COGNITIVE ABILITIES THAT UNDERLIE MATHEMATICS ACHIEVEMENT:

A HIGH ABILITY PERSPECTIVE

by

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ABSTRACT

In this study the cognitive abilities underlying math excellence among children are examined, with a focus on children of high mathematical ability. The relationship between cognitive functioning—as defined by the Cattell-Horn-Carroll (CHC) theory—and academic achievement among children who excel in mathematics is explored in order to understand whether strong math skills correspond to any "typical" cognitive ability profile(s). Results suggest that Short-Term Memory, Working Memory, and Visual/Spatial Thinking are significant predictors of strong and specific achievement in math calculation skills, whereas Fluid Reasoning is a significant predictor of strong and specific achievement in math reasoning. The results outlined in this study may supplement the existing research body relating to the full range of mathematics ability.

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CHAPTER ONE:

Introduction

The mastery of mathematics is important to academic success at school. Today's children are brought up in a technologically-oriented society, in which knowledge of arithmetic and mathematical problem solving are key for many aspects of life, including education attainment and future employment prospects. Despite reported risks and negative life outcomes of individuals who experience difficulties in mathematics, research on the successful mastery of mathematics among school children has lagged behind research on other academic subjects, such as reading (Geary & Hoard, 2001; Gersten, Jordan, & Flojo, 2005; Light & DeFries, 1995). Furthermore, the literature associated with the study of cognitive traits underlying math proficiency is fragmented and inconsequential, covering only a narrow range of math domains and operations and, for the most part, omitting key cognitive abilities required for the understanding of mathematical proficiency (Floyd, Evans, & McGrew, 2003; Proctor, Floyd, & Shaver, 2005).

The literature is particularly sparse when pertaining to children of high ability. Generally, gifted children have received scant attention in the professional literature from psychologists and educators (Winner, 1996). We know far more about intellectual deficiency and learning problems than we do about giftedness. In the words of Ellen Winner (2000, p. 3):

> Psychologists have always been interested in the deviant.... Research on retardation is more advanced and more integrated into the field of psychology than is research on giftedness.... It also surely reflects the view that retardation is a problem researchers may eventually learn to alleviate, whereas gifts are privileges to be admired or envied rather than problems in need of solution.

Yet, a better understanding of one end of the ability spectrum can shed light on the other.

According to Robinson, Zigler, and Gallagher (2000, p. 1):

> Professionals in the fields of mental retardation and giftedness have much to teach each other.... Examining the commonalities and differences between the fields in social issues, definitions, developmental differences from the norm, values and policy issues, and educational and long-term implications deepens insights about both normal and deviant development.

Thus, research results relating to high achieving children in math, in addition to filling a gap in our understanding of exceptional children and contributing to an important and understudied research area, could also advance research relating to low achievers in mathematics.

In this study the cognition of math achievement among high achieving children was examined, building upon and integrating research results in two areas: (a) the cognition of mathematics, and (b) students with high ability. Complementing and adding upon recent work by Proctor, Floyd and SHaver (2005) who studied the cognitive traits underlying low mathematics achievement, this study examined the cognitive profiles of high achievers in mathematics, using as a framework the Cattell-Horn-Carroll (CHC) theory psychometric theory of intelligence. CHC theory integrates the Cattell-Horn Gf-Gc theory (Horn & Cattell, 1966), which distinguished "fluid" from "crystallized" intelligence, and Carroll's Tri-Stratum theory (Carroll, 1993) that depicts intelligence as a three-level structure: narrow, broad, and general. CHC theory was first put to practice in the Woodcock-Johnson, Third Edition (WJ III) assessment and interpretation battery (McGrew, 2001; Schrank, Flanagan, Woodcock, & Mascolo, 2002) and was then extended to encompass cross-battery assessments (Flanagan, Genshaft, & Harrison, 1997; Flanagan & Harrison, 2005; McGrew, 1993; Woodcock, 1990).

Using CHC theory as a framework, the cognitive profiles of children were examined in order to find out if there are typical cognitive profiles that characterize high achievement in mathematics. Given the view that high achievement and underachievement represent two opposing portions of the ability spectrum, the cognitive-profiles characteristic of high ability in mathematics would be expected to constitute a mirror image of those characteristic of mathematical deficiency. In other words, an increased level of certain cognitive factors—those most strongly associated with math achievement—could underlie high mathematical ability, whereas a decreased level of the same cognitive factors could underlie low mathematical ability. Therefore, a better understanding of cognitive abilities underlying math proficiency could broaden our understanding of math underachievement and thus be indirectly instrumental in devising and optimizing programs that address mathematical deficiencies.

Definition of Key Terms

Several key concepts pertaining to this study—cognitive ability, high ability, and math achievement—are defined below, as their meaning is often misunderstood or the terms are used in different ways by researchers in the fields of education and psychology.

Cognitive Abilities. While contemplating how the brain represents information and its processing, several questions come to mind, such as: how do humans acquire and manipulate knowledge, and what underlies mental abilities and disabilities. Answers to these questions pertain to the study of cognition. Cognitive ability describes the process and results of information processing, including perception, conceptualization, and problem solving. The term is frequently used in psychological assessment as a synonym for intelligence—a term that has multiple and diverse definitions. In this study the term "cognitive abilities" is used in a manner consistent with the primary cognitive-assessment instrument—the Woodcock-Johnson III Tests of Cognitive Abilities (WJ III COG, Woodcock, McGrew & Mather, 2001a))—and the CHC Theory that underlies it.

High Ability. High-ability individuals represent the positive anchor of the ability spectrum. They are usually thought of as "intuitive" and "perceptive," and excel in problem solving and in the ability to generalize and verbalize concepts (Sriraman, 2003). Definitions of intellectual high ability vary, ranging from Humphreys' (1985) traditional definition of highly able individuals as occupying the high end of the behavioural dimension of intelligence, to Sternberg's (2001) view of high ability as a form of expertise manifest in faster and more accurate problem-solving capabilities, improved knowledge organization, automation, information-processing skills, analytical capabilities, and the ability to adapt a solution to real-world constraints. Non-cognitive factors such as intuition, (Sak, 2004), and affective and motivational attributes (Porath, 1996; 2000), are often thought of as an important part of high ability, but are difficult to assess objectively. Therefore, a narrow operational definition is often used to categorize individuals with high ability—the intellectually gifted—using an arbitrary cutoff of standardized intelligence test or achievement test scores, such as above 120 (Roid, 2003) or above 130 (Zigler & Farber, 1985). For the purpose of this study, children with moderately high ability and children with very high achievement in mathematics are defined as those performing with a standard score of 115 or higher (84th percentile) and those performing with a standard score of 125 or higher (95^{th} percentile) respectively, on clusters assessing mathematics achievement on the Woodcock-Johnson III, Tests of Achievement (WJ III ACH, Woodcock, McGrew & Mather, 2001b).

Mathematics Achievement. Math achievement for schoolchildrenchildren signifies a certain level of attainment in mathematics expected for a given age or grade level and demonstrated by performance on tests assessing math skills. Assessment instrument publishers such as the authors of the WJ III ACH (Woodcock, McGrew, & Mather, 2001b), have created tests that assess children's arithmetic and problem-solving skills in comparison to age or grade peers. The present study focused on two areas of achievement in mathematics as measured on the WJ III: math calculation skills which involves the application of mathematical operations and basic axioms to solve computational problems; and math reasoning, which involves the use of knowledge of math operations and quantitative concepts to solve novel mathematical problems.

Cognitive Ability Profiles. In this study, cognitive ability profiles comprise one or more broad Cattell-Horn-Carroll (CHC) cognitive abilities that were found significant predictors of a measure of mathematical achievement (math achievement clusters and tests) and are ranked according to their importance as unique predictors of the mathematics measure. Multiple Regression Analysis is used in the present study to yield cognitive ability profiles corresponding to groups of participants of various achievement levels and regression condition, with focus on children who display normative strength in math calculation skills and math reasoning. The cognitive profiles obtained in this study are compared with results reported in the literature. Of a special interest are results reported by Proctor, Floyd and Shaver (2005) who examine the broad CHC cognitive profiles of children who display normative weakness in math calculation skills, using statistical tests grouped under the rubric of Profile Analysis (Tabachnick & Fidell, 2001) to compare the group profiles of low achievers in mathematics with average achievers. Proctor et al.'s have used the Parallelism test to determine if the patterns of highs and lows on the CHC factor clusters were similar across groups; the Flatness test to determine profile scatter; and the Levels test to determine if low achieving groups scored significantly lower than average achievers on the set of CHC factor clusters.

Purpose of the Study

In this study the correlation between cognitive abilities and high achievement in mathematics was investigated. Specifically, the role that the abilities measured by the WJ III COG (McGrew, 2001) played in achievement for children of high mathematics ability—as measured by the WJ III ACH—was explored and compared to that of children of average mathematics achievement. The study's sample of major focus was further limited to children who were no more than average readers, in order to focus on normative achievement strength specific to math, rather than on children with generally high overall ability. A non restricted sample was also used for comparison.

The following research questions were addressed:

1. Which CHC abilities, as measured by the WJ III COG [specifically, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Short-Term Memory (Gsm), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), and Visual/Spatial Processing (Gv)], and additional clusters, as measured by the WJ III COG [specifically, Working Memory (Gwm[1]), are the best predictors of math calculation skills, as measured by the WJ III ACH cluster [specifically, Math Calculation Skills] and tests comprising that cluster [specifically, Calculation and Math Fluency], for the group of students who excel in math calculation skills?

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[1] An arithmetic mean of the Numbers Reversed and Auditory Working Memory tests was used to estimate the Working Memory cluster score.

2. Which CHC abilities, as measured by the WJ III COG [specifically, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Short-Term Memory (Gsm), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), and Visual/Spatial Processing (Gv)], and additional clusters, as measured by the WJ III COG [specifically, Working Memory (Gwm)], are the best predictors of math reasoning, as measured by the WJ III ACH cluster [specifically, Math Reasoning] and tests comprising that cluster [specifically, Applied Problems and Quantitative Concepts], for the group of students who excel in math reasoning?

3. Which CHC abilities—as measured by the WJ III COG [specifically, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Short-Term Memory (Gsm), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), and Visual/Spatial Processing (Gv)],and additional clusters, as measured y the WJ III COG [specifically, Working Memory (Gwm)[1]]—are the best predictors of math calculation skills, as measured by the WJ III ACH Math Calculation Skills cluster, and which CHC abilities are best predictors of math reasoning as measured by the WJ III ACH Math Reasoning cluster, for the group of students who excel in both math calculation skills and in math reasoning?

4. Which CHC abilities—as measured by the WJ III COG [specifically, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Short-Term Memory (Gsm), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), and Visual/Spatial Processing (Gv)], and additional clusters, as measured by the WJ III COG [specifically, Working Memory (Gwm)[1]]—are the best predictors of math calculation skills, as measured by the WJ III ACH cluster [specifically, Math Calculation Skills] and tests

comprising that cluster [specifically, Calculation and Math Fluency], and which CHC abilities are best predictors of math reasoning, as measured by the WJ III ACH cluster [specifically, Math Reasoning] and tests comprising that cluster [specifically, Applied Problems and Quantitative Concepts], for the group of students who are average performers in math calculation skills, math reasoning, and reading?

Significance of the Present Study

The present study is important for several reasons. It contributes to the research examining the CHC broad cognitive abilities associated with math achievement and contributes to the scant data relating to an exceptionality group, providing further understanding of whether a typical set of CHC broad cognitive abilities, or profile (s), characterize children with mathematical giftedness. Furthermore, understanding the group and individual profiles of children with mathematical strength could also contribute to understanding the profiles of children with mathematical weakness, as both exceptional groups represent two extremes of the ability continuum. Thus, results of this study could supplement the existing research body relating to the full range of mathematics ability and identify empirically-supported correlates between child cognitive traits and math achievement that could benefit clinicians and researchers in diagnostics and educational practices (Flanagan, Ortiz, Alfonso, & Mascolo, 2002; Sternberg & Grigorenko, 2002).

CHAPTER TWO:

Review of Relevant Literature

State of The Art of Mathematics-Related Studies

The mastery of mathematics is a key element of modern society. A sizeable population of school children shows difficulties in math; by some accounts, 10-15% of the school-age population experience mathematical difficulties (Badian, 1999; Fleischner & Manheimer, 1997), where 5-8% reportedly show remarkable deficiency in math achievement (Geary, 2004; Proctor, Floyd, & Shaver, 2005). This statistic presents a challenge to a society that demands at least minimal math competency for success in formal schooling, daily living (e.g., filling tax forms, understanding one's bank statements, or figuring out a sale price), and employment (Geary & Hoard, 2001; Light & DeFries, 1995). Nevertheless, math achievement has not been studied to the same extent as reading proficiency (Gersten et al., 2005). In particular, not enough is known about cognitive factors that underlie mathematical disability (Geary, 2005; Rourke & Conway, 1997) and even less is known about cognitive factors that underlie mathematical giftedness.

Different reasons could account for the relative scarcity in math-related studies. To begin with, in comparison to reading proficiency, for which phonological awareness serves as a predominant prerequisite for skill mastery (Gersten et al., 2005), the field of mathematics studied at school encompasses a multifaceted array of subdomains that tends to grow in complexity at advanced levels of math, including numerical calculations, word problems, and geometry, and with specialized applications such as the estimation of mathematical quantities, the interpretation of graphs and charts, and the handling of time and money. Of the wide array of math domains, most research-to-date involving the cognition of mathematics has tackled only a narrow set of competencies related to numerical and arithmetical operations, such as counting and simple arithmetic (Geary, 2004).

In comparison, little research has been conducted on children's ability to solve more complex arithmetic problems (Russell & Ginsburg, 1984) and not enough is known about most mathematical domains, such as geometry and algebra (Geary, 2005). Because different subsets of mathematics could pose different demands on the child's cognitive traits, many variables must be considered in studying math achievement in order to reach meaningful conclusions relating to math proficiency (Geary, Hoard, & Hamson, 1999), thus introducing a wider range of variability in the results (Geary, Hamson, & Hoard, 2000).

The complexity of the field may explain the relative lack of consensus among researchers in math cognition and the fragmented research body, which lacks a common conceptual framework for cognitive traits that underlie math achievement. This is further complicated by environmental variables such as home and school settings (Walberg, 1984) and math instruction (Carnine, 1991; Russell & Ginsburg, 1984), all of which have been shown to affect math achievement. Studies analyzing the cognitive traits underlying math proficiency among low-achieving children—the predominant research group—may be further complicated by a lack of consensus on what constitutes math difficulties. Generally, a difficulty is perceived when a child's performance on a math achievement test falls below a certain cutoff point (e.g., the bottom 10th, 25th, 31st, 35th, or 45th percentile), or if the learning disability in math is perceived to be biologically-based (Mazzocco, 2001). Even less agreement has been reached on a definition of mathematical giftedness.

Reported Cognitive Abilities Underlying Math Disability

The cognitive abilities on which most investigations of math proficiency have focused so far are: *retrieval* from *long-term memory* (Geary, 1990; 1994; Geary, Brown, & Samaranayake, 1991); *working memory*—holding and manipulating information in short-term memory (Conway & Engle, 1994; Geary, 1994; Hitch & McAuley, 1991; Passolunghi & Siegel, 2004; Shafir & Siegel, 1994; Swanson, 1994); and *processing speed*—the speed with which we perform simple cognitive operations (Bull & Johnston, 1997; Geary, 1994). Recent studies (Floyd, Evans & McGrew, 2003; Proctor et al., 2005) have also investigated the significance of *fluid reasoning* – the ability to solve problems and reason with novel information – with regard to numerical information. According to Geary (2005), mathematical disability can result from deficits in information representation or processing. For example, children with a mathematical learning disability are often characterized by poor conceptual understanding of some aspects of counting, resulting in delayed competencies in the use of counting to solve arithmetic problems, and poor skill at detecting and, thus, correcting counting errors (Geary, 2004; Geary, Bow-Thomas, & Yao, 1992). Overall, opinions about the etiology of mathematical difficulties, on the one hand, or mathematical strength, on the other, vary widely.

The most consistently-reported findings in the literature regarding children with a math learning disability are difficulties in storing arithmetic facts, such as number combinations, or in accessing them from long-term memory to solve simple arithmetic or word problems (Barrouillet, Fayol, & Lathuliere, 1997; Garnett & Fleischner, 1983; Geary, 1990; 1993; Geary et al., 1991; Hanich, Jordan, Kaplan, and Dick, 2001; Jordan & Montani, 1997; Jordan, Hanich, & Kaplan, 2003; Ostad, 1997; 1998; 2000; Russell & Ginsburg,1984). Recent studies have suggested that there may be two forms of retrieval deficit (Barrouillet et al., 1997; Geary et al., 2000). One appears to involve a straightforward deficit in the ability to retrieve facts from a semantics-based long-term memory network. The second results from a disruption of the retrieval process due to difficulties in inhibiting the retrieval of irrelevant information (Passolunghi & Siegel, 2004). Furthermore, differences are reported in the strategic and memory-based processes used by children with math learning disability and their typically-achieving peers (e.g., Barrouillet et al., 1997; Geary, 1990; Gross-Tsur, Manor, & Shalev, 1996; Jordan & Montani, 1997; Ostad, 1997; 1998).

A mathematics learning disability could also manifest as a deficit in conceptual or procedural competencies in mathematics due to underlying deficits in the *central executive* or in the information representation or manipulation systems of the language or visuospatial domains (Geary, 2005). The central executive controls the *attentional* and *inhibitory* processes used during problem solving. Children with math learning disability have some form of working memory deficit (Hitch & McAuley, 1991; McLean & Hitch, 1999; Passolunghi & Siegel, 2004; Siegel & Ryan, 1989; Swanson, 1993) that appears to involve information representation and manipulation and reduced inhibition of non-target and irrelevant information in memory (Passolunghi & Siegel, 2001; 2004). These children tend to show impaired performance in nonverbal working memory (Siegel & Ryan, 1989) and slow processing speed (Geary & Brown, 1991). McGrew et al. (1997) also report a strong predictive relationship of working memory and processing speed to success in math calculation skills. Passolunghi and Siegel (2001) report a relationship between short-term memory, working memory, inhibitory control, and math reasoning where the latter is affected by the ability to reduce the accessibility of irrelevant information in memory. Wilson and Swanson (2001) have also reported a relationship between mathematics ability and working memory across a broad age span. LeFevre et al. (2005) also report a strong connection between working memory and mathematics that explains the well-documented effects of the problem complexity paradigm (why problems involving a larger number of steps are more difficult than those with fewer steps) in terms of working memory load contributing to math disability in children. Studies on mathematics anxiety—a negative reaction to math and to mathematical situations—also suggest cognitive consequences affecting performance, especially given the heavy demand on working memory resources (Ashcraft & Ridley, 2005).

There is much disagreement on the role of *visual-spatial abilities* in mathematics proficiency, to some extent reflecting a lack of consensus on the definition of visual-spatial thinking (Dehaene et al., 1999; Geary, 1996), suggesting that deficits in visual-spatial abilities could translate into a learning disability manifesting in certain areas of geometry and affecting the child's ability to solve complex word problems. On the other hand, Prevatt and Proctor (2003) report that college students affected by a math difficulty displayed, on average, a relative strength in visual-spatial processing and a relative weakness in long-term retrieval. Barnes, Chant, and Landry (2005) also report that visual-spatial skills of subjects with a neuro-developmental disorder do not appear related to deficits in multi-digit calculations. Furthermore, Floyd et al. (2003) report insignificant correlations between visual-spatial thinking and mathematics proficiency for the general population..

Relatively little has been reported in the literature about cognitive abilities underlying math *problem-solving* skills. Children with a math learning disability show developmental delays in the adoption of complex procedures and reductions in their ability to detect procedural errors (e.g., Ohlsson & Rees, 1991). McGrew et al. (1997) report a strong and ongoing relationship between *general intelligence* and math across all developmental levels and a strong relationship between *fluid reasoning* and the ability to solve applied mathematics problems, and fluid reasoning appears strongly associated with applied mathematical reasoning tasks such as problem solving, number reasoning and algebra. *Estimation* of mathematical values—an important element of mathematical cognition that requires the application of prior knowledge and flexible translation from one numerical representation to another (Siegler & Booth, 2005)—is reportedly correlated with *fluid reasoning* and *visual-spatial* skills (Barnes et al., 2005). Indeed, Hanich et al. (2001) report that children with a math learning disability have a limited ability to make arithmetic approximations, regardless of their reading or language capabilities and show deficits associated with *problem-solving speed*, particularly in solving story problems and orally-presented ones. Children show substantial individual differences in computational estimation (Dowker, Griffiths, Harris, & Hook, 1996), that correlate positively and often substantially to IQ (Reys, Rybolt, Bestgen, & Wyatt, 1982) and to various measures of math achievement (Siegler & Booth, 2004). According to Siegler and Booth (2005), problems in estimation skills appear to stem from difficulties in attaching numbers to magnitudes. Dixon (2005) shows that children can and do access previously solved problems when asked to generate mathematical solutions to new problems. Mapping the structure (schemata) of the current problem to that of the stored problem requires conceptualization of the information retrieved and is thus strongly related to the cognitive factor of *fluid reasoning*.

In Search of Cognitive Profiles for Low Mathematics Achievement

In a recent interesting study, Proctor et al. (2005) examine the question of whether there exists a "typical" cognitive profile—expressed in terms of performance on seven CHC cognitive factors—that characterizes low achievement in mathematics. Achievement in mathematics has been studied in two areas: math calculation skills—the application of mathematical operations and basic axioms to solve mathematical problems; and math reasoning—the ability to solve mathematical problems by using knowledge about math operations and quantitative concepts. Proctor et al. (2005) reports that a common cognitive profile of children with normative weaknesses in math reasoning appear to show weaknesses in specific cognitive abiliites (Fluid Reasoning and, perhaps, Comprehension–Knowledge). On the other hand, there may not be a unique profile of cognitive abilities for children with normative weaknesses in math calculation skills. As mentioned above, literature reports suggest a correlations between specific cognitive abilities and low achievement in math calculation skills. Thus, additional information is therefore warranted to better understand the cognition of math proficiency.

In summary, researchers have reported associations between math calculation skills and long-term memory, working memory, processing speed, executive processes and visual-spatial abilities, although different studies have assessed different arithmetic functions, and some findings, especially relating to visual-spatial abilities, are inconsistent. There is less information in the literature on cognitive abilities that underlie math reasoning, although fluid reasoning, long-term memory, general intelligence and, according to some accounts, visual/spatial abilities, are cited as correlates of mathematical problem solving.

High-Ability Perspective

Atypical individuals such as children with developmental disability and intellectually-gifted children, while representing different ends of the ability spectrum, appear to operate under similar stresses, having to cope with the perception of being deviant from the norm (Zigler & Farber, 1985). Such individuals have always been of special interest to psychologists (Winner, 2000), drawing professionals to use similar conceptual models in representing both exceptionality groups. Yet, in contrast to the focus in recent decades by psychologists and educators on low-achieving or learning-disabled children, understandably triggered by societal obligation to better diagnose and help remediate their performance, not enough research has been conducted on high-ability children (e.g., Whalen, 1999; Winner, 1996; 2000). The latter have been considered by researchers, educators, and the public-at-large as privileged and able to fend for themselves by virtue of their giftedness and, thus, not a group requiring the attention of researchers and educators.

The literature concerning children of high mathematics ability is scarce and there is little agreement among researchers as to cognitive factors underlying mathematical gifts. Several studies comparing gifted children to average and learning-disabled children suggest a higher variability in the performance of gifted children along different domains, than that observed among non-gifted children (Horowitz & O'Brien, 1985; Keating & Bobbitt, 1978; Marr & Sternberg, 1986; Siegler & Kotovsky, 1986; Sternberg & Davidson, 1986). Thus, some authors report that gifted children show significant discrepancies between verbal and performance IQ scores (Benbow & Minor, 1990; Lewis, 1985; Silver & Clampit, 1990; Winner, 2000. In a study on CHC factor clusters of the WJ III COG, Krasner (1998) conducted a profile analysis of gifted and non-gifted individuals, in terms of the seven CHC factors utilized in the tests. The gifted group scored, on average, consistently higher than the non-gifted group across the set of CHC factor clusters, although Krasner (1998) found no significant intra-cognitive differences among the clusters for either the gifted or non-gifted group. At present, research on the cognition of mathematics is scarce and researchers appear divided in their views accounting for mathematical giftedness.

Dark and Benbow (1991) report that excellence in mathematics is associated with exceptional *visual-spatial* skills utilized in mathematical problem solving, suggesting that spatial abilities underlie mathematical giftedness (Benbow & Minor, 1990; Benbow et al, 1983; Gardner, 1983; Hermelin & O'Connor, 1986; Krutetskii, 1976). It is reasonable to assume that spatial abilities are instrumental in solving higher-level algebraic word problems through the diagramming of important relationships in the problems. Recent research on the spatial representation of numbers suggests that processing numerical magnitudes and processing *visual-spatial information* are functionally connected, that the meaning of numbers is spatially coded in the brain, and that spatial associations are attached to numbers as part of our strategic use of knowledge and skills (Fias & Fischer, 2005). According to Hadamard, (1996), Einstein and other great mathematicians explicitly emphasized the role of visuo-spatial imagery on their mathematical ideas.

According to Geary and Brown (1991), the primary difference observed when comparing math calculation skills of children of high mathematics ability with average children was the effectiveness of *retrieving* from *long-term memory*, information used to solve counting problems, resulted in fewer counting trials and a lower proportion of counting errors for the gifted group relative to the non-gifted group. Geary's (1990) parallel report of the difficulties characteristic of children with math learning disability to store and access information from long-term memory, as discussed earlier, could lend support to viewing the cognitive factors underlying mathematical giftedness as a mirror image of those underlying mathematical challenge.

According to Dark and Benbow (1990; 1991), children of high mathematics ability surpass verbally-precocious children in *working-memory* manipulation. Thus, mathematical talent includes a superior ability to represent and manipulate information in *short-term memory.* Gifted children have reportedly demonstrated a higher level of *processing speed*—the rate with which they execute basic elementary processes—compared to less able peers (Geary & Brown, 1991; Keating & Bobbitt, 1978). On the other hand, gifted examinees on the Wechsler Intelligence Scale for Children (WISC-III) were reportedly slow to respond on timed tests (Kaufman, 1994), suggesting that *processing speed* may not be correlated with giftedness. The latter could be consistent with the relatively low *g*-loading of processing speed (Carroll, 1993) and could explain a lesser role in giftedness.

Fluid reasoning appears strongly connected to mathematical and other intellectual giftedness. Roid (2003) reports that children characterized as cognitively gifted in the standardization sample for the Stanford Binet Intelligence Scales, Fifth Edition (SB5), show a relative strength in fluid reasoning and *quantitative reasoning* compared to their full-scale IQ, and a relative weakness, compared to their IQ, in working memory. The significance of fluid reasoning to giftedness is corroborated by Montague and van Garderen (2003), who report that mathematically gifted students scored significantly higher than average and learning-disabled students in their ability to estimate discrete quantities, and in their knowledge and use of *estimation strategie*—a skill consistent with abstraction and reasoning abilities. Furthermore, mathematically talented students were reportedly better than other groups at translating linguistically presented mathematical information into an equation form necessary for reaching a correct solution (Dark & Benbow, 1990), thereby demonstrating developmentally mature strategy approaches to problem-solving tasks (Siegler & Kotovsky, 1986). In comparison to children with math disability who tend to select developmentally immature strategies, Friedman and Shore (2000) note a difference in the extent to which children of high mathematics ability invoke different strategies and the *fluency* and *speed* with which they are used, although gifted learners do not seem to use strategies that are novel to others.

Pesenti (2005) notes that children of high mathematics calculation ability rely on perfect *knowledge* of basic arithmetic operations, perfect familiarity with complex calculation algorithms, increased number-specific *short-term memory* capacities, and increased number-specific *long-term memory* capacities, and also on the application of automated algorithms and careful *monitoring and control.* Thus, gifted children differ from others along several cognitive traits and in the extent to which they draw on a repertoire of intellectual skills available to others.

In summary, high mathematics ability has been correlated with visual-spatial skills, long-term memory, short-term memory, working-memory, fluid reasoning, quantitative reasoning, knowledge, strategy formulation, monitoring and control, and processing speed, although—as in studies pertaining to low-achieving children—contradictory results (e.g., as to correlation with visual-spatial skills and processing speed) are sometimes reported. Geary and Brown (1991) suggest that further studies are needed to determine how various cognitive traits might differentially contribute to the differences noted among achievement groups on tasks of varying complexity and in wider domains of mathematics. Studies of high ability children, in addition to advancing our knowledge of this little-studied exceptionality group, could strengthen our understanding of the cognitive abilities that underlie low mathematical achievement, the specific cause of which remains elusive.

CHC Theory: A Cognitive Framework for Studying Math Proficiency

Many questions have been raised concerning the nature of representations and processes underlying mathematical cognition (Fayol & Seron, 2005). It is not always easy to distinguish between a child's difficulty to master complex material and an actual *cognitive disability*— a disability that impedes learning despite appropriate instruction (e.g., Fuchs, Fuchs, & Prentice, 2004; Geary et al., 1991)—and to assess the relative impact of environment upon genetic factors

(Light & DeFries, 1995; Shalev et al., 2001). Similar considerations would apply to mathematical giftedness.

Early efforts to establish an association between psychological processes and academic achievement were hindered by vague definition, and measurement, of such processes (Hale et al, 2001). Progress was also encumbered by the tendency of many early researchers to concentrate on limited features of mathematics proficiency (e.g., Hanich et al., 2001) and to disregard cognitive traits that are significant to mathematical proficiency, such as *inductive fluid reasoning* (Floyd, Evans, & McGrew, 2003; Proctor et al., 2005). The latter could reflect reliance in earlier studies on assessment instruments that offered limited scope of the ability spectrum.

In recent years, the Cattell–Horn–Carroll (CHC) theory of cognitive abilities (Carroll, 1993; Horn & Cattell, 1966; McGrew, 1993; Schrank et al., 2002; Woodcock, 1990; Woodcock et al., 2001) has had a significant impact on the measurement of cognitive abilities and the interpretation of intelligence test performance. CHC theory has thus been instrumental in the development of assessment instruments that cover a wide range of cognitive traits. It is a hierarchical framework describing human cognition, comprising three strata. Stratum III (general intelligence) represents a child's overall level of cognition and corresponds to the overall, or global score (IQ), in a test of cognitive functioning. Stratum II consists of a number of broad cognitive abilities representing a child's functioning in a broad cognitive area such as visual-spatial skills, processing speed and fluid reasoning. Stratum I consists of over 70 narrow cognitive abilities representing more specific traits. Because of the width and depth of coverage of human cognition offered by CHC theory, and because it is a psychometric theory of intelligence based on assumptions that "the structure of intelligence can be discovered by analyzing the interrelationships between scores on mental ability tests" (Davidson & Downing, 2000), CHC theory appears to be an excellent framework for a better understanding of the cognitive processes underlying mathematical deficiency and giftedness alike (Phelps, McGrew, Knopik & Ford, 2005). Table A.1 in Appendix A provides a more detailed description of cognitive abilities under CHC Theory.

The strong framework provided by CHC theory is complemented by the recent availability of reliable and valid measures of cognitive processes that can provide specific evaluations for a wide range of traits and an analysis of strengths and weaknesses (Kavale, Kaufman, Naglieri & Hale, 2005) that may be applicable to the diagnosis of mathematical learning disability and giftedness. CHC theory was first put to practice in the WJ-R assessment and interpretation battery (McGrew & Woodcock, 2001; Schrank et al., 2002) and serves as the foundation of the Woodcock-Johnson III, Tests of Cognitive Abilities (Woodcock, McGrew & Mather, 2001a).

CHAPTER THREE:

Methodology

Purpose of the Study

In this study the cognitive abilities underlying math excellence among children were examined. The relationship between cognitive functioning—as defined by the Cattell-Horn-Carroll (CHC) theory—and academic achievement among children who excel in mathematics was explored, using the Woodcock-Johnson Third Edition, Tests of Cognitive Abilities (WJ III COG; Woodcock, McGrew & Mather, 2001a) and Tests of Achievement (WJ III ACH; Woodcock, McGrew & Mather, 2001b) to assess students' math proficiency in a variety of domains. The focus of the study was on an understudied exceptionality group—children with high ability in areas of mathematical achievement —and it aims to provide an understanding of whether there exists a "typical" CHC broad cognitive ability profile corresponding to strong math skills. This study attempts to build on and complement results found for low mathematics achievers (e.g., Geary, Hamson & Hoard, 2000); Geary & Hoard, 2001; Hanich, Jordan, Kaplan & Dick , 2001; Proctor, Floyd & Shaver., 2005).

In exploring the relationship between cognitive functioning and academic achievement among children who excel in both fields of mathematics (math calculation skills and math reasoning), this study emphasized specific math proficiency. Thus the sample of major focus was further limited to children who were no more than average readers – in order to focus on normative achievement strength specific to math rather than on generally high ability children. Profiles of children who excelled in mathematics but were no more than average readers were, in turn, compared to those of children who performed at an average level in both areas of mathematics and reading.

Research Questions

The following specific research questions were addressed:

1. Which CHC abilities, as measured by the WJ III COG [specifically, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Short-Term Memory (Gsm), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), and Visual/Spatial Processing (Gv)], and additional clusters, as measured by the WJ III COG [specifically, Working Memory (Gwm[2]), are the best predictors of math calculation skills, as measured by the WJ III ACH cluster [specifically, Math Calculation Skills] and tests comprising that cluster [specifically, Calculation and Math Fluency], for the group of students who excel in math calculation skills?

2. Which CHC abilities, as measured by the WJ III COG [specifically, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Short-Term Memory (Gsm), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), and Visual/Spatial Processing (Gv)], and additional clusters, as measured by the WJ III COG [specifically, Working Memory (Gwm)[2]], are the best predictors of math reasoning, as measured by the WJ III ACH cluster [specifically, Math Reasoning] and tests comprising that cluster [specifically, Applied Problems and Quantitative Concepts], for the group of students who excel in math reasoning?

3. Which CHC abilities—as measured by the WJ III COG [specifically, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Short-Term Memory (Gsm), Fluid

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[2] An arithmetic mean of the Numbers Reversed and Auditory Working Memory tests was used to estimate the Working Memory cluster score.

Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), and Visual/Spatial Processing (Gv)],and additional clusters, as measured y the WJ III COG [specifically, Working Memory (Gwm)[2]]—are the best predictors of math calculation skills, as measured by the WJ III ACH Math Calculation Skills cluster, and which CHC abilities are best predictors of math reasoning as measured by the WJ III ACH Math Reasoning cluster, for the group of students who excel in both math calculation skills and in math reasoning?

4. Which CHC abilities—as measured by the WJ III COG [specifically, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Short-Term Memory (Gsm), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), and Visual/Spatial Processing (Gv)], and additional clusters, as measured by the WJ III COG [specifically, Working Memory (Gwm)[2]]—are the best predictors of math calculation skills, as measured by the WJ III ACH cluster [specifically, Math Calculation Skills] and tests comprising that cluster [specifically, Calculation and Math Fluency], and which CHC abilities are best predictors of math reasoning, as measured by the WJ III ACH cluster [specifically, Math Reasoning] and tests comprising that cluster [specifically, Applied Problems and Quantitative Concepts], for the group of students who are average performers in math calculation skills, math reasoning, and reading?

Procedures

Participants.

Participants in this study were drawn from the school-age portion of the Woodcock-Johnson III standardization sample (ages 6 to 18) on the basis of their performance on three clusters of the WJ III Tests of Achievement (WJ III ACH; Woodcock, McGrew & Mather, 2001b), specifically: Math Calculation Skills, Math Reasoning, and Broad Reading. Cognitive profiles of participants consisted of their scores on seven clusters of the Woodcock-Johnson III Tests of Cognitive Abilities (WJ III COG; Woodcock, McGrew & Mather, 2001a), specifically: Comprehension-Knowledge, Long-Term Retrieval, Short-Term Memory, Fluid Reasoning, Processing Speed, Auditory Processing, and Visual/Spatial Processing. An additional cluster assessing working memory was developed for this study using the arithmetic mean of the scores on the Numbers Reversed and Auditory Working Memory tests from the WJ III since the WJ III Working Memory cluster was not available in the data set provided for this study.

The groups of participants in the present study were selected from the Woodcock-Johnson III standardization sample on the basis of their scores on the Math Calculation Skills and Math Reasoning clusters and on their performance on the Broad Reading cluster from the WJ III ACH. High achieving participants of major focus in this study were further restricted to those who were no more than average readers, in order to focus on normative achievement strength specific to mathematics. In selecting children who excelled specifically in math, it was judged preferable to exclude above-average readers rather than those with above-average overall cognitive functioning, because the latter approach may have depleted the sample of intellectually-gifted children, and would thus represent an overly-restrictive sample for this study. Participants who excelled in mathematics, regardless of reading skills, were also selected as comparison groups.

The following participant groups were studied, representing high achievers in mathematics as well as average achievers:

High achievers in math calculation skills. The following groups were selected. Participants with an age-based standard score of 115 or above on the Math Calculation Skills cluster and a score of 110 or below on the Broad Reading cluster (moderately high and specific achievers); Participants with an age-based standard score of 125 or above on the Math Calculation Skills cluster and a score of 110 or below on the Broad Reading cluster (very high and specific achievers); participants with an age-based standard score of 115 or above on the Math Calculation Skills cluster, regardless of reading skills (moderately high achievers); and participants with an age-based standard score of 125 or above on the Math Calculation Skills cluster, regardless of reading skills (very high achievers).

High achievers in math reasoning. The following groups were selected. Participants with an age-based standard score of 115 or above on the Math Reasoning cluster and a score of 110 or below on the Broad Reading cluster (moderately high and specific achievers); participants with an age-based standard score of 125 or above on the Math Reasoning cluster and a score of 110 or below on the Broad Reading cluster (very high and specific achievers); participants with an age-based standard score of 115 or above on the Math Reasoning cluster, regardless of reading skills (moderately high achievers); and participants with an age-based standard score of 125 or above on the Math Reasoning cluster, regardless of reading skills (very high achievers).

High overall math achievers. The following groups were selected: Participants with an age-based standard score of 115 or above on the Math Calculations Skills cluster, a score of 115 or above on the Math Reasoning cluster and a score of 110 or below on the Broad Reading cluster (moderately high and specific achievers); participants with an age-based standard score of 125 or above on the Math Calculations Skills cluster, a score of 125 or above on the Math Reasoning cluster and a score of 110 or below on the Broad Reading cluster (very high and specific achievers); participants with an age-based standard score of 115 or above on the Math Calculations Skills cluster, a score of 115 or above on the Math Reasoning cluster (moderately high achievers); and participants with an age-based standard score of 125 or above on the Math Calculations Skills cluster, a score of 125 or above on the Math Reasoning cluster (very high achievers).

Average achievers. This group of participants had an age-based standard score ranging from 90 to 110 on the Math Calculation Skills cluster, a score ranging from 90 to 110 on the Math Reasoning cluster, and a score between 90 and 110 on the Broad Reading cluster.

Instruments

The Woodcock-Johnson III, Tests of Cognitive Abilities – Third Edition (Woodcock, McGrew, & Mather, 2001a and the Woodcock-Johnson III, Tests of Achievement – Third Edition (Woodcock et al., 2001b) were used in this study. Reliability estimates and validity evidence supporting the use and interpretation of these clusters are presented in McGrew and Woodcock (2001). Overall, the reliability characteristics of the WJ III meet or exceed basic standards. Specifically, median reliabilities for clusters are mostly centered around .90 or higher, thus sufficing for making informed decisions. Median test reliabilities are mostly .80 or higher. Table B.1 in Appendix B reports median reliabilities and standard errors of measurement for clusters for the WJ III COG, and clusters and tests WJ III ACH, used in the present study. As is evident, most clusters' reliabilities are .90 or higher.

The validity of the WJ III is ascertained based on test content, developmental patterns, internal structure, and relationship to other measures. Content validity of the WJ III COG and the WJ III ACH rely on CHC theory where WJ COG clusters correspond to CHC theory broad abilities. Internal structure validity or construct validity rely on factor analytic models. The WJ III COG is shown to correlate well with other tests that measure similar constructs where the General Intellectual Ability (GIA) global scores show correlations of approximately .70 with global scores of other test batteries. The WJ III ACH is also shown to measure academic skills and abilities similar to other achievement test batteries, showing a correlation of .65 with the Wechsler Individual Achievement Test (WIAT) and .79 with the Kaufman Test of Educational Achievement (KTEA).

The WJ III tests and clusters are also shown to minimize test bias relating to race, gender, and ethnic groups. Additional information about the WJ III Standardization sample and on participants' characteristics is provided in the WJ III Technical Manual (McGrew & Woodcock, 2001). Table B.1 in Appendix B lists psychometric properties of the WJ III COG and WJ III ACH clusters and tests used in this study.

WJ III tests of Cognitive Abilities. The WJ III COG (Woodcock, McGrew & Mather, 2001a) is an individually administered battery that contains twenty tests, each measuring a different aspect of a broad ability, or a narrow cognitive ability, as described in Stratum I of the Cattell-Horn-Carroll (CHC) model of intellectual abilities. Individual test scores are combined into clusters, providing measures of cognitive functioning in domains of cognition that correspond to broad abilities described in Stratum II of the CHC model [specifically: Comprehension-Knowledge, Visual/Spatial Thinking, Auditory Processing, Fluid Reasoning, Processing Speed, Long-Term Retrieval, and Short-Term Memory] (McGrew & Woodcock, 2001). In addition, "clinical" clusters provide measures of cognitive functioning in domains of interest to researchers and clinicians. CHC cluster scores are then combined into two measures of the child's overall cognitive functioning to provide global scores (IQ scores: GIA-Std—for the WJ III COG standard battery or GIA-Ext—for the WJ III COG extended battery) that correspond to Stratum III (general intelligence) of the CHC model. The psychometric properties (median cluster reliability and standard error of the mean) for WJ III COG clusters are provided in Appendix B.

WJ III Tests of Achievement. The WJ III ACH (Woodcock, McGrew & Mather, 2001b) is comprised of twenty-two tests that measure several curricular areas including: reading, oral language, mathematics, written language, and academic knowledge. The present study examined two achievement clusters in the area of mathematics: 1) the Math Calculation Skills cluster assessing mathematical computation skills and the application of mathematical operations and basic axioms to solve mathematical problems, and 2) the Math Reasoning cluster assessing the child's ability to understand and solve novel mathematical problems using knowledge about math operations and quantitative concepts. In addition, the tests that comprise the Math Calculation Skills cluster (Calculation and Math Fluency) and the tests comprising the Math Reasoning cluster (Applied Problems and Quantitative Concepts) were used individually, to depict more specific math skills. The Broad Reading cluster assessing the child's reading skills was used, together with performance on math clusters, to select participants who exhibited specific math strength. The WJ III COG and ACH tests yield raw scores that were then converted into standard scores, based on a mean of 100 and a standard deviation of 15. The psychometric properties (median cluster reliability and standard error of the mean) for relevant WJ III ACH clusters and tests are provided in Appendix B, Table 1 (Schrank, McGrew & Woodcock, 2001). Appendix C provides information on the characteristics of WJ III Tests of Achievement in the area of mathematics (Table C.1), and sample items of the four WJ III ACH Math tests used in this study.

Analysis

Multiple Regression Analyses (MRA) using the Statistical Package for Social Sciences (SPSS, Nie, 1975), version 11, was used to assess the relationship between the above-mentioned

WJ III COG cognitive clusters and two clusters of the WJ III ACH [specifically, Math Calculation Skills and Math Reasoning], and the tests that comprise these clusters [specifically, Calculation, Math Fluency, Applied Problems, and Quantitative Concepts], in order to determine which cognitive clusters were the best predictors of math calculation skills and math reasoning among several groups of students who excelled in these areas, in comparison to an average group.

Data cleanup and assumption verification. In preparation for MRA analysis, the data were examined in order to rule out the existence of incomplete cases or outliers, and the assumptions of Multiple Regression Analysis were checked. The data were not found to include extreme outliers. Incomplete cases were deleted; assumptions of regression analysis were verified by the examination of residual plots, histograms, and P-P plots, and examination of the Durban Watson Statistics for the various groups of participants (Miles & Shevlin, 2001, Chapter 4).

Developmental trends. Participant groups in this study represent a wide range of ages (6 to 18). Therefore, it was noteworthy to examine developmental trends among children and determine whether cognitive and achievement scores are relatively stable over the age range studied. Thus, prior to presenting results for the specific groups studied , means scores and standard deviations pertaining to CHC clusters of the WJ III COG and relevant achievement clusters of the WJ III ACH, were recorded, and plotted, for the entire population studied (the WJ III standardization sample, ages 6-18), after deletion of incomplete cases.

Stepwise Regression. Stepwise regression analysis (Miles & Shevlin, 2001) was used to determine which cognitive traits were the best predictors of math calculation skills and math reasoning among groups of students who excelled in math calculation skills and math reasoning.

For comparison, stepwise regression analysis was conducted similarly for average achievers. Stepwise regression is a hierarchical form of multiple regression analysis, combining forward inclusion and backward removal of predictors. It is used to find the most parsimonious model – one that explains the most variance in the dependent variable in terms of the fewest number of independent variables (Miles & Shevlin, 2001, pp38). Despite its effectiveness, the results of stepwise regression must be used with some caution (Cohen & Cohen, 1983), due to the risk of obtaining different results given different data sets and a different order of variables' entry and removal. The limitations of stepwise regression are further discussed in Chapter 5.

For each dependent variable (math achievement cluster and test), stepwise regression was initially used to examine whether the model was significant, to calculate R^2 – the proportion of variance in the dependent variable (math cluster or test) accounted for by a composite of predictors (the CHC cognitive abilities and working memory) that signifies the effect size in regression analysis (Miles & Shevlin, 2001, pp 120). Thereafter, for individual predictors, stepwise regression was used to report standardized regression coefficients (βs)—the values of raw regression analysis coefficients standardized by the ratio of the standard deviations for the independent variable and the standard deviation for the dependent variables (Miles & Shelvin, 2001, pp 227). As indicated, the dependent variables in this study were the two cluster scores from the WJ III ACH [Math Calculations Skills and Math Reasoning] and test scores [Calculations, Math Fluency, Applied Problems, Quantitative Concepts] comprising these clusters, jointly assessing the students' math proficiency. The independent, or predictor variables in this study were the seven CHC cluster scores from the WJ III COG [Comprehension Knowledge, Long-Term Retrieval, Visual/Spatial Thinking, Auditory Processing, Fluid

Reasoning, Processing Speed and Short-Term Memory] and one WJ III COG cluster developed for this study [Working Memory], jointly assessing the students' cognitive functioning.

For each math achievement cluster, initially the entire model was reviewed for statistical significance by using the Multiple Regression Analysis F-Test to determine the model significance (Miles & Shevlin, 2001, pp 37). If the model was significant, its effect size was inferred from R^2 and Cohen's (1988) convention was used to determine whether the effect was small ($R^2 = .02$), medium ($R^2 = .13$), or large ($R^2 = .26$). Thereafter, the standardized regression coefficients (βs) were examined for each predictor. Changes in R^2 – as variables were entered and removed during Stepwise Regression, and corresponding F-Test values for the change, were also recorded. Finally, those predictors that were found significant at the .05 level ($p<.05$) were ranked according to the absolute values of their standardized regression coefficients (β values).

Research Question 1: Identifying cognitive abilities underlying math calculation skills. Multiple Regression Analysis was used to determine the extent to which the performance of students in the High Math Calculation Skills groups on the WJ III ACH Math Calculations Skills cluster, the Calculation test and Math Fluency test, could be predicted by their performance on the seven clusters from the WJ III COG [Comprehension Knowledge, Long-Term Retrieval, Visual/Spatial Thinking, Auditory Processing, Fluid Reasoning, Processing Speed, and Short-Term Memory] and/or Working Memory. This was conducted by:

(a) regressing the seven cognitive CHC clusters on the Math Calculation Skills cluster, Calculations test and Math Fluency test, using stepwise regression to examine which cognitive abilities significantly predict excellence in math reasoning;

(b) repeating the Stepwise Regression by replacing the Short-Term Memory cluster (Gsm) with the Working Memory additional cluster (Gwm). An arithmetic mean of the Numbers

Reversed and Auditory Working Memory tests was used to estimate the Working Memory cluster score;

(c) examining the overall model significance and effect size; and

(d) reporting standardized regression coefficients (β values) and effect size for statistically significant predictors and rank-ordering significant predictors of math calculation skills according to absolute β values.

Research Question 2: Identifying cognitive abilities underlying math reasoning. Multiple Regression Analysis was used to determine the extent to which the performance of students in the High Math Reasoning groups on the WJ III ACH Math Reasoning cluster and the Applied Problems and Quantitative Concepts tests, could be predicted by their performance on the seven clusters from the WJ III COG [Comprehension Knowledge, Long-Term Retrieval, Visual/Spatial Thinking, Auditory Processing, Fluid Reasoning, Processing Speed, Short-Term Memory] or Working Memory. This was conducted by:

(a) regressing the cognitive CHC clusters (and/or working memory) on the Math Reasoning cluster, the Applied Problems test and the Quantitative Concepts test, using Stepwise Regression to examine which cognitive abilities significantly predict excellence in math reasoning;

(b) repeating the stepwise regression by replacing the Short-Term Memory cluster (Gsm) with the Working Memory additional cluster (Gwm). An arithmetic mean of the Numbers Reversed and Auditory Working Memory tests was used to estimate the Working Memory cluster;

(c) examining the overall model significance and effect size; and

(d) reporting the standardized regression coefficients (β values) and effect size for statistically significant predictors and rank-ordering significant predictors of math reasoning according to absolute β values.

Research Question 3: Identifying cognitive abilities underlying math skills among overall high achievers in mathematics. Multiple Regression Analysis was used to determine the extent to which the performance of students in the High Overall Math Achievers group on the WJ III ACH Math Calculations Skills cluster and the Math Reasoning cluster could be predicted by their performance on the seven clusters from the WJ III COG [Comprehension Knowledge, Long-Term Retrieval, Visual/Spatial Thinking, Auditory Processing, Fluid Reasoning, Processing Speed, and Short-Term Memory] and/or Working Memory. This was conducted by:

(a) regressing the cognitive CHC clusters (and/or working memory) on the Math Calculation Skills cluster, and on the Math Reasoning cluster, using Stepwise Regression to examine which cognitive abilities significantly predict excellence in math calculation skills and math reasoning;

(b) repeating the stepwise regression by replacing the Short-Term Memory cluster (Gsm) with the Working Memory additional cluster (Gwm). An arithmetic mean of the Numbers Reversed and Auditory Working Memory tests was used to estimate the Working Memory cluster;

(c) examining the overall model significance and effect size; and

(d) reporting standardized regression coefficients (β values) and effect size for statistically significant predictors and rank-ordering significant predictors of math calculation skills according to absolute β values.

Research Question 4: Identifying cognitive abilities underlying math skills among average achievers. Multiple Regression Analysis was used to determine the extent to which the performance of students in the Average group on the WJ III ACH Math Calculations Skills cluster, the Calculation test and Math Fluency test, the Math Reasoning cluster, the Applied Problems test and the Quantitative Concepts test could be predicted by their performance on the seven clusters from the WJ III COG [Comprehension Knowledge, Long-Term Retrieval, Visual/Spatial Thinking, Auditory Processing, Fluid Reasoning, Processing Speed, and Short-Term Memory] or Working Memory. The analysis was conducted by:

(a) regressing the cognitive CHC clusters (and/or working memory) on the Math Calculation Skills cluster and on the Math Reasoning cluster using Stepwise Regression to examine which cognitive abilities significantly predict excellence in math calculation skills and in math reasoning;

(b) repeating the stepwise regression by replacing the Short-Term Memory cluster (Gsm) with the Working Memory additional cluster (Gwm). An arithmetic mean of the Numbers Reversed and Auditory Working Memory tests was used to estimate the Working Memory cluster;

(c) examining the overall model significance and effect size; and

(d) reporting standardized regression coefficients (β values) and effect size for statistically significant predictors and rank-ordering significant predictors according to absolute β values.

CHAPTER FOUR

Results

The purpose of this chapter is to present the results of the investigation for several groups of children of varying mathematics ability (high achievers in math calculation skills, high achievers in math reasoning, overall high math achievers, average achievers), correlating their performance on measures of cognitive ability, using the Woodcock-Johnson III Tests of Cognitive Abilities (WJ III COG), and on measures of mathematics achievement, using the Woodcock-Johnson III Tests of Achievement (WJ III ACH). Initial descriptive analysis is presented first, followed by detailed data with regard to each of the research questions presented in Chapter Three, and then followed by summaries of the results in tabular form. Discussion and implications of the findings will be presented in Chapter Five.

Initial Descriptive Analysis

Table 4.1 provides descriptive data (means and standard deviations) for cognitive and achievement clusters of the WJ III entire school-aged standardization sample, after cases with missing relevant data have been deleted. Standard scores have a mean of 100 and standard deviation of 15.

Table 4.1

Means and Standard Deviations of WJ III COG and ACH Scores for Entire School-age WJ III sample (n = 2555)

WJ III Cluster Scores	M	SD
WJIII Cognitive Scores		
General Intellectual Ability – Extended	102.65	14.584
Comprehension-Knowledge (Gc)	102.00	14.70
Long-Term Retrieval (Glr)	103.00	14.08
Visual-Spatial Thinking (Gv)	101.78	14.54
Auditory Processing (Ga)	102.62	14.59
Fluid Reasoning (Gf)	101.92	14.84
Processing Speed (Gs)	101.36	14.53
Short-Term Memory (Gsm)	102.07	14.80
Working Memory (Gwm)	101.29	12.42
Achievement Scores		
Broad Reading Skills	102.85	14.08
Math Calculation Skills	102.51	14.33
Math Reasoning	101.76	13.42

Note. The Working Memory cluster score was estimated as the arithmetic mean of two test scores assessing working memory: Numbers Reversed and Auditory Working Memory. Standard scores have a mean of 100 and standard deviation of 15.

Developmental Trends

To examine children's developmental trends for the entire standardization sample, Figure 4.1 shows mean scores for the seven CHC cognitive clusters of the WJ III COG, and working memory, plotted by age and figure 4.2 shows relevant mean scores for several achievement clusters and tests, plotted by age, for the WJ III ACH. Figure 4.1 demonstrates that children's cognitive scores are fairly uniform over time. Minor trends visible for the present sample include an elevation in long-term retrieval (Glr) and auditory processing (Ga) at ages 7 and 14, a small elevation in processing speed (Gs) at age 8 and 18, a peak in short-term memory (Gsm) at ages 8 and 14, and an elevation in comprehension knowledge (Gc) during ages 11-14. Achievement scores on math and reading clusters are fairly uniform over age, with some lowered values between ages 10 and 15.

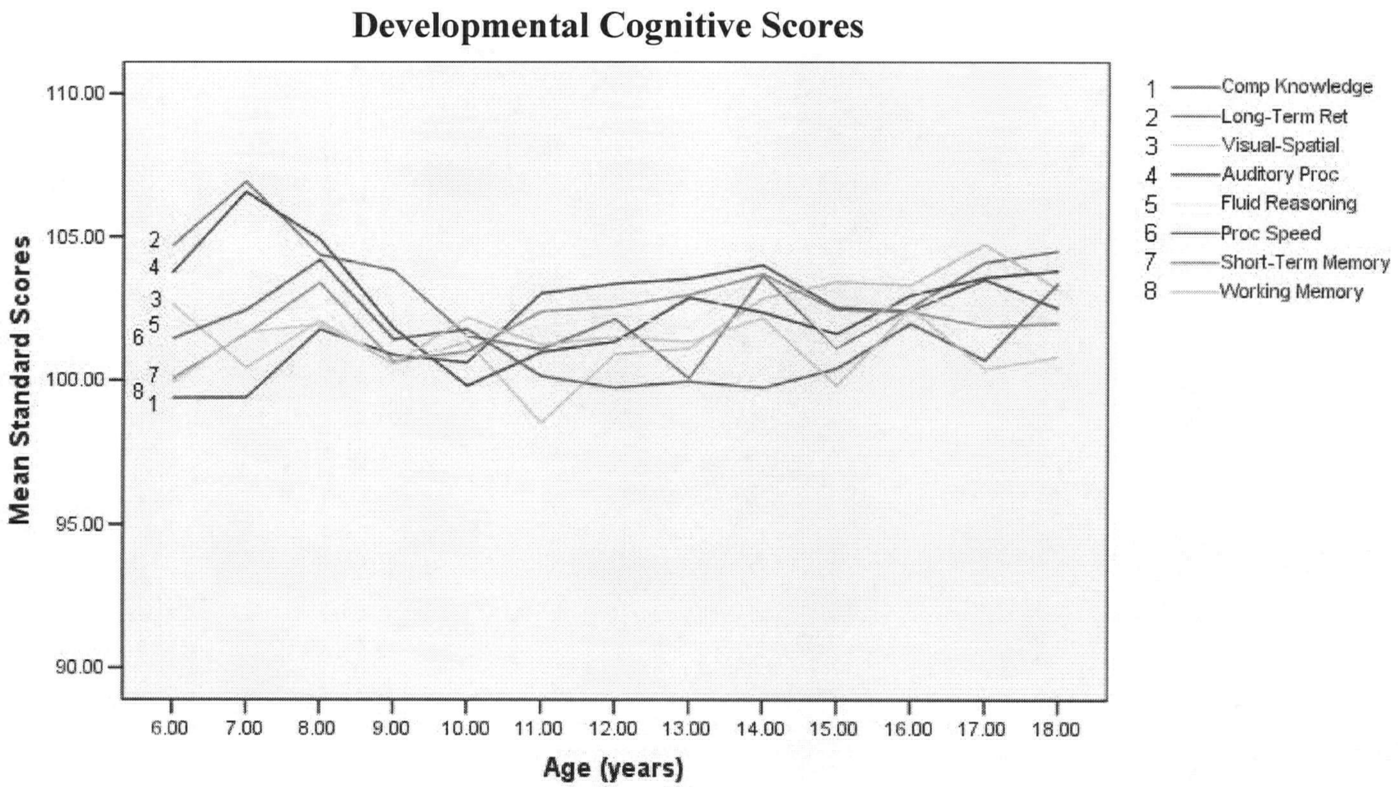

Figure 4.1: Cognitive Scores by Age

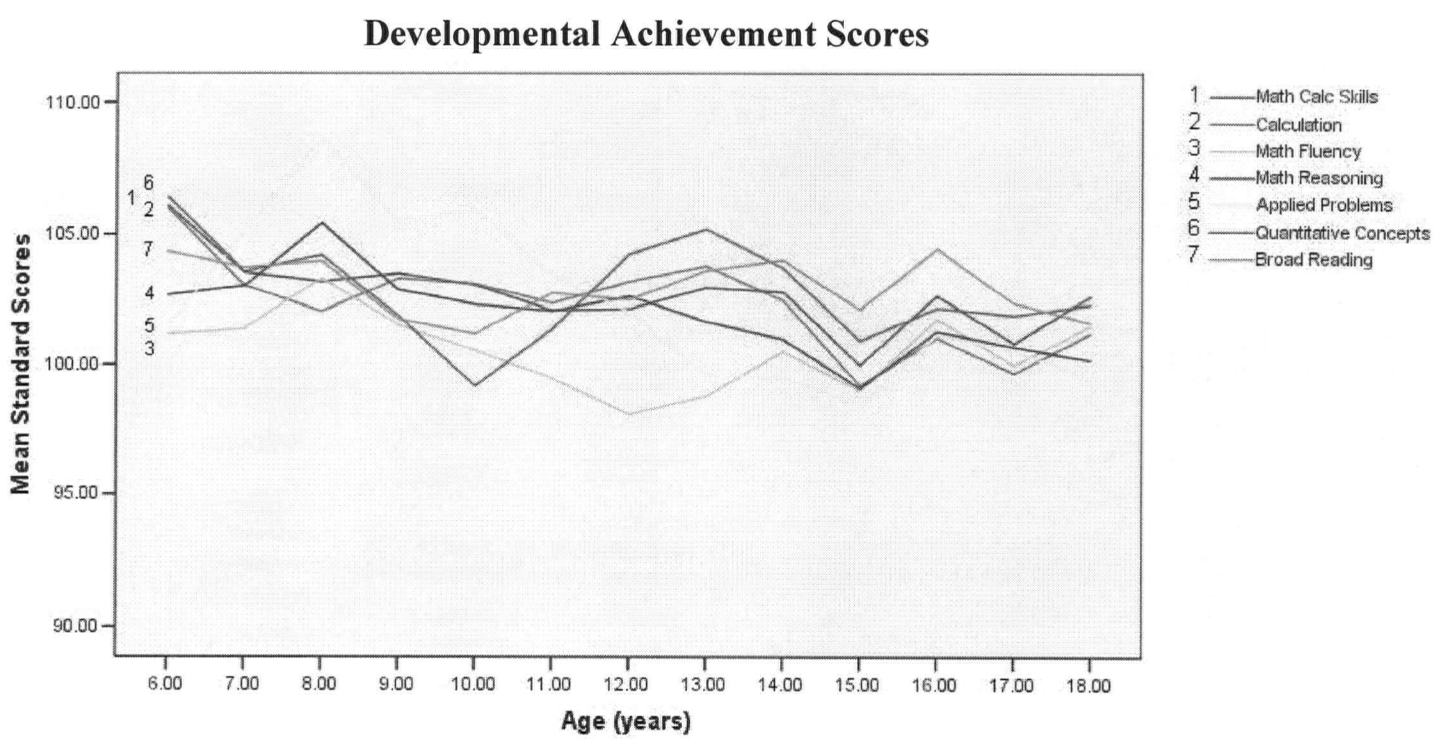

Figure 4.2: Achievement Scores by Age

Research Question 1: Cognitive Abilities that Best Predict Excellence in Math Calculation Skills

In this question, the WJ III COG cognitive ability clusters, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), and Visual-Spatial Processing (Gv), Short-Term Memory (Gsm) , and additional cluster, Working Memory (Gwm) were analyzed through individual multiple stepwise regression analyses to find the best predictors of the Math Calculation Skills cluster, the Calculation test and the Math Fluency test from the WJ III ACH. The prediction of each achievement cluster and test was analyzed for four groups of children selected on the basis of their math and reading achievements. For each regression conducted, if the model was found significant, the standardized regression coefficients (β), the change in R^2, and the F value for the change in R^2 after entry of each independent variables, were presented for each significant predictor, and β values were used to order relevant predictors according to their relative importance. Cohen's (1988) convention was used to determine effect size.

Moderately High Achievers in Math Calculation Skills Not Excelling in Reading. This group consists of 172 children who show high achievement in math calculation skills on the WJ II ACH (SS≥115) but are no more than average readers (SS≤110 on Broad Reading). Table 4.2 provides descriptive information summarizing relevant cognitive and achievement scores for this group. The results for stepwise regression analyses on this group are presented in Tables 4.3, 4.4, and 4.5, where the dependent variables are the Math Calculation Skills cluster, the Calculation test and the Math Fluency test respectively. Each table presents two regressions: (a) when the seven CHC cognitive factors are used as predictors and; (b) where the Working

Memory cluster is used instead of the Short-Term-Memory cluster to represent immediate awareness.

The stepwise regression analysis revealed that for the *Moderately High Achievers in Math Calculation Skills Not Excelling in Reading* group, when the dependent variable was the Math Calculation Skills cluster and Short-term Memory was used to describe immediate awareness, the only significant predictor for success in math calculation skills was Short-Term Memory (Gsm), $F(1, 170) = 9.00$ =, $p < .05$, $R^2 = .050$ – signifying a small effect size. When the regression was repeated and the Short-Term Memory cluster was replaced by the Working Memory cluster upon entering the independent variables, the only significant predictor for success in math calculation skills was Working Memory (Gwm), $F(1, 170) = 5.42$, $p < .05$, $R^2 = .031$ – signifying small effect size.

When the dependent variable was the Calculation test and Short-term Memory was used to describe immediate awareness, the significant predictors for success in math calculation skills, ordered according to the absolute values of their standardized regression coefficients, were Visual/Spatial Thinking (Gv), followed (inversely) by Processing Speed (Gs) and then by Fluid Reasoning (Gf), $F(3,168) = 10.82$, $p < .05$, $R^2 = .162$, signifying a medium effect size. Identical results were obtained when Short-Term Memory was replaced by Working Memory upon entering the independent variables.

When the dependent variable was the Math Fluency test and Short-term Memory was used to describe immediate awareness, the significant predictors for success in math fluency ordered according to the values of their standardized regression coefficients, included Processing Speed (Gs), followed (inversely) by Visual/Spatial Thinking (Gv), and then (inversely) by Long-Term Retrieval (Glr), $F(3, 168) = 10.54$, $p < .05$, $R^2 = .398$, signifying a large effect size. Identical results were obtained when Short-Term Memory was replaced by Working Memory upon entering the independent variables.

Table 4.2

Means and Standard Deviations of WJ III COG and ACH Cluster Scores for Moderately High Achievers in Math Calculation Skills Not Excelling in Reading (n = 172)

Cluster	M	SD
<u>WJIII Cognitive Scores</u>		
General Intellectual Ability – Extended	104.84	11.05
Comprehension-Knowledge (Gc)	102.48	12.13
Long-Term Retrieval (Glr)	104.33	12.41
Visual-Spatial Thinking (Gv)	102.65	13.48
Auditory Processing (Ga)	103.36	14.27
Fluid Reasoning (Gf)	105.63	13.55
Processing Speed (Gs)	103.22	11.28
Short-Term Memory (Gsm)	104.02	12.65
Working Memory (Gwm)	103.73	10.96
<u>WJ III Achievement Scores</u>		
Broad Reading Skills	101.83	6.81
Math Calculation Skills	121.36	5.35
Calculation Test	121.34	8.00
Math Fluency Test	112.27	10.35
Math Reasoning	109.27	11.52
Applied Problems Test	109.81	13.66
Quantitative Concepts Test	109.61	11.56

Note. The Working Memory cluster score was estimated as the mean of the Numbers Reversed and Auditory Working Memory tests scores. Standard scores have a mean of 100 and standard deviation of 15.

Table 4.3

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Calculation Skills Cluster

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1. Short-Term Memory (Gsm)	.244	.050	.050	8.90
(b) Working Memory Used				
1. Working Memory (Gwm)	.296	.032	.031	4.80

$p < .05$

Table 4.4

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Calculation Test

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1. Fluid Reasoning (Gf)	. 226	.072	.072	13.21
2. Processing Speed (Gs)	-.231	.112	.40	7.56
3. Visual/Spatial Thinking (Gv)	.237	.162	.50	10.02
(b) Working Memory Used				
1. Fluid Reasoning (Gf)	.226	.072	.072	13.21
2. Processing Speed (Gs)	-.231	.112	.040	7.56
3. Visual/Spatial Thinking (Gv)	.237	.162	.050	10.02

$p < .05$

Table 4.5

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Fluency Test

Predictor	β	R^2	Change in R^2	F for Change
<u>(a) Short-Term Memory Used</u>				
1. Processing Speed (Gs)	. 318	.065	.065	11.81
2. Visual/Spatial Thinking (Gv)	-.215	.126	.061	11.86
3. Long-Term Memory (Glr)	-.185	.158	.032	6.39
<u>(b) Working Memory Used</u>				
1. Processing Speed (Gs)	.318	.065	.065	11.81
2. Visual/Spatial Thinking (Gv)	-.215	.126	.061	11.86
3. Long-Term Memory (Glr)	-.185	.158	.032	6.39

$p < .05$

Very high achievers in math calculation skills not excelling in reading. This group consists of 40 children who show very high achievement in math calculation skills on the WJ III ACH (SS≥125) but are no more than average readers (SS≤110 on the Broad Reading cluster). Table 4.6 provides descriptive information summarizing relevant cognitive and achievement scores for this group. The results for stepwise regression analyses on this group are presented in Tables 4.7, 4.8, and 4.9, where the dependent variables are the Math Calculation Skills cluster, the Calculation test and the Math Fluency test respectively.

The stepwise regression analysis revealed that when the dependent variable was the Math Calculation Skills cluster and Short-term Memory was used to describe immediate awareness, the only significant predictor for success in math calculation skills was Visual/Spatial Thinking (Gv),

$F(1, 38) = 8.28$, $p < .05$, $R^2 = .179$ signifying a medium effect size. Identical results were obtained when the Short-Term Memory cluster was replaced by the Working Memory cluster.

When the dependent variable was the Calculation test and Short-term Memory was used to describe immediate awareness, the significant predictors for success in calculation skills, ordered according to the absolute values of their standardized regression coefficients, were Visual/Spatial Thinking (Gv), followed (inversely) by Processing Speed (Gs), $F(2, 37) = 9.54$, $p < .05$, $R^2 = .340$ – signifying a large effect size. Identical results where obtained when Short-Term Memory was replaced by Working Memory upon entering the independent variables.

When the dependent variable was the Math Fluency test and Short-term Memory was used to describe immediate awareness, the significant predictors for success in math fluency ordered according to the absolute values of their standardized regression coefficients, included, Processing Speed (Gs), followed (inversely) by Visual/Spatial Thinking (Gv), and then followed (inversely) by Long-Term Retrieval (Glr), $F(3, 36) = 8.15$, $p < .05$, $R^2 = .404$, signifying a large effect size. Identical results were obtained when Short-Term Memory was replaced by Working Memory.

Table 4.6

Means and Standard Deviations of WJ III COG and ACH Cluster Scores for Very High Achievers in Math Calculation Skills Not Excelling in Reading (n = 40)

Cluster	M	SD
WJ III Cognitive Scores		
General Intellectual Ability -Extended	107.07	13.87
Comprehension-Knowledge (Gc)	103.80	14.95
Long-Term Retrieval (Glr)	106.10	13.43
Visual-Spatial Thinking (Gv)	104.82	15.87
Auditory Processing (Ga)	103.75	17.61
Fluid Reasoning (Gf)	103.75	14.39
Processing Speed (Gs)	107.32	12.72
Short-Term Memory (Gsm)	107.57	14.54
Working Memory (Gwm)	106.16	11.21
WJ III Achievement Scores		
Broad Reading Skills	101.62	8.14
Math Calculation Skills	129.27	3.78
Calculation Test	128.95	8.60
Math Fluency Test	117.05	11.83
Math Reasoning	114.80	12.36
Applied Problems Test	116.20	14.27
Quantitative Concepts Test	113.75	12,56

Note. The Working Memory cluster score was estimated as the mean of the Numbers Reversed and Auditory Working Memory tests scores. Standard scores have a mean of 100 and standard deviation of 15.

Table 4.7

Stepwise Regression for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Calculation Skills Cluster

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1. Visual/Spatial Thinking (Gv)	.423	.179	.179	8.28
(b) Working Memory Used				
1. Visual/Spatial Thinking (Gv)	.423	.179	.179	8.28

$p < .05$

Table 4.8

Stepwise Regression for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Calculation Test

Predictor	β	R^2	Change in R^2	F for Change
a) Short-Term Memory Used				
1. Visual/Spatial Thinking(Gv)	.497	.212	.212	10.22
2. Processing Speed (Gs)	-.360	.340	.128	7.19
b) Working Memory Used				
1. Visual/Spatial Thinking (Gv)	.497	.212	.212	10.22
2. Processing Speed (Gs)	-.360	.340	.128	7.19

$p < .05$

Table 4.9

Stepwise Regression for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Fluency Test

Predictor	β	R^2	Change in R^2	F for Change
a) Short-Term Memory Used				
1. Processing Speed (Gs)	. 541	.237	.065	11.82
2. Visual/Spatial Thinking (Gv)	-.313	.327	.061	4.94
3. Long-Term Memory (Glr)	-.279	.404	.032	4.67
b) Working Memory Used				
1. Processing Speed (Gs)	-.541	.237	.237	11.82
2. Visual/Spatial Thinking (Gv)	-.313	.327	.090	4.94
3. Long-Term Memory (Glr)	-.279	.404	.077	4.67

$p < .05$

Moderately high achievers in math calculation skills regardless of reading skills. This group consists of 371 children who show high achievement in math calculation skills on the WJ II ACH (SS≥115), regardless of reading achievement. Table 4.10 provides descriptive information summarizing relevant cognitive and achievement scores for this group. The results for stepwise regression analyses on this group are presented in Tables 4.11, 412 and 4.13, Each table presents two regressions: (a) when the seven CHC cognitive factors are used as predictors and; (b) where the Working Memory cluster is used instead of the Short-Term-Memory cluster to represent immediate awareness. Stepwise regression analysis on this group was conducted with the Math Calculation Skills cluster, the Calculation test and the Math Fluency test as the dependent variables.

When the dependent variable was the Math Calculation Skills cluster and Short-term Memory was used to describe immediate awareness, the significant predictors for success in math calculation skills, ordered according to the absolute values of their standardized regression coefficients, were Fluid Reasoning (Gf), followed by Processing Speed (Gs) and then followed by Visual/Spatial Thinking (Gv), $F(3, 367) = 14.94$, $p < .05$, $R^2 = .109$, signifying a medium effect size. Identical results were obtained when Short-Term Memory was replaced by Working Memory upon entering the independent variables.

When the dependent variable was the Calculation test and Short-Term Memory was used to describe immediate awareness, the significant predictors for success in calculation skills, ordered according to the absolute values of their standardized regression coefficients, were Fluid Reasoning (Gf), followed by Visual/Spatial Thinking (Gv), then followed (inversely) by Processing Speed (Gs), and then by Comprehension Knowledge (Gc), $F(4, 366) = 13.86$, $p < .05$, $R^2 = .132$ – signifying a medium effect size. Identical results were obtained when Short Term Memory was replaced by Working Memory upon entering the independent variables.

When the dependent variable was the Math Fluency test and Short Term Memory was used to describe immediate awareness, the significant predictors for success in math fluency, ordered according to the absolute values of theeir standardized regression coefficients, included Processing Speed (Gs), followed (inversely) by Visual/Spatial Thinking (Gv), $F(2, 368) = 8.15$, $p < .05$, $R^2 = .155$, signifying a medium effect size. Identical results were obtained when Short Term Memory was replaced by Working Memory.

Table 4.10

Means and Standard Deviations of WJ III COG and ACH Cluster Scores for Moderately High Achievers in Math Calculation Skills Regardless of Reading Skills (n = 371)

Cluster	M	SD
WJ III Cognitive Scores		
General Intellectual Ability -Extended	113.53	13.11
Comprehension-Knowledge (Gc)	110.12	13.76
Long-Term Retrieval (Glr)	109.42	12.45
Visual-Spatial Thinking (Gv)	106.08	14.76
Auditory Processing (Ga)	108.12	14.95
Fluid Reasoning (Gf)	110.89	13.40
Processing Speed (Gs)	110.06	14.25
Short-Term Memory (Gsm)	109.95	13.24
Working Memory (Gwm)	109.15	11.27
WJ III Achievement Scores		
Broad Reading Skills	114.0135	12.97
Math Calculation Skills	124.1752	7.48
Calculation Test	122.1644	9.00
Math Fluency Test	117.1375	12.28
Math Reasoning	114.3100	12.27
Applied Problems Test	114.5526	14.18
Quantitative Concepts Test	115.0809	13.02

Note. The Working Memory cluster score was estimated as the mean of the Numbers Reversed test score and the Auditory Working Memory test score. Standard scores have a mean of 100 and standard deviation of 15.

Table 4.11
Stepwise Regression for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Calculation Skills Cluster

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1. Fluid Reasoning (Gf)	.184	.068	068	26.93
2. Processing Speed (Gs)	.162	.098	.030	12.14
3. Visual/Spatial Thinking (Gv)	.112	.109	.011	4.55
b) Working Memory Used				
1. Fluid Reasoning (Gf)	.184	.068	068	26.93
2. Processing Speed (Gs)	.162	.098	.030	12.14
3. Visual/Spatial Thinking (Gv)	.112	.109	.011	4.55

$p < .05$

Table 4.12

Stepwise Regression for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Calculation Test

Predictor	β	R^2	Change in R^2	F for Change
a) Short-Term Memory Used				
1. Fluid Reasoning	.211	.077	.077	30.90
2. Visual/Spatial Thinking(Gv)	.172	.100	.022	9.17
3. Processing Speed (Gs)	-.162	.122	.022	9.11
4. Comprehension Knowledge (Gc)	.114	.132	.010	4.22
b) Working Memory Used				
1. Fluid Reasoning (Gf)	.211	.077	.077	30.90
2. Visual/Spatial Thinking(Gv)	.172	.100	.022	9.17
3. Processing Speed (Gs)	-.162	.122	.022	9.11
4. Comprehension Knowledge (Gc)	.114	.132	.010	4.22

$p < .05$

Table 4.13

Stepwise Regression for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Fluency Test

Predictor	β	R^2	Change in R^2	F for Change
a) Short-Term Memory Used				
1. Processing Speed (Gs)	.404	.145	.145	62.53
2. Visual/Spatial Thinking (Gv)	-.104	.155	.010	4.45
b) Working Memory Used				
1. Processing Speed (Gs)	.404	.145	.145	62.53
2. Visual/Spatial Thinking (Gv)	-.104	.155	.010	4.45

$p < .05$

Very high achievers in math calculation skills regardless of reading skills. This group consists of 142 children who show very high achievement in math calculation skills on the WJ III ACH (SS ≥125), regardless of reading achievement. Table 4.14 provides descriptive information summarizing relevant cognitive and achievement scores for this group. The results for stepwise regression analyses on this group are presented in Tables 4.15, 4.16 and 4.17 where the dependent variables are the Math Calculation Skills cluster, the Calculation test and the Math Fluency test respectively. Each table presents two regressions: (a) when the seven CHC cognitive factors are used as predictors and (b) where the Working Memory cluster is used instead of the Short Term Memory cluster to represent immediate awareness.

When the dependent variable was the Math Calculation Skills cluster and Short Term Memory was used to describe immediate awareness, the significant predictors for success in math calculation skills, ordered according to the absolute values of their standardized regression coefficients, were Visual/Spatial Thinking (Gv), followed by Fluid Reasoning (Gf) and then by

Processing Speed (Gs), $F(3, 138) = 9.42$, $p < .05$, $R^2 = .170$, signifying a medium effect size. Identical results were obtained when Short Term Memory was replaced by Working Memory upon entering the independent variables.

When the dependent variable was the Calculation test and the Short-term Memory was used to describe immediate awareness, the significant predictors for success in calculation, ordered according to the absolute values of their standardized regression coefficients, were Fluid Reasoning (Gf), followed by Visual/Spatial Thinking (Gv) and then (inversely) by Processing Speed (Gs), $F(3, 138) = 8.50$, $p < .05$, $R^2 = .156$, signifying a medium effect size. Identical results were obtained when Short Term Memory was replaced by Working Memory upon entering the independent variables.

When the dependent variable was the Math Fluency test and the Short- Term Memory was used to describe immediate awareness, the only significant predictor was Processing Speed (Gs), $F(1, 140) = 29.15$, $p < .05$, $R^2 = .172$, signifying a medium effect size. Identical results were obtained when Short Term Memory was replaced by Working Memory upon entering the independent variables.

Table 4.14

Means and Standard Deviations of WJ III COG and ACH Cluster Scores for Very High Achievers in Math Calculation Skills Regardless of Reading Skills (n = 142)

Cluster	M	SD
WJ III Cognitive Scores		
General Intellectual Ability -Extended	116.63	14.19
Comprehension-Knowledge (Gc)	112.79	14.10
Long-Term Retrieval (Glr)	109.65	12.44
Visual-Spatial Thinking (Gv)	108.16	15.50
Auditory Processing (Ga)	108.12	15.47
Fluid Reasoning (Gf)	113.92	14.15
Processing Speed (Gs)	112.90	15.15
Short-Term Memory (Gsm)	111.85	15.16
Working Memory (Gwm)	111.13	12.13
WJ III Achievement Scores		
Broad Reading Skills	116.92	13.98
Math Calculation Skills	131.64	6.83
Calculation Test	128.34	9.42
Math Fluency Test	123.42	13.20
Math Reasoning	119.83	12.47
Applied Problems Test	120.23	13.95
Quantitative Concepts Test	120.20	13.59

Note. The Working Memory cluster score was estimated as the mean the Numbers Reversed test score and the Auditory Working Memory test score. Standard scores have a mean of 100 and standard deviation of 15.

Table 4.15

Stepwise Regression for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Calculation Skills Cluster

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1. Fluid Reasoning (Gf)	.195	.105	.105	16.34
2. Visual/Spatial Thinking (Gv)	.210	.144	.040	6.48
3. Processing Speed (Gs)	.170	.170	.026	4.28
(b) Working Memory Used				
1. Fluid Reasoning (Gf)	.195	.105	.105	16.34
2. Processing Speed (Gs)	.210	.144	.040	6.48
3. Visual/Spatial Thinking (Gv)	.170	.170	.026	4.27

$p < .05$

Table 4.16

Stepwise Regression for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Calculation Test

Predictor	β	R^2	Change in R^2	F for Change
a) Short-Term Memory Used				
1. Visual/Spatial Thinking (Gv)	.241	.092	.092	14.13
2. Fluid Reasoning(Gf)	.248	.122	.030	4.78
3. Processing Speed (Gs)	-.196	.156	.034	5.60
b) Working Memory Used				
1. Visual/Spatial Thinking (Gv)	.241	.092	.092	14.13
2. Fluid Reasoning(Gf)	.248	.122	.030	4.78
3. Processing Speed (Gs)	-.196	.156	.034	5.60

$p < .05$

Table 4.17

Stepwise Regression for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Fluency Test

Predictor	β	R^2	Change in R^2	F for Change
a) Short-Term Memory Used				
1. Processing Speed (Gs)	.415	.172	.172	29.15
b) Working Memory Used				
1. Processing Speed (Gs)	.415	.172	.172	29.15

$p < .05$

Research Question 2: Cognitive Abilities that Best Predict Excellence in Math Reasoning

In this question, the WJ III COG cognitive ability clusters, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), and Visual-Spatial Processing (Gv), Short-Term Memory (Gsm) , and the additional cluster, Working Memory (Gwm) were analyzed through individual multiple stepwise regression analyses to find the best predictors of the Math Reasoning cluster, the Applied Problems test and the Quantitative Concepts test from the WJ III ACH.. The prediction of each achievement cluster and test was analyzed for four groups of children selected on the basis of their math and reading achievements. For each regression conducted, if the model was found significant, the standardized regression coefficients (β), the change in R^2, and the F value for the change in R^2 after entry of each independent variables, were presented for each significant predictor, and β values were used to order relevant predictors according to their relative importance. Cohen's (1988) convention was used to determine effect size.

Moderately high achievers in math reasoning not excelling in reading. The Moderately High Achieving group consists of 156 children who show high achievement in math reasoning on the WJ II ACH (SS≥115) but are no more than average readers (SS≤110 on the Broad Reading cluster). Table 4.18 provides descriptive information summarizing relevant cognitive and achievement scores for this group. The results for The results for stepwise regression analyses on this group are presented in Tables 4.19 and 4.20, where the dependent variables are the Math Reasoning cluster and the Quantitative concepts test. Each table presents two regressions: (a) when the seven CHC cognitive factors are used as predictors and (b) where the Working Memory cluster is used instead of the Short Term Memory cluster to represent immediate awareness.

The stepwise regression analysis revealed that for the Moderately High ability group, when the dependent variable was the Math Reasoning cluster and Short-Term Memory was used to describe immediate awareness, the only significant predictor for success in math calculation skills was Fluid Reasoning (Gf), $F(1, 154) = 9.28$ =, $p < .05$, $R^2 = .057$, signifying a small effect size. Identical results were obtained when the Short-Term Memory cluster was replaced by the Working Memory cluster.

The model was not significant at the .05 statistical significance level when Stepwise Regression was run, using the Applied Problems test as the dependent variable, whether Short-Term Memory or Working Memory was used to describe immediate awareness.

When the dependent variable was the Quantitative Concepts test and Short-term Memory was used to describe immediate awareness, the only significant predictor for success in quantitative reasoning was Fluid Reasoning (Gf), $F(1, 154) = 6.07$, $p < .05$, $R^2 = .038$, signifying a small effect size. Identical results were obtained when Short Term Memory was replaced by Working Memory upon entering the independent variables.

Table 4.18

Means and Standard Deviations of WJ III COG and ACH Cluster Scores for Moderately High Achievers in Math Reasoning Not Excelling in Reading (n = 156)

Cluster	M	SD
<u>WJ III Cognitive Scores</u>		
General Intellectual Ability -Extended	110.81	9.45
Comprehension-Knowledge (Gc)	109.88	9.41
Long-Term Retrieval (Glr)	108.89	11.96
Visual-Spatial Thinking (Gv)	105.88	14.31
Auditory Processing (Ga)	105.06	15.19
Fluid Reasoning (Gf)	112.71	11.42
Processing Speed (Gs)	102.19	11.98
Short-Term Memory (Gsm)	106.27	12.96
Working Memory (Gwm)	105.33	10.83
<u>WJ III Achievement Scores</u>		
Broad Reading Skills	105.05	6.21
Math Calculation Skills	110.78	11.22
Calculation Test	111.79	12.56
Math Fluency Test	104.26	10.96
Math Reasoning	120.03	5.36
Applied Problems Test	122.61	8.04
Quantitative Concepts Test	118.11	8.58

Note. The Working Memory cluster score was estimated as the mean of the Numbers Reversed test score and the Auditory Working Memory test score. Standard scores have a mean of 100 and standard deviation of 15.

Table 4.19

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Reasoning Cluster

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1.Fluid Reasoning (Gf)	.238	.057	.057	9.28
(b) Working Memory Used				
1. Fluid Reasoning (Gf)	.238	.057	.057	9.28

$p < .05$

Table 4.20

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Quantitative Concepts Test

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1. Fluid Reasoning (Gf)	. 195	.038	.038	6.07
(b) Working Memory Used				
1. Fluid Reasoning (Gf)	.195	.038	.038	6.07

$p < .05$

Very high achievers in math reasoning not excelling in reading. This group consists of 23 children who show very high achievement in math reasoning on the WJ II ACH (SS≥125) but are no more than average readers (SS≤110 on the Broad Reading cluster). Table 4.21 provides descriptive information summarizing relevant cognitive and achievement scores for this group. The results for stepwise regression analyses on this group, using the Math Reasoning cluster, the Applied Problems test, and the Quantitative Concept test, were not significant at the .05 statistical significance level. This is consistent with the small size of this group (23 cases), compared to the number (7) of predictors.

Table 4.21

Means and Standard Deviations of WJ III COG and ACH Cluster Scores for Very High Achievers in Math Reasoning Not Excelling in Reading (n = 23)

Cluster	M	SD
WJ III Cognitive Scores		
General Intellectual Ability -Extended	113.43	8.99
Comprehension-Knowledge (Gc)	114.87	7.40
Long-Term Retrieval (Glr)	111.61	11.98
Visual-Spatial Thinking (Gv)	110.43	15.47
Auditory Processing (Ga)	105.17	12.80
Fluid Reasoning (Gf)	117.09	10.96
Processing Speed (Gs)	99.61	9.31
Short-Term Memory (Gsm)	106.52	11.11
Working Memory (Gwm)	107.89	10.37
WJ III Achievement Scores		
Broad Reading Skills	104.78	4.53
Math Calculation Skills	119.61	10.32
Calculation Test	121.17	14.34
Math Fluency Test	108.56	8.80
Math Reasoning	130.43	5.44
Applied Problems Test	132.04	6.70
Quantitative Concepts Test	127.91	8.31

Note. The Working Memory cluster score was estimated as the mean of two the Numbers Reversed test score and the Auditory Working Memory test score. Standard scores have a mean of 100 and standard deviation of 15.

Moderately high achievers in math reasoning regardless of reading skills. This group consists of 401 children who show high achievement in Math Reasoning on the WJ II ACH (SS>≥115) regardless of reading achievement. Table 4.22 provides descriptive information summarizing relevant cognitive and achievement scores for this group. The results for stepwise Regression analyses on this group are presented in Tables 4.23, 4.24 and 4.25 where the dependent variables are the Math Reasoning cluster, the Applied Problems test and the Quantitative Concepts test. Each table presents two regressions: (a) when the sever CHC cognitive factors are used as predictors and (b) where the Working Memory cluster is used instead of the Short-Term-Memory cluster to represent immediate awareness.

When stepwise regression analysis for this group was conducted with the Math Reasoning cluster as the dependent variable, and Short-Term Memory was used to describe immediate awareness, the significant predictors for success in math reasoning, ordered according to the absolute values of their standardized regression coefficients, were Fluid Reasoning (Gf), followed by Comprehension Knowledge (Gc), $F(2, 398) = 29.47$, $p < .05$, $R^2 = .129$, signifying a medium effect size. When Short Term Memory was replaced by Working Memory, the significant predictors for success in math reasoning, ordered according to the absolute values of their standardized regression coefficients, were Fluid Reasoning (Gf), followed by Comprehension Knowledge (Gc), and then followed by Working Memory (Gwm), $F(3, 397) = 23.51$, $p < .05$, $R^2 = .151$ signifying a medium effect size.

When the dependent variable was the Applied Problems test and Short-Term Memory was used to describe immediate awareness, the only significant predictor was Short-Term Memory (Gsm), $F(1, 399) = 6.76$, $p < .05$, $R^2 = .017$, signifying a small effect size. When Short-Term Memory was replaced by Working Memory upon entering the independent variables, the only significant predictor was Working Memory (Gwm), $F(1, 399) = 13.89$, $p < .05$, $R^2 = .034$, signifying a small effect size.

When the dependent variable was the Quantitative Concepts test and Short-Term Memory was used to describe immediate awareness, the only significant predictor was Short-Term Memory (Gsm), $F(1, 399) = 830.10$, $p < .05$, $R^2 = .675$, signifying a large effect size. When Short Term Memory was replaced by Working Memory upon entering the independent variable, the significant predictors for success in quantitative concepts, ordered according to the absolute values of their standardized regression coefficients, included Working Memory (Gwm) followed (inversely) by Long-Term Retrieval (Glr), $F(2,398) = 505,27$, $p < .05$, $R^2 = .718$, signifying a large effect size.

Table 4.22

Means and Standard Deviations of WJ III COG and ACH Cluster Scores for Moderately High Achievers in Math Reasoning Regardless of Reading Skills (n = 401)

Cluster	M	SD
WJ III Cognitive Scores		
General Intellectual Ability -Extended	117.39	11.12
Comprehension-Knowledge (Gc)	114.92	10.92
Long-Term Retrieval (Glr)	112.24	12.22
Visual-Spatial Thinking (Gv)	108.67	14.93
Auditory Processing (Ga)	109.80	15.30
Fluid Reasoning (Gf)	115.32	11.75
Processing Speed (Gs)	108.95	14.61
Short-Term Memory (Gsm)	111.30	13.94
Working Memory (Gwm)	109.78	11.50
WJ III Achievement Scores		
Broad Reading Skills	114.48	12.31
Math Calculation Skills	115.35	12.92
Calculation Test	114.34	13.05
Math Fluency Test	110.34	13.92
Math Reasoning	121.84	6.42
Applied Problems Test	123.64	8.73
Quantitative Concepts Test	121.12	9.37

Note. The Working Memory cluster score was estimated as the arithmetic mean of two test scores assessing working memory: Numbers Reversed and Auditory Working Memory. Standard scores have a mean of 100 and standard deviation of 15.

Table 4.23

Means and Standard Deviations of WJ III ACH Cluster Scores for Moderately High Achievers in Math Reasoning Regardless of Reading Skills (n = 401)

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1. Fluid Reasoning (Gf)	.253	.092	.092	40.38
2. Comprehension Knowledge (Gc)	.199	.129	.037	16.93
(b) Working Memory Used				
1. Fluid Reasoning (Gf)	.211	.092	.092	40.38
2. Comprehension Knowledge (Gc)	.161	.129	.037	16.94
3. Working Memory (Gwm)	.161	.151	.022	10.23

$p < .05$

Table 4.24

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Applied Problems Test

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1. Short-Term Memory (Gsm)	.129	.017	.017	6.72
(b) Working Memory Used				
1. Working Memory (Gwm)	.183	.034	.034	13.89

$p < .05$

Table 4.25

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Quantitative Concepts Test

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1. Fluid Reasoning (Gsm)	.822	.675	.675	830.10
(b) Working Memory Used				
1. Working Memory (Gwm)	. 869	.714	.714	994.61
2. Long-Term Memory (Glr)	-.072	.718	.005	6.42

$p < .05$

Very high achievers in math reasoning regardless of reading skills. This group consists of 102 children who show high achievement in math reasoning on the WJ III ACH (SS≥125), regardless of reading achievement. Table 4.26 provides descriptive information summarizing relevant cognitive and achievement scores for this group. The results for stepwise regression analyses on this group are presented in Tables 4.27 and 4.28 where the dependent variables are the Math Reasoning cluster and the Quantitative Concepts test. Each table presents two regressions: (a) when the sever CHC cognitive factors are used as predictors and (b) where the Working Memory cluster is used instead of the Short-Term Memory cluster to represent immediate awareness.

When stepwise regression analysis for this group was conducted with the Math Reasoning cluster as the dependent variable, and Short-Term Memory was used to describe immediate awareness, the significant predictors for success in math reasoning, ordered according to the absolute values of their standardized regression coefficients, were Fluid Reasoning (Gf), followed (inversely) by Long-Term Retrieval (Glr), $F(2, 99) = 4.56$, $p < .05$, $R^2 = .047$, signifying a small effect size. Identical results were obtained when Short Term Memory was replaced by Working Memory, upon entering independent variables.

When the dependent variable was the Applied Problems test, the Stepwise Regression analysis was not significant whether Short-Term Memory or Working Memory were used to describe immediate awareness.

When the dependent variable was the Quantitative Concepts test and Short-Term Memory was used to describe immediate awareness, the only significant predictor was Fluid Reasoning (Gf), $F(1,100) = 12.36$, $p < .05$, $R^2 = .110$, signifying a moderate effect size. The results were identical when Short-Term memory was replaced by Working Memory when entering the independent variables.

Table 4.26

Means and Standard Deviations of WJ III COG and ACH Cluster Scores for Very High Achievers in Math Reasoning Regardless of Reading Skills (n = 102)

Cluster	M	SD
<u>WJ III Cognitive Scores</u>		
General Intellectual Ability -Extended	123.23	11.17
Comprehension-Knowledge (Gc)	119.90	11.54
Long-Term Retrieval (Glr)	115.92	11.62
Visual-Spatial Thinking (Gv)	112.19	16.98
Auditory Processing (Ga)	113.41	16.20
Fluid Reasoning (Gf)	120.24	11.31
Processing Speed (Gs)	111.89	15.76
Short-Term Memory (Gsm)	115.36	14.42
Working Memory (Gwm)	114.72	11.18
WJ III Achievement Scores		
Broad Reading Skills	120.15	12.34
Math Calculation Skills	124.52	13.70
Calculation Test	122.86	14.33
Math Fluency Test	116.52	15.46
Math Reasoning	131.08	5.22
Applied Problems Test	131.98	8.30
Quantitative Concepts Test	130.46	8.57

Note. The Working Memory cluster score was estimated as the mean of the Numbers Reversed test score and the Auditory Working Memory test score. Standard scores have a mean of 100 and standard deviation of 15.

Table 4.27

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Reasoning Cluster

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1 Fluid Reasoning (Gf)	.262	.047	.047	4.99
2. Long-Term Memory (Glr)	-.197	.084	.037	3.98
(b) Working Memory Used				
1 Fluid Reasoning (Gf)	.262	.047	.047	4.99
2. Long-Term Memory (Glr)	-.197	.084	.037	3.98

$p < .05$

Table 4.28

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Quantitative Concepts Test

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1. Fluid Reasoning (Gf)	.332	.110	.110	12.37
(b) Working Memory Used				
1. Fluid Reasoning (Gf)	.332	.110	.110	12.37

$p < .05$

Research Question 3: Cognitive Abilities that Best Predict Excellence in Math Calculation Skills and in Math Reasoning for Overall High Math Achievers

In this question, the WJ III COG cognitive ability clusters, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), and Visual-Spatial Processing (Gv), Short-Term Memory (Gsm) , and additional cluster, Working Memory (Gwm) were analyzed through individual multiple stepwise regression equations to find the best predictors of the Math Calculation Skills cluster and the Math Reasoning cluster from the WJ III ACH.. The prediction of each achievement cluster and test was analyzed for four groups of children selected on the basis of their math and reading achievements. For each regression conducted, if the model was found significant, the standardized regression coefficients (β), the change in R^2, and the F value for the change in R^2 after entry of each independent variables, were presented for each significant predictor, and β values were used to order relevant predictors according to their relative importance. Cohen's (1988) convention was used to determine effect size.

. **Overall moderately high achievers in math not excelling in reading.** This group consists of 52 moderately high achievers in both math calculation skills and math reasoning that are no more than average readers (SS≥ 115 on Math Calculation Skills and SS≥ 115 on Math Reasoning and SS ≤110 on the Broad Reading clusters). Table 4.29 provides descriptive information summarizing relevant cognitive and achievement scores for this group. The results for stepwise Regression analyses on this group are presented in Tables 4.30 and 4.31 where the dependent variables are the Math Calculation Skills and the Math Reasoning Clusters. Each table presents two regressions: (a) when the sever CHC cognitive factors are used as predictors and (b) where the Working Memory cluster is used instead of the Short-Term-Memory cluster to represent immediate awareness.

The stepwise regression analysis revealed that when the dependent variable was the Math Calculation Skills cluster and Short-term Memory was used to describe immediate awareness, the only significant predictor for success in math calculation skills was Short-Term Memory (Gsm), $F(1, 50) = 4.80$, $p < .05$, $R^2 = .088$, signifying a small effect size. The model was not significant when Short-Term Memory cluster was replaced by the Working Memory cluster.

When the dependent variable was the Math Reasoning Skills cluster and Short-term Memory was used to describe immediate awareness, the only significant (inversely correlated) predictor was Processing Speed (Gs), $F(1, 50) = 10.29$, $p < .05$, $R^2 = .171$, signifying a medium effect size. Identical results were obtained when Short Term Memory was replaced by Working Memory upon entering the independent variables.

Table 4.29

Means and Standard Deviations of WJ III COG and ACH Cluster Scores for Overall Moderately High Achievers in Math Not Excelling in Reading (n = 52)

Cluster	M	SD
WJ III Cognitive Scores		
General Intellectual Ability -Extended	113.92	10.05
Comprehension-Knowledge (Gc)	110.81	10.56
Long-Term Retrieval (Glr)	111.00	11.52
Visual-Spatial Thinking (Gv)	108.58	13.59
Auditory Processing (Ga)	107.23	16.84
Fluid Reasoning (Gf)	115.75	11.84
Processing Speed (Gs)	104.40	11.40
Short-Term Memory (Gsm)	109.98	13.46
Working Memory (Gwm)	107.18	11.05
WJ III Achievement Scores		
Broad Reading Skills	104.87	5.23
Math Calculation Skills	123.33	5.93
Math Reasoning	122.63	6.60

Note. The Working Memory cluster score was estimated as the arithmetic mean of two test scores assessing working memory: Numbers Reversed and Auditory Working Memory. Standard scores have a mean of 100 and standard deviation of 15.

Table 4.30

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Calculation Skills Cluster

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1.Short-Term Memory (Gsm)	.296	.088	.088	4.80
(b) Working Memory Used				
Model Not Significant				

$p < .05$

Table 4.31

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Reasoning Cluster

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1.Processing Speed Gs)	-.413	.171	.171	10.29
(b) Working Memory Used				
1.Processing Speed Gs)	-.413	.171	.171	10.29

$p < .05$

Overall very high achievers in math not excelling in reading. This group consists of 9 very high achievers in both math calculation skills and math reasoning that are no more than average readers (SS≥ 125 on Math Calculation Skills and SS≥ 125 on Math Reasoning and SS ≤110 on the Broad Reading clusters). Table 4.32 provides descriptive information summarizing relevant cognitive and achievement scores for this group. Probably because of the small sample size (9 cases), The results for stepwise regression analyses on this group were not significant when the dependent variable was the Math Calculation Skills cluster or the Math Reasoning cluster, whether the Short-Term Memory or the Working Memory clusters were used to represent immediate awareness.

Table 4.32

Means and Standard Deviations of WJ III COG and ACH Cluster Scores for Overall Very High Achievers in Math Not Excelling in Reading (n = 9)

Cluster	M	SD
WJ III Cognitive Scores		
General Intellectual Ability -Extended	113.11	9.12
Comprehension-Knowledge (Gc)	118.33	8.35
Long-Term Retrieval (Glr)	111.00	10.87
Visual-Spatial Thinking (Gv)	107.89	15.02
Auditory Processing (Ga)	107.00	16.66
Fluid Reasoning (Gf)	114.56	11.59
Processing Speed (Gs)	95.78	10.02
Short-Term Memory (Gsm)	107.78	9.86
Working Memory (Gwm)	109.83	8.48
WJ III Achievement Scores		
Broad Reading Skills	106.78	3.42
Math Calculation Skills	128.89	5.84
Math Reasoning	131.67	6.46

Note. The Working Memory cluster score was estimated as the arithmetic mean of two test scores assessing working memory: Numbers Reversed and Auditory Working Memory. Standard scores have a mean of 100 and standard deviation of 15.

Overall moderately math achievers in math regardless of reading skills. This group consists of 190 moderately high achievers in both math calculation skills and math reasoning, regardless of their reading skills (SS≥ 115 on Math Calculation Skills and SS≥ 115 on Math Reasoning clusters). Table 4.33 provides descriptive information summarizing relevant cognitive and achievement scores for this group. The results for stepwise regression analyses on this group are presented in Tables 4.34 and 4.35 where the dependent variables are the Math Calculation Skills and the Math Reasoning Clusters. Each table presents two regressions: (a) when the sever CHC cognitive factors are used as predictors and (b) where the Working Memory cluster is used instead of the Short-Term-Memory cluster to represent immediate awareness.

The stepwise regression analysis revealed that when the dependent variable was the Math Calculation Skills cluster and Short-term Memory was used to describe immediate awareness, the only significant predictor for success in math calculation skills was Processing Speed (Gs), $F(1, 188) = 11.68$, $p < .05$, $R^2 = .058$, signifying a small effect size. When Short Term Memory was replaced by Working Memory, the significant predictors for success in math calculation skills, ordered according to the absolute values of their standardized regression coefficients, were Processing Speed (Gs) followed by Working Memory (Gwm), $F(2, 187) = 8.79$, $p < .05$, $R^2 = .086$ – signifying a moderate effect size.

When the dependent variable was the Math Reasoning cluster and Short-Term Memory was used to describe immediate awareness, the significant predictors for success in math reasoning, ordered according to the absolute values of their standardized regression coefficients, were Fluid Reasoning (Gf) followed by Comprehension Knowledge (Gc), $F(2, 187) = 15.09$, $p < .05$, $R^2 = .139$, signifying a medium effect size. Identical results were obtained when Short Term Memory was replaced by Working Memory upon entering the independent variables.

Table 4.33

Means and Standard Deviations of WJ III COG and ACH Cluster Scores for Overall Moderately High Achievers in Math Regardless of Reading Skills (n = 190)

Cluster	M	SD
WJ III Cognitive Scores		
General Intellectual Ability -Extended	117.39	11.12
Comprehension-Knowledge (Gc)	116.57	11.69
Long-Term Retrieval (Glr)	113.36	11.66
Visual-Spatial Thinking (Gv)	109.95	15.50
Auditory Processing (Ga)	111.19	15.38
Fluid Reasoning (Gf)	117.55	11.47
Processing Speed (Gs)	112.81	14.52
Short-Term Memory (Gsm)	114.59	13.69
Working Memory (Gwm)	112.54	11.24
WJ III Achievement Scores		
Broad Reading Skills	128.44	8.18
Math Calculation Skills	126.23	8.81
Math Reasoning	124.34	7.35

Note. The Working Memory cluster score was estimated as the arithmetic mean of two test scores assessing working memory: Numbers Reversed and Auditory Working Memory. Standard scores have a mean of 100 and standard deviation of 15.

Table 4.34

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Calculation Skills Cluster

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1.Processing Speed (Gs)	.241	.058	.058	11.62
(b) Working Memory Used				
1.Processing Speed (Gs)	.188	.058	.058	11.63
2. Working Memory (Gwm)	.175	.086	.028	5.65

$p < .05$

Table 4.35

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Reasoning Cluster

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1.Fluid Reasoning (Gf)	.289	.107	.107	22.49
2. Comprehension Knowledge (Gc)	.183	.139	.032	6.90
(b) Working Memory Used				
1.Fluid Reasoning (Gf)	.289	.107	.107	22.49
2. Comprehension Knowledge (Gc)	.183	.139	.032	6.90

$p < .05$

Overall very high achievers in math regardless of reading skills. This group consists of 56 very high achievers in both math calculation skills and math reasoning, regardless of reading skills (SS≥ 125 on Math Calculation Skills and SS≥ 125 on Math Reasoning clusters).

Table 4.36 provides descriptive information summarizing relevant cognitive and achievement scores for this group. The results for stepwise Regression analyses on this group are presented in Table 4.37 where the dependent variable is the Math Calculation Skills cluster. Each table presents two regressions: (a) when the sever CHC cognitive factors are used as predictors and (b) where the Working Memory cluster is used instead of the Short-Term-Memory cluster to represent immediate awareness.

The stepwise regression analysis revealed that when the dependent variable was the Math Calculation Skills cluster and Short-Term Memory was used to describe immediate awareness, was not significant. When Short Term Memory was replaced by Working Memory, the only significant predictor for success in math calculation skills was Working Memory (Gwm), $F(1, 54) = 4.52$, $p < .05$, $R^2 = .077$ – signifying a small effect size..

The Stepwise Regression was not significant when the dependent variable was the Math Reasoning cluster, whether the Short-term Memory or the Working Memory clusters were used to describe immediate awareness.

Table 4.36

Means and Standard Deviations of WJ III COG and ACH Cluster Scores for Overall Very High Achievers in Math Regardless of Reading Skills (n = 56)

Cluster	M	SD
WJ III Cognitive Scores		
General Intellectual Ability -Extended	124.96	10.62
Comprehension-Knowledge (Gc)	120.55	10.88
Long-Term Retrieval (Glr)	114.96	10.63
Visual-Spatial Thinking (Gv)	111.57	16.51
Auditory Processing (Ga)	113.45	16.33
Fluid Reasoning (Gf)	121.46	10.96
Processing Speed (Gs)	114.93	16.64
Short-Term Memory (Gsm)	117.91	14.41
Working Memory (Gwm)	116.71	10.88
WJ III Achievement Scores		
Broad Reading Skills	122.87	11.90
Math Calculation Skills	134.37	8.64
Math Reasoning	131.84	5.70

Note. The Working Memory cluster score was estimated as the arithmetic mean of two test scores assessing working memory: Numbers Reversed and Auditory Working Memory. Standard scores have a mean of 100 and standard deviation of 15.

Table 4.37

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Calculation Skills Cluster

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
Not Significant				
(b) Working Memory Used				
1.Working Memory (Gwm)	.278	.077	.077	4.52

$p < .05$

Research Question 4: Cognitive Abilities that Best Predict Excellence in Math Calculation Skills and Math Reasoning for Average Achievers

In this question, the WJ III COG cognitive ability clusters, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), and Visual-Spatial Processing (Gv), Short-Term Memory (Gsm) , and the WJ III COG additional cluster, Working Memory (Gwm) were analyzed through individual multiple stepwise regression analyses to find the best predictors of the Math Calculation Skills cluster and the Math Reasoning cluster, the Calculation test, the Math Fluency test, the Applied Problems test and the Quantitative Concepts test from the WJ III ACH for average achievers. The prediction of each achievement cluster and test was analyzed for a group of children selected on the basis of their math and reading achievements. For each regression conducted, if the model was found significant, the standardized regression coefficients (β), the change in R^2, and the F value for the change in R^2 after entry of each independent variables, were presented for each significant predictor, and β values were used to order relevant predictors according to their relative importance. Cohen's (1988) convention was used to determine effect size.

Average achievers. This group consists of 588 average achievers in math calculation skills, math reasoning and reading (SS≥ 90 and SS ≤110 on the Broad Reading, Math Calculation Skills and Math Reasoning clusters). Table 4.38 provides descriptive information summarizing relevant cognitive and achievement scores for this group. The results for stepwise regression analyses on this group where the dependent variables are the Math Calculation Skills cluster, the Calculation test and the Math Fluency test are presented in Tables 4.39, 4.40, and 4.41. Results where the dependent variables are the Math Reasoning Cluster, the Applied Problems test and the Quantitative Concepts test are presented in Tables 4.42, 4.43 and 4.44. Each table presents two regressions: (a) when the sever CHC cognitive factors are used as predictors and (b) where the Working Memory cluster is used instead of the Short-Term Memory cluster to represent immediate awareness.

The stepwise regression analysis revealed that when the dependent variable was the Math Calculation Skills cluster and Short-Term Memory was used to describe immediate awareness, the only significant predictor for success in math calculation skills was Processing Speed (Gs), $F(1, 586) = 28.59$, $p < .05$, $R^2 = .047$, signifying a small effect size. Identical results were obtained when Short-Term Memory cluster was replaced by the Working Memory cluster.

When the dependent variable was the Calculation Test and Short-Term Memory was used to describe immediate awareness, the significant predictors for success in math reasoning, ordered according to absolute the values of their standardized regression coefficients, were Comprehension Knowledge (Gc) followed by Fluid Reasoning (Gf), $F(2, 585) = 9.58$, $p < .05$,

$R^2 = .032$, signifying a small effect size. Identical results were obtained when Short Term Memory was replaced by Working Memory upon entering the independent variables.

When the dependent variable was the Math Fluency Test and Short-Term Memory was used to describe immediate awareness, the significant predictors for success in math reasoning, ordered according to absolute values of their standardized regression coefficients, were Processing Speed (Gs), followed (inversely) by Comprehension Knowledge (Gc), $F(2, 585) = 74{,}68$, $p < .05$, $R^2 = .203$, signifying a large effect size. Identical results were obtained when Short Term Memory was replaced by Working Memory upon entering the independent variables.

When the dependent variable was the Math Reasoning Skills cluster and Short-Term Memory was used to describe immediate awareness, the significant predictors for success in math reasoning, ordered according to absolute values of their standardized regression coefficients, were Comprehension Knowledge (Gc) followed by Fluid Reasoning (Gf) and then followed by Long-Term Retrieval (Glr), $F(3, 584) = 36.67$, $p < .05$, $R^2 = .162$, signifying a medium effect size. Identical results were obtained when Short Term Memory was replaced by Working Memory upon entering the independent variables.

When the dependent variable was the Applied Problems test and Short-Term Memory was used to describe immediate awareness, the significant predictors for success in applied problems, ordered according to absolute values of their standardized regression coefficients, were Fluid Reasoning (Gf) followed by Long-Term Memory (Glr), $F(2, 585) = 11.31$, $p < .05$, $R^2 = .037$, signifying a small effect size. Identical results were obtained when Short Term Memory was replaced by Working Memory upon entering the independent variables.

When the dependent variable was the Quantitative test and Short-term Memory was used to describe immediate awareness, the significant predictors for success in quantitative concepts, ordered according to the values of their standardized regression coefficients, were Comprehension Knowledge (Gc), followed by Long-Term Memory (Glr), and then followed by Fluid Reasoning (Gf), $F(3, 584) = 32.53$, $p < .05$, $R^2 = .143$, signifying a medium effect size. Identical results were obtained when Short Term Memory was replaced by Working Memory upon entering the independent variables

Table 4.38

Means and Standard Deviations for WJ III COG and ACH Cluster Scores for Average Achievers (n = 588)

Cluster	M	SD
WJ III Cognitive Scores		
General Intellectual Ability -Extended	101.23	9.52
Comprehension-Knowledge (Gc)	101.01	10.57
Long-Term Retrieval (Glr)	102.91	12.42
Visual-Spatial Thinking (Gv)	101.17	13.30
Auditory Processing (Ga)	102.54	13.24
Fluid Reasoning (Gf)	100.93	12.05
Processing Speed (Gs)	99.72	12.38
Short-Term Memory (Gsm)	101.59	12.83
Working Memory (Gwm)	100.61	10.29
WJ III Achievement Scores		
Broad Reading Skills	100.28	5.46
Math Calculation Skills	99.89	5.59
Calculation Test	100.35	7.24
Math Fluency Test	97.89	9.63
Math Reasoning	100.18	5.61
Applied Problems Test	100.71	7.92
Quantitative Concepts Test	100.65	7.94

Note. The Working Memory cluster score was estimated as the arithmetic mean of two test scores assessing working memory: Numbers Reversed and Auditory Working Memory. Standard scores have a mean of 100 and standard deviation of 15.

Table 4.39

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Calculation Skills Cluster

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1.Processing Speed (Gs)	.216	.047	.047	28.59
(b) Working Memory Used				
1.Processing Speed (Gs)	.216	.047	.047	28.59

$p < .05$

Table 4.40

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Calculation Test

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1.Comprehension Knowledge (Gc)	.114	.023	.023	14.09
2. Fluid Reasoning (Gf)	.099	.032	.008	4.97
(b) Working Memory Used				
1.Comprehension Knowledge (Gc)	.114	.023	.023	14.09
2. Fluid Reasoning (Gf)	.099	.032	.008	4.97

$p < .05$

Table 4.41

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Fluency Test

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1.Processing Speed (Gs)	.389	.158	.158	109.59
2.Comprehension Knowledge (Gc)	-.214	.203	.046	33.67
(b) Working Memory Used				
1.Processing Speed (Gs)	.389	.158	.158	109.59
2.Comprehension Knowledge (Gc)	-.214	.203	.046	33.67

$p < .05$

Table 4.42

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Math Reasoning Cluster

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1.Comprehension Knowledge Gc)	.200	.102	.102	66.56
2. Fluid Reasoning (Gf)	.180	.143	.041	28.34
3. Long-Term Retrieval (Glr)	.150	.162	.019	13.08
(b) Working Memory Used				
1.Comprehension Knowledge Gc)	.200	.102	.102	66.56
2. Fluid Reasoning (Gf)	.180	.143	.041	28.34
3. Long-Term Retrieval (Glr)	.150	.162	.019	13.08

$p < .05$

Table 4.43

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Applied Problems Test

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
2. Fluid Reasoning (Gf)	.132	.028	.028	17.13
3. Long-Term Retrieval (Glr)	.101	.037	.009	5.35
(b) Working Memory Used				
2. Fluid Reasoning (Gf)	.132	.028	.028	17.13
3. Long-Term Retrieval (Glr)	.101	.037	.009	5.35

$p < .05$

Table 4.44

Stepwise Regressions for: (a) 7 WJ III Ability Clusters, (b) with Working Memory Substituting Short-Term Memory, on Quantitative Concepts Test

Predictor	β	R^2	Change in R^2	F for Change
(a) Short-Term Memory Used				
1.Comprehension Knowledge Gc	.213	.099	.099	64.17
2. Long-Term Retrieval (Glr)	.142	.128	.029	19.45
3. Fluid Reasoning (Gf)	.142	.143	.016	10.63
(b) Working Memory Used				
1.Comprehension Knowledge Gc	.213	.099	.099	64.17
2. Long-Term Retrieval (Glr)	.142	.128	.029	19.45
3. Fluid Reasoning (Gf)	.142	.143	.016	10.63

$p < .05$

Summary of Results

Tables 4.45 summarizes and compares the means and standard deviations of cognitive abilities (CHC clsuters and working memory) for specifically high achievers in math calculation skills, math reasoning and overall math – the focus groups of this study – and those of average achievers,. Tables 4.46, 4.47, 4.48 and 4.49 summarize results of the stepwise regression analyses pertaining to research questions 1, 2, 3, 3 and 4 respectively. Each table provides – for a given group of participants and dependent variable used – the list of significant predictors, ordered according to absolute values of the standardized regression coefficients (βs), and the effect size according to Cohen's (1988) convention. Column 1 depicts the dependent variables used in the stepwise regressions. Columns two and three provide the results with Short-Term memory used to signify immediate awareness, whereas columns four and five provide the results with working memory used to signify immediate awareness. The summary tables facilitate several comparisons that are further discussed in Chapter 5. These include a comparison of the cognitive abilities underlying math calculation skills and math reasoning among different groups of participants representing different achievement levels in mathematics (moderately high, very high and average), and broad reading. Furthermore, the summary tables facilitate a comparison of significant cognitive abilities when short-term memory versus working memory signifies immediate awareness. Finally, a comparison of the cognitive abilities underlying specifically high math achievement but no more than average reading skills versus cognitive abilities underlying high math achievement regardless of reading skills is also facilitated.

In the summary tables, the following notation is used to describe the significant predictors, signifying CHC abilities (Carroll, 1993) and working memory (CHC abilities are further described in Table A.1, Appendix A):

Comp Knowledge = Comprehension Knowledge

Long-Term Ret = Long-Term Retrieval

Visual-Spatial = Visual/Spatial Thinking

Auditory Proc = Auditory Processes

Fluid Reasoning = Fluid Reasoning

Proc Speed = Processing Speed

Short-Term Memory = Short-Term Memory

Working Memory = Working Memory

MHS MCS = Moderatley High and Specific Achievers in Math Calculation Skills

VHS MCS = Very High and Specific Achievers in Math Calculation Skills

MHS MR = Moderatley High and Specific Achievers in Math Reasoning

VHS MR = Very High and Specific Achievers in Math Reasoning

MHS Math = Moderatley High and Specific Achievers in Overall Math

VHS Math = Very High and Specific Achievers in Overall Math

Average = Average Achievers

Math Calc Skills = Math Calculation Skills cluster

SS = Standard Score

n = sample size

In describing the magnitude or strength of the association between the dependent and independent variables in thus study, Cohen's (1988) convention was used to determine effect size:

Small = effect size has an absolute value of approximately 0.02

Medium = effect size has an absolute value of approximately 0.13

Large = effect size has an absolute value of approximately 0.26

NS = corresponding regression model was not significant.

Table 4.45

Cognitive Ability Cluster Means and Standard Deviations for High and Specific Math Achievement Groups and Average Achieving Group

	MHS MCS (n = 172)		VHS MCS (n = 40)		MHS MR (n= 156)		VHS MR (n =23)		MHS Math (n = 52)		VHS Math (n = 9)		Average (n =588)	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Cluster														
Math Calc. Skills	SS≥115		SS≥125						SS≥115		SS≥125		110≥SS≥90	
Math Reasoning					SS≥115		SS≥125		SS≥115		SS≥125		110≥SS≥90	
Broad Reading	SS≤110		SS≤110		SS≤110		SS≤110		SS≤110		SS≤110		110≥SS≥90	
Comp Knowledge	102.48	12.13	103.80	14.95	109.88	9.41	114.87	7.40	110.81	10.56	118.33	8.35	101.01	10.57
Long-Term Ret	104.33	12.41	106.10	13.43	108.89	11.96	111.61	11.98	111.00	11.52	111.00	10.87	102.91	12.42
Visual-Spatial	102.65	13.48	104.82	15.87	105.88	14.31	110.43	15.47	108.58	13.59	107.89	15.02	101.17	13.30
Auditory Proc	103.36	14.27	103.75	17.61	105.06	15.19	105.17	12.80	107.23	16.84	107.00	16.66	102.54	13.24
Fluid Reasoning	105.63	13.55	107.75	14.39	112.71	11.42	117.09	10.96	115.75	11.84	114.56	11.59	100.93	12.05
Proc Speed	103.22	11.28	103.32	12.72	102.19	11.98	99.61	9.31	104.40	11.40	95.78	10.02	99.72	12.38
Short-Term Memory	104.02	12.65	107.57	14.54	106.27	12.96	106.52	11.11	109.98	13.46	107.78	9.86	101.59	12.83
Working Memory	103.73	10.96	106.16	11.21	105.33	10.83	107.89	10.37	107.18	11.05	109.83	8.48	100.61	10.29

Note: See definitions of column and raw names above.

Table 4.46
Research Question 1: Summary of Significant Predictors, Effect Size, for High Achievers in Math Calculation Skills

	Short Term Memory Used		Working Memory Used	
Dependent Variable	Effect Size	Significant Predictors Ranked	Effect Size	Significant Predictors Ranked
(a) Moderately High Achievers in Math Calculation Skills Not Excelling in Reading				
Math Calculation Skills Cluster	Small	Short-Term Memory	Small	Working Memory
Calculation Test	Medium	Visual-Spatial, Proc Speed, Fluid Reasoning	Medium	Visual-Spatial, Proc Speed, Fluid Reasoning
Math Fluency Test	Medium	Proc Speed, Visual-Spatial, Long-Term Ret	Medium	Proc Speed, Visual Spatial, Long Term Ret
(b) Very High Achievers in Math Calculation Skills Not Excelling in Reading				
Math Calculation Skills Cluster	Medium	Visual-Spatial	Medium	Visual-Spatial
Calculation Test	Large	Visual-Spatial; Proc Speed	Large	Visual-Spatial; Proc Speed
Math Fluency Test	Large	Proc Speed, Visual-Spatial, Long-Term Ret	Large	Proc Speed, Visual-Spatial; Long-Term Ret
(c) Moderately High Achievers in Math Calculation Skills Regardless of Reading Skills				
Math Calculation Skills Cluster	Medium	Fluid Reasoning, Proc Speed, Visual-Spatial	Medium	Fluid Reasoning, Proc Speed, Visual-Spatial
Calculation Test	Medium	Fluid Reasoning, Visual-Spatial, Proc Speed, Comp Knowledge	Medium	Fluid Reasoning, Visual-Spatial, Proc Speed, Comp Knowledge
Math Fluency Test	Medium	Proc Speed, Visual-Spatial	Medium	Proc Speed, Visual-Spatial
(d) Very High Achievers in Math Calculation Skills Regardless of Reading Skills				
Math Calculation Skills Cluster	Medium	Visual-Spatial, Fluid Reasoning, Proc Speed	Medium	Visual-Spatial, Fluid Reasoning, Proc Speed
Calculation Test	Medium	Fluid Reasoning, Visual-Spatial, Proc Speed	Medium	Fluid Reasoning, Visual-Spatial, Proc Speed
Math Fluency Test	Medium	Proc Speed	Medium	Proc Speed

Table 4.47
Research Question 2: Summary of Significant Predictors, Effect Size, for High Achievers in Math Reasoning

	Short-Term Memory Used		Working Memory Used	
Dependent Variable (WJ III ACH Cluster/Test)	Effect Size	Significant Predictors Ranked	Effect Size	Significant Predictors Ranked
(a) Moderately High Achievers in Math Reasoning Not Excelling in Reading				
Math Reasoning Cluster	Small	Fluid Reasoning	Small	Fluid Reasoning
Applied Problems Test	NS		Non Significant	
Quantitative Concepts Test	Small	Fluid Reasoning	Small	Fluid Reasoning
(b) Very High Achievers in Math Reasoning Not Excelling in Reading				
Math Reasoning Cluster	NS		NS	
Applied Problems Test	NS		NS	
Quantitative Concepts Test	NS		NS	
(c) Moderately High Achievers in Math Reasoning Regardless of Reading Skills				
Math Reasoning Cluster	.Medium	Fluid Reasoning, Comp Knowledge	Medium	Fluid Reasoning, Comp Knowledge, Working Memory
Applied Problems Test	Small	Short-Term Memory	Small	Working Memory
Quantitative Concepts Test	Large	Short-Term Memory	Large	Working Memory, Long-Term Ret
(d) Very High Achievers in Math Reasoning Regardless of Reading Skills				
Math Reasoning Cluster	Small	Fluid Reasoning, Long-Term Ret	Small	Fluid Reasoning, Long-Term Ret
Applied Problems Test	NS		NS	
Quantitative Concepts Test	Medium	Fluid Reasoning,	Medium	Fluid Reasoning

Table 4.48

Research Question 3: Summary of Significant Predictors, Effect Size, for Overall High Math Achievers

	Short Term Memory Used		Working Memory Used	
Dependent Variable (WJ III ACH Cluster/Test)	Effect Size	Significant Predictors Ranked	Effect Size	Significant Predictors Ranked
(a) Overall Moderately High Achievers in Math Not Excelling in Reading				
Math Calculation Skills Cluster	Small	Short-Term Memory	NS	
Math Reasoning Cluster	Medium	Proc Speed	Medium	Proc Speed
(b) Overall Very High Achievers in Math Not Excelling in Reading				
Math Calculation Skills Cluster	NS		NS	
Math Reasoning Cluster	NS		NS	
(c) Overall Moderately High Achievers in Math Regardless of Reading Skills				
Math Calculation Skills Cluster	Small	Proc Speed	Medium	Proc Speed, Working Memory
Math Reasoning Cluster	Medium	Fluid Reasoning, Comp Knowledge	Medium	Fluid Reasoning, Comp Knowledge
(d) Overall Very High Achievers in Math Regardless of Reading Skills				
Math Calculation Skills Cluster	NS		Small	Working Memory
Math Reasoning Cluster	NS		NS	

Table 4.49

Research Question 4:Summary of Significant Predictors, Effect Size, for Average Achievers

	Short Term Memory Used		Working Memory Used	
Dependent Variable (WJ III ACH Cluster/Test)	Effect Size	Significant Predictors Rank-ordered	Effect Size	Significant Predictors Rank-Ordered
Math Calculation Skills Cluster	Small	Proc Speed	Small	Proc Speed
Calculation	Small	Comp Knowledge, Fluid Reasoning	Small	Comp Knowledge, Fluid Reasoning
Math Fluency	Large	Processing Speed, Comp Knowledge	Large	Processing Speed, Comp Knowledge
Math Reasoning Cluster	Medium	Comp Knowledge, Fluid Reasoning, Long-Term Ret	Medium	Comp Knowledge, Fluid Reasoning, Long-Term Ret
Applied Problems Test	Small	Fluid Reasoning, Long-Term Ret	Small	Fluid Reasoning, Long-Term Ret
Quantitative Concepts Test	Medium	Comp Knowledge, Long-Term Ret, Fluid Reasoning	Medium	Comp Knowledge, Long-Term Ret, Fluid Reasoning

CHAPTER FIVE

Discussion

The purpose of this chapter is to expand the discussion of the results of the investigation regarding the cognitive abilities that are related to, and best predict high achievement in mathematics. This study also provides concurrent validity information for the CHC cognitive abilities most important for high academic achievement, as well as the utility of the WJ III for this population.

An expansion of the findings in Chapter Four is discussed per each research question in this chapter. Implications of these conclusions as they relate to research results published in the literature will be explored, and important contributions to the literature will be outlined. Strengths and limitations of this study and directions for future research will also be discussed.

Initial Descriptive Analyses

Comparison of the performance of the entire sample of school-aged children on the WJ III (after the removal of incomplete cases) to that expected based on standard scores with a mean of 100 and a standard deviation of 15, revealed that most observed means and standard deviations for WJ III COG and WJ III ACH clusters were slightly higher than would be expected. It is possible that the deleted cases with missing relevant data may have had slightly biased mean scores.

As was discussed in Chapter 4, children's cognitive scores are fairly uniform over time, thus it is reasonable to use mean scores over age groups (6 to 18). Cognitive scores show relatively elevated levels in long-term retrieval (Glr) and auditory processing (Ga) at ages 7 and 14, in processing speed (Gs) at ages 8 and 18, in short-term memory (Gsm) at ages 8 and 14, and in comprehension knowledge (Gc) at ages 11 to 14. Achievement scores in math and reading clusters are also fairly uniform per age with lower levels from ages 10 to 15, possibly reflecting reduced effort as children approach puberty.

Prediction of math calculation skills from cognitive abilities for high achievers: Discussion and implications. In this question, the WJ III COG cognitive ability clusters, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Short-Term Memory (Gsm), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), Visual-Spatial Processing (Gv), and the WJ III COG additional cluster, Working Memory (estimated as the mean of two test scores assessing working memory) were analyzed through individual multiple stepwise regression analyses to find the most important cognitive abilities from the WJ III ACH for the Math Calculation Skills cluster. This was calculated at several levels of achievement and specificity in math calculation skills. Specifically, regression analyses were performed on: (a) a group of children exhibiting moderately high and specific proficiency in math calculation skills; (b) a group of children exhibiting very high and specific proficiency in math calculation skills; (c) a group of children exhibiting moderately high non specific proficiency in math calculation skills; and (d) a group of children exhibiting very high non specific proficiency in math calculation skills. To further refine the correlation between cognitive abilities and math calculation skills, the relationship between cognitive abilities and scores on the Calculation and Math Fluency tests comprising the Math Calculation Skills cluster were also performed on the same groups. Results for this group are summarized in Table 4.46 in Chapter 4.

In the examination of cognitive abilities that best predict *moderately high and specific* proficiency in math calculation skills, it was found that Short Term Memory (Gsm), or Working memory (Gwm) – when the regression was run with the Working Memory cluster replacing Short Term Memory, appear the most significant predictors of the Math Calculation Skills cluster, although with a small effect size. This is consistent with Dark and Benbow's (1990; 1991) stipulation that children of high mathematics ability outperform verbally precocious children *in* working-memory manipulation. At the test level, Visual/Spatial Thinking (Gv), followed (inversely) by Processing Speed (Gs), and then followed by Fluid Reasoning (Gf), in this order, appear the best predictors, with a medium effect size, of the Calculation test (solving grade-level math problems), whereas Processing Speed (Gs), and then (inversely) Visual/Spatial Thinking (Gv), followed by (inversely) Fluid Reasoning (Gf), appear the best predictors, also with a medium effect size, of the Math Fluency test (fluently solving simple arithmetic problems).

In the examination of cognitive abilities that best predict *very high and specific* math calculation skills, it was found that Visual/Spatial Thinking (Gv) appears the most significant predictor of very high and specific proficiency in math calculation skills with a medium effect size, although an examination of Table 4.45 – summarizing CHC cognitive abilities group mean scores – shows that the mean Visual/Spatial Thinking score for very high achievers, whereas higher than the corresponding score for average achievers, is not particularly high, compared to other CHC cognitive abilities for this group. Visual/Spatial Thinking (Gv), followed (inversely) by Processing Speed (Gs), appear the best predictors of the Calculation test, with a large effect size, whereas Processing Speed (Gs) followed (inversely) by Visual/Spatial Thinking (Gv), and then followed (inversely) by Long-Term memory (Glr), appear the best predictors of the Math Fluency test, also with a large effect size. Thus it appears that performance at a higher level of math calculation skills places higher demands on visual/spatial abilities, supporting reports in the literature on the role of Visual/spatial abilities in arithmetic among mathematically gifted populations (Benbow & Minor, 1990; Benbow., Stanley, Kirk, & Zonderman, 1983; Gardner, 1983; Hermelin & O'Connor, 1986; Krutetskii, 1976). Results at the test level demonstrated the importance of processing speed as predictor of the (timed) Math Fluency test but not at the cluster level. This is in line with inconsistent reports on the role of processing speed in the high ability literature, some of which has demonstrated higher capabilities in processing speed for gifted children compared to less able peers (Geary & Brown, 1991; Keating & Bobbitt, 1978) whereas other reports question the role of processing speed in math proficiency (Kaufman, 1994).

For children with *moderately high non specific* proficiency in math calculation skills, at the cluster level, it was found that Fluid reasoning (Gf), Processing Speed (Gs), and Visual/Spatial Thinking (Gv), in this order, are best predictors of math calculation skills, with a medium effect size, whereas Visual/Spatial Thinking (Gv), Fluid reasoning (Gf) and Processing Speed (Gs), in this order, are best predictors of *very high non specific* proficiency in math calculation skills, with a medium effect size.

In summary, the cognitive abilities found to be best predictors of math calculation skills are generally consistent with those reported in the literature. The role of visual/spatial abilities as best predictors of math calculation skills – for very high achievers – is noteworthy, given inconsistent literature about its predictive power for low math achievers. Appendix C provides examples of WJ III ACH math tests items with Visual/Spatial cemands.It may be surprising that Processing Speed is not a significant predictor of the Math Calculation Skills cluster for the groups of specific high math achievers studied, although it is consistently the best predictor of the time-restricted Math Fluency test. However, examination of mean scores on the Math Calculation Skills cluster, the Calculation test and the Math Fluency test suggest that the Math Calculation Skills cluster scores for the groups studied appear to more closely resemble scores on the (not timed) Calculation test than to scores on the (timed) Math Fluency test, and thus be less correlated to Processing Speed. As noted, different cognitive profiles appear to predict strength in math calculation skills among the groups studied, possibly reflecting different cognitive demands at different levels of math calculation skills, but also possibly reflecting a considerable cognitive variability within and across groups. For example, the General Intellectual Ability – Extended (GIA-Ext) score for the Moderately High Achievers in Math Calculation Skills Not Excelling in Reading group was 104.65 – a level that more closely represents average rather than high ability, whereas the group of Very High Achievers in Math Calculation Skills Regardless of Reading Skills scored in the High Average range (GIA-Ext=116.63). These numbers are mean scores, possibly covering a high range of participants, of average to high cognitive abilities, respectively.

Prediction of math reasoning from cognitive clusters for high achievers: Discussion and implications. In this question, the WJ III COG cognitive ability clusters, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Short-Term Memory (Gsm), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), Visual-Spatial Processing (Gv), and the WJ III COG additional cluster, Working Memory (estimated as the mean of two test scores assessing working memory), were analyzed through individual multiple stepwise regression analyses to find the most important cognitive abilities for the Math Reasoning cluster from the WJ III ACH. This was done at several levels of achievement and specificity in math reasoning. Specifically, regression analyses were performed on: (a) a group of children exhibiting moderately high and specific proficiency in math reasoning; (b) a group children exhibiting very high and specific proficiency in math reasoning; (c) a group of children exhibiting moderately high non specific proficiency in math reasoning; and (d) a group of children exhibiting very high non specific proficiency in math reasoning. To further refine the correlation between cognitive abilities and math reasoning, the relationship between cognitive abilities and scores on the Applied Problems and Quantitative Concepts tests comprising the Math Reasoning cluster were also performed on the same groups. Results for this group are summarized in Table 4.47 in Chapter 4.

In the examination of cognitive abilities that best predict *moderately high and specific* proficiency in math reasoning, it was found that Fluid Reasoning (Gf) appears the only significant predictors of math calculation skills at the cluster level, although with a small effect size. Fluid Reasoning (Gf) was also the only significant predictor of achievement on the Quantitative Concepts test, also with a small effect size. (Results on the Applied Problems test were not statistically significant). Results for *very high achievers* in math reasoning were not significant, probably because of the small sample size.

For children with *moderately high non specific* proficiency in math reasoning it was found that Fluid reasoning (Gf) and Comprehension Knowledge (Gc), in this order, are best predictors of the Math Reasoning cluster, and that Fluid reasoning (Gf), Comprehension Knowledge (Gc) and Working Memory (Gwm), in this order, are best predictors of math reasoning – when the regression was run with the Working Memory cluster replacing Short Term Memory, both with a medium effect size. For *very high non specific* achievers in math reasoning, it was found that Fluid reasoning (Gf) followed (inversely) by Long-Term Memory (Glr), are best predictors of the Math Reasoning cluster, with a small effect size.

In summary, in accordance with results reported in the high ability literature (Dark & Benbow, 1990; 1991; Montague & van Garderen, 2003; Roid, 2003), fluid reasoning is the most significant predictor of math reasoning, followed by Comprehension Knowledge (Proctor et al., 2005) for moderate achievers, and inversely by Long-Term Retrieval for very high achievers.

Another interesting observation is that all the groups representing high achievement in math reasoning are characterized by higher general intellectual ability than those groups representing corresponding achievement level in math calculation skills, possibly defining a more cohesive sample. Thus, excellence in fluid reasoning appears to distinguish a more gifted group showing very high achievement in mathematics.

Prediction of math skills from cognitive abilities for overall high achievers: Discussion and implications. In this question, the WJ III COG cognitive ability clusters, Comprehension-Knowledge (Gc), Long-Term Retrieval (Glr), Short-Term Memory (Gsm), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), Visual-Spatial Processing (Gv), and the WJ III COG additional cluster, Working Memory (estimated as the mean of two test scores assessing working memory) were analyzed through individual multiple stepwise regression analyses to find the most important cognitive abilities for the Math Calculation Skills cluster, and for the Math Reasoning cluster from the WJ III ACH for overall high achievers in math. This was done at several levels of achievement and specificity in mathematics. Specifically, regression analyses were performed on: (a) a group of children exhibiting moderately high and specific proficiency in mathematics; (b) a group children exhibiting very high and specific proficiency in mathematics; (c) a group of children exhibiting moderately high non specific proficiency in mathematics; and (d) a group of children exhibiting very high non specific proficiency in mathematics. This was performed only at the cluster level. Results for this group are summarized in Table 4.48 in Chapter 4.

In the examination of cognitive abilities that best predict *overall moderately high and specific* achievement in *math calculation skills*, it was found that Short-Term Memory (Gsm) appears the only significant predictors of math calculation skills, with a small effect size. This result is consistent with that obtained for the group of moderately high and specific achievers in math calculation skills (Research Question 1). In examining cognitive abilities that best predict *moderately high and specific* proficiency in math reasoning, it was found that Processing Speed (Gs) appears the only significant but inversely correlated predictor of math reasoning, with a medium effect size.

For overall moderately high non specific achievers in mathematics it was found that Processing Speed (Gs) is the only predictor of the Math Calculation Skills cluster – when Short-Term Memory represented immediate awareness; and that Processing Speed (Gs), and Working Memory (Gwm), in that order, were the best predictors of Math Calculation skills,– when Working Memory replaced Short-Term Memory, both with a medium effect size. It was also found that Fluid reasoning (Gf) and Comprehension Knowledge (Gc), in this order, are best predictors of math reasoning, with a medium effect size, for the same group. These results are consistent with the high ability literature as discussed in Chapter 2. For very high achievers in math, regardless of reading skills, Working Memory (Gwm) was found to be the only predictor of Math Calculation skills, with a small effect size (the regression used to predict math reasoning was not significant). The significance of working memory as a predictor of excellence in math calculation skills for high ability participants was discussed above (see Research Question 1).

In summary, results for this group are generally consistent with those reported in the literature. However, this group – consisting of high achievers in both math calculation skills and math reasoning who do not excel in reading – is probably quite exceptional in the general population.

Prediction of math skills from cognitive abilities for average achievers: Discussion and implications. In this question, the WJ III COG cognitive ability clusters, Comprehension-

Knowledge (Gc), Long-Term Retrieval (Glr), Short-Term Memory (Gsm), Fluid Reasoning (Gf), Processing Speed (Gs), Auditory Processing (Ga), Visual-Spatial Processing (Gv), and the WJ III COG additional cluster, Working Memory (estimated as the mean of two test scores assessing working memory) were analyzed through individual multiple stepwise regression analyses to find the most important cognitive abilities for the Math Calculation Skills and Math Reasoning clusters and for the Calculation, Math Fluency, Applied Problems and Quantitative Concepts tests from the WJ III ACH, for the group of students who are average achievers in math calculating skills, math reasoning and reading.. Results for this group are summarized in Table 4.4 in Chapter 4.

In the examination of cognitive abilities that best predict average proficiency in math calculation skills it was found that Processing Speed (Gs) appears the only significant predictor of math calculation skills for average achievers, with a small effect size. At the test level, Comprehension Knowledge (Gc) and Fluid Reasoning (Gf), in this order appear the best predictors, with a small effect size, of the Calculation test, whereas Processing Speed (Gs) followed (inversely) by Comprehension Knowledge (Gc), appear the best predictors, with a large effect size, of the Math Fluency test,.

In the examination of cognitive abilities that best predict average proficiency in math reasoning it was found that Comprehension Knowledge (Gc), Fluid Reasoning (Gf), and Long-Term Retrieval (Glr), in that order, appear the only significant predictors of math reasoning, at the cluster level, with a medium effect size. It was also found that Fluid Reasoning (Gf), and Long-Term Retrieval (Glr), in that order, were significant predictors of achievement on the Applied Problems test, with medium effect size, whereas Comprehension Knowledge (Gc), Long-Term Retrieval (Glr) and Fluid Reasoning (Gf), in that order, were significant predictors of the Quantitative Concepts test, with a medium effect size. In summary, significant cognitive predictors for the Math Calculation Skills cluster and the Math Reasoning cluster for average achievers were generally consistent with findings in the literature.

It is interesting to note that in the present study, the correlation between cognitive abilities and math calculation skills varies with achievement level: Processing Speed was found to be the only predictor of math calculation skills for *average* achievers, whereas Short-Term Memory or Working Memory, as well as Visual-Spatial Thinking were found to be significant predictors of math calculation skills for *moderately high* and *very high* achievers respectively (Research Question 1).Examination of cluster and test scores for average achievers shows that they score similarly on the Calculation test (not timed) and the Math Fluency test (timed), and their scores on the Math Calculation Skills cluster correlate well with their scores on both the Calculation test and the Math Fluency test., Therefore it is not surprising the Processing Speed is a significant predictor of the Math Calculation Skills cluster. On the other hand, higher achievers in math calculation skills tend to score better on the Calculation test than on the Math Fluency test and their scores on the Math Calculation Skills cluster appear to correlate better with results on the not timed Calculation test (possibly reflecting weighted averaging algorithms of the WJ III), thus Processing Speed appears a better predictor of the Math Calculation Skills cluster for average achievers than for high achievers.

Another noteworthy observation is that for *average* achievers, Math Reasoning appears correlated to Comprehension Knowledge, Fluid Reasoning and Long-Term Retrieval, all of which are reported in the literature as significant predictors of math reasoning (e.g., Geary, 1990, 1994; Proctor et al., 2005) whereas Fluid Reasoning (Gf) is the sole significant predictor for higher achievers in math reasoning. This confirms observations in the literature (Dark &

Benbow, 1990; 1991) about the significance of fluid reasoning as the most important predictor of success in mathematics among very high achievers.

Summary of Implications

A comparative examination of the results across the groups studied gives rise to the following observations:

High achievers in math calculation skills vs. high achievers in math reasoning. Generally, the cognitive functioning of children who excelled in math reasoning, as represented by mean General Intellectual Ability – Extended (GI-Ext) scores, was significantly higher than cognitive functioning of those who excelled in math calculation skills, both at the moderate and very high levels of achievement, with mean GIA-Ext scores of children excelling in math calculation skills falling in the Average to High Average range whereas mean GIA-EXT scores of those excelling in math reasoning fall in the High Average to Superior range. It is possible that the consistency with which fluid reasoning (Gf) predicts excellence in math reasoning may be due, in part, to the relative homogeneity of the groups of high achievers in math reasoning, representing primarily high ability children. In comparison, high scores in math calculation skills appear to be associated, on average, with lower cognitive abilities that include average to high-ability participants (more heterogeneous groups) whose strength in math calculation skills may be due, in part, to ecological factors (e.g., home and school environment, teaching quality), leading to more variable predictors.

High achievers in math calculating skills vs. average achievers. At the cluster level, Short-Term Memory (Gsm) or Working memory (Gwm) were found best predictors among moderately high achievers in math calculation skills whereas Visual/Spatial Thinking (Gv) was the sole predictor of excellence in math calculation skills among very high achievers, although the mean score in Visual/Spatial Thinking for very high achievers – whereas higher than for average achievers – is not particularly high compared to other CHC cognitive abilities for this group. In comparison, Processing Speed (Gs) was found the sole predictor of math calculation skills among average readers. This suggests that more advanced math calculation problems seem to place higher demands on visual/spatial abilities, and to a lesser extent, on immediate awareness. In comparison, less advanced math calculation problems seem to place larger demands on processing speed.

High Achievers in Math Reasoning vs. Average Achievers. Fluid Reasoning (Gf) was a consistent predictor of math reasoning among high achievers whereas Comprehension Knowledge *(*Gc), Fluid Reasoning (Gf) and Long-Term Memory (Glr) were all predictors of math reasoning among average achievers. This could point to more unique cognitive profiles predicting strength in math reasoning among high achievers compared to average achievers. The results also suggest that advanced math reasoning problems place high demands on *Fluid Reasoning* although fluid reasoning is also important for less advanced math reasoning problems.

Overall high achievers in mathematics vs. high achievers in specific areas of math. Results predicting achievement in math calculation skills for overall high achievers point to Short-Term Memory (Gsm) – for high achievers in math who do not excel in reading, and Processing Speed (Gs) – for high achievers in math regardless of reading skills, thus resembling results obtained for high achievers in math calculation skills (research questions 1). Results predicting achievement in math reasoning – for high achievers in math in math who do not excel in reading, point to Processing Speed (Gs) as inversely predicting math reasoning,. Results for overall high achievers regardless of reading skills, are as expected (Processing speed predicting math calculation skills and Fluid Reasoning and Comprehension Knowledge predicting math reasoning)..

Moderately high achievers vs. very high achievers in math. In the area of math calculation skills, different cognitive profiles were obtained for moderately high achievers and very high achievers, suggesting that Visual/Spatial Thinking (Gv) may be more important to solving very demanding math calculation, whereas Short-Term Memory (Gsm) or Working memory (Gwm) were more important in solving moderately demanding math calculation problems. In the area of math reasoning, results were not significant at very high level of achievement, possibly because fewer participants met the thresholds, resulting in a much smaller group.

Spherically High Math Achievers Restricted by Reading vs. Not restricted by Reading. The groups of high achievers in math who don't excel in reading represent more specific expertise in math. This may explain more unique sets of cognitive predictors for specifically high achievers in math (emphasis on quantitative skills), compared to those who excel in math regardless of reading skills (broader set of skills).

Short-term memory vs. working memory representing immediate awareness. Because the present data did not include scores on the Working Memory cluster, an arithmetic mean of two tests assessing working memory was used to estimate the working memory clusters. This and the f act that Short-Term Memory and Working Memory share a WJ III COG test (Numbers Reversed) may explain, in part, the relatively similar results obtained when working memory replaced short-term memory as representing immediate awareness, although for some groups, Working Memory, but not Short-Term Memory was shown as a significant predictor of math achievement. In other words, the WJ III COG Short-Term Memory cluster represents memory span as well as working-memory capability.

Current results for high achievers in math vs. results reported for underachievers. In the areas of math calculation skills, Proctor et al. (2005) reports that ""When students who performed poorly on math calculation were compared to the average achieving group, no difference in the overall level of performance across abilities was indicated. The comparative analyses also showed that none of the specific cognitive abilities "separated" the low-achievement group from the average-achieving group". In comparison, the present study shows different cognitive predictors of math calculation skills among moderately high achievers, very high achievers and average achievers. Given that the groups of *moderately high achievers* in math calculation skills in this study have demonstrated overall cognitive scores in the average range, as discussed above, it is possible that Proctor et al.'s sample of *moderately low achievers* in math calculation skills might have also represented children closer to the average range, thus failing to show a significant difference between low achievers and average achievers in math calculation skills. In the area of *math reasoning*, both the present study and results reported by Proctor et al (2005) point to Fluid Reasoning (Gf) as the most significant predictor of high achievement. Thus, in this area of mathematics, the cognitive profile characteristic of high math achievement can be thought of as a mirror image of the cognitive profile characteristic of low math achievement. This conclusion is consistent with Zigler and Farber (1985).

Current results vs. publications in the literature. The cognitive abilities found in this study to be best predictors of math calculation skills are generally consistent with those reported in the literature for high ability populations (e.g., Dark& Benbow, 1990, 1991; Geary & Brown, 1991; Montague & Van Garderen, 2003). The significance of Visual/Spatial Thinking (Gv) as best predictor of math calculation skills for very high achievers – although mean scores in Visual/Spatial Thinking for this group, whereas higher than for average achievers, are not particularly high – may reflect inconsistent reports on the role of Visual/Spatial Thinking in the literature on low math achievement (e.g., Geary and Brown, 1991). The relatively low predictive power of Processing Speed (Gs) in this study compared to results reported by some authors in the high ability literature (Geary & Brown, 1991) may reflect, in part, weighted averaging algorithms on the WJ III (this is suggested by the apparent stronger correlation – in the samples studied – of Math Calculation Skills cluster scores with scores on the Calculation test that is not constrained in time, compared to scores on the Math Fluency test that is constrained in time). Results on the importance of Fluid Reasoning (Gf) on achievement in math reasoning are consistent with those reported in the literature for high ability individuals (e.g., Dark & Benbow, 1990; 1991; Proctor et al., 2005).

Contributions to the Field

The present study makes several contributions to practice in education and psychology. First, the current study provides a greater understanding of the cognitive abilities underlying mathematics skills, an area that merits additional research (Geary & Hoard, 2001). Next, the present study provides information on the differential nature of cognitive abilities that underlie math achievement for highly able children – a population generally neglected by researchers in educational psychology and education (Winner, 1996, 2000). Thus, this work could be instrumental in devising special programs for children who excel in mathematics.

Contributions to the Field of School Psychology

Results of this study suggest that the profiles of high achievers in mathematics, at least in the area of math reasoning, can be thought of as mirror images of the cognitive profiles characteristic of mathematical deficiency. Therefore, results outlined in this study may supplement the existing research body relating to the full range of mathematics ability, thus benefiting clinicians and researchers in diagnostics and educational practices (Flanagan, Ortiz, Alfonso, & Mascolo, 2002; Sternberg & Grigorenko, 2002). The current study also adds to the accumulating knowledge on using a CHC theoretical perspective and may provide insight on how the WJ III can be more effectively utilized for highly-able children.

Limitations of the Study

The present study was limited by a number of factors: (a) cases with relevant missing data were deleted from the samples assessed, reducing the accuracy of reported results; (b) as reported by Cohen and Cohen (1983), there is a risk associated with Stepwise Regression of obtaining different results given different data sets and a different order of variables' entry and removal; (c) some of the samples used in this study – especially those selecting high achievers in mathematics who are no more than average readers – represent distributions that deviate somewhat from normality, limiting the accuracy of regression analysis; (d) the grouping of math achievement scores on the WJ III ACH into two clusters: Math Calculation Skills and Math Reasoning, may not generalize to results obtained with other test batteries; (e) the cluster and test scores used represent means over a large range of ages (6 to 18) that despite the relative stability of scores over time, may result in lower effect sizes; and (f) using an arithmetic mean of tests assessing working memory as an estimation of the working memory cluster may compromise the accuracy of the results.

Strengths of the Study

This study enhances our understanding of high ability children and provides a better understanding of the cognitive abilities that underlie achievement in math calculation skills and math reasoning, both at the cluster and test level. The math clusters and tests used in the study provide wide coverage of math achievement skills that is important for an accurate assessment of the complex and multi-faceted domain presented by mathematics. In addition, the study provides a comparison of the cognitive abilities underlying: (a) different levels of math skills (moderately high, very high and average); (b) different levels of specificity of math skills (by restricting or not restricting participants according to reading scores); and (c) strength in specific areas of math (math calculation skills and math reasoning) versus overall strength in math. Finally, the study also attempts to compare the relative significance of short-term memory versus working memory to mathematics achievement.

Implications for Future Research

This study attempted to examine the cognitive abilities underlying math achievement for highly able children who excel in mathematics. To accomplish that, the focus of the study was on participants selected on the basis of high scores on measures of math achievement (specifically, the Math Calculation Skills and Math Reasoning clusters of the WJ III ACH) who were further restricted by their reading scores (on the Broad Reading cluster of the WJ III ACH). However, restricting participants to high achievers in math who do not excel in reading may have resulted in samples that were too restrictive, by eliminating participants with cognitive abilities (e.g., working memory) that are characteristic of high overall reading skills. It is proposed that future research in this area should include groups that are less restrictive. Selected participants could consist of high achievers in math that do not excel in word recognition rather than those who do not excel in all aspects of reading (by using, for example, the Basic Reading cluster, rather than the Broad Reading cluster of the WJ III ACH to select specifically high achievers in math). Another suggestion for future studies is to perform complementary analyses for both specifically high achievers in math and specifically low achievers in math – at different levels of math achievement, to better understand the extent to which the cognitive abilities underlying high math achievement represent a mirror image of cognitive abilities underlying low math achievement.

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APPENDIX A:

Cognitive Abilities and Working Memory

Table A.1:

CHC Factors and Working Memory

CHC Cluster/Working Memory	Ability
Comprehension-Knowledge (Gc)	Measures the breadth and depth of knowledge of a culture, the ability to reason using lexical (word) knowledge, the depth of general verbal knowledge
Long-Term Retrieval (Glr)	Measures the ability to ability to store information and fluently retrieve it later through association and consolidation of information in memory
Visual-Spatial Processing (Gv)	Measures the ability to the ability to generate, perceive, analyze, synthesize, manipulate, transform, and think with visual patterns, including the ability to store and recall visual configurations
Auditory Processing (Ga)	Measures the perception and processing of auditory input, the ability to analyze, synthesize, and discriminate auditory stimuli, blend sounds

Fluid Reasoning (Gf)	Measures the ability to form concepts and solve problems creatively using unfamiliar information or novel procedures, analogies, and inferences.
Processing Speed (Gs)	Measures the ability to perform simple, automatic, cognitive tasks quickly, while maintaining focus and attention. Mental quickness
Short-Term Memory (Gsm)	Measures the ability to hold information in immediate awareness
Working Memory (Gwm)	Measures the ability to apprehend, hold, and manipulate information in immediate awareness and then use it within a few seconds

APPENDIX B:

Psychometric Specifications

Table B.1

Median Cluster Reliability Statistics and standard error of the mean (SEM) for Relevant WJ III Tests of Cognitive Abilities and WJ III Tests of Achievement

WJ III Cluster/Test	Reliability	SEM
Cognitive Clusters		
Comprehension-Knowledge (Gc)	0.95	3.35
Long-Term Retrieval (Glr)	0.88	5.20
Visual/Spatial Thinking (Gv)	0.81	6.54
Auditory Processing (Ga)	0.91	4.50
Fluid Reasoning (Gf)	0.95	3.35
Processing Speed (Gs)	0.92	4.24
Short-Term Memory (Gsm)	0.88	5.20
Academic Achievement Clusters		
Basic Reading Skills	0.95	3.35
Math Calculation Skills	0.91	4.50
Math Reasoning	0.95	3.35
Academic Achievement Tests		
Calculation	0.86	5.65
Math Fluency	0.90	4.83
Applied Problems	0.93	4.08
Quantitative Concepts	0.91	4.50

Note: The SEM values are provided in standard score units.

APPENDIX C:

WJ III Tests of Achievement, Characteristics and Examples of Math Tests:

Table C.1

Characteristics of WJ III Tests of Achievement in the Areas of Mathematics

Test	Curricular Area	Test Requirement
Calculation	Math achievement Number fluency	Performing various mathematical calculations
Math Fluency	Math achievement	Performing various math calculations
Applied Problems	Quantitative reasoning, math achievement, knowledge	Performing math calculation in response to orally presented problems
Quantitative Concepts	Math knowledge, quantitative reasoning	Identifying math terms and formulae, identifying number patterns

Note: This table is derived from Schrank, McGrew & Woodcock, (2001).

Examples of WJ III ACH Math Tests' Items

In the following examples, older and more highly achieving children tend to reach higher numbered items than younger and less highly achieving children.

Calculation Test

Item 10: 8+9=

Item 20: 503 – 254 =

Item 30: 120 * (3/2) =

2^3 =

Math Fluency Test

Item 10: 3-1 =

Item 50: 5+6 =

Item 100: 6*2 =

Item 150: 7*7 =

Note: the Math Fluency test is restricted to 3 minutes.

Applied Problems Test

Item 10: (Picture of 4 cans). Point to the picture and say: "If you take away two cans, how many would be left?"

Item 20: (Picture of a click.) Point to picture on subject's page and say: "What tiem does this clock say?".

Item 30: Run your finger across item n subject's page and say" "Thomas walks thirteen blocks to school, Isabelle walks six blocks, and Antonio walks eight blocks. Howe many more blocks does Thomas walk than Antonio?".

Item 60: Point to item and say: "A rectangle six centimeters wide has a diagonal ten centimeters long. Find the perimeter.

Quantitative Concepts Test

Item 10A: What number comes between three and five?

Item 20A: Name the four basic arithmetic operations.

Item 30: 3! =

Item 10B: Tell the number that goes in the blank space. 4 7 10. __

Item 20B: Tell the number that goes in the blank space. 1 3 7 15 . __

Examples of WJ III ACH Math Tests' Items with Visual/Spatial Demands

Calculating Test

Item 41: f(f) = (Џh^2)/8

f'(h) =

Item 45: Integral from 0 to 1 of X^2d_x =

Applied Problems Test

Item 52: Picture of a triangle. Point to picture and say: "What is the length of the red side on this triangle?

Item 60: Point to item and say: "A rectangle six centimeters wide has a diagonal ten centimeters long. Find the perimeter

Quantitative Concepts Test

Item 7. Picture of buildings of various hight. Point to the *highest* building. Now point to the *lowest* building. Item 27: Picture of applies. Point to the item on subject's page and say: "If you had one third of these apples, how many would you have?