THE VALIDITY OF THE SIMPLIFIED LIMIT DESIGN METHOD FOR THE DESIGN OF STRUCTURES

by

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ABSTRACT

Practice in the field of limit design has tended to place certain restrictions on structural loading patterns in order to simplify the calculations involved in the limit design procedure. The loads considered in this simplified approach are assumed to either remain constant and fixed, or if they vary then this is to be in such a manner that their magnitudes stand in a constant relationship one to the other.

Actual structural loadings seldom satisfy these restrictive conditions and the question naturally arises as to whether or not this simplified limit design procedure is valid for general use in practical design problems in which external loads may be wholly independent in their individual actions.

This question is investigated in the present paper through the examination of several practical forms of structure which portray the more adverse conditions of independent and variable loading to be met in practice. These structures are, respectively, single and double bay gable bents of lightweight construction, and two forms of multispan bridge girders.

The study indicates that all of these structures are able to support the ultimate loads predicted by the simplified limit

design method; the actual ultimate loads exceeding the predicted values by up to twenty percent.

It is concluded that structural failure in practice can always be expected to occur within acceptable limits of the ultimate load capacity as predicted by the simplified method.

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THE VALIDITY OF THE SIMPLIFIED LIMIT DESIGN METHOD FOR THE DESIGN OF STRUCTURES

SECTION 1

INTRODUCTION

Much thought has been devoted in recent times toward the development of structural design techniques employing an Ultimate Load or Limit Design concept, in which a problem structure is approached in terms of its total load support capacity at failure, rather than in terms of the internal distribution of stress under working loads as with the classical elastic theory.

Briefly stated, these limit design techniques have been developed on the supposition that the failure of an initially rigid structure occurs as the result of the formation of localized yield points, or 'plastic hinges', at certain key locations in its members, which transform the structure into a collapse mechanism at its ultimate load. Plastic behaviour of the structure is fundamental to the theory and the application of limit design is confined to structural materials whose stress-strain characteristics exhibit a marked plastic range and for which a constant resisting moment at each hinge may be assumed over a wide range of angular displacement of the hinge. The method is inapplicable to materials devoid of plasticity or where the plastic range is of limited extent. Some doubt also exists under certain circumstances as to the suitability of the method when a strain-hardening region, within the plastic range, is absent.

The properties of structural steels, and particularly mild steel, fulfil the desired plastic requirements almost ideally, as evidenced by the typical stress-strain relationship for mild steel shown in Figure 1(a) and the M-ϕ relationship shown in Figure 1(b), and the theory thus finds its greatest application in the field of steel structures. The exact relationship between M and ϕ in Figure 1(b) applies to the mild steel beam illistrated in the figure (one actually taken from a gable bent example to be later studied in detail) and the closeness of this exact relationship, over a wide range in angle ϕ, to the constant moment value assumed in limit design of 2705 ft. kips is well demonstrated.

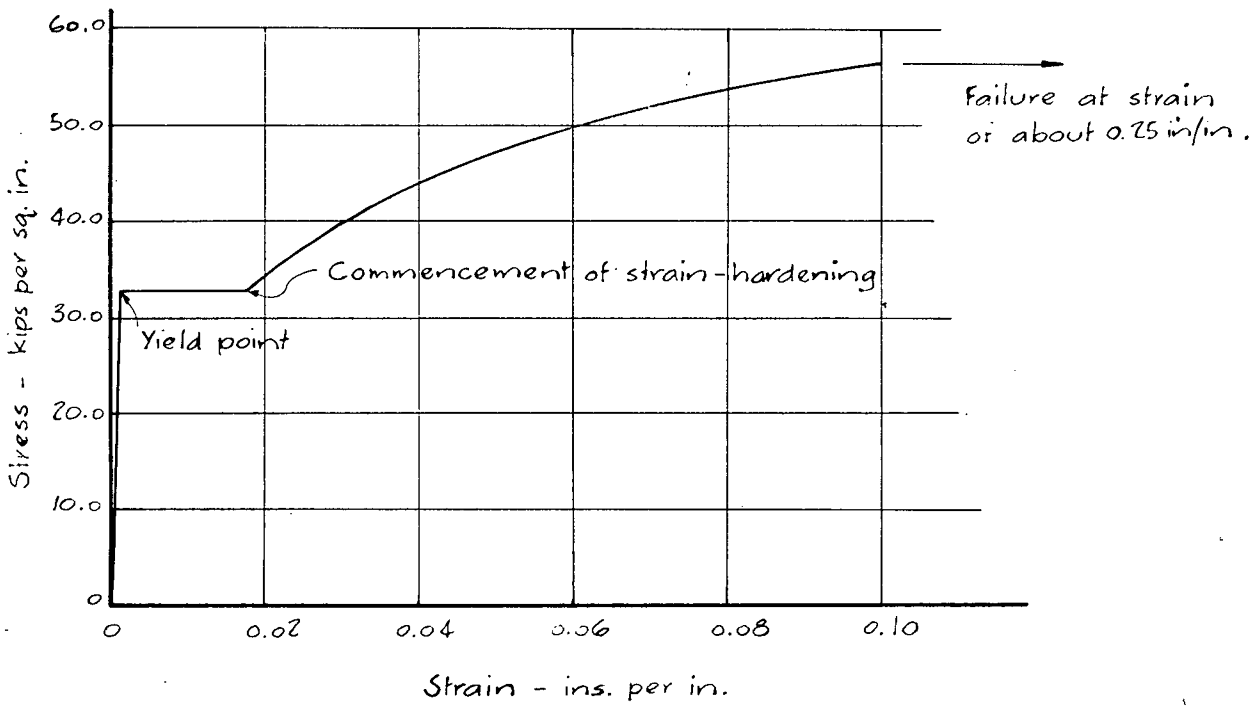
The design procedure employed in the limit design concept consists, essentially, in proportioning the members of the structure so that the collapse load of the whole structure, or any critical portion thereof, is equal to the appropriate working loads multiplied by a certain load factor, with this load factor so selected as to provide a sufficient margin of safety against failure of the whole structure or part.

In cases where the working loads are of constant magnitude the limit design procedure is direct and simple as, irrespective of the indeterminacy of the structure, elastic analysis can be entirely avoided. On the other hand, in cases where the working loads act and vary independently the limit design procedure becomes fairly complex and a solution to the problem requires the evaluation of elastic moments in the structure as the first step in an elasto-plastic analytical procedure.

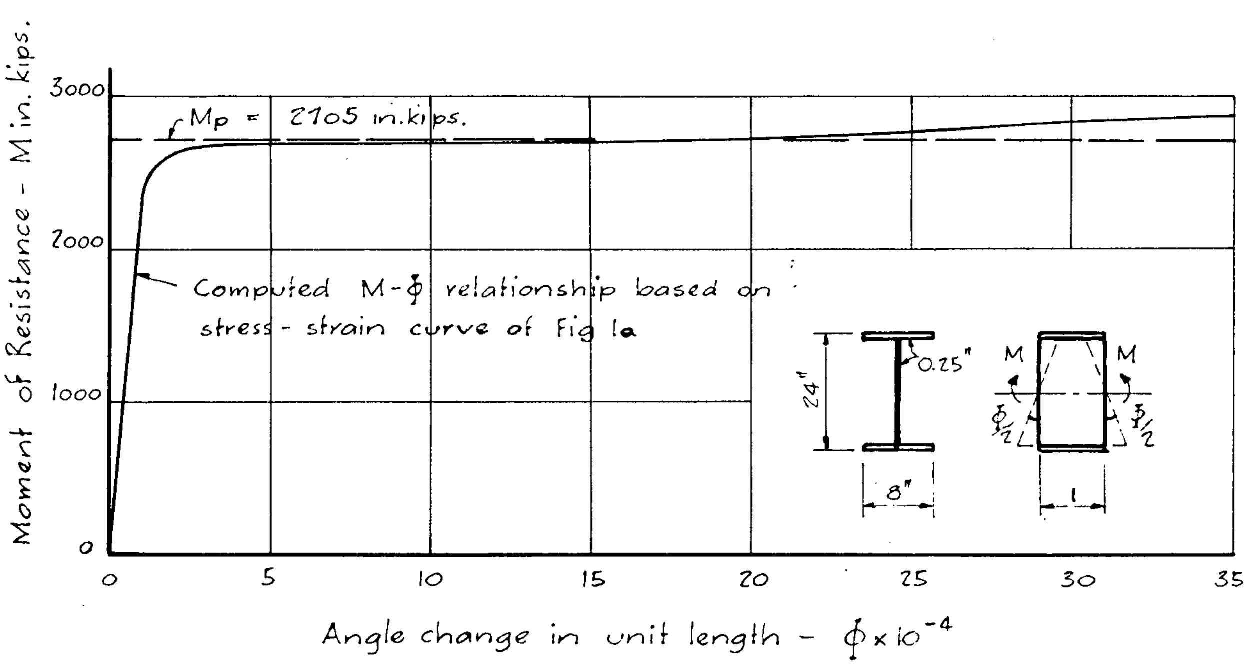
In order to avoid the latter more complex 'variable' load design procedure it is becoming an accepted practice to ignore the true independent nature of most structural loadings and to assume that all of the loads applied to a structure behave in the 'constant' manner appropriate to the simpler design method. This assumption of constant loading is in many cases a poor approximation to the actual load state existing and the question naturally arises as to whether or not this simplified design method can be reasonably accepted as valid for all cases of structural design likely to be met in practice.

It is known that variable loading can be severer on a structure than constant loading, and, in theory, a structure designed in accordance with the constant load method may fail prematurely, that is before its designed load capacity is reached, if it is actually subjected to variable loads of the same design magnitude. The question raised above concerning the validity of the general use of the simplified method for practical design purposes thus becomes one of whether in practice a premature collapse condition can actually exist, and if so, whether it reaches serious proportions or whether it can always be expected to fall within acceptable limits of the load capacity of the structure as indicated by the simplified method.

This question has been given very little critical attention to date and the purpose of this present study is to attempt to obtain an indication of the likelihood and possible extent of the premature collapse condition through the examination of several practical forms of structure which are purposefully selected as examples of the more adverse conditions of loading to be expected in practice.



(a) Typical stress-strain curve for Mild Steel.



(b)

Figure 1.

SECTION 2

GENERAL THEORETICAL CONSIDERATIONS

2.01 Conditions Governing Structural Failure

A. For Case of Constant Loading

The frame shown in Figure 2 is acted upon by fixed working loads of magnitude W, and under these particular loads it will be assumed that the structure deforms elastically thus creating certain elastic moments at all sections of its members.

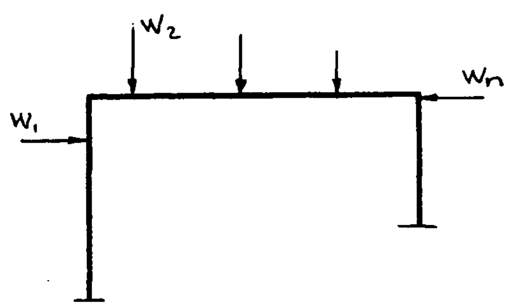


Figure 2

If the loads are increased in magnitude, with a constant ratio remaining between their values, the elastic moments will be proportionately increased, and this proportionality of loads and moments will continue, as the loads further increase, until at some load value the elastic limit stress of the frame material is reached at some critical section. Yielding commences in the outer fibres of this section and develops through the section until at some higher load value a fully mobilized plastic hinge is created which offers a constant moment of resistance to further deformation.

At a still higher load value the elastic limit stress will be reached at some other key point of the structure and a second hinge created. Additional hinges are similarly developed until under the failure load sufficient hinge points exist to transform the initially rigid structure into a mechanism able to offer no additional resistance to complete collapse.

This is the simple mechanics of the constant load theory. If now a virtual displacement of the structure in the form of the collapse mechanism is considered an equation of virtual work can be written relating the total internal work done at the hinge points, in terms of plastic moments M_p and angles of rotation θ, to the total external work done by the failure loads of magnitude F.W in moving through the distances δ of their virtual displacements. Thus:

$$\Sigma M_p.\theta \quad = \quad \Sigma F.W.\delta \qquad (1)$$

This equation constitutes the basic expression of the simplified method, in which, given the proportions of the frame and the geometry of the collapse mechanism, the true failure value of the loads, and thence the factor F by which the original working loads were increased to the failure value, can be readily determined. In the analysis of a given structure a number of possible mechanisms may exist and the true collapse mode will be the one resulting in the smallest value of load factor F.

Now, referring back temporarily to the original condition indicated by Figure 2 with the frame subjected to working loads W and with certain elastic moments M existing at the key points and writing a similar equation to (1) above for an identical virtual displacement, in terms of working loads and moments:

$$\Sigma M.\theta = \Sigma W.\delta \qquad (2)$$

Again increasing these loads to the failure value F.W by multiplying both sides of equation (2) by F:

$$\Sigma F.M.\theta = \Sigma F.W.\delta ,$$

amd equating this expression with (1), gives:

$$\Sigma M_p.\theta = \Sigma F.M.\theta \qquad (3)$$

The plastic moments M_p occuring in these equations should be considered as being positive. Elastic moments M at each hinge are likewise positive if they occur in the same sense as M_p, elastic moments of opposite sense to M_p must therefore be treated as negative.

Load factor F may be obtained from equation (3), given the properties of the structure and the distribution of elastic moments within it. This equation thus provides an alternative basis to that of equation (1) for obtaining the failure load factor for the constant loading condition.

Rewriting equation (3), the required quantity F may be expressed in the following form:

$$F = \frac{\Sigma M_p.\theta}{\Sigma M.\theta} \qquad (4)$$

Now, of all possible mechanisms whereby the structure may collapse the true failure mode, that is the one requiring the smallest value of F, is obviously obtained when the terms of the numerator are smallest in comparison with those of the denominator. Two variables are involved in these terms, namely, moments and angles of rotation. Variations in the latter are continuous and usually gradual and it is thus reasonable to expect that the hinge points of the true failure mechanism will be associated with sections of the frame where minimum values of $\frac{M_p}{M}$ occur.

This association of hinge points of the constant load mechanism with minimum values of the quantity $\frac{M_p}{M}$ is found to be closely followed in numerous examples which have been considered. Such minima may exceed in number the hinges required for the development of a mechanism but each is a key point of the structure representing a possible hinge point of a failure mechanism. It will be noted that in cases where M_p remains constant along a member of the structure the key points of this member will be associated with those sections where maximum values of elastic moment occur.

Equation (3) may also be written in the form:

$$\Sigma M_p.\theta \; - \; \Sigma F.M.\theta \; = \; 0$$

$$\text{or,} \quad \Sigma(M_p - F.M).\theta \; = \; 0 \qquad (5)$$

Now, it will be remembered that equation (3) represents the structure in the failure state under the application of loads F.W. To obtain the above equation (5) the quantities F.M.θ have been subtracted from both sides of equation (3) and this subtraction has, in effect, removed the failure loads and left the structure in a zero load state. Equation (5) thus refers to the completely unloaded structure and must be the virtual work expression for this state. The quantities (M_p - F.M) are, then, residual moments remaining at the hinge points of the structure under this zero load state.

B. <u>For Case of Variable Loading</u>

Consider now the structure shown in Figure 2 subjected to the more general form of loading in which certain of the applied loads are assumed as fixed and constant, and the remainder are capable of independent variation. This is the variable loading situation which arises in practical structures. The assumed fixed loads are 'dead' loads of magnitude W_D and they create elastic moments in the structure of magnitude M_D at any section. The remaining loads are 'live' loads of magnitude W_L and the numerically greatest moment created at any section by their independent action will be referred to as M_L.

Failure of the structure occurs in the form of a mechanism when dead and live loads are increased in magnitude by load factors F_D and F_L, respectively.

The zero load state of the frame at this condition of failure is such that:

$$\Sigma(M_p - F_L.M_L - F_D.M_D).\theta \quad = \quad 0, \qquad (6)$$

this being the work expression for a virtual displacement of the structure in the form of the failure mechanism. The convention for the signs of moments is the same as that earlier established, namely, all plastic moments M_p are considered as positive and the elastic moments M_L and M_D are likewise positive if they occur in the same sense as M_p, if of the opposite sense then they are negative.

The live load factor F_L is now required and this may be readily obtained by rearranging the terms of equation (6), thus:

$$F_L = \frac{\Sigma(M_p - F_D.M_D).\theta}{\Sigma M_L.\theta} \tag{7}$$

The true failure mode for variable loading therefore requires that terms involving $(M_p - F_D.M_D).\theta$ are smallest with respect to those involving $M_L.\theta$, and thus key points of the structure for variable loading can be expected to be associated with sections where minimum values of the quantity $\frac{(M_p - F_D.M_D)}{M_L}$ occur.

Now, equation (6) defines the zero load state for the structure subjected to variable loading and it will be evident from inspection of this equation that the residual moments remaining at the hinge points in this case are equal to $(M_p - F_L M_L - F_D M_D)$. As live loads $F_L.W$ are repeatedly and independently applied the structure will eventually 'shakedown' to this pattern of residual moments. This shakedown procedure may involve a number of applications of the loads, each of these earlier cycles of loading necessitating a readjustment of internal stress from the key points where overstress initially occurs to adjacent sections of the structure. This transfer of stress takes place through increments of plastic yielding at these key points, with each increment of yield being associated with an increment of permanent deflection of the structure as a whole, and these deflections continue until the shakedown state is reached. If loads slightly greater than the true failure value are applied the structure is unable to reach the shakedown state and the increments of permanent deflection continue indefinitely, with the structure deforming as a mechanism, until complete collapse eventually results. This type of failure under variable loading is therefore referred to as one of 'incremental collapse'.

A second form of failure under variable loading must also be investigated. It was previously stated, in defining the quantities involved in equation (6), that M_L represents the numerically greatest elastic moment which could occur at any section of the structure. Both positive and negative values of M_L will generally exist at each key point and the quantity involved in equation (6) will be either the positive or negative amount depending on the sense of the hinge rotation at the key point. The total range in live load moment at any section will be the algebraic summation of these positve and negative amounts. This total range in moment $\overline{M}_L$ at a section conceivably may exceed the elastic resistance of that section, and if this condition occurs the opposite extremes of live load moment will produce an inelastic yielding at the section first in one direction and then in the other. This alternating plastic yield in turn results in increments of permanent strain which must eventually lead to the failure of the section. This condition is described as one of 'plastic fatigue', or, more commonly, 'alternating plasticity'. It obviously may occur at any section of the structure.

Now, if the moment at which yield stress is first reached in the outer fibres of the section is referred to as M_y then the total elastic resistance of the section, under reversals of moment, is twice this moment value, that is $2.M_y$.

The condition of alternating plasticity will develop when the following equality exists:

$$F_L'.\overline{M}_L = 2.M_y \tag{8}$$

from which,

$$F_L' = \frac{2.M_y}{\overline{M}_L} \tag{9}$$

The critical section of the structure at which this condition of failure is first reached evidently is where the smallest value of $\frac{M_y}{\overline{M}_L}$ occurs. This state is attained at a live load factor equal to F_L'.

C. Summary of Failure Conditions

In briefly summarizing the findings sofar reached in these theoretical considerations it may be said that in the case of variable loading, that is in the general case to be met in practice in which both fixed dead loads and independently varying live loads are encountered, two states of failure must be considered.

Firstly, incremental collapse of the structure in the form of a mechanism, the condition of the structure at the point of incipient collapse being defined by equation (6), as follows:

$$\Sigma(M_p - F_L.M_L - F_D.M_D).\theta = 0, \qquad (6)$$

the terms of this equation extending to all hinge points of the failure mechanism.

Secondly, alternating plasticity, involving only one section* of the structure, and for which incipient failure is defined by the equality stated in equation (8), thus:

$$F_L'.\overline{M}_L = 2.M_y \qquad (8)$$

The true failure load for the structure under the condition of variable loading will correspond to the smallest value obtainable for either F_L or F_L'. Both states of failure require elastic analysis of the structure for the determination of moments M_D, M_L, and $\overline{M}_L$.

The simplified case of constant loading may be thought of as replacing the dead and live loads, to which the structure is actually subject, by an equivalent system of constant loads. These constant loads being restricted to simultaneous action and

* If general symmetry of structure and loading exists then more than one section of the structure may fail under alternating plasticity.

if they vary in magnitude this must be in such a fashion that a constant ratio always exists between them. In this equivalent system the loads are thus of the same nature and the same load factor must be applied to each and every one. The condition of the structure at the point of incipient collapse in this case is expressed by equation (1), as follows:

$$\Sigma M_p.\theta \quad = \quad \Sigma F.W. \tag{1}$$

or, alternatively, in the form given by equation (3), thus:

$$\Sigma M_p.\theta \quad = \quad \Sigma F.M.\theta \tag{3}$$

The terms of these equations extending to each hinge point of the failure mechanisms, amd to each load item in the case of equation (1). Equation (3) requires an elastic analysis of the structure for the determination of moments M, but this is not necessary for equation (1).

The true failure load for this condition of constant loading will correspond to the smallest value obtainable for load factor F.

2.02 Premature Collapse

Having established the conditions governing the failure of structures subjected, on the one hand to actual variable loading, amd on the other hand to an assumed constant loading, it is now only necessary to make a comparison between F_L and F in order to determine the extent of premature collapse. If ratios of $\frac{F_L}{F}$ of less than 1.0 are obtained this will indicate that premature collapse can take place, and the magnitude of this ratio will indicate the severity of the premature collapse condition. Such a comparison of failure load values can be put on a practical basis by selecting practical forms of structure for analysis, but before this is attempted here it is desirable to further extend the previous theoretical evaluations.

Firstly, consider the criteria established for the location of key points in structures subject to mechanism failure. For constant loading the criterion is that these key points will be at sections of the structure where minimum values of $\frac{M_p}{M}$ occur. For variable loading the criterion is minimum values of $\frac{M_p - F_D.M_D}{M_L}$ but inspection of this quantity will show that these minimum values occur when the combined effects of the elastic moment terms are largest with respect to M_p. As critical values for M, M_L, and M_D are commonly a maximum in the same vicinity in a structure these separate criteria may amount to substantially the same thing and normally it can be expected that the key points of the structure for both loading conditions will also be in the same vicinity. In fact in limit design it is usual to assume identical key points for both loadings.

If it is also assumed here temporarily that the same critical mechanism applies for both forms of loading then the ratio $\frac{F_L}{F}$ for mechanism failure may be written as:

$$\frac{F_L}{F} = \frac{\Sigma M.\theta}{\Sigma M_L.\theta} \cdot \frac{\Sigma(M_p - F_D.M_D).\theta}{\Sigma M_p.\theta} ,$$

and making use of known relationships this expression may be readily resolved into the following form*:

$$\frac{F_L}{F} = \frac{\Sigma M_{Lc}.\theta}{\Sigma M_L.\theta}\left[1 + \frac{F-F_D}{F} \cdot \frac{\Sigma W_D.\delta}{\Sigma W_L.\delta}\right] \qquad (10)$$

in which,

$\Sigma M_{Lc}.\theta$, and $\Sigma W_L.\delta$, are the internal and external work, respectively, of live loads W_L applied in the manner of constant loading,

M_L, as previously defined, are the numerically greatest moments produced by these same live loads acting independently as variable loads, and $\Sigma M_L.\theta$ is therefore the work done by these moments, and

$\Sigma W_D.\delta$ is the external work done by dead loads W_D.

Now, the interest in equation (10), and the reason for introducing it, lies in the light it sheds in isolating the main features or factors which influence the load factor ratio $\frac{F_L}{F}$.

* See Appendix 1.

Two such factors predominate in the equation and on these the ratio is directly dependent. These factors are:

1. $\frac{\Sigma W_D . \delta}{\Sigma W_L . \delta}$. This factor compares the external work done by loads W_D and W_L. The effect of this factor is such that the larger the dead loads are with respect to live loads the more favourable will be the result on ratio $\frac{F_L}{F}$, and the less the likelihood of premature collapse.
2. $\frac{\Sigma M_{Lc} . \theta}{\Sigma M_L . \theta}$. Compares the internal work done at hinge points by the live loads acting respectively in the manner of constant loads and as independent variable loads. This factor evidently measures the relative extremes of live load moment created in the structure under these different states of loading. The more variable the actual live loads are the smaller will be the value of the internal work ratio and the more adverse will be the effect on $\frac{F_L}{F}$, hence the greater the likelihood of premature collapse.

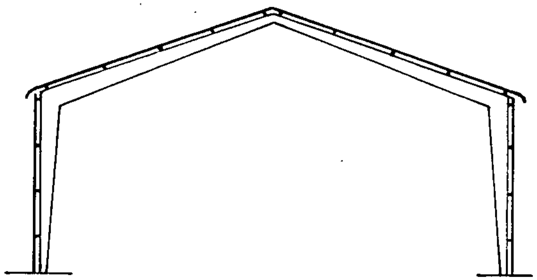
Conditions which can therefore be expected to tend to produce a lower limit to the load factor ratio will be those in which the live loads are highly variable and in which dead loads are as small as possible. Although these conclusions are based on an identical form of mechanism failure it will be evident that the same conclusions are relevant to the case of failure through different mechanism forms and also to that of failure by alternating plasticity.

Now, these findings present a logical basis on which to select the structures for analysis in the present study. As the purpose of the study is to establish the likelihood and possible extent to which premature collapse may be expected to occur in practice it is obviously logical to choose for analysis forms of structure which satisfy the loading requirements found necessary for small values of $\frac{F_L}{F}$.

Two forms of structure which exhibit such loading characteristics are the gable bents shown in Figures 3(a) and 3(b)[1]. Dead weight is minimized in both cases by the use of lightweight roofing construction and tapered frame members, and live loads are highly variable - consisting of independent gravity and reversible sidesway forces. The single bay and double bay bents were also specifically chosen of similar form to illistrate the effects of different degrees of structural redundancy.

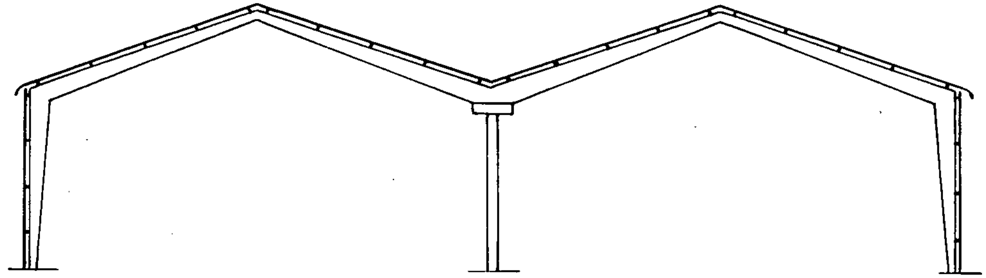
These gable bents, then, can be expected to indicate the probable order of the lower limit to the premature collapse condition. Now, it would also be of practical interest to examine the effect on $\frac{F_L}{F}$ of variations in the dead to live load ratio and this is best done by selecting a second type of structure in which the live loads are highly variable, as with the bents, but in which the dead weight is quite large in magnitude in comparison with the applied live loads. A suitable example of such a loading condition is the multispan plate girder highway bridge shown in Figure 3(c), in which the moving vehicle loads cause highly variable live moments but these loads are considerably smaller in magnitude than the combined dead weight of the steel girder system and concrete deck.

Detailed analyses of the bents and bridge girders will be made in the remaining sections of this paper. The several structures are actual designs, and dimensions and other details used in the study have been obtained from the fabrication and construction drawings. The original designs were based on the elastic theory and although a comparison of plastic and elastic designs is beyond the scope of this paper the original loading specifications and design assumptions will be also employed in the plastic analyses in order that such a comparison for these structures could be independently made.



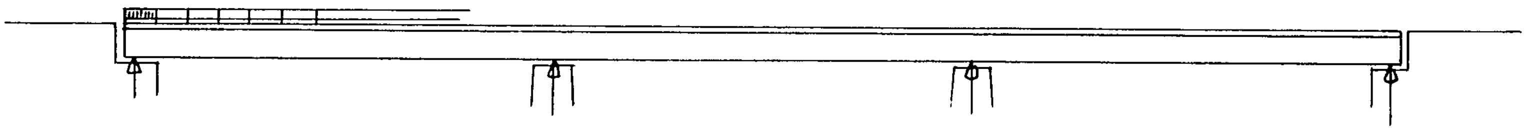
(a)

Single Bay Gable Bent.



(b)

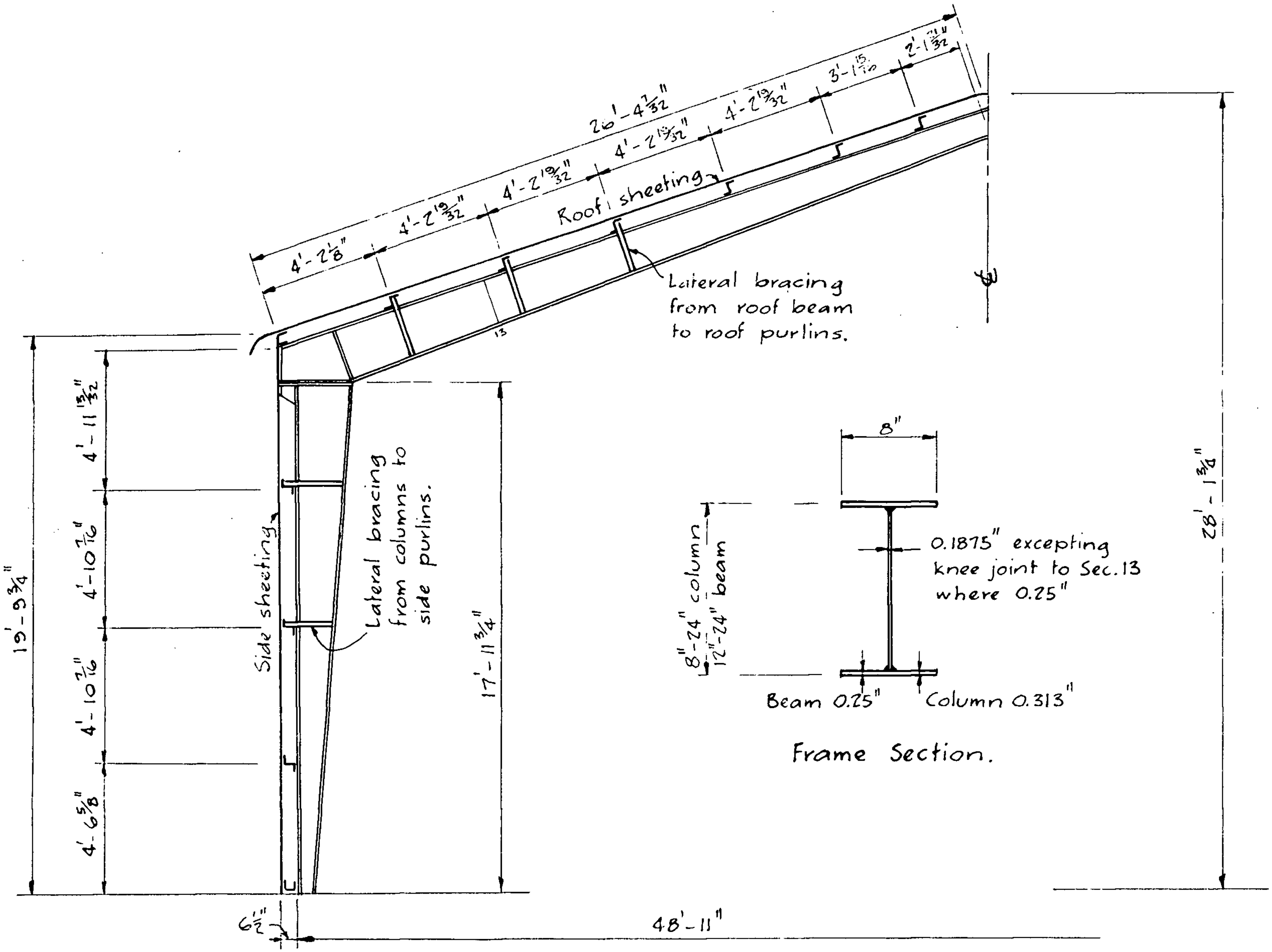
Double Bay Gable Bent.



(c)

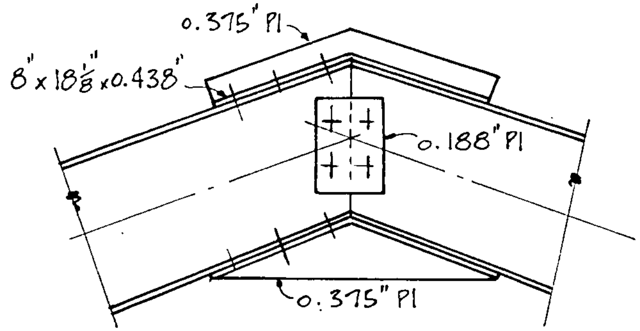
Multispan Plate Girder Highway Bridge.

Figure 3

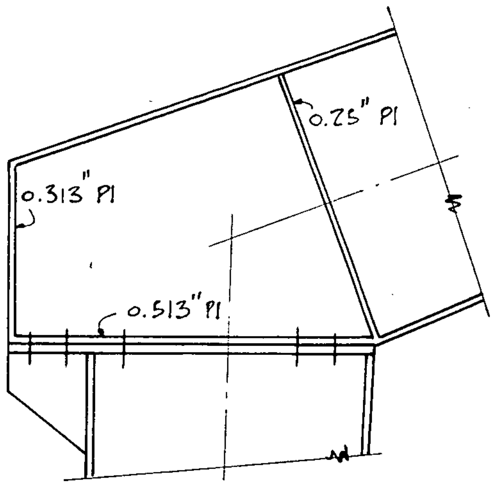


26'-4 7/32"
4'-2 1/8"
4'-2 19/32"
4'-2 19/32"
4'-2 19/32"
4'-2 19/32"
3'-1 15/16
2'-1 11/32"
Roof sheeting
Lateral bracing from roof beam to roof purlins.
13
4'-11 13/32"
4'-10 7/16"
4'-10 7/16"
4'-6 5/8"
19'-9 3/4"
Side sheeting
Lateral bracing from columns to side purlins.
17'-11 3/4"
28'-1 3/4"
6 1/2"
48'-11"
8"
8"-24" column
12"-24" beam
0.1875" excepting knee joint to Sec.13 where 0.25"
Beam 0.25"
Column 0.313"
Frame Section.

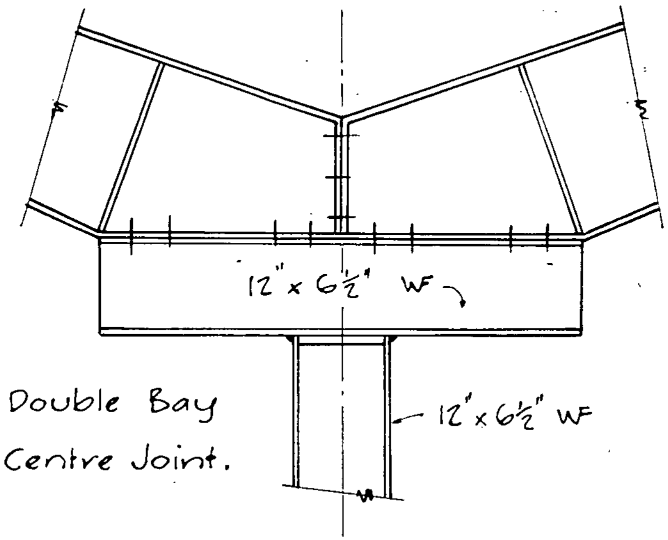
Figure 4.



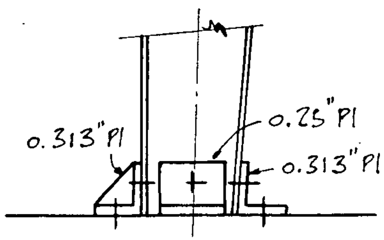
Roof Joint



Knee Joint



Double Bay
Centre Joint.



Column Base

Figure 5

Note: ¾" φ high strength steel bolts throughout.

Table I

Properties of Frame Sections

Frame Section No.	Depth of Section ins.	Area of Section sq. ins.	Elastic Properties			Plastic Properties	
			I in.^{4}	Z_e in.^{3}	M_y in. kips	Z_p in.^{3}	M_p in. kips
A	8.00	6.39	80.5	20.2	665	21.8	720
1	8.83	6.65	99.4	22.6	745	24.5	805
2	10.49	6.85	145	27.5	910	30.1	1005
3	12.15	7.17	199	32.8	1080	35.8	1180
4	13.81	7.48	264	38.2	1260	41.8	1380
5	15.47	7.79	340	44.0	1450	48.3	1590
6	17.13	8.10	425	49.5	1635	54.8	1805
7	18.79	8.41	520	55.3	1825	61.6	2030
8	20.45	8.72	627	61.5	2035	68.7	2265
9	22.11	9.03	751	67.9	2240	76.0	2505
10	23.77	9.44	882	74.2	2450	83.7	2760
$B_{col.}$	24.00	9.38	904	75.3	2490	84.8	2800
B_{beam}	24.00	9.88	839	69.7	2300	82.6	2705
11	23.10	9.80	811	68.5	2260	77.6	2560
12	22.47	8.12	708	62.9	2070	74.6	2460
13	21.85	7.83	581	54.7	1800	64.6	2130
14	20.01	7.65	506	60.6	1670	57.3	1895
15	18.78	7.43	439	46.7	1540	52.7	1740
16	17.54	7.19	376	42.9	1415	48.2	1590
17	16.31	6.79	320	39.2	1295	43.8	1445
18	15.08	6.74	268	35.5	1170	39.6	1305
19	13.85	6.51	222	32.1	1060	35.6	1175
20	12.63	6.38	181	28.6	945	31.6	1045
C	12.00	6.16	160	27.0	890	29.7	980

SECTION 3

ANALYSES OF SELECTED STRUCTURES

3.01 Analysis of Single Bay Gable Bent

A. General

Frame sections and detailed dimensions of the bent are shown in Figures 4 and 5. The column and roof beams are of built-up welded section, as shown in Figure 4, and the properties of these are given in Table 1. High strength bolted connections are employed throughout and details of these connections are shown in Figure 5. Lateral stiffeners for the inside flanges of columns and beams are located in Figure 4 and take the form of angle braces between flanges and side and roof purlins. The spacing of frames is 20'-0" on centre.

The original design of the bent was to AISC - 1952 specifications. Loading conditions were figured as follows:

(1) Dead loading plus 30.0 pounds per square foot live loading on the full horizontal projection of the structure.

(2) Dead loading plus 20.0 pounds per square foot wind loading acting on the full vertical projection of the structure in one or other horizontal direction.

(3) Dead loading plus 30.0 pounds per square foot on the horizontal projection plus 20.0 pounds per square foot in either direction on the vertical projection.

Design loadings were computed on the basis of these conditions with dead loads assumed distributed uniformly over the roof span and estimated thus:

Roof beams	@ 1.4 lbs. per sq. ft.
Roof purlins	@ 1.1 " " " "
Roof sheeting	@ 1.0 " " " "
Total dead weight	3.5 lbs. per sq. ft.

giving for:

Design condition (1),

uniform load on full horizontal projection

$= (30.0 + 3.5).20.0 \quad = \quad 670$ lbs. per lin. ft.

Design condition (2),

wind load on full vertical projection

$= (20.0).20.0 \quad = \quad 400$ lbs. per lin. ft.

plus dead load

$= (3.5).20.0 \quad = \quad 70$ lbs. per lin. ft.

Design condition (3),

wind load on vertical projection

$= \quad 400$ lbs. per lin. ft.

uniform load on horizontal projection

$= \quad 670$ lbs. per lin. ft.

B. Elastic Analysis

A step by step derivation of the elastic analysis is unnecessary for the purposes of this study. The procedures used are standard and well known. Moment distribution methods were employed, with the fixed end moments, stiffness coefficients, and carry-over factors required for the analysis determined by means of the column analogy method. Both supports were assumed as hinged.

The properties of the frame elements used in these calculations are given for reference purposes in Table 11, and in the accompanying Figure 6. The symbols used carry their usual meanings.

Resulting moments at sections throughout the frame are given in Table 111 for the individual gravity and wind loadings. Shears and thrusts are not considered as investigation has shown that their effects are small and can be entirely neglected in the study.

Moment diagrams for each loading condition, plotted from the values tabulated in Table 111, are shown in Figures 7, 8, and 9. For these diagrams the moments are plotted, in the conventional manner, on the side of the frame at which they create compressive stress.

The severest fibre stress created by the applied loadings is worthy of note although this data is not required for the present purpose. The severest stress condition is found to occur in the roof beams at the knee joints where the combined stress figures at 26.6 ksi. The value permitted by the AISC Code, for wind and all other loadings, equals 26.7 ksi (= 20.0 ksi plus

Table 11 Properties of Cable Bent Elements.

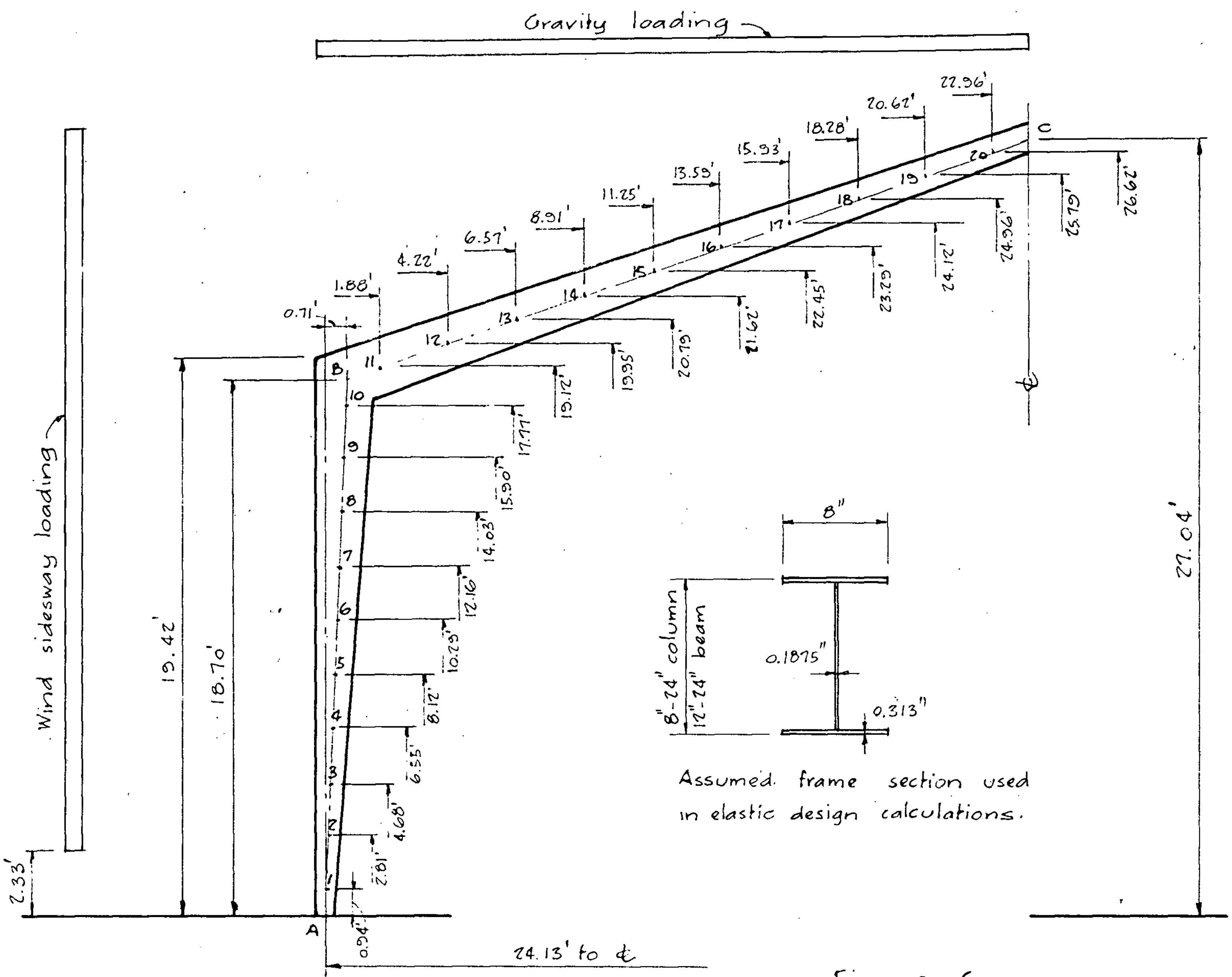
Frame Section No.	Frame Element Δ_s ft.	x ft.	y ft.	d ins.	I ins.^{4}	I ft.^{4}	$\frac{\Delta_s}{I}$	$\frac{y \cdot \Delta_s}{100 I}$	$\frac{y^2 \cdot \Delta_s}{100 I}$	Vertical Loading M_s ft. kip	Vertical Loading $\frac{M_s \cdot y \cdot \Delta_s}{100 I}$	Wind Loading, Left of ℄ M_s ft. kip	Wind Loading, Left of ℄ $\frac{M_s \cdot y \cdot \Delta_s}{100 I}$	Wind Loading, Right of ℄ M_s ft. kip	Wind Loading, Right of ℄ $\frac{M_s \cdot y \cdot \Delta_s}{100 I}$
1	1.87	0.04	0.94	8.83	99.4	.00478	391	3.68	3.5	1.0	3.7	23	85	0.3	1.1
2	1.87	0.11	2.81	10.49	145	.00698	268	7.52	21.1	2.7	20.3	69	518	0.8	6.0
3	1.87	0.18	4.68	12.15	199	.00958	195	9.15	42.8	4.32	39.5	112	1025	1.4	12.8
4	1.87	0.25	6.55	13.81	264	.0127	147	9.64	63.1	6.01	57.8	151	1453	1.9	18.3
5	1.87	0.32	8.42	15.48	340	.0164	114	9.60	80.8	7.67	73.6	187	1797	2.4	23.0
6	1.87	0.39	10.29	17.13	425	.0205	91.3	9.40	96.8	9.33	87.6	220	2070	2.9	27.2
7	1.87	0.46	12.16	18.79	520	.0250	74.8	9.10	110.6	11.00	100	248	2260	3.4	31.0
8	1.87	0.53	14.03	20.45	627	.0302	61.9	8.70	122.1	12.66	110	274	2380	4.0	34.8
9	1.87	0.60	15.90	22.11	751	.0361	51.8	8.24	130.1	14.32	118	297	2450	4.5	37.1
10	1.87	0.68	17.77	23.77	882	.0424	44.1	7.83	139.0	15.98	125	314	2460	5.0	39.2
11	2.49	1.88	19.12	23.10	876	.0421	59.2	11.31	216	43.70	495	319	3610	14.1	160
12	2.49	4.22	19.95	22.47	776	.0378	66.6	13.00	265	93.10	1240	308	4100	31.6	420
13	2.49	6.57	20.79	21.85	685	.0330	75.5	15.7	326	137	2150	294	4620	49.2	773
14	2.49	8.91	21.62	20.01	599	.0288	86.8	18.7	405	177	3310	283	5300	66.7	1245
15	2.49	11.25	22.45	18.78	520	.0251	99.3	22.3	500	209	4670	270	6030	84.4	1880
16	2.49	13.59	23.29	17.54	448	.0216	115	26.8	624	236	6330	256	6870	102	2740
17	2.49	15.93	24.12	16.31	381	.0183	136	32.8	792	259	8500	241	7900	119	3900
18	2.49	18.28	24.96	15.08	321	.0155	161	40.1	1000	275	11050	224	8990	137	5500
19	2.49	20.62	25.79	13.85	266	.0129	192	49.5	1277	286	14170	209	10350	155	7670
20	2.49	22.96	26.62	12.63	217	.0105	236	62.9	1678	292	18360	192	12100	172	10820

Table III Moments for Single Bay Bent.

Frame Section No.	Dead load Moment in. kips	Live load Moment in. kips			Moments for Constant load Analysis in kips.			Plastic Moment M_p in kips
		Gravity Loading	Wind from left	Wind from right	Condition 1	Condition 2	Condition 3	
3_L	− 32	− 271	+ 361	− 166	− 303	+ 329	+ 58	1180
5_L	− 57	− 489	+ 587	− 299	− 546	+ 530	+ 41	1590
7_L	− 82	− 707	+ 743	− 432	− 789	+ 661	− 46	2030
9_L	− 108	− 922	+ 835	− 565	− 1030	+ 727	− 195	2505
B_L	− 127	− 1085	+ 865	− 665	− 1212	+ 738	− 347	2705
11_L	− 108	− 930	+ 817	− 638	− 1038	+ 709	− 221	2560
13_L	− 43	− 367	+ 642	− 531	− 410	+ 599	+ 232	2130
15_L	+ 3	+ 22	+ 455	− 425	+ 25	+ 458	+ 480	1740
17_L	+ 35	+ 301	+ 257	− 318	+ 336	+ 292	+ 593	1445
19_L	+ 47	+ 401	+ 36	− 211	+ 448	+ 83	+ 484	1175
C	+ 42	+ 356	− 131	− 131	+ 398	− 89	+ 267	980
19_R	+ 47	+ 401	− 211	+ 36	+ 448	− 164	+ 237	1175
17_R	+ 35	+ 301	− 318	+ 257	+ 336	− 297	+ 18	1445
15_R	+ 3	+ 22	− 425	+ 455	+ 25	− 422	− 400	1740
13_R	− 43	− 367	− 531	+ 642	− 410	− 574	− 941	2130
11_R	− 108	− 930	− 638	+ 817	− 1038	− 746	− 1676	2560
B_R	− 127	− 1085	− 665	+ 865	− 1212	− 792	− 1877	2705
9_R	− 108	− 922	− 565	+ 835	− 1030	− 673	− 1595	2505
7_R	− 82	− 707	− 432	+ 743	− 789	− 514	− 1221	2030
5_R	− 57	− 489	− 299	+ 587	− 546	− 356	− 845	1590
3_R	− 32	− 271	− 166	+ 361	− 303	− 198	− 469	1180

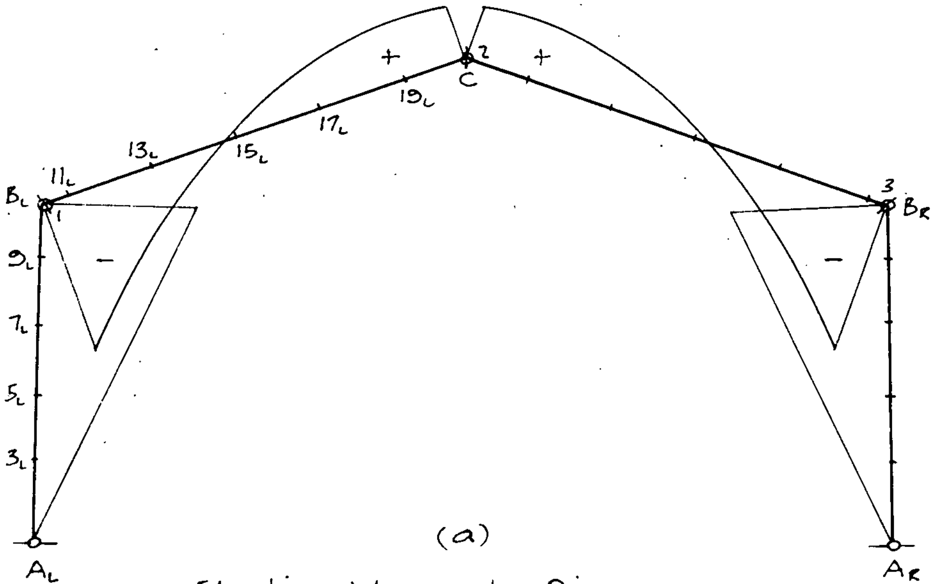
Table IV Moments for Single Bay Bent.

Frame Section No.	Dead Load Moments in. kips	Live Load Moments for Variable Load Analysis in. kips			Plastic Moments M_p, in. kips	Yield Moments M_y, in. kips
		Greatest Positive M_L	Greatest Negative M_L	Range in Moments $\tilde{M}_L$		
3_L	− 32	+ 361	− 437	798	1180	1080
5_L	− 57	+ 587	− 788	1375	1590	1450
7_L	− 82	+ 743	− 1132	1875	2030	1825
9_L	− 108	+ 835	− 1487	2322	2505	2240
B_L	− 127	+ 865	− 1750	2615	2705	2300
11_L	− 108	+ 817	− 1568	2385	2560	2260
13_L	− 43	+ 642	− 898	1540	2130	1800
15_L	+ 3	+ 477	− 425	902	1740	1540
17_L	+ 35	+ 558	− 318	876	1445	1295
19_L	+ 47	+ 437	− 211	648	1175	1060
C	+ 42	+ 356	− 131	487	980	890
19_R	+ 47	+ 437	− 211	648	1175	1060
17_R	+ 35	+ 558	− 318	876	1445	1295
15_R	+ 3	+ 477	− 425	902	1740	1540
13_R	− 43	+ 642	− 898	1540	2130	1800
11_R	− 108	+ 817	− 1568	2385	2560	2260
B_R	− 127	+ 865	− 1750	2615	2705	2300
9_R	− 108	+ 835	− 1487	2322	2505	2240
7_R	− 82	+ 743	− 1132	1875	2030	1825
5_R	− 57	+ 587	− 788	1375	1590	1450
3_R	− 32	+ 361	− 437	798	1180	1080



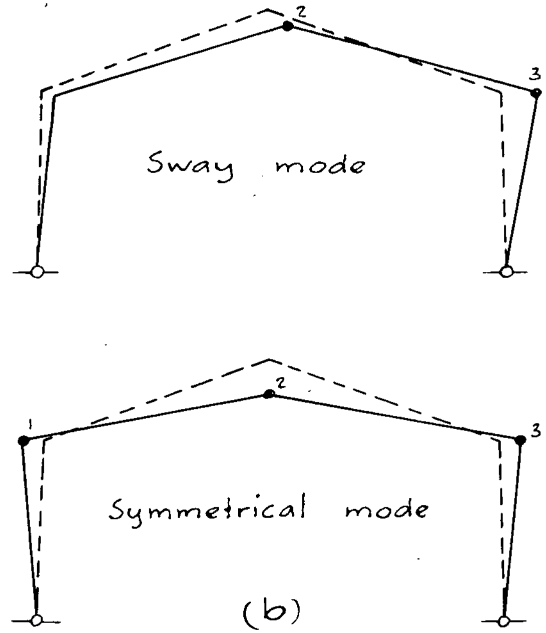
Gravity loading
Wind sidesway loading
A
B
C
2.33'
19.42'
18.70'
0.94'
0.71'
1.88'
4.22'
6.57'
8.91'
11.25'
13.59'
15.93'
18.28'
20.62'
22.96'
2.81'
4.68'
6.55'
8.12'
10.25'
12.16'
14.03'
15.90'
17.77'
19.12'
19.95'
20.79'
21.62'
22.45'
23.29'
24.12'
24.96'
25.79'
26.62'
27.04'
24.13' to ℄
8"
0.1875"
0.313"
8"-24" column
12"-24" beam
Assumed frame section used in elastic design calculations.

Figure 6



(a)

Elastic Moment Diagram



(b)

Collapse Mechanisms

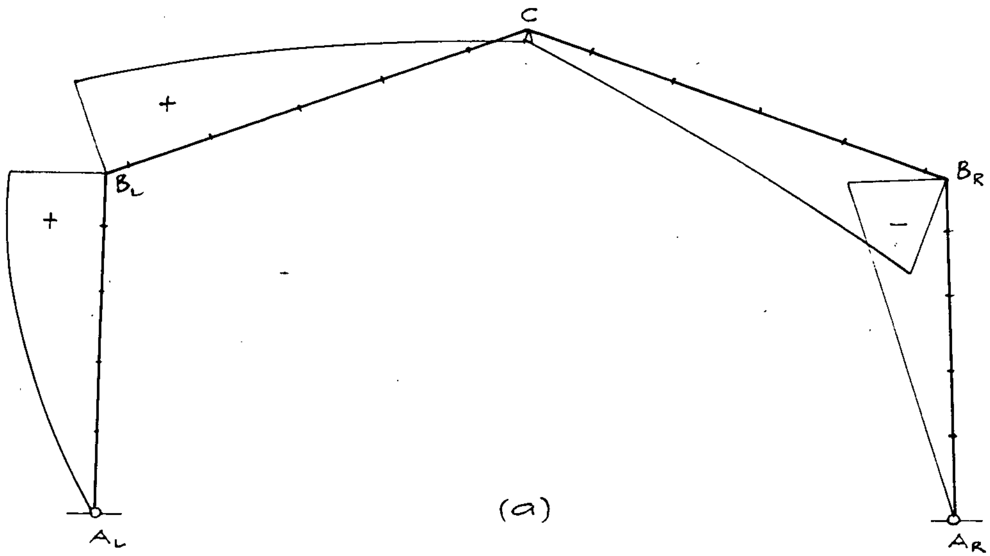
NOTES:

1. Elastic moments are plotted on the compression side of the frame.
2. Scale of moments – 1 in = 1500 in kips.
3. Key points of frame shown thus – ⌖²
4. Hinge points of mechanisms shown thus – •²

DESIGN CONDITION 1

Loading: 670 lbs per ft on horizontal projection

Figure 7



Elastic Moment Diagram

NOTES :

1. Elastic moments are plotted on the compression side of the frame.

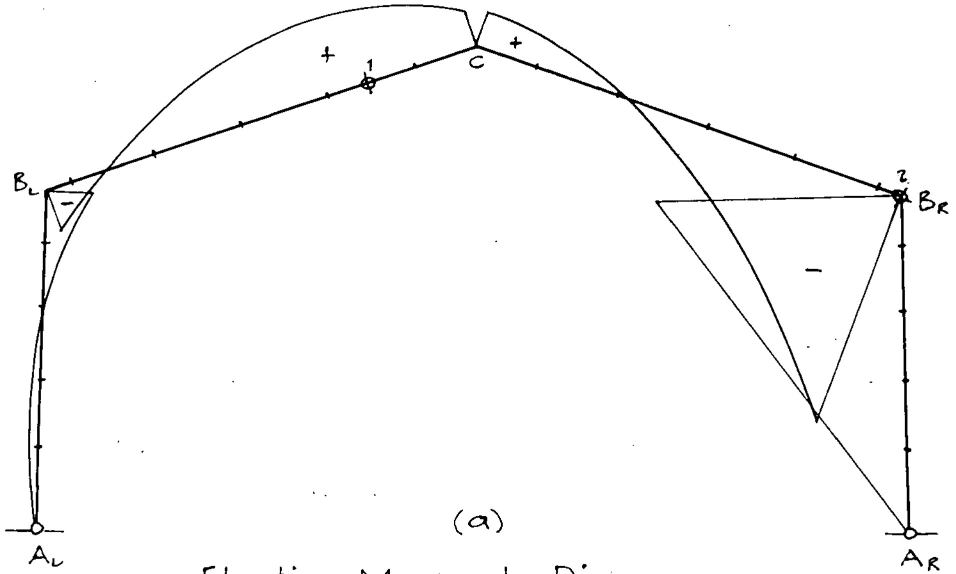
2. Scale of moments – 1 in = 1500 in kips.

DESIGN CONDITION 2

Loadings: 70 lbs per ft. on horizontal projection.

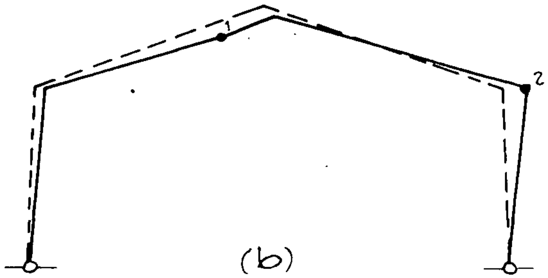
400 lbs per ft. on vertical projection (acting from left)

Figure 8



(a)

Elastic Moment Diagram



(b)

Collapse Mechanism

NOTES

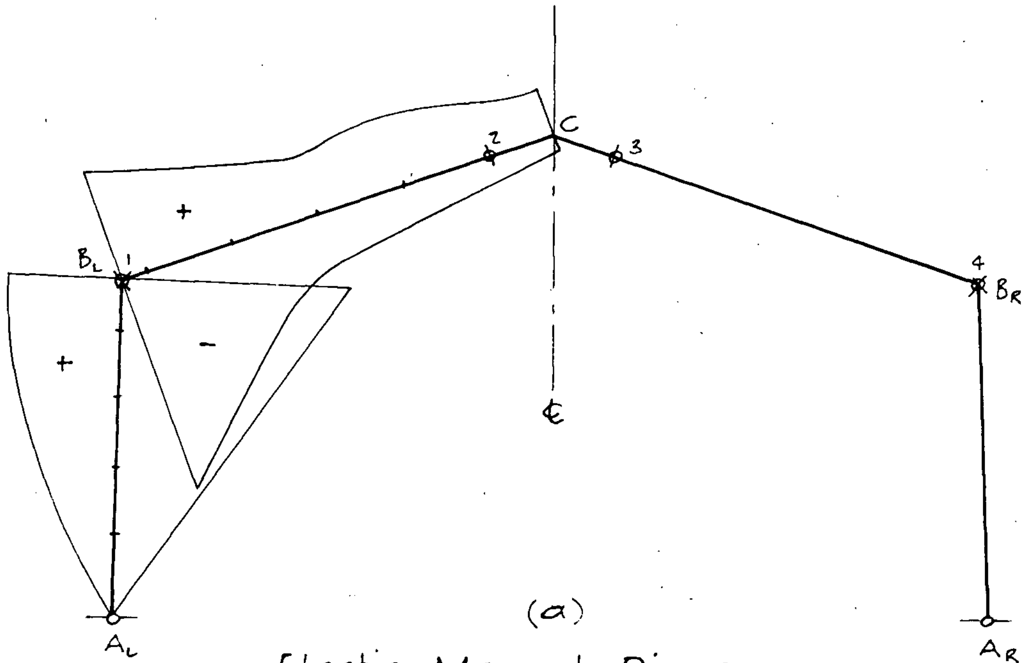
1. Elastic moments are plotted on the compression side of the frame.
2. Scale of moments - 1 in = 1500 in kips
3. Key points of frame shown thus - ϕ^2
4. Hinge points of mechanism shown thus - $\bullet^2$

DESIGN CONDITION 3

Loadings: 670 lbs per ft on horizontal projection.

400 lbs per ft on vertical projection (acting from left)

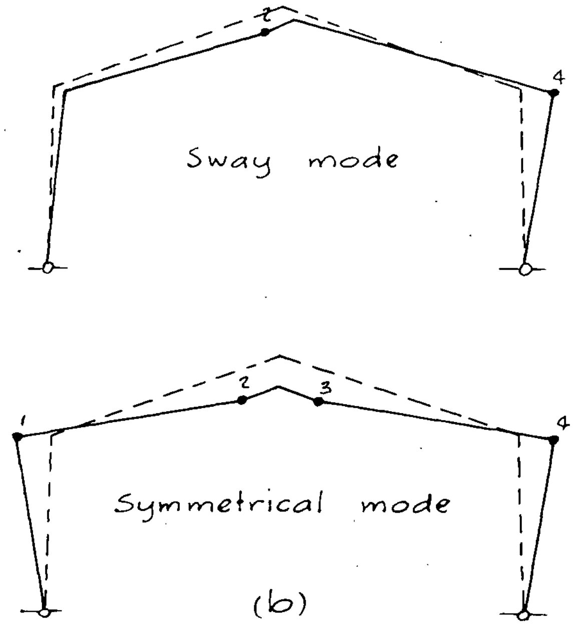
Figure 9



(a)

Elastic Moment Diagram

(showing positive and negative values of M_L)



(b)

Collapse Mechanisms

NOTES:

1. Scale of moments – 1 in = 1500 in kips.
2. Key points of frame shown thus – ⌀[2]
3. Hinge points of mechanisms shown thus – •[2]

VARIABLE LOADING CONDITION

Loadings: 600 lbs per ft. on horizontal projection.

400 lbs per ft. on vertical projection (acting from either direction)

Figure 10

33 1/3 percent as per Clause 15(e) of the Code). Thus the actual stress falls just within the allowable value. Elastic values for shearing stress throughout the frame are well within acceptable limits.

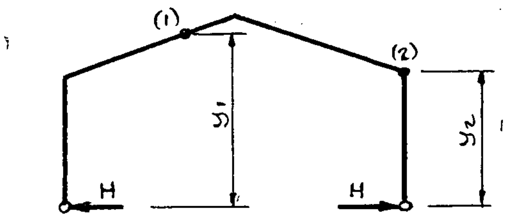
C. Limit Design Analysis by Constant Load Method

From previous considerations it is known that two methods are available for the determination of the load capacity of the bent under an assumed constant load condition. These are, firstly, the virtual work equality of equation (1) relating the internal and external work done under the failure loads during a displacement of the structure in the form of a collapse mechanism, and, secondly, by means of the zero load equation (5) defining the internal work done by the residual moments at the hinge points during a mechanism displacement.

Now, as the variations in elastic moment throughout the bent have been determined already the latter procedure will be adopted here as it presents a far more direct means of solution than does the former method.

First consider the frame in general terms, the number of hinge points necessary to develop a mechanism in general will be two, and if these hinge points are located at, say, frame sections (1) and (2), refer diagram below, equation (5) becomes:

$$F = \frac{M_{p1} . \theta_1 + M_{p2} . \theta_2}{M_1 . \theta_1 + M_2 . \theta_2}$$



where M_{p_1}, M_1, and θ_1, are moments and angular deformations at section (1), and M_{p_2}, M_2, and θ_2, are like quantities at section (2).

The residual moments at these sections, as defined by equation (5), are $(M_{p_1}-F.M_1)$ at section (1), and $(M_{p_2}-F.M_2)$ at section (2), with moments signs throughout conforming to the previously established convention.

The diagram shows the forces acting on the unloaded bent and it is at once evident from equilibrium requirements that reactions H must be equal and opposite. Thus the residual moments must be direct functions of ordinate 'y', and these moments can now be written as:

$$M_{p_1} - F.M_1 = y_1.H ,$$

and,
$$M_{p_2} - F.M_2 = - y_2.H ,$$

and solving for F gives:

$$F = \frac{M_{p_1}.(1/y_1) + M_{p_2}.(1/y_2)}{M_1.(1/y_1) + M_2.(1/y_2)} \qquad (11)$$

Angular deformations θ_1 and θ_2 are thus inversely proportional to ordinates y_1 and y_2, respectively. Quantities M_p, M, and y, are known for all frame sections and the value of F corresponding to any two sections can be readily determined. It is now only necessary to locate the sections resulting in the smallest value of F in order to establish the correct failure mechanism and the correct failure load.

The two hinges will not develop simultaneously and the first one to form will be at the section where yield stress is first reached in the frame, that is where the value of $\frac{M}{Ze}$ is a maximum. For design condition (3), as represented by the moment diagram in Figure 9, the maximum value of $\frac{M}{Ze}$ occurs at the knee joint section B_R and this therefore is one hinge point. The correct position for the second hinge can now be readily established by trial by means of equation (11). Such a procedure fixes the second hinge at between sections 18_L and 19_L. Relative values of rotation θ for the two hinges are figured to be 0.770 and 1.000, for θ_1 and θ_2 respectively, and thus F, obtained from equation (5), is as follows:

$$
\begin{array}{llll}
M_{p1}.\theta = 1240(0.770) = & 950, & M_1.\theta = 522(0.770) = & 403, \\
M_{p2}.\theta = 2705(1.000) = & 2705, & M_2.\theta = 1877(1.000) = & 1877, \\
\hline
\text{Numerator} = & 3655, & \text{Denominator} = & 2280,
\end{array}
$$

and, $$F = \frac{3655}{2280} = 1.605$$

This method has established the exact failure mechanism and has produced the true minimum value for the failure load.

If, on the other hand, the approximate method suggested previously, of locating the hinges at sections of minimum $\frac{M_p}{M}$, is applied it is found that the hinges are located at knee joint B_R

and at section 18_L, giving a load factor of 1.61. The difference between the exact and approximate methods is thus of insignificant consequence in this case. Other cases have also been considered and the difference between the two results has been found to be small in every instance. On the basis of these findings the approximate method for hinge point location will be adopted in the remainder of this study.

Now, in addition to loading condition (3), considered above, it is also necessary to evaluate the effects of loading condition (1), as shown in Figure 7. Sections of minimum $\frac{M_p}{M}$ are also indicated in the figure. The general symmetry of the moment diagram is at once evident and this fact immediately suggests the possibility of a symmetrical failure mode with hinges forming at all three key points, 1, 2, and 3 (see Figure 7). For this mechanism relative values of θ are found to be, $\theta_1 = \theta_3 = 1.000$, and $\theta_2 = 1.382$, and solving for F:

$$
\begin{array}{lcrclcr}
2705(1.000) & = & 2705\ , & \quad & 1212(1.000) & = & 1212\ , \\
980(1.382) & = & 1355\ , & & 392(1.382) & = & 550\ , \\
2705(1.000) & = & 2705\ , & & 1212(1.000) & = & 1212\ , \\
\hline
\text{Numerator} & = & 6765\ , & & \text{Denominator} & = & 2974\ ,
\end{array}
$$

and, $$F = \frac{6765}{2974} = 2.27.$$

That the sway mechanism with hinges at points 1 (or 3), and 2, produces an identical value for F will be apparent from the relationship between Θ's for the two mechanisms.

Thus limiting values for F for loading conditions (1) and (3) are 2.27 and 1.61, respectively. Comparing these values it would appear that condition (3) is critical, however this condition involves all external loads including wind and a smaller load factor can be used for this case than for condition (1) which excludes the wind loadings. Recommended design values for load factors conforming to AISC requirements are as follows:

(a) 1.41, for all forces including wind,

(b) 1.88, for all forces excluding wind.

The actual additional margins of safety provided by the bent over and above these recommended values are 1.135 and 1.21 for conditions (3) and (1), respectively, and the former condition therefore definitely governs.

The failure load factor for the bent under the loading assumptions of the constant load method is therefore 1.61, corresponding to an additional margin of safety over specification requirements of 1.135. The failure mechanism is as shown in Figure 9.

D. Limit Design Analysis for True Variable Loading

The actual loading to which the bent is subjected consists in a constant dead load and a random pattern of applications of gravity live load and wind, and the moments which must be taken into account in the variable loading analysis will be those resulting from each of these distinctly independent loadings. As wind is involved in this load pattern the specification requirement for load factor will be 1.41.

It will be clear that the case of gravity load without wind need not be considered (and certainly not with a higher safety requirement) in this variable loading analysis as gravity alone represents just a portion of this total actual loading.

Now, as previously discussed, two modes of failure are possible under variable loading. Firstly, the structure may fail as a mechanism through incremental collapse with true failure load obtaining when the following expression is minimized:

$$F_L = \frac{\Sigma(M_p - F_D.M_D).\theta}{\Sigma M_L.\theta} \qquad (7)$$

Secondly, failure may occur through alternating plasticity at a localized section of the structure with failure given when the following expression is minimized:

$$F_L' = \frac{2.\underline{M_y}}{\overline{M_L}} \qquad (9)$$

Solutions for these equations can be readily obtained once moments M_L and $\overline{M_L}$ are known. Both the wind and gravity loads are involved in these terms and the magnitude of the contribution due to wind will be composed of the maximum load acting on one side of the bent and an equal or lesser wind effect acting independently as a reversed loading from the opposite direction.

The magnitude of this reversed wind effect is not actually specified for the bent, nor is it required in the elastic analysis where the loading criterion need only be in terms of the maximum wind acting in one direction. The reversed wind effect is also of no interest for the previous constant load analysis for which the same criterion applies.

The severest possible condition involves a reversed wind equal in magnitude to the maximum loading of 20.0 lbs. per sq. ft. Values of M_L and $\overline{M_L}$ have been computed for this reversed loading and these values are given in Table lV. The variation in M_L throughout the bent, for both positive and negative amounts, is shown in Figure 10, together with the location of sections where minimum values of the quantity $\frac{(M_p - F_D . M_D)}{M_L}$ occur, a dead load factor of 1.25* being employed in the values for the latter quantity.

The symmetry of this figure again immediately suggests the probability of a symmetrical failure mechanism with hinges at points 1, 2, 3, and 4, of the figure. Hinges 1 and 4 being at sections B_L and B_R, respectively, and hinges 2 and 3 at sections 19_L and 19_R, respectively. Relative values of θ are figured to

* This factor of 1.25 is based on an evaluation of probable errors involved in the quantities entering into the design procedure when dead loads only are present. Details of this evaluation will not be included here, but reference is made to papers on the subject of Factors of Safety and particularly to the paper by A. Freudenthal, entitled "Safety and Probability of Structural Failure", Proceedings, No. 468, ASCE, August 1954.

be, $\theta_1 = \theta_4 = 1.380$, and $\theta_2 = \theta_3 = 1.000$. Only one half of the frame need be considered and F_L, obtained from equation (7), becomes:

$$\begin{array}{lrlr}
(2705 - (1.25)127)1.380 & = 3510\ , & (1750)1.380 & = 2413\ , \\
(1175 - (1.25)47)1.000 & = 1115\ , & (437)1.000 & = 437\ , \\
\hline
\text{Numerator} & = 4625\ , & \text{Denominator} & = 2850\ ,
\end{array}$$

and, $$F_L = \frac{4625}{2850} = 1.62.$$

The corresponding sway mechanism with hinges at points 1 and 3 (or 2 and 4) also produces a value for F_L of 1.62 as will again be evident from the similarity in the relative values of θ. The load factor for incremental failure of the bent is thus 1.62.

Considering now the possibility of alternating plasticity. It is evident from equation (9) that the smallest value for F_L' occurs when $\frac{M_y}{\overline{M}_L}$ is minimized, and reference to Table lV shows that the smallest value of this quantity is obtained in the roof beam at the knee joints where M_y and $\overline{M}_L$ are 2300 and 2615 in. kips, respectively. Thus the load factor for failure by alternating plasticity is:

$$F_L' = 2.\ \frac{2300}{2615} = 1.76.$$

Load factor F_L is smaller, at 1.62, and therefore incremental failure is critical for the bent; this representing the true live load factor to produce failure of the bent under actual loading conditions, as envisaged by the variable load method of analysis. As previously noted a specification load factor of 1.41 is appropriate to the live loading involved and the additional safety margin over this specification requirement is $\frac{1.62}{1.41}$, equals 1.15. The collapse mechanism may take the form of either the symmetrical mode or the sway mode, as shown in Figure 10.

3.02 Analysis of Double Bay Gable Bent

A. General

The double bay bent to be considered is a standardized design employing outside column and roof members of identical section and dimensions with those used for the single bay bent. Reference is made to Figures 4 and 5 for the dimensions and details of these sections. A vertical 12" × $6\frac{1}{2}$" × 36.0 lb. rolled wide-flange section replaces the variable section fabricated columns for the central support, and details of the connection between the wide-flange and roof members is shown in Figure 5. All other connections and stiffener arrangements are as for the single bay bent. The spacing of frames is 20'-0" on centre.

Design requirements are also as described for the previous structure, and loading conditions are identical and, briefly, as follows:

Design condition (1),

uniform load on horizontal projection of 670 lbs. per lin.ft.

Design condition (2),

wind load on vertical projection of 400 lbs. per lin. ft.
plus dead load of 70 lbs. per lin. ft.

Design condition (3),

wind load on vertical projection of 400 lbs. per lin. ft.
plus load on horizontal projection of 670 lbs. per lin. ft.

B. Elastic Analysis

Procedures employed in the elastic analysis are as previously described for the single bay bent. All supports are assumed hinged.

Resulting moments at sections throughout the frame are given in Table V for individual gravity and wind loadings. Shears and thrusts are not shown as their effects are again small and can be neglected in the limit design analysis.

Moment diagrams for each loading condition, plotted from the tabulated results are shown in Figures 11, 12, and 13. For these diagrams moments are plotted on the side of the frame at whch they create compressive stress.

Fibre stresses are not required for this study but it is interesting to note that the severest stress occurs in the roof beams at the central joint and is equal to 24.6 ksi. This is appreciably less than the allowable value but the condition is tolerated in the interests of standardization of components.

C. Limit Design Analysis by Constant Load Method

With elastic moments known a solution for load factor F is again most readily obtained in terms of equation (5). As with the single bay bent loading conditions (1) and (3) only need be considered.

Investigating loading condition (1). The general symmetry of Figure 11 indicates that a symmetrical failure mechanism is to

Table V Moments for Double Bay Bent.

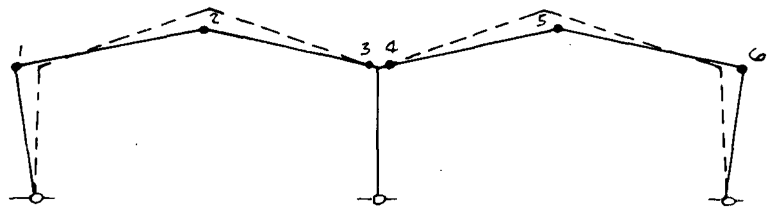
Frame Section No.	Dead Load Moments in. kips	Live Load Moments in. kips			Moments for Constant Load Analysis, in. kips			Plastic Moment M_p in. kips
		Gravity Loading	Wind from L.	Wind from R.	Condition 1.	Condition 2.	Condition 3.	
3_{LL}	- 35	- 304	+ 280	- 89	- 339	+ 245	- 59	1180
5_{LL}	- 61	- 527	+ 429	- 161	- 588	+ 368	- 159	1590
7_{LL}	- 92	- 779	+ 529	- 234	- 871	+ 437	- 342	2030
9_{LL}	- 120	- 1032	+ 554	- 306	- 1152	+ 434	- 598	2505
B_L	- 141	- 1214	+ 542	- 360	- 1355	+ 401	- 812	2705
11_{LL}	- 109	- 936	+ 499	- 340	- 1045	+ 390	- 546	2560
13_{LL}	- 45	- 390	+ 391	- 288	- 435	+ 346	- 44	2130
15_{LL}	–	–	+ 254	- 237	–	+ 254	+ 254	1740
17_{LL}	+ 26	+ 222	+ 113	- 196	+ 248	+ 139	+ 361	1445
19_{LL}	+ 35	+ 295	- 40	- 134	+ 330	- 5	+ 290	1175
C_L	+ 30	+ 258	- 163	- 96	+ 288	- 133	+ 125	980
19_{LR}	+ 29	+ 247	- 164	- 4	+ 276	- 135	+ 112	1175
17_{LR}	+ 19	+ 161	- 165	+ 135	+ 180	- 146	+ 15	1445
15_{LR}	- 11	- 97	- 166	+ 265	- 108	- 177	- 274	1740
13_{LR}	- 63	- 537	- 167	+ 396	- 600	- 230	- 767	2130
11_{LR}	- 130	- 1112	- 168	+ 528	- 1242	- 298	- 1410	2560
D_{LR}	- 164	- 1406	- 168	+ 580	- 1570	- 332	- 1738	2705
$D_{col.}$	–	–	- 748	+ 748	–	- 748	- 748	1682
D_{RL}	- 164	- 1406	+ 580	- 168	- 1570	+ 416	- 990	2705
11_{RL}	- 130	- 1112	+ 528	- 168	- 1242	+ 398	- 714	2560
13_{RL}	- 63	- 537	+ 396	- 167	- 600	+ 333	- 204	2130
15_{RL}	- 11	- 97	+ 265	- 166	- 108	+ 254	+ 157	1740
17_{RL}	+ 19	+ 161	+ 135	- 165	- 180	+ 154	+ 315	1445
19_{RL}	+ 29	+ 247	- 4	- 164	+ 276	+ 25	+ 272	1175
C_R	+ 30	+ 258	- 96	- 163	+ 288	- 66	+ 192	980
19_{RR}	+ 35	+ 295	- 134	- 40	+ 330	- 99	+ 196	1175
17_{RR}	+ 26	+ 222	- 196	+ 113	+ 248	- 170	+ 52	1445
15_{RR}	–	–	- 237	+ 254	–	- 237	- 237	1740
13_{RR}	- 45	- 390	- 288	+ 391	- 435	- 333	- 723	2130
11_{RR}	- 109	- 936	- 340	+ 499	- 1045	- 449	- 1385	2560
B_R	- 141	- 1214	- 360	+ 542	- 1355	- 501	- 1715	2705
9_{RR}	- 120	- 1032	- 306	+ 554	- 1152	- 426	- 1438	2505
7_{RR}	- 92	- 779	- 234	+ 529	- 871	- 326	- 1105	2030
5_{RR}	- 61	- 527	- 161	+ 429	- 588	- 222	- 749	1590
3_{RR}	- 35	- 304	- 89	+ 280	- 339	- 124	- 428	1180

Table VI Moments for Double Bay Bent.

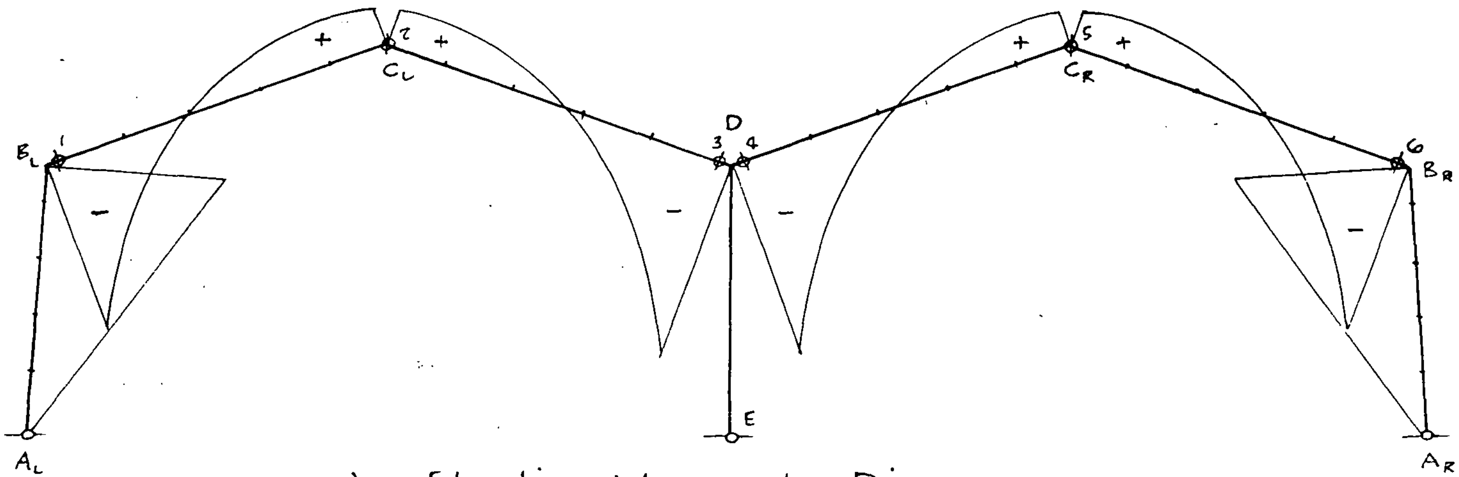
Frame Section No.	Dead Load Moments, in. kips	Live Load Moments for Variable Load Analysis, in. kips			Plastic Moments M_P, in. kips	Yield Moments M_y, in kips
		Greatest Positive M_L	Greatest Negative M_L	Range in Moments $\bar{M}_L$		
3_{LL}	- 35	+ 289	- 393	682	1180	1080
5_{LL}	- 61	+ 429	- 688	1117	1590	1450
7_{LL}	- 92	+ 529	- 1013	1542	2030	1825
9_{LL}	- 120	+ 554	- 1338	1892	2505	2240
B_L	- 141	+ 542	- 1574	2116	2705	2300
11_{LL}	- 109	+ 499	- 1276	1775	2560	2260
13_{LL}	- 45	+ 391	- 678	1069	2130	1800
15_{LL}	-	+ 254	- 237	491	1740	1540
17_{LL}	+ 26	+ 335	- 196	531	1445	1295
19_{LL}	+ 35	+ 295	- 134	429	1175	1060
C_L	+ 30	+ 258	- 163	421	980	890
19_{LR}	+ 29	+ 247	- 164	411	1175	1060
17_{LR}	+ 19	+ 296	- 165	461	1445	1295
15_{LR}	- 11	+ 265	- 263	528	1740	1540
13_{LR}	- 63	+ 396	- 704	1100	2130	1800
11_{LR}	- 130	+ 528	- 1280	1808	2560	2260
D_{LR}	- 164	+ 580	- 1574	2154	2705	2300
$D_{col.}$	-	+ 748	- 748	1496	1682	1515
D_{RL}	- 164	+ 580	- 1574	2154	2705	2300
11_{RL}	- 130	+ 528	- 1280	1808	2560	2260
13_{RL}	- 63	+ 396	- 704	1100	2130	1800
15_{RL}	- 11	+ 265	- 263	528	1740	1540
17_{RL}	+ 19	+ 296	- 165	461	1445	1295
19_{RL}	+ 29	+ 247	- 164	411	1175	1060
C_R	+ 30	+ 258	- 163	421	980	890
19_{RR}	+ 35	+ 295	- 134	429	1175	1060
17_{RR}	+ 26	+ 335	- 196	531	1445	1295
15_{RR}	-	+ 254	- 237	491	1740	1540
13_{RR}	- 45	+ 391	- 678	1069	2130	1800
11_{RR}	- 109	+ 499	- 1276	1775	2560	2260
B_R	- 141	+ 542	- 1574	2116	2705	2300
9_{RR}	- 120	+ 554	- 1338	1892	2505	2240
7_{RR}	- 92	+ 529	- 1013	1542	2030	1825
5_{RR}	- 61	+ 429	- 688	1117	1590	1450
3_{RR}	- 35	+ 289	- 393	682	1180	1080

NOTES:

1. Elastic moments are plotted on the compression side of the frame.
2. Scale of moments – 1 in. = 1500 in. kips.
3. Key points of frame shown thus – ϕ^{2}
4. Hinge points of mechanism shown thus – $\bullet^{2}$



(b) Collapse Mechanism



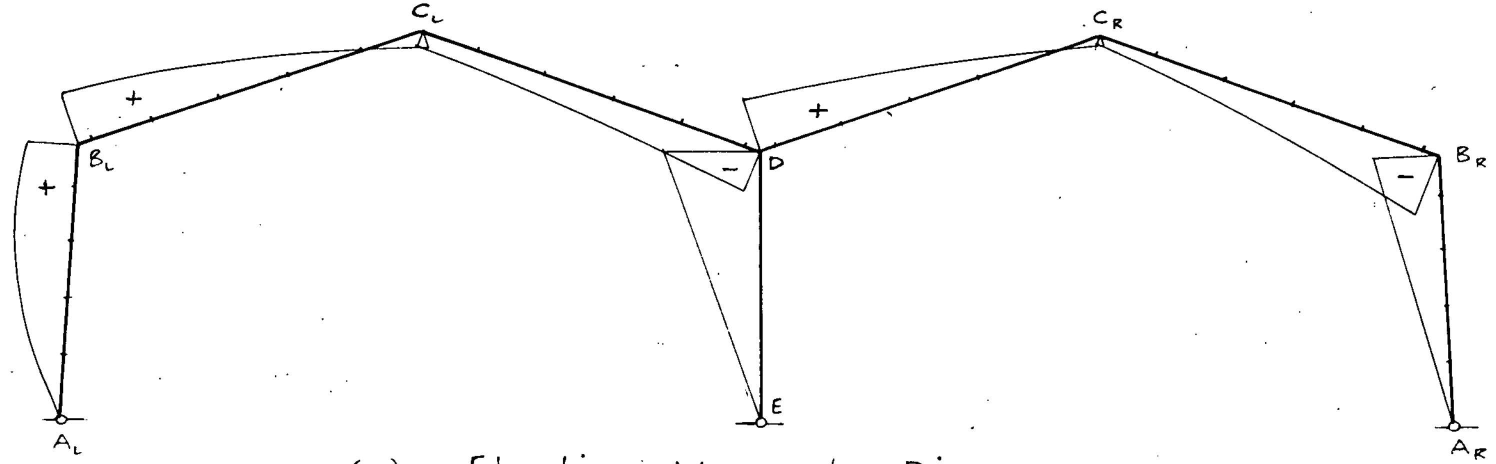
(a) Elastic Moment Diagram

DESIGN CONDITION 1 Loading: 670 lbs per ft. on horizontal projection.

Figure 11

NOTES : 1. Elastic moments are plotted on the compression side of the frame.

2. Scale of moments - 1 in = 1500 in. kips.



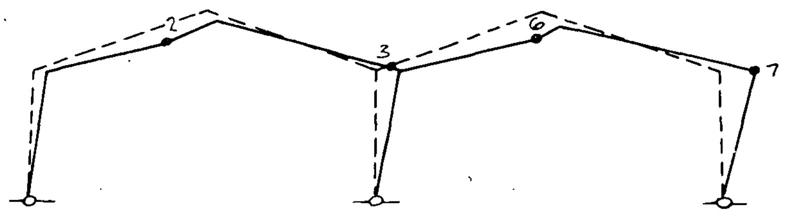
(a) Elastic Moment Diagram

DESIGN CONDITION 2 Loading: 70 lbs per ft. on horizontal projection
400 lbs per ft. on vertical projection (acting from left)

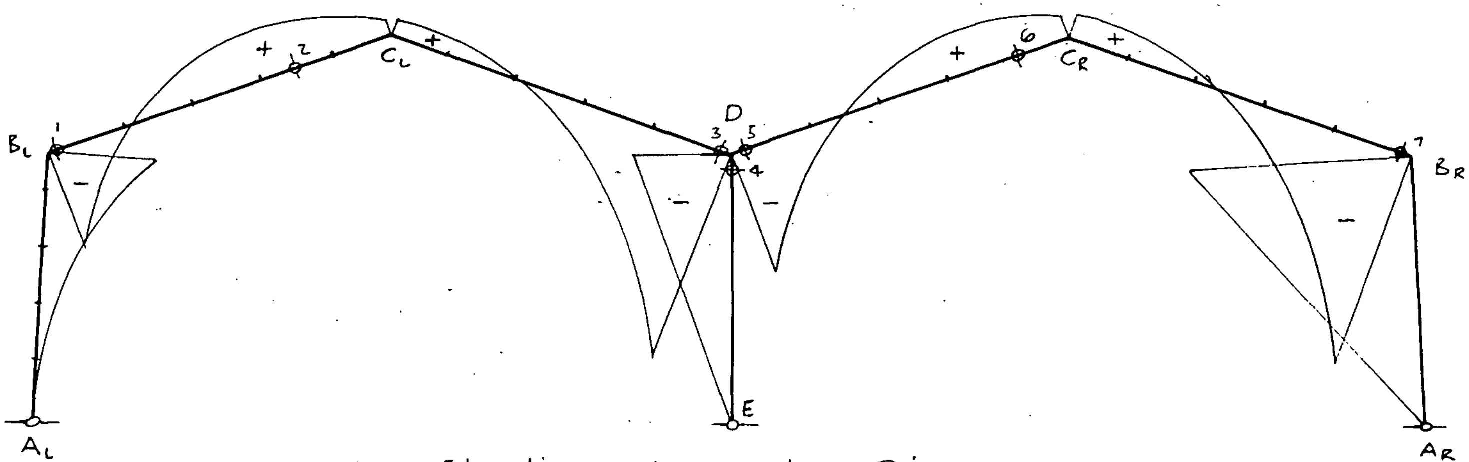
Figure 12

NOTES:

1. Elastic moments are plotted on the compression side of the frame.
2. Scale of moments – 1 in = 1500 in kips.
3. Key points of frame shown thus – ⌀ 2
4. Hinge points of mechanism shown thus – • 2



(b) Collapse Mechanism



(a) Elastic Moment Diagram

DESIGN CONDITION 3

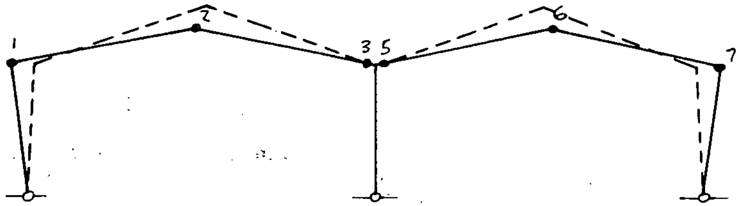
Loadings: 670 lbs per ft. on horizontal projection

400 lbs per ft. on vertical projection (acting from left)

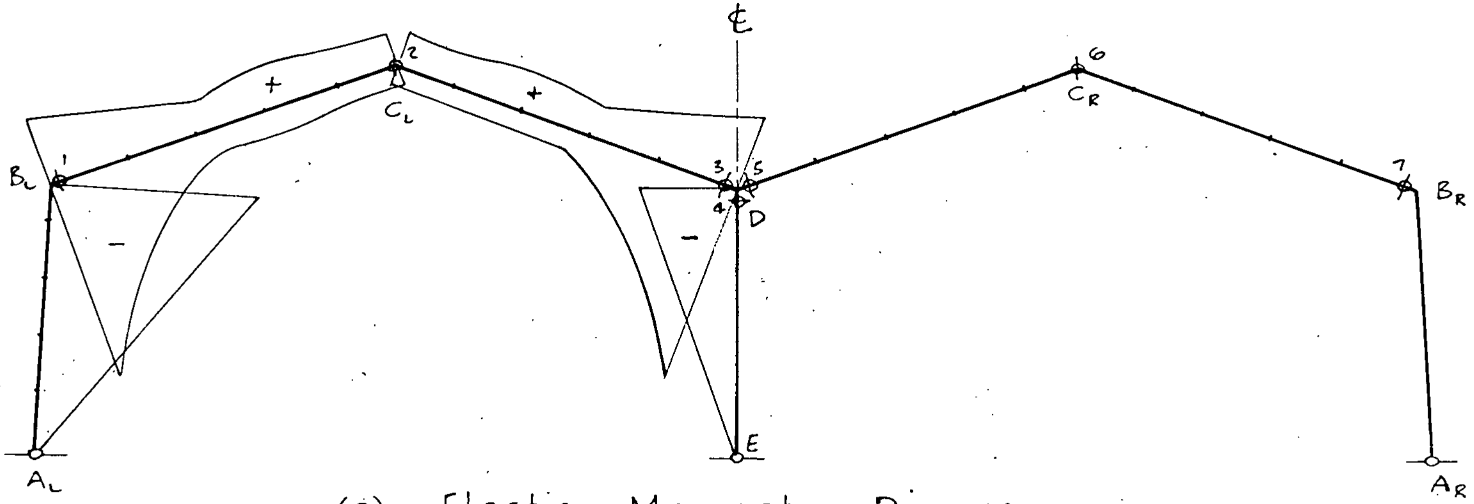
Figure 13

NOTES :

1. Scale of moments – 1 in = 1500 in kips.
2. Key points of frame shown thus – $\oplus^2$
3. Hinge points of mechanism shown thus – $\bullet^2$



(b) Collapse Mechanism



(a) Elastic Moment Diagram
(showing positive and negative values of M_L)

VARIABLE LOADING CONDITION

Loadings: 600 lbs per ft. on horizontal projection
400 lbs per ft. on vertical projection
(acting from either direction)

Figure 14

be expected. It has been determined that maximum elastic stress occurs in the roof beams at the central joint and, as yielding must first occur here, plastic hinges will be created simultaneously at these sections. Additional sections at which minimum values of $\frac{M_p}{M}$ occur are located at joints B and C and it will be fairly evident from inspection of Figure 11 that the critical mechanism will conform to the mode indicated in the same figure. Only one half of the structure on either side of centre line need be considered in evaluating F. The relative values of hinge rotation may be figured from the geometrical concept of instantaneous centres, as follows:

Relative frame dimensions and radii of rotation of frame elements are shown in Figure 15, from which,

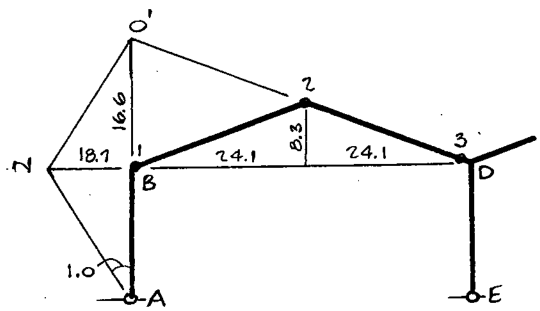


Figure 15.

$$B\hat{O}'N = \frac{18.7}{16.6} = 1.127$$

$$\text{thus, } \theta_1 = 1.0 - 1.127 = 2.127$$

$$\theta_2 = (1.127).2 = 2.254$$

$$\theta_3 = BO'N = 1.127$$

Now, substituting these values for θ, with the appropriate values for M_p and M, into equation (5), gives:

$$2705(2.127) = 5750, \qquad 1355(2.127) = 2880,$$

$$980(2.254) = 2210, \qquad 288(2.254) = 650,$$

$$2705(1.127) = 3050, \qquad 1570(1.127) = 1770,$$

Numerator = 11010 , Denominator = 5300 ,

thus, $F = \frac{11010}{5300} = 2.08.$

Check analysis of the alternative mechanisms confirms this as the true value of load factor for loading condition (1). It will be noted that on account of the symmetrical mode of failure the number of hinges involved in the mechanism is greater than the four required for a general collapse of the frame.

Considering now loading condition (3). Sections of the structure at which minimum values of $\frac{M_p}{M}$ occur are located in Figure 13. It should first be observed that hinges can develop at only two of the three key points at joint D, as will be evident if the statical equilibrium of this joint is considered; further, one of these hinges must be at the key point identified as 3 as yield stress is first reached in the frame at this section. A second hinge must be at key point 7 as stress here is equal to that at key point 3 and these two hinges will therefore develop simultaneously.

With these facts recognized the simplest procedure is now to regard the total mechanism for the frame as the compounding of two elementary mechanisms with one of these in each bay. A brief review of the previous analysis of the single bay bent will reveal that the critical total mechanism must consist of hinges at key points 2, 3, 6, and 7, as shown in Figure 16.

Again using the concept of instantaneous centres, refer

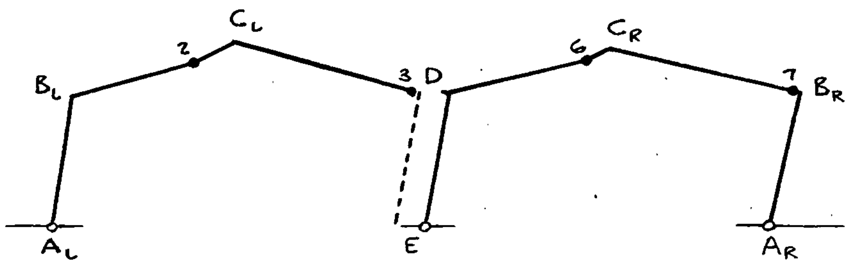


Figure 16.

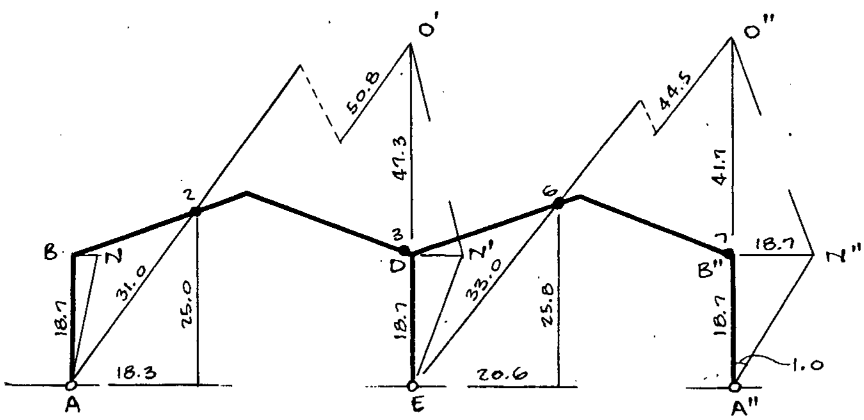


Figure 17.

Figure 17, the angles of rotation at hinge points are figured as follows:

$$B''\widehat{O''}N'' = \frac{18.7}{41.7} = 0.449$$

$$D\widehat{E}N' = \frac{44.5}{33.0}(0.449) = 0.605$$

$$D\widehat{O'}N' = \frac{18.7}{47.3}(0.605) = 0.239$$

$$B\widehat{A}N = \frac{50.8}{31.0}(0.239) = 0.391$$

thus,

$$\theta_2 = 0.391 + 0.239 = 0.630$$

$$\theta_3 = 0.605 + 0.239 = 0.844$$

$$\theta_6 = 0.605 + 0.449 = 1.054$$

$$\theta_7 = 1.000 + 0.449 = 1.449$$

and substituting these values of θ, and the appropriate values of M_p and M, into equation (5), gives:

1310(0.630)	= 825 ,		320(0.630)	= 205 ,	
2705(0.844)	= 2285 ,		1738(0.844)	= 1465 ,	
1175(1.054)	= 1240 ,		272(1.054)	= 287 ,	
2705(1.449)	= 3920 ,		1715(1.449)	= 2483 ,	
Numerator	= 8270 ,		Denominator	= 4440 ,	

thus, $$F = \frac{8270}{4440} = 1.86.$$

Failure load factors for loading conditions (1) and (3) are therefore 2.08 and 1.86, respectively. However allowance must be made for the different forms of loading involved in these conditions, as previously discussed in the case of the single bent. The additional margins of safety involved in the above factors, over and above the appropriate recommended values, figure at $\frac{2.08}{1.88} = 1.11$, for loading condition (1), and $\frac{1.86}{1.41} = 1.32$, for loading condition (3).

The governing loading condition for the frame under the assumptions of the constant load method is thus actually condition (1). The critical load factor is therefore 2.08, and the corresponding failure mechanism is as shown in Figure 11.

D. Limit Design Analysis for True Variable Loading

The actual pattern of loading to be considered is as previously discussed for the single bay bent, and as with the earlier bent a reversed wind loading of 20.0 lbs. per sq. ft. is assumed. The variations in maximum positive and negative moments corresponding to this condition are tabulated in Table Vl and plotted in Figure 14.

The general symmetry of this moment diagram is at once evident and this fact again suggests the probability that the critical mechanism will also be symmetrical in form. Such a failure mode involves a total of six hinges at key points 1, 2, 3, 5, 6, and 7. Only one half of the frame need be considered and, from previously, the relative values of hinge rotation θ at points 1, 2, and 3, figure at $\theta_1 = 2.127$, $\theta_2 = 2.254$, and $\theta_3 = 1.127$, and the equation for F_L becomes:

$$
\begin{array}{rclrcl}
2.127(2705 - (1.25)141) & = & 5380\ , & (1574)2.127 & = & 3350\ , \\
2.254(980 - (1.25)30) & = & 2120\ , & (258)2.254 & = & 580\ , \\
1.127(2705 - (1.25)162) & = & 2820\ , & (1574)1.127 & = & 1780\ , \\
 & & \overline{} & & & \overline{} \\
\text{Numerator} & = & 10320\ , & \text{Denominator} & = & 5710\ ,
\end{array}
$$

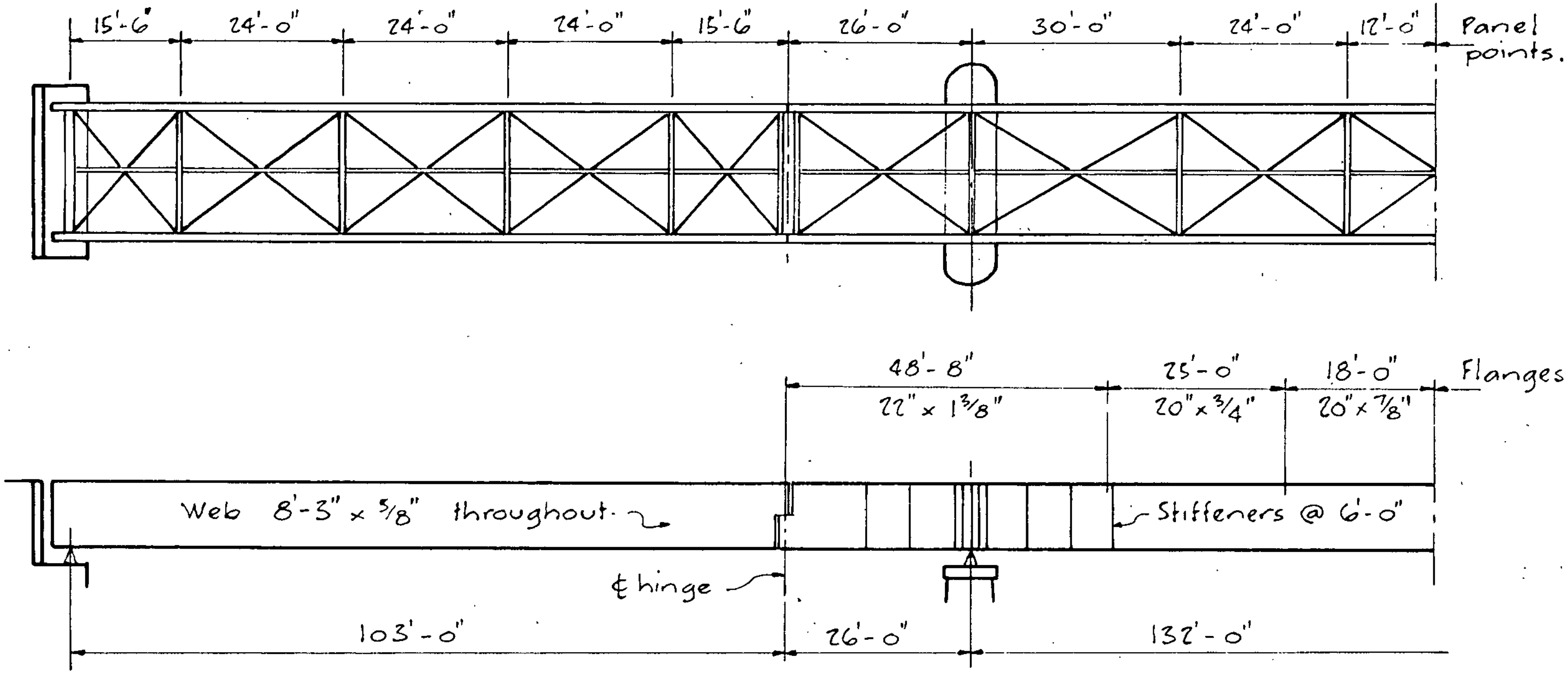
from which, $F_L = \dfrac{10320}{5710} = 1.80.$

Further consideration of the alternative modes of failure involving other possible combinations of key points, proves this to be the true mechanism for incremental collapse.

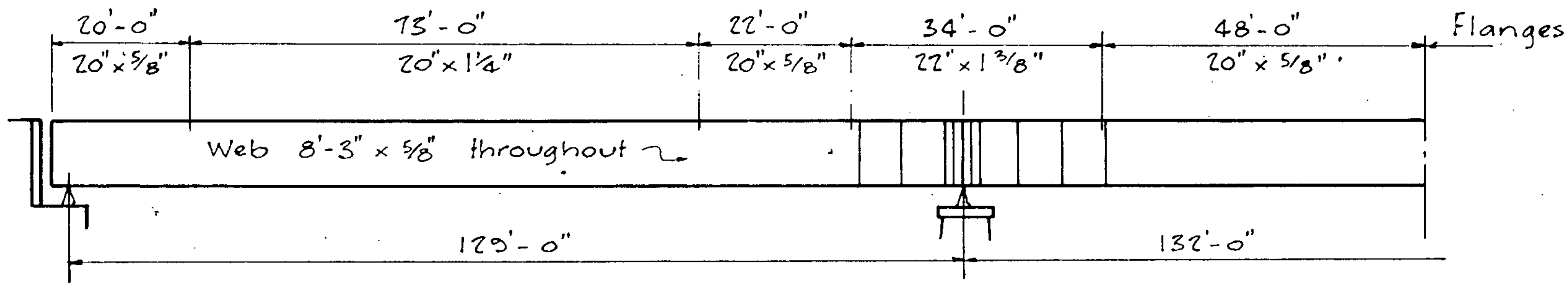
For failure through alternating plasticity on the other hand reference to Table V1 shows that the smallest value of $\frac{M_y}{\overline{M}_L}$ is associated with the central column joint D where values of My and $\overline{M}_L$ are 1515 and 1496 in. kips, respectively. The limiting value for F_L' from equation (9) is thus:

$$F_L' = \frac{1515}{1496}(2) = 2.03$$

The critical failure load for the condition of variable loading is thus produced through incremental collapse and corresponds to a failure load factor of 1.80. The additional margin of safety provided by the bent, over and above specification requirements is $\frac{1.80}{1.41}$, equals 1.28. The mode of failure is as shown in Figure 14.

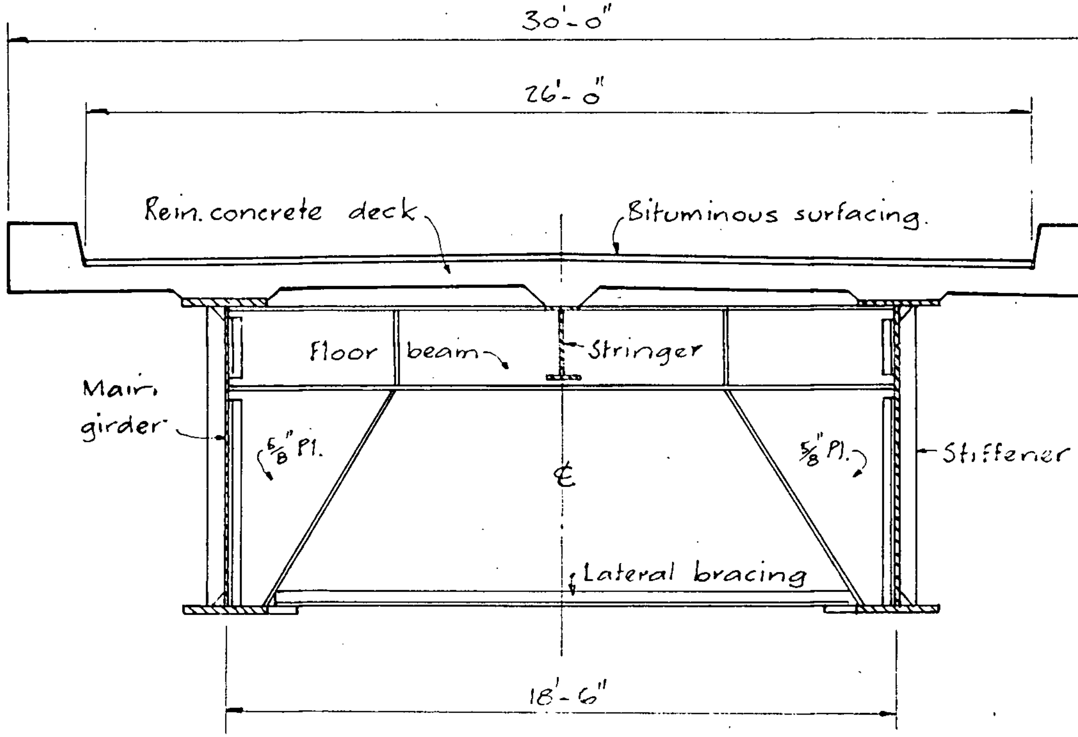


Details of Cantilever Girder Design



Details of Continuous Girder Design

Figure 18



Typical section showing cross frame

Figure 19

3.03 Analysis of Bridge Girders

General

The complete three span, two lane, highway bridge, from which two examples to be later considered are taken, is shown in general arrangement in Figure 3(b). The two examples chosen for study involve:

1. The actual bridge, consisting of two cantilever girders which overhang the central piers and support simple girder spans at each end of the bridge via bearings at the outer ends of the cantilevers.
2. A hypothetical continuous girder variant, which consists of twin girders continuous throughout the entire length of the bridge.

General structural details of the plate girders and deck of the actual bridge are shown in Figures 18 and 19. The main girders are of built-up welded section throughout, consisting of ASTM A-7 structural steel webs and flanges. The deck is composed of cast-in-place concrete. No longitudinal shear connectors are provided between deck and girders and composite action is therefore not allowed for in the designs.

In the case of design 1, involving the cantilever girders, interest will be confined to the portions of the cantilevers between the central supports in which reversals in moment take place due to the movement of the live loads. The simple end beams are of no special interest from the point of view of the study. The cantilever girders are statically determinate and are therefore not subject to incremental collapse, however as a result of the reversals in moment they are subject to alternating plasticity and must be examined for this mode of failure.

The continuous girders of design 2 are statically indeterminate and may be subject to failure through either incremental collapse or alternating plasticity. Both of these possibilities must therefore be considered for this case.

Both the actual and hypothetical structures are designed to withstand the standard AASHO H20-S16 loading in each lane, to the following specifications:

(a) CSA Specifications for Steel Highway Bridges, 1952,

(b) AASHO Standard Specifications for Highway Bridges,

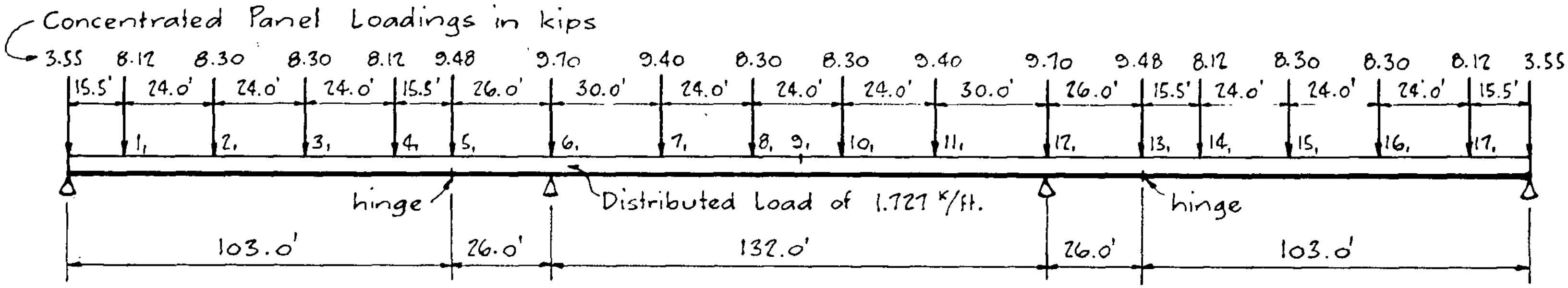
(c) ACI Building Code Requirements for Reinforced Concrete.

In all instances where alternative requirements occured in these several specifications the severest condition was accepted as governing.

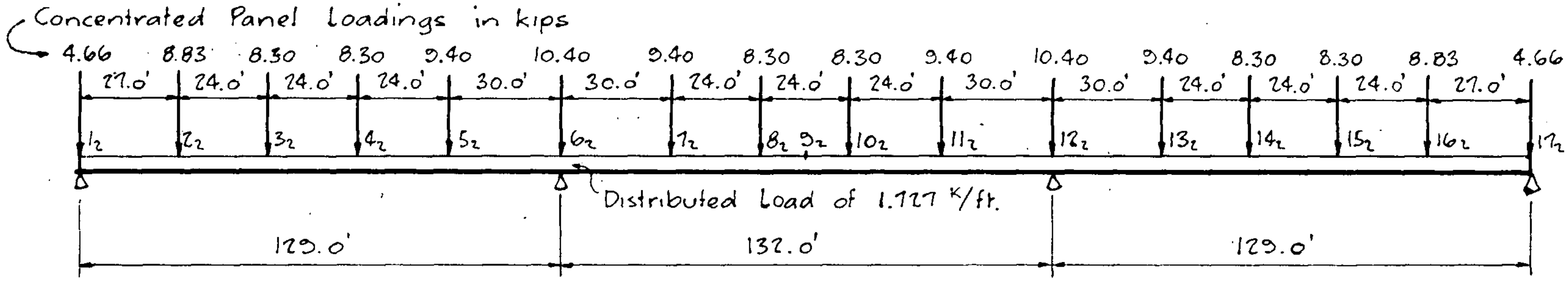
Table VII Moments for Cantilever Girder.

Girder Panel No.	Maximum Bending Moments in ft. kips due to:				Maximum Total Moments, ft. kips.		Maximum Live Load Moments, ft. kips		Maximum Range in Live Load Moments ft. kips
	Dead loading	Live Loading in Span ① *	Live Loading in Span ②	Live Loading in Span ③ *	Positive Moments (cols. 2+4)	Negative Moments (cols 2+3+5)	Positive Moments (col. 4)	Negative Moments (cols. 3+5)	
Col. 1.	Col. 2.	Col. 3.	Col. 4.	Col. 5.	Col. 6.	Col. 7.	Col. 8.	Col. 9.	Col. 10.
6.	- 3565	- 3010	-	-	-	- 6575	-	- 3010	3010
7.	- 415	- 2330	+ 2605	- 615	+ 2190	- 3360	+ 2605	- 2945	5550
8.	+ 785	- 1780	+ 3610	- 1110	+ 4395	- 2105	+ 3610	- 2890	6500
9.	+ 900	- 1505	+ 3700	- 1360	+ 4600	- 2050	+ 3700	- 2865	6565
10.	+ 785	- 1230	+ 3610	- 1605	+ 4395	- 2105	+ 3610	- 2890	6500
11.	- 415	- 680	+ 2605	- 2100	+ 2190	- 3360	+ 2605	- 2945	5550
12.	- 3565	-	-	- 2715	-	- 6575	-	- 3010	3010

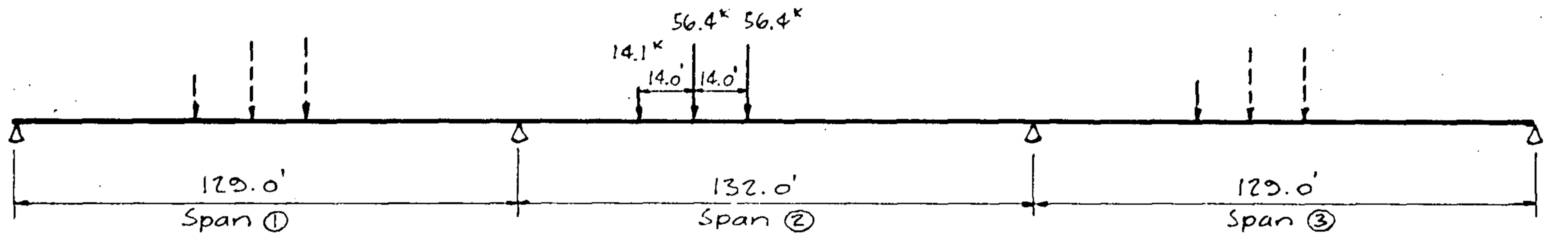
* Note: As axle loadings shown in Figure 20(c) are reversible, maximum moments shown in column 3 may be transposed, with inversion, to column 5. Column 5 moments are similarly transferable, with inversion, to column 3. Maximum moments shown in columns 6, 7, 8, 9, and 10, are for the severest of these axle loading conditions.



(a) Dead Loadings of Cantilever Girder Design

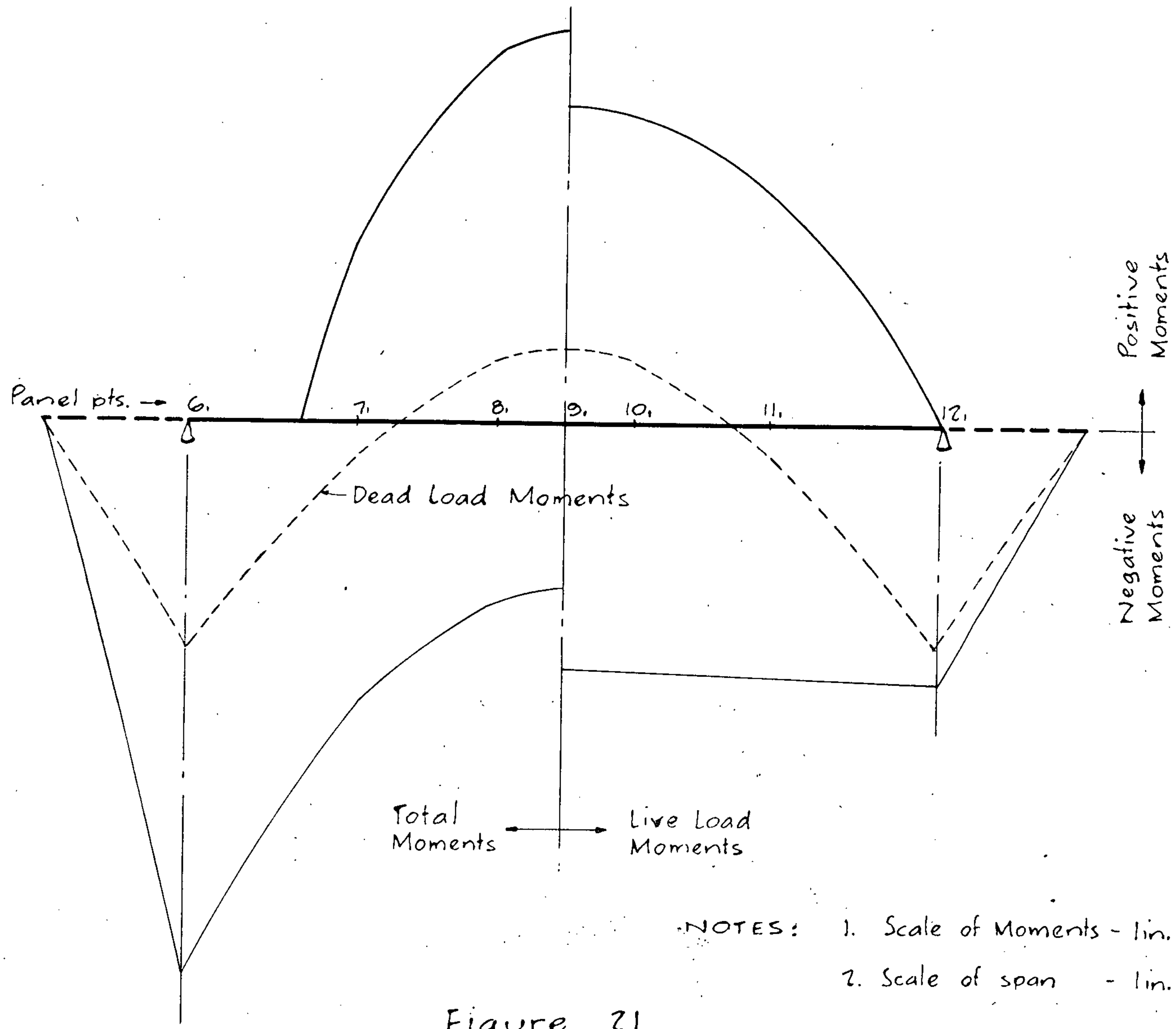


(b) Dead Loadings of Continuous Girder Design



(c) Live Loadings for Girder Designs

Figure 20



NOTES: 1. Scale of Moments - 1 in. = 2000 ft. kips

2. Scale of span - 1 in. = 30 ft.

Figure 21

3.031 Analysis of Cantilever Girder Design

A. Elastic Analysis

Dead weights of the steel system and concrete deck for the cantilever design are shown in Figure 20(a). The intensities and form of live load considered is shown in Figure 20(c), this conforming to the AASHO H20-S16 pattern of loading and including the specified impact allowance.

The combinations of these basic loadings which were assumed in the elastic design for maximum positive and negative moments over the centre span portion of the cantilevers are as follows:

(1) For maximum positive moments, the entire dead load system of the cantilevers and end beams (Figure 20(a)), together with the live load (Figure 20(c)) applied between the centre span supports only.

(2) For maximum negative moments, the entire dead load system of the cantilevers and end beams (Figure 20(a)), together with the action of live loads (Figure 20(c)) applied in both outer spans simultaneously.

Maximum elastic moments derived from these combinations of load are figured to be:

(1) Maximum positive moment, at midspan = + 4600 ft kips,

(2) Maximum negative moments, at supports = - 6575 ft kips.

The extreme flexural stress permitted by specification for

A-7 steel is 20.0 ksi. Actual stress realized at the midspan section is figured at 20.1 ksi, and for the support sections at 19.9 ksi.

B. Limit Design Analysis by Constant Load Method

As the centre span is statically determinate only one hinge is required for the development of a failure mechanism and limit design analysis for constant loading simply consists in the determination of the critical plastic moment of resistance for the span, which will be either at the midspan section or at the support sections, and comparing this with the appropriate applied moment.

Considering firstly the maximum positive moment condition at midspan. This moment is 4600 ft. kips and thus this value can be substituted for M in equation (5). The plastic moment of resistance M_p is also involved in this equation and its value may be calculated as follows:

Plastic moment of web $= (99.0).0.625(33.0)\frac{1}{12} = 4200$ ft. kips

" " " flanges$= (99.88)20.0(0.875)33.0\frac{1}{12} = 4800$ " "

thus, total plastic moment of section $= 9000$ ft. kips

As only one hinge is involved the value for θ in both numerator and denoninator of the equation for F is, of course, the same and thus the equation becomes:

$$F = \frac{9000}{4600} = 1.96$$

Similarly for the maximum negative moment condition at the supports. The value for M in this case is 6575 ft. kips and for M_p is 12560 ft. kips, and thus:

$$F = \frac{12560}{6575} = 1.91$$

The value of F at the support sections is the smaller and this is therefore the limiting value.

It is interesting to here note that as there is no essential difference between this limit design analysis of the girder and the procedure of elastic design for statically determinate structures it is not surprising that these values of load factor for the girder are not much different from the limiting load factor requirement of the AISC Code, of 1.88.

C. Limit Design Analysis for True Variable Loading

As previously mentioned the failure of the girder under variable loading can only be associated with the state of alternating plasticity. For this state the failure load factor is given by equation (9).

The largest range in live load moment occurs at the mid-span section and is equal to 6565 ft. kips, refer Table V11. The correct value for M_y must be computed on the basis of a linear stress distribution across the girder section varying from zero at the neutral axis to the yield stress value of 33.0 ksi at the extreme fibres, and for the midspan section this is equal to 7530 ft. kips. Thus at midspan:

$$F_L' = (2)\,\frac{7530}{6565} = 2.30$$

The girder section 18 ft. from midspan, where flange areas are reduced from 20"$\times\frac{7}{8}$" to 20"$\times\frac{3}{4}$", should also be examined for failure. In this case the range in moments is equal to 6365 ft. kips and the yield moment is equal to 6880 ft. kips, thus:

$$F_L' = (2)\,\frac{6880}{6365} = 2.14$$

The limiting load factor for failure under variable loading is therefore 2.14 with failure occuring, in the state of alternating plasticity, at points 18 ft. from midspan in the reduced flange area sections of the girder.

Table VIII Moments for Continuous Girder.

Girder Panel No.	Dead Load Moments, ft. kips	Live Load Moments, ft. kips			Total Moments, ft kips		Plastic Moments, M_p, ft. kips	Yield Moments, M_y, ft. kips
		Maximum Positive	Maximum Negative	Range in Moments	Maximum Positive	Maximum Negative		
2_2	+ 2130	+ 2220	− 275	2495	+ 4350	−	11100	9580
3_2	+ 2755	+ 3095	− 520	3615	+ 5850	−	11100	9580
4_2	+ 2180	+ 2785	− 770	3555	+ 4965	−	11100	9580
5_2	+ 440	+ 1865	− 1015	2880	+ 2305	− 575	7610	6190
6_2	− 3495	+ 420	− 2960	3380	−	− 6455	12560	10920
7_2	− 330	+ 1520	− 1240	2760	+ 1190	− 1570	7610	6190
8_2	+ 875	+ 2355	− 1240	3595	+ 3230	− 365	7610	6190
9_2	+ 985	+ 2425	− 1240	3665	+ 3410	− 255	7610	6190
10_2	+ 875	+ 2355	− 1240	3595	+ 3230	− 365	7610	6190
11_2	− 330	+ 1520	− 1240	2760	+ 1190	− 1570	7610	6190
12_2	− 3495	+ 420	− 2960	3380	−	− 6455	12560	10920
13_2	+ 440	+ 1865	− 1015	2880	+ 2305	− 575	7610	6190
14_2	+ 2180	+ 2785	− 770	3555	+ 4965	−	11100	9580
15_2	+ 2755	+ 3095	− 520	3615	+ 5850	−	11100	9580
16_2	+ 2130	+ 2220	− 275	2495	+ 4350	−	11100	9580

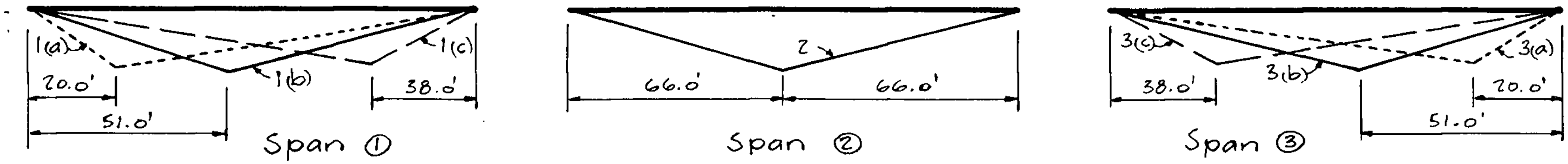
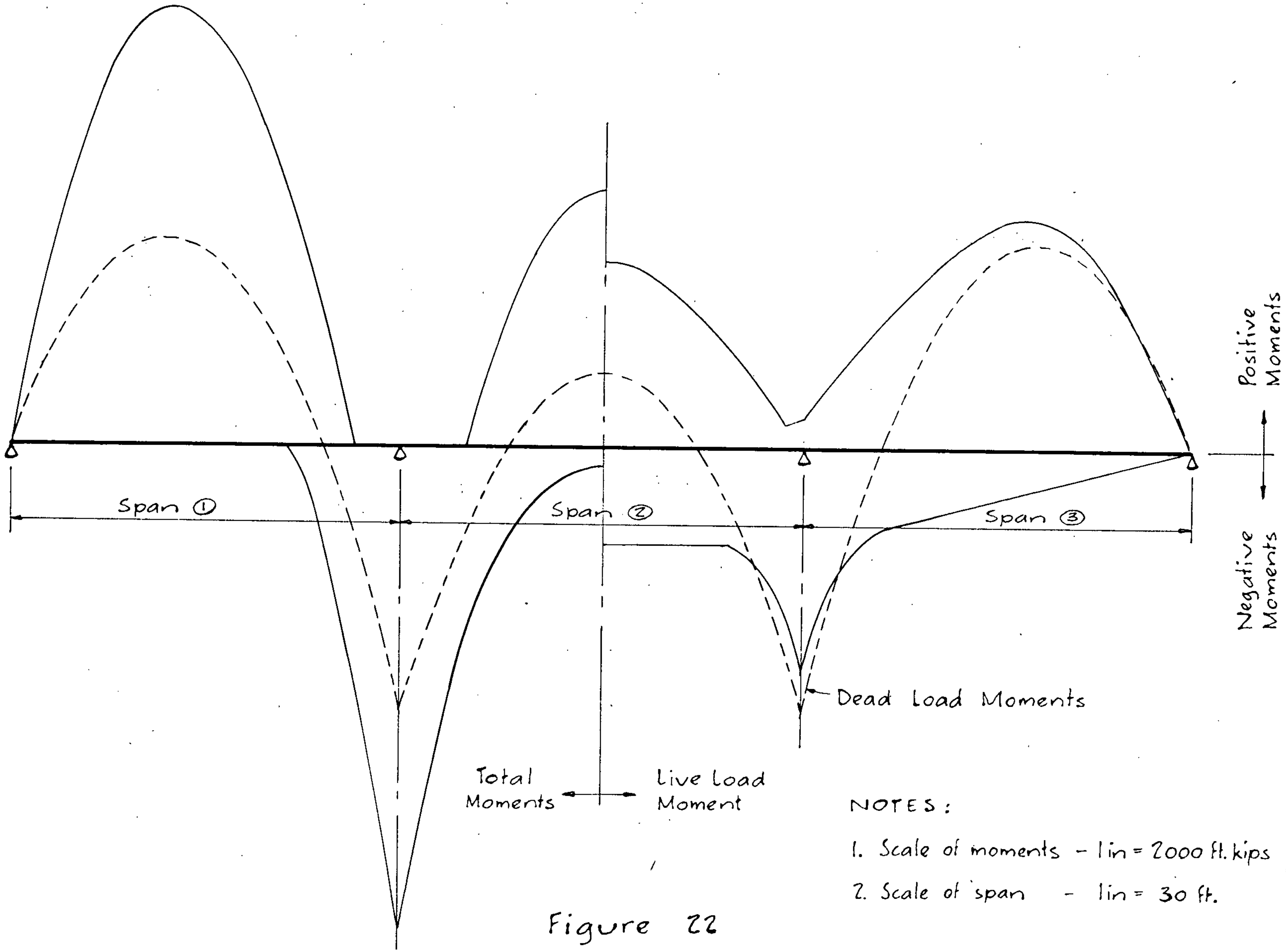


Figure 23 Failure Modes for Continuous Girder.

Table IX Derivation of Load Factors for Continuous Girder.

Failure Mode	Plastic Moments M_p, ft. kips			Total Elastic Moments M, ft. kips			Hinge Rotation, θ			Sum of Numerator Terms, $M_p \cdot \theta$	Sum of Denominator Terms, $M \cdot \theta$	Load Factor F
	Hinge Pt. 1	Hinge Pt. 2	Hinge Pt. 3	Hinge Pt. 1	Hinge Pt. 2	Hinge Pt. 3	Hinge Pt. 1	Hinge Pt. 2	Hinge Pt. 3			
1(a)	7610	12560		3300	3535		1.00	0.139		9350	3790	2.47
1(b)	11100	12560		5850	4260		1.00	0.395		16050	7530	2.13
1(c)	7610	12560		3300	4150		1.00	0.808		17710	6650	2.66
2	12560	7610	12560	3940	3410	3940	1.00	2.00	1.00	40340	14700	2.74

Failure Mode	Moments $(M_p - F_D \cdot M_D)$ ft. kips			Live Load Moments M_L, ft. kips			Hinge Rotation, θ			Sum of Numerator Terms, $(M_p - F_D M_D)\theta$	Sum of Denominator Terms, $M_L \cdot \theta$	Live Load Factor F_L
	Hinge Pt. 1	Hinge Pt. 2	Hinge Pt. 3	Hinge Pt. 1	Hinge Pt. 2	Hinge Pt. 3	Hinge Pt. 1	Hinge Pt. 2	Hinge Pt. 3			
1(a)	5550	8200		1640	2960		1.00	0.139		6640	2100	3.16
1(b)	7650	8200		3095	2960		1.00	0.395		10890	4265	2.55
1(c)	6250	8200		2200	2960		1.00	0.808		12870	4590	2.81
2	8200	6380	8200	2960	2425	2960	1.00	2.00	1.00	29160	10770	2.71



NOTES:

1. Scale of moments - 1 in = 2000 ft. kips
2. Scale of span - 1 in = 30 ft.

Figure 22

3.032 Analysis of Continuous Girder Design

A. Elastic Analysis

Structural details of the continuous girder design are shown in Figures 18 and 19. Dead weights of the steel system and concrete deck are shown in Figure 20(b). The H20-S16 vehicle loading used in the design is, of course, identical with that considered for the previous case, refer Figure 20(c).

The procedure used in the elastic analysis of the continuous girders involves the preparation of influence lines for critical sections of the girder, and the construction of an envelope of maximum positive and negative moments across each span. Details of the calculations necessary for this analysis need not be given here but the final moment envelopes, for both live loading and combined live and dead loadings, are shown in Figure 22. Maximum moments at panel points of the girders are also tabulated in Table V111.

Extreme fibre stresses at critical sections of the girders, noted here for sake of interest, are figured to be:

1. Near mid-section of outer spans - 20.1 ksi,
2. At mid-section of centre span - 18.2 ksi,
3. At centre supports - 19.5 ksi.

B. Limit Design Analysis by Constant Load Method

The key points of the girders are associated with sections where minimum values of $\frac{M_p}{M}$ occur, and a reference to Figure 19 will indicate that a number of such sections must exist across the girders. The number of hinge points required for the development of a failure mechanism on the other hand may be any integer between two and five, depending on whether failure is partial and limited to only one span or whether all spans are involved in a total collapse of the girder.

The limit design analysis on first sight would thus appear to be a formidable task involving all possible combinations of key points in either partial or total collapse. The problem however is considerably simplified once the elementary modes of failure of the system are identified and the procedure of analysis then becomes one of investigating each elementary mode in turn and superimposing these modes for all likely combinations of failure.

The elementary failure modes, involving each span individually, are readily identified and these are shown in Figure 23. There are several key points in each outer span and there are thus several alternative elementary modes for each of these spans. Now, it will be seen from the form of the elementary mechanisms that any possible complex mechanism, involving some combination of the elementary modes, must exhibit a failure load factor with a value which stands somewhere between the limiting load factors appropriate to each of the elementary modes which are combined. The load factor of the complex mechanism therefore cannot possibly be less than the smallest load factor obtained for the separate elementary modes. It is thus evident that the critical mechanism for the girders must be an elementary mode and only these forms need therefore be considered.

Thus for elementary mode 1b in span (1)(or mode 3b in span (3)) the point of maximum total moment near the midpoint of the girder, which is a section of minimum $\frac{M_p}{M}$, is located 51 ft. from the end of the girder where the moment is equal to +5850 ft. kips. The corresponding moment at support B, with live load so placed to give the above maximum positive moment, figures at -4260 ft. kips. Values of hinge rotation at the two points are calculated as follows:

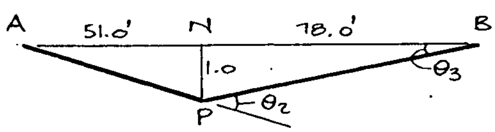


Figure 24

$$\widehat{PAN} = \frac{1.0}{51.0} = 0.0196$$

$$\widehat{PBN} = \frac{1.0}{78.0} = 0.0128$$

$$\theta_2 = 0.0196 - 0.0128 = 0.0324$$

$$\theta_3 = \widehat{PBN} = 0.0128$$

thus relative values of θ_2 and θ_3 are 1.000 and 0.395, and load factor F determined by means of equation (5) becomes:

11100(1.000)	=	11100 ,	5850(1.000)	=	5850 ,
12560(0.395)	=	4950 ,	4260(0.395)	=	1680 ,
Numerator	=	16050 ,	Denominator	=	7530 ,

and thus, $F = \frac{16050}{7530} = 2.13$

Similar calculations required for all of the other elementary mechanisms are given in Table 1X. The value of 2.13 obtained for the failure load factor in mechanism 1b is limiting and this thus represents the critical load factor for the girders under the assumptions of the constant load method.

C. Limit Design Analysis for True Variable Loading

Considering firstly the state of incremental collapse. The envelopes of maximum positive and negative live load moments shown in Figure 22 represent the variations in positive and negative values of M_L across the girders. This figure also shows the variation in dead load moment M_D. Now, it will be evident from inspection of this figure that the key points of the girders for the variable loading condition, sections of minimum $\frac{M_p - F_D . M_D}{M_L}$, are at the same locations as those earlier established for constant loading and therefore the elementary modes of failure for the present form of loading will be exactly the same as those considered previously. Also, the earlier comments on the load factor of combined mechanisms will also apply equally to incremental collapse and thus analysis can again be limited to these same elementary modes.

Thus for elementary mechanism 1b, elastic moments at the midspan hinge, as obtained from Figure 19, are M_D = 2755 ft. kips, M_L = 3095 ft. kips, and for the hinge at support B, M_D =

3495 ft. kips, M_L = 2960 ft. kips. Thus load factor F_L becomes:

$$\begin{array}{lrlr} (11100 - (1.25)2755)1.000 = & 7650\ , & (3095)1.000 = & 3095\ , \\ (12560 - (1.25)3495)0.395 = & 3240\ , & (2960)0.395 = & 1170\ , \\ \hline & 10890\ , & & 4265\ , \end{array}$$

$$\text{and,} \quad F_L = \frac{10890}{4265} = 2.55$$

The alternative elementary mechanisms are analysed in Table 1X and reference to this table will show that the limiting value of F_L for failure is that obtained above for mode 1b.

Considering now the state of alternating plasticity. The largest value for $\overline{M}_L$ occurs at the midpoint of the centre span and as this coincides with the smallest value for M_y for the girder this section must be the critical one for alternating plasticity. The values of these moments at this section are 3665 and 6190 ft. kips, respectively, and thus:

$$F_L' = (2)\,\frac{6190}{3665} = 3.38$$

The smallest load factor obtained for the girders is 2.55 and this is therefore the true load factor for the variable loading condition.

Table X Summary of Results.

Item		Load Factor Values for the Following Structures:			
		Single Bay Gable Bent	Double Bay Gable Bent.	Cantilever Bridge Girder	Continuous Bridge Girder
1	Governing Failure Load Factor 'F' for structure under Constant Loading.	1.61	2.08	1.91	2.13
2	Limiting Load Factor (for design)	1.41	1.88	1.88	1.88
3	Ratio of 'F' value to Limiting Design Value	1.14	1.11	1.02	1.13
4	Failure Load Factor 'F_L' for structure under Variable Loading	1.62	1.80	2.14	2.55
5	Limiting Load Factor (for design)	1.41	1.41	1.88	1.88
6	Ratio of 'F_L' value to Limiting Design Value	1.15	1.28	1.14	1.36
7	Premature Collapse Ratio $\frac{F_L}{F}$ (corrected) $= \frac{\text{item 6}}{\text{item 3}}$	1.01	1.15	1.12	1.20

SECTION 4

INTERPRETATIONS AND FURTHER CONSIDERATIONS

4.01 Interpretation of Analytical Results

A summary of the results obtained in the previous analyses is to be found in Table X, but before proceeding with a discussion of these results it is desirable to clarify several previous conceptions.

The first point to be mentioned is the necessity for further elaboration in the definition of load factor F. The smallest numerical value obtained for this load factor is not necessarily the governing value for failure load, as was found in the case of the double bay bent. Combinations of working loads containing wind forces have a different safety requirement (1.41 for the AISC Code) from combinations which exclude wind forces (1.88), and full allowance must be made for this in establishing the governing value of F. Thus for the double bay bent example the smallest value of F is 1.86 with wind and other forces acting, but the governing value is actually that which applies to the condition of gravity loads acting alone, that is, 2.08. The governing value of F must therefore be thought of in terms of the margin of safety over limiting code requirements rather than as the numerically smallest value of load factor obtainable.

Then again, the same considerations will apply in determining the premature collapse ratio $\frac{F_L}{F}$, and the necessity for substituting appropriate margins of safety for the F_L and F values, when different safety requirements are involved, will be apparent.

The importance of recognizing these facts is well illistrated in the results obtained for the double bay bent. If the smallest numerical values of load factor are considered F_L and F are 1.80 and 1.86, respectively, and the ratio is 0.97. The corrected ratio on the other hand is actually 1.15 and an apparent premature collapse condition is in fact the reverse when proper account is made of the safety requirement aspect.

Returning now to Table X. The correct premature collapse ratios are shown in item 7 of the tabulation. It will be at once evident that these values are all greater than 1.0 and thus premature collapse is not a factor in any of the structural examples considered. The smallest value obtained is that of 1.01 for the single bay bent and the largest value is 1.20 for the continuous girders.

The lightweight structures may be said to exhibit ratios which, on the average, are lower than those for the high dead-weight structures, as is of course to be expected from equation (10), and it would seem reasonable to conclude that where structures are subjected to highly variable loadings the actual collapse load will be somewhere between zero to twenty percent greater than the failure load predicted by the constant load method, depending on the proportion of live to dead loads present with the smaller dead loads tending to produce the smaller differences in failure load values.

It will be noted that the more highly redundant structures of both types portray ratios which are definitely larger than those for the corresponding simpler forms, although this could be coincidental as there appears to be no theoretical basis for a tendency one way or the other.

Finally, it should be recognized that the values obtained for F_L, and thus also for the premature collapse ratio, are, in three out of the four cases considered, dependent on the value chosen for the dead load factor F_D. Dead loading is not involved in the failure of the cantilever girders and as far as the gable bents are concerned, in which dead weight is low, any change in the value of F_D would have immaterial effects; however this is not so in the case of the continuous girders. A smaller factor for dead loads, over those used for live loads, must be acknowledged as justifiable in view of the fact that the magnitude of these loads is known with far greater certainty than is the magnitude, and also doubtful behaviour, of live loads. The value used in this study for F_D, of 1.25, recognizes this fact and this value is entirely consistent with the somewhat higher values recommended by code for live loadings. However, apart from this logical basis for the use of a factor of 1.25 it is interesting to reason the effect of the choice of a higher F_D

value. If the figure of 1.41 is selected (this value is used as a live load factor and it is difficult to see how F_D could reasonably be greater than this) then the live load failure factor F_L for the continuous girders works out to be 2.39, instead of 2.55. Now, the constant load failure factor is 2.13 and thus the premature collapse ratio is reduced from 1.20 to 1.12 with the increase in F_D; however this reduced value has not altered the sense of the result for the continuous girders as far as the present study is concerned for the ratio is still greater than 1.0, indicating that premature collapse does not take place.

Arguments regarding the value to be assigned to F_D are thus largely academic as far as the final purpose of this paper is concerned for no significant change can be grought about in the premature collapse ratios for any of the four structural examples considered.

4.02 Further Considerations Regarding Premature Collapse

Premature collapse is therefore not a factor in the structural examples considered, and as these structures were specifically chosen to represent severe cases of loading, it would perhaps seem reasonable to extrapolate this finding to the extent of covering all problems of structural design likely to be met in practice. It must however be agreed that even severer, and not at all exceptional, cases are very likely to fall within the scope of design. The smallest premature collapse ratio recorded for the four cases considered is that of 1.01, for the single bay bent, and it would not therefore take very much to create the severer conditions leading to a ratio of less than 1.0. Thus a small degree of premature collapse, based on the theoretical evaluation of load factor, must be accepted as highly probable within the range of practical design problems.

Insofar as these theoretical evaluations are concerned it must be remembered that certain basic assumptions are involved in the derivation of load factor equations. One such assumption is that regarding the magnitude of the resisting moment at hinge points which was earlier referred to as the plastic moment and for which a constant value was assumed throughout the entire range of angle change at the hinge. This constant value for the moment is idealized, as will be readily appreciated if reference is made to Figure 2, and entirely neglects the increase in moment which results if strains at the hinge section extend into the strain-hardening region. Now, it has been theoretically demonstrated by exact methods of analysis[2] that the earliest hinge points formed in a structure will almost invariably extend into this strain-hardening region and these hinges therefore must offer moments of resistance which are greater than the 'plastic' value M_p. It follows that the true load capacity of a structure at failure, that is when the final hinge becomes fully mobilized and the mechanism state materializes, must be somewhat greater than the computed limit design value. This fact has been demonstrated in test cases involving incremental collapse in which experimental continuous beams were subjected to variable concentrated loads, and the reported results show load increases of up to nineteen percent[3]. It is doubtful that increases of this order can be expected in all instances of incremental collapse, and no increase is allowable under conditions involving alternating plasticity where failure is associated with the elastic state at only one section of the structure, nevertheless, in general, some increase in actual failure load is indicated over and above the computed value for F_L.

A second assumption embodied in the theoretical evaluations is that the working loads applied to the structure for both constant and variable loading conditions must be of identical magnitude if exactly comparable failure load factors are to be obtained. Now, this assumption is also, strictly, not quite valid as with the constant loading condition only one application of the failure load is necessary in order to produce the mechanism state resulting in failure of the structure, whereas under variable loading failure takes place only after a number of repeated cycles of the failure loads are applied, and this is so whether failure occurs through incremental collapse or alternating plasticity. In terms of the probability of load occurence there is, then, justification for either using working loads for variable loading which are somewhat less than those used for constant loading, or, (for it does not matter which approach is considered) using a safety requirement which is slightly less for the former condition than for the latter. Such smaller working loads (or smaller safety requirement) would again have the effect of increasing the true load capacity of the structure above the computed value.

As a result of these two influences the actual collapse load of a structure will be greater than that indicated by the theoretical value of F_L, and this increased load capacity must, of course, also proportionately increase the premature collapse ratio $\frac{F_L}{F}$.

Now, this resulting increase in the load factor ratio will tend to offset the effect of a severer loading condition (in reducing the ratio) and it would seem not unreasonable to conclude that in actual fact in problems of practical design it is highly unlikely that true ratios of less than 1.0 will occur.

SECTION 5

CONCLUSIONS

The following general conclusions are indicated by the present study:

1. The theoretical findings suggest that structural failure in practice can always be expected to occur within acceptable limits of the value for ultimate load computed by means of the simplified limit design method, and the method would therefore appear to be entirely valid for design purposes.

2. For practical structures the actual ultimate load will generally range between a value equal to, to perhaps more than twenty percent greater than, the value predicted by the simplified method; the exact relationship for a particular structure depending on the proportion of live to dead load present and the variable quality of the live loads.

3. In computing the theoretical collapse loads of a structure subjected to the constant and variable forms of loading it is essential to appreciate the fact that different safety requirements exist for different combinations of live loading. As a result of these differing safety requirements it appears that the smallest numerical values for load factor frequently will not represent the governing conditions for

design.

A smaller safety requirement for dead loads, as against live loads, would also appear to be justifiable.

SECTION 6

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APPENDIX 1

In terms of mechanism failure the load factors F and F_L, for constant and variable loading respectively, are given by equations (4) and (7), as follows:

$$F = \frac{\Sigma M_p . \theta}{\Sigma M . \theta} \tag{4}$$

$$F_L = \frac{\Sigma (M_p - F_D . M_D) . \theta}{\Sigma M_L . \theta} \tag{7}$$

and thus ratio $\frac{F_L}{F}$ becomes:

$$\frac{F_L}{F} = \frac{\Sigma M . \theta}{\Sigma M_L . \theta} . \frac{\Sigma (M_p - F_D . M_D) . \theta}{\Sigma M_p . \theta}$$

Expanding the right hand side of this expression and replacing $\Sigma M_p . \theta$ by $F . \Sigma M . \theta$:

$$\frac{F_L}{F} = \frac{\Sigma M . \theta}{\Sigma M_L . \theta} . \frac{F \Sigma M . \theta - F_D \Sigma M_D . \theta}{F \Sigma M . \theta}$$

now, $\Sigma M . \theta = \Sigma M_{LC} . \theta + \Sigma M_D . \theta$, and thus :

$$\frac{F_L}{F} = \frac{1}{\Sigma M_L . \theta} . \left[\Sigma M_{LC} . \theta + \Sigma M_D . \theta - \frac{F_D}{F} \Sigma M_D . \theta \right]$$

$$= \frac{\Sigma M_{LC} . \theta}{\Sigma M_L . \theta} . \left[1 + \frac{F - F_D}{F} . \frac{\Sigma M_D . \theta}{\Sigma M_{LC} . \theta} \right]$$

$$\frac{\Sigma M_{LC} . \theta}{\Sigma M_L . \theta} . \left[1 + \frac{F - F_D}{F} . \frac{\Sigma W_D . \delta}{\Sigma W_L . \delta} \right]$$