HYDROELASTIC STUDIES OF BLUFF CYLINDERS

by

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Date June, 1965
Abstract

Uniform water flow past cylinders of various cross-sections produces oscillating lift and drag forces. In the tests made in this investigation the cylinders were elastically mounted on a cantilever of known stiffness. The experiments were carried out to obtain a series of two-dimensional vibration records which were then analysed to find:

i) the variation of the types of lateral and longitudinal vibrations,

ii) the variation of the amplitude of lateral and longitudinal vibrations with respect to Reynolds numbers,

iii) the relation between frequency of lateral and longitudinal vibrations and the velocity of flow,

iv) Strouhal number for both lateral and longitudinal vibrations and to relate them with the corresponding Reynolds numbers of the flow,

v) Coefficient of lift and drag allowing for magnification factor.

The range of investigation was within \( Re = 2.1 \times 10^3 \) and \( Re = 7.25 \times 10^4 \). The cantilever system including the model was 38 inches long, cylinders of various cross-sectional shapes, such as circular, truncated-conical, D-shaped, square and rectangular were tested.
For the circular cylinders, the natural frequency of vibration was found to vary between 2.78 to 3.57 cycles/sec. and the damping coefficients varied between 0.068 to 0.12. For circular cylinders, the resonance of longitudinal vibration occurred at \( \frac{V}{V_{y \text{res}}} = 0.42 \).

The most interesting result, which has not been reported elsewhere, is that the longitudinal amplitude was re-excited by the lateral vibration and started rising again to a second maximum. For the circular cylinders, maximum values of coefficient of drag and of coefficient of lift were found to be 3.50 and 4.70 respectively even after allowing for magnification, Strouhal number variation in the lateral direction was found to be between 0.185 and 0.14. For circular cylinders, up to the point where lateral resonance occurred, the frequency of the excitation in the longitudinal direction was found to be twice that in the lateral direction. Although the natural frequency of vibration was the same for both lateral and longitudinal direction, the Strouhal number was not constant, but varied within a narrow range. Thus, the velocity of resonance in the lateral direction was not twice that of the velocity of resonance in the longitudinal direction but was 2.3 times instead.

For the D-section, the flat face normal to flow was found to be the most unstable of all orientations. The flat face parallel to flow gave the least coefficients of drag and lift out of all the various shapes investigated.
The Strouhal number variation in the lateral direction for the circular cylinders, for the D-shaped cylinder with flat face parallel to the flow, and for rectangular cylinders with 2" face normal to the flow was the same. The maximum value of the coefficient of lift was higher for circular cylinders than that for any other shapes investigated.
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I. Introduction:

It is common experience that bluff cylinders may vibrate when elastically mounted in a flow of water. In most cases, this vibration has been related to a resonance with the eddies being shed by the model to form a karman vortex path in the wake. Some cylinders, however, will begin to oscillate under conditions in which the frequency of vortex shedding is far remote from any elastic natural frequency of the cylinder. These bluff cylinders may be termed hydro-dynamically unstable.

In this series of tests, the models were allowed to have two degrees of freedom in order to study the longitudinal and transverse oscillations simultaneously. Efforts were made to ensure that the flow was as two dimensional as possible. The oscillograms of vibrations were recorded and analysed for Reynolds numbers varying from $2.1 \times 10^3 - 7.25 \times 10^4$. Measurements included temperature, velocity, amplitude, and the frequency records for circular, truncated-conical, D-shaped, square and rectangular cylinders.

In an earlier phase of the same project Y.M. Chow (20) experimented with circular cylinders using aluminium and steel cantilevers. He also investigated the effects on eddy shedding by putting wires of various sizes at various positions of the models. His thesis contains a fair amount of information on the two dimensional structural vibrations induced by fluid flow past a circular cylindrical body.
Parkinson (2) puts the existing theories relating to fluid dynamic causes of oscillations into three categories.

In one, the oscillating nature of the flow past the cylinder, as indicated by the wake, causes a periodic force on the cylinder which is the source of oscillation. According to him shear stresses have negligible contribution to the direct determination of fluid forces on the cylinder for flow speeds of engineering interest. It is the surface distribution of fluid pressure that gives rise to the torque or the transverse component of force which is responsible for transverse excitation on an elastically mounted bluff cylinder. However, shear stresses, have a strong indirect effect, since it causes the flow separation and influences the wake geometry. Thus in the flow past the circular cylinder viscous shear causes a flow retardation in a thin boundary layer next to the upstream surface. This retarded flow cannot remain on contact with the surface in the face of increasing pressure near the transverse diameter, and it separates. The position of the points of separation is dependent on the ratio of inertial to viscous stresses in the boundary layer and is determined by the Reynolds number. For square and rectangular cylinders having sharp edged, separation occurs at these edges, and as such there is much less dependence of wake properties on Reynolds number, unless reattachment occurs at some points downstream. Parkinson (2) finds that separated flow starts to re-attach to
the side surface of the rectangle when b/h exceeds 2.5, and
the reattachment is complete for b/h greater than 3.

In second category, the hydro-dynamic properties of
the cylinder as a function of its attitude give rise to
instability. For an elastically-mounted bluff cylinder, the
obvious source of excitation is the karman vortex street in
its wake. All bluff-cylinders generate a wake which exhibit a
dominant vortex formation frequency over most of the Reynolds
number range of engineering interest. This periodic structure
of the wake causes a periodic pressure variation over the
cylinder surface, and a resultant periodic transverse force
and torque, which might excite vibration of the elastic system.

In the third category, the random fluctuations of a
turbulent wake cause buffeting of the cylinder, and can excite
a response in the elastic system. There is one thing common
in all these cases, namely that the forces on the bodies are
determined by conditions in the separated wake.

Types of Vibration

Elastically mounted cylinders may exhibit various
types of vibrations in uniform flow, depending on the flow and
the cylinder properties. These are -

1. Vortex-induced vibration,
2. Response to random excitation,
3. Flutter,
4. Plunging or twisting oscillation of an unstable section.
1. Vortex-induced vibration

This category of vibration is an important consideration in the design of smoke stacks, towers, periscopes and other similar structure; it occurs when the frequency at which vortices are being shed alternately from each shear layer approaches a natural frequency of the system. If the magnitude of structural damping is sufficiently low, oscillation will ensue. The relation between the size of the cylinder $d$, velocity of flow $V$ and frequency of vortex shedding $f$, is given by the strouhal number $S$ defined as:

$$ S = \frac{fd}{V} $$

2. Random Excitation

At very high Reynolds numbers where the wakes are relatively disorganised, the excitation of a cylinder by the random forces exerted by the wakes has to be taken into consideration. It has been observed that oscillations do occur under these conditions and the problem has been considered for air by Fung (4) and Davenport (9). Their approach has been to consider a linear elastic system subjected to a force random in time. The response of the cylinder was then calculated from the power spectrum of the force -- and the mechanical impedance of the elastically mounted cylinder.
3. **Flutter**

Flutter occurs when two modes of vibrations, plunging and twisting, are excited simultaneously at a frequency usually between the natural frequencies for pure twisting or pure plunging. This has been thoroughly investigated as a possible mechanism for the oscillation of ice coated transmission lines and suspension bridges. The forces involved in flutter are generally larger than for any other form of oscillation.

4. **Plunging Oscillation**

Plunging, sometimes called galloping, is a transverse oscillation of the cross-section, without twisting, normal to the direction of stream flow and is exhibited by several non-circular sections. Only one degree of freedom is excited, oscillations occurring above a minimum starting speed and the amplitude increasing with water velocity.

**Classification of vibrations**

Eduard Naudascher (1) classifies vibrations into forced, self-excited and self-controlled categories.

*Forced vibration* is sustained by an alternating force that exists independently of the vibration and persists even when the oscillation of the body is stopped. Here the frequency
of the alternating force is independent of the natural frequency of the body. When the frequency of the disturbing force is less than that of the natural vibration of the system, the forced vibrations and the disturbing force are always in the same phase. When the frequency of the disturbing force exactly coincides with the frequency of the vibrating system, the magnification factor and the amplitude of forced vibration become maximum. This is the condition of resonance. When the frequency of the disturbing force increases beyond the frequency of free vibration, magnification factor and amplitude start diminishing. Here the forced vibrations and the disturbing force run more and more out of phase, depending on the degree of damping.

Self-excited vibration is sustained by an alternating force which is created by the vibration itself and vanishes when the body ceases to oscillate. Here the alternating force is automatically resonant with the natural frequency of the body and amplification will continue as long as the energy, generally drawn from a continuous energy source, exceeds the work done by damping forces.

Example: vibration of electric transmission lines.

It happens usually when a rather strong transverse wind is blowing and the temperature is around 32°F, i.e. when the weather is favourable for formation of sleet on the wire.
Self-controlled vibration: In this case the body motion influences the frequency of the alternating force and the amplification usually exceeds that corresponding to the normal resonance condition. In case of an oscillating cylinder, the edges from which the free shear layers originate are displaced as well and assist in orienting and phasing the eddies. The greater the displacements of the cylinder, the greater will this latter influence be as compared to the former. As such the eddies will be increasingly governed, in both strength and frequency, by characteristics of the oscillation of the structure. If the alternating side thrust is caused by the eddies through counter-circulation, this influence may lead to structural self-control.

The quantitative effect of the control depends primarily on the profile geometry and, to a lesser extent, on the elasticity and the damping of the structural support.

In flow-induced vibrations, the alternating force is set up by an oscillating flow pattern and is transmitted along the boundaries of the structure by a fluctuating fluid pressure or shear. The resulting structural oscillation will hence be forced, self-controlled or self-excited, depending on whether the oscillating flow pattern is unchanged by, influenced by, or the result of the motion of the structure, respectively.
II. Summary of Additional Published Papers in this Field:

1. Eduard Naudascher: 'on the role of flow induced vibrations'. He makes an excellent classification of the types of vibration. He points out that in a vibrating structure, the dominant frequency of the eddy formation has to be in agreement with that of the structural vibration. This observation has led to the common misconception that the vibration is necessarily the result of the eddy motion, as a consequence of which flow-induced vibrations are often treated as being "forced" without regard to self-control or self-excitation.

Periodic eddy formation from free shear layers:

A free shear layer is produced wherever flow separates from a boundary. It is an unguided layer with pronounced transverse velocity gradient and vorticity, gradually increasing in thickness as it diffuses downstream. Of main interest with respect to vibration phenomena in flows involving eddy formation from free shear layers are the possible mechanisms that cause the initial disturbance to be periodic.

Basically, this control can be (i) purely hydrodynamic in nature, in which case the alternating mechanism acts internally within a flow confined by nonelastic boundaries; (ii) hydro-elastic, in which case the alternating mechanism is set up or influenced by the motion of elastically
restrained boundaries. This distinction is essential in regard to the classification of the structural vibration which may arise from the periodic pattern of flow.

In general, the following sources of periodicity may be distinguished:
1. Oscillating solid boundaries,
2. Interaction of adjacent free shear layers,
3. Feed-back mechanism,
4. Extraneous control & buffeting.

1. Oscillating solid boundaries: The most effective disturbance that can be imposed on a free shear layer is the oscillation of the edge from which the layer originates. Near this point the layer is so thin as to approach the quality of a surface of discontinuity. The slightest motion of the body from which shear layer originates will exert a definite control on the eddy formation.

Classification of Vibrations:
(1) forced,
(2) self controlled,
(3) self-excited,

(1) forced vibration - is sustained by an alternating force that exists independently of the vibration and continues to be there even when the oscillation of the body is stopped.
Here the frequency of the alternating force is independent of the natural frequency of the body, and the amplification depends upon an accidental proximity to resonance.

(2) **self-excited vibration** - is sustained by an alternating force which is created by the vibration itself and disappears when the body ceases to oscillate. Here the alternating force is automatically resonant with the natural frequency of the body and amplification will continue as long as the energy, generally drawn from a continuous energy source, exceeds the work done by damping forces.

(3) **self-controlled**: Very often an alternating force exists, without, but not independently of the oscillation of the body. Here the body motion influences the frequency of the alternating force and the amplification usually exceeds that corresponding to the normal resonance condition.

In flow-induced vibrations, the alternating force is set up by an oscillating flow pattern and is transmitted along the boundaries of the structure by a fluctuating fluid pressure or shear. The resulting oscillation will as such be forced, self-controlled, or self-excited, depending on whether the oscillating flow pattern is unchanged by, influenced by or the result of the motion of the structure, respectively. Most structural vibrations due to free-shear-layer flow must be expected to be at least self-controlled
because of the great sensitivity of shear layers with respect to oscillating boundaries. When the influence of the oscillating structure upon the flow becomes so strong that the alternating forces are produced merely by the relative flow patterns which accompany the structural vibration, the latter will be self-excited, i.e. automatically at resonance.
2. G.V. Parkinson: in Aspects of the Aeroelastic Behaviour of Bluff Cylinder, discusses three forms of aeroelastic instability:
(1) vortex-induced vibration,
(2) turbulence induced vibration,
(3) galloping vibration.

Although shear has little direct effect on the cylinder force, it has a powerful indirect effect, since it causes the flow separation and influences the wake geometry. In the flow around the circular cylinder viscous shear causes a retarded flow in a thin boundary layer next to the upstream surface. This retarded flow cannot remain attached to the surface in the face of the increasing pressure near the transverse diameter and it separates from the surface. The position of these separation points depends on the ratio of inertial and viscous stress in the boundary layer, and this is determined by the Reynolds number of the flow.

For cylinders with sharp edged cross-sections, such as the rectangle, separation always occurs at these edges, and there is much less dependence of wake properties on Reynolds number, unless reattachment occurs. The boundary layer after separation from the cylinder becomes a free shear layer, through which the velocity varies from a local maximum at the outer edge to a local minimum at the inner edge. The pressure is nearly constant through the layer, and
the decrease in local maximum velocity at outer edge represents a decrease in total head of the flow caused by the vorticity or rotation, of the fluid in the shear layer.

It is reasonable to believe all rectangular cylinder sections would show the same drag coefficient and strouhal number as the flat plate, since the flow separates at the front edges of a rectangle, just as it does for the flat plate. However, a very long rectangle will cause the separated flow to re-attach to the sides, reducing the wake width and the drag. The study of the effect of the aspect ratio \( d/h \) of a rectangular cross-section on the vortex formation frequency is very significant. Even if the separating shear layers for all rectangular cylinders start out the same way, their history after separation is strongly influenced by the length of the cylinder lying in the wake - called afterbody length. In particular, the afterbody influences the formation of the vortices by occupying part of the space behind the separation points in which the formation occurs, and inhibiting the communication between the two shear layers. Dr. Parkinson points out that the separated flow starts to re-attach to the side surface of the rectangle when \( d/h \) exceeds 2.5 and this reattachment is complete for \( d/h > 3.0 \).
3. Problems of lift and vibrations are discussed by Hunter Rouse in the proceedings of the fourth Hydraulic conferences (June 12-15, 1949). According to the basic theory of potential flow, a body immersed in a moving fluid will be subject to balanced longitudinal and lateral pressures so long as the innermost streamlines conform to the body profile. The actual longitudinal force exerted upon a body may be explained, and in certain cases evaluated, in terms of the potential theory through additional use of the theory of boundary-layer separation.

![Dimensional Pattern of Irrotational Flow Around a Cylinder](image-url)

(a)

Fig. Two. Dimensional Pattern of Irrotational Flow Around a Cylinder
Irrotational vortex with constant circulation,

A combination of the corresponding velocity potentials through simple addition will yield a pattern as above.
It will be noted that the addition of circulation increases the velocity on the one side of the cylinder and decreases it on the other side, the resulting change in pressure giving rise to a lateral force in the direction of the decreased spacing of streamlines. Since the exact details of the pattern depend upon the magnitude of the ratio $\frac{V}{V_0}$, it should be evident that the cross thrust on a given cylinder in a given flow may be controlled over a considerable range simply by changing the circulation.

The circulation around the cylinder at any instant will be equal and opposite to the algebraic sum of the initial circulations of all vortices which have been formed. Since each vortex has the initial strength $\pm V$, then the circulation around the cylinder varies continuously from $-\frac{V}{2}$ to $+\frac{V}{2}$.

![Diagram](image)

**Fig.** Development of the vortex trail in the wake of a stationary cylinder.
The net result is an oscillating side thrust upon the cylinder in a direction away from the last vortex. The approximate magnitude of the force per unit length of the cylinder may be evaluated from the equation \( \tau_L = \rho V_0 \frac{V_v}{2} \) and \( V_v \approx 2.8 \gamma (V_0 - V_v) \approx 1.7 V_0 d \).

It must be noted here that the effective diameter of the cylinder is increased by the overall displacement which it undergoes during vibration, the sum of the diameter and the displacement then being the characteristic length term in the various relationships. Such lateral displacement will tend to increase both the period of oscillation and the force which is exerted.
4. Fluctuating lift and drag acting on a cylinder in a flow at super-critical Reynolds numbers - Y.C. Fung.

He gives some information about the fluctuating lift and drag acting on circular cylinders in flows at supercritical Reynolds numbers. His investigations cover both stationary cylinder and cylinder subjected to forced oscillations of specific amplitudes and frequencies.

The most direct measurement of the fluctuating forces was made by Drescher in water tunnels in the Reynolds number range $10^4$ to $1.13 \times 10^5$. The transient pressure distribution was recorded by multiple manometers in 12 orifices equally spaced around a circumference at the mid sections of the models. It was found that the amplitude of the oscillatory lift coefficient varies with time and lies between 0.6 to 1.3; similarly, the steady component of the drag coefficient lies between 0.95 to 1.2. It was shown that at higher Reynolds numbers, approaching the transitional regime, the fluctuating forces are non harmonic; they appear to come in successive bursts.

**Experimental Arrangement**

The test model had a diameter of 12.65 in. and a length of 6 ft. It was supported between two vertical channels that spanned the tunnel and were tied to an unusually heavy floor beam beneath the working section.
Results and Discussion

The lift coefficient $C_L$ on a stationary cylinder

The root-mean square value remains nearly constant for large Reynolds numbers, but increases as the Reynolds numbers decrease into the transition region. It is to be recalled that for subcritical Reynolds numbers an amplitude of $C_L$ ranging from 0.45 to 0.9 has been given by various authors.

The Drag coefficient $C_D$ on a stationary cylinder

The scatter is larger than that of the $C_L$, probably because of a larger contribution by a resonance condition. The mean value or the steady component has a value of approximately 0.25 for supercritical Reynolds number. This value agrees reasonably well with Delany and Sorensen at $R = 0.5 \times 10^6$.

Fung draws a tentative conclusion that for Reynolds number larger than 600,000 for strouhal number (based on forcing frequency) less than 0.12, and for a ratio of oscillation amplitude to cylinder ratio of less than 1:12, the lift coefficient is not influenced by the forced oscillation to any large extent.

Experiments by some aircraft companies have shown that the end effects of the cylinder can be very important in determining the amplitude of the oscillation of cantilevered structures.
5. On a circular cylinder in a steady wind at transition Reynolds numbers - John S. Humphreys.

Humphreys presents some results of an experimental investigation of forces associated with the subsonic flow of air around a circular cylinder in a wind tunnel. He studies the oscillating forces due to the downstream vortex street for Reynolds number in the critical range $4 \times 10^4$ to $6 \times 10^5$. The general eddy formation phenomenon seems to fall into four regions in terms of Reynolds number $Re = \frac{\rho U_0 d}{\mu}$, (where $U_0$ is the free-stream velocity, $d$ the cylinder diameter, $\rho$ density and $\mu$ viscosity), which are not separated by clear boundaries but by transition zones that can, within limits, be altered by individual experimental conditions. These regions can be referred to by the terms symmetric, regular, irregular, and supercritical. The symmetric range extends from $Re$ about 5 to about 40, and is characterised by a lack of any oscillatory features. In the regular range, with $Re$ between 50 and 140, the vortex motion is laminar and persists for a long distance downstream before being washed out by viscous diffusion.

The irregular region, extending over a wide range of $Re$ from about 300 to $2 \times 10^5$ and containing many cases of engineering interest, is different in several respects. The wake contains considerable energy in the form of random
turbulent fluctuations, and this can be explained in terms of a transition to turbulence of the free vortex layer that is still laminar as it leaves the cylinder. Due to mixing, the subsequent eddies of the turbulent fluid now die much faster.

The oscillating lift and drag on circular cylinders are determined from measurements of the fluctuating pressure on the cylinder surface in the range of Reynolds number from $4 \times 10^3$ to just above $10^5$.

Recently renewed interest has followed on Lighthill's publications (1952, 1954) on aerodynamic sound. At least fifteen papers were so far published concerning the acoustical concomitant of vortex shedding known as Aeolian tones. Lighthill's theory was applied to the problem of Aeolian tones by Curle (1955) and Phillips (1956). They show that in the presence of a rigid body the sound field is dominated at low mach numbers by dipole radiation attributable to the fluctuating pressure at the surface of the body.

Applying dimensional analysis to the fundamental equation of Curle and Phillips, it can be shown, that the acoustic intensity at large distances from a cylinder of finite length in the plane of symmetry bisecting the cylinder at right angles, is given by -

$$I \propto \frac{\rho \sin^2 \phi}{\alpha^3 \gamma^2} \cdot \bar{U}^2 \bar{L}^2 \bar{C}_L^2$$

Bars means with respect to time,
\( \varphi \) = free stream density,
\( \alpha \) = free stream speed of sound,
\( \gamma, \sigma \) = Polar co-ordinates,
\( \Theta = 0 \) being the upstream direction,
\( U \) = free stream velocity,
\( S = \frac{Nd}{U} \) = strouhal number,
\( d \) = cylinder diameter,
\( N \) = vortex shedding frequency,
\( \lambda \) = a representative length of the cylinder which will depend on the phase variation along the cylinder length,
\( C_L \) = lift coefficient,

This equation relates the intensity of Aeolian tones to the parameters of the flow.

**The lift and drag:**

As the pressure is in phase on one side of the cylinder and out of phase with that of the other side, the lift on the cylinder could be calculated by integrating the pressure. He found that the assumption of no change in the angular distribution of phase or intensity down to the lowest Reynolds numbers investigated seemed likely to produce a large error in the lift.

Even if the angular distribution of intensity changes to a sino curve, the resulting change in the lift is only about 10\%.
The fluctuating component of the drag may be determined from the second harmonic pressure intensities if an assumption is made about the phase. The most logical assumption is that the second harmonic pressure is of the same phase over all the cylinder. The resulting drag coefficients are between 10 and 13 times less than the lift coefficients.
7. Experiments on the flow past a circular cylinder at very high Reynolds number - Anatol Roshko.

This is one of the classical papers on this subject. Various experimental measurements of drag coefficient $C_D$ at subcritical Reynolds numbers are in fairly good agreement as to the values of $C_D$, but in the supercritical range, i.e. after the transition to low values of $C_D$, there is little agreement, except that $C_D$ lies between values 0.2 and 0.4. Relatively large discrepancies here may be due to difficulties in measurement or more sensitivity of the flow to the conditions of the experiment. The measurements of Delany and Sorensen (1953) at Reynolds numbers up to $2 \times 10^4$ exhibit a multivaluedness which they attribute to changes in the flow from a symmetric to an unsymmetrical type, higher values of $C_D$ occurring in the unsymmetrical flow. Other authors claim to observe no asymmetries, or about 0.2. At critical and supercritical Reynolds numbers there are only two sets of measurements made so far. The early ones by Relf and Simmons (1924) indicate that in the transition range there is a predominant frequency in the wake, which they called 'aperiodic', compared to the 'accurately periodic' flow at $R < 10^5$. Their measurements show that the strouhal number of these frequencies increases as the drag coefficient $C_D$ decreases.
Delany & Sorensen (1953) obtained measurements at still higher Reynolds numbers ($10^6$ to $2 \times 10^6$) using a pressure pick-up in the wake close behind the cylinder. The shedding frequencies, which were determined from the predominant frequencies on an oscillograph record, show considerable scatter; the values of $S$ are between 0.35 and 0.45.

From these two sets of measurements, it would appear that $S$ rises rapidly in the interval $R = 2 \times 10^5$ to $10^6$, and that there may be a rapid decrease at about $2 \times 10^6$.

**Drag Coefficient:** $C_D$ increases in the range $10^6 < R < 3.5 \times 10^6$, from a value of about 0.3 to about 0.7, and then levels off at the latter value.

**Use of splitter plate**

Roshko used a 'splitter plate' on the centre line behind the cylinder. It spanned the $8 \frac{1}{2}$ ft. height of the test section, extended 4 ft. along the centre line, and was 2 inch thick, being made up of two pieces of plywood bolted together.
Arrangement of cylinder in the wind tunnel

The main effects of the splitter plate are as follows:

(i) The shedding is suppressed; there is no indication of a peak in the spectrum.

(ii) The plot of $C_D$ indicates a small decrease in $C_D$ at values of $R$ above $3.5 \times 10^6$, but no effect at all at lower values.

(iii) There is no significant effect on the pressure distribution. At higher dynamic pressures severe flutter developed at the trailing edge of the splitter plate. The reason for this flutter is not clear; apparently it was not connected with vortex shedding off the cylinder, since this was suppressed by the plate. Possibly shedding off the trailing edge of the splitter plate itself had an effect.
Transitions and characteristic range:

The lower or critical, transition at $2 \times 10^5 < R < 5 \times 10^5$, from high to low values of $C_D$ is followed by another (upper) transition, at $10^6 < R < 35 \times 10^6$, to a new plateau on which the coefficients have the following mean values: $C_D = 0.70$, $S = 0.267$.

the range on either side of the lower transition 'subcritical' ($C_D = 1.2$) and 'supercritical' ($C_D = 0.3$) respectively. He coined the term 'transcritical' to the plateau following the upper transition ($C_D = 0.7$).

Effect of the splitter plate

In experiments at subcritical Reynolds numbers it was found that a splitter plate has a strong influence on the flow. Briefly, the flow changes from one with alternate shedding to a steady symmetrical flow in which the separation streamline reattaches on the splitter plate, forming two closed circulation regions on either side of the plate.

Nature of transitions:

The author proposes the following classification - at subcritical Reynolds numbers the separation is laminar, in the supercritical range there is a laminar separation bubble followed by turbulent separation, and in the transcritical range the separation is purely turbulent.
Two categories of instability problems - They are self-excited vibrations and forced vibrations.

Self-excited vibrations - Here the alternating force that amplifies and sustains the oscillation is created or controlled by the oscillation itself. The alternating force is automatically resonant with the natural frequency of the oscillations. The galloping of transmission lines and the oscillations of flexible bridge spans are considered in this classification.

Forced vibration - Here the alternating force that initiates, amplifies, and sustains the vibration exists independently of the vibration and persists even when the vibratory motion is stopped. The frequency of the alternating force is independent of the natural frequency of the vibration, and amplification depends upon accidental resonance or proximity to resonance. Vibrations caused by vortex shedding fall into this category.

It has been found that vibrations identified with vortex shedding also involve automatic synchronism, or control of impulse frequency by vibration frequencies, once the vibrations have been initiated. This broadens the base
of similarity and correspondence between the two types of instability and strengthens the prospect of ultimately reducing all instability phenomena to a common basic explanation and analysis.

**Strouhal number:**

The dimensionless ratio \( S = \frac{fd}{V} \approx 0.20 \), first determined in 1878 by V. Strouhal as 0.185, is known as the strouhal number. This dimensionless ratio, \( \frac{fd}{V} \) is the basic parameter in the formulation of all vibration phenomena related to the velocity of fluid flow. From the known value of the Strouhal number, the eddy frequency is determined for any value of \( \frac{V}{d} \),

\[
f = S \frac{V}{d} \approx 0.20 \frac{V}{d}
\]

The Strouhal number varies with the form of the section and with the Reynolds number of the flow. For Reynolds numbers between 500 and 200,000, \( S \) may be taken as practically constant at \( S \approx 0.20 \). At higher values of \( R \), \( S \) increases steeply. At \( R = 700,000 \), \( S \approx 0.30 \). (For a flat plate held normal to the flow, \( S \approx 0.145 \) at \( R = 3,000 \)).

A common misconception needs correction. The oscillations of the cylinder (or other section) are not caused or produced by the vortices. The vortices in the wake
are merely counters, markers, or footprints providing a convenient physical and mathematical trail from which the circulation about the cylinder and the consequent lateral forces acting on the cylinder may be inferred, formulated and completed.

**Prevention and cure of Aerodynamic instability:**

1. By augmenting the rigidity of the structure,
2. By augmenting the positive damping,
3. By modifying the cross-section.

Almost any method of increasing rigidity is also directly effective in augmenting structural damping.

The variation of Strouhal number:

With circular cylinder, the vortex street first becomes apparent at $R = 50$ and remains regular and stable for a considerable distance downstream. Roshko observes that this wake regime holds up to $R = 150$. The Strouhal number in this range rises from an initial value of 0.12 to 0.18 at $R = 150$.

In the range of Reynolds number between 500 and $2 \times 10^5$ the Strouhal number is fairly constant at values, recorded differently by different observers, between 0.18 and 0.21 - close to the value of 0.185 originally recorded by Strouhal in 1878.

A further change in the wake takes place at about $R = 3 \times 10^5$. This is the critical Reynolds number, marked by a noticeable drop in the coefficient of drag $C_D$. Delany & Sorensen's wind tunnel results indicate a peak Strouhal number of 0.43 at $R = 1.5 \times 10^6$, falling off slightly at $R = 2 \times 10^6$.

Results derived from wind-tunnel tests on spring mounted cylinders by Penzien suggest that peak vibrations occur at a Strouhal number in the region 0.17 - 0.21, even at higher Reynolds number.
Preventive Measures

Several methods for preventing or inhibiting oscillations of tall stacks have been suggested.

(i) Modification of aerodynamic shape - Modifying the cylindrical shape of the stack-structures by projecting fins, by helical "straks" wound at a certain pitch and spacing around the outside circumference of the structure, and by perforated shields surrounding the stack. These all appear to be effective by inhibiting the buildup of the periodic eddies, by fixing the position of the separation points or by introducing a phase difference in the eddies being shed along the length of the stack. It remains to be seen whether structures fitted with these devices experience more oscillation due to the randomly shed eddies.

(ii) Increase of structural damping: Increase in structural damping decreases the amplitude of vibration. This may be effected by providing a lining of 'gunite' or other such vibration absorbing material, by using rivetted joints in preference to welding, and by using external damping forces such as guy wires anchored to automotive shock absorbers. Aluminium structures may have appreciably higher damping properties than an equivalent steel or concrete structure.

(iii) Modification of the mass-made configuration: Vibration amplitude can be reduced by increasing mass. Marsh describes an application of this where, to prevent wind excitation of the tall arc-lamp standard at Bienne, heavy steel plates were added to the top of the structure.
10. The supersonic and low-speed flows past circular cylinders of finite length supported at one end - D.M. Sykes.

This study of the flow past circular cylinders of finite length was undertaken to determine the extent to which the flow over the central portion of such cylinders could be considered to be two dimensional and also to determine the main properties of the flows near the ends of the cylinders.

It was found that the flows could be divided into three regions (a) the central region, (b) near the free end, (c) near the fixed end or root. The flows in regions (b) and (c) were independent of one another and, in the supersonic stream, independent of the length of the cylinder, provided the length to diameter ratio exceeded 4. The end regions were each about two diameters in extent. In the low-speed stream the whole flow was found to be dependent on the length to diameter ratio and this effect was attributed to changes in the local fluctuating flow field.
Conclusions drawn were -

(1) The presence of neighbouring cylinders may increase the drag force of a cylinder by 75% or decrease it to less than minus 15%.

(2) Lift forces of 75% of the single cylinder drag force may result from the presence of neighbouring cylinders.

(3) The average drag force on a cylinder may be increased by as much as 45% if outside, or decreased by 80% if inside, the wake of close neighbours.

(4) Fluctuating drag forces are caused mostly by eddies from neighbouring cylinders.

(5) One neighbour has approximately the same effect as two.

(6) Neighbours more than 10 diameters from a test cylinder cause only minor drag & lift disturbance.

Experimental investigations were carried out for the purpose of studying certain fundamental characteristics of the steady separated flow past a circular cylinder. Attention was paid to the variation of these characteristics. With increasing Reynolds number, in the hope that the asymptotic nature of such steady flows for the case of very large Reynolds numbers could thereby be inferred. Such an investigation became possible when it was found that the steady separated flow past a cylinder could be artificially stabilised up to a Reynolds number of about 300, without significantly distorting other characteristics of the flow, by the presence of a wall effect and by the use of a splitter plate within the wake.

Significant results are -

(i) The rear pressure coefficient reaches the value of approximately -0.45 at $R = 25$ and remains unchanged up to $R = 177$.

(ii) The pressure drag coefficient of the cylinder as a function of the Reynolds number is given by

$$C_{D, p} = 0.62 + 12.6/R \quad \text{for } 10 \leq R \leq 177$$
(iii) The character of the wake bubble behind the circular cylinder remains unchanged as the Reynolds number is increased and the vortex type circulation persists.

(iv) The wake-bubble elongates with increasing Reynolds number in such a way that its length is proportional to the Reynolds number $6 \leq R \leq 280$. 
Ozker and Smith have presented a new concept of the dynamic response of tall stacks to wind action. By measuring the dynamic response of a full scale stack to a wide range of wind velocities they concluded that wind induced vibrations of a stack are 'self-excited' rather than forced. With this type of oscillation the frequency of vortex shedding is controlled by the response of the stack itself. That is, the stack vibrates at its own natural frequency at all times and is independent of wind velocity.

The vibration amplitude is generally not constant during a condition of steady wind. Instead the oscillation increases rapidly and then decreases in a similar manner. While this behaviour is somewhat random, it does indicate that energy is suddenly fed into the system from the airstream. Some of this energy is dissipated through structural and aerodynamic damping, while the remainder is suddenly fed back into the airstream.

One possibility for this phenomenon is that there is sufficient periodic shedding of vortices on alternate sides of the cylinder to build up the vibration. After a number of cycles the shedding becomes out of phase with response, thus reducing the vibration. Another possibility is that at these high Reynolds numbers an alternating force is produced due to flow changes resulting from an alternate shifting of the separation point on the boundary layer.
Shedding of vortices in this case would be considered aperiodic. This shifting of separation point can be alternately in and out of phase with response.

**Strouhal number:** The change from 'laminar' to 'turbulent' flow in the critical range results in a sudden reduction of drag coefficient and an increase in Strouhal number.

**Amplitude Response:**

1. Maximum response in general does not continue to increase with wind velocity, but does reach a maximum at certain critical velocities.

2. There appears to be a certain critical stiffness or rigidity for which vibration due to wind is most severe.

3. Stress produced by vibration of a cylinder normal to the direction of the wind can be much larger than those produced by the steady drag force provided the right combination of wind velocity, structural stiffness, and structural geometry is present.

**Effect of surface roughing on dynamic response:**

While roughness or projections from a smooth cylinder increase the steady drag load it does leave a stabilizing effect on vibration normal to wind.
Various ways suggested for suppressing the vibrations -

(1) By increasing the flexural stiffness of the member so that its critical velocity is above the range of moderate winds;

(2) By reducing the effective length of the member through the introduction of intermediate struts;

(3) By use of damping devices to restrict the amplitude of vibration;

(4) By attaching "spoilers" to the members that serve to disrupt the flow near the surface and to interfere with the regular formation of vortices, thereby destroying the cause of the vibrations.

Investigations were limited to a study of tubular aluminium members in the form of circular cylinders. Types of test members considered are as follows -

1. Stationary rigid-body cylinders,
2. Spring-supported rigid-body cylinders,
3. End supported flexural members - fixed,
4. End supported flexural members - partially fixed,
5. End supported flexural members - pinned,
6. Cantilever flexural members.

Cylinder diameters range from 1\(\frac{1}{2}\)" to 6".
Cautions on the use of spoilers: Spoilers are not recommended for Reynolds numbers below $1 \times 10^4$ or above $4 \times 10^5$, because it was in this range that spoiler optimization was carried out. Fluid flow has greater ability to over ride protuberances at the lower R-values. At high Reynolds numbers the lift forces on bare cylinders are quite erratic without the addition of spoilers, and phase A (tests on stationary cylinders) tests revealed no differences in the oscillograph records for super critical R-values with and without spoilers. Spoilers are not recommended for members with diameters smaller than 2" or for cantilevers with $l/d$ greater than 20. ($d =$ diameter of cylinder or member in inches).
15. Vortical traces for flow around vibrating cylinders

- L.P. Smirnov and M.A. Pavliluna.

Aluminium powder was used to make the current visible - which permitted a clear picture to be obtained of the flow around the cylinder and gave data concerning the structure of the flow only on the free surface of the liquid. The method used of making it visible by electrolizing the water, permitted the flow around the cylinder at a certain depth under the free surface to be studied. The method is as follows: - By causing a continuous current to circulate between electrodes (one electrode was formed in a lead plate placed on the bottom of the channel, and the other in a copper wire with the end split, placed in the water at a depth of 1\textquoteleft.97 - 3\textquoteleft.15 beneath the free surface), a large quantity of small bubbles of hydrogen and oxygen is obtained which moves at once with the current and makes the flow near the cylinder visible. At a depth of 1\textquoteleft.97 - 3\textquoteleft.15 under the surface, the stratum of liquid must be illuminated by a reflector with a voltaic arc.
III. Theory:

Hydro-dynamic oscillations of a cylinder:

When water flows steadily past a stationary submersed or partly submersed cylinder, eddies are shed periodically from the sides of the cylinder. At moderate Reynolds numbers the eddies are shed regularly, forming the well known Von Karman vortex trail. Every time an eddy is shed, an unbalanced lateral force acts on the cylinder and if the cylinder is free to vibrate laterally, the alternating lateral forces may impose on it a forced vibration. Referring to figure 1, if $d$ is the diameter of the cylinder, $b$ the width between the two rows of eddies, $l$ the distance between successive eddies in the same row, $V$ the relative stream velocity, $v$ the speed of translation of the eddies relative to the cylinder, the established relations are:

$$\frac{b}{d} = 1.3, \quad \frac{b}{l} = 0.3, \quad \frac{l}{d} = 4.3, \quad \frac{V}{v} = 0.86$$

Von Karman established analytically that $\frac{b}{l}$ is a constant independent of Reynolds number and of the cause of the wake. Roshko has made wind tunnel studies on vortex streets behind circular cylinders in a Reynolds number range between 40 to 10,000. He found that the formation and shape of the vortex streets are strongly dependent upon the magnitude of Reynolds number.
The eddy frequency then reduces to

\[ f = \frac{v}{l} = \frac{0.86V}{4.3d} = 0.2 \frac{V}{d}. \]

The non dimensional ratio \( S = \frac{fd}{V} = 0.2 \), is known as the Strouhal number. The Strouhal number varies with the form of the model and with the Reynolds number of the flow.

Strouhal number, \( S = \frac{d}{l} \frac{V}{V}. \)

This shows that \( S \) and \( f \) vary inversely with the eddy spacing \( l \).

Steinman (8) gives an expression for drag, \( F_D = \)

\[ F_D = \frac{C}{2} V^2 \left[ 1.587 \left( \frac{u}{V} \right) - 0.628 \left( \frac{u}{V} \right)^2 \right] \]

where \( u \) is the eddy speed relative to the stream.

If the circulation around the cylinder is \( \Gamma \), uniform stream velocity \( V \), the lateral force \( F_L \) per unit length of the cylinder is given by the relation \( F_L = \frac{\rho}{m} \Gamma V. \)

Von Karman found that the circulation of an eddy in the vortex trail is

\[ \Gamma = 2.83 lu \approx 1.71 Vd. \]

The shedding of an eddy into the wake causes an equal and opposite circulation or increment of circulation around the cylinder. The net circulation around the cylinder at any instant is the algebraic sum of the circulations of all the vortices in the wake. If the progressively increasing strength of the alternate vortices during the initial period
of acceleration, before reaching the steady state, is represented by a series of the form -
\[1, -2, 3, -4, 5, -6, \ldots \pm \Gamma\] it is evident that the maximum circulation about the cylinder in the steady state is \[\pm \frac{1}{2} \Gamma\].

Therefore \[F_L \approx 1.71 \left(\frac{v^2}{2}\right) d\]. This lateral force is bigger than the drag, \(F_D\).

As alternate eddies are released in the double-row wake, \(F_L\) is a periodically alternating force, with frequency \(f\), acting on the cylinder. The force \(F_L\) acts to the right when the eddy is shed from the left of the circular section and to the left when the eddy is shed from the right of the section. This periodically alternating force \(F_L\) is able to initiate oscillations from a state of rest, also of amplifying them until a steady state of oscillation is attained. Weaver (14) observes that when an elastically supported cylinder or a flexural member is vibrating in resonance with periodic vortex forces, the vibration itself causes an increase in the magnitude of the lift force. The additional hydrodynamic force is probably associated with a periodic shifting of the points of separation of the cylinder, resulting in increased circulation.
Effects of vibrations on wake and on eddy shedding frequency:

Steinman (8) holds that when the cylinder is in transverse vibration due to the action of its vortex trail, it sheds an eddy at or near each end of its amplitude range. As a result the normal width \( b \) of the vortex street is increased by the total amplitude \( 2A \) or by the ratio, \( \gamma = \frac{b + 2A}{b} \).

Hence, \( F_D = \gamma F_{D0} \), \( \Gamma = \gamma \Gamma_0 \), \( F_L = \gamma F_{L0} \),

\( S = \frac{1}{\gamma} S_0 \), \( f = \frac{1}{\gamma} f_0 \).

Suffix 'o' refers to original state without modification.

When the boundary layer changes from laminar to turbulent, the wake becomes narrower and consequently \( s \) and \( f \) are increased but the drag is reduced.

Fig. 1. Development of the vortex trail in the wake of a stationary cylinder.
Self-excited oscillations:

Some sections, when exposed to steady fluid flow will build up rapidly amplifying oscillations transverse to the flow direction which are at the structural natural frequency, but not necessarily at eddy frequency. Such sections are called unstable sections. Some unstable sections are so called hard oscillators in that they need a disturbance to initiate oscillations. Examples are --

Fig. 2. Vortex trail in wake of oscillating cylinder.
D-section with flat face towards the stream flow, T-section with head toward the flow direction, a deep H-section, a deep U-section etc. Here the amplification is produced by drawing energy from the flow. The oscillation frequency is the natural frequency of the oscillating system, and is independent of the stream velocity. Any rise in flow velocity increases the amplification. At high flow velocity, the oscillation may become 'catastrophic'. Steinman (8) observes that these self-excited oscillations are always transverse to the direction of the fluid flow. Oscillations in the direction of the flow are damped.

The distinguishing feature of the unstable sections is the direction of the resultant force. An upward inclined fluid flow against the stationary section will produce a downward resultant. The downward motion of the section compounded with horizontal flow produces a relatively upward angle of attack. As such an unstable section is subjected to a downward resultant whenever the section is moving down and an upward resultant whenever the section is moving up, hence the oscillations are amplified.

Referring to Fig. 4, the slope of the static lift graph has successfully been used to determine the vertical stability or instability of the section. A static lift graph with a central range of positive slope signifies a hydrodynamically stable section. The steeper the slope,
the greater is the vertical stability. When the central range of the static lift graph has a negative slope, it shows that the section is hydrodynamically unstable. Steeper negative slope shows greater vertical instability. The drawbacks of determining the hydrodynamic characteristics of a section by means of a static test (i.e. the model is held stationary at successive angles of incidence) are firstly that it does not account for the cumulative amplification of oscillations and secondly it disregards the effects of acceleration. It is generally proved to be a conservative test.

Fig. 3. Instability of a D section.
Forced vibrations with viscous damping:

When a structure is acted upon by an external disturbance, force vibration results. The forces that come into play would be external force, inertia force, elastic
restoring force of the structural system and damping forces.

Assuming that a disturbing force $P \sin \omega t$ is acting on the vibrating body, the motion of the system can be described by the equation -

$$\ddot{x} + \omega_n^2 x = \frac{W}{g} - (W + kx) - c\dot{x} + P \sin \omega t.$$  

using $\omega_n^2 = \frac{kg}{W}$

$$\frac{cF}{W} = 2n,$$

The equation reduces to

$$\ddot{x} + 2n \dot{x} + \omega_n^2 x = \frac{P g}{W} \sin \omega t.$$  

Solution of the equation is found to be as follows -

$$x = e^{-\frac{\pi t}{\omega_n}} (c_1 \cos \omega_n t + c_2 \sin \omega_n t) + A \sin \omega t + B \cos \omega t.$$  

The first member of the right side represents the free damped vibration, the two other terms having the same frequency as the disturbing force represents forced vibration.

Using the rotating vector diagram the following expression for the forced vibration is obtained -

$$x \text{dynamic} = \frac{P}{m \omega_n^2} \frac{1}{\sqrt{1 - (\frac{\omega}{\omega_n})^2}^2 + \left[ 2 \times \frac{\omega}{\omega_n} \right]^2} \sin (\omega t - \phi).$$

$$x \text{dynamic} = \frac{1}{m \omega_n^2} \frac{1}{m} \sin (\omega t - \phi).$$
Where \( M \) is the magnification factor,

\[
M = \frac{1}{\sqrt{1 - \left(\frac{\omega}{\omega_n}\right)^2}^2 + \left[2 \gamma \frac{\omega}{\omega_n}\right]^2}
\]

\[
P \sin (\omega t - \alpha) = \frac{x}{\sqrt{m^2 + \omega_n^2}} \frac{\omega_n^2}{M}
\]

It can be seen that when \( \omega \) approaches \( \omega_n \), if the damping in the system is small, even a small oscillatory force such as that produced by eddy shedding can build up large amplitude vibrations. The mathematical model is not exact for finite amplitude vibrations because eddy frequency is a function of amplitude of vibration. This dependence of eddy frequency on amplitude becomes apparent in the re-excitation of the longitudinal vibration at lateral resonance velocities.

The starting equation of motion was -

\[
m \ddot{x} + c \dot{x} + kx = P \sin \omega t.
\]

or \( \ddot{x} + 2 \gamma \dot{x} + \omega_n^2 x = \frac{P}{m} \sin \omega t. \)

Referring to the system used -

\( P \sin \omega t = \) is an impressed force

\( x = \) the deflection of the model at any time ,

\( c = \) coefficient of damping,

\( k = \) stiffness of the system used,

\( \omega_n = \) the natural frequency of the system,

\( m = \) mass of the system.
Damping in case of the circular cylinders was found to be linear. In case of D-shaped, square and rectangular cylinders the first cycles in the damping curves were marked by rapid drops but thereafter these were linear. Hence the damping in all the cases investigated were taken as linear. So the equation has the characteristics of the system used.
IV. Apparatus and Instrumentation:

4.1 Flume:

A rectangular flume 46'-0" long, 2'-6" wide, 3'-0" high with glass walls was used for the experiments. It had two devices for regulating the flow, an intake valve and a tail water gate. Two wire meshes, each one 2'-6" wide and 3'-0" high with 1/4" square holes were placed at 3'-0" apart near the upstream end of the flume to stabilise the flow.

Test section:

The test section was located 26 ft. downstream from the intake point. To increase the velocity of flow and also to minimise the boundary layer effect, a parabolic entrance contraction was installed in this section. A point gauge was used to measure the depth of flow in the test section. Practically constant water depth was maintained for all the tests, the velocity of the flow being varied by altering the tail gate and altering the valve opening at inlet.

At high velocity the water drop at the entrance of the section was appreciable and the standing wave with white water appeared behind the model. These critical flow conditions put a limit on the maximum velocity of flow that could be used.

4.2 Models:

1. 2" diameter hollow brass cylinder,
2. 2" diameter wooden cylinder,
3. 2" x 2" wooden square cylinder,
4. 1" x 2" wooden rectangular cylinder,
5. 1" radius D shaped wooden cylinder,
6. 1¾" - 2¾" truncated-conical cylinder.
All models were 12" long.

Wooden cylinders were made of B.C. Douglas Fir and painted with two coats of 'Monamel Marine Finish' (product of General Paint Corporation of Canada Limited) to make them waterproof. 3/8" diameter holes were drilled through the longitudinal axis of each model to accommodate the bolt. The bolt in turn was screwed to the cantilever to form a unit. At both ends of the model, 6" diameter circular discs were provided as splitter plates to avoid the formation of secondary vortices. A parabolic shaped streamlined hollow brass ring was used in the portion of the cantilever submerged under water.

Criteria for selection of the cantilever:

The relation \( S = \frac{fd}{V} \) was used as a guide to the selection of a material and size suitable for a cantilever. Attempts were made to see that the lateral resonance takes place within the velocity range available in the test section. At resonance condition, frequency of the forcing function and the natural frequency of vibration of the cantilever coincide. Using \( S = 0.185 \) for a circular cylinder, the above relation
becomes -

\[ 0.185 = \frac{\text{fnd}}{V} \]

But the maximum velocity attainable in the flume was only about 4.5 ft./sec. Hence a cantilever with lower natural frequency of vibration and smaller section was desired. But allowable working stress of the material put a limit to the minimum size of the cantilever that could be used.

From the relation:

\[ f = \frac{1}{2\pi} \sqrt{\frac{3EIG}{W L^3}} \text{ cycles/sec.} \]

The length of the cantilever system including the model was 38 inches. To get lesser natural frequency of vibration a material with smaller modulus of elasticity was required. The elastic modulus of aluminium is one third of that of steel. This results in the lower stresses due to misfit, temperature changes, impact and imposed deformation. Hence \( \frac{1}{2}" \times \frac{1}{2}" \) square aluminium section was used for the purpose.

4.3 Instrumentation:

Strain-gauges of type C12, 141 with overall gauge length of 0".503 and overall width of 0".25 were used on four faces of the cantilever 4" below the point of clamping. GA-1 contact cement (Budd cement kit) was used for bonding the strain gauges with the aluminium cantilever. (For details refer to strain-gauge literature, Budd Instrument Limited, 170 Donway Street, Don Mills, Ontario). One separate set of strain-gauges was used for temperature compensation and stabilisation.
Two channel brush pen recorders were used to facilitate the recording of lateral and longitudinal vibrations simultaneously so that their amplitude and frequency could be determined. Two brush amplifiers were used to magnify the two signals before feeding into the oscillograph. The amplitude of the signal could be adjusted by changing the attenuations and the zero could also be adjusted by means of the pen centering levers.

Three chart drive recording speeds were available: 5, 25 and 125 mm. per sec.

**Measuring the velocity of flow:**

A mini-flow meter with dekatron counter unit (Armstrong Whitworth Equipment) was used to measure the velocity of flow. Noting the number of revolutions in a fixed time interval the corresponding velocity was obtained from the calibration chart supplied by the manufacturer. This velocity was further checked by the weighing tank and from the known area of flow. The velocity was again checked by the U type mercury manometer connected to an orifice flow meter in the main supply line, but at very high velocities the range of the manometer was inadequate. The pipe leading water to the weighing tank was unable to discharge efficiently at high flows, a part of the water went directly to the sump. So the weighing tank was also unsuitable for high flows and the velocities had
to be determined only by the miniature flow meter. The water temperature was recorded and the corresponding value of kinematic viscosity was taken from the American Civil Engineering Practice, Volume II by Abbett.
V. Experimental Procedure and Results:

5.1 Sequence of Tests:

Experiments were carried out in the order given below:

\( \frac{1}{2}'' \times \frac{1}{2}'' \) square Aluminium cantilever was used.

1. 2" circular hollow brass cylinder,
2. 2" circular wooden cylinder,
3. 1\( \frac{1}{4} \)" - 2\( \frac{1}{4} \)" truncated-conical wooden cylinder,
4. 1" radius D shaped wooden cylinder,
   (a) flat face away from flow,
   (b) flat face normal to flow,
   (c) flat face parallel to flow.
5. 2" x 2" square wooden cylinder,
6. 1" x 2" rectangular wooden cylinder,
   (a) 1" face normal to flow,
   (b) 2" face normal to flow.

All tests were run from the lowest flow for which the recorder was sensitive, to the highest flow for which the flow in the section was sufficiently sub-critical to avoid surface disturbances. The speed of the pen recorder was set to 5 mm/sec. first, then switched over to 25 mm/sec.
Test procedure:

5.2 Calibration:

The amplitude of the vibrating cylinder was calibrated by giving a known deflection at the mid point of the model. The corresponding deflections were recorded simultaneously by pen recorders. Because the bending of the aluminium cantilever was within elastic limit, and because the cantilever was vibrating in its fundamental mode, the amplitude was assumed to be proportional to the amount of strain measured by the gauges. For static loading, force calibration was done by applying a known pulling force at the mid point of the model by a miniature spring balance while the recorder was operating. For dynamic loading, especially near resonance, the applied force and cantilever response are not equal, there being an amplification of response near resonance, and a diminution well above resonance.

The frequency of the free damped vibration in still water was found by vibrating the test body in position. The cylinder was displaced from its equilibrium position in a longitudinal direction and on being released it vibrated at its natural frequency. The amplitude of vibration in still water gradually dies away because of damping. The procedure was repeated for the lateral direction. All these were done in still water.
Observations of the recorded damping curves in still water show that in case of 2\textquotedbl diameter hollow brass and wooden cylinders the damping is linear. For the D shaped cylinder, with the configuration of flat face normal to the flow, damping curve in the lateral direction is linear but in the longitudinal direction damping curve shows a marked drop at the beginning and then follows a linear relationship.

For the square cylinder, the damping curve shows a departure from linearity at very start, later on follows a linear relationship.

For the rectangular cylinder, damping curve with 1\textquotedbl face normal to flow in the longitudinal direction it is linear, in the lateral direction there is a departure from linearity at the initial stage, later on it follows a linear relationship.

From these damping curves in still water both the natural frequency and the damping coefficient, were determined from the relations -

\[
X = e^{-\xi \omega n t} \left[ A \cos \omega n \sqrt{1 - \xi^2} t + B \sin \omega n \sqrt{1 - \xi^2} t \right]
\]

where \( \xi = \frac{c}{2m \omega n} \)

\[
\delta = \text{logarithmic decrement} = \log_e \frac{x_1}{x_2}.
\]

\[
\delta = \log_e \left( e^{-\xi \omega n t} \frac{\omega n}{\omega n \sqrt{1 - \xi^2}} \right)
\]

\[
\delta = -\xi \omega n t + \xi \omega n t + \frac{2 \pi \xi \omega n}{\omega n \sqrt{1 - \xi^2}} = \frac{2 \pi \xi}{\sqrt{1 - \xi^2}}.
\]
We get the value of $\delta$ from $\delta = \log \frac{x_1}{x_2}$, where $x_1$ and $x_2$ are successive amplitudes of damped free vibration. $\xi$ is unknown.

$$\delta^2 = \frac{4 \pi^2 \xi^2}{1 - \xi^2}$$

$$\xi = \sqrt{\frac{1}{\frac{4 \pi^2}{\delta^2} + 1}}$$

$$\frac{C}{2 \pi m \omega_n} = \sqrt{\frac{1}{\frac{4 \pi^2}{\delta^2} + 1}}$$

$$c = 2 \frac{W}{g} 2 \pi \xi n \sqrt{\frac{1}{\frac{4 \pi^2}{\delta^2} + 1}}$$

$$= \frac{4 \pi W \xi n}{g} \sqrt{\frac{1}{4 \pi^2 \xi^2 + 1}}$$

$$\xi n = \frac{1}{\gamma} = \frac{\omega_n}{2 \pi}.$$  \hspace{1cm} \omega_n = 2 \pi \xi n \cdot$$

$\delta$ and $\xi n$ are known from damping curve, $W$ is known, so the damping coefficient $c$ is calculated.
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**TABLE II.** 2" dia. wooden cylinder with 1/2" square aluminium cantilever.

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TABLE IV.  1" radius D shaped cylinder (flat face away from the flow) with 1/2" square aluminium cantilever

fnx = 3.16 cps  fny = 3.68 cps

Vyres = 3.14 fps
**TABLE V.** 1" radius D shaped cylinder
(flattened face normal to flow)

* fnx = 3.16 cps   fny = 3.68 cps

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with 1/2" square aluminium cantilever

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**TABLE VI.** 1" radius D shaped cylinder (flat face parallel to the flow)

| fnx = 3.68 cps | fny = 3.16 cps | fnx = 3.68 cps | fny = 3.16 cps |

with 1/2" square aluminium cantilever

Vxres = Vyres = 2.64 fps
<p>| Run No. | V (fps) | Re x10^5 | f_x | f_y | s_x | s_y | Ax (in) | Ax (in) | Ax (in) | Ax (in) | f_x/Vxres | f_y/Vyres | V/Vyres | Mx | D | Cp | My | FL | CL |
|--------|--------|-----------|-----|-----|-----|-----|---------|---------|---------|---------|-----------|-----------|----------|--------|----|---|---|----|----|----|
| 1      | 0.83   | 0.112     | 2.86| 0.545| 0.04 | 0.04 | 0.436   | 0.98    | 0.585   | 4.78    | 0.062     | 0.160     | 1.15     | 0.003 | 0.028 | |
| 2      | 0.88   | 0.122     | 2.86| 0.542| 0.04 | 0.04 | 0.464   | 0.98    | 0.620   | 5.70    | 0.087     | 0.180     | 1.24     | 0.003 | 0.025 | |
| 3      | 0.96   | 0.133     | 2.81| 0.488| 0.08 | 0.08 | 0.505   | 0.964   | 0.675   | 13.9    | 0.011     | 0.074     | 25      | 0.007 | 0.04 | |
| 4      | 1.03   | 0.143     | 2.86| 0.464| 0.09 | 0.09 | 0.542   | 0.98    | 0.725   | 25      | 0.004     | 0.016     | 66.6    | 0.004 | 0.016 | |
| 5      | 1.29   | 0.178     | 2.94| 0.380| 0.15 | 0.15 | 0.680   | 1.008   | 0.910   | 0.112   | 0.036     | 0.010     | 1.0.0   | 0.036 | 0.106 | |
| 6      | 1.42   | 0.196     | 3.06| 0.360| 0.18 | 0.18 | 0.746   | 1.050   | 1.000   | 4.78    | 0.062     | 0.160     | 1.15     | 0.397 | 1.01 | |
| 7      | 1.53   | 0.212     | 2.60| 0.283| 0.23 | 0.08 | 0.89    | 0.805   | 0.366   | 1.08    | 5.70      | 0.087     | 1.24     | 0.458 | 0.955 | |
| 8      | 1.70   | 0.235     | 2.66| 0.260| 0.30 | 0.05 | 0.91    | 0.895   | 0.441   | 1.19    | 11.60     | 0.043     | 1.29     | 0.750 | 1.32 | |
| 9      | 1.85   | 0.256     | 2.79| 0.251| 0.49 | 0.08 | 0.96    | 0.975   | 0.471   | 1.30    | 25.00     | 0.022     | 1.32     | 0.120 | 0.20 | |
| 10     | 1.90   | 0.266     | 2.86| 0.125| 0.47 | 0.08 | 0.98    | 1.000   | 0.490   | 1.34    | 25.00     | 0.022     | 1.32     | 0.120 | 0.20 | |
| 11     | 1.98   | 0.278     | 2.98| 0.250| 0.60 | 0.23 | 1.02    | 1.040   | 0.490   | 1.39    | 25.00     | 0.028     | 1.32     | 0.900 | 1.38 | |
| 12     | 2.21   | 0.310     | 3.08| 0.232| 0.66 | 0.30 | 1.05    | 1.160   | 0.473   | 1.56    | 10.00     | 0.090     | 1.29     | 1.015 | 1.25 | |
| 13     | 2.34   | 0.329     | 3.18| 0.226| 0.90 | 0.15 | 1.09    | 1.230   | 0.504   | 1.65    | 5.25      | 0.245     | 1.34     | 1.330 | 1.45 | |
| 14     | 2.48   | 0.348     | 3.34| 0.224| 0.90 | 0.15 | 1.14    | 1.300   | 0.524   | 1.75    | 3.33      | 0.445     | 1.38     | 1.290 | 1.26 | |
| 15     | 2.72   | 0.382     | 3.46| 0.212| 0.90 | 0.10 | 1.18    | 1.430   | 0.514   | 1.92    | 2.57      | 0.695     | 1.36     | 1.530 | 1.24 | |
| 16     | 3.06   | 0.430     | 3.75| 0.204| 0.95 | 0.10 | 1.27    | 1.610   | 0.535   | 2.16    | 1.56      | 1.21      | 1.40     | 1.800 | 1.15 | |
| 17     | 3.42   | 0.480     | 3.00| 0.146| 0.57 | 0.60 | 1.40    | 1.800   | 0.520   | 2.40    | 16.60     | 0.168     | 1.37     | 2.260 | 1.16 | |
| 18     | 3.57   | 0.500     | 3.01| 0.140| 1.35 | 0.38 | 1.72    | 1.88    | 0.505   | 2.52    | 16.60     | 0.161     | 1.34     | 2.540 | 1.20 | |
| 19     | 3.97   | 0.557     | 3.40| 0.143| 2.00 | 0.20 | 2.02    | 2.09    | 0.555   | 2.80    | 2.86      | 1.370     | 1.45     | 2.750 | 1.05 | |
| 20     | 4.24   | 0.595     | 3.42| 0.135| 1.65 | 0.30 | 1.96    | 2.23    | 0.590   | 2.98    | 2.70      | 1.210     | 1.53     | 2.53  | 0.85 | |</p>
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**TABLE VIII.** 1" x 2" rectangular cylinder with 1/2" square aluminium cantilever
(1" Face normal to the flow)

fnx = 3.571 cps  fny = 3.125 cps

Vxres = 1.98 fps  Vyres = 1.68 fps
### TABLE IX. 1" x 2" rectangular cylinder with 1/2" square aluminium cantilever
(2" face normal to flow)

fnx = 3.571 cps   fny = 3.125 cps   Vyres = 3.6 fps

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<td>18</td>
<td>3.60</td>
<td>0.537</td>
<td>3.80</td>
<td>0.282</td>
<td>0.143</td>
<td>1.10</td>
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<td>4.00</td>
<td>0.276</td>
<td>0.138</td>
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TABLE X.

Natural frequency of damped free vibration in still water:

<table>
<thead>
<tr>
<th>Serial No.</th>
<th>Model</th>
<th>Material</th>
<th>Configuration</th>
<th>Logarithmic decrement S</th>
<th>Coefficient of damping C</th>
<th>fnx</th>
<th>fny</th>
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<tbody>
<tr>
<td>1</td>
<td>2'' dia. circular</td>
<td>brass</td>
<td></td>
<td>0.215</td>
<td>0.121</td>
<td>2.78</td>
<td>2.78</td>
</tr>
<tr>
<td>2</td>
<td>2'' dia. circular</td>
<td>wood</td>
<td></td>
<td>0.131</td>
<td>0.068</td>
<td>3.125</td>
<td>3.125</td>
</tr>
<tr>
<td>3</td>
<td>2'' x 1'' conical</td>
<td>wood</td>
<td></td>
<td>0.148</td>
<td>0.080</td>
<td>3.571</td>
<td>3.571</td>
</tr>
<tr>
<td>4</td>
<td>1'' radius D shape</td>
<td>wood</td>
<td>flat face away</td>
<td>8x = 0.57</td>
<td>0.248</td>
<td>3.160</td>
<td>3.680</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>flat face normal</td>
<td>8y = 0.378</td>
<td>0.192</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>flat face normal</td>
<td>8x = 0.50</td>
<td>0.218</td>
<td>3.16</td>
<td>3.68</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>flat face parallel</td>
<td>8y = 0.405</td>
<td>0.206</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2'' x 2'' square</td>
<td></td>
<td></td>
<td>0.292</td>
<td>0.139</td>
<td>2.92</td>
<td>2.92</td>
</tr>
<tr>
<td>6</td>
<td>1'' x 2'' rectangle</td>
<td></td>
<td>1'' face normal</td>
<td>8x = 0.14</td>
<td>0.076</td>
<td>3.571</td>
<td>3.125</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2'' face normal</td>
<td>8y = 0.57</td>
<td>0.231</td>
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</table>
### Table XI.

**Velocity of Resonance**

<table>
<thead>
<tr>
<th>Serial No.</th>
<th>Model</th>
<th>Material</th>
<th>Configuration</th>
<th>$V_x$ res</th>
<th>$f_x$ res</th>
<th>$Re$ res $x 10^5$</th>
<th>$V_y$ res</th>
<th>$f_y$ res</th>
<th>$Re$ res $x 10^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2&quot; dia circular</td>
<td>brass</td>
<td></td>
<td>1.50</td>
<td>3.02</td>
<td>0.236</td>
<td>3.45</td>
<td>2.92</td>
<td>0.545</td>
</tr>
<tr>
<td>2</td>
<td>2&quot; dia circular</td>
<td>wood</td>
<td></td>
<td>1.29</td>
<td>3.18</td>
<td>0.187</td>
<td>3.08</td>
<td>2.90</td>
<td>0.440</td>
</tr>
<tr>
<td>3</td>
<td>2½&quot; - 1½&quot; conical</td>
<td>wood</td>
<td></td>
<td>1.31</td>
<td>3.65</td>
<td>0.171</td>
<td>3.13</td>
<td>3.06</td>
<td>0.409</td>
</tr>
<tr>
<td>4</td>
<td>1&quot; radius D shaped</td>
<td>wood</td>
<td>flat face away</td>
<td>Erratic</td>
<td></td>
<td></td>
<td>3.14</td>
<td>3.75</td>
<td>0.455</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>flat face normal</td>
<td>Erratic</td>
<td></td>
<td></td>
<td>3.00</td>
<td>3.34</td>
<td>0.435</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>flat face parallel</td>
<td></td>
<td></td>
<td></td>
<td>2.64</td>
<td>4.00</td>
<td>0.388</td>
</tr>
<tr>
<td>5</td>
<td>2&quot; x 2&quot; square</td>
<td>wood</td>
<td></td>
<td>1.90</td>
<td>2.86</td>
<td>0.266</td>
<td>1.42</td>
<td>3.06</td>
<td>0.196</td>
</tr>
<tr>
<td>6</td>
<td>1&quot; x 2&quot; rectangle</td>
<td>wood</td>
<td>1&quot; face normal</td>
<td>1.98</td>
<td>3.52</td>
<td>0.141</td>
<td>1.68</td>
<td>3.34</td>
<td>0.120</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>2&quot; face normal</td>
<td>Erratic</td>
<td></td>
<td></td>
<td>3.60</td>
<td>3.08</td>
<td>0.537</td>
</tr>
</tbody>
</table>
5.4 Analyses of experimental results

Frequencies and amplitudes are found from the simultaneous oscillation records. Graphs were plotted with amplitude versus Reynolds numbers. Strouhal numbers were calculated from the relation \( s = \frac{f_d}{V} \), graphs for strouhal number versus Reynolds number were plotted for lateral and longitudinal vibrations to show the nature of variation. Graphs with \( \frac{f}{f_n} \) versus \( \frac{V}{V_{res}} \) were plotted to facilitate the study of amplification, degree of control or self-excitation present. Lateral resonance occurred at the frequency of eddy shedding, which coincided with the natural frequency in the lateral direction. Similarly, longitudinal resonance occurred at longitudinal natural frequency of the system.

Range of spread of resonance in various cases:

1. 2" dia. hollow brass cylinder,

Longitudinal resonance predominates from

\[
\frac{f_x}{f_{nx}} = 0.922 \text{ to } \frac{f_x}{f_{nx}} = 1.17.
\]

Lateral resonance predominates from

\[
\frac{f_y}{f_{ny}} = 0.79 \text{ to } \frac{f_y}{f_{ny}} = 0.98.
\]

Lateral frequency \( f_y \) begins to decrease.
2. 2" dia. wooden cylinder:

Longitudinal resonance predominates in the neighbourhood of $\frac{f_x}{f_{nx}} = 1.02$.

Lateral resonance predominates from $\frac{f_y}{f_{ny}} = 0.92$ to $\frac{f_y}{f_{ny}} = 1.108$.

3. Truncated-conical cylinder:

Longitudinal resonance predominates from $\frac{f_x}{f_{nx}} = 0.992$ to $\frac{f_x}{f_{nx}} = 1.02$

Lateral resonance predominates from $\frac{f_y}{f_{ny}} = 0.858$ to $\frac{f_y}{f_{ny}} = 1.02$

4. D-shaped cylinder:

Configuration (1) flat face away from flow.

Lateral resonance predominates from $\frac{f_y}{f_{ny}} = 0.985$ to $\frac{f_y}{f_{ny}} = 1.05$.

Configuration (2) flat face normal to flow.

Lateral resonance predominates in the neighbourhood of 0.917.

Configuration (3) flat face parallel to flow.

Longitudinal resonance predominates in the neighbourhood of $\frac{f_x}{f_{nx}} = 1.09$ and that the lateral resonance predominates in the neighbourhood of $\frac{f_y}{f_{ny}} = 1.27$. 
5. **2" x 2" square cylinder:**

Longitudinal resonance predominates from

\[
\frac{fx}{fnx} = 0.98 \text{ to } \frac{fx}{fnx} = 1.05,
\]

and that the lateral resonance predominates from \(\frac{fy}{fny} = 1.008\) to \(\frac{fy}{fny} = 1.05\).

6. **1" x 2" rectangular cylinder:**

Configuration (1) 1" face normal to flow

Longitudinal resonance predominates in the neighbourhood of \(\frac{fx}{fnx} = 0.986\) and the lateral resonance predominates over \(\frac{fy}{fny} = 1.07\) and \(\frac{fy}{fny} = 1.13\).

Configuration (2) 2" face normal to flow

Lateral resonance predominates from

\[
\frac{fy}{fny} = 0.985 \text{ to } \frac{fy}{fny} = 1.02.
\]

Range of resonance in some cases may spread little beyond the range specified here, because the test runs were not performed at such close steps with this end in view. Except in the case of lateral resonance for 2" dia. hollow brass cylinder, all other range of dominance of resonance condition conform to the ideal theoretical situations in standard texts.

Prior to longitudinal resonance the longitudinal response occur in same phase with the oscillating drag. When the frequency of longitudinal excitation approaches natural frequency of the elastic system, the structure
vibrates near natural frequency. Then the amplitude goes on increasing to its maximum. Beyond this velocity, response and drag run out of phase and the amplitude starts decaying.

Lift and drag were calculated by using the relations:

\[ P = \frac{X_d y m \omega n^2}{M} \]

\[ M = \text{magnification factor} = \frac{1}{\sqrt{1 - (\frac{\omega}{\omega_n})^2} + \left( 2 \frac{\xi}{\omega_m} \frac{\omega}{\omega_n} \right)^2} \]

\[ \xi = \frac{6}{2m \omega n} \]

Coefficient of lift is calculated from:

\[ C_L = \frac{F_L}{\frac{1}{2} \rho u^2 A} \]

where \( F_L = \text{lift force}, \)

\( A = \text{Area of projection of the model on a plane normal to the flow direction}, \)

Coefficient of drag is calculated from:

\[ C_D = \frac{F_D}{\frac{1}{2} \rho u^2 A} \]
VI. Discussion of Results and Conclusions:

6.1 Discussion:

Group A: 2" dia. hollow brass cylinder,
         2" dia. wooden cylinder,
         Truncated-conical cylinder,

Reason for testing 2" dia. wooden cylinder:

The weight of the 2" dia. wooden cylinder was 2.66 lbs. and that of the 2" dia. hollow brass cylinder was 3.25 lbs. Although the two models were similar in size and shape, the wooden cylinder was tested to see how the reduction in mass affects the hydroelastic behaviour of the system. Because the difference in mass was small the difference in characteristics of vibrations was not very marked. Analyses of results obtained for the brass and wooden cylinders are presented separately.

Longitudinal and lateral vibrations

In all these cases, longitudinal vibrations appear at \( Re = 0.08 \times 10^5 \). Then the amplitude of vibration starts building up and the frequency of the forcing function gradually approaches natural frequency of vibration of the system. Resonance occurs at about \( Re = 0.2 \times 10^5 \). Beyond resonance,
amplitude goes on decaying with the increase of Reynolds number so that at about $Re = 0.3 \times 10^5$ the amplitude dies out. At this stage, the lateral oscillations become conspicuous [Fig. 9, 18, 267]. Longitudinal vibrations re-appear at $Re = 0.31 \times 10^5$ and start building up.

**Explanation for the re-excitation of the longitudinal vibration beyond $Re = 0.31 \times 10^5$.**

Steinman (8)* observes that if the cylinder is in transverse vibration due to the action of its vortex trail, it sheds an eddy at or near each end of its amplitude range. Consequently the normal width of the vortex street is augmented by the total amplitude. Accepting this explanation when the lateral oscillation is large, increase in apparent diameter of the model lowers the eddy shedding frequency to the frequency required to excite the longitudinal oscillations. Beyond $Re = 0.3 \times 10^5$, longitudinal vibration is thus re-excited by the lateral vibration. Beyond this stage, as Reynolds number increases, the amplitude of lateral vibration increases. Lateral resonance occurs at about $Re = 0.45 \times 10^5$. At this stage the amplitude of lateral vibration and the amplitude of longitudinal vibration both become maximum. From this stage, as Reynolds number increases further, the amplitude

---

* Number inside the bracket refers to the bibliography.
in both directions starts decaying. Up to Re = 0.3 x 10^5, the frequency of the forcing function in the longitudinal direction is twice that in the lateral direction. But beyond this point this relation does not strictly hold. This may be attributed to the interactions of the oscillations in the two directions. Water velocity at lateral resonance is nearly twice that of the longitudinal resonance. For example, for the 2" dia. brass model, Vy res = 2.30 Vx res, for the 2" dia. wooden model, Vy res = 2.38 Vx res, while for the truncated-conical wooden model, Vy res = 2.39 Vx res.

**Strouhal number versus Reynolds number**

**Longitudinal Direction**

In all these cases it has been found that Sx starts with a value of 0.83 at Re = 0.085 x 10^5. From then on it starts gradually diminishing until it reaches a value of 0.08 at Re = 0.60 x 10^5.

**Lateral direction:**

The two circular cylinders, the 2" dia. brass and the 2" dia. wooden, which have masses of 0.101 and 0.083 respectively, show exactly the same type of variation of the Strouhal number against Reynolds number. The range of variation of the Strouhal number for these two cases is from 0.185 to 0.14. Davenport (9) gives a list of observed records
of Strouhal numbers on cylindrical structures taken by many different observers in many different situations. In that list Strouhal number is shown to be varying from 0.28 to 0.15 for different observers, but most values centre around 0.185. There is a perfect agreement of our results with the observed records on prototype made of reinforced concrete, concrete, brick, rivetted steel, welded steel and welded aluminium.

The truncated conical model with a mass 0.076 has shown a greater frequency as compared with the regular cylindrical models for the same Reynolds number. This in turn has increased the Strouhal number. Strouhal number in this case has shown a variation between 0.256 and 0.121 for the same Reynolds number range as above cases.

\( \text{Fig. 12, 21, 297} \)

**Types of vibration:**

In the lateral direction, Strouhal number in the working range is practically constant. The following relation holds for this range -

\[
\frac{f_s d}{V} = S = S_{res} = \frac{f_{nd}}{V_{res}}
\]

\[
\frac{V}{V_{res}} = \frac{f_s}{f_n}.
\]
This relation can be represented by a straight line making an angle of 45° with the horizontal. This represents the forced vibration due to excitation by the eddy shedding force on a cylinder.

The relationship between frequency ratio \( \left( \frac{f}{f_{n}} \right) \) and velocity ratio \( \left( \frac{V}{V_{\text{res}}} \right) \) specifies the type of vibration occurring in the test. When the velocity of flow is low the amplitude is small and the velocity remains much less than the velocity of resonance for both longitudinal and lateral directions. Vibrations in this range will be of forced type. For circular sections, as soon as the velocity ratio \( \left( \frac{V}{V_{\text{res}}} \right) \) approaches 0.4 the longitudinal vibration starts, the lateral vibration starts later (only when \( \frac{V}{V_{\text{res}}} = 0.4 \)). During the initial stages of lateral vibration, longitudinal vibration effects the lateral vibration. The points in this range on the region appear on the figures as controlled vibration. When the velocity ratio \( \left( \frac{V}{V_{\text{res}}} \right) \) approaches 1.2, the longitudinal vibration disappears, and the curve of the controlled vibration degenerates into that of the forced vibration. Beyond velocity ratio \( \left( \frac{V}{V_{\text{res}}} \right) 1.25 \), longitudinal vibration is re-excited by lateral vibration and the vibration in the two directions builds up prominently. In this range the vibration in one direction is effected by that of the other direction and vice versa. Vibration in this range appears as
controlled type. A lot depends on the value of the natural frequency and the magnitude of structural stiffness. It was found that for circular sections, frequency in the longitudinal direction is twice that of the frequency in the lateral direction. If for example the natural frequency in lateral direction is twice that of longitudinal direction, then lateral and longitudinal resonance might coincide. This condition occurred in the case of D-shaped cylinder with flat face parallel to the flow because of the different cross-sectional area in each direction. If the stiffness is less, the amplitude around longitudinal resonance will be too large, so that, due to the fast shifting of separation points and the increasing rate of eddy generation, the flow pattern at rear of the cylinder becomes almost random. The nature of vibration then alters. Since the models tested were of different shapes which altered the natural frequency of vibration and the damping coefficients -- each case had to be studied on the individual merit. A detailed discussion is presented for each series of tests in the rest of this chapter.

Types of vibrations occurring:

Model 1. 2" dia. brass cylinder:

Fig. 6 and Fig. 7 show the types of vibrations. At low flows the amplitudes in either direction remain small and
velocity of flow remains far from the velocity of resonance in longitudinal and lateral directions. Vibrations in this zone appear as forced type. In longitudinal directions forced vibrations are prevalent up to \( \frac{V}{V_{x \text{res}}} = 0.9 \). As the stiffness of the system is small, the amplitudes in the lateral directions are large. In lateral direction, forced vibrations are prevalent up to \( \frac{V}{V_{y \text{res}}} = 0.90 \). Beyond \( \frac{V}{V_{x \text{res}}} = 0.9 \), longitudinal vibrations are effected by the lateral vibrations. Beyond \( \frac{V}{V_{x \text{res}}} = 0.9 \) the longitudinal vibrations are of the self-excited type. Beyond \( \frac{V}{V_{y \text{res}}} = 0.9 \), the amplitude of the longitudinal vibrations become sufficiently high so as to effect the forcing function which in turn effect the flow pattern and as a consequence the lateral vibration is effected. Lateral vibration beyond \( \frac{V}{V_{y \text{res}}} = 0.9 \) appears to be of the controlled type. It would be expected that, because the frequency of eddy shedding is a function of amplitude of lateral vibration, a certain degree of control will exist whenever the amplitude of vibration is significant compared with cylinder diameter.

Model 2: 2" dia. wooden cylinder:

In fig. 17 from \( \frac{V}{V_{x \text{res}}} = 0.4 \) to \( \frac{V}{V_{x \text{res}}} = 1.1 \) the vibration appears to be self-excited. Beyond \( \frac{V}{V_{x \text{res}}} = 1.1 \), the lateral vibration grows up and it effects the longitudinal vibration. Amplitude of the lateral vibration is much
bigger than that of the longitudinal vibration. Longitudinal vibration beyond \( \frac{V}{V_{x \text{res}}} = 1.1 \) is predominantly of controlled type. In fig. 16, from \( \frac{V}{V_{y \text{res}}} = 0.45 \) to \( \frac{V}{V_{y \text{res}}} = 0.70 \), the vibrations appear to be of forced type. Because in this range the amplitude of the longitudinal vibration is not big enough to effect the flow pattern seriously and to influence the lateral vibration, beyond \( \frac{V}{V_{y \text{res}}} = 0.70 \), lateral vibrations are of controlled type. Here the mutual effect of lateral and longitudinal vibrations is quite marked.

Model 3: Truncated-conical model:

In fig. 25, from \( \frac{V}{V_{x \text{res}}} = 0.5 \) to \( \frac{V}{V_{x \text{res}}} = 1.1 \), longitudinal vibration appears as self-excited. Beyond \( \frac{V}{V_{x \text{res}}} = 1.1 \), lateral vibrations build up and the magnitude lateral amplitude becomes much more than that of the longitudinal vibration. Hence the effect of lateral vibration on the longitudinal vibration becomes prominent. Hence beyond this point longitudinal vibrations are of controlled type. In lateral direction \( \text{[Fig. 24]} \) amplitude of lateral vibration at the beginning comes out to be equal in magnitude to that of the longitudinal amplitude. Hence up to \( \frac{V}{V_{y \text{res}}} = 0.8 \), vibrations are of controlled type.

From \( \frac{V}{V_{x \text{res}}} = 1.3 \) to 1.6, the longitudinal amplitude becomes practically zero, and the curve of the controlled
vibration in the lateral direction drops down to the forced vibration. From then on as the longitudinal amplitude grows up the influence of it on the lateral vibration becomes more marked again. Hence the lateral vibration above \( \frac{V}{V_y \text{ res}} = 0.9 \) becomes controlled type again.

Coefficient of lift versus Reynolds number:

Magnification factor is taken into consideration in the calculations of coefficient of lift \((C_L)\) and coefficient of drag \((C_D)\). In the expression for magnification factor \( \frac{w}{wn} \) term occurs in the denominator. At the exact peak point of resonance \( w \) would be equal to \( wn \), so the magnification factor would be the highest.

In case of 2" dia. brass cylinder, \( C_L \) rises up from a value 0.63 until the maximum value of \( C_L = 4.7 \) is reached at \( Re = 0.378 \times 10^5 \), then on it gradually decreases until finally it drops to a value of 0.11 at \( Re = 0.646 \times 10^5 \). \( \text{Fig. 14} \). In the case of the 2" dia. wooden cylinder, the mass is reduced and the magnification factor is therefore increased compared with the brass cylinder. As a consequence the lift coefficient is diminished, \( C_L \) gradually rising up from a value 0.308 at \( Re = 0.234 \times 10^5 \) until reaching a maximum of \( C_L = 1.75 \) at \( Re = 0.429 \times 10^5 \), then drops to 0.189 at \( Re = 0.476 \times 10^5 \) \( \text{Fig. 23} \).
For the truncated conical model, \( C_L \) gradually rises up from a value \( C_L = 0.079 \) at \( Re = 0.224 \times 10^5 \) until it reaches a maximum \( C_L = 2.36 \) at \( Re = 0.354 \times 10^5 \) and then gradually drops down to a value \( 0.254 \) at \( Re = 0.57 \times 10^5 \) \( \text{Fig. 31} \).

In all these cases there is a striking similarity of the graphs plotted with \( C_L \) versus \( Re \).

**Coefficient of drag versus Reynolds number:**

2" dia. hollow brass cylinder:

Maximum value of \( C_D = 1.96 \) and minimum value of \( C_D = 0.04 \). Maximum values of \( C_L \) and \( C_D \) occur at \( Re = 0.378 \times 10^5 \). There is a second maximum of \( C_D = 1.89 \) at \( Re = 0.552 \times 10^5 \) \( \text{Fig. 15} \).

For the 2" dia. wooden cylinder and the truncated-conical model, maximum value of \( C_D = 3.4 \) at \( Re = 0.4 \times 10^5 \) minimum value of \( C_D = 0.04 \). \( \text{Fig. 23, 31} \).

**Group B**

(i) D-shaped model:

**Flat face away from the flow**

Unlike the circular cylinder, the lateral vibration appears first. Vibration in the longitudinal direction appears later \( \text{Fig. 33} \) and is excited by the lateral vibration.
At very high velocities, the lateral vibration shows a beat phenomenon. The lateral resonance for the 2" dia. wooden cylinder occurs at \( \text{Re} = 0.44 \times 10^5 \) and the full maximum amplitude is 5". In this case lateral resonance occurs at \( \text{Re} = 0.455 \times 10^5 \) and the full maximum amplitude was 1".8. There was no sign of longitudinal resonance occurring anywhere. Longitudinal vibration was erratic and the magnitude of the amplitude remained less than \( \frac{1}{2}" \) all along /Fig. 34/. Galloping would not normally appear at this stage and it would be interesting to be able to test at high Reynolds numbers to see whether the vibrations cease or build up.

**Variation of Strouhal number with Reynolds number**

\( S_x \) starts with a value of 0.243 at \( \text{Re} = 0.348 \times 10^5 \) and gradually drops to a value \( S_x = 0.191 \) at \( \text{Re} = 0.506 \times 10^5 \) /Fig. 34/. The maximum value of \( S_y = 0.292 \) at \( \text{Re} = 0.31 \times 10^5 \) and the minimum value of \( S_y = 0.188 \) at \( \text{Re} = 0.568 \times 10^5 \). As compared to the 2" dia. wooden cylinder, the variation of \( S_x \) here is much less but the values of \( S_y \) are higher.

**Types of vibrations:**

Lateral vibrations from \( \frac{V}{V_y \text{ res}} = 0.65 \) to \( \frac{V}{V_y \text{ res}} = 1.0 \), are of the self-excited type /Fig. 38/. Longitudinal vibration is much less than the lateral vibration. As such the influence of the longitudinal vibration upon the lateral vibration is
not prominent. Later vibrations beyond \( \frac{V}{V_{y\text{res}}} = 1.0 \) appear as the forced type.

Coefficient of lift versus Re:

\[ C_L = 0.027 \text{ at } Re = 0.322 \times 10^5 \text{ and reaches a maximum value of } 1.04 \text{ at } Re = -0.604 \times 10^5 \] \( \text{Fig. 37} \). Maximum value of \( C_L \) is lower than any of the cases described under group A.

Flat face normal to and facing the flow:

Lateral oscillations appear earlier than the longitudinal vibration. Longitudinal vibrations are quite irregular and are effected by the lateral vibration all along. Lateral resonance occurs at \( Re = 0.435 \times 10^5 \) and the magnitude of amplitude is 6 inches \( \text{Fig. 39} \). This amplitude is bigger than that occurring in any of the cases described so far. The amplitude of the longitudinal vibration remained less than 4.75 inches \( \text{Fig. 39} \).

So this orientation of the D-shaped model exhibits higher amplitudes of vibration than with the flat face away from the flow.

Variation of Strouhal number with Re:

\( S_x \) starts with a value \( 0.316 \text{ at } Re = 0.260 \times 10^5 \) and rises until it reaches a maximum of \( 0.425 \text{ at } Re = 0.38 \times 10^5 \), and then drops to a value \( S_x = 0.37 \text{ at } Re = 0.435 \times 10^5 \) \( \text{Fig. 42} \).
Maximum value of $S_y = 0.333$ at $Re = 0.254 \times 10^5$ and minimum value of $S_y = 0.185$ at $Re = 0.435 \times 10^5$. $S_y$ max. is higher than the $S_y$ max. for the case where flat face is away from the flow.

**Types of vibrations:**

Because of the configuration of the body, any movement of it changes the forcing function and thus the flow pattern. Vibrations when the amplitude of vibration is large therefore exhibit a degree of control or self-excitation at $Fig. \ 387$.

**Coefficient of lift ($C_L$) versus $Re$:**

For the D-shape $C_L$ starts rising from a value $C_L = 0.049$ at $Re = 0.254 \times 10^5$ and reaches a maximum $C_L = 2.12$ at $Re = 0.318 \times 10^5$, and then drops down to $C_L = 1.3$. $Fig. \ 437$. Maximum value of $C_L$ here is twice the value of $C_L$ max. found for the case where the flat face is kept away from the flow. This value is even a bit higher than the maximum value of $C_L$ found for the 2" dia. wooden cylinder.

**Flat face parallel to the flow:**

Longitudinal and lateral oscillations appear simultaneously at $Re = 0.178 \times 10^5$ and start building up gradually. Longitudinal and lateral resonances occurred at
the same velocity of flow at \( Re = 0.388 \times 10^5 \). \[ \text{Fig. 46} \]

Amplitudes of vibrations are the smallest here of all the cases discussed so far. At \( Re = 0.405 \times 10^5 \) the two longitudinal and lateral vibrations are reduced to zero. From then on, as velocity of flow increases, the vibrations also gradually increase. The vibration in one direction is henceforth effected by the vibration in the other direction.

**Strouhal number versus Reynolds number:**

\[ S_x \text{ is maximum and equal to } 0.206 \text{ at } Re = 0.178 \times 10^5 \]

and gradually comes down until \( S_x = 0.051 \) at \( Re = 0.725 \times 10^5 \). Maximum value of \( S_y = 0.206 \) at \( Re = 0.178 \times 10^5 \) and minimum value of \( S_y = 0.02 \) at \( Re = 0.686 \times 10^5 \). Of all the orientations of the D section, the value of \( S_y \) in this case is the smallest. \[ \text{Fig. 48, 49} \]

The nature of the graphs for \( S_x \) versus \( Re \) and \( S_y \) versus \( Re \) are identical. Values of \( S_x \) and \( S_y \) are also practically identical.

**Types of vibrations:**

Velocity of resonance in both lateral and longitudinal directions is the same, and the amplitudes of vibrations in both directions are of equal magnitude and display the same type of variation with respect to Reynolds numbers. So from
\[
\frac{V}{V_{\text{res}}} = 0.46 \text{ to } \frac{V}{V_{\text{res}}} = 1.0, \text{ vibrations in either direction are of controlled type. From } \frac{V}{V_{\text{res}}} = 1.00 \text{ to } \frac{V}{V_{\text{res}}} = 1.1, \text{ amplitudes in both directions become negligibly small and as such influence of vibrations on changing the flow pattern is negligible. In this range vibrations in both directions appear as forced type. Beyond this range amplitudes in both directions again build up and vibrations in both directions appear as controlled type.} \quad \text{[Fig. 44, Fig. 45].}
\]

**Coefficient of lift versus Reynolds number:**

\[C_L\] gradually rises up from a value, \(C_L = 0.047\) at \(Re = 0.232 \times 10^5\) until it reaches a maximum \(C_L = 0.55\) at \(Re = 0.388 \times 10^5\). Then \(C_L\) drops down to a value \(C_L = 0.013\) at \(Re = 0.46 \times 10^5\), but rises up to a second maximum of \(C_L = 0.510\) at \(Re = 0.725 \times 10^5\). \[\text{[Fig. 50].}\] The maximum value of \(C_L\) remains less than any of the cases described in group A and also for the cases where flat face of the D-section was normal to flow and away from the flow.

**Coefficient of drag versus Re:**

Maximum value of \(C_D = 0.213\) occurs at \(Re = 0.388 \times 10^5\) and the minimum value of \(C_D = 0.005\) occurs at \(Re = 0.405 \times 10^5\). Maximum value of \(C_D\) in this case is much less compared to that occurring in the case of 2" dia. wooden cylinder.
2" x 2" square cylinder:

Lateral vibrations appear first and start building up. As velocity increases oscillations in both directions go on increasing. Amplitude of the longitudinal and the lateral vibrations remained less 2 inches. \( \text{Fig. 57} \).

**Strouhal number versus Re:**

Maximum value of \( S_x = 0.283 \) occurred at \( Re = 0.212 \times 10^5 \) and gradually comes down to \( S_x = 0.135 \) at \( Re = 0.595 \times 10^5 \). \( \text{Fig. 567} \).

Maximum value of \( S_y = 0.545 \) occurred at \( Re = 0.112 \times 10^5 \) and the minimum value of \( S_y = 0.068 \) occured at \( Re = 0.595 \times 10^5 \). Maximum value of \( S_y \) occuring in this case is higher than that occured in any of the cases described so far.

**Types of vibrations:**

From \( \frac{V}{V_y \text{ res}} = 0.6 \) to \( \frac{V}{V_y \text{ res}} = 1.0 \), later vibrations are self-excited type. Beyond \( \frac{V}{V_y \text{ res}} = 1.0 \), amplitudes of the longitudinal vibrations become large enough to effect the vibrations in the lateral directions. Lateral vibrations beyond \( \frac{V}{V_y \text{ res}} = 1.0 \) appear clearly as controlled type. Longitudinal vibrations up to \( \frac{V}{V_x \text{ res}} = 1.0 \) are of forced type. Beyond \( \frac{V}{V_x \text{ res}} = 1.0 \) longitudinal vibrations are of controlled type. \( \text{Fig. 51, Fig. 527} \).
Coefficient of lift versus Re:

$C_L$ gradually rises up from a value of $C_L = 0.025$ at $Re = 0.122 \times 10^5$ until it reaches a maximum of $C_L = 1.45$ at $Re = 0.329 \times 10^5$. Above $Re = 0.329 \times 10^5$ the value of $C_L$ remains practically constant. \(\text{\textsuperscript{[Fig. 527]}\textsuperscript{[Fig. 527]}}\). This value of $C_L$ maximum is less than that which occurred in the case of the D-section with flat face normal to the flow.

Coefficient of drag versus Re:

All values of $C_D$ lie between 1.1 and 0.037. Maximum value of $C_D$ remains less than that for cases described under group A. $C_D$ max. here is higher than that which occurred in the D section with flat face parallel to flow.

1" x 2" rectangular cylinder:

1" face normal to flow

Longitudinal vibrations appear first at $Re = 0.031 \times 10^5$ and start building up. At $Re = 0.044 \times 10^5$ longitudinal vibrations disappear. At this stage lateral vibrations appear and go on building up. Lateral resonance occurs at $Re = 0.120 \times 10^5$. At this point longitudinal vibrations reappear. Longitudinal resonance occurs at $Re = 0.141 \times 10^5$, and from then on amplitudes of the longitudinal and lateral vibrations go on diminishing as
velocity of the flow increases. Maximum amplitude of the longitudinal vibration is 0.14 inches and that of the lateral vibration is 0.66 inches (Fig. 60, 61). These values are the smallest of all so far discussed.

**Strouhal number versus Re:**

Maximum value of $S_x = 0.99$ occurred at $Re = 0.021 \times 10^5$ and then drops to a value of 0.066 at $Re = 0.32 \times 10^5$ (Fig. 64).

Maximum value of $S_y = 0.365$ occurred at $Re = 0.044 \times 10^5$ and the minimum value of $S_y = 0.144$ occurred at $Re = 0.171 \times 10^5$ (Fig. 64). $S_y$ max occurring here is lower than that which occurred in case of 2" x 2" square model but higher than that occurred in any other cases discussed so far.

**Types of vibrations:**

Longitudinal vibrations from $V/V_{x_{res}} = 0.6$ to $V/V_{x_{res}} = 1.0$ is of controlled type, from $V/V_{x_{res}} = 1.0$ to $V/V_{x_{res}} = 1.5$ is of forced type and over $V/V_{x_{res}} = 1.5$ it is of controlled type again (Fig. 58). Lateral vibrations from $V/V_{y_{res}} = 0.5$ to $V/V_{y_{res}} = 1$ are of controlled type. Beyond this point amplitude of the lateral vibration reduces until $V/V_{y_{res}} = 2.0$. In this range lateral vibrations are of forced type. Above $V/V_{y_{res}} = 2.0$ amplitude of the lateral
vibration increases and the lateral vibrations above \( \frac{V}{V_{y \text{ res}}} = 2.0 \) fall under controlled type. [Fig. 57].

**Coefficient of lift versus Re:**

\[
C_L = 0.965 \text{ at } Re = 0.044 \times 10^5 \text{ and then it gradually comes down to } C_L = 0.108 \text{ at } Re = 0.107 \times 10^5.
\]

Then again it rises up to \( C_L = 1.48 \) at \( Re = 0.189 \times 10^5 \). [Fig. 65]. This value of \( C_L \text{ max.} \) is equal to that in the case of 2" x 2" square model.

**Coefficient of drag versus Re:**

\[
C_D = 0.038 \text{ at } Re = 0.107 \times 10^5 \text{ and rises up to a value } C_D = 0.381 \text{ at } Re = 0.226 \times 10^5 \text{ then it drops to a value } C_D = 0.005 \text{ at } Re = 0.304 \times 10^5. \quad [\text{Fig. 65}].
\]

**2" Face normal to the flow:**

Vibrations do not occur until \( Re = 0.240 \times 10^5 \). From then on the lateral oscillations appear. Longitudinal vibrations occur only at \( Re = 0.405 \times 10^5 \). The longitudinal amplitude remains less than 1.2 inches. This value is higher than that which occurred for the 1" face normal to the flow. Maximum amplitude of the lateral vibrations is 2.86 inches. This value is higher than that which occurred in the case where the 1" face was normal to the flow. [Fig. 66, 68].
Longitudinal vibrations only occur when they are excited by the lateral vibrations and they have no regularity. For all velocities, the frequency in longitudinal direction is twice the frequency in the lateral direction.

**Strouhal number versus Reynolds number:**

Strouhal number in the longitudinal direction varies between 0.36 and 0.26 and that in the lateral directions varies between 0.18 and 0.12. [Fig. 7](#). Maximum value of $S_y$ is less than that which occurred when the 1st face was normal to the flow. The value of $S_y$ corresponds to those in group A.

**Types of vibrations**

The amplitude of longitudinal vibration remains very low compared to that of the lateral vibrations. Up to $\frac{V}{V_y \text{res}} = 0.6$, amplitude of the lateral vibration is very small. Hence from $\frac{V}{V_y \text{res}} = 0.4$ to $\frac{V}{V_y \text{res}} = 0.6$, lateral vibrations are of forced type. Above $\frac{V}{V_y \text{res}} = 0.6$, amplitude of lateral vibrations are increased and from $\frac{V}{V_y \text{res}} = 0.6$ to $\frac{V}{V_y \text{res}} = 0.90$ lateral vibrations are of controlled type. Beyond $\frac{V}{V_y \text{res}} = 0.9$, lateral vibrations are of self-excited type.
6.2 Conclusions:

From the experimental work presented here, it may be concluded:

1. For the circular cylinders, the velocity of resonance for the lateral vibration is 2.3 times the velocity of resonance for the longitudinal direction. Vibrations in either the longitudinal or lateral directions only occurred for a small range of velocity either side of resonance. This is therefore basically a resonance phenomenon although a certain degree of controlled vibration was also observed.

2. For the circular cylinders, the longitudinal vibration disappears after resonance in the longitudinal direction \( \frac{V}{V_{x \text{ res}}} = 1.2 \) but it is re-excited by the lateral oscillations above \( \frac{V}{V_{x \text{ res}}} = 1.25 \). In this higher velocity range the amplitudes of the longitudinal vibrations build up and attain a second maximum. No reference has been found in the literature to this re-excitation of the longitudinal mode by the lateral mode and it is believed that this is the first report of such a phenomenon. The re-excited longitudinal vibration might be classified either as a self-excited vibration or as a controlled vibration.

3. For the circular cylinders, the Strouhal numbers for lateral vibrations vary within narrow limits of 0.185 and 0.140.
4. For the circular cylinders, the Strouhal number, determined from the structural response frequency for the longitudinal direction is of the order of 0.83 at Re = 0.085 x 10^5 and gradually drops down to 0.08 at Re = 0.600 x 10^5. Note that the Strouhal numbers quoted, which are deduced from structural response, may not reflect the actual eddy shedding Strouhal number. Further work should be done to determine the corresponding wake Strouhal numbers.

5. For the circular cylinders, C_L max = 4.7 with a magnification factor of the order of 2. The maximum value of the coefficient of drag for the 2" dia. wooden cylinder was found to be 3.32 whereas C_D max. for the 2" dia. hollow brass cylinder was 1.89. The increased mass and smoothness of the brass cylinder resulted in lowering the value of the coefficient of drag.

6. For the D-section, the flat face normal to the flow is found to be the most unstable of all orientations. Of all the cylinders tested and of the different orientations of the D shaped cylinders, the flat face parallel to the flow gives the least coefficients of lift and drag.

7. For the D-shaped cylinder, with the circular face upstream, the variation of Strouhal number in the lateral direction is the same as that found with the circular face downstream; the range of variation being 0.33 to 0.185.
For the flat face parallel to the flow, the Strouhal number variation for lateral vibrations is similar to that of the circular cylinders.

8. The Strouhal number variation for the rectangular cylinder with the 2" face normal to the flow shows a similar variation to that of the circular cylinders, namely from 0.18 to 0.12.

9. For the two rectangular cylinders tested the coefficient of lift reduces as the after body length (dimension parallel to the flow) decreases. The maximum value of $C_L$ for 2" x 2" square section is 1.45 with magnification factor of 1.34 and that for 1" x 2" rectangular section with 2" face normal to flow is 1.28 with magnification factor of 1.41. Maximum value of $C_L = 2.54$ for 1" x 2" rectangular cylinder when 1" face is normal to flow and max. $C_L = 1.28$ when 2" face is normal to flow. Cylinders of different ratios of width to after body length should be tested to confirm this trend.
VII. Appendix:

7.1 Virtual mass of a solid moving through fluid:

The total kinetic energy of the solid and of the fluid set in motion by it, can be regarded as the kinetic energy of a solid of the same dimensions, but of increased mass, the increase being known as the virtual mass.

For the cylinder moving with a velocity $U$, through a fluid, initially at rest, the fluid velocity at any point has the magnitude

$$V = \frac{N}{\gamma^2} = \frac{a^2}{\gamma^2} U.$$ The centre of the cylinder is at the origin at that instant. Total kinetic energy of the fluid per unit length of cylinder is

$$T' = \int_\gamma^\infty \frac{1}{2} \nu^2 \, d\gamma.$$

$$= \rho \pi a^4 U^2 \int_\gamma^\infty \frac{d\gamma}{\gamma^3}.$$

$$= \frac{1}{2} \rho \pi a^2 U^2 = \frac{1}{2} m' U^2$$

where $m' = \rho \pi a^2$, the mass of fluid with a volume equal to the cylinder volume. Total kinetic energy of the fluid and the cylinder is

$$T = \frac{1}{2} (M + m') U^2.$$

In accelerating or retarding the cylinder, since the work done equals the change in total kinetic energy, the effective mass
to be considered is the actual mass plus the virtual mass, and the additional resistance to accelerative force is -

\[ F' = m' \frac{du}{dt} \]
Bibliography


Nomenclature

\( \text{Re} = \text{Reynolds number} \left(\frac{Vd}{\nu}\right) \)

\( d = \text{diameter of circular cylinder} \)

\( \nu = \text{kinematic viscosity} \ (\text{ft}^2 \text{ per sec.}) \)

\( V, u = \text{velocity of flow (fps)} \)

\( S = \text{Strouhal number} \left(\frac{fd}{V}\right) \)

Suffix \( x = \text{direction parallel to the flow, also referred to as longitudinal} \)

Suffix \( y = \text{direction normal to the flow, also referred to as lateral} \)

\( f = \text{frequency of the forcing function} \)

\( f_n = \text{natural frequency of the system} \)

\( P = \text{force} \)

\( F_D = \text{maximum hydro-elastic vibrating force in longitudinal direction (Drag force)} \)

\( F_L = \text{maximum hydroelastic vibrating force in lateral direction (Lift force)} \)

\( C_D = \text{coefficient of drag} = \frac{F_D}{\frac{1}{2} \rho u^2 A} \)

\( C_L = \text{coefficient of lift} = \frac{F_L}{\frac{1}{2} \rho u^2 A} \)

\( A = \text{amplitude of vibration} \)

\( V_{\text{res}} = \text{velocity of resonance} \)

\( \rho = \text{density of water} \)

\( t = \text{time} \)

\( k = \text{spring constant} \)
\( b \) = dimension of the rectangular cylinder in a direction parallel to flow.

\( h \) = dimension of the rectangular cylinder in a direction normal to flow.

\( \omega = (\frac{k}{M})^{\frac{1}{2}} \) = circular frequency of free undamped oscillation of system,

\( m \) = mass of oscillating system,

\( \delta \) = logarithmic decrement of damping.

\( M \) = magnification factor.
Figure 5: Flume

**Plan**
- I-beams
- Strain gauge
- Water surface
- Splitter plate
- Streamlined ring
- 12" long model
- 1/2" square Al. cantilever

**Sectional Elevation**
- Curved contraction
- Test section
- 15"
- 30"
- 3'
- 1'
- 2'
- 25'

*Fig. 5 Flume*
Fig. 6. 2" dia. hollow brass cylinder with ¾" square aluminium cantilever. Variation of type of vibration.
Fig. 7. 2" dia. hollow brass cylinder with 3" square al. cantilever. Variation of type of vibration.
Fig. 8. 2" dia. hollow brass cylinder, with 6" square al. cantilever.
Fig. 9. 2" dia. hollow brass cylinder with \( \frac{5}{4} \)" square al. cantilever.
Longitudinal amplitude vs. Re
Fig. 10. 2" dia. hollow brass cylinder with ½" square aluminium cantilever. $f_n = 2.78$ c.p.s.
Fig. 11. 2" dia. hollow cylinder with 1" square al. cantilever. fx versus Re

fn = 2.78 cps
Longitudinal resonance

Lateral resonance

Fig. 12. 2" dia. hollow cylinder with 1/2" square al. cantilever. Sy versus Re

Reynolds No. (Re)
Fig. 13. 2" dia. hollow brass cylinder with 1/4" square al. cantilever.

Sx versus Re

Longitudinal resonance

Lateral resonance
Fig. 14. 2" dia. hollow brass cylinder with 1/8" square al. cantilever

Coefficient of lift versus Reynolds number

Longitudinal resonance

Lateral resonance
Fig. 15. 2" dia. hollow brass cylinder with \( \frac{3}{4} \)" square al. cantilever

Coefficient of drag versus Reynolds number

Longitudinal resonance

Lateral resonance
Fig. 16. 2" dia. wooden cylinder

Variation of types of vibration

Self-excited vibration

Forced vibration

Controlled vibration

\[ \frac{f_y}{f_n} \text{ vs. } \frac{V}{V \text{res}} \]
Fig. 17. 2" dia. wooden cylinder

Variation of types of vibration

Forced vibration
Self-excited vibration
Controlled vibration

\[ \frac{f_x}{f_n} \]

\[ \frac{V}{V_{x \text{res}}} \]
Fig. 18. 2" dia. wooden cylinder. Amplitude vs Reynolds No.
Fig. 19. 2" dia. wooden cylinder.

Lateral frequency vs. velocity.

Experimental values

$L = 0.169$

Longitudinal resonance

Lateral resonance

$V$ in f.p.s.
Fig. 20. 2" dia. wooden cylinder
Longitudinal frequency vs. Reynolds number.
Fig. 21. 2" dia. wooden cylinder

Strouhal number vs. Reynolds number
Fig. 22. 2" dia. wooden cylinder
Strouhal No. (Sx) vs. Re.

Longitudinal resonance

Lateral resonance
Fig. 23. 2\textsuperscript{nd} dia. wooden cylinder
Coefficient of drag vs. Re.

Longitudinal resonance

Lateral resonance

Coefficient of drag

Coefficient of lift
Fig. 24. Truncated-conical cylinder.

Variation of type of vibration.

Forced vibration

Self-excited vibration

Controlled vibration

$V/V_{res}$

$fy/fn$
Fig. 25. Truncated conical cylinder.

Variation of type of vibration.

- Forced vibration
- Controlled vibration
- Self-excited vibration
Fig. 26. Truncated-conical cylinder.
Amplitude vs. Reynolds number

Longitudinal amplitude

Lateral amplitude

Longitudinal resonance

Lateral resonance

Amplitude (max) - in inch

Re
Fig. 27. Truncated-conical cylinder.
Lateral frequency vs. velocity

Longitudinal resonance
Lateral resonance

Corresponds to $f_{ny}$

$S = 0.20$
Fig. 28. Truncated-conical cylinder.

Longitudinal frequency vs. velocity.

(0,0) 1 2 3 4 5 6 7 8
Fig. 29. Truncated-conical cylinder.
Strouhal No. vs. Reynolds No.
Fig. 30. Truncated conical cylinder. Strohhal No. (Sx) vs. Re.

corresponds to
$f_{nx} = 3.571 \text{ c.p.s.}$
Fig. 31. Truncated-conical cylinder. Coefficient of drag and lift vs. Re.
Fig. 32. D-shaped cylinder

Flat face away from flow.

Variation of the type of vibration.

Forced vibration

Self-excited vibration

Controlled vibration
Lateral resonance

Fig. 33. D-shaped cylinder
Flat face away from
flow

Lateral amplitude vs. Re
Fig. 34. D-shaped cylinder
Flat face away from flow

Longitudinal amplitude vs. Re
Lateral frequency \( (f_y) \) corresponds to \( f_y \approx 3.68 \)

Longitudinal frequency \( (f_x) \)

Fig. 35. D-shaped model
Flat face away from flow

Frequency vs. velocity
Fig. 36. D-shaped cylinder
Flat-face-away from flow

Strouhal Number vs. Re

- Lateral resonance

Strouhal Number (Sy)
Strouhal Number (Sx)
Fig. 37. D-shaped cylinder. Flat face away from flow Coefficient vs. Re

Lateral resonance

Strouhal Number (Sx)

Re

(0,0) .1 .2 .3 .4 .5 .6 .7 .8x10^5

0.5 0.6 0.7 0.8 1.0 1.5 2.0 2.5 3.0
Forced vibration
Self-excited vibration
Controlled vibration

Fig. 38. D-shaped cylinder
Flat face normal to flow.
Variation of the type of vibration

\[ \frac{V}{V_{y \text{ res}}} \]

0.20 0.40 0.60 0.80 1.00 1.20 1.40 1.60

\( \frac{f_y}{f_{y^*}} \)
Fig. 39. D-shaped cylinder
Flat face normal to flow
Amplitude vs. Re
Fig. 40. D-shaped cylinder
Flat face normal to flow
Lateral frequency vs. velocity
Fig. 41. D-shaped cylinder
Flat face normal to flow
Longitudinal frequency vs. velocity

(0,0) .1 .2 .3 .4 .5 .6 .7 .8x10^5
Fig. 42. D-shaped cylinder 
flat face normal to flow 
Strouhal No. vs. Re.
Fig. 43. D-shaped cylinder
flat face normal to flow

Coefficient of lift vs. Re

Lateral resonance
Variation of the type of vibration

Fig. 44. D-shaped cylinder flat face parallel to flow

Controlled vibration

Forced vibration

Self-excited vibration
Controlled vibration / Self-excited vibration

Forced vibration

Fig. 45. D-shaped cylinder flat face parallel to flow

Variation of type of vibration

\[ \frac{f_x}{f_{nx}} \]

\[ \frac{V}{V_{x res}} \]
Amplitude vs. Re.

Resonance, both lateral and longitudinal

Longitudinal amplitude

Lateral amplitude

Fig. 46. D-shaped cylinder flat face parallel to flow

Amplitude vs. Re.
Fig. 47. D-shaped cylinder
flat face parallel to flow

Frequency vs. velocity

- Longitudinal frequency ($f_x$)
- Lateral amplitude

corresponds to $f_{ny} = 3.16$ CPS
Fig. 48. D-shaped cylinder flat face parallel to flow

Strouhal No. (Sy) vs. Re.

Resonance, lateral & longitudinal
Fig. 49. D-shaped cylinder
flat face parallel to flow

Strouhal No. (Sx) vs. Re

Resonance, lateral & longitudinal

Re

(0,0) 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8x10^5
Fig. 50. D-shaped cylinder, flat face parallel to flow

Coefficient of Drag and Lift vs. Re

Resonance, both lateral and longitudinal

Coefficient of Lift ($C_L$)

Coefficient of Drag ($C_D$)

Re

(0,0) .1 .2 .3 .4 .5 .6 .7 .8 x $10^5$
Fig. 51. 2" x 2" square cylinder

Variation of type of vibration

Forced vibration

Self-excited vibration

Controlled vibration
Fig. 52. 2" x 2" square cylinder.

Variation of type of vibration
Fig. 53. 2" x 2" square cylinder
Amplitude vs. Reynolds No.

Longitudinal amplitude
Lateral amplitude (Ay)
Corresponds to $\nu_{ny} = 2.92$ CPS

Fig. 54. 2" x 2" square cylinder.
Frequency vs. velocity
Fig. 55. 2" x 2" square cylinder

Frequency (fx) vs. velocity

Corresponds to fnx 2.92 CPS
Fig. 56. 2" x 2" square cylinder
Strouhal No. vs. Re
Fig. 57. 2" x 2" square cylinder

Coefficient of drag and lift vs. Re

Coefficient of Drag ($C_D$) vs. Reynolds Number ($Re$)

- Coefficient of Lift ($C_L$)
- Longitudinal resonance
- Lateral resonance
Fig. 5.1 1" x 2" rectangular cylinder
1" face normal to flow

Variation of type of vibration

Forced vibration
Self-excited vibration
Controlled vibration

\( f_y / f_{ny} \) vs. \( V / V_{y \text{ res}} \)
Forced vibration

Self-excited vibration

Controlled vibration

Fig. 59. 1" x 2" rectangular cylinder
1" face normal to flow

Variation of type of vibration

\[
\frac{f_x}{f_{nx}} \quad \text{vs} \quad \frac{V}{V_x \text{ res}}
\]
Fig. 60. 1" x 2" rectangular cylinder
1" face normal to flow

Lateral amplitude vs. Re

Lateral resonance
Fig. 61. 1" x 2" rectangular cylinder
1" face normal to flow

Longitudinal amplitude vs. Re

Longitudinal resonance
Fig. 62. 1" x 2" rectangular cylinder
1" face normal to flow

Frequency (fx) vs. velocity, V

Corresponds to fnx = 3.571
Fig. 63. 1\textquotedbl x 2\textquotedbl rectangular cylinder
1\textquotedbl face normal to flow
Frequency (fy) vs. velocity, V

Corresponds to 
fy = 3.125 CPS
Fig. 64. 1" x 2" rectangular cylinder
1" face normal to flow
Strouhal No. vs. Re

(0,0)  .1  .2  .3  .4  .5  .6  .7  .8 x 10^5

Longitudinal resonance
Lateral resonance

Strouhal No.
Fig. 65. 1" x 2" rectangular cylinder
1" face normal to flow

Coefficients of lift and drag vs. Re
Fig. 66: 1" x 2" rectangular cylinder
2" face normal to flow
Variation of type of vibration
Fig. 67. 1\" x 2\" rectangular cylinder
2\" face normal to flow

Lateral amplitude vs. Re

Lateral resonance
Fig. 68. 1" x 2" rectangular cylinder
 2" face normal to flow

Longitudinal amplitude vs. Re
Corresponds to lateral resonance \( f_y = 3.125 \) CPS

Fig. 69. 1" x 2" rectangular cylinder
2" face normal to flow

Frequency \( (f_y) \) vs. Velocity
Fig. 70. 1" x 2" rectangular cylinder
2" face normal to flow

Frequency (fx) vs. velocity
Fig. 71. 1" x 2" rectangular cylinder
2" face normal to flow

Strouhal No. vs. Reynolds No.

Strouhal No. (Sx)

Strouhal No. (Sy)

lateral resonance

(0,0) .1 .2 .3 .4 .5 .6 .7 .8x10^5

Re
Fig. 72. $\frac{1}{\text{in}} \times 2''$ rectangular cylinder, $2''$ face normal to flow

Coefficient of lift vs. Re.

Lateral resonance
PLATE 1. Models
PLATE 2. Test Section and the Model
PLATE 3. Flow Meter, Amplifiers and Oscillograph
PLATE 4. Cylinder Vibrating Near Longitudinal Resonance