Selection and Organization of Subjective Contours

By

YOKO ANDO

B.Sc., University of Guelph, Canada, 1989

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

in

THE FACULTY OF GRADUATE STUDIES

DEPARTMENT OF COMPUTER SCIENCE

We accept this thesis as conforming

to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

April 1994

Department of Computer Science

The University of British Columbia
Vancouver, Canada

Date April 27, 1994

Abstract

Subjective contours are physically invisible borders drawn on certain images that can nevertheless be seen by humans. This is because the human vision system makes assumptions on the occlusion of objects. The study of subjective contours is important for helping us understand more about the human visual perception. The purpose of this thesis is to understand the perception of subjective contours and to detect subjective contours by computer. The previous subjective contour detection systems limit the subjective contours they can detect by restricting the locations on the figures where the subjective contours can be seen and by using the consistent subjective surface orientation. In this thesis, we consider the overall organization of subjective contours. We do not put the restriction on the subjective surface orientation because we view the subjective contour as a boundary separating the two regions locally.

A model for subjective contour detection is presented based on four criteria: no prior knowledge is necessary to detect a subjective contour; a subjective contour is a special type of occluding contour; the shape of a subjective contour is determined by the viewing condition; and it is possible to have many subjective contour organizations from one image. The rules for subjective contour organization are described and the model explains different types of subjective contour organizations.

There are three stages in the computer implementation of subjective contour detection. The first stage is preprocessing of figures where the real contours are segmented according to their curvature discontinuities by Lowe's curve partition method. The next stage is local processing in which each real contour segment selects all the potential subjective contours and their connecting real contour segments. The final stage is global processing to organize the real and subjective contours which can be seen at the same time. Many subjective contour images are tested and good results are produced.

Contents

List of Tables

List of Figures

Acknowledgement

I would like to thank my supervisor Dr. Jim Little for his guidance and encouragement throughout my work on this thesis. I would like to acknowledge the graduate students, faculties and staffs of the University of British Columbia Department of Computer Science providing me good working environment. Special thanks to Ms. Xiaomei Han for her friendship, Mr. Xun Li for his help in research of computer vision, and Mr. George Phillips for his expertise on using Latex and converting image file formats. This work was supported by the Natural Sciences and Engineering Research Council of Canada, and the University of British Columbia Department of Computer Science. I would like to thank my mother and my father for their support.

Chapter 1

Introduction

A contour in general is located on a division of two adjacent regions. When two adjacent regions have different brightnesses or colours, we see a real contour. Examples of real contours are the boundary or border of an object, the outline of a figure, and an edge. By comparison, we can also see a contour in the image with a certain arrangement of figures where the two adjacent regions have no real difference but an apparent brightness difference. This type of contour is called a *subjective contour* [Kanizsa, 1976] because it is physically not present in an image but is provided by human visual perception. The goal of this thesis is to understand the perception of subjective contours and to detect subjective contours by computer.

1.1 Subjective Contour Characteristics

The use of subjective contour is an effective technique for art because it can increase brightness without physical gradient [Meyer and Petry, 1987]. When there is a limitation on use of colours on a material such as coins or woodcuts, an artist can either outline or use subjective contours to draw an object. The subjective contour technique on a drawing makes the object looks more intense than the background colour, and thus provides additional colours to the art work. This technique has been recognized for a long time in the art world. Recently, subjective contours have been used widely in designing the logos of organizations.

The psychological study of subjective contours began about a century ago started by Fred-

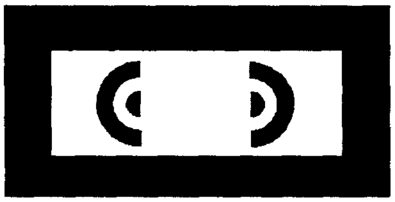


Figure 1.1: Schumann's Figure

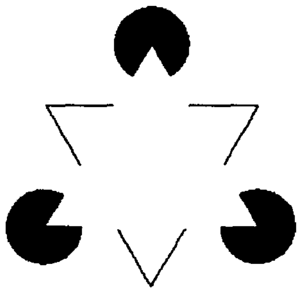


Figure 1.2: Kanizsa Triangle

erich Schumann. Figure 1.1 is one of the example presented in [Schumann, 1904]; Schumann observed a white rectangle in the center of the image with sharply defined contours that are physically not present. In 1955 Gaetano Kanizsa published an article on subjective contours [Kanizsa, 1955]. Figure 1.2 is a popular subjective contour image depicted by [Kanizsa, 1955] and is called Kanizsa triangle. Kanizsa noticed the enhancement of brightness, the sharp apparent edges, and the depth discontinuities in his figures. There have been many theories of subjective contour perception proposed since 1970 but there is no settled theory today, which indicates that subjective contours are a difficult and complicated subject to study.

The following characteristics are observed after examining many subjective contour examples; also see [Kanizsa, 1976] and [Meyer and Petry, 1987] for detail. A subjective contour is normally noticed as an edge or border delimiting a discrete change in apparent brightness. Usually, there is a surface, called a *subjective surface* [Kanizsa, 1976], bounded by a subjective contour. A subjective surface is a region with the same physical quality as the background, but with a different visual quality that makes it to stand out from its background. This visual quality is caused by brightness enhancement, i.e., the surface adjacent to a darker or brighter coloured surface seems to be more intensive then its physical colour. For example, the subjective surface looks brighter than the background when the foreground figures have darker colour than the background; on the other hand, the subjective surface looks darker than the background when the foreground figures have brighter colour than the background. A figure is an object in the image, a foreground is part of an image seen closer to the viewer, and a background is an area where the foreground figures are seen against in the image.

A subjective surface is usually considered a two dimensional figure and interpreted as a opaque surface parallel or tilted to the image plane. [Brady and Grimson, 1981] have different opinions about the dimensionality of subjective surfaces and they proposed that subjective surfaces are natural three dimensional surfaces.

A subjective contour can only be observed on the background coloured area; it cannot be seen by itself. In order to see a subjective contour, there must be a pair of *supporting edges* [Ullman, 1976] on each end of the subjective contour and the subjective contour continues through the supporting edges. A supporting edge is part of an outline of a figure merged to form the subjective contour or a tip of a figure that touches the subjective contour to support its shape. A figure with supporting edges is called an *inducing element* [Meyer and Petry, 1987]. There are three types of inducing elements: *blob*, *line*, and *dot* [Brady and Grimson, 1981]. A blob is a region with foreground colour, a line is a thin strip with foreground colour, and a dot is one spot with foreground colour.

Unlike real contours, subjective contours cannot always be perceived in any images. There must be some evidence of discontinuities in the inducing elements to produce a subjective contour. Discontinuities in the inducing elements or a gap between aligned or continuous edges from two inducing elements suggest that those inducing elements are occluded by an object. Moreover, [Gregory, 1972] explains unlikely gaps are due to eclipsing or occlusion by some near opaque object or surface. A subjective surface is perceived as an occluding surface in front of those inducing elements, and the inducing elements appear to be part of larger figures that continue behind the subjective surface. In this thesis, a subjective contour is considered an occluding contour of an opaque object which has the same colour as the background.

1.2 Subjective Contour Classification

A new classification scheme for subjective contours is presented in the following two subsections. Subjective contours are classified into two categories, edge-based and tip-based, depending on the type of inducing elements that appear in the image. The distinction between the edge-based and tip-based subjective contour is based on whether the supporting edge can suggest the direction of the subjective contour shape or not. Any subjective contour image can be simplified to a black-and-white image separated by the foreground figures and the background colour. Through this thesis black is used as the foreground colour and white is the background colour.

Edge-based Subjective Contour

The *edge-based subjective contour* is formed in a blob-based image where parts of the blobs merge to shape the subjective contour. At the end of the blob supporting edge, there is a tangent direction that continues through the subjective contour. The characteristics of the edge-based subjective contour is that the supporting edges suggest the subjective contour shapes.

A blob can be regular (Figure 1.3(a), on page 5), or irregular in shape (Figure 1.3(b)). It can have a concave angle corner (Figure 1.3(a) and Figure 1.3(b)) or a curved corner of both concave and convex shape (Figure 1.3(c)). A subjective contour can have the appearance of corner (Figure 1.3(d)) when viewed from far distance; let's call this type of subjective contour a *cornered subjective contour.* In addition to blobs, line ends and dots can also enhance the shapes of subjective contours if they can help to support the subjective contour shapes (Figure 1.3(e), Figure 1.3(f), and Figure 1.3(g)).

A subjective surface does not always appear opaque with a more intensive colour than the background but it can have some patches on it (Figure 1.3(k)). A subjective contour can be closed (Figure 1.3(a)) or opened (Figure 1.3(h)) depending on the subjective surface associated with it. The *overlaid subjective contours* are seen as one subjective surface occluding another to produce multiple depth levels (Figure 1.3(i) and Figure 1.3(j)). In this kind of image, the

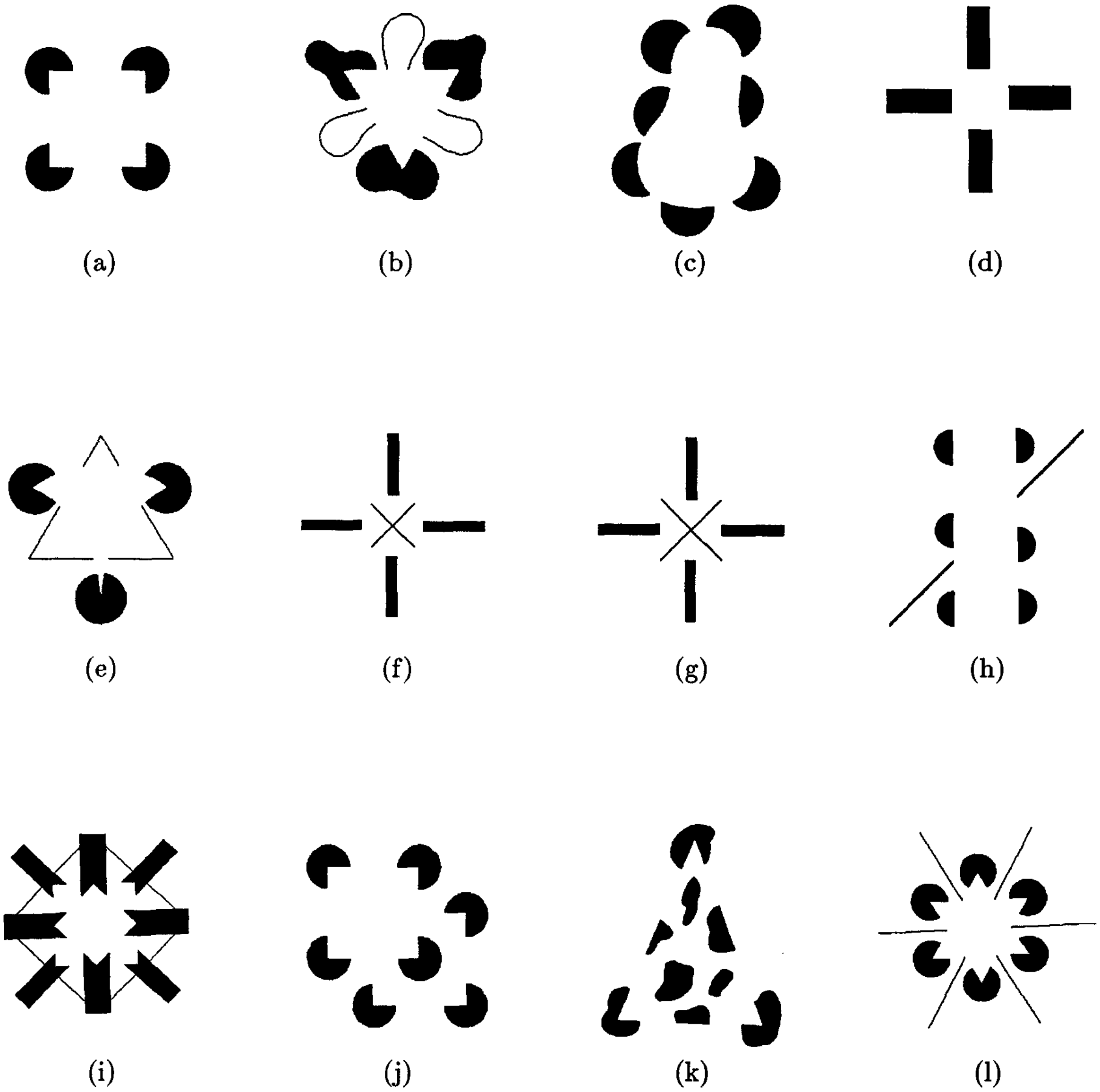


Figure 1.3: Edge-based Subjective Contours (See Appendix A for sources of figures.)

surface seen on the top looks brighter than the surfaces below it.

The *reversible subjective contours* are seen on an image with perceptually ambiguous subjective contours and their configurations can be perceptually organized in many ways. Each of the subjective contour organization that emerges from the image has a unique set of supporting edges and brightness associated with the subjective contours. Still, only one interpretation of subjective contour organization can be perceived at a time; besides, the perception shifts from one organization to alternative ones. For instance, Figure 1.3(l) shows two simultaneous subjective figures, two triangles, that are depth reversible with respect to each other.

Tip-based Subjective Contour

The *tip-based subjective contour* is formed in an image with lines where the ends of the lines touch the subjective contour (Figure 1.4(a) on page 7). There is no tangent at the line end because the dimension of the line end is just a point. Since the line end is unable to suggest the subjective contour shape, many interpretations of subjective contour shapes are possible when the two ends of lines are connected by a subjective contour.

Whenever there is an interrupted black line on a white background, the discontinuity may be caused by an interposed white figure that is whiter than the background. The end of a line can be viewed as sudden line termination and it includes the tip of a blob with sharp angle (Figure 1.4(b)). The bent part of a straight line can be interpreted as the intersection of two straight lines; thus, it contains two line ends at the bend (Figure 1.4(c)).

Each of Figures 1.4(a) and 1.4(d) appears as a subjective surface with its subjective contour interrupting a radiating and random line set respectively. A line can be straight (Figure 1.4(a) and Figure 1.4(d)) or curved (Figure 1.4(i) and Figure 1.4(j)) and a subjective contour can be open (Figure 1.4(d) and Figure 1.4(i)) or closed (Figure 1.4(a) and Figure 1.4(j)). Figure 1.4(g) and Figure 1.4(h) show a curved or straight subjective contour between the misaligned line segments terminating the lines on both sides of the subjective contour. Those images give a sense of two adjoining surfaces and a subjective contour is perceived where the two surfaces meet. Dots can help to determine the shape of subjective contours; compare Figure 1.4(e) and

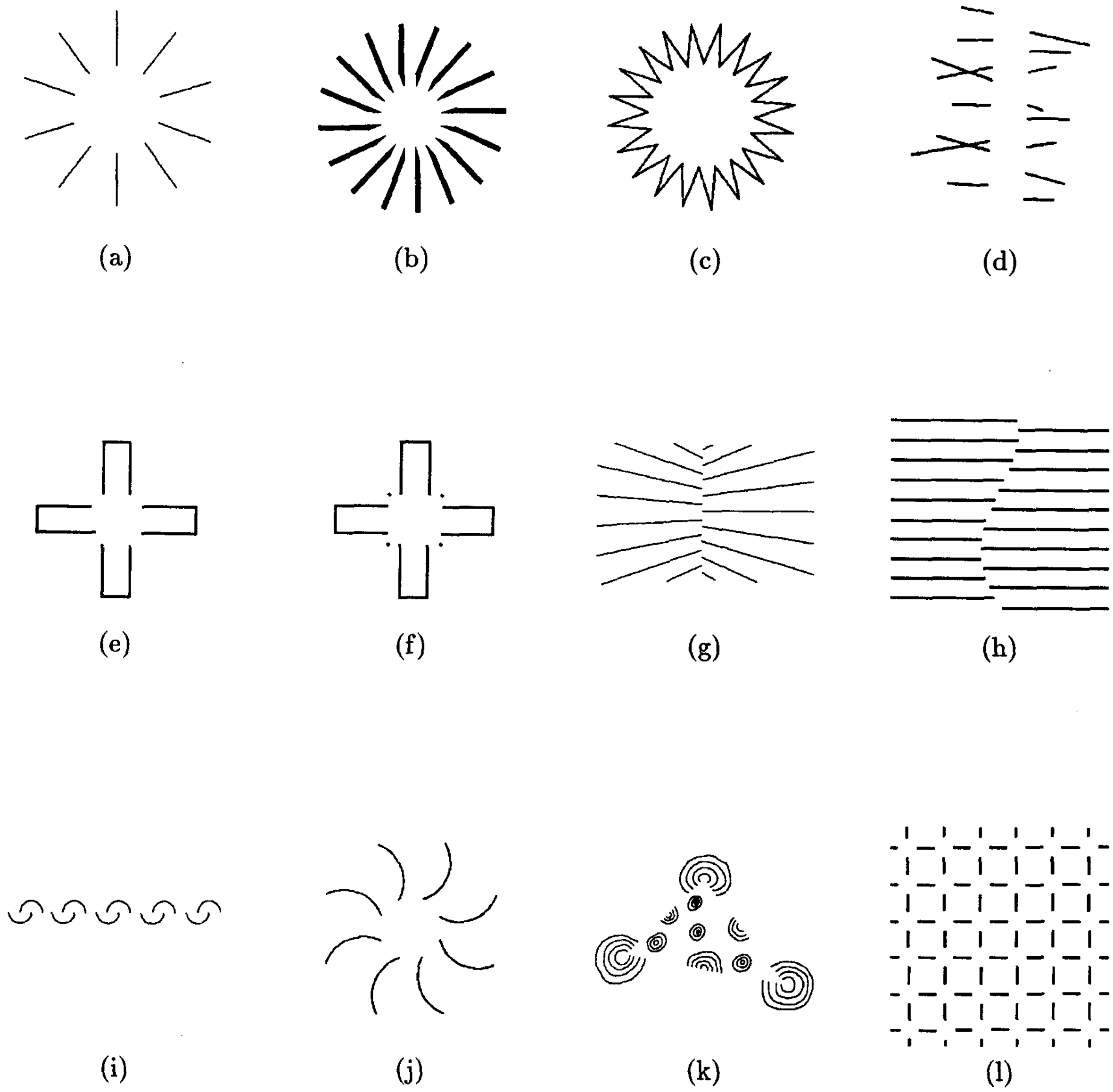


Figure 1.4: Tip-based Subjective Contours (See Appendix A for sources of figures.)

Figure 1.4(f). In addition, a subjective surface can have patches on it (Figure 1.4(k)).

The *Ehrenstein pattern* shown on Figure 1.4(l) demonstrates the change of organization for the tip-based subjective contour by pattern orientation. [Zucker and Cavanagh, 1985] have shown that depending on the orientation of the Ehrenstein pattern, two different patterns of subjective figures can be seen: either a rectangle array of discs, or a grid of stripes by rotating the Ehrenstein pattern by 45° and looking at it from the side of the pattern.

1.3 Thesis Motivation

There has been a lot of interest in research into subjective contours in the areas of psychology, physiology, and computer vision. The blood vessels and blindspots interrupt the retinal image [Kawabata, 1984], and some objects overlap the natural image [Barrow and Tenenbaum, 1978]. Image processing often detects the contours separated by gaps, and the object boundaries are not complete; see [Canny, 1983], for example. In contrast, a human can effortlessly connect the interrupted contours and notice the interposition of objects. Subjective contour perception is an extreme case of contour perception because a subjective contour is a physically invisible occluding contour. By studying the subjective contours, we can identify many clues and constraints for contour perception. Therefore, understanding subjective contours is important for understanding human visual perception.

Ordinary contour detection algorithms do not detect subjective contours because they can only deal with physically measurable image qualities. Nevertheless, there are some partial solutions to subjective contour detection. For example, the shape of a subjective contour can be found if we know which two supporting edges are connected by the subjective contour [Ullman, 1976], or the subjective contour configurations can be computed if the supporting edges and the location of the subjective surfaces are known [Williams, 1990].

In this thesis, the type of the subjective contour images we handle are black-and-white, still, monocular, and edge-based subjective contour images. We consider subjective contour perception based on the following four criteria derived from the properties of subjective contours:

1. No prior knowledge of object shape is required to detect subjective contours. In this thesis prior knowledge of subjective contours is not necessary because it is related to learning. A subjective contour can be seen without prior knowledge of its shape.

2. A subjective contour is a special type of an occluding contour. A subjective contour is an invisible outline of a subjective surface which occludes the inducing elements. A subjective contour is a border of figure-ground that separates the subjective surface and the background. There is a T-junction where the inducing element is occluded by the subjective surface.

3. The perceived shape of a subjective contour depends on the figural configuration and the observer's viewing distance. The supporting edges can be found at curvature discontinuities along the inducing element outline. A subjective contour is locally seen between the two supporting edges, and the curvature continuity of a subjective contour through supporting edges complies with the curvature continuities of the supporting edges. Also, the observer's viewing distance limits the size of the subjective contour that can be seen.

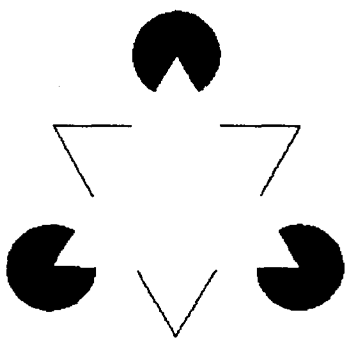
4. All images that produce subjective contours also have alternative contour organizations. If we choose one subjective contour organization—the dominant contour organization—over many subjective contour organizations, there is always another subjective contour organization—the alternative contour organization—based on the real and subjective contours which are not chosen at the previous organization. The alternative contour organization recovers the contours occluded by the surfaces which associated with the subjective contours that are selected in the dominant organization.

Currently, no single system can decide supporting edges and the overall subjective contour organization and its alternative organizations for various subjective contour images. The purpose of this thesis is to demonstrate some techniques for subjective contour detection on a computer to meet the above four subjective contour perception criteria. In this thesis, we detect subjective contours by considering all the supporting edge candidates and by finding the overall organization of the subjective contours. There is no restriction on subjective surface orientation because the model focuses on the contour separating the two regions—subjective surface and the background—and not the subjective surface itself. The rule for contour organization is presented to discover the dominant contour organization and the underlying contour organization occluded by the subjective surfaces which are selected in the dominant contour organization.

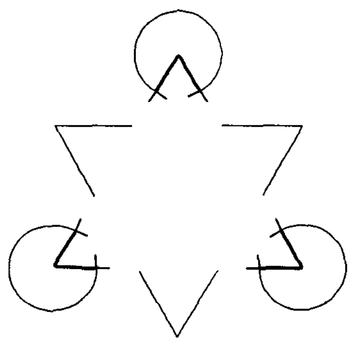
There are three stages in the computer implementation to detect subjective contours: preprocessing, local subjective contour selection, and global contour organization. The first stage is to locate and segment physically present contours according to their curvature discontinuity by Lowe's curve partition method. The segmented contours within one blob are grouped and ranked as supporting edge candidates. To illustrate the implementation stages, let Figure 1.2 on page 2 be the input image to detect subjective contours. Figure 1.5(b) on page 11 shows that the segmented edges with thick blob outlines are stronger supporting edge candidates than the thin ones. A segment of ten pixels length tangent to each endpoint of the blob outline segment is extended where there is a possibility of contour continuation beyond the real edge.

In the next stage, all the potential subjective contours originated from each blob supporting edge candidate to another edges that satisfy the subjective contour selection criteria are selected and weighted. The subjective contour selection criteria apply the same curvature continuity measure for real and subjective contours, and limit the gap size between the two supporting edge endpoints. A subjective contour continues from one supporting edge endpoint to the other. The maximum gap size allowed between any two supporting edges is set to 130 pixels apart in Figure 1.5.

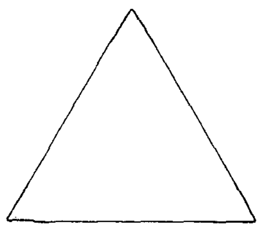
After finding all the potential subjective contours, the final stage is global processing to organize the real and subjective contours that can be seen at the same time. Figure 1.5(c) and (d) show two resulting subjective contour organizations. Figure 1.5(c) is the dominant contour organization because each subjective contour is supported by the stronger supporting edges and it is perceived immediately. Figure 1.5(d) is an alternative contour organization that uses the real and subjective contours not used in the dominant subjective contour organization. The



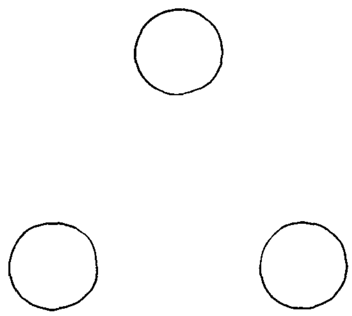
(a) Input image



(b) After preprocessing



(c) Dominant Organization



(d) Alternative Organization

Figure 1.5: An Example of Test Results

alternative contour organization completes the blob outlines that were occluded by a middle white subjective surface.

The algorithms for local subjective contour selection and global contour organization have been developed and the whole system for subjective contour detection has been implemented. There is no restriction on the shape of subjective contour; therefore, straight subjective contours as well as curved subjective contours can be found by this system. The method emphasizes which supporting edges to choose in each contour organization. The exact shape of the subjective contour is not the main concern in this thesis because it depends on the viewing condition. A number of subjective contour images have been tested and the system produces good results on a large variety subjective contour images.

1.4 Thesis Organization

This thesis comprises six chapters. In the next chapter, previous research on subjective contours is reviewed; theories and hypotheses, and related computer systems are discussed. A model for finding subjective contours based on the local assumption of occlusion, and global contour organizations depending on the observer's viewing distance is proposed in Chapter 3. Chapter 4 presents the computer implementation of the model presented in Chapter 3. Results and discussions of applying the computer program to a number of subjective contour images are given in Chapter 5. The last chapter, Chapter 6, draws conclusions about subjective contour detection on a computer system and suggests some future research possibilities.

Chapter 2

Previous Research

Research into subjective contours can be found in the area of psychology, physiology, and computer vision. The first section of this chapter summarizes subjective contour theories that are mainly psychological and physiological. The next three sections survey related computer vision techniques. Image segmentation techniques can detect real contours but they cannot detect subjective contours. In Section 2.2, the shortcomings of image segmentation techniques applied to subjective contour detection including additional constraints upon occluding contours are described. If we know which two supporting edges produce a subjective contour, then the shape of the subjective contour can be computed. Shape completion, in Section 2.3, computes the shape of a contour connecting two edges. The computer systems capable of detecting subjective contours are described in Section 2.4.

2.1 Subjective Contour Theories

There are basically four theories about subjective contour perception: the Gestalt theories, the cognitive theories, the physiological theories, and perceptual organization. For summary see [Halpern and Salzman, 1983], [Parks, 1984], and [Meyer and Petry, 1987].

The Gestalt theories explain that subjective contour perception is a result of spontaneous and preattentional organization. [Kanizsa, 1955] has reasoned that incomplete blob inducing elements have the tendency to complete and they converge into simpler and more stable regular figures on completion. Consequently, subjective contours are due to perceiving a surface superposed on those figures. [Minguzzi, 1987] has extended Kanizsa's explanation of figural completion to figural continuation—tendency to continue. His approach of subjective contour perception takes line, dot, and irregular blob inducing elements into account.

The cognitive theories express that subjective contours are suggested by the inducing elements configuration and produced as a result of perceptual hypothesis. The figural-cue hypotheses explain that a subjective contour is perceived as a result of responses to partial figural cues such as figure-ground interpretation of inducing elements [Rock and Anson, 1979] or gaps due to occluding objects [Gregory, 1972]. [Coren, 1972] has proposed the depth-cue hypothesis, and he demonstrated that a plane is perceived with the impression of depth by monocular depth cues such as interposition, and the edges of the perceived plane form a subjective contour; therefore, subjective contour is perceived as a result of depth organization. Each cognitive theory requires assumptions of cues derived from specific object knowledge, and the subjective contour arises when the observer perceptually reorganizes the configuration according to those cues. The cognitive theories cannot present multiple subjective contour organizations because once it is cued, only one hypothesis is proposed and it will be tested for the feasibility of the solution.

The physiological theories concern with the physiological responses of subjective contour perception. The brightness-contrast hypotheses pay attention to the brightness enhancement associated with the subjective surface due to the contrast of the inducing elements spread to fill the region; for detail see [Day and Jory, 1978], [Frisby and Clatworthy, 1975], [Jory and Day, 1979], and [Kennedy, 1979]. The hypotheses explain that the subjective contour is perceived after the perception of brightness difference between the subjective surface and its background. [Smith and Over, 1975] have presented experimental results that show the orientation-sensitive edge detector cells in the human visual system can detect subjective contours which continue in the same direction as the physically present edges. Networks for segmentation by [Grossberg and Mingolla, 1985] and for non-linear contour-sensing by [Shapley and Gordon, 1985] are some of the neural functional models which describe subjective contour formation.

Neuron response to subjective contours are found in visual cortex cells of monkeys [von der Heydt, Peterhans, and Baumgartner, 1984] and cats [Redies, Crook, and Creutzfeldt, 1986]. The physiological theories can provide only one interpretation of subjective contour perception because the neural inhibitory interactions predetermine the object shape.

There are always some counter examples reported that make each explanation of subjective contour perception incomplete; for example, [Day and Kasperczyk, 1983a] to Kanizsa's Gestalt theory, [Day and Kasperczyk, 1983b] to Coren's depth-cue hypothesis, and [Parks, 1980] to the brightness-contrast hypotheses. Currently there is no single theory that can fully explain the subjective contour perception. However, some experimental results from [Halpern, 1981] and [Halpern, Salzman, Harrison, and Widaman, 1983] suggest that those theories are correlated and the better explanations of subjective contour perception can be obtained by combining many theories.

Perceptual organization is one of the human vision abilities to notice grouping and structures without prior knowledge of its contents. Perceptual organization is definitely applied on subjective contour perception especially the figure-ground separation. [Bradley and Dumais, 1975] have stressed the importance of reversible subjective contours that perceptually can be organized in many ways. In a reversible subjective contour image, the observer can only see one subjective contour organization at a time but has the ability to shift organization spontaneously.

Perceptual organization stands somewhere in between physiological and cognitive theories because it is not totally the early stage or the late stage of visual processing and the subjective contour organization can be shifted by will. This thesis applies the perceptual organization idea with a help of figural-cues to finding subjective contours.

2.2 Image Segmentation

The purpose of image segmentation in computer vision is to divide an image into regions based on some properties. There are two approaches to image segmentation: region segmentation and edge detection. The region segmentation technique is an indirect way to find contours because it first segments the image into regions, and then the contours of the regions are extracted. The edge detection technique directly computes contours from the image. For detail survey of segmentation techniques refer to [Haralick, Mackworth, and Tanimoto, 1989] and Chapter 10 of [Rosenfeld and Kak, 1981].

A subjective contour is a special type of contour; thus, a comprehensive segmentation technique should be able to detect both real and subjective contours. However, current simple image segmentation techniques cannot detect subjective contours as they are limited by the assumption on the type of contours they want to detect. The human visual system can effortlessly distinguish a subjective contour and its background where the physical quality of the area is uniform. On the other hand, image segmentation occurs where the intensity changes in the image. This is why image segmentation fails to detect subjective contours because the techniques are based on using a physically measurable image quality only. Nevertheless, image segmentation is useful in finding physically present inducing element contours.

The grouping of regions or edges can give better segmentation results when it is based on assumptions on the characteristics of the image we are dealing with. The improved region segmentation methods impose the assumption of occlusion to perform region grouping for the reason that the fragmented regions are due to occlusion. [Darrell and Pentland, 1991] have grouped together regions of objects with homogeneous intensity that are disjoint due to occlusion in a multiple layer description but they didn't try to recover the shape under the occlusion. [Nitzberg and Mumford, 1990] have emphasized the T-junctions, and described a model that segments an image into regions and finds the occlusion relation of objects by estimating the depth of the objects. They noted that the line ends and corners are degenerated form of T-junctions but narrowed the explanation of subjective contours to missing outlines in between the two aligned edges or among several aligned line ends.

Edge detection finds locations in the image where there are sudden changes in intensities. Edge detection sometimes gives object boundaries with many gaps due to shadows, occlusions, and the blur image. To overcome the fragmented object boundaries as a result of edge detection, edge grouping can be performed on the edge detection results. Edge grouping requires some assumptions on the object shape: usually convexity and smooth object outline. [Huttenlocher and Wayner, 1991] have made assumptions on the local geometric properties of the object as convex and found a method for identifying groups of intensity edges that are likely to result from the same convex object as global phenomenon in an image. [Trytten and Tuceryan, 1991] have performed curvilinear grouping of edges to detect object boundaries by generating hypotheses, used energy minimization curve to measure the grouping of edges, and selected the best hypothesis. [Ullman and Sha'ashua, 1988] have looked at salient structures in the image, and connected the gap between the two edges by evaluating curvatures and curvature variations. This method has no distinction between the gap due to occlusion and the gap due to the image characteristics; consequently, both types of gap is filled when the edges across the gap are continuous.

The edge grouping method only deals with edge segments as its inputs. The blob outlines consist of edge segments but line ends or dots have no edge segments. Hence, the blob inducing elements are the only concerns when applying the method to finding the subjective contours. We have to segment the blob outline where there are curvature discontinuities before performing the edge grouping which groups edges over different figures. We can use curve partitioning techniques to segment the blob outline. Refer to [Asada and Brady, 1986], [Freeman and Davis, 1977], [Lowe, 1989], and [Rosenfeld and Johnston, 1973], for curve partitioning techniques.

The edge grouping method requires assumptions on the object shape being either convex or smooth curve; as a result, prior knowledge of the object shape is needed since the method has no distinction between occluding and non-occluding contours. The edge grouping method finds both the subjective contours and non-subjective contours at the same time. The cornered subjective contours are not found by the edge grouping method because the method connects only continuous gaps. The method gives only one contour organization to result in no alternative contour organization. Furthermore, the method is not making use of the border information for figure-ground separation. In conclusion, the image segmentation technique is not adequate to detect subjective contours.

2.3 Shape Completion

A shape completion method is a local process for computing the shape of a missing contour that joins two real contours. The missing contour is due to occlusion of an object or due to a shadow, or is a subjective contour. The two real contours are seen as part of a continuous edge separated by a gap, and are seen as a part of the missing contour. The missing contour passes through the endpoint of each real contours assuming smooth connections. Shape completion aims at making the interpolated curve and the entire segment of contours as smooth and continuous as possible.

In the subjective contour situation, after the curve partitioning of the figure outlines, if we know which two supporting edges are connected, then the shape of a subjective contour can be computed mathematically. [Ullman, 1976] interprets that a subjective contour consists of two circles, tangent to the source point, and joined smoothly and with minimal total curvature. [Rutkowski, 1979] improves Ullman's method and uses a cubic polynomial to estimate the subjective contour shape. [Brady and Grimson, 1980] minimize quadratic variations in the continuous curve, and [Horn, 1983] investigates many curves which have small integral of the square of curvature. [Webb and Pervin, 1984] argue that a subjective contour is a parabola or a straight line. Each shape completion method evaluates the subjective contour by minimizing some measures of the total curvature.

The shape completion of subjective contour requires prior knowledge of the missing contour shape as a smooth contour and obviously with no corner. Moreover, it does not tell which two supporting edges are connected, has no limit on the gap size between the supporting edges, and is processed locally so it does not organize the entire results. Also, there is no consistent measure used between the continuity of real and subjective contours. In summary, the shape completion of subjective contours is useful only if the two supporting edges and the direction in which they connect are known.

2.4 Subjective Contour Detection Systems

The computer system that detects subjective contours accepts as input a subjective contour image and output the subjective contours in an organized manner. The system simulates the model of human subjective contour perception process. Four such systems based on cognitive theory, physiological theory, and perceptual organization are discussed in this section.

[Williams, 1990] takes [Rock, 1983]'s view of human perception as problem solving and describes the mechanics of occlusion of one surface by another using a set of integer linear constraints. In his method, a gap in an image contour is completed by a non-local grouping process with specific knowledge of the surface and occlusion. His model deals with straight sections of subjective contours; as a result, no curved subjective contours and cornered subjective contours are detected. He puts a global restriction on maintaining consistent orientation of subjective surface that is adjacent to the blobs, although this rule does not always apply especially for the patched subjective surfaces. There is no alternative organization possible in his method; in contrast, he presents a feasible solution which contains subjective contours and a non-feasible solution which has no subjective contour. Williams makes use of region and edge information in a global sense, but using the global information limits the type of subjective contours his method can detect.

The following two models are based on the neurophysiological findings from the monkey visual cortex by [von der Heydt, Peterhans, and Baumgartner, 1984]. [Skrzypek and Ringer, 1992] believe subjective contour detection is an early stage of visual processing and present a neural network model to find occluding surface completion. [Heitger and von der Heydt, 1993] present a bottom-up computational model with no feedback loops that combine one and two dimensional image information to find occluding contours and their figure-ground directions.

In [Skrzypek and Ringer, 1992]'s model, the image features that appear to contribute to the subjective contour perception such as line ends and corners are extracted before being input to the neural network. They simulate the primate visual system to correspond one spatial position to a single receptor cell. In one spatial position, the signals from response of real and subjective contours at different orientation are combined. In order to perform perceptual completion of a subjective contour within the receptive field, the both sides of a gap must have a real contour. The subjective contour perception is locally ambiguous, and to overcome the problem they include the surface gradient in local hypothesized completion. The responses at each spatial position are combined at layers of general contour neurons to achieve a steady state. The receptive field for both a real and subjective contour respond in straight line; therefore, only straight subjective contours across the gap are detected in this model.

[Heitger and von der Heydt, 1993] consider both the edge-based and the tip-based subjective contours originated from each supporting edge and combined the responses to perform grouping of supporting edges. The subjective contour responses are convolved with two polar separable integration kernels in the opposite direction in many orientations. The strength of contour combines contrast defined boundaries and grouping responses. At each orientation, the strength of contour is calculated. The position of a subjective contour is extracted by local maxima of the strength of contour at combined orientations, and as a result only one interpretation of subjective contour shape is possible. They group pairs of supporting edges where each pair connects a subjective contour. Grouping happens if the interpretation of occlusion is consistent across the two supporting edges.

[Heitger and von der Heydt, 1993]'s model assumes that blob and line inducing elements are located in the background relative to the subjective surface even though this is not always the case. They assume a consistent direction of occlusion along the subjective contour, and this assumption restricts the subjective contours the model can detect. They extend the model to find curved subjective contours by allowing combinations of two convolution kernels meeting at a small non-zero angle. The dominant and alternative contour organizations are detected at the same time when the two edges at a corner originate subjective contours because either edge could be interpreted as an occluding edge. As they stated this problem, "the model cannot resolve ambiguities and tends to complete also the background."

[McCafferty, 1990] presents perceptual organization which groups many low level representation by calculating Gestalt laws of organization as an energy minimization problem. His grouping method is based on the bounding envelope of the figure and requires prior expectation of object shape to find a weight function for each parameter corresponding on each of the Gestalt laws. He demonstrates the perceptual organization techniques on subjective contour detection by reducing corners into dots at the curvature discontinuities or line ends into dots, and performs grouping of dots. In order to use this method, the location of supporting edges must be known prior to find the subjective contour. The alternative contour organization is not possible in this method. Moreover, the boundary information is lost when corners and line ends are converted into dots, and there is no concern about occlusion in his method.

All the above subjective contour detection systems limit the supporting edges to be the concave corners of blobs and the line ends. They have to know which data points would help to shape the subjective contour, and only one subjective contour organization is possible from the given set of supporting edges; therefore, no alternative contour organization is possible. The additional constraint derived from occluding contours is to make the subjective surface orientation consistent with the real contours and the subjective contours, and this constraint also limits the number of possible contour organizations in the systems described above.

In this thesis, we try to overcome some of the limitations imposed on the previous subjective contour detection systems by concentrating on subjective contour organization. The supporting edge types that can find subjective contours can be expanded by ranking all the supporting edge candidates by the likelyhood of becoming the supporting edge that supports a subjective contour. The ranking of supporting edges helps to select the supporting edges and the connecting subjective contours. Our approach to the subjective contour organization is first find the dominant contour organization, and then find the alternative contour organization using the supporting edges and subjective contours that were not used in the dominant contour organization. In this way, both the prominent subjective contours and their occluded contours can be recovered in the different contour organizations. The restriction on the consistent orientation of the subjective surface in relation to the subjective contour can be relaxed because our approach focuses on the subjective contour, and locally the contour can be seen as separating the two regions: the subjective surface and the background.

Chapter 3

Detection of Subjective Contours

Usually the shapes of the objects cannot be seen completely due to the interposition of the objects or the shadows. Humans can perceive such shapes by imposing the assumption that occlusion has occurred. Thus, what are the grouping and partitioning criteria for images used by the human visual system? This chapter attempts to answer the above question by studying the subjective contour which represents the boundary of subjective surface. The emphasis is placed on the local boundary properties such as supporting edges, T-junctions, and shape of subjective contours, and the global contour organizations. Whether the subjective surface is seen as a two dimensional plane parallel to the image plane or three dimensional shape, the dimensionality of the subjective surface is not the concern in this thesis because our focuses is on the boundary of subjective surface.

3.1 Assumptions

No prior knowledge of object shape is needed to detect subjective contours. Also, knowledge about shape of subjective contour is not required to perceive subjective contours in general. In addition, the preset knowledge from learning can help to enhance the subjective contour detection. [Rock, 1983] found that the observer can perceive subjective contours even with the unfamiliar inducing elements once he or she is aware of the possibility of perceiving the certain subjective contour shapes in the image. However, the preset knowledge about the shape of

Figure 3.1: Two Interpretations on a Tip-Based Subjective Contour Image

subjective contour is not a crucial factor because subjective contour in general can be seen without learning.

Any subjective contour image can be simplified to black and white colour with no textures and patterns. A black and white image is a simplification of the real world scene: use one colour, white, as the foreground figure and the other colour, black, as the background colour. Two colours are sufficient to describe any image locally because the real contour exists where the intensity changes suddenly. There are some figures with gray scales or different textural qualities that can induce subjective contours. To simplify those images into black and white images, we can choose some dominant texture qualities or gray scales and replace them with the foreground colour and set the other texture qualities as the background colour depending on the focuses of the observer. For example, to convert the image with white and black figures and gray background, we can convert the figures to black colour and the background to white colour.

The scope of images we are dealing in this model are the edge-based subjective contour images. An edge-based subjective contour is connected to two supporting edges where a subjective contour continues from the endpoint of one supporting edge to the endpoint of another supporting edge. The supporting edge pair that supports a subjective contour is mostly the blob outline segments. For a variation of the edge-based subjective contour, one of the supporting edges can be a tip only if the tip can help to shape the subjective contour. A tip is either a line end or a dot. Therefore, in the edge-based subjective contour image, at least one of the supporting edges that continues through the subjective contour has to be part of the blob outline. This condition is important for the edge-based subjective contour because the supporting edges give direction to the subjective contour shape, unlike the tip-based subjective contour which can be shaped without the supporting edges guidance.

The subjective contours seen on the tip-based subjective contour image can be interpreted in many ways. Any subjective contour shape connecting the two line ends is possible to perceive. Figure 3.1 on page 23 shows examples of two different interpretations of subjective contours on the tip-based subjective contour image. The difference in perceiving the subjective contour originating from a blob outline segment and a line end is that the former has a tangent direction at the end of the segment that can predict the direction of subjective contour shape, but the latter has no tangent to suggest the direction of the subjective contour shape.

3.2 Occluding Contours

An occluding contour is the contour of an object that is occluding other objects. A subjective contour is a special type of an occluding contour because the subjective contour is the outline of a subjective surface which occludes the inducing elements. An inducing element is a figure partially occluded by the subjective surface.

A subjective surface has the same physically measurable colour as the background. Accordingly, its border on the background is indistinguishable from the background and is invisible. Still, the interruption of an inducing element surface suggests that there is an object occluding the inducing element. A supporting edge is located where the subjective surface occludes the inducing element, and is part of the border of a subjective surface.

There are two types of relations between a subjective surface and an inducing element. The first case is a subjective surface located adjacent to an inducing element: region (A) in Figure 3.2 on page 25 is a subjective surface and region (B) is a background relative to the subjective surface. It is interpreted as a subjective surface occluding a blob. The second case is an inducing element being part of a subjective surface: region (B) is a subjective surface

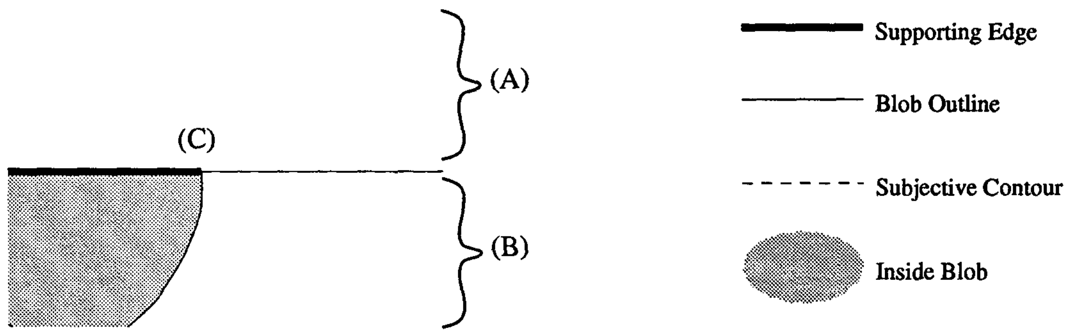


Figure 3.2: T-junction on a Subjective Surface and an Inducing Element

and region (A) is a background in Figure 3.2. In this case, the inducing element is seen as a patch on the subjective surface at the border. In summary, occlusion of an inducing element always involves two surfaces, and a T-junction is seen where the two surfaces meet. In the first case, the two surfaces are a subjective surface and an inducing element; in the second case, a background and an inducing element. In both cases, the inducing element ends suddenly at a point (C) in Figure 3.2 where the supporting edge and the subjective contour meet. The supporting edge and subjective contour together form an occluding edge, and the inducing element disappears behind the occluding surface. In general, the border of a subjective surface consists of supporting edge and subjective contour alternating.

Sometimes the orientation of the subjective surface in relation to the subjective contour is unclear because of ambiguity in the arrangements of the inducing elements. We don't have to know on which side of the subjective contour the subjective surface is located. The inducing element outline, the supporting edge, and the subjective contour together form a T-junction independent of the subjective surface orientation. Also, our focuses is on the subjective contours and there is only one contour separating the two regions—the subjective surface and the background. This is in accordance to [Rock, 1983]'s view of figure-ground separation that in studying the reversible figures he values contours which define the borders of figure-ground and not foreground and background surfaces, while those surfaces can be found locally. A contour is locally seen as one side belonging to the figure and the other belonging to the background. The figure is determined by the biased membership of the contour to one region and not the other. Therefore, the inducing element can be on either side of the region sharing an edge along the subjective surface border. Hence, by looking along a subjective contour, the inducing element can be on either side of the subjective contour.

3.3 Shape of Subjective Contour

The perceived shape of a subjective contour depends on the figural configuration and the viewing condition. A subjective contour is supported by two supporting edges. An identification of supporting edges is a strong cue to find a subjective contour. A supporting edge can be found by locating the curvature discontinuity in an inducing element outline. The same curvature continuity measure is applied to a subjective contour whose curvature continuity continues through the supporting edges. The observer's viewing distance also limit the gap size between the two supporting edges connected by a subjective contour.

3.3.1 Curvature Continuity of a Curve

Normally an object is considered convex and has slow change in the outline shape. If there is a sudden change in the outline shape then the section must be occluded by another object. The supporting edge lies along the occluded contour of the inducing element. The supporting edges in a blob can be found by first locating the sudden terminating section of the blob, and then segmenting the blob outline.

A vertex separating the two segments, called a *breakpoint*, belongs to one endpoint of each segment. There are two endpoints at a breakpoint. For example, the blob outline before curve segmentation is shown in Figure 3.3(a) on page 27, and the blob outline after curve segmentation is shown in Figure 3.3(b). In Figure 3.3(b), point a is an endpoint of segment ad and point b is an endpoint of segment bc. Both endpoints a and b are referring to a same vertex A in Figure 3.3(a) which is a breakpoint.

The other endpoint sharing a breakpoint with an endpoint is called a *samepoint*. For

(a) Before curve segmentation (b) After curve segmentation

Figure 3.3: Breakpoints and Endpoints on the Blob Outline

example, the endpoint b is samepoint of the endpoint a and vice versa. There are two endpoints in a segment: each located at one end of the segment. The endpoint at the other end of the segment is called an *otherpoint* with respect to the endpoint at one end. For example, the otherpoint of a is endpoint d, and the otherpoint of b is endpoint c.

To calculate a breakpoint, we need to segment the blob outline. We use the curve partition method developed by [Lowe, 1989] in this model. Lowe's curve partition method finds the locations of tangent discontinuities, which are the breakpoints in this model, on a curve. His method first performs Gaussian smoothing the curve and correcting the shrinkage of the curve over at some range of standard deviation of Gaussians. Next, the smoothed curve is segmented when the rate of change of curvature is above the preset threshold. This is Lowe's curvature continuity measure. If the change of curvature along a curve is below the threshold then the curve is continuous under the given standard deviation of Gaussian smoothing. If there is a point on a curve where the change of curvature is above the threshold then there is a discontinuity in the curve tangent, and the procedure breaks the curve at this point. For different scales of smoothing, a given curve might have different places of curve tangent discontinuities. Finally, the process selects the longest smoothed curve segments over the same curves.

3.3.2 Supporting Edge Type

There are four types of supporting edge candidates: SUPPORT, NON-SUPPORT, UNDECIDED, and TIP. All supporting edge candidates are classified as one of the four types. The

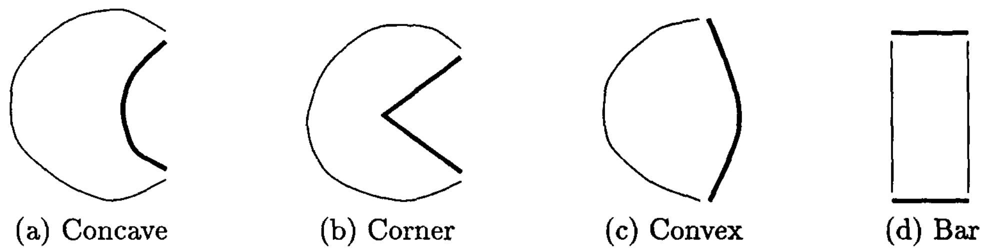


Figure 3.4: Supporting Edge Candidates

distinction is made depending on the strength of supporting edge which affects the connecting subjective contour type and the contour organization.

The blob outline segments have three types of supporting edge candidates: SUPPORT, NON-SUPPORT, and UNDECIDED. The SUPPORT type supporting edge candidate indicates a very strong supporting edge. In contrast, the NON-SUPPORT type supporting edge candidate is the edge adjacent to a SUPPORT type supporting edge candidate which they share a breakpoint, and indicates a weaker supporting edge. The edge that cannot rank as neither SUPPORT nor NON-SUPPORT type supporting edge candidate is classified as the UNDECIDED type supporting edge candidate. Line ends and dots are also qualify as supporting edge candidates and they are classified as the TIP type supporting edge candidates.

A blob supporting edge is an occluding edge whose shape belongs to the subjective surface outline. Figure 3.4 on this page shows some examples of segmented blob outlines. The SUPPORT type supporting edge candidates are drawn in thick lines, and the NON-SUPPORT type supporting edge candidates are drawn in thin lines in Figure 3.4. The concave section of a blob suggests a stronger supporting edge; see Figure 3.4(a) for an example. Moreover, a concave corner on a blob suggests the strongest supporting edge because the L-junction on the blob indicates that there is a convex object occluding the blob with its corner on the blob; see Figure 3.4(b) for an example. Both the concave section and the concave corner of a blob are ranked as the SUPPORT type supporting edge candidates.

If a blob is convex and its outline can be separated into two segments, then the shorter segment is likely to be the interrupted section of the blob by a concave subjective surface; see Figure 3.4(c) for an example. The longer segment of a convex blob outline has larger curvature than the shorter segment. If we choose the longer segment as a supporting edge, then the subjective surface would be more concave than the one which passes the shorter segment. This is not likely to happen. The subjective surface with smaller curvature is more likely to be seen than the one with larger curvature due to the assumption that the object usually has slow change in the outline shape.

A blob must be thick enough to be able to show that there is some surface in front to occlude the blob. The short sides of a bar suggest occlusion because it indicates the sudden termination of a bar (see Figure 3.4(d)), and are ranked as the SUPPORT type supporting edge candidates.

If a blob outline can be segmented into three, then either one or two sides of a blob can be supporting edges. We rank the all three segments as the UNDECIDED type supporting edge candidates because the strength of the supporting edges are ambiguous in this case. Also, for the blob outline with four segments, if the adjacent edges are alternating shorter than the average length and longer than the average length, then the longer segments are ranked as the SUPPORT type supporting edge candidates. In addition, all the segments on the blob outlines that cannot qualify as neither SUPPORT nor NON-SUPPORT type supporting edge candidates are ranked as the UNDECIDED type supporting edge candidates.

A line can be considered as a blob with very narrow rectangle. The line width is so thin that there is no suggestion of occlusion on the line side. However, the line end can be considered as the shorter side of a rectangle with very narrow edge. The end of line suggests that the line is interrupted by a surface in front of the line. Therefore, it indicates a supporting edge at the line end. A dot can be interpreted as a very short line. There is no hint about which section of a dot is occluded by a subjective surface; therefore, any section of a dot can be occluded by a subjective surface. There is a hierarchy of strength of inducing elements in which blobs are the strongest inducing element and then lines and dots. If there are some blobs producing subjective contours, then the tips act as supplement to those contours, and they can contribute only if they help to strengthen the shape of subjective contours.

3.3.3 Subjective Contour Type

After all the supporting edge candidates are found, we can locally find the potential subjective contours that connecting two supporting edges. The length and orientation of each supporting edge, and the separation between the two supporting edges determine the presence and shape of a subjective contour. Hence, there are three subjective contour selection criteria being applied to locally find the potential subjective contours originating from one endpoint in this model. The first subjective contour selection criterion is that only the endpoint from the blob supporting edge candidate can initiate the subjective contour connection. The supporting edge can extend itself only if it is a blob outline segment. The tips—line ends and dots—have no direction themselves and the orientation of the subjective contour at the tip depends on the orientation of the blob supporting edge at the other end of the subjective contour.

Let's call the maximum separation allowed between the two supporting edges connected by a subjective contour the *maximum gap size*. The second subjective contour selection criterion is that the gap size between the two endpoints must be less the maximum gap size. The maximum gap size is set by the observer's viewing distance. If the observer is viewing the subjective contour image from the close distance, then the observer can only see a small portion of the image at a time, which leads to the narrow gap between the two supporting edges. On the other hand with the same subjective contour image, if the observer is viewing from the far distance, then the observer can see the large portion of the image, which leads to the large gap between the two supporting edges. Therefore, the farther the viewing distance, the larger the gap can be.

For example, in Figure 3.5 on page 31, the thick borders indicate supporting edges and the dotted lines show the minimum distances between the two connecting endpoints. The distance between the endpoints a and b is shorter than the distance between the endpoints a and c. If the maximum gap size is set longer than the length of ac, then there are two subjective contour possibilities originating from the supporting edge endpoint a; one to the supporting edge with endpoint b and the other to the supporting edge with endpoint c. If the maximum gap size

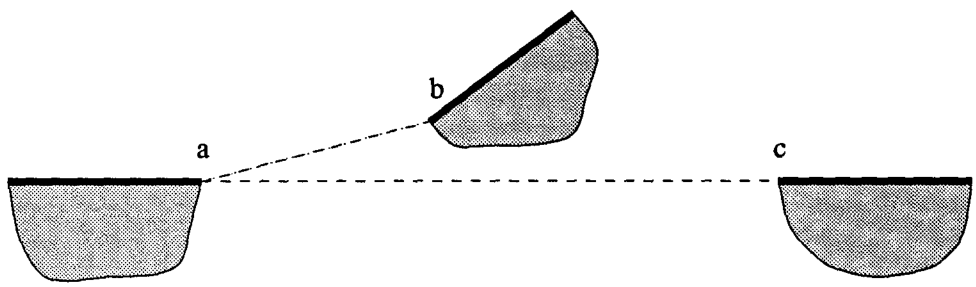


Figure 3.5: Separation Between Supporting Edges

is set longer than the length of ab, but shorter than ac, then there is one subjective contour possibility originating from the supporting edge endpoint a connected to the supporting edge with endpoint b.

The third subjective contour selection criterion is that the subjective contour must be curvature continuous through the supporting edges. A subjective contour is seen as a missing contour connecting the two supporting edges. The entire subjective contour segment—a supporting edge, follow by a subjective contour, and another supporting edge—belongs to part of an object outline: a subjective surface. The object outline is usually considered as a smooth curve when there is no corner. This quality also applies to the entire subjective contour segment, and to the subjective contour itself.

A supporting edge is already a smooth continuous curve segment resulting from applying Lowe's curve partition method on the blob outline. The same curve continuity measure applies to determining the continuation of a subjective contour. When we look at a potential subjective contour connecting from one blob supporting edge to another supporting edge within the maximum gap size, the connecting subjective contour must continue smoothly from the supporting edge as if the supporting edge were extending itself.

The shape of subjective contour can be found by interpolating the gap between the two supporting edges guided by the blob supporting edge extensions as continuous manner. The continuation of the entire segment can be checked against the curvature continuity measure to decide whether to connect or disconnect the curve. Note that the exact shape of the subjective contour is not known because the information is missing and it cannot be recovered. In this thesis, the main concern is the overall organization of the subjective contour and we are much interested in which supporting edges to connect rather than the exact shape of perceived subjective contours. Nevertheless, the subjective contour shape determines which supporting edge pair to choose.

We consider three types of subjective contours: CURVE, LINEAR, and STRAIGHT. The distinction is made because of the supporting edge type and the strength of subjective contour which affects the contour organization. A weight is assigned to each potential subjective contour to select the subjective contour among many potential subjective contours originated from one endpoint.

The CURVE subjective contour is supported by blob supporting edges and the entire subjective contour segment continues as a smooth curve. The connection between the supporting edge and the subjective contour is smooth as they share the tangent at the connecting point. The CURVE subjective contour is approximated by a Bezier curve using three points—two endpoints from each supporting edge and an interception point of endpoint tangents. The supporting edge extensions are identical to the tangents of the endpoints with the direction of extending the supporting edge. For an example in Figure 3.6(a) on page 33, point a and c are endpoints and point b is an interception point of endpoint tangents. A Bezier curve continues tangent to the two endpoints and is smoothly curved from one endpoint to the others. Therefore, the Bezier curve is sufficient to show the general direction and shape of a CURVE subjective contour.

In the extreme case of CURVE subjective contour, the subjective contour can be straight line when the two supporting edges are colinear. The LINEAR subjective contour is supported by the blob supporting edges, and the two supporting edges and the connecting subjective contour are aligned in a straight line. For example in Figure 3.6(c) or (d), the subjective contour is the LINEAR subjective contour when the angles $a'ab$ and $b'ba$ are small enough that we consider the two supporting edges are colinear.

The STRAIGHT subjective contour is a subjective contour connecting an endpoint of a

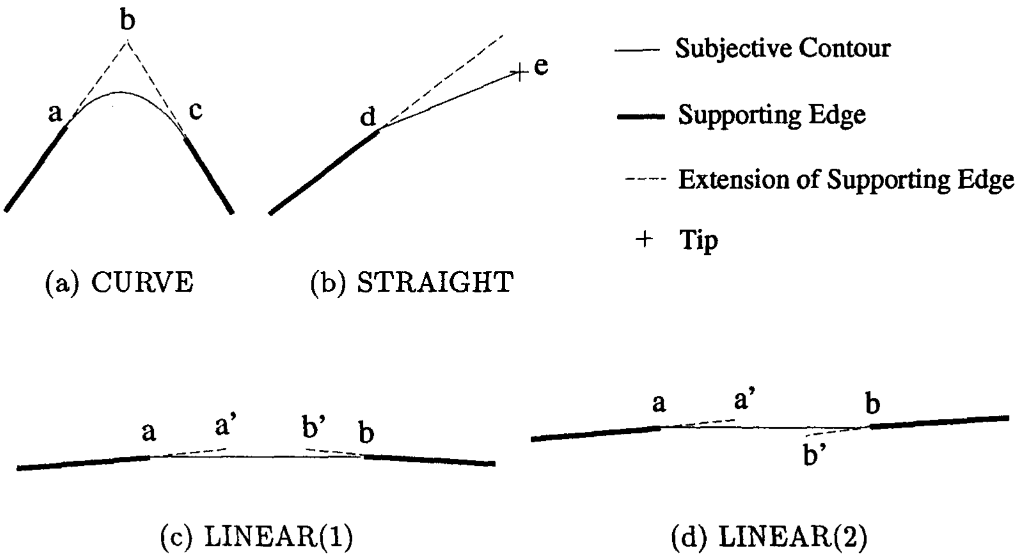


Figure 3.6: Subjective Contour Type

blob supporting edge to a tip, and uses a straight line to approximate its shape. We do not expect such subjective contour with the large curvature because the curve with larger curvature would not be continuous by the Lowe's curvature continuity measure. Still, the straight line approximation of a subjective contour has to be smoothly continued from one supporting edge to another. For example, in Figure 3.6(b) the edge segment consists of the supporting edge with endpoint d and the subjective contour de goes through Lowe's curve partitioning to smooth the edge over the different scales of smoothing and applies the curvature continuity measure on the curve especially at point d. If the entire curve is curvature continuous, then the subjective contour de is a potential subjective contour.

The weight of the subjective contour is useful for choosing one subjective contour from many potential subjective contours with the same subjective contour type originating from one endpoint. We define that the smaller the weight, the stronger the subjective contour is. The stronger subjective contour means that it is more noticeable than other subjective contours; therefore, it is more likely to be perceived. The potential subjective contour with smaller gap size between the two supporting edge endpoints is stronger than the one originating from the same endpoint with larger gap size because the length of former subjective contour is shorter than the latter when we compare the potential subjective contours with similar curvature. The potential subjective contour with less curvature is more prominent than the potential subjective contour originating from the same endpoint with larger curvature of the equal length for the reason that the subjective contour closer to a straight line is stronger.

3.4 Contour Organizations

There are possibilities of many subjective contour organizations emerging from one image. *A contour chain* is a continuous chain of alternating real and subjective contours. One subjective contour organization consists of many contour chains. The contour organization can be viewed as each organization contains set of supporting edges and subjective contours that can be seen at the same time. Usually, first we see a dominant contour organization which is stable and prominent subjective contour organization, and has long contour chains. Then sometimes we see the alternative contour organization using the real edges not used in the dominant contour organization and with the associated subjective contours. The dominant contour organization can be found by making use of the stronger figural cues. For example, a blob inducing element outline is segmented into at least two edges; one belongs to the original blob outline, and the other belongs the border of a subjective surface that composes a contour chain.

In order to find the overall organization of subjective contours in an image, it is important to identify the rules we use to perceive the subjective contour. At the smallest unit of subjective contour organization is a subjective contour segment: a subjective contour and its connecting supporting edges. The next unit is a contour chain in one organization which contains many subjective contours and many supporting edges connecting one after another. The third unit is a contour organization that collects many contour chains which can be grouped in one contour organization. In the last unit, there are many contour organizations possible from one image with the given viewing distance. The subjective contour organization can be obtained by finding the smallest unit first and then combine them to get the larger unit. The followings are some rules for finding each unit.

A supporting edge is connected to a subjective contour. Among the potential subjective contours originating from one supporting edge endpoint, we can only perceive one subjective contour emerging from the endpoint and connecting to another supporting edge. The subjective contour with the smaller curvature and supported by the longer and thicker supporting edges is easier to perceive. When there are many potential subjective contours originating from one endpoint, the LINEAR subjective contours are more likely to be perceived than the CURVE subjective contours, and the CURVE subjective contours are more likely to be perceived than the STRAIGHT subjective contours. Among the subjective contours with the same subjective contour type, the subjective contour with smaller weight is more likely to be perceived for the reason of the smaller gap size between the two supporting edges and smaller curvature on the subjective contour.

A contour chain can be either open or closed. Each subjective contour is supported by two supporting edges; accordingly, the both ends of a contour chain have to be supporting edges if a contour chain is open. One subjective contour is supported by two supporting edges and each endpoint on the supporting edge has at most one subjective contour emerging. At one breakpoint, there is at most one subjective contour allowed to be seen at a time. There are two endpoints at a breakpoint and sometimes both endpoints have potential subjective contour connections. However, we can only see one subjective contour at a time because one of the two adjacent supporting edges is used in the dominant contour organization and the other could be used in the alternative contour organization.

In general, the endpoints of two adjacent supporting edges at one breakpoint cannot be in the same contour organization. If we select one subjective contour belonging to a supporting edge then the adjacent supporting edge cannot have subjective contour originated in the same contour organization. The subjective contour must have two supporting edges, and a tip can be considered as a special type of supporting edge but at most one tip is allowed in one subjective contour. If there is a subjective contour connecting to a tip, then it has to be the end of one contour chain because a tip has no indication of direction and it cannot initiate a subjective

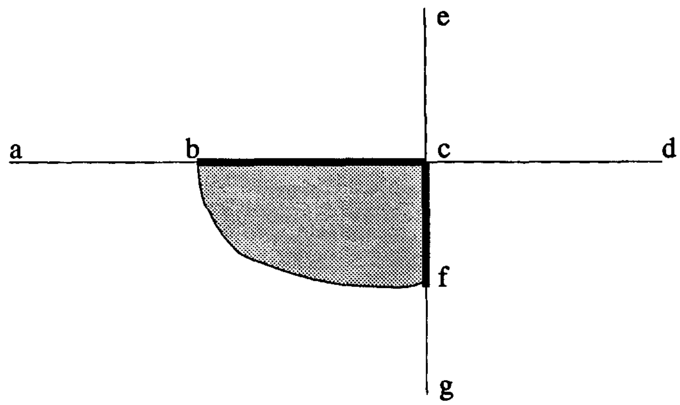


Figure 3.7: Organization of Contour Chains

contour.

Let a contour chain which we are currently working on be a *current contour chain.* When we follow the current contour chain in one direction and it connect to a supporting edge, we have to check the validity of the current contour chain on both endpoints of the supporting edge. The first endpoint of a supporting edge on the current contour chain is called *firstpoint*, and it is an endpoint connecting towards the supporting edge. The second endpoint of a supporting edge on the current contour chain is called *secondpoint*, and it is an endpoint connecting towards the subjective contour if it exists. Let *eitherpoint* refers to either the firstpoint or the secondpoint. To illustrate on Figure 3.7 on this page, when the current contour chain continues from g to f, and it connects to the supporting edge fc next. Point f is the firstpoint and point c is the secondpoint in the supporting edge fc when the current contour chain continues. The samepoint of eitherpoint has one of the following status: either current contour chain, other contour chain of same organization, other contour chain of different organization, or not yet belong to any contour chain. Depending on the current contour direction on the eitherpoint and the status of the samepoint of eitherpoint, the current contour chain can either continue, become one of the contour chain of the alternative organization, or has conflict. These decisions are listed in Table 3.1 on page 37, where x represents the eitherpoint and x' represents the samepoint of

x' belongs to	x has potential subjective contour	status of current contour chain
no organization	yes	continue
	no	continue
current contour chain	yes	conflict
	no	continue
other contour chain of same organization	yes	alternative contour chain
	no	continue
other contour chain of different organization	yes	continue
	no	continue

Table 3.1: Decision of Current Contour Chain Continuation

eitherpoint.

The current contour chain has conflict within itself if x' belongs to the current contour chain and x has some potential subjective contours originating. There are two supporting edges at a breakpoint; however, we can have at most one subjective contour originating from a breakpoint in one contour organization. The conflicted contour chain is excluded from any contour organization. The current contour chain becomes one of the contour chain of the alternative organization when x' belongs to other contour chain of the same organization, and x has potential subjective contour originating, for the same reason as the conflicted current chain. The current contour chain may continue for all the other cases of x and x'.

At most two contour chains can intersect at a tip in one organization. A tip is always located at the end of a contour chain. However, a tip can connect two subjective contours that belong to the same contour organization, even though a tip has no direction by itself and the orientation of the subjective contour at a tip is depending on the supporting edge tangent at the other end of a subjective contour.

There are many subjective contour organizations in one image. Different contour organization uses different set of supporting edges and subjective contours. Therefore, the set of supporting edges and subjective contours that construct one subjective contour organization cannot be used in another organization. However, the edges that were not used in the former organization can be used to find the alternative contour organization. If we choose one organization, there is always alternative contour organization associated with it because only one of the two adjacent edges at a corner that each of which with potential subjective contour originating at the corner can be seen at a time. The alternative contour organization represents the continuous contours lying underneath the subjective surfaces which are selected in the dominant contour organization.

In this model, there is no restriction on the contour organization about subjective contours crossing each other because the depth of the subjective contours are not considered. An overlaid subjective contour image is produced when a subjective contour is on the top of other subjective contours. The depth of subjective surfaces in the contour organization is ambiguous when the subjective contours of different contour chain crosses each other at the subjective contour.

A reversible subjective contour is seen on an overlaid subjective contour image. It happens when there are more than one depth interpretation of the subjective contour organizations, and each interpretation of the subjective contour organizations is equally likely to be perceived. Still, there are dominant and alternative contour organizations existing within each interpretation of the subjective contour organization. In a reversible subjective contour image, it is impossible to see more than one contour organization at a time but the perception shifts from one organization to the other. The reversible subjective contours are depth ambiguous subjective contours because there are not enough depth information and the depth of the subjective contours shift from one subjective contour organization to the other. There is a brightness associated with the subjective figures; the figure on the top have more intensive colour than the background.

The silhouette image is the image with overlaid figures which had reversed the foreground and the background colour, and as a result the figures become the background. The shapes of the original figures can be recovered by the subjective contour organizations; however, the depth of the figures are ambiguous, so each contour organization are equally dominant. A silhouette image can show that the perception of the subjective figures and partly occluded objects uses the same curve grouping and segmentation process.

Chapter 4

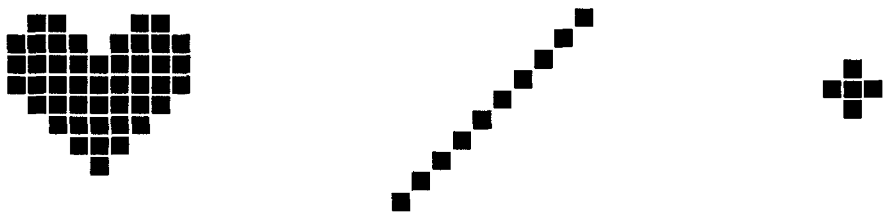
Computer Implementation

How can we make a computer determine the presence of subjective contours? In this chapter, the computer implementation of the subjective contour detection model presented in the previous chapter is described. The subjective contours resulting from the blob inducing elements are the main concern in this implementation. A tip, either a line end or a dot, can become useful only if it supports the shape of a subjective contour.

Based on our model, there are three stages in detecting subjective contours: preprocessing, local processing, and global processing. The first section describes preprocessing—finding the image features, segmenting the blob outline by Lowe's curve partition method, and ranking the supporting edge candidates. The second section presents the local processing for finding all the potential subjective contours originating from an endpoint of one supporting edge candidate to another supporting edge. The final section describes subjective contour organization in the global sense.

4.1 Preprocessing

The input to the subjective contour detection system is a black-and-white image. The input image is obtained by scanning the image using a Macintosh computer running Digital Darkroom and connected to an Abaton scanner. Also, some images are hand drawn using IslandPaint drawing software on the SPARCstation. As a result, there is no need to reduce image noise.



(a) Blob (b) Line (c) Dot

Figure 4.1: Three Figure Types

There are three types of figures–*blob*, *line*, and *dot*–in the image; see Figure 4.1 for examples. A *blob* is a region with foreground colour, a *line* is a connected thin strip whose width is one pixel wide with foreground colour, and a *dot* is a spot represented by one pixel surrounded by four neighbours with foreground colour in horizontal and vertical direction. Each figure in the input image is not connecting or crossing others.

4.1.1 Extract Features

In the preprocessing stage, first we have to find the the physically present local properties of the image such as outline of blobs, location of lines and dots. The input to this step is a subjective contour image. The output is a linked list of figure records with each record consists of figure type, number of points in the edge, and an array of edge points. Thus, each figure record represents the feature of one figure. The outline of a blob, a line, or a dot is an edge in a sense, and each adjacent point on the edge is one pixel apart.

The outline of a blob can be found by identifying pixels at the border and inside of the blob. A pixel inside the blob is surrounded by four horizontal and vertical neighbours with foreground colour pixels and more than one diagonal neighbour with foreground colour pixel. The pixel at the border of the blob, which is adjacent to the background, has less than four horizontal and vertical neighbours with foreground colour pixels, and is adjacent to one or more pixels inside the blob. Therefore, the outline of a blob can be found by marking the border of the figure, and linking those points in the direction of the figure on the right and the background on the left until it is connected back to the starting point.

A line can be easily distinguished from the blob because a pixel in the middle of a line has two neighbours in any direction and a pixel at the end of a line has one neighbour. Starting from a pixel at one end of a line, linking follows and connects the points until it reaches the other end of a line. A dot is located at the middle pixel surrounded by four horizontal and vertical neighbours.

4.1.2 Lowe's Curve Partitioning

The connected points of blob outlines, lines, and dots are detected in the previous step. The curve partitioning of the blob outline based on [Lowe, 1989] is performed next in the preprocessing stage. The simplest input to Lowe's method is the connected points of an open edge—the edge that does not connect back to itself. His method involves three main functions as follows:

1. Smooth and shrinkage correct a curve.
2. Segment a curve at curvature discontinuity.
3. Select smoothed curve segments on the edge.

First, smooth the edge by Gaussians with the standard deviation σ starting from $\sqrt{2}$ pixels to $8\sqrt{2}$ pixels by increments of $\sqrt{2}$ as a default. Each smoothed curve is shrinkage corrected and curvature on each curve point is calculated. The change of curvature on each curve point is calculated by taking the difference in curvature form the previous point to the next point κ'. Multiply κ' by σ^2 to get the scale invariant rate of change of curvature. We can use a single threshold *maximum change of curvature* to check against the scale invariant change of curvature on all the curve points for the smoothed curves with different Gaussian of σ. The maximum change of curvature is set to 0.2 as a default.

Next, for each point on the curve starting from a point after half the Gaussian mask size, and ending at the point before half the Gaussian mask size, check the curvature continuity of each smoothed curve by the following curvature continuity measure:

$$\sigma^2\kappa' < maximum\ change\ of\ curvature \tag{4.1}$$

Segment the curve when a point on the curve does not satisfy the Equation 4.1. The end of the curve segment is extended to approximate the points truncated by Gaussian smoothing.

Finally, we consider all the smoothed curves with different σ of Gaussian smoothing, and select the curve segments that covers the largest length of the input edge. Once a curve segment is selected, the sections of all the smoothed curves overlapping the selected segment are removed from the segment selection process. The segment selection process continues until collection of the selected segments can describe the smoothed edge.

Lowe [personal communication] indicates that it should be easy to apply smoothing to the closed curves but we have not done so here. The sequence of Lowe's curve partitioning steps on the open edges is modified to segment closed edges—blob outlines. We apply the curve partitioning process for the open edges twice on the close edges. The idea is that first we arbitrarily open the closed edge and apply the curve partition method to find the breakpoints along the edge, and next use one of the breakpoints to open the original edge and apply the method again to segment the edge.

Input to the curve partition method is a list of figure records. If the figure type of a record is blob, then we can perform the Lowe's curve partitioning on its edge points along the blob outline. The outline of a blob is a closed edge but the list of edge points representing the blob outline opens the closed edge at an arbitrary point on the blob outline. The point which opens the closed edge becomes the starting point as well as the ending point on the input edge; however, the point might not be the true breakpoint. If there is one resulting segment from the curve partitioning, then the entire edge is curvature continuous. For more than one resulted segment, we choose one endpoint of the longest interval that is not the start or end on the input edge, and choose the corresponding point on the original input edge as a starting point for the new edge. The new edge continues until the end of the original edge, connects to the start of the original edge, and continues till the new starting point is reached. Note that the

(a) Concave corner: $\overrightarrow{P_1P_3}$ to $\overrightarrow{P_1P_2}$ is turning clockwise

(b) Convex corner: $\overrightarrow{P_1P_3}$ to $\overrightarrow{P_1P_2}$ is turning counter-clockwise

Figure 4.2: Concave and Convex Corner of a Blob

starting point and ending point of the new edge are referring to the same breakpoint. Input the new edge to the second round of the curve partitioning process, and output the segmented and smoothed blob outline that is segmented at the location of curvature discontinuities which are usually the corners.

The output is a list of figure records. The blob figure record gains two extra entries—number of segments in a figure and a list of segment records. Each segment record consists of the standard deviation of Gaussian smoothing σ, the number of points in the segment, and a list of point records. Each point record contains location of smoothed point after curve smoothing, tangent at the point, curvature of the point, and the original point location.

4.1.3 Ranking Real Edges

In the third step of the preprocessing, the concave corner on the segmented blob outlines resulting from Lowe's curve partitioning needs to be grouped together as one segment, the segments are ranked into different types of supporting edge candidates, and the data structure is prepared for the next processing stage. The output data structure consists of a list of endpoint records. Each endpoint is uniquely named by an endpoint number. Each endpoint record contains the endpoint number, the location of the endpoint, the supporting edge points which endpoint belong to, σ of Gaussian smoothing of the supporting edge, the supporting edge type, the otherpoint number, and the samepoint number.

The grouping of concave corners becomes useful in the next processing stage because the concave corner cannot have any subjective contours originating at the corner since the subjective contour cannot be seen on the figure. If there are more than three edges in a blob outline, then we have to determine whether a blob has concave corners. Three vertices are taken from the two adjacent edges along the blob outline, the first breakpoint $P_1 = (x1, y1)$ from the first point on the first edge, the second breakpoint $P_2 = (x2, y2)$ from the vertex that is shared by the endpoints of the two edges, and the third breakpoint $P_3 = (x3, y3)$ from the last endpoint on the second edge. Note that a blob outline is connected in the direction where the figure is in the right side and the background in its left. A concave corner can be found by taking the cross product of $\overrightarrow{P_1P_3}$ and $\overrightarrow{P_1P_2}$ as follows.

$$side = \overrightarrow{P_1P_3} \times \overrightarrow{P_1P_2} = (x3 - x1)(y2 - y1) - (x2 - x1)(y3 - y1) \tag{4.2}$$

If *side* is greater than zero, $\overrightarrow{P_1P_3}$ to $\overrightarrow{P_1P_2}$ is turning clockwise, and there is a concave corner; see Figure 4.2(a) on page 43 for an example. Otherwise, the two edges do not from a concave corner; see Figure 4.2(b) for an example. We have to group the two segments of the concave corner into one segment.

If a blob outline is segmented into one edge, then the blob outline is either continuous or have one concave or convex corner. This type of segment cannot have subjective contour connection, and must be eliminated from the supporting edge candidate. If a blob outline is segmented into more than one edges, then each edge is qualified as a supporting edge candidate. There are three types of blob supporting edge candidates: SUPPORT, NON-SUPPORT, and UNDECIDED. Refer to Section 3.3.2 for details on classifying the supporting edge candidates. The lines and dots are ranked as the TIP type supporting edge candidates.

We have to prepare the output data structure for the next processing stage, that is based on the endpoints. A unique number is assigned to each endpoint. Each blob supporting edge candidates have two endpoints—one from each end of the edge segment; therefore, we have to create two endpoint records for each supporting edge candidate on the blob outline. Those two endpoints are from the same edge segment and they are otherpoints to each other. The edge segments located adjacent to each other on the blob outline share one breakpoint; however, there are two endpoints at the breakpoint, and those endpoints are samepoints to each other. A line has two endpoints—one from each end of the line, and a dot has one endpoint. The endpoint of either the line or the dot has no otherpoint and samepoint.

4.2 Local Subjective Contour Selection

The preprocessing result gives a list of endpoint records. All the endpoints are qualified to have a potential subjective contour connection because the endpoints that do not qualify to produce a subjective contour are eliminated in the previous process. This section describes the next stage—the local process to select all the potential subjective contours emerging from each endpoint which satisfy the selection criteria, and to assign a weight to each subjective contour. The potential subjective contour is limited by the viewing condition, one of which is determined by the maximum gap size l. Also, the curvature continuity of subjective contour must comply with that of its supporting edges.

The output of this stage is a list of endpoint records with a linked list of subjective contour records attached to the endpoint record if there are some potential subjective contours originating from the endpoint. The subjective contour record consists of the endpoint it is connected to, the subjective contour type, the weight, and a list of points describing the subjective contour shape.

4.2.1 Extend Edge

Only the endpoint from the blob outline can initiate the subjective contour connection, i.e., the endpoints with supporting edge types SUPPORT, NON-SUPPORT, and UNDECIDED are the endpoints from the blob outline. This is the first subjective contour selection criterion. For each qualifying endpoint, check to see if there is a subjective contour connecting to another endpoint. The second subjective contour selection criterion is that the gap between the two endpoints must be less than the maximum gap size l. If there is a pair of endpoints satisfying the condition, then we can calculate the shape of a subjective contour connecting the two endpoints.

The third subjective contour selection criterion is that the subjective contour must be curvature continuous through the supporting edges. Refer to Section 4.2.2 for the calculation of the shape of subjective contour. If the subjective contour satisfies all the above conditions, then a potential subjective contour connection between the two endpoints is found. We can calculate the weight of the subjective contour; refer to Section 4.2.3 for the weight calculation. The subjective contour record for each endpoint connecting the subjective contour is created, and it is attached to each endpoint record.

4.2.2 Shape of Subjective Contour

There are three types of subjective contours resulted in this implementation: LINEAR, CURVE, and STRAIGHT. The LINEAR subjective contour is supported by the blob supporting edge pair, and the angles between the extended part of supporting edges and the line connects the two endpoints are within 5° each. There is a straight subjective contour connecting the two endpoints of supporting edges, and the two supporting edges and a subjective contour are assume to be connected as a straight line; hence, we assume the subjective contour to be curvature continuous with supporting edges. For example, if the angle of α and β are both within 5° on either supporting edge pair on Figure 4.3 on page 47, then a LINEAR subjective contour is found which is connecting the supporting edge pair.

The CURVE subjective contour is also supported by the blob supporting edge pair, but does not qualify as a LINEAR subjective contour, and the extension of the two supporting edges intersects. To find the CURVE subjective contour, do the following. First, the shape of subjective contour is approximated by a Bezier curve using three points: two from supporting edge endpoints and one intersecting point from the extension of the two supporting edges. Refer to Appendix B for detailed calculation of the Bezier curve. Second, perform Lowe's curve smoothing with shrinkage correction using the bigger value of σ associated with either supporting edge. Third, for each point on the Bezier curve, apply Lowe's curvature continuity measure. The curvature continuity of the supporting edges and their extension to the subjective

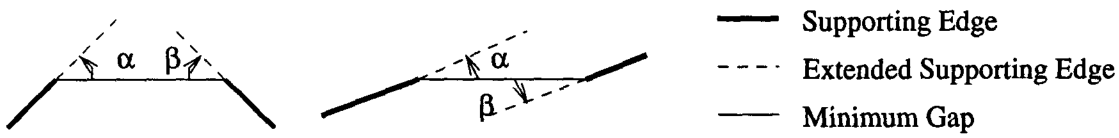


Figure 4.3: α and β Angles

contour do not have to be checked because the Bezier curve is tangent to both supporting edges. Finally, if the Bezier curve is curvature continuous, then accept the subjective contour; otherwise, reject the subjective contour connection.

The STRAIGHT subjective contour is supported by a blob supporting edge and a tip. We approximate the shape of the subjective contour connecting the blob supporting edge and the tip to a straight line. In order to accept this subjective contour, we have to check the curvature continuity from the blob supporting edge to the subjective contour. First, we have to find the points on the subjective contour by sampling the points along the straight line between the endpoint and the tip with one pixel apart. Second, the subjective contour is smoothed in order to get the curvature on the subjective contour to check the curvature continuity. Use the σ from the blob supporting edge to perform Lowe's Gaussian curve smoothing with shrinkage correction on the subjective contour. Third, check the curvature continuity from the blob supporting edge to the subjective contour especially around the endpoint connecting the supporting edge and the subjective contour. The reason is that the supporting edge is curvature continuous already and the subjective contour is a straight line; therefore, the only concern is the curvature continuity around the endpoint. Finally, if the whole segment—the supporting edge and the subjective contour—is curvature continuous, then accept the subjective contour; otherwise, reject the subjective contour connection.

4.2.3 Weighing Subjective Contour

The weight of subjective contour is compared among the subjective contours with same type. The smaller the weight on the subjective contour, the more likely that the subjective contour is selected in the contour organization. α and β are angles between the line connecting endpoint to endpoint line and the extended supporting edges in radian; see Figure 4.3 on page 47 for the illustrated examples. m is the minimum gap size between the two endpoints of a subjective contour. The weight of a subjective contour is calculated as follows:

$$weight = m \times (1 + |\alpha|) \times (1 + |\beta|) \tag{4.3}$$

We want to have the weight smaller when both the gap size between the supporting edge pair connecting a subjective contour and the curvature of the subjective contour are smaller. The magnitude of subjective contour curvature is approximated by α and β angles. Each of α and β is the difference in angle between the straight subjective contour orientation and the supporting edge orientation from different supporting edge. α is 0 when the subjective contour is straight, and α is larger when the curvature of the subjective contour is larger. Similar analysis is applied for the value of β. We use α and β as penalty of not being straight subjective contour and multiply the weight by $(1 + |\alpha|)$ and $(1 + |\beta|)$.

For a variation, we can put the larger weight on the subjective contours that are not horizontal or vertical. Let's say that the STRAIGHT subjective contour on horizontal and vertical orientation is more likely to be detected than the similar subjective contour in the diagonal. Let γ be the acute angle between horizontal and the STRAIGHT subjective contour. γ is $0°$ when the subjective contour is horizontal and $90°$ when the subjective contour is vertical. We can use $\sin 2\gamma$ as a weight for horizontal and vertical orientation because it gives 0 if the subjective contour is horizontal or vertical, and 1 if the subjective contour is diagonal in $45°$.

$$weight = m \times (1 + |\alpha|) \times (1 + |\beta|) \times (1 + \sin 2\gamma) \tag{4.4}$$

4.3 Global Contour Organization

The previous processing stage finds all the potential subjective contours emerging from each endpoint. The next stage, the final stage of subjective contour detection, is described in this section. This section presents how to find and group contour chains in different organizations selected among many supporting edge candidates and many potential subjective contours. The algorithm chooses one best subjective contour for each endpoint among many subjective contour possibilities. The dominant contour organization and the alternative contour organization are found as a result. This stage is a global process because all the supporting edges and subjective contours are considered for each contour organization.

4.3.1 Data Structure

The input to this processing stage is a list of endpoint records. There is a linked list of subjective contour records attached to the endpoint record if there are some potential subjective contours originating from the endpoint. The output is a linked list of organizations where each organization contains many contour chain records. Each contour chain record consists of linked list of segment records. Each segment record describes the segment type, and the points on the segment. The segment type can be either real or subjective contour. The linked list of segment records is circularly connected if the contour chain is closed, and not circularly connected if the contour chain is open.

We need two data structures for the intermediate process to know the dynamic change in the subjective contour connections and the endpoint status. The entry to the *link table* describes whether the potential subjective contours connecting from one endpoint to the other are selected or not. An *endpoint table* shows the status of each endpoint: whether it is available or belongs to some contour organization.

One real contour endpoint has many potential subjective contours originating from it. However, once the real contour is selected in a contour chain, there is only one subjective contour emerging from an endpoint. Therefore, we need to keep track of which potential subjective contours are selected in the contour chain and which is not used in the contour chain. The link table is constructed for this purpose.

A link table is a matrix of endpoints where the row shows where the subjective contour originating from and the column shows where the subjective contour is connected to. Each matrix entry contains the subjective contour type, the weight, and the connection status. The connection status can be either ABLE, TEMP-DISABLE , and DISABLE. Initially the matrix is constructed from the input, and each entry in the matrix has the connection status either

ABLE if there is a potential subjective contour connection, or DISABLE if there is no subjective contour connection. TEMP-DISABLE is used to temporary disable the subjective contour connection when working on the current contour chain. When the current contour chain selects a subjective contour from endpoint a to b then all the ABLE connection status in the link table that connected from endpoint a to the other endpoints except to b must change the connection status to TEMP-DISABLE. Similarly, change the connection status in the link table from endpoint b to the other endpoints except a.

Each subjective contour organization uses different set of real contours, and we use the endpoint table to keep track of each endpoint status—CURRENT, AVAILABLE, ORGANIZED, ALTERNATIVE, and UNAVAILABLE. The AVAILABLE status means that the endpoint is available to use in the current contour chain, and the UNAVAILABLE status means that the endpoint is not available for the contour organization, and the ALTERNATIVE status means that the endpoint is not available for the current organization but can be used in the other organization. The CURRENT status means that the endpoint is used in the current contour chain, and the ORGANIZED status gives organization number of the previously found contour chain.

4.3.2 Finding a Contour Chain

To find a contour chain, first we have to find a starting point on the contour chain. Next, we follow the contour chain in one direction until it reaches back to the starting point or the end of the contour chain. If the contour chain ends at one direction, we have to find the contour chain at the other direction from the starting point, too. We follow the contour chain in the other direction until it reaches the end of the contour chain. As we are following the current contour chain, the endpoint table and the link table are updated. At the end a contour chain is found with its status. The status of a contour chain is either VALID, DISCARD, or ALT-CHAIN. We keep the VALID contour chain in the current organization as a result.

The starting point for a contour chain is chosen among the AVAILABLE blob endpoints from the endpoint table. If there are some endpoints of SUPPORT type supporting edge candidates, then among them the endpoint with the best subjective contour connection become the starting point. If there is no endpoint of SUPPORT type supporting edge candidates but there are some endpoints of NON-SUPPORT type supporting edge candidates, then among them the endpoint with the best subjective contour connection become the starting point. Similarly, for there is no endpoint of NON-SUPPORT type supporting edge candidates but there are some endpoints of UNDECIDED type supporting edges. The better subjective contour connection means the LINEAR subjective contour is a stronger subjective contour than the CURVED subjective contour and the STRAIGHT subjective contour. Among the better subjective contours of the same subjective contour type, choose the one with the smallest weight as the best subjective contour.

After a starting point is selected, we can follow the contour chain in either direction from the starting point. Depending on the direction, first the endpoint connects to a real or a subjective contour. When the endpoint connects to the real contour, its otherpoint follows. If the endpoint connects to the subjective contour across the gap between the two figures then the endpoint of another figure follows. In the case of the endpoint connects to the subjective contour that completes the blob outline, the otherpoint on the blob supporting edge follows.

Each contour segment—real or subjective contour—selected in the current contour chain that connects one endpoint to another is recorded in the segment record. As we follow the current contour chain and find the segment, the segment record is created and inserted in the appropriate position on the linked list of segments that describes the current contour chain. The position to insert a segment record depends on the direction of investigation along the contour chain, i.e., if the current contour chain is examined in the initial direction, then insert the segment record at the end of the linked list, and if it is examined in the other direction, then insert the segment record at the beginning of the linked list.

There is only one subjective contour originating from an endpoint if the endpoint has a subjective contour connection. The best subjective contour is chosen among all the potential subjective contours originating from an endpoint, and is chosen in similar way as choosing the starting point for a contour chain.

When a real segment is added in the current contour chain, the two endpoints of the segment must change their status to CURRENT in the endpoint table. When a subjective contour segment is added in the current contour chain, the two otherpoints of the endpoints which are connected to the subjective contour must change their status to ALTERNATIVE in the endpoint table. If the other endpoint of the connecting subjective contour or the connecting supporting edge has the UNAVAILABLE endpoint status, then set the current contour chain status to DISCARD and stop searching for the current contour chain.

The status of a current contour chain can be found while the contour chain continues to search its segments and when the contour chain reaches its end. Initially the current contour status is set to VALID. The VALID current contour status can be overwritten when the current contour chain status changes to ALT-CHAIN or DISCARD. The simplest end of contour that gives the VALID current contour chain status occurs when the subjective contour connects to the starting point, to the tip, or the supporting edge that has no subjective contour connection. Refer to Table 3.1 on page 37 for the current contour chain status where the *status of current contour chain* entry in the table *continue* means the VALID contour chain status, *conflict* means the DISCARD contour chain status, and *alternative contour chain* means the ALT-CHAIN contour status. In the table, x is an endpoint of the expanding part of the current contour chain and x' is the samepoint of x. In the x' *belongs to* entry in the table, *no organization* means x' endpoint status is AVAILABLE, *current contour chain* means x' endpoint status is CURRENT, and *other contour chain of same organization* and *other contour chain of different organization* mean x' endpoint status is ORGANIZED.

4.3.3 Grouping of Contour Chains

The grouping of contour chains results in one contour organization, and there can be many contour organizations in one image. Each organization has organization number assigned to it. For each contour organization do the following to group contour chains:

1. While there is a starting point repeat this step.

   1.1 Find a contour chain and its status.

1.2 Check the status of the contour chain and update tables.

1.2.1 If the status is VALID, add the contour chain to the current organization.

1.2.2 If the status is ALT-CHAIN, the contour chain is an alternative contour chain.

1.2.3 If the status is DISCARD, discard the contour chain.

2. Update the endpoint table to prepare for the next contour organization.

To find out whether there is a starting point or not, we have to search through the endpoint table to see if there is any AVAILABLE endpoints from the blob outline. The previous section, Section 4.3.2, describes how to find a contour chain and its status. After a contour chain and it status is found, process the contour chain according to the status and update the endpoint table and the link table. If the contour chain status is VALID, then add the contour chain to the current organization; otherwise, do not add the contour chain to the current organization. Refer to Table 4.1 on this page to update the endpoint table and the link table. Note that we have to change the link table back in the state before selecting the contour chain when the contour chain status is not VALID; however, the contour chain status reflects on the endpoints along the contour chain in the endpoint table. When all the contour chains in one organization are found then the ALTERNATIVE endpoints in the endpoint table must change their endpoint status back to AVAILABLE, so that we can use the endpoints in the next contour organization.

contour chain status	update CURRENT in endpoint table to	update TEMP-DISABLE in link table to
VALID	ORGANIZED	DISABLE
ALT-CHAIN	ALTERNATIVE	ABLE
DISCARD	UNAVAILABLE	ABLE

Table 4.1: Decision of Endpoint Table and Link Table Updates

Chapter 5

Results and Discussion

The subjective contour selection and organization algorithms were implemented in the C programming language on a SPARCstation 2 running under the UNIX operating system. The algorithms were tested on a large variety of subjective contour images such as subjective contours with a white subjective surface, subjective contours with patches on a subjective surface, subjective contours induced by bars, and subjective contours with overlaid subjective surfaces. Note that most of the images tested have been depicted in previous research on subjective contours. See Appendix A for sources of figures.

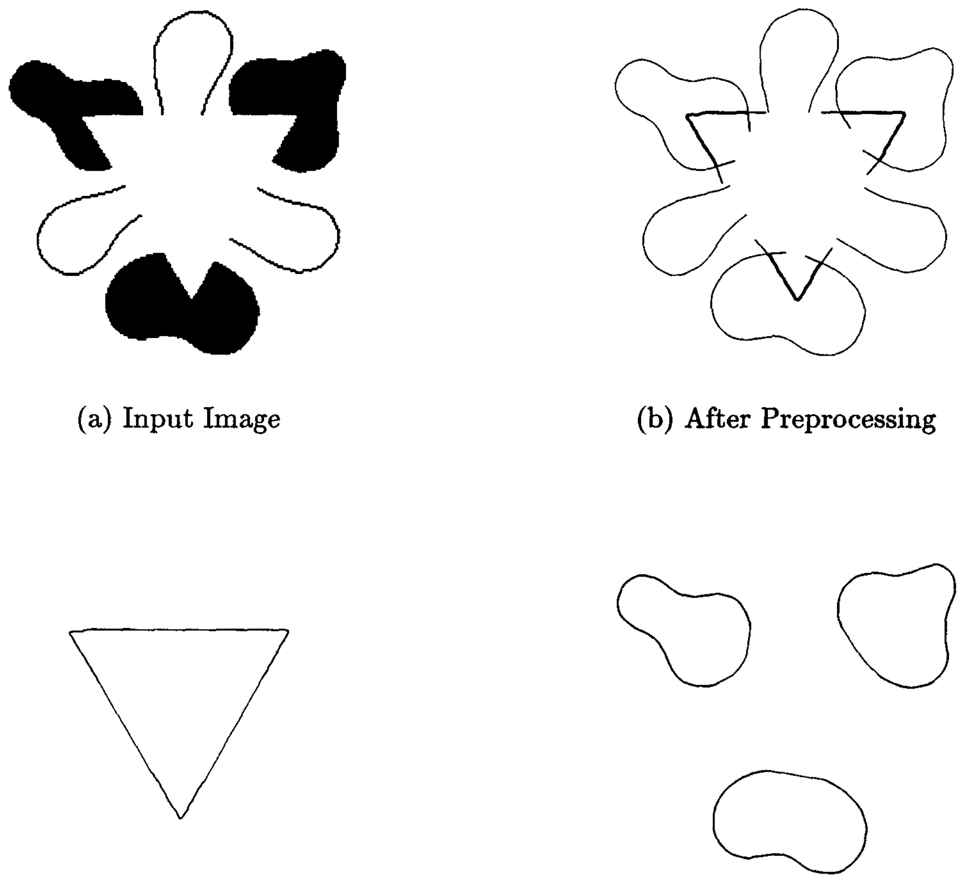
In this chapter, the input images, the intermediate processing results, and the final results are shown in the figures. Each set of figures has three or more figures depending on the number of resulting subjective contour organizations; part (a) is the original image, part (b) shows the result of preprocessing, part (c),(d) and (e),(f) present the subjective contour organizations. The original image is a scanned or hand drawn binary image. The preprocessing of the input image results in a curve partition and a ranking of the edges. The thick blob outline indicates that it is a SUPPORT type supporting edge candidate. The thin blob outline is a NON-SUPPORT type supporting edge candidate if the blob has some SUPPORT type supporting edge candidates, and a UNDECIDED type supporting edge candidate if the blob has no SUPPORT type supporting edge candidate. A segment of ten pixels length tangent to each endpoint of the blob outline segment is extended where there is a possibility of contour continuation beyond the real edge. The length of the extended contour is reflected in the scale of the image, i.e., the longer the extension, the larger the scale of the image. The weight of each potential subjective contour is calculated using Equation 4.3 on page 48 except for Figure 5.21 which uses Equation 4.4 on page 48.

The subjective contour organization results are shown where a thin line is a subjective contour and a thick line is a real contour. l is the maximum gap size, and σ is the standard deviation for Gaussian smoothing. The first subjective contour organization found by the system is usually the dominant contour organization because each contour chain chooses the available strongest potential subjective contour as the starting point. The next subjective contour organization found by the system is usually the alternative contour organization that uses the real and subjective contour that did not use in the previous organization.

5.1 Subjective Contours with a White Subjective Surface

This section examines the typical and the simplest subjective contour examples: subjective contours with a white subjective surface in the middle of the subjective contour image. The resulting subjective contours can be straight (Figure 5.1 and Figure 5.2) or slightly curved (Figure 5.3 to Figure 5.5) and each contour chain is closed. The maximum gap size is set longer than the distance between the endpoints of the two adjacent SUPPORT type supporting edge candidates across the gap; therefore, the blob supporting edge connects to another blob supporting edge and the line ends in between the gap are ignored. The ends of lines or dots become useful if they can help to shape the subjective contours (see Figure 5.7(d)).

Figure 5.1, Figure 5.2, Figure 5.4, and Figure 5.5 contain blobs with concave corners, and the blobs can be regular or irregular shapes (e.g., Figure 5.1(a)). In the alternative contour organization shown in part (d) of Figure 5.1, Figure 5.4, and Figure 5.5, the blobs are connected to themselves. Outline of all the blobs in Figure 5.2 and some blobs in Figure 5.3 could not connect to themselves because the extension of both ends of a NON-SUPPORT type supporting edge candidate never intersect. The blobs in Figure 5.3 could not connect to themselves to complete the outlines, even though some of them appear that the extension of both ends of



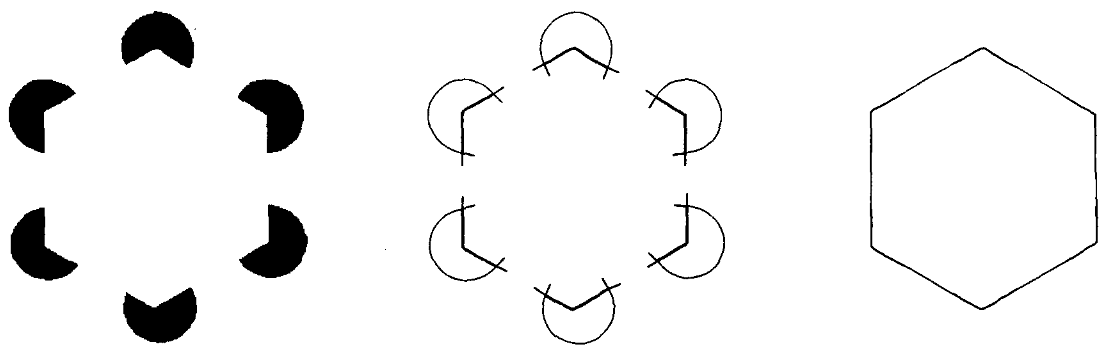
(a) Input Image

(b) After Preprocessing

(c) One contour chain ($l = 70$)

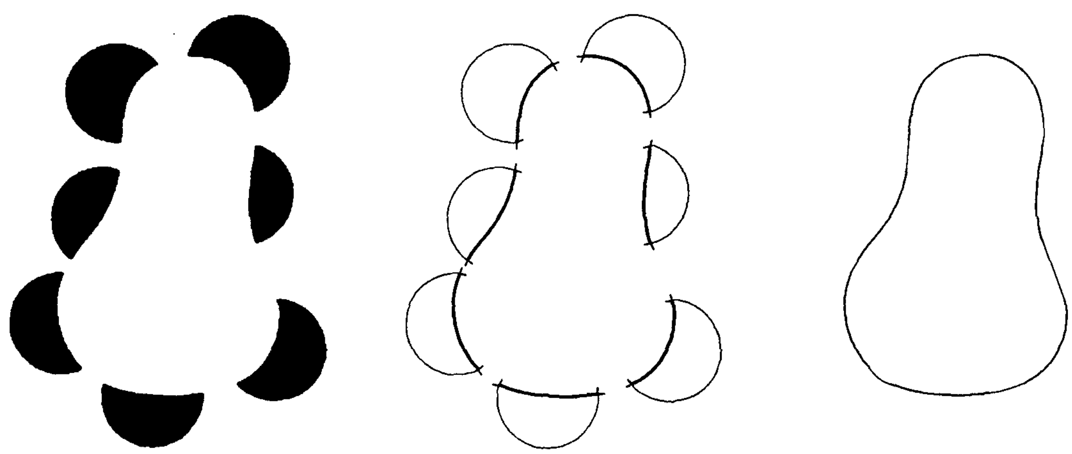
(d) Three contour chains ($l = 70$)

Figure 5.1: Test Results of Subjective Contours with a White Subjective Surface (1)



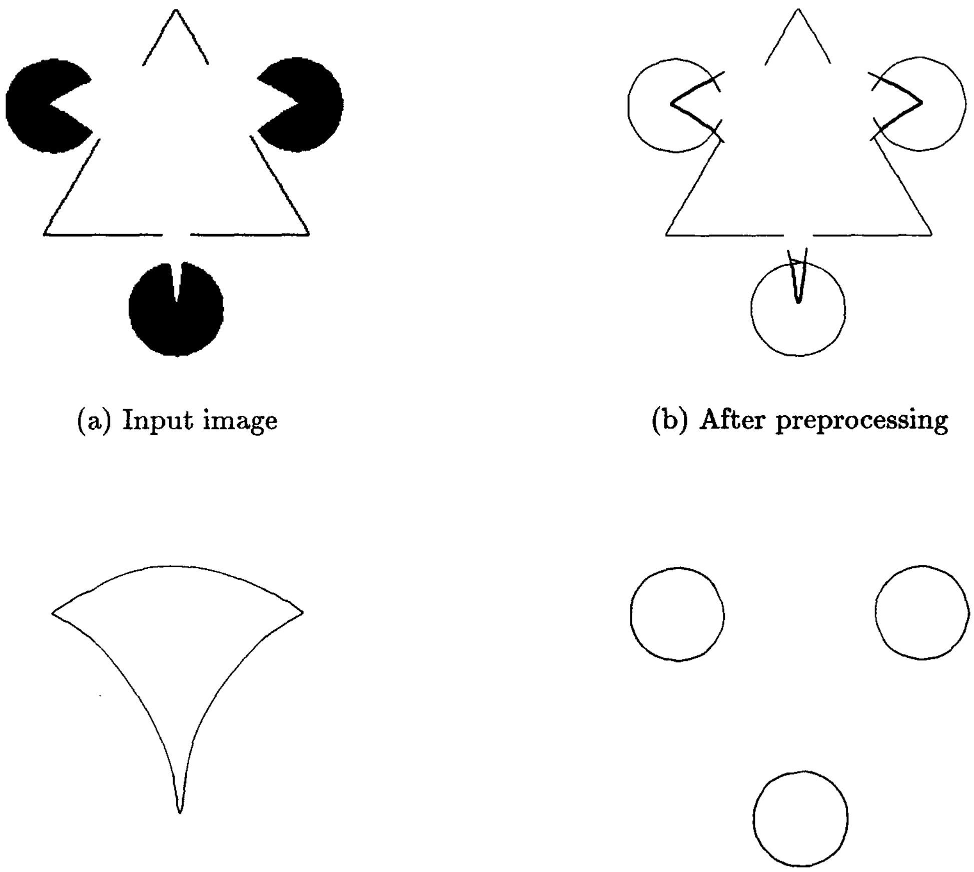
(a) Input image (b) After preprocessing (c) One contour chain ($l = 50$)

Figure 5.2: Test Results of Subjective Contours with a White Subjective Surface (2)



(a) Input image (b) After preprocessing (c) One contour chain ($l = 110$)

Figure 5.3: Test Results of Subjective Contours with a White Subjective Surface (3)



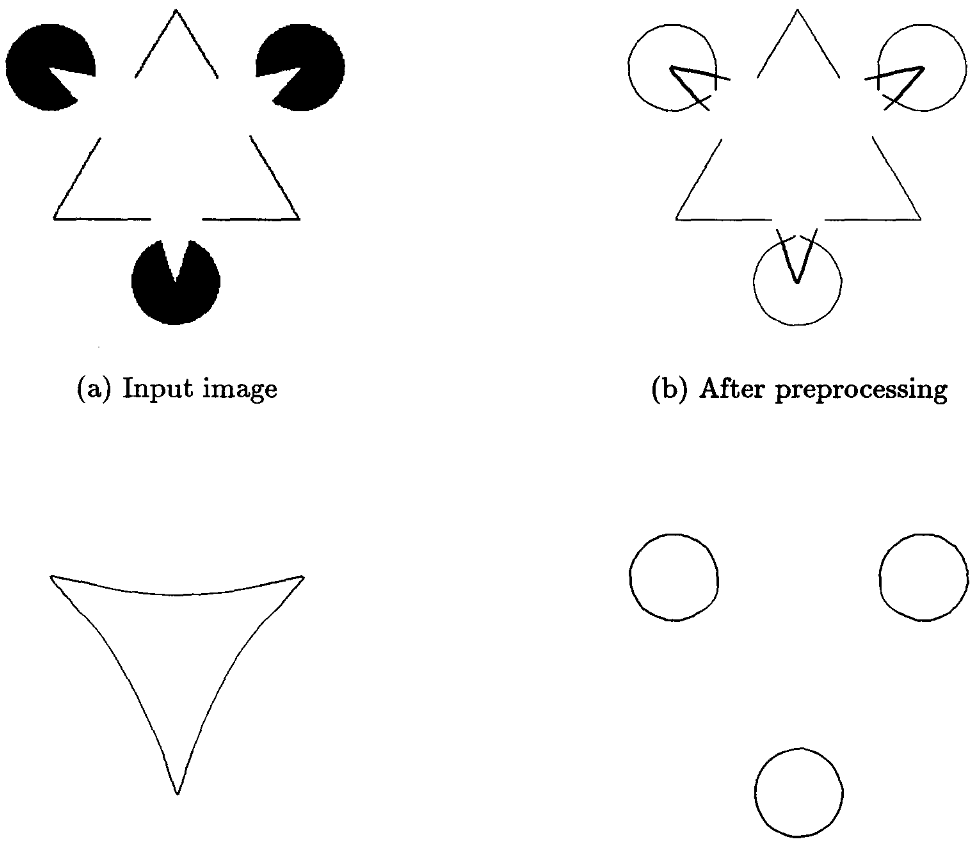
(a) Input image

(b) After preprocessing

(c) One contour chain ($l = 120$)

(d) Three contour chains ($l = 120$)

Figure 5.4: Test Results of Subjective Contours with a White Subjective Surface (4)



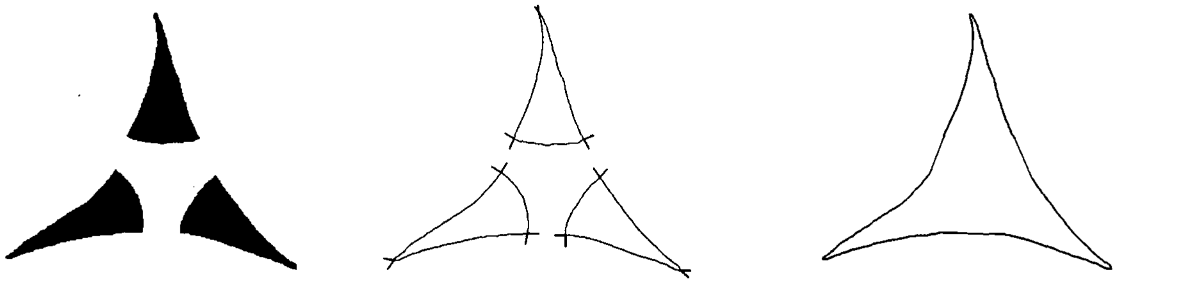
(a) Input image

(b) After preprocessing

(c) One contour chain ($l = 100$)

(d) Three contour chains ($l = 100$)

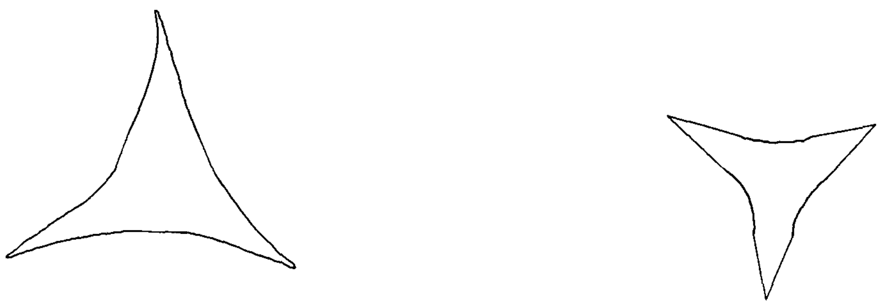
Figure 5.5: Test Results of Subjective Contours with a White Subjective Surface (5)



(a) Input image (b) After preprocessing (c) Three contour chains ($l = 60$)

Figure 5.6: Test Results of Subjective Contours with a White Subjective Surface (6)

(a) Input image (b) After preprocessing



(c) Three contour chains ($l = 80$) (d) Three contour chains($l = 80$)

Figure 5.7: Test Results of Subjective Contours with a White Subjective Surface (7)

each blob outline could connect smoothly, because of the maximum gap size being set narrower than the gap between the two end points of each supporting edge.

Figure 5.3(c) shows curved subjective contours without corners. Figure 5.4(c) and Figure 5.5(c) shows curved subjective contours with corners in the blobs. All of the curved subjective contours are continuous in accordance with the contour continuity of their supporting edges.

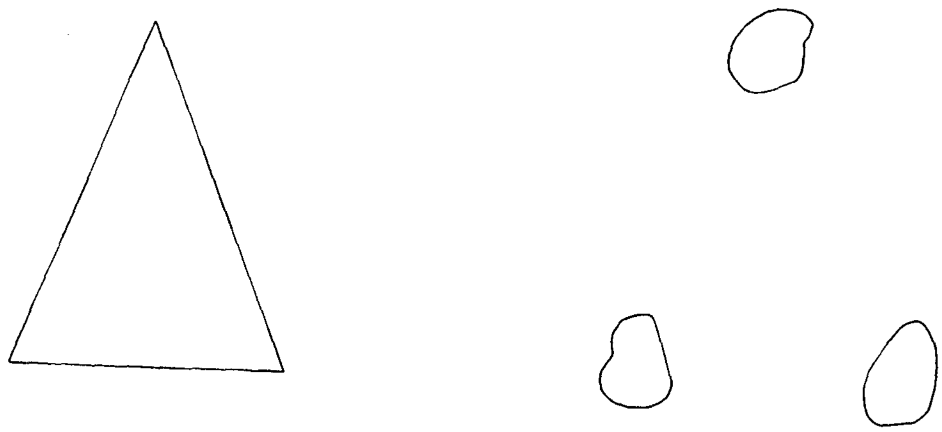
Figure 5.6(a) shows a figure which can be interpreted as a concave curved triangle with a missing middle surface. The shape of the middle white surface cannot be recovered because what would be the supporting edges for this subjective surface would not connect to each other due to the orientations of the extension to the supporting edges that could not be smoothly connected. The only possible contour organization is to connect the missing sides of the concave curved triangle as in Figure 5.6(c). If we add three dots to guide the corner locations for the middle white surface as in Figure 5.7(a), the contour organization for the subjective surface is possible as shown in Figure 5.7(d) providing that the maximum gap size is long enough to cover the length from the endpoint of each supporting edge to the connecting dot.

5.2 Subjective Contours with Patches on a Subjective Surface

Subjective contour images with patches on the subjective surfaces are examined in this section. The resulting dominant contour organization has straight subjective contour with inducing elements on both side of the contour chain, has corner on the blobs, and has a closed contour chain. The preprocessing eliminates the blobs with one continuous contour (compare part (a) and part (b) of Figure 5.8 and Figure 5.10) because such blobs cannot originate subjective contours.

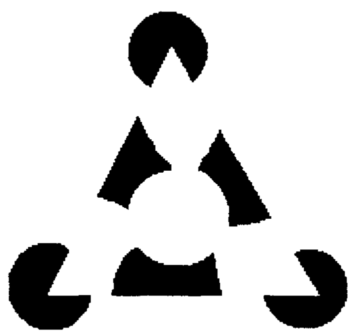
Figure 5.8(c) shows the dominant contour organization of a triangle shape with its corners on the blobs. It is seen as a white triangle surface with some black patches on it. Figure 5.9(c) shows the dominant contour organization of a circle inside a triangle contour chain. The circle contour chain uses part of the triangle's patches as supporting edges. The alternative contour

(a) Input image (b) After preprocessing

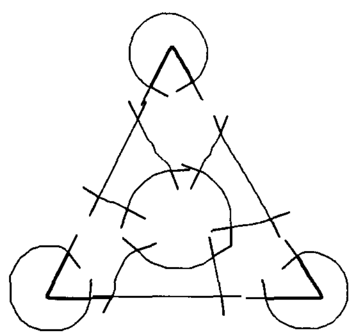


(c) One contour chain ($l = 260$) (d) Three contour chains ($l = 260$)

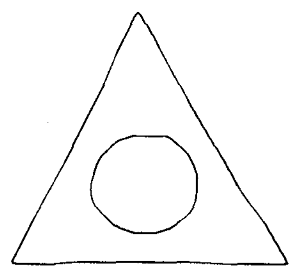
Figure 5.8: Test Results of Subjective Contours with Patches on a Subjective Surface (1)



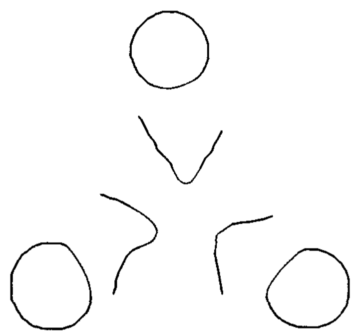
(a) Input image



(b) After preprocessing

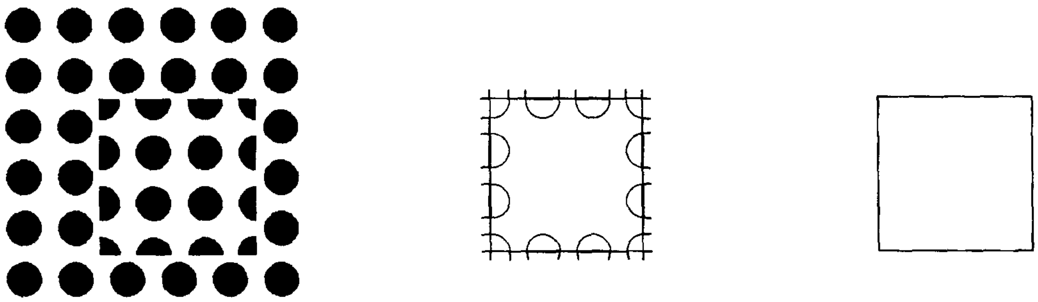


(c) Two contour chains ($l = 50$)



(d) Six contour chains ($l = 50$)

Figure 5.9: Test Results of Subjective Contours with Patches on a Subjective Surface (2)



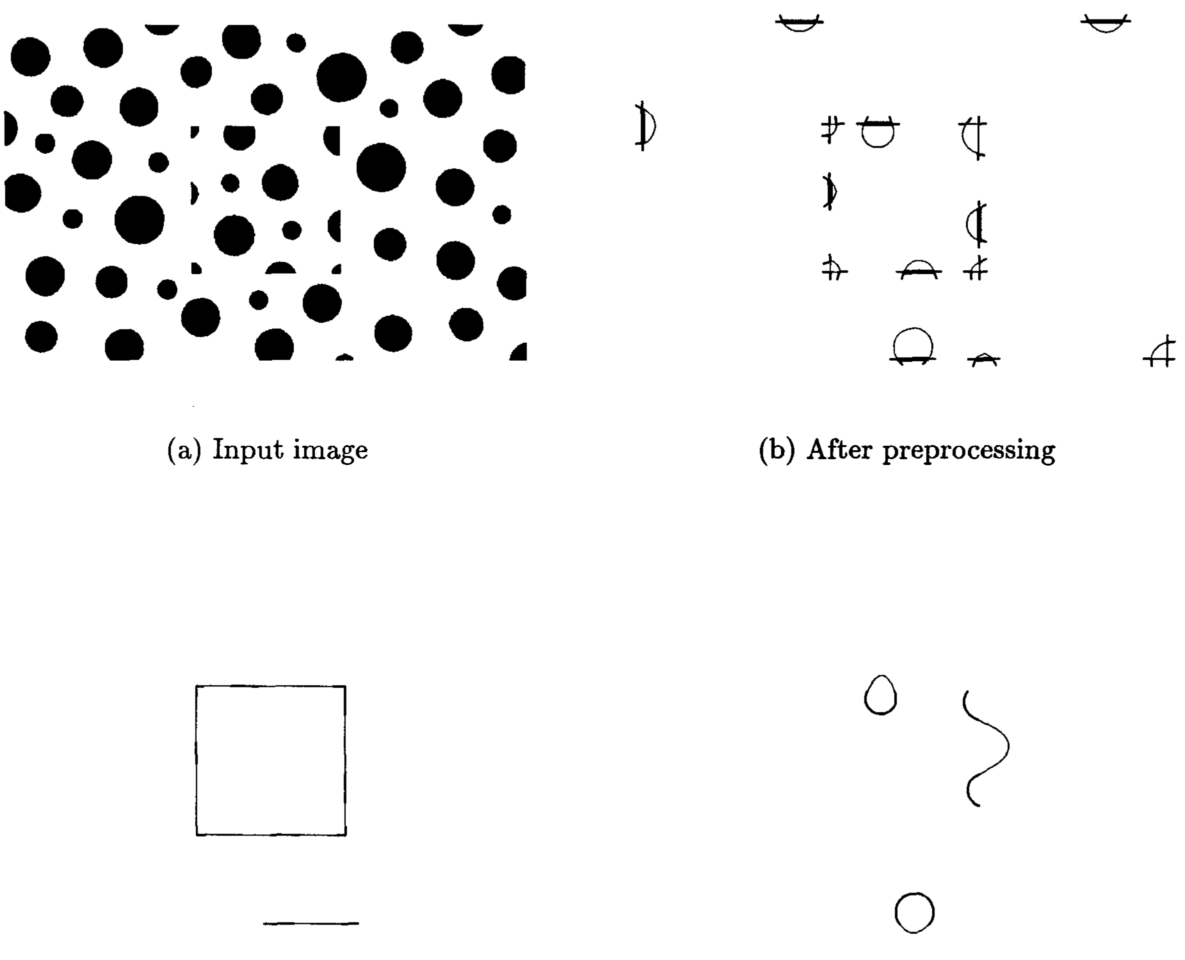
(a) Input image (b) After preprocessing (c) Four contour chains ($l = 22$)

Figure 5.10: Test Results of Subjective Contours with Patches on a Subjective Surface (3)

organization, as in Figure 5.9(d), uses the edges that were not chosen in the dominant contour organization, and as a result six contour chains are found; three closed contour chains that complete the blob outlines, and three contour chains that are connected across the concave corner of each blob satisfying the curvature continuity measure set with the real edges.

Figure 5.10(a) has regular figural configuration, and a square subjective surface with regular circle patches in the middle of the image is perceived. The square subjective surface can be seen as located on the top image plane, or alternatively as located behind the image plane seeing through a square hole. The impression of the subjective surface depth can be shifted at will because it is ambiguous. However, the location of a contour separating the square and the rest of the figure will not change. The four corners of the subjective square are defined by the convex corner of the blobs. These blobs are seen as if the circle blobs are cut into the corner shape or the circle blobs are covered by white surfaces with concave corners. Four contour chains are found corresponding to the four sides of a square because each convex corner consists of two supporting edges (see Figure 5.10(c)). No alternative contour organization is found because large areas on the blobs on the border of the subjective contours are occluded that the system couldn't recover any blobs.

Figure 5.11(a) has similar figural configuration as Figure 5.10(a) except the circle patchs



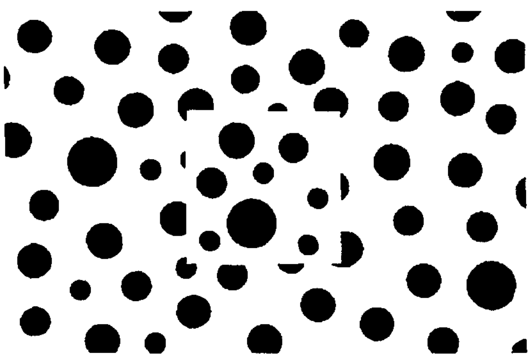
(a) Input image

(b) After preprocessing

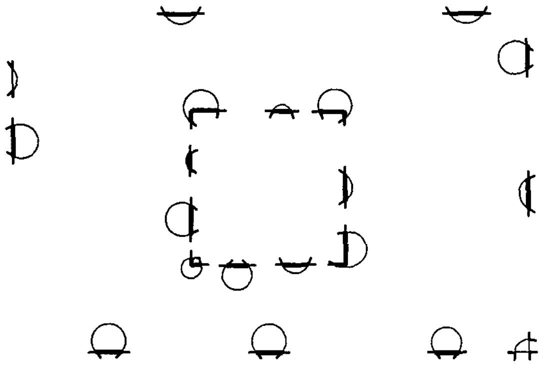
(c) Five contour chains ($l = 100$)

(d) Three contour chains ($l = 100$)

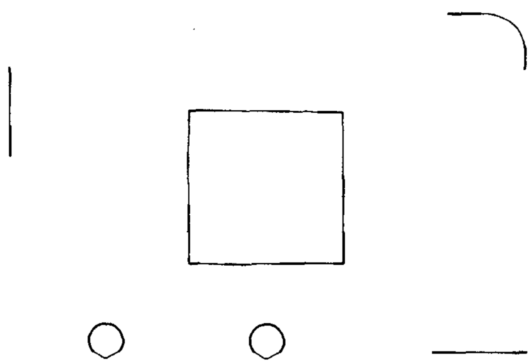
Figure 5.11: Test Results of Subjective Contours with Patches on a Subjective Surface (4)



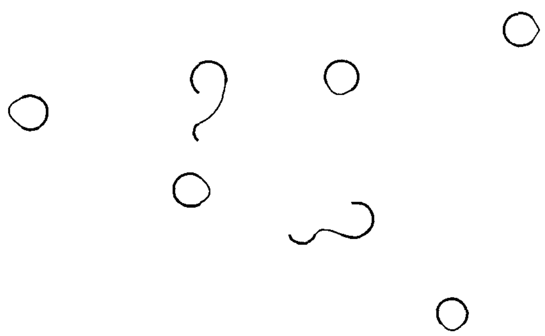
(a) Input image



(b) After preprocessing



(c) Six contour chains ($l = 100$)



(d) Seven contour chains ($l = 100$)

Figure 5.12: Test Results of Subjective Contours with Patches on a Subjective Surface (5)

in various sizes. The square subjective surface gives the impression that it is located behind the image plane seeing through a square hole. Five contour chains are found in the dominant contour organization with the given maximum gap size (see Figure 5.11(c)), and some more contour chains are found in the alternative contour organization where the shape of some circle patches at the borders are recovered (see Figure 5.11(d)).

Figure 5.12(a) has similar figural configuration as Figure 5.11(a) except the four corners of subjective square are defined by the concave corners on the blobs, and these blobs are seen as if there were the white square corners occluding the round blobs. The square subjective surface has some round patchs, and it is perceived as located on the top of the image plane. Six contour chains are found in the dominant contour organization with the given maximum gap size (see Figure 5.12(c)). The middle square subjective contour is our focuses, and the rest of the contour chains are organized around the image border. The middle square is described by one contour chain because the two segments on each concave corner on the blob are grouped into one supporting edge and the subjective contour continues through the supporting edges. Some more contour chains are found in the alternative contour organization (see Figure 5.12(d)). However, we would not normally perceive the contour chains that each subjective contour connecting to the two adjacent blob outlines.

5.3 Subjective Contours Induced by Bars

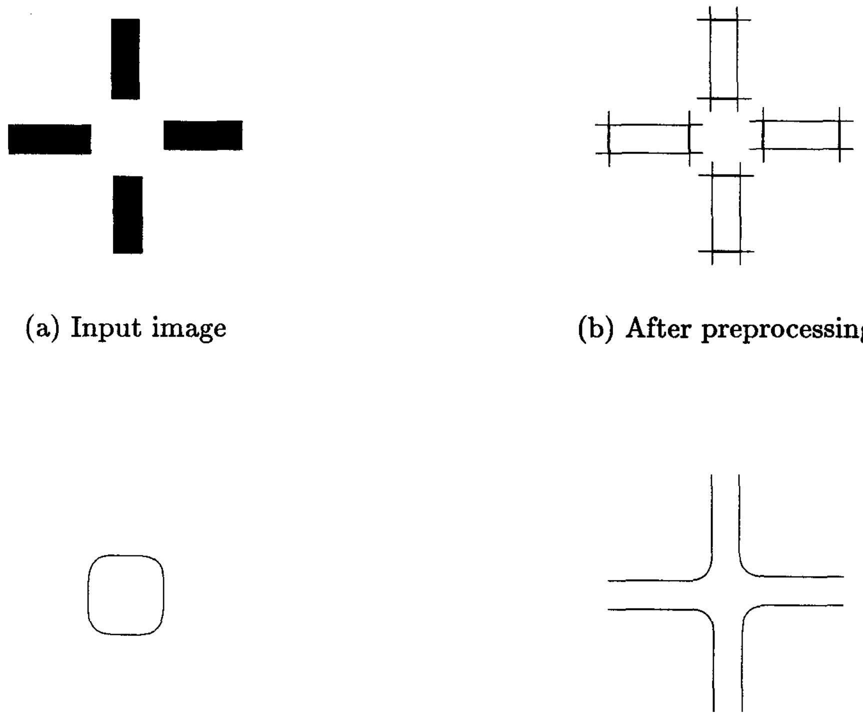
Subjective contours emerging from bars are tested in this section. The strength of bar sides determine whether the subjective contour organization is dominant or not. The formation of cornered subjective contour is suggested. Also, similarity of bar and line is discussed.

Figure 5.13(a) on page 69, the basic subjective contour figure used in this section, has four bars arranged in a cross shape with an opening in the middle. The two shorter sides of each bar are ranked as the SUPPORT type supporting edge candidates because they suggest the sudden termination of a bar. The dominant contour organization is found in the middle of the image as shown in Figure 5.13(c), and the alternative contour organization using the longer sides of bars as supporting edges is shown in Figure 5.13(d). Figure 5.14(a) arranged horizontal and vertical bars not aligned with the square shape opening in the middle, and it also produces the reasonable subjective contour organizations (see part (c) and part (d) of Figure 5.14).

To demonstrate that the shape and strength of subjective contour depends on inducing element configuration, each bar in Figure 5.13(a) is thickened to make the gaps between adjacent bars close to the center of the image narrower as in Figure 5.15(a). The central white area is seen as a square with round corners, and the corners appear sharper when the gap size is smaller; compare Figure 5.13(c) and Figure 5.15(c). The reason is that the Bezier curve with the bigger curvature will fit into the smaller gap size providing that the subjective contour is curvature continuous.

In general, when the object is in a distance, then the smaller σ for Gaussian smoothing of the object outline is sufficient compared to the same object, because of the farther object looks smaller and less detailed. In the far distance, the increase in the change of curvature on the subjective contour is compensated for the smaller σ, in order for all the points on the curve to have the change of curvature below the maximum change of curvature. Refer to Equation 4.1 on page 42 for the curvature continuity measure. To achieve the maximum change of curvature allowed in the far distance object, the smaller σ on the Gaussian smoothing of the object outline is compensated by the larger change of curvature on both the subjective contours and the supporting edges. The large change of curvature on a subjective contour means that the subjective contour can have larger curvature than the subjective contour in the near distance; therefore, the subjective contour with sharper corner could be seen. Moreover, the subjective contour appears stronger when the supporting edges are longer and each gap in between the two supporting edges is narrower. It is possible to see a cornered subjective contour in Figure 5.15(a) because the effect of extension of the supporting edges is very strong.

Figure 5.16(a) arranges bars in the way that some bar supports two chains in one organization. A line can be considered as a special type of a bar when the bar is very narrow. The subjective contour organization of lines will be similar to the dominant contour organization using the shorter sides of the bars as supporting edges (see Figure 5.16(c)). However, the shape



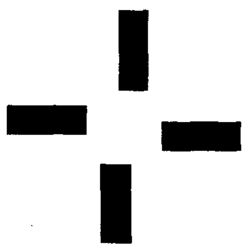
(a) Input image

(b) After preprocessing

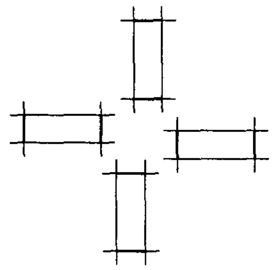
(c) One contour chain ($l = 30$)

(d) Four contour chains ($l = 30$)

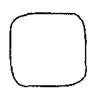
Figure 5.13: Test Results of Subjective Contours Induced by Bars (1)



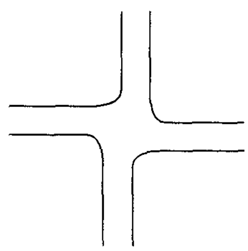
(a) Input image



(b) After preprocessing

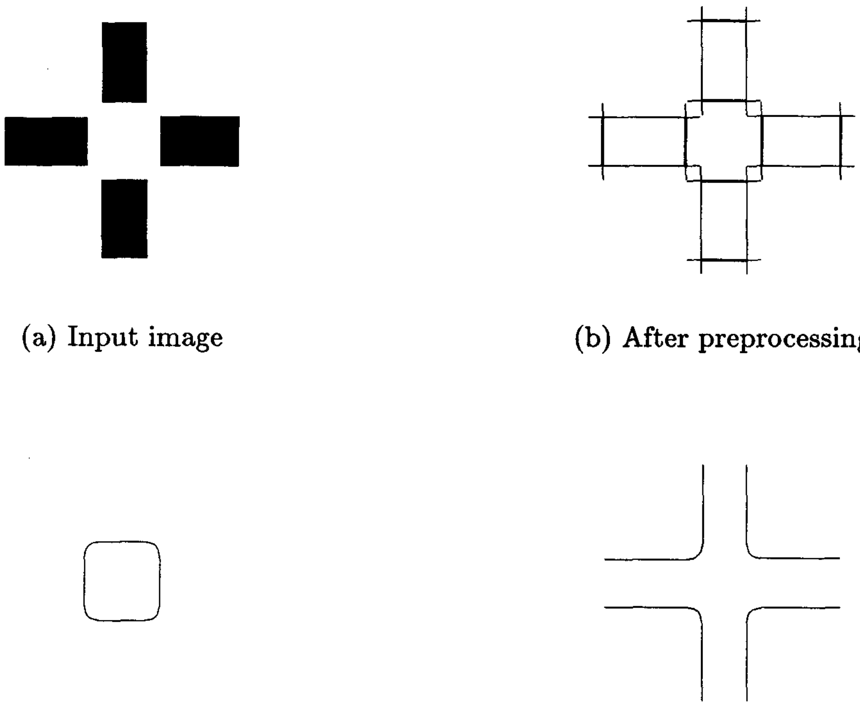


(c) One contour chain ($l = 30$)



(d) Four contour chains ($l = 30$)

Figure 5.14: Test Results of Subjective Contours Induced by Bars (2)



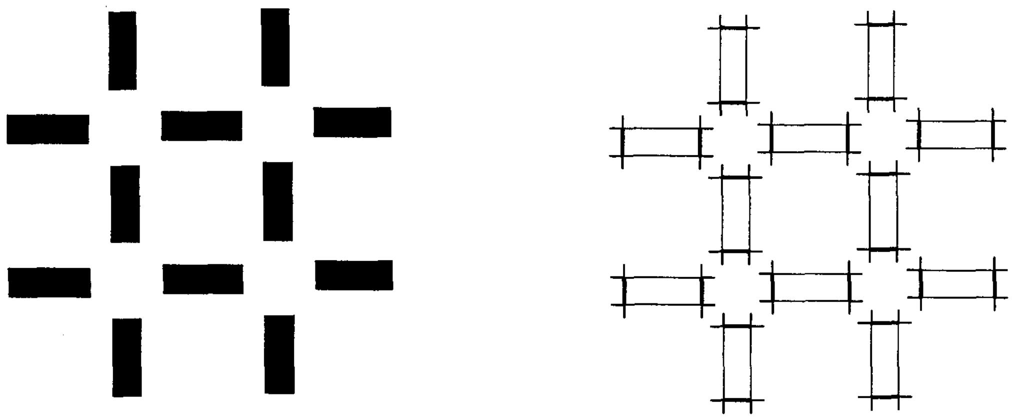
(a) Input image

(b) After preprocessing

(c) One contour chain ($l = 30$)

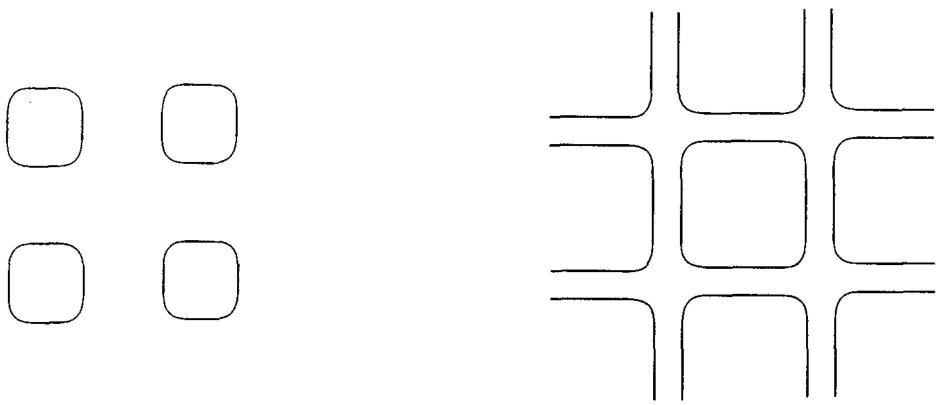
(d) Four contour chains ($l = 30$)

Figure 5.15: Test Results of Subjective Contours Induced by Bars (3)



(a) Input image

(b) After preprocessing



(c) Four contour chains ($l = 30$)

(d) Nine contour chains ($l = 30$)

Figure 5.16: Test Results of Subjective Contours Induced by Bars (4)

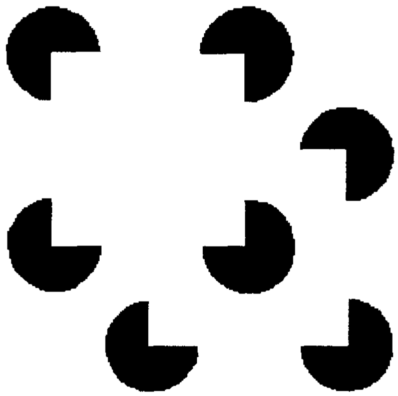
of subjective contour formed by the line ends is not easy to predict because the line end has no length; in fact, it is one pixel wide. Therefore, there is no suggestion about orientation of subjective contour at the line end. Also, unlike a line, the long sides of a bar can become supporting edges for an alternative contour organization (see Figure 5.16(d)).

5.4 Subjective Contours with Overlaid Subjective Surfaces

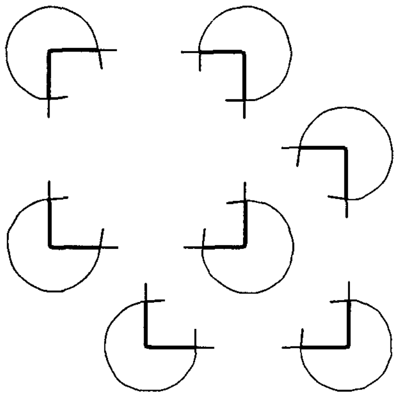
In this section, the more complicated subjective contours are examined. The overlaid subjective contour images (see Figure 5.17(a) to Figure 5.20(a)) show different depth level of the subjective surfaces in one contour organization. The reversible subjective contour image (see Figure 5.21(a)) alternates two or more different depth interpretation of the subjective contour organizations, and the subjective contour perception is unstable. The silhouette image (see Figure 5.22) is the image with reverse of the foreground and the background colour in order to recover the shapes of the original figures by subjective contour organizations. The different maximum gap size, which can decide the shape of subjective contours, are demonstrated in Figure 5.19, Figure 5.20, and Figure 5.21.

Figure 5.17 and Figure 5.18 have two dominant square subjective surfaces with the square located at top left corner superimposed on the square located on the bottom right corner and partially occluding circles in their corners. Their alternative organizations are circles. Looking at the result shown in Figure 5.17(c), the bottom subjective square does not touch the border of the top subjective square because there is no supporting edge for the bottom subjective square at the border of the top subjective square, and no effort has been made to extend the subjective contour until it touches another contour chain in this implementation. In contrast, Figure 5.18(c) shows the bottom subjective square continuing until it touches the top subjective square because there are supporting edges for the bottom subjective square which touch the top subjective square.

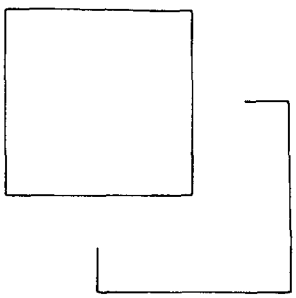
Figure 5.20 is similar to Figure 5.19 except that additional lines shape the subjective contours. Part (c) and part (d) in both figure uses the maximum gap size short enough to covert



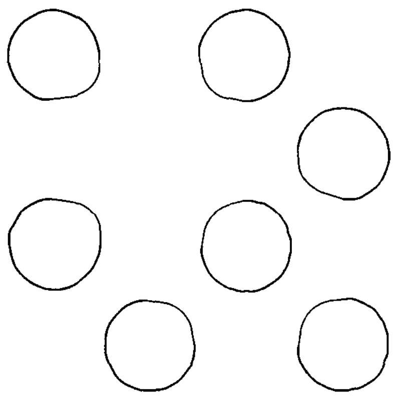
(a) Input image



(b) After preprocessing

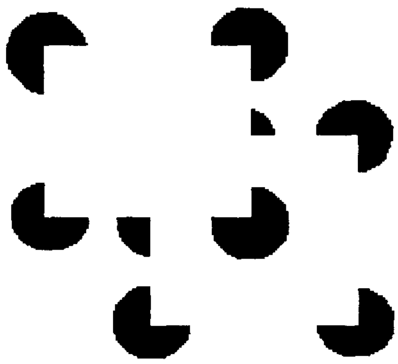


(c) Two contour chains ($l = 60$)

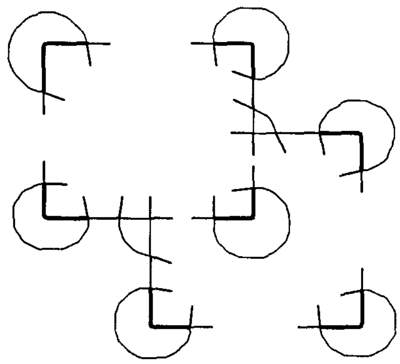


(d) Seven contour chains ($l = 60$)

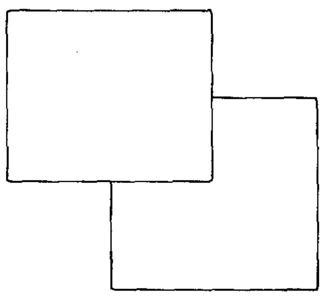
Figure 5.17: Test Results of Subjective Contours with Overlaid Subjective Surfaces (1)



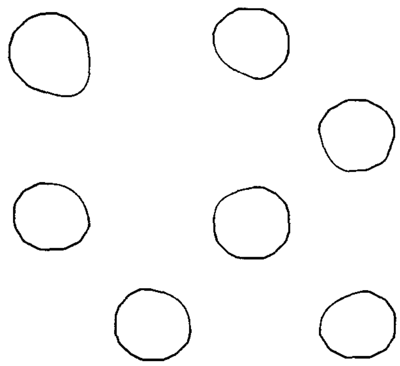
(a) Input image



(b) After preprocessing

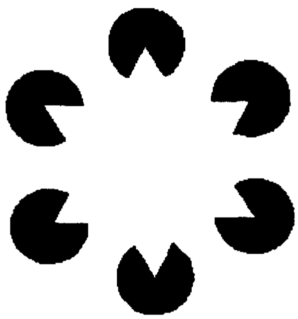


(c) Two contour chains ($l = 60$)

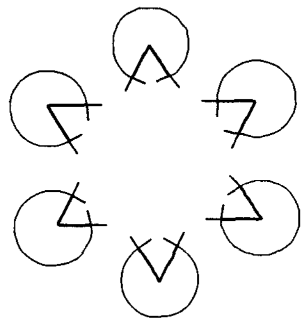


(d) Three contour chains ($l = 60$)

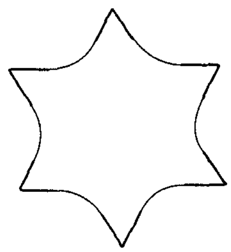
Figure 5.18: Test Results of Subjective Contours with Overlaid Subjective Surfaces (2)



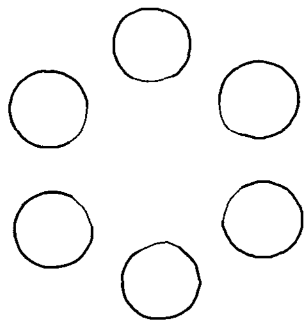
(a) Input image



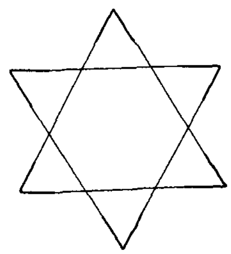
(b) After preprocessing



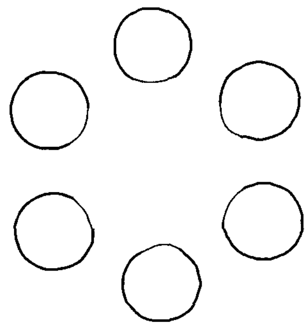
(c) One contour chain ($l = 40$)



(d) Six contour chains ($l = 40$)

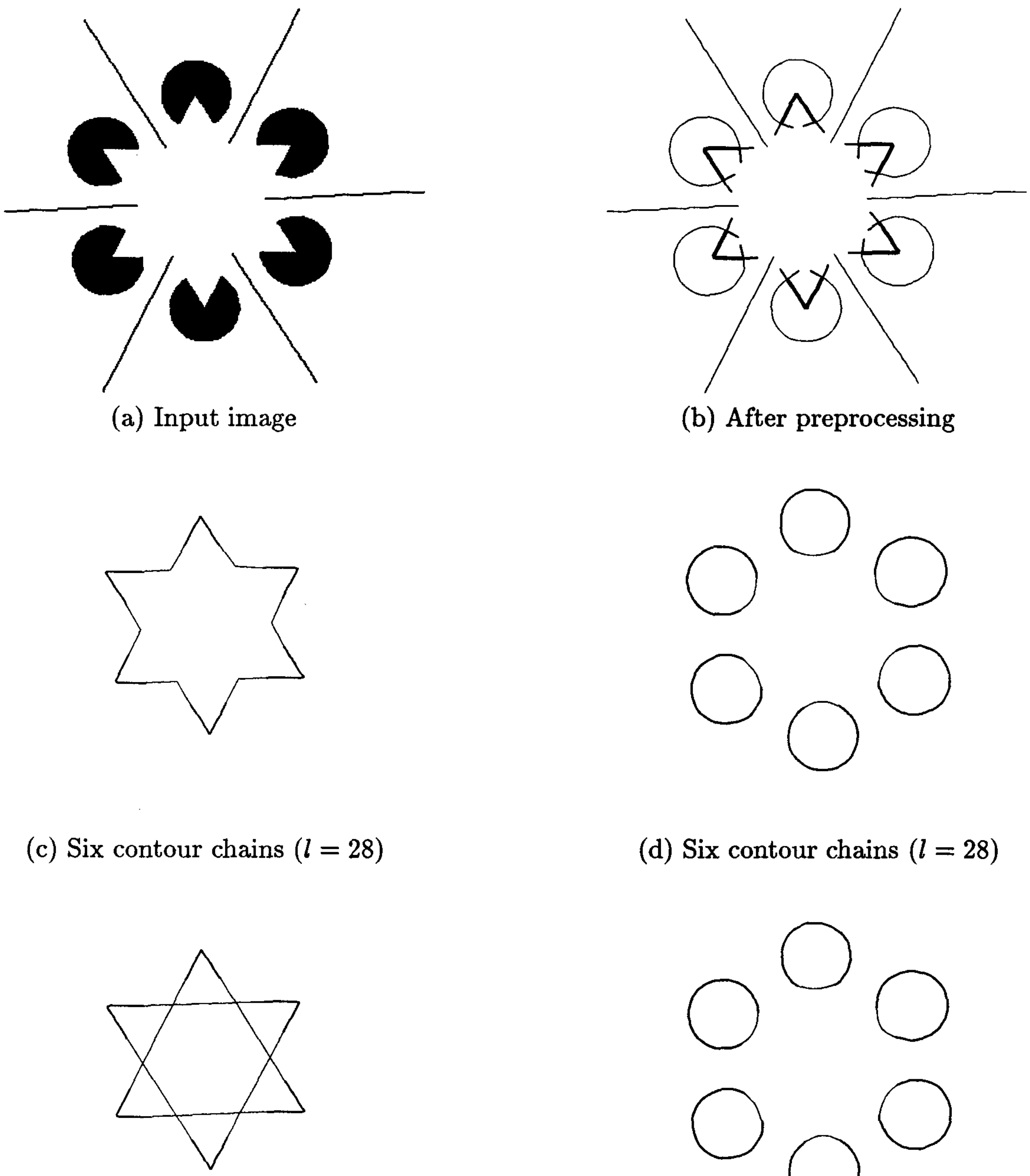


(e) Two contour chains ($l = 100$)



(f) Six contour chains ($l = 100$)

Figure 5.19: Test Results of Subjective Contours with Overlaid Subjective Surfaces (3)



(a) Input image

(b) After preprocessing

(c) Six contour chains ($l = 28$)

(d) Six contour chains ($l = 28$)

(e) Two contour chains ($l = 100$)

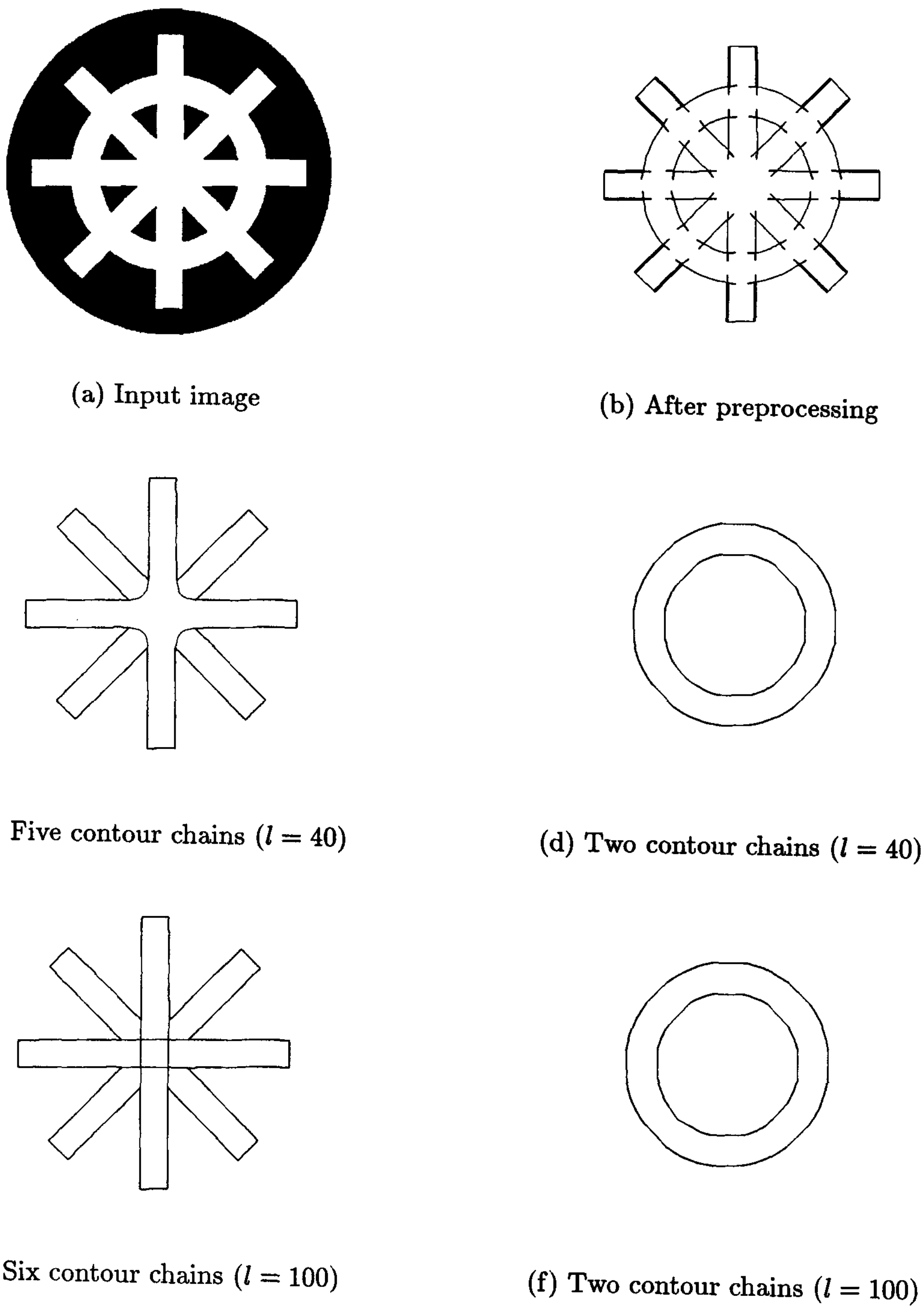
(f) Six contour chains ($l = 100$)

Figure 5.20: Test Results of Subjective Contours with Overlaid Subjective Surfaces (4)

the distance from one blob supporting edge to the adjacent blob supporting edge or line end. When the subjective contour is connected from one blob supporting edge to adjacent blob supporting edge as in Figure 5.19(c), the CURVE subjective contour is formed. The contour chain stops when it touches the line end and in Figure 5.20(c) shows corner at a line end as a result. When their maximum gap size is long enough to connect two aligned supporting edges across the gap and produce a LINEAR subjective contour as in part (e) of both figures, two triangle shape contour chains are found. Since the two contour chains intersect each other on subjective contours, the result shows two triangles in the same organization; those contour chains do not share the same supporting edges nor supporting edges from two contour chains are adjacent to each other. The depth order of these two subjective surfaces are ambiguous since either subjective surface can be on top of other subjective surface. The alternative organization (see part (f)) recovers the shape of the occluded blobs.

Figure 5.21(a) is an example of reversible subjective contour because the subjective contour image can be organized in many ways changing the arrangement of depth. The central white region is overlap with four bars if long maximum gap size being used, and two crosses one on top of others when shorter maximum gap size is used. Figure 5.21(c) is an example of using shorter maximum gap size when adjusting the weight to select the horizontal and vertical subjective contours first. The horizontal and vertical parts of a contour chain are connected by curved subjective contours near the center of the image. The cross in the bottom, oriented at 45°, resulted in four chains because of occlusion from the cross shape subjective surface on its top, and the supporting edges for the top cross are adjacent to the supporting edges for the bottom cross. The alternative contour organization is shown in Figure 5.21(d). There is a white subjective circle in the middle of the image touching the blob tips. This type of subjective contour is organized by the tip-based subjective contour, and it could not be found by the system because the system detects the edge-based subjective contour only.

The priority to select horizontal and vertical subjective contours is just one way of finding the subjective contour organizations. The other reversible subjective contour arrangement can reverse the depth of the two crosses in Figure 5.21(c) that a diagonal cross on the top of the



(a) Input image

(b) After preprocessing

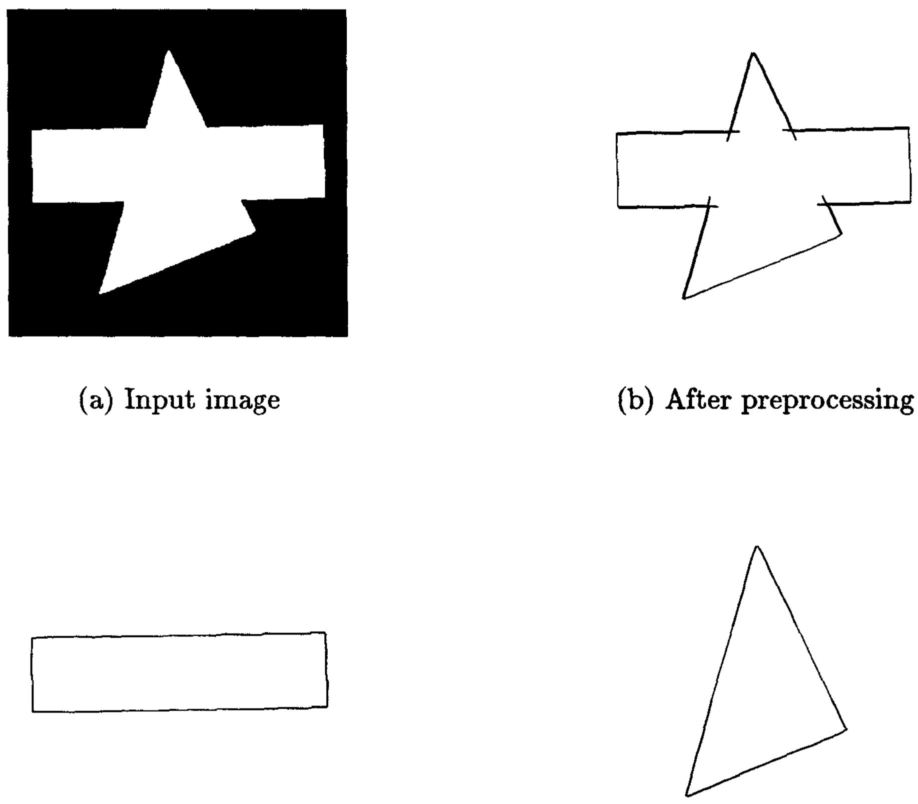
(c) Five contour chains ($l = 40$)

(d) Two contour chains ($l = 40$)

(e) Six contour chains ($l = 100$)

(f) Two contour chains ($l = 100$)

Figure 5.21: Test Results of Subjective Contours with Overlaid Subjective Surfaces (5)



(a) Input image

(b) After preprocessing

(c) One contour chain ($l = 150$)

(d) One contour chain ($l = 150$)

Figure 5.22: Test Results of Subjective Contours with Overlaid Subjective Surfaces (6)

horizontal and vertical cross. Each reversible subjective contour organization uses a some set of supporting edges but the subjective contour connections are arranged in the different way.

Figure 5.21(e) is an example of using longer maximum gap size when selecting the horizontal and vertical subjective contours first. Two contour chains in the bar shape are found first and its shows that they are crossing each other because the model does not try to label the depth when a subjective contour crosses another subjective contour. The other two contour chains that shape diagonal bars are discontinued under the horizontal and vertical subjective bars on the top. The alternative organization is shown in Figure 5.21(f). When using the long maximum gap size, the four bars can be on any depth order because this image produces reversible subjective contour figures.

Figure 5.22(a) shows the silhouette of two figures—a triangle and a rectangle—overlaid. There is no depth information specifying whether the triangle is in front of or behind the rectangle. This kind of image is an extreme case of overlaid objects where all objects have the same colour; consequently, we cannot distinguish each object from the colour information. The only clue to distinguish each object is the outline continuity. The subjective contours cannot be seen on the figure; however, we can see subjective contours on the background coloured area. Therefore, we reverse the figure and the background of such image to recover the overlaid objects. A silhouette is the shadow of such objects; hence, it simply put the objects on the background and make the background of the objects as foreground colour because the shadow area is suppose to be the foreground in the silhouette image. The white part of the image is ambiguous in depth whether the white triangle in front or the white rectangle in front. The system first selects a rectangle subjective contour organization because this contour chain has the shortest LINEAR type subjective contour. Note that each of the two adjacent corners on the rectangle are grouped as one supporting edge because the two segments on each concave corner is grouped together and one segment in the middle is shared by both corners. The alternative organization gives a triangle shaped subjective contour chain. In the silhouette image, there is no dominant contour organization and the subjective contour organization shifts from one to the other because the depth of the subjective surfaces are ambiguous.

Chapter 6

Conclusions

Until today, not many computer vision systems are capable of detecting subjective contours. Each system limits its input and the subjective contours it deals with, and some parameters must be entered to adjust the system. In this thesis, we present a model of subjective contour detection system based on the approach that finds boundary of subjective surfaces and performs subjective contour organization with less limitations on the input images and the subjective contours that can be detected. In particular, we use figural cues to find supporting edges and apply the perceptual organization to find subjective contours. This thesis presents steps involved in detecting subjective contours. Moreover, a new classification scheme of subjective contours is presented in Chapter 1. Based on the classification, the edge-based subjective contour is investigated.

A subjective detection model is presented based on four criteria: no prior knowledge is necessary to detect subjective contour; a subjective contour is a special type of occluding contours; the shape of a subjective contour is determined by the viewing condition; and it is possible to have multiple subjective contour organizations from one image. The model emphasizes contours rather than surface because perception of subjective contour is local phenomenon of surface perception. The main concern in this thesis is the overall organization of the subjective contours and our focuses is on which supporting edges to connect rather than the exact shape of a subjective contour. The rules for subjective contour organization are described and the model explains different types of subjective contour organizations.

The algorithms for local subjective contour selection and global contour organizations have been developed. The observer's viewing distance is translated into the maximum gap size allowed between the two supporting edges that are connected by a subjective contour. The consistent curvature continuity measure is used to find the contour discontinuities on the curve to segment the real and subjective contours. The factors affecting the perception of subjective contours are identified and incorporated in the algorithm.

The computer implementation of subjective contour detection is performed in three stages: the preprocessing, local subjective contour selection, and global contour organization. The preprocessing identifies figures, and the blob outlines are segmented according to their curvature discontinuity by Lowe's curve partition method. The local processing processes each supporting edge which selects potential subjective contours depending on the maximum gap size. The global processing chooses the subjective contours among the potential subjective contours and groups the supporting edges and the subjective contours into contour organizations.

The implementation of subjective contour detection system is limited to detecting the subjective contour on the black-and-white image which we can perceive subjective contours without preset knowledge of the object shapes, i.e., we use the subjective contour image with strong subjective contour effect. The straight subjective contours as well as the curved subjective contours can be found by the system. The shape of a curved subjective contour is approximated by the Bezier curve. The gap size between the two supporting edge endpoints and the orientation of the supporting edges with respect to the subjective contour are considered into the weight of the subjective contour. The smaller the gap size and the less curvature between the supporting edges, the stronger the subjective contour is.

Many subjective contour images have been tested on the subjective contour detection system. The straight as well as the curved subjective contours have been detected, and the dominant and alternative contour organizations are found in each image. In addition, we have demonstrated the effects of additional line ends that help to shape the subjective contours, and the results of different maximum gap size that could find different subjective contour organizations. The system sometimes gives the different interpretation of the contour organizations, especially the alternative contour organizations, then the perception of human observer due to the assumptions used in the system. In general, the system produces good results.

The subjective contour detection system is capable of selecting and grouping subjective contours based on the four subjective contour detection criteria. The system can find the subjective contour regardless of the inducing element orientations along the subjective contour; therefore, it gives the flexibility to find the subjective contour with patches on the subjective surface. Moreover, the inducing element outline, the supporting contour, and the subjective contour together form a T-junction indicating occlusion that is independent of the subjective surface orientation. The similarity of lines and bars are discussed, and the formation of curved subjective contour is suggested. The subjective contours might cross each other in one contour organization because the model does not label the depth of the surfaces. The system is robust because the slight change in the maximum gap size would not change the contour organizations. It requires the large change in the maximum gap size to result in the different contour organization if it is possible to have some different contour organizations.

The immediate application of the subjective contour detection system is to uncover camouflaged figures in the image. Camouflaged figures have textures and patterns similar to the surroundings and they appear to be part of the surroundings. However, often the pattern on the boundary of a camouflaged figure and its background are misaligned, and we see the occlusion of the background by a surface. In fact, the subjective figure in general is a camouflaged figure. Therefore, the same clues to detect subjective contours also apply to finding camouflaged figures. The other application of the subjective contour detection system is to separate two or more occluded objects with similar textures and colours, or to separate the overlapping shadows. The former case is similar to detecting the reversible subjective contours, and the latter case is similar to detecting the subjective contours on the overlapping silhouettes of many objects.

The subjective contour detection system can be extended in the following three areas: the strength of subjective contours, the tip-based subjective contours, and the depth labeling of subjective contours. The strength of subjective contour is dependent on the length of supporting edges, the figure thickness perpendicular to the supporting edge, and orientation of the supporting edge pair that connect to the subjective contour. The longer the supporting edge, the thicker the inducing element along the supporting edge, and the smaller the curvature between the supporting edge pair, the stronger subjective contour it produces. We haven't considered the blob thickness perpendicular to the supporting edge in the strength of subjective contour, so this is one future work possibility. In addition, we can explore more about the strength of subjective contour due to its orientation, i.e., the subjective contour is stronger in the horizontal and vertical direction. The brightness associated with the subjective surface also determines the subjective contour strength but the brightness measure is rather subjective. In summary, the strength of subjective contour adds more clues to the subjective contour detection.

In this implementation, line ends and dots are treated as tips because the image is based on the blobs and and distinguishing between tips is not important. In contrast, we have to distinguish between line ends and dots in the image based on line ends because line ends can produce subjective contours but more than one dot cannot produce a subjective contour. To modify the system to deal with a tip-base subjective contour image, we make two endpoints at the line end to consider the line end as if it was a very thin bar. There is no direction at the line end, so either a T, Y, or arrow junction is formed at a line end when there is a subjective contour connecting to the line end.

In a certain image, the implementation result gives the subjective contours crossing each other in one contour organization. This happens because there is no depth labeling of subjective contours. In most cases, the depth of subjective contours are ambiguous when they are crossing each other, and the depth of each subjective surface shifts as the contour organization changes. One possible improvement to the subjective contour detection system is to apply the depth labeling to the subjective surfaces, and we can present the combination of different depth of the subjective surfaces in many contour organizations. The subjective contour uses the same depth information as its subjective surface. Depth labeling of the subjective contours will improve subjective contour organizations.

The subjective contour detection system can interact with the perceptual organization to notice grouping and structures in the image; in particular, the figure-ground separation in which the system gives the contours separating the two regions. Moreover, the subjective contour detection is located in the intermediate-level of computer vision because it uses the processing results from low-level vision system that gives the local features and processes the data into global description about the object boundaries. The subjective contour detection system can be connected to high-level vision systems to produce the description of the input image.

In conclusion, perception of a subjective contour is understood in this thesis as a perception of an invisible occluding contour. The subjective contour detection system is implemented and many subjective contours are detected. Although the system is not capable of detecting every types of subjective contour, we found some clues and constraints for contour perception by modeling the system. Human can recover the partially occluded object shape by completing the missing outline, and subjective contour is an extreme case of occluding contour perception. Therefore, understanding subjective contour is important for understanding human visual perception. We should continue research in the topic of subjective contours to have better understanding of contour perception.

Bibliography

[Asada and Brady, 1986] Haruo Asada and Michael Brady. The curvature primal sketch. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 8(1):2–14, January 1986.

[Barrow and Tenenbaum, 1978] H. G. Barrow and J. M. Tenenbaum. Recovering intrinsic scene characteristics from images. In A. R. Hanson and E. M. Riseman, editors, *Computer Vision Systems*, pages 3–26. Academic Press, 1978.

[Bradley, 1987] Drake R. Bradley. Cognitive contours and perceptual organization. In Susan Petry and Glenn E. Meyer, editors, *The perception of illusory contours*, pages 201–212. Springer-Verlag, 1987.

[Bradley and Dumais, 1975] Drake R. Bradley and Susan T. Dumais. Ambiguous cognitive contours. *Nature*, 257:582–584, October 16 1975.

[Brady and Grimson, 1980] Mike Brady and W. E. L. Grimson. Shape encoding and subjective contours. *First Annual National Conference on Artificial Intelligence*, pages 15–17, 1980.

[Brady and Grimson, 1981] Michael Brady and W. Eric L. Grimson. The perception of subjective surfaces. Technical Report 666, MIT Artificial Intelligence Laboratory, November 1981.

[Canny, 1983] J. F. Canny. Finding edges and lines in images. Technical Report 720, MIT Artificial Intelligence Laboratory, 1983.

[Coren, 1972] Stanley Coren. Subjective contours and apparent depth. *Psychological Review*, 79(4):359–367, 1972.

[Darrell and Pentland, 1991] Trevor Darrell and Alex Pentland. On the representation of occluded shapes. In *IEEE Conference on Computer Vision and Pattern Recognition*, pages 728–729, 1991.

[Day and Jory, 1978] R. H. Day and M. K. Jory. Subjective contours, visual acuity, and line contrast. In John C. Armington, John Krauskopf, and B. R. Wooten, editors, *Visual Psychophysics and Physiology*, pages 331–340. Academic Press, 1978.

[Day and Jory, 1980] R. H. Day and M. K. Jory. A note on a second stage in the formation of illusory contours. *Perception and Psychophysics*, 27(1):89–91, 1980.

[Day and Kasperczyk, 1983a] R. H. Day and R. T. Kasperczyk. Amodal completion as a basis for illusory contours. *Perception and Psychophysics*, 33(4):355–364, 1983.

[Day and Kasperczyk, 1983b] Ross H. Day and Richard T. Kasperczyk. Illusory contours in line patterns with apparent depth due to either perspective or overlay. *Perception*, 12:485–490, 1983.

[Freeman and Davis, 1977] Herbert Freeman and Larry S. Davis. A corner-finding algorithm for chain-coded curves. *IEEE Transactions on Computers*, pages 297–303, March 1977.

[Frisby and Clatworthy, 1975] John P. Frisby and Jeremy L. Clatworthy. Illusory contours: curious cases of simultaneous brightness contrast? *Perception*, 4:349–357, 1975.

[Gillam, 1987] Barbara Gillam. Perceptual grouping and subjective contours. In Susan Petry and Glenn E. Meyer, editors, *The perception of illusory contours*, pages 268–273. Springer-Verlag, 1987.

[Gregory, 1972] R. L. Gregory. Cognitive contours. *Nature*, 238:51–52, July 7 1972.

[Grossberg and Mingolla, 1985] Stephen Grossberg and Ennio Mingolla. Neural dynamics of perceptual grouping: textures, boundaries, and emergent segmentations. *Perception and Psychophysics*, 38(2):141–171, 1985.

[Halpern, 1981] Diane F. Halpern. The determinants of illusory-contour perception. *Perception*, 10:199–213, 1981.

[Halpern and Salzman, 1983] Diane F. Halpern and Billie Salzman. The multiple determination of illusory contours: 1. a review. *Perception*, 12:281–291, 1983.

[Halpern, Salzman, Harrison, and Widaman, 1983] Diane F. Halpern, Billie Salzman, Wayne Harrison, and Keith Widaman. The multiple determination of illusory contours: 2. an empirical investigation. *Perception*, 12:293–303, 1983.

[Haralick, Mackworth, and Tanimoto, 1989] R. M. Haralick, A. K. Mackworth, and S. L. Tanimoto. Computer vision update. Technical Report TR89-12, Department of Computer Science, U.B.C, June 1989.

[Heitger and von der Heydt, 1993] Friedrich Heitger and Rüdiger von der Heydt. A computational model of neural contour processing: figure-ground segregation and illusory contours. In *IEEE 4th International Conference on Computer Vision*, pages 32–40, 1993.

[von der Heydt, Peterhans, and Baumgartner, 1984] R. von der Heydt, E. Peterhans, and G. Baumgartner. Illusory contours and cortical neuron responses. *Science*, 224:1260–1262, June 1984.

[Horn, 1983] B. K. P. Horn. The curve of least energy. *ACM Transactions on Mathematical Software*, 9:441–460, December 1983.

[Huttenlocher and Wayner, 1991] Daniel P. Huttenlocher and Peter C. Wayner. Finding convex edge groupings in an image. In *IEEE Conference on Computer Vision and Pattern Recognition*, pages 406–412, 1991.

[Jory and Day, 1979] Max K. Jory and Ross H. Day. The relationship between brightness contrast and illusory contours. *Perception*, 8:3–9, 1979.

[Kanizsa, 1955] Gaetano Kanizsa, Translated by Walter Gerbino. Quasi-perceptual margins in homogeneously stimulated fields. In Susan Petry and Glenn E. Meyer, editors, *The perception of illusory contours*, pages 40–49. Springer-Verlag, 1987.

[Kanizsa, 1976] Gaetano Kanizsa. Subjective contours. *Scientific American*, 234:48–52, April 1976.

[Kanizsa, 1979] Gaetano Kanizsa. *Organization in vision: essays on gestalt perception.* Praeger, 1979.

[Kawabata, 1984] N. Kawabata. Perception at the blind spot and similarity grouping. *Perception and Psychophysics*, 36(2):151–158, 1984.

[Kellman and Loukides, 1987] Philip J. Kellman and Martha G. Loukides. Object perception and subjective contours. In Susan Petry and Glenn E. Meyer, editors, *The perception of illusory contours*, pages 151–164. Springer-Verlag, 1987.

[Kennedy, 1978a] John M. Kennedy. Illusory contours and the ends of lines. *Perception*, 7:605–607, 1978.

[Kennedy, 1978b] John M. Kennedy. Illusory contours not due to completion. *Perception*, 7:187–189, 1978.

[Kennedy, 1979] John M. Kennedy. Subjective contours, contrast, and assimilation. In Calvin F. Nodine and Dennis F. Fisher, editors, *Perception and Pictorial Representation*, pages 167–195. Praeger Publishers, 1979.

[Kennedy, 1987] John M. Kennedy. Lo, perception abhors not a contradiction. In Susan Petry and Glenn E. Meyer, editors, *The perception of illusory contours*, pages 253–261. Springer-Verlag, 1987.

[Lowe, 1989] David G. Lowe. Organization of smooth image curves at multiple scales. *International Journal of Computer Vision*, 3:119–130, 1989.

[McCafferty, 1990] James D. McCafferty. *Human and machine vision: computing perceptual organisation.* Ellis Horwood, 1990.

[Meyer and Petry, 1987] Glenn E. Meyer and Susan Petry. Top-down and bottom-up: The illusory contour as a microcosm of issues in perception. In Susan Petry and Glenn E. Meyer, editors, *The perception of illusory contours*, pages 3–20. Springer-Verlag, 1987.

[Minguzzi, 1987] Gian Franco Minguzzi. Anomalous figures and the tendency to continuation. In Susan Petry and Glenn E. Meyer, editors, *The perception of illusory contours*, pages 71–75. Springer-Verlag, 1987.

[Nitzberg and Mumford, 1990] Mark Nitzberg and David Mumford. The 2.1-d sketch. In *IEEE 3rd International Conference on Computer Vision*, pages 138–144, 1990.

[Parks, 1980] Theodore E. Parks. Subjective figures: some unusual concomitant brightness effects. *Perception*, 9:239–241, 1980.

[Parks, 1984] Theodore E. Parks. Illusory figures: A (mostly) atheoretical review. *Psychological Bulletin*, 95(2):282–300, 1984.

[Redies, Crook, and Creutzfeldt, 1986] C. Redies, J. M. Crook, and O. D. Creutzfeldt. Neuronal responses to borders with and without luminance gradients in cat visual cortex and dorsal lateral geniculate nucleus. *Experimental Brain Research*, 61:469–481, 1986.

[Richardson, 1979] Barry L. Richardson. The nonequivalence of abrupt and diffuse illusory contours. *Perception*, 8:589–593, 1979.

[Rock, 1983] Irvin Rock. *The logic of perception.* The MIT Press, 1983.

[Rock and Anson, 1979] Irvin Rock and Richard Anson. Illusory contours as the solution to a problem. *Perception*, 8:665–681, 1979.

[Rosenfeld and Johnston, 1973] Azriel Rosenfeld and Emily Johnston. Angle detection on digital curves. *IEEE Transaction on Computers*, pages 875–878, September 1973.

[Rosenfeld and Kak, 1981] Azriel Rosenfeld and Avinash C. Kak. *Digital picture processing*, volume 1, 2. Academic Press, 1981.

[Rutkowski, 1979] Wallace S. Rutkowski. Shape completion. *Computer Graphics and Image Processing*, 9:89–101, 1979.

[Sambin, 1987] Marco Sambin. A dynamic model of anomalous figures. In Susan Petry and Glenn E. Meyer, editors, *The perception of illusory contours*, pages 131–142. Springer-Verlag, 1987.

[Schumann, 1904] F. Schumann, Translated by Anne Hogg. Contributions to the analysis of visual perception–first paper: some observations on the combination of visual impressions into units. In Susan Petry and Glenn E. Meyer, editors, *The perception of illusory contours*, pages 21–34. Springer-Verlag, 1987.

[Shapley and Gordon, 1985] Robert Shapley and James Gordon. Nonlinearity in the perception of form. *Perception and Psychophysics*, 37(1):84–88, 1985.

[Skrzypek and Ringer, 1992] Josef Skrzypek and Brian Ringer. Neural network models for illusory contour perception. *IEEE*, pages 681–683, 1992.

[Smith and Over, 1975] Andrew Smith and Ray Over. Tilt aftereffects with subjective contours. *Nature*, 257:581–582, October 16 1975.

[Trytten and Tuceryan, 1991] Deborah A. Trytten and Mihran Tuceryan. Segmentation and grouping of object boundaries using energy minimization. In *IEEE Conference on Computer Vision and Pattern Recognition*, pages 730–731, 1991.

[Ullman, 1976] Shimon Ullman. Filling-in the gaps: the shape of subjective contours and a model for their generation. *Biological Cybernetics*, 25:1–6, 1976.

[Ullman and Sha'ashua, 1988] Shimon Ullman and Amnon Sha'ashua. Structural saliency: the detection of globally salient structures using a locally connected network. Technical Report 1061, MIT Artificial Intelligence Laboratory, July 1988.

[Webb and Pervin, 1984] Jon A. Webb and Edward Pervin. The shape of subjective contours. *Proceedings of the National Conference on Artificial Intelligence*, pages 340–343, August 1984.

[Williams, 1990] Lance R. Williams. Perceptual organization of occluding contours. *International Conference on Computer Vision*, pages 133–137, December 1990.

[Zucker and Cavanagh, 1985] Steven W. Zucker and Patrick Cavanagh. Subjective figures and texture perception. *Spatial Vision*, 1(2):131–139, 1985.

Appendix A

Sources of Figures

Figure	Reference
Figure 1.1	[Schumann, 1904], Figure 2.7, page 26.
Figure 1.2	[Kanizsa, 1955], Figure 4.11, page 44.
Figure 1.3a	[Kanizsa, 1976], figure at middle in middle illustration on page 51.
Figure 1.3b	[Kanizsa, 1976], figure at right in bottom illustration on page 49.
Figure 1.3c	[Kanizsa, 1976], figure at left in bottom illustration on page 49.
Figure 1.3d	[Sambin, 1987], Figure 14.19a, page 141.
Figure 1.3e	[Kanizsa, 1976], figure at left in middle illustration on page 49.
Figure 1.3f	[Minguzzi, 1987], Figure 7.3a, page 72.
Figure 1.3g	[Minguzzi, 1987], Figure 7.3b, page 72.
Figure 1.3h	[Kanizsa, 1976], figure at right in top illustration on page 50.
Figure 1.3i	[Kanizsa, 1955], Figure 4.16, page 45.
Figure 1.3j	[Parks, 1980], Figure 3.(a), page 240.
Figure 1.3k	[Brady and Grimson, 1981], Figure 26, page 29.
Figure 1.3l	[Bradley and Dumais, 1975], Figure 3, page 583.
Figure 1.4a	[Kennedy, 1978a], Figure 1.(a), page 606.
Figure 1.4b	[Kennedy, 1987], Figure 28.2, page 255.
Figure 1.4c	[Kennedy, 1978b], Figure 1.(a), page 188.
Figure 1.4d	[Gillam, 1987], Figure 30.8, page 272.
Figure 1.4e	[Day and Jory, 1980], Figure 2A, page 89.
Figure 1.4f	[Day and Jory, 1980], Figure 1, page 89.
Figure 1.4g	[Brady and Grimson, 1981], Figure 18, page 18.
Figure 1.4h	[Kanizsa, 1979], Figure 12.12, page 204.
Figure 1.4i	[Gillam, 1987], part of Figure 30.10 turned side way, page 272.
Figure 1.4j	[Parks, 1984], Figure 5.(b), page 291.
Figure 1.4k	[Minguzzi, 1987], Figure 7.5, page 73.
Figure 1.4l	[Zucker and Cavanagh, 1985], Figure 1. left, page 132.

Figure	Reference
Figure 1.5a	[Kanizsa, 1955], Figure 4.11, page 44.
Figure 5.1	[Kanizsa, 1976], figure at right in bottom illustration on page 49.
Figure 5.2	[Richardson, 1979], Figure 4a, page 593.
Figure 5.3	[Kanizsa, 1976], figure at left in bottom illustration on page 49.
Figure 5.4	[Kanizsa, 1976], figure at left in middle illustration on page 49.
Figure 5.5	[Kanizsa, 1976], figure at right in middle illustration on page 49.
Figure 5.6	[Brady and Grimson, 1981] Figure 23 left, page 27.
Figure 5.7	[Brady and Grimson, 1981] Figure 23 right, page 27.
Figure 5.8	[Brady and Grimson, 1981], Figure 26, page 29.
Figure 5.9	Hand drawn subjective contour image.
Figure 5.10	Hand drawn subjective contour image.
Figure 5.11	[Kanizsa, 1979], Figure 10.11, page 179.
Figure 5.12	[Kanizsa, 1979], Figure 10.9, page 177.
Figure 5.13	[Sambin, 1987], Figure 14.19a, page 141.
Figure 5.14	[Sambin, 1987], Figure 14.19b, page 141.
Figure 5.15	Based on Figure 5.14, thicken the bars.
Figure 5.16	Based on Figure 5.14, multiply the bars.
Figure 5.17	[Parks, 1980], Figure 3.(a), page 240.
Figure 5.18	Hand drawn subjective contour image.
Figure 5.19	[Bradley and Dumais, 1975], Figure 3 without lines, page 583.
Figure 5.20	[Bradley and Dumais, 1975], Figure 3, page 583.
Figure 5.21	[Kellman and Loukides, 1987], Figure 3c colour inverted, page 160.
Figure 5.22	[Bradley, 1987], Figure 22.6a, page 206.

Appendix B

Bezier Curve

A Bezier curve described in this section takes three control points, P_0, P_1, and P_2. The curve originates from point P_0, and does not always passes through point P_1, and terminates at point P_2. Furthermore, the curve is tangent to P_0P_1 at point P_0 and P_1P_2 at point P_2

Let *bigx* be the largest x coordinate among the three control points and *bigy* be the largest y coordinate among the three control points. Let *count* be the bigger value of *bigx* and *bigy*. The *interval* to calculate the curve point is $1/count$.

For each k value form 0.0 to 1.0 with increment of *interval*, calculate dx and dy as follows:

$$dx = (1.0 - k)^2 P_{0x} + 2k(1.0 - k)P_{1x} + k^2 P_{2x}$$

$$dy = (1.0 - k)^2 P_{0y} + 2k(1.0 - k)P_{1y} + k^2 P_{2y}$$

ix, and iy are round numbers for dx and dy respectively. We would like to have the points on the curve one pixel apart; therefore, we record a point (ix, iy) on the curve only if it is one pixel apart from the previous point on the curve.