VALIDATION OF A PIAGET-BASED HIERARCHY LEADING TO NUMBER CONSERVATION

by

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Abstract

The main purpose of this dissertation was to validate a developmental hierarchy of component abilities underlying number conservation. This hierarchy or sequence of component abilities was derived from Piaget's theory of children's intellectual development. A secondary purpose of this dissertation was to investigate the extent to which predictions can be made about the performance of children on the proposed sequence of tasks, as a function of the specific Piagetian stages of number conservation, in which they have been classified.

The proposed developmental hierarchy consisted of seven tasks. These were constructed to conform to Piaget's conception of what they were purported to measure. The tasks in the predicted hierarchy were: Construction of Equivalence ...> Cognitive Shift ...> Hindsight-Foresight ...> Multiplication of Relations / Multiplication of Classes ...> Conservation of Number ...> Conservation of Ordinal Correspondence, where ...> indicated developmental precedence. These seven tasks were administered to all subjects. The predicted direction in the performance on the proposed hierarchy of Piaget's three stage groups in number conservation was: Stage III > Stage II > Stage I, where > indicates superior performance.

One hundred and fifty-nine children, aged four to seven, participated in the study: 53 Nursery, 53 Kindergarten and 53 Grade One children. The results indicated partial support of the proposed sequence of component abilities underlying number conservation. The results also indicated that predictions regarding the performance on the hierarchy of Piaget's Stage III children in number conservation were substantiated except for the prediction on Multiplication of Classes.

The predictions regarding the performance on the hierarchy of Piaget's Stage I and Stage II children in number conservation were not substantiated because the results did not attain statistical significance. However, they were consistently in the predicted direction.

This dissertation points to the fruitfulness of developmental research for educators in its practical implications for building preschool and primary curricula. Moreover there are implications for special education of mentally handicapped children, as well as for children with arithmetic learning disorders of a specific kind, namely, absence of the concept of one-to-one correspondence and absence of conservation concepts in number and/or quantity.

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CHAPTER I

INTRODUCTION

The main purpose of this dissertation experiment is to validate a hierarchy of component abilities underlying number conservation. This hierarchy or sequence of component abilities has been derived from Piaget's developmental theory. A secondary purpose seeks to answer the following question: to what extent can predictions be made about the performance of children who were classified according to Piaget's three categories of Stage I (non-conservers), Stage II (transitionals), and Stage III (conservers) in number conservation, on the proposed sequence of tasks in this study.

Synopsis of Piaget's Theory on Children's Attainment of Number Conservation

Piaget's theory focusses on the cognitive development of the young child. In his theory, the principle of conservation assumes a vital position because Piaget sees it as a vehicle for understanding the development of logical thinking. He defines conservation thus: "We call conservation the invariance of a characteristic despite transformations of the object or of a collection of objects possessing this characteristic." Piaget (1968, p. 978).

Piaget uses the conservation task as a means of demonstrating unequivocally what he refers to as the difference between operational thought and pre-operational thought. "Operational thought" is a term Piaget uses to indicate a special kind of logical thinking. Essentially, this kind of thinking revolves around "operations". Piaget defines an "operation" as any action which is internalizable as thought.

"Psychologically, operations are actions which are internalizable, reversible, and co-ordinated into system characterized by laws which apply to the system as a whole. They are actions, since they are carried out on objects before being performed on symbols. They are internalizable, since they can also be carried out in thought without losing their original character of actions. They are reversible as against simple actions which are irreversible. In this way, the operation of combining can be inverted immediately into the operation of dissocating, whereas the act of writing from left to right cannot be inverted to one of writing from right to left without a new habit being acquired differing from the first. Finally, since operations do not exist in isolation they are connected in the form of structured wholes. Thus, the construction of a class implies a classificatory system and the construction of an asymmetrical transitive relation, a system of serial relations, etc." Piaget (1953, p. 8). Moreover, Piaget adds that: "From the point of view of psychology, the criterion for the appearance of such operational systems is the construction of invariants or concepts of conservation." (Ibid). He therefore considers children who conserve, as showing operational thought and children who do not conserve as showing pre-operational thought. The importance Piaget attaches to the attainment of the conservation principle in the cognitive development of the child is self-explanatory in the light of the above statements.

In his well-known studies of conservation, Piaget describes its gradual evolution in three stages. These stages refer to a within analysis of a narrow area called conservation, as contrasted to stages referring to broader between-categories analyses such as sensory-motor, pre-operational, concrete-operational and formal operational stages.[1] From Piaget's (1952) descriptions, the Stage I child

[1] Piaget's theory is a stage theory in that he segments children's cognitive development into periods called "stages" for descriptive and theoretical purposes. On a descriptive level Piaget intends "stages" to designate consistent demonstration of certain behaviour, for example, children in the pre-operational period demonstrate failure to conserve. On a theoretical level Piaget hypothesizes development of specific cognitive structures to correspond to specific "stages", for example, in the concrete-operational period, children are hypothesized to possess the cognitive structures called "groupings" and there are eight "groupings" in total, (cf. Kessen, 1962; Flavell, 1963; Piaget, 1953; 1971). Usually Piaget's stage-concept is considered to refer to broad developmental periods as in the sensory-motor stage; the pre-operational stage; the concrete-operational stage and the formal operational stage. However this is not the only context Piaget refers to in using his stage-concept. There is another context though much more narrow, where Piaget uses "stage". He uses "stage" to describe children's sensory-motor development, (six stages), and children's cognitive development in conservation, (three stages). Thus it is important to know the context wherein Piaget uses "stage" in order to ascertain which kind of developmental periods he is referring to.

shows an absence of logical thinking (conserving ability). Piaget points out that the Stage I child can only make global comparisons because perceptual cues (quality) and quantity hold the same meaning for him, (Piaget, 1952; Zimilies,1963). Thus "more" means the "taller" column; the "longer" row or the "denser" row. Moreover the Stage I child tends to fixate on one dimension at a time. For example, in the conservation of mass experiment where a plasticene ball is transformed into a sausage shape, the Stage I child will focus on only one dimension: "It's longer, so there's more in the sausage." If the elongation of the plasticene continues, there will come a point when he will say: "No, now it's too thin, so there's less." Now he is thinking about the width, but he forgets length, because according to the theory, he cannot co-ordinate two dimensions. In contrast, the Stage III child judges the two quantities as equivalent and can explain his judgment. He can conserve mass because he no longer is hampered by an inability to co-ordinate two dimensions. He realizes that when you elongate the plasticene, you make it thinner, and when you make it shorter, you make it thicker. Once he discovers the interdependence between the two dimensions, he begins to think in terms of the transformation rather than in terms of the final (static) configuration and, thus, can conserve.

The Stage II child is intermediate between the Stage I and the Stage III child. He shows rudimentary evidence of logical thinking, that is, he begins to co-ordinate the two dimensions. He will, for example, oscillate between width and length and he will discover that they are related. He will even show "empirical reversibility"[2] (Piaget, 1952; 1968; Inhelder et al., 1966). However because he has not yet attained the kind of logical inferential thinking as embodied in conservation tasks, the Stage II child still bases his thinking essentially on perceptual cues. Hence

[2] "Empirical reversibility" refers to the possible return to the original spatial array or original position.

when a distractor cue is deliberately manipulated as in the transformation of the shape of the plasticene ball, he will readily revert to pre-operational thinking, that is, non-conserving thought.

The significance of conservation behaviour cannot be over-emphasized. It is universally attained, (cf. The Marinique study, Ginsburg and Opper, 1969; Lloyd, 1971). Piaget's studies in conservation of substance, mass, number, weight, and volume have been widely replicated both in England and North America, (cf. Flavell, 1963). Moreover researchers have documented the observation that children do not attain these various conservations of mass, number, length, etc., simultaneously. Typically they attain conservations of mass, number and length before they attain conservations of weight, volume and area. (Goldschmid, 1967).

Piaget terms such developmental lags "horizontal decalage" or "time lags" in translation. The meaning of "horizontal decalage" is best expressed in the function Piaget assigns to it. This function pertains to a finding Piaget and his collaborators observed and which has since their initial observation, been much replicated. Piaget found that children tend to attain conservation of mass before conservations of weight and volume. Typically there appears to be approximately about two years' delay between the child's attainment of conservation of mass and his attainments of conservations of weight and volume. Piaget considers the reasons for such "horizontal decalages" to be:

"Time lags are always due to an interaction between the person's structures on the one hand, and the resistances of the object on the other. The object may be flowers which offer little resistance; one places them on the table, and one makes a bunch of them. But there are other objects which offer more resistance, as for instance the birds. One cannot put them on the table. Some resistances of objects are unpredictable. When one encounters them, one can explain them, but always after the event. It is not possible to have a general theory of the resistances." Piaget (1971, p. 11).

"Horizontal decalage" is thus a posthoc explanation for empirical findings which do not support Piaget's predictions of simultaneous development of "operations" involving similar cognitive structural laws. Its status as a viable concept in Piaget's theory remains controversial, although attempts have been made to defend it, (cf. Pinard and Laurendeau, 1969).

Review of Literature on Number Conservation

Review of Replication Studies in Number Conservation

Of the topics Piaget covered in his investigations of the conservation principle, conservation of number has provoked much subsequent research. Wohlwill and Lowe (1962) credited this to its empirically substantiated relation with education,(for example, arithmetic), and to the fact that the elements used are discrete units, identifiable by the corresponding integer, before and after spatial transformation of one row of elements.

Conservation of number is defined by Piaget as a specific subtopic within his general definition of conservation. "We call 'conservation' the invariance of a characteristic despite transformations of the object or of a collection of objects possessing this characteristic. Concerning number, a collection of objects 'conserves' its number when the shape or disposition of the collection is modified, or when it is partitioned into subsets." Piaget (1968, p. 978).

Various replication studies on number conservation have been carried out. These contain many methodological and statistical refinements, (Dodwell, 1960, 1962; Elkind, 1961a, 1961b; Almy *et al*., 1966; and Rothenberg, 1969). Moreover there have been various attempts to induce number conservation among non-conservers, (cf. Brainerd and Allen, 1971; Brainerd, 1974b). Both replication and training studies with mental retardates have also been reported, (cf. Hood, 1962; Lister, 1969, 1970; and Brown, 1973).

The replication studies have provided general substantiation of Piaget's observations of children's progressive attainment of number conservation. However, many doubts have been expressed by these investigators about Piaget's emphasis on growth of mental structures as the sole determinant of children's ability to think logically. Thus Hood (1962) drew attention to Piaget's neglect of individual differences and the influences of environment on children's cognitive development. Similarly Churchill's (1958a, 1958b) earliest training study questioned Piaget's dismissal of counting skills as facilitative to children's attainment of number conservation. Churchill's position was echoed by Renwick (1963).

The contribution of these early replication studies and Churchill's training study can be summarized as:

(1) providing general substantiation of Piaget's observations of children's behaviour in number conservation tasks

(2) underlining the need for methodological and statistical rigour in Piagetian research

(3) indicating areas which need further investigation, for example, the role of experiential factors on number conservation ability and the possibility of accelerating acquisition of number conservation.

Reversibility Training Studies in Number Conservation

Piaget (1953) has argued that the two forms of operational reversibility (inversion and reciprocity)[3] are necessary for attainment of conservation in children. This led to experimental manipulation of reversibility training by various investigators in their attempts to induce number conservation among non-conserving children. Some investigators manipulated the factor of reversibility alone, while others manipulated multiple factors, including reversibility. The former type of studies will be reviewed first.

[3] Inversion refers to reversibility by negation of the action just occurred, for example, the act of spacing out a row of elements can be negated either physically or mentally by condensing the elements in the row. The original spatial arrangement is thus restored. Reciprocity refers to reversibility by compensation of relational differences. For example, the child conceives that the increase in length in the row of pennies is balanced by (compensated by), the increase in density in the row of sweets. In effect, one increase cancels out the other with the result that the sets remain equivalent in number. This compensation is one form of reversibility. Since the increase in length counteracts the increase in density, the result is a return, or a reversal, to the original situation of equal number.

Training Studies Involving Reversibility Only

Wallach and Sprott (1964) used dolls and beds to show subjects how they could pair the dolls and the beds to recover the initial one-to-one correspondence even though the perceptual array of dolls was out of alignment with the beds. The authors reported success with such a procedure. However, Wohlwill and Lowe (1962) pointed out that in Wallach and Sprott's training procedure, addition and subtraction of either a doll or a bed was also used. Thus the reversibility training was confounded by these additions and subtractions.

Roll (1970) trained children to conserve number using the reversibility training procedure reported by Wallach and Sprott (1964) and by Wallach et al., (1967), that is, dolls and beds without the addition and subtraction procedure. Roll refined the reversibility procedure by Wallach and her cohorts in two ways. In the post-test, he used materials which were more clearly differentiated from the material on which he trained his subects. In addition, he also used a counter-suggestion method to evaluate the effects of training.

Roll found his training succeeded in inducing number conservation among eleven out of sixteen initially non-conserving children. However this finding is somewhat clouded by the fact that only four out of sixteen trainees showed awareness of the logic underlying conservation, that is, identity, reversibility, compensation. Thus, on strict Genevan criteria, his finding may be suspect. But Roll pointed out that nine of the eleven conservers resisted counter-suggestion to revert to non-conservation. On this basis he considers his training procedure effective.

If Roll (1970) had used a delayed post-test after three weeks or more, in addition to his immediate post-tests, we might be in a better position to evaluate the effectiveness of his training procedure, in terms of durability of the acquired conservation.

Training Studies Involving Reversibility and Other Factors

Wallach, Wall and Anderson (1967) ran a study in which they attempted to tease out the confounding factor in Wallach and Sprott's (1964) study. In this experiment, they compared the relative efficacy in inducing number conservation between the two procedures: reversibility training vs. training in addition and subtraction. They reported reversibility training was superior to the addition and subtraction procedure. However, it is not clear if the effective induction of number conservation in Wallach et al.'s (1967) study could be attributed to reversibility training. Judging from the subjects' explanations to justify conservation, there is some basis for attributing their conservation attainment to another factor. This factor appears to be the subjects' resistance to misleading cues. The evidence was sufficient to make Wallach et al. interpret their finding as a joint function of reversibility (training) and non-reliance on misleading cues. However there is doubt as to whether they could maintain the contribution of reversibility training to their results. It is recalled that only one of two of their reversibility training procedures was effective. In contrast to the doll-reversibility training, their liquid reversibility training was ineffectual in inducing number conservation. Moreover reversibility training with dolls and beds was confounded with the factor of provoked correspondence, inherent in the stimuli Wallach et al. used, (cf. Schnall et al., 1972).

Bearison (1969) provided non-conservers with measurement experiences, which focussed on the conservation of continuous quantities in terms of numeration and comparison of discrete units of liquid quantity. The equipment in Bearison's experimental training task consisted of two containers filled with liquid and two sets of beakers. His subjects were allowed to watch and then participate in the actual pouring of liquid from one container to one set of beakers and returning the liquid from the beakers to the container. They were taught to ascertain equality and/or inequality of amount of liquid by counting the number of beakers which the specific container fills. Training was terminated after subjects attained the designated acquisition level. Post-tests of conservation one month later showed 71% specific transfer (continuous quantity) and 47-65% non-specific transfer (discontinuous and continuous area, mass, length, number, discontinuous quantity). Post-tests seven months later indicated an increase of transfer effects (63% - 81%). Although Bearison (1969) interpreted his data in terms of the development of a quantitative set supplanting the existence of a perceptual set, he did state that certain measurement operations are hypothesized to be the effective source of this perceptual-quantitative shift. The measurement operations Bearison referred to were the ones he used in his procedure, and they essentially are "empirical analogues" of reversibility, (cf. Brainerd and Allen, 1971a). Thus in effect, Bearison's successful induction of number-conservation reflects the contribution of reversibility and a cognitive shift of attention.

Rothenberg and Orost (1969) induced number conservation in children by using a conglomerate of available training techniques,(for example, Wallach and Sprott's (1964) reversibility training; Wohlwill and Lowe's (1962) technique of reinforced counting; and Gruen's (1965) technique of verbal pre-training), as well as manipulating amount of individual attention given by the experimenter and peer instruction. The authors were successful in their experimental induction of number conservation and interpreted their results to indicate the feasibility of their training approach.

Schnall <u>et al</u>. (1972) criticized previous reversibility training studies as training children on what Piaget terms "empirical reversibility". Piaget (1952) showed children who demonstrate knowledge of empirical reversibility (empirical return) were unable to conserve liquid or number. In so doing, Piaget illustrates the crucial role of "operational reversibility" as contributing to the child's ability to conserve. "Operational reversibility" refers to logical reversibility where any operation implies its reverse by virtue of the system in which they both function, for example, the operation of addition is reversible by subtraction. It appears then that reversibility as manipulated in previous training studies on number conservation, is a far cry from Piaget's notion of operational reversibility.

In an attempt to provide a closer approximation to the kind of logical reversibility Piaget has in mind, Schnall *et al*. used a sensory-motor analogue of reversibility of thought where bi-directional tension exists as the central aspect of the situation. Such a sensory-motor analogue, they reasoned, embodies implicitly and simultaneously, an operation and its negation rather than the juxtaposition of two unrelated acts. Schnall *et al*. used a strip of elastic as the sensory-motor analogue where its being stretched out by the subject respresents the operation and where its being pulled back against subject's grip represents or implies negation (of the subject's act of stretching).

Schnall *et al*. gave 80 five to seven year-old non-conserving children three trials on a conservation of quantity task. Subjects were asked standard conservation questions on each of the three trials. There were four conditions. One of these was the elasticity condition involving bi-directional tension, the other three conditions represented various degrees of empirical reversibility:

(1) the experimenter spreads one row of elements by hand,

(2) subject spreads elements by hand,

(3) subject spreads elements by means of a non-elastic cardboard device.

Results indicated the elasticity condition alone led to significant induction of conservation judgments. The authors interpreted their data to support Piaget's theoretical distinction between empirical reversibility and operational reversibility in relation to conservation.

Schnall et al.'s (1972) experiment is reported here at greater length for two reasons: First, although it is a sensory-motor analogue of reversibility, it appears to be an analogue which is closer to Piaget's notion of reversibility. Secondly, the experiment obtained relatively clear-cut results despite the absence of correction or reinforcement of correct response. On an immediate post-test, subjects in the elasticity condition alone showed conservation judgments. Moreover the number of conservation judgments increased progressively from trial one to trial three of the experiment among subjects in the elasticity condition.

It appears that authors of reversibility training studies assumed they were manipulating "reversibility" in inducing number conservation. In actual fact, they used demonstrations of "empirical reversibility" as their training procedure. They had trained their subjects to see that objects (dolls and beds) after spatial displacement, can be returned to the initial one-to-one alignment in the absence of addition or removal of the elements in each set.

"Empirical reversibility", the possible return to the original spatial array or original position, differs vastly from "mental reversibility" (operational reversibility) which refers to reversible mental activities that a Stage III child can perform. Piaget (1952) repeatedly emphasized this difference because knowledge of possible empirical return to the original display does not avail in the child's conserving number, whereas mental reversibility contributes to his conserving number.

How does mental reversibility relate to or contribute to the child's conserving behaviour? To answer this question, one has to examine what Piaget theorizes as the psychological processes underlying children's conservation behaviour.

According to Piaget, the child fails to conserve number because he has not grasped the interdependent relationship between length and density cues. For the child to perceive this relationship, he needs to attain decentration, reversible thought and multiplication of relations.

The role of decentration in conservation is best captured by a depiction of its opposite, centration. Piaget says the pre-operational child centers (fixates) on one dimension, for example, length, and ignores the other dimension, density. He does not make full use of all the visual information available. Moreover he is centred on final states, for example, the final state or the product of the transformation (that is, either the elongated row or the condensed row). He ignores the transformations which intervene between the original display (the original state) and the final display (the end state). "Centration" therefore prevents the child from co-ordinating the dimensional relations. (Piaget, 1967, p. 81; Flavell, 1963; Ginsburg and Opper, 1969, pp. 151-152). To attain conservation, the child must first undergo the psychological process of decentration. He must attend to both dimensional cues simultaneously, thus making use of all the available information. He must also become decentred on the final states and move his mental focus onto the transformational states. "The child first perceives by means of simple, one-way actions with centration on the states (and, above all, on the final states) without the decentration which alone permits the conceptualization of 'transformations' as such." Piaget (1967, p. 79).

The development of mental reversibility bears directly on the child's "conceptualization of transformations". The child has to be able to recall the transformations which occurred, in order mentally to reconstruct the original state of the row of objects which he knows has the same number of objects as the standard. Thus if the child can recall or mentally re-enact the transformations as well as be able to mentally reverse the transformations, he can picture how a row of objects would look like before and after transformations. Moreover he can perform any deformation of the row he wants, that is, space it out or close it up, since whatever action he performs on it, he can cancel it and restore the original state simply by mentally reversing the intervening transformation.

Piaget argues that unless the child is capable of such reversible thought, he will only be capable of empirical reversibility. The latter does not suffice to enable a pre-operational child to conserve because he cannot shake off dominance by perceptual factors. He has not as yet brought them under the control of mental actions (reversible thought) which can compensate for apparent discrepancies in visually perceived information. His inability here is a direct function of his rudimentary development of mental reversibility, (cf. Inhelder and Piaget, 1958; Ginsburg and Opper, 1969).

Thus in the light of the above exposition of Piaget's theory of children's attainment of conservation, the attempts at accelerating children's ability to conserve via reversibility training appear misdirected. The respective investigators have failed to heed Piaget's distinction between empirical reversibility and mental reversibility.

The relevance of these reversibility training studies to the present dissertation resides in the preceding criticism, because from the standpoint of Piaget's theory, none of the reversibility tasks, including the elasticity task used by Schnall et al., satisfy the criterion of construct validity of "mental reversibility". The present writer is thus alerted to the necessity of constructing a reversibility task which would conform to Piaget's ideas of "mental reversibility". These ideas had been expressed in diverse sources, (cf. Piaget, 1962; Inhelder and Piaget, 1964; Piaget and Inhelder, 1966).

The review on reversibility training studies is presented to show that in the context of Piaget's theory of children's attainment of conservation, it seems more pertinent to pursue questions such as:

(1) Are these Piagetian notions really pre-requisites to a child's attainment of number conservation? (These notions being: decentration, mental reversibility and multiplication of relations).

(2) Do they form a hierarchy of component abilities underlying number conservation?

No direct study has been attempted in regard to the first question. However there have been three studies which investigated Piaget's predictions on the relationship between compensation and conservation. However these involved conservation of quantity rather than conservation of number, (cf. Bruner et al., 1966, Larsen and Flavell, 1970; Gelman and Weinberg, 1972).

Bruner's colleague, Susan Carey (1966) devised an ingenious experiment to investigate the role of compensation as an important factor in conservation of quantity. She had 19 four and five year old non-conservers in the experiment, which consisted of five tests. A partly filled standard beaker was placed before the subject. Next to the standard beaker was placed an empty, identical beaker designated as subject's glass. He was then shown a series of five pairs of beakers with instructions to choose that one in each pair that would give him just the amount of water necessary for his glass to match the experimenter's glass. The correct choice was always of the type that shows compensation, for example, choice of a wider beaker with a lower water level or a narrower one with a higher water level.

Carey found that half of the four-year-olds' choices and half of the five-year-olds' choices conformed to her expectation, that is, they chose the "correct" responses as defined above. On the basis of her findings, Bruner and Carey stated that compensation is, like reversibility, "irrelevant to conservation". Brainerd (1972a) responded to Bruner's contention that necessity without sufficiency somehow entails "irrelevance", by pointing out that such thinking constitutes an error in deductive reasoning, (cf. Wason, 1966, p. 146). Thus Bruner and Carey's views cannot be taken without further empicial research.

Larsen and Flavell (1970) were interested in seeing if children compensate before they conserve. Thus they incorporated this interest in a study designed to investigate verbal factors in compensation performance, and they found little evidence that compensation necessarily precedes or accompanies conservation, or vice-versa.

Gelman and Weinberg (1972) undertook the same kind of investigation as Larsen and Flavell (1970). However Gelman and Weinberg's experimental design has some fine methodological features, for example, they observed different assessments of compensation had been made in previous studies and concluded that it was necessary to determine the effect of varying the compensation assessment task. They also noted the need to determine criteria on subjects' performance in compensation tasks.

In a study which used different compensation tests and criteria in order to clarify the conflicting interpretation of previous work, Gelman and Weinberg succeeded in obtaining some very interesting data. The authors found that even using a non-verbal measure of compensation relations, (the act of pouring liquid), estimations of compensation relations appeared a harder task than conservation of liquid. More importantly, the authors found that if subjects were given a variety of tasks designed to measure their ability to compensate relations, they showed <u>some</u> degree of this ability, specially conservers. However they could not obtain the same result by using any <u>single</u> measure.

These two studies by Larsen and Flavell (1970) and Gelman and Weinberg (1972) suggest the tenuous relation between compensation and conservation. The only data supportive of the claim that children attain compensation relations before they attain conservation comes from Piaget and Inhelder. However Larsen and Flavell (1970) pointed out that Piaget and Inhelder used a criterion which might have been biased in favour of their hypothesis. In the light of the preceding, it is interesting to note a study by Curcio, Kattef, Levine and Robbins (1972). The authors selected children who could anticipate compensation relations without conservation and then gave them conserving training with discontinuous quantity, (seeds were used instead of water). The authors reported success in inducing conservation in subjects who demonstrated anticipation of compensation relations. This study illustrates a crucial point which has already been made, namely, that Curcio et al. assumed as valid Piaget's theory of compensation leading to conservation. If Piaget were right, then the significance of Curcio et al.'s study would have been questionable. After all, if compensation relations were precipitators of conservation of continuous quantity, then showing children who possess concept of compensation relations are more susceptible to conservation training than children who do not possess the same concept, does not add any significant information to Piagetian research. To be fair to Curcio et al. (1972), they did acknowledge Larsen and Flavell's (1970) findings and qualified the conclusions of their results by suggesting that compensation relations may not be an important precipitator in conservations such as length and number, (cf. Curcio et al., p. 264). However this interpretation distorts Piaget's theory on conservation attainment in children. Piaget intends for his theoretical construct of Multiplication of Relations (compensation) a general application. Multiplication of Relations is hypothesized to precipitate all conservations, be it conservation of mass, or of number, or conservation of length, weight, and volume, (cf. Piaget, 1964).

The author has been unable to find any studies which related to the second question of whether Piagetian notions form a hierarchy of component abilities underlying number conservation.[4] However there have been several studies directed at

[4] There are however studies on hierarchies of early number concepts, which do not ensue from Piaget's theory, (cf. Wohlwill, 1960; D'Mello and Williamsen, 1970; Wang et al., 1971).

validating the order of concept-acquisition in Piaget's concrete-operational period, (cf. Brainerd and Brainerd, 1972; McManis, 1969; Brainerd, 1973b), and one study directed at validating the developmental sequence of certain number concepts within Piaget's theory, (Siegel, 1971).

Brainerd and Brainerd (1972) investigating the order of acquisition of number and quantity conservation, used a within-subject ordinal analysis to establish developmental sequence and four levels of increasingly stringent criteria to determine absence or presence of the concept in question. They obtained results which showed that number conservation developmentally precedes liquid conservation.

Certain investigators explored the extent to which Piaget's hypothesis of the developmental synchronism between conservation and transitivity in weight and length could be validated. Lovell and Ogilvie (1961) compared the occurance of conservation and transitivity of weight in children. They reported that 53% of the non-conservers were able to perform the operation of transitivity.

Using a matched-group design where normal and retarded subjects were matched on mental age, McManis (1969) tested his subjects for conservation and transitivity of weight and length. McManis took care to avoid methodological confounds noted by Smedslund with regard to transitivity of length. The results showed that conservation developed prior to transitivity, with more retardates between mental ages seven to ten being in a transitional stage of the sequence.

Brainerd (1973b) investigated the developmental sequence in conservation, transitivity, class inclusion of length and weight. He pointed out the propensity towards committing Type II errors in previous research in this area. These researchers had tended to use countervailing perceptual illusions, for example, Muller-Lyer illusion and the size-weight illusion, to weed out intransitive subjects who might otherwise be classified as transitive. However this procedure also netted transitive subjects. Brainerd took care to avoid this source of Type II error. Moreover he also avoided a second source of Type II error by basing his evaluations of subjects on judgment responses alone.

In two studies employing 240 children, Brainerd (1973b) obtained results which showed this developmental sequence: transitivity ----> conservation ----> class inclusion in the concept areas of length and weight, (where ----> indicates developmental precedence).

The preceding section has particular relevance to the present dissertation in that it presents the methodology of data-analysis in Brainerd and Brainerd (1972) and also yields results which are contradictory to those obtained by other investigators, (cf. Brainerd, 1973b).

The findings of these studies point to two observations: (1) the importance of guarding against Type II errors in research with young children, (2) the importance of recognizing possible developmental decalage between children's ability to conserve and their ability to succeed in class inclusion. Both of these observations are pertinent to this dissertation. The former affects consideration of criteria in assessing absence or presence of component abilities underlying number conservation, while the latter affects consideration of various alternatives to data-interpretation.

Siegel (1971) tested directly the sequential hierarchy of the cognitive abilities described by Piaget (1952): magnitude discrimination of continuous and discontinuous quantities, recognition of equivalence between sets of objects in spatially similar arrays, conservation, ordination, seriation and addition. She found simultaneity of development of magnitude discriminations of continuous and discontinuous quantities. Recognition of spatial equivalence was found to follow the preceding magnitude discriminations. Thence conservation of number, ordination, seriation and addition were found to follow each other. Thus for the main part, Siegel (1971)'s findings substantiate Piaget's theory. The only exception is her discovery of the simultaneity in development of magnitude discrimination of both continuous and discontinuous quantities.

It is pertinent to note that each of the concepts thus sequenced in Siegel's study are within themselves composite of component cognitive factors, for example, the conservation of number. Hence the basic question of sequencing component abilities underlying number conservation in the form of a hierarchy, remains open. This point applies in particular to component abilities hypothesized by Piaget. Therefore the present dissertation addresses itself to this question. It aims to compose Piagetian notions and constructs hypothesized to lead to number conservation, in a sequential hierarchy. The validation of such a hierarchy would yield direct application to curriculum planning in kindergarten and grade 1. Moreover it would apply equally to facilitating the teaching of mentally-handicapped children in attaining this particular concept. Such an approach towards concept-analysis, if sufficiently validated, may be used with other Piagetian concepts, and therefore increases the educational applicability of Piaget's theory.

<u>Rationale of the Hierarchy</u> (Please look at Table 1 on the following page simultaneously.)

Piaget (1952) used construction of equivalence as an initial step to discriminate between children with global ideas of quantity and those with more differentiated ideas of quantity. In this task[5], the examiner lays down a row of objects one by one and asks the subject to make an equivalent row, matching the examiner's in amount. Piaget (1952, 1964b) showed that the Stage I child cannot succeed here. For any given set of objects the examiner lays out in a row, the Stage I child always puts out either more or less than in the model. Piaget attributes such lack of

[5]Any task mentioned in "Rationale of the Hierarchy" refers to a representative sample from the domain of tasks purported to measure a particular concept.

TABLE 1

Proposed Hierarchy of Developmental Abilities

Underlying the Concept of Number Conservation

Conservation of ordinal correspondence	Piaget (1952) Schreck (1971)
Conservation of number	Piaget (1952)
Multiplication of relations and classes	Piaget (1952) Inhelder & Piaget (1958)
Mental reversibility	Piaget (1952; 1962) Inhelder & Piaget (1958)
Cognitive Shift from using perceptual cues to using quantitative cues	Zimiles (1963) Wohlwill (1960) Bearison (1969) Gelman (1969) Piaget (1952)
Construction of equivalence	Piaget (1952)

Entering behaviour:

language competence concept of same-different and more	Griffith et al. (1967) Rothenberg and Orost (1969); Fleishman et al. (1966)
methodological concerns	Zimiles (1963; 1966) Rothenberg (1969) Siegel and Goldstein (1969) Gelman (1972)

Note.--Predictions on subjects' performance on the hierarchy: (1) Stage III children (conservers) are predicted to succeed in all tasks. (2) Stage II children (transitionals) are predicted to succeed in the first two tasks (C. Equivalance and C. Shift), but to fail the rest excepting Hindsight-Foresight where some of them are predicted to pass. (3) Stage I children (non-conservers) are predicted to fail on all tasks.

correspondence to the global concepts of quantity the Stage I child possesses in his cognitive repertoire. His primitive notion of quantity enables only primitive (global) comparions of quantity. However Stage II and Stage III children were shown to succeed in this task, (cf. Piaget, 1952). Piaget indicated that Stage II children succeed here because they use spatial positional parallel cues. Typically they would place each individual object right below one belonging to the standard row. Thus Stage II children matched the standard not because they have substantial understanding of the concept of quantitative equivalence, but because they were guided by positional or parallel cues. Piaget (1952) bore out this inference when he condensed one row and asked Stage II children about the quantitative equivalence between the two rows. He found they typically denied equivalence under such transformation of the array. In contrast, Stage III children not only succeeded in this task, they also demonstrated independence of positional or parallel cues. Typically they made a matching row whose elements were not in spatial positional correspondence with those in the standard. Either they used their eyes or fingers to ascertain the quantitative equivalence by one-to-one correspondence during their performance, or they counted the right number of objects in the standard before embarking on building their row, (cf. Piaget, (1952). The equivalence task appears to discriminate consistently between Stage I and Stage II, Stage III children. As such, it seems an appropriate choice as the first step in the present proposed hierarchy. It is recalled that the latter attempts to sequence component abilities leading to number conservation. Because success in number conservation indicates the child has attained the concept of quantitative invariance despite perceptual distortions of an array, it follows that at the opposite end of this continuum, the child has not attained the same concept of quantitative invariance owing to poor differentiation between quantitative and perceptual cues. Hence construction of equivalence is proposed as the first step in the proposed hierarchy. It is predicted that Stage I children will fail here whereas Stage II and Stage III children will pass this task.

Cognitive shift from use of perceptual cues to number cues is proposed as the next step in the hierarchy. Stage I children will tend predominantly to use length cues, (cf. Piaget, 1952; Gelman, 1969). Stage II children can make use of density cues as well as length cues. Hence they are not likely to make errors that Stage I children make. If two rows are equal in length, they will look to density cues to make their decision on quantitative equivalences (Pufall and Shaw, 1972). However if length and density both vary independently of each other, Stage II children would fall back on length cues, (Pufall and Shaw, 1972, 1973). Only Stage III children would succeed on the latter either by one-to-one correspondence or by counting, because they wouldn't be easily misled by perceptual disarrays. Thus this Cognitive Shift Task aims to separate out Stage I, Stage II and Stage III children by their performance, which would reflect or indicate the extent of their reliance on either perceptual cues or quantitative cues. It is placed here on the proposed hierarchy because it seems a logical sequelae to construction of equivalence. The latter aims to depict the initial differentiation of quantitative notions from its perceptual aspects among the children. The Cognitive Shift Task is designed to show the continual development of this growing separation between perceptual and quantitative cues as the child strives towards clearer notions of quantity. The continual development towards increasing reliance on quantitative cues instead of perceptual cues is important because it contributes towards number conservation, (Piaget, 1952; Gelman, 1969; Bearison, 1969).

This proposed Cognitive Shift Task is essentially a form of static equivalence task. Estimation of static equivalence as an experimental task has often been used, (Zimiles, 1966; Siegel, 1971; Pufall and Shaw, 1972; 1973). It is predicted that Stage I children would have the lowest scores here. Stage II children are predicted to fail on items where length and density vary independently. Stage III children are predicted to have the highest scores.

The development of mental reversibility is proposed as the third step in the hierarchy. The role of mental reversibility in number conservation has already been explained. It is placed here on the proposed hierarchy because Piaget stated explicitly that mental reversibility heralds or paves the way for the development of multiplication of relations and classes, (Inhelder and Piaget, 1968; Piaget, 1967). Mental reversibility is a necessary but insufficient factor in the child's attainment of number conservation. It must be integrated with multiplication of relations to bring about number conservation in the child. It is placed after Cognitive Shift because it involves the development of special kinds of mental activities rather than a mental set, as appears to be the case in Cognitive Shift. It is predicted that no Stage I children will possess mental reversibility. Some Stage II children should pass because this would merely indicate their cognitive progress towards number conservation and substantiate Piaget's theory that mental reversibility is a necessary but insufficient factor. All Stage III children should pass this task.

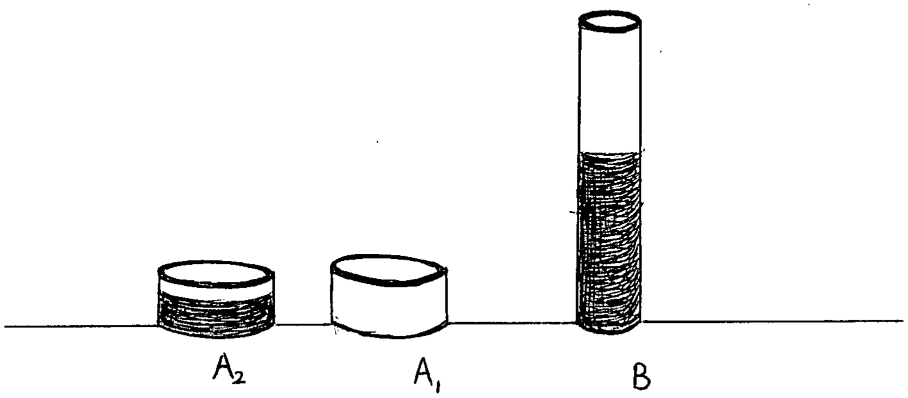
Multiplication of relations and multiplication of classes are proposed as the fourth step in the hierarchy. They constitute the last factor contributing to the child's conservation of number.

Multiplication of relations is part of a developmental cognitive system (structure) Piaget labels "logical multiplication". Piaget (1952, p. 244) defines the latter as an expression of the fact that two or more attributes are considered simultaneously. If the simultaneous comparison of attributes centres on similarities or sameness, then it leads to "Multiplication of Classes". Take the example of classifying a bunch of red round beads, red square beads, blue round beads and square beads. The child puts together beads that share a common attribute: red against blue, and then further subdivides the red and blue beads as follows:

	round	square
red		
blue		

He thus arrives at a 2 by 2 classification plan.

However if the simultaneous comparison of attributes centres on differences between attributes, then it leads to "Multiplication of Relations". Take the example of a typical water conservation problem. Here the child has to compare two quantities from the points of view of height and cross-section. (Sometimes number of glasses must be considered in a variation of the typical water-jar problem). The child must attend to relations of difference: the taller jar against the shorter jar; the narrow jar against the wider jar and integrate such relations of difference (differential relationships), in order to arrive at a realization that the perceptual differences in level of liquid he sees, are a function of relational differences in the height and width of the vessels. In other words, the child can see that in A_1, the shortness of the vessel is compensated for by its width. Similarly, the narrowness of B is compensated for by its height.



Both multiplication of relations and multiplication of classes are theoretical constructs. In conservation, the former construct holds direct relevance. Piaget (1952) documents extensively its important role vis-a-vis number conservation. However the concept of classes is involved in the development of the number concept. Moreover as mentioned previously, both constructs are related in the cognitive development of "logical multiplication", (Inhelder and Piaget, 1958; Toussaint, 1974a; 1974b). Thus the two are proposed together as the fourth step on this hierarchy.

Separate tasks are designed to measure multiplication of relations and multiplication of classes. In the light of Piaget's theory, it is hypothesized that multiplication of classes develops at approximately the same time as multiplication of relations. Concerning the present experiment, it is predicted that no Stage I or Stage II children would pass these two tasks. However, it is predicted that Stage III children will pass them.

Conservation of number is the next step on the proposed hierarchy. In the light of the previous discussion on how decentration, mental reversibility and multiplication of relations bring about conservation, its proposed placement here on the hierarchy is deemed appropriate. It is predicted that only Stage III children will pass it.

The last item on the proposed hierarchy is conservation of ordinal correspondence. Ordinal correspondence is a one-to-one correspondence between two equal sets, based on a relation of corresponding magnitude of some kind, for example, the fifth largest element in one set is paired with the fifth largest element in the other set. Conservation in this instance simply means maintaining the ordinal one-to-one correspondence despite spatial transformations of one set within the original array. This task is placed here on the hierarchy, after the number conservation task, because it involves an advanced form of seriation even though the same construct which enabled the child to co-ordinate the two dimensions, namely, multiplication of relations, underlies it, (Piaget, 1953; Boyle, 1969). Thus although conservation, seriation and conservation of ordinal correspondence are abilities demonstrated by the child in the concrete-operational period, they are not demonstrated simultaneously, for example, conservation ability is demonstrated before others, (Piaget, 1952). This is due to the occurrence of horizontal decalage, which has been defined earlier.

This last step in the hierarchy is of theoretical interest. Piaget says that when a child uses one-to-one correspondence to check the quantitative equality of the rows, he shows a grasp of the concept of a unit. This is because one-to-one correspondence involves isolating a pair of objects from each set successively and then grouping the ones covered by one-to-one correspondence. Piaget says this ability to simultaneously group together and separate discrete objects demonstrates the child's grasp of the concept of a unit. Moreover the same ability, hence the same concept of a unit,underlies successful performance in conservation of ordinal correspondence. The latter task consists of eight blue sticks varying in size from one inch to nine inches and eight red sticks varying in size from eight to fifteen inches. These two sets of sticks were lined up in one-to-one correspondence. Then one set is deliberately spatially displaced and the child's task is to point to the red stick which goes with the fourth largest blue stick which is, of course, the fourth largest red stick. The child can solve the problem by making a one-to-one correspondence of the largest blue stick with the largest red stick, the second largest blue stick with the second largest red stick, etc. Or he can solve the problem by counting. Whichever the case, solution of the problem depends on the ability to simultaneously group together and separate discrete objects.

On the basis of the above, it is hypothesized that no non-conservers can pass both conservation of number and conservation of ordinal correspondence since non-conservers have not attained the concept of a unit. Whereas a significantly large proportion of conservers will pass these two tasks. Some conservers may not pass conservation of ordinal correspondence, owing to developmental lag (horizontal decalage), since ordination is shown to follow conservation developmentally. (Piaget, 1952).

Summary of Hypotheses

A. The predicted sequence of component abilities underlying number conservation consists of:

Construction of Equivalence ----> Cognitive Shift ----> Hindsight-Foresight ----> {Multiplication of Relations / Multiplication of Classes} ----> Conservation of Number ----> Conservation of Ordinal Correspondence.

(where ----> indicates developmental precedence).

B. The predictions concerning children's performance on the proposed hierarchy as a function of specific Piagetian stages:

Hypothesis (1) on Construction of Equivalence: Stage II and Stage III children are predicted to succeed here whereas Stage I children are predicted to fail.

Hypothesis (2) on Cognitive Shift. Stage III children are predicted to score highest whereas Stage I children are predicted to score poorest. Stage II children are predicted to perform at a level intermediate between Stage I and Stage III children.

Hypothesis (3) on Hindsight-Foresight. Predictions here are same as those in hypothesis (2).

Hypothesis (4) on Multiplication of Relations and Multiplication of Classes. Only Stage III children are predicted to succeed here. Stage I and Stage II children are predicted to fail.

Hypothesis (5) on Conservation of Number. Predictions here are same as those in hypothesis (4).

Hypothesis (6) on Conservation of Ordinal Correspondence. Only Stage III children are predicted to succeed in this task.

To sum up, Hypotheses (1) to (6) take the general direction in prediction regarding children's performance on the proposed hierarchy: Stage III > Stage II > Stage I, where > indicates more correct responses, hence superior performance.

CHAPTER II

METHOD

Subjects

Subjects were 159 children aged four to seven years. They were drawn from nursery, kindergarten and Grade One classes in the school district of North Vancouver, B.C. Within each pre-school and academic division, the size of subjects was equal (n=53). Care was taken to obtain subjects from the same locale. For example, the two schools which provided the kindergarten and grade one subjects were separated from each other by half a mile. The four nurseries which provided pre-school subjects were within five to fifteen minutes' drive from them.

The mean age of the nursery subjects was four years, six months. The mean age of kindergarten subjects was five years, seven months. The mean age of the grade one subjects was six years, six months.

The Verbal Intelligence and Performance Intelligence of the subjects were assessed. This was purposed to see if subjects had similar levels of vocabulary and visual-motor functions. The Peabody Picture Vocabulary Test was used to measure Verbal Intelligence. The Wechsler Intelligence Scale for Children (WISC) was used to measure Performance Intelligence among Kindergarten and Grade One subjects. More specifically, four items of the WISC were administered: Picture Completion, Block Design, Object Assembly and Coding. The Wechsler Preschool and Primary Scale of Intelligence (WPPSI) was administered to Nursery subjects. More specifically, four items of the WPPSI were administered: Animal House, Picture Completion, Geometric Design, and Block Design.

General Administration Procedure

Each subject took three experimental sessions to complete the seven experimental tasks. In the first session, he/she was given Construction of Equivalence, Cognitive Shift, and Hindsight-Foresight; in the second session, Multiplication of Relations and Multiplication of Classes; and in the last session, Conservation of Number and Conservation of Ordinal Correspondence. The respective sessions lasted about 15 minutes, 20 minutes, and 15 minutes. Within the first session, the order of tasks administration was randomized, and within the remaining two sessions the order of task administration was counter-balanced, the immediately succeeding subject never had the same presentation order of tasks as the subject who preceded him.

Care was also taken to ensure that at every phase of the experiment, all three groups of Nursery, Kindergarten and Grade One children were included, that is, at no time did the experiment concentrate on one group of subjects exclusively.

Tasks: Construction of Equivalence[6]

Materials

Green chips of different set-size were used in this task which consisted of three trials, each trial involving a specific set-size of chips. Set-size of six, ten and fourteen were used for the respective trials.

Procedure

The experimenter sat opposite the subject at a table. Engaging the subject's attention, the experimenter said to the subject: "Watch me," laying out a row of

[6] With the exception of Cognitive Shift, Conservation of Number and Conservation of Ordinal Correspondence, all task illustrations are put in the Appendix.

six chips for the first trial. When that was done, the experimenter gave the following instructions: "See what I have done? I have put out some chips here. (indicating the row of chips), I want you to put out as many chips as I have." The subject was allowed as much time as desired. When the subject indicated that he had finished, the experimenter ascertained this was the case before brushing aside the two rows of chips, recording the response and setting out the row of ten chips for the second trial. Both instructions and procedure for the second and third trials (ten and fourteen chips respectively) of this task were exactly the same as described above. For each trial, the subject was supplied with 25 chips.

Scoring

The subject received one point for each successful matching of the standard row in a trial. There were altogether three trials, thus the range of possible scores for this task was 0-3.

Task: Cognitive Shift

This task consisted of four sets of bingo chips pasted on strips of light cardboard paper with set-size of five, six, seven, and eight respectively. Each set consisted of four strips, (see illustration on next page). The practice set consisted entirely of pink chips. This was not the case for the experimental sets. For the latter experimental sets, two strips contained blue chips, two contained pink chips in every set. Decision on which strip had what colour-chips was strictly random, (see illustration). Moreover each stimulus in a set was numbered, the number being recorded on the back of the cardboard strip so that only the experimenter saw it. Stimulus number (1) was the referent stimulus in every set, used in comparisons with other stimuli strips in the same set. Actual measurements of cardboard paper used in practice set were (L x B): 17.7 cm. x 3 cm., in experimental set (1), L x B = 20.2 cm. x 3 cm.; in experimental set (2), L x B = 20 cm. x 3 cm.; and in experimental set (3), L x B = 25.9 cm. x 3 cm. (L denotes length, B denotes breadth).

Procedure

The experimenter began with three practice trials on the set of five chips. Stimuli-pairs 1 and 2; 1 and 3; 1 and 4 were randomly presented one pair at a time to the subject. At each presentation of a stimuli-pair, the subject received the following instructions: "We are going to play another game. Look at these." (The experimenter presented, for example, stimuli-pair 1 and 2 of the first practice trial). "Do they have the same number of chips? Do they have as many as each other?" The subject was allowed free time to decide. The experimenter did not give the subject any feedback on his response.

For the experimental session proper, the following pairing of stimuli within the three sets of stimuli of set-size six, seven and eight were presented: stimuli-pair 1 and 2; 1 and 3; 1 and 4. The reasons for choosing such pairs were stated under rationale of the hierarchy in the previous chapter. The verbal questions used for the practice trials with stimuli of set-size of five, were used for the experimental trials. The presentation order of stimuli-pairs was randomized within each set of stimuli-pairs, as was the positioning of the referent stimulus-strips, (for example, the referent stimulus-strip was never placed in the same position on top, or below the other on two sucessive presentations).

Scoring

The subject received one point for each correct response. Thus possible scoring range here consisted of 0-9.

Task: Mental Reversibility (Hindsight-Foresight)

This task consisted of one practice-trial and three experimental trials. Four sets of stimuli were used. The stimuli consisted of picture cards whose specific

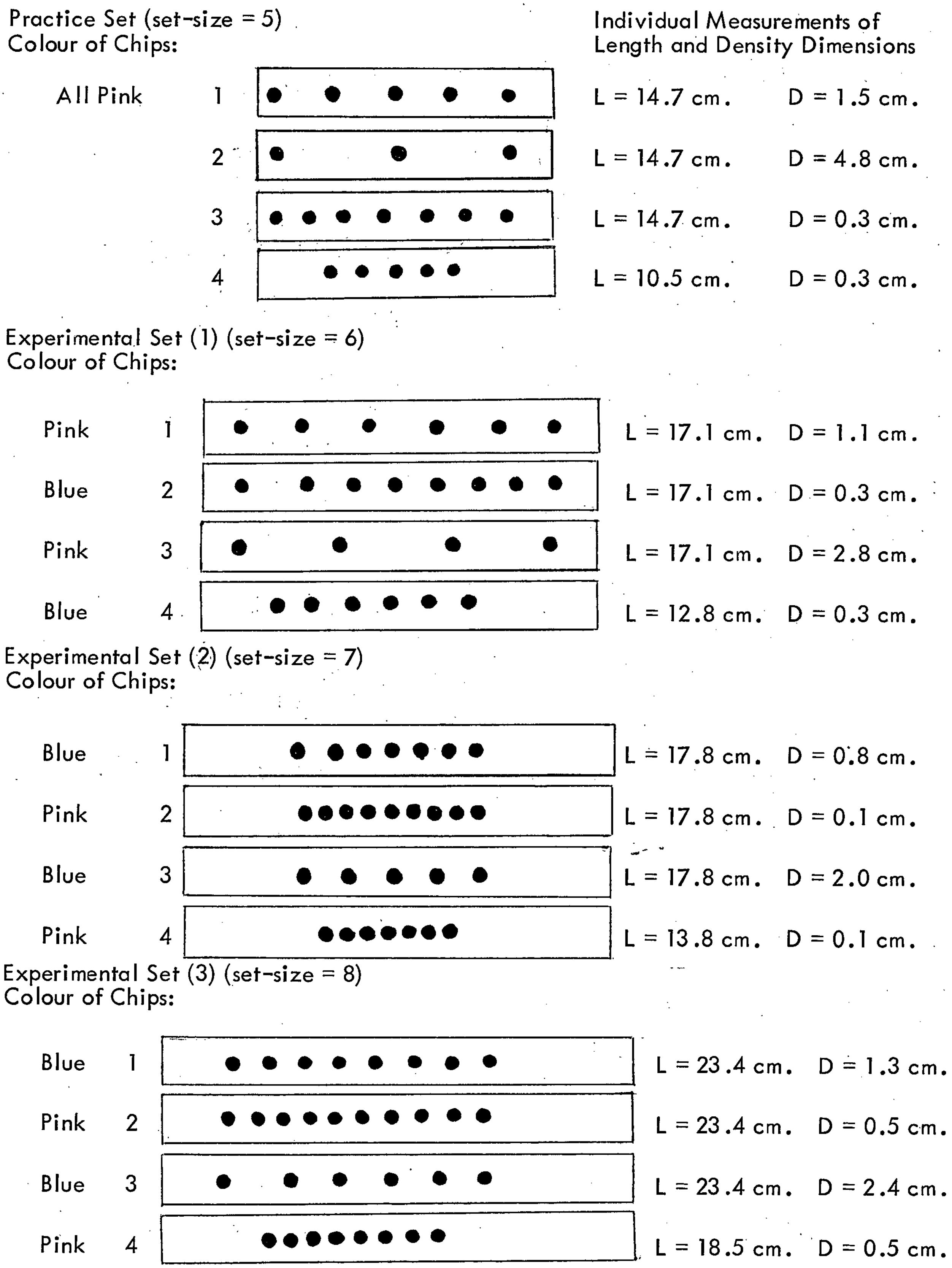


Fig. 1. Illustration of Stimuli in Cognitive Shift Task

measurement and contents are detailed in figures A, B, C, and D in the Appendix. The set for the practice trial contained five cards. The set-size of stimuli choice-cards for the three experimental trials were five for kindergarten and pre-school subjects, while that for grade one subjects was six. The rationale for differential set-size of stimuli choice cards used arose from Inhelder and Piaget (1964a) where they found set-size of six induced more random behaviour among four to five year olds.

For each set of stimuli, the subject was given two cue cards, (the positioning of which is clearly shown in figures A, B, C, and D in the Appendix), and asked to fill in the missing cards in order to complete the story. Specific instructions accompanied each set of stimuli.

For the practice-trial, the experimenter said to the subject: "Now we are going to make a story together." She then put out the first cue cards (see figure A in the Appendix). "You see, Charlie Brown and Lucy are going to play a football game, and they are standing here thinking about how to play it. Then Charlie Brown says to her: 'I know how to play it. You hold the footabll and I'll run up to kick it'." Simultaneously the experimenter put out the second cue card. "So Lucy held the ball while Charlie Brown came rushing up to kick it." The experimenter put out a blank card for this part of the story and explained to the subject the picture depicting this was missing. She then continued: "Guess what happened when Charlie Brown got near the ball to kick it? Lucy tricked him. She pulled the ball from him and poor Charlie Brown slipped and fell and flew into the air, looking very scared." The experimenter exposed the third cue card while telling the story, pointing out how Charlie Brown flew into the air, and looking very scared. "And he's going to come down and hit his head on the ground with a loud noise." The experimenter put out another blank card and explained to the subject that it signified a missing card. "Now look at these picture cards carefully." The experimenter displayed the choice cards. Pointing to the first blank, the experimenter said: "Which picture shows you Lucy is holding the ball and Charlie Brown is rushing up to kick it?" The experimenter directed the subject to look at each choice card before permitting him to make the response. The subject was to select a choice-card from a total of four.

The experimenter let the subject try out on hiw own. If he chose the correct choice card, the experimenter would praise him. If the subject made a wrong choice, the experimenter corrected the subject. The same procedure was applied to the subject's locating the right choice card for the second blank card. No help or feedback was given the subject after this practice trial.

Instructions for "The little man." "See this little man? At the start of the story, he's got his hands down, next to his sides. At the end of the story, he's got his hands all the way up over his head. Now can you find the pictures that should go in here (pointing to the three blank cards) to show me how he got his hands all the way up over his head?"

Instructions for "The growing plant." "See this tiny little plant. It's going to grow into a beautiful big flower. But that's going to take time, right? Now you find me the pictures that go in here (pointing to blanks) which will show me how the little plant grows into a beautiful, big flower."

Instructions for "The fish." "See this fish? He's swimming along and he sees this worm on the hook. He wants to eat it. So he swam closer and closer to it and gobbled it up. But guess what happens after he ate the worm? He finds he's got the hook in his mouth and he can't get out of it. He got caught. Can you find me the pictures to go in here (pointing to the blanks) that will show me how he got closer and closer to the worm to eat it and ends up with the hook in his mouth?"

Scoring

For every correct response-choice, the subject receives one point.

Task: Multiplication of Relations

This task comprised of three subtasks. Each of the latter took the form of a 3 x 3 matrix, with cells all filled but for the three diagonal cells. The subject's task was to fill in the three diagonal cells with the appropriate choice cards.

Materials

A total of 45 picture cards were used, each measuring 5.1 cm x 6.4 cm. Each subtask involved 15 such picture cards. The first subset depicted green turtles which varied consistently in size and orientation. The second subset depicted geometric designs which could be grouped according to colour and form. The last subset consisted of little rectangles measuring 2.5 cm x 3 cm each. These rectangles varied consistently in brightness and thickness. Figures E, F, and G in the Appendix show the three subtasks of Multiplication of Relations.

Procedure

First cells one and two in the matrix (see figure E in the Appendix) were filled and the subject asked to verbalize their contents, and to compare and contrast them. Then cell three was filled and the subject was asked to relate it to the first two cards in terms of their differences and similarities. Next cell four was filled and the whole procedure of comparison and contrast with the other cards repeated. Finally cells five and six were filled and the same procedure of comparison and contrast with the rest of the cards was repeated.

The subject was then asked to select one choice card out of three to fill in an empty cell. For example, for cell X, the experimenter lined up three choice cards. After the subject had chosen his response card and inserted it in place, the experimenter removed the remaining two cards. A new row of three choice cards for cell Y were then presented. Care was taken to ensure that the position of the correct stimulus card varied from the preceding position of correct choice card for X. After the subject filled the middle empty cell, the experimenter removed the remaining stimuli and presented a new set of stimulus cards. Again the correct choice card varied in position from either cell X or cell Y.

The stimuli for the above experimental trials are illustrated in figures D, E, and F in the Appendix.

Task: Multiplication of Classes

Materials and Procedure

This task consisted of three subtasks. Each subtask consisted of two sets of pictures classifiable according to shape or colour. Each set numbered either eight or six cards in total; containing two subsets of four or three. Each of these subsets of pictures was pasted on cardboard paper. The purpose of subdividing equally the eight pictures in each set and pasting them on cardboard was to permit the respective subsets of stimuli to be presented in the form of a cross, such that where they intersected, there was an empty space. This space was just sufficient for placement of one choice card. (See illustrations of Multiplication of Classes in Appendix).

Figure H illustrates the stimulus array for the first subtask. The column consisted of green objects, while the row consisted of leaves of various colours. First, the subject was shown the two vertical columns of pictures of green objects. (Each column contains four pictures). The experimenter elicited from the subject the common quality here, that is, green colour. Next, the experimenter showed the two rows of pictures of leaves of diverse colours and elicited from the subject their common quality, that is, leaf(shape). Then the experimenter said to the subject (while putting stimuli columns and rows into the form of a cross): "See, they meet in the middle and a picture goes in here, right where they meet. It must go with all the green objects AND all the leaves. It has to match the green objects AND match the leaves. Which one should go in there?" The experimenter presented the five choice cards one by one, saying, "Is it this one? Or this one? Or this one? ..." The correct choice card was the green leaf.

The second subtask consisted of drawings of flowers of different colours in the row and yellow objects in the column. The correct choice card was the picture of a yellow flower.

The last subtask consisted of dogs drawn in black and white for the row and blue objects for the column. The correct choice card was a drawing of a blue dog. This choice of a blue dog was purposedly designed to see if children would follow logical deductions and ignore commonsense, that is, there are no blue dogs in real life.

Scoring

The subject received one point for each correct choice card. Possible range of scores here was 0-3.

In both tasks of multiplication of relations and multiplication of classes, only two dimensions were varied. This is because in the conservation paradigm, dimensional changes occur in height and width; or length and density. Moreover restricting dimensional changes to two sources enable direct and clear interpretation of the subject's performance. However, the multiplication of relations task involved a 3 x 3 matrix rather than a 2 x 2 matrix. This was designed to avoid solution on a purely perceptual basis which was found to occur in 2 x 2 matrices. (cf. Inhelder and Piaget, 1964a).

Task: Conservation of Number

Materials

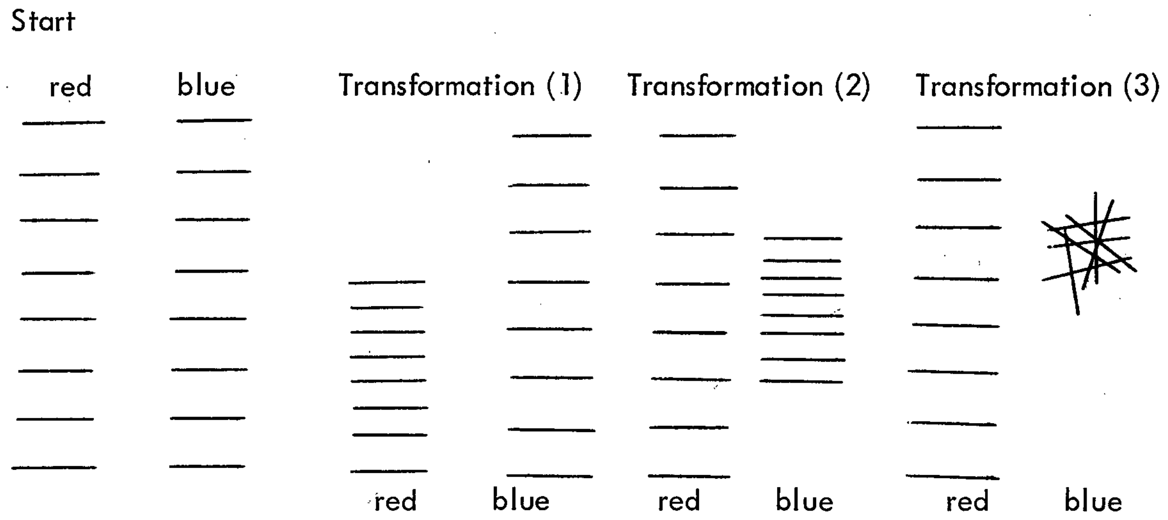
This task involved two sets of eight sticks each, measuring 16.2 cm in length, 2.5 cm in width and 0.4 cm in height. One set was painted light blue, the other was painted bright red.

Procedure

There were three trials. At the start of each trial, the two sets of sticks were lined up in spatial one-to-one correspondence and the subject's estimation of their equality ascertained. Then the experimenter performed the transformation in front of the subject and asked the subject two questions. (Q's): "Do we have the same number of sticks now?" After the subject gave his response, the experimenter asked: "Does one of us have more?" The subject's responses were recorded and the subject was only given a score of one point if he gave two consistently correct answers to the two questions. (cf. Rothenberg, 1969). Following the two questions, the experimenter returned the sticks to their original spatial one-to-one correspondence before starting the next trial.

This procedural format was adhered to throughout the three trials. The following diagram illustrated the three transformation trials which took place.

Placement of Sticks in Number Conservation



Placement of Sticks in Conservation of Ordinal Correspondence

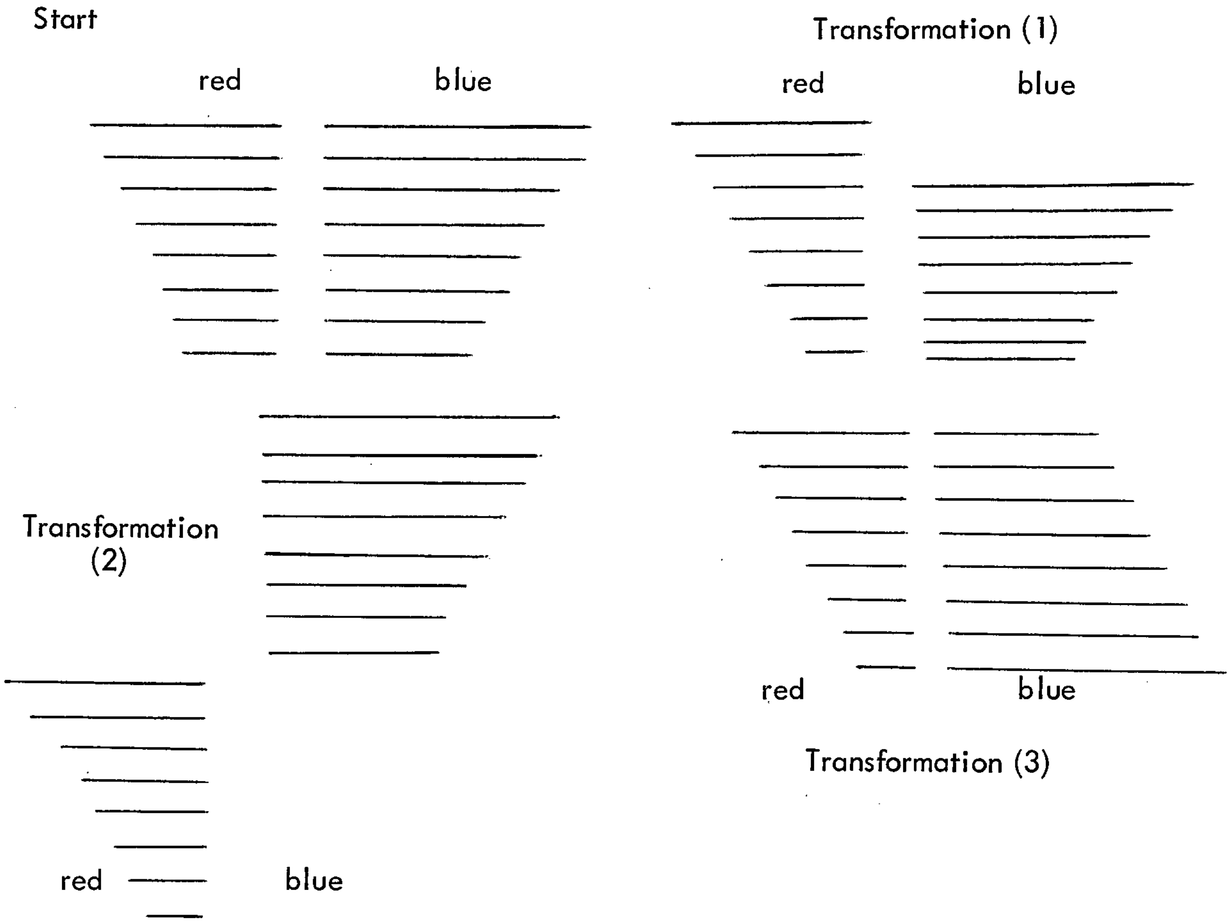


Fig. 2. Illustration of Placement of Sticks in Conservation of Number Task and in Conservation of Ordinal Correspondence Task.

Task: Conservation of Ordinal Correspondence

Materials

The stimuli consisted of eight red sticks and eight blue sticks, varying in length by 2.54 cm from 19.05 cm to 36.83 cm. To ensure that the subjects match the red sticks with the blue ones on the basis of relative size (for example, the fifth longest blue stick with the fifth longest red stick), it is essential that the overlap between the range of lengths of the blue and red sticks be minimal.

Procedure

The experimenter first laid out the red sticks on the subject's left with the smallest red stick close to the subject. Then the experimenter said to the subject: "See these red sticks. They get bigger and bigger, don't they? Show me the smallest red stick. Good. Show me the biggest red stick. Good. Now see these blue sticks?" The experimenter lays them out parallel to the series of red sticks. "They are a lot bigger than the red sticks, aren't they? But they get bigger and bigger too. Show me the biggest blue stick. Good. Show me the smallest blue stick. Good."

Calling the subject by the name, the experimenter continued with the instructions. "The smallest red stick goes with the smallest blue stick. They are partners." The experimenter touched the pair of smallest sticks. "The biggest red stick goes with the biggest blue stick." The experimenter touched the pair of largest sticks. "And these go with each other." The experimenter pointed to the remaining six pairs of red and blue sticks.

To ascertain if the subject understood the one-to-one correspondence between the two sets of sticks, the experimenter randomly pointed to one stick and asked the subject to find its partner. This was repeated on another randomly chosen stick. If the subject failed, the experimenter would explain the instructions again and test the subject's understanding subsequently. When the experimenter was satisfied with the subject's understanding of the task requirements and instructions, she proceeded with the experiment proper. The latter consisted of three transformations as shown in the diagram below. (1) The experimenter pushed the blue sticks closer together so that the biggest blue stick was opposite the third biggest red stick while the smallest blue stick remains opposite the smallest red stick. (2) The experimenter pushed the blue sticks away from the child until the smallest blue stick was above the largest red stick. (3) The experimenter reversed the blue series, that is, putting the biggest blue stick opposite the smallest red stick.

After each transformation, the experimenter asked, pointing at either the fourth, fifth or sixth smallest blue stick according to the specific order for the respective transformation: "Which red stick goes with this blue stick? Find the partner for this blue stick." Care was taken to randomize order of the blue sticks for which the subject must find the partners. Thus for transformation (1), the order of blue sticks pointed to by the experimenter, was fourth, sixth and fifth. That for transformation (2), the order was sixth, fourth and fifth. The order for the last transformation was fifth, fourth and sixth.

Scoring

The subject received three points per transformation if he succeeded matching all of the three sticks the experimenter designated. Thus the total maximum score on the three transformations amounted to nine points. The possible range of score here was 0-9.

Statistical Note

For analyses of variance, all scores were transformed to percentages, because the seven tasks did not have a common scoring range. Some tasks were scored from 0-3; the remaining were scored from 0-9.

For the simplex analysis, no transformation was necessary, hence the original scores were used.

CHAPTER III

RESULTS AND DISCUSSION

Pilot Study

A pilot study was undertaken to investigate (1) content validity of the task, (2) level of task difficulty, and, (3) suitability of instructions. A total of 50 children participated in the pilot, (Nursery = seven, Kindergarten = 17, and Grade One = 26). The availability of subjects determined the number of children in each academic division. Thus the small sample of Nursery pilot subjects reflected the difficulty in obtaining Nursery children for immediate use.

The experimental tasks, with the exception of Conservation of Number and Conservation of Ordinal Correspondence, underwent several major changes before their final formats elicited stable responses from the subjects. Some children served as subjects on two tasks, some more, depending on what tasks the experimenter had constructed or re-constructed.

The results of the pilot study indicated the following. Nursery children demonstrated consistent and total failure on the three tasks in Construction of Equivalence excepting one child. The Nursery children had all just turned four. Kindergarten and Grade One children had no problem here. These results conformed to expectation. The same was true for the Cognitive Shift task which appeared to be a good discriminator between the conservers and non-conservers in number conservation.

Hindsight-Foresight appeared to have been rather hard for Nursery children. However, they managed to achieve two to three credit points out of a total of nine. This was considered to be a satisfactory finding since Piaget's theory predicts Stage I children to fail in anticipating a sequence of events. All of the Nursery children fell into Piaget's category of Stage I in number conservation.

The same task appeared to have been relatively easy for Grade One children. The rate of success was high here although perfect scores of nine out of nine were not the rule. Again these results conformed to expectation because Piaget's theory predicts that conservers would possess the ability to anticipate transformations.

The Kindergarten children performed at an intermediate level between the Nursery and Grade One groups.

Regarding Multiplication of Relations and Multiplication of Classes, the results showed that conservers always succeeded in both of them. Put differently, these two tasks were always passed together by conservers. Non-conservers on the other hand, tended to fail in both. This finding supported Inhelder and Piaget's (1964) predictions.

The results of the Conservation of Number task showed the usefulness of Rothenberg's (1969) question format. The experimenter was able to check the subjects' response consistency in answering Rothenberg's two short questions. Moreover both judgment and explanation criteria were used to determine whether the child conserved number or not. It was interesting to note that conservers could readily explain their judgment responses here. It was also observed that conservation in number appears an all or none matter. Children either failed or passed consistently.

Conservation of Ordinal Correspondence appeared to be the hardest task. This finding was considered satisfactory because it was placed at the end of the proposed hierarchy.

Background Information

To ascertain if subjects were functioning at similar levels in verbal intelligence and non-verbal intelligence, the following analyses were performed. A one-way Analysis of Variance was performed on the 159 subjects' scores on the Peabody Picture Vocabulary Test. These were verbal I.Q. scores so that chronological age has been compensated for. The results indicated that the three groups of subjects, that is, Nursery, Kindergarten and Grade One, did not differ significantly from one another. ($F \leqslant 1$, df 2, 156; $p \ngtr .05$). Similarly, a one-way Analysis of Variance was performed on all the subjects' I.Q. scores on four items of the Wechsler Intelligence Scale for Children (WISC) or the Wechsler Preschool and Primary Scale of Intelligence (WPPSI). The results indicated that the groups did not differ significantly from each other, ($F = 1.715$, df 2, 156; $p > .05$).

Correlations between I.Q. measures, (Verbal I.Q. being the Peabody P.V. Test; Performance I.Q. measure being the WISC/WPPSI), and the seven tasks were obtained. Table A in the Appendix shows the correlations ranged from very low negative to very low positive. Correlations between I.Q. measures and the sum of the seven taks were also obtained. The same table shows these correlations to be very low negative. The only comment on Table A this writer has to make concerns the significant negative correlation between Construction of Equivalence and Performance I.Q. among Kindergarten children. This finding was unexpected because there is no theoretical basis to hypothesize significant relations between Piagetian tasks and intelligence tests. "The most reliable tests of intellect are not directed toward the adaptive properties of thought, but are instead tests of a particular conception of intelligence." (Feldman, Lee, McLean, Pillemer and Murray, 1974). The observed finding could possibly be accounted for by the particular sample of Kindergarten children. This suggestion receives support from the observation that in general the correlations obtained on this appeared to be much more extreme than either the Nursery group or the Grade One group.

On Simplex Analysis

Guttman's simplex analysis is designed to see if a set of variables can be arranged in a simple rank order from the least complex to the most complex.

Guttman's notion of complexity is not to be confused with or equated to difficulty in learning. He defines his notion of complexity as follows:

"Suppose we are given n tests $t_1 \ldots t_n$ which differs only on a single complexity factor. T_1 is the least complex, t_2 next; it requires everything t_1 does and more. Similarly, t_3 is more complex than t_2, requiring everything t_2 does and more. In this case, t_3 is clearly also more complex than t_1." (Guttman, 1958, p. 269).

Moreover the simplex intercorrelation pattern is said to assume a specific form. The largest non-diagonal element in any row (column) will be next to the main diagonal of that row (column), and these elements will decrease as they depart from the main diagonal either to the left or to the right (upwards or downwards). A set of tests or tasks whose observed intercorrelations satisfy such a condition is said to form a simplex. They are said to have a simple order of complexity.

The present data failed to demonstrate the simplex pattern in which the correlations tend to decrease in absolute magnitude as they get further from the diagonal of the matrix. This suggests that a simple order of complexity as defined by Guttman, cannot be found with the data.

Major Analyses

Ordinal Analyses

Using scores of five out of nine for tasks[7] scored out of 0-9, and two out of three for tasks scored 0-3 as passing criteria, a 2 x 2 contingency pass-fail table was constructed for each pair of tasks. Such pair-wise comparison of the experimental tasks resulted in a 7 x 7 matrix table. Thus for each of the three groups, Nursery, Kindergarten and Grade One groups, individual 7 x 7 matrix tables were constructed. In addition, there was one 7 x 7 matrix table constructed from combined data of the three groups.

Within each matrix, (cf. Tables 2-5), the binomial test was used to test the significance of the various relationships. Readers should note that for each group of Nursery, Kindergarten and Grade One children, and for the pooled data, there were 42 simultaneous contrasts on which the binomial test was performed. The appropriate conceptual unit of error rate attached to these simultaneous contrasts was error rate per comparison, (cf. Kirk, 1968, pp. 81-83). In an effort to control Type I error, p was designated at $< .01$.

Table 6 shows the resulting sequences. The obtained sequences show a certain amount of variation from group to group despite some clear consistency. Such variations reflect the distribution of non-conservers and conservers of number in the composition of the respective groups. Table 7 illustrates the distribution of conservers, non-conservers and transitional children in number conservation, as well as criteria for classification.

[7] Tasks scored out of 0-9 consisted of Cognitive Shift, Hindsight-Foresight, Multiplication of Relations and Conservation of Ordinal Correspondence; tasks scored out of 0-3 consisted of Construction of Equivalence, Multiplication of Classes and Conservation of Number.

From the table of obtained sequences, Table 6, it appears that Construction of Equivalence emerges prior to all other items in both Nursery and Kindergarten groups. However in the Grade One group, Construction of Equivalence, Cognitive Shift and Conservation of Number emerge simultaneously. This appears to reflect a performance ceiling effect, since 46 of 53 Grade One children passed both Construction of Equivalence and Cognitive Shift tasks with only one child failing both; in the case of Construction of Equivalence and Conservation of Number, 40 of them passed both tasks with again one child failing both; and lastly 38 of them passed both Cognitive Shift and Conservation of Number with only one child failing both. Hence it would be spurious to conclude from Grade One data that this cluster of items suggest developmental synchronism of Construction of Equivalence, Cognitive Shift, and Conservation of Number. Rather it would be more judicious to fall back on data from the two younger groups and infer that Construction of Equivalence emerges before all the other items.

Cognitive Shift task emerges after Construction of Equivalence but before the rest of the items. This is evidenced in the Nursery group. In the Kindergarten group, Cognitive Shift task appears simultaneously with Conservation of Number. However an examination of the data here shows that such concurrence was a function of performance ceiling effedts among Kindergarten conservers, (see diagram below).

Kindergarten Conservers

		Conservation of Number: FAIL	PASS
Cognitive Shift	PASS	0	25
	FAIL	0	6

Kindergarten Non-Conservers

		Conservation of Number: FAIL	PASS
Cognitive Shift	PASS	11	0
	FAIL	11	0

TABLE 2

Developmental Relationships Among the Seven Tasks
For 159 Nursery, Kindergarten and Grade One Children

		Construction of Equivalence		Cognitive Shift		Hindsight-Foresight		Multiplication of Relations		Multiplication of Classes		Conservation of Number		Conservation of Ordinal Correspondence	
		Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass
Construction of Equivalence	P			**36	95	**58	73	**51	76	**76	55	**57	74	**93	38
	F			19	9	25	3	28	4	22	6	24	4	26	2
Cognitive Shift	P	** 9	95			**38	65	**35	68	**56	47	**38	65	**69	34
	F	19	36			45	11	44	12	42	14	44	12	50	6
Hindsight-Foresight	P	** 3	73	**11	65			21	55	**35	41	22	54	**43	33
	F	25	58	45	38			57	26	66	17	59	24	76	7
Multiplication of Relations	P	** 4	76	**12	68	26	55			**45	43	26	54	**48	32
	F	28	51	44	35	57	21			53	18	55	24	71	8
Multiplication of Classes	P	** 6	55	**14	47	**17	41	**18	43			*23	38	*39	22
	F	22	76	42	56	66	35	53	45			58	40	80	18
Conservation of Number	P	** 4	74	**12	65	24	54	24	54	*40	38			**42	36
	F	24	57	44	38	59	22	55	26	58	23			77	4
Conservation of Ordinal Correspondence	P	** 2	38	** 6	34	** 7	33	** 8	32	*18	22	** 4	36		
	F	26	93	50	69	76	43	71	48	80	39	77	42		

p <.01 *
p <.001 **

TABLE 3

Developmental Relationships Among the Seven Tasks
For Nursery Children

		Construction of Equivalence		Cognitive Shift		Hindsight-Foresight		Multiplication of Relations		Multiplication of Classes		Conservation of Number		Conservation of Ordinal Correspondence	
		Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass
Construction of Equivalence	P			**20	14	**25	6	**19	8	**25	6	**27	4	**30	1
	F			13	6	20	2	24	2	18	4	21	1	22	0
Cognitive Shift	P	** 6	14			**13	6	**12	7	*18	3	**17	2	**19	0
	F	13	20			32	2	31	3	26	6	32	2	33	1
Hindsight-Foresight	P	** 2	6	** 2	6			5	3	5	3	7	1	* 9	0
	F	20	25	32	13			38	7	39	6	41	4	44	0
Multiplication of Relations	P	** 2	8	** 3	7	7	3			6	1	10	0	*10	0
	F	24	19	31	12	38	5			40	6	38	5	42	1
Multiplication of Classes	P	** 4	6	* 6	3	6	3	6	1			7	3	*10	0
	F	18	25	26	18	39	5	40	6			41	2	42	1
Conservation of Number	P	** 1	4	** 2	2	4	1	5	0	2	3			4	0
	F	21	27	32	17	41	7	38	10	41	7			49	0
Conservation of Ordinal Correspondence	P	** 0	1	** 1	0	** 1	0	** 1	0	** 1	0	0	0		
	F	22	30	33	19	44	8	42	10	42	10	49	4		

$p < .01$*
$p < .001$**

TABLE 4

Developmental Relationships Among the Seven Tasks
For Kindergarten Children

		Construction of Equivalence		Cognitive Shift		Hindsight-Foresight		Multiplication of Relations		Multiplication of Classes		Conservation of Number		Conservation of Ordinal Correspondence	
		Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass
Construction of Equivalence	P			**15	35	**24	26	**20	30	**26	24	**20	30	**38	12
	F			2	1	3	0	3	0	2	1	2	1	3	0
Cognitive Shift	P	** 1	35			**17	19	**25	11	**16	20	11	25	**26	10
	F	2	15			10	7	13	4	12	5	11	6	15	2
Hindsight-Foresight	P	**24	26	* 7	19			8	18	11	15	7	19	**16	0
	F	3	0	10	17			15	12	18	9	15	12	25	2
Multiplication of Relations	P	** 0	30	** 4	11	12	18			8	16	9	21	**20	10
	F	3	20	13	25	15	8			20	9	13	10	21	2
Multiplication of Classes	P	** 1	24	** 5	20	9	15	8	20			9	16	*17	8
	F	2	26	12	16	18	11	16	9			13	15	24	4
Conservation of Number	P	** 1	30	6	25	12	19	10	21	15	16			**20	11
	F	2	20	11	11	15	7	13	9	13	9			21	1
Conservation of Ordinal Correspondence	P	** 0	12	** 2	10	** 2	0	** 2	10	* 4	8	** 1	11		
	F	3	38	15	26	25	16	21	20	24	17	21	20		

$p < .01$ *
$p < .001$ **

TABLE 5

Developmental Relationships Among the Seven Tasks
For Grade One Children

		Construction of Equivalence		Cognitive Shift		Hindsight-Foresight		Multiplication of Relations		Multiplication of Classes		Conservation of Number		Conservation of Ordinal Correspondence	
		Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass	Fail	Pass
Construction of Equivalence	P			2	46	* 9	41	*12	38	**25	25	10	40	**25	25
	F			1	4	2	1	1	2	2	1	1	2	1	2
Cognitive Shift	P	2	46			* 9	40	**26	22	**25	23	10	38	**24	24
	F	1	4			3	1	3	2	2	3	1	4	2	3
Hindsight-Foresight	P	* 1	41	* 1	40			8	34	**19	23	8	34	**19	23
	F	2	9	3	9			4	7	9	2	3	8	7	4
Multiplication of Relations	P	* 2	38	* 2	22	7	38			**16	24	7	33	**18	22
	F	1	12	3	26	4	8			12	1	4	9	8	5
Multiplication of Classes	P	** 1	25	** 3	23	** 2	23	** 1	24			* 7	19	17	9
	F	2	25	2	25	9	19	12	16			4	23	23	4
Conservation of Number	P	2	40	4	38	8	34	9	33	*23	19			**18	24
	F	1	10	1	10	3	8	4	7	4	7			8	3
Conservation of Ordinal Correspondence	P	** 2	25	** 3	24	** 4	23	* 5	22	4	9	** 3	24		
	F	1	25	2	24	7	19	8	18	23	17	8	18		

$p < .01$*
$p < .001$**

TABLE 6

Summary of Developmental Sequences

Sequence (a) from pooled data:

Construction of Equivalence ...> Cognitive Shift ...> (Hindsight-Foresight) (Multiplication of Relations) (Conservation of Number) ...> Multiplication of Classes ...> Conservation of Ordinal Correspondence.

Sequence (b) from Nursery data:

Construction of Equivalence ...> Cognitive Shift ...> (Hindsight-Foresight) (Multiplication of Relations) (Conservation of Number) (Multiplication of Classes) ...> (Conservation of Ordinal Correspondence) (Conservation of Number)

Sequence (c) from Kindergarten data:

Construction of Equivalence ...> (Cognitive Shift) (Conservation of Number) ...> (Hindsight-Foresight) (Multiplication of Relations) (Conservation of Number) (Multiplication of Classes) ...> Conservation of Ordinal Correspondence.

Sequence (d) from Grade One data:

(Construction of Equivalence) (Cognitive Shift) (Conservation of Number) ...> (Hindsight-Foresight) (Multiplication of Relations) (Conservation of Number) ...> Multiplication of Classes ...> Conservation of Ordinal Correspondence

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Note.--...> Denotes Developmental Precedence
( ) Denotes Developmental Simultaneity

TABLE 7

Distribution of Conservers, Non-Conservers and Transitionals In Number Conservation Among Nursery, Kindergarten and Grade One Children

	Piagetian Stages in Number Conservation			
	(Stage I)[a]	(Stage II)[b]	(Stage III)[c]	
Groups	Non-Conservers	Transitionals	Conservers	
Nursery	21	27	4	
Kindergarten	2	20	30	
Grade One	1	10	40	
Total	24	57	74	155

Note.--Four children were found who could not be classified into Piaget's stages because they were conservers in number who did not pass Construction of Equivalence. Hence 155 + 4 = 159 subjects.

[a] Stage I subjects: Failure on both Construction of Equivalence and Conservation of Number.

[b] Stage II subjects: Pass Construction of Equivalence but fail Conservation of Number.

[c] Stage III subjects: Pass both Construction of Equivalence and Conservation of Number.

It is recalled that performance ceiling effects also accounted for the simultaneous occurrence between Cognitive Shift and Conservation of Number in the Grade One group. Hence it appears that Cognitive Shift does emerge as the second step in the sequence as predicted.

Concerning the cluster of (Hindsight-Foresight), the developmental
(Multiplication of Relations)
(Conservation of Number)
order appears equivocal. This item-cluster was found consistently across all groups. The possible reasons for its occurrence are as follows. In the Nursery group, this cluster reflects performance floor or depression effects. Among Nursery children 38 out of 53 failed both Hindsight-Foresight and Multiplication of Relations, while only three passed both; 38 out of 53 failed both Multiplication of Relations and Conservation of Number with none passing both; and 41 out of 53 failed both Hindsight-Foresight and Conservation of Number with only one passing both. However, both the Kindergarten and Grade One data attest to the equivocal nature of the developmental sequence. Altogether the data suggests the tasks may have been too hard for the Nursery children whereas they may have been insufficiently hard for the Kindergarten and Grade One children. In either case, it resulted in lack of discriminating power to detect any developmental order among the tasks. Thus the adequacy of task construction here is called into question.

The mastery of Multiplication of Classes appears to emerge after Construction of Equivalence, Cognitive Shift, Hindsight-Foresight, Multiplication of Relations and Conservation of Number in the case of the Grade One group. It is of interest to note that 16 Grade One children passed Multiplication of Relations and failed Multiplication of Classes while there was only one case in reverse. Moreover among Grade One children, 23 out of 53 conserved number but failed

Multiplication of Classes, while there were seven cases in the reverse. These two findings suggest some form of decalage between abilities Piaget theorized to emerge and develop synchronously in the concrete-operational period. However the same findings are not shown in the Nursery and Kindergarten data. Among the latter two groups, no discernable order appeared among Hindsight-Foresight, Multiplication of Relations, Conservation of Number and Multiplication of Classses. Measurement insensitivity accounts for the lack of discernable developmental order of the four tasks among the two younger groups. The same explanation cannot justifiably be applied to the Grade One group, of whom 79% are conservers. It seems more valid to fall back on the Grade One data and infer that mastery of Multiplication of Classes does emerge after mastery of number conservation, Hindsight-Foresight and Multiplication of Relations. This is because the majority of these children are by definition of performance on the number conservation task, within Piaget's category of concrete-operational period of intellectual development. If there were any decalages among the concrete-operational skills, it would be demonstrable among this group. Hence the Grade One data will be taken to provide the basis for inferring that Multiplication of Classes emerge after Multiplication of Relations and Conservation of Number.

Lastly, Conservation of Ordinal Correspondence emerges as the last step in the sequence among the Kindergarten and Grade One groups. However it was found to occur simultaneously with Conservation of Number among the Nursery children. It would be spurious to infer from the Nursery data simultaneous development of these two abilities. Rather it indicates for Nursery children, the two tasks were equally difficult. Table 2 shows that four out of 53 Nursery subjects conserved number but failed Conservation of Ordinal Correspondence with no subject showing the reverse. Thus it appears more valid to infer from Kindergarten and Grade One data that Conservation of Ordinal Correspondence emerges as the last step on the proposed sequence as predicted.

In the light of the above results, it can be seen readily that sequence (a) which is constructed from pooled data of the three groups, reflects the various contributions of the individual group sequences. More specifically, it maintains the surfacing of the dominant trends in the data, for example, the developmental order of Construction of Equivalence ...> Cognitive Shift ...> the rest of the items; and the developmental order of (Multiplication of Relations) ...>
(Conservation of Number)
Multiplication of Classes ...> Conservation of Ordinal Correspondence.

(Where ...> indicates developmental precedence).

Results From Analysis of Variance on Groups and Tasks

A three (Groups) x seven (Tasks) factorial design with repeated measures on the second factor was run. Results from the Groups x Tasks Analysis of Variance are summarized in Table 8. The main effects of Groups and Tasks are highly significant. (F for Groups = 84.751; df 2,156; $p < .001$; F for Tasks = 51.869, df 6,936; $p < .001$). The Groups x Tasks interaction was also highly significant. (F = 4.876; df 12,936; $p < .001$).

The presence of the significant interaction dictates a further analysis by tests of simple main effects, to locate the sites of significant Groups x Tasks interaction. Table 9 summarizes the results of the Analysis of Variance for simple main effects. It can be seen that significant interactions are located at the following sites: between Nursery and Kindergarten children on Construction of Equivalence and on Conservation of Number. On both tasks, Kindergarten surpassed Nursery children significantly. However, Kindergarten and Nursery children did not differ in their performance on the remaining tasks.

The same Analysis of Variance for Simple Main effects shows that Grade One children gave significantly superior performance than Nursery children on all seven tasks. However Grade One children did not differ in performance from Kindergarten children on any of the seven tasks. Figure 3 depicts graphically the performance of the Nursery, Kindergarten and Grade One children on the seven tasks.

The results of multiple comparison of performance among the three groups is shown in Table 10. Using the Tukey procedure, it was found that Grade One children gave significantly superior performance on the seven tasks than Nursery children. Similarly Kindergarten children surpassed Nursery children. However Kindergarten children did not differ from Grade One children in task performance.

The results in Table 11 show that all children found Conservation of Ordinal Correspondence the hardest and Construction of Equivalence the easiest. They also found the task of Multiplication of Classes harder than Cognitive Shift. They seemed to find the rest of the tasks approximately similar in difficulty.

TABLE 8

Analysis of Variance of the Performance of Nursery, Kindergarten and Grade One Children on the Seven Tasks

Source	df	MS	F	p
Groups (G)	2	163365.700	84.751	$< .001$
Error	156	1927.603		
Tasks (T)	6	37893.660	51.869	$< .001$
G x T	12	3562.057	4.876	$< .001$
Error	936	730.563		

TABLE 9

Analysis of Variance for Simple Main Effects

Source	df	MS	F	p
Nursery vs. Kindergarten Children on Various Tasks				
Task 1	2	14721.132	7.737	$< .007$
Task 2	2	5479.480	2.843	
Task 3	2	9099.245	4.721	
Task 4	2	5256.085	2.727	
Task 5	2	7971.698	4.136	
Task 6	2	33543.820	17.402	$< .007$
Task 7	2	2693.000	1.397	
Kindergarten vs. Grade One Children on Various Tasks				
Task 1	2	83.821	< 1	
Task 2	2	7700.519	3.995	
Task 3	2	4296.746	2.229	
Task 4	2	1395.944	< 1	
Task 5	2	256.745	< 1	
Task 6	2	4716.980	2.447	
Task 7	2	6545.660	3.396	

TABLE 9

(continued)

Source	df	MS	F	p
Grade One vs. Nursery Children on Various Tasks				
Task 1	2	17028.302	8.834	$<$ .007
Task 2	2	26171.604	13.577	$<$ .007
Task 3	2	25925.519	13.450	$<$ .007
Task 4	2	12075.472	6.265	$<$ .007
Task 5	2	11089.670	5.753	$<$ .007
Task 6	2	63418.349	32.900	$<$ .007
Task 7	2	17632.217	9.147	$<$ .007
Error	156	1927.603		
Tasks at Nursery Level	6	14201.986	19.440	$<$ .016
Tasks at Kindergarten Level	6	18276.487	25.017	$<$ .016
Tasks at Grade One Level	6	12538.661	17.163	$<$ .016
Groups x Tasks	12	3562.057	4.876	$<$ .016
Error	936	730.563		

Note.--Attached to the Test of Simple Main Effects is the same per family error rate as that allotted to the overall F-ratio. This is accomplished by testing each of the simple main effects ratios for Groups and Tasks at $\frac{.05}{7} = .007$ and $\frac{.05}{3} = .016$ levels of significance, respectively.

This procedure divides the overall α for a main-effects test evenly among the collection of simple main-effects tests. Thus, the critical values for $\alpha = .05$ for tests involving Groups and Tasks are $F_{.007}$; df 2,156; =5.024 and $F_{.016}$; df 6,936; =2.625, respectively. (Kirk, 1968, p. 181).

TABLE 10

Comparison of Nursery, Kindergarten and Grade One Performance Totals[a] Using the Tukey Procedure

Group Means	Differences Among Means Nursery X=28.961	Kindergarten X=55.944	Grade One X=70.290
Nursery X=28.961		26.983*	41.329*
Kindergarten X=55.944			14.346
Grade One X=70.290			

$p < .05$*

[a] Totals = Sums of Seven Tasks.

TABLE 11

Differences of Means of All Subjects on the Seven Tasks

Using the Tukey Procedure

Tasks	Differences Among Means						
	Conservation of Ordinal Correspondence X=27.812	Multiplication of Classes X=45.492	Conservation of Number X=48.008	Hindsight-Foresight X=49.895	Multiplication of Relations X=51.292	Cognitive Shift X=61.215	Construction of Equivalence X=78.406
Conservation of Ordinal Correspondence X=27.812		17.680*	20.196*	22.083*	23.480*	33.403*	50.594*
Multiplication of Classes X=45.492			2.516	4.403	5.800	15.723*	32.914*
Conservation of Number X=48.008				1.887	3.284	13.207	30.398*
Hindsight-Foresight X=49.895					1.397	11.320	28.511*
Multiplication ofRelations X=51.292						9.923	27.114*
Cognitive Shift X=61.215							17.191*
Construction of Equivalence X=78.406							

$p < .05$*

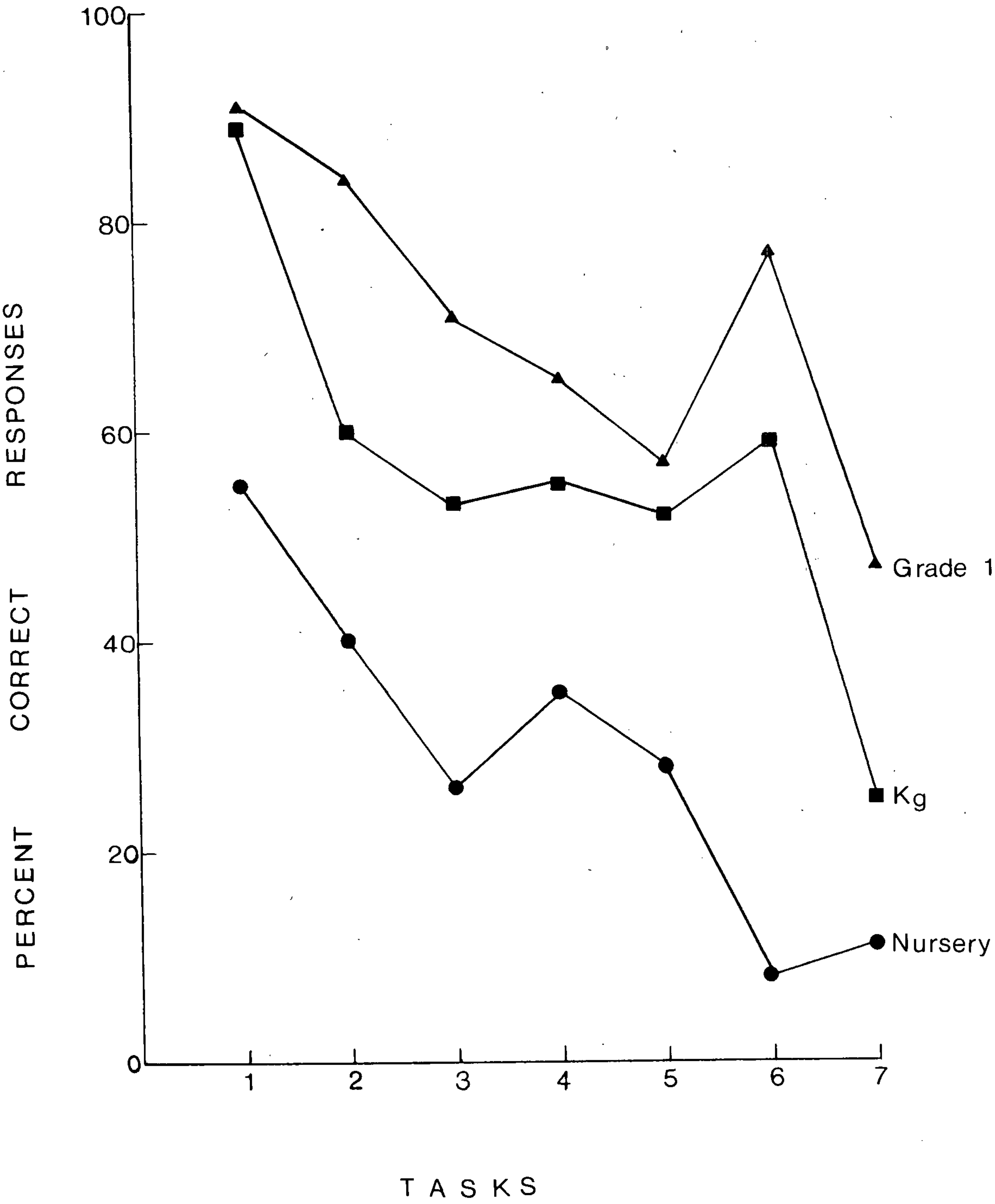

Fig. 3 . Performance of Nursery, Kindergarten and Grade One Children on the Seven Tasks of the Hierarchy.

Results From Discriminant Analysis

It is recalled that six hypotheses were made predicting children's performance on each of the tasks on the hierarchy as a function of specific Piagetian stages of number conservation. To ascertain which of these six hypotheses were substantiated by the data, a discriminant analysis followed by a one-way multivariate analysis of variance were run.

A discriminant analysis was run after three groups were set up a priori on the basis of subjects' performance on Construction of Equivalence and Number Conservation. Group (1) consisted of subjects who failed both of these tasks. These are Piaget's Stage I children. Group (2) consisted of subjects who passed Construction of Equivalence but failed Conservation of Number. These are Piaget's Stage II children. Group (3) consisted of subjects who passed both tasks. These are Piaget's Stage III children. It is pertinent to note that such classification conforms to Piaget's (1952) depiction of the three stages of number conservation. A discriminant analysis was run on these three groups, using subjects' scores on the remaining five tasks, (that is, Construction of Equivalence and Number Conservation were not included in the discriminant analysis).

A step-wise discriminant analysis was executed, wherein all five variables (tasks) were entered into the function in order of relative significance. The analysis was conducted in this fashion for two purposes: to determine which of the five indicators significantly differentiate the three groups, and to determine the minimum set of indicators required to differentiate the three groups. At the first step of the discriminant analysis, that indicator was entered which was most effective in differentiating the three groups, the memberships of which were known a priori. At each subsequent step, that variable was entered into the function which made the most significant additional statistical contribution, after partialling out the effect(s) of the variable(s) included in the earlier step(s) in the analysis. A summary of the results of this analysis is given in Table 12.

From the results in Table 12, it is clear that any one of the five variables (tasks), of itself, significantly differentiates the three groups. (See the first column of p-values). It is interesting to note that there is some slight overlap of variance in the set of five variables (tasks), for after four of them, (Hindsight-Foresight, Conservation of Ordinal Correspondence, Cognitive Shift and Multiplication of Relations), are entered into the discriminant function, no other variable (task) makes a further significant contribution, ($p = > .05$ for Multiplication of Classes; see second column of Table 12).

TABLE 12

Significance of the Differences Between
Three Piagetian Stages in Number Conservation
on Five Tasks

Tasks	Initial Significance[a] p	Significance Level[b] at Entry p
Hindsight-Foresight	< .001	< .001
Conservation of Ordinal Correspondence	< .001	< .001
Cognitive Shift	< .001	< .001
Multiplication of Relations	< .001	< .05
Multiplication of Classes	< .01	> .05

Note.--Sample sizes for the three stage groups (Stages I, II, III) were respectively: 24, 57 and 74.

[a] Probability associated with the F-ratio for the discriminant function when it included only the indicated variable (task).

[b] Probability associated with the F-ratio for the further contribution of the indicated variable, given the prior variables in the discriminant function. In other words, the probability indicates whether or not each successively entered variable adds to the discrimination of the three stage groups. It can be seen that after entry of Hindsight-Foresight, Conservation of Ordinal Correspondence, and Cognitive Shift and Multiplication of Relations, Multiplication of Classes is redundant in terms of aiding the discrimination among Piaget's three stages in number conservation.

Results From One-Way Multivariate Analysis of Variance

The results of comparison of means on Cognitive Shift, Hindsight-Foresight, Multiplication of Relations, Multiplication of Classes, and Conservation of Ordinal Correspondence among the subjects are shown in Table 14. These means were derived from the one-way multivariate analysis of variance, (MANOVA). The results indicate clearly that (1) Stage III children consistently surpassed in performance Stage I and Stage II children in Cognitive Shift, Hindsight-Foresight, Multiplication of Relations, and Conservation of Ordinal Correspondence; (2) No significant differences in task-performance were found between Stage I and Stage II children; (3) Multiplication of Classes is the only task on which Stage III children did not differ from Stage I or Stage II children in performance.

The above findings suggest partial substantiation of all the predictions regarding children's performance on the proposed hierarchy as a function of specific stages in Piaget's theory. It is recalled that hypotheses (2), (3), (4), and (6) all predict a performance expectation of Stage III > Stage II > Stage I where > indicates superior performance. Thus only predictions regarding the overall performance superiority of Stage III children received empirical substantiation, an exception being the case of Multiplication of Classes. The remaining portions within the respective hypotheses on Cognitive Shift, Hindsight-Foresight, Multiplication of Relations, and Multiplication of Classes, Conservation of Ordinal Correspondence, where Stage II children were predicted to surpass Stage I children, did not receive any empirical substantiation. However, it is observed that the performance means on the five tasks of the three groups of Stage I, Stage II, and Stage III subjects did differ in the expected direction. This trend is clearly observed in Table 13.

Finally hypothesis (1) on Construction of Equivalence and hypothesis (5) on Number Conservation remain untested because the subjects' performance scores on these two tasks were used to set up the three a priori stage groups. Had data analyses been possible by means of simplex analysis, information pertaining to these two hypotheses would have been forthcoming.

TABLE 13

Performance Means on Five Tasks

Among Piaget's Three Stage Groups in Number Conservation

Stages	Performance Means				
	Cognitive Shift	Hindsight-Foresight	Multiplication of Relations	Multiplication of Classes	Conservation of Ordinal Correspondence
Stage I	2.83	2.04	3.00	0.792	0.875
Stage II	4.77	3.49	4.00	1.21	1.11
Stage III	6.93	6.09	5.62	1.66	4.15

Note.--Means are consistently in the predicted direction, that is, Stage III > Stage II > Stage I.

TABLE 14

Differences Between Means on Five Tasks

Among Stage I, Stage II and Stage III Children

Tasks	Comparisons					
	Stage I vs. Stage II	p	Stage I vs. Stage III	p	Stage II vs. Stage III	p
Cognitive Shift	1.94	> .05	4.10	< .05	2.16	< .05
Hindsight-Foresight	1.45	> .05	4.05	< .05	2.60	< .05
Multiplication of Relations	1.00	> .05	2.62	< .05	1.62	< .05
Multiplication of Classes	0.418	>.05	0.868	>.05	0.45	>.05
Conservation of Ordinal Correspondence	0.235	>.05	3.275	< .05	3.04	< .05

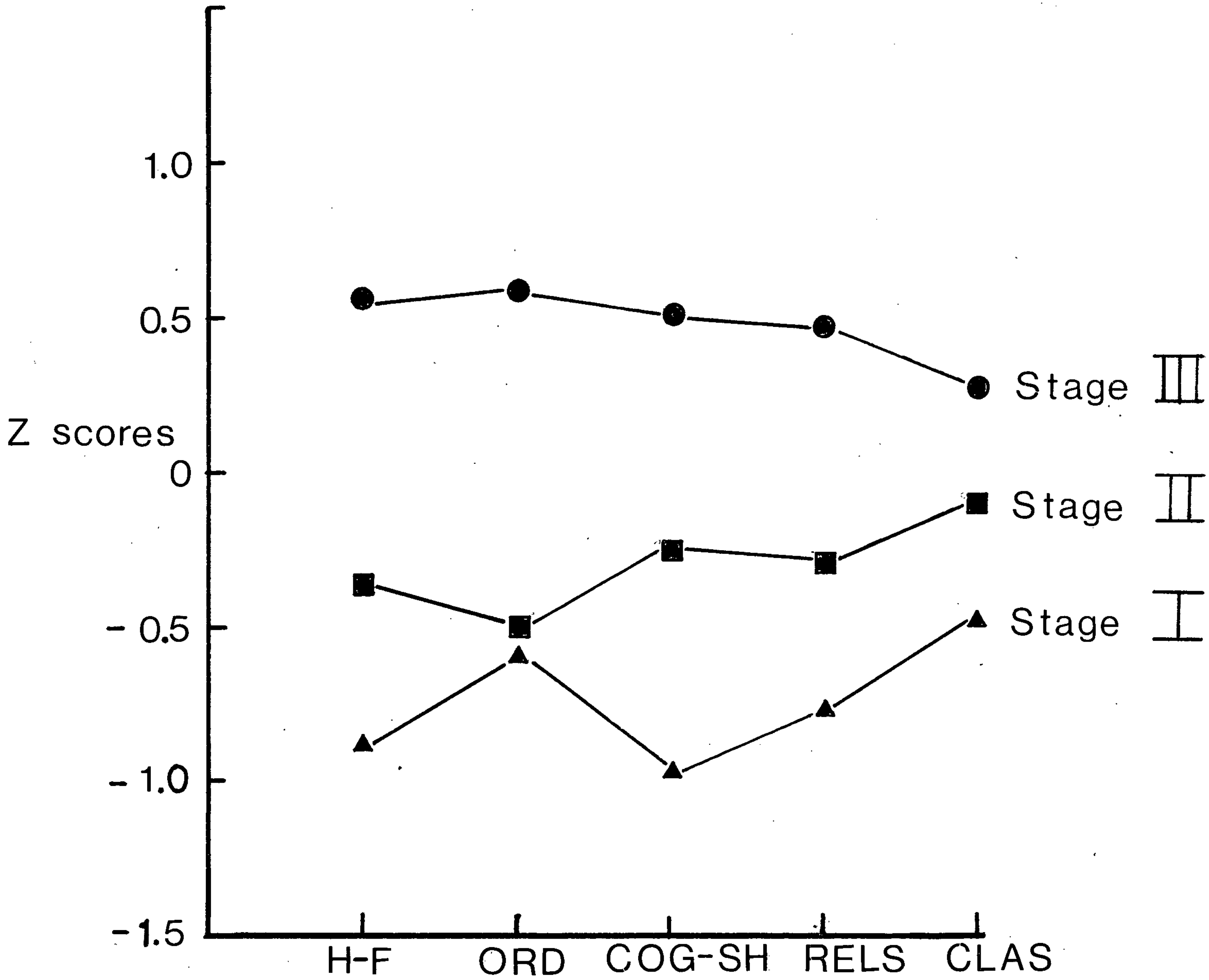

Fig. 4. Profiles of Piagetian Stage I, Stage II and Stage III groups in Number Conservation on Five Tasks. (Order of tasks on horizontal axis indicates Direction of Discriminatory power of tasks, with Hindsight-Foresight being the most discriminating of the groups).

Summary of Results

Regarding the main purpose of inquiry on the sequence of component abilities underlying number conservation, the pooled data from the Nursery, Kindergarten and Grade One groups indicated the following sequence:

Construction of Equivalence ...> Cognitive Shift ...>

(Hindsight-Foresight)
(Multiplication of Relations) ...> Multiplication of Classes ...>
(Conservation of Number)

Conservation of Ordinal Correspondence.

(where ...> indicates developmental precedence).

Regarding the secondary purpose of inquiry on the subjects' performance on the proposed hierarchy as a function of specific Piagetian stages, the data indicated the following:

(1) With one exception, Stage III subjects surpassed both Stage I and Stage II subjects in Cognitive Shift, Hindsight-Foresight, Multiplication of Relations, and Conservation of Ordinal Correspondence. The exception concerned Multiplication of Classes where Stage III subjects did not differ significantly in performance from the others.

(2) Stage I and Stage II subjects did not differ significantly from one another in performance. However, the obtained data showed a consistent trend of Stage II children performing better than Stage I children despite non-attainment of statistical significance.

It is recalled that in all the hypotheses tested, the predicted direction of performance superiority was: Stage III > Stage II > Stage I. Thus only partial substantiation of four hypotheses were obtained. These hypotheses pertained to Cognitive Shift, Hindsight-Foresight, (Multiplication of Relations), and
(Multiplication of Classes)

Conservation of Ordinal Correspondence. The hypotheses on Construction of Equivalence and Conservation of Number were not tested for reasons previously detailed in this chapter.

CHAPTER IV

DISCUSSION

The purpose of this dissertation was to validate a sequence of component abilities underlying number conservation, within the framework of Piaget's theory. The predicted sequence of component abilities was:

Construction of Equivalence ...> Cognitive Shift ...>

Hindsight-Foresight ..> {Multiplication of Relations / Multiplication of Classes} ...> Conservation of Number ...> Conservation of Ordinal Correspondence. The obtained sequence from pooled data suggests the following development order:

Construction of Equivalence ...> Cognitive Shift ...>

(Hindsight-Foresight)
(Multiplication of Relations) ...> Multiplication of Classes ...>
(Conservation of Number)

Conservation of Ordinal Correspondence. Thus the empirically derived sequence deviates from the predicted sequence in the finding of the cluster of Hindsight-Foresight; Multiplication of Relations and Conservation of Number, and in the finding of Multiplication of Relations and Conservation of Number preceding Multiplication of Classes.

Methodological Considerations

The cluster of Hindsight-Foresight (H-F), Multiplication of Relations and Conservation of Number was consistently found across all groups. After careful consideration of possible reasons for its occurrence in the previous chapter, it was decided that this cluster reflects possible measurement insensitivity. It is recalled that the H-F task requires the child to select three out of five response cards in order to fill in a five-card sequence. In the arrangement of each of the three 5-card sequences, two cards were given as cue cards, occupying the first and last card-position of the H-F sequence for two subtasks; and occupying the first and third position for one subtask. Thus the child basically has to attend to seven cards. Conceivably this may be beyond the attention span of most 4-year olds. Hence in retrospect, the H-F task used may have been inadvertently too hard for them. It appears to require certain refinements so as to provide measurement information on the performance of the younger subjects in the sample. These methodological refinements would consist of some subtasks which are scaled down more to the performance level of Nursery children. For example, H-F tasks which consist of 3-card sequences, where initially Nursery children have to select one response card out of three; and then they have to select two response cards out of four, etc. The lengths of the H-F sequence would progressively increase from 3-card sequences to 4-card sequences before continuing to the H-F tasks used in this experiment. In this way, not only would the measurement sensitivity of the task be increased, but some additional information on the 4-year olds' progressive development from static imagery to transformational imagery might be obtained.

The H-F task appears to be at a suitable level of difficulty for the kindergarten subjects.

For the older Grade 1 children, the H-F task should have included subtasks which require anticipatory transformational imagery in two dimensions and three dimensions, (cf. Toussaint, 1974b). The second and third H-F subtasks of this experiment can be considered as depicting concommittant transformations in two dimensions, (the "Little Plant" growing in height and size; the "Fish" moving closer to bait and widening the mouth). But to succeed in either, the subject need only use one set of cues. In short, he could solve it using cues from one dimension alone. The subtasks therefore are not specific enough in performance criteria of the two dimensions. Hence two dimensional and three dimensional changes should have been incorporated as extended subtasks to the current ones in order to provide more exhaustive measurements of the performance of these older subjects. If the range of tasks were more comprehensive vis-a-vis the age range sampled, more clear-cut results might have been obtained regarding the developmental order of H-F, Multiplication of Relations and Conservation of Number. It would seem unwise, however, to stipulate the use of abstract geometric stimuli. This is because young children need stimuli which are meaningful to them in some way in order to arouse their interest and maintain their motivation.

Similar criticisms are levied against the Multiplication of Relations task. This task which consists of 3 x 3 matrices may present more difficulty to 4-year-olds than 2 x 2 matrices, where the child has to fill in only one card. In a 3 x 3 matrix, he has to fill in three cards, (the diagonal). Again the performance demands may exeed their attentionspan, since the children have to attend to nine response cards in each decision-making operation. For the older children, specially the grade 1 children, the fill-in procedure required for these matrices may have been quite easy because of extraneous factors in the task, for example perceptual factors of size, brightness, etc., which increase the probability of correct choices of response cards. For them, the Multiplication of Relations task should have included other performance criteria, for example, construction of the whole matrix, given one card out of nine; and transposition of the matrix, (cf. Bruner, 1966; Toussaint, 1974a).

It is believed that such design and methodological refinements would shed more light on the developmental order of H-F, Multiplication of Relations and Conservation of Number.

One may wonder why the experimenter had not anticipated such methodological problems, especially after a pilot study of 50 subjects. It is recalled that the pilot sample of subjects performed well according to predictions. The 4-year-olds were able on the average to obtain a score of 2 -3 on H-F and Multiplication of Relations. The older children did not seem to find H-F or Multiplication of Relations unusually easy. However, in the experimental sample, 24 out of 53 of the Nursery subjects scored below two correct responses on H-F, whereas 26 out of 53 Grade 1 subjects scored seven or more correct responses. The Kindergarten group performed at a level intermediate between Nursery and Grade 1 groups. Similar trends are observed for the experimental subjects' performance on Multiplication of Relations. Nineteen out of 53 Nursery subjects scored at or below two correct responses on Multiplication of Relations while 24 out of 53 Grade 1 subjects scored at seven or more correct responses. These discrepancies between pilot data and obtained data may be attributable to size of pilot sample. The seven pilot Nursery subjects might not have been adequately representative of the population of Nursery children at large. However, sampling error appears to be a minor factor when one considers the over-all data from this experiment. In retrospect, all the discrepancies between the pilot data and obtained data might have been absent had the experimenter increased the variety of subtasks to cover a wider range of task difficulty. It is important to note that the writer is <u>not</u> suggesting the present tasks of H-F and Multiplication of Relations have been complete failures. Rather she cannot obtain maximal information on the development order between H-F and Multiplication of Relations using only the current subtasks of H-F and Multiplication of Relations. Hence she has suggested using these subtasks of H-F and Multiplication of Relations again, but embedded among other H-F and Multiplication of Relations subtasks. These should be at one end of the continuum of task difficulty, more easy than the current ones and at the other end of the continuum of task difficulty, more difficult than the current ones. With this improvement in task construction the writer believes chances are more optimal in measuring or tapping a developmental order between the two conceptual domains of H-F and Multiplication of Relations.

On Developmental Decalages

The data on the hypothesized synchrony of Multiplication of Relations and Multiplication of Classes suggests that Multiplication of Classes develops after Multiplication of Relations despite Piaget's theory which stipulates synchronic development of these inter-related structural systems. Moreover, the data are also contrary to that of Mackay, Fraser and Ross (1970) which is the only study which shows that Multiplication of Classes developmentally precedes Multiplication of Relations. However, Mackay, Fraser and Ross' study confused Bruner's definition of transposition which is basically designed to apply to Multiplication of Relations and not to Multiplication of Classes tasks, (cf. Brainerd, 1974). In addition they had used different groups of subjects for the respective treatments of reproduction and transposition of matrices, rather than a complete within-group repeated measures design. For these reasons, Mackay, Fraser and Ross' findings have little bearing on the current finding.

The task remains to analyze the observed decalage between Mutliplication of Relations and Multiplication of Classes by eliminating rival interpretations. These rival hypotheses suggest that task variables are responsible for the obtained developmental asynchrony between Multiplication of Relations and Multiplication of Classes. (Tragakis, private communication). The first of these suggests that the observed decalage results from the lack of equivalence between performance requirements of the two tasks, in that the Multiplication of Relations task contains perceptual or "infralogical" factors whereas the Multiplication of Classes task involves more abstract reasoning. It may be contended that such lack of equivalence in performance requirements results in Multiplication of Relations being easier than Multiplication of Classes. However, this contention is refuted by Inhelder and Piaget, who stated that these perceptual ("infralogical") factors are as much present in the Simple Intersection task as in the Multiplication of Relations task. "True, there is an important perceptual factor in matrix tests; but it is also present in this experiment (Simple Intersection), especially when the two rows form a cross, even if it is less compelling." (Inhelder and Piaget, 1964, p.179).

The second rival interpretation to that of developmental decalage suggests that the obtained results are due to lack of equivalence in task-formats. It is recalled that the Multiplication of Relations task consisted of three 3 x 3 matrices while the Multiplication of Classes task consisted of three simple intersection tasks. It may be contended that such differences in task-formats call for differential information-processing among the subjects. In short, this rival interpretation suggests the obtained data of decalage is a function of interaction between subjects and task-formats, (cf. Wohlwill, 1963; 1966; Klair and Wallace, 1970; 1972). The importance of task equivalence needs no emphasis, Wohlwill (1963; 1966) first points out the role of the "encoding process" on children's performance on Piagetian tasks. He criticized Piaget for being oblivious to possible interactions between subjects and tasks. Subsequently Klair and Wallace (1970; 1972) took up the lead from Wohlwill and built an information-processing model of children's cognitive behaviour. Thus the different task-formats used in the present experiment may have contributed to the children's differential performances. Despite the persuasiveness of this rival interpretation, there is empirical evidence to refute it.

Hooper first observed that children understood and could solve 3 x 3 matrix problems of Multiplication of Relations before they could solve 3 x 3 matrix problems of Multiplication of Classes. Subsequently, he was able to replicate this finding in a large-scale study of the development of children's classification abilities, (cf. Brainerd, 1974 g). Brainerd (1974 g) also observed the same sequence as Hooper in a number development study. Brainerd's subjects were more successful with 3 x 3 matrix Multiplication of Relations tasks than they were with 3 x 3 matrix Multiplication of Classification tasks. Further empirical support of the developmental precedence of Multiplication of Relations over Multiplication of Classes comes from the following study. Hooper and Burke-Merkel, (cf. Brainerd, 1974 f), conducted a training experiment which involved instruction in both 3 x 3 matrix Multiplication of Relations and 3 x 3 matrix Multiplication of Classes tasks. He found that 3 x 3 Multiplication of Relations matrix training was more successful than either Multiplication of Classes training or combined Multiplication of Relations and Multiplication of Classes training. Hooper's finding is consistent with the developmental findings considered in the preceding studies.

In the light of Hooper's and Brainerd's work, a resolution or interpretation of the present data in terms of developmental priority of Multiplication of Relations over Multiplication of Classes appears justifiable, because it seems the present finding accrues to a significant trend that has already been observed. The fact that these findings are very new does not necessarily detract from their significance because the data have been repeatedly and independently replicated. Had there not been such independent replications and Hooper's training study, the writer would consider replication of the present data regarding Multiplication of Relations and Multiplication of Classes necessary before interpreting it as decalage.

Lastly Brainerd (1974 g) notes that although he had discussed data from Hooper and himself that consistently support the priority of Multiplication of Relations, the data also indicate that the gap between double seriation (3 x 3 Multiplication of Relations matrix) and double classification (3 x 3 Multiplication of Classes) is fairly small, probably six months to a year in the average child. "From a psychometric standpoint, this means that the double seriation double classification is not nearly as robust as the transitivity/conservation/class inclusion sequence discussed earlier. Thus, while one must commit fairly crude measurement errors to mask the latter sequence, small methodological perturbations will suffice to mask the former." (Brainerd, 1974 g, p. 10). The preceding quotation may well explain why the same trend of developmental priority of Multiplication of Relations over Multiplication of Classes, had not been observed among the younger subjects in Nursery and Kindergarten.

In the event that the use of differential task formats for these two tasks is considered a methodological oversight, the following defence is made. The two tasks have been made deliberately different for fear of possible proactive/retroactive inhibition effects between task-formats (particularly for the 4-year olds), if matrix-type tasks were used in both Multiplication of Relations and Multiplication of Classes. Moreover Shantz (1967) had argued for the need to control correlation obtained from similarity of task structures (formats) which may inflate the amount of apparent correlation between two cognitive functions.

Thus it appears that the two rival interpretations to that of developmental decalage between Mutliplication of Relations and Multiplication of Classes are relatively insubstantial. It is suggested that the obtained relationship between Multiplication of Relations and Multiplication of Classes be perceived as a genuine case of decalage. This conclusion is reinforced by the asynchronism between Multiplication of Classes and Conservation of Number.

The last aspect where the obtained sequence deviates from the predicted sequence, concerns the developmental order between Multiplication of Classes and Conservation of Number. Piaget has postulated necessary developmental synchronism among all the mental structures that children reputedly attain in the concrete-operational period of intellectual growth. Such developmental synchronism is purported to be mandatory in two ways:

(1) operational thought underlies all of these cognitive structures and its attainment once achieved, "pervades" all concrete-operational "items", (cf. Flavell, 1963).

(2) the "stage" concept rests on such developmental synchronism along inter-related structures. Absence of empirical substantiation of developmental synchronism would undermine drastically the whole theoretical perspective of structuralism from which Piaget conceptualizes children's cognitive development.

Returning to the present data, there are 40 conservers in Grade 1; 30 conservers in Kindergarten and four in Nursery. Of these, 19 out of 40 (45%); 16 out of 30 (52%); and 1 out of 4 (25%) succeeded in the Multiplication of Classes task. In short, 36 out of 74 succeeded in both Multiplication of Classes and Conservation of Number; while 38 out of 74 failed Multiplication of Classes but passed Conservation of Number. This suggests some form of decalage between these two abilities in children's cognitive development. No substantial explanations in terms of task or instruction can be given for this finding because the verbal instructions for Multiplication of Classes cannot be considered more complex than those for Conservation of Number; and the task-requirements appear equally clear. Nor could an explanation be found in the composition of subjects who evidenced this decalage between conserving ability and ability to succeed in Multiplication of Classes. This group of subjects was composed of children from three out of four Nurseries, from the two schools of Kindergartens and Grade 1's. This clearly shows that the observed decalage was not concentrated in one particular school nor one particular grade. The single Nursery which was not represented in the preceding distribution is due to the absence of conservers in it. In view of the low proportion of conservers among the Nursery group, (n = 4 out of 53), this non-represented Nursery does not affect the present interpretation of decalage between Multiplication of Classes and Conservation of Number.

On Piagetian Stages as Predictors of Performance of Hierarchy of Experimental Tasks

A secondary purpose of this study was to discover the extent to which predictions can be made about children's performance on the proposed hierarchy as a function of specific Piagetian stages. This inquiry led to additional analysis of the data. Discriminant analysis results indicated that the categories of subjects, set up a priori, can be discriminated on the referent dimensions of Hindsight-Foresight Cognitive Shift, Conservation of Ordinal Correspondence and Multiplication of Relations. However, the same did not occur with Multiplication of Classes. This finding indicates that all children performed similarly here, regardless of stage-category, suggesting that even for conservers, Multiplication of Classes were difficult. Collaborative evidence can be obtained from the binomial results where 38 out of 74 Stage III subjects failed Multiplication of Classes.

The results of the one-way multivariate analysis of variance (MANOVA) suggest that substantial performance differences were found between Stage III and Stage I subjects and between Stage III and Stage II subjects. No statistically significant performance differences were obtained between Stage I and Stage II subjects, although Stage II subjects consistently performed better than Stage I subjects. Had there been significant differences found between Stage I and Stage II subjects, a strong case for stage differentiation or categorization could have been made. As the MANOVA results stand, one may ask whether Piaget's Stage II category in number conservation is superfluous. Put differently, the question is, should Piaget simply dichotomize children into conservers vs. non-conservers, combining into the category of non-conservers both Stage I and Stage II children. An affirmative answer to this question appears premature for two reasons. (1) Although statistical significance was not obtained in performance comparison between Stage I and Stage II subjects, Stage II subjects were found to perform consistently better than Stage I subjects. (2) Given tasks other than those used in this experiment, or given improved versions of tasks in this experiment, we may or may not find Stage II subjects significantly better than Stage I. Either way, more empirical information is needed. Thus as affirmative answer to the question raised is not justified by the present data. Further empirical investigation is required to answer the question.

Theoretical Implications

The current results touch on certain theoretical points. First, within the obtained developmental sequence, one may explore the possible relationships between the cognitive items in the light of Flavell's (1972) paper. Secondly, the results of decalages between Multiplication of Relations and Multiplication of Classes and between Conservation of Number and Multiplication of Classes in the obtained sequence, render mandatory an examination of their effects on Piaget's stage theory. However, this examination will be confined to the present experimental design and purposes, because the implications on Piaget's stage concept are a by-product of the present data.

(1) Theoretical implications from the obtained developmental sequences. Flavell (1972) drew attention to the need to categorize obtained $X_1 \ldots\ldots X_2$ sequences in order to give meaning to the item relationship within any developmental sequence empirically derived. The categories he espouses draw heavily

on theoretical and logical analyses of Piaget's theory, Werner, and other psycholinguists. An attempt is made to apply Flavell's "Inclusion category" to the obtained cluster of H-F, Multiplication of Relations and Conservation of Number and to restrict the interpretation to the cluster among conservers. Only this attempt at theoretical speculation will be made, because it appears that item-relationships here are most interpretable in terms of Flavell's theoretical paper. Theoretical speculation here may enhance understanding of the possible processes that occur between items, thereby enabling us to attain a fuller picture of the dynamics of children's cognition, (cf. Flavell, 1972).

Flavell (1972) defines the Inclusion category of conceptualizing the relationship between a sequence of abilities X_1 -- X_2 as follows:

"The basic scheme of an Inclusion relation is as follows. Item X_1 begins its development toward functional maturity. At some period during this development it starts to become interconnected or co-ordinated with one or more other items to constitute some larger cognitive whole X_2. Whatever else one may be able to say about the specific relationships among X_1, X_2, and any other items which might jointly compose X_2 in such cases, it would at least always be true to say that X_1 becomes 'included in X_2', in the same sense that a subroutine is 'included in', or forms a part of, a computer program". (Flavell, 1972, p.305). Flavell gives at a molar level, the following example of an Inclusion relation; Piaget's sensory-motor ----- concrete ----- formal stage sequence. He gives at a molecular level the following examples: (A) "The infant who had for some time previously been able to push objects (one isolated schema) and to grasp objects (another isolated schema) is now able to push aside a pillow in order to grasp a desired object hidden behind it. In our terminology, a hitherto separate pushing (X_1) has now become included or subsumed as a subroutine within a larger behavioural unit (X_2). Example(B). Piaget believes that the act of transitive reasoning directly calls upon or "recruits" the child's concrete-operational understanding of serial relations : for instance, his knowledge that in the series A $>$ B $>$ C, B is at one and the same time greater than A and less than C". (Flavell, 1972, pp. 304-308).

In the light of the above Inclusion view, the cluster of H-F, Multiplication of Relations, Conservation of Number found among conserving children may be interpreted thus: H-F develops first and when this ability in children attains sufficient functional maturity, meaning when it attains concrete operational status, it becomes interconnected or co-ordinated with another item (Multiplication of Relations) and together they lead to the attainment of conservation and become annexed in a bigger Cognitive whole (conservation concepts). Thus the child's conservation ability constitutes the whole which has annexed H-F and Multiplication of Relations.

Two points here deserve attention. Implied is the causal relationship between H-F, Multiplication of Relations on the one hand and conservation ability on the other hand. Flavell appears to restrict such causal implications in the Inclusion view because these would blur the distinction between the Inclusion view and another single-category of his, the Mediation view of relations between items. However, he seems to permit an Inclusion view to have overtones of causality provided such implicated causality is justifiable. Speaking of a previous Inclusion example of object-naming being included in a long rehearsal process: "It seems reasonable to say that the development of object-naming skills 'was partly responsible for' (helped cause, etc.) the development of a rehearsed strategy. In contrast, one would have to defend any claim that the genesis of enactive representation and a real-external conception of dreams 'helped cause', respectively, symbolic representation and an unreal-internal conception of dreams.

Mediation is aligned with Inclusion in this respect." (Flavell, 1972, p. 310).

Secondly an Inclusion relationship suggests that the items thus integrated together act on one another in reciprocal relations. The child's ability to conserve is, in accordance with the Inclusion view, seen to reinforce his reliance on compensation relations to estimate quantity equivalence rather than relying on perceptual cues. The reciprocal relationship between Hindsight-Foresight and conserving ability is embodied in the following quotations: "Let us take the case of the ball of clay lengthened into a sausage shape. We have seen how hard the younger subjects find it to accept that the sausage is not only longer, but at the same time thinner. But once they are able to anticipate the correlative elongation and narrowing by means of images, they will be able to understand the operational compensation all the better. True, (as we recalled above), such anticipation presupposes an operational conservation framework at least in process of formation. But the fact remains and this is all we wished to bring out, that the image, once rendered anticipatory by the operations, in turn facilitates the functioning of the operations." (Piaget and Inhelder, 1971, p. 378). Further quotation: "The situation is indeed quite different when the images become anticipatory under the influence of the operations. The image then constitutes an auxilliary that is not only useful to, but in many instances necessary for the functioning of the operations. After having structured and fashioned it in their own likeness, the operations in fact come to depend on the image." (Ibid, p. 378). This "dependence" comes from the precision that mental imagery provides. "Once the child is capable of the image he will be able to arrive at a more <u>precise</u> deduction of the transformations themselves. In such cases, what happens is that the image becomes anticipatory under the influence of the operations and then serves them as a supporting base." (Ibid, p. 379). The above quotations illustrate clearly how and why Piaget and Inhelder perceive a reciprocal relationship between Hindsight-Foresight and Conservation ability in children's cognitive functions.

The third reason for recommending an Inclusion interpretation for the obtained cluster of Hindsight-Foresight, Multiplication of Relations and Conservation of Number is that it can incorporate other empirical findings in its fold. Bruner and his colleagues (1966) demonstrated that children may cognize co-variation between two dimensions without conserving. Although Bruner et al. interpreted their data to show irrelevance of compensation as a mediating source for conservation, one could incorporate his data into the Inclusion view. One could treat the child's ability to co-ordinate two dimensions without deducing conservation from his dimensional co-ordination, to indicate his Multiplication of Relations ability as being too immature yet to become serviceable as a "subroutine" or an integral part of conservation ability. (cf. Flavell, 1972, p. 305). Similarly, static imagery could be perceived along this line.

Thus using an Inclusion interpretation, one can arrive at a meaningful though speculative analysis of the item-relationships within the cluster of Hindsight-Foresight, Multiplication of Relations and Conservation of Number among conservers. However it is important to point out that the cluster of Hindsight-Foresight, Multiplication of Relations and Conservation of Number indicates that the problem of identifying a developmental order among them remains unsolved. This author suggests repeating measurement on these items with additional subtasks and with methodological refinement and elaboration on these, on the same non-conserving subjects at approximately the time they were last measured, (that is, January 1975), to see if any developmental order could be discerned among these three tasks.

(2) Theoretical implications from the obtained developmental sequence.

It is recalled that the obtained developmental sequence shows decalages between Multiplication of Relations and Multiplication of Classes; and between Conservation of Number and Multiplication of Classes where Multiplication of Classes had been found to trail behind the others. Such a finding raises problems for Piaget's theoretical stipulation of developmental synchronism among abilities that the child supposedly attains within the concrete-operational period of intellectual development. One may contend perhaps that within the concrete-operational period, the child's cognitive development in the domain of conservation concepts and in the domain of class-concepts is one of continual growth and stability. Thus, such obtained asynchronism may indicate that conserving children in this sample have only just begun to master certain operations and that presumably when the children progress further, such asynchronism would decrease, as functional maturity and stability in these domains are attained.

The idea of continual growth and stability of children's cognitive functions receives endorsement from Emmerich (1966; 1968). Flavell (1971) even considers growth beyond a 'stage' a feasible idea. However, such considerations eventuate in watering down Piaget's concept of stage, because they contradict Piaget's notion of 'structure d'ensemble' which is his main theoretical tenet in defining a 'stage' in his theory, (whether 'stage' refers to broad categories, for example, sensory-motor period or a narrow area, for example, number conservation).

The theoretical notion of 'structure d'ensemble' represents Piaget's view of children's cognitive development. For example, he perceives that the child in the concrete-operational period attains certain operations. Typical of these is conservation of substance, number, etc. These operations in Piaget's opinion, are not developed in the child in isolation as separate entities. Rather operations unite to form a structure, ("grouping"). Sets of operations lead to formation of different structures. (In the concrete-operational period, Piaget has specified that the child develops eight structural groupings). These structures are in turn closely inter-related. Thus the concrete-operational period of the child's intellectual growth represents to Piaget, the development of these eight inter-related structures, among which are Multiplication of Relations and Multiplication of Classes. None of them is ever perceived as developing in splendid isolation. Piaget believes firmly that the elements in children's cognitive structural development march in unison with one another, across a broad front. This is the essence of his theoretical notion of "structural d'ensemble".

The theoretical elegance of "structure d'ensemble" has not been matched by an abundance of equally elegant or neat empirical findings substantiating its existence in the real world. It has so far received only partial substantiation, (cf. Toussaint, 1974a; 1974b; Bynum, Thomas and Weitz, 1972). More importantly, the notion of "structure d'ensemble" embodies a criterion which proves to be a bane to Piaget's theory in more aspects than one. This criterion stipulates structural developmental synchronism of cognitive operations of different structures within the same broad period, for example, concrete-operational, (cf. Pinard and Laurendeau, 1969). That this criterion wrought havoc with Piaget's theory is shown by:

(a) the vagueness of the criteria of developmental synchronism, (cf. Flavell, 1971) where he raised the question of diverse possible meanings of developmental synchronism: where does it occur in the course of development between two cognitive items?

(b) the persistent findings of asynchronism between related structures in the concrete-operational period, (cf. Flavell, 1970; Brainerd, 1974g and current data in this dissertation).

The consequences of (a) and (b) on the stage-concept in Piaget's theory are rather devastating, as evidenced in the following quotation:

"If these highly general operations are also bound together into structures (with these structures in turn tightly interlinked), then one would likewise expect developmental asynchronisms across operations. As soon as a child can master any task requiring one operation, therefore, he should be able to master any other task requiring any other operation, whether it belongs to the same grouping or not. To the extent that developmental reality fails to accord with this ideal picture, that is, it presents numerous synchronisms within and between operations, to that extent would such key Piagetian expressions as "stage", "operation" and "structure" become imprecise and even misleading." (Flavell, 1970, p. 1038). Brainerd (1974g) substantiates Flavell's perceptive comment in his most recent paper. Brainerd (1974g) has marshalled ample empirical evidence to pinpoint how poorly the synchronous emergence prediction of the "structure d'ensemble" principle has fared in conjunction with three groups of concrete-operational skills. These three groups of concrete-operational skills are (a) transitivity/conservation/class inclusion; (b) double classification/double seriation (exemplified by 3 x 3 matrices in Multiplication of Relations and Multiplication of Classes); and (c) ordinal, cardinal and natural number concepts. Brainerd has shown that asynchronous emergence of stage-related skills appears to be the rule rather than the exception, and suggests that the specific asynchronies he discussed are not isolated idiosyncratic phenomena. Rather he thinks they are part of some underlying pattern in the growth of human logic that we do not yet fully comprehend, (cf. Brainerd, 1974g).

Brainerd has also expressed the same misgivings about Piaget's "structure d'ensemble" as one of the major defining attributes of Piaget's stages of mental growth from a psychometric perspective. In a theoretical paper, Brainerd states:

> "... in order to observe the predicted n-modal distribution for theory S (Stage Theory), we must observe during each (age) subrange a unimodal distribution of the stage-defining traits appropriate to that subrange. In each case, the mode corresponds to a high percentage of the stage-defining traits appropriate to that subrange. Such a mode will be observed for any given subrange only in the event that the stage-defining traits appropriate to that subrange emerge abruptly at the beginning of the subrange.
>
> Thus, the stage hypothesis can be verified empirically for any given stage theory only if the traits defining each posited stage appear abruptly at the onset of the chronological interval the stage purports to cover."
>
> Brainerd, (1974f, pp. 21-22)

The theoretical issue of stage cannot be settled in this study because of its complexity and because this study was not designed toward that end. However the controversies over "structure d'ensemble" as one defining attribute of Piaget's stage concept are registered. The decalages observed in the present data are interpreted to support the contention that "structure d'ensemble", hence the stage concept, needs to be re-vamped, as suggested by Flavell (1970; 1971) and Brainerd, (1974e; 1974f; 1974g). However, ultimate settlement of the status of stage as a theoretical notion remains for future research and more dialogue between those who want to salvage Piaget's stage concept as a theoretical notion, (cf. Wohlwill, 1973[8]; Pinard and Laurendeau, 1969), and others who consider Piaget's stage concept has dubious worth.

The Relation of the Current Findings to Previous Research Findings

The observed developmental sequence of Construction of Equivalence as the first step in children's cognitive understanding of number conservation concurs with previous research where matching quantity between sets has been found as the first

[8] Wohwill has now left the area of Child Development and joined environmental psychology. His 1973 book where he defends the stage-concept is his last piece of work in Child Development. Piaget has thus lost an interpreter in this continent.

validated step in hierarchies of early mathematic concepts, (cf. Wang et al., 1971; D'Amello and Williamsen, 1970). It also adds empirical data to the validation of Piaget's (1952) observations. The finding of Cognitive Shift as the second step may help explain the success of training studies which concentrated on this factor, (cf. Gelman, 1969; Bearison, 1969). Moreover it vindicates Zimiles' (1963) emphasis on research in the role of this cognitive shift of attention.

Educational Implications

Early childhood education. Many nursery educational projects have incorporated Piaget's ideas, (cf. Weikart, 1971; Kamii and Devries, 1973; Lavatelli, 1970). Lavatelli (1970a; 1970b) has produced a systematic curriculum in early child-education. However, examination of Lavatelli's curriculum indicates that it does not always conform to Piaget's theory.

The data of this study may well be of use to teachers of nursery children and Kindergarten and Grade One children, to complement their educational curricula.

Implications for special education (I). Children's number-conserving ability has been found to correlate with their performance in arithmetic, (Williams, 1958; Dodwell, 1961; Hood, 1962; Steffe, 1966; and Wheatley, 1970).

> "Although a causal relationship between conservation ability and arithmetic achievement in children is not implied (in these studies by the obtained positive correlations), the fact that conservers as a group scored significantly higher in arithmetic achievement test warrants attention. This fact would seem to indicate that conservation is an important factor in learning arithmetic at the first grade level."
>
> Wheatley, (1970, p. 299)

On the basis of his own conclusions, Wheatley (1970) has gone to the extent of devising a Number Concept Test which contains six items on number conservation, six on counting, 12 on cardination, two on one-to-one correspondence, and one on conservation of length. He suggests that knowledge of conservation could readily be obtained by teachers and used for planning first grade curricula. Subsequent readiness activities could be developed for those scoring low on conservation, (cf. Hood, 1962; Wheatley, 1970). Other researchers such as Wallach and Sprott (1964) indicate that such attempts would be profitable. The tasks in this experiment can certainly be used to provide readiness activities for non-conservers.

One may well ask how is number conserving ability related to arithmetic performance in children.

> "A child may be able to count two sets of objects and arrive at the same answer, but still maintain there is more in the spread out set. The spatial configuration of what he sees (perception) triumphs over the intellectual idea of the conservation of number. It is the conservation concept that he does not yet have and which is necessary for a real understanding of number. <u>Number by its very nature is invariant. This the (non-conserving) child does not understand."</u>
>
> Copeland, (1970, p. 65)

Also, he says:

> "To understand number, the child must first develop its basic characteristic of invariance or conservation. For how is the child to comprehend the meaning of number if he thinks there is more or less if a set of objects is re-arranged."
>
> Ibid, (p. 66)

Similar ideas have been expressed by Wheatley (1970). Wheatley explains why non-conserving first graders are likely to experience difficulty in understanding the concept of addition:

> "When the teacher pushes together a set of two objects with one of three objects and says: 'See, now there are five, so 2 + 3 = 5,' the non-conserver is not going to see the '=5' since the objects were moved and he believes this changes the number property. The child may learn to parrot 2 + 3 = 5, but he will not be able to understand what it means, since the objects have been moved and to him this changes the number. The child is also likely to have great difficulty applying addition to any problem situation.
>
> Wheatley, (1970, p. 294)

The pertinence of Copeland and Wheatley's analyses of the role of number conservation in children's arithmetic learning is supported by the work of Johnson and Myklebust (1967).

Johnson and Myklebust (1967) reported specific observations on children who can understand and use spoken language, who can read and write, but who fail to understand mathematic principles and processes, or fail to learn to calculate. These children are considered to have arithmetic learning disorders, (cf. Johnson and Myklebust, 1967, pp. 244-253). The authors have found such children to possess in varying degrees thirteen arithmetic disorders, (Ibid, p. 252). Foremost on the list is the absence of the concept of one-to-one correspondence and sixth on the list is conservation of quantity. Johnson and Myklebust elaborated on these two arithmetic disorders: "Inability to establish one-to-one correspondence. The number of children in a room cannot be related to the number of seats, nor an estimate made of how many forks to place on a table at which four people are to eat." And, "Inability to grasp the principle of conservation of quantity. Some dyscalculics are not able to comprehend that ten cents is the same whether it consists of two nickles, one dime, or ten pennies, or that a one-pound block of butter is the same as four one-quarter pound sticks." Johnson and Myklebust, (1967, p. 252).

The present experimental data indicates that the Construction of Equivalence, that is, establishing one-to-one correspondence, appears to be the first step in children's attainment of number conservation. Piaget (1952) has demonstrated the same, and refers to children who fail it as Stage I children. Thus the children Johnson and Myklebust described who lack concept of one-to-one correspondence, are likely to do poorly in number conservation. Moreover Steffe (1966) and Wheatley (1970) found a positive relationship between poor number conserving ability and poor arithmetic achievement among children. It is recalled that the children Johnson and Myklebust described, do perform poorly in arithmetic, (Johnson and Myklebust, 1967). By linking such children's absence of one-to-one correspondence to number conservation, one enhances understanding of their problem with conservation of quantity, (discontinuous quantity as reported by Johnson and Myklebust). Both concepts involve quantitative thinking and both depend on the same cognitive structures in the child, viz. Hin ight-Foresight, Multiplication of Relations, (cf. Piaget, 1952; 1964; 1967). It is recalled that Piaget views the child's development of conservation concepts among various contents to be a wholistic development. He maintains that the child does not develop conservation concepts or related concepts of classes and relations in isolation of one another. Thus it would be dissonant with Piaget's conception of children's cognitive development if a child with arithmetic learning disabilities attained all other concepts of conservation, for example, conservation of number, length, etc., with the single exception of conservation of quantity, given the time necessary to cover "horizontal decalages". In the light of Piaget's theory and empirical replications of his conservation studies, it is reasonable to suggest that children with arithmetic learning disabilities who lack conservation of quantity are likely to lack at least some other concepts of conservation, among which number conservation is one such possibility. This suggestion appears in the findings of Steff (1966) and Wheatley (1970), which have already been described.

In view of Johnson and Myklebust's observations, the present experimental findings and tasks lend themselves to remedial diagnostic use, (cf. Berry, 1968). The remedial tutor could use tasks similar to Construction of Equivalence and Cognitive Shift for children who lack the concept of one-to-one correspondence. For children who perform poorly at number conservation, the remedial tutor could use tasks similar to those in this experiment to see (a) where the child is, (b) commence remedial training in number conservation, using tasks similar to those in this experiment. It is recalled that the developmental order of Hindsight-Foresight, Multiplication of Relations and Conservation of Number was equivocal in this experiment, since they were found to appear in a cluster across all three groups of Nursery, Kindergarten and Grade One children. However, a suggestion of reciprocal interactions regarding cognitive items of Hindsight-Foresight, Multiplication of Relations and Conservation of Number among conservers has been put forth, (cf. Flavell, 1972). Hence the equivocal developmental order should not detract from the usefulness of the tasks of Hindsight-Foresight and Multiplication of Relations in remedial teaching of children with arithmetic learning disorders who lack one-to-one correspondence and conservation of quantity, and those who perform poorly at number conservation as described by Wheatley and Steffe.

The importance of training reversibility thought has been pointed out by Copeland (1970).

> "The youngster has achieved reversibility in that if one of two equal sets is re-arranged such as from a row of objects to a pile or heap and the other from a heap to a row, he realizes that the number of each set has not changed, that is heap to row is the same as from row to heap as far as the number of the set is concerned.

```
                    o                     o
o o o o o o  =     o o        and        o o    =  o o o o o o
                  o o o                 o o o
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> Reversibility is also necessary for the 'additive' concept. If a child knows 3 + 2 = 5, can he also solve 5 = + 2 or 3 + = 5? Many children are 'taught' addition when they have not yet reached the stage of reversibility of thought necessary for the conservation of number concepts involved in such problems. It isn't surprising that first grade teachers find it difficult to teach these ideas."
>
> Copeland, (1970, p. 73)

The function of the present data therefore pertains to remediation of basic pre-number concepts among children with arithmetic disabilities who evidence either absence of one-to-one correspondence or poor number-conserving ability.[9] In so doing, the present data serves to restore, and to rebuild a solid foundation for such children in their arithmetic learning.

Implications for Special Education (II)

Relation to mental retardation. Kirk (1958) did a study showing pre-school training benefitted mental retardates in non-institutionalized as well as institutionalized children aged three to six. The effects were seen in increase in I.Q. as measured by the Stanford-Binet, the Kulman and in a social maturity measure, the Vineland. Such effects were found to be durable at the end of the children's attendance of a year's public schooling of Grade One or in a special class.

Kirk's study showed the importance of pre-school training for mentally retarded children. At present there are pre-school training programs for mentally retarded children. Some of these directly employ Piagetian notions, (cf. Weikart et al., 1970, Weikart, 1971). Where the pre-school program for retardates is not specifically Piagetian, some notions are found to be closely akin to Piagetian concepts, (cf. Waite, 1972, pp. 162, 222, 281). Waite (1972) uses "simple seriation" to teach mentally retarded children to place chips in a row progressing from lightest to darkest or reverse. She also uses "simple classification" and simple measurement problems.

The tasks in the present experiment such as Construction of Equivalence, Cognitive Shift and Multiplication of Relations can easily be modified and incorporated

[9]The distinction between these two concepts rests on the basis that children who do not conserve may in some cases know one-to-one correspondence, that is, Piaget's Stage II children.

into Waite's training program to complement the section on "Quantitative Concepts" in her book.

There has been some specific attempts to use a Piagetian approach to teach arithmetic for the mentally retarded, (cf. Stephens, 1971). Stephens (1971) advocates concentrating efforts to enable the retardates to attain sequence of abilities basic to mathematics rather than to drill them in counting, addition or subtraction. These "basic, number-relevant capabilities" are Piaget's notions of one-to-one correspondence and seriation. Since mental retardates are found to be able to attain "concrete-operational concepts", (cf. Stephens, 1971; Lister, 1969; 1970), this writer suggests training them to conserve in number, quantity and conservation of length, because these concepts are relevant to their daily living. Stephens also points out the relevance of training flexibility and reversibility of thought in the retardate, (Ibid, pp. 7-9). However she has not provided any suggestions on how to approach this, even though she made it clear that she has in mind Piaget's notion of flexibility and reversibility of thought. The present writer believes tasks based on her Hindsight-Foresight tasks will be serviceable to promoting flexibility and reversibility of thought among retardates.

To summarize, the present writer considers her experimental tasks and findings are directly serviceable to (1) preschool training programs for mentally retarded children. Incorporation of the present experimental tasks can only enrich such nursery programs. (2) Hindsight-Foresight tasks or variations of it are directly serviceable to promoting mental flexibility and reversibility among retardates. In view of Stephens' (1971) statements, that cognitive development does proceed in educable retardates well into late adolescence, the Hindsight-Foresight tasks here can be used for adult retardates as well as for young retardates.

The relevance of the present dissertation experiment to education and special education has been discussed. It appears opportune to conclude with the following comments.

> "Teacher preparation, particularly for special education, would do well to emphasize the process of learning and the effects upon it of primary disorders and their secondary overlay. Rather than learning a specific method for each subject to be taught, teachers need to learn how to analyze the structure of subject matter so they can identify the readiness components for mastering it, then evaluate methods and materials for filling gaps in readiness and for meeting the requirements of each learning style."
>
> Freidus, (1966, p. 123)

The child's attainment of number conservation is an act of learning through his own interactions with environmental objects. By acting on them, he does not merely derive knowledge of the properties of objects, such as weight of the object. More importantly, he derives from his own actions on objects logico-mathematical knowledge, of which the concept of conservation is one. (Piaget, 1964).

Freidus also states: "The process of learning is most profitably studied in relation to normal child development." (Ibid). Stephens (1971) expresses the same idea: "For teacher to supply the pupil with the appropriate learning situation requires a developmental analysis of task as well as of pupil." (Stephens, 1971, p. 4). The pertinence of developmental hierarchy or sequence to remediation has also been emphasized by Copeland (1970) and Koopman (1971).[10]

Lastly, the relevance of developmental hierarchies depends on the emphasis on the understanding of basic cognitive processes in children, and this understanding must precede curriculum building and specially the development of remedial programs. Where educators have plunged in vis-a-vis remedial programs

[10] In a lecture on Learning Disabilities.

and curricula building without attending to questions on basic cognitive development in children, their attempts appear to have short-term usefulness. This is because research findings suggest that most of their premises on which they have built their programs, appear in retrospect, premature. It is hoped that through more interdisciplinary co-operation between developmental research and education and special education, we would be able to build curricula and remedial programs on a more sound base.

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APPENDIX A

<u>Brief Review of Number Conservation Training Studies Using Procedures Other than Reversibility Training</u>

There is abundant documentation in the literature of other approaches to induce number conservation in children. These consist of: (1) the addition and subtraction training approach, (2) the learning approach which can be subdivided into two procedures and (3) the perceptual discrimination approach.

The addition and subtraction training procedure was used by Wohlwill and Lowe (1962); Wallach <u>et al</u>., (1967) and Winer (1968). In general these attempts to induce number conservation in children using addition and subtraction training have not been successful. However, Brainerd (1974) reports success by Japanese investigators using a methodologically refined addition and subtraction training procedure.

The two learning approaches in inducing number conservation in children consist of the following: The first learning approach was used by Kingsley and Hall (1967) and Lister (1969; 1970). These investigators followed Gagne's model of learning. Gagne (1965) put forth the idea that any criterion behaviour, for example, conservation behaviour, subsumes a hierarchy of subtasks or subskills. Mastery of such pre-requisite subtasks eventuates in the child's ability to succeed in the criterion task. In line with the given example, this means to conserve. Gagne's additive model is built on discrimination-learning and memory and emphasizes that mastery of all the subtasks <u>suffice</u> to bring on the desired behaviour in the child for which he undergoes the given training. Kingsley and Hall (1967); Lister (1969; 1970) followed Gagne's model and from logical task analyses, derived respectively a list of pre-requisite subtasks underlying conservation of length and weight in Kingsley and Hall's study; and a list of pre-requisite subtasks for weight and volume conservation in Lister's studies. The respective researchers trained their subjects on such lists. They reported success in training by such a learning approach.

The second learning approach is more closely akin to discrimination learning in experimental psychology where discrimination learning involved the use of a discriminative stimulus. (S_D). Gruen (1965); Halford and Fullerton (1970) used this training approach. Halford and Fullerton's procedure serves as a good illustration to distinguish this learning approach from the one described in the preceding paragraph. These investigators used a discrimination task to induce number conservation. This discriminative task consisted of a set which was shown to equate numerically the standard set but which had since been re-arranged (transformed). The authors trained subjects to use this discriminative task to select other sets which would match the standard. Functionally then, this discriminative task occupied the role of a discriminative stimulus, (S_D). Halford and Fullerton (1970) reported successful induction of number conservation among their subjects using this training procedure.

The last approach is training via a perceptual discrimination approach. It is based on the idea that the typical conservation task presents children with misleading cues, and that conservation involves learning to ignore such misleading perceptual information. Such a position basically does not contradict Piaget's views, since he too believes that children must overcome misleading perceptual cues if they were to attain conservation. A conflict in interpretation arises over how the misleading perceptual cues are to be overcome. For Bruner (1966), it appears that a verbal formula shields the child from the illusion of non-conservation. Bruner thinks the child's ikonic mode of thinking has to conflict with his symbolic mode of thinking, thus producing an "intraperceptual conflict". It is from this intraperceptual conflict that children attain conservation. (Ikonic refers to visual imagery, perception; enactive refers to motor, and symbolic refers to language). For Gelman (1967; 1969), it is the inculcation of a quantitative set or more sophisticated perceptual discrimination, that facilitates children's acquisition of conservation.

Studies using these respective training approaches have not been reviewed in the Section of Literature review because they do not bear on the present dissertation. However they are mentioned here for two reasons: (1) to complete literature survey in induction of number conservation in children, and more importantly, (2) to show interpretations different from Piaget's theory of children's absence of conserving ability.

TABLE A

Correlations Between I.Q. Measures and the Seven Tasks

Tasks

I.Q.	Construction of Equivalence	Cognitive Shift	Hindsight-Foresight	Multiplication of Relations	Multiplication of Classes	Conservation of Number	Conservation of Ordinal Correspondence	Sum of Seven Tasks
Pooled Data from Nursery, Kindergarten and Grade One Group								
Verbal I.Q.	-0.109	-0.056	-0.113	-0.105	-0.050	-0.098	-0.013	-0.102
Performance I.Q.	-0.112	-0.004	-0.064	-0.095	-0.128	-0.099	-0.168	-0.124
Results from Nursery Group Only								
Verbal I.Q.	-0.157	-0.211	-0.006	0.081	-0.044	-0.119	-0.163	-0.160
Performance I.Q.	0.010	-0.021	0.001	0.007	-0.129	-0.087	-0.206	-0.080
Results from Kindergarten Group Only								
Verbal I.Q.	-0.280	-0.122	-0.254	-0.250	0.002	-0.105	-0.118	-0.250
Performance I.Q.	-0.652	-0.088	-0.179	-0.053	-0.184	-0.189	-0.167	-0.252
Results from Grade One Group Only								
Verbal I.Q.	-0.032	0.006	-0.269	-0.252	-0.150	-0.259	0.030	-0.221
Performance I.Q.	0.156	0.153	-0.007	-0.238	-0.088	-0.055	-0.211	-0.146

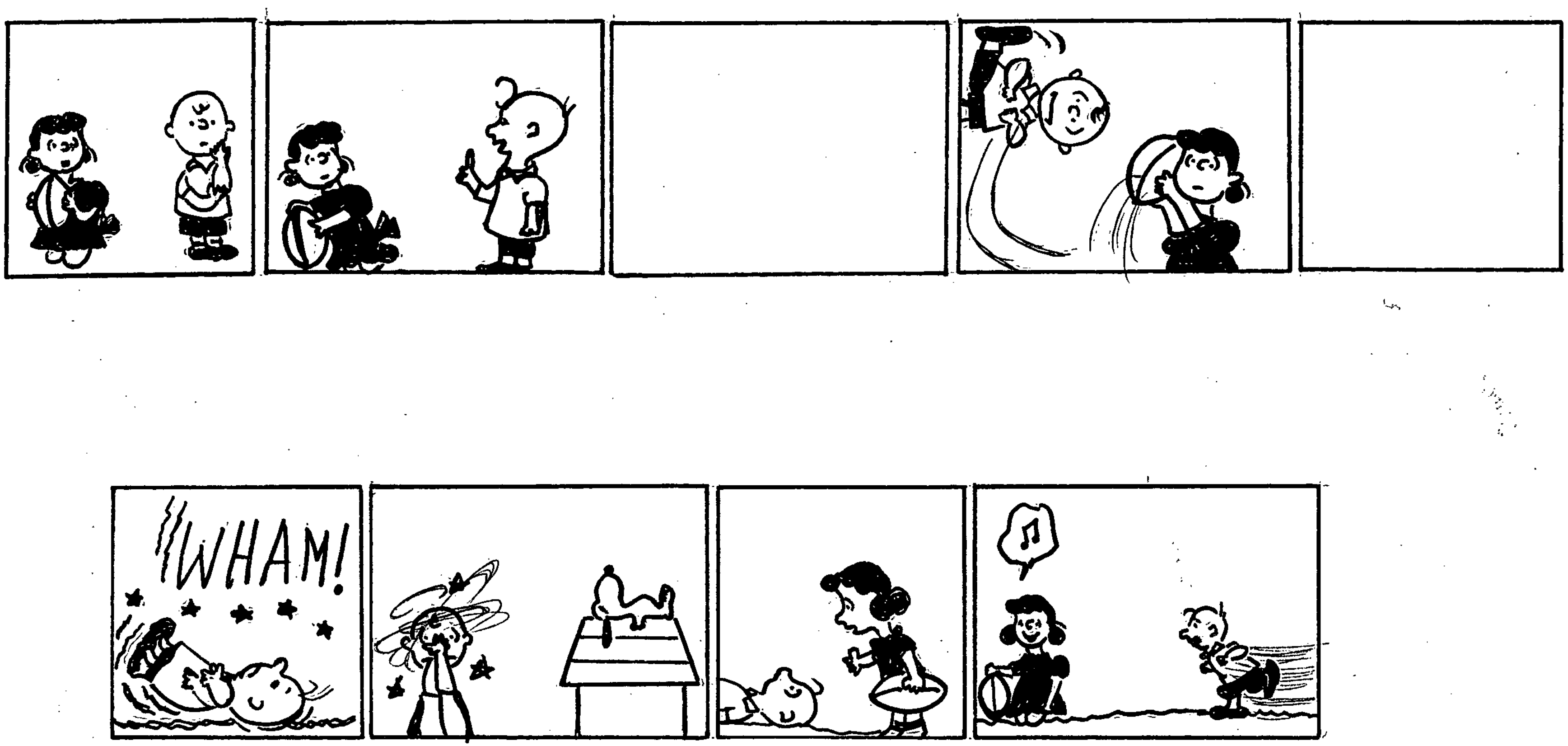


Fig. A. Practice Trial in Hindsight-Foresight.

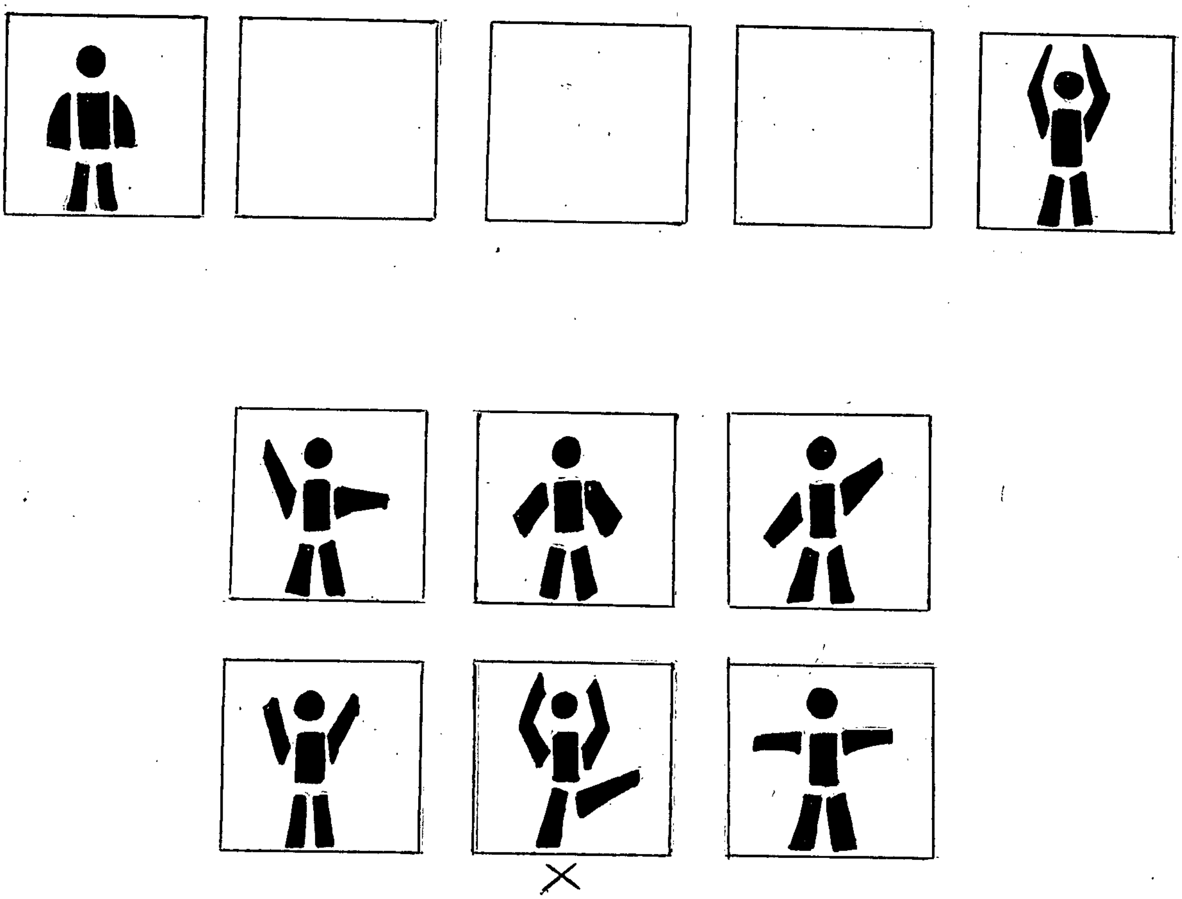


Fig. B. First subtask of Hindsight-Foresight: The Little Man.
Actual measurement of stimuli was 5 cm x 5 cm card.
Card "X" was not presented to pre-schoolers and kindergarten subjects because they had been found unable to handle more than five items of pictorial stimuli. (Inhelder and Piaget, 1964a).

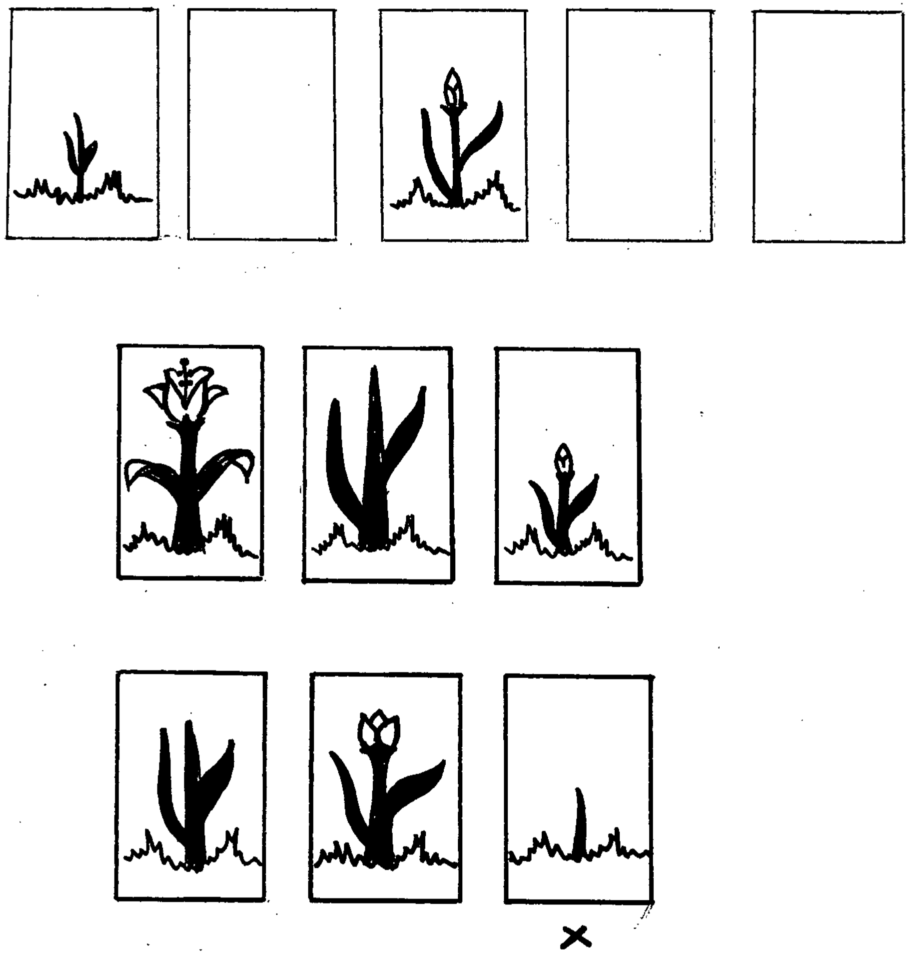


Fig. C. Second subtask of Hindsight-Foresight: The Growing Plant.
Actual measurement of stimuli was 9.4 cm x 5.1 cm card.
Card "X" was not presented to pre-school and kindergarten subjects.

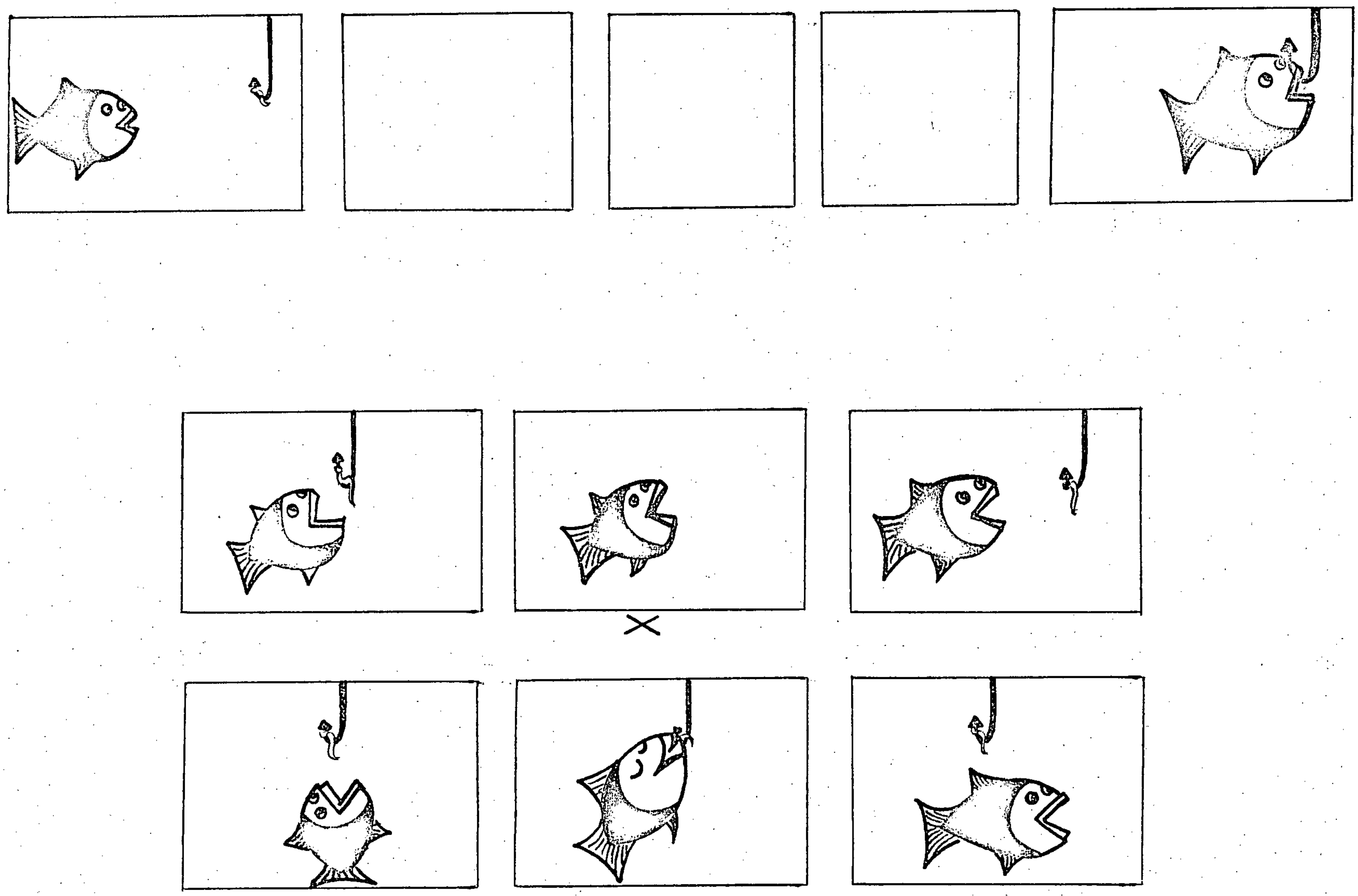


Fig. D. Third subtask of Hindsight-Foresight: The Fish.
Actual measurement of stimuli was 10.6 cm x 6.5 cm. Card "X" presented to Grade One subjects only.

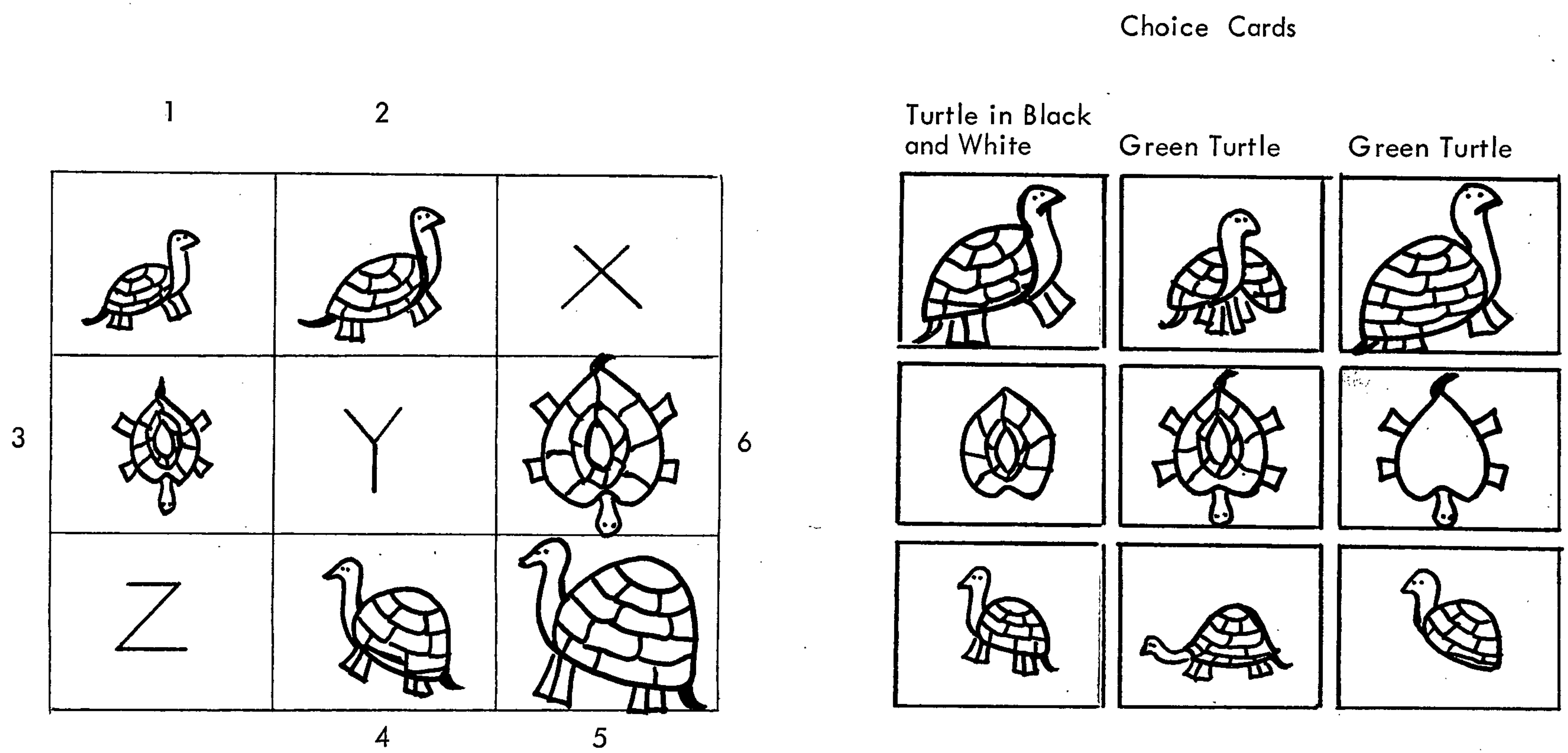
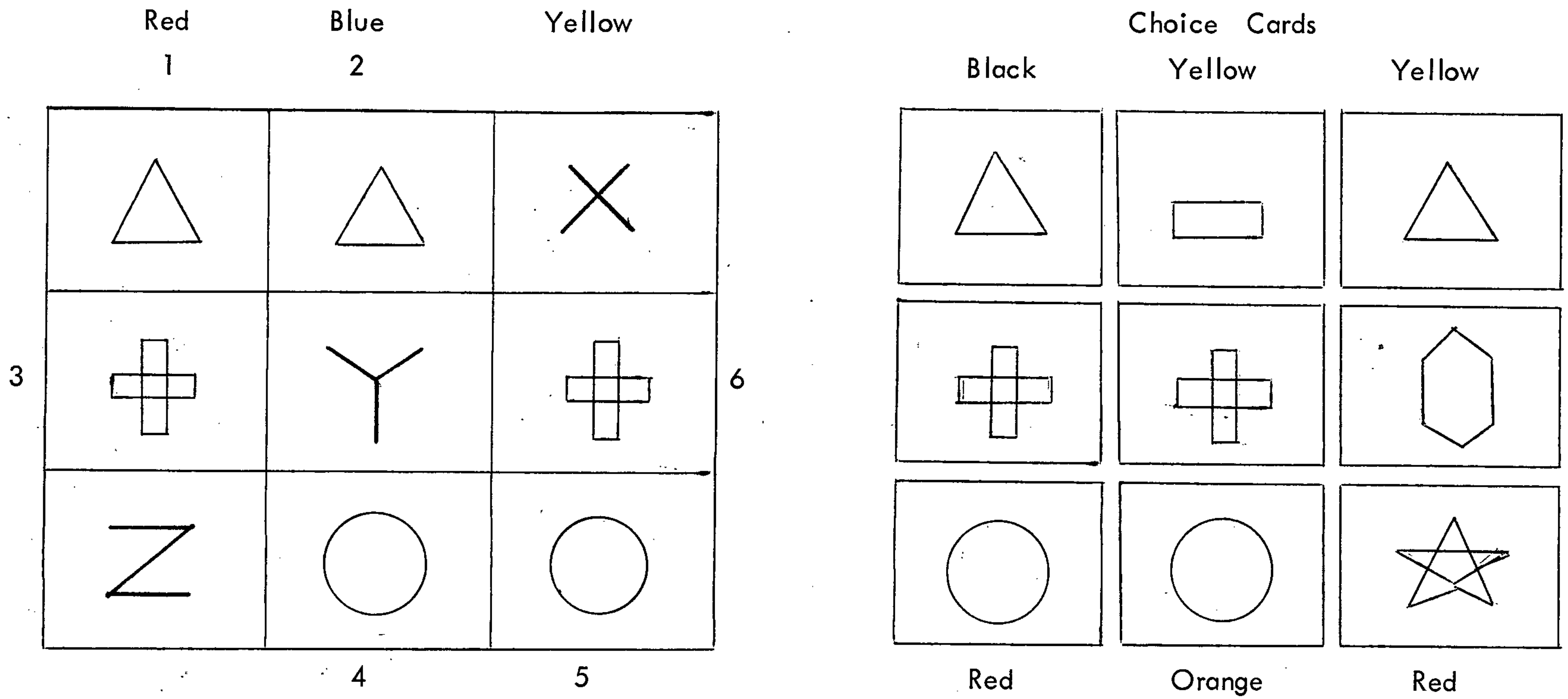
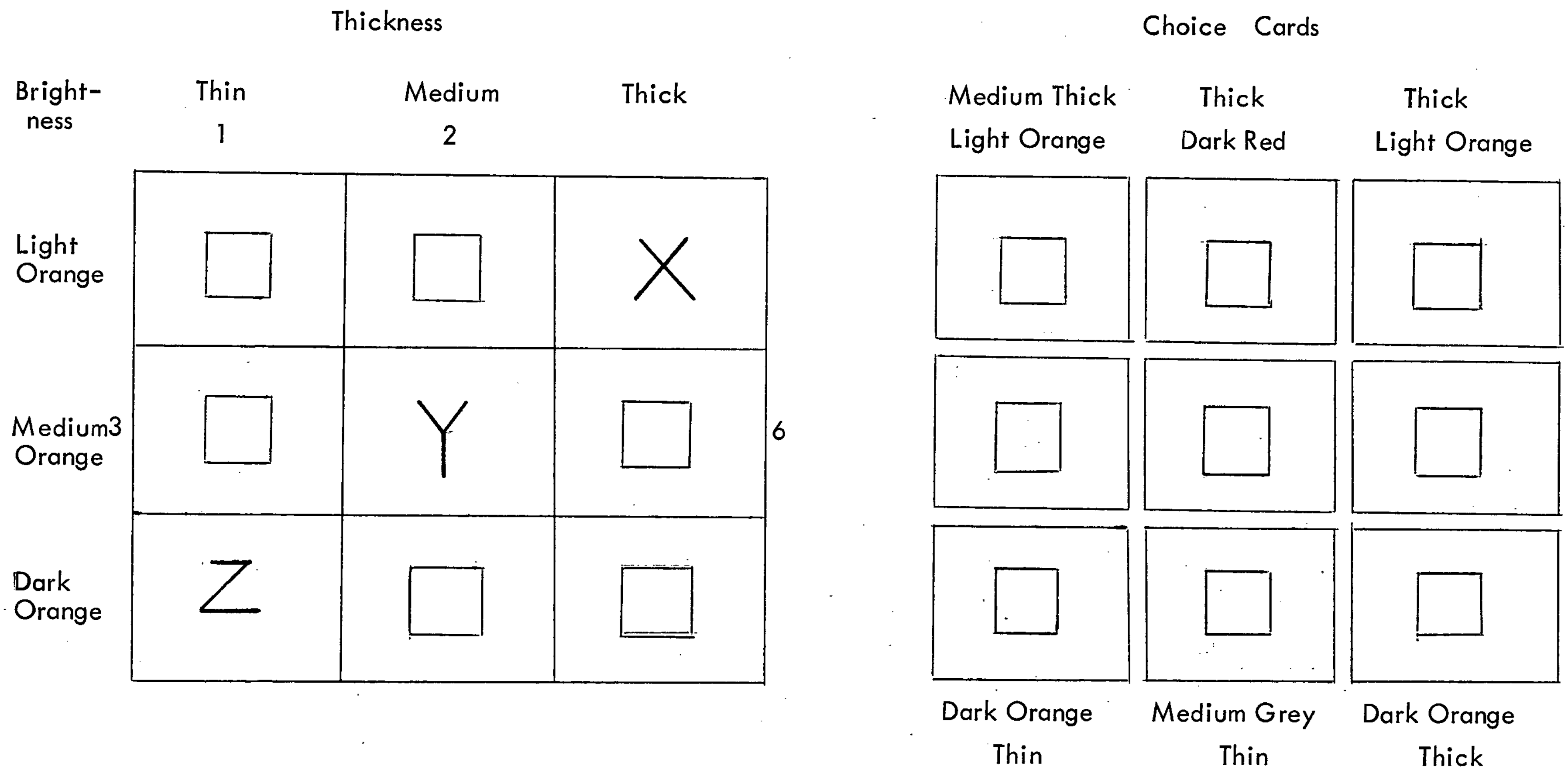


Fig. E. First subtask of Multiplication of Relations.
Dimensions varied were size and orientation. Actual measurement of stimuli was 5.1 cm x 6.4 cm card.



Colour of choice cards from left to right in second row = purple, blue, blue.

Fig. F. Second subtask of Multiplication of Relations. Dimensions varied were shape and colour. Actual measurement of stimuli was 5 cm x 6.3 cm card.



Brightness and thickness of choice cards from left to right in second row = medium green and medium thick; medium orange and medium thick; medium orange and thick.

Fig. G. Third subtask of Multiplication of Relations.
Dimensions varied were brightness and thickness.
Actual measurement of stimuli was 5.1 cm x 6.3 cm card.

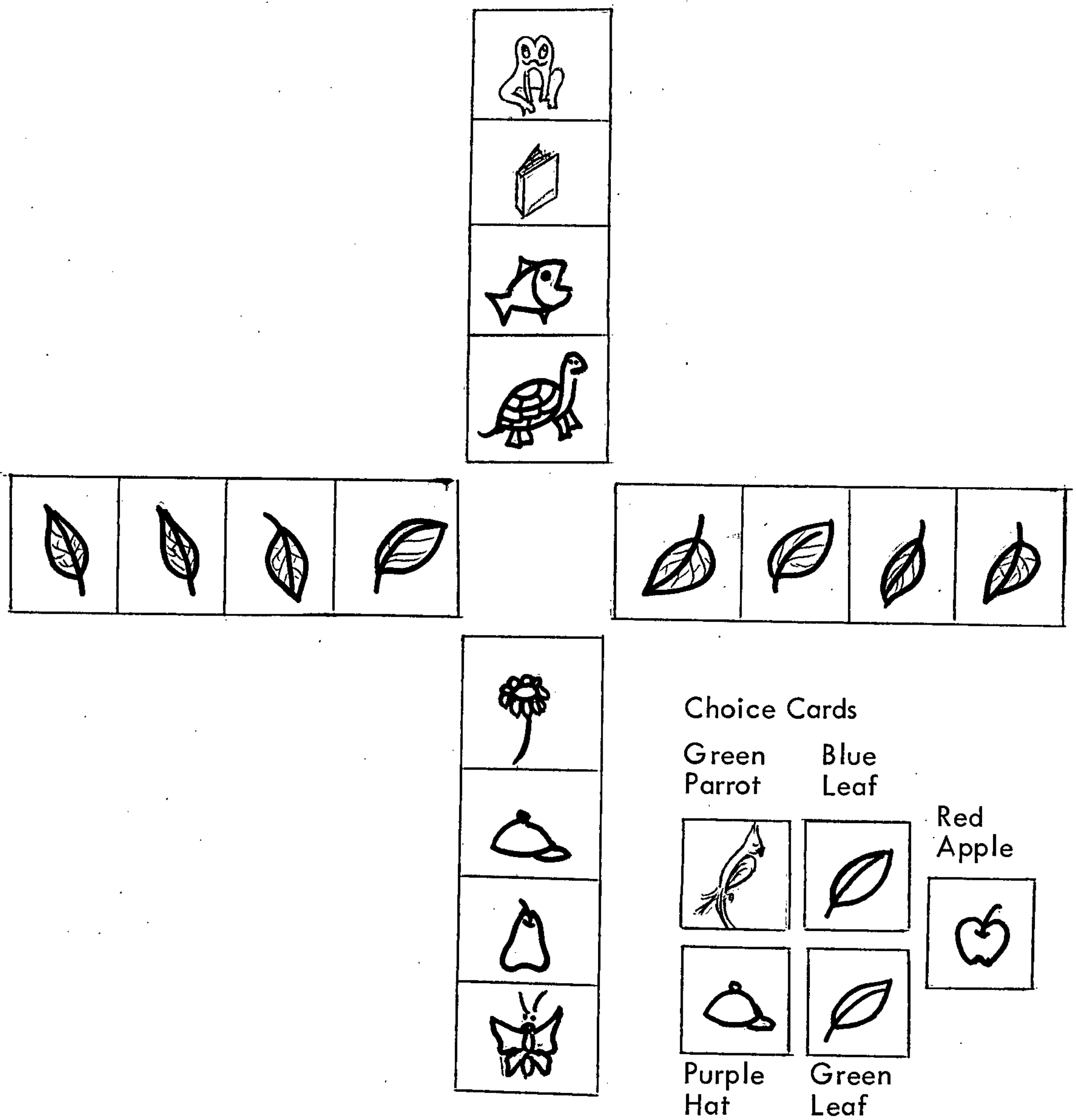


Fig. H . First subtask of Multiplication of Classes. Column consisted of green objects while row consisted of leaves of various colours.

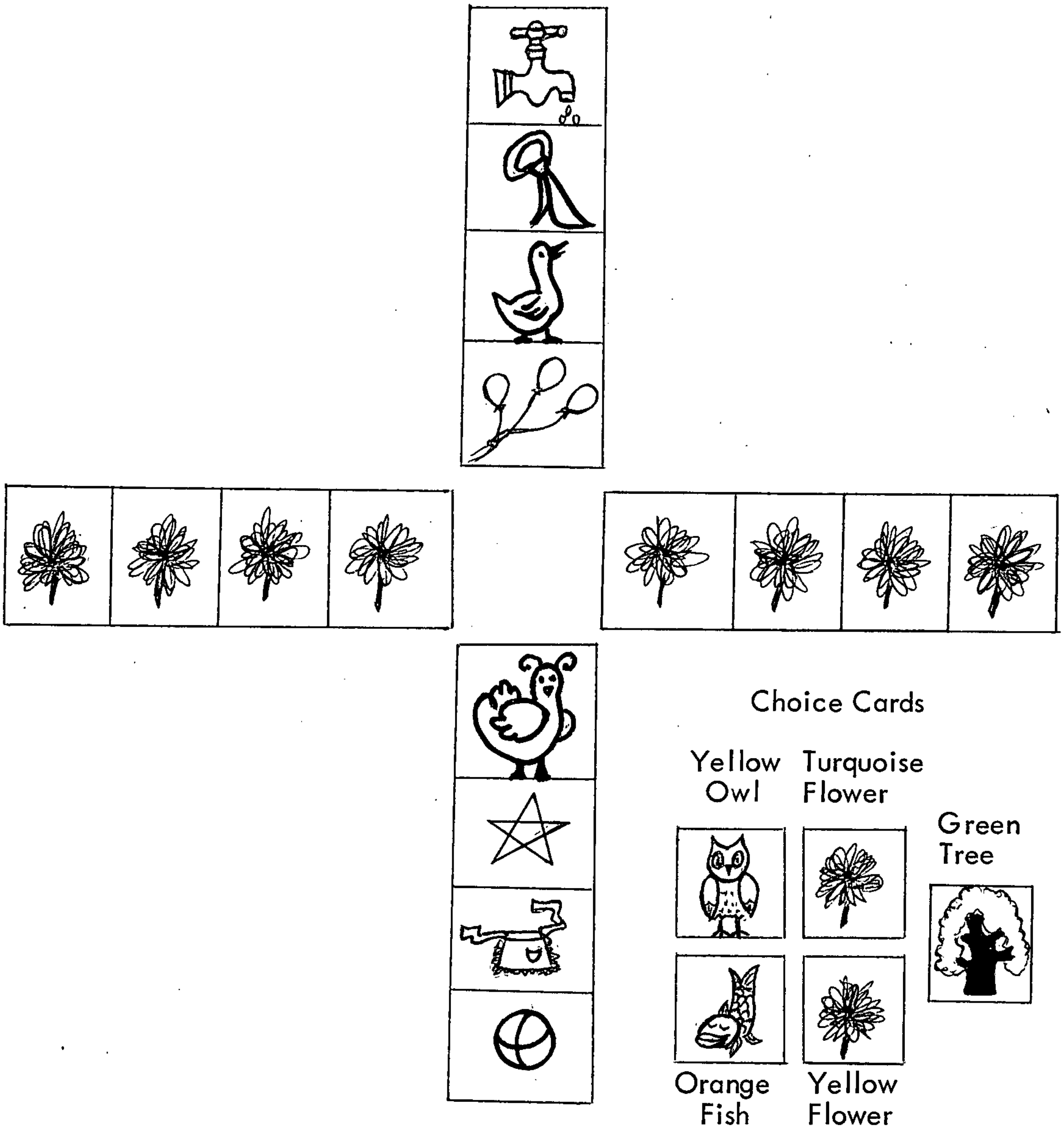


Fig. 1. Second subtask of Multiplication of Classes. Column consisted of yellow objects while row consisted of flowers of various colours.

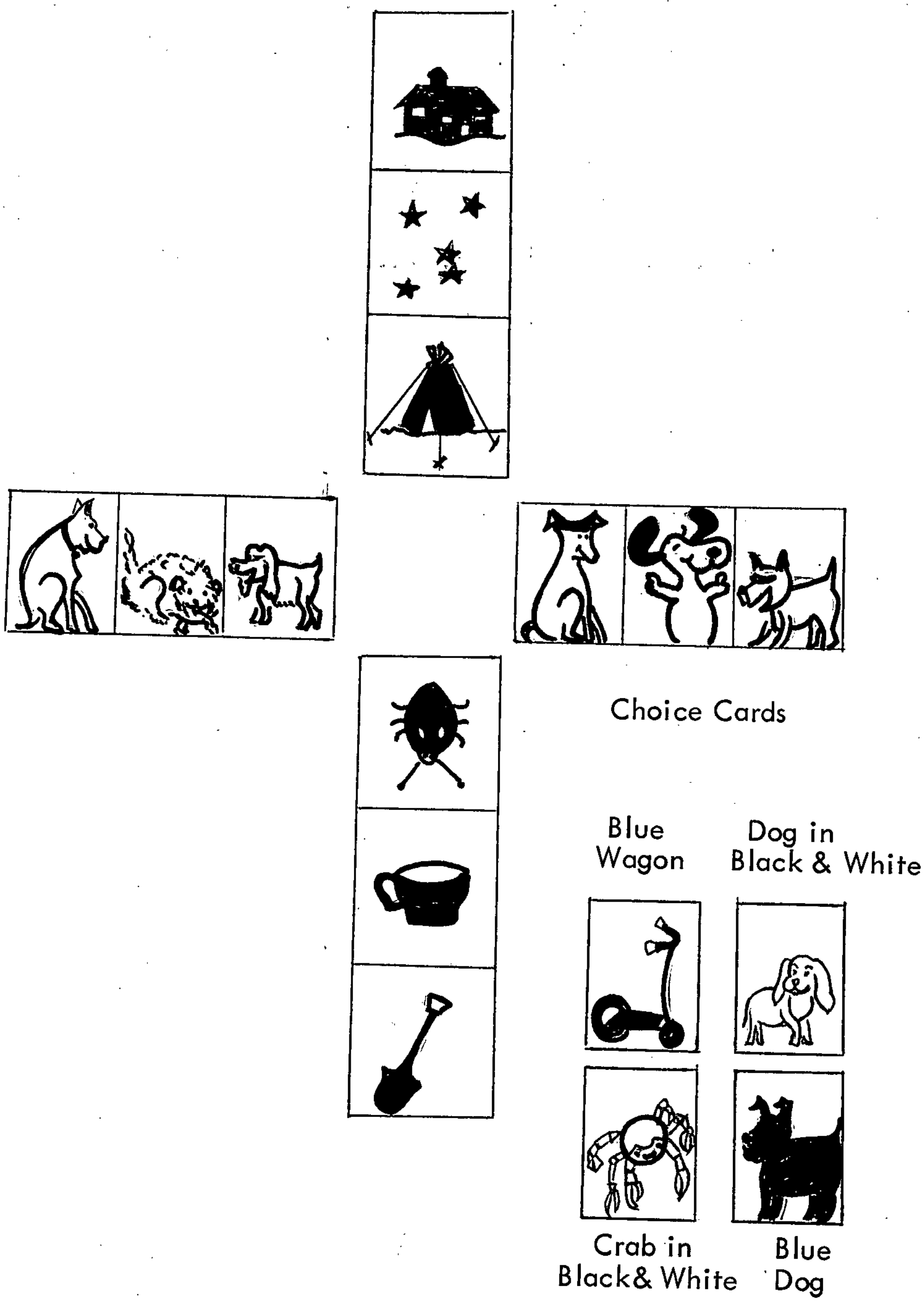


Fig. J. Third subtask of Multiplication of Classes. Column consisted of blue objects while row consisted of dogs in black and white.