WAVE-PARTICLE INTERACTION
AROUND THE LOWER HYBRID RESONANCE

by

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ABSTRACT

Wave-particle interaction in the ionosphere is studied theoretically for wave frequencies around the lower hybrid resonance (LHR) frequency. Expressions are derived by two methods for the growth rate of whistler-mode waves propagating in a magneto-active plasma penetrated by a tenuous beam of nonthermal particles. The first method employs the electrostatic dispersion equation; the second uses the full-wave dispersion equation which reduces to the electrostatic one for large values of refractive index. The equilibrium distribution function for the plasma is Maxwellian, and that for the diffuse streaming particles is also Maxwellian, but is shifted by a streaming velocity parallel to the background magnetic field. The first method assumes that the temperatures are isotropic, while the second assumes that the distributions are characterized by the perpendicular and parallel temperatures, $T_{\perp}$ and $T_{\parallel}$. The growth-rate expressions are fairly general, but numerical calculations are performed for the case of a cold plasma consisting of electrons, H^{+} , He^{+} , and O^{+} ions and a beam of nonthermal electrons.

The growth-rate expression obtained using the electrostatic dispersion equation shows that waves propagating slightly off the direction perpendicular to the background magnetic field can grow due to the Landau instability process which is excited by high energy (~ 10 keV) electrons streaming along the direction of the magnetic field of the earth. The growing wave thus triggered is shown to have a frequency band with a

sharp lower cutoff at the LHR frequency and an upper limit at the electron cyclotron frequency or electron plasma frequency, whichever is lower.

The previous growth-rate expression is generalized by making use of the full-wave dispersion equation. It is shown that there are two regions in propagation angle θ where the Landau instability may occur. The "electrostatic" region lies just below the resonant angle and, separated by a region of damping, the "low- θ " region lies above $\theta = 0$. The growth-rate values calculated in the "electrostatic" region correspond to the values obtained in the previous calculation. Generally, the maximum growth rate is larger in the "electrostatic" than in the "low- θ " region. It is also seen that with increasing frequency the "electrostatic" maximum growth rate increases monotonically and the cyclotron instabilities become important at frequencies above about ten times the LHR frequency. The influence of the following parameters on the growth rate is also examined: temperature ratio $T_{\parallel}/T_{\perp}$, streaming velocity of the nonthermal particles, and the ratio of the kinetic energy in the streaming motion to the thermal energy of the streaming electrons.

The theory presented is applied to LHR noise bands discovered by the Canadian Alouette I satellite. It is shown that many features are in good agreement. Other observations, such as auroral hiss, also have features which suggest that the theoretical work may be relevant to these types of ionospheric noise.

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CHAPTER 1

INTRODUCTION

1.1 Brief Background

Many new types of signals were observed for the first time by instruments carried by rockets or satellites into the ionosphere and above. The first observations of plasma resonances in the ionosphere were made during rocket tests of the topside sounder technique (Knecht et al., 1961; Knecht and Russell, 1962). Similar as well as additional (Lockwood, 1963) resonances were detected with the Alouette topside sounder satellite which was launched on September 29, 1962 into a circular, polar orbit at a height of 1000 km. The sounder sweeps from 0.5 to 12.0 MHz in 18 sec. These resonances appear on ionograms as persistent responses at certain characteristic frequencies often lasting for several milliseconds. The characteristic frequencies were found by Calvert and Goe (1963) to be: the electron plasma frequency Π_e , multiples of the electron cyclotron frequency Ω_e , the upper hybrid resonance frequency ω_{UH} and its harmonic $2\omega_{UH}$. The resonance at the electron cyclotron frequency is not always observed and, when it is, is much weaker than that at the second or third harmonic. Calvert and Goe (1963) suggested that the major resonances at the electron plasma frequency and the upper hybrid resonance frequency are caused by electrostatic oscillation of the ionospheric plasma along and across the earth's magnetic field respectively. The others he suggested were the result of cyclotron resonances. Fejer and Calvert (1964)

subsequently extended and unified the explanation of resonance phenomena, attributing all the observed resonances to electrostatic oscillations of the ionospheric plasma. They stated that the resonant frequencies Π_e and Ω_e correspond to oscillations approximately along the ambient magnetic field; ω_{UH} , $2\Omega_e$, $3\Omega_e$, etc., to oscillations across the field. In order to obtain this result, they adapted for the special conditions of the ionosphere previous work on electrostatic oscillations (Landau, 1946; Bernstein, 1958). Other relevant references are: Tonks and Langmuir (1929), Bohm and Gross (1949), Gross (1951), and Gordeyev (1952). Fejer and Calvert also stated that the absence of responses corresponding to intermediate angles is accounted for by the greater group velocity of the electrostatic waves, carrying the energy away more rapidly. The group velocity is near zero for frequencies near the resonances. Subsequently Crawford et al. (1967) made studies of the group delay of a wave packet propagating between two antennas. Theoretical predictions of the delay were found to agree closely with laboratory plasma measurements. They reported experimental results on a laboratory plasma simulating the ionospheric resonances.

In addition to the topside sounder, a broad-band very-low-frequency (VLF) receiver was included in the payload of the satellite. The receiver covers the VLF band from 400 Hz to 10 kHz and is connected to a 150-ft. dipole antenna. An unusual type of signal detected by the VLF receiver is bands of noise characterized by a sharp lower frequency cutoff which

usually increased with decreasing latitude of the satellite (Barrington and Belrose, 1963). The lower cutoff frequency ranges from about 5-10 kHz and the bandwidth is several kHz or less. Brice and Smith (1964) showed that many features of the noise band lead to the conclusion that its lower cutoff frequency is the lower hybrid resonance (LHR) frequency ω_{LH} for the ambient plasma. Hence the name LHR noise bands. It will be shown later that, generally, the characteristics of LHR noise bands appear to be identical to those of electrostatic waves propagating nearly perpendicular to the earth's magnetic field. Since the appearance of LHR noise bands is distinctly different at middle and northern latitudes, the two types of noise have been called midlatitude and polar LHR noise. Certain types of midlatitude LHR noise seem to be similar to the resonance at the upper hybrid resonance frequency observed by the topside-sounder technique. Polar LHR noise bands have a diurnal variation suggesting a relationship with electron precipitation (McEwen and Barrington, 1967).

Jørgensen (1968) has suggested that polar LHR noise is equivalent to auroral hiss. He states that the hiss is generated in the auroral regions of the magnetosphere and that the mechanism of generation is closely connected to very intense fluxes of electrons with energies of the order of several keV.

1.2 Purpose and Scope of Thesis

The purpose of this thesis is to present a theory based on wave-particle interaction for the origin of various

ionospheric noise which have electrostatic characteristics and are related directly to particle precipitation. The theory may be summarized in the following two points. The first is that an electrostatic wave is excited in the ionosphere. The second is that a type of Landau instability process due to a stream of higher-energy particles is responsible for the excitation of the electrostatic waves. Emphasis is placed on electron precipitation since this is observed frequently with ionospheric noise, especially in the auroral zone. The theory seems quite appropriate to explain the origin of polar LHR noise, the original motivation for this work.

The growth rates for the electrostatic waves are obtained by two methods. The first employs the dispersion equation for electrostatic waves (large refractive index) while the second uses the general dispersion equation which reduces to the former for large values of refractive index. This provides a check on the work and also determines the effect of restricting one method to large values of refractive index. The growth rate is obtained directly from the electrostatic dispersion equation. For the full-wave analysis, the growth rate is obtained by calculating the wave energy and the power transfer between the streaming particles and the propagating wave. The second analysis is much more complicated than the first and it was expedient to use a computer to perform the numerical calculations based on the more complex expressions. Most of the growth rate calculations were

for frequencies just above ω_{LH} . However, some calculations were made at frequencies much greater than ω_{LH} . In addition, a few calculations were made just below the lowest ion gyrofrequency which is much less than ω_{LH} .

A short chapter provides more details of LHR noise. It deals with the possible application of the theory presented herein to explain the generation of LHR noise as well as other ionospheric noise.

The theory is a linearized one based on the collisionless Boltzmann equation. The electrostatic analysis is limited to those waves with wave growth rates much smaller than their oscillation frequencies. Relativistic effects are excluded. Although the analyses are quite general concerning the streaming particles, only streaming electrons have been studied in detail. Streaming protons, for example, may also be investigated but this study is beyond the scope of this work. Also beyond the scope of this work is a non-linear analysis.

CHAPTER 2

ELECTROSTATIC WAVE ANALYSIS

2.1 Landau Instabilities in Electrostatic Waves Propagating Nearly Perpendicular to the Background Magnetic Field

Lower hybrid resonance is known to take place in plasma waves propagating perpendicular to the background magnetic field. In a cold plasma which consists of electrons and only one positive ion species, the lower hybrid resonance frequency ω_{LH} is given approximately by the following formula (see Eq. (2-17), p. 32 of the text by Stix (1962) which will be referred to as Ref. I):

$$\frac{1}{\omega_{LH}^2} = \frac{1}{\pi_i^2} + \frac{1}{\Omega_i \Omega_e} \quad (2-1)$$

where π_i is the ion plasma frequency and Ω_i and Ω_e are the ion and electron cyclotron frequencies respectively. Of the two wave modes propagating perpendicular to the background magnetic field, that which is called the extraordinary (X) wave (p. 18, Ref. I) shows resonance as the wave frequency ω approaches the lower hybrid resonance frequency from below; i.e., the square of the refractive index, v^2 , tends to positive infinity. Just above the lower hybrid resonance frequency, the X wave is an imaginary wave; v^2 is negative and tends to negative infinity as the wave frequency ω approaches ω_{LH} from above. (See Eq. (2-15), p. 32, Ref. I. Note that $0 < \omega_{LH}^2 < \omega_L^2 < \omega_{UH}^2 < \omega_R^2$ in that equation.) In

Ref. I (p.p. 223-224), it is shown that if the refractive index v takes a large value, the wave is electrostatic, i.e., the electric field of the wave is almost parallel to the wave number vector $\vec{k}$ and the wave magnetic field is very small. Accordingly, the X wave is an electrostatic wave around the lower hybrid resonance frequency.

As shown by the theory of Landau instabilities (see e.g., p.p. 132-136, Ref. I), an electrostatic wave grows if there is a stream of charged particles which flows in the direction of wave propagation with a mean velocity almost equal to but slightly greater than the wave phase velocity. In the case of an electrostatic wave propagating exactly perpendicular to the background magnetic field, it cannot grow if the streaming particles flow in the direction of the background magnetic field as assumed in this thesis. However, the Landau instability process can take place for an electrostatic wave propagating off the perpendicular direction (if such a wave exists at all).

The condition for the Landau instability process, viz that the mean velocity of the streaming particles in the direction of wave propagation is almost equal to the wave phase velocity, is given as follows (see Fig. 1),

$$\frac{\omega}{k} \approx V_s \cos \theta \quad (2-2)$$

The wave vector $\vec{k}$ with magnitude k is taken in the x-z

Fig. 1. Reference diagram showing the relationship between the direction of the streaming motion of the non-thermal particles and the propagation of the electrostatic wave.

plane with the z axis taken parallel to the background magnetic field $\vec{B}_0$. The angle between the wave number vector $\vec{k}$ and the background magnetic field is denoted by θ and the mean velocity of the streaming particles by V_s . The phase velocity of an electrostatic wave is relatively small as the refractive index is required to take a large value.

The streaming particles, however, might be of relatively higher energy and therefore the above condition for the Landau instability process is satisfied only for smaller values of $\cos \theta$, i.e., only for θ near (but not equal to) 90° . Since the relevant mode of the plasma wave at $\theta = 90^\circ$ is electrostatic around the lower hybrid resonance frequency, waves with θ near 90° are expected to be electrostatic for fre-

quencies which are not too much different from the lower hybrid resonance frequency. Note in the CMA diagram (Fig. 2-1 of Ref. I) that the X mode suffers a destructive transition between regions 10 and 9 and between regions 3 and 2 but undergoes a reshaping transition between regions 8b and 11 when the lower hybrid resonance is crossed (p. 37, Ref. I). It will be shown in this thesis that an electrostatic wave in regions 7 and 8 can grow due to the Landau instability process.

Since the X mode is also electrostatic for frequencies near the upper hybrid resonance frequency, it is expected that the Landau instability process might work for those waves as well and one might also observe upper hybrid resonance noise using an appropriate receiver. This noise band is expected to have a sharp upper frequency cutoff at the upper hybrid resonance frequency (in contrast to LHR noise) because resonance occurs in region 3 of the CMA diagram.

2.2 Basic Equations for Weakly Unstable Electrostatic Waves in Magneto-Active Plasmas

In this section, the dispersion equation for electrostatic waves in the upper ionosphere will be derived. The background plasma is assumed to consist of electrons and three species of positive ions, viz, protons and singly charged helium and oxygen ions. Each of the four species of gases, the electron gas and the three ion gases, is assumed to have a Maxwellian velocity distribution and the same temperature,

denoted by T . The temperature of each gas is assumed to be isotropic; the temperature in the direction along the background magnetic field is thus equal to that in the direction perpendicular to it. In addition to these background gases, it is assumed that there is a gas of nonthermal charged particles of one kind which streams along the direction of the background magnetic field. The mean streaming velocity is denoted by V_s . The distribution function for the streaming particles is also assumed to be Maxwellian with an isotropic temperature T_s as viewed from a reference system moving with the streaming velocity V_s . The number density of each of the five species of gases, the four background gases plus the streaming particles, is assumed to be uniform in space in the unperturbed state. Collisions between particles are ignored. The background magnetic field is assumed to be uniform in space, and its direction is taken along the z axis while the wave vector $\vec{k}$ is taken in the x - z plane.

The dispersion equation for electrostatic waves given in Ref. I (Eq. (9-103), p. 225) holds under conditions more general than in our case. From the conditions mentioned above and Eq. (8-34) of Ref. I (p. 178), the dispersion equation may be written as

$$\begin{aligned}
 k_x^2 + k_z^2 + 2\beta_e^2 \pi_e^2 e^{-\lambda_e} \sum_{n=-\infty}^{\infty} I_n(\lambda_e) \left\{ 1 - 2\alpha_o^{(e)} S(\alpha_n^{(e)}) \right\} \\
 + \sum_{(ions)} 2\beta_j^2 \pi_j^2 e^{-\lambda_j} \sum_{n=-\infty}^{\infty} I_n(\lambda_j) \left\{ 1 - 2\alpha_o^{(j)} S(\alpha_n^{(j)}) \right\}
 \end{aligned}$$

$$\begin{aligned}
& + 2\beta_s^2 \pi_s^2 e^{-\lambda_s} \sum_{n=-\infty}^{\infty} I_n(\lambda_s) \left\{ 1 - 2\alpha_0^{(s)} S(\alpha_n^{(s)}) \right\} \\
& + i 2\beta_e^2 \pi_e^2 e^{-\lambda_e} \sum_{n=-\infty}^{\infty} \pi^{\frac{1}{2}} \frac{k_z}{|k_z|} I_n(\lambda_e) \alpha_0^{(e)} e^{-(\alpha_n^{(e)})^2} \\
& + i \sum_{(\text{ions})} 2\beta_j^2 \pi_j^2 e^{-\lambda_j} \sum_{n=-\infty}^{\infty} \pi^{\frac{1}{2}} \frac{k_z}{|k_z|} I_n(\lambda_j) \alpha_0^{(j)} e^{-(\alpha_n^{(j)})^2} \\
& + i 2\beta_s^2 \pi_s^2 e^{-\lambda_s} \sum_{n=-\infty}^{\infty} \pi^{\frac{1}{2}} \frac{k_z}{|k_z|} I_n(\lambda_s) \alpha_0^{(s)} e^{-(\alpha_n^{(s)})^2} = 0
\end{aligned}
\tag{2-3}$$

This is the dispersion equation for plane electrostatic waves where the wave quantities are assumed to vary as $\exp i(k_x x + k_z z - \omega t)$ with respect to time and space. The same notation as that in Ref. I is used except for β_e , β_j , and β_s , which are defined as follows,

$$\beta_e^2 = \frac{m_e}{2\kappa T}, \quad \beta_j^2 = \frac{m_j}{2\kappa T}, \quad \beta_s^2 = \frac{m_s}{2\kappa T} \tag{2-4}$$

where κ is Boltzmann's constant, and m_e , m_j , and m_s are the masses of the electrons, ions, and streaming particles respectively. The symbols $\alpha_n^{(e)}$, $\alpha_n^{(j)}$, and $\alpha_n^{(s)}$ ($n=0, \pm 1, \pm 2, \dots$) are defined as follows:

$$\alpha_0^{(e)} = \frac{\omega}{k_z} \beta_e, \quad \alpha_n^{(e)} = \frac{\omega + n\Omega_e}{k_z} \beta_e \tag{2-5.1}$$

$$\alpha_o^{(j)} = \frac{\omega}{k_z} \beta_j, \quad \alpha_n^{(j)} = \frac{\omega + n\Omega_j}{k_z} \beta_j \quad (2-5.2)$$

$$\alpha_o^{(s)} = \frac{\omega - k_z V_s}{k_z} \beta_s, \quad \alpha_n^{(s)} = \frac{\omega - k_z V_s + n\Omega_s}{k_z} \beta_s \quad (2-5.3)$$

The notation k_z is for the z component of the wave vector $\vec{k}$. The electron, ion, and streaming particle cyclotron frequencies are taken positive and denoted by Ω_e , Ω_j , and Ω_s ; the corresponding plasma frequencies are represented by Π_e , Π_j , and Π_s . The symbols λ_e , λ_j , and λ_s are defined as follows,

$$\lambda_e = \frac{1}{2} k_x^2 \Omega_e^{-2} \beta_e^{-2}, \quad \lambda_j = \frac{1}{2} k_x^2 \Omega_j^{-2} \beta_j^{-2}, \quad \lambda_s = \frac{1}{2} k_x^2 \Omega_s^{-2} \beta_s^{-2} \quad (2-6)$$

where k_x is the x component of the wave vector.

The third, fourth, and fifth terms in the dispersion equation represent the reactive terms due to the background electron gas, the background ion gases, and the streaming particles respectively. The sixth and seventh terms represent Landau damping ($n = 0$) and cyclotron damping ($n \neq 0$) by the background electron gas and the background ion gases. The last term gives the Landau and cyclotron instabilities or damping due to the streaming charged particles.

In order to simplify Eq. (2-3), the case is considered where the value of the parameter λ is much smaller than 1 for every kind of particle. If λ is much smaller

than 1 for thermal oxygen ions (O^+), it is so for the other thermal particles because λ is proportional to the particle mass. Numerical values of several ionospheric parameters in the auroral zone are given in Table I. If it is assumed that the temperature in the ionospheric region of interest is 1000° K , values of the β parameters for thermal particles are as given in Table II. The numerical values of the cyclotron frequencies are also given in the table. In the calculation of these cyclotron frequencies, the strength of the local magnetic field at an altitude of 1000 km has been assumed to be 0.38 gauss. The corresponding ground magnetic field strength is 0.58 gauss, a typical value in the auroral zone. From the numerical values for β_{O^+} and Ω_{O^+} , it can be shown that the condition $\lambda_{O^+} \ll 1$ is satisfied if the following condition is met,

$$|k_x| \ll 3.18 \times 10^{-3} \text{ cm}^{-1} \quad (2-7)$$

For waves propagating nearly perpendicular to the background magnetic field, $2\pi/|k_x|$ is almost equal to the wavelength ℓ , and the above condition can be rewritten as

$$\ell \gg 19.7 \text{ m} \quad (2-8)$$

Eq. (9-101) of Ref. I (p. 224) can be used with the cold plasma dielectric tensor to give an approximate value for the opposite limit for k_x . For frequencies close to the LHR frequency

TABLE I

Numerical values of ionospheric parameters at
an altitude of 1000 km in the auroral zone.

Strength of the earth's magnetic field = $B_0 = 0.38$ gauss

Number density of thermal electrons = $N_e = 10^4 \text{ cm}^{-3}$

Number density of nonthermal electrons = $N_s = 1 \text{ cm}^{-3}$

Mean streaming velocity of nonthermal electrons = $V_s =$
 $6 \times 10^9 \text{ cm/sec}$

Electron plasma frequency = $\pi_e = 5.64 \times 10^6 \text{ rad/sec}$

Typical value of LHR frequency = $\omega_{LH}/2\pi = 5 \text{ kHz}$

Typical value of the mean ion plasma frequency = $\sqrt{\sum_{\text{ions}} \pi_j^2} =$
 $4.11 \times 10^4 \text{ rad/sec}$

TABLE II

Numerical values of the cyclotron frequencies and the
 β -parameters at an altitude of 1000 km in the auroral zone.

	Cyclotron frequency (rad/sec)	Inverse of the mean thermal velocity ($\text{cm}^{-1}\text{-sec}$)
Electron	$\Omega_e = 6.69 \times 10^6$	$\beta_e = 5.74 \times 10^{-8}$
Proton (H^+)	$\Omega_{H^+} = 3.64 \times 10^3$	$\beta_{H^+} = 2.46 \times 10^{-6}$
Helium (He^+)	$\Omega_{He^+} = 9.15 \times 10^2$	$\beta_{He^+} = 4.92 \times 10^{-6}$
Oxygen (O^+)	$\Omega_{O^+} = 2.29 \times 10^2$	$\beta_{O^+} = 9.84 \times 10^{-6}$

(where the propagation angle is near $\pi/2$) the Stix cold plasma dielectric tensor component P dominates the others, S and D , and the refractive index is given by $v \approx k_x c / \omega$, where c is the speed of light. It can then be seen from Eq. (9-101) that $v \gg 180 \omega_{LH} / \omega$. This leads to

$$|k_x| \gg 1.88 \times 10^{-4} \text{ cm}^{-1} \quad (2-9)$$

and $\lambda \ll 334 \text{ m} \quad (2-10)$

From the above figures it can also be shown that the approximate range for the refractive index is given by

$$180 \ll v\omega / \omega_{LH} \ll 3030 \quad (2-11)$$

It is assumed that the streaming particles are electrons with an average streaming velocity $V_s = 6 \times 10^9 \text{ cm/sec}$ (corresponding to a kinetic energy of 10 keV). This assumption is made simply because electrons with energy of that order are known to be related to various geophysical phenomena. It is not implied that the 10-keV energy is optimum for triggering the instability process under consideration. Although this energy is mildly relativistic, relativistic effects have been neglected in this analysis. The relativistic results can be obtained to a first approximation by making approximate changes in the effective mass of the streaming electrons. In our case we are dealing with the factor $[1 - (V_s/c)^2]^{1/2} = 98\%$,

so that the relativistic correction is small. Also, other kinds of streaming charged particles such as protons are not excluded. If each streaming electron is assumed to have a thermal energy comparable to the kinetic energy of the streaming motion (i.e. 10 keV), the value of β_s is 2.04×10^{-10} cm⁻¹-sec. Then the parameter λ_s is much smaller than 1 if

$$|k_x| \ll 1.93 \times 10^{-3} \text{ cm}^{-1} \quad (2-12)$$

This is approximately satisfied if the condition in equation (2-7) is satisfied.

The contribution from the streaming electrons to the reactive term in the dispersion equation can be ignored since their number density is much smaller than that of the thermal electrons. In this thesis, the number density of the thermal electrons is assumed to be 10^4 cm^{-3} , and that of the streaming electrons 1 cm^{-3} , being comparable to that in the case of a typical auroral display. With this choice of particle densities, the fifth term in equation (2-3) can be shown to be negligible compared to the third term.

The function $I_n(\lambda)$ is a modified Bessel function (see p. 175, Ref. I). For $\lambda \ll 1$, the values of $I_n(\lambda)$ decrease very rapidly for increasing (in absolute value) n , as can be seen from the following formula (Watson, 1922; Whittaker and Watson, 1940; Tearazawa, 1960):

$$I_n(\lambda) = \sum_{m=0}^{\infty} \frac{1}{m! (n+m)!} \left(\frac{\lambda}{2}\right)^{n+2m} \quad (2-13)$$

In equation (2-3), only those terms are retained which are second order or less in λ . Accordingly, the summations with respect to n should be taken up to $|n| = 2$.

The parameter $\alpha_0^{(e)}$ is assumed to be much larger than 1 in absolute value. Since we are interested in frequencies higher than the proton cyclotron frequency and lower than the electron cyclotron frequency, $|\alpha_n^{(e)}|$ for $n \neq 0$ is also much greater than 1 if $|\alpha_0^{(e)}| \gg 1$. Since the frequency range of interest is about 5 - 10 kHz, the condition $|\alpha_0^{(e)}| \gg 1$ is satisfied if

$$|k_z| \ll 1.80 \times 10^{-3} \text{ cm}^{-1} \quad (2-14)$$

The absolute value of the parameter $\alpha_0^{(j)}$ for each ion species is also much greater than 1 if inequality (2-14) is satisfied. This is because the value of α_0 is proportional to the square root of the particle mass.

Since $|\alpha_n| \gg 1$, we expand the function $S(\alpha_n)$ into the following asymptotic series (equation (8-45), p. 180, Ref. 1)

$$S(\alpha_n) = \frac{1}{2 \alpha_n} + \frac{1}{2 \cdot 2 \alpha_n^3} + \frac{1 \cdot 3}{2 \cdot 2 \cdot 2 \alpha_n^5} + \dots \quad (2-15)$$

Since $\omega = \omega_r + i\omega_i$ has an imaginary part which is much smaller (in absolute value) than its real part, so does α_n . Therefore, the function U in the above formula in Ref. I is taken equal to zero. For thermal electrons, $|\alpha_n^{(e)}| \gg 1$ for any n as already mentioned, and the expansion in Eq. (2-15) can always be used no matter what value n may take. For the frequency range of interest, the function $(1 - 2\alpha_0^{(e)} S(\alpha_n^{(e)}))$ is almost equal to 1 except when $n = 0$, and accordingly it is safe to ignore terms with $|n|$ greater than 2 in Eq. (2-3). On the other hand, for thermal ions, $\alpha_n^{(j)}$ with negative n decreases at first as $|n|$ increases. For such an n satisfying

$$\omega_r + n\Omega_j \approx 0 \quad (2-16)$$

$\alpha_n^{(j)}$ could be smaller than 1. Therefore, it must be investigated carefully whether the ion terms with $|n|$ greater than 2 in Eq. (2-3) can be neglected or not. This point has been studied numerically, and it has been found that those terms of higher order are negligible.

Since slowly growing or decaying waves are of interest here ($|\omega_i| \ll \omega_r$), we first consider only the reactive terms in Eq. (2-3), ignoring the reactive term due to the streaming particles (the fifth term in Eq. (2-3)). Regarding $1/\alpha_n^2$ as a first-order term and retaining only those terms which are second-order or less in λ and/or $1/\alpha_n^2$, and then

regarding the damping or instability terms (the last three terms in Eq. (2-3)) as a small perturbation and applying an iterative method, we obtain:

$$\begin{aligned}
 & k_x^2 \left(1 + \sum \frac{\pi^2}{\Omega^2 - \omega_r^2} \right) + k_z^2 \left(1 - \sum \frac{\pi^2}{\omega_r^2} \right) \\
 & - k_x^4 \sum \frac{3\kappa T}{m} \frac{\pi^2}{(\Omega^2 - \omega_r^2)(4\Omega^2 - \omega_r^2)} - k_z^4 \sum \frac{3\kappa T}{m} \frac{\pi^2}{\omega_r^4} \quad (2-17) \\
 & + k_x^2 k_z^2 \sum \frac{\kappa T}{m} \frac{\pi^2 (\Omega^4 - 3\Omega^2 \omega_r^2 + 6\omega_r^4)}{\omega_r^2 (\Omega^2 - \omega_r^2)^3} = 0
 \end{aligned}$$

and

$$\omega_i = - \frac{\omega_r}{2} \frac{\Delta(\omega_r)}{H(\omega_r)} \quad (2-18)$$

where the function $H(\omega_r)$ is given by

$$\begin{aligned}
 H(\omega_r) &= k_x^2 \sum \frac{\pi^2 \omega_r^2}{(\Omega^2 - \omega_r^2)^2} + k_z^2 \sum \frac{\pi^2}{\omega_r^2} \\
 &+ k_x^4 \sum \frac{3\kappa T}{m} \frac{\pi^2 \omega_r^2 (2\omega_r^2 - 5\Omega^2)}{(\Omega^2 - \omega_r^2)^2 (4\Omega^2 - \omega_r^2)^2} + k_z^4 \sum \frac{3\kappa T}{m} \frac{2\pi^2}{\omega_r^4} \quad (2-19) \\
 &+ k_x^2 k_z^2 \sum \frac{\kappa T}{m} \frac{\pi^2 (\Omega^4 + 3\omega_r^4) (4\omega_r^2 - \Omega^2)}{\omega_r^2 (\Omega^2 - \omega_r^2)^4}
 \end{aligned}$$

The summation notation in Eqs. (2-17) and (2-19) indicates that contributions from all species of thermal particles be taken. In our case, there are four terms - one due to thermal electrons and three to the three kinds of thermal ions. The last three terms in Eq. (2-3) are represented by $i\Delta(\omega_r)$ where the argument ω_r indicates that the real part of ω be taken.

For frequencies which are much higher than the ion cyclotron frequencies but much lower than the electron cyclotron frequency, Eqs. (2-17) and (2-19) may be approximated as follows,

$$\begin{aligned}
 & k_x^2 \left(1 - \frac{\pi_e^2}{\Omega_e^2} - \frac{1}{\omega_r^2} \sum_{\text{ions}} \pi_j^2 \right) + k_z^2 \left(1 - \frac{1}{\omega_r^2} (\pi_e^2 + \sum_{\text{ions}} \pi_j^2) \right) \\
 & - k_x^4 \left(\frac{3}{4} \frac{KT}{m_e} \frac{\pi_e^2}{\Omega_e^4} + \frac{1}{\omega_r^4} \sum_{\text{ions}} \frac{3KT}{m_j} \pi_j^2 \right) - k_z^4 \frac{1}{\omega_r^4} \sum \frac{3KT}{m} \pi^2 \\
 & + k_x^2 k_z^2 \left(\frac{KT}{m_e} \frac{\pi_e^2}{\omega_r^2 \Omega_e^2} - \frac{1}{\omega_r^4} \sum_{\text{ions}} \frac{6KT}{m_j} \pi_j^2 \right) = 0
 \end{aligned} \tag{2-20}$$

and

$$H(\omega_r) = k_x^2 \left(1 + \frac{\pi_e^2}{\Omega_e^2} \right) + k_z^2$$

$$\begin{aligned}
& + k_x^4 \left(-\frac{3}{4} \frac{\kappa T}{m_e} \frac{\pi_e^2}{\Omega_e^4} + \frac{1}{\omega_r^4} \sum_{\text{ions}} \frac{3\kappa T}{m_j} \pi_j^2 \right) + k_z^4 \frac{1}{\omega_r^4} \sum \frac{3\kappa T}{m} \pi^2 \\
& + k_x^2 k_z^2 \frac{1}{\omega_r^4} \sum_{\text{ions}} \frac{6\kappa T}{m_j} \pi_j^2
\end{aligned} \tag{2-21}$$

It can be seen that $H(\omega_r)$ is always positive. The only negative term on the right-hand side of the above equation is much smaller than the first term because of the condition $\lambda_e \ll 1$.

In Eq. (2-20), each of the last three terms is proportional to the temperature T and is much smaller than the other terms. If these small terms are ignored, the dispersion equation (2-20) can be simplified as follows,

$$\omega_r^2 = \frac{k_x^2 \sum_{\text{ions}} \pi_j^2 + k_z^2 (\pi_e^2 + \sum_{\text{ions}} \pi_j^2)}{\left(1 + \frac{\pi_e^2}{\Omega_e^2}\right) k_x^2 + k_z^2} \tag{2-22}$$

The terms which are proportional to the temperature T are also much smaller compared to the other terms in Eq. (2-21) and therefore the expression for the function $H(\omega_r)$ can be

simplified to give,

$$H(\omega_r) = \left(1 + \frac{\pi_e^2}{\Omega_e^2}\right) k_x^2 + k_z^2 \quad (2-23)$$

2.3 Electron Plasma Waves Propagating along the Background Magnetic Field

In the case of longitudinal plasma waves propagating parallel to the background magnetic field, the magnetic field has no effect on the motion of charged particles, and therefore the solution obtained in the previous section should reduce to that for electrostatic waves in the absence of the background magnetic field.

Let us assume that the ions are infinitely heavy in order to consider electrostatic waves in an electron plasma. Then the following result from Eq. (2-17) is obtained,

$$\frac{\omega_r^2}{k_z^2} = \frac{\pi_e^2}{k_z^2} + \frac{3\kappa T}{m_e} \frac{\pi_e^2}{\omega_r^2} \quad (2-24)$$

Since the second term on the right-hand side of this equation is much smaller than the first ($|\alpha_0^{(e)}| \gg 1$), ω_r in the second term can be replaced by π_e^2 , resulting in the well-known formula for electron plasma oscillations

(see, for example, Eq. (3-17) in the text by Spitzer (1962)).

The function defined by Eq. (2-19) becomes as follows in the present case,

$$H(\omega_r) = k_z^2 \frac{\pi_e^2}{\omega_r^2} + k_z^4 \frac{6\kappa T}{m_e} \frac{\pi_e^2}{\omega_r^4} \quad (2-25)$$

The second term on the right-hand side is much smaller than the first. The frequency ω_r is almost equal to π_e as can be seen in Eq. (2-24). The function $H(\omega_r)$ is, therefore, almost equal to k_z^2 . In the calculation of $\Delta(\omega_r)$, the contribution from the ions (the seventh term in Eq. (2-3)) is ignored. In the sixth and last terms, it can be seen that only the terms with $n = 0$ are nonzero. Then finally the growth rate ω_i is obtained as follows,

$$\omega_i = -\pi^{\frac{1}{2}} \frac{\omega_r}{k_z |k_i|} \left\{ \pi_e^2 \beta_e^3 v_z e^{-\beta_e^2 v_z^2} + \pi_s^2 \beta_s^3 (v_z - V_s) e^{-\beta_s^2 (v_z - V_s)^2} \right\}_{v_z = \frac{\omega_r}{k_z}} \quad (2-26)$$

This expression for the growth rate can also be derived from the formula given in Ref. I (equation (14), p. 136). In our case, the electron distribution function $f(v)$ in Ref. I is given by

$$f(v_z) = \frac{1}{N_e + N_s} \left\{ \frac{N_e \beta_e}{\pi^{\frac{1}{2}}} e^{-\beta_e^2 v_z^2} + \frac{N_s \beta_s}{\pi^{\frac{1}{2}}} e^{-\beta_s^2 (v_z - V_s)^2} \right\} \quad (2-27)$$

The plasma frequency π_e^2 in the formula in Ref. I should be replaced by $\pi_e^2 + \pi_s^2$ in our notation. With these substitutions, it can be seen that the formula given in Ref. I leads to the expression given in Eq. (2-26).

2.4 Electrostatic Waves Excited around the LHR Frequency

In the case of propagation perpendicular to the background magnetic field ($k_z = 0$), the dispersion equation for frequencies which are much higher than the ion cyclotron frequencies but much lower than the electron cyclotron frequency becomes,

$$1 + \frac{\pi_e^2}{\Omega_e^2} - \frac{1}{\omega_r^2} \sum_{ions} \pi_j^2 - k_x^2 \left(\frac{3}{4} \frac{\kappa T}{m_e} \frac{\pi_e^2}{\Omega_e^4} + \frac{1}{\omega_r^4} \sum_{ions} \frac{3\kappa T}{m_j} \pi_j^2 \right) = 0 \quad (2-28)$$

The last term in this equation is much smaller than the second and third terms. If the last term is ignored, it can be seen that the frequency ω_r determined from the above equation (or alternatively from Eq. (2-22)) is equal to the particular frequency ω_{LH} defined as follows,

$$\frac{1}{\omega_{LH}^2} = \frac{1}{\sum_{ions} \pi_j^2} \left(1 + \frac{\pi_e^2}{\Omega_e^2} \right) \quad (2-29)$$

This equation gives an approximate expression for the lower hybrid resonance frequency and can be rewritten in the form given by Eq. (6) in the paper by Brice and Smith (1965) by

using the wave frequency f in Hz instead of the angular frequency ω_r in rad/sec.

If the last term in Eq. (2-28) is taken into account, the frequency ω_r is given approximately by

$$\omega_r \cong \omega_{LH} + \omega' \quad (2-30)$$

where the correction term ω' is given by

$$\frac{\omega'}{\omega_{LH}} = \frac{1}{2} \frac{k_x^2}{\sum_{ions} \Pi_j^2} \left(\frac{3KT}{4m_e} \frac{\Pi_e^2 \omega_{LH}^2}{\Omega_e^4} + \frac{1}{\omega_{LH}^2} \sum_{ions} \frac{3KT}{m_j} \Pi_j^2 \right) \quad (2-31)$$

In the case when k_z is not zero but very small, i.e., the propagation is slightly off the perpendicular direction, the frequency ω_r is determined from Eq. (2-20) and is given approximately by

$$\omega_r \cong \omega_0 + \omega'' \quad (2-32)$$

where the frequency ω_0 is the solution of Eq. (2-28) (the case of perpendicular propagation) and is approximately equal to the right-hand side of Eq. (2-30). The correction term ω'' is given by

$$\frac{\omega''}{\omega_0} = \frac{k_x^2}{k_x^2} \frac{-1 + \frac{1}{\omega_0^2} \left(\Pi_e^2 + \sum_{ions} \Pi_j^2 - k_x^2 \frac{KT}{m_e} \frac{\Pi_e^2}{\Omega_e^2} \right) + \frac{1}{\omega_0^4} \left(k_x^2 \sum_{ions} \frac{3KT}{m} \Pi^2 + k_x^2 \sum_{ions} \frac{6KT}{m_j} \Pi_j^2 \right)}{2 \sum_{ions} \frac{\Pi_j^2}{\omega_0^2} + \frac{4}{\omega_0^4} k_x^2 \sum_{ions} \frac{3KT}{m_j} \Pi_j^2} \quad (2-33)$$

The right-hand side of Eq. (2-33) is seen to be always positive. Inside the bracket of the second term in the numerator, the last term is much smaller than the first term, π_e^2 , because of the condition $\lambda_e \ll 1$. The second term is also much smaller than the first term. Since π_e , the electron plasma frequency, is much higher than the lower hybrid resonance frequency ω_{LH} , the second term in the numerator which is approximately equal to π_e^2/ω_{LH}^2 is much greater than the first term, -1. Equation (2-33) can be considerably simplified by ignoring the smaller terms which are proportional to the temperature T , giving the following result,

$$\frac{\omega''}{\omega_{LH}} \approx \frac{1}{2} \frac{k_z^2}{k_x^2} \frac{\pi_e^2}{\sum_{ions} \pi_j^2} = \frac{1}{2} \frac{\pi_e^2}{\sum_{ions} \pi_j^2} \cot^2 \theta \quad (2-34)$$

This equation can also be derived from Eq. (2-22) on the following condition,

$$\cot^2 \theta \ll \left(\sum_{ions} \pi_j^2 \right) / \pi_e^2 \quad (2-35)$$

In our case, the values in Table I lead to $89^\circ 35' < \theta < 90^\circ 25'$. Since ω'' is positive for $k_z \neq 0$, the frequency ω_r is greater than ω_0 ($\cong \omega_{LH}$) for off-perpendicular propagation ($\theta \neq \frac{\pi}{2}$), and tends to ω_0 as the direction of propagation approaches the perpendicular direction ($\theta \rightarrow \frac{\pi}{2}$). The frequency ω_r increases very rapidly as the propagation

angle θ deviates from the perpendicular direction, since π_e^2 is much larger than $\sum_{\text{ions}} \pi_j^2$.

In Eq. (2-18) which gives the growth rate, the function $H(\omega_r)$ is always positive, and therefore $\Delta(\omega_r)$ should be negative in order that the growth rate ω_i be positive. As mentioned already, the function $\Delta(\omega_r)$ consists of three parts, $\Delta_1(\omega_r)$, $\Delta_2(\omega_r)$, and $\Delta_3(\omega_r)$ which correspond to the seventh, eighth and ninth terms in Eq. (2-3) respectively. The functions $\Delta_1(\omega_r)$ and $\Delta_2(\omega_r)$ are always positive and only the function $\Delta_3(\omega_r)$, the contribution from the streaming particles, can be negative. The function $\Delta_3(\omega_r)$ is given by

$$\Delta_3(\omega_r) = 2\beta_s^3 \pi_s^2 e^{-\lambda_s} \sum_{n=-\infty}^{\infty} \pi^{\frac{1}{2}} \frac{k_z}{|k_z|} I_n(\lambda_s) \frac{\omega_r - k_z V_s}{k_z} \exp \left\{ -\beta_s^2 \left(\frac{\omega_r - k_z V_s + n\Omega_s}{k_z} \right)^2 \right\} \quad (2-36)$$

As can be seen from the exponential factor (inside the summation sign), a representative term is very small unless the following condition is satisfied,

$$\omega_r - k_z V_s + n\Omega_s \approx 0 \quad (2-37)$$

For $n = 0$, this becomes

$$\omega_r - k_z V_s \approx 0 \quad (2-38)$$

which gives the same condition as Eq. (2-2). For frequencies near the lower hybrid resonance frequency ω_{LH} , the value of k_z which satisfies this condition is approximately given by

$$k_z \approx 5.24 \times 10^{-6} \text{ cm}^{-1} \quad (2-39)$$

if we take $\omega_r/2\pi \approx \omega_{LH}/2\pi = 5 \text{ kHz}$ and $V_s = 6 \times 10^9 \text{ cm/sec}$. The value of k_z is within the limit imposed by inequality (2-14). For $n = 1$, the value of k_z to satisfy the condition in Eq. (2-37) is about $1.12 \times 10^{-3} \text{ cm}^{-1}$ which is barely within the limit given by inequality (2-14). For $|n| \geq 2$, there are no values of k_z within the limit set by condition (2-14). In order that the electrostatic wave take a frequency near the lower hybrid resonance frequency, k_x^2 must be much larger than k_z^2 as seen from Eq. (2-34). For a given value of k_z^2 , k_x^2 is smaller for a higher wave frequency. For a frequency about twice as large as the lower hybrid resonance frequency (viz, $\omega_r \approx 2\omega_{LH}$; such a frequency may even be larger than the upper frequency limit of an LHR noise band), the value of $|k_x|$ is about $6.44 \times 10^{-4} \text{ cm}^{-1}$ for $k_z \approx 1.05 \times 10^{-5} \text{ cm}^{-1}$ (the case for $n=0$) and $7.00 \times 10^{-2} \text{ cm}^{-1}$ for $k_z \approx 1.14 \times 10^{-3} \text{ cm}^{-1}$ (the case for $n=1$). The value of k_x in the latter case is far outside the limit imposed by inequality (2-7). Therefore, only the term with $n=0$ is important in the function $\Delta_3(\omega_r)$.

Using the condition that $\lambda_s \ll 1$ and retaining the $n=0$ term only, we obtain

$$\Delta_3(\omega_r) \approx 2\pi^{\frac{1}{2}} \beta_s^3 \Pi_s^2 \frac{\omega_r - k_z V_s}{|k_z|} \exp\left\{-\beta_s^2 \left(\frac{\omega_r - k_z V_s}{k_z}\right)^2\right\} \quad (2-40)$$

In order that $\Delta_3(\omega_r)$ be negative (the growth rate be positive), the following condition must be met,

$$\omega_r - k_z V_s < 0 \quad (2-41)$$

Equation (2-38) and the above inequality show that to cause the instability process, the projection of the streaming velocity of the nonthermal particles in the direction of wave propagation should be in the same sense as the wave phase velocity (i.e., V_s and ω_r/k_z should have the same sign) and the projected streaming velocity $V_s \cos \theta$ should be almost equal to but slightly greater than the wave phase velocity.

From expression (2-23) for the function $H(\omega_r)$ and Eq. (2-40), the growth rate ω_i is approximately given by

$$\omega_i \approx \pi^{\frac{1}{2}} \Pi_s^2 \omega_r \frac{\frac{k_z V_s - \omega_r}{|k_z|} \beta_s^3 \exp\left\{-\beta_s^2 \left(V_s - \frac{\omega_r}{k_z}\right)^2\right\}}{\left(1 + \frac{\Pi_s^2}{\Omega_e^2}\right) k_x^2 + k_z^2} \quad (2-42)$$

The inequality sign is used since the contributions from $\Delta_1(\omega_r)$ and $\Delta_2(\omega_r)$ to ω_i are negative. As far as frequencies near the lower hybrid resonance frequency are concerned, $\Delta_1(\omega_r)$, the contribution from the thermal electrons, may be unimportant since the lower hybrid resonance frequency is much smaller than the electron cyclotron frequency. The quantity $\Delta_1(\omega_r)$ is significant only for a frequency ω_r that is near the electron cyclotron frequency or its harmonics. However, the quantity $\Delta_2(\omega_r)$, the contribution from the thermal ions, could be significant since the frequency ω_r could take a value at a certain harmonic of an ion cyclotron frequency. Generally, the contribution is not significant. If the temperature of each thermal background gas is assumed to be zero, both $\Delta_1(\omega_r)$ and $\Delta_2(\omega_r)$ are zero, and the inequality sign can be dropped in expression (2-42).

For the purpose of numerical calculation, it is convenient to rewrite expression (2-42) in a dimensionless form. The unit for frequencies is taken as ω_{LH} , the lower hybrid resonance frequency. The unit for wave numbers is defined as follows,

$$k_o = \frac{\omega_{LH}}{V_s} \quad (2-43)$$

Expression (2-42) can then be rewritten as

$$\omega_i' \approx \frac{\pi^{\frac{1}{2}} \Pi_s'^2 \left(\frac{3}{2} \phi^2\right)^{\frac{3}{2}} \omega_r'}{\left(1 + \frac{\Pi_e'^2}{\Omega_e'^2}\right) \tan^2 \theta + 1} \frac{V_s}{|V_s|} \frac{k_z'}{|k_z'|} \frac{1 - \frac{\omega_r'}{k_z'}}{k_z'^2} \exp\left\{-\frac{3}{2} \phi^2 \left(1 - \frac{\omega_r'}{k_z'}\right)^2\right\} \quad (2-44)$$

where $\omega_r' = \omega_r / \omega_{LH}$ $\omega_i' = \omega_i / \omega_{LH}$ (2-45.1)

$\Pi_e' = \Pi_e / \omega_{LH}$, $\Omega_e' = \Omega_e / \omega_{LH}$, $\Pi_s' = \Pi_s / \omega_{LH}$ (2-45.2)

and $k_x' = k_x / k_0$ $k_z' = k_z / k_0$ (2-46)

and the dimensionless quantity $\beta_s V_s$ has been expressed in terms of a parameter ϕ^2 which is the ratio of the kinetic energy in the streaming motion to the thermal energy of the streaming electrons,

$$\beta_s^2 V_s^2 = 3/2 \phi^2 \quad (2-47)$$

and $\tan^2 \theta = k_x^2 / k_z^2$ is obtained from Eq. (2-22). From this equation, the relationship between the frequency ω_r' and the angle θ has been graphed in Fig. 2 for our case.

Differentiation of Eq. (2-44) with respect to k_z' shows that for given values of frequency ω_r' and ϕ , the maximum and minimum growth rates occur when k_z' satisfies

$$k_z'^3 + \frac{3}{2} \omega_r' (\phi^2 - 1) k_z'^2 - 3 \phi^2 \omega_r'^2 k_z' + \frac{3}{2} \phi^2 \omega_r'^3 = 0 \quad (2-48)$$

Fig. 2. Frequency dependence of the electrostatic waves on the angle of propagation θ near ω_{LH} , the angular frequency of the lower hybrid resonance. The dimensionless frequency is defined by $\omega'_r = \omega_r / \omega_{LH}$, where ω_r is the real part of the complex angular frequency ω .

Calculations were made for ω'_i where the values of the constants used are those given in Section 2-2. Figure 3 shows the growth rate ω'_i plotted as a function of the parallel component of the wave number vector, k'_z , for three values of frequency ω'_r . The value of the parameter ϕ is unity, that is, the kinetic energy in the streaming motion and the thermal energy of the streaming particles both equal 10 keV. Note the change in scale of ω'_i for positive and negative k'_z . It is clear from the curves that damping occurs ($\omega'_i < 0$) when $k'_z < \omega'_r$. In this damping region, ω'_i has two minima and passes through zero at $k'_z = 0$. For $k'_z > \omega'_r$, the growth rate increases to a maximum and then decreases rapidly. A

Fig. 3. Growth rate vs. parallel wave number for $\phi = 1$.

Note the change in scale for $k_z' < 0$. See Eqs. (2-43), (2-45), and (2-46) for definitions of the dimensionless co-ordinates. The parameter ϕ^2 is the ratio of the kinetic energy in the streaming motion to the thermal energy of the streaming electrons.

general feature is the increasing magnitude of the maximum and the minima values of ω'_i for increasing frequency.

Figure 4 is a similar plot of the growth rate ω'_i versus k'_z except that the parameter ϕ is varied with $\omega'_r = 1.1$. For negative k'_z , the magnitude of ω'_i diminishes rapidly with increasing ϕ . In the region of positive k'_z , it is clear that the larger the value of ϕ , the larger is the maximum value of ω'_i and the narrower the wave number bandwidth. When $\phi = 1$, the maximum value of ω'_i is 8.04×10^{-6} at $k'_z = 1.523$. Then from Eq. (2-22) is obtained $k'_x = 568$. This gives $k_x = 2.98 \times 10^{-3} \text{ cm}^{-1}$ which is just within the limit imposed by inequality (2-7) but just outside that given by inequality (2-12). The upper bound of the maximum growth rate is 0.253 sec^{-1} and the growth time (the e-folding time of the wave amplitude) is about 3.96 sec. When $\phi^2 = 10$, the maximum value of ω'_i is 1.53×10^{-4} at $k'_z = 1.290$. Also, $k_x = 2.53 \times 10^{-3} \text{ cm}^{-1}$ and the maximum growth rate is 4.81 sec^{-1} , about twenty times greater than in the previous case with $\phi = 1$. Finally, when $\phi^2 = 100$, the following values are obtained: the maximum value of $\omega'_i = 1.95 \times 10^{-3}$, $k'_z = 1.163$, $k_x = 2.28 \times 10^{-3} \text{ cm}^{-1}$, and a maximum growth rate of 61.3 sec^{-1} . For $\phi \neq 1$, equation (2-12) must be modified to $|k_x| \ll 1.93 \times 10^{-3} \phi$. Thus, when ϕ^2 is 10 or 100, the value of k_x is within the limit imposed by both inequalities (2-7) and (2-12), where the latter has been appropriately modified.

Fig. 4. Growth rate vs. parallel wave number for $\omega_r' = 1.1$.

Note change in scale for negative values of k_z' . Curves are not drawn for $\phi^2 = 10$ and 100 when $k_z' < 0$ since values of ω_i' are negligible here.

The above calculations show that the growth times for frequencies near ω_{LH} are in the order of $1 - 10^{-2}$ sec for values between 1 and 100 of ϕ^2 , the ratio of the streaming kinetic energy to the thermal energy of the streaming electrons. If the value of ϕ^2 is much greater than unity, then the growth time is very small.

From equations (2-34) and (2-42) and Fig. 3, it can be seen that as the wave frequency ω_r tends to ω_{LH} , the lower hybrid resonance frequency, the growth rate ω_i tends to zero. Also, from Eq. (2-22) can be obtained the upper limiting value of ω_r' for electrostatic waves. The frequency ω_r increases monotonically with decreasing θ , giving at $\theta = 0$, $\omega_r^2 = \pi_e^2 + \sum_{\text{ions}} \pi_j^2$ so that we obtain

$$\omega_{\text{LH}}^2 \leq \omega_r^2 \leq \pi_e^2 + \sum_{\text{ions}} \pi_j^2 \quad (2-49.)$$

for the frequency range of the electrostatic waves. The maximum growth rate given by Eqs. (2-44) and (2-48) increases with frequency ω_r and near the upper limit at $\theta = 0$ takes a value of the order of $\omega_i' \approx 10^{-2}$ for $\phi^2 = 1$. It should be mentioned that in the derivation of these equations it had been assumed that $\omega_r^2 \ll \Omega_e^2$, so that care must be taken if π_e is of the same order as Ω_e (as it is in our case).

A basic limitation to the validity of the growth rate expression given by Eq. (2-44) is that the refractive index must be large. But Eqs. (2-22) (or (2-34)) and Fig. 2

show that the wave frequency ω_r becomes smaller as the propagation angle θ is larger. To meet the Landau instability condition (Eq. (2-2)), therefore, the wave phase velocity should be larger (the refractive index v smaller) for a higher wave frequency ω_r (holding the streaming velocity V_s constant). If the refractive index v becomes small enough, the wave may no longer be considered electrostatic.

CHAPTER 3

FULL-WAVE ANALYSIS

3.1 Power Absorption in a Collisionless Plasma

The previous chapter has indicated that an extension in the analysis would be useful where no restrictions are placed on the refractive index v . This extension may be made by employing the generalized dispersion equation instead of the electrostatic one. This generalized equation is then used to obtain the wave electric field which is then inserted into the growth rate expression; the growth rate is a function of the power transfer and the wave energy. The restrictions in the electrostatic analysis that λ and $1/\alpha_n$ are much less than unity are removed in the full-wave analysis.

The assumptions are similar to those made in the previous chapter. A tenuous stream of particles impinges upon a background plasma. All species of particles have a Maxwellian velocity distribution, albeit with various parallel and perpendicular temperatures, $T_{\parallel}$ and $T_{\perp}$, and a parallel drift velocity V_s in the case of the streaming particles. As before, the number densities of the background electrons and positive ions plus the streaming particles are assumed to be uniform in space in the unperturbed state; the background magnetic field B_0 is assumed uniform in space also.

The power absorption by the particles per unit volume P_a is found by an expression in Ref. I (Eq. (9-54), p. 205),

$$P_a = \sum_k P_k = \sum_k \left[\frac{N_k Z_k e c \epsilon}{4 B_0} (\vec{E} \cdot \vec{M}^* \cdot \vec{E}^* + \vec{E}^* \cdot \vec{M} \cdot \vec{E}) \right]_k \quad (3-1)$$

where c is the velocity of light, N_k is the number density of particles with charge of magnitude $Z_k e$, and ϵ_k is the sign of the charge, ± 1 . $\vec{M}$ is the mobility tensor and $\vec{E}$ is the wave electric field; parameters appropriate to particles of the k th type are to be used in evaluating $\vec{M}$. The summation is over all plasma components and the streaming particles. Equation (3-1) may be expanded and expressed in terms of the components of the wave electric field and mobility tensor as,

$$\begin{aligned} P_k = \sum_{n=-\infty}^{\infty} P_{kn} &= \sum_{n=-\infty}^{\infty} \frac{N_k Z_k e c \epsilon}{4 B_0} \left[|E_x|^2 (M_{xx}^n + cc) \right. \\ &+ |E_y|^2 (M_{yy}^n + cc) + |E_z|^2 (M_{zz}^n + cc) \\ &+ (E_y E_x^* - E_x E_y^*) (M_{xy}^n - cc) \\ &+ (E_x E_z^* + E_x^* E_z) (M_{xz}^n + cc) \\ &\left. + (E_z E_y^* - E_z^* E_y) (M_{yz}^n - cc) \right] \quad (3-2) \end{aligned}$$

where the asterisk denotes the complex conjugate and cc denotes the complex conjugate of the companion term within the brackets. The subscript k has been omitted for simplicity of notation. The components of the tensor $\vec{M}^n$ are

given by (see Eq. (9-8), p. 188, Ref. I),

$$M_{xx}^n = - \frac{\Omega \epsilon \kappa T_{\perp}}{2mk_z} A_{xx}^n \langle \psi \rangle_n$$

$$M_{yy}^n = - \frac{\Omega \epsilon \kappa T_{\perp}}{2mk_z} A_{yy}^n \langle \psi \rangle_n$$

$$M_{zz}^n = - \frac{\Omega \epsilon}{2k_z} A_{zz}^n [n \langle v_z \phi \rangle_n - \langle v_z \psi \rangle_n] \quad (3-3)$$

$$M_{xy}^n = \frac{i \Omega \kappa T_{\perp}}{2mk_z} A_{xy}^n \langle \psi \rangle_n$$

$$M_{xz}^n = - \frac{\epsilon \kappa T_{\perp} k_x}{2mk_z} A_{xz}^n \langle v_z \psi \rangle_n$$

$$M_{yz}^n = - \frac{i \kappa T_{\perp} k_x}{2mk_z} A_{yz}^n \langle v_z \psi \rangle_n$$

$$M_{yx}^n = - M_{xy}^n$$

$$M_{zx}^n = M_{xz}^n$$

$$M_{zy}^n = - M_{yz}^n$$

where Ω is the cyclotron frequency, m the mass, k_x and

k_z the x- and z-components of the wave vector $\vec{k}$, κ the Boltzmann constant, and v_z the particle velocity component along the z-direction. We have defined

$$A_{xx}^n = 2 e^{-\lambda} I_n \frac{n^2}{\lambda}$$

$$A_{yy}^n = 2 e^{-\lambda} \left[\left(\frac{n^2}{\lambda} + 2\lambda \right) I_n - 2\lambda I_n' \right]$$

$$A_{zz}^n = -2 e^{-\lambda} I_n$$

(3-4)

$$A_{xy}^n = -2 e^{-\lambda} n [I_n - I_n']$$

$$A_{xz}^n = -2 e^{-\lambda} \frac{n}{\lambda} I_n$$

$$A_{yz}^n = 2 e^{-\lambda} [I_n - I_n']$$

where I_n and I_n' are the modified Bessel function of the first kind and its derivative. The formula for $I_n(\lambda)$ has already been given in Eq. (2-13). Since, in this section, we allow arbitrary λ , it is useful to have the asymptotic expansion for $I_n(\lambda)$. When $-3\pi/2 < \arg \lambda < \pi/2$ we have (p. 373, Whittaker and Watson, 1940)

$$I_n(\lambda) \sim \frac{e^\lambda}{\sqrt{2\pi\lambda}} \left[1 + \sum_{r=1}^{\infty} (-1)^r \frac{[4n^2-1^2][4n^2-3^2]\cdots[4n^2-(2r-1)^2]}{r! (8\lambda)^r} \right]$$

$$+ \frac{e^{-(n+\frac{1}{2})\pi i} e^{-\lambda}}{\sqrt{2\pi\lambda}} \left[1 + \sum_{r=1}^{\infty} \frac{[4n^2-1^2][4n^2-3^2]\cdots[4n^2-(2r-1)^2]}{r! (8\lambda)^r} \right] \quad (3-5)$$

where the second series is negligible when $|\arg \lambda| < \pi/2$. In the computer calculations of Sec. 3.3 a subroutine (BESIK) provided by the UBC Computing Centre was used to evaluate $I_n(\lambda)$. For values of λ greater than 88.029692 this subroutine cannot be used but another subroutine can be written based on the asymptotic formula of Eq. (3-5). It was not found necessary to write this second subroutine at this time. Some recurrence relations for the modified Bessel function are:

$$I_{n-1} - I_{n+1} = \frac{2n}{\lambda} I_n(\lambda) \quad (3-6)$$

$$\lambda I_n'(\lambda) + n I_n(\lambda) = \lambda I_{n-1}(\lambda) \quad (3-7)$$

The general expression for the bracketed factors in the mobility tensor components in Eq. (3-3) are given in Ref. I (Eqs. (9-12) and (9-13), p.p. 190-191). We are interested here in some of the relations which may be reduced after some algebraic manipulation:

$$\langle \theta \rangle_n + cc = - \frac{4\sqrt{\pi} k_z}{\omega |k_z|} \left(\frac{m}{2\kappa T_{||}} \right)^{\frac{3}{2}} \left[\omega - k_z V_s + n\Omega \left(1 - \frac{T_{||}}{T_{\perp}} \right) \right] \exp(-\alpha_n^2) \quad (3-8)$$

$$\langle v_z \Theta \rangle_n + cc = - \frac{4\sqrt{\pi}}{\omega |k_z|} \left(\frac{m}{2\kappa T_{||}} \right)^{\frac{3}{2}} (\omega + n\Omega) \left[\omega - k_z V_s + n\Omega \left(1 - \frac{T_{||}}{T_{\perp}} \right) \right] \exp(-\alpha_n^2) \quad (3-9)$$

$$n \langle v_z \Phi \rangle_n - \langle v_z \Psi \rangle_n + cc = \frac{4\sqrt{\pi}}{k_z |k_z|} \left(\frac{m}{2\kappa T_{||}} \right)^{\frac{3}{2}} \frac{(\omega + n\Omega)^2}{\omega} \left[\omega - k_z V_s + n\Omega \left(1 - \frac{T_{||}}{T_{\perp}} \right) \right] \exp(-\alpha_n^2) \quad (3-10)$$

where V_s is the macroscopic streaming velocity in the z -direction. It can be easily shown that Eqs. (3-3), (3-8), (3-9), and (3-10) lead to:

$$M_{xx}^n + cc = C A_{xx}^n$$

$$M_{yy}^n + cc = C A_{yy}^n$$

$$M_{zz}^n + cc = - C A_{zz}^n \frac{m}{\kappa T_{||}} \left(\frac{\omega + n\Omega}{k_z} \right)^2$$

(3-11)

$$M_{xy}^n - cc = - i \epsilon C A_{xy}^n$$

$$M_{xz}^n + cc = C A_{xz}^n \frac{k_x}{k_z} \frac{\omega + n\Omega}{\Omega}$$

$$M_{yz}^n - cc = i \epsilon C A_{yz}^n \frac{k_x}{k_z} \frac{\omega + n\Omega}{\Omega}$$

where we have defined for convenience,

$$C = 2 \sqrt{\pi} \frac{\Omega \epsilon \kappa T_{\perp}}{\omega m |k_z|} \left(\frac{m}{2 \kappa T_{\parallel}} \right)^{\frac{3}{2}} \left[\omega - k_z V_s + n \Omega \left(1 - \frac{T_{\parallel}}{T_{\perp}} \right) \right] \exp(-\alpha_n^2) \quad (3-12)$$

Inserting the expressions (3-11) and (3-12) into Eq. (3-2) leads to a simple expression for the power absorption per unit volume by particles of the k th species.

$$P_{kn} = M_n \exp(-\alpha_n^2) \quad (3-13)$$

where

$$M_n = \frac{\pi^2}{16 \sqrt{\pi} |k_z| \omega} \left(\frac{m}{2 \kappa T_{\parallel}} \right)^{\frac{1}{2}} |E_x|^2 \frac{T_{\perp}}{T_{\parallel}} \left[\omega - k_z V_s + n \Omega \left(1 - \frac{T_{\parallel}}{T_{\perp}} \right) \right] G_n \quad (3-14)$$

and

$$\begin{aligned} G_n = & A_{xx}^n + \frac{|E_y|^2}{|E_x|^2} A_{yy}^n - \frac{|E_z|^2}{|E_x|^2} \frac{A_{zz}^n}{\lambda} \left[\frac{k_x}{k_z} \frac{\omega + n \Omega}{\Omega} \right]^2 \\ & - \frac{E_y E_x^* - E_x E_y^*}{|E_x|^2} i \epsilon A_{xy}^n + \frac{E_x E_z^* + E_x^* E_z}{|E_x|^2} A_{xz}^n \frac{k_x}{k_z} \frac{\omega + n \Omega}{\Omega} \\ & + \frac{E_z E_y^* - E_y E_z^*}{|E_x|^2} i \epsilon A_{yz}^n \frac{k_x}{k_z} \frac{\omega + n \Omega}{\Omega} \end{aligned} \quad (3-15)$$

Thus, summing P_{kn} over n and k yields the total power absorption per unit volume by all charged particles, P_a .

3.2 Wave Energy and Electric Field

The energy W of the wave may be obtained from Eq. (3-7) of Ref. I (p. 48)

$$W = \frac{1}{16\pi} [\vec{B}^* \cdot \vec{B} + \vec{E}^* \cdot \frac{\partial}{\partial \omega} [\omega \vec{K}_h] \cdot \vec{E}] \quad (3-16)$$

where $\vec{B}$ and $\vec{E}$ are the complex Fourier amplitudes of the wave magnetic and electric fields and $\vec{K}_h$ is the Hermitian part of the dielectric tensor.

The magnetic field components may be expressed in terms of the electric field components by the Maxwell relation

$$\nabla \times \vec{E} = - \frac{1}{c} \frac{\partial \vec{B}}{\partial t} \quad (3-17)$$

which becomes after Fourier analysis in space and time

$$i \vec{k} \times \vec{E} = i \frac{\omega}{c} \vec{B} \quad (3-18)$$

Thus we have

$$\frac{\omega}{c} B_x = - k_z E_y$$

$$\frac{c}{\epsilon} B_y = k_z E_x - k_x E_z \quad (3-19)$$

$$\frac{c}{\epsilon} B_z = k_x E_y$$

since we have chosen a co-ordinate system such that $k_y = 0$, without loss of generality. Therefore the first term in the brackets of Eq. (3-16) becomes

$$\vec{B}^* \cdot \vec{B} = v_z^2 |E_x|^2 + v^2 |E_y|^2 + v_x^2 |E_z|^2 - v_x v_z (E_z^* E_x + E_z E_x^*) \quad (3-20)$$

where v_x and v_z are the x- and z-components of the refractive index $\vec{v} = \vec{k}c/\omega$.

The expressions for the power absorption P_a and the wave energy W may be evaluated once the wave electric field components are determined. These components may be obtained from Eq. (9-18) of Ref. I (p. 193),

$$\begin{bmatrix} -v_z^2 + K_{xx} & K_{xy} & v_x v_z + K_{xz} \\ K_{yx} & -v_x^2 - v_z^2 + K_{yy} & K_{yz} \\ v_x v_z + K_{zx} & K_{zy} & -v_x^2 + K_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0 \quad (3-21)$$

which is the matrix form, with $k_y = 0$, of the relation

$$\vec{v} \times (\vec{v} \times \vec{E}) + \vec{K} \cdot \vec{E} = 0 \quad (3-22)$$

where $\vec{K}$ is the dielectric tensor which, in turn, is given by

$$\vec{K} = \vec{1} + i \sum_k \frac{\epsilon_k n_k^2}{\omega \Omega_k} \vec{M}(k) \quad (3-23)$$

where $\vec{1}$ is the unit dyad and the mobility tensor has already been given in Eqs. (3-3). From Eq. (3-21) we may obtain for the electric field component ratios,

$$\frac{E_y}{E_x} = \frac{(v_x v_z + K_{xz}) K_{yx} - K_{yz} (-v_z^2 + K_{xx})}{K_{yz} K_{xy} - (v_x v_z + K_{xz}) (-v_x^2 - v_z^2 + K_{yy})} \quad (3-24)$$

$$\frac{E_z}{E_x} = \frac{K_{xy} K_{yx} - (-v_x^2 - v_z^2 + K_{yy}) (-v_z^2 + K_{xx})}{(-v_x^2 - v_z^2 + K_{yy}) (v_x v_z + K_{xz}) - K_{xy} K_{yz}} \quad (3-25)$$

These ratios may be used to express the power absorption P_a and the wave energy W in terms of $|E_x|^2$. Then is obtained the ratio P_a/W which is proportional to the growth rate ω_i .

Equation (3-1) indicates that contributions are made to P_{kn} from all the plasma components as well as the tenuous stream of particles. We may simplify considerably the analysis by assuming a cold plasma so that contributions are made to

P_{kn} from only the streaming particles. In addition, simple expressions may be obtained for the electric field components from the cold plasma form of Eq. (3-21)

$$\begin{bmatrix} S - v_z^2 & -iD & v_x v_z \\ iD & S - v_x^2 & 0 \\ v_x v_z & 0 & P - v_x^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0 \quad (3-26)$$

where

$$S = \frac{1}{2} (R + L), \quad D = \frac{1}{2} (R - L) \quad (3-27)$$

$$R = 1 - \sum_k \frac{\pi_k^2}{\omega^2} \left(\frac{\omega}{\omega + \epsilon_k \Omega_k} \right) \quad (3-28)$$

$$L = 1 - \sum_k \frac{\pi_k^2}{\omega^2} \left(\frac{\omega}{\omega - \epsilon_k \Omega_k} \right) \quad (3-29)$$

$$P = 1 - \sum_k \frac{\pi_k^2}{\omega^2} \quad (3-30)$$

These expressions are:

$$\frac{E_y}{E_x} = \frac{iD}{v^2 - S} \quad (3-31)$$

$$\frac{E_z}{E_x} = \frac{v_x v_z}{v_x^2 - P} \quad (3-32)$$

Putting these values into G_n in Eq. (3-15) gives the power absorption P_k as a function of wave frequency ω , propagation angle θ , and the physical parameters of the plasma and the streaming particles. Some algebraic manipulation yields

$$G_n = \frac{L_n}{\lambda} \left[\frac{D^2}{(\nu^2 - S)^2} (n^2 + \lambda^2) + R_n^2 \right] - L'_n \frac{2\epsilon D}{\nu^2 - S} R_n \quad (3-33)$$

where

$$R_n = \frac{\epsilon D}{\nu^2 - S} \lambda + \frac{\nu^2 \sin^2 \theta}{\nu^2 \sin^2 \theta - P} \frac{\omega + n\Omega}{\Omega} - n \quad (3-34)$$

$$L_n = 2 e^{-\lambda} I_n \quad (3-35)$$

$$L'_n = 2 e^{-\lambda} I'_n \quad (3-36)$$

It should be borne in mind that λ , ϵ , Ω are parameters of the streaming particles which alone contribute to the power absorption. The values of S , D , R , L , and P are functions of the wave frequency and the cold plasma constituents only.

The wave energy W also takes a simple form when a cold plasma is assumed. The second term of Eq. (3-16) can be expressed as:

$$\begin{aligned} \vec{E}^* \cdot \frac{\partial}{\partial \omega} (\omega \vec{K}_h) \cdot \vec{E} &= \vec{E}^* \cdot \vec{T} \cdot \vec{E} \\ &= |E_x|^2 T_{xx} + |E_y|^2 T_{yy} + |E_z|^2 T_{zz} \end{aligned}$$

$$\begin{aligned}
& + (E_y E_x^* - E_x E_y^*) T_{xy} + (E_x E_z^* + E_x^* E_z) T_{xz} \\
& + (E_z E_y^* - E_z^* E_y) T_{yz}
\end{aligned} \tag{3-37}$$

In the case of a cold plasma, the expression for the tensor $\overleftrightarrow{T}$ may be found to be,

$$\begin{aligned}
T_{xx} &= \frac{\partial}{\partial \omega} (\omega S) = 1 + \sum_k \Pi_k^2 \frac{\omega^2 + \Omega_k^2}{(\omega^2 - \Omega_k^2)^2} \\
T_{yy} &= T_{xx} \\
T_{zz} &= \frac{\partial}{\partial \omega} (\omega P) = 1 + \sum_k \frac{\Pi_k^2}{\omega^2} \\
T_{xy} &= \frac{\partial}{\partial \omega} (-i\omega D) = 2i \sum_k \frac{\epsilon_k \omega \Omega_k \Pi_k^2}{(\omega^2 - \Omega_k^2)^2} \\
T_{yx} &= -T_{xy}
\end{aligned} \tag{3-38}$$

where, again, the summation over k excludes the tenuous beam of particles. From Eqs. (3-16), (3-20), (3-37) and (3-38) can be obtained the expression for the wave energy W ,

$$\begin{aligned}
\frac{16 \pi W}{|E_x|^2} &= \nu^2 \left[\frac{D^2}{(\nu^2 - S)^2} + \left(\frac{\nu^2 \sin^2 \theta}{\nu^2 \sin^2 \theta - P} - 1 \right)^2 \cos^2 \theta \right] \\
&+ T_{xx} + T_{yy} \frac{D^2}{(\nu^2 - S)^2} + T_{zz} \left(\frac{\nu^2 \cos \theta \sin \theta}{\nu^2 \sin^2 \theta - P} \right)^2 \\
&+ T_{xy} \frac{2iD}{\nu^2 - S}
\end{aligned} \tag{3-39}$$

3.3 Growth Rate and Computer Results with Emphasis near the LHR Frequency

From the results of Sections 3.1 and 3.2 may be obtained the wave growth rate by computing the energy balance. This is accomplished by equating the rate of increase in energy for the particles to the rate of decrease in energy in the plasma wave,

$$\frac{dW}{dt} = 2 \omega_i W = - P_a \quad (3-40)$$

Therefore the growth rate is given by

$$\omega_i = -\frac{1}{2} \frac{P_a}{W} \quad (3-41)$$

This expression was programmed for computer calculation (see Appendix I) where the relevant ionospheric parameters given in Table I were used. Since temperature anisotropy has been introduced, the parameters for the streaming particles λ_s and $\alpha_n^{(s)}$ become

$$\lambda_s = \frac{1}{2} \frac{T_\perp}{T_\parallel} k_x^2 / (\Omega_s^2 \beta_\parallel^2) \quad (3-42)$$

$$\alpha_n^{(s)} = \frac{\omega - k_z V_s + n \Omega_s}{k_z} \beta_\parallel \quad (3-43)$$

where

$$\beta_\parallel^2 = \frac{m_s}{2 \kappa T_\parallel} \quad (3-44)$$

and $\beta_{||}$, $T_{||}$ and $T_{\perp}$ refer to the streaming particles.

Finally, the solution for the dispersion equation for the wave propagating through the background cold plasma is given in Ref. I (Eq. (1-26), p. 12); we are interested in the whistler-mode solution whose wave-normal surface is a dumbbell lemniscoid. This solution is

$$v^2 = \frac{B' - F}{2A} \quad (3-45)$$

$$\text{where} \quad A = S \sin^2 \theta + P \cos^2 \theta \quad (3-46)$$

$$B' = RL \sin^2 \theta + PS (1 + \cos^2 \theta) \quad (3-47)$$

$$F^2 = (RL - PS)^2 \sin^4 \theta + 4 P^2 D^2 \cos^2 \theta \quad (3-48)$$

These equations complete the set required to perform the computer calculation. The background plasma is assumed to consist of electrons and singly-charged positive ions of hydrogen, helium, and oxygen. The fractional abundance of oxygen ions is assumed to be 90%, a reasonable value in the polar ionosphere (Barrington et al., 1965; Barrington and McEwen, 1967). This value and the electron density given in Table I lead to number densities of 2.05×10^2 and $7.95 \times 10^2 \text{ cm}^{-3}$ for the H^+ and He^+ ions because of charge neutrality and $S = 0$ at $f_{LH} = 5 \text{ kHz}$. The streaming particles, as before, are taken to be electrons with a number density much less than

that of the background electrons (Table I). With these numerical values, normalized growth rate $\omega'_i = \omega_i/\omega_{LH}$ versus propagation angle θ plots were obtained with the parameters: normalized frequency $\omega'_r = \omega_r/\omega_{LH}$, the ratio of the parallel and perpendicular temperatures of the streaming electrons $T_{||}/T_{\perp}$, and the ratio of the kinetic energy in the streaming motion to the thermal energy of the streaming electrons, ϕ^2 . (Eq. (2-47) is modified by: $\beta_s \rightarrow \beta_{||}$ and $3/2 \rightarrow 1/2 + T_{\perp}/T_{||}$.)

Figure 5 shows the normalized growth rate versus θ with $\omega'_r = 1.1$, 2, and 5 where $T_{||}/T_{\perp} = 1$ and $\phi^2 = 1$. Note that the scales are different for negative and positive ω'_i . It shows that with increasing θ , ω'_i increases from a small positive value about zero to a maximum, decreases through zero to exhibit one or two negative minima before becoming positive again, reaches a positive maximum, and drops to zero at $\theta = \theta_{res}$ where θ_{res} is given by

$$\tan^2 \theta_{res} = -\frac{P}{S} \quad (3-49)$$

This "electrostatic" region for positive ω'_i near θ_{res} occurs over an extremely small interval in θ and consequently the curve plots on the figure as a straight line here. (The reason for the "electrostatic" designation will become clear shortly.) For these frequencies ω_r , only the Landau term where $n = 0$ is significant and contributions from the cyclotron terms ($n \neq 0$) to $P_a = P_s = \sum_{n=-\infty}^{\infty} P_{sn}$ are negligible. The cyclotron terms do not become significant until ω'_r is about 10.

Fig. 5. Growth rate vs. propagation angle θ for 3 frequencies.

Therefore Fig. 5 shows that Landau instabilities occur for low values of θ as well as for values near θ_{res} , and that these two regions are separated by a region where Landau damping occurs. Note also that the instability region for θ above $\theta = 0$ ("low- θ " region) diminishes with increasing ω_r' . It has been found to have disappeared at about $\omega_r' = 10$. The resonance angle θ_{res} also decreases with increasing ω_r' , becoming zero when $P = 0$, that is, when the frequency is given by

$$\omega_r^2 = \pi_e^2 + \sum_{\text{ions}} \pi_j^2 \quad (3-50)$$

Figure 5 also indicates that the maximum growth rate near θ_{res} increases with frequency. Figure 6 is a plot of this maximum growth rate as a function of frequency showing this behaviour as well as the lower cutoff at the lower hybrid resonance frequency ($\omega_r' = 1$). The positive values of ω_i' near $\theta = 90^\circ$ are related to the positive values of ω_i' plotted in Figs. 2 and 3 which are based on the electrostatic dispersion equation. A comparison can therefore be made between the maximum growth rates $(\omega_i')_{\text{max}}$ obtained from the electrostatic and the general dispersion equation. These values were taken from Figs. 2 and 6 or calculated if necessary and are listed in Table III.

Values for $(\omega_i')_{\text{max}}$ computed using the general dispersion equation were taken from Fig. 6 and are listed in the fourth column of Table III. Corresponding values for the

Fig. 6. Maximum growth rate vs. frequency ω'_r .

electrostatic case were taken from Fig. 2 or calculated and are listed in the fifth column of this table. The last column gives the ratios of the corresponding $(\omega'_i)_{\max}$. The departure from unity of these ratios reflects the effect of the assumptions made in the electrostatic case, such as $\lambda \ll 1$ and large refractive index v . In the table, the ratio of the maximum growth rates approaches 1 as λ becomes much less than 1.

Figures 7 and 8 show the variation of refractive index v and λ on the propagation angle θ for three values of ω'_r . In the calculation for λ , the parameters $T_{||}/T_{\perp}$ and ϕ^2 both are taken to be 1 for Fig. 8; the refractive index v , however, is here independent of these two parameters since they describe the streaming particles assumed

TABLE III

Maximum growth rates and associated values

ω_r'	ν	λ	$(\omega_i')_{\max}$ $\times 10^{-6}$	electrostatic $(\omega_i')_{\max, \text{elect}}$ $\times 10^{-6}$	$(\omega_i')_{\max, \text{elect}}$
					$(\omega_i')_{\max}$
1.1	1851	1.22	3.13	8.04	2.57
2	523	0.323	44.8	63.0	1.40
5	186	0.256	151	202	1.34
10	91	0.243	312	413	1.32

Fig. 7. Refractive index vs. propagation angle θ for 3 frequencies.

Fig. 8. λ vs. propagation angle θ for 3 frequencies.

to be tenuous in the analysis. It can be seen in Fig. 7 that the refractive index is in general lower for higher frequencies except near θ_{res} , where $v \rightarrow \infty$, since the higher frequencies have lower values of θ_{res} as noted before in Fig. 5. The parameter λ can be seen in Fig. 8 to increase monotonically with θ , approaching infinity as θ approaches θ_{res} . At a given value of θ , λ is always greater for higher frequencies. It has also been observed that if either ϕ^2 or $T_{\parallel}/T_{\perp}$ is increased, then the value of λ decreases.

Figures 5, 6, and 8 in this chapter showed the variation of growth rate and λ with propagation angle θ for frequencies $\omega'_r = 1.1, 2, 5$ (Figs. 5 and 8) and the maximum growth rate as a function of ω'_r where the other two parameters had values $\phi^2 = 1$ and $T_{\parallel}/T_{\perp} = 1$. Let us now consider the effect of these two parameters, keeping $\omega'_r = 1.1$.

The dependence of ω'_i on ϕ^2 is shown in Fig. 9. Three curves for $\phi^2 = 1, 2, \text{ and } 3$ are plotted where $\omega'_r = 1.1$ and $T_{\parallel}/T_{\perp} = 1$. Note that, as before, the scales are different for positive and negative ω'_i . One curve only was drawn near θ_{res} , where all the curves reach zero growth rate, to avoid a tangle of lines here. Generally, an increase in ϕ^2 leads to an increase in the magnitude of the excursions from the horizontal axis. For example, the values of $(\omega'_i)_{\text{max}}$ are 3.13×10^{-6} , 11.9×10^{-6} , and 24.0×10^{-6} for $\phi^2 = 1, 2, \text{ and } 3$.

Next, the dependence of ω'_i on the temperature

anisotropy is depicted on Figs. 10 and 11. Curves are drawn for three values of $T_{||}/T_{\perp}$: 0.5, 1, and 2. The other parameters are $\omega_r' = 1.1$, $\phi^2 = 1$. In Fig. 10, only one curve is drawn near θ_{res} for the same reason as in the previous figure. But Fig. 11 shows the three curves in the immediate vicinity of θ_{res} and demonstrates the very narrow interval of θ in which the Landau instability occurs. Figs. 10 and 11 show that the two positive maximum growth rates are greater for smaller $T_{||}/T_{\perp}$, i.e., for streaming particles with more thermal energy in the parallel than in the perpendicular direction. Although not shown here, it was noted that this behaviour with $T_{||}/T_{\perp}$ is reversed for the "electrostatic" maximum growth rate if the parallel thermal energy is kept constant and the perpendicular thermal energy is varied.

It had been assumed in Table I that the streaming electrons have a streaming velocity of 6×10^9 cm/sec corresponding to an energy of 10 keV. It is well-known that streaming electrons of 6 keV energy are prevalent too (McIlwain, 1960). Therefore calculations were made for ω_i' with $\omega_r' = 1.1$, $\phi^2 = 1$, $T_{||}/T_{\perp} = 1$ for a number of values of V_s . Three curves are plotted in Fig. 12 for values of $V_s = 4.6 \times 10^9$, 6×10^9 , and 7.3×10^9 cm/sec. Again, only one curve for $V_s = 6 \times 10^9$ cm/sec was drawn near θ_{res} where all the curves meet at zero growth rate. The values for $(\omega_i')_{max}$ near θ_{res} are nearly the same for the three curves. However of the two minima in each curve, the one with θ

Fig. 10. Growth rate vs. propagation angle θ for 3 temperature ratios.

Fig. 11. Growth rate vs. propagation angle θ for 3 temperature ratios.

Fig. 12. Growth rate vs. propagation angle θ for 3 values of streaming velocity V_s .

closer to θ_{res} is greater in absolute value for smaller V_s . For the next minima (which are drawn in Fig. 12) the reverse is true and the absolute value of the minimum growth rate is smaller for smaller V_s . In the "low- θ " region, the maximum growth rate decreases with decreasing V_s . Another feature to be noted is the decrease in the region for Landau instability for low θ values as V_s decreases. In fact, as V_s decreases below 4.6×10^9 cm/sec (6 keV) to about 2×10^9 cm/sec (1 keV), this region vanishes altogether leaving only the region near θ_{res} (the "electrostatic" region) in which the Landau instability occurs. As V_s is decreased still further this "electrostatic" region with positive growth rate also vanishes, leaving only Landau damping of the wave to take place. Thus there is a minimum energy for the streaming particles below which the Landau instability process cannot be triggered.

The results of the computer calculation show that positive growth rates occur for frequencies above the lower hybrid resonance and that there exists a sharp lower cutoff at ω_{LH} . Although emphasis has been placed on frequencies near the LHR frequency, positive growth rates were obtained in the "electrostatic" region for frequencies well above the LHR frequency where cyclotron damping or instability becomes significant. The upper limit is given either by the electron cyclotron frequency or the electron plasma frequency, whichever is lower.

To demonstrate that the equations are fairly general

and to determine whether instabilities can be excited at other frequencies, computer calculations were made at frequencies much less than the lower hybrid resonance frequency. For example, Fig. 13 shows the results of such a calculation when the parameters are the same as those used for Fig. 5 except that now the frequency is $\omega_r' = 7 \times 10^{-3}$. This frequency is just below the lowest ion gyrofrequency, the oxygen ion gyrofrequency, which is much less than ω_{LH} . In this calculation, only the Landau instability term ($n = 0$) was found to be significant. The figure shows that Landau instability may take place at all propagation angles below θ_{res} . Such results may be relevant to ion cyclotron whistlers (Gurnett et al., 1965) and noise observed around the proton cyclotron frequency (Harvey, 1967).

Fig. 13. Growth rate vs. propagation angle θ for a frequency just below the oxygen ion cyclotron frequency.

CHAPTER 4

APPLICATION OF THEORY TO RELEVANT IONOSPHERIC NOISE

4.1 LHR Noise4.1.1 Observations and Characteristics of LHR Noise(a) General Features

It has been mentioned already in Chapter 1 that the lower-frequency cutoff of the LHR band has been identified as the frequency of the lower hybrid resonance at the location of the satellite and that the characteristics of LHR noise bands appear to be related to electrostatic waves propagating nearly perpendicular to the earth's magnetic field. What then are these characteristics? They have been reported in papers by a number of authors. Some of these are papers by: Barrington and Belrose (1963), Brice and Smith (1964), Brice et al. (1964), Brice and Smith (1965), and McEwen and Barrington (1967). The last two papers in this list are particularly important contributions.

LHR noise has been seen in the upper ionosphere by the VLF receiver of Alouette I but, it is claimed, never by a VLF receiver on the ground (Brice and Smith, 1964). The noise is observed in the frequency range from about 5 to 10 kHz with a bandwidth which varies from one to several kHz. The lower frequency cutoff of the band varies consistently with latitude, increasing in frequency with decreasing latitude of the satellite. These LHR noise bands have also been found in Injun 3 VLF recordings, although not so often as in Alouette recordings. Also

they have been observed on OGO II records; the LHR noise bands triggered by fractional-hop whistlers are found to be among the most intense signals observed on these records (Laaspere et al., 1967). Both the Alouette I and the OGO II satellites use electric dipole antennae whereas the Injun 3 satellite employs a magnetic loop antenna. Research is still in progress on the phenomenon since the LHR frequency is a function of the positive-ion composition of the plasma and is a potentially valuable diagnostic tool for investigation of the ionosphere.

(b) Polar LHR Noise

The appearance of LHR noise bands is distinctly different at middle and northern latitudes and accordingly the two types of noise have been called midlatitude and polar LHR noise (McEwen and Barrington, 1967). Polar LHR noise bands have relatively large bandwidths and a cutoff frequency which is erratic and exhibits rapid fluctuations (for invariant latitudes above 70°), although on the average it is lower at higher latitudes. The fluctuations are believed to be due to variations in the electron density of the medium. The noise is usually continuous, lasting for several minutes.

Polar LHR noise has a maximum occurrence in the region 70 - 80° invariant latitude, where the noise bands are observed as much as one half of the time. Although it has no seasonal variation, it has a diurnal variation and shows two maxima in frequency of occurrence, one slightly before midnight and another secondary peak about 0600-0800 hours local time. These diurnal

maxima are similar to those for auroral activity and particle precipitation and lead one to postulate a possible relation between polar LHR noise bands and particle precipitation.

(c) Midlatitude LHR Noise

Midlatitude LHR noise bands exhibit smooth lower-frequency cutoffs consistently, unlike polar LHR noise bands. Also, they show a wide variation in appearance. They range from short wedge-shaped signals directly related to whistlers (referred to as "whistler-triggered" emissions) to continuous emissions of unknown origin. The latter emissions account for about three quarters of the observed midlatitude LHR noise. Whistlers are observed on most of the records with continuous emissions so that it is not possible to resolve whether the steady midlatitude noise bands are triggered by whistlers or not. The whistler-triggered emissions have durations from a fraction of a second to several seconds, and indicate that frequencies near the lower cutoff frequency are enhanced for longer periods than the higher frequencies. Short as well as long fractional-hop whistlers are observed to trigger LHR emissions with occurrences of almost equal frequency. The triggering long fractional-hop whistlers are rarely observed on corresponding ground-based recordings. On the other hand, none of the whistlers with echoes are seen to trigger LHR emissions. Consequently it has been suggested that whistlers with echoes are ducted, whereas the whistlers which trigger LHR emissions are non-ducted. Also, recently Brice (private communication, 1967) has suggested

that these whistler-triggered emissions are not emissions but are non-ducted whistlers propagating with very large wave-normal angles where the group velocity becomes quite small and the time delay large.

Midlatitude LHR noise has a maximum in occurrence at 50-60° invariant latitude. It is observed mainly during the months of June-October and primarily at night, with maxima at approximately the same times as polar LHR noise. Generally, there exists good correlation between occurrence of whistlers recorded on the ground and midlatitude LHR noise.

4.1.2 Comparison of Theory with LHR Noise Observations

In an electrostatic wave, the electric field of the wave is almost parallel to the wave vector and accordingly, the magnetic field of the wave is very small. This is one of the characteristics of LHR noise for it has been reported that LHR noise is observed more often by Alouette I using an electric dipole antenna with the VLF receiver than by the Injun 3 satellite which employs a loop antenna. Also, LHR noise bands triggered by fractional-hop whistlers have been found to be among the most intense signals observed by the OGO II satellite which also uses an electric dipole antenna. In addition, from the conclusion that whistlers which trigger LHR noise are non-ducted while those that do are ducted, it was suggested that triggering is more likely for signals propagating with large wave-normal angles (Brice and Smith, 1965).

(The wave-normal angle referred to by these authors is the same as the propagation angle θ defined in this thesis.) We have already seen in Chapter 3 that the electrostatic region is in the vicinity of $\theta = 90^\circ$ for frequencies near the lower hybrid resonance frequency.

It had been mentioned earlier that the quantity $\Delta_2(\omega_r)$, the contribution from the thermal ions, could be significant for ω_r at a harmonic of an ion cyclotron frequency. At these values of ω_r the maximum contribution from the thermal ions is proportional to $(\lambda_j/2)^n/(n!)$. Since the smallest values of n are 9, 35, and 138 for the hydrogen, helium, and oxygen ions respectively, only the contribution from the protons are significant. Generally, this contribution leads to a value for $\Delta_2(\omega_r)$ which is less than that for $\Delta_3(\omega_r)$. However, under certain conditions (for example, if ϕ^2 is small) the situation may be reversed and damping results at the proton cyclotron frequency harmonics just above the lower hybrid resonance frequency. This damping might result in absorption bands within the LHR noise band. It would be interesting to look for this effect in spectrograms.

It had been noted that as the wave frequency ω_r tends to ω_{LH} , the lower hybrid resonance frequency, the growth rate ω_i tends to zero. This explains the observational fact that an LHR noise band has a sharp lower frequency cutoff at the lower hybrid resonance frequency.

We had seen in Sec. 2.4 that the growth times are in the order of $1 - 10^{-2}$ sec for values between 1 and 100 of ϕ^2 , the ratio of the streaming kinetic energy to the thermal energy of the streaming electrons. If the value of ϕ^2 is much greater than unity, then the growth time is very small and the total duration of the signal may also be very small for the following reason. According to the quasi-linear theory of Landau instabilities for an electrostatic wave propagating in an electron plasma with no external magnetic field, the wave amplitude does not grow indefinitely as predicted by the linear theory but decays finally with time, with a decay time comparable to the maximum initial growth time (Bernstein and Engleman, 1966). Therefore, the total duration is comparable to the growth time in order of magnitude. This mechanism is one of a number of possible generation mechanisms for whistler-triggered LHR noise whose duration ranges from a fraction of a second to several seconds.

The appearance of LHR noise of long duration seems to indicate that conditions are such that the quasi-linear theory no longer applies and a wave introduced into a growing instability no longer decays with time but develops into random noise (Kadomtsev, 1965). It is not known how an unstable wave chooses between these two possibilities in the time-evolution of the signal. This is a problem which needs investigating in the non-linear theory of LHR noise. Since polar LHR noise is continuous and is related to particle precipitation, our theoretical work appears most relevant to this type of LHR noise.

4.1.3 Discussion

The theory developed in Chapter 2 predicts the sharp lower frequency cutoff in the LHR noise band, but fails to explain the existence of the upper frequency limit (though it is observed less clearly than the lower frequency cutoff). As mentioned before, the observed LHR frequency ranges from about 5 - 10 kHz, and the bandwidth is about several kHz or less, i.e., the upper frequency limit is usually less than twice the LHR frequency. The growth rate ω_i calculated from equation (2-44) becomes neither zero nor negative as the wave frequency ω_r becomes several times as large as the lower hybrid resonance frequency ω_{LH} .

The generalized analysis was performed since it was conjectured the upper frequency limit might be obtained by placing no restrictions on the refractive index. Also, Kennel (1966) had shown that a whistler can be unstable over a significant cone of propagation angles θ when energetic resonant electrons have a sufficiently hard energy spectrum, and a pitch angle anisotropy corresponding to more electron energy perpendicular than parallel to the magnetic field. Growth rate curves were not drawn beyond $\theta = 80^\circ$ in Fig. 3 of his paper because an approximation in the analysis broke down beyond this point. However, it appeared that some of the curves may re-enter the growth region at some value of θ between 80° and 90° before terminating at zero growth rate at $\theta = 90^\circ$. This suggested that the upper frequency limit of the LHR band might be

determined by extending the analysis given in Chapter 2 or in the paper by Kennel (1966) with appropriate modifications. However, the upper frequency limit was found to occur at either the electron plasma frequency or the electron gyrofrequency, whichever was smaller. These frequencies are well above the LHR frequency and could not explain the upper frequency limit of LHR noise.

One possible explanation for the upper frequency limit may lie in the method of measurement or the instrumentation involved. For example, the signals may be so intense close to the LHR frequency so that the AGC levels are such that higher frequencies in the band are not observed. Or there may be some physical process involved which may not always yield an upper frequency limit. For example, the precipitated flux of electrons may have an energy spectrum quite different from that assumed in the theoretical work in Chapters 2 and 3.

A stream of electrons, each of which has an energy comparable to that of auroral particles precipitating into the upper ionosphere along the direction of the earth's magnetic field, is assumed to be the cause of an instability process. At high latitudes, it appears reasonable to assume that such an electron stream exists. In fact, nearly monoenergetic electrons which appear to be suitable for exciting the Landau instability process proposed in this paper had been detected by McIlwain (1960). Very recently (Albert, 1967a; Albert, 1967b), nearly monoenergetic electrons with peak energy between 10 -

14 keV have been observed during a visible aurora. It has also been proposed (Evans, 1967) that these electron beams may excite various plasma oscillations and that conditions may be favourable for the growth and subsequent decay of some of these waves in a time of about 0.1 sec. The spectral form of nearly monoenergetic electron streams can be represented by a Maxwell-Boltzmann energy distribution, but characterized by a paucity of high-energy electrons. The parameter ϕ^2 , the ratio of the kinetic energy in the streaming motion to the thermal energy of the streaming particles, is then much greater than unity and therefore these electrons may fulfill the requirements of the theoretical analysis presented in this paper, for example, the electrons have large enough values for V_s and ϕ^2 to give instabilities. However, although different in appearance from that at northern latitudes, LHR noise with continuous emissions is observed at midlatitudes also. At these latitudes, a possible source of electrons might be from the precipitation of higher-energy particles from the trapped radiation zones.

It may be worth-while considering other possibilities as the cause of the instability process. One possibility is that the instability might be caused by anisotropy in the particle velocity distribution in one or all of the component gases (such as anisotropy in the pitch angle distribution of the charged particles or differences in the temperatures parallel and perpendicular to the background magnetic field). Electrostatic wave instabilities due to anisotropic temperature have been investigated by Harris (1961).

Finally, another possible generation mechanism for the short wedge-shaped signals known as "whistler-triggered" emissions is suggested. The proposed mechanism is a resonance similar to that producing the strong upper hybrid resonance (abbreviated UHR) spikes in Alouette topside sounder ionograms. Since the topside sounder sweeps the frequency range from 0.5 MHz to 12.0 MHz which is well above the LHR frequency, LHR spikes are not observed on these ionograms. However, a whistler which sweeps the frequency range about the LHR frequency may play the same role as the topside sounder and yield LHR spikes which appear as the wedge-shaped signals observed on the VLF sonograms. Taking typical values for the UHR spike of 5×10^{-3} sec and 1.5 MHz for the duration and frequency respectively, the ratio of the duration to the wave period ($2\pi/\omega_{UH}$) is of the order of 1000 cycles. It is interesting to notice that this ratio is comparable to that for the LHR noise, for taking a typical value of $f_{LH} = 5$ kHz yields an LHR duration of the order of 1 sec. For comparison, the observed duration of "whistler-triggered" LHR noise ranges from a fraction of a second to several seconds.

4.2 Other Related Observations

Other observations have been made, besides the LHR noise bands, of ionospheric noise which may also be related to the theory presented in this thesis. These observations may be related to the instabilities for small wave-normal angles as well as those in the "electrostatic" region. Some of them,

unlike the LHR noise bands, do not exhibit an upper frequency limit so close to the LHR frequency.

Gurnett (1966) made a study of VLF hiss observed by the Injun 3 satellite during its 10-month lifetime. He described the frequency spectra of the VLF hiss events chosen, the occurrence in latitude and local time, and the relationship between the VLF hiss and energetic charged particle fluxes. Typically the frequency spectrum of the VLF hiss extended from a lower frequency limit of about 2 - 4 kHz to above the upper frequency limit of the VLF receiver (8.8 kHz). One form of VLF hiss has a lower frequency cutoff which changes systematically with latitude. The cutoff frequency first decreases with increasing latitude, reaches a minimum of about 2 - 4 kHz, then increases with increasing latitude thereby giving a V-shaped appearance on the spectrogram. The VLF hiss was found to occur in a zone about 7° wide in latitude centred on 77° invariant latitude at 1400 magnetic local time (MLT). A diurnal variation was found with occurrence predominant during local afternoon and evening, from 1200 to 2400 MLT. This differs from the diurnal occurrence of ground-based VLF observations (Jørgensen, 1966) with a maximum during the evening (about 1900 to 2100 MLT). This difference led Gurnett to suggest that the satellite VLF hiss occurring during the local afternoon (1200 to 1800 hours MLT) may not be observed on the ground. The occurrence of VLF hiss was also found to be associated with the intense fluxes of soft electrons with energy of about 10 keV

often found during the early evening.

Jørgensen (1966) refers to two kinds of hiss, namely hiss below about 2 kHz, which is a morning or day phenomenon, and hiss above about 2 kHz which is almost exclusively seen at night, except at high latitudes. The wide-band hiss above 2 kHz is sometimes called auroral hiss because of its association with auroral arcs and bands. Jørgensen (1968) stated that the upper frequency limit for auroral hiss is not known and that hiss at 500 kHz is often observed at Byrd Station. This statement appears to be inconsistent with another statement made in his paper, viz that the emission named polar LHR by some workers is equivalent to the emission called auroral hiss by others, since polar LHR noise has an upper frequency limit close to the LHR frequency. Jørgensen postulates that auroral hiss may be generated by incoherent Cerenkov radiation from electrons with energies of the order of 1 keV. It is interesting to note that Cerenkov radiation is produced by a charged particle moving in a plasma if the component in the wave normal direction of the particle's velocity along the background magnetic field equals the phase velocity of the wave. This is equivalent to Eq. (2-2), the condition for the Landau instability process. However, the Landau process is a collective interaction between a beam of particles and the propagating wave whereas the Cerenkov emission is from individual particles traversing a plasma. Jørgensen's analysis assumes a wave normal angle of zero in section 2 of his paper and may be inappropriate in its present form for polar LHR noise which appears to be

related to large wave normal angles, as noted previously. The Cerenkov radiation theory proposed by Jørgensen and the theory proposed in this thesis both predict an upper frequency limit equal to the electron gyrofrequency at the bottom in the ionosphere. However, the Landau instability process defines a lower cutoff at the LHR frequency whereas the Cerenkov radiation is also generated at frequencies below the LHR frequency, although the power is several orders of magnitude below the power generated above ω_{LH} .

The characteristics of auroral hiss suggest that perhaps this type of VLF emission may originate from electrostatic waves excited around the lower hybrid resonance frequency by the Landau instability process analyzed in this thesis. As an electrostatic wave generated in the ionosphere presumably cannot propagate out of the ionosphere, it must be converted into an electromagnetic wave somewhere in the ionosphere in order to be observed as a VLF emission on the ground. Such a conversion might take place if the plasma density distribution is nonuniform in space. A process whereby an electrostatic wave is converted into an electromagnetic wave when propagating in a nonuniform plasma or when penetrating a surface of discontinuity in the plasma density distribution has been investigated for a type of radio wave emission from the sun (Tidman, 1960; Tidman and Weiss, 1961; Tidman and Boyd, 1962). Even if the plasma density distribution is uniform in the unperturbed state, conversion of an electrostatic wave to an electromagnetic

wave can take place as a result of a nonlinear wave-wave interaction process in the electrostatic wave (Tidman and Dupree, 1965; Tidman, 1965). In the case of auroral hiss type VLF emissions, there is evidence that such a surface of discontinuity or a steep gradient in the plasma density distribution does exist in the ionosphere.

Aurora-associated VLF hiss emissions are received at a polar ground station only in the pre-breakup period (the N-1 phase (Morozumi and Helliwell, 1966)) during the course of auroral activity (Morozumi, 1962; Harang and Larsen, 1964). The auroral activity during this period is characterized by quiet arcs or bands. It has been observed that hiss type VLF emissions are always received simultaneously with the occurrence of arc or band type of aurorae (Morozumi, 1962). An arc extends a very long way (1000 km or more) nearly in the magnetic east-west direction (Störmer, 1955), but is very thin in the north-south direction. Quiet arcs are usually characterized by this thin-blade structure. The thickness (or rather thinness) of the structure is about a few hundred meters or even less (Davies and McCormac, 1967). The distribution of luminosity against height has a very sharp maximum in the height range, normally from about 100 - 110 km (Störmer, 1955). It is believed that an arc is caused by charged particles (presumably electrons) streaming along the direction of the earth's magnetic field. As suggested by the location of the sharp maximum in the luminosity-height distribution, the ionization

within an arc is most enhanced presumably in the lower ionospheric region (100 - 110 km). Because of the thin-blade structure, the ionization in the lower ionosphere is surmised to change very abruptly across the boundaries of an arc. It would be interesting to make simultaneous observations of LHR noise in the ionosphere, particularly in the lower region by a space vehicle and VLF emissions of auroral hiss type at a ground station.

Another interesting observation has been made by Barrington and McEwen (1966) which may be the same as the V-shaped VLF hiss studied by Gurnett (1966). They reported an interesting noise burst phenomenon at high latitudes, that appears to be related in some way to the LHR noise bands. Because of their appearance, they have been termed VLF splashes. Generally, they are V-shaped and the width of the V varies from a couple of seconds to a few minutes. Often, the V-shaped noise bursts seem to grow out of or replace a continuous LHR noise band. In addition, the occurrence pattern for the splashes is not inconsistent with an association between splashes and auroral particle precipitation.

CHAPTER 5

SUMMARY AND CONCLUSIONS

Wave-particle interaction around the lower hybrid resonance frequency has been investigated in this thesis. Expressions for the growth rate of whistler-mode waves propagating in a plasma penetrated by a tenuous beam of nonthermal particles were obtained using the electrostatic dispersion equation and the full-wave dispersion equation given in the text by Stix (1962). All the species of particles involved were assumed to have a Maxwellian velocity distribution. In the case of the streaming particles, the distribution was shifted by a streaming velocity parallel to the background magnetic field. The electrostatic analysis assumed isotropic temperatures and the full-wave analysis assumed that the distributions are characterized by two temperatures, parallel and perpendicular to the background magnetic field. The expressions derived for the growth rate are somewhat general, but in the numerical calculations we considered the case of streaming electrons interacting with whistler-mode waves propagating in a cold magneto-active plasma consisting of electrons and H^+ , H_e^+ , and O^+ ions.

The growth-rate expression obtained by using the electrostatic dispersion equation showed that the propagating waves can grow due to the Landau instability process for frequencies above the lower hybrid resonance frequency. The instability region has a sharp lower-frequency cutoff at the

LHR frequency and an upper frequency limit at the smaller of the electron cyclotron or electron plasma frequency. With increasing frequency within this instability region, the maximum growth rate increases monotonically.

The full-wave analysis gave an instability region in the "low- θ " region above $\theta = 0$ in addition to that in the "electrostatic" region near resonant angle θ_{res} which corresponds to the instability found from the electrostatic-wave analysis. The growth rates in the "low- θ " region are not as large as those in the "electrostatic" region and become negative (damping) before the latter with decreasing streaming velocity V_s . The cyclotron instabilities or damping ($n = \pm 1, \pm 2, \dots$) become important at frequencies greater than about ten times the LHR frequency. It was also observed that the growth rates decrease with increasing $T_{\parallel}/T_{\perp}$ in both the "low- θ " and the "electrostatic" region. An increase in ϕ^2 (the ratio of kinetic energy in the streaming motion to the thermal energy of the streaming electrons) generally leads to an increase in the magnitude of Landau instability or damping.

The theory developed in this dissertation was then applied to various relevant ionospheric noise. Many characteristics of the lower hybrid resonance (LHR) noise bands discovered by the Alouette I satellite suggest that the theory can explain their generation. This is true particularly in the case of polar LHR noise. However, the polar LHR noise

has an upper frequency limit very close to the LHR frequency whereas the theory gives one many orders of magnitude higher. The method of measurement or some physical process may be the cause of the observed upper frequency limit. Other types of noise, such as auroral hiss with an upper frequency limit greater than 500 kHz, have also been discussed. Briefly, this thesis has presented a theory which has features which agree with many ionospheric noise observations.

Future work based on the theory given here could investigate growth rates at frequencies much lower or much higher than the LHR frequency. For example, the computer program was used to show that an instability can occur below the oxygen ion (O^+) cyclotron frequency. Also, different ionospheric compositions could be studied.

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APPENDIX I

COMPUTER PROGRAM TO CALCULATE GROWTH RATE OF
WHISTLER-MODE WAVESFortran Source List

```

C      GROWTH RATE OF WHISTLER-MODE WAVES
C
C      INPUT DATA FOR PLASMA COMPONENTS AND STREAMING PARTICLES
C      DIMENSION XM(10),SGN(10),DENS(10),ZP(10),ZC(10),ZC2(10)
C      DATA ECG,CVEL,BOLTZ/4.80286E-10,2.99793E10,1.38044E-16/
C      PI = SNGL(3.14159265359D+0)
C      SCALEF = 10. * 2.302585
C      INPUT IN CGS UNITS
C      A. NUMBER OF SPECIES INVOLVED
C      B. MASS (GM), SIGN OF CHARGE (+1 OR -1), AND NUMBER DENSITY
C      (1/CC) OF EACH SPECIES
C      C. BACKGROUND MAGNETIC FIELD (GAUSS), STREAMING VELOCITY
C      (CM/SEC), ENERGY RATIO OF STREAM, PARALLEL/PERPENDICULAR
C      TEMPERATURE OF STREAMING PARTICLES, LOWER HYBRID
C      RESONANCE FREQUENCY (HZ), AND HIGHEST ORDER OF
C      CYCLOTRON INSTABILITY CONSIDERED
C      D. FREQUENCIES--INITIAL, INCREMENTAL, AND FINAL (NORMALIZED
C      BY THE LHR FREQUENCY)(INCREASING)
C      E. WAVE NORMAL ANGLES--INITIAL, INCREMENTAL, AND FINAL
C      (DEGREES) (DECREASING FROM RESONANT ANGLE)
66      READ (5,1) NSPEC
1       FORMAT (I3)
        DO 2 I=1,NSPEC
2         READ (5,3) XM(I),SGN(I),DENS(I)
3         FORMAT (E15.5, F4.0, E10.3)
        READ (5,4) B0,SVEL,PHI2,TRATIO,FREQLH,N
C      N IS THE HIGHEST ORDER OF CYCLOTRON INSTABILITY CONSIDERED
4         FORMAT (5E10.3,I3)
        READ (5,5) OMEGAS, OMEGAI, OMEGAF
5         FORMAT (3E10.3)
        READ (5,6) THETAS, THETAI, THETAF
6         FORMAT (3E10.3)
C      THETA VALUES INPUT ARE IN DEGREES
C      CALCULATE CONSTANTS FROM INPUT DATA
        WLH = 2.* PI * FREQLH
        WLH2 = WLH * WLH
        ECG4P = 4. * ECG * ECG * PI
        BETA = SQRT ((0.5 + 1./TRATIO) * PHI2)/SVEL
        DO 7 I=1,NSPEC
        ZC(I) = ECG * B0/(XM(I)*CVEL)
        ZC2(I) = ZC(I) * ZC(I)
7        ZP(I) = ECG4P * DENS(I)/XM(I)
C      FIND THE TWO UNKNOWN DENSITIES (NEG) AND COMPUTE THEIR VALUES
C      THEY ARE FOUND FROM CHARGE NEUTRALITY AND S = 0 AT THE LOWER

```

```

C      HYBRID (LH) RESONANCE FREQUENCY.
      Y1 = 0.
      Y2 = 0.
      INSP = NSPEC - 1
      DO 9 I = 1, INSP
        J = I
        IF (DENS(I))10,9,9
9       CONTINUE
10      NJ = J
        NIX = NJ + 1
        DO 12 I = NIX, INSP
          K = I
          IF (DENS(I))13,12,12
12         CONTINUE
13        NI = K
          DO 16 I = 1, INSP
            IF (I-NJ)14,16,14
14           IF (I-NI)15,16,15
15          Y1 = Y1 + SGN(I) * DENS(I)
            Y2 = Y2 + DENS(I) / (XM(I)*(WLH2-ZC2(I)))
16         CONTINUE
            AI = 1./(XM(NI)*(WLH2-ZC2(NI)))
            AJ = 1./(XM(NJ)*(WLH2-ZC2(NJ)))
            DENS(NJ) = (SGN(NI)*(1./ECG4P-Y2)+AI*Y1)/(SGN(NI)*AJ
            - -SGN(NJ)*AI)
            DENS(NI) = (-1./SGN(NI))*(Y1 + SGN(NJ)*DENS(NJ))
C      PRINT DATA AT BEGINNING OF OUTPUT
      WRITE (6,70) NSPEC
70      FORMAT (7H0NSPEC=,I3)
      DO 500 I=1, NSPEC
500     WRITE (6,71) XM(I), SGN(I), DENS(I)
71      FORMAT (6H MASS=,1PE12.5,5X,6H SIGN=,0PF4.0,5X,9H DENSITY=,
      -1PE10.3)
      WRITE (6,72) B0, SVEL, PHI2, TRATIO, FREQLH, N
72      FORMAT (4H B0=,E10.3,5X,6H SVEL=,1PE10.3,5X,8H PHI**2=,
      -1PE10.3,5X,11H TPAR/PERP=,1PE10.3,5X,8H FREQLH=,1PE10.3,
      -5X,3H N=,I3)
      WRITE (6,73) OMEGAS, OMEGAI, OMEGAF
73      FORMAT(12H OMEGA INIT=,1PE12.3,5X,6H INCR=,1PE12.3,5X,
      -7H FINAL=,1PE12.3)
      WRITE (6,74) THETAS, THETAI, THETAF
74      FORMAT(12H THETA INIT=,2PE12.3,5X,6H INCR=,1PE12.3,5X,
      -7H FINAL=,2PE12.3)
      IF (DENS(NJ)) 67,64,64
67      WRITE (6,68)
68      FORMAT (74H THE VALUES FOR THE INPUT DENSITIES AND THE
      -LH FREQUENCY ARE INCOMPATIBLE./)
      GO TO 66
64      IF (DENS(NI)) 67,65,65
65      ZP(NJ) = ECG4P * DENS(NJ)/XM(NJ)
      ZP(NI) = ECG4P * DENS(NI)/XM(NI)
      WRITE (6,69)

```

```

69  FORMAT (70H0THE SCALE IS TO BE ADDED TO THE EXPONENTS FOR
    -POWER AND GROWTH RATES.)
    WRITE (6,90)
90  FORMAT(/6H THETA,10X,17H REFRACTIVE INDEX,2X,8H LAMBDA ,
    -8X,6H POWER, 9X,7H ENERGY, 4X,14H NORM GROWTH R,3X,
    -12H GROWTH RATE, 4X,6H SCALE//)
C    CALCULATE FREQUENCY-DEPENDENT PARAMETERS
    W = OMEGAS * WLH
34  W2 = W * W
    OMEGAP = W/WLH
    WRITE (6,80) OMEGAP
80  FORMAT ( 22H NORMALIZED FREQUENCY=,1PE10.3/)
    R=1.
    XL =1.
    P = 1.
    HXX=1.
    HXY = 0.
    DO17 I= 1,INSP
    R = R -ZP(I)/(W * (W + SGN(I)* ZC(I)))
    XL =XL -ZP(I)/(W * (W - SGN(I)* ZC(I)))
    P = P - ZP(I)/W2
    HXX = HXX + ZP(I)*(W2+ZC2(I))/((W2-ZC2(I))*2 )
17  HXY = HXY + W*SGN(I)*ZC(I)*ZP(I)/((W2-ZC2(I))*2 )
    HZZ = 2. - P
    S = (R + XL)/2.
    IF (P) 77,77,75
75  WRITE (6,76)
76  FORMAT (15H P IS POSITIVE.)
    GO TO 33
77  IF (S) 81,81,83
81  WRITE (6,82)
82  FORMAT (15H S IS NEGATIVE.)
    GO TO 33
83  D = (R - XL)/2.
C    CALCULATE ANGLE-DEPENDENT PARAMETERS
    THETAR = ATAN2(SQRT(-P),SQRT(S))
    THETAR = THETAR * 180. /PI
    IF (THETAR-THETAS)19,19,18
18  THETA = THETAS
    GO TO 20
19  THETA = THETAR - THETAI
20  THETA = THETA * PI/180.
    XCOS = COS(THETA)
    XSIN = SIN(THETA)
    XCOS2= XCOS**2
    XSIN2= XSIN**2
    A = S * XSIN2 + P *XCOS2
    B = R * XL*XSIN2 + P*S*(1.+XCOS2)
    F = SQRT(((R*XL-P*S)*XSIN2)**2 + 4.*P*P*D*D*XCOS2)
C    REFRACTIVE INDEX AND PARAMETERS
    RIN2 = (B-F)/(2.*A)
    RIN = SQRT (RIN2)
    XK = RIN * W /CVEL

```

```

      XKZ =XK * XCOS
      XKX =XK * XSIN
      XLAMBD = (XKX/(ZC(NSPEC)*BETA))*2 /(2.*TRATIO)
      IF (XLAMBD - 88.029692) 56,54,54
54    THETA = THETA * 180./PI
      WRITE (6,55) XLAMBD , THETA
55    FORMAT (34H0LAMBDA EXCEEDS 88.029692, LAMBDA=,
-E13.4,5X,7H THETA=,2PE15.6)
      GO TO 38
56    V = SGN(NSPEC) * D /(RIN2 - S)
      Y = RIN2 * XSIN2/(RIN2 * XSIN2 - P)
C    CALCULATE MODIFIED BESSEL FUNCTIONS I WITH SUBROUTINE BESIK
C    COULD ADD ASYMPTOTIC FORMULA FOR MODIFIED BESSEL FUNCTION FOR
C    LAMBDA GREATER THAN 88.029692, IF REQUIRED LATER
      DIMENSION BIN(100)
      CALL BESSEL (BIN0,BIN,N,XLAMBD )
      DIMENSION BINP(100)
      CALL BESSP (BINP0,BINP,N,XLAMBD,BIN0,BIN)
C    CALCULATE POWER TRANSFER
      DIMENSION PKN(100), ALPHA(100), ALPHA2(100), ZMN(100)
      JJ = 1
      NN = 0
      ZMNC = ZP(NSPEC)*BETA/(TRATIO*16.*SQRT(PI)*ABS(XKZ)*WLH)
21    IF (NN)23,22,23
22    XLN = 2.* EXP(-XLAMBD )*BIN0
      XLNP = 2.* EXP(-XLAMBD )*BINP0
      GO TO 24
23    NNA = IABS(NN)
      XLN = 2. * EXP(-XLAMBD )*BIN(NNA)
      XLNP= 2. * EXP(-XLAMBD )*BINP(NNA)
24    ZNN = NN
      RN = V * XLAMBD + Y*(W+ZNN*ZC(NSPEC))/ZC(NSPEC) -ZNN
      GN = (XLN/XLAMBD )*(V*V*(ZNN*ZNN+XLAMBD**2 ) + RN*RN)-
-2.*XLNP*V*RN
      ZMN(JJ) = ZMNC * (W-XKZ*SVEL + ZNN*ZC(NSPEC))*
-(1.-TRATIO))*GN/W
      ALPHA (JJ) = (W-XKZ*SVEL+ZNN*ZC(NSPEC))*BETA/XKZ
      IF (JJ-1) 58,57,58
57    IJK = ALPHA(JJ)**2/SCALEF
      ISCALE = -10 * IJK
      ZIJK = IJK
58    ALPHA2(JJ) = ALPHA(JJ)**2 - ZIJK * SCALEF
      PKN (JJ) = ZMN(JJ) * EXP (-ALPHA2(JJ))
      JJ = JJ + 1
      IF (NN)26,25,27
25    NN = NN + 1
      GO TO 28
26    NN = -(NN-1)
      GO TO 28
27    NN = -NN
28    IF (N+1-NN)21,30,21
30    PSP = 0.
      NTOT = 2*N+1

```

```

DO 31 I=1,NTOT
31  PSP = PSP + PKN(I)
C    CALCULATE ENERGY
    ENERGY = (RIN2*V*V + RIN2*(1.-Y)**2 *XCOS2 + HXX*(1.+V*V)-
-4.*HXY*SGN(NSPEC)*V + HZZ*Y*Y*XCOS2/XSIN2)/(16.*PI)
C    CALCULATE GROWTH RATE (NORMALIZED AND REAL)
    GROWTH = - PSP/(2.*ENERGY)
    REGROW = GROWTH * WLH
    WRITE (6,35) (PKN(I),I=1,NTOT)
35  FORMAT (32H          PKN (I=0,1,-1,2,-2,...)=,1P10E12.3)
    THETA = THETA * 180./PI
C    OUTPUT WAVE NORMAL ANGLE (DEGREES), REFRACTIVE INDEX, LAMBDA,
C    POWER TRANSFER (DIMENSIONLESS), WAVE ENERGY (DIMENSIONLESS),
C    GROWTH RATE (NORMALIZED BY LH FREQUENCY), GROWTH RATE (1/SEC),
C    AND THE SCALE FACTOR
    WRITE (6,37) THETA,RIN,XLAMBD,PSP,ENERGY,GROWTH,REGROW,ISCALE
37  FORMAT (2PE15.6, 1P6E15.4, I15 )
38  THETA = THETA - THETAI
    IF (THETA -THETAF)33,20,20
33  W = W + OMEGAI*WLH
    IF (W - OMEGAF*WLH)34,34,66
39  STOP
END

```

```

SUBROUTINE BESSEL (BIN0,BIN,N,XLAMBD )
DIMENSION BIN(N)
M = N-1
BIN0 = BESSI0(XLAMBD)
IF (XLAMBD - .01) 102, 102, 101
102 BIN(1) = XLAMBD/2.
    BIN(2) = XLAMBD**2/8.
GO TO 103
101 BIN(1)=BESSI1(XLAMBD)
    BIN(2)=BIN0 - 2.*BIN(1)/XLAMBD
103 IF (N.LT.3) RETURN
DO 100 I= 2,M
    AIX =I
100 BIN(I+1)=BIN(I-1)-2.*AIX * BIN(I)/XLAMBD
    RETURN
END

```

```

SUBROUTINE BESSP(BINP0,BINP,N,XLAMBD,BIN0,BIN)
DIMENSION BINP(N)
DIMENSION BIN (N)
BINP0 = BIN(1)
BINP(1)=BIN0 -BIN(1)/XLAMBD
IF (N.LT.2) RETURN
DO 200 I=2,N
    AIY =I
200 BINP(I)= BIN(I-1)-AIY*BIN(I)/XLAMBD
    RETURN
END

```


APPENDIX II

PARTIAL LIST OF SYMBOLS USED

A	cold plasma parameter
A_{ij}	constant
B'	cold plasma parameter
B	wave magnetic field
B_0	background magnetic field
C	constant
c	velocity of light
D	cold plasma parameter
E	wave electric field
e	charge of electron
e	as subscript or superscript, refers to background electrons
F	cold plasma parameter
f	frequency
I_n	modified Bessel function
i	imaginary number
j	as subscript or superscript, refers to background ions
$\hat{K}$	dielectric tensor
$\hat{K}_h$	Hermitian part of dielectric tensor
$\vec{k}$	wave (number) vector
L	cold plasma parameter
ℓ	wavelength
$\hat{M}$	mobility tensor
M_n	constant

m	mass
N	number density
n	order of Landau and cyclotron instability or damping
P	cold plasma parameter
P_a, P_k	power absorption
R	cold plasma parameter
R_n	constant
S	cold plasma parameter
$S(\alpha_n)$	function of complex error function
s	as subscript or superscript, refers to streaming particles
$T, T_{ }, T_{\perp}$	temperature
T_{ij}	constant
t	time
V_s	streaming velocity
v_z	particle velocity in z-direction
W	wave energy
x, y, z	coordinates
Z	atomic number
α_n	Stix parameter
β	inverse of the mean thermal velocity
ϵ	sign of charge (± 1)
θ	propagation angle (or wave-normal angle)
κ	Boltzmann constant

λ	Stix parameter
ν	refractive index
Π	plasma frequency
Σ	summation
ϕ	square root of ratio of the kinetic energy in the streaming motion to the thermal energy of the streaming electrons
Ω	cyclotron frequency
ω	angular frequency