DUAL PERMEABILITY MODELING
OF
FR ActURED MEDIA

by

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Dual permeability is introduced as an approach to modeling flow and transport through fractured media. The approach allows a large reduction in the number of fractures that are represented explicitly in a discrete fracture network. The most important fractures in terms of fluid flow are identified using their physical characteristics. These fractures form a sub-network that divides the entire fracture network into smaller domains. The fractures of the smaller domains are approximated. The approximations do not rely on continuum assumptions. They are determined individually and independently for each small domain, resulting in a parallel structure to the calculations.

An exploratory model is developed for steady state fluid flow and solute transport in two dimensional fracture networks and compared to a discrete fracture model that represents all of the fractures explicitly. Individual fractures are represented by finite lines with constant hydraulic transmissivity. Solute transport calculations use particle tracking. The dual permeability model is shown to provide acceptably accurate solutions while reducing the maximum number of simultaneous equations by well over an order of magnitude.

The dual permeability approach takes advantage of the hydraulic behavior of highly heterogeneous media. Channeling of flow that develops in such media is used to reduce errors introduced through the sub-continuum approximations. The dual permeability approach works better when the fracture system has a broad range in the scale of fracturing as demonstrated by transport calculations for fracture networks with a high degree of clustering or with a fractal nature.

A study of sub-REV fracture networks reveals that averaged flow through fractured media can be represented using tensor notation in much smaller domains than an REV. At these scales, heterogeneity within a fracture domain causes the tensor representation to be asymmetric, unlike the continuum representation of granular porous media. Hydraulic heads within a heterogeneous fracture
network do not vary in a smooth fashion. The hydraulic head reflects the distribution of heterogeneity within the domain. The approximations developed for the dual permeability model required that the boundary conditions imposed on the small domains reflect heterogeneity both within the domains and external to them.
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CHAPTER 1

SIMULATION OF FRACTURE NETWORKS

AT THE FIELD SCALE

1.1 INTRODUCTION

Understanding heterogeneous systems at a scale where the heterogeneity can not be represented explicitly nor is amenable to homogenization through simple volume averaging is an active research topic in the hydrogeologic community. Many problems involving field scale simulation of flow in fractured rock may lie in this region between the applicability of effective continuum models and the practicality of explicit models. Solute transport exhibits even greater sensitivity to heterogeneity than does fluid flow and is less likely to be amenable to analysis using effective continuum models. It is not clear that the considerable advances that have been made in the past few years in understanding flow and transport in heterogeneous porous media[ e.g. Gelhar and Axness, 1983; Dagan, 1988; Neuman, 1993] can be applied to fracture dominated media. Given this uncertainty, there is a need to pursue techniques that may allow the application of discrete fracture modeling at scales where the representation of each fracture is made impossible because of an overwhelming number of fractures.

The dual permeability concept that is the focus of this thesis may provide a mechanism to develop a simulation capability within the transition region between conventional discrete fracture modeling and the treatment of fractured rock as an equivalent porous medium. The primary tenants of the dual permeability approach are: 1) a relatively small number of fractures within a fracture system dominate the large scale hydraulic head distribution; 2) these same fractures carry the majority of the flow within the system; 3) the vast majority of fractures within the model domain have a subsidiary influence; and 4) the influence of these subsidiary fractures is, however, still important to the overall hydraulic behavior of the
fracture system because of their shear numbers and the effects of these fractures on the transport behavior of the system.

Figure 1.1 illustrates how a few fractures can dominate the flow system. In Figure 1.1a, fracture traces are shown for a two-dimensional representation of a fracture network. In this particular fracture network, the lengths of the fractures vary widely. The longer fractures also tend to have larger apertures. In Figure 1.1b, where the widths of lines are proportional to the flow rates within fractures, the long, wide fractures dominate the flow system. Fractures with flow rates less than 3% of the largest flow rate are not drawn in Figure 1.1b, revealing that most of the fractures carry relatively little flow. Figures 1.1c shows the fracture traces of a sparse fracture network that has a smaller range of fracture lengths than the network in Figure 1.1a. The corresponding flow rates are shown in Figure 1.1d. In this network, connections between fractures control the dominant flow paths suggesting that fracture connectivity can also strongly influence the character of flow in fractured media.

In both of these networks, a few interlinked fractures form dominant flow channels. However, these channels do not provide routes by which the majority of the fracture network is bypassed. Instead a clear interaction between the channels and the rest of the fracture network can be seen in both Figure 1.1b and Figure 1.1d by noting the variation of the flow rates along these channels. The flow in these channels tends to swell and then diminish indicating flow to and from other fractures in the network. This happens because groups of highly transmissive fractures form local clusters that do not interlink. These clusters are connected by individual or groups of less transmissive fractures.

The importance and degree of channeling within fracture networks is dependent on the variability of fracture transmissivity and fracture connectivity in a complex manner. Densely fractured rock with a large variation in fracture transmissivity will tend to have flow concentrated in fewer fractures than densely fractured rock with more uniform fracture transmissivity. In contrast, within sparsely fractured rock, flow dominance may be more dependent on the way fractures intersect than on the transmissivity
Flow distribution within a fracture network.  a) Fracture traces of a network with two scales of fracturing.  b) Flow rates within fractures depicted as line width.  Fractures with flow rates less than 3% of the largest flow rate are not drawn.  c) Fracture traces of a sparse network.  d) Flow rates within the sparse network.
of those fractures.

The name "dual permeability" arises from the structure that follows from flow dominance. Permeability is incorporated in a dual permeability model in two different ways. The dominating fractures are modeled explicitly while the majority of fractures are represented, in some manner, as a group. The phrase "in some manner" leaves a large degree of latitude to the actual implementation of the dual permeability concept. The dual permeability structure is illustrated in Figure 1.2. This figure shows the selection of a few fractures as the dominant ones. The fractures were selected on the basis of both their transmissive character and their connections to other fractures; not by whether they actually dominate the flow regime which is unknown at the time of selection. These fractures are used to define a geometric structure within the fracture network. The properties of the intervening regions of the fracture network that are defined by this structure are represented in the model without explicit representation of the individual fractures within the region.

Dual permeability modeling is an attempt to accommodate the heterogeneity of fractured rock in field-scale simulation models. The long term goal of this area of research is to provide a predictive capability that can be applied at the field scale. The goals of this thesis are more limited. These goals are: to translate the dual permeability concept into a flow and transport simulation model; to identify strengths and weaknesses of the concept; to identify features of the concept that may limit its application to fractured rock investigations; and to use both the model itself and the issues which arise from its implementation to further our understanding of flow and transport in fractured media.

1.2 THEORETICAL MOTIVATION

Flow and transport in fractured rock is strongly influenced by the three-dimensional aspects of the fracture network in most rock bodies. True three-dimensional fracture network models are presently limited to scales far smaller that most field scale problems and to scales below those required to
Figure 1.2: Structure of the dual permeability model concept. The fracture network is subdivided into small blocks by finding a network of dominant fractures. The hydraulic properties of the fractures within the small blocks, such as the one isolated in this schematic representation, are approximated in the representation of the blocks.
adequately address the issues of heterogeneity. At this time only two-dimensional representations can both capture the fundamental aspects of fractured rock heterogeneity and allow the representation of enough discrete features so that the transition region, up to the scale of continuum behavior, can be explicitly represented. Therefore, the emphasis of this research must be on concepts that arise in accommodating heterogeneity in flow and transport processes rather than in providing a practical simulation model for flow and transport in fractured rock.

The critical aspects of the hydrology of fractured rock that are captured in two-dimensional representations are: 1) the extreme contrast between the permeability of a fracture and that of the host rock; 2) the strong local anisotropy introduced by the narrowness of fractures in comparison to their extent in the other dimensions; 3) the important role that connections of fractures play in the network scale hydraulic behavior of a fracture system; and 4) the size of some fractures may be so large that on an individual basis they noticeably effect the fluid flow in the rock mass.

Another important aspect of heterogeneity in geologic media, and in fractured rock in particular, is the usual high degree of ignorance that exists about the hydraulic properties at specific locations within the flow domain. Discrete fracture models require a physical description of a fracture network in terms of the location of the fractures and the space between the fracture walls. The identification of all the features of a fracture network needed to create an explicit representation of a discrete fracture model is impossible. Even in the most simple situations, only a crude knowledge of most of the fractures will be known. Information from bore-holes or exposed surfaces may give some information about the locations of a small subset of the fractures, but the information will not be of sufficient detail to describe the entire fracture network. Geophysical techniques may be able to identify the locations of very large fractures or zones of intense fracturing, but they can not provide the detail that is needed for discrete fracture modeling. Most of the information about the fracture system must come from characterization of the
fracture system in an indirect way; from investigations of bore-holes and surface exposures that are expected to be representative of the interior of the rock.

There are many possible approaches to accommodate the basic lack of knowledge about the structure of a fracture system. Of these, two stand out as being generally accepted concepts. One is through repeated simulations of possible configurations of the fracture system, commonly called Monte Carlo analysis. The other approach is to develop large scale (macroscopic) representations of the hydraulic and transport properties of the fracture system. These representations account for the average effects of heterogeneity but are insensitive to specific small scale heterogeneity. Macroscopic representations may themselves be based on stochastic analysis and may be used in a Monte Carlo fashion to represent heterogeneity on a scale that is larger than the scale at which the average properties are defined.

Macroscopic representations are appropriate only if the details of small scale heterogeneity are not important to the larger scale hydraulic behavior of the fracture network. The scale at which macroscopic representations become appropriate has become known as the scale of the representative elementary volume (REV). The REV concept is fundamental to the issues addressed in this thesis. The concept has been so well established in hydrogeology that a review here would be overly pedantic. The reader is referred to Bear [1972] for a general discussion and Long et al. [1982] for specifics related to fluid flow in fractured media. The subject of the REV is discussed further in Chapter 5.

The discrete fracture network models used in a Monte Carlo procedure require a complete description of the network of fractures within the rock mass. A set of equally possible realizations are generated according to the probability distributions that describe the actual fracture system. Each realization will contain features that are consistent with the statistical representation of the fracture system, but do not exist in the actual fracture system. The essence of the Monte Carlo procedure is to create enough realizations to adequately estimate the frequency distribution of possible hydraulic behavior of the statistical representation, thereby reducing the influence of unlikely occurrences in individual realizations.
This distribution can be used to anticipate the likely hydraulic behavior of the fracture system and to identify possible behavior that may have important consequences. An important aspect of this process is that it allows an analyst to associate a likelihood with possible but improbable hydraulic characteristics.

The Monte Carlo process can also be described using the more abstract concept of information. Figure 1.3 is included to aid in this description. The fracture network of a discrete fracture model contains a great deal of detailed geometrical information. A well understood fracture system will be characterized mostly though a statistical representation in terms of probability distributions of fracture physical characteristics. The Monte Carlo realization process creates the required detailed information of the fracture network from the statistical representations, step one in Figure 1.3. The discrete fracture model then transforms the detailed geometrical information about the fracture network into detailed information about the hydraulic behavior of the fracture network, such as the solute concentration as a function of time at locations within the fracture network. Unfortunately, most of this detailed information is highly unlikely as a representation of the hydraulic behavior at a specific point within the simulation domain and it must be weighted properly in the overall analysis. Forming a distribution over many realizations reduces the importance of information from any individual realization so that the essential behavior that is consistent with the statistical representation of the fracture system is enhanced over the multitude of unlikely events that are imbedded in the detailed hydraulic behavior.

Monte Carlo analysis is a tool that can be used to accommodate uncertainty in the knowledge of a fracture network. A byproduct of using Monte Carlo analysis is a tolerance toward some of the errors that are introduced into the predictions of hydraulic behavior through the flow and transport models, in addition to errors introduced through ignorance of the physical system. If the variations in predicted behavior introduced through modeling errors are small compared to variations inherent in the generation of fracture network realizations, then the errors are not important to the overall results. In other words, if the errors introduced by the discrete fracture model are small compared to the variability introduced
Figure 1.3: The Monte Carlo process. On the right are the stages of a Monte Carlo analysis of a fracture system. The boxes are processes. The area between boxes are stages resulting from preceding process and/or input to the next process. On the left, the Monte Carlo process has been abstracted to an information processing procedure.
through random sampling of the statistical representation of the fracture system geometry they will not influence the post-averaging results of the Monte Carlo process.

The same argument cannot be used to dismiss biases in the hydraulic behavior introduced by a discrete model. Biased errors introduced through a discrete fracture model will produce biased average behaviors and biased predictions. If, for example, the discrete fracture model consistently underestimates the transport time through individual fractures then transport predictions will be biased toward faster movement of solute. The use of Monte Carlo analysis would not correct for this discrepancy.

The conspicuous lack of detailed information about fracture networks at the field scale that is characteristic of hydrologic studies of fractured media makes the Monte Carlo process a standard approach to understand the hydraulic behavior of these systems. Conventional discrete fracture models provide an exact transformation of the physical description of a fracture network into a hydrologic one. Unbiased errors in the results are tolerable because of the averaging process required in the Monte Carlo procedure. This tolerance provides an opportunity to intentionally allow errors in the fracture network model if additional capabilities are derived through the process. The major capability of interest at this time in fracture network modeling is to be able to increase the scale of the problem that can be solved.

This view of the Monte Carlo process provides justification for approximating the majority of fractures in a dual permeability model, not in a manner that best retains the hydraulic behavior of the fracture network, but in a manner that allows the greatest number of fractures to be modeled while still avoiding the introduction of biases. There should be a clear distinction here between the approximation in terms of equivalent porous media modeling and approximation in terms of dual permeability modeling. In equivalent porous media modeling, the detailed hydraulic behavior is considered unimportant to the large scale behavior of the system. In dual permeability modeling, the detailed behavior is considered important to the large scale behavior of a single realization, but not important to end use of the modeling results.
1.3 Thesis Overview

We have already seen how accommodating the lack of knowledge of a fracture network at the field scale influences the conceptual framework of this thesis. An important consideration, therefore, is understanding what constitutes a reasonable degree of knowledge about a fractured rock mass. Chapter 2 addresses this issue with a review of how fracture system information is acquired. The information is needed to identify physical characteristics of fractures that are usually included in discrete fracture models; the subject of Section 2.3. The chapter also includes a summary of some of the models of fracture network geometry that could be used to incorporate the interrelationships of fractures into discrete fracture models.

The fundamental modeling assumptions that are the basis of discrete fracture modeling used in this thesis are presented in Chapter 3. The validity of many of these assumptions is investigated and discussed. Chapter 3 includes a description of the fracture network generation procedure that defines the fracture geometry for each fracture network realization. The chapter ends with a model description for the discrete fracture model that is used to benchmark the dual permeability model.

Chapter 4 presents the dual permeability model. This presentation uses the basic model description of Chapter 3 as a starting point. An overview of the model structure is provided first and is followed by more detailed descriptions. An evaluation of the dual permeability model in terms of a Monte Carlo investigation shows that the dual permeability concept is a viable method of investigating the hydraulic behavior of fracture networks. The dual permeability model is compared to an alternate strategy of selectively ignoring fractures in a network in Section 4.6. The chapter concludes with some of the alternative implementations that were either tried or show promise. The last section focuses on the lessons that were learned from these trials that are important considerations in sub-REV models of fracture networks.
Chapter 5 turns to a characterisation of flow in fracture networks. This chapter presents a study of the influence of fracture network heterogeneity on flow at sub-REV scales. The study is concerned with representation of the aggregate flow behavior of a fracture network below the REV scale. The study demonstrates that flow does not depend on the direction of the hydraulic gradient in an erratic manner, as has been implied by earlier studies. Instead, the flow dependence behaves in a predictable manner that can be represented using a tensor relationship.

The concluding chapter provides an assessment of the dual permeability model and interprets the lessons learned from this one model in terms of the dual permeability concept. The chapter also pulls together some of the disparate results from Chapters 4 and 5 into a discussion of some important aspects of representing highly heterogeneous media below the scale of continuum behavior.
CHAPTER 2

FRACTURE SYSTEM CHARACTERIZATION:

A REVIEW

2.1 INTRODUCTION

This chapter describes the characterization of the physical attributes of fractures in a manner that can be useful to numerical simulation of fluid flow and solute transport within a fractured rock mass. Physical characterization is divided here into two subjects. 1) The statistical description of the physical properties of individual fractures. 2) The description of the relationships of individual fractures to each other; referred to in this thesis as structural modeling. This division is not always straightforward and is not common in the relevant literature, but I believe it is useful.

The description of the physical characteristics of fractures is a necessary part of any attempt to understand the hydraulic properties of fractured media through simulation of the hydraulic behavior of discrete fracture networks. It is not, however, central to the research described in this thesis. The material presented in this chapter is a synthesis of some of the relevant literature on the topic, from the perspective of flow and transport simulation using discrete fracture networks.

The description of the physical characteristics of fracture networks has been referred to as joint system modeling. Published descriptions of joint system models often include both the properties of individual fractures and the interrelationships between them. The two aspects are intentionally separated here without indication of the original linkage. Most joint system models have been presented as three-dimensional models. The focus in this discussion is on representation of a fractured rock in two dimensions. In some instances however, three-dimensional aspects are included.
An overview of methods used to acquire data about fractures is given in Section 2.2. One of the
topics discussed in Section 2.2 is the difficulty in acquiring detailed information on the locations and
characteristics of individual fractures within a rock mass. The scarcity of information about individual
fractures in a fracture network is a critical feature of fracture system modeling and, as has been mentioned
in Chapter 1, has influenced the course of this research. In Section 2.3 the characteristics of individual
fractures are reviewed first, then the section focuses on relationships between fractures. Section 2.4
discusses a limited number of structural models which represent the relationships between fractures in a
quantitative manner.

2.2 SOURCES OF FRACTURE SYSTEM INFORMATION

One of the most basic aspects of characterizing the hydraulic behaviour of fractured rock is the
scarcity of data that can be readily obtained about the geometry of individual fractures. This is true
despite the tremendous variety of techniques that have been developed to measure and characterize
fractures. Measurement of fracture properties can be divided into two categories; direct and inferred.
Direct measurements are the physical measurements of the fracture geometry, such as fracture orientation
or trace length. Inferred measurements require interpretation of the influences of fracture geometry on
the measured parameter. An estimate of fracture transmissivity from analysis of pressure response to an
injection test is an example of inferred measurements. Such an estimate requires assumptions about the
shape of the fractures and about the interconnection of the fracture intersected by the borehole with other
fractures within the rock body.

Both measurement categories can be further sub-divided based on the type of acquired information.
One type of information is specific, such as the location of individual fractures within a borehole. The
other type of information is representative. An example of representative information is data from the
measurement of fracture orientations on a surface exposure. These measurements give no specific
information about the fractures within the rock mass, but may be indicative of those fractures. The
distinction between the terms specific information and representative information is not the information itself but in how it is used. Specific information is used to identify the features of individual fractures in a model. Representative information is used to identify the features of populations of fractures.

2.2.1 SOURCES OF SPECIFIC INFORMATION

Specific information is primarily available from borehole investigations. A large number of tools have been developed to investigate the walls of boreholes. Some of these tools, such as callipers or acoustic televiewers, provide direct information on fracture spacing, orientation and aperture. Cores from the drilling of boreholes can also be used to provide this information. Correlating cores with other borehole surveys may be useful in screening out fractures that have been created by the drilling process and that should not be used in characterizing the fracture system in the surrounding rock. Geophysical techniques, such as natural gamma logs, epithermal neutron logs, electrical resistivity logs, and radar surveys, provide inferred information on the location and possibly the extent of individual fractures or zones of intense fracturing.

Hydraulic tests using one or more boreholes can be conducted to gain understanding of the hydraulic properties of localized regions of the fracture system and can give an indication of the connectivity of the fractures near the wellbore (e.g. Hsieh and Shapiro [1994]).

Geophysical techniques can be separated into borehole techniques, cross-borehole tomographic techniques and regional techniques. Cross-borehole and regional techniques do not have the resolution to provide the kind of detailed information about the fracture system that is available from boreholes and surface exposures. Cross-borehole and regional techniques can, however, provide data, such as regions of intense fracturing or the general direction of solute transport, that can be used to constrain discrete fracture models.
Borehole geophysical techniques can give indications of the extent of fractures in the immediate vicinity of the boreholes. Geophysical methods can also identify the existence of water or injected tracer in nearby fractures. This information can be used to establish which fractures are actually participating in the flow system and to give an indication of the channelling of flow within individual fractures [Olsson et al., 1991]. Tomographic imaging of multiple seismic or radar measurements made between boreholes or from boreholes to tunnels can give an indication of the existence of fractures or, more easily, zones of intense fracturing [Majer et al., 1990; Black et al., 1991].

On a scale larger than can be investigated through borehole investigations, measurements of hydraulic head, flow rates into open wellbores or tunnels, or the movement of natural tracers may provide specific information that could be useful in establishing constraints on larger scale behaviour. These data may provide checks on the large scale properties of the fracture network such as effective hydraulic conductivity. In making such checks, recognition of the influence of fracture system heterogeneity on the measured data is important. Transport experiments provide data that may also prove useful. However, the uncertainty that accompanies the geometric description of a fracture network composed of more than a few fractures makes interpretation of the results of these tests difficult.

The amount of specific information about the fractures, from even the most thorough site investigation, would not be enough to fully define a discrete fracture network that could be used in a flow and transport simulation. We are forced to use structural models and individual fracture statistics based on representative information to synthetically create the specific information required by discrete fracture flow and transport models. Representative information is, therefore, critical to modeling of the hydraulic behavior of fracture systems.
2.2.2 SOURCES OF REPRESENTATIVE INFORMATION

Representative information often is gathered by both borehole investigations and surface surveys. The surfaces can be either outcrops, exposures from excavations, or the walls of tunnels. In all cases, changes in internal stresses within the rock mass resulting from the exposure of the surfaces may induce additional fracturing that make the surfaces unrepresentative of the fracture system that exists within the rock. Recent fractures and fractures that were not hydraulically active may be identifiable through the study of surface coatings or rock alteration near the fracture. Cores provide samples for measuring fracture roughness and fracture surface coatings that may be useful in delineating fracture sets [e.g. Herbert 1991]. Surface coatings may also be useful in identifying which fractures were hydraulically active prior to drilling a borehole.

The expression of fractures on a surface is called a trace. Procedural techniques for carrying out trace surveys are well established [e.g. Priest and Hudson 1976, Baecher and Lanney 1978, La Pointe and Hudson 1985, Blin-Lacroix and Thomas 1990]. Surface surveys can give only partial information on fracture shape. An understanding of the shape of fractures can be acquired through careful excavation of a fracture [Dershowitz and Einstein 1988].

2.3 STATISTICAL CHARACTERIZATION OF FRACTURES

Table 2.1, modified from Dershowitz and Einstein [1988], presents a list of identifiable fracture characteristics that are appropriate for statistical parameterization and are candidates for incorporation into joint system models. The characteristics are divided into three groups: 1) characteristics of individual fractures, 2) fracture density which is a composite characteristic of all the fractures, and 3) characteristics of the relationships of individual fractures. Most of these characteristics are discussed in the following sub-sections.
<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Orientation</strong></td>
<td>Orientation of the fracture plane.</td>
</tr>
<tr>
<td><strong>( Attitude )</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Spacing</strong></td>
<td>Distance between intersection points produced by a line perpendicular to the mean orientation of the fracture set where a set consists of fractures that are parallel or sub-parallel to each other.</td>
</tr>
<tr>
<td><strong>Fracture Size</strong></td>
<td>Extent of fracturing; generally expressed as trace length on two-dimensional surfaces such as outcrops or as the surface area of individual fractures.</td>
</tr>
<tr>
<td><strong>Shape</strong></td>
<td>Shape of boundaries can be polygonal, circular, elliptical or irregular.</td>
</tr>
<tr>
<td><strong>Planarity</strong></td>
<td>Character of the surface with respect to an ideal plane. Deviations from planer surfaces can be on several scales.</td>
</tr>
<tr>
<td><strong>Roughness</strong></td>
<td>Deviation from planarity &lt; 1 mm</td>
</tr>
<tr>
<td><strong>Waviness</strong></td>
<td>Deviation from planarity &gt; 1 mm</td>
</tr>
<tr>
<td><strong>Aperture</strong></td>
<td>The distance between fracture surfaces.</td>
</tr>
<tr>
<td><strong>Density</strong></td>
<td>Number of fractures per unit area or volume. A measurable indication of density such as total trace length per unit area or total surface area per unit volume may be used as a surrogate.</td>
</tr>
<tr>
<td><strong>Persistence</strong></td>
<td>The classifications persistent, sub-persistent, and non-persistent are used to indicate the degree of connectedness of the network.</td>
</tr>
<tr>
<td><strong>Termination</strong></td>
<td>The tendency of a fracture to end along the surface of another fracture. Commonly measured in outcrops in terms of the fraction of ends that terminate against another fracture.</td>
</tr>
<tr>
<td><strong>Co-planarity</strong></td>
<td>Co-planarity indicates that two or more fractures exist in the same plane.</td>
</tr>
</tbody>
</table>

**Table 2.1:** Fracture characteristics. The first seven characteristics describe individual fractures. The last three entries are properties describe the interrelationships of fractures. Density separating the two categories does not belong in either category. Modified from *Dershowitz and Einstein* [1988].
2.3.1 ORGANIZATION OF FRACTURES INTO SETS

Pollard and Aydin [1988] state "Curiously, joints never occur alone, but as a series of subparallel fractures defining a set." They also state that "Joint patterns comprising more than one joint set are common in nature." Multiple sets may arise from a single deformation episode or from different events. The most common method of classifying fractures into sets is based on the orientation of the fractures. Unfortunately orientation alone can be a poor guide to classification. Often, the sets are not well defined [Barton and Hsieh 1989, see also Long and Billaux 1987]. Fractures with similar orientations may not have been created in the same deformation episode and may belong to sets with much different characteristics. Termination and cross-cutting relationships give indications that fractures of similar orientation belong to different sets. Surface coatings have also been used to relate individual fractures with specific deformation episodes, allowing different sets with similar orientations to be distinguished. These different sets may have different aperture characteristics.

2.3.2 ORIENTATION

Fracture orientation indicates the plane through space that tends to correspond with a fracture surface. The orientation measure that is most convenient to work with is the direction of the fracture pole, the line normal to the plane best aligned with the fracture surface. If the fracture surface is wavy, then the orientation is dependent on where the measurement is made. Generally, orientation measurements are accurate to ±2° [Barton and Hsieh, 1987; see also Chernyshev and Dearman, 1991]. Poles are usually plotted on a Schmidt hemispherical projection [e.g. Davis, 1984]. Fractures of the same set tend to cluster on these diagrams. Identification of sets can be aided through the use of clustering algorithms that will separate fracture pole data into sets [e.g. Shanley and Mahtab, 1975].

Once sets have been identified, the orientations of fractures of each set must be characterized in terms of statistical distributions. Dershowitz [1984] lists six candidate distributions that can be used to
represent the orientation of a set statistically: Uniform, Fisher, Elliptical Fisher, Bingham, Fisher-Bingham, and Bivariate Normal. These distributions are all bi-variate because orientation is represented by two angles in the three-dimensional representations of interest to Dershowitz. Based on data from 25 sites with varying geology, he concluded that none of the distributions could consistently fit the field data at the 95% confidence level indicating that an appropriate description for fracture orientations at a particular rock body should not be a generic description but should be instead developed from the investigations of the rock. The Fisher distribution is most commonly used because it is analogous to a normal distribution on hemispherical projections [Dershowitz, 1984]. In two dimensions, orientation is defined by a single angle which can be represented by a normal distribution, albeit not necessarily accurately.

2.3.3 SPACING

The spacing of intersections with fractures along a line on an exposed surface has been studied extensively. The line is commonly referred to as a scan-line and the density of intersections along the line is called scan-line density. Spacing is an important issue in fracture system modeling because different structural models may have characteristic spacings. Spacing can therefore be used to assess the appropriateness of the different structural models. Nine investigations of fracture spacing are reported in Kulatilake's [1988] review of fracture system statistical models. Of the nine, four papers [Steffen et al. 1975, Bridges 1975, Barton 1977, Sen and Kazi 1984] reported that spacing follows a lognormal distribution. Four papers reported that spacing follows a negative exponential distribution [Call et al. 1976, Priest and Hudson 1976, Baecher et al. 1977, Wallis and King 1980]. Einstein et al. [1979] reported that 80% of the distributions they investigated were best fit with a negative exponential distribution and that 20% were best fit with a lognormal distribution. Of the surveys reported by Kulatilake, only Baecher et al. and Einstein et al. used goodness of fit tests to select the best distributions. Kulatilake et al. [1993] present a survey of fracture spacing in the Stripa mine. These authors divided the fracture network into sets based on fracture orientation prior to determining the spacing for each set. They
determined that a gamma distribution fit the data better than negative exponential, log-normal or normal distributions based on goodness of fit tests.

The lognormal and negative exponential distributions are very similar except for the frequency of small values. Chernyshev and Dearman [1991] suggest that the studies that have reported lognormal distributions may be neglecting the smallest fractures because they are not readily visible to the human eye. As a caution, Priest and Hudson [1979] demonstrate that a mixture of sets composed of evenly spaced fractures will combine to form intersections with a scan-line that have a negative exponential distribution. Gillespie et al. [1993] provide a clear illustration of this phenomena. These authors demonstrate that a scan-line survey that superficially indicates a negative exponential distribution of fracture spacing also clearly supports the conclusion that fractures of a particular set are roughly equally spaced and that this spacing is best represented using a normal distribution (a conclusion that seems obvious from a visual inspection of the fracture traces).

The conclusion that is supported by many studies of fracture spacing is that, within crystalline rock, fractures occur randomly within a rock mass. The work of Gillespie et al. [1993] casts a shadow on this conclusion. The results of surveys that do not first distinguish fractures by set must be questioned. Following the logic of Chernyshev and Dearman [1991], original deformations result in a uniform distribution of fractures with a relatively narrow range of orientations. Creation of fractures during subsequent deformation episodes are affected by the preexisting fractures. These subsequent fractures are often regularly spaced with densities and orientations that are effected by the spacing between the pre-existing fractures; causing clustering. The influence of pre-existing fractures causes a much greater variation in the orientations of the subsequent fractures than the original fractures.

The recent interest in the fractal nature of geologic media has expanded the debate on fracture spacing. It is possible that the results of spacing surveys have been constrained by the distribution models that are considered. Barton and Larson [1985] performed a trace survey of three pavements at Yucca
Mountain in Nevada. They concluded that the fracture trace distributions can be represented fractally, with very similar coefficients for each pavement. Barton [1993] also found a power law length distribution in the fracture traces observed on road cuts at the Mirror Lake site in New Hampshire. Geier et al. [1989] investigating a trace survey conducted along the floor of the TSE drift in Stripa, Sweden, found that the distribution of trace lengths fit a power law distribution. The success of Barton and Larson, Barton, and Geier et al. in identifying fractal related characteristics in fractured rock gives support to the fractal structural models. However, an examination of the techniques used to test for fractal behaviour by Gillespie et al. [1993] found that the techniques used in the above surveys do not discriminate fractal from non-fractal distributions well. These authors concluded that they found no evidence to support fractal distributions of fracture joints but that some data of spacing of faults support fractal distributions. The issue of how fractures are spaced in a rock mass is far from settled.

2.3.4 Size

The length of traces on outcrops and other surface exposures has been extensively studied. Trace length measurements are biased in three ways: 1) Traces are truncated along the edges of the exposure surface. 2) The probability of intersection of a trace with a scan-line is inversely proportional to the trace length. 3) Data is generally not collected below some length cut-off. Both Baecher and Lanney [1978] and Priest and Hudson [1981] provide techniques to correct length estimates of the traces for these biases. Chung [1986] provides a computer algorithm for this purpose.

Kulatilake [1988] reviews nine studies of trace length. In five of these studies, the investigators found that a lognormal distribution adequately represented the distribution of trace lengths [McMahon 1974, Bridges 1975, Barton 1977, Baecher et al. 1977, Einstein et al. 1980]. In the other four studies, a negative exponential distribution was used to fit the data [Robertson 1970, Call et al. 1976, Cruden 1977, Priest and Hudson 1981]. Only Baecher et al. and Einstein et al. compared the goodness of fit between the two distributions. The degree of bias correction prior to fitting the trace lengths to a
probability distribution differs in these studies. Priest and Hudson [1981] have shown that a negative exponential distribution of true trace lengths will result in a lognormal distribution of trace lengths that intersect a scan-line. This may have influenced the results of some of the surveys cited by Kulatilake.

The trace length distribution gives an indication of the size of fractures within a rock mass. Large fractures are better represented in the exposures unless small fractures preferentially exist close to the surface exposures as a result of stress relief. Fractures will also be better represented in the surface exposures if they intersect the exposure at large angles. These biases are correctable. However, the corrections depend on assumptions about the shape of fractures. Research on the relationship of trace lengths to fracture dimensions has taken two paths: 1) Trace length distribution predictions have been made starting with assumptions of fracture shape and the distribution of fracture size [Baecher and Lanney 1977, Baecher et al. 1977, Dienes 1979, Warburton 1980a, 1980b and Kulatilake and Wu 1986], and 2) The distribution of fracture sizes has been estimated from assumptions of fracture shape and distributions of trace lengths [Dienes 1979 and Kulatilake and Wu 1986]. The first approach can be used to test hypotheses of fracture shape and size. The second approach can be used to estimate fracture size distributions from trace length data as demonstrated by Kulatilake et al. [1993].

2.3.5 SHAPE

Fracture propagation and the resulting fracture shape is a complex topic that is touched only briefly here. Because of the major simplification that we must make to model fracture systems in two dimensions, the details of fracture shape are relatively unimportant to this thesis. Pollard and Aydin [1988] state "Although little is known about joint shape in massive rocks, rib marks suggest that an elliptical geometry may be common." Rib marks are ridges or furrows on fracture surfaces that seem to radiate away from what is believed to be the location of fracture initiation. All current structural models assume that fractures are planar. This assumption seems quite good except along the edges of many fractures where hackle marks may form at a small angle to the original fracture plane.
There are two dominant forms of fracture shape models, disc and polygonal. Polygonal models are well supported by surface exposures where the rock is broken into a multitude of separate blocks. Disc models seem more appropriate for rock masses that are not completely subdivided by fracturing [Dershowitz, 1984]. These discs are usually assumed to be either circular or elliptical.

2.3.6 APERTURE AND TRANSMISSIVITY

The transmissivity of a fracture is strongly dependent on the spatially varying separation of a fracture’s surfaces, i.e. the volume of the fracture, and on the possible existence of material filling this volume. The existence of infilling material has a strong influence of the transmissivity of a fracture. For the moment, consider fractures to be free of material between the fracture surfaces. Flow between two faces with a constant aperture separating them is related to the cube of the aperture if laminar flow exists within the fracture. Under natural conditions, flow in fractured rock is nearly always laminar. The assumption that fractures have constant aperture is often made to take advantage of this cubic relationship even though it is well recognized that the surfaces of fractures are not constant and rarely smooth. They are rough and have spatially varying apertures. Indeed, to transfer stress across the surface of a fracture, the fracture must be in contact at some points. A constant aperture fracture would be in contact everywhere. It is estimated that for crystalline rocks only about 3% of the surface area needs to be in contact to transfer stress [Chernyshev and Dearman 1991]. Pyrak-Nolte et al. [1987] found 10-15% of the surface area of granitic cores to be in contact for self-propped conditions and 30-40% for normal stresses of up to 85 MPa.

Studies of the effects of aperture variation on flow in individual fractures have demonstrated that flow is dominated by constrictions [Tsang and Tsang, 1987]. Aperture roughness and the resultant channeling of flow induces dispersion into the transport of solute through a single fracture. Numerical studies of the affects of aperture roughness have also demonstrated that most of the volumetric flow in fractures tends to occur in a few channels [Moreno et al., 1988; Tsang et al., 1988]. Observations of and
experiments on seepage into tunnels support the concept of channeling of flow in fractures [Neretnieks et al., 1987].

The importance of fracture aperture on the hydraulic properties of a fracture has spawned a plethora of techniques to determine apertures. Direct measurement of aperture from fracture traces on surface exposures, cores, bore-holes, and cores can be performed using a wide variety of tools. Such measurements have not been proven reliable in providing an estimate of the magnitude of apertures of in situ fractures. Bianchi and Snow [1968] used surface aperture measurements to suggest that apertures are log-normally distributed. Chernyshev and Dearman [1991] reviewed a number of Russian studies of fracture aperture distribution. These studies involving thousands of individual measurements resulted in the conclusion that in some instances the apertures were better represented with log-normal distributions and in other cases were closer to normal distributions.

Profilometers and optical methods are used to measure the roughness distribution on exposed fracture surfaces [Brown, 1987; Huang et al., 1988; Miller et al., 1990; Voss and Shotwell, 1990]. Injection of plastics into fractures prior to excavation can yield accurate information on the distribution of aperture [Gale, 1987; Pyrak-Nolte et al., 1987; Gentier et al., 1989; Cox et al., 1990]. Differences between in situ rock stress and the stress during injection can have a large influence on the apertures measured by the injection technique and apertures within the rock body. One technique that does not require the separation of fracture surfaces to inspect the aperture distribution is tomographic imaging of radioactive tracers [Wang et al., 1990].

Investigations of the effect of stress on flow through fractures have been carried out by subjecting a fractured rock to a controlled confining pressure [Raven and Gale, 1985; Pyrak-Nolte et al., 1987]. These studies are important because in situ stress conditions vary and may not be duplicated in laboratory tests or may be disturbed by wellbores. If corrections can be developed for stress variation then laboratory or surface measurements of aperture may provide useful data for discrete fracture models. As it is, the
present state of knowledge about fracture apertures and the present technology developed to characterize them makes the accuracy of direct incorporation of measured aperture data into discrete fracture network models suspect.

So far this discussion of apertures and fracture transmissivity has been based on the assumption that fracture volumes are filled only with fluid. The presence of mineral precipitation along fracture surfaces is easily accommodated in this discussion by defining the surface of a fracture as the boundary of the liquid volume rather than the boundary of the intact rock. In some instances, a fracture may be filled with fault gouge or clays that make the fracture less permeable than the host rock. Such fractures will not contribute to the hydraulic behavior of a fracture network. Fractures may also be filled with material that is more permeable than the host rock. The local transmissivity of these fractures is dependent on the permeability of the filling material and directly proportional to the fracture aperture rather than aperture cubed.

Effective apertures or the transmissivity of in situ fractures must be inferred from hydraulic data such as borehole packer tests, flow into tunnels and regional flow and head gradient estimates. Hydraulic tests of packed off intervals within a wellbore provides the most direct source of transmissivity information [e.g. Novakowski, 1989]. A large body of literature has been published on the subject of hydraulic testing of fractured rock using wellbores and the subsequent evaluation of results [Barker, 1988; Karasaki et al., 1988; Black et al. 1991]. Attempts to find an equivalent single aperture measure to describe the velocity of solute progressing though fractures have found that they are usually an order of magnitude, or more, larger than the equivalent aperture needed to match flow rates using a single aperture [Abelin et al., 1985; Novakowski et al., 1985; Tsang and Tsang, 1987] However, Raven et al. [1988] found the opposite relationship for monzonitic gneiss at Chalk River, Ontario. Silliman [1989] has explained this apparent contradiction in terms of different definitions of equivalent aperture. The heart of the matter is that no single equivalent aperture is appropriate for both flow and transport. In addition,
Smith et al. [1987] have demonstrated, using numerical modeling, that estimation of effective hydraulic apertures from pump tests is largely a measure of the aperture in the proximity of the wellbore and is not representative of the entire fracture surface.

Flow into tunnels or regional flows provide data at a larger scale than is obtainable through wellbore testing. These data can be used to compare and calibrate aggregate behaviour of discrete network models. Further calibration can be achieved through comparisons to tracer tests. These calibration procedures are constrained by uncertainties in the boundary conditions of physical experiments and in the expense of carrying out in situ experiments at large scales.

In summary, because of the sensitivity of both flow and transport to fracture apertures and because of the difficulty in estimating aperture/transmissivity, the uncertainty in model results introduced by aperture/transmissivity uncertainty dominates other sources of uncertainty in discrete fracture modeling [e.g. Herbert et al., 1991]. The problem is so serious that some believe that discrete fracture modeling is not a viable approach [Neuman, 1987].

To this rather negative assessment of the problems of incorporating aperture or transmissivity data into discrete fracture models, I must add the success of Herbert et al. [1991] in simulation of flow at Stripa, Sweden. These authors used data from single borehole hydraulic tests that are described in detail by Black et al. [1991]. Fracture sets were identified and the geometric characteristics of each set were estimated from bore-hole investigations. Despite this information, Herbert et al. decided that transmissivity estimates of the entire population of fractures would be used for all of the fracture sets, instead of using the specific transmissivity estimates for each set. A lognormal distribution was assumed over most of the transmissivity range but a linearly decreasing probability of highly transmissive fractures was imposed on the tail of the distribution. The mean transmissivity used in the distribution was estimated to be within 1.5 orders of magnitude of the true mean transmissivity of the fractures within the rock mass, based on comparisons of various fitted distributions of transmissivity. This uncertainty overstates the
uncertainty with respect to flow through a network of fractures because the estimated variance of the fitted distributions was inversely proportional to the fitted mean. Both larger transmissivity variation and larger mean transmissivity lead to larger flows through a network of fractures.

A discrete fracture model based on these estimates was constructed by Herbert et al. [1991] and used to demonstrate the scale of continuum behaviour and to provide permeability estimates for REV-sized finite element blocks. Continuum representations of these blocks were later used in a larger scale flow model. The uncertainties in the mean flow though these blocks for a known hydraulic head gradient were estimated to be a factor of two for dense networks and a factor of ten for sparse networks. Uncertainty in transmissivity was cited as the dominant cause of the uncertainty in flow. However, to have estimated the flow rates to within a factor of two within the highly fractured regions of the Stripa site should be considered a significant achievement.

2.3.7 FRACTURE DENSITY

For modeling purposes, fracture density is most usefully represented as fracture centers per unit volume or unit area. Density is usually determined from trace surveys and is usually reported in terms of intersections per meter of scan-line, scan-line density. Scan-line density contains both fracture length information and information about the density of individual fractures. The density of intersections of a fracture set with a scan-line has a correctable bias as a function of the angle between the scan-line and the mean pole of the fracture set [Terzaghi, 1965].

2.3.8 PERSISTENCE

Persistence of a fracture system is a measure of the degree of interconnection of the individual fractures. It is an important characteristic of a fracture system that should be reproduced by the fracture system model. The fracture system of a rock mass that is so intensely fractured that the rock is separated into blocks that are bounded on all sides by fracture surfaces is classified as persistent. If the fracturing
is so sparse that the fracture system does not connect sufficiently to provide a continuous pathway for fluid migration through the rock mass, then the fracture system is classed as non-persistent. In between these two extremes is the condition where fractures are well connected in the sense of forming a continuous network but not so dense as to actually form distinct blocks of rock. The sub-persistent classification is not relevant in two dimensions because any continuous pathway will divide the rock into separate blocks. A fracture system that is sub-persistent in three dimensions will appear either persistent or non-persistent in any specific two-dimensional cross-section. The transformation of this qualitative measure of persistence into a measurable quantity has not been standardized. Dershowitz [1984] discusses a number of methods to measure persistence-like properties.

One term closely allied with persistence is connectivity. Connectivity has been introduced as one measure of the degree of persistence of a fracture network. Connectivity has been quantified as the average number of intersections of a fracture with other fractures [Robinson, 1984; Charlaix et al., 1986, Hestir and Long, 1990]. Hestir and Long [1990] and Pike and Seager [1974] have shown that a connectivity of 3.6 intersections per fracture is often related to the percolation threshold of two-dimensional Poisson fracture network models (See section 2.4.3 for the definition of a Poisson fracture network model). The term percolation threshold is associated with the minimum density of fracturing of a persistent fracture network (sub-persistent in 3D). Hestir and Long [1990] have also demonstrated a functional dependence of fracture network hydraulic conductivity on connectivity.

2.4 DISCRETE FRACTURE STRUCTURAL MODELS

2.4.1 STRUCTURAL MODELS

Structural models are a vehicle to incorporate knowledge of the fracture system that goes beyond a statistical representation of individual fracture properties. The way individual fractures are connected
is a very important aspect of a fracture system. Not surprisingly, the development history of a fracture system has a strong influence on the resulting connectivity of the system.

It is through structural models that knowledge of fracture genesis and the possible episodic history of the creation of the fracture system can be incorporated into the mathematical description of fracture systems. Knowledge of the episodic history of deformation may provide dependencies of younger fractures on preexisting fractures. This knowledge is reflected in different models of how the size and locations of younger fractures are controlled by preexisting fractures.

The following section briefly describes a few of the structural models that have been proposed. These are listed in Table 2.2. The emphasis in this section is on rationale behind each model and on the interrelationships between fractures that are represented by the models. Some of the structural models

<table>
<thead>
<tr>
<th>Structure Model</th>
<th>Key Features</th>
<th>Used in this Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular Spacing</td>
<td>Continuous fractures</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>Equal spacing</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Parallel fractures</td>
<td></td>
</tr>
<tr>
<td>Poisson</td>
<td>Location and orientation independent</td>
<td>Yes</td>
</tr>
<tr>
<td>Parent-Daughter</td>
<td>Spatially correlated density variation</td>
<td>No</td>
</tr>
<tr>
<td>Truncated Poisson</td>
<td>Poisson with truncated fractures</td>
<td>Yes</td>
</tr>
<tr>
<td>Hierarchical</td>
<td>Location and truncation dependencies</td>
<td>No</td>
</tr>
<tr>
<td>War Zone</td>
<td>Poisson with truncated fractures</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>Zones of high fracture density</td>
<td></td>
</tr>
<tr>
<td>Fractal</td>
<td>Power law fracture spacing</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>Power law length distribution</td>
<td></td>
</tr>
<tr>
<td>Colonnade</td>
<td>Hexagonal or distorted hexagonal fracture pattern</td>
<td>No</td>
</tr>
</tbody>
</table>

Table 2.2: Key features of structural models.
mentioned in this section are used to generate fracture networks for the studies presented in later chapters. The implementation of these models is described in Chapter 3. Most structural models have been proposed for incorporation into three-dimensional discrete network models to be used for hydrogeologic models and/or rock mechanics models. While the proposed models have generally included assumptions about the statistical characterization of the individual fracture properties, the organization used here separates the structural models from assumptions of individual fracture characteristics.

2.4.2 UNIFORM SPACING

Uniform spacing models assume that fractures are continuous and regularly spaced. The periodic nature of these models provides geometric symmetries that allow analytical analysis of the behavior of the fracture networks in fewer dimensions than those of the network. Models based on uniformly spaced fractures have been used in many studies and are the basis of a large number of numerical models. Of historical note are the early orthogonal networks of Irmay [1955], Childs [1957], and Snow [1965]. In particular, the dual porosity modeling approach of Warren and Root [1963] using a uniform spacing model has been extremely influential in the petroleum industry [Pruess and Narashimhan, 1982; Nelson, 1987; Zimmerman et al., 1993]. Dual porosity models, in general but not exclusively, focus on the pressure response within well-bores from pumping transients rather than transport processes. Pressure response is less sensitive to the way fractures interconnect than is solute transport.

Even for transport models, regular spacing is in some instances well justified. For example, fractures in sedimentary rock fractures are often regularly spaced and the spacing often scales with the thickness of the fractured layer [Pollard and Aydin 1988]. The use of continuous fractures is harder to justify. For most fractured rock uniform spacing models do not adequately represent the heterogeneous character of the fracture networks.
2.4.3 Poisson Model

Poisson structural models are based on the assumption that the location and orientation of individual fractures are independent of all other fractures. In a Poisson model, fracture centers are assumed to be distributed randomly throughout the rock mass with a constant expected density. There is substantial evidence to support the assumption of the independence of fracture location in many crystalline rocks [Snow, 1968; Priest and Hudson, 1976; Call et al., 1976; Wallis and King 1980; Chernyshev and Dearman, 1991]. This evidence, supporting the independence of fracture location, does not mean that a consensus has been reached on the applicability of Poisson models. Chernyshev and Dearman [1991] state that the first episode of fracturing in crystalline rock tends to show independence of location, but fractures created between existing fractures are often regularly spaced.

A three-dimensional Poisson model of circular or elliptically shaped fractures was proposed by Baecher et al. [1977]. Long et al. [1985] used the Beaecher Poisson model with circular fractures to create a three-dimensional flow model. Two-dimensional Poisson models have been used in hydrogeological models by Long et al. [1982], Schwartz et al. [1983], Long and Witherspoon [1985], Herbert et al. [1991], and Hestir and Long [1990] among others.

Figure 2.1 shows a two-dimensional fracture network generated by a Poisson structural model. The network is composed of two fracture sets. The mean orientations of the sets are orthogonal but each set has a standard deviation in orientation of 10°. The scan-line density of each set is 1.5 fractures per meter.

2.4.4 Truncated Poisson Model

A truncated Poisson model recognizes the tendency for fractures to stop growing when they encounter another fracture. In a truncated Poisson model, fractures of some sets are preferentially terminated when they intersect fractures of assumed older sets. The truncated fracture network shown in
Figure 2.1: Two-dimensional Poisson fracture system representation.
Figure 2.2 has the same statistical properties as in Figure 2.1, except for the specification of a preferential termination of the sub-horizontal set at 50% of the intersections of the set on the vertical set. The appearance of the two fracture networks is quite different.

*Geier et al.* [1989] report that 1/3 to 2/3 of fracture terminations are truncations at preexisting fractures in 9 out of ten sites where truncation data was collected. At the other location, 15% of the fractures were truncated. *Dershowitz et al.* [1991] report 75% of terminations in densely fractured zones and 44% of terminations in less densely fractured rock were truncations. The probability that a fracture termination is a truncation at another fractures is related to but not the same as the truncation probability at each fracture intersection which is used to describe fracture truncation in the truncated Poisson model. *Barton* [1994] presents data from Yucca Mt., Nevada and the Mirror Lake site in New Hampshire showing truncation probabilities of 50-80% at Yucca Mt. and 45-55% at Mirror Lake.

### 2.4.5 WAR ZONE MODEL

The war zone model, as proposed by *Geier et al.* [1989], attempts to incorporate the internal geometry of shear zones as described by *Segal and Pollard* [1983] into a Poisson structural model. These shear zones have a high density of conjugate shear fractures between subparallel faults. The war zone model was originally proposed as a three-dimensional model. Figure 2.3 shows a network generated from a two-dimensional interpretation of *Geier et al.*'s proposal. The war zones are evident as the areas with a higher concentration of fracture traces.

Fracture generation in the war zone model starts with the creation of independent fracture sets. From this initial fracture system, "war zones" are identified between closely spaced subparallel fractures. The zones are identified on the basis of three characteristics: 1) The angle between neighboring fracture planes, 2) the percentage of the fractures areas that overlap between the fractures, and 3) the average distance between the fractures. Secondary fractures are created with a higher density within the war zones.
Figure 2.2: Truncated Poisson fracture network.
Figure 2.3: Fracture network of a war zone structural model.
than outside the zones.

2.4.6 PARENT-DAUGHTER MODEL

The Parent-Daughter model was introduced by Long et al. [1987] to represent the fracture system at the Fanay-Augeres uranium mine in France. Trace surveys of an 80m drift wall indicated that the fractures often occurred in clusters with similar orientation. The clustering could not be represented with a structural model that assumes that the statistics of the fracture network are constant throughout the rock mass, as is the case with the Poisson model. The Parent-Daughter model incorporates spatial variability in the distribution of fracture locations by using a Poisson model to generate fractures in small sub-regions. The values of the fracture system statistical parameters vary between sub-regions, but are spatially correlated with the values of the neighboring regions.

Figure 2.4 was presented in Long et al. [1987]. It shows a 100m by 100m domain containing 65,740 fractures. The sub-regions for this domain were 10m on a side. Both the fracture density and mean length varied as a function of the sub-region. At first glance the density variation may not seem to be apparent. Close inspection reveals a number of sub-regions that are clearly different than their neighbors.

2.4.7 HIERARCHICAL MODEL

Lee et al. [1990] describe a structural model that accommodates both spatial clustering of fractures and the dependence of younger fracturing on preexisting fractures. The model generates locations of the first fracture sets through a random process referred to as a doubly stochastic Cox process [Diggle, 1983]. Subsequent sets may incorporate a dependence of the fracture location on traces of the first sets, including truncation. The hierarchical model is an expansion of an earlier model of Conrad and Jacquin [1975].
Figure 2.4: Parent-Daughter fracture network generated from Fanay-Augeres fracture system. From Long et al. [1987]. The network is 100m by 100m, 25 times the area of figures 2.1, 2.2 and 2.3.
2.4.8 Fractal Model

Geier et al. [1989] present a fractal-based model they refer to as the Levy-Lee model. In this model fracture centers are created sequentially by a Levy Flight process. The size of the fractures are proportional to the distance to the previous fracture. The distance, $L'$, between fractures is selected from the cumulative distribution:

$$P_L(L' > L) = L^{-D}$$

(2.1)

where $D$ is a specified fractal dimension. A fracture network generated from a Levy-Lee structural model is shown in Figure 2.5. The fractal dimension used to create the network was 1.3. It is evident from Figure 2.5 that the Levy-Lee structural model produces a fracture network in which there is a tendency for fractures to form clusters. A larger fractal dimension would have resulted in tighter clusters of fractures. Geier et al. determined that a fractal dimension of 2.5 (1.5 in two dimensions) was appropriate for trace survey performed on the floor of the Stripa TSE drift. Barton and Larson [1985] have calculated a fractal dimension between 1.12 and 1.16 from various trace survey maps. Barton [1994] calculated a power law distribution of fracture trace lengths to be 2.41 at the Mirror Lake site in New Hampshire. Barton and Hsieh [1985] report power law distributions in the range of 0.8 to 1.3 for trace length surveys at Yucca Mt. Nevada and fractal dimensions for spacing between 1.6 and 1.8. The Levy-Lee model uses the same exponent for fracture trace length and spacing.

2.4.9 Colonade Model

The colonade structure of large basalt flows that form long columns of roughly hexagonal cross-section may be the most appropriate fracture system for two-dimensional representation because of the symmetry along the axis of the column. Khaleel [1989] used a mesh generation scheme to define a two-dimensional pattern of fracturing that represented fractures of the colonade structure in the flow interiors.
Figure 2.5: Fracture network of a Levy-Lee structural model.
of the Columbia River Basalt Group. The patterns Khaleel achieved are shown in Figure 2.6.

Dershowitz [1984] describes Voronoi tessellation [Voronoi, 1908] as a method to achieve similar patterns. The Voronoi mosaic tessellation uses a Poisson point process. It is based on the center of the rock blocks rather than on the center of the fractures as in a Poisson structural model. From the centers, the rock blocks are "grown" outward at variable rates. Growth stops at locations where two blocks meet. The resulting boundaries between the blocks define fracture surfaces.

Although colonnade structural models are appropriate for two-dimensional modeling, they are not appropriate for this research. These fractures systems lack the flow dominating fractures that are considered essential to the dual permeability concept. As Khaleel [1989] demonstrates, these fractures systems are amenable to representation using continuum approaches at large scales.

2.5 DISCUSSION OF FRACTURE SYSTEM CHARACTERIZATION

If viewed as a representation of fractured rock as a whole, much of the data and interpretations of fracture systems are contradictory. There is undoubtedly no one set of either statistical parameters or structural models that best represent fracture systems. Fracture networks are most likely a combination of randomness and regularity that can not be completely captured by either a statistical description nor a fixed set of structural model rules. There seems to be a trend toward more sophisticated structural models that are attempting to combine these two features [e.g. the hierarchical model of Lee et al., 1990]. It may be that structural model development has entered a stage of rapid proposal, investigation, rejection and replacement with a more advanced proposal. This model development requires increasingly more sophisticated fracture characterization surveys.

While considerable progress has been made in understanding the hydraulic behavior of individual fractures in the past ten years, the basic models are still in a state of flux. It is still not clear how to relate aperture and fracture density measurements from boreholes and surface surveys to undisturbed rock. To
Figure 2.6: Colonnade structural model from Khaleel [1989].
make this connection, a greater understanding of the affects of stress variation and of fracture shapes is needed. Of particular importance is the need to understand the nature of rough walled fractures and the affects of aperture variations on both flow and transport.

Four critical problems need to be solved in order to make field-scale, discrete fracture models an effective tool for assessing contaminant transport in fractured media. These problems are: 1) the lack of field-scale three-dimensional fracture network flow and transport models, 2) the uncertainty in extrapolating borehole and surface measurements to undisturbed rock, 3) the primitive state of single fracture flow and transport models, and 4) the lack of understanding of influence of connectivity and heterogeneity at field-scales. The relative importance of these issues to the hydraulic behavior of fractured rock at the field-scale is not known. It may be that one or more of these problems will prevent the discrete network approach to modelling fractured media from ever becoming an effective tool at the field-scale. The current limitations of discrete fracture models prevent us from resolving these issues.

The emphasis of this thesis focuses on the fourth topic. The other three areas of uncertainty are addressed in the following manner: 1) fracture systems are approximated as two-dimensional, 2) a fracture system characterization of a specific site is not used - instead statistical characterizations and structural models are used in a generic sense to emulate the way fracture system information is incorporated into discrete fracture models, and 3) a constant aperture model is used to model individual fractures, isolating the affects of network scale heterogeneity from the effects of single fracture heterogeneity. Hopefully, the methods developed to incorporate network scale heterogeneity in two dimensions will provide insight to incorporating network scale heterogeneity in three-dimensional models. These methods are not likely to be dependent on the assumption of constant aperture fractures.
CHAPTER 3

FRACTURE NETWORK GENERATION

AND FLOW AND TRANSPORT REPRESENTATION

IN A DISCRETE FRACTURE MODEL

3.1 INTRODUCTION

Details of the discrete fracture model developed for this thesis are presented in this chapter. The discrete fracture model provides flow and transport simulations that are used to assess the performance of the dual permeability model. Both the discrete fracture and dual permeability models are based on the same assumptions describing the hydraulic properties of fracture networks. Both models share a common algorithm to generate fracture realizations. This chapter begins with a description of this procedure. The mathematical formulations of the simulated hydraulic processes are presented next. With the basic groundwork common to both of these models described, the discrete fracture model is briefly described. The dual permeability model is described in Chapter 4.

3.2 DEFINITION OF A FRACTURE NETWORK

Fracture networks are approximated using a two-dimensional representation. The following assumptions and approximations describe how fractures are represented:

- Fractures are represented as straight lines of finite length.

- The aperture of each fracture is assumed constant and with no infilling, resulting in a constant transmissivity along its length.
• Fractures that overlap are assumed to be interconnected with no change in the aperture of either fracture.

• Fractures are assumed to extend equally and symmetrically out of the plane of simulation.

• The host matrix is treated as non-porous and impermeable.

The fractures for a single realization are generated using a selected structural model from specified statistical distributions of the characteristics of individual fractures. The fracture system description is assumed to be constant within the domain, with no dependence on depth or other varying stress conditions. The width, length, orientation and center of each fracture are generated stochastically using these probability distributions. These properties may be modified based on the rules of the selected structural model. Additional fractures, or possibly the entire network, may be specified explicitly.

3.3 FRACTURE NETWORK GENERATION

The procedure for creating a fracture network realization is quite similar to others described in the literature [e.g. Long et al., 1982]. There are some differences in the way fractures near the edges of the modeled region are treated, but these differences are fairly minor. A flow chart of the fracture generation procedure is presented as Figure A.2 in the appendices. Four of the structural models that were described in Chapter 2 have been implemented. These are the Poisson, truncated Poisson, war zone, and Levi-Lee models. The Poisson, truncated Poisson, and war zone models form a sequence of models, in that each contains the previous model in the sequence as a sub-model.

The generation of a fracture network realization has two components: fracture creation and fracture network reduction. Fracture creation is the positioning and establishment of the physical properties of individual fractures within the fracture generation region. Fracture reduction is the elimination of
fractures, or segments of fractures, that cannot contribute to flow through the domain because they lack connections to other fractures. The reduced fracture network is subsequently referred to as the hydraulically active network because the fractures of this network are all connected to the regional flow regime.

An attempt has been made to implement the discrete fracture and dual permeability models in such a way that the data inputs are statistical parameters that could be estimated from a rock body. The data inputs are:

- the number of fracture sets,
- scan-line density of each set,
- fracture length distribution of each set,
- fracture orientation distribution of each set,
- fracture aperture distribution of each set, and
- for structural models that incorporate fracture truncation, dependence of sets on previous sets, and the fracture intersection truncation probability for fractures of dependent sets on the fracture faces of earlier sets

3.4 Implementing the Structural Models

3.4.1 Poisson Model

The Poisson structural model is based on the assumption that the density of fractures is constant through the domain and that the position of any one fracture is independent of all other fractures. Each fracture in the Poisson model is generated independently. The number of fractures that are generated for each set is controlled by the scan-line density, the size of the model domain, the orientation distribution, and the length of the fractures for that set. Length enters the determination of the number of fractures that
are generated for each realization in two ways; first in the conversion of scan-line density to the number of fractures per unit area, and second in determining the distance beyond the flow domain within which fractures are generated.

Scan line density is used as a measure of fracture density for each fracture set. If the fractures of a set are all parallel, then the scan line density for the fracture set is

\[ \lambda_s = \rho \bar{l} \]  \hspace{1cm} (3.1)

where \( \rho \) is the density of fracture centers and \( \bar{l} \) is the average length of the fractures. If the fractures vary about the mean orientation, then Equation 3.1 must be adjusted. The correction, from Terzaghi [1965], is

\[ \lambda_s = \rho \bar{l} E \]  \hspace{1cm} (3.2)

where

\[ E = \int_0^{2\pi} \cos(\theta) g(\theta) d\theta \]  \hspace{1cm} (3.3)

\( \theta \) is the deviation of the fracture orientation from the mean orientation, and \( g(\theta) \) is the probability density function of fracture orientations. \( E \) has been included as a look-up table in the fracture generation routine as a function of the standard deviation of orientation, \( \sigma_\theta \), assuming a normal distribution of orientation about the mean. The number of fractures generated for each set is given by
\[ n_f = \frac{\lambda}{kE(\sigma_0)} W_x W_z \]  

where \( W_x \) and \( W_z \) are the dimensions of the fracture generation region. The fracture generation region extends in each direction beyond the intended model domain by \((2+\alpha)\max(\bar{t}_i)\), where \( i \) indicates the fracture set. This extension incorporates most of the fractures that may be centered outside the flow domain yet protrude into it. It is impossible to include all fractures that may protrude into the model domain because of the finite probability of fractures larger than the largest specified generation region. The parameter \( \alpha \) is used to avoid an artificial increase in fracture density along the boundaries of the domain. The use of \( \alpha \) will be explained in a later paragraph.

The generation of each fracture proceeds by defining the fracture midpoint from a uniform distribution of midpoints over the fracture generation region. Fracture length, aperture and orientation are then independently selected from specified distributions. The fracture length may have a negative exponential distribution, be log-normal, or be fixed as a constant. The aperture may be selected from normal, log-normal, or negative exponential distributions. Orientation may have a uniform distribution or be sampled from a normal distribution. Unless stated otherwise, the distributions used for this study are; a negative exponential length distribution, log-normal aperture distribution and normal orientation distribution.

The fracture network of Figure 3.1 was generated using a Poisson structural model. The two fracture sets of Figure 3.1 were generated using the parameter definitions of Table 3.1.

3.3.2 TRUNCATED POISSON MODEL

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Figure 3.1: Fracture network generated from a Poisson structural model. The parameter definitions for the two fractures sets are given in Table 3.1.
The truncated Poisson structural model adds fracture truncation to the Poisson structural model. Younger fractures may stop growing when they intersect the face of pre-existing fractures. The tendency for more recent fractures to stop growing when a preexisting fracture is encountered can be measured in terms of a probability of truncation. Older fractures never truncate on the faces of younger fractures; an observation that is often used to establish the relative ages of fractures. The age relationship is incorporated in the truncated Poisson model by specifying truncation dependencies.

<table>
<thead>
<tr>
<th></th>
<th>Number of fracture sets</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fracture set 1</strong></td>
<td></td>
</tr>
<tr>
<td>1.5 fr/m</td>
<td>Fracture density</td>
</tr>
<tr>
<td>2. m</td>
<td>Mean length</td>
</tr>
<tr>
<td>6x10(^{-5}) m</td>
<td>Mean aperture</td>
</tr>
<tr>
<td>0.</td>
<td>Standard deviation of log(_{10}) aperture</td>
</tr>
<tr>
<td>90°</td>
<td>Mean orientation</td>
</tr>
<tr>
<td>10°</td>
<td>Standard deviation of orientation</td>
</tr>
<tr>
<td><strong>Fracture set 2</strong></td>
<td></td>
</tr>
<tr>
<td>1.5 fr/m</td>
<td>Fracture density</td>
</tr>
<tr>
<td>2. m</td>
<td>Mean length</td>
</tr>
<tr>
<td>6x10(^{-5}) m</td>
<td>Mean aperture</td>
</tr>
<tr>
<td>0.</td>
<td>Standard deviation of log(_{10}) aperture</td>
</tr>
<tr>
<td>0°</td>
<td>Mean orientation</td>
</tr>
<tr>
<td>10°</td>
<td>Standard deviation of orientation</td>
</tr>
</tbody>
</table>

Table 3.1: Fracture system definition for the fracture network of Figure 3.1.
In the structural model, fractures that belong to sets that are dependent on pre-existing fractures are truncated at intersections with pre-existing fractures with probability $T_{\text{r}_{ij}}$, the truncation probability of the fractures of set $i$ on the faces of the fractures of set $j$. The procedure proceeds as follows. First, the fractures of the entire fracture network are generated independently. The lengths of the fractures to be truncated are increased to account for the anticipated truncation of these fractures. Then, for each fracture of the truncated sets, the truncation procedure starts at the midpoint of the fracture and works toward the two ends, testing for truncation at each intersection. A fracture that is truncated at an intersection ends at that intersection with all intersections with other fractures beyond the truncation being eliminated. It can be shown that the fraction of the length lost to fractures of set $j$ because of the intersection of these fractures with the fractures of set $i$ can be written as:

$$F_{ij} = \frac{\lambda_{s_i} T_{\text{r}_{ij}}}{4} \int_{0}^{\pi} \int_{0}^{\pi} \sin(|\theta_i - \theta_j|) f(\theta_i) f(\theta_j) d\theta_i d\theta_j$$  \hspace{1cm} (3.5)$$

where $f(\theta)$ is the probability density function of fracture orientation. $F_{ij}$ is approximated in the truncation routine by

$$F_{ij} = \frac{\lambda_{s_i} T_{\text{r}_{ij}}}{4} \sin(|\bar{\theta}_i - \bar{\theta}_j|)$$  \hspace{1cm} (3.6)$$

unless the two mean angles are equal. The total fraction of length lost to truncation is

$$F_j = \sum_i F_{ij}$$  \hspace{1cm} (3.7)$$

This truncation loss is a fraction of the fracture length that is initially generated. The fracture length prior to truncation, $\bar{e}_j$, must be
for the post truncated mean fracture length, $\bar{l}$, and scan-line density to be consistent with the specifications for the set.

Figure 3.2 presents a truncated Poisson fracture network both prior to truncation, panel a, and after truncation, panel b. The second set is truncated on the first set with a truncation probability of 0.5. The parameters for the fracture network in Figure 3.2b are the same as for Figure 3.1. The pre-truncation average length of the second set has been increased by a factor of four to obtain the desired post truncation length and scan line density resulting in a different appearance of the pre-truncated network in Figure 3.2a from that of the network in Figure 3.1.

### 3.3.3 War Zone Model

War zones are areas of more intense fracturing than in the rest of the fracture network. The war zone model was proposed by Geier et al. [1989] for three dimensional fracture networks. A two dimensional version of the war zone model has been created for this thesis for two reasons; 1) the war zone model has two scales of fracturing, 2) the well defined war zones have very different hydraulic characteristics than the region surrounding them. Both of these features fit well into the structure of the dual permeability model described in Chapter 4. It has been recognized that most fractured rock has fractures on many scales and the interaction of fractures at different scales may be an important aspect of fractured rock hydrology; an aspect that is missing in the Poisson model. An example of a war zone fracture network has been presented in Figure 2.3.

War zones exist between pairs of closely spaced sub-parallel fractures as depicted in Figure 3.3a, modified from Geier et al. [1992]. Once the war zones are identified, additional fractures are then created.
Figure 3.2: Fracture network generated from truncated Poisson structural model. a) The network prior to truncation. b) The truncated fracture network. The parameter definitions are given in Table 3.1. The truncation probability of the sub-horizontal set is 0.5.
Figure 3.3: War zone definitions. a) Identification of war zones between close, subparallel, overlapping fractures (modified from Geier et al. [1989]). b) Definition of war zone characteristic parameters $A_1$ and $A_2$ are the fracture lengths, $A_w$ is the overlap between fractures, $A_c$ is the average distance between fractures.
within the zones. The fracture pairs defining war zones are selected based on the basis of three criteria: "parallelness", "largeness", and "closeness". Parallelness \( W_p \) is measured as

\[
W_p = \cos^2(\alpha_w)
\]

(3.9)

where \( \alpha_w \) is the angle between the two fractures. The largeness \( W_L \) of a war zone is a measure of the fractional overlap between the fractures:

\[
W_L = \frac{2A_w}{A_1 + A_2}
\]

(3.10)

where:

\( A_w \) is the projected overlap between the two fractures

\( A_1 \) is the projection of the first fracture

\( A_2 \) is the projection of the second fracture.

The projection is onto a line that bisects the difference in angle between the two fractures as shown in Figure 3.3b. The closeness \( W_c \) is:

\[
W_c = \max \left( 1 - \frac{2A_c}{A_w}, 0 \right)
\]

(3.11)

where \( A_c \) is the average distance between the overlapping portions of the two fractures. A region is classified as a war zone if

\[
W_p Z_p + W_L Z_L + W_c Z_c > 3
\]

(3.12)

where \( Z_p, Z_L, \) and \( Z_c \) are empirically defined parameters based on investigation of a fracture system. Geier et al. suggested that 1.5 was adequate for each of these parameters for the Stripa SCV Project site. These values have been used for all of the war zone structural models used in this thesis.
War zones are areas of increased fracturing. The dependent fracture sets are first generated through the entire fracture generation region using the input scan line density as in the Poisson model. Then, more fractures are added to each of the war zones. The resulting scan line density within the war zone becomes

\[ \lambda_{wn} = k_{wn} \lambda_s \]  

(3.13)

where \( k_{wn} \) is the war zone intensity factor. An intensity factor of 4.0 has been used for all war zones. Fractures that intersect the bounding fractures of a war zone are truncated at the intersection. These new fractures may retain the orientation distribution specified in the input or they can be normally distributed about the orientation perpendicular to the projection line.

3.3.4 LEVY-LEE MODEL

A fracture generation algorithm has been developed based on the Levy-Lee model proposed by Geier et al. [1989]. As with the war zone model, the Levy-Lee model is a useful tool to investigate issues of multiple scales of fracturing. The Levy-Lee flight process is used to generate a fracture network that has a continuum of scales, both in terms of fracture length and in the spacing between fractures.

The report of Geier et al. is not a formally published document and describes the model in conceptual rather than detailed form. Therefore, the implementation described in this section may deviate from the intentions of the original creators. As described by Geier et al., the spatial distribution of fractures generated using the Levy-Lee model is fractal. Fracture lengths are proportional to the spacing distance, resulting in a power law distribution of fracture length. A power law distribution less than -1, which is recommended, must be truncated at some minimum fracture spacing. The resulting minimum length determines the scan-line density of the generated fractures.
The Levy-lee flight process results in a random walk in the locations of the generated fractures. Each new fracture center is located a random distance and random orientation away from the previous fracture. The distance is determined by dividing a minimum spacing distance by a uniform random number that has been raised to the power law exponent. The displacement orientation is uniformly distributed between 0° and 360°. When the locations move outside of the fracture generation region, the fracture is relocated randomly within the region; resulting in a series of over-printed random walks. Fractures are generated until the specified scan-line density is reached. In the realizations that are used in this study, the fractures are relocated 30 to 40 times. The Levy-Lee structural model generated highly clustered fracture systems with power law length distributions but the spacing distributions are probably consistent with a power law distribution over a limited range of scale. The actual spacing distributions have not been determined.

Geier et al. proposed a correlation between fracture spacing and orientation. A linear relation between the standard deviation of fracture orientation and the spacing distance is used for this study. This relationship results in sub-parallel long fractures with little orientation variability. Smaller fractures have much greater variability in orientation and are highly clustered. Clustering results in concentrations of highly connected fractures embedded in a larger scale framework.

A fracture network generated using the Levi-Lee model has been presented earlier in Figure 2.5. The fracture sets have the same orientation and fracture scan-line densities as those used for the Poisson model of Figures 3.1. The mean length of the fractures in Figure 2.5 is half of that in Figure 3.1. The minimum fracture length of the Levy-Lee model is proportional to the specified mean fracture length. A fractal dimension of 1.3 and a minimum spacing distance of 0.25m were used to generate the network.
3.4 Fracture Network Reduction

Once generated, fractures go through a series of screening processes that reduce the number of fracture segments that define the fracture networks of the flow and transport models. Fracture segments that are eliminated cannot have fluid flowing through them and thus are not important hydraulically. Eliminating these fracture segments makes the solution of the hydraulic head distribution easier to determine. A flow chart of the fracture reduction process is included as Figure A.3 in the appendices.

Figure 3.4 is used to aid the explanation of the fracture reduction procedure. Figure 3.4 shows three rectangular regions and a number of fractures. The outer rectangle defines the fracture generation region that is used to place fracture centers. Fracture centers are shown as dots in the figure. The next largest rectangle defines the region within which fractures are needed to provide a hydraulic connection between the fractures of the model domain and the surrounding fracture network. This rectangle is referred to below as the influence boundary. The model domain, or flow domain, is drawn as the inner rectangle.

The first screening process is to discard fractures that are not likely to be needed to provide a hydraulic connection between the fractures of the model domain and the surrounding fracture network. The portions of fractures that are outside the influence boundary, a distance $a*\max(q)$, from the model domain are discarded. These fractures are shown as dashed lines in Figure 3.4.

The next stage in the reduction process is to find all the intersections of each fracture with other fractures. Fractures that do not intersect other fractures or which have only one intersection cannot be a pathway for flow. These fractures are discarded from further consideration. In addition, fractures that cannot be connected through two distinct pathways to the outer edge of the zone of influence cannot have fluid flowing in them and are discarded. The portions of each fracture that are not between fracture intersections and/or intersections with the domain boundary are eliminated by shortening the fractures.
Figure 3.4: Region boundaries used in the fracture reduction procedure. The fracture generation region defines the outer limit of fracture center placement. The boundary influence region defines the limit of the region where fractures connect to form a hydraulically active fracture network. The model domain identifies the extent of the flow domain.
Fracture segments removed at this stage are shown as both shaded thick lines and thin lines in Figure 3.4.

The procedure described in the two preceding paragraphs is repeated using the flow domain boundary rather than the outer edge of the zone of influence. The fractures that remain after the removal and truncation process make up the hydraulically active network, the solid thick lines in the figure. The hydraulically active network contains all of the fracture segments that are flow pathways. If the screening process were perfect then the remaining fractures would be limited to the active network. It is not. Some fracture segments are retained that are not flow pathways, but are hydraulically connected to the active network as is the group of fractures labelled A in Figure 3.4. These fracture segments slow the flow solution but do not influence the results.

The use and rationale for the first pass through the screening process based on truncation at the influence boundary requires some explanation. In Figure 3.4 the shaded thick fractures would be retained if the parameter $\alpha$ were zero. These fractures form pathways that cross the domain boundary but do not intersect the influence boundary. For sparse fractures systems, setting $\alpha$ to zero results in an increase in fracture density and hence hydraulic conductivity near the edges of the domain. In the study described in Chapter 5, the average hydraulic conductivity of the domain was sensitive to this increase in permeability; especially when fractures such as the otherwise isolated fracture in the lower left corner of Figure 3.4, labelled B, were included. For dense fracture systems, changing the influence boundary has little effect. The reduced fracture network of the Poisson fracture network presented in Figure 3.1 is shown in Figure 3.5.

3.6 Modeling of Fluid Flow and Solute Transport

3.6.1 Fluid Flow

The fluid flow model is based on the following set of assumptions.

1) parallel sided, unfilled fractures
Figure 3.5: Reduced fracture network of the network shown in Figure 3.1.
2) non-porous, impermeable host rock

3) fully saturated conditions

4) steady state, laminar flow

5) Fluid density, \( \rho \), and viscosity, \( \mu \), are assumed to be insensitive to any pressure, temperature or solute concentration variations within the flow domain. The values used for all simulations are a fluid density of 998.0 kg/m\(^3\) and a viscosity of 0.001 kg/m-s.

The assumption of laminar flow is supported by investigations of the Reynolds number representation of flow within the fractures. The Reynolds number is a dimensionless measure of the ratio of inertial forces to viscous forces. The Reynolds number for a fracture is given by

\[
Re = \frac{\rho V b}{\mu}
\]  

(3.14)

where \( V \) is the average fluid velocity, \( b \) is the fracture aperture, \( \rho \) is the fluid density, and \( \mu \) is the viscosity. In circular pipes, the flow is laminar if \( Re \) is less than \( 2 \times 10^3 \) [Sabersky et al., 1971]. A similar threshold should apply to fractures. The fracture network realization of the fracture system defined by Table 3.1 and shown in Figure 3.1 was used to estimate typical Reynolds numbers for the simulations that are performed during the course of this research. Figure 3.6a shows the distribution of Reynolds numbers for the flow solution obtained by applying a constant hydraulic head gradient of 0.01 m/m along the boundaries of the fracture network. The maximum Reynolds number is \( 10^3 \) indicating the assumption of laminar flow is well supported.
Figure 3.6: Reynolds and Péclet number distributions of flow and transport processes in single fractures. Distributions are from solution to the fracture network shown in 1. The hydraulic head gradient was 0.01. a) Reynolds number. b) Péclet numbers calculated for longitudinal diffusion. c) Weighted Péclet numbers calculated for transverse diffusion as defined by Equation 3.23. d) Longitudinal dispersion coefficients. e) Péclet numbers calculated for fracture intersections.
Under laminar flow conditions the average flow within a fracture is described by the cubic law for parallel-sided fractures [Snow, 1965]:

\[ Q = \frac{-\rho gb^3}{12\mu} \frac{\partial h}{\partial L} \]  

(3.15)

where

- \( Q \) = volumetric flow per unit thickness of the model,
- \( g \) = gravitational constant,
- \( \frac{\partial h}{\partial L} \) = hydraulic head gradient along the fracture.

For a fracture with a constant aperture the hydraulic head gradient is constant between fracture intersections. Equation 3.15 can be rewritten

\[ Q_{ij} = \frac{-\rho gb^3}{12\mu} \frac{\Delta h_{ij}}{\Delta L_{ij}} \]  

(3.16)

where \( Q_{ij} \) is the flow from intersection \( i \) to intersection \( j \), \( \Delta h_{ij} \) is the hydraulic head difference between intersections \( i \) and \( j \) and \( \Delta L_{ij} \) is the distance between them. An equivalent expression is

\[ Q_{ij} = -T_{ij} \frac{\Delta h_{ij}}{\Delta L_{ij}} \]  

(3.17)

where \( T_{ij} \) is the transmissivity of the fracture between the two intersections.

Equation 3.17 is conceptually more satisfying than Equation 3.16. The use of Equation 3.17 allows the assumption of constant aperture to be relaxed. That is

\[ T_{ij} = \frac{\rho g}{12 \mu} b^5 \]  

(3.18)
The constant fracture aperture, \( b \), in Equation 3.16 is replaced by an effective hydraulic aperture, \( b_h \), that yields the appropriate \( T \) over the length of the fracture segment between intersections. The effective hydraulic aperture is defined as [Silliman, 1989]

\[
b_h = \left[ \frac{1}{\Delta L_y} \int_0^L \frac{1}{b^3(q)} \, dq \right]^{-1/3}
\]

(3.19)

where \( t \) is a position along the fracture segment. To be fully consistent with the notion of the fracture aperture representing an effective hydraulic aperture, \( T \) need not be the same for each segment fracture, unlike the constant aperture assumption for each fracture in this model.

By symmetry, no flow occurs into or out of the plane of the simulation. There are also no sources or sinks for fluid such as wells. At each fracture intersection the fluid mass must be conserved. The flow into each intersection must be equal to the flow out of the intersection. The mass balance equation for each intersection is

\[
0 = \sum_i -\frac{T_{ij}}{\Delta L_y} (h_i - h_j)
\]

(3.20)

In equation 3.20, the index \( j \) is limited to interior intersections but index \( i \) can also include fracture intersections with the fracture network boundary. The set of equations 3.20 for all intersections within the flow domain can be written in matrix notation as

\[
A \ddot{x} + \ddot{b} = 0
\]

(3.21)

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where $\tilde{A}$ is a symmetric matrix with diagonal coefficients $\sum_i \frac{T_{ij}}{\Delta L_{ij}}$ and off diagonal coefficients $\frac{-T_{ij}}{\Delta L_{ij}}$.

$x$ is a vector of hydraulic heads for each intersection, and $\bar{b}$ is the vector containing boundary conditions for each intersection.

Intersections that are directly connected to the domain boundary have boundary conditions defined by

$$b_i = -T_{in} \frac{h_n}{\Delta L_{in}} \quad (3.22)$$

where $T_{in}$ is the transmissivity of the fracture segment connecting intersection $i$ to the boundary location $n$ and $h_n$ is the hydraulic head specified at $n$. The other components of $\bar{b}$ are all zero.

The $x$ that satisfies Equation 3.21 defines the hydraulic head at all intersections of the fracture network and through Equation 3.17 defines the flow in each fracture segment. In the simplest terms, a central mission of the research has been to investigate methods of finding an approximate solution to Equation 3.21 that avoids the direct use of matrix $\tilde{A}$, in hopes of increasing the size of fracture networks that can be studied.

3.6.2 SOLUTE TRANSPORT

At the level of individual fractures, two processes interact to effect the movement of solute. One process, advection, is driven by flow within the fractures. The other process, diffusion, is driven by solute concentration gradients. It is assumed that the solute concentration is always small so that the perturbation of fluid density by the solute has no influence on the advective process.
The character of solute transport in a fracture network is strongly dependent on the relative importance of advection compared to diffusion at the scale of individual fractures. For example, if advection dominates the flow within intersections of fractures then the solute from one inlet fracture segment will not mix into the fluid of another inlet fracture. If diffusion dominates then the fluids do mix across streamlines in an intersection and solute may enter fractures that disperse the solute to a much greater extent.

Diffusion may be an important process at the scale of individual fractures because of the sensitivity of network scale behavior to these local conditions. The relative importance of advection compared to diffusion is typically measured in terms of a dimensionless Péclet number. The Péclet number can be defined by

\[ Pe = \frac{VL}{D} \]  

(3.23)

where \( V \) is the fluid velocity, \( L \) is a representative distance, and \( D \) is the solute diffusion coefficient. A representative diffusion coefficient of \( 10^{-9} \text{ m}^2/\text{s} \) is used in the Péclet number calculations. The Péclet number can be thought of as the ratio of the time needed for solute to diffuse a distance \( L \), over the time needed to be moved by advection across that distance. If the Péclet number is much greater than one, then diffusion can be neglected in modeling transport. I am not aware of an accepted value of the Péclet number that would indicate negligible diffusion. The number is probably between 100 and 1000. If the Péclet number is much less than one, diffusion dominates and advection can be ignored.

The advection distance may be different than the diffusive distance in some situations. In this situation, diffusion will dominate if the Péclet number is much greater than the ratio of the diffusive distance to the advective distance. To retain one as the indication of transition region between advection and diffusion, the Péclet number can be weighted by the inverse of the distance ratio.
For transport along a fracture segment, $V$ is taken as the average velocity within the fracture $(Q_i / b_i)$, and $L$ is the length of the fracture segment, $L_{ip}$. Figure 3.6b presents the Péclet numbers for transport along the fractures for the network of Figure 3.1. Most of the Péclet numbers are greater than 1, but roughly half of the Péclet numbers are less than 100 indicating that longitudinal diffusion along a fracture influences solute transport in many fractures.

The fluid velocity within a fracture is not uniform. The fluid near the center of the fracture moves faster than fluid near the fracture walls. The distribution of the velocity introduces dispersion into the movement of solute through fractures. The dispersion is influenced by the interaction of diffusion across the streamlines within the fracture as the solute moves along the fracture segment. The relative importance of diffusion across the width of the fracture relative to advection along the fracture is characterized using a weighted Péclet number defined by

$$ Pe_b = \frac{Vb}{b D} L \quad (3.24) $$

If the weighted Péclet number is less than 8 then the transverse distribution of tracer is nearly constant across the fracture [Hull et al., 1987]. The weighted Péclet number calculated for the network of Figure 3.1 (shown in Figure 3.6c) provides assurance that transverse diffusion homogenizes the tracer across the width of the fracture. Under these conditions dispersion is Fickian and can be described by a longitudinal dispersion coefficient defined as [Hull et al., 1987]

$$ D_l = \frac{(Vb)^2}{210D} \quad (3.25) $$

Figure 3.6d shows the longitudinal dispersion coefficient calculated for the same network as the other figures. The figure reveals that this longitudinal dispersion coefficient is much less than molecular diffusion under these conditions and can be neglected.
In the discrete fracture and dual permeability models, both diffusion and longitudinal dispersion are assumed to be negligible in all fractures. It is not clear to what extent that neglecting diffusion influences temporal dispersion at the scale of the fracture network. By neglecting solute longitudinal dispersion in individual fractures, temporal dispersion caused by the variation of fracture characteristics and the way fractures are connected can be investigated more clearly.

In a parallel sided fracture, the time to move between intersections is given by

\[ t_y = \frac{b \Delta L_y}{Q_y} \]  

(3.26)

As mentioned earlier, the assumption of parallel sided fractures is not supported by fracture investigations. This assumption requires some discussion because it is an important limitation of the models used for this thesis. In a rough fracture, the time to move between intersections is given by

\[ \Delta t_y = \frac{1}{Q_y} \int_0^{\Delta L_y} b(t) dt \]  

(3.27)

After substituting Equation 3.16 for Q, an effective aperture for transport could written as [Silliman, 1989]

\[ b_T = \left[ \frac{1}{b_h^3 \Delta L_y} \int_0^{\Delta L_y} b(t) dt \right]^{-1/2} \]  

(3.28)

This effective aperture is not equal to the effective hydraulic aperture, \( b_h \), calculated in Equation 3.19. To represent the effective aperture of a rough walled fracture, the representation should be different for the flow process and the transport process. The direct linkage of the flow and transport processes through

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the assumption of constant aperture fractures may have an influence on the predictions of network scale dispersion. For a more thorough discussion of effective apertures, see Silliman [1989] and Tsang [1992].

Fracture aperture roughness also introduces dispersion into the transport process [Moreno et al., 1985; Tsang and Tsang, 1987]. Dispersion arises from the tortuous paths solute takes through a fracture plane. No attempt to model dispersion within a single fracture is made here. Dispersion could potentially be introduced through the use of statistical transport times through a single fracture in a manner similar to the three dimensional flow and transport model described by Nordqvist et al. [1992]. These authors first created a library of possible transport times by stochastically generating a large number of two dimensional single fractures. Transport times through individual fractures in the fracture network model were determined by sampling from the library. The relative importance of dispersion in a rough-walled fracture compared to dispersion from fracture network heterogeneity is presently unknown.

Another important source of dispersion in fracture networks arises from mixing at fracture intersections. Diffusion across stream lines in fracture intersections causes solute to enter fractures that it otherwise would not enter if it did not diffuse. For fracture intersections a Péclet number can be defined as

\[ \text{Pe} = \frac{r V}{D} \]  

where \( r \) is defined as

\[ r = \frac{\sqrt{b_1^2 + b_2^2}}{2} \]  

and \( b_1 \) and \( b_2 \) are the apertures of the intersecting fractures. The distribution of fracture intersection Péclet numbers is shown in Figure 3.6e. These Péclet numbers indicate that transport in nearly all intersections of this simulation are diffusion dominated. A complete mixing model of transport in diffusion dominated fractures assumes that the solute is homogenized within the fracture intersection but leaves the intersection
into the downstream fractures in proportion to fluid advection. Berkowitz et al. [1993] have shown that a condition of complete mixing will never be reached because diffusion causes much of the solute to enter the downstream fractures before the solute reaches a state of homogenization. The resulting mixing is less than complete. To model this transport behavior would require a diffusive transport model that incorporated the exact geometry of the fracture intersections. No such model exists at present and the development of such a model for this work is not justified by the anticipated gain in accuracy of modeling network dispersion.

The transport model is based on a particle tracking algorithm. It is similar to the particle tracking model described by Schwartz et al. [1983]. It is assumed that the aggregate behavior of a large number of particles mimics the advection of a dilute, non-reactive solute. Particles are assumed to move through fractures at the average fluid velocity within the fractures (plug flow). There are no explicit diffusion or dispersion terms in the particle tracking algorithm. Diffusion of solute into the host matrix and into non-flowing fractures is assumed to be negligible. The complete mixing model is used at fracture intersections.

Figure 3.7 is used to explain the salient features of the transport algorithm. In Figure 3.7, four fracture intersections are shown along with the connecting fracture segments. Particles that pass through intersection 1 may be transported though the fracture segments that carry flow away from intersection 1. The fracture segment that the particle enters is chosen randomly (the complete mixing assumption) with a probability equal to the ratio of the flow through the fracture segment to the total flow through the intersection. In Figure 3.7, a particle that enters intersection 1 at time $t$ from fracture segment 1 has a 70 percent chance of traveling to intersection 4 and a 30 percent chance of moving towards intersection 2. A particle traveling from intersection 1 to intersection 2, will arrive at intersection 2 at time $t + \Delta t_{1,2}$. Where $\Delta t_{1,2}$ is
Figure 3.7: Detail of fracture segments and intersections. The parallel lines represent fracture segments. The dots represent fracture intersections, where the solute is completely mixed. The Q’s are relative flow rates within the segments.
Here $Q_{1,2}$ is the flow rate determined from Equation 3.16, $b_{1,2}$ is the aperture, and $\Delta L_{1,2}$ is the length of the fracture segment respectively.

The source of solute is based on a pulse injection. All particles are assumed to arrive at the upstream edge of the fracture domain at the same time, $t=0$. Transport for more complex situations can be determined from the breakthrough histories of the pulse injection by convolving the breakthrough histories with the desired concentrations at the upstream boundaries as a function of time.

Particles are introduced into the domain either from a point source or a line source along an upstream boundary. A uniform concentration is assumed along line sources. Particles start at each fracture intersecting a line source with probabilities that are equal to the fraction of total flow across the line source that is carried by the intersecting fracture segment.

A single particle is moved at a time in the algorithm. This one particle is moved exclusively until it reaches another edge of the domain boundary. The positions of the particles at specific times may be recorded to provide the data for particle position plots and dispersion calculations. The location and time of arrival at a downstream boundary may be recorded, providing particle arrival histories at specific locations and spacial distributions of arrival along the boundaries.

3.7 BOUNDARY CONDITIONS

The models developed during this research are all limited to either specified head boundary conditions or a combination of specified head and impermeable boundary conditions. The head specified along permeable boundaries is consistent with a constant hydraulic head gradient along the boundaries of the flow domain. Figure 3.8 depicts the hydraulic head specified along each boundary for a gradient.
Figure 3.8: Application of a constant hydraulic head gradient along the boundaries of the flow domain.
oriented $30^\circ$ to boundary 1. The designations of the boundaries shown in the figure are used throughout this thesis. The boundary conditions used for these models becomes important in Chapter 5.

3.8 Program Discrete

The program Discrete was developed to provide a reference calculation that the dual permeability model could be tested against. The structure of Discrete is outlined in the flow chart labeled Figure 3.9. The basic structure, as with any of the models described in this thesis, is fracture network generation and reduction, solution for the hydraulic head distribution, and particle transport. The hydraulic head distribution is determined by solving Equation 3.21 using a conjugate gradient iterative solver for sparse matrices that uses incomplete Cholesky decomposition to precondition the $\tilde{A}$ matrix.
Program Discrete

Fracture Network Realization

Fracture Network Reduction

Determine Flow Equations

Solve Flow Problem

Determine Fracture Intersection Flow Splits and Fracture Residence Times

Particle Tracking Transport Simulation

Figure 3.9: Flow chart of reference program, Discrete. Procedures with an * in the upper right corner are expanded in the indicated figure of Appendix A.
CHAPTER 4

DUAL PERMEABILITY:

IMPLEMENTATION AND EVALUATION

4.1 INTRODUCTION

As discussed in Chapter 1, the dual permeability flow and transport model is based on the concept that in many cases a relatively small number of the fractures dominate the flow system. The critical assumption underpinning the dual permeability model is that these fractures should be treated in detail, whereas the majority of fractures in the network should be approximated, not neglected. The name dual permeability is intended to be evocative of this two-level treatment of the fracture system.

Two groupings of fractures are defined in the dual permeability model. Group 1 contains fractures that are anticipated to dominate flow through the fracture system. This group forms the primary fracture network. Group 2 contains all the other fractures in the network. Most group 1 fractures are identified in the model through a set of rules that link fracture characteristics with large flow rates. Additional fractures are added to the group to ensure that group 1 fractures form a connected network. Finally, a few more fractures may be added to group 1 to force the regions defined by these fractures to be within a maximum size constraint. Figure 4.1 demonstrates the subdivision of the fracture network. The entire network is shown in Figure 4.1a. Figure 4.1b is a schematic of the primary fracture network that has been identified for this network. Identifying a primary fracture network requires inferences about the features of a fracture network that influence flow dominance, including scale effects that are critical to understanding flow and transport in fractured media.

The regions enclosed by the primary fracture network are called network blocks. Figure 4.1c shows a network block that has been defined by the primary fracture network of Figure 4.1b. This block
Figure 4.1: Dual permeability model structure. a) The hydraulically connected fracture network. b) The primary fracture network. c) A network block consisting of internal fractures and the bounding primary fractures is labeled B in Panel b of 1 near the location (15,32).
is located near the point (15m, 30m) and is labeled B in Figure 4.1b. Network blocks define a sub-network of fracture segments that include both those primary fracture segments on the boundaries of the block and all other fracture segments located within the block boundaries.

Before discussing individual model components in detail, I will provide an overview of the method used to create a dual permeability model. Figure 4.2 is a flow chart that shows how division of the fracture network into primary fractures and network blocks is used to create a flow and transport model. Creating a realization of the fracture network and subsequent fracture network reduction is the first stage of forming a dual permeability model. The next stage is to find a primary fracture network and to define the internal sub-networks of all the network blocks. The effective hydraulic conductance of each of the network blocks is calculated and then combined with the transmissivity of the fractures of the primary fracture network to form an approximate set of flow equations for the entire fracture network. This set of equations is solved to provide an estimate of a coarse-scale hydraulic head distribution that is defined at each intersection of the primary fracture network. One advantage of solving for the hydraulic head distribution at this coarse-scale is that the number of equations is reduced from the number needed to describe the head distribution in the entire network. Using the hydraulic head estimates at the primary fracture intersections as boundary conditions, a detailed flow solution that provides hydraulic head estimates throughout the fracture network is calculated by treating each network block as an independent sub-domain. Thus, the flow through the entire network is determined but the calculations have been broken up so that the flow equations for the entire network (Equation 3.21) are not solved directly.

At this point in the modeling process, the flow solution could be used in conjunction with the transport model described in Chapter 3. Instead, the dual permeability structure is also used to create a solute transport model. Particle tracking is used to simulate transport in the dual permeability model. Unlike the particle tracking routine described in Chapter 3, particles are moved across the regions defined by the network blocks in single steps. These large steps are made possible by precalculating residence
Dual Permeability Program

Fracture Network \* A.2
Realization \* A.3

Primary Fracture Network Selection \* A.6

Determine Effective Conductance of Network Blocks \* A.7

Incorporate Network Blocks into Primary Network to Form Flow Model \* A.7

Solve Flow Model for Hydraulic Head Distribution

Find Network Block Residence Time Distributions \* A.8

Form Transport Model from Network Blocks and Primary Fractures \* A.9

Particle Tracking Transport Simulation \* A.11

Figure 4.2: Flow chart of the dual permeability model.

79
time distributions for all possible transport paths through the blocks. Residence time distributions link locations of inflow to locations of outflow of each network block. For each of these links, the probabilities of particle arrival at discrete times are determined from the flows calculated individually for each block.

Sections 4.2 through 4.4 provide a detailed description of the components of the dual permeability model. In some instances, the results of dual permeability simulations are presented to help explain certain elements of the model. In Section 4.5, the flow and transport behavior calculated using the dual permeability model is compared with that of Discrete. In addition, Section 4.5.4 presents an efficiency evaluation of the model in comparison to Discrete. In Section 4.6, dual permeability simulations are compared to Discrete simulations in which some of the less important fractures have been completely neglected. This comparison facilitates a discussion of differing modeling philosophies; one in which the less important fractures are approximated, and the other in which these fractures are neglected. A discussion of alternatives to the methods described in Sections 4.2 through 4.4 are presented in Section 4.7.

4.2 Selection of Primary Fractures

4.2.1 Primary Fracture Selection Criteria

Primary fractures are either those fractures that are anticipated to dominate local flow conditions; those that have been explicitly defined as important fractures through the program input; or those that are required to link the primary fracture network together creating network blocks. Flow dominance is a subjective term. A fracture is considered to dominate the local flow if the flow within the fracture is large in proportion to the average flow rates within fractures in the immediate vicinity of the fracture. Both fracture transmissivity and fracture connectivity influence the tendency for a fracture to dominate the flow regime in its vicinity.
The initial stage of the selection process for identifying primary fractures is based strictly on the geometric characteristics of individual fractures. The individual characteristics of fractures are: position, length, orientation, and aperture (transmissivity). Of these characteristics, the orientation of a fracture does not seem to be strongly related to flow dominance. The position of a fracture relative to the boundaries of the domain can be important, however the interaction of the boundary conditions with the behavior of the fracture network is an artifact of limited domain sizes and should not be used as the basis of selecting primary network fractures.

A set of seven criteria have been examined to determine an appropriate method to choose primary fractures. Each of these criteria weight the relative importance of transmissivity and length differently. The seven selection criteria are listed in Table 4.1. The criteria are used to rank the fractures in an ordered list. Using the ranking, a specified percentage of the total number of fractures are identified as primary fractures. This fraction is specified in the model definition but may be increased during the selection process if an adequate primary fracture network cannot be found using the specified fraction (see Section 4.2.2).

<table>
<thead>
<tr>
<th>Criteria Number</th>
<th>Criteria Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Aperture</td>
</tr>
<tr>
<td>2</td>
<td>Length</td>
</tr>
<tr>
<td>3</td>
<td>Aperture with minimum length</td>
</tr>
<tr>
<td>4</td>
<td>Aperture * length</td>
</tr>
<tr>
<td>5</td>
<td>Aperture² * length</td>
</tr>
<tr>
<td>6</td>
<td>Aperture³ * length</td>
</tr>
<tr>
<td>7</td>
<td>Aperture⁶ * length</td>
</tr>
</tbody>
</table>

Table 4.1: Primary fracture selection criteria.
The symbol * indicates multiplication.
The primary fractures must form a completely connected network. Each end of every primary fracture must have a flow pathway through other primary fractures that leads eventually to a domain boundary. Additional fractures are added to those selected strictly by individual geometric properties in order to provide the required interconnection. These fractures are selected strictly due to connectivity, without regard to transmissivity.

An additional factor is introduced into the selection process for primary fractures by a need to limit the size of the tables used to calculate and store the particle residence time distributions of the network blocks. Network blocks that have too many internal fracture intersections are subdivided using procedures described in detail in Section 4.2.2.

In Section 4.5.2, a suite of seven fracture systems are used to investigate aspects of the dual permeability model, including the merit of the various selection criteria. Results from the investigation of one of these fracture systems (fracture system 1 of Table 4.5) are summarized in Table 4.2. These results are included here to provide some perspective on issues that need to be considered in determining the primary fracture network. An example of a fracture network realization for this fracture system is shown later in Figure 4.10. A hydraulic head gradient parallel to the bottom boundary is imposed on the boundaries of the fracture network. Particles are introduced over a ten meter region centered at the middle of the left boundary and are transported to the downstream boundary.

The comparison is based on three measures; the total flow across the domain boundaries, the time of the first particle arrival at the downstream boundary of the fracture network, and the time of the 50th percentile particle arrival. Table 4.2 presents a comparison of the average deviation of the results of a dual permeability calculation from the Discrete. The last column of Table 4.2 lists the percentage of realizations where an acceptable primary fracture network could not be found using the selection criteria. One hundred realizations are used in this analysis.
<table>
<thead>
<tr>
<th>Criteria</th>
<th>Flow Error ( % )</th>
<th>Time Delay Initial ( % )</th>
<th>Time Delay Median ( % )</th>
<th>P.F.N. Failure ( % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-8.2</td>
<td>2.5</td>
<td>9.1</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>-9.9</td>
<td>1.8</td>
<td>8.9</td>
<td>3</td>
</tr>
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<td>3</td>
<td>-5.7</td>
<td>0.7</td>
<td>5.2</td>
<td>11</td>
</tr>
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<td>1.6</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>-4.5</td>
<td>1.0</td>
<td>4.0</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 4.2: Primary fracture selection criteria test. The results are based on one hundred realizations. The comparisons are calculated for individual fracture networks, that is the results do not include fracture network variability. Flow error is based on the total flow across the domain boundaries. The time delays of the dual permeability model are for initial solute arrival and median particle arrival. P.F.N Failures indicate the number of realizations where the primary fracture selection routine gave up trying find an acceptable network.

The table shows that there is reasonably close agreement between the two models and that the agreement is sensitive to the selection criteria. If the median arrival time is given emphasis in comparing the selection criteria, then criterion six, product of length and transmissivity, is the best selection criteria. The last column of the table reveals that an acceptable primary fracture network could not be identified for some of the fracture network realizations.

It is emphasised that acceptability of a primary fracture network is not based on the results of the flow and transport calculations. Unacceptable primary fracture networks either require too many primary fractures or have network blocks that are too large and cannot be reduced in size. An upper limit is imposed on the number of primary fractures in order to limit the number of coarse-scale flow equations. The need to limit the network block size is discussed later in this chapter. Failure to find an acceptable primary fracture network was more common using criteria based primarily on transmissivity, i.e. criteria
1 and 3. Criteria that exclusively weight either transmissivity or length do not work as well as criteria that weight both connectivity and transmissivity.

A second category of failures in primary fracture selection has not been listed in Table 4.2 because these failures are not sensitive the selection criteria. The selection algorithm makes a mistake in interpreting the fracture network geometry in roughly 5% of the realizations. These mistakes are usually associated with the accuracy of the trigonometric calculations. Despite extensive efforts to make the calculations of fracture geometry both precise and robust, some geometric calculations, such as determining what network blocks contains which portions of a particular fracture, are not totally reliable.

Creating an algorithm to select the primary fracture network has proven to be one of the most difficult problems in developing the dual permeability model. A complex algorithm evolved from these difficulties, yet the number of failures indicated in Table 4.2 demonstrates that the algorithm is still not flexible enough to deal with all of the arbitrary fracture geometries that arise from stochastically generated realizations.

4.2.2 PRIMARY FRACTURE SELECTION ALGORITHM

This section describes, in detail, the major steps in the fracture selection procedure. A simplified flow chart of the primary fracture selection routine is given in Figure 4.3.

The first fractures to be included as primary fractures are those that may have been preselected. These fractures may be known a priori and are being used to condition the fracture network, or they may be fractures that have been selected by the modeler from those fractures generated through the network realization process. The latter case may arise if an initial attempt of the primary fracture selection algorithm failed to find an adequate primary fracture network. In this case, the pre-selection of specific fractures may enable the selection process to create a network that incorporates these key fractures. Pre-selection of primary fractures is an option that allows greater control of the primary fracture network, but
Primary Fracture Network Selection

Increase Percentage of Primary Fractures

Select Primary Fractures From Individual Characteristics

Find Primary Fracture Intersections. Connect Ends Using Added Primary Fracture Segments.

Remove Unconnected Primary Fractures

Define Network Block Geometry

For Each Block

Test Block Size

OK

Too Large

Divide Block

No

Any New Primary Fractures

Yes

For Each Block

Find Internal Network Block Fractures and Intersections

Test Block Size

OK

Too Large

Divide Block

Select New Primary Fracture

Select New Primary Fracture

Can't Divide

Select New Primary Fracture

Can't Divide

Block Failure

Block Define Network

Figure 4.3: Flow chart of the selection of a primary fracture network.
is not required to form the network. Pre-selection of primary fractures has not been used in any of the simulations reported in this thesis.

The next group of fractures are selected from the generated fracture network using one of the criteria listed in Table 4.1. The first step in this process is to order the individual fractures based on their physical characteristics using the specified selection criteria. Fractures are added to the primary fracture network from this ordered list until these fractures reach a fixed percentage of fractures in the entire network. The appropriate percentage is dependent on the characteristics of the fracture system and the selection criteria used. An initial guess for this percentage is supplied by the modeller. As shown in the flow chart, this percentage may be increased during the selection process if the initial attempts to form a primary fracture network are unsuccessful. The initial primary fracture percentage was set to 1.4% in the cases presented in Table 4.2. Selection criteria that weight transmissivity more heavily tend to require more primary fractures.

After the primary fractures have been selected based on individual fracture characteristics, additional fractures are added to completely connect the primary fracture network. Each primary fracture must be connected at each end to another primary fracture. Starting at the end of a primary fracture, the intersections with other fractures are investigated. If the intersecting fracture is a primary fracture then the other end or next primary fracture is checked. If the intersecting fracture is not a primary fracture then the intersecting fracture is checked to see if it is connected to another primary fracture. If it is connected to another primary fracture then the segment of fracture between the two primary fractures is itself made a primary fracture segment. If the intersection does not lead to a connection to another primary fracture then the next closest intersection to the end is checked. This procedure continues until either a link is made or the end of the fracture is reached. The checking procedure is repeated for both sides at each end of all of the primary fractures. The connection procedure may result in some primary fractures that are
not connected to the rest of the primary fracture network or to fractures that are connected at only one location. These fractures are removed from the primary fracture network.

Once a primary fracture network is formed from the primary fractures, it is no longer meaningful to describe the network in terms of individual fractures. Instead, the number of fracture segments that connect the intersections becomes a much more meaningful description. For example, a primary fracture network selected for one of the realizations used to obtain the results presented in Table 4.2 contained 58 primary fractures and 734 primary fracture segments. Of these 734 segments, 151 segments are portions of fractures that were included to provide the necessary connectivity. In this realization, 4,035 fractures survived the fracture reduction procedure (Section 3.3).

Once the primary fracture network has been determined then the boundaries of each network block are identified. The last stage in the primary fracture selection routine is to subdivide network blocks that are too large. The number of intersections within the block is estimated by assuming that the density of intersections is constant across the model domain. The total number of intersection is calculated in the fracture network generation portion of the program. Any block that is estimated to contain too many intersections is divided by finding a fracture that connects two boundaries of the block. The fracture that comes closest to the centroid of the block and has the largest transmissivity is added to the primary fracture network. If no one fracture connects two sides of the block then the block is searched for two fractures that provide the connection. If this procedure also fails, then the fracture that is ranked highest using selection criteria 5 and is at least partly within the network block is added to the primary fracture network. Additions of new primary fractures necessitates the redetermination of the interconnected primary fracture network and subsequent identification of network blocks.

If, after a number of fractures have been added to the primary fracture list without successful identification of a primary fracture network, then the percentage of initial primary fractures can be increased. This outer loop in the procedure can continue until the requested percentage exceeds an upper
limit on the primary fractures. An increase in the limit on primary fractures tends to both reduce the number of failures to find an adequate primary fracture network and increase the accuracy of the dual permeability model, but increase the computer memory requirements of the model.

4.3 DUAL PERMEABILITY FLOW MODEL

Determining the flow within the fracture network is accomplished in three distinct stages. The first stage is to find a hydraulic conductance approximation that can be used to represent the effective hydraulic conductance of the network of fracture segments that are contained within each of the network blocks. These approximations are then combined with the conductance of the primary fractures to form the coarse-scale flow problem. The hydraulic head distribution that results from solution of the coarse-scale flow problem provides the boundary conditions that are used to estimate the flow within the fractures represented by the network blocks.

4.3.1 EFFECTIVE CONDUCTANCE OF NETWORK BLOCKS

The hydraulic conductances of the fractures within the network blocks are approximated using effective conductances for the blocks. The effective conductance of each network block is calculated independently. Figure 4.4 is used to illustrate the concept of effective conductances. This figure shows a schematic drawing of a single network block. In the figure, the segments of primary fractures that define the boundaries of the block are represented but the internal fractures are not drawn. At the ends of each segment of a primary fracture, filled circles represent the intersections of the bounding primary fracture segments. Between each pair of intersections is an open circle. These are referred to as fracture midpoint nodes. Fracture midpoints are located half way between the two ends of the fracture segment. The drawing also shows two kinds of arrows that are labeled "two way connections" and "flow to midpoint". These arrows will be explained shortly. First, however, is an explanation of the fracture midpoint nodes and the role they play in the effective conductance representation of the network block.
Two way Connections

Flow to Midpoint

Fracture Midpoint

Primary Fracture Intersection

Figure 4.4: Effective conductance approximation for network blocks.
The fracture midpoint nodes provide an estimate of the hydraulic head within the primary fracture segments. This head estimate is needed to calculate the flow from each primary fracture intersection into (or out of) each primary fracture segment. This flow is needed in the transport algorithm to determine the probability that a solute particle will enter the respective fracture segments from the primary fracture intersection. A flow balance exists at the midpoint nodes that corresponds to the net flow into the primary fractures. This flow consists of two terms; 1) the net flow exchange between the primary fracture and the interior of the network block, and 2) flow entering the primary fracture segment directly through its ends, the primary fracture intersections. The outer edges of the primary fracture segments are treated as impermeable boundaries for the individual network block flow calculations, as depicted in the Figure 4.4.

Two types of arrows are used in Figure 4.4 to emphasize the difference in the hydraulic connections of the midpoint nodes from those of the primary fracture intersections. These arrows have been drawn for one of the midpoint nodes. Double headed arrows depict the flow exchange between the intersections of the primary fractures and the primary fracture segments.

Single headed arrows, drawn as dashed lines, represent the hydraulic connection between the primary fracture intersections and a primary fracture segment through the fracture segments that are within network block. The single headed arrows depict connections that are "unidirectional". That is, the hydraulic connection between the primary fracture intersections and the primary fracture segment is included in the mass balance of the primary fracture midpoint nodes, but is not included in the mass balance of the primary fracture intersections. There is no direct flow between the primary fracture intersections and the network block unless a fracture within the network block directly connects to the intersection. The flow balance equations for the primary fracture intersections only include the direct flow exchange with the primary fracture segments through the ends of the segments (i.e. the double headed connections). The use of unidirectional connections ensures that the correct apportionment of the volumetric flow into (out of) the primary fractures from the network blocks is maintained.
In order to gain a full description of the effective conductance between all primary fracture intersections and midpoint nodes, multiple simulations of flow through the fracture network in the network block are required. A separate flow problem, which focuses on the hydraulic connections of an individual primary fracture intersection, is solved for each primary fracture intersection of the network block. Figure 4.5 shows how these flow calculations are used to determine the hydraulic connection through the network blocks of the primary fracture intersection and the midpoint nodes. For each calculation, Dirichlet boundary conditions are imposed at all of the primary fracture intersections. All of the boundary heads are assumed to be 0.0 m except at one intersection. Here, a unit head of 1.0 m is imposed. Flow through fractures within the network blocks is calculated using these boundary conditions.

The flow calculations are performed in isolation from other blocks. The solution for flow within each network block neglects any influence of fractures within neighboring network blocks on the hydraulic head distribution within the primary fractures. The boundaries just outside the primary fractures segments are treated as impermeable except at the primary fracture intersections.

The flow through each intersection of a primary fracture with the internal fractures is summed to form a response of the primary fracture midpoint to a unit change in the hydraulic head at the primary fracture intersection. This unit response is the effective conductance between the primary fracture intersection and the primary fracture midpoint. A single flow calculation provides the effective conductances of all of the primary segment midpoint nodes to one primary fracture intersection. Combining the effective conductances into a matrix for the series of flow solutions creates a local stiffness matrix for the block that is analogous to the local stiffness matrices of a finite element model.

If the primary fractures truly dominate the flow system then, almost by definition, flow exchange between the primary fractures and the network blocks is small relative to the flow along the primary fractures and hence has little influence on the hydraulic head distribution along the fractures. For these fractures, neglecting flow into neighboring network blocks is a reasonable approximation. Unfortunately,
Internal fracture flow into Primary fracture segment

Flow to Midpoint

\[ \sum \]

Unit Head  \( \bullet = \text{Zero Head} \)

Figure 4.5: Calculation of the hydraulic connection between primary fracture intersections and primary fracture segment midpoints.
the fractures added to the primary fracture network to ensure that the network is completely connected almost always include fractures that do not truly dominate the local flow system. A significant flow error can result from forcing the flow, that should cross a primary fracture into an adjacent block, to remain in the fracture until a primary fracture intersection is reached.

A heuristic modification has been added to the calculation of effective conductances to reduce the influence of primary fractures with low transmissivity on the calculation of flow within the network blocks. In the flow calculations of a network block, the transmissivities of the primary fracture segments are adjusted by adding the transmissivity of all fractures that connect each segment to adjacent network blocks. If the primary fracture transmissivity is large relative to the combined transmissivities of these fractures, then the addition is minor. If the primary fracture transmissivity is not large, then the addition of additional hydraulic transmissivity to the primary fractures can have a significant influence. One can view this addition as a relaxation of the assumption that the outer boundaries of the network block are impermeable. The conductance of neighboring network blocks is approximated by adding to the transmissivity of the primary fracture segments. The addition of extra transmissivity is a recognition that much of the flow into the primary fracture crosses the fracture into the adjacent block.

Table 4.3 presents the results from flow and transport simulations of the same fracture networks used to study the primary fracture selection criteria. In these cases, however, the transmissivities of the primary fracture segment were not increased for the network block conductance calculations. A comparison of the results in Table 4.3 to those of Table 4.2 demonstrates that increasing the transmissivity of primary fractures in the network block conductance calculations significantly improves the performance of the dual permeability model.
<table>
<thead>
<tr>
<th>Criteria</th>
<th>Flow Error (%</th>
<th>Time Delay Initial (%)</th>
<th>Time Delay Median (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-17.8</td>
<td>16.1</td>
<td>26.9</td>
</tr>
<tr>
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<td>-12.6</td>
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<td>7</td>
<td>-10.9</td>
<td>8.7</td>
<td>14.7</td>
</tr>
</tbody>
</table>

Table 4.3: Primary fracture selection criteria test with unmodified network blocks. The networks were identical to those used for Table 4.2. The errors are calculated for individual fracture networks, that is the results do not include fracture network variability. Flow error is based on the total flow across the domain boundaries. The time errors are for initial solute arrival and median particle arrival.

4.3.2 HYDRAULIC HEAD DISTRIBUTION

The coarse-scale flow problem is formed by combining the local network block stiffness matrices with the two way connections between the midpoint nodes of the primary fracture segments and primary fracture intersections at each end of the segments (See Figure 4.4). The hydraulic head values at the primary fracture intersections provided by the coarse-scale flow calculations are used as boundary conditions for the network blocks. One more set of calculations of flow within each of the network blocks provides a fine scale description of flow within the entire fracture network.

While the fine scale hydraulic head distribution calculated within the network blocks is unique, it is not single valued along the primary fracture segments. Adjacent network blocks share common
primary fracture segments. The hydraulic head distribution along these common primary fracture segments will be different in each block. This disagreement in the value of the hydraulic head is acceptable because it is the flow rates within the network blocks that are used for solute transport, not the hydraulic head values. After the flow calculations, the flow distribution is known throughout the network blocks and at each primary fracture intersection. The final step in calculating flow within the fracture network involves the determination of the flow rates within the primary fracture segments.

The flow rates within the primary fractures are determined by forcing mass conservation at each intersection of the primary fracture with fractures of the network blocks. The flow entering each primary fracture segment from the intersections of primary fractures is calculated using Equation 3.17 applied to the half of the primary fracture segment between intersection and the fracture midpoint. The flow from the primary fracture intersection is determined strictly from the results of the coarse-scale flow model, ensuring a mass balance at each primary fracture intersection.

The calculation proceeds along the primary fracture segment from the primary fracture intersection with the highest hydraulic head to that with the lowest. At each intersection of the primary fracture segment with the internal network block fractures, the flow into the intersection that was calculated in the network block flow calculations is added to the flow in the fracture segment.

Calculation of flow downstream of the last intersection determines the flow into the primary fracture intersection with the lowest value of hydraulic head. A comparison of flow rate at the end of the fracture segment calculated using the above procedure and the flow rate determined by applying Equation 3.17 provides a check of mass conservation between the network blocks and the primary fractures.

The coarse-scale / fine-scale formulation of the flow problem within the fracture network has the advantage that the largest set of flow equations that must be solved simultaneously is either the size of the largest network block or the size of the coarse-scale flow model for the primary fracture network.
This advantage is reduced by the disadvantage of the need to solve for the network block flow several times to obtain the effective conductance of the network blocks and the overhead burden of defining the primary fracture network geometry. For small domains all calculations can be performed using a direct solver. Large domains require an iterative solver. This issue is pursued in more detail in Section 4.5.4.

4.4 DUAL PERMEABILITY TRANSPORT MODEL

4.4.1 TRANSPORT THROUGH A NETWORK BLOCK

The primary fracture network / network block structure is also used in the particle tracking portion of the dual permeability model. Network blocks are represented in the particle transport routine by residence time distributions. The residence time distributions are tables of probability that link locations of inflow to the network blocks to locations of outflow. For each inlet-outlet pair there may be many paths a particle may take through the network block. Different paths lead to different residence times within the block. Each inlet / outlet pair has a set of residence times, each of which have different probabilities of occurrence. This leads to a three dimensional table of probabilities. One dimension is needed for the inlet locations. One dimension is needed for the outlet locations. And, one dimension is needed to represent the different times of arrival. The values in the table are pairs of probabilities and arrival times. A front tracking method introduced by Shimo and Long [1987] has been modified to determine the residence times and their associated probabilities of occurrence.

The Shimo and Long front tracking method follows the rise in concentration along a fracture network from a step function injection of solute at an upstream boundary. The rise in concentration downstream of the injection location is followed from intersection to intersection in descending order of hydraulic head at the intersections. The concentration at an intersection is the sum of the concentrations at intersections that are directly upstream, with appropriate time delays in the upstream concentrations to account for the travel times through the fracture segments. Figure 4.6, from Shimo and Long [1987],
Figure 4.6: Front tracking procedure of Shimo and Long [1987] showing the combining of upstream solute concentrations to form the concentration as a function of time at a fracture intersection. From Shimo and Long [1987].
depicts this combination of concentrations.

The implementation used here replaces the step function rise in concentration with the pulse injection of a single particle. The increase of solute concentration at a downstream intersection at a given time becomes the probability that the particle will pass through the intersection at that time.

Figure 4.7a, which shows four fracture intersections that are part of a larger fracture network within a network block, is used to illustrate the formation of residence time distributions for a network block. The intersections are drawn as filled circles. Fractures are presented as parallel lines. Arrows give the direction of flow within the fracture segments. The variables $Q_j$ in the figure are the flow rates within the fracture segments connecting the intersections. The rest of Figure 4.7 contains plots of the possible particle arrival time probabilities at these four intersections, for a particle that is injected into the fracture network upstream of the first intersection. To illustrate the calculation, panel b presents an assumed probability of arrival of solute at intersection 1 from an unidentified upstream location. The two values, at $t_1$ and $t_2$, indicate that there are two paths from the inlet location to intersection 1 that are not shown in Panel a. In Panel c, the residence time distribution of intersection 2 is displayed. The probabilities at intersection 2 are $30\%$ of those at intersection 1 because $30\%$ of the flow through intersection 1 goes to intersection 2. The times $t_1$ and $t_2$ have increased from $t_1$ and $t_2$ by the particle travel time from intersection 1 to intersection 2. Panel e, for intersection 3, has a similar explanation. Panel d, representing intersection 4, shows four probability values resulting from the combination of possible paths through intersection 2 and intersection 3. The offset in time of $t_1$ from $t_3$ is caused by the difference in travel time along the two paths.

Figure 4.7 demonstrates an increase in the number of distinct probability-time pairs as the number of potential paths from the block inlet to outlet increases. The number of computations required to establish the residence time distribution increases in proportion to the increase in the number of probabilities. This behavior places a practical limit on the number of intersections that can exist in a
Figure 4.7: Calculation of network block residence times. a) Schematic of four fracture intersections and their contiguous fracture segments. The intersections are drawn as dots. The intersecting fractures are presented as parallel lines. Arrows give the direction of flow within the fracture segments. The variables $Q_{ij}$ are relative flow rates within the fracture segments connecting the intersections. b) Probability of a particle arriving at the intersection 1 at specific times. c) Probabilities for intersection 2. d) Probabilities for intersection 3. e) Probabilities for intersection 4.
network block. The limit is highly dependent on the actual flow distribution within the network block, however 400 intersections seems to be reasonable upper limit to the size of the blocks without some form of reduction in the number of distinct arrival times.

Two techniques, again following Shimo and Long, are used to reduce the number of distinct arrival times at the outlets from the network block. One technique is to combine probabilities of arrival for times that are close together. The second technique is to remove low probability arrivals by combining them with higher probability arrival events at an intersection. If the probability of a particle arriving at a intersection from an inlet is below a specified threshold then the probability-time pair is combined with another pair that is the closest in time. For both of these techniques, the mean time is used to represent both paths with a probability that is the sum of the two probabilities. The mean time is determined by weighting the two times by their respective probabilities.

The two techniques can have a significant influence on the efficiency of the particle tracking algorithm. This point is demonstrated using a fracture network composed of two orthogonal sets of continuous fractures. While the trace map of this fracture network in Figure 4.8a looks like evenly ruled graphpaper, transmissivity variations make the network heterogeneous. Both Discrete and the dual permeability model were used to determine transport through the domain. The input to the dual permeability model was specified so that there were no primary fractures selected for the fracture network. All of the fractures were included in a single network block.

Table 4.4 contains statistics from various dual permeability simulations of transport through the domain in which the techniques for enhancing efficiency were applied. The first column of the table identifies the simulation by number. The second column contains values for the minimum probability threshold (p_lim) of distinct probability-time pairs. The third column lists the value of the maximum time difference (t_diff) between probability-time pairs that are combined into a single pair at an intersection. The fourth column lists the number of distinct probability-time pairs needed to represent the inlet-to-outlet
Figure 4.8: Comparisons of particle arrival rates of the dual permeability model using a single network block and Discrete. a) The fracture network model as a single network block. b) Arrival at peak location with no storage reduction. c) Arrival at peak location with $p_{lim} = 0.01$. d) Arrival at peak location with $p_{lim} = 0.0001$ and $t_{diff} = 0.001$ days.
residence time distribution for the block. The final column is the time needed to form and solve the flow problem, form the transport model, and then move 50,000 particles using an SGI R4000 computer.

The first simulation had both \( p_{\text{lim}} \) and \( t_{\text{diff}} \) set to zero. There are almost 710,000 distinct arrival time, inlet-outlet combinations for this extremely well connected network. The time needed for the transport calculation was 730s for the dual permeability model and 12.5s for the program Discrete. Figure 4.8b shows the cumulative arrival of particles at the location where the greatest number of particles crossed the downstream boundary. These arrival patterns are not significantly different.

Setting \( p_{\text{lim}} \) to \( 10^{-6} \) had only a minor influence on these results. The probability limit was increased by an order of magnitude for each successive run up to a value of \( 10^{2} \). At the highest value the number of distinct points in the residence time distribution was reduced to 2,780 and the time of the dual permeability model simulation was reduced to 2.8s. Figure 4.8c shows a noticeable discrepancy in the cumulative arrival curves, but the results are still quite close. For the rest of the test, the minimum time difference, \( t_{\text{diff}} \), was increased while \( p_{\text{lim}} \) remained \( 10^{4} \) in order to evaluate the influence of combining the probabilities of similar times. For run 7, the minimum time difference was 0.0001 days. At this level, the dual permeability model took 7.3s and needed 42,000 points for the residence time distribution of the block. The arrival curve was nearly identical to the curve from Discrete.

The final run used a minimum time difference of 0.001 days, the same size time interval used to collect the particle arrival rates. This run required only 2.9s of computer time. The results of the dual permeability model shows a raggedness in Figure 4.8d, signifying that the large minimum time difference has shifted the arrival of a significant number of particles. The value of \( t_{\text{diff}} \) is larger than the time interval between points used to represent the cumulative arrival causing a stair step pattern to the dual permeability results.
The use of minimum probabilities and time differences had a substantial influence on both the memory requirements and computation time of these simulations. Both the time and memory requirements varied by a factor of more than 200. The particle arrival curves did not show a strong sensitivity to the value of $p_{lim}$. The use of large values of $t_{diff}$ only had a noticeable influence if $t_{diff}$ was larger than the time interval used to determine the particle arrival rate. In no case were the results strongly influenced by the use of $p_{lim}$ and/or $t_{diff}$. A substantial performance increase can be obtained through the use of these approximations without the need for tremendous caution in their application.

In summary, this limited evaluation of the use of residence time distributions to simulate transport through network blocks indicates that a substantial time savings can be obtained with negligible
degradation in the transport predictions. Later, in Section 4.5.4, it is shown that in practice the time
needed to set up the residence time distribution and then move 50,000 particles is about 50% of the time
needed by the particle tracking algorithm of Discrete.

4.4.2 PARTICLE MOVEMENT

Figure 4.9 is a flow chart for the particle movement routine. I will explain the particle tracking
routine by following a particle. The top of the flow chart represents the introduction of a particle at the
domain boundary. For this discussion, the boundary location is at one end of a primary fracture, but it
could also be a network block inlet location. The particle enters the fracture. The time that the particle
took to reach the first intersection of the primary fracture with a fracture of a network block is recorded
as the cumulative transport time. At each intersection of the primary fracture with fractures of the network
blocks, a random number is used to test whether the particle enters the network block. The probability
of entering the block is based on the fraction of flow through the node that enters the block (i.e. complete
mixing).

If the particle continues to move down the fracture instead of entering the block, the cumulative
transport time of the particle increases as each intersection along the primary fracture segment is reached.
After the last intersection of the segment is passed, the particle enters a primary fracture intersection.
From here the particle transfers to a new primary fracture segment, again based on relative flow rates from
the intersection.

When a particle transfers from an intersection along a primary fracture segment to a network
block, the intersection corresponds to an inlet location of the network block. A random number is
generated from the uniform distribution between 0 and 1. Starting with the exit location that is the most
probable, the probabilities of a particle leaving an exit location are summed until the next location would
cause the sum to exceed the random number. This location is the exit location from the block for this
Particle Tracking Simulation

Figure 4.9: Flow chart of the particle tracking transport algorithm.

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particle. The sum is subtracted from the random number to create a new random number that is uniformly distributed between 0 and the probability of exiting at the selected exit location. The procedure of summing probabilities, in the order of decreasing probabilities, is repeated for the probability-time pairs until the random number would again be exceeded. The time of the last pair is selected as the residence time for the particle in the network block. The exit location corresponds to a network block intersection on some downstream bounding primary fracture segment of the network block.

The particle movement through the newly entered primary fracture segment proceeds from the selected intersection. The movement may be into the adjoining network block or along the primary fracture. The particle movement continues until a domain boundary is reached. When the boundary is reached, the location and total transport time of the particle is recorded, providing particle arrival histories at the domain boundaries.

The distributions of particles at a common time can be obtained from the particle transport routine. Each time a particle moves, the time is tested against a list of recording times. If the addition to the total transport time causes that time to exceed one of the times on the list then the position of the particle is estimated from a linear interpolation of its position at the beginning and end of the particular movement. Along a primary fracture the interpolated position is precise. Within network blocks, the precision of the interpolated position is limited by the size of the block. The interpolated position is along a line connecting the inlet location to the outlet location. The actual travel path would rarely coincide with this estimate.

4.5 DUAL PERMEABILITY MODEL EVALUATION

The purpose of the dual permeability model evaluation is to assess the degradation in the accuracy of using a discrete fracture representation when the model is transformed from a "complete" discrete fracture model to a dual permeability model. Discrete represents that complete discrete fracture model.
Any differences between the hydraulic characteristics of a hypothetical fractured rock mass represented by the fracture system statistical characterization and those predicted by *Discrete* are beyond the scope of this evaluation. The calculations using *Discrete* are treated as the truth.

The evaluation is composed of four components. The first component is a comparison between the dual permeability model and *Discrete* of flow and transport through a single realization of a fracture network. Detailed results from one realization are used for three purposes: 1) to reiterate the major features of the dual permeability model, 2) to give an indication of types and magnitude of errors that are introduced through the decomposition of the network into primary fractures and network blocks, and 3) to demonstrate information that can be obtained from the model.

The second and most important component in this evaluation is in the context of a Monte Carlo study. A premise of the dual permeability concept was that the variability and inherent uncertainty in the physical characteristics of a fractured rock body strongly compels a stochastic investigation of the fracture system through a Monte Carlo analysis. Hence, it is from the results of a Monte Carlo investigation that one should judge the merits of the dual permeability model. While the single realization comparison is limited to one network, the Monte Carlo evaluation uses a wide variety of fracture systems because the performance of the dual permeability model depends, to a degree, on the nature of the fracture system.

A study of the influences of network block size on the accuracy of the dual permeability model is presented as the third component of the evaluation. The question addressed is: "Does the accuracy of the dual permeability model degrade quickly as the scale of the network blocks increases?". This is a minor consideration compared to the broader issue of the efficacy of the dual permeability concept yet it must be addressed in order to put the results presented in the second and fourth components of this evaluation in perspective.
The fourth component of this evaluation looks at the advantages of the dual permeability model rather than focusing on the inaccuracies that it introduces. It addresses the basic questions of: 1) "How large of a fracture network can be simulated?", and 2) "How long does it take to solve the problem?". These investigations compare two variations of the dual permeability model, that have different memory storage requirements and speed attributes, to Discrete. The two variations of the dual permeability model are distinguished by the solver used to find the coarse-scale head distribution.

4.5.1 A SINGLE REALIZATION

The fracture system chosen for this demonstration is one that was known to provide excellent results. Therefore, the results presented here, in some ways, represent a best case scenario. However, the specific fracture network was not selected from a suite of realizations. The results presented in this section should not be considered abnormally accurate. They are in fact, slightly less accurate than an "average" realization for this fracture system. The realization was created from fracture system 1 of Table 4.5 using a Poisson structural model. The dimensions of the flow domain were 40m by 40m. The fracture trace map of the 8,400 fractures within the flow domain is shown in Panel a of Figure 4.10. In Panel b, the reduced fracture network of just over 4,000 fractures is shown. Flow rates calculated for a hydraulic head gradient of 0.01 in the horizontal direction of the figure are presented as line widths in Figure 4.11. Only fractures with flow rates above 5.1x10^{-11} m³/s are drawn, giving the appearance that some fractures are isolated from the rest of the network. The cumulative flow across the boundary on the right was 3.2x10^9.
Figure 4.10: Fracture traces of the one realization of fracture system 1 of Table 4.5. a) The complete fracture network. b) The reduced fracture network.
Fracture traces of the one realization of fracture system 1 of Table 4.6. b) The reduced fracture network.
Minimum flow drawn = 3.0e-11 m$^3$/s

Figure 4.11: Fracture flow rates within the fracture network of Figure 4.10. Fractures are drawn with a line width proportional to the flow rates calculated using *Discrete*. The scaling of flow rate to line width is given by the scale below the main figure. Only flow rate above 3.0x10^-11 are drawn. The hydraulic head gradient has a magnitude of 0.01 and is oriented at 0° from the lower boundary.
<table>
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<td>2</td>
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* Scan-line density outside of war zones. The actual scan-line density is larger.
† Minimum fracture length generated from a power law distribution with an exponent of -1.3.

Table 4.5: Fracture system statistics for the dual permeability evaluation. Entries in the table are only made when the parameter differs from system 1.
m$^3$/day. The flow was largely controlled by the first two of four fracture sets, which both have a 10m mean fracture length and have higher transmissivity than the rest of the fractures in the network.

Fifty-seven fractures were initially selected to be primary fractures using the selection criteria based on the product of length and aperture, selection criteria 4. The primary fracture network shown in Figure 4.12 resulted from forming an interconnected network composed of these 57 fractures and other fracture segments needed to define closed network blocks. The primary fracture percentage was initially specified as 1.4% and was not increased during the primary fracture selection process. The interconnected network has 518 primary fracture segments and divides the fracture network into 175 network blocks. The coarse-scale hydraulic head distribution is defined by 302 locations in the dual permeability model compared to 13,787 locations required to define the head distribution within the entire network. The coarse-scale head distribution required the solution of 731 simultaneous equations. Recall that the number of equations is greater than the number of hydraulic head locations in the coarse-scale hydraulic head distribution because of the inclusion of the primary fracture midpoints in the flow equations.

Figure 4.13 is a plot of the flow rates within the primary fracture network. The plot is made using the same scale of line widths as in Figure 4.11. Only the flow in the primary fractures is shown. Most, but not all, of the fractures that have larger flow rates in the Discrete simulation are represented in the primary fracture network. While the general flow pattern is similar in Figures 4.11 and 4.13, there are some noticeable differences. Primary fractures tend to have greater flow rates in this dual permeability model than in Discrete. This tendency for large flows results in a five percent higher net flow through the dual permeability flow domain and earlier particle arrival at the downstream boundaries. In most of the realizations encountered in the Monte Carlo portion of the evaluation, the calculated flow using the dual permeability model was less than the flow calculated using Discrete. So, the estimation of higher flow in the dual permeability model is atypical, but the primary fractures do tend to have high flow rates in most realizations.
Figure 4.12: Primary fracture network defined for the fracture network of Figure 4.10.
Figure 4.13: Flow rates within the primary fractures of the dual permeability model.

Minimum flow drawn = 3.0e-11 m³/s
The net fluid flow across the domain boundaries for both models is presented in Table 4.6. The first row lists the net flow through the entire domain. The other four rows of the table list the net flow across each boundary. The flow difference for each boundary is of the same order of magnitude and does not correlate strongly with the net flow across the boundary.

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<th>Difference (m³/s)</th>
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<td>3.3x10⁻¹⁰</td>
</tr>
</tbody>
</table>

Table 4.6: Net flows across the flow domain boundaries of the single realization comparison.

Fifty thousand particles were used to simulate transport in both the dual permeability model and Discrete. The particles were introduced into the fracture network over a range from 15m to 25m along boundary 2. Figure 4.14a depicts the particle arrival locations along the downstream boundary in a perspective plot. The height of the columns represent the cumulative number of particles that crossed the boundary at the location of the column. The columns representing data from Discrete are shaded. Panel b is a plot of the cumulative arrival along boundary 4. The solid line is a plot of the particle arrivals predicted by the program Discrete. The dashed line represents the particle arrivals predicted by the dual permeability model. The abscissa in Panel b corresponds to position along boundary 4 as defined in Figure 4.11. The arrow above the columns in Panel a points in the direction of increasing distance along boundary 4. The ordinate in Panel b is the cumulative number of particles that crossed the boundary with increasing distance along the boundary. The particles crossed the downstream boundary 4 over a range of 10m to 30m as shown in Figure 4.14. The results presented in Figure 4.14 show that dispersion
Figure 4.14: Particle arrival distribution at the downstream boundary of the fracture network of Figure 4.10. a) A perspective view. Particles were released over the interval from 15 m to 25 m along upstream boundary. The percentage of particles that crossed the boundary at a specific location is presented as columns along the downstream boundary. b) The particle arrivals are integrated from the lower right corner of the flow domain to the upper right corner. The arrow above the columns in Panel a points in the direction of integration.
perpendicular to the hydraulic head gradient was closely matched but that there were variations in the
number of particles that crossed the boundaries at any particular location. The results are typical in that
the particle transport predictions of the models differ in the details but retain the same general behavior.

Figure 4.15 contains graphs of particle arrival as a function of time. In Panels a and b, the arrival
of particles at boundary 4 through the fracture located 20.2 m from bottom is plotted. Panel a shows the
rate of arrival and Panel b shows the cumulative arrival over time. Both plots are in terms of the
percentage of the 50,000 particles injected. The general timing of the particle arrival is quite similar.
*Discrete* predicted a larger number of particles crossing the boundary at this location.

In Panels c and d, the particle arrival rate and cumulative arrival at boundary 4 are presented for
all of the particles that crossed the boundary. Panels c and d are a better representation of the general
transport behavior of the network than the results from a particular point. Panels c and d both reveal a
slightly larger longitudinal dispersion in the particle transport of the dual permeability model. The fastest
particles reach boundary 4 slightly earlier than in *Discrete*. Slower particles arrive a bit later. In general,
the shapes of the arrival curves are very similar.

Particle transport is characterized using the mean and standard deviation of particle positions in
Figure 4.16. Using statistical moments of solute plume behavior is a popular method of characterizing
transport behavior in heterogeneous porous media (*e.g.* Graham and McLaughlin [1989]). The dual
permeability model is not well suited to gathering these statistics. Position statistics are influenced by the
residence time distribution method of modeling transport through the network blocks. Particle locations
within the network blocks are not known. The positions used to collect the statistics presented in Figure
4.16 are estimated by linearly interpolating the particle positions within the network blocks from the
particle entrance location and time and the downstream exit location and time. The linear interpolation
introduces inaccuracy into the position estimates. It should be noted, however, that fracture network
heterogeneity introduces variability in the particle statistics at the scale of the network blocks. The
Figure 4.15: Particle arrival curves for entire downstream boundary and at the 14m location. The curves represent the arrival along the entire boundary. a) The rate of particle arrival at the downstream boundary. b) Cumulative arrival over time at the downstream boundary. c) Particle arrival rate at 14m. d) Cumulative arrival at 14m.
Figure 4.16: Particle location statistics. a) Mean particle position parallel to the gradient direction. b) Standard deviation of particle location parallel to the gradient direction. c) Mean particle position along the axis perpendicular to the gradient direction. d) Standard deviation in particle position perpendicular to the gradient direction.
uncertainty in particle statistics introduced by the network block structure occurs at the same scale or smaller than the large variability in plume statistics caused by fracture network heterogeneity.

Panel a in Figure 4.16 shows the average particle position along the axis of boundary 1. The positions are tracked only to 100 days. The first particle reached boundary 4 at 115 days, limiting the validity of the position statistics to the period prior to this time. Even after 100 days, most of the particles are near the particle source region. Panel a shows the mean particle position. Because at these times most particles are still concentrated in a few network blocks, the mean positions determined from the dual permeability model and Discrete are not in as close agreement as one would expect from the results presented in Figure 4.15. Panel b confirms the increased longitudinal dispersion in the dual permeability model. Panel c, a plot of the mean particle position transverse to the regional hydraulic gradient, shows a disagreement in position that is not evident in the cumulative arrival along boundary 4 (Figure 4.14). The affect is also evident in the standard deviation of particle positions that are transverse to the gradient direction shown in Panel d. As the plume of particles spreads away from the source region, and grows in size, the influence of uncertainty in the particle positions within network blocks would become less evident.

The comparison made for this single realization demonstrates that the dual permeability model is in close agreement with Discrete, at least for this particular fracture network. A more appropriate comparison is how the results of a series of simulations might affect the estimates of expected behavior of a fracture system.

4.5.2 MONTE CARLO EVALUATION

4.5.2.1 INTRODUCTION

For the Monte Carlo evaluation, the performance of the dual permeability model was evaluated using the seven fracture systems defined in Table 4.5. The evaluation is based on one hundred transport
calculations for each fracture system. The seven systems represent a broad spectrum of differing heterogeneity. The fractures systems were chosen in part to test the importance of a clearly definable dominant fracture type on the performance of the model and to test whether the dual permeability model is influenced by changing the fracture system structural model. The fracture systems were also chosen to explore the robustness of both the dual permeability model and, by inference, the concept of flow dominance. Of particular interest is the relationship of the range in the scale of fracturing to flow dominance and transport characteristics.

The fracture systems used in the evaluation are briefly reviewed in this paragraph. They are described in more detail later in separate sub-sections. Fracture system 1, composed of four fracture sets, was designed to provide easily identifiable primary fracture networks of long and highly transmissive fractures. Fracture network realizations are generated using a Poisson structural model. The fracture system has two dominate length scales because two of the fracture sets have mean lengths of ten meters and the other two have mean fracture lengths of one meter. In fracture system 2, the sets of long fractures are retained, but the aperture statistics have been adjusted so that the long fractures are not necessarily the most transmissive; all of the fracture sets have the same transmissivity statistics. The long fracture sets are removed completely in fracture system 3, leaving a fracture system composed of two sub-orthogonal sets. The scan-line density and mean transmissivity of the fractures is the same for fracture system 3 as for fracture system 2, but the distribution of lengths is different. Fracture system 4 is the antithesis of what the dual permeability concept was designed to simulate. In this fracture system, the transmissivity variability has been completely removed, leaving all the fractures with the same transmissivity. Fracture system 5 returns to the general characteristics of the first fracture system, but the realizations are generated using a war zone structural model rather than a Poisson model. The two long fracture sets define the war zones. The long length and near parallelism of these fractures define large narrow zones of dense fracturing. The sixth fracture system is based on the fracture sets of fracture system 3 using a war zone structural model to generate the fracture network realizations. In this network, the war zones are clusters
of high-density fracturing rather than the linear features of the other war zone fracture system. The seventh system is implemented with a Levy-Lee structural model. This fracture system has the same fracture density and transmissivity variation as fracture system 3 but, because of the Levy-Lee model, the fractures have a broad spectrum of fracture length, with shorter fractures being tightly clustered.

Before discussing the results for each fracture system in detail, I will provide a brief overview of conclusions that can be drawn from the entire exercise. Transport characteristics are the focus of the comparisons because particle transport is a sensitive indicator of both errors in the flow solution and in the particle tracking routine. I believe that the differences between transport characteristics of the dual permeability and Discrete are dominated by differences in the flow solutions. As demonstrated in Section 4.4.1, the transport portion of the dual permeability model introduces little error into the calculations. The major conclusions drawn from this evaluation are:

1) The dual permeability model requires a fracture network where the flow rates within some fractures are much higher than most of the other fractures that they intersect. Given this situation, the existence of relatively long fractures is crucial in the creation of the primary fracture network.

2) Transport predictions of the dual permeability model are not strongly sensitive to the selection criteria listed in Table 4.1. However, selection criteria for identifying the primary fracture network that use both fracture transmissivity and length are usually better than selection criteria that focus primarily on one of these characteristics.

3) Selection criteria 5 and 6 always resulted in average net flows through the domain that were within five percent of the average net flow calculated using Discrete, except for fracture systems 3 and 4 which fail to meet condition 1 above. These criteria also led to an estimate of the mean arrival time at the downstream boundary that was also within five percent of the Discrete results.
4) The fastest particles tend to move faster in the dual permeability model than in Discrete but most particles tended to be retarded in the dual permeability model introducing a slight increase in temporal dispersion. The simulations for fracture system 2 are an exception to this, as discussed later.

5) Primary fracture networks could not be determined for all realizations. Failure to find a primary fracture network is highly dependent on both the selection criteria and the nature of the network. Using selection criteria that weight transmissivity more strongly is less likely to result in a successful search for a primary fracture network.

4.5.2.2 Fracture System 1

Fracture system 1 fits the assumptions that underlie the dual permeability concept better than any of the other fracture systems listed in Table 4.5. That is, the hydraulic behavior of this fracture system is dominated by a small proportion of fractures with high transmissivity that form a well connected network by themselves. The fracture system is based on a Poisson structural model with four fracture sets. There are two scales of fracturing in fracture system 1. Two of the fracture sets are composed of fractures that are much longer and wider than the fractures of the other two sets, but these sets make up only some five percent of the total number of fractures. The mean orientations of the two sets are orthogonal. This fracture system was used earlier to investigate the primary fracture network selection criteria and for the single realization comparison. A fracture trace map and flow distribution for a realization of this fracture system were presented in Figures 4.10 and 4.11.

A comparison flow and transport predictions of the dual permeability model and Discrete for fracture system 1 has already been presented in Table 4.2 for each of the selection criteria. Table 4.2 shows that the differences in mean behavior in terms of both flow and transport is small and that these differences are sensitive to the selection criteria. Figure 4.17 presents this comparison in terms of the
Figure 4.17: Average cumulative particle arrival at downstream boundary for each of the primary fracture network selection criteria used in dual permeability models of fracture system 1 of Table 4.6. The curves are limited to the initial arrival time and the 25th, 50th, 75th and 90th percentile arrival times. The solid line represents the cumulative arrival for 70 realizations of the fracture system using *Discrete*. The dashed lines represent the corresponding average cumulative arrivals using the dual permeability model. The correlation between line type and selection criteria is given in the upper left of the figure.
cumulative arrival of particles along the downstream boundary and provides a visual indication of how dispersion is influenced by the dual permeability model.

In Figure 4.17 the average time when the leading edge of the particle plume crosses the boundary is the first point plotted for the cumulative arrival. Other points on the curve represent the average times when 25%, 50%, 75% and 90% of the particles have reached the boundary. The results from Discrete are plotted as a solid line. The results from the dual permeability model using the various primary fracture network selection criteria are plotted as dashed lines that are defined by the legend in the upper left portion of the figure. SC.1 refers to selection criteria 1.

The data presented in Figure 4.17 is slightly different than the data presented in Table 4.2. Table 4.2 lists the average differences for individual fracture network realizations where a primary fracture network could be found. For each selection criteria the realizations may have been slightly different. In Figure 4.17, the cumulative arrivals are averages using only the 70 realizations for which all of the selection criteria resulted in an acceptable primary fracture network.

The initial particle arrival at the downstream boundary is close to the Discrete prediction, with some of the selection criteria resulting in earlier arrivals and some resulting in later arrivals. After 15% of the particles have crossed the downstream boundary all of the dual permeability simulations tend to lag behind the Discrete results. Use of any of the selection criteria provided good transport predictions. The use of selection criteria 1, 2, and 3 which primarily use just one geometric property tend to lead to longer delays in the particle arrival than the use of selection criteria 5 or 6 which use both length and transmissivity. The increasing time lag in the dual permeability results is an indication of an increase in temporal dispersion that was described for the single realization. Calculated in terms of the half width of the arrival curve, the difference between the 75th and 25th percentile arrival, the dual permeability model increases the temporal dispersion of the particles by 3 to 7 percent.
The distribution of the initial, 50th, 75th, and 90th percentile arrival times, using selection criteria 6, along with the corresponding distribution from Discrete are presented as bar charts in Figure 4.18. The data from Discrete are lightly shaded. The dual permeability results are shaded with thicker and more widely-spaced lines. An example of Discrete is shown between Panels c and d and an example of the dual permeability plot is shown between Panels a and b. Panel a represents the initial arrival times. Panel b shows the median arrival time. Panel c contains the 75th percentile times and Panel d the 90th percentile arrival. This figure clearly shows that the inaccuracies introduced by the dual permeability model are small compared to the variations in arrival times that are inherent to the fracture system. The variability in transport through different realizations is large. This variability is a feature that is lost in the averaging that is used for the rest of the figures presented in this section. Recall from the discussion in Chapter 1 that if inaccuracies introduced by approximating the fracture system would not change the conclusions that an analyst might draw from a Monte Carlo investigation of the fracture system, then the decrease in accuracy is not important. The conclusions about transport in fracture system 1 drawn from the distributions of arrival times from the dual permeability model as presented in Figure 4.18 would, in all likelihood, not be significantly different than those drawn from the Discrete results.

The final figure presented for fracture system 1, Figure 4.19, is the arrival data from the dual permeability model for the case where the transmissivities of the network block boundary fractures were not adjusted in the effective hydraulic conductance calculations of the network blocks. Is clear from Figures 4.17 and 4.19 that the adjustment of the boundary fractures, in the effective conductance calculations, is required for an accurate dual permeability model.

4.5.2.3 Fracture System 2 - Common Aperture Statistics

The major difference between fracture system 2 of Table 4.5 and fracture system 1 is that mean transmissivity of the four fracture sets is the same in fracture system 2, whereas the fractures of the longer fracture sets of system 1 were also considerably wider. The transmissivity of all of the fractures are not
Figure 4.18: The distributions of the initial, 50th, 75th and 90th arrival times using selection criteria 6 in the dual permeability model and for the corresponding realizations of the fracture system using Discrete.
Figure 4.19: Average cumulative particle arrival at downstream boundary, when the transmissivity of the boundary fractures of the network blocks are not increased to compensate for flow losses through crossing fractures.
all the same, however. Each fracture set has a standard deviation of the log10 aperture of 0.3. In fracture system 2, the long fractures do not necessarily dominate the local hydraulic behavior of the fracture system. Figure 4.20 shows the flow distribution in one fracture network of fracture system 2. The network differs from the network shown in Figure 4.10 only in the transmissivity of the fractures. The arrangement of the fractures is identical. A comparison of Figure 4.20 with Figure 4.11 shows that a larger number of fractures are drawn in Figure 4.20, indicating that the flow in fracture system 2 is less concentrated within channels than the flow in fracture system 1.

The cumulative downstream particle arrival curves for *Discrete* and for the dual permeability model using various selection criteria are presented in Figure 4.21. With the decrease in the transmissivity of the long fractures, particles remain in the fracture network longer in fracture system 2 than they did in fracture system 1. The average cumulative arrival times predicted by the dual permeability results tend to be shifted forward roughly 5% to 10% relative to *Discrete*.

Again, all of the selection criteria resulted in good transport predictions. Selection criteria 1, which used only fracture transmissivity, was not included in figure 4.21 because of unsuccessful primary fracture network searches for a large number of realizations. The use of a minimum fracture length of 1.5m overcame this problem using selection criteria 3. In contradiction to the conclusions drawn from fracture system 1, it is not clear whether any of the selection criteria are superior. The use of selection criteria 2 and 3 resulted transport predictions that were just as accurate as those using selection criteria 6. The performance of the dual permeability model using selection criteria 4 relative to the model using selection criteria 6 presented in figures 4.19 and 4.21 may indicate that the modification to the transmissivity of the primary fractures in the effective conductance calculations is too large for fracture system 2 but too small for fracture system 1.

The good performance of the dual permeability model for fracture system 2 indicates that a fracture system is not required to have a sub-system of long, high transmissivity fractures in order for
Figure 4.20: Flow plot of one realization of fracture system 2. The fracture trace map for this realization is identical to the map shown for fracture system 1 in Figure 4.10.

Minimum flow drawn = 2.0e-11 m³/s
Figure 4.21: Average cumulative particle arrival at the downstream boundary for 100 realizations of fracture system 2.
some fractures to dominate the flow in the fracture system and that the selection criteria were able to identify the most important fractures.

4.5.2.4 Fracture System 3 - Combined Fracture Sets

Fracture system 3 of Table 4.5 focuses on the importance of connectivity in the primary fracture network and, by inference, in the dominance of local flow. Fracture system 3 is composed of only two fracture sets and has a only one scale of fracturing. Each fracture set retains the vertical and horizontal scan-line density, 2.8 fractures per meter, and mean length, 3.6 m, as the composite of the two corresponding fracture sets in fracture systems 1 and 2. A fracture network from this system is shown in Figure 4.22a. Fracture system 3 lacks the long, nearly orthogonal fractures of systems 1 and 2. Based on a visual comparison, the plot of flow rates in Figure 4.22b has a degree of channeling that is intermediate to that of fracture systems 1 and 2. In the upper right of the figure a few fractures with extremely large flows are evident. The flow rates in these fractures are influenced by the proximity of the fractures to the boundaries of the fracture network. These large flows are not important to the evaluation of the dual permeability model but are a concern in the development of models sub-REV fracture networks. The causes and effects of this interaction with the boundaries are pursued further in Chapter 5.

The dual permeability model is much less effective in simulating transport in fracture system 3 than for the earlier fracture systems, as shown in Figure 4.23. The curves in Figure 4.23 were plotted from 52 common realizations. Because of the difficulty in establishing the primary fracture network for each realization using selection criteria that tend to ignore fracture length, only selection criteria 2, 4, 5, and 6 are presented for fracture system 3. All of these criteria resulted in transport simulations that lagged behind the results of Discrete. Using criteria 4, 5, or 6, the median arrival time is delayed 20% from the mean arrival predicted by Discrete. The net flows through the fracture network are roughly 10% lower in the dual permeability simulations. Using selection criteria 2, the corresponding errors are closer to 50%
Figure 4.22: Fracture trace map and flow plot for one realization of fracture system 3. a) The fracture trace map. b) Flow rates within fractures.
Minimum flow drawn = 3.3e-11 m^3/s

Figure 4.22: Cont. Fracture trace map and flow plot for one realization of fracture system 3. b) Flow rates within fractures.
Figure 4.23: Average cumulative particle arrival at the downstream boundary for 52 realizations of fracture system 3.
delay in the cumulative arrival times and a 20% flow error.

The large difference in the performance of the dual permeability model revealed by the investigations of fracture systems 2 and 3 may indicate that a larger scale structure must exist in the fracture network for the creation of the dual permeability model to be effective. It may be that the flow channels that are evident in Figure 4.22 include too few fractures and thus are not well enough connected to be represented properly by the primary fracture network structure. Flow dominance may be so local in this fracture system that it cannot be adequately represented using the conductance approximations of the network blocks.

4.5.2.5 FRACTURE SYSTEM 4 - UNIFORM APERTURES

Fracture system 4 is the antithesis of fracture system 1. This system has neither a wide distribution in fracture length nor variation in fracture transmissivity. The negative exponential distribution in fracture length provides a range in length scales but the larger lengths of sets 1 and 2 of fracture system 1 is lacking. Other than the variation in fracture transmissivity, fracture system 4 is identical to fracture system 3. The fracture network shown in Figure 4.22 also serves as an example of fracture system 4. Dominant flow channels do not form in fracture system 4 as shown in flow plot of Figure 4.24. There are variations in the flow rates of different fractures but these are small.

Since there is no transmissivity variation in fracture system 4, only selection criteria 2 was used to find the primary fracture networks in the dual permeability model. The dual permeability model is less effective simulating fracture system 4 than simulating fracture system 3. The average net flow rate calculated for the fracture networks is 20% less in the dual permeability simulations compared to Discrete. As shown in Figure 4.25, the downstream arrival times average a delay of roughly 30% in the median arrival time. The test of fracture system 4 provides further evidence that the dual permeability concept is limited to fracture systems that have dominant flow pathways.
Flow through one realization of fracture system 4, a fracture system without variation in fracture aperture. The fracture trace map is identical to Figure 4.22a.

Minimum flow drawn = 4.5e-12 m³/s
Figure 4.25: Average cumulative particle arrival at the downstream boundary for 100 realizations of fracture system 4.
4.5.2.6 A WAR ZONE MODEL; FRACTURE SYSTEM 5

The bimodal range of fracture lengths that exists in fracture systems 1 and 2 is an inherent feature of the war zone structural model (Sections 2.4.5 and 3.4.3). In the war zone model there is a length scale defined by the fracture set parameters and a length scale defined by the size of the war zones.

Fracture system 5 is nearly the same as fracture system 1, except that a war zone structural model is used to generate the fracture network realizations. The war zones are defined by the two sets of longer fractures. The other two sets define the fractures created within the war zones. Some of these fractures in these latter two sets are generated without respect to the war zones and others are generated exclusively within each war zone. The density of these two fracture sets was reduced so that the sets have the same scan-line densities in fracture system 5 as in fracture systems 1 and 2. Figure 4.26a is a trace map of a realization from fracture system 5. The particular arrangement of war zones makes the fracture density appear quite blocky in this figure. There are large regions of both high and low densities of fractures. The flow plot presented in Figure 4.26b shows the strong influence that the region of low fracture density in the lower right of the domain has on the flow pattern. Much of the flow entering the lower part of the domain is drained off by a highly transmissive fracture that defines the boundary of the low density region.

The averaged transport characteristics of the war zone fracture networks are nearly identical to those of fracture system 1; an indication that the larger fractures truly dominate the flow within these fracture systems. For the selection criteria that were tested, the choice of selection criteria have little influence on the dual permeability results unless fracture transmissivity is completely ignored, as shown in Figure 4.27.
Figure 4.26: Fracture trace map and flow plot for one realization of fracture system 5, a war zone version of fracture system 1. a) The fracture trace map. b) Flow rates within fractures.
Minimum flow drawn = 9.8e−11 m³/s

Figure 4.26: Cont. Fracture trace map and flow plot for one realization of fracture system 5, a war zone version of fracture system 1. b) Flow rates within fractures.
Figure 4.27: Average cumulative particle arrival at the downstream boundary for 76 realizations of fracture system 5.
Fracture system 6 closely mimics fracture system 3 except for the use of a war zone structural model to generate the realizations. Fractures of the first fracture set define the war zones in fracture system 6. The second fracture set represents the later fractures that are created preferentially within the war zones. Figure 4.28a presents the fracture trace map for one realization of this fracture set. Comparing Figures 4.28a and 4.24a reveals that the war zone fracture system has a much greater variation in the clustering of the fractures. There is a clumpiness to the distribution of fractures in Figure 4.28a that is not evident in Figure 4.24. The size of the war zones is related to the size of the fractures that define them, leading to war zones of a smaller scale in fracture system 6 compared to those of fracture system 5. The difference in the relative size of the fractures within the war zones compared to the war zone defining fractures is much smaller in fracture system 6 compared to fracture system 5. As was done for fracture system 5, the scan-line density of set 2 was reduced so that the scan-line density of all the set 2 fractures is equivalent to the scan line density of set 2 in fracture system 3. The war zone model, however, creates fractures that have smaller lengths than the fractures defined by the set parameters.

There is a similarity between the flows rates depicted in Figure 4.28b for fracture system 6 and the flow rates of Fracture system 3, in part because the fractures of fracture set 1 are identical in the two realizations. The flows in fracture system 6 appear more channeled than in fracture system 3, but the differences are slight. More importantly, these channels are more continuous in fracture system 6 than in fracture system 3.

Despite the similarity in the flow diagrams for fracture systems 3 and 6, the performance of the dual permeability model for these two fracture systems is radically different. As shown in Figure 4.29, the arrival times predicted using selection criteria 4, 5 or 6 are within five percent of the arrival times predicted using Discrete. The difference may be a result of fundamental differences in the nature of the flow systems in fractures systems 3 and 6; namely, the larger scale of dominate flow channels with respect
Figure 4.28: Fracture trace map and flow plot for one realization of fracture system 6, a war zone structural model version of fracture system 3. a) The fracture trace map. b) Flow rates within fractures.
Flow in the Major Carriers

Minimum flow drawn = 3.3e-11 m³/s

Figure 4.28: Cont. Fracture trace map and flow plot for one realization of fracture system 6, a war zone structural model version of fracture system 3. b) Flow rates within fractures.
Figure 4.29: Average cumulative particle arrival at the downstream boundary for 45 realizations of fracture system 6.
Reviewing the first six fracture system indicates the bimodal length scales of fracture systems 1, 2, 5, and 6 leads to flow dominance that has a larger scale than the majority of fractures and that this situation is well suited to simulation using the dual permeability concept. In fracture systems 1, 2, and 5, two fracture scales were defined explicitly through the fracture set definitions. In fracture system 6, and to a lesser degree in fracture system 5, the two scales were an inherent feature of the war zone structural model.

4.5.2.8 THE LEVI-LEE MODEL; FRACTURE SYSTEM 7

Fracture system 7 is based on the Levy-Lee structural model (Sections 2.4.8 and 3.4.4). The fracture system is composed of two orthogonal fracture sets. The scan-line density of each set is the same as in fracture system 3. The fracture spacing follows a negative power law distribution with an exponent of -1.3. Fracture lengths and the variation of orientation are linearly related to the fracture spacing. The distribution of fractures is highly clustered as shown in Figure 4.30a. Shorter fractures are more highly concentrated than the longer fractures making the larger scale structure defined by the longer fractures more evident in Figure 4.30a than any of the other fracture systems. Channeling with the Levy-Lee fracture system (Figure 4.30b) is much stronger than in fracture system 1 (Figure 4.11).

The transport predictions of the dual permeability model are in close agreement with Discrete for the Levy-Lee fracture system (Figure 4.31). Using selection criteria 4 the dual permeability model is within 2 percent of Discrete for all of the mean arrival times other than the initial arrival. All of the results show small increases in the temporal dispersion of the particles.

There is no correlation between fracture transmissivity and length in the Levy-Lee fracture system. The excellent agreement of the transport predictions of the dual permeability model with those of Discrete may indicate that the large scale connectivity of the long fractures is more important to flow dominance.
Figure 4.30: Fracture trace map and flow plot for one realization of fracture system 7, generated from the Levi-Lee structural model. a) The fracture trace map. b) Flow rates within fractures.
Figure 4.30: Cont. Fracture trace map and flow plot for one realization of fracture system 7, generated from the Levi-Lee structural model. b) Flow rates within fractures.

Minimum flow drawn = 3.9e\(-11\) m\(^3\)/s
Figure 4.31: Average cumulative particle arrival at the downstream boundary for 70 realizations of fracture system 7.
than the linkage of highly transmissive fractures. The structure of a fracture network generated using the Levy-Lee model meets the requirements of the dual permeability model well because the flow dominance is strongly associated with fracture connectivity and hence length. As with two scales of fracturing, the dual permeability concept is well suited to the broad range of fracture scales in the Levy-Lee structural model.

4.5.2.9 POTENTIAL BIAS INTRODUCED BY FAILURE TO FIND PRIMARY FRACTURE NETWORK

The dual permeability solute transport calculations for fracture system 1 are only presented for the 70 realizations for which all seven selection criteria resulted in an acceptable primary fracture network. For this fracture system, there were 130 other realizations where the dual permeability model could not determine a primary fracture network for one or more of the selection criteria. The failures to find an acceptable primary fracture network occurred in different realizations for different selection criteria, suggesting a weak correlation between the failures of most of the selection criteria. However, failure to identify an acceptable primary fracture network must be dependent on the nature of the fracture network. A correlation may exist between the failure of the primary fracture selection algorithm and the transport characteristics of the fracture network. If so, neglecting these networks introduces a bias into sampling of the fracture system which affects the flow and transport characteristics of both the *Discrete* calculations and those of the dual permeability model. This bias is unrelated to the accuracy of the dual permeability model for individual networks, the purpose of the present evaluation. The bias may be important if the results of a Monte Carlo study are to be considered representative of the fracture system.

The possibility of bias introduced by failure to find an acceptable primary fracture network was investigated by comparing the arrival statistics for realizations that were used in the Monte Carlo evaluation to those that were rejected because one or more of the selection criteria caused the primary fracture selection algorithm to fail. The difference in mean arrival times of the 70 realizations of the program *Discrete* were compared to the other 130 realizations using a Student's t test. The comparison
indicated that the mean arrival times of the 70 realizations were earlier than the 130 other realizations, but these differences could be due to random fluctuation. The difference in the averages is less than the delay introduced by the dual permeability model. Therefore, arrival time calculations of the dual permeability model for the 70 realization are delayed in comparison to the Discrete results for all 200 realizations, but less so than presented in Figure 4.17.

The investigation of bias was also performed for the other fracture systems. The comparisons for most of the fracture systems (3, 4, 5 and 7) do not indicate a statistically significant bias in the realizations used for the Monte Carlo analysis. However, the results from the 100 realizations of fracture system 2 are biased towards earlier times. More detailed comparisons were made by comparing successful to unsuccessful realizations for each selection criteria. These individual comparisons indicate that the bias in the fracture system 2 results is predominantly due to section criteria 3 and 6 which weight fracture aperture most heavily. The bias introduced using these selection criteria decreases the average arrival times by roughly five percent. The data for fracture system 6, Figure 4.29, is also biased toward earlier arrival times. The differences in the mean arrival times between the realizations used in Figure 4.29 and all of the realizations are larger than the delays introduced by the dual permeability model. Thus, the dual permeability arrival times shown in Figure 4.29 are earlier than the mean arrival times of Discrete using all of the realizations. If the realizations that were rejected because of selection criteria 6 were added to those used in the Monte Carlo evaluation then the bias would be reduced to below a statistically significant level.

This analysis of bias is based on too few realizations to be considered definitive. It also relies on an assumption of a normal distribution in the arrival times, even though the data presented in Figure 4.18 indicates that the arrival times are not normally distributed about the mean. The analysis does demonstrate a possible bias introduced by primary selection failures is a potential source of error introduced by the dual permeability model; an error that is not reduced through the use of Monte Carlo analysis. This bias
is independent of the inaccuracies introduced by the dual permeability calculations for individual realizations.

4.5.2.10 SUMMARY OF MONTE CARLO EVALUATION

This evaluation of the dual permeability model demonstrates that the technique can provide accurate predictions of flow and transport through fracture networks composed of thousands of individual fractures. The failure of the dual permeability model to adequately simulate the hydraulic behavior of fracture systems 3 and 4 indicates that the existence of a flow structure that is somewhat larger than most of the fractures is required for the modeling approach to work. The success of the dual permeability model in simulating fracture systems 2 and 6 indicates that the fractures that dominate flow do not have to be extremely different than the majority of fractures, nor are they necessarily those that are the most transmissive. Indeed, the distinction between the flow regimes of fracture system 6 and fracture system 3 is subtle.

The dual permeability approach requires that the fractures within the fracture network vary in transmissivity. This is not a constraint that conflicts with our general view of fracture systems. The approach also requires a larger separation of fracture lengths than is defined by a negative exponential distribution. Certainly some fractured rock have broader length ranges than this but some may have smaller ranges. It is uncertain at this time whether the length constraint seriously limits the applicability of the dual permeability approach.

4.5.3 ACCURACY AS A FUNCTION OF NETWORK BLOCK SIZE

One of the controlling factors of the primary fracture network is the limit on the maximum size of the network blocks. The number of primary fracture intersections and hence the memory storage requirement of the model decreases with increasing network block size. In this section, the influence of maximum block size on model accuracy is investigated. The maximum network block size used in the
Monte Carlo investigations of section 4.5.2 was never more than 600 fracture intersections and never less than 400. One hundred realizations of fracture system 1 using selection criteria 6 were repeated using block size limits of 200, 400, 600, 800, 1000, and 1500 fracture intersections. The results presented in Figure 4.32 show that the accuracy of the dual permeability model is influenced by the maximum block size. The figure presents the average percentile arrival times that are familiar from the Monte Carlo investigations. Allowing for some variations due to limited sample size, the results show a monotonic decrease in accuracy as the network block size increases.

The legend in the upper left of the figure relates the block size limit to the different curves. After the arrow in the legend, the number fractures that were selected to be primary fractures based on their geometric characteristics is listed as a percent of the total number of fractures in the flow domain. The total number of fractures that are at least partly included in the primary fracture network is larger because of the addition of connecting fractures to the network (see Section 4.2). The entire primary fracture network includes about one and a half to three times as many fractures than the percentages listed. The number of primary fractures is directly related to the block size limit. Recall that the primary fracture selection routine attempts to limit the number of primary fractures to the specified percentage. For the 800 and 1500 block sizes the specified number of primary fractures was large enough to bias the final number of primary fractures slightly. Only a small percentage of the fractures in a network are included in the primary fracture network. As this percentage increases, the results become more accurate. In the limit, as the number of primary fractures approaches all of the fractures in the fracture network, the flow and transport predictions of the dual permeability model would approach those of Discrete.

4.5.4 TIME AND MEMORY

The fundamental purpose of the dual permeability approach is to provide a modelling capability that allows the investigation of fracture systems that cannot be studied using the conventional discrete fracture approach because of computer resource limitations. These may be limitations of either execution
Figure 4.32: Average cumulative particle arrival for different network block size limits. The curve legends indicate the maximum number of fracture intersections included in a network block. Following the arrows in the legend, the percentage of fracture that were selected to be primary fractures is listed.
time or memory storage requirements. An implicit assumption is made here that some form of approximation must be introduced in order to simulate flow and transport at the field scale. The emphasis of the model developed for this thesis, however, has been on investigating the dual permeability concept, not in maximizing computational efficiency.

The execution time and memory storage requirements of the dual permeability model and Discrete were determined for a series of fracture networks of increasing scale. The scale is measured in terms of the number of fracture intersections that remain after the fracture network is reduced to only the fracture segments that carry flow. The number of fracture intersections is the number of equations that are required to solve for flow within Discrete. Calculations were performed, for fracture system 1 using selection criteria 6 and a maximum block size of 800, on a Hewlett-Packard 735 workstation.

Table 4.7 presents four sets of data. Two sets of data are presented for the dual permeability model. One of these sets is for a model that uses a sparse direct solver to find the coarse-scale head distribution. The other dual permeability data set is from calculations using a sparse, conjugate gradient iterative solver based on LU decomposition, DILUCG. The results of Discrete are presented next. The final set of data is for a modified version of fracture system 1 where the variability of the log10 of the aperture has been increased 0.5. For this latter case two results are presented, one for the dual permeability model using the direct solver and one for Discrete. The dual permeability model using DILUCG did not converge.
The first column of the table lists the dimensions of the domain. The second column is the size of the domain measured in terms of the number of fracture intersections in the reduced fracture network. The total computer memory requirement is next, followed by the memory needed for the flow solution. The memory required by the dual permeability model for the residence time distributions of the network blocks is listed under the heading Trans. The memory required for the flow solution listed for the dual permeability models is the maximum of either the coarse flow solution or the largest network block flow calculation.

The dual permeability model that used the direct solver requires more memory than either of the other two models. The program requires about 40% more memory for each calculation than does Discrete. The dual permeability model using the iterative solver requires a bit less, about 20% less than Discrete. The memory requirements of the network block residence time distributions are about 10-15% of the total requirements of the dual permeability model. This requirement is significant but does not dominate the total memory needed.

The total time required to calculate both flow and transport is listed in the sixth column. The following columns list estimates of the times taken to perform specific functions within the simulations. The first of these is the time needed to create and then reduce the fracture network to just the fractures that have flow within them. This procedure dominates the execution time of the larger networks. The next two columns list the time needed to determine the primary fracture network, PFN, and to find the effective conductances of the network blocks, NB. Both dual permeability models use identical procedures for this stage of the calculations. The time estimates are approximate and should be considered accurate to roughly 10% of the times listed. The table shows that determining the primary fracture network requires less than 10% of the total simulation time. Finding the effective conductances of the network blocks is a time consuming process, however. The next column lists the time used to calculate the coarse-scale head distribution in the dual permeability models and the time used to calculate the full head
distribution in *Discrete*. For large domains the dual permeability model is significantly faster than *Discrete* if the direct solver is used and significantly slower if the iterative solver is used. The time required for the iterative solution of the coarse-scale flow solution increases with domain size at a much faster rate than the ICCG routine used within *Discrete*, a feature that may be caused by the lack of symmetry in the coarse-scale flow equations.

The next two columns of the table list the times used in the transport calculations. These time are based on using 50,000 particles. The column titled RTD is the time needed to calculate both the flow distribution within the network blocks of the dual permeability model and the time required to create the residence time distribution tables. The time to calculate the residence time distribution table is about 10% of the total. The transport column lists the time used to calculate particle movement. The dual permeability model uses approximately one half to one third of the time to move particles, but the transport time is small compared to the total simulation time. The time needed to set up the residence distribution tables in the dual permeability model is roughly 10% of the time spent moving the particles.

The final two rows of Table 4.7 are data from calculations of a modified fracture network. For this network the standard deviation of the log$_{10}$ aperture was increased from 0.2 to 0.5 for all sets of fracture system 1. This standard deviation in aperture is large but not unreasonable for fractured rock. The dual permeability model using the direct solver is not sensitive to this change in the fracture system. Iterative solvers are sensitive to this change. The dual permeability model using DILUCG did not converge in 10,000 iterations. *Discrete* did converge but the time required by the ICCG solver increased from 1,286s to 5,660s. The dual permeability model with the direct solver took about one third of the time need by *Discrete* to solve the problem.

For fracture system 1, the dual permeability model using the direct solver does not have a significant computational advantage over *Discrete* even though the calculation of the coarse-scale head distribution and subsequent determination of flow in the network blocks requires much less time than the
solution of the hydraulic head distribution for the entire network. The time spent determining the primary fracture network and calculating the effective conductances of the network blocks more than compensates for this time advantage. The time requirements of the iterated solver of dual permeability model increases more rapidly than those of Discrete making this version much slower for the simulation of flow in large domains. The last set of calculations indicates that the dual permeability model can have a significant advantage for some problem definitions.

The reader is reminded that the development of the current dual permeability has not emphasised computational efficiency. Some improvements to the current model that could have a significant influence of model efficiency are suggested in Sections 4.7.5 and 4.7.6. However, the true purpose of model developed for this thesis is to provide a tool to explore the dual permeability concept and the importance of flow dominance to forming sub-continuum approximations. The current model serves as a platform from which further approximation of fracture network flow and transport behavior can be isolated from the more basic influences of separating a fracture network into blocks.

4.6 COMPARISON TO REDUCED FRACTURE SIMULATION

How do the inaccuracies introduced through the dual permeability model compare to other methods of approximating the fracture network? One such approach is to reduce the fracture network by ignoring small aperture fractures. This method was used by Herbert et al. [1991] in their three dimensional simulations of the Stripa test site (See Section 2.3.6.). Herbert et al. investigated the influences on both flow and transport of simulating only the largest fractures. They found that using 70% of the fractures reduced the net flow though a cube 12.5 on a side by 10%. Using only 30% of the fractures reduced the net flow by 30%. The selection of fractures to be retained in the model was based both on the length of fractures and on the aperture. The fracture system at the Stripa site was modeled using a standard deviation in natural logarithm of aperture in the range of 0.8 (0.35 log10). This variation in aperture is slightly larger than the aperture variations of the fracture systems listed in Table 4.5. More
importantly, the fracture system simulated by Herbert et al. is denser and more highly connected than the networks simulated in this study which makes the fracture system of Herbert et al. less sensitive to fracture reduction than the fracture systems used in this investigation.

The transport simulations at Stripa were made using 30% of the fractures in the fracture network. The applicability of the transport model was tested using 5m cubes. The results of these tests indicated a general agreement in the transport times between the complete and reduced network calculations, with less dispersion in the reduced fracture network. A more specific influence of the fracture reduction on transport could not be determined from the results published in Herbert et al. [1991] because approximations to particle transport within individual fractures were also included in the reduced network simulations. These single fracture particle transport approximations were not included in the complete network simulations.

In this section, the dual permeability model is compared to the simulation strategy of reducing the number of fractures in the networks by eliminating the least transmissive fractures. For each of the six fracture systems with variable transmissivity, transport sensitivity to fracture reductions of 10%, 25%, 50%, and 75% were investigated, using fracture domains of 40m by 40m. The dual permeability model represents about 2% of the fractures using the primary fracture network and approximates the rest using the network blocks.

In Figure 4.33, the results reducing the fracture networks of fracture system 1 are presented, using dashed lines as labeled in the upper left corner of the figure. In addition, the results from using the full fracture system are plotted using a solid line and the dual permeability model results, for selection criteria 6, are shown using a dotted line. Only the smaller fracture sets were reduced for this fracture system leaving the dominant fractures unaffected. This reduction strategy contains an a priori recognition of the importance of the long fractures. All of the reduced networks have initial arrival times that are close to the initial arrival of the full networks. Reducing the number of fractures causes later particles to arrive
Figure 4.33: Fracture system 1 mean particle arrival times for fracture networks that have been reduced by removing small aperture fractures of fracture sets 3 and 4.
earlier than they do in the full networks; decreasing the temporal dispersion of the particles. This result is in agreement with the findings of Herbert et al., but is in contrast to the dual permeability model which has a slight increase in temporal dispersion. Except for the initial arrival time, the cumulative arrival times of the dual permeability model are closer to the full network than the arrival times of any of the reduced fracture simulations.

The results from reducing fracture system 2, which has the same transmissivity statistics for each of the four fracture sets, are presented in Figures 4.34 and 4.35. In Figure 4.34, only fracture sets 3 and 4, the short fractures, are subject to fracture reduction. For the smaller reductions in the number of fractures, 10% and 25%, the influences on transport are similar to those of fracture systems 1. With a 50% reduction, and more clearly with a 75% reduction, a different behavior is evident. The arrival times using a 75% reduction in the number of fractures can be later than using a 50% reduction. This reversal in trends is an indication that fractures are being removed that provide important connectivity to the fracture network, a feature that is more clearly displayed in the investigation of fracture system 3 (Figure 4.36).

Applying the fracture reduction to all of the fracture sets results in the cumulative arrival curves shown in Figure 4.35. Removing the longer low transmissivity fractures results in a significantly larger reduction in dispersion. A comparison of Figure 3.34 and 4.35 indicates that although the long fractures have a more significant impact on dispersion, the highly transmissive fractures that were removed from sets 3 and 4 instead of these long fractures have greater influence on flow in the dominant channels of the full network. It should be noted that for these and all other simulations in this section, realizations that resulted in greater than 10% of the particles crossing the top or bottom boundaries were not used. The cumulative arrival times are based solely on particles that crossed boundary 4, the right side. Rejecting realizations in which these constraints were violated led to a slightly different set of realizations for Figures 3.34 and 3.35. Note that the cumulative arrival times of the dual permeability model match
Figure 4.34: Fracture system 2 mean particle arrival times for fracture networks that have been reduced by removing small aperture fractures of fracture sets 3 and 4.
Fracture System 2 — All sets reduced

Figure 4.35: Fracture system 2 mean particle arrival times for fracture networks that have been reduced by removing small aperture fractures of all fracture sets.
those of *Discrete* for the full fracture system closer than the cumulative arrival times of a 10% reduction in the fracture network at later arrival times but not for the early times in both Figure 4.34 and 4.35.

Figure 4.36 presents the arrival curves for fracture system 3. The difference in character between the influence of a 10% and 25% reduction in the number of fractures and the 50% and 75% reductions is a vivid demonstration that at some point removing fractures starts to influence the dominant flow channels. When this occurs the major influence on transport of removing fractures changes from primarily decreasing dispersion to substantially delaying the arrival of particles. The dual permeability model which contains less than five percent of the fractures in the primary fracture network provides a better representation of transport compared to a simulation where 25% of fractures remain after fracture reduction, but for less drastic reductions predictions of the dual permeability model are worse than those from the reduced networks.

The fracture reduction of fracture system 5 which used a war zone structural model was limited to the fracture sets than were preferentially created in the war zones. This method of reducing the fracture network recognizes the important role that fractures that define the war zones play in connecting the zones together. As shown in Figure 4.37, this war zone fracture system was less sensitive to the reduction in fractures than fracture system 1.

For fracture system 6, fracture reduction was applied to all the fracture sets which resulted in the loss of some war zones. Fracture removal was applied prior to the identification of war zones. Because of the way the fracture networks are generated, the 100 realizations used to determine the cumulative arrival curves shown in Figure 4.38 differ for each curve. This introduces a variation between curves that does not exist in the other figures of this section. I do not believe that the influences on the plots are significant, but it may explain why even the 10% and 25% fracture reductions result in a slight delay in the initial arrival times. This delay may instead indicate that this fracture system was sensitive to a minor loss of connectivity between war zones. The initial arrival times for the 50% reduction are delayed by

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Figure 4.36: Fracture system 3 mean particle arrival times for fracture networks that have been reduced by removing small aperture fractures of both fracture sets.
Figure 4.37: Fracture system 5 mean particle arrival times for fracture networks that have been reduced by removing small aperture fractures of war zone dependent sets 3 and 4.
Figure 4.38: Fracture system 6 mean particle arrival times for fracture networks that have been reduced by removing small aperture fractures of both sets.
about 30% from the full network, much more than can be attributed to variations between realizations. With a 75% reduction in the number of fractures, the 25th, 50th, 75th, and 90th percentile arrival times are in close agreement with the full network. Based on an interpretation of Figure 4.36 from fracture system 3, the arrival times are increasing rapidly with increasing fracture reduction at the 75% level and, by coincidence, agree with the full network at this reduction level. For this fracture system, the dual permeability model is superior to a fracture reduction strategy.

Fracture reduction on fracture system 7 which uses a Levi-Lee structural model has a greater influence of the estimated dispersion than on any of the other fracture systems. The cumulative arrival times displayed in Figure 4.39 show that the average initial arrival times are not influenced by fracture reduction, even with 75% of the fractures removed. Recall from Figure 4.30 that flow through this fracture system is strongly dominated by long wide fractures that span the domain. One interpretation of the arrival times presented in Figure 4.39 is that although fracture reduction leaves the dominant flow paths intact the interaction between these dominant fractures and the less insignificant fractures has been lost. The dual permeability model contains far fewer fractures within the primary fracture network than remain in even the most highly reduced fracture networks but, since the other fractures are approximated within the network blocks, the essential interaction between the dominant fractures and the rest of the fracture network is still retained.

In summary: 1) Reducing the fracture network reduces particle dispersion leading to earlier arrival times for later particles, 2) The initial arrival times are not significantly influenced by fracture reduction based on aperture until the 50% reduction is reached, 3) The dual permeability model tends to result in closer agreement in the cumulative arrival times compared to the arrival times of a 10% reduction in the fracture network; fracture system 3 excepted. 4) The interaction between the dominant fractures and the least important fractures is lost through fracture reduction but not in the dual permeability model.
Figure 4.39: Fracture system 7 mean particle arrival times for fracture networks that have been reduced by removing small aperture fractures.
4.7 ALTERNATE APPROACHES TO NETWORK BLOCK FLOW MODELS

4.7.1 INTRODUCTION

In the introductory chapter, it was emphasised that the dual permeability model presented in this chapter is one implementation of the dual permeability concept; there are a variety of possible approaches to structuring the hierarchy of primary fractures and network blocks and to approximating the flow and transport behavior in the fracture networks within the network blocks. The present formulation of the network blocks (Section 4.3) requires repeated solution of the network block flow equations to fully define the hydraulic characteristics of the blocks. The calculated flows have been demonstrated to provide enough accuracy to be used for solute transport. Early attempts to formulate a flow model for the network blocks were based on computationally more efficient algorithms, but these formulations were either not compatible with the particle tracking transport algorithm or suffered from unacceptable inaccuracy at the level of individual realizations.

The first of these formulations is presented in Section 4.7.2. Network blocks were represented within the dual permeability model using finite elements with a conductivity defined by the tensor representation of conductance for each block. The tensor representation presented in Chapter 5 emerged from the development of this network block model. This network block flow model is presented to explain why a more specialized formulation was needed and to reinforce the notion that the model presented in this chapter does not define the dual permeability concept.

A third network block approach is presented in Section 4.7.3. This approach is similar to the present model except that the head distribution along the bounding primary fractures was treated as a boundary condition to the network blocks. The head gradient along the boundaries of the blocks was assumed to vary linearly between the primary fracture intersections. The failure of this model to provide adequate transport predictions points to some serious limitations in sub-REV scale modeling that rely on
boundary conditions that are not sensitive to heterogeneity within the domain. This issue is pursued further in the concluding chapter of this thesis.

A fourth alternative is presented in Section 4.7.4. The major innovation in this alternative, over the present model of Section 4.3, is the use of iteration to improve the estimate of the hydraulic head provided by the flow model. The flow solution provided by this alternative is identical, within numerical precision, to that provided by Discrete. However, the number of iterations needed to reach an adequate flow solution is larger than the number of iterations that Discrete requires to reach a flow solution. This alternative has not been explored in great detail. An improvement in primary fracture selection, in the algorithm used in the iteration, or the use of parallel processors may make iteration of the head distribution an attractive alternative.

4.7.2 Finite Element Network Block Model

The REV needed to represent flow may be smaller than the REV needed to represent transport. A dual permeability model that represents the fracture network as a continuum for the purposes of establishing regional flow rates but discretely for the purposes of estimating local flow rates and solute transport has the potential to be extremely efficient. The network block formulation described in this section uses triangular finite elements with homogeneous hydraulic conductivity to represent the conductance of the network blocks and line elements to represent primary fractures. See Baca et al. [1983] for a description of line element representation of fractures. The line elements were located along the edges of the network blocks.

The triangular elements were formed using the centroid of a network block as one vertex of all the triangles within the block. The other vertices of a triangle were located at the intersections of the primary fracture segments that define the boundaries of the blocks. An equivalent conductance matrix (stiffness matrix) was then formed by combining the finite element stiffness matrices. The centroid of
each of the blocks, which is not a primary fracture intersection, was removed in the procedure that formed the coarse-scale flow model.

To obtain the conductivity for the triangular finite elements, each network block was tested using constant hydraulic head boundary conditions along the boundaries of the network block that were consistent with a regional gradient oriented first at 0° and then at 90°. The directional flow through the blocks was measured by summing the directional components of flow at each boundary fracture as described in procedure 1 of Section 5.4.2.

The use of triangular finite elements required that the network blocks be convex. By convex, I mean that the shape of a network block must be such that any line passing through the block would intersect the block boundaries at only two locations. To ensure convexity some boundaries of the blocks were defined by straight lines connecting primary fractures rather than always being actual fractures. These boundary lines were treated as primary fractures of zero aperture.

The finite element representation of the network blocks was used only to estimate the hydraulic heads at the primary fracture intersections, just as the coarse-scale flow model is presently used. Fine scale flow within individual network blocks was determined by using the hydraulic head values of the primary fracture intersections as boundary conditions for the network blocks. Linear head gradients along the primary fractures were assumed. The use of linear gradients along the network block boundaries is consistent with the assumptions made in approximating the network blocks using triangular finite elements with linear basis functions, but is incompatible with the particle transport algorithm used to determine the residence time distributions of the network blocks.

Combining the flows from the network blocks with flows calculated for the primary fractures led to local mass balance errors that were unacceptable for use with the particle tracking transport algorithm. Efforts to eliminate these local mass balance errors by modifying the common hydraulic heads along the
zero aperture boundary lines of adjacent network blocks led to the creation of fictitious circular flow patterns. The finite-element based formulation of network block conductance does not provide an acceptable flow estimate for the particle tracking routine described in section 4.5. It may provide an adequate flow estimate if a transport algorithm that is less sensitive to local mass balance errors were used. One possibility might be a continuum based transport routine such as the transport model proposed by Schwartz and Smith [1988].

4.7.3 EFFECTIVE CONDUCTANCE WITH LINEAR BOUNDARY CONDITIONS

In a second network block model, the effective conductance of the network blocks was calculated directly, in order to increase the accuracy of the network block flow approximation compared the use of finite elements. The technique was the same as the one described in Section 4.3 except that the network block flow model did not include the primary fractures. Instead, a linear hydraulic head gradient was assumed to exist along the primary fractures.

This formulation successfully eliminated local mass balance errors. Despite this fact, this network block model led to poor transport characteristics when viewed over the entire fracture network domain. The problem with this model formulation is a fundamental difficulty in forming sub-REV flow models. There is a strong correlation between local hydraulic head gradients and local heterogeneity. In fractured media, if the correlation is ignored on the boundaries of modeling blocks then excessive flow rates may be estimated within large aperture fractures that intersect the boundaries. Further implications of ignoring internal heterogeneity in specifying the boundary conditions are pursued in Chapter 5 and addressed in a discussion in Section 6.1.

4.7.4 ITERATED FLOW SOLUTION (DOMAIN DECOMPOSITION)

The formulation of the dual permeability model rests on the subdivision of the fracture network into distinct parts. This concept relates the dual permeability approach to the concepts of domain
decomposition. The term domain decomposition encompasses numerous techniques that are based on the separation of domains. See for example Chan et al. [1988] and Keyes et al. [1992]. To my knowledge, none of these techniques use approximate solutions based on a non-iterated coarse-scale calculation. Therefore, I am using the term domain decomposition to describe only the technique of iterating the primary fracture hydraulic head distribution to reach a hydraulic head solution that agrees, within numerical precision, with that of *Discrete*.

The composition of the network blocks differs from the model described in Section 4.3. In the standard dual permeability formulation, the network blocks include the primary fractures along the block boundaries. Thus, two contiguous network blocks may have different hydraulic heads at locations that represent the same intersection of fractures. The dual permeability formulation forces a mass balance at these points but not continuity of hydraulic head. In the iterated procedure continuity of hydraulic head is maintained by excluding the primary fractures from the network blocks and using the entire hydraulic head distribution along the primary fractures to define the boundary conditions of the blocks.

A mass balance error is calculated at each intersection of the boundary primary fracture segments with fractures within the network block. The error is the sum of the flows entering the intersections from the network blocks and flows entering the intersection from the primary fracture segment. The mass balance error is the error residual used in a conjugate gradient iteration procedure. Local conservation of mass at each intersection along the primary fracture ensures that the hydraulic head solution is consistent with *Discrete*. The solution for hydraulic head within the model domain must be found through iteration because the primary fracture hydraulic head distributions depend on the flows calculated within the network blocks.

Figure 4.40 is a flow chart of the iteration procedure. The first step is to provide an initial estimate of hydraulic head in the primary fractures by creating the coarse-scale model described earlier. The hydraulic head calculated along the boundaries of two adjacent network blocks is averaged to obtain
Figure 4.40: Flow chart of the conjugate gradient algorithm used to iterate the hydraulic head solution along primary fractures.
a head distribution along each primary fracture segment. The network block flows are recalculated using these averaged hydraulic heads as new boundary conditions.

The conjugate gradient iteration approach applied to domain decomposition was developed in a variational framework by Glowinski and Wheeler [1988] and applied in a finite element domain decomposition model of groundwater flow by Beckie et al. [1993]. The conjugate gradient method used in the dual permeability model follows the application of Beckie et al. As pointed out by Beckie et al., solution by conjugate gradient based domain decomposition (DDCG) is much slower than by a conjugate gradient technique with convergence acceleration enhanced by preconditioning the entire $\tilde{A}$ matrix using incomplete Cholesky decomposition (ICCG). These authors found the DDCG technique to take on the order of 45 times as long as ICCG, for the problems they investigated. For the one fracture network used to test this model, the iterated hydraulic head solution of the dual permeability model took approximately 27 times as long to obtain a flow solution as Discrete, which uses an ICCG solver. The iterated solution uses a direct solver to obtain flow solutions within the network blocks which probably explains its relative speed in comparison to the results of Beckie et al. They were able to reduce the solution times to 5 to 10 times the ICCG solution time by applying a multigrid-based convergence acceleration technique. Multigrid acceleration was not attempted in the dual permeability model because of difficulty applying the multigrid technique to the irregular geometry of the network blocks.

For the investigations addressed in this thesis, computer execution time rather than memory storage requirements have been the constraining resource. Because the initial investigations of the iterated version indicated that this version of the dual permeability was much slower than Discrete, it has not been pursued to the point where its utility in comparison to the non-iterated version or Discrete can be conclusively established. For other problems, where only a few realizations of a large model domain are required, the lower memory requirements of the iterated version may be an advantage over Discrete. The memory storage requirements for the coarse-scale hydraulic head distribution are smaller for the iterated version
than for the non-iterated version of the dual permeability model using an iterative solver to find the coarse-scale head distribution. It is unclear at this point what the relative speed of these two versions would be for extremely large domain sizes. Therefore, while the iterated or domain decomposition approach to the dual permeability model has been abandoned for this research, there is a possibility that it has the potential to be a useful tool in investigating extremely large fracture domains.

4.7.5 IMPROVEMENTS TO THE CURRENT NETWORK BLOCK FLOW MODEL

The memory usage and problem execution time study of Section 4.5.4 have revealed two aspects of the effective conductance approximation that show obvious room for improvement. The first aspect is the time required to determine the effective conductance of each block. The second aspect is the slow convergence of the iterative solution of the coarse-scale head distribution.

In Table 4.7, the time required to determine the effective hydraulic conductances of the network blocks is much larger than the time required to calculate the coarse-scale head distribution. Additional investigations, that were not reported in Section 4.5.4, have shown that longer times are required to determine the effective conductances of fewer network blocks with many primary fracture intersections along their boundaries than to find the conductances of more blocks but with fewer primary fracture intersections along each block boundary. A substantial reduction in these times may be brought about by determining the block conductances in a single calculation rather than by performing an separate flow calculation for each primary fracture intersection as is described in Section 4.3.1. To perform this calculation an effective means of inverting a portion of the complete stiffness matrix describing the network block flow equations must be determined. This inversion need not be exact; the effective conductance of a network block is already an approximation because of the way the block boundary conditions are treated.
The equations that describe the effective conductance of a network block are non-symmetric. The non-symmetry arises from the difference in the equations describing the hydraulic head response of the primary fracture intersections from the equations of the midpoints. The non-symmetry of the coarse-scale flow equations can have a large influence on the time required to find the coarse-scale head distribution using an iterative solver and roughly doubles the memory storage requirements using the direct solver. In hindsight, it may be more efficient to create the effective hydraulic conductance approximation using only the primary fracture intersections. The flow into each end of the primary fracture segments would then need to be determined from the final flow calculations the network blocks.

4.7.6 PARALLEL COMPUTING AND THE DUAL PERMEABILITY MODEL

I would be remise in not pointing out that the structure of the dual permeability approach is inherently parallel and in not speculating on the ramifications of the application of parallel computing techniques to the dual permeability model. The independence of the effective conductance calculations for the network blocks allows these calculations to be done simultaneously. This stage of the dual permeability model could be reduced from the most time consuming part of the coarse-scale flow calculation to an insignificant portion through the use of parallel processors. The calculation of the residence time distributions of the network blocks are also independent and, thus, inherently parallel. The particle transport calculation is another aspect of the model that is inherently parallel. Finally, the fracture generation and reduction procedures could be divided into regional sub-problems that could be solved independently, or nearly so. The use of parallel processing in the dual permeability approach would tremendously speed the calculations with very little change to the way the models are structured.

4.8 SUMMARY OF DUAL PERMEABILITY MODEL

In this Chapter, one implementation of the dual permeability modeling concept has been presented in detail. The model was shown to be effective at simulating both flow and transport within fracture
networks where a primary fracture network of hydraulically important fractures could be identified. A crucial feature of this implementation is the approximation of the effective conductance of a small region of the fracture network.

The effective conductance of the network blocks is used in a calculation of a coarse-scale hydraulic head distribution within the fracture network. The hydraulic conductance matrix that describes the equivalent conductance through the network block from a primary fracture intersection to a fracture midpoint is not an approximation. The network block conductance retains all of the complex hydraulic conductance structure of the fractures within the network block in a condensed form. However, it was found that this description of the network block was not enough to fully capture the mutual influences of the small portion of the fracture network that the block represents and the rest of the entire network on hydraulic behavior. The way boundary conditions are applied to the network blocks is the essential approximation in the network block model.

Neither impermeable boundaries nor completely open boundaries with linear fixed head gradients were adequate descriptions of the network block boundary conditions. The current network block model uses "responsive" boundary conditions. There are two important features to these boundary conditions; 1) the hydraulic head on the boundaries is allowed to respond to the heterogeneity of the internal fracture network, and 2) the equivalent of a drain is modeled along the boundaries of the network blocks. The drain accounts for the hydraulic conductance of the bounding primary fractures and includes a rough approximation of the hydraulic conductance of fractures that are near, but still exterior, to the network block. Thus, the boundary conditions of the network block reflect, in a crude sense, the heterogeneity of the fracture network that surrounds the network blocks. Figures 4.17 and 4.19 demonstrate that the recognition of the exterior fracture network is an important aspect of the network block model.

The network block model allows the fracture network to be divided and modeled in independent steps rather than being treated as a whole; potentially providing a computational advantage at the cost of
decreased accuracy in the flow and solute transport calculations. The computational advantage is limited to special situations in the model presented in this chapter. In Section 4.7.5, some changes are proposed that may increase the computational efficiency of the model.

A key feature in the dual permeability concept is the identification of a primary fracture network. This network defines the geometry of the network blocks. The primary fracture network provides an important hydraulic buffer between adjacent network blocks, allowing flow in the network blocks to be calculated independently of other network blocks. An algorithm has been developed that, in conjunction with the selection criteria, determines which fractures should be included in the primary fracture network. It was found that both the transmissivity and connectivity of fractures are important characteristics for the fractures of the primary fracture network.

The algorithm for determining the primary fractures evolved into a complex element of the dual permeability model. The complexity is potentially a serious liability that may limit the extension of the dual permeability model to three dimensions. However, much of the complexity involved in defining the primary fracture network could be avoided if the primary fracture network were generated first and then the network blocks were filled with the less important fractures. While the primary fracture selection process is complex, it is also relatively fast compared to defining the fracture geometry and solving for flow within the fracture network.

In summary, the model described in this chapter has been used both to explore the dual permeability concept and to increase understanding of the influence of flow channeling on solute transport in fracture networks. The primary fracture selection algorithm is able to identify a primary fracture network in a wide variety of fracture networks - shedding light on the importance of fracture connectivity. The dual permeability approach has been shown to be effective in simulating flow and transport in fracture networks where a small subset of fractures tend to dominate the flow field. An important characteristic of the model is that the losses in the accuracy of flow and transport simulations can be reduced to
insignificance compared to the inherent uncertainty in the fracture networks. While the model presented in this chapter neither captures all of the important physical processes that control flow and transport in fracture media nor exhibits a decided advantage over conventional discrete fracture network models, the results of this first investigation into sub-continuum approximations of fractured media are sufficiently promising to warrant additional research to identify methods for improving computational efficiency and efficacy of the modeling approach.
CHAPTER 5

A TENSOR DESCRIPTION OF FLOW

5.1 INTRODUCTION

Are fractured media special cases of porous media? Do fractured media behave differently from media dominated by primary porosity? Conversely, can the framework used to describe flow and transport in porous media adequately describe the behavior of fractured media? These issues are central to a conceptual understanding the hydraulic properties of fractured media and to the development of modeling tools that can be used to represent fluid flow and solute transport in fractured media. Measurements of the hydraulic properties of fractured media typically exhibit a high degree of spatial variability. Fractured media are heterogenous over a broad range of scales. To some, this heterogeneity forms the basis for the supposition that fractured media are fundamentally different from porous media. To others the difference is one of degree. This latter group argues that the tools being developed to describe flow in heterogeneous granular porous media can adequately capture the behavior of flow in fractured media.

A number of studies of fractured media have focused on determining the conditions needed to enable fractured media to be treated as classical porous media. To meet these conditions, a fractured medium must behave as a uniform continuum at the scale of interest. The conventional concept used to describe this behavior is that of the representative elemental volume (REV), the volume over which the detailed structure of the medium must be averaged to represent the hydraulic properties in terms of a much simpler, uniform continuum. The scale of the REV is dependent on the property of interest. For predictions of volumetric flow through a fracture network, at the scale of an REV, the flow is insensitive to the precise structure of the local fluid pathways within the averaging volume.

In field investigations, hydraulic conductivity measurements from borehole pressure tests tend to show a high degree of variability, suggesting that the portion of the fracture system investigated from
single boreholes is below the scale of an REV [e.g. Raven 1986, Neuman 1987]. Hsieh and Neuman [1985] developed a technique using cross-borehole pressure measurements to sample a larger volume of a rock mass than is possible using single boreholes. The cross-borehole measurements yield directional hydraulic conductivity estimates. The technique combines the measurements from a series of closely spaced cross-borehole tests. Even though the measurements of single borehole or individual cross-borehole tests may indicate sub-REV behavior, the entire volume tested in the series may be larger than the REV for flow. If the volume that is tested is larger than an REV, the directional hydraulic conductivity estimates provide a basis for constructing a porous medium model of the fracture system.

Hsieh et al. [1985] and Hsieh [1987] present case studies of using cross-borehole pressure tests to define the directional hydraulic conductivity of fractured rock. Two criteria were used to examine whether the scale of an REV for flow had been reached. At the Oracle site in Arizona, individual pressure responses of the cross-borehole tests could be fit to the analytical solution for a point source in an isotropic porous media of infinite extent, indicating that the flow in the vicinity of the wellbores was not dominated by individual fractures. These tests gave indications that the fracture system is well connected and not extremely heterogeneous at the scale of the packer tests. Given this behavior Hsieh et al. then demonstrated that, although the measurements show considerable scatter, directional conductivity measurements from cross-borehole tests were consistent with a tensor description of the hydraulic conductivity. Thus, at the test scale of 10m, they concluded that it is reasonable to describe the hydraulic properties of the fractured Oracle granite in terms of a uniform, anisotropic porous medium. Neuman [1987] later noted that the tensor description of the directional hydraulic conductivity may change if cross-borehole data were obtained at a larger scale because the cross-borehole data from the Oracle site covered a depth interval that is smaller than the estimated vertical correlation length of hydraulic conductivity derived from the evaluation of single-borehole tests.
Hsieh [1987] describes a similar set of pressure tests carried out at Mirror Lake in New Hampshire. Although the site is densely fractured, single-borehole packer tests revealed that most of the fractures are not transmissive. The permeable fractures had a large variation in hydraulic conductivity. In the Mirror Lake tests, pressure responses of individual cross-borehole tests could not be fit to the analytical solution for a uniform porous media, indicating flow was predominantly limited to a small number of fractures. A hydraulic conductivity tensor was not defined. These results indicate that the Mirror Lake site is extremely heterogeneous at the scale of both the single and cross-borehole tests and that the volume tested by the cross-borehole tests was below the scale of an REV for flow.

Numerical studies have been used to address the issue of whether, or at what scale, a fractured medium can be treated as a uniform continuum. The investigations of Long et al. [1982], Long and Witherspoon, [1985], and Hestir and Long [1990] have established an understanding of the important criteria that control hydraulic conductivity in two-dimensional fracture system representations. Of the many contributions of these papers, two are of particular relevance: 1) Long et al. [1982] established techniques to determine whether a fracture network was above or below the scale of an REV. 2) Long et al. [1982] surmised that sparse fracture systems may not be amenable to modeling as a continuum at any scale.

One of the criteria used to establish the scale of an REV in numerical studies has been the smoothness of the hydraulic conductivity ellipse. In the study of Long et al. [1982], and more recent investigations by Dershowitz and Einstein [1987], Khaleel [1989], and Odling and Webman [1991], plots of the inverse of the square root of the directional hydraulic conductivity as a function of the angle of the hydraulic head gradient \(1/\sqrt{K(\theta)}\) are erratic below the scale of the flow REV for the fracture network. In each of these studies, the model domain was rotated within the fracture system as a means of changing the hydraulic gradient, which was aligned with the domain boundaries. A consequence of the rotation of the model domain is that the fracture network within the model domain changes. As the size of the
domain increases, directional hydraulic conductivity estimates will converge toward a smooth curve that plots as an ellipse. Sensitivity of the directional hydraulic conductivity to rotation of the model domain provides a simple and direct indication that the domain is below the scale of an REV for flow.

While rotation of the domain boundaries provides a technique to establish the REV size for a fractured medium, it does not provide a clear indication of the properties of the medium below the scale of the REV. The observations that for some fracture networks the REV size can be larger than the capacity of current fracture network models and that other networks may not have an REV at any scale indicates the need for further understanding of sub-REV scale behavior.

An alternative procedure is examined in this chapter for evaluating the size of an REV that is based on calculations using a fixed model domain. This approach highlights aspects of flow in fractured media that are observed when only the direction of the hydraulic head gradient changes rather than both the gradient and the fracture domain. Such an approach permits an evaluation of the scale-dependent properties of a hydraulic tensor describing fluid flow through the fracture network and allows an estimation of variability in the directional aspects of the hydraulic conductivity. Perhaps more importantly, the properties of fracture networks that are revealed through this simple change in the treatment of model boundaries may affect our view of fractured media in relation to granular porous media.

Many researchers have come to accept the notion that directional hydraulic conductivity measured below its REV scale is inherently erratic in fractured media. This chapter will demonstrate that: 1) at the sub-REV scale, flow through a two-dimensional fracture system can be represented by a tensor equation (i.e. the directional hydraulic conductivity tensor does not behave in an erratic manner); 2) this tensor can be asymmetric - a result of averaging the flow through a heterogeneous medium; 3) while a radial plot of the inverse square root of the directional hydraulic conductivity with respect to the hydraulic gradient may be a smooth ellipse, one cannot conclude from this that the fracture system necessarily behaves as a porous medium.
5.2 Conductivity Ellipse and Mohr's Circle

At the continuum scale, the hydraulic conductivity of an anisotropic porous medium is a smoothly varying function of the direction of the hydraulic head gradient. There are many possible methods of representing anisotropic hydraulic conductivity. Four methods are used here. First, the tensor can be described in matrix notation. Second, a graphical representation of a hydraulic conductivity ellipse can be defined in terms of the direction of the specific discharge vector, \( \theta_s \). Third, the inverse hydraulic conductivity ellipse can be defined in terms of the direction of the hydraulic head gradient, \( \theta_h \). Fourth, the hydraulic conductivity tensor can be represented graphically as a Mohr’s circle [Kraemer, 1990]. Each of the three graphical representations is briefly reviewed in this section.

To define the hydraulic conductivity ellipse, the hydraulic conductivity in a direction parallel to the specific discharge vector can be written [Hsieh et al., 1985]

\[
K_s = \frac{1}{n_i n_j (K^{-1})_{ij}}
\]  

(5.1)

where \( n_i \) and \( n_j \) are directional cosines of flow along the \( i \) and \( j \) axes, respectively. The term \( (K^{-1})_{ij} \) represents the elements of the inverse of the hydraulic conductivity tensor. Summation over both \( i \) and \( j \) is assumed in Equation 5.1 and in the following equations with two or more symbols with common indices that are multiplied together. Defining

\[
p_i = \sqrt{K_s} n_i
\]  

(5.2)

as coordinates in conductivity space results in
Equation 5.3 defines an ellipse in two dimensions, an ellipsoid in three dimensions, where the semiaxes are aligned with the principal directions of hydraulic conductivity.

To define the inverse conductivity ellipse, the hydraulic conductivity in a direction parallel to the hydraulic head gradient is defined as

\[ K_g = K_j n_j n_i \]  

(5.4)

where \( n_j \) and \( n_i \) are direction cosines of the hydraulic gradient along the \( j \) and \( i \) axes, respectively. Defining a new set of coordinates

\[ \hat{p}_1 = \frac{n_i}{\sqrt{K_g}} \]  

(5.5)

results in

\[ 1 = K_j \hat{p}_j \hat{p}_i \]  

(5.6)

Using Equation 5.5, \( 1/\sqrt{K_g(\theta)} \) plots as an ellipse with respect to the direction of the hydraulic head gradient, rather than the direction of flow which is used to plot \( \sqrt{K_g(\theta)} \).

The hydraulic conductivity tensor of a homogeneous porous medium is symmetric, that is \( K_{ij} = K_{ji} \). As shown later, the tensors describing flow through fracture domains below the scale on an REV may not be symmetric. The hydraulic conductivity ellipses (Equations 5.3 and 5.6) are not sensitive indicators of asymmetry in the hydraulic conductivity tensor. Kraemer [1990] has presented a graphical method, that is sensitive to asymmetry, based on a Mohr’s circle and used it to characterize the relationship between the direction of flow and the direction of the hydraulic head gradient. Kraemer incorporates
asymmetries in the hydraulic response of the fracture network, a feature that distinguishes his work from the more familiar Mohr's circle development of Bear [1972].

The coordinates of the Mohr's circle are the components of flow normalized per unit hydraulic head gradient and aligned parallel and orthogonal to the direction of the head gradient. This set of coordinates is:

\[ \bar{p}_s = \frac{q_s}{|\nabla h|}, \quad \bar{p}_t = \frac{q_t}{|\nabla h|} \] (5.7)

where \( q_s \) is the specific discharge in the direction of the hydraulic head gradient, \( q_t \) is the specific discharge in the direction normal to the head gradient, and \( |\nabla h| \) is the magnitude of the head gradient.

Following Kraemer, the equations describing the coordinates are

\[ \bar{p}_s = \frac{K_{11}+K_{22}}{2} + \frac{K_{11}-K_{22}}{2} \cos(2\theta_s) + \frac{K_{12}+K_{21}}{2} \sin(2\theta_s) \]
\[ \bar{p}_t = \frac{K_{12}-K_{21}}{2} - \frac{K_{12}+K_{21}}{2} \cos(2\theta_s) + \frac{K_{11}-K_{22}}{2} \sin(2\theta_s) \] (5.8)

These coordinates define a circle centered on the location

\[ \frac{K_{11}+K_{22}}{2}, \quad \frac{K_{12}-K_{21}}{2} \] (5.9)

The radius, \( \ell \), of the circle is

\[ \ell = \sqrt{\left( \frac{K_{11}-K_{22}}{2} \right)^2 + \left( \frac{K_{12}+K_{21}}{2} \right)^2} \] (5.10)
If the hydraulic conductivity tensor is symmetric, then the Mohr's circle will be centered along the axis parallel to the gradient direction. Figure 5.1 depicts a Mohr's circle plot for an asymmetric tensor. The $\tilde{P}$ axis is horizontal. The $\tilde{P}_x$ axis is vertical. There are four points on the circle that are of special interest. The first two, labelled n, are the center of gradients. The two center of gradients points are located at $(K_{22} - K_{21})$ and $(K_{11}, K_{12})$. The center of gradients are useful because a line drawn through one of these points and any other point on the Mohr's circle will be at an angle with respect to the $\tilde{P}$ axis that is equivalent to the angle formed by the direction of the hydraulic head gradient and one of the coordinate axes of the hydraulic conductivity tensor. The reader is referred to Kraemer [1990] for a proof of the properties of the center of gradients.

The other two points of interest, labelled a and b in Figure 5.1, are the intersections of the Mohr's circle with the $\tilde{P}$ axis. These points correspond to the situation where the flow direction is parallel to the hydraulic gradient. These directions are referred to as the principal directions, $\theta_a$ and $\theta_b$ in Figure 5.1, and each of the directional hydraulic conductivities along these directions is called a principal hydraulic conductivity. If the hydraulic conductivity tensor is symmetric then the principal directions are orthogonal to each other and a hydraulic gradient aligned along these directions will give the maximum and minimum specific discharges. It follows that these directions also define the alignment of the major and minor axes of the hydraulic conductivity ellipse. The principal directions are not orthogonal if the tensor is asymmetric, nor do they correspond with the major and minor axes of the directional hydraulic conductivity ellipses, which are always orthogonal.

5.3 Flow in a Fracture Network: An Example

Figure 5.2 depicts the flow in a fracture network that is typical of the fracture systems that were investigated for this chapter. Figure 5.2a shows the fracture network. The fracture network realization
Figure 5.1: Hydraulic conductivity drawn as a Mohr's circle. The K's correspond to the elements of the hydraulic conductance tensor. The points label n are the center of gradients. $\theta_a$ and $\theta_b$ define the principal directions.
Figure 5.2: Trace map and flow plot of one fracture network used to investigate hydraulic conductance. a) Fracture trace map. b) Flow rates within fractures.
Minimum flow drawn = 5.8e-12 \text{ m}^3/\text{s}

Figure 5.2: Cont. Trace map and flow plot of one fracture network used to investigate hydraulic conductance. The hydraulic head gradient is oriented along the bottom boundary. Fractures with flow rates less than 5.8x10^{-12} \text{ m}^3/\text{s} are not shown.
was generated using a Poisson structural model. The fracture system has two orthogonal fracture sets. The statistical parameters of the fractures sets are defined in Table 5.1 under the column for fracture system \( a \). In Figure 5.2b the fracture network is redrawn with line widths proportional to the flow rates within each fracture. Only flow rates greater than \( 5.8 \times 10^{-12} \text{ m}^3/\text{s} \) are drawn, making some of the fractures appear to be isolated from the rest of the network. One can see that the flow is not evenly distributed through the fracture network and that there are local channels of high flow.

5.4 HYDRAULIC CONDUCTANCE TENSOR

To avoid the use of the term hydraulic conductivity, which is a continuum property, the flow behavior is described here in terms of a conductance across the domain boundaries when referring to the flow properties of a sub-REV flow domain. The conductance is a description of an averaged flow behavior. For a single realization of a fracture system, an equation describing flow across each of the domain boundaries can be written:

\[
q_m = -\tilde{K}_{mx} \frac{\partial h}{\partial x} - \tilde{K}_{mc} \frac{\partial h}{\partial z} \tag{5.11}
\]

where \( m \) corresponds to the boundaries as designated in Figure 5.2, and the terms \( \tilde{K}_{mj} \) are hydraulic conductances. Two calculations of flow using unit hydraulic gradients oriented 0° and 90° from the \( x \) axis provide enough information to determine the eight components \( \tilde{K}_{mj} \) of \( \tilde{K} \). With the gradient aligned with the \( x \) axis, the volumetric flow across the four boundaries determines the values \( \tilde{K}_{mx} \). With the gradient aligned with the \( z \) axis, the volumetric flow across the four boundaries determines the values \( \tilde{K}_{mc} \). In
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Table 5.1: Fracture system definitions for conductance study. Entries in the table are only made when the parameter differs from system \( a \).
expanded form, Equation 5.11 is

\[
\begin{pmatrix}
q_1 \\
q_2 \\
q_3 \\
q_4
\end{pmatrix} = -
\begin{pmatrix}
\bar{K}_{1x} & \bar{K}_{1z} \\
\bar{K}_{2x} & \bar{K}_{2z} \\
\bar{K}_{3x} & \bar{K}_{3z} \\
\bar{K}_{4x} & \bar{K}_{4z}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial h}{\partial x} \\
\frac{\partial h}{\partial z}
\end{pmatrix}
\]

(5.12)

Note that below the scale of an REV, flow across parallel boundaries on either side of a network domain will not necessarily be equal. For example, the flow across boundary 2 of Figure 5.2b is larger than the flow across boundary 4. Also of note in Figure 5.2b is the tendency for high flow rates near the boundaries, as demonstrated by the short wide lines along boundaries 1 and 3. These high flow rates do not influence \( \bar{K} \) because flow that enters and leaves along the same domain boundary does not contribute to the conductance calculation, only the net flow across a boundary is used in the analysis. An example of a very large flow that is excluded is represented by the large spot under the boundary 3 label of Figure 5.2b.

The eight component tensor provides a complete description of the average fluid flow across the boundaries imposed on the fracture domain, for any orientation of the hydraulic gradient. In terms of describing average flows through a given volume of rock, it fully characterizes sub-REV behavior. Arguments can be made that one need not proceed any further in attempting to simplify the eight component tensor. However, in this form it is not amenable to comparison with the more traditional four-component tensor used in porous media studies.

It is possible to reduce the conductance terms in Equation 5.12 to a two by two conductance tensor, \( K' \), by working with the average flows across parallel boundaries of the flow domain. To do this, a second set of specific discharges is defined.
For a unit hydraulic gradient first aligned in the positive x direction, and then in the positive z direction, the components of the conductance tensor $K'$ are calculated using the relation

$$
\begin{pmatrix}
q_x \\
q_z 
\end{pmatrix} = -
\begin{bmatrix}
\frac{K'_{xx}}{2} & \frac{K'_{xz}}{2} \\
\frac{K'_{zx}}{2} & \frac{K'_{zz}}{2}
\end{bmatrix}
\begin{pmatrix}
\frac{\partial h}{\partial x} \\
\frac{\partial h}{\partial z}
\end{pmatrix}
$$

Equation 5.14 can also be written in a form that more clearly shows its relationship to the eight component tensor

$$
\begin{pmatrix}
q_x \\
q_z 
\end{pmatrix} = -
\begin{bmatrix}
\frac{K''_{2x} + K''_{4x}}{2} & \frac{K''_{2z} + K''_{4z}}{2} \\
\frac{K''_{1x} + K''_{3x}}{2} & \frac{K''_{1z} + K''_{3z}}{2}
\end{bmatrix}
\begin{pmatrix}
\frac{\partial h}{\partial x} \\
\frac{\partial h}{\partial z}
\end{pmatrix}
$$

Below the scale of an REV, the off-diagonal terms in Equation 5.14 may not be equal.

The values $\tilde{K}$ and $K'$ are unique to each realization of the fracture system. These are the conductance values that preserve the total flow across each of the domain boundaries; that is, they are the equivalent homogeneous values that describe the average flow across the fracture network. The tensor $\tilde{K}$ replicates the different flows across parallel boundaries, while $K'$ replicates the average flow across parallel boundaries. For a given fracture density, $K'$ should approach the fracture system's equivalent porous medium conductivity tensor as the size of the fracture network approaches the scale of the REV.

The components of $\tilde{K}$, and $K'$, are computed using only two orientations of the hydraulic gradient. The ability of $\tilde{K}$ to replicate the flow across the domain boundaries for any orientation of the hydraulic gradient has been verified. The verification was performed by comparing the flow across each boundary,
calculated using Equation 5.11, with the observed flow rates in a corresponding discrete network simulation. The \( q_m \) for gradient orientations of 0° to 180° in 15° increments were calculated, for ten fracture network realizations generated from each of five of the fracture systems listed in Table 5.1. In each case, the use of \( \hat{K}_{ab} \) predicted the flow rates across each of the boundaries to within the accuracy of the numerical results. The tensor \( \hat{K} \) contains all the information that is needed to predict the flow across the four boundaries and thus, the average flow through the fracture network from which it was derived, for any orientation of the hydraulic gradient.

As noted earlier, directional conductance of a fracture network can be plotted either as a function of the direction of the specific discharge vector \( (K_s) \), or the direction of the hydraulic gradient \( (K_g) \). At or above the scale of the flow REV, the first method illustrates the hydraulic conductivity ellipse for an equivalent porous medium, the second method the inverse hydraulic conductivity ellipse. Below the scale of a flow REV, these methods result in elliptical figures that characterize the directional conductance of the fracture network domain. Panel a in Figure 5.3 shows a plot of the inverse hydraulic conductance, \( 1/\sqrt{K_s(\theta_g)} \), determined for the network in Figure 5.2. The directional hydraulic conductance as a function of the gradient direction has been defined

\[
K_g(\theta_g) = \frac{|q| \cos(\theta_s-\theta_g)}{|\nabla h|}
\]  

(5.16)

where \( |q| \cos(\theta_s-\theta_g) \) is the flow in the direction of the gradient, and \( \theta_s \) is

\[
\theta_s = \cos^{-1} \frac{q_x}{|q|}
\]  

(5.17)

Values of the inverse hydraulic conductance as a function of \( \theta_s \) can be estimated in two ways: (1) from the conductance tensor \( K' \), and (2) from solutions of the discrete network model. Method (2) provides
Figure 5.3: Hydraulic conductance ellipses and Mohr's circle for the fracture network of Figure 5.2.
a) Hydraulic conductance ellipse.  b) Inverse hydraulic conductance. Xs are plotted from the hydraulic conductance tensor $K'$, listed below the hydraulic conductance ellipse. $\Box$s are plotted from the results of the numerical simulations. c) Hydraulic conductance tensor with the principal axes listed below. d) Mohr's circle plot of the hydraulic conductance.

Conductance Tensor $K'$ (m/sec)

\[
\begin{bmatrix}
2.42e-08 & -3.01e-09 \\
-1.14e-08 & 1.15e-08
\end{bmatrix}
\]

Principal Directions
$
\theta_1 = 37.2$ Degrees
$
\theta_2 = 101.3$ Degrees

Major axes
155.8 Degrees
65.8 Degrees

Flow perpendicular to head gradient (m/sec)
Flow parallel to head gradient (m/sec)
a check on values calculated directly from the conductance tensor. Both methods proceed by first determining \( q \) in the direction of the hydraulic gradient, and then solving for \( K_g \), for various orientations of the hydraulic gradient.

The X's on Figure 5.3 are the conductance values calculated using the tensor \( K' \). These values have been plotted at 5° increments for a hydraulic gradient orientation from 0° to 360°. The □'s are values of \( K_g \) determined from the flow solution provided by the discrete network model. The components of the hydraulic conductance tensor \( K' \) are listed in Panel c of Figure 5.3. The tensor is nonsymmetric. These results indicate: (1) a smooth and regular pattern is seen in the directional dependence of the hydraulic conductance, as opposed to the erratic behavior that occurs when the model domain is rotated, (2) the nonsymmetric conductance tensor correctly predicts the directional conductivity of the fracture network, and (3) the regular nature of \( K_g \) occurs even though the network is below the scale of an REV. The asymmetry in the conductance tensor is one indication that the flow domain is not at the scale of an REV. Additional evidence for sub-REV behavior will be cited later in the examination of this fracture system as the scale of the flow domain is changed.

Panel b of Figure 5.3 shows a plot of the hydraulic conductance as a function of the direction of the specific discharge vector. The X's are conductance values of \( K_g \) using the tensor \( K' \). The □'s are the values of \( K_g \) determined from the flow solution provided by the discrete network model. The directional conductance is plotted with respect to the direction of specific discharge, which is calculated as

\[
K_g(\theta_g) = \frac{|q|}{|h| \cos(\theta_g - \theta_q)}
\]  

(5.18)

The behavior of \( K_g(\theta_g) \) is consistent with that observed for \( K_g(\theta_q) \).
The orientations of the major and minor axes of the conductance ellipse are listed in Panel c. These values can be determined graphically, or as done in a later sensitivity study, by calculating the direction of the hydraulic gradient, $\theta_m$, that yields maximum/minimum values of $q$. The direction $\theta_m$ is given by

$$
\theta_m = \tan^{-1}\left( \frac{b}{2 \left( \frac{b^2}{4} - 1 \right)} \right)
$$

(5.19)

where

$$
b = \frac{K_{xx} - K_{yy} + K_{yy} - K_{xx}}{K_{xx}K_{yy} + K_{yy}K_{xx}}
$$

(5.20)

The Mohr's circle representation of the conductance tensor is shown on Panel d of Figure 5.3. The □ symbols are the values of $(\theta_c, \phi_c)$ calculated using the conductance tensor. The values lie on a circle, with no indication of an erratic behavior as a function of orientation. The offset of the center of the circle from the $\phi_c$ axis is an indication of the asymmetry in the conductance tensor for this fracture network. The principal directions of the conductance tensor can be calculated graphically from the Mohr's circle (Figure 5.1), or algebraically by finding eigenvectors associated the roots of the characteristic equation for the hydraulic conductance tensor

$$
\det[K' - \lambda I] = 0
$$

(5.21)

The principal directions for this realization of the fracture network are listed in Panel c of Figure 5.3. Note that they are not orthogonal to each other, differing from orthogonality by 25.9°, and that the principal directions are not aligned with the major and minor axes of the directional hydraulic conductance ellipse. These features again point to sub-REV behavior.
5.5 Short Circuit Fractures

The boundary flows used in calculating the directional conductances can be sensitive to the presence of fractures that directly connect two boundaries, such as found near the left corner of Figure 5.2b. With values of hydraulic head being specified at both ends of these fractures, flow through these "short circuits" is effectively independent of the connectivity of the fracture network. If these fractures have a high transmissivity, then the flow through them will dominate the calculation of the average flows across the domain boundaries. As shown in a calculation that follows, these short-circuiting fractures have no effect on the characteristic smooth variation observed in the directional conductance. They do however increase the magnitude of the measured conductance, and the degree of asymmetry in the conductance tensor.

As a demonstration of the affects of short circuiting fractures, a conductance tensor was determined for a 2m by 2m flow domain composed of the fracture network in the upper right hand corner of Figure 5.2. Figure 5.4 shows the hydraulic conductance tensor, ellipses and Mohr’s circle for this block. Of the four fractures in the small network that directly connect adjacent boundaries, two have transmissivities greater than the expected average for their set. These fractures were modified in two experiments. The transmissivities are first set to the average value for the fracture network. In the second experiment, the transmissivity of these fractures is increased rather than reduced.

In the first experiment, the change corresponds roughly to a reduction in transmissivity by a factor of two for both of the fractures. The conductance tensor, principal directions and semi-major axes of the conductance ellipses are presented in Panel a of Figure 5.5. The result is less than a ten percent decrease in the absolute magnitude (determinant) of the conductance tensor, but a change from 29° to 21° in the deviation from orthogonality of the principal directions of the conductance tensor. The plot of the Mohr’s circle in Figure 5.5b reveals a small shift of the center toward the $\bar{p}_s$ axis.
Flow parallel to head gradient (m/sec)

Conductance Tensor $K'$ (m/sec)

\[
\begin{bmatrix}
4.92 \times 10^{-8} & -4.98 \times 10^{-9} \\
5.91 \times 10^{-9} & 2.71 \times 10^{-8}
\end{bmatrix}
\]

Principal Directions

$\theta_1 = 164.1$ Degrees
$\theta_2 = 103.5$ Degrees

Major axes
1.2 Degrees
91.2 Degrees

Figure 5.4: Hydraulic Conductance for the 2m by 2m region in the upper right corner of Figure 5.2.

a) Hydraulic conductance ellipse. b) Inverse hydraulic conductance. Xs are plotted from the hydraulic conductance tensor $K'$, listed below the hydraulic conductance ellipse. □s are plotted from the results of the numerical simulations. c) Hydraulic conductance tensor with the principal axes listed below. d) Mohr's circle plot of the hydraulic conductance.
Figure 5.5: Hydraulic conductance tensor and Mohr's circle for the modified versions of the same fracture network as Figure 5.4. a) The hydraulic conductance tensor when two of the short circuiting fractures have been reduced to the mean transmissivity of the fracture set. b) Mohr's circle plot. c) The hydraulic conductance tensor for a doubling of the apertures of the corner cutting fractures. d) Mohr's circle plot for the doubled aperture case.
Figure 5.5c is a listing of the conductance tensor, principal directions and semi-major axes of the conductance ellipses when the apertures of the two fractures are doubled rather than reduced. The transmissivity of the two fractures increased by a factor of eight. The magnitude of the conductance tensor more than doubles in this case. The deviation from orthogonality of the principal directions increases to 76° and the center of the Mohr’s circle moved far from the \( \hat{g} \) axis. The modification also results in a 10° shift in the orientation of the semi-major axes, an effect that was not evident when the transmissivities are reduced. Although not presented here, the agreement of the flow calculations from the tensor description and the numerical model remains exact.

These two experiments conducted on a small domain demonstrate the sensitivity of the conductance tensor to fractures that directly connect two boundaries of the flow domain. If these fractures have transmissivities near or below the mean transmissivity then their influence on the conductance tensor may be noticeable but is not significant. If these fractures are extremely wide then the affect may be dramatic. In the second experiment, the fracture transmissivities were roughly sixteen fold greater than the mean of the sets and accounted for more than half the flow though the small domain. For a fracture set with a standard deviation of \( \log_{10} \) transmissivity of 0.6 as in fracture system \( a \) of Table 5.1, there is less than a 0.1 percent probability that any one short circuiting fracture would have such a large transmissivity. However, for fracture system \( d \), with a standard deviation in \( \log_{10} \) transmissivity of 1.5, over five percent of such fractures would have transmissivities greater than sixteen times the mean. Within the flow domain the influence of these wide fractures is reduced by the narrower fractures that connect to them. The flow through short-circuiting fractures is not influenced by other fractures. The large flow through these fractures is a result of imposing a constant hydraulic head gradient along the domain boundaries. These short-circuiting fractures are not a representative measure of the directional conductance at the scale of the model domain.
One method of reducing the influence of the short circuiting fractures would be to allow the hydraulic head values on the boundary to respond to the high flows. This approach has been applied through different methods in Hestir and Long's study [1990] and by Herbert et al. [1991]. These authors impose constant head boundary conditions on a larger extension of the fracture domain that surrounds the flow domain they investigate, allowing the head gradient along the boundaries of the flow domain to be influenced by heterogeneity both inside and outside the flow domain. In the case of a highly transmissive fracture crossing the corner of the flow domain, the local hydraulic head gradient within the fracture would most likely be smaller than the regional gradient and the flow within the fracture would be smaller than if the regional gradient were applied to the flow domain boundaries. The approaches of Hestir and Long and Herbert et al. result in hydraulic head variations along the boundaries that respond to heterogeneity both interior and exterior to the model domain.

While the imposition of a linear hydraulic head gradient is more restrictive than an approach that allows the boundary heads to vary, in order to investigate the properties of a hydraulic conductance tensor we must be able to uniquely define the hydraulic heads along the boundaries of the domain when specifying the gradient direction. To reduce the influence of high transmissivity fractures that form short circuits, their transmissivity is reset to be equal to the mean value of the fracture set. Note that none of the fractures were modified in the network presented in Figure 5.2, nor were any fractures modified in the testing of the eight component tensor. During the sensitivity study presented later, the number of fractures that were modified was monitored. It will be shown that a small percentage of the fractures were affected.

Returning to the broader issue of flow behavior in fractured media, the efficacy of the conductance tensor to describe the variation of flow with respect to gradient direction demonstrates that flow in a two-dimensional fracture network can be examined in a way that does not lead to erratic variation in directional conductance at a scale smaller than the REV for flow. The erratic variation documented in earlier studies occurs as a result of changes in fracture geometry in the corner regions of the flow domain.
as the model domain is rotated within the larger fracture network. Given boundary conditions that are consistent with the assumption of a uniform regional hydraulic gradient, it is possible to describe the average flow behavior in sub-REV domains using a tensor notation for hydraulic conductance. Unlike tensors describing flow in a homogeneous medium, the four-component conductance tensor formed by averaging the flow across the boundaries of the domain may not be symmetric for domains smaller than a flow REV.

5.6 ASYMMETRY IN CONDUCTANCE MEASURES

The conductance tensor relates the average flow through a fracture network to the direction of the hydraulic gradient. There are many ways to average flow and the method chosen can affect the properties of the conductance tensor. In using Equation 5.12, the $q_{in}$ are determined by summing the volumetric flow from each fracture crossing the boundary and dividing by the length of the boundary. By doing so, any fluid that enters and then leaves the same boundary is neglected. The direction that fluid is moving in individual fractures is lost in this averaging process. The net flow direction through the model domain can be estimated, but is limited by the averaging process.

To guide a discussion on the affect of averaging on the estimated direction of flow, we can partition the total flow into 12 groups on the basis of the two boundaries the flow crosses (Figure 5.6). We know the magnitude of the flow and the identity of the two boundaries of each group. The two faces are used to define a flow direction. The 12 groups define eight unique flow directions as shown in Figure 5.6. As an example, flow entering boundary 2 and leaving across boundary 3 would be assigned a flow direction of 45°. The net flow direction is the vector sum of these eight directions with each vector weighted by the magnitude of flow between the appropriate faces. The eight directions are basis vectors for the flow measurement. If all of the flow entered boundary 2 and left across boundary 3 then the estimated flow direction can only be 45°. It is shown below that if the net flow direction is not 45° then a tensor representation of the measured flow direction is asymmetric. It is also demonstrated that a four component
Figure 5.6: Diagram of the flow direction defined by fluid crossing two domain boundaries. The flow direction is given by the arrows pointing from the flow entrance boundary to the flow exit boundary.
tensor representing flow in a "homogeneous" fracture network will be symmetric.

5.6.1 ASYMMETRY IN SIMPLE FRACTURE NETWORKS

Consider a square domain with one fracture connecting boundary 2 to boundary 3. The boundary conditions on the domain are the same as in the numerical simulations, a constant hydraulic head gradient along the boundaries of the domain. Applying Equation 3.17, the flow along the fracture is

\[ Q = -T_f \frac{dh}{d\theta} \cos(\theta_f - \theta) \]  

(5.22)

where \( \theta_f \) is the orientation of the fracture. The average boundary flows are \( q_m = \frac{Q}{W} \) for \( m = 2, 3 \) where \( W \) is the length of a boundary. Substituting for \( q_m \) in Equation 5.11 results in

\[ \frac{-T_f \frac{dh}{d\theta} \cos(\theta_f)}{W} \cos(\theta_f - \theta) = -K_{mx} \frac{dh}{d\theta} \cos(\theta_f) - K_{mx} \frac{dh}{d\theta} \sin(\theta_f) \]  

(5.23)

Using the trigonometric relation

\[ \cos(A - B) = \cos(A)\cos(B) - \sin(A)\sin(B) \]  

(5.24)

leads to

\[ \tilde{K}_{mx} = \frac{T_f}{W} \cos(\theta_f) \]  

(5.25)

\[ \tilde{K}_{mx} = \frac{T_f}{W} \sin(\theta_f) \]

From Equations 5.14 and 5.15, the four component tensor is
For a single fracture connecting boundaries 2 and 3, $K'$ will be symmetric if and only if $\theta_f$ is $45^\circ$. Now let the fracture connect boundaries 1 and 3 instead of 2 and 3. The four component tensor becomes

$$K' = \begin{bmatrix} 0 & 0 \\ \frac{T_f \cos(\theta_f)}{2W} & \frac{T_f \sin(\theta_f)}{2W} \end{bmatrix}$$

(5.27)

$K'$ will be symmetric if and only if $\theta_f$ is $90^\circ$. From these two cases, it is clear that $K'$ will be symmetric if and only if the fracture is oriented parallel to the direction of flow as given by the basis vector for the two faces connected by the fracture.

Now consider a network composed of $n$ parallel fractures. Let the aperture of each fracture vary such that the transmissivity of each fracture is $\frac{T_f}{n}$. First consider the case of $n=2$ where one fracture connects boundaries 2 and 3 and the other connects boundaries 1 and 3. For this network the four component tensor is

$$K' = \begin{bmatrix} \frac{T_f \cos(\theta_f)}{4W} & \frac{T_f \sin(\theta_f)}{4W} \\ \frac{3T_f \cos(\theta_f)}{4W} & \frac{3T_f \sin(\theta_f)}{4W} \end{bmatrix}$$

(5.28)

$K'$ will be symmetric if $\theta_f$ is $\tan^{-1}(3)$. Next let the fractures be evenly spaced and let $n \to \infty$.

If $45^\circ < \theta_f < 90^\circ$ then

$$\frac{n \cos(\theta_f)}{\sin(\theta_f) + \cos(\theta_f)}$$

of the fractures connect boundary 2 to boundary 3.
\[
\frac{n \sin(\theta) - \cos(\theta)}{\sin(\theta) + \cos(\theta)} \text{ of the fractures connect boundary 1 to boundary 3, and}
\]
\[
\frac{n \cos(\theta)}{\sin(\theta) + \cos(\theta)} \text{ of the fractures connect boundary 1 to boundary 4. The factor } \sin(\theta) + \cos(\theta) \text{ accounts for the change in fracture spacing required by the constraint of a fixed number of fractures, } n, \text{ within the domain.}
\]

For this network,

\[
K'_{xx} = \frac{T_f \cos^2(\theta)}{2W \sin(\theta) + \cos(\theta)}
\]
\[
K'_{yy} = \frac{T_f \cos(\theta) \sin(\theta)}{2W \sin(\theta) + \cos(\theta)}
\]
\[
K'_{xx} = \frac{T_f \cos(\theta) \sin(\theta)}{2W \sin(\theta) + \cos(\theta)}
\]
\[
K'_{yy} = \frac{T_f \cos(\theta) \sin(\theta)}{2W \sin(\theta) + \cos(\theta)} \left( \frac{\cos^2(\theta)}{(\sin(\theta) + \cos(\theta))} + \frac{(\sin(\theta) - \cos(\theta)) \cos(\theta)}{\sin(\theta) + \cos(\theta)} \right)
\]
\[
K'_{xx} = \frac{T_f \cos(\theta) \sin(\theta)}{2W \sin(\theta) + \cos(\theta)} + \frac{\sin(\theta) \cos(\theta)}{\sin(\theta) + \cos(\theta)}
\]
\[
K'_{yy} = \frac{T_f \cos^2(\theta)}{2W \sin(\theta) + \cos(\theta)} + \frac{\sin(\theta) \cos(\theta)}{\sin(\theta) + \cos(\theta)}
\]

Combining terms results in

\[
K' = \begin{bmatrix}
\frac{T_f \cos^2(\theta)}{2W \sin(\theta) + \cos(\theta)} & \frac{T_f \cos(\theta) \sin(\theta)}{2W \sin(\theta) + \cos(\theta)} \\
\frac{T_f \cos(\theta) \sin(\theta)}{2W \sin(\theta) + \cos(\theta)} & \frac{T_f \sin^2(\theta)}{2W \sin(\theta) + \cos(\theta)}
\end{bmatrix}
\]

\[
K' \text{ is symmetric for all } \theta.
\]

These networks indicate that a network of only a few fractures leads to an asymmetric conductance tensor unless the fracture orientation coincides with the sum of the basis vectors defined by the boundary connections. Two networks composed of the same fractures, but positioned differently within a domain, can result in different conductance descriptions. For example a domain that contains ten equally spaced
fractures with equal transmissivities and all oriented at 60° could have principal directions of conductance oriented 56° and 150° or 62° and 150° depending on the positioning of the fractures within the network. Note that for the ten fractures, the deviation of the principal directions from orthogonality is less than 5°. If the network is composed of many parallel fractures of equal transmissivity, then the conductance tensor would be very close to being symmetric.

These simple examples lead to a supposition that the asymmetry in $K'$ is a result of the flow averaging process. The last example indicates that the $K'$ tensor of a homogeneous medium will be symmetric. Asymmetry in $K'$ is a product of an interaction between the way the direction of flow through the domain is calculated and inhomogeneities within the flow domain. The asymmetry is affected by heterogeneity within the fracture network and is, therefore, an indicator of sub-REV behavior.

5.6.2 ALTERNATIVE FLOW MEASURES

If the asymmetry in the conductance tensor is a result of averaging flow across the flow boundaries, can a flow averaging procedure be found that does not introduce asymmetry into the conductance measurements? In this section three alternative measurements of flow through the fracture network domain are investigated. These are:

1) Summation of the vector components of the flow across each boundary such that

$$q_i = \frac{1}{2(w_x+w_y)} \sum_j Q'_j$$

(5.31)

where $j$ represents locations of flow across the domain boundaries and $i$ is an ordinal direction.

2) Summation of the normal component of flow across each boundary.

3) A magnitude of flow defined by the net volumetric inflow, and a flow direction defined as the direction from the centroid of the flow weighted inlet locations to the centroid of the flow weighted outlet locations.
A sequence of four networks are used to evaluate each of these four average flow measures. The first network contains only one fracture oriented 60° to boundary 1 (Figure 5.7, Panel a). The second network contains two fractures that intersect the domain boundary at the same locations as the fracture in the first network (Panel b). The apertures of these fractures are larger than the aperture of the single fracture such that the magnitude of flow within the fractures would be equivalent to the single fracture if the boundary conditions of the two networks were equivalent. The third network contains three fractures (Panel c). Again the intersections with the boundaries and fracture apertures are such that for equivalent boundary conditions the magnitude of flow through this network would be equivalent to the flow through the first two networks. The two fractures intersecting the domain boundaries are parallel in this network resulting in flow in the two fractures being equal but in opposite directions. The last network has the same set of three fractures as the third network plus an additional fracture that connects the network to boundary 4 (Panel d).

As we have seen before, the first network results in an asymmetric tensor when flow is calculated using the original flow measure. The flow in this case is always calculated to be at 45° to boundary 1. The other three measures provide an estimated flow direction of 60° to boundary 1 and yield a symmetric hydraulic conductivity tensor. When applied to the second network, measures 1 and 2 yield identical results to the original measure, that is flow at 45°. Measure 3 yields an average flow direction of 60° and a symmetric tensor.

When applied to the third network, measure 1 results in zero flow and no conductance, a result that is not acceptable. Measure 2 provides an estimated net flow direction of 30° rather than 60°. Applied to the third network, measure 3 results in a 60° flow direction, consistent with the other networks. Measure 3, which works wonderfully for networks that have only one entrance and one exit, fails to produce consistent results for more complicated situations, as shown below for the fourth network. Figure 5.8 presents the conductance tensor and graphical plots from the results of measure 3 applied to the fourth
Figure 5.7: Fracture networks used to investigate alternative flow measures. a) One 60° fracture. b) Two orthogonal fractures. c) Three fractures, -30°, 60°, and 150°. d) Four fractures with three intersections with the boundary.
Figure 5.8: Conductance ellipses and Mohr's circle using alternative flow measurement 3 to measure the flows through network 4 of Figure 5.7. The figure demonstrates the inconsistent nature of this flow measurement.
network of these examples. The graphical plots reveal that the conductance tensor developed from gradient directions of 0° and 90° does not adequately represent the flow for other gradient directions. Figure 5.8 reveals that the flow as determined by measure 3 can no longer be described using a tensor representation.

Although each of the three alternate flow measures can be considered correct for the case of a single fracture, they are all inferior to the measure used in this research. Measures 1 and 3 are inadequate for our purposes because of the problems identified above. Measure 2 is sensitive to the orientation of the fractures that intersect the domain boundaries. The measurement changes for each of the three networks, indicating a sensitivity to the angle of the fractures intersecting the boundaries. A small change in any of these networks, such as a short wide fracture intersecting the network near the domain boundary, could change the calculated flow direction. This sensitivity to the local conditions on the boundary could result in a gross misrepresentation of the bulk of the domain, a situation that is not acceptable for this research. The flow averaging procedure used in the conductance tensor calculations is superior to all of the alternates considered in this section even though it has been demonstrated to be a contributing factor in the asymmetry of the conductance tensors.

5.7 PARAMETER STUDIES

5.7.1 INTRODUCTION

This section describes the results of simulations carried out to investigate the sensitivity of the conductance tensor and the principal axes to changes in the fracture system statistics and to the scale of measurement. Using these observations, can the tensors $\bar{K}$ or $K'$ be used as a guide in establishing the minimum scale of a REV? A number of criteria can be examined to test whether a fracture network at a given scale of observation can be approximated by a uniform continuum. Long et al. [1982] established the use of the deviation of the directional hydraulic conductivity from a best-fitting ellipse, where
directional conductivity is determined by rotating the model domain within the fracture system. As demonstrated earlier, plots of $K_p(\theta_p)$ and $K_r(\theta_r)$ are smooth and regular below the scale of a REV when the hydraulic head gradient is rotated about a fixed fracture network domain. The behavior of the eight-component tensor $\mathbf{K}$ (Equation 5.12) could be examined to see if there are equal flows across parallel boundaries, as expected if the domain encompasses an REV for the fracture system. Instead, the flow behavior is analyzed in terms of $K''$ because of the large body of work that has focused on the four component tensor (Equation 5.14). The following three criteria are used to assess whether or not the scale of observation is above the scale of the REV. The first focuses on the conductance tensor of individual networks, the latter two emphasize how the tensor varies with changes in the scale of measurement.

1) The tensor $K''$ should be symmetric, and the principal axes should be orthogonal. The deviation from orthogonality of the principal axes is used as the measure of asymmetry. Two other measures of asymmetry have been investigated. In Appendix B, the results of these measures are presented and compared to the results using the orthogonality of principal axes as a measure of asymmetry.

2) If the scale of the fracture network domain is above that of a flow REV, then the tensor $K''$ should not change appreciably with an increase in the scale of measurement.

3) In that $K''$ is a description of the average hydraulic conductivity of the fracture network, then at the scale of an REV, $K''$ should show only small variations between different realizations of the fracture system. This independence refers to the magnitude of the conductance, the orientation of the principal directions of the conductance tensor, and the orientation of the major and minor axes of the hydraulic conductance ellipse. The variation in the orientation of the major and minor axes of the directional hydraulic conductance ellipse will be presented, as it is intuitively easier to visualize than the orientation of the principal directions. If $K''$ is not reasonably independent of specific realizations, this behavior suggests that the local scale inhomogeneities remain
important in determining the average hydraulic properties of the network at the scale of the analysis.

The fracture systems listed in Table 5.1 have been used to study the sensitivity of the hydraulic conductance tensor to fracture system parameters. The fracture systems were created to study the influence of six parameters on the nature of the fracture system conductance as listed in Table 5.2. The influence of each of these parameters on the conductance is discussed below under separate sub-headings.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fracture Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmissivity</td>
<td>a b c d and j k</td>
</tr>
<tr>
<td>Fracture density</td>
<td>a e</td>
</tr>
<tr>
<td>Mean fracture length</td>
<td>a f</td>
</tr>
<tr>
<td>Mean fracture orientation</td>
<td>a h</td>
</tr>
<tr>
<td>Orientation variance</td>
<td>a j n and g h i</td>
</tr>
<tr>
<td>Isotropy</td>
<td>k m</td>
</tr>
</tbody>
</table>

Table 5.2: Fracture systems used to investigate parameter sensitivity.

For each of the fracture systems listed in Table 5.1, conductance tensors are calculated for 150 realizations of a fracture system for square domains 5m, 10m, 20m, and 40m on a side. The number of realizations that could be investigated was constrained by computer resource time limitations. The tensor determination for a 40m x 40m domain requires from 40 minutes, for fracture system a, to eight hours, for fracture system d, on an SGI R4000 computer. The progression of domain sizes provides an indication of the behavior of the conductance as the domain approaches the size of an REV. For the density of open fractures used in this study, the 5m by 5m domains are on the order of the size one might expect to investigate using a single borehole pressure test. These small domains show a very large variation in the
hydraulic conductance measures. An appreciation for the domain size, relative to fracture density, can be gained by referring back to the network shown in Figure 5.2.

As mentioned in the Section 5.5, short circuiting fractures that directly connect two boundaries of a flow domain may inappropriately dominate the average flow through the domain. The calculated flow through these fractures is not consistent with the notion that calculated flow is related to the size of the domain because the fixed head boundary conditions for these fractures are imposed over a short distance. It is not reasonable to assume that the flow rates through these fractures could be maintained through the fractures immediately surrounding the domain. The effect has been reduced by adjusting the transmissivity of these fractures; effecting about 0.5% of the fractures generated in the smallest domains (5m by 5m) and 0.01% of the fractures in the largest domains (40m by 40m). The adjustment of most of these fractures has a small effect because most of these fractures have transmissivities that are less than one standard deviation from the average transmissivity. The occasional short-circuiting fracture has a transmissivity three or four standard deviations above the average and can have a tremendous impact on the average flow through the domain.

Despite the reduction of the transmissivities of the short circuiting fractures, anomalously large values of conductance still occur in a small number of the fracture network realizations. These large values may be attributed to the interconnection of two or more large transmissivity fractures that, combined, also provide a direct connection between adjacent domain boundaries near a corner. As an additional filter of these small scale features, fracture network realizations that have log values of the magnitude of the conductance tensor that are more than four standard deviations above the mean log conductance values have been removed from the data base. As with discarding the short circuiting fractures, these realizations were not used because the calculated flow rates are highly unlikely to be consistent with the possible flows in the surrounding fracture network. Given the large conductances of these infrequent events, to have retained these values would have biased the averages of the fracture systems towards these small scale
phenomena and masked the essential character of the scale dependence of the fracture systems. At most three realizations have been discarded for any domain because of anomalously large conductances. In addition some realizations of fracture system e were discarded because, for this sparse system, no fractures connected the boundaries of the domain resulting in non-conducting networks. Because our conductance measure is in log conductance a conductance of zero could not be used in the averages.

5.7.2 Transmissivity Variance

Fracture systems b, a, c and d have standard deviations in \( \log_{10} \) transmissivity of individual fractures of 0., 0.6, 1.0, 1.5 respectively. Figure 5.9a presents a plot of the average maximum directional hydraulic conductance for these simulations. The average conductance decreases as the size of the flow domain increases, a clear indication that for these fracture systems even the largest domains are below the REV scale of flow.

The decrease in conductance observed in Figure 5.9a may be related to channeling similar to that evident in Figure 5.2b. If the flow is highly channelized and dominated by a few pathways, then the flow through these pathways would be limited by the narrowest fracture in the pathway. Dominate pathways that connect opposite boundaries require more fractures as the domain size increases. Pathways requiring more fractures are less likely to contain only the largest transmissivity fractures of the transmissivity distribution. On statistical grounds, the conductance should decrease with increasing domain size.

Figure 5.9a also shows that the average measured conductance decreases with increasing transmissivity variation. It is important to note that the average transmissivity of each set is constant in these studies, not the average of the log transmissivity. As the transmissivity variance increases fewer of the fractures have transmissivities that are larger than the average. Following the logic of the previous paragraph, the decrease in log conductance with respect to transmissivity variation shown in Figure 5.9a is consistent with the notion of dominant pathways and channeling of flow.
Figure 5.9: Hydraulic conductance sensitivity to transmissivity variation. a) \( \log_{10} \) mean of the maximum directional conductance. b) \( \log_{10} \) standard deviation of the maximum directional conductance. c) Standard deviation of the direction of the principal axes of the conductance tensor. d) Orthogonality deviation of the principal directions of conductance.

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Figure 5.9b presents the standard deviation in the distribution of the log10 of the maximum directional hydraulic conductance (SDK). As one would expect, the SDK decreases with increasing domain size and with smaller transmissivity variation of individual fractures.

Fracture systems with a large transmissivity variation show a large variation in the direction of the major axes of the conductance ellipse. The variation decreases with increasing domain size (Figure 5.9c). Even in the 40m by 40m realizations, systems e and d have large variations in the direction of maximum conductance. Clearly, the scale of a REV increases with increasing variability in the individual fracture transmissivities. It is also evident that these domains are smaller than a flow REV.

Figure 5.9d presents an asymmetry measure in terms of the deviation from orthogonality of the principal directions of the conductance tensor. Figure 5.9d shows that the asymmetry decreases with domain size and that the asymmetry is dependent on the variance of fracture transmissivity. If one sets an arbitrary threshold of five degrees to be significant, then only system b in the largest realization does not have significant asymmetry. To put a five degree threshold in perspective, consider the channelized flow in the fracture network depicted in Figure 5.2b, a single realization of fracture system a. The orthogonality deviation in Figure 5.3, from the network of Figure 5.2, is sixteen degrees.

5.7.3 Fracture Density

The scan-line density was halved in fracture system e, from 3 fractures per meter to 1.5 fractures per meter, for each fracture set. Fracture system e is close to the percolation threshold and is quite sparse. The investigation of fracture system e demonstrates a strong sensitivity of the conductance to fracture density in a system near the threshold of percolation. Comparing system e with a, in Figure 5.10a, reveals that not only does e have a lower mean conductance, as one would expect, but that the conductance decreases much more rapidly with domain size than it does for the other fracture systems. This is consistent with the notion of channel-dominated flow and the consequent sensitivity to pathways composed
Figure 5.10: Hydraulic conductance sensitivity to fracture density, e, length, f, and isotropy, k and m, variation. a) $\log_{10}$ mean of the maximum directional conductance. b) $\log_{10}$ standard deviation of the maximum directional conductance. c) Standard deviation of the direction of the principal axes of the conductance tensor. d) Orthogonality deviation of the principal directions of conductance.
of larger than average transmissivity. As the domain size increases, fewer clusters of highly transmissive fractures are large enough to connect the domain boundaries. Individual realizations of system e contain cases that do not have any fractures that connect the domain boundaries; these cases are not included in the conductance calculations.

The orientation of the fracture sets is the same in fracture system e and the reference system a. However, the standard deviation in the orientation of the major axes of the conductance ellipse indicates that for fracture system e the distribution of the major axes varies so widely as to be reasonably approximated by a uniform distribution. The difference in the variation of the orientation of major axes between systems e and a is an indication that the lack of connectivity due to the sparseness of system e is more important to the direction of the maximum conductance than the orientation variability of the fracture sets. System e has a relatively large degree of asymmetry at all the scales investigated (Figure 5.10d), which is consistent with the supposition that the sparseness of system e is an important feature and that asymmetry is linked to a few dominant flow pathways.

5.7.4 Fracture Length

The mean fracture length of the first fracture set of fracture system f is doubled from that in system a to 2m. The mean length of the set 2 fractures remains at 1m. Since the scan-line density of system f is the same as the scan-line density of system a, the density of fractures expressed in terms of numbers of fracture centers per unit area is smaller in fracture system f than in the reference system. The conductance statistics for fracture system f are presented in Figure 5.10. The longer fractures in system f, in comparison to the reference system a, mean that fewer fractures are required to form a dominant channel. The result is a larger mean maximum conductance that decreases less rapidly. The SDK is roughly equivalent. The major axes of the conductance ellipses are more closely aligned with the x axis than those of system a. For the 40m by 40m domain, system f has a standard deviation in the major axes direction of 5 degrees. The longer fractures in set 1 also result in less asymmetry in the conductance
In summary of these characteristics, while system $f$ has longer fractures and a lower density in terms of areal density, this system has a higher connectivity and a smaller REV than the reference system.

5.7.5 ORIENTATION

The effects of the orientation of the fracture sets on hydraulic conductance have been investigated using two groupings of the fracture systems. Both groups, $j$, $a$ and $n$ and $g$, $h$ and $i$, have systems with standard deviations about the mean orientation of 0°, 10°, and 30° respectively. The mean orientations of the group $g$, $h$ and $i$ are 30° for set 1 and 60° for set 2. The group $j$, $a$ and $n$ are orthogonal fracture systems. Figure 5.11 contains the statistical results from these systems. The non-orthogonal systems $g$, $h$, and $i$ have a lower connectivity, resulting in smaller conductances and slightly larger SDKs. Fracture system $i$ has such a large variation in fracture orientation that the lack of orthogonality in the mean orientation yields a conductance behavior that similar to the sub-orthogonal reference simulation. While the mean conductance of the non-orthogonal systems are sensitive to the variability in orientation, the orthogonal systems show limited sensitivity to this parameter. As might be expected, the larger variation in fracture orientation tends to result in larger variations in the direction of major axes of the conductance ellipses. System $j$, with no orientation variability, is an exception. System $j$ exhibits a slightly larger directional variability in the major axes of the conductance ellipse than the reference system. Figure 5.11d does not reveal a clear link between asymmetry of the conductance tensor and the variance in fracture orientation. The asymmetry is larger for the sub-orthogonal fracture systems.

5.7.6 ISOTROPY

Fracture system $m$ is composed of two orthogonal sets with identical statistical parameters, leading to an isotropic hydraulic conductance. In this case, the conductance ellipse is nearly circular. The orientation of the major axis of the conductance ellipse and the deviation in the principal directions of the conductance tensor from orthogonality are sensitive to small variations in the flow through the domain.
Figure 5.11: Hydraulic conductance sensitivity to mean fracture orientation, and orientation variation.

a) $\log_{10}$ mean of the maximum directional conductance.  
b) $\log_{10}$ standard deviation of the maximum directional conductance.  
c) Standard deviation of the direction of the principal axes of the conductance tensor.  
d) Orthogonality deviation of the principal directions of conductance.
The large variations shown in Figure 5.10c and d for fracture system \( m \) are not indicative of sub-REV behavior. The small variation in the mean log conductance with domain size, the magnitude of the standard deviation in log \( k \), and its small variation with domain size, all indicate that the flow REV for system \( m \) is the smallest of all those examined.

### 5.7.7 Measurement Scale

Figures 5.9, 5.10 and 5.11 show very consistent dependencies on scale. The mean conductances and the SDKs all decrease with an increasing scale of measurement. The variation in the direction of the major axes of the conductance ellipse and the degree of asymmetry in the conductance tensor also decrease with domain size. Of all the fracture systems investigated, system \( m \) seems to have the smallest REV scale for flow. This scale is near or slightly larger than 40m. Hestir and Long [1990] have proposed an empirical formula to estimate the scale of a REV for two-dimensional Poisson networks with zero transmissivity variance. System \( m \) is the only fracture system in this study that satisfies the assumptions adopted in Hestir and Long’s analysis. Applying their formula to fracture system \( m \) predicts the scale of the REV to be close to 50m, in apparent agreement with the conclusions drawn from Figure 5.10.

### 5.8 Sensitivity to Fracture Network Structure

The sensitivity investigation in Section 5.7 focused on the effects of changes in the characteristics of the fracture sets on the hydraulic conductance tensor of a fracture network. The fracture networks were generated using a Poisson structural model. In this section, the effects of using different structural models to generate the fracture networks are investigated. The following two questions are addressed. 1) Do the sensitivities to parameter changes differ with the structural model? 2) How does the scale of a REV depend on the structural model?

Two structural models are examined; the war zone model and the Levy-Lee model. These two structural models were described in Sections 3.3.3 and 3.3.4, respectively. For both structural models, the
fundamental changes in the nature of the fracture systems can be described as an increased clustering of fractures and in a broadening of the length scale of the fractures. The length scales of fractures of the poisson model are determined by the parameters of the fracture sets. In the war zone model, fracture length scales are determined by the size of the war zones in addition to the fracture set parameters. The Levy-Lee model has a broader range in the scale of the fracture lengths than either of the other two models.

5.8.1 WAR ZONE FRACTURE SYSTEMS

War zones can be though of as regions where pre-existing fractures have changed the local stress conditions promoting a higher frequency of fracturing within the zones. A war zone network of fracture system \( a \) is shown in Figure 5.12a. The distribution of fractures is more heterogeneous than in the corresponding Poisson model (Compare with Figure 5.2a). To apply the war zone structural model to fracture system \( a \), it is assumed that fracture set 1 represents the pre-existing fractures and fracture set 2 represents later fractures. Fractures of set 2 are more dense than a corresponding Poisson network within the war zones, but outside of the war zones they are less dense. The lower density of fractures outside the war zones follows from an assumption that the scan-line density for fracture set 2 includes both the fractures created within the war zones and those outside the war zones. The specified scan-line density of set 2 was halved in order to keep the overall scan-line density of fracture set 2 equal to the scan-line density in the Poisson model.

The assumption that the scan-line density includes the war zones is arbitrary. It is also possible to assume that the measured scan-line density does not include war zone fractures, in which case the decrease in fractures outside the war zones would not occur. The issue is mentioned because many of the differences in the results of the war zone sensitivity from those of the Poisson structural model can be attributed to the lower fracture density outside of the war zones.
Figure 5.12: War zone fracture network. a) the trace map. b) Flow rates within fractures.
Figure 5.12: War zone fracture network  b) Flow rates within fractures.

Minimum flow drawn = 3.8e-12 m³/s
A diagram of flow through the war zone network is shown in Figure 5.12b for a hydraulic head gradient oriented parallel to boundary 1. Flow appears to be concentrated in fewer channels in Figure 5.12b, compared to the flow plot of the Poisson network shown in Figure 5.2b. War zones are often a part of these channels. In war zones, such as the one near the location (18,9), the flow tends to disperse and then reconcentrate as the flow leaves the zone.

5.8.1.1 SENSITIVITY TO TRANSMISSIVITY VARIATION

The changes in hydraulic conductance with the scale of measurement, that are seen from the war zone model (Figure 5.13), are similar to those observed from the Poisson model. These results, which are plotted for fracture systems with different standard deviations in the log of the fracture transmissivity, should be compared with the corresponding calculations for the Poisson model shown in Figure 5.9. The sensitivity of the hydraulic conductance to increases in fracture transmissivity variance observed in Figure 5.13 are also similar to the sensitivity of the Poisson based fracture systems. However, the magnitudes of the hydraulic conductance are lower in the war zone based networks than in the corresponding Poisson networks and the variability between realizations is larger. These differences are much like the change that occurred when the fracture density was decreased in the Poisson fracture system.

The results of halving the fracture density of the Poisson fracture system $a$ to form fracture system $e$ are presented in Figure 5.10. The differences are more pronounced in Figure 5.10 than the difference between the war zone generated fracture system $a$ and the Poisson generated fracture system $a$, consistent with a larger decrease in fracture density from fracture system $a$ to fracture system $e$ compared to the fracture density decrease outside the war zones. The changes between fracture system $a$ and fracture system $e$ can be used as a reference in understanding the effects of the increased fracture clustering in the war zone model.
Figure 5.13: Hydraulic conductance sensitivity to transmissivity variation of war zone generated fracture networks. a) log₁₀ mean of the maximum directional conductance. b) Log₁₀ standard deviation of the maximum directional conductance. c) Standard deviation of the direction of the principal axes of the conductance tensor. d) Orthogonality deviation of the principal directions of conductance.
The variation in orientation of the major axes of the hydraulic conductance tensor for the war zone fractures systems shows a slight increase over that of the corresponding Poisson fracture systems, but not the large increase in the variation in orientation of fracture system e. Evidently, the change in fracture density outside the war zones is not enough to make the connectivity of the fractures a stronger influence in the orientation of the conductance than the higher transmissivity in the first fracture set, as is the case in fracture system e.

In comparing the war zone and Poisson models, the deviation from orthogonality of the principal directions of conductance is somewhat higher for fracture systems a and b but virtually the same for the fracture systems with greater aperture variability. This behavior may indicate a transition from a case where connectivity is the dominant form of heterogeneity in fracture systems a and b, to one where transmissivity variability dominantly influences the conductance.

In summary, the basic response of the hydraulic conductance of a fracture network to changes in transmissivity variations is not effected by the introduction of the war zone model. The first fracture set, which is the same in the war zone networks and the Poisson networks, controls the orientation of the conductance tensor but the network scale connectivity provided by the second fracture set has been reduced by concentrating the fractures in the war zones. This reduction in connectivity lowers the average effective conductance of the network and causes a greater variability in the hydraulic conductance between different fracture networks. The war zone fracture systems have a larger scale of an REV than the corresponding Poisson fracture systems.

5.8.1.2 NON-ORTHOGONAL FRACTURE SYSTEMS

The averages calculated from the hydraulic conductance tensors of the war zone generated fracture networks with non-orthogonal fracture sets are presented in Figure 5.14. Fracture systems g, h, and i have fracture sets with mean orientations that differ by 30° rather than 90° as in fracture system a. The
Figure 5.14: Hydraulic conductance sensitivity to mean fracture orientation, and orientation variation of war zone fracture networks. a) $\log_{10}$ mean of the maximum directional conductance. b) $\log_{10}$ standard deviation of the maximum directional conductance. c) Standard deviation of the direction of the principal axes of the conductance tensor. d) Orthogonality deviation of the principal directions of conductance.
standard deviation of orientation distribution for fracture systems \( g \), \( h \), and \( i \) is 0°, 10°, and 30° respectively. More war zones and war zone fractures are created in models with less variation in the fracture orientation. Fracture system \( g \) has roughly 20% more war zones than system \( h \) and fracture system \( i \) has about 50% less. The density of fractures outside the war zones is the same for each fracture system. Thus, the scan-line density of fractures of set 2 is approximately 10% higher in fracture systems \( g \) and about 25% less in fracture system \( i \) compared to the corresponding Poisson fracture systems.

The sensitivity of the hydraulic conductance to changes in the variability in the orientation of the fractures of the non-orthogonal fracture systems is the same in the war zone fracture systems as in the Poisson model based fracture systems. Indeed, unlike the orthogonal fracture systems the non-orthogonal war zone fracture systems have only a slight sensitivity to the change in structural models. The most noticeable difference is a lesser sensitivity of the average hydraulic conductance to domain size in the war zone fracture systems and an increase in the variability in the magnitude of the hydraulic conductance that decreases more rapidly with domain size. The average magnitude of the conductance tensor is slightly larger at the 40m domain size in the war zone systems but somewhat smaller in the smallest domains.

The similarity in the changes of these fracture systems to the war zone structural model indicates that the differences in the frequency of the war zones is relatively unimportant. The magnitude of the hydraulic conductance in fracture system \( h \) is much less sensitive to the clustering in the war zone model than is the orthogonal networks of fracture system \( a \). The variability in the magnitude of the conductance, however, increases about the same degree as the reference system. Fracture system \( h \) has a slight increase in the variability in orientation of the major axes but little change in the deviation between principal conductance directions. The shift in the relative magnitude of the mean conductance can’t be explained by the decrease in fracture density outside the war zones. It may be due to increased connectivity within the war zones themselves in fracture system \( h \).
Differences in the overall density of fractures is not a dominate influence of the conductance of fracture systems g, h and i. Each of these fracture systems have a slight decrease in the magnitude of the hydraulic conductance. The variability of the conductance increases with all three fracture systems, with fracture systems of less orientation variability having a larger variance. It may be that the variation is being effected by the relative high conductivity in the war zone regions but the effect may also be due to the decrease in fracture density outside of the war zones.

The standard deviation in the orientation of the principal directions of the hydraulic conductance tensor shows much less variation between these non-orthogonal fracture networks generated using the war zone structural model, when compared to networks generated with the Poisson model. The fracture systems with little orientation variation are more sensitive to the decrease in fracture density outside the war zones. The deviation from orthogonality of the principal directions of the conductance tensor is not appreciably effected by the structural model change for these fracture systems.

In summary, the major influence of the greater clustering of the fractures in the war zone model seems to be from the reduction in the density of fracturing outside of the war zones required to maintain a constant overall fracture density. The characteristics of fracture systems g, h, and i indicate that the variability in local conductivity of the war zones may increase the variability in hydraulic response of a network at all scales. The scale at which the war zone fractures systems may be regarded as a continuum is larger than the scale of the REV when a Poisson structural model is used to generate the networks.

5.8.2 LEVY-LEE FRACTURE SYSTEMS

The predominant feature of fracture networks generated using the Levy-Lee structural model is that a few fractures completely span the fracture network domain no matter what is the scale of the domain. One realization of the Levy-Lee fracture system is shown in Figure 5.15a. This figure shows that the fracture network has local clusters of fractures that are connected by a few long fractures, at this
Figure 5.15: Levy-Lee fracture network. a) Fracture trace map. b) Flow rates within fractures.
Figure 5.15: Cont. Levy-Lee fracture network. b) Flow rates within fractures. Line widths are proportional to the flow rate.

Minimum flow drawn = \(9.7 \times 10^{-12} \text{ m}^3/\text{s}\)
scan-line density. These long fractures tend to control flow through the network as shown in Figure 5.15b where three wide, long fractures carry most of the flow across the fracture network. The net flow through the fracture network shown in Figure 5.15b, and in general for the Levy-Lee fracture system, is mostly dependent on the apertures of the few fractures that span the network. The remaining flow is dependent on a small number of additional connected fractures.

The scan-line density of regions within the trace map of Figure 5.15a has a large variation. This is an important feature that motivated a change in the method used to generate fracture networks for this portion of the study. All of the fracture networks for the Levy-Lee fracture systems were generated in a 45m x 45m domain. This ensures that the scan-line density of the smaller domains fluctuates in a manner that is representative of the Levy-Lee structural model. The standard method of generating the fracture network ensures that the scan-line density within the flow domain is close to the specified scan-line density.

5.8.2.1 SENSITIVITY TO TRANSMISSIVITY VARIATION

The most striking feature of the network conductance of the Levy-Lee fracture system is that the mean conductance has little if any dependence of the variability of the transmissivity as shown in Panel a of Figure 5.16. The mean conductance of the fracture networks is strictly a function of the mean transmissivity of the individual fractures because the flow through the fracture networks is controlled by individual fractures. The fracture network conductance of the Levy-Lee structural model also has a different dependence on domain size than the other fracture networks. In Panel a, the mean conductance of the networks shows little change with domain size. The slight decrease in conductance with increasing domain size may be due to the limited fracture generation region. Long fractures are probably under represented in the larger domain sizes.
Figure 5.16: Maximum directional hydraulic conductance sensitivity to transmissivity variation of Levy-Lee generated fracture networks. a) $\log_{10}$ mean of the maximum directional conductance. b) $\log_{10}$ standard deviation of the maximum directional conductance. c) Standard deviation of the direction of the principal axes of the conductance tensor. d) Orthogonality deviation of the principal directions of conductance.
The mean conductance of the Levy-Lee model is not totally independent of scale because of the dependence on fracture orientation variation with fracture length. Recall that the standard deviation in the orientation of the fractures generated using the Levy-Lee model is inversely proportional to the fracture length. It appears that this feature changes the character of the Levy-Lee generated fracture networks between the 5m and 10m domain scale. The 5m x 5m fracture networks may also be effected by the 0.5m minimum fracture length in the Levy-Lee model.

As one would expect from a stronger dependence on individual fractures, the standard deviation of conductance of the Levy-Lee fracture networks shown in Panel b is dependent on the variability of the fracture transmissivity and is larger than that of the other fracture systems. Neither the variability in the direction of the conductance (Panel c) nor the orthogonality deviation of the principle directions of conductance (Panel d) show significant differences in their dependence on transmissivity variation or scale from the Poison fracture systems (Figure 5.9). None of the data presented in Figure 5.16 indicates a sensitivity to the variability in scan-line density. That the variability in the scan-line density of the smaller fracture networks is not reflected in Figure 5.16 is a further indication that the conductance of the Levy-Lee fracture networks is dominated by a few long fractures.

5.9 Discussion

In this chapter, an approach to evaluating the scale of an REV for fluid flow in fractured media has been presented that provides a tensor characterization of sub-REV flow. The tensor representation fully describes the dependence of average fluid flow on the orientation of the hydraulic gradient. The magnitude of the hydraulic conductance tensor has an inverse dependence on the scale of the flow domain. Asymmetries in the conductance tensor representations of flow behavior are sensitive to heterogeneity within the flow domain.
It should be emphasized that sensitivity to rotation of the model domain, as used in previous investigations, is a clear indication that the scale of the domain is below the scale the REV. The erratic behavior noted by previous investigators provides a sensitive indication that the domain is below the scale of an REV. The importance and use of the erratic behavior below the scale of an REV has led to the assumption on the part of many people that the erratic behavior is an inherent property of fractured media and that it distinguishes fractured media from granular porous media. The dependence of flow in fractured media on the direction of the hydraulic head gradient is not inherently erratic but that it can appear erratic using a specific method of analysis. Thus, while the hydraulic behavior of a fractured medium does have some special attributes that arise from its geometric structure, the medium behaves in a manner analogous to a granular porous medium.

This issue brings us back to the field evaluations of the cross-borehole method described by Hsieh et al. [1985]. A direct comparison of cross-borehole directional hydraulic conductance measurements to the numerical calculations of directional hydraulic conductance for two-dimensional fracture networks is not possible. Not only is the difference in dimensionality important but the method used to characterize the flow behavior influences the conclusions one draws, as demonstrated by both the smooth behavior of the conductance tensor and the asymmetry introduced by the flow averaging. Fitting the square root of directional conductance measurements to an ellipse is not, in and of itself, an indication that the scale of a REV has been reached in both two and three-dimensional fracture networks. Even though flow through an individual realization of a fracture system can be described by a conductance tensor, different fracture networks of the same fracture system can have very different tensor descriptions.

In a field setting there is only one fracture network. However, a local determination of directional hydraulic conductance samples only a portion of the network. If the volume sampled in the investigation is a REV of the fracture network then, if the macroscopic character of the fracture system does not change spatially, the results of the investigation can be applied to the rest of the network. If the volume sampled
is below the scale of a REV then the results of the study described in this chapter indicate that a hydraulic conductivity representative of the fracture network may differ in both magnitude and in the orientation of anisotropy from the hydraulic conductance determined for the sampled volume. If determinations of directional hydraulic conductivity from field experiments, such as the cross-borehole pressure measurement technique, are best represented by an asymmetric hydraulic conductivity tensor then the asymmetry may be an indication that the averaging volume of the field experiments is smaller than an REV. To make the link between tensor asymmetry and sub-REV behavior requires an understanding of the relationship of asymmetry in the conductance tensor of a measured volume of rock to the method of analysis used to determine the directional hydraulic conductivity.

5.10 SUMMARY

1. Flow through fractured media below the REV scale can be described using tensor notation. The tensor representation fully describes the average flow behavior of the fracture network. For fracture networks defined with fixed boundaries and constant hydraulic head gradient boundary conditions, plots of the square root of directional hydraulic conductance as a function of the direction of flow, and plots of the square root of the inverse of the directional hydraulic conductivity as a function of the direction of the hydraulic gradient, are smooth and predictable from the hydraulic conductance tensor. The erratic behavior of conductance estimates noted in previous papers [Long et al., 1982; Long and Witherspoon, 1985; Dershowitz and Einstein, 1987; Khaleel, 1989; Hestir and Long, 1990; Odling and Webman, 1991] is not a fundamental property of flow through fracture systems. The erratic behavior is a result of the rotation of the model domains that are used in these studies to change the direction of the hydraulic gradient. As such, it provides a useful method for evaluating the scale of an REV for flow.

2. The hydraulic conductance tensors for the two-dimensional representations of fracture systems modeled in this paper need not be symmetric. The asymmetry is, however, an artifact of the
averaging process used to measure flow across the domain boundaries. The asymmetry is an
indication of sub-REV behavior.

3. A consistent scale dependence is observed in the hydraulic behavior of the two-dimensional
fracture systems below the scale of an REV. Measurements of mean conductances, the standard
deviation in maximum directional conductance and the deviation from orthogonality of principal
directions of the conductance all decrease with an increase in domain size. These dependencies
may be different in three dimensional fracture systems.

4. The variation in the orientation of the major axes of the conductance ellipse, calculated from
multiple realizations of the same fracture system, is a sensitive indicator of the scale of an REV.
CHAPTER 6

CONCLUDING COMMENTS

6.1 MODELING FRACTURE NETWORKS BELOW THE REV SCALE

A constant theme has run through earlier chapters. The theme has been the hydraulic behavior of fractured regions smaller than the scale appropriate for equivalent homogeneous continua representation. It is axiomatic to say that the heterogeneity in the hydraulic characteristics of fractures within a sub-REV domain must be considered; it is not clear how closely the heterogeneity must be retained. The importance of the strong interaction of internal heterogeneity and the boundaries conditions of the domain is a less obvious but equally important issue.

In Chapter 5, it was demonstrated that the erratic character attributed to fractured media below the scale of the REV is not an inherent property of fractured media but rather a hypersensitivity to small changes in the fracture network in the corners of a fracture domain. The sensitivity is amplified by boundary conditions that do not respond to fractures that cross domain boundaries twice in a short distance. The tensor relationship developed in Chapter 5 shows that the flow within fracture networks is not erratic and behaves in a smooth and predictable manner. At the same time, the asymmetric nature of the hydraulic conductance tensors further demonstrates that ignoring the influence of heterogeneity, be it internal or external, creates behavior in the model domain that is not representative of a natural situation.

The elimination of corner cutting fractures in some of the networks in Chapter 5 was required to keep the hydraulic behavior of a relatively large domain from being obscured by high flow rates through short fracture segments that connected with boundaries with fixed hydraulic heads. While the boundary conditions could be considered reasonable for the domain as a whole, the lack of response of these boundary conditions to those short-circuiting fractures could not be considered reasonable. Hestir and Long [1990] and Herbert et al. [1991] addressed the sensitivity of fractured media to the boundary
conditions by limiting their analysis of a fracture network to an interior region; buffering the flow domain boundaries from the specified boundary conditions with an intervening zone of the fracture network.

Network block fractures could not be eliminated to rid the network blocks of short-circuit fractures, nor could the network blocks be completely buffered from their neighbors. The treatment of the boundary conditions in the dual permeability model evolved toward boundary conditions that not only respond to the heterogeneity inside the network blocks but also incorporated some sense of the heterogeneity of the fractures surrounding the network blocks. The ability of the dual permeability model to simulate flow and transport in a wide variety of fracture networks demonstrates that the method used to characterise the network blocks retains the most important characteristics of these sub-REV regions.

There are three essential features to the way the boundary conditions of the network blocks are treated. The first is the partial buffering from the rest of the fracture network that the primary fractures provide. These fractures are selected, in part, because they are anticipated to have a strong influence on their local flow regime. It was initially thought that this feature alone would be sufficient to allow a linear hydraulic head gradient to be assumed within these fractures. The second major feature of the network block boundary conditions is inclusion of the primary fractures in the effective conductance representation of network blocks. This approach allows the hydraulic head distribution along these fractures to responded to the heterogeneity within the network blocks. It is appropriate to consider the primary fractures as part of the boundary conditions. As such, they provide a responsive aspect to the boundary conditions. The third feature is the enhancement of the transmissivity of the primary fractures as a crude method of including the heterogeneity of the surrounding fracture network in the boundary conditions. These three aspects of the way boundary conditions have been treated in the network blocks are the key element to effectively incorporating these sub-REV domains in the dual permeability model.
6.2 DUAL PERMEABILITY MODELING

The main thrust of this thesis has been the proposal and investigation of a new concept of modeling fluid flow and solute transport in fractured media at the field scale. The main proposition of the concept is that it is acceptable to increase the uncertainty of the predicted hydraulic behavior of the rock mass through approximations to the hydraulic behavior of sub-continuum regions of a fracture network, because the knowledge of the fracture network of any rock body will, in practice, have a large degree of uncertainty. These approximations are used to determine a coarse-scale hydraulic head distribution within the fracture network. Solute transport properties of the sub-continuum regions are determined using the coarse-scale head distribution as boundary conditions to the regions. The transport properties of the sub-continuum regions are used in a model of transport for the entire fracture network.

An implementation of this concept has been presented in Chapter 4. The emphasis in the development of the dual permeability model has been on maximizing the accuracy of the flow and transport predictions in a model that contains the essential features of a dual permeability model. These features are: 1) the subdivision of the fracture network into smaller domains based on the heterogeneity of the fracture network, 2) sub-continuum models of flow and transport within these small domains, 3) coarse-scale models of flow and transport that utilize approximations of the small domains, 4) the independence of the small domains from adjacent domains.

The dual permeability model succeeds in reproducing the hydraulic behavior of a wide variety of fracture systems as demonstrated by the agreement in the particle transport predictions of the dual permeability model and Discrete for most of the fracture systems examined in Chapter 4. The model seems to be particularly well suited to the Levy-Lee structural model which uses a power law model of fracture scaling. The model is limited to fracture networks that have a broad range of fracture lengths as demonstrated by the poor agreement for fracture system 3. The two aspects of the dual permeability approach that proved to be the most difficult to address were how to subdivide the domain and how to
approximate the hydraulic conductance of these domains. The approximation of the network blocks was discussed in the previous section. The development of an automated procedure to subdivide the fracture domain on the basis of the geometric properties of the fractures has been a difficult task. The routine that has been developed is very complex and is not universally successful in creating an acceptable primary fracture network. It is this phase of the dual permeability model that may be the most difficult transition to three dimensional fracture system modeling. However, the routine is able to treat a wide variety of two dimensional fracture systems and is fast relative to the other tasks within the dual permeability program.
NOTATION

\( A \) overlap area of two fractures in war zone model
\( \bar{A} \) matrix representing the conductances in a flow equation
\( b \) fracture aperture
\( \bar{b} \) right hand side of the flow equation
\( D \) diffusion coefficient or fractal dimension
\( E \) Conversion factor from average fracture length per square meter to scan line density
\( F \) Fraction of length lost to truncation
\( g \) unit vector in the direction of the hydraulic head gradient
  also acceleration of gravity
\( h \) hydraulic head
\( k_{war} \) war zone intensity factor
\( K \) hydraulic conductivity or conductance
\( \bar{K} \) 4 element by 2 element hydraulic conductance tensor
\( K' \) 2 element by 2 element hydraulic conductance tensor
\( K_{ij} \) hydraulic conductance tensor components
\( L \) distance
\( \ell \) radius of the Mohr's circle
\( \bar{\ell} \) mean fracture length
\( n_i \) direction cosine between the \( i^{th} \) axis and the average flow direction
\( \bar{n}_i \) direction cosine between the \( i^{th} \) axis and hydraulic gradient
\( p \) co-ordinates of conductivity ellipse
\( \bar{p} \) co-ordinates of inverse conductivity ellipse
\( \bar{\bar{p}} \) co-ordinates of the Mohr's circle
\( Pe \) Péclet number
\( q \) specific discharge
\( Q \) flow rate in an individual fracture
\( Re \) Reynolds number
\( s \) unit vector in the direction of average flow
\( t \) time
\( T \) transmissivity
NOTATION

- $T_r$: fracture truncation probability
- $V$: average fluid velocity
- $\mathbf{\hat{x}}$: vector of hydraulic heads
- $\mathbf{\hat{x}_i}$: unit vector along the $i^{th}$ axis
- $W$: length of domain boundary
  
  also parameter measure of war zone model
- $\alpha$: offset coefficient for the influence boundary
- $\alpha_w$: angle between fractures in war zone model
- $\lambda_s$: scan-line density
- $\mu$: Fluid viscosity
- $\rho$: fluid density or fracture midpoint density
- $\sigma$: standard deviation
- $\theta$: Orientation (counterclockwise rotation angle from x direction)

Subscripts
- $c$: closeness in war zone model
- $i,j$: number index
- $f$: fracture
- $g$: hydraulic gradient
- $h$: hydraulic
- $L$: largeness in war zone model
- $p$: parallelness in war zone model
- $t$: transverse to hydraulic gradient
- $T$: transport
- $m$: boundary face (1..4) or boundary node index
- $s$: flow
- $x$: x axis
- $z$: z axis
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Appendix A contains two series of flow charts that describe the program *Discrete* and the dual permeability program. Four charts (Figures A.1 - A.4) provide an overview of *Discrete*.

Seven additional charts (Figures A.5 - A.11) are used to present the more complicated dual permeability program. The first chart in each series is an overview of the structure of the entire program. Tasks that are described in more detail in later flow charts are indicated with the appropriate figure number in the upper right corner of the task description box. The flow charts describing the fracture network generation process are identical for both models.

The flow chart figures are:

- Figure A.1: The structure of *Discrete*;
- Figure A.2: Fracture realization process;
- Figure A.3: Fracture network reduction;
- Figure A.4: Particle tracking in *Discrete*;
- Figure A.5: The structure of the dual permeability program;
- Figure A.6: Primary fracture network selection;
- Figure A.7: Formation of the dual permeability coarse scale flow model;
- Figure A.8: Determination of the network block residence distributions;
- Figure A.9: Formation of the transport model;
- Figure A.10: Calculation of the primary fracture transport characteristics;
- Figure A.11: Particle tracking algorithm.
Program Discrete

- Fracture Network Realization
- Determine Flow Equations
  - Solve Flow Problem
  - Determine Fracture Intersection Flow Splits and Fracture Residence Times
  - Particle Tracking Transport Simulation

Figure A.1: The structure of Discrete.
Figure A.2: Fracture realization process.
Reduce Network

Remove Intersections Outside Boundaries

Identify Fractures Connected to Boundaries

Is the Fracture Connected to a Boundary by the Intersecting Fracture?

Yes

Mark Fracture as Connected

Have new Connections been Found?

Yes

No

Mark Intersection to be Deleted from Intersecting Fracture

Remove Deleted Intersections

Truncate Retained Fractures at Outer Intersections

For each fracture not connected to two boundaries

For each intersecting fracture

Figure A.3: Fracture network reduction.
Particle Tracking Simulation

For each Particle

Introduce Particle at Injection Node

Randomly Select an Exit Fracture

Add time to Reach Next Intersection

Is Time Later than Position Monitoring Time?

Yes

Add to Position Statistics

No

Is the Intersection on the Boundary?

Yes

Save Particle Exit Time and Location

No

Add to Position Statistics

Figure A.4: Particle tracking in Discrete.
Dual Permeability Program

1. Fracture Network Realization
2. Primary Fracture Network Selection
3. Determine Effective Conductance of Network Blocks
4. Incorporate Network Blocks into Primary Network to Form Flow Model
5. Solve Flow Model for Hydraulic Head Distribution
6. Find Network Block Residence Time Distributions
7. Form Transport Model from Network Blocks and Primary Fractures
8. Particle Tracking Transport Simulation

Figure A.5: The structure of the dual permeability program.
Primary Fracture Network Selection

Increase Percentage of Primary Fractures

Select Primary Fractures From Individual Characteristics

Find Primary Fracture Intersections. Connect Ends Using Added Primary Fracture Segments.

Remove Unconnected Primary Fractures

Define Network Block Geometry

Test Block Size

For Each Block

Find Internal Network Block Fractures and Intersections

Test Block Size

For Each Block

Select New Primary Fracture

Select New Primary Fracture

Figure A.6: Primary fracture network selection.
Form Dual Permeability Flow Model

Allocate Space for Stiffness Matrix

Form Network Block Stiffness Matrix

Assign Unit Head
Set others to Zero Head

Solve Flow Equation

Determine Flow into each Primary Fracture Segment

Add Flows as conductance terms to Dual Permeability Stiffness Matrix

Add Conductance of Primary Fracture Segments to Stiffness Matrix

Figure A.7: Formation of the dual permeability coarse scale flow model.
Form Transport Model

Figure A.8: Formation of the transport model.
Recalculate Network Block Stiffness Matrix

Solve Flow Problem

Calculat Flow at each Boundary Node

Order Internal Nodes by Head

Determine the Inlet and Outlet Boundary Nodes

Determine Flow Splits at each Internal Node

Find Arrival time at Node Connected to Boundary Inlet

Calculate Probabilities and Times of Entering Node from Upstream Nodes

Calculate Probabilities and Arrival Times at Outlets

Figure A.9: Determination of the network block residence distributions.
Calculate Transport Characteristics of Primary Fracture Segment

Calculate Flow at Upstream End from Coarse Scale Heads

Calculate Time to Reach Intersection

Calculate Probability of leaving Primary Fracture

Calculate Flow to Next Intersection

Calculate Flow at Downstream End from Coarse Scale Heads

Do Downstream Flow Calculations Agree?

No → Stop

Yes

For each intersection with a Network Block Fracture

Figure A.10: Calculation of the primary fracture transport characteristics.
Particle Tracking Simulation

For each Particle

Introduce Particle at Injection Node

Randomly Select an Exit Fracture

Add time to Reach Next Intersection

Exit at intersection to a Network Block?

Select Exit Location from Network Block and add Residence Time

Is the Node a Boundary Node?

End of Fracture?

Locate Exit on Primary Fracture Segment

Save Particle Exit Time and Location

Is the Node a Boundary Node?

Yes

No

Figure A.11: Particle tracking algorithm.
APPENDIX B

ASYMMETRY MEASUREMENT STATISTICS

In Appendix B, three alternative methods of statistically characterizing the sensitivity of conductance tensor asymmetry to fracture network geometry and the scale of the flow domain are investigated. These statistics are:

1) The deviation in degrees from orthogonality of the principal directions of the conductance tensor.

2) The ratio of the positive difference in the off-diagonal elements of the conductance tensor to average of the diagonal elements of the tensor.

3) The ratio of the positive difference in the off-diagonal elements to the radius of the Mohr's circle.

As noted earlier, the principal directions of a symmetric tensor are orthogonal. The first statistic is motivated by considering asymmetry as a rotation of the principle directions of the conductance tensor. The second statistic can be thought of as the fraction of the magnitude of the conductance that contributes to asymmetry. The third measure treats asymmetry as a fraction of the anisotropy in the conductance.

Figure B.1 merges the average orthogonality deviation for all of the cases studied into a single figure. As a measure of sub-REV behavior, the deviation from orthogonality behaves in a similar manner to the variation in the orientation of the major axes of the conductance tensor. It appears to be slightly more erratic than the orientation of the major axes. The orthogonality deviation tends to decrease as the domain size increases, and more fractures are included in the flow domain. The exceptions are cases m and e which are insensitive to domain size, a characteristic that is shared with the standard deviation of the orientation of the major axes of the conductance tensor.
Figure B.2 shows the average ratio of the difference in the off-diagonal elements to the average of the diagonal elements of the conductance tensor, relative to domain size. This measure is sensitive to increasing domain size in all cases. Of particular interest in comparing measure 1 and measure 2 are the results for cases \( m \) and \( n \). Case \( m \) along with case \( e \) show the largest asymmetry using measure 1. In contrast, case \( m \) has the lowest ratio of off-diagonal element difference to average diagonal elements. Case \( n \) shows a similar shift from a high value for measure 1 to a median value for measure 2. Case \( m \) is isotropic and case \( n \) is nearly so because of the variation in the mean orientation of the fractures of each set. The ratio of the difference in the off-diagonal elements to the average of the diagonal elements of the conductance tensor is a superior measure of sub-REV behavior compared to the orthogonality deviation of the principal directions of hydraulic conductance.

Figure B.3 shows the results of averaging the ratio of difference the off-diagonal elements and the radius of the Mohr's circle. These plots do not show a strong correlation between the measured values and domain size. The highest individual values result from networks that are nearly isotropic, that is networks with a small Mohr's circle radius. The ratio of difference the off-diagonal elements and the radius of the Mohr's circle is the poorest measure of sub-REV behavior of the three we have investigated. The link between asymmetry and anisotropy is poor.
Figure B.1: Average deviation of the orthogonality of the semi major and semi minor axis of the conductance ellipsi. a) Sensitivity to transmissivity variation. b) Sensitivity to fracture density, e, length, f, and isotropy, k and m, variation. c) Sensitivity to mean fracture orientation, and orientation variation.
Figure B.2: Average ratio of the difference in the off-diagonal elements to the average of the diagonal elements of the conductance tensor. a) Sensitivity to transmissivity variation. b) Sensitivity to fracture density, e, length, f, and isotropy, k and m, variation. c) Sensitivity to mean fracture orientation, and orientation variation.
Figure B.3: Average ratio of the asymmetry offset in the Mohr's circle to the radius of the Mohr's circle. a) Sensitivity to transmissivity variation. b) Sensitivity to fracture density, e, length, f, and isotropy, k and m, variation. c) Sensitivity to mean fracture orientation, and orientation variation.