CONCEPTIONS OF TEACHING MATHEMATICS:

THE IDEAS OF SIX PRE-SERVICE SECONDARY MATHEMATICS TEACHERS

by

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Abstract

"I am a math teacher." This simple statement evokes a plethora of images. Elucidating pre-service teachers' personal meanings for this statement adds to the knowledge base for teacher education. Knowing their own beliefs and ideas about teaching also provides teacher candidates a basis from which to consider alternative conceptions.

In this investigation, six pre-service teachers of secondary mathematics were interviewed about their ideas of teaching mathematics at the conclusion of their extended practicum. All had bachelor's degrees with a mathematics specialty; three had pursued graduate study in pure or applied mathematics. Results show all six teacher candidates had different conceptions of teaching mathematics. Some emphasized the content of mathematics; others emphasized mathematical processes. Some gave priority in their teaching perspective to student development and cognitive skills. Those who had studied graduate level mathematics expressed a desire to make math interesting and stimulating for students. Results support the constructivist ideas that conceptions are personally formed and past experiences are influential in shaping them.

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Chapter One: Introduction

Pre-service teachers have already constructed their own ideas of what it means to teach when they enter a teacher education program. Lortie (1975) first used the term "apprenticeship of observation" [p 61] to describe how those who choose this occupation have actually spent 12 to 16 years or more watching practitioners of the profession they have decided to enter. He points out that this is not a true apprenticeship, however, in that as a student, they have only seen the show from one narrow vantage point, rather than having viewed the entire kaleidoscope which comprises a teacher's full responsibilities. He goes on to claim that the vision of teaching formed during this period, skewed though it is, will very likely persist for that individual.

Stability of Conceptions

These initial conceptions of teaching have been found to be relatively stable for education students throughout their pre-professional program (Brown & Borko, 1992; Goodman, 1988; Borko, Lalik & Tomchin, 1987; Tabichnick and Zeichner, 1984). In his work with pre-service teachers, Goodman (1988) termed these "intuitive screens":

> "...their pre-professional images formed an 'intuitive screen' through which they interpreted their professional education....[In] the [teacher education] program, these intuitive screens gave them an orientation point from which they made sense out of the activities and ideas presented to them....When exposed to new ideas or experiences, students tended to act first on an intuitive rather than an intellectual level. No matter how logical an idea seemed, if it directly contradicted a student's intuitive screen, it was usually rejected....Most students tended to be influenced by those people or experiences that legitimated their existing 'intuitive screen.'" [pp 130-131]

In other words, Goodman found the teacher education program solidified and/or clarified these pre-professional images rather than introduce new images of teaching. He also found that those who did experience conceptual change underwent a good deal of unrest as they were giving up their incipient ideas.

Influence of Conceptions

For mathematics teachers, their ideas of teaching seem to be strongly influenced by what they perceive as the essence of the subject (Thompson, 1984). These ideas in turn influence their actions in the classroom and flavour the messages about mathematics they give their students (Dossey, 1992; Thompson, 1992). From the opposite side of the desk, students who learn mathematics from teachers with different beliefs about its nature and different purposes for teaching it may have very different classroom experiences (Brophy, 1991). This highlights the significance of investigating teachers' ideas about mathematics and its teaching; namely, to acquire knowledge of the beliefs that potentially drive their actions in the classroom and to gain insight into their goals and purposes for teaching.

Teacher Education Research

Borko, Lalik and Tomchin (1987) observed that teacher education programs were largely informed by research on experienced teachers rather than teacher candidates. They "found only a few studies that addressed student teachers' conceptions of teaching" [p 78] and identified the need for more research in exploring this population's

conceptions of teaching and relating these to teaching behaviour. Further, Lampert (1988) stated that there existed only a "small amount of research that is specific to teacher education in mathematics" [p 161] and most of that was concerned with observable behaviours rather than teacher thinking. For example, Valli (Valli & Agostinelli, 1993) followed one individual through his teaching experiences before, during and after his teacher education program and used such criteria as amount and type of questioning, clarity of boardwork and amount of lesson and unit planning to evaluate his progress.

Since those statements were made, more studies have explored pre-service teachers' conceptions. However, most of these have dealt with elementary candidates and have centred around specific content. The studies range from exposing fallacies in, and thus lack of, mathematical understanding on certain topics such as division (e.g. Tirosh and Graeber, 1990; Ball, 1990b) to uncovering beliefs that knowing math is doing math according to set rules and canons and producing correct answers (e.g. Civil, 1990). Intervention studies on elementary candidates have also been carried out. For example, Wilcox, Schram, Lappan, & Lanier (1991) instituted a "learning community" in a mathematics content course in an attempt to initiate conceptual change in their students' ideas of the nature of mathematics.

Research on secondary pre-service teachers is even more limited (Ball & McDiarmid, 1990). Little is known about their views of teaching mathematics though some content issues have been identified. For example, Ball's (1990b) study investigated secondary as well as elementary prospective teachers and found a lack of conceptual

understanding of division in both groups. Zaslavsky and Peled (1996) found conceptual difficulties with the commutative and associative properties of binary operations in the understandings of 20 prospective teachers.

Research Questions

Those entering the teaching profession cannot be considered a cohort with homogeneous backgrounds, particularly in the present societal milieu of constant change and uncertain economics. Students in a pre-professional program vary in age, come from other professions, and have further academic qualifications. It is likely they will have varying conceptions of teaching mathematics.

To help fill the gap in what is known about the kinds of ideas this population holds, the primary research question in this study is: what are the individual conceptions of teaching mathematics held by a group of pre-service teachers of secondary mathematics?

Some of the subjects in this study were pursuing, or had completed, graduate work in mathematics. This gave rise to a subquestion: for those with graduate studies in mathematics, are their conceptions qualitatively different from those who hold only a baccalaureate?

This study set out to explore and consequently describe the conceptions of teaching mathematics of a group of pre-service teachers of secondary mathematics. While the results are not intended to convey the entire landscape of possible conceptions, they do provide a comprehensive picture of these prospective teachers' ideas, thoughts,

attitudes, and goals related to the teaching of mathematics. The primary intent of this research is to add to the knowledge base that informs mathematics teacher education. In addition, the study provided the teacher candidates with what Brown, Cooney and Jones (1990) have described as "encounters in which teachers are not taught methodologies but are encouraged to find out what they believe to be important." [p 653]

Chapter 2: The Literature

Constructivism in Mathematics Education

The influence of constructivism in mathematics education research which began approximately 30 years ago has increased steadily over the past decade (Steffe & Kieren, 1994). As a relatively new perspective, constructivism was the focus of intense debate with regard to its relevance and usefulness in informing mathematics education research compared to the traditional process-product paradigm (see Brophy, 1986a; Confrey, 1986; Brophy, 1986b). Now, it has become prominent to the extent that a constructivist perspective is being used as the basis for a tentative theory of intellectual development (Confrey, 1994), as grounds for an emerging theory of learning (Thompson, 1991) and as a foundation for emerging models of teaching and teacher education (see Simon, 1995b; Steffe & D'Ambrozio, 1995; Simon, 1995a). It is even being construed as "politically correct" (Cobb, 1994a)!

The basic premise of constructivism is that individuals create their own knowledge rather than receiving it from an external source. What form this knowledge takes and how it is created have been issues of debate among constructivist theorists.

Von Glasersfeld (1989, 1990) highlighted the assertion that the knowledge created by an individual was unique and not a replication of an external, objective reality when he formulated "radical constructivism" (von Glasersfeld, 1990, p 28). He argued that knowledge was personal because it was formed as a result of an individual's

experiences and how that person made sense of those experiences. Because sense-making by its nature is individual and internal, he also argued there can be no precise way of knowing someone else's knowledge. We can only have our own understandings of what we think someone else knows. Communication among individuals then becomes interpretative acts using shared meanings rather than information transfer (Confrey, 1994). This counters the realist view which purports that there is knowledge which exists independently of human experience and that learning is acquiring a personal understanding or "knowing" of that external knowledge.

Another debate focusses on how the knowledge is constructed. One perspective views it as the activity of the individual where knowledge construction occurs through comparing and contrasting new experiences to ideas already held. Konold and Johnson (1991) call this psychological constructivism. In the language of cognitive psychology, ideas or constructs are held in mental structures called "schema." Recognition occurs when there are many similar characteristics between an individual's schema and a new encounter. However, when there are sufficient differences that the new situation does not "fit" with existing schema, then there is re-organization or creation of new schema to accomodate the new circumstances. This mental development to account for new circumstances constitutes learning. It occurs consciously or subconsciously, and is continual. Learning or knowledge construction can be described as developing "increasingly viable descriptions of the phenomena that we come into contact with" (Erickson, 1987, p 9).

The other stance follows a sociocultural perspective. Those proponents claim knowledge formation for the individual occurs as a result of participating in social interactions. Knowledge has meaning only in the context of these interactions; it is socially constructed through shared understandings. Learning, or personal knowledge construction, is the process of the individual internalizing what was observed on the external plane (e.g. Cobb, Wood, & Yackel, 1991) and making personal sense of it. Through this internalization, the person "learns" and becomes enculturated into the social group.

Cobb (1994b) has suggested these two perspectives are complementary, one becoming the background to the other depending on the specific problems and issues being researched. This approach allows for creation of knowledge by the individual through social interactions.

Despite debate over its philosophical aspects, adherents to constructivism have changed the course of research in mathematics education. The focus has shifted from investigations whose results are based on measurable quantities, so-called "scientific studies," to "field-based studies" (Romberg and Carpenter, 1986) where the thoughts and ideas of the principals are examined, from those with an "analytic perspective" to those with a "humanist perspective" (Brown, Cooney and Jones, 1990), from "a process-product paradigm" to "a focus on teachers' thinking and decision-making processes" (Thompson, 1992). By allowing for alternative understandings, constructivism has opened the way for investigation into multiple conceptions.

How an individual conceives of teaching mathematics is the focus of this study. That import is given to personal understandings on

teaching is a consequence of my belief in a constructivist epistemology. There is no ontological reality to teaching. There is no one right way to teach, nor to conceive of teaching. What an individual believes to be teaching mathematics must have been constructed from his/her experiences and beliefs.

The Role of Existing Beliefs

The beliefs of teachers may not be integrated and may not be articulated (Thompson, 1984). Nevertheless, knowing what they are is important for researchers as well as for the teachers themselves. Teachers' thoughts substantially influence their actions in the classroom (Clark and Peterson, 1986). The researcher gains insight into what the teacher is thinking and is able to apply this context to results of other studies.

> "...we can make sense of [research] findings only in relation to the psychological context in which the teacher plans and decides....[where] the psychological context is thought to be composed of a mixture of only partially articulated theories, beliefs and values about his or her role and about the dynamics of teaching and learning." [Clark and Peterson, 1986, pp 286-287]

Not only do teachers' beliefs affect their actions in the classroom, they also influence their students' beliefs. In one study of ten fifth-grade teachers in four schools, Ford (1994) found their beliefs about problem solving in mathematics was paralleled by their students' beliefs; both groups described problem solving as applying computational skills applied to real-life situations. In another study, Brown (1992) also found students' images of mathematics linked strongly with their teachers' images of mathematics. The images were varied; mathematics was perceived as the content in a set of books, as rules and

ideas used to solve problems, as easy after being sorted out, and as enjoyable.

The teacher also benefits from this type of research activity through increased self-awareness of personal beliefs about teaching. Nickson (1988) has suggested that teachers' beliefs may be counterproductive to curricular change and development but, on the other hand,

> "they may come to appreciate the effect these [beliefs] have on their performance as teachers of mathematics and, if necessary, call them into question." [p 246]

Knowing their own beliefs regarding students, school, and their images of themselves as teachers is equally important for teacher candidates. This metacognition provides a foundation to compare alternative teaching perspectives which arise during their preparation program (Kagan, 1992). Raymond and Santos (1995) state even more emphatically, "Confronting and challenging preservice teachers' beliefs should be an integral part of teacher education" [p 68].

Research with pre-service teachers has yielded evidence of different interpretations for the same concept. Using interviews and observations, Goodman (1988) studied 12 pre-service elementary teachers and found two broad practical philosophies of teaching: "Teaching as a Problem of Control" and "Teaching as the Facilitation of Children's Growth" (p 123). Cooperation, authority, and autonomy were three concepts which emerged as "guiding images" (p 124) for the first philosophy, teaching as a problem of control. Interpretations of these concepts differed amongst the teacher candidates even though they used the same terms. For example, to nine of his subjects, cooperation meant

pupils followed the rules laid out by the teacher. To the others, it meant meant working together harmoniously. In both instances, the desired result was a smoothly operating classroom. However, the contrast in philosophy is obvious.

Similarly, there were two distinct interpretations of authority: institutional authority and personal authority. The first was granted by status: I am the teacher. The second was granted by the pupils: I am your friend and we respect each other. Goodman (1988) concludes,

> "While on the surface these students [teachers] expressed a common philosophy, they uniquely interpreted the images within each perspective, and thus their practical philosophy of teaching differed significantly." [p 129]

Feiman-Nemser and Buchmann (1989) take a stronger stand on the role existing beliefs play in the learning of pre-service teachers. They give as an example two elementary candidates' perspectives of the issue of equal access to knowledge where "both candidates relied on personal experience that was limited and subject to bias" [p 372]. In Janice's case, because her perspectives went unchallenged, her notions that ethnic minorities were not capable of the same academic achievement as non-ethnic students were actually reinforced by her interpretation of her course readings. On the other hand, Sarah, who came from a disadvantaged background, was frustrated at not being able to provide more for her pupils.

As Kagan (1992) reported in a review of 27 studies dealing with the professional growth of elementary and secondary pre-service teachers, the ideas of teaching they had formulated were strong. Their personal beliefs of what constituted good teaching and their particular images of themselves as a teacher had been derived from "personal

biographies," i.e. from the images of exemplary teachers they had remembered and their past experiences as students. In concert with findings of earlier studies, most notably, Tabachnick & Zeichner (1984), it was reported these beliefs and images often remained unchanged during the program. In the one study (Florio-Ruane & Lesmire, 1990, cited in Kagan, 1992) where conceptual change did occur, it was attributed to the unique nature of that particular methods course.

The stability of the ideas of teacher candidates underscores the importance of investigating their conceptions. For teacher education programs to make a difference and for personal professional growth, teacher candidates need to be made aware of their ideas. This knowledge can then provide a basis for reflection from which change can be effected.

The Nature of Mathematics

Because how mathematics is perceived may strongly affect instructional practice (Dossey, 1992; Ernest, 1991), this section looks at various characterizations of mathematics.

Following the historical development of the discipline, Davis and Hersh (1981) have delineated three broad philosophies regarding the nature of mathematics: Platonist, formalist and fallibilist.

The Platonist philosophy holds mathematics to be an ideal, a reality with immutable truths set apart from and above human existence. Mathematical principles are discovered rather than invented. They exist independently of human knowledge and mathematicians work to locate and give voice to these truths.

The formalist philosophy states mathematics is comprised of axioms, definitions and theorems. These are manifested via symbols which have been created and formulas and rules which have been invented. Even though these may be used to give accurate descriptions and predictions of real-world phenomena, these applications are not the primary purpose for the creation of mathematics. The formulas and mathematical models in themselves are merely symbols strung together whose meaning is derived from sets of rules (axioms) devised by and agreed upon by that cadre of professionals known as mathematicians. In this sense, mathematics is constructed. It is a field of logical deduction in which conclusions can withstand rigorous proof.

The third philosophical school on the nature of mathematics is the fallibilist. This school holds the belief that mathematics is mutable, a field in constant flux as it responds to new questions and challenges. Mathematics is not rigid, its rules and definitions not "set in stone." Its precepts can and do change through debate and discussion by mathematicians.

Though these philosophical perspectives have been derived from the work of mathematicians, they are useful in describing the thoughts of teachers about mathematics. A teacher's view of mathematics could lead to the use of instructional techniques and strategies in the classroom consistent with that view. For example, if a teacher believes mathematics to be a fixed, non-changing entity to be revealed to students, "presenting," which carries a connotation of transmitting knowledge to learners, could be a favored teaching technique. By contrast, a teacher who believes mathematics to be a dynamic and

changing body of knowledge could emphasize that aspect. Someone who believes mathematics to be derived through a consensus of opinions could value the contributions of individual students in classroom discussions. Another teacher with a formalist philosophy may present content in a structured format "calling on set theoretical language and conceptions" (Hersh, 1986 cited in Dossey, 1992, p 42).

Research on Conceptions of Teaching Mathematics

Research specific to conceptions of teaching mathematics held by pre-service teachers of secondary mathematics has been difficult to locate.

Wilson (1994) investigated the evolving beliefs of one prospective secondary teacher, Molly, in the context of a course which integrated pedagogy and mathematical content. He examined how her understanding of functions changed and the relationship between this understanding and her views of mathematics and teaching mathematics. He found that at the beginning of the coursework, Molly had a narrow view of functions and this view was consistent with her general view of mathematics - that it was a collection of procedures to be applied to obtain correct answers. Molly's conceptual understanding broadened and deepened during the course; toward the end she was able to identify real-world applications of functions and use various forms of functions under different circumstances.

How did this increased understanding affect her view of teaching mathematics? Molly believed her view had changed significantly. However, Wilson (1994) claimed she had simply found new ways to reduce

the monotony of classroom lectures. She had learned new instructional techniques, not formulated a changed perspective to offer "a more vibrant and meaningful image of mathematics. Thus, although [she began] to see alternative ways of teaching mathematics, her views of mathematics and mathematics teaching were still relatively narrow." [p 367]

Two other studies which figure prominently in the literature - Brown (1985) and Owens (1987) - are both unpublished doctoral dissertations. This has meant gleaning descriptions of the studies through secondary sources.

Brown (1985) investigated the changes in one pre-service teacher's conception of mathematics teaching as he became socialized into the teaching profession. Conceptions of mathematics, beliefs about the appropriate goals and tasks for a math classroom, and beliefs about the "relative responsibilities of teacher and students concerning motivation, discipline, and evaluation" (Brown & Borko, 1992, p 225) comprised the analytical framework. What she found was that pupils' conceptions of mathematics teaching had a strong influence in changing his ideas of teaching. However, "he did not change some of his most basic general conceptions of mathematics and teaching" (Brown & Borko, 1992, p 225), pointing once again to the stability of conceptions.

Owens (1987) sought to describe the constructs of four pre-service teachers regarding mathematics and teaching mathematics. He found their perspectives of mathematics were largely influenced by pre-college experiences. Additionally, the teacher candidates found the mathematical perspective of college-level math courses incompatible with

their own. These findings confirm Ball and McDiarmid's (1990) claim that pre-college education figures prominently in content understanding for prospective secondary teachers.

A General Model for Describing Conceptions of Teaching

Pratt (1992) has offered a framework for describing conceptions of teaching. The elements used to describe the conceptions and hence establish their distinctive differences are: teacher, learners, content, context, ideal. The first three are germane to any teaching situation. The fourth, context, defines the organizational setting (e.g. type of school, type of program, type of class) and includes any external factors that could influence the teaching and/or learning, such as impending examinations. "Ideal" was used to describe the "purposes of adult education" (p.205) and can be construed as the central or overriding goal, implicit or explicit, for the teacher. Pratt's research was a study in adult education and the framework was derived from interviews of over 250 teachers in 5 countries and across a variety of contexts such as post-secondary institutions, government and military classes, religious education, and training programs in business and industry.

Pratt (1992) delineated five different conceptions of teaching: Engineering - Delivering Content; Apprenticeship - Modeling Ways of Being; Developmental - Cultivating the Intellect; Nurturing - Facilitating Personal Agency; and Social Reform - Seeking a Better Society. The distinctions among the conceptions arise from differing emphases in the relationships between the elements.

The Engineering conception has as its distinctive feature delivering content. It is teacher-centered and information transmission is a primary focus. Learners are viewed as non-problematic. Energies are concentrated on improving techniques for content delivery. Content is often broken down into discrete objectives and learning measured by completion of tasks, skills gained and/or competencies acquired.

The Apprenticeship conception, like the Engineering conception, has the teacher and content as dominant elements. However, the teacher in this case is not only the content expert, he is also seen to be an exemplar of the values of the content. The idea of modelling, or being a master craftsman, conveys this implication that the whole person embodies the content as much as he has cognition of it. Hence, "knowledge [is] passed on through role modelling" (p. 212) and learning constitutes acquiring both information and the values inherent within that corpus of knowledge. Examples of this conception were most often found in such areas as vocational apprenticeships and medical internships.

The Developmental conception is characterized by the teacher's desire to further the cognitive development of her learners. The focus of exercises and the role of curriculum is to stimulate and enhance learners' ways of thinking since the central belief is that individuals possess the capacity to develop increasingly abstract and complex forms of thought. Inquiry-oriented exercises, whether to arouse curiosity or produce disequilibrium, are a preferred mode of instruction.

The personal worth of the individual student is paramount in the Nurturing conception. A strong bond between the teacher and learner is

evident. The teacher exhibits a high degree of concern for the personal welfare of the student. The student's self-knowledge and development, indicated by an increasingly positive self-concept and internal locus of control is considered more important than content knowledge. Content knowledge is seen as being a vehicle through which the learner's self-identity can be developed.

In the conception labelled Social Reform, a specific ideology drives the activities of the teacher. This ideology may be religious, political or derived from a different, but equally strong, set of convictions and beliefs. It is clearly articulated and, being at centre stage for the teacher, becomes the guiding principles by which he conducts the class and relates to the learners. The teacher has a mission to enlighten the learner and solicit support for the ideology, thereby creating a "better world." Examples are some forms of environmental and feminist education.

The Framework for This Study

Thompson (1992) described a conception of teaching mathematics as including but not limited to "desirable goals of the mathematics program, [a teacher's] own role of teaching, the students' role, appropriate teaching activities, desirable instructional approaches and emphases, legitimate mathematical procedures and acceptable outcomes of instruction" [p 135]. Undergirding these is a philosophy of teaching mathematics.

For the scope of this study, my framework to describe the conceptions of pre-service mathematics teachers consisted of the

conception of the nature of mathematics and the conception of teaching mathematics where the latter encompasses an underlying philosophy, a goal for teaching mathematics, a perspective on the role of the teacher and a conception of students. In developing the framework, I drew heavily on the work of Pratt (1992). The appeal of this model lies in its usefulness in bringing to the fore the elemental relationships between the teacher and student, the teacher and content, and the student and content.

The tentative frameworks offered by Ball (1988) and Ernest (1989, 1991) to study conceptions of mathematics teachers also informed my work. Ball (1988) outlined 3 strands: knowledge of and about mathematics, ideas of math teaching and learning, feelings about oneself in relation to mathematics. The knowledge component was comprised of content knowledge and pedagogical content knowledge, i.e. "the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p 6). An example of an idea of math teaching would be "teaching as telling," prominent in her study of elementary teacher candidates. In addition, through her work with this cohort, Ball found apprehension of mathematics significant and hence included personal feelings with respect to content in her framework.

Ernest (1989, 1991) used similar components in his framework. His broad categories were knowledge, beliefs and attitudes of the math teacher. The knowledge category was subdivided into knowledge of: mathematics, other subject matter, teaching mathematics (math pedagogy and math curriculum), classroom organization and management for math teaching, the context of teaching mathematics (school and students), and

education (educational psychology, educational aims, mathematics education). The category of beliefs was subdivided into beliefs of: conception of the nature of mathematics, models of teaching and learning mathematics, and principles of education. Finally, the category of attitudes was subdivided into an attitude toward mathematics and an attitude toward teaching mathematics.

By inquiring into perspectives of content as well as instructional perspectives, this study goes beyond looking at general views of teaching to investigating particular views of teaching mathematics.

Chapter 3: Methodology

Noddings (1990) has dubbed constructivism a "post-epistemological position" which yields an important methodological perspective. The constructivist paradigm necessitates a qualitative research methodology (Denzin & Lincoln, 1994) for it is only by allowing the subject to represent his own views using his own words and not those imposed by the researcher that insight into his understandings of a concept can be gained.

Since the aim of this study was to allow individuals to express their personal perspectives on the teaching of mathematics, I selected the general interview guide technique (Patton, 1990). This semi-structured approach ensured that the same topics delineated in the framework were covered with each person yet it gave room for flexibility to expand upon certain topics or explore others. It also allowed for interaction for probing and clarification.

Development of the Protocol

An initial set of questions and potential probes on mathematics as a discipline, mathematics as a school subject, message about mathematics, the role of the teacher, view of the student, and underlying goals of teaching mathematics were composed.

Contrast can be effective for illuminating and clarifying ideas, so that was incorporated into the protocol. An example is "Has your view of teaching changed over the year?" Noting that mathematics and English are often juxtaposed (Ball & McDiarmid, 1990) the question "How

would you compare teaching mathematics to teaching other subjects, like English?" was also included.

Whether underlying philosophies are explicit or implicit, Goodman (1988) points out "the roots of their perspectives [are] visual" [sic] [p 124]. With this in mind, I selected "Views of teaching and learning" (Appendix A) as representing some common notions of teaching to use as a potential probe during the interviews.

The Pilot Studies

Two pilot projects were carried out prior to the main study. Their general purpose was to refine my interview technique as well as validate the interview questions.

The first pilot study.

A preliminary pilot study was conducted with two volunteer students from the elementary B.Ed. program at the University of British Columbia. Both students were interviewed in my office at the university for 35-45 minutes. Each interview was audio-taped and transcribed.

These interviews allowed me to practice my questioning and my listening. While transcribing the tapes, I noted the comparative lengths of questions and answers, and any distracting comments I made. This prepared me to listen more closely and make fewer inconsequential comments for the next set of interviews. I also reviewed the data cursorily which gave me preliminary experience in looking for categories of ideas.

The context of these discussions was teaching mathematics to elementary aged children. While these were elementary teacher candidates and hence represented a different group from those in the

main study, the data showed an interesting contrast in ideas of teaching mathematics. One saw mathematics as being an inextricable part of other content areas. She described using interdisciplinary projects to teach mathematics to her young pupils through integration with other subjects. The other person believed mathematics could only be taught in isolation from other subjects using a standard drill and practice format.

The second pilot study.

For the second pilot investigation, four volunteers from the B.Ed. secondary math program were chosen. These interviews, conducted in different locations, lasted 35-45 minutes. They were audio-taped and transcribed.

With this set of interviews, I learned to probe and clarify more effectively. For example, during the interview I listened for the words "it" or "that" used as the subject of a sentence and sought to clarify the concept being referred to. Through repetition and increasing familiarity with the protocol, I relied less on the written questions before me and focussed more on what the interviewee was saying. This resulted in a more conversational interview style.

I developed a graphic template of the questions I would use in the main study (Appendix B). Its non-linear format was useful for making notes during the interview. I also decided to begin all interviews of the main study with "What does it mean to you to teach math?" During the second pilot, this question generated the most illustrative answers from which I could follow-up with further questioning.

The results of this pilot study showed substantial differences in ideas of teaching mathematics and suggested that there also would be differences in the ideas of the pre-service teachers in the main study.

The Main Study

The program.

The Bachelor of Education program at the University of British Columbia (UBC) for secondary mathematics is a twelve-month program consisting of course work and two practice teaching experiences. The first practicum occurs during the fall semester for two weeks and the second occurs during the spring semester for nineteen weeks. Students are assigned to the same school for both practica.

The sample.

Seven students from the secondary math B.Ed. program at UBC were selected. Three were male, four were female. All had a four-year B.Sc. specializing in mathematics, a requirement for entrance. Of these seven, one had a Masters degree in statistics and two were working towards Masters degrees in pure mathematics. The fourth had a combined English and mathematics specialty and the fifth had a combined Spanish and mathematics specialty. The final two had a major in mathematics only.

The seven who comprised the sample was the group I supervised as faculty advisor. For that class of 41 math specialists, I was one of three faculty advisors. To ease travelling during the practica, I was assigned those whose teaching schools clustered about a main highway; so, the decision was geographically based. Six were assigned to secondary schools (Grades 8-12); one was assigned to a junior secondary

school (Grades 7-9). In selecting these prospective teachers for the study I considered them "typical" cases (Miles & Huberman, 1994) who were convenient to access.

As faculty advisor, it was my responsibility to act as liaison between them and their receiving institution, to ensure a smooth working relationship between them and their school advisor, to lend advice and support for this phase of their professional training, and to evaluate their introductory teaching adventures formatively and summatively.

This role provided advantages and disadvantages to the research. The obvious disadvantage was the power connotation inherent through my role as faculty advisor. Rather than speaking freely, the teacher candidates could have felt intimidated to give "right" answers, i.e. answers I might be looking for. To mitigate this effect, I had informed them of my interest in using them for this study when we first met and I had emphasized that I too was a student, as they were. I told them what I was learning was how to interview and how to analyze interview data, and the topic that interested me was ideas about teaching mathematics. Through this "equal footing" explanation, I hoped to establish rapport with the group. I also informed them that they could "opt out" of the interview with me at any time with no consequence to their practicum evaluation. (The fact that I am still in touch with most of the individuals on a personal basis attests, in my view, to the mutual comfort level we attained.)

I conducted classroom visits with each teacher candidate about every two weeks. We always debriefed after the classroom observations, and the summaries of our discussions were written in the monthly

formative evaluations. When the final, summative evaluations were composed at the conclusion of the practicum, they contained no surprises to the teacher candidates as they were simply summaries of the conversations we had been having. In the Pass/Fail grading scheme for the practicum experience, all received a "Pass."

By working with them during both practica, I was able to establish a comfortable relationship with each person. Our ease with each other was an advantage to the research. Appendix D, an excerpt from one of the transcripts, provides a sense of the flavour of the interviews. Our familiarity meant that during the interview, either of us could evoke specific incidences which the other person also knew of to highlight or provide an example of the topic we were pursuing.

The data collection.

Each person was interviewed once. The interviews were conducted during the final week of the nineteen-week practicum and the following week, after each person had been given his/her final evaluation.

The first two interviews were conducted in the practicum schools. One occurred at the end of the school day and the other during a spare period. Although both were successful interviews, the situations seemed to constrain the subjects' responses in that they focussed on the immediacy of the previous class which tended to overshadow the broader thoughts and feelings regarding teaching in general. In one case, we became increasingly distracted toward the end of the interview by anticipating the bell ringing to signal the beginning of the next period.

Thereafter, interviews were conducted in cafes. These venues were more conducive to bringing out the pensiveness fruitful in investigating conceptions. Problems with background noise were minor and did not hamper the interview process.

Each interview began with the question "What does it mean to you to teach math?" From the response, the interview travelled its own path to cover areas pertinent to the investigation. Subjects were probed for examples, clarification, and further thoughts on the topic. Those who did not have a clear personal conception of teaching in the form of an image, metaphor or motto were shown the diagrams "Views of teaching and learning" (Appendix A).

The interviews were audio-taped. Each lasted from 25 to 40 minutes. I took field notes using the schematic developed from the second pilot study and made additional notes after each interview. Appendix C shows how the schematic was used.

The data processing and analysis.

I transcribed the tape of each interview. Pseudonyms were given to each teacher candidate. One tape was not usable as the subject had inadvertently pulled the microphone out of the recorder. This occurred while he was adjusting his position. The microphone apparently had moved just enough to disconnect but not enough to fall out, so its dislocation was not noticed until several weeks later, during transcription. Since the purpose of the study was to look at snapshots of individuals' conceptions taken at the same point during their year-long program, I decided not to arrange a substitute interview. This

left six cases for analysis. Appendix D provides a sample excerpt from one of the interviews.

The transcripts were analyzed using the technique of pattern coding (Miles and Huberman, 1994). I read each twice. Then codes corresponding to categories dealing with mathematics, its teaching and/or its learning were written beside the appropriate "chunk" (Miles & Huberman, 1994, p 56) in the entire interview. As I generated these, I also wrote memos; these corresponded to code notes and theoretical notes (Strauss and Corbin, 1990). Each coded entry on the transcript was then transferred to a separate index card with all the information necessary to re-locate it in the raw data. Each transcript generated approximately 40 to 60 cards. These cards were then sorted by category. For each person, I wrote a summary of the ideas contained on the cards for each category.

To increase the trustworthiness of the data, I began the process of conducting member checks (Lincoln & Guba, 1985) of these summaries. I was able to locate four of the six subjects and sent each a copy of the first draft of my summary of his/her interview. Each person was asked to read it and tell me whether I had captured his/her thoughts fairly and accurately. Three were returned. Their written comments indicated no changes were necessary. Two suggested including one more point; these were incorporated.

A description of each person's conceptions was written with supporting evidence from the data. These descriptions constitute chapter 4. Analyzing components within each case gave rise to

interesting comparisons across the cases. These comparisons are included in chapter 5.

Chapter 4: Results

In this chapter, I describe each subject's academic background, conception of mathematics, and view of teaching mathematics. Because of the semi-structured approach of the interviews, the descriptions for each case vary slightly, allowing for the different foci of each teacher candidate to come to the fore as they did during the individual interviews.

Yvette

Academic Background

Yvette had a 4-year B.Sc. with a major in mathematics. She is a "traditional" student, i.e. one who went directly into a B.Sc. program after completing high school, and subsequently on to a B.Ed. program.

Conception of Mathematics

Yvette saw mathematics as an inflexible core of content and its problems as having one answer which could be arrived at through a variety of methods. Sciences also fit this category and contrasted with arts subjects.

> "...math is possibly, not as...compared to an arts type course, it's not as flexible in some respects. I mean you can be flexible in the way you approach a problem. Lots of kids might solve problems differently, but, there's usually one answer....But for English, say you're writing an essay, you can always, you can approach things differently, and there's usually not an answer..." [p 9]

> "...I prefer um, the sciences for example, because there seems to be an answer [small laugh], you when you're asked a question, you're finding an answer....I do like the idea that, here's a question, there's an answer. Try and find it." [p 5-6]

Yvette had a positive disposition towards mathematics. She enjoyed it a great deal during high school and attributes that to one particular teacher. She also enjoyed the mathematics courses she took in university, "...but I started to not see the relevance in them any more, so I guess that's where I kind of got lost." [p 5] Her enjoyment of the subject came from the challenge she experienced in solving problems and satisfaction she found in getting the "right answer."

> "But I also liked, math was challenging, yet I was able to succeed at it and I found that that was satisfying for me, to know that this was, um, you're given a problem and it's hard, and I, I was challenged and I WANTED to figure it out....for me, I enjoyed the challenge, of math and um, I felt a satisfaction to get an answer at the end." [p 5]

View of Teaching Mathematics

To Yvette, teaching mathematics meant presenting content using a set of effective presentation skills. These skills needed to accommodate the different sensory modalities that define different learning styles.

> "...[teaching] begins with introducing new concepts to students, um, teaching something or introducing something that's different to them, that they've never experienced before, that maybe they possibly have, but teaching it in a different way." [p 1]

> "I think that you should try and have a variety of, of, um, methods, so, to learn math best. I think that some students understand by being shown how to do something, um, both, so visual, some sort of visual presentation, um, orally, explain, trying to explain things in different ways....I think that trying to have a variety because not all the students are gonna learn the same way..." [p 2]

She believed the student was responsible for his own learning.

> "My role would be to, um, introduce the new material, but then to um, kind of like a, um, a manager or a facilitator, someone who facilitates their learning, so although I may not be responsible for every individual's learning...I can, um, try to spark something in them to have them learn. Um, so my role would be a facilitator." [p 2]

Her teaching perspective carried over to all subjects. She described similarities in teaching mathematics and English.

> "I think that there's a lot of similarities, just in teaching anything because you, um, it's usually you're teaching something that's new to them, and you want them to practice it, have some guided practice maybe, and, um, work on it individually, work on it maybe in groups. I'm, even in English, say English and math, you might teach them something new, or read a new book, but then work on it together and discuss it, just the same way you would discuss a math problem or work on it together....I think that just by what I remember, of being in those kind of [English] classes a lot of things ARE [sic] the same, you listen to the teacher demonstrate something new, you work on it with other students, you discuss it....The same basics." [p 9]

Showing the relevance of a topic to students was important to Yvette. If a topic could be made to seem relevant, then students would be more motivated to learn and some of the discipline problems would disappear. Yvette cited the importance of relevance to understanding in her own math history: "There were some math courses in university that...I started to not see the relevance in them any more, so I guess that's where I kind of got lost." [p 5] She measured herself using this standard. "I think that there are times where I could have more often showed the relevance, or showed, um, um, just been more encouraging perhaps, but I have especially lately tried really hard to, to show when things are useful." [p 4]

Yvette had not developed a motto or metaphor for teaching and thus I showed her the "Views of teaching and learning." She chose #2, the guide and the traveller, as most representative of her conception of teaching.

> "...I think 2's a good example 'cause it's the guide and the traveller and the guide is, um, obviously guiding the traveller [laughter] along and showing them different things....the person standing was the guide and the, the fellow lying down was the traveller. Um, just 'cause it kind of looks like they're in front, they're pointing everything out, but the traveller's not just an innocent bystander, so to speak. The traveller is

> actively involved....and I think that relates to teaching because as a teacher you can guide the students and that's what you're there to do, guide them and show them different things, but they are actively a part of it in this exploring idea...in the classroom, I suppose, you're exploring mathematics." [p 12]

Characteristics of a good math teacher.

Yvette touched upon four qualities she thought were important for a good teacher of mathematics. These were a desire to teach math, the ability to have a good rapport with students, the knowledge to explain concepts, and good presentation skills.

> "I think you have to want to teach math..." [p 6]

> "[A good math teacher is s]omeone who can, I guess, motivate these students to learn, but, um, at the same time it has to be someone, um as part of that, the person has to be able to relate well to the students, have a good rapport, um, so that the students want to listen and are, are actively involved. Um...just things like if you mumble or if you don't explain things carefully, um, that would make for um, a difficult teacher to understand..." [p 7]

> "...You need to know how to explain things, else, um, else I don't know how you could teach it....If you don't know what you're teaching, then when kids come up with tricky questions like they do, you're not gonna be able to explain it, and your, your explanations for things aren't gonna make too much sense to them!" [p 7]

When asked to elaborate on these aspects, she focussed again on what she believes teaching to be all about - clear portrayal of concepts.

> "...if I graduated from high school and I did well in math, and I know, you know, all the math Grade 8 through 12, and, it's possible that I could be a good math teacher if it's just natural instinct to, um, you have the communication skills maybe....[To be] a good teacher. Um, good communication skills, um, someone you can understand, speaks clearly, um, tries hard to have the students learn, tries to demonstrate things in different ways, not doing it on the board but also saying something, doing something actively." [p 8]

She was able to elaborate on what clear presentation meant, e.g. speaking clearly, but was unable to give examples of how a teacher "tries hard to have students learn." This underscores Yvette's strong belief in teaching as a set of good delivery skills.

View of students.

Yvette viewed students as having differences in abilities and shaped her goal of teaching around this. She would help each student to achieve what he/she could achieve.

> "My goal, I guess, is to, that the students will achieve, some sort of um, level higher than where they're at....So, my goal can't be the same for every student I don't think, but, um, different students achieve at a different level." [p 1]

She also saw student differences in disposition towards mathematics. Though problem-solving was a stimulus for her, Yvette acknowledged that not all students enjoyed this type of challenge.

> "...I was challenged and I WANTED to figure it out. Now I'm sure all students aren't like that, but for me, I enjoyed the challenge...there are definitely is some that I can tell think the same way, that they enjoy a challenge and they, they kind of like the idea that, um, this is a, you know, a tricky question, let's try and find the answer." [p 5-6]

During her practicum, she became aware of other factors and people influencing the lives of the teenagers. This increased her sympathy towards the students, but did not affect the way she taught.

> "...you realize that the student has so many outside factors that you don't know about, or you may or may not know about, and that really affects a student's life....I guess it doesn't really change the way I teach. It just changes the way I, I try to understand them...So, but I don't know if my teaching, my teaching style, I don't think, has been affected by that." [p 13]

Conception of learning math.

When asked how math is best learnt, Yvette focussed again on different learning styles. She mentioned the need to accommodate different sensory modalities to help individual students learn, and how student practice of math questions was important.

She claimed "the right mind set" [p 9] was important for success in math. She had difficulty elaborating on this concept:"...it's more

scientific, I don't know! It's more mathematical!" [p 9] Later, she described it as active involvement in the problem.

> "...they get down to work, they're, they're thinking about a problem, they read it, and they, they ask questions that show that they're actively, um, involved in what they're doing, whereas someone else probably wouldn't even ask a question if they're not actively involved in what they're doing. So, a mind set would be an active mind. Someone that's ready to think." [p 10]

It is interesting to note that while Yvette claimed that the teaching of math was similar to the teaching of other subjects, learning math required a "right," mathematical mind set.

Shannon

Academic and Personal Background

Shannon had an Honours B.Sc. (4-year) in mathematics. Before entering the B.Ed. program she tutored privately. She had also worked full-time in a restaurant and subsequently became the manager. After completing her teacher preparation program, Shannon's plans were to return to her home province and find a teaching job or help her husband manage a restaurant.

Conception of Mathematics

Shannon described mathematics as having both theoretical and practical components. She clearly favoured the practical aspect.

> "In university, I hated theory....I like applied math. I don't like theoretical math." [p 5]

To distinguish between the two, Shannon characterized them:

> "Theory to me is memorization. When I had an exam in my, in my courses where it was theory, I had to memorize all the proofs, and to me when you apply, you learn how to USE [sic] those proofs....my fourth year course in math, I really enjoyed. It was, a whole thing was about, uh, simulation? And going to the GM [General Motors] plant? And watching how the robots...make cars...trying to find a system that would put more output of

cars....I think that's excellent....I think that's more important." [p 5-6]

For Shannon, there was a black and whiteness to mathematics.

"I like seeing a right or wrong answer. And you know math is one or the other." [p 7]

Shannon saw a distinct difference between the mathematics in university and that taught in high school.

"[W]hat you learn in university is totally different than what you learn in high school." [p 10]

View of Teaching Mathematics

The student-teacher relationship was primary for Shannon. Her goal was to make a difference to each student, particularly lower achievers.

"...I wanna make a difference to each student and especially the ones that aren't doing well." [p 20]

"I want to make a difference to each one...the ones that are problem children are the ones I'm, more concerned about?" [p 21]

For her, successful teaching meant being able to "cover every student." [p 11] She worried that when she had her own classroom, she wouldn't find enough time to approach each one, and some would not get attention from her. She tried establishing individual relationships with each student during her practicum but found this task daunting.

SH: [In the beginning of the practicum] I thought...I had to do everything. I realized you can't do everything. You can't expect yourself to do everything.
I: What would be examples of the everything?
SH: Of everything? Meaning help every student in that class, go to every individual student. [p 11]

When probed for what she wanted to accomplish with each student, her response was "I want them to pass..."[p 19] Elaborating on one particular student who had poor attendance and discipline problems, she said, "I want to see him pass. I want to see him pass the course, 'cause I would feel like I'm the failure, which maybe is wrong." [p 19]

She also gave two examples of students who were "problem kids" but whom she saw as having potential to succeed in school. She attempted to strike up a relationship with each. [p 19-20]

For Shannon, not only was more math learnt by the student when she worked with them individually, but a firmer relationship could be built in this manner.

> "I get a lot more done with one-to-one. And they understand it more than if I do it in front of the whole class. And it's impossible to do that, unfortunately...I wish I could do it to every student...It's just the way they even act and, like even when I help students here at their seats they're st-, they're happy when they understand it....you don't have time to go around. [p 9]

Model for teaching.

An exemplar for Shannon was the protagonist in Stand and Deliver, a movie she viewed in one of her teacher education courses. He worked with his students the way she wanted to work with hers and he represented a model for her to follow. She saw a parallel between her students and his in the movie.

> "So this teacher [in the movie] took it upon himself and he made math so enjoyable that the kids even came in on WEEKENDS!...And I used that motto, I loved that movie, and I loved how he could relate to those kids. My Grade 10's are identical to those kids....And that's why I'm buddy-buddy with them because HE was and he got through with them that way. " [p 13]

Shannon related even more with the movie when there was a teachers' strike during her practicum. This meant classes were cancelled and consequently, some students wanted extra help outside regular hours. She saw this alliance as epitomizing her goal of aiding students through establishing a personal relationship.

> "...my Grade 10's remind me of them because one of the students when we were allowed to help [during the strike] he was here with me 'til 6 o'clock....Now how many students will do that? And this kid is a 50% average. And I felt great." [p 14]

Shannon believed a non-authoritarian relationship enhanced student learning. In being friendly, however, she experienced problems with classroom control.

> "I can't be buddy-buddy with them, because then they take advantage of me. So I don't know where my limits are....Because I find when I'm their friend helping them, they learn more. And then, and then they take advantage sometimes -" [p 14]

When she was shown the diagrams in "Views of Teaching and Learning," she decided none of them conveyed her conception of teaching as clearly as the movie.

Conception of teacher.

Shannon's conception of a teacher was an omniscient person who was expected to be so in front of students and peers. "I sometimes wonder, and that shouldn't come out of me" [p 3] and "I don't like hesitating 'cause I want them to learn math" [p 6] was how she described what her behaviour in front of students should be.

Shannon believed those in positions of authority should have all the answers and make them available to others. She elaborated on this when discussing what to tell students who asked why Euclidean proofs were studied:

> "...what's the reason they need geometry, I don't know an answer. This is not the right answer, but I tell them well in order to graduate, to go to university, you need your grade 12. So you need geometry. That's not a good enough answer. I wish they also teach us how to tell the students the reason for each [topic], without me making up something?....I used to have a teacher say it's because it makes you think. Is that the answer? I don't know." [p 3]

And later in the interview,

> SH: Because like I said I don't know the reason why we learn geometry. I wish I did. And I wish they would tell us more, of how to...'cause I know it's my own opinion and I don't know what that is for geometry...
> I: Ok, where do you think you can find that out?
> SH: Hopefully, from uni-, from profs will tell me! [p 6]

In this excerpt, Shannon laid out a hierarchical pathway for relaying information. The professors would tell her why geometry was taught so she in turn could tell the students.

The idea of the teacher being all-knowing put great pressure on her. She was concerned her ignorance of content or content-related issues would cause her to lose face with other staff once she became a certified teacher.

> "You know I, I know I was lucky I could go to [school advisor]...and say I don't understand this question....But when I become my own teacher can I go to other teachers? And they said yes. I would feel more weird. I know I'm a student [teacher], so I can do that, but as a teacher, can I do that? To me, I'll look stupid." [p 8]

> "...I was terrified 'cause I forgot everything....To me I don't remember this stuff that I learned in high school and I feel stupid." [p 10]

The teacher was not allowed to wonder, guess, or not know. This put a great strain on her as she tried to accomplish this ideal.

> "[The students are] coming here so you can talk to them, listen to them, teach them....You have to make yourself...even though you don't feel well, you have to get across that this is important, even though you might not think it is, personally, but you gotta get to, to them that it is important." [p 18]

Consistent with the notion of omniscience, she held in disdain those in the profession she perceived to be not all-knowing.

> "When I had an exam in my, in my [university mathematics] courses where it was theory, I had to memorize all the formulas,...all the proofs....but not even the prof knows that by heart, and yet they expect the students to. I don't agree with that." [p 5]

<u>Requirements to teach mathematics</u>.

> "Patience. A lot of patience." [p 9]

Shannon's replies to this section of the interview seemed to be influenced by the rambunctious nature of her students, and by the difficulty she had making the transition from successful private

tutoring to teaching a class. She described the job: "[B]eing a teacher you always have to have your wits to yourself. You can't have a day where you don't feel well. You have to make yourself [teach]." [p 17]

Teaching methods.

Shannon voiced strong opposition against teaching for understanding as advocated by the teacher education program. During her practicum, she noticed how many students could not do simple arithmetic without their calculators. She was adamant that they should be drilled on number facts, her rationale being that was the way she had learnt them and the number facts had stuck in her memory.

> "...the students they can't even add, and multiply, and subtract; it's disgusting....what was it when I learned how to add and subtract? Why did it stay with me compared to these students?....And about making calculators - they can't use them anymore. Make them think again. Go back in time....[for] adding and subtracting - drilling. I'm sorry, I think that's the best way. I was drilled multiplying. And I learned it." [p 1-2]

Teaching, for Shannon, meant direct instruction, telling. Cooperative methods, as she understood from her school advisors, were considered "extra" and a waste of time.

> "...you're being told that you can't do cooperative learning because this curriculum has to be covered by this time....And it's like, I'm sorry you only have this amount of time to cover this math. If you do cooperative learning, you won't have time to cover it. That's what I'm being told by these guys. And if they tell you to do it, but then when I attempt it, oh you've wasted a class. That wasn't as...as productive as if you taught a lesson and let them do the work....even though this way I know, with cooperative learning you can see they really understand it." [p 16]

In comparison to teaching English, Shannon asserted that teaching mathematics was more constraining, because in English "you can get away with so much more." [p 15] To elaborate, she cited watching a student teacher of English show movies in class. Shannon felt trapped by the curriculum which had to be covered, and felt the only way to do it was

through direct instruction. Other, more student-centred techniques she wished to use were considered to be extracurricular and superfluous according to her interpretation of what her school advisors wanted.

Her goal was to show students applications of mathematics, her favourite aspect of the discipline. She described an undergraduate math course she thoroughly enjoyed when she went to a local automotive plant and studied the output of the robotics assembly line. The term project for the course was to develop a system to increase the efficiency of the line. "And I thought, that is great even for high school. To take the kids to the General Motors plant to see why certain math is applied, and to understand technology that's coming out." [p 5]

Other Issues

Dilemma with content.

Shannon felt very uncomfortable that she was a university math major, but could not remember the junior high school level mathematics she was to teach. "I feel stupid. And I don't understand that." [p 10] She blamed the teacher education program at UBC for not preparing her, i.e. not reviewing more content with the teacher candidates. She was adamant that content review be a component of teacher preparation. "...I think the Faculty of Education should be teaching us math teachers, reviewing everything we have to teach in high school....I had to do all the homework, because I didn't understand it. Instead of teaching us - , a lot of the stuff we learned in our concentration courses was useless." [p 8]

Dilemma with student relationships.

Shannon wanted to be a friend to the students, but was told by her school advisors that as a teacher, she had to be "mean." Since her view of the teacher-student relationship leaned heavily toward students, she allowed them to have a great deal of control of the flow of the lesson and the class. Then, being young adolescents, they would take advantage of this, and classes would become unmanageable. Shannon did have problems with class management, "...and I don't know where my mid-point is." [p 14] She was troubled by not finding a workable balance between being a friend to the students and being the teacher, the authority figure.

Perspective on authority.

Shannon had a very strong belief that those in authority should know all the answers. For Shannon, there were right answers and wrong answers, and "they" who knew the right answers should tell "us" (the others). This attitude resonates with Perry's (1970) description of duality in his scheme of intellectual and ethical development. This played out for Shannon on several fronts. First, the professors at UBC were the authorities in the teacher preparation program and they were the ones who should tell the pre-service teachers all they needed to know. Second, when the pre-service teachers were in the classroom in front of students, they in turn became the authorities for these younger ones. Therefore they needed to know all the answers to students' questions. This caused Shannon great worry.

Even her conviction to her goal in teaching, "...to make a difference to each one" [p 21], created dissonance with this fundamental perspective. "And I, I don't know if that's right or wrong. I don't

know...I don't know if that's correct to be thinking that way or not." [p 21] Underlying her teaching philosophy, she had an extremely strong belief in a right versus wrong way of thinking and was searching for the correct one.

Doug

Academic Background

Doug was the only subject with a liberal arts degree (B.A.). His math specialty qualified him for the B.Ed. program in secondary mathematics.

Conception of Mathematics

Doug believed mathematical knowledge was created through negotiation and agreement. Convenient and useful definitions were formed by scholars in the field and evolved continually as need arose. He cited the function as an example.

> "I mean math is based on definition, and I think ah, an important part is to show [students] that, that we create these definitions as people. We make up this, here's a definition, right? And ah, we could just as easily say the definition is this....a function used to have more than one possibility. For instance the square root...was at one time considered a function, right?...now in order to make it a function we say the positive square root. So...it's changed over time because it became a more convenient definition, right? So we define these things....It's convenient to locate the centre of the circle at the centre position at (0,0) but if, I mean it could just as easily have been (1,1) or something. We just choose whatever it suits us at the time." [p 3]

Doug enjoyed the structure and orderliness of mathematical content.

> "[T]here's a certain cleanliness to math, or purity to math, to the, to the order and structure and everything, and how it's laid out? You know, it's very clean that way? And I like that? Very logical? I think that, you know like, that math is beautiful idea, of the rationality, and simplicity I think I really like." [p 9]

The common perception of mathematics as a tool of science and engineering troubled him. He believed it masked pure mathematics as a discipline unto itself. In a broader context, he feared society as a whole was slowly losing its aesthetic side.

> ...[M]athematics is a, is often seen not as a discipline, necessarily by itself. Or as a, just a way of thinking, but it's seen as a part of science, and, and engineering....all our ways of looking at the world, our, our philosophy, our social studies, our economics, psychology, they've all, they've all started to call themselves sciences, like soft sciences, right?...I think that may be cheapening each of those other areas, you know?...I think we lose sight of other ways of looking at psychology....I don't necessarily believe that it is a science. Or economics, as a science, right?...Things become valid when they're scientific, or they become valid when we can back them up with numbers and so on. I don't necessarily agree with any of that, and yet, there's a danger in teaching math is that it's seen as a big part of that side of the world." [p 13-15]

View of Teaching Mathematics

To Doug, teaching mathematics meant teaching reasoning, thinking and communication skills through mathematical content. He saw the order in mathematics as conducive to teaching thinking and fostering the ability to communicate well thought-out ideas. Communication entailed not only interpersonal communication, i.e. the exchange of ideas with others, but also communication with oneself.

> "Because I think that math is about...putting things in an ordered fashion...thinking in a structured ordered manner...and so that lends itself to both thinking and to communication." [p 1]

> "...you're teaching people the thinking and reasoning skills conceptually, and patterns, and putting things together and so on?" [p 4]

> "I think the thing that we're trying to do is to teach them to think and to communicate....And I also think that one of the most important things about learning how to communicate is learning how to communicate with yourself...thinking within yourself, putting your own feelings or notions or whatever into being able to play with them as a thought...and that's a part of communication as much as me telling you or us discussing something." [p 1]

He placed teaching in a broader perspective, one that embodied the purpose of education.

> "...we're teaching values....I think that teaching of values - I think the construction of values is something that we're all involved in, or may be involved in to, for our own values. And we're s-, and if there's democratic education, or any of a number of things we're teaching people to question, to think for themselves and to start to create at some point, to give them the tools necessary to create values, to create their own values, rather than accept...whatever that doesn't allow them to question, to create." [p 2]

This over-riding goal of education was linked to his view of what it meant to be a person, and though Doug never elaborated on his perspective of human development, he clearly saw it as an elemental component of life, both his own and his students'.

> "But...the primary, primary goal though is, ah to help people develop as human beings, you know? or to grow as human beings and that's true for me too." [p 10]

Conception of teacher.

Doug believed a math teacher was a role model for adulthood. As such, the teacher would model the qualities he believed important to foster in students: the ability to reason, the willingness to learn, the willingness to make mistakes.

> "...let's say at this particular age level, I mean these are people that are coming out of childhood learning to be adults, right? So we're, so even if it's just as a role model, you know, and maybe as a poor one, but as the only one that they see in a, out of a group of 30 people, um, you know, we're teaching things like what it means to be an adult, or what it means to function socially...and things like that." [p 2]

> "Well, I think that's a worthwhile role model. Someone who questions....If I get presented with a question, and I try and pretend that there's one way to answer it, or that there's one right answer to every mathematical question, then I've created a certain role model, but I would call it a negative one. If I show that...I can make mistakes and learn from that, or if I show that I can think through things and try or...I see different people coming up with different answers and I accept different answers, then I'm showing a very different person, a very different belief in what is ideal and right, whether there's a right and a wrong, or whether there's many different rights or many different

possibilities....So rather than teaching them what is right and modeling that, I'm teaching them the ability to question." [p 3]

Doug wanted to show students that making decisions involved ambiguity and that answers were not always clear-cut. He wanted them to realize that "the" answer was not always available and was in fact sometimes the result of making many changes, outright mistakes and going up blind alleys. He also wanted them to see that decisions could be made through consensus after reasonable pros and cons of an argument were weighed. This form of critical thinking was what he hoped to engender through his teaching, and for him it distinguished a good teacher from a bad one.

"A good teacher is a good student, someone who shows an ability to learn and a willingness to learn, demonstrates the qualities that you want to see and that, that you're encouraging in a student, in a positive approach to life maybe....so someone who's rigid and thinks knowledge is a certain thing...makes a bad teacher." [p 8]

"...I think you have to be willing like to, to make mistakes, or, or have, show that something a little is hard and you have to think about it." [p 5]

His work with and interest in students from alternative programs caused him to consider his interaction and influence on them as a teacher even more deeply.

"Like, like I questioned why I was teaching mathematics when, you know, I got people coming into the room whose mother just kicked them out of the home. And all these various things going on, and I'm saying, I'm thinking like, I'm teaching, you know, quadrilaterals, and what the hell does that have to do with this person's life? But, in actual fact, I think it did make a difference at that point in time, to do that, you know?" [p 10]

Doug's primary focus was on teaching, rather than teaching mathematics; the content of his classes was secondary.

"No, I don't think it's that important that they end up at the end of grade 12 with a huge knowledge of math. I think it's important that they end up with an ability to reason, and an ability to communicate. But I think, ah, I think for the most part, it's likely that the two will be linked. You know like, not that if they can't do fractions they can't [think], but as they gain the ability to do fractions, they gain the ability to think." [p 6]

> "...I don't think you teach reasoning skills specifically, you teach a course of something and through studying that course they learn reasoning skills..." [p 7]

He advocated more logic in the math curriculum, believing that the skills acquired could be used by students to deliberate pressing and controversial issues of the day.

> "...I'd love to be teaching at some point and say, teach a little bit of basis for some kind of ah, logical skills? of say a way of breaking up an argument or so, you know the sort of logic that you do in a, in a philosophy course? Just a very basic level and them give them a, a, an argument on something that's current, like logging, you know in some small town in B.C. or whatever, and have them break up the argument, you know? And, and therefore apply some of those thinking and reasoning skills directly to something you're involved in." [p 6]

Thus, mathematics was a vehicle to help students develop their thinking. Doug made it clear that mathematical content took a back seat to the larger goal of developing the ability of critical analysis.

> "So clearly, I mean if they didn't learn math, but they, they were able to sit down and, and reason with me over something and argue cohesively about something, that would be much, much better than being able to sit and memorize the steps to multiply fractions and yet not be able to in the long run, you know, critically analyze somebody else's statement, you know, or to question what we see in the media or something like that, based on, you know, rational reasoning." [p 10]

However, he voiced a caveat in being a teacher of mathematics. One could be seen as an advocate of the "scientific" way of looking at things; this in fact contradicted his personal world view.

> "But I, I find that a danger in teaching mathematics is that you end up encouraging this scientific view of the world....This has got to do with the fact that mathematics is a, is often seen not as a discipline, necessarily by itself. Or as a, just a way of thinking, but it's seen as a part of science, and, and engineering and da da da da da. So, by encouraging people in math to be successful mathematically...it can be interpreted as encouraging them to go on into the science which alone isn't wrong, but may also encourage this view of the world, you know, which is that the world operates on scientific principles, or, and so on." [p 14]

Here, Doug contrasted his goal for teaching with other goals. Whereas his primary focus was to foster and encourage thought and development in

adolescents, he feared that, due to the popularly perceived nature of mathematics, his teaching could be seen as encouraging students to accept the increasingly "scientific" view of the world.

Teaching methods.

Though Doug was a strong advocate of teaching for conceptual understanding, he believed there was also a place for teaching procedures. He felt as students worked through both aspects, the interplay would generate more conceptual understanding and help students develop the skills of analysis and synthesis.

> "you need to teach people the procedures and this and that, and you teach them the conceptions and everything else, but I think over the long run of practice and, and going in between those two different sides of it, or practising the procedural terms, they, they pick up a greater understanding of both." [p 4]

Doug regarded the secondary math curriculum overly geared toward provincial exams, leaving little time for creativity in the classroom. Not only was the curriculum set and each unit neatly supplied by a textbook chapter, but the types of questions generated were all procedural. Doug contrasted this inflexibility with Social Studies, English, and Graphic Arts, where he saw the teacher with more freedom to develop her own units and design a variety of student activities.

> "some subjects afford far more opportunity....in Socials you almost create your own unit every time it seems to me....[whereas in math] here's a unit, and her's the textbook. And the textbook chapter is the unit, you know? And there's very little freedom..." [p 11]

Other Issues

Adolescent Development.

Doug had a special interest in teaching "alternative" adolescents, i.e. those whose emotional problems precluded them from joining a

regular classroom. He believed math class could be used as one point of structure in an otherwise chaotic life.

> "...that person might need a, a sense of order and structure, and things, and so on, which a regularity in the classroom gives them, the discipline in the classroom gives them....and it gives something to think about outside of themselves." [p 10]

Student teaching.

Doug felt constrained in class because of his status as a pre-service teacher. He felt the evaluative aspect of the practicum forced him to be the opposite type of teacher from what he wanted to depict.

> "Well, if you're a student teacher like me, and, you're just trying to play into what someone else expects....I think we can [teach thinking skills] just as simply by, as showing a willingness to make mistakes in front of them, you know. And showing a willingness to not have the answer, like-, and I think that's a danger of student teaching is that you feel like you have to be prepared and you have to walk in there and you can't make mistakes and all the rest of it." [p 5]

World view.

Throughout his descriptions of teaching and of the teacher, Doug was consistent in his philosophy of helping students develop their thinking abilities so that they could analyze and subsequently choose values for themselves. He contrasted two views:

> "Knowledge is a top-down thing, there IS an answer, we're looking for various truths, or there are truths, you know whether they're Biblical, or mythological, or scientific, or ah, or whether you know, we just create our own perceptions of the world as we want to, and ah, you know, we use our abilities the way we choose to." [p 8]

Doug fit his philosophy of teaching mathematics in with his larger picture of life.

> D: You know what the danger is in teaching math, I think? For me, the scientific view of looking at the world is mythology? And, it's you know, it's as valid?
> I: Mythology. You mean constructed?
> D: Yeah. It's as valid, and it's a valid way of looking at the world, but it's as valid as the Greek mythology, right? And we look at the Greek mythology and think, what a bunch of stories that weren't true, but in the same way we're just

> making up our own models for the world using scientific language, but we really buy into it...a danger of teaching math is that you teach, or you lead people into believing that, that much more? So if you teach people thinking skills you can teach them to question and answer that for themselves, and if they want, to the extent that they want to buy into it, or want to use scientific mythology, that's fine" [p 14]

Ray

Academic and Personal Background

Ray had a B.Sc. in Applied Mathematics. He had also completed 2 years of an Engineering degree and some graduate level mathematics courses. Prior to entering the B.Ed. program he tutored privately and worked as an actuarial assistant in an insurance firm.

Conception of Mathematics

Ray's conception of mathematics emphasized its utilitarian nature. He saw mathematics as a tool for other fields. "...we used math in physics, we use math in biology, use math in every other [science]" [p 4]

Ray believed mathematics was accessible to all and could be fun for all, especially if the topics were relevant to common events. "It can be accessible to every single kid out there." [p 10]

View of Teaching Mathematics

> "I think teaching is 20% teaching and...80% dealing with students in different ways and their problems....you deal with kids on an individual basis....80% of your time is interacting with the kids, and, ah, getting to know the kids, being their friend, try to be a role model and steer in the right direction." [p 1-2]

For Ray, the teacher took on multiple roles of counsellor, guide, friend and parent depending on the type of class as well as individual students.

> "Role as a teacher...depends on different grade levels, but I think you're there to guide them in the 'right' direction, not as

> a parent, not even as a teacher. I guess you take different roles as a teacher." [p 2]
>
> "At one point you are the teacher, you have discipline rules, at the next point, you're a parent, at the next point you're a friend, you're a counsellor. Ah...it's just amazing how the different roles come upon you." [p 13]

The most prominent of these for Ray was counsellor, for he found himself both before and after class dealing with a variety of students who were going through diverse problems. "So you know you take the time to sit down with these kids and move them aside....I guess [you] get to know the kids." [p 2]

View of Students.

Ray's ideas of teaching were largely influenced by his 9A/10A class, of whom he spoke a great deal. 9A/10A was a terminal math class composed of grades 9 and 10 students, a class the department head had fought to establish. Ray and the department head team-taught this class. He described them as "22 students with different behavioural problems" [p 1] and though he spoke of the challenges they presented he never claimed they drained his enthusiasm or energy. Instead, he spoke repeatedly of the personal problems each student was having and how those inhibited their learning. He saw them as individuals with emotional and behavioural problems, kids who came from dysfunctional families and acted out their frustrations in school.

> "...and I realized that, even after you lectured them you just sat there and helped out individual students and most of them would react after 5, 10 minutes, they'd get up, throw something, or they'd have a temper tantrum. So, ok, for that class you actually taught 20% of the time. 80% of the time you would actually deal with them regarding their emotional behaviours....You are dealing, uh, with a variety of problems." [p 1]

(When Ray returned my draft summary of his comments, he noted that for "regular" students, the ratio of teaching time to time spent dealing with behavioural problems was probably 50:50.)

Ray spoke of other students as mechanistically attending school. He gave this as the reason most students were disinterested in mathematics.

> "I think most of them feel they have to come to school, do the assigned homework, go, come back the next day. They find that that's their job...and I think that's why they have a dislike for...any subject." [p 5]

In addition, he believed students saw math as "...something quite hard, and a terror, and a waste of time, has no use." [p 3] However, he claimed students had an "unknown curiosity about mathematics" [p 5] and that once the connection of math to daily life was made, they would be interested in the subject. According to Ray, students needed to see the usefulness of mathematics to break down their emotive barriers.

Though he claimed there was beauty in math, he stressed that "kids have to appreciate the beauty of algorithms at a later stage." [p 10] Relating his own experience, he stated he didn't see "the beauty of algorithms" [p 10] until 3rd year university, when he could "make the connection....But in high school, they have the opportunity [to make the connection] because you are the teacher, or as a counsellor, or as a friend who's there to help them make the connection." [p 10]

Goals of a math teacher.

A math teacher needed to "hook kids onto school" [p 4] by "hooking them onto math." [p 14] Ray believed that by showing students the usefulness and relevance of mathematics they would become interested in the subject matter.

"...kids [need] to understand that math is used in every aspect of our life, in every subject of life, whether it's music for notes...for physics...out on the football field, average in class, keeping scores, ah, finding percentages. It's just the daily knowledge, and most kids don't have that....They have to have a reason and I think that's what a lot of kids lack. A reason. Why are they learning this." [p 9]

"Again, if it doesn't pertain to them I think they see math and science, sciences as a tool to go on to higher education and university, so if they're not going to go, why bother learning it? That has to be changed. They have to be taught the usefulness in math even if you don't go onto university, in your daily life, in whatever occupation you're in, dealing with the bank, buying a car, going to the grocery store..." [p 7]

The teacher also needed to combat psychological barriers the students might have regarding mathematics by making classes enjoyable.

"But they have to take this [math] class which was bestowed upon them by the school. Which is sad, so you try and make the class fun for them." [p 3]

"...you can use math to boost their confidence level, for kids who have no way of passing, who failed before. You share, you show them the usefulness of math during class time." [p 3]

Qualities of a math teacher.

"So I think the attributes of a good math teacher number one is to make the kids love math...understand why math is there, what use math is." [p 5]

To Ray, a teacher of mathematics needed to be a person of mathematics.

"...if you just keep somebody who is not a math teacher to teach math, they don't understand the beauty and the appreciation of the daily life, they can't convey that message across...you have to be able to give out that message to the kids that you are having a wonderful time doing this...you have to love doing mathematics too." [p 11]

He gave himself as an example of someone who was always looking for applications and opportunities of using mathematics.

"...somebody who enjoys it always looks for examples. For example, if you're driving on a long trip, uh, what I tend to do sometimes is unconsciously look at the sign posted as you go along on the highway that says one mile, two miles, three miles. And look at my speedometer and try and do a mess of calculations. And you do it unconsciously because you enjoy doing it." [p 12]

Teaching methods.

Ray advocated methods which would give students that "connection" to the real world. He suggested guest speakers. He envisioned inviting sanitation workers, secretaries and cashiers to tell the class how they used math in their work. During his practicum he had planned a field trip to a baseball game where the students had an assignment to complete. (Unfortunately, the game was rained out.) He also suggested having students devise their own math problems based on their experiences. He called these "positive problems" [p 8].

Playing games and using puzzles, riddles, and problems of intrigue were other methods of "hooking them onto math" [p 14]. He stated a teacher would acquire a bank of these through experience, and claimed to have already collected quite a few which he used. He tried to stay away from lecturing. He felt that having life experience and "tricks" to share with the students would allow for more variety in class and move away from a traditional lecture style.

Metaphor.

Ray had developed his own metaphor for teaching. It was of the teacher as a gardener and students as plants under his supervision. This metaphor highlighted student individuality. The care and concern he felt for them is evident in his description.

> "I think a gardener would be a great metaphor. Because you know if you have a nursery, you get plants and you nurture them, because when they're at a young age that's when they're most susceptible for disease or to fall off and you know as a gardener you, you tend to them in different forms. In different flowerbeds, you fertilize them differently. Yeah, I think that's a very good analogy of it....So I think you're constantly tending for their best use....Every plant is totally different....As a gardener you're constantly watching out for, and I think what I meant by disease was you know, kids falling off the wagon. They lose interest and you know you try and change your methods of planting or I mean you're always adapting....you care for them,

you nurture them....And I think that, that pertains to being a teacher too if you think about it." [p 14-15]

Other Issues

Conceptual change.

Ray's view of teaching mathematics had changed with his practicum experience from one which was content-oriented to one which was student-oriented.

"Let's see, yeah, my views have changed. Ah, drastically, I think....my previous assumption was you just go, you teach, you leave, you mark papers. But no that's not it. Teaching is more than that. Teaching is, it's a multitude of jobs." [p 13]

Victoria

Academic and Personal Background

Victoria had a 3-year B.A. with Distinction in Economics, and a 4-year B.Sc. in Mathematics achieved one year after her B.A. An M.Sc. in mathematics was in progress. She called herself a "professional student." [p 6]

Victoria was a whiz kid in mathematics from public school through university and graduate school. She skipped grades and was able to learn math effortlessly, absorbing it. "...you've always been able to put me somewhere with the textbook and I can learn the course faster than you can teach it to me." [p 13] As a graduate student in UBC's mathematics department she had taught an undergraduate course for those with no math background who were planning to enroll in a B.Ed. program.

Conception of Mathematics

Victoria differentiated lower level mathematics from graduate level mathematics. She stated at the lower level it was all known and formulaic; it exemplified the scientific nature of math. Graduate level

math, however, involved "fuzzy, grey areas" [p 7] which required intuition and creativity to achieve results; this was the artistic side of math.

> "...where [the public school is] at in math is a known area and it's not changing all that much....you're not going to suddenly have equations of lines changing desperately. I mean it's known what a line is, and how to solve it. [p 7]
>
> "...as an undergrad you have the right or wrong and I mean you have, you'll have, you're doing a proof? And there's different ways of doing the proof, but you know you're trying to prove the same thing in the end....Whereas when you get into grad math, you start getting into...the fuzzy grey areas there, you're getting into the edges of math rather than, rather than this known area....you're just still discovering, you're still creating it, ok? It's still being created.' [p 6-7]
>
> "I think [mathematics] is a little of each [art and science], and I think the shame of it is we teach it too much as a science...[M]ost people as an undergrad have only seen it as a science. And it's creative, you have to be creative to be a good mathematician." [p 19]

Victoria claimed students generally saw math as rigid, "this cold hard object" [p 1], totally externalized, and difficult to comprehend. In contrast she saw mathematics, in particular her sub-specialty of discrete mathematics, as fun and explicable to the lay-person as "glorified connect-the-dots." [p 6] She went further to claim that elementary teaching usually rewards memorization in mathematics, a trait not directly leading to becoming a good mathematician. In higher years where understanding rather than memorization was required, the transition often proved difficult for the student who was formerly successful doing memory work. [p 9]

Victoria claimed mathematics' aesthetic quality was often overshadowed by its utilitarian nature for pragmatic purposes.

> "...we're the building blocks of other sciences....we're creating tools, basically....for me I find math a joy and beauty and everything else in math, but nobody's going to fund, no company's going to go out there and, um, fund mathematicians or say that

> mathematician's are at all useful unless they actually create something, ok?...creating a tool that will be useful in the real world." [p 8]

Conception of Mathematician

Victoria formulated her ideas of a mathematician through algebraists she had met as a graduate student at Queen's University. All were creative, artistic and musical, "things that you don't think a mathematician should be." [p 20]

> "They all play an instrument of some sort. Um, they all are involved, go to the theatres, they all do this, very artsy...stuff." [p 20]

She described the mathematician's work as checking for meaning and understanding; when dealing with a problem it must make sense "logically and intuitively." [p 10]

View of Teaching Mathematics

Victoria had two goals in teaching mathematics. The first was to dispel the notion of a mathematician as a "geek" or "nerd," and the second was to change the commonly perceived nature of mathematics as being rigid and unattainable for the average person.

> "...teaching mathematics is...making math human rather than having it this cold hard object that they [students] have to get over and deal with and survive rather than something they enjoy." [p 1]

> "...just making it [math] very, ah, touchable? If that makes sense? Making, making math as I said, very human..." [p 2]

> "...what I would like to do is I'd like to make math fun..." [p 18]

> "And I think you're sort of saying that hey, I'm human and I'm doing math and I enjoy math. Therefore, you are human and you, you can enjoy math too. you don't have to see this person that with the, very stereotypical, like the glasses with the tape on it, the geek, the geek pack with the pen leaking through and you know, the pocket protector type thing....Breaking that stereotype." [p 13]

She saw the fear of math in the university students she had taught and became concerned that this fear would transfer to the children they

would eventually teach. A scenario she depicted for this was a forced half-hour of instruction in math versus a one-and-a-half hour of music or art instruction in the elementary classroom. Victoria wanted to help them and others overcome their fear and paranoia of mathematics and have them enjoy the subject. She advocated getting the message to students that "math can be enjoyed by humans" [p 13] and dispel the stereotype of the "math nerd", passing on instead a positive attitude and fostering a willingness to explore math.

Metaphor for teaching.

Victoria had begun to develop a metaphor for teaching. Teacher as Painter, and Student as Painting was how she conceptualized it. The student was a painting where the teacher/painter was adding on to the oeuvre, and not a sculpture where the master artist would be chipping away substance during the creation of the work.

Her metaphor emphasized greatly the composition of the student. When the student first entered your classroom, he or she embodied the prepped painting, carrying the work of others on the canvas. These others were the parents, friends, the home, and life in general. These had all left their impressions on the canvas. The painting was an accumulation of the influence of every event in the student's past; each individual had left their mark on the student's canvas, each person was helping to create it.

> "I like the idea of a painter?...the student is the painting you're quote unquote creating. Mind you it's a painting that comes to you prepped and started, ok?...So they have their home life, their parents, their friends, all this, all this is all turning into the painting and adding to the painting, ok? So what's happening is you, you're given a painting that's prepped and started, and you, you and the student are helping work create more of it. Like you're adding on to it. And hopefully positively, ok? And the reason I like painting is because you are

adding on. You're not, it's not like sculpting where you're taking away..." [p 15]

This was a strong conception for Victoria as shown by the trepidation she felt in the role of teacher. It intimidated her that she had the opportunity to design, create, and draw on the painting. The influence could be negative as well as positive and the power inherent in that was frightening.

> "...that's the scary part of teaching, is your ability to affect the lives is not just positive. It can be negative. And that, that's something that, to some extent, scares me." [p 17]

She recalled a high school math teacher who was a strict disciplinarian. She remembered the physical force he used with various members of the class and little else. This may have had a bearing on how she perceived the role of the teacher.

Requirements for a teacher of mathematics.

The ability to foresee student difficulties and generate explanations was important for a teacher of mathematics.

> "I think the real goal and the real importance isn't so much actually physically knowing the math?...So you'll have people...who understand the stuff so well that they can't see where the problems could be, lie? and they can't explain the problems. And I think that's what is important for a good math teacher, is to see where the difficulty is going to come up and what is the really important CONCEPT to get across, ok? What do they [the students] really need to understand about this?" [p 12]

Confidence in the subject matter was key. The number of university courses it took to achieve this was not important. The fact that the person becoming a teacher of math had no fear of the subject, was. "You need to enjoy math and not fear it [because]...it transmits." [p 14]

She enumerated other requirements as well: a sense of humour, knowledge and use of alternative approaches/methods. She also commented that those who do well in mathematics would not have empathy for the

slower learners. This is notable when we recall that Victoria described herself as extremely quick to learn mathematics.

To Victoria, understanding adolescent development "[w]ith what [the students are] going through at this stage in their life" [p 3] was another important component of a good math teacher.

During her practicum she had the opportunity to witness someone she considered a model teacher. She was particularly impressed by him because he was a substitute, "[a]nd you know with substitutes the kids go absolutely ape on." [p 18] She described him as having "a presence," a calm and controlled manner which meant business to the kids. She made the point that he was not overly strict nor demanding, just "accepting them for who they were." [p 18] That was what she wanted to emulate.

She claimed that she herself was "still very black and white" [p 17] and rigid, and would like to change to "just accept...the students without wanting to change them too desperately." [p 17] What she saw in her favour was her other interests: gardening, dance, music. "...math teachers must have quote unquote a personality, ok?" [p 20] This notion reinforces her goal for making math human and "touchable."

Teaching methods.

The word "teach" meant to convey information through lecturing to Victoria. While she preferred not to "teach," she claimed it was at times necessary to do so. Her reasons for not liking this method was that it alienated and promoted the image of mathematics as this "cold, hard thing" [p 3] that she was trying to break.

Alternative methods she considered effective were cooperative ones: pairing, paired quizzes, warming up the class with review

questions. Another strategy she liked to use was to have each person solve a question on his/her own, then form a group with others and convince the others in the group that his/her answer was correct. With the belief that verbalization aided understanding, Victoria advocated group work for her classes. The process concretizes the algorithm used by the student. The steps become nailed down so that if an error is made, the student can more easily discern where it occured.

View of students.

Student development was a concern of Victoria's. She illustrated this when she described one of her teaching techniques. "Walk me through it" [p 4] was a phrase she used to encourage students to more fully explain their answer. On the surface to the class, this was done so that the student would explain his/her steps to the others. However, Victoria used it as a tool to facilitate growth of the student. It "is occasionally painful but...at the same point they're going, hey I can do this, this is good." [p 4] So, in addition to her goal of helping students overcome their fear of math, she also saw value in fostering their growth in self-esteem.

Other Issues

Students' view of mathematics.

Victoria contrasted the black and whiteness of the secondary curriculum with the students' lives. In the throes of emotional undulations with identity crises and deep familial problems such as divorcing parents, it would be difficult for them to accept and assimilate something so concrete. Math was completely different from Life.

"...it makes math this thing where they can't understand because so much of their life, I mean with their [parental] separations, with what they're going through at this stage in their life....there are grey areas in their life, and a lot of things are subject to change. And in math we're giving them a set, a carved in stone rules basically, and saying learn these, do this. And it's, it's such contrast to the rest of their lives that it sometimes makes, it alienates them from it, I think." [p 3-4]

Mary

Academic Background

Mary had a B.Sc. in mathematics and an M.Sc. in Statistics. She had taught introductory statistics to medical students as a graduate teaching assistant. Her secondary schooling occurred in Vietnam, and so during the interview she made comparisons between the curriculum she took and the one she taught.

Conception of Mathematics

To Mary, math was not "crunching numbers" nor did it really involve content. Rather, it was process: logical thinking, solving problems, deducing and reasoning, logic. At a higher level, i.e. graduate work, it became philosophy.

"[T]o me, math is not so much what you know, um, it's, it's how to think, how to solve problems, and how to deduce things and reason. It's, it's logic. If you're talking about math at a much more advanced level, then it's almost philosophy....Just sitting in class, and you know, killing an hour, and getting out of there, crunching numbers, completely meaningless. To me that's not math." [p 1]

Mary's attraction to mathematics was the element of problem-solving for she enjoyed doing puzzles, and "figuring things out." She likened it to doing crossword puzzles, and enjoyed the stimulation and challenge that came with exploring different ways to solve puzzles and problems. To her, that was the fun in math.

"I enjoyed mathematics not so much because of...the actual subjects or areas of mathematics that I had come across as much as

it is a problem-solving thing for me. It's like a puzzle. I love doing puzzles, I love solving problems....I like homework...where you have to go home and crack your head over some problems. [The university professor and I] we'd exchange ideas on how many different ways you can solve a problem. That was challenging for me. That was fun." [p 2]

View of Teaching Mathematics

Teaching students to learn how to think and how to learn were the major goals of Mary's teaching. These skills would prepare students for coping with problems they would encounter in their adult life.

"...to me it's very important for people to learn how to think logically...To me, [math] is not like teaching a kid how to add, so much as teaching him how to think about solving a problem....allowing them to do their own thinking and coming up with solutions and solving things...and that should they encounter other problems in life, not necessarily to do with mathematics, that they are able to solve them you know." [p 1]

"...the most important part of learning mathematics is that the kids are able to think and tackle the problem." [p 9]

"I don't think it's as important to push as much content as it is to teach kids how to think" [p 10]

In fact, the philosophy of teaching mathematics as developing students' thinking skills carried over for Mary to teaching in general. She saw no difference between teaching mathematics and teaching English. Both meant helping students develop skills of analysis, strategizing and solving problems. The linear, logical process used to write a persuasive essay was the same one used to formulate a proof in Euclidean geometry, according to Mary.

"[Teaching] really isn't about mathematics. That it is that they're learning how to think. How to put things in order...and make strategies for things, and so on so forth. It's not so much about the content as much as about, you know, how to, how to solve a problem. It's like any other course...To me, they're all the same...you know for example, to write an essay. You still have to come up with a topic that you want to write about, or an argument, set up how you're going to argue your point....It's just a different, um, medium, of solving a problem." [p 13]

"...the more I teach the more I realize that teaching math has nothing to do with math. It has to do with teaching....You go from English to biology to math to physics to, uh, Spanish. It's all developing a way of learning, showing kids how to learn." [p 14]

Another goal of teaching mathematics for Mary was to change the popular perception of mathematics as boring, and number-crunching.

> "I have seen a lot of kids growing up hating mathematics and hating arithmetic, or - . Just dry. Just sitting in class, and you know, killing an hour, and getting out of there, crunching numbers, completely meaningless. To me, that's not math." [p 1-2]

Consistent with the philosophy of problem-solving as the purpose of teaching, she described a teacher as "somebody who poses questions that...gets their curiosity up and, is able to guide them through solving a problem..." [p 7]

In enumerating qualities she believed necessary for a mathematics teacher, Mary stated several times that a teacher should have a lot of energy, possibly mirroring the energy she felt she needed during her practicum. Enthusiasm, confidence, the ability to handle questions and knowledge to explain the concept thoroughly were other characteristics. Being able to have fun with the students, getting them interested and participating were also listed. Time was a critical element. The good teacher required a lot of time for proper lesson preparation. The good teacher would "think very carefully about teaching the concept and how you're gonna get kids to think about it." [p 4]

Mary was shown the diagrams (Appendix A) representing possible conceptions of teaching. However, none of them seemed to capture the essence of her beliefs, that teaching math was teaching problem-solving.

Teaching methods.

Mary began her practicum believing that drilling facts was the most efficient way of teaching. When she noticed how students were unsuccessful in solving the diversity of Euclidean proof problems in Grade 10 geometry she changed her ideas.

> "That comes from Grade 10 geometry. 'Cause every problem's different, right?....and you see that going from one problem to another, they just, they, they're stumped right away. You know. They don't have the ability to think on their own....Like if I had not taught the geometry part I would not have realized that, I don't think." [p 10-11]

Now she disagreed with simply presenting a concept to students. Instead, she believed good teaching entailed a good lesson plan, one which was carefully laid out, which had logic and flow, and which lead students to think about the topic by using pertinent examples. Mary believed an effective lesson plan would lead students to understanding, and it was worth spending hours to develop. "Your most productive hour is the hour in which you do your lesson plan" [p 14] and a good one "will come out very naturally, flow, and things will work out very well." [p 8]

> "Um, how would I recognize [a good lesson plan]? The flow of the lesson, the logic....you can tell when a lesson plan has been carefully thought out, that there was examples leading up to a certain idea, and then get kids to think about it and then, and finally cluing in on their own, what is going on, why that is, certain things happen, as opposed to just giving the concept to the kids." [p 5]

View of students.

Mary enjoyed the students. She enjoyed that the Grade 9 students were still "full of personality" [p 16] and found them "so cute." [p 11] The fact that they could be quite outspoken in class did not perturb her. She attributed this to their intrinsic curiosity. She considered disciplining a waste of time even though it was at times necessary.

Some of the discipline problems, in Mary's opinion, came from the students being "spoonfed," [p 7] rather than being pushed to think independently. Mary believed that students had an inherent ability to strategize (think) and if they were given the opportunity to develop

this, content understanding would naturally follow. To her, students had lost their ability to think beyond a given example because they were accustomed to doing mathematics via rote procedures. "...they don't even...think they're capable of solving something unless they have seen it before...And that's a shame." [p 9]

> "I don't care about linear equations. I care if the kids can recognize it right away, and are they having fun, are they, you know, forgetting they are learning about math, and their natural [curious] selves come out." [p 15]

> "I mean if you allow them time to think, you'd be amazed..." [p 7]

Other Issues

Conceptual Change.

Because Mary had emigrated to British Columbia after completing high school in Vietnam, she was able to compare the two mathematics curricula. Her observations were a factor in changing her ideas of teaching from a drill and practice, learn-the-facts approach to one that sought to develop thinking abilities.

> "...I think kids here compared to other countries are quite weak in mathematics, um, and they're not pushed hard enough. They're not allowed to think. And they are being just, they're just spoonfed all the time....I had just finished high school in my country, I came here, um - don't laugh, it was really easy. I had to do a Grade 12 equivalency all over again. It was a joke! You know, I could do [that] when I was in Grade 9 in my own country." [p 6-7]

> "I'm very used to being drilled....and having taught during my practicum I realized that there, there is another way of teaching." [p 9-10]

Another factor in effecting a change in her ideas of teaching was teaching geometry, as explained above.

Teaching preparation and duties.

Mary spent much time preparing her lessons because she believed this was the most important aspect of her job. In fact, she was so

concerned that students understood the concept being taught she didn't seem to balance her time.

> "You know [school advisor] told me I spent way too much time trying to perfect the lesson, trying to make it fun for the kids and he said I can't always do that because I'll drain myself right out of it." [p 6]

Mary resisted carrying out other duties normally associated with teaching: administrative details, marking attendance, following up on non-attendance, dealing with parents and counsellors. She considered it a waste of her preparation time and her teaching time.

Summaries

To encapsulate their ideas and facilitate comparison, the conceptions of these six prospective teachers are summarized.

Yvette: Content Presentation

"I teach math." Yvette's notion of teaching mathematics centred about content presentation. She felt students were responsible for their own learning, and her role was to present the material effectively. This meant using a variety of techniques to favour different learning styles.

Shannon: A Buddy

Shannon's conception of teaching mathematics focused on building a one-to-one relationship with her students. This was evidenced by her desire to form a relationship with each, her frustration at not being able to accomplish this, her claim that helping those who did poorly was a high priority, and her choice of a role model - someone portrayed as valuing his students over all.

Shannon had a strong and pervasive sense of duality, i.e. there existed a right way versus a wrong way. She believed those in authority should have correct answers for all questions. At UBC, those were the professors; in the classroom, it was herself.

Shannon favoured the utility of mathematics and her goal in teaching was to impart that applicability. She advocated drilling as the most effective way to teach number facts.

Doug: A Role Model

Doug's conception was driven by his belief that students were in a developmental phase before adulthood. Being a teacher meant being a role model for them and displaying the principles he valued. For example, by showing them he sometimes stumbled when solving a mathematics problem he believed he was demonstrating that making mistakes and learning from them was part of everyone's life.

Doug believed mathematical knowledge was created and constantly changing. He wanted to share this perspective with students. He also believed mathematics had another purpose for troubled teenagers. He claimed the rigidity and routineness in the mathematics they were learning provided them a form of stability.

Ray: Multiple Roles, A Gardener

Ray viewed teaching mathematics as involving several different roles including that of counsellor, parent and friend as well as teacher. His goal was for kids to love learning math and he believed this could be achieved if students found mathematics useful, fun, and relevant. On a broader scale, he believed the teacher's role was to

"hook them to school." Ray's metaphor for teaching was a gardener taking care of plants.

Victoria: Changing Images, A Painter

Victoria's goal was to dispel the notion that mathematics was a "cold, hard object" practiced only by geeks and nerds. She wanted to instil in her students that doing math required creativity and intuition as well as a sense of logic. She felt lower level math was too process-oriented to address the creation of mathematics. Thus, the true nature of mathematics could not be realized. School math was different from real math.

Victoria believed the rigidity and certainty in school mathematics alienated students because it contrasted with uncertainties in other areas of their life. She carried this complexity of life into her metaphor for teaching. The teacher was a painter; the student the tapestry being worked on. Others had worked on it earlier - former teachers, parents, friends, family - and now, as one of the student's teachers, it was her turn to contribute to the tapestry.

Mary: Problem-Solving

Mary's conception of teaching math was consistent with her ideas of mathematics as a discipline, her belief in the purpose of teaching mathematics and her vision of a math teacher. Essentially, "it really isn't about math" but it was about critical thinking and problem solving in preparation for life. Consequently, she infused her teaching with problems relevant to the students and encouraged students to stretch their thinking beyond the rote and look for strategies beyond text examples. She herself found problem solving and doing puzzles the most

enjoyable elements of mathematics and wanted her students to experience this as fun too. Teaching mathematics is teaching thinking and teaching problem solving would be Mary's axiom.

Chapter 5: Conclusions, Other Issues & Recommendations

Conclusions

The first research question for this study was: what are the conceptions of teaching mathematics held by individuals preparing to teach secondary mathematics?

The results show clear differences in the conceptions of teaching mathematics for each person. Perspectives ranged from an emphasis on the individual student to highlighting content. Those pre-service teachers who emphasized the student differed in whether significance was placed on cognitive skills or psychological development. Those who emphasized content wanted to demonstrate various aspects of mathematics - that it involved critical thinking, that it was relevant to students' lives, and that it was fun.

Though each person's conception of teaching mathematics encompasses several aspects which cannot be slotted exclusively into a single category, it is nevertheless useful to be able to describe the dominant features of their view. Pratt's (1992) model for describing conceptions is helpful in this regard.

Yvette exhibited an Engineering approach to teaching mathematics. Her objective in teaching was to deliver the content of her subject in the most effective way. Doug and Shannon, whose primary emphasis in teaching mathematics was to foster self-esteem and cultivate development in their students, typified a Nurturing conception. For them, helping students to mature and gain self-confidence was an important objective.

Shannon identified a short-term goal for the students in that she wanted to help them pass the course whereas Doug was clear he was preparing his students for adulthood. Doug's attention to his students' reasoning ability also illustrated Pratt's (1992) Developmental conception where "cultivating the intellect" (p 213) dominates. Mary's perception also typified the Developmental conception. She focussed on preparing students for their future when she taught; her emphasis lay in developing their ability to solve problems by thinking critically through mathematical questions. Ray's view of teaching mathematics showed aspects of the Apprenticeship and Nurturing conceptions. He believed that math was present everywhere and he wanted students to realize this. In this way, he hoped to engender a deeper appreciation for the subject in his students. He also demonstrated concern for students by giving them individual attention he felt they needed. He claimed at least half the job of teaching had nothing to do with content, but entailed "being there" in other capacities for the teenagers.

Though Pratt's (1992) model is useful for describing conceptions of teaching in general, it lacks subject specificity. What needs to be included are the different aspects of mathematics on which teachers focus when they teach. Victoria's idea of the teacher as a painter displayed aspects of a Nurturing conception, but her primary goal was content-focussed; she wanted to dispel negative stereotypes of mathematics and mathematicians. Ray believed he could show students the fun in mathematics and how it was commonly used and this would "hook" them onto the subject and onto school. Mary also sought to show

students that mathematics was relevant by formulating problems which pertained to their everyday activities.

The second research question was: do those with graduate studies in mathematics hold a qualitatively different conception of teaching mathematics from those with only a baccalaureate?

Mary, Ray, and Victoria had completed all or part of a graduate degree in mathematics. Yvette and Shannon had bachelor of science degrees and Doug had a liberal arts degree with a mathematics specialty. Specifically, all six teacher candidates had varying conceptions unique to the individual. However, what distinguished Mary's, Ray's and Victoria's descriptions of teaching mathematics was how they wanted to share their enjoyment of the subject with their students.

All three indicated a desire for students to have fun doing mathematics. Victoria's goal in teaching mathematics was to dispel the commonly-held image of mathematics as objective and difficult to comprehend. She stated wanting to make math "human." Ray believed that being a teacher meant being other things too, like a counsellor and a friend. However, in drawing students to school, he wanted to draw them to mathematics as well, and he believed that forming mathematical situations that students could relate to and they would find fun would accomplish this. Mary's goal for teaching was to foster the ability of her students to think critically. This is similar to Doug's goal for teaching and in fact, both stated that teaching students to reason was more important than having students learn content. However, Mary also wanted to specifically change students' perceptions of math class and mathematics. She wanted them to find math interesting and exciting for

how it could be applied rather than being only number-crunching and boring. Mary, Ray and Victoria believed that mathematics was accessible and could be enjoyable for all students.

Yvette and Shannon, on the other hand, did not express such inclinations. While they both described aspects of mathematics which they enjoyed, there was no reference to "turning kids on" to the subject itself. Rather than trying to instil a love for math, they were more oriented toward getting students through the material by being an effective facilitator, in the case of Yvette, and through being an empathetic friend, in the case of Shannon.

Doug also did not express generating a love for math as part of his teaching goal.

Besides having different views of teaching, the teacher candidates also varied in their view of mathematics. For some, there appeared to be a strong link between their view of mathematics and their expressed views of teaching.

Shannon and Yvette held Platonist views of mathematics. Yvette stated how she enjoyed finding the answer when it came to doing mathematics, implying there was one answer. Shannon claimed math had a right or wrong answer, and even described theoretical mathematics as memorization. They both saw mathematics as being a body of content and they understood their job as a teacher as helping students learn that content. Yvette's focus on using various methods to convey topics and Shannon's propensity to drilling are consistent with a Platonic view of math.

Doug held a fallibilist view. He considered mathematics to be created through human endeavour and subject to change. As such, mathematics was dynamic and fallible. He was very clear in connecting this belief with what he wanted to convey through his teaching. For example, he felt that by showing students he too made mistakes, they would learn that life in general, and mathematics in particular, was not predetermined.

Mary and Victoria also viewed math as a dynamic subject. Mary characterized mathematics as process rather than content and graduate level mathematics as philosophy. Her conception of teaching was one in which she encouraged students to use the thinking and reasoning processes she believed inherent in the subject. Victoria described doing mathematics as requiring creativity and intuition. She explicitly stated she wanted to show students this aspect of mathematics. However, a relationship between this view of mathematics and the teaching methods she described is not evident. This lack of an apparent relationship can possibly be attributed to Victoria's understandings of instructional techniques and the differentiation she made between school math (all known) and non-school math (still being created): how could you teach something that is completely known by using instructional methods meant to promote creativity?

Ray had a utilitarian view which was supported by his academic background. Correspondingly, his view of teaching included demonstrating and using many applications to show students this aspect of mathematics.

These links between the interviewees' perspectives of mathematics and their views of teaching mathematics support the research findings. In particular, Thompson (1984), who studied three inservice teachers, found "the teachers' beliefs, views, and preferences about mathematics and its teaching, regardless of whether they are consciously or unconsciously held, play a significant, albeit subtle, role in shaping the teachers' characteristic patterns of instructional behaviour." (p 124) She suggested the connection between their view of math and their actual teaching behaviour may have been diminished by various institutional and other constraints on their practice. Similarly, for the prospective teachers in this study, when they embark on their career, they may find the exigencies of a new, full-time position will not allow them to teach in ways that are fully compatible with their beliefs.

Other Issues

In addition to their perspectives on teaching mathematics, I noted seven areas of interesting comparison among the teacher candidates.

Adolescence

All the teacher candidates became more cognizant of adolescence during their practicum. Questions regarding this age group were not scripted into the interviews, yet every person made comments indicating they were greatly affected by their teenaged students. Adolescence had became a reality embodied in kids with behavioural problems, peer pressure, identity crises and unstable families. They became aware that they were dealing with, not just a particular age, but with the

psychological tasks of that age group. The teacher candidates felt deeply about the pupils they were working with.

School Math Compared to "Real" Math

Victoria and Mary, both of whom had undertaken graduate studies, commented that school math was not equivalent to "real" math. In other words, the mathematics of the school curriculum did not involve the same techniques and engender the same types of understandings as what professional mathematicians worked with and developed. Shannon expressed dismay that the mathematics she was to teach in her junior high classes was unfamiliar to her, someone who had majored in mathematics at university.

School Mathematics in Students' Lives

Doug, who worked with troubled youth, saw mathematical algorithms as therapeutic. He believed the repetition and routineness of the procedures were a source of stability and certainty for the teens. Math class gave them an opportunity to focus on something completely different and separate from the roiling turmoil of their personal lives.

Victoria, by contrast, worried that the black-and-whiteness of school math would alienate students. She reasoned that the rigidity of mathematical procedures was opposite to the tumult of their adolescence. This would give them even greater cause to distance themselves from adults and authority figures.

Teaching Mathematics Compared to Other Subjects

> "It's not so much about the content as about you know, how to solve a problem. It's like any other course....To me, they're all the same." [Mary, p 13-14]

"...a lot of things ARE the same, you listen to the teacher demonstrate something new, you work on it with other students, you discuss it...the same basics." [Yvette, p 9]

"...I think the thing we're trying to do is teach them to think and to communicate. English is obvious for that, but I think math has one of the greatest potentials for that." [Doug, p 1-2, paraphrased]

These excerpts indicate these teacher candidates had ideas of teaching which were not linked to content. They felt their personal philosophy of teaching held true for all subjects.

Teacher's Role in the Mathematics Classroom

Doug and Shannon had opposing views of what impression the teacher of mathematics should give students. Doug believed in showing students it was acceptable to make mistakes by having them see a teacher make mistakes during class. In contrast, Shannon believed the math teacher should present herself as knowing all the answers and not even hesitating to think about them.

Conceptual Change

Mary and Ray spoke of how their conceptions of teaching mathematics had changed during the practicum. Mary identified the teaching of Euclidean proofs as the cause for her change. For Ray, it was his close contact with the students in his terminal math class.

The results of this study support the underlying belief that ideas of teaching are personal and have been constructed through past experiences and attitudes. For example, Shannon believed the drill approach was the best way to learn number facts because that was how she was taught them. Her previous work in managing a restaurant may also have influenced her belief that being able to make mental calculations

quickly was very important. Ray, who studied applied mathematics in actuarial and engineering courses, wanted to emphasize applications in his teaching. Doug had a background in liberal arts which may have contributed to his epistemological stance; his belief in knowledge construction carried over to his perspective in teaching mathematics.

The results also support in part the literature which indicates the beliefs of teacher candidates are stable. The views of these prospective teachers were expressed at the end of their practica, near the conclusion of their program. They had gone through the same teacher preparation program, yet their ideas of teaching mathematics were different from each other and the influence from each of their backgrounds is clear.

However, their ideas are not fixed and this study shows that the practicum does have an influence in the construction of their conceptions. Most notably, Mary and Ray acknowledged a change in their conception of teaching resulting from their practica experiences. In addition, all the teacher candidates expressed a greater awareness of adolescence because of teaching. Whether or not this awareness directly affected their actions in the classroom (Yvette claimed it did not), it contributes to their knowledge and experience and may influence their future teaching.

Recommendations

Based on the findings of this study, four recommendations are offered.

1. *Pre-service teachers be given a range of experiences during their practica to expose them to different types of students as well as different types of topics.*

Research on the influence of practica on prospective teachers' conceptions show varying results (Brown & Borko, 1992), and there is strong evidence that conceptions remain unchanged (Kagan, 1992). However, two subjects in this study clearly identified a change in their teaching perspective as a result of their practica experiences. While it is impossible for teacher candidates to have infinite experiences during a finite practicum, increasing the breadth of their experience to include different types of classes (e.g. academic and non-academic) and varying mathematical topics (e.g. geometry and algebra) will expose them to different contexts under which they may more closely examine personal ideas of teaching. Wilson (1994) states:

> "by giving [pre-service teachers] opportunities to reflect on their own conceptions while learning (or re-learning) mathematics that they will have to teach themselves [s]uch experiences will allow them to make accommodations in their belief systems to first acknowledge alternative perspectives of mathematics and mathematics teaching, and then (perhaps) embrace them." [p 368]

2. *More studies on conceptions of prospective secondary mathematics teachers be carried out.*

With research that points to the existence of teacher candidates' beliefs on teaching prior to their entering a professional program (e.g. Feiman-Nemser & Buchmann, 1989), the stability of these perspectives during the program (e.g. Tabichnick & Zeichner, 1985), and their influence on actions in the classroom after the program (e.g. Thompson, 1984), it becomes important to know these beliefs and ideas.

In the small group which comprised the sample for this study, major differences in perspectives of teaching mathematics were found. The interviewees had gone through the identical teacher preparation program yet their ideas were different. This strongly suggests additional perspectives would have been laid open if the sample had consisted of different individuals. Further, using a larger sample could have resulted in some commonalities in conceptions.

Ideas about teaching math can be investigated using a variety of techniques in addition to the type of interview used in this study. For example, teacher candidates could be asked to: prepare mini-lessons on selected topics; respond to episodes depicting various types of student encounters and/or different teaching styles; observe a videotaping of their own teaching. In each of these, they could be probed for the reasons behind their responses - what they liked and didn't like about others' teaching, what led them to make the decisions they made, what thoughts guided their actions. In this way, teacher candidates can explore and articulate their own ideas while they abide within the supported environment of a preparation program. Teacher educators can also use these structured opportunities to build on existing conceptions, challenging them, if necessary, but ultimately, helping the pre-service teachers to build integrated and appropriate conceptions of teaching mathematics.

3. *Longitudinal studies be carried out for individual pre-service and novice teachers.*

Brown and Borko (1992) have noted that most studies on teacher development have used several teachers rather than a single teacher. However, longitudinal investigations during the teacher preparation program such as Wilson's (1994) inquiry would help identify evolving thoughts and beliefs. "Snapshots" of conceptions could be taken through analysis of interviews and journal writings of teacher candidates, as well as using methods mentioned above, at various times during the program. In addition, following the same person from a preparation program through to when she has accumulated some substantive teaching experience, conceptual development, factors influencing it, and how conceptions relate to her actions in the classroom can be explored. One example of such a study is the case of Fred, a novice teacher studied by Cooney (1985) and Brown (1985) who describe his dilemma when his view of teaching mathematics conflicts with that of his students.

Longitudinal studies of teacher development should include theories of adult development as part of their framework. These would provide a more holistic context for understanding changes in ideas. Owens (1987), for example, used Perry's (1970) scheme of intellectual development and the construct of authority to investigate the beliefs of prospective teachers. However, he too applied it to a small sample of prospective teachers rather than using it in a longitudinal context.

4. *Investigations of conceptions of teaching mathematics be carried out with other populations of teachers and learners.*

Knowing students' conceptions of teaching mathematics could provide insight into classroom interactions. Thompson (1992) has

claimed that none of this type of research exists. However, in Brown's (1985) study, students' ideas were investigated as a potential factor in the socialization of a novice teacher. Those ideas were found to be influential in changing, at least superficially, the beliefs and actions of the teacher.

My particular interest lies in Adult Basic Education (ABE). These instructors are charged with teaching pre-college level content to adults. Typically, their background entails a B.Ed. degree and several years' teaching experience. Instructors new to the field will usually have teaching experience only at the secondary level. Knowing how they conceive of teaching mathematics to adults and how they perceive their role would be important information.

ABE students begin their program with experiences of school, mathematics, and teachers remembered from high school. Elucidating their ideas of mathematics and its teaching would provide knowledge through which an instructor could improve her instructional approach. International students bring different cultural notions of teaching and the teacher to a Canadian classroom. Exploring these could uncover potential sources for conflict or misunderstandings in the classroom.

Researching teacher candidates' conceptions of teaching math yields knowledge for mathematics teacher education programs; it also serves personal teacher development. Those who have been involved think about their ideas, their minds piqued by the researcher's questions. One candidate explicitly stated after her interview that she had never considered her own ideas before and the interview helped her think in

different ways about teaching. It is hoped that this study provoked the minds of the others as well.

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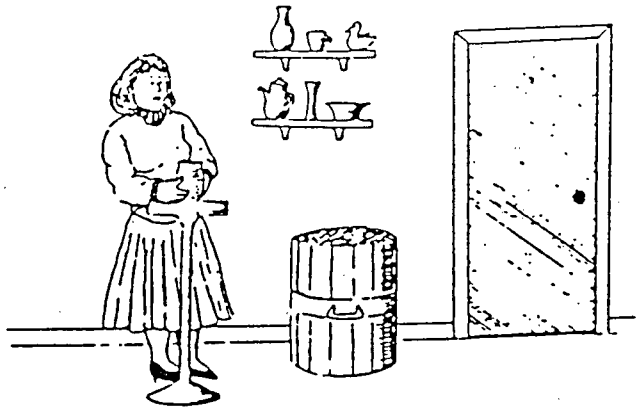
APPENDIX A

"Views of teaching and learning"

1 VIEWS OF TEACHING AND LEARNING:

1



A piece of clay is being moulded by the potter.

2

A guide and a traveller are moving through hilly terrain. There are a lot of hills, and one or two are very tall indeed. The view of the landscape changes as they ascend the higher ground.

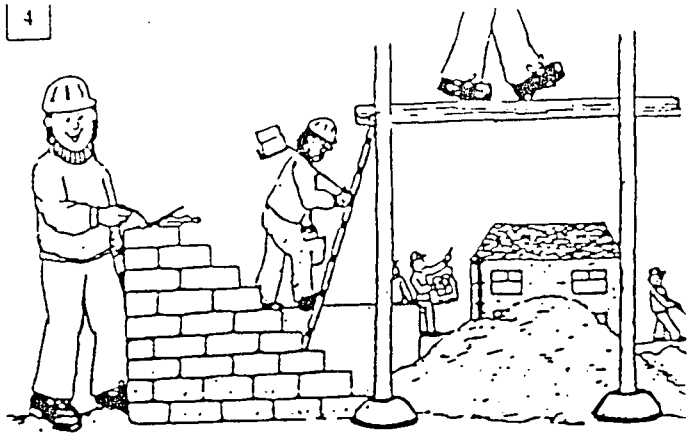
1.6

3

A petrol pump attendant is putting petrol into a car. The driver, who sometimes uses self-service petrol stations, will soon be able to drive away.

1.7

4



Many people work on the building site. They are involv in clearing, carrying, building, planning and supervisin

5

A child is throwing stones into a pond, and watching ripples spread outwards.

6

A gardener, surrounded by a range of garden equipm is tending some of the different types of plant in garden. He prefers the garden as it is, but realises there are many types of garden.

APPENDIX B

Sample questions

What does it mean to you to teach math?

TEACHER
How would you describe your role as a math teacher?
Attributes of a good math teacher

MATHEMATICS
How would you characterize/describe math?
What message about math do you want to give?

STUDENTS
What is a student's job?
How do they learn math?

PEDEGOGICAL CONT. KNOWLEDGE
Can anyone teach math?
What (k, s, a) do they need?
What are effective teaching strategies?
Cf other subjects

PHILOSOPHY
motto, metaphor
diagrams
(philosophy)

Have your views of teaching math changed from the beginning of the year?

APPENDIX C

Sample notes

BA — 3
Eco.
BSc. Hon.
MSc.
Discrete
B.Ed.

(1) Philosophy

What does it mean to you to teach math?

TEACHER

(2) How would you describe your role as a math teacher?
Attributes of a good math teacher

(12) art or science?

MATHEMATICS (4) Discrete math.
How would you characterize/describe math?
What message about math do you want to give?

(5) Math in H.S.

(6) love of math.

(3) STUDENTS
What is a student's job?
How do they learn math?

PEDEGOGICAL CONT. KNOWLEDGE
Can anyone teach math?
What (k, s, a) do they need?
What are effective teaching strategies?
Cf other subjects

"concept"

PHILOSOPHY
motto, metaphor
diagrams
(philosophy)
Have your views of teaching math changed from the beginning of the year?

(10) painting

post-discussion on "maturity" + life

Appendix D

Sample Transcript Excerpt

This excerpt is taken from pages 5 to 9 of the interview with Victoria. She and the interviewer are identified by V and I respectively.

I: Ok, ok. Now, you have a degree in mathematics?

V: Um, my second degree, yep.

I: Ok, what, what kind of, what's your academic background?

V: Um, my first degree is a Bachelors, a Bachelors of Arts with Distinction in Economics. My second degree is a Bachelors of Science Honours in Mathematics. My third degree that I've been working on is my Masters of Science in, um, discrete mathematics, and my fourth degree, which I'm doing right now, is my Bachelor of Education, in mathematics.

I: Ok, so your B.A. in Economics is a 3 or 4-year?

V: 3-year.

I: 3-year. And your B.Sc. Honours?

V: 4-year.

I: 4-year? Ok. And what was your major in that?

V: Um, mathematics. Um, other one's, economics.

I: Ok, so, really you could describe yourself as a mathematician.

V: Yeah.

I: Ok. How would you characterize [V: or professional student (laughter)] No...how would you characterize math?

V: Ok, you're going to have to do a bit more of.... What do you mean by characterize math?

I: What adjectives would you use to describe mathematics?

V: Ok, I'm going to start an' use, I'm gonna take a look at my area in discrete mathematics and, I actually think, I think it's fun? um, and I think a way to do it would be, I mean I describe, I can describe discrete mathematics to anybody walking on the street and they can understand it because it's glorified, what I do is glorified connect-the-dots. Everyone knows how to connect the dots! Ok. We're just gonna do it a heck of a lot more and a little bit tougher, basically. I do hypergraphs which is multiple lines, and this type of thing. Um...so, I guess it's, it's different because when you're an, as an undergrad you have the right or wrong and I mean you have, you'll have, you're doing a proof? And there's different ways of doing the proof, but you know you're trying to prove the same thing in the end. I mean you're going from, um, if epsilon delta type thing, ok prove this is continuous, prove it's discontinuous. You can't have something that's well, it's sort of continuous in this area. It's either continuous or not. Whereas when you get into grad math, you start getting into things, um, sort of the proofs of - well, we don't know if it's true or not yet, ok? We're trying to prove it, and it's up, it's up to you to make sure you're making the right choices and so it might mean intuitively, you sort of go, well it SHOULD happen... Well, if it should

happen then prove it. Um, and they don't know. And I mean you're in the fuzzy grey areas there, you're getting into edges of math, [I: oh really?] rather than, rather than this known area. 'Cause you're starting off, I mean calculus is pretty much known, ok? You move out further, and further, and my, I do hypergraphs or, um, set theory and, I'm, I was working under one of the top guys in the field. Um, and it's sort of, a very, it's, it's, it's, you're just still discovering, you're still creating it, ok? It's still being created. And like that to me is neat because I know the math that we teach in high schools or like go out, it's been here forever. I mean our parents did this, our grandparents did this. Nothing's changed. Whereas kind of, no, it's not that, it's changing all the time.

I: Is it changing at the level that the students are learning about math or, or can you con-, convey that to the students? Is it reasonable to convey that to them?

V: Um, hmm. I think what's more reasonable to convey to them is how the advent of technology is changing how we do math. I mean as far as, well it's nice to know how multiplication and all that works, but I don't expect you to sit, to have to sit down and longhand do 158 million dot etc. etc. etc. I mean a horrible number times whatever divided into something because you punch it into your calculator. So it's not really worth your time and effort because I know you can do it, so that, so we'll just use the calculator to do it, ok? Whereas we used to have to use slide rules, etc, etc. to prove or to go through that. Well now, we don't have to, now we're not asking pure "do it" type questions, we're now asking questions that actually require more thought.

I: Ok, we. Who's we?

V: Um, we being mathem-, being teachers in general of math. [I: ok...] We're not, we're not asking, we're not, we're not gonna give a grade 12 exam and tell students to multiply this by this or do long division on this. Long division, by the way was something that at one point in time only 5 people in the world could do, ok? And it's the same thing as with negative numbers. Negative numbers are something, um, that many mathematicians wrote thesises [theses] on and wrote papers on why negative numbers can't exist. So that, I thought that was kind of neat. But, um, I think it's hard to because where they're at in math is a known area and it's not changing all that much. I think you can bring some of the new stuff in possibly, but it, it, what they're dealing with, is not, I mean, you're not going to suddenly have equations of lines changing desperately. I mean it's known what a line is, and how to solve it. We're not having that much change happening there. The changes in how math is being taught are more technology-based.

I: Ok. That's really intriguing. I'm really glad you can add that kind of perspective on it. Because a lot of - [laughter] no, truly, because a lot of, um, student teachers, well, people who are getting their B.Ed.'s don't have an in-depth mathematical background as you do. [laughter] Can we, ok, we'll go back to that [V: ok] because I want to continue, of course, that teaching aspect. And I'd like to go back to the field of mathematics.

V: Ok, I've sidetracked on that.

I: No, no, that's fine. That's perfect. Um, you made one statement and you said, well math is created or, or discovered, and is it one or both, or what?

V: Um, I think the new theories are proposed, I guess, I mean, you're, you're making hypothesis, um, and therefore that's a discovery, it's a discovery of the hypothesis. Then you do the proof, and that's creating. I mean the two are tied together, I don't know if you can really distin-, distinguish between them. Because you're not, what we've done, is we've set, we've set in stone to some extent the basis on which math is built. And now other scientists are using this math, ok? So we're, we're the building blocks of other sciences.

I: For example, algebra, [V: um -] some finite math at the basic level...

V: At the basic level. And I mean, I mean chemistry use-, uses us a lot, biology uses us, physics uses us, and I mean a lot of our math, the higher level, we're trying to closely approximate what they're doing in physics, and what they're doing in chemistry. We're trying, we're trying to get the stuff written before they need to use it, so when they need to use it they can just pull this thing out and go, great this is a tool I need to use. We're creating tools, basically.

I: Is that the purpose of, of developing mathematics?

V: Um, yeah, basic-, I mean, math, for me I find math a joy and beauty and everything else in math, but, nobody's going to fund, no company's going to go out there and, um, fund mathematicians or say that mathematicians are at all useful unless they actually create something, ok? I'm not saying that every mathematician's out there is going to create something wonderful or anything, but you need to have somebody occasionally creating a tool that will be useful in the real world. Otherwise, math will never get funding and people will kind of go, well, my computer, I will put this into my computer. I don't need to know how to do this, and that's, that's dangerous.

I: Let's talk about humans and math.

V: [laughter] Ok!

I: What does it take for someone to be good at math? And if you think it's different at different levels, please say so.

V: Oh, definitely. Um, definitely very different going from, I'm just going to look at it, um, in elementary school, it's pure memorization.