Seismic Imaging of the Canadian Upper Mantle

by

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Abstract

Upper-mantle structure beneath Canada is investigated at both large and small scales, using the broadband, three-component data set of the Canadian National Seismograph Network. In the large-scale study, 500 surface-wave waveforms are processed using partitioned wave form inversion, yielding an S-wave velocity model of the Canadian upper mantle with resolution down to 400 km. This is the first surface-wave based 3-D model of Canada and offers improved resolution over the body-wave model of Grand (1994). The model displays two large-scale anomalies: a high-velocity structure beneath the Canadian shield and platform associated with the cratonic keel, and pronounced low velocities beneath the Cordillera. The velocity contrast between these features is strong (~ 10%) and sharp, occurring over distances of 600 km or less. High velocities persist to depths of ~ 250 km beneath the North American craton, which I interpret to represent the base of the continental keel. Moderately low velocities beneath the St Lawrence valley region and Labrador may be related to intracontinental volcanism and rifting, respectively.

At smaller scales, seismic discontinuities in the upper mantle lead to the scattering of teleseismic $P$ waves into $Ps$ conversions and free-surface multiples. The teleseismic $P$-wave coda thus provides constraints on small-scale, layered upper-mantle structure, including anisotropy and layer dip. I develop an efficient high-frequency method for characterizing upper mantle structure beneath a single station in the presence of interface dip and anisotropy, using it to model synthetic data and seismograms from CNSN station YKW3 (Yellowknife, NWT). In order to automate the recovery of upper-mantle models from CNSN datasets, I combine the forward-modeling scheme with the Monte Carlo-based neighborhood algorithm of Sambridge (1999) to invert receiver-function data. The combined approach is applied to synthetic waveforms, successfully constraining layer velocities and thicknesses, the degree and orientation of anisotropy, and the dip of interfaces. Applied to real data, the method provides strong constraints on crustal velocities and thicknesses beneath stations GAC (Quebec), SADO (Ontario), and ULM (Manitoba). In addition, it is used to determine the orientation of anisotropic fabric beneath YKW3, and the structural dip of the Cascadia subduction zone beneath station PGC (southern Vancouver Island).
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for Veronica
Chapter 1

Imaging the Canadian upper mantle

1.1 Introduction

The composition and evolution of the continental lithosphere is a field of increasingly active research in the Earth sciences. Discriminating between different lithospheric models requires an understanding of the thermal and chemical structure of the upper mantle beneath continents (Jordan, 1978, 1988; Anderson, 1979, 1990; Pari and Peltier, 1996). An important step toward gaining such an understanding may be made by using seismological techniques to obtain information about lithospheric structure at multiple scales, and comparing the results with surface geologic constraints. The Canadian landmass, containing the largest Precambrian shield on Earth juxtaposed against three Phanerozoic orogenic belts and the Cascadia and Alaskan subduction zones, is an excellent target for the investigation of continental upper mantle. Moreover, the existence of nearly a
decade's worth of broadband seismic data from Canadian arrays permits a variety of analyses to be undertaken leading to constraints on the nature of the Canadian upper mantle.

I begin this chapter with a brief review of seismic techniques for characterizing upper-mantle structures, followed by summaries of existing constraints on continental lithosphere in general and the Canadian upper mantle in particular, in order to indicate how my research relates to larger problems and the existing literature. Note that throughout this thesis, I will take the term "upper mantle" to exclude the transition zone, and thus refer only to that portion of the mantle between the Moho and the 410 km discontinuity (Ringwood, 1975).

1.2 Seismic techniques for upper-mantle studies

Seismic techniques for imaging or otherwise constraining upper mantle structure may be broken down in two ways: by source (active or passive) and by interaction type (transmission-based or scattering-based). The first distinction refers simply to whether the seismic source is anthropogenic (active-source) or earthquake-generated (passive-source). In the former case, the geometry of the experiment may be carefully controlled. The second categorization is based on the nature of the interaction between seismic waves and Earth structure (e.g. Bostock, 1999). Transmission-based methods record the integrated effect of seismic structure on a wave traveling a direct path from source to receiver as a single mode type (e.g. travel times for body waves, and frequency-
dependent phase delays or dispersion for surface waves), thereby constraining the large-scale structure of the region being sampled. Scattering methods consider the interaction of seismic waves with small-scale structure (discontinuity surfaces, or inhomogeneities of a scale comparable to the wavelength of the seismic wave). The resulting secondary phases (i.e. reflections or diffractions) provide constraints on the nature of small-scale inhomogeneities in the sampled region. Transmission and scattering-based techniques therefore provide complementary information about Earth structure.

The active-source techniques used for upper-mantle studies are larger-scale versions of the standard techniques used in oil and gas exploration: seismic refraction and seismic reflection. Seismic refraction (figure 1.1a) is primarily a transmission-based method providing information on lateral velocity variations and layer thicknesses, and has been successful in measuring velocities in the uppermost mantle. Several such studies have been performed in Canada through the LITHOPROBE program (Clowes et al., 1992). The seismic reflection technique records waves back-scattered at near-vertical angles from interfaces and other inhomogeneities, and exploits high data redundancy to obtain detailed images (figure 1.1b). Some profiles have employed large vibrator arrays and long listening times, and located substantial coherent reflectors in the upper mantle (e.g. Cook et al., 1999; more Canadian examples are given in section 1.4).

Due to the global propagation of energy from earthquakes with moment magnitudes above $\sim 5.5$, and the many possible geometries associated with earthquake seismology, earthquake seismograms contain a substantial number of arrivals which sample the upper
mantle (figure 1.2), and therefore are amenable to a considerable variety of relevant passive-source techniques. These methods divide into the above-mentioned transmission-based and scattering-based categories.

Conceptually, the simplest transmission-based technique is the analysis of body-wave (P- and S-wave) travel times, as compared to the predictions of a standard Earth model. In the teleseismic case, corrections are made for source-side contributions, with the result that positive/negative travel time residuals are associated with slow/fast receiver-side upper mantle. Canadian examples of this residual travel time analysis for distant (teleseismic) earthquakes are Buchbinder and Poupinet (1977) and Wickens and Buchbinder (1980). An analogous technique in long-period seismology involves the measurement of frequency-dependent phase delays of surface waves, which provide information on the one-dimensional velocity structure along the earthquake-to-receiver path (e.g. Hashizume, 1976).

Given geographically comprehensive databases of phase measurements from body and
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Figure 1.2: Two examples of three-component seismograms, annotated to show the different waves commonly used in analysis of crust and upper-mantle structure. Both traces were recorded at Yellowknife, Northwest Territories. Trace (a) is a magnitude 6.3, 10 km event from the Philippines (95.5° from the station) recorded on March 11, 1997; trace (b) is a magnitude 6.4, 33 km deep event from off the coast of Costa Rica (57.0° from the station) recorded on July 21, 2000.
surface waves (e.g. travel times, dispersion curves), it becomes possible to invert many
data simultaneously to obtain well-constrained models of three-dimensional Earth struc­ture, a process known as tomography (Aki et al., 1977). Tomography can be performed at
different scales, and using different seismic phases, to obtain models ranging from global
to regional in extent. Current global tomographic models employ a variety of data sets,
including $P$- and $S$-wave travel times (e.g. Bijwaard and Spakman, 2000), surface-wave
dispersion (e.g. Trampert and Woodhouse, 1996), normal-mode spectra (e.g. Ishii and
Tromp, 1999), and data sets incorporating multiple data types (e.g. Su and Dziewonski,
1997). Regional tomography is generally performed using one of three types of data.
Surface-wave data are useful in producing continental-scale models of the crust and up­per mantle, as in van der Lee and Nolet (1997). Second, body wave travel times from
seismc events recorded on dense regional arrays image the upper mantle down to the
transition zone in still finer detail (a Canadian example is Bank et al., 1998). Finally,
local-earthquake travel times may be employed to image the mantle if seismicity persists
to mantle depths, (such as at subduction zones, e.g. Masson and Delouis, 1997).

Another transmission-based technique used to investigate the upper mantle is shear­wave splitting (Silver and Chan, 1991; Vinnik et al., 1992), which usually employs the
$SKS$ phase (figure 1.2). The $SKS$ phase arrives in the upper mantle as an $S$-wave which
is polarized in the radial plane (assuming the lower mantle is largely isotropic), due to
its passage through the fluid outer core as an acoustic wave. If the mantle beneath the
recording station is azimuthally anisotropic (i.e. has directionally-dependent velocity),
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the SKS splits into two plane-polarized components. The temporal separation of these components is a measure of the degree of anisotropy, while their respective polarizations indicate the fast and slow directions of the anisotropic material. SKS does not, however, constrain the vertical extent of the anisotropy.

More detailed characterization of isotropic and anisotropic upper-mantle structure requires the use of scattering-based techniques. One successful method used to map mantle stratigraphy is the stacking and modeling of ScS reverberations (Revenaugh and Jordan, 1991). The transverse-component ScS phase and its higher-order multiples (ScS-ScS, etc.) are reflections between the surface and the core-mantle boundary, which decay slowly due to total reflection at both boundaries. These phases may be stacked in order to enhance precursors and postcursors to the main arrival, resulting from mantle layering. The result is a high-resolution one-dimensional profile of mantle reflectivity, although the region of horizontal averaging is extensive.

The highest resolution, upper-mantle imaging techniques employing earthquake sources are based on the scattering of higher-frequency P- and S-waves. Teleseismic receiver functions (e.g. Langston, 1979; Cassidy and Ellis, 1991), the most commonly applied of these techniques, are source-normalized recordings of the P-coda that reveal the scattering of the incident P-wave into Ps conversions and multiples by upper-mantle structure. Receiver functions are most commonly interpreted in terms of one-dimensional, isotropic layered-Earth models. However, given sufficiently large teleseismic data sets and/or recordings at multiple receivers, it is possible to obtain 2-D and 3-D structural
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constraints. Dense linear arrays of temporary broadband stations yield 2-D models of the upper mantle and transition zone, through either moveout-corrected stacking (e.g. Dueker and Sheehan, 1997) or, increasingly, true depth migration (Ryberg and Weber, 2000) and inversion (Bostock and Rondenay, 1999). In addition, large databases of events recorded at individual permanent stations provide, in some cases, sufficient back-azimuth and epicentral-distance sampling to resolve anisotropic and dipping structures (Bostock, 1998; Farra and Vinnik, 2000).

1.3 Overview of mantle structures in continental regions

A growing body of research into continental upper-mantle structure in recent years is leading to an understanding of the mantle structures which typify continental regions, and their relationship to overlying crustal geology. Expansive regions of high seismic velocity extending to depths of 250 km or more beneath Precambrian shields and adjacent platforms are well-resolved features in global tomography models (Masters et al., 1982; Woodhouse and Dziewonski, 1984; Grand, 1994; Fischer and van der Hilst, 1999). These high-velocity regions, believed to represent low mantle temperatures, are generally known as cratonic roots or keels, and the existence of an age relationship between root thickness and crustal age is a matter of current debate. Jordan (1978, 1988) suggests that the roots originated through advective thickening during orogenesis and continental assembly. In
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this process, the roots develop a more refractory composition, allowing them to translate coherently with overlying crust. Moreover, an associated density decrease serves to neutralize the negative buoyancy induced by low temperatures and further enhance the root’s high-velocity signature. A variant of the advective thickening model involves the imbrication of slabs of Archean oceanic lithosphere, which are believed to have been positively buoyant due to higher temperatures in the Archean mantle (Helmstaedt and Schulze, 1989; Hoffman, 1990; Abbott, 1991). Anderson (1990) argues, conversely, that the anomalies are not bound to the continents, but instead are dynamic features representing regions of downwelling, a thesis supported by the calculations of Pari and Peltier (1996). In contrast to continental roots, orogenic belts are frequently underlain by regions of low-velocity upper mantle, believed to be high-temperature and possibly partially molten (Silver and Chan, 1991). These two extremes of upper mantle velocity anomaly are sometimes found in immediate juxtaposition (e.g. Zielhuis and Nolet, 1994a, in which the high-velocity mantle of the Russian Basin abuts against low-velocity mantle underlying the Pannonian basin).

SKS splitting results have demonstrated that upper-mantle anisotropy is ubiquitous in continental regions (e.g. Silver and Chan, 1991). Variations in the degree of splitting and in the orientation of the fast direction have been interpreted in two ways. One school of thought holds that shear wave splitting is a consequence of asthenospheric crystal alignment resulting from absolute plate motion and associated mantle flow (Vinnik et al., 1992). Another viewpoint considers the anisotropic signal to arise primarily through
accumulated strain in the lithosphere (Silver and Chan, 1988), while more recent work has implied that a contribution from both factors is required (Silver and Savage, 1994; Fouch et al., 2000). However, as the near-vertical path of the SKS generally records only the vertically averaged anisotropy over the full mantle column, it is not suitable for the recovery of detailed information about finer anisotropic layering in the upper mantle. Observation of such layering requires the use of scattering-based methods.

Abrupt mantle velocity transitions between the Moho and the 410-km discontinuity have long been known to exist (Lehmann, 1961; Hales, 1969), and have been extensively cataloged using ScS reverberations (Revenaugh and Jordan, 1991). Bostock (1999) gives a review of these features as observed beneath continental lithosphere. The principal continental discontinuities to be widely observed are the Hales discontinuity, at a depth of ~60 km, which has traditionally been ascribed to the spinel/garnet phase transition (Hales, 1969; Revenaugh and Jordan, 1991), and the Lehmann discontinuity (ranging from 210 to 300 km depth), which has been attributed to either a transition between anisotropic lithosphere and more isotropic asthenosphere or a mechanical boundary within the lithosphere (Leven et al., 1981; Revenaugh and Jordan, 1991; Karato, 1992; Gaherty and Jordan, 1995). Recently, a correspondence between some shallow mantle discontinuities and ancient, underplated former oceanic lithosphere in the continental root has been suggested on the basis of correlations with surface geology, significant dip on some interfaces (Bostock, 1998), and anisotropic (~10 km thick) layering interpreted to be stranded oceanic crust.
1.4 A brief summary of previous Canadian upper-mantle studies

Due to the wide variety of tectonic environments represented by the Canadian landmass, including the largest Precambrian shield on Earth (figure 1.3), Canada is of particular interest for the study of continental mantle structures and their association with crustal geology. In addition to a general knowledge of the large-scale continental upper mantle obtained from the aforementioned studies, a number of specific constraints on the Canadian upper mantle have been obtained through both seismic and other geophysical studies. Studies of $P$ and $S$ teleseismic station residuals have revealed that the Canadian shield and Canadian Cordillera correspond to significant high and low upper-mantle velocity anomalies, respectively (Buchbinder and Poupinet, 1977; Wickens and Buchbinder, 1980), an observation supported by one-dimensional surface-wave modeling (Brune and Dorman, 1963; Hashizume, 1976; Wickens, 1977; Nakada and Hashizume, 1983).

The cratonic and Cordilleran velocity anomalies are prominent features of global tomographic models. The model of Grand (1994), based on a travel time inversion of body-wave data, is the highest-resolution $S$-velocity model currently available for Canada (figure 1.4), and resolves considerable detail in lateral structure. The regional, surface-wave based models of Alsina et al. (1996) and van der Lee and Nolet (1997) image portions of the cratonic root and adjacent mantle below the coterminous United States, but do not attain good resolution of structure beneath Canada due to limited
path coverage. More narrowly-focused regional teleseismic experiments have also yielded tomographic images of the upper-mantle, uncovering substantial local velocity anomalies attributed to hot-spot tracks and local upwellings (Bank et al., 1998; Frederiksen et al., 1998; Rondenay et al., 2000).

At individual stations, SKS measurements have revealed that anisotropy in the upper mantle is a common feature across a wide variety of Canadian tectonic environments (Cassidy and Bostock, 1996; Ji et al., 1996; Bank et al., 2000b). Receiver-function studies at permanent stations have located fine layering in the upper mantle at a number of locations (Cassidy 1995a, 1995b). Exploiting the large data set available from the Yellowknife Array, Bostock (1997, 1998) used teleseismic Ps conversions to examine
Figure 1.4: Four horizontal sections through an S-velocity image of the Canadian upper mantle, from the model of Grand (1994; updated model, pers. comm.) The dark high-velocity region centered on the Canadian shield preferentially underlies Precambrian crust.
upper-mantle structure beneath the Slave craton, and obtained indications of anisotropic layering, including features termed (in order of increasing depth) $H$, $X$, and $L$. The $H$ and $X$ features were tentatively attributed to shingled slabs from shallow, buoyant Archean subduction (cf. Helmstaedt and Schulze, 1989; Abbott, 1991), while the more sharply-dipping $L$ was ascribed to steeper subduction from the Proterozoic.

Active-source data sets from LITHOPROBE have also contributed to our understanding of the Canadian upper mantle. A reflection study by Cook et al. (1999) imaged spectacular, sharply dipping upper-mantle reflectors beneath the Archean Slave province, which are likely related to the $H$ and $L$ layers observed by Bostock (1998). Calvert et al. (1995) located a similar Archean mantle suture in reflection data from the Superior province. Refraction/wide-angle reflection data sets have been used to place further constraints on layering and velocity variations in the uppermost mantle. The 1995 Deep Probe experiment, which employed unusually large shots (up to 18,000 kg) and offsets (up to 3000 km), and was targeted at the upper mantle along a line from Yellowknife, NWT to New Mexico, observed a deep Moho and thick high-velocity lower crust beneath the Wyoming province and Medicine Hat Block (Henstock et al., 1998; Gorman, 2000). Wide-angle reflected energy from within the mantle on several Deep Probe sections has been modeled as relict subducted oceanic crust associated with collisional sutures on the borders of the Medicine Hat block with the Hearne and Wyoming provinces; in addition, an upper-mantle waveguide resulting from scattering bodies between 100 and 140 km has been detected beneath the study area (Gorman, 2000). Refraction profiles across the
Trans-Hudson Orogen (a Paleoproterozoic suture zone) detected high mantle velocities immediately below the Moho, as well as a zone of layered mantle beneath the central orogen between 75 and 158 km, interpreted to represent imbrication of eclogite and peridotite during orogenic thickening of the lithosphere (Hajnal et al., 1997; Németh and Hajnal, 1998). Fernandez Viejo et al. (1999) analyzed wide-angle reflections from a line crossing the Wopmay orogen and Slave province, and detected significant upper-mantle reflectivity consistent with the results of Cook et al. (1999). The collective implication is that thickening of the mantle lithosphere is ubiquitous beneath Precambrian orogens.

Further constraints on the Canadian upper mantle have been obtained using non-seismic methods. The magnetotelluric (MT) technique, which utilizes extremely low-frequency electromagnetic waves, is the only electromagnetic method to reliably image the mantle (Jones, 1999). A conductive mantle region beneath Vancouver Island was detected using MT and associated with fluids fluxed from the subducting Juan de Fuca plate (Kurtz et al., 1986). Vertical transitions in upper-mantle conductivity have been found to correspond with major upper-mantle discontinuities in north-central Ontario (Schultz et al., 1993), including, possibly, the lithosphere-asthenosphere boundary. Moreover, lateral transitions corresponding to tectonic boundaries have also been identified. Boerner et al. (1999) have documented an order of magnitude decrease in upper mantle conductivity (at depths of 100-250 km) between the Archean Rae-Hearne province and adjacent Paleoproterozoic crust. Resistive electrical lithosphere decreases in thickness from 350-400 km beneath the Slave province to 50-100 km beneath the Proterozoic Fort Simpson terrane.
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(Jones et al., 1999). Thus, MT provides evidence for a correspondence between crustal provinces (defined by age) and underlying upper mantle. Heat flow studies (Hyndman and Lewis, 1995, 1999; Lewis and Hyndman, 1998) have been used to define mantle thermal regimes in the Canadian Cordillera and adjacent shield and platform. These authors locate a sharp thermal boundary in the southern Cordillera, corresponding to the Rocky Mountain Trench, with unusually high mantle temperatures apparent west of the boundary. However, the heat flow measurements provide little constraint on the depth of thermal anomalies. Gravity measurements exhibit a similar insensitivity to the depth of anomalies; nonetheless, by invoking assumptions with respect to isostasy, gravity measurements and topography may be used to determine the elastic thickness of the lithosphere. Wang and Mareschal (1999) performed such calculations for the Canadian Shield, locating particularly strong and thick lithosphere in the Hearne Province and the Trans-Hudson Orogen.

Mantle xenoliths are fragments of upper-mantle rock brought to the surface by volcanism, and thus represent a direct sampling of the upper mantle. The mantle xenoliths found embedded in kimberlites and some alkaline basalts are often found to be little altered, particularly in kimberlites, which are rapidly emplaced. These xenoliths are therefore among the best indicators of upper-mantle geochemistry. A number of xenolith-containing kimberlites and basalts exist in Canada. Studies of Cordilleran xenoliths suggest that the upper mantle is hot, somewhat depleted, younger in age than the overlying crustal domains, and has in places been partially melted (Shi et al., 1998; Peslier et al.,
2000). By contrast, mantle xenoliths beneath the Slave province are indicative of cool upper mantle, with a thermal lithosphere at least 180 km thick (MacKenzie and Canil, 1999; Schmidberger and Francis, 1999; Russell and Kopylova, 1999).

1.5 Project overview and potential impact

A major impetus for my thesis research is the large data set available from the Canadian National Seismograph Network (CNSN), which has been continuously recording broadband three-component seismic data since 1992. The existence of this large, high-quality data set opens up the possibility of more comprehensive and detailed analysis of Canadian upper mantle structure than has been hitherto possible, at both continental and regional scales.

Prior to my work, a published continental-scale surface-wave tomographic model of the Canadian upper mantle did not exist. In chapter 2, I make use of the tomographic technique of partitioned waveform inversion (Nolet, 1990), which uses surface-wave records to produce smooth S-wave velocity models, even in regions of limited seismicity, such as the majority of Canada. Applied to the CNSN data set, it provides a model comparable to that of van der Lee and Nolet (1997) (who mapped upper-mantle structure below the coterminous United States) for the Canadian landmass, thus together providing good coverage of the entire North American continent, and constraining the geometry and nature of the mantle anomalies beneath the Canadian Shield and Cordillera.
CHAPTER 1. IMAGING THE CANADIAN UPPER MANTLE

The extensive archive of teleseismic P-wave data recorded at individual CNSN stations, along with the high degree of back-azimuth sampling resulting from Canada’s position in relation to global seismicity, afford an ideal opportunity to investigate mantle stratigraphy at a detailed scale. However, this endeavor requires more sophisticated analysis techniques than are generally available. The conventional method of receiver-function analysis generally employs a forward-modeling approach that assumes horizontal, isotropic layering. However, the CNSN data set shows in many instances clear evidence of anisotropy and dipping interfaces (e.g. in the Slave province, [Bostock, 1998], and in Cascadia, [Cassidy, 1995b]). A general ray tracer for 3-D anisotropic media could be used to model these data; however, the high degree of user input required by the shooting methods used in ray-tracing render this approach unattractive for venturing beyond forward-modeling into inversion. Therefore, in chapters 3 and 4, I present a new approach to receiver-function modeling that allows interface dip and layer anisotropy without requiring ray tracing. It is computationally inexpensive, relatively easy to implement, and a suitable candidate for use in an inversion technique.

Inversion of receiver functions for Earth structure is known to be highly non-linear, and therefore not easily amenable to solution via linearization. However, the number of model parameters required is generally low; therefore, non-linear sampling techniques are viable candidates for exploring the model space and determining what constraints the data place on the model. In chapter 5, I adopt the neighborhood algorithm of Sambridge (1999), which is fast, conceptually simple, and requires little tuning of parameters. Ap-
plying this method to CNSN data, I recover constraints on the fine-scale structure of the upper mantle beneath individual Canadian stations.

Combining the two methods of surface-wave tomography and receiver-function modeling, it becomes possible to place tight constraints on the nature of upper-mantle structures beneath the Canadian landmass. These results should contribute to an understanding of the origin of mantle anomalies underlying the Canadian Shield and Cordillera, and, by extension, the global history of the continental upper mantle.
Chapter 2

A tomographic image of the Canadian upper mantle *

2.1 Data set: the Canadian National Seismograph Network

The Canadian National Seismograph Network (CNSN), operated by the Geological Survey of Canada, comprises 29 broadband three-component digital seismographs in operation since 1992. (North and Basham, 1993) CNSN data used in this study were supplemented with data from 12 broadband and long-period IRIS stations in use since the mid-1980's to provide a comprehensive coverage of the Canadian landmass (figure 2.1), with a slight bias toward western Canada and a lower density of instruments in

*This chapter is an expanded version of Frederiksen et al., 2000
the north of the country. A full list of stations used is given in table 2.1. As the majority of the Canadian landmass is seismically quiescent, the event set consists primarily of earthquakes from the Pacific Coast, Alaska and the Aleutians, and the Atlantic and Arctic ridges, circumscribing the western, northern, and eastern flanks of the country (figure 2.2). Thirty-four events of magnitude 4.5 or greater were used (table 2.2), for a total of 500 seismograms, producing 5500 linear constraints on the upper-mantle velocity structure. This represents the vast majority of available high-quality broadband data, although some redundant Alaskan events were not included due to the high seismicity in that region.

Figure 2.1: Station locations and boundaries of major tectonic provinces and adjacent areas, superimposed on a grey-shade plot of topography over the Canadian landmass. White circles and squares represent CNSN and IRIS stations, respectively.

Figure 2.1: Station locations and boundaries of major tectonic provinces and adjacent areas, superimposed on a grey-shade plot of topography over the Canadian landmass. White circles and squares represent CNSN and IRIS stations, respectively.
Table 2.1: Stations used in the surface-wave inversion.

<table>
<thead>
<tr>
<th>Station</th>
<th>Type</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALE</td>
<td>IRIS broad-band</td>
<td>82.50°N</td>
<td>62.34°W</td>
<td>Alert, N.W.T.</td>
</tr>
<tr>
<td>BBB</td>
<td>CNSN broad-band</td>
<td>52.18°N</td>
<td>128.11°W</td>
<td>Bella Bella, B.C.</td>
</tr>
<tr>
<td>COL</td>
<td>IRIS broad-band</td>
<td>64.90°N</td>
<td>147.79°W</td>
<td>College Outpost, Alaska</td>
</tr>
<tr>
<td>COR</td>
<td>IRIS broad-band</td>
<td>44.58°N</td>
<td>123.30°W</td>
<td>Corvallis, Oregon</td>
</tr>
<tr>
<td>DAWY</td>
<td>CNSN broad-band</td>
<td>64.06°N</td>
<td>139.39°W</td>
<td>Dawson City, Yukon</td>
</tr>
<tr>
<td>DLBC</td>
<td>CNSN broad-band</td>
<td>58.43°N</td>
<td>130.02°W</td>
<td>Dease Lake, B.C.</td>
</tr>
<tr>
<td>DRLN</td>
<td>CNSN broad-band</td>
<td>49.25°N</td>
<td>57.50°W</td>
<td>Deer Lake, Newfoundland</td>
</tr>
<tr>
<td>EDM</td>
<td>CNSN broad-band</td>
<td>53.22°N</td>
<td>113.34°W</td>
<td>Edmonton, Alberta</td>
</tr>
<tr>
<td>FCC</td>
<td>IRIS broad-band</td>
<td>54.72°N</td>
<td>102.97°W</td>
<td>Flin Flon, Manitoba</td>
</tr>
<tr>
<td>FRC</td>
<td>CNSN broad-band</td>
<td>63.74°N</td>
<td>68.54°W</td>
<td>Iqaluit, Nunavut</td>
</tr>
<tr>
<td>GAC</td>
<td>CNSN broad-band</td>
<td>45.70°N</td>
<td>75.47°W</td>
<td>Glen Almond, Québec</td>
</tr>
<tr>
<td>GDH</td>
<td>IRIS long-period</td>
<td>69.25°N</td>
<td>53.53°W</td>
<td>Godhaven, Greenland</td>
</tr>
<tr>
<td>HRV</td>
<td>IRIS broad-band</td>
<td>42.50°N</td>
<td>71.55°W</td>
<td>Harvard, Massachusetts</td>
</tr>
<tr>
<td>INK</td>
<td>CNSN broad-band</td>
<td>68.30°N</td>
<td>133.52°W</td>
<td>Inuvik, N.W.T.</td>
</tr>
<tr>
<td>LMN</td>
<td>CNSN broad-band</td>
<td>45.85°N</td>
<td>64.80°W</td>
<td>Caledonia Mtn., New Brunswick</td>
</tr>
<tr>
<td>LMQ</td>
<td>CNSN broad-band</td>
<td>47.54°N</td>
<td>70.32°W</td>
<td>La Malbaie, Québec</td>
</tr>
<tr>
<td>LON</td>
<td>IRIS long-period</td>
<td>46.75°N</td>
<td>121.80°W</td>
<td>Longmire, Washington</td>
</tr>
<tr>
<td>MBC</td>
<td>CNSN broad-band</td>
<td>76.24°N</td>
<td>119.36°W</td>
<td>Mould Bay, N.W.T.</td>
</tr>
<tr>
<td>MOBC</td>
<td>CNSN broad-band</td>
<td>53.19°N</td>
<td>131.89°W</td>
<td>Morsbey, B.C.</td>
</tr>
<tr>
<td>OZB</td>
<td>CNSN broad-band</td>
<td>49.96°N</td>
<td>125.49°W</td>
<td>Mt. Ozzard, B.C.</td>
</tr>
<tr>
<td>PGC</td>
<td>CNSN broad-band</td>
<td>48.65°N</td>
<td>123.45°W</td>
<td>Sidney, B.C.</td>
</tr>
<tr>
<td>PHC</td>
<td>CNSN broad-band</td>
<td>50.70°N</td>
<td>127.43°W</td>
<td>Port Hardy, B.C.</td>
</tr>
<tr>
<td>PMB</td>
<td>CNSN broad-band</td>
<td>50.51°N</td>
<td>123.07°W</td>
<td>Pemberton, B.C.</td>
</tr>
<tr>
<td>PNT</td>
<td>CNSN broad-band</td>
<td>49.31°N</td>
<td>119.61°W</td>
<td>Penticton, B.C.</td>
</tr>
<tr>
<td>RES</td>
<td>CNSN broad-band</td>
<td>74.68°N</td>
<td>94.90°W</td>
<td>Resolute, Nunavut</td>
</tr>
<tr>
<td>RSNT</td>
<td>IRIS long-period</td>
<td>62.47°N</td>
<td>114.59°W</td>
<td>Yellowknife, N.W.T.</td>
</tr>
<tr>
<td>RSNY</td>
<td>IRIS long-period</td>
<td>44.54°N</td>
<td>74.52°W</td>
<td>Adirondack Mtns., N.Y.</td>
</tr>
<tr>
<td>RSON</td>
<td>IRIS long-period</td>
<td>50.85°N</td>
<td>93.70°W</td>
<td>Red Lake, Ontario</td>
</tr>
<tr>
<td>RSSD</td>
<td>IRIS long-period</td>
<td>44.12°N</td>
<td>104.03°W</td>
<td>Black Hills, South Dakota</td>
</tr>
<tr>
<td>SADO</td>
<td>CNSN broad-band</td>
<td>44.76°N</td>
<td>79.14°W</td>
<td>Sadowa, Ontario</td>
</tr>
<tr>
<td>SCHQ</td>
<td>CNSN broad-band</td>
<td>54.83°N</td>
<td>66.83°W</td>
<td>Schefferville, Quebec</td>
</tr>
<tr>
<td>SCP</td>
<td>IRIS long-period</td>
<td>40.79°N</td>
<td>77.86°W</td>
<td>State College, Pennsylvania</td>
</tr>
<tr>
<td>ULM</td>
<td>CNSN broad-band</td>
<td>50.24°N</td>
<td>95.87°W</td>
<td>Lac du Bonnet, Manitoba</td>
</tr>
<tr>
<td>WALA</td>
<td>CNSN broad-band</td>
<td>49.05°N</td>
<td>113.91°W</td>
<td>Waterton Lake, Alberta</td>
</tr>
<tr>
<td>WHY</td>
<td>CNSN broad-band</td>
<td>60.65°N</td>
<td>134.88°W</td>
<td>Whitehorse, Yukon</td>
</tr>
<tr>
<td>YKW3</td>
<td>CNSN broad-band</td>
<td>62.56°N</td>
<td>114.61°W</td>
<td>Yellowknife, N.W.T.</td>
</tr>
</tbody>
</table>
Figure 2.2: Epicenters of events used in the surface-wave inversion.
Table 2.2: Events used in the surface-wave inversion, with the number of stations recording each event.

<table>
<thead>
<tr>
<th>Date and time</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Depth</th>
<th>Stations</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1982/01/09 12:53:51</td>
<td>47.97°N</td>
<td>66.66°W</td>
<td>10 km</td>
<td>2</td>
<td>New Brunswick</td>
</tr>
<tr>
<td>1983/03/30 18:06:16</td>
<td>61.56°N</td>
<td>140.13°W</td>
<td>15 km</td>
<td>1</td>
<td>Southwest Yukon</td>
</tr>
<tr>
<td>1983/10/07 10:18:46</td>
<td>43.93°N</td>
<td>74.26°W</td>
<td>13 km</td>
<td>5</td>
<td>New York</td>
</tr>
<tr>
<td>1983/10/28 19:51:24</td>
<td>44.06°N</td>
<td>113.91°W</td>
<td>10 km</td>
<td>5</td>
<td>Idaho</td>
</tr>
<tr>
<td>1985/12/23 19:37:54</td>
<td>62.06°N</td>
<td>124.30°W</td>
<td>10 km</td>
<td>4</td>
<td>Southwest NWT</td>
</tr>
<tr>
<td>1986/01/31 16:46:42</td>
<td>41.63°N</td>
<td>81.11°W</td>
<td>10 km</td>
<td>2</td>
<td>Ohio</td>
</tr>
<tr>
<td>1987/12/13 21:05:03</td>
<td>74.36°N</td>
<td>93.31°W</td>
<td>10 km</td>
<td>3</td>
<td>Central Arctic</td>
</tr>
<tr>
<td>1988/11/25 23:46:04</td>
<td>48.11°N</td>
<td>71.18°W</td>
<td>28 km</td>
<td>1</td>
<td>Saguenay, Quebec</td>
</tr>
<tr>
<td>1989/12/25 14:24:33</td>
<td>60.08°N</td>
<td>73.44°W</td>
<td>5 km</td>
<td>3</td>
<td>NW Quebec</td>
</tr>
<tr>
<td>1994/06/30 00:43:10</td>
<td>85.56°N</td>
<td>88.49°E</td>
<td>10 km</td>
<td>15</td>
<td>W of Severnaya Zemlya</td>
</tr>
<tr>
<td>1994/10/27 19:51:24</td>
<td>40.55°N</td>
<td>125.53°W</td>
<td>10 km</td>
<td>23</td>
<td>Off Oregon coast</td>
</tr>
<tr>
<td>1995/03/14 17:33:50</td>
<td>54.77°N</td>
<td>161.33°W</td>
<td>35 km</td>
<td>23</td>
<td>Aleutians</td>
</tr>
<tr>
<td>1995/04/17 07:14:34</td>
<td>33.76°N</td>
<td>38.57°W</td>
<td>10 km</td>
<td>25</td>
<td>Atlantic</td>
</tr>
<tr>
<td>1995/06/21 20:24:14</td>
<td>51.10°N</td>
<td>130.51°W</td>
<td>10 km</td>
<td>7</td>
<td>S of Queen Charlottes</td>
</tr>
<tr>
<td>1995/12/08 07:41:12</td>
<td>72.64°N</td>
<td>3.48°E</td>
<td>10 km</td>
<td>19</td>
<td>SW of Svalbard</td>
</tr>
<tr>
<td>1995/12/30 02:07:16</td>
<td>63.21°N</td>
<td>150.60°W</td>
<td>137 km</td>
<td>21</td>
<td>Alaska</td>
</tr>
<tr>
<td>1996/03/20 22:22:42</td>
<td>51.95°N</td>
<td>30.01°W</td>
<td>10 km</td>
<td>21</td>
<td>Atlantic</td>
</tr>
<tr>
<td>1996/06/08 05:26:09</td>
<td>58.35°N</td>
<td>31.98°W</td>
<td>10 km</td>
<td>25</td>
<td>East of Greenland</td>
</tr>
<tr>
<td>1996/06/23 19:14:19</td>
<td>51.49°N</td>
<td>178.12°W</td>
<td>33 km</td>
<td>25</td>
<td>Aleutians</td>
</tr>
<tr>
<td>1996/06/22 16:47:14</td>
<td>75.81°N</td>
<td>134.61°E</td>
<td>10 km</td>
<td>23</td>
<td>Laptev Sea</td>
</tr>
<tr>
<td>1996/08/20 00:11:02</td>
<td>77.86°N</td>
<td>7.56°E</td>
<td>10 km</td>
<td>23</td>
<td>Svalbard</td>
</tr>
<tr>
<td>1996/09/29 10:48:18</td>
<td>64.78°N</td>
<td>17.56°E</td>
<td>10 km</td>
<td>23</td>
<td>Iceland</td>
</tr>
<tr>
<td>1996/12/09 11:28:47</td>
<td>29.85°N</td>
<td>42.85°W</td>
<td>10 km</td>
<td>24</td>
<td>Atlantic</td>
</tr>
<tr>
<td>1997/07/03 21:51:28</td>
<td>61.95°N</td>
<td>167.39°E</td>
<td>10 km</td>
<td>13</td>
<td>Kamchatka</td>
</tr>
<tr>
<td>1997/04/19 15:26:33</td>
<td>78.44°N</td>
<td>125.82°E</td>
<td>10 km</td>
<td>21</td>
<td>Laptev Sea</td>
</tr>
<tr>
<td>1997/06/27 04:39:52</td>
<td>38.27°N</td>
<td>26.70°W</td>
<td>10 km</td>
<td>21</td>
<td>Atlantic</td>
</tr>
<tr>
<td>1997/07/14 23:41:59</td>
<td>59.54°N</td>
<td>76.30°W</td>
<td>18 km</td>
<td>2</td>
<td>NW Quebec</td>
</tr>
<tr>
<td>1997/11/06 02:34:30</td>
<td>46.79°N</td>
<td>71.41°W</td>
<td>23 km</td>
<td>7</td>
<td>Southern Quebec</td>
</tr>
<tr>
<td>1997/12/06 08:06:48</td>
<td>64.90°N</td>
<td>88.08°W</td>
<td>10 km</td>
<td>14</td>
<td>NW Hudson's Bay</td>
</tr>
<tr>
<td>1998/02/16 23:53:19</td>
<td>52.71°N</td>
<td>33.67°W</td>
<td>10 km</td>
<td>17</td>
<td>Atlantic</td>
</tr>
<tr>
<td>1998/03/06 05:47:40</td>
<td>36.06°N</td>
<td>117.63°W</td>
<td>2 km</td>
<td>19</td>
<td>California/Nevada border</td>
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<tr>
<td>1998/06/25 22:51:58</td>
<td>50.26°N</td>
<td>130.02°W</td>
<td>10 km</td>
<td>18</td>
<td>Queen Charlotte Fault</td>
</tr>
<tr>
<td>1998/10/14 01:36:19</td>
<td>60.71°N</td>
<td>44.04°W</td>
<td>10 km</td>
<td>21</td>
<td>S tip of Greenland</td>
</tr>
</tbody>
</table>
CHAPTER 2. CANADIAN UPPER MANTLE TOMOGRAPHY

Seismograms for a typical event are shown in figure 2.3. Rayleigh waves naturally separate into the low-frequency, late-arriving fundamental mode, which typically contains most of the Rayleigh-wave energy, and earlier-arriving Rayleigh overtones known as higher modes. The fundamental-mode Rayleigh waves typically display high signal-to-noise ratio, while the amplitude of the higher modes is dependent on the event, epicentral distance, and radiation pattern. As the surface-wave technique used is based on the assumption that surface waves propagate laterally in the vertical source-receiver plane through a horizontally-averaged one-dimensional model, the overly ray-like energy arriving early in the wavetrain and out-of-plane scattered energy following the fundamental Rayleigh mode must be excluded. My data set therefore consists of a restricted time and frequency range within the CNSN dataset.

2.2 Technique: partitioned waveform inversion

The partitioned waveform inversion technique of Nolet (1990) consists of a two-stage process, in which (i) the full long-period waveform of each seismogram is inverted to recover a one-dimensional (1-D) average S-velocity model representing structure along the source-receiver path, and (ii) the full set of linear constraints derived from the 1-D models is inverted to recover a 3-D model on a Cartesian grid (van der Lee and Nolet, 1997). Dividing the problem in this fashion restricts the non-linearity inherent to seismic tomography to a one-dimensional inversion, allowing the three-dimensional calculation to be robust and inexpensive. However, careful attention must be paid to the waveform-
March 6, 1998 California earthquake, magnitude 5.2

Station

<table>
<thead>
<tr>
<th>WALA</th>
<th>PGC</th>
<th>PNT</th>
<th>PMB</th>
<th>BBB</th>
<th>ULM</th>
<th>DLBC</th>
<th>YKW3</th>
<th>WHY</th>
<th>KAPO</th>
<th>SADO</th>
<th>DAWY</th>
<th>INK</th>
<th>LMQ</th>
<th>SCHQ</th>
<th>RES</th>
<th>LMN</th>
<th>FRB</th>
<th>DRLN</th>
</tr>
</thead>
</table>

Figure 2.3: Vertical-component seismograms for a typical event, filtered to show long-period energy, trace-normalized, and displayed in order of increasing epicentral distance.
fitting process in order to produce accurate and reliable constraints.

In this study, the 1-D inversion was performed by fitting synthetic seismograms computed via summation of the first 18 Rayleigh modes. I fit the fundamental and higher-order modes in separate time windows, in order to accommodate their differing frequency content and signal-to-noise ratio (figure 2.4). Only 45% of the seismograms had usable higher-mode energy, due to noise and overly long paths (for which the higher modes were too ray-like); higher-mode fits were more frequent for western Canadian paths. Initial time windows were chosen based on group velocity: 3.4 to 4.1 km/s for the fundamental mode, in order to avoid fitting late-arriving scattered waves, and 4.1 to 5 km/s for the higher modes, in order to fit the SS and SSS phases at appropriate distance ranges, but without including overly ray-like deep-bottoming body waves. I then adjusted these time windows based on visual inspection of the traces in different frequency bands, in order to exclude all suspect (possibly scattered) energy. Frequency windows were chosen on a case-by-case basis, depending on the spectrum of the data and the behavior of the inversion; typical values used are 0.01-0.04 Hz for the fundamental Rayleigh mode and 0.02-0.06 Hz for the higher modes. The 1-D inversions were performed with respect to base models with differing crustal thickness and velocity, depending on the region sampled by each source-receiver path: a model with a 35-km thick continental crust was used for most central continental paths, while 25 and 30-km crusts were used for coast-parallel paths and paths including both continent and ocean. Primarily oceanic paths were inverted using models with oceanic crustal thickness ranging from 11 to 20 km, depending on the
age of the region sampled. Deviations in $S$-velocity (the parameter to which Rayleigh waves are most sensitive) were represented by ten triangular basis functions, along with an eleventh parameter representing crustal thickness. A similar parameterization was used by van der Lee and Nolet (1997).

I used the 1-D models produced by waveform fitting to derive path-averaged linear constraints on 3-D upper-mantle structure, in the manner described by van der Lee and Nolet (1997); the resulting linear constraints include depth resolution information. After testing several parameterizations, I selected, based on station spacing and the expected resolution of the model, a grid of 279,310 nodes spaced 150 km apart horizontally and 75 km vertically. Within the grid, $S$ velocity was expressed as a perturbation from a one-dimensional background model. Moho depth was included as an additional parameter in the inversion, in order to prevent crustal thickness variations from being mapped into upper-mantle structures. However, due to tradeoffs between crustal thickness and velocity, the crustal-thickness map is not considered sufficiently well resolved to be directly interpreted. Based on the density of path coverage, I applied a 300-km lateral smoother to the model (applied also to the Moho depth using the fixed scaling employed by van der Lee and Nolet [1997]), and selected an appropriate level of damping of the smoothed model from the model amplitude/data misfit tradeoff curve (figure 2.5; van der Lee and Nolet, 1997).
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October 7, 1983 New York event, recorded at station RSNT

March 6, 1998 California event, recorded at station DAWY

Figure 2.4: Two sample waveform fits (solid, observed; dashed, calculated), showing the division into fundamental Rayleigh mode and higher mode windows, and the recovery of a one-dimensional model (dashed; base model is solid). Source (diamond) to receiver (triangle) travel path is indicated in the inset.
Figure 2.5: Tradeoff between data misfit and model norm. Values near the “kink” in the curve are representative of the true misfit. The arrow indicates the tradeoff used in the final value.
2.3 Coverage and resolution

The resolution of the final three-dimensional model is controlled by the set of waveforms used to produce it. Vertical resolution is determined by the vertical resolving power of each waveform fit, which is a function of the signal-to-noise ratio, the path length, the inclusion or exclusion of higher modes (which sample greater depths than the fundamental mode), and the frequency and time windows used. The horizontal resolution is determined geometrically by the density and directionality of path coverage. A map view of 2-D path coverage is shown in figure 2.6. The path coverage is clearly denser in western and southern Canada, and the higher-mode coverage is stronger in the west. In addition, there is a slight dominance of northwest-southeast paths, indicating the likely direction of any structural smearing in the recovered model.

The standard method for evaluating the resolution of tomographic models is the synthetic resolution test. It consists of a three-stage process. First, a model of idealized Earth structure is constructed. Second, a synthetic data set is derived from this input model through forward modeling, using the same geometry (source and receiver placement) as the real data. Finally, the synthetic data set is inverted using the same parameters as were used for the real data; the degree of resemblance between the input and output models is taken to be a measure of the resolving power of the real data set. Different input models may be used to test different aspects of a data set’s resolution. I tested two input-model cases: a constant model and a checkerboard.

A constant-model resolution test uses an input model consisting of a static perturb-
Figure 2.6: Event-receiver energy paths and model grid for this experiment, divided into (a) higher-mode fits and (b) traces in which only the fundamental mode could be used. Small grey dots represent the model grid nodes.
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ation from the background velocity, applied over the whole model. Recovery of a static perturbation is essentially a test of coverage, although it also serves as an "optimistic" measure of resolution of large-scale structure. In this case, a positive S-velocity perturbation of 300 m/s was used. Figure 2.7 presents a series of depth sections through the input and recovered velocity models. The results show good lateral coverage over the Canadian landmass, with strong recovery of the anomaly down to 450 km depth. Some loss of amplitude is present in the less well-recovered regions of the continent, particularly in the northeast of the model where resolution appears to be lost below 350 km. Outside the region of station coverage, there is, as expected, little resolution.

In order to obtain a more conservative estimate of the resolution of the final model, I performed a checkerboard resolution test. An input model of 500-km-thick positive and negative 500 m/s anomalies, spaced 900 km apart, was forward modeled to produce a synthetic set of 1-D constraints. I added Gaussian random noise at a level slightly above the error indicated by the tradeoff curve of the real data (2.8 in scaled units, as opposed to 2.64 for the real data), and then inverted the synthetic data using the same parameters as used for the real data set. The input and output models are compared in figure 2.8. Recovery of lateral structure was good over the entire Canadian landmass in the upper levels of the model (50-200 km depth), although true amplitudes are better recovered in the west than in eastern Canada, due to the greater density of stations. There is little resolution south of 45°, beneath western Alaska, or offshore of British Columbia, where there is no station coverage and few crossing paths. Below 200 km, resolution west
Constant-model resolution test

Figure 2.7: Plan sections (at a series of depths) through the input and output models of a constant-model resolution test.
of 90° remains good, but some northwest-southeast smearing and a loss of amplitude become evident further east. This is, in part, a consequence of the stronger excitation of higher modes by subcrustal Alaskan and Aleutian events than by shallow Atlantic and continental events.

Overall, I consider the model's resolution to be interpretable to a depth of 400 km, with the caveat that NW-SE smearing and some underestimation of anomaly amplitudes are likely below 200 km east of 90°, particularly in northeastern Canada. Finer details of the model are likely to be interpretable west of Hudson's Bay. Generally speaking, recovered anomalies under-represent the input anomalies, which will also be the case for the real data. An additional limiting factor on resolution is the assumption of isotropic structure, which could lead to individual path fits producing distorted 1-D models due to unmodelled anisotropy. Fortunately, the good directional sampling of the paths used can be expected to prevent anisotropy from distorting large-scale structure, although some small-scale artifacts may be produced.

\section{Model}

The preferred model is shown in figures 2.9 and 2.10, plotted as a percentage of $S$-velocity deviation from the background model (model MC35, a model with a 35 km thick continental crust based on the model PEMC of Dziewonski et al., 1975). The maximum range of the anomalies is approximately $\pm 9\%$ at 100 km depth, decreasing to $\pm 1.5\%$ at 400 km. Two large-scale structures occupy the bulk of the well-resolved volume within
Figure 2.8: Checkerboard resolution test results. The input anomalies are rectangular prisms to 500 km depth in the Cartesian 3-D grid, and so are somewhat distorted in this geographical projection.
the model. The first of these is a large high-velocity anomaly underlying the Canadian shield and platform, with well-defined boundaries on the north, east and west sides. It reaches a maximum velocity perturbation of approximately +9%. If its base is taken to be the 2% contour, the anomaly reaches a maximum depth of ~ 250 km, which is well within the depth range of good resolution determined by the checkerboard test. The other prominent anomaly visible in my model is a relatively narrow, intense (up to -9% perturbation) low-velocity feature, underlying the western portion of the Cordillera. Its contrast with the high-velocity cratonic mantle is sharp (10% variation over ≤600 km), and it appears to persist to approximately 250 km depth. A pronounced southward widening in this feature to a depth of at least 150 km, coinciding with the Canada-U.S. border, is well resolved. The Moho model (figure 2.11) is, in my opinion, too poorly resolved to be interpreted, although there is some correlation with the receiver-function results of Bank et al. (2000a).

A number of smaller-scale features of this model are sufficiently well resolved to warrant interpretation. Moderately low velocities (to -2% perturbation) underlie the northern Appalachians, northeastern Quebec and Labrador. Although resolution there is limited, there are suggestions of a high-velocity anomaly beneath central Alaska. Finally, the large high-velocity anomaly beneath the Canadian Shield shows both internal structure (including a “dent” along the Cordilleran boundary at about 60°N by 120°W, and another beneath the St. Laurence valley, ~ 45°N by 77°W) and basal topography.

The principal features of this model are visible in other tomographic S-wave velocity
Figure 2.9: Plan sections through the final S-wave velocity model. The grey scale changes between the 150 and 200 km sections.
Figure 2.10: (a) Cross-sections through the final model, with substantial vertical exaggeration. (b) Locations of cross-sections.
models of this region. The global model of Grand (1994) displays the same large-scale features as my model (high velocities beneath the shield and platform, low velocities beneath the Cordillera) but is more coarsely parameterized in the vertical direction and displays a smoother high-velocity anomaly (figure 2.12). Overall anomaly amplitudes are considerably lower in Grand’s model (±6% perturbation at most), and the boundary between Cordillera and craton is more diffuse. The model of van der Lee and Nolet (1997) resolved many of the same features as my model in the vicinity of the Canada-U.S. border, including the sharp widening of the sub-Cordilleran anomaly south of the border and the moderately low velocity anomaly beneath the Appalachians.
2.5 Interpretation

2.5.1 Lithospheric root

I interpret the large high-velocity anomaly beneath the Canadian shield and platform to represent a lithospheric root underlying the stable portion of North America. Such features are a well-documented characteristic of Precambrian shields in global tomographic models (Polet and Anderson, 1995) although there has been some disagreement as to their thickness (Lerner-Lam and Jordan, 1987; Anderson, 1990). Comparable features have been observed in regional models beneath North America (Grand, 1994; van der Lee and Nolet, 1997), Europe (Zielhuis and Nolet, 1994a), and Australia (Simons et al., 1999) using the partitioned waveform inversion technique.

The resolution of my model is sufficient to delimit the lateral extent of the Canadian continental root at its western, northern, and eastern boundaries, while the southern edge lies well south of the Canadian border and has been documented by van der Lee and Nolet (1997). The western limit abuts on the Cordilleran anomaly. South of 60°N, in particular, the boundary is sharp and corresponds closely to the Cordilleran deformation front (figure 2.12). However, the boundary shifts slightly west of the Cordilleran front north of 60°N and becomes more diffuse. The northern limit is somewhat smeared (due to a scarcity of Arctic stations) and its sharpness is not well resolved, but the edge of the anomaly appears to correspond to the limit of the Arctic archipelago. The root's eastern edge is well-resolved, and is located substantially inboard of the continental margin,
beneath the Grenville province, as it is truncated by moderately low velocities beneath the Appalachians and northeastern Quebec.

The depth extent of the root warrants some consideration, due to the active debate regarding the thickness of continental keels. The base of my observed anomaly is not sharply defined (figure 2.9); however, the lateral coherence of the high-velocity anomaly disappears below 250 km, which approximately corresponds to the positive 2% velocity-perturbation contour. As my resolution is acceptable down to 400 km over most of the model, I conclude that the average thickness of the cratonic root beneath stable Canada is approximately 250 km. This is consistent with the thickness observed by Grand (1994), although the coarser vertical parameterization and contrasting body-wave sensitivity of Grand’s model obviates a direct comparison, and is also compatible with the root observed beneath the northern USA by van der Lee and Nolet (1997). By comparison with surface-wave tomography in other environments, the cratonic root beneath Proterozoic central Australia has been observed to be 250-300 km thick (Fischer and van der Hilst, 1999; Simons et al., 1999), while the higher velocities beneath the Baltic Shield relative to western Europe extend to at least 140 km depth (Zielhuis and Nolet, 1994a).

The high-velocity anomaly I observe exhibits significant internal lateral variation in both thickness and intensity, in contrast to the smoother anomaly observed by Grand (1994; figure 2.12). Interpreting the detailed topography of the base of the lithospheric root is complicated by the laterally variable recovery of anomaly amplitude and decline in lateral resolution observed at depth in the resolution test. With these caveats in
Figure 2.12: (a) 100-km section through the final velocity model, with tectonic boundaries indicated. (b) An equivalent section through the model of Grand (1994; updated model, pers. comm.)
mind, I take the positive 2% contour as a proxy for the root base, and observe two regions where the lithosphere appears to be unusually thick (>250 km): beneath the northern Wopmay orogen (northeastern Yukon, northwestern Northwest Territories; see figure 2.10, section D-D'), and beneath the northwestern Superior province and eastern Trans-Hudson Orogen (northern Manitoba; section B-B'). The highest velocities in the 100-km section (figure 2.12) are observed beneath the Rae-Hearne, Superior, and northern Trans-Hudson tectonic provinces, of which the Superior and Rae-Hearne are of Archean age. This high-velocity area, which frames Hudson’s Bay, also overlies the Manitoba region of increased root thickness. It is bounded by more moderate root velocities to the south (a resolution effect due to the lack of path coverage south of the border) and to the west, beneath the Archean Slave province and the Wopmay orogen. There are also suggestions of finer-scale variations, the robustness of which is questionable. In general, there does not appear to be a direct relationship between the thickness of the lithospheric root beneath the Precambrian domains and the age of the overlying crust.

2.5.2 Cordilleran anomaly

I interpret the low-velocity anomaly beneath the Cordillera to indicate the presence of warm mantle beneath the orogenic belt. The presence of low-velocity anomalies beneath recent orogens is well recognized in regional and global tomographic models. Evidence from shear-wave splitting and and heat flow suggests that such anomalies represent the presence of young, hot upper mantle (Sclater et al., 1980; Silver and Chan, 1991). The
North American Cordillera anomaly is observed in the models of Grand (1994), Alsina et al. (1996), and van der Lee and Nolet (1997). Based on the high amplitude (-10%) of the observed feature, Van der Lee and Nolet interpret the U.S. extension of this anomaly to represent the presence of partial melt and/or water in the upper mantle. However, the anomaly they observe is influenced by extension in the Basin and Range province and is therefore not directly comparable.

The anomaly observed here is very strong (exceeding -8%) in the southern Cordillera, where the most pronounced contrast lies at the southern limit of resolution, and becomes weaker and more diffuse to the north. The lowest velocities detected lie beneath the western portion of the southern Cordillera, centered on coastal British Columbia. Velocity variations beneath the southern Cordillera correlate fairly well with the active-source-derived lithospheric cross-section of Clowes et al. (1995), in which the lowest uppermost-mantle velocities are found beneath the southern Coast belt, while the lithosphere thickens considerably east of the Omineca belt. The anomaly's depth extent decreases northward, ranging from 250 km in the south (-2% contour; figure 2.10, profile F-F') to 150 km or less in the northern Cordillera. Partial melt is a possibility beneath the southern Cordillera, due to the high amplitude of the anomaly and the presence of substantial recent volcanism. However, I consider extensive partial melt in the northern Cordillera to be less likely due the low amplitude of observed S- and P-velocity anomalies and interpreted thermal variations (Frederiksen et al., 1998). The anomaly's southern and western boundaries are generally resolution-determined, and its northern boundary
is diffuse, presumably reflecting a gradation in mantle temperature, as the low velocities fade away in the northern Yukon. To the east, the boundary ranges from diffuse in the northern Cordillera (where it appears to coincide with the Tintina fault; see figure 2.12) to very sharp in the southern Cordillera, where the anomaly extends eastward, once at \( \sim 55^\circ\text{N} \), and again at the U.S. border (\( \sim 45^\circ\text{N} \)). This latter widening is also present in the model of van der Lee and Nolet (1997).

The eastern boundary of the southern Cordilleran anomaly is remarkable for its sharpness. It lies in the best-resolved section of my model, where recovery of amplitudes is good, and displays a total velocity contrast of 10% over a distance of 500-600 km at 100 km depth. Comparable mantle velocity contrasts beneath major boundaries between Precambrian shields and Phanerozoic crust have been observed in Australia (Zielhuis and van der Hilst, 1996; Simons et al., 1999) and Europe (Zielhuis and Nolet, 1994a, 1994b).

In Australia, the Tasman line represents the contrast between the Archean-Proterozoic craton and the Phanerozoic rocks of eastern Australia. This line corresponds well with a boundary between fast and slow mantle domains, which possesses an S-velocity contrast of approximately 6% at 100 km depth (Simons et al., 1999). The Tornquist-Teissyre Zone (TTZ), a major northwest-southeast lineament across eastern Europe, represents the contact between Precambrian provinces (the Baltic shield and Russian platform) and the Phanerozoic regions of western Europe. The TTZ coincides with a major boundary between fast and slow mantle in the European tomographic model of Zielhuis and Nolet (1994a) which at its strongest point displays a 12% contrast between the Russian...
platform and the Pannonian basin at the 80-km level (Zielhuis and Nolet, 1994b).

Zielhuis and Nolet (1994b) calculate a heat conduction scale length of approximately 100 km for hot mantle accreted at 400 Ma, indicating that the boundary they observe at the TTZ is consistent with a thermal contrast. Performing the same calculation for the Canadian Cordillera, I calculate the characteristic distance of heat conduction

\[ L = \sqrt{\kappa t} \approx 70 \text{ km}, \]

where the thermal conductivity \( \kappa \) is taken to be \( 10^{-5} \text{ m}^2/\text{s} \), and the time \( t \) is taken to correspond to the initiation of Rocky Mountain orogenesis at 160 Ma (Price, 1994). With the comparable European boundary in mind, I interpret the sharp eastern boundary of the southern Cordilleran mantle anomaly to represent a thermal boundary between distinct mantle domains, resulting from the emplacement of hot, young upper mantle during the Cordilleran orogenesis. Support for this hypothesis accrues from heat-flow studies (Hyndman and Lewis, 1995; Lewis and Hyndman, 1998), in which an increase in heat flow is documented west of the Rocky Mountain Trench in the southern Cordillera, while the corresponding increase to the north is located well east of the Cordilleran deformation front. Although it is possible that the two mantle domains are chemically distinct, this is not required by the steepness of the observed contrast.

2.5.3 Other features

Beneath the Canadian termination of the Appalachian orogen and the eastern Grenville province (southeastern Quebec and the Atlantic provinces), I observe a moderately low-
velocity feature, which matches well with the low velocities observed beneath the Appalachians in the models of Grand (1994) and van der Lee and Nolet (1997). As I map only a small portion of the Appalachians, it is difficult to distinguish between generally low Appalachian mantle velocities (presumably orogenic in origin) and the feature related to the Great Meteor and Monteregean hotspot described by van der Lee and Nolet (1997) and Rondenay et al. (2000), although the presence of low velocities west of the Appalachians implies the presence of the latter. The boundary between the Appalachian anomaly and the craton is much more diffuse than the Cordilleran boundary (and the TTZ in eastern Europe, which is closer in age; Zielhuis and Nolet, 1994b), perhaps due to the disruption of the Appalachian mantle signature by Atlantic rifting. I also observe moderately low velocities beneath northeastern Quebec and Labrador, at the northern end of the Grenville tectonic province (figure 2.11). A similar feature is visible in Grand’s (1994) model, although the low velocities are stronger in my model, and penetrate further inland. I interpret these low velocities to represent the effect of recent (80-50 Ma) rifting along a failed arm in the Labrador Sea (McWhae, 1981; Srivastava and Tapscott, 1986); a similar mechanism is suggested for easternmost Australia (Zielhuis and van der Hilst, 1996).

Subducting lithosphere is frequently identified with substantial high-velocity anomalies in tomographic models (e.g. Lebedev et al., 1997). Two subduction zones lie within the area of coverage of my model. I therefore expect to sample the Pacific slab, descending beneath Alaska, and the Juan de Fuca slab, converging with southern B.C. and the
U.S. Pacific Northwest at the Cascadia subduction zone. The Pacific slab is probably responsible for the high-velocity anomalies observed beneath Alaska below 150 km, although it lies within a region of the model that is susceptible to extensive smearing and therefore its configuration cannot be interpreted. The Cascadia slab, which lies within a well-resolved region of the model, does not produce a high-velocity anomaly; it is equally absent from the $S$-velocity models of Grand (1994) and van der Lee and Nolet (1997), although it appears as a fairly weak high-velocity anomaly in the $P$-velocity model of Bostock and VanDecar (1995). As the Juan de Fuca Ridge is close to the subduction zone (less than 400 km), I expect the Juan de Fuca slab to be unusually warm and thin, and therefore to represent little or no $S$-velocity perturbation at the scale of this study.

### 2.6 Principal conclusions

Through use of partitioned waveform inversion, I have imaged the Canadian upper mantle with reasonable resolution down to 400 km. I observe the root underlying the Canadian shield to represent a strong high-velocity anomaly whose base is approximately 250 km deep, with significant internal structure. I believe my estimate of the root’s thickness to be more accurate than those obtained in previous body-wave models (e.g. Grand, 1994), due to the absence of vertical smearing in surface-wave tomography. The root is bounded at its western margin by a strong low-velocity anomaly underlying the Canadian Cordillera, the southern portion of which is likely influenced by the presence of partial melt. The boundary between high-velocity cratonic keel and low velocities in the southern Cordillera
is very sharp, and is consistent with the juxtaposition of hot and cold material beneath the Cordillera and shield. In addition, more minor low-velocity anomalies beneath the Appalachian orogen and Labrador may be related to intracontinental volcanism and rifting, respectively. My model extends results of previous surface-wave studies north of the coterminous United States and provides a characterization of mantle structure beneath northern North America with improved depth resolution relative to the body-wave model of Grand (1994).
Chapter 3

A modeling technique for upper-mantle teleseismic waves in the presence of dip and anisotropy *

3.1 Introduction: existing techniques

Two seismic modeling methods capable of handling anisotropy are currently in common use: the reflectivity technique and ray theory. Anisotropic reflectivity approaches (Booth and Crampin, 1983; Fryer and Frazer, 1984; Levin and Park, 1997; Thomson, 1997) are restricted to 1-D structures and are computationally expensive. 3-D ray tracing (e.g. Červený, 1972, Kendall and Thompson, 1993), relies on a high-frequency approxima-

*Material in this chapter was published in condensed form as Frederiksen and Bostock, 2000.
tion of the wavefield, and can handle non-horizontal interfaces. However, it generally requires shooting or bending techniques to isolate the correct ray trajectory, and user input to avoid complications due to triplications. In this chapter, I present a computationally inexpensive, high frequency method for modeling teleseismic wave propagation in homogeneous, plane layered, anisotropic dipping structures that does not require ray tracing.

3.2 Basis: Diebold’s travel-time equation

In the general case, determining a ray-theoretical seismic travel time requires knowledge of the correct trajectory of the ray, obtained through ray tracing. However, in the case of certain symmetric model geometries, the travel time may be determined in a simplified manner, without requiring knowledge of the ray’s lateral position. The classic example of this is the case of a planar, horizontally layered medium, which is completely symmetric along its horizontal axes. Diebold (1987) demonstrated that some features of the horizontally-layered case may be generalized to the case of arbitrarily dipping layers, and that, as a consequence, travel times in such a medium may be calculated directly, without recourse to ray tracing.

The travel-time equation of Diebold (1987) is an analytic expression for the travel time along an individual ray between source and receiver through a medium comprising isotropic plane-dipping layers in three dimensions. As suggested by Richards et al. (1991), this equation is extensible to generalized ray theory and can provide the basis
for an efficient ray-theoretical seismic modeling technique. Diebold’s formula, derived for isotropic media, is written in the notation of Richards et al. (1991) as

\[ T = -p_A \cdot X_A + p_B \cdot X_B + \sum_j (\xi_{a_j} + \xi_{b_j}) z_j \quad (3.1) \]

where \( p_A \) and \( p_B \) are the horizontal slownesses of the ray at the source and receiver, respectively; \( X_A \) and \( X_B \) are distance vectors from an arbitrary vertical reference line to the source and receiver, \( z_j \) is the thickness of layer \( j \) at the vertical reference line, and \( \xi_{a_j} \) and \( \xi_{b_j} \) are the downgoing and upgoing vertical slownesses in layer \( j \) (figure 3.1). The vectors \( X_A \) and \( X_B \) are parallel to the interfaces on which the source and receiver, respectively, are located\(^1\) (taking them to be horizontal vectors as described in Richards et al. [1991] is erroneous; see Aldridge, 1992).

In the teleseismic case, for a single station, it is convenient to approximate the incoming wavefront \((P, S\) or \(SKS)\) as a plane wave with known slowness vector. I begin by re-deriving an analog of Diebold’s equation for the teleseismic case using simple vector notation and demonstrate that, as suggested by Hake (1986), the principles behind Diebold’s equation are applicable to anisotropic media.

\(^1\)If the source and receiver are not located on true interfaces, imaginary interfaces with no velocity contrast may be introduced.
Figure 3.1: Geometry of the vectors used in the travel-time equation of Diebold (1987; equation 3.1) for a reflected phase. The dashed line is the vertical reference line at which layer thicknesses $z_i$ are defined.
3.3 Derivation of travel-time equation in the presence of anisotropy

We consider a teleseismic (plane) wave front incident from below upon a medium consisting of dipping, planar and possibly anisotropic layers (figure 3.2). The phase slowness vector $\mathbf{p}$ is perpendicular to the wavefront (and, in the isotropic case, parallel to the ray); its length represents the reciprocal of the phase velocity of the wave (Červený, 1972). According to ray theory, the travel time $t$ along a ray $\Gamma$ from the base of a general model to a receiver at the surface may therefore be written as

$$t = \int_{\Gamma} \mathbf{p} \cdot d\mathbf{l}.$$  \hfill (3.2)

In the isotropic case, $\mathbf{p}$ is parallel to $\Gamma$, and so this reduces to $t = \int_{\Gamma} p \, dl$. In a medium consisting of homogeneous layers, the path reduces to a series of straight-line segments, and the travel time may be expressed as

$$t = \sum_{i=1}^{N} t_i - \tau = \sum_{i=1}^{N} \mathbf{p}_i \cdot \mathbf{l}_i - \tau$$  \hfill (3.3)

where $\mathbf{p}_i$ is the phase slowness vector for the $i$th ray segment, $\mathbf{l}_i$ is the ray segment represented as a vector, and $\tau$ is a time shift to a common reference time, chosen to be the time when the plane wave sweeps across the base of the layer stack immediately below the station (point B in figure 3.2). Note that this notation differs from Diebold (1987). The number of segments $N$ is greater than or equal to the number of layers $L$,
and depends on the phase under consideration; for example, a first-order multiple in the topmost layer makes three transits of the layer, and so has \( N = L + 2 \).

Figure 3.2: Teleseismic ray-path decomposition for a transmitted phase in a three-layer model, simplified to two dimensions.

Introducing the assumption that the layer boundaries are planar, the ray length vector \( \mathbf{l}_i \) in any layer may be decomposed into vertical and interface-parallel portions \( \mathbf{z}_i \) and \( \mathbf{s}_i \) (figure 3.3). By Snell’s law, the component of phase slowness parallel to an interface is preserved across the interface; hence, the travel time in a single layer may be written

\[
 t_i = \mathbf{p}_i \cdot \mathbf{l}_i = -s_i \cdot \mathbf{p}_i - z_i \cdot \mathbf{p}_i + s_{i+1} \cdot \mathbf{p}_i = -s_i \cdot \mathbf{p}_i - z_i \cdot \mathbf{p}_i + s_{i+1} \cdot \mathbf{p}_{i+1}. \tag{3.4}
\]
CHAPTER 3. ANISOTROPIC TELESEISMIC MODELING

The time correction $\tau$ may be decomposed in a similar fashion into interface-parallel and wavefront-parallel components, with the latter making no contribution to the travel time; hence,

$$\tau = p_b \cdot l_b = -s_1 \cdot p_b + w_b \cdot p_b = -s_1 \cdot p_b = -s_1 \cdot p_1.$$  \hfill (3.5)

Figure 3.3: A single ray segment, decomposed into vertical and interface-parallel components.

Inserting expressions (3.4) and (3.5) into (3.3) yields

$$t = \sum_{i=1}^{N} (-s_i \cdot p_i - z_i \cdot p_i + s_{i+1} \cdot p_{i+1}) + s_1 \cdot p_1,$$  \hfill (3.6)

which, as $s_{N+1} = 0$, simplifies to

$$t = -\sum_{i=1}^{N} p_i \cdot z_i = \sum_{i=1}^{N} \xi_i z_i$$  \hfill (3.7)

where $\xi_i$ is the absolute value of the vertical slowness component for segment $i$ and $z_i$ is the thickness of the layer corresponding to segment $i$. 
This travel-time formula is a summation of components that can be calculated directly, without resorting to ray tracing. The required vertical slownesses are obtained through application of Snell's law and the calculation of the system matrix for an anisotropic medium, as shown in the next section.

### 3.4 Calculation of amplitudes

Calculating the vertical slowness in each layer amounts to determining the independent ray modes of propagation within an anisotropic medium (Červený, 1972). I follow the conventional approach for the layered case (Woodhouse, 1974; Guest et al., 1993).

Assume that the elastic tensor $c_{ijkl}$ and the density $\rho$ are known. If we adopt a coordinate system such that $x_1$ and $x_2$ lie within the interface and $x_3$ is perpendicular to the interface our ray has just crossed (figure 3.4), then we know the two interface-parallel components $p_1$ and $p_2$ of the phase slowness $p$ (due to Snell's law). It remains then simply to calculate $p_3$. Since we are dealing with plane waves in plane layered media it is appropriate to use the plane wave reflection and transmission coefficients. The equations of motion away from the source may be written in the frequency domain as

$$\frac{dy}{dx_3} = i\omega A y$$

(3.8)

where $y$ is the 6-vector of displacements and tractions, and $A$, the system matrix for the medium, is determined by the elastic constants, density, and interface-parallel slowness, all of which are known quantities.
Specifically, the $6 \times 6$ system matrix $A$ is expressed in terms of the $3 \times 3$ matrices $C_{ij}$, $T$, and $S$, by the expression

$$A = \begin{pmatrix} T^T & C_{33}^{-1} \\ S & T \end{pmatrix}$$

(3.9)

where $C_{ij}$, $T$, and $S$ are given by

$$[C_{ij}]_{kl} = c_{ijkl}$$

(3.10)

$$T = \sum_{\alpha=1}^{2} p_{\alpha} C_{\alpha 3} C_{33}^{-1}$$

(3.11)
The eigenvalues of the system matrix are the interface-perpendicular slownesses \( p^{(n)}_p \) for six possible modes (upgoing and downgoing versions of the \( qP \), fast \( qS_1 \), and slow \( qS_2 \) waves). Thus, we may form slowness vectors which may be rotated into the standard coordinate system, giving the desired vertical slownesses \( \xi \). The eigenvectors are the values of \( y \) for each of these modes, and may also be rotated into the new system.

In order to calculate the reflection and transmission coefficients, we simply take \( y \) to be constant over the interface (by continuity of tractions and displacements). If \( N \) is the matrix of column eigenvectors, the scattering matrix between layers 0 and 1 is given by \( Q = N_0^{-1} N_1 \). If we divide \( Q \) into four \( 3 \times 3 \) submatrices \( Q_{ij} \), we may write the reflection and transmission matrices (where e.g. \( T_U \) represents the transmission of an upgoing wave; see Kennett, 1983) in the form

\[
\begin{pmatrix}
T_D & R_D \\
R_U & T_U
\end{pmatrix} =
\begin{pmatrix}
Q_{22}^{-1} & Q_{12} Q_{22}^{-1} \\
-Q_{22}^{-1} Q_{21} & Q_{11} - Q_{12} Q_{22}^{-1} Q_{21}
\end{pmatrix}.
\]

(3.13)

Combined with the travel-time formula, this amplitude-calculation technique allows us to calculate the travel time and amplitude of any phase arising from an incident plane wave, and thereby to build up a seismogram by assembling all phases of interest. A displacement impulse-response seismogram \( u(t) \) comprising \( M \) phases, each consisting of \( N_i \) segments, may then be written as
CHAPTER 3. ANISOTROPIC TELESEISMIC MODELING

\[ u(t) = \sum_{i=1}^{M} \mathbf{F}_i \mathbf{w}_i \left( \prod_{j=1}^{N_i} S_{ji}^i \right) \delta(t - \sum_{k=1}^{N_i} \xi_{ik}^i z_k^i). \]  (3.14)

where \( S_{ji}^i \) is the appropriate reflection/transmission coefficient for phase \( i \) and the interface preceding segment \( j \), \( \mathbf{F}_i \) is the free-surface transfer matrix appropriate to phase \( i \) (which depends on the slowness vector in the uppermost layer; see e.g. Kennett, 1991), and \( \mathbf{w}_i \) is an upgoing \( 3 \times 1 \) wave vector with one non-zero element corresponding to the wavetype of the final segment of phase \( i \). No ray tracing is required, and the resulting travel times and amplitudes are exact within the usual assumptions inherent in ray theory. Note that the formulation applies to both directly transmitted waves and phases involving multiple reflection. Equation 3.14 is based on the assumption that all phases are known to reach the surface, and that corresponding eigenvectors have the same indices in successive layers. In practice, this requires an additional verification step.

3.5 Implementation and caveats

Due to the problem of determining whether a given phase reaches the surface, and the possibility of inconsistent eigenvector polarizations leading to mismatched phases, I do not generate seismograms directly using equation 3.14. Instead, the method for producing impulse-response seismograms is best expressed as an algorithm, as follows.

1. Fix slowness vector and amplitude at model base.

2. For each desired phase:

   (a) List layer and mode of propagation (\( P/qP \), \( SV/qS_1 \), or \( SH/qS_2 \)) of each segment.
CHAPTER 3. ANISOTROPIC TELESEISMIC MODELING

i. Determine next interface encountered by ray.

ii. Project slowness vector onto interface.

iii. If the slowness vector is oriented away from the interface, the ray does not intersect the interface; this phase is trapped, and should be ignored.

iv. If the layer above or below the interface is anisotropic, rotate its elastic tensor into layer-parallel coordinates.

v. Calculate system matrices above and below interface.

vi. Perform eigenvalue decomposition on system matrices and assemble reflection/transmission matrices.

vii. Check that the eigenvector polarization is consistent with the previous phase.

viii. Calculate slowness vector and amplitude of next ray segment.

(b) For each segment, starting with the lowermost:

(c) Check that the final slowness vector points upward; if not, the phase does not reach the surface, and should be ignored.

(d) Calculate surface displacement using free-surface transfer matrix.

(e) Calculate travel time using equation 3.7.

(f) Insert appropriate spike into synthetic impulse response.

3. Filter spike series or convolve with wavelet to form seismogram.

In practice, some of the required quantities involved may be calculated in advance and tabulated. In particular, the rotation of the elastic tensor $c_{ijkl}$ is time-consuming, but need only be done once per model; as well, if multiple phases are being calculated, only the segments which differ from the preceding phase require recalculation. Although this algorithm is a direct calculation, involving no iteration, it is important to note that the number of possible phases increases exponentially with the number of layers. Considering only transmitted phases, $L$ layers may transmit $3^L$ possible phases. However, not all phases will have significant amplitude, and it is my usual practice to exclude double
CHAPTER 3. ANISOTROPIC TELESEISMIC MODELING

P-to-S conversions (phases that convert from (quasi-) P to (quasi-) S and back), as the resulting amplitude is negligible. This reduces the number of possible transmitted phases in the incident-P case to $\sum_{i=1}^{L} 2^{L-i}$. I seldom consider multiples beyond first-order, for similar reasons. The program was implemented twice: once in MATLAB, to refine the algorithm, and once in FORTRAN 77. The FORTRAN version is faster than the MATLAB implementation by a factor of $\sim 20$.

The limitations of this method result from its underlying assumptions. Employing ray theory does not seem to be a source of error at the frequencies used, as I find the synthetics for horizontally-layered models to be essentially identical to those produced by reflectivity. In addition to the obvious restrictions of ray theory and homogeneous layers, the assumption that the incident wave may be approximated as planar restricts its utility to teleseismic waves in the upper mantle and crust, where the curvature of the wavefront is minimal (note, however, that by integrating over a range of slownesses, it is possible to construct the response to more general wavefields). Furthermore, the planar, dipping-layer assumption is reasonable for single stations where lateral sampling is limited, but it may be inadequate when considering array data in the presence of more general (e.g. folded) layering. Moreover, a steep interface ($\sim 50^\circ$ or more) at depth can present problems, as updip arrivals may miss it entirely due to a pinch-out, while downdip arrivals may arrive above the interface, rather than below.
Chapter 4

Modeling $P$ and $S$ upper-mantle response: scenarios and Canadian examples *

4.1 Overview of previous studies

Previous workers have investigated the effects of anisotropy and layer dip on teleseismic wave propagation separately, but not in combination (Langston, 1977; Cassidy, 1992; Levin and Park, 1998; Savage, 1998). Both factors lead to back-azimuthal variations in impulse response, involving travel-time and amplitude, as well as the rotation of $P$-$SV$ energy onto the transverse component. Hexagonal anisotropy with a horizontal...
symmetry axis (i.e. azimuthal anisotropy) typically produces four-lobed back-azimuthal patterns with 180° symmetry. However, this symmetry breaks as dip increases, producing a dominantly two-lobed (360°) pattern for 45° inclination (Levin and Park, 1998). An isotropic dipping layer will produce a similar 360° periodicity in azimuth, and also causes rotation of energy from an initial $P$ or $SV$ arrival onto the transverse component. This latter effect is only reproducible in laterally homogeneous anisotropic layers when the uppermost layer is anisotropic. Modeling of the teleseismic $S$ response has been restricted largely to simplified predictions of shear-wave splitting parameters for horizontally layered cases (e.g. Silver and Savage, 1994; Rümpker and Silver, 1998).

4.2 Modeling $Ps$ conversions: interaction of dip and anisotropy

To examine the interaction of anisotropy and layer dip, I consider two classes of models (figure 4.1): wedge models, consisting of an isotropic crust underlain by a wedge of anisotropic upper mantle, and slab models, in which a dipping anisotropic layer is embedded in isotropic ambient mantle and overlain by an isotropic crust. Fixed properties of the models are given in table 4.1. As the number of independent variables in these models is large, I fix the thickness of the layers, assume hexagonal symmetry and a prolate phase-velocity surface in the anisotropic layers, and vary only the interface dip and the trend and plunge of the fast anisotropic axis. The anisotropy is parameterized as a percentage
variation in \(P\)- and \(S\)-wave velocity in the manner described by Farra et al. (1991), while the remaining parameter \(\eta\) (which constrains the shape of the velocity surface between its extremes) is fixed at a value of 1.03. As in all synthetics in this paper, the filtered impulse responses have been transformed into the \(P-SV-SH\) wave vector domain as defined by the horizontal slowness of the incident wave in the underlying half-space, to separate different phases more clearly. This procedure would be equivalent to eliminating the premultiplication by \(F_i\) in equation 3.14 if the model were horizontally layered. In the dipping-layer situation, the procedure involves replacing \(F_i\) with the transfer matrix appropriate for the half-space slowness. Patterns detected in the \(P\), \(SV\), and \(SH\) components are similar to those visible in the vertical, radial, and transverse components, respectively. However, presentation in this “quasi” wave vector domain provides a clearer separation of the principal phases than the displacement domain does, improves the visibility of low-amplitude phases (e.g. \(Sp\) conversions), and is also my preferred approach to the display and analysis of real data (see next section).

Figure 4.2 shows the filtered impulse responses of variants of the wedge model to an incident teleseismic \(P\) plane wave, plotted as a function of back azimuth for a fixed incident horizontal slowness (0.05 s/km, equivalent to an incidence angle of 24\(^\circ\)). From left to right, the three models plotted have anisotropic fast axes oriented west, north, and north with a 30\(^\circ\) plunge, while the base of the wedge dips 30\(^\circ\) N. The \(P\) and \(SV\) components of these models are largely unaffected by the anisotropy, in part due to the presence of crustal multiples that mask the \(Ps\) conversion from the base of the anisotropic
Figure 4.1: The simple wedge (left) and slab (right) structures used in forward modeling.

Table 4.1: Properties of synthetic models, defined as in Farra et al. (1991) for a hexagonally-symmetric medium: \( z \) is the layer thickness, \( \alpha \) and \( \beta \) are directionally-averaged \( P \)- and \( S \)-velocities, \( \rho \) is density, and anisotropy is given as a percentage velocity deviation from the average. \( \alpha \) and \( \beta \) are in m/s, \( \rho \) in kg/m\(^3\), \( z \) (thickness) in km. The percent anisotropy is the same for \( P \) and \( S \); the remaining parameter \( \eta \) is fixed at 1.03.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Wedge model</th>
<th>Slab model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( z )</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>6540</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>8100</td>
</tr>
<tr>
<td>3</td>
<td>—</td>
<td>8100</td>
</tr>
<tr>
<td>4</td>
<td>—</td>
<td>8100</td>
</tr>
</tbody>
</table>
wedge at \( \sim 31 \) s reference time. The wedge-base conversion is only faintly visible on the \( SV \) component (mainly in the third panel). The \( SH \) component is more complex than \( SV \). In particular, the direct \( P \) arrival appears in a two-lobed pattern, an effect produced by the dipping interface, as indicated by the fact that its polarity remains unchanged after a \( 90^\circ \) rotation of the anisotropic symmetry axis. By contrast, the character of the Moho \( Ps \) conversion is anisotropy-controlled, and the symmetry axis' orientation controls the polarity of its four-lobed pattern. The pattern loses its \( 180^\circ \) symmetry in the dipping-axis case. The \( Ps \) conversion off the base of the wedge is the most complex, in that it displays a dip-controlled travel-time variation, a polarity pattern governed by the fast-axis location, and a doubled arrival indicating the splitting of the shear wave into \( qS_1 \) and \( qS_2 \) phases. The back-azimuthal behavior of the multiples appears to be dictated by the conversion interface (Moho or free surface); thus, \( Pp_{mP} \) (not converted) and \( Pps_{ms} \) (free-surface conversion) display two-lobed patterns, while \( P_{ms}, Pp_{ms}, \) and \( Ps_{ms} \) (Moho conversions) are four-lobed.

Figure 4.3 shows \( P \) impulse responses for four variants of the slab model. The first three models contain a slab dipping \( 30^\circ \) to the north, with the anisotropic symmetry axis trending east, north, and north plunging \( 30^\circ \), respectively. The fourth model has a horizontal slab in which the fast axis plunges \( 30^\circ \) to the north. As before, the main evidence of dip and anisotropy is on the \( SH \) component, due to the high amplitudes of crustal multiples on the \( SV \) and \( P \) components. The slab has parallel top and bottom interfaces with identical velocities above and below, and therefore transmitted phases that
Figure 4.2: Impulse response to incident $P$-wave at 0.05 s/km horizontal slowness (equivalent to an incidence angle of 24°), for three variants of the wedge model. Each trio of panels represents a back-azimuth suite of three-component seismograms rotated into a $P$-$SV$-$SH$ domain (see text for details); white regions are positive arrivals, black are negative arrivals. Timing of phases in this and subsequent figures is relative to the time when the incident plane wave crosses the deepest interface below the station; see figure 3-2.
do not convert within the slab are not rotated onto the transverse component, because the upper interface cancels the effect of the lower interface. The principal features of note are therefore the $P_s$ conversions produced by the top and bottom slab boundaries. These conversions form a pair of oppositely-signed arrivals with identical back-azimuthal dependence. The two horizontal-axis models differ only in polarity; pointing the axis downdip alters the effective anisotropy to near-vertical rays and so changes the amplitude pattern. A horizontally layered model with identical anisotropy displays a similar amplitude pattern, but is distinguishable through its lack of significant travel-time variation in back azimuth, a difference which may be difficult to detect in real data.

An interesting question that arises from this modeling is whether pathological cases exist where interface dip and anisotropy are practically indistinguishable to $P_s$ conversions. Azimuthal or near-azimuthal anisotropy produces a diagnostic four-lobed $SH$-component pattern in back azimuth; however, an axis of symmetry that dips at $\sim 45^\circ$ produces a two-lobed pattern reminiscent of the effect of a dipping interface (Levin and Park, 1997). Figure 4.4 shows $P_s$ conversions from two highly dissimilar models: a 5% anisotropic, laterally homogeneous 10 km thick lower crust whose axis of symmetry dips 55$^\circ$ to the north, sandwiched between isotropic crust and mantle (left panel), and a high-velocity, 10 km thick lower-crustal wedge whose upper and lower interfaces dip 15$^\circ$ to the north and south, respectively (right panel.) The only apparent difference in polarity pattern between the resulting impulse-response panels is the rotation of direct $P$ that occurs only in the presence of layer dip. Furthermore, the back-azimuthal variation in $P_s$
Figure 4.3: Impulse response to incident $P$-wave at 0.05 s/km horizontal slowness ($24^\circ$ incidence), for three variants of the slab model.
travel time differs in phase by $180^\circ$ between the two models. These distinctions might be difficult to isolate in real data.

These modeling results show that the interaction between layer dip and anisotropy can be complicated, even for simple models. For $Ps$ conversions, crustal multiples are a problem in that they mask upper-mantle conversions. Most accessible information concerning anisotropy is contained in the amplitudes of the transverse component, while layer dip dominates back-azimuth travel-time variations. Transverse-component polarity shifts are useful in locating the horizontal orientation of the anisotropic axis of symmetry, although its dip is more difficult to constrain. Note also that in more complicated situations where two or more anisotropic layers are considered, the polarity crossovers do not, in general, exhibit such a simple relation to axis orientation.

### 4.3 Sp conversions and shear-wave splitting

Modeling teleseismic $S$ impulse responses requires knowledge of the polarization of the incident wave. With this in mind, I have calculated impulse responses for incident $SV$ and $SH$-polarized waves. Figure 4.5 shows the incident-$SV$ and incident-$SH$ responses for the wedge model, with a basal dip of $30^\circ$ to the north and the anisotropic fast axis pointing downdip. Notable features are the splitting of the $S$-wave into two arrivals, and the $Sp$ conversion off the wedge base. The direct $S$ shows the $qS_1 - qS_2$ split in the anisotropic layer in the form of two separate arrivals with a significant travel-time difference, forming a four-lobed back-azimuthal pattern with energy rotation into and out
Figure 4.4: A demonstration of how, in the case of a near-vertical incident wave, the $P$ impulse response of a flat anisotropic model can resemble that of a model with dipping layers. The line graphs at the bottom of the figure compare $SV$ and $SH$ amplitudes (relative to $P$) for the first-arriving conversion. The models are described in the text; the incident slowness used was 0.05 s/km.
of the $P-SV$ plane, and little evidence of a dip effect except in the faint rotation of the direct $S$ phase onto the $P$ component. $Sp$ conversions originate at both the Moho and the base of the wedge. The former is faint and shows dip-controlled polarity, while the latter is also weak, displays considerable back-azimuthal variation in travel time, and shows a two-lobed amplitude pattern. The corresponding slab model displays similar features (figure 4.6; also with a downdip symmetry axis). Double arrivals (top and bottom of slab) are visible on the $Sp$ conversion, as well as a very small split in the transmitted $S$.

$S$-wave impulse responses contain information on anisotropy in the form of shear-wave splitting, and in addition, $Sp$ conversions are strongly sensitive to layer dip. However, this sensitivity may prove difficult to observe due to their low amplitudes and the ubiquitous presence of signal-generated noise in the form of the $P$-coda.

### 4.4 Application to data from the Yellowknife array

The Yellowknife Array (YKA), Northwest Territories, Canada, has recorded three-component data at four broadband stations since 1989. It is well-situated with respect to global seismicity, in that it records teleseismic events from a broad range of back azimuths (figure 4.7). Bostock (1998) analyzed a comprehensive $Ps$ data set from the YKA and found evidence of both anisotropic layering and dipping interfaces in the upper mantle. The YKA $P$ and $S$ data set thus provides a useful test for my modeling technique.

Figure 4.7 displays the full set of $Ps$ data for one of the YKA stations (YKW3), binned, transformed into the $P-SV-SH$ domain as described earlier, and source-normali-
Figure 4.5: SV and SH impulse responses of the downdip-axis variant of the wedge model, at an incident slowness of 0.09 s/km (equivalent to an incidence angle of 24°). Note the limited back-azimuth window (100 – 250°) in which the Sp arrival from the dipping interface appears. Outside this window, the phase is supercritical.
Figure 4.6: $SV$ and $SH$ impulse responses at an incident slowness of 0.09 s/km (24° incidence), for the downdip-axis variant of the slab model.
Figure 4.7: Deconvolved $P$ data (a,c) from YKA station YKW3 (Bostock, 1998) compared to the filtered synthetic impulse response (b,d) of the model described in table 4.2. Data from events of similar back azimuth and epicentral distance are binned together; the bins are sorted by counting outward from the stations in successive back-azimuth swathes. Map in (f) shows the number of traces in each bin, while plot (e) relates the bin number to back azimuth. Principal phases are indicated on the synthetic. The $SV$ component (a,b) is dominated by multiples $Pp_m s$ and $Pps_m s$, while the $SH$ component (c,d) is dominated by the anisotropic upper-mantle features $H$, $X$ and $L$. The travel-time misfit of $Pps_m s$ ($\sim 20$ s) is a result of a tradeoff between $P_s$ and $S_p$ results and may be indicative of a more complex Moho than presented in the model.
zed. Source normalization is performed through simultaneous deconvolution of all seismograms within a given bin as described in Bostock (1998). The bin separation is chosen to afford an approximately uniform sampling in slowness of the incident $P$-wave. Approximately 361 seismograms were used to generate the $SV$ and $SH$ panels, each comprising 143 impulse responses. I display these responses sorted in an order determined by counting outward from the station in successive back-azimuthal swathes, with the geographic distribution of bins shown in figure 4.7f. The $SV$ and $SH$ impulse response panels (figures 4.7a,c) are compared to synthetics (figures 4.7b,d) developed through trial and error with good qualitative agreement. Table 4.2 presents the model used.

Table 4.2: Properties of YKW3 model. All anisotropic layers have horizontal symmetry axes, and all interfaces are horizontal, with the exception of layer 9, whose upper and lower interfaces strike at $-70^\circ$ and dip $25^\circ$, and whose symmetry axis plunges $25^\circ$.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness (km)</th>
<th>$\alpha$ (m/s)</th>
<th>$\beta$ (m/s)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>% anis.</th>
<th>Axis trend (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36</td>
<td>6200</td>
<td>3650</td>
<td>2600</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
<td>8100</td>
<td>4500</td>
<td>3500</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>3 (H)</td>
<td>10</td>
<td>8100</td>
<td>4500</td>
<td>3500</td>
<td>5.5</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>41</td>
<td>8100</td>
<td>4500</td>
<td>3500</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>5 (X)</td>
<td>5</td>
<td>8100</td>
<td>4500</td>
<td>3500</td>
<td>5.5</td>
<td>165</td>
</tr>
<tr>
<td>6 (X)</td>
<td>10</td>
<td>8100</td>
<td>4500</td>
<td>3500</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>7 (X)</td>
<td>5</td>
<td>8100</td>
<td>4500</td>
<td>3500</td>
<td>5.5</td>
<td>75</td>
</tr>
<tr>
<td>8</td>
<td>49</td>
<td>8100</td>
<td>4500</td>
<td>3500</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>9 (L)</td>
<td>5</td>
<td>8150</td>
<td>4530</td>
<td>3500</td>
<td>5.5</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>---</td>
<td>8150</td>
<td>4530</td>
<td>3500</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

The principal features of the model are as follows: an isotropic 36 km thick crust
overlies a mantle in which layers of hexagonally symmetric anisotropic fabric are embedded. The first anisotropic layer (H) occurs at 80 km depth over a 10 km thick interval with horizontal axis pointing east. Cross-overs in polarity near 310° back azimuth on the $SV$ component, and at 180° and 270° on the $SH$ component, constrain the orientation of this axis. A pair of 5 km thick anisotropic layers separated by 10 km of isotropic material occur at 131 km depth, reproducing the feature marked as X. The anisotropy axis orientations of the upper and lower layers at 165° and 75°, respectively, are required to reproduce this complex arrival if the anisotropy is assumed to be prolate (i.e. the fast direction is assumed to be the symmetry axis). At 193 km depth a final anisotropic layer (L) dips at 25° to the northeast. The nature and dip of these layers are consistent with the interpretation that they represent the remains of ancient subducted oceanic crust. As this model differs little from the one in Bostock (1998), it will not be discussed further here.

Analysis of $Sp$ data is somewhat more complicated due to i) the source-dependent polarization of the incident $S$-wave; ii) higher levels of signal-generated noise (see, e.g., Vinnik and Romanowicz, 1991; Bock, 1994), and iii) low energy levels at frequencies above 0.2 Hz from all but very deep events. The data set is therefore greatly reduced in numbers relative to $P$. I have confined my attention to two of the best events available for YKA and detailed in Table 4.3. Source-dependent polarization precludes effective use of deconvolution procedures, because each $S$ trace results from two impulse responses and two source functions ($SV$ and $SH$). Instead, I transformed $S$ seismograms into the $P-SV$-
CHAPTER 4. P AND S UPPER-MANTLE RESPONSE

SH wave vector domain as defined by the horizontal slowness predicted from a 1-D Earth model, and estimated the incident SV and SH waveforms by taking windows around the direct S-arrival on the SV and SH components. I convolved these waveform estimates with P impulse responses derived from the model in Table 4.2 and summed the convolved traces to form a synthetic P-component for comparison with the data. Figure 4.8 shows this comparison. The Moho Sp conversion is strong, and there are indications that the mantle structure modeled for P contributes significantly to the response. In particular, the structure termed H appears to be responsible for the reasonable match on the Izu-Bonin event between -5 and -10 s whereas deeper structure (X) near 131-141 km depth matches signal near -15 s for the Afghanistan event. Note also that the H structure produces impulse responses for SV and SH in the Izu-Bonin event that partially cancel when summed. A better fit to these data was achievable, but at the expense of the fit to the Ps data. It is clear from the P component data in figure 4.8 that, even for these high quality S events, high levels of signal-generated noise pose a significant limitation to retrieval of mantle structure using S.

Table 4.3: Events used for Sp conversion analysis at station YKW3.

<table>
<thead>
<tr>
<th>Region</th>
<th>Date</th>
<th>Time</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Depth</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Izu-Bonin Trench</td>
<td>96/06/26</td>
<td>03:22:03</td>
<td>27.73°N</td>
<td>139.75°E</td>
<td>469 km</td>
<td>5.5</td>
</tr>
<tr>
<td>Afghanistan</td>
<td>94/06/30</td>
<td>09:23:21</td>
<td>36.33°N</td>
<td>71.13°E</td>
<td>227 km</td>
<td>6.1</td>
</tr>
</tbody>
</table>
Figure 4.8: Two examples of $Sp$ conversion data fits, using events from Afghanistan and the Izu-Bonin Trench recorded at station YKW3 and described in table 3. The uppermost panel shows the $SV$ and $SH$ arrivals; the next superimposes the real and synthetic $P$ component. The lowermost two panels show the impulse responses and convolved $Sp$ synthetics due to $SV$ and $SH$ polarization components, respectively.
CHAPTER 4. P AND S UPPER-MANTLE RESPONSE

4.5 Conclusions

The forward modeling technique presented here shows promise for extracting lithospheric structure from $P$- and $S$-wave data sets. As the example from station YKW3 illustrates, the technique should be particularly useful for analysis of data at individual, 3-component, broadband stations where underlying lateral heterogeneity may be accurately modeled in terms of anisotropic, dipping layers. Because the approach is ray-based and does not require raytracing, it is sufficiently economical to be used as the basis for non-linear waveform inversion schemes. The development of such a scheme is the subject of the next chapter.
Chapter 5

Inverting the upper-mantle teleseismic response at a single station using the neighborhood algorithm

5.1 Justification: why a search technique?

The teleseismic $P$-coda records the scattering of an incident $P$-wave by crustal and upper-mantle structure. As a result, it contains information on the crustal and upper-mantle structure beneath the station, particularly after the influence of the source is removed through deconvolution, producing a so-called “receiver function”. The most common
approach to extracting structural information from receiver functions is simple forward
modeling (as was used, for example, in section 4.4). However, automated data fitting
through inversion of receiver function data is becoming increasingly common, despite the
fact that receiver functions present difficulties for standard inversion techniques.

The principal difficulties in receiver-function inversion are nonlinearity and nonu-
niqueness (Ammon et al., 1990). The scattering-based nature of receiver functions is
the source of much of the nonlinearity. Even under the assumption of a horizontally
layered medium, the travel time of an individual $Ps$ conversion depends on the depth
of the scattering interface as well as the velocities above it, in a manner that does not
easily approximate to a weighted sum of model parameters. If layers and interfaces are
permitted to dip, the nonlinearity becomes more acute.

Nonuniqueness in receiver-function inversion arises due to an inherent tradeoff be-
tween parameters. If we consider a layered Earth model, a $Ps$ conversion occurring at
the base of layer $l$ arrives at time $t_l = \sum_{i=1}^{l} \left( \xi_i^s - \xi_i^p \right) z_i$ relative to the arrival time of the
direct $P$, where $\xi_i^s$ and $\xi_i^p$ are $S$- and $P$-phase slownesses in layer $i$, respectively, and $z_i$ is
the layer thickness (see equation 3.7). Assuming that the layers in question are horizontal
and isotropic, the travel time becomes $t_l = \sum_{i=1}^{l} \left( \sqrt{\beta_i^{-2} - p_z} - \sqrt{\alpha_i^{-2} - p_z} \right) z_i$, where $\beta_i$
and $\alpha_i$ are layer $S$- and $P$-wave velocities and $p_z$, the incident horizontal slowness, is
constant in all layers. $Ps$ conversion travel times are thus dependent on layer thicknesses
and both seismic velocities, and phase travel times in a single receiver function are only
capable of constraining one of these parameters if the other two are held fixed. For an
interface which is sufficiently reflective to produce observable multiples (e.g. the Moho),
two parameters may be constrained (generally the layer thickness and the Poisson’s ratio,
which depends on $\alpha/\beta$). If multiple high-quality receiver functions with significantly dif­
ferent slownesses are available, the moveout of the conversions provides an independent
constraint. The amplitudes of conversions relative to direct $P$ deliver some additional
information, although amplitudes in real data are considerably less robust than travel
times.

The receiver-function inversion problem may be cast as follows: given a set of receiver-
function data, and a parameterization for an Earth model, search for a model or set of
models for which the fit to the data is optimal in some sense. Viewed in this fashion, the
available techniques may be classified into three categories: undirected searches, directed
searches using gradients, and directed searches that do not employ gradients. Undir­
ected searches comprise grid searches and Monte Carlo techniques, and involve finding
the misfits of a large set of models, either chosen systematically or generated randomly,
from which good-fitting models are chosen; such searches are the most explorative of the
model space, at the expense of efficiency. Gradient-based directed searches rely upon
linearization of the problem and the computation of Fréchet derivatives; such searches
are efficient for problems that are not highly non-linear, but do not explore the model
space and may diverge if a suitable starting model is not adopted. Gradient-free dir­
ected searching occupies a middle ground with respect to efficiency and exploration; such
searches are generally Monte Carlo-based, but exploit information from previous models
in the selection of new models. This category includes genetic algorithms, simulated annealing, and the neighborhood algorithm used here.

A grid search obviously offers the most comprehensive coverage of the model space. The computational expense of a grid search rapidly becomes unmanageable for large numbers of parameters, however, and so the method is seldom used to recover more than two or three parameters. Grid searches have been used to recover crustal thickness and average Poisson's ratio beneath southern California from receiver functions (Zhu and Kanamori, 2000), although their method employed stacking along moveout curves rather than waveform matching. Monte Carlo inversion permits the use of a somewhat larger number of parameters; Paulssen et al. (1999) generated 30,000 six-parameter models in order to constrain crustal structure beneath St. Petersburg, Russia. Neither of these methods is suitable for the examination of complex upper-mantle structures, due to the limited number of variable parameters that can be accommodated.

Linearized inversion has been applied to the receiver function problem by a number of researchers, despite problems with nonlinearity and nonuniqueness. True linear inversion assumes that the data represent weighted averages of the model parameters, where the weights are known as the data kernels. For the non-linear case, the model parameters are taken to be perturbations around a base model, allowing the problem to be treated as linear provided the perturbation is small. Although repeated shifts of the base model allow convergence, linearized inversion is prone to converging on local minima. In addition, the non-linearity becomes more acute if layer depth and dip are included as parameters.
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Calculating Fréchet kernels may be done fairly efficiently in the horizontally-layered case (Randall, 1994); however, in the presence of dip, calculating Fréchet kernels must be done by brute force (by perturbing each parameter in turn) and can be computationally expensive.

The inversion technique of Ammon et al. (1990) addresses nonlinearity and nonuniqueness problems by restricting the parameterization to flat, isotropic fixed-thickness layers, and inverting only for the S-wave velocity using propagator-based forward modeling. As well, inversions are performed for a series of randomly-perturbed starting models, yielding a set of recovered models which fit the data. Similar techniques have been used by other researchers to recover structure beneath Tien Shan (Kosarev et al., 1993), Germany (Kind et al., 1995), the Philippines (Besana et al., 1995), and China (Mangino et al., 1999). Zhang and Langston (1995) successfully recovered dipping structure beneath Belgium by combining linearized inversion and grid search approaches. S-wave velocities were inverted, while a grid search was performed for the strike and dip of the layering. Tomfohrde and Nowack (2000) stabilized the linearized inversion by filtering data in expanding frequency bands and alternately inverting for layer thickness and velocity, to recover Taiwanese crustal structure. Another means of stabilizing linearized receiver-function inversion is to incorporate other types of data. This is usually done implicitly by incorporating a priori knowledge from controlled-source or tomographic images into initial models. For example, Julià et al. (2000) presented a method for the explicit joint inversion of receiver functions and surface-wave dispersion curves.
Global directed-search optimization techniques strike a middle ground between linearized inversion and undirected searching in their approach to inverse problems. The two most widely used methods, genetic algorithms and simulated annealing, are based on analogies with natural systems: evolutionary biology and thermodynamics, respectively. Genetic algorithms employ a population of models, in which models with low misfits are more likely to reproduce into the next generation, through steps analogous to cross-breeding and mutation (Sambridge and Drijkoningen, 1992). As a result, successive “generations” (iterations) tend to converge toward high-fitting solutions, albeit with an adjustable degree of random exploration of the model space. The technique has been successfully applied to receiver-function inversion beneath Australia (Shibutani et al., 1996; Clitheroe et al., 2000) and Russia (Levin and Park, 1997). Simulated annealing is a related technique based on a probabilistic random walk which favors the vicinity of good-fitting models, subject to a “cooling schedule” in which the sampling density function is tightened as the inversion proceeds (Sambridge, 1999). I am not aware of any published application of simulated annealing to teleseismic receiver functions.

Both simulated annealing and genetic algorithms avoid many of the difficulties inherent in linearized inversion of receiver functions. As nonlinearity does not present major difficulties, the choice of model parameterization used is somewhat less critical, and (unlike the pure Monte Carlo case) a realistic number of parameters may be recovered. Although neither method is guaranteed to recover the global minimum misfit, they are less prone to being trapped in local minima than the linearized inversion method, and do
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not require the calculation of Fréchet kernels. Both techniques, however, require careful adjustment of multiple parameters in order to tune the algorithm for a particular problem: the binary representation of the model space, the population size, and the reproductive, crossover and mutation probabilities in the genetic-algorithm case, and the cooling schedule and probability density functions in the simulated-annealing case. This complexity is rooted in the algorithms' origins as analogies to natural systems. I have instead elected to apply a new algorithm (the neighborhood algorithm) developed by Sambridge (1999), which is based purely on geometry, and is therefore elegant, conceptually simple, has only two tunable parameters, and promises to be fast and robust. Sambridge (1999) applied the neighborhood algorithm to synthetic receiver function data in the horizontally layered, isotropic case; I apply it to real data in the presence of anisotropy and interface dip. The algorithm is presented in detail in the next section.

5.2 The natural-neighbor method

The goal of a global-optimization search routine is to concentrate Monte Carlo searching on models that are in some sense similar to models that previously have been found to have low misfit. The neighborhood algorithm of Sambridge (1999) replaces the complex methodologies used in genetic algorithms and simulated annealing with the notion of a geometric neighborhood. The presentation given here is summarized from Sambridge (1999).

Consider a bounded two-dimensional parameter space, over which we intend to min-
imize some misfit function \( f(x_1, x_2) \), where \( x_1^{\text{min}} < x_1 < x_1^{\text{max}} \) and \( x_2^{\text{min}} < x_2 < x_2^{\text{max}} \).

If we generate some number \( N \) of randomly-selected points within the model space, the remaining unsampled space may be divided into *neighborhoods*: regions of points which are closer to one sampled point than to any other sampled point. The neighborhood of each sampled point forms a convex polygon known as a *Voronoi cell* (Voronoi, 1908; Okabe *et al.*, 1992; figure 5.1a). If the function \( f(x_1, x_2) \) has been determined for every sampled point, we can select \( M \) points with low values of \( f \) in whose neighborhoods we wish to search further. By repeatedly generating \( N \) new points within the selected neighborhoods, and then selecting the \( M \) most desirable cells for further searching, it becomes possible to iterate to any desired level of precision.

The method outlined above requires a more rigorous presentation before it may be used in practice for a model space of arbitrary dimension. I begin by formally defining the Voronoi cell. Suppose we have sampled a set of \( N \) points \( m_1, \ldots, m_N \) in a bounded parameter space of dimension \( D \). The parameter space may then be divided into \( N \) Voronoi cells, each cell being associated with one of the sampled points. The \( i \)th Voronoi cell \( V(m_i) \) contains a given point \( x \) if and only if (Sambridge, 1999)

\[
V(m_i) = \left\{ x \left| \|x - m_i\| \leq \|x - m_j\|, \forall j \in 1, \ldots, N, i \neq j \right. \right\}.
\]

The distance between points \( \|x - m_i\| \) is only meaningful if the parameters constituting the elements of \( x \) and \( m_i \) are non-dimensional. This may be accomplished in a general way by imposing
Figure 5.1: An example of searching a two-dimensional parameter space using Voronoi cells. a) The first iteration: Voronoi-cell boundaries around a randomly generated set of points. In this case, we search further around the three best points (heavy outline). b) Iterating further: more points (asterisks) are randomly generated within the earlier-chosen cells, and the best three are selected for further examination. [After Sambridge, 1999.]
where $C_m$ is a matrix that non-dimensionalizes the model space, such as an imposed model covariance matrix. In practice, it is simpler to ignore covariance and divide each parameter by a scale factor in advance. My general practice has been to divide each factor by its range of variation, thus assigning equal significance to relative variations in all parameters.

The definition of the Voronoi cell (equation 5.1) does not provide a simple means for determining the boundaries of a particular cell. In fact, determining the boundaries of a Voronoi cell in a parameter space of arbitrary dimension is a difficult problem. However, it is not necessary to completely specify the boundaries of a cell in order to generate points within it, if the points are generated by a random walk.

\[ \| x - m_i \| = \sqrt{(x - m_i)^T C_m^{-1} (x - m_i)} \]  

Figure 5.2: A five-step random walk in a two-dimensional Voronoi cell. The length of each step is chosen randomly from a uniform distribution along the axis segment lying within the cell (grey lines).
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Figure 5.2 illustrates the generation of points within a Voronoi cell by a random-walk algorithm. For each step in the walk, it is only necessary to find the endpoints of the segment along a particular coordinate axis lying within the cell, and then generate the step length from a uniform distribution between the segment endpoints. The endpoints may be found (somewhat inefficiently) by stepping away from the starting point along the desired axis until the nearest sampled point (and therefore the Voronoi cell) changes.

A more efficient method is to assume that the boundary between two cells is given by $||\mathbf{x} - \mathbf{m}_i|| = ||\mathbf{x} - \mathbf{m}_j||$, and then solve for the intersection on the desired axis for all possible pairings of cells, retaining the shortest interval; see Sambridge (1999) for details.

The overall algorithm then may be expressed as follows:

1. Establish the bounds of the parameter space in which to search.

2. Within the desired bounds, generate $N$ uniformly sampled points using a quasi-random number generator.

3. Calculate the value of the misfit function for each of these points.

4. Repeat the following procedure for the desired number of iterations:

   (a) Sort all of the misfit values determined thus far, and retain the $M$ points with the lowest values.

   (b) Generate $N$ new points within the Voronoi cells associated with the $M$ chosen points, using a random-walk method. The Voronoi cells are determined from all of the calculated points.

   (c) Calculate the misfit function for each of the newly-generated points.

5. Sort all of the calculated points by misfit value. The best-fitting point may be taken as an estimate of the global minimum, or the entire population of sampled points may be examined.
The only parameters requiring tuning, apart from the number of iterations, are the sample sizes $N$ and $M$. The ratio $\frac{M}{N}$ controls the balance between Monte Carlo exploration of the model space and exploitation of existing points; a low ratio tends to produce faster convergence at the expense of thorough coverage. The value of $N$ determines how frequently the Voronoi cells are recalculated; overly low values of $N$ can potentially impair the robustness of the sampling. In practice, I have chosen to fix $N$ at two to four times the number of model dimensions.

5.3 Implementation issues: the data misfit function

In order to apply this technique to the receiver function problem, the model space, data set, and misfit function need to be defined. I assume the subsurface to be parameterized as a series of layers, each of which is parameterized in terms of thickness, density, average $P$-wave velocity and average $S$-wave velocity. If anisotropy is to be included in the model for a given layer, the parameters used are percentage $P$- and $S$-velocity anisotropy (expressed as a deviation from the average velocity), and trend and plunge of the axis of symmetry of the velocity surface (the anisotropy is assumed to be hexagonally symmetric*). If recovery of interface dip is desired, the surfaces of interest are assigned strike and dip parameters. This amounts to the same parameterization as was used for the forward problem in chapter 4 (see section 4.2).

*The remaining parameter $\eta$ required to uniquely specify the elastic tensor has a more minor effect, and is held fixed.
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The data set is a set of receiver functions recorded at a single station, for a range of different back azimuths and epicentral distances. Both the radial and transverse components are included, which may optionally be transformed into the wave vector domain (as described in chapter 5). Forward modeling is performed using the FORTRAN implementation of the ray-theoretical method presented in chapter 4, which can model all of the desired parameters to efficiently produce synthetic seismograms. Computed arrival times and amplitudes are mapped into seismograms as Gaussian pulses of a specified width; this is equivalent to forming a spike series and applying a low-pass filter. Adaptation of Malcolm routines, used by permission.

The remaining technical issue to be addressed is the misfit function. If the model and data are represented by vectors \( \mathbf{m} \) and \( \mathbf{d} \), respectively, the calculation of synthetic data \( \mathbf{s} \) using the forward model may be represented as \( \mathbf{s} = g(\mathbf{m}) \). The misfit \( \phi \) is then determined by \( \phi = f(\mathbf{d}, \mathbf{s}) = f(\mathbf{d}, g(\mathbf{m})) \), where \( f \) is the misfit function comparing the two sets of traces. The neighborhood algorithm described above does not make use of the value of the data misfit, requiring only the ordering of models with respect to misfit. As a consequence, there is considerable freedom of choice for the misfit function \( f \).

I have implemented and tested five different misfit functions, all of which adhere to the same general template. The data consist of \( N_t \) traces, each of which contains \( N_s \) samples and up to 3 components (allowing for the possibility of including the \( P \) component). The data may then be stored in a three-component array as \( d_{ijk} \), where \( i \) is the trace index, \( j \) is the sample index, and \( k \) is the component index. Similarly, I represent the
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synthetic data as $s_{ijk}$. I allow for the possibility of weighting the misfit by trace and by component through the introduction of trace weight factors $w_i^t$ ($i = 1, \ldots, N_t$) and component weights $w_k^c$ ($k = 1, \ldots, 3$). Setting the appropriate component weight to zero allows for the elimination of a particular component from the misfit calculation.

The simplest form of misfit is the difference of two traces. The $l_2$-norm (root-mean-square) misfit may be expressed as

$$f_{l2} (d_{ijk}, s_{ijk}) = \frac{\sum_{i=1}^{N_t} w_i^t \sqrt{\sum_{j=1}^{N_s} \sum_{k=1}^{3} w_k^c (d_{ijk} - s_{ijk})^2}}{N_s \left( \sum_{i=1}^{N_t} w_i^t \right) \sqrt{\sum_{k=1}^{3} W_k^c}}. \quad (5.3)$$

The denominator is not essential for the neighborhood inversion, but is included in order to make results comparable between problems. A variant of this misfit function was used by Sambridge (1999) for synthetic receiver-function data. The classic Cartesian distance used in the $l_2$-norm misfit is not necessarily ideal for comparing waveforms, due to its heavy weighting of larger differences, which may de-emphasize smaller arrivals as well as exaggerating the effect of noise spikes. An alternative measure is the $l_1$ (absolute value) norm, leading to the $l_1$-norm difference misfit function

$$f_{l1} (d_{ijk}, s_{ijk}) = \frac{\sum_{i=1}^{N_t} \sum_{j=1}^{N_s} \sum_{k=1}^{3} w_i^t w_k^c \left| d_{ijk} - s_{ijk} \right|}{N_s \left( \sum_{i=1}^{N_t} w_i^t \right) \left( \sum_{k=1}^{3} W_k^c \right)}. \quad (5.4)$$

Although the difference misfits given by equations 5.3 and 5.4 work well on synthetic data, they are dependent on the amplitudes of $d_{ijk}$ and $s_{ijk}$ being comparable. In principle receiver functions are scaled to the amplitude of the incident $P$-wave. However, in
practice, the amplitudes of receiver-function phases are sufficiently variable in real data
to cause problems with difference misfits. Therefore, it seems advisable to scale the real
and synthetic data by their respective amplitudes, in this manner:

\[
\tilde{d}_{ijk}^{l_2} = \frac{d_{ijk}}{\sqrt{\sum_{j=1}^{N_t} \sum_{k=1}^{3} d_{ijk}^2}}
\]

\[
\tilde{d}_{ijk}^{l_1} = \frac{d_{ijk}}{\sum_{j=1}^{N_t} \sum_{k=1}^{3} |d_{ijk}|}
\]

Using these scaling functions, we may introduce scaled versions of the \( l_1 \) and \( l_2 \) difference
norms, which are

\[
f_{l_2}^{sc} (d_{ijk}, s_{ijk}) = f_{l_2} \left( \tilde{d}_{ijk}^{l_2}, \tilde{s}_{ijk}^{l_2} \right)
\]

\[
f_{l_1}^{sc} (d_{ijk}, s_{ijk}) = f_{l_1} \left( \tilde{d}_{ijk}^{l_1}, \tilde{s}_{ijk}^{l_1} \right)
\]

The scaled \( l_1 \)- and \( l_2 \)-difference misfit functions perform rather better on real data
than their unscaled equivalents. However, they do not address the remaining issues of
frequency mismatch between real and synthetic data, variability in amplitude due to
deconvolution limitations, and the presence of unmodelled phases. In the event that
the pulse width differs between the real and synthetic traces, or the real data contain
pulses not modeled by the synthetic (such as noise bursts, or additional phases), a cross-
correlation-based misfit measurement should be more reliable; as well, cross-correlation
places less emphasis on amplitudes than on travel times, which are more robustly re-
covered features of receiver functions. Waveform cross-correlations are unbounded; as
the neighborhood algorithm requires a function which tends toward zero for good fits, I employ the correlation coefficient $R$, which runs from -1 to 1, and define the misfit to be $1 - R$. Thus, I have elected to use

$$f_c (d_{ijk}, s_{ijk}) = 1 - \frac{\sum_{i=1}^{N_t} \sum_{j=1}^{N_s} \sum_{k=1}^{3} w_i^k w_i^k d_{ijk} s_{ijk}}{\sqrt{\sum_{i=1}^{N_t} \sum_{j=1}^{N_s} \sum_{k=1}^{3} w_i^k w_i^k d_{ijk}^2} \sqrt{\sum_{i=1}^{N_t} \sum_{j=1}^{N_s} \sum_{k=1}^{3} w_i^k w_i^k s_{ijk}^2}}, \quad (5.9)$$

a misfit function which has proved to be highly effective for real data. The correlation misfit, and variations thereof, may be calculated more efficiently in the frequency domain if the frequency band of interest is limited (Levin and Park, 1997; Frazer and Sun, 1998). However, its time-domain equivalent remains substantially less computationally expensive than the forward-modeling step, and has proved to be adequate for my purposes.

### 5.4 Synthetic examples

#### 5.4.1 Synthetic modeling procedure

The next step is to evaluate the method’s performance on synthetic data produced from known models. I generate synthetic data from a series of models using the high-frequency method presented in chapter 4, to explore recovery of 1-D isotropic structure, anisotropy, and dip.
5.4.2 Isotropic models

The simplest isotropic model I tested was a two-layer crust/mantle model with a horizontal Moho (figure 5.3). A single receiver function for this model (figure 5.4) was inverted using a deliberately broad (explorative rather than efficient) set of search parameters \((N = 30, M = 15, 40\) iterations). The parameters allowed to vary were the \(P\)- and \(S\)-wave velocities of the two layers, and the crustal thickness; the allowed ranges and best-fitting model are shown in figure 5.3. The results, plotted in figure 5.5, demonstrate the unresolvable tradeoff between parameters inherent in a single trace. As the linear trends on figures 5.5b and 5.5c demonstrate, the data are primarily sensitive to the velocity-depth tradeoff and the \(P/S\) velocity ratio (or, equivalently, the Poisson’s ratio). This is a consequence of the travel times of the observed phases, which are, from equation 3.7, \(t_{Ps} = (\xi_S - \xi_P)z\) for the Moho \(Ps\) conversion, \(t_{Pps} = (\xi_S + \xi_P)z\) for the first free-surface multiple, and \(t_{Pss} = 2\xi_Sz = t_{Ps} + t_{Pps}\) for the second free-surface multiple, \(\xi_S\) and \(\xi_P\) being the \(S\) and \(P\) vertical slownesses and \(z\) being the crustal thickness. As these travel times only provide two independent constraints on the crustal parameters, the crustal structure cannot be fully characterized from a single trace.

However, a spread of seismograms generated for different slownesses but the same model (figure 5.6) contains additional information from the variation of the travel-time and amplitude with respect to the incident slowness. Inverting this model using a more efficient set of parameters \((N = 12, M = 3, 70\) iterations), the search converges to a final model which is very close to the initial model (figure 5.7), with the exception of the
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Figure 5.3: Models used in the inversion for crustal structure from a single trace. The bounds of the search are shaded, the best-fit model is solid, and the true model used to generate the synthetic data is dashed.

Figure 5.4: Input SV-polarized data and best-fitting synthetic for the single-trace Moho model. The fit is essentially exact.
Figure 5.5: A plot of the tradeoff between individual model parameters for the single-trace Moho model. Panels (a) through (d) show all generated models, for successive pairs of model parameters. The grey shade of each dot indicates the misfit (low misfits being black). The best-fit model is indicated by a diamond, and the true model by a star. Panel (e) shows the misfit for successively-calculated models, thus indicating convergence of the algorithm; the vertical axis serves as a scale bar for the shaded dots used in the other panels. Further figures of this type will follow the same pattern.
mantle $P$-velocity (figure 5.7d). It appears, then, that given high-quality $P$-coda data for a broad sampling of slownesses, the travel times and amplitudes of the transmitted $Ps$ conversion and the strong Moho multiples provide sufficient information to constrain all crustal parameters. In the presence of realistic noise levels, however, only the best-constrained parameters are likely to be well-recovered. For Moho inversions, these will be the bulk Poisson's ratio and thickness of the crust (as evidenced by figure 5.7a).

Figure 5.6: SV-polarized data for the Moho model at seven values of slowness, along with recovered data from the best-fit model. The input and recovered data are essentially identical.

In order to examine the recovery of subcrustal layer properties, I constructed a model containing a layer of decreased mantle velocity beneath the crust (figure 5.8d), and generated synthetic data using the same set of slownesses as used in the Moho section. I then fixed the properties of the crust and the mantle half-space, searching only for the mantle layer's thickness and seismic wave velocities. For the inversion, I excluded multiples from the base of the layer, since multiply reflected arrivals from such weak
Figure 5.7: Results of a neighborhood inversion of the data set in figure 5.6, shown as a distribution of misfits (shades of grey) over model parameters for all models sampled, as well as the best-fit (diamond) and true (star) models. Panel (f) shows the boundaries of the search (shaded), the true model (dashed), and the best recovered model (solid).
velocity contrasts are seldom visible in real data. The inversion parameters used were quite explorative \((N = 60, M = 30, 15\) iterations). The results are shown in figure 5.8. The important observation to make from these results is that while the subcrustal layer’s S-wave velocity and thickness are well constrained, the P-wave velocity is not (figure 5.8a, 5.8b). This is presumably a consequence of the lack of supporting information from multiply-reflected arrivals (multiples). In the absence of strong multiples, it seems advisable to fix the P-velocity and invert only for layer thicknesses and S-velocities.

5.4.3 Detecting anisotropy

Recovery of anisotropic parameters is one of my primary motivations for exploring receiver function inversions using the neighborhood technique. As a first test of the method’s effectiveness for this purpose, I constructed a three-layer model with horizontal interfaces in which the uppermost mantle layer is 5% anisotropic, the anisotropy having hexagonal symmetry and a horizontal axis. In order to examine anisotropy, the data set must comprise a range of back azimuths. The slownesses and back azimuths used for this model are plotted in figure 5.9, while figure 5.10 displays the resulting synthetic data. Inverting for subcrustal properties (i.e. using a fixed crustal model, while allowing both mantle layers to vary), I find that the average S velocity and thickness of the layer are well-constrained, as in the isotropic case (figure 5.11a). The anisotropic axis orientation is well constrained within the limits of back-azimuthal sampling (figure 5.11d), as is the degree of S-wave anisotropy (figure 5.11c). The P-wave anisotropy is less
Figure 5.8: Recovery of properties from a subcrustal layer (panels a through c), along with the true (dashed; star) and best recovered (solid; diamond) models (panel d; range of search is shaded).
well constrained, much like the $P$-velocity of subcrustal isotropic layers. This appears to indicate that $S$ anisotropy should be the primary target of field-data investigations. In addition, there appears to be little sensitivity to variations in plunge of the anisotropic axis (figure 5.11d).

![Figure 5.9: Set of geometric parameters (horizontal slownesses and back azimuths) used in the anisotropic-layer inversion. These parameters were selected to cover a range of back-azimuths and slownesses in relatively few traces.](image)

5.4.4 Constraining interface dip

The dip of mantle interfaces is another possible target for this technique. In order to examine the recovery of anisotropic, dipping layering, I generated synthetic data for the same wedge and slab models considered in chapter 4 (table 4.1). A set of back-azimuths
and slownesses was selected to cover a range of both parameters (figure 5.12); the same set of trace geometries was used for both models. Furthermore, I assumed the crustal parameters were known (e.g. from an earlier round of inversion), and so searched only for mantle properties. Although all first-order phases were calculated for the input data, including multiple reflections from subcrustal interfaces, multiples from interfaces other than the crust were not included in the inversion, as the amplitudes of such phases in real data tend to be low (a consequence of their longer paths, and hence greater degree of amplitude loss through scattering, compared to transmitted phases).

The input data section for the anisotropic wedge model is shown in figure 5.13. These data were inverted using an efficient parameter set \((N = 25, M = 4, 50\) iterations), yielding the results in figure 5.14. The strike and dip of the base of the anisotropic wedge were recovered accurately, as was the \(S\) anisotropy of the wedge and the absence of dip
Figure 5.11: Results of anisotropic inversion. The best-fit model (diamond) compares well with the true model (star) for most parameters, with the exception of the $P$ anisotropy.
Figure 5.12: Distribution of back-azimuths and slownesses used to generate synthetic data for the anisotropic wedge and slab models. The values were selected to cover the range of back-azimuth and slowness found in real data, while avoiding post-critical slownesses.
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on its upper surface.

Figure 5.13: Input data section for the anisotropic wedge inversion.

Using the same trace geometry, synthetic data were generated for the anisotropic-slab model and then inverted (figure 5.15). Using efficient parameters \((N = 30, M = 3)\), 120 iterations were required for convergence; the results are shown in figure 5.16. The orientation and anisotropy of the slab, and the parallelism of its sides, were successfully determined. The only parameter that was not accurately recovered was the slab thickness (figure 5.16b), which had a \(\sim 2\) km error, probably due to the frequency maximum of 0.5 Hz used in the modeling (which translates to a \(P\) wavelength of 16 km). Resolving depth variations significantly smaller than the teleseismic wavelength will probably not be possible in real data.
Figure 5.14: Generated models for wedge inversion, plotted by misfit. The diamond and star represent the best-fit model and the true model, respectively.
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![SV-polarized traces and SH-polarized traces](image)

Figure 5.15: Input data section for the anisotropic slab inversion.

5.5 CNSN data examples

5.5.1 Crustal properties and Moho depth

Many of the same structures discussed in the previous section may be examined using data from the Canadian National Seismograph Network. The simplest test of the neighborhood inversion technique is the examination of crustal thickness and velocities. Bank et al. (2000a) have investigated the thickness and Poisson's ratio of the crust beneath CNSN stations using the stacking technique of Zhu and Kanamori (2000), thus providing independent measurements using the same data sets, to which my results may be compared.

In order to test the recovery of crustal properties from real data, I have selected three stations which exhibit strong Moho arrivals (i.e. the direct $Ps$ conversion and free-surface multiples). Stations GAC and SADO are located in southwestern Quebec...
Figure 5.16: Generated models for slab inversion, plotted by misfit. The diamond and star represent the best-fit model and the true model, respectively.
CHAPTER 5. NEIGHBORHOOD INVERSION OF TELESEISMIC RESPONSE

and southeastern Ontario, respectively; station ULM is in southeastern Manitoba. All of these stations exhibit both a sharp $P_s$ conversion at the Moho (figure 5.17), and two well-developed free-surface multiples (the first converting from $P$ to $S$ at the Moho, the second at the free surface). As this inversion assumes only laterally homogeneous, isotropic properties, back-azimuthal variations and the $SH$ component provide no information. Therefore, I stacked the $SV$-polarized receiver-function traces from the chosen stations in slowness bins, collapsing all azimuthal information while enhancing phases arising from horizontal, isotropic structure. Prior to stacking, the data were filtered from 0.01 to 0.5 Hz, corresponding to the frequency band with highest signal to noise levels.

Results of the crustal-parameter inversions are shown in figure 5.18. Inversions were performed using the following parameters: $N = 20, M = 5$ and 50 iterations for GAC, $N = 30, M = 8$ and 25 iterations for SADO, and $N = 30, M = 8$ and 40 iterations for ULM. Although Moho depth and crustal $P$- and $S$-velocities are the only interpretable parameters, the upper-mantle velocities were also permitted to vary. The best-constrained parameter is the crustal Poisson’s ratio ($\sigma$), which I calculated from the $P$- and $S$-velocities ($\alpha$ and $\beta$) using the formula $\sigma = \frac{(\alpha/\beta)^2-2}{2\alpha/\beta}$. The crustal thickness is somewhat less tightly constrained, perhaps due to the upper frequency limit used, while the crustal velocities display considerable tradeoff. Nonetheless, the correspondence with Bank et al.’s (2000a) results is very good.
Figure 5.17: SV-polarized receiver-function data (stacked by slowness) for three CNSN stations (left) and the corresponding best-fit synthetic data sections (right). The visible phases are the Moho $Ps$ conversion and two crustal multiples.
Figure 5.18: Moho-inversion model parameter plots for three CNSN stations. The best-fit model for each station is indicated by a diamond. In panels (a1), (b1), and (c1), the best-fit model is compared to Bank et al.'s (2000a) values of crustal thickness and Poisson's ratio (stars) for the same three stations. Note that for station SADO (b1), the correspondence between my results and those of Bank et al. is essentially exact.
5.5.2 Constraining anisotropy beneath the Slave province

As the anisotropic structure beneath station YKW3 (Yellowknife, Northwest Territories) is well-constrained (Bostock, 1998; this document, section 4.4), data from YKW3 provide a good test of the neighborhood inversion method’s ability to detect upper-mantle anisotropy in real data. For testing purposes, inversion was restricted to the layer identified as H (figure 4.7), which is interpreted to be a layer of anisotropic upper-mantle fabric with a horizontal symmetry axis at azimuth ~ 100° (approximately ESE). I fixed the crustal properties at the values used in forward modeling, allowing the thickness and S-velocity of upper mantle layers to vary, along with the degree and orientation of anisotropy in the H layer.

Figure 5.19 shows the SV- and SH-polarized receiver-function data from YKW3, along with the best-fit synthetic data section. The cross-overs in polarity on the transverse component provide good constraints on the orientation of the H layer's anisotropic axis, as borne out by the results (figure 5.20). Judging from the results, it is safe to conclude that the H layer does not exhibit a significant isotropic contrast with the layers above and below it. The layer's anisotropic axis, which I assumed to be horizontal, has a best-fit orientation of 89° east of north. Although no rigorous error has been computed, this value appears to be constrained within ±10° on the basis of the distribution of good-fitting models. The P-wave anisotropy is not constrained at all, and was assigned a fixed value of ±4% for the final inversion. The percentage of S anisotropy converged to a value of ±7.7%, and is well constrained by the large amplitudes of SH-component
arrivals related to the H layer.

Figure 5.19: Real and synthetic data for station YKW3, windowed to highlight the presence of the anisotropic H layer. See figure 4.7 for the full data window and the relationship between trace number and back azimuth/slowness values.

5.5.3 Locating dipping layers beneath Vancouver Island

The CNSN station PGC, in Sydney, British Columbia, is located near the southern tip of Vancouver Island. It overlies the Cascadia subduction zone, approximately 200 km landward of the subduction deformation front. Forward modeling of receiver functions from this station by Cassidy (1995a) revealed a sequence of low-velocity layers in the lower crust and upper mantle, the lowermost of which (a ~ 5 km thick low-velocity
Figure 5.20: Inversion results from modeling the H layer at station YKW3. The best-fitting model is indicated by white diamonds.
layer at approximately 47 km depth) was interpreted to represent the oceanic crust of the subducting Juan de Fuca plate. Cassidy’s model (figure 5-23) is 1-D and isotropic; however, the subducting plate almost certainly exhibits substantial dip given its tectonic setting. Station PGC thus provides a useful opportunity to test the recovery of dipping structures in real data.

As a first approximation of PGC structure, I stacked the $SV$-polarized PGC receiver-function data in slowness bins (figure 5.21), and inverted them for 1-D Earth structure. The number of layers and range of parameters to search were chosen based on a simplification of the model of Cassidy (1995a); layer thickness and $S$-velocity were allowed to vary, while $P$-velocities for all layers but the uppermost were fixed from Cassidy’s model.

![Data and Synthetic](image.png)

**Figure 5.21:** Trace-normalized $SV$-component seismograms for station PGC, stacked in slowness bins, as compared to synthetic data from the best-fitting recovered model. Unmodelled energy between 0-2 seconds in the data results from shallow structure that I did not include in the inversion.

Results of the inversion are shown in figure 5.22. Although there is some veloc-
ity/depth tradeoff, both parameters appear fairly well constrained, except in the case of layer 5 (between \textbf{E} and \textbf{JdF}). The best-fit model (figure 5.23) agrees well with Cassidy’s (1995a) model, except for the fifth and sixth layers. Layer 6 (\textbf{JdF} in figure 5.23), which Cassidy interpreted to be part of the Juan de Fuca slab, is located 5 km deeper in my model, while the layer above the slab is thicker and faster. However, as these are the layers most likely to have dipping boundaries, further refinement of the model requires inversion for interface orientation.

In order to examine the dip of interfaces beneath PGC, it is necessary to consider the transverse component and the back-azimuthal variation of the data. The set of back-azimuths and slownesses represented by the data set is shown in figure 5.24, and the unstacked data set is shown in figure 5.25. The strong variation in conversion amplitude with back azimuth and the presence of substantial coherent energy on the \textit{SH} component imply the presence of dipping interfaces in the uppermost mantle.

For the dip inversion, I fixed the layer depths and velocities at the values determined by 1-D inversion, and allowed the strike and dip of the lowermost four interfaces to vary. The resulting best-fit synthetic section corresponds well to the observed data on the \textit{SV} component (figure 5.25), although there are problems with the \textit{SH}-component fit for traces 1-40. The full set of modeling results is shown in figure 5.26, and the best-fit model in table 5.1. A second inversion was performed, in which the lowermost layers were allowed to be anisotropic (table 5.1); although some anisotropy in the \textbf{JdF} layer was recovered, the resulting best-fit synthetic was not noticeably different from
Figure 5.22: 1-D inversion results for station PGC. The best-fitting model is indicated by a diamond.
Figure 5.23: Preferred one-dimensional isotropic model for station PGC compared to the model of Cassidy (1995a). C, E and JdF are the low-velocity layers identified by Cassidy. JdF is believed to represent the oceanic crust of the Juan de Fuca slab (Cassidy, 1995a) while C and E have been interpreted to be layers of underplated sediment (Clowes et al., 1987; Cassidy and Ellis, 1993).
Figure 5.24: Distribution of event bins for station PGC: a) back azimuth versus bin number; b) a map view of the bin locations, with the number of traces per bin indicated. Small circles represent 30° epicentral-distance intervals.
Figure 5.25: Unstacked PGC receiver-functions (numbered as in figure 5.24), with the best-fitting synthetic. The amplitude of the $SH$ component is about half of the $SV$ amplitude.
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the one recovered assuming isotropic layering, and the interface orientations were not significantly affected. I conclude that there is substantial dip in the subcrustal layering beneath station PGC, and that although anisotropy may be present, the available data set does not require it.

![Graphs showing results of inverting for interface dip beneath station PGC. The best-fit model is indicated by white diamonds.](image_url)

Figure 5.26: Results of inverting for interface dip beneath station PGC. The best-fit model is indicated by white diamonds.

The "E" layer is a common feature of Vancouver Island LITHOPROBE reflection pro-
Table 5.1: Best-fit recovered models for station PGC, with and without anisotropy. Layer 6 is interpreted to be the crustal portion of the Juan de Fuca slab; layers 2 and 4 are low-velocity layers identified by Cassidy (1995a).

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files (Clowes et al., 1987), where it appears as a highly reflective zone dipping north-eastward ~ 18°, coinciding with a conductive zone detected using the magnetotelluric technique (Kurtz et al., 1986). Clowes et al. (1987) interpret E to represent a zone of underplating of highly sheared, fluid-saturated oceanic sediments. Through forward modeling of receiver functions from central Vancouver Island and the adjacent mainland, Cassidy and Ellis (1991, 1993) determined E to strike N~ 80°W and dip ~ 7° northeast beneath station ALB-B (central Vancouver Island), with similar orientation and ~ 10 km of deepening at station LAS (strait of Georgia), and a splitting of the layer into two low-velocity zones at station EGM (coastal mainland). Beneath station PGC, Cassidy (1995a), using back-azimuthal amplitude variations in receiver functions, interpreted E to dip less than 10°.

Based on my isotropic results, the strike and dip of both interfaces of the E low-velocity layer (figure 5.26a,b) are well constrained. In my models, E represents a low-velocity wedge whose upper interface dips northward (or possibly northeastward; see figure 5.26a) by ~ 14°, while its lower surface is approximately horizontal (with a recovered dip of 7°). This orientation of the upper interface is consistent with the above-mentioned earlier work, while the northward-thinning wedge geometry I interpret for E is consistent with an accretionary origin for the layer.

My modeled values for the orientation of the JdF layer, which presumably represents the oceanic crust of the Juan de Fuca slab, (JdF) are less tightly constrained (figure 5.26c,d). In particular, the orientation of the slab base shows large variations between
models with good fits (figure 5.26d), and is not considered to be robust. The dip of the upper interface (figure 5.26c) is constrained to be approximately 12°; however, there is considerable variation in its strike. Cassidy and Ellis (1991) determine JdF to strike at N60°W and dip 15° beneath ALB-B, while Cassidy (1995a) calculated a strike of N50°W ±30° and a dip of 15-20° for the slab beneath PGC. The ~ 12° dip I detect for the upper surface of JdF is compatible with these earlier results. My measured strike of N67°E, however, is 90° away from its expected orientation, requiring the slab to dip southeast rather than northeast. In part, this is likely to be a consequence of uneven back-azimuthal coverage for PGC (figure 5.24), resulting in a high degree of uncertainty on the strike of layers which are poorly oriented relative to the event set. In addition, the poor recovery of JdF strike may be a reflection of the poor $SH$-component fit at later travel times (figure 5.25), possibly resulting from nonplanar structure in the vicinity of JdF.

As these examples show, neighborhood inversion of receiver-function sections is a viable technique for recovering upper-mantle anisotropy and interface dip. The principal limitations are those of the forward-modeling technique, as non-planar interfaces cannot be recovered; otherwise, the limitations of data sets for resolving upper-mantle parameters may be explored. Future development of this technique will focus on improvements in forward-modeling efficiency and in the data misfit function, as well as statistical interpretation of the full set of models produced in order to obtain a better understanding of the errors on the recovered model parameters.
Summary and conclusions

The accumulated data set of the Canadian National Seismograph Network contains a wealth of information on Canadian upper mantle structure. In order to generate upper-mantle velocity models from this data set, I have applied an established technique to recover large-scale structure, and developed a new method for recovery of smaller-scale structure.

By applying partitioned waveform inversion to surface waves from the CNSN data set, I have generated a large-scale, tomographic S-velocity image of the upper mantle beneath the Canadian landmass, in which structure is resolved down to 400 km. My model is the first surface-wave based model to provide good coverage of Canada, and provides improved depth resolution compared to existing body-wave models. A pronounced high-velocity mantle root is present beneath the Canadian shield to a depth of 250 km, which shows evidence for internal velocity variations. The Canadian Cordillera is underlain by a pronounced low-velocity anomaly which strengthens to the south and may reflect small degrees of partial melting. The boundary between the southern Cordilleran anomaly and cratonic keel is sharp, and I have interpreted it to represent a thermal boundary.
between hot and cold mantle domains beneath the Cordillera and shield, respectively. Less pronounced low-velocity anomalies beneath Labrador and the Appalachian orogen were also detected, and tentatively interpreted to represent the effects of rifting in the northeast Atlantic and the passage of hotspots beneath the northeastern Appalachians.

Finer-scale imaging of the upper mantle is possible through the analysis of teleseismic receiver functions, which record the scattering of incident \( P \) waves into \( Ps \) conversions and multiples by crustal and upper-mantle structure. As the relationship between Earth structure and the resulting receiver functions is complex, a modeling tool is needed to interpret such data. Extending the results of Dielbold (1987) and Richards et al. (1991) to teleseismic geometries and anisotropic media, I have developed a novel forward-modeling technique for teleseismic waves in the presence of anisotropy and dipping interfaces, using a geometrical simplification of ray theory. The result is a fast, robust method that overcomes some of the limitations of established techniques. I have extensively tested my technique, applying it both to synthetic geometries and to real \( P \) and \( S \) data from the CNSN station YKW3 (Yellowknife, Northwest Territories).

Having developed an efficient forward modeling technique for receiver functions, one can invert comprehensive, broadband data sets from individual CNSN stations. As the receiver-function problem is highly nonlinear, and the number of model parameters is small, I have employed a Monte Carlo-based directed search technique, bearing some resemblance to genetic algorithms and simulated annealing. The method I have used is the neighborhood algorithm of Sambridge (1999), a geometric technique that is concep-
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tually simple and robust. Pairing the neighborhood inversion technique with my forward modeler, and developing an appropriate misfit function for teleseismic data sets, I have successfully inverted real and synthetic data for layer thickness, velocity, anisotropy and dip. In applying the approach to individual CNSN stations, I have successfully recovered Moho depth and crustal velocities beneath stations in Quebec, Ontario and Manitoba, as well as constraining the anisotropic-axis orientation of an upper-mantle layer beneath YKW3. Beneath station PGC (Sydney, B.C.), I have constrained the dip of the subducting Juan de Fuca plate and an overlying low-velocity zone. On this basis, I conclude that the tools developed in this thesis should prove useful in characterizing detailed lithospheric structure beneath a growing body of permanent stations constituting the global networks.


BIBLIOGRAPHY


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