (1971) reported that female students who consider mathematics useful are more apt to take advanced mathematics courses.

Fennema (1979) found that female students who view mathematics as not useful and have little confidence in learning mathematics tend to avoid mathematics courses.

Mathematics anxiety. Mathematics anxiety refers to a fear of failure when students learn the content of mathematics (Tobias, 1980). He argued that timed tests, right answers, and difficult word problems lead students to mathematics anxiety. Lazarus (1974) described that

> The student then actively turns away from mathematics, opts for nonmathematical courses whenever possible, and rapidly develops fatalistic attitudes about his/her problems with mathematics, fully expecting to do badly. This attitude itself, quite apart from other factors, can seriously impair performance. (p. 16)

Both males and females can be afflicted by mathematics anxiety, but females appear to be more anxious toward mathematics (e.g., Fennema, 1982; Bander & Betz, 1981) and suffer more from mathematics anxiety (e.g., Burton, 1979; Osen, 1974; Tobias, 1980). Mathematics anxiety affects students' perception of mathematics. For example, females with low anxiety perceive mathematics as more useful than those with high anxiety (Kincaid & Austin-Martin, 1981). Mathematics anxiety also affects students' choice of mathematics-related careers. For example, Hackett (1985) reported that mathematics anxiety both directly and indirectly influences the choice of mathematics-related college majors.

Mathematics anxiety has been suggested as one of the most important predictors of mathematics participation. Armstrong (1980) showed that mathematics anxiety was the most important determinant of the number of mathematics courses that the 13-year-old students intended to take. Pedro, et al. (1981) indicated that mathematics anxiety is an important factor, second only to the usefulness of mathematics, to determine the election of high school mathematics. Chisholm (1980) found that mathematics anxiety is one of the significant predictors of mathematics dropout. Eccles and Jocobs (1986) suggested that "math anxiety appears to be a key social/attitudinal variable that might account for sex differences in achievement and enrollment in mathematics courses" (p. 375). Hembree's (1990) meta-analysis showed that highly anxious students take fewer mathematics courses in high school and rarely express plans to pursue careers related to mathematics or science. Many researchers have concluded that not only is dropout the prominent means of coping with mathematics anxiety, it is also an antecedent as evidenced by significant correlations between mathematics anxiety and mathematics preparation in high school (e.g., Betz, 1978; Frary & Ling, 1983; Hackett, 1985; Hendel, 1977; Tobias, 1980).

On the other hand, some researchers argue that there is no sufficient evidence that a relationship exists between mathematics anxiety and mathematics dropout (e.g., Harriss, Galassi, & Galassi, 1984). For example, Armstrong (1980) demonstrated that career and education plans decided the number of mathematics courses that high school seniors intended to take. Theoretically, Aiken (1966) considered mathematics anxiety a "relative" of general attitude toward mathematics, only being more visceral and exciting. Pearson (1980) even equated mathematics anxiety and mathematics dropout. Sherman (1983) found that women avoid mathematics not because of anxiety with respect to ability, but because of the sex role strain and the potential conflict between professional and wife/mother roles. Lips (1984) believes that the emphasis on concepts such as mathematics anxiety as explanations for women's absence in mathematics seems to unreasonably assume that women have strongly negative self-concepts in areas of quantitative ability.

Belief about self. Bandura's (1977a) social learning or self-efficacy theory concerns the reciprocal relationships among personal, behavioral, and environmental factors. The interactions among these factors make individuals aware of what behaviors bring about desired changes, what environmental conditions necessitate certain behaviors, and what outcomes are contingent on their actions (Bandura, 1977a; Norwich, 1987; Stipek & Weisz, 1981). Given that an outcome is considered contingent on a certain course of action, an individual must evaluate whether he or she is able to make necessary responses. The belief that one is able to perform certain behaviors required to bring about a desired outcome is referred to as "self-efficacy", which is a personal cognitive factor presumed to mediate between an individual's behaviors and factors in his or her environment. Many factors, such as previous performance, cues from relevant others, and levels of emotional arousal, contribute to one's judgments about his or her efficacy (Bandura, 1977b, 1982). Often self-efficacy helps determine whether a person will attempt the necessary course of action. Once the action is in place, self-efficacy affects the persistence especially when difficulties are encountered (Norwich, 1987; Stipek & Weisz, 1981). Individuals with low self-efficacy are less likely to have an attempt to compete and are more likely to give up in case of frustration. Because of these effects of self-efficacy, it is considered a relevant variable in the examination of achievement-related academic activities.

Self-esteem is often considered in research on dropping out of school. School dropouts have lower self-esteem than school graduates (e.g., Bachman, Green, & Wirtanen, 1971; Bachman, O'Malley, & Johnson, 1978; Hunt & Woods, 1979; Yudin, et al., 1973). A tentative conclusion, however, derived from more recent studies is that self-esteem may not play a critical role in students' decision to drop out of school. For example, using the High School and Beyond (HS&B) data, Ekstrom, et al. (1986) reported that self-esteem items that measure whether students have a positive attitude toward themselves or feel of equal worth in comparison to others showed no practical or significant differences between school stayers and dropouts, although self-esteem items that examine whether students are satisfied with themselves, or have much to be proud of, showed significant differences in favor of stayers. The HS&B data also indicated that dropouts, non-college bound students, and college-bound students all increased their sense of self-esteem in a positive direction over a period of three years (Wehlage & Rutter, 1986). This is true for Hispanics, Blacks, and Whites, and the change was significant for all but Black dropouts. Dropouts began with slightly higher self-esteem than non-college-bound students, and the difference grew over time even after dropouts left school. Dropouts had the same overall gain in self-esteem as the college-bound. Students who were similar to dropouts in some respects, but did not drop out of school, had less growth in self-esteem than either dropouts or the college-bound.

Self-confidence in mathematics has been stressed as one of the most important predictors of mathematics achievement and participation (e.g., Kloosterman, 1988; Schoenfeld, 1989; Sherman, 1983). Sherman and Fennema (1977) reported a strong relationship between self-confidence as a learner of mathematics and the pursuit of advanced mathematics courses. Self-confidence is related to the psychological construct, locus of control: students with high levels of self-confidence attribute success to ability or effort, and failure to the lack of effort. These students are highly motivated because they believe that future success is controllable and contingent on their action. Kloosterman (1988) found that the pattern of causal attributions accounted for 17% of the variance in self-confidence scores, indicating that students' self-confidence in mathematics can be explained in part by their perceptions of the causes behind academic successes and failures. Females express significantly less self-confidence in mathematics than males even when their actual abilities are the same (Sherman, 1983). Singer and Stake (1986) found that the female level of self-confidence drops evidently after a failure experience in mathematics.

Students who are confident in their abilities to learn mathematics are more likely to enroll in mathematics courses when enrollment becomes optional (Kloosterman, 1988). Lower levels of self-confidence in mathematics among women may, therefore, be one explanation for the lack of female participation in higher levels of mathematics courses (Fennema, 1980). Female students' negative self-judgment of their mathematics performance in early high school years also leads to a decline in female students' participation in mathematics later in high school (Catsambis, 1994). Self-efficacy in mathematics was found to be predictive of mathematics anxiety, mathematics course-taking plans, and the selection of a mathematics-related college major (Hackett, 1985). Self-efficacy in mathematics was even found to be a more important determinant of mathematics-related majors than mathematics achievement (Hackett & Betz, 1989).

Other psychological factors. There is some evidence that certain personality factors lead students to mathematics dropout (Van Blerkem, 1986). Chisholm (1980) found that "expectancy motivation" is one of the significant predictors of mathematics participation. Students who have not succeeded in mathematics classes due to learning and behavior problems, sensory handicaps, and physical and health impairments tend to drop out of mathematics (Kasten & Howe, 1988). Age, IQ, temperament, level of maturity have also been offered intuitively as influential factors, but they individually account for little of the variance in mathematics participation for the majority of students (Chisholm, 1980).

The Effects of Ability

Kasten and Howe (1988) indicated that achievement of more than one year below grade level in reading or mathematics for those in grades 1 to 7, and more than two years for those in grades 8 to 12, can make typical mathematics students stop taking further mathematics courses, whereas achievement of more than one year below grade level in mathematics can make nominal mathematics students avoid further contact with mathematics. Therefore, mathematics achievement seems the most important measure in this set of factors.

That academic achievement has direct, substantial effects on the completion of secondary school has been well-documented (e.g., Reitzes & Mutran, 1980; Stryker, 1981; Williams, 1972; Yogev, 1981). For example, low academic achievement is associated with high dropout rates (Ekstrom, et al., 1986; Wehlage & Rutter, 1986). Findings, however, are not entirely consistent on the relationship between mathematics achievement and participation in mathematics courses. Neale (1969) conducted a survey of several correlation studies, and reached the conclusion that mathematics achievement may not account for much of the variance in mathematics participation. Mathematics achievement was listed as one of the secondary factors affecting mathematics participation (Armstrong & Kahl, 1979). Sherman and Fennema (1977) found that many male students, though from the lower half of mathematics achievement distribution, tend to continue their mathematics courses. The sex gap in mathematics achievement may also not correlate with that in mathematics participation (Sherman, 1980).

Other studies, however, counter these conclusions. Armstrong (1980) claimed that there is a strong relationship between mathematics achievement and mathematics participation. Students' mathematics grades also highly correlate with mathematics participation (Armstrong, 1980). Lee and Bryk (1989) reported that higher average level of academic achievement relates to higher average level of academic course-taking. Knaupp (1973) argued that students who dislike mathematics because they do poorly in mathematics usually avoid further contact with mathematics. The number of years that students enroll in mathematics courses was found to be related to their mathematics achievement (Minnesota State Department of Education, 1976). For minority students, the level of mathematics achievement is one of the most serious barriers to mathematics participation (Catsambis, 1994; Green, 1978). Mathematics achievement has also been offered as a major explanation of sex differences in mathematics participation and choices of mathematics-related careers (Wise, 1978). Lee and Bryk (1988) concluded that mathematics achievement is the most important predictor of students' enrollment in advanced mathematics courses.

The causal link between mathematics achievement and mathematics participation still remains unclear (Oakes, 1990). Benbow and Stanley (1980) rejected the hypothesis that the male-female gap in mathematics achievement comes from differences in mathematics course-taking. Lee and Ware (1986) indicated that the effect of course enrollment in mathematics on SAT-Mathematics performance is weak. Lee and Bryk (1988) found that the effect of the 10th-grade mathematics achievement on the 11th-grade advanced mathematics course-taking is much stronger than the effect of the 11th-grade advanced mathematics course-taking on the 12th-grade mathematics achievement.

On the other hand, some researchers have shown that sex differences in mathematics participation contribute substantially to the achievement gap in mathematics between males and females (e.g., Berryman, 1983; Fennema, 1980; Fennema & Sherman, 1977a; Oakes, 1990; Pallas & Alexander, 1983; Smith & Walker, 1988; Wise, Steel, & MacDonald, 1979). For example, Fennema (1977, 1979) argued that if the amount of time spent in learning mathematics is equated for males and females, educationally significant sex differences should disappear. Fennema and Sherman (1977b) showed that at ages of 9 and 13 when students have similar educational and mathematical backgrounds, they have similar mathematics achievement as well. After controlling for the amount of coursework in mathematics, sex differences in mathematics achievement become trivial (deWolf, 1981). Fennema (1982) concluded that poor mathematics performance of female students can be attributed to sex-related differences in the selection of mathematics courses.

The Effects of School Factors

Kasten and Howe (1988) concluded that both typical mathematics dropouts and nominal mathematics students are affected by academic track, curriculum and instruction, school climate, peer effects, school academic expectations, and the lack of effective programs to work with students at risk of dropping out. Teachers have great influence on students' mathematics participation (Armstrong & Kahl, 1979). With teacher encouragement, students are more likely to take elective mathematics courses (Armstrong, 1980; Kaczala, 1980; Smith, 1980). Barrington and Hendrick (1989) reported that teachers' comments on elementary school records of students differentiate graduates from dropouts. There is also evidence that teachers who perceive a student as capable in mathematics will urge "perseverance" at the point of discouragement; while teachers who see a student as lacking mathematics abilities permit the student to give up mathematics courses (Weiner, 1972).

A number of studies on sex differences in mathematics have shown that mathematics teachers pay more attention to male students in mathematics classes, thus discouraging mathematics participation of female students (e.g., Becker, 1981; Fennema, 1979). Ernest (1976) found that 41% of mathematics teachers believe that male students generally do better than females in mathematics. Such beliefs activate sexist expectations with differential preference by teachers, leading to differential reinforcement (Fennema, 1974). In fact, educational expectations of teachers have been shown to be one of the best predictors of mathematics persistence (Gemmill, Bustoz, & Montiel, 1982).

Peer relationships, or interactions between students, are also proposed to explain mathematics participation. Because role models affect one's behaviors (Beane & Lipka, 1980), the inappropriate perception of how role models act in and feel about mathematics may lead one to less-than-successful experiences in mathematics (Burton, 1984). The New York State Education Department (1986) considered mathematics dropout partly a result of the lack of peer support in the learning of mathematics. Aiken (1976) indicated that college students are attracted to others with similar attitude toward mathematics. Hallinan and Williams (1990) found that peer influences on educational aspirations and outcomes vary with the ethnic and sex composition of students' friends, and that interracial friendships are beneficial to the aspirations of both Black and White students. In sum, a student who sees that his or her friends do not take mathematics courses may well avoid them also. However, in comparison with attitude toward mathematics, or the influence of parents and teachers, peer influences tend to become secondary (Armstrong & Kahl, 1979).

Curriculum and instruction are also considered for their roles in mathematics participation, because they often do not foster desired attitudes, aspirations, skills, and understanding relating to mathematics (Kasten & Howe, 1988). Mathematics curriculum seldom makes students, especially minority students, appreciate the role of mathematics in everyday life and the value of mathematics to their future schooling and careers (Beane, 1988). Mathematics instruction frequently suffers from various problems (see Kasten & Howe, 1988):

1. The usual classroom routine is not effective for developing new concepts.

2. The pace is wrong for many students.

3. Drill and practice are ineffective.

4. Diagnosis and treatment of errors are often superficial.

5. Instruction does not provide sufficient hand-on experiences.

When these problems go unnoticed in mathematics classes, some students are discouraged and may avoid taking further mathematics courses beyond the minimum requirement of the school. Moreover, compared with other school subjects, more of the skill development in mathematics is cumulative. Consequently, if students experience over one year of poor mathematics instruction, they may fail to learn the knowledge necessary for further development in mathematics.

Sex Differences in Mathematics Participation

In general, women represent approximately 52% of the population, 49% of the total employment in professional and related occupations, yet only 15% of the scientific and engineering workforce (National Science

Foundation, 1988). Educational literature examines this imbalance from the perspective of sex differences in mathematics and science participation, and female participation in mathematics is always at the heart of contemporary concerns about sex differences in mathematics education (Burton, 1979; Cohen & Cohen, 1980; Moore & Smith, 1985; Pallas & Alexander, 1983).

Ernest (1976) and Sells (1973) detected a huge discrepancy in high school mathematics background between male and female freshmen. These documents stimulated a number of investigations on the nature of sex differences in the course-taking pattern of high school mathematics. Although occasional studies indicated some improvement in female participation in advanced mathematics courses (e.g., Rallis, 1986), the vast majority of research in this area suggested that the change has been painfully slow (e.g., Meece & Parsons, 1982; Sells, 1980; Sherman & Fennema, 1977). Overall, sex differences are considered substantial in mathematics participation, though might not be as large as those reported by Sells (e.g., Pallas & Alexander, 1983; Cohen & Kosler, 1991; Educational Testing Service, 1979; Ernest, 1976; Fennema, 1977; Fennema & Sherman, 1977a).

There is no lack of mixed research findings in this area. Some national surveys revealed that sex differences are not appreciable in terms of participation in less advanced mathematics courses (e.g., Armstrong, 1981; Fennema & Carpenter, 1981). These researchers claimed that mathematics dropout is more likely to happen when students face more advanced mathematics courses. Male students clearly outnumber females in taking advanced mathematics courses. Female students seem to be under considerable psychological or sociological pressure in taking mathematics courses. For example, female students indicate an intent to take additional mathematics courses more often than their male peers, but they actually take fewer (Fennema & Sherman, 1977a). Lee and Ware (1986) noticed that most small-scale studies where stronger sex differences are observed are based on college-bound students, suggesting that sex differences in mathematics course-taking may be greater among academically able students.

There is a sizable literature speculating the reason why female students drop out of mathematics. A few researchers believe that female students avoid the study of mathematics because of the properties of the school (e.g., Casserly, 1980; Marret & Gates, 1983). The stronger tendency in the literature, however, is to examine sex differences in mathematics participation from psychological and sociological perspectives. Female students' attitude toward mathematics has received much attention in the examination of sex differences in mathematics dropout. Female students are more likely to have negative attitudes toward mathematics (Armstrong, 1980; Brush, 1980; Fennema & Sherman, 1977a; Sherman, 1981, 1982b, 1983; Sherman & Fennema, 1977; Wise, 1978), and negative attitude toward mathematics substantially undermines mathematics participation of female students (see Catsambis, 1994). Male students rate mathematics as useful and having better practical value in earning a living (Cohen & Kosler, 1991). Female students, on the other hand, are less convinced that mathematics will be useful to them in the future (Fox, 1975; Kaczala, 1980; Smith, 1980). Haven (1971) found that the two most significant predictors of mathematics course-taking in high school for high ability girls are the perception of usefulness of mathematics in the future and the desire to have a greater impact in the natural than social sciences.

Furthermore, female students tend to be particularly at risk of dropping out of mathematics because mathematics has frequently been viewed as a male domain. Instructional materials, as well as family, peer, and teacher behaviors and expectations, have frequently reinforced this negative belief (Kasten & Howe, 1988). As a result, female students exhibit low self-confidence in their mathematics abilities (Sherman, 1981, 1982b, 1983). In addition, mathematical abilities and aptitudes of female students are found to be attributable to female mathematics dropout (Brush, 1980; Sherman, 1981, 1983; Stallings & Robertson, 1979). Their images of scientists and themselves also influence their behavior in mathematics participation (Brush, 1979, 1980; MacCorquodale, 1984).

Finally, female students receive less encouragement from their parents, teachers, and friends to take mathematics courses (Casserly, 1980; Fox, 1977; Sherman, 1982b; Stallings & Robertson, 1979). Hardeman and Laquer (1982) identified educational practices that discourage young women from pursuing the study of mathematics as sexism in the school, sex stereotyping in mathematics textbooks, and the behavior of mathematics teachers in the classroom. Good, Sikes, and Brophy (1973) showed that mathematics teachers give high achieving males much more attention in mathematics classes than any other group of female students. Females are also more likely than males to seek career advice from counselors regarding advanced mathematics courses (Harway, Astin, Shur, & Whitely, 1976). However, some counselors admit to discouraging female students from taking high-level mathematics courses, based on their own stereotyped views of sex abilities (Meece & Parsons, 1982). Collier (1989), in a review of 100 general psychology textbooks, revealed that 91% of them present as a fact that men have greater mathematical ability than women. Therefore, some national reports such as Everybody Counts: A Report on the Future of Mathematics Education (National Research Council, 1989) believe that sex differences in mathematics are predominantly due to the accumulated effects of sex-role stereotypes in family, school, and society.

Summary

This review attempts to put in perspective the theoretical framework underlying mathematics participation. The literature review shows that sex, SES, mathematics achievement, attitude toward mathematics, mathematics anxiety, and self-esteem appear to have the potential to make an important contribution to mathematics participation. Specifically, there seem to be sex differences in mathematics participation in favor of male students. Common reasons in the research literature that explain sex differences in mathematics participation include (see, for example, Leder, 1992):

1. Various psychological disadvantages of women relating to mathematics such as negative attitude, low confidence, and high anxiety.

2. School-level differences such as the different treatment of male and female students in mathematics classes.

3. General social inequalities between men and women such as social image, social stereotyping, and type of career.

Numerous studies have shown that SES exerts considerable influences on educational attainment in general, as well as all kinds of dropout behaviors in particular (e.g., Ekstrom, et al., 1986; Bachman, Green, & Wirtanen, 1971; Rumberger, 1983).

Willms (1992) demonstrated that the measures of prior academic achievement and cognitive ability are particularly important control variables, even more important than the measures of SES and student characteristics, in the examination of students' schooling outcomes. Attitude toward mathematics seems to have a substantive relationship with mathematics participation in that negative attitude toward mathematics undermines mathematics participation. Female students show more negative attitude toward mathematics than males. Researchers are divided with respect to the relationship between mathematics anxiety and mathematics participation. However, it is relatively consistent that both male and female students develop mathematics anxiety, with females being at a higher level. Mathematics anxiety also has more negative effects on female's than male's mathematics learning. The research literature seems to imply that there is a relationship between self measures relating to mathematics and mathematics participation. Lower levels of self-confidence in mathematics is one explanation for the lack of mathematics participation, especially of female students.

The major shortcoming in the research of mathematics participation is the lack of longitudinal scope. For example, although sex and SES affect mathematics participation, there is no clear evidence whether their effects are consistent throughout students' academic careers in high school. In addition, there have been no systematic inquiries into the relationship of mathematics achievement, attitude toward mathematics, mathematics anxiety, and self-esteem to mathematics participation. Studies are limited in number, but do demonstrate that those variables are potential determinants of course enrollment in mathematics. This study will examine the roles of sex, SES, mathematics achievement, attitude toward mathematics, mathematics anxiety, and self-esteem in mathematics participation, and trace how these variables affect mathematics participation of high school students over the five-year period from grade 8 to 12. The probabilities of dropping out of mathematics courses in each year over the five-year period will be calculated, conditional on the above background variables. The changing role of each background variable in mathematics participation over the entire secondary schooling will be illustrated, and the differential characteristics of groups of students regarding mathematics participation will also be discussed. This study, therefore, will fill in many gaps in the research on mathematics participation.

Chapter 3

Methodology

This study will examine two types of mathematics participation. The first is participation in advanced mathematics courses, of which most are electives. The aim is to examine mathematics participation beyond the basic mathematics requirement for graduation. The results can be used to address issues such as who is persistent in pursuing advanced mathematics courses. The second type of participation does not distinguish between basic and advanced mathematics courses, and therefore depicts an overall picture of mathematics participation. The results can be used to address issues such as the time when high school students are at the greatest risk of dropping out of mathematics courses. In terms of analytic methods, these two types of mathematics participation can be handled in the same way. The models will examine the same set of independent variables, and employ the same statistical procedures. The only difference is the dependent variable, with enrollment in advanced courses as one variable, and overall enrollment as the other. The statistical analyses were performed with SPSS/PC+, and the SPSS Axum program was used to generate statistical graphs (Norusis/SPSS Inc., 1992).

Sample of the Study

The data for this study were drawn from the Longitudinal Study of American Youth (LSAY), a national six-year panel study of mathematics and science education in public middle and high schools in the United States. Miller and Hoffer (1994) prepared a user's manual that contains a detailed description of LSAY. LSAY contains two sets of public schools: one is a national probability sample of 52 high schools and 52 middle schools; the other is a special sample of 8 high schools and 8 middle schools in school districts with outstanding elementary science programs. LSAY started in the fall of 1987 with samples of about 60 seventh graders, referred to as "cohort 2", and 60 tenth graders, referred to as "cohort 1", from each of sixty localities across the United States. The seventh and tenth graders were followed for six years. The current study employs data of cohort 2 from the 1987 - 88 school year (the seventh grade) to the 1992 - 93 school year (the twelfth grade). The total sample includes 3116 students.

Description of Variables

Variables in LSAY cover student social and psychological background, students' schooling experiences, and school organizational and social context. As described in Chapter 1, the independent variables examined in this study include the following:

Student sex is based on students' reports. In LSAY, students' sex was checked against the students' first name with miscodings being corrected. Sex is renamed as female, and used as a dummy variable in this analysis, coded 0 for males and 1 for females. This variable is then centered on its mean.

Parental socioeconomic status (SES) is a continuous composite variable based on parents' reports of their education and occupational status, and students' reports of household possessions. Although the theoretical construct of SES is well defined (see Duncan, 1961), the accuracy of its measurement may sometimes be problematic depending on the channels of data collection. For example, some studies use information from students on their parents' education and occupation, resulting in the measure of SES being not very accurate. Because LSAY collected socioeconomic information from both parents and students, the measure of SES is far more accurate than a SES measure based only on students' information. In this study, SES is considered relatively stable over a period of five years. SES is standardized with a mean of 0 and a standard deviation of 1.

Prior mathematics achievement (from grade 7 to 11) is used to examine its effect on mathematics participation in the next school year. In LSAY, mathematics achievement was obtained in the fall of each of the years the students were in school. In the first five years (grade 7 to 11), items from the National Assessment of Educational Progress (NAEP) were used. LSAY selected NAEP items based on three skill dimensions: (a) simple recall and recognition, (b) routine problem-solving, and (c) more complicated problem-solving. The reliabilities of mathematics achievement are 0.86 in the 7th grade, 0.91 in the 8th grade, 0.92 in the 9th grade, 0.94 in the 10th grade, and 0.95 in the 11th grade. Test scores are actually formula scores that have been controlled for difficulty, reliability, and guessing, on the basis of item response theory (IRT) (see Crocker & Algina, 1986). As a result, test scores can be compared across test forms and grade levels. Prior mathematics achievement is standardized in this study.

Prior attitude toward mathematics[1] (from grade 7 to 11) is a continuous composite variable measuring the extent to which students (a) like mathematics, (b) perceive mathematics to be useful in everyday life, and (c) are confident in learning mathematics. This variable is in a metric of 0 - 12, with high values indicating positive attitude toward mathematics. The reliabilities of attitude toward mathematics are 0.69 in the 7th grade, 0.66 in the 8th grade, 0.67 in the 9th grade, 0.72 in the 10th grade, and 0.76 in the 11th grade. Prior attitude toward mathematics is standardized in this study.

Prior mathematics anxiety (from grade 7 to 11) measures two dimensions: (a) perceived ability to study mathematics, and (b) anxiety toward mathematics. LSAY provides a continuous composite variable of mathematics anxiety in a metric of 0 - 8, with high values indicating high levels of mathematics anxiety. The reliabilities of mathematics anxiety are 0.62 in the 7th grade, 0.68 in the 8th grade, 0.71 in the 9th grade, 0.71 in the 10th grade, and 0.71 in the 11th grade. Prior mathematics anxiety is standardized in this study.

Prior self-esteem (from grade 7 to 11) is a continuous composite variable measuring (a) attitude toward self, (b) self-confidence, and (c) level of self-satisfaction. This variable is in a metric of 0 - 24, with high values indicating high levels of self-esteem. The reliabilities of self-esteem are 0.66 in the 7th grade, 0.70 in the 8th grade, 0.74 in the 9th grade, 0.78 in the 10th grade, and 0.80 in the 11th grade. Prior self-esteem is standardized in this study.

Mathematics participation is the dependent variable. It comes from students' reports about the enrollment status in mathematics courses in each year when students were in school. Students are one of the best data sources regarding course enrollment in mathematics. However, this measure of mathematics participation would have been more accurate if LSAY had used school records to verify the course enrollment status of each student. It was coded as a dummy variable with 0 denoting not taking mathematics courses and 1 denoting taking mathematics courses. Note that some students might not take mathematics courses in a particular year but later enroll in mathematics courses. Mathematics dropout thus refers to the grade in which students stopped taking mathematics courses altogether. For example, putting the enrollment status of a student from grade 8 to 12 in a sequence, the course-taking pattern of 10100 means that the student took mathematics in grade 8, did not take mathematics in grade 9, did take mathematics in grade 10, and dropped out of mathematics in grade 11. For the purpose of survival analysis this student would figure in the analysis in the same way as a student with a 11100 pattern.

Advanced mathematics is based on the LSAY coding of mathematics courses. Except for low 7th-grade mathematics, low 8th-grade mathematics, basic mathematics (9th - 12th grade), vocational mathematics, consumer mathematics, and mathematics (NEC), all other mathematics courses are considered advanced in this study. These courses include average and high 7th-grade mathematics, average and high 8th-grade mathematics, Geometry (including honors), Pre-algebra, Algebra I and II (including honors), Trigonometry (including honors), Analytic Geometry, Calculus, and Statistics and Probability. What constitutes advanced mathematics is often relative and . This study choses not to define advanced mathematics as such that only college-bound students consider to take. Rather, the definition of advanced mathematics actually separates the most basic mathematics courses from the rest. The practical benefit of including some average levels of mathematics courses as advanced mathematics is that students may be encouraged and thus reduce their anxiety toward advanced mathematics.

Among the independent variables, sex and SES are time-invariant variables which are constant over the entire five years between grade 7 and 11. Prior mathematics achievement, prior attitude toward mathematics, prior mathematics anxiety, and prior self-esteem are time-varying variables which fluctuate over the five years of observation.

Statistical Rationale

This study employs a set of statistical procedures generally referred to as survival analysis or event history analysis (see Allison, 1984; Willett & Singer, 1991; Yamaguchi, 1991). Allison (1984) has delineated several statistical dimensions that differentiate and determine various methods of survival analysis:

1. Distributional versus regression methods;
2. Repeated versus nonrepeated events;
3. Single versus multiple kinds of events;
4. Parametric versus nonparametric methods;
5. Discrete versus continuous time.

Note that the event of mathematics dropout can be considered a single and nonrepeated event.

The statistical model for describing students' academic careers is based on the logistic regression method that examines how mathematics participation depends on a linear function of explanatory variables. The logistic regression analysis is appropriate for modeling the likelihood or probability of an event because of its capability of linearizing the dichotomous outcome and predicting independent effects (see Aldrich & Nelson, 1984). The logistic technique permits fitting a logistic model to the available data to test whether independent variables reliably predict the occurrence of an event. Furthermore, the logistic regression model can produce meaningful inferences for both discrete and continuous predictors. Statistical results of logistic regression can also be interpreted with less technical terms such as odds and likelihood.

The logistic regression analysis is a nonparametric method in that it makes "few if any assumptions about the distribution of event times" (Allison, 1984, p. 14). The nonparametric nature of the model is preferable in this study because researchers know little about the characteristics of the distribution concerning mathematics participation. This study will examine the shape of survival and hazard probabilities over the six-year period between grade 7 and 12. A survival probability is the proportion of individuals who have not experienced an event until a time point. A hazard probability is the proportion of individuals who experience an event during a certain period. Thus, this study provides insight into the potential parametric form of the distribution of mathematics participation.

Because this study measures time in years, the time unit is too large to be treated as continuous time (see Allison, 1984). Therefore, discrete-time logistic regression analysis is employed, as outlined in Allison (1982, 1984). Considering simplicity a desirable trait, Miller and McGill (1984) state that "a model for estimating future enrollments should be simple to minimize the amount of time spent in data collection, yet accurate enough for decision making" (p. 32). The survival analysis model, therefore, will include only significant predictors of mathematics participation. Significant variables are used to define a series of groups of students for a more comprehensive analysis of mathematics participation.

Statistical Procedures

The statistical analysis comprises three major steps: (a) analysis of the survival and hazard function over the six-year period from grade 7 to 12, (b) logistic regression analysis for each year of the five-year period from grade 8 to 12, (c) survival analysis over the five-year period. These are described below:

Analysis of the survival and hazard functions. In survival analysis, the time until an event is experienced is referred to as the survival time. A distribution of survival times is characterized with a survival function, $S(t)$, which is the probability that an individual survives beyond time t. Equivalently, $S(t)$ specifies the proportion of individuals who have not experienced the event during the period preceding time t. The graph of $S(t)$ versus t is called a survival curve. Usually, it begins at $S(t) = 1$ for $t = t_0$ (i.e., all individuals have not yet experienced the event), and declines towards S(t) = 0 as t increases. The rate of decline corresponds to the rate at which individuals have experienced the event.

One of the goals of a survival analysis is to detect especially risky time periods. The slope of a survival function or curve indicates the extent to which the individuals experience the event between one time point and the next. A sharp plunge of the survival function implies that a large proportion of the individuals who survive until the end of one time point experience the event before the end of the next. However, a researcher who uses this visual approach to identify risky time periods may overlook potentially important variations in slope, especially if the researcher is attempting to compare the survival functions for two or more groups (Willett & Singer, 1991).

The hazard function H(t) allows one to identify the fluctuations in the slope of a survival curve more precisely. The hazard probability refers to the proportion of individuals in the risk set (the pool of the individuals available at the beginning of a certain period) who experience the event during the period. A distribution of these probabilities or proportions over time forms the hazard function H(t). The graph of H(t) versus t is called a hazard curve. Because the hazard function is based on the group remaining at the end of the preceding period, the most risky periods can be accurately identified. A comparison of hazard probabilities from different periods is also meaningful.

This study will examine the survival function of mathematics participation over the six-year period between grade 7 and 12. It will also examine the hazard function of mathematics participation over the six-year period to precisely pinpoint the grade(s) in which students were most likely to stop taking mathematics courses. A useful starting point for the analysis will be to graph the survival and hazard functions for the entire cohort of students, and separately for males and females.

Logistic regression analysis. The standard logistic regression model for predicting the likelihood or probability that an event, u, occurs, from a single independent variable, is described as (see Bock, 1975)

$$P(u=1)=\frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}$$

where β_0 is the intercept parameter, β_1 is the slope parameter, X is the independent variable, and e is the base of the natural logarithms. β_0 and β_1 are estimated from the data. The model can include a number of independent variables. A more general form of the logistic regression model is expressed as

$$P(u=1)=\frac{e^{Z}}{1+e^{Z}}$$

or equivalently

$$P(u=1)=\frac{1}{1+e^{-Z}}$$

where Z is the linear composite of independent variables

$$Z=\beta_0+\beta_1 X_1+\beta_2 X_2+\ldots+\beta_p X_p$$

The interpretation of logistic regression analysis is similar to that of multiple regression analysis. β_0 represents the intercept (the unadjusted hazard probability), and the other parameters β_i ($i = 1, 2, \ldots p$) denote the expected change in the dependent variable for a one-unit increase in the independent variable, X_i, given that other independent variables in the model are held constant. The likelihood or probability that the event, u, does not occur, denoted as $P(u = 0)$, is estimated as

$$P(u = 0) = 1 - P(u = 1)$$

Logistic regression results can also be expressed as the <u>odds</u> of an event occurring which is defined as

$$\frac{P(u = 1)}{P(u = 0)} = e^Z$$

or

$$\frac{P(u = 1)}{1 - P(u = 1)} = e^Z.$$

The result is then the expected change in the odds of an event occurring for a one-unit increase in the independent variable X_i.

In this study, a series of logistic regression analyses are used to predict the likelihood of mathematics participation, and to discern the variables useful in making the prediction. One logistic regression analysis is performed for each year of the five-year period between grade 8 and 12. The linear composite of independent variables is:

$$\begin{aligned} Z = \; & \beta_0 + \\ & \beta_1(\text{female}) + \end{aligned}$$

β_2(SES) +

β_3(prior mathematics achievement) +

β_4(prior attitude toward mathematics) +

β_5(prior mathematics anxiety) +

β_6(prior self-esteem)

The odds of mathematics participation is then

$$\frac{\text{P(taking mathematics courses)}}{\text{1 - P(taking mathematics courses)}}$$
$$= e^{\beta_0} e^{\beta_1(\text{female})} e^{\beta_2(\text{SES})} \ldots e^{\beta_6(\text{prior self-esteem})}$$

For example, if β_1 were zero, then e^{β_1} would be one, indicating that females had the same odds of taking mathematics courses as males. If β_1 were negative, then e^{β_1} would be less than one, indicating that females had smaller odds of taking mathematics courses than males. If β_1 were positive, then e^{β_1} would be greater than one, indicating that females had greater odds of taking mathematics courses than males.

Results of logistic regressions can be used to address the first research question. For example, after estimating the coefficients in the model, one can estimate the probability of dropping out of mathematics courses in grade 11 for a female student with high SES, given particular levels of achievement, attitude, anxiety, and self-esteem in grade 10. The probability of dropping out of mathematics courses is one minus that of taking mathematics courses. The logistic regression model will also report statistically significant variables which best predict the survival probability in each year of the five years, providing insight into the second research question. For example, one can discern whether SES is significant, and if so, whether its effects are constant across grades 8 to 12. An examination of the five logistic regression models will show the pattern of students' decisions to take or avoid mathematics courses over the five-year period, thereby addressing the third research question.

Survival analysis. Nested logistic regression models are used to synthesize statistical information obtained separately over the five-year period and perform a combined analysis of mathematics participation. Although the nature of the analysis is still logistic regression, there is special treatment for time-varying variables. The procedure is that "for each unit of time that each individual is known to be at risk, a separate observational record is created" (Allison, 1984, p. 18). It is preferable to refer to these observations as person-years in this study. For example, students who drop out of mathematics courses in year 2 (grade 8) contribute 2 person-years. Students who do not drop out of mathematics courses (from grade 8 to 12) contribute the maximum of 5 person-years.

The first model is a base-line model which assumes that the hazard probability β_0 is constant across the five-year period. The general form of the linear function in such a model is denoted as

$$Z = \beta_0 + \beta_1 X_1 + \ldots + \beta_m X_m + \beta_{m+1} X_{m+1}(t) + \ldots \beta_{m+n} X_{m+n}(t)$$

where $X_1, \ldots X_m$ are time-invariant variables and $X_{m+1}(t), \ldots X_{m+n}(t)$ are time-varying variables. In this study, the linear function Z is therefore specified as

$$
\begin{aligned}
Z = \; & \beta_0 + \\
& \beta_1(\text{female}) + \\
& \beta_2(\text{SES}) +
\end{aligned}
$$

β_3(prior mathematics achievement)(t) +

β_4(prior attitude toward mathematics)(t) +

β_5(prior mathematics anxiety)(t) +

β_6(prior self-esteem)(t)

where t denotes grade 8 to 12.

The second model allows the hazard probability β_0 to be different across the five years. The procedure is to create a set of dummy variables, one for each of the last four years (grades 9 to 12) (see Allison, 1984). The hazard probability in grade 8 is then the base for comparison. The general form of the linear function in such a model is described as

$$Z = \beta_0(t) + \beta_1 X_1 + \ldots + \beta_m X_m + \beta_{m+1} X_{m+1}(t) + \ldots \beta_{m+n} X_{m+n}(t)$$

where $\beta_0(t)$ represents the time-varying hazard probability. In this study, the above linear function Z is specified as

Z = β_0(t) +

β_1(female) +

β_2(SES) +

β_3(prior mathematics achievement)(t) +

β_4(prior attitude toward mathematics)(t) +

β_5(prior mathematics anxiety)(t) +

β_6(prior self-esteem)(t)

where t denotes grade 8 to 12.

The likelihood-ratio chi-square test is used to compare and evaluate the fit of the data for the nested models (Allison, 1984). The test statistic is twice the positive difference between the log-likelihoods of the two models. This statistic has an asymptotic chi-square distribution under the null hypothesis of no difference between the two models, with the degrees of freedom being the number of constraints that distinguish the two models. If the chi-square test is not significant, then the two models fit the data equally well, and the simpler model with invariant hazard probabilities is preferable. If the chi-square test is significant, the model with different hazard probabilities is preferable.

The final survival model contains only statistically significant variables. One can use this model to identify characteristics of students who have either minimum or maximum hazard of dropping out of mathematics courses. Because the model takes into account time-varying variables, it is also possible to examine issues such as whether maintaining a top position in mathematics achievement over time or growing in mathematics achievement over time leads to more active mathematics participation.

Similar to the notion in multilevel regression analysis (e.g., Bryk & Raudenbush, 1992), this study introduces what is called a "typical" student. A typical student refers to one with nationally average characteristics. Specifically, with the above linear composite of the independent variables, a typical student is one who has nationally average SES, prior mathematics achievement, prior attitude toward mathematics, prior mathematics anxiety, and prior self-esteem. One can also envisage a group of nationally average students which has the same proportion of males and females as the national sample. The statistical procedure to produce information on a typical student is to center the background variables such that zero values represent the characteristics of the typical student. A hazard profile, namely the hazard probabilities $\beta_0(t)$, of the typical student can therefore be established, which provides a national measure of mathematics participation.

Treatment of Missing Data

Missing data can be substantial in a panel design like LSAY simply because of the long duration of data collection. To maintain a reasonable proportion of original data is always a challenge for users of longitudinal surveys. This study must cope not only with missing data on the independent variables but also with missing data on the dependent variable, which are traditionally deleted in regression analyses (see Cohen & Cohen, 1983). As the dependent variable, mathematics participation was coded based on the historical pattern of mathematics course-taking status for each student.

Some students are known to drop out of mathematics courses in, for example, grade 12, but have missing data on mathematics participation in grades 10 and 11. The question is whether one should (a) delete the individuals from the data, (b) consider them to have dropped out in grade 10 (i.e., 11000), (c) consider them to have dropped out in grade 12 (i.e., 11110), or (d) establish a rule that is a compromise between (b) and (c). About 15% of the cases in this study presented this problem. The treatment emphasized in this study was option (d); that is, to regard the first half of the missing data as taking mathematics courses and the other half as not taking mathematics courses. In the above example, therefore, students are coded as dropping out of mathematics courses in grade 11 (i.e., 11100). If students also have missing data in grade 9, the treatment is to regard the first two-thirds of the missing data as taking mathematics courses and the last one-third as not taking mathematics courses. Therefore, students are coded as dropping out of mathematics courses in grade 11 (i.e., 11100).

Cohen and Cohen's (1983) method was used to handle missing data on the independent variables. Cohen and Cohen (1983) represent the existence of missing data with dummy indicator variables. The theoretical base is that if missing data on, for example, the independent variable X are missing randomly, then the mean of the dependent variable Y for subjects with complete data would be similar to the mean of Y for subjects with missing data. A simple statistical procedure to test whether such an expectation is sustainable is to create a dummy indicator variable X_a which is coded zero for subjects with complete data and 1 for subjects with missing data. After replacing missing data on X with the mean of X, one regresses Y on both X and X_a.

With this strategy, the regression coefficient β associated with X will be equivalent to the simple regression coefficient that would be obtained from subjects with complete data. The regression coefficient β_a associated with X_a denotes the difference in coefficients between subjects with complete data and those with missing data. Therefore, if β_a is not significant, then β is also representative of subjects with missing data. If β_a is significant, then a measure of the difference in coefficients is obtained.

Evaluation of Statistical Models

There are a number of methods to evaluate how well a survival model fits the data. This study employs three of these methods. The basic approach is to compare the model-predicted outcomes to the observed outcomes. A histogram is used in which the frequency distribution of model-predicted probabilities is separately presented for participants and dropouts in a scale of 0 to 1. If the survival model fits the data well, a polarization between participants and dropouts in terms of their frequency distribution should be observed. In other words, the frequency distribution of participants should not overlap with that of dropouts. If the distribution of participants moves to the "0" side and overlaps with the distribution of dropouts, then the model has problems in classifying participants. If the distribution of dropouts moves to the "1" side and overlaps with the distribution of participants, then the model has problems in classifying dropouts.

Another measure of model-data-fit is the goodness-of-fit statistic, which compares the observed probabilities to those predicted by the model (see Norusis/SPSS Inc., 1992). The goodness-of-fit statistic is defined as

$$Z^2 = \sum \frac{\text{Residual}_i^2}{P_i(1-P_i)}$$

where the residual is the difference between the observed value Y_i and the predicted value P_i. For example, if the probability of taking mathematics courses is estimated to be 0.75 ($Y_i = 0.75$) for a participant ($P_i = 1$), the residual is then 1 - 0.75 = 0.25. If the probability of taking mathematics courses is estimated to be 0.35 ($Y_i = 0.35$) for a dropout ($P_i = 0$), the residual is then 0 - 0.35 = -0.35. Therefore, the goodness-of-fit statistic is the sum of residual components for all students. Without references, however, this sum statistic is difficult to interpret. The average goodness-of-fit statistic is more interpretable; it is defined as:

$$\text{Average goodness-of-fit statistic} = \frac{Z^2}{N}$$

where N is the number of students in the model.

To understand this average goodness-of-fit statistic, consider a special case in which the probability of taking mathematics courses is predicted to be

0.50 for a participant. As a result, the average goodness-of-fit statistic has a value of 1. That is, if the predicted probability is in the middle between participation (P = 1) and dropout (P = 0), the average goodness-of-fit statistic is 1. If the predicted probability is closer to 1 for the participant (close to his or her real status of mathematics participation), the average goodness-of-fit statistic is less than 1. If the predicted probability is closer to 0 for the participant (away from his or her real status of mathematics participation), the average goodness-of-fit statistic is greater than 1. Similar reasoning also applies to dropouts. Therefore, a good statistical model shows an average goodness-of-fit statistic of less than 1, indicating that the model is able to predict the likelihood of mathematics participation of a student which is, on average, close to his or her real status of mathematics participation.

This study also employs another method, the goodness-of-fit index, to assess whether or not a survival model fits the data. It examines how likely the sample results actually are, given the parameter estimates. Note that the major goal of building a statistical model is to choose parameter estimates that would make the observed results as likely as possible (Norusis/SPSS Inc., 1992). Because the probability or likelihood of the observed results, given the parameter estimates, is always less than 1, it is traditionally convenient to use -2 times the log of the likelihood (-2LL) as a measure of how well the estimated model fits the data (see Allison, 1984). A better model produces a high likelihood of the observed outcomes, resulting in a smaller value for -2LL.

Model-data-fit is always relative, and, therefore, comparisons between models are important. This study includes two goodness-of-fit indices. The first is the base-line index for the model that contains only the constant. The second is the index for the (full) model with all the independent variables.

The difference between the two indices is referred to as the model chi-square with the degrees of freedom being the number of variables in the full model (see Norusis/SPSS Inc., 1992). If the model chi-square is significant, the full model fits the data significantly better. If the model chi-square is not significant, the full model needs improvement or modification.

Chapter 4

Results

This chapter reports the results of statistical analyses. It contains two major parts. The first part presents the results of the survival analysis pertaining to mathematics participation; the second part displays the results of the survival analysis pertaining to participation in advanced mathematics. In each part, the results are arranged in terms of (a) survival and hazard probabilities; (b) factors influencing participation; (c) logistic regression models predicting participation by year; (d) survival analysis models predicting participation; (e) longitudinal effects of factors on participation; (f) the adequacy of the statistical models. Additional statistical analyses are performed on participation in advanced mathematics because the effects are more dramatic.

Survival Analysis of Mathematics Participation

Survival and Hazard Probabilities[2]

Figure 1 presents the survival curve of mathematics participation: the sample survival probability is plotted against the grade level. The results indicate that the likelihood of mathematics participation is 100% from grade 7 to 9, and only 2 in 1000 drop out of mathematics in grade 10. Mathematics dropout becomes evident in grade 11. The likelihood of continuing mathematics is approximately 93% in grade 11. The highest risk of dropping out of mathematics is in the transition to grade 12. The likelihood of mathematics participation is approximately 64% in grade 12. Therefore, if 100 students were randomly selected in the 7th grade and followed until the 12th grade, 93 students would still be participating in mathematics in the 11th

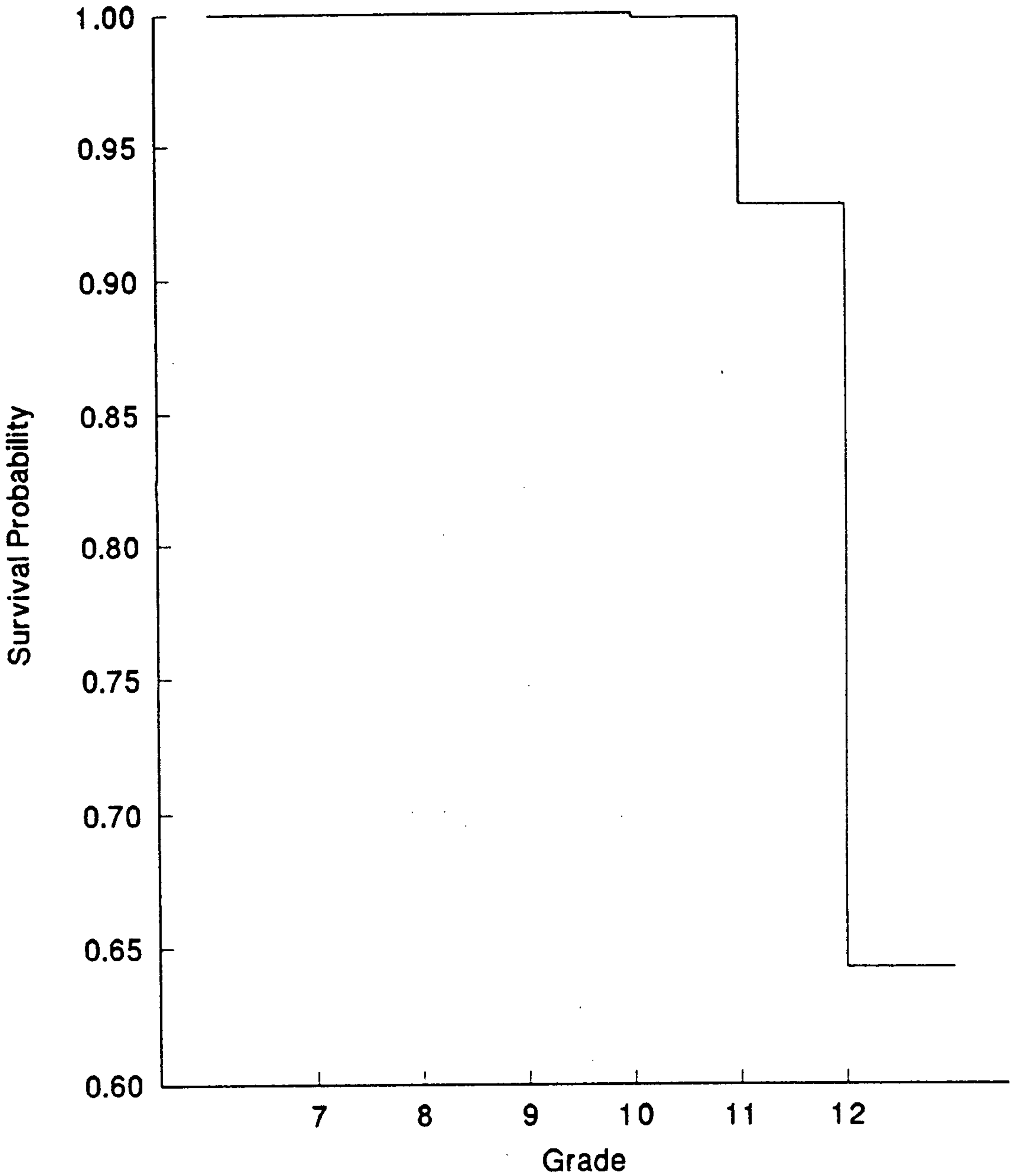

Figure 1. The survival function of mathematics participation (sample survival probability versus grade level).

grade, and 64 of them would still be enrolling in a mathematics course in the 12th grade.

Figure 2 presents the hazard curve of mathematics participation: the sample hazard probability is plotted against the grade level. There are no dropouts in mathematics from grade 7 to 9. The dropout rate in mathematics is trivial, with approximately 2 in 1000, in grade 10. An evident hazard probability of 7% is observed in grade 11, however. Among all the students available to take mathematics courses in the 11th grade (note that these students have successfully enrolled in a mathematics course in the 10th grade), 7% of them fail to participate in grade 11. The highest hazard probability of 31% appears in grade 12. Among all the students available to take mathematics courses in the 12th grade (note that these students have successfully enrolled in a mathematics course in the 11th grade), 31% of them fail to participate in grade 12. The transition from grade 11 to 12, therefore, is the point at which students are most at risk of dropping out of mathematics.

The survival curves of males and females are graphed separately in Figure 3 to show sex differences in mathematics participation. Males and females have the same survival pattern with regard to mathematics participation. The likelihood of mathematics participation is 100% for both males and females from grade 7 to 9. The trivial differences in mathematics participation between males and females in grades 10 and 11 are of no practical significance, although both males and females avoid mathematics courses more evidently in grade 11. Sex differences in mathematics participation appear in grade 12: females have a higher risk of dropping out of mathematics than males. Suppose 100 males and 100 females were randomly selected in the 7th grade and followed until the 12th grade. About 93 males and 93 females would still be participating in mathematics in the 11th grade,

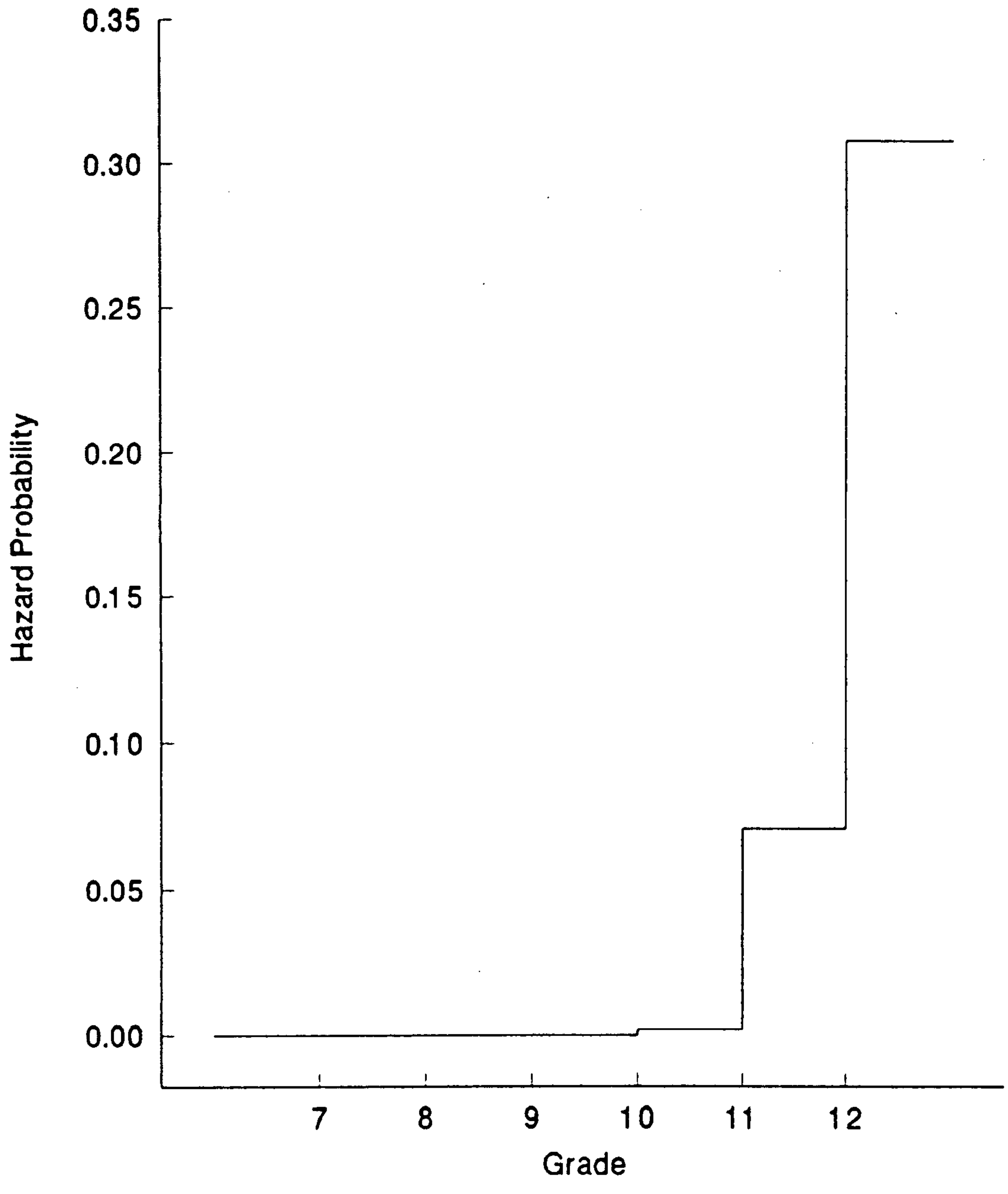

Figure 2. The hazard function of mathematics participation (sample hazard probability versus grade level).

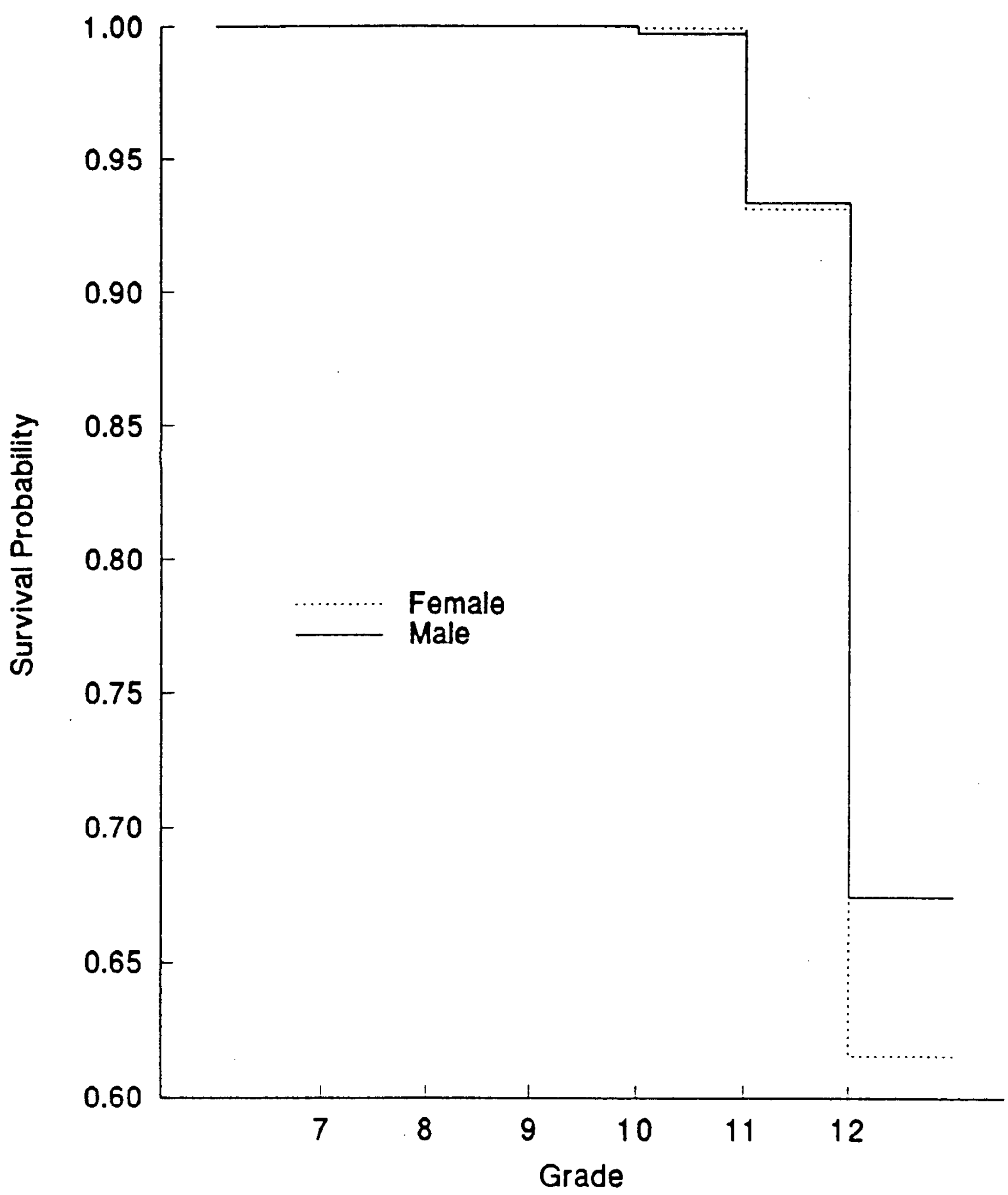

Figure 3. The survival function of mathematics participation (sample survival probability versus grade level), by sex.

but in the 12th grade, about 67 males and 62 females would still be enrolling in a mathematics course.

The hazard curves of males and females are graphed separately in Figure 4 to illustrate the extent to which males and females drop out of mathematics in each grade. Males and females have the same hazard pattern concerning mathematics participation. The hazard probability of mathematics participation is zero for both males and females from grade 7 to 9. The difference in hazard probability between males and females is trivial in grade 10. Females have a slightly higher hazard probability in the 11th grade, but a substantially higher hazard probability in the 12th grade, than males. Among all the male and female students available to take mathematics courses in the 11th grade (note that these students have successfully enrolled in a mathematics course in the 10th grade), about 6% of males and 7% of females fail to participate. Among all the male and female students available to take mathematics courses in the 12th grade (note that these students have successfully enrolled in a mathematics course in the 11th grade), about 28% of males and 34% of females fail to participate. Therefore, the transition from grade 11 to 12 is the most risky point in which both males and females are likely to stop taking a mathematics course, with females being at higher risk of dropping out of mathematics.

Factors Influencing Mathematics Participation

Table 1 shows the means and standard deviations of five variables for participants and dropouts. Differences in means between participants and dropouts are tested for statistical significance based on their 95% confidence intervals (see Glass & Stanley, 1970). Levene's test is used to examine equality of variances between participants and dropouts (see Glass & Stanley, 1970). Mathematics participants have statistically significantly higher SES than

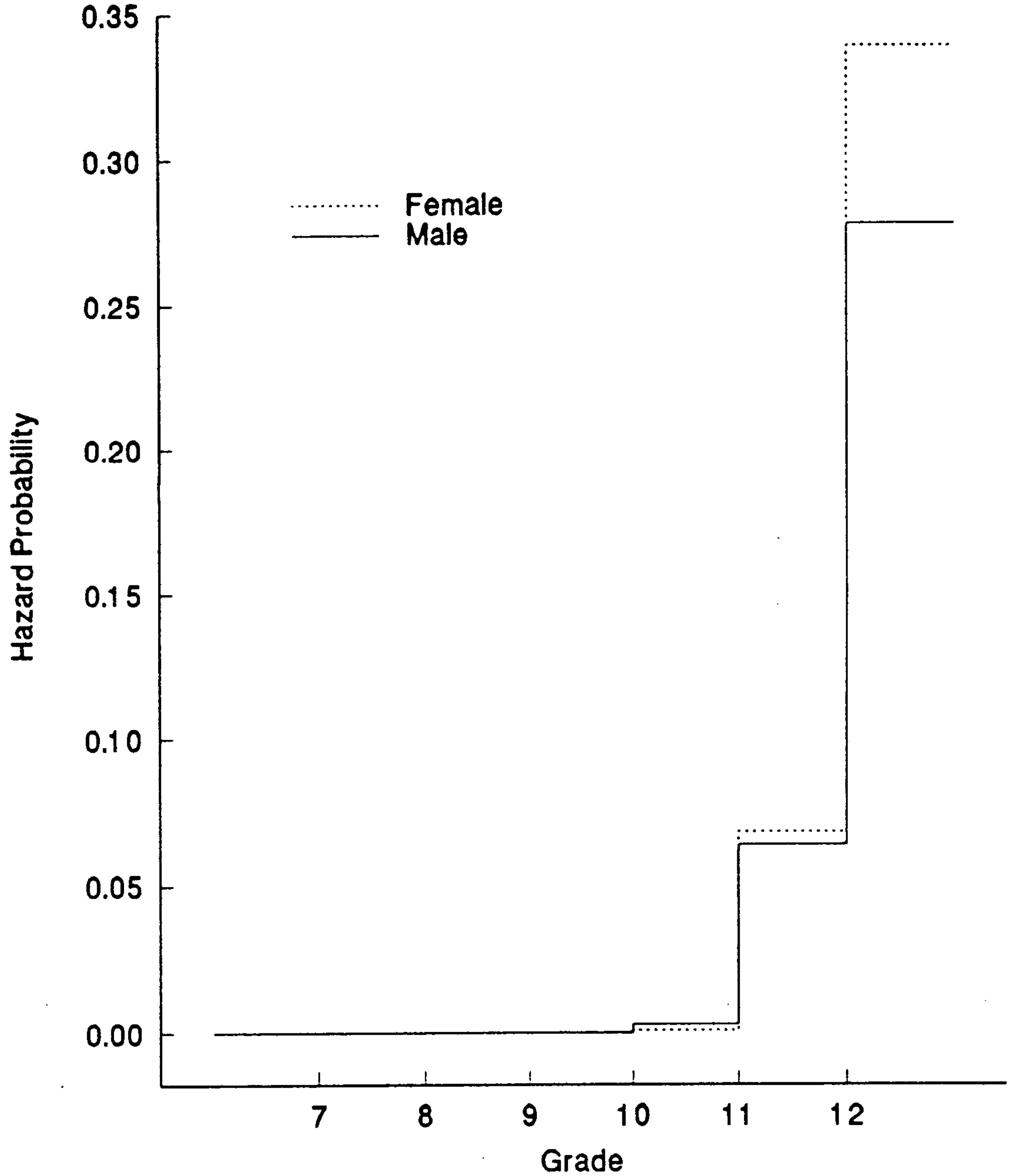

Figure 4. The hazard function of mathematics participation (sample hazard probability versus grade level), by sex.

Table 1

Means and Standard Deviations of Socioeconomic Status, Prior Mathematics Achievement, Prior Attitude toward Mathematics, Prior Mathematics Anxiety, and Prior Self-Esteem for Participants and Dropouts in Mathematics, by Grade and Sex

	Grade 11		Grade 12	
Group	M	SD	M	SD
	Socioeconomic Status			
Females				
Participants	41.62	16.37	42.81	16.06
Dropouts	36.26	13.61	40.57	16.76
Males				
Participants	42.50	16.33	43.68	16.88
Dropouts	35.03	14.44	40.71	15.93
	Prior Mathematics Achievement			
Females				
Participants	61.09	12.40	64.92	13.03
Dropouts	56.63	10.39	60.26	13.12
Males				
Participants	60.30	14.73	64.46	16.36
Dropouts	53.18	11.93	59.20	14.32
	Prior Attitude toward Mathematics			
Females				
Participants	7.68	2.32	7.71	2.27
Dropouts	7.06	2.70	7.20	2.28

Table 1 (continued)

Males				
Participants	8.22	2.17	**8.30**	2.18
Dropouts	7.71	2.11	**7.34**	2.29
		Prior Mathematics Anxiety		
Females				
Participants	2.96	1.99	2.94	2.04
Dropouts	3.39	2.06	3.19	1.92
Males				
Participants	2.88	1.85	2.91	1.84
Dropouts	3.27	1.93	3.11	1.82
		Prior Self-Esteem		
Females				
Participants	17.11	**3.95**	17.15	4.10
Dropouts	16.61	**2.98**	17.17	3.69
Males				
Participants	**17.33**	3.66	17.36	3.70
Dropouts	**16.38**	3.62	16.84	3.77
		Sample Size		
Participants	2267		1424	
Dropouts	160		632	

Note. There are no dropouts in mathematics in grades 8 and 9. The dropout rate in mathematics (0.1%) is trivial in grade 10. For each sex, statistically significant differences in means and standard deviations between participants and dropouts are bold in each grade. The significance level is 0.05.

mathematics dropouts. Male dropouts are as variable in SES as male participants, whereas female dropouts are statistically significantly less variable in SES than female participants in grade 11. Participants score statistically significantly higher in prior mathematics achievement than dropouts. Female dropouts are as variable in prior mathematics achievement as female participants, whereas male dropouts are statistically significantly less variable than male participants in grade 11.

Mathematics participants have statistically significantly higher scores in prior attitude toward mathematics than mathematics dropouts in the 12th grade. Male dropouts are as variable in prior attitude toward mathematics as male participants, while female dropouts are statistically significantly more variable than female participants in the 11th grade. Neither the means nor the standard deviations of prior mathematics anxiety are statistically significantly different between participants and dropouts. The means of prior self-esteem are not statistically significantly different between female participants and female dropouts, while male participants show statistically significantly higher prior self-esteem than male dropouts in grade 11. Male dropouts are as variable in prior self-esteem as male participants, while female dropouts are statistically significantly less variable than female participants in grade 11.

Logistic Regression Models Predicting Mathematics Participation by Year

Table 2 shows the effects of various variables on the likelihood of mathematics participation. The results are from separate logistic regression analyses for grades 11 and 12. The first two columns display estimates of the coefficients and their standard errors separately. The third column, Exp, denotes the regression result in terms of $\underline{e}$ raised to the power of each

Table 2

Statistical Estimates for the Survival Model Predicting the Probability of Mathematics Participation in Grades 11 and 12

	Grade 11			Grade 12		
Explanatory variables	Effect	SE	Exp	Effect	SE	Exp
Female (D)	0.03	(0.21)	1.03	-0.32**	(0.11)	0.73
Socioeconomic status	0.52***	(0.12)	1.69	0.17**	(0.06)	1.18
Prior mathematics achievement	0.28**	(0.11)	1.32	0.29***	(0.06)	1.33
Prior attitude toward mathematics	0.26**	(0.10)	1.30	0.29***	(0.06)	1.34
Constant	2.91***	(0.12)		0.64***	(0.06)	

Note. * $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$. Statistical analyses are not performed for grades 8 to 10 because there are no dropouts in mathematics in grades 8 and 9 and the dropout rate in mathematics (0.1%) is trivial in grade 10. D denotes dummy variables. SE denotes standard errors. Exp denotes the regression results in terms of e raised to the power of each effect. Prior mathematics anxiety and prior self-esteem are deleted from the model because of their consistent nonsignificance in both grades. Prior mathematics achievement (in grade 11) has substantial missing data (21%) which are treated through the Cohen's method (Cohen & Cohen, 1983) (Effect = 0.02, SE = 0.15, $p = 0.92$). The difference in effect is therefore not statistically significant between students with achievement data and those without achievement data.

effect. It indicates the expected change in the odds of mathematics participation for each one-unit change in an independent variable, given that other variables in the model are held constant. Because all the independent variables are standardized, a one-unit change in each variable corresponds to an approximate change of 34% in terms of percentile rank. The models in Table 2 are the best-fitting models from a number of models tested. Other logistic regression models tested whether there were statistically significant interactions between sex and SES, prior mathematics achievement, and prior attitude toward mathematics. These interactions were not statistically significant. Also, prior mathematics anxiety and prior self-esteem were not statistically significant at either grade level.

There are no detectable sex differences in mathematics participation in grade 11. Sex, however, is a statistically significant predictor at the 0.01 level of mathematics participation in grade 12; that is, it is unlikely (less than 1 time in 100) that the observed sex differences in mathematics participation occurred by chance alone. On average, the likelihood of female participation in mathematics is about 73% of the likelihood of male participation in grade 12, with other variables in the model held constant. Note that $1/0.73 = 1.37$ is the effect for males because sex is a dummy variable (female = 1; male = 0). In other words, males are about 37% [(1/0.73) - 1] more likely than females to take a mathematics course in grade 12.

SES is statistically significant at the 0.001 level in grade 11 and at the 0.01 level in grade 12. In grade 11, each one-unit increase in SES increases the likelihood of mathematics participation by an estimated 69%, given that other variables in the equation are statistically controlled. That is, if the SES of two students is one-unit apart, the one with higher SES is 69% more likely to take a mathematics course than the one with lower SES. In grade 12, each one-unit increase in SES increases the likelihood of mathematics participation by an estimated 18%, given that other variables in the equation are statistically controlled. If the SES of two students is one-unit apart, the one with higher SES is 18% more likely to take a mathematics course than the one with lower SES. Therefore, students with higher SES are at lower risk of dropping out of mathematics, but the effect of SES on mathematics participation decreases substantially from grade 11 to 12.

Prior mathematics achievement is a statistically reliable predictor of mathematics participation at the 0.01 level in grade 11 and at the 0.001 level in grade 12. Controlling for other variables in the model, each one-unit increase in prior mathematics achievement raises the likelihood of mathematics participation by 32% in grade 11 and by 33% in grade 12. If two students differ in their prior mathematics achievement by one-unit, the one with higher prior mathematics achievement is 32% more likely in grade 11 and 33% more likely in grade 12 to enroll in a mathematics course than the one with lower prior mathematics achievement. Thus, poor mathematics achievement increases the risk of dropping out of mathematics in the next school year, and the effect of prior mathematics achievement on mathematics participation is almost constant in grades 11 and 12.

Prior attitude toward mathematics is statistically reliable in predicting mathematics participation at the 0.01 level in grade 11 and at the 0.001 level in grade 12. With other variables held constant, each one-unit increase in prior attitude toward mathematics increases the likelihood of mathematics participation by 30% in grade 11 and by 34% in grade 12. If the prior attitude toward mathematics of two students differs by one unit, the one with higher prior attitude toward mathematics is 30% more likely in grade 11 and 34% more likely in grade 12 to take a mathematics course than the one with lower prior attitude toward mathematics. On the whole, positive attitude toward mathematics decreases the risk of dropping out of mathematics in the next school year, and the effect of prior attitude toward mathematics on mathematics participation appears similar in grades 11 and 12.

Note that the cumulative (or joint) effects of statistically significant variables on mathematics participation are substantial. For example, if two students are one-unit apart in SES, prior mathematics achievement, and prior attitude toward mathematics, the one with higher characteristics is more than twice ($1.18 \times 1.33 \times 1.34$) as likely to enroll in a mathematics course in grade 12 as the one with lower characteristics. If the one with higher characteristics happens to be male and the one with lower characteristics female, the male is then close to 3 times $[(1/0.73) \times 1.18 \times 1.33 \times 1.34]$ more likely to take a mathematics course in grade 12 than the female.

The constant is also statistically significant at the 0.001 level in both grades. It represents the hazard rate for the typical student (with nationally average characteristics) in each grade. The probability of taking a mathematics course for the typical student is 95% $\{1/[1 + \exp(-2.91)]\}$ in grade 11 and 65% $\{1/[1 + \exp(-0.64)]\}$ in grade 12. Therefore, the typical student is at the greatest risk of dropping out of mathematics in grade 12.

Because SES, prior mathematics achievement, and prior attitude toward mathematics were standardized on the full national sample, one can compare the effects across these variables. Prior mathematics achievement and prior attitude toward mathematics have similar effects on mathematics participation, and the effects are also nearly constant in both grades. However, SES has a much stronger effect than either of these variables in grade 11, but a weaker effect in grade 12. Hypotheses pertaining to whether these observed differences in coefficients are statistically reliable are tested in the full survival model discussed below.

Survival Analysis Models Predicting Mathematics Participation

Table 3 shows the results derived from the survival models which include time-varying variables of prior mathematics achievement and prior attitude toward mathematics[3]. Note that Model 2 which assumes different hazard rates between grade 11 and 12 fits the data significantly better than Model 1 which assumes invariant hazard rates between grade 11 and 12. Therefore, Model 2 is accepted as the better model for estimating the probability of mathematics participation. As noted in the table, grade 12 is a dummy variable (grade 12 vs. grade 11) in Model 2, thus a statistically significant interaction between grade 12 and a variable indicates that the effect of the variable is statistically significantly stronger in grade 12 than 11.

The variable, female, interacts statistically significantly with grade 12 (Effect = -0.50, $p < 0.05$), indicating that sex differences on mathematics participation appear mainly in grade 12. The likelihood of female participation in mathematics is about 68% [exp(0.12 - 0.50)] of the likelihood of male participation, holding other variables constant in the model. Expressed another way, males are 47% [(1/0.68) - 1] more likely than females to take a mathematics course in grade 12.

The interaction between SES and grade 12 is statistically significant (Effect = -0.46, $p < 0.001$), indicating that the effect of SES on mathematics participation decreases significantly from grade 11 to 12. Students are 79% more likely in grade 11 and 13% [exp(0.58 - 0.46) - 1] more likely in grade 12 to enroll in a mathematics course with each one-unit increase in SES, given that other variables in the model are statistically controlled.

Table 3

Statistical Estimates for the Survival Models Predicting the Probability of Mathematics Participation (4,488 Person-Years)

	Model 1			Model 2		
Explanatory variables	Effect	(SE)	Exp	Effect	(SE)	Exp
Female (D)	-0.16	(0.10)	0.85	0.12	(0.20)	1.13
Socioeconomic status	0.21***	(0.05)	1.24	0.58***	(0.11)	1.79
Prior mathematics achievement (T)	0.18***	(0.05)	1.19	0.32***	(0.05)	1.37
Prior attitude toward mathematics (T)	0.31***	(0.05)	1.37	0.28***	(0.05)	1.32
Grade 12 (D)				-2.16***	(0.13)	-
Female by grade 12				-0.50*	(0.23)	0.61
Socioeconomic status by grade 12				-0.46***	(0.13)	0.63
Constant	1.67***	(0.05)		2.97***	(0.12)	
Log-likelihood	-1387.94			-1194.96		

Note. * $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$. Model 1 assumes invariant hazard rates from grade 11 to 12. Model 2 assumes different hazard rates from grade 11 to 12. D denotes dummy variables. T denotes time-varying variables. SE denotes standard errors. Exp denotes the regression results in terms of e raised to the power of each effect. Grade 12 (D) represents the relative hazard rate in grade 12 in comparison to grade 11.

Table 3 (continued)

Prior mathematics anxiety (T) and prior self-esteem (T) are deleted from both models because of their consistent nonsignificance. Prior mathematics achievement (T) by grade 12 (D) and prior attitude toward mathematics (T) by grade 12 (D) are deleted from Model 2 because of their nonsignificance. The doubled, positive difference in log-likelihood between Model 1 and 2 is statistically significant (Chi-square = 385.96, $\underline{df}$ = 3, $\underline{p}$ = 0.000). Model 2 therefore fits the data significantly better.

Neither prior mathematics achievement nor prior attitude toward mathematics has a statistically significant interaction with grade 12. Therefore, both variables have similar effects on mathematics participation in grades 11 and 12. Holding other variables in the model constant, each one-unit increase in prior mathematics achievement increases the likelihood of mathematics participation by 37%, and each one-unit increase in prior attitude toward mathematics increases the likelihood of mathematics participation by 32%. Thus, the role of prior attitude toward mathematics is as important as that of prior mathematics achievement in mathematics participation.

As time-varying predictors, prior mathematics anxiety and prior self-esteem were deleted from the survival models because they were not statistically significant. Therefore, prior mathematics anxiety and prior self-esteem make little difference in mathematics participation, after controlling for the other variables in the model.

The constant is statistically significant at the 0.001 level. It represents the hazard rate of the typical student (with nationally average characteristics) in grade 11 against which the hazard rate of the typical student in grade 12 is compared. Statistical results show that the hazard rate in grade 12 is significantly higher than it is in grade 11 (Effect = -2.16, $\underline{p} < 0.001$). The typical student is 95% $\{1/[1 + \exp(-2.97)]\}$ likely in grade 11 and 69% $\{1/[1 + \exp(-2.97 + 2.16)]\}$ likely in grade 12 to take a mathematics course. Therefore, the transition from grade 11 to 12 represents the most risky point in which the typical student is likely to stop taking a mathematics course.

Longitudinal Effects of Factors on Mathematics Participation

Because the final survival model (Model 2) includes time-varying variables of prior mathematics achievement and prior attitude toward mathematics, it provides an opportunity to examine how longitudinal changes in mathematics achievement and attitude toward mathematics affect mathematics participation. Based on the results of Model 2 in Table 3, the following algebraic operations can be performed. In the 11th grade, the odds of mathematics participation is

$$\text{Odds (taking a mathematics course in grade 11)} = e^{Z_1}$$

where

$$Z_1 = 2.97 + 0.12(\text{female}) + 0.58(\text{SES}) + 0.32(\text{mathematics achievement in the 10th grade}) + 0.28(\text{attitude toward mathematics in the 10th grade}).$$

Similarly, the odds of mathematics participation in the 12th grade is

$$\text{Odds (taking a mathematics course in grade 12)} = e^{Z_2}$$

where

$$\begin{aligned} Z_2 &= (2.97 - 2.16) + (0.12 - 0.50)(\text{female}) + (0.58 - 0.46)(\text{SES}) + 0.32(\text{mathematics achievement in the 11th grade}) + 0.28(\text{attitude toward mathematics in the 11th grade}) \\ &= 0.81 - 0.38(\text{female}) + 0.12(\text{SES}) + 0.32(\text{mathematics achievement in the 11th grade}) + 0.28(\text{attitude toward mathematics in the 11th grade}). \end{aligned}$$

With attitude toward mathematics in the model held constant (same values in grades 10 and 11), the longitudinal effect of prior mathematics achievement on mathematics participation can be described as

$$\frac{\text{Odds (taking a mathematics course in grade 12)}}{\text{Odds (taking a mathematics course in grade 11)}} = e^{Z_2 - Z_1}$$

where

$$Z_2 - Z_1 = -2.16 - 0.50(\text{female}) - 0.46(\text{SES}) + 0.32(\text{difference in mathematics achievement between grade 11 and 10})$$

Therefore, the odds ratio indicates that each one-unit increase in mathematics achievement from grade 10 to 11 raises the odds of taking a mathematics course in grade 12 by an estimated 38% [exp(0.32) - 1], given that other variables in the model are statistically controlled. More important, the longitudinal effect of prior mathematics achievement on mathematics participation depends on sex and socioeconomic background. In other words, the same change in prior mathematics achievement from grade 11 to 12 results in different effects on mathematics participation, conditional on students' sex and socioeconomic background: males have stronger effects than females and students from lower SES have stronger effects than those from higher SES, holding prior attitude toward mathematics constant in grades 11 and 12. Therefore, males from low socioeconomic background are in the best position to improve their odds of mathematics participation, given the same change in prior mathematics achievement over grades.

For example, if a male student from average socioeconomic background (with a standardized score of zero in SES) maintains his level of mathematics achievement from grade 10 to 11, the odds of his taking a mathematics course in grade 12 is an estimated 12% $\{1/[1 + \exp(-Z_2 + Z_1)]$ where $-Z_2 + Z_1 = 2.16 + 0.50 \times 0 + 0.46 \times 0 - 0.32 \times 0 = 2.16\}$ of the odds in grade 11. If he improves his level of mathematics achievement by one-unit from grade 11 to 12, the odds of his taking a mathematics course in grade 12 is an estimated 16% $\{1/[1 + \exp(-Z_2 + Z_1)]$ where $-Z_2 + Z_1 = 2.16 + 0.50 \times 0 + 0.46 \times 0 - 0.32 \times 1 = 1.84\}$ of the odds in grade 11. If a female student has the same conditions, the odds of her taking a mathematics course in grade 12 is approximately 7% $\{1/[1 + \exp(-Z_2 + Z_1)]$ where $-Z_2 + Z_1 = 2.16 + 0.50 \times 1 + 0.46 \times 0 - 0.32 \times 0 = 2.66\}$ of the odds in grade 11 given her maintaining her level of mathematics achievement from grade 10 to 11, and approximately 10% $\{1/[1 + \exp(-Z_2 + Z_1)]$ where $-Z_2 + Z_1 = 2.16 + 0.50 \times 1 + 0.46 \times 0 - 0.32 \times 1 = 2.34\}$ of the odds in grade 11 given an improvement in her level of mathematics achievement by one-unit from grade 10 to 11. Therefore, improving the level of mathematics achievement over grades results in a better chance of mathematics participation than maintaining the level of mathematics achievement over grades. Equivalently, worsening the level of mathematics achievement over grades decreases the likelihood of mathematics participation. Note also that, with the same socioeconomic background, males who maintain their level of mathematics achievement over grades are still more likely to participate in mathematics than females who improve their level of mathematics achievement over grades.

The same algebraic operations can be performed to examine the longitudinal effect of prior attitude toward mathematics on mathematics participation:

$Z_1 - Z_2 = -2.16 - 0.50(\text{female}) - 0.46(\text{SES}) + 0.28(\text{difference in attitude toward mathematics between grade 11 and 10}).$

Similar conclusions can be reached. The same change in attitude toward mathematics from grade 10 to 11 has different effects on mathematics participation for students, conditional on sex and socioeconomic background. Males have stronger effects than females, and students from lower SES have stronger effects than those from higher SES, with prior mathematics achievement held constant. Each one-unit increase in attitude toward mathematics from grade 10 to 11 increases the odds of taking a mathematics course in grade 12 by an estimated 32% [exp(0.28) - 1]. Improving the level of attitude toward mathematics over grades results in a better chance of mathematics participation than maintaining the level of attitude toward mathematics achievement over grades. Worsening the level of attitude toward mathematics over grades decreases the likelihood of mathematics participation. With similar SES, males who maintain their attitude toward mathematics over grades are still more likely to enroll in a mathematics course than females who improve their attitude toward mathematics over grades.

The Adequacy of the Statistical Models

Three model-data-fit methods were used to examine how the statistical models fitted the data: (a) the histogram method, (b) the goodness-of-fit statistic, and (c) the goodness-of-fit index. The model-data-fit results of the statistical models predicting mathematics participation are listed in Appendices A and B. The histogram of the logistic model for the 11th grade puts together the frequency distribution of model-predicted probabilities for both participants and dropouts (see Appendix A-1). As can be seen, the distribution of participants is located on the "1" (participation) side, indicating that the model is successful in classifying participants in mathematics. The distribution of dropouts, however, moves away from the "0" (dropout) side, and overlaps with the distribution of participants, indicating that the model is not very successful in classifying dropouts in mathematics. Because the number of dropouts is much smaller than that of participants, the model, overall, is successful in correctly classifying the majority of the students. For example, if the probability of 0.50 is arbitrarily chosen so that students with probabilities above 0.50 are classified as model-predicted participants and students with probabilities below 0.50 are classified as model-predicted dropouts, then the model correctly classifies 93% of the students. Projecting this figure to the entire population shows that if 100 students were randomly selected, the model would be able to precisely classify 93 of them based on their sex, socioeconomic background, and scores on prior mathematics achievement and prior attitude toward mathematics.

The average goodness-of-fit statistic of the logistic model is less than 1 (see Appendix B). This suggests that the model, on average, is capable of predicting the likelihood of mathematics participation of students, and its prediction is close to the real status of students in mathematics participation. Note that the histogram method and the goodness-of-fit statistic cannot provide any relative measures of how well the statistical models fit. As mentioned in Chapter 3, however, model-data-fit is always relative. Therefore, the goodness-of-fit index is used as a comparative method of evaluation. The assessment, then, focuses on the comparison between the initial and full models. The model chi-square (see Appendix B) indicates that the full model (with all explanatory variables in the equation) significantly improves the model-data-fit at the 0.001 level over the initial model (with only a constant in the equation); that is, it is unlikely (less than 1 time in 1000) that the observed improvement in model-data-fit occurs by chance alone.

The logistic model for the 12th grade is also successful in classifying participants in mathematics (see Appendix A-2). Although it is better than the logistic model for the 11th grade in terms of classifying dropouts in mathematics, it is still not as strong as one might like. If the probability of 0.50 is arbitrarily used as the standard, then the model correctly classifies 66% of the students. Projecting this figure to the entire population indicates that if 100 students were randomly selected, the model would be able to precisely classify 66 of them based on their sex, SES, and scores on prior mathematics achievement and prior attitude toward mathematics. The average goodness-of-fit statistic of the model is 2 (see Appendix B), which is an indication of the weakness of the model. The improvement of the full model is statistically significant at the 0.001 level over the initial model (see Appendix B).

For the survival model containing time-varying variables, the histogram shows success in predicting participants in mathematics, but weakness in predicting dropouts in mathematics (see Appendix A-3). If the probability of 0.50 is arbitrarily chosen as the standard, then the model correctly classifies 83% of the students. Projecting this figure to the entire population shows that if 100 students were randomly selected, the model would be able to precisely classify 83 of them based on their sex, SES, and scores on prior mathematics achievement and prior attitude toward mathematics. The average goodness-of-fit statistic of the survival model is less than 1 (see Appendix B), indicating that the model, on average, is able to predict the likelihood of mathematics participation of students, and its prediction is close to the real status of students in mathematics participation.

The improvement of the full survival model is statistically significant at the 0.001 level over the initial model (see Appendix B).

Note that the model's inability to identify dropouts may not be due to flaws in model specification. Each independent variable has its absolute and relative effects on the dependent variable. For example, that prior self-esteem is individually significant indicates that its absolute effect is strong on the probability of dropping out of mathematics. However, if the effect of prior self-esteem is removed, the probability may still be affected significantly by other variables. That is, when prior self-esteem is collectively among other variables, its relative effect is not as strong as it appears to be individually. This consideration of absolute versus relative effect should be kept in mind when examining the adequacy of statistical models. In the case of the model for grade 12, for example, if nonsignificant variables had remained in the model, the model-data-fit data would have been much more satisfactory. However, the aim of this study (perhaps of all statistical modeling) is not to achieve a maximum model-data-fit on purpose, but to establish a context in which collectively significant variables indicate their theoretical importance and relationship. Therefore, nonsignificant variables are removed from the model to maintain the theoretical significance of the model.

Overall, the results of model-data-fit are considered reasonable. The concern is that neither the logistic models nor the survival model is very successful in predicting dropouts in mathematics. However, the validity of this analysis is not threatened by this weakness because the logistic and survival models replicate each other (see Lee & Bryk, 1988). That is, the major conclusions derived from the logistic models are replicated in the survival model.

Survival Analysis of Participation in Advanced Mathematics

Survival and Hazard Probabilities

Figure 5 illustrates the survival curve of participation in advanced mathematics, in which the sample survival probability is graphed against the grade level. The likelihood of participation in advanced mathematics is 99% in the 7th grade, 98% in the 8th grade, 96% in the 9th grade, 94% in the 10th grade, 85% in the 11th grade, and 55% in the 12th grade. If 100 students were randomly selected in the 7th grade and observed until the 12th grade, then 99 of them would still be participating in an advanced mathematics course in the 7th grade, 98 in the 8th grade, 96 in the 9th grade, 94 in the 10th grade, 85 in the 11th grade, and 55 in the 12th grade. Therefore, the decline in participation in advanced mathematics becomes significant in grade 11, and the transition from grade 11 to 12 represents the point at which students are most likely to stop taking an advanced mathematics course.

Figure 6 illustrates the hazard curve of participation in advanced mathematics, in which the sample hazard probability is graphed against the grade level. The hazard probability of participation in advanced mathematics is slight from grade 7 to 10. Among students who are available to participate in advanced mathematics in the 7th grade, about 1% of them fail to participate. Among students who are available to participate in advanced mathematics in the 8th grade (these students have successfully enrolled in an advanced mathematics course in the 7th grade), about 1% of them fail to participate. For all the students available to participate in advanced mathematics in the 9th grade (these students have successfully enrolled in an advanced mathematics course in the 8th grade), about 2% of them fail to enroll. For all the students available to participate in advanced mathematics in the 10th grade, about 2% of them fail to enroll. A much higher hazard

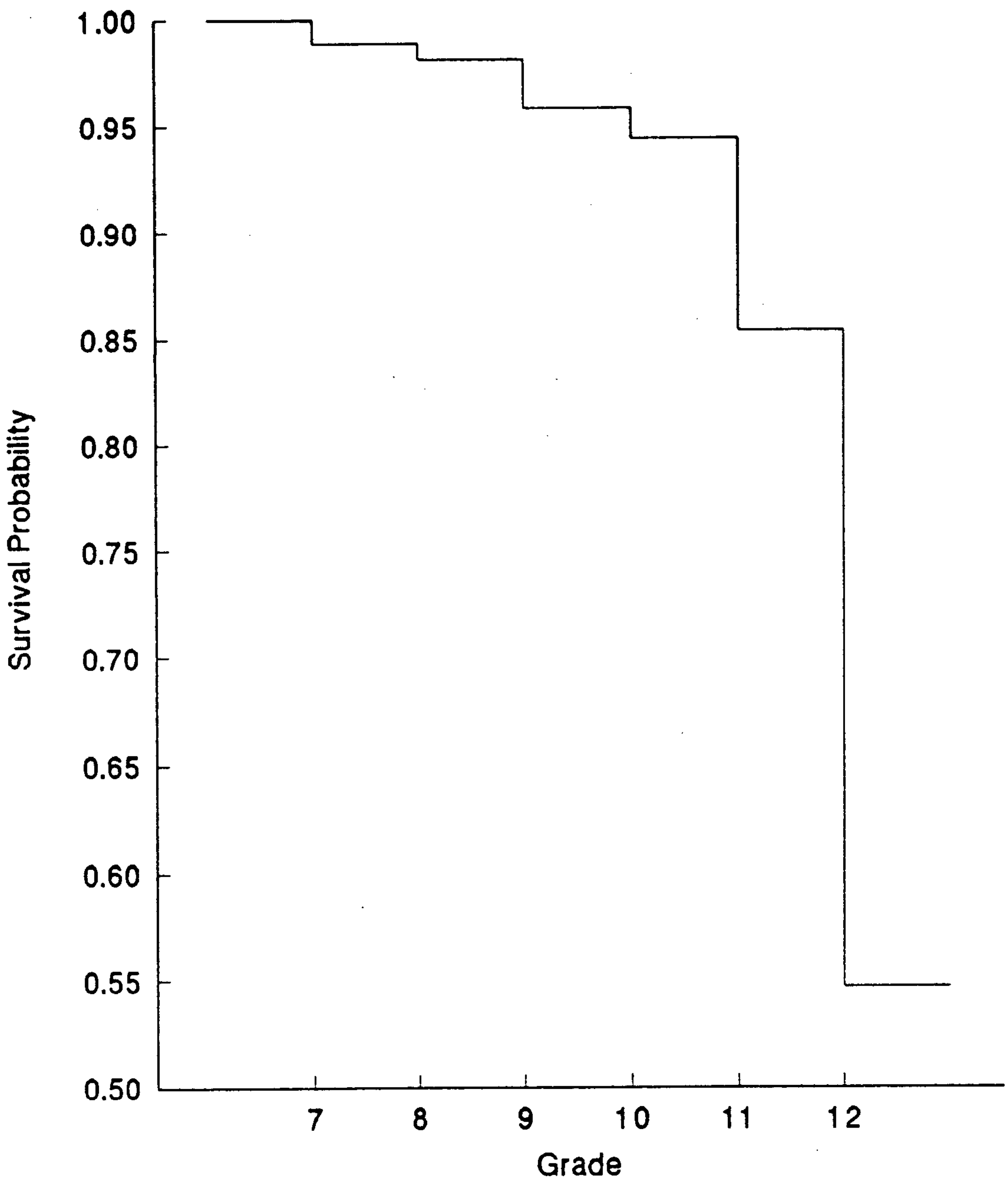

Figure 5. The survival function of participation in advanced mathematics (sample survival probability versus grade level).

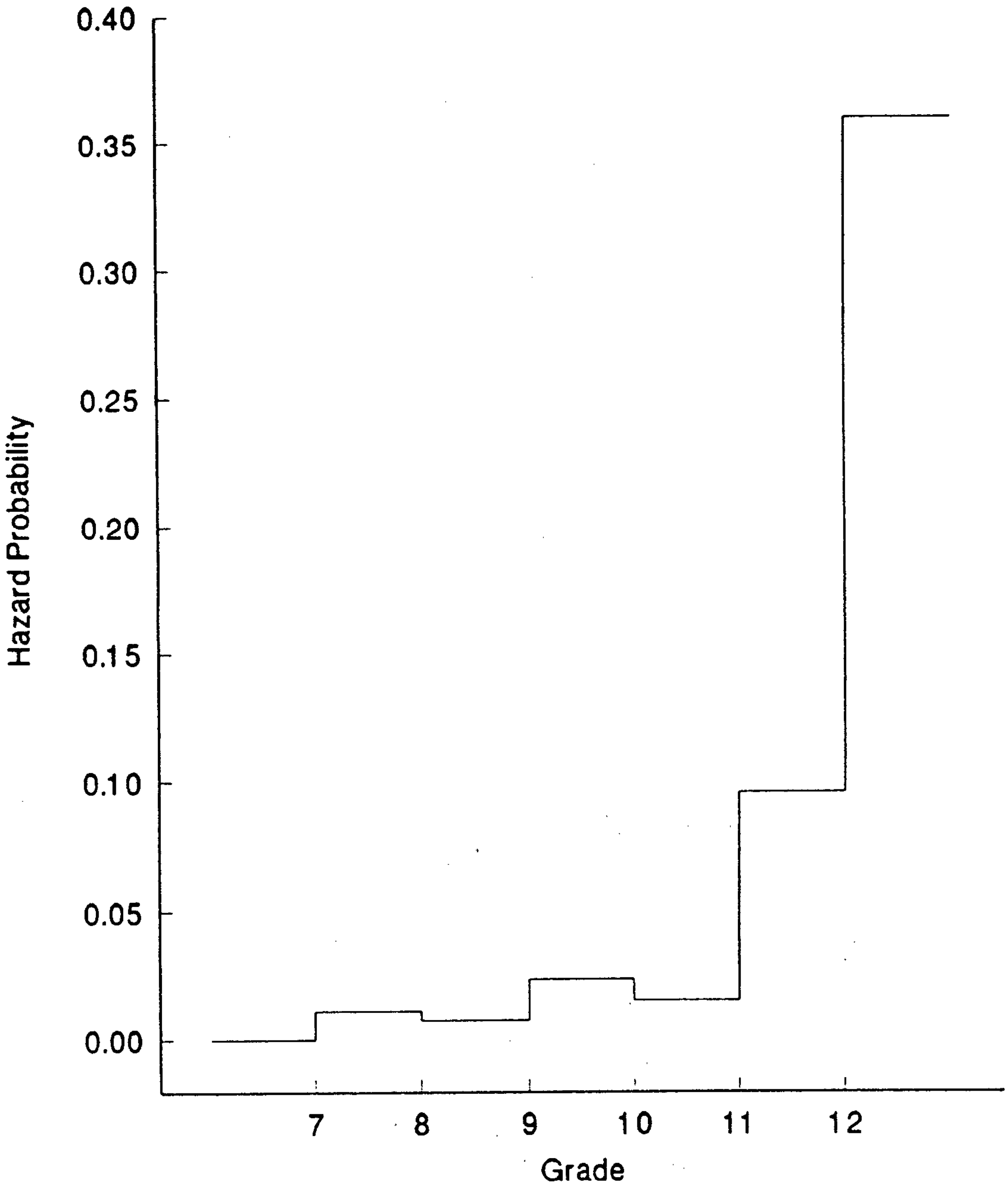

Figure 6. The hazard function of participation in advanced mathematics (sample hazard probability versus grade level).

probability appears in grade 11. Among students who are available to participate in advanced mathematics in the 11th grade, about 10% of them fail to participate. The highest hazard probability appears in grade 12. For all the students available to participate in advanced mathematics in the 12th grade, about 36% of them fail to enroll. Therefore, the transition from grade 11 to 12 represents the point at which students are most likely to drop out of advanced mathematics.

Male and female survival curves are demonstrated separately in Figure 7 to examine sex differences in participation in advanced mathematics. Males and females have the same survival pattern with regard to participation in advanced mathematics. The likelihood of participation in advanced mathematics is similar between males and females from grade 7 to 11, although both males and females drop out of advanced mathematics more evidently in grade 11. Therefore, there are no obvious sex differences in participation in advanced mathematics prior to grade 12. In grade 12, females show a higher risk of dropping out of advanced mathematics than males. Suppose 100 males and 100 females were randomly selected in the 7th grade and observed until the 12th grade, about 99 males and 99 females would still be participating in advanced mathematics in the 7th grade; 98 males and 99 females in the 8th grade; 95 males and 96 females in the 9th grade; 94 males and 95 females in the 10th grade; 86 males and 85 females in the 11th grade; 58 males and 52 females in the 12th grade. In sum, both males and females are at the greatest risk of dropping out of advanced mathematics in grade 12, and females appear to have an even higher risk than males.

Male and female hazard curves of participation in advanced mathematics are demonstrated separately in Figure 8 to compare the extent to which males and females drop out of advanced mathematics in each grade.

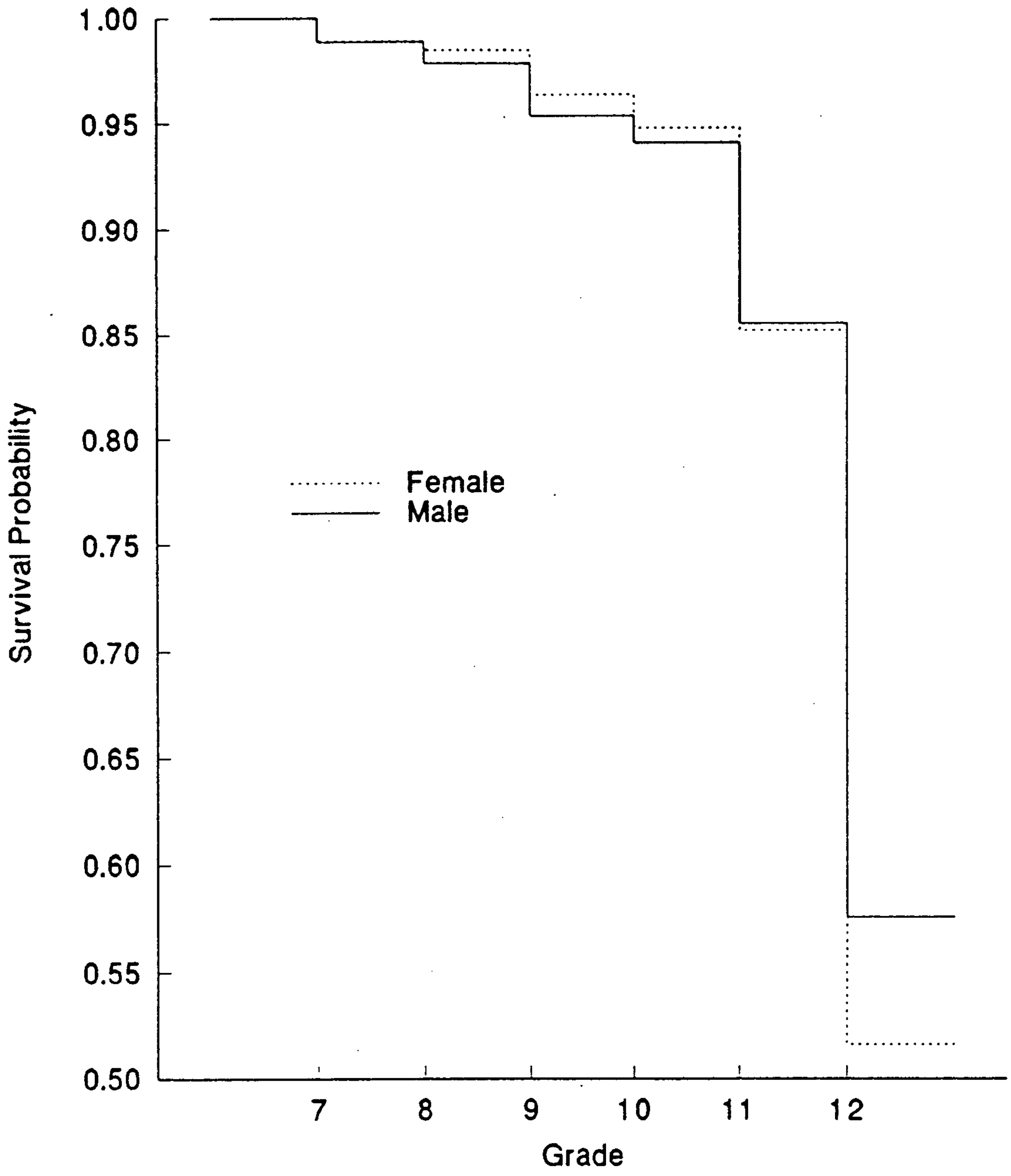

Figure 7. The survival function of participation in advanced mathematics (sample survival probability versus grade level), by sex.

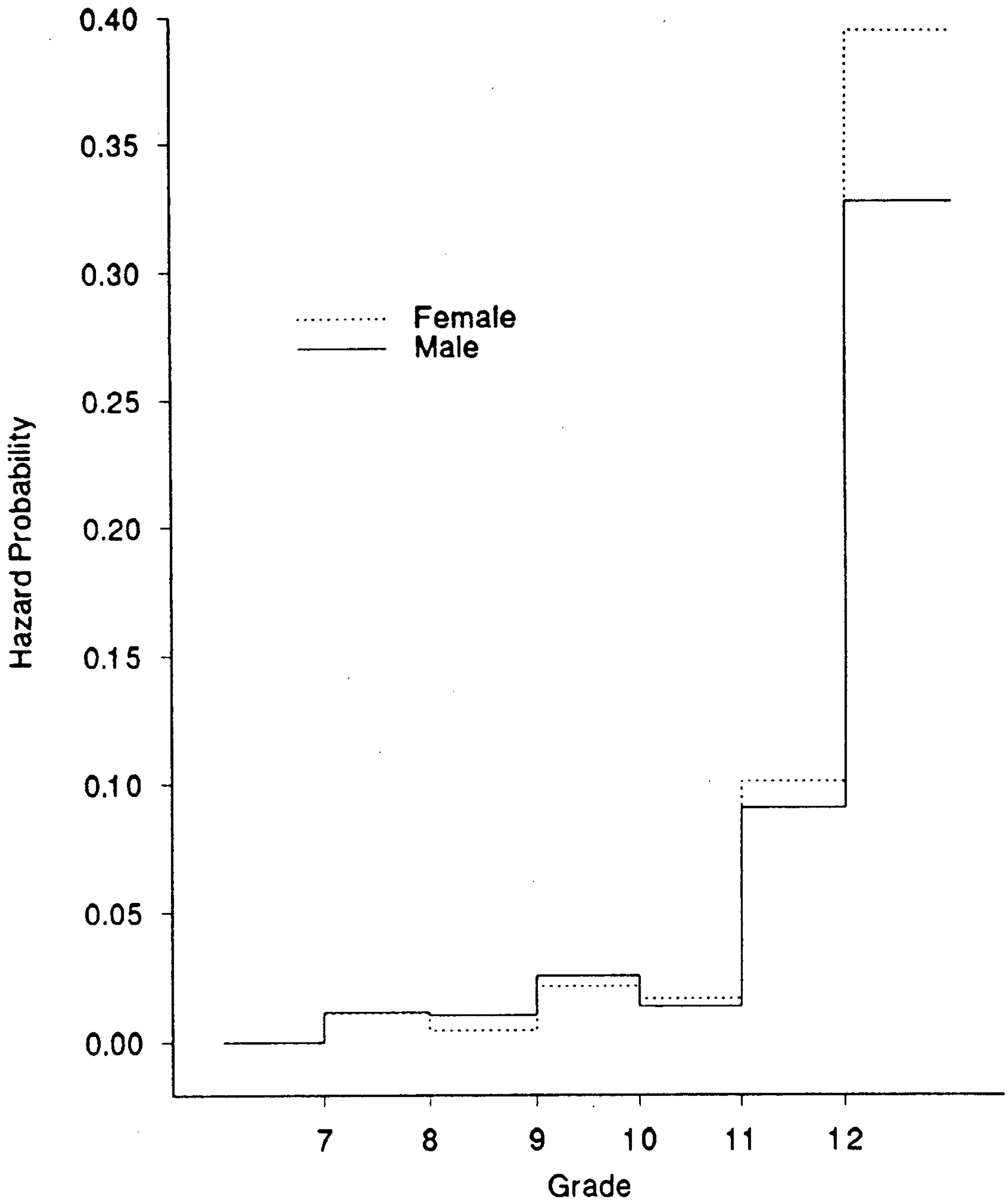

Figure 8. The hazard function of participation in advanced mathematics (sample hazard probability versus grade level), by sex.

Males and females have the same hazard pattern concerning participation in advanced mathematics. Among male and female students who are available to take advanced mathematics courses in the 7th grade, about 1% of males and 1% of females fail to participate. Among male and female students who are available to take advanced mathematics courses in the 8th grade (these students have successfully enrolled in an advanced mathematics course in the 7th grade), about 1% of males and 1% of females fail to participate. Among male and female students who are available to take advanced mathematics courses in the 9th grade, about 3% of males and 2% of females fail to participate. For male and female students available to participate in advanced mathematics in the 10th grade, about 1% of males and 2% of females fail to enroll. For male and female students available to participate in advanced mathematics in the 11th grade, about 9% of males and 10% of females fail to enroll. For male and female students available to participate in advanced mathematics in the 12th grade, about 33% of males and 39% of females fail to enroll. Therefore, the hazard of female dropout in advanced mathematics is slightly lower in grades 7 to 9, yet slightly higher in grades 10 and 11, than the hazard of male dropout. Particularly, females have a much higher hazard probability than males in the 12th grade. In sum, the transition from grade 11 to 12 is the most risky point in which both males and females are likely to stop taking an advanced mathematics course, and females are at a higher risk than males.

Factors Influencing Participation in Advanced Mathematics

Table 4 shows the means and standard deviations of various variables for participants and dropouts in advanced mathematics. Participants in advanced mathematics have statistically significantly higher SES than dropouts in advanced mathematics in each grade except grade 10 in which the

Table 4

Means and Standard Deviations of Socioeconomic Status, Prior Mathematics Achievement, Prior Attitude toward Mathematics, Prior Mathematics Anxiety, and Prior Self-Esteem for Participants and Dropouts in Advanced Mathematics, by Grade and Sex

	Grade 8		Grade 9		Grade 10		Grade 11		Grade 12	
Group	M	SD	M	SD	M	SD	M	SD	M	SD
					Socioeconomic Status					
Females										
Participants	**41.55**	16.53	**41.90**	16.35	41.79	16.34	**42.38**	16.42	**43.92**	15.94
Dropouts	**31.52**	11.78	**33.47**	13.42	36.73	10.19	**37.90**	14.17	**40.93**	16.93
Males										
Participants	**42.09**	16.60	**42.38**	16.55	42.76	16.42	**43.28**	**16.49**	**45.01**	16.61
Dropouts	**33.43**	16.11	**31.01**	14.13	35.08	14.55	**38.11**	**13.49**	**40.70**	16.70
					Prior Mathematics Achievement					
Females										
Participants	**51.61**	9.13	**53.85**	**9.75**	**58.31**	10.83	**62.73**	11.71	**67.00**	12.31
Dropouts	**42.93**	7.95	**43.88**	**5.98**	**50.87**	11.55	**54.19**	11.72	**62.56**	11.99

Table 4 (continued)

Males										
Participants	**50.44**	**10.34**	**52.31**	11.73	57.21	12.85	**62.00**	14.12	**66.62**	15.70
Dropouts	**39.82**	**6.54**	**44.35**	10.12	52.90	14.08	**54.23**	12.54	**59.03**	15.06
Prior Attitude toward Mathematics										
Females										
Participants	8.40	2.14	8.34	2.15	8.15	2.23	**7.73**	2.30	**7.85**	2.26
Dropouts	8.39	2.36	8.02	2.68	7.87	2.11	**6.62**	2.53	**6.93**	2.24
Males										
Participants	8.69	2.11	**8.73**	2.15	**8.58**	1.96	**8.36**	**2.07**	**8.47**	**2.12**
Dropouts	8.77	1.94	**7.25**	2.65	**6.95**	2.42	**7.36**	**2.50**	**7.24**	**2.33**
Prior Mathematics Anxiety										
Females										
Participants	2.71	1.97	2.64	1.91	2.87	1.97	**2.88**	1.97	**2.89**	2.08
Dropouts	3.22	1.88	3.03	2.60	3.06	1.61	**3.84**	2.14	**3.21**	1.89
Males										
Participants	**2.89**	2.06	2.76	2.06	3.07	1.90	**2.80**	**1.81**	**2.82**	1.90
Dropouts	**3.61**	1.66	3.29	2.01	2.38	1.56	**3.47**	**2.13**	**3.27**	1.79

Table 4 (continued)

	Prior Self-Esteem									
Females										
Participants	16.63	3.67	17.03	3.86	16.87	3.98	17.16	3.96	17.19	4.11
Dropouts	15.12	4.09	16.18	3.68	15.67	3.10	17.12	3.44	17.35	3.74
Males										
Participants	**16.66**	3.74	**17.31**	3.73	17.11	3.66	17.46	3.62	**17.63**	3.69
Dropouts	**15.01**	3.39	**15.46**	3.27	15.70	3.72	16.85	3.67	**16.67**	3.81
	Sample Size									
Participants	2747		2401		2320		1915		1245	
Dropouts	63		40		31		215		731	

Note. For each sex, statistically significant differences in means and standard deviations between participants and dropouts are bold in each grade. The significance level is 0.05.

means of SES are not statistically significantly different between participants and dropouts. Dropouts are as variable in SES as participants (the exception that male participants are statistically significantly more variable than male dropouts in grade 11 might occur by chance).

Participants in advanced mathematics score statistically significantly higher in prior mathematics achievement than dropouts in advanced mathematics (the exception that the means of prior mathematics achievement are not statistically significantly different between male participants and male dropouts in grade 10 might occur by chance). Male participants are statistically significantly more variable in prior mathematics achievement than male dropouts in grade 8, and female participants are statistically significantly more variable than female dropouts in grade 9. Dropouts are as variable in prior mathematics achievement as participants in grades 10 to 12.

Female participants in advanced mathematics have statistically significantly more positive prior attitude toward mathematics than female dropouts in advanced mathematics in grades 11 and 12, whereas male participants have statistically significantly more positive prior attitude toward mathematics than male dropouts in grades 9 to 12. Female dropouts are as variable in prior attitude toward mathematics as female participants, whereas male dropouts are statistically significantly more variable than male participants in grades 11 and 12.

Female dropouts in advanced mathematics have statistically significantly higher prior mathematics anxiety than female participants in advanced mathematics in grades 11 and 12, while male dropouts have statistically significantly higher prior mathematics anxiety than male participants in grades 8, 11 and 12. Dropouts are as variable in prior mathematics anxiety as participants (the exception that male dropouts are statistically significantly more variable than male participants in grade 11 might occur by chance).

Neither the means nor the standard deviations of prior self-esteem are statistically significantly different between female participants and female dropouts in advanced mathematics. Male participants have statistically significantly more positive prior self-esteem than male dropouts in grades 8, 9 and 12, and male dropouts are as variables in prior self-esteem as male participants.

Logistic Regression Models Predicting Participation in Advanced Mathematics by Year

Table 5 presents the effects of various variables on the likelihood of participation in advanced mathematics. The results are from separate logistic regression analyses for grades 8 to 12. The first two columns are estimates of the coefficients and their standard errors. The third column, Exp, denotes the regression result in terms of e raised to the power of each effect. It indicates the expected change in the odds of participation in advanced mathematics for each one-unit change in a variable, given other variables in the model held constant. Note that, because all the independent variables are standardized, a one-unit change in each variable corresponds to an approximate change of 34% in terms of percentile rank. The models presented in Table 5 are the best-fitting models from a number of models tested. Other logistic regression models tested whether there were statistically significant interactions between sex and SES, prior mathematics achievement, prior attitude toward mathematics, and prior mathematics anxiety. These interactions were statistically nonsignificant. Note, also, that prior self-esteem was statistically nonsignificant across grades 8 to 12.

Table 5

Statistical Estimates for the Survival Model Predicting the Probability of Participation in Advanced Mathematics from Grades 8 to 12

	Grade 8			Grade 9			Grade 10			Grade 11			Grade 12		
Explanatory variables	Effect	(SE)	Exp	Effect	(SE)	Exp	Effect	(SE)	Exp	Effect	(SE)	Exp	Effect	(SE)	Exp
Female (D)	0.19	(0.32)	**1.21**	-0.48	(0.36)	**0.62**	0.50	(0.47)	**1.65**	0.14	(0.18)	**1.15**	-0.36**	(0.12)	**0.70**
Socioeconomic status	0.32	(0.17)	**1.37**	0.57**	(0.21)	**1.77**	0.34	(0.25)	**1.40**	0.28**	(0.10)	**1.33**	0.21***	(0.06)	**1.24**
Prior math achievement	1.14***	(0.20)	**3.11**	0.72***	(0.19)	**2.06**	0.41*	(0.20)	**1.51**	0.32***	(0.07)	**1.38**	0.25***	(0.05)	**1.28**
Prior attitude toward math	-0.27	(0.16)	**0.77**	0.17	(0.16)	**1.19**	0.41	(0.22)	**1.50**	0.34***	(0.09)	**1.40**	0.49***	(0.06)	**1.63**
Prior math anxiety	-0.10	(0.15)	**0.90**	0.01	(0.18)	**1.01**	0.57*	(0.28)	**1.76**	-0.23*	(0.10)	**0.80**	0.13	(0.07)	**1.14**
Constant	3.83***	(0.18)		4.23***	(0.22)		4.53***	(0.28)		2.31***	(0.11)		0.60***	(0.07)	

Note. * $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$. D denotes dummy variables. SE denotes standard errors. Exp denotes the regression results in terms of e raised to the power of each effect. Prior self-esteem is deleted from the model because of its consistent nonsignificance across grades 8 to 12. Prior mathematics achievement (in grade 11) has substantial missing data (27%) which are treated through the Cohen's method (Cohen & Cohen, 1983) (Effect = 0.12, SE = 0.16, $p = 0.46$). The difference in effect is therefore not statistically significant between students with achievement data and those without achievement data.

Prior to grade 12, the variable, female, is not a statistically significant predictor of participation in advanced mathematics. There are, however, statistically significant sex differences in participation in advanced mathematics in grade 12 at the 0.01 level, which means that it is unlikely (less than 1 time in 100) that the observed sex differences occurred by chance alone. The likelihood of female participation in advanced mathematics is about 70% of the likelihood of male participation in grade 12, given that other variables in the model are statistically controlled. Equivalently, males are about 43% [(1/0.70) - 1] more likely to take an advanced mathematics course than females in grade 12.

The effect of SES on participation in advanced mathematics is statistically significant at the 0.01 level in grades 9 and 11 and at the 0.001 level in grade 12. With other variables in the model held constant, each one-unit increase in SES increases the likelihood of participation in advance mathematics by 77% in the 9th grade, 33% in the 11th grade, and 24% in the 12th grade. Suppose the SES of two students is one-unit apart, the one with higher SES is 77% more likely in the 9th grade, 33% more likely in the 11th grade, and 24% more likely in the 12th grade to enroll in an advanced mathematics course than the one with lower SES. Therefore, although the risk of dropping out of advanced mathematics is much higher if a student comes from low socioeconomic background, the (statistically significant) effect of SES on participation in advanced mathematics decreases over grades.

Prior mathematics achievement is the single predictor in the model that is statistically significant across all grades (at the 0.05 level in grade 10 and at the 0.001 level in other grades). If other variables in the model are held constant, each one-unit increase in prior mathematics achievement raises the likelihood of participation in advanced mathematics by 51% in the 10th grade, 38% in the 11th grade, and 28% in the 12th grade. If two students differ in their prior mathematics achievement by one-unit, the one with higher prior mathematics achievement is 51% more likely in the 10th grade, 38% more likely in the 11th grade, and 28% more likely in the 12th grade to take an advanced mathematics course. Note that the effect of prior mathematics achievement on participation in advanced mathematics is far stronger in grades 8 and 9. Suppose the prior mathematics achievement of two students is one-unit apart, the one with higher prior mathematics achievement is more than 3 times as likely in grade 8 and more than 2 times as likely in grade 9 to enroll in an advanced mathematics course than the one with lower prior mathematics achievement. Thus, although students with low prior mathematics achievement have a substantially higher risk of dropping out of advanced mathematics, there is a clear pattern that the effect of prior mathematics achievement on participation in advanced mathematics decreases over grades.

The effect of prior attitude toward mathematics on participation in advance mathematics becomes statistically significant in grades 11 and 12 (at the 0.001 level). Each one-unit increase in prior attitude toward mathematics increases the likelihood of participation in advanced mathematics by 40% in grade 11 and 63% in grade 12, once other variables in the model are statistically controlled. If two students differ in their prior attitude toward mathematics by one-unit, the one with higher prior attitude toward mathematics is 40% more likely in grade 11 and 63% more likely in grade 12 to take an advanced mathematics course than the one with lower prior attitude toward mathematics. Thus, in grades 11 and 12, students with lower prior attitude toward mathematics have a substantially higher risk of dropping out of advanced mathematics, and the effect of prior attitude toward mathematics on participation in advanced mathematics increases from grade 11 to 12.

The effect of prior mathematics anxiety on participation in advanced mathematics is statistically significant at the 0.05 level in grades 10 and 11. Note that mathematics anxiety is measured in such a manner that higher values indicate higher levels of mathematics anxiety. Therefore, prior mathematics anxiety has a positive effect on participation in advanced mathematics in grade 10. One explanation for this is that a lot of students become highly anxious about mathematics in the 10th grade, yet still participate in advanced mathematics in that grade (dropout rate is 2% as shown in Figure 6). This can result in a positive association between prior mathematics anxiety and participation in advanced mathematics. The effect of prior mathematics anxiety on participation in advanced mathematics is negative in grade 11. Suppose the prior mathematics anxiety of two students is one-unit in difference, the one with lower prior mathematics anxiety is 80% more likely to take an advanced mathematics course in grade 11 than the one with higher prior mathematics anxiety, after other variables are statistically controlled.

The constant is statistically significant at the 0.001 level across the five grades. It represents the hazard rate of the typical student with nationally average characteristics. The probability of participation in advanced mathematics for the typical student is an estimated 98% $\{1/[1 + \exp(-3.83)]\}$ in the 8th grade, 99% $\{1/[1 + \exp(-4.23)]\}$ in the 9th grade, 99% $\{1/[1 + \exp(-4.53)]\}$ in the 10th grade, 91% $\{1/[1 + \exp(-2.31)]\}$ in the 11th grade, and 65% $\{1/[1 + \exp(-0.60)]\}$ in the 12th grade. Therefore, the transition from grade 11 to 12 represents the most risky point at which the typical student is likely to stop taking an advanced mathematics course.

Note that the joint (or cumulative) effects of statistically significant predictors can be substantial. For example, suppose two students are one-unit apart in their SES, prior mathematics achievement, and prior attitude toward mathematics, the one with higher characteristics is more than 2.5 times ($1.24 \times 1.28 \times 1.63$) as likely to enroll in an advanced mathematics course in grade 12 as the one with lower characteristics. If the student with higher characteristics happens to be male and the other student female, the male is then almost 4 times $[(1/0.70) \times 1.24 \times 1.28 \times 1.63]$ as likely to take an advanced mathematics course in grade 12 than the female.

Because SES, prior mathematics achievement, prior attitude toward mathematics, and prior mathematics anxiety were standardized on the full national sample, their effects can be compared across grades and variables. Prior mathematics achievement seems to be the most important predictor of participation in advanced mathematics in the early grades, whereas prior attitude toward mathematics seems to be the most important predictor in the later grades. The role of SES in participation in advanced mathematics seems particularly important in the early grades, and prior mathematics anxiety appears to play a role in participation in advanced mathematics in the middle grades. These hypotheses are tested for statistical significance in the survival analysis model presented later.

Graphical Analysis of Participation in Advanced Mathematics in Grade 12

Figure 9 describes sex differences in participation in advanced mathematics across different socioeconomic backgrounds. The range of SES is between the 10th and 90th percentile in each sex. These two percentiles are chosen because there are few students both below the 10th percentile and above the 90th percentile. Cutting these extremes allows one to obtain a more

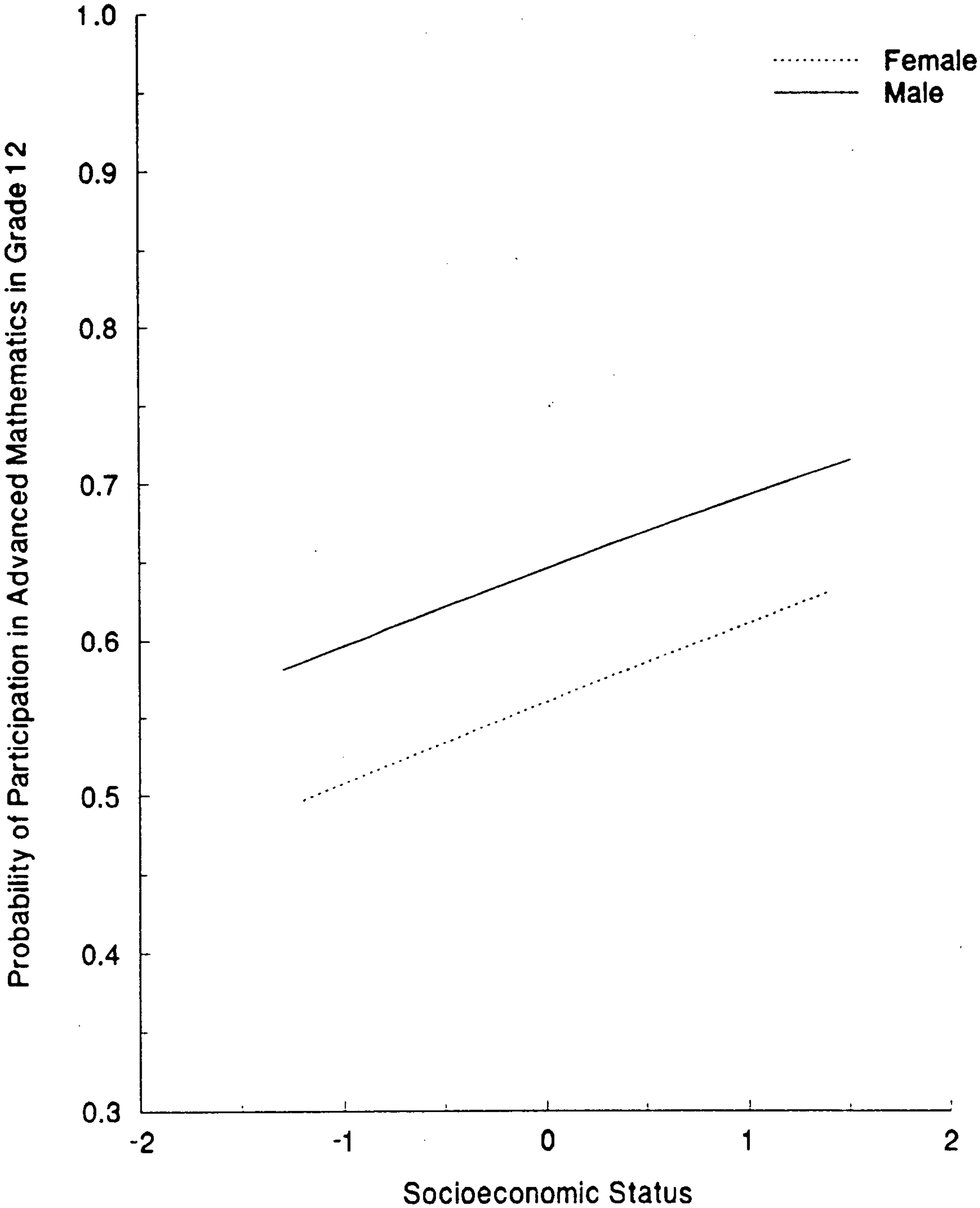

Figure 9. The effect of socioeconomic status (SES) on participation in advanced mathematics in the 12th grade, by sex. The range of SES is from the 10th to 90th percentile in each sex.

accurate description of the relationship between participation and SES for each sex. Note that this rationale is also used in the rest of the figures.

Both male and female students from lower socioeconomic background have a higher risk of dropping out of advanced mathematics in grade 12. On average, students (both males and females) with highest SES are approximately 15% more likely to participate in advanced mathematics in grade 12 than students with lowest SES, controlling for prior mathematics achievement, prior attitude toward mathematics, and prior mathematics anxiety. With the same socioeconomic background, males are more likely to take an advanced mathematics course in grade 12 than their female counterparts. Note that the male and female curves appear parallel or have a common slope. Therefore, there is no interaction effect between sex and SES. This implies that sex differences in participation in advanced mathematics are similar across differing levels of SES. On average, with similar socioeconomic background, males are approximately 10% more likely than females to enroll in an advanced mathematics course in grade 12, controlling for prior mathematics achievement, prior attitude toward mathematics, and prior mathematics anxiety.

Figure 10 describes how students differ in participation in advanced mathematics given their prior attitude toward mathematics and prior mathematics achievement. To generate this figure, a percentile score of prior attitude toward mathematics is calculated for each student, and the 20th, 40th, 60th, and 80th percentiles are used as cut-off points to create five groups or percentile ranges. Students are then classified into these five groups according to their percentile scores on prior attitude toward mathematics. The range of prior mathematics achievement is between the 10th and 90th percentile in each group.

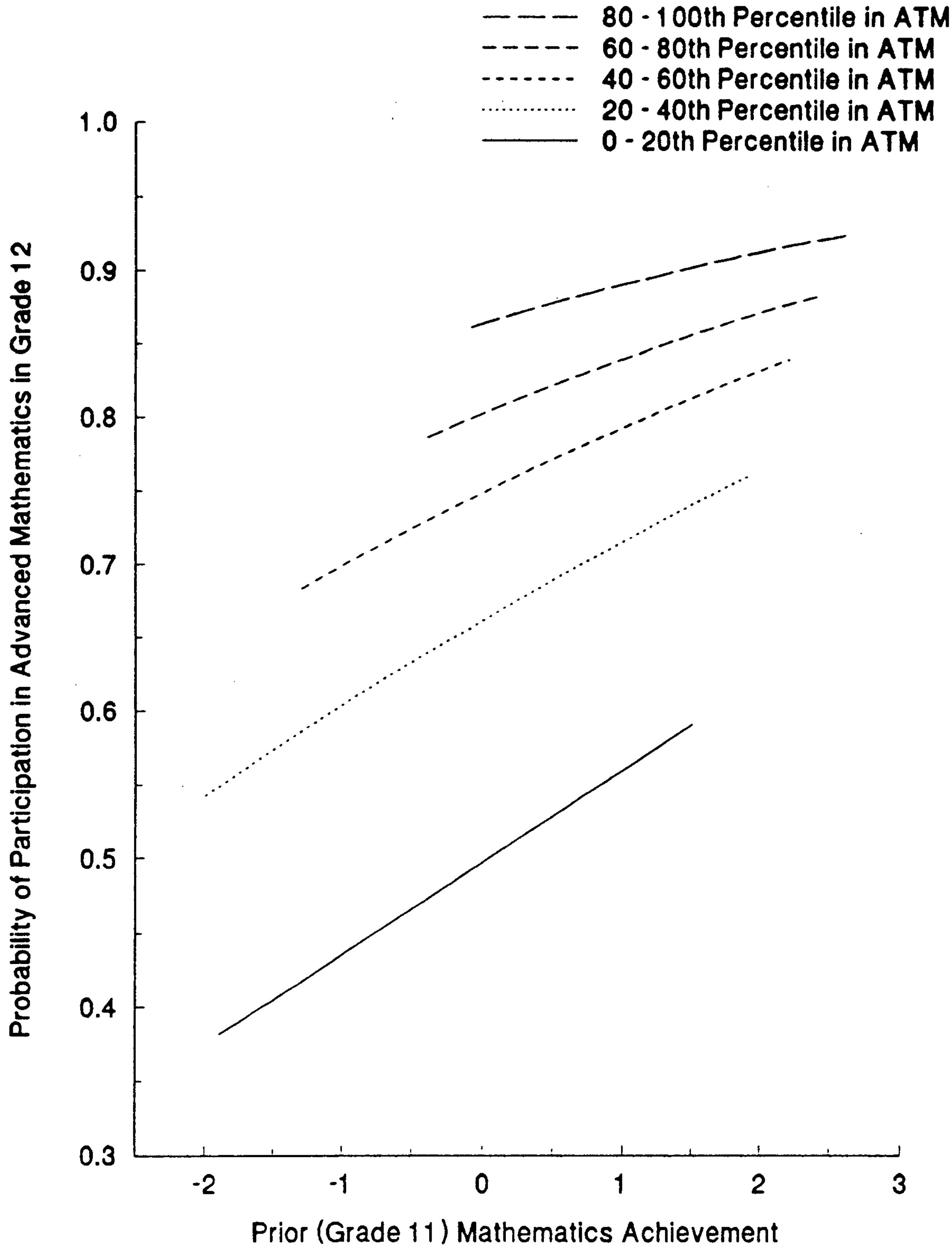

Figure 10. The effect of prior (grade 11) mathematics achievement on participation in advanced mathematics in the 12th grade, by prior (grade 11)

Figure 10 (continued)

attitude toward mathematics. The 20th, 40th, 60th, and 80th percentiles of prior attitude toward mathematics are used to create five groups or percentile ranges. Students are classified into these five groups according to their prior (grade 11) attitude toward mathematics. The range of prior (grade 11) mathematics achievement is from the 10th to 90th percentile in each group.

For each group of students, the risk of dropping out of advanced mathematics is higher for students who have lower prior mathematics achievement. Note that the slope of the curve becomes flatter from the bottom curve to the top curve. This indicates that for students with the worst prior attitude toward mathematics, their prior mathematics achievement is able to make a substantial difference in participation in advanced mathematics. That is, in this group, students with higher prior mathematics achievement are more likely to enroll in an advanced mathematics course than those with lower prior mathematics achievement. Controlling for sex, SES, and prior mathematics anxiety, among students who have the worst prior attitude toward mathematics, the one with the highest prior mathematics achievement is about 20% more likely to enroll in an advanced mathematics course in grade 12 than the one with the lowest prior mathematics achievement. For students with the best prior attitude toward mathematics, the effect of prior mathematics achievement on participation in advanced mathematics is much less significant. Controlling for sex, SES, and prior mathematics anxiety, among students who have the best prior attitude toward mathematics, the one with the highest prior mathematics achievement is about 5% more likely to take an advanced mathematics course in grade 12 than the one with the lowest prior mathematics achievement.

Given the same level of prior mathematics achievement, higher prior attitude toward mathematics substantially increases the likelihood of participation in advanced mathematics. Among students with lower prior mathematics achievement, the effect of prior attitude toward mathematics on participation in advanced mathematics is stronger. Note that the distance among the five curves is relatively larger in the lower distribution of prior mathematics achievement than the higher distribution of prior mathematics achievement. Consider one student in the 20 to 40th percentile in prior attitude toward mathematics and the other in the 40 to 60th percentile. If their prior mathematics achievement is both low, the one with higher prior attitude toward mathematics is approximately 11% more likely to participate in advanced mathematics than the one with lower prior attitude toward mathematics, controlling for sex, SES, and prior mathematics anxiety. On the other hand, if their prior mathematics achievement is both high, the one with higher prior attitude toward mathematics is approximately 6% more likely to take an advanced mathematics course than the one with lower prior attitude toward mathematics, controlling for sex, SES, and prior mathematics anxiety.

To notice the fact that prior attitude toward mathematics is a more important predictor of participation in advanced mathematics than prior mathematics achievement in grade 12, suppose the prior mathematics achievement of two students is one-unit in difference, the one with higher prior mathematics achievement is actually less likely to enroll in an advanced mathematics course than the one with lower prior mathematics achievement, if the former has lower prior attitude toward mathematics than the latter. For example, suppose the former has prior mathematics achievement of 1 (standardized score) and is in the 0 to 20th percentile range in prior attitude toward mathematics, and the latter has prior mathematics achievement of 0 (standardized score). If in the 20 to 40th percentile range in prior attitude toward mathematics, the latter is approximately 10% more likely to take an advanced mathematics course in grade 12 than the former, controlling for sex, SES, and prior mathematics anxiety. If in the 80 to 100th percentile range in prior attitude toward mathematics, the latter is more than

30% as likely to enroll in an advanced mathematics course in grade 12 as the former, controlling for sex, SES, and prior mathematics anxiety.

Figure 11 describes how males and females with differing socioeconomic backgrounds differ in participation in advanced mathematics given their prior mathematics achievement. To create such a figure, a percentile score of SES is calculated for each student, and the 20th, 40th, 60th, and 80th percentiles are used as cut-off points to generate five groups or percentile ranges for each sex. The range of prior mathematics achievement is between the 10th and 90th percentile in each group.

Both male and female students with lower prior mathematics achievement have a higher risk of dropping out of advanced mathematics regardless of their socioeconomic background. For example, for females in the 0 to 20th percentile range in SES, the one with the highest prior mathematics achievement is approximately 20% more likely to participate in advanced mathematics in grade 12 than the one with the lowest prior mathematics achievement, controlling for prior attitude toward mathematics and prior mathematics anxiety. Take another example, for males in the 60 to 80th percentile range in SES, the one with the highest prior mathematics achievement is approximately 20% more likely to participate in advanced mathematics in grade 12 than the one with the lowest prior mathematics achievement, controlling for prior attitude toward mathematics and prior mathematics anxiety.

With the same level of prior mathematics achievement, both males and females from higher socioeconomic background are more likely to participate in advanced mathematics in grade 12. Note that all the curves appear parallel or have a similar slope. This indicates that socioeconomic differences in participation in advanced mathematics are similar across

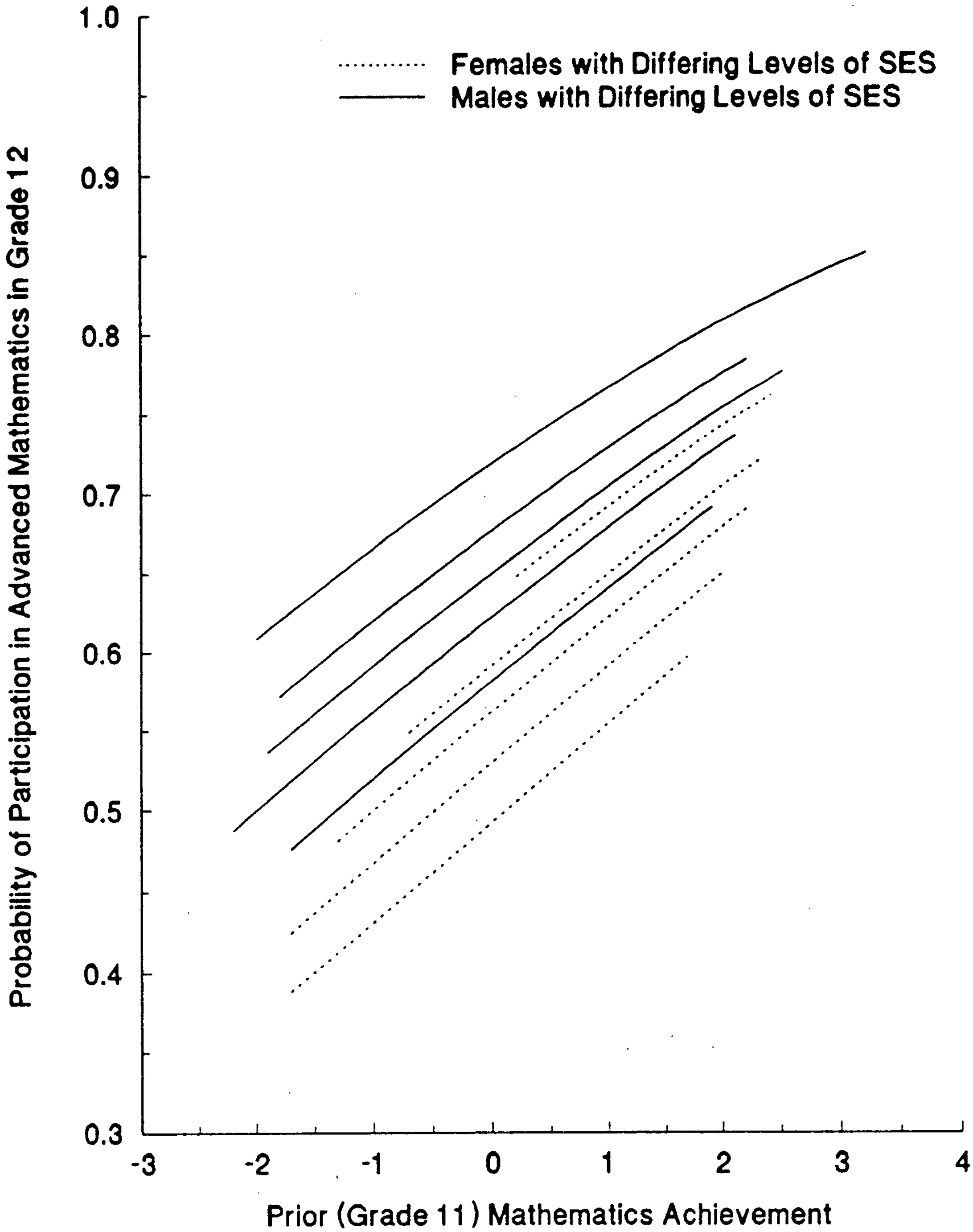

Figure 11. The effect of prior (grade 11) mathematics achievement on participation in advanced mathematics in the 12th grade, by sex and

Figure 11 (continued)

socioeconomic status (SES). The 20th, 40th, 60th, and 80th percentiles of SES are used to generate five groups or percentile ranges for both males and females. Students are cross-classified into ten groups according to their sex and SES. The range of prior (grade 11) mathematics achievement is from the 10th to 90th percentile in each group.

different levels of prior mathematics achievement for each sex. For example, for both males and females with similar prior mathematics achievement, students from the highest SES are about 15% as likely to enroll in an advanced mathematics course in grade 12 as students from the lowest SES, controlling for prior attitude toward mathematics and prior mathematics anxiety.

Note that the male-female distance (distance between analogous male and female curves) in each socioeconomic group is roughly alike. This implies that sex differences in participation in advanced mathematics are similar across socioeconomic groups. For example, males in the 0 to 20th percentile range in SES are approximately 10% more likely to enroll in an advanced mathematics course than their female counterparts, controlling for prior attitude toward mathematics and prior mathematics anxiety. Such sex differences in enrollment in advanced mathematics also apply to students in other percentile ranges of SES.

The graph provides new insight into sex differences in participation in advanced mathematics with a consideration of socioeconomic background. For example, with the same level of prior mathematics achievement, even males from the lowest socioeconomic background are more likely to participate in advanced mathematics in grade 12 than females from the average socioeconomic background, controlling for prior attitude toward mathematics and prior mathematics anxiety. For another example, with the same level of prior mathematics achievement, even males from the average SES are more likely to enroll in an advanced mathematics course in grade 12 than females from the highest SES, controlling for prior attitude toward mathematics and prior mathematics anxiety.

Figure 12 describes how males and females with differing socioeconomic backgrounds differ in participation in advanced mathematics given their prior attitude toward mathematics. To produce this figure, a percentile score of SES is calculated for each student, and the 20th, 40th, 60th, and 80th percentiles are used as cut-off points to create five groups or percentile ranges for each sex. The range of prior attitude toward mathematics is between the 10th and 90th percentile in each group.

Both male and female students with lower prior attitude toward mathematics have a higher risk of dropping out of advanced mathematics regardless of their socioeconomic background. For example, for females in the 0 to 20th percentile range in SES, the one with the highest prior attitude toward mathematics is approximately 40% more likely to enroll in an advanced mathematics course in grade 12 than the one with the lowest prior attitude toward mathematics, controlling for prior mathematics achievement and prior mathematics anxiety. Consider another example, for males in the 60 to 80th percentile range in SES, the one with the highest prior attitude toward mathematics is approximately 30% more likely to participate in advanced mathematics in grade 12 than one with the lowest prior attitude toward mathematics, controlling for prior mathematics achievement and prior mathematics anxiety.

With similar prior attitude toward mathematics, both males and females from higher socioeconomic background are more likely to take an advanced mathematics course in grade 12. Note that all the curves appear parallel or show a similar slope. This indicates that socioeconomic differences in participation in advanced mathematics are similar across different levels of prior attitude toward mathematics for each sex. For example, for both males and females with similar prior attitude toward

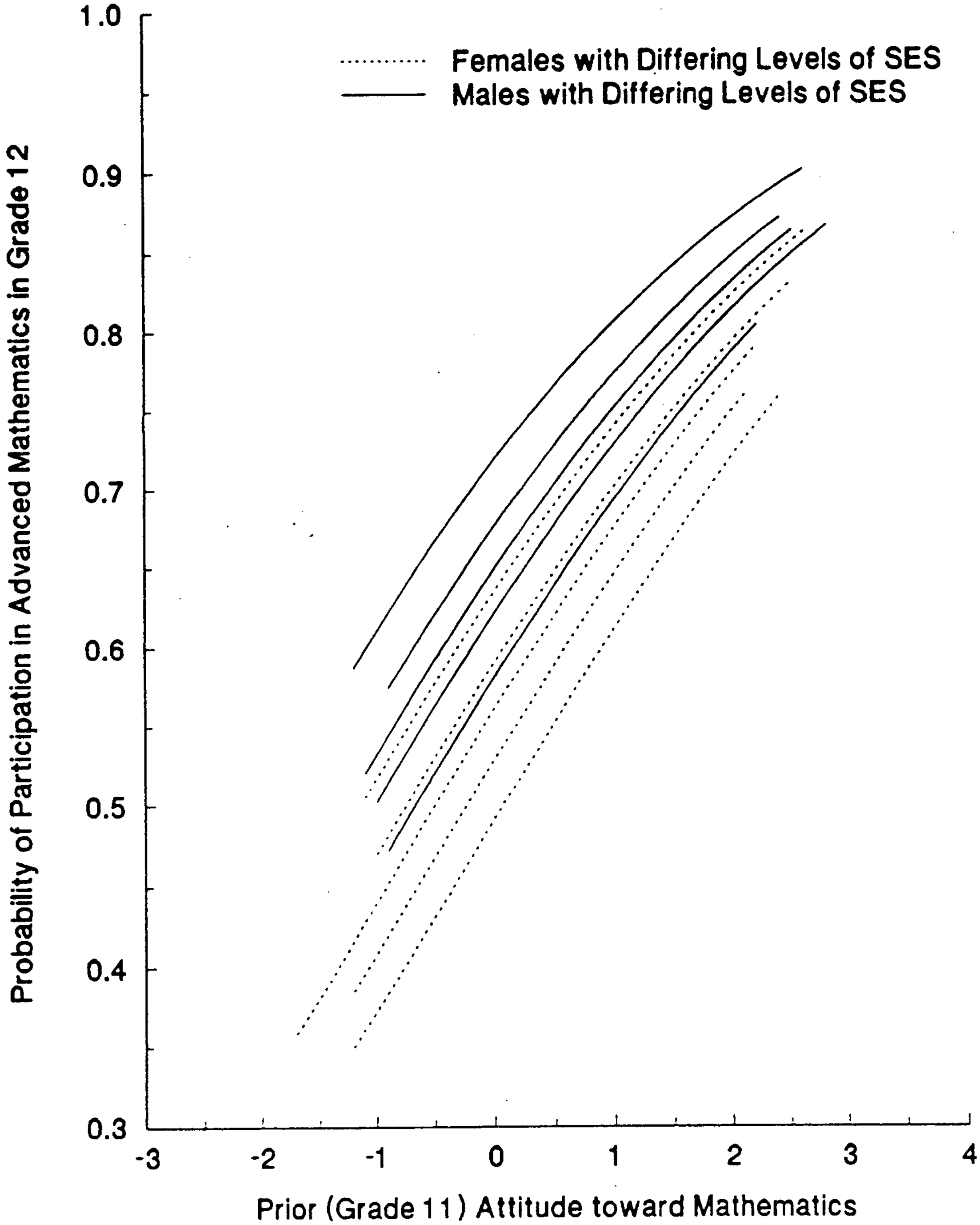

Figure 12. The effect of prior (grade 11) attitude toward mathematics on participation in advanced mathematics in the 12th grade, by sex and

Figure 12 (continued)

socioeconomic status. The 20th, 40th, 60th, and 80th percentiles of SES are used to create five groups or percentile ranges for both males and females. Students are cross-classified into ten groups according to their sex and SES. The range of prior (grade 11) attitude toward mathematics is from the 10th to 90th percentile in each group.

mathematics, students from the highest SES are approximately 15% as likely to participate in advanced mathematics in grade 12 as students from the lowest SES, controlling for prior mathematics achievement and prior mathematics anxiety.

Note that the male-female distance (distance between analogous male and female curves) is similar in each socioeconomic group. This implies that sex differences in participation in advanced mathematics are equivalent across socioeconomic groups. For example, in the 0 to 20th percentile range in SES, males are approximately 10% more likely to enroll in an advanced mathematics course than females, controlling for prior mathematics achievement and prior mathematics anxiety. Such sex differences in participation in advanced mathematics are also observed in other percentile ranges of SES.

Figure 12 also provides new insight into sex differences in participation in advanced mathematics with a consideration of socioeconomic background. For example, with the same level of prior attitude toward mathematics, even males from the lowest socioeconomic background are more likely to take an advanced mathematics course in grade 12 than females from the average socioeconomic background, controlling for prior mathematics achievement and prior mathematics anxiety. For another example, with the same level of prior attitude toward mathematics, even males from the average SES are more likely to enroll in an advanced mathematics course in grade 12 than females from the highest SES, controlling for prior mathematics achievement and prior mathematics anxiety. Note also that, in comparison to Figure 11, the curves in Figure 12 have steeper slopes. This means that prior attitude toward mathematics has a stronger effect on participation in advanced mathematics than prior mathematics achievement.

Survival Analysis Models Predicting Participation in Advanced Mathematics

Table 6 presents the results derived from the survival analysis models that contain time-varying variables. Model 2 which assumes different hazard rates from grade 8 to 12 is accepted as the model for estimating the probability of participation in advanced mathematics because it fits the data significantly better than Model 1 which assumes invariant hazard rates across grades 8 to 12. Note that the effects of prior mathematics anxiety and prior self-esteem are not statistically significant in both survival models.

The survival model takes sex differences in grades 8 to 11 as the base-line effect again which sex differences in grade 12 are compared. The base-line effect is not statistically significant. However, the variable, female, interacts statistically significantly with grade 12 (Effect = -0.42, $p < 0.05$), suggesting that sex differences in participation in advanced mathematics appear mainly in grade 12. With other variables in the model statistically controlled, females are 71% [exp(0.08 - 0.42)] as likely to enroll in an advanced mathematics course in grade 12 as males are. Equivalently, males are 41% [(1/0.71) - 1] more likely than females to participate in advanced mathematics in grade 12. Therefore, females are at significantly higher risk of dropping out of advanced mathematics in grade 12.

The survival model uses the effect of SES on participation in advanced mathematics in grades 10 to 12 as the base-line effect. This effect is statistically significant at the 0.001 level. The effects of SES in grades 8 and 9 are compared respectively with the base-line effect. In comparison to grades 10 to 12, the effect of SES is similar in grade 8, but significantly stronger in grade 9 (Effect = 0.39, $p < 0.05$). Each one-unit increase in SES raises the likelihood of participation in advanced mathematics by an estimated 90% [exp(0.25 + 0.39)] in grade 9, given other variables in the model held constant. That is, if the

Table 6

Statistical Estimates for Survival Models Predicting the Probability of Participation in Advanced Mathematics (11,957 Person-Years)

	Model 1			Model 2		
Explanatory variables	Effect	(SE)	Exp	Effect	(SE)	Exp
Female (D)	0.02	(0.08)	1.02	0.08	(0.13)	1.08
Socioeconomic status	0.30***	(0.04)	1.34	0.25***	(0.05)	1.28
Prior mathematics achievement (T)	-0.22***	(0.04)	0.80	1.41***	(0.25)	4.09
Prior attitude toward mathematics (T)	0.59***	(0.04)	1.81	-0.10	(0.16)	0.90
Grade 9 (D)				-0.91*	(0.41)	-
Grade 10 (D)				-0.78	(0.41)	-
Grade 11 (D)				-2.98***	(0.35)	-
Grade 12 (D)				-4.73***	(0.35)	-
Female by grade 12				-0.42**	(0.18)	0.66
Socioeconomic status by grade 8				0.10	(0.19)	1.11
Socioeconomic status by grade 9				0.39*	(0.19)	1.48
Prior mathematics achievement by grade 9				-0.72*	(0.31)	0.49
Prior mathematics achievement by grade 10				-0.88*	(0.34)	0.42

Table 6 (continued)

Prior mathematics achievement by grade 11				-0.96***	(0.27)	0.38
Prior mathematics achievement by grade 12				-1.13***	(0.26)	0.32
Prior attitude toward mathematics by grade 9				0.45*	(0.21)	1.56
Prior attitude toward mathematics by grade 10				0.24	(0.27)	1.27
Prior attitude toward mathematics by grade 11				0.48**	(0.18)	1.62
Prior attitude toward mathematics by grade 12				0.60***	(0.17)	1.83
Constant	2.71***	(0.05)		5.32***	(0.34)	
Log-likelihood	-2317.07			-1694.23		

Note. * $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$. Model 1 assumes invariant hazard rates across grades 8 to 12. Model 2 assumes different hazard rates from grades 8 to 12. D denotes dummy variables. T denotes time-varying variables. SE denotes standard errors. Exp denotes the regression results in terms of e raised to the power of each effect. Grades 9 (D) to 12 (D) represent relative hazard rates in grades 9 to 12 in comparison to grade 8. Prior mathematics anxiety (T) and prior self-esteem (T) are deleted from both models because of their consistent nonsignificance. The doubled, positive difference in log-likelihood between Models 1 and 2 is statistically significant (Chi-square = 1245.68, $df = 15$, $p = 0.000$). Model 2 therefore fits the data significantly better.

SES of two students is one-unit apart, the one with higher SES is 90% more likely to participate in advanced mathematics in grade 9 than the one with lower SES. In grades 10 to 12, however, each one-unit increase in SES raises the likelihood of participation in advanced mathematics by an estimated 28%. Suppose that the SES of two students is one-unit apart, the one with higher SES is 28% more likely to enroll in an advanced mathematics course than the one with lower SES. In sum, students from disadvantaged socioeconomic background have a substantially higher risk of dropping out of advanced mathematics, and the role of SES in participation in advanced mathematics is significantly more important in the early grades.

The survival model takes the effect of prior mathematics achievement on participation in advanced mathematics in grade 8 as the base-line effect. This effect is statistically significant at the 0.001 level. The effects of prior mathematics achievement on participation in advanced mathematics in other grades are compared respectively with the base-line effect. The interaction between grade and prior mathematics achievement is statistically significant at the 0.05 level in grades 9 and 10 and at the 0.001 level in grades 11 and 12 (Effect = -0.72, -0.88, -0.96, and -1.13 respectively). Thus, the effects of prior mathematics achievement on participation in advanced mathematics become weaker from grade 9 to 12. Once other variables in the model are statistically controlled, each one-unit increase in prior mathematics achievement increases the likelihood of participation in advanced mathematics by 4 times in the 8th grade, 2 times [exp(1.41 - 0.72)] in the 9th grade, 70% [exp(1.41 - 0.88)] in the 10th grade, 57% [exp(1.41 - 0.96)] in the 11th grade, and 32% [exp(1.41 - 1.13)] in the 12th grade. If two students differ in their prior mathematics achievement by one-unit, the one with higher prior mathematics achievement is 4 times more likely in the 8th grade, 2 times more likely in the 9th grade, 70% more likely in the 10th grade, 57% more likely in the 11th grade, and 32% more likely in the 12th grade to participate in advanced mathematics. In sum, although the risk of dropping out of advanced mathematics is substantially higher for students scoring at the lower distribution of mathematics achievement, there is an evident decrease in the effect of prior mathematics achievement on participation in advanced mathematics from grade 8 to 12.

The survival model also takes the effect of prior attitude toward mathematics on participation in advanced mathematics in grade 8 as the base-line effect. This base-line effect is not statistically significant. The effects of prior attitude toward mathematics from grade 9 to 12 are compared respectively with the base-line effect. In comparison to grade 8, the effects of prior attitude toward mathematics are stronger in grade 9 (Effect = 0.45, $p < 0.05$), grade 11 (Effect = 0.48, $p < 0.01$), and grade 12 (Effect = 0.60, $p < 0.001$). The effect of prior attitude toward mathematics is similar between grade 8 and 10. Once other variables in the model are statistically controlled, each one-unit increase in prior attitude toward mathematics raises the likelihood of participation in advanced mathematics by 42% [exp(-0.10 + 0.45)] in the 9th grade, 46% [exp(-0.10 + 0.48)] in the 11th grade, and 65% [exp(-0.10 + 0.60)] in the 12th grade. Thus, if the prior attitude toward mathematics of two students is one-unit apart, the one with higher prior attitude toward mathematics is 42% more likely in the 9th grade, 46% more likely in the 11th grade, and 65% more likely in the 12th grade to take an advanced mathematics course. On average, students who have lower prior attitude toward mathematics are at substantially higher risk of dropping out of advanced mathematics, and the effect of prior attitude toward mathematics on participation in advanced mathematics increases across grades.

The constant is statistically significant at the 0.001 level. It indicates the hazard rate of participation in advanced mathematics for the typical student with nationally average characteristics in grade 8 against which the hazard rates of the typical student in other grades are compared. The typical student is 100% {1/[1 + exp(-5.32)]} likely to enroll in an advanced mathematics course in grade 8, given other variables in the model statistically controlled. The hazard rate in grade 9 is significantly higher than that in grade 8 (Effect = -0.91, $p < 0.05$); once other variables in the model are held constant, the typical student is 99% {1/[1 + exp(- 5.32 + 0.91)]} likely to take an advanced mathematics course in grade 9. Because of this likelihood of 99%, the statistically higher hazard rate in grade 9 makes little practical difference in participation in advanced mathematics between grade 8 and 9. The hazard rate in grade 10 is similar to that in grade 8, and, thus, the typical student is about 100% likely to enroll in an advanced mathematics course in grade 10. The hazard rates in the 11th grade (Effect = -2.98, $p < 0.001$) and the 12th grade (Effect = -4.73, $p < 0.001$) are significantly higher than that in the 8th grade. Once other variables in the model are held constant, the likelihood of participation in advanced mathematics for the typical student is 91% {1/[1 + exp(-5.32 + 2.98)]} in grade 11 and 63% {1/[1 + exp(-5.32 + 4.73)]} in grade 12. Therefore, the typical student with nationally average characteristics is at the greatest risk of dropping out of advanced mathematics in the 12th grade.

<u>Longitudinal Effects of Factors on Participation in Advanced Mathematics</u>

Containing prior mathematics achievement and prior attitude toward mathematics as time-varying variables, the survival model is suitable to examining the longitudinal effects of prior mathematics achievement and prior attitude toward mathematics on participation in advanced mathematics. Because the last two years of high school are most important in regard to participation in advanced mathematics, the examination focuses mainly on grades 11 and 12. Based on the results of Model 2 in Table 6, the following algebraic operations are performed. In the 11th grade, the odds of participation in advanced mathematics is

$$\text{Odds (taking an advanced mathematics course in grade 11)} = e^{Z_1}$$

where

$$\begin{aligned} Z_1 &= (5.32 - 2.98) + 0.08(\text{female}) + 0.25(\text{SES}) + (1.41 - 0.96)(\text{mathematics achievement in the 10th grade}) + (0.48 - 0.10)(\text{attitude toward mathematics in the 10th grade}) \\ &= 2.34 + 0.08(\text{female}) + 0.25(\text{SES}) + 0.45(\text{mathematics achievement in the 10th grade}) + 0.38(\text{attitude toward mathematics in the 10th grade}). \end{aligned}$$

Similarly, the odds of participation in advance mathematics in the 12th grade is

$$\text{Odds (taking an advanced mathematics course in grade 12)} = e^{Z_2}$$

where

$$\begin{aligned} Z_2 &= (5.32 - 4.73) + (0.08 - 0.42)(\text{female}) + 0.25(\text{SES}) + (1.41 - 1.13)(\text{mathematics achievement in the 11th grade}) + (0.60 - 0.10)(\text{attitude toward mathematics in the 11th grade}) \\ &= 0.59 - 0.34(\text{female}) + 0.25(\text{SES}) + 0.28(\text{mathematics achievement in the 11th grade}) + 0.50(\text{attitude toward mathematics in the 11th grade}). \end{aligned}$$

With attitude toward mathematics in the model held constant (same values in grades 10 and 11), the longitudinal effect of prior mathematics achievement can then be described as

$$\frac{\text{Odds (taking an advanced mathematics course in grade 12)}}{\text{Odds (taking an advanced mathematics course in grade 11)}} = e^{Z_2 - Z_1}$$

where

$Z_2 - Z_1$ = - 1.75 - 0.42(female) - 0.17(mathematics achievement in the 11th grade) + 0.12(attitude toward mathematics in the 11th grade) + 0.28(difference in mathematics achievement between grade 10 and 11).

Therefore, the odds ratio indicates that each one-unit increase in mathematics achievement from grade 10 to 11 increases the odds of taking an advanced mathematics course in grade 12 by an estimated 32% [exp(0.28) - 1]. More important, the longitudinal effect of prior mathematics achievement on participation in advanced mathematics depends on sex, mathematics achievement in the 11th grade, and attitude toward mathematics in the 11th grade. In other words, the same change in prior mathematics achievement from grade 11 to 12 has different effects on participation in advanced mathematics according to students' sex and their mathematics achievement and attitude toward mathematics in grade 11. Males have stronger effects than females. So do students who have low mathematics achievement in the 11th grade or students who have high attitude toward mathematics in the 11th grade. Therefore, males with low mathematics achievement but high attitude toward mathematics are in the best position to improve their odds of participating in advanced mathematics, given the same change in prior mathematics achievement over grades.

For example, if a male student with average mathematics achievement and attitude toward mathematics (standardized scores of zero) maintains his level of mathematics achievement from grade 10 to 11, the odds of his taking an advanced mathematics course in grade 12 is approximately 17% {$1/[1 + \exp(-Z_2 + Z_1)]$ where $-Z_2 + Z_1 = -1.75 - 0.42 \times 0 - 0.17 \times 0 + 0.12 \times 0 + 0.28 \times 0 = -1.75$} of the odds in grade 11. If he improves his mathematics achievement by one-unit from grade 11 to 12, the odds of his taking an advanced mathematics course in grade 12 is approximately 23% {$1/[1 + \exp(-Z_2 + Z_1)]$ where $-Z_2 + Z_1 = -1.75 - 0.42 \times 0 - 0.17 \times 0 + 0.12 \times 0 + 0.28 \times 1 = -1.47$} of the odds in grade 11. If a female student has the same conditions, the odds of her taking an advanced mathematics course in grade 12 is 11% {$1/[1 + \exp(-Z_2 + Z_1)]$ where $-Z_2 + Z_1 = -1.75 - 0.42 \times 1 - 0.17 \times 0 + 0.12 \times 0 + 0.28 \times 0 = -2.17$} given that she maintains her level of mathematics achievement over grades, and 15% {$1/[1 + \exp(-Z_2 + Z_1)]$ where $-Z_2 + Z_1 = -1.75 - 0.42 \times 1 - 0.17 \times 0 + 0.12 \times 0 + 0.28 \times 1 = -1.89$} given that she improves her level of mathematics achievement by one-unit over grades. Therefore, improving the level of mathematics achievement over grades results in a better likelihood of participation in advanced mathematics than maintaining the level of mathematics achievement over grades. Worsening the level of mathematics achievement over grades decreases the likelihood of participation in advanced mathematics. Note also that, with the same achievement and attitude background in mathematics, males who maintain their level of mathematics achievement over grades are actually more likely to participate in advanced mathematics than females who improve their level of mathematics achievement over grades.

The same algebraic operations are performed to examine the longitudinal effect of prior attitude toward mathematics on participation in advanced mathematics:

$Z_2 - Z_1$ = - 1.75 - 0.42(female) - 0.17(mathematics achievement in the 11th grade) + 0.12(attitude toward mathematics in the 11th grade) + 0.38(difference in prior attitude toward mathematics between grade 11 and 12).

Similar conclusions are reached. Each one-unit increase in attitude toward mathematics from grade 10 to 11 increases the odds of taking an advanced mathematics course in grade 12 by an estimated 46% [exp(0.38) - 1]. The same change in attitude toward mathematics from grade 10 to 11 shows different effects on participation in advanced mathematics for students, conditional on sex as well as mathematics achievement and attitude toward mathematics in grade 11. Male students, students with low mathematics achievement in grade 11, and students with high attitude toward mathematics all have stronger effects. Therefore, males with low mathematics achievement but high attitude toward mathematics are in the best position to improve their odds of participation in advanced mathematics, given the same change in prior attitude toward mathematics. Improving the level of attitude toward mathematics over grades results in a better likelihood of participation in advanced mathematics than maintaining the level of attitude toward mathematics over grades. Worsening the level of attitude toward mathematics over grades decreases the likelihood of participation in advanced mathematics.

Note that the longitudinal effect of mathematics achievement is 0.28, whereas the longitudinal effect of attitude toward mathematics is 0.38. This indicates that the change (for example, one-unit of standard deviation) in attitude toward mathematics over grades results in more dramatic change in participation in advanced mathematics than the same amount of change in mathematics achievement over grades.

The Adequacy of the Statistical Models

The model-data-fit results of the statistical models predicting participation in advanced mathematics are listed in Appendices C and D. In case of the logistic model for the 8th grade, the histogram shows that the model is successful in identifying participants in advanced mathematics, but not very successful in identifying dropouts (see Appendix C-1). Because the number of dropouts is much smaller than that of participants, the model, overall, is considered successful in correctly predicting the enrollment status of the majority of the students. For example, if the probability of 0.50 is arbitrarily chosen so that students with probabilities above 0.50 are classified as model-predicted participants and students with probabilities below 0.50 are classified as model-predicted dropouts, then the model correctly identifies 98% of the students. Projecting this figure to the entire population shows that if 100 students were randomly selected in the 8th grade, the logistic model would be able to precisely classify 98 of them based on their sex, socioeconomic background, and scores on prior mathematics achievement, prior attitude toward mathematics, and prior mathematics anxiety.

The average goodness-of-fit statistic of the logistic model is less than 1 (see Appendix D). This suggests that the model, on average, is capable of making a prediction of the likelihood of participation in advanced mathematics of students which is close to their actual status of participation in advanced mathematics. As to the goodness-of-fit index (see Appendix D), the model chi-square indicates that the full model with all explanatory variables in the equation significantly improves the model-data-fit at the 0.001 level over the initial model with only constant in the equation.

The histograms of the logistic models for grades 9 to 11 show similar results (see Appendices C-2 to C-4). Overall, the models are successful in correctly identifying the majority of the students. For example, with the probability of 0.50 as the standard, 98% of the students in the 9th grade, 99% of the students in the 10th grade, and 90% of the students in the 11th grade are correctly identified. The average goodness-of-fit statistic is less than 1 across all three models (see Appendix D), indicating that each model, on average, is capable of making a prediction of the likelihood of participation in advanced mathematics of students which is close to their actual status of participation in advanced mathematics. The improvement of the full model is statistically significant over the initial model for all three models (see Appendix D). The logistic model for the 12th grade is much better in predicting dropouts than any other logistic models. With the probability of 0.50 as the standard, the model correctly identifies 68% of the students. As shown in Appendix D, the average goodness-of-fit statistic is 1, and the improvement of the full model is statistically significant at the 0.001 level over the initial model.

The histogram of the survival model with time-varying variables shows high success in predicting participants in advanced mathematics and some success in predicting dropouts (see Appendix C-6). Overall, the survival model is successful in correctly identifying the majority of the students. With the probability of 0.50 as the standard, 93% of the students are precisely predicted based on their sex, socioeconomic background, and scores on prior mathematics achievement and prior attitude toward mathematics. The average goodness-of-fit statistic is less than 1 (see Appendix D), indicating that the survival model, on average, is able to make a prediction of the likelihood of participation in advanced mathematics of students which is close to their actual status of participation in advanced mathematics. The improvement of the full survival model is statistically significant at the 0.001 level over the initial model (see Appendix D).

In sum, although both the logistic and survival models show some weaknesses in identifying dropouts in advanced mathematics, overall the model-data-fit of the logistic and survival models is considered reasonable. Note also that the logistic and survival models replicate each other in terms of major statistical results. As mentioned before, this cross-validation between models is important in judging the functions of each model.

Chapter 5

Discussion and Conclusions

Principal Findings of the Study

Parents, educators, and policymakers have been concerned about the inadequate mathematics preparation of high school students because of its negative effects on students' future educational and occupational opportunities. This study simultaneously examined mathematics participation and participation in advanced mathematics of secondary students in the United States.

Mathematics Participation. This study reveals the following major findings regarding mathematics participation:

1. Students are most likely to drop out of mathematics in grade 12.

Although mathematics dropout becomes evident in grade 11, the highest risk of dropping out of mathematics is in grade 12. An estimated 64% of students are persistent in taking mathematics courses from grade 7 to 12. The typical student with nationally average characteristics is 95% likely in the 11th grade and 69% likely in the 12th grade to enroll in a mathematics course. The transition from grade 11 to 12 represents the point at which the typical student is most likely to stop taking a mathematics course.

2. There are significant sex differences in mathematics participation in grade 12.

As a matter of fact, sex is the most important predictor of mathematics participation in grade 12. The survival model shows that males are 47% more likely than females to take a mathematics course in grade 12. Projecting this figure to the entire population means that the rate of male to female participants is 1.47.

3. The effect of SES on mathematics participation weakens from grade 11 to 12.

Mathematics dropout is more likely to occur for students with lower socioeconomic background. SES is the most important predictor of mathematics participation in grade 11, but the effect of SES on mathematics participation decreases substantially in grade 12.

4. Prior attitude toward mathematics is as important as prior mathematics achievement in mathematics participation.

Poor mathematics achievement increases the risk of dropping out of mathematics. The effect of prior mathematics achievement on mathematics participation is similar for the grade 10 to 11 transition and the grade 11 to 12 transition. A lower score on the scale of attitude toward mathematics also increases the risk of dropping out of mathematics. The effect of prior attitude toward mathematics on mathematics participation is also similar for the grade 10 to 11 transition and the grade 11 to 12 transition. Moreover, prior attitude toward mathematics and prior mathematics achievement have similar effects on mathematics participation.

5. The longitudinal effect of prior mathematics achievement or prior attitude toward mathematics on mathematics participation depends on students' sex and socioeconomic background.

Changes in prior mathematics achievement and prior attitude toward mathematics over grades have the strongest effects on males from low socioeconomic background. For example, the same improvement in prior mathematics achievement will increase mathematics participation of males from low SES more dramatically than females from high SES. On the other hand, the same decrease in prior mathematics achievement will be associated with more mathematics dropouts among males from low socioeconomic background than females of any socioeconomic background.

With the same socioeconomic background, males who maintain their level of mathematics achievement or attitude toward mathematics over grades are still more likely to participate in mathematics than females who improve their level of mathematics achievement or attitude toward mathematics over grades. In general, improving the level of mathematics achievement or attitude toward mathematics over grades results in better mathematics participation than maintaining the level of mathematics achievement or attitude toward mathematics over grades. Also, worsening the level of mathematics achievement or attitude toward mathematics over grades decreases the likelihood of mathematics participation. Common sense or belief states that if a student increases his or her mathematics achievement and attitude toward mathematics, he or she is more likely to take mathematics courses. The above results are an example of research providing evidence to confirm and support common sense or belief.

Participation in Advanced Mathematics. Overall, high school students appear to be taking more mathematics courses (National Center for Education Statistics, 1995). The improvement in participation in advanced mathematics during the past decade, however, has been trivial. Although, in comparison to 1982, 1987, and 1990, high school graduates in 1992 were more likely to take a mathematics course at the level of Algebra I or higher (National Center for Education Statistics, 1994), consistently less than 20% of graduates since 1982 took Trigonometry, Pre-calculus, or Calculus (National Center for Education Statistics, 1993). Female under-representation in these courses is even more serious (Educational Testing Service, 1994). The National Education Longitudinal Study (NELS) (see Ingels, et al., 1989) data also demonstrate that very few students have taken advanced mathematics courses such as Trigonometry, Pre-calculus, and Calculus in the 9th and 10th grades (Scott & Ingels, 1992). Regarding participation in advanced mathematics, this study reveals the following major findings:

1. Students are at the greatest risk of dropping out of advanced mathematics in grade 12.

The decline in participation in advanced mathematics becomes evident in grade 11, and students are most likely to stop taking an advanced mathematics course in grade 12. An estimated 55% of students are persistent in taking advanced mathematics courses from grade 7 to 12. The typical student with nationally average characteristics is 91% likely in the 11th grade, and 63% likely in the 12th grade to enroll in an advanced mathematics course. Therefore, the typical student is at the greatest risk of dropping out of advanced mathematics in the transition from grade 11 to 12.

2. There are significant sex differences in participation in advanced mathematics in grade 12.

There are no significant sex differences in participation in advanced mathematics prior to grade 12. The NELS data also show that male and female sophomores are equally likely to have completed advanced level mathematics coursework (Ingels, et al., 1994). Females are at significantly higher risk of dropping out of advanced mathematics in grade 12, however. The survival analysis model demonstrates that males are 41% more likely to participate in advanced mathematics than females. Projecting this figure to the entire population indicates that the rate of male to female participants in advanced mathematics is 1.41.

3. Sex differences in participation in advanced mathematics are similar across different levels of SES.

Males are more likely than females to take an advanced mathematics course in grade 12 regardless of the level of socioeconomic background. Furthermore, the male advantage over female is also constant across students with differing socioeconomic backgrounds. It is estimated that, with the same socioeconomic background, males are about 10% more likely than females to take an advanced mathematics.

4. SES plays a critical role in the early grades for participation in advanced mathematics.

Although the risk of dropping out of advanced mathematics is much higher if a student comes from lower socioeconomic background, the significant effect of SES on participation in advanced mathematics decreases over grades. That is, the role of SES is more important in the early grades. The NELS data also indicate that sophomores from the highest SES quartile are more than twice as likely to have completed an advanced mathematics sequence (a year or more of any combination of Algebra II, Trigonometry, Pre-calculus or Calculus) than students from the lowest SES quartile (Ingels, et al., 1994).

5. The effect of prior mathematics achievement on participation in advanced mathematics decreases consistently from grade 8 to 12.

Note that prior mathematics achievement is the single factor that has significant effects on participation in advanced mathematics in every grade from 8 to 12. Dropouts in advance mathematics are more likely to be students scoring at the lower end of the distribution of mathematics achievement. Lee and Bryk (1988) also reported that sophomore mathematics achievement is the most important determinant of students' enrollment in advanced mathematics courses in all three curriculum tracks (academic, general, and vocational). However, the present study reveals that there is an evident decrease in the effect of prior mathematics achievement on participation in advanced mathematics from grade 8 to 12. Therefore, prior mathematics achievement is the most important factor in the early grades for participation in advanced mathematics.

6. <u>The effect of prior attitude toward mathematics on participation in advanced mathematics increases from grade 11 to 12.</u>

In grades 11 and 12, students with lower prior attitude toward mathematics have a substantially higher risk of dropping out of advanced mathematics. Other researchers have also found that attitude toward mathematics is predictive of the intention to continue to participate in mathematics courses once enrollment becomes optional (e.g., Thorndike-Christ, 1991). The present study extends their findings, showing that the effect of prior attitude toward mathematics on participation in advanced mathematics increases from grade 11 to 12. Prior attitude toward mathematics is the most important factor in the later grades for participation in advanced mathematics.

7. <u>The effect of prior mathematics achievement on participation in advanced mathematics varies across different levels of prior attitude toward mathematics, and vice versa.</u>

For students with lower prior attitude toward mathematics, prior mathematics achievement makes a substantial difference in participation in advanced mathematics in grade 12. But for students with higher prior attitude toward mathematics, the effect of prior mathematics achievement on participation in advanced mathematics is less significant. Similarly, among students with lower prior mathematics achievement, the effect of prior attitude toward mathematics on participation in advanced mathematics is stronger in grade 12. But among students with higher prior mathematics achievement, the effect of prior attitude toward mathematics is less significant.

8. Socioeconomic differences in participation in advanced mathematics are similar across different levels of prior mathematics achievement or prior attitude toward mathematics.

With the same level of prior mathematics achievement or prior attitude toward mathematics, both males and females from higher socioeconomic background are more likely to participate in advanced mathematics in grade 12. Moreover, the advantage of high SES over low SES is also constant across both male and female students with differing levels of prior mathematics achievement or prior attitude toward mathematics.

9. There is a male advantage in participation in advanced mathematics even when there is a male disadvantage in socioeconomic background.

With the same level of prior mathematics achievement or attitude toward mathematics, even males from low socioeconomic background are more likely to participate in advanced mathematics in grade 12 than females from average socioeconomic background, and males from average SES are more likely to enroll in an advanced mathematics course in grade 12 than females from high SES.

10. The longitudinal effect of prior mathematics achievement or prior attitude toward mathematics on participation in advanced mathematics depends on sex, mathematics achievement, and attitude toward mathematics.

Changes in prior mathematics achievement and prior attitude toward mathematics over grades have the strongest effects on males with low mathematics achievement but high attitude toward mathematics. With the same socioeconomic background, males who maintain their level of mathematics achievement or attitude toward mathematics over grades are still more likely to participate in advanced mathematics than females who improve their level of mathematics achievement or attitude toward mathematics over grades. Generally speaking, improving the level of mathematics achievement or attitude toward mathematics over grades results in better participation in advanced mathematics than maintaining the level of mathematics achievement or attitude toward mathematics over grades. Worsening the level of mathematics achievement or attitude toward mathematics over grades decreases the likelihood of participation in advanced mathematics.

11. Prior attitude toward mathematics has a stronger longitudinal effect on participation in advanced mathematics than prior mathematics achievement.

In comparison to mathematics achievement, the same amount of change in attitude toward mathematics results in a more dramatic change in participation in advanced mathematics.

Policy Implications of the Study

The findings of this study have significant implications for the development of policies or programs that tackle the dropout problem in mathematics. Based on the findings, this section goes beyond the data to derive some implications useful as cues for educational reform.

1. The collective efforts of various policies or programs are indispensable.

No single policy or program is able to meet the needs of the diverse population of students who are at risk as mathematics dropouts. This study implies that four types of programs are needed: (a) programs that encourage students from disadvantaged socioeconomic background to remain in mathematics; (b) programs that encourage female students to remain in mathematics; (c) programs that help students improve their attitude toward mathematics; (d) programs that help ensure that a core set of foundation skills is attained at each grade level.

Because this study shows that female students and those from low socioeconomic background are most likely to drop out of mathematics, especially advanced mathematics, programs ought to give attention to the most vulnerable group — female students from disadvantaged socioeconomic background. Policies should be developed that help students realize that inadequate preparation in high school mathematics seriously limits their future career choices. This concern must be fully emphasized among female students and students from low socioeconomic background. Counseling programs that are currently available in many schools should take a critical role in reinforcing this message.

2. Identification of potential dropouts in mathematics should be carried out prior to the 11th grade.

Because the decline in participation in mathematics, especially advanced mathematics, becomes significant in grade 11, it is important to identify potential dropouts in mathematics prior to the 11th grade. Students' attitude toward mathematics and their mathematics achievement should be monitored regularly and used as indicative signs for implementation of educational interventions. The findings of this study support the use of statewide programs of early mathematics readiness testing in junior high schools. The purpose of the testing is to encourage students to take mathematics courses in the senior grades in order to better prepare for college majors and the labor market (see Payne, 1992). There is evidence now that such programs are influential in promoting students to take senior-level mathematics courses (Payne, 1992).

3. It is appropriate to carefully raise the mathematics requirement for graduation.

It has been recommended that all students be required to take three years of mathematics in high school regardless of ability levels (National Commission on Excellence in Education, 1983; National Council of Teachers of Mathematics, 1989; National Science Board, 1983), with an additional year of mathematics for college-bound students (National Council of Teachers of Mathematics, 1989). This study demonstrates that the vast majority of students in the United States have already met these recommended standards in mathematics.

Although 42 states have raised standards for high school graduation since 1983, there are still calls for increases in graduation requirements and programmatic changes (e.g., Gill, 1988). This study shows that students stop taking mathematics courses in the later grades not due to a lack of ability, but to their low attitude toward mathematics. The role of attitude toward mathematics is more important than that of mathematics achievement for mathematics participation in the later grades of high school. Note that the effect of mathematics achievement on mathematics participation declines substantially from grade 8 to 12. Therefore, affective, rather than cognitive, problems of students contribute to mathematics dropout in the later grades of high school. In other words, students do not drop out of mathematics because they lack the cognitive abilities required for further mathematics courses, but because they do not desire to take further mathematics courses. Raising mathematics standards for high school graduation can significantly improve students' preparation in high school mathematics. Completion of four years of high school mathematics is commonly considered a prerequisite for many college mathematics, science, and statistics courses. Therefore, there should be different levels of compulsory mathematics courses until, at least, the 11th grade.

Note that performance and participation in college are related to performance and participation in high school (Deboer, 1984). For example, the number of high school mathematics and science courses is the most important determinant of whether students take quantitative courses at the undergraduate level (Marion & Coladarci, 1993). The High School and Beyond (HS&B) data suggest that college mathematics course entry level is positively and significantly related to whether students complete a mathematics course in the 12th grade (Payne, 1992). The HS&B data also indicate that advanced mathematics courses (Algebra III, Pre-calculus, and Calculus) relate strongly to achievement in college calculus (Stribling, 1990). The benefit of increasing mathematics standards for high school graduation in students' college studies should be appreciable from the above research findings, given that the present study suggests that raising the mathematics requirement for graduation may not necessarily create cognitive difficulties for the majority of students.

4. <u>Policies should be developed to help teachers, guidance counselors, and administrators obtain necessary training in identifying the key elements of academic persistence and in designing strategies to develop them in students.</u>

Good counseling, by teachers, counselors, and administrators, can provide a realistic picture of the relationship between students' present course actions and their future career options. However, many teachers, counselors, and administrators may not be familiar with the issue of academic persistence, and may lack strategies for developing students' academic persistence. Mathematics teachers are in the most convenient position to quickly observe changes in students' attitude toward mathematics and mathematics achievement because of their close contact with students. They need to be sensitive to early signs of mathematics dropout and be able to implement remedial strategies. Teacher education programs should take this concern into consideration.

5. There should be parent education programs that help parents become aware of the importance of mathematics to their children's futures.

Although this study did not include any variables relating to parents (except their SES), the important signs of mathematics dropout such as negative attitude toward mathematics and declining mathematics achievement are easily visible to parents. Schools alone may not be able to solve the dropout problem in mathematics. Effective cooperation between schools and families are as critical in mathematics participation as in any other educational issue. Therefore, programs should be developed that offer suggestions and activities for parents to use in nurturing their children's interest in mathematics and encouraging appreciation of mathematics for their future career choices.

6. There is a need for effective school monitoring systems which indicate to educators potential mathematics dropouts.

School monitoring systems use reliable instruments to measure various schooling outcomes of students including attitude and achievement (see Willms, 1992). Therefore, teachers and administrators can detect the first indicative signs of mathematics dropout with greater confidence. Consequent educational interventions can then be more focused. Regular collection and analysis of data within schools can not only discern students at risk of dropping out of mathematics but also trace the pattern of change in attitude toward mathematics and mathematics achievement as they relate to mathematics participation. This information helps school staff to work out prevention programs for future students. Policies should be developed to help schools establish basic, effective monitoring systems.

At the regional level, programs such as comprehensive statewide minimum competency testing should be encouraged. There are such programs in, for example, New York and Texas where all students take the test as well as in California where a "matrix sampling" method is used (Ju, 1992). Data from these programs can be used to predict individual schools that may need support and encouragement in mathematics participation.

Limitations of the Study

The major limitation of this study is the use of the relatively "thin" measures of mathematics anxiety and self-esteem. Note that the mathematics anxiety scale in LSAY contains only two items, and the LSAY self-esteem scale contains only six items. When only two items are used to measure mathematics anxiety, it is possible for students with differing status of mathematics participation to have the same responses. The variable, mathematics anxiety, then, loses its predictive power on mathematics participation because it fails to distinguish students with different status of mathematics participation. There could have been greater confidence in the finding that the effect of either prior mathematics anxiety or prior self-esteem is trivial on mathematics participation, if more sophisticated measures of mathematics anxiety and self-esteem had been used. For example, the Mathematics Anxiety Rating Scale (MARS) (see Suinn, Edie, Nicoletti, & Spinelly, 1972) provides far more reliable measures of mathematics anxiety.

The sample size of 3116 is also considered small from the perspective of a national survey. This may undermine to some degree the confidence of projecting the statistical estimates to the entire population simply because the range of students' educational characteristics in different schools, districts, regions, and states may not be fully represented by this relatively small pool of students, even though random sampling was employed. Nevertheless, the steering function of these data is still appreciable given the extensive waves (successive collections) of data that cover the entire secondary school career of students.

Finally, because one of the purposes of this study is to examine mathematics participation in each grade from 8 to 12, grade cohorts are used in statistical analysis. There are advantages, however, in using age cohorts, rather than grade cohorts (see Willms, 1992). For example, there is the issue that some variables may have already done their "filtering job" before students ever get to grade 8.

Recommendations for Further Research

Outlined below are several extensions of this study that would enhance a thorough examination of mathematics participation:

1. Examination of the roles of specific components of both attitude toward mathematics and mathematics achievement in mathematics participation.

The current measures of attitude toward mathematics are considered crude approximations to the "true" attitude (see Leder, 1987). Moreover, the multiplicity of meaning given to the concept of attitude toward mathematics is the primary culprit of the inconsistencies in the literature on attitude toward mathematics (Anderson, 1981). The best solution for these problems, before more advanced attitudinal measures are developed, is to measure specific attitudes toward certain mathematical components, rather than a generalized attitude toward mathematics as a whole (Aiken, 1970b). For example, attitude toward mathematics is comprised of three subscales in LSAY: (a) interest, (b) utility, and (c) ability. That prior attitude toward mathematics is the most important predictor of mathematics participation in grade 12 does not mean that these three components are equally critical in mathematics participation.

A better specification of attitude toward mathematics would increase the reliability and validity of attitudinal measures. It would also help identify the core factors in attitude toward mathematics that shapes the general role of attitude in mathematics participation. Consider one example. Mathematics educators believe that most students lose their interest in mathematics during high school for various reasons. As an attitudinal variable, interest in mathematics may, then, have more predictive power of mathematics participation than utility or ability. Also, whether the perceived usefulness of mathematics or the personal assessment of mathematics abilities affects more the decision of mathematics participation is not clear at this point. These issues require further investigation.

Anderson (1981) also suggested that measures of mathematics achievement be area-specific (e.g., arithmetic, algebra, geometry, etc.) because a generic achievement measure of mathematics is usually problematic, especially when the measurement is not accurate. Because prior mathematics achievement is an important predictor of mathematics participation, area-specific measures of mathematics achievement should be taken into account. The aim is not only to improve the reliability and validity of mathematics achievement tests, but also to discern the core areas in mathematics that shape the role of mathematics achievement in mathematics participation. For example, geometry was found to be among the key "gatekeeper" courses for college admission (Ingels, et al., 1994).

2. Examination of the effects of race and curriculum tracking on mathematics participation.

A measure of race or ethnicity is not available in the LSAY data. However, the research literature has demonstrated ethnic differences in mathematics participation (see, for example, Ayabe, 1982; Anick, et al., 1981; Carter & Segura, 1979; Leap, et al., 1982; Moore & Smith, 1985; Pallas & Alexander, 1983; Rendon & Triana, 1989). The measure of curriculum tracking in LSAY had substantial missing data, and, therefore, was not used in this study. However, the effect of curriculum tracking on academic participation is well-known (see, for example, Gamoran, 1987; Kulik & Kulik, 1984; Lee & Bryk, 1988; Massachusetts State Department of Education, 1986). These variables are important because they may be related to students' attitudes toward mathematics and therefore affect mathematics participation.

3. Examination of more affective variables on mathematics participation.

The finding that prior attitude toward mathematics outweighs prior mathematics achievement in mathematics participation in the later grades of high school projects the impression that affective factors may be more important than cognitive factors during the most risky period of dropping out of mathematics. This certainly calls for an inclusion of other affective outcome variables. For example, future career plans and the selection of college majors may be predictive of mathematics participation particularly when enrollment becomes optional. There has been some evidence that students' course experiences in science and mathematics differ according to their intended career destinations (see Westbury, 1988).

4. Examination of the effect of schools on mathematics participation.

The weaknesses of the logistic and survival models in discerning dropouts in mathematics indicate that participants and dropouts are likely to have similar characteristics with regard to socioeconomic background, prior mathematics achievement, and prior attitude toward mathematics. This suggests that other variables should be examined for improvement of prediction. Because this study has considered major individual level variables, the expectation is that some school level variables can help identify mathematics dropouts. For example, Lee and Bryk (1988) showed that students in Catholic schools take more mathematics courses than those in public schools, indicating that the type of school, one of the school composite variables, is predictive of mathematics participation. Indeed, some school level variables such as disciplinary climate and school average socioeconomic background may be predictive of mathematics participation.

5. Distinction among students who have the same enrollment pattern in mathematics.

This study does not distinguish a grade 12 student taking a 12th grade level mathematics course from a grade 12 student taking a 10th grade level mathematics course. They are all coded as participants in mathematics in grade 12. There are reasons, however, to believe that these two students exhibit quite different course-taking behaviors in mathematics although they all participate in mathematics in grade 12. The distinction is important because, in spite of the same enrollment pattern, it is still possible for students to have different mathematics preparation. That is, a 12th grader taking a 12th grade level mathematics course is much better prepared for college studies or for the labor market than a 12th grader taking a 10th grade level mathematics course.

6. Treatment of mathematics dropout as a repeated event.

This study considers mathematics dropout as a single event. That is, mathematics dropout refers to the grade in which a student stops taking mathematics courses altogether. According to this definition, as long as a student enrolls in a mathematics course in grade 12, he or she is considered persistent in mathematics during the entire secondary schooling. Statistical techniques, however, allow mathematics dropout to be modeled as a repeated event (see Allison, 1984). That is, a student is treated as a dropout everytime when he or she does not take a mathematics course in a particular grade. Therefore, a student may drop out of mathematics several times during his or her secondary school career. This treatment enables one to have more precise estimates of mathematics participation for each particular grade.

7. Examination of the relationship between school dropout and mathematics dropout.

Obviously, when a student drops out of school, he or she drops out of mathematics as well. What is not obtained at this moment is the relative measure of mathematics participation that rules out the extent to which students drop out of school. There is evidence now that 64% of students are persistent in taking mathematics courses from grade 7 to 12. If, for example, 70 % of students stay in school from grade 7 to 12, mathematics participation does not seem to be troublesome because mathematics dropout is merely a reflection of school dropout. On the other hand, if, for example, 90% of students stay in school from grade 7 to 12, mathematics dropout is, then, a problem in and of itself.

Note that a comparison between school dropout and mathematics dropout is a very crude method to understand the effect of school dropout on mathematics participation. Survival analysis provides more precise mathematical methods to model the relationship between school dropout and mathematics dropout. One avenue is to consider school dropout as a competitive risk to mathematics participation (see Chiang, 1984). In this manner, one can remove the effect of school dropout as a competitive risk, and obtain a "pure" measure of mathematics participation. This kind of investigation would, therefore, be able to expose further the problems of mathematics participation.

8. The use of multilevel survival models to examine the effects of school factors.

In the last 10 years, researchers have developed multilevel statistical modeling to simultaneously estimate the effects of individual- and school-level variables (see Bryk & Raudenbush, 1992). The idea underlying these models is that a separate regression is fitted for each school. These regression models yield a mean score for each school, with adjustment for students' background. They also produce measures of equality, such as the differential between males and females in their performance, or the relationship between achievement and social class. The models then use the individual school estimates (e.g., adjusted mean scores or measures of equality) as dependent measures in a model that attempts to explain variation among schools with measures of schooling processes (see Gamoran, 1991; Lee & Smith, 1993; Willms, 1992).

The idea of multilevel survival analysis is to first construct a simple survival model using survival modeling techniques, and then combine survival analysis and multilevel regression analysis by fitting the simplified survival model to the data for each school, and modeling the likelihood that a student will drop out of mathematics on a number of student-level characteristics. The statistical program for multilevel modeling is now capable of modeling dichotomous dependent variables. Goldstein (1996) describes multilevel survival modeling as one of the most important extensions of multilevel modeling.

Footnote

1. The two items used to measure mathematics anxiety were originally included as a subscale in the composite variable of attitude toward mathematics in the Longitudinal Study of American Youth (LSAY). In the current study, if the composite variables of attitude toward mathematics and mathematics anxiety were both used without modification, collineariility would occur due to the fact that the composite variable of mathematics anxiety is a part of the composite variable of attitude toward mathematics. One simple, workable strategy to resolve this problem is to delete the two items used to measure mathematics anxiety from the composite variable of attitude toward mathematics. Note that this procedure alters the metric of the composite variable of attitude toward mathematics from 0 - 16 to 0 - 12.

2. The survival function and the hazard function may look like they are "inverses" of each other. However, they are fundamentally different. The survival probability is the proportion of initial cohort of individuals surviving through each of several successive periods, whereas the hazard probability is the proportion of survivors from the last period experiencing the event during the current period. Therefore, given one function (either survival or hazard), it is impossible to compute the other function directly. These functions also serve different purposes. The survival function aims to describe the pattern of survival probabilities over the time of investigation, whereas the hazard function aims to identify the most risky periods during the time of investigation.

3. Statistical results in Table 3 are likely to be different from those obtained in separate logistic regression analyses. For example, statistical estimates concerning mathematics participation for grade 12 in the survival

analysis model are slightly different from those in the logistic regression analysis for grade 12 (see Table 2). The reason for this is that the survival model in Table 3 includes time-varying variables such as prior mathematics achievement and prior attitude toward mathematics. Estimates regarding mathematics participation in grade 12 are generated in the survival model with prior mathematics achievement and prior attitude toward mathematics being statistically controlled not only in grade 12 but also in grade 11. The logistic model for grade 12, on the other hand, only controls prior mathematics achievement and prior attitude toward mathematics in grade 12.

Therefore, the survival model adds a longitudinal perspective to statistical estimates with considerations of individual variation over time in time-varying variables such as prior mathematics achievement and prior attitude toward mathematics. In other words, even though the focal point is on one particular grade, the effect of prior mathematics achievement and prior attitude toward mathematics in the other grade is taken into account. In sum, the survival model provides more statistical controls because of its longitudinal nature, and, therefore, produces more accurate statistical estimates.

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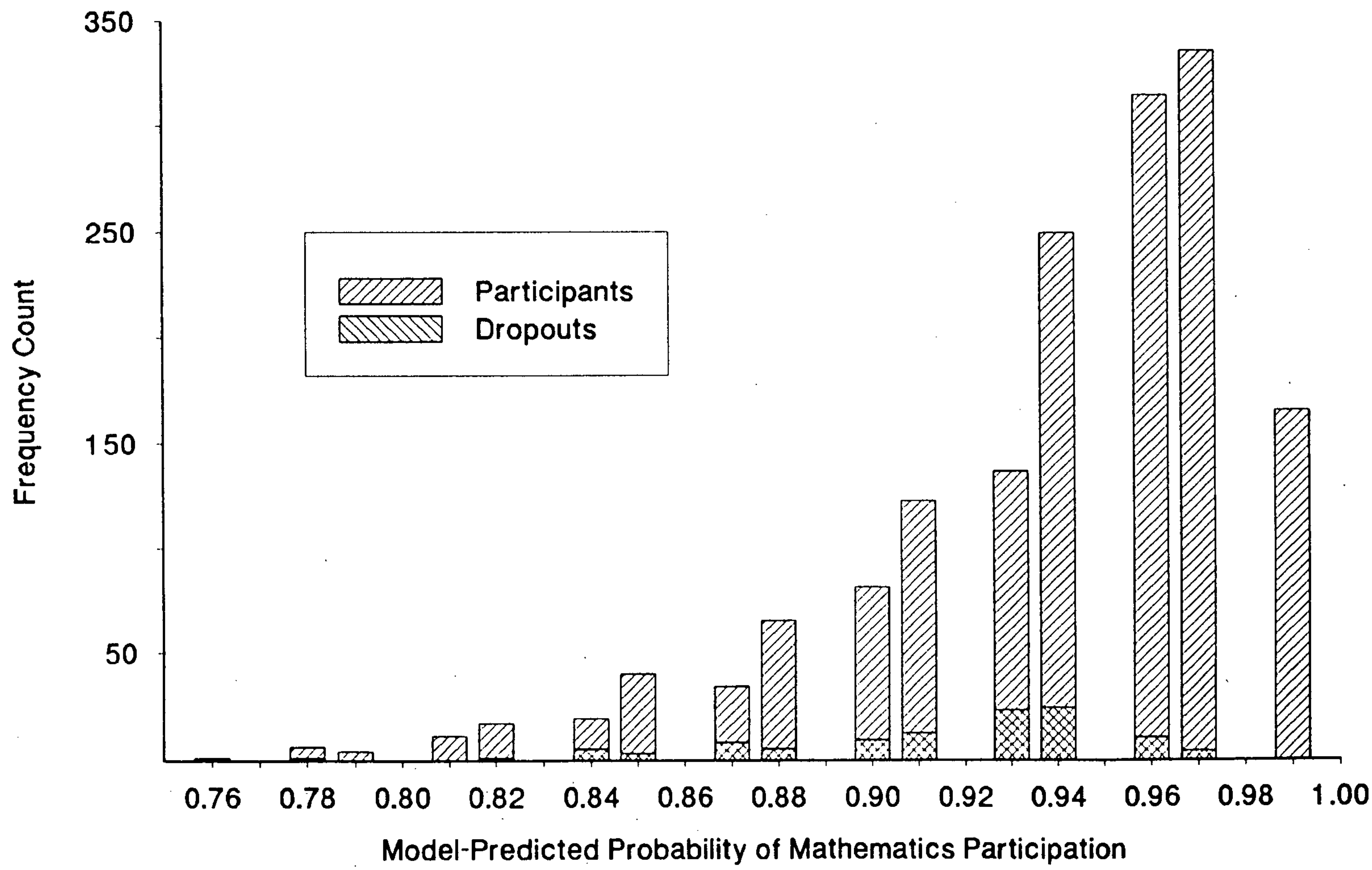
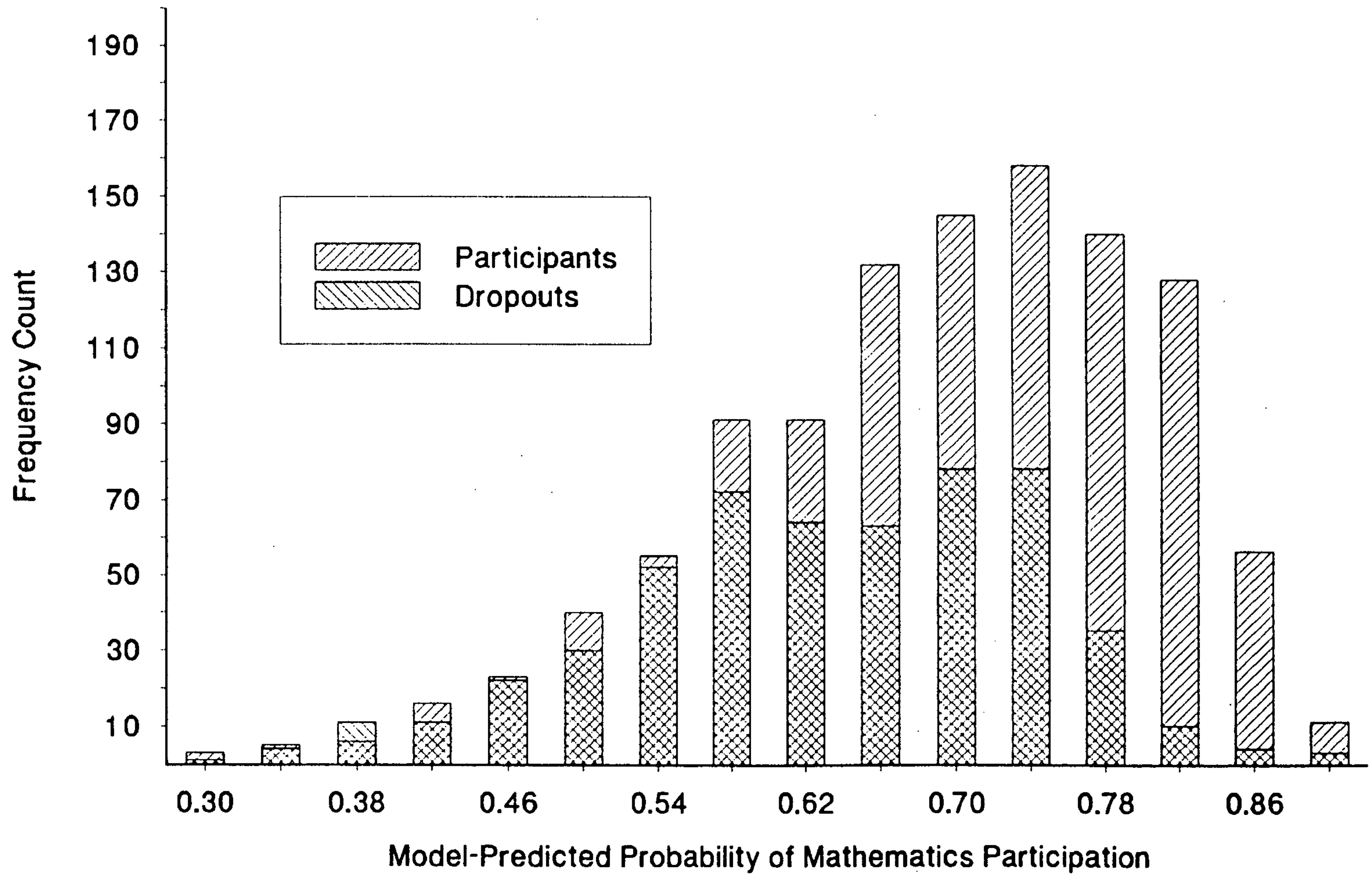
Appendix A - 1

Histogram of the Logistic Regression Model Predicting Mathematics Participation in Grade 11



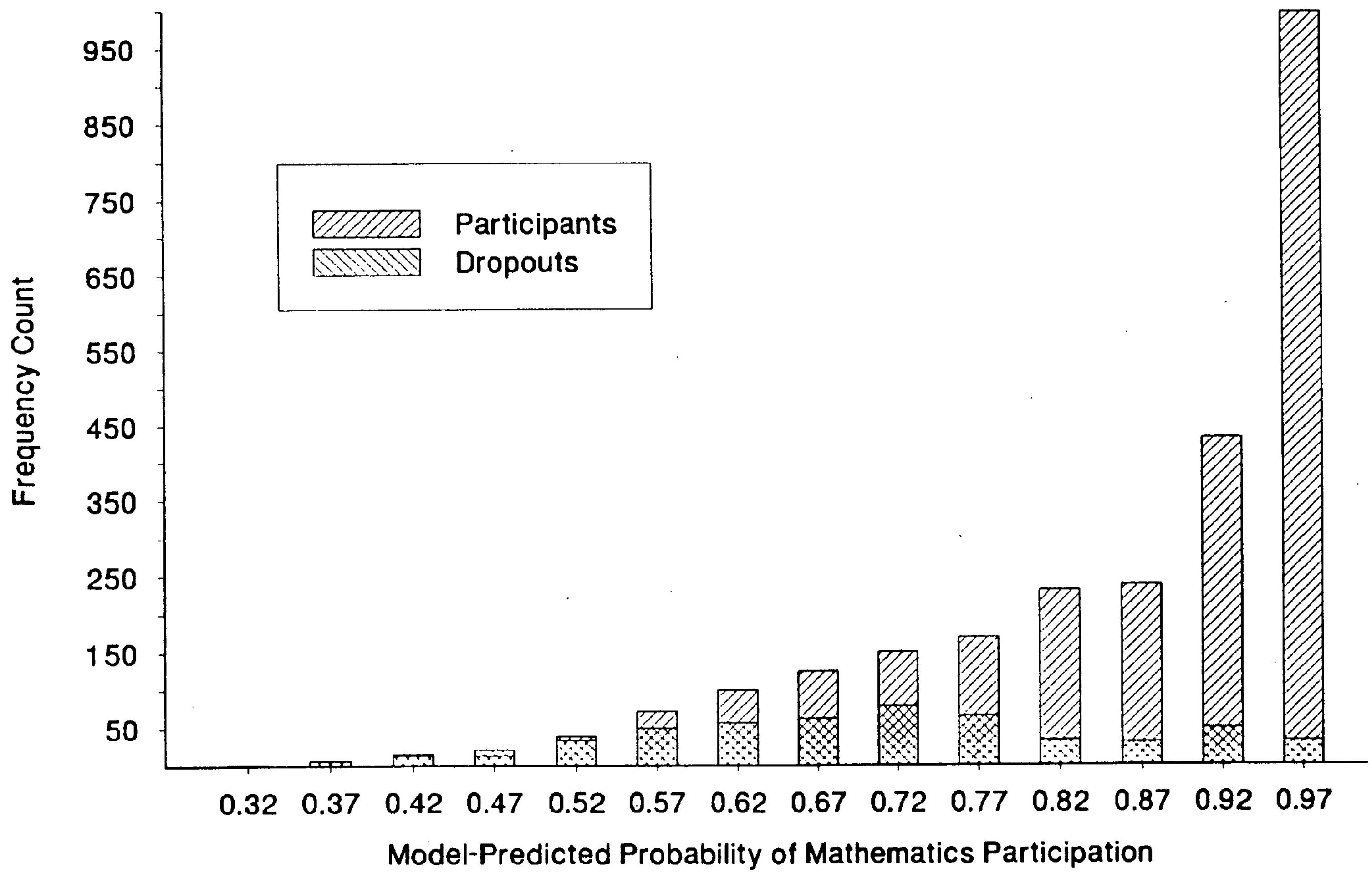
Appendix A - 2

Histogram of the Logistic Regression Model Predicting Mathematics Participation in Grade 12



Appendix A - 3

Histogram of the Survival Analysis Model Predicting Mathematics Participation



Appendix B

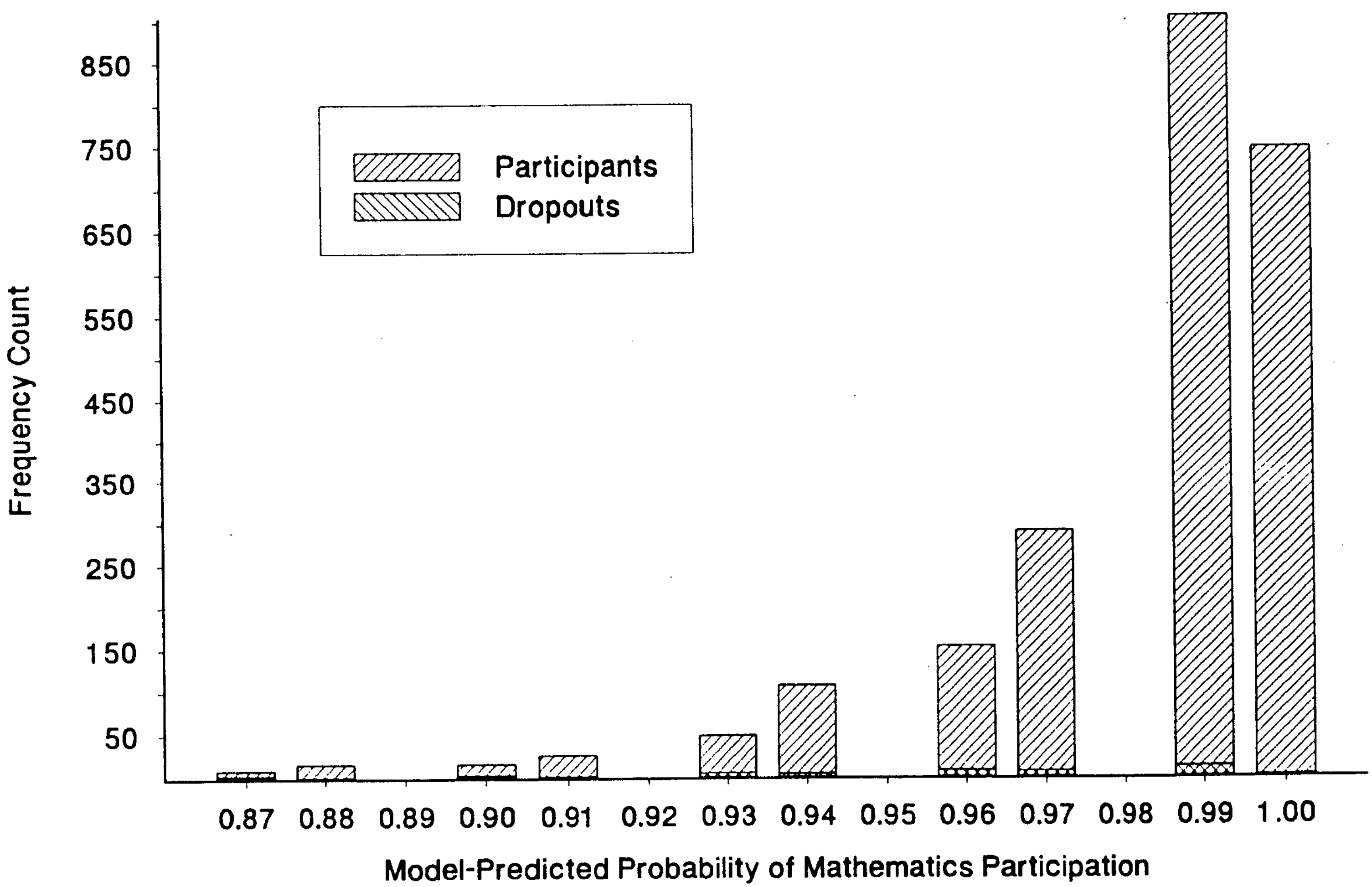
Model-Data-Fit Statistics on Logistic and Survival Models of Mathematics Participation

Fitting index	Logistic model: Grade 11	Logistic model: Grade 12	Survival model
Goodness-of-fit statistic			
Sum statistic	1528.50	3084.31	2980.88
Average statistic	0.84	2.01	0.94
Goodness-of-fit index			
-2LL of the initial model	788.80	2072.00	2874.95
-2LL of the full model	742.91	1974.69	2389.92
Model chi-square (improvement)	45.89***	97.31***	485.03***

Note. *** $p < 0.001$. -2LL denotes -2 log likelihood. The degree of freedom (in model chi-square) for the logistic model is 5. The degree of freedom (in model chi-square) for the survival model is 7.

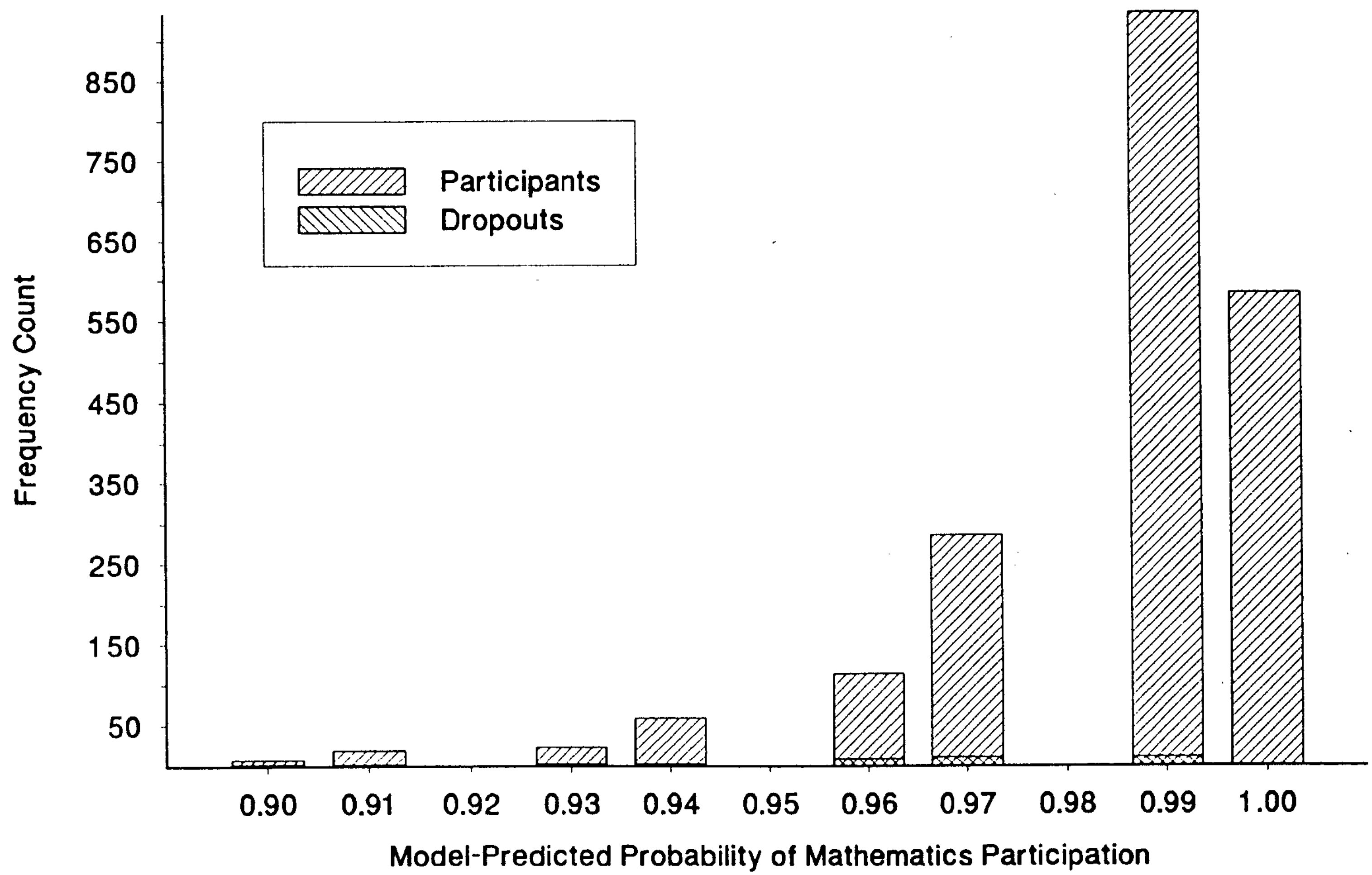
Appendix C - 1

Histogram of the Logistic Regression Model Predicting Participation in Advanced Mathematics in Grade 8



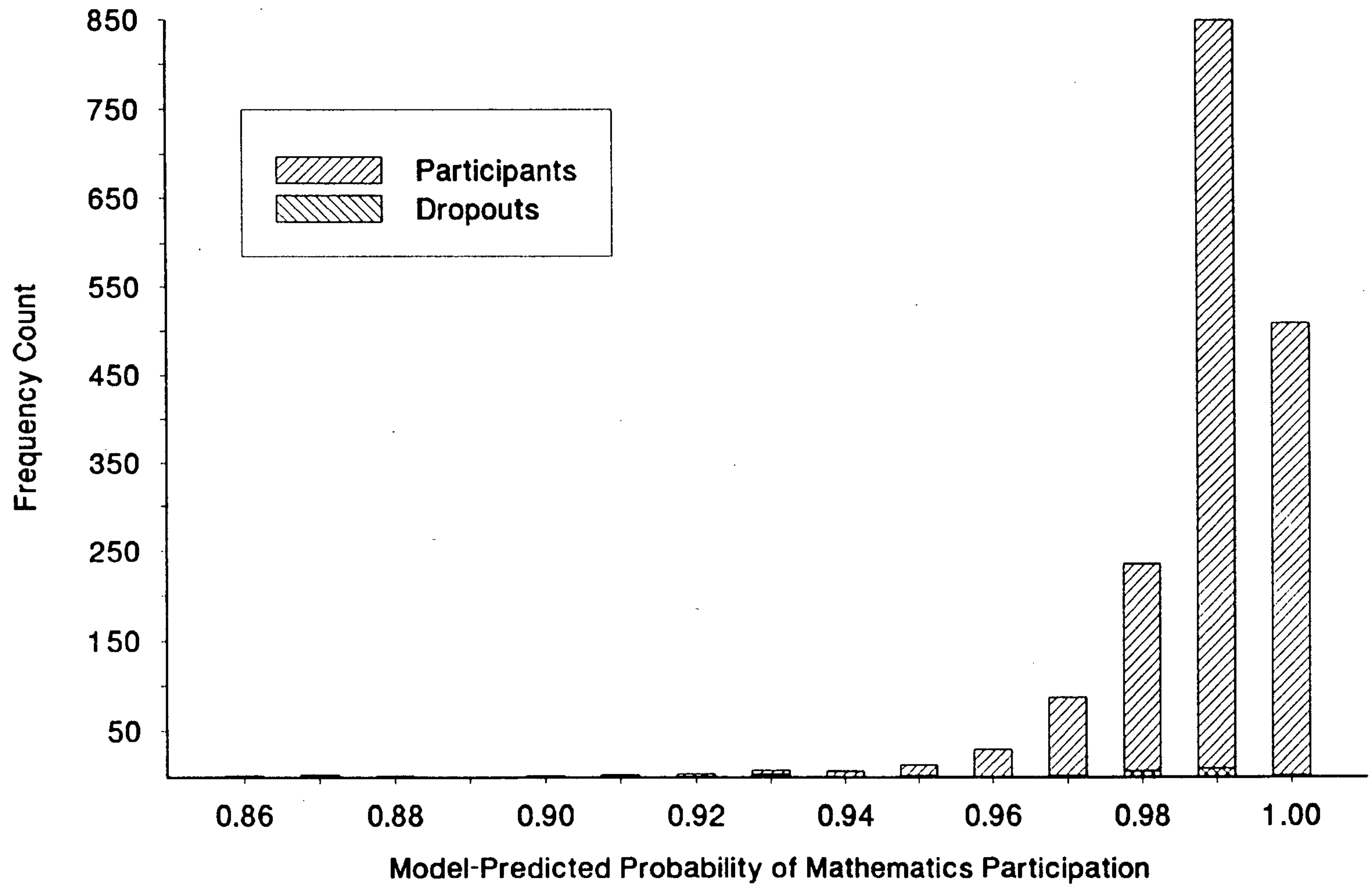
Appendix C - 2

Histogram of the Logistic Regression Model Predicting Participation in Advanced Mathematics in Grade 9



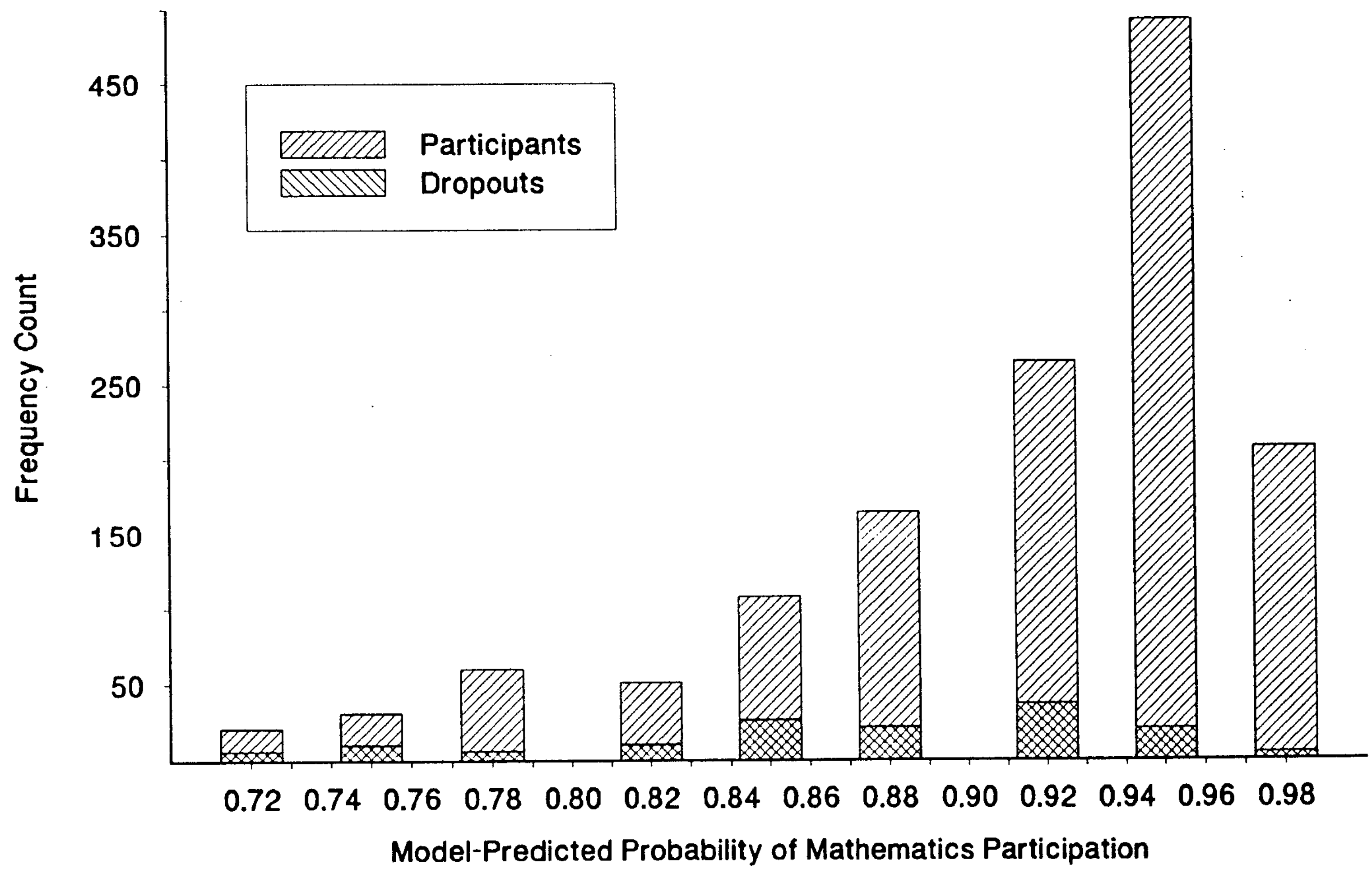
Appendix C - 3

Histogram of the Logistic Regression Model Predicting Participation in Advanced Mathematics in Grade 10



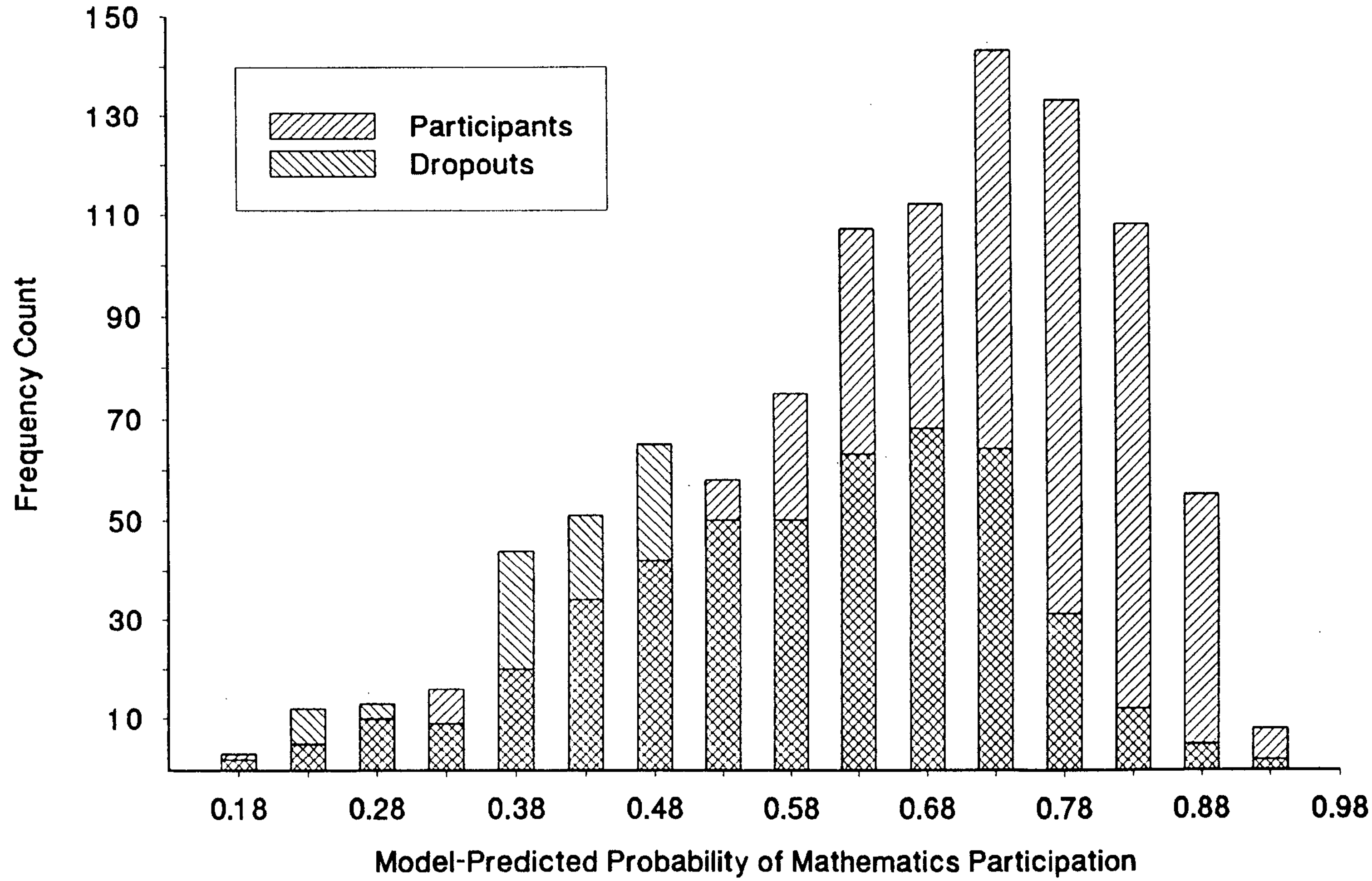
Appendix C - 4

Histogram of the Logistic Regression Model Predicting Participation in Advanced Mathematics in Grade 11



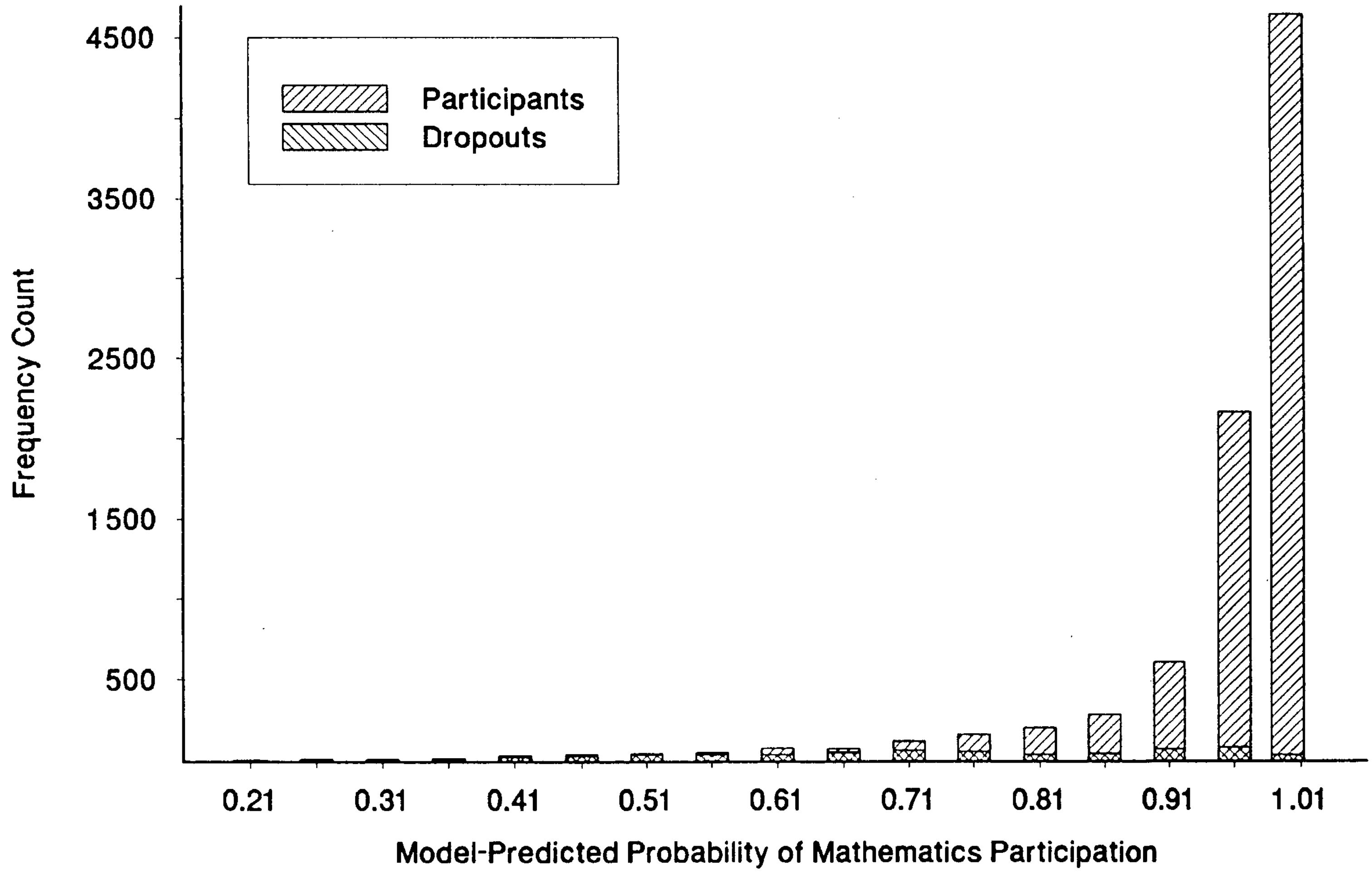
Appendix C - 5

Histogram of the Logistic Regression Model Predicting Participation in Advanced Mathematics in Grade 12



Appendix C - 6

Histogram of the Survival Analysis Model Predicting Participation in Advanced Mathematics



Appendix D

Model-Data-Fit Statistics on Logistic and Survival Models of Participation in Advanced Mathematics

	Logistic model					Survival
Fitting index	Grade 8	Grade 9	Grade 10	Grade 11	Grade 12	model
Goodness-of-fit statistic						
Sum statistic	1950.78	1714.04	1733.92	1485.63	1461.16	8390.30
Average statistic	0.81	0.82	0.94	0.89	1.06	0.91
Goodness-of-fit index						
-2LL of the initial model	445.80	343.87	220.93	989.05	1929.51	4924.21
-2LL of the full model	392.41	309.69	205.31	907.55	1775.43	3388.45
Model chi-square (improvement)	53.39***	34.18***	15.62**	81.50***	154.08***	1535.76***

Note. * $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$. -2LL denotes -2 log likelihood. The degree of freedom (in model chi-square) for the logistic model is 5. The degree of freedom (in model chi-square) for the survival model is 19.