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Date 96 Aug 29
ABSTRACT

This thesis focuses on a mother's perceptions of her younger daughter's mathematical activity and thought, and how this view affects other areas in the mother's life. I am the mother, and Jaclyn (Jackie) is my daughter. How, when, and what mathematizing occurred in our home-life interactions is explored through the use of ethnographic case study methods. The data collection started when Jaclyn was three-and-a-half years old and continued until she was five-and-a-half, during which time I was a full-time teacher of 8- to 10-year-olds. The analysis, interpretation, and author reflections began immediately and continued long after the anecdotes were recorded.

I provide explanatory glimpses of the mathematical relationships developed and explored by myself and my daughter as we created our ways of relating to each other and the world we both inhabit, particularly our home environment. I reflect upon these incidents to interpret them and to highlight the mathematical thinking and ways of mathematizing inherent in them, as well as to examine the mathematics that can develop out of them.

This work offers an in-depth look at what 'found' mathematics is like in one child's home. Because the whole context of learning is a factor in how and what people learn, I provide descriptions of the learning situation and the relationship between myself and my daughter. Jaclyn's developing awareness of a social and mathematical world is communicated through her words and actions. These are described and interpreted through my perceptions.

This study also looks at the multiple, and often invisible, interactions among the roles of mother, teacher, and researcher. I am all three, in different ways, at different times. Mediation strategies in the home usually reflect my sense of mothering, but on many occasions it is possible to see my sense of both teaching and researching informing the role of mother. On other occasions, the mother influences the thinking of the teacher/researcher. The reciprocity of attentive educational functioning in the home and in the school is worthy of notice. On many occasions, reflections about Jackie's words and/or actions elicited self-reflection on my teaching practice, and such reflection comprises one central element of the study.

(ii)
# TABLE OF CONTENTS

ABSTRACT.......................................................................................................................... ii

TABLE OF CONTENTS ........................................................................................................ iii

ACKNOWLEDGEMENT ........................................................................................................ vi

DEDICATION ........................................................................................................................ vii

CHAPTER 1: INTRODUCTION: LOOKING BACKWARDS THROUGH THE MIRROR .................... 1

- FROM PARTS TO A WHOLE ............................................................................................. 5

THIS STORY HAS A PURPOSE ......................................................................................... 8

WHY IS THIS IMPORTANT? .............................................................................................. 10

SOME CONTRIBUTIONS .................................................................................................... 11

ANECDOTAL RESEARCH COLLECTION ........................................................................ 12

ANECDOTES: TRANSFORMATIONS AND ANALYSIS .................................................. 15

BOUNDARIES OF THIS ACCOUNT ................................................................................ 16

CHAPTER 2: WHAT OTHERS HAVE TO SAY ................................................................. 19

CASE STUDIES AND QUALITATIVE RESEARCH ............................................................ 20

ASPECTS OF MATHEMATICS AND MATHEMATIZATION ........................................ 27

STUDIES OF YOUNG CHILDREN MATHEMATIZING: CASE STUDIES AND MORE ........ 31

CHAPTER 3: HOW I GATHERED MY DATA ..................................................................... 38

A DESCRIPTION OF MYSELF AS THE RESEARCH INSTRUMENT .............................. 38

WHAT KIND OF ANECDOTES WERE COLLECTED? ..................................................... 41

(iii)
Word play games ......................................................... 149
HOME VERSIONS OF COMMERCIAL GAMES ...................... 154
Games with cards .................................................... 155
Games with dice ....................................................... 158
COMMERCIAL GAMES ............................................. 168
STORIES AS GAME SOURCES ...................................... 172
IN SUMMARY: PLAYING THE CHAPTER AGAIN .................... 177
CHAPTER 7: "IT FEELS LIKE INFINITY" ............................. 180
A LOOK AT THE PARTS ................................................ 181
A LOOK AT THE WHOLE: CONNECTING THE OPPORTUNITIES ...... 191
A LOOK AT THE WHOLE: EXTENDING AND EXPANDING FROM THE OPPORTUNITIES ........................................... 193
IN SUMMARY: CONCLUSIONS WITHOUT CLOSURE ............ 196
CHAPTER 8: LESSONS I LEARNED FROM JACLYN ................ 199
REVIEWING THE ORGANIZING FRAMEWORK ...................... 200
REVISITING THE RATIONALE ....................................... 201
SUMMARY? CONCLUSIONS? IMPLICATIONS? ...................... 202
MATHEMATIZING AS IT FEATURES IN THIS THESIS .............. 214
AND SO IT CONTINUES ................................................ 216
BIBLIOGRAPHY .......................................................... 217

(v)
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To my younger daughter, Jaclyn, I give simple and profound thanks for her willing participation and active involvement in this study.
DEDICATION

I dedicate this thesis to Jaclyn Phillips. You gave me yourself. You were a happy, enthusiastic, and curious partner in this work. Through you, and often because of you, I saw Mathematics in forms and places that it had never existed for me before.
CHAPTER 1: INTRODUCTION:
LOOKING BACKWARDS THROUGH THE MIRROR

While I was still trying to make sense of what I was really attempting to do in this research study, I came across a book that caused me to come to a full stop. As a result of reading parts of it, I completely revised my thinking about my study.

Walkerdine and Lucey (1989), in their book *Democracy in the Kitchen* which provides a critical re-analysis of a corpus of mother-daughter (four-year-olds) nursery school and home interactions first collected and analysed by Tizard and Hughes (1984), describe their personal response as researchers to some of their data in blatantly emotional terms. Through their eyes, I felt the strong orienting presence of cultural context, point of view, and the frequently unquestioning acceptance of what is familiar and normal in interpretative research.

We systematically read the working-class transcripts. [...] A feeling of confident recognition came over us. We knew where we were. We then set about reading the transcripts of the middle-class mothers and daughters. What we felt is almost unspeakable. Helen felt contempt and Valerie hate. And both with a deep intensity. All the class hatred of our childhood, mixed with anger and disdain, hurt and humiliation, envy and longing, wailed out in those moments. We felt such things about perfectly ordinary afternoons in perfectly ordinary middle-class homes, afternoons that women with middle-class
backgrounds would doubtless identify in the same unproblematic way that we recognised the working-class ones, with a kind of 'yes, that's how it is'. But something inside us smouldered, a pain that we would rather not remember and could not easily articulate. (pp. 8-9)

Reading further into this book, I was constantly reminded of the range of possible interpretations for the same incident. This caused me to reflect deeply upon my own research. I had been noticing and recording activities and interactions that occurred between my daughter and myself which I sometimes actively mediated as 'mathematical', and which I sometimes simply observed.

At this stage, I had been watching my daughter and recording signs of a developing sense of mathematics for two years. Jaclyn was then five years old, and only a few weeks previously, I had realized that soon, with kindergarten approaching, I would close my data collecting, for thesis purposes, and start more formally the processes of analyzing, defining, interpreting and communicating my discoveries about her mathematical world. These thoughts, framed as a poem given below, were what I wrote at that point. The words express the turmoil that I was feeling as I started to make sense of my work. I originally feared that this research would be too easy, too superficial, too lacking in rigour, and suddenly I felt overwhelmed by it all. How would I ever find a way to have it all come together?
I am beginning to see how very naive I have been.

How can I claim to know her world?
I am beginning to feel very frightened and insecure.

What am I really learning about?
I am beginning to think I have set myself an impossible task.

First, know thyself.

I am just beginning ...

... when I thought I was near the end.

I have only now realized that I have been looking at myself, as one of the shapers of my daughter’s mathematical world, and not looking directly at her mathematical world.

Who said that all long journeys begin with a single step? I began this journey, somewhat blindly, many steps ago: looking for mathematical elements in my young daughter’s world; looking, as I explained in an earlier paper, for “natural math, in a natural relationship, in a natural setting” (Phillips, 1992). I produced a whole file full of examples; wonderful anecdotes that I thought would allow me to demonstrate the development of mathematical power, of mathematical communication, of mathematical reasoning, and of mathematical knowledge (N.C.T.M., 1989) in the world of Jaclyn from ages three-and-a-half to five-and-a-half.

I wanted to see how much mathematics, and of what variety, she knew. I wanted to see how she had learned about this mathematics. I was sure that I would recognize Mathematics. I told myself it was simply a matter of being attuned to what I was looking for. For now, I can say that this is the least of what I found.
This thesis began with an assignment that I was given in my very first graduate level mathematics education course. I was to prepare and present a case study of a young child’s learning of mathematics. I didn’t know what constructivism was, nor had I heard of sociocultural theories of learning. I remembered Piaget from long years ago, in undergraduate psychology courses, and was amazed at the range of his influence still today. I recalled hearing about Vygotsky in a linguistics course that I have taken. It seemed a lifetime ago. I was only beginning to learn the vocabulary that mathematics education uses to define early mathematical experiences.

Although I had been an elementary teacher for twenty years, I had not thought about the mathematics that might occur prior to school. When my first daughter, Robin, was of pre-school age, I had concentrated on developing her love of literature. After all, some research studies showed a high correlation between being read to, being surrounded by books, being encouraged to talk about ideas for stories and being encouraged to make up stories, with success in school. My background was in English and psychology. I believed that I could help create a setting that would result in school success for my, now older, daughter. Did I do anything to encourage her mathematical development? Probably – but not consciously.

However, with the case study assignment, I decided to see if I could find examples of mathematics in the world of my younger daughter, Jaclyn.
The initial purpose of this research study was to explore my young daughter, Jaclyn’s, mathematical world. The research, however, has grown beyond this conception. It has also become a story of how I see and recognize mathematics, and of how reflections in one area extend into, and influence, other areas. I am noticing how my various roles fit together to support and extend each other, as well as on occasion to conflict or contradict. I have become more than the sum of all my parts. What started as a rather schizophrenic look at myself (mother/teacher) has become a more blended, less boundaried look at my different roles. I am (at the least) a mother, teacher, and researcher; all rolled into one. (Since drafting this, I have become aware of Ainley’s (1994, 1996) attempts to explore a similar set of tensions.)

When I saw myself as a mother/teacher, but not yet a researcher, I failed to recognize the influence that I might have in mathematics for my daughter. I failed to identify myself as one of those primary ‘mathematical elements’ at work in her world. I am sure that she was mathematizing long before I was aware of it. Indeed, from the very moment of conception, she has been dividing and multiplying at the cellular level.

When she was one year old, I remember her sorting blocks by colour and size before seeing how high she could stack them. I do not recall attending to the mathematics I now see inherent in this activity, but I do recall a friend who was watching her saying, “Oh, my little guy
wouldn’t have the co-ordination to do that. Little girls are so much more persistent.” Was she being persistent or was she simply good at balancing?

Why do I recall this incident? I believe in part it is because it was commented upon. My awareness of the incident was mediated. I see ‘drawing attention to’ as one power in research such as this. The ‘normal’ becomes marked and is therefore made memorable, and it is hence rendered available for further study and reflective consideration. (See Mason (1996, pp. 20-22) for an exposition of his view of ‘disciplined noticing’ with its sequence of notice – mark – record and a discussion of its role in mathematics education research.)

With the case study assignment that I referred to, a second potential schism appeared: I became a mother/teacher/researcher. I started to see the mathematical possibilities in the world that I was both presenting to, and sharing with, Jaclyn – and in the world she was presenting to and sharing with me. Slowly, I began to identify mathematical elements and foci in some of our routine activities. I noticed that I ‘naturally’ drew her attention to number relationships, categories, size and weight comparisons, and patterns. To a certain extent, this case study assignment comprised my ‘pilot study’.

I identified, to my own satisfaction, that the world that Jaclyn and I shared was rich with possibilities to attend to informal mathematics. In the context of our ‘regular’ life, mixed in with all of our ‘regular’ activities, I was able to identify Mathematics. Upitis (1990) asserts,
not least through the title of her book *This Too Is Music*, a broader, more inclusive, sense of ‘the musical’. Where once I recognized only possibilities for literature, I was now able additionally to locate mathematics and say, “This too is math”. Where once I saw games and art, I was also able to say, “This too is math”. Where once I was aware of bath and bedtime routines, I now saw that, “This too is math”.

At this stage of my account-giving, I realize that I can no longer say, “When I was a mother ..., teacher ..., researcher ...,” with such clear delineation. I am who I am, and when I act it is as me. However, on reflection, I can draw out some of the lines that separate these roles in me and can see how context might have affected the response I made. This is a turning point. From feeling that I reacted in my appropriate role to situations, I now believe that I act as myself, perhaps stressing one role more than another, according to the situation. A subtle difference, perhaps, but an important one.

This research has attuned me to the possibilities of mathematical learning available prior to school. It has also alerted me to the wealth of mathematical knowledge that parents and guardians can have about their child if they have been aware participants in their child’s mathematical knowledge acquisition. It also leads me to conjecture that even if such adults were unaware of the mathematics being explored, they could still be effective participants in activities that upon later questioning elicited mathematical knowledge – knowledge that at the time was hidden below the surface of conscious awareness.
For example, I was watching a friend's two year old son playing with a tape measure, walking around the kitchen, seemingly measuring things and occasionally saying a number and commenting on whether what he was measuring was “bigger”. My friend noticed my attention to this and said, “He loves playing with his dad's tools. He always has to have a hammer or tape measure or screwdriver around.” I commented that it was interesting that he was comparing measures; that he knew numbers were somehow needed and that size was featured. My friend said that she had never noticed that before, she had only noticed that he seemed to know how to use the tools. The elements of the mathematical practice of measuring was there, but her awareness of it was not. I believe that such awareness can also be elicited after the fact, by questioning, and can serve as a discussion point when parents are interviewed about their child's experiences in mathematics.

**THIS STORY HAS A PURPOSE**

This research, as it is reported here, is about how I see, and perceive, my young daughter's mathematical world. It is also about the way that this perceiving has affected my mathematical world. I explore my participation, describe my awarenesses, and explain the mathematics that I see evolving. I reflect on the ways that being a mother and a teacher and a researcher have informed each other, and I include observations concerning Jackie's presence and role in these situations.
The research questions addressed are the ones that developed as the research was happening. I originally thought that I would be exploring whether mathematics developed prior to formal schooling; and if so, which particular areas of mathematics developed between a mother and pre-school daughter in an informal home setting. Although these questions are addressed, they are no longer central to my main theme.

Rather, my focus has shifted towards how I participated in the shaping of my daughter's mathematical world and how she participated in the shaping of mine. To do this, I look at what was mediated, and report how it looked, sounded, and felt. I conjecture that my young daughter has been developing a mathematical world, and her words and actions, which I report, provide evidence for this claim.

Through reflection, I have found that all three of my major roles (mother/teacher/researcher) are always present, but one may frequently be more in evidence in a situation than another. Also, through reflection on the opportunities for mathematizing, I learned more about my own learning. As Mason (1996) has observed:

The overt product of research is usually some supported assertion(s). A covert product of research is a transformation in the perspective and thinking of the researcher. Undoubtedly, one of the most significant effects of any piece of research in education is the change that takes place in the researcher. (p. 58)
Although the questions that inform my inquiry overlap one another, it is possible to state three areas of searching.

- How might mathematizing with a pre-schooler look in a home environment?
- How might reflecting on such identified mathematized events affect my teaching and my thinking about my teaching in a school setting with elementary-age children?
- How do the roles of mother/teacher/researcher interact and inform each other?

These questions are not weighted equally in this account. The first two questions frame the study, whereas the third is more in the background – present at all times, but not often directly commented upon.

**WHY IS THIS IMPORTANT?**

Education in British Columbia (B.C.) is moving towards a more fluent transition between home and school learning. Informal learning as it occurs prior to entry into kindergarten is being looked at as a model for elementary education. There is a trend towards attempting to teach in a more integrated, child-centred way. The above statements are supported by *The Primary Document* (B.C., 1990) once mandated by the B.C. Ministry of Education. However, without clear documentation of what it means to learn informally at home, this transition is basically left to the imagination of educators. Others too
are looking at this area (Tizard and Hughes, 1984; Young Loveridge, 1991; Anderson, 1991).

The concept of mediation is also addressed in this specific context. How is mathematics mediated in this home, between this mother/teacher and this daughter? In the relationship being examined, it seemed that mediation was often reciprocal. Most often, mediation involved spoken words, occasionally written symbols, and sometimes simply attentive silence. It is interesting, I believe, to see how little or how much mediation occurred in different contexts.

If we, as educators, are to teach children where they are, according to need rather than simply following the advice that I once had given to me, assume they know nothing, then I believe teachers need a clearer picture of the types of learning and range of experiences that children may possess upon entering school. Teachers can benefit from exposure to accounts of experiences that children might have had, and an opportunity to reflect on possible similarities and differences between home and school learning situations.

**SOME CONTRIBUTIONS**

Some of the specific contributions that this research account makes are of a personal nature. First, it confirmed, for me, that it is possible to see a large and varied mathematical world developing prior to formal schooling. It helped to increase my perception of the role that mediation plays in shaping a mathematical orientation and attitude, and it increased my awareness about the home-school mathematical continuum that I want for my daughter. In carrying out this
research, I became increasingly aware of the mathematical potential in my home environment. I also became increasingly aware of how Jaclyn uses mathematics in her life, including her emotional life.

This study adds to the knowledge base provided by case studies in the field of early mathematics. Through it, others have access to the mathematical context, activities, and discourse shared by one mother and her child. Others again can view this as an example of the possibilities for mathematical acculturation between a mother and her child in their middle-class Canadian home. Yet others still might use it to explore and explain aspects of various theories of mathematical learning.

ANECDOOTAL RESEARCH COLLECTION

The research for this account follows qualitative, case work guidelines, and is informed by an ethnographic, participant-observer perspective. (For an extensive account of this perspective in mathematics education, see Eisenhart, 1988.) Incidents that I saw as involving mathematics were written down as soon as possible after they occurred. Pens and pads of paper were kept throughout my house to aid this. I tried to pay particular attention to words and hand actions. I tried to note aspects of the complete context of the events. By ‘context’, I mean not only where and when incidents occurred, but also the feelings and atmosphere surrounding them.

Some artifacts relating to the events were kept. I wish I had kept more. At first, I simply did not realize the richness of the scribblings that my daughter and I had produced, or the value of keeping her
bits and pieces of cut paper. In this type of research, an opportunity missed is an opportunity gone for ever. Although opportunities for similar mathematizing recur, the exact situation never repeats itself: neither a play back nor another exact trial was possible.

I usually perceived the mathematical in a situation after the event had started, but before it was completely over. However, there were some events that I myself mathematized fully only during my reflections after the incident. The closest that I can get to the data is recollection of the experience or mental replay of the events, and my reflections on the incident. There was no way that, during the bulk of the data collection, I could have audio or video taping available. I did not know when or where an incident might occur. I was the recording device and was entirely responsible for pressing the record button in my mind (and soon thereafter on paper). In this non-trivial sense, it was I who determined what was perceived as mathematical in these reported anecdotes.

On various occasions I tested my accuracy. Sometimes a friend or Jackie's dad were around when an incident occurred. I would write the core dialogue as accurately as I could, and record the contextual information and then play/say it back to the other observer. In each case, they agreed that my representation was accurate. Sometimes, when I really wanted the exact words and thought I might have lost them, I would ask Jackie what she had said. And sometimes she asked me on her own account to read back what I had written.
van den Brink (1982) refers to this use of subject calibration and confirmation of real-time data collection with young children as “mutual observation”, and proposes it as “a refinement of ‘soft’ methods of research, particularly the clinical interview” (p. 29). He adds, “By making children aware of one’s method, one creates a relaxed atmosphere and has them take an active part in one’s work” (p. 29).

This type of checking usually occurred in context, for instance, when Jaclyn said something that contradicted what I thought was an earlier point. For example, I once asked, “But didn’t you just say the ball was higher?”; she then replied, “No, I said the flower was higher unless the ball was bouncing”. In this incident, the contradiction that I heard might have provided mediation for something that she thought she had said, but actually had not, or it might have indeed been my misunderstanding. Whichever it was, the point I am making here is that the data is as accurate as this method of gathering allowed it to be.

I also tested myself a few times by setting up a recording session and first writing down my ‘in-my-head’ playback, and then replaying the actual tape. All these checks convinced me that I was an accurate recording device, provided the session was not too long. As will become evident, most situations that occurred were fairly brief and concise. The relatively few longer sessions (particularly in Chapter 6 on games) were often recorded as my personal recollection, sometimes with key phrases, but generally without the exact dialogue of our conversation.
ANECDOTES: TRANSFORMATIONS AND ANALYSIS

What I was faced with at the end of my anecdote collection time was a file full of loose bits of paper that I needed to type into my computer. Because of the nature of this research, I was aware of some of the trends that I had noted when I was experiencing the events themselves. Discovery analysis, to a certain extent, occurred throughout the recording of events, and the readings that I was engaged in over the time of the research. What I did next was use an analytical technique that Glaser and Strauss call “constant comparison” (cited in McMillan and Schumacher, 1993, p. 487) to aid noticing pattern trends or themes. Themes appropriate to my emerging research questions were noticed and extracted for reporting. The reporting takes the format of a narrative that presents actual quotations and contexts in conjunction with my reflections and interpretations.

I believe strongly in the educational value of anecdotes. Many times, the story that is used to illustrate a point in a speech or a paper, is what I take away with me for future use and reflection. It is often the story that connects what is being presented to my lived life, and it is often from reflecting on the story that I develop my significant meaning.
BOUNDARIES OF THIS ACCOUNT

My conjectured boundaries for this piece of research are closely tied to its strengths. I set the stage by presenting imagined (almost catechismic) questions and my responses. I present these boundaries here in part as I feel they delineate further research areas.

The mother is involved in the data collection. How can she be really objective? Won't she see things through her mother lenses and not her researcher lenses?

Of course. This is a study of what a mother who is also a teacher and also a researcher sees as mathematical in her ‘natural’ relations with her daughter. It is about the shaping of mathematics in the context of this home. This type of work requires informed parental involvement. Who else could do this work? This study is not about creating a child mathematical genius, nor is it about the ‘good’ or ‘bad’ mother. It is about personal reporting of personal events that are recorded and interpreted through reflection.

As Stake (1994) says:

It is the researcher who decides what is the case’s own story, or at least what of the case’s own story he or she will report. More will be pursued than was volunteered. Less will be reported than was learned. (p. 240)

Being a researcher is my newest role. In it, I combine all of the ‘me’ that was in existence prior to this addition. But, because of the new
role, I may choose to stress and ignore things differently from the way that I might have before.

Since this study is about one particular relationship, what meaning will it have in general? How can these specific findings be of any use? How can they be replicated?

For my defence of this boundary, I refer to a quotation of Stephen Brown's that Pimm (1987) cites:

One incident with one child, seen in all its richness, frequently has more to convey to us than a thousand replications of an experiment conducted with hundreds of children. Our preoccupation with replicability and generalizability frequently dulls our senses to what we may see in the unique unanticipated event that has never occurred before and may never happen again. (p. 194)

Won't readers be inclined to interpret these incidents in their own way? How will you know that you are being understood?

I will attempt to report in enough detail to allow the reader to feel (as if they were) present during the incidents. In my data collecting, redundancy and multiple-interaction techniques have been built in. This should reduce the danger of misinterpretation, without denying the possibility of re-interpretation.

I also want to acknowledge that I see reader interpretation as beyond my scope. As Stake (1994) adds:
The reader too will add and subtract, invent and shape - reconstructing the knowledge in ways that leave it differently connected and more personally useful. (p. 241)

Indeed, this is one of the powers of narrative accounts.

As I have mentioned, this is not an attempt to present Jaclyn as an exemplary mathematical child. It is not, for me, a developmental study. Nevertheless, I have included Jackie's age with each anecdote, in an attempt not to distract those who find they wish to know this information.

Before I turn to describing my data and how I collected it, I will present in the next chapter some of the literature that I found useful as background and framing reading for my study.
CHAPTER 2: WHAT OTHERS HAVE TO SAY

In teaching, there is a belief that reading is more effective if work has been done to prepare the reader for what is to come. However, it is also possible to overwhelm by too much detail, drowning readers in a sea of references and thereby leaving them unsure as to which are felt to be of greater or lesser significance. There are a number of strategies that can be used to contend with this difficulty. The one I have chosen here is to divide my literature review into two parts.

I will briefly present in this chapter the literature that I drew on in exploring and researching the topic as a whole, and which helped to frame the general account. Then, within the body of the detailed findings (Chapters 4-7), I make reference to pieces of writing which relate or refer to specific phenomena similar to those which I report. Because of this decision, a considerable portion of my literature review is distributed throughout the text of the narrative account, at appropriate places, as the story unfolds.

I look here at three main types of literature.

First, I briefly discuss material on qualitative methods of case study, ethnography, and narrative reporting. I have looked at guidelines and hints about how to conduct, analyze and interpret the data that I have collected in order to produce a well-supported study. Besides gaining practical information from this search, I have also found strong statements that support what I am doing.
Second, I look at accounts which discuss the process of mathematization and how children develop mathematical knowledge.

Third, I look at case studies of young children in mathematics education. I have also sought case studies in other fields which have a similar framing context to this study. In this search, I have also pursued some other published accounts that, although not specifically case studies, inform this study and help to provide an increased sense of context because they look at young children in a mathematical setting.

CASE STUDIES AND QUALITATIVE RESEARCH

Eisenhart (1988), in her seminal paper ‘The ethnographic research tradition and mathematics education research’, writes of similarities and differences between these two research arenas. She contrasts the assumptions and sort of questions that many mathematics education researchers ask with those posed by educational anthropologists (whom she typifies as, “those who conduct research in the traditions of cultural anthropology”), observing that the latter, “have been trained to assume that human behavior and human learning are responsive to a context that is interpreted by participants and that is dominated by social relationships” (p. 101).

Underlying the techniques and approaches of cultural anthropology is a philosophical position termed ‘interpretivism’, and she claims, “from this perspective meanings and actions, context and situation are inextricably linked and make no sense in isolation from one another”, adding, “Because intersubjective meanings are implicit, the
ways in which beliefs and actions make sense may only be accessible to insiders” (p. 103).

Among a number of interpretivist research goals, she identifies the following:

the researcher must learn how to behave appropriately in that world and how to make that world understandable to outsiders, especially in the research community. Thus, the researcher must be involved in the activity as an insider and able to reflect on it as an outsider. (p. 103)

Eisenhart’s account captures precisely both the framing and tensions of my study. In it, I am a participant observer, both involved in and detached from the activity I am engaged in studying. By choosing to study a context in which I am an intimate insider (indeed possibly one of the most intimate settings imaginable), I am confident I have good access to insider meanings, encountered in a setting of security, honesty and empathy.

But my greater challenge is to reflect on them as sufficient of an outsider in order to gain a perspective separate from that of a ‘mere’ participant, and also to communicate successfully with an outside research community. Eisenhart goes on to claim, “an adult researcher trying to understand the mathematical concepts of young children will find full participation in the children’s world of mathematics difficult” (p. 105). I hope through this study to show how I strove to overcome such difficulties.
Eisenhart concludes by drawing attention to an issue which is of concern to me:

the knowledge about schools in general and mathematics in particular that students learn and use at home and in peer groups is rarely considered relevant to mathematics education researchers even though these processes have been shown to be very important in structuring opportunities for students to learn in school. (p. 111)

This last observation links to a further issue about my study in relation to home-knowledge and mother-knowledge. Learning that takes place prior to school, in the home, tends not to be valued by teachers (see, for example, Tizard and Hughes, 1984; Merttens, 1994; Morgan and Merttens, 1994). Tizard and Hughes conclude their study by commenting that:

What concerns us, however, is the assumption that professionals know how parents should interact with, and educate, their children. This is also the assumption underlying most parent education courses, [...] There is no real evidence that parents need to interact with children in any particular way; often the advice offered seems to be based on ideas about what a 'good' middle class parent does. Even more worrying is advice which seems intended to make parents behave like teachers, for example, by suggesting that they ask their children 'open-ended' and 'stimulating' questions, and teach them colour, size and shape names.
Our study suggests that the exchange of views and questions, equally balanced between adult and child, that makes up conversation at home is better attuned to young children’s needs than the question-and-answer technique of school. It also suggests that no particular home context or activity is especially ‘educational’. It is the concern of the parent for the child to understand, and the child’s own curiosity and persistence, which promote learning. (pp. 266-267)

Mother-knowledge in particular tends to be under-valued and accorded an inferior position when compared with other more ‘authoritative’ forms of knowing. This latter claim is developed in Tyler’s (1994) doctoral dissertation, in which she claims that, “it is not understood in the schools that mother/teachers gain knowledge from their mothering experience” (p. 295). Interestingly, although herself a mother/teacher, Tyler chose to exclude her experiences and ideas from her study. While not in the same vein as Tyler’s work, my case study also offers, among other things, an account of one aspect of my ‘maternal’ knowledge and what it might offer teachers in elementary schools.

Case study research can be viewed as ‘soft’ research. However, many well-known and respected researchers value and indeed highlight the pliability and involvement which this softness allows. Yin (1989) defines a case study as:
an empirical study that investigates a contemporary phenomenon within its real-life context; when the boundaries between phenomenon and context are not clearly evident; and in which multiple sources of evidence are used. (p. 23)

I needed to identify seeing mathematical learning as a phenomenon in order to fit this specification. Because I found that this was possible, then the rest of Yin's definition was also applicable to my research work. Many types of activities were occurring in my relationship with my older daughter Robin and with Jaclyn prior to the start of this work, but I was not focused on mathematics and mathematical ways of thinking and perceiving, so in an important sense I did not 'see' them. The act of looking for them made me more aware of opportunities to interact with Jaclyn in a mathematical way. In this way, I believe that the very act of projectively 'seeing' mathematical possibilities in a situation can be viewed as a phenomenon worthy of study.

Yin warns of "the demands of a case study on a person's intellect, ego, and emotions" (p. 62), and adds:

> Engagement, enticement and seduction - these are unusual characteristics of case studies. To produce such a case study requires an investigator to be enthusiastic about the investigation and to want to communicate the results widely.

(p. 151)

As I write this, I can only say that I feel this is an understatement. This research is among the most difficult that I have done. In
particular, deciding what to leave unsaid has at times ripped at my soul. It all seemed so important and intricately connected. Establishing the boundaries of the study as I developed the themes proved much harder for me than some of my earlier quantitative research where I easily and unproblematically established these parameters prior to conducting the study. Also, from what I have read, I find that ‘engagement, enticement, and seduction’ are usual characteristics of case studies.

Stake (1994) argues for case studies in the following manner:

> With broader purview than that of crafters of experiments and testers of hypotheses, qualitative case researchers orient to complexities connecting ordinary practice in natural habitats to the abstractions and concerns of diverse academic disciplines. (p. 239)

Other forms of research are not being devalued here. My point is that case studies allow a different type of looking, enabling the researcher to look at the ‘ordinary’, the everyday, through academic lenses. As I quoted in the previous chapter, Stake also acknowledges the difficulty in reporting personal meaning. Other interpretations are to be expected in case study work, but the possibility of misinterpretation can be minimised by clear descriptions. In order to be clear, conciseness sometimes needs to be sacrificed.

Bissex (1987) carried out a five-year study of her own son from ages 5 to 10, concerned with his development as a writer and reader. She cites Rawlings as saying, “A man [sic] may learn a deal of the general
from studying the specific, whereas it is impossible to know the specific by studying the general” (p. 11). Bissex extends this thought when she states that, “The process of observing even a single individual sensitizes us that much more to other individuals. [...] In other words, the process of seeing in a certain way is generalizable” (p. 14).

This is a very powerful claim and provided great encouragement for me.

As Eisner (1985) proposes, we must let ourselves become people who look and see. His idea of ‘connoisseurship’ is very applicable here. First, I had to become aware; second, I had to look, experience and record; third, I had to organize these experiences in a manner that would allow me to see/understand what was occurring. Fourth, as I reflected on the anecdotes, I grew aware of ways that their force and influence extended into other areas. My final step is writing my thoughts for others who might also like to learn more about how to see in my ‘certain way’, as well as vicariously to see what I saw.

van den Brink (1990) also draws attention to observation, claiming, “Observation is not only watching carefully, it is reflecting on what has been observed”. He goes on to note “that one can arrive at different kinds of truth via different senses” (p. 36). The role of the different senses and of observation are very prominent in this study.
Bissex (1987) claims:

We need to discuss not only what we see, but how we see, realizing that choices exist. Research methods provide selective lenses, sharpening our focus on some things while excluding others from view. (p. 17)

Her stated opinion seems strikingly mathematical to me. The forces implicit in Gattegno's (1970) phrase ‘stress and ignore’ are very evident here. He wrote:

There is one universal functioning without which nothing is noticed. This is the stressing and ignoring process. [...] To stress and ignore is the power of abstraction that we as children use all the time, spontaneously and not on demand, [...] And teachers insist that we teach abstraction to children through mathematics at the age of twelve. (pp. 27-28)

ASPECTS OF MATHEMATICS AND MATHEMATIZATION

Underlying all of my work for this thesis is my sense of the mathematical. It is implicit in those events that I have singled out as anecdotes to report; it is tacitly present whenever I attend to something that arose as having mathematical elements, features or possibilities, and was therefore felt worthy of note. One of my current beliefs about mathematics is that it is not ‘out there’, waiting to be picked up. It needs to be brought into being by specific human acts of mathematizing. And young children need to become aware of
these possibilities, through their own mental actions and through the
directing attention of an already-sensitized peer or adult.

This does not mean that someone is not doing mathematics unless
they are aware of it. As I stated earlier, much mathematical activity
happens regularly in homes without conscious awareness on the part
of the participants. But for that activity to be part of a mathematical
framework, it needs to be mathematized consciously. Finally, there is
no general agreement about what constitutes mathematics: for
instance, current debates in ethnomathematics (see e.g. Ascher and
D'Ambrosio, 1994) are pushing hard at conventional boundaries of
'the mathematical'.

I have used the term 'mathematization' in the foregoing as a process
term distinct from, but closely related to, mathematics. Wheeler
(1974, 1975, 1978, 1982) has written extensively on the nature of
mathematization, discussing when and under what conditions it leads
to mathematics. At one point, the subtitle of this thesis was
'mathematizing my daughter's world', reflecting my growing
awareness of my role in directing her attention to mathematical
possibilities. I believe that she too is mathematizing, but is doing so
in the influential presence of my own mathematical awareness. This
study, in part, is a study of mathematization.
Wheeler (1978) offered the following as a possible activity that mathematicians could do for mathematics education. They could:

- describe the mental processes that produce mathematics - i.e. help us understand how any mathematics comes into being. [...] In common with many others I adopt the word “mathematization” to refer to the mental processes which produce mathematics. [...] it can be detected most easily in situations where something not obviously mathematical is being converted into something which obviously is. (pp. 147, 149-50)

His concern is with a possible classroom pedagogy based on mathematization, and calls for a “serious, subtle, study of mathematization”, in the hope that it “will help all of us in two respects”:

- it will help connect the inner mathematical experience with its outer, objective, form; and, in the classroom, it will help us to handle the individuality and spontaneity of students who are coming to terms with the most impersonal subject we ask them to learn. (pp. 155-156)

Bishop (1988) has written about “environmental activities and mathematical culture” (p. 20), and explores the idea of “mathematics as a cultural phenomenon” (p. 21). In his search for cross-cultural similarities and contrasts, he identifies six activity categories “which lead to the development of mathematics” (p. 22). These are: counting, measuring, locating, designing, playing and explaining. I am
interested to see that instances of all of these activities occur in my data.

Bishop's categories are not uniform in any sense, mingling as they do the social, cultural, and more overtly mathematical. At the end of Chapter 3, I detail the contextual range of my anecdotes and the breadth of mathematical arenas included. Although I have not chosen to use his categories as my chapter themes, some are closely interwoven in the story that I tell.

Most directly relevant to this thesis are counting (particularly in Chapters 4 and 5) and designing and playing games (Chapter 6), while explaining is evident throughout. On the theme of playing, Bishop observes:

> Playing may seem initially to be a rather strange activity to include in a collection of activities relevant to the development of mathematical ideas, until one realises just how many games have mathematical connections. (p. 42)

> It was generally surprising to me to learn how relatively little writing existed about the relevance for education of games and playing, from a cultural perspective. [...] There is no doubt in my mind, [...] that playing is a crucial activity for mathematical development. (pp. 47-48)

When I characterize my study as a study in mathematization, I am seeing it as an exploration of this sort into the early contributory experiences of both my and my daughter's sense of 'the
mathematical'. But I also found that despite it not being a classroom study, it had some striking effects on my own ways of being in the classroom.

STUDIES OF YOUNG CHILDREN MATHEMATIZING:
CASE STUDIES AND MORE

Anderson (1991) posed the following questions:

When do children begin to mathematize? How and why do they engage in mathematical activity? What drives or motivates a young child to make sense of her environment? What do these early mathematical ideas look like? (p. 47)

She has explored 'emergent mathematics' (a neologism coined in parallel to emergent writing; Atkinson, 1992) in the form of a case study of her daughter, Terri. Anderson uses the themes of spontaneity, competence, and questioning to explore the learning experiences that the paper is discussing.

I find these categories very interesting, and have noticed their emergence in my study too. Anderson’s paper also strongly urges further research into the field of emergent mathematics. She writes:

To understand emergent mathematics fully, further research is required to document commonalities, differences and idiosyncrasies among children’s early mathematical experiences in home and preschool environments. (p. 55)

My study aims to add to this research area.
Higginson (1980) talks about his three-year-old daughter, Kate, and their encounters with the mathematical. He states that, "[she] is a fairly normal child as far as her interest and ability in things mathematical are concerned" (p. 12). Like me, he is making no claims of genius under construction in his paper. He itemizes some of Kate's mathematical interests at this age. These include saying double-digit numbers, trying to make sense of digital clocks, and reading written numerals. Each of these were also a part of Jackie's and my world. He also comments that she counts the "one, two, threes of her daily life" (p. 8) and recognizes that only a small number of materials are used for Kate's "mathematical edifice" (p. 12).

Higginson concludes that:

It is no accident that much of the pioneer work in developmental psychology has been based on observations made by researchers of their own children. Cognitive growth is accretive; by its nature development is a gradual process. To appreciate it fully, therefore, takes time and a situation where trust has been established. [...] Certainly it causes one to abandon forever any idea that mathematics education begins on the first day of school. (p. 12)

Higginson points to a need in education research that still exists today as much as it did some fifteen years ago when he was writing:

Where they act spontaneously in natural settings children can show surprising abilities and we need to know much more about the circumstances which elicit these responses. (p. 13)
Zeman (1987) writes about teaching his son, Ethan, mathematics. He feels that he has "been observing how he [Ethan] has been thinking about the mathematics that we have been dealing with" (p. 191). Unlike myself, this researcher believes that he can describe, "the frames that he [Ethan] developed to cope with the mathematics that he was presented" (p. 193). While I am concerned that I cannot speak to Jaclyn's actual understanding, others are sometimes not so reticent. I cannot specify her understanding, but try to present my perceptions of it.

Pimm (1987) addresses the issue of regular words being used to mean or refer to specific, non-regular ideas in mathematics. He discusses why it is necessary to build a "mathematics register" and suggests ways that this can be done (pp. 75-110). It is important to pay attention to the language that we use for mathematics and to help novices build their own register of terms. Pimm (1993) claims:

Part of learning mathematics is learning to use language like a mathematician, and more subtly learning how to mean like one. But the demands of mathematical expression place severe pressures on natural language. (p. 35)

I have looked for examples of natural language being used 'to mean' elements in mathematics, and for examples of Jaclyn developing a mathematics register within her growing command of English. Although the issue of the development of mathematical vocabulary is not addressed in a separate chapter, several incidents relating to her
developing mathematics register are discussed (two instances include the terms ‘take away’ in Chapter 4 and ‘infinity’ in Chapter 7).

Walkerdine (1988) has developed her ideas of learning in mathematics in part as a result of considering transcripts of thirty-six children, from an even mix of working-class and middle-class backgrounds. From these transcripts she develops many claims. She notes that because mathematics shares much of its vocabulary with regular, everyday English, there are ample opportunities for children to misunderstand what a word is signifying. She maintains, and I agree, that language needs to be mediated for specialized usage. It cannot be assumed that the child, or indeed anyone learning a word in a new context, will easily adopt or even be aware of the new meaning. Since much of the evidence for mathematical thinking in this thesis occurs in verbal form, it has been interesting to look for previously-learned words or expressions (such as ‘and’ or ‘take away’) being used in new ways.

Walkerdine’s analysis of word frequency and usage attracts me, as does her attention to context. She focuses in detail on use of size terms, especially in home practices as contrasted with school practices. As I write this, I recall Jaclyn’s strong differentiation between ‘smallest’ and ‘littlest’ (see below); words that for me were virtually synonymous. For Jackie, the distinction in size marked by these two words was as different as the sound of the words were to me. So, Walkerdine’s book has, among other things, provided a context for my attending to the language that was used, both by Jaclyn and myself, in the situations recorded.
I end this chapter with the first instance of an anecdote from my data in order to attune the reader to what is to come as well as illustrate Walkerdine's and Pimm's ideas. (Throughout the thesis, anecdotes are presented in bold typeface, with actions given in square brackets.)

Jaclyn (3 years 8 months) and I are reading *One Snowy Night* at bedtime and we are on the page where lots of animals are lined up on the stairs.

M: Which one do you think is the smallest?

J: That one. [She points to Badger, the largest one, on the top stair.]

M: I think this one's the smallest. [I point to the smallest mouse.]

J: No. That's the littlest.

M: But 'littlest' and 'smallest' mean the same.

J: Then what's this one? [She points to Badger.]

M: He's the biggest.

J: Oh.

In answering questions, people are often not only attending to the question at hand, but are also trying to predict the next question or the way that the conversation will flow. I wonder if Jaclyn were anticipating the question after “Which one is the smallest?” being
"Which one is the tiniest?" rather than what I had in mind, "Which one is the biggest?".

Another look at this brief interchange makes me wonder if Jaclyn felt slightly tricked at the way things turned out. When all the animals were within the ‘small’ range, why would I want to start comparing bigness? ‘Big’ is for giants and monsters, and, in another context, for ‘big girl behaviour’. I was quite willing to accept ‘small’ and ‘little’ as synonymous; Jaclyn was not. I felt dissatisfied at the end of this exchange and pursued it the following night (see Phillips and Anderson, 1993). I ascertained that Jaclyn had developed a more detailed classification system for ‘small’ than I was allowing in my thinking. I was prepared for ‘smallest’ leading into ‘biggest’, whereas her system went ‘smallest, littlest, tiny’. It turned out that my ‘biggest’ was her ‘smallest’. No wonder we were confused.

While thinking about this incident again, I am struck by the definiteness of Jaclyn. She strongly asserts that Badger is the smallest, she corrects me when I refer to the size of the mouse, and she challenges me when I attempt to influence her belief about small and little. Language has a lot of power in mathematizing at this age. It has the power both to clarify and to confuse.

As a teacher and mother, I need to listen carefully to the words that are being used. I need to listen not only to my meaning, but to all the other meanings that I am not intending. This one incident has exemplified the miscommunication possible when basic everyday words are being used for size comparison. Words with multiple
meanings deployed in multiple contexts need to be used with care and awareness. I know that I often assume a general understanding of common terms. This incident demonstrates that understanding my usage relies on knowing my question plan, and does not indicate, as I had initially surmised, that Jaclyn had no context for ‘smallest’.

Finding I can misunderstand this response from my own child means it is likely there are many times when I have unknowingly misunderstood her. It is even more likely that there are times when, unawarely, I have misinterpreted a child in my class. Both formal and informal sessions are full of incidents where confusion has been caused by differentiated meanings for the same word.

I once had a student who was very puzzled when I asked him, “How many trips did the child make?” in response to this problem:

A child was moving his books from his bedroom to the basement and was able to carry ten books each trip. If he had 142 books, how many trips did he need to make?

He said he just didn’t get it, so I started to explain the procedure of moving books from one spot to another. When he still looked confused, I had him act it out and ‘we’ determined that he would need to make 15 trips. Finally, he blurted out in exasperation, “But I did not even trip once”. As with Walkerdine’s (1988) class-based analysis of size and quantity terms, drawing attention as she does to the regulating and economic significance of terms such as ‘more’, the psychologically charged nature of ‘tripping’ for a young child is of significance here.
CHAPTER 3: HOW I GATHERED MY DATA

A DESCRIPTION OF MYSELF AS THE RESEARCH INSTRUMENT

Earlier, in Chapter 1, I referred to a book by Walkerdine and Lucey that highlighted the reactions of these two researchers as they read some transcripts. Their emotionally-charged responses caused me to think about myself as a research tool; one that not only records, but also interprets, based on the sum of who I am and what my life has been like. I believe that this is important background information to include in a piece of work such as this. So, I will try briefly to tell some things about me.

What needs to be told so that readers can better understand the sense-maker’s lens? Some of my life as it existed during the data collecting is talked about in a later paragraph. But, on re-reading my work, I find that there is nothing about my own childhood influences; and some of these need to be stated. My childhood was undoubtedly influenced by the adults in it and the circumstances of it.

I am a white, heterosexual female raised in a working-class home by parents of Scottish and English heritage. My mother stayed at home, often undertaking child care to earn extra money. My father worked as a longshoreman until he retired. For recreation, he was a miner, loving to get out into the bush and stake claims.

I always enjoyed school, and attended large public schools in a stable part of the city. I did well in school and always tried to please both my parents and my teachers. I usually succeeded in this. I do not
recall being pushed to excel at anything; there seemed to be no expectations placed on me. What I accomplished, ranging from non-noteworthy to outstanding, was generally accepted as good enough. Whatever I did was accepted as the way it was; no rewards, no punishments.

What is it like for me as a middle-class adult, raised in a working class family, to read the transcripts of my own daughter’s middle class ‘ordinary’ life? Recall the Walkerdine and Lucey quotation I cited at the beginning of Chapter 1. Was there any ‘pain’ and a deep, almost forgotten ‘something’ that ‘smouldered’ inside me as it had for them?

I am not aware of things I would rather not remember surfacing - but perhaps I did not let them. What I am most conscious of is that reading their writing caused me to reflect upon the fact that markedly different perspectives were possible and produced a heightened awareness that I, as the viewing lens, one of the key participants and the major interpreter of incidents, was a very complex research tool. I tell all this so the reader can better understand – intellectually and emotionally – my living perspectives as an involved writer of this account.

In my daily interactions as a mother with my young daughter, many opportunities to point out items and issues of interest occurred. As a mother, I often had to decide whether I would direct attention to an aspect of an incident or not. This usually depended upon the time available, my state of mind, the urgency with which Jaclyn presented the activity and her degree of insistence.
As the mother of a teenager and a pre-schooler, at that time married to a busy executive who often was called away at short notice for business, a full-time elementary school teacher, and a part-time graduate student involved in several research projects, time was often a very critical factor. I mention this to set the temporal context of this research. Not only were the anecdotes collected over two years, but each day in those years was a very full day. There was no time to contrive situations to explore mathematics. The events recorded happened without deliberate staging. The mathematics occurred in the situation, though the reflections and mathematizing often continued long after.

This fits Goetz and LeCompte’s characterisation of ethnographic research as concerned with producing a “holistic depiction of uncontrived group interaction over a period of time, faithfully representing participant views and meanings” (cited in Eisenhart, 1988, p. 99). The ‘group’ was small: in virtually all the anecdotes I report, Jaclyn and I are the only two present.

Deciding to pay attention to mathematizing in my relationship with my younger daughter, not surprisingly, raised my awareness of mathematics as it occurred in our interactions. After we started an activity, I would often sense an opportunity to probe for mathematics; but, sometimes the precise mathematical focus was only available to me on reflection about an incident. The research context is the where and how (broadly interpreted) of the mathematizing events. I look at incidents where mathematics
occurred naturally and informally, between Jaclyn and myself, in our home environment.

Many of the activities where mathematics was mediated and/or noted were situated around either bathtime or bedtime. These times have a built-in significance because they happen each day and, if I am home, they are usually times available for me to spend exclusively with my young daughter. Bedtime was also noteworthy because it was a time when Jaclyn chose the books we read and/or the games we played. Although I like to think that the timing of bath and bed are under my control, this research has shown me how much influence Jaclyn actually has over these contexts. This will become more evident to the reader as the vignettes are presented.

WHAT KIND OF ANECDOTES WERE COLLECTED?

The anecdotes are a collection of incidents in which I saw mathematizing in the context of the home. Mathematizing, as was discussed in the previous chapter, I view as using mathematical ideas and practices to help make meaning of and/or see connections in a situation. Mathematizing can be seen as an optional way of viewing a non-mathematical situation. Meaning could sometimes be constructed differently, but I have noted situations where mathematics was either mediated or invoked as the sensitising lens for the construction.

Many times I could have stressed the sentence structure in a poem, or discussed the artwork in terms of colour and line. Other times I might have identified the scientific elements of a story or picture, or
I could have drawn on concepts of family, fantasy, or almost anything at all. If I thought that mathematics was obviously situated in the context, then it was the mathematics I primarily pursued. This does not mean that I never stressed these other elements; they too were a welcomed and encouraged part of our conversations. But, in the incidents reported here, I chose to stress the mathematics, and ignore much of the rest.

Mathematical mediation was available as a tool for both mother and daughter. Jaclyn and I each chose mathematical elements and themes, on many occasions, to extend the social context of the situation. The home environment included the house we lived in, but it also included the car, shopping trips and recreational situations where the conditions of the data collection were met. Instances of these latter situations are fewer because the mathematics occurring in them was often more directed, thereby not meeting the criterion of unplanned informality, and the situations also often included other people and so did not meet my ideal criterion of mother/daughter focus.

HOW WERE THE ANECDOTES COLLECTED?

Over a two-year period, I kept written records of incidents that included mathematical thinking, communication, and activity. The process that I used entailed ensuring that there were always piles of blank paper and pencils available to make a quick record. As soon as possible after a mathematizing incident occurred, the conversation and the context were recorded on paper. These bits of paper were
kept in a file and it was my intention, as researcher, to enter them into my computer at regular intervals. This procedure worked well for the first four months. Anecdotes that had been collected during the week were attended to on the weekend. After this, the entering process broke down, and the bits of paper were stored in a file until the end of the collection period.

Anecdotes were collected and maintained in terms of who said and did what and when and why. What impressed me, as I later looked at these bits of paper with hastily written anecdotes, was how clearly the incidents themselves return to me. I had mentally marked and physically recorded enough data in sufficient detail to allow me to reconstruct the situations and speak about the mathematics in them (up to four years later). The strength, and longevity, of these anecdotal vignettes has proved very powerful.

On one occasion, in the summer of 1994, I presented some of my research notes to a class of graduate students for their comments. Upon questioning, there were some details that I could not recollect, but on reflection, these were details that I probably did not attend to at the time and therefore would not have had access to even immediately after. An example of one of these was, “Which hand was she using when she pointed?” My intuitive answer to this was to say, “her right hand”, because Jaclyn is right-handed now. However, there was a long time during which she seemed to show no particular hand preference and where I paid no attention except to notice that she sometimes used one and sometimes the other. Hand preference was not something that I attended to in these situations.
The main value of this experience of presenting to my fellow students was the feeling of resonance that I gained from comparing their interpretations of the data with my own. I also felt confirmed that I had recorded enough information in a clear enough manner to validate my method.

Recording devices such as videos or tape recorders were not used. I was both interacting with and observing my child naturalistically. In my home, it is not natural to use such sophisticated technology to record everyday, informal events. Me writing on paper was such a common occurrence. In fact, I am one of those people who often says, "I wish I had my camera!", but almost never do. Even for events where it would seem more normal to have a recording device (e.g. a birthday), I often think about it too late, and with some regret, after the fact.

**HOW THE ANECDOTES WERE CATEGORISED**

When Jaclyn started kindergarten, some of the conditions necessary for this study's anecdote collecting came to an end. Jackie was no longer a pre-schooler. It was time for me to find a way to present the story of our mathematizing over the past two years.

Because I was a participant in all of the incidents, I had an on-going awareness of what areas of mathematics had been addressed, what types of situations I had recorded, and the different mediation strategies that occurred. This was like the "discovery analysis" stage noted in McMillan and Schumacher (1993, p. 484). I had some ideas,
by virtue of being involved as a participant, of themes that might be developed.

Initially, I thought I might present all the data chronologically and, by means of a coding system, demonstrate the variety of mathematics and mathematizing that occurred in this study. I had analysed my pilot study, described in the course assignment mentioned in Chapter 1, chronologically, identifying the salient mathematics featured in each anecdote.

In this way, I believed I would be able to chart not only the variety of mathematics that occurred, but also the relative frequency of topics. By looking at the incidents that contained several different areas of mathematics within them, I would also be in a position to make a tentative statement about the possibilities of mathematical integration and home learning.

I ended up carrying out the initial categorisation of all the anecdotes following this system. As I read each one, I wrote down what its major mathematical foci were. Categories I used included: counting by twos, repeating a pattern, naming a colour pattern, counting to thirteen, decomposing numbers, number composition to ten, cardinality, the use of 'none', and attention to zero.

As I did this, I started to combine some of the more specific classificatory headings into more general categories. For example, 'none'/ 'nothing' and 'zero' became one theme, as did the various aspects of counting, of measuring, of games and of fractions. Some of the original themes, such as 'cardinality' became more specific. For
instance, as I noticed a recurring interest of Jaclyn's in the number 'four', I noted 'fourness' as a possible theme.

This categorising process of allocating incidents to headings took a considerable length of time, and many of the anecdotes continued to fit into two or three possible themes. I became more aware of how each incident presented a different face according to the focus of theme I placed it in (another instance of stressing and ignoring at work). I realised I could not present all the data nor explore all the possible avenues.

During the process of deciding on my chosen chapter themes, I considered and even started writing about incidents falling under the following broad themes, which are still prevalent throughout much of this story. At one point, I planned to deal with them in one go, but because of their ubiquity, I found discussion of them needed to be intertwined throughout this account.

- the language of questioning;
- mathematization;
- home-school links;

There were other, more particular topics, which although not chosen as chapter themes either, give an indication of the breadth of mathematizing incidents.
• zero, none, nothing;

• the significance of particular numbers, e.g. favourite numbers, the role of 'four' in the data;

• the spoken numeration system and ways of representing number;

• ordinals, including the use of 'last';

• fractions;

• geometry, including symmetry, shape combinations and names;

• measurement;

• telling time;

• money;

• problem-solving strategies.

Throughout this thesis, these 'roads not taken' are often present, but are not the focal point of the theme. The stressing of certain themes, while not completely ignoring others, is decidedly at play in my selecting and writing of the anecdotal reflections.

Before I finally selected my main thesis themes, I shortlisted (on the basis of both frequency on the one hand and interesting anomalies on the other) the themes of: fingers, sound, games, infinity, measurement, four, and problem solving. I went through my
anecdote file attending to how each incident contained elements of some of these themes (a very few actually contained them all). I placed a copy of each anecdote relevant to a theme in a corresponding data theme bank (seven in all), in part to ensure that I did not inadvertently lose any data when I made my final theme selection.

It took me many trials to decide on a final list of themes that I would present. As I mentioned, I entertained a number of themes in addition to the ones noted above. I also looked at ones that represented mathematical attitudes and at categorisations developed by others such as Bishop (1988) and the N.C.T.M. (1989) in its Standards documents. In the end, I decided to present the anecdotes in a story that told both of sense and sense-making. This, I felt, was more in keeping with the way that the mathematics had occurred in this particular home environment.

My final blended themes of fingers/touch (Chapter 4), sound/count/pattern (Chapter 5), games/sound/practice (Chapter 6) and infinity/feeling (Chapter 7) I felt blended aesthetic and mental senses involved in mathematizing. They included some episodes I felt were unique or anomalous in some way, as well as incidents that are far more representative of our regular middle-class home life.

These sense and sense-making themes were still a long way from their final form as presented here. I found that, although it was fairly easy to extract anecdotes for the infinity theme (in large part because I decided only to select ones which actually included the
word ‘infinity’), I had some difficulty deciding whether certain anecdotes would be best placed in fingers, sound or games, containing as they did elements of two or even all three themes.

I decided to extract the fingering anecdotes first, because they seemed to be so abundant. Many anecdotes have not been reported here in order to avoid repetitiveness. As I wrote about the anecdotes I had chosen, I found that, although elements of fingers and touch were prevalent, sub-themes of questioning and sight were also regularly present. I decided to stress touch/fingers and comment on questioning. Sight I did not stress, because it seemed such a given partner to touch for a sighted child. These appear in Chapter 4.

When I attempted to analyse sound and games in the same manner as fingering, I was unable to do it. There proved too many overlaps between these two themes. Also, I gradually realised the role of practice and the skill of counting were intermingled in the two. Dealing with sounds, games, counting and practice in one chapter proved far too unwieldy. On re-reading the sound incidents, I saw a possibility of a chapter about patterns. By selecting anecdotes according to patterns of counting and patterns of sound, I found that most of the remaining anecdotes fitted well into the games chapter. This resulted in the original sound chapter being split into two. Part of it is in Chapter 5, with a focus on patterning and counting, and part of it is in Chapter 6, with a focus on games and practice.

I also read some more research writing about counting, and divided my counting pattern incidents into the categories of ‘transitive and
‘intransitive’ (see Pimm, 1995; it is also discussed in Chapter 5). I found that many of the transitive counting incidents were less sound related than I had originally believed, and fitted better into the theme of games or one I was not to develop: problem-solving strategies. Those latter anecdotes were not used in this thesis.

In organising the anecdotes for the games chapter (Chapter 6), I originally thought they would blend into one long chapter about the role of practice in game playing. But I soon differentiated ‘games’ into invented ones and those using commercial materials, and others which fell in between.

Chapter 8 presents a summary of what I feel I have learned.

The final breakdown of this thesis into chapters was the upshot of a long and arduous process. I found that each time I returned to my work, often having to leave it for other commitments, on occasion for a considerable length of time, I needed to reorganise it some. Even when I was not actually writing, I was working on some anecdote or classification scheme in my head, endeavouring to find ways to present a sensible, systematic account of the two years’ worth of incidents.

There are numerous overlaps between chapters that, although not stressed are not ignored either. As mentioned earlier, there are many roads not taken. And although the data presented in the next four chapters are primarily about number, there were many non-numerical incidents throughout the two years: a full range of mathematizing occurred.

50
HOW ARE THE ANECDOTES REPORTED?

I wrote down the conversations as they occurred between Jaclyn and myself, and started writing my reflections. I offered my written reports to others (e.g. supervisory committee members, colleagues and friends) to read and discuss with me. I did not intend the analysis/interpretation stage to be entirely a solitary process. Only Jackie and I know the entire context, and even for the two of us the specifics differ. Others had access to the words and actions and, therefore, to some sense of the meaning, and could make observations that my immersed view sometimes did not permit me to see, and this undoubtedly had some influence on my final analysis.

The anecdotal data is presented here as a series of themes written as narrative pieces. Included in the stories are dialogue excerpts that state both the words and actions of the incident, provide a description of the context, and then present an analysis of the perceived mathematics and of our interactions, and offer reflections on the vignettes. Without leaving the main focus of mathematics, I have tried to produce a warm, living piece of work that will encourage others to become involved. Each chapter has been written to stand on its own, but I hope the entire account will be read.

As I wrote this thesis, I became increasingly aware of how much I was exposing both Jaclyn and myself. I became somewhat uneasy, wondering how much more I might reveal than I intended. While it is undoubtedly true that, "every text that is created is a self-statement, a bit of autobiography, a statement that carries an
individual signature" (Smith, 1994, p. 286), the very nature of this thesis study goes well beyond the norm in this regard.

After all, I am presenting a very exclusive situation for public examination. Should I have tried to protect the identity of my daughter and myself? Because of the nature of thesis writing, I could not protect my own identity without creating many exceptions. This, I felt, was not necessary. By my spoken and written words, by my actions and non-actions, I am constantly exposing myself to others. Those who know me personally, know my family. For them, my younger daughter already has a name and an identity. I cannot disguise her to this audience. For other readers, I do not imagine what her name is matters. It is her words and actions that are significant. These I had already decided to share.

So, for me, the question is, "Do I have that right?" Asking for her informed consent was not a meaningful action. I have been able to answer myself affirmatively in this respect, or I would not have begun. I mean Jaclyn no harm, and my intent is not to prejudice others who might meet her. More strongly put, I judge that the situation created by my research has been mostly beneficial or, at worst, neutral for her. It has provided her with a forum for getting my undivided attention, alerted her to mathematics and has given an additional context where she is really listened to. In fact, one of her day-care teachers once remarked to me that Jackie was "rather unusual", because, "Jackie expected to be listened to, and she expected her ideas to count". My research has provided one context which I believe has contributed significantly to this expectation.
CHAPTER 4: “I'M JUST FINGERING IT OUT!”
THE USE OF TOUCH IN MATHEMATIZING

In this chapter, I want to explore the use of touch, specifically with fingers, in Jaclyn's early mathematizing. My primary focus will therefore be the use and significance of fingers in emergent mathematics. The context (the where and how) as well as the actual events (the what) will be of importance. I will present the mathematizing that I, rather than Jaclyn, recognised in each of the segments. In fact, it often seemed to me that she was more attuned to the social than the mathematical of these situations. In writing about these incidents, I will also reflect on aspects of them that seem to have significance for me as a teacher, most specifically the form and purpose of questions and who asks them. By no means have all examples of finger use been used for this chapter; it is a theme which runs through many anecdotes throughout the thesis, illustrating well the adage 'my fingers are an extension of my brain' (cited in Pimm, 1995, p. 14).

The contexts for each of the vignettes that I am about to discuss are specifically different and yet generally the same. The sameness of each situation includes mother/daughter, trust, pleasure, caring, and choice. They also include fingers and, predominantly, aspects of numbering and counting. In addition to this, the use and role of questioning is a strong sub-theme in this chapter.

Before exploring this first set of anecdotes, I would like to provide some brief background. As a teacher, I have long been aware of
children who hang on to finger counting and finger manipulating when doing arithmetic. I have wondered at their reluctance to let go. There have been times when I have been subtly directed to stop them. Parents have admonished, “You know, Johnny is still using his fingers. I was sure you’d get him out of it.” Colleagues have commented, “When I used to teach grade 4 [the grade I often teach] I never let them use their fingers. They can become a crutch.”

Try as I might, and this was not too passionately, I could not get students to stop using their fingers. I could only get them to conceal their use. Use of finger calculating did not stop, it just disappeared from sight. This I refuse to see as a desirable goal. As with most things, children use their fingers until they no longer need to use them. Even then, it is not a case of use/not use, but rather one of degree of use. Indeed, I find that I still use my fingers for some reckoning.

However, to allow or actually to encourage the use of fingers is one of the tensions in mathematics. At what point should students be solely reliant on mental constructs, pencil and paper and other calculating tools? Mathematics educators want children to understand, yet are often too quick to have them discard the tools from which they build their understanding. More than thirty years ago, Gattegno (1963a) pointed to a similar problem:

We spend a long time establishing counting as the basis for addition and, when we have succeeded, we forbid children to use it and insist upon answers being given without counting. It
is no wonder that some children never recover from such treatment and that others develop a resentment against a subject in which, as soon as a skill has been acquired, it becomes shameful to use it. (p. 4)

In his recent book, Pimm (1995) describes at length disparate sources of mathematical understanding through the human senses of touch and sight. In particular, he distinguishes among counting fingers, counting with fingers, and counting on fingers. He writes:

If I am counting fingers, then they are to be treated as objects like any others susceptible of being counted. I can also use them to ‘show’ numbers such as eight or five. [...] If I am counting with my fingers, then they are serving quite a different purpose: they are part of my counting mechanism, helping to guide my attention in assisting the process of ‘attaching’ number names to objects, temporarily baptising them. [...] In this sense, counting is initially most importantly about touch. [...] Finally, and contrastingly, if I am counting on my fingers, as well as the implied reliance on them [...], I am using them as placeholders for whatever number names I choose. [...] They are serving as dynamic physical symbols for the process of numeration itself, as I move around the number name sequence. (pp. 15-16)

It is with these distinctions in mind that I start to explore some of the anecdotes.
FINGERS AND COUNTING

Jaclyn (3 years 7 months) selects the story *The Three Little Pigs* for us to read together at bedtime. The cover has a picture of a wolf positioned horizontally above the three pigs. She places her three middle fingers on the pigs and declares:

J: One, two, three pigs and one long wolf

As she says ‘wolf’, she places her index finger sideways onto the wolf.

This incident is indicative of one routine that she and I adhere to. At bedtime, Jaclyn usually chooses the book that she would like us to read together and we spend a few minutes looking at the cover before going to the story inside. When I first thought about the words and actions in this incident, I thought that the mathematical elements were categorisation (pigs and wolves), counting using one-to-one correspondence with fingers, and an idea that the word ‘long’ had some connection to ‘across’.

On further reflection, however, I also saw her separated touch of the distinct finger tips contrasted with her use of the length of her index finger: fingers used for both counting and measuring. But there is also the hint of equivalence. I wondered whether Jaclyn was seeing three pigs as equal to one wolf.

Finger tips are used to represent pigs, length of index finger to represent the wolf: size correlations remain. Different
representations for one include: one finger tip, one index finger. One pig is not the same as one wolf. ‘One’ does not always signify the same thing.

Jackie may, of course, have simply been ‘comparing’ pigs and wolves. To what extent she was comparing remains unavailable to me. We had to get to the story! Another tension for teachers is how much can you talk about a story without interfering with the plot, the telling itself?

As we read The Three Little Pigs, there is a picture with a marbles game in it.

J: How many green?

M: Why don’t you see?

J: One, two, three, and one hiding.

M: How many’s that?

J: This many.

At this point, Jaclyn holds up four fingers, but doesn’t name the amount.

I know she can intransitively count aloud to four, she has done it many times before. Why is she seemingly reluctant to name four now, instead opting simply to ‘show’ the number physically? Perhaps it has something to do with it being the final number in the count. Perhaps it is because the fourth one is not completely there, it is
'hiding'. Only a bit of it can be seen inside the marbles pouch. Perhaps four fingers are seen as one group and she is feeling uncertain about "one, two, three, [four]" meaning the 'same' as "[one group of] four": it is after all a procedural oddity of counting that 'four' is a particular and temporary 'baptism' of one of the parts as well as the name accorded to the whole.

Perhaps she is still working on the cardinality for four. I find myself increasingly aware of the layers of meaning that I am trying to expose with Jaclyn. Unlike the pearl, however, these layers are meant not to cloak an irritant, but to ease it. For Jaclyn, using her fingers was, perhaps, a way to show what she knew while she worked at further sense-making.

I do not answer her question to me, her opening gambit. Instead, I deflect back to her with another question/suggestion, "Why don't you see?" - a teacher move? A quite separate issue arose when I found myself wondering why I stopped this exchange here. Was it because of continuity for the story line, or because Jaclyn seemed uninterested, or because time before bed was running out? As mother, I am often unaware of my motivation for timing. It seemed like we had talked enough.

As a teacher, in class, how do I know when to end a discussion period? Is it when we have run out of new things to say, when the children get restless, or when the bell goes? Often it can be any one or a combination of these. But, the ideal time seems to be when there are still ideas waiting to be shared, just after the point that all have
joined in by really attending to the discussion, but before even one of them has left it. Why? It makes it easier to return to. Often after an opportunity to reflect on a discussion or a problem, we get to a deeper level of involvement and achieve different understandings than would have sufficed earlier.

In class, how do I interpret the actions of a child who does not verbalise an answer, one who will give hand answers only? If the child is capable of speech, do I 'make' him talk or do I accept his chosen method of participating? In my career as a teacher, I have so far taught two students who have been elective mutes. Although capable of speech in different situations, neither of these children would speak at school. Their stories and what I did to encourage them to speak is not within the scope of this thesis, but the way their disposition affected their mathematics learning is.

In both cases, I found that mathematics was the subject least negatively affected. Both of these students wanted to learn and were willing to use their fingers to communicate. Each easily learned, and used, active participation techniques to share their thinking and their answers. For example, I would say, "Show me the number in the thousand's place", and the number of fingers that matched the answer would be shown. Similarly, I sometimes said, "Write whether you believe the answer is <, >, =, or ? (meaning "don't know") on a piece of paper and hold it up when I say, "Show me".

My early interest in writing to communicate in mathematics (see Phillips, 1996; Phillips and Crespo, 1996) stems from my work with
these two students. If they would not talk, how could I see what they were thinking when their thinking was too complex for finger answers?

At bathtime, Jaclyn (3 years 7 months) is telling me about her day.

J: At school [day-care], we have a cake on our birthday.

M: Oh, really? I didn’t know that. [She’s only been attending since the beginning of the month.]

J: On my birthday I’ll be this many. [Shows four fingers.]

M: Right. You’ll be four. How many are you now?

J: This many. [Shows three fingers.]

M: Right. How many did you used to be?

Jaclyn pauses, plays with her fingers and then holds up two.

M: Right, last year you were two. How many were you before that?

Jaclyn holds up three fingers.
I smile a wry smile at my questioning ‘technique’. [Before she told me she was two, she had told me she was three! Two different ‘befores’.]

Finger displays are standing in for words but are also representing, holding, showing specific numbers. They are visible rather than audible, prior to written numerals. Are they also representing a sense of the whole that cannot yet be named? Reading this transcription makes me wonder about the language used too. Are fingers being used to signify birthdays? Years or age are not mentioned. On my birthday, I’ll be four [fingers]? No wonder Jaclyn is reluctant to use words! I am attempting to draw attention to a basic learning situation in mathematics. What should be used to signify what? I am certain that Jaclyn did not think that she would be four fingers on her birthday, but I did not help her see that she would be four years either. I assumed that she knew that we counted birthdays in years and yet neither one of us mentioned it. After all, when asked ‘how old are you?’, the answer is simply ‘four’, a bare number name, an intransitive count, where the answer is more a noun than an adjective.

Jaclyn seemed to know how to represent, and calculate, numbers on her fingers. She did not seem comfortable with saying the number words, or perhaps the fingers were easier and seemed enough to her. Touch with sight before speech?

In our discussion, Jaclyn was once again the initiator of the topic. She had information to give that I could genuinely respond to: I really
did not know about cakes on birthdays at the new day-care. In terms of Ainley's (1988a) classification of questioning purposes, mine are 'genuine' questions, relatively unusual within a classroom context, less so perhaps at home once a child has a wealth of experiences away from her mother. The norm in school is 'testing' questions, ones where the asker already knows the answer and is using the question to find out whether the respondent does. Although the next context I offer is geometric, the theme of styles and purposes of questioning continues.

I am getting Jaclyn (3 years 7 months) ready for her bath the day after the previous anecdote.

M: [I'm looking at the bathroom floor.] Do you see any squares here?

J: Yes, over there. [Points to the heat grate.]

M: Do you see any others?

J: No.

M: What about those? [I point to some tiles that form a border.]

J: Yes, those are squares too. Is that a square too? [Touches window ledge.]

M: No, that's a rectangle. [I make a circle with my index finger and thumb.] What's this?
J: A triangle.

M: It's a circle.

J: [Makes a triangle using both her index fingers and thumbs.] What's this?

M: Hmm ... could it be a ...

J: Triangle!!!

M: Great, Jackie, we've seen a triangle, a circle, and squares.

J: Yeah.

Much of the communication here was presented through the use of hands and fingers. Jaclyn started by pointing, not naming or describing. I followed her lead and continued my questions by pointing. I initiated the shape-making and she followed my lead. So often our interaction takes the form of a dance. We each get a chance to lead and we each get a chance to follow. The same is true of words and the asking of questions.

I was struck here by Jaclyn's interest in using her fingers to create shapes, to call them into being the same way she had used fingers to conjure numbers. I was also interested in her uses of questioning in comparison with mine. Sometimes the mediation that I use involves asking questions and guiding her looking. Often what I do, in my mother role, can be labelled 'providing exposure'. I was not specifically trying to offer Jaclyn a geometry lesson, but I was trying
to increase her awareness of shapes and patterns that are around us, in our home environment. I found myself using ‘testing’ questions. Did I assume that Jaclyn was attuned to the differences between this purpose and that of my earlier usage? The surface forms are actually indistinguishable.

At the time, I wondered what made her jump in to answer her own question about the shape she was making? As I re-read this incident, I am looking for meaning making. I want to know whether she could identify the shapes she had seen by name and from a name, and my questions were to this end: classic teacher testing questions.

Jaclyn’s first question seemed a genuine check on vocabulary; her last one, which echoes my words exactly, I felt was an invitation for me to answer a question to which she already knew the answer. So much so that she answered her own question when I seemed to be taking too long (I was dissimulating and certainly believed she knew that it was a triangle). Jackie had shown that she had some understanding about a triangular shape. Even though it seemed to me that she made a mistake in naming a circle as a triangle, she was quick to show she really knew what a triangle looked like, and in the process revealed she knew about the pragmatics of questioning for different purposes.

This makes me reflect on my role as teacher. If I am the only one asking (testing) questions, then do I really get to see how much my students understand? Does a wrong answer mean no understanding? How can I find out? Jaclyn was keen to show me that she knew
something about a triangle. She used the word and she made the shape with her fingers. I found out that she had some knowledge to build on.

How often do the children in my class get the chance to show what they do know? I know that there are times when an incorrect answer, or the lack of the right word label gets in the way of my understanding or ascertaining what they know. I must remember to give lots of opportunities for showing, and for 'showing off'. I do not think Jackie minded her 'mistake', but I am confident she enjoyed her success at both making and identifying her triangle. I also learned that she has figured out that sometimes people ask questions that they already know the answers to.

Just prior to dinner two days later, Jaclyn (3 years 7 months) and I have been playing cards, and with them, for about twenty minutes. Jackie has just managed, after several attempts, to construct a copy of a closed rectangular shape I had made using four cards. She had been trying to make use of four cards, but previous to this success she kept having one card over, resulting in a triangular shape.

J: There! I can do it again. [Proceeds to make it three more times.]

M: Can you count the cards?
J: [Uses her index finger.] One, two, three, four, five.  
[Touches the third card twice.]

M: Try again.

J: One, two, three, four, five. [She leaves the one we've  
been counting and makes a new one.]

M: Can you count this one?

J: One, two, three, four, five. [Again, she counts the  
third card as “three, four” while tapping it.]

M: Let’s spread them out and count them. [Does so.]

J: One, two, three, four. [Touches as she counts.]

M: Let’s put them back. [I count “one, two, three, four”  
as I rearrange them.]

J: I’m finished now, Mommy, you keep playing. [Goes  
off.]

Just prior to this, Jaclyn had watched me make a shape from four  
cards, like this:
But I had returned the cards to the pack. She tried to copy my shape with the same four cards, but kept making a triangular one, using only three.

She seemed surprised that each time she tried she had one card left over. The sense of touch I noticed was one of using fingers for placement. In the part reported here, Jackie used her fingers to try to count the cards. She touched the cards as she counted:
Jaclyn did not seem to be making a transitive counting mistake. (See van den Brink, 1984, for a catalogue of things that can go wrong with transitive or, as he calls it, ‘quantity’ counting.) She was not over­
counting, she did not skip a number. Each time, she seemed to count the third card with two touches on purpose.

I believe that her attention was being diverted from card objects to rectangular shapes, and she switched between counting objects and counting shapes. In order to count successfully, you have to know what counts. However, when the cards were spread out, she counted only the objects, using her fingers as a way of co-ordinating one card with one count word. I now also realise that I first asked her to count the cards, but later asked her simply to “count this one”. What my ‘one’ is is less specific. Spreading the cards out destroys both the
configuration and the ‘ghost’ inner shape brought into being by the placing of the cards.

Jaclyn did not seem concerned about counting five ‘things’ in the closed shape and four with them laid out in a row. She was aware that four cards were used, or at least that she had used up all the cards needed. I can conjecture that she was counting shapes in one and objects in the other, so it was unproblematic because she was not associating, as I was, the two counts as producing numbers that should be the same. As she left me to keep playing, I wondered if she thought that I was the one who needed more practice.

But I was also struck by her persistence in practising making the shape four times in all at the outset, making no comment on it, simply assuring herself that she could do it fluently.

This incident also now makes me ponder why I would push a ‘game’ so far. We had already played cards for twenty minutes, and Jaclyn was probably tired and/or hungry. As mother, I usually am more sensitive to ‘circumstantial’ timing. As teacher, I often try to push more content or experience than can be comfortably assimilated. Why? I tell myself that it provides a preview of lessons to come. But, I wonder, how many students in my class would like to tell me to keep at it by myself while they change to something new.

So, I see another tension in teaching arising. How long to maintain a lesson: until the students are exhausted, or until the topic is exhausted, or, to try to stop while there is still enough momentum
and interest to get started the next time? At home, Jaclyn has the freedom to leave. Students at school, traditionally, do not.

We decide to read a book and Jaclyn (3 years 8 months) chooses *There was an Old Woman who Swallowed a Fly.*

J: How many animals?

M: Let's see: 'one'-horse, 'two'-cow, 'three'-dog, 'four'-cat, 'five'-bird, 'six'-spider, 'seven'-fly. [Rather than pointing to the pictures and counting in turn, I use my fingers to produce the numbers, while saying the appropriate picture word aloud. This is why I have used single quotation marks here.]

J: [Joins in with me (counting using her fingers also) at 'two'-cow.]

The section/page that we were discussing is right at the end of the story. I genuinely did not know how many animals there were. The arrangement on the page is one where each succeeding animal envelops the previous ones, to suggest the 'swallowing' in the rhyme.

This is the first time that I have been aware of counting with Jaclyn in this way. The system can be described as one where the spoken number word corresponds to the finger display, as well as to the type of animal listed.

Fingers represent both number words and animal words. In this, the counting seems doubly removed. I was interested when Jaclyn
picked up this system and counted along with me. Could she sustain this on her own? I really do not know, but she certainly was capable of double representative counting in this ‘scaffolded’ atmosphere. (See Bruner, 1986, for a discussion of this notion.)

Jaclyn (3 years 9 months) and I are sitting at the kitchen table in the morning playing with plastic tops off milk bottles.

J: I’ll see if I can make a circle.

J: There, I made one.

J: Do you want me to make one for you?

M: Okay.

J: There.

M: How many are in the circle? [Mine, which has eight.]

J: One, two, three, four, five, six, seven, eight.

M: How many in your circle? [Hers has only seven.]

J: One, two, three, four, five, six, seven, eight.

M: You counted this one twice.

J: Oh. [She puts her finger on one of the tops and counts round, using the other hand.] One, two, three, four, five, six, seven.
It is hard to count something that has no marked beginning. She also may have thought hers and mine were both the same, and so should have had the same number in them, therefore managed to find a way to count that made this so. It is interesting that Jaclyn chose to use her finger as a place holder rather than use another strategy. She could have spread the tops into a line or stacked them up. By using a finger as place-holder, she still had to remember whether she started by counting the held item or not. It is important to note that although she acted on my information, she did not merely accept it as fact. She thought of a way to check her count.

I also realise I gave her more information than was necessary. I indicated which one she had counted twice, rather than merely saying she had counted one twice or must have counted one twice. Yet I did not identify the fact that the one she had counted twice was her first/last.

Jaclyn could name a shape and represent it and enumerate the pieces. In this anecdote, circles are being used to make circles. In other situations, I have noticed the relationship between the shape of the pieces and the final shape constructed, as with her using cards to create a rectangular shape. Also, I noticed that this time she did not count the inner shape as part of the circle count (as I believe she did in the card counting anecdote described earlier). The inner circle here was much larger than the circles used to create it. Perhaps this accounts for the difference.
Jaclyn (3 years 8 months) has a fever so we are seeing how many Tylenol she should take.

M: Let me read the special directions. It says you can have two when you’re sick. [I deliberately intend to dump more than two into my hand: three come out.] Take two.

J: [She takes two.] Put that back.

In this incident, Jackie is using her fingers as takers. She took two tablets without counting and directed me to put the remaining one back. One of the common ways of introducing subtraction involves the idea and language of ‘take away’ and directing children to notice what is left. Here, the reverse was true. She paid attention to the two she took away (which were important) and I did as I was told and put the remaining one back.

As I re-look at the conversation, I now see that I was also setting up the care needed around medicine. I deliberately read the instructions aloud carefully, directed the taking of an exact amount, and specified “when you’re sick”. Prior to this we had used a thermometer to confirm the fever. More numbers. One of the few times that I am really concerned about accuracy in the home is in regard to illness and medicine. Jaclyn is on her way to understanding this. She echoed the form of my imperative statement and did not argue, in a way she might well have with candy, about whether she could have all three. I wanted her to know of a social contract concerning medicine, one that involves accuracy and non-abusive use. I made the contract
explicit and clear, and mathematics was used as a regulating factor in this.

Jaclyn (3 years 10 months) and I have been reading a book called *Subtraction*. In it, various creatures are eaten or hidden to show the subtraction context of 'taking away'. For example, there are nine fish and a pelican scoops up five, leaving four. Jaclyn enjoys this and seems to be able to predict the answer. The last page is a series of standard written arithmetic questions involving numerals and the subtraction sign. The answer is also under a flap.

M: This one says ‘One take away one is?’.

J: One.

M: Show me one finger. [I fold it down to ‘take it away’.] See. One take away one is zero. Let’s look under the flap. ... Zero.

J: No, it’s really one. See. One take away one. [She covers the ‘-1’.] It leaves one.

M: I see it your way. What’s eight take away two?

J: Eight. See. [Again she covers the ‘-2’ in the book.]

Although this book has no real plot, it is one that Jaclyn enjoys. We often make up stories to go with the text provided. The book has moveable bits and so it is fun to use. It is a doing book, one that we ‘do’ a little differently each time. We use our hands and fingers to
move and cover bits, as well as to count objects. In this incident, 'taking away' did not result in anything disappearing. Jackie's fingers could not lift the numbers off the page as they had the tablets out of my hand in the previous anecdote.

In this 'take away', her fingers covered the number, as she had used them earlier in the book to show fish from a larger group being eaten in a predator's mouth, leaving a resultant smaller group. When I showed her how to fold a finger down to show 'take it away', there was no disagreement with the fingering, but Jackie did seem to disagree that this folding of fingers was a way of doing what was asked on the page. Of course, she may have been offering me a second way to see the number sentence.

I was intrigued by Jaclyn's interpretation of subtraction. It made sense to me, even if it was not based in the mathematics of numbers of things, but with the 'mathematics' of number symbols being acted on. But much pencil and paper manipulation in arithmetic is precisely about 'working on the symbols as if they were the objects'. And the image of 'take away' as 'cover up' made as much or as little sense with fish pictures as number symbols. When we subtract, we do not cause something to cease to exist, we transform it. Yet, many of the examples offered to young children can represent disappearance, e.g. giving away, eating, losing, and hiding. When Jaclyn and I have used objects and subtracted by removing part of the total, I thought that she was on her way to developing a model for subtraction. We have eaten our way down to zero many times. The amount remaining always gets smaller.
Why was there no conflict with number subtraction? Was it seen as something different from object subtraction? If $8 - 2$ is viewed as ‘8, cover the 2, then 8 remains’, I was not sure what to do here. Should I bring out more objects? Trying to demonstrate using her fingers did not yield the results that I was looking for. I believe that Jaclyn realises that there can often be more than one right answer, and that the important thing about answers is that you can show them. She did not seem concerned that the two answers were different. It was okay for mine to be zero and for hers to be one. This seems similar to her earlier lack of concern at the two differing counts for the cards. Given the situation, and my ability to make her sense of it ("I see it your way"), this was okay for me too.

I do not think that I was letting Jackie construct a false impression of subtraction, I was enjoying her showing of one meaning for ‘take away’. I think that subtracting numbers is the same as subtracting objects for her. To subtract an object, you hide it in some way; to subtract a number, you hide it too. The fact that they yield different results proved non-problematic for her at this stage of her mathematizing.

At what point, I wonder, would I want to mediate the potential arithmetic in this situation. Will it be necessary for me to provoke Jaclyn to see 8 as eight ones, or as eight things if she is going to operate on it. This is very close to the constructing/deconstructing process that is basic to mathematics. From first seeing only individual items, to learning one-to-one correspondences, to constructing cardinality, she needs to reverse the process in order to
operate on it algorithmically. Seeing the symbols '8' and '-2' as the objects is not mathematically useful in subtraction, though seeing '18' as being made up of two pieces, a one and an eight, is useful.

Connections between the two need to be formed. This connection has been noted at the manipulative level of objects, but not at the symbolic level. My instinct is to wait for Jaclyn to acquire more experience with subtraction. She needs to be ready to see a conflict in the results between $8 - 2 = 6$, and eight objects, take away two of them, leaves six. She still needs to learn how to interpret an arithmetic sentence in the conventional way.

Jaclyn (3 years 11 months) has crawled into bed with me in the morning. I'm really tired still because I had a late night.

**J:** This is five fingers, right? [Holds up one hand.]

**M:** Yes, it is.

**J:** If I put down my thumb, it's four. If I put down my finger, it's three. If I put down this finger, it's two. If I put down this finger, it's one. If I put down this finger, it's zero. Zero is the same as none.

It is so cosy and somehow safe to share things in bed. Jaclyn loves to come and crawl in with me for an early morning chat. Often, I encourage her simply to cuddle in and be quiet. She has learned, however, that if she talks about the digit display on the bedside clock (recall Higginson, 1980) or about anything to do with shapes or
numbers or problems, that she has a willing ear. In this incident, she demonstrated an ability to count backwards by naming the number of fingers on one hand left up when others were folded down. She chose to stress the remaining fingers and not the ones being folded.

This incident results in a more typical subtraction model than the one demonstrated in the last example, or in the Tylenol instance where she was attending to the amount taken. I also noted that Jaclyn called no fingers standing ‘zero’. My sense (confirmed by my notes) is that she had been referring to ‘nothing’ or ‘none’ previously. Here, she tied the two representations of no fingers standing together as ‘zero’ and ‘none’, and indicates synonymous meaning. It is possible she had just worked this out and had come to tell/show me.

After dinner, I’m giving Robin (Jaclyn’s big sister) and her friend some questions to practise for a mathematics midterm exam. Jaclyn (4 years) is watching.

J: Give me one too, Mommy.

M: If you had three bears and one bunny, how many animals would you have?

J: [Puts up four fingers.]

M: Right.

J: Another one.

M: You have three fish and three monkeys, how many things?
J: This many. [Holds up one hand.]

M: Let's see. Here's three fish and here's one, two monkeys. Where's the third?

J: He's hiding. Monkeys can be naughty.

M: I see. [Laugh.]

Why did Jaclyn feel the need to make up a story for her error? Was it because her big sister and her sister's friend were in the room? Was this a case of face-saving? I even wonder if the error was not a way to direct my attention back to her. She was not used to my giving Robin attention for/with school work. This was not because it did not occur, but because I generally help Robin after Jaclyn is in bed. Among other things, mathematics is used by Jaclyn as an attention-getting and attention-maintaining device.

Jaclyn has a good sense of humour and it shows up here. She could also have thought of the 'naughty monkey' story because it triggered a memory of the counting chant she knew, 'Five little monkeys jumping on the bed'.

Initially in this episode, Jaclyn showed me four fingers for three bears and one bunny. These were fingers of one hand, and were shown all at once, not as three and one. However, they were only shown: the number word 'four', as in an earlier incident, was not spoken. Fingers were being used to represent the total amount of objects in a way that shows both one-to-one correspondence and, I believe, cardinality. Maybe this is about naming or labelling too: four
what? What common name could she give to a collection of bears and a bunny, or, even more so, to a collection of fish and monkeys?

In the second question, even though Jaclyn was not right arithmetically, I did not confront her directly with the error. I referred her back to her fingers as a way of checking. I expected her to say, “Oh, I need one more. Six.” As I read this again I notice my using the ordinal number ‘third’. This did not cause any apparent confusion.

Another thought that I would like to explore has become available to me. Why did I give Jaclyn such easy questions? I knew that she could do three and one, and that on earlier occasions she had answered three plus three accurately. Were my questions merely a way to keep her occupied in a manner that would require no intervention on my part? After all, my main focus here was Robin and her mid-term practice.

How do children at school respond to work that is too easy? Why do I sometimes give them work that is less than challenging? Should each minute count as an opportunity for new learning or is it okay to coast some of the time? No answers at this time, but I am becoming more aware of this.

This is just after dinner, Jackie (4 years 3 months) and I are in the kitchen, while I’m cleaning up the dishes after the evening meal.
J: Mom, look [holds up two fingers in one hand, one in the other] and [switches to one in the first hand and two in the other] and [shows three in one hand and none in the other].

M: Oh, two and one is three.

J: And ...

M: One and two is three.

J: And ...

M: And three and zero is three.

J: And ...

M: [I'm stretched here.] They're all three!

At this point, Jaclyn goes straight into one of her favourite games, discussed more in Chapter 6.

J: Let's play, "What plus one?". You say "What plus one?".

M: What plus one?

J: Five and four is ...?

M: Let me see ... five and four is nine.

J: My turn. What plus one?

M: Two and two and two.
J: Two plus two. Four. [She gives this answer without pausing.]

M: I said “two and two and two”.

J: [Thinking ...]

M: Like this. [I show her two and two on one hand and hold up two fingers of my other hand.]

J: Don’t do it! I’m just fingering it out!

J: See. [She holds up five fingers on one hand and one finger on the other, and counts them by touching her nose with each finger that is held up.] One, two, three, four, five, six. It’s six!

M: Great!

Jaclyn has used her fingers to show these different compositions of the number three: (2,1), (1,2), (3,0). She has also guided me to play her game of “What plus one?” When the number combination was too high to count on one hand, Jackie used her nose as an additional counting ‘finger’ to count with while her other fingers were being used to count on (combining two of the senses described at the outset of this chapter).

I noticed that while figuring out the answer for ‘two and two and two’, she had changed my representation to five and one. In this exchange, she was also clearly stating the cardinality for six. Not simply leaving six fingers up or ending her count at six. This is
different from other instances given earlier in this chapter, where cardinality was shown (perhaps), but not definitively stated.

Not only was Jaclyn playing with many names for finger representations of number, but she really took the lead in the exchanges. I was busy, and she once again used mathematics to hook me in. I only became aware of her doing this a year and a half after the beginning of this study. Going back over the data shows me that this has been going on for a long time.

Jaclyn likes leading. She has directed my attention towards her, she has started with the other names for finger representations of numbers, and she has initiated the game that she calls ‘What plus one?’, a name she coined herself. Although none of the tasks have ‘plus one’ in them, Jackie seems to have generalized her thinking of ‘and’ to include ‘plus’. She has also extended the use of ‘what’ to mean a number, and ‘one’ to be an example of another number. She even accepted my ‘two plus two plus two’. (See Chapter 6 for an instance where Jaclyn would only let me give two addend questions.) This experience can later be drawn on and extended into a more formal look at variables.

This incident also shows the strength in wanting to do something on her own (a resonant word in all the current discussion about students ‘owning’ mathematics). I thought Jackie needed help, but all she really needed was time to ‘finger’ it out. How often do children at school get thwarted by an overly ‘helpful’ teacher or classmate? How
frustrating it can be to be shown one way to do something while you are working on constructing your own method.

Once trust and willingness to help are established in the classroom, I also need to learn to be patient and to create an atmosphere where my help can be rejected (see also Ainley and Goldstein, 1988 for a discussion of this in the context of Logo mathematics projects in schools). I need to pay close attention in order to distinguish what is a request for help, and what is simply a requirement for time. All learners need the opportunity to 'finger'/figure something out. All learners have the right both to request and to refuse 'help'.

IN SUMMARY: FINGERINGS

In this chapter, I have discussed some incidents of Jaclyn’s mathematizing that included the use of fingers. Examples of fingers being used to count with, to count on, to represent, to hold places, to show cardinality, as takers and coverers of objects, as representatives of comparative size and direction, to manipulate, and to stress importance were presented. The contexts for these events included nightly bathtime, bedtime, and weekend mornings.

Several of the tensions involved in and around teaching were also discussed. Should teachers encourage the use of fingers in mathematics? How long should a lesson be? How much to talk about a story while reading it aloud? When is it all right to give easy work? How to balance challenging work with the need to attend to only some children at a time? How many times must something be done
before recognising that understanding has been demonstrated? Is understanding a continuum? When, how, and what to mediate?

I have seen many young, and not so young, children use their fingers to do mathematics. Whether they are used as place holders, as the counter, as the objects being counted, as symbols, the relationship between fingers and mathematics seems to be very strong. This relationship dates back to early times and is prevalent in many cultures (see Bishop, 1988).

Incidents noted are ones that I saw as being particularly relevant to the construction of Jaclyn’s mathematical world. However, after writing this, I continue to realise I have been somewhat naive in collecting my data. Initially, I was looking for signs that I thought represented her understanding of mathematics. I soon adjusted this thinking to say that I was looking for examples of times that I had helped her construct some meaning from a situation by mediating the mathematics. I now propose that what I have been doing is observing Jaclyn’s and my interactions and whenever I noticed her doing something that might be construed as mathematics, I mediated the mathematics as I saw it. I also propose that through finger use and the power of naming, Jaclyn has been mathematizing to make more sense of her world.

Like Jackie, I too am just fingering it out.
CHAPTER 5: SOUND COUNTS AND SOUND PATTERNS

Music is the pleasure the human soul experiences from counting without being aware it is counting.

(Gottfried Leibniz, in Pappas, 1995, p. 41)

Educational linguist Michael Stubbs has claimed: “There is a sense in which, in our culture, teaching is talking” (1983, p. 17). Although speech provides a major teaching tool, attentive listening, as a mode of learning, is not highly regarded by most educators. (For a detailed exploration of this theme, see Davis, 1996.) It is commonly felt that to tell someone something is not as powerfully educational as letting them engage in activity, in order ‘really’ to learn.

There is an often-quoted Chinese proverb (for instance, in the formative 1960s Nuffield primary mathematics materials in the U.K.), “I hear and I forget, I see and I remember, I do and I understand”. I feel this is often taken too literally by teachers in their zeal to provide the ‘best’ learning situations. Juxtaposed to this is the English proverb “take care of the pence and the pounds will take care of themselves”, which has been subtly transformed into, “take care of the sounds and the sense will take care of itself” (cited in Pimm, 1995, p. 9)

In this chapter, I want to build on the opportunity to learn through listening: both listening to sounds and to the absence of sound. Sometimes, the listening is self-listening, attending to what the speaker is saying as well as saying it (Lawler, 1990, writes on a similar theme) and sometimes it is listening to others. I am not
concerned with ‘passive’ listening as often occurs during the giving of explanations or telling; I am interested in active listening as a means of learning. I will be highlighting instances where such listening, either alone, or in conjunction with other senses, seems an important component of Jaclyn’s learning/sense-making. Often the two look the same, much as genuine questions can sound the same as test questions.

As a travel companion to listening, I want to acknowledge practice. Often, with Jaclyn, the times when listening was really important involved repetitive practice of something that she was learning. And, often this something involved counting or some other patterning. If one thinks of music, an area where listening is recognisably important, and imagines that sort of attention to listening being paralleled in mathematics, my intent becomes clearer. Music takes careful listening and involves learning how to listen; so, I maintain, does much of early mathematical acquisition. Although the main emphasis in this chapter is sound and its particular role in counting and patterning, I will also be attending (in a more minor way) to the role of practice. The next chapter, on games, however, will emphasise and extend this discussion.

COUNTING

Counting has been a theme explored by many researchers, and endeavouring to explain the nature of early counting has been an area central to several studies (e.g. Steffe et al., 1981; Fuson, 1988; van den Brink, 1984). A number of terms have been used to mark a
distinction between the counting of objects and the saying of number words in order. These include “number word sequences and countable items” used by Steffe et al. (p. 13), and “acoustic counting and quantity counting” used by van den Brink (p. 2). Possibly because of my background in the area of English, I prefer to use the terms “intransitive and transitive counting” introduced by Pimm (1995, p. 64). They also have the advantage, admittedly as do those of van den Brink, of signalling both practices as different sorts of counting.

Transitive counting is specified as counting that involves counting something; intransitive counting is ‘just’ counting: that is, saying the sequence of number words in order. I will also add another type of counting, one that I will call ‘internal counting’ to this lexicon of words to describe aspects of counting. As will become evident, these distinctions are not clear-cut, and sometimes more than one element can be present.

Internal counting is demonstrated when a non-word pattern of sounds is repeated, or when a number is assigned to a quantity after simply a look at the object group (sometimes called ‘subitizing’ in the literature). Although these two types of internal counting are quite different, I am reluctant at this time to try to coin terms for each of them. I will only be looking at counting that involved listening as a major component. Other chapters look at certain incidents calling on counting which do not rely heavily on sound.

88
In other chapters, I have written spoken numbers in words, as being the most accurate representation of what was said. For convenience, however, in this chapter I have used numerals in most cases, simply because there are so many long counting sequences.

*Intransitive counting*

Jaclyn (3 years 7 months) and I are having a conversation about how old I am, and I show her by flashing my fingers and counting aloud.

M: 10, 20, 30, 40, plus 4 [I end with four fingers displayed.]

J: Like this: 10, 40, 40, 40, 4. [She flashes four times and ends with four fingers.]

It struck me that Jaclyn said the same amount of number groupings that I had said, even though she did not give them the same names. Jaclyn certainly had prior knowledge of ten and four? 'Forty' would be a familiar sound, whereas, 'twenty' and 'thirty' are newer sounds, except for the '-ty' part that she heard and repeated. Or, was she unable to recollect the names, but since she had listened, could replicate the chunks? This is similar to the thumping sequence (the 'Bambi' story) that is discussed later in this chapter. Sound was a factor in remembering the type of words and rhythm of the number phrase. Next time, she might attend more closely to the names because she already has the sound pattern of counting by tens.
This incident reminds me of learning a language. Often I cannot remember the exact words that I need, but I can remember the flow or rhythm of what I want to say. If I have nerve enough to say a nonsense phrase in the way that the phrase I want to say sounds, a more proficient speaker will often recognise what I want to say and can help me out. If I can take care of the sounds, the sense can be achieved. Early counting is like this. Children first learn the words that allow counting; then they learn to let these words symbolise the objects being counted.

This same type of occurrence is frequent in emergent writing. As a teacher, I often get real insights into children’s perceived meanings or non-meanings when they write what they apparently hear. Examples from stories handed in to me for editing include ‘pink tales’ for ‘pig tails’ and ‘Chester drawers’ for ‘chest of drawers’. Do pigs have tails? What does a pig’s tail have to do with hair in two bunches? Other examples, slightly different in linguistic character, include 'ascared' used instead of ‘afraid’, and ‘unthaw’ used instead of ‘thaw’. In these latter examples, the incorrect words actually make better sense in some ways. And, mathematically speaking, one of the more interesting examples I have come across was ‘wrecked angle’.

Students do not expect or seem to need everything to make sense and this is a strength of their learning. They can still function while they wait for sense making to occur: this seems to be true for both language and mathematics learning. This trust in eventual meaning is a necessary attitude to have in mathematics. It allows learning to move forward to the point where it can make more sense.
Jackie (4 years 4 months) and I are waiting in the car for Jackie’s dad to return from shopping. He has just dashed in to get something. As we wait, I ask Jackie about her (intransitive) counting.

M: Let’s see how high you can count.

J: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 14, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 21, 26, 27. There! How high can you count, Mommy?

M: Really high, but it would take too long. Let’s just count to thirty, okay.

J: Okay.

J and M: (We count from 1 to 30 together. She can do it with me.)

This provides a good example of the power of ‘scaffolding’. Jaclyn cannot count successfully to thirty by herself, yet has no difficulty counting with me. It is also a reminder of the strong, supportive role of sound and listening. It is similar to the times when I can sing along with a song on the radio, but cannot sing it by myself, even though at any given instant I know ahead of time what the next few words will be. Lyrics that seem so well known can nevertheless become lost without the support of the song itself.

Although Jackie has missed saying ‘thirteen’, she has kept the right number of word sounds by repeating fourteen. (There is a sense in
which the sounds themselves provide the ‘things’ being counted, though children will sometimes count with syllables e.g. ‘se-ven’ being two sounds when counting objects; van den Brink, 1984.) I find it interesting that thirteen used to be the number she counted to consistently, and now it is the one that she often skips and twenty by itself does not feature at all. At this time, I cannot offer an explanation for this. It is also noteworthy that ‘twenty’ is not named, even though Jackie has demonstrated a knowledge of ‘twenty’ when it is combined with another number word.

Jaclyn (4 years 5 months) is sitting counting (intransitively, aloud) to herself. I take notice.

J: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 14, 15, 16, 17, 17, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, ...

then she goes on to say a series of numbers in the twenties.

As in the previous anecdote, Jackie did not go back to teens or units once she reached twenty-nine, nor used the names ‘twenty’ or ‘thirty’. She said twenty-one through twenty-nine, but did not name twenty by itself. I think this is because she knows the number pattern one through nine. Twenty by itself has no pattern to fit into, the decades perhaps simply seeming unrelated words for her.

As a teacher, this struck me as odd, because I thought that counting by tens was really easy. Intransitive counting by tens is one of the first count-by’s that is practised in school after counting by one is established. Perhaps these names are hard to remember because
they are not yet part of an established pattern. Jackie also indicates that she knows the right number of chunks for the numbers in the teens, although both ‘thirteen’ and ‘eighteen’ are unsaid in this incident.

At this time, Jaclyn was consistently counting fourteen twice and omitting thirteen. I think one of the fourteen was acting as a placeholder for thirteen. Jackie seemed quite happy to sit and rehearse: she was not looking for feedback. This may be an attention-seeking activity, but it is also one that sustains itself, seemingly just for the pleasure of counting. The power of repetition and rehearsal is being downplayed in today’s ‘for understanding’ type of mathematics teaching. There is still an important place for it. I also notice that Jackie started her counting sequence at zero. This is unusual for her, since she usually starts at one. The importance of zero and naming it as ‘none’, or ‘nothing’ is apparent throughout this collection of anecdotes, and, although I note it here, it is one of the areas I am not developing in this thesis.

Jackie likes to sit and practise and often her practice is out loud. Does the sound help by giving feedback? Does sound help to keep her company? Does the sound provide reinforcement? Perhaps any and all of these are true.
Jaclyn (4 years 4 months) and I are driving to day-care in the morning.

J: Four, three, two, one. ... Five, four, three, two, one. That's easy. Remember when I could only go 'three, two, one'?

M: Yes. Can you start at six?

J: Six, five, thr [almost says "three"], four, three, two, one.

M: Seven?

J: Seven, six, four, I mean five, four, three, two, one.

M: Eight?

J: Eight, seven, six, fo [almost says "four"], five, four, three, two, one.

M: Nine?

J: Nine, eight, seven, six, five, four, three, two, one.

M: Ten?

J: Ten, nine, eight, seven, six, five, four, three, two, one.

[These are all said deliberately without rushing or speeding up.]
Jackie noticed and remarked upon the growth in her ability to count backwards. I am interested to see that she was able to reflect on her own growth. When prompted, she counted backwards perhaps twice as far as she originally intended. I believe that the sound of counting was providing feedback. It let Jackie hear when she was saying a number word out of sequence and allowed her to correct herself. She was using sound both as a way of displaying her counting prowess and hearing her counting sounds helped her to co-ordinate and control her count. She was also rehearsing earlier reverse sequences by subordinating them to each new task (see Hewitt, 1996).

Children often love to hear stories about when they were little or younger. Such stories in our family often start: “When you were much littler, I remember ...”. Jackie is telling her own story about counting, and letting me hear it. I do not know what the origin of her self-comparison was; I was only able to hear the verbalised assessment, “Remember when I could only go ‘three, two, one’?”.

Both Jackie and the students in my classes value having benchmarks to compare themselves with. Being aware of one’s own learning and being able to make self-comparisons is an area of meta-cognition that I encourage at school, and one that is shaped at home by “remember when” stories.

Control does not seem to be an issue often. In our case, sometimes Jackie controls, sometimes I do, and sometimes the external environment does. More often, however, it seems to be an interaction of the three that results in the opportunities being seen and seized.
When Jackie was younger, I was more often the mathematical opportunist; being more aware of the possibility of mathematizing than Jackie. As we continued to explore experiences using our mathematical lenses, I noticed that Jackie was increasing her ability to focus activities mathematically. She became more able to distinguish mathematical situations and draw upon them as means of both pleasing me and getting my focused attention while satisfying herself at the same time.

**Internal counting**

Jaclyn (3 years 7 months) has selected *Bambi* as her bedtime story. I am reading the story to her and we are at the scene where Bambi meets Thumper. I explain that Thumper probably got his name because he likes to thump his foot. I use my hand to demonstrate.

(/ means one thump)

M: Like this ///

J: Can I do that?

M: Sure.

J: ///

M: Jaclyn, can you thump just like me? ////

J: ////

M: Try this //
J: ////////// Like that?

M: Yes, nice and loud like that, but can you do it the same number of times as me? Let's try.

M: ///

J: ///

M: ////

J: ///</

M: /////

J: ///</

M: //////

J: ///</

M: //

J: //

M: Great! You can do it exactly like me. [We hug and rub noses and laugh about how silly that was.]

Later, Thumper slides on the ice and makes thumping noises there.

M: /////

J: ///// [After the thumps, she spontaneously hugs me and rubs noses.]

This is an instance where sound was used to enable patterning by copying. In order to remember the model thumping, I had to count
internally. In order to know if Jaclyn's were the same, I had to count as I listened. I also listened to the steady beat of her response – It shared the same tempo as my original thumps.

I wonder whether Jackie counted like this, as I did, as she made the thumps, or if she did something else – heard a pattern perhaps, or an unnamed number of thumps. I have no access to her thought processes as I write this, and am uncertain if asking her at the time would have provided me with an answer. My point is that hearing and repeating was a way to draw attention to number and to beat. It was also a way for us to have a bit of fun and show affection; two important components of home learning. Pleasure and learning are so often companions.

Jackie (4 years 3 months) and I were reading *Jack and the Beanstalk* as her bedtime story. We are at the part when the giant is coming.

M: Thump, thump, thump.

The next time this part comes up:

M: Thump ...

J: [Joins me with ...] Thump, thump. You need to say it three times.

It seems that Jaclyn is not just using the rhythm of repetition, but has the ability to count internally. She was able to add on the amount needed, two more thumps, and state what she felt was wrong; you
need three thumps in all. I must have paused too long and she felt that I was stopping after saying only one thump. Here the presence of my silence, the absence of expected sound, was her cue. This alerts me to the potential comfort of routine and repetition. Anybody reading to a young child who has ever changed the wording in a much-loved and much-read story will probably identify with this.

Again, there is a sense of equality in the interchange. At home, with me, Jackie can freely correct and explain. At school, there is often a sense that the teacher is the authority and that students need to be very careful if ever they offer a correction. Yet her offering allows me to see/understand a good deal about where Jackie is in her thinking and in her mathematical formation. At school, the same can be true.

_Transitive counting (almost)_

Jackie (3 years 8 months) and I are outside on the deck, throwing a beachball back and forth, playing catch.

M: Let's see if we can count while we play catch. Shall we try to go to a hundred?

J: Yeah!

We count together until we get to sixteen, then it is obvious that mine is the guiding voice.

M: 20, 21, 22,
J: [then chimes in] 23, 24, 25, 26. [She listens while I continue a few more and then once again joins in.] 34, 35, 36, 39. [She hears the difference between her number and mine, 37, and listens again before picking up counting after I say 41.]

J: [Counting with me] 42, 43, 44, 45, 46, 47, 48, 49. [At this, she gives a big smile and listens for all of the 50s until 61, then she jumps ahead of my voice and we alternate.]

J: 62
M: 63

J: 64
M: 65

J: 66
M: 67

J: 68
M: 69

Jaclyn did not give a number for 70. She paused and listened. This was probably because she did not know what to say, but possibly because she was distracted by dropping the ball. It is important to remember that we were playing catch as our main activity. Once the ball seemed to be getting away fairly frequently there was a lot of
chasing to get it. Attention to, and concentration on, counting was easily broken.

At home, learning can result from a whole activity in a whole context. The counting was made part of the game, but was not the main focus. The pleasure of being together as well as the hand-eye co-ordination of ball catching and throwing were equally important.

Note the success evidenced by Jackie’s smile after joining the counting of the 40s. She seemed confident that she knew the counting pattern. Then, she listened throughout the fifties seemingly to confirm that the pattern continued. We then counted in an alternating manner, different from our previous duets, for the sixties.

She did not yet know the names of the new decades (from twenty), but could count according to pattern once the decade was identified. She knew to listen for the naming word and then could quite capably go on. Knowing what to listen for is a big part of this type of learning. Jackie was aware that the sound patterns changed and that by listening after the ‘nine’ part, or at any time when she heard a difference between her words and mine, she could pick-up the pattern again.

We continued catching, but both the game and the counting were falling apart. I continued the count and Jaclyn occupied herself catching and chasing. At “100, ta da!”, we stopped and I went to make lunch. Jaclyn went outside to see if her dad would play catch.
A waning attention to sound seems to be a key step between intransitive counting and transitive counting. The catching of the ball could simply be a calibrating trigger to say the next number word, or it could be seen as counting the number of 'catches'. As Jackie moved more toward transitive counting, the sounds and sequence of number words became automatic and attention seemed to be more on the strategies of keeping track of objects that had been counted and separating these from those objects still to be counted. The move to transitive counting is not smoothly linear.

Within the course of this study, Jackie moved back and forth between intransitive and transitive counting. She counted both ways, sometimes for practice and at other times to discover an amount. Sometimes she counted intransitively but incorrectly. Sometimes she counted transitively but incorrectly: wanting to know 'how many' does not guarantee the helpful number sounds are necessarily available.

Transitive counting

Jaclyn (3 years 7 months) has selected *Bambi* as her bedtime story. Before I begin reading to her we look at the cover picture.

J: How many animals?

M: How many?

J: One, two, three, four, five. [She points and counts aloud.]
M: Who’s the biggest?

J: Bambi. [She says and touches the picture.]

M: Who’s the highest?

J: That one. [Touches a bird on a branch above Bambi’s head.]

In this incident, I made no comment on Jackie’s correct counting of the animals. Both she and I knew that she could count them, using one-to-one correspondence and counting out loud. Her question to me, “How many?”, was almost a ritual we used to discuss the pictures. The uttering of this question provoked an opportunity to talk about a subject of mutual interest. This was why I chose to deflect her question back to her rather than answer it myself.

I found that once counting has moved beyond intransitive counting, neither Jackie nor I paid it much attention. Further to this, it seems that counting became a type of background activity once transitive counting had been established. The counting in this episode provides an example of this. However, this is not black and white because ease of transitive counting varies substantially with the size of the quantity, whether movement is involved, the complexity of the shape(s) being counted, and whether a pattern is apparent or not.

I was curious to see how the comparative terms ‘biggest’ and ‘highest’ would be interpreted by Jackie. I wondered if she might be confused by the language because these words can sometimes be used interchangeably, and the ‘high’ notes are ones to the right on a
piano, yet all the keys are at the same level above the floor. Walkerdine (1988) speaks frequently of everyday language and changed contexts sometimes causing confusion in mathematical understanding.

When I arrive home from school Jaclyn (3 years 8 months) is 'reading' a book.

J: Look, Mommy, I can count the trees. [She waits for my attention.]

J: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 14.

Thirteen seems to be a 'break' for Jackie lately. I have been hearing this without recording each incident. However, after noticing this a few times I decided to pay attention to her counting sequences for a while. I wanted to hear how they changed and developed. (Previous examples of this have been presented in the subsection headed 'intransitive counting'. They indicate that the omission of thirteen continued for some months.) I did notice, however, that she was saying one number for one tree, operating as an enumerator of trees. She was demonstrating a knowledge of one-to-one correspondence even though the number sequence was not accurate. (For further discussion on this, see van den Brink, 1984.)

Jackie was practising counting by herself when I came home. No one asked her to do it, no one enforced the activity. She knew, though, that reading a book and doing mathematics was a good way often to gain my attention. She had been rehearsing, and one reason, I
believe, was to gain my attention by showing me her counting when I got home.

In addition, Jackie seemed to desire to account for objects. She has learned that knowing quantities is often useful for the types of conversations she and I frequently have; for example, making comparisons about more and less, and how much or how many. I think she also found counting a pleasurable activity. She seemed to enjoy quantifying her world, and would count items even when no one else was seemingly listening or paying attention.

Jaclyn (3 years 8 months) and I are looking at the back cover of the book *One Snowy Night* that we have just read.

M: Can you count the snowflakes, Jackie?

J: [She counts from 1-10] Mommy, there’s lots and lots of snowflakes.

M: Oh, can you count how many?

J: No, there’s too many lots.

M: Yes, there are probably over one hundred.

J: Yeah, lots of lots.

I was interested to see how high Jackie would count, and whether she would have a strategy for keeping track of which snowflakes she had counted. Jackie seemed to realise, after counting to ten, that there were too many to count and she was not interested in
developing a counting schema for larger numbers. Reading into her novel phrases “too many lots” and “lots of lots”, I can now hypothesize the beginning of grouping or chunking for counting. These interrelations, however, were not pursued by either Jackie or myself at this time. I am still thinking about Jackie’s use of the word ‘of’ in “lots of lots”. She earlier said “lots and lots”, which seems simply cumulative; ‘of’, for me, signals multiplication.

Transitive counting as a problematic activity

There were many cases in this study when transitive counting became a problem-solving exploration. In Chapter 4, I referred to some incidents that involved transitive counting. In one of these, Jaclyn was counting plastic milk bottle tops that had been arranged in a circle shape. She over-counted the number, counting one of the tops twice. When I said, “You counted this one twice”, she then put a non-counting finger on a starting place, and successfully counted round using a finger of her other hand to count with. Although she was using sound to say the numbers aloud, I do not believe that sound was the most crucial element in getting the count right.

In this instance, Jackie needed to recognise her problem of over-counting within a closed circular shape, find a strategy, and test it out. Transitive counting required her to attend to more than the sound of the number words. The number words Jackie needed to identify the quantity of tops were already automatically available, so her attention was able to be on the one-to-one allocation of number word to object.
Another instance of this occurred when Jaclyn (3 years 8 months) and I were visiting my mother in the hospital. We had taken my mom down to the cafeteria for a snack and a change of atmosphere. Jaclyn has chosen some cheese puffs to eat.

M: Here, I’ll dump some out.

J: That’s a lot. Let’s count.

J: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 18.

M: 14! Wow, that is a lot!

J: You can have some.

M: Let’s share like this. One for you, one for me, one for you, one for me, ... [Jaclyn takes over until the pile is divided.]

M: We each have seven.

J: Are we the same? [She pairs them up end-to-end.]

M: [After her action.] Yes.

M: Now, look at this. [I spread mine out.] Do I have more, or do we both still have the same?

J: Let’s see. [She moves hers out to touch mine.] The same.
Jackie's question in this incident is like the one when she asked me to name a triangle (discussed in Chapter 4]. As there, she knew the answer to “Are we the same?” and showed me before I could answer. Though Jackie is counting the cheezies one to one, her counting words break as an accurate count at “fourteen”. She says “eighteen”. In this case, I simply stated the correct amount and she accepted the number. When she heard me say fourteen, I believe it fitted her number counting knowledge. She did not need to check or confirm this with me. Hearing the correct number seemed enough. When she heard “fourteen” after she had said “thirteen”, she did not insist that it was “eighteen”, nor did she recount, both things she has done at other times. I believe that hearing the sequence ‘... 11, 12 13, 14’ sounded right. I wonder whether she even heard herself say “eighteen”.

After Jackie had paired our snack bits up, I was unable to resist a Piagetian conservation-type activity. However, because Jackie chose to ‘check’ before she gave her response, I am uncertain about her thinking for this task. For Jackie, answering the question required a problem-solving strategy. She surprised me. She did not simply say that they were the same or different, and she did not use oral transitive counting. Instead, she chose pairing as a way to check the quantities: one-to-one correspondence between sets of the same things, rather than one-to-one correspondence between objects and spoken sounds (the number words). Her strategy allowed her to show that the amount was the same without using numbers or sound. This is one of the few times that I was really aware of acting
like a conventional researcher: I was doing a standard, classic even, research task. But Jackie did not know that: to her I was still simply being mom. In this incident, because I was aware of Piagetian conservation, I was surprised by Jaclyn’s problem-solving approach. She did not see the question as needing a yes/no answer: she saw it as another invitation to explore.

In an article about using young children as informants, Hatch (1990) elaborates on some areas that are problematic in classroom interviews. He writes:

Based on data collected in preschool and kindergarten studies, four problems that can threaten the quality of interviews with young children are identified: the adult-child problem, the right-answer problem, the pre-operational thought problem, and the self-as-social-object problem. (p. 251)

These areas, he maintains, are problematic if researchers try to interview children as if they were adults. He cautions researchers to interview children after they have established a personal relationship, and to interview them asking questions that they can answer which will have a base in their direct concrete experiences.

As a researcher looking at her own daughter, many of these problematic situations are avoided (another strength of being a genuine participant in an ethnographic study), though the issue of questioning (in relation to ‘the right answer’) is alive for me. Nevertheless, Hatch’s work provided me with a reminder that Jaclyn’s perspective is one based on her experiences, whereas mine
will be based on a much wider range. Those perspectives will overlap but are different. For instance, there are recorded incidents when we have both participated in ostensibly the ‘same’ experience, but have viewed it quite differently (see, for instance, the anecdote about her invented word game ‘yes means no and no means yes’ in Chapter 6).

SOUND PATTERNS

I am by the telephone, in the kitchen, and I have been talking to a friend to find another friend’s new number. I’m writing it into my book, and Jaclyn (3 years 9 months) has been listening and watching.

J: What’s your number, Mommy?

M: 264 02 64 [“two six four, zero two, six four”].

J: What’s yours, Daddy?

D: 264 02 64

J: What’s mine?

M: 264 02 64

J: They’re the same! 264 02 64

I was struck by two things, first that Jackie could recall the number strings enough to tell that they were the same, just by hearing them. And, secondly, that I did not know if she really knew her phone number, or was just able to rattle it off out of short-term auditory memory. The context of asking questions is again Jackie’s way of
being included in our adult conversation. She actually took over and guides the talk for awhile. It is unclear why she asked three questions. Could it be because she truly did not realise that we all have the same number, or that she wanted to include everyone, or that it confirmed it as something we all share, or that she liked to hear the sound of the repeated responses, or because she was simply checking? Did the spoken emphasis I transcribed as an exclamation mark indicate surprise? Perhaps it indicated pleasure in her expected answers being given, thereby ‘proving’ that she was right.

Telephone numbers have a certain sound to them. People usually say them with a special rhythm that makes them easier to recall than with an equal cadence of seven digits. For example, with this particular phone number, I found that people had trouble writing it down if I said, “2, 6, 4, ... 0, 2, 6, 4.”. The two sides seemed too similar and people always checked if they had heard right. If I said, “2, 6, 4 ... 0, 2, ... 6, 4 or 2, 6, 4, ... 0 ... 2, 6, 4”, this problem did not occur. Jaclyn repeated the rhythmic cadence as well as the digits when she confirmed that all three were the same.

I wonder if she would have been able to repeat a series of seven numbers that were given with different rhythms or that were part of a series that she had not heard before. Telephone numbers are not numbers in the arithmetic counting sense - they are not the transitive count of anything. But, they are a series of number words, said in a predetermined order; and, like the early numbers in intransitive counting, they need to be memorised.
It is late in the morning and Jaclyn (3 years 9 months) and I are playing at the table, using milk tops. I make an equilateral triangle that has three tops in the first row, then two in the one beneath it, then one below that. Jaclyn looks over and becomes interested in what I’m doing.

M: See. Three, two, one. Can you do one by yourself? [I mess mine up.]

J: Three, two, one. [She says this almost as a ritual incantation as she places her tops, but her shape does not look the same as mine did and she makes another.]

J: Three, two, one. [This shape has a triangular three at the top, then two, then one.]

J: It’s still different.

Her speech strategy of repeating “Three, two, one” was a good one; it just did not happen to work here. This sounds like counting down, but it is really remembering the number pattern I stated. Jackie tried to build on the strategy I presented. She invoked verbal language as well as sight memory. Perhaps, without the verbal, even the number of tops used might have varied. Sound is much more powerful than I expected earlier in this collection. With all the anecdotes before me, I am able to acknowledge that hearing what is happening is very significant, both for Jackie and myself.

Although Jaclyn had retained a mental picture that allows her to judge that her shapes were not the same as the original one, she
could not seem to reconstruct the desired shape from her mental image. This is often the case during early exposure to something; you can tell that it does not look right, or sound right, but cannot see how to fix it. If I had not messed up all of my design, if just the top three, for example, had been left, would that have provided enough of a trace to allow reconstruction? In fact, I wonder why I did not just leave mine alone for her to see and use if she needed.

Why was it important for me to see if she could do it by herself? In school mathematics, children are often shown what to do and then immediately expected to be able to do it by themselves. As a teacher, I need to remember to let students see models, work from partial models, and work together. The road to ‘by yourself’ is a long one (and sometimes not worth taking).

Jaclyn (3 years 11 months) and I have earlier been making a paper chain for a birthday decoration, and now Jackie’s having a bath before bed. I’ve been rushing in and out of the room, and have stopped to see if she’s ready to get out.

M: Well, are you all clean? Ready to get out?

J: Yes, I used water then soap, water then soap, then water. That’s a pattern. Maybe we should draw that.

M: How would you?

J: Like this (around on tub) ^^^^0^^^0^^^ Then I’d cut them into strips and make a chain.
Jackie saw a connection between her love of patterns and the artwork she has just been doing for her birthday decorations. She also developed a way to represent her spoken word pattern as non-standard symbols. Applied mathematics has early beginnings, and 'transfer', in this instance, seemed natural.

Many times, at least at school, children do not readily make connections or transfer learning from one situation to another. Often connections that seem obvious to an adult can seem two completely different concepts to a student. For example, I had a student write in his math journal that at first he found rounding to the nearest hundred really hard, but now it’s easy, "But I still can’t do rounding to thousands. It’s really tricky".

However, connections are sometimes made. I have found that mathematics/art combines very well and seems to provide fertile ground for transfer of learning as long as the mathematics has been made visible, and not camouflaged as merely magical designs. In the above example, Jackie made the connection between the patterning of colours she had been doing in her birthday chains, and the pattern of washing she’d used to bathe herself. She connected from sound pattern to drawn symbol pattern to a chain that could be made to represent her activity.

I want to emphasise that the sound of the pattern was heard. Jackie told me her pattern first, and then suggested that she could draw it. Sound often precedes representation for Jaclyn. I need to keep aware of this. I am really only now becoming appreciative of the scope of
sound and listening, in conjunction with doing, as a contribution for learning mathematics.

An area of the school curriculum that often works similarly is activity that involves tiling and tessellations. Rena Upitis and I found this to be true in the math/arts projects that we did during the 1992-93 school year. (This work is reported in Upitis, Phillips and Higginson, 1996.) We learned that when the mathematics concepts we were going to extend through a project were well established through discussion and activity, then the project became meaningful and provided many references to both art and math. Whereas, if the concepts were not well established, the project became more craft-like and discussion was more at the doing of art level, and it was harder to get meaningful talk about the math involved.

We also found that the children who had more experiences with the relevant mathematics prior to applying it to a project made the most successful products. Often activity and words can be combined in patterns that provide an easier transfer of knowledge. Speaking and listening can bridge the gap between contexts and can promote concept extensions.

Jaclyn (3 years 11 months) is trying to convince me it is time to get up.

J: When are you getting up, Mom?

M: At 8:30.

J: What will it look like?
M: Eight-three-zero.

J: Oh. It’s eight-one-four.

J: It’s eight-two-one.

J: It’s eight-five-five.

M: What? It went eight-two-one to eight-five-five?

J: Yes. ... I mean no! It’s eight-two-two. Now it’s eight-two-three.

J: We’re waiting for the middle ‘two’ to be a three.

M: Right.

Those digital twos and fives proved really confusing for Jaclyn (as they were for Higginson’s (1980) daughter Kate). I believe that my response helped her to rethink her interpretation in two ways. First, I sounded incredulous and that would have alerted her to an error, and then I echoed her statements back to her. I think that hearing the numbers may have helped her to see her confusion of two and five. What she heard me say back to her was not what she expected. She knew enough about number patterns to recognise the big jump taken when her attention was drawn to it.

Sounds can be very important for those instances when visual cues and/or visual memory is not yet reliable. As indicated here, the spoken word combined with a visual symbol can provide a stronger learning environment than either one on its own.
Jackie was also using a strategy to identify when my time to get up had arrived. She knew that the middle number was the critical one. She is starting to be aware that not all numbers are used equally, that sometimes a certain position is more important than another. She is starting to learn the value of attending to which part needs her focus.

Jaclyn (4 years 11 months) and I are in the kitchen. I am preparing dinner and Jackie is playing with mini-cookie bears left over from her lunch.

J: Mommy, come look at what I made. [She has made a pattern with her bears. She has alternated white and brown bears as well as alternated them head up, feet up.]

M: Oh, I see a pattern. Do you?

J: Yes. White, brown, white, brown, white, brown, white, brown.

M: There's another pattern. Can you see it?

J: Yes. [But she does not elaborate another pattern.]

M: Just look at these ones. They go: head up, ...

J: [Takes over immediately ...] head down, head up, head down, ...

M: Wonderful. Two patterns in one.
I wanted Jackie to realise the complexity that there can be in patterns. At my first prompt, Jackie was willing to believe that there was another pattern, but she did not appear to find one. I notice that she did not try to correct me here. Often, if I have indicated something that she does not believe is so, she will tell me. So, she must be willing to believe that there is more to patterning than she knows at the moment. She is willing to allow me to guide her observations. Here, when I started to prompt the pattern verbally that I saw, she did not even need a full chunk, she picked up on the pattern characteristic after hearing just one of the pair.

Prompts are valuable in extending ideas. If a child is ready, or 'set' in the psychological meaning of the word, then a prompt or hint opens the door to new learning. If the child is not ready, then I have found that the prompt will likely be met with a blank look, a puzzled inquiry, or will be ignored.

Sound is very strong for pattern recognition, as is an understanding of what a pattern might sound like. I am referring to Jackie's response to my "head up" lead. She took over with, "head down". She might also have said "feet up" or "head not up," but, to be seen as patterning in the school sense of the word, she needed to say something that could alternate in a rhythm with what had already been said. Patterning using sight and sound is a skill that is built on in mathematics, and it is an activity that Jackie enjoys doing enough to do it by herself.
Jaclyn (5 years 1 month) and I are in the car driving to my sister’s. While we are driving, we often play games such as ‘I Spy’, or ‘What plus what?’ (see Chapter 6) to help pass the time. Jaclyn is writing on a pad of paper as we drive.

J: What’s eleven?

M: A one and a one.

J: What’s twelve?

M: A one and a two.

J: What’s thirteen?

M: A one and a three. Do you see a pattern? We had one and one, one and two, one and three.

J: Next will be one and four. Then one and five, then one and six, then one and seven. [Jaclyn writes these and then says] Mom, can you read these numbers?

M: No, I’m driving, but I think they go to sixteen.

J: No, to seventeen.

M: Great. I’d like to read them when we get to Auntie’s, okay?

J: Okay.

Notice that I told Jackie that eleven was a one and a one, and not one ten and one one. She was writing, and I chose what seemed to be the
simplest way to tell her what to write, and did not use the opportunity to try to develop an understanding of place value and how eleven can be interpreted in that model. I am realising that patterning can be a way of understanding, and can be a tool for prompting concept extension.

Jaclyn used my prompts of "Do you see a pattern?" and a few spoken examples to enable herself to recite the next numbers in her written sequence. She also demonstrated transfer of this knowledge to standard form representation and naming. She spoke of her written numbers going to seventeen, not to a one and a seven. I am appreciating the complexity of written number representation and valuing the role that patterning, both oral and written, can have in this.

Jackie (5 years 4 months) and I are getting ready to go out. She is talking to me while I'm brushing her hair. (In what follows, 'ti' and 'ta' are spoken sounds.)


J: That's one of my patterns, Mommy. Did you get it?

Patterns can be visual and/or oral and/or tactile. Patterns can be seen, in this case, as both musical and mathematical. I think that Jaclyn is using vocabulary that expresses her awareness of this. She has had a Kodaly music lesson today, and typically, the group listens to and makes up rhythms and compositions. Jackie realises that this
is a pattern, a term that she and I use both when we are speaking musically and mathematically; and she is aware of the connection between the two.

Integration is a big theme in education at the moment. Theories suggest that exploring a topic in many contexts is a self-reinforcing way to learn. At home, learning is practically always integrated and in context. That is one of the major differences between home and school learning. At school, we are often in the position of creating contexts for integration to occur.

This usually entails identifying a theme and building a variety of activities around it. At home, the themes present themselves and are immersed in the activities that produce them. Perhaps at school we should be less quick to provide packages of integrated units; we might try building on what is already there and let the students, each in their own way, provide their own integration context. Life is like that, and one of the aims of current mathematics education, as stated repeatedly in this province's integrated resource package for mathematics K to 7 (B.C., 1995), is to make learning reflect 'real' life.

It is just after dinner and Jaclyn (5 years 5 months) is asking me how to spell the names of some friends she wants to invite over.

J: How do you spell Roberta?

M: R-O-B-

J: Rob, hey, that's like Robin.
This looks like language, but with my mathematizing awareness, I can see that it also patterns like mathematics: that is, it contains sound elements of mathematical pattern. Once again, it is clear that at home the links that occur are often between subjects, not just within subjects. The same skill that lets Jackie hear the sameness in the spelling of Roberta (her friend's name) and Robin (her sister's name) will allow her to hear similarity in, for example, one hundred twenty-one starting the same as one hundred twenty-two. This, I believe, is an important understanding for me to have. Sound links language and mathematical patterns so well, and this is something I can build on and emphasise when teaching.

IN SUMMARY: LEARNING, LISTENING AND SENSE-MAKING

These anecdotes provided me with an opportunity to see, and reflect on, some of the sounds of mathematizing. It appears that Jaclyn often used sound as an entry into mathematical sense-making. Her use of sound was often game-like and so was mine. We made patterns with sound rhythm, sound alerted us to sameness and differences; and sound drew attention to transferred knowledge. The chapter title actually contains a veiled claim that I believe the anecdotes have borne out: that sound matters and that it allows a sound basis for patternning.

Throughout this chapter, there have been stories that helped me see the usefulness of sound as a teaching and learning tool. Jaclyn used sound to help identify and continue a pattern. She used it to help replicate patterns. Jaclyn needed to learn what to listen for in order
to use sound effectively. She learned to listen for expected rhythms, breaks in sound, and the overall pattern. If one of these was not what was anticipated, then Jackie was alerted to a possible problem.

Additionally, this chapter has shown how equal the opportunity for learning by leading and learning by following is at home. Jaclyn had no difficulty correcting me or explaining to me where I went wrong. Enjoyment and affection were key elements in home learning and the focus was usually provided by the overall activity and not the specific skill.

Sound was seen to support visual memory, short-term memory, and learning how to see. Sound was a factor in practice, self-reasoning, and meta-cognition. Quite significantly, sound often preceded understanding and representation of that understanding. And, in the case of counting, once a level of understanding was sufficient to allow the counting sequence to be automatic, then the role of sound seemed to be less important. That is, it seemed that once comfort was attained within the counting sequence, then sound became merely present rather than a necessary condition. Then, the positioning of fingers and/or the value of sight became the senses chosen to support mathematical counting situations. Accurate counting became more of a problem-solving situation: recall Jackie placing her finger on the milk tops to enable accurate counting.

Listening and speaking, combined with seeing and/or doing, is a powerful learning strategy. In education, the voice is often the sound of being told what to attend to rather than the sound of what is
attended to being voiced. Saying what is noticed and acting on patterns of sounds is, it seems to me, a good way to build the opportunity for sense-making. Practice is often the repetition of known sounds, and can be seen as self-reinforcing and self-sustaining. Jaclyn builds practice into her own play; and in the next chapter, on games, I want to look more completely at the games Jaclyn played that enabled her to practise and employ mathematical ideas.
How do people learn a new skill? What roles do watching, listening, speaking and doing have in learning? I have been exploring some of the aspects of learning through the senses by reflecting on the anecdotes I have collected. Over the two years, I became increasingly aware of the power of the different senses in mathematics. These observations have also provided me with some insights into the role of practice.

The time that is spent in practising something we wish to learn is often very enjoyable and rewarding. When we practise with a friend, there is often lots of laughing, talking and fun. When we practise by ourselves there is often intense concentration and periods of steady repetition. What drives the repetition? What prevents boredom? What is the motivation?

By looking again at some of the anecdotes I have collected of Jaclyn practising, I have become more aware of the self-motivating reward inherent in self-directed practice, of a cycle of practice. Sometimes this practice seemed to be just for its own sake, sometimes it was a method that Jaclyn used to self-talk her way through a new skill/concept, and sometimes it appeared to be a rehearsal before showing what she knew.

In many ways, I find that rehearsal and practice are blending for me. I used to think that practice was drill toward a non-specific goal and that rehearsal was drill for a specific goal, but I find that these
definitions are often blended into one another at home. Drill at home is often rehearsal for sharing a new learning. Practising through rote drill and memorisation are not in favour as ways to learn at school; yet I have observed that children, and Jackie in particular, build repetitive practice into their own conduct.

I want to look at the games Jaclyn and I played. Which of these are games for practice and which are for some other purpose? What does 'practice' look like in a home setting? How are commercial games and non-commercial games used? I want to look at the role of my mediation in these games; and in some of them, the role that mediation had in making the activity into a game. What differences and similarities are there between home and school practice; between enforced, regulated practice and spontaneous, independent rehearsal?

In this chapter, I will look at the game development that Jaclyn was engaged in, how she often made up games that allowed her to develop skills that she used, among other things, to get both my attention and approval. She invented games that she knew would entice me into her play world. She quickly learned how to categorise the mathematical from the non-mathematical, and used this knowledge in her gaming, particularly for situations when I was otherwise occupied and she wanted to engage my focus. Jaclyn's playing of pre-made games changed gradually yet radically over the course of this study, and the mathematics in her game playing also changed in ways that I found to be of interest.
As part of this chapter, I also want to look at the specific development of the playing of commercial games. Young Loveridge (1991) looked at the playing of ‘Snakes and Ladders’ among other games, as part of her studies on factors that can be used to predict success in mathematics. In her study, she reports how:

Games were developed for use in the intervention program which would give children substantial amounts of experience with those number skills which had previously been identified as being strong predictors of later success: i.e. forming sets of given sizes, numeral recognition, enumeration and pattern recognition [...] Games were selected according to whether or not they involved the use of the number skills being targeted. (p. 67)

She also refers to the role of practice in other studies of children using games to improve number concepts: “Hughes (1986) also found dice games to be a very effective way of providing children with a great deal of counting and counting out” (p. 65).

Young Loveridge adds that:

Children were quite happy to do a lot of counting over a short period, because they were highly motivated by the game. Games have an advantage over other means of instruction in that they occur within a meaningful social contest.” (p. 65)

Although I am not using games as an instructional intervention strategy, I also found that Jackie was motivated by the social
situation of games such as 'Snakes and Ladders', and that she would happily play them for a long time (often longer than I really wanted). Jackie also enjoyed creating or helping to create the adaptations that we played. This differs from Young Loveridge's study where the adapted versions were presented already modified to the children.

Hewitt (1993) reports himself watching his two-year-old nephew, Robert, as the latter learned to use a computer mouse to play a game of 'Patience' on the computer. The process Hewitt describes is one of guided learning. He noticed the interventionist technique of his brother Chris, one that involved gradually relinquishing control once a more independent skill level was achieved. Hewitt discusses the game's relation to 'understanding':

There is no prerequisite that Robert understands. Understanding comes through participating. Chris ensures that Robert experiences the playing of the game rather than having it demonstrated or explained. This requires Robert to place a level of trust in Chris. Robert does not know, at the outset, what he is required to do and yet he is prepared to put trust in his father and allow himself to be taken on a journey into the unknown. [...] After the learner has been doing this content for some time, [his] attention can be directed to what [he is] doing, thus forcing an awareness of the mathematical content. (p. 32)

Continuing this idea, he adds that: "Robert may be unaware that he has learnt anything. It may require a separate, reflective, activity for
him to become aware of what he has been doing for some time” (p. 32).

The point here seems to be that children can successfully participate in activities if they are guided and given rehearsal, even though they may not be aware of the understanding that they are developing. Simply by watching his nephew, Hewitt was able to identify a learning situation that could inform both educators and parents about the nature of play and games. Guided involvement is supported by his explanation.

I am also interested in Hewitt’s idea of a reflective activity to aid his nephew to become aware of his understanding. I have identified occurrences of spontaneous reflection in Jaclyn. I am not aware of my actually mediating her reflection, but in her very sense-making she has demonstrated reflectiveness.

The most extensive and powerful account of the mathematics of games, although set in a teaching context, is to be found in a book chapter by Ainley (1988b). As well as challenging many fondly-held teacher beliefs, she also analyses the different ways in which the mathematical ideas interact with the structure and purpose of the games themselves. Ainley observes:

at most, games can help children to learn mathematics. [...] if teachers use games in the hope that the games will teach their pupils particular pieces of mathematics, they will be sadly disappointed. (p. 243)
The most effective mathematical games are those in which the structure of the game is based on mathematical ideas, and where winning the game is directly related to understanding the mathematics. (p. 241)

Ainley also writes of the complex relation between adult and child purposes in mathematics, and contrasts them with reading. When I write about my games with Jaclyn, the variety of purposes achieved is worth attending to.

When children learn to read, they can straight away begin to use reading in the same way, and for the same purposes, that adults do; they can read for pleasure, to get information, etc., and there is a wide range of children's books and comics designed for them to do just that. The same is true to some extent of writing, children can write for their own pleasure, to communicate with others, to label their possessions, etc. [...] When children learn mathematics, however, there is very little that they can do with it, except to complete exercises set by someone else. Mathematical games are one way of providing the equivalent of children's books and comics; within a game there is a context for using some mathematics that you have learned, and that context is real for the children because they can engage with it and the outcome matters to them.

(pp. 243-244)

My study offers an abundance of occurrences when Jackie used her mathematical knowledge, and few of these involved completing
exercises (presumably in a book) set by someone else. I offer these as an addition to set exercises. I agree with Ainley, however, that games provide an environment where mathematics can be used in a genuine way.

Ainley also refers to specifically mathematical ways of thinking in context to which game playing can afford access.

Within a mathematical game, many situations will occur where making predictions (based on mathematical knowledge) is clearly a valuable strategy. [...] Within a game, conjecturing is both natural and safe; games can provide opportunities to talk explicitly about the process of conjecturing. [...] Typically, teachers encourage children to check their own results, but young children find this difficult to do, and can often see little purpose in doing it. In the context of a game, there is a clear purpose for checking your own conjectures; once a move has been made, it cannot be undone. (pp. 244-246)

Finally, Ainley writes about the interactions that games afford the teacher, both to participate alongside their students in the same activity and to observe their thinking in a 'natural' setting.

It would be a pity if the teacher never spent time joining in with the games, or even just watching what is going on, since she would then miss a rare opportunity to observe her pupils doing real mathematics. [...] When children are playing games their thinking is much more transparent. Their actions reveal much about their thinking strategies.
In the context of playing any mathematical game, it is easy and natural for a teacher to question children about their thinking. (p. 248)

And, I will add, that these questions can be genuine and not testing ones. Both with Jackie and the students in my class, I have been able to elicit information about their strategies without raising defensiveness. In a game situation, children are more likely to share their ideas freely, and I believe this is because they do not perceive themselves to be in a situation where making a mistake means 'getting it wrong'.

Many of Ainley's observations were pertinent to my commercial game playing with Jaclyn. We both got engaged in these games: I could observe Jackie's thinking at work and I could ask her about what she was going to do and why. But game playing had two additional effects on the format of this thesis.

First, the games took much longer than the brief anecdotes I have reported until now. And second, because I too was engaged in playing the games, it became difficult for me to attend to what was happening with Jackie. In consequence, I was far less confident about being able to recall the detailed spoken interchanges between us, with the result that a number of the anecdotes reported in the second part of this chapter are descriptions of contexts and conversations and do not include exact spoken dialogue.
INVENTED GAMES

The games that Jackie invented covered many mathematical strands. The areas that I am highlighting in this section mainly concern adding and subtracting to compose and decompose new numbers, patterning to add and to enable counting on, and methods of keeping track of numbers in order to score and be able to skip numbers in a sequence. In addition to this, I will be drawing on some of the word games that Jackie played that had mathematical qualities.

Games of composition and decomposition

It is dinner time and Jaclyn (4 years 2 months) is talking while I work.

J: Two and two makes four. Three plus one makes four. One plus two plus two makes five. Two and two and one makes five.

M: [I do not comment ... but I notice that she’s using her fingers and that her eyes are big.]

J: Mom, write down: “What plus one?”.

Five and one.

Eight and nine makes ten.

Three and two, 1, 2, 3, 4, 5, five!

Three fingers and three fingers. [Touches each to nose while counting] 1, 2, 3, 4, 5, 6.
Eight and nine. Eighty-nine. No, eight and nine.

R and T.

C and eight.

M: What does that make?

J: I don’t know. You do.

M: No, I don’t.

J: Yes, you know.

Here I was witnessing a long series of musings, of playing with numbers and combinations of numbers. Jackie was enjoying her game of using number words, and letters. I was surprised when she started branching out into mixes of letters and numbers, and just letters. So, I asked her about it. (See also the discussion of questioning in Chapter 4.) Her insistence that I knew how to make sense of things that she was doing is quite telling. She expected that there is sense there, and she did not seem concerned that she could not express it or know it. Jackie expected that her combinations have meaning even if she cannot identify all of the meanings presently. She believed that I knew, even when I said that I did not. I expect, with a bit of creativity we could have come up with a representation for R and T and for C and eight that would have pleased us both. She was right in thinking that even if I did not know, I could know.

I find her willingness to trust that sense is there absolutely critical to her developing mathematical thinking. Many students whom I see
floundering with mathematical concepts were those who did not expect it to make sense. For example, when they subtract, sometimes they get a right answer and sometimes they do not. They do not generally know why, they may not even care why. It is as if mathematics is just like that for them - inconsistent. They seldom pause to reflect on their solutions, they merely do something and wait for someone else, or an answer book, to tell them if they are correct. When their answer is not correct, they tend not to analyse what went wrong; they either give up or do it again in the same manner as before.

Jackie was rehearsing various ways of combining numbers. She used the words 'and' and 'plus' as synonyms. She uses 'makes' sometimes, but was happy leaving it out too. Eight and nine presented a difficulty. She knew the sequence eight, nine, ten but realised that it is not really what she needed for 'eight and nine makes __'. She used her sight, fingers and nose for counting out loud to help with lower digit adding, but did not seem to have a strategy for dealing with sums that involve digits adding two-hand digits. She tried combining the digits to make a new number eight and nine. Eighty-nine. But, she indicated that this was not what she intended either.

Why did not I step in and help? I was busy. I was watching her while I prepared dinner. I could have suggested that she use milk tops, and if that strategy made sense to her, she would have asked to get them out. Mostly I was interested in where she would go with this practice session. She moved on ... to letters and letter number combinations. Jackie was clearly playing with sounds and sense. I
was happy that I had left her experimenting alone. I, once again, earned insight by attentive silence.

It is Mother’s Day, in the afternoon. We are in the car, returning after a day spent sailing. Jaclyn (4 years 3 months) and I are sitting in the back.

J: What’s one and one?

M: Two. What’s two and two?

J: Four. What’s five and five?

M: Ten. What’s one and two?

J: Three. What’s eight and eight?

M: Sixteen. What’s three and two?

J: Five. What’s ten and ten?

M: Twenty. What’s one and two and three?

J: No. Go ‘what’s m and m?’.

M: Okay. What’s three and four?

J: I don’t know.

M: Let’s see. [We use our fingers. I put three up on one hand and four on the other.]

J: 1, 2, 3, 4, 5, 6, 7. Seven.
Later, the same day as above, we continue:

**M**: What's one and one and two?

**J**: Four. [No fingers, no pause, no objection.]

**J**: What's one and one and one and one and one and one and one makes ...

**M**: Seven.

I was trying to write much of this in the car while we were moving. We were also playing while we were driving, so I did not closely watch which of these were done with fingers, and which without. But I did note that there was a mixture. Jaclyn did not want me to change the game to become three addends. I was interested when she corrected my attempt to change the game by using letters “m and m” to put me back on track. I think she was indicating that this was only an example. If she had used numbers, then I probably would have given an answer. She wanted the concept clarified, not a question answered.

She also used two letters that were the same. This is consistent with her preference for using double digits. The sound of the exchanges was what she was drawing my attention to. She wanted me to notice ‘__ and __’ was what she was offering to play; not ‘__ and __ and __’. Using letters and sound made sense; it got the message across that her rules were for a two-addend game. It also revealed a highly algebraic mode of thinking.
However, later in the day she was willing to let me try three addends again, and showed that she could answer. She even asked me a multiple ‘and’ question. Since she presented so many ‘ones’ to me, it made me wonder if two ‘ands’ were perceived by her as complex. That is, I wonder if anything more than one ‘and’ is considered the other type of question.

Later that same month, Jaclyn asked to play a game she called ‘What plus one?’ (see Chapter 4). When she first asked to play ‘What plus one?’, I was unsure whether she meant the same game as ‘What and what?’ which is described in the next anecdote. Her first question asked me to find the sum of two numbers, so I decided at the time she was using a different name for the same game.

As I write this, however, I am now less sure. Perhaps I was over-generalising. I notice now that the questions she posed for ‘What and what?’ use the same digits, e.g. two and two, eight and eight, ten and ten. Even when she extended beyond two digits, it was: “one and one and one and ...”. Whereas, for ‘What plus one?’, she asked, “five and four is ...”. Perhaps for her there are two distinct games that I am confusing. In the anecdote prior to this one, Jackie asked me to write down “What plus one?” and gave a series of addition questions: five and one, eight and nine, three and three, R and T, C and eight. All except one of these used different addend pairs. Perhaps this was her intended difference and I failed to see it until three years later. Jaclyn attempted to block me or alert me to my confusion sometimes, and at others went along with me. Interestingly, this is what I do with her also.
The game itself is one of practice. Jaclyn is practising number combinations seemingly with no particular goal other than the amusement and accomplishment of finding the answer, the pleasure of creating questions, and holding my attention. When children are taking the lead in an activity, I have seen a reluctance to let an adult try to change the rules (Phillips and Crespo, 1996). Jackie was willing to follow my change and even extend it when it was my game, but she did not want me to change her game.

Children, and indeed most adults, seem to feel that there is an unstated contract about playing by the rules. The rules that are set out at the beginning are not to be changed during the play for that session of the game. You need to start over to make a change. This is not consistently true as I have sometimes seen games changed to become easier, harder or quicker with mutual agreement of the players. But, when the child has made the game, there is great reluctance to let someone else change it, without negotiation. At home and at school, this is one of the times when children will stand up to adults and deny adult authority.

Further to this, however, there are times when Jaclyn will happily follow my lead in a game. One of these times is when we play games that involve making other combinations for the same number. As in the above incident of it being okay for me to change the rules since I was the lead voice in the new game session, so Jaclyn will play along with me if I initiate an easily picked up change in a game. In the following incident, there is an example of this.
Jaclyn (4 years 3 months) is in the bath and I am sitting talking to her.

J: Let's play what and what. I'll go first. What's one and one?

M: Two. What's two and two?

J: [Without hesitation] Four. What's six and six?

M: Twelve. What's two and two and one?

J: [No hesitation] Five.

Jackie had now incorporated two or three addends as being part of the same game. She seemed to realise that we are still playing her game. Perhaps she had identified what the critical aspects of the game were, or perhaps she was indulging my 'not seeing the rules'? Maybe she was fine with this because she readily knew the answer. It started with one of her favourite doubles and then added one. I find it interesting that often her addends are doubles. I wonder if she asks them because she knows the answers or if it is a pattern that she is investigating?

Jackie has also given this game the name of 'What and what?'. This is the first time that I have noticed this. It is useful to have a name to identify games being played; it helps to make them easier to recall and they become more real and established. This is not trivial: "... naming is one of the fundamental activities of mathematics" (Pimm, 1995, p. xiv).
Jaclyn (4 years 3 months) and I are looking at a book, and it has a design of flowers on a page. We are playing 'How many?'. This is a game where the answer is sometimes known and sometimes needs to be found out, usually by counting.

J: How many?

M: Six. See three and three makes six.


I was explaining my strategy to Jackie. She seemed to understand my way of looking at the shape. Then, she added her own 'look', and explained it to me. Jackie realised that there is often more than one way of viewing something. Here she was showing that a number can be composed differently. She also felt, at many times, that we were equals in our conversations. We both got to ask questions, give answers, make and offer explanations. Often her strategy echoed mine. I see some of my pen-pal research findings mirroring this observation (Phillips, 1996). A student will often adopt the writing style of his older pen-pal even when he will not follow his older pen-pal's diversions away from his topics.
Patterning games

We are on a family holiday and Jaclyn (4 years 6 months) and I are playing a game she has made up.

J: Mommy, I say six, so you say seven. I say four, so you say ...?

M: Five. If I say nine, you say ...?

J: Ten. Do another.

M: If I say three, you say ...?

J: Four. But, I can’t say zero.

This game seemed to come out of nowhere. I do not know what triggered either her imagination or her memory. This game, like the other, was initiated by giving me an example and a prompt; ways that I often promote pattern games myself, both at home and school.

The remark about zero does not seem to fit. Just the previous month, as I reported in Chapter 5, she had started a counting sequence from zero, so she was aware that at least ordinally one comes after zero. I wonder where this observation came from: yet the pattern we are following is one of naming the next number. Perhaps she was thinking that zero will never be an answer in this game, because, if we were always to give the next higher number, the only way she could use zero is as a starter, not an answer. As far as I know, she had no knowledge of -1 at this time. Waiting seems to be best in
these situations. Next time a little more will be revealed. But, like Jaclyn, I am finding a trust that sense-making will eventually occur.

It is 9:30 a.m. I am still in bed after arriving late. Jackie (5 years 5 months) comes to join me in bed. We chat about what she’s been doing over the last two days. [She was with me in Lisbon, but left before me.] Then she starts a game.

J: What’s twenty plus three?

M: Twenty-three. Why?

J: I want to play that game.

M: Okay, what’s twenty plus six?

J: Twenty-six.

M: Wow. How did you know that?

J: You just did it, so I copied you.

M: So, twenty and three is twenty-three, so twenty and six is twenty-six?

J: Yes. What’s twenty and eight?

M: Twenty-eight. What’s thirty and two?

J: Thirty-two.

M: Great. Should we get up now?

J: Yeah!
Jaclyn has introduced a new game here. She heard the response to her ‘twenty plus three’ as ‘twenty-three’, and both identified a pattern and created a new game. I too understood the game and the patterning from one example. I was unsure how she got her answer of twenty-six and she told me she copied me. She was not adding in the arithmetic sense, she was adding by combining the sounds of the number words.

Patterns, in math, are very strong. Sometimes children can see/hear the patterns and can give correct responses. The question that I struggle with is: “Do they ‘really’ understand, even if they can continue a pattern?” This question has been mentioned in the previous chapter, and, as stated there, I have come to believe that pattern recognition is one kind of understanding. Like many other kinds of understanding, it can be very time consuming to generate from scratch, so it is not always the best/most efficient method of understanding. It is, however, a type of understanding that brings a feeling of joy and well-being. It is akin to discovery.

I believe that it is really necessary to understand a pattern to use it, though you may not understand all of the processes and concepts that can be associated with it. Conversely, it seems that you can understand at the process level and have little appreciation of the patterning, and experience no joy of discovery. I am not advocating one over the other, but I am encouraging that both of these, and many more types, of understanding be part of learning, knowing, and representing mathematics. And, I wish to emphasise, as in the previous chapter, that there is a strong case for valuing patterns as a
learning tool in mathematics. When Jackie builds a game out of a number pattern that she has recognised, I know she is applying both her sense of game development and her mathematical knowledge. She is mathematizing. More will be said about mathematizing in the concluding chapter.

Games involving keeping track

We are out sailing and Jackie (4 years 5 months) has made a paper fortune teller. She is not using it in the traditional way of only being able to choose one of the numbers written on it, but is allowing the choice of any number.

J: What number do you want?

M: Five.

J: 1, 2, 3, 4, 6, 7, 8, 9, 10.

M: Where's the five? I said 'five'.

J: Oh yeah, I forgot. 1, 2, 3, 4, 5. Now what number?

M: Fifteen.

J: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18.
Oh no! 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.
Now what number?

M: Twenty-one.
Jackie is having a good time tricking me. She really fooled me the first time that she skipped a number. She may have fooled herself too. But, after that, the game of skipping the number that I had asked for was established. All the time that she was counting, she was busily switching her fortune teller back and forth, but she made no attempt to open a pocket and tell my fortune. The game of skipping my number seemed to provide enough fun and focus. This also provides another example, similar to the triangle question discussed in Chapter 4, of Jackie showing off her skill by answering the unspoken questions of ‘Where’s the fifteen?’ and 'Where's the twenty-one?'

It was interesting that her counting was strong enough to allow her to skip a number without skipping a beat. I had no idea that her counting, and counting on, was so solid. She was demonstrating her ability to count an abstract thing, i.e. movement, transitively. There is, however, the feel and ‘roteness’ of an intransitive count. She was also relying on me to be a good, careful listener. Otherwise, her joke would not have been as effective.

Jackie expects me to listen to her, especially when she is performing mathematically. It did not seem important that I point out her skipped number, as I had the first time it occurred; indeed, she did not give me the opportunity. Attending to her count, appreciating the
joke, and supplying the next number seemed to be my role. Jaclyn was capable of counting, and keeping track of the number she wanted to skip.

Keeping track of scores and turns can be very complicated, even in a relatively simple game.

The whole family is in the car driving home from dinner out. Jackie (5 years 4 months) and I are in the back and we are playing the ‘Hide the Penny’ game that Jackie invented for car trips. (The only time that she organises playing this is in the car.) In this game, we take turns guessing which hand the other person has hidden a penny in. Jackie makes up the rewards and consequences for guessing right or wrong, and changes these rules often; almost each session of the game has a rule change of some sort, but the game retains its major characteristics of hiding money and guessing.

J: You guessed right, so you get two turns.

M: Okay, guess.

J: That one. [She points to the hand that she thinks I have the penny in.]

M: Right.

J: Now I get two turns, but you finish your other turn first.
M: Okay. Which hand?

J: That one.

M: [I turn my hand so she sees the coin.]

J: Right. Now I get four turns because I have two from before.

M: Okay.

J: Which hand?

M: That one.

J: No, but I still have three left. [Softly says: “Three and one are four”.] Which hand?

M: That one.

J: Yes. Now you get two turns, but not until my two turns are over. I still have two turns left.

Jackie is demonstrating an ability to do mental arithmetic. She is keeping track of her turns, and is juggling her turns and my turns to keep them in sequence. When she said: “Three and one are four”, her voice was fast and quiet, as if she was checking her answer with herself, and not as if she was offering proof to me. I noticed that she was using addition to confirm the number of turns she had left.

The ability to reason with oneself, to participate in such self-talk, is a skill for mathematics that I want to develop in myself and others, at
home and at school. It is one way of being aware of one’s own thinking, and checking for understanding. If children would do more of this type of talking, either to themselves or to their partners, I believe that it would have an impact on the amount of mathematics that is understood. Sharing thoughts is powerful, whether orally or in writing, and whether it is to oneself or to another. My classroom work with children as researchers serves to confirm these beliefs (Klawe and Phillips, 1995).

Scoring is one of the early ways that mathematics proves itself of use to children. Through the need to keep score they learn how to tally, how to add, how to keep running totals, and how to decide who has the most points. In the case of this game, Jackie’s scoring is quite complex because she is adding on turns won and subtracting turns as they are used up, for two people, in her head. She is using spoken words to help her calculate and keep track. If something is said, while being done, retention and utility seem high. This point has been emphasised in the previous chapter.

Word play games

I want to present some of the games that did not involve number words. I think that these games are relevant and important because I believe that strong language patterning and sequencing are transferable to, and form a necessary part of, mathematical strategies and memory development.
I am home after a class at U.B.C. and Jackie (3 years 11 months) is talking to me while I eat my dinner.

J: Mom, do you want to hear a story?

M: Sure.

J: Some dogs have floppy ears, some dogs don’t.

Some dogs have short legs, some dogs don’t.

Some dogs have short tails, some dogs don’t.

Some dogs have fleas in their eyes, some dogs don’t.

Some dogs have spots on them, some dogs don’t.

Some dogs sleep in a house, some dogs don’t.

Some dogs sleep with their brothers and sisters.

Later she asked me to read her the story.

M: I can’t read it, we didn’t write it down. I’d have to remember.

J: I’ll help. You say it after me. “Some dogs have floppy ears, ... .”

We said her whole story, and at the end, I said:

M: Some dogs are big and some dogs aren’t.

J: No, it’s not the same as mine. It’s not the right end!
At that time, there had been quite a few language incidents that seemed to tie with mathematics. This one seemed very much like making a pattern and noticing which one is different, either because it does not fit or it has been added on. I was trying to put a math context of size into Jackie’s poem, but it was rejected because it was not the right ending. Jackie had a pre-set word pattern that she would not allow me to alter. I am used to playing with word order, but Jackie takes a ‘mathematical’ view in her word pattern.

This play between us reminds me of those school problem-solving activities used to teach kids to discard irrelevant information. They need to decide what does not belong by noting what they do not need. This also supports my earlier claims in this chapter that children do not like their rules changed in the same session of a game. Perhaps if I had negotiated being able to add on to her poem this ending would have been all right.

In the next anecdote Jackie (4 years 11 months) is telling her father a story as I come home from a late class. The story is about a king, and a knight, and ... (several characters) who went to visit the prince, but they never got to see him and always had to come back the next day. When the next day comes, they add a new person to their entourage. The story starts on Sunday and works through to Saturday, when the prince is found at home. By Saturday, the seven days of the week have been named, and there are seven characters, presented in order, who need to be recalled.
Cumulative stories, such as the one in this game, are great for developing skills that are useful in mathematics. For example, sequencing, adding on, and visualisation can all be used in recalling the story pattern. Also, this type of linear accumulation is similar to our number system's place value. You need what is already in place in order to make sense of the information you have. A game like this necessitates careful listening and concentration from the speaker and the audience. Both players have to attend to the details to ensure the success (pleasure, in this case) of the game.

As I listened to Jackie's story, I remembered one that my dad used to tell me about a dog that would not cross a bridge and all the animals and items that a farmer had to use to force it across. I loved his story, and can recall his voice and action as he told it to me. As he lay dying, I sat with him and my older daughter Robin (then aged three) and told her this story I had so often heard. My dad laughed with joy to hear me, and to see Robin's eyes light up at the sequencing.

Memory is a tool that needs to be developed, and although educators are stressing understanding more than rote recall, there is a very real place for memory in mathematics. Memory and understanding often work together. Listening to Jackie recall her story made me think of things that can be done to develop memory: tasks like cumulative stories, poetry recitations, chanting and story telling. All can play a useful role in preparing the mind for mathematics. Pattern and repetition are strong reinforcers of memory.
Jaclyn (5 years 5 months) is watching me and talking as I prepare dinner. She is playing a word game of opposites.

J: Mom, yes means no and no means yes.

M: I see. Would you like to swim?

J: No.

M: Okay, where's your suit?

J: Are you coming too?

M: No, I have some work to do for dinner.

J: You said 'no'. Remember 'no' means 'yes'.

M: Sorry, I forgot. Yes. What does 'I don't know' mean if 'yes' means 'no' and 'no' means 'yes'?

J: I don't know.

M: So, 'no' means 'yes' and 'yes' means 'no' and 'I don't know' means 'I don't know'.

J: Say that again. [Giggling.]

Jackie loves the word play here, and I enjoy the math play. We can be seen to be in the land of “if a, then b” in the yes means no language. Early algebra involves being able to rename knowns and unknowns. I believe there is a close link with this type of substitution language game. Language and mathematics have many overlaps, so that often two learnings are acquired for the price of
one, one part being language related and the other mathematics. I am not certain that this was one of those times, but I do know that Jackie often uses her perceptions of mathematics to get my attention. So, I believe that, at least initially, Jackie was in the realm of mathematics.

I think that she saw a pattern in her language game and interpreted all patterns as math based. I was very busy and initially tried to distract her with swimming. Then I briefly played along with her game. At the end, I believe she was enjoying the word play because she asked to hear it again. I think she liked the sound that the phrases made. She may have been holding the meanings, especially considering that she generated them, and been enjoying the patterning in the language alongside. Again, the fun of the exchange is important. Even though I was busy, I was able to engage with her because my dinner preparation work was almost automatic for me, and speaking and listening did not require the use of my hands or take up any of the preparation space.

**HOME VERSIONS OF COMMERCIAL GAMES**

I will look here at home versions of some of the traditional games that we played. One of these, ‘Memory’ cards, is a home-made adaptation of the commercially available game. (‘Memory is sometimes known as ‘Concentration’.) The others, ‘Snakes and Ladders’ and ‘Dominoes’, use commercial products that we changed to suit ourselves. The rules we made were adapted to suit our level of
play, the time we had available, and whether we were playing to win or simply playing to play.

*Games with cards*

Jaclyn (3 years 9 months) and I bought some stickers while we were out and Jackie is thinking of something to do with them.

J: Can we make cards with my stickers?

M: Okay. What do we need? Hmm, some cardboard. Does Daddy have some cardboard from his shirts?

[Jackie comes back with one cardboard piece.]

M: Okay, get the scissors and let’s cut some cards. [We do this by folding and cutting until we have eight cards.]

M: Now let’s put one sticker on this, and one on this. [Jackie puts the stickers on, using a pattern sequence of a pair of cards with one on each, then a pair of cards with two on each, until we each have pairs of cards to call one, two, three, and four.]

I was curious to see if Jackie would try to make the stickers on the cards go in similar patterns. I wondered if her prior experience with cards and dice (see the following anecdotes) would influence her placements. It appeared to me, however, that she randomly stuck
them down, paying careful attention only to the number of stickers on each card.

We used these cards to play two games, 'Who has more?' and 'Memory'. For the first game we turned our cards over one at a time and whoever had the most stickers on their card got to take both cards. We played this until one of us did not have any cards left and then we started again. 'Memory' was played the traditional way with all the cards face down. We turned up one card and then had one try to find its match. Whether we were successful or not we just had one turn at a time. If you were successful, you got to take the pair.

Jackie really enjoyed these games and we played for about half an hour. She quickly caught on to the rules, and soon developed strategies for remembering in the 'Memory' game. For example, if I turned over a card second that had previously had its partner turned, she would immediately turn my last card over first in her turn and then search for where she thought the partner was. I realise that this strategy is actually 'backwards' from my usual one of remembering the last one seen and trying to turn its partner over first, but she seemed to feel confident once she had turned over the one she knew.

I think that developing strategies for problem solving is of key importance. One can always discard a plan in favour of a new one at a later date, but the knowledge that one has a plan is powerful. And, even though I was modelling a strategy that I felt was superior to hers, Jackie did not take it on. I believe that this is because she
either saw both strategies as the same, or she liked to know that the one she turned over matched the one that was in her head from short term memory. Perhaps getting the first card overturned to agree with what you thought it was, is the first step in this game. Perhaps this is the primary level of the game of 'Memory'.

Jackie and I created the cards for these games by folding, cutting and putting on the stickers. Doing this involved us in a discussion of how many cards we were making as well as determining how many cards of each kind were needed. By adapting the games that we were playing, we created new versions of the games. Creation is a big part of the whole picture. Creating uses for math and creating or finding the materials for the math is well worth attention. I have found that my classroom students, particularly the girls, will stay at a math project or a computer game much longer and with more focus if there is an element of creation involved (Upitis, Phillips and Higginson, 1996, and Phillips, 1996).

Almost a year and a half later, Jackie (5 years 2 months) and I were playing 'Memory' with half a traditional card deck and Jackie used the strategy of picking up the match seen earlier by first attending to the least recently seen member of the pair. As noted above, earlier when we played this game she also tried to make pairs that she had seen before, but then she would pick up the card seen most recently and often have trouble finding the other one. This way, when she missed the first card, she was not giving the other player the advantage of having seen the other card recently, just prior to their
turn. In addition, she was able to use newer knowledge perhaps to match up the card she overturned.

She may have been copying me, but I think that she would not do this if it had not made sense to her. In the earlier example, she did not copy me. Using a strategy that you have seen someone else employ is one way of learning. But, in order to know what to copy, and to use a strategy well, it has to make some sense. Yet, it is not a given that just because a learner sees something and has it explained that he will adopt it; he may make an adaptation as Jaclyn did in the earlier episode if he is not ready to understand the strategy as presented, or, as Jackie has done in other circumstances, he may ignore it completely.

Games with dice

Jaclyn (3 years 8 months) is watching her older sister, Robin, rolling a die, idly, while watching television. I pass by and notice what’s happening and say the numbers as they are rolled. Robin joins me in this and, after naming a few more, I go to the kitchen to begin dinner. Jaclyn starts to join in with Robin, but it is obvious that she is guessing. Her numbers are rarely the same as those I hear Robin say and she even puts in calls of “forty, eighteen, and twelve”.

This was clearly not a number-recognition game until I intervened. But, once I started to name the numbers, it almost became one. ‘Almost’, because Jaclyn did not yet have the dot configurations on the die connected to a number. It was interesting to be able to step
back here and watch the interaction. Jaclyn was looking at Robin as if there was magic in her answers. She did not appear to make any connection between the pattern or dots on the die and the number said. It seems that she decided to see if she could impress, too, by saying bigger numbers.

Jaclyn only gave numbers: she had figured out that this was a number saying game. She knew that letters, for example, were not written on the die; and she knew that this was a number-giving set from the words that Robin and I spoke. She waited for each roll of the die to give a response and in this way indicated that she understood there was a relationship, and perhaps a connection, between the die roll and the word said.

Jackie (3 years 9 months) wants to play a game and has selected 'Snakes and Ladders'. We begin by rolling the die and she now seems able to identify the configurations as number values. Occasionally, she calls the five a four, but gets the others. I am not aware of her doing any counting. This is quite a contrast from a month earlier when she seemed to think identifying the number representation was magic.

During the past month Jackie has built in many opportunities to roll die and to make sense of the dots on the top. She started by counting the number of dots but soon became aware of the pattern of the dots. She now seems to use pattern recognition as her way of knowing the number being represented.
Counting for ‘Snakes and Ladders’ did not follow a pattern for Jackie. She rolled the die, said her number and counted in any direction that she could in order to reach either a ladder or a snake. She did not pay attention to the number order on the board, nor to the fact that I was only counting one square for each number when it was my turn. She liked going up ladders and down snakes, and down ladders and up snakes equally well. Soon I gave up ‘modelling’ correct play and just joined her in trying to hit either a snake or a ladder.

Young Loveridge (1991) attended to this differently:

Our adaptations of Snakes and Ladders included reducing the number of squares to 30 in one version and 50 in the other version. Instead of presenting the squares in the form of a grid, the squares were laid out in the form of a winding path in order to overcome problems of directionality. [...] The other major problem with the commercial versions of Snakes and Ladders is that there is often a very long snake which runs from almost the end of the game to somewhere near the beginning. This can be an extremely frustrating experience for five-year-olds. (p. 68)

Young Loveridge chose to change the traditional board game, altering the number of squares by decreasing them and making the movement easier to follow by creating a path. They even changed what Jackie loved most, the long snake that she, using her rules, could use to go up or down. I did, however, persevere with one square for each number counted, hoping to get her to see this
connection. It did not happen during this game, but maybe next time she will be more prepared to attend to it.

There has been much work done on learning to count, as cited in the previous chapter on sound. Jackie’s ability to count using a one-to-one correspondence, though in place for other contexts (see Chapter 4 also), is disrupted here. I believe this is because her game goal of getting to snakes and to ladders is taking her attention.

After playing our game, I realised the complexity of items in it. For example, counting on from a midpoint is unusual when you are saying 1, 2, 3, .... In fact, at one point, when I rolled a one and moved one space ahead, Jaclyn told me, “That’s not one, one’s back there” and she pointed to the square with the 1 in it. The idea of counting one square for one die-point was also difficult, not because of one-to-one correspondence, but because the ‘fun’ of getting to a snake or ladder was too hard to delay. Even when she realised she had to land in or pass through each square en route to a snake or a ladder, Jaclyn was using sound as a way of reaching her goal. She would take as long to say her number string as it took to reach the desired snake or ladder. Sound was used as the spacing mechanism, not the squares on the board.

This leads to the next ‘difficulty’, namely, the game seemed more fun to play without the intended goal of reaching 100. It seems that an end goal, being a space marked 100, was not a reward for Jaclyn at this time, and neither was winning important.
It is bedtime and Jackie (4 years 8 months) and I are playing ‘Snakes and Ladders’.

J: (Has rolled a six.) Six ... 1, 2, 3, 4, 5. [Looks at the die.] Oh, yeah, six. [Moves one more place and says:] Six.

Jackie has recognised the pattern for six on the die. This is almost a year later than the above incident and she no longer needs to count the dots. She has also built in self-checking. I do not know why she stopped at five, but when she relooked at her roll she was able to remember that she had only moved five places and realised that she could move one more. She knew that six was one more than five and moved ahead a space automatically.

Games are a powerful way to build in practice. I have known this for a long time, but what I am appreciating more now is the less obvious learning that takes place: in this case, self-checking a turn. I try hard, and often to no avail, to get students in my class to check their answers, especially to see if they make sense. Perhaps games are, for me at least, an opportunity that I need to explore further for teaching general skills and attitudes. I already appreciate the atmosphere of challenge and the socialisation that games promote. Perhaps it is important to pursue games as an avenue into ‘good’ work habits also.

Jaclyn (4 years 8 months) and I are playing ‘Snakes and Ladders’. Jaclyn rolled the die, looked and moved the correct number of squares without counting out loud.
Jaclyn is now able to move on the board in the traditional manner and I find that it makes the game more difficult for me to just ‘sort of’ follow. I need to pay closer attention to what she is rolling and to her moves if she is not counting aloud. When I move without counting, she protests. So, we agree to count out loud.

Silence does make one more attentive and alert. Without verbal cues, the situation is quite different. Silence forces one to focus and attend differently (see also Chapter 5).

This is a continuation of the above ‘Snakes and Ladders’ game. Jackie and I are each on the same square. This is part of our agreed-upon rules. We did not like sending each other back to the start when we landed on the other’s square, so we agreed that we could both share a space.

J: [Rolls a five.] 1, 2, 3, 4, 5. Your turn.

M: [Rolls a six.] I’ll just go one ahead of you because you had five. Okay?

J: Okay, because six is one more than five.

I really wondered if Jackie would be able to follow my thinking. Even though she showed that she knew six was one more than five a few turns ago, I wondered if this context might be too different to allow her to apply her knowledge. I figured that if she protested I could always count mine out. She seemed quite happy with my plan and even offered her explanation of why it was okay. This kind of
dialogue really excites me because it allows me access to Jackie’s thinking.

At school, I will often give a prompt to a student just to get them past a sticky bit. If they take the prompt and act on it then I believe that they understood the concept that the hint was linked to. If they look blankly back at me, then I know that I need to provide something that is more basic. I then believe that they are not quite ready for the mental leap that I was proposing.

I consider this type of dancing to be the essence of good teaching and of good parenting. Often, to give too much is less valuable for learning because it often ends up with children not listening, either because they think they already know it (so feel bored, or insulted); or because they are not ready for the explanation and end up not listening as self-defence (from feeling confused or stupid). The challenge is to supply just enough information so that extension of learning is encouraged. More is said about this in the following anecdote.

_Dominoes_

Jackie (5 years) and I are playing with a set of dominoes that she got for her birthday two days earlier. We just dump the blocks out and are looking at them and arranging them. We decide that we could play a game by sharing the dominoes and then taking turns matching up the dots. This goes well. Jackie decides that the blank meant ‘zero’ and that you need to match it with another blank one. I do not
try to explain that a blank could sometimes be used for any number.

At times, it is not necessary to have the whole picture, or to play by the real rules. When we first played ‘Fish’, ‘Memory’, and ‘Snakes and Ladders’, we made our own rules. This is another instance of starting where it feels comfortable and extending when the opportunity arises.

In class, I often feel like I am cheating the kids if I do not give the whole explanation of something. When I think about this, it seems likely that I am sometimes the only one who conceptually reaches the end of the explanation. It is important to know when to stop. Stretching is good, but not past breaking point. In this case, the breaking point is the point where attention breaks off. Sometimes I tell myself that it does not matter if the students do not really understand, at least they are getting exposure. I believe this, but will add the condition that exposure should not lead to sunburn. The exposure offered should be within the realm of making some sense, otherwise it is probably better left until some ground work is in place. And, I cannot provide all of the groundwork. Each student needs to prepare his own ground. In order to learn from an explanation, some experience and prior knowledge need to be in place. Explanations, it seems, only have meaning if they can fit into a framework.

It is bedtime and Jackie (5 years 1 month) has chosen to play dominoes as her before bed activity. Since she first got
the game for her birthday a month ago, Jaclyn and I have been playing ‘Dominoes’ according to our own rules. The resulting game looks like a ‘Scrabble’ board. Tonight we decide to read the rules and play it the suggested way.

Reading the rules reveals that a blank could be used for any number and that the dominoes should be placed end-to-end. Jackie seems to understand and we are playing well until she tries to use a blank/six to join a three/three pieces. I told her that she could not join her six to a three.

J: Let me count. 1, 2, 3, 4, 5, 6. The other one is 1, 2, 3, ... 4, 5, 6.

M: They both have six dots, but this one is three and three. See.

J: Yes, But it’s 1, 2, 3, 4, 5, 6.

M: But, it’s on both parts. It’s really 1, 2, 3 and 1, 2, 3.

J: Oh. ... And this one’s 1, 2, 3, 4, 5, 6 and zero.

M: Yes.

J: But this is blank, so it’s okay.

M: Right. You can turn it and use it for three.

This is another one of those times where the learning entailed learning how to see. Jackie needed to know what was important to pay attention to and needed to learn how to adapt her understanding
of numbers to the rules of this game. Flexibility and the desire to play seem key ingredients, as does the ability to share what is being thought.

The learning how to see, how to read the dominoes, was aided by sound. Looking had permitted Jackie to combine her dots into a group of six; sound helped her to identify the separateness and see two groups of three. She heard the difference and was able to extend this to show that her blank and six could be read as zero and six; she also showed that she understood the rule, new to us, that a blank could be used for any number.

Sound, used in conjunction with sight and doing, once again provided the needed difference between understanding and just following the rules. Sound in a pattern, given as an example, helped to emphasise the break between the two parts of the domino in a way that only sight and my explanation had not. The type of sound that I am promoting in mathematics is often linked with pattern and repetition; it is not the sound of lecture or uninvited teacher-explanation.

If sound is only used to convey information in the form of explanation, without examples, without active listening, then it is not very valuable for learning. The learner needs to be able to attend to the sound patterns and make meaning in mathematics. Sound, in context with active participation, is powerful for extending learning and for creating awareness of new learning possibilities.
As a teacher I am often in the position of providing missing bits of information that will, perhaps, bridge a gap that a student is struggling with. I am aware that telling is not often effective; yet, I sometimes tell anyway hoping that the child is in active listening mode. If this is the case, and the student has a framework for the words, then telling can be a good sound prompt.

As a teacher, I need to be able to help students engage in active listening and I need a way to assess whether active listening is occurring, because if students are actively listening, then they can often construct from what I, or anyone else, is willing to offer. Telling is as valid in school as it is at home, and need not always be seen as bad teaching practice. Sorting this out will take more work on my part. I am learning so much by reflecting on these anecdotes; and I am still surprised by how much this case study with Jackie is casting light on my teaching style.

COMMERCIAL GAMES

Some of the games we played by the given rules right from the start. No lead-up versions seemed necessary.

Jaclyn (5 years) and I are playing 'Dinosaur Bingo'. This is a letter recognition game played on a three by three grid. A spinner is used to randomly select letters.

M: How many ways can you get a straight line?
J: One, two, three, four. (She has used her finger to show me the lines that run parallel to the outside edges of her bingo card.)

M: I can see eight. (I show her the ways, including the plus shape and two diagonals.) Can you see all eight?

J: Yes.

Later she has four pieces arranged like this:

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1 2 3
4 5 6
7 8 9
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J: I almost have a bingo three ways.

M: Which three ways? [She shows me.]

M: I see one more. [I trace it for her with my finger.]

J: Oh yeah.
Twice in this portion of the game, I have extended Jackie’s seeing. Each time she was able to follow what I was showing her. If I had not been there, she might not have discovered these for herself. Yet, she could construct them with my demonstrations.

This is another way to prompt. Extending what is seen to include what was previously not noticed is part of teaching. Once again I find myself thinking about how much and when. By showing Jackie that there were more lines for a bingo, I was setting up a learning situation. She was either going to be able to follow my demonstration, or not. Since she appeared to, and even demonstrated a diagonal in one of her three ways of almost having a bingo, I was confident that she would see the fourth route easily.

Multiple ways of seeing is an important skill to have. In problem solving it is important to be able to use many different strategies. It is also important to be able to follow other people’s explanations. Part of the ability to do this is the ability to understand that there’s more than one way to do many things, and the ability to visualise possibilities from incomplete data. This simple game provided Jackie with experience of open-minded looking, and at seeing possibilities. It also acquaints her with some of the properties of a rectangle.

Jaclyn (5 years) and I are playing ‘Sleeping Grump’. This is a co-operative game that involves rolling a die for moves and each player acquiring a complete set of objects.

I thought that the game would be a little complicated to play, but Jackie already understood about rolling and reading the die and
knew how to move spaces, so we just had to discuss our strategy for each getting a complete set of items before the giant woke up.

Although this game seemed complicated to explain, it was fairly easy to play. It was important that Jackie had game-playing experiences already. We worked co-operatively to get our items, and discussed the best strategy to avoid using all the pieces that would wake the giant. It was fun to be learning a new game with Jackie. Neither of us had seen this game before, and so we were genuinely trying to figure it out together. I am glad that I decided not to read the rules ahead of time. We had a lot of discussion about what some of the rules meant that would not have occurred if I had predetermined what I thought they meant. Agreeing on, and establishing, conventions is an important part of mathematics and games readily give practice in this type of negotiating of meaning.

In class, too, it is important to have opportunities where everyone is learning something new. In my room, it has been learning how to play the games on our computers and learning how to deal with program ‘bugs’ that seem to crop up. I am not the expert, and the children very quickly realise this. Even though we have rules about checking with me before certain operations are carried out, the students know that this is a formal check for permission, and not a check to see whether their decision is right or not. For each game and each problem that occurs, there seems to be a different class expert, and I am not an expert on each of the situations.
STORIES AS GAME SOURCES

One of the things we do every night at Jackie’s bedtime is select a story and read it. Jackie chooses and I read. We each take turns, irregularly rather than alternating, asking questions and inventing games to play as the story is being read. Most often these activities/questions involve the illustrations, rather than the plot, in the stories.

It is just before Jackie’s (5 years 2 months) bedtime and we are reading the story One Watermelon Seed. In this book there are numerous fruits and vegetables just made for counting. Jaclyn counted easily from one to ten, and then the decades from ten to one hundred with my lead. She also counted by one to fifty, and then said:

J: I don’t want to count these ones.

When we are playing games, or reading stories, Jackie participates in what she wants to and feels free to stop when she wants to. At school, children quickly learn that they have to do something when it “needs to be done” and are not free to decide when they do not want to do something. This becomes increasingly the case the further along they go.

This work with Jackie is strengthening my belief that even when teachers and parents know what has been presented, it is often a long time before it is fully known what was, or was not, understood.

Jaclyn (5 years 2 months) and I are reading One Snowy Night before bedtime.
J: Let's play, 'Do you see any shapes?'. Do you see any circles?

While listening to stories, Jackie automatically looks for other activities to do. These asides do not seem to detract from the themes of the books we read, but perhaps this is because favourite books are read over and over. It is interesting that, although the words to a story get memorised, the activities that we do are variable. Jackie does not draw attention to the same things on the same pages, and neither do I. We seem to have an unspoken contract that encourages many different activities for each page that we choose to stop on. Whereas the story contract seems to be 'read the same words each time' unless we have agreed simply to tell the story. We sometimes do this when it is too late to read the whole thing.

Jackie is leading us into a game she names 'Do you see any shapes?' and has prompted me to look for circles. As previously mentioned, she often initiates a game with a question or a question and an example. Circles seem to recur often.

Again it is bedtime, and I am reading *Tawny Scrawny Lion* to Jaclyn (5 years 2 months). We are looking at a double page that has pictures of the rabbits and the lion.

J: Five rabbits and five rabbits. [Five on each page.]

M: If I cover these [five], how many are left?

J: Five.
M: Great! How did you know?

J: Well there's ten all together and there's five here so you have five because five and five is ten. Now I'll do it. [She covers the lion.]

M: Ten rabbits left.

I continue reading the story, but Jackie insists ...

J: Do one more.

M: How many have I covered?

J: Two.

M: How do you know?

J: Because I see three and five.

I cover some on both pages.

J: I see three and one. You covered two and four.

M: How many is that?

J: [She counts silently using her nose, touches two fingers on one hand and four on the other hand.] Six.

This seems to be developing into fairly complex arithmetic. From recognising how many are left, we move to how many are covered and the way Jackie interprets the question asked leads her to regard each page individually and give two answers that she has figured out.
at close to the same time. She seems to really enjoy this because she wanted to continue with the activity rather than return to the story.

This may be a case of wanting to get enough practice in so that she felt that she would remember for next time. I am noticing that I do not have to push her to be persistent. She keeps at things until she meets her own level of satisfaction, whenever possible. On the occasions when I have wanted to continue and she has had enough, she has left me to ‘play’ on my own. For an example of this, refer back to Chapter 4.

Jaclyn is really willing to share her thinking with me. I wonder about some of her processing. For the first question, I thought that she would tell me that she knew that there were five covered, so five had to be left. She had just announced five and five. But, she offered the additional information that there were ten in all, and seemed to base her thinking around this. I am uncertain about this. Was she anticipating a question about five and five totalling ten, and that was why ten was in her thinking, or did ten really play a role in how she figured this out. I would have needed to ask about this, and I did not. Why not? Surely her response puzzled me at the time and not just in retrospect. Perhaps I expected further play to reveal her thinking to me. I have found that too many questions during play detract from the game – both for Jaclyn and myself.

Continuing with the story, Jackie has covered a whole page and asked me how many are covered? I am recalling from memory.
M: Five animals.

J: (lifting her hand) Two and two and one. Yes, that's five animals.

Jackie was looking at the picture representation of the animals, and responded to me with their groupings, and immediately confirmed that there were five. She did not seem to add, nor did she pair her fingers with the pictures. I wonder if just saying the number words triggered a naming for five. Once again it seems that the use of sound is aiding sight. Was she confirming the number for me or for herself? It strikes me that I am the main person that she does this sort of chatting to. It is not her method of choice for getting her father's or her sister's attention. She has different ways with each of them. Thinking of this leads me to reflect on school situations and some of the differences children face.

Each year at school, with a different teacher, brings a different emphasis. This is one of the strengths of the school system. If children are exposed to a keen interest in one area, they often develop a liking and interest too. Even one person, for one year, can influence a student to pursue an area that they might not otherwise. One person can make a difference.

One of the first things that I do with a new class of students is have them make graphs about their interests, likes and dislikes. Usually, in September, about four children will list mathematics as one of their favourite subjects. When this same survey is done at the end of the year, usually about twenty will list it. Of course, there may be
some element of wanting to please the teacher, but I think that we know each other well enough by the end of the year that my instructions to be honest are heeded. I also get indications of their changing interest by reading their math journals throughout the year. As children get more involved in mathematical thinking, they usually get increasingly hooked on the subject and begin to develop a mathematical awareness that permeates other subjects.

IN SUMMARY: PLAYING THE CHAPTER AGAIN

This chapter has looked at ways that Jaclyn and I used games for our mathematical and personal enjoyment. Jaclyn played games for many reasons. Sometimes the mathematizing that occurred was in the game itself, sometimes as a result of the game. Usually both of us were involved in the game play, but on occasion Jackie played by herself, for her own ends.

Playing games absorbed both of us; we immersed ourselves regularly in the social context of game play. Between us we chose and negotiated what to play and how we would play it. We both contributed (individually and collectively) to game design and in deciding what the end of the game would look like. Jaclyn frequently took the lead in positioning herself as an inventor, creator or adapter of games. She would happily set the conditions of play, as in “No, go ‘What’s m and m?”’, as well as determine the rules and the time frame.

The games were played for challenge and engagement. Often there was an element of skill and/or memory involved, but despite our
markedly different levels, we found things to interest us both. On many occasions, I was watching for signs of more complex strategies and more sophisticated question asking. I believe there were times when she was both observing and participating as well. For example, the progress of her strategies in 'Memory' could have come from observing my actions as I played the game. I witnessed her increasing ability to read a die and move pieces in a board game. I heard her keep score. By playing games with Jaclyn, I had some intimate access to her thinking, as she did to mine.

The games were played in the kitchen, bathroom, bedroom, in the car and on a boat. All of these venues, at the time of playing, exemplified the feel and atmosphere of being 'at home'. Competition seemed less important than leadership in terms of determining the rules and development of play. A game tended to be played over and over, much as favourite stories were re-read night after night, until it was time to move on. When returning to an old game, it was usually restarted at the skill level where we had left off, and the skill level frequently increased quickly from there. The challenge in these games was frequently self-imposed and not inherent in the game.

Jaclyn often listened to herself and watched her own moves - in a word, she self-monitored. I also mediated, usually with brief hints or suggested strategies, or showing particular moves with reasons, or by making comments about the play, and, when necessary, being 'the reader of the written instructions'.
In the classroom, I feel games tend to have a more clearly delineated purpose of skill development. They are played for a particular reason with a specific goal in mind. They often comprise an introductory activity, intended to ‘hook’ children in, to ‘warm them up’ or to ‘review’ a skill or concept prior to extending it. Often games become ‘extra’ activities for those who finish early (Wassermann, 1990). As such, players are often instructed to be quiet so they do not disturb others who are still working.

The idea that games can provide a way to work while playing is one I find I have to teach the children in my class, in order to counteract their entrenched entering belief that games are ‘just for fun’. (Walkerdine, 1982, has written on the disjuncture between ‘work’ and ‘play’ in elementary classrooms.) At home, such splits between ‘work’ and ‘fun’ seem more easily blurred.

For me, this work on games has been a real eye-opener. Although I value and enjoy games at school, I do not usually take part as a player with the students in my classroom. At school, I have tended to supervise several groups playing, or to direct the play of the class in a whole-class game, or to teach a group about something usually not even related to the game while the others are playing.

By playing with Jackie, I have had access to some of her thinking that I could not have known otherwise. Realising this has encouraged me to create time to sit and play games with my students - for fun, for challenge, and for the opportunity it affords to know them better.
CHAPTER 7: “IT FEELS LIKE INFINITY”

Deep, deep infinity! Quietness. To dream away from the tensions of daily living; to sail over a calm sea at the prow of a ship, toward a horizon that always recedes; stare at the passing waves and listen to their monotonous soft murmur; to dream ways into unconsciousness.

(Maurits Escher, in Pappas, 1995, p. 54)

Infinity is an abstraction that is not based on an actual known. The very nature of the infinite makes it unknowable in real terms, it cannot even be concretely modelled. It can only be thought about, hypothesized, and symbolically represented. Endeavouring to grasp the idea of infinity is striving to grasp the ungraspable.

Holt (1989) writes:

Kids talk about infinity as if it were a number, but it isn’t. The word infinite means endless or boundless. You can’t get to the end, or to the edge, because there isn’t one; no matter how far up you go, you can keep on going. Not an easy idea, maybe, for a six year old, or even most adults, to grasp. (p. 72)

Infinity is a central concept in mathematics. Gattegno is attributed with having said that ‘mathematics is shot through with infinity’, referring to its core involvement in things mathematical. In searching my notes/data, I have found that Jaclyn has been intensely interested in infinity throughout the latter part of the study. One claim of this thesis is about my involvement in Jackie's developing
sense of infinity as an important, centrally mathematical sense, to be used alongside the more familiar human physical senses of touch, sight and sound.

Jaclyn has a fascination with the notion of infinity. Other researchers have documented similar interest in young children. Anderson (1993a) has written that “the infinity of numbers” (p. 29) has been explored with her daughter, Terri. Holt (1989) tells of an instance of a parent writing to him about her son’s question, “What is the number next to infinity?” (p. 72).

I will now present some of the ideas about infinity that Jackie and I explored. I will begin by presenting the incidents in isolation and will then discuss the overall experience by reflecting on the whole. This reflecting will be done in two stages. First, I will connect the opportunities that arose for exploring infinity with Jackie, and then I will extend these reflections to a newly-formed response to infinity.

A LOOK AT THE PARTS

Jackie (4 years 6 months) and I are on a sailboat.

J: Mommy, do you know the biggest number?

M: It’s bigger than thousands, it’s bigger than millions, it’s bigger than billions. Do you know what a big number that would be?

J: The biggest number is infinity.
Infinity is perceived, by Jackie, as having a numerical value, it is the name of a number the way a million is, and its value is that of the biggest number. Even in grade four, some children find it hard to conceive of an unending number system. Many will still say, “Infinity plus one”. Some will try to explain infinity by saying, “It’s when you take the biggest number and then keep on counting.” Even those that say, “There’s no such number as infinity”, sometimes fail to convince me, because this ‘answer’ is too quick, too unthought. When pushed, they often say something like ‘infinity goes on and on’, and this does provide a partial understanding.

I was quite intrigued to hear Jackie questioning me about infinity. I wish I had asked her where she heard about infinity because I do not recall an introductory discussion or an off-hand use of the word. Her answer was given with such certainty that it seemed untimely to pursue a reconstruction of the concept at the time. Many times, at home, I choose to wait.

It is bedtime. Jackie (4 years 11 months) and I are reading the story *The Country Mouse and the City Mouse*.

J: I see one duck.

M: I see one butterfly.

J: I see five chicks.

M: I see one striped chicken.

J: I see infinity hundred grass!
Jackie seems to understand when the use of infinity is appropriate. However, I think she still sees it as meaning "too many to count", and does not really 'know' the concept of infinite. To her, 'hundred' is a big amount, so 'infinity hundred' must be beyond the imaginable. In this respect, the power of the infinite is touched upon.

In support of Holt's claim that I cited earlier (though I disagree with his belief that infinity is not a number), Jaclyn also seems to be using 'infinity' as a number word: 'infinity hundred' has the same structure as 'six hundred' or 'nineteen hundred', and it was striking that she did not simply say 'infinity grass'. I wonder whether she believed 'infinity hundred' to be more than 'infinity'. Linguistically, she used 'grass' in the singular (unlike 'chicks'), neither pluralising via 'blades of grass' nor with the possible plural 'grasses'.

What interests me is that she has invoked 'infinity' at all. There were many other options available to her. She might have used an approximator 'about' (see Rowland, 1995) and then given a number; she might have said 'millions', or she might have asked me how many grass blades I thought were there.

When I think about the spoken ideas that children offer, I find that in order to understand the most that I can about their learning, I often have to 'hear' what was not said. Awareness of the whole realm of possibilities helps me to more accurately assess their knowledge. Similarly, I had to notice that she did not offer other large numbers, indicating an uncountable amount, she used the term 'infinity' (though not simply by itself, but in relation to another number
word). By choosing to use it, she simultaneously chose not to use other methods to explain the large number of blades of grass.

At night, I am counting Jaclyn’s (5 years) chicken pox on her back and she’s counting the ones she can see on her front, as we cover/touch them with lotion.

J: It’s so itchy, Mom.

M: I know.

J: And it hurts.

J: How many are on my back? It feels like sixty-three. No. ... more. It feels like infinity. ... I keep feeling more itches.

Jaclyn’s favourite numbers at the moment are sixty-one or sixty-three. She is using these where she once used ‘lots’. Sixty-three is just within her counting range at this time and seems to signify a big number. However, this incident is about more than counting. It is about exploring infinity. Does an itch go on and on and on? Or is it separate itches, so many that they cannot be counted. There are always more, so any attempt to count them leaves some uncounted. How can a hurt be expressed?

I remember my uterine contractions being measured during Jaclyn’s birth. Some of them had an intensity so great that their peaks were above the scale. Was that similar to what Jaclyn was feeling? Hurt and itching so intense that it had no number, no reference points.
I was using counting of the chicken pox spots as a way of distracting Jaclyn’s attention away from her discomfort: using mathematics to soothe and to distract. People are often asked to count backwards from ten as they are being anaesthetized. There can be something very comforting in the repetition of numbers and patterns. Comfort can come from being able to rely on a system to stay the same. In this case, both Jaclyn and I received comfort from the regularity. Mathematics provided us with a way to communicate about what she was feeling. It provided a way for her to explain her discomfort and for me to realize it: a system that can be counted on.

It’s 8:30 in the evening, and Jackie (5 years 5 months) is in the bath.

J: Mom, come look at my big, bubble mountain.

M: Wow! I wonder how many bubbles you have there.

J: One hundred fifty twelve.

M: Hmm. If you had one less, if one bubble popped, how many would there be?

J: One hundred fifty twelve.

M: What about if you had one more bubble?

J: One hundred fifty twelve.

M: Really? Still the same.

J: [Pauses] No, it would be infinity.
M: I see.

J: Yeah, the biggest number.

Jackie is using the same non-standard number that she used two days earlier. Then, she asked me, “What comes after one hundred, fifty, twelve”. I had replied, “It depends on what you mean by one hundred, fifty, twelve. It could be one hundred fifty thirteen, or one hundred sixty-three, or fifteen thousand thirteen, or one hundred fifty twelve plus one.” At that time, Jackie was not willing to try to explain what she meant, and I was left with the feeling that she was just rolling number words off her tongue. I thought that the sequence was probably random play with number words. When the same sequence appeared again, I wanted to hear what she would say the next number would be. I was hoping for some clues to her numeration system. I wanted to understand where one hundred fifty twelve was coming from.

Rather than another number sequence, I got ‘infinity’ which she labels ‘the biggest number’. She must think that a number like one hundred fifty twelve is really big, too big to count to perhaps. One method that Jackie has used to indicate a large amount has been a linking of number words one after another. I have noticed, in earlier incidents, that numbers that represent large amounts have needed to sound long. (This too is an example of emergent mathematics: it is generally true that bigger numbers have more number words in them, and she may have identified this structural regularity.)
Frequently, in such situations, children acquire the form first; Ainley, 1995).

It also provides an interesting linking between the form of an utterance and its meaning. Pimm (1987, p. 191) cites Papandropoulou and Sinclair's work on young children separating properties of the language from properties of the objects named by the language, where they give the example of four- and five-year-old children offering 'train' (a long object) when asked for a long word, and offering 'primula' (a small object) when asked for a short word.

The previous summer we had been walking along a beach and Jackie (4 years 5 months) was collecting pebbles and shells. She asked me how many I thought she had and I replied, "Over a hundred". To this, she responded, "It's sixty twenty seventy". When I suggested that it was probably around one hundred ten, she asserted, "It's way more. ... It's seventy eighty sixteen fourteen." In this case, she was definitely using number names to create a representation of a large amount. The longer the name, the larger the number, and the more it could represent. The current incident, though more standard sounding and more consistent, fits soundly with this sense of form.

In bubble baths, the bubble density and size of bubbles are constantly changing. Some pop, others appear because of movements and swishes. As the pile gets older and smaller, the bubbles also get smaller in size. The process of getting smaller is often one of subdivision. I wonder if the number actually increases for awhile. I
know that bubbles seem to form more easily in hotter water than in colder. Does Jackie know these things too? If so, the whole idea of bubble counting would have a feel of the undo-able. It is indeed possible that the number of bubbles could remain the same even if one were added and one were removed. The operations of adding and subtracting would not make sense, just as the act of bubble-counting would not fit Jackie's current counting system. Claiming infinity seems like a wonderful way out of this dilemma. So, although Jackie identified infinity as the biggest number, she is, perhaps, using it here to name things that are uncountable because they are in a state of flux.

Jackie (5 years 5 months) and I are walking along the sea front in Cascais, Portugal.

J: We are sure walking a long way.

M: How far do you think we can walk?

J: Infinity steps ... no I can't say ... there's always more.

M: Let's try. [We begin to count, alternating turns. Jackie started at one and I did the even numbers until we got to 46. At this point, she said 48 as her next number, and I did not correct this. She managed with even numbers, including the decades, until 70 ...]

J: If I knew the numbers we could count on and on. ... Infinity.
[We continue our walk, silently, and Jackie picks up the conversation a bit later ...]

J: You know Grandma [my mother] will still be able to help us forever. She can see us here, but we can't see her. She's invisible. Being dead is like infinity.

M: Hmm. It goes on and on.

Jackie has identified a link between counting, patterns and infinity. A pattern is recognised as such because of its ability to regenerate again and again, forever. She seems to realize that if she knew all of the key words, she could count on and on. She knows the pattern for add one more, she does not know all the names for the increasing powers of ten.

Tears well up whenever I think of this incident. It was so beautiful. Partly the emotion was because of my still-rawness over my mother's relatively recent death, partly because of the memory of the scenery, including the feel of the breeze and the salty smell of the air. I am reminded of the line from a Yoruba prayer - "Death counting, continually counting, continually counting, does not count me" - which Pimm (1993) invokes in his lamentation for a dead colleague and friend Rita Nolder.

Invisible death and infinite death, closely linked in sound and in perceived meaning. Infinity is invisible. Infinity is one of the great ideas that mathematics has to offer. It allows discussion of the unseeable, of what is invisible from our current vantage point. I had
never linked these two concepts before. I will always link them now. Jaclyn's comments, whether connected for herself or not, jolted my perceptions and my meaning making.

For me, the walk we were having was of a reasonable length, and it provided a relaxing break from the Psychology of Mathematics Education (P.M.E.) annual conference that I was attending. For her, it must have seemed like we were going to walk forever. The shore between Cascais and Estoril is one continuous walkway, extending beyond vision, with new things always apparent as you close in on the direction you are walking.

In our case, we were just walking ... toward an unknown destination. I wonder whether counting made the unknown more familiar to Jackie. I wonder if it were comforting to be able to add some normalcy to a situation that was completely new. New scenery, different food, different language, and new transportation modes. (At home, we do not use train travel and subways.)

Using the ritual and the familiar to bridge the gap between known and unknown, between not new and new, is common territory both to teachers and to parents. The idea of providing comfort through routine is well established. This speaks to the importance of repetition and tradition (see Pimm, 1995). These are areas that are losing ground in modern teaching of mathematics courses. Memory, drill, and rote learning are being swathed in negativeness.

In mathematics, as in other areas of the curriculum, we need to safeguard our traditions as well as look for new and exciting
innovations. It is not a case of either/or exclusion, but another instance of inclusion. As a mathematics educator, I want to include as much as possible in my program. Comfort is worth keeping; links between generations of math-doers are worth preserving.

A LOOK AT THE WHOLE: CONNECTING THE OPPORTUNITIES

These opportunities for mathematizing have shown a look at infinity. The interpretative possibilities are infinite. For me, this is both exciting and daunting. From the infinite it is now my task to present the variations that strike me, that cause me, at this moment, to stop and pause and reflect.

Reflection is cyclical. Each time I go through these segments, I stress and ignore differently. However, reflection presents a finite view of infinity. Although infinite reflections are possible, they are limited in each individual by time, interest, and death. However, the power of the written word is one that extends our thoughts beyond death and into the living world of others. In this way, reflections are regenerative and endless ... Infinite.

For me, the key ideas that were raised in these opportunities were infinity as:

• a way of saying the biggest number;

• a way of describing the invisible;

• a feeling that continues;

• a way of representing the count of things in flux;
• a comforting mechanism;
• a constantly-repeating pattern;
• a way of dealing with the impossible;
• a way of giving perpetual action to mathematics.

When I think back on my ideas of the infinite prior to this, I recognise that my use of infinity was as a way of saying that something repeats itself forever, or gets increasingly smaller or larger forever. For example, to students I have often said, “Infinity means you can count forever because you can always add one more”. I really thought that this was explanation enough. For my own view, I was slightly broader, allowing that infinity was a way of adding boundlessness to forever. Infinity has always seemed more than forever to me. I view ‘forever’ as linear, whereas infinity allowed me to express concentric foreverness.

But, now my view is extended, amplified, intensified, and includes feelings and emotions. Mathematics is not a simple black and white statement of facts as it is often perceived by others. A teaching colleague once told me, “At least math is easy to grade. The answer is either right or wrong.” Where would my current musings on infinity fit in such a curriculum?

How can I help kids see that infinity is infinite, that it is more than a convenient way of naming an unknowable quantity? How can I add the notion of genuine uncountability and immeasurability to the concept of the infinite? As a teacher I want my students to feel the
ongoingness of infinity and not just use it to represent a count that is impossible because of finite limitations like knowledge of names and/or time available. How can I help each of my students, in the words of the poet William Blake, to 'hold infinity in the palm of [their] hand'?

A LOOK AT THE WHOLE:
EXTENDING AND EXPANDING FROM THE OPPORTUNITIES

The concept of infinity is one way that mathematics is integrated with the world of poetry and philosophy. Pappas (1994) expresses this same oneness when she states:

Infinity has stimulated imaginations for thousands of years. It is an idea drawn upon by theologians, artists, philosophers, writers, scientists, mathematicians [...] Infinity has taken on different identities in different fields of thought. (p. 44)

As mathematicians we are often viewed as narrow and boring. How grand to have notions such as infinity to elevate/extend this myopic perception. Infinity stimulates the imagination and the emotions, for me, more than any other single mathematical concept. This was not always the case. The power of reflective writing in the field of mathematics is exponential, and the reflections that occurred as a result of reporting Jackie's interest in infinity have been substantial.

Holt (1989) ends his section on infinity by suggesting that:
[we] talk about infinite instead of infinity. There is no such thing, or mathematical idea, as infinity. There is just the adjective infinite, meaning [...] without an end or an edge.
(p. 74)

I would like to oppose this view. It is the word ‘infinity’ that children use and hold dear. Where do they hear it first? Why is it so fascinating? Is it intuitively understood? I will digress slightly here to introduce an idea on intuition offered by Gattegno (1963b).

No one can boast that he is able to dispense with intuition, but there is a tendency in mathematics to minimize it and be suspicious of it. I should like to suggest to readers that the educationalist should be the last to do without it, and indeed should learn to make a wise and more circumspect use of it.
(p. 56)

Infinity holds appeal for children and adults alike. It is an area of mathematics that encourages and even necessitates mathematizing.

Once relationships are observed, a big forward stride has been taken, proof both to the power of the mind and also to the concept of infinity. [...] Mathematics, like poetry, satisfies the deeper senses in us all; both maintain us in ever changing contact with the ever changing, the unformed yet the formed and subtle. (Gattegno, 1963b, p. 98)

At the Queen’s/Gage mathematics conference, August, 1994, the notion of infinity was raised as one of the key concepts in a
discussion of what mathematics has to offer. Pappas (1994) writes, "In addition to teasing our minds, infinity is an indispensable mathematical tool" (p. 46). I agree, and suggest that the study of the infinite is an area for universal exploration. I believe that infinity transcends cultures, ages, and ability. Infinity also unites mathematics with the sensitivities of other creative arts.

Novelist Peter Høeg (1994) writes about what he terms Cantor's 'Hotel Infinity':

What delights me about this story is that everyone involved, the guests and the owner, accept it as perfectly in order to carry out an infinite number of operations so that one guest can have peace and quiet in a room of his own. That is a great tribute to solitude. (p. 9)

This author has drawn a connection between infinity, charity, and solitude. Now, I have another new connection/relationship to ponder.

I enjoy playing with words and meanings. It appeals to my intuitive sense of the close relationship that mathematics has to language. For example, both are communicative symbol systems that rely on individual sense making for their effectiveness. One of the things that I have noticed about gaining meaning in mathematics is that I can sometimes achieve this by exploring language.

One of the word-plays that I hope to explore in my understanding of infinity is to look at other words with the negating prefix ‘in-’. It seems that if I take several of these words and roll them together, I
might have some closer understanding of what is involved in infinity. Here is a beginning list for further, but future, connections. Indescribable, inaccurate, inaccessible, invisible, incessant, intangible, and intuition. (Pimm (1994) writes of a child in a mathematics class sliding between ‘infinity’ and ‘infidelity’.) Infinity is indeed a joyous concept, one that all mathematically literate people should hold with intimate familiarity, and yet with abundant differences. Imagination is required and awareness is needed.

I close this section with a poem of Proclus ca 450AD, cited in *Images of Infinity* (Hemmings and Tahta, 1992). This delightful piece combines the known and unknown aspects of the existence of the infinite, and describes the imagination’s ability to recognise by its very failure to understand.

The infinite exists in the imagination:
not the object of knowing imagination
but of imagination that is uncertain about its object,
suspends further thinking
and calls infinite all that it abandons.

Just as sight recognises darkness
by the experience of not seeing
so imagination recognises the infinite
by not understanding it. (p. 13)
IN SUMMARY: CONCLUSIONS WITHOUT CLOSURE

Earlier in this chapter, I wondered how the concept of infinity could be explored by children. Specifically, how can I take advantage of the opportunities that will be presented in the future to guide Jackie's developing ideas of the infinite? What are some of the things that I might do to ensure an ever widening knowledge of infinity?

Doing this study has led me to believe that I need to stay away from definitions of infinity that include examples/statements like, “A line is infinite. It goes on forever in both directions.” Why? Because children see/visualize lines non-mathematically. They understand a line to be a mark on a page. They understand the mathematical term for a line segment as if it were a line. Everyday language connotations get in the way of a line being a useful example of the infinite. I also hope to avoid merely stating, as I have in the past, that the number system is infinite because you can always add one more. Why? Because explanations like this tend to confirm kids’ notions of infinity as the biggest number. To compound this, this type of comment does not even lead to the foreversness of the small; it seems to support a limited idea of infinity as uni-directional. I’ve thought some about what I don’t think will help; now what might help?

Jackie has helped me to realize that infinity is more easily felt than it is understood. As a result of this, I offer that in order to build knowledge of the infinite, feelings and emotions should be invoked. Just thinking about the itchiness of chicken pox helped me to connect
intuitively to infinity. Realizing that patterns, flowing out in two and three dimensions, provided a less linear entry to infinity helps me recognise the power of tessellations and fractals in exploring infinity.

Even writing this chapter, knowing that there are often as many (or more) interpretations as there are people willing to interpret, provides a glimpse into the infinite. As Bishop (1988) suggests:

> mathematical thinking is concerned essentially with imagination and not with manufacture, and that our imagination is fed by feelings and beliefs, just as much as it is by figures and objects. (p. 42)

Infinity requires imagination and willingness to suspend understanding. To understand the infinite, in the traditional sense of understanding, limits the concept. I propose that infinity does not require understanding, but that knowledge of it is gained through personal exploration. Closure is neither required nor desired. Awareness allows increasing inclusions. Exploring how infinity is represented in other fields contributes to mathematics' use of the infinite.
CHAPTER 8: LESSONS I LEARNED FROM JACLYN

I believe it is possible to learn from those who do not specifically intend to teach. I learn from those to whom I pay active attention, and Jackie has been one of those people. In paying attention to her, I did more than watch; I consciously attended. I was aware of reflecting on her behaviour, actions, and speech long after the events that initially drew my attention had occurred. I am aware that sometimes these reflections led to a long, introverted look at myself. And, it is this looking inward that provided the environment necessary for this thesis.

Initially I was unconvinced that mathematics was even taking place before school, and uncertain about what it might look like if it were. I wondered if it would be disguised, and, if so, what the guise would be. Would I be able to find it?

Having found early in my looking that mathematics was easily accessible within my home, within Jackie’s and my regular-life relationship, I started to seek other ‘truths’. Finding them, in this context, was not easy. I was soon confronted with my belief that I was finding mathematics simply by watching Jackie; I learned this was not entirely true. I was really finding mathematizing incidents by attending to Jackie and her world through the lenses of my awareness ... my awareness of mathematics, often perceived in subject-neutral events.

I also struggled hard to find a way to keep emotionally uninvolved so that my research would remain ‘pure’; realising eventually that
this was impossible. Each hour of reflection resulted in more personally-involved research, an involvement that accumulated.

Reflection also led me to realize that I was using ideas in my classroom teaching that I had gleaned from attending to Jackie. Thinking about her was allowing me once again to see elements of learning behaviour that had been trained out of me as a teacher: elements such as who is in control, who needs to be asking the questions, who needs the freedom to say that this is enough for today, and many more. These are thoughts that I want to explore one by one in these conclusions. What are the specific things that I learned by doing this research? How did Jackie, intentionally or not, help me to learn them?

**REVIEWING THE ORGANIZING FRAMEWORK**

The use of particular senses and specific sense-making has been the organizing lens for this thesis. I have stressed the use of touch and sound as partners to sight; senses that feed external stimuli to the mathematizer. There is a sense of play throughout that often provides a unifying link across incidents, whether playing with words, objects, roles, or feelings.

I have also looked at the development of a sense of infinity, and at the sense of comfort that can sometimes be provided by mathematics. Making sense of events through mathematics and finding a way to extend this sense-making to other contexts has also been part of the structure. Additionally, creating and sensing multiple uses for mathematics has been explored.
In these ways, the organizing structure has allowed both the utility and the aesthetics of mathematics to be perceived, and although this is not a primary theme, it is one that permeates the thesis as it influences the foundations of mathematics itself.

REVISITING THE RATIONALE

Within this thesis I have looked at each of the following areas, some with more attention than others. As I explained in the opening paragraphs, a number of the questions and themes that were initially in the foreground became less prominent, and others emerged to take their place. Although my initial foci moved into the thesis background, they nevertheless gave rise to my selection of these anecdotes, which became my thesis context and raw data.

However, it was the newer questions which then became the filter for making sense of these selected anecdotes. A personal story unfolded and will continue to unfold each time I, or indeed others, look at the words and actions I recorded. Some lines from the poet Bronwen Wallace (1991) resonate for me regarding this:

Always, I am amazed at what we tell, how much faith we put in it. Never really knowing who is listening, how they’re going to take it, where. (p. 21)

As I close this work, I see the following as background themes:

Does mathematizing occur at home?
How does it happen?
Where does it take place?
When does it occur?
What type of mathematics is involved?

Can education flow from the home to the school?

Given that my answers to the two main questions above were readily available as ‘yes’, and that the inner ones proved to be less and less interesting to me, the next stage of exploratory themes emerged and included the questions re-searched throughout this thesis:

What are the interactions between the roles of mother, teacher, researcher?

How are these roles interdependent?

When are the roles independent?

Do reflections about the mathematizing Jackie and I are doing lead to reflections about school and my classroom teaching?

Can reflections from one context be used to explore actions and thoughts in another context?

Given the same topic but different contexts, how much and what kinds of knowledge can be transferred?

SUMMARY? CONCLUSIONS? IMPLICATIONS?

To summarize, to conclude, and to list implications is possible only within the conditions of ‘for now’, ‘for me’, ‘at this time’. With this in
mind, I now present the more specific findings of this thesis. Rather than referring to the learnings in each chapter, I feel that it will be more meaningful to present these as learnings from the thesis as a whole. So often my recognitions, once focused in awareness by a particular incident, emerge in many other contexts. Reflection has tended to generalize my awareness across chapters. What started as a spool of thread has become a woven cloth.

To begin, I want to offer a reading frame by providing a poem by William Carlos Williams:

To make a start
out of particulars ...
Sniffing the trees,
just another dog
among a lot of dogs.

(quoted in Jardine, 1992, p. xv)

One of my findings is that mathematics arose in a context where mathematics was often not the main focus. Mathematizing was able to occur, for example, when having a bath, reading a story, preparing dinner, playing catch, creating pictures, or driving home. As a mother, I did things with Jackie and, in the doing, sometimes found mathematics emerging. Mathematizing occurred in a context where the mathematical was not the only, or necessarily, the prime focus.

Mathematics emerged much more often than I initially expected it would. Once my awareness was attuned to looking for mathematizing
occurrences, I found that I noticed incidents that I might not have otherwise. In fact, I noticed the mathematical possibilities in activities that were similar to ones I had done with my older daughter, completely oblivious of the potential mathematics therein. So, the necessity of having an awareness of mathematics which then serves as a vehicle for its own emergence in another person is a finding of this study.

Once I became aware that opportunities for mathematics existed in home situations, I noticed that no special equipment was necessary for mathematization. Jackie and I used things that we already had. Examples included paper and pencils, storybooks, milk bottle tops, cards and board games.

The fact that Jaclyn could readily gain my attention through introducing mathematizing strategies points to her knowledge of a framework that I recognise as mathematical in her world. She even used mathematics to get my focus for the non-mathematical. In the following incident, Jackie used the vocabulary of geometry as a way to open the door that led to asking for violin lessons.

Jaclyn (5 years 5 months) and I have been eating dinner and having a general conversation about our day. As I leave to go study and work on a paper I am writing, Jackie draws me into an activity by dangling a hexagonal carrot.

M: I've got to go study now.

J: First, can you draw me a ... hexagon?
M: [I intended to say, “No”, but couldn’t resist ‘hexagon’.] Why a hexagon?

J: Because I want to remember one.

M: Okay. The important thing is you need six sides that join up like a stop sign. [I draw one.] Or like this. [I draw an irregular hexagon while counting the sides aloud.] Or any one with six sides that join.

J: Can you draw an oval?

M: [I draw one.]

J: No, one that you play like this. [She moves her fingers at the side of her mouth.]

M: Oh, you mean an oboe.

J: No, an oval.

M: They sound alike and if you mean the instrument that you can play, then the word is oboe. I’m not too sure if I can draw one, but it’s something like ... [I draw one.]

J: Can I take violin lessons?

M: Sure. From Sally’s mom?

J: Yeah.
Jackie certainly knows how to get my attention. I notice the conversation route starts at hexagon and flows from oval/oboe to violin lessons. I believe that creative 'math-speak' and attention seeking was in action here. Jackie knows that the reward for her mathematical pursuits is my attention. She needed my attention to ask her question about violin lessons, yet was unsure whether talk about violins would be powerful enough to divert me from my goal of leaving to study.

I was completely taken in, even to the point of helping her to remember hexagons by sharing some of my strategies about the shape. If she had, indeed, said 'oboe' instead of 'oval', I probably would have still heard 'oval'. My mind was set on mathematical shapes, not musical instruments.

Jackie became very good, very early in my quest for our natural mathematics opportunities, at categorizing the mathematical from the non-mathematical. It took me longer to notice the attention-getting and attention-keeping aspects of this than it did for her to make the distinction. I learned that focused attention is very motivating for Jaclyn. I also learned that this demand for careful attention extended to other settings for her as well as for me.

Recall at the end of Chapter 3, I mentioned her day-care teacher telling me that Jaclyn (4 years 11 months) expected to be listened to and expected her opinions to be valued. That teacher also mentioned that Jackie could focus and concentrate on activities for extended
periods of time. Focused concentration and persistence are evident in the mathematizing that we did at home.

Another finding of this study was that Jackie could transfer learnings from one context to another, whether the contexts were both within mathematics, across subjects, or between different situations. (For references to the issue of transfer, see, for example, the math and art connections in Chapter 5.) Jackie also became expert at asking questions, discussing and representing what she was doing and thinking, and at getting me to understand her point of view. In other words, the world of two-way discourse was open to us.

Connected to this was my coming to know the amount of control Jackie had over our activities. She usually controlled the length of time that we would stay with our activity, whatever form it was taking. There are incidents in Chapter 5, among other places, that speak to this. I found that if Jackie grew tired of an activity, she told me to continue by myself; if she wanted to continue, she seemed endlessly able to think up questions to keep me engaged.

As mentioned in the preceding paragraph, most of the incidents of mathematizing were very short. Many of them were but brief asides within the activity we were doing. I learned to appreciate the strength of powerful mathematizing opportunities, no matter what their length. Time does not seem to be the same issue for learning at home as it is in the bell-regulated school where I teach. Neither is there a rule at home that mathematics must be done in a pre-established time slot. At home, there was no pressure to cover a
curriculum, no need to feel that everything must be done. This allowed the freedom of stopping whenever either Jackie or I wanted, and also permitted us to return to it at any time.

The contrast between home and school was, for me, quite dramatic. After reflecting on these constraints of time and curriculum, I found that I was more able, when practical, to encourage my students at school to timetable their own work periods, stopping when they needed a break, and continuing longer with a subject when they were not ready to stop.

Although most of the incidents reported here were less than five minutes in duration, some of them lasted up to half an hour. Many, especially bedtime stories, involved a dancing in and out of mathematics and story plot as the story itself moved toward its end. Some of the stories involved mathematics, either obviously or less so, in their plot. For example, *Jim and the Beanstalk* had opportunities for measurement and size comparisons within its text and illustrations.

In these cases, the dance was in and out of mathematical topics situated inside and outside the story, as well as the plot line that held the story together. I learned that mathematics integration can take many forms, and that it need not be staged, nor highly analyzed. And, I extended this to confirm for myself that, in the classroom, too, integration can happen within each individual child; it does not need to be contrived through overtly and obviously planned theme development.
Another finding was that, frequently, at home Jackie and I were wandering in and out of what at school would be called 'strands'. At home, rather than strands, the focus was more on the activities themselves. These activities often involved different types of opportunities for mathematics. For example, in one we might be doing the activity but also be counting; in the next, discovering a pattern; in the next, drawing shapes; in another, categorizing items. The activity was the base and the mathematics came out of it. At school this is often flip-flopped.

In the traditional presentation of topics at school, one topic is worked on at a time, and activities are used to support, develop, and review the mathematics. I have noticed that I now often mix topics without necessarily connecting them, rather in the spirit of taking advantage of teachable moments. If something occurs in Science or in Art that also involves mathematics, I take the time to offer the math to the students. As a result of this research, I find that I am more available to play the game of “What’s the math here?” I also teach the basic numerical operations together more often than as discrete mathematical processes. I elicit connections among them by asking, for example, “If you know 12 x 20 is 240, what else do you know?”

Another area where my learning with Jackie has influenced my thinking about school is in the area of practice. Jackie often built her own drill into games and activities because it engaged her. Examples of this are ample in Chapters 4 and 6 particularly, but can be found in the other chapters too. Practice, for Jackie, was not monotonous; it
was usually challenging and it was fun, and often it was self-initiated and self-guided. Her interests tended to come in chunks.

Just as she would have a favourite story for a few days, she also had favourite games and activities, and she seemed to want to stay with them until satisfied. She kept at something until whatever need she was filling was satisfied, and then she went on to the next challenge or activity, with or without breaks, until that one was finished. She often left these activities for quite a time and then returned, ready to add to her previous experiences. Her retention from incident to incident was extremely good. She did not seem to need to go back and review.

What did this say to me as a teacher? I have been trained to arrange 'review' for most of the things that I teach. I have also been taught that children get bored very quickly with the same activities and so it is necessary to change activities frequently. I decided to ask my students at school about this (in May, 1995). They told me that they liked to be able to engage with something until they felt like stopping. They also said that when they really understood something that it was boring to go over it again and again, and that doing it over and over did not make them learn it better if they already knew how. Sometimes, though, even when they thought they knew something, if they left it, they needed help in remembering what to do. In this case, they felt that they probably did not have enough time with it in the first place.
When I think of all thirty children and about their individual experiences with topics being taught and with their confidence levels and ability to judge when they really know something, teaching anything at all seems like a daunting task. Yet, somehow, I watch and listen and ask questions and, to some extent at least, seem to be able to determine who needs what, when. Sometimes I am not sure how much understanding a child has, but I have learned that they can help me. I have found that, in an atmosphere of openness and honesty, the children are really very capable of assessing their own progress. They simply need to know the criteria upon which to base their judgement. I would never have thought to seek children’s opinions about such matters prior to my reflections on this thesis. Listening to Jackie has taught me better how to listen, to ask, and to have confidence in those who are learning. Finding myself trusting Jackie as a self-aware learner is another ‘finding’ of my study.

I have also gained an appreciation for attentive silence. Chapters 5 and 6 highlight this particularly, but again, the value of "silent participation" (Anderson, 1993b, p. 7) is present throughout much of this work. At school, one of the strategies for successful questioning is allowing enough ‘wait time’. This gives children, it is believed, an opportunity to formulate their answers and think of connections. Paying attention to the power of spoken words and sound patterns has taught me to think of silence as more than a break between a question and a response. It has taught me to appreciate the power of the breaks from sound as important elements in presenting and recognizing patterns, in establishing an atmosphere of respect for
thoughtful opinions and for focusing attention on sight strategies. Although sound was shown to be valuable and necessary in many of the incidents, silence is important for more reasons than I originally thought.

In addition to silence, there were some times of near silence; times when I used a prompt or hint to suggest a route or strategy for Jackie to follow. I found that it did not take much to bring her into my line of reasoning if she were ready for it. I also found that too many words about a topic, when she was not ready to hear them, resulted in the words being discounted or ignored. I learned that Jackie did not accept hints that made no sense to her. Yet, at times, she was willing to suspend belief and continued an activity based on her trust that it would make sense later.

Patterns and trusting to pattern has been a theme of this thesis. Jackie seemed to appreciate that pattern recognition and pattern continuance were good tools for sense-making. Watching Jackie increased my respect for patterning as a mathematical element. It also increased my understanding of different ways of knowing a topic. For example, I no longer think of intransitive counting to ten as ‘just’ counting to ten. I appreciate that the sequence of digits is an important pattern to establish before trying to build place value concepts. Incidents where Jackie learned which digit to pay attention to (one example, telling digital time, was discussed in Chapter 5) made me more aware of how sight, sound and pattern combined to make more sense of one way that time can be represented. I now
realize that creating a structure made from alternating colours of bottle tops is not ‘just’ a simple pattern.

Being able to string a simple pattern together, and being aware of the possibility of its infinite continuation, comprise powerful mathematical knowings. In addition to this, I have learned from reflecting on my anecdotes to appreciate the feeling of infinity, the comfort of something that goes on and on in all ways possible. I have written about this extensively in Chapter 7. The soothing capacity of repetition was another learning I have reported in this study. Recall how counting chicken pox as I touched them with medication was comforting. Counting was a distraction from the itch. The ability of mathematics to occupy her focus of attention, thereby distracting from another attention-seeking sensation, was a new thought for me.

Perhaps most important, I learned to value creativity. Looking back over my thesis I was struck with the importance and prevalence of creation in the mathematizing incidents reported. Jaclyn and I created games, cards, questions, stories, songs, patterns, and a way of relating that was immersed in the mathematical. I want to turn now to a final look at this immersion in mathematizing.
MATHEMATIZING AS IT FEATURES IN THIS THESIS

[Mathematization] can be detected most easily in situations where something not obviously mathematical is being converted into something which obviously is.
(Wheeler, 1978, p. 3)

What did such situations look like in this thesis?

Some of these situations included figuring out how a symbol worked to represent what it was showing. An instance of this was Jackie figuring out what the dots on a die meant. In game play, it also meant understanding how the play moved the game towards its goal. It even meant agreeing on what the goal should be, as in ‘Snakes and Ladders’.

Mathematization meant knowing “How many reds?” and “Can you draw me a hexagon?” were types of mathematical questions. It meant realizing that ‘red, green, yellow, red, green, yellow’ was a type of pattern, and that it was a pattern that could continue forever, with or without enough milk bottle tops. It meant knowing that the pattern could go on and on by using the words only. It meant noticing that the 2, for example, in the digital time 8:21 was the important number to watch if you were waiting for 8:30.

Mathematization connected with the meta-cognitive awareness inherent in a statement such as, “Four, three, two, one. ... Five, four, three, two, one. That’s easy. Remember when I could only go ‘three, two, one’?” It included the action in showing that five could be
represented with three fingers and two fingers as well as two fingers and two fingers and one finger.

I knew mathematization was at work when I saw the mathematics in Jackie’s poem about dogs, and the word play where ‘yes’ meant ‘no’ and ‘no’ meant ‘yes’.

When I realized that Jackie was using content areas of mathematics to gain and hold my attention, I knew she was mathematizing for her own ends - here, her deciding what is Mommy attending to? She was using her knowledge of math and not-math to capture my focus, extend our nightly storytimes or playtimes, and to gain entry into topics she wanted to open.

Inventing, or creating versions of, games that used numbers, pairings, words, and drawings all pointed to mathematization. Transferring knowledge of one activity to another exemplified mathematization.

Mathematization probably started out mostly as my domain. I learned to recognize the mathematical in regular at-home activities. I had not initially been aware of the math in most of these earlier, but as I learned to listen and watch, I found more and more occurrences. At the same time, as mentioned above, Jackie learned to identify the types of activities that would hold my attention, so probably started doing more of these. A mathematization cycle was created.

But I would be doing us both a disservice to suggest that Jaclyn was mathematizing only to interest and control the attention of a
significant adult. She was also deeply caught up in and engaged in
the mathematical on her own account – for the challenge and the
intellectual and emotional pleasure it provided.

AND SO IT CONTINUES ...

Jackie has started school and I am very interested in the
mathematics she is doing (and not doing). As she approaches the age
of the children I usually teach, I shall be interested to see how
mathematization at home might create even more tensions in me;
tensions between the mathematics teacher I am at school and the
mathematizing mother I find I have become at home.

Having become aware of mathematics in the home, it is not possible
to stop being aware. I find that though my anecdote collecting has
slowed down, it has not stopped. I still write down some of the
mathematical thinking we do together, but not as systematically as
when I was noticing, marking and recording it for this thesis. I still
put samples of our math-related activities into my mathematization
file. I still collect and read articles about home and school
mathematics. I am hooked on reflecting about mathematization
opportunities both at home and at school.
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217


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