OPTIMUM TURBINE GATE OPERATION TO MINIMIZE SPEED CHANGE IN AN HYDRAULIC TURBINE

by

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ABSTRACT

This thesis examines the influence of different gate operation curves on the surplus or deficiency of energy input to a hydraulic turbine accompanying a sudden change of load on the turbine. A general solution to the problem is obtained by evaluating the energy input to the penstock and the energy conversion within the penstock during transient conditions. The results show that for given maximum pressure rise or drop, a considerable reduction in the surplus or deficiency of energy input to the turbine can be obtained by use of a suitable gate operation curve. At the same time it is possible to reduce the hydraulic oscillations in the system.

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INTRODUCTION

Many of the complicated processes of modern industry require electric power whose frequency is maintained within very close tolerances ($\pm$ 1/10 cycle). In any large electrical system, any major increase or decrease in the magnitude of the load on the system can be considered instantaneous (such as would be caused by the loss of a transmission line). Furthermore, the distribution of this load change between the various generators is also instantaneous when compared to the speed of response of the turbine governor. The resulting change in system frequency is approximately proportional to the square root of the difference between the combined input to the turbines and the combined output of the connected generators; and inversely proportional to the inertia of the whole system (i.e. including connected load). To keep the frequency change (or corresponding generator speed change) within specified limits for an assumed major load change it is necessary to either increase the system inertia or decrease the difference between turbine input and generator output; the choice being mainly one of economics.

In a hydraulic turbine-generator arrangement this input-output difference is a result of the finite speed of response and sensitivity of the turbine governor and the inertia of the fluid supplying energy to the turbine. This paper is concerned with the problem of minimizing this input-output difference in a hydraulic turbine under transient conditions. It is assumed that the governor sensitivity and speed of response, and the allowable penstock pressure rise and drop are specified and that the only variable quantity is the turbine gate closure curve. It is further assumed that the penstock is of constant dimensions, the reservoir elevation is constant, the turbine head-discharge curves for the rated speed are known and any influence of turbine speed changes on the head-discharge relationships can be neglected (with a frequency change of $\pm$ 1/10 cycle this last assumption is quite justified).

Although no optimum gate operation curves are derived in this paper, an example is worked showing that the optimum closure curve for limiting pressure rise derived by E. Ruus (6)[1] is a reasonably practical curve for minimizing the input-output energy difference.

The approach in this paper is based on the graphical solution of the Allievi chain equations developed by Schnyder and Bergeron (1), combined with an evaluation of the energy conversion taking place in the penstock under transient conditions. Although energy methods are not usually used in rapidly varied flow problems because of difficulty in evaluating friction losses, it is shown that in this case the losses can be neglected.

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1. Numbers in the parenthesis refer to the Bibliography.

CHAPTER I

ELEMENTS OF THE SCHNYDER-BERGERON GRAPHICAL SOLUTION OF THE ALLIEVI WATERHAMMER EQUATIONS

The theory derived in this paper is based on the graphical method of waterhammer analysis developed by Schnyder and Bergeron. This method is a graphical solution of the equation representing conditions at the turbine gate simultaneously with the conjugate waterhammer equations developed by Allievi.

According to Allievi the conditions in the penstock at the turbine gate can be represented by

$$V = A_{gi} \sqrt{2gH_i}$$

or

$$v = \tau_i \sqrt{h_i} \qquad \text{(in relative form)}$$

For different values of A_g (or τ) the H-V (or h-v) plot is a parabola.

The conjugate waterhammer equations developed by Allievi are

$$H - H_o = \mp \frac{a}{g} (V - V_o)$$

or

$$h - 1 = \mp 2\rho (v - 1) \qquad \text{(in relative form)}$$

CHAPTER II

THEORY OF ENERGY CONVERSION IN THE PENSTOCK

2.1 General

If conservation of energy is applied to the system shown in figure 1, then for any given time interval

$$\hat{E}_g = E_i + E_c - E_{f1} - E_{f2} \tag{1}$$

where: E_g is the energy output through the gate in the time interval;

E_i is the energy input to the penstock from the reservoir in the time interval;

E_c is the change of energy in the penstock, i.e. the difference between the change in kinetic energy and the change in energy, stored as strain energy, in the fluid and in the penstock walls. (E_c is positive if the net energy contained in the fluid and in the penstock walls is reduced);

E_{f1} is the steady state friction loss;

E_{f2} is the energy dissipated as friction during, and as a result of, any change of energy form in the penstock.

The steady state energy dissipation E_{f1} is usually a small fraction of the total energy (at most 10%) and therefore can be neglected safely. i.e. $E_{f1} = 0$ for all calculations in this paper.

Any energy changes in the penstock must be initiated from outside the penstock and must be in the form of either pressure or velocity changes. At the intake there is no regulating device and the reservoir elevation is assumed constant, therefore, any change in the energy content of the penstock must be initiated by changes in velocity at the gate (which are a function of gate movement). If the velocity at the gate is changed, then according to Allievi there is an associated pressure change and this pressure change travels, at high velocity, toward the intake. To say there is a velocity change means the kinetic energy of the fluid is changed; similarly, a pressure change means a change in the amount of energy stored as strain energy in the fluid and in the penstock walls. Any difference between the change in kinetic energy and the change in strain energy must appear at the gate. This energy, E_c, is positive if the net energy of the fluid and penstock walls is reduced. (With this convention a decrease in kinetic energy is positive and a decrease in strain energy is positive).

Finally consider the term E_{f2} which represents the losses in the conversion process which produces E_c. In a waterhammer process, with the exception of a small length of penstock adjacent to the gate, there is essentially no change in the direction of flow of a fluid particle, so that the flow is essentially vortex free. As it is the creation of vortices in a fluid which allows the dissipation of energy over a longer period, the friction losses due to the conversion process must approximately equal zero (i.e. $E_{f2} \cong 0$).

The energy equation can then be written to a good approximation as:

$$E_g = E_i + E_c \tag{2}$$

Example

$a = 3200$ ft/sec $\qquad H_o = 100$ ft

$A = 1/62.4$ ft^2 $\qquad V_o = 20$ ft/sec

$L = 3200$ ft

Consider an incremental change of velocity of

$$\Delta V = -1 \text{ ft/sec}$$

The associated pressure rise is

$$\Delta H = -\frac{a}{g}\Delta V \doteq -\frac{3200}{32}\ (-1) = 100 \text{ ft}$$

The time required for this pressure wave to travel the length of the penstock is

$$t = \frac{L}{a} = \frac{3200}{3200} = 1 \text{ sec}$$

At instant $t = \frac{L}{a}$ the pressure is constant along the penstock and is

$$H = H_o + \Delta H = 100 + 100 = 200 \text{ ft}$$

The velocity in the penstock is

$$V = V_o + \Delta V = 20 + (-1) = 19 \text{ ft/sec}$$

The energy output through the gate during this $t = \frac{L}{a}$ seconds is

$$E_{g1} = w\ A\ H\ V\ \frac{L}{a} = 62.4\ \frac{1}{62.4}\ 200\ (19)\ 1 = 3800 \text{ ft-lbs}$$

The energy input to the penstock during this time is

$$E_{i1} = w\ A\ H_o\ V_o\ \frac{L}{a} = 62.4\ \frac{1}{62.4}\ 100\ (20)\ 1 = 2000 \text{ ft-lbs}$$

The change in the kinetic energy of the fluid in the penstock is

$$\Delta KE_1 = \frac{1}{2}\frac{w}{g}\ A\ L\ (\ V_o^2 - (V_o + \Delta V)^2\ )$$

$$\Delta KE_1 = \frac{1}{2}\ \frac{62.4}{32}\ \frac{1}{62.4}\ (3200)\ (20^2 - 19^2) = 1950 \text{ ft-lbs}$$

The difference between the energy output and the energy input is

$$E_{g1} - E_{i1} = 3800 - 2000 = 1800 \text{ ft-lbs}$$

This is different from the change in kinetic energy of the fluid; therefore, the strain energy stored in the fluid and in the penstock wall is

$$\Delta PE_1 = (E_{g1} - E_{i1}) - \Delta KE_1 = 1800 - 1950 = -150 \text{ ft-lbs}$$

(Note an increase in stored energy is considered negative).

From t = 1 to t = 2 seconds the wave is returning to the gate. At t = 2 seconds the conditions in the penstock are

$$H = H_o = 100 \text{ ft}$$

$$V = V_o + 2\Delta V = 18 \text{ ft/sec}$$

The energy output through the gate in this time is

$$E_{g2} = w\ A\ H\ V \frac{L}{a} = 3800 \text{ ft-lbs} = E_{g1}$$

The energy input to the penstock in this time is

$$E_{i2} = w\ A\ H_o\ (V_o + 2\Delta V) \frac{L}{a} = 62.4 \frac{1}{62.4} 100\ (18)\ 1$$

$$= 1800 \text{ ft-lbs}$$

The change in the kinetic energy of the fluid is

$$\Delta KE_2 = \frac{1}{2} \frac{w}{g} A\ L\ \left((V_o + \Delta V)^2 - (V_o + 2\Delta V)^2 \right)$$

$$= \frac{1}{2} \frac{62.4}{32} \frac{1}{62.4} 3200\ (19^2 - 18^2) = 1850 \text{ ft-lbs}$$

Since the change in pressure is equal and opposite to that of the first $\frac{L}{a}$ time interval the change in stored energy is

$$\Delta PE_2 = - \Delta PE_1 = - (-150) = 150 \text{ ft-lbs}$$

As a check:

$$E_g = E_i + E_c = E_i + \Delta KE + \Delta PE$$

$$E_g = 3800 = 1800 + 1850 + 150 = 3800 \text{ ft-lbs}$$

Note that the net energy output of the wave in the first $\frac{L}{a}$ interval is

$$E_{c1} = \Delta KE_1 + \Delta PE_1 = 1950 + (-150) = 1800 \text{ ft-lbs}$$

In the second $\frac{L}{a}$ interval the net output is

$$E_{c2} = \Delta KE_2 + \Delta PE_2 = 1850 + 150 = 2000 \text{ ft-lbs}$$

The difference between the two wave outputs is

$$E_{c2} - E_{c1} = 2000 - 1800 = 200 \text{ ft-lbs}$$

Also,

$$E_{i2} - E_{i1} = 1800 - 2000 = -200 \text{ ft-lbs}$$

These two values exactly cancel so that as far as the gate is concerned, the energy output of the incident and reflected waves from the reservoir is constant. (At the gate end of the penstock no velocity change is noticed until the wave returns to the gate).

2.2 The Energy Balance for an Infinitesimal Incremental Gate Movement

Any continuous gate operation can be approximated by a series of instantaneous infinitesimal, incremental movements. Consider the change in power output at a penstock gate due to one of these increments occurring at time t, when the head and velocity at the gate are H and V respectively. (Note that the conditions are not necessarily those of steady state). If the time interval δt is made small enough so that no waves which have been reflected from the reservoir are within a distance $2\ \delta x = 2a\ \delta t$ of the gate then within δx, H and V will be constant during the time interval δt. In this distance δx, the change in the kinetic energy of the fluid due to the wave is

$$\delta KE = \frac{1}{2} M\ (V + \delta V)^2 - \frac{1}{2} M\ V^2$$

$$= \frac{1}{2}\ \frac{w}{g}\ A\ (\delta x)\ (V^2 + 2V\,\delta V + \delta V^2 - V^2)$$

$$= \frac{1}{2}\ \frac{w}{g}\ A\ (\delta x)\ \ (2V\,\delta V + \delta V^2) \qquad (3)$$

Since the change was infinitesimal

$$\delta V << V$$

and as a result

$$\delta V^2 \cong 0$$

and

$$\delta KE = \frac{w}{g}\ A\ (\delta x)\ \ (V\ \ \delta V) \qquad (4)$$

As a decrease in kinetic energy is to be considered positive, a minus sign must be added to make the energy and power outputs of the wave of correct sign. Therefore, the energy output of the wave due to the change in kinetic energy is

$$\delta E_K = -\ \delta KE = -\frac{w}{g}\ A\ (\delta x)\ \ (V\ \delta V) \qquad (5)$$

The change in stored energy (see figure 2) in distance δx consists of two parts: a) the work done in expanding the penstock and b) the work done in compressing the fluid.

a) The work done in expanding the penstock

From Hookes law

$$\delta R = \frac{w R^2}{s E} \delta H$$

The work done on the pipe wall in expanding from R to R + δR is equal to the strain energy increase S_e stored in the pipe wall. For infinitesimal values of δH and δR this is

$$S_e = F \delta R = w\, 2 \pi R \;\delta x\; H \;\frac{w R^2}{s E} \delta H$$

$$S_e = \frac{w^2\, 2 \pi R\, (\delta x)\, R^2}{s E} \;(H \;\delta H) \qquad (6)$$

b) The work done in compressing the fluid

In the length δx the length of the water column is changed by δx^1. From Hookes law

$$\delta x^1 = \frac{w}{K} (\delta x)\; \delta H$$

The work done, S_f, in compressing the fluid is equal to the strain energy increase stored in the fluid. For infinitesimal values of δx^1 and δH this is

$$S_f = F \;\delta x^1 = w \pi R^2 H \frac{w}{K} (\delta x) \;\delta H$$

$$S_f = w^2 \pi R^2 \frac{\delta x}{K} \;(H \;\delta H) \qquad (7)$$

The total change in strain energy in distance δx resulting from the wave is

$$S_T = S_e + S_f \qquad (8)$$

By rearranging and adding equations 6 and 7 we obtain

$$S_T = \left\{ w^2 \pi R^2 \;(\delta x)\, \frac{2\,R}{s\;E} + w^2 \pi R^2 \;(\delta x)\, \frac{1}{K} \right\} (H \;\delta H)$$

Further rearrangement and multiplication by $\frac{g^2}{g^2}$ yields

$$S_T = \frac{w}{g} \pi \;R^2 \;\; \delta x \;\left\{ \left(\frac{2\,R}{s\;E} + \frac{1}{K} \right) \frac{w}{g} \right\} g^2 \;\; (H \;\delta H)$$

But by Allievi the wave velocity is

$$a = \frac{1}{\sqrt{\frac{w}{g}\left(\frac{2\,R}{s\;E} + \frac{1}{K}\right)}}$$

and therefore

$$\left(\frac{2\,R}{s\;E} + \frac{1}{K}\right) \frac{w}{g} = \frac{1}{a^2}$$

Furthermore the cross-sectional area of the penstock is

$$A = \pi \;R^2$$

Substitution of these values yields

$$S_T = \frac{w}{g} \;\; A \;(\delta x) \;\; \frac{g^2}{a^2} \;\; (H \;\delta H) \qquad (9)$$

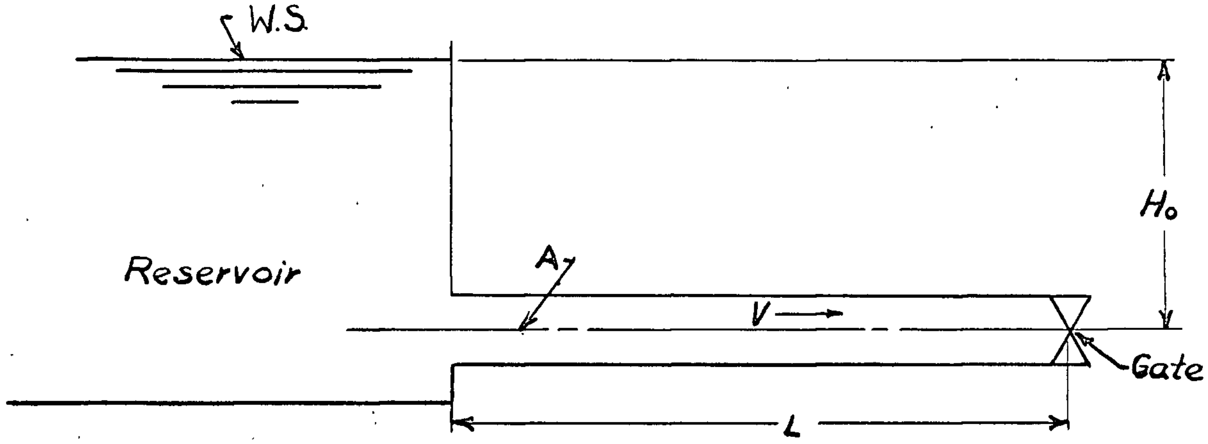


FIG 1 THE BASIC HYDRAULIC SYSTEM

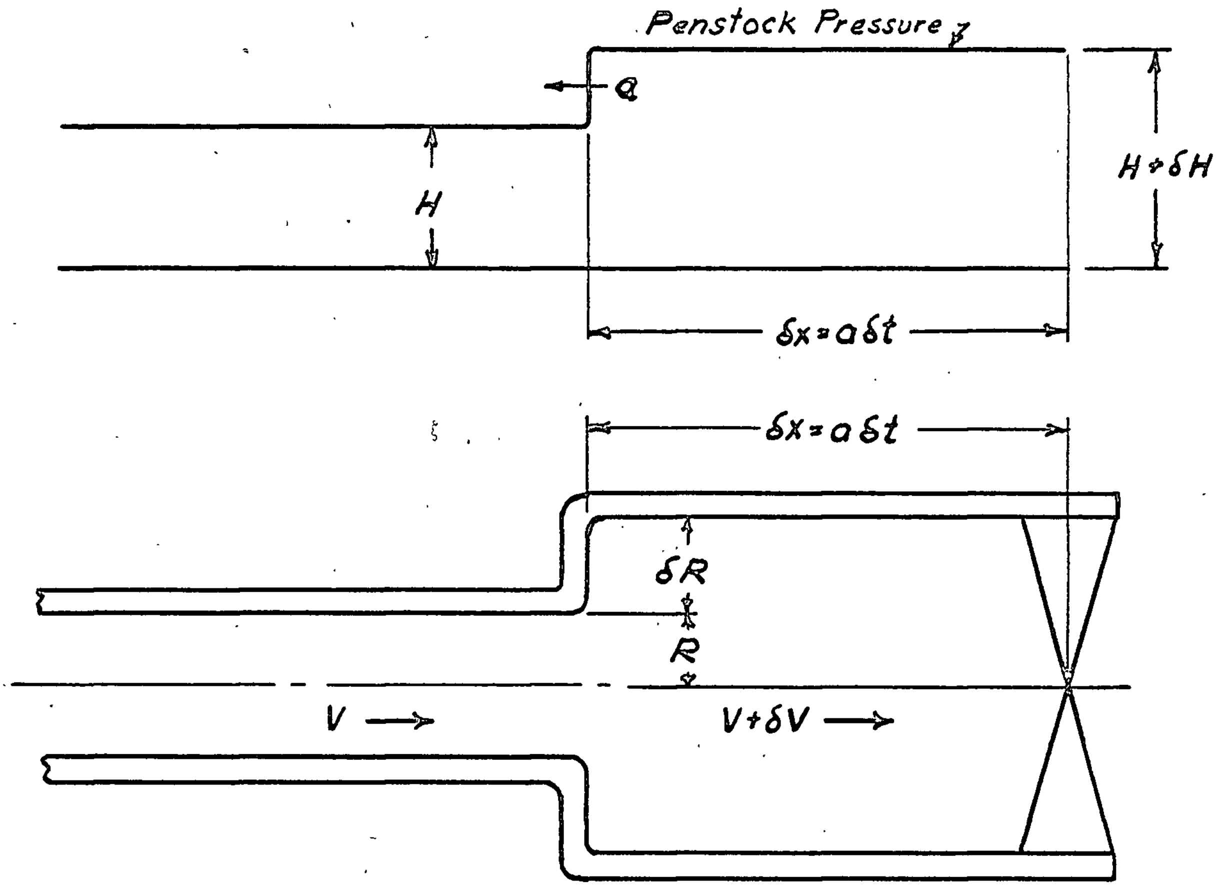


FIG 2 CONDITIONS IN THE PENSTOCK RESULTING FROM AN INFINITESIMAL WAVE

With a decrease in strain energy considered positive, the energy output of the wave due to the change in strain energy is

$$\delta E_P = - S_T = - \frac{w}{g} A \quad (\delta x) \quad \frac{g^2}{a^2} \quad (H \delta H) \tag{10}$$

The total energy output of the wave is

$$\delta E_c = \delta E_P + \delta E_K \tag{11}$$

Substitution of the results of equations 5 and 10 into equation 11 yields

$$\delta E_c = -\frac{w}{g} A \; (\delta x) \quad (V \delta V + \frac{g^2}{a^2} H \delta H) \tag{12}$$

Substitution of $H = H_o h$ and $V = V_o v$ into equation 12 and rearrangement yields

$$\delta E_c = -\frac{w}{g} A \; (\delta x) \; H_o \; V_o \frac{g}{a} \left(\frac{h}{V_o} \frac{g}{a} \delta H + \frac{a}{g} \; \frac{v \; \delta V}{H_o}\right)$$

With $\qquad \delta V = - \frac{g}{a} \; \delta H$

and $\qquad \delta V = \quad V_o \delta v$

$$\delta v = - \frac{g}{a} \; \frac{\delta H}{V_o}$$

with $\qquad 2\rho = \dfrac{a \; V_o}{g \; H_o}$

it follows that

$$\delta E_c = \frac{w}{g} \; A \; H_o \; V_o \frac{g}{a} \; (h - 2\rho v) \; \delta v \; \; \delta x \tag{13}$$

The power output of the wave is

$$\frac{\delta E_c}{\delta t} = \frac{w}{g} A H_o V_o \frac{g}{a} (h - 2\rho v) \quad \delta v \quad \frac{\delta x}{\delta t}$$

But

$$\frac{\delta x}{\delta t} = a$$

and therefore

$$\frac{\delta E_c}{\delta t} = P_W = w A H_o V_o (h - 2\rho v) \quad \delta v \qquad (14)$$

Consider now the graphical solution to a waterhammer problem shown in figure 3. An incremental wave created at the gate when conditions there are represented by point "b" in the figure, will, as it travels towards the reservoir, encounter head and velocity relationships that fall on the line b-c at all times. The slope of the line b-c is 2ρ and therefore the equation of the line must be

$$h = 2\rho v + \text{constant (C)}$$

or

$$h - 2\rho v = C \qquad (15)$$

In equation 14 δv is a constant, since

$$\delta v = - \frac{g}{a} \quad \delta h$$

and δh is unchanged as the wave travels along the penstock of constant

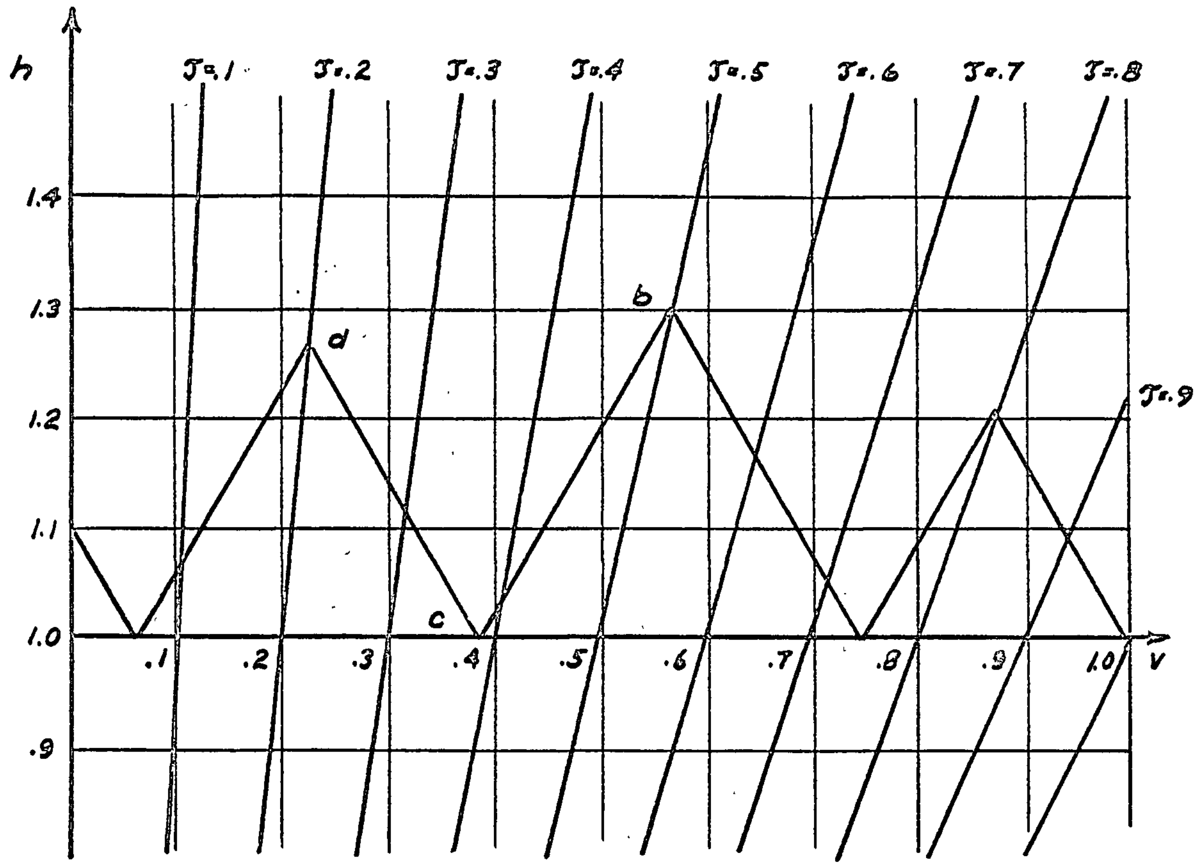


FIG 3 A TYPICAL GRAPHICAL SOLUTION TO A WATERHAMMER PROBLEM

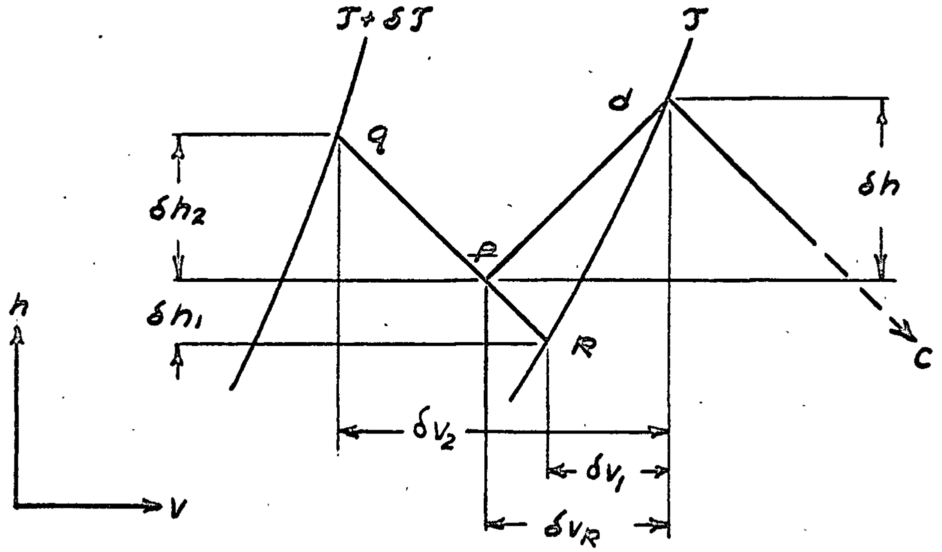


FIG 4 AN ENLARGED VIEW OF THE WATERHAMMER CHART SHOWING THE CREATION OF A WAVE

wall thickness. Therefore substitution of equation 15 into equation 14 yields

$$P_W = w\, A \cdot H_o\, V_o\, C\, \delta v = \text{constant} \tag{16}$$

The net difference between the change in kinetic energy and the change in strain energy resulting from the wave travelling toward the reservoir is a constant at any point in the penstock.

At the reservoir the conditions are represented by point "c" figure 3. If the time interval δt after the wave is reflected from the reservoir is made small enough so that no waves that have been created at the gate are within a distance $2\,\delta x = 2\,a\,\delta t$ of the reservoir, then within δx, H and V will be constant during the time interval δt. In this distance δx the change in the kinetic energy of the fluid due to the reflected wave is

$$\delta E_{KR} = -\frac{w}{g}\, A\, (\delta x)\, (V)\, \delta V \tag{17}$$

The energy output due to the change in strain energy is as before

$$\delta E_{PR} = -\frac{w}{g}\, A\, (\delta x)\, \frac{g^2}{a^2}\, H\, \delta H \tag{18}$$

The net energy output of the reflected wave is

$$\delta E_{cR} = \delta E_{PR} + \delta E_{KR}$$

By making the same substitutions as before, with the exception that for a reflected wave

$$\delta V = \frac{g}{a}\, \delta H$$

we obtain by the addition of equations 17 and 18

$$\delta E_{cR} = \frac{w}{g} \; A \; (\delta x) \; \frac{g}{a} \; H_o \; V_o \; (-h - 2\rho v) \; \delta v \tag{19}$$

and

$$\delta P_{WR} = \frac{\delta E_{cR}}{\delta t} = w \; A \; H_o \; V_o \; (-h - 2\rho v) \; \delta v \tag{20}$$

The change in power input from the reservoir due to the arrival and reflection of the wave is

$$\delta P_R = w \; A \; H_o \; V_o \; (2h) \; \delta v \tag{21}$$

where $h = 1$.

The wave leaving the reservoir, where conditions are given by point "c", figure 3, will, from the principles of graphical analysis, encounter a pressure-velocity relationship that falls on line c-d in figure 3. The slope of this line is $- 2\rho$ and therefore its equation is

$$- h = 2\rho v + K$$

or

$$- h - 2\rho v = K \tag{22}$$

Since this line and the line given by equation 15 pass through the common point "c", (figure 3) whose coordinates are

$$h = 1, \quad v = v$$

the value of K can be determined in terms of C. From equations 14 and 22,

$$2\rho v = C - 1$$
$$2\rho v = K + 1$$

and therefore $\qquad K = C - 2$

The equation of the line c-d is then

$$- h - 2\rho v = C-2 \tag{23}$$

Substituting equation 23 into equation 20,

$$P_{WR} = w\ A\ H_o\ V_o\ (C-2)\ \delta v \tag{24}$$

From the same reasoning as for the incident wave, P_{WR} must be a constant as the reflected wave returns to the gate.

Also $$P_W - P_{WR} = w\ A\ H_o\ V_o \left\{ C\ \delta v - (C-2)\ \delta v \right\}$$

$$P_W - P_{WR} = w\ A\ H_o\ V_o\ (2\delta v) \tag{25}$$

But if h=1, the condition for the reservoir, is substituted in equation 21 then:

$$\delta P_R = w\ A\ H_o\ V_o\ (2\delta v) = P_W - P_{WR} \tag{26}$$

Thus the difference in power outputs between the incident and reflected waves is exactly compensated by the change in power input from the reservoir, so that as far as the gate is concerned, the power output of the wave is a constant throughout its life span of $2\frac{L}{a}$ seconds. At the gate no change in input resulting from this particular wave is noticed until this reflected wave reaches the gate, at which point the wave ceases to exist and a new wave is formed.

Consider the wave as it arrives at the gate. Figure 4 shows an enlarged graphical solution of the problem for point "d" in figure 3. At the instant before the wave arrives h and v are given by point d. The instant the wave arrives the head is altered by δh and the velocity by

δv so that conditions at the gate are represented by point p. At this instant the wave ceases to exist and therefore its power output ceases also. However conditions at point p are unstable as the relative gate opening is $\mathcal{T}$ and the head and discharge must be related by $v = \mathcal{T}\sqrt{h}$. Therefore a new wave must be formed at the gate so that conditions are such as are given by "R" in figure 4; or, if at this instant an incremental closure $\delta\mathcal{T}$ takes place, by point "q" figure 4. In either case the new wave formed will produce a constant power output, as far as the gate is concerned, during its $\frac{2L}{a}$ life span.

The power increase (which may be of positive or negative sign) at the gate due to the cessation of the old wave and the creation of the new wave is given by the sum of the change in power due to the cessation of the old wave and the change in power due to the creation of the new wave and is

$$\delta P = P_{WN} - P_{WRO} \qquad (27)$$

(Where "N" denotes the new wave and "O" the wave that just ceased).

From equation 20 and figure 4 it follows that

$$-P_{WRO} = w\ A\ H_o\ V_o\ (h_d + 2\rho v_d)\quad \delta v_R \qquad (28)$$

If

$$h_d \cong h_p$$

and

$$v_d \cong v_p$$

then for a wave created by a reflection only it follows from equation 14 that

$$P_{WN} = w\ A\ H_o\ V_o\ (h_d - 2\rho v_d)\ (\delta v_1 - \delta v_R) \tag{29}$$

Substitution of equations 28 and 29 into equation 27 yields

$$\delta P = w\ A\ H_o\ V_o \left\{ (h_d - 2\rho v_d)\ (\delta v_1 - \delta v_R) + (h_d + 2\rho v_d)\,\delta v_R \right\}$$

or

$$\delta P = w\ A\ H_o\ V_o \left\{ (h_d - 2\rho v_d)\,\delta v_1 + 2\rho(2v_d)\,\delta v_R \right\} \tag{30}$$

But, $-2\rho\ \delta v_R$ is the pressure wave "δF" which originated from the gate $\frac{2L}{a}$ seconds earlier.

$$\delta P = w\ A\ H_o\ V_o \left\{ (h_d - 2\rho v_d)\ \delta v_1 - 2v_d\ (\delta F) \right\} \tag{31}$$

Similarly if an incremental closure takes place to point q figure 4

$$\delta P = w\ A\ H_o\ V_o \left\{ (h_d - 2\rho v_d)\ \delta v_2 - 2v_d\ (\delta F) \right\} \tag{32}$$

δv_1 and δv_2 are the net changes in velocity taking place at the gate at time t. Therefore neglecting subscripts and dividing equations 31 and 32 by δt we obtain

$$\frac{\delta P}{\delta t} = w\ A\ H_o\ V_o \left\{ (h - 2\rho v)\ \frac{\delta v}{\delta t} - 2v\ \frac{dF}{\delta t} \right\} \tag{33}$$

or in the limit as $t \rightarrow 0$

$$\frac{dP}{dt} = w\ A\ H_o\ V_o \left\{ (h - 2\rho v)\ \frac{dv}{dt} - 2v\ \frac{dF}{dt} \right\} \tag{34}$$

Note: This result can be obtained directly by the following derivation:

$$P = w\ A\ H_o\ V_o\ hv$$

$$\frac{dP}{dt} = w\ A\ H_o\ V_o\left\{h\ \frac{dv}{dt} + v\ \frac{dh}{dt}\right\}$$

$$\frac{dh}{dt} = -2\rho\frac{dv}{dt} - 2\ \frac{dF}{dt}$$

$$\frac{dP}{dt} = w\ A\ H_o\ V_o\left\{(h-2\rho\, v)\ \frac{dv}{dt} - 2v\ \frac{dF}{dt}\right\}$$

CHAPTER III

APPLICATION OF ENERGY PRINCIPLES TO THE DETERMINATION OF OPTIMUM GATE OPERATION

3.1 General

Consider the h-v diagrams shown in figures 5a and 5b and the corresponding plots of power output versus time shown in figures 6a and 6b. τ_S and τ_F are the initial and final gate positions. P_S and P_F are the initial and final steady state power outputs through the turbine gates. T_C is the total time of gate motion for a gate operation between τ_S and τ_F and T_F is the time for the power output to first cross the steady state P_F line. As this point is not reached until conditions at the gate are given by h_F, v_F then $T_F \geqslant T_C$.

The excess energy output resulting from a gate operation is represented by the shaded areas of figure 6, which are

$$\Delta E_g = \int_0^{T_F} (P - P_F)\, dt = \int_0^{T_F} P\, dt - P_F T_F \tag{35}$$

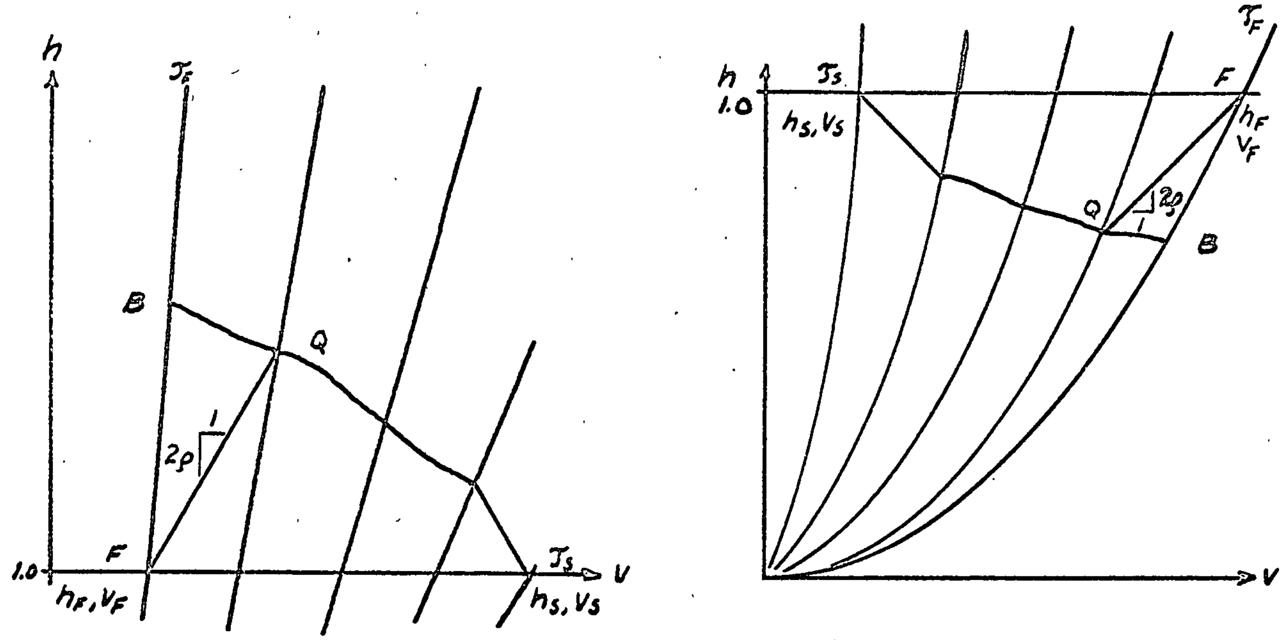
The derivative of equation 2 with respect to time yields

$$P = \frac{dE_g}{dt} = \frac{dE_i}{dt} + \frac{dE_c}{dt} \tag{36}$$

Therefore

$$\Delta E_g = \int_0^{T_F} \frac{dE_i}{dt}\, dt + \int_0^{T_F} \frac{dE_c}{dt}\, dt - P_F T_F \tag{37}$$

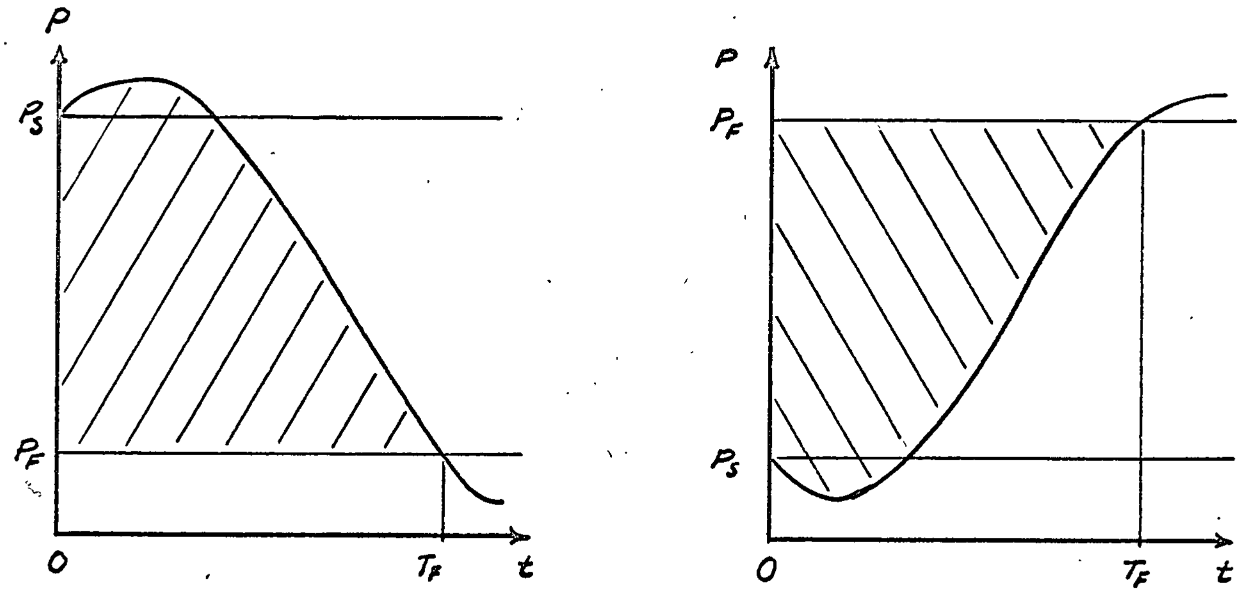
$$E_g = \Big[E_i + E_c \Big]_0^{T_F} - P_F T_F \tag{38}$$



a) Gate Closure

b) Gate Opening

FIG 5 TYPICAL GRAPHICAL SOLUTIONS TO WATERHAMMER PROBLEMS



a) Gate Closure

b) Gate Opening

FIG 6 POWER INPUT TO THE TURBINE DURING TRANSIENT CONDITIONS

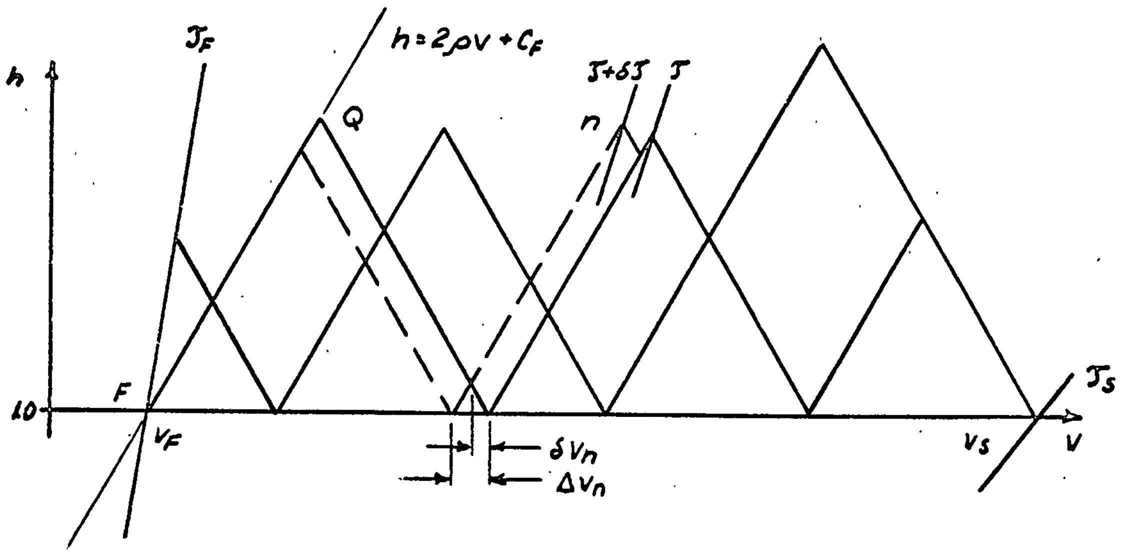
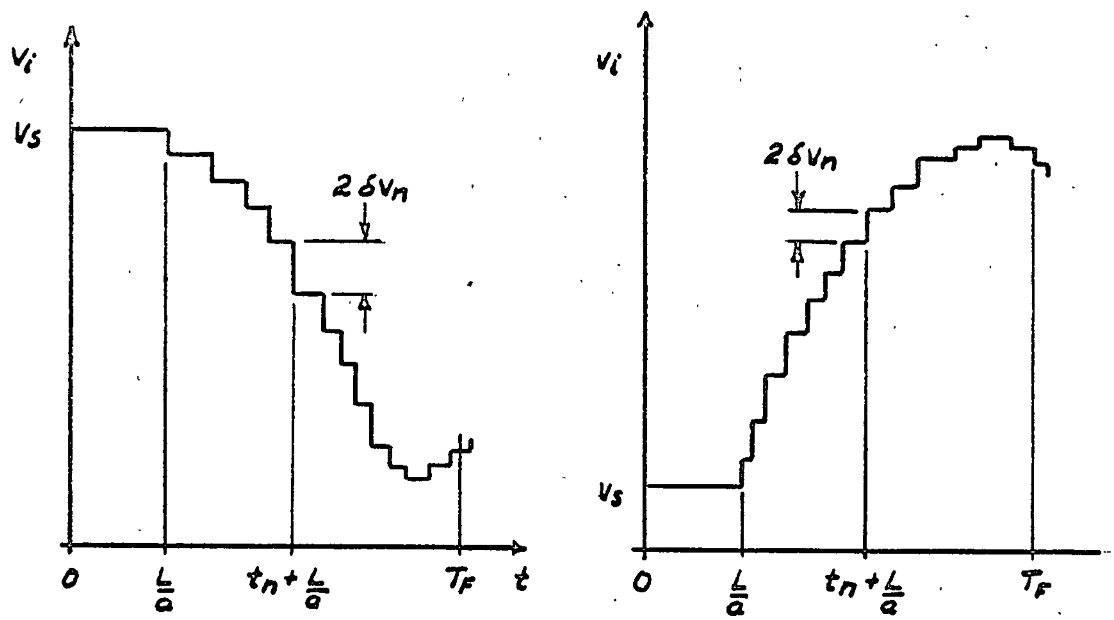


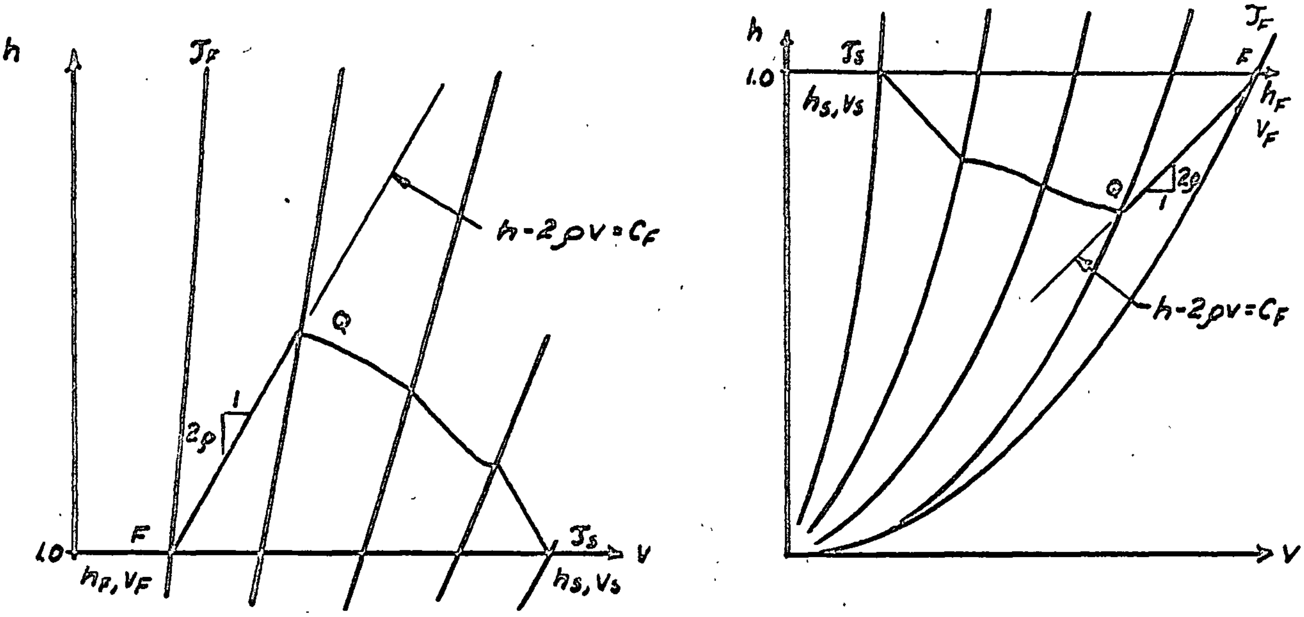
FIG 7 THE HEAD-VELOCITY CONDITIONS FOR AN INFINITESIMAL WAVE



a) Gate Closure

b) Gate Opening

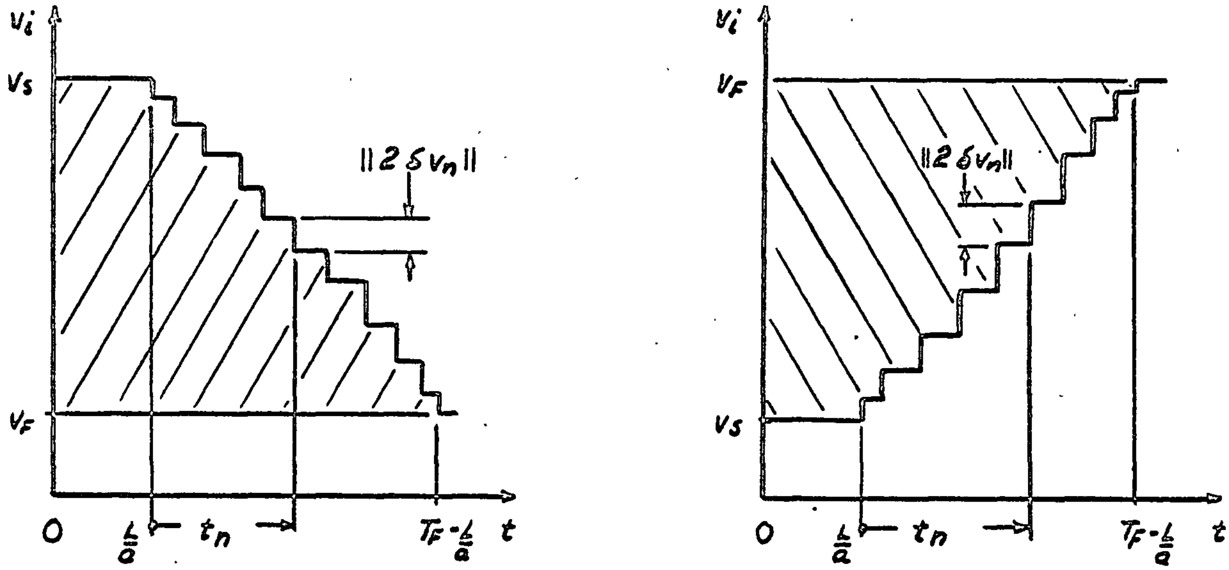
FIG 8 INPUT VELOCITY TO THE PENSTOCK



a) Gate Closure b) Gate Opening

FIG 9 GRAPHICAL SOLUTIONS TO WATERHAMMER PROBLEMS

$$\text{WHEN} \quad \sum_{n=Q+1}^{F} C_n \, \delta v_n \, (T_F - t_n) = 0$$



a) Gate Closure b) Gate Opening

FIG 10 PLOT OF INPUT VELOCITY TO THE PENSTOCK FOR THE

$$\text{CONDITION} \quad \sum_{n=Q+1}^{F} C_n \, \delta v_n \, (T_F - t_n) = 0$$

E_c is the total net energy output resulting from all waves originating at the gate between t=0 and $T=T_F$. If a continuous gate movement is approximated by a series of infinitesimal incremental movements then

$$E_c \Big]_0^{T_F} = \sum_{n=1}^{F} (E_W + E_{WR})_n$$

where for a wave travelling from the gate to the reservoir

$$E_{Wn} = P_{Wn} \ (t - t_n) \tag{39}$$

and for a wave travelling from the reservoir to the gate

$$E_{WRn} = P_{WRn} \ (t - (t_n + \frac{L}{a})) \tag{40}$$

t_n is the time of origin of the wave at the gate.

Note that

$$P_{Wn} = 0 \ \text{ if } \ (t - t_n) > \frac{L}{a}$$

and

$$P_{WRn} = 0 \ \text{ if } \ (t - (t_n + \frac{L}{a})) > \frac{L}{a}$$

(This follows from the original assumptions made in deriving P_W and P_{WR})

3.2 Evaluation of E_c between t=0 and $t=T_F$.

All the waves created up to point Q of figure 5 will have a full life span of $\frac{2L}{a}$ sec so that

$$E_{Wn} = P_{Wn} \ (\frac{L}{a}) \tag{41}$$

$$E_{WRn} = P_{WRn} \ (\frac{L}{a}) \tag{42}$$

Substitution of equations 14 and 20 into equations 41 and 42 yields

$$E_{Wn} = w\ A\ H_o\ V_o\ (h-2\rho\,v)\ \delta v_n \frac{L}{a} \tag{43}$$

$$E_{WRn} = w\ A\ H_o\ V_o\ (-h-2\rho\,v)\ \delta v_n \frac{L}{a} \tag{44}$$

If the constant terms $h-2\rho\,v$ and $-h-2\rho\,v$ are evaluated when $h=1$ i.e. at the reservoir, then

$$E_{Wn} + E_{WRn} = w\ A\ H_o\ V_o \frac{L}{a}\,(1-2\rho\,v - 1 - 2\rho\,v)\,\delta v_n$$

$$= w\ A\ H_o\ V_o \frac{L}{a}\,(-4\rho\,v)\ \delta v_n \tag{45}$$

Figure 7 shows an enlarged picture of the wave occurring at $t=t_n$. The total energy output of this wave is given by equation 45. But from figure 7,

$$\delta v_n = \frac{\Delta v_n}{2} \tag{46}$$

thus

$$\sum_{n=1}^{Q} (E_W + E_{WR})_n = w\ A\ H_o\ V_o \frac{L}{a} \sum_{n=1}^{Q} (-4\rho\,v)\ \frac{\Delta v_n}{2} \tag{47}$$

and

$$\lim_{n\to\infty} \sum_{n=1}^{Q} (-4\rho\,v)\ \frac{\Delta v_n}{2} = \int_{v_S}^{v_F} -2\rho\,v\ dv$$

$$= \rho\,(v_S{}^2 - v_F{}^2) \tag{48}$$

Substituting equation 48 into equation 47 and substituting $\rho = \dfrac{aV_o}{2gH_o}$

$$\sum_{n=1}^{Q} (E_W + E_{WR})_n = \frac{w}{2g} A L V_o^2 (v_S^2 - v_F^2) = \Delta KE \tag{49}$$

This is the total kinetic energy change of the fluid between the initial and final steady state conditions. Therefore the total energy output of all waves produced up to the waterhammer line of slope 2ρ passing through the final steady state v_F, h_F point is equal to the change in kinetic energy of the fluid in the penstock in going from the initial to the final steady state points and is independent of the way in which the gate is operated. This may be seen more easily by referring simultaneously to figures 7 and 4. If for each wave reaching the gate after $t = t_F - \frac{2L}{a}$ the gate is adjusted to point p of figure 4, no new waves will be created at the gate after $t = T_F - \frac{2L}{a}$ and the pressure-discharge relationship will fall along line Q-F of figure 7. At $t=T_F$, $P_i = \frac{dE_i}{dt} = P_g = \frac{dE_g}{dt}$

that is $h_i = h_g = h_o$, $v_i = v_g$

and therefore

$$\frac{dE_c}{dt} = 0$$

With $\frac{dE_c}{dt} = 0$ there can be no waves in the penstock and no additional strain energy. The total energy output must then be

$$E_g = E_i + E_c = E_i + \Delta KE$$

where ΔKE is the change in the kinetic energy of the fluid resulting from the velocity change from v_S to v_F. Therefore

$$E_c = \Delta KE = \sum_{n=1}^{Q} (E_W + E_{WR})_n$$

Now consider the energy output per wave of all the waves originating between $t=t_Q$ and $t = T_F - \frac{L}{a} = T_R$. All the waves produced in this time interval will be reflected by the intake but will not have enough time to return all the way to the gate. Therefore

$$E_{Wn} = w\ A\ H_o\ V_o\ (h-2\rho\, v)\ \delta v_n\ \frac{L}{a} \tag{50}$$

and

$$E_{WRn} = w\ A\ H_o\ V_o\ (-h-2\rho\, v)\ \delta v_n\ \left\{ T_F - (t_n + \frac{L}{a})\right\} \tag{51}$$

The total energy output up to time T_F due to waves originating in the time interval $t = T_Q$ to $t = T_R$ is then

$$\sum_{n=Q+1}^{R} (E_W + E_{WR})_n = \sum_{n=Q+1}^{R} w\ A\ H_o\ V_o\ (h-2\rho\, v)\ \delta v_n\ \frac{L}{a}$$

$$+ \sum_{n=Q+1}^{R} w\ A\ H_o\ V_o\ (-h-2\rho\, v)\,\delta v_n \left\{ T_F-(t_n + \frac{L}{a})\right\} \tag{52}$$

The energy output of each wave originating between T_R and T_F, none of which reach the reservoir is given by

$$E_{Wn} = w\ A\ H_o\ V_o\ (h-2\rho\, v)\ \delta v_n\ (T_F - t_n) \tag{53}$$

and

$$E_{WRn} = 0 \tag{54}$$

The total energy output of the waves originating in this time interval is then

$$\sum_{n=R+1}^{F} (E_W + E_{WR})_n = \sum_{n=R+1}^{F} w\ A\ H_o\ V_o\ (h-2\rho\, v)\ \delta v_n\ (T_F - t_n) \qquad (55)$$

The total energy output of the waves created in the time interval t = 0 to $t = T_F$ is then

$$E_c = \sum_{n=1}^{F} (E_W + E_{WR})_n$$

or

$$E_c = \sum_{n=1}^{Q} (E_W + E_{WR})_n + \sum_{n=Q+1}^{R} (E_W + E_{WR})_n + \sum_{n=R+1}^{F} (E_W + E_{WR})_n \qquad (56)$$

Substitution of the results of equations 49, 52 and 55 into equation 56 yields

$$E_c = \Delta KE + w\ A\ H_o\ V_o \Bigg\{ \sum_{n=Q+1}^{R} (h-2\rho\, v)\ \delta v_n\ (\tfrac{L}{a})$$

$$+ \sum_{n=Q+1}^{R} (-h-2\rho\, v)\ \delta v_n\ (T_F - (t_n + \tfrac{L}{a}))$$

$$+ \sum_{n=R+1}^{F} (h-2\rho\, v)\ \delta v_n\ (T_F - t_n) \Bigg\} \qquad (57)$$

Substituting for convenience the results of equations 15 and 23, which are,

$$C_n = h - 2\rho v$$

and

$$C_n - 2 = -h - 2\rho v$$

into equation 57 we get

$$E_c = \Delta KE + w\ A\ H_o\ V_o \left\{ \sum_{n=Q+1}^{R} C_n\ (\delta v_n)\ \frac{L}{a} \right.$$

$$+ \sum_{n=Q+1}^{R} (C_n - 2)\ \delta v_n\ (T_F - (t_n + \frac{L}{a}))$$

$$\left. + \sum_{n=R+1}^{F} C_n\ \delta v_n\ (T_F - t_n) \right\} \qquad (58)$$

Rearrangement of the term

$$\sum_{n=Q+1}^{R} (C_n - 2)\ \delta v_n\ (T_F - (t_n + \frac{L}{a}))$$

yields

$$\sum_{n=Q+1}^{R} (C_n - 2)\,\delta v_n\ (T_F - (t_n + \frac{L}{a})) = \sum_{n=Q+1}^{R} (-2)\,\delta v_n\ (T_F - (t_n + \frac{L}{a}))$$

$$+ \sum_{n=Q+1}^{R} C_n\ \delta v_n\ (T_F - t_n) + \sum_{n=Q+1}^{R} C_n\ \delta v_n\ (- \frac{L}{a}) \qquad (59)$$

Furthermore

$$\sum_{n=Q+1}^{R} c_n \delta v_n (T_F - t_n) + \sum_{n=R+1}^{F} c_n \delta v_n (T_F - t_n) = \sum_{n=Q+1}^{F} c_n \delta v_n (T_F - t_n) \tag{60}$$

Substitution of the results of equations 59 and 60 into equation 58 yields

$$E_c = \Delta KE + w\ A\ H_o\ V_o \left\{ \sum_{n=Q+1}^{R} (-2)\ \delta v_n \left(T_F - \left(t_n + \frac{L}{a}\right)\right) + \sum_{n=Q+1}^{F} c_n \delta v_n (T_F - t_n) \right\} \tag{61}$$

3.3 Evaluation of the energy input to the Penstock for the time interval $t = 0$ to $t = T_F$.

The wave originating at the gate with an associated velocity change δv_n causes a velocity change of $2\delta v_n$ at the reservoir. Therefore, referring to figure 8, the energy input to the penstock during the time interval $t = 0$ to $t = T_F$ is given by

$$E_i = w\ A\ H_o\ V_o \left\{ h_o\ v_S\ T_F + \sum_{n=1}^{Q} h_o\ (2\delta v_n) \left(T_F - \left(t_n + \frac{L}{a}\right)\right) + \sum_{n=Q+1}^{R} h_o\ (2\delta v_n) \left(T_F - \left(t_n + \frac{L}{a}\right)\right) \right\} \tag{62}$$

Note that any waves leaving the gate after $t = T_R = T_F - \frac{L}{a}$ will have no effect on the input to the penstock during the time interval $t = 0$ to $t = T_F$. As a result, the last term of equation 62 is only summed to $n = R$. Remember too that for gate closure δv_n is negative.

The first term of equation 62 yields

$$w\ A\ H_o\ V_o\ (h_o\ v_S)\ T_F = P_S\ T_F \tag{63}$$

Furthermore, referring to figure 7 note that

$$\sum_{n=1}^{Q} 2\ \delta v_n = v_F - v_S \tag{64)a}$$

(This means that the final steady state velocity is first reached at the reservoir at time $T_F - \frac{L}{a}$). Using this result we obtain

$$w\ A\ H_o\ V_o \sum_{n=1}^{Q} h_o\ 2\,\delta v_n\ (T_F - (t_n + \frac{L}{a}))$$

$$= w\ A\ H_o\ V_o \left\{ h_o\ (T_F - \frac{L}{a}) \sum_{n=1}^{Q} 2\ \delta v_n + \sum_{n=1}^{Q} 2h_o\,\delta v_n\ (-t_n) \right\}$$

$$= w\ A\ H_o\ V_o \left\{ h_o\ (T_F - \frac{L}{a})\ (v_F - v_S) + \sum_{n=1}^{Q} h_o\ 2\,\delta v_n\ (-t_n) \right\}$$

$$= (P_F - P_S)\ (T_F - \frac{L}{a}) + w\ A\ H_o\ V_o \sum_{n=1}^{Q} h_o\ 2\,\delta v_n\ (-t_n) \tag{64b}$$

Letting $h_o = 1$, which is the condition at the reservoir, and substituting the results of equations 63 and 64b into equation 62 yields

$$E_i = P_S T_F + (P_F - P_S)\left(T_F - \frac{L}{a}\right) + w\,A\,H_o\,V_o \left\{ \sum_{n=1}^{Q} 2\,\delta v_n\,(-t_n) + \sum_{n=Q+1}^{R} 2\,\delta v_n \left(T_F - \left(t_n + \frac{L}{a}\right)\right) \right\}$$

or

$$E_i = (P_S - P_F)\frac{L}{a} + P_F T_F + w\,A\,H_o\,V_o \left\{ \sum_{n=1}^{Q} 2\,\delta v_n\,(-t_n) + \sum_{n=Q+1}^{R} 2\,\delta v_n \left(T_F - \left(t_n + \frac{L}{a}\right)\right) \right\} \qquad (65)$$

3.4 Evaluation of the Excess Energy Output

The excess energy output ΔE_g can now be found by substituting the results of equations 61 and 65 into equation 38 which is

$$\Delta E_g = \Big[E_i + E_c \Big]_0^{T_F} - P_F T_F$$

and upon substitution yields

$$\Delta E_g = - P_F T_F + (P_S - P_F)\frac{L}{a} + P_F T_F + w\,A\,H_o V_o \left\{ \sum_{n=1}^{Q} 2\,\delta v_n\,(-t_n) + \sum_{n=Q+1}^{R} 2\,\delta v_n \left(T_F - \left(t_n + \frac{L}{a}\right)\right) \right\}$$

(cont'd)

$$+ \Delta KE + w\ A\ H_o\ V_o \left\{ \sum_{n=Q+1}^{R} (-2)\ \delta v_n\ (T_F - (t_n + \frac{L}{a})) \right.$$

$$\left. + \sum_{n=Q+1}^{F} C_n\ \delta v_n\ (T_F - t_n) \right\} \qquad (66)$$

Cancellation and rearrangement of terms yields the general equation for ΔE_g which is

$$\Delta E_g = (P_S - P_F)\ \frac{L}{a} + \Delta KE + w\ A\ H_o\ V_o \left\{ \sum_{n=1}^{Q} 2\ \delta v_n\ (-t_n) \right.$$

$$\left. + \sum_{n=Q+1}^{F} C_n\ \delta v_n\ (T_F - t_n) \right\} \qquad (67)$$

If the gate is operated so that during the interval $t_n = T_F - \frac{2L}{a}$ to $t_n = T_F$ no new waves are formed then the h-v diagram will be as shown in figure 9. In this case we have

$$\delta v_n \Big]_{Q+1}^{F} = 0$$

and therefore

$$\sum_{n=Q+1}^{F} C_n\ \delta v_n\ (T_F - t_n) = 0 \qquad (68)$$

Equation 67 is then minimized by minimizing the absolute value of (refer to figure 10)

$$\sum_{n=1}^{Q} 2\,\delta v_n\,(-t_n)$$

which is the shaded area of figure 10 and can be written as

$$\sum_{n=1}^{Q} \left\| 2\,\delta v_n \right\| (t_n)$$

All other terms of equation 67 are constant for a given gate operation between $\mathcal{T}_S$ and $\mathcal{T}_F$. (Note in the term

$$\sum_{n=1}^{Q} 2\,\delta v_n\,(-t_n)$$

the maximum value of t_n is

$$t_n = T_Q = T_F - \frac{2L}{a}$$

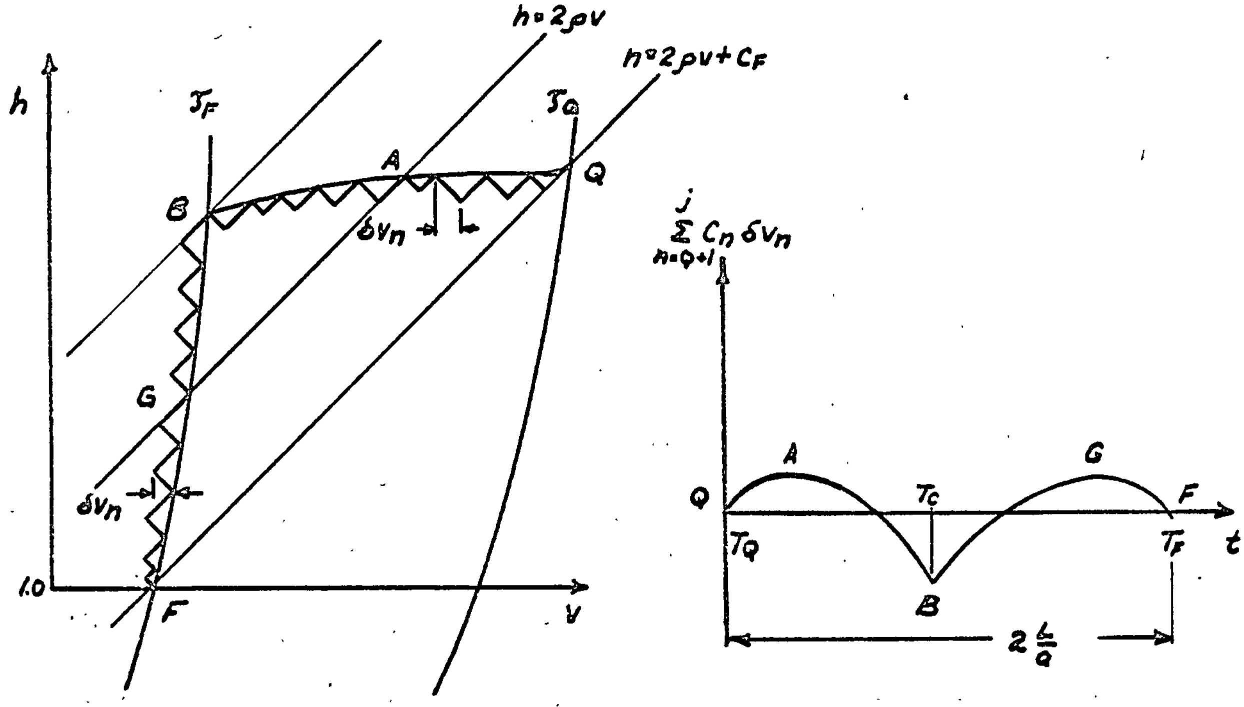
and for gate closure δv_n is negative. As a result

$$\sum_{n=1}^{Q} 2\,\delta v_n\,(-t_n)$$

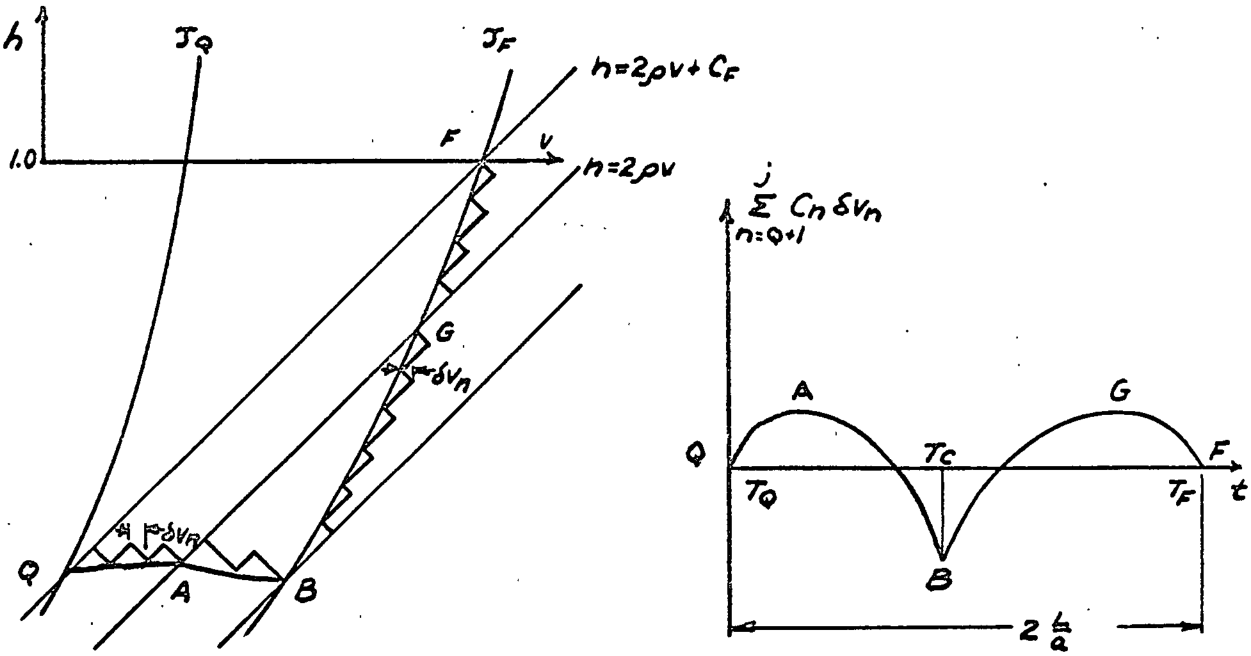
is positive). Obviously from figure 10 the term

$$\sum_{n=1}^{Q} \left\| 2\,\delta v_n \right\| (t_n)$$

is a minimum if $\left\| 2\,\delta v_n \right\|$ is as large as possible when t_n is as small

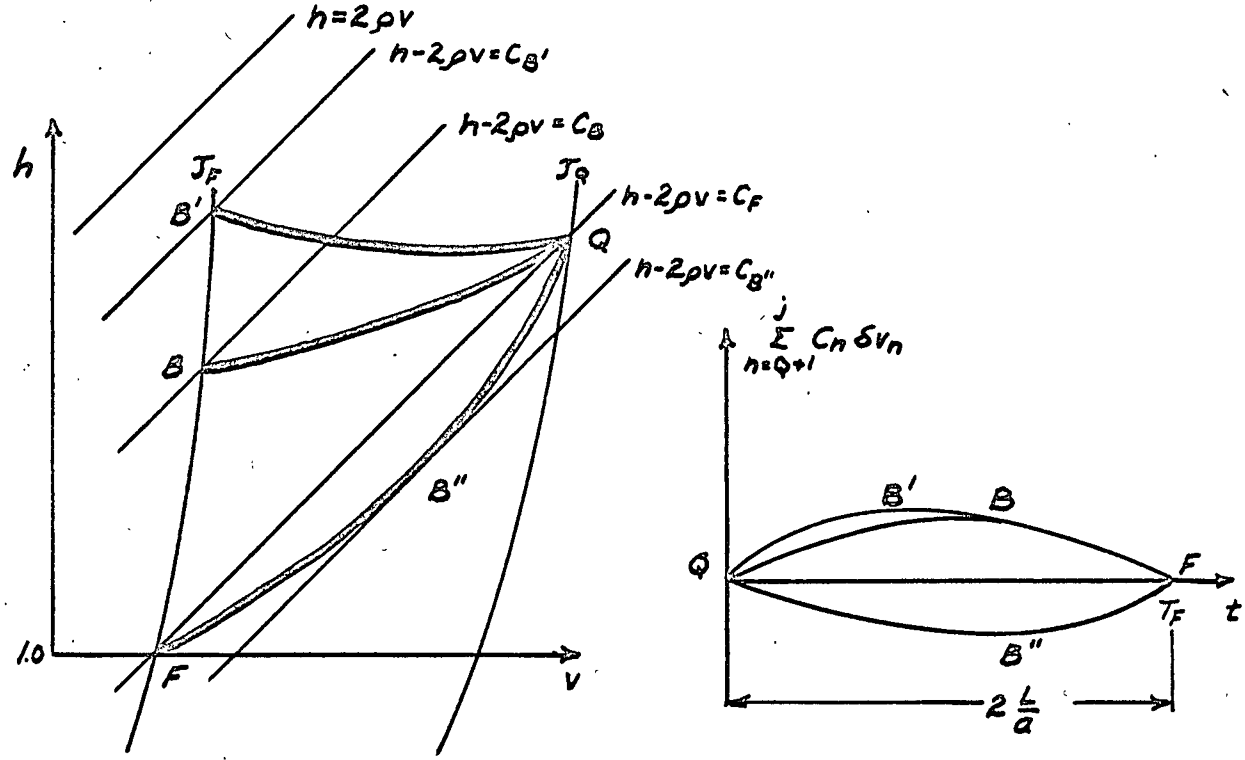


a) Gate Closure

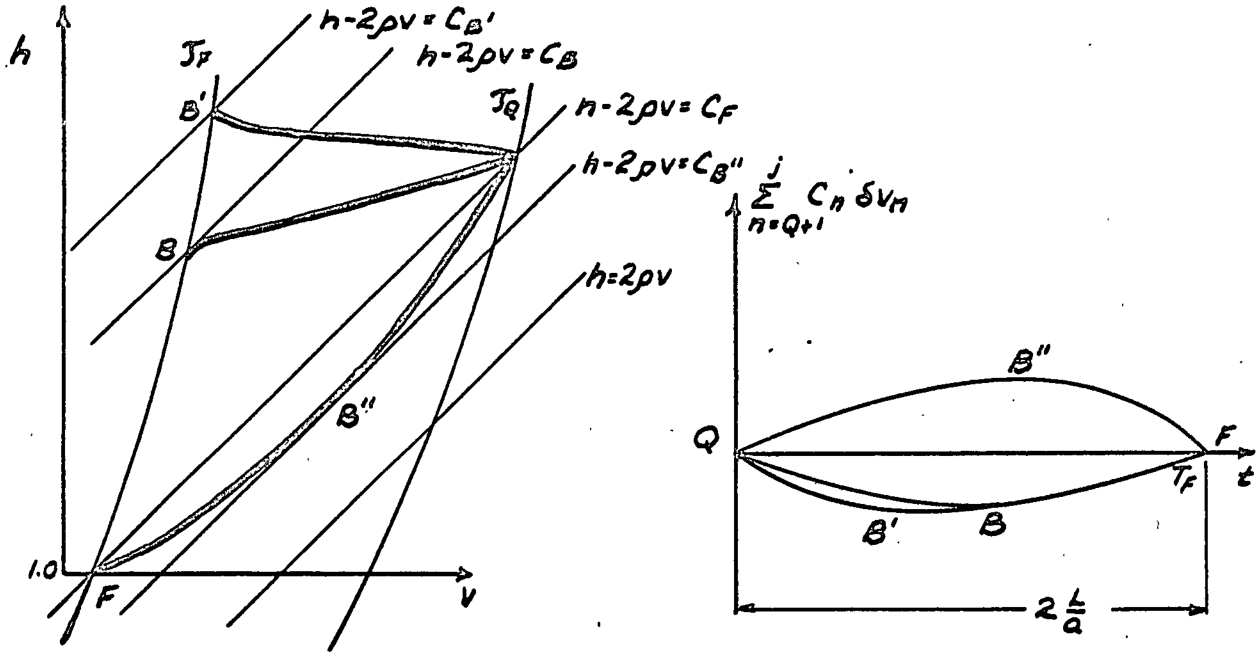


b) Gate Opening

FIG 11 WATERHAMMER CHARTS AND SURPLUS ENERGY PLOTS FOR THE TIME INTERVAL $t = T_Q$ TO $t = T_F$

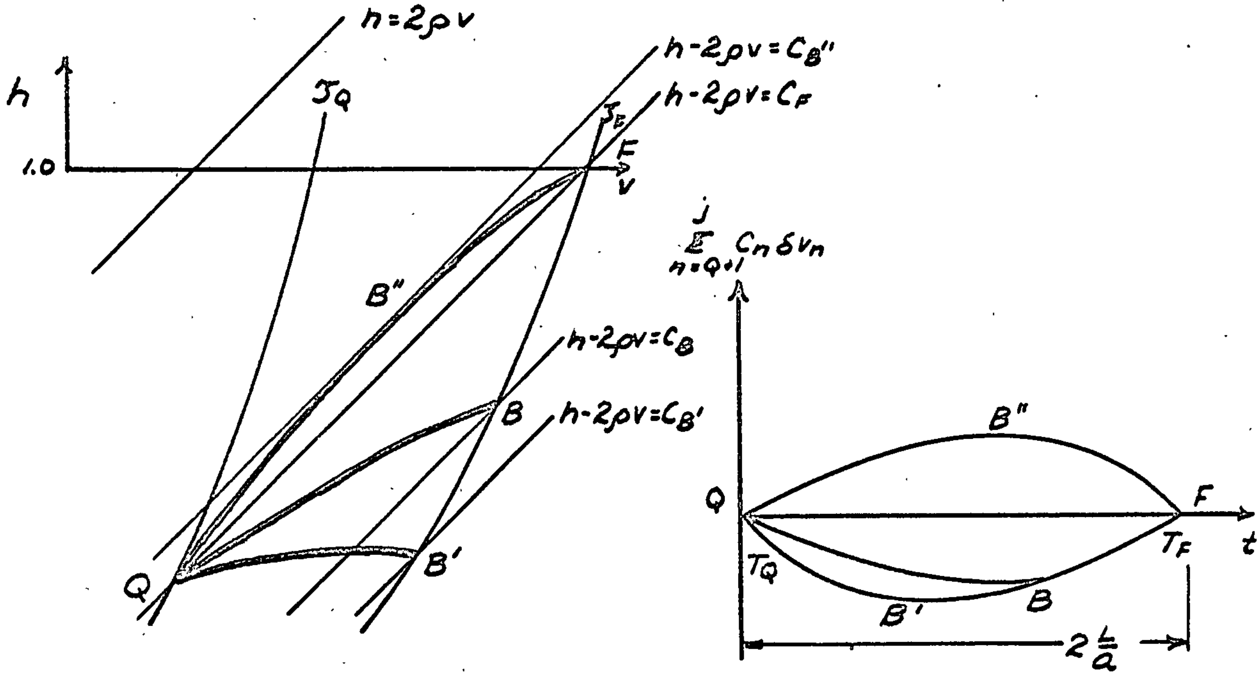


a) Line $h = 2\rho v$ to the Left of $\mathcal{J}_F$

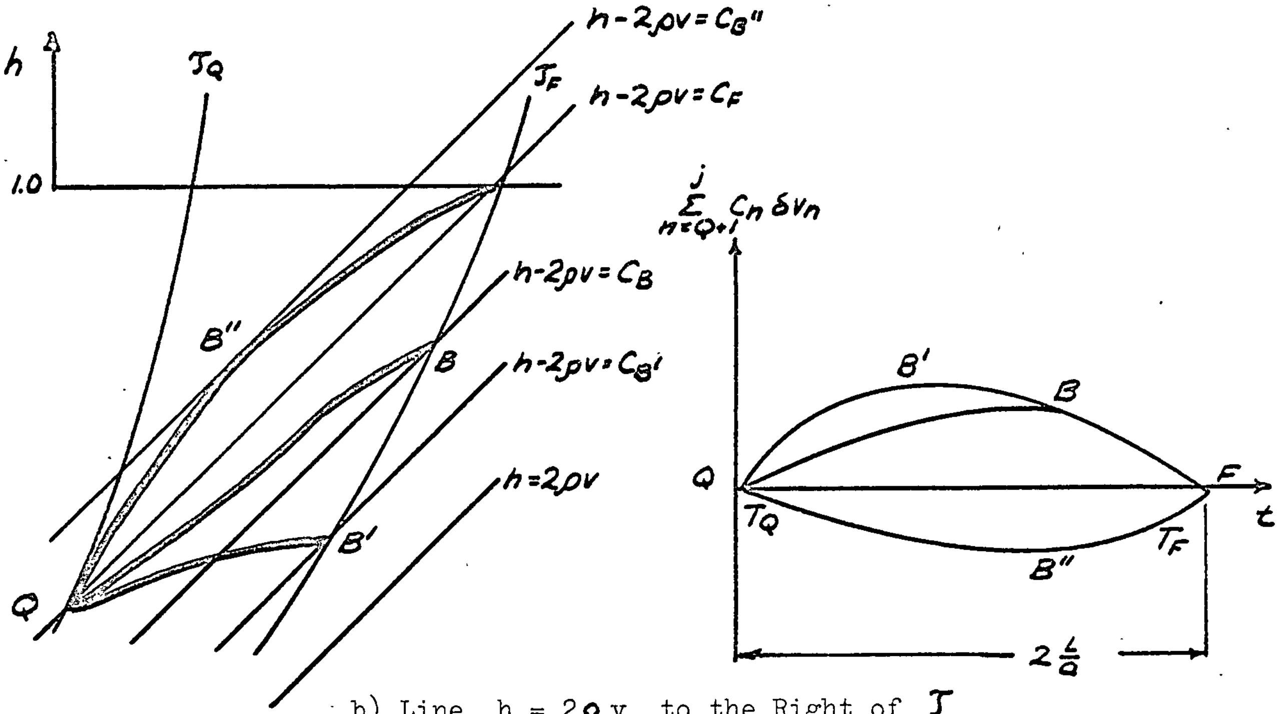


b) Line $h = 2\rho v$ to the Right of $\mathcal{J}_F$

FIG 12 WATERHAMMER CHARTS AND SURPLUS ENERGY PLOTS FOR THE TIME INTERVAL $t = T_Q$ TO $t = T_F$ - GATE CLOSURE



a) Line $h = 2\rho v$ to the Left of $\mathcal{T}_F$



b) Line $h = 2\rho v$ to the Right of $\mathcal{T}_F$

FIG 13 WATERHAMMER CHARTS AND SURPLUS ENERGY PLOTS FOR THE TIME INTERVAL $t = T_Q$ TO $t = T_F$ - GATE OPENING as possible since

$$\Delta v = - \frac{1}{2\rho} \Delta h$$

This means that the maximum velocity and head changes should be made as soon as possible for this form of gate operation.

Now consider the general equation 67 with:

$$\delta v_n \Big]_{n=Q+1}^{n=F} \neq 0$$

The only difference in E_g between this case and the case where:

$$\delta v_n \Big]_{n=Q+1}^{n=F} = 0$$

is the term $\sum_{n=Q+1}^{F} C_n \, \delta v_n \cdot (T_F - t_n)$. This term is just the sum of the energy outputs of all waves created between $t_n = T_Q$ and $t_n = T_F$, AS SEEN BY THE GATE. (Remember from the discussion of an incremental wave that the rate of energy output of a single wave is as far as the gate is concerned, a constant throughout its life span).

If $\sum_{n=Q+1}^{j} C_n \, \delta v_n$ is plotted against time, then changing the axis of integration yields

$$\int_0^{T_F} \left(\sum_{n=Q+1}^{j} C_n \delta v_n \right) dt \cong \sum_{n=Q+1}^{F} C_n \delta v_n \,(T_F - t_n) \qquad (69)$$

The qualitative results of such plots for several gate operations are shown in figures 11, 12 and 13. Note in these figures that the optimum method of gate operation after $t = T_Q$ depends upon the relative position of the line $h = 2\rho v$ so that in certain instances it is advantageous to reduce the rate of gate operation in the time interval $t = T_Q$ to $t = T_F$ if $\left\| \Delta E_g \right\|$ is to be kept to a minimum.

3.5 Ideal Gate Closure Curves

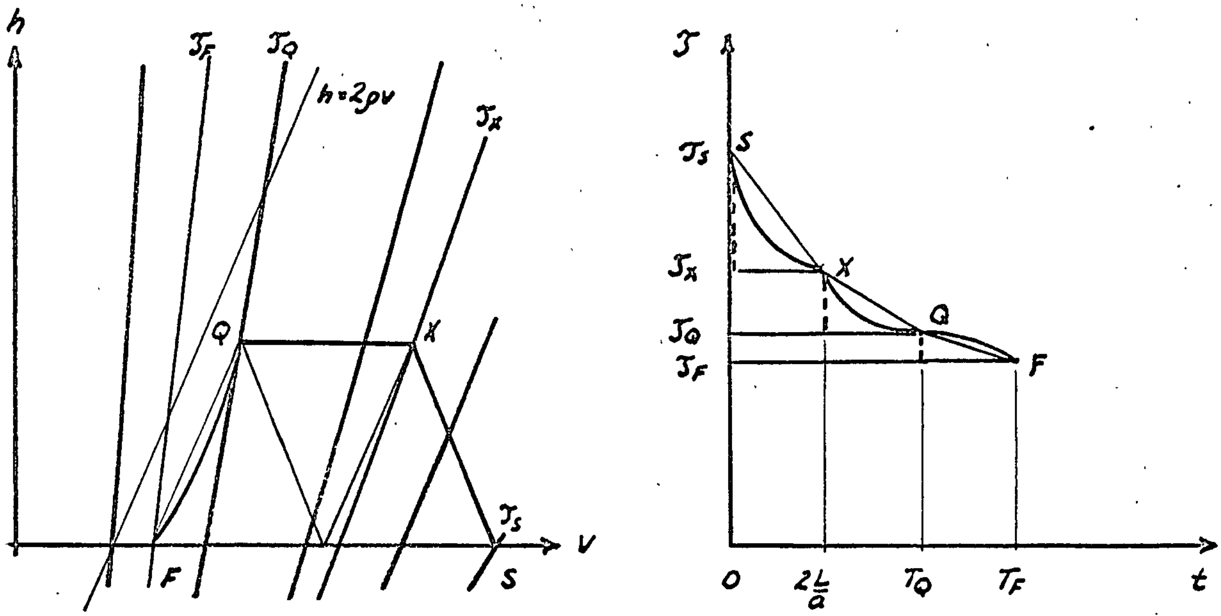
For gate closure, from the condition that the maximum head and velocity changes be made as soon as possible so as to reduce the energy input, the maximum allowable head (h_m) must be reached at $t = 2\frac{L}{a}$ and maintained at least until $t = T_Q$. This will minimize the term

$$\sum_{n=1}^{Q} 2\,\delta v_n\,(-t_n)$$

of equation 67 (refer to figure 10a). If the line $h = 2\rho v$ is to the left of point Q on the h-v diagram, then the term

$$\sum_{n=Q+1}^{F} C_n\,\delta v_n\,(T_F - t_n)$$

which is negative, is maximized by decreasing the rate of gate closure after $t = T_Q$. Figure 14 shows the h-v diagram and the gate closure curve for such a case. Note that to maintain a constant head at the gate between $t = 2\frac{L}{a}$ and $t = T_Q$, the closure curve in this time interval is specified by the closure curve in the time interval $t = 0$ to $t = 2\frac{L}{a}$ (i.e. point X specifies point Q etc.). If the extreme case of figure 14b is



a) Waterhammer Chart   b) Gate Closure Curve

FIG 14 WATERHAMMER CHART AND GATE CLOSURE CURVE FOR OPTIMUM GATE OPERATION

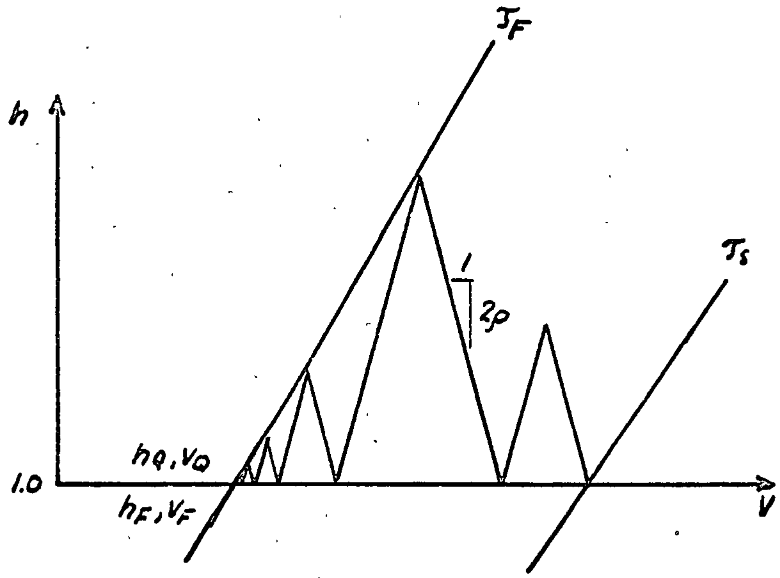


FIG 15 WATERHAMMER CHART FOR GATE CLOSURE: $2\rho > \left.\frac{dh}{dv}\right]_{\mathcal{T}_F}$

taken, it becomes a series of instantaneous closures taking place at $t = 0, \frac{2L}{a}, T_Q$.

As instantaneous closure is impossible and a closure of the form shown in figure 14b extremely difficult to duplicate, the most reasonable, form of gate closure would seem to be that made up of a series of straight line segments joining points S, X, Q, F of figure 14b. However this curve has the disadvantage that if closure should be started from a point other than $\mathcal{T}_S$, a higher than allowable pressure rise may result (6). It is possible to derive a gate closure curve for which the maximum waterhammer occurs at $t = \frac{2L}{a}$ and for all other times $h \leq h_m$. As the closures under consideration are always less than full gate (and never to $\mathcal{T} = 0$) this curve could be a reasonable solution.

3.6 Ideal Gate Opening Curves

For gate opening ΔE_g is a negative quantity so that to make $\| \Delta E_g \|$ as small as possible the term

$$\sum_{n=1}^{Q} 2\,\delta v_n\,(-t_n)$$

of equation 67, which is negative must be made as small as possible and the term

$$\sum_{n=Q+1}^{F} c_n\,\delta v_n\,(T_F - t_n)$$

should be made positive and as large as possible. With these factors in mind, results of similar form to gate closure may be obtained.

3.7 Ideal Gate Operation When $2\rho > \left(\frac{dh}{dv}\right)$ $\mathcal{T}$ = const.

For many cases the slope of the waterhammer line is greater than the slope of the $\mathcal{T}_F$ line. Such a case is shown in figure 15. Points Q and F coincide so that we have

$$\sum_{n=Q+1}^{F} c_n \delta v_n (T_F - t_n) = 0 \tag{70}$$

The general equation then reduces to

$$\Delta E_g = (P_S - P_F)\frac{L}{a} + \Delta KE + \sum_{n=1}^{Q} 2\,\delta v_n (-t_n) \tag{71}$$

From figure 15 it is obvious that the input velocity is reduced to the final steady state velocity by reaching $\mathcal{T}_F$ as rapidly as possible.

With gate opening the same results apply so that the minimum absolute value of ΔE_g is obtained by reaching $\mathcal{T}_F$ as rapidly as possible.

CHAPTER IV

EVALUATION OF DIFFERENT GATE OPERATION CURVES

4.1 Instantaneous Partial Closure

In figure 16b a very rapid gate closure is shown as a series of small increments. If Δt is very small then the closure can be considered instantaneous. The point Q is located as shown in figure 16a. For all the waves formed by the closure to $\mathcal{T}_F$, $t_n \cong \Delta t \cong 0$. For all the waves formed by the reflection of returning waves $t_n \cong \frac{2L}{a} + \Delta t \cong \frac{2L}{a}$. With $T_F = \frac{2L}{a}$ for instantaneous closure, we have in the general equation

$$\sum_{n=1}^{Q} 2\,\delta v_n\,(-t_n) \cong 0 \tag{73}$$

and

$$\sum_{n=Q+1}^{F} c_n\,\delta v_n\,(T_F - t_n) = \sum_{n=Q+1}^{B} c_n\,\delta v_n\,(T_F - t_n) + \sum_{n=B+1}^{F} c_n\,\delta v_n\,(T_F - t_n)$$

$$\cong \frac{2L}{a} \sum_{n=Q+1}^{B} c_n\,\delta v_n \tag{74}$$

The energy output of the waves formed between n = Q + 1 and n = B can be evaluated by referring to figure 16a and noting that on the line h = 1

$$\delta v_n = \frac{\Delta v_n}{2} \tag{75}$$

and

$$C_n = 1 - 2\rho v \tag{76}$$

From this we obtain

$$\frac{2L}{a} \sum_{n=Q+1}^{B} C_n \; v_n = \frac{2L}{a} \sum_{n=Q+1}^{B} (1-2\rho v) \frac{\Delta v}{2} \cong \frac{2L}{a} \int_{v_F}^{v_1} (1-2\rho v) \frac{dv}{2} \tag{77}$$

or

$$\frac{2L}{a} \sum_{n=Q+1}^{B} C_n \delta v_n \cong \frac{L}{a} \left[v - \rho v^2 \right]_{v_F}^{v_1} \tag{78}$$

v_1 is located as shown in figure 16a (Note that as $\Delta t \rightarrow 0$ equations 77 and 78 become exact. Substitution of the results of equations 73 and 78 into equation 67 yields

$$\Delta E_g = (P_S - P_F) \frac{L}{a} + \Delta KE + w \; A \; H_o \; V_o \frac{L}{a} \left[v - \rho v^2 \right]_{v_F}^{v_1} \tag{79}$$

Furthermore we have

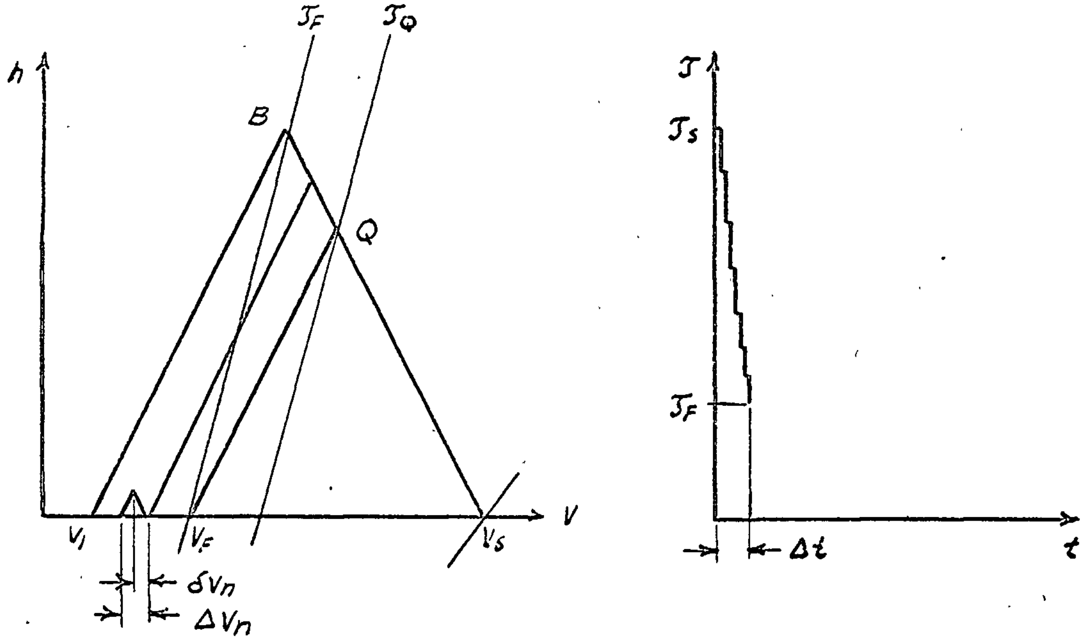
$$P_S = w \; A \; H_o \; V_o \; (h_o \; v_S) \tag{80}$$

$$P_F = w \; A \; H_o \; V_o \; (h_o \; v_F) \tag{81}$$

and

$$\Delta KE = \frac{1}{2} \frac{w}{g} \; A \; L \; V_o^2 \; (v_S^2 - v_F^2) \tag{82}$$

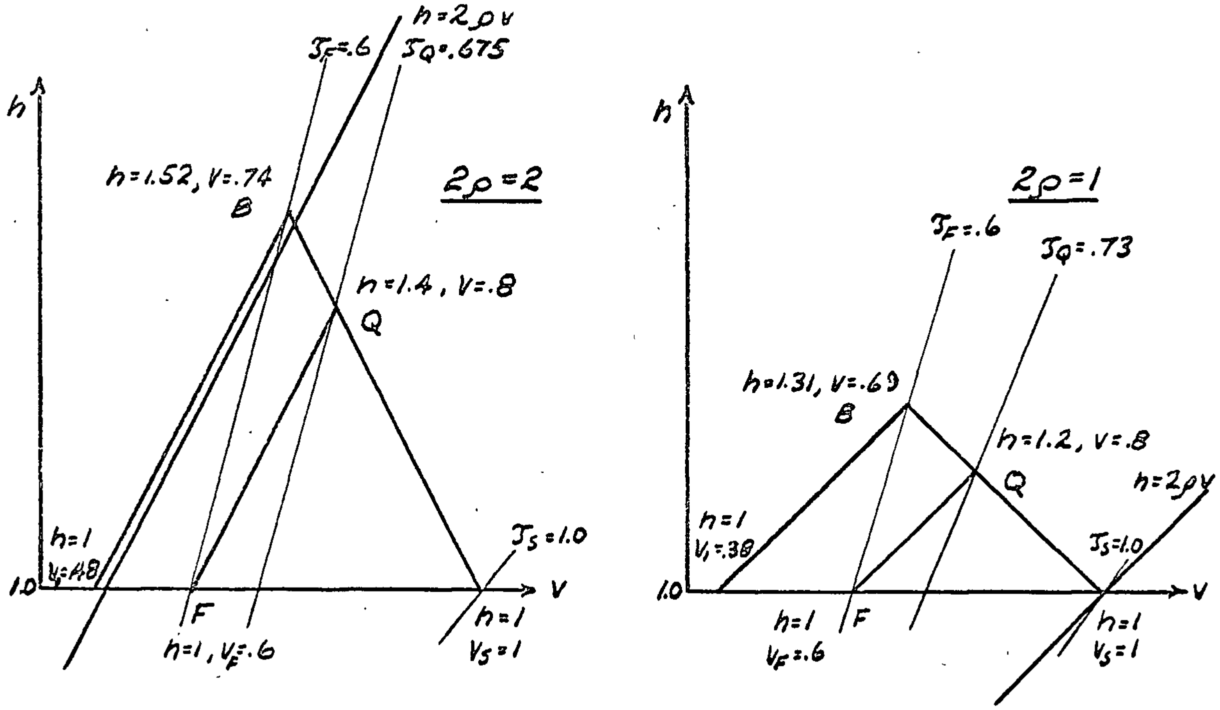
Substituting equations 80, 81 and 82 into equation 79 and letting $h_o = 1$



a) Waterhammer Chart b) Gate Closure Curve

FIG 16 WATERHAMMER CHART AND GATE CLOSURE CURVE

INSTANTANEOUS GATE CLOSURE



a) $2\rho = 2$ b) $2\rho = 1$

FIG 17 WATERHAMMER CHARTS FOR DIFFERENT VALUES OF ρ

- INSTANTANEOUS GATE CLOSURE

we get

$$\Delta E_g = w\ A\ H_o\ V_o\ \frac{L}{a}\ (v_S - v_F) + \frac{1}{2}\ \frac{w}{g}\ A\ L\ V_o^{\ 2}\ (v_S^{\ 2} - v_F^{\ 2})$$

$$+ \ w\ A\ H_o\ V_o\ \frac{L}{a}\ \Big[v - \rho v^2 \Big]_{v_F}^{v_1} \qquad (83)$$

Example

$A = 1/62.4\ ft^2$

$L = 3220$ ft

$a = 3220$ ft/sec

$\rho = 1$

$V_o = 20$ ft/sec

$H_o = 1000$ ft

$\mathcal{T}_S = v_S = 1$

$\mathcal{T}_F = v_F = .6$

For the given data the surplus energy resulting from an instantaneous closure will be calculated using equation 83. From the data

$$w\ A\ H_o\ V_o\ \frac{L}{a} = \frac{62.4}{62.4}\ 1000\ (20) = 20{,}000 \text{ ft lbs.}$$

From the h-v diagram shown in figure 17a, $v_1 = .48$. Substituting values into equation 83 yields

$$\Delta E_g = 20{,}000\ (1 - .6) \ + \ \frac{1}{2}\ \frac{62.4}{32.2}\ \frac{1}{62.4}\ 3220\ (20^2)\ (1^2 - .6^2)$$

$$+ \ 20{,}000\ \left\{ (.48 - .48^2) - (.6 - .6^2) \right\}$$

$$\Delta E_g = 8{,}000 + 12{,}800 + 200 = 21{,}000 \text{ ft-lbs}$$

As a check the power output during $t = 0$ to $t = \frac{2L}{a}$ is from figure 17a

$$\frac{2L}{a}\ P_B = w\ A\ H_o\ V_o\ (h_B\ v_B) = 2\ (20{,}000)\ 1.52\ (.74) = 45{,}000 \text{ ft lbs.}$$

The desired power output in this interval is given by

$$\frac{2L}{a} P_F = \frac{2L}{a} w \; A \; H_o \; V_o \; (h_F \; v_F) \; = 2 \; (20,000) \; .6 = 24,000 \text{ ft-lbs}$$

As a result we have

$$\Delta E_g = \frac{2L}{a} \; (P_B - P_F) = 45,000 - 24,000 = 21,000 \text{ ft-lbs}$$

If in the above example an instantaneous closure had been made to $\tau = \tau_Q = .675$ at $t = 0$ and from $\tau_Q = .675$ to $\tau_F = .60$ at $t = \frac{2L}{a}$ then no waves would be formed after point Q. This gives

$$\sum_{n=Q+1}^{B} c_n \; \delta v_n = 0$$

and as a result

$$\Delta E_g = 8,000 + 12,800 = 20,800 \text{ ft-lbs}$$

This amounts to about a 1% saving in ΔE_g, and a 25% reduction in pressure rise when compared to the first case. Furthermore there are no residual oscillations in the hydraulic system. In this example the line $h = 2\rho v$ was essentially to the left of the area representing the zone of gate operation, thus permitting the above mentioned savings.

<u>Example</u>

$$A = 2/62.4 \text{ ft}^2 \qquad V_o = 10 \text{ ft/sec}$$

$$\rho = .5$$

The remaining data is the same as the previous example. The h-v diagram for this case is shown in figure 17b.

Substituting into equation 83 we get

$$\Delta E_g = 20,000 \; (1 - .6) + 10,000 \; (1^2 - .6^2)$$

(cont'd)

$$+ 20,000 \left\{ (.38 - .5\ (.38)^2) - (.6 - .5\ (.6)^2) \right\}$$

$$\Delta E_g = 8,000 + 6,400 - 2,200 = 12,200 \text{ ft-lbs}$$

As a check, from figure 17b we get

$$\Delta E_g = \frac{2L}{a}\ (P_B - P_S) = 40,000\ (1.31(.69) - 1(.6)) = 12,200 \text{ ft-lbs.}$$

In this example if closure is made in two steps; τ_S to τ_Q at $t = 0$; and τ_Q to τ_F at $t = \frac{2L}{a}$ then the value of ΔE_g is increased by 16% i.e.

$$\Delta E_g = 8,000 + 6,400 + 0 = 14,400 \text{ ft-lbs.}$$

The effect of being to the left of the line $h = 2\rho v$ is very marked on the term $\sum_{n=Q+1}^{F} C_n\ \delta v_n\ (T_F - t_n)$.

In comparing the two examples which were for the same initial and final steady-state power outputs the effect of reducing the term ΔKE by reducing the initial velocity should be noted. Also the necessary increase in penstock area accompanying the velocity reduction, results in an increased capacity of the penstock to store energy thus resulting in further savings. These savings are accompanied by the disadvantage of hydraulic oscillations.

4.2 Gate Closure which Yields the Smallest Value of Maximum Waterhammer

This form of closure was developed by E. Ruus (6), and for a given pipe line constant ρ and closure time T_C gives the minimum possible value of maximum waterhammer. It is shown in the derivation (6) that the change in penstock water velocity at the turbine gate and at the intake

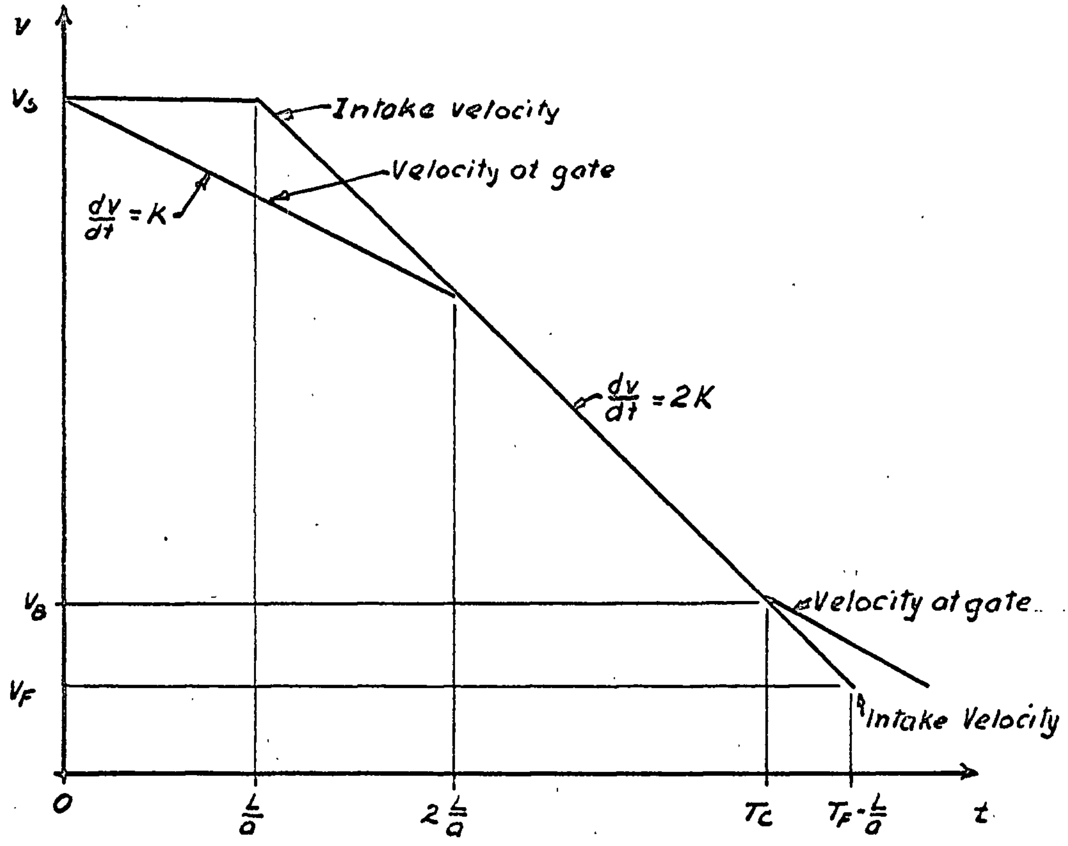


FIG 18 WATER VELOCITY AT THE INTAKE AND AT THE GATE

- OPTIMUM GATE CLOSURE

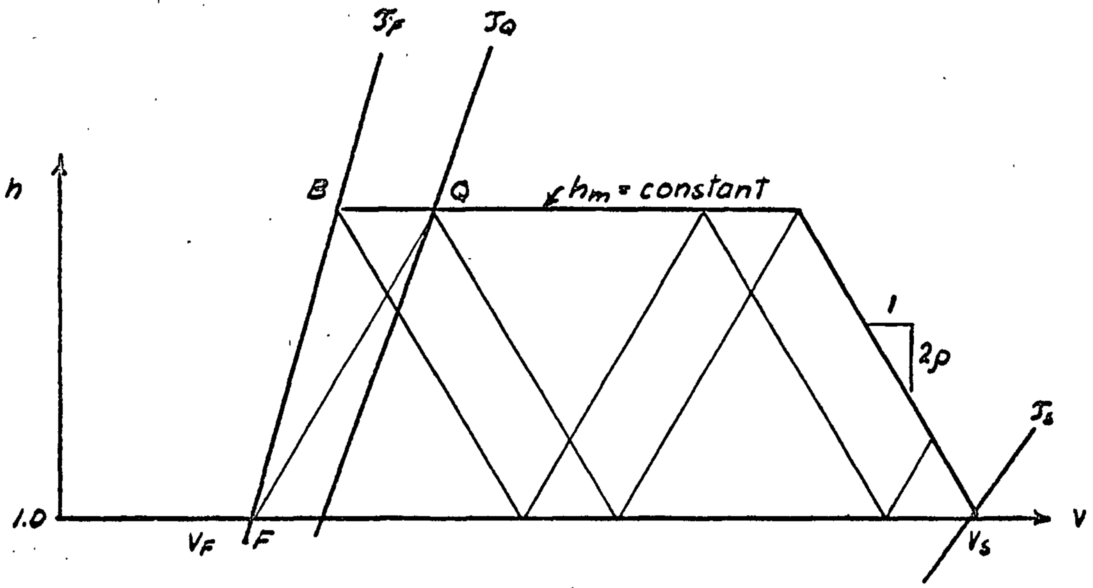


FIG 19 WATERHAMMER CHART FOR OPTIMUM GATE CLOSURE

vary in the time interval $t = 0$ to $t = T_C$ as shown in figure 18. The h-v diagram is shown in figure 19.

If for the general form of the waterhammer chart shown in figure 19, ρ and h_m are the specified variables, then the correct shape of closure curve can be obtained. This curve has many of the characteristics of the optimum curve for keeping ΔE_g as low as possible. A general derivation of this curve for partial gate movements is given below.

At the gate during the first $\frac{2L}{a}$ time interval (refer to figure 18), the rate of change of velocity is given by

$$\frac{dv}{dt} = K \tag{84}$$

The head-discharge relation for the gate is

$$v = \mathcal{T}\sqrt{h} \tag{85}$$

Furthermore from Allievi's first chain equation we have

$$h - 1 = -2\rho\,(v - v_S) \tag{86}$$

Combining the results of equations 85 and 86 we obtain

$$v = \mathcal{T}\sqrt{1 - 2\rho(v - v_S)} \tag{87}$$

Integration of equation 84 and substitution of the result into equation 87 yields

$$Kt + v_S = \mathcal{T}\sqrt{1 - 2\rho(Kt + v_S - v_S)} \tag{88}$$

or

$$\mathcal{T} = \frac{Kt + v_S}{\sqrt{1 - 2\rho Kt}} \tag{89}$$

K is determined from equation 86 by substituting $h = h_m$ at $t = \frac{2L}{a}$ and dividing the equation by $\frac{2L}{a}$. The result is

$$\frac{h_m - 1}{-2\rho\left(\frac{2L}{a}\right)} = \frac{v - v_S}{\frac{2L}{a}} = \frac{dv}{dt} = K$$

or

$$K = \frac{h_m - 1}{-4\rho\frac{L}{a}} \tag{90}$$

For the closure between time $t = \frac{2L}{a}$ and $t = T_C$ we have

$$v = \tau\sqrt{h_m} \tag{91}$$

and therefore

$$\frac{dv}{dt} = 2K = \frac{d\tau}{dt}\sqrt{h_m} \tag{92}$$

or

$$\frac{d\tau}{dt} = \frac{2K}{\sqrt{h_m}} \tag{93}$$

Integration of equation 93 with respect to time yields

$$\tau = \frac{2K}{\sqrt{h_m}}\left(t - \frac{2L}{a}\right) + \tau_1 \tag{94}$$

τ_1 is determined by substituting $t = \frac{2L}{a}$ into equation 89. The result yields

$$\tau_1 = \frac{K\frac{2L}{a} + v_S}{\sqrt{1 - 2\rho K\frac{2L}{a}}} = \frac{K\frac{2L}{a} + v_S}{\sqrt{h_m}} \tag{95}$$

Finally substitution of this result into equation 94 yields

$$\tau = \left(2Kt - 2K\frac{L}{a} + v_S\right)\frac{1}{\sqrt{h_m}} \tag{96}$$

The gate closure time T_C is determined as follows. From figure 18 the total velocity change at the gate in time T_C is given by

$$K \frac{2L}{a} + (T_C - \frac{2L}{a}) 2K = v_B - v_S \tag{97}$$

From figure 19 we have

$$v_B = \mathcal{T}_F \sqrt{h_m} \tag{98}$$

and

$$v_S = \mathcal{T}_S \sqrt{1} = \mathcal{T}_S \tag{99}$$

Combining the results of equations 97, 98 and 99 we obtain

$$T_C = \frac{\mathcal{T}_F \sqrt{h_m} - \mathcal{T}_S}{2K} + \frac{L}{a} \tag{100}$$

The general form of the closure curve as given by equations 89 and 96 is shown in figure 20. The results are general and apply to gate opening as well as gate closure. However, for each value of $\mathcal{T}_S$ the curve is different. (If closure were started from $\mathcal{T}(\frac{2L}{a})$ of figure 18, h_m would be exceeded (6).)

To evaluate ΔE_g for this form of gate operation it is necessary to know T_Q. From figure 19 we have

$$v_Q - v_F = \frac{1}{2\rho} (h_m - 1)$$

or

$$v_Q = \frac{1}{2\rho} (h_m - 1) + v_F \tag{101}$$

also

$$v_F = \mathcal{T}_F \tag{102}$$

$$v_Q = \mathcal{T}_Q \sqrt{h_m'} \tag{103}$$

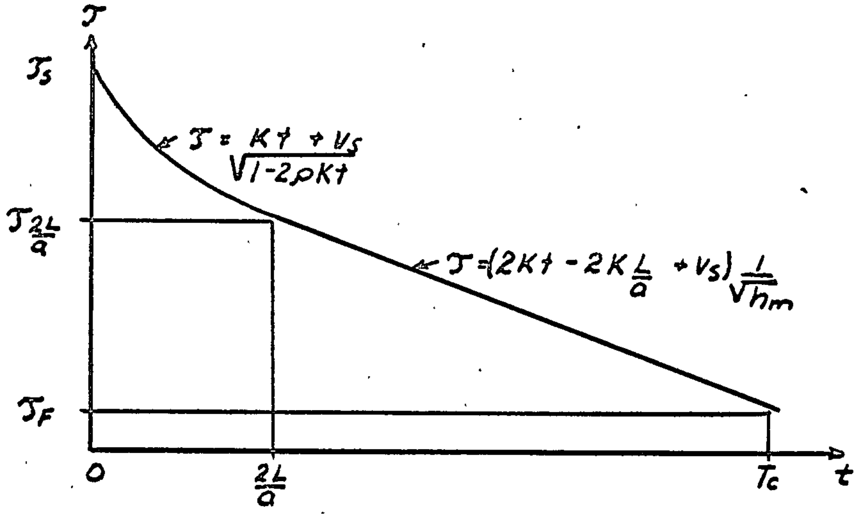


FIG 20 OPTIMUM GATE CLOSURE CURVE

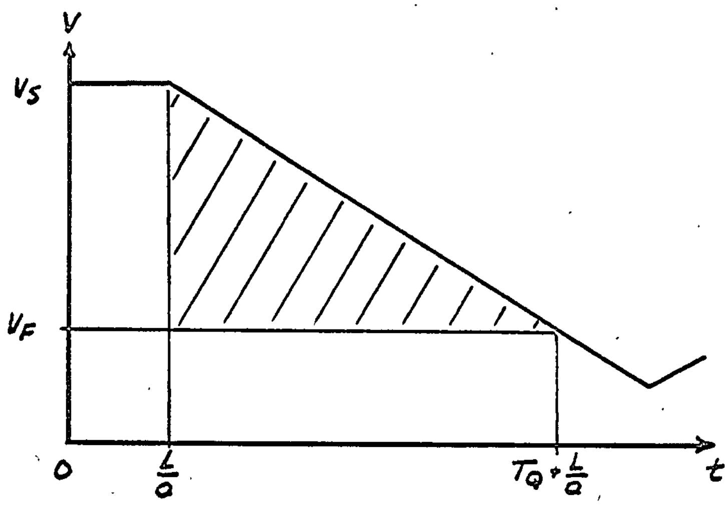


FIG 21 WATER VELOCITY AT THE INTAKE - OPTIMUM GATE OPERATION

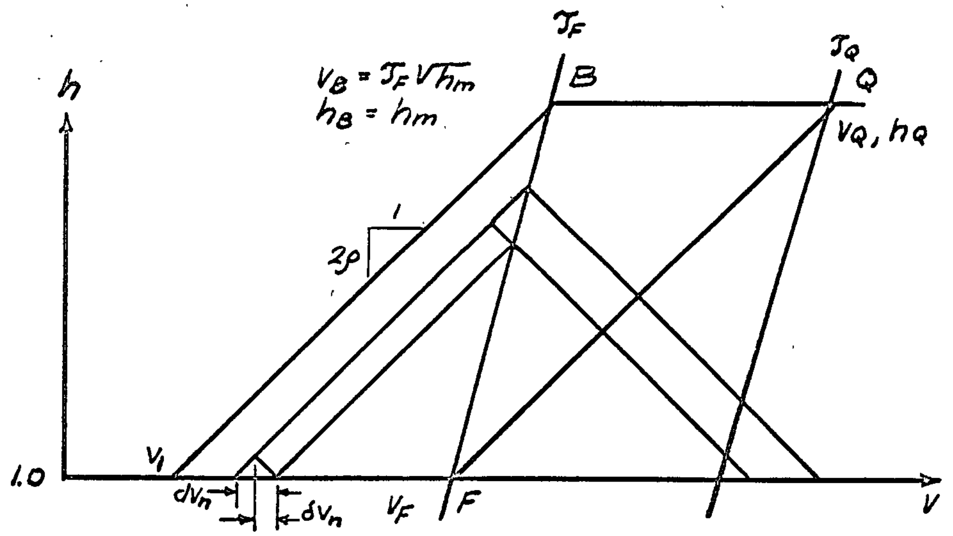


FIG 22 WATERHAMMER CHART FOR THE INTERVAL $t = T_Q$ TO $t = T_F$

- OPTIMUM GATE OPERATION

Combining the results of equations 101, 102 and 103 we obtain

$$\mathcal{T}_Q = \frac{1}{\sqrt{h_m}} \left(\frac{1}{2\rho} (h_m - 1) + \mathcal{T}_F \right) \tag{104}$$

But from equation 96 we have

$$\mathcal{T}_Q = (2\ K\ T_Q - 2\ K \frac{L}{a} + v_S) \frac{1}{\sqrt{h_m}} \tag{105}$$

Equating equations 104 and 105 we obtain

$$\frac{1}{2\rho} (h_m - 1) + \mathcal{T}_F = 2\ K\ T_Q - 2\ K \frac{L}{a} + v_S \tag{106}$$

or

$$\frac{1}{2\rho} \frac{h_m - 1}{2K} + \frac{\mathcal{T}_F - \mathcal{T}_S}{2K} = T_Q - \frac{L}{a} \tag{107}$$

Substituting

$$2K = \frac{h_m - 1}{2\rho \frac{L}{a}}$$

into the first term of equation 107 and rearranging we get

$$T_Q = \frac{\mathcal{T}_F - \mathcal{T}_S}{2K} \tag{108}$$

It is now possible to evaluate ΔE_g for this optimum form of gate operation. Consider the term

$$\sum_{n=1}^{Q} 2\ \delta v_n\ (-t_n)$$

of the general equation for ΔE_g. According to figure 21 this area is

$$\sum_{n=1}^{Q} 2\,\delta v_n\,(-t_n) = (v_S - v_F)\,\frac{T_Q}{2} \qquad (109)$$

(Recall from equation 64a that

$$\sum_{n=1}^{Q} 2\,\delta v_n = v_F - v_S \qquad)$$

Substituting equation 108 into 109 and letting

$$v_S = \mathcal{T}_S$$

$$v_F = \mathcal{T}_F$$

we obtain

$$\sum_{n=1}^{Q} 2\,\delta v_n\,(-t_n) = (\mathcal{T}_S - \mathcal{T}_F)\left(\frac{\mathcal{T}_F - \mathcal{T}_S}{4K}\right) = -\,\frac{(\mathcal{T}_S - \mathcal{T}_F)^2}{4K} \qquad (110)$$

The term

$$\sum_{n=Q+1}^{F} c_n\,\delta v_n\,(T_F - t_n)$$

of the general equation can be evaluated if it is assumed that $\mathcal{T}_F$ is a straight line between the points h_m, v_B and h_o, v_F. (For $\Delta h \cong .6$ this is a reasonable approximation.) This term can be written as

$$\sum_{n=Q+1}^{F} c_n\,\delta v_n\,(T_F - t_n) = \sum_{n=Q+1}^{B} c_n\,\delta v_n\,(T_F - t_n)$$

$$\sum_{n=B+1}^{F} c_n\,\delta v_n\,(T_F - t_n) \qquad (111)$$

The term

$$\sum_{n=Q+1}^{B} c_n \,\delta v_n \,(T_F - t_n)$$

can be evaluated along the line h = 1 (refer to figure 22) by considering the facts that

$$C_n = 1 - 2\rho v$$

$$\delta v_n = \frac{dv}{2}$$

Furthermore, since $\frac{dv}{dt} = K$, we have from geometry (refer to figure 22)

$$T_F - t_n = T_F - \left(T_Q + \frac{T_C - T_Q}{v_B - v_Q}\,(v - v_Q)\right) \tag{112}$$

Furthermore from geometry we have

$$v_1 - v_F = v_B - v_Q \tag{113}$$

$$v - v_F = v - v_Q \tag{114}$$

Combining equations 112, 113 and 114 we obtain

$$T_F - t_n = T_F - \left(T_Q + \frac{T_C - T_Q}{v_1 - v_F}\,(v - v_F)\right) \tag{115}$$

As a result we have

$$\sum_{n=Q+1}^{B} c_n \,\delta v_n \,(T_F - t_n) \cong \int_{v_F}^{v_1} (1 - 2\rho v)\left[T_F - \left(T_Q + \frac{T_C - T_Q}{v_1 - v_F}\,(v - v_F)\right)\right]\frac{dv}{2} \tag{116}$$

Similarly, since δv_n is a constant between n = B + 1 and n = F (as a result of the straight line approximation)

$$\sum_{n=B+1}^{F} C_n \delta v_n \, (T_F - t_n) \cong \int_{v_1}^{v_F} (1 - 2\rho v) \left\{ T_F - \left(T_C + \frac{T_F - T_C}{v_F - v_1} (v - v_1)\right)\right\} \frac{dv}{2} \tag{117}$$

Rearrangement of equation 117 and reversal of the limits of integration yields

$$\sum_{n=B+1}^{F} C_n \delta v_n \, (T_F - t_n) \cong -\int_{v_F}^{v_1} (1 - 2\rho v) \frac{T_F - T_C}{v_1 - v_F} (v - v_F) \frac{dv}{2} \tag{118}$$

Adding equations 116 and 118 and cancelling terms we get

$$\sum_{n=Q+1}^{F} C_n \delta v_n \, (T_F - t_n) \cong \frac{T_F - T_Q}{2} \; \frac{v_1}{v_1 - v_F} \int_{v_F}^{v_1} (1 - 2\rho v) \, dv - \frac{T_F - T_Q}{2\,(v_1 - v_F)} \int_{v_F}^{v_1} (1 - 2\rho v) \, v \, dv \tag{119}$$

Upon substitution of

$$T_F - T_Q = \frac{2L}{a}$$

the solution of equation 119 becomes

$$\sum_{n=Q+1}^{F} C_n \delta v_n \, (T_F - t_n) \cong \frac{L}{a} (v_1 - v_F) \left[\frac{1}{2} - \frac{\rho}{3} (2\, v_F + v_1) \right] \tag{120}$$

From figure 22 we have

$$v_F = \mathcal{T}_F \tag{121}$$

$$v_1 = \mathcal{T}_F \sqrt{h_m} - \frac{1}{2\rho} \quad (h_m - 1) \tag{122}$$

Substitution of the results of equations 121 and 122 into equation 120 yields

$$\sum_{n=Q+1}^{F} c_n \delta v_n (T_F - t_n) \cong \frac{L}{a} \left[\mathcal{T}_F \left(\sqrt{h_m} - 1 \right) - \frac{1}{2\rho} (h_m - 1) \right] \left[\frac{1}{2} - \frac{\rho}{3} \left(\mathcal{T}_F \left(\sqrt{h_m} + 2 \right) - \frac{1}{2\rho} (h_m - 1) \right) \right] \tag{123}$$

or

$$\sum_{n=Q+1}^{F} c_n \delta v_n (T_F - t_n) = \frac{L}{a} \left(\frac{\sqrt{h_m} - 1}{12\rho} \right) \left[2\rho \mathcal{T}_F - \sqrt{h_m} - 1 \right] \left[h_m + 2 - \mathcal{T}_F (2\rho) \left(\sqrt{h_m} + 2 \right) \right] \tag{124}$$

Equation 124 is valid for gate closure and gate opening of the form given by equations 89 and 96 provided

$$T_Q > 2 \frac{L}{a}$$

and

$$2\rho < \left. \frac{dh}{dv} \right]_{\mathcal{T}_F}$$

For the remaining terms in the general equation for ΔE_g we have

$$(P_S - P_F) \frac{L}{a} = w \ A \ H_o \ V_o \frac{L}{a} \quad (\mathcal{T}_S - \mathcal{T}_F) \tag{125}$$

and

$$\Delta KE = \frac{1}{2} \frac{w}{g} \quad A \ L \ V_o^2 \quad (\mathcal{T}_S^2 - \mathcal{T}_F^2) \tag{126}$$

or

$$\Delta KE = w\ A\ H_o\ V_o \frac{L}{a}\ (\rho)\ (\mathcal{T}_S^{\,2} - \mathcal{T}_F^{\,2}) \tag{127}$$

Furthermore upon substitution of

$$K = \frac{h_m - 1}{-4\rho \frac{L}{a}}$$

into equation 110 we obtain

$$\sum_{n=1}^{Q} 2\,\delta v_n\,(-t_n) = \frac{L}{a}\ \frac{\rho}{h_m - 1}\ (\mathcal{T}_S - \mathcal{T}_F)^2 \tag{128}$$

Substitution of the results of equations 124, 125, 127 and 128 into the general equation for ΔE_g which is

$$\Delta E_g = (P_S - P_F)\frac{L}{a} + \Delta KE + w\ A\ H_o\ V_o \left\{ \sum_{n=1}^{Q} 2\,\delta v_n\ (-t_n) + \sum_{n=Q+1}^{F} c_n\,\delta v_n\ (T_F - t_n) \right\} \tag{129}$$

yields

$$\Delta E_g = w\ A\ H_o\ V_o \frac{L}{a} \left\{ (\mathcal{T}_S - \mathcal{T}_F) + \rho(\mathcal{T}_S^{\,2} - \mathcal{T}_F^{\,2}) + \frac{\rho}{h_m - 1}\ (\mathcal{T}_S - \mathcal{T}_F)^2 + \frac{\sqrt{h_m - 1}}{12\rho}\left[2\rho\,\mathcal{T}_F - \sqrt{h_m} - 1 \right]\left[h_m + 2 - \mathcal{T}_F\ (2\rho)\ (\sqrt{h_m} + 2)\right] \right\} \tag{130}$$

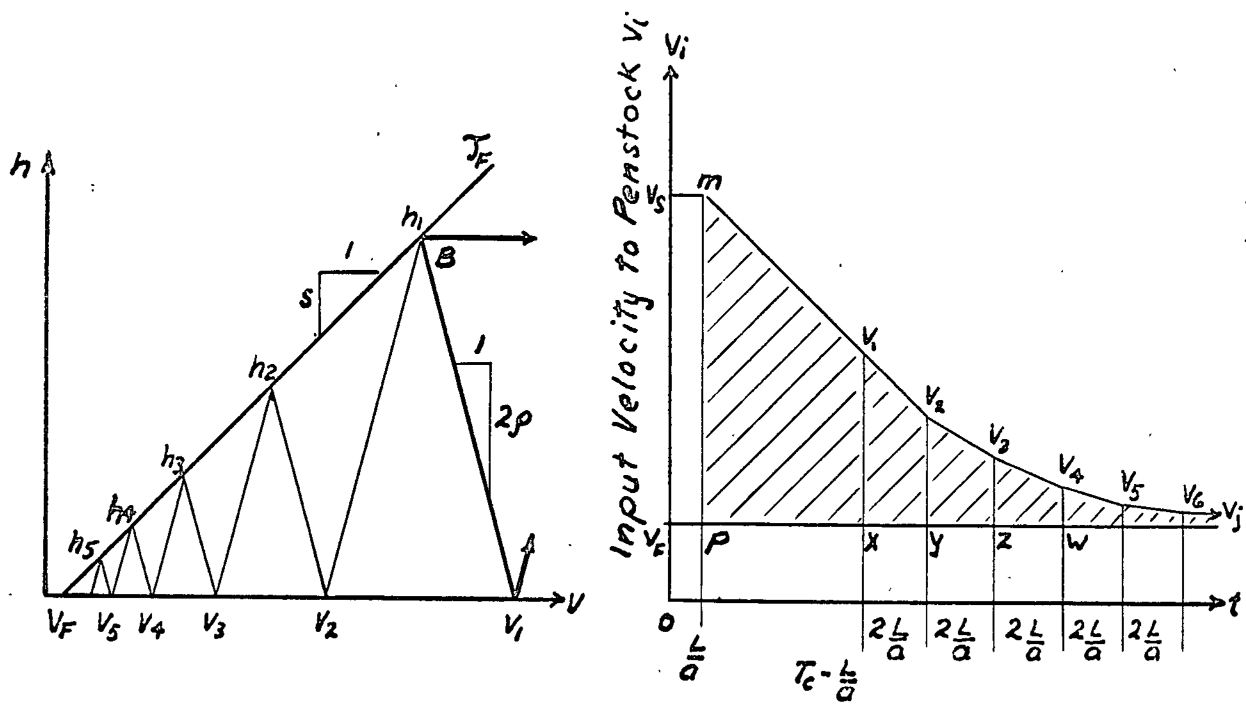


FIG 23 WATERHAMMER CHART AND INPUT VELOCITY FOR GATE CLOSURE: $2\rho > \left.\frac{dh}{dv}\right]_{\mathcal{T}_F}$

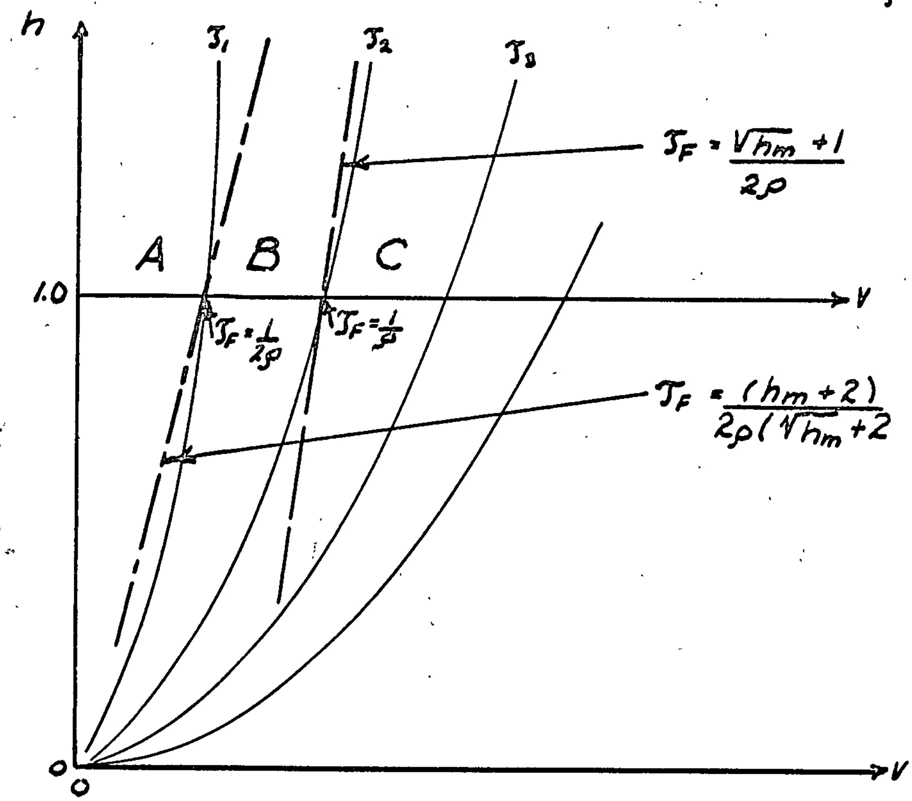


FIG 24 WATERHAMMER CHART DEFINING THE REGIONS FOR OPTIMUM GATE OPERATION: $t = T_Q$ TO $t = T_F$

This equation for ΔE_g is subject to the limitations previously mentioned. For the case where

$$2\rho > \frac{dh}{dv}\Big]_{\tau_F}$$

ΔE_g can be evaluated from figure 23 where the assumption has again been made that τ_F is a straight line. The shaded area of figure 23b is as before

$$\text{Area } mpv_j = \sum_{n=1}^{Q} 2\,\delta v_n\,(-t_n) \tag{131}$$

Furthermore

$$\text{Area } mpxv_1 = \frac{1}{2}\left(T_C - \frac{2L}{a}\right)\left((v_S - v_F) + (v_1 - v_F)\right) \tag{132}$$

$$\text{Area } v_1xyv_2 = \frac{L}{a}\left((v_1 - v_F) + (v_2 - v_F)\right) \tag{133}$$

$$\text{Area } v_2yzv_3 = \frac{L}{a}\left((v_2 - v_F) + (v_3 - v_F)\right) \tag{134}$$

From figure 23a we have

$$v_1 - v_F = \frac{1}{2\rho}(h_1 - 1) + \frac{1}{s}(h_1 - 1) = \left(\frac{1}{2\rho} + \frac{1}{s}\right)(h_1 - 1) \tag{135}$$

$$v_2 - v_F = -\frac{1}{2\rho}(h_1 - 1) + \frac{1}{s}(h_1 - 1) = \left(\frac{1}{s} - \frac{1}{2\rho}\right)(h_1 - 1) \tag{136}$$

$$v_2 - v_F = \frac{1}{2\rho}(h_2 - 1) + \frac{1}{s}(h_2 - 1) = \left(\frac{1}{s} + \frac{1}{2\rho}\right)(h_2 - 1) \tag{137}$$

$$v_3 - v_F = -\frac{1}{2\rho}(h_2 - 1) + \frac{1}{s}(h_2 - 1) = \left(\frac{1}{s} - \frac{1}{2\rho}\right)(h_2 - 1) \tag{138}$$

etc.

Dividing equation 138 by equation 137 and equation 136 by equation 135 we obtain

$$\frac{v_3 - v_F}{v_2 - v_F} = \frac{(\frac{1}{s} - \frac{1}{2\rho})\ (h_2 - 1)}{(\frac{1}{s} + \frac{1}{2\rho})\ (h_2 - 1)} = \frac{(\frac{1}{s} - \frac{1}{2\rho})\ (h_1 - 1)}{(\frac{1}{s} + \frac{1}{2\rho})\ (h_1 - 1)} = \frac{v_2 - v_F}{v_1 - v_F} = R \tag{139}$$

Therefore we have

$$v_3 - v_F = R\ (v_2 - v_F) \tag{140}$$

$$v_2 - v_F = R\ (v_1 - v_F) \tag{141}$$

Substitution of the results of equations 140 and 141 into equation 134 yields

$$\text{Area } v_2yzv_3 = \frac{L}{a} R\ ((v_1 - v_F) + (v_2 - v_F)) \tag{142}$$

$$= R\ (\text{Area } v_1xyv_2) \tag{143}$$

etc.

Therefore we have

$$\sum_{n=1}^{Q} 2\ \delta v_n\ (-t_n) = \text{Area } mpxv_1 + \text{Area } v_1xyv_2\ (\sum_{j=0}^{\infty} R^j) \tag{144}$$

However

$$2\rho > s$$

and as a result

$$R < 1$$

This means that

$$\sum_{j=0}^{\infty} R^j = 1 + R + R^2 + \ldots. + R^j = \frac{1}{1-R} \tag{145}$$

and

$$\sum_{n=1}^{Q} 2\,\delta v_n \;(-t_n) = \text{Area } mpxv_1 + \frac{1}{1-R}\text{ Area } v_1xyv_2$$

Referring to figure 23a with $h_1 = h_m$ we have

$$v_1 = \mathcal{T}_F\sqrt{h_m} + \frac{1}{2\rho}(h_m - 1)$$

and

$$v_1 - v_F + (v_2 - v_F) = 2\,\mathcal{T}_F(\sqrt{h_m} - 1)$$

so that

$$\sum_{n=1}^{Q} 2\,\delta v_n \;(-t_n) \simeq \frac{1}{2}\left(T_C - 2\frac{L}{a}\right)\left(\mathcal{T}_S + \mathcal{T}_F\sqrt{h_m} + \frac{1}{2\rho}(h_m - 1) - 2\,\mathcal{T}_F\right)$$

$$+ \frac{1}{1-R}\left(2\,\frac{L}{a}\right)\mathcal{T}_F(\sqrt{h_m} - 1) \tag{146}$$

where

$$R = \frac{\mathcal{T}_F\dfrac{\sqrt{h_m} - 1}{h_m - 1} - \dfrac{1}{2\rho}}{\mathcal{T}_F\dfrac{\sqrt{h_m} - 1}{h_m - 1} + \dfrac{1}{2\rho}} = \frac{2\rho\,\mathcal{T}_F - (\sqrt{h_m} + 1)}{2\rho\,\mathcal{T}_F + (\sqrt{h_m} + 1)} \tag{147}$$

$$\frac{1}{1-R} = \frac{2\rho\,\mathcal{T}_F + (\sqrt{h_m} + 1)}{2(\sqrt{h_m} + 1)} \tag{148}$$

and

$$T_C - \frac{2L}{a} = \frac{\mathcal{T}_F \sqrt{h_m} - \mathcal{T}_S}{2K} - \frac{L}{a} \tag{149}$$

$$= \frac{L}{a} \left\{ (\mathcal{T}_F \sqrt{h_m} - \mathcal{T}_S) \left(\frac{-2\rho}{h_m - 1} \right) - 1 \right\} \tag{150}$$

Substituting these values into the equation for ΔE_g which is in this case given by equation 71, we obtain

$$\Delta E_g = WAH_o V_o \frac{L}{a} \Bigg\{ (\mathcal{T}_S - \mathcal{T}_F) + \rho(\mathcal{T}_S^2 - \mathcal{T}_F^2)$$

$$+ \frac{1}{2} \left[(\mathcal{T}_F \sqrt{h_m} - \mathcal{T}_S) \left(\frac{-2\rho}{h_m - 1}\right) - 1 \right] \Bigg[\mathcal{T}_S + \mathcal{T}_F \sqrt{h_m}$$

$$+ \frac{1}{2\rho} (h_m - 1) - 2\mathcal{T}_F \Bigg]$$

$$+ \left[\frac{2\rho \mathcal{T}_F + \sqrt{h_m} + 1}{\sqrt{h_m} + 1} (\mathcal{T}_F)(\sqrt{h_m} - 1) \right] \Bigg\} \tag{151}$$

It can be shown that when

$$\mathcal{T}_F = \frac{\sqrt{h_m} + 1}{2\rho} \tag{152}$$

equations 130 and 151 coincide. If

$$\mathcal{T}_F < \frac{\sqrt{h_m} + 1}{2\rho} \tag{153}$$

equation 130 applies. If

$$\mathcal{T}_F > \frac{\sqrt{h_m} + 1}{2\rho} \tag{154}$$

equation 151 applies.

From equations 124 and 152 it is seen that for any gate operation

$$\sum_{n=Q+1}^{F} c_n \, \delta v_n \, (T_F - t_n) = 0$$

if

$$\tau_F \geqslant \frac{\sqrt{h_m} + 1}{2\rho} \tag{155}$$

or if

$$\tau_F = \frac{(h_m + 2)}{2\rho\,(\sqrt{h_m} + 2)} \quad . \tag{156}$$

For values of τ_F given by

$$\frac{\sqrt{h_m} + 1}{2\rho} > \tau_F > \frac{h_m + 2}{2\rho\,(\sqrt{h_m} + 2)} \tag{157}$$

$\sum_{n=1}^{Q} 2 \, \delta v_n \, (-t_n)$ is a positive quantity for gate closure and a negative quantity for gate opening. For values of τ_F given by

$$\tau_F < \frac{h_m + 2}{2\rho\,(\sqrt{h_m} + 2)} \tag{158}$$

$\sum_{n=1}^{Q} 2 \, \delta v_n \, (-t_n)$ is a negative quantity for gate closure and a positive quantity for gate opening. These results are shown in figure 24. Only for gate operation into region B of figure 24 would be advantageous to

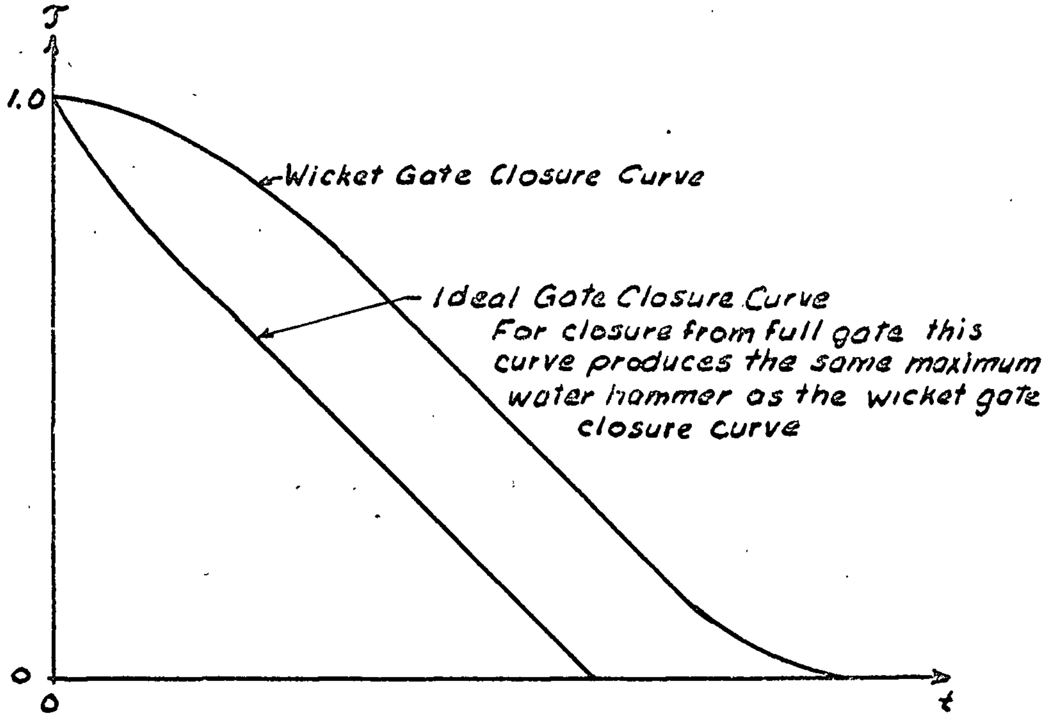


FIG 25 COMPARISON OF WICKET GATE CLOSURE CURVE AND OPTIMUM CLOSURE CURVE

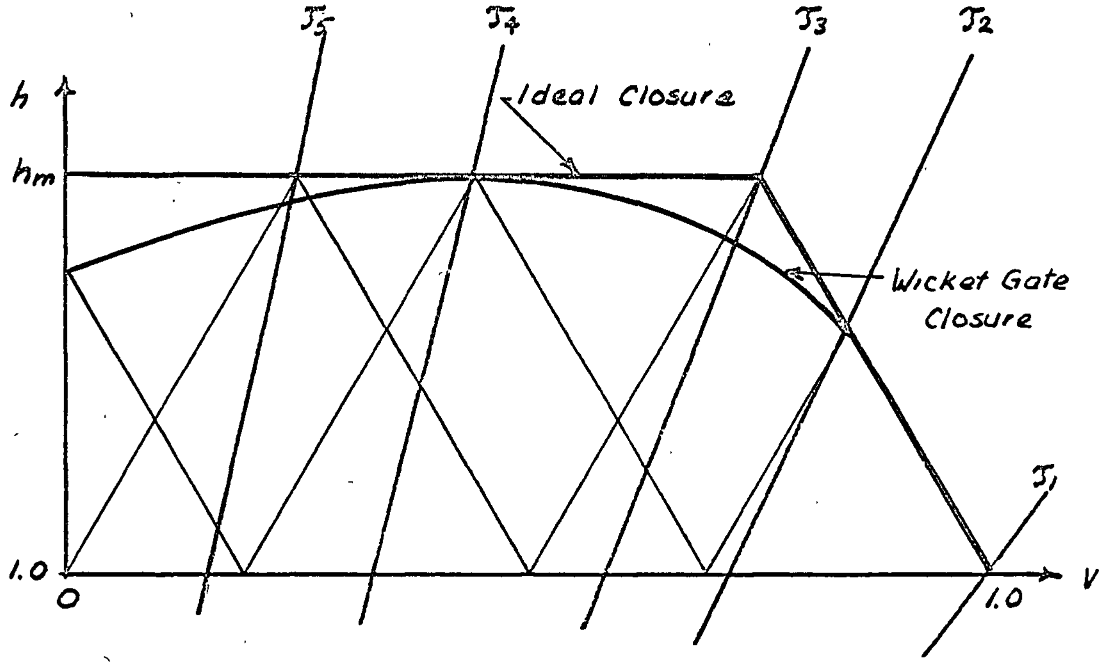


FIG 26 WATERHAMMER CHART COMPARING WICKET GATE AND OPTIMUM CLOSURE CURVES

reduce the rate of gate operation after $t = T_Q$. For gate operation into regions A or C of figure 24 gate operation should be at the maximum rate.

It is interesting to note from figure 24 that for all but low values of ρ ($\rho < 1.5$) the optimum gate operation is that which leaves no hydraulic oscillations in the system. (The normal range of gate operation is between $\tau = .3$ and $\tau = 1$. Below $\tau = .3$ the machine is running at speed no load).

Close examination of equations 130 and 151 shows that the effect of ρ on ΔE_g is very pronounced while that of h_m is less important. Therefore in design a reduction in ρ (although increasing the diameter of a penstock) may possibly result in economic savings because of the reduction in head rise for the same specified ΔE_g (particularly if decreased steady state friction losses are taken into account).

The relative importance of ρ and h_m show that the worst values of ΔE_g will be obtained at rated head.

4.3 Turbine Wicket Gate Closure Curve

A typical turbine wicket gate closure curve is shown in figure 25 and the associated h - v diagram for closure from full gate is shown in figure 26. This type of closure begins slowly so that the term

$\sum_{n=1}^{Q} 2\,\delta v_n\,(-t_n)$ of the general equation for ΔE_g is much larger than

it would be for the optimum closure curve shown in figure 25. The remaining values in the equation for ΔE_g would be reasonably similar for the two closures. Note that for gate operations from full gate to partial gate, (which are important for their effects on the electrical frequency even if they do not cause maximum speed deviation), the effect of the initial slowness of the wicket gate closure is even more magnified when the ratios of the values of ΔE_g for each type of closure are considered. This is because most of the difference between the two values of ΔE_g occurs in the first 3 $(\frac{2L}{a})$ seconds in the example shown. Use of a good closure curve can result in savings of 20 - 30% in the value of ΔE_g for the same head rise.

4.4 Use of an Upstream Gate in a Closing Operation

During the first $\frac{L}{a}$ seconds after the initiation of gate closure the power input to the penstock is unchanged. If closure were initiated at the upstream end of the penstock, simultaneously with closure at the turbine, ΔE_g could be substantially reduced along with a reduction in h_m. Figures 27, 28 and 29 show the gate closure curves, power input to the penstock, and power output from the penstock, and the h - v diagram for a case where closure is made in 12 $\frac{L}{a}$ seconds at the downstream gate and 7 $\frac{L}{a}$ seconds (to .05 gate) at the upstream gate. (The upstream closure is more rapid because the initial effect at large gate openings is small).

The head discharge equations for different gate openings of the upstream gate are given by:

$$h - 1 = v^2 \left(\frac{1}{\Psi} - 1\right)^2 \frac{\rho V_o}{a} \tag{159}$$

where Ψ is the relative gate opening (3).

In figure 29, the head rise when the downstream valve only is used is 2.5 times greater than the head rise due to combined up and downstream closures. In figure 28, ΔE_g is approximately 20% less for the case of combined closures when compared to downstream closure only.

Use of an upstream valve in the closing operation obviously can produce a large reduction in ΔE_g and in waterhammer and may avoid the use of a pressure regulator. The butterfly valve and spherical valve are not suited to this type of operation and a needle valve would present tremendous design problems. Figure 30 shows an alternative which should eliminate most of the problems of the other valves and might warrant some investigation. Aside from the problem of a suitable valve there are several operating problems; particularly that of failure of the upstream valve. A possible solution to this problem is to mount an electrical contact on the upstream valve that would be connected to a solenoid controlling part of the fluid supply to the downstream servomotor piston. If the upstream valve failed to operate, the electrical contact would remain open, the solenoid would then remain closed and the rate of closure of the downstream valve would be limited to a safe value. Another problem is the actual operating sequence of the two valves. (It would be uneconomic to leave the upstream gate in its partially closed position). If the downstream gate were closed to its final steady state position and the upstream valve closed to say $\Psi = .05$, then by maintaining the upstream valve in this position until the initial value of ΔE_g were nearly cancelled by the losses due to the upstream valve, the upstream valve could then be slowly opened to full gate. The net speed change of the turbine would be zero and hydraulic oscillations would be reduced.

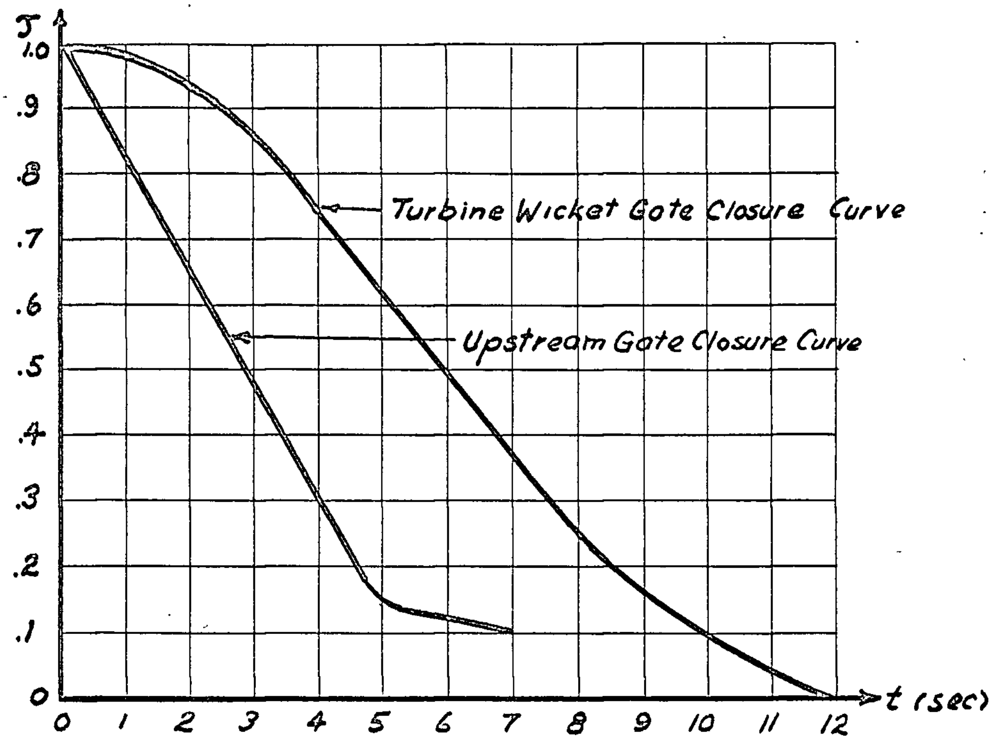


FIG 27 CLOSURE CURVES FOR SYNCHRONIZED GATE OPERATION

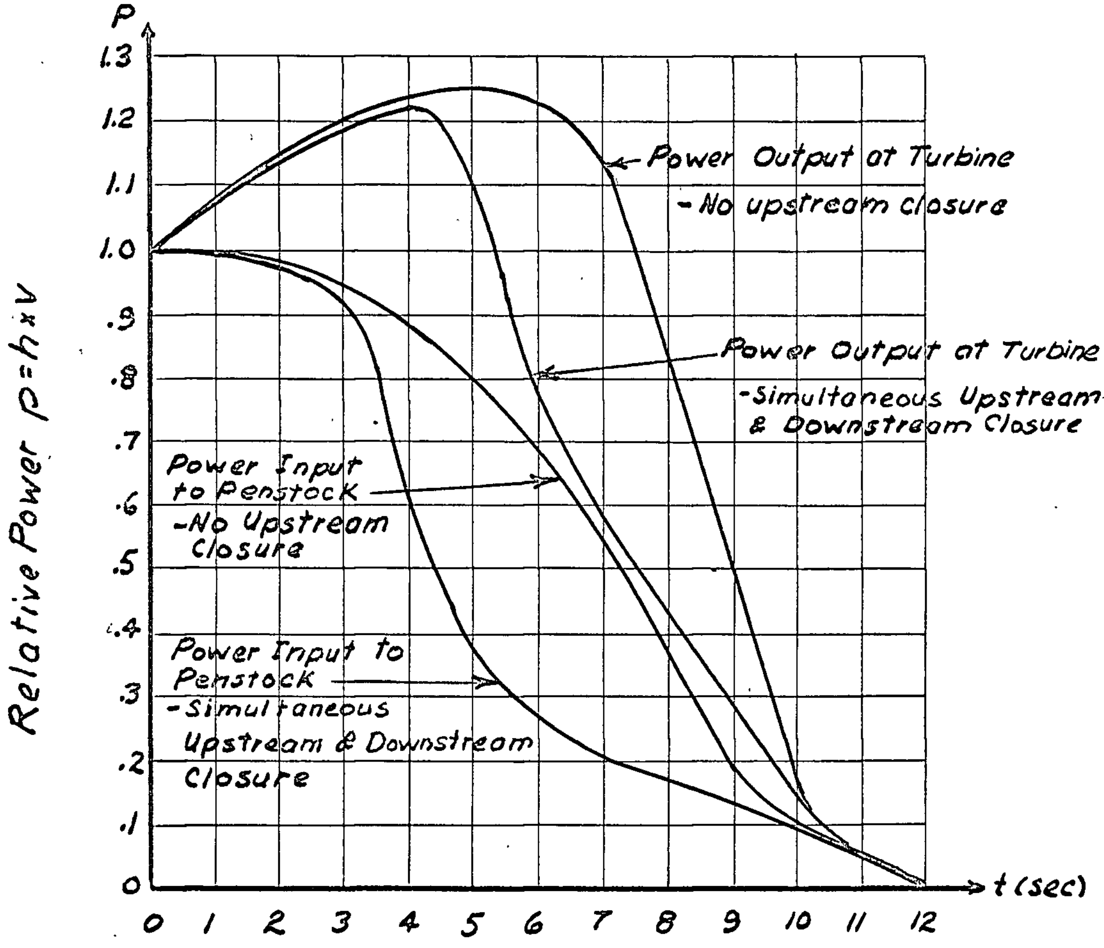


FIG 28 COMPARISON OF POWER OUTPUTS FOR SINGLE AND SYNCHRONIZED GATE OPERATION

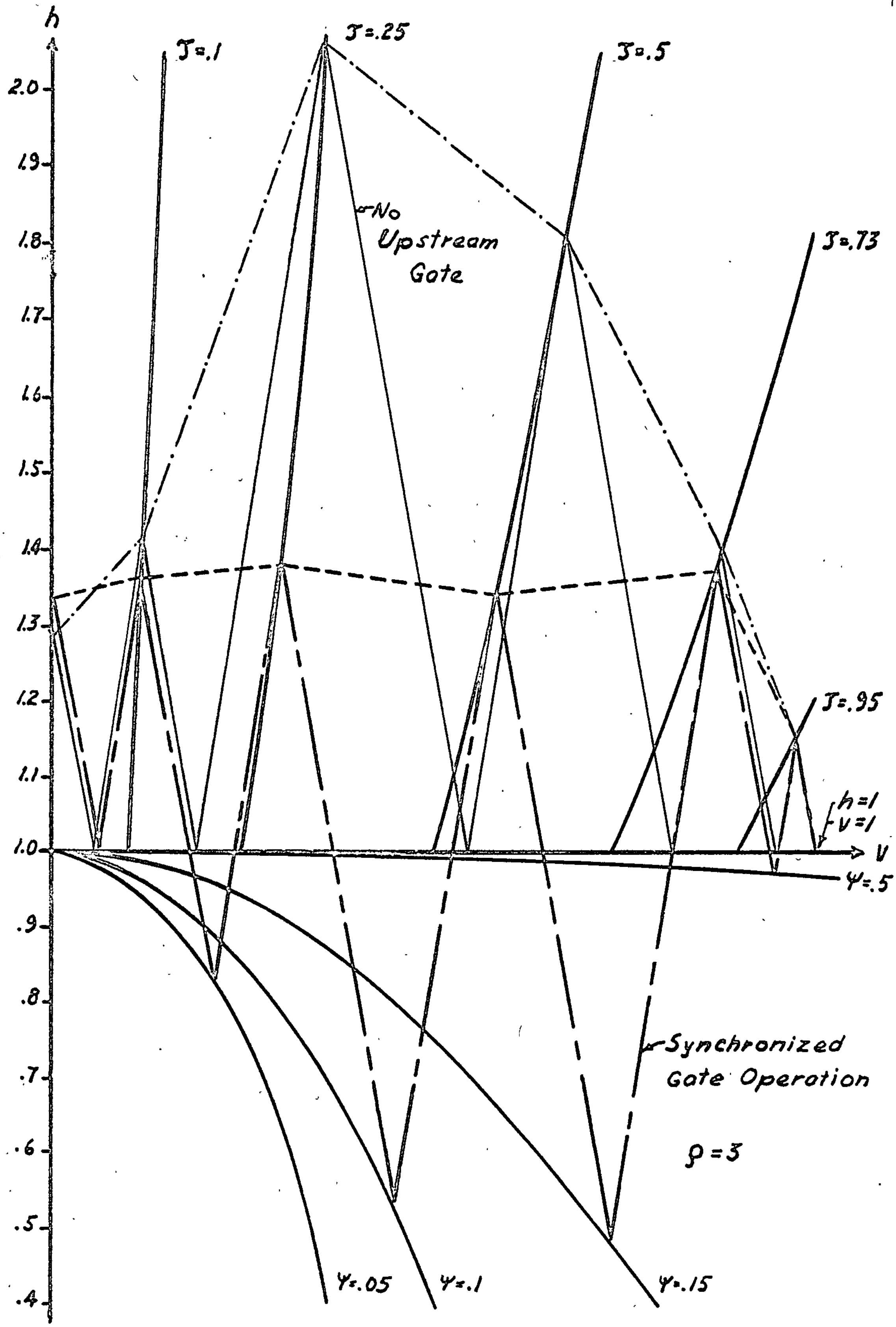


FIG. 29 WATERHAMMER CHART COMPARING SINGLE AND SYNCHRONIZED GATE OPERATION

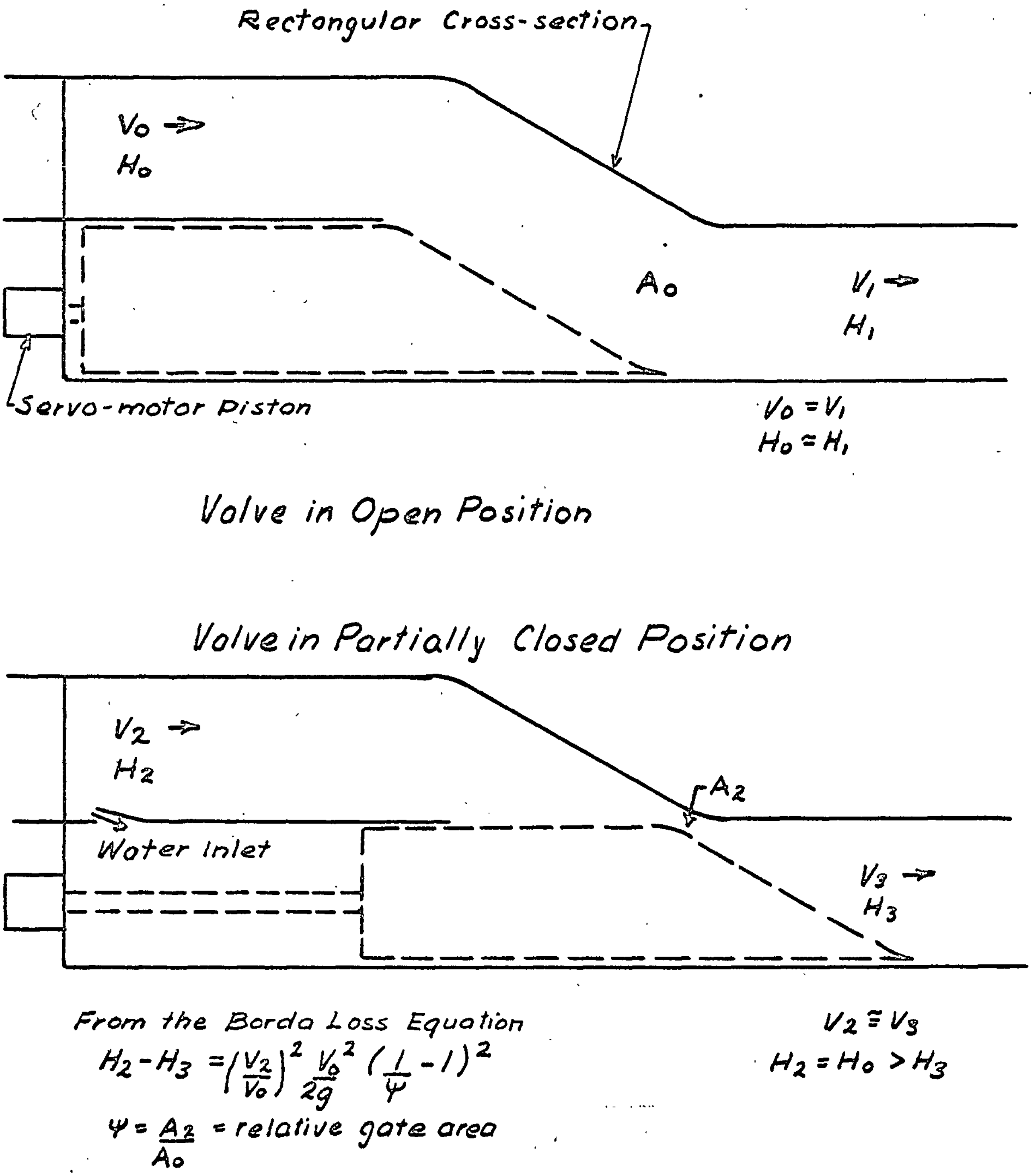


FIG 30 AN UPSTREAM VALVE

CHAPTER V

CONCLUSIONS

5.1 The Use of the Energy Method of Solution.

Although many text-books state that energy methods are not applicable to problems of rapidly varied flow, it cannot be denied that in certain instances energy methods can lead to valuable interpretations. If the process causing the rapid flow variations is adiabatic (as it usually is - even in the case of an hydraulic jump) and if the accelerations of the flow take place in the direction of the stream-lines (so that no turbulence - which is the major source of energy loss - takes place) then the energy losses, which usually make solutions by energy methods impossible, can be neglected. If such is the case the use of energy methods is justifiable. The waterhammer process meets the above requirements and so energy problems may be solved directly.

Use of the energy method for an incremental gate movement shows that as the pressure wave travels along the penstock, any change in kinetic energy is accompanied by a change in the energy, stored as strain energy, in the fluid and in the penstock, and the net change of energy at any point in the penstock is a constant for a given wave as it travels in one direction along the penstock.

5.2 The Importance of the Line $h = 2\rho v$ on the Waterhammer Chart.

The energy method explains why, under certain conditions of gate operation, the change in power output is initially opposite to that desired. If the point representing the conditions at the start of a gate operation is to the right of the line $h = 2\rho v$ on the waterhammer chart, then any waves related to an originating point located to the right of this line will cause a change in the kinetic energy of the fluid that is greater than the amount of energy that the penstock and the fluid can store. Therefore, if the gate operation is one of closure, there will be an excess of energy equal to the difference between the absolute value of the change in kinetic energy and the absolute value of the change in stored energy. This excess appears at the gate and the result is an increased power output. If the gate operation is one of opening, an energy deficiency equal to the difference between the absolute value of the change in kinetic energy and the absolute value of the change in stored energy will result. Unless ρ is very small this means that for most gate operations the initial power change will be opposite to that desired.

5.3 The Importance of the Line $\tau = \frac{\sqrt{h} + 1}{2\rho}$ on the Waterhammer Chart.

For any given relative head h this line defines where the slope of the τ curve is equal to the slope of the waterhammer line (2ρ). This line is always to the right of the line $h = 2\rho v$. Any gate operation finishing to the right of the line $\tau = \frac{\sqrt{h} + 1}{2\rho}$ must maintain the maximum value of $\frac{d\tau}{dt}$ that is consistent with the maximum allowable head deviation, if the absolute value of ΔE_g is to be kept to a minimum. (The term $\sum_{n=Q+1}^{F} C_n \delta v_n (T_F - t_n)$ in the general equation for ΔE_g does not exist to the right of the line $\tau = \frac{\sqrt{h} + 1}{2\rho}$.)

5.4 The Importance of the Area on the Waterhammer Chart Between the Lines $h = 2\rho v$ and $\tau = \frac{\sqrt{h} + 1}{2\rho}$

In this area any waves created will cause an energy output opposite to that desired. As a result it is desirable to create the minimum number of waves possible in the $\frac{2L}{a}$ seconds before the final steady state power output is first reached. (Up to the point $T_F - \frac{2L}{a}$ the energy output is fixed by the total change in the kinetic energy of the fluid between the initial and final steady states, and the energy input to the penstock. Thus it is desirable to decrease the creation of waves only in the last $\frac{2L}{a}$ seconds before reaching steady state power output.) This is equivalent to reducing the rate of gate operation in this interval. Although the resultant reduction in the absolute value of ΔE_g due to the reduced closing rate may be small, the hydraulic oscillations in the system will be greatly reduced, thus leading to greater system stability. The example of instantaneous gate closure demonstrates the idea that in cases where a partial gate operation takes place in a time less than $\frac{2L}{a}$ seconds, it may be quite advantageous from the point of head rise as well as energy output and hydraulic stability to reduce the rate of gate closure after a certain interval.

5.5 The Importance of the Term $\sum_{n=Q+1}^{F} C_n \delta v_n (T_F - t_n)$ in the General Equation for ΔE_g.

In the discussion of the gate closure which yields the smallest value of maximum waterhammer

$$\sum_{n=Q+1}^{F} C_n \delta v_n (T_F - t_n)$$

was evaluated to be:

$$\sum_{n=Q+1}^{F} c_n \delta v_n (T_F - t_n) = \left(\frac{\sqrt{h_m} - 1}{12\rho}\right)\left[2\rho \mathcal{T}_F - (\sqrt{h_m} + 1)\right]\left[(h_m + 2) - 2\rho \mathcal{T}_F (\sqrt{h_m} + 2)\right]$$

If the variations in head during the last $\frac{2L}{a}$ interval of closure are not too great then this equation would be a reasonable approximation for any type of gate operation. If so then it is immediately obvious that unless ρ is very small or $\sqrt{h_m}$ is very large this term is of negligible value when compared to terms of the order of $\rho(\mathcal{T}_S^2 - \mathcal{T}_F^2)$ which is representative of the change in kinetic energy between the initial and final steady states. Therefore the main advantage of making the absolute value of this term as small as possible in the area between the lines $h = 2\rho v$ and $\mathcal{T} = \frac{\sqrt{h+1}}{2\rho}$ of the waterhammer chart, is the reduction of hydraulic oscillations. Even to the left of the line $h = 2\rho v$ the advantages of reduced hydraulic oscillations may well outweigh any reduction of ΔE_g that would be obtained by keeping $\frac{d\mathcal{T}}{dt}$ at a high value.

5.6 <u>The Importance of the Term</u> $\sum_{n=1}^{Q} 2\delta v_n (-t_n)$ <u>in the General Equation for</u> ΔE_g.

The last section showed that for a wide range:

$$\sum_{n=Q+1}^{F} c_n \delta v_n (T_F - t_n) = 0$$

If such is the case then:

$$\Delta E_g = (P_S - P_F)\frac{L}{a} + \Delta KE + \left\{\sum_{n=1}^{Q} 2\,\delta v_n(-t_n)\right\}\left\{(WAH_o V_o)\right\}$$

From this equation, the importance of changing the energy input to the penstock as rapidly as possible is obvious ($WAH_o V_o \sum_{n=1}^{Q} 2\,\delta v_n(-t_n)$ is the variable term in the energy input equation). Furthermore this equation gives an approximate method for comparing the relative merits of different closure curves since the rate of change of velocity at the intake is related to the rate of change of velocity at the gate. We can then write

$$\sum_{n=1}^{Q} 2\;\delta v_n(-t_n) = f\left(\frac{d\mathcal{T}}{dt}\right)$$

Therefore those closure curves which have a high initial value of $\frac{d\mathcal{T}}{dt}$ can be expected to produce smaller values of ΔE_g. In fact the plot of input power is usually very similar in form to the gate closure curve (compare figures 28 and 27 for the case of no upstream closure).

5.7 The Use of an Upstream Gate in Closing Operations.

The example showing the effect of combined up and downstream gate closure showed that many advantages may be gained by the use of this type of synchronized closing operation. As there is usually a valve on the upstream end of a penstock, it seems well worthwhile to make greater use of it, particularly in cases where a pressure regulator is necessary. The possible economic advantages of one less valve and its accompanying energy dissipator, along with reduced penstock costs and energy savings make further investigation of this possibility of definite usefulness.

5.8 General Comments

Because of the generally small value of $\sum_{n=Q+1}^{F} C_n \delta v_n (T_F - t_n)$ much stability can be gained with negligible effect on ΔE_g if turbine gates are always regulated in a manner such that the hydraulic oscillations are reduced to as near zero as possible. To this end it might be of some advantage to govern hydraulic turbines on the basis of the change in load on the generator instead of on the basis of speed deviation. (For small load changes the turbine gate is often regulated on the basis of load - in such cases the turbine speed governor is not in action.) If governing were based on the amount of load change for large load variations a form of programmed gate operation might be used (i.e. for each load change there would be a predetermined gate operation pattern). The result would be better speed regulation and increased system stability.

Good speed regulation in general is determined by the ratio of the total energy supplied to the prime-mover, to the kinetic energy of motion of the fluid supplying the energy to the prime-mover. Thus a steam turbine is comparatively easy to govern - the ratio of the total energy of a pound of steam to its kinetic energy of motion is usually far greater (particularly with the present high temperature, high pressure, steam plants) than that of the highest head hydro-power plants.

In a hydro-power plant the total energy supplied varies as the product HV while the kinetic energy of the fluid varies as V^2. The ratio of these two energies is:

$$\frac{\text{Total Energy}}{\text{Kinetic Energy}} \propto \frac{HV}{V^2} = \frac{H}{V}$$

But $\frac{1}{2\rho} = \frac{gH_o}{aV_o}$

Therefore ρ is a measure of the ratio of the total energy of the fluid to the kinetic energy of the fluid. Furthermore, from the fact that the line $h = 2\rho v$ indicates under what conditions the delivery system (i.e. the penstock) can store an amount of energy greater than an accompanying change of kinetic energy, (at which point good governing becomes possible) it is seen that ρ is a very important factor in turbine governing. (This was demonstrated in the discussion of the gate closure curve which yields the minimum value of maximum waterhammer). A low value of ρ is not only indicative of a high total energy to kinetic energy ratio, but also increases the area on the waterhammer chart in which good governing is possible. Because of its effect on the pressure head rise for a given gate operation (the lower the value of ρ, the lower the maximum pressure deviation) increasing ρ may be more economical than usually believed, particularly if the savings due to decreased steady state friction losses are included.

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APPENDIX I

SYMBOLS, ABBREVIATIONS AND UNITS

a	-	waterhammer wave velocity	ft/sec
A	-	cross-sectional area of penstock	ft^2
D	-	penstock diameter	ft
E	-	modulus of elasticity of penstock wall	lbs/ft^2
E	-	energy	ft-lbs
F	-	force	lbs
g	-	acceleration of gravity	ft/sec^2
H	-	total pressure head	ft
H_o	-	steady state pressure head	ft
δH	-	infinitesimal change in pressure head	ft
ΔH	-	total change in pressure head	ft
h	-	relative pressure head $= \frac{H}{H_o}$	-
K	-	bulk modulus of fluid	lbs/ft^2
L	-	penstock length	ft
P	-	power	ft-lbs/sec
R	-	penstock radius	ft
S	-	work	ft-lbs
t	-	time - general	sec
T	-	specified time interval	sec
V	-	velocity of the fluid in the penstock	ft/sec
V_o	-	full gate velocity at $H = H_o$	ft/sec
v	-	relative velocity $= \frac{V}{V_o}$	-
w	-	unit weight of water	lbs/ft^3
ρ	-	pipe line constant $= \frac{aV_o}{2gH_o}$	-
τ	-	relative gate area	-