LATERAL STABILITY OF RECTANGULAR BEAMS

BY

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We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA
1966
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ABSTRACT

This thesis presents a method for finding the lateral buckling load of a straight beam loaded perpendicular to its longitudinal axis with loads applied at any eccentricity. The beam can be restrained at any number of points along its length against deflections and rotations.

The beam is divided into a number of segments and joints for which equations involving statics, elasticity, and continuity are written. The resulting group of nonlinear simultaneous equations is solved for several magnitudes of the loading pattern for which a buckling load is desired. A graph of the determinant of the structure matrix versus the load level yields the critical load.

Included in the thesis is a complete listing of the computer program used in the solution technique. It was written for use on the IBM 7040 installation at the University of British Columbia. Included also are curves giving the lateral buckling stress of a beam simply supported at one end, and cantilevered over a flexible column at the other end, and whose top flange is laterally restrained. The beam carries a uniform load together with a concentrated load at the end of the cantilever. In addition, some data is included on the effect of tension flange bracing on the buckling load.
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<td>XDET1</td>
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<td>$I_y$</td>
<td>XI2</td>
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<td>$J$</td>
<td>XJ</td>
<td>torsional constant $J = th^3(1 - .63t)/(n)$</td>
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<td>$L$</td>
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<td>Q₁</td>
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<td>Q₂, Q₃</td>
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<td>s</td>
<td>slenderness ratio = $\sqrt{\frac{Lh}{t^2}}$</td>
<td></td>
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<tr>
<td>t</td>
<td>T width of beam</td>
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<tr>
<td>uᵢ</td>
<td>horizontal component of displacement of the centre of gravity of the beam</td>
<td></td>
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<tr>
<td>Vᵦ₁</td>
<td>shear force parallel to &quot;b&quot; axis (right face of joint i)</td>
<td></td>
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<tr>
<td>Vᵦ₂</td>
<td>shear force parallel to &quot;c&quot; axis (right face of joint i)</td>
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<tr>
<td>vᵢ</td>
<td>vertical component of displacement of the centre of gravity of the beam</td>
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<tr>
<td>w₁</td>
<td>uniform vertical load applied along segment i</td>
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<td></td>
<td>W₁(K)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>W₂(K)</td>
<td></td>
</tr>
<tr>
<td>βᵢ</td>
<td>angle of twist of beam at joint i, measured from initial position</td>
<td></td>
</tr>
<tr>
<td>β</td>
<td>generalized horizontal deformation</td>
<td></td>
</tr>
<tr>
<td>θ</td>
<td>generalized rotation</td>
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</tr>
<tr>
<td>θᵤᵢ</td>
<td>slope of beam in the xz plane at joint i</td>
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<tr>
<td>θᵥᵢ</td>
<td>slope of beam in the xy plane at joint i</td>
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<td>kcr</td>
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Vancouver, B.C.
January, 1966
INTRODUCTION

The solutions available for the elastic lateral buckling problem are limited to those beams with simple loads and support conditions [1]. A search of the literature failed to yield any general differential equations for the lateral buckling of a beam continuous over supports and arbitrarily braced. The lack of such a set of equations led the author to look more deeply into the problem, with the result that a general solution procedure, involving the electronic computer, was developed.

The method developed by the author is suitable for the analysis of a continuous rectangular beam under almost any combination of constraints, horizontal and vertical loads perpendicular to the longitudinal axis, and moments about any principal axis.

At present, the following limitations exist as to what type of problem can be analysed:

1. The beam must be of constant, rectangular cross-section.
2. Loads parallel to the longitudinal axis cannot be applied.
3. Pin joints cannot exist anywhere except at the extreme ends of the beam.

---

a. Numbers in square parentheses refer to the bibliography.
FIGURE 1. COORDINATE AXIS
DERIVATION OF EQUATIONS

2.1 ASSUMPTIONS

1. The trigonometric functions used in the derivation of the equations were approximated in accordance with the small deflection theory.

2. The curvatures in the ac and the ab planes, Fig. (1) were assumed to be equal to the curvatures in the xz and the xy planes respectively. This was in accordance with the small deflection theory.

3. Deformations of the beam due to shear stresses were neglected.

4. The material must be elastic, that is, it must obey Hooke's Law.

5. Loads applied vertically were assumed to remain vertical when the beam was in its deflected position. Similarly, horizontally applied loads were assumed to remain horizontal.

2.2 DIFFERENTIAL EQUATIONS

Two sets of co-ordinate axes, Fig. (1), are used in developing the required set of differential equations. The xyz system, with its origin at the centre of gravity of the left end of the beam, refers to the beam in its unstressed, undeformed position. From this system are measured u, the horizontal displacement of the centre of gravity of the beam, v, the vertical displacement of the centre of gravity,
and $\beta$, the rotation of the cross-section of the beam. The co-ordinate system based on the $abc$ axis has its origin at the centre of gravity of the beam section in its deformed position at the location $x = x$. Because the beam in its deformed position is curved, the orientation of these axes varies with $x$.

The differential equations were written for a beam with no discontinuities. This was in anticipation of the solution procedure which would require that the beam be divided into a large number of equal lengthed segments over which no discontinuities would exist, and joints, where all discontinuities due to concentrated loads or to constraints would be applied. From the free body diagram of the beam segment $dx$, drawn in its deformed position, Fig. (2a), five equations of statics were obtained. The internal forces were defined to be always acting in the plane of the deformed member. The following equations were obtained:

1. Sum of moments about the "a" axis at $x = x$, yields,

$$\frac{-dM_a}{dx} + M_c \frac{d^2u}{dx^2} + M_b \frac{d^2v}{dx^2} + w(h_1 + e_1 \beta) = 0$$

(1)

2. Sum of moments about the "b" axis at $x = x$, yields,

$$\frac{dM_b}{dx} + M_c \frac{d\beta}{dx} + M_a \frac{d^2v}{dx^2} + V_c + w(h_1 + e_1 \beta) \frac{dv}{dx} = 0$$

(2)

3. Sum of moments about the "c" axis at $x = x$, yields,

$$\frac{-dM_c}{dx} + M_b \frac{d\beta}{dx} - M_a \frac{d^2u}{dx^2} - V_b - w(h_1 + e_1 \beta) \frac{du}{dx} = 0$$

(3)
FIGURE 2a. FREE-BODY DIAGRAM OF BEAM SEGMENT

FIGURE 2b. POSITIVE DIRECTIONS FOR MOMENTS AND SHEARS
4. Sum of shears parallel to the "b" axis at \( x = x \), yields,

\[
\frac{dV_b}{dx} - V_c \frac{dB}{dx} + w = 0
\]

(4)

5. Sum of shears parallel to the "c" axis at \( x = x \), yields,

\[
\frac{dV_c}{dx} + V_b \frac{dB}{dx} - w \beta = 0
\]

(5)

The symbols \( h_1 \) and \( e_1 \) designate the horizontal and vertical eccentricities of the load \( w \), as measured from the centre of gravity of the cross-section. In deriving the five equations, only terms of the first order of magnitude (\( M, V, u, v, \beta \)), \( \frac{dM}{dx}, \frac{dv}{dx}, \frac{dv}{dx}, \frac{d\beta}{dx}, \frac{d^2u}{dx^2}, \frac{d^2v}{dx^2} \) were retained.

Since the five equations contained eight unknowns, three further equations were required for a unique solution. The three additional independent equations were obtained from elasticity considerations and are:

\[
EI_y \frac{d^2u}{dx^2} = -M_b
\]

(6)

\[
EI_z \frac{d^2v}{dx^2} = M_c
\]

(7)

\[
GJ \frac{d\beta}{dx} = -M_a
\]

(8)

The positive directions for moments is shown in Fig. (2b).

2.3 DEVELOPMENT OF A SOLUTION PROCEDURE

Due to the nonlinear nature of the equations involved, created by the existence of terms made up of products of forces multiplied by deformations, a numerical solution technique was used. The numerical technique was especially advantageous
since its use made possible the development of a very efficient system for adjusting the equations to accommodate constraints and concentrated loads.

The beam was subdivided into a large, even number of segments of equal length, each segment being bounded by joints. The joints are not to be taken as mechanical joints; rather, they are mathematical points at which all constraints and concentrated forces are to be applied. In this way, the segments were kept completely free of discontinuities. A set of algebraic equations was written for each joint and segment. The full group of equations was then solved on a computer.

With no discontinuities existing on the segment, and assuming that the segment size is small enough so that only negligible errors arise when a differential is replaced by a difference, as for example $dV$ being replaced by $V_{\text{right}} - V_{\text{left}}$, then the previous set of differential equations can be converted into an equivalent set of algebraic equations. Eqns. (1) to (5) and Eqn. (8) contain only first order differentials and therefore all the differentials occurring in these equations can be expressed by the end values of the one segment under consideration. Eqns. (6) and (7), however, are second order differential equations and therefore more than two points are required to express the function algebraically. To overcome this, two additional equations are included, defining two new unknowns. These equations are:
\[
\frac{du}{dx} = \theta u_{(\text{average})} \quad (9)
\]
\[
\frac{dv}{dx} = \theta v_{(\text{average})} \quad (10)
\]

Setting derivatives equal to the average of the end slopes of the segment has often been used in similar applications, with successful results. To systemize the solution, all joints are numbered sequentially beginning with 1 at the left end, while the segment number is set equal to the lesser of the joint numbers bordering it. In the following set of algebraic equations the subscript \(i\) refers to the joint number, and the superscripts \(R\) and \(L\) refer to the right and left faces of the joint respectively. Averaging of moments and shears occurring in the nonlinear terms was not done, rather, the value at the left end of the segment (right face of the joint) was always used. Referring to Fig. (3), the following set of equations was obtained:

\[
M_{a_1}^R - M_{b_1}^R \theta_{R}^R v_{1} - M_{c_1}^R \theta_{R}^R u_{1} - w_1e_1\beta_{1}^R
\]

\[
= M_{a_1}^L v_{1} + M_{b_1}^L \theta_{L}^L v_{1} + 1 - M_{c_1}^L \theta_{L}^L u_{1} + 1 + w_1dh_{1} \quad (1a)
\]

\[
M_{a_1}^R R + M_{b_1}^R + M_{c_1}^R \beta_{1}^R - V_{c_1}^R d - w_1d(h_{1} + e_{1}^R)\theta_{R}^R
\]

\[
= M_{a_1}^R \theta L v_{1} + 1 + M_{b_1}^L + 1 + M_{c_1}^L \beta_{1}^L + 1 \quad (2a)
\]
FIGURE 3. FREE-BODY DIAGRAM OF A JOINT AND A SEGMENT
\[ M^R_{a_1} \theta^R_{u_1} - M^R_{b_1} \beta^R_{1} + M^R_{c_1} - V^R_{b_1} d - w_1 d(h_1 + e^R_{1} \theta^R_{u_1}) = 0 \]

\[ = M^L_{a_1} \theta^L_{u_1} - M^L_{b_1} \beta^L_{1} + M^L_{c_1} + w_1 d \] (3a)

\[ v^R_{b_1} - v^R_{c_1} \gamma^R_{1} = v^L_{b_1} + w^R_{c_1} \beta^L_{1} + v_1 d \] (4a)

\[ v^R_{c_1} + v^R_{b_1} \gamma^R_{1} + w_1 d \theta^R_{1} = v^L_{c_1} + v^R_{b_1} \beta^L_{1} + v_1 d \] (5a)

\[ \theta^R_{u_1} - M^R_{b_1} d^{(2E1y)} = \theta^L_{u_1} + M^L_{b_1} + d^{(2E1y)} \] (6a)

\[ \theta^R_{v_1} + M^R_{c_1} d^{(2E1z)} = \theta^L_{v_1} - M^L_{c_1} + d^{(2E1z)} \] (7a)

\[ \beta^R_{1} - M^R_{a_1} d^{(2GJ)} = \beta^L_{1} + M^L_{a_1} + d^{(2GJ)} \] (8a)

\[ u^R_{1} + \theta^R_{u_1} d^{(2GJ)} = u^L_{1} - \theta^L_{u_1} + d^{(2GJ)} \] (9a)

\[ v^R_{1} + \theta^R_{v_1} d^{(2GJ)} = v^L_{1} - \theta^L_{v_1} + d^{(2GJ)} \] (10a)

This block of equations summarized in matrix notation gives:

\[ F^R_{Z^R_1} = F^L_{Z^L_{1+1}} + B^R_1 \] (11)

\( Z^R_1 \) and \( Z^L_{1+1} \) are solution vectors containing the unknowns, Figs. (4) and (5). In the case of \( Z^R_1 \), the unknowns are terms on the left hand side of Eqns. (1a) to (10a) which possess the subscript 1 and the superscript R, while for \( Z^L_{1+1} \) the unknowns
\[ F_i^R \times Z_i^R \]

\[ N1 = w_i e_{li} d \]
\[ N2 = -M_{ai} - w_i (h_{li} + e_{li} \beta_i^R) d \]
\[ N3 = Vb_i^R + w_i d \]

**FIGURE 4.** MATRIX $F_i^R$ AND VECTOR $Z_i^R$
\[ \begin{bmatrix} R_{MC_i} & R_{Ma_i} \\ R_{Mb_i} & R_{Ma_i} \\ R_{Mc_i} & R_{Ma_i} \\ R_{Mb_i} & R_{Ma_i} \\ R_{Vc_i} & R_{Ma_i} \\ R_{Vb_i} & R_{Ma_i} \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} d_{26} \\ d_{26} \\ d_{26} \\ d_{26} \\ d_{26} \\ d_{26} \end{bmatrix} \]

\[ \begin{bmatrix} \mathbf{F}_{i+1}^L \times \mathbf{Z}_{i+1}^L \end{bmatrix} \]

**FIGURE 5. MATRIX \( \mathbf{F}_{i+1}^L \) AND VECTOR \( \mathbf{Z}_{i+1}^L \)**
are the terms on the right hand side of the equations that possess the subscript $i+1$ and the superscript $L$. The coefficients of the terms that are contained in the solution vector $Z_1^R$ make up the square matrix $F_1^R$, Fig. (4), and the coefficients of the terms that are contained in the vector $Z_{i+1}^L$ make up the square matrix $F_{i+1}^L$, Fig. (5). In the case of the nonlinear terms, the deformation portion was considered to be the coefficient. This arrangement was chosen since the solution technique used (section 4.2) necessitated guessing the values of the nonlinear coefficients and it was felt that the forces could be guessed more exactly than deformations, especially for a beam with a complex bracing arrangement.

Since two arrangements of the nonlinear terms were possible, the solutions could conceivably converge to two different critical loads. In all test cases considered, however, results always converged to the anticipated critical load. Two terms, $w_i h_1 d$ and $w_i d$, occurring in the first and fourth elements of the vector make up the load vector $B_i$.

A set of algebraic equations can be written for each joint. Acting on the joint faces are sets of internal forces equal in magnitude and opposite in direction to the forces acting on the faces of the adjacent elements, Fig. (3). Acting directly on the joint are all the externally applied forces. The equations have been limited to the cases in which $u$, $v$, $\beta$, $\theta_u$ and $\theta_v$ are continuous over the joint, that is, the value of the deformation is the same on the right and on the
left face of the joint. A discussion on the adjustment of the equations to accommodate forces of unknown magnitude that are created by any constraints is included in the next section.

The following set of joint equations result:

\[
M^L_{a_1} = M^R_{a_1} - M_{o_{c_1}} \theta^R_{u_1} - M_{o_{b_1}} \theta^R_{v_1} - M_{o_{a_1}} - P_{o_1}(e^R_{2_{b_1}} + h_{2_1}) \\
- H_{o_1}(e^R_{3_{b_1}} - e^R_{3_{b_1}})
\]

\[
M^L_{b_1} = M^R_{b_1} + M_{o_{b_1}} - M_{o_{b_1}} + M_{o_{a_1}} \theta^R_{v_1} + P_{o_1}(e^R_{2_{b_1}} + h_{2_1})\theta^R_{v_1} \\
+ H_{o_1}(e^R_{3_{b_1}} - e^R_{3_{b_1}})\theta^R_{v_1}
\]

\[
M^L_{c_1} = M^R_{c_1} - M_{o_{c_1}} - M_{o_{b_1}} \theta^R_{u_1} + M_{o_{a_1}} \theta^R_{u_1} + P_{o_1}(e^R_{2_{b_1}} + h_{2_1})\theta^R_{u_1} \\
+ H_{o_1}(e^R_{3_{b_1}} - e^R_{3_{b_1}})\theta^R_{u_1}
\]

\[
V^L_{b_1} = V^R_{b_1} + P_{o_1} - H_{o_1} \beta^R_{b_1}
\]

\[
V^L_{c_1} = V^R_{c_1} - H_{o_1} - P_{o_1} \beta^R_{b_1}
\]

\[
u^L_{1} = u^R_{1}
\]
This block of equations summarized in matrix form give:

\[ Z^L_i = P_i Z^R_1 + C_i \]  

(22)

\( Z^R_1 \) and \( Z^L_1 \) are the solution vectors as previously defined, and \( P_i \) is a square matrix of coefficients, Fig. (6). \( C_i \) is the load vector consisting of all linear terms made up of applied loads.

To decrease the number of simultaneous equations that have been created by the segmentation of the beam, each equation of type (11) has been combined with an appropriate equation of type (22) to give:

\[ F^R_{i+1} = F^L_{i+1} (P_{i+1} Z^R_{i+1} + C_{i+1}) + B_i \]

or expanded,

\[-F^R_{i+1} + F^L_{i+1} P_{i+1} Z^R_{i+1} = -F^L_{i+1} C_{i+1} - B_i \]  

(23)

The right hand side of Eqn. (23) after matrix multiplication and addition contains only linear terms made up of the applied loads. Since there is one more joint than segment, the group
\begin{equation}
N_4 = -P_0 j e_2 i + h_3 i \beta_R R, h_2 i + h_1 i) + H_0 j (e_3 i - h_3 i \beta_R R)
\end{equation}

\begin{equation}
N_5 = M_0 j + P_0 j (e_2 i + h_2 i) - P_0 j h_1 i R - H_0 j e_3 i R
\end{equation}

\begin{equation}
N_6 = -M_0 j - P_0 j h_2 i R - H_0 j e_3 i R
\end{equation}

\begin{figure}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline
$\Theta_V$ & $\Theta_U$ & $\beta_R$ & $V_i$ & $V_u$ & $V_C$ & $V_b$ & $M_C$ & $M_B$ & $M_Q$ & $M_R$ \\
\hline
\hline
\end{tabular}
\end{figure}

\begin{figure}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline
$N_4$ & $-M_0 j - P_0 j$ & $\beta_R$ & $h_1 i$ & $h_2 i$ & $h_3 i$ & $-P_0 j$ & $H_0 j$ & $N_5$ & $N_6$ & $-M_0 j$ \\
\hline
\hline
\end{tabular}
\end{figure}

\begin{figure}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline
$O$ & $0$ & $0$ & $0$ & $0$ & $-H_0 j$ & $P_0 j$ & $-M_0 j$ & $-M_0 j$ & $-M_0 j$ & $N_6$ \\
\hline
\hline
\end{tabular}
\end{figure}

\begin{figure}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline
$\Theta_V$ & $\Theta_U$ & $\beta_R$ & $V_i$ & $V_u$ & $V_C$ & $V_b$ & $M_C$ & $M_B$ & $M_Q$ & $M_R$ \\
\hline
\hline
\end{tabular}
\end{figure}
of matrix equations will be made up of \( N - 1 \) equations of type (23), where \( N \) is the number of joints, and one equation of type (22), that being for joint number one.

With ten algebraic equations making up each matrix equation, the resulting set of \( 10N \) algebraic equations contains \( 10(N+1) \) unknowns. The additional ten unknowns can be obtained from a consideration of the left side of joint number one and the right side of joint number \( N \). Considering joint number one, and remembering that the external loading is applied directly on the joint rather than on its face, the following five equations will always hold:

\[
\begin{align*}
M_{a1}^L &= 0 \\
M_{b1}^L &= 0 \\
M_{c1}^L &= 0 \\
V_{b1}^L &= 0 \\
V_{c1}^L &= 0
\end{align*}
\]

A similar set holds for the right side of joint number \( N \).

The ten equations could have been incorporated into the main body of equations, but for convenience in programming the various types of constraints, this was not done.
2.4 CONSTRAINTS - GENERAL

A constraint shall be defined to be a restriction to the movement of the cross-section of a beam, in a particular direction, at the joint under consideration. Constraints can exist at any or all joints, and from a combination of them the effects of bracing or boundary conditions can be created. Since constraints can be applied only at joints, only the matrix \( P_j \), or in the case of Eqn. (23), the matrix \( \Phi P_1 \) will be affected by their inclusion. Before listing the constraints that can be accommodated by the solution procedure, it will be advantageous to discuss in general the effects of one particular type on the original set of equations.

Consider the case of preventing any vertical movement of the centre of gravity of the beam at joint \( i \). In equation form this is:

\[
v_i = 0
\]

Including this equation in the previous ten written for joint \( i \) gives eleven equations, but only ten unknowns. However, because vertical movement is restricted, a vertical reaction of unknown magnitude must be present to do this. This force becomes the eleventh unknown. To accommodate this force the first ten equations are rewritten, this time, however, instead of including only the vertically applied force \( P_{0i} \), two forces \( P_{0i} \) and \( P_{0i}(\text{unknown}) \) are applied. The equations are similar to the previous set except that wherever a function of \( P_{0i} \) existed.
alone in the $P_i$ matrix, there now exist functions of $P_{Q_i}$ and $P_{Q_i}(\text{unknown})$. In addition, where $P_{Q_i}$ existed in the $C_i$ vector, $P_{Q_i}(\text{unknown})$ would now also exist. However, because it is an unknown, it was removed from the $C_i$ vector and was placed in the solution vector to become the required eleventh term. The coefficients of $P_{Q_i}(\text{unknown})$ create an eleventh column for the $P_i$ matrix, making it an $11 \times 11$ square matrix. Similar manipulations are used to accommodate the other types of constraints.

2.5 SUMMARY OF CONSTRAINT TYPES

The following constraints can be applied to the beam, either alone or, with the exception of certain groups that are listed at the end of the section, in combinations to form boundary conditions.

1. $u_i = c_{1_i}$

This is the case in which the centre of gravity of the beam, at joint $i$ is restricted in its horizontal movement to the specific displacement $c_{1_i}$. A concentrated horizontal force at $i$ is included as an additional unknown.

2. $u_i = c_{2_i} \beta_i$ (where $c_{2_i}$ is a constant)

This is the case in which the horizontal displacement of the centre of gravity of the beam at joint $i$ is directly proportional to the angle of twist. A
concentrated horizontal force acting at a distance \( c_2 \) above the centre of gravity is included as an additional unknown. If, for example, the top flange of the beam cannot move horizontally, all other movements being possible, then for small angles of twist, \( c_2 = \frac{h}{2} \).

3. \( v_1 = c_{3_1} \)

This case is similar to case (1) except that the vertical displacement is restricted to a movement \( c_{3_1} \) and a concentrated vertical force becomes the additional unknown.

4. \( v_1 = -c_{4_1} \beta_1 \) (where \( c_{4_1} \) is a constant)

This case is similar to case (2) except that the vertical displacement is directly proportional to the angle of twist and a concentrated vertical force becomes the additional unknown.

5. \( \beta_1 = c_{5_1} \)

This is the case in which the twist at joint \( i \) is specified to be \( c_{5_1} \) radians. A twisting moment at \( i \) becomes the additional unknown.

6. \( \theta_{u_1} = c_{6_1} \)

This is the case in which the slope of the beam in the \( XZ \) plane, at joint \( i \), is specified to be \( c_{6_1} \) radians. A bending moment about the weak axis at \( i \) becomes the additional unknown.
21.

7. \( \theta_{v1} = c \gamma_1 \)

This is the case in which the slope of the beam in the XY plane, at joint 1, is specified to be \( c \gamma_1 \) radians. A bending moment about the strong axis at 1 becomes the additional unknown.

The final cases relate to a beam supported on a flexible column, Fig. (7). The beam must be attached to the column in a manner such that the movement of the centre of gravity of the beam at joint 1 can be related directly to the movement of the column cap.

The stiffness approach is used to derive the necessary equations. A unit horizontal displacement is given to the column cap, all other movements being held fixed, and the forces developed at the cap are calculated. Next, a unit rotation is applied at the cap, all other movements being held fixed, and again the forces developed are calculated. Neglecting vertical deformations, the equations relating forces and deformations at the column cap are:

\[
H_{\text{column}}^{o1} = a\delta + b\theta \\
M_{\text{column}}^{o1} = b\delta + c\theta
\]

where \( a, b, c \) are stiffness coefficients.

8. In this case, the beam sits on a column that is fixed at the base, Fig. (7a). The stiffness co-efficients are:
FIGURE 7. BEAMS RESTING ON VARIOUS COLUMN SUPPORTS
\[ a = \frac{12E c_1 I c_1}{L c_1^3} \]

\[ b = \frac{6E c_1 I c_1}{L c_1^2} \]

\[ c = \frac{4E c_1 I c_1}{L c_1} \]

Equations (31) and (32) are obtained by substituting the above co-efficients (but with negative signs) into Eqns. (29) and (30) and then relating \( S \) and \( \theta \) to the centre of gravity of the beam, Fig. (7e). Negative signs were required since the forces were derived with reference to the column.

\[ H_{01}(\text{unknown}) + \frac{12E c_1 I c_1 u_1}{L c_1^3} - \frac{6E c_1 I c_1 h(1 + \frac{L c_1}{h})B_1}{L c_1^3} = 0 \quad (31) \]

\[ M_{0a1}(\text{unknown}) - \frac{6E c_1 I c_1 u_1}{L c_1^2} + \frac{E c_1 I c_1 h(3 + 4L c_1)B_1}{L c_1^2} = 0 \quad (32) \]

Case (8) must always be used in conjunction with either Case (3) or (4) since vertical movement is always restricted.

9. This case is similar to Case (8) except that the column is pinned at the base, Fig. (7b). The equations become:

\[ H_{01}(\text{unknown}) + \frac{3E c_1 I c_1 u_1}{L c_1^3} - \frac{5E c_1 I c_1 h(1 + 2L c_1)B_1}{h} = 0 \quad (33) \]
\[ M_{a1}(\text{unknown}) = \frac{3E_c I_{c1} u_1}{L_{c1}^2} + \frac{5E_c I_{c1} h(1 + 2L_{c1})^2}{h} = 0 \quad (34) \]

Case (9) must also be used in conjunction with either Case (3) or (4) since vertical movement is always restricted.

10. Case (10) is Case (8) combined with Case (2), Fig. (7c). Here, the top flange of the beam is prevented from moving horizontally and therefore the co-efficient \( c_2 \) equals .5h. It has been included as a special case because the boundary condition technique used in the computer program precludes any possibility of combining these two cases in the usual manner. The inclusion of this special case makes it very easy to adjust the program to accommodate the cases shown in Figs. (7d) and (7e). (Refer to Appendix III).

The equations become:
\[ H_{o1}(\text{unknown}) = \frac{6E_c I_{c1} h(2 + L_{c1})}{h} = 0 \quad (35) \]
\[ M_{a1}(\text{unknown}) + \frac{2E_c I_{c1} h(3 + 2L_{c1})}{h} = 0 \quad (36) \]

Case (10) must always be used in conjunction with Case (3) or (4).

Because of the method by which the individual cases have been programmed, the following combinations are not possible at a particular joint:
1. Case (1) cannot be combined with Case (2).
2. Case (3) cannot be combined with Case (4).
3. Cases (8), (9), (10) cannot be combined with one another.
4. Case (1) or (2) cannot be combined directly with Case (9) or (10).
5. Case (5) cannot be combined with (8), (9) or (10).

Except for these, practically all boundary conditions can be developed from a combination of not more than five of the fundamental cases. To accommodate these five possible additional equations in the computer program, all the matrix equations were augmented by the introduction of five dummy equations of the form:

$$ a_t = a_t \quad (t = 1, 2, 3, 4, 5) \quad (37) $$

If a particular boundary condition or bracing system exists at joint i, then the equations that define this condition replace an equal number of dummy equations.

**SOLUTION OF SIMULTANEOUS EQUATIONS**

3.1 TRIPLE BLOCK MATRICES [2]

The triple block procedure was used to solve the large group of simultaneous equations. Its main advantages were the ease with which the process could be programmed and the simplicity with which the programme could be made to accept the necessary modifications to the equations caused by the constraints. To use the triple block method, the structure
MATRIX $A_1$ - 5x5 Unitary matrix obtained from Eqns. (20) to (28) written for joint 1.

MATRIX $A_2$ - 5x5 Unitary matrix written for the 5 Eqns. of type (37)

MATRIX $A_3$ - 5x5 Unitary matrix obtained from Eqns. (24) to (28) written for joint N.

All other blocks have been described in the text.

FIGURE 8. EXAMPLE OF STRUCTURE MATRIX FOR BEAM BROKEN INTO 4 SEGMENTS (5 JOINTS)
matrix was broken up into a number of equal sized square matrices (blocks) as shown in Fig. (8). The partitioning had to be such that every row and column of the partitioned matrix contained not more than three non-zero blocks, except for the first and the last which could contain two non-zero blocks. All other blocks were zero matrices. The 30 x 30 block size was chosen for reasons that will be discussed later.

Since the procedure used in obtaining the critical load required the value of the determinant of the structure matrix, it was necessary to investigate the standard triple block approach in somewhat more detail than could be found in the literature. Referring to Fig. (9) for the notation to be used, a step by step analysis of the triple block procedure follows.

The original structure matrix $T$ can always be formed from the product of a lower triangular matrix $L$ and an upper triangular matrix $U$. Furthermore, the diagonal blocks of the lower triangular matrix can be made to consist of unitary matrices. For this arrangement, the elements of the blocks making up each triangular matrix are:

\[ u_{l,l} = a_{l,l} \]
\[ u_{i,i} = a_{i,i} - a_{i+1}u_{i-1,i} - 1, i-1 \]
\[ u_{i, i+1} = a_{i,i+1} \]
\[ u_{i, i-1} = a_{i,i-1} \]

(38) (39) (40)
FIGURE 9. TRIPLE BLOCK MATRIX NOTATION (4 x 4 matrix used for convenience)
29.

The postscript \(-1\) implies the matrix operation of inversion and the subscript \(i\) refers to the block locations in the partitioned matrix, Fig. (9). A disadvantage of the triple block procedure is that an inversion is required. Such an operation on the computer is very time consuming, especially if a large block must be inverted. Because the matrix \(u_{1,1}\) is to be inverted, it must be non-singular, that is, all rows (columns) must be completely independent of one another. All off-diagonal blocks may be singular.

Letting \(Z\) and \(W\) denote respectively the solution vector and the load vector for the entire beam, the matrix equation for the beam can be written as:

\[ TZ = W \]  \hspace{1cm} (41)

where \(T\) is a \(15(N+1) \times 15(N+1)\) structure matrix.

Letting

\[ T = LU \]  \hspace{1cm} (42)

and substituting into Eqn. (41) yields:

\[ LUZ = W \]  \hspace{1cm} (43)

Furthermore, letting

\[ X = UZ \]  \hspace{1cm} (44)

and substituting \(X\) into Eqn. (41) yields:

\[ LX = W \]  \hspace{1cm} (41a)

Since \(L\) is triangular it is a very simple process to solve for \(X\). Solving, the following set of equations is obtained:
These values are substituted into Eqn. (44) and again, because  
$U$ is triangular, the following results are easily obtained:

$$Z_{\frac{N+1}{2}} = u_{\frac{N+1}{2}}, \frac{N+1}{2}X_{\frac{N+1}{2}}$$

$$Z_i = u_{i,i}(X_i - a_i, i+1Z_{i+1}), \ (i = \frac{N-1}{2}, \ldots, 1)$$

3.2 SIZE OF BLOCKS

Since partitioning of the structure matrix breaks up the set of equations into arbitrary parts, it is possible that while the structure matrix as a whole is non-singular, the diagonal blocks themselves are not. Because every diagonal block has to be inverted, its size must be such that singularity would not occur under any set of constraints. It was possible to decide upon a suitable size for matrix $u_{1,1}$ by inspection since its inversion was not preceded by any matrix operations. Whether the same size would be satisfactory for the other diagonal blocks was not certain. However, in all the tests that were carried out, not once did the $30 \times 30$ block become singular.

Of the block sizes that would have been satisfactory, the smallest was chosen to decrease the computation time.
DETERMINATION OF THE CRITICAL LOAD

4.1 APPLICATION OF LOADING

When a loading pattern consisting of several loads is applied to the beam, all loads are applied simultaneously. Otherwise, due to the non-linear characteristics of the system, a different buckling load will be obtained.

If the load pattern acting on a beam consists either of a set of concentrically applied vertical loads or a set of concentrically applied horizontal loads then a value for the buckling load will probably not be attained, since the non-linearity of the system will not be triggered. To initiate the non-linearity, it is necessary to either rely on round-off errors that creep in during computations, or do one of the following:

1. Apply all or a portion of the loading at some small eccentricity.

2. Apply a small auxiliary couple.

Because all calculations were carried out on the computer with double precision (sixteen significant figures), round-off errors would be insignificant, and therefore a great many iterations (see Section 4.2) would be required before the beam stopped behaving linearly. It is therefore impractical to rely on round-off errors. The alternate methods have been found satisfactory when loads were applied with eccentricities of up to twenty percent of the beam width, or when auxiliary couples of about one percent of the magnitude of the principal bending moment were applied. Larger values of eccentricities and couples were not investigated.
4.2 DETERMINANT METHOD FOR FINDING CRITICAL LOAD

The procedure used for calculating the lateral buckling load evolves from the fact that the value of the determinant of the structure matrix at buckling is equal to zero. The general procedure involves obtaining the value of the determinant for three different magnitudes of the load pattern for which a solution is required. From these values the determinant versus load curve, Fig. (10a) is approximated in the general region of the buckling load by a second degree polynomial which can be solved for the critical load.

Before the actual procedure used for obtaining the buckling load is discussed, an explanation as to how the value of the determinant can be obtained from the triple block procedure shall be given. For this, the following results from linear algebra are utilized. Firstly, the value of the determinant of the product of m non-singular matrices is equal to the product of the determinants of the m original matrices [3]. Secondly, the value of the determinant of a triangular matrix, partitioned into square blocks of equal size, is equal to the product of the determinants of the individual diagonal blocks [3], [4]. In the case of the triple block procedure, since the L matrix has only unitary blocks on its main diagonal, the value of the determinant is equal to the product of the determinants of the diagonal blocks of the U matrix. Since a determinant was required in the inversion procedure used in the computer programme, it was a simple matter to gather up
the determinants of the individual $u_{i,j}$ matrices and multiply them together to obtain the determinant of the structure matrix.

The actual procedure used for calculating the critical load will now be explained. Let $Q_n$ be a generalized load that is applied to the beam, and $D_n$ the value of the determinant of the structure matrix corresponding to that applied $Q_n$. An initial guess $Q_1$ is made as to the value of the critical load. In addition, since the structure matrix is non-linear, the terms $M_{a_1}$, $M_{b_1}$, $M_{c_1}$, $V_{b_1}$, $V_{c_1}$ and $\beta_1$ corresponding to $Q_1$ as well as the forces created by the constraints are guessed. An attempt is made to guess these forces accurately. Accordingly, when a vertical load is applied to the beam, $M_{c_1}$ and $V_{b_1}$ are guessed to be their linear values. Using these guesses, the computer solves for the solution vector $Z$. If the initial guesses happened to be correct, then they would be equal to their counterparts in the $Z$ vector. In general, however, this will not be the case. From preliminary investigations into the choice of a solution technique, it was found that even though the non-linear terms were not equal to the corresponding terms in the solution vector, the value of the determinant was within three per cent of its true value if the values of the linear moments and shears were used as initial guesses. This was discovered during the initial stages of the development of the procedure when a full iteration technique was used to find the rates of convergence of the various items. The iteration technique consisted of substituting as new guesses for the
non-linear items, the appropriate terms from the solution vector, and then repeating the process until every guessed value was within one per cent of its value in the solution vector. Because the value of D could be obtained quite accurately without iterating, the major portion of the iteration scheme was removed and the technique was somewhat modified.

The value of D₁ and the solution vector Z, both corresponding to Q₁, were calculated without any iterating. A second value of loading, Q₂, equal to 1.2Q₁ was automatically applied to the beam. The non-linear items were taken to be equal to the appropriate terms from the first solution vector increased by twenty per cent. The computer programme again calculated a solution vector and a corresponding determinant D₂. As before, iteration was not carried out. Note, however, that because terms from the first solution vector were used as guesses, the value of D₂ should be closer to its true value than was D₁.

To guess a load Q closer to the critical load, the determinant-load curve, Fig. (10a), was approximated by a straight line passing through the two points defined by (Q₁, D₁) and (Q₂, D₂). The intersection of this line with the horizontal axis yielded Q₃, which has the value:

$$Q_3 = KQ_2 = \left( \frac{5}{6} + \frac{D_1}{6(D_1-D_2)} \right)Q_2$$  \hspace{1cm} (47)

With this load, and appropriate terms from the solution vector multiplied by K, a third solution vector and a determinant D₃ were calculated. Without changing the load, the terms from
this solution vector were used as guesses for an additional trial, and a fourth determinant $D_4$ was obtained. This extra cycle was carried out to obtain a more exact value for the determinant, since its magnitude near the critical load is comparatively small.

Using the three points $(Q_1, D_1)$, $(Q_2, D_2)$ and $(Q_3, D_4)$ a polynomial of the form:

$$Q = AD^2 + BD + C$$

(48)

can be written to approximate the determinant-load curve. $A$, $B$, and $C$ are constants which can be evaluated from the three sets of available information. Setting $D = 0$, as is the condition at $Q_{cr}$, the value of the critical load equals the constant $C$, whose value is:

$$C = Q_{cr} = Q_3 \left(1 + \frac{D_1D_2D_4}{(D_4 - D_1)(D_4 - D_2)(1.2D_1 - D_2)}\right)$$

(49)

If the value of $Q_3$ was indeed close to $Q_{cr}$ then the result is quite exact.

4.3 THE EFFECT OF THE INITIAL GUESS ON RESULTS

The initial guess as to the magnitude of the critical load is a most important consideration for obtaining an accurate solution.

If the initial guess is too low, less than about $.1Q_{cr}$, then the result will usually be inaccurate. This is because at small loads the beam behaves almost linearly, and is somewhat erratic in its attempt to take on its buckled shape.
FIGURE 10a. PLOT OF DETERMINANT VERSUS LOAD

FIGURE 10b. EFFECTIVE RANGE FOR INITIAL GUESS OF CRITICAL LOAD
In this case, $D_1$ and $D_2$ are practically equal, and since their difference is required in Eqn. (47), a small error in either would tend to make the value of $Q_3$ too different from $Q_{cr}$, thereby making any extrapolation inaccurate, (Section 4.2). Furthermore, there was no attempt during the first two cycles to iterate onto the exact solution, and therefore, as described earlier, the value of the determinant could be in error by about three per cent. Since the value of the difference of two determinants may also be small for small loads, the possible error is obvious. It must be noted that the use of approximate values for $D_1$ and $D_2$ affects results only if the guess is too low. With larger loads the non-linearity is magnified, causing the values of the determinants to be quite different from one another, and since $D_1$ and $D_2$ are much larger than the value of the determinant near the critical load, an error in them has little effect on establishing the approximate equation of the determinant-load curve near the critical load.

It has been found that if the values of $D_1$ and $D_2$ are within about eight per cent of one another, then the value obtained for the critical load will be unreliable. The reason for this is that such values for $D$ means that either the guess as to the value of the critical load is too low, or that the beam is extremely resistant to buckling, both of which imply that the beam acts almost linearly, (a linear system has only one value for its determinant). Because Eqn. (47) requires the difference $D_1 - D_2$, which in the present case is very small,
a slight error in either determinant will cause a major change in the location of $Q_3$. A further test on the results can be based on the value of $D_4$. If the value of $D_4$ is less than about $1D_1$, or $.5D_2$, then a good result may be expected, since only a small amount of extrapolation is required to reach the horizontal axis.

An especially good result will be attained if $D_4$ is small in comparison to $D_1$ and $D_2$ and negative. Normally, the best results will be obtained if the value of $Q_1$ can be guessed at not less than $.25Q_{cr}$. If the value of the initial guess is greater than $Q_{cr}$, but in the range defined by $S$ in Fig. (10b), then the same conclusions as to accuracy can be applied.

The stiffer that a beam is against buckling, the greater will be the possibility that a second or even a third trial will be required to determine the buckling load. This is due to the nature of the determinant-load curve. If the beam is very stiff, then the curvature of the curve near the horizontal axis is very great, and in fact, if the beam cannot buckle, then the curve would not even cross the axis. On the other hand, if the beam is susceptible to buckling, then the change in slope near the horizontal axis is almost zero, and it crosses as a straight line. Because of the great curvature, the intersection with the horizontal axis is at a very flat angle, the curve must be approximated with much greater accuracy for satisfactory results. To overcome this problem, the computer programme can be revised to develop the determinant-load
curve exactly. This is done by inputting an initial guess $Q_1$ and iterating until the values in the solution vector equal those in the previous solution vector. The load and appropriate terms in the solution vector would then be increased by some percentage and the procedure repeated. This would be continued until enough points were obtained to establish the critical load.

4.4 COMPUTATION TIME

The total time required to run one case on the IBM 7040 installation at the University of British Columbia can be obtained from the following approximate formula:

$$\text{TIME (minutes)} = 0.57(N+1) + 2.0$$

This equation is developed for the case in which an object deck is used. Without such a deck an extra six minutes should be added to the computation time.

It was found that the computation time was negligibly affected by the complexity of the loading and the number of constraints. It depended almost entirely on the number of joints, the time to read the object deck, and the time it took to start the machine. The latter two items make up the two minutes that were added to the equation.

EXAMPLES

To emphasize the value of the theory that has been developed, several numerical examples of a more or less general
nature have been included. Furthermore, to make the information presented more general, it has been summarized in dimensionless form whenever convenient.

The critical stress, \( G_{cr} \), must be a function of the following items:

\[
G_{cr} = \text{function } [E, G, L, h, t]
\]  

(50)

In addition, it will depend upon the type of loading and the type of boundary conditions. Since the six items in Eqn. (50) contain only the dimensions of force and length, they are linked by any four dimensionless parameters which contain all six terms. One set of parameters is:

\[
\frac{G_{cr}}{E} = \text{function } \left[ \frac{L}{h}, \frac{t}{h}, \frac{G}{E} \right]
\]  

(51)

The following three cases illustrate how \( \frac{G_{cr}}{E} \) varies with these parameters.

CASE 1.

This is the case of a simply supported beam loaded at its neutral axis by either a concentrated load applied at the centre line of the span, or a uniformly distributed load, Fig. (11). Constraints are applied at the centre of the span which prevent twist of the section, but allow horizontal and vertical movement. It seems reasonable to assume that the behaviour of this beam is governed by the same parameters as a simply supported beam with no diaphragm. Equations of this simpler case [6] are:

\[
P_{cr} = \frac{16.3}{L^2} \sqrt{E I y J}
\]  

(concentrated load)
FIGURE II. RESULTS FOR CASE I.
\[
w_{cr} = \frac{28.3 \sqrt{EIyGJ}}{L^3} \text{ (uniformly distributed load)}
\]

These can be transformed to:

\[
1000 \frac{\sigma_{cr}}{E} = 4200 \frac{\sqrt{G}}{S^2} \sqrt{E} \text{ (concentrated load)}
\]

and

\[
1000 \frac{\sigma_{cr}}{E} = 3540 \frac{\sqrt{G}}{S^2} \sqrt{E} \text{ (uniformly distributed load)}
\]

where

\[S = \sqrt{\frac{Lh}{t^2}}\]

and is called the slenderness ratio. These equations show that \(\frac{\sigma_{cr}}{E}\) depends directly on \(\sqrt{G/E}\), a result that cannot be predicted by dimensionless analysis. Furthermore, \(\frac{\sigma_{cr}}{E}\) depends not on \(L/h\) and \(t/h\) independently, but rather on \(S\).

Fig. (11) shows the results for Case 1. plotted against \(S\) (for \(G/E = 0.58\)) and compared with \(\sigma_{cr}\) for a simply supported beam with no diaphragm. As expected, \(\sigma_{cr}\) is considerably higher when a diaphragm has been added. However, more computer runs are necessary to firmly establish that \(\frac{\sigma_{cr}}{E}\) is in fact inversely proportional to \(S^2\) and \(\sqrt{E/G}\).

CASE 2.

Because very little data exists that quantitatively describes the extent to which continuous bracing of the tension flange against horizontal displacement by a roof deck affects the magnitude of the lateral buckling load, the beneficial affects of such bracing is usually disregarded in design. It was decided to investigate this procedure with the idea of
determining whether a thorough investigation would be in order. For this case, the constraints making up the bracing are assumed pinned to the tension flange so as to offer no torsional restraint.

It was anticipated that a simply supported beam whose ends are prevented from twisting, and whose tension flange is braced against horizontal displacement would be governed by the same parameters as that of a plate with similar boundary conditions, Fig. (12). It was desired to analyse the beam separately for equal end moments, a concentrated load, and a uniformly distributed load, the latter two acting on the free edge of the beam. Since no exact solution could be found for a plate loaded in a like manner, an energy solution was developed with which results could be compared. \( \sigma_{cr} \) can be found by equating the work done by the external loads during buckling to the increase in strain energy \( V \), where

\[
V = \frac{1}{2} \int_{0}^{L} \int_{-h/2}^{h/2} \left[ D_{xx} W_x^2 + 2D_{xx} W_x W_y + D_y W_y^2 + 4D_{xy} W_x W_y \right] \, dx \, dy \quad [5]
\]

Here, \( W \) is the displacement in the z direction and \( D_{xx}, D_x, D_y, D_{xy} \) are stiffness co-efficients. The expression assumed for the deflected shape was:

\[
W = k\left(\frac{h}{2} - y\right) \sin\frac{\pi x}{L}
\]

where \( x \) and \( y \) are measured from the centre of gravity of the left end of the beam, Fig. (12), and \( k \) is a constant. The values of \( D_{xx} \) and \( D_{xy} \) are respectively \( Et^3/12 \) and \( Gt^3/12 \), while the values of \( D_x \) and \( D_y \) are not required since \( W_{yy} = 0 \). The values obtained for the work done by a concentrated load,
Figure 12. Results for Case 2
a uniformly distributed load, and equal end couples, are respectively, $P k^2 h/2$, $w k^2 h^2 a/4$, and $M k^2 h^2 a h/4$. The resulting equations are:

$$\frac{L_{cr}}{E} = \frac{n^2 (t)}{4 h^2} \frac{G}{E} \left[ 1 + \frac{2}{12} \left( \frac{h}{L} \right)^2 \frac{E}{G} \right]$$  \hspace{1cm} \text{(concentrated or uniformly distributed loads)}

$$\frac{L_{cr}}{E} = \frac{2 (t)}{h} \frac{G}{E} \left[ 1 + \frac{2}{12} \left( \frac{h}{L} \right)^2 \frac{E}{G} \right]$$  \hspace{1cm} \text{(equal end couples)}

These equations show that for a plate in which $h/L$ is small, $\frac{L_{cr}}{E}$ depends directly on $G/E$ and $(t/h)^2$ and is independent of $h/L$, results that cannot be predicted by dimensionless analysis. The reason why the equation is the same for either a uniformly distributed load or a concentrated load is that the energy solution is only approximate, since the deflected shape $W$ is only a guess.

Referring to Fig. (12), it is seen that the computer results for a concentrated load and for equal end moments compare almost exactly with those of the energy solution, while the result for a uniformly distributed load does not. The reason for this may be that the assumed deflection function was incorrect for the uniformly distributed load.

All cases showed that the tension flange bracing substantially increased the critical buckling stress over that of an unrestrained beam and it is felt, therefore, that this is an area in which further investigation would be valuable.
CASE 3.

In a great many instances, the roofs of many of the smaller industrial and storage buildings are constructed of timber decking supported on a series of bents made up of glued laminated beams resting on columns located such as to divide the width of the building into three equal bays. The centre span of such an arrangement often consists of a simply supported beam resting on a cantilevered projection of the sidespan. Such a system, Fig. (13), braced along the top flange to prevent horizontal displacement of that flange, is investigated to determine the effects of various beam to column connections on the critical stress.

Because only a limited number of computer runs were made, not enough to consider all the variables, no new theory is presented. The critical stress is plotted against the slenderness ratio $S$ only to give some basis for comparing the various connections.

Connections to cause the actions shown in Figs. (7d) and (7e) and in addition the case of a diaphragm located at the column line and preventing horizontal and vertical movement, as well as twist of that section, were studied. Fig. (13) expresses the importance of a proper beam to column connection. When the beam is attached as in Fig. (7e), the critical stress, for the range of slenderness ratios investigated, remained a constant. The bounding case, that of the beam with a diaphragm, was found to have a critical stress many times greater than the
For the points plotted $\beta = .3, \ b/h = 1/8, \ CI = .025$
$\alpha = .2, \ CL/d = 5.1$
$G/E = .58, \ E/CE = 1.0$

FIGURE 13. RESULTS FOR VARIOUS TYPICAL BEAM-COLUMN CONNECTIONS
latter condition. This case is not affected by the column properties, and any set of constraints that restricts the beam in the same manner can be substituted. When the beam is connected as in Fig. (7d), the results lie between the two previous sets of values. The condition that the critical stress increases with increased slenderness ratio can be explained by the fact that as the beam is increased in length while its section properties remain constant, the relative stiffness of the column is increased, until in the limit, it becomes equivalent to the diaphragmed case.

The graphs adequately express the importance of the need for a proper beam to column connection.

CONCLUSIONS

The intention of this thesis was to develop some theory by which it would be possible to obtain the critical buckling load of a beam loaded and constrained in a most general way. With the aid of an electronic computer, this has been accomplished. The procedure involving an approximation to the load-determinant curve proved to be accurate, as well as relatively fast. In all cases tested, whenever a comparison could be made against existing solutions [1], the results were found to be within one per cent of the published values, with only a single trial required.

The determination of the buckling load of a relatively stiff beam usually required more than one trial. This is
because the load-determinant curve exhibits very large curva-
tures in the vicinity of the horizontal axis, and the interpo-
lation scheme is relatively poor. Great accuracy is necessary
if a good result for $Q_{cr}$ is to be expected. To alleviate this
problem, the programme can be easily modified to develop the
curve almost exactly. Such a procedure, in fact, was used
during the preliminary stages of development, at which time
it was done to study the rates of convergence of the various
non-linear terms. Though the latter technique is most accurate,
in most instances it is uneconomical to use because of the
computation time involved, about eight minutes for each point
calculated to define the curve.

The included examples show just a few of the many
facets of the lateral buckling problem that can be studied by
programming the equations developed in the theory.

Extension of the equations can lead to the inclusion
of axial forces, various cross-sections, and possibly decreasing
computation time.
BIBLIOGRAPHY


APPENDIX I - DATA INPUT-OUTPUT

In the data input description of Table I, it has been assumed that the reader is familiar with computer programming. To further clarify the procedure, an example, Figs. (14) and (15) have been included. For convenience of description, the beam has been divided into only four segments.

All information input into the programme is in units of feet and kips.

The information that the programme prints out includes the following:

1. The value of the critical load acting at each joint. If the critical load is made up of more than one type of load, as for example a concentrated load and a uniformly distributed load, then both values are printed out.

2. The values of the determinants $D_1, D_2, D_3, D_4$ - these are printed out to be used as a check on the accuracy of the resulting critical load.
<table>
<thead>
<tr>
<th>SET</th>
<th>DATA</th>
<th>FORMAT</th>
<th>DETAILS</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.1</td>
<td>N, E, G, D, T, H</td>
<td>F10.0</td>
<td>- 1 card</td>
</tr>
<tr>
<td>G.2.1</td>
<td>XMA(K), XMB(K), XMC(K), VB(K), VC(K), BETA(K)</td>
<td>F10.0</td>
<td>- N-1 cards, one for each joint from 1 to N-1.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- each card contains a guess as to the magnitude of the item at the joint</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>under consideration caused by the initial load Q1.</td>
</tr>
<tr>
<td>G.2.2</td>
<td>BETA(N)</td>
<td>F10.0</td>
<td>- 1 card</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- card contains a guess as to the value of at joint N caused by the initial load Q1.</td>
</tr>
</tbody>
</table>

One set of G.3 data cards is required for each joint

<p>| G.3.1| (NBC(IK), I=1,10), JLN(K)                | 1115   | - 1 card per joint.                                                     |
|      |                                           |        | - the first ten fields indicate which constraints, 1 to 10 respectively, as described in Section 2.3.3 exist at the joint. If, for example, constraint types 3 and 8 exist, then a 1 is placed in fields 3 and 8, while zeros are placed elsewhere (card 7 of example). |
|      |                                           |        | - the 11th field indicates whether cards G.3.4, G.3.5, G.3.6, G.3.7 are to be included. If the integer 2 (card 7 of example) is placed in the field, then all four cards are to be included. If any other integer is used, then only G.3.7 is to be included (card 14 of example). |</p>
<table>
<thead>
<tr>
<th>SET</th>
<th>DATA</th>
<th>FORMAT</th>
<th>DETAILS</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.3.2</td>
<td>BC(I,K)</td>
<td>F10.0</td>
<td>- 1 card for each integer 1 occurring on card G.3.1. It gives the value of ( \alpha ), (Section 2.5). The value 0.0 is to be used if a 1 occurs in fields 8, 9, or 10, (card 9 of example).</td>
</tr>
<tr>
<td>G.3.3</td>
<td>CI(K), CL(K), CE(K)</td>
<td>3F10.0</td>
<td>- 1 card is read when a 1 occurs in any of fields 8, 9, 10 of G.3.1. It gives the physical properties of the column, (card 11 of example).</td>
</tr>
<tr>
<td>G.3.4</td>
<td>P1(K), E21(K), H21(K), P2(K), E22(K), H22(K)</td>
<td>6F10.0</td>
<td>- 1 card contains data describing any vertically applied load as well as guesses as to value of the vertical force created by any constraints that may be acting.</td>
</tr>
<tr>
<td>G.3.5</td>
<td>H1(K), E21(K), H21(K), P2(K), E22(K), H22(K)</td>
<td>6F10.0</td>
<td>- 1 card contains data describing any horizontally applied load as well as guesses as to the value of the horizontal forces created by any constraints that may be acting.</td>
</tr>
<tr>
<td>G.3.6</td>
<td>XMOA(K), XMOB(K), XMOC(K), XMOA2(K), XMOB2(K), XMOC2(K)</td>
<td>6F10.0</td>
<td>- 1 card contains data describing applied moments, and guesses as to value of any moments created by any constraints that may be acting.</td>
</tr>
<tr>
<td>SET</td>
<td>DATA</td>
<td>FORMAT</td>
<td>DETAILS</td>
</tr>
<tr>
<td>-------</td>
<td>-------------------------------</td>
<td>--------</td>
<td>-------------------------------------------------------------------------</td>
</tr>
<tr>
<td>G.3.7</td>
<td>W1(K), E1l(K), H1l(K), W2(K), E12(K), H12(K)</td>
<td>6F10.0</td>
<td>- 1 card (for joint 1 to N-1 only)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- data for vertically applied, uniformly distributed load along segment K.</td>
</tr>
<tr>
<td>G.4.1</td>
<td>ITT(L), L=1,6</td>
<td>6I5</td>
<td>- 3 cards</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- describes which externally applied loads are to be increased during each cycle.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>If a zero is placed in the field, then that particular loading is not increased.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>If a 1 is placed in a field, then that loading is increased.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 in field 1--increase XMOA(K)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 &quot; &quot; 2-- &quot; XMOB(K)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 &quot; &quot; 3-- &quot; XMOC(K)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 &quot; &quot; 4-- &quot; Pl(K)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 &quot; &quot; 5-- &quot; Hl(K)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 &quot; &quot; 6-- &quot; W1(K)</td>
</tr>
</tbody>
</table>
COLUMN - fixed at base
- CL(I) = column length = 12.0 ft
- CE(I) = modulus = 260,000 ksf
- CI (I) = inertia = .025 ft^4

BEAM N = 5 joints
E = 260,000 ksf
G = 150,000 ksf
D = 9.0 ft (segment length)

INITIAL GUESS
PI(3) = 10 (acting at top of beam at an eccentricity H2l(3) = .25 ft)
XMOC(I) = XMOC(5) = 30 k-ft
WI(I) = WI(2) = WI(3) = WI(4) = .1 k/ft (acting at centre of gravity of beam)

FIG. 14 EXAMPLE USED FOR DESCRIBING DATA INPUT PROCEDURE
FIG. 15 CODING SHEET FOR EXAMPLE DESCRIBING DATA INPUT
APPENDIX II - ADJUSTMENT OF PROGRAMME FOR SPECIAL CASES

The computer programme written to solve the lateral stability problem was written in Fortran IV language for the IBM 7040 computer installation located at the University of British Columbia. A copy of the programme is included in Appendix III. A list of the relevant symbols is included in the Notation section at the beginning of the paper.

To include the column connections shown in Figs. (7d) and (7e), a series of revisions must be made to the original programme. In both cases, the new cases replace constraint type 10.

To develop the condition shown in Fig. (7d), the lines labelled a., b., c., d. in the programme listing must be replaced successively by the following:

930 \[ A3(6,8) = -3.0 \times CE(N2) \times CI(N2) \times H \times (1. + CL(N2)/H)/CL(N2)^3 \]
\[ A3(8,8) = -A3(6,8) \times CL \]

934 \[ A3(21,23) = -3.0 \times CE(N3) \times CI(N3) \times H \times (1. + CL(N3)/H)/CL(N2)^3 \]
\[ A3(23,23) = -A3(21,23) \times CL \]

To develop the condition shown in Fig. (7e), the following revisions must be made. Replace the statements labelled a., b. successively by:

930 \[ A3(6,6) = P2(N2)/CL(N2) \]
\[ A3(6,8) = -0.5 \times P2(N2) \times H/CL(N2) \]

and immediately after b., add:
\[ A1(26,13) = 0.0 \]
\[ A1(27,10) = A1(27,10) - XMOA2(N2) \]
Replace the statements labelled c., d. successively by:

\[ A3(21,21) = \frac{P2(N3)}{CL(N3)} \]
\[ A3(21,23) = -0.5 \times \frac{P2(N3)}{H/CL(N3)} \]

and immediately after d., add:

\[ A3(11,28) = 0.0 \]
\[ A3(12,25) = A3(12,25) - XMOA2(N3) \]
\[ A3(13,24) = A3(12,25) \]
\[ A3(13,28) = 0.0 \]
\[ A3(19,25) = A3(19,25) - D2 \times XMOA2(N3) \]
\[ A3(20,24) = A3(20,24) + D1 \times XMOA2(N3) \]

Remove statements labelled e., f.

Replace statements labelled g., h. by the single statement:

\[ IF(NBC(5,K).EQ.0.AND.NBC(8,K).EQ.0.AND.NBC(9,K).EQ.0) \text{GOT0947} \]

Similarly replace the pairs of statements labelled i., j.; k., l.; m., n.; o., p.; and q., r. successively by the statements:

\[ IF(\text{... as above}) \text{GOT0952} \]
\[ IF(\text{... as above}) \text{GOT01024} \]
\[ IF(\text{... as above}) \text{GOT01029} \]
\[ IF(\text{... as above}) \text{GOT01035} \]
\[ IF(\text{... as above}) \text{GOT01040} \]
Start
Read Data Card G.1
Calculate Constants
Read Data Card Set G.2
Calculate terms for Matrix a_11
Fig. 9
Adjust a_11 for Boundary Conditions
Invert a_11, obtain its determinant and store inverted matrix on tape
Calculate terms for matrix a_\text{j} (j<i)
Calculate terms of 1st load vector w_1
Adjust w_1 for boundary conditions
Read data card set G.3 for next 2 joints
2nd, 3rd, 4th run
1st run
Calculate terms of next load vector w_1
Adjust w_1 for boundary conditions
Calculate terms of next matrix a_{\text{i}}
Calculate terms of next matrix a_{\text{j}} (j>i)
Adjust matrix a_{\text{j}} and a_{\text{j}} for boundary conditions
Store a_{\text{j}} on tape
Calculate x_i (Eqn. 45) during process obtain determinant of u_{\text{j}}
Backsubstitution to calculate Z, Eqn. (46) and determinant of structure matrix
1st run
Read card G.4, increase applied loads and appropriate terms of load vector by 20%
2nd run
Read card G.4, increase applied loads and appropriate terms in load vector Z by % calculated by Eqn. (49)
3rd run
Adjust appropriate terms of load vector Z
4th run
Print 4 determinants and critical load
Stop

FIG. 16 FLOW DIAGRAM FOR COMPUTER PROGRAM
SIBFTC STAB

LATERAL STABILITY OF RECTANGULAR BEAMS

N = NUMBER OF JOINTS = 1 + NUMBER OF SEGMENTS

DIMENSION NBC(10,23), BC(10,23), JLN(23), ITT(6)

DOUBLE PRECISION W1(21), E1(21), H1(21), W2(21), E2(21), H2(21)

DOUBLE PRECISION P1(21), E2(21), H2(21), P2(21), E2(21), H2(21)

DOUBLE PRECISION H1(21), E3(21), H3(21), P3(21), E3(21), H3(21)

DOUBLE PRECISION XMOA(21), XMBO(21), XMOC1(21), XMOC2(21), XMOC3(21)

DOUBLE PRECISION XMOA2(21), XMOC2(21), CE1(21), CL(21), CI(21), VB(21), VC(21)

DOUBLE PRECISION A(30,30), A2(30,30), A3(30,30), A3(30,30), AX(5,30)

DOUBLE PRECISION NZX(5,30), P(30,11), XP(30,11), XR(30,11), Z(30,11)

DOUBLE PRECISION ZM(30,11), BETA(21), XMA(21), XMB(21), XMC(21)

DO 100 K = 1, NM

READ 100, N, F, G, D, T, H

CONTINUE

READ 100, N, F, G, D, T, H

100 FORMAT (I10, 5F10.0)

XI1 = 0.08333*T*H**3

XI2 = 0.08333*H**3

XJ = 4.*XI2*(1.*G*3/T/H)

D1 = 5*D/(E*XI1)

D2 = 5*D/(E*XI2)

DJ = 5*D/(G*XJ)

COUNT = 0.0

NM = N-1

DO 200 K = 1, NM

READ 102, XMA(K), XMB(K), XMC(K), VB(K), VC(K), BETA(K)

102 FORMAT (6F10.0)

200 CONTINUE

READ 120, BETA(N)

120 FORMAT (F10.0)

READ 103, (NBC(I,1), I = 1, 10), JLN(1)

103 FORMAT (11I5)

DO 202 I = 1, 10

IF (NBC(I,1).EQ.0) GO TO 202

DO 202 I = 1, 10

IF (NBC(I,1).EQ.0) GO TO 202
READ 101, BC(I,1)
FORMAT (5F10.0).
CONTINUE
IF (NBC(8,1).EQ.1.OR.NBC(9,1).EQ.1.OR.NBC(10,1).EQ.1) GO TO 236
GO TO 245
236 READ 104, CI(1), CL(1), CE(1)
FORMAT (3F10.0)
GO TO 245
READ 102, P1(1), E21(1), H21(1), P2(1), E22(1), H22(1)
READ 102, H1(1), E31(1), H31(1), H2(1), E32(1), H32(1)
READ 102, XMOA(1), XM0B(1), XMOC(1), XMOA2(1), XM0B2(1), XMOC2(1)
READ 102, W1(1), E1(1), H1(1), W2(1), E2(1), H2(1)
READ 103, (NBC(I,2), I=1,10), JLN(2)
DO 300 I=1,10
IF (NBC(I,2).EQ.1) GO TO 300
READ 101, BC(I,2)
300 CONTINUE
IF (NBC(8,2).EQ.1.OR.NBC(9,2).EQ.1.OR.NBC(10,2).EQ.1) GO TO 237
GO TO 238
READ 104, CI(2), CL(2), CE(2)
238 DO 295 I=1,10
IF (NBC(I,2).EQ.1) GO TO 297
295 CONTINUE
IF (JLN(2).EQ.2) GO TO 297
P1(2) = 0.0
E21(2) = 0.0
H21(2) = 0.0
E22(2) = 0.0
H22(2) = 0.0
H1(2) = 0.0
E31(2) = 0.0
H31(2) = 0.0
E32(2) = 0.0
H32(2) = 0.0
XMOC(2) = 0.0
P2(2) = 0.0
H2(2) = 0.0
XMOA2(2) = 0.0
XMOB2(2) = 0.0
XMOC2(2) = 0.0

GO TO 296

297 READ 102, P1(2), E21(2), H21(2), P2(2), E22(2), H22(2)
READ 102, H1(2), E31(2), H31(2), H2(2), E32(2), H32(2)
READ 102, XMOA(2), XMOC(2), XMOA2(2), XMOC2(2), XMOC2(2)
READ 102, W1(2), E11(2), H11(2), W2(2), E12(2), H12(2).

296 CONTINUE
DET = 0.0
ADET = 0.0

DO 882 I = 1, 30
DO 882 J = 1, 30

882 A(I,J) = 0.0
A(1,1) = 1.0
A(2,2) = 1.0
A(3,3) = 1.0
A(4,4) = 1.0
A(5,5) = 1.0
A(6,6) = 1.0
A(7,7) = 1.0
A(8,8) = 1.0
A(9,9) = 1.0
A(10,10) = 1.0
A(11,11) = -1.0
A(11,23) = P1(1)*E21(1)+H1(1)*H31(1)
A(11,24) = -XMOC(1)
A(11,25) = -XMOC(1)
A(12,2) = -1.0
A(12,17) = 1.0
A(12,23) = XMOC(1)
A(12,25) = XMOC(1)+P1(1)*E21(1)+BETA(1)+H21(1)
A(13,3) = -1.0
A(13,18) = 1.0
A(13,23) = -XMOC(1)
| A(13,24) | A(12,25) |
| A(14,4)  | -1.0     |
| A(14,19) | 1.0      |
| A(14,23) | -H1(1)   |
| A(15,5)  | -1.0     |
| A(15,20) | 1.0      |
| A(15,23) | -P1(1)   |
| A(16,6)  | 1.0      |
| A(16,21) | 1.0      |
| A(17,7)  | -1.0     |
| A(17,22) | 1.0      |
| A(18,8)  | 1.0      |
| A(18,23) | -1.0     |
| A(19,9)  | 1.0      |
| A(19,24) | -1.0     |
| A(20,10) | 1.0      |
| A(20,25) | 1.0      |
| A(21,26) | 1.0      |
| A(22,27) | 1.0      |
| A(23,28) | 1.0      |
| A(24,29) | 1.0      |
| A(25,30) | 1.0      |
| A(26,16) | 1.0      |
| A(26,23) | -D*(W1(1)*E11(1)+W2(1)*E12(1)) |
| A(26,24) | +XMC(1)  |
| A(26,25) | XMB(L)   |
| A(27,17) | -1.0     |
| A(27,20) | D        |
| A(27,23) | -XMC(1)  |
| A(27,25) | XMA(1)+(W1(1)*(H11(1)+E11(1))*BETA(1)) |
|         | +W2(1)*(H12(1)+E12(1)*BETA(1))|D |
| A(28,18) | -1.0     |
| A(28,19) | D        |
| A(28,23) | XMB(1)   |
| A(28,24) | A(27,25) |
| A(29,19) | -1.0     |
| A(29,23) | VC(1)    |
| A(30,20) | -1.0     |
A(30,23) = -VB(1) - (W1(1) + W2(1)) * D

NN = 1

NBC2 = 1

GO TO 900

900 N2 = 2*(NN-1)

N3 = N2 + 1

IF (NN . GT. 1) GO TO 901

DO 880 I = 1, 30

DO 880 J = 1, 30

880 A3(I.J) = A(I.J)

GO TO 913

901 DO 902 I = 1, 10

IF (NBC(I,N2).EQ.1) GO TO 903

902 CONTINUE

GO TO 913

903 IF (NBC(1,N2).EQ.0 . AND. NBC(2,N2).EQ.0) GO TO 905

A1(26,8) = A1(26,8) + H2(N2) * H32(N2)

A1(26,11) = -E32(N2)

A1(27,10) = A1(27,10) + H2(N2) * (E32(N2) - H32(N2) * BETA(N2))

A1(28,9) = A1(27,10)

A1(29,8) = A1(29,8) - H2(N2)

A1(30,11) = -1.0

A3(6,6) = 1.0

A3(6,11) = 0.0

A3(3,8) = A3(3,8) + DJ * H2(N2) * H32(N2)

A3(3,11) = -E32(N2) * DJ

A3(4,10) = A3(4,10) + D2 * H2(N2) * (E32(N2) - H32(N2) * BETA(N2))

A3(5,9) = A3(5,9) - D1 * H2(N2) * (E32(N2) - H32(N2) * BETA(N2))

IF (NBC(2,N2).EQ.1) GO TO 904

GO TO 905

904 A3(6,8) = +BC(2,N2)

12 905 IF (NBC(3,N2).EQ.0 . AND. NBC(4,N2).EQ.0) GO TO 907

A1(26,8) = A1(26,8) - P2(N2) * E22(N2)

A1(26,12) = -H22(N2)

A1(27,10) = A1(27,10) + P2(N2) * (E22(N2) * BETA(N2) + H22(N2))

A1(28,9) = A1(27,10)

A1(29,12) = 1.0
\[
\begin{align*}
A_1(30,8) &= A_1(30,8) - P_2(N_2) \\
A_3(3,8) &= A_3(3,8) - D_1*P_2(N_2)*E_22(N_2) \\
A_3(3,12) &= -H_22(N_2)*D_J \\
A_3(4,10) &= A_3(4,10) + D_2*P_2(N_2)*(E_22(N_2)*BETA(N_2) + H_22(N_2)) \\
A_3(5,9) &= A_3(5,9) - D_1*P_2(N_2)*(E_22(N_2)*BETA(N_2) + H_22(N_2)) \\
A_3(7,7) &= 1.0 \\
A_3(7,12) &= 0.0 \\
\text{IF (NBC(4,N_2) EQ 11) GO TO 906.} \\
\text{GO TO 907} \\
A_3(7,8) &= +B_C(4,N_2) \\
\text{GO TO 907} \\
A_3(8,8) &= 1.0 \\
A_3(8,13) &= 0.0 \\
\text{IF (NBC(6,N_2) EQ 0) GO TO 909} \\
A_1(26,10) &= A_1(26,10) - XMOA2(N_2) \\
A_1(27,10) &= A_1(27,10) + XMOA2(N_2) \\
A_1(28,9) &= A_1(28,9) + XMOA2(N_2) \\
A_3(3,13) &= -D_J \\
A_3(4,10) &= A_3(4,10) + D_2*XMOA2(N_2) \\
A_3(5,9) &= A_3(5,9) - D_1*XMOA2(N_2) \\
A_3(8,8) &= 1.0 \\
A_3(8,13) &= 0.0 \\
\text{IF (NBC(6,N_2) EQ 0) GO TO 909} \\
A_1(26,10) &= A_1(26,10) - XMOB2(N_2) \\
A_1(27,10) &= A_1(27,10) + XMOB2(N_2) \\
A_1(28,8) &= A_1(28,8) - XMOB2(N_2) \\
A_3(3,10) &= A_3(3,10) - D_J*XMOB2(N_2) \\
A_3(4,14) &= -D_2 \\
A_3(5,8) &= A_3(5,8) + D_1*XMOB2(N_2) \\
A_3(9,9) &= 1.0 \\
A_3(9,14) &= 0.0 \\
\text{IF (NBC(7,N_2) EQ 0) GO TO 910} \\
A_1(26,9) &= A_1(26,9) - XMOC2(N_2) \\
A_1(27,8) &= A_1(27,8) + XMOC2(N_2) \\
A_1(28,15) &= 1.0 \\
A_3(3,9) &= A_3(3,9) - XMOC2(N_2)*D_J \\
A_3(4,8) &= A_3(4,8) + XMOC2(N_2)*D_2 \\
A_3(5,15) &= D_1 \\
A_3(10,10) &= 1.0 \\
A_3(10,15) &= 0.0 \\
\text{IF (NBC(8,N_2) EQ 0, AND NBC(9,N_2) EQ 0, AND NBC(10,N_2) EQ 0) GO TO 913} \\
\end{align*}
\]
\[ A(26,8) = A(26,8) + H2(N2) \cdot H3(N2) \]
\[ A(27,10) = A(27,10) + XMOA2(N2) + H2(N2) \cdot (E32(N2) - H32(N2) \cdot BETA(N2)) \]
\[ A(28,9) = A(27,10) \]
\[ A(29,8) = A(29,8) - H2(N2) \]
\[ A(30,11) = -1.0 \]
\[ A(26,13) = -1.0 \]
\[ A(32,8) = A(32,8) + DJ \cdot H2(N2) \cdot H32(N2) \]
\[ A(33,11) = -E32(N2) \cdot DJ \]
\[ A(33,13) = -DJ \]
\[ A(34,10) = A(34,10) + D2 \cdot H2(N2) \cdot (E22(N2) - H32(N2) \cdot BETA(N2)) \]
\[ 1 + D2 \cdot XMOA2(N2) \]
\[ A(35,9) = A(35,9) - D1 \cdot H2(N2) \cdot (E22(N2) - H32(N2) \cdot BETA(N2)) \]

IF \( NBC(8,N2).EQ.1 \) GO TO 911.
IF \( NBC(9,N2).EQ.1 \) GO TO 912.
IF \( NBC(10,N2).EQ.1 \) GO TO 930.

911
\[ A(3,6) = +12 \cdot CE(N2) \cdot CI(N2) / CL(N2)^2 \cdot 3 \]
\[ A(3,8) = -6 \cdot CE(N2) \cdot CI(N2) / CL(N2)^2 \cdot H \cdot (1 + CL(N2) / H) / CL(N2)^2 \cdot 3 \]
\[ A(3,8) = -6 \cdot CE(N2) \cdot CI(N2) / CL(N2)^2 \cdot H \cdot (1 + CL(N2) / H) / CL(N2)^2 \cdot 2 \]
GO TO 913.

912
\[ A(3,6) = +3 \cdot CE(N2) \cdot CI(N2) / CL(N2)^2 \cdot 3 \]
\[ A(3,8) = -15 \cdot CE(N2) \cdot CI(N2) / CL(N2)^2 \cdot H \cdot (1 + 2 \cdot CL(N2) / H) / CL(N2)^2 \cdot 3 \]
\[ A(3,8) = -3 \cdot CE(N2) \cdot CI(N2) / CL(N2)^2 \cdot 2 \]
GO TO 913.

930
\[ A(3,6) = -6 \cdot CE(N2) \cdot CI(N2) / CL(N2)^2 \cdot H \cdot (2 + CL(N2) / H) / CL(N2)^2 \cdot 3 \]
\[ A(3,8) = 2 \cdot CE(N2) \cdot CI(N2) / CL(N2)^2 \cdot H \cdot (3 + 2 \cdot CL(N2) / H) / CL(N2)^2 \cdot 2 \]

913 DO 914 I = 1, 10
IF \( NBC(I,N3).EQ.1 \) GO TO 915.
CONTINUE.
915 GO TO 925.

917 IF \( NBC(1,N3).EQ.0 \) AND \( NBC(2,N3).EQ.0 \) GO TO 917.
\[ A(11,23) = A(11,23) + H2(N3) \cdot H32(N3) \]
\[ A(11,26) = -E32(N3) \]
\[ A(12,25) = A(12,25) + H2(N3) \cdot (E32(N3) - H32(N3) \cdot BETA(N3)) \]
A3(13,24) = A3(12,25)

A3(21,23) = A3(18,23) + DJ*H2(N3) + H32(N3)

A3(18,27) = -H22(N3)*DJ

A3(18,23) = A3(18,23) - DJ*P2(N3)*E22(N3)

A3(19,25) = A3(19,25) + D2*H2(N3)*(E32(N3) - H32(N3)*BETA(N3))

A3(20,24) = A3(20,24) - D1*H2(N3)*(E32(N3) - H32(N3)*BETA(N3))
A3(20,24) = A3(20,24) - D1*XMOA2(N3)

920 IF (NBC(6,N3) .EQ. 0) GO TO 921
A3(11,25) = A3(11,25) - XMOB2(N3)
A3(13,23) = A3(13,23) - XMOB2(N3)

A3(12,29) = -1.0
A3(24,24) = 1.0
A3(25,29) = 0.0

921 IF (N3 .EQ. 1) GO TO 922
A3(18,25) = A3(18,25) - XMOB2(N3)*DJ
A3(19,29) = -D2
A3(20,23) = A3(20,23) + XMOB2(N3)*D1

922 IF (NBC(7,N3) .EQ. 0) GO TO 922
A3(11,24) = A3(11,24) - XMOC2(N3)
A3(12,23) = A3(12,23) + XMOC2(N3)
A3(13,30) = -1.0
A3(25,25) = 1.0

A3(25,30) = 0.0
IF (N3 .EQ. 1) GO TO 922
A3(18,24) = A3(18,24) - DJ*XMOC2(N3)
A3(19,23) = A3(19,23) + XMOC2(N3)*D2
A3(20,30) = D1
1 -D1*XMOA2(N3)

923  IF (NBC(8,N3),EQ,1) GO TO 924
IF (NBC(9,N3),EQ,1) GO TO 933
IF (NBC(10,N3),EQ,1) GO TO 934

924  A3(21,21) = +12.*CE(N3)*CI(N3)/CL(N3)**3
A3(21,23) = -6.*CE(N3)*CI(N3)*H*(1./CL(N3)/H)/CL(N3)**3
A3(23,21) = -6.*CE(N3)*CI(N3)/CL(N3)**2
A3(23,23) = CI(N3)/CL(N3)**2

GO TO 925

933  A3(21,21) = +3.*CE(N3)*CI(N3)/CL(N3)**3
A3(21,23) = -15.*CE(N3)*CI(N3)*H*(1./CL(N3)/H)/CL(N3)**3
A3(23,21) = -3.*CE(N3)*CI(N3)/CL(N3)**2
A3(23,23) = 15.*CE(N3)*CI(N3)*H*(1./CL(N3)/H)/CL(N3)**2

GO TO 925

934  A3(21,23) = -6.*CE(N3)*CI(N3)*H*2.*CL(N3)/H)/CL(N3)**3
A3(23,23) = 2.*CE(N3)*CI(N3)*H*(3./CL(N3)/H)/CL(N3)**2

925  IF (NN,GT,1) GO TO 926

926  GO TO (272,273),NBC2
272  CONTINUE

DO 881 I=1,30
DO 881 J=1,30

881  A(I,J) = A3(I,J)
CALL INV (A,30,30,DET,COND)
WRITE (1) A
ADDET = DET

DO 883 I=1,30
DO 883 J=1,30

883  A2(I,J) = 0.0
A2(1,21) = -1.0
A2(1,24) = -5*D
A2(2,22) = -1.0
A2(2,25) = -5*D
A2(3,16) = DJ
A2(3,23) = -1.0
A2(4,17) = D2
A2(5,18) = -D1
A2(4,24) = -1.0
A2(5,25) = -1.0
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>P(1,1)</td>
<td>= 0.0</td>
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</tr>
<tr>
<td>P(2,1)</td>
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<tr>
<td>P(3,1)</td>
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<td></td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>P(5,1)</td>
<td>= 0.0</td>
<td></td>
</tr>
<tr>
<td>P(6,1)</td>
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<td></td>
</tr>
<tr>
<td>P(7,1)</td>
<td>= 1.0</td>
<td></td>
</tr>
<tr>
<td>P(8,1)</td>
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<td></td>
</tr>
<tr>
<td>P(9,1)</td>
<td>= 1.0</td>
<td></td>
</tr>
<tr>
<td>P(10,1)</td>
<td>= 1.0</td>
<td></td>
</tr>
<tr>
<td>P(11,1)</td>
<td>= XM0A(1)+P1(1)*H21(1)+H1(1)*F31(1)</td>
<td></td>
</tr>
<tr>
<td>P(12,1)</td>
<td>= XM0B(1)</td>
<td></td>
</tr>
<tr>
<td>P(13,1)</td>
<td>= XM0C(1)</td>
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</tr>
<tr>
<td>P(14,1)</td>
<td>= P1(1)</td>
<td></td>
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<tr>
<td>P(15,1)</td>
<td>= H1(1)</td>
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</tr>
<tr>
<td>P(16,1)</td>
<td>= 0.0</td>
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</tr>
<tr>
<td>P(17,1)</td>
<td>= 0.0</td>
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<tr>
<td>P(18,1)</td>
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<tr>
<td>P(19,1)</td>
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<tr>
<td>P(20,1)</td>
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<tr>
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<td>= 1.0</td>
<td></td>
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<tr>
<td>P(22,1)</td>
<td>= 1.0</td>
<td></td>
</tr>
<tr>
<td>P(23,1)</td>
<td>= 1.0</td>
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</tr>
<tr>
<td>P(24,1)</td>
<td>= 1.0</td>
<td></td>
</tr>
<tr>
<td>P(25,1)</td>
<td>= 1.0</td>
<td></td>
</tr>
<tr>
<td>P(26,1)</td>
<td>= XM0A(2)+P1(2)*H21(2)+H1(2)<em>F31(2)+D</em>(W1(1)*H1(1)+W2(1))</td>
<td></td>
</tr>
<tr>
<td>P(27,1)</td>
<td>= XM0B(2)</td>
<td></td>
</tr>
<tr>
<td>P(28,1)</td>
<td>= XM0C(2)</td>
<td></td>
</tr>
<tr>
<td>P(29,1)</td>
<td>= -P1(2)-D*(W1(1)+W2(1))</td>
<td></td>
</tr>
<tr>
<td>P(30,1)</td>
<td>= H1(2)</td>
<td></td>
</tr>
<tr>
<td>NN</td>
<td>= 1</td>
<td></td>
</tr>
<tr>
<td>NBC1</td>
<td>= 1</td>
<td></td>
</tr>
<tr>
<td>GO TO 500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>N2 = 2*(NN-1)</td>
<td></td>
</tr>
<tr>
<td>N3 = N2+1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IF (NN EQ 1) GO TO 512</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
DO 501 I=1,10
   IF (NBC(I,N2),EQ,1) GO TO 502
501 CONTINUE
   GO TO 502
502 IF (NBC(1,N2),EQ,0) GO TO 503
   P(6,NN) = BC(1,N2)
503 IF (NBC(2,N2),EQ,0) GO TO 504
   P(6,NN) = 0.0
504 IF (NBC(3,N2),EQ,0) GO TO 505
   P(7,NN) = BC(3,N2)
505 IF (NBC(4,N2),EQ,0) GO TO 506
   P(7,NN) = 0.0
506 IF (NBC(5,N2),EQ,0) GO TO 507
   P(8,NN) = BC(5,N2)
507 IF (NBC(6,N2),EQ,0) GO TO 508
   P(9,NN) = BC(6,N2)
508 IF (NBC(7,N2),EQ,0) GO TO 509
   P(10,NN) = BC(7,N2)
509 IF (NBC(8,N2),EQ,0) GO TO 510
   P(6,NN) = 0.0
510 IF (NBC(9,N2),EQ,0) GO TO 511
   P(6,NN) = 0.0
511 IF (NBC(10,N2),EQ,0) GO TO 512
   P(8,NN) = 0.0
512 DO 513 I=1,10
   IF (NBC(I,N3),EQ,1) GO TO 514
513 CONTINUE
   GO TO 524
514 IF (NBC(1,N3),EQ,0) GO TO 515
   P(21,NN) = BC(1,N3)
515 IF (NBC(2,N3),EQ,0) GO TO 516
   P(21,NN) = 0.0
516 IF (NBC(3,N3),EQ,0) GO TO 517
   P(22,NN) = BC(3,N3)
517 IF (NBC(4,N3),EQ,0) GO TO 518
P(22,NN) = 0.0

518 IF (NBC(5,N3) .EQ. 0) GO TO 519
    P(23,NN) = BC(5,N3)

519 IF (NBC(6,N3) .EQ. 0) GO TO 520
    P(24,NN) = BC(6,N3)

520 IF (NBC(7,N3) .EQ. 0) GO TO 521
    P(25,NN) = BC(7,N3)

521 IF (NBC(8,N3) .EQ. 0) GO TO 522
    P(21,NN) = 0.0
    P(23,NN) = 0.0

522 IF (NBC(9,N3) .EQ. 0) GO TO 523
    P(21,NN) = 0.0
    P(23,NN) = 0.0

523 IF (NBC(10,N3) .EQ. 0) GO TO 524
    P(21,NN) = 0.0
    P(23,NN) = 0.0

524 GO TO (274,275), NBC1

274 DO 219 I = 1, 30
    XP(I,1) = P(I,1)
    CONTINUE

COUNT = COUNT + 1.0
NMM = N + 1
DO 203 K = 3, NMM, 2
    IF (COUNT .GT. 1.) GO TO 261
    READ 103, (NBC(I,K), I = 1, 10), JLN(K)
    DO 204 I = 1, 10
        IF (NBC(I,K) .EQ. 0) GO TO 204
        READ 101, BC(I,K)
    CONTINUE

204 IF (NBC(8,K) .EQ. 1 .OR. NBC(9,K) .EQ. 1 .OR. NBC(10,K) .EQ. 1) GO TO 239
    GO TO 248

239 READ 104, CI(K), CL(K), CE(K)

248 DO 249 I = 1, 10
    IF (NBC(I,K) .EQ. 1) GO TO 246
    CONTINUE

249 IF (JLN(K) .EQ. 2) GO TO 246
    P1(K) = 0.0
E21(K) = 0.0
H21(K) = 0.0
E22(K) = 0.0
H22(K) = 0.0
H1(K) = 0.0
E31(K) = 0.0
H31(K) = 0.0
E32(K) = 0.0
H32(K) = 0.0
XMOA(K) = 0.0
XMOC(K) = 0.0
XMOB(K) = 0.0
XMOA2(K) = 0.0
XMOB2(K) = 0.0
XMOC2(K) = 0.0

GO TO 260

246 READ 102, P1(K), E21(K), H21(K), P2(K), E22(K), H22(K)
READ 102, H1(K), E31(K), H31(K), H2(K), E32(K), H32(K)
READ 102, XMOA(K), XMOB(K), XMOC(K), XMOA2(K), XMOB2(K), XMOC2(K)
IF (K.EQ. N) GO TO 261

260 READ 102, W1(K), E11(K), H11(K), W2(K), E12(K), H12(K)
READ 103, (NBC(I,K+1), I=1,10), JLN(K+1)
DO 262 I=1,10
IF (NBC(I,K+1).EQ.0) GO TO 262
READ 101, BC(I,K+1)

262 CONTINUE
IF (NBC(8,K+1).EQ.1.OR.NBC(9,K+1).EQ.1.OR.NBC(10,K+1).EQ.1) GO TO 263
GO TO 264

263 READ 104, CI(K+1), CL(K+1), CE(K+1)
DO 265 I=1,10
IF (NBC(I,K+1).EQ.1) GO TO 266
CONTINUE
IF (JLN(K+1).EQ.2) GO TO 266

265 P1(K+1) = 0.0
E21(K+1) = 0.0
H21(K+1) = 0.0
IF(K.EQ.N) GO TO 261
GO TO 267
266 READ 102,X1(K+1),E21(K+1),H21(K+1),P2(K+1),E22(K+1),H22(K+1)
READ 102,X1(K+1),E21(K+1),H21(K+1),E31(K+1),H31(K+1),E32(K+1),H32(K+1)
READ 102,XMOA(K+1),XMOB(K+1),XMOC(K+1),XMOA2(K+1),XMOB2(K+1),XMOC2(K+1)
267 READ 102,X1(K+1),E21(K+1),H21(K+1),W2(K+1),E12(K+1),H12(K+1)
261 CONTINUE
NE = (N+1)/2
NN = (K+1)/2
P(1,NN) = 0.0
P(2,NN) = 0.0
P(3,NN) = (XMOA(K-1)+P1(K-1)*H21(K-1)+H1(K-1)*E31(K-1))#DJ
P(4,NN) = XMOB(K-1)*D2
P(5,NN) = -XMOC(K-1)*D1
P(6,NN) = 1.0
P(7,NN) = 1.0
P(8,NN) = 1.0
P(9,NN) = 1.0
P(10,NN) = 1.0
P(11,NN) = XMOA(K)+P1(K)*H21(K)+H1(K)*E31(K)+(W1(K-1)*H11(K-1)+W2(K-1)*H12(K-1))*D
\[ P(12,NN) = \text{XMOB}(K) \]
\[ P(13,NN) = \text{XMC}(K) \]
\[ P(14,NN) = -P1(K) - D*(W1(K-1) + W2(K-1)) \]
\[ P(15,NN) = H1(K) \]
\[ P(16,NN) = 0.0 \]
\[ P(17,NN) = 0.0 \]
\[ P(18,NN) = (\text{XMOA}(K) + P1(K) * H21(K) + H1(K) * E31(K)) * DJ \]
\[ P(19,NN) = \text{XMOB}(K) * D2 \]
\[ P(20,NN) = -\text{XMC}(K) * D1 \]
\[ P(21,NN) = 1.0 \]
\[ P(22,NN) = 1.0 \]
\[ P(23,NN) = 1.0 \]
\[ P(24,NN) = 1.0 \]
\[ P(25,NN) = 1.0 \]

IF (NN.EQ. NE) GO TO 268
\[ P(26,NN) = \text{XMOA}(K+1) + P1(K+1) * H21(K+1) + H1(K+1) * E31(K+1) + \]
\[ (W1(K) * H11(K) + W2(K) * H12(K)) * D \]
\[ P(27,NN) = \text{XMOB}(K+1) \]
\[ P(28,NN) = \text{XMC}(K+1) \]
\[ P(29,NN) = -P1(K+1) - D*(W1(K) + W2(K)) \]
\[ P(30,NN) = H1(K+1) \]
GO TO 269

268
\[ P(26,NN) = 0.0 \]
\[ P(27,NN) = 0.0 \]
\[ P(28,NN) = 0.0 \]
\[ P(29,NN) = 0.0 \]
\[ P(30,NN) = 0.0 \]

269
\[ NBC1 = 2 \]
GO TO 500

275 CONTINUE
DO 884 I = 1, 30
DO 884 J = 1, 30

884
\[ A3(I,J) = 0.0 \]
\[ A3(1,6) = 1.0 \]
\[ A3(1,9) = -0.5*D \]
\[ A3(2,7) = 1.0 \]
\[ A3(2,10) = -0.5*D \]
\[ A3(3,1) = DJ \]
\[ A3(3,8) = 1.0 + (-P1(K-1) * E21(K-1) + H1(K-1) * H31(K-1)) * DJ \]
\[ A3(3,9) = -XMOC(K-1) * DJ \]
\[ A3(3,10) = -XMOC(K-1) * DJ \]
\[ A3(4,2) = D2 \]
\[ A3(4,8) = XMOC(K-1) * D2 \]
\[ A3(4,9) = -1.0 \]
\[ A3(4,10) = (XMOA(K-1) + P1(K-1) * (E21(K-1) * BETA(K-1) + H21(K-1)) + H1(K-1) * H31(K-1)) * DJ \]
\[ A3(5,3) = -D1 \]
\[ A3(5,8) = XMOC(K-1) * D1 \]
\[ A3(5,9) = -A3(4,10) * D1 / D2 \]
\[ A3(5,10) = 1.0 \]
\[ A3(6,11) = 1.0 \]
\[ A3(7,12) = 1.0 \]
\[ A3(8,13) = 1.0 \]
\[ A3(9,14) = 1.0 \]
\[ A3(10,15) = 1.0 \]
\[ A3(11,1) = -1.0 \]
\[ A3(11,8) = -D * W1(K-1) * E11(K-1) + W2(K-1) * E12(K-1) \]
\[ A3(11,9) = +XMOC(K-1) \]
\[ A3(11,10) = +XMOC(K-1) \]
\[ A3(11,16) = 1.0 \]
\[ A3(11,23) = -P1(K) * E21(K) + H1(K) * H31(K) \]
\[ A3(11,24) = -XMOC(K-1) - XMOC(K) \]
\[ A3(11,25) = -XMOC(K-1) - XMOC(K) \]
\[ A3(12,2) = -1.0 \]
\[ A3(12,5) = D \]
\[ A3(12,8) = -XMOC(K-1) \]
\[ A3(12,10) = -XMA(K-1) + D * W1(K-1) * (H11(K-1) + E11(K-1) * BETA(K-1)) \]
\[ 1 = +D * W2(K-1) * (H12(K-1) + E12(K-1) * BETA(K-1)) \]
\[ A3(12,17) = 1.0 \]
\[ A3(12,23) = XMOC(K-1) + XMOC(K) \]
\[ A3(12,25) = XMA(K-1) + XMOC(K) + P1(K) * (E21(K) * BETA(K) + H21(K)) \]
\[ 1 = +H1(K) * (E31(K) - H31(K) * BETA(K)) \]
\[ A3(13,3) = -1.0 \]
\[ A3(13,4) = D \]
\[ A3(13,8) = XMOC(K-1) \]
| A3[13, 9] | = A3[12, 10] |
| A3[13, 18] | = 1.0 |
| A3[13, 23] | = -XMB(K-1) - XMOB(K) |
| A3[14, 4] | = -1.0 |
| A3[14, 8] | = VC(K-1) |
| A3[14, 19] | = 1.0 |
| A3[14, 22] | = -VC(K-1) - H1(K,L) |
| A3[15, 5] | = -1.0 |
| A3[15, 8] | = -VB(K-1) - D*(W1(K-1) + W2(K-1)) |
| A3[15, 20] | = 1.0 |
| A3[15, 23] | = +VB(K-1) - P1(K) |
| A3[16, 6] | = -1.0 |
| A3[16, 9] | = -5*D |
| A3[16, 21] | = 1.0 |
| A3[16, 24] | = -5*D |
| A3[17, 7] | = -1.0 |
| A3[17, 10] | = -5*D |
| A3[17, 22] | = 1.0 |
| A3[17, 25] | = -5*D |
| A3[18, 1] | = DJ |
| A3[18, 8] | = -1.0 |
| A3[18, 16] | = DJ |
| A3[18, 23] | = 1.0 - P1(K)*E21(K)*DJ + H1(K)*H31(K)*DJ |
| A3[18, 24] | = -XMOC(K)*DJ |
| A3[18, 25] | = -XMOB(K)*DJ |
| A3[19, 2] | = D2 |
| A3[19, 9] | = -1.0 |
| A3[19, 17] | = D2 |
| A3[19, 23] | = XMOC(K)*D2 |
| A3[19, 24] | = 1.0 |
| A3[19, 25] | = (XMOA(K) + P1(K)*(E21(K)*BETA(K) + H21(K))) + H1(K)*(E31(K) - H31(K)*BETA(K)))*D2 |
| A3[20, 3] | = -D1 |
| A3[20, 10] | = -1.0 |
| A3[20, 18] | = -D1 |
| A3[20, 23] | = XMOB(K)*D1 |
\[
\begin{align*}
A_1(27,2) &= 1.0 \\
A_1(27,8) &= XMC(K-2) + XMOC(K-1) \\
A_1(27,10) &= XMA(K-2) + XMOA(K-1) + P1(K-1) + E21(K-1) \times \text{BETA}(K-1) \\
1 &= H21(K-1) + H1(K-1) + E31(K-1) - H31(K-1) \times \text{BETA}(K-1) \\
A_1(28,3) &= 1.0 \\
A_1(28,8) &= -XMB(K-2) - XMOB(K-1) \\
A_1(28,9) &= A_1(27,10) \\
A_1(29,4) &= 1.0 \\
A_1(29,8) &= -VC(K-2) - H1(K-1) \\
A_1(30,5) &= 1.0 \\
A_1(30,8) &= VB(K-2) - P1(K-1) \\
\text{NBC2} &= 2 \\
\text{GO TO 900} \\
\end{align*}
\]

**273 CONTINUE**

**WRITE (2) A1**

**DO 276 J = 1, 30**

\[
AX(1,J) = -A(21,J) - 0.5 \times D \times A(24,J)
\]

**AX(2,J) = -A(22,J) - 0.5 \times D \times A(25,J)**

**AX(3,J) = D1 \times A(16,J) - A(23,J)**

**AX(4,J) = D2 \times A(17,J) - A(24,J)**

**276 AX(5,J) = -D1 \times A(18,J) - A(25,J)**

**DO 886 I=1, 5**

**DO 277 J=1, 30**

**AXX(I,1) = 0.0**

**DO 886 I=1, 5**

**AXX(I,1) = -AX(I,26)**

**AXX(I,2) = -AX(I,27)**

**AXX(I,3) = -AX(I,28)**

**AXX(I,4) = -AX(I,29)**

**AXX(I,5) = -AX(I,30)**

**AXX(I,8) = AX(I,26) \times A(26,8) - AX(I,27) \times A(27,8) - AX(I,28) \times A(28,8) - AX(I,29) \times A(29,8) - AX(I,30) \times A(30,8)**

**AXX(I,9) = AX(I,26) \times A(26,9) - AX(I,28) \times A(28,9)**

**AXX(I,10) = AX(I,26) \times A(26,10) - AX(I,27) \times A(27,10)**

**AXX(I,11) = AX(I,26) \times A(26,11) - AX(I,30) \times A(30,11)**

**AXX(I,12) = AX(I,26) \times A(26,12) - AX(I,29) \times A(29,12)**

**AXX(I,13) = AX(I,26) \times A(26,13)**

**AXX(I,14) = AX(I,27) \times A(27,14)**
277  \( A_{XX(I,15)} = -A_{X(I,28)} \cdot A_{1(28,15)} \)

\[
\begin{align*}
\text{DO } 893 & 
\end{align*}
\]

\[
\begin{align*}
\text{DO } 893 & \quad J = 1, 30 \\
\text{IF } (I \quad . \quad \text{GE} \quad . \quad 6) & \quad \text{GO TO } 894 \\
A(I,J) & = A_{3(I,J)} + A_{XX(I,J)} \\
\text{GO TO } 893 \\
894 & 
\end{align*}
\]

\[
\begin{align*}
A(I,J) & = A_{3(I,J)} \\
\end{align*}
\]

892 \text{ CONTINUE}

\[
\begin{align*}
\text{CALL INV (A,30,30,DET,COND)} \\
\text{WRITE (1) A} \\
\end{align*}
\]

\[
\begin{align*}
\text{ADET} & = \text{ADET} \cdot \text{DET} \\
\text{DO } 278 & \quad I = 1, 30 \\
278 & 
\end{align*}
\]

\[
\begin{align*}
\text{XPX(I,NN)} & = 0.0 \\
\text{DO } 279 & \quad I = 1, 5 \\
279 & 
\end{align*}
\]

\[
\begin{align*}
\text{XPX(I,NN)} & = \text{XPX(I,NN)} + A_{X(I,J)} \cdot \text{XP(J,NN-1)} \\
\text{DO } 280 & \quad I = 1, 30 \\
280 & 
\end{align*}
\]

\[
\begin{align*}
\text{XP(I,NN)} & = \text{P(I,NN)} - \text{XPX(I,NN)} \\
\text{CONTINUE} \\
\text{GO TO } (1001,1002,1003,1004), \text{NRUN} \\
1001 & \quad \text{XDET1 = ADET} \\
1001 & \quad \text{PRINT 1006,XDET1} \\
1006 & \quad \text{FORMAT(1X,34HVALUE OF DETERMINANT FIRST RUN IS,E16.8/)} \\
1002 & \quad \text{GO TO 1005} \\
1002 & \quad \text{XDET2 = ADET} \\
1007 & \quad \text{PRINT 1007,XDET2} \\
1007 & \quad \text{FORMAT(1X,34HVALUE OF DETERMINANT SECOND RUN IS,E16.8/)} \\
1003 & \quad \text{GO TO 1005} \\
1003 & \quad \text{XDET3 = ADET} \\
1008 & \quad \text{PRINT 1008,XDET3} \\
1008 & \quad \text{FORMAT(1X,34HVALUE OF DETERMINANT THIRD RUN IS,E16.8/)} \\
1004 & \quad \text{GO TO 1005} \\
1004 & \quad \text{XDET4 = ADET} \\
1009 & \quad \text{PRINT 1009,XDET4} \\
1009 & \quad \text{FORMAT(1X,34HVALUE OF DETERMINANT FOURTH RUN IS,E16.8/)} \\
1005 & \quad \text{GO TO 1013} \\
1005 & \quad \text{CONTINUE}
\end{align*}
\]
BACKSPACE 1
READ (11) A
DO 887 I = 1,30
  887 Z(I,NE) = 0.0
DO 888 I = 1,30
  888 Z(I,NE) = Z(I,NE) + A(I,J)*XP(J,NE)
NNE = NE - 1
DO 281 KK = 1, NNE
  K = NNE - KK + 1
IF (K .LT. NNE) GO TO 282
BACKSPACE 2
READ (2) A1
GO TO 283
  282 BACKSPACE 2
  BACKSPACE 2
  READ (2) A1
  283 DO 889 I = 1,30
    889 ZM(I,K) = 0.0
DO 890 I = 1,30
  DO 890 J = 1,30
    890 ZM(I,K) = ZM(I,K) + A1(I,J)*Z(J,K + 1)
DO 284 I = 1,30
  284 ZM(I,K) = XP(I,K) - ZM(I,K)
BACKSPACE 1
BACKSPACE 1
READ (1) A
DO 891 I = 1,30
  891 Z(I,K) = 0.0
DO 892 I = 1,30
  DO 892 J = 1,30
    892 Z(I,K) = Z(I,K) + A(I,J)*ZM(J,K)
  CONTINUE
DO 281
    281 CONTINUE
GO TO (1010, 1011, 1012, 1013), NRUN
  1010 READ 116, (ITT(L), L = 1, 6)
  116 FORMAT (1X, 6I5)
DO 303 K = 1, N
  303 IF (ITT(1), EQ, 0) GO TO 304
XMOA(K) = 1.2*XMOA(K)

304 IF (ITT(2) .EQ. 0) GO TO 305
XMOB(K) = 1.2*XMOB(K)

305 IF (ITT(3) .EQ. 0) GO TO 306
XMOC(K) = 1.2*XMOC(K)

306 IF (ITT(4) .EQ. 0) GO TO 307
P1(K) = 1.2*P1(K)

307 IF (ITT(5) .EQ. 0) GO TO 308
H1(K) = 1.2*H1(K)

308 IF (K .EQ. N) GO TO 303
IF (K .EQ. N) GO TO 303
W1(K) = 1.2*W1(K)

303 CONTINUE

NMN = N - 1
DO 310 K = 2, NMN, 2
NB = (K + 2) / 2
XMA(K) = 1.2*Z(1, NB)
XMB(K) = 1.2*Z(2, NB)
XMC(K) = 1.2*Z(3, NB)
VB(K) = 1.2*Z(4, NB)
VC(K) = 1.2*Z(5, NB)
BETA(K) = 1.2*Z(8, NB)
1.EQ.0.AND. NBC(1, K) .EQ. 0 .AND. NBC(2, K) .EQ. 0 .AND. NBC(3, K) .EQ. 0 .AND. NBC(9, K) = 1.EQ.0 .AND. NBC(2, K) .EQ. 0 .AND. NBC(3, K) .EQ. 0 .AND. NBC(9, K) .EQ. 0 .AND. NBC(10, K) .EQ. 0
945 IF (NBC(1, K) .EQ. 0 .AND. NBC(10, K) .EQ. 0) GO TO 945
H2(K) = 1.2*Z(11, NB)

946 IF (NBC(3, K) .EQ. 0 .AND. NBC(4, K) .EQ. 0) GO TO 946
P2(K) = 1.2*Z(12, NB)

947 IF (NBC(6, K) .EQ. 0) GO TO 948
XMOA2(K) = 1.2*Z(13, NB)

948 IF (NBC(7, K) .EQ. 0) GO TO 949
XMOB2(K) = 1.2*Z(14, NB)

949 CONTINUE

310 CONTINUE
DO 311 K = 1, N, 2
NN = (K+1)/2

XMA(K) = 1.2*Z(16,NN)
XMB(K) = 1.2*Z(17,NN)
XMC(K) = 1.2*Z(18,NN)
VB(K) = 1.2*Z(19,NN)
VC(K) = 1.2*Z(20,NN)
BETA(K) = 1.2*Z(23,NN)

IF (NBC(1,K).EQ.0 .AND. NBC(2,K).EQ.0 .AND. NBC(8,K).EQ.0 .AND. NBC(9,K).EQ.0 .AND. NBC(10,K).EQ.0) GO TO 950
H2(K) = 1.2*Z(26,NN)

950 IF (NBC(3,K).EQ.0 .AND. NBC(4,K).EQ.0) GO TO 951
P2(K) = 1.2*Z(27,NN)

951 IF (NBC(5,K).EQ.0 .AND. NBC(8,K).EQ.0 .AND. NBC(9,K).EQ.0) GO TO 952
XM0A2(K) = 1.2*Z(28,NN)

952 IF (NBC(6,K).EQ.0) GO TO 953
XM0B2(K) = 1.2*Z(29,NN)

953 IF (NBC(7,K).EQ.0) GO TO 954
XM0C2(K) = 1.2*Z(30,NN)

954 CONTINUE

CL3 = 5./6. + XDET1/(6.*(XDET1-XDET2))

READ 116, (ITT(L), L = 1, 6)
DO 1015 K = 1, N

1015 IF (ITT(1).EQ.0) GO TO 1016
XMOA(K) = CL3*XMOA(K)

1016 IF (ITT(2).EQ.0) GO TO 1017
XM0B(K) = CL3*XMOB(K)

1017 IF (ITT(3).EQ.0) GO TO 1018
XM0C(K) = CL3*XMOC(K)

1018 IF (ITT(4).EQ.0) GO TO 1019
PI(K) = CL3*PI(K)

1019 IF (ITT(5).EQ.0) GO TO 1020
H1(K) = CL3*H1(K)

1020 IF (ITT(6).EQ.0) GO TO 1015

CONTINUE
GO TO 1014
1015 CONTINUE
NMN = NMN - 1
DO 1021 K = 2, NMN, 2
NB = (K + 1) / 2
XMA(K) = CL3*Z(1, NB)
XMB(K) = CL3*Z(2, NB)
XMC(K) = CL3*Z(3, NB)
VB(K) = CL3*Z(4, NB)
VC(K) = CL3*Z(5, NB)
BETA(K) = CL3*Z(8, NB)
IF (NBC(1, K) .EQ. 0 .AND. NBC(2, K) .EQ. 0 .AND. NBC(8, K) .EQ. 0 .AND. NBC(9, K) .EQ. 0 .AND. NBC(10, K) .EQ. 0) GO TO 1022
1022 H2(K) = CL3*Z(11, NB)
P2(K) = CL3*Z(12, NB)
XMOA2(K) = CL3*Z(13, NB)
1023 IF (NBC(3, K) .EQ. 0 .AND. NBC(4, K) .EQ. 0 .AND. NBC(9, K) .EQ. 0 .AND. NBC(10, K) .EQ. 0) GO TO 1024
1024 IF (NBC(5, K) .EQ. 0 .AND. NBC(8, K) .EQ. 0 .AND. NBC(9, K) .EQ. 0 .AND. NBC(10, K) .EQ. 0) GO TO 1025
1025 IF (NBC(11, K) .EQ. 0 .AND. NBC(12, K) .EQ. 0 .AND. NBC(13, K) .EQ. 0 .AND. NBC(14, K) .EQ. 0 .AND. NBC(15, K) .EQ. 0) GO TO 1026
1026 IF (NBC(15, K) .EQ. 0 .AND. NBC(16, K) .EQ. 0 .AND. NBC(17, K) .EQ. 0 .AND. NBC(18, K) .EQ. 0 .AND. NBC(19, K) .EQ. 0 .AND. NBC(20, K) .EQ. 0 .AND. NBC(21, K) .EQ. 0) GO TO 1027
1027 H2(K) = CL3*Z(26, NN)
P2(K) = CL3*Z(27, NN)
1028 IF (NBC(15, K) .EQ. 0 .AND. NBC(16, K) .EQ. 0 .AND. NBC(17, K) .EQ. 0 .AND. NBC(18, K) .EQ. 0 .AND. NBC(19, K) .EQ. 0 .AND. NBC(20, K) .EQ. 0 .AND. NBC(21, K) .EQ. 0) GO TO 1029
1029 CONTINUE

DO 1026 K = 1, N, 2
NN = (K + 1) / 2
XMA(K) = CL3*Z(16, NN)
XMB(K) = CL3*Z(17, NN)
XMC(K) = CL3*Z(18, NN)
VB(K) = CL3*Z(19, NN)
VC(K) = CL3*Z(20, NN)
BETA(K) = CL3*Z(23, NN)
IF (NBC(1, K) .EQ. 0 .AND. NBC(2, K) .EQ. 0 .AND. NBC(8, K) .EQ. 0 .AND. NBC(9, K) .EQ. 0 .AND. NBC(10, K) .EQ. 0) GO TO 1027
1027 H2(K) = CL3*Z(26, NN)
P2(K) = CL3*Z(27, NN)
1028 IF (NBC(15, K) .EQ. 0 .AND. NBC(16, K) .EQ. 0 .AND. NBC(17, K) .EQ. 0 .AND. NBC(18, K) .EQ. 0 .AND. NBC(19, K) .EQ. 0 .AND. NBC(20, K) .EQ. 0 .AND. NBC(21, K) .EQ. 0) GO TO 1029
1029 CONTINUE
1 NBC(10,K) .EQ. 0) GO TO 1029
XMOA2(K) = CL3*Z(28,NN)
1029 IF (NBC(6,K) .EQ. 0) GO TO 1030
XMOB2(K) = CL3*Z(29,NN)
1030 IF (NBC(7,K) .EQ. 0) GO TO 1026
XMOC2(K) = CL3*Z(30,NN)
1026 CONTINUE
GO TO 1014
1012 NMN = N - 1
DO 1031 K = 2, NMN, 2
NB = (K + 2) / 2
XMA(K) = Z(1,NB)
XMB(K) = Z(2,NB)
XMC(K) = Z(3,NB)
VB(K) = Z(4,NB)
VC(K) = Z(5,NB)
BETA(K) = Z(8,NB)
IF (NBC(1,K) .EQ. 0.AND. NBC(2,K) .EQ. 0.AND. NBC(8,K) .EQ. 0.AND. NBC(9,K) .EQ. 0.AND. NBC(10,K) .EQ. 0) GO TO 1033
H2(K) = Z(11,NB)
1033 IF (NBC(3,K) .EQ. 0.AND. NBC(4,K) .EQ. 0) GO TO 1034
P2(K) = Z(12,NB)
1034 IF (NBC(5,K) .EQ. 0.AND. NBC(6,K) .EQ. 0.AND. NBC(9,K) .EQ. 0.AND. NBC(10,K) .EQ. 0) GO TO 1035
1NBC(10,K) .EQ. 0) GO TO 1035
XMOA2(K) = Z(13,NB)
1035 IF (NBC(6,K) .EQ. 0) GO TO 1036
XMOB2(K) = Z(14,NB)
1036 IF (NBC(7,K) .EQ. 0) GO TO 1031
XMOC2(K) = Z(15,NB)
1031 CONTINUE
DO 1037 K = 1, N, 2
NN = (K + 1) / 2
XMA(K) = Z(16,NN)
XMB(K) = Z(17,NN)
XMC(K) = Z(18,NN)
VB(K) = Z(19,NN)
VC(K) = Z(20,NN)
BETA(K) = Z(23,NN)
IF (NBC(1,K) .EQ. 0 .AND. NBC(2,K) .EQ. 0 .AND. NBC(8,K) .EQ. 0 .AND. NBC(9,K) .EQ. 0 .AND. NBC(10,K) .EQ. 0) GO TO 1038

H2(K) = Z(26,NN)

1038 IF (NBC(3,K) .EQ. 0 .AND. NBC(4,K) .EQ. 0) GO TO 1039

P2(K) = Z(27,NN)

1039 IF (NBC(5,K) .EQ. 0 .AND. NBC(8,K) .EQ. 0 .AND. NBC(9,K) .EQ. 0 .AND. NBC(10,K) .EQ. 0) GO TO 1040

XMOA2(K) = Z(28,NN)

1040 IF (NBC(6,K) .EQ. 0) GO TO 1041

XMOB2(K) = Z(29,NN)

1041 IF (NBC(7,K) .EQ. 0) GO TO 1037

XMOC2(K) = Z(30,NN)

1037 CONTINUE

GO TO 1014

1014 NRUN = NRUN + 1

GO TO 955

1013 PRINT 1054

1054 FORMAT (1X, 74H*******THE FOLLOWING ARE VALUES OF THE CRITICAL LATERA
1L BUCKLING LOAD********, /)

S1 = XDET1/(XDET4-XDET1)
S2 = XDET2/(XDET4-XDET2)
S3 = XDET4/(1.2*XDET1-XDET2)
ADJ = 1. + 2*S1*S2*S3

READ 116, (ITT(L), L=1,6)

DO 1042 K = 1, N

IF (ITT(1) .EQ. 0) GO TO 1043

XMOA(K) = ADJ*XMOA(K)
PRINT 1048*XMOA(K), K

1043 IF (ITT(2) .EQ. 0) GO TO 1044

XMOB(K) = ADJ*XMOB(K)
PRINT 1049*XMOB(K), K

1044 IF (ITT(3) .EQ. 0) GO TO 1045

XMOC(K) = ADJ*XMOC(K)
PRINT 1050*XMOC(K), K

1045 IF (ITT(4) .EQ. 0) GO TO 1046

P1(K) = ADJ*P1(K)
PRINT 1051*P1(K), K
1046 IF (ITT(5) .EQ. 0) GO TO 1047
   HI(K) = ADJ * HI(K)
   PRINT 1052, HI(K), K
1047 IF (ITT(6) .EQ. 0) GO TO 1042
   W1(K) = ADJ * HI(K)
   PRINT 1052, W1(K), K
1048 FORMAT (1X, 15HCritical XMOC =, F16.8, 10H AT JOINT, I5)
1049 FORMAT (1X, 15HCritical Xmoc =, F16.8, 10H AT JOINT, I5)
1050 FORMAT (1X, 15HCritical XMOC =, F16.8, 10H AT JOINT, I5)
1051 FORMAT (1X, 15HCritical P =, F16.8, 10H AT JOINT, I5)
1052 FORMAT (1X, 15HCritical H =, F16.8, 10H AT JOINT, I5)
1053 FORMAT (1X, 15HCritical W =, F16.8, 10H AT JOINT, I5)
1042 CONTINUE
   GO TO 298
955 CONTINUE
   COUNT = COUNT + 1.0
   REWIND 1
   REWIND 2
   GO TO 299
298 CONTINUE
   STOP
END
$IBFTC INV

SUBROUTINE INV (A, N, M, DET, COND)
DOUBLE PRECISION A(2), IP(100)
DOUBLE PRECISION AIK, AMAX, CSUMA, CSUMB, EPS, T, DET, COND
MN1 = M*(N-1)
EPS = 1.E-20
DO 41 I = 1, N
   IP(I) = I
   C IP(K) WILL KEEP TRACK OF WHICH ROW ENDS UP AS K*TH PIVOT ROW
12 DET = 1.
11 C FIRST PART OF COND
   KSW = 1
   GO TO 260
5 C INVERSION STARTS

M5-2 6
M5-2 7
M5-2 8
M5-2 9
M5-2 10
M5-2 11
M5-2 12
M5-2 13
M5-2 14
M5-2 15
DO 199 K = 1,N
MK = M*K - M
KK = K + MK
C FIND MAXIMUM ELEMENT IN K*TH COLUMN
IMAX = K
AMAX = DABS(A(KK))
IF(K.GE.N) GO TO 65
KP_ = I + 1
DO 60 I = KP,N
IK = I + MK
AIK = DABS(A(IK))
IF(AIK.LE.AMAX) GO TO 60
AMAX = AIK
IMAX = I
CONTINUE
IF(AMAX.LT.EPS) GO TO 300
LAST = MN1+K
C INTERCHANGE ROWS K AND IMAX
IF(K.EQ. IMAX) GO TO 100
DET = -DET
IMAXJ = IMAX
DO 75 KJ = K,LAST,M
T = A(KJ)
A(KJ) = A(IMAXJ)
A(IMAXJ) = T
IMAXJ = IMAXJ + M
J = IP(K)
IP(K) = IP(IMAX)
IP(IMAX) = J
C COMPUTE DETERMINANT
DET = DET*A(KK)
C DIVIDE K*TH ROW BY A(K,K)
T = 1./A(KK)
A(KK) = 1.0
DO 140 KJ = K,LAST,M
A(KJ) = A(KJ)*T
SUBTRACT A(I,K) TIMES THE K*TH ROW FROM THE OTHER ROWS

IF(I.EQ.K) GO TO 199
IK = I + MK
T = A(IK)
A(IK) = 0.0
IJ=I
DO 190 KJ=K*LAST*M
A(IJ) = A(IJ) - T*A(KJ)
190 IJ = IJ + M
199 CONTINUE

RESTORE PROPER COLUMN ORDER IN THE INVERSE.

MK = M*K - M
COLUMN NOW OCCUPYING K*TH POSITION IS ACTUALLY COLUMN ...

IF(J.EQ.K) GO TO 250
RELOCATE COLUMN K TO ITS FINAL POSITION J
MJ = M*K - M
DO 225 I = 1,N
IK = I + MK
T = A(IJ)
A(IJ) = A(IK)
225 A(IK) = T
ADJUST IP RECORD
IP(K) = IP(J)
IP(J) = J
AND CHECK NEW K*TH COLUMN.
GO TO 210
CONTINUE
KSW = 2
CALCULATE COND
CSUMB = 0.0
DO 270 I=1,N
LAST = MN1+I
DO 270 IJ=I, LAST, M
   T = A(IJ)
   CSUMB = CSUMB + T*T
   GO TO (45, 275), KSW
270   CONTINUE
275   COND = DSQRT (CSUMA*CSUMB)/FLOAT(N)
   RETURN
   M5-2 79
   M5-2 80
   M5-2 83
   M5-2 84
   M5-2 85
   M5-2 86
   M5-2 88
   M5-2 90

PROCEDURE FOR SINGULAR OR NEARLY SINGULAR MATRIX.

300 PRINT 310, K, AMAX
310 FORMAT (6H0, STEPI3, 9H PIVOT = 1PE15.8, 25H, INVERSION DISCONTINUED)

1) DET = 0.
    COND = 1.E38
    RETURN
    END

RETURN

M5-2 79
M5-2 80
M5-2 83
M5-2 84
M5-2 85
M5-2 86
M5-2 88
M5-2 90