THE ESTIMATION OF PACK-ICE MOTION IN DIGITAL SATELLITE IMAGERY BY MATCHED FILTERING

by

MICHAEL JOHN COLLINS

B.Sc.Eng.(Survey), The University of New Brunswick, 1981

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE

in

THE FACULTY OF GRADUATE STUDIES
DEPARTMENT OF OCEANOGRAPHY

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

March 1987

© Michael John Collins, 1987
In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Oceanography

The University of British Columbia
1956 Main Mall
Vancouver, Canada
V6T 1Y3

Date April 10 1987
Abstract

This thesis addresses the problem of computationally estimating the motion of pack ice in sequential digital satellite images. The problem is posed in terms of linear filter theory and is solved by minimizing the error variance. The intuitive use of cross correlation and edge detection are shown to flow naturally from this approach. The theoretical framework also allows a geometric intuition into the action of the filter which is not possible through ad hoc methods. The noise corrupting the filtering process is investigated and the filter is implemented through both a first order method common to image processing, and a more sophisticated second order approach from computational vision. The class of imagery for which the filtering system is appropriate is discussed and the images chosen for the experiments are shown to be representative of this class. The experimental results reveal the power of the system in estimating ice motion, and some analysis of the derived motion is performed by comparison to a simple theory of wind-driven ice motion. The failings of the system are discussed and improvements are suggested.
Contents

Abstract ii

List of Tables iv

List of Figures v

Acknowledgement vi

1 Introduction 1
  1.1 Arctic Sea Ice ........................................... 1
    1.1.1 Physical Extent ...................................... 1
    1.1.2 Drift Characteristics ................................. 3
    1.1.3 General Characteristics of Ice Dynamics .......... 6
    1.1.4 The Role of Remote Sensing ........................... 7
  1.2 Outline of The Research ................................. 9
    1.2.1 Previous Work ........................................ 9
    1.2.2 Organization of the Thesis ........................... 10

2 Theory 12
  2.1 Statement of the Problem ............................... 12
  2.2 Derivation of the Optimum Filter ........................ 13
  2.3 Suboptimal Matching ..................................... 20
  2.4 Summary .................................................. 22

3 Implementation 23
  3.1 Questions of Noise ....................................... 23
    3.1.1 A Noise Model ....................................... 24
  3.2 The Whitening Filter ..................................... 27
    3.2.1 The First Order Filter ............................... 31
    3.2.2 The Second Order Filter .............................. 32
  3.3 The Matching Filter ...................................... 35
    3.3.1 Optimum Window Size ................................ 37
    3.3.2 Error Correction ..................................... 38
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>The channel bandwidths of the AVHRR on the NOAA satellites used in this study.</td>
<td>42</td>
</tr>
<tr>
<td>4.2</td>
<td>AVHRR images used in this study.</td>
<td>47</td>
</tr>
<tr>
<td>B.1</td>
<td>comparison between the $u_a$ and $u_m$ measurements for image pair 1 (June</td>
<td>99</td>
</tr>
<tr>
<td></td>
<td>05 – 06 1984), where $e$ is the magnitude of the error vector and $\alpha$ is its angle.</td>
<td></td>
</tr>
<tr>
<td>B.2</td>
<td>comparison between the $u_a$ and $u_m$ measurements for image pair 1 (June</td>
<td>104</td>
</tr>
<tr>
<td></td>
<td>06 – 07 1984), where $e$ is the magnitude of the error vector and $\alpha$ is its angle.</td>
<td></td>
</tr>
<tr>
<td>B.3</td>
<td>comparison between the $u_a$ and $u_m$ measurements for image pair 3 (June</td>
<td>109</td>
</tr>
<tr>
<td></td>
<td>11 – 12 1984).</td>
<td></td>
</tr>
<tr>
<td>B.4</td>
<td>comparison between the $u_a$ and $u_m$ measurements for image pair 4 (June</td>
<td>114</td>
</tr>
<tr>
<td>B.5</td>
<td>comparison between the $u_a$ and $u_m$ measurements for image pair 5 (July</td>
<td>119</td>
</tr>
<tr>
<td>B.6</td>
<td>comparison between the $u_a$ and $u_m$ measurements for image pair 6 (Au­-</td>
<td>124</td>
</tr>
<tr>
<td></td>
<td>gust 06 – 07 1985).</td>
<td></td>
</tr>
</tbody>
</table>
List of Figures

1.1 Arctic sea ice extent in February (top) and September (bottom) based on 75% concentration limits from Hibler (1980). ........................................ 3
1.2 300-month time series of the departure from the monthly means of the area covered by arctic sea ice, showing unsmoothed values (a) and 24-month running means (b). from Walsh and Johnson (1971). ................. 4
1.3 Ice drift in the Arctic ocean. The arrows indicate the average direction of ice movement in each region. Several locations are marked by letters: A-The New Siberian Islands; B-the Bering Strait; C-Franz Joseph Land; D-Spitzbergen and P-North Pole. from Pounder (1965). ....................... 5

3.1 Autocorrelation functions $R_n$ for AVHRR images (top) and SAR images (bottom). ................................................................. 26
3.2 A small portion of rev.1439 showing some significant image features (top). Profiles showing brightness gradient from image and operator responses to the image features (bottom). .................................................. 30
3.3 Mask used for a generic $3 \times 3$ operator. ................................................. 32
3.4 One dimensional slice through the (a) impulse response and (b) frequency response of the $\nabla^2 g$ filter. ................................................................. 35

4.1 Spectral albedos over snow and melt ponds: (a) dry snow (b) wet new snow, (c) melting old snow, (d) partially refrozen melt pond, (e) early season melt pond (10 cm deep), (f) mature melt pond (10 cm deep) on MY ice, (g) melt pond on FY ice (5 cm deep), (h) old melt pond (30 cm deep) on MY ice. Grenfell and Maykut (1977). .......................... 43
4.2 Spectral albedos over bare sea ice: (a) frozen MY white ice, (b) melting MY white ice, (c) melting FY white ice, (d) melting FY blue ice. Grenfell and Maykut (1977). .......................... 44
4.3 If the wind stress $\tau_a$ balances $\tau_w + C$, then a larger stress $\tau_a'$ turned to the right of $\tau_a$ is required to balance $\tau_w + C + \nabla \cdot \sigma$. Thorndike and Colony (1982). ........................................ 49

B.1 AVHRR band 1 - June 05 1984. ................................................................. 87
B.2 AVHRR band 1 - June 06 1984. ................................................................. 88
B.3 AVHRR band 1 - June 07 1984. ................................................. 89
B.4 AVHRR band 1 - June 11 1984. ................................................. 90
B.5 AVHRR band 1 - June 12 1984. ................................................. 91
B.6 AVHRR band 1 - June 13 1984. ................................................. 92
B.7 AVHRR band 1 - July 30 1985. ................................................. 93
B.8 AVHRR band 1 - July 31 1985. ................................................. 94
B.9 AVHRR band 1 - August 06 1985. .............................................. 95
B.10 AVHRR band 1 - August 07 1985. .............................................. 96
B.11 Automatically measured displacements from June 05-06 1984. .... 97
B.12 Manually measured displacements from June 05-06 1984. ........... 98
B.13 Displacements predicted from theory for June 05-06 1984 along with
the 24 hour averaged isobars for same period. .............................. 100
B.14 Residual displacements (see text) for June 05-06 1984 along with the
24 hour averaged isobars for same period. .................................. 101
B.15 Automatically measured displacements from June 06-07 1984. ....... 102
B.16 Manually measured displacements from June 06-07 1984. ........... 103
B.17 Displacements predicted from theory for June 06-07 1984 along with the
24 hour averaged isobars for same period. .................................. 105
B.18 Residual displacements (see text) for June 06-07 1984 along with the
24 hour averaged isobars for same period. .................................. 106
B.19 Automatically measured displacements from June 11-12 1984. ....... 107
B.20 Manually measured displacements from June 11-12 1984. ........... 108
B.21 24 hour averaged isobars for June 11-12 1984. ......................... 111
B.22 Automatically measured displacements from June 12-13 1984. ....... 112
B.23 Manually measured displacements from June 12-13 1984. ........... 113
B.24 Displacements predicted from theory for June 12-13 1984 along with the
24 hour averaged isobars for the same period. .............................. 115
B.25 Residual displacements (see text) for June 12-13 1984 along with the
24 hour averaged isobars for the same period. .............................. 116
B.26 Automatically measured displacements from July 30-31 1985. ....... 117
B.27 Manually measured displacements from July 30-31 1985. ........... 118
B.28 Displacements predicted from theory for July 30-31 1985 along with the
24 hour averaged isobars for the same period. .............................. 120
B.29 Residual displacements (see text) for July 30-31 1985 along with the
24 hour averaged isobars for the same period. .............................. 121
B.30 Automatically measured displacements from August 06-07 1985. ..... 122
B.31 Manually measured displacements from August 06-07 1985. ........ 123
B.32 Displacements predicted from theory for August 06-07 1985 along with
the 24 hour averaged isobars for the same period. ........................ 125
B.33 Residual displacements (see text) for July 30-31 1985 along with the 24
hour averaged isobars for the same period. .............................. 126
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.34</td>
<td>Sample cross-correlation function for typical window for the June 11-12 1984</td>
</tr>
<tr>
<td>B.35</td>
<td>Sample cross-correlation function for typical window for the June 11-12 1984</td>
</tr>
<tr>
<td>C.1</td>
<td>SEASAT SAR - revolution 1439 - October 5 1978.</td>
</tr>
<tr>
<td>C.2</td>
<td>SEASAT SAR - revolution 1482 - October 8 1978.</td>
</tr>
<tr>
<td>C.3</td>
<td>Location of SEASAT SAR image pair used in this study.</td>
</tr>
<tr>
<td>C.4</td>
<td>Manually measured ice displacements computed from SAR pair (courtesy of D.A. Rothrock, University of Washington).</td>
</tr>
<tr>
<td>C.5</td>
<td>Manual measurements of ice displacement. (a) shows displacements as vectors, (b) shows displacements as a deforming grid (see text), square denotes outline of undeformed grid.</td>
</tr>
<tr>
<td>C.6</td>
<td>Convolution of Sobel operator with rev. 1439.</td>
</tr>
<tr>
<td>C.7</td>
<td>Convolution of Sobel operator with rev. 1482.</td>
</tr>
<tr>
<td>C.8</td>
<td>Convolution of $V^2g$ operator with rev. 1439.</td>
</tr>
<tr>
<td>C.9</td>
<td>Zero crossings from rev. 1439.</td>
</tr>
<tr>
<td>C.10</td>
<td>Convolution of $V^2g$ operator with rev. 1482.</td>
</tr>
<tr>
<td>C.11</td>
<td>Zero crossings from rev. 1482.</td>
</tr>
<tr>
<td>C.12</td>
<td>Automatically measured displacements from first order operator experiment.</td>
</tr>
<tr>
<td>C.13</td>
<td>Automatically measured displacements from second order operator experiment.</td>
</tr>
<tr>
<td>C.14</td>
<td>Errors in the first order experiment.</td>
</tr>
<tr>
<td>C.15</td>
<td>Errors in the second order experiment.</td>
</tr>
<tr>
<td>C.16</td>
<td>Detail 1 from the lower left corner of rev. 1439 (top) and rev. 1482 (bottom).</td>
</tr>
<tr>
<td>C.17</td>
<td>Detail 2 from the upper right corner of rev. 1439 (a) and rev. 1482 (b).</td>
</tr>
</tbody>
</table>
Acknowledgements

I would like to acknowledge the support of my supervisor, Bill Emery, whose unflagging enthusiasm for this project was essential to its fruition. Ron Ninnis gave me the idea to pursue this topic and also provided me with a great deal of support during my first year of study. Peter Lawrence and Jim Riemer of the Dept. of Electrical Engineering and Paris Vachon of Oceanography were always ready to discuss the nuances of digital filtering. Bob Woodham introduced me to computational vision and stressed the importance of the theoretical approach which has, in no small way, inspired my work. Drew Rothrock of the University of Washington introduced me to the community of ice motion researchers and our discussions were invaluable. The research described in this thesis was performed at the Laboratory for Computational Vision in the Dept. of Computer Science and the Satellite Oceanography Laboratory in the Dept. of Oceanography and I thank Dan Razzell of the former and Denis Laplante and Paul Nowlan of the latter for their assistance with many technical details. Don Hodgins and Don Dunbar of Seaconsult Ltd. helped to obtain and process the data necessary for the production of the geostrophic wind field. I must also acknowledge the tireless efforts of Mark Majka without whom this thesis would still be a series of \texttt{TeX} error messages. My parents were always supportive of my return to school. Finally, without the love and friendship of Ramona I would be adrift.
Chapter 1

Introduction

1.1 Arctic Sea Ice

A remote frozen blanket of sea ice covers approximately 10% of the northern hemisphere and 13% of the southern hemisphere and modifies their fundamental character accordingly. It displays dramatic spatial and temporal variability and yet it is only in the last twenty years that a concerted effort by oceanographers, glaciologists and material scientists has lead to a quantum leap forward in our understanding of the behavior and properties of this inscrutable material.

The harsh environment of the polar and sub-polar regions severely restrict the spatial and temporal coverage which may be obtained through surface-based sea-ice experimental programmes. Yet it remains a scientific priority to monitor these regions due to their profound effect on global weather systems, on oceans and on climate. The advent of satellite-borne remote sensing has provided a solution to this problem and a time-series of the ice data may be built up with a variety of sensors.

1.1.1 Physical Extent

The maximum amount of ice covering the Arctic has been estimated to be $15 \times 10^6$ km$^2$ (Walsh and Johnson, 1979; Nazarov, 1963) with a seasonal variability of 20–25% (maximum in February, minimum in August) as shown in figure 1.1. An interesting feature of this figure is the presence of ice year-round off the northeastern coast of
CHAPTER 1. INTRODUCTION

Greenland. This is due to its being the main channel of ice flow out of the Arctic basin.

The extent of the ice may vary substantially from year to year as can be seen in figure 1.2. The general trend is that the ice extent has increased over the last twenty years which appears to agree with surface air temperatures (Walsh, 1970). The extent of ice on the north slope of Alaska was investigated by Barnett (1979), who produced a time-series of ice severity, a statistically derived quantity. While he attempted no explanation of the variability of this index, he noted a strong correlation between the severity and the atmospheric pressure changes over the duration of his study. Rogers (1978) studied the interannual variability of ice in the Beaufort Sea and found that the air temperature, in the form of thawing degree days, was the parameter which correlated most highly with the summertime ice margin.
CHAPTER 1. INTRODUCTION

1.1.2 Drift Characteristics

The general drift of ice in the Arctic basin is well known and the circulation consists of a central gyre and a transpolar drift stream, as shown in figure 1.3. The recirculating nature of the Beaufort gyre makes it possible for ice to survive the summer melt and stay in the basin for many years which explains why some of the thickest ice is found in this region.

Drift rates in the Arctic can vary over days, months and years, but a representative figure could be taken from, for instance, Dunbar and Wittman (1962) - about 6 km/day. This figure falls between extreme values of 20 km in one day and many weeks of no
Figure 1.2: 300-month time series of the departure from the monthly means of the area covered by arctic sea ice, showing unsmoothed values (a) and 24-month running means (b). from Walsh and Johnson (1971).
Figure 1.3: Ice drift in the Arctic ocean. The arrows indicate the average direction of ice movement in each region. Several locations are marked by letters: A-The New Siberian Islands; B-the Bering Strait; C-Franz Joseph Land; D-Spitzbergen and P-North Pole. from Pounder (1965).
motion (Thorndike and Colony, 1979). Dunbar and Wittman have observed that it took ten years for an ice island to make a complete circuit of the Beaufort gyre. While the drift of ice stations gives us general drift characteristics, more complete information on ice drift has been gained from the U.S. Navy navigation satellites (Thorndike and Colony, 1979), and laser surveying equipment (Hibler, 1980). These methods show that velocity variations are greatest at low frequencies (0.2 day^{-1}), but are also statistically significant at 2.0 day^{-1}. This high frequency variability is more pronounced in the summer and has been explained by McPhee (1978) as inertial oscillations resulting from a balance between the inertia of the ice and the Coriolis force. These 12 hour oscillations are prominent in a typical seasonal velocity spectrum from the summer (Thorndike and Colony, 1979).

1.1.3 General Characteristics of Ice Dynamics

Recent research in sea ice dynamics has substantially improved our understanding of the nature of ice drift. Sea ice, that is, ice of sea origin (pack ice) rather than land origin (icebergs and ice islands), forms a floating three-dimensional continuum that moves in response to stresses exerted on it by the wind from above and the sea from below. This continuum is also able to sustain substantial internal stresses which result from external forcing. It is clear that to understand the physics of ice drift and deformation we must first understand the way ice interacts with its environment. The study of ice interaction in the central ice pack was the primary objective of the Arctic Ice Dynamics Joint Experiment (AIDJEX) (Maykut, Thorndike and Untersteiner, 1972; Untersteiner, 1978) while the Marginal Ice Zone Experiment (MIZEX) is focused on the complex dynamics at the various ice edges or marginal ice zones.

Sea ice dynamics deals with the way momentum is transferred through the sea-ice-atmosphere system. It is generally agreed that the large scale dynamics of this complex system can be characterized by the following elements (Hibler, 1984, 1980):

1. A momentum balance describing the ice drift, including: air and ocean stresses;
CHAPTER 1. INTRODUCTION

Coriolis force; internal ice stress; inertial forces and ocean current effects.

2. An ice rheology which relates the stress experienced by the ice to the deformation and strength of the ice.

3. An ice strength which can be found in studies of the mechanical properties of the ice (Mellor, 1983; Michel, 1978), but for modelling purposes is often a function of the so-called ice thickness distribution (Hibler, 1979).

1.1.4 The Role of Remote Sensing

Satellite borne remote sensing has made major contributions to our knowledge of the sea ice cover of the polar oceans and is now a standard operational tool in both polar regions. The sensors currently in use all make unique contributions to our knowledge of the ice and conversely they all have drawbacks which limit their usefulness. Any future operational system should make use of some combination of two or more satellite data.

The TIROS-N series of operational weather satellites operated by the National Oceanic and Atmospheric Administration (NOAA) use an Advanced Very High Resolution Radiometer (AVHRR) which operates in the visible and Infrared (IR) bands. This sensor has proven to be extremely useful for studying the ice over scales of hundreds of kilometers. At present there are two such satellites in orbit and images of the arctic may be collected several times per day. While the pixel resolution of approximately 1.5 km excludes studies of smaller scale dynamics, the imagery affords a synoptic picture of a large region. This imagery combined with some geophysical forcing data provides an excellent research tool. The major drawback of these sensors is that both the visible and IR bands will not see through clouds and the former is also restricted to daylight use. These restrictions are severe especially in the arctic which is often cloudy and has little or no light from November to March. In spite of these restrictions, however, the Satellite Oceanography Laboratory at U.B.C. has a total of 104 separate image pairs that it has collected since it began receiving arctic data in late 1983. One sequence
of images August 1985 covers 10 consecutive days in 10 images, and has been used
to study the yearly ice-arching phenomenon in Amundsen Gulf (Collins, 1986). This
strongly suggests that this type of sensor is essential to arctic research.

Microwave frequencies may also be sensed passively and the passive microwave
sensors are enjoying increasing popularity. The Electrically Scanning Microwave Ra­
diometer (ESMR) on board NASA’s Nimbus-5 satellite and the Scanning Multifre­
quency Microwave Radiometer (SMMR) on board the Nimbus-7 have extremely large
coverage and are not affected by clouds or darkness. They have proven very useful for
mapping ice concentrations in a single scene and variations in the extent of sea ice over
both polar regions over several scenes. The very large footprint, or ground resolution
of the sensor (~ 25km) has confined its usage to global studies.

The sensor which has been touted by many as having the greatest potential for sea
ice research is imaging radar, especially Synthetic Aperture Radar (SAR). Features
within a SAR image appear bright to the extent that they are strong scatterers of
the pulses of microwave energy emitted by the SAR system. Sea ice features are
characterized by a wide range of scattering coefficients so that there is usually good
contrast between multiyear ice (ice which has survived at least one summer), first year
ice (which has not yet been through a summer), new ice (ice which is less than 10
cm thick), ridges and rubble fields (piles of ice blocks and fragments resulting from
convergence or shear between ice floes) and open water. The sensitivity of SAR to
variations in the ice/water surface scattering combined with its fine spatial resolution
makes it very useful for ice type characterization as well as for studies of ice kinematics
and dynamics. The SAR has the added advantage of being an all-weather all-season
system. A particular disadvantage of the system is that it has proven useful only in
small regions since it lacks the useful synoptic scale of the NOAA satellites. The fine
spatial resolution scales of the SAR have rarely been matched by observations of forcing
phenomena, making it of limited use in small scale ice dynamics studies. The resolution
of the SAR has, however, made it the most useful data set for tracking ice motion in
CHAPTER 1. INTRODUCTION

successive images.

1.2 Outline of The Research

The study of sea ice dynamics has been greatly enhanced by remote sensing. Early sea ice studies involving remote sensing used photographic images only (Nye and Thomas, 1974; Nye 1975a, 1975b), which involved extremely time-consuming photogrammetric techniques in order to yield quantitative results. Recent studies have all used digital imagery which lends itself very well to purely computational analysis. This thesis seeks to construct a theoretical framework within which a purely computational pack-ice motion estimation system may be developed. This framework is the well-developed theory of matched filtering. The motion estimation problem is first posed as a filtering problem which is then solved by optimizing a particular filter performance characteristic common in estimation problems. The derived filters are equivalent to intuitive and thus far, ad hoc, techniques. The filtering system is tested on several AVHRR image pairs and one SAR image pair.

1.2.1 Previous Work

The problem of estimating sea ice motion from remotely sensed imagery has traditionally been solved via manual, hence very time-consuming, procedures. ¹ This manual method was first applied to SAR imagery by Hall and Rothrock (1981). The approach taken was to match recognizable features in the ice field which was imaged from two successive orbits. The SAR data used was optically processed and hence was subject to resolution degradation and geometric distortions incurred during the processing. The displacement errors resulting from the use of this imagery were reduced by Leberl, Raggam, Elachi and Campbell (1983) by using radargrammetric methods to improve positional accuracy. This latter study, however, still relied on the manual tracking of

¹ Hibler, Tucker and Weeks, 1975; Crowder, McKim, Ackley, Hibler and Anderson, 1974; Shapiro and Burns, 1975
ice features. In fact the authors went so far as to say that as more accurate data became available, it was the operators ability to recognize ice features that would be the limiting factor on measurement accuracy. This assertion has been ignored by subsequent attempts to automate the procedure yet as we shall see is very true.

A subsequent paper by Curlander, Holt and Hussey (1985) used digitally correlated SAR data which substantially reduced radiometric and geometric errors. The measurement of ice displacements was facilitated by the use of digital data; however, the system still relied completely on the visual identification of ice features by an operator, and despite the claim of limited operator intervention, the displacements between the images used in the present study required ten hours of operator time. Fily and Rothrock (1985, 1986) completely automated the measurement procedure by using cross correlation which they borrowed from the cloud tracking work of Leese and his co-workers (Leese and Epstein, 1963; Leese, Novack and Clark, 1971, 1972; Phillips and Smith, 1972). However, while Leese et al computed their cross correlations using fast Fourier transforms (FFT), Fily and Rothrock computed them directly. They used a coarse-to-fine strategy to successively increase the accuracy of the displacement measurements while drastically reducing the computations. This approach was also adopted by Vesecky, Samadami, Daida, Smith and Bracewell, 1986a, 1986b, 1987. They have also investigated the effects of rotation and shear on the area correlation method and have successfully applied feature matching techniques to SEASAT SAR images.

Some of the results from this thesis have already been published in the first and, to date, the only published ice motion study using AVHRR imagery (Ninnis, Emery and Collins, 1986).

1.2.2 Organization of the Thesis

Chapter 2 poses the motion estimation problem as a filtering problem and derives the optimal filter by solving an appropriate optimization problem. This chapter also gives a brief review of some other sub-optimal matching schemes and uses the theory
of abstract function spaces to provide some geometric intuition into the action of the optimal and sub-optimal filters and defines a simple relationship between them.

Chapter 3 deals with the implementation of the derived filter. It refers to a discussion of the relevant features of white vs. Markov noise included in appendix A and proposes a noise model to be used in the study. Based on the previous noise discussion the noise model is shown to agree with the data. The noise model used is implemented as a two-stage filter for the SAR data where the first stage is an edge enhancing prefilter. An analytic form for this prefilter is derived and two implementations of it are discussed in 3.2. The chapter closes with a discussion of the implementation of the matching filter.

Chapter 4 reports the results of the experiments and is divided into a discussion of the AVHRR results and of the SAR results. The SAR results are compared to a manually measured grid of displacements and the results are analysed. The AVHRR results are compared to a some scattered manually measured displacements and the ice motion derived from the geostrophic wind. These results are analysed to reveal some important properties of the motion of the ice.

Chapter 5 addresses some issues raised by the research, reviews the general results and suggests directions for future work.
Chapter 2

Theory

2.1 Statement of the Problem

The problem which we wish to solve is the automated measurement of the field of motion of Arctic pack ice in sequential satellite images. We demand that the automated system which is to be implemented is an optimal solution to an appropriate estimation problem. We begin with two discrete image functions \( m'(x, y) \), imaged at time \( t_0 \), and \( f(x, y) \), imaged at time \( t_1 \), of finite domain, whose range is \([0,255]\). The images are separated in time by an amount, \( \Delta t = t_1 - t_0 \), that will allow all significant changes to be modeled as translations. This is a constraint both on the time interval between images and on the image pair themselves since many image pairs will be unsuitable for this technique due to small scale rotations or a large amount of deformation. First we focus on a particular area in the ice pack by operating on \( m'(x, y) \) with a boxcar function which is zero outside some region and unity inside. This operation yields a subimage, \( m(x, y) \), which is reduced in spatial extent. We also assume that, while somewhere within the function \( f'(x, y) \) there is an instance of \( m(x, y) \), the former function is corrupted with additive stationary Gaussian noise, \( n(x, y) \). We may narrow the region of \( f' \) in which we are interested by operating on this function with a second window function. This subimage is denoted \( f \). Hence we have the following two signals of interest:

- subimage 1 \( m(x, y) \) imaged at time \( t_0 \)
• subimage 2 \( f(x, y) = m(x + x_0, y + y_0) + n(x, y) \) imaged at time \( t_1 \)

where \((x_0, y_0)\) is the displacement of the ice within \( f' \) over the time interval, \( \Delta t \). An important assumption which we use is that the motion of the ice within \( f' \) is homogeneous, ie. that the ice is moving at the same speed and direction. As shall presently be seen this is equivalent to imposing a large signal-to-noise ratio (SNR).

The problem to be solved is: find the filter function, \( h(x, y) \), such that when it is convolved with \( f(x, y) \) the maximum value of the output will yield the best estimate of the displacement of \( m(x, y) \). Further, we wish the estimate to be \textit{optimal}, that is we wish to minimize the error, \( \epsilon \), between the \textit{true} displacement, \((\hat{x}, \hat{y})\), and the \textit{estimated} displacement, \((\hat{x}, \hat{y})\). The most commonly used performance criteria in estimation problems is the mean-squared error, \( E(\epsilon^2) \). It is assumed that the error, which is a random variable, has zero mean, \( E(\epsilon) = 0 \), hence the mean square error is the variance of \( \epsilon \).

The matched filter is a general result and no restrictions are placed on the colour, or frequency distribution, of the noise. In the case of white noise the filter is simply a correlator. Due to the linearity of the filter the coloured noise result may be broken into two filters: the first whitens the noise and the second is a correlator.

The matched filter is certainly not the only solution used in detecting image signals of known form. Another body of suboptimal matching techniques, each based on a particular measure of match or mismatch, are briefly reviewed.

The section closes with a brief summary on the key features of the theory to be carried forward into the implementation.

\section*{2.2 Derivation of the Optimum Filter}

At the filter input there may appear a combination of the signal of known form \( m(x, y) \) and the noise \( n(x, y) \) which is a stationary random process. Hence at the filter input
we have:

\[ f(x, y) = m(x, y) + n(x, y) \]  

(2.1)

This signal is passed through a linear filter \( \mathcal{H} \) with transfer function \( H(u, v) \), where \( u, v \) are spatial frequencies or wavenumbers. At the output of \( \mathcal{H} \) we have:

\[ \varphi(x, y) = \mu(x, y) + \nu(x, y) \]  

(2.2)

where \( \mu(x, y) \) is the result of passing the useful signal \( m(x, y) \) through \( \mathcal{H} \), and \( \nu(x, y) \) is the result of passing the noise \( n(x, y) \) through \( \mathcal{H} \).

We shall now derive the optimal signal parameter estimation filter which to a large extent follows Mc Gillem and Svedlow (1977). The derivation begins by expanding \( \mu(x, y) \) in a second-order Taylor series about the true lag position \((\bar{x}, \bar{y})\)

\[
\mu(x, y) \approx \mu(\bar{x}, \bar{y}) + \mu_x(\bar{x}, \bar{y})(x - \bar{x}) + \mu_y(\bar{x}, \bar{y})(y - \bar{y}) \\
+ \mu_{xy}(\bar{x}, \bar{y})(x - \bar{x})(y - \bar{y}) + \frac{1}{2}\mu_{xx}(\bar{x}, \bar{y})(x - \bar{x})^2 \\
+ \frac{1}{2}\mu_{yy}(\bar{x}, \bar{y})(y - \bar{y})^2 + \ldots
\]  

(2.3)

where

\[
\mu_x(\bar{x}, \bar{y}) = \frac{\partial \mu(x, y)}{\partial x} \bigg|_{x=\bar{x}, y=\bar{y}}
\]

(2.4)

Here we have assumed that \((x - \bar{x})\) and \((y - \bar{y})\) are small enough so that all higher order terms may be neglected. The necessary condition for a maximum at \((\bar{x}, \bar{y})\) is

\[
\frac{\partial \mu(\bar{x}, \bar{y})}{\partial x} = \frac{\partial \mu(\bar{x}, \bar{y})}{\partial y} = 0
\]  

(2.5)

substituting (2.3) and (2.5) into (2.2) we obtain

\[ \varphi(x, y) = \mu(\bar{x}, \bar{y}) + \mu_x(\bar{x}, \bar{y})(x - \bar{x}) + \mu_y(\bar{x}, \bar{y})(y - \bar{y}) + \mu_{xy}(\bar{x}, \bar{y})(x - \bar{x})(y - \bar{y}) \\
+ \frac{1}{2}\mu_{xx}(\bar{x}, \bar{y})(x - \bar{x})^2 + \frac{1}{2}\mu_{yy}(\bar{x}, \bar{y})(y - \bar{y})^2 + \nu(x, y) \]  

(2.6)

The filter that we seek will generate a maximum at the estimated lag position \((\hat{x}, \hat{y})\), hence we shall introduce the necessary condition for a maximum at this point

\[
\frac{\partial \varphi(x, y)}{\partial x} = \mu_{xy}(\hat{x}, \hat{y})(\hat{y} - \bar{y}) + \mu_{xx}(\hat{x}, \hat{y})(\hat{x} - \bar{x}) + \nu_x(\hat{x}, \hat{y}) = 0
\]

\[
\frac{\partial \varphi(x, y)}{\partial y} = \mu_{xy}(\hat{x}, \hat{y})(\hat{x} - \bar{x}) + \mu_{yy}(\hat{x}, \hat{y})(\hat{y} - \bar{y}) + \nu_y(\hat{x}, \hat{y}) = 0
\]  

(2.7)
if we arrange these equations in terms of \((\hat{x} - \bar{x})\) and \((\hat{y} - \bar{y})\), we can write the error in the lag as follows
\[
(\hat{x} - \bar{x}) = \frac{\mu_{zy} \nu_y - \mu_{yy} \nu_z}{\mu_{zx} \mu_{yy} - \mu_{zy}^2}
\]
(2.8)
\[
(\hat{y} - \bar{y}) = \frac{\mu_{zy} \nu_z - \mu_{zz} \nu_y}{\mu_{zx} \mu_{yy} - \mu_{zy}^2}
\]
where the arguments \((\hat{x}, \hat{y})\) and \((\bar{x}, \bar{y})\) have been left out for compactness of notation.

The variance of the error may be found by taking the mathematical expectation of \((\hat{x} - \bar{x})^2\) and \((\hat{y} - \bar{y})^2\) where it is assumed that \(E[\hat{x} - \bar{x}] = E[\hat{y} - \bar{y}] = 0\)

\[
(\hat{x} - \bar{x})^2 = \frac{\mu_{zy}^2 \nu_y^2 - 2 \mu_{zy} \mu_{yy} \nu_z \nu_y}{[\mu_{zx} \mu_{yy} - \mu_{zy}^2]^2}
\]
(2.9)
\[
(\hat{y} - \bar{y})^2 = \frac{\mu_{zy}^2 \nu_y^2 - 2 \mu_{zy} \mu_{zz} \nu_z \nu_y + \mu_{zz} \nu_y^2}{[\mu_{zx} \mu_{yy} - \mu_{zy}^2]^2}
\]

These relationships are greatly simplified if we assume \(\mu_{zy} = 0\). In this case the variance expressions become
\[
(\hat{x} - \bar{x})^2 = \frac{\nu_y^2}{\mu_{zx}^2}
\]
(2.10)
\[
(\hat{y} - \bar{y})^2 = \frac{\nu_y^2}{\mu_{yy}^2}
\]

\textit{McGillem and Svedlow (1977)} have suggested that a reversible linear transformation may be applied to the \((x, y)\) coordinate system so that in the new coordinate system \(((x_1, y_1), \text{say}),\) the term \(\mu_{zy}(\hat{x}_1, \hat{y}_1) = 0\) this new coordinate system and the reverse transformation can be applied to return to the original space. Hence there is no loss of generality in assuming \(\mu_{zy} = 0\).

We may rewrite (2.10) in their equivalent integral forms
\[
(\hat{x} - \bar{x})^2 = \frac{\int \int \int h(\alpha, \beta) h(x, y) R_{xx}(\alpha - x, \beta - y) \, d\alpha \, d\beta \, dx \, dy}{\left[ \int \int h(\alpha, \beta) m_{xx}(\hat{x} - \alpha, \hat{y} - \beta) \, d\alpha \, d\beta \right]^2}
\]
(2.11)
\[
(\hat{y} - \bar{y})^2 = \frac{\int \int \int h(\alpha, \beta) h(x, y) R_{yy}(\alpha - x, \beta - y) \, d\alpha \, d\beta \, dx \, dy}{\left[ \int \int h(\alpha, \beta) m_{yy}(\hat{x} - \alpha, \hat{y} - \beta) \, d\alpha \, d\beta \right]^2}
\]
where \( R(x, y) \) is the autocorrelation function of the input noise \( n(x, y) \), the subscripts denote partial differentiation with respect to the corresponding variables. Given these expressions for the variance one would like to find a filter, \( h(x, y) \), that minimizes \((\hat{x} - \bar{x})^2\) and \((\hat{y} - \bar{y})^2\). The derivation may be stated in the following equivalent form (Luenberger, 1969).

\[
\text{minimize : } I(h) = \int \int \int h(\alpha, \beta)h(x, y)R_{zz}(\alpha - x, \beta - y) \, d\alpha \, d\beta \, dx \, dy
\]

subject to : \( J(h) = \left[ \int \int h(\alpha, \beta)m_{zz}(\hat{x} - \alpha, \hat{y} - \beta) \, d\alpha \, d\beta \right]^2 = K_1^2 \)

where \( K_1 \) is a constant and \( I(h) \) and \( J(h) \) are functionals of the filter \( h(x, y) \). This is a constrained minimization problem and is referred to as the isoperimetric problem in the calculus of variations (Weinstock, 1974).

The method of solution is to form an augmented functional, which is the sum of \( I(h) \) and a parametric constant, \( \lambda \), times \( J(h) \) and then minimize the augmented functional. The appropriate functional in our case is

\[
L(h) = I(h) - \lambda J(h) = \int \int \int h(\alpha, \beta)h(x, y)R_{zz}(\alpha - x, \beta - y)
- \lambda h(\alpha, \beta)h(x, y)m_{zz}(\hat{x} - \alpha, \hat{y} - \beta)m_{zz}(\hat{x} - \alpha, \hat{y} - \beta) \, d\alpha \, d\beta \, dx \, dy
\]

to simplify the notation let us make the following substitution

\[
Q(x, y, \alpha, \beta) = R_{zz}(\alpha - x, \beta - y) - \lambda m_{zz}(\hat{x} - \alpha, \hat{y} - \beta)m_{zz}(\hat{x} - \alpha, \hat{y} - \beta)
\]

then we may rewrite the augmented functional as

\[
L(h) = \int \int \int h(\alpha, \beta)h(x, y)Q(x, y, \alpha, \beta) \, d\alpha \, d\beta \, dx \, dy
\]

The necessary condition for \( L(h) \) to be a minimum is that the variation of the augmented functional be zero (ie. that \( L \) is stationary). Hence

\[
\delta L(h) = \int \int \int [\delta h(\alpha, \beta)h(x, y) + h(\alpha, \beta)\delta h(x, y)]Q(x, y, \alpha, \beta) \, d\alpha \, d\beta \, dx \, dy
\]

\[
= 2 \int \int \left\{ \int h(\alpha, \beta)Q(\alpha, \beta, x, y) \, d\alpha \, d\beta \right\} \delta h(x, y) \, dx \, dy
\]

\[
= 0
\]
the latter result is due to the symmetry of $Q$ with respect to $\alpha, \beta$ and $x, y$. If (2.16) is equal to zero then

$$\int \int h(\alpha, \beta)Q(\alpha, \beta, x, y) \, d\alpha \, d\beta = 0 \quad (2.17)$$

by expanding $Q$ in (2.17) we have

$$\int \int R_{zz}(x - \alpha, y - \beta)h(\alpha, \beta) \, d\alpha \, d\beta = \lambda_1 K_1 m_{zz}(\bar{x} - x, \bar{y} - y) \quad (2.18)$$

This can be solved by taking the Fourier Transform of both sides:

$$\int \int \int R_{zz}(x - \alpha, y - \beta)h(\alpha, \beta)e^{-j2\pi(ux+vy)} \, dx \, dy = \lambda_1 K_1 \int \int m_{zz}(\bar{x} - x, \bar{y} - y)e^{-j2\pi(ux+vy)} \, dx \, dy \quad (2.19)$$

the frequency domain equivalent of this is

$$-4\pi^2 u^2 S_n(u, v)H(u, v) = -4\pi^2 u^2 \lambda_1 K_1 M^*(u, v)e^{-j2\pi(u\bar{z}+v\bar{y})} \quad (2.20)$$

solving this for the filter transfer function $H(u, v)$ we obtain

$$H(u, v) = \lambda_1 K_1 e^{-j2\pi(\bar{z}+\bar{y})} \frac{M^*(u, v)}{S_n(u, v)} \quad (2.21)$$

which is the definition of the matched filter derived earlier multiplied by an arbitrary constant factor $\lambda_1 K_1$. Hence the filter which minimizes the error variance along the $x$ direction is the matched filter. The minimization of $(\bar{y} - \bar{y})^2$ follows the same path where we are minimizing (from 2.11)

$$\mu_{yy}^2 = \int \int \int h(\alpha, \beta)h(x, y)R_{yy}(\alpha - x, \beta - y) \, d\alpha \, d\beta \, dx \, dy \quad (2.22)$$

subject to

$$\mu_{yy}^2 = [\int \int h(\alpha, \beta)m_{yy}(\bar{x} - \alpha, \bar{y} - \beta) \, d\alpha \, d\beta]^2 = K_2^2 \quad (2.23)$$

The minimization problem yields the following solution for $H(u, v)$

$$H(u, v) = \lambda_2 K_2 \frac{M^*(u, v)e^{-j2\pi(u\bar{z}+v\bar{y})}}{S_n(u, v)} \quad (2.24)$$
INTER 2. THEORY

This is the transfer function of the *Matched Filter* which simultaneously minimizes the error variance in both the $x$ and $y$ directions.

The physical meaning of (2.24) is quite simple, the larger the *amplitude spectrum* of the useful signal and the smaller the *power spectrum* of the noise in the frequency interval $(u, v) \rightarrow (u + du, v + dv)$ the more the optimum filter $H$ passes the frequencies in this interval.

If the noise is *white*, that is its power spectral density is constant

$$S_n(u, v) = S_0 = \text{constant}$$

This means the noise spectrum is evenly distributed over all frequencies. The optimum filter corresponding to the white noise case called the *Matched Filter*, (although this name is often applied to the coloured noise case as well). The transfer function is given by

$$H(u, v) = e^{-j(ux_0 + uy_0)}M^*(u, v) \quad (2.25)$$

hence $H(u, v)$ is completely determined by the form of the signal $m(x, y)$ to which it is matched. Equation (2.25) is obtained by making the substitution $c = S_0$ in equation (2.24).

The impulse response of the matched filter defined by (2.25) is

$$h(x, y) = \frac{1}{2\pi} \int \int e^{j(ux + uy)}H(u, v) \, du \, dv \quad (2.26)$$

or

$$h(x, y) = m(x - x_0, y - y_0) \quad (2.27)$$

hence the action of the matched filter is described by the following

$$\mu(x, y) = \int \int f(x', y')m(x' - x + x_0, y' - y + y_0) \, dx' \, dy' \quad (2.28)$$

If we consider (2.1) the matched filter forms the cross-correlation between the useful signal $m(x, y)$ and the input function $f(x, y)$. It happens that the signals we are dealing
with have finite domain, thus $m$ is zero outside some bounded region $A$. We also assume for the moment that $m$ is small compared to $f$, although it will become clear later that this is a necessary condition for the correct calculation of the correlation. We thus compute (in abbreviated notation) $\int \int m f$ for all spatial lags of $m$ with respect to $f$. By the Schwartz inequality we have

$$\frac{\int \int_A m(x, y) f(x + \alpha, y + \beta) \, d\alpha \, d\beta}{\sqrt{\int \int_A m^2(x, y) \, dx \, dy \int \int_A f^2(x, y) \, dx \, dy}} \leq 1 \tag{2.29}$$

This is the normalized cross correlation which is the form of the matched filter we shall use. It should be noted that while $\int \int m^2$ is constant, $\int \int f^2$ changes with each lag position and must be recomputed. A further normalization is accomplished by removing the mean value from each image signal before (2.29) is computed. This makes the (2.29) a normalized cross covariance function which in statistical parlance is the Pearson product moment correlation (Brownlee, 1960).

The normalization of the correlation function makes the matching measure more robust against bright spots in $f$. These bright spots heavily influence the correlation by dramatically increasing the sum of products in a small area. However, while the normalization can reduce these effects it cannot remove them completely.

A better understanding of the mismatch error variance can be obtained by substituting the filter transfer function into our original expressions for the error variance (2.11) and transforming to the frequency domain.

$$\frac{(\hat{x} - \bar{x})^2}{\delta^2} = \left[ \frac{-\mu_{xy}^2}{\delta v^2 \rho} + \delta u^2 \rho \right]^{-1}$$

$$\frac{(\hat{y} - \bar{y})^2}{\delta^2} = \left[ \frac{-\mu_{xy}^2}{\delta v^2 \rho} + \delta v^2 \rho \right]^{-1} \tag{2.30}$$
where

\[
\delta u = \left[ 4\pi^2 \int \int u^2 \frac{|M(u,v)|^2}{S_n(u,v)} \, du \, dv \right]^{-1/2}
\]

\[
\int \int \frac{|M(u,v)|^2}{S_n(u,v)} \, du \, dv
\]

= effective signal bandwidth of the input signal in the \(x\) direction \hspace{1cm} (2.31)

\[
\delta v = \left[ 4\pi^2 \int \int v^2 \frac{|M(u,v)|^2}{S_n(u,v)} \, du \, dv \right]^{-1/2}
\]

\[
\int \int \frac{|M(u,v)|^2}{S_n(u,v)} \, du \, dv
\]

= effective signal bandwidth of the input signal in the \(y\) direction

and

\[
\rho = \int \int \frac{|M(u,v)|^2}{S_n(u,v)} \, du \, dv
\]

\[
\int \int \frac{|M(u,v)|^2}{S_n(u,v)} \, du \, dv
\]

= output signal-to-noise ratio for correlated noise \hspace{1cm} (2.32)

Here the variance is expressed in terms of the SNR and a quantity known as the effective or mean squared bandwidth. We may simplify (2.30) if we assume that \(\mu_{xy}\) is zero. This assumption would apply if the spectral shape of the noise and the useful signal was similar and could be modeled as differing by only a constant (McGillem and Svedlow, 1976). The variance expressions may then be written:

\[
(\hat{x} - \bar{x})^2 = \frac{1}{\delta u^2 \rho} \quad \text{and} \quad (\hat{y} - \bar{y})^2 = \frac{1}{\delta v^2 \rho}
\]

(2.33)

Hence for a given SNR there is an inverse relationship between bandwidth and variance. This is a reaffirmation of the classic tradeoff between the spatial extent and bandwidth of a signal.

### 2.3 Suboptimal Matching

While the matched filter is the optimal matching system it is not without its competitors. In fact there are many ways of measuring the degree of match or mismatch between two image functions \(m\) and \(f\) over some region \(A\). The most popular measures (Rosenfeld and Kak, 1982; Ballard and Brown, 1982) are as follows:
CHAPTER 2. THEORY

1. Maximum of the Absolute Differences (MAD) \[ \max_A |m - f| \]

2. Sum of the Absolute Differences (SAD) \[ \int \int_A |m - f| \]

3. Mean Square Difference (MSD) \[ \int \int_A (m - f)^2 \]

These expressions are all distance measures or metrics. The measure is computed for each for each lag position and the minimum is chosen as the correct match location. Their use is embraced by a field known generally as Template Matching and the principal motivation for using them is to reduce the computational load implied by the matched filter.

If the image functions are viewed as points in an infinitely dimensioned function space, then the distance between two points in any particular space are measured with the metric underlying that space. The metrics underlying the three distance measures given above as well as the matched filter are members of the same class of metrics, the $L_p$ Norms. Duornichenko, (1981) made a study of the effects of changing the metric underlying cross correlation. He found that for analytic functions representative of image signals both the MAD and SAD are unsuitable measures.

The MAD is used very infrequently in practice. The SAD is much more popular and is the cornerstone of the Sequential Similarity Detection Algorithm (SSDA) (Barnea and Silverman, 1972). This algorithm reduces the number of high accuracy calculations by randomly selecting the areas of the search image to be compared with the reference image. In the SSDA if the accumulated absolute differences exceeds some threshold before all calculations within it have been completed, processing for that particular lag position ceases and the number of pixels examined is recorded as a rating for that lag position. After all possible positions have been examined, the position with the highest rating is declared the best match. Both a constant and a monotonically decreasing (from the centre of $A$) threshold were used. Use of the SSDA resulted in significant savings in computation time for the registration of images from the NOAA meteorological satellite ITOS-3 (Barnea and Silverman, 1972). However the match measure defined the SAD
is unsuitable for most imagery (Dvornychenko, 1981).

The MSD match may be rewritten as:

\[ \int \int (m - f)^2 = \int \int m^2 + \int \int f^2 - 2 \int \int mf \]  

(2.34)

the first term represents the reference image energy which is a function of the particular reference, not of lag position and thus remains constant until a new reference is chosen. The second term is the search image energy and is constant for each lag position. Hence maximizing \( \int \int (m - f)^2 \) is equivalent to minimizing \( \int \int mf \) for a given \( \int \int m^2 \) and \( \int \int f^2 \). Thus we see that the MSD measure is equivalent to the matched filter for the white noise case.

### 2.4 Summary

In this chapter we pose the problem we wish to solve as a estimation problem. The solution to this problem is an optimal filter which is obtained by minimizing the error variance. The transfer function of this optimal filter is shown to be:

\[ H(u, v) = \lambda_2 K_2 e^{-j2\pi(u\bar{s}+v\bar{v})} \frac{M^*(u, v)}{S_n(u, v)} \]  

(2.35)

For the case of white noise the filter is a correlator. For the case of coloured noise the filter, due to it’s linearity, may be broken into two filters. The first is an inverse filter that shapes the signal spectrum according to the power spectrum of the noise. This step yields a white noise signal and is, hence, called a whitening filter, \( H_w(u, v) \), and the second step is a correlator, \( H_m(u, v) \), hence,

\[ H(u, v) = H_w(u, v)H_m(u, v) \]
Chapter 3
Implementation

3.1 Questions of Noise

Noise is a somewhat ambiguous term when standing on its own. It may be conveniently defined as some process, either random or structured, which is corrupting or masking a particular process or function of interest. For instance the snow which appears in television pictures is representative of a whole class of noise and its removal was one of the first applications of digital signal processing (Graham, 1962). The noise belonging to this class is comprised by random uncorrelated additive errors whose formal characterization is Gaussian stationary white noise.

In this study the noise which is of interest, and that which is modeled is the noise which corrupts the systems ability to detect the motion of the ice. This is fundamentally different from the noise which corrupts the imaging process. In the case of AVHRR this latter type of noise corrupts the radiometers ability to sense the ice and is well modeled as additive and white. In the case of SAR this noise is due to the coherent nature of the processing and is referred to as speckle noise. This noise being a function of range is multiplicative (the brighter the region the noisier it is) and is potentially very dangerous to the detection system. However the digital processor developed at the Jet Propulsion Laboratory includes a means for significantly improving this noise situation (Curlander et al, 1982; Porcello et al, 1976) and the low pass filtering used in this study further improves the noise (Lee, 1981). Hence at the 200m resolution
used the effects of the speckle are effectively nonexistent and the image noise may be modeled as in the AVHRR.

Two important theoretical models of noise are briefly reviewed in Appendix 1. Both models allow a complete description of the noise with second order statistics and both yield analytic expressions for the autocorrelation function which is subsequently compared to that from the images themselves.

3.1.1 A Noise Model

The noise model used in this study is similar to that used in a series of papers from the Laboratory of Applications in Remote Sensing (LARS) at Purdue University (McGillem and Svedlow, 1976,1977; Anuta and Davillou, 1982) on the registration of LANDSAT agricultural scenes. In these studies the subimage at time $t_1$ is modeled as signal, while the subimage at time $t_2$ is modeled as signal plus additive noise, where the temporal changes between $t_1$ and $t_2$ are assumed to be contained in the noise. This means that samples of the noise may be obtained by computing $f - m$ at the correct match location between them. Figure 3.1 show the autocorrelation functions, $R_n$, of $f - m$, where $f - m$ is the average of many samples of $f - m$ between the two images. It is clear that $R_n$ from the two AVHRR image pairs approximates a delta function which suggests a white noise model is appropriate for this imagery. Figure 3.1 (bottom) shows $R_n$ from the SAR image pair as well as the function specified by the Markov model derived in Appendix 1.1.

$$R(\tau_x, \tau_y) = K^2 \exp\{-\alpha_x|\tau_x| + \alpha_y|\tau_y|\}$$  (3.1)

The correspondence between the two curves strongly suggests that the Markov model is appropriate for the SAR imagery and hence the image signal must be whitened with the noise spectral density.

It should be noted that in computing the averaged difference image, $f - m$, the *ergodic* assumption was made. This assumption equates spatial and ensemble averages
and allows for the computation of certain statistical averages such as the mean and autocorrelation function from a single finite segment of data.
Figure 3.1: Autocorrelation functions $R_n$ for AVHRR images (top) and SAR images (bottom).
3.2 The Whitening Filter

The transfer function of the optimal estimation filter was found in chapter 2, and was given by:

\[ H(u, v) = c e^{-j2\pi(uu_0 + vv_0)} \frac{M^*(u, v)}{S_n(u, v)}. \]  

(3.2)

If \( S_n(u, v) \) satisfies \( \epsilon \leq S_n \leq M \) for some \( \epsilon > 0 \) and \( M < \infty \) and the noise autocorrelation function satisfies \( \sum |R_n| < \infty \), then \( S_n \) is given by the following (Woods, 1984):

\[ S_n(u, v) = c \frac{1}{|H_w(u, v)|^2}. \]  

(3.3)

In fact by modeling the noise as a Gaussian stationary Markov process we avoid all the difficulties of inverse filtering that are implied by (3.3) since the inverse of its spectral density is well defined. If we assume that the Markov process is separable then the spectral density of the noise is given by the Fourier transform of (3.1):

\[ S_n(u, v) = \frac{4\alpha\beta K}{(\alpha^2 + 4\pi^2u^2)(\beta^2 + 4\pi^2v^2)}. \]  

(3.4)

substitution of this expression into (3.2) followed by factorization into complex conjugates yields the whitening filter:

\[ H_w(u, v) = \frac{1}{2K\sqrt{\alpha\beta}}(\alpha\beta + \beta j2\pi u + \alpha j2\pi v - 4\pi^2uv) \]  

(3.5)

the spatial domain representation of this is:

\[ h_w(x, y) = \frac{\alpha\beta\delta(x, y) + \beta \frac{\partial}{\partial x} + \alpha \frac{\partial}{\partial y} - \frac{\partial^2}{\partial y\partial x}}{2A\sqrt{\alpha\beta}} \]  

(3.6)

It is clear that this filter acts as a differential operator and in what follows I shall focus on this basic function rather than on attempting to discretize the function. Differential operators have been standard tools in image processing for locating edges or brightness changes in digital imagery. SAR imagery is, in general, replete with edges that correspond to significant features in the ice pack, such as:

1. The boundary between open water or very thin ice (nilas) and the actual ice floes
2. boundaries between multiyear ice and younger ice

3. locations of pressure ridges and rubble fields within the ice floes.

The SAR imagery used in this study is from an L-band SAR whose features generally tend to be of type 1 or 3. The discrimination between thick first year and older ice has only been successful with higher frequency SAR such as X and C-band or a combination of X/C and L-band SAR (Lyden et al, 1984; Burns et al, 1984; Holmes et al, 1984). An example of the SAR signatures of types 1 and 3 features may be seen in figure 3.2. Hence by operating on the imagery with a differential operator, the second stage of the matched filter operates on the significant image features rather than on the raw brightness values.

Differential operators may be broken roughly into two classes (Horn, 1986):

1. First order operators
   
   (a) brightness gradient
   
   \[ g(x, y) = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) \]  
   
   (3.7)

2. Second order operators
   
   (a) squared gradient
   
   \[ g(x, y) = \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 \]  
   
   (3.8)

(b) Laplacian
   
   \[ g(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \]  
   
   (3.9)

(c) quadratic variation
   
   \[ g(x, y) = \left( \frac{\partial^2 f}{\partial x^2} \right)^2 + 2 \left( \frac{\partial^2 f}{\partial x \partial y} \right) \left( \frac{\partial^2 f}{\partial y \partial x} \right) + \left( \frac{\partial^2 f}{\partial y^2} \right)^2 \]  
   
   (3.10)

The brightness gradient and the Laplacian are both linear while the other two are both non-linear. The brightness gradient is the only directional operator which
produces results which depend on the orientation of the edge, implying a need for the application of a series of orientation-specific operators at any given point. The second order operators are all rotationally symmetric, treating edges at all angles equally, however of the three only the Laplacian retains the sign of the brightness difference across the edge meaning that it alone would allow the reconstruction of the original image from the edge image. The brightness gradient and the Laplacian have become the most popular differential operators for very different reasons.
Figure 3.2: A small portion of rev.1439 showing some significant image features (top). Profiles showing brightness gradient from image and operator responses to the image features (bottom).
CHAPTER 3. IMPLEMENTATION

An important characteristic of these two operators is their frequency response or transfer function given, in the $x$ direction, by:

- **Brightness gradient**
  \[
  G_1(u, v) = -j2\pi u F(u, v)
  \]

- **Laplacian**
  \[
  G_2(u, v) = -4\pi^2(u^2 + v^2) F(u, v)
  \]

It is evident that the transfer function of $G_1$ increases linearly while that of $G_2$ increases exponentially. This has an important impact on the discrete implementation of the operators. It should be remembered that the noise model discussed in §3.1.1 is assumed to represent the temporal changes between the images; in fact, each image is also corrupted with white noise. While both $G_1$ and $G_2$ enhance the high frequency components of the image noise, $G_2$ has a tendency to get lost in the noise especially when its impulse response has small support (spatial extent). One of the earliest published edge detector was a $2 \times 2$ implementation of $G_1$ by (Roberts (1965), this operator along with several small support ($5 \times 5$ and less) implementations of $G_1$ (Prewitt, 1970; Abdou and Pratt, 1979; Shaw, 1979) have shown that it is robust against noise in many applications.

3.2.1 The First Order Filter

The $3 \times 3$ Sobel edge detector given a reasonable SNR has been shown to be an accurate measure of edge presence (Davis, 1975; Abdou and Pratt, 1979; Kittler, 1983; Iannino and Shapiro, 1979). The operator is defined in terms of two $3 \times 3$ masks which measure the edge signal in both the $x$ and $y$ directions. If the $3 \times 3$ mask is centered on the pixel defined in figure 3.3 as $e$, then the edge signals in the $x$ and $y$ directions at the pixel $e$ are defined by:

\[
\begin{align*}
  e_x &= [c + Kf + i] - [a + Kd + g] \\
  e_y &= [a + Kb + c] - [g + Kh + i]
\end{align*}
\]

(3.13)
the magnitude of the edge signal is:

\[ E = \sqrt{e_x^2 + e_y^2} \]  

(3.14)

while the edge angle is given by:

\[ \gamma = \arctan \frac{e_y}{e_x} \]  

(3.15)

where K has been shown to be a function of the angle (Kittler, 1983) and varies between 2.0 and 3.0. In this study we use edge magnitude only and use the standard value of \( K = 2.0 \) (Ballard and Brown, 1982).

The discrete implementation of the whitening filter based on the Sobel operator involves the computation of \( E \) at each pixel in the images defined in chapter 2 as, \( f \) and \( m \). The response of the Sobel operator to a typical one dimensional image slice is shown in figure 3.2.

3.2.2 The Second Order Filter

The ability of the first order filter to withstand lower SNR's is somewhat gratuitous since numerical differentiation is not, in general, robust against noise. In fact numerical differentiation is known to be an *ill-posed* problem (Torre and Poggio, 1984; Poggio and Torre, 1984). An important criterion for defining well-posed problems is that the solution depends continuously on the initial data (this is equivalent to saying that the solution is robust against noise). This criterion is violated in numerical differentiation.
which can be disrupted by small amounts of noise. The method for transforming an
ill-posed problem into one which is well-posed is known as regularization (Tikhonov and
Arsenin, 1977). The theory of regularization has been applied to edge detection by
Torre and Poggio (1984) who found that the appropriate regularizing operator was a
low-pass filter. This is an intuitive result since from looking at (3.11) it would appear
that a low-pass filter would be necessary to stabilize the frequency response of the
second order differential operator. This is also a very general result and the specific
shape of the filter is constrained by two issues:

1. intensity changes in the imagery are generally localized in the spatial domain.

2. a single operator cannot detect intensity changes at different spatial scales and
   it is therefore desirable to restrict the scale at which intensity changes take place
   in the output of the filter.

These two issues are conflicting but may be optimally satisfied by the Gaussian
smoothing function (or roll-off low-pass filter) whose impulse response, \( g \), and transfer
function, \( G \) are given (in one dimension) by:

\[
g(x) = K_1 \exp\left\{-\frac{x^2}{2\sigma^2}\right\} \\
G(\omega) = K_2 \exp\left\{-K_3 2\sigma^2 \omega^2\right\}
\]  

(3.16)

where \( K_1, K_2 \) and \( K_3 \) are scaling constants. When this filter is combined with the
Laplacian function we have the so-called \( \nabla^2 g \) operator given, in two dimensions, by,

\[
\nabla^2 g(x, y) = K[2 - (r^2/\sigma^2)] \exp\left\{-r^2/2\sigma^2\right\}
\]  

(3.17)

where \( r^2 = x^2 + y^2 \) and \( K \) is a scaling constant. This filter is an approximately bandpass
filter which detects edges of a particular resolution. The one dimensional profile of this
filter is shown in figure 3.4.

The band of frequencies passed by the filter may be selected by changing the value
of \( \sigma^2 \). By using particular values for this parameter a number of wavenumber chan-
nels may be produced. Each channel corresponds to a particular filter, and hence
edge, resolution. This approach has been the core of the theory of computational vision spearheaded by the late David Marr of M.I.T. \((Marr, 1976,1977,1982; Marr and Poggio, 1977,1978; Marr and Hildreth, 1979; Poggio, 1985)\).

A quantity which is used as a measure of zero-crossing strength is the \textit{slope} \((Grimson and Hildreth, 1985)\). It is defined to be the magnitude of the gradient of the bandpassed images at the location of the zero-crossing. Thus

\[
slope = \left| \nabla (\nabla^2 g \otimes I) \right|
\]

where \(I\) denotes the image. The zero crossings were located using an efficient predicate-based algorithm described in \textit{Huertas and Medioni} (1986). This algorithm inspects each \(3 \times 3\) neighborhood in the convolved image and matches it to one of eleven allowable zero crossing predicates which define a total of 24 edges positions. The contrast and width of the intensity change may be computed explicitly using the slopes of the zero-crossings from operators with two different resolutions. In this study a single resolution was used whose frequency response matched the spatial resolution of the intensity changes of interest. The value of \(\sigma\) used in the study was \(\sqrt{2}\) which is a tradeoff between high bandwidth (lower number of filter coefficients) and low aliasing error (due to the sampling of a continuous function) \((Clark and Lawrence, 1984)\). The edge magnitudes for the SAR images are shown in Appendix C where they have been scaled between 0 and 255 and shown as grey scale images.

Two recent papers have highlighted an important feature of the zero crossings of the \(\nabla^2 g\) filter \((Berzins, 1984; Clark, 1986)\). It has been shown that some of the edges asserted by the filter do not correspond to any significant changes in image intensity. These edges have been referred to as \textit{phantom edges} and Clark provides an excellent analysis of these features. He shows that if the image function is modeled as a white normally distributed function, the phantom edges are fewer by a factor of three and a half and weaker by a factor of eight than the authentic edges. The model used to represent the image in order to derive these quantitative results is not particularly well-suited to most images. However it is evident that the strength of the authentic
edges is much greater than that of the phantom edges and the classification function proposed by Clark was not implemented in this study.

3.3 The Matching Filter

As mentioned in chapter 2 the matching system systematically divides the images into smaller subimages where it is assumed that the motion of the ice within each subimage is homogeneous. This methodology has previously been applied to the detection of cloud motion (Leese and Novack, 1971) and updating automatic guidance systems (Lo and Gerson, 1979; Mostafavi, 1979; Steding and Smith, 1979a). While the approach may seem somewhat brute-force, this systematic division is what makes the system fully automatic. It removes the necessity for a decision mechanism as to which piece of ice or ice feature to track and which to ignore. These kinds of decisions are either made by the operator (Curlander and Holt, 1985), in which case the system is no longer automatic, or they are computational. The process of making decisions computationally involves compiling a list of rules and as regards ice motion this would require a better knowledge of radar and radiometer ice signatures and of the geographic variability of
ice dynamics than is presented available. A general discussion of these issues is included in chapter 5. The cross correlation is made computationally more efficient by using the convolution theorem which equates convolution in the spatial domain to multiplication in the wavenumber domain \((Cooley et al, 1967)\),

\[ m(x, y) \otimes f(x, y) = M^*(u, v)F(u, v) \]

This is a standard result which has been harnessed many times in correlation-based image registration \((Anuta, 1970)\) and cloud-motion tracking \((Leese and Novack, 1971)\). However care must be exercised in using this theorem since the Fourier transform assumes the signal is periodic and convolutions implemented in this way are subject to wrap-around error in which the correlation coefficients at each spatial lag will be biased with the exception of zero lag \((Blackman and Tukey, 1958)\). This problem is easily rectified, however, by choosing signal segments of different lengths. The matching system processes the imagery according to the following sequence of events:

1: The image \(m(x, y)\) is subdivided into \(N_f \times N_f\) pixel filter windows, \(m_w(x, y)\), and the succeeding image \(f(x, y)\) is subdivided into corresponding \(N_s \times N_s\) pixel search windows, \(f_w(w, y)\).

2: The window means were subtracted and the variances were computed. In the case of \(f_w(x, y)\) we have an array of variances corresponding to each location of \(m_w\) within \(f_w\).

3: The filter and search windows are padded with zeros to the next highest power of 2 and the FFT's of \(m_w\) and \(f_w\) (denoted \(M_w\) and \(F_w\) resp.) are computed.

4: The product, \(R_{mf}(u, v) = M_w^*(u, v)F_w(u, v)\) is formed and the inverse FFT, \(r_{mf}\), is computed.

5: The unbiased portion of \(r_{mf}\) (correlations which correspond to positions where \(m_w\) lies entirely within \(f_w\)) is extracted and the maximum value within this unbiased array is located.
3.3.1 Optimum Window Size

The choice of the filter window size has an important effect on the performance of the system. The size must balance two opposing criteria: it must be small enough to be increase the probability that the motion within the window is pure translation; and it must be large enough to ensure a good distinction between a false match and a correct match. The window sizes used were based on a small amount of experimentation with different values and is therefore rather subjective. However, since the ice motion is potentially discontinuous and the scales of rotation and deformation are unknown a more analytic approach such as found in a group of papers from Systems Control Inc is not possible. This latter research was devoted to automatically updating unmanned airborne vehicle guidance systems and derived optimal filters and windows for registration based on a single rotation angle applied to the whole scene, where this rotation was predictable.

The AVHRR filter window used agrees with some error analysis done by Fily and Rothrock (1986b) while the SAR filter window agrees with some experimentation with window sizes done by Vesecky et al, (1986b). The filter window sizes used are as follows:

- AVHRR $N_f = 22 \times 22$ pixels
- SAR $N_f = 30 \times 30$ pixels

The choice of the search window size is based on the maximum amount of ice motion expected in the region. This is a difficult issue to resolve since the magnitude and variability of ice motion varies according to season and geographic location. However the fact that this method approximates all motion as translations combined with the assumption that motion within the filter windows is homogeneous restricts the use of this system, as presently implemented, to winter scenes of the central ice pack. It is well known that the two major forces in this regime are the wind and water stress.

\footnote{Mostafavi and Smith, 1978a,b; Smith and Mostafavi, 1979; Mostafavi, 1979,1981; Steding and Smith, 1979a,b}
(Rothrock, 1979; Hibler, 1984). Of these, the former especially causes the ice to react quickly to its influence. Hence a knowledge of the prevailing current system and surface pressure field would considerably improve the system's ability to initiate itself (Holt, 1986). In the absence of this information a maximum amount of ice motion was input. This value was chosen to be reasonable both in terms of the amount of computation it implied and the number of different image pairs to which it could be applied. A maximum magnitude of 4 km per day was used which translates into the following search window sizes:

- AVHRR \( N_s = 32 \times 32 \) pixels
- SAR \( N_s = 128 \times 128 \) pixels

It should be noted at this point that another force which plays a major role in the dynamics of the ice cover is the complex and poorly understood interaction of the ice with itself. This force depends not only on the material characteristics of the ice, which vary considerably according to the age of the ice, but also on the presence of an immovable object such as a coastline. The increased influence of this force over the dynamics of the ice implies a much larger potential for discontinuous motion which in turn will elude the detection system.

### 3.3.2 Error Correction

As has been discussed earlier, the matched filter can often give erroneous results due to its dependence on the energy of the image signals. The prewhitening of the SAR images which produces an edge image means that, in this case, the filter is dependent on edge energy rather than on image energy. The false matches stem from the fact that the matched filter correlates areas only. These false matches may be detected in a post-processing step which examines the following evidence:

1. global statistics of the correlation coefficients;
2. consistency in the local field of motion.

With regard to the first, the mean and standard deviation of the correlation coefficients are computed and a vector is considered in error if it is more than one standard deviation from the mean. Consistency in the local displacement field is also interpreted in terms of statistical deviation. First we compute the mean and standard deviation of the $x$ and $y$ displacements in the $5 \times 5$ neighborhood around the vector being processed. A displacement is considered inconsistent if it deviates by more than one standard deviation from the mean.

The displacement tagged as incorrect by either of the above decisions is recomputed by using the displacements of its eight nearest neighbors as queues in searching the original unbiased correlation array. The maximum correlation coefficient is chosen from the $5 \times 5$ neighborhood surrounding each search queue. This process yields eight coefficients from which the largest is chosen as the replacement for the erroneous displacement. It was found that the recomputation process was most effective when implemented in two passes, the first pass used both decisions while the second pass used only the motion consistency decision. This two pass process allowed the system to recover from certain noisy neighborhoods which would normally have escaped from the first pass. Hence the matching system consists of three steps, the matching step and two recomputation steps.

In some cases the deformation of the ice cover will be too great and the system's estimate of the ice motion will be poor. In this study the quality of the correlation coefficient was estimated statistically. In order to estimate the statistical significance of the correlation coefficients some simple analysis, similar to Emery et al (1986), was performed. The spatial decorrelation scales of the imagery were first derived by averaging the autocorrelation functions from all of the $N_f \times N_f$ windows in the imagery and finding the distance from the central peak to the nearest zero-crossing. The total number of independent points or, effective degrees of freedom, $N^*$, of the correlation computations, was obtained by dividing the total number of observations by the decor-
relation scale. Finally $N^*$ is used to compute the correlation coefficients which are significant at the 95% significance level. The null hypothesis, that the population correlation coefficient is zero, is tested by computing the following statistic (Brownlee, 1960):

$$
\frac{r\sqrt{n - 2}}{\sqrt{1 - r^2}}
$$

(3.19)

This value is referred to a $t$ table with $(n - 2)$ degrees of freedom.
Chapter 4
Experiments

4.1 AVHRR data

The AVHRR images used in this study were received and processed at the Satellite Oceanography Laboratory in the Department of Oceanography at the University of British Columbia. The raw satellite data was earth-located and geometrically rectified using high quality ephemeris data supplied by the United States Navy who independently track the NOAA satellites. The satellite orbit is modelled as a ellipse. The radius of the earth is calculated for the centre of the region of interest which is assumed to be locally spherical. The spherical pixel coordinates are then mapped to a plane using the Lambert conformal conic (LCC) map projection with one standard parallel. The final correction step is a translation of the image based on the location of one ground control point (GCP). This navigation procedure was described by Emery and Ikeda (1984) where a one pixel accuracy was reported. The resolution of the navigated pixels is 1.1 km at nadir and changes slowly away from this point. There is some spatial distortion at great distances from the nadir, however these situations were avoided in this study. The navigated digital images cover $870 \times 870$ km ($512 \times 512$ pixels) and are centered at $72^\circ$N, $127^\circ$W.

The frequency bands sensed by the AVHRR instrument are shown in table 4.1. The image sequences used in this study are shown in table 4.2. During the winter months (October – May) the small amount of daylight necessitates the use of the thermal
Table 4.1: The channel bandwidths of the AVHRR on the NOAA satellites used in this study.

<table>
<thead>
<tr>
<th>channel</th>
<th>wavelength ((\mu m))</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.58 - 0.68</td>
<td>visible - red</td>
</tr>
<tr>
<td>2</td>
<td>0.73 - 1.10</td>
<td>near-infrared</td>
</tr>
<tr>
<td>3</td>
<td>3.55 - 3.99</td>
<td>extremely noisy - not used</td>
</tr>
<tr>
<td>4</td>
<td>10.3 - 11.3</td>
<td>thermal-infrared</td>
</tr>
<tr>
<td>5</td>
<td>11.5 - 12.5</td>
<td>thermal-infrared</td>
</tr>
</tbody>
</table>

infrared (IR) channels to view the ice cover. Images from this period often exhibit uniform pixel intensities over the ice pack reflecting the near uniform temperatures of the blanket of snow. The presence of leads are the only clearly distinguishable feature in winter scenes due to the great difference in the thermal signatures of ice/snow and water. Because the computational method developed in this study is essentially an area correlation, it is not useful for studying most winter scenes of the ice.

The onset of the summer melt season is characterized by a decaying snow cover, the exposure of bare ice and the formation of melt ponds over much of the ice surface. The albedo of the exposed sea ice, which relates directly to the amount of solar radiation backscatter, depends most strongly on the internal structure of the near-surface layers. There are substantial differences in the overall internal structure between multi-year (MY) ice and first-year (FY), which is ice that has not undergone a summer melt season.

In contrast to the relatively high albedo of the bare ice, melt ponds are low albedo areas where a large amount of solar radiation is absorbed. Melt ponds reach a maximum extent shortly after the disappearance of the snow cover, when they may cover up to 50% of the ice. Following this maximum, pond coverage on the multiyear ice decreases as some ponds drain and others deepen. *Grenfell and Maykut (1977)* have broken bare first year ice into two categories: i) melting white ice (ice whose surface lies above the local water table) and ii) blue ice (ice saturated with, but not covered by, melt
CHAPTER 4. EXPERIMENTS

Figure 4.1: Spectral albedos over snow and melt ponds: (a) dry snow (b) wet new snow, (c) melting old snow, (d) partially refrozen melt pond, (e) early season melt pond (10 cm deep), (f) mature melt pond (10 cm deep) on MY ice, (g) melt pond on FY ice (5 cm deep), (h) old melt pond (30 cm deep) on MY ice. Grenfell and Maykut (1977).
CHAPTER 4. EXPERIMENTS

Figure 4.2: Spectral albedos over bare sea ice: (a) frozen MY white ice, (b) melting MY white ice, (c) melting FY white ice, (d) melting FY blue ice. Grenfell and Maykut (1977).

water. Surface conditions on MY ice differ from FY ice in two respects: i) melts ponds cover a smaller area but are substantially deeper and ii) white ice, rather than blue ice, covers the unponded areas due to greater surface relief and better drainage (Grenfell, 1979). Grenfell and Maykut made extensive high resolution measurements of the albedo over FY ice, MY ice, melt ponds and snow. These measurements are summarized in figures 4.1 and 4.2. It is evident from these curves that the magnitude and shape of the spectral albedo function depends strongly on the amount of water in the near-surface layer of the ice. Two features which impact on this study are the significant overlap in albedo between: i) unfrozen melt ponds and blue ice and ii) frozen melt ponds and FY white ice.

The image intensities in band 1 are proportional to the albedo of the underlying surface. Their absolute values are not significant because the signal has been attenuated by the atmosphere in an unknown way. Grenfell and Maykut suggest that in the absence
of leads the regional albedo may be approximated by a simple combination of melt pond/blue ice and white ice albedos. The albedo of leads is much lower than either of these two categories and they can saturate a pixel even their size is smaller than the resolution of the instrument. The band 1 images used in this study show great variations in pixel intensity over the ice pack. These variations reflect the differing amounts of melt water at the surface of the ice and is robust over the time periods used.

4.1.1 Results

Verification of the automatically measured displacements \( (u_a) \) was achieved through comparison to: (a) manually measured displacements \( (u_m) \) and (b) displacements predicted by a theory of wind-driven ice motion \( (u_p) \). The graphic results of this section are all contained in Appendix B. The first data set was gathered by sitting at an image display system and measuring the displacement of invariant features (IFs) in the ice. The IFs are sharp intensity changes (edges) which are perceived by the operator as being significant features in the ice pack. At the scale of the imagery these ice pack features are most often large leads, since ridge lines and rubble fields are never 1 km wide. By quickly and repeatedly viewing each image in a pair we can easily perceive the optical flow of the scene. This is a qualitative perception of the field of motion which the human visual system can perform under remarkable conditions. In order to measure the motion field necessary for scientific and operational purposes, however, we must choose one pixel which appears to be an IF and find it in the next image. The accuracy of such a measurement is very difficult to estimate. The operator obtains an excellent sense of the orientation of the displacement from the optical flow. A conservative estimate of the error in the manual measurements in this study is 0.5 pixels, half that of the navigation. In many cases, especially when the IF is robust, the measurement is essentially errorless. In cases when the IF was not as distinct, the displacement was measured a number of times and the mean of these measurements was recorded. In
these cases the standard error was very small (less than 0.1). This apparently small measurement uncertainty, however, does not correctly reflect the ambiguity which often exists in making measurements in the central pack. The images of the summer ice pack contains a great deal of spatial variability due to the aforementioned variability of surface conditions. There are often very large areas of the pack where there are no leads and no other physical features which produce the intensity discontinuities that a human operator needs to make accurate manual measurements. The fact that an operator has such difficulty measuring motion in these situations underlines the utility of the automated system.

The density of the manual measurements is very much a function of how many significant edges could be measured with at least the same accuracy as the navigation. This led to a sparse distribution of $u_m$. It was felt that due to this paucity of data the interpolation of the irregularly distributed data onto a regular grid would introduce too much error into $u_m$. The comparison between $u_a$ and $u_m$ is therefore carried out in a series of tables which are shown in Appendix B. From these tables we may cull the following representative figures:

- $|u_m| - |u_a| = 0$ pixels in 48% of the measurements
- $|u_m| - |u_a| \leq 1$ pixels in 94% of the measurements

It is evident from these figures that the automatic measurements represent the manual measurements to within the uncertainty imposed by the navigation and the manual measurement. It should be emphasized that $u_a$ and $u_m$ are two very different measurements. We have no way of manually measuring spatial correlation. As will be investigated more closely in the next section, small amounts of motion shear could yield a discrepancy between $u_a$ and $u_m$. If the pixel chosen as the IF is on one side of a shear and the cross-correlation from the automatic scheme contains the shear, then $u_a$ and $u_m$ will differ. Because of the resolution of the imagery this difference be small.

As mentioned earlier the area correlation is only appropriate in the central ice pack.
where the motion of the filter windows may be approximated as translations. For this reason the values of \( u_a \) are not shown at the edge of the ice pack which is a MIZ nor are they shown for Amundsen Gulf (between Banks Island and Cape Parry). The results of the method are also not shown in the top left hand corner of the images since the quality of the images in this area is seriously degraded by dropouts (scan lines during which there were reception problems).

The image pairs analysed were:

1: June 05-06 1984
2: June 06-07 1984
3: June 11-12 1984
4: June 12-13 1984
5: July 30-31 1985
6: August 06-07 1985

and are shown in Appendix B. It should be noted that the images shown in the figures
have been enhanced, by a contrast stretch \((\text{Gonzalez and Wintz, 1977})\) to bring out the ice features. The fields of \(u_a\) and \(u_m\) for each pair are shown in Appendix B.

Area correlation methods have often been accused of yielding a broad correlation function, making the selection of a single maxima difficult. Figure B.34 and B.35 shows two typical examples of the correlation function from the third pair. While there is a certain amount of spatial structure to these functions, the peaks clearly rise above the background level resulting in well-defined maxima.

As discussed in Chapter 3, a simple test was performed to determine the statistical significance of the cross-correlation coefficients. The effective degrees of freedom is computed using the decorrelation length scales derived from the averaged autocorrelation functions of the imagery. These functions were very nearly isotropic with a decorrelation length of 10 pixels (about 15 km). In the \(22 \times 22\) window used, there were 484 data points, hence there were 48 effective degrees of freedom. From a \(t\) table we find that coefficients greater than 0.24 are statistically significant at the 95% level of confidence. In the plots of \(u_a\) shown in Appendix B, vectors that are below the significance level are plotted in gray, while those above are in black.

One of the primary uses of the synoptic field of ice displacements derived from remotely sensed imagery is in verifying the results of dynamic models of ice motion \((\text{Coon, 1980})\). The AIDJEX project, which investigated the motion and deformation of the ice pack in the Beaufort Sea, used primarily LANDSAT imagery, supplemented by some NOAA and aircraft data. The modelling group within AIDJEX used the 80 m resolution LANDSAT imagery to generate an ice thickness distribution for the model area and also to test the momentum balance and constitutive relations used in the model. Unfortunately the spatial and spectral resolution of the NOAA AVHRR imagery does not allow us to produce the accurate thickness distribution required by the AIDJEX model and its close relative, the INTERRA ice model\(\text{(Leavitt, Sykes and Wong, 1981; Hall, 1980)}\).

The force whose functional form is known the least and whose spatial and temporal
Figure 4.3: If the wind stress $r_a$ balances $r_w + C$, then a larger stress $r_a'$ turned to the right of $r_a$ is required to balance $r_w + C + \nabla \cdot \sigma$. Thorndike and Colony (1982).

Variability is the highest arises due to the spatial variations in the state of stress $\sigma$ in the ice cover. This force is given by $\nabla \cdot \sigma$ and, although it cannot be measured directly, it can be estimated indirectly as a residual if the other terms in the force balance are known (Rothrock, Colony and Thordike, 1980).

Thorndike and Colony (1982) investigated the following linear relationship between the ice velocity $u$, the geostrophic wind $G$ and the mean ocean current $\bar{c}$

$$u = AG + \bar{c} + \epsilon$$

(4.1)

where $A$ is a complex constant and the vectors $u$, $G$, $\bar{c}$ and $\epsilon$ are represented as complex numbers. In this equation, $\epsilon$ represents that part of the ice motion which is neither constant nor a linear function of the geostrophic wind. It includes the effects of measurement error, internal ice forces, time dependent currents, etc. The complex constant $A$ consists of a scaling factor and a turning angle,

$$A = |A| e^{-i\theta}$$

The validity of the relationship was tested against observations of the Arctic data buoy array deployed by the Polar Science Centre of the University of Washington. Using two
full years of ice motion and geostrophic wind data, *Thorndike and Colony (1982)* found that after removing the mean current, the geostrophic wind accounted for 70% of the variance of the ice velocity. Using shorter time series they investigated the seasonal and geographic differences of $|A|$ and $\theta$. They found a marked seasonal difference as follows,

$$A = \begin{cases} 
0.0077 \ e^{-15^\circ} & \text{winter, spring} \\
0.0105 \ e^{-118^\circ} & \text{summer} \\
0.0080 \ e^{-16^\circ} & \text{fall} 
\end{cases}$$

This seasonal difference is most likely a combination of two effects:

- the relationship between the surface wind and the geostrophic wind changes in the summer with the surface stress on the ice being 10-20% larger and 6° further to the right given the same geostrophic wind (*Albright, 1980*),

- internal ice stress play a larger role in the dynamics of the ice cover in the winter. The net effect of these stresses is to dissipate energy and oppose ice velocity as shown in figure 4.3. With this added effect, a larger wind stress and a smaller turning angle is required to balance the forces (*Thorndike and Colony, 1982*).

There were no geographic variations observed except near coastlines where, again, two effects could exist.

- One of the data buoys drifted near the Canadian Archipelago where the correlation between the variance of the ice velocity and $G$ dropped to less than 0.5. It was postulated that ice stress gradients play a large role near coastlines and that (4.1) may only be applied in regions 400 km from the coast.

- Several of the buoys drifted near the east coast of Greenland where the ice-wind correlation again became small. The cause was believed to be the large mean and time-dependent currents which exist in the area.

The ice velocities predicted by Thorndike and Colony were subtracted from the automatically measured ice displacements in order to compare the residuals with the
known mean current in the Beaufort Sea and the anticipated effect of the internal ice stresses based on an inspection of the imagery.

The surface pressure charts were obtained from the Arctic weather office in Edmonton. These charts contained 24 hour averaged isobars which are drawn on the plots of the predicted ice motion and the residuals shown in Appendix B. A regular grid of surface pressures and geostrophic winds were computed at Seaconsult Ltd. using standard techniques. The error in the geostrophic wind estimates are approximately 2-4 m/s (Thorndike and Colony, 1982). In his study of geostrophic winds for AIDJEX, Albright (1980) used only those winds in excess of 5 m/s since below this level the uncertainty approaches 100%. Another problem with winds less than 5 m/s is that the assumptions that led to (4.1) break down and the relationship becomes non-linear. The last consideration is the effect of orographic steering of the winds. This is an extremely important effect for in the western Beaufort Sea due the presence of the Brooks Range of mountains which affects the surface wind within a region of at least 50 km from the coast (Dickey, 1961; Kozo and Robe, 1986). Our study area is far removed from this range of mountains which ends abruptly at the McKenzie River. The topography of the southern mainland is quite flat and will have no effect on the wind. The southern tip of Banks Island (Nelson Head) contains one peak whose height is at the lower limit of topographic influence and steering may be a factor in this area. It is highly unlikely, however, that any steering which may be present will affect the winds over the pack and orographic effects are therefore ignored.

In his extensive study of the circulation of the Canada Basin, Newton (1973) confirmed, through his computation of the dynamic height, that the long-term mean flow is the anticyclonic Beaufort gyre. He also computed a simple steady, frictionless vorticity balance which he compared to the dynamically more complex numerical model of Galt (1973) to conclude that the wind stress curl was the important driving mechanism. The mean atmospheric circulation over the Canada Basin is dominated by a high centered at approximately 78°N, 140°W. Newton computed the magnitude of the
wind stress curl and found that the predominant feature was a region of large negative curl west of the Canadian Archipelago. The transfer of the wind's momentum to the ocean is heavily damped by the ice cover and the magnitudes of the flow varies from 1 cm/s near the centre of the gyre to 3-5 cm/s near Banks Island. The centre of the gyre according to Newton’s dynamic height is approximately 76°N, 145°W which is about 150 km northwest of the images shown at the beginning of Appendix B. The current at any point will, of course, contain a component of the stationary mean flow and a component from each of a variety of time dependent flows. The latter class of flows are dominated by highly energetic mesoscale eddies. These eddies are strongly baroclinic with a subsurface velocity maximum in the steep density gradient between 30 and 200 m (Manley, 1981). It has been shown that the kinetic energy of the central Arctic ocean resides almost entirely in these eddies with the mean flow contributing less than 1%. While the eddies are usually under-sampled, their diameter appears to be 10 km, and it has been estimated that they cover 10-20% of the western Arctic ocean (Hunkins, 1981). While the eddies of the Atlantic have their velocity maxima at the surface, the Arctic eddies have their maxima at a much deeper level (about 120 m) due to the presence of the ice cover. While these eddies lose some of their momentum and vorticity to the ice cover, the time scale of this transfer is much longer than that represented by the image sequences used in this study. Hence any ocean circulation component of the residual field will represent the mean flow. The anticipated flow is that of the lower right corner of the gyre, that is south past Banks Island, turning clockwise and then westerly.

The first image sequence (June 5/6/7 1984) is characterized by large pressure gradients giving rise to a predicted motion field towards the northwest. The ice has obviously responded to this period of strong wind forcing as is evidenced by the fields of $u_a$. The residual field shows a pattern which resembles the mean ocean circulation with magnitudes away from the coast that correspond to those observed (3-5 cm/s) (McPhee, 1980). Since the area immediately west of Banks Island is not a region of mean flow
CHAPTER 4. EXPERIMENTS

intensification (Newton, 1973; Semtner, 1986), the large residual magnitudes in this region are most likely a manifestation of the internal ice stress. The observed ice motion may be resolved into a component parallel to the coast (northward) and a component perpendicular to the coast (westward). It is well known that ice cannot support tensile stress (Mellor, 1973), hence the perpendicular component is unopposed. The parallel component gives rise to a shear stress which is opposed by $\nabla \cdot \sigma$. An inspection of the images shows that the northwest corner of Banks Island is jutting out into the ice field and hindering the northward component of ice motion. This region is also characterized by the largest residual magnitudes. There is a large band of open water separating the pack from the southern coastline and the residuals in this area are likely due to the mean surface circulation of the ocean.

The second sequence of images (June 11/12/13 1984) is characterized by very weak pressure gradients. The pressure field is so weak in the June 11/12 pair that it confounded the interpolation software and the predicted ice motions are not given. The very small geostrophic wind field is below the threshold necessary for (4.1) to be useful. The predicted motion is shown for June 12/13, but, again, the magnitudes of the uncertainties have risen to the level of the computed winds, as is evidenced by the spurious vectors. The measured motion is in the form of two gyres which are almost closed. The pressure system shows some north-eastward motion as the gradients strengthen in this direction. The ice gyre centre (taken to be the region of no motion) also shows signs of moving in a north-eastward direction. It would appear that, in spite of the light geostrophic winds, the ice is strongly coupled to the atmospheric pressure systems on these short time scales. The residual field is rather difficult to interpret since, given the arguments used in the previous paragraph, the direction appears to be opposite from what is expected. Since the computed geostrophic winds are of the same order as their uncertainty, the computation of the residuals may not be a useful exercise.

The next pair of images (July 30/31 1985) is separated by only 4.5 hours, and contains the kind of weather system that strikes fear into the heart of anyone involved
in summertime offshore Arctic activity (i.e. drilling rigs). The pressure gradients and
the geostrophic winds are very large and the latter flows south-easterly, pushing the
entire pack onto the mainland. In the short time interval between images the ice
moved 1 km towards the east. The ice in the vicinity of Amundsen Gulf, which is more
unconsolidated than the pack, appears to respond more directly to the wind forcing,
and the small residuals in this area suggest that the wind is dominating the force
balance. The residuals over the pack are quite uniform and show no hint of the mean
ocean circulation. The perpendicular (southward) component of the wind results in a
compressive stress in the ice cover while the parallel (eastward) component gives rise
to a shear stress. The residuals show that $\nabla \cdot \sigma$ is acting in opposition to the former,
which is a reasonable result since it would be the larger of the two. A longer interval
may have shown a west-ward component as well.

The last sequence (August 6/7 1985) contains a region of fairly high pressure gradi­
ents beside one which is almost homogeneous. The predicted motion in this latter area
is, as expected, useless. The pattern of the residuals in the central pack again reflects
the mean circulation with magnitudes that are reasonable except along the western­
most isobar and at the southern extremity of the measured field. The motion of the
ice may again be resolved into a parallel (shear stress producing) and a perpendicular
(tensile stress producing) component. While the northward residuals do not agree with
what is expected from the 24 hour averaged forcing, it may be a delayed or continuing
response to a previous atmospheric forcing event.

4.2 SAR data

The SAR imagery used in this study was imaged by SEASAT, a satellite dedicated
to observing the global oceans with a number of microwave sensors (Jordan, 1980).
SEASAT, in spite of a very short lifespan (June 28 - October 10, 1978), generated a
data set so large that it is still being evaluated. The SAR was a 1.27 GHz (L-band)
radar, which, while not being an optimal frequency for sea ice type discrimination,
CHAPTER 4. EXPERIMENTS

proved quite useful. The images used in this study are from revolutions 1439 (October 5) and 1482 (October 8) and are centered at about 74.5°N, 174°W (figure C.3). The images cover ~ 100 x 100 km containing about 4000 x 4000 pixels at a resolution of 25 m per pixel. The images were provided by the Jet Propulsion Laboratory where they were earth located and geometrically and radiometrically rectified (Curlander, 1982; Curlander et al, 1985).

It was felt that the resolution of 25m was unnecessarily high and the images were filtered with a half-band low-pass filter (Reimer, 1986; Clark and Lawrence, 1985) and sub-sampled until they were in a 512 x 512 pixel format. These images have a 200 m resolution and improved noise statistics (Lee, 1981; Porcello, Massey, Innes and Marks, 1976). The images were then filtered with the first and second order differential operators described in chapter 3 (figures C.6-C.11) and submitted to the motion detection system.

For a winter scene of sea ice, the radar scattering coefficient which describes the backscattered radar signal is known to be controlled by comparable contributions of surface roughness, volume refractive-index inhomogeneities and surface morphology (Carsey, 1985). Previous studies using various frequency bands during winter and spring, have shown that frequencies above 9 GHz (X- and K-band) have the ability to discriminate between different ice types, while frequencies below 1.5 GHz (L-band) do not (Onstott and Gogineni, 1985). Returns in the latter band, however, easily distinguished between flat areas of the pack and more prominent features such as pressure ridges and rubble fields (Onstott, Moore and Weeks, 1979). The water surface in the leads has a very weak return and is easily distinguishable in the two images used in this study. In some cases, however, wind roughened water will have a very high return, and the signature of large leads, will change dramatically, due to a change in the local wind field. The image pair used did not fall into this category and the image intensities which reflect the return strength are thought to be invariant over the 3 day interval between images.
CHAPTER 4. EXPERIMENTS

The manually computed displacements, \( u_m \) (shown in figure C.4) were computed by University of Washington scientists using software described in Curlander et al (1985). The density of these measurements (779 in all) allowed for a relatively accurate interpolation to a regular grid (figure C.5) which was performed by Fily and Rothrock (1985, 1986) who estimated the error in \( u_m \) to be ±3 pixels (0.075 km).

4.2.1 Results

The automatically measured displacement fields from the first order and second order operators are shown in figures C.12 and C.13. The error in these fields is found by simply computing \( e = u_m - u_a \) at each grid point. The field of errors is shown in figure C.14 (first order) and figure C.15 (second order). It is evident that the second order filter improves upon the results of the first order filter, if only slightly. A further improvement may well be gained by using authentic zero crossings only as discussed in chapter 3. Another possibility which warrants further investigation is using both the magnitude and direction of the zero-crossings.

Figure C.5 represents the ice motion as a deforming two-dimensional grid where the displacements are applied to each vertex in the grid. This plot gives a different, and in many ways a more relevant sense of the ice motion, since it describes the scene as a deforming solid rather than disconnected displacements. One characteristic of the motion which is immediately evident from looking at figure C.5 is that the entire scene has rotated by about 4°. The lack of any systematic element in the fields of \( e \) demonstrates the robustness of the matching system against this amount of rotation. While this amount of rotation would be small for the rapidly deforming MIZ it is a very reasonable value for the perennial ice cover (Colony and Thorndike, 1981, 1984), where this method may be applied. The second interesting feature of the field of motion in C.5 is that there are several regions which rotate as solid bodies as evidenced by the lack of grid deformation within them. These regions are separated by narrow corridors of deformation or motion shear. An inspection of the raw images will show that the
deformation corresponds to areas where leads are either opening or closing in a highly discontinuous fashion. Figure C.15 shows that the errors in the matching system are limited to these regions of deformation. This observation was also made by Fily and Rothrock, (1986) in their study of the same SAR image pair. It is appropriate to examine two of these areas more closely to determine the nature of these deformations and why area correlation is not always an effective technique.

The first area, denoted *detail 1* (figure C.16), is from the lower left of the images and gives rise to the line of error vectors which the matched filter did not improve. Though it is certainly not obvious from looking at the images, detail 1 contains two ice fields moving as solid bodies. The 25 km. long floe marked $A$ in figure 11b and lead $E$ are within the northern field while lead $C$ is in the southern field. The different motions of these two fields has joined a number of disconnected leads into the long lead denoted $D$ in 11b. This long lead terminates in lead $B$ which appears to remain undeformed. This is a classic case of a discontinuous ice motion field which is so common in Arctic pack ice. It is clear that both matched filtering and cross correlation are able to estimate the solid body motion on either side of $D$. It is, however, inevitable that the windows of the area correlation will straddle the discontinuity and produce erroneous estimates. In the manual measurement, features along the edges of the two motion fields were chosen and the discontinuity was resolved. Since $D$ was formed from a number of leads in C.16a, the digital edges delineating the ice-water boundaries of these leads remain in C.16b. It is these digital edges which must be detected, extracted and matched if an automated system is to successfully resolve the motion.

*Detail 2*, shown in figure C.17, is from the upper right corner of the images and exhibits a very different kind of deformation field. Rather than a simple discontinuity, this detail shows three fields moving in different directions. These directions are shown roughly in C.17b. The small scale rotation of the ice in the upper left of the detail opened up lead $J$, while the motion of the field to the immediate right has greatly reduced the size of lead $I$ and closed leads $K$ and $L$. The discontinuity between the
two right hand fields is easily seen by tracking the feature marked $G$ which is within the middle field, and floe $F$ which belongs to the right hand field. The main result of this discontinuous motion is the enlargement of lead $H$. The edge detection step was able to improve the performance of the area correlation by enhancing features in the ice pack which remained intact. These features include: (a) the many ridge lines running through the ice which remain in spite of the leads opening between them and (b) the many small leads whose shape remains unaltered. In every region of the motion discontinuity in the imagery the matched filter was able to enhance significant features in the ice, either ridge lines or lead boundaries, and dramatically reduce the errors induced by matching the raw image intensities. However to fully resolve discontinuous motion the matched filter system must be supplimented by a system that can extract and match features.

The statistical significance of the correlation coefficients was also computed for the SAR vectors. The decorrelation length scale of 3 pixels (about 600 m) was observed. The total number of data points was $30 \times 30 = 900$, hence the effective degrees of freedom is approximately 265. From this we find that all coefficients greater than 0.13 are significant at the 95% level. This test is, however, incapable of tagging erroneous vectors. This is illustrated in figures C.14 and C.15 where vectors whose coefficients are below the significance level are plotted in gray.
Chapter 5
Discussion

5.1 Estimation versus Detection

In this study the problem of computing ice motion was posed as a signal estimation problem and the matched filter was found to be an optimal solution. However, the matched filter is more often found in connection with signal detection problems (Sewartz and Shaw, 1975, Stark and Tuteur, 1979), and it is therefore necessary to compare the two problem types and justify our choice.

Signal detection is characterized by the common problem of detecting the presence or absence of a particular type or class of signals in the presence of noise. The theory of detection concerns itself with either maximizing the probability of detecting a signal when it appears or minimizing the probability of mistaking noise for signal when it is absent. One or other of these criteria must be chosen since both cannot be optimized simultaneously. The optimization leads to a likelihood ratio which in turn leads to a decision threshold which is used to determine whether or not the signal is present in the input data.

Signal estimation involves estimating the values of a signal from a given data sample. The signal, in this case, is known to be present but due to noise and a variety of other obscuring phenomena the data samples scatter about the actual signal values. A very simple example of estimation is line-fitting. In this case the line is the signal whose slope and intercept are sought such that it best fits the data. Estimation theory requires
the definition of a quantitative measure of goodness-of-fit. The most commonly used measure, and that which is justifiable on statistical grounds, is the mean-square error.

The problem could have been posed as a detection problem in which case the optimal system would have been derived by maximizing the probability of detecting a signal when a signal is present while keeping the false-alarm probability at a tolerable level. This is similar to a radar-type detection problem where the false-alarm probability can be specified based on the physics of the problem. The specification of such a parameter in the present problem has no physical meaning and it was decided to avoid this situation.

5.2 Confidence Estimates

In the present study the confidence of the motion estimates was evaluated in two different ways. In the AVHRR imagery the vector fields were smooth and no post-processing was necessary. The output of the system, namely the cross-correlation coefficients, represent the estimation error. In both the SAR and AVHRR the 95% confidence threshold was computed for the coefficients and vectors which fell below this level were flagged. This process was, however, incapable of finding vectors which were obviously wrong.

In the SAR case the first motion estimates were quite noisy and a post-processing step was instituted to smooth the field. As was stated two criteria were used to find bad vectors; deviation from the global statistics of the coefficients and deviation from the local statistics of the motion. No interpolation was done in this step, the original correlation function was searched and the replacement vector was selected from it.

Another possible means for evaluating quantitative confidence in the vectors is the shape of the correlation function around the peak. While several people have shown plots of this function for illustrative purposes (Smith and Phillips, 1972; Vesecky et al, 1986a, b; Emery et al, 1986), none have attempted to analyse them. This is not to say that it is not on people’s minds (Burns, Curlander, Vesecky - personal communication).
CHAPTER 5. DISCUSSION

Vesecky et al have shown the effect of window size on the shape of the function, where, as might be expected, the larger the window the smoother and broader the function. The computation of certain representative shape characteristics will certainly provide more information with which we can evaluate the goodness of the vector estimate. It may prove to be more useful than simply evaluating the peak itself, however, it will certainly be at the expense of computer time.

5.3 Computational Efficiency

The computations by the system described in this study are performed at one resolution only. The scheme is made more efficient by using the FFT; however, the system is extremely computation intensive. The system will provide substantial savings in time for AVHRR applications but is not yet practical for SAR images due to the necessity of very large search windows.

The hierarchica1 system used by Fily and Rothrock (1985, 1986) and Vesecky et al (1986a, 1987) does provide a more efficient means of computing cross-correlations for SAR imagery. In this system the image is low-pass filtered and sub-sampled a number of times thus producing an image pyramid. The motion estimation is first performed on the coarsest resolution images and the vectors are used to guide the estimation at the next highest level. This passing of information between levels means that the sizes of the filter and search windows may be greatly reduced in size leading to much less computations and large savings in time. The critical stage is the first level since errors at this level can propagate to the next level and, due to the much smaller search space, produce incorrect estimates. Another important consideration which has not been directly addressed in the aforementioned studies is aliasing due to incorrect low-pass filtering and sub-sampling. The low-pass filter must be a half-band filter in order that the resulting image may be sub-sampled correctly. A particularly efficient implementation of this is given by Clark and Lawrence (1984).
5.4 System Evaluation and Future Research

The major assumptions built into the system are that the motion within each filter window may be approximated by pure translations (ie no discontinuities) and that the rotation of each window is small. While these assumptions are restrictive, they are obeyed in the majority of scenes of the central ice pack. The small number of AVHRR images used are felt to be approximately representative of typical pack ice conditions and further testing would not disprove the success of the filter in detecting the majority of the ice motion for this application. The SAR image pair is quite well-behaved, however, and it is not known, due to the paucity of data, whether it is truly representative of central pack-ice conditions. The results will presently be examined to determine methods of improvement, but it is evident that the matched filter should form the core of any larger integrated system. This conclusion has recently been confirmed by NASA's ice motion algorithm working group (Holt, 1986). Another important consideration is that the system not only automates what a human operator can do but provides measurements of ice motion in areas of the image where it is extremely difficult to obtain manual measurements.

Improvements to the system may be suggested by looking at where the two main assumptions break down. The pure translation assumption will fail (i) at the ice edge, (ii) at locations of motion shear and (iii) where the ice motion exhibits divergence or convergence. Since leads manifest themselves as edges in the digital imagery a possible method for resolving the disparate motion of these features is to use feature-matching instead of area-matching in these areas. A recent thesis by Szczechowski (1986), was successful at detecting significant edges in AVHRR imagery using relaxation techniques, which at least demonstrates that these features can be extracted. There have been several studies involving edge extraction and matching in remotely sensed imagery (Little, 1980; Medioni and Nevatia, 1984; Majka, 1982), but the features extracted have all had a strongly linear nature, in contrast to the highly deformed edges in the
SAR imagery. The AVHRR edges are generally more linear and these methods may be applied more directly to this imagery. Vesecky et al (1986a) have been quite successful at extracting and matching edges in the SAR image pair used in this study. However the derived field duplicates the area correlation and does not provide any more information regarding the existing motion discontinuities. The edges which exist between the rigid plates are highly deformed and it does not appear hopeful that any edge detection algorithm could extract any edge segments in this area that are long enough to be useful for matching purposes. A possible approach may be some sort of local texture analysis to yield, for instance, the spatial distribution of ice versus open water (Fily and Rothrock, 1985) since ice in highly deforming areas will be heavily interspersed with open water. A cue that significant deformation is taking place would be large changes in the ratio of ice to open water.

The assumption of small rotations will fail at or near the MIZ which has been identified as the most variable and dynamic region of the entire ice pack. But rotations which will cause an area-correlation to fail may also occur during storms exhibiting large wind stress curl. While cross-correlation is strictly valid only for pure translation, small rotations can still be handled. Vesecky et al (1986b) have experimented with the effect of rotation on the correlation function and found that for a $32 \times 32$ window, the autocorrelation peak falls from 1.0 at 0 degrees rotation to about 0.6 at 5 degrees and 0.2 at 10 degrees. One way to make the area correlation robust against large rotations is to compute and match rotation- and scale-invariant statistical moments within the filter and search windows (Hu, 1962; Sadjadi and Hall, 1978). The presence of rotation generally signals the presence of deformation which preempts the systematic decomposition of the image into windows. A more appropriate approach may be to segment the image based on some texture measure and compute the aforementioned statistics for these homogeneous regions. Vesecky et al (1986b) manually extracted ice floes from other SEASAT SAR imagery and computed the series of moments advocated by Wong and Hall (1978). These moments, however, were very ambiguous measure-
mements of the underlying scene and were unable to discriminate between the correct ice floe and other incorrect scenes.

The image processing system at the Jet Propulsion Laboratory presently uses a human operator to identify polygons within the scene which move as rigid pieces. The next step is to perform this step computationally. A useful way to cue the whole system to potentially dangerous rotations is to extract the strongest edges in the scene and match these. If these features rotate significantly then there should be some sort of decision mechanism to send the processing in a direction which is more robust against rotations and deformations. A stand-alone system will have a decision network as its spine. It will have a knowledge base containing various types of information: ice type signatures, last detected position of the ice edge, etc and a palette of matching methods from which to choose. The core of the system will be the matched filter and those scenes which are relatively undeformed will yield the densest grid of observations.
Bibliography

ABDOU I. E. and W. K. PRATT (1979) Quantitative design and evaluation of enhancement/thresholding edge detectors. *Proceedings of the IEEE, 67, 753-763*


AHLNAS K. and G. WENDLER (1979) Sea ice observations by satellite in the Bering, Chukchi and Beaufort Sea. *Sixth International conference on Port and Oceans under Arctic Conditions*, 313-320


DOOB J. L. (1953) Stochastic Processes, John Wiley and Sons,


GRENFELL T. C. and G. A. MAYKUT (1977) The optical properties of ice and snow in the Arctic basin. *Journal of Glaciology, 18, 445-463*


HOLT B. and S. A. DIGBY (1985) Processes and imagery of first year fast ice during the melt season. *Journal of Geophysical Research, 90, 5,045-5,062*


HUECKEL M. H. (1971) An operator which locates edges in digital pictures. *Journal of the Association for Computing Machinery, 18, 191-203*

HUECKEL M. H. (1973) A local visual operator which recognizes edges and lines. *Journal of the Association for Computing Machinery, 20, 634-647*


LEES J. A. and E. S. EPSTEIN (1963) Application of two-dimensional spectral analysis to the quantification of satellite cloud photographs. *Journal of Applied Meteorology*, 2, 629-644


MARR D. (1976) Early processing of visual information’. Philosophical Transactions of the Royal Society of London, B275, 483-519


MELSA J. L. and A. P. SAGE (1973) An Introduction to Probability and Stochastic Processes Prentice-Hall, 403 pp


PHILLIPS D. R., E. A. SMITH and V. E. SUOMI (1972) Comments on "An automated technique for obtaining cloud motion from geosynchronous satellite data using cross correlation". *Journal of Applied Meteorology, 11*, 752-754


REIMER J. (1986) personal communication


76
STARK H. and F. B. TUTEUR (1979) Modern Electrical Communications, Prentice-Hall, 601 pp


THOMAS J. B. (1969) Statistical Pattern Recognition, John Wiley and Sons

TIKHONOV A. N. and V. Y. ARSENIN (1977) Solutions of Ill-Posed Problems, John Wiley and Sons, 258 pp


TURIN G. L. (1960) An introduction to matched filters. IRE Transactions, IT-6, 311-320


Appendix A

Review of Two Important Noise Processes

A.1 Gaussian Stationary White Noise

The most important characteristic of both types of noise reviewed in this appendix is that they are random processes. It is well known that random processes are, formally speaking, an indexed family of random variables \( x(i,j) \) (Papoulous, 1984). This family of random variables is completely characterized by its \( n \)th order probability distribution functions, which, in general, may be a function of the indices \( i \) and \( j \), which in this case are associated with spatial position. At each point in the image the noise is described by a particular probability distribution function

\[
P(x; i, j) = \text{probability}[x(i, j) \leq x]
\]

where \( x(i, j) \) denotes the random process and for a specific \( (i, j) \) \( x(i, j) \) is a random variable. The density function is given, as usual, by:

\[
p(x; i, j) = \frac{\partial P(x; i, j)}{\partial x}
\]

The function \( P(x; i, j) \) is called a first order distribution function of the process \( x(i, j) \). Given two spatial positions \( (i_1, j_1) \) and \( (i_2, j_2) \), the interdependence between the two random variables \( x(i_1, j_1) \) and \( x(i_2, j_2) \) is given by their joint probability distribution
function:

\[ P(x_1, x_2; i_1, j_1, i_2, j_2) = \text{probability}[x(i_1, j_1) \leq x_1, x(i_2, j_2) \leq x_2,] \]  

(A.2)

This is called a second order distribution of the process \( x(i, j) \). The corresponding density function is given by:

\[ p(x_1, x_2; i_1, j_1, i_2, j_2) = \frac{\partial^2 P(x_1, x_2; i_1, j_1, i_2, j_2)}{\partial x_1 \partial x_2} \]

Similarly the \( n \)th distribution function is given by

\[ P(x_1, \ldots, x_n; i_1, j_1, \ldots, i_n, j_n) = \text{probability}[x(i_1, j_1) \leq x_1, \ldots, x(i_n, j_n) \leq x_n]. \]  

(A.3)

for any \( n \) and \( i_1, j_1, \ldots, i_n, j_n \). In what follows I shall make the assumption of continuous random variables in giving statistical expressions, noting that the discrete extension is straightforward.

While a complete description of a random process requires a knowledge of its \( n \)th order probability functions this is not usually possible. In this case we may characterize the random process by statistical averages of the random variables. Such averages are called ensemble averages and consist of the moments of the process where the \( m \)th moment of the first order probability distribution of \( x \) is defined by (Davenport and Root, 1958):

\[ E[x^m(i, j)] = \int_{\infty}^{\infty} x^m p(x; i, j) \, dx \]  

(A.4)

where \( E \) denotes mathematical expectation. Clearly the moment of interest is the first moment which defines the mean of the process:

\[ E[x(i, j)] = m_x(i, j) \]  

(A.5)

In general the mean of the product of two random variables is not equal to the product of the mean (ie. \( E[x_{ij}x_{kl}] \neq E[x_{ij}]E[x_{kl}] \)). If this is the case, however, the random variables are said to be linearly independent or uncorrelated.
Another useful statistical measure is the first order $m^{th}$ central moment of the process defined by:

$$
\mu_z(i, j) = E[(x(i, j) - m_x(i, j))^m] = \int_{-\infty}^{\infty} (x - m_x(i, j))^m p(x; i, j) \, dx
$$

(A.6)

The central moment of interest is given by $m = 2$ which defines the variance (denoted $\sigma^2_{z_1j}$) of the process. While the first order moments (i.e. mean and variance) are important characterizations of a random process they provide only a small amount of information about the process. A more precise description of a random process is provided by its second order moments (Yaglom, 1962). The second order first moment is also known as the autocorrelation function and is given by:

$$
R(i_1, j_1, i_2, j_2) = E[x(i_1, j_1), x(i_1, j_1) \leq x_1,]
$$

$$
= \int_{-\infty}^{\infty} x_1 x_2 p(x_1, x_2; i_1, j_1, i_2, j_2) \, dx_1 \, dx_2
$$

(A.7)

Similarly the second order second central moment is known as the autocovariance function and is defined by:

$$
C(i_1, j_1, i_2, j_2) = E[(x(i_1, j_1) - m_x(i_1, j_1))(x(i_2, j_2) - m_x(i_2, j_2))]
$$

$$
= R(i_1, j_1, i_2, j_2) - m_x(i_1, j_1)m_x(i_2, j_2)
$$

(A.8)

The autocorrelation function measures the dependence between different values of a random process at different points in space. Hence it describes the spatial variation of a random signal.

As has been pointed out, the statistical properties generally vary in space. In a stationary process, however, the statistical properties are invariant to a shift in spatial origin. Stationarity of order one means the first order density function is independent of spatial position $(i, j)$ that is $p(x; i, j) = p(x)$, i.e

$$
E[x(i, j)] = m_x = \text{constant} \quad E[(x(i, j) - m_x)^2] = \sigma_x^2
$$

(A.9)

If we denote spatial difference by $(\alpha, \beta)$, stationarity of order two must be such that:

$$
p(x_1, x_2; i_1, j_1, i_2, j_2) = p(x_1, x_2; \alpha, \beta)
$$

(A.10)
Hence the autocorrelation and autocovariance depend only on spatial difference:

\[ R(\alpha, \beta) = E[x(i + \alpha, j + \beta)x(i, j)] = R(-\alpha, -\beta) \]
\[ C(\alpha, \beta) = E[(x(i + \alpha, j + \beta) - m_x)(x(i, j) - m_x)] \]  

(A.11)

A random process is called stationary in the strict sense if it is stationary of order \( n \), that is its \( n \)th order probability density remains the same if the indices \( i, j \) are shifted along the spatial axes. This implies that all first order distribution functions are identical (i.e., do not depend on \( i \) and \( j \)) and all second order distribution functions depend only on the difference \( i - \alpha \) and \( j - \beta \).

For most practical situations the random processes that are encountered are not stationary in the strict sense, yet their first order moments are constant and the autocorrelation functions satisfy 3.11. In this case the processes are said to be stationary in the wide sense. Hence wide sense stationary processes are stationary of order two.

It should be noted that the aforementioned moments do not specify the random process completely unless the probability distribution functions describing the process are all Gaussian. For independent random variables which may not have Gaussian distributions we may appeal to the central limit theorem to impose Gaussian statistics on our problem (Yaglom, 1962).

Finally if all the random variables in a random process are uncorrelated then we have (Oppenheim and Schafer, 1975)

\[ m_z = 0 \]
\[ R(k, l) = \delta(k, l) \]  

(A.12)

where \( \delta(k, l) \) is the Dirac delta function defined as usual. This leads to a flat power spectrum (2.15) and the process is referred to as white noise. As noted by Yaglom (1962), (2.15) corresponds to an unbounded power spectrum and hence infinite average noise power, thus white noise is useful only as a mathematical idealization. The utility of this idealization stems from the fact that the frequency response of any physically realizable system is always zero outside some finite interval. Hence when a random process \( n(x, y) \), whose spectral density \( S_n(u, v) \) is constant over some finite region \( \Lambda \),
appears at the input of such a system there is no need to describe in detail how \( S_n(u,v) \) falls off at high frequencies and we may treat it as being constant.

### A.2 Gaussian Stationary Markov Noise

When a stochastic process is \textit{Markov} many problems are considerably simplified. Recall that the characterization of random processes normally requires the knowledge of the \( n \)th order probability distribution functions for each random variable. However the assumption that the variables are jointly Gaussian reduces the description to simply a knowledge of the mean and covariance. The Markov property makes a similar reduction in effort possible. In what follows the notation is simplified by using one dimensional processes to develop the ideas, where the extension to two dimensions is a straightforward extension. Also several assertions regarding the Markov process are given without proof, these proofs may be found in most standard texts on stochastic processes \textit{(Papoulus, 1984; Melsa and Sage, 1973)}.

A stochastic process is called Markov for all \( i,j \), where \( i \) and \( j \) are subscripts denoting spatial position, and \( i,j = 1,\ldots,n \) if,

\[
p(x_n|x_{n-1}, x_{n-2}, \ldots, x_1) = p(x_n|x_{n-1}) \tag{A.13}
\]

ie. the conditional distribution of \( x_n \) given all past states \((x_{n-1}, \ldots, x_1)\) equals the conditional distribution assuming only the most recent state\((x_{n-1})\).

A central characteristic of a Markov process is that joint probability densities may be expressed as products of so-called \textit{transition probability densities}, ie.

\[
p(x_1, x_2, \ldots, x_n) = p(x_n|x_{n-1})p(x_{n-1}|x_{n-2}) \ldots p(x_2|x_1)p(x_1) \tag{A.14}
\]

that is the joint distribution of \( x_1,\ldots,x_n \) is determined in terms of \( p(x_1) \) and the transitional densities \( p(x_n|x_{n-1}) \).

In a Markov process the past is independent of the future under condition of the present. This means that if \( n > r > s \) then, assuming \( x_r \), the random variables \( x_n \) and
\(x_s\) are independent of one another ie.

\[
p(x_n, x_s | x_r) = p(x_n | x_r)p(x_s | x_r)
\]  

(A.15)

A Markov process is called *homogeneous* if the conditional density, \(p(x_n | x_{n-1})\), is independent of \(n\). A Markov process is called stationary if it is homogeneous and all the random variables, \(x_n\), have the same density functions.

If a process \(x(t)\) is Gaussian Markov with zero mean then its autocorrelation satisfies

\[
R(t_3, t_2)R(t_2, t_1) = R(t_3, t_1)R(t_2, t_2) \quad \forall t_3 > t_2 > t_1.
\]  

(A.16)

If the process is stationary then,

\[
R(t_3 - t_2)R(t_2 - t_1) = R(t_3 - t_1)R(0)
\]  

(A.17)

or by substituting \(t_3 - t_2 = \lambda_1\) and \(t_2 - t_1 = \lambda_2\) we may write,

\[
R(\lambda_1 + \lambda_2) = R(\lambda_1)R(\lambda_2)R^{-1}(0).
\]  

(A.18)

The only continuous function that satisfies (3.18) \(\forall \lambda_1, \lambda_2\) is an exponential of the form *(Doob, 1953)*,  

\[
R(\tau) = R(0)e^{-c|\tau|}.
\]  

(A.19)

Hence the autocorrelation of a Gaussian stationary Markov process must be an exponential. This is an important result which has been used successfully in image processing where the two dimensional equivalent of (2.20) is used *(Pratt, 1975)*

\[
R(\tau_x, \tau_y) = K^2 \exp \left\{ -\sqrt{\alpha_x^2 \tau_x^2 + \alpha_y^2 \tau_y^2} \right\}
\]  

(A.20)

where \(K\) is an energy scaling constant and \(\alpha_x, \alpha_y\) are scaling constants. In many cases the Markov process is assumed to be of separable form in which case we may write

\[
R(\tau_x, \tau_y) = K^2 \exp \left\{ -\alpha_x |\tau_x| + \alpha_y |\tau_y| \right\}
\]  

(A.21)
Appendix B

AVHRR Graphic Results
Figure B.1: AVHRR band 1 - June 05 1984.
Figure B.2: AVHRR band 1 - June 06 1984.
Figure B.3: AVHRR band 1 - June 07 1984.
Figure B.1: AVHRR band 1 - June 11 1984.
Figure B.2: AVHRR band 1 - June 12 1984.
Figure B.3: AVHRR band 1 - June 13 1984.
Figure B.1: AVHRR band 1 - July 30 1985.
Figure B.2: AVHRR band 1 - July 31 1985.
Figure B.3: AVHRR band 1 - August 06 1985.
Figure B.1: AVHRR band 1 - August 07 1985.
Figure B.2: Automatically measured displacements from June 05-06 1984.
Figure B.3: Manually measured displacements from June 05-06 1984.
### Table B.1: Comparison between the $u_a$ and $u_m$ measurements for image pair 1 (June 05 – 06 1984), where $e$ is the magnitude of the error vector and $\alpha$ is its angle.
Figure B.4: Displacements predicted from theory for June 05-06 1984 along with the 24 hour averaged isobars for same period.
Table B.1: comparison between the $u_a$ and $u_m$ measurements for image pair 1 (June 06 - 07 1984), where $e$ is the magnitude of the error vector and $\alpha$ is its angle.
Figure B.1: Residual displacements (see text) for June 05-06 1984 along with the 24 hour averaged isobars for same period.
vector scale: — 17.00 km

Figure B.2: Automatically measured displacements from June 06-07 1984.
Figure B.3: Manually measured displacements from June 06-07 1984.
Figure B.4: Displacements predicted from theory for June 06-07 1984 along with the 24 hour averaged isobars for same period.
Figure B.5: Residual displacements (see text) for June 06-07 1984 along with the 24 hour averaged isobars for same period.
### APPENDIX B. AVHRR GRAPHIC RESULTS

<table>
<thead>
<tr>
<th>measured displacement</th>
<th>error $e = u_m - u_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>automatic</td>
<td>manual</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>
Figure B.1: Automatically measured displacements from June 11-12 1984.
Figure B.2: Manually measured displacements from June 11-12 1984.
Table B.1: comparison between the $u_a$ and $u_m$ measurements for image pair 3 (June 11 – 12 1984).
Figure B.3: 24 hour averaged isobars for June 11-12 1984.
Figure B.4: Automatically measured displacements from June 12-13 1984.
Figure B.5: Manually measured displacements from June 12-13 1984.
Table B.1: comparison between the $u_a$ and $u_m$ measurements for image pair 4 (June 12 – 13 1984).

<table>
<thead>
<tr>
<th>measured displacement</th>
<th>error $e = u_m - u_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>automatic</td>
<td>manual</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>-3</td>
<td>3</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
</tr>
</tbody>
</table>

| mean errors | $\bar{x}$ | $\bar{y}$ | $\bar{e}$ | $|e|$ |
|--------------|-----------|-----------|------------|-----|
|              | 0.0       | 0.0       | 6.1        | 0.1 |
APPENDIX B. AVHRR GRAPHIC RESULTS

Figure B.1: Displacements predicted from theory for June 12-13 1984 along with the 24 hour averaged isobars for the same period.
Figure B.2: Residual displacements (see text) for June 12-13 1984 along with the 24 hour averaged isobars for the same period.
Figure B.3: Automatically measured displacements from July 30-31 1985.
vector scale: $- 17.00 \text{ km}$

Figure B.4: Manually measured displacements from July 30-31 1985.
Table B.2: comparison between the $u_a$ and $u_m$ measurements for image pair 5 (July 30 – 31 1985).
Figure B.1: Displacements predicted from theory for July 30-31 1985 along with the 24 hour averaged isobars for the same period.
Figure B.2: Residual displacements (see text) for July 30-31 1985 along with the 24 hour averaged isobars for the same period.
vector scale: — 17.00 km

Figure B.3: Automatically measured displacements from August 06-07 1985.
Figure B.4: Manually measured displacements from August 06-07 1985.
### Table B.1: Comparison between the $u_a$ and $u_m$ measurements for image pair 6 (August 06 – 07 1985).

<table>
<thead>
<tr>
<th>measured displacement</th>
<th>error $e = u_m - u_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>automatic</td>
<td>manual</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>-5</td>
<td>-5</td>
</tr>
<tr>
<td>-2</td>
<td>6</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>-2</td>
<td>5</td>
</tr>
<tr>
<td>-3</td>
<td>6</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>-3</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>mean errors</th>
<th>$\bar{x}$</th>
<th>$\bar{y}$</th>
<th>$\bar{\alpha}$</th>
<th>$\bar{e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>-0.7</td>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>
Figure B.5: Displacements predicted from theory for August 06-07 1985 along with the 24 hour averaged isobars for the same period.
Figure B.1: Residual displacements (see text) for August 06-07 1985 along with the 24 hour averaged isobars for the same period.
Figure B.2: Sample cross-correlation function for typical window for the June 11-12 1984
Figure B.3: Sample cross-correlation function for typical window for the June 11-12 1984
Appendix C

SAR Graphic Results
Figure C.1: SEASAT SAR - revolution 1439 - October 5 1978.
Figure C.2: SEASAT SAR - revolution 1482 - October 8 1978.
APPENDIX C. SAR GRAPHIC RESULTS

Figure C.3: Location of SEASAT SAR image pair used in this study.

Figure C.4: Manually measured ice displacements computed from SAR pair (courtesy of D.A. Rothrock, University of Washington).
Figure C.5: Manual measurements of ice displacement. (a) shows displacements as vectors, (b) shows displacements as a deforming grid (see text), square denotes outline of undeformed grid.
Figure C.1: Convolution of Sobel operator with rev. 1439.
Figure C.2: Convolution of Sobel operator with rev. 1482.
Figure C.3: Convolution of $\nabla^2 g$ operator with rev. 1439.
Figure C.1: Zero crossing magnitudes from rev. 1439.
Figure C.2: Convolution of $\nabla^2 g$ operator with rev. 1482.
Figure C.3: Zero crossing magnitudes from rev. 1482.
Figure C.1: Automatically measured displacements from first order operator experiment.
Figure C.2: Automatically measured displacements from second order operator experiment.
Figure C.3: Errors in the first order experiment.
vector scale: $\rightarrow$ 4.00 km

Figure C.4: Errors in the second order experiment.
Figure C.5: Detail 1 from the lower left corner of rev. 1439 (top) and rev. 1482 (bottom).
Figure C.6: Detail 2 from the upper right corner of rev. 1439 (top) and rev. 1482 (bottom).