STUDENT UNDERSTANDING OF THE KINEMATIC QUANTITIES OF ANGULAR SPEED AND ANGULAR ACCELERATION

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ABSTRACT

This study describes first and second year university physics students’ understanding and reasoning of the concepts, angular speed and angular acceleration.

The analysis was based on student responses to various tasks presented to them during one hour long interviews. These responses were characterized from a phenomenographic research perspective developed by Marton (1981) and his colleagues at Gothenburg University in Sweden.

The findings of the study are described by categories of description and by categories of reasoning. Categories of description characterize; students’ conceptualizations of angular speed from different frames of reference, and the ways in which students make comparisons of the angular speeds of two objects. Categories of reasoning characterize the ways in which students were thought to reason about the concepts of angular speed and angular acceleration in several task settings.

Interpretation of these findings are discussed with reference to the role a typical introductory physics textbook may have had in shaping the way in which students think about these angular kinematic concepts. Finally, instructional implications and directions for future research are given.
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CHAPTER ONE
THE PROBLEM

1.10 Introduction and background to the study

This chapter provides an overview of the research in physics education as it relates to this study, an introduction to the problem investigated by this study, a description of the study's specific research questions and a rationale for the study.

A large amount of work to date has been done in the field of physics education by researchers who have been primarily interested in student conceptualizations of physical concepts. These investigations have ranged over such diverse areas as; heat and temperature (Erickson, 1979), motion (Saltiel & Malgrange, 1980), gravity (Gunstone & White, 1981), statics (Minstrell, 1982), electricity (Shipstone, 1984), vectors (Aguirre & Erickson, 1986), optics (Goldberg & McDermott, 1987), sound (Linder & Erickson, 1989), and quantum mechanics (Fischler & Lichtfeldt, 1991).

These studies and others like them have provided evidence that many students before, during or after instruction hold conceptions of physical phenomena at variance with currently accepted principles or theories of physics. Some of these have been found to be similar to those of scientists of the past (Nussbaum, 1983; McCloskey, 1984; Baxter, 1989; and

The lack of success of these efforts may in part be attributable to the following beliefs: a belief that a conception (metaphorically or literally speaking) is a stable mental entity inside the head of a student; a belief that student reasoning is theory-like; and a belief that the context of a situation plays a minor role in influencing student's thinking.

Some of these beliefs will be discussed later, however, with respect to the later belief, that context plays a minor role in influencing students' thinking, the early work by the cognitive psychologist Jenkins, (1974) is germane. In his numerous investigations of memory in higher mental processes Jenkins concluded, as some educators have yet to recognize, that:

What is remembered in a given situation depends on the physical and psychological context in which the event was experienced, the knowledge and skills that the subject brings to the context, the situation in which we ask for evidence for remembering, and the relation of what the subject remembers to what the experimenter demands. (p793)

A philosophical and methodological foundation which reflects in part this contextualized nature of student thinking, as expressed by Jenkins is described by Marton's (1981) research perspective of phenomenography. This perspective
provides a framework by which to characterize the ways in which students understand a scientific concept which is embedded in the context of a task setting.

These ways of understanding a scientific concept may also be influenced by the historical and cultural context associated with the individual as it relates to the task environment, (see for e.g. Hewson's, 1985, notion of conceptual ecology). What is less apparent, however is the influence of the "mental context" generated as a result of one's own thinking, as one processes; images, sights, and sounds, that appear and disappear from consciousness, from moment to moment during the interview.

Concepts are not viewed (from the perspective of this researcher) as stable entities in the minds of students. Concepts, like memories, are viewed "..not like filed letters stored in cabinets...(but rather) ..like melodies realized by striking the keys on a piano" (Wechsler, 1963 p151). The context of the task situation plays a critical role in the way these "keys" are struck. Thus the term student conceptualization is preferred to the term student conception because the former term tends to infer a mental activity rather than a static mental entity as the later term does.

This study by drawing partly on the research in cognitive psychology, artificial intelligence, and science education, may offer some new directions in the investigation of students' beliefs about scientific concepts. However, any new direction is not without risk, as "One must make a start in
any line of research, and this beginning almost always has to be a very imperfect attempt...We are almost always con-
demned to experience errors in order to arrive at truth" (Diderot, D. circa 1750).

1.20 The problem investigated

This study investigated students' understanding of the kinematic quantities, angular speed and angular acceleration. Both the products and the processes of students' thinking about these concepts were characterized in terms of a set of categories of description. These categories were developed, in a manner to be discussed in Chapter Three, by Marton (1981, 1984).

In general terms this study investigated four areas. These were:

1. A student's ability to recognize the importance of establishing a frame of reference from which to measure the angular speed of an object.
2. A student's ability to make comparisons between the angular speed of two objects.
3. A characterization of students' reasoning about the concept of angular speed.
4. A characterization of students' reasoning about the concept of angular acceleration.

These four areas are framed in terms of the specific research questions outlined in the next section.
1.30 **Specific research questions**

1. What are students' conceptualizations of the angular speed of an object from different frames of reference?

2. What is the nature of students' conceptualizations as they make *comparisons* of the angular speed of two objects moving in semicircular paths of differing radii of curvature?

3. What is the nature of students' reasoning as they address several task problems involving the concept of *angular speed*?

4. What is the nature of students' reasoning as they address several task problems involving the concept of *angular acceleration*?

1.40 **Rationale for the problem**

The objective of this study was to better understand how students think about and use the concepts of angular speed and angular acceleration. This type of study is important for the following reasons.

Firstly, from a pedagogical point of view, for a physics teacher to be more effective in communicating the concepts of rotational motion such as angular speed it is important to have a better understanding of how students are likely to interpret present instructional attempts at communicating
these concepts by way of the classroom and physics textbook. Such an understanding helps in designing new instructional strategies.

Secondly, this study addresses students' understanding of concepts which themselves constitute a very important area of physics. Anyone planning a career in physics or engineering must have a good background knowledge in the area of kinematics and dynamics of rotational motion.

Thirdly, this study fills a gap in the literature of mechanics as it pertains to students' conceptions of rotational motion. Both a survey of the literature and a review of papers presented at two major conferences at Cornell University (1983, 1987) dealing with students conceptions in science show a lack of research into students' conceptualizations of angular speed and angular acceleration.

Finally, it is hoped that some new directions in the field of science education will follow from studies such as this. Such new directions may result in studies which not only offer a description of what students understand about scientific concepts, but offer a more detailed description of how such understandings develop, and how within a particular context, students reason about such scientific concepts. This is necessary if more effective instructional strategies are to be developed.
1.50 Overview of dissertation chapters

This study is described over a span of five chapters. Chapter Two, which follows provides; a literature review which enables the reader to situate this study with other relevant studies in the field, a context by which the development of an appropriate methodological framework was chosen for this study, and a background for the reader in which to better understand the discussion of the findings of the study that follows.

Chapter Three sets forth the methodology used in this study, part of which is based on the phenomenographic perspective developed by Marton (1981). The writings of Marton (1981, 1988), MacCracken (1988), and Lincoln & Guba (1987) have contributed to the development of a specific methodological description which addresses the issues of trustworthiness and generalizability of the findings of the study.

Chapter Four describes the analysis of the data. The data which consists of segments of interview transcripts are arranged and presented in such a manner so as to permit the reader to validate the characterization of various student responses which have been constructed from the data. Such characterizations serve to describe both the content of student thinking and the processes of student thinking in the context of the task setting.

Chapter Five presents a summary of the findings of this study, followed by a discussion of the findings. Some of the findings which describe the ways in which students think
about angular speed and angular acceleration are related to the treatment of these angular kinematic concepts as presented in a typical introductory physics textbook.

The contribution and implication of the findings to the field of science education are also reviewed in Chapter Five, and the implications for instruction and for future research are discussed.
CHAPTER TWO

LITERATURE REVIEW

2.00 Introduction

This chapter describes the literature review from three different contexts. The first context consists of a review of studies that have investigated students’ understanding of motion as it relates to this study. This review will help situate this study in terms of other similar research work.

The second context consists of a review of the literature which has informed the perspective of this researcher as to the way in which individuals acquire and construct their knowledge.

Lastly, the third context comprises a review of the literature as it relates to instructional strategies. This review will serve to inform the discussion of instructional recommendations that appear in Chapter Five.

2.10 A review of motion studies

An extensive amount has been written about students’ ideas of physical phenomena that exist prior to, during and after instruction in the physical sciences. Some of the topic areas that have been studied are listed in the introduction to Chapter One. Other areas that relate more directly to the study of rotational motion, however, are few in number, and as yet no study has been done of student’s
understanding of rotational kinematics after instruction at the first and second year university level.

The studies of motion have been grouped for reference under various headings. These headings are: cognitive and cognitive development studies, problem solving studies, kinematic studies, frames of reference studies, and "situated cognition" studies. Each of these groups of studies will be discussed in the order aforementioned.

2.11 Cognition and cognitive development studies

The age at which children begin to use linear and rotational terms to describe motion has been investigated by Levin and Gardosh (1987). Students by the age of ten were found to be able to describe motion in rotational or linear terms and by the age of twelve could describe motion in both terms at the same time for the same object. The choice of descriptors for the motion whether in linear or rotational terms was however dictated by the feature of the motion which was most salient to the student.

Students enrolled at the University of California were tested by Ehri & Muzio (1974) for cognitive style in solving a rotational problem. Cognitive style was defined in this study as being field dependent or independent as measured by an embedded figure test\(^1\). Students in the study were asked

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1. This test asks students to identify specific figures that are hidden in a more complex figure or field. The ability to separate the required figures from the different fields is a measure of field independence.
questions about the relative speeds of horses turning on a merry-go-round. The results revealed that nearly all the students who were strongly field dependent could not solve the merry-go-round problem while those who were strongly field independent could. The researchers interpreted their results as providing evidence consistent both with cognitive style distinctions and with Pascual-Leone’s (1970) information processing model.

2.12 A problem solving study

Barowy and Lochhead (1981) conducted a study with about thirty university students shortly after they had completed the section on rotational kinematics of a non calculus based physics course. These researchers found that, in many problems involving torque, students’ attention was focused on preserving the superficial structure of the problem and as such were unable to disregard or modify perceptually unimportant features of the problem situation. It was concluded that: "many students who have reasonable intuitions about qualitative physics are unable to apply them correctly because they cannot isolate critical features of the problem" (p15).

2.13 Kinematic studies

Studies of the kinematic concepts of velocity and acceleration are of particular importance to this dissertation as these concepts are directly related to the concepts of angu-
lar speed and angular acceleration. Hence they will be discussed in more detail than the preceding studies.

Trowbridge (1980) investigated student's understanding of the concept of velocity\(^2\). Students taking an introductory physics course at the University of Washington were asked to make a comparison of the speeds of two objects in three different contexts. A description of these three contexts which follows, will be later contrasted to the responses of students in this study with respect to questions about the comparative angular speeds of two objects. The three tasks used by Trowbridge were: the speed comparison task 1, speed comparison task 2, and a task consisting of written examination questions.

In the speed comparison task 1, a ball, A (see diagram below) was made to roll horizontally at a constant speed (from left to right) while at the same time a similar ball, B, was made to roll up an inclined plane starting with an initial speed greater than ball A.

![Diagram of speed comparison task 1](image)

**Figure 1:** Speed Comparison Task 1. Motion is from left to right.

2. The discussion to follow unless otherwise indicated is based on Trowbridge's (1979) Doctoral Dissertation.
The horizontal component of the motion of the two balls (each ball having different starting positions) were juxta-posed such that the balls would appear to pass each other twice. At the points of passing the balls did not have the same speed, yet Trowbridge found that a significant number of students responded to the points of passing as the location where the balls had the same speed.

In the speed comparison task 2, a ball, C was made to roll down an inclined plane, starting from rest (see figure #2), while another ball, B was made to roll up an inclined plane as was done in task 1.

![Figure 2: Speed Comparison Task 2: Motion is from left to right.](image)

In this task the motion of the two balls was such that they never passed each other (note: the balls have different starting positions). However, the balls did at one point have the same speed. A number of students stated that the balls could not have the same speed because they never met or lined up with each other. This response was similar to some of the responses obtained in the speed comparison task 1. One such response which illustrates a possible source
in every day experience of such a conceptualization as described is illustrated by the following excerpt from the interview transcripts.

[Transcript notation I: interviewer, S: Student]

I: Do they ever have the same position along this track? (Balls are released)

S: There are two positions, right around here and here. So does that mean they have the same speed then?

I: What do you think?

S: Well, it's hard to say. It seems like they would. ....Yeah, they are, because like when I'm driving on the freeway...You know how you hate to have someone directly on the side of you...I assume that even though he might have been behind me, that he's caught up, and now he's neck and neck with me for a little while, so he must be going the same speed, even though he keeps on passing me. So at the time when we're together, we're probably going the same speed.  

A third task given the students was a set of examination questions. The written answers to these questions showed that students had difficulty with the concept of speed at an instant. One of the exam questions asked whether it was always true if: when two objects both reach the same position at the same clock reading, then they must have the same speed. Explanation, including an example (if the statement was false) was required of the student. One response to this question quoted below illustrates the student's inability to conceptualize the motion of the position of an object at an instant in time.

S: Objects can not really have speed for an instant;
for speed to be calculated, there must be an interval of time. For an instant the objects have no speed, just a location. (p 127)

Trowbridge maintained it is essential to be able to discriminate between instantaneous and average speed if the student is to understand the numerical meaning of $\Delta v$, which is crucial for an understanding of acceleration. Using a similar task set as previously described Trowbridge investigated student understanding of the concept of acceleration.

Trowbridge found that a common (incorrect) procedure for deciding which of two balls had a greater acceleration was to conclude that ball A had a greater acceleration than ball B because it was catching up to ball A. This would indicate a failure to differentiate between velocity and acceleration. Another (incorrect) procedure used by students in deciding which ball had the greater acceleration was to make judgments based on the final speeds of the balls alone. From observations of these procedures used by students, Trowbridge concluded that even though most students could define acceleration in an apparently acceptable manner after instruction, they could not use the definition to determine a procedure for comparing the accelerations of two moving objects.

Like Trowbridge, Reif's (1987) investigation of students' understanding of acceleration found that a significant number of students lacked the procedural knowledge of how to interpret the formula that defines acceleration ($a = \frac{dv}{dt}$) in problems that went beyond those ordinarily discussed in
physics courses. Most students could not for example figure out the acceleration of a pendulum bob at various points in its swing or the acceleration of a car that slows down while moving in a curved path.

2.14 Frame of reference studies

Frame of reference studies have particular relevance to this study because students being interviewed in this study were sometimes asked to respond to questioning about the motion of an object from a particular frame of reference.

A study by Saltiel and Malgrange (1980) of fifty first year university students found that students had difficulties with problems involving frames of reference. Such problems would, for example, require the student to think about the distance travelled and velocities of two observers, themselves in relative motion in three different contexts. These contexts involved a comparison of the motion as seen by an observer on a river bank versus an observer in a boat moving parallel to the river bank; an observer in a boat crossing a river versus an observer in a boat crossing the same river at a different orientation; and in a different setting, an observer on the ground versus an observer moving along on a moving pavement (as might be found at an airport terminal).

3. Problems involving motion along a straight line or of circular motion at a constant speed.
From these problems the researchers found that students would freely combine velocities in different frames without any special consideration for the frame of the observer, even though students acknowledged the existence of differing frames of reference. It was believed by the researchers that students thought not in terms of reference frames because they saw "real" motion as being an intrinsic property of the moving body, independent of the observer, and having a physical cause, whereas "apparent" motion is an illusion devoid of physical reality. The researchers hypothesized that the use of such terms as absolute or relative velocity in the classroom may have given the students the idea that some velocities have a deeper, more physical meaning than others.

Erickson & Aguirre (1985) in their study of students' perceptions of motion from differing frames of reference found that grade ten students experienced difficulties describing such motion. In one of the tasks students were asked to describe the paths traced by a moving cart as seen from two frames of reference, one moving relative to the other. A significant number of students described the path of the moving cart as being the same irrespective of the location of an observer in either of the two frames. And, the speed of the moving cart was considered to be a constant independent of the observer's motion with respect to the cart.
The second context that is germane to this study, consists of the literature which focuses on describing the manner in which individuals construct their knowledge. This context is described in two parts. First, a review of studies is undertaken which have characterized learning or cognition as dependent on the situation in which it occurs. And second, a review of studies is undertaken so as to provide a philosophical basis for the assumptions being made about the way in which individuals construct their knowledge.

2.15 Situated Cognition

Claxton (1981) questioned those that believed: "...that once you are 'in possession' of some information, or have 'acquired' an ability, they may be retrieved, in principle, whenever they are subsequently relevant. Once in your head, information and ability take on a life of their own, unrestrained by the incidental details of the context in which, and the purpose for which, they were originally registered" (p68). Claxton claimed as did Hills (1988), that the way in which children learn to understand and make sense of their own everyday experiences is inherently contextualized with respect to its purpose and occasion of use.

Rogoff and Lave (1984) have likewise argued that cognitive skills and the ability to orchestrate them is not an abstract, context-free competence. They suggest that the manner in which a person interprets the context in any par-
ticular activity may be important in facilitating or blocking the application of skills learned in one context to another. While it is possible that a level of generality of skills or knowledge may be achieved in certain instances and must be achieved in other instances if we are to function, it should not be assumed from the outset, say Rogoff and Lave, that such generality is usually the case.

Perkins and Salomon (1989) in answering the question: "Are cognitive skills context bound?", have suggested a synthesis in which a rich contextualized knowledge base operates in conjunction with generalized heuristics to enhance one's ability to solve problems.

Gallagher (1992) in a discussion of the role of hermeneutics in education puts forth the argument made by Husserl (the German philosopher) that everything comes to be known within a context and never in isolation. It is the context, he argues, which enables one to make sense out of the "unknown" thing.

Brown, Collins and Duguid (1989) claim that all knowledge is like a language. "Its constituent parts index the world and so are inextricably a product of the activity and situations in which they are produced" (p33). In support of their argument they cite such diverse areas of study as: Miller and Gildea's (1987) work on vocabulary teaching and Schoenfeld's work on problem solving in mathematics (1985).

1985, Clough and Driver, 1986) support the view that reasoning is contextualized as evidenced by student responses on tasks and problems which differ only in surface features or perceptual features (Driver, Guesne, and Tiberghien, 1985). This situation specificity of knowledge has been commented upon by Roncato and Rumiati, (1986) in their study of the concept of equilibrium, by Reif (1986) in his study of graduate students' and professors' attempts at solving non-standard textbook physics problems, and by Erickson and Aguirre (1987) who found that the: ".. task context appeared to play an extremely important role in terms of the way in which a student both frames and responds to a problem" (p17).

A perspective from which to view knowledge as inherently contextualized is offered by phenomenography. Rather than viewing students as holding concepts or other mental entities (as schemata) in their heads which have been abstracted from the context from which they arose, a more dynamic, contextualized, relational view of knowledge between the observer and that which is observed has been adopted by Marton (1981). However, while recognizing the importance of context, Marton (1988), believes that we do not have to "buy" the whole context; rather, it is our task he says to discern its most significant aspects. Which aspect is relevant in an interview setting (for example) will depend on the relationship of each aspect of the context to a particular interviewer's question as judged from the viewpoint of the interviewee.
Not only do some researchers in science education view knowledge as inherently contextualized but also some in the field of artificial intelligence (A.I.). Shank and Seifert (1985) have tried to represent knowledge as a structure composed of scripts, where a script represents a sum of our experiences, as the restaurant script for restaurants. The context of the situation would cue an appropriate script. Some of this work with scripts has informed the debate over whether or not student knowledge is theory-like.

Research with scripts was not able to answer some questions. One such question is: how is it possible that there exist many situations in which we as humans know what to do in novel contexts without having any situation specific script for that situation? "There are many situations which, as individuals, we approach without any clear beliefs or sets of expectations." (Driver and Erickson, 1983, p42). Therefore, what kind of cognitive system can exhibit this degree of flexibility without being straight jacketed by a specific context?

The answer according to McClelland and Rumelhart (1987) is that the knowledge base of the human system does not consist of scripts, but a set of atomistic bits of knowledge that configure themselves dynamically in each context to form tailor-made scripts. These elementary bits of knowledge can be actively constructed by the mind to form a coherent assembly called schemata. "Schemata are not explicit entities" say McClelland and Rumelhart, "but rather
are implicit in our knowledge and are created by the very environment that they are trying to interpret—as it is interpreting them" (p21).

Descriptions of the details of the knowledge system as described by Rumelhart and McClelland are unfortunately at a microlevel of neuronal functioning or processing that is not appropriate for the study of pedagogy. There have been many descriptions to characterize student knowledge at an appropriate level of detail, such as: misconceptions (Helm, 1980; Strike, 1984), alternate conceptions (Clement, 1984), conceptual frameworks (Driver and Erickson, 1983), alternate frameworks (Watts, Gilbert and Pope, 1982; Nussbaum and Novick, 1982), alternate theories (Hanson, 1984), intuitive theories (McCloskey, Kaiser, & Proffitt, 1986), mini-theories (Claxton, 1982), situation-specific knowledge (Roncato and Rumiati, 1986; Reif, 1986), and "knowledge in pieces" (DiSessa, 1988). One reason for such differences in the way in which student's knowledge has been portrayed may be due to the differing methodologies that have been used (Driver, 1989). Nevertheless, even if one accepts the methodology and research data of others there is still the problem of interpretation.

DiSessa (1986), for example has strongly argued against characterization of knowledge elements in terms of theories, claiming that it is possible to accept McCloskey's strongest data without accepting his conclusions. DiSessa has chosen instead to characterize the intuitive elements of student's
sense of mechanism as being minimal abstractions, *phenomenological primitives*, of common events.

The characterization of student knowledge as being either theory-like or not theory-like has been debated by other researchers. For example, in a case study of a student learning about the structure of matter and the way in which the student's ideas developed during instruction, Scott (1991) claimed that any prior conceptions the student had about the structure of matter did not exist in a coherent, generalizable, theory-like form.

However, Hills (1989) drawing on the work of the philosopher Churchland (1979), supports the theory-like view of common sense knowledge. Hills claims that students' theories represent an organized collection of propositions, based on experiences, which allow the student to categorize objects and events, form explanations of events, and predict future events. In other words, students' "untutored" beliefs about natural phenomena are common sense theories which share much in common with scientific theories. However, McClelland (1984) states that: "To suppose that children are scientists of a sort when they think about such phenomena (as heat conduction) seems to me to misconstrue totally the meaning and purpose of science" (p1).

It could be argued that a significant amount of what children do know about the world, as for example their knowledge of the concept zebra *cannot* be represented by a theory consisting of *only* propositional elements as: a zebra
is a four legged animal with black and white markings. The problem with propositional knowledge is illustrated by the following figure.

Figure 3: "Zebra", from Lefrancois, (1975)

This figure is meant to suggest that we clearly know more about the concept zebra than can be specified by a set of propositional elements which might be described by some theory.

The foregoing argument is only one part of a much larger debate over the status of the nature of student's knowledge about the world. Clearly such a debate is important and in the next section two important philosophical perspectives that have informed this debate, phenomenography and constructivism will be discussed.
2.20 **Phenomenography**

Phenomenography is a view of how people understand phenomena. Rather than describing a phenomenon which Marton (1981, 1984) called a *first order perspective*, phenomenography characterizes the way in which individuals may understand and experience a phenomenon, called a *second order perspective*.

Phenomenography as a research perspective has drawn upon two philosophical principles. First, the principle of intentionality, whereby all mental acts are directed towards something beyond ourselves; we cannot disconnect ourselves from the world "out there". And a second principle, an ontological principle of constitutionality where the individual and the world are considered as one, a unity, in a relationship together. It is this relationship which the phenomenographer seeks to describe, as opposed to the constructivist, who according to Marton (1989) views the individual and the world as two distinct entities, a subjective world created by the individual and the objective world "out there".

Conceptions are not seen by the phenomenographer as individual mental constructs residing in the mind of the individual by which we understand and experience the world out there. It is the ways the individual experiences the lived *in* (not "out there") world which become the elementary units of analysis. The analysis of these relationships, ways of
perceiving the world of which we are a part, are reconstructed and characterized as *categories of description*.

Which category of description is ascribed to an individual is not fixed. Individuals may move from one category to another in any one context or across contexts. The sum of all possible categories form a set (an "outcome space" in phenomenography) which is considered by the phenomenographer to be stable and generalizable to other settings.

Besides phenomenography, another perspective from which to view how individuals experience and understand their world is offered by constructivism.

### 2.30 Constructivism

It is not an easy task to define what constructivism is. There is no agreed upon definition. To quote Kelly (1970):

> Personal construct theory (constructivism) has...been categorized by responsible scholars as an educational theory, a learning theory, a psycho-analytic theory...a logical positivistic theory...a reflective theory, and no theory at all! (p10)

Noddings (1990) claims that to say knowledge is constructed tells us nothing about truth, knowledge, the justification of belief or the nature of evidence. And, Von Glasersfeld (1990), in the last few years has taken to calling his constructivist orientation a theory of knowing rather than a 'theory of knowledge'.

Despite the lack of clarity as to what precisely the term constructivism should encompass, there is general agreement on one central principle. Personal knowledge is not pas-
sively received but is formed by active construction by the individual of information that comes to one's senses.

For some the "...suggestion that people are active in producing knowledge seems...to be 'trivially true'” (Strike, 1987 p483). And for Von Glasersfeld (1987), this principle alone represents a trivial notion of constructivism.

A second principle of constructivism that is needed according to Von Glasersfeld, is that the function of cognition is adaptive and serves to organize our experiential world, rather than the "discovery of ontological reality". This is to say that we are not trying to discover the reality outside ourselves as much as we are trying to make sense of experiences as we perceive them.

A consequence of adopting the first (central) principle of constructivism is that one's understanding of constructivism is itself a construction. And, while this construction may be idiosyncratic and therefore private it does not mean that a public meaning of constructivism is impossible. Constructivists do not deny the possibility of the existence of shared meanings amongst individuals. They merely assert that for a shared meaning to exist to the maximum degree possible, one cannot simply ignore the personal constructs of others when engaged in the act of communicating.

Despite the difference in language between the constructivist term conception and the phenomenographer's use of the term categories of description they can share similar meanings under one condition, in which the individual's re-
sponses across a variety of task settings are identifiable with one and only one category of description (i.e. a stable conceptualization). Under such a circumstance the constructivist notion of conception comes close in meaning to that of the phenomenographer's category of description, despite the differing ontological viewpoints of each perspective. Or, to put it another way, a stable conceptualization may be thought of as the Constructivist notion of conception.

Three instructional models compatible with this constructivist perspective are reviewed below.

2.40 Instructional strategies

Three differing instructional models will be briefly introduced followed by a discussion of each approach. One approach to instruction is based on (or similar to) the conceptual change model of Hewson (1981, 1982, 1989). This model requires students to modify existing conception(s) by either a process of: conceptual capture, by recognizing that the new conception fits into an existing conception (similar to Piaget's notion of assimilation); or by conceptual exchange, where the new conception replaces another already existing conception (similar to Piaget's notion of accommodation).

A second approach to instruction, is based on Niedderer's (1987) model which postulates that students need not necessarily replace their existing conceptions but recognize the
difference between everyday thinking and scientific thinking.

A third approach to instruction emphasizes strategies which build upon existing student intuitions in order to have the student adopt a scientific viewpoint. Such an approach is exemplified by Brown and Clement (1989, 1991).

The first approach, based on Hewson's conceptual model requires students to experience dissatisfaction with their existing conception(s). Dissatisfaction may be achieved in two ways. One is by creating conceptual conflict where an individual fails to explain or predict an event; or in the second way, the individual finds a present conception somehow inadequate.

If an individual is to understand the new (scientific) conception being "presented" during instruction it must be intelligible, that is comprehensible. And if the conception is intelligible then it should at least be plausible, namely (in the case of a theory or proposition) it could be true. Lastly, the conception should be fruitful for the individual. The acceptance of the new conception should in some way benefit the person, as for example it might contribute to a more coherent theory of physical events or phenomena.

In the second approach to instruction, Niedderer's model proposes that students should come to view their own explanation of phenomena (which are usually found to be localized or contextualized), as being qualitatively different to what a scientist seeks of his or her own theories. The student
and scientist may, and usually do, have different objectives in arriving at or developing their own "theories". The scientist, unlike the student, seeks a greater generality. Thus, while it is desirable to have the student move towards a scientific conception it is acceptable for the student to have or hold alternate (non-scientific) conceptions of phenomena or events concurrently.

In the third approach to instruction based on Brown and Clement’s (1987) model, an attempt is made to build on students' existing valid physical intuitions in order to change their existing conceptions. Students are helped to make intuitive sense of scientific theory by the use of special types of analogies called bridging analogies. A bridging analogy helps the student "see" that a teacher generated analogy is valid in viewing a phenomena or event from a scientific perspective.

From these three instructional models a background is possible from which to discuss instructional implications as they arise from this study. The instructional implications will be dealt with in Chapter Five following the analysis in Chapter Four, and a discussion of the methodology, which is discussed next, in Chapter Three.
3.00 Introduction

This chapter describes in order: the subjects, the interview setting, the methodological framework, and the tasks that were used in this study.

The basis from which to examine students' understanding of the angular concepts, angular speed and angular acceleration was provided by a set of interviews with fifteen university students. The subjects of these interviews are described next.

3.10 Subjects

The fifteen subjects that volunteered to participate in this study were first and second year physics students all of whom had prior instruction in the concepts of angular speed and angular acceleration at the time of the interviews. Of the 15 students interviewed, 4 were Physics 115 students, 8 were Physics 120 students and 3 were Physics 216 students.

Physics 115 students when interviewed were in the last term of a first year calculus based physics course in waves, electricity and mechanics. Physics 120 students were also taking a similar course to physics 115 in its coverage of topics; however the depth of coverage of mechanics is
greater in physics 120 than in physics 115\(^1\). Physics 216
students were taking a second year course in mechanics.

3.20 The interview setting

Each of the interviews lasted approximately one hour and
was audio and video recorded. The interviews were conducted
in a small room having a table and two chairs. The student
was seated at one chair and the interviewer at the other.
The table was used to support the various task equipment or
materials that were presented to the student during the
course of the interview. Interviews were held on the campus

The interviews with the students were designed around
d four tasks. Each task interview required a student to view
the motion of one or more objects moving in a path from a
specified frame of reference. The interview protocol was
semi-structured consisting of some questions called focus
questions which were asked of each student. These focus
questions served to keep the discussion between the student
and interviewer centered on a particular research question. 
Other questions asked by the interviewer functioned in two
ways.

One function was to seek simple verification of the re-
searcher's understanding of a statement made by the student
in response to a focus question. And a second function was

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1. Physics 120 is restricted to those students who have done
well in their previous high school grade 12 physics and
grade 12 math.
to probe issues that could not be anticipated beforehand, but which were deemed relevant by the student as to the way he or she thought about some aspect of the task situation.

3.30 Methodological framework

Following Marton's (1981, 1984) approach for analyzing qualitative data, categories of description were generated to describe ways in which students conceptualize or understand the concepts of angular speed and angular acceleration. These categories of description together form part of the "outcome space" which collectively describe the ways in which angular speed and angular acceleration were conceptualized.

The process by which the researcher constructs categories is not "linear", that is the process of creating categories of description does not proceed by a mechanistic, rule based procedure. Categories of description are tentatively formed after several readings of the transcripts only to be modified or changed upon further reading or reflection upon what has been read. This process is not unlike the following description of the hermeneutical cycle:

When we consider the task of interpretation...we have to say that our increasing understanding of each sentence and of each section [of a text], an understanding which we achieve by starting at the beginning and moving forward slowly is always provisional. It becomes more complete as we are able to see each larger section as a coherent unity. But as soon as we turn to a new part we encounter new uncertainties and begin again, as it were, in the dim morning light. It is like starting all over, except that as we push ahead the new material illumines everything we have already treated..

(Schleiermacher, 1977 p198)
Our understanding is never complete, however. "Not only do we never understand an individual view exhaustively, but what we do understand is always subject to correction". And, the process of forming categories of description like the hermeneutic cycle, becomes a spiral, which is never in a way complete: "We can never claim that there are no other ways of seeing a phenomenon than the ones we know of" (Marton, 1989, p37). However, at some point during the process a set of categories does emerge which serves to establish a framework from which to describe and interpret student responses.

In a like manner a similar discussion could also apply to categories of reasoning as outcomes of the study.

3.40 Categories of reasoning

Marton created the term categories of description to characterize how students conceptualize or understand concepts or ideas. This researcher has created the term categories of reasoning to characterize some of the forms of reasoning students engaged in. Categories of description thus serve to characterize the content of what students have said while categories of reasoning characterize what are thought to be their thought processes. For example, during the interview a student might respond to a question about the angular speed of an object in terms of the object’s (linear) speed. This response, of associating the word

2. Schleiermacher (1977), Hermeneutics, p149
angular speed with speed was characterized as a form of reasoning, a word association.

The phrase "word association" as used in this study does not mean what a psychologist understands by this term. The most important difference is that in psychology word association usually requires students to list words they think about when given a target word and this is not (as the reader will note later) the same procedure employed in this study. Word associations in this study are inferred from student responses. Other categories of reasoning have also been characterized as forms of association, namely: symbolic association and contextual association, both of which will be discussed later.

Finally, how is the reader who has not participated in the process of creation of these categories of description or categories of reasoning to have trust in such descriptions? The answer to follow is given for categories of description but applies equally well for categories of reasoning.

3.50 Trustworthiness

There are three distinct elements which serve to "triangulate", or establish trust in findings of studies like these. These elements are signified by three words: process, product and person, which are described respectively.

The process of documenting a description of; the tasks, the task setting, the subjects, the manner in which the in-
Interviews were conducted, excerpts from the transcripts plus an entire interview transcript, and the process by which the categories of description were created, all serve to establish trust in the findings of the study.

A product of this study is the categories of description. To the extent that this product, these categories "ring true" to the reader, that the reader may find a recognizable truth by virtue of his or her own experiences, serves to establish a trust in the outcomes of the study.

Lastly, the person, as researcher provides yet another check on the trustworthiness of the study. Who did the research, not just the content of the research itself, may be viewed as a criterion for judging knowledge claims. "In other words, scientists often regard it as proper to judge the man rather than the knowledge claim" (Mulkay, 1985, p67). But there is also a sense in which the person as researcher may be judged other than perhaps by previously established work in the field.

"It is precisely because the qualitative researchers are working in their own culture that they can make the...interview do such powerful work. It is by drawing on their understanding of how they themselves see and experience the world that they can supplement and interpret the data they generate" (McCracken, 1988, p11-12). In this way this researcher may claim to have access into the world of the science student and thereby establish a degree of trustworthi-
ness in a way that the foregoing elements, process, and product do not.

Even though conditions for trustworthiness of the study may be established, there still remains a question as to the extent the outcomes of this study may be transferred to other contexts. This is the issue of generalizability.

3.60 Generalizability

In the physical sciences the justified belief that the laws of nature have a universality of application provides a rationale to generalize to other places and times. In the field of statistics, a priori assumptions, such as normality of population distributions, provide a rationale to generalize from the sample to the population. But, in the case of the individual interviews in which the object of study is to describe the way in which students conceptualize their world, such a rationale as is used in the physical sciences or statistics is not possible. The purpose of qualitative research is: "not to discover how many, and what kinds of people share a certain characteristic...It is the categories...not those who hold them, that matter. In other words, qualitative research does not survey the terrain, it mines it." (MaCracken, 1988, p17).

In qualitative research a different type of generalizability is possible, a naturalistic generalizability, where the outcomes of the research are seen to be: "...in harmony
with the reader’s experience and thus to that person a natural basis for generalization” (Stake, 1978, p5).

However, the issue of generalizability need not be framed as making a choice between two polar opposites, between a scientific view and a specific, idiographic view. Instead, "...we are dealing with a continuum, the two ends of which do not begin to encompass all of the possibilities that exist" (Lincoln and Guba, 1985, p122-3). Where on this continuum of transferability do the outcomes of this study lie? This depends on both the researcher and the reader.

It is the obligation of the researcher to describe the categories of description and the context in which the investigation was conducted in such a manner as to reasonably allow anyone else interested in transferring the findings of the study to another situation to have a basis upon which to make appropriate judgments. And, it is the role of the reader to judge the degree of fit that might exist between the context of the study and the context to which the reader wishes to apply the findings of the study. Together the researcher and the reader provide the elements necessary to achieve a transferability or a naturalistic generalizability of the study’s findings.

3.70 The Tasks

The tasks used in this study are identified by the shape of the path of the moving object shown to the student during a particular task. The tasks to be described in order are;
the oval path task, the semi-circular path task, the elliptical path task, and the irregular path task.

A discussion of how a physicist might respond to these tasks is provided in Appendix III. This will provide the reader who may be less familiar with these concepts a basis for better understanding some of the students' conceptualizations outlined in Chapter Four.

3.71 **The oval path task**

In this task a toy train travelled around an oval track at a constant linear speed. The student who was seated at a table upon which the train moved was asked to respond to questions from the position of a toy observer located at the center of the oval and from a position close to where the student sat.

The track was approximately one meter across at its furthest points. The following diagram of the oval track shows the points of the track which were used as references points during the interview.

![Figure 4: Diagram of oval path](image-url)
A, B, C, D are reference points used in the interview to help identify a point or section of the track. X, is the position of the student relative to the track.

**Focus questions**

Having identified to the student the position from which he or she was to act as the observer, the focus questions asked were:

1. Does the train have an angular speed between positions B and D ...(and other parts of the track)? Why?
2. Does the train have an angular acceleration? If so, where and why?

As a reminder, these focus questions are only some of the questions that were asked during the task. Other questions asked depended in part on the responses given by a particular student. Such questioning (see dialogue given in Chapter Four or in the Appendix) was directed at investigating unexpected or unclear responses, or investigating a new direction in a student’s thoughts that might offer new insights.

The student was then asked questions about the motion of objects moving in semi-circular paths.

3.72 **The semi-circular path task**

In this task pingpong balls were made to travel down two rail tracks shaped into semicircles of differing radii of
curvature. The inner track had a radius of curvature of 46 cm, and the outer track had a radius of curvature of 56 cm. The starting point of each track was elevated 4 cm above the end point.

First, one ball was released from the top position which is labelled the "starting point" of the outer track in the diagram to follow. After the ball had travelled a short distance it triggered a photogate which in turn triggered a solenoid causing a second ball located 15 cm from the top of the inner track to be released from its position on the track.

The release position of the second ball was so designed that both balls arrived at the same time at the end of each track. A video recording was made of the motion of the balls on the track. This was done because it was difficult to consistently reproduce the movement of the balls. This video was shown to the student after the student saw several demonstration runs. Another advantage of the video recording was that the motion of two balls could be shown in slow motion. Thus making it easier for the student to make comparative judgments of the balls angular speeds. Despite this visual aid some students still had difficulty in making judgments of the relative angular speeds. Due to this problem it was decided not to include in the task a question about the relative angular acceleration of the balls. However, a question regarding the angular acceleration of one ball was asked.
Figure 5: Diagram of semi-circular paths

A-G: are used as reference points.

X, is the position of the student as the observer.

The focus questions.

The focus questions were:

Do the two balls ever have the same angular speed?
If so where and why?

Does this ball (pointing to one of the balls) have an angular acceleration? Why?

As with the previous task additional questions were asked which sought to enhance the student response(s) that were given to the focus questions.

In the task that is described next the motion of an object moving in an elliptical path was used to further investigate the ways in which students understood the concepts of angular speed and angular acceleration.
3.73 The elliptical path task

The elliptical path made by a moving object was represented by a diagram to the student. The object itself was indicated by a dot located on the elliptical path. The student was informed that the object was to be thought of as moving around this path at a constant linear speed. If during the interview it appeared as though a student might have forgotten or not have adequately grasped the motion of the object then the object's motion was again explained.

The following diagram of the path (not the one actually shown the student) is shown here for reference. The diagram shown to the student appears in the appendix.

![Diagram of elliptical path]

**Figure 6**: Diagram of elliptical path

A-E; A'-E': are reference points.

X's are observer positions.

**Focus questions**

The focus questions for this task were:

1. Does the ball have an angular speed? Why?
2. Is the angular speed of the object a constant? Why?

3. If the angular speed is not a constant then where is the angular speed a maximum and where is it a minimum as seen by someone standing at the:
   i) centre of the ellipse?
   ii) outside, near the edge of the ellipse?

   As with previous tasks additional follow up questions were asked. The last task to be discussed was used to investigate students' understandings of the motion of an object moving in an irregular path.

3.74 The irregular path task

   In this task a diagram of an object moving in an irregular path was shown to the student. It is assumed that most if not all of the students interviewed would not have seen physics problems which required them to analyze the motion of an object moving at a constant speed in such an irregular path.

   The diagram of the path which follows is provided for reference. As before, the diagram of the actual path shown to the student is illustrated in the appendix.

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3. As judged by the problems in physics textbooks and from talking with physics instructors, the students would not have done kinematic problems in which an object moves in an irregular path.
A-H: are reference points used during the interview.

X, is the position of an observer.

**Focus questions:**

The focus questions were:

1. Does the object moving in this path have an angular speed? Why?

2. If yes (to question #1), then does the object have a constant angular speed? If not, where is the maximum and minimum angular speed? Explain.

As before, additional questions were asked of the students as was done in the other task interviews.

In summary four different tasks were presented to students during the interviews. Each of these tasks involved an object moving in a particular path. These paths were the oval, semi-circular, elliptical, and irregular.
Focus questions sought to elicit student reasoning about and conceptualizations of the concepts angular speed and angular acceleration. Other questions served to clarify a response or explored an interesting or unexpected response which was thought to be important as to how the student viewed the task situation.

By a process not unlike the hermeneutic cycle, categories of description and reasoning emerged from repeated analysis of the interview transcripts. The particular interviewer-student dialogue which led to particular categories being formed is part of the analysis that is described in the next chapter.
CHAPTER FOUR

ANALYSIS

4.00 Introduction

As discussed earlier the purpose of this study has been twofold. First, to investigate the ways students conceptualize angular speed and angular acceleration. And second, to investigate the nature of student reasoning as inferred by their responses to questioning about these angular concepts.

With respect to the first purpose, the analysis of the first two research questions from a phenomenographic perspective has led to the construction of categories of description that serve to describe students' conceptualizations of angular speed and angular acceleration. This analysis was based on student responses to questions about: an object's motion from differing frames of reference (research question one) and to comparisons of the motions of two objects from a fixed frame of reference (research question two).

With respect to the second purpose, the analysis of the last two research questions used similar analytical techniques to those used for the first two research questions. However, these techniques led to the construction of categories of reasoning which characterize the reasoning used
by students in responding to questioning about angular speed and angular acceleration.

The analysis of all four research questions (as described in chapter one) is organized into a structure which is explained below.

4.10 The structure of the analysis

The analysis of the interview data has been organized into the following format:

1. Research Question
   The research question is stated.

2. Introduction
   An introduction to the analysis is presented.

3. Overview of findings
   An overview of the research findings is given.

4. Category title
   A category is identified by a label called a category title. A category may be further broken down into subcategories.

5. Category description
   A description of the characteristics that identify a specific category or subcategory.

6. Category examples
   Excerpts from the transcripts that support the interpretation of a group of responses as belonging to a specific category.

7. Discussion of findings
   A discussion of the findings of the analysis of each research question.
4.11 Dialogue format

Any dialogue quoted in the analysis will use the letter "I" to represent the interviewer. A student will be represented by another letter. Square brackets, [ ] at the end of the quote contain information that is used to reference the student identity and the physics course he or she was enrolled in at the time of the interview. For example, [120-5] refers to student "#5" who was enrolled in physics 120, a first year honors physics class.

4.20 Summary of findings

The analysis of student responses to the first research question on reference frames revealed five conceptualizations. These conceptualizations fall into two broad categories in which the student’s response is either associated with the dependence or independence of the observer’s position or of some preferred position, and the path of the object.

The analysis of the second research question resulted in finding two major methods that students use to make comparisons of the angular speed of two objects moving in semicircular paths. These methods were: to compare the arc length swept out by each object in a fixed time, or to compare the relative position of the objects at an instant in time.

Analysis of the third research question focused on a description or characterization of the forms of reasoning students use in responding to questioning about the angular speed of an object. Student reasoning in this study was
best characterized by the concept of association. An association characterizes a way of thinking about the concept of angular speed. Several forms of association were identified as being: word, symbolic and contextual, each of which will be discussed, respectively.

Word association is the inferred association of the words: angular, speed or other words, with the word: angular speed that influenced a student's thinking about the angular speed of an object.

Symbolic association is the inferred association of the symbolic form of angular speed, \( \omega \) in terms of other symbols, namely formulae, which influenced a student's thinking about the angular speed of an object.

Finally, contextual association is the association of some aspect of the context of the task setting with some other non-task context which influenced the student's thinking about the angular speed of an object.

The analysis of the student responses to questioning about an object's angular acceleration (the fourth research question) has led to findings similar to that of research question three. The influences upon student reasoning were again characterized by forms of association. However, the particular details of each form was different as the concepts being investigated (angular speed versus angular acceleration) were different. The details of these forms of association for the last two research questions are discussed in the analysis to follow.
Research Question #1

What are students' conceptualizations of the angular speed of an object from different frames of reference?

4.30 Introduction to research question #1

When measuring any kinematic quantity such as angular speed it is necessary to have an understanding of the need for establishing a frame of reference. Angular measurements required for measuring the angular speed of an object are normally taken from an agreed upon frame of reference. Otherwise, it is possible to arrive at apparently conflicting or ambiguous statements about the angular speed of an object. Thus it was important to investigate student thinking about reference frames.

During the course of the interviews it was judged that not only the position of the observer but the shape of the object's path was relevant to the way the student conceptualized the task. Thus the conceptualizations are divided into two major groupings labeled: POSITION and PATH. Each grouping has several identifiable conceptualizations which are described later.

In the dialogue and the discussion which follows, the origin of the coordinates of a reference frame for an object moving in an elliptical or oval path is sometimes described as being located at the geometric centre. The term "geometric centre" is meant to refer to or indicate the midpoint
of a line segment connecting the two extrema points (i.e. the distance between these two points is a maxima) of an elliptical or oval curve\(^1\). For some students this geometric center held special significance, and is discussed in the following overview of the findings.

4.3.1 **Overview of findings of research question \#1**

From an analysis of student dialogue five different categories of description were constructed to reflect the various ways in which students responded to the task questions. These conceptualizations were grouped under two major headings depending on whether the particular conceptualization could be characterized as being associated with the *position* from which to measure angular speed or characterized in terms of the object's *path*. The following are the five conceptualizations grouped by the two headings of *position* and *path*.

**POSITION:**

1. The angular speed of an object is independent of the position of the observer.

2. An object may not have an angular speed if the position of the observer is not enclosed by the path of the object.

3. The angular speed of an object should be measured from a preferred position.

**PATH:**

4. An object moving in a *rectilinear* path doesn’t have an angular speed or it doesn’t make sense to talk

\(^1\) While this is not a mathematical definition it does serve as an operational definition.
about the angular speed of such an object.

5. An object moving in an irregular path doesn’t have an angular speed or it doesn’t make sense to talk about the angular speed of such an object.

These five different conceptualizations will be referred respectively as first, second, third, fourth and fifth "beliefs" in the analysis which follows. A description of these beliefs are organized according to the format as described in the introduction.

Category title: POSITION

Category description:

The measurement of angular speed is independent of the position of the observer, dependent on whether or not the observer is enclosed by the path of the object, or dependent on some preferred position.

Subcategory title:
Position: First belief

Subcategory description:

The angular speed of an object is independent of the position of the observer.

Category examples

Example #1: Position: First belief (elliptical path)

P: I think you would get the same (angular speed) for both (observers) because actually when it comes down to it even if I can still only see the balls motion along, within one plane I can still see the amount of time it takes for it (the object) for instance to reach one maximum of the ellipse to the other.

I: Suppose you’re looking down at it (the ellipse) just a little?
P: Just a little, that makes things even more simple.

I: Just the way you are right now, looking down at it.

P: Right.

I: Now, as opposed to you and a person standing here (at the center) would you both report back the same angular speed?

P: The same angular speed, I would say so, yeah.

I: Ok.

P: Assuming not so far away the speed of light makes no difference.

I: So it doesn’t matter where you’re standing or where your sitting where you are?

P: That’s right.

The angular speed of an object moving in an elliptical path is thought by this student to be independent of the observer position because (as suggested by the dialogue), the time for the object to move from one position to another is the same for both observers irrespective of the observer’s position.

Example #2: Position: First belief (elliptical path)

I: ..what would you reply if you were that observer standing at the outside (edge of ellipse) there? Would the ball have an angular speed?

F: Yes it does, it is still moving in an angle, angular direction, circular direction.

I: Now would the angular speed that you would give me for some part of the path be the same as if you were standing there (pointing at the "geometric center" of the ellipse)? Would it make any difference?

F: It’s the same because it’s travelling over the same angle for the same amount of time it travels over the same angular distance.

I: Angular distance being?
F: A full revolution. Travels that revolution in the same time whether or not I am in the middle or outside (of the elliptical path).

Again, this dialogue suggests that the angular speed of an object moving in a (uniform) curvilinear path is believed to be independent of observer position. The next example illustrates a different position related belief for motion in which the observer is enclosed by the path of the object.

Subcategory title

Position: Second belief

Subcategory description

An object may not have an angular speed if the position of the observer is not enclosed by the path of the object.

Subcategory example

Example #1: Position: Second belief (elliptical path)

I: You are standing at the centre and this object (indicated by a red ball moving in an elliptical path) is going around you at eye level--then does the red ball have an angular speed?

D: Yes.

I: Because?

D: Because it is moving around me. [115-1]

The student is then asked what his response would be if he were standing on the outside of the elliptical path.

I: Does it (the object) have an angular speed as seen from your position?

D: Uh-no.

I: Ok, because it is not going around you? (possibly a leading question but it was meant to repeat the reasoning given by the student a few lines earlier)
D: Uh-huh.

In a similar task setting a student became somewhat confused as to how to analyze motion in an elliptical path. Consequently, it was decided to ask this student a question about the angular speed of an object moving in a simpler path, a circle, from the viewpoint of an observer standing outside the circle. The task is still however referenced with respect to the elliptical path.

Example #2: Position: Second belief (elliptical path)

H: I don't know if it has angular (speed)--it is moving through the angle but--(long pause).

I: What bothers you the most about it?. The fact that the person (observer) is not at the center, or the fact that he is outside the circle?

H: The fact that he is outside the circle. Like with the last one (elliptical motion at a constant speed), I can't refer it to anything I've done.

I: So, not too clear that you can talk about an object having an angular speed from an observer outside (the circle)? That's not too clear?

H: Yeah.

I: But (observer) inside, no problem?

H: Inside you can see it moving around the angle like that, but if you are outside I don't know.

I: So in the inside you said you can see it is moving around an angle. What happens if your are here, still inside the circle but half way between the bottom and the top, does it (the object) have an angular speed?

H: Yeah, it is still-(pause)-still going through an angle.
This dialogue suggests that the student had difficulty in deciding if an object moving in a circular path would for an observer standing outside the circular path have an angular speed. Other students may have "avoided" this difficulty because they believed the angular speed of an object should be measured from a preferred position. The existence of such a preferred position characterizes the third belief.

Subcategory title
Position: Third belief
Subcategory description
The angular speed of an object should be measured from a preferred position.

Subcategory examples
Example #1: Position: Third belief  (elliptical path)

I: So the angular speed depends on your position.

G: Uh-huh

I: So what does it mean to say that something has an angular speed if we can say its different...different depending on where you we are standing--so how can we talk about an angular-?

G: We have to have some fixed reference point to where you are referring to-where it is moving from.

I: You often see in a text the angular speed is-or given the angular speed and-

G: Yeah, usually you can just assume where the position is they're referring to.

I: Suppose we were talking about an ellipse. An object going round an ellipse and I said the angular speed was-

G: Yeah.
I: Two radians per second or something. What would you say?

G: I would just assume that they were talking from the middle.

I: The middle—the geometric "center"?

G: Right. [216-3]

In the next example a student thought that in order to determine the angular speed of the toy train moving in an oval path it would be necessary to measure the "radius" from the "center" of the oval path (the center being a preferred position).

Example #2: Position: Third belief (oval path)

I: Does the train have an angular speed between points A and B?

A: Yes.

I: Because?

A: Because, eh it is a change in the effective radius of the train...

I: Where would you measure the radius you mentioned—is that what you said?

A: Yeah, we would have to measure from the center of the track (student indicates the "geometric center" as defined earlier in the introduction of the analysis) [120-12]

The need to measure the angular speed from some preferred or "standard" position is again illustrated in a different context in the following example.

Example #3: Position: Third belief (elliptical path)

J: You have to measure things according to a certain standard. It is all relative. If you are moving at 30 km/hr and you see a car coming at you at 30 km/hr it seems to be coming at you at 60 km/hr. You would be
fine if you measured everything according to you moving at 30 km/hr, but you just measure everything relative to the earth.

I: Ok, so it is to some standard: the standard being the center of--?

J: Of the circle

I: What about an ellipse, where is the standard there, do you think?

J: Uh, it would be the center of rotation. Uh, there are two centers of rotation of an ellipse isn't there? Eh, just the center where the observer is here. [115-2]

Another student during a discussion of how to measure the angular speed of an object moving in an elliptical path briefly made reference to the preferred observer position as being the most logical point from which to measure the angular speed.

Example #4: Position: Third belief (elliptical path)

I: If an observer were standing outside the ellipse does that ball have an angular speed?

R: Yes.

I: Because?

R: Because the angle is changing.

I: Ok.

R: But there are times when I guess it would be zero.

I: Would it be the same--would the angular speed be the same as a person who is in the middle?

R: No.

I: Ok, what you’re saying is that depending on where the observer is the angular speed will differ.

R: Yeah.

I: So there are many possibilities for the angular speed depending on where the observer is located. So what does it mean then to talk about the angular speed of
an object if there are so many possibilities, perhaps hundreds of possibilities?

R: Well, whenever you’re using it to figure something out you would have to specify the most logical point the center here.

I: But, why is it logical?

R: Well I’d guess that is premature. You would have to know what you were trying to do. [216-10]

The next two beliefs characterizes those responses where students decide on the basis of the object’s path whether or not it is possible to say an object has an angular speed.

Subcategory title

Path: Fourth belief

Subcategory description

An object moving in a rectilinear path doesn’t have an angular speed or it doesn’t make sense to talk about the angular speed of such an object.

Subcategory examples

Example #1: Path: Fourth belief (oval path)

I: Does the train from your position, from where you looking have an angular speed between points A and B? (A to B is a straight section of the track)

D: No

I: No. Ok, and why?

D: It’s not rotating through an angle it is moving straight. [115-1]

The student was then asked without changing his position if the train had an angular speed between another straight section of track, D-C which runs parallel to A-B.

I: Does the train have an angular speed between D and C?
D: No.

I: Why?

D: Same as AB moving straight.

Again, the response is the same as before, that an object moving in a straight line whether or not the observer is in the same line as the motion of the object, cannot have an angular speed.

Example #2: Path: Fourth belief (oval path)

I: Does the train have from the position where you are sitting, from where you are—does the train have an angular speed between C and A?

L: Yes,

I: Because?

L: Because there is a radius and it is moving in circular motion.

I: Does the train have an angular speed between points A and B?

L: Uh, no it does not.

I: Because?

L: Because it is not moving in a circular motion.

I: And does it have an angular speed between B and D?

L: Yes, it does.

I: Because?

L: Because it is moving in a circular motion.

I: And finally between D and C does it have an angular speed?

L: No, because it is moving in a straight line. [120-11]
As with the previous example, an object moving in a straight line is believed by the student not to have an angular speed.

**Example #3:** Path: Fourth belief (oval path)

I: *Does it (the train) have an angular speed between A and B?*

P: No.

I: *Because?*

P: Because it is moving in a straight line.

I: *Between B and D?*

P: Yes.

I: *Because?*

P: Well, because it is not moving in a straight line. It’s curving.

I: *Between D and C does it have an angular speed?*

P: No.

I: *Because?*

P: Because it is moving in a straight line. [120-5]

The next example of the fourth belief is illustrated for an object moving in an *elliptical* path.

**Example #4:** Path: Fourth belief (elliptical path)

I: *That's your position right there at the center. You can imagine your the observer.*

(The investigator further describes the task pointing out that the ball is moving at a constant speed in an elliptical path)

I: *Does the ball have an angular speed from your position there in the middle? (Student is aided by thinking of himself in the middle of a football field with somebody running around the outside track).*
J: Yeah (there is an angular speed).

I: Again, why would you have said that - there is an angular speed - if you were to explain to somebody?

J: Again it is not travelling in a straight line it is travelling through an angle. [115-2]

These foregoing examples illustrate the belief that the angular speed of an object moving in a rectilinear path does not exist. However a similar belief was also evident by the student responses to motion in a closed *irregular* path instead of a rectilinear path.

**Subcategory title**

**Path: Fifth belief**

**Subcategory description**

An object moving in an irregular path doesn’t have an angular speed or it doesn’t make sense to talk about the angular speed of such an object.

**Example #1: Path: Fifth belief (irregular path)**

I: *Does it make any sense to talk about that object having an angular speed?*

M: Where am I?

I: *Suppose you were right here (pointing to a position inside the path).*

(The student then explains with some lack of clarity his thinking. And eventually concludes by saying:)

M: *So, I am trying to say that if you are discussing angular speed, I don’t think you should.* [115-15]

**Example #2: Path: Fifth (irregular and elliptical paths)**

I: *Can we talk about that ball (moving in an irregular path) having an angular speed?*
K: We'd have a very hard time—in relation to the center it would be rather difficult.

(and in reference to the ellipse)

I: I am interested in a person on the outside of the ellipse. Can you talk about the angular speed? Does it make a difference (where the observer is)?

K: I'd think he would have trouble calculating the angular velocity of the object because he wouldn't be really sure where the center is... [120-6]

Example #3: Path: Fifth belief (irregular path)

I: How about an irregular shape. Suppose I asked you about the angular speed there. What would it mean if I said the angular speed is--?

R: It would mean nothing. [216-10]

Example #4: Path: Fifth belief (irregular path)

I: Does it make sense to talk about it (the ball moving in an irregular path) having an angular speed?

K: I don't know if it makes sense. [120-7]

These foregoing examples illustrates the difficulty students have with the concept of angular speed for motion of an object in an irregular path. A summary of this finding and previous findings so far discussed follows.

4.32 Discussion of findings of research question #1

Five important conceptualizations ("beliefs") about angular speed have been categorized. These have been grouped in terms of whether or not the angular speed of an object was conceptualized in terms of position: angular speed is dependent or independent of the position of the observer, or dependent on some preferred position. Or in terms of path: the angular speed of an object is dependent on the shape of
the path of the object. The context of the task setting vis-a-via the position of the observer and the shape of the object’s path played an important role in the manner in which the student responded to questioning about the angular speed of an object.

The belief that the motion of an object is independent of the observer’s position has been observed by Saltiel and Malgrange (1980) in their work on student’s ways of reasoning in elementary kinematics. They noted that students see motion as being an intrinsic property of the moving body and not dependent on the position of the observer. In addition Saltiel and Malgrange noted that students would sometimes seek to measure the velocity from some preferred position.

Velocities are only very loosely defined with respect to frames. This probably corresponds to the fact that, in everyday life, a particular frame is very often highly favoured for practical reasons...(p75)

While these findings of Saltiel and Malgrange have parallels to this study, other findings of this study have no parallel with other research work in the domain of kinematics. These (other) findings are, the belief that an object moving in a rectilinear path doesn’t have an angular speed, and the belief that an object doesn’t have an angular speed if the position of the observer is not enclosed by the path of the object.

Besides characterizing the ways in which angular speed is conceptualized by students from different frames of refer-
ence or observer positions it was also of interest to characterize the ways in which students make *comparisons* between the angular speeds of two objects. This is the focus of the next research question.
Research Question #2

What is the nature of students’ conceptualizations when making comparisons of the angular speed of two objects moving in semicircular paths of differing radii of curvature?

4.40 Introduction to research question #2

The purpose of this question was twofold. Firstly, to find out how students compare the angular speed of two balls, each of which is made to roll down an inclined semicircular track from the perspective of an observer located at the "center" of this (semi) circular path. And secondly, to compare the findings of this task with the work of Trowbridge’s (1980) study on the ways students compare the speed of two balls rolling down inclined planes.

4.41 Overview of findings of research question #2

Most students were able to effect a comparison of the angular speed of the two balls rolling down inclined semicircular tracks. Students frequently stated that at some time, or position, or over some part of the track, the two balls did have the same angular speed. From their responses two major ways of comparing the angular speeds of objects moving in semicircular paths were categorized. These categories to be discussed respectively are entitled: Arc Length Equivalence and Position Equivalence beliefs.
Category title

Arc Length Equivalence

Category description

Two objects (balls) are believed to have the same angular speed if they travel in the same arc in the same amount of time over some part of their motion.

Category examples

Example #1: Arc Length Equivalence

I: Do the two balls have the same angular speed?

K: I think at point C they have the same angular speed. ..sorry, they meet at point C and during the time between point C where they meet first and point E - (student meant point G and corrects himself) - they actually meet again at point G when they drop off. Between point C and point G they cover the same arc in the same time so they have the same angular speed (emphasis added)

Another physics 120 student quoted below (identified by the same letter "K" but not the same student [#6] as above) replied in a similar manner.

Example #2: Arc Length Equivalence

I: Do the two balls ever have the same angular speed?

K: Yeah, from the formula they should.

I: The formula?

K: D-theta: dt, from that point of view..the same arc-length is cut out in the same time.

While the arc length equivalence belief was used by some students in deciding whether or not the two balls had the same angular speed another form of comparison, position equivalence, was also used.
Category title

Position Equivalence

Category description

Two objects are believed to have the same angular speed if they have the same position at the same time on some part of their paths.

Category examples

Example #1: Position Equivalence

I: Do the balls ever have the same angular speed? Take your time—if it is not clear as to whether at any point or at any time they have the same angular speed—tell me.

J: I think the angular speed would have to be the same, the angular speed is the same.

I: For?

J: For both of them

I: Over what period, over the whole track, over part of the track? I'm just trying to clarify.

J: Over the whole track.

I: Because?

J: It's mainly because they both end up at the same—they both drop off the track at the same time.

The student is asked later how he would explain to someone else that the angular speeds of the two balls were the same in order to clarify the student's earlier remark.

I: How would you explain to someone that the angular speed is the same, because—? What was it about their motion—?

J: It's mainly because they end up at the same time. The outer one's velocity was larger but they did end up at the same time.
Another student (example #2) refers to the place where the balls are close as being the position where they have the same angular speed.

**Example #2: Position Equivalence**

I: *Do the two balls ever have the same angular speed?*

Y: They do

I: *Over what part of the motion?*

Y: Near the end.

I: *Why?*

Y: They are close to each other.

Both statements by this and the previous student would indicate that since the two balls arrived at the end of the track (the same position) at the same time then they must have the same angular speed. The student in the following example further illustrates this belief.

**Example #3: Position Equivalence**

I: *Do the two balls ever have the same angular speed?*

M: Yes..from the point of the observer the balls have the same angular speed when they are in line with each other with respect to the observer.

I: *And that would be at? Where does that happen?*

M: That happens right at the end at point (interviewer interrupts and says point G to confirm this is the point being mentioned by the student) point G.

I: *They have the same angular speed because they are in a line?*

M: Because they are in line.

I: *Any other points?*

M: It looks like (looking at the video replay)- it looks
like as if, around C. They're in line at C and G. [115-15]

Again, this student like the previous students believed that the two balls have the same angular speed if the balls have the same position (are in "line") even if this is just for an instant in time. In the discussion to follow it is clear that the "position equivalence" belief is not just isolated to the comparison of angular speeds.

4.42 Discussion of findings of research question #2

The analysis of the data revealed two forms of conceptualizations used to effect comparisons between the angular speed of two objects moving in semicircular paths.

One form of conceptualization, **Arc Length Equivalence** is characterized as follows: two objects have the same angular speed if they move in the same arc over a certain time period.

The other form of conceptualization, **Position Equivalence** is characterized as follows: two objects have the same angular speed if they have the same position at the same time. In Trowbridge's (1980) study in which two balls were made to roll down inclined planes at slightly differing accelerations students were asked if the two balls ever had the same speed. To this question a substantial number of students responded that they did at the "instant of passing". To quote Trowbridge:

..failure on the speed comparison tasks was almost invariably due to improper use of a position criterion to determine relative velocity. (p1027)
This observation parallels the one in this study in which some students were found to use a form of position equivalence in making comparisons, not between the speed of the objects (as in Trowbridge's study) but between the angular speed of the objects.

Trowbridge also observed that in another task where the balls do not pass each other but at some point have the same speed that:

..logical arguments did not necessarily influence student responses. Even after satisfactorily describing the speed of ball B as decreasing to zero and the speed of ball C as increasing from zero, some students observing the demonstration would still claim that the speeds were never the same since the balls never passed each other. (p 1024)

While no such logical arguments were presented by the interviewer to any student in this study, Trowbridge's observation suggests that some students are strongly committed to a belief of "position equivalence".
Research Question #3

What is the nature of students’ reasoning as they address several task problems involving the concept of angular speed?

4.50 Introduction to research question #3

Previous analysis of the content of student responses as described in research question #1 has led to categories of description. The bringing of a different type of analysis to research question #3 (as described earlier in chapter three) has led to categories of reasoning being used to characterize student thinking.

Research question three was designed to investigate the ways in which students reason or utilize various strategies in thinking about the concept of angular speed. These reasoning strategies which collectively are called categories of reasoning, are individually described by the concept of association. Association (as used in this study) represents a simple mental link made by the student with words, symbols, events or experiences to some aspect of the task situation. The concept of association is not meant to describe all aspects of student thinking, nor is any one particular type of association (to be discussed) assumed to be representative of the reasoning of any one student.

4.511 Overview of findings of research question #3

The categories of reasoning that were constructed to characterize students’ thinking about an object’s angular
speed were described by three forms of associations. Each form of association represents either a weak or a strong link in the student’s memory to: the words angular speed, the symbol for angular speed, or to the context of the task environment. These three forms of associations have been categorized as: word associations, symbolic associations, and contextual associations, and are discussed respectively.

4.511 Word association

A word association is characterized by the words "angular speed" being associated with the word angular or speed. A student in responding to a question about the angular speed of an object might think for example in terms of the speed of the object (instead of its angular speed). This form of reasoning is described as an association and is labelled for reference first by the type of association, "word association", followed by the words which are being associated, angular speed with speed to give the label: "word association::angular speed:speed". This particular form of association, is illustrated next.

Subcategory title
Word Association::angular speed:speed

Subcategory description

Because the word speed is associated with the term angular speed, a question about the angular speed of an object moving in a curvilinear path is interpreted by a
student as if the *speed* of the object is being asked about rather than its *angular* speed.

**Example #1** Word Association::angular speed:speed

(oval path)

I: *Ok, is the angular speed a constant?*
(A train is moving around the oval track)

J: It would be - wait (pause) - yeah.

I: *Because?*

J: Because it is going around at the same speed. [216-4]

**Example #2:** Word Association::angular speed:speed

(oval path)

I: *Is the angular speed between A and C the same as between B and D?*

P: Uhm, well, its speed would be because assuming uniform velocity of the train eh, its velocity would be different because its going in a different direction of course. Velocity is continuously changing when you’re going around the corner, its acceleration--it has acceleration right, but its speed, its angular speed would be the same, I would assume. [120-5]

**Example #3:** Word Association::angular speed:speed

(irregular path)

I: *Does it make any sense to talk of angular speed?*

L: I don’t think so. Because I think about just magnitude. So it is a constant speed (referring to the object moving in an irregular path).

Since the student referenced the magnitude as influencing his thinking it was decided to investigate this student’s response to a question about the angular *velocity* of the object.

I: *Suppose I had talked about angular velocity?*

L: Yeah, I’d say it has an angular velocity.
I: *Because?*

L: Uh, it is moving through these different angles and it varies. Right?  

For this student, different angles like the different direction of the velocity vector are associated with a change in angular velocity.

Not only did the word *speed* influence the way some students thought about angular speed but so did the word *angular*.

**Subcategory title**

*Word Association:* angular speed: angular

**Subcategory description**

The word *angular* is associated with the term *angular speed*. The word *angular* suggests to the student that an object *must* move in a curved path in order to have an *angular* speed.

**Subcategory examples**

**Example #1:** *Word Association:* angular speed: angular (irregular path)

A physics 120 student when thinking about angular speed was reminded of an angular momentum problem in her textbook. While the word angular momentum was mentioned it is clear from the dialogue that follows that it is the word *angular* and her interpretation of this word that guided her thinking about the angular speed (or angular velocity as used by the student) of an object moving in an irregular path.

H: I know that anything can have angular momentum depending on the reference point..somebody explained (it) to me-I
was having some trouble understanding the idea of the angular momentum—cause a little diagram in the book (a diagram in her physics text book) . . . there was an object . . . I couldn’t understand how something that just happened to be moving along (in a straight line) had an angular velocity when it wasn’t actually rotating or going round in a circle. [120-9]

This comment by the student revealed a barrier to the interpretation of angular speed as a consequence of the association of the word angular to the student’s conception of what the term angular means to her, namely something that is moving in a path that is not straight. Two other examples of this form of association follow.

Example #2: Word Association:: angular speed: angular (oval path and irregular path)

I: Ok, does it (the train) have an angular speed between points A and B? (a straight section of track)

P: No.

I: Because?

P: Well, because it is moving in a straight line. It has got a linear velocity.

Later, when shown an object moving in an irregular path the student response was:

P: Well, uhm—all things aside, if all I could observe is this ball as it moves along this path, I could say with certainty that it has an angular speed because any time it is moving in a curved path in which it is changing direction it has an angular speed. [120-5]

Example #3: Word Association:: angular speed: angular semicircular path)

I: Let me talk about speed and angular speed, how would you compare the two?

J: Ok, well when I think of speed I think of it more as in a linear term, like straight forward in a line, and angular speed would be like a wheel turning. [216-14]
Not only do words themselves influence student thinking but so do symbols. This influence is characterized as a symbolic association, by which students link the symbol for angular speed, the Greek letter omega, $\omega$ to formulas that have the term, $\omega$ in them.

4.512 **Symbolic association**

In general the use of formulas by students in solving various task problems may not be surprising (see for eg. Reif, 1986 and Trowbridge, 1980). But it is not the use of formulas per se that is of interest but which formulas (even if incorrect) are used while others are not.

The use of formulas by these students as they think about their responses to questioning about the angular speed of an object is indicated by the following brief quotes.

**H:** I am trying to think in terms of formulas [120-9]

**J:** I was thinking about the formula [115-2]

**G:** I am trying to think in terms of an equation [216-3]

The most commonly cited formula (or equation) that a student referenced was $\omega = \frac{v}{r}$ for uniform circular motion ($v = v_{tangential}$) which along with other forms of symbolic association are illustrated by the examples that follow.$^2$

---

2. The use of a particular formula by a student for $\omega$ is stated as reflecting what the student said, even though the formula may not be appropriate.

The reader should note that in the special font being used, $\omega$ is a scalar, and $\omega$ is a vector quantity.
Subcategory title

Symbolic Association::Formulas

Subcategory description

The angular speed, $\omega$ is interpreted via a symbolic association with a formula(e) in which the symbol $\omega$ is a term.

Subcategory examples

In the example to follow student "J" (quoted above) made several references to the use of formulas. One such reference is illustrated.

Example #1: Symbolic Association:: Formulas: $\omega = \frac{v}{r}$ (elliptical path)

J: I was thinking in terms of the formula.

I: *The formula being?*

J: Velocity equals $r$ times $\omega$. So, $\omega$ would be $v$ over $r$ so if the radius was greater the angular speed would actually be less. [115-2]

Example #2: Symbolic Association: Formulas: $\omega = \frac{v}{r}$ (elliptical path)

I: *Is the angular speed (of the ball) a constant?*

P: Eh, no it's not

I: *Why?*

P: Because its radius isn't a constant. It also changes.

I: *Where is its (the ball's) maximum and minimum (angular speed) then? It must have a maximum and a minimum, if not a constant.*

P: Yeah, uhm I'll just start getting the-eh-formula here. Its angular speed will say omega because that is a convention, is equal to, proportional to $v$ or is equal to $v$ over its radius, so its nice to have a constant speed. [120-5]
Of those students that used the formula, \( \omega = v/r \) few indicated an awareness that "\( v \)" in the formula is the tangential component of the velocity of an object moving in a curvilinear path (see the kinematics of curvilinear motion in the footnote below\(^3\)). The awareness of this fact is represented as aspect #1 of symbolic association and is illustrated below.

**Subcategory title**

**Symbolic Association::Formulas:** \( \omega = v/r \): aspect #1

**Subcategory description**

A distinction is made between the velocity, \( v \) and its components: \( v_{\text{tangential}} \) and \( v_{\text{radial}} \) when using the formula \( \omega = v/r \).

**Subcategory examples**

**Example #1:** Symbolic Association: Formulas: \( \omega = v/r \): aspect #1

(I): *Does the train have an angular speed between points A and B? (a straight section of track)*

(D): From here?

(I): *Where you are sitting.*

(D): Well, what angular speed is--is the velocity perpendicular to a radius from where I am standing and from where I am standing here this is my radius, and there is no velocity perpendicular to it.

---

3. For an object moving in a curvilinear path the velocity vector can be broken up into two vector components. One component, the tangential component, has magnitude \( v_t \). The other component, the radial component, has magnitude \( v_r \). For uniform circular motion the speed, \( v \) is equal to the magnitude of the tangential component of the velocity since the radial component is zero. Under such a circumstance the angular speed, \( \omega \) may be written as \( v/r \) rather than \( v_t/r \).
I: *How about between D and C? (a curved section of track)*

D: Yeah, because from where I am standing it does. It is going this way which is partially a velocity this way and partially a velocity that way and this times the radius here is its angular speed.

I: *And how about between C and A? Same type of reason.*

D: Sure, yeah.

This student was aware of the need to take components of the velocity and not the velocity of the object into account when computing the angular speed. This form of reasoning was also illustrated when this student attempted the ellipse task:

I: *Is the angular speed a constant?*

D: (long pause)-huh-em

I: *Well you can tell me some of the considerations that are going through your mind.*

D: Some of the considerations that are going through my mind: when you are at point A here its all its velocity is perpendicular and it is very close-uhm-when you are over here it is taking a sharper turn (pause)-did you ask me if it was constant?

I: *(nods head)*

D: No, it’s probably not constant too many things changing, the radius changes, the angle of the velocity compared to the radius changes because it is not a circle it’s not perpendicular all the time.

Another student (in the example to follow) also made an effort to use velocity components, in order to compute the angular speed of the object moving in an elliptical path.
Example #2: Symbolic Association: Formulas: \( \omega = \frac{v}{r} \): aspect #1
(elliptical path)

M: And going back on that ellipse when you asked how I would measure the angular speed.

I: Ok, let’s go back.

R: At B the angular speed would have to be the perpendicular component of velocity divided by the radius, so not just the speed divided by the radius. [120-10]

The use of the term "perpendicular component" by the student is interpreted as being \( v_{\text{tangential}} \). Again an awareness by the student of the need to distinguish between various components of the velocity and the velocity itself is evidenced in this response. However, some students failed to make a distinction between the velocity and its components. This failure to make such a distinction is labeled as aspect #2 of symbolic association and is illustrated by the following examples.

Subcategory title
Symbolic Association::Formulas: \( \omega = \frac{v}{r} \): aspect #2

Subcategory description

No distinction is made between the magnitude of the velocity, \( v \) and the magnitude of the object’s \( v_{\text{radial}} \) or \( v_{\text{tangential}} \) components when using the formula \( \omega = \frac{v}{r} \).

Subcategory examples

Example #1: Symbolic Association: Formulas: \( \omega = \frac{v}{r} \): aspect #2
(elliptical path)

I: ..can we talk about angular speed (of an object going around in an ellipse)?

K: Yeah.
I: And why?

K: From definition of \( v = \omega \times r \) so \( \omega = v/r \), and there is change. Radius is changing, the angular speed is slower at E and faster at A (corresponding respectively to points labeled on the elliptical path where r is at its maximum and minimum).

I: Why faster at A?

K: Because r is smaller, \( \omega \) is faster. [120-7]

This response by the student suggests that the speed of the object is equated with its velocity. No distinction is made by this student between \( v_{\text{radial}} \) or \( v_{\text{tangential}} \) when computing the angular speed of the object via the formula \( \omega = v/r \). The following example of a student quoted earlier also illustrates this lack of distinction.

Example #2: Symbolic Association:Formulas: \( \omega = v/r \): aspect #2 (elliptical path)

P: I'll just start getting the-eh-formula here. Its angular speed will say omega because that is a convention is equal to, proportional to v, or is equal to v over its radius, so it's nice to have a constant speed. The radius however changes continuously, so at the point where it is furthest away from the center, where its radius is the greatest, its--

I: E to E'?

P: Yeah, from observer to E or E'. Its semimajor axis, uhm, it will have its smallest angular velocity because again its inversely, angular velocity is inversely proportional to the distance, the radius. [120-5]

Again no distinction is made by the student between the velocity, v and its components. Sometimes a student would associated the angular speed, \( \omega \) with a formula for the angular momentum or energy of a rotating body (since both formulas have the symbol, \( \omega \)). This form of association whereby another formula other than \( \omega = v/r \) was used by the
student is considered to be a subcategory of symbolic association and is referenced as: "other" formula(s).

Subcategory title

Symbolic Association:: Formulas: "other"

Subcategory description

The use of a formula other than $\omega = v/r$ to "measure" the angular speed of an object.

Example #1: Symbolic Association::Formulas: "other"

(elliptical path)

I: So you are not so clear how you might calculate the angular speed of an object moving in an ellipse.

K: I should be though, because that was what our last midterm was about...normally I use angular momentum and energy considerations.

I: To calculate?

K: Eh, to calculate omega (the angular speed).

[120-7]

Example #2: Symbolic Association::Formulas: "other"

(elliptical path)

I: How would you measure the angular speed at point C.?

S: (a long pause)

I: What are some thoughts going through your mind?

S: .. energy conservation and the fact that "I", the inertial moment would be changing over here...linear velocity would be constant so "I" times $\omega$ would be constant, so if calculate change in "I", (a pause) $mr^2$ that would probably give you the change in $\omega$, I think.

[120-8]

Not only did the association of angular speed with formulas influence student thinking but so did the context of the task also influence student responses. This influence is
characterized as a **contextual** association and is discussed next.

4.513 **Contextual association**

In this form of association some aspect of the context provides a cue by which the student is influenced in his or her thinking about the angular speed of an object. This cue is usually interpreted in the form of some *analogy* as the examples to follow illustrate.

**Subcategory title**

**Contextual Association**: analogy

**Subcategory description**

The context of the task setting cues an analogy for the student. Students were found to use a variety of analogies. The analogies that were used by students for analyzing the angular speed of an object were: planetary motion, circular motion, simple harmonic motion, and the coriolis force, each of which will be respectively discussed.

The analogy of elliptical motion to *planetary* motion for one student evoked the use of Kepler's laws. This student (quoted in the next example) tried to use Kepler's, "equal area law" for deciding where the maximum angular speed of an object moving in an elliptical path would occur.

**Subcategory examples**

**Example #1:**

Contextual Association::analogy:planetary motion (elliptical path)

L: I am just using one of Kepler's laws.
I: Which is?

L: Which is in a certain time interval the area that the object sweeps out—the sector (?) area—the area from the center is equal at all time intervals...

Some other students who made an analogy to planetary motion found the recasting of elliptical motion into one of planetary motion perplexing. Because, the object in the "ellipse task" was moving around at a constant speed unlike that for planetary motion.

Example #2:

Contextual Association::analogy:planetary motion (elliptical path)

M: You made those restrictions (referring to the constant speed of the object)...it shoots out what I know about—about—eh—space, elliptical orbits, foci, speeds and everything. Now you are saying the speed is a constant and now you're telling me I am in the middle. So, fine, so—eh.

I: Oh, because you were thinking of planetary motion?

M: Right.

Another student during the interview also had difficulty in evaluating the use of his analogy of comparing elliptical motion to circular motion.

Example #3:

Contextual Association::analogy:circular motion (elliptical path)

I: Is the angular speed (of the object moving in the elliptical path) a constant?

K: No, because the radius varies in each case.

(A few moments later the interviewer returns to the question of how to measure the angular speed at a certain point on the ellipse.)

I: If you have to measure the angular speed at point C.

K: Need location of foci.
I: *One here and here.*

K: The instantaneous angular speed?

I: *Yes, what are some of your thoughts?*

K: I am trying to think of it in terms of a circle as
in sweeping out an area.. but the strange thing is the
radius varies. Ellipses are not really my speciality.
[120-6]

For an object moving in an *irregular* path some students
also made use of the analogy to circular motion. These stu-
dents when asked how to measure the angular speed (having
previously acknowledged that it made sense to do so) for an
object moving in an irregular path proceeded to approximate
the radius of curvature at various points on the object’s
path by drawing circles of varying radii (a diagram of this
is illustrated in the example to follow). These attempts
are interpreted as being an effort by the student to find a
value for the radius, \( r \) in the formula, \( \omega = v/r \). The
magnitude of the radius vector that was used was not (as it
should have been) the distance from the observer to the ob-
ject but rather the radius of curvature at some point on the
path where the object was located.

Example #1:

*Contextual Association:* analogy: circular motion
(irregular path)

L: Yeah, I’d say it has an angular velocity.

I: *Because?*

L: Because, uhm it is moving through these different
angles and well they vary right?

I: *And what does that make change?*
L: It changes the radius.

I: *What radius?*

L: The centre of motion.

I: *Ok, so is the radius at this position different than the radius at another position?*

L: Yeah, I think you have to--(interviewer puts down a transparent overlay on the object's path with various points marked for reference)

I: *(interviewer having interjected)--you can give me an example.*

L: Ok, for example if it is moving through (the point) C, the radius will probably be somewhere around here.

I: *So you would construct what?*

L: You kind of construct--can construct--(interviewer interjects)

I: *You can draw on that (the overlay).*

Student then proceeds to draw a circle as shown below.

![Circle](image)

r: indicates the r, the student was trying to measure.

(The circle drawn by the student was either within or outside the irregular path depending on the sign of the radius of curvature of the irregular path at a particular point.)

I: *So at what points will its angular velocity be greater?*

L: It will be greater at the sharp ones, because the radius (of the circle) will be shortest. [120-11]
The student went on to indicate that point "F" (see diagram of irregular path) has a minimum angular velocity because the curve is almost straight here (and the radius of the circle would therefore be very large).

**Example #2:**
Contextual Association: analogy: circular motion (irregular path)

P: Well, again linear speed, you would try and find some way to look at its radius of movement and that's not an easy thing to do. These are completely irregular curves but basically at a given instant you can find a circle which approximates its curve.

I: *Ok, let's take a point C or D or E.*

P: Well, at point C you can look at a very small circle here.

I: *Which is inside the path* (interview describing in words what the student has indicated by hand on the overlay).

P: Which is inside the path, right. Because it is a small circle it has a very small radius of curving. Again, because angular speed is inversely proportional to the radius and whenever the radius is the smallest that's where your going to have your greatest angular speed.

**Example #3:**
Contextual Association: analogy: circular motion (irregular path)

Q: I think we can say that at any point the ball has an instantaneous angular speed that-uhm-would be-we'd have to extrapolate a circle from a small arc.

I: *Suppose you want to measure the angular speed of that ball. How would you go about that?*

Q: We'd have to extrapolate. Take the arc there and put it into a circle, and..(interviewer interjects)

I: *And what circle would that be? Could you just show?*

Q: In this direction.

I: *Ok, in that direction* (student indicates an arc on the outside of the irregular path)
When the student was asked to indicate where the maximum and minimum angular speed would be the student response was:

Q: The highest angular speed would be achieved at the points that have very small radius of this virtual circle that I've been talking about. [120-12]

Besides the analogy to circular motion sometimes an analogy would be made to *simple harmonic motion*.

**Example #4:**

Contextual Association: analogy: simple harmonic motion (elliptical path)

P: ..If I know that, if I know it's an ellipse and I know it's going to hit its lowest velocity at the two semi-major axises. And when it does that it will appear to kind of slow down and speed up and slow down again. It's like *simple harmonic motion* again. So I can simply time it (the object) takes for it to go from one extreme to the other. (emphasis added) [120-5]

While simple harmonic motion (SHM) along a straight line is related to uniform circular motion it is inappropriate to link SHM to elliptical motion.

Finally, another analogy which was cued by the context of the task setting was to the *coriolis force*. In the example illustrated below a student was shown a diagram of an object moving in a circle after having experienced difficulty in responding to a question about an object's angular speed during the "elliptical" path task.

**Example #5:**

Contextual Association: analogy: coriolis force ("elliptical" path)

I: *Let me go back to something that might be a bit more familiar.* (Interviewer shows the student a diagram of a ball moving around a circular path)
H: Oh, yes that is lovely.

I: Here is a circle, and you're in the middle. Does it (the ball) have an angular speed?

H: Yes.

(The same question was later repeated for an observer standing outside the circle)

I: If that observer stands outside (the circle) can that observer still talk about that object having an angular speed?

H: Well--

I: Or (to reword the question) does he (the observer) have to be inside (the circle) to talk about it (the angular speed), or can he be outside "looking in".

H: I think-(pause)-, ok when you asked me that (question) the first thing that came into my head was the coriolis effect. Because, uhm, when something is moving on the earth that is rotating around it--it appears to have an extra component to the way it moves...

I: When did you take coriolis? Just recently, or--?

H: A few weeks ago.

I:..., and this reminds you of coriolis?

H: I am just thinking of coriolis because the frame of reference is changed, like the frame from which you are looking at is changed.

(emphasis added) [120-9]

The interpretation of this student response is that the changing of the position of the observer from one frame of reference to another acted as a cue. This in turn reminded the student of something discussed a few weeks ago in a physics class about (rotating) reference frames (as the earth) and the coriolis "effect".
The different types of contextual association and other forms of student's reasoning so far discussed are now summarized in the following review of the research findings.

4.52 Discussion of findings of research question #3

The concept of association was found to best characterize the categories of student reasoning as judged by student responses to questioning about the angular speed of an object moving in different paths. The associations students made were described as: word, symbolic, and contextual; association, each of which will be reviewed respectively.

A word association is an association of the word angular speed with other words. These "other" words were found to be: angular and speed. A student for example in responding to a question about the angular speed of an object moving in a curvilinear path might respond as if the question asked was about the speed of the object and not its angular speed. Likewise, a student might respond by associating the word angular with non linear motion. Consequently an object moving in a straight line is not seen as having an angular speed (irrespective of the observer's position) by the student. Even though the words speed and angular influenced student reasoning it is in principle possible that other word associations may have influenced student reasoning. Not only did word associations influence student's thinking about angular speed but so did the symbolic association of \( \omega \), the angular speed with formulae having this symbol.
The most common formula that students explicitly mentioned was \( \omega = \frac{v}{r} \) for determining an object’s angular speed. A common error in the use of this formula was the failure to distinguish between the speed, \( v \) and the magnitude of the velocity components (\( v_{\text{radial}} \) and \( v_{\text{tangential}} \)) of an object’s motion. This error was found among students at all levels, physics 115, 120, and 216. And while such errors are not the central interest of this research question it is nonetheless of interest from an instructional point of view.

Momentum or energy formulae having the symbol \( \omega \) as a term were also recalled by some students. Such formulae, were less commonly referred to, perhaps because no information or cue was given about the mass, moment of inertia or energy of the object.

Finally, the third form of association that was identified was contextual association. In this form of association some aspect of the context of the task setting cues a memory of an event, some piece of knowledge, or experience the student has had. The context may for example remind the student of a picture in a physics textbook, a physics lecture, or a personal experience. And (as noted in the literature review) Erickson and Aguirre (1987) have observed that the "..task context appears to play an extremely important role in terms of the ways in which a student both frames and responds to a problem" (p17).
Analogies, a form of contextual association were found to be made to circular motion, planetary motion, simple harmonic motion and the coriolis force. While some students recognized the appropriateness of some of their analogies others did not. The failure by some students to evaluate the appropriateness of an analogy has also been observed by Clement (1987) in his work on analogical reasoning in the field of mechanics.

In summary these three associations word, symbolic or contextual serve to describe the form of reasoning "strategies" students are thought to engage in, at an appropriate level of detail (for pedagogical purposes). While it may appear that such associations represent a description of "surface" level thinking it is important to emphasize that such forms of thinking as contextual association can be very effective. A great deal of what experts are able to do says Simon (1985) is because of an ability to recognize cues in familiar situations. However, experts unlike novices usually are able to correctly evaluate the appropriateness of their initial response(s) to such cues.

The diagram on the next page summarizes the relationship amongst the forms of association which have been discussed.
The question as to whether student's reasoning about the concept of angular acceleration is similar to that for angular speed is the subject of the next research question to follow.
Research Question #4

What is the nature of students' reasoning as they address several task problems involving the concept of angular acceleration?

4.60 Introduction to research question #4

Reif (1987) investigated the reasoning processes of experts and novices as they interpreted the concept of acceleration in various task settings. He described the knowledge of being able to find the acceleration of a particle moving in an arbitrary path as being procedural knowledge. The procedural knowledge necessary to find the acceleration of a particle is represented as five steps:

1) Identify the velocity $v$ of the particle at the time of interest.

2) Identify the velocity $v'$ of this particle at a slightly later time $t'$.

3) Use vector subtraction to find the velocity change, $\Delta v = v' - v$ of the particle during the short time interval, $\Delta t = t' - t$.

4) Divide $\Delta v$ by $\Delta t$ to find the ratio $\Delta v/\Delta t$.

5) Imagine that the time $t'$ is chosen sufficiently close to the time $t$ so that the time interval $\Delta t$ becomes infinitesimally small. Find the limiting value $dv/dt$ of the ratio $\Delta v/\Delta t$ and call it the "acceleration $a$ of the particle at the time $t$".

Reif observed that student novices in their reasoning about kinematic quantities as acceleration, rarely use any form of procedural knowledge, instead relying heavily on various pieces of knowledge. These knowledge pieces stated as propositions are for example: if the speed of a particle
changes then its acceleration cannot be zero; if a particle moves in a circle its acceleration is directed towards the center (only true if the particle is travelling at a constant speed), and if a particle has a velocity that is zero or a constant then its acceleration must be zero (incorrect). It was from observations such as these that Rief concluded that student knowledge was fragmented, a description echoed by DiSessa (1986) who characterized the intuitive physical dynamical knowledge of students novices as "knowledge in pieces".

Reif's investigation of student understanding of the concept of acceleration is extended in this study to the domain of angular acceleration. Before proceeding with an overview of the findings to research question #4 it will be useful to review some elements of the concept of angular acceleration.

Angular acceleration is symbolized by the Greek letter alpha, $\alpha$ and is for the purpose of this study treated as a scalar (non-vector) quantity, $\alpha$. Angular acceleration is defined as the limiting change in the vector, angular speed, $\omega$ with respect to time, $t$ which expressed symbolically is $d\omega/dt$. Again, for the purpose of this study only the magnitude, $|d\omega/dt|$ or $\alpha$ is of interest.

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4. Physicists use the name speed for the magnitude of the vector quantity velocity. Unfortunately, there is no special name for the magnitude of the vector quantity acceleration or angular acceleration.
4.61 Overview of findings of research question #4

The reasoning students used in answering questions about the angular acceleration of an object moving in a variety of paths is again characterized by the concept of association. As before, three forms of association were found: word, symbolic and contextual, each of which will be described respectively.

Word association is a simple or a complex association of words. The words associated by students with the words angular acceleration were inferred from their responses. For example, a student might state that an object moving in an oval path at a constant speed does not have an angular acceleration because it is never increasing its speed. While the student never used the word acceleration (by itself), it was nevertheless assumed that a mental reference to this word was made by the student, as such a reference provides a coherent explanation of what was said.

A student may go through a series of mental steps in arriving at a response to a question about the angular acceleration of an object. These "steps of reason" may at times be represented by a sequence of inferred associations. Such a sequential order of steps is indicated by an arrow, "-->" and the non verbalized association is placed in brackets, "( )" to give for example:

\[ \text{angular acceleration} \rightarrow (\text{acceleration}) \rightarrow \text{change in speed} \]

which symbolically can be rewritten as:

\[ \alpha \rightarrow (a) \rightarrow \Delta s \]
Again, this sequence is meant to represent a hypothesized ordering of mental steps characterized by associations, of the student's reasoning process. Using this approach the analysis of other response(s) to the interviewer's questions when appropriate to do is summarized as a sequence of word, symbolic, and contextual associations. As before, symbolic association manifests itself in the use of formulae, while contextual association may manifest itself in the use of analogy.

The following illustrates possible sequential associations of word, symbolic and contextual associations.

**WORD associations:**
1. \( \alpha \longrightarrow (a) \longrightarrow \Delta s \)
2. \( \alpha \longrightarrow (a) \text{ or } a \longrightarrow \Delta v \)

**SYMBOLIC associations:**
3. \( \alpha \longrightarrow \text{formula} \)

**CONTEXTUAL associations:**
4. \( \alpha \longrightarrow \text{analogy} \)

Sometimes a multiple of these sequences are inferred to have occurred during a student's response to a question.

Beginning with word associations the following sequences of word associations and the detailed findings and supporting evidence for each sequence follows.
Word Associations

Subcategory title

Word Association: angular acceleration -> acceleration ---> change in A speed

Subcategory description

The term angular acceleration is associated with the word acceleration. In turn the word acceleration is associated with a change in the speed of the object.

Subcategory examples

In the examples to follow (unless otherwise indicated) the object (a toy train) moving in an oval path.

Example #1:

Word Association: angular acceleration -> (acceleration) ---> change in A speed

I: Does the train ever have an angular acceleration?

K: (pause) No, because during the track you are never increasing the speed. [120-6]

Example #2:

Word Association: angular acceleration -> (acceleration) ---> change in A speed

I: Does the train have an angular acceleration?

A: No, because the speed doesn’t change.

I: Angular speed?

A: The angular speed doesn’t change. [120-12]

Example #3:

Word Association: angular acceleration -> (acceleration) ---> change in A speed

I: Does the train ever have an angular acceleration?

G: (after a false start). It is clear to me that it doesn’t have an angular acceleration. And why?
It is just not—it is not like increasing-going around faster and faster. [216-3]

For some students a change in angular speed was associated with a change in angular acceleration.

Example #4:

Word Association: angular acceleration->acceleration- -->(Δ speed)--→Δ angular speed

I: Does the train have an angular acceleration?

D: Sure.

I: Where?

D: Uh, most places except right along here (section AB of track), it is kind of obvious because its angular speed is zero here and it has an angular speed here so it must have accelerated somewhere. [120-13]

Example #5:

Word Association: angular acceleration->acceleration- -->(Δ speed)--→Δ angular speed

I: Does the train have an angular acceleration?

F: Acceleration is the change in velocity and angular speed. So, here it has angular speed around the curve, but on the straight track there is no angular speed. I would say the angular speed is changing obviously. [115-8]

Acceleration was also associated with a change in velocity, rather than a change in speed and is represented as a subcategory of word association.

Subcategory title

Word Association: angular acceleration->acceleration- --→ Δ velocity

Subcategory description

The term angular acceleration is associated with the word acceleration. In turn the word acceleration is associated with a change in the velocity of the object, however it was
usually a change in the direction of the velocity vector that led some students to conclude that an object had an acceleration and thus an angular acceleration.

**Subcategory examples**

**Example #1:**

Word Association: angular acceleration->(acceleration)->\(\Delta\) velocity-change in direction

\[\text{I: } \ldots \text{does that train have an angular acceleration?}\]

\[\text{K: Yeah, because it is a vector..magnitude and direction. There is a change in direction. [120-7]}\]

**Example #2:**

Word Association: angular acceleration->(acceleration)->\(\Delta\) velocity-change in direction

\[\text{I: Now, does the train ever have an angular acceleration?}\]

\[\text{J: It would have an angular acceleration because the velocity vector is changing as it is going around from point B to D, and C to A. [216-4]}\]

Sometimes a student would recall a formula in helping to decide if an object had an angular acceleration. The use of a formula as described earlier is a form of **symbolic association**.

**Symbolic association**

**Subcategory title**

**Symbolic Association::angular acceleration::formula**

**Subcategory description**

The *angular acceleration*, of an object is associated with its symbolic form, namely a formula (or formulae). The for-
mula used by a student may at times have been inappropriately used in the context given.

Subcategory examples

In the subcategory example, example (#1) to follow a student cites a formula for the angular acceleration of an object moving uniformly in a circular path of radius \( r \), namely, \( \alpha = \frac{a}{r} \) (only true if it is understood that \( a = a_t \) tangential in the context given). The subsequent example (#2) a student cites (an incorrect) formula \( \alpha = \frac{\omega}{r} \) to find the angular acceleration of an object.

Example #1: Symbolic Association:Formula-> \( \alpha = \frac{a}{r} \) (semicircular path)

I: How about angular acceleration which you were just about to mention.

K: Yeah, because alpha is a over r so if there is a change in acceleration there has to be a change--there has to be a change in angular acceleration. There has to be an angular acceleration.

Example #2: Symbolic Association:Formula-> \( \alpha = \frac{\omega}{r} \) (semicircular path)

I: Let's talk about angular acceleration. What things come to your mind when I talk about angular acceleration?

L: I think about the angular speed and the radius and formula.

I: Which is?

L: Angular acceleration equals \( \omega \) over \( r \).

Another form of association that was identified was contextual association. This form of association is characterized by the student relating an event, experience or some
piece of knowledge with some aspect of the context of the task setting.

**Contextual association**

**Subcategory title**

**Contextual Association:** circular motion

**Subcategory description**

The task context cues some physical knowledge that an object moving in a circular or curved path accelerates.

**Subcategory example**

**Example #1:** Contextual association: circular motion

(oval path)

I: *Does the train have an angular acceleration? Or, is it hard to tell?*

L: (a pause) uhm, it is hard to tell. I think it does have angular acceleration because it is moving in a circular motion depending on which points from C to A and...(emphasis added). [120-11]

**Example #2:** Contextual Association: circular motion

(semicircular path)

I: *Does the ball have an angular acceleration down the track?*

P: Uh, yeah it does continuously because the track is curved. [120-5]

Another form of contextual association was an association between acceleration a kinematic quantity and force, a dynamical quantity, for an object moving in a circular like motion, as the following examples illustrate.
Example #1:
Contextual Association::circular motion:force
(semicircular path)

I: *Now, how about angular acceleration?*

M: Always has (angular acceleration), because it is keeping within the course of the track in order to do that it must have a *force* to allow the ball to stay in. (emphasis added) [115-15]

Example #2:
Contextual association::circular motion:force
(irregular path)

I: *Let's go back to the irregular path for a moment. If I were to ask you does the ball have an angular acceleration--you said it would be very difficult to judge, would that be correct? Does it make sense to talk about the ball having an angular speed?*

K: Well, it makes sense to talk about the ball having an angular speed, but to talk about the ball having an angular acceleration it implies a *force* being acted upon it and there is nothing pushing the object around the "circle" at a constant speed. (emphasis added) [120-6]

Sometimes the context cued a personal analogy for the student. Such a form of contextual association was evidenced in the response to a question about the angular acceleration of a train moving in an oval path.

Example #3
Contextual association::(personal) analogy:force

M: *I think the key factor for angular acceleration is that the train--how the train feels...*

I: *By the train feels, you would be referring to what?*

M: Uh.

I: *Do you think about yourself inside the train or--?*

M: Yeah, basically. If you are inside the train I would consider that the same thing.

I: *And how does this feeling translate into--?*
M: Let's say it wasn't a train, let's say it was another person and I was to run around the track. I would have to—if I wanted to run around at a constant speed I would always have to make my left foot-like-put more force on my left foot to turn right.

Another student (cited earlier) made a specific reference to the coriolis force.

Example #4: Contextual association: coriolis force (semicircular path)

I: Does that ball (rolling down the inclined path) have an angular acceleration?

L: Yes.

I: And because?

L: Because, again its moving in circular motion and because of the coriolis force, eh the coriolis force is angular acceleration times radius, so-- (emphasis added)

I: Therefore?

L: Angular speed times radius (student "correcting" previous statement) and therefore angular acceleration.

Even though the student's formula for the coriolis force is incorrect, it is the association (cued by the context of the task) to the coriolis force that is noteworthy.

Some hint as to the reason for the student thinking about the coriolis force (or acceleration) is given by the following dialogue.

I: Do you want to explain that to me a little bit?

L: Coriolis acceleration?

I: Yeah.

L: How I understand it, I understand it to be, eh-uhm, when an object is on a frame that's rotating.

I: When an object is on a frame that's rotating (repeating student's statement), uh-ha.
L: For example, the ball on the earth, on the table and
the table is on the earth and well the frame would be
say the earth, well initial frame--the bigger frame
that's the earth and the earth is rotating, so that
causes a coriolis force.

The ball's motion on the "rotating" table was seen by the
student to experience a coriolis force. This suggests that
the student was reminded (by the context of the task set-
ting) of a physics lecture in which rotating reference
frames were discussed. While this is an inappropriate anal-
ogy, it does nevertheless represent an interesting example
of contextual association.

Finally, a student response may exhibit more than one
form of association as the following example illustrates.

Example of a multiple association:

Contextual association: circular motion--> (acceleration)
Word association: (acceleration)--->angular acceleration
Symbolic association: formula--> $\alpha = \frac{v^2}{r}$

(oval path)

I: Does it have an angular acceleration between C and A?
(the semi-circular end of the oval track)

L: I think it does.

I: Because?

L: Because it is moving in a circular motion and angular
acceleration is $v^2/r$. So if that is radius
and that's velocity then I think it does have angular
acceleration. [120-11]

While it is possible to represent the flow of reasoning
differently than shown above, it is reasonable to character-
ize such a flow as some sequence of associations.

This and other characterizations of student reasoning are
summarized in the next section.
4.62 **Discussion of findings of research question #4**

Student responses to questioning about the angular acceleration of an object have been described in terms of associations. Three forms of associations have been characterized. These are: **word**, **symbolic** and **contextual**, each of which will be discussed respectively.

**Word** association is the association of particular words with the term angular acceleration. Angular acceleration was associated with the words: acceleration, force, a change of velocity, or a change in speed. A consequence of using this form of reasoning is that a student might for example decide that an object had an angular acceleration on the basis of whether or not the object was accelerating or if a force was acting upon it.

**Symbolic** association is the association of the symbol $\alpha$, angular acceleration with other symbols, namely formulae. Formulae that were recalled by students were: $\alpha = a/r$, $\alpha = \omega/r$, and $\alpha = v^2/r$. A consequence of this form of reasoning is that a student may decide that an object has an angular acceleration on the basis of a particular formula, even though the formula may be incorrect or incorrectly interpreted.

Finally, **contextual** association, is the association of some event, experience, or personal knowledge that is cued by the context of the task setting. A student may reason for example on the basis of analogy to: planetary motion or the coriolis force, the force exerted by one's foot, in
deciding whether or not an object has an angular acceleration. Clement (1987), in reference to personal analogies (as the force exerted by one’s foot) noted that: "approximately half of the analogies (made) were personal analogies referring to some sort of body action" (p6).

It should be remembered that neither contextual association or any other form of association uniquely characterizes the reasoning process of any one student. A student may use several forms of reasoning even though on a particular occasion only one form of reasoning may be exhibited. The possible pathways of student reasoning by association are illustrated by the diagram below.

Figure 9: Reasoning paths for angular acceleration
5.00 Introduction

The conclusions of this study are presented in this chapter for all research questions. Each research question will be presented in the order in which the questions were analyzed in Chapter Four followed by a brief summary of the findings of each question. After the findings for all research questions have been presented a discussion of these findings, instructional recommendations, and recommendations for future research will be given respectively.

5.10 Conclusions of the Study

The following is a short summary, without comment, of the findings of each research question.

Research Question #1

What are students' conceptualizations of the angular speed of an object from different frames of reference?

The findings to research question #1 can be expressed in terms of five categories of description which were constructed to describe student conceptualizations of angular speed from different frames of reference. These conceptualizations were grouped in terms of whether or not the object's angular speed was independent of the object's path or
observer position. Accordingly, the summary of the findings to research question one are:

**Position:**

1. The angular speed of an object is independent of the position of the observer.

2. An object may not have an angular speed if the position of the observer is not enclosed by the path of the object.

3. The angular speed of an object should be measured from a preferred position.

**Path:**

4. An object moving in a rectilinear path doesn’t have an angular speed or it doesn’t make sense to talk about the angular speed of such an object.

5. An object moving in an irregular path doesn’t have an angular speed or it doesn’t make sense to talk about the angular speed of such an object.

The findings of research question one were based on tasks which involved the angular speed of one object. The findings of the next research question were based on a task which involved the angular speeds of two objects.

**Research Question #2**

What is the nature of students’ conceptualizations as they make comparisons of the angular speed of two objects moving in semicircular paths of differing radii of curvature?

Students were characterized as using two different methods in comparing the angular speeds of two balls, each one rolling down separate semicircular inclined planes. These methods were: i) Arc Length Equivalence and ii) Position Equivalence. The student using a method of Arc Length
Equivalence believed that the two balls had the same angular speed if they traveled the same arc (angular) distance in the same amount of time. While the student using a method of Position Equivalence believed that two balls had the same angular speed if they had the same (radial) position at the same time.

Later, the findings of this research question will be commented upon as it relates to the work of Trowbridge (1980) upon which this research question was developed.

**Research Question 3**

What is the nature of students' reasoning as they address several task problems involving the concept of angular speed?

The findings of research question three (based on a similar set of tasks as described in research question one) characterizes student reasoning as a form of association. Three forms of association were constructed to describe student responses, these were: word association, symbolic association or contextual association.

**Word association** is characterized by an association of the words *angular speed* with *angular, speed* or other words. For example, a student in responding to a question about an object's angular speed might respond as if the question asked was about the object's *speed* rather than its *angular speed*. Consequently, a student might conclude that an object moving in a curvilinear path had a constant *angular speed* if it had a constant *speed*. 
Symbolic association is characterized by the association of the symbol, \( \omega \), with other symbols, which is expressed as some formula containing the symbol \( \omega \). These formulae as given by the students were either kinematic in form, \( \omega = v/r \) or dynamic in form, as \( I\omega \) equals some constant. Sometimes a no distinction was made between \( v \), \( v_{\text{tangential}} \) and \( v_{\text{radial}} \) of the object in the use of some of these formulae.

Lastly, contextual association is characterized by the association of circular motion, planetary motion, simple harmonic motion and the coriolis "effect", to some aspect of the context of the task setting.

The fourth and last research question of this study sought to extend the investigation of students' reasoning about angular speed to angular acceleration.

Research question #4

What is the nature of students' reasoning as they address several task problems involving the concept of angular acceleration?

The categories of association which were developed as a result of student responses to the above question were characterized in the same way as were the findings to research question three. These categories of association being: word, symbolic and contextual.

Word association in the context of the fourth research question is the association of the words angular acceleration.
tion with the words acceleration, force, and a change in, velocity or speed. For example, a student might conclude that an object moving in a curvilinear path had a constant angular acceleration if it had a constant acceleration based on a form of word association between the term angular acceleration and the word acceleration.

**Symbolic association** is the association of the symbol, α with other symbols, as formulae, which as given by the students were: \( \alpha = a/r \), \( \alpha = \omega/r \) and \( \alpha = v^2/r \). For example, a student might decide, based on the (incorrect) formula \( \alpha = v^2/r \), if an object had or had not an angular acceleration.

Finally, **contextual association** is the association of physical analogies as planetary motion or of personal analogies as the force one feels in one’s foot in going around a curved track, to some aspect of the context of the task setting.

5.20 **Discussion of Findings**

The contributions of this study to the field of science education and the manner in which the findings of this study are situated with respect to the work of others in the field will be discussed with respect to each research question.

The findings of research question one are a unique contribution to the field of science education, as no other study has yet documented students’ conceptualizations of angular speed. The findings of research question one have
also extended Saltiel and Malgrange’s (1980) work of students’ reasoning in the domain of elementary kinematics.

Saltiel and Malgrange observed that students regarded velocity as being loosely associated with reference frames, "Almost never do students think in terms of frames" (p79). This independence of the observer's position parallels one of the conceptualizations characterized in research question one, that the angular speed of an object is independent of the position of the observer. Such a conceptualization might be intrepreted as a belief that angular speed is an intrinsic property of the object. If this were so it would concur with Trowbridge’s interpretation of his findings that students conceptualized velocity as being an intrinsic property of the object independent of the observer’s motion or position.

Finally, a conceptualization as the angular speed of an object should be measured from a preferred position, could (in part) be explained by Saltiel and Malgrange observation that in everyday life a particular frame is very often highly favoured, because for practical reasons it is often unnecessary to think explicitly about reference frames.

The findings of research question two have also extended Trowbridge’s (1980) findings in his investigation of the methods used by students to compare the relative speeds of two objects. The finding that some students state that two balls have the same angular speed if they have the same position at the same time parallels Trowbridge’s finding that
according to some students, two objects have the same velocity when they have the same position.

The findings of research question three and four represent a contribution to the field of study called situated cognition. The words and images used in the context of a task setting (part of the situatedness of the task) appeared to influence the way in which a student conceptualized the angular speed and angular acceleration of an object. Such influences were interpreted as arising from some aspect of the task setting being associated with some prior knowledge or experience the student might have had.

For example, the construct word association, offers a way of viewing student reasoning as being influenced by the words used in the task dialogue. The word angular for example was sometimes associated with an arc or curved path. This form of association is hypothesized to be responsible for the conceptualization that an object moving in a rectilinear path does not have an angular speed.

Another construct, symbolic association, offers another way of viewing student reasoning as being influenced by the association of the symbol, $\omega$, with a group of symbols, a formula. It is to be expected that students might have difficulties with concepts such as angular speed if they use this form of reasoning without being able to operationalize\(^1\)

\(^1\) The use of this term is to be understood as being the procedural knowledge which allows a student to find the angular speed by geometric construction in a similar manner to that defined by Reif (1987) for acceleration (see Chapter Four).
the concept of angular speed as defined by the formula \( \omega = |d\theta/dt| \). This interpretation would be consistent with Reif’s (1987) finding that students often lack the procedural knowledge of how to interpret the formula that defines the concept of acceleration, \( a = dv/dt \).

Finally, the construct *contextual association* offers a way of viewing student reasoning as being influenced by the context of the task setting (excluding those that may be described by word or symbolic association). Reasoning by contextual association, as for example the association of the elliptical path with planetary motion, contrasts sharply with "spontaneous analogical reasoning" defined by Clement (1987).

Clement, in defining this concept of analogical reasoning, *excluded* the use of *trivial* strategies involving surface similarities of a problem. The characterization of analogical reasoning in this study does not exclude such strategies, nor are such "strategies" considered trivial. Being reminded of something, whether or not it occurs by words (as in word association) or by symbols (as in symbolic association) or by surface similarities (as in contextual association), with some aspect of a problem situation is thought to be an important way in which we frame, think about, or *understand* a problem.

Schank and Seifert (1985) assert that our *understanding* of a situation means being reminded of the thing in memory
that is closest to the experience we are currently having.

To illustrate this principle Schank (voicing a constructivist theme) recalls as a public lecturer the following episode:

I used to be infuriated with the person in the audience who said, "Isn't this just like the work of X?" I'd be mad. It wasn't like the work of X. How could they not see that? Now I say, "Ha! You've just proved my theory. You've understood what I said in terms of the things closest to it in your memory." (p73)

This view of understanding supports one characterization of reasoning as a form of contextual association. Such a characterization of reasoning is consistent with the view that how a person acts and responds in a situation will be inherently contextualized, as posited by Brown, Collins and Duguid (1989).

Reasoning, whether by word, symbolic or contextual association, provides a powerful way in which to view how students are able to understand both familiar and novel situations without the need to theorize, or even the desirability of theorizing about what to do or how to act next. For example, if a very large rock is suddenly thrown towards you, you might, given the context of the situation, quickly move aside to avoid injury without "theorizing" about what to do. Usually this action would be considered appropriate even though later you might find out it was a "Styrofoam" rock and you were not in danger of being injured.

Or, if as a physics student you are shown a picture of an object moving around an ellipse you might remember planetary
motion, this being the "closest" thing available to you in your memory to make sense out of the situation, even though later you may realize this was an inappropriate analogy to make.

While theorizing by individuals does occur, and needless to say some of us spend a great deal of time doing this, it is nevertheless possible to explain a great deal of our actions, especially the actions of students, without positing that reasoning is theory-like. In this regard the findings of this study support and extend to student reasoning about kinematic quantities the position held by DiSessa (1987) that student reasoning about dynamical quantities need not be theory-like.

Besides relating the findings of this study to the work of other researchers in the field, these findings can be related to the way in which concepts of angular speed and angular acceleration have been treated by authors of physics textbooks. It is hypothesized that some of the findings arising from this study can partly be explained by the treatment of kinematic and dynamic quantities as exemplified in the first year physics text Fundamentals of Physics, by Haliday and Resnick, 2nd edition.

In the introductory chapter to angular kinematics, the concept of angular speed is first examined for a rigid body in simple planar rotational motion. From examination of

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2. A rigid body is one in which the relative positions between the particles that make up the body remain at a constant distance.
this introductory chapter and subsequent chapters in the
text certain observations about the content have been made.

These observations are as follows:

i) The introductory chapter's title is *Rotational Kinematics*.

ii) Angular speed, $\omega$ is often measured in *rotations* or *revolutions* per second$^3$.

iii) Textbook problems are given of *rotating* wheels, *rotating* phonograph tables and other *rotating* objects revolving about a *central* axis.

iv) While some mention is made of reference frames, there are no problems (or examples) which require (or show) the calculation of the *angular speed*, $\omega$ of an object from differing *frames of reference*.

v) The motions of the planets are portrayed as moving in *elliptical* orbits about the sun in which the sun is taken to be at rest at one focus.

vi) A special emphasis in the section of the text which describes the relationship between linear and angular kinematics is given to the relationship $v = \omega r$, where $v$ is the speed of a particle in *circular* motion.

vii) The foregoing section of the text (vi) gives the relationships $a_T = \omega r$, $a_r = v^2/r$ and $a_r = \omega^2 r$.

All the problems given in the text (except for two) are examples of objects moving at a *constant* angular acceleration. The two exceptions to this are problems which require the student to find the angular acceleration of an object from a functional relationship which expresses the object's angular displacement as a function of time.

viii) *Simple harmonic motion*, (SHM) is represented as a projection of uniform *circular* motion.

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3. At least eight of the review problems (on p176-177) refer to $\omega$ having units of rev/sec. For example problem #13 begins with the following words: "A uniform disk rotates about a fixed axis starting from rest and accelerating with constant angular acceleration. At one time it is rotating at 10 rev/sec. After completing 60 more complete revolutions its angular speed is 15 rev/sec....".
From these observations it is possible to infer that students may be influenced in their understanding of angular speed and angular acceleration in the ways given below. Each way is referenced with respect to the research finding (r.f.) of the corresponding research question (#1, #2 or #4) and to the specific text observation or observations (i, ii, ...viii) so that the reader may judge as to the validity of the inference being made.

a) Objects must revolve around something to have an angular speed. [r.f.#1: i, ii, iii]

b) Objects moving in a rectilinear path do not have an angular speed. [r.f.#1: i, ii, iii]

c) Angular speed is to be measured (if possible) from a preferred position, the geometric centre or in the case of elliptical motion from a foci. [r.f.#1, iii, v]

d) Angular speed is independent of the observer's frame of reference. [r.f.#1, iv]

e) No distinction is made between v, and vtangential when students use the formula, v = ωr [r.f.#3, iv]

f) Inability to state the correct expression that defines the angular acceleration, α of an object. Or, an inability to find the angular acceleration of an object. [r.f.#4, vii]

g) Inappropriate analogies are made to simple harmonic motion and planetary motion. [r.f.#4, v, viii]

If, as inferred by the foregoing discussion, the influence of the textbook does help shape students conceptualizations of angular concepts as described, then there exists a basis upon which to make instructional recommendations as to the way the textbook presents these concepts.
Also, conceptualizations as described in parts a and b as: *objects moving in a rectilinear path do not have an angular speed*, can have instructional implications beyond the study of kinematics to the study of dynamics. To quote a student cited earlier (in Chapter Four):

H: ...I was having some trouble understanding the idea of the angular momentum...I couldn't understand how something that just happened to be moving along (in a straight line) had an angular velocity when it wasn’t actually rotating or going round in a circle. [120-9]

And, even when a student recognizes a difficulty with how they understand a situation, he or she may be unable to proceed. Again, to quote a student cited earlier (in Chapter Four):

M: You made those restrictions (referring to the constant speed of the object in an elliptical orbit)...it shoots out what I know about-eh-space, elliptical orbits...and everything.

Such difficulties as this including the various conceptualizations of angular speed and angular acceleration which are at variance with accepted scientific interpretations can only be addressed by instruction.

5.30 Implications for Instruction

Based on the findings of this study and a reading of the textbook it is possible to make certain instructional recommendations. However, the recommendations should be considered as being tentative, as it is not certain that any recommended change will result in overcoming the difficulties so far mentioned or in altering inappropriate ways in which
a student thinks about or conceptualizes the concepts of angular speed and angular acceleration.

Some of these recommendations which follow may not apply to the reader's particular situation. For example, a recommendation to change some feature of the textbook may not apply to a physics teacher who is presently using a textbook which is in some important respects different from the one upon which these recommendations are based. Or, an instructional recommendation may not be compatible with an individual's teaching practice. It is for the reader to judge the appropriateness of the recommendations to his or her own particular situation.

**Recommendation #1**

In the introduction to the concept of angular speed, the units of angular speed, \( \omega \) should be measured in radians per second *exclusively* \(^4\), and the symbol \( v \) or \( f \) should be used for revolutions, cycles or vibrations, per second (thus \( \omega \) would equal \( 2\pi v \) or \( 2\pi f \)). Even though \( v \), \( f \) and \( \omega \) do have the same dimensions, the use of *revolutions per second* for \( \omega \) may give the impression to some students that the angular speed of an object moving in a closed curvilinear path can be determined solely from the time it takes an object to complete one or part of one *revolution*.

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4. Units of angular speed as measured in radians per unit of time need not have as the unit of time the second. However, it is standard practice to use the second since it is a basic unit of the *cgs, mks* and English systems.
Recommendation #2

Students need more practice in differentiating between the average angular speed and the instantaneous angular speed. In many problems the average angular speed and the instantaneous angular speed have the same value. This may lead to a difficulty for the student who may associate the symbol, \( \omega \), with the average speed as well as the instantaneous speed.

It may help if a number of problems are given to the student in which the instantaneous angular speed \( \frac{d\theta}{dt} \) of the object is not equal to the average angular speed \( \frac{\Delta \theta}{\Delta t} \). This would for example, prevent a student from simply finding the angular speed of an object moving in a circle (at a non-uniform speed) by measuring the time for the object to make one revolution.

Recommendation #3

It is recommended that the student be shown how to find, using explicit geometric procedures, the angular speed of a particle. Problems which require the student to use such a procedure should be given. For example, a problem might be to find with reference to an observer, "B", the angular speed of some particle at point "A" moving at a constant speed along a path which is not definable by any analytic expression.

This recommendation parallels that given by Reif (1987) in his study in the domain of kinematics. Reif suggested that one should teach explicitly the formal knowledge speci-
fying a concept, by teaching both the descriptive knowledge of the concept and an explicit procedure for interpreting the concept.

**Recommendation #4**

Students should have ample opportunity to compare the angular speed of a particle from differing frames of reference. Since the treatment of reference frames is sometimes given little emphasis in the topic area of kinematics it would be advantageous to have more examples throughout this part of the course (as judged by the emphasis given in the textbook) which illustrate the procedure of making measurements from differing frames of reference. Otherwise, students may think about reference frames in isolation, partly because it is seldom an issue that needs to be explicitly considered in most problems.

**Recommendation #5**

The axis of rotation of a rigid body is usually "made" to pass through its geometric center, as illustrated by textbook examples of: rotating grindstone wheels or phonograph records. In the study of particle motion a student may extend this idea to think that the angular speed for an object's motion in a closed path is to be taken to be at some "geometric center" (assuming such a center is identifiable).

Thus it is recommended that during the study of kinematics that the student be presented with a number of problems in
which the axis of rotation of a rigid body does not pass through the geometric center.

**Recommendation #6**

Extending the discussion in recommendation #5, it is of note that some students view either the focus or geometric centre of an ellipse or a circle as being a *preferred* position from which to make measurements of the angular speed of an object. This may be attributable to the manner in which planetary motion and rigid body rotation is treated in the textbook.

Planetary motion in physics is often used to illustrate Kepler's Laws in conjunction with that of conservation of angular momentum. Since the mass of the sun is very much greater than the mass of any planet, the position of the *center of mass* (of the sun-planet system) is very close to the position of the center of mass of the sun. This makes it difficult for the student to differentiate the physical significance between the position of the center of mass of the sun-planet system and the center of mass position of the sun. Furthermore, the student may think that the *focus* of the elliptical path traced out by the planet has special significance, *only* because it is a special geometric position and not because it is special as a consequence of general physical laws, and a central force which varies inversely with the square of the distance between the sun and the planet.
Therefore, it is recommended that students be made aware of this distinction between some special geometric position of an orbit (either a focus or the "center" of the elliptical orbit) and how, if appropriate to do so (because of physical laws) it relates to the center of mass position.

5.40 Implications for future research

Four recommendations for future research based on the findings of this study are as follows:

1) A subject for future research is the investigation of the frequency distribution of conceptualizations of angular speed. It would be useful to know (given the task context) the relative frequency of students (based on a properly chosen sample from the general student population) exhibiting a given conceptualization as for example, an object moving in a rectilinear path doesn’t have an angular speed. Knowing the relative frequencies of various inappropriate conceptualizations helps design curriculum materials which aim to address the most frequent of these conceptualizations in a particular instructional setting.

2) One subject for future research might be to test the effectiveness of any recommendation suggested in this study in a classroom setting. Any one recommendation by itself may not be effective in the classroom unless applied in conjunction with another recommendation or instructional strategy.
A recommendation to make explicit the procedural knowledge by which to find the angular speed of a particle, for example, is not in itself sufficient for students to achieve an understanding of the concept of angular speed. Because "...it is the interrelationships (within a much larger scientific knowledge structure) which provides the concept with a rich 'contextual meaning' extending far beyond its formal definition" (Reif, 1986 p27).

Thus it is for future researchers to decide, with due regard to these interrelationships and to the context of the classroom setting, the effectiveness of any specific recommendation.

3) In an investigation of high school physics teachers, Berg and Brewer (1991) observed that few teachers were aware of student alternate conceptions about rotational motion and gravity. It thus might be of interest to compare what physics teachers believe are students' conceptualizations of angular speed and angular acceleration with the findings of this study.

4) Finally, a language needs to be developed which characterizes the reasoning activities students engage in when analyzing problems or tasks presented to them. The appropriate level of detail conveyed by such a language should reflect the purpose for which it is to be used, in this case pedagogy.
For example, the use of the expression reasoning by association in this study serves to characterize student reasoning at a sufficient level of detail without the need to use terms that describe the neurological functioning of the brain.

In conjunction with the need to describe reasoning at an appropriate level of detail is the need to describe memory (again at an appropriate level of detail) associated with the act of reasoning.

What do students retain of their formal learning experiences? And why do some learning experiences get recalled in one situation but forgotten in another? The answers to these questions are incomplete, despite the thousands of memory experiments that psychologists have done, "It is difficult to find even a single study, ancient or modern, of what is retained from academic instruction" (Neisser, 1982 p5). It is suggested that psychologists alone should not be doing memory experiments; some researchers in science education should be, especially if a comprehensive account (and hence a language) of student's reasoning processes is thought to be a desirable goal.
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APPENDIX I
A SAMPLE OF A COMPLETE INTERVIEW TRANSCRIPT
Subject [120-5]

The Oval Path Task

(Student is shown a toy train moving around an oval track and is asked to imagine that he is located at the position of a toy observer placed at at the center of the oval track)

I: If I were to ask that observer at the center of the track does the train have an angular speed, what would your response be?

P: At point A or just in general?

I: Ok, does it have an angular speed between points A and B? (The train).

P: No.

I: Because?

P: Well, because it is moving in a straight line. It has got a linear velocity.

I: Ok, and secondly, does the train have an angular speed between points B and D as seen by that observer?

P: You (observer) can turn around?

I: Yes

B. Uh-yeh, yes it would because its heading is changing even if its actually instantaneous velocity is not-its heading is changing so it would appear to have an angular velocity.

I: It would appear to have an angular velocity?

P: Well, it does have an angular velocity.

I: Because, its heading?

C. Its heading is changing-its changing direction its on an arc.

1. This task is again discussed in review: see p.149
I: Is there an angular speed between points D and C?

P: Uhm-relative to his position-no. The train has in the course of going around this course has changed its direction, you know continuously while its on the curves and not so continuously while its on the straight a ways.

I: So its not changing?

P: No, its not really changing between D and C. So in effect you can say it has a linear velocity and that none of its velocity is angular.

I: And between C and A?

P: It has angular speed.

I: For the same reason. (a reference to an earlier student response re: angular speed between B and D)

P: Yes.

I: Between B and D because its--changing?

P: Changing direction continuously.

I: Continuously?

P: Right.

I: Is the angular speed between A and C the same as between B and D?

P: Uhm, well, its speed would be because assuming uniform velocity of the train eh, its velocity would be different because its going in a different direction of course. Velocity is continuously changing when going around the corner its acceleration--it has acceleration right, but its speed, its angular speed would be the same, I would assume.

I: Does the train ever have an angular acceleration?

P: Between points B and D and between points C and A it has angular acceleration.

I: Because?

P: Uhm, well because it is constantly changing eh-angular acceleration isn’t dependent necessarily on whether it is changing, increasing or decreasing its linear velocity. It has an acceleration because it is continually changing its direction and acceleration is defined to be a change in velocity and since velocity is dependent both on head-
and on speed on magnitude-its a vector quantity there-fore acceleration can also occur when you are simply changing your direction.

I: So, therefore?

P: It has an angular acceleration.

I: Ok, good.

(A toy observer is now placed at the edge of the track, close to where the student is sitting)

I: If I were to place the observer just about where you are sitting but your head is lined up with AB (section of the track). If you were at that position and I were to ask: does the train have an angular speed between A and B, your response would be?

P: No.

I: Because.

P: It doesn’t appear to be moving laterally or any other way.

I: And, between B and D does it have an angular acceleration?

P: Well actually you can argue that it wouldn’t appear to be. But it does. It would necessarily appear to be, because as I am looking at it would appear to accelerate. It would appear to accelerate linearly-uhm say I have no depth perception then eh..

(Interviewer interrupts and refocuses the questioning by starting again with the question of the train’s angular speed.)

I: Does it have an angular speed between A and B?

P: No

I: Because?

P: Because it is moving in a straight line.

I: Between B and D?

P: Yes.

I: Because?

P: Well, because it is not moving in a straight line. Its
I: Between D and C does it have an angular speed?

P: No.

I: Because?

P: Because it is moving in a straight line.

I: And between C and A?

P: Yes, because it is not moving in a straight line its curving.

I: Is the angular speed between C and A the same as between D and C?

P: Yes it is.

I: And that’s because?

P: Because in spite of the change in direction its got the same magnitude of speed.

I: Ok, good.

The Semicircular Path Task

(Interviewer releases one ping ball down the track. Before asking about angular speed the objects speed and acceleration are investigated first.)

I: First question I want to ask you, is that ping pong ball accelerating down the track? (ping pong ball is made to roll down the track)

P: Yes.

I: And the reason it has an acceleration down the track.

P: Well, because it is at a slight slope so it is also being subjected to the acceleration due to gravity-and eh-well that’s basically it-you can break it into a couple of components, but its effectively on a slope this way and it actually eh there is some lateral--well you can break it down into gravitational acceleration downwards but the slope actually changes because of the curve but that’s sort of trivial.

I: Now, does it also appear to have an acceleration. Suppose it was dark and all I could see was a flourescent ball.
P: Sure you can, there is a distinct increase in its rolling. It seems to be rolling at a faster and faster rate.

I: Ok, same would be true if I roll the ball on the outer track, it has an acceleration? (ball is shown rolling down outer track)

P: Yeh, it does.

I: Same reason?

P: For similar reasons, yeah.

I: Does the ball have an angular acceleration down the track?

P: Uh-yeh it does continously because the track is curved.

I: Ok.

(Student is shown a video recording of each ball going down the track separtely then together. The mechanism of release of one ball is explained as being triggered by the passage of the other ball through a photogate.)

I: First thing I would like to talk about is speed. If you could compare the speed of these two balls. You can refer to inner (ball) and outer one so I know which one you are talking about.

P: Are you speaking of linear speed, now?

I: Speed.

(Interviewer has cards on which are marked speed, angular speed, acceleration, angular acceleration. Interviewer shows student these cards, and selects the one written "speed" placing it on the table)

P: Well, in terms of this video tape it appears that-- (pause)

I: I’ll play (video tape) again.

P: Yeah play it one more time. They (the balls) are both reaching the end at the same time I notice-uh-what that indicates to me is that obviously they can’t be travelling the same speed because one ball is farther along the track than the other but, uh-at the same time because the inner track has a smaller radius-uh-the linear velocities will be different so- (student watchs replay of tape)

P: Yeah, it is sort of the instance in which you get when
you watch speed skaters on the ice in which one appears to start further ahead on a track and one may look to be ahead but in fact their are effectively neck and neck and this becomes apparent when they hit a situation of being at a parallel point, perpendicular to the tracks, the two tracks.

I: Let me ask this: does one always have a greater speed or a less speed or at times do they have the same speed?. Does the outer ball--

P: Uh, let’s see, in terms of just motion the angular acceleration is independent of the other accelerations so the speed of the balls will be--the outer ball has more time in which to build up speed.

I: When you look at the video tape do the two balls ever have the same speed?

P: No.

I: Ok, therefore one must have a greater speed

P: Right, I would assume the outer ball did.

I: And the reason? Because it appears, or because you worked it out somehow, is there some other factors not just appearance?

P: Just based on appearances is appears to have a faster speed simply because it was rolling for a longer period of time.

I: Ok.

P: I may be completely over looking some factor.

I: What is the strongest factor that you say it has?

P: Well, it has been rolling longer and basically that is what I am basing it on. It has a greater distance to roll and its acceleration does vary with distance the longer you roll the more quicker you are going to be going.

I: Ok, now what I would like to ask you--do the two balls ever have the same angular speed? (Interviewer rewinds the video tape so student may see again)

I: Do the two balls ever have the same angular speed and why? (tape is replayed again for the second time)

P: Uh-again I would say no, uh, never simultaneously the same speed.
I: The same angular speed?

P: The same angular speed, because effectively both balls are gaining their angular speed from like converting potential energy of their height into kinetic energy. And some of that goes into translational and some goes into angular. And I would say not because again the first one you can relate angular speed to linear speed. The first ball has been rolling for a greater distance. Again I am trying to figure this out in terms of what I know of theory and the fact that one track is of a smaller radius than the outer track indicates that there might be an instance in which they happen to be--but I can't observe any point which they have the same speed simultaneously.

I: And for angular speed, you're speaking?

P: Both (speed and angular speed).

I: And for them to have the same angular speed, for them to appear to have the same angular speed what would have to happen in your mind, is it clear to you?

P: It isn't. They would have to have the same linear speed at the same time because obviously you are dealing with a smaller radius. One ball can have a higher velocity on this track and this ball can have a smaller velocity on this (outer) track. They can still have the same angular speed. It is not necessarily apparent from the video tape, judging by just watching it. But, even if they never had the same velocities simultaneously it is conceivable there will be a point in which uhm the total angular speed determined by the linear velocities at a certain point of the track will be the same between the two because they have different radii. But I can't observe that, it is not easy for me to see.

I: Is it easy to observe under any conditions the angular speed of the two balls going down.

P: In effect you could, you are dealing with acceleration on a curve you could do the old simple harmonic motion experiment from observing it from a two dimensional point of view, and eh-one dimensional point of view and watching the two balls on their plane as they move you could simply eliminate all this mismatch of going around corners and things and would be seeing the balls as they travel straight. But it would be difficult to observe their angular speed because they are going to be in different places at different times.

I: Now, do the two balls have the same acceleration?
P: Yes, well-yes they do they have the same downward acceleration.

I: Ok, because?

P: Well basically because 9.8 m/sec squared is uniform for all objects-uh-lets see-because they might not necessarily be at the same angle then you are not guaranteed they are going down the ramp at the same speed but in effect the components of their motion indicate that downwards they are going to be pulled at 9.8 m/s.

I: So what you are saying is that the force acting on both the balls is the same.

P: Is the same.

I: Therefore?

P: Therefore the acceleration is going to be the same, assuming they have the same mass

I: Let me talk about angular acceleration. Do they have the same angular acceleration?

P: They wouldn't because their radii are different-uh, however there could be instances, again as I said before, instances in which they have the same angular acceleration because you know-

I: So they can have the same acceleration but they won't have in this case the same angular acceleration.

P: Not necessarily they might but-there could be instances in which that was the case but it would be an instance.

I: Why would it be the case that they might as opposed to not?

P: Well-again it is a situation where it is not clear, because uhm-angular acceleration is dependent on the radius and on the linear velocity of the two balls which means if at any point the two balls had velocities such that with their different radii they got the same acceleration it would only be for an instant, because ehm-their velocities are linearly are changing as they accelerate due to gravity. Right. See their velocities either way are not staying the same they are in a constant state of downward acceleration plus they are in a constant state of angular acceleration. Right. So the angular acceleration depending on linear velocity also depends on their acceleration due to gravity so they would never consistently have the same angular accelera-
tion. They might for a brief instant have the same angular acceleration simultaneously.

I: And, it is hard to say where?

P: It would be very difficult to say where. I can't judge.

I: Because they are several things you have to think about?

P: There are several things you have to watch at once-yeah because they are going through two different types of accelerations at the same time.

I: One is being--the two acceleration being? To repeat.

P: One due to gravity and the other due to the angular motion about the track

I: And where does that point? Where does the angular--due to the track? One due to gravity is down.

(student digresses a bit from the question being asked)

P: They are going around the track. Right. Which means their direction is constantly changing which means they have an angular acceleration. Because again, velocity being a vector quantity depends on the direction as well as magnitude.

I: And where does that acceleration point?

P: Uhm, oh I see what you are saying, ok. Inwards along the radii of the track.

I: Ok, so the one due to gravity points--?

P: Points down.

I: And, the one due to--

P: the angular motion points inwards towards the center along the radius.

I: Good, and that's the factors you consider?

P: Yeah.

I: And which it is difficult to decide--not easy to calculate?

P: It would be easy enough to calculate but I am just trying to judge by observation-and-I can say with relatively certainty their is probably a point but not necessarily a point at which they have the same angular acceleration
The Elliptical Path Task

I: Ok.

Here I have an ellipse and this ball. This blue ball is going around this ellipse at a constant speed of one meter per second.

P: Ok.

I: And assuming here you are an observer standing in the middle.

P: Uhm-huh.

I: Now, does the ball have an angular speed? Does it make sense to talk about the ball having an angular speed? And why?

P: Yes, again because it is continually changing its direction throughout the entire path it follows.

I: Is the angular speed a constant?

P: Eh, no it’s not.

I: Why?

P: Because its radius isn’t constant. It also changes.

I: Where is its maximum and minimum (angular speed) then? It must have a maximum and a minimum if not a constant.

P: Yeah, uhm I’ll just start getting the-eh-formulae here. Its angular speed will say omega because that is a convention is equal to, proportional to v or is equal to v over its radius, so its nice to have a constant speed. The radius however changes continuously so that at the point where it is farthest away from the center, where its radius is the greatest, its--

I: E to E’?

P: Yeah, from observer to E or E’. Its semimajor axis uhm, it will have its smallest angular velocity because again its inversely, angular velocity is inversely proportional to the distance, the radius.

I: Does the ball ever have an acceleration-just acceleration?

P: Just plain acceleration. Uhm, yeah because again--well
it maintains a constant speed its direction changes-like
I said velocity is a vector quantity, so it is continu-
ally changing.

I: Does it have an angular acceleration?

P: Well it must because its angular speed is never the same.

I: And that’s throughout the entire--?

P: And that’s throughout the entire ellipse.

I: And the same as with acceleration?

P: Yeah.

I: Now what would you do if I asked (you) to measure the
angular speed at point C (on ellipse)? What infor-
mation--I’ve told you that it (the object) is travelling
at a constant speed at one meter per second--given any
other information that you would like.

P: Just the radius.

I: Suppose the radius was 2 meters between the observer and
(point) C.

P: Well I’d know that omega, the angular speed would be
equal to the constant speed over 2. So it would be one
meter per second over two, so that’s eh--see is that
expressed in radians yeah I think that’s right that would
be half a radian per second.

I: Ok, that’s fine.

**The Irregular Path Task**

I: I’ll show you another picture of an object travelling at
a constant speed. Does the ball have an angular speed?
Or, does it make sense to talk about a ball travelling
in such an irregular path as having an angular speed?

P: Well, if it is on a track, I am assuming it is on a track
then it can certainly have an angular speed.

I: What happens if not on a track? Why does it have to be
on a track?

P: There has to be some force, either friction or something
which keeps it on a track or keeps it on this path.
Otherwise, it wouldn’t do that, nothing behaves this way
in and of itself.
I: That's important for you that it has a force acting, in order for you to determine whether it has an angular speed?

P: Well, uhm—all things aside—if all I could observe is this ball as it moves along this path, I could say with certainty that it has an angular speed because any time it is moving in a curved path in which it is changing direction it has an angular speed. The key question if you are to determine its angular acceleration you would need to know what sort of force is acting on it.

I: Is the angular speed the same as it goes around that track?

P: Not a chance.

(Interviewer puts a plastic overlay on the irregular path figure. The overlay has marks for reference on it.)

I: Point X is where you are standing. If the angular speed is not a constant, if I were to ask that person at point X if the angular speed is a constant, or if ask does it have an angular speed, what would his response be? It does have an angular speed?

P: Yeah.

I: And would it be a constant?

P: No.

I: And where abouts given the letters (of reference) I've shown you, where would you think the maximum and minimum angular speed would be?

P: Oh—(pause)—again it's hard to get away from this intuitive sense of the ball rolling due to gravity—because eh.

I: Just imagine this is in outer space, and there could be forces as you said acting on it to make it move in this direction, but you have no sense—

P: So, no linear acceleration. Uhm, as long as there is no linear acceleration. Uhm, well again linear speed, you would try to find some way to look at its radius of movement and of course that’s not an easy thing to do. These are completely irregular curves but basically at a given instant you can find a circle which approximates its curve.

I: Ok, let's take a point C or D or E.

P: Well, at point C you can look at a very small circle
I: Which is inside the path?

P: Which is inside the path, right. Because it is a small circle it has a very small radius of curving, again because angular speed is inversely proportional to the radius and wherever the radius is the smallest that’s where your going to have your greatest angular speed.

I: So, point C is greater (the angular speed) than point D?

P: Yeah, certainly, because obviously D is a very shallow curve. And it’s probably got a radius some where about the observer.. (inaudible)

I: How would you compare G and F?

P: Well, G would have-again its weird, the forces are now this way -it would be outside the curve and G would have a greater angular speed than at F.

I: Ok, good.

(This completes the task set. The interviewer now asks "review" questions and some further exploratory questions.)

Review

I: Back to the train. Here is the observer at the center. and you said the angular speed of the train as it was going around, what was it between A and B?

P: Angular speed would be zero.

I: Because?

P: Because it is going in a straight line.

I: Then I asked you, the angular speed betweed B and D. Is it a constant?

P: Pretty well, this looks a pretty close semicircle to me. I would say so.

I: And does the train have an angular acceleration between C and D or A and B?

P: No.

I: Does the train have an acceleration between A and B?

P: No, not as far as I can tell, the magnitude of its
velocity is a constant.

I: Ok. And when it (the train) is over on the track between B and D, does it have an angular acceleration?

P: Angular acceleration?

I: Sorry, acceleration.

P: Yeah, it would because its heading is changing. The magnitude of its velocity would remain the same, its constantly changing direction, so--

I: So the train does have an acceleration?

P: Does have an acceleration.

I: And it doesn't have between A and B because it is going in a straight--

P: going in a straight line.

I: And between B and D it points towards, the acceleration?

P: Towards, inwards along the radii.

I: So, it has an acceleration, does it have an angular acceleration?

P: No, it wouldn't.

I: It doesn't have an angular acceleration because?

P: Well, because its rate of going around this curve (B to D) is uniform.

I: So it doesn't have angular acceleration?

P: No.

I: Now, so it has an acceleration but not an angular acceleration?

P: Yeh-uhtm-right because again its radii-its velocity remains the same, I mean its linear velocity remains the same, its angular speed-well uhm-its angular speed is the same because it is a uniform curve so if its angular speed is the same then it wouldn't really have an angular acceleration. But although that doesn't make altogether a lot of sense anyway, because if it didn't have an angular acceleration it wouldn't be going in a curve-uhtm-its got an acceleration inwards along the radii.

I: Well, let me ask you (student interjects).
P: But that's just its acceleration in effect. It's what keeps it on the curve its not speeding up, its actual rate of going around isn't increasing. Right.

I: What is your definition of angular acceleration?

P: Well let's see, it's the rate at which angular velocity is changing.

I: So if the angular velocity changes then?

P: Then you have angular acceleration. In this case the angular velocity changes at B and at D as it straightens out. But along this track-

I: So does it have at any point on the track have an angular acceleration?

P: Well right at A,B,C,D-because it suddenly goes into a curve.

I: Ok.

(Interviewer returns to the task where an object moves in an elliptical path. Diagram of elliptical path is again shown to the student).

I: What happens if you were standing outside the ellipse just like where you are over here looking at it. And, your at one edge of this ellipse, and this ball is going around. Can you still talk of angular speed? Or, does it make any difference whether you are at this point at the center of the ellipse or there (at the outside edge of the ellipse)?

P: As long as I can perceive the ball is changing direction and going along a path that is not simply not one dimensional then I can speak of angular velocity as long as I can accurately measure the radius or at least the relative radii that I can see.

I: If I were to ask you, and all you could see is this florescent ball going around this path, for the angular speed, and if I were to ask an observer standing at the center for the angular speed, would you and the other person report back the same answer for the angular speed? Or, would they get different values?

P: I think you would get the same for both, because actually when it comes down to it even if I can only see the balls motion along within one plane I can still see the amount of time it takes for it for instance to reach one maximum
of the ellipse to the other.

I: Suppose your looking down at it just a little.

P: Just a little, that makes things even more simple.

I: Just the way you are right now, looking down at it.

P: Right.

I: Now, as opposed to you and a person standing here (at the center) would you both report back the same angular speed?

P: The same angular speed, I would say so, yeah.

I: Ok.

P: Assuming not so far away the speed of light makes no difference.

I: So it doesn't matter where you standing or where your sitting where you are?

P: That's right

I: And would both of you (two observers) measure the angular speed in the same way? Would that be correct?

P: Well, probably. Again like depending on what the circumstances are if you can only see the ball and really can't see how its track is-uh it doesn't make a lot of difference because he can still see how much time it takes to complete one cycle for the ball to return back to the same position. And if you can time that you can effectively time the number of revolutions per second, or whatever.

I: And so you would figure out time of revolution for this, for this ball to go around once and then--?

P: I would simple time it. I'd see the ball at a given spot.

I: And that, so that would be a certain number of degrees or radians per unit of time?

P: Yeah, but also simply, it would appear at a certain point to hit its lowest velocity. I know that, if I know it's an ellipse and I know that its going to hit its lowest velocity at the two semimajor axes and when it does that it will appear to kind of slow down and speed up, and slow down again. Its like simple harmonic motion again, so I can simple time it takes for it to go from one extreme to the other.
I: Now, you talked about angular velocity. Is angular velocity a vector?

P: Yeah, well yeah, velocity is always a vector.

I: Then, where does the angular velocity vector point at point C (a point marked on the ellipse)?

P: Uh-damn-u-hm-well actually that's interesting I think with angular velocity you are dealing with vectors that are multiple-cross multiple-I am trying to think this is the part I am a bit shaky on-I think-(pause)-it's not-because a vector has to-it's straight-this is moving in a curved path. I don't know whether you actually employ a right hand rule here or not-u-hm-something tells me from my recollection it's not-it's nothing that I can say intuitively but something is telling me that it actually points downwards-u-hm-in effect along the axis.

I: Points down perpendicular to the plane of the table?

P: Yeah.

I: So where the angular velocity (vector) points and where the velocity points is not the same?

P: No.

I: Is that what your saying?

P: Yeah.

I: And the direction of the velocity is changing but the direction of the angular velocity is not?

P: That's right.

I: Ok, I'm just seeing what that consequence of it changing (the vector changing direction), but the magnitude--

P: The magnitude of the angular velocity changes.

I: As it goes around.

P: Yeah.

(Interviewer asks about definition of some kinematic concepts so far discussed)

I: How would you define speed?

P: Speed is merely the magnitude of velocity, independent of direction.
I: And velocity is?

P: Velocity is a vector quantity that describes the motion of an object in terms of its speed and direction—truly a textbook definition.

I: And angular speed?

P: Angular speed, uhm—would simply be the rate at which an object completes a number of cycles, like an object in—when something is in angular motion it is in rotation or revolving around something. It is the rate, the amount of arc it can complete in a certain amount of time.

I: Angular velocity?

P: Uhm, would be a measure again of its motion in terms of both angular speed and in terms of its direction.

I: What is the definition of acceleration?

P: It is the rate of change of velocity.

I: Angular acceleration?

P: The rate of change of angular velocity.

I: Ok. Just the rate of change. So you just take the difference. So if I want to know the acceleration, acceleration is the change in velocity?

P: Oh, linear velocity.

I: Rate of change?

P: The rate of change of linear velocity.

I: Or velocity?

P: Yeah, and angular acceleration is the rate of change of angular velocity.

I: I just want to know how you think about—what comes to your mind when you say angular.

P: Well, angular I simply mean curved motion. Curved motion in which you have a vector that pulls it in towards the center of some curve.

I: Ok, good.

(END of INTERVIEW).
APPENDIX II

DIAGRAMS OF PATHS USED IN TASKS

The path diagrams are illustrated in the following order.

1. The oval path path...........p 155
2. The semicircular paths.......p 156
3. The elliptical path...........p 157
4. The irregular path...........p 158
OVAL PATH
SEMI-CIRCULAR PATHS
APPENDIX III
THE PHYSICS OF THE TASK SETTINGS

The following is a description of the responses a physicist might give to the focus questions (as presented in Chapter Three) of each task. The responses given are descriptive avoiding where possible the use of formulae.

The Oval Path Task

Question: Does the train have an angular speed between B and D and other parts of the track? Why?

Response: From the position of the observer (X in the diagram of Figure 4) the train does not have an angular speed along the straight section of track. Because, from the viewpoint of the observer the angle the train makes with respect to the observer is not changing with respect to time. However, when the train moves along the other straight section of track CD the train will have an angular speed because, the angle with respect to the observer is changing with respect to time. On both curved sections of track the train will have an angular speed, as seen from the observer’s position since the angle the train makes with respect to the observer is changing with respect to time.

Question: Does the train have an angular acceleration? If so where and why?

Response: Along the straight section of track, AB the angular speed is not changing with respect to time, it has a constant value of zero as measured by the observer, hence the angular acceleration is zero since the angular speed is a constant. At all other places the train will have an angular acceleration which is not zero, because the angular speed is continually changing.
**The Semi-circular Path Task**

Question: *Do the two balls ever have the same angular speed? If so where and why?*

Response: Yes, the two balls appear to have approximately the same angular speed somewhere between C and G. Because, for a short moment in time they seem to sweep out the same angle in the same time from my position as observer (X in diagram Figure 5).

Question: *Does this ball (pointing to one of the balls) have an angular acceleration? Why?*

Response: Yes, the ball does have an angular acceleration. The ball appears to be steadily increasing its angular speed as it rolls faster and faster down the inclined plane. At first the angle swept out as measured by me in say a second is very small, but as the ball gets closer towards the end of the track the angle swept out in a second is larger than before. Thus the angular speed is increasing, therefore the ball has an angular acceleration.

(Note: other more formal arguments are possible)

**The Elliptical Path Task**

Question: *Does the ball have an angular speed? Why?*

Response: Yes, the ball does have an angular speed. Because the ball from my position as observer either inside or outside the elliptical path (as indicated by the X’s in Figure 6) is changing its angle with respect to me as it moves around the elliptical path.

Question: *Is the angular speed of the object a constant?*

Response: No, the angular speed is not a constant. From the observer at the center of the elliptical path the point A is closer than the point E. In a very short time interval the ball will move through a greater angle as seen by me when it passes point A compared to point E in the same time interval.

---

1. "*Is the angular speed a constant?*" is meant to be interpreted as asking: "Does the object have the same angular speed no matter where you measure the angular speed?"
Question: *If the angular speed is not a constant then where is the angular speed a maximum and where is it a minimum as seen by someone standing at the:*

i) centre of the ellipse?

ii) outside, near the edge of the ellipse?

Response: The angular speed will be maximum near points A and A' because these points are closest to the centre of the ellipse. Since the ball is travelling at a constant speed then in a small time period the ball will as seen by an observer at the centre of the ellipse sweep out a greater angle in a given time period than when the ball is at any other point(s).

For the observer outside, near the edge of the ellipse (as indicated by the X in the diagram). Since the ball will travel from E to A', and from E' to A in the same time and since the angle EXA' is greater than the angle E'XA, then the angular speed will be greater (and will reach a maximum) between EA' than between E'A as seen by the observer.

The Irregular Task

Question: *Does the object moving in this path have an angular speed? Why?*

Response: Yes, the object has an angular speed. Because from the position of the observer (X in Figure 7) the ball sweeps out an angle with respect to time.

Question: *If yes, then does the object have a constant angular speed? If not, where is the maximum and minimum angular speed? Explain.*

Response: The object does not have a constant angular speed, because the angle swept out per unit time is not the same. Since the object has a constant speed, in a small time interval the object will move through a greater angle when it is close to points A or G as compared to points B or F. It also would appear without doing actual measurements that at points A or G the object will have a maximum angular speed, and a minimum angular speed near points B or F.
The Formulae

For the reader's reference some of formulae that pertain to circular motion are given below.

**Angular Speed, \( \omega \)**

\[
\omega = \lim_{\Delta t \to 0} \frac{|\Delta \theta/\Delta t|}{|d\theta/dt|}^2
\]

In uniform circular motion, i.e. motion around a circle at a constant speed, then:

\[v = v_{\text{tangential}} = \omega r \quad v_{\text{radial}} = 0\]

**Angular Acceleration, \( \alpha \)**

\[
\alpha = \lim_{\Delta t \to 0} \frac{|\Delta \omega/\Delta t|}{|d\omega/dt|}^3
\]

In uniform circular motion:

\[a_{\text{tangential}} = 0 \quad a_{\text{radial}} = \omega^2 r = v^2/r\]

\(a_{\text{radial}} \) is the centripetal acceleration

If the object is increasing its speed at a constant rate around a circle, (and because it is a circle) the angular speed will be increasing at a constant rate, i.e. \( \alpha = \) constant, then:

\[a_{\text{tangential}} = \alpha r \quad a_{\text{radial}} = \omega^2 r \quad \text{where } \omega = \omega(t)\]

(The previous example of uniform circular motion is a special case, where \( \alpha = 0 \))

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2. For planar motion (as described in this study), \( |d\theta/dt| = d\theta/dt \).

3. For planar motion (as described in this study), \( |d\omega/dt| = d\omega/dt \).