ANALYTIC MODELS
FOR
CARRIER SENSE MULTIPLE ACCESS NETWORKS
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We accept this thesis as conforming
to the required standard.

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ABSTRACT

A computer network using non-persistent and 1-persistent Carrier Sense Multiple Access protocols is considered. The terminals in this network are connected to a common bus. Previous analytic models for such networks have assumed the electrical distance between terminals to be constant and equal to the maximum length of the cable. This gives a lower bound for system throughput.

We propose a more accurate model for an uncontrolled CSMA network with infinite users. Our model does not assume identical propagation delay between all terminals. Instead, it considers the distribution of the terminals along the cable to be uniform. Simulation results show that this model is more accurate than previous models, especially when the propagation delay and the offered traffic are high.

Another uncontrolled, infinite user CSMA model is also described. This model gives fairly accurate results for networks with small end to end propagation delay and is simpler to formulate than existing models of comparable accuracy.
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CHAPTER 1

INTRODUCTION

The Network Structure

This thesis is concerned with the modelling of the Carrier Sense Multiple Access (CSMA) network. The network considered consists of a collection of terminals connected to a common cable. Whenever a terminal has a packet to transmit, it senses the channel (cable) for the presence of any carrier and then acts according to the protocol being used. The protocols considered are non-persistent CSMA and 1-persistent CSMA. A collision occurs if two or more terminals transmit at the same time. When this happens the information in the colliding packets is destroyed. The terminals involved in a collision wait a random amount of time before trying again. This process is repeated until the terminals have successfully transmitted their packets.

Performance evaluation of CSMA network

A CSMA network, like any engineering system, must meet certain design and performance specifications. It is therefore necessary to identify the system parameters that have a non-negligible influence on performance and the manner in which these parameters are related to the performance indices. This emphasizes the need for developing tools and techniques to evaluate the performance of CSMA networks. Two techniques that are commonly used are simulation and analytic modelling. In this
thesis we are concerned with the latter technique.

The performance of a network is related to the system (i.e., the system hardware characteristics, topology, protocol etc.) as well as to the work load that drives the system. A good model should accurately represent both the work load and the system's characteristics. Let us look at the features that characterize these in the CSMA network described above.

**Work Load Characterization**

The work load in a CSMA network is typically characterized by:

- The distribution of packet lengths.
- The distribution of inter-arrival times of packets.

For accurate description of the work load, both these distributions should be characterized as a function of the position of the terminals in the network. For example, suppose there are two terminals engaged in file transfer activity and the total network work load is mainly due to these two terminals. Let us examine the throughput of the network. If these two terminals are located near one another then one terminal can learn about the others's transmission faster than if they were separated by a larger distance. This would reduce the probability of collisions. Thus the throughput of the network would be higher the closer the terminals are together. Similarly, it can be shown that the throughput of the network would be higher if both these terminals are located near the middle of the cable than if they were located towards either end.
of the cable.

The order in which the packets arrive is not important in characterizing the network workload. Though this order might affect the throughput in isolated cases, the main goal in modelling the network is to estimate the performance indices statistically.

To our knowledge there exists no analytic model that considers the distribution of packet lengths and inter-arrival times as a function of the location of terminals in the network. These distributions are always assumed to be independent of the location of terminals. In fact in most models the distribution of packet lengths is also ignored and packets are assumed to be of constant length. As well, the interarrival times are invariably taken to be independent and identically distributed. Generally, they are assumed to be exponentially distributed as the memoryless property makes the mathematics tractable and the distribution is found to approximate real world behaviour reasonably well.

**System Characterization**

We view the System to be characterized by:

- **The protocol**, that is the set of formal conventions governing the communication between terminals.

- **The retransmission policy**, which defines the action to be taken when a packet suffers one or more collisions.

- **The control policy**, which defines the action to be taken to maintain the network throughput above a certain specified level.
The location of the terminals in the network.

It is difficult to distinguish between new and retransmitted packets in an analytic model. Thus most models assume the packet interarrival times -- for both new packets and those that are being retransmitted -- to be exponentially distributed.

The analysis of the network with general distribution on the locations of terminals is also very difficult. All the earlier models have ignored this distribution and assumed the electrical distance between any terminal pair to be constant and equal to the distance between the furthest two terminals (or the maximum length of the cable). Though unrealistic, the analysis of the network with this assumption gives a lower bound on the throughput.

The Thesis

This work is mainly concerned with improving the accuracy of CSMA models by considering the terminals to be uniformly distributed along the cable.

Overview

Chapter 2 contains a historical review of packet switching in radio channels. It describes the analytic modelling work done earlier and discusses some important results that have been obtained. It ends with an outline of some of the areas that need further research.

Chapter 3 describes a very simple analytic model using
Markov chains for analysing CSMA networks. This model gives fairly accurate results for small networks.

Chapter 4 contains the main work of this thesis. It describes the model that analyses CSMA network without assuming identical propagation delay between terminals. This chapter is divided into three sections. The first section analyses non-persistent CSMA. The second section analyses 1-persistent CSMA and the last section describes a simulation model used to estimate the effect of approximations made in our model. This section also compares the results of our model with those obtained by previous models.

Chapter 5 concludes this thesis and discusses the significance of the results obtained.
CHAPTER 2

2.0 HISTORICAL REVIEW

During the early 1970's, N. Abramson and his colleagues at the University of Hawaii developed a radio packet switching system known as THE ALOHA SYSTEM for linking computers. They were interested in connecting the computers at the seven campuses of The University of Hawaii without using telephone lines as the phone lines were expensive and unreliable. Today, many more sophisticated techniques for linking computers through radio channel are available. Most of these techniques have evolved from the ALOHA SYSTEM. Therefore, in many ways, their characteristics resemble those of the Aloha System. Because of its relative simplicity and because the performance of the Aloha System forms a basis with which to compare the performance of the more advanced techniques, we shall briefly review the characteristics and the performance of this system.

2.1 The ALOHA SYSTEM

Abramson's basic idea-- known as THE PURE ALOHA, was the following: Whenever a terminal (or computer) has a packet to transmit, it transmits the whole packet. If while this terminal is transmitting another terminal also starts to transmit then there is a collision and the information is destroyed. In the absence of collision the receiving terminal correctly receives the packet addressed to it (provided there are no transmission errors). Since a
transmitting terminal can listen to what it is sending, it can easily detect collision if one occurs. In the event of collision, the colliding stations wait a random amount of time (depending on the retransmission policy being used) before trying again.

Abramson \cite{1,2} was also the first to analyse the performance of the Pure Aloha System. For an infinite user system, the maximum achievable channel utilization for Pure Aloha was computed to be $1/2e$ or about 18.4%.

In 1972, Roberts\cite{21} described a method to double the utilization of an ALOHA System. In his method, which is known as the SLOTTED ALOHA, the time axis is divided into discrete intervals, the length of each interval being equal to the transmission time of a packet. A terminal can transmit only at the beginning of each slot. Therefore, whenever a terminal has a packet to transmit, it must wait till the beginning of the next slot. With this method the maximum achievable channel utilization for an infinite user population can be shown to be $1/e$ or about 36.8%.

It has been shown that the throughput of an ALOHA System is maximum when all the packets are of equal length\cite{9}.

Kleinrock and Lam \cite{13} have analysed the stability of the Slotted Aloha System. Using Fluid Flow approximation method, they found that an ALOHA Channel is always in one of three possible states: Stable, Unstable or Saturated. A stable state has only one equilibrium point. It has relatively high throughput and few backlogged users.
(Backlogged users are those whose packets have collided and are waiting for retransmission.) A saturated channel also has one equilibrium point in which nearly all the users are backlogged. An unstable channel exhibits bistable behavior. It has two equilibrium points. One equilibrium point has high throughput and few backlogged users, while the other equilibrium point is marked with low throughput and large number of backlogged users. For an unstable channel any throughput-delay performance results obtained under the steady state assumption are valid only for a finite period of time. That is, after some finite time, the system enters the unstable region and moves towards the equilibrium point in which nearly all the users are trying to retransmit collided packets and nearly every transmission results in a collision.

Simulation studies\cite{13} have shown that an uncontrolled infinite population Aloha System is always unstable.

It can be mathematically shown that an Aloha Channel which is stable for stationary input rate (i.e., both the number of users and the probability of generating new packets per user are constant), can become unstable for time varying input with the same mean input rate. Furthermore, if the input is stationary, an unstable channel can always be made stable by increasing the mean retransmission delay. However, this also leads to higher average transmission delay \cite{13}.

Kleinrock and Lam have defined a stability measure called FET for unstable channels. FET is the average first
exit time into the unsafe region starting from an initially empty channel. An unsafe region is one from where the system begins to move towards the low throughput and high backlogged equilibrium point.

2.2 **CSMA PROTOCOLS**

In situations where all the users are relatively close, that is, if the propagation delay is short compared to the packet transmission time, a terminal can sense the channel for the presence of carrier before transmitting a packet. This can significantly reduce the number of collisions and thus improve the channel utilization. Such protocols, in which a terminal listens for the carrier before transmitting are called Carrier Sense Multiple Access protocols (CSMA).

CSMA protocols can be classified into two categories: Non-Persistent CSMA and p-Persistent CSMA.

**Non-Persistent CSMA**

A ready terminal monitors the channel and then acts in the following manner:
- If the channel is sensed to be idle, it transmits the packet.
- If the channel is sensed to be busy, the terminal schedules the transmission of the packet to some later time according to some algorithm. At this new point in time it repeats the above algorithm.
In a slotted non-persistent CSMA the time axis is divided into slots of length equal to the maximum propagation delay between any two terminals. A terminal can transmit only at the beginning of a slot. When a terminal has a packet to transmit, it waits till the beginning of the next slot, then senses the channel and acts according to the protocol described above.

1-Persistent CSMA

1-persistent CSMA is a special case of p-persistent CSMA where p equals one. In this protocol, a ready terminal, after sensing the channel behaves as follows:
- If the channel is sensed to be idle, it transmits the packet with probability one.
- If the channel is sensed to be busy, it continues to sense the channel till it becomes idle and then transmits with probability one. That is, it persists in transmitting.

A slotted version of 1-persistent CSMA can be defined in a manner similar to the slotted non-persistent CSMA.

p-Persistent CSMA

In a p-persistent CSMA (p<1), the time axis is divided into slots of size equal to the maximum propagation delay. All users are synchronized and may transmit only at the beginning of a slot. When a ready terminal has a packet to transmit, it waits until the beginning of the next slot. At this point it senses the channel and operates in the following manner:
- If the channel is sensed to be idle, it transmits the packet with probability $p$. Or, with probability $(1-p)$ the transmission is deferred to the beginning of the next slot. If at the beginning of the next slot the channel is again detected idle, the above process is repeated. Otherwise the terminal acts as if a collision has occurred and reschedules the transmission to some later point in time. (According to some retransmission policy).

- If the channel is sensed to be busy, it continues sensing the channel till it becomes idle. At this point in time, it acts in a manner described in the above step.

Intuitively, it appears that the 1-persistent CSMA should give lower delay under light load when compared to non-persistent and $p$-persistent CSMA ($p<1$), while the latter protocols should give higher throughput than the former under conditions of heavy load. This is indeed found to be true\cite{12}.

2.3 TOBAGI'S ANALYSIS OF CSMA PROTOCOLS

Tobagi and Kleinrock \cite{12 to 16, 26 to 31} have extensively analysed CSMA protocols. In this section we shall present their results for an infinite population model for unslotted non-persistent and 1-persistent protocols. Their model does not assume existence of any control policy to ensure that the performance does not degrade beyond some threshold limit under conditions of heavy load. We shall
begin by stating the assumptions used in their model.

2.3.1 ASSUMPTIONS
- All packets are of constant length.
- There are an infinite number of independent users who collectively form a Poisson source.
- Each user is assumed to have at most one packet requiring transmission at any one time (including any previously blocked packet).
- The random delay after a collision is uniformly distributed and large compared to the packet transmission time.
- The propagation delay is identical between all terminals.
- Transmission errors (other than that due to collisions) are assumed to be very few and are therefore neglected.
- There is no capture effect.

Suppose two terminals transmit simultaneously. If the signal from one terminal is much stronger than the other (due, for example to its proximity to the receiving terminal), then the receiving terminal might not be able to detect the weaker signal. This phenomenon, known as capture effect, is assumed not to exist.
- Each terminal can sense the transmission of all terminals.
- A terminal may not transmit and receive simultaneously.
- Sensing the channel can be done instantaneously.
- The channel for message is different from that for the acknowledgements.
the system.

T - Transmission time in seconds for each packet.

S - Average number of packets generated per transmission time. Therefore, under steady state, S is also the channel throughput and the channel utilization.

G - Mean traffic offered to the channel (packets per transmission time T). This includes the new packets and the previously collided packets.

τ - Propagation delay between terminals in seconds.

We would like to express the results normalized with respect to the packet transmission time. Therefore we choose T=1 and define a=τ /T to be the normalized propagation delay.

2.3.3 THE RESULTS

Non Persistent CSMA

\[ S = \frac{G \cdot \exp(-aG)}{G \cdot (1 + 2a) + \exp(-aG)} \]  \hspace{1cm} (2.1)

1-Persistent CSMA

\[ S = \frac{G \cdot (1 + G + aG \cdot (1 + G + aG/2)) \cdot \exp(-G \cdot (1 + 2a))}{G \cdot (1 - 2a) - (1 - \exp(-aG)) + (1 + aG) \cdot \exp(1 + a)} \]  \hspace{1cm} (2.2)

p-persistent CSMA

\[ S(G, p, a) = \frac{(1 - \exp(-aG)) \cdot (Ps \cdot P_0 + Ps \cdot (1 - P_0))}{(1 - \exp(-aG)) \cdot (a \cdot t \cdot P_0 + a \cdot t \cdot (1 - P_0) + 1 + a) + a \cdot P_0} \]  \hspace{1cm} (2.5)

where Ps, Ps', t, t', and P_0 are defined in {12}. 
where $P_s$, $P_s$, $t$, $t$, and $\Pi_0$ are defined in \{12\}.

The throughput of uncontrolled CSMA channel tends towards zero as the load on the channel increases. Therefore in a practical system there must exist some control policy to maintain the throughput of the channel above some threshold value. Tobagi has analysed the stability and throughput for slotted non-persistent CSMA channel for various control policies. His analysis is basically an extension of Kleinrock and Lam's work on the stability of Aloha System \{28\}.

Kleinrock and Tobagi originally analysed CSMA protocols for ground radio. The situation they had in mind was a number of mobile users within line of sight trying to communicate with one another. In this situation, the assumption of identical propagation delay between all users is not bad.

Now consider the case in which all terminals are fixed and connected to a common bus. In this case some terminals are closer to the transmitting terminal than others. There are less chances of collision between terminals that are close together than between terminals that are farther apart. Therefore for networks with large end to end normalised delay, the assumption of identical propagation delay (equal to the end to end propagation delay) between terminals does not seem to be very accurate.
2.4 CARRIER SENSE MULTIPLE ACCESS WITH COLLISION DETECTION (CSMA/CD)

The throughput of CSMA protocols can be further improved if the colliding terminals are able to truncate their transmission once they detect a collision. This leads to a saving in the bandwidth and therefore to better channel utilization. Such protocols are known as carrier sense multiple access with collision detection or CSMA/CD. Ethernet is an example of a system using this protocol.

CSMA/CD protocols can also be classified as non-persistent and p-persistent (For their definitions, see Section 2.3). The characteristics of CSMA/CD protocols are similar to those of CSMA protocols. However, they are more stable and yield better throughput than the latter.

Tobagi and Hunt \cite{13} have analysed the stability and throughput of uncontrolled slotted non-persistent CSMA/CD protocol. Herr and Nute \cite{11} have analysed the performance of uncontrolled non-persistent and slotted 1-persistent CSMA/CD protocols.

2.4.1 We have stressed the importance of control policy. Most working systems make use of some control policy to stabilize and to ensure some minimum level of performance. (The control policy could be distinct from the retransmission policy or be integrated into it as in the case of Ethernet). Hence, what is needed is a model that can also take into account the effect of the control and retransmission policies.
The first attempt in this direction was made by Metcalfe and Boggs\cite{18}. They suggested a very simple model that surprisingly gave quite accurate results. They showed that for a system with $Q$ terminals waiting to transmit, the maximum channel throughput is achieved when each terminal transmits with probability $1/Q$. The truncated exponential backoff strategy used in Ethernet tries to approximate this $1/Q$ behaviour.

In 1979, Almes and Lazowska\cite{4} developed an infinite user model that used Markov Chains to represent different states of the system. The throughput at each state is calculated using Metcalfe and Boggs's model. The performance predicted by this model has been shown to be in close agreement with the actual measured results on the Ethernet\cite{24}.

Gelenbe and Mitrani\cite{10} have developed a technique somewhat similar to that of Almes and Lazowska's. However, instead of considering the states of the network (as in the case of Almes and Lazowska's model), they have focussed on the states of an isolated terminal. It is a more detailed model and therefore contains many more states. The number of states for an average sized network in their model is so large that it is nearly impossible to solve it exactly. Therefore they have abandoned exact analysis and resorted to the use of approximation techniques.

2.4.2 We have discussed the simplest and the most popular versions of CSMA protocols. There exists various other types
of CSMA protocols that resolve contention without creating a collision. We shall not attempt to describe collision-free CSMA protocols. Interested readers may refer to \cite{14}.

2.5 SOME RESEARCH ISSUES

- The performance of CSMA or CSMA/CD protocols is closely related to the propagation delay between terminals, the nature of retransmission policy and the control policy. The propagation delay would in general not be identical between all terminals. The control policy could be integrated into the retransmission policy. A research direction would be to analyse the system taking the propagation delay and the two policies into account.

- The exponential backoff retransmission strategy used in some CSMA/CD systems (e.g., Ethernet) has a last-in-first-out flavour. That is, the new packets tend to get transmitted before those that have suffered collisions. One interesting topic of research is to find out whether it is possible to avoid this effect and yet maintain the $1/Q$ strategy.

- The work load in the existing models is considered to be independent of the location of terminals. It would be useful to develop a model that could represent the work load as a function of the terminal locations.
CHAPTER 3

AN ALTERNATE SOLUTION OF 1-PERSISTENT CSMA

In their paper [12], Tobagi & Kleinrock have analysed the performance of 1-persistent CSMA under the assumptions stated in section 2.3.1. Here we present an alternate solution using the same assumptions. The chief (and probably the only) merit of this solution is that it is much simpler than the former method.

Suppose a packet arrives while some transmission is in progress. It is well known (from paradox of residual life [15]) that the probability that this packet arrives in a longer transmission period is higher than the probability that it arrives in a shorter transmission period. Analysis of the network taking this fact into account is fairly complex [12]. In this chapter we show that if we ignore this fact and assume that a packet is equally likely to arrive in any transmission period, then the analysis becomes very simple. The results of this model are valid for small normalized propagation delay. For normalized propagation delay equal to 0.01, which is a typical value for many practical systems, the results obtained using this model are within 1.6% of those obtained using Tobagi's analysis for $G \leq 5.0$. As the normalized propagation delay increases, the difference in the throughput predicted by the two models also increases.

The Model

Consider the packet transmission time to be one unit and
the end to end propagation delay to be 'a' units (all units of
time are normalised by the packet transmission time).

Let \( t \) denote the time a packet is transmitted immediately
upon arrival into an idle channel. If another packet arrives
between \( t \) and \( t+a \), the channel will still appear to be idle and
this packet will also be transmitted. This will create a
collision. If no packet arrives during \( t \) and \( t+a \), then the first
packet will be successfully transmitted.

In the event of a collision, let \( t+Y \) be the time of arrival
of the last packet arriving between \( t \) and \( t+a \) (see Fig3.1). Thus, the length of a successful transmission period is \( 1+a \) and
the length of an unsuccessful transmission period is \( 1+Y+a \). (Note:
in the following we shall abbreviate a transmission period as
T.P.)

Any packet arriving after the first \( a \) seconds of a
transmission period will sense the channel to be busy and must
wait until the channel is sensed idle, at which time they all
would be transmitted simultaneously.

We need to calculate \( Y \), the mean value of \( Y \).

\[
\Pr(Y \leq y) = \Pr(\text{at least one arrival occurs in the first } y \text{ seconds}
\quad \& \quad \text{no arrival occurs during the next } a-y \text{ seconds| at least one arrival occurs in the first } a \text{ seconds})
\]
\[
= \exp(-G(a-y)) \cdot (1 - \exp(-Gy))
\]
\[
1 - \exp(-aG)
\]

\[
\Pr(Y > y) = 1 - \frac{\exp(-G(a-y)) - \exp(-aG)}{1 - \exp(-aG)}
\]
\[
= \frac{1}{1 - \exp(-aG)} \cdot (1 - \exp(-G(a-y)))
\]
or \( Y = \int_{y=0}^{y=a} \frac{\Pr(Y > y)}{1 - \exp(-a*G)} \cdot \frac{1}{1 - \exp(-a*G)} \cdot (a - (1/G) \cdot (1 - \exp(-a*G))) \)  

(3.1)

We now construct a Markov's model with three states (Fig.3.2): Success state, Failure state and Idle state. The Success state represents a successful transmission, The Failure state represents an unsuccessful transmission and the Idle state corresponds to an idle channel. The state transition probabilities are quite obvious (that is, their derivation does not require much effort) and are shown below:

\( P_{is} = \Pr(\text{transition from Idle to Success state}) \)
\( = \Pr(\text{no packet arrives during the first a seconds}) \)
\( = \exp(-a*G) \)  

(3.2)

\( P_{if} = \Pr(\text{transition from Idle to Failure state}) \)
\( = 1 - P_{is} \)  

(3.3)

\( P_{si} = \Pr(\text{transition from Success to Idle state}) \)
\( = \Pr(\text{no arrival during packet transmission time}) \)
\( = \exp(-G) \)  

(3.4)

\( P_{ss} = \Pr(\text{transition from Success to Success state}) \)
\( = \Pr(\text{one arrival during the packet transmission time}) \)
\( \cdot \Pr(\text{no arrival during the next a seconds}) \)
\( = G \cdot \exp(-G) \cdot \exp(-a*G) \)
\( = G \cdot \exp(-(1+a)*G) \)  

(3.5)

\( P_{sf} = \Pr(\text{transition from Success to Failure state}) \)
\( = 1 - P_{si} - P_{ss} \)  

(3.6)

\( P_{fi} = \Pr(\text{transition from Failure to Idle state}) \)
\( = \Pr(\text{no arrival during an unsuccessful T.P.}) \)
\( = \exp(-(Y+1)*G) \)  

(3.7)
\[ Pfs = \Pr(\text{transition from Failure to Success state}) \]
\[ = \Pr(\text{one packet arrives during the unsuccessful T.P.}) \]
\[ \times \Pr(\text{no packet arrives during the next a second}) \]
\[ = (Y+1)G\exp(-(Y+1)G)\exp(-aG) \]
\[ = (Y+1)G\exp(-(Y+1+a)G) \quad (3.8) \]

\[ Pff = \Pr(\text{transition from Failure to Failure state}) \]
\[ = 1 - Pfi - pfs \quad (3.9) \]

Let \( Ps, Pf \) and \( Pi \) be the probability of being in Success, Failure and Idle state respectively. These probabilities are related to the state transition probabilities by the following set of equations.

\[ Ps = Ps*Pss + Pf*Pfs + Pi*Pis \quad (3.10) \]
\[ Pf = Ps*Psf + Pf*Pff + Pi*Pif \quad (3.11) \]
\[ Pi = Ps*Psi + Pf*Pfi \quad (3.12) \]

Only two out of the above three equations are independent. Therefore we introduce another constraint:

\[ 1 = Ps + Pf + Pi \quad (3.13) \]
\[ \text{or } Pi = 1 - Ps - Pf \quad (3.14) \]

Substituting (3.14) in (3.10) and (3.12) gives:

\[ Ps*(Pss-Pis-1) + Pf*(Pfs-Pis) = -Pis \quad (3.15) \]
&
\[ Ps*(Psi+1) + Pf*(pfi+1) = 1 \quad (3.16) \]

Applying Cramers rule we get

\[ Ps = \frac{-Pis*(Pfi+1)-(Pfs-Pis)}{(Pss-Pis-1)*(Pfi+1)-(Psi+1)*(Pfs-Pis)} \quad (3.17) \]
\[ \text{Pf} = \frac{(Pss - Pis - 1) + Pis \cdot (Psi + 1)}{(Pss - Pis - 1) \cdot (Pfi + 1) - (Psi + 1) \cdot (Pfs - Pis)} \]  
\[ (3.18) \]

\[ \text{Pi} = 1 - Pf - Ps \]  
\[ (3.14) \]

Let

\( Ls = \) Mean duration of success state = \((1+a)\) seconds.

\( Lf = \) Mean duration of Failure state = \((1+a+Y)\) seconds.

\( Li = \) Mean duration of idle state

\( = \frac{1}{G} \) (Assuming poisson arrival)

The mean throughput, \( S \), can be obtained using the following equation.

\[ S = \frac{Ps}{Ls \cdot Ps + Lf \cdot Pf + Li \cdot Pi} \]  
\[ (3.19) \]

Tables 3.1 and 3.2 compare the performance predicted by this model with that predicted using Tobagi's model for \( a=0.01 \) and \( a=0.10 \) respectively. Observe that the throughput predicted by this model is always less than or equal to that of Tobagi's model. This is because in this model a packet is equally likely to arrive in all transmission periods. This causes the probability of collision after a short transmission period to be as high as that after a long transmission period. On the other hand Tobagi's model considers that a packet is more likely to arrive in a longer transmission period than in a shorter one. Therefore in Tobagi's model the probability of collision after a short transmission period is less than the corresponding probability in the model described here, although the
probability of multiple collision after a long transmission period is more in Tobagi's model.
Table 3.1
\( a = 0.01 \)

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<tr>
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<td>0.1360</td>
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</tr>
<tr>
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<td>0.0621</td>
</tr>
<tr>
<td>5.0100</td>
<td>0.0368</td>
<td>0.0377</td>
</tr>
</tbody>
</table>

**NOTE:**
- **S1** = Throughput predicted by our model
- **S2** = Throughput predicted by Tobagi's model
- **G** = Offered traffic
### TABLE 3.2

\( a = 0.10 \)

<table>
<thead>
<tr>
<th>G</th>
<th>S1</th>
<th>S2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0100</td>
<td>0.0100</td>
<td>0.0100</td>
</tr>
<tr>
<td>0.2100</td>
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<td>0.1936</td>
</tr>
<tr>
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</tr>
<tr>
<td>0.6100</td>
<td>0.4117</td>
<td>0.4128</td>
</tr>
<tr>
<td>0.8100</td>
<td>0.4461</td>
<td>0.4490</td>
</tr>
<tr>
<td>1.0100</td>
<td>0.4456</td>
<td>0.4510</td>
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<tr>
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<td>0.3871</td>
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<tr>
<td>5.0100</td>
<td>0.0169</td>
<td>0.0200</td>
</tr>
</tbody>
</table>

**NOTE:**

- S1 = Throughput predicted by our model.
- S2 = Throughput predicted by Tobagi's model.
- G = Offered traffic.
CHAPTER 4

ANALYSIS OF CSMA PROTOCOLS ASSUMING TERMINALS ARE DISTRIBUTED
UNIFORMLY ALONG THE BUS

The performance of CSMA networks is related to the
propagation delay between terminals which, in turn, depends on
their relative locations. It is difficult to analyse the network
with any general distribution of terminal locations. Therefore
the earlier models assumed the propagation delay between
terminals is constant and equal to the propagation delay between
the furthest two terminals (or, the length of the cable).

The time taken by one terminal to learn about the
transmission from another increases as the distance between them
increases. Thus the probability of collision between a pair of
terminals would be higher the further apart the terminals are
located. Therefore, the assumption of identical propagation
delay (equal to that over the length of the cable) over-
estimates the proportion of collisions or under-estimates the
actual throughput. The magnitude of the error made by this
assumption increases as the network length increases.

In this chapter we present models that do not make the
assumption of identical propagation delay between terminals.
However, inorder to simplify our analysis we assume that the
terminals are uniformly distributed along the cable.

It may be argued that the earlier models could be used for
networks with large propagation delay by using the mean value of
propagation delay instead of the maximum propagation delay.
While this formulation may give more accurate results, this assumption is still not very accurate since the performance indices are related to the propagation delay in a highly nonlinear fashion.

The assumption of identical propagation delay is not a source of inaccuracy in slotted versions of CSMA protocols. In these protocols, time is divided into slots of duration equal to the maximum round-trip propagation delay. All terminals are synchronized and forced to transmit only at the beginning of a slot. Thus all terminals learn about a transmission by the end of the slot. Therefore the probability of collision between any pair of terminals is not a function of their relative location. For this reason we consider only the unslotted non-persistent and 1-persistent CSMA protocols in this chapter.

We now state the assumptions made in the following analysis.

General Assumptions
(For both 1-persistent and non-persistent CSMA models.)

- There are an infinite number of terminals uniformly distributed along the cable. (The assumption of uniform distribution is not critical. Our analysis can also be applied to some other distribution of terminals along the cable.)

- The terminals collectively form a poisson source.

- A terminal can listen to all other terminals connected to the cable.
Transmission errors (other than those due to collisions) and capture effect are neglected.

Transmission time is the same for all packets.

The channel for acknowledgement is different from that for the message. This means that an acknowledgement is immediately received after a successful transmission as there is no contention for the acknowledgement packets.

The exact analysis of CSMA protocols under the above assumptions is very difficult if not impossible. However we are able to obtain an expected value of throughput which is reasonably close to the actual value. Our approach is in some sense similar to that of Tobagi and Kleinrock {12}.

In the following analysis, unless stated otherwise, we have used only normalized units. All measures of time have been divided by the packet transmission time. Thus, the transmission time for a packet is one normalized unit. Similarly, all measures of distance have been divided by the product of the velocity of light per second and the packet transmission time.

We shall now recollect an important theorem from Probability Theory.

Theorem:1 Suppose \( \{N(t), \, t \geq 0\} \) is a poisson process and \( m \) events take place in the interval from 0 to \( t \). Then these \( m \) events have a joint uniform distribution in the interval \((0,t)\).
Proof: Here we present an informal proof. The reader is referred to [22] for a more rigorous proof.

\[ \Pr(\text{all events in } \leq y \text{ sec.} | \text{m events have occurred in } t \text{ sec}) = \Pr(\text{m events in } y \text{ sec}) \* \Pr(0 \text{ event in } t-y \text{ sec}) \]

\[ = \frac{(\lambda * y)^m * \exp(-\lambda * y) * \exp(-\lambda *(t-y))/m!}{(\lambda * t)^m * \exp(-\lambda * t)/m!} \]

\[ = \left( \frac{y}{t} \right)^m \]

(Q.E.D.)
SECTION 4.1
Non-persistent-CSMA

RESULT

The throughput, $S$, for non-persistent CSMA is:

$$S = \frac{yn}{(aG)^* \exp(-aG/4)*erf(\sqrt{aG}/2)}$$

$$1+9a/8+(2/(a^2G^2))*[\exp(-Ga/2)-\exp(-Ga)]$$

where $a$ is the normalized end to end propagation delay in seconds, $G$ is the total offered traffic in packets per second and erf is the error function given by:

$$erf(x) = (2/\sqrt{\pi}) \int_0^x \exp(-t^2).dt$$

Derivation

Let $S_1$ and $S_2$ be any two terminals on the cable (see Figure 4.1) and $a_{12}$ be the propagation delay between them. Suppose a packet arrives at time $t$ at terminal $S_1$ and at time $t+\Delta$ at terminal $S_2$. If the propagation delay between $S_1$ and $S_2$, i.e., $a_{12}$, is greater than $\Delta$ then the terminal $S_2$ would sense the channel to be empty and immediately transmit the packet, thus creating a collision. If $a_{12} < \Delta < a_{12}+1$ then $S_2$ would sense the channel to be busy and reschedule its transmission to some later time. Else, if $\Delta > a_{12}+1$, the channel would be sensed as free and $S_2$ would immediately transmit its packet.

The normalized length of the cable is $a$ units. However, when terminal $S_1$ transmits, the effective vulnerable period, i.e. the period during which collisions can occur, is only $\max(a_1, a-a_1)$ seconds, where $a_1$ is the normalized distance
\( a_1 < a/2. \)

**Fig 4.1**

**Fig 4.2**
between terminal S1 and one end of the cable. (see figure 4.1). This example shows that the vulnerable period is a function of the location of the terminal that is transmitting. Therefore, inorder to calculate the mean throughput, we need to know the mean effective vulnerable period. We shall denote the vulnerable period by $a'$ and the mean effective vulnerable period by $\bar{a}'$.

Consider Figure 4.2. The vertical arrows indicate the time of packet arrivals. The dotted line at the end of the transmission period indicates the propagation delay between the transmitting terminal and the terminal that transmits next. It is, however, possible for the signal from two terminals to be simultaneously present on the cable. This could be the case when two terminals that have packets to transmit are very near to one another. As soon as one terminal completes its transmission, the second terminal begins transmitting before the signal from the first terminal has reached the other end of the cable. The length of the dotted line at the end of a transmission period is $a_0$ seconds, (Figure 4.2) where $a_0$ is the propagation delay between the last transmitting terminal and the terminal that transmits next. The mean value of $a_0$ is $\bar{a}'/2$, because on an average the next terminal to start transmission would be $a'/2$ normalized units away from the last transmitting terminal.

Let $t+Y$ be the time of arrival of the last packet arriving between $t$ and $t+a$. The interval between $t$ and $t+Y+1+a_0$ is called the transmission period(T.P). Due to the nature of this protocol there can be atmost one successful transmission in a transmission period. Therefore, for this protocol, a transmission period is the same as the busy period. In between
two transmissions the channel is empty and this is known as the idle period. A busy period followed by an idle period constitutes a cycle.

Let \( \bar{B} \) be the mean duration of a busy period. \( \bar{I} \) be the mean duration of an idle period and \( \bar{U} \) be the average utilization in a transmission period, i.e., the mean number of successful transmissions per transmission period.

Using renewal arguments:

Mean throughput = \( S = \frac{\bar{U}}{\bar{B} + \bar{I}} \) \hspace{1cm} (4.1)

\( \bar{B} = 1 + \bar{Y} + a'/2 \) \hspace{1cm} (4.2)

Assuming inter-arrival times to be exponentially distributed:

\( \bar{I} = 1/G \) \hspace{1cm} (4.3)

where \( G \) is the offered traffic to the channel. \( G \) includes both new packets and retransmitted packets.

\( \bar{U} = P_0 = \text{Probability of no collision during a transmission period.} \)

Calculation of \( P_0 \)

Suppose a terminal at one extreme end of the cable (i.e., the terminal at the origin in Figure 4.3), transmits into an empty channel. We wish to calculate the probability that it suffers no collision. In the following discussion, by packet arrival at a terminal, we mean a packet becomes ready for transmission at the terminal.
Figure 4.3

Figure 4.4.
Let the maximum normalized propagation delay = a
and
A_m = m packets arrivals during the a seconds following the
transmission of the packet.

\[ \Pr(\text{no collision}|\text{terminal at origin transmits}) = \sum_{m=0}^{\infty} \Pr(A_m) \cdot \Pr(\text{no collision}|A_m) \]  
(4.4)

Assuming exponential interarrival time

\[ \Pr(A_m) = (aG)^m \cdot e^{(-aG)} \quad (m \geq 0) \]  
(4.5)

To calculate \( \Pr(\text{no collision}|A_m) \), see Figure 4.3. We
represent each packet arrival (ready for transmission) by a
point on this graph whose coordinates are specified by (terminal
at which arrival occurred, time of arrival). The ordinate is the
normalized time axis and the abscissa represents the normalized
cable axis. That is, the locations of the active terminals on
the cable are mapped on to the abscissa.

The point at which the two axis intersects in Figure 4.3
represents the terminal that has transmitted. We call a graph
such as Figure 4.3, the characteristic graph of the transmitting
terminal.

Consider a point such as A (S1,t1), that lies above the
diagonal OQ in Figure 4.3. The packet from the terminal
associated with this characteristic graph would reach terminal
S1 at time tS1. Since tS1 is less than t1, the terminal at S1
would sense the channel to be busy at time t1 and reschedule its
transmission. Now consider a point such as B (S2,t2), that lies
below the diagonal OQ. The signal from the terminal at the
origin would reach S2 at time ts2. Since t2 is less than ts2, terminal S2 would sense the channel to be idle at time t2. Therefore the packet would be transmitted and a collision would occur.

From the above discussion, we see that if the coordinate of any packet arrival is below the diagonal in the characteristic graph, then there would be a collision. Otherwise the terminal would sense the channel to be busy and would take an action according to the protocol being used.

If m packets arrive during a seconds, then from Theorem 1 the time of arrival of these m packets is uniformly distributed between 0 and a seconds. If in addition, the terminals are also uniformly distributed along the cable, then the coordinates of the m arrivals are uniformly distributed inside the square PQRO of Figure 4.3.

\[
\Pr(\text{no collision} | \text{A}^m) = \Pr(\text{coordinates of all m packets lie inside the upper triangle of Figure 4.3}) = \left(\frac{1}{2}\right)^m \quad (4.6)
\]

Substituting (4.5) and (4.6) into (4.4)

\[
\Pr(\text{no collision} | \text{terminal at the origin transmits}) = \sum_{m=0}^{\infty} \left(\frac{(aG)^m \exp(-aG)}{m!}\right) * \left(\frac{1}{2}\right)^m = \exp(-aG/2) * \sum_{m=0}^{\infty} \left(\frac{(aG/2)^m \exp(-aG/2)}{m!}\right) = \exp(-aG/2) \quad (4.7)
\]

(4.8)
= \exp(-\text{vulnerable period} \times \text{offered traffic}/2) \quad (4.9)

Now, let us assume that a terminal, \( S \), at a distance of \( x \) normalized units (where \( x > a/2 \)) away from the origin transmits. (see Figure 4.4)

Let \( P_01 \) be the probability of no collision from terminals located to the left of \( S \) and let \( P_02 \) be the probability of no collision from terminals to the right of \( S \).

Consider the segment left of \( S \)

The vulnerable period = \( x \)

Offered traffic = \( xG/a \)

Therefore from equation (4.9) we have

\[ P_01 = \exp\left(-\frac{Gx^2}{2a}\right) \quad (4.10) \]

Similarly

\[ P_02 = \exp\left(-\frac{G(a-x)^2}{2a}\right) \quad (4.11) \]

Probability that a terminal located between \( x \) and \( x+dx \) from the origin transmits = \( dx/a \)

\[ P_0 = \Pr(\text{no collision}) = \left(\frac{1}{a}\right) \int_0^a P_01 \times P_02 \, dx \]

\[ = \left(\frac{1}{a}\right) \int_0^a \exp\left(-\frac{G}{2a}\right) (x^2 + a^2 + x^2 - 2ax) \, dx \]

\[ = \left(\frac{1}{a}\right) \int_0^a \exp\left(-\frac{G}{a}(x^2 - ax + (a/2)^2 + (a/2)^2)\right) \, dx \]

\[ = \left(\frac{1}{a}\right) \exp\left(-\frac{aG}{4}\right) \int_0^a \exp\left(-\left[\frac{x-a/2}{\sqrt{a/G}}\right]^2\right) \, dx \]

Let \( (x-a/2)/\sqrt{(a/G)} = z \)

Then

\[ P_0 = \frac{1}{aG} \exp\left(-\frac{aG}{4}\right) \int_{-\frac{\sqrt{aG}}{2}}^{\frac{\sqrt{aG}}{2}} \exp(-z^2) \, dz \]

Or \( P_0 = \sqrt{\pi}/(aG) \times \exp(-aG/4) \times \text{erf}(\sqrt{aG}/2) \) \quad (4.12)

where \( \text{erf} \) denotes the error function.
FIGURE 4.5

TERMINAL TRANSMITS
Let us digress for a moment. Equation (4.12) is an important relation that gives the probability that a packet transmitted into an empty channel will not encounter a collision. This equation should be compared with (4.13)

$$P_0 t = \exp(-aG)$$

(4.13)

that gives the corresponding probability of success in Tobagi's analysis, when identical propagation delay is assumed between all terminals.

We shall now determine the distribution of $Y$ and its mean value $\bar{Y}$.

**Distribution For $Y$**

Let us assume that at time $t=0$ second, a packet arrives at a terminal located $x$ normalized units away from one end of the cable, $0 \leq x \leq a/2$. Assume further that the channel is completely idle and the packet gets immediately transmitted. Figure 4.5 shows the characteristic graph for this terminal. The cable segment to the left of the transmitting terminal is denoted as segment I and that to its right as segment II.

$$\Pr(Y \leq y | \text{terminal at } x \text{ transmits})$$

$$= \Pr(Y \leq y | \text{last packet to arrive in VP arrives in segment I})$$

* $\Pr(\text{last packet to arrive in VP arrives in segment I})$

$$+ \Pr(Y \leq y | \text{last packet to arrive in VP arrives in segment II})$$

* $\Pr(\text{last packet to arrive in VP arrives in segment II})$

Where VP denotes the vulnerable period, i.e., the time period following a transmission during which collisions can occur.
Now

A packet arriving in segment I can cause collision if and only if it arrives during the first x seconds of the transmission period.

A packet arriving in segment II can cause collision if and only if it arrives during the first a-x seconds of the transmission period.

For $x \leq a-x$,

\[ \Pr(\text{Packet arriving in segment II causes collision}) \geq \Pr(\text{Packet arriving in segment I causes collision}) \]

or

\[ \Pr(Y \leq y \mid \text{last packet to arrive in VP arrives in segment I}) \geq \Pr(Y \leq y \mid \text{last packet to arrive in VP arrives in segment II}) \]

This implies that

\[ \Pr(Y \leq y \mid \text{terminal at } x \text{ transmits}) \]

\[ \geq \Pr(Y \leq y \mid \text{last packet to arrive in VP arrives in segment II}) \]

\[ = \Pr(\text{no packet arrives in interval } (a-x-y) \text{ seconds}) \]

\[ = \exp(-G(a-x-y)) \cdot [\delta(y-0) - \delta(y-a-x)] \]

\[ + \delta(y-a-x) \]

where $\delta(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$

Removing the condition on x

\[ \Pr(Y \leq y) = \left(\frac{2}{a}\right) \int_{0}^{a/2} \exp(-G(a-x-y)) \cdot \delta(y) \cdot dx \]

\[ + \left(\frac{2}{a}\right) \int_{0}^{a/2} \left(1 - \exp(-G(a-x-y))\right) \cdot \delta(y-(a-x)) \cdot dx \]

(4.14)
Now \((2/a) \int_{0}^{a/2} \exp(-G(a-x-y)) \cdot \delta(y) \cdot dx\)
\[= \frac{2}{(aG)} \cdot [\exp(-G(a/2-y)) - \exp(-G(a-y))] \cdot \delta(y) \quad (4.15)\]

Let \(I_1 = (2/a) \int_{0}^{a/2} [1-\exp(-G(a-y-x))] \cdot \delta(y-(a-x)) \cdot dx\)

Note that \(\delta(y-(a-x))\) implies that \(y-(a-x) \geq 0\)

Or \(x \geq a-y\) \quad (4.16)

Using inequality (4.16) in conjunction with the two limits of integration, we get

\(a-y \geq 0\) which implies \(y \leq a\)

\(a-y \leq a/2\) which implies \(y \geq a/2\)

Therefore

\[I_1 = \left\{ (2/a) \int_{y-a}^{a/2} [1-\exp(-G(a-y-x))] \cdot dx \right\} \cdot \delta(y-a/2)\]
\[= \left\{ (2/a) \cdot y-1 + \frac{2}{(aG)} \cdot [1-\exp(-G(a/2-y))] \right\} \cdot \delta(y-a/2) \quad (4.17)\]

Substituting (4.17) and (4.15) in (4.14) we have

\[\Pr(Y \leq y) \geq \frac{2}{(aG)} \cdot [\exp(-G(a/2-y)) - \exp(-G(a-y))] \cdot \delta(y)\]
\[+ \left\{ (2/a) \cdot y-1 + \frac{2}{(aG)} \cdot [1-\exp(-G(a/2-y))] \right\} \cdot \delta(y-a/2) \quad (4.18)\]

\[\Pr(Y > y) \leq 1-\frac{2}{(aG)} \cdot [\exp(-G(a/2-y)) - \exp(-G(a-y))] \cdot \delta(y)\]
\[-\left\{ (2/a) \cdot y-1 + \frac{2}{(aG)} \cdot [1-\exp(-G(a/2-y))] \right\} \cdot \delta(y-a/2) \]

Now

\[\bar{Y} = \int_{0}^{a} \Pr(Y > y) \cdot dy\]
\[\leq \int_{0}^{a} \left[ 1-\frac{2}{(aG)} \cdot [\exp(-G(a/2-y)) - \exp(-G(a-y))] \right] \cdot dy\]
\[-\int_{0}^{a} \left[ (2/a) \cdot y-1 + \frac{2}{(aG)} \cdot [1-\exp(-G(a/2-y))] \right] \cdot dy\]

On simplifying we have
\[
\bar{y} \leq \frac{3a}{4} - 1/G + \left(\frac{2}{(aG^2)}\right)[\exp(-Ga/2) - \exp(-Ga/2)] \quad (4.19)
\]

Calculating the mean vulnerable period, \( a \)

Let a terminal at location \( x \) (0 \( \leq x \leq a/2 \)) transmit.

The effective vulnerable period = \( a - x \)

Therefore \( \bar{a}' = \frac{2}{a} \int_0^{a/2} (a-x) \, dx \)

Or \( \bar{a}' = \frac{3a}{4} \quad (4.20) \)

The Throughput

Substituting \( (4.20), (4.19), (4.12), (4.3) \) & \( (4.2) \) into \( (4.1) \), we have

\[
S \geq \frac{\pi}{(aG)} \left[ \exp(-aG/4) \right. \\
\left. + \left( \frac{2}{(aG^2)} \right)[\exp(-G*a/2) - \exp(-G*a)] \right] \quad (4.21)
\]

Equation \( (4.21) \) gives the lower bound on throughput for non-persistent CSMA. Figure 4.6 is the plot of \( S \) vs \( G \) for various values of \( a \). Figures 4.9 to 4.14 compare the results of this model with those of Tobagi's model for \( a=0.01 \) to \( a=1.00 \).
NON-PERSISTENT CSMA
RESULT

The throughput, \( s \), for 1-persistent CSMA is:

\[
s = \frac{G \cdot P_0 \left( q_0 + \left( 1 + \bar{y} \right) G \cdot \hat{q}_0 \right)}{(1 + a \cdot \bar{y}) G + q_0}
\]

where

\[
P_0 = \frac{\pi}{(a \cdot G)} \cdot \exp(-a \cdot G/4) \cdot \text{erf}(\sqrt{a \cdot G}/2)
\]

\[
q_0 = \left\{ 4/a \cdot G + 1 \right\} \exp(-G(1 + a/2)) - \left\{ 4/a \cdot G + 2 \right\} \exp(-G(1 + a))
\]

\[
\hat{q}_0 = \left\{ 1/(1 + \bar{y}) \right\} \left\{ \exp(-G(1 + a/2)) \right\} \{ 1 + a/4 + 1/G + 4/(a \cdot G) + 2/(a \cdot G^2) \}
\]

\[
\bar{a}_0 = (3a/4) \left\{ 1 - (1/G(1 + \bar{y})) \left\{ 1 - \exp(-G(1 + \bar{y})) \right\} + 0.5 \cdot \exp(-G(1 + \bar{y})) \right\}
\]

\[
\bar{y} = 3a/4 - 1/G + (2/(a \cdot G^2)) \left\{ \exp(-Ga/2) - \exp(-Ga) \right\}
\]

\( a \) = End to end propagation delay.

\( G \) = Total offered traffic.

DERIVATION

An approach to calculate the throughput for \( \tau \)-persistent CSMA is to take a terminal at location \( x \). Then calculate the throughput of terminals between \( x \) and \( x + dx \) -- computing all the relevant parameters with respect to \( x \). Finally by integrating the result from zero to \( a \), the desired throughput can be obtained. The final integration, is however, very complex. We therefore did not use this technique. Ours is a simpler and an approximate method. Instead of calculating the exact mean value of the final expression of throughput, we computed the mean of
Fig 4.7

SUCCESSFUL TRANSMISSION PERIOD

SUCCESSFUL TRANSMISSION PERIOD

UNSUCCESSFUL TRANSMISSION PERIOD

BUSY PERIOD

IOLE PERIOD

BUSY PERIOD
its individual factors.

Consider Figure 4.7. The vertical arrows in this figure show the time of packet arrivals. Let \( t \) be the time of arrival of a packet that finds the channel to be empty and is immediately transmitted and \( t + \gamma \) be the time of transmission of the last packet between \( t \) and \( t + \alpha' \).

Suppose some terminal, \( S \), starts to transmit a packet. While \( S \) is transmitting, packets arrive at terminals \( S_1, S_2, \ldots, S_n \). All these terminals sense the channel to be busy and wait till they sense the channel to be free at which time they all transmit. If \( S_k \) is the most distant terminal from \( S \), then the dotted line (in Figure 4.7) at the end of a transmission period denotes the propagation delay, \( a_0 \), between \( S \) and \( S_k \). If \( n = 0 \), that is, no packet arrive while \( S \) is transmitting, then \( a_0 \) is equal to the propagation delay between \( S \) and the terminal at which the next packet arrives.

If one or more packets arrive during a transmission period, then the next transmission period begins immediately after the current transmission period ends. Therefore in this protocol, sequence of adjoining transmission periods are separated by periods of inactivity (Figure 4.7). We define a busy period to be the time between \( t \) and the end of that transmission period during which no packet arrive for transmission. We define an idle period to be the period between two busy periods during which the channel is idle. A busy period followed by an idle period constitutes a cycle.

Let \( \bar{B} \) be the expected duration of a busy period and \( \bar{I} \) be the expected duration of an idle period. Let \( B' \) denote the time
in a busy period during which a terminal senses the channel to be busy. It is a sequence of random length \( Z = 1 + \gamma \) separated by intervals of \( a' \) seconds. From the "paradox of residual life", we know that a packet is more likely to arrive in a longer time segment than in a shorter one [15]. Let \( \hat{Z} \) be the time segment in which the arrival occurred. Let \( q_0 \) be the probability that no arrival occurs in \( \hat{Z} \) and \( q_0 \) be the probability that no arrival occurs in any arbitrary time segment \( Z \).

Let us consider the state of the channel when a packet arrives at any random terminal \( S \). The channel could be in one of the following three states:

1. The channel is completely idle. There is no signal on the cable. The packet, therefore, immediately gets transmitted. The probability that this packet is successfully transmitted is given by \( P_0 \) (equation 4.12).

\[
P_0 = \frac{\sqrt{\pi} \left( aG \right)}{\left( aG \right)^{\frac{1}{4}}} \exp\left( -aG/4 \right) \text{erf}\left( \sqrt{aG}/2 \right)
\]

(4.12)

2. Station \( S \), at which the packet arrives, senses the channel to be idle. However, some other terminal has started transmitting, but its carrier has not yet reached \( S \). The probability of success in this case is zero.

3. Station \( S \) senses the channel to be busy, that is, some other station is transmitting on the channel. The packet gets successfully transmitted if and only if no other packet arrives for transmission during the current transmission period and the packet suffers no collision during the next transmission period. That is, the probability of success is
Using renewal arguments, the probability that a packet finds the channel idle is $\bar{I}/(\bar{B}+\bar{I})$ and the probability that it finds the channel busy is $\bar{B}/(\bar{B}+\bar{I})$, where $\bar{B}$ is the expected duration of $\bar{B}$. The probability of success can then be written as:

$$P_s = \bar{I} \cdot \bar{P}_0 + \bar{B} \cdot \bar{q}_0 \cdot \bar{P}_0$$

$$\bar{B}+\bar{I} \quad \bar{B}+\bar{I}$$

(4.22)

Under Poisson assumption

$$\bar{I} = 1/G$$

(4.23)

Let us now calculate $\bar{a}_0$, the expected duration of $\bar{a}_0$.

**Expected duration of $\bar{a}_0$**

A terminal at location $x$ is within max($x$, $a-x$) normalized units of any remaining terminal. We assume, for the sake of mathematical simplicity, that all these remaining terminals are uniformly located between 0 and max($a$, $a-x$). This assumption gives a lower bound on throughput, as shown in section 4.1.

Denote by $a$ the period max($a$, $a-x$). In section 4.1 we showed that

$$\bar{a}' = (3a)/4$$

(4.20)

Assuming that some terminal $S$ is transmitting, $m$ packets arrive for transmission. Let us calculate the expected propagation delay between $S$ and the terminal furthest away from $S$ at which a packet has arrived. From Theorem 1:

$$\Pr(\text{max propagation delay} \leq x_m | m \text{ packets arrive}) = (x_m/\bar{a}')^m$$

or $\bar{x}_m = \int_{x=0}^{\bar{a}'} [1-\Pr(\text{max propagation delay} \leq x_m | m \text{ packets arrive})] \, dx$
\[ \int_0^a \{1 - (x_m/\alpha') \} \, dx = m\alpha'/ (m+1) \]

Now \( a_0 = (1/2) \cdot \Pr(0 \text{ packets arrive during a TP}) + \sum_{m=1}^{\infty} \bar{x}_m \cdot \Pr(m \text{ packets arrive during a TP}) \)

\[ \Pr(m \text{ packets arrive during a TP}) = (G^*(1+\bar{Y}))^m \cdot \exp(-G^*(1+\bar{Y}))/m! . \]

Let \( G_1 = G^*(1+\bar{Y}) \).

Therefore
\[ a_0 = a' \left\{ (1/m+1) \cdot G_1^m \cdot \exp(-G_1)/m! \right\} + 0.5 a' \cdot \exp(-G_1) \]
\[ = a' G_1 \cdot \exp(-G_1) \cdot \left\{ \sum_{m=0}^{\infty} \frac{G_1^m}{(m+1)!} \right\} + 0.5 a' \cdot \exp(-G_1) \]

Let \( n = m+1 \).

We have,
\[ a_0 = a' \frac{d}{dG_1} \left\{ (1/G_1) \sum_{n=0}^{\infty} G_1^n \right\} + 0.5 a' \cdot \exp(-G_1) \]
\[ = a' \frac{d}{dG_1} \left\{ (1/G_1) (\exp(G_1)-1) \right\} + 0.5 a' \cdot \exp(-G_1) \]
\[ = a' \left\{ -(1/(G_1)) (1-\exp(-G_1)) \right\} + 0.5 \cdot \exp(-G_1) \]

Substituting back the value of \( G_1 \)
\[ a_0 = a' \left\{ 1 - (1/(G(1+\bar{Y}))) \right\} (1 - \exp(-G(1+\bar{Y}))) + 0.5 \cdot \exp(-G(1+\bar{Y})) \}
\]
or
\[ \bar{a}_0 = (3a/4) \left\{ 1 - (1/(G(1+\bar{Y}))) \right\} (1 - \exp(-G(1+\bar{Y}))) + 0.5 \cdot \exp(-G(1+\bar{Y})) \}
\]

(4.24)

To obtain \( \bar{B} \) and \( a_0 \) we need to obtain the Laplace transform of the probability density function of \( Y \).

Laplace transform of \( f(y) \)

Laplace transform of \( f(y) \) is defined by
\[ FY(s) = \int_0^\infty \exp(-sy)f(y) \, dy \quad (4.25) \]

\( F(Y) \) was obtained in Equation (4.17):

\[ F(Y) = \Pr(Y \leq y) \]
\[ \approx \frac{2}{(aG)}\{\exp(-G(a/2-y)) - \exp(-G(a-y))\} \delta(y) \]
\[ + \{(2/a)y - 1 + (2/(aG))(1 - \exp(G(a/2-y)))\} \delta(y - a/2) \quad (4.17) \]

Differentiating Equation (4.17)

\[ f(y) = \frac{2}{(aG)}G\{\exp(-G(a/2-y)) - \exp(-G(a-y))\} \delta(y) \]
\[ + \frac{2}{(aG)}\{\exp(-Ga/2) - \exp(-Ga)\}I_0(y) \]
\[ + \frac{2}{a}\{1 - \exp(-G(a/2-y)/2)\} \delta(y - a/2) \quad (4.26) \]

where \( I_0 \) denotes the impulse function. Substituting (4.26) into (4.25), we have

\[ FY(s) = \frac{2}{a} \int_0^\infty \exp(-sy)\{\exp(-G(a/2-y)) - \exp(-G(a-y))\}, dy \]
\[ + \frac{2}{(aG)}\{\exp(-Ga/2) - \exp(-Ga)\} \]
\[ + \frac{2}{a} \int_{a/2}^\infty \{1 - \exp(-G(a/2-y))\}\exp(-sy), dy \]

On integrating and simplifying, we have

\[ FY(s) = \frac{2}{(aG)}\{\exp(-aG/2) - \exp(-aG)\} \]
\[ + \frac{2}{(aG-s)}\{\exp(-as/2) - \exp(-as) - \exp(-aG/2) + \exp(-aG)\} \]
\[ + \frac{2}{(as)}\{\exp(-as/2) - \exp(-as)\} \quad (4.27) \]

Let us now determine \( q_0 \), the probability that no arrival occurs in \( 1+Y \leq Z \).

Determining \( q_0 \)

Let \( q_m = \Pr(m \text{ packets arrive in time } 1+Y \text{ seconds}) \)

Let \( Q(z) \) denote the generating function of \( q_m \), i.e.,

\[ Q(z) = \sum_{m=0}^{\infty} q_m z^m \]

Let \( m1 \) packets arrive during a period of 1 second and let \( m2 \) packets arrive during \( Y \) seconds. If \( Q1(z) \) and \( Q2(z) \) are the
generating function for \( m_1 \) and \( m_2 \) respectively, then

\[
Q(z) = Q_1(z)Q_2(z) \quad (4.28)
\]

\[
Q_1(z) = \sum_{l=0}^{\infty} z^l G_l \exp(-G)/l!
= \exp(G(z-1))*\left(\sum_{l=0}^{\infty} (zG) \exp(-zG)/l!\right)
\]
or \( Q_1(z)= \exp(G(z-1)) \quad (4.29) \)

\[
Q_2(z) = FY(G(1-z)) \quad (4.30)
\]

For the derivation of Equation (4.30) see [15].

Substituting (4.27) into (4.30)

\[
Q_2(z) = \frac{2}{(aG)}\{\exp(-aG/2)-\exp(-aG)\}
+\frac{2}{(aG(1-z))}\{\exp(-aG(1-z)/2)-\exp(-aG(1-z))\}
+\frac{2}{(aGz)}\{\exp(-aG(1-z)/2)-\exp(-aG(1-z))-\exp(-aG/2)
\]

or

\[
Q(z) = \exp(G(z-1))*\left\{\frac{2}{(aG)}\{\exp(-aG/2)-\exp(-aG)\}
+\frac{2}{(aG(1-z))}\{\exp(-aG(1-z)/2)-\exp(-aG(1-z))\}
+\frac{2}{(aGz)}\{\exp(-aG(1-z)/2)-\exp(-aG(1-z))-\exp(-aG/2)
\right\} \quad (4.31)
\]

The probability of zero packet being accumulated in \( 1+Y \) seconds is given by \( Q(z)|z=0 \). That is

\[
q_0 = Q(z)|z=0
= \exp(-G)*\left\{\frac{2}{(aG)}\{\exp(-aG/2)-\exp(-aG)\}
+\frac{2}{(aG(1-z))}\{\exp(-aG(1-z)/2)-\exp(-aG(1-z))\}
+\frac{2}{aG}\{\exp(-aG(1-z)/2)-\exp(-aG(1-z))-\exp(-aG/2)+\exp(-aG)\}\right\}/aGz
\]

The limit for the last term can be obtained by differentiating the numerator and the denominator. That is,

\[
\lim_{z \to 0} \frac{2\{\exp(-aG(1-z)/2)-\exp(-aG(1-z))-\exp(-aG/2)+\exp(-aG)\}}{aGz}
\]
\[
\lim_{z \to 0} 2(\frac{aG}{2})\exp(-aG(1-z)/2) - aG\exp(-aG(1-z))/aG = \exp(-aG/2) - 2\exp(-aG)
\]

Therefore
\[
q_0 = \{1+4/(aG)\}\exp(-G(1+a/2)) - \{2+4/(aG)\}\exp(-G(1+a)) \quad (4.32)
\]

We can now calculate the expected duration of \(B\) and \(\bar{B}\).

**Expected duration of \(B\) and \(\bar{B}\)**

The expected duration of a transmission period is \(1+a_0+\bar{Y}\)
where \(a_0\) can be obtained from Equation (4.24) and \(\bar{Y}\) can be obtained from Equation (4.19).

It is easy to see that the number of transmission periods in a busy period is geometrically distributed with mean \(1/q_0\).

Therefore
\[
\bar{B} = (1+a_0)/q_0 + \sum_{j=1}^{\infty} \bar{Y}(1-q_0)^j
\]
\[
= (1+a_0+\bar{Y})/q_0
\]

\(\bar{B} = (1+a_0+\bar{Y})/q_0 - a_0/q_0 = (1+\bar{Y})/q_0 \quad (4.33)
\]

Since the average number of transmission periods in a busy period is \(1/q_0\)
\[
\bar{B'} = (1+a_0+\bar{Y})/q_0 - a_0/q_0 = (1+\bar{Y})/q_0
\]

\(\bar{B'} = (1+a_0+\bar{Y})/q_0 - a_0/q_0 = (1+\bar{Y})/q_0 \quad (4.34)
\]

To calculate \(P_s\) in Equation (4.22), we need to compute the value of \(q_0\).

**Determining \(q_0\)**

In Equation (4.25) we computed the probability density function for \(Y\).

\[
f(y) = \frac{2}{(aG)}\{\exp(-aG/2) - \exp(-aG)\}I_0(y) + \frac{2}{a}\{\exp(-G(a/2-y)) - \exp(-G(a-y))\}\delta(y) + \frac{2}{a}\{1 - \exp(-G(a/2-y))\}\delta(y-a/2)
\]

\(0 \leq y \leq a\)
where \( I_o \) denotes the impulse function and \( \delta \) the unit step function.

The probability density function of \( z = 1 + y \) is then

\[
f_z(x) = \frac{2}{aG} \left\{ \exp(-G/2) - \exp(-G) \right\} I_o(x-1) + \frac{2}{a} \left\{ \exp(-G(a/2+1-x)) - \exp(-G(a+1-x)) \right\} \delta(x-1) + \frac{2}{a} \left\{ 1 - \exp(-G(a/2+1-x)) \right\} \delta(x-(1+a/2))
\]

\[1 \leq x \leq 1+a\]

The probability density function of \( \hat{Z} \), the segment in which the arrival occurs is given by \( \{15\} \).

\[
f_z(x) = \frac{x f_z(x)}{\hat{Z}}
\]
or

\[
f_z^\wedge(x) = \frac{1}{1+Y} \left\{ \frac{2}{aG} \left\{ \exp(-G/2) - \exp(-G) \right\} I_o(x-1) + \frac{2}{a} \left\{ x \exp(-G(a/2+1-x)) - x \exp(-G(a+1-x)) \right\} \delta(x-1) + \frac{2}{a} \left\{ x - x \exp(-G(a/2+1-x)) \right\} \delta(x-(1+a/2)) \right\}
\]

\[1 \leq x \leq 1+a\]

The probability that no arrival occurs in the interval \( \hat{Z} \) is

\[
q_0 = \int_{x=1}^{1+a} \exp(-Gx) f_z(x) . dx
\]

\[= \frac{1}{1+Y} \left\{ \exp(-G) \left\{ \frac{2}{aG} \left\{ \exp(-G/2) - \exp(-G) \right\} + \frac{2}{a} \left\{ \int_{1}^{1+a} x \exp(-G(a/2+1-x)) - x \exp(-G(a+1-x)) . dx \right\} + \frac{2}{a} \left\{ \int_{1}^{1+a} x \exp(-Gx) - x \exp(-G(a/2+1-x)) . dx \right\} \right\} \right\} \]

On simplifying equation (4.35) we get

\[
q_0 = \frac{1}{1+Y} \left\{ \exp(-G(1+a/2)) \left\{ 1+a/4+1/G+4/(aG)+2/(aG^2) \right\} - \exp(-G(1+a)) \left\{ 2+a+2/G+4/(aG)+2/(aG^2) \right\} \right\}
\]

(4.35)

(4.36)

Throughput

Substituting (4.33), (4.34) & (4.36) in (4.22) we have
\[ P_s = \frac{(1/G)P_0 + ((1+\bar{Y})/q_0)P_0q_0}{(1+a_0+\bar{Y})q_0 + 1/G} \]

\[ = \frac{q_0P_0}{(1+a_0+\bar{Y})G+q_0} + \frac{((1+\bar{Y})Gq_0)^\wedge}{(1+a_0+\bar{Y})G+q_0} \]

\[ = \frac{P_0}{(1+a_0+\bar{Y})G+q_0} \{q_0 + (1+\bar{Y})Gq_0\} \]  \hspace{1cm} (4.37)

The throughput, \( s \), is given by

\[ s = G*Ps \]

or \( s = G*P_0 \frac{\{q_0 + (1+\bar{Y})Gq_0\}}{(1+a_0+\bar{Y})G+q_0} \]  \hspace{1cm} (4.38)

Figure 4.8 shows \( S \) vs \( G \) for various values of \( a \). Figures 4.15 to 4.20 compare the throughput predicted by this model with that predicted by Tobagi's analysis for different values of \( a \) ranging from 0.01 to 1.00.
FIGURE 4.8

1-PERSISTENT CSMA

THROUGHPUT

OFFERED TRAFFIC $\lambda$
SECTION 4.3
Model Validation & Comparison

In sections 4.1 and 4.2, the following approximations were made:

- The distribution obtained for $Y$ (the interval between the start of the first and the last transmission in a transmission period) is approximate (assumed to be equal to its upper bound).

- The distribution of $Y$ is used to obtain the expressions for $q_0$ and $\hat{q}_0$ in the analysis of 1-persistent CSMA. Therefore the expressions for $q_0$ and $\hat{q}_0$ are also approximate.

- The method used to analyse 1-persistent CSMA yields an approximate result because instead of obtaining the exact mean value of the final expression for throughput, we have computed the sum of the mean of its factors without considering the interaction among them.

In this section we present simulation models to estimate the magnitude of error introduced by the above approximations. We assume that there are an infinite number of terminals uniformly distributed along the cable and that the total offered traffic to the channel follows a Poisson distribution. The basic structure of the simulator is the same for both non-persistent and 1-persistent CSMA. We first describe this basic structure.
and then the difference between the simulators for non-persistent CSMA and 1-persistent CSMA.

The Basic Simulator Structure

This is an event driven simulator written in the PASCAL programming language. Four major events are identified.

1. PACKET ARRIVAL: The time of packet arrival. Each packet arrival is also associated with the terminal at which the arrival occurred.

2. PACKET TRANSMISSION POSSIBLE: Check if it is possible to transmit the packet at the current time.

3. SIGNAL DEAD: The time at which the whole of the signal due to a packet transmission reaches both ends of the cable.

4. END SIMULATION: The time to stop the simulation run.

We maintain two queues. An Event queue, which keeps track of the next event that is going to take place and a Transmission queue, which contains a list of the terminals that are currently transmitting.

The main loop of the simulator checks for the event that is to take place next:

- If the next event is Packet Arrival, the routine Pr-arrival checks if the packet that has arrived can be immediately transmitted. It also determines the time and the terminal of the next packet arrival.

- If the next event is Packet Transmission Possible, the routine Pr-trans-poss(at,et,stn) determines whether the terminal, stn, can sense any carrier at time, et. If the terminal cannot sense any carrier, the packet that arrived
at time 'at' is allowed to be transmitted. Otherwise the routine determines the time to again check for transmission possible if 1-persistent CSMA protocol is being used.

- If the next event is Signal Dead, the terminal whose carrier has reached all the other terminals is deleted from the transmission queue.
- If the next event is End Simulation, the simulation is stopped and simulation statistics are printed out.

The program for 1-persistent CSMA simulator is given in Appendix I. The only difference between this simulator and the simulator for nonpersistent CSMA is that the ELSE block in procedure Pr-trans-poss (lines 424-428) is not present in the latter.

If it is desired to simulate Tobagi's model (identical propagation delay between all terminals), then lines 186 and 187 in routine Ch-tr-poss should be modified to read

186 IF et < (t+a+epsilon) THEN y := TRUE
187 ELSE IF et+epsilon >= (t+a+1.0) THEN y := TRUE

and line 280 in Procedure ch-coll should be changed to

280 IF (t+a+epsilon) >= et THEN

The constant epsilon has been introduced to take care of truncation and roundoff errors.

Range of Simulation Output

The 1-persistent simulator was run ten times. Each run was for 10000 simulated seconds. In each case a different random sequence was used by changing the value of the seed in the
random number generating procedure. Table 4.1 lists the output of the ten runs for (a=0.21 & g=0.41) and (a=0.41 &g=0.41). The results obtained were:

Case 1: a=0.21  G=0.41
Mean = 0.33889
Standard deviation = 0.00538171

Case 2: a=0.41  G=0.41
Mean = 0.32009
Standard deviation = 0.00775557

Validation of Simulator
1. The simulator was validated by chronologically following its output.
2. The simulator was modified, as described above, to simulate Tobagi's model. Its output was then compared with Tobagi's theoretical results. The two were found to be in excellent agreement (Table 4.2 and Table 4.3).

Comparison
Figures 4.9 to 4.20 contain the simulation output, results of our model and results of Tobagi's model. Figures 4.9 to 4.14 contain the non-persistent CSMA graphs for a=0.01 to a=1.00. On close scrutiny we notice that:
1. Our analysis is in excellent agreement with the simulation output.
2. As end to end propagation delay increases, the throughput predicted by Tobagi's analysis becomes significantly less than the simulation output. This shows that the assumption
of identical propagation delay between all terminals is not satisfactory for systems where the end to end propagation delay is large.

Figures 4.15 to 4.20 show the graphs for 1-persistent CSMA for \( a = 0.01 \) to \( a = 1.00 \). We notice that the remarks made above are also applicable here.

**SUMMARY**

We have introduced a new technique to analyse the throughput of non-persistent and 1-persistent CSMA protocols, when the terminals are connected to a common bus. The use of characteristic graph enables us to obtain the probability of no collision by taking the distribution of terminals into account. The assumption of uniform distribution of terminals greatly simplifies our analysis. However, with the aid of the characteristic graph, it is possible to obtain the probability of no collision, and thus the throughput, for some other distribution of terminals too.

In order to estimate the error introduced by the assumptions in our analysis, simulation models for both non-persistent and 1-persistent CSMA were constructed. Our analysis was found to be in excellent agreement with the simulation results.

A comparison between our results and those of Tobagi shows that for small end to end propagation delay (\( a \) less than 0.01), the results obtained through both methods are quite close. But,
as the end to end propagation delay increases, the assumption of identical propagation delay significantly underestimates the actual throughput. In this case, our method is a more accurate indicator of throughput.
### TABLE 4.1

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**mean** = 0.33889, 0.32009  
**standard deviation** = 0.00538171, 0.00775557

s1 corresponds to a = 0.21 & G = 0.41  
s2 corresponds to a = 0.41 & G = 0.41
Table 4.2
Validation of Tobagi's Non-persistent Simulator

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S - Simulator's Throughput.
T - Theoretical Throughput.
Table 4.3  
Validation of Tobagi's 1-persistent Simulator

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<td>0.81</td>
<td>1.61</td>
<td>0.0420</td>
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</tr>
</tbody>
</table>

S - Simulator's Throughput.
T - Theoretical Throughput.
FIGURE 4.9

NON-PERSISTENT CSMA

a = 0.01
FIGURE 4.10

NON-PERSISTENT CSMA

\[ a = 0.20 \]

THROUGHPUT

\[ \text{OFFERED TRAFFIC G} \]

- SIMULATION
- OUR MODEL
- TOBAGI'S MODEL
FIGURE 4.11

NON-PERSISTENT CSMA

\( a = 0.40 \)
NON-PERSISTENT CSMA

\[ a = 0.60 \]
FIGURE 4.13

NON-PERSISTENT CSMA

\[ a = 0.80 \]
FIGURE 4.14

NON-PERSISTENT CSMA

\[ a = 1.00 \]
FIGURE 4.15

1-PERSISTENT CSMA

\[ a = 0.01 \]
FIGURE 4.16

1-PERSISTENT CSMA

$a = 0.20$
FIGURE 4.17

1-PERSISTENT CSMA

\[ a = 0.40 \]
FIGURE 4.18

1-PERSISTENT CSMA

$a = 0.60$
FIGURE 4.19

1-Persistent CSMA

\[ a = 0.80 \]

Throughput vs. offered traffic.
FIGURE 4.20

1-PERSISTENT CSMA

$\alpha = 1.00$
CHAPTER 5

CONCLUSIONS

In this thesis we have developed two models for analysing uncontrolled CSMA networks. The first model described though very simple is less accurate than Tobagi's model. Its range of validity is quite small (normalized propagation delay less than 0.1 seconds). However, it gives fairly accurate results for most practical networks. The second model analyses CSMA networks in which all terminals are connected to a common bus. This model does not assume identical propagation delay between terminals. To our knowledge this is the first model that does not make this assumption.

The two models discussed in this thesis, as well as Tobagi's model, present the final results in a closed form. Hence, the computational effort required to calculate the throughput for a given value of offered traffic and propagation delay is nearly the same for all three models. However, the complexity and therefore the effort required to understand and to formulate the models varies. The model described in Chapter 3 is relatively very simple and easy to understand, while that in Chapter 4 is slightly more complex than Tobagi's model as it also considers the distribution of terminals along the cable.

The assumption of steady state was made in our analysis. Let us examine the validity of this assumption. In Fig. 4.6 and 4.8 it can be observed that as G (offered traffic) increases, the slope of the throughput vs. offered traffic curve becomes
negative. Tobagi has shown [28] that when the slope of the operating point in the throughput vs. offered traffic curve is negative, the system becomes unstable and the throughput soon goes to zero. He has also shown that an uncontrolled infinite user CSMA network is intrinsically unstable. That is, after some time, nearly all the terminals in the network are trying to retransmit collided packets, and practically every transmission is suffering a collision. At this point some external action (such as restricting transmission) is necessary to bring the system back into normal operation. In light of this result our analysis should be interpreted to have been obtained under quasi-steady state conditions.

In most working networks some control policy would be in effect in the region in which the slope is negative. The magnitude of the negative slope is an indicator of the rate at which the system goes to the unstable equilibrium point once it enters the unstable region. The point at which the slope becomes negative is also important. The greater the value of offered traffic at which this point is located, the stabler is the system [28].

The exact analysis of CSMA networks is very difficult because:

i) The system is intrinsically unstable. Therefore the assumption of stationary probability distribution is not really true (although it has been found to be a good approximation).

ii) The assumption that packet interarrival times are independent is also not very true. When a packet suffers collisions, there is a definite relationship between the time of
its first, second and successive transmissions. As the value of offered traffic increases, the proportion of retransmitted packets also increases. Therefore the validity of this assumption decreases with increase in offered traffic.

iii) The work load of the network should be ideally represented as a function of the location of the terminals. However there exists no known method of representing the work load in this fashion.

The model described in this thesis represents the network more accurately than the previous models. The throughput predicted by it was observed to be more accurate than that predicted by Tobagi's model when compared with the simulation output, particularly when the normalized propagation delay and the offered traffic are high. We would finally like to test this model with the measurement results obtained from a working system.
REFERENCES


18. R.M. Metcalfe and D.R. Boggs, "Ethernet: Distributed Packet Switching for Local Computer Networks", Commun. of ACM, July


APPENDIX I

Simulation Program For 1-persistent CSMA

PROGRAM one_persistent;

CONST maxtime = 100.0;{simulation termination time}
epsilon=0.00000001;
forever = FALSE;

TYPE event_type=(pkt_arrival,pkt_trans_pos,signal_dead,
end_sim);
link_event=Eventelt;
{structure of the elements in the event_queue}
eventelt=
RECORD
next:link_event;
ev_type:event_type;
stn:real;
at,et:real
END;
link_tran=Tran_q_elt;
{structure of the elements in the transmission_queue}
tran_q_elt=
RECORD
next:link_tran;
at,et:real;
stn:real;
c:boolean
END;

VAR x,a,g:real;
i,j:integer;
head_f_ev:link_event;
head_f_tr:link_tran;

FUNCTION randomU(x:real):real;FORTRAN 'RAND';

PROCEDURE new_ev(var p:link_event);
{If there are element of type eventelt in the program
buffer_pool it gets the record from there,otherwise it
calls new(p)}
BEGIN
IF head_f_ev=NIL THEN new(p)
ELSE BEGIN
p := head_f_ev;
head_f_ev := head_f_ev@.next;
END;
END;

PROCEDURE new_tr(var p:link_tran);
BEGIN
 IF head_f_tr=NIL THEN new(p)
 ELSE
 BEGIN
   p := head_f_tr;
   head_f_tr := head_f_tr@.next;
   END;
 END;

PROCEDURE disp_tr(p:link_tran);

{disp_tr & disp_ev returns empty records to the program
 buffer_pool}

BEGIN
 p@.next := head_f_tr;
 head_f_tr :=p;
 END;

PROCEDURE disp_ev(p:link_event);

BEGIN
 p@.next := head_f_ev;
 head_f_ev := p;
 END;

PROCEDURE get_ready;

{initializes the program buffer pool}

BEGIN
 x := random(2.71828);
 head_f_ev:=NIL;
 head_f_tr:=NIL;
 END;

PROCEDURE simulate;

LABEL 99;

VAR head_event:link_event;
 head_tran:link_tran;
 ev_type:event_type;
 tot_idle:real; {total time system idle}
 tot_delay:real; {tot. delay suffered by the packets}
 stn:real; {station number}
 et:real; {event time}
 at:real; {packet arrival time}
 channel_idle_time:real;
 {time when channel will become free of signal}
 tot_succ:integer; {total number of succ. trans.}
 tot_coll:integer; {total number of coll.}
PROCEDURE cr_event(evtype:event_type;stn,at,et:real);

{Creates an event of type evtype and inserts this event into the event queue. The elements in the event queue are arranged in the order of increasing event time}

VAR p,k,r:link_event;

BEGIN
  new_ev(p);
  p@.ev_type := evtype;
  p@.stn := stn;
  p@.at := at;
  p@.et := et;
  r := NIL; k:=head_event;
  WHILE (k =NIL)AND(p@.et>k@.et) DO
    BEGIN
      r:=k;
      k:=k@.next;
    END;
    p@.next := k;
    IF r=NIL THEN head_event := p
    ELSE R@.next := p;
  END;

PROCEDURE initialize;

{Initializes the simulator for a new simulation run}

VAR station:real;

BEGIN
  head_tran:=NIL;
  head_event :=NIL;
  tot_succ:= 0;
  tot_coll:=0;
  channel_idle_time:=0.0;
  tot_idle:=0.0;
  station := random(0.0)*a;
  cr_event(pkt_arrival,station,0.0,0.0);
  cr_event(end_sim,0.0,maxtime,maxtime);
END;

PROCEDURE reset;

{Returns the elements in the event queue and transmission queue to the program buffer}

VAR k:link_tran;
  l:link_event;

BEGIN
  k:=head_tran;
  WHILE head_tran =NIL DO
    BEGIN

head_tran:=head_tran@.next;
disp_tr(k);
k := head_tran;
END;
l := head_event;
WHILE head_event=NIL DO
BEGIN
    head_event := head_event@.next;
disp_ev(l);
l := head_event;
END;
BEGIN
PROCEDURE ch_tr_poss(st,t,et,stan:real;var y:boolean);
{Check transmission possible
The constant 'epsilon' is introduced to remove the
effect of roundoff errors}
VAR distance:real;
BEGIN
    distance := abs(stn-st);
    IF et<=(t+distance+epsilon) THEN y:= TRUE
    ELSE IF et+epsilon>=(t+distance+1.0) THEN y:= TRUE
    ELSE y := FALSE;
END;
PROCEDURE pr_arrival(ev_±ime,station:real);
{It is activated when a packet arrives at a station.
It creates an event to check if transmission is possible
at the time of arrival. It also determines the time and
the station for the next packet arrival. The time is
chosen from an exponential distribution the station from
a uniform distribution.}
VAR u:real;
BEGIN
    writeln('pkt_ar stn=',station:8:4,'time=',ev_time:8:4);
cr_event(pkt_trans_poss,station,ev_time,ev_tIme);
u := random(0.0);
ev_time:= -(1/g)*ln(u)+ev_time;
u := random(0.0);
station := a*u;
cr_event(pkt_arrival,station,ev_time,ev_time);
END;
PROCEDURE error(n:integer);
BEGIN
    writeln('error',n:2);
goto 99;
END;
PROCEDURE get_nxt_ev(var evtype: event_type; var stn,at,
et:real);

{Gets the first element from the event queue.}

VAR p:link_event;

BEGIN
  p := head_event;
  evtype:=p@.ev_type;
  stn := p@.stn;
  at := p@.at;
  et := p@.et;
  head_event := p@.next;
  disp_ev(p);
END;

FUNCTION cl_tf_ch(stn,et:real):real;

{Calculate time free channel:
 Calculates the time when the station stn would sense
 the channel free}

VAR tmax,s1,t,crosstime:real;
  k:link_tran;

BEGIN
  tmax:=0.0;
  k := head_tran;
  WHILE k = NIL DO
    BEGIN
      s1 := k@.stn;
      t := k@.et;
      IF (abs(s1-stn)+t) < et THEN
        BEGIN
          crosstime := t+abs(s1-stn)+1.0;
          IF tmax<crosstime THEN tmax := crosstime;
        END;
    k := k@.next;
    END;
  IF tmax=0.0 THEN error(3);
  cl_tf_ch :=tmax;
END;

PROCEDURE ch_coll(et,stn:real;var c:boolean);

{This is activated when a packet is transmitted on the
 channel. It checks for collisions. If there are collision
 it makes the 'c' field of the colliding stations in the
 transmission queue--TRUE & also returns the boolean
 variable c as TRUE.}

VAR k:link_tran;
  s1,t:real;

BEGIN
  C := FALSE;
k := head_tran;
WHILE K = NIL DO
BEGIN
  s := k@.stn;
  t := k@.et;
  IF t+abs(stn-s)+epsilon>=et THEN
    BEGIN
      c := TRUE;
      k@.c := TRUE;
      END;
      k := k@.next;
    END;
  END;
PROCEDURE in_tr_q(at,et,stn:real; c:boolean);
{Inserts an element into the transmission queue.}
VAR p:link_tran;
BEGIN
  new_tr(p);
p@.at := at;
p@.et := et;
p@.stn := stn;
p@.c := c;
p@.next := head_tran;
  head_tran := p;
END;
PROCEDURE cl_s_dt(et,stn:real;VAR dt:real);
{Calculates signal dead time.}
BEGIN
  IF stn<(a/2) THEN dt := et+a-stn+1.0
  ELSE dt := et+stn+1.0;
END;
PROCEDURE del_tq(at,stn:real;var c:boolean);
{Deletes an element from the transmission queue. 'c' indicates whether the element deleted had collided or no
VAR r,k:link_tran;
BEGIN
  k := head_tran;
  r := NIL;
  WHILE (k = NIL)AND((k@.at = at)OR(k@.stn = stn)) DO
    BEGIN
      r := k;
      k := k@.next;
    END;
IF \( k = \text{NIL} \) THEN error(2)

ELSE

BEGIN

\( c := k@.c; \)

IF \( r = \text{NIL} \) THEN head_tran := k@.next

ELSE \( r@.next := k@.next; \)

\( \text{disp_tr}(k); \)

END;

END;

PROCEDURE pr_end_sim;

{This is invoked when it is time to terminate simulation. It prints out the results of the simulation. If at the simulation termination time, there are some packets being transmitted, then the effect of these packets is removed from the simulation statistics.}

VAR earliest_tr:real;

c:link_tran;

BEGIN

IF channel_idle_time < maxtime THEN

BEGIN

\( \text{tot_idle} := \text{tot_idle} + \text{maxtime} - \text{channel_idle_time}; \)

\( \text{earliest_tr := maxtime}; \)

END

ELSE

BEGIN

\( k := \text{head_tran}; \)

IF \( k = \text{NIL} \) THEN error(1)

ELSE earliest_tr := k@.at

WHILE \( k = \text{NIL} \) DO

BEGIN

IF \( k@.at < \text{earliest_tr} \) THEN earliest_tr := k@.at

\( k := k@.next; \)

END;

END;

reset;

writeln('a=',a);

writeln('g=',g:8:4);

writeln('tot_succ=',tot_succ:8);

writeln('tot_coll=',tot_coll:8);

writeln('tot_idle=',tot_idle:8:4);

writeln('tot_delay=',tot_delay:8:4);

writeln('tot_time_period=',earliest_tr:8:4);

GOTO 99;

END;

PROCEDURE pr_signal_dead(at,et,stn:real);

VAR c:boolean;n:integer;

BEGIN

\( \text{del_tq}(at, stn, c); \)

IF \( c \) THEN begin \( \text{tot_coll} := \text{tot_coll} + 1; \ n := 1 \) end

ELSE begin \( \text{tot_succ} := \text{tot_succ} + 1; \ n := 0 \) end;
writeln('sig dead from stn=',stn:8:4,'at=',at:8:4,'coll
92
n:3);  
END;
PROCEDURE pr_trans_poss(at,et,stn:real);
{Checks whether it is possible to transmit from station,s
at time et.}
VAR k:link_tran;
possible,y,c:boolean;
t,st,dt:real;
BEGIN
possible := TRUE;
k := head_tran;
WHILE (k =NIL) and possible DO
BEGIN
st := k@.stn;
t := k@.et;
ch_tr_poss(st,t,et,stn,y);
IF y THEN k := k@.next
ELSE possible := FALSE;
END;
IF possible THEN
BEGIN
writeln('pkt transmitted stn=',stn:8:4,'at=',at:8:4,
et=',et:8:4);
ch_coll(et,stn,c);
in_tr_q(at,et,stn,c);
IF channel_idle_time<et THEN
  tot_idle := tot_idle+et-channel_idle_time;
cl_s_dt(et,stn,dt);
IF channel_idle_time<dt THEN channel_idle_time :=dt;
cr_event(signal_dead,stn,at,dt);
tot_delay := tot_delay+et-at;
END
ELSE
BEGIN
  t := cl_tf_ch(stn,et);
cr_event(pkt_trans_poss,stn,at,t);
END;
END;
BEGIN
initialize;
REPEAT {main simulation loop}
  get_nxt_ev(ev_type,sn,at,et);
  CASE ev_type of
    pkt_arrival : pr_arrival(et,stn);
    pkt_trans_poss: pr_trans_poss(at,et,stn);
    signal_dead:pr_signal_dead(at,et,stn);
    end_sim:pr_end_sim
  END;
UNTIL FOREVER;
END;
BEGIN{main program}
get_ready;
a := 0.21;
g := 0.41;
simulate;
END.
End of file
$1.71, $1.72T

$SIGNOFF