BOUNDARY CONDITIONS FOR ANALYSIS OF WATERHAMMER
IN PIPE SYSTEMS

by

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ABSTRACT

The transient flow in pipe networks is represented by a pair of quasi-linear hyperbolic partial differential equations. The method of characteristics is used to transform these equations to a set of ordinary differential equations, which are then solved by a first order finite difference technique using suitable boundary conditions.

The main purposes of these investigations are:

1) To derive suitable boundary conditions or boundary condition equations for valves, sprinklers, surge tanks and air chambers, and

2) To investigate the effect of these boundary conditions on the transient flow in pipe systems.

Several numerical examples are solved on the digital computer using the method of characteristics. The results are compared with those obtained by the graphical method.

Although in this thesis the developed boundary conditions are used to study the transient response of the irrigation pipe systems, the boundary conditions, without any modification, can be used to determine the transient conditions in water supply pipe networks or in pipes carrying other liquids.
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NOTATION

The following symbols are used in this thesis:

\[ A = \text{cross-sectional area of pipe, in } \text{ft}^2; \]

\[ A_g = \text{area of opening of a valve, in } \text{ft}^2; \]

\[ A_s = \text{cross-sectional area of surge tank, in } \text{ft}^2; \]

\[ A_a = \text{cross-sectional area of air chamber, in } \text{ft}^2; \]

\[ a = \text{velocity of pressure wave, in ft/sec}; \]

\[ C_d = \text{coefficient of discharge of valve or sprinkler}; \]

\[ D = \text{inside diameter of pipe, in ft}; \]

\[ E = \text{modulus of elasticity, in } \text{lbs/ft}^2; \]

\[ f = \text{Darcy-Weisbach friction factor}; \]

\[ H = \text{transient state piezometric pressure head above datum at the beginning of a time interval, in ft}; \]

\[ H_P = \text{transient state piezometric pressure head above datum at the end of a time interval, in ft}; \]

\[ H_o = \text{steady state piezometric pressure head above datum, in ft}; \]

\[ H_f = \text{friction loss in a pipe, in ft}; \]

\[ h = \text{relative head } \frac{H}{H_o}; \]

\[ H_{orfo} = \text{orifice throttling loss corresponding to discharge } q, \text{ in ft}; \]

\[ H_{orf} = \text{orifice throttling loss corresponding to discharge } q_o, \text{ in ft}; \]

\[ I = \text{mass moment of inertia of pump and motor, in } \text{lb-ft-sec}^2; \]
L = length of pipe, in ft;
N = rotational speed of pump, in revolutions/minute;
N_R = rated rotational speed of pump, in rev/min;
p = pressure at a point, in lbs/ft^2;
Q_P = transient state discharge, in ft^3/sec;
Q_o = initial steady state discharge, in ft^3/sec;
q = transient state orifice discharge, in ft^3/sec;
T = pump input torque, in lb-ft;
T_R = rated pump input torque, in lb-ft;
t = time, in seconds;
V = transient state velocity in pipe at the beginning of a time interval in ft/sec;
V_P = transient state velocity in pipe at the end of a time interval, in ft/sec;
V_o = initial steady state velocity in pipe, in ft/sec;
v_{air} = transient state volume of air in air chamber at the beginning of a time interval, in ft^3;
v_P_{air} = transient state volume of air in the air chamber at the end of a time interval, in ft^3;
v_o_{air} = initial steady state volume of air in air chamber, in ft^3;
WR^2 = moment of inertia of rotating parts of motor, pump and entrained water, in lb-ft^2;
x = distance along the pipeline, in ft;
Z = elevation above datum, in ft;
α = relative pump speed, \( \frac{N}{N_R} \);

β = relative pump torque, \( \frac{T}{T_R} \);

γ = specific weight of liquid, in lbs/ft³;

θ = angle of slope of pipe;

η = efficiency of pump;

\( \eta_R \) = efficiency of pump at rated conditions;

\( \varepsilon \) = density of liquid, in lb-sec²/ft⁴;

τ = ratio of effective gate opening to full gate opening; and

Δt = time increment, in seconds.

Subscripts:

The subscript, j, refers to pipe while 1, 2, ... n, n+1 represent pipe sections.
INTRODUCTION

If the flow in a pipe is changed from one steady state condition to another, the intermediate temporary unsteady flow is defined as transient flow.

While investigating the transient behaviour of a system it is usually necessary to choose the main parameters first. The subsequent analysis verifies whether the transient pressures are within the prescribed limits. If not, some parameters are altered and the analysis is repeated. This procedure is continued until the desired transient response is achieved.

The continuity and momentum equations which govern the unsteady flow through pipes form a set of non-linear, hyperbolic partial differential equations. By making certain simplifying assumptions or neglecting the non-linear terms the following methods have been developed to solve these equations:

a) **Arithmetical Method**: This method\textsuperscript{13} neglects friction losses and assumes the pipeline horizontal. An account is kept of all the reflections in the system. This method is almost obsolete because it is tedious, time-consuming and approximate.

b) **Graphical Method**: This method has been developed\textsuperscript{3,12} by neglecting the non-linear terms of the above-mentioned equations. However, the friction losses can be considered, if desired, by assuming a

** These numbers refer to the Bibliography.
hypothetical obstruction located either at the upstream or the down­stream end of the pipe. The friction loss due to the obstruction is taken equal to that of the entire pipeline. Although more accurate solutions can be obtained by assuming a number of obstructions along the pipeline, this complicates the graphical analysis.

The graphical method has the advantages that the phenomenon of waterhammer can be visualized and simple systems can be easily analyzed. The disadvantages are as follows:

i) Assumptions regarding friction losses are inaccurate.

ii) Large systems cannot be analyzed.

iii) Many boundary conditions can be analyzed only by trial and error.

Because of these disadvantages, this method is being replaced by numerical methods suitable for computer analysis.

c) Analytical Methods: In these methods the continuity and momentum equations are linearized by neglecting the non-linear terms of lesser importance (see Section 1.2) and considering the friction losses proportional to velocity. Wood\textsuperscript{22} used Heaviside's operational calculus to solve the resulting linearized equations. This method gives only surge pressures and velocities. Rich\textsuperscript{14} solved the linearized equations by using Laplace-Mellin transformations. Unlike Wood's method, this method gives total pressures and velocities. Streeter and Lai tried to solve the series obtained by this method on the digital computer. They reported\textsuperscript{20}: 
"However, there are still some disadvantages in using the operational mathematics, even with aid of the digital computer. The transformations and inverse transformations involve difficult mathematical manipulations and often result in tedious series. It has certain limitations because of necessary approximations; it is sometimes difficult to determine constants for some given boundary conditions and consequently is not flexible in application."

d) **Method of Characteristics:** In this method all the non-linear terms of the continuity and momentum equations are retained. The partial differential equations are converted into ordinary differential equations by the method of characteristics. These equations are then solved by a finite-difference technique. Besides the relative ease to obtain any desired accuracy, this method has the following advantages:

i) Large and complex systems can be analyzed.

ii) It is very fast because the equations derived can be easily solved on a digital computer.

iii) Once a programme has been written for a system, similar systems can be analyzed by changing the data cards.

Because of the above advantages, this method is used in this study.

While introducing the method of characteristics in the field of waterhammer, Streeter and Lai presented boundary conditions for a few simple systems. In this thesis, boundary conditions are developed for reservoirs, sprinklers, valves, surge tanks and air chambers. Transient conditions initiated by opening or closing a valve, by pump failure or by changes in the water surface elevation of a reservoir, are analyzed. A number of examples are solved and the results presented in Appendices A to C.
1.1 BASIC EQUATIONS FOR UNSTEADY FLOW THROUGH PIPES:

The velocity and pressure of moving fluids in pipes are governed by the momentum and continuity equations. The pipes are considered to be made up of "compartments". The term compartment is used to denote a length of pipe having a uniform diameter. It is assumed that flow is one dimensional in each compartment and the pressure and velocity are uniform at the cross section of a compartment.

The subscripts $x$ and $t$ indicate partial differentiation with respect to distance and time. For example,

$$H_x = \frac{\partial H}{\partial x},$$

$$H_t = \frac{\partial H}{\partial t},$$

in which $H$ is total pressure head in feet of water.

The momentum equation for flow through a pipe which is inclined or horizontal, tapered or straight, slightly or highly deformable, is given by

$$gH_x + V_t + V V_x + \frac{fV |V|}{2D} = 0,$$

in which $g$ is acceleration due to gravity, $V$ is velocity of the fluid,
f is the Darch-Weisbach friction factor, H is the total pressure head above the datum line, D is the inside diameter of the pipe, and
\[ \frac{fV|V|}{2D} \] is the frictional force of the fluid. The absolute sign is introduced to take into account any change in the direction of velocity. Thus the frictional force is always opposite to the direction of velocity.

For a pressure change of 100 lbs./in\(^2\) and for a waterhammer wave velocity of 2000 ft/sec, the change in the density of water is approximately \(3.6 \times 10^{-3}\) lbs-sec\(^2\)/ft\(^4\). Thus, changes in the density of water can be neglected. Considering the density as constant, the continuity equation is given by

\[ \frac{a^2}{g} V_x + H_t + V [H_x + \sin \theta] = 0, \]  
(1.2)
in which \(\theta\) is the angle the centreline of the pipe makes with the horizontal axis (measured positive downwards), and \(a\) is the velocity of the waterhammer wave.

1.2 BASIC CONSIDERATIONS:

Eqs. (1.1) and (1.2) can be re-written in the form

\[ (V_t + V V_x) + gH_x + \frac{f}{2D} V |V| = 0 \]  
(1.3)
and

\[ H_t + \frac{a^2}{g} V_x + V [H_x + \sin \theta] = 0. \]  
(1.4)

The above equations form a set of simultaneous, non-linear first order partial differential equations. Since the non-linear terms
V V_x, V H_x and \(\frac{f}{2D} V |V|\) involve only the first power of the derivative, the equations are classified as quasi-linear. These equations may be further classified as elliptic, parabolic or hyperbolic as follows:

The most general representation of a pair of first order, quasi-linear, partial differential equations in two independent variables can be expressed by the following single matrix equation:

\[
\frac{\partial}{\partial t} \begin{pmatrix} V \\ H \end{pmatrix} = - [A(V,H)] \frac{\partial}{\partial x} \begin{pmatrix} V \\ H \end{pmatrix} - \{B(V,H)\}.
\]

The square matrix \([A]\) and the column vector \(\{B\}\) are functions of the unknowns \(V\) and \(H\). In the particular case represented by Eqs. (1.3) and (1.4), these matrices are function of \(V\) only and are given by the following equations:

\[
[A(V)] = \begin{pmatrix} V & g \\ \frac{a^2}{g} & V \end{pmatrix}
\]

and

\[
\{B(V)\} = \begin{pmatrix} \frac{f}{2D} V |V| \\ V \sin \theta \end{pmatrix}.
\]

The eigen-values, \(\lambda\), of matrix \(A\) determine the type of the set of equations. The characteristic matrix of \(A\) is given by

\[
\begin{pmatrix} V-\lambda & g \\ \frac{a^2}{g} & V-\lambda \end{pmatrix}.
\]

Hence, the characteristic equation of matrix \(A\) can be written as

\[
(V-\lambda)^2 - \frac{a^2}{g} g = 0.
\]
Solving this equation for $\lambda$, one obtains

$$\lambda_1 = V + a \quad \text{and} \quad \lambda_2 = V - a.$$  

Since $V$ and $a$ are real, both the eigen-values are real and distinct.

Eqs. (1.3) and (1.4) are, therefore hyperbolic.

1.3 CHARACTERISTIC EQUATIONS:

To simplify Eqs. (1.3) and (1.4), non-linear terms of lesser importance are neglected in this section. The next section, however, deals with these equations without neglecting any term.

Eqs. (1.3) and (1.4) can be written in a simplified form as

$$L_1 = g H_x + V_t + \frac{f}{2D} V |V| = 0 \quad (1.5)$$

and

$$L_2 = H_t + \frac{a^2}{g} V_x = 0. \quad (1.6)$$

Multiplying Eq. (1.6) by an unknown multiplier $\lambda$ and adding the result to Eq. (1.5), one obtains

$$L_1 + \lambda L_2 = \lambda \left[ g \frac{H}{\lambda} H_x + H_t \right] + \left[ V_t + \frac{\lambda a^2}{g} V_x \right]$$

$$+ \frac{f}{2D} V |V| = 0. \quad (1.7)$$

Any two real and distinct values of $\lambda$ give two equations in $V$ and $H$ which are equivalent to Eqs. (1.5) and (1.6) in all respects.

If

$$V = V(x,t)$$

and

$$H = H(x,t)$$
are solutions of Eqs. (1.5) and (1.6), then

\[ \frac{dV}{dt} = V \frac{dx}{dt} + V_t \]  

(1.8)

and

\[ \frac{dH}{dt} = H \frac{dx}{dt} + H_t. \]  

(1.9)

Let

\[ \frac{dx}{dt} = \frac{g}{\lambda} = \frac{\lambda a^2}{\lambda} = \lambda, \]  

(1.10)

so that

\[ \lambda = \pm \frac{g}{a}, \]  

(1.11)

and

\[ \frac{dx}{dt} = \pm a. \]  

(1.11)

Then, it follows from Eqs. (1.7) to (1.10) that

\[ \lambda \frac{dH}{dt} + \frac{dV}{dt} + \frac{f}{2D} V \|V\| = 0. \]  

(1.12)

By substitution of these values of \( \lambda \) into Eq. (1.12), the following characteristic equations are obtained:

\[ \begin{align*}
\frac{g}{a} \frac{dH}{dt} + \frac{dV}{dt} + \frac{f}{2D} V \|V\| &= 0, \\
\frac{dx}{dt} &= \pm a. \\
- \frac{g}{a} \frac{dH}{dt} + \frac{dV}{dt} + \frac{f}{2D} V \|V\| &= 0.
\end{align*} \]  

(1.13)  

(1.14)  

(1.15)  

(1.16)
Eqs. (1.13) and (1.15) are valid along the curves \( \frac{dx}{dt} = a \) and \( \frac{dx}{dt} = -a \). Mathematically, these curves represent lines along which the derivatives of velocity and pressure may have discontinuities. Physically, they represent the paths on the \( x-t \) plane along which the waterhammer waves are propagated (See Fig. 1.1).

A first order finite-difference technique may be used to solve Eqs. (1.13) to (1.16). Let the transient conditions start at time \( t = t_0 \). Then, at the points A and B (Fig. 1.1), \( V, H, x \) and \( t \) are equal to the known steady state values. These quantities are, however, unknown at the point P, where characteristics through the points A and B intersect each other. Note that on the \( x-t \) plane, the point P represents time \( t = t_0 + \Delta t \), where \( \Delta t \) is the time interval.

The subscripts A, B and P are used to denote quantities at the points A, B and P. In the finite-difference form, Eqs. (1.13) to (1.16) can be written as,

\[
\begin{align*}
(V_P - V_A) + \frac{E_a}{a} (H_P - H_A) + \frac{f}{2D} \left| \frac{V}{|V|} \right| (t_P - t_A) &= 0 \quad \text{C}^+ \\
(x_P - x_A) &= a (t_P - t_A) \quad \text{(1.17)} \\
(V_P - V_B) - \frac{E_a}{a} (H_P - H_B) + \frac{f}{2D} \left| \frac{V}{|V|} \right| (t_P - t_B) &= 0 \quad \text{C}^- \\
(x_P - x_B) &= -a (t_P - t_B) \quad \text{(1.18)}
\end{align*}
\]

By solving Eqs. (1.17) to (1.20) simultaneously, four unknowns for the point P can be determined.
The pipe length, \( L \), is divided into \( N \) equal reaches, so that

\[
\Delta x = \frac{L}{N}
\]

in which \( \Delta x \) is the length of a reach. Now, if the time interval, \( \Delta t \), is selected such that

\[
\Delta t = t_P - t_A = t_P - t_B
\]

and

\[
\Delta t = \frac{\Delta x}{a}
\]

then the characteristics lines intersect one another at the intermediate sections, as shown in Fig. 1.2. The velocity and pressure head at time \( t = t_0 \) being known, \( V_P \) and \( H_P \) at the interior section \( i \) at time \( t = t_0 + \Delta t \) can be determined from eqs. (1.17) and (1.19). Solving Eqs. (1.17) and (1.19) simultaneously, one obtains

\[
V_{P_i} = 0.5 \left[ V_{i-1} + V_{i+1} + \frac{a}{g} (H_{i-1} - H_{i+1}) \right]
- \frac{f \Delta t}{2D} \left[ V_{i-1} \left| V_{i-1} \right| + V_{i+1} \left| V_{i+1} \right| \right]
\]

and

\[
H_{P_i} = 0.5 \left[ H_{i-1} + H_{i+1} + \frac{a}{g} (V_{i-1} - V_{i+1}) \right]
- \frac{a f \Delta t}{g 2D} \left[ V_{i-1} \left| V_{i-1} \right| - V_{i+1} \left| V_{i+1} \right| \right]
\]

The velocity and pressure at the end points are computed from the boundary conditions. Now, the values of \( V \) and \( H \) at \( N+1 \) sections are known. By proceeding in this manner, the computation can be performed up to the specified time.
At \( t = t_0 \), \( \Delta t \), and \( \Delta x \).

**CHARACTERISTIC CURVES**

FIG. 1.1

- Initial boundary condition
- Determined by solving equations H7 and H9 simultaneously
- Determined by solving equation for \( C^+ \) and equation for end condition simultaneously
- Determined by solving equation for \( C^- \) and end condition simultaneously

**GRID CHARACTERISTIC**

FIG. 1.2
1.4 **GENERAL CHARACTERISTICS METHOD:**

In section 1.3, simplified continuity and momentum equations were used. This is almost adequate for calculating the transient conditions in metal pipes. In this section, a general solution for the continuity and momentum equations is obtained in which all the terms of the aforesaid equations are retained. For a piping system of two or more pipes, it is necessary that the same time increment be used for all the pipes, so that the end conditions common to the pipes may be obtained from a set of simultaneous equations. The method of specified time intervals, which involves linear interpolation, is used.

---

The momentum and continuity equations may be written as

\[ L_1 = g H_x + V V_x + V_t + \frac{f V |V|}{2D} = 0. \] (1.23)

and

\[ L_2 = H_t + \frac{a^2}{g} V_x + V H_x + V \sin \Theta = 0. \] (1.24)

Multiplying Eq. (1.24) by \( \lambda \) and then adding it to Eq. (1.23), one obtains

\[ L_1 + \lambda L_2 = \lambda [H_x (V + \frac{g \lambda}{\lambda}) + H_t] + (V_x - (V + \frac{a^2}{g} \lambda) + V_t] \]

\[ + \lambda V \sin \Theta + \frac{f V |V|}{2D} = 0. \] (1.25)

Let

\[ \frac{dV}{dt} = V + \frac{g \lambda}{\lambda} = V + \frac{a^2}{g} \lambda, \]

so that

\[ \lambda = \pm \frac{g}{a}, \]
and
\[ \frac{dx}{dt} = V \pm a. \] \hspace{1cm} (1.26)

By using these relations, Eq. (1.25) takes the form
\[ \lambda \frac{dH}{dt} + \frac{dV}{dt} \lambda V \sin \theta + \frac{f}{2D} V = 0. \] \hspace{1cm} (1.27)

It follows from Eqs. (1.26) and (1.27) that
\[ \left\{ \begin{array}{l}
\frac{dH}{dt} + \frac{dV}{dt} \lambda V \sin \theta + \frac{f}{2D} V = 0, \\
\frac{dx}{dt} = V \pm a,
\end{array} \right\} \hspace{1cm} (1.28) \]

\[ \left\{ \begin{array}{l}
\frac{dH}{dt} + \frac{dV}{dt} \lambda V \sin \theta - \frac{f}{2D} V = 0, \\
\frac{dx}{dt} = V - a.
\end{array} \right\} \hspace{1cm} (1.30) \]

and
\[ \frac{dx}{dt} = V - a. \hspace{1cm} (1.31) \]

Because \( V = V(x,t) \), the characteristic lines \( C^+ \) and \( C^- \), given by Eqs. (1.29) and (1.31), plot as curves on the \( x-t \) plane (See Fig. 1.3).

Eqs. (1.28) to (1.31) can be written in the following finite-difference forms:

\[ (V_P - V_R) + \frac{\xi}{a} (H_P - H_R) + \frac{\xi}{a} V_R \sin \theta (t_P - t_R) \]
\[ + \frac{f}{2D} V_R |V_R| (t_P - t_R) = 0. \hspace{1cm} (1.32) \]

\[ (x_P - x_R) = (V_R + a) (t_P - t_R). \hspace{1cm} (1.33) \]

\[ (V_P - V_S) - \frac{\xi}{a} (H_P - H_S) - \frac{\xi}{a} V_S \sin \theta (t_P - t_S) \]
\[ + \frac{f}{2D} V_S |V_S| (t_P - t_S) = 0. \hspace{1cm} (1.34) \]
14.

\[(x_p - x_s) = (v_s - a)(t_p - t_s) \tag{1.35}\]

Eq. (1.32) to (1.35) can be solved numerically by using a grid of characteristics or specified time intervals. The former method is advantageous when either \(v\) or \(a\) varies considerably with \(x\) and \(t\), as in the case of highly deformable tubes. In the latter method \(x_p\) and \(t_p\) are assigned definite values, leaving \(x_p\) and \(v_p\) as the two unknowns. This method is useful for most of the waterhammer problems and is used here.

Since the conditions at the points A, B and C (Fig. 1.3) are known, they can be determined at the points R and S by linear interpolation. Thus

\[
\frac{x_c - x_R}{x_c - x_A} = \frac{v_c - v_R}{v_c - v_A}.
\]

But

\[x_p = x_c,
\]

and

\[x_c - x_A = \Delta x.
\]

By using these relation, the above equation takes the form

\[
x_p - x_R = \frac{v_c - v_R}{v_c - v_A} \Delta x \tag{1.36}
\]

Since \(v_R\) is much smaller than the waterhammer wave velocity, \(a\), it can be neglected. By neglecting \(v_R\) in Eq. (1.33) and then combining it with Eq. (1.36) one obtains

\[
a \Delta t = \frac{v_c - v_R}{v_c - v_A} \Delta x \tag{1.37}
\]
The grid mesh ratio, \( \theta' \), is defined by the expression

\[
\theta' = \frac{\Delta t}{\Delta x}.
\]

It follows from Eq. (1.37) that

\[
a \cdot \theta' (V_C - V_A) = V_C - V_R,
\]

so that

\[
V_R = V_C - a \cdot \theta' (V_C - V_A).
\]  \hspace{1cm} (1.38)

Similarly,

\[
H_R = H_C - a \cdot \theta' (H_C - H_A).
\]  \hspace{1cm} (1.39)

\[
V_S = V_C - a \cdot \theta' (V_C - V_B).
\]  \hspace{1cm} (1.40)

\[
H_S = H_C - a \cdot \theta' (H_C - H_B).
\]  \hspace{1cm} (1.41)

By solving Eqs. (1.32) and (1.34) simultaneously, the following expressions for \( V_p \) and \( H_p \) are obtained:

\[
V_p = 0.5 \left[ V_R + V_S + \frac{g}{a} (H_R - H_S) - \frac{g}{a} \Delta t \sin \theta (V_R - V_S) \right.

\]

\[
- \frac{f \Delta t}{2D} (V_R V_R + V_S V_S) \right] .
\]  \hspace{1cm} (1.42)

\[
H_p = 0.5 \left[ H_R + H_S + \frac{a}{g} (V_R - V_S) - \Delta t \sin \theta (V_R + V_S) \right.

\]

\[
- \frac{a}{g} \frac{f \Delta t}{2D} (V_R V_R - V_S V_S) \right] .
\]  \hspace{1cm} (1.43)

At the boundary points, either Eq. (1.32) or Eq. (1.34) or both are used together with the conditions imposed by the boundary. Eqs. (1.32) and (1.34), (henceforth called the negative characteristic equation and the positive characteristic equation), can be rewritten in the following forms:
METHOD OF SPECIFIED TIME INTERVALS

FIG. 1.3

CHARACTERISTICS AT THE BOUNDARIES

FIG. 1.4
The negative characteristic equation:

$$v_p = C_1 + C_2 H_p \quad (1.44)$$

in which

$$C_1 = v_s - C_2 H_s + C_2 v_s \sin \theta \Delta t - FF \cdot v_s \left|v_s\right|, \quad (1.45)$$

$$C_2 = \frac{a}{g}, \quad (1.46)$$

and

$$FF = \frac{f \Delta t}{2D}. \quad (1.47)$$

The positive characteristic equation:

$$v_p = C_3 - C_2 H_p \quad (1.48)$$

in which

$$C_3 = v_R + C_2 H_R - C_2 v_R \Delta t \sin \theta - FF \cdot v_R \left|v_R\right|. \quad (1.49)$$

Note that $C_2$ and $FF$ represent pipe-constants. The values of $C_1$ and $C_3$ are constant during each time step.

1.5 CONVERGENCE AND STABILITY OF THE METHOD OF FINITE DIFFERENCES:

The finite-differences scheme developed in the preceding section is convergent if the exact solution of the difference equations approaches that of the original differential equation, as the grid mesh ratio tends towards zero. If the round-off error due to representation of the irrational numbers by a finite number of significant digits grows or decays as the solution progresses, the scheme is said to be unstable or stable respectively. It has been determined that convergence implies stability and stability implies
Methods for determining the convergence or stability criteria for the non-linear equations are extremely difficult. However, analytical studies for the convergence and stability can be made by linearizing the basic equations. Since, the non-linear terms are relatively small, it is reasonable to assume that the criteria applicable to the simplified equations are also valid for the original non-linear equations.

Using the procedure proposed by O'Brien and considering linearized equations, Perkins has proved that for the process to be stable

\[
\frac{\Delta t}{\Delta x} < \frac{1}{a}
\]

This shows that the characteristics through the point P (Fig. 1.3) should not fall outside the segment AB. For a neutrally stable scheme, \(\frac{\Delta t}{\Delta x} = \frac{1}{a}\).

The criteria for convergence indicate that the most accurate solutions are obtained in this case. Thus, the convergence and/or stability criterion for the finite-difference scheme based on the method of characteristics is given by the expression

\[
\frac{\Delta t}{\Delta x} \leq \frac{1}{a}
\]

(1.51)

### 1.6 SELECTION OF THE TIME INCREMENT FOR A COMPLEX PIPING SYSTEM:

For a complex piping system of two or more pipes, it is necessary that for all the pipes, the same time increment be used. Using the interpolation method outlined above, it is theoretically possible for one to merely
satisfy the limitation on the grid mesh ratio in each pipe and proceed with the solutions. However, with the linear interpolation, the larger the interpolation i.e. the larger the quantity \((\Delta x \cdot \Delta t)\), the less accurate the solution is likely to be. The convergence considerations indicate\(^{11}\) that the most accurate solutions are obtained when \(\Delta x = \Delta t\). Therefore, time increment, \(\Delta t\), is selected such that the computing time is not excessive and the interpolation is applied over a minimum distance. The time increment and the number of reaches, \(N_j\), in each pipe may be selected as follows:

i) Select number of reaches \(N_1\) in the shortest pipe of length \(L_1\).

ii) Compute the time increment, \(\Delta t\), from the equation,

\[
\Delta t = \frac{L_1}{(V_1 + a_1) N_1}
\]

where \(V_1\) and \(a_1\) represent the velocities of water and waterhammer wave in the pipe.

iii) Then, the number of reaches, \(N_j\), in the j\(^{th}\) pipe is the smallest integer such that

\[
N_j = \frac{L_j}{(V_j + a_j) \Delta t}
\]
CHAPTER II

BOUNDARY CONDITIONS

The boundary condition for various pipeline geometries and appurtenances in the piping systems are developed in this chapter. The minor losses such as the entrance loss, the loss at a change in the cross-section are neglected. By slight modifications and alterations, these can, however, be considered.

Although in the computer programme, developed in the later sections, three dimensional subscripted variables are used to describe the conditions at a point; for simplicity, only two subscripts are used in this chapter. The first subscript denotes the pipe number while the second refers to a section. The pipes are assumed horizontal. The centre-line of a pipe is considered as the datum line. The velocity and the piezometric pressure head above the datum line at the beginning and at the end of a time interval are designated by $V$, $H$, $VP$ and $HP$ respectively.

To determine the velocity, $VP$, and the pressure head, $HP$, at a boundary point, Eq. (1.44) or Eq. (1.48) or both are solved simultaneously with the conditions imposed by the boundary.

In the piping system, the initial steady state flow direction is considered positive. For a pipe in which initial steady state velocity is zero, positive flow direction is arbitrarily assumed.
An appurtenance on a pipe should be located at the end of a reach into which the pipe is divided. If in the actual system such is not the case, it is assumed to be located at the nearest end of the reach.

2.1 RESERVOIR OF CONSTANT WATER LEVEL:

a) Reservoir located at the upstream end (Fig. 2.1a):

At the junction of the pipe and the reservoir,
\[ HP_{j,1} = H_{res} , \]
in which \( H_{res} \) is the height of the water surface in the reservoir above the centreline of the pipe.

The negative characteristic equation for section \((j,1)\) is given by
\[ VP_{j,1} = C_{1j} + C_{2j} HP_{j,1} , \]
in which \( C_{1j} \) and \( C_{2j} \) are the constants for pipe \( J \). For the expressions for these constants, see Eqs. (1.45) and (1.46).

From the above two equations, it follows that
\[ VP_{j,1} = C_{1j} + C_{2j} H_{res} . \] (2.1)

b) Reservoir at the downstream end (Fig. 2.1-b):

At the junction of the pipe and the reservoir,
\[ HP_{j,n+1} = H_{res} . \]
The positive characteristic equation for section \((j,n+1)\) is given by
\[ VP_{j,n+1} = C_{3j} - C_{2j} HP_{j,n+1} . \]
(a) Reservoir at the upstream end

(b) Reservoir at downstream end

(c) Reservoir at an intermediate section

FIG. 2.1
From the above two equations, it follows that

$$V_{P,j,n+1} = C_3^j - C_2^j H_{res}$$  \hspace{1cm} (2.1)

c) Reservoir at an intermediate section (Fig. 2.1-c):

In this case,

$$H_{P,j,n+1} = H_{P,j+1,1} = H_{res}$$  \hspace{1cm} (2.3)

The positive characteristic equation for section \((j,n+1)\) is given by

$$V_{P,j,n+1} = C_3^j - C_2^j H_{P,j,n+1}$$

By virtue of Eq. (2.3), the above equation becomes

$$V_{P,j,n+1} = C_3^j - C_2^j H_{res}$$  \hspace{1cm} (2.4)

The negative characteristic equation for section \((j+1,1)\) can be written as

$$V_{P,j+1,1} = C_1^{j+1} + C_2^{j+1} H_{P,j+1,1}$$

which, upon combining with Eq. (2.3), yields

$$V_{P,j+1,1} = C_1^{j+1} + C_2^{j+1} H_{res}$$  \hspace{1cm} (2.5)

2.2 DEAD END AT THE DOWNSTREAM END (Fig. 2.2):

The velocity at a dead end is always zero. Hence, in this case

$$V_{P,j,n+1} = 0$$  \hspace{1cm} (2.6)

The value of \(H_{P,j,n+1}\) can now be determined from the following positive characteristic equation for section \((j,n+1)\):

$$V_{P,j,n+1} = C_3^j - C_2^j H_{P,j,n+1}$$  \hspace{1cm} (2.7)
From Eqs. (2.6) and (2.7), it follows that
\[ HP_{j,n+1} = C_{3j}/C_{2j} \]  \hspace{1cm} (2.8)

2.3 SERIES CONNECTION (Fig. 2.3):

In this section, boundary conditions for a junction of two pipes having different diameters are developed. However, without any modifications, these results can be applied to a junction of two pipes having the same diameter but different slopes, wall thicknesses or wall-material or any combination of these variables.

For the steady state pressure head and velocity at the junction, the following equations can be written:

The continuity equation:
\[ V_{P,j,n+1} A_j = V_{P,j+1,1} A_{j+1} \]  \hspace{1cm} (2.9)
in which \( A_j \) and \( A_{j+1} \) are the cross-sectional areas of pipe \( j \) and pipe \( j+1 \).

The equation for common pressure head:
\[ HP_{j,n+1} = HP_{j+1,1} \]  \hspace{1cm} (2.10)

The positive characteristic equation for section \( (j,n+1) \):
\[ V_{P,j,n+1} = C_{3j} - C_{2j} HP_{j,n+1} \]  \hspace{1cm} (2.11)

The negative characteristic equation for section \( (j+1,1) \):
\[ V_{P,j+1,1} = C_{1j+1} + C_{2j+1} HP_{j+1,1} \]  \hspace{1cm} (2.12)

It follows from Eqs. (2.9) to (2.12) that
DEAD END
FIG. 2.2

SERIES CONNECTION
FIG. 2.3
(C₃ᵢ - C₂ᵢ Hₚᵢ,n+1) Aᵢ = (C₁ᵢ+1 + C₂ᵢ+1 Hₚᵢ,n+1) Aᵢ₊₁,
so that
(C₂ᵢ Aᵢ + C₂ᵢ+1 Aᵢ₊₁) Hₚᵢ,n+1 = C₃ᵢ Aᵢ - C₁ᵢ+1 Aᵢ₊₁.

Simplification of the above equation, yields

\[ Hₚᵢ,n+1 = \frac{C₃ᵢ Aᵢ - C₁ᵢ+1 Aᵢ₊₁}{C₂ᵢ Aᵢ + C₂ᵢ+1 Aᵢ₊₁}. \]  \hspace{1cm} (2.13)

Now the values of Hₚᵢ₊₁, VPᵢ,n+1 and VPᵢ₊₁,n+1 can be determined from Eqs. (2.10) to (2.12).

2.4 BRANCH CONNECTION:

The following derivation applies to a junction of m pipes. Fig. (2.4-a) shows the assumed initial steady state flow directions. A bar on the subscript indicates a pipe in which the assumed steady state flow is towards the junction.

For the velocity and pressure head at the junction, the following equations can be written:

The continuity equation:

\[
A₁ Vₚ₁,n+1 + A₃ Vₚ₃,n+1 + \ldots + Aᵢ Vₚᵢ,n+1 + \ldots + Aⱼ Vₚⱼ,n+1 + \ldots + Aₘ Vₚₘ,n+1
\]

\[ = A₂ Vₚ₂,₁ + A₄ Vₚ₄,₁ + \ldots + A₊₁ Vₚ₊₁,₁ + \ldots + \]

\[ + Aₘ Vₚₘ,₁ \] \hspace{1cm} (2.14)
The equation for common pressure head:

\[ HP_{1,n+1} = HP_{2,1} = HP_{3,n+1} = \ldots = HP_{j,n+1} = \ldots = HP_{m,1} \]

The positive characteristic equations for the pipes in which the assumed steady state flow is towards the junction:

\[ VP_{1,n+1} = C_{31} - C_{21} HP_{1,n+1} \]
\[ VP_{3,n+1} = C_{33} - C_{23} HP_{3,n+1} \]

The negative characteristic equations for the pipes in which the assumed steady state flow is away from the junction:

\[ VP_{2,1} = C_{12} + C_{22} HP_{2,1} \]
\[ VP_{j+1,1} = C_{1j} + C_{2j} HP_{j+1,1} \]
\[ VP_{m,1} = C_{1m} + C_{2m} HP_{m,1} \]

Solving the above equations simultaneously, one obtains
Note: Arrows indicate initial steady state flow direction.
\[ H_{j+1,n+1} = \{(C_{31} A_{j1} + C_{32} A_{j2} \cdots + C_{3j} A_{j}) + \ldots + C_{3} A_{j} \} \]

\[ - (C_{12} A_{j2} \cdots + C_{1,j+1} A_{j+1} + \ldots + C_{1} A_{1})/ \]

\[ \{C_{21} A_{j1} + C_{22} A_{j2} \cdots + C_{2j} A_{j} + C_{2,j+1} A_{j+1} + \ldots \]

\[ \ldots + C_{2} A_{1}\} . \]  

(2.15)

The remaining \((2m-1)\) unknowns can be determined from the remaining \((2m-1)\) equations.

Example:

For a branch connection of three pipes, in which the initial steady state flow directions are as shown in Fig. (2.4-b), Eq. (2.15) reduces to

\[ H_{j,n+1} = \frac{C_{31} A_{j1} - C_{11} A_{j+1} - C_{1} A_{1}}{C_{21} A_{j1} + C_{22} A_{j+1} + C_{2} A_{1}} . \]  

(2.16)

The values of the remaining unknowns can be determined from the following equations:

\[ H_{j+1,1} = H_{j+2,1} = H_{j,n+1} . \]  

(2.17)

\[ V_{j,n+1} = C_{3} A_{j} - C_{2} A_{j} \]  

(2.18)

\[ V_{j+1,1} = C_{1} A_{j+1} + C_{2} A_{j+1} \]  

(2.19)

\[ V_{j+2,1} = C_{1} A_{j+2} + C_{2} A_{j+2} \]  

(2.20)

2.5 THE SPRINKLERS:

A sprinkler is an orifice on an irrigation pipe. The distance between two consecutive sprinklers on a pipe should be either more than or equal to the value of \(\Delta x\) for the pipe.
Boundary conditions for the following cases are developed:

a) Sprinklers located at the junction of two pipes having different diameters and wall thicknesses (Fig. 2.5-a):

For the steady state conditions,

\[
Q_{\text{osp}} = C_{\text{do}} A_{\text{sp}} \sqrt{2gH_{\text{osp}}}, \quad (2.21)
\]

in which \(Q_{\text{osp}}\) is the steady state discharge of the sprinkler, \(A_{\text{sp}}\) is the cross-sectional area of the sprinkler, \(C_{\text{do}}\) is the coefficient of discharge and \(H_{\text{osp}}\) is the steady state pressure head at the sprinkler. If \(Q_{\text{sp}}^{P}\) denotes the transient state discharge through the sprinkler, \(H_{\text{sp}}^{P}\) is the transient state pressure at the sprinkler and if it is assumed that unsteady flow through the sprinkler follows the steady state law, then

\[
Q_{\text{sp}}^{P} = C_{\text{d}} A_{\text{sp}} \sqrt{2gH_{\text{sp}}^{P}}, \quad (2.22)
\]

By dividing Eq. (2.22) by Eq. (2.21) and assuming that the coefficient of discharge is constant, one obtains

\[
\frac{Q_{\text{sp}}^{P}}{Q_{\text{osp}}} = \frac{C_{\text{d}}}{C_{\text{do}}} \frac{A_{\text{sp}}}{A_{\text{sp}}} \sqrt{\frac{2gH_{\text{sp}}^{P}}{H_{\text{osp}}}} \cdot
\]

Let

\[
C_{\text{sp}} = \frac{Q_{\text{osp}}}{\sqrt{H_{\text{osp}}}} \cdot
\]

Then,

\[
Q_{\text{sp}}^{P} = C_{\text{sp}} \sqrt{H_{\text{sp}}^{P}} \cdot \quad (2.23)
\]

This equation is called the characteristic equation for a sprinkler.
For the velocity and pressure head at sprinkler, the following equations can be written:

The positive characteristic equation for section \((j, n+1)\):

$$V_{P,j,n+1} = C_3^j - C_2^j H_{P,j,n+1} \quad (2.24)$$

The negative characteristic equation for section \((j+1, 1)\):

$$V_{P,j+1,1} = C_1^{j+1} + C_2^{j+1} H_{P,j+1,1} \quad (2.25)$$

The continuity equation:

$$V_{P,j,n+1} A_j = Q_{P,sp} + V_{P,j+1,1} A_{j+1} \quad (2.26)$$

Common pressure head:

$$H_{P,j,n+1} = H_{P,j+1,1} = H_{P,sp} \quad (2.27)$$

It follows from Eqs. (2.23) to (2.27) that

$$(C_3^j - C_2^j H_{P,j,n+1}) A_j = C_{sp} \sqrt{H_{P,j,n+1}} + (C_1^{j+1} + C_2^{j+1} H_{P,j,n+1}) A_{j+1}.$$  

Let

$$H_{sq} = \sqrt{H_{P,j,n+1}}.$$  

Then, rearrangement of the terms of the above equation yields

$$(C_2^j A_j + C_2^{j+1} A_{j+1}) H_{sq}^2 + C_{sp} H_{sq} - (C_3^j A_j - C_1^{j+1} A_{j+1}) = 0.$$  

Solving this quadratic equation for \(H_{sq}\), one obtains

$$H_{sq} = \left[-C_{sp} \pm \sqrt{C_{sp}^2 + 4(C_3^j A_j - C_1^{j+1} A_{j+1})(C_2^j A_j + C_2^{j+1} A_{j+1})}\right]/\{2(C_2^j A_j + C_2^{j+1} A_{j+1})\}.$$  

Put

$$C_k = 2(C_2^j A_j + C_2^{j+1} A_{j+1})/ C_{sp}.$$  

If the negative sign with the radical term is discarded, the expression for \(H_{sq}\)
becomes

\[ H_{sq} = \left( -1 + \sqrt{1 + \frac{2C_k}{C_{sp}}} \left( \frac{C_3}{A_j} - \frac{C_1}{A_{j+1}} \right) \right) / C_k. \]  

(2.28)

The values of \( H_{j,n+1} \) and \( H_{j+1,1} \) are obtained from the following equations:

\[ H_{j,n+1} = H_{j+1,1} = \left( H_{sq} \right)^2. \]  

(2.29)

Having determined \( H_{j,n+1} \) and \( H_{j+1,1} \), the values of \( V_{j,n+1} \) and \( V_{j+1,1} \) can be determined from Eqs. (2.24) and (2.25).

b) **Sprinklers on a pipe of constant diameter and wall thickness**

(Fig. 2.5-b):

In order to have not more than three subscripts (the maximum allowed on the IBM 7040/44) the sections on a distributor are designated as shown in Fig. 2.5-b. If the section on the upstream side of a sprinkler is designated as \( m \), then the section downstream of it is designated as \( m+1 \).

In this case,

\[ A_j = A_{j+1} \]

and

\[ C_{2,j} = C_{2,j+1} \]

By virtue of these relations, Eq. (2.28) reduces to

\[ H_{sq} = \frac{-1 + \sqrt{1 + \frac{2A_j C_k}{C_{sp}}} \left( \frac{C_3}{C_{j+1}} - \frac{C_1}{C_{j+1}} \right) / C_{sp}}{C_k}. \]  

(2.30)

in which

\[ C_k = \frac{4C_2 A_j}{C_{sp}}. \]
(a) SPRINKLER AT THE JUNCTION OF TWO PIPES

(b) SPRINKLER ON A PIPE

FIG. 2.5
The values of $H_{p,j,n}$, $V_{p,j,n}$, and $V_{p,j,n+1}$ can be determined from the following equations:

$$H_{p,j,n} = H_{p,j,n+1} = H_{sq}^2.$$  

$$V_{p,j,n} = C_{3,j} - C_{2,j} H_{p,j,n}. \quad (2.32)$$

$$V_{p,j,n+1} = C_{1,j} + C_{2,j} H_{p,j,n+1}. \quad (2.33)$$

**c) Sprinkler at the downstream end of a pipe:**

In this case,

$$A_{j+1} = 0, \quad C_{2,j+1} = 0.$$  

By using these relations, Eq. (2.28) can be written as

$$H_{sq} = \frac{-1 + \sqrt{1 + 2c_k \frac{C_{3,j} A_j}{C_{sp}}}}{c_k}, \quad (2.31)$$

in which

$$C_k = \frac{2c_{2,j} A_j}{C_{sp}}.$$  

Now the values of $H_{p,j,n+1}$ and $V_{p,j,n+1}$ can be determined from the following equations:

$$H_{p,j,n+1} = H_{sq}^2. \quad (2.32)$$

$$V_{p,j,n+1} = C_{3,j} - C_{2,j} H_{p,j,n+1}. \quad (2.33)$$

**2.6 THE VALVES:**

The boundary conditions for four different locations of the valves
are developed in this section. Various types and combinations of the valves used to limit the transient pressures within an allowable range upon pump failure, are discussed in Section 2.9.

The effective gate opening, \( \tau \), is a function of either time, \( t \), or pressure, \( p \) (henceforth called the \( \tau-t \) or \( \tau-p \) relationship). This relationship is given either by a formula or by a set of numerical values. In the latter case, the given values are stored in the computer and for an intermediate value of \( t \) or \( p \), the value of \( \tau \) is determined by parabolic interpolation.

**Formulae for the effective gate opening:**

(a) **The \( \tau-t \) relationship:**

For example, \( \tau-t \) relationship for gate closure may be given by

\[
\tau = (1 - \frac{t}{t_c})^m,
\]

in which \( t_c \) is the time required for total gate closure.

(b) **The \( \tau-p \) relationship:**

This relationship is specified in the case of a spring loaded valve. This type of valve opens when the pressure, \( p \), at the valve exceeds \( p_{in} \), is fully open when \( p \geq p_{fin} \), and closes automatically when \( p < p_{fin} \). The values of \( p_{in} \) and \( p_{fin} \) depend upon the stiffness of the spring. Assuming a \( \tau \sim p \) curve of degree \( m \), (Fig. 2.6-b), \( \tau \) at any pressure \( p \), \((p_{in} < p < p_{fin})\), is given by

\[
\tau = k (p - p_{in})^m,
\]

in which \( k \) is a constant.
\( \tau - t \) RELATIONSHIP FOR A VALVE

**FIG. 2.6 (a)**

<table>
<thead>
<tr>
<th>( t )</th>
<th>0.0</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau )</td>
<td>1.0</td>
<td>0.9</td>
<td>0.7</td>
<td>0.5</td>
<td>0.3</td>
<td>0.1</td>
<td>0.0</td>
</tr>
</tbody>
</table>

\( \tau - p \) RELATIONSHIP FOR A VALVE

**FIG. 2.6 (b)**
It is clear from Fig. 2.6-b that at $p = p_{\text{fin}}$, $t = 1$. Hence

$$l = k (p_{\text{fin}} - p_{\text{in}})^m,$$

so that

$$k = (p_{\text{fin}} - p_{\text{in}})^{-m} \quad (2.37)$$

Elimination of $k$ from Eqs. (2.36) and (2.37), yields

$$t = \frac{(p - p_{\text{in}})}{(p_{\text{fin}} - p_{\text{in}})^m} \quad (2.38)$$

If the values of $p_{\text{in}}$ and $p_{\text{fin}}$ are given in lbs/in$^2$ and the transient pressure head, $H$, is given in feet of water, then

$$p = 0.433H,$$

in which $p$ is the transient pressure in lbs/in$^2$. Substitution of this value of $p$ into Eq. (2.38) gives

$$t = \left(\frac{0.433H - p_{\text{in}}}{p_{\text{fin}} - p_{\text{in}}}\right)^m \quad (2.39)$$

**CASE I. VALVE AT THE DOWNSTREAM END (Fig. 2.6-c):**

If the valve is discharging into atmospheric pressure, then for the steady state conditions, the gate equation can be written as

$$V_{o_j} A_j = \left(C_d A_g \sqrt{2gH_{o_j,n+1}}\right) \quad (2.40)$$

in which the subscript zero denotes the steady state values, $A_g$ is the area of opening of the valve, and $C_d$ is the coefficient of discharge. By designating the transient state values by the subscript $P$, gate equation for the valve for the transient state conditions can be written as
\[ VP_j A_j = \left( C_d A_e \right)_p \sqrt{2gHP_{j,n+1}}. \]  
(2.41)

By dividing Eq. (2.41) by Eq. (2.40) and making the substitution

\[ \tau = \frac{(C_d A_e)_p}{(C_d A_e)_0}, \]

one obtains

\[ VP_{j,n+1} = \frac{\tau v_{o,j}}{\sqrt{H_{o,j,n+1}}} HP_{j,n+1}. \]

By squaring both sides of the above equation, one gets

\[ VP_{j,n+1}^2 = \tau^2 C_j^2 C_v HP_{j,n+1}. \]  
(2.42)

in which

\[ C_v = \frac{v_{o,j}^2}{C_j^2 H_{o,j,n+1}}. \]

The positive characteristic equation for section \((j,n+1)\) is given by

\[ VP_{j,n+1} = C_3 j - C_2 j HP_{j,n+1}. \]  
(2.43)

Let \( C_4 = \tau^2 C_v \). Note that \( C_4 \) is constant during each time step. Then, elimination of \( HP_{j,n+1} \) from Eqs. (2.42) and (2.43) yields

\[ \frac{1}{C_4} VP_{j,n+1}^2 + VP_{j,n+1} - C_3 j = 0. \]

Solving the above equation for \( VP_{j,n+1} \) and neglecting the negative sign with the radical term, one obtains

\[ VP_{j,n+1} = \frac{1}{2} C_4 (-1 + \sqrt{1 + 4 C_3 j / C_4}). \]  
(2.44)
The value of $HP_{j,n+1}$ can now be determined from either Eq. (2.42) or Eq. (2.43).

When the valve is fully closed i.e. $\tau = 0$, then $VP_{j,n+1} = 0$.

By virtue of the above remark, it follows from Eq. (2.43) that

$$HP_{j,n+1} = \frac{C3_j}{C2_j}$$  \hspace{1cm} (2.45)

CASE II. VALVE AT THE JUNCTION OF TWO PIPES (Fig. 2.6-d):

Considering the positive flow direction as shown in Fig. 2.6-d the boundary conditions for the valve are developed. During the transient state, these can also be used for the reverse flow. But this requires special attention in the computer programme to avoid the possibility of taking the square root of a negative number.

For the transient state conditions at the valve, the following equations can be written:

The positive characteristic equation for section $(j,n+1)$:

$$VP_{j,n+1} = C3_j - C2_j HP_{j,n+1}$$  \hspace{1cm} (2.46)

The negative characteristic equation for section $(j+1,1)$:

$$VP_{j+1,1} = C1_{j+1} + C2_{j+1} HP_{j+1,1}$$  \hspace{1cm} (2.47)

The continuity equation:

$$VP_{j,n+1} A_j = VP_{j+1,1} A_{j+1}$$  \hspace{1cm} (2.48)

The gate equation:
(c) VALVE AT THE DOWNSTREAM END OF A PIPE

(d) VALVE AT THE JUNCTION OF TWO PIPES

FIG. 2.6
\[ VP_{j,n+1} = \frac{\tau V_{o1}}{\sqrt{\Delta H_0}} \sqrt{HP_{j,n+1} - HP_{j+1,1}} , \]  

(2.49)

in which

\[ \Delta H_0 = H_{oj,n+1} - H_{oj+1,1} . \]

From Eq. (2.49), it follows that

\[ VP_{j,n+1}^2 = \frac{\tau^2 V_{o1}^2}{\Delta H_0} (HP_{j,n+1} - HP_{j+1,1}) . \]

By combining Eqs. (2.46) and (2.47) with the above equation, one obtains

\[ VP_{j,n+1}^2 = \frac{\tau^2 V_{o1}^2}{\Delta H_0} \left\{ \frac{C_{3j} - VP_{j,n+1}}{C_{2j}} - \frac{VP_{j+1,1} - C_{1j+1}}{C_{2j+1}} \right\} . \]

Substitution of \( VP_{j+1,1} \) in terms of \( VP_{j,n+1} \) from Eq. (2.48) yields

\[ VP_{j,n+1}^2 = \frac{\tau^2 V_{o1}^2}{\Delta H_0} \left\{ \frac{C_{3j}}{C_{2j}} - \frac{VP_{j,n+1}}{C_{2j}} - \frac{A_j}{A_{j+1}} \frac{VP_{j,n+1}}{C_{2j+1}} \right\} \]

\[ + \frac{C_{1j+1}}{C_{2j+1}} \]  

(2.50)

Let

\[ C_{4j} = \frac{\tau^2 V_{o1}^2}{C_{2j} \Delta H_0} \text{ and } C_{4j+1} = \frac{\tau^2 V_{o1}^2}{C_{2j+1} \Delta H_0} . \]

Then Eq. (2.50) takes the form

\[ VP_{j,n+1}^2 = C_{3j} C_{4j} - C_{4j} VP_{j,n+1} - \frac{A_j}{A_{j+1}} C_{4j+1} VP_{j,n+1} \]

\[ + C_{1j+1} C_{4j+1} , \]
so that

\[ V_{P,j,n+1}^2 + \left( C_{4,j} + \frac{A_j}{A_{j+1}} C_{4,j+1} \right) V_{P,j,n+1} = \left( C_{3,j} C_{4,j} + C\_{1,j+1} C_{4,j+1} \right) \]

Solving this equation for \( V_{P,j,n+1} \) and neglecting the negative sign with the radical term, one obtains

\[ V_{P,j,n+1} = \frac{1}{2} \left\{ \left( -\left( C_{4,j} + \frac{A_j}{A_{j+1}} C_{4,j+1} \right) \right) \right. \\
+ \sqrt{ \left( C_{4,j} + \frac{A_j}{A_{j+1}} C_{4,j+1} \right)^2 - 4\left( C_{3,j} C_{4,j} + C\_{1,j+1} C_{4,j+1} \right) } \right\} 
\]

(2.51)

Now the values of \( H_{P,j,n+1}, \ H_{P,j+1,1} \) and \( V_{P,j+1,1} \) can be determined from Eqs. (2.46) to (2.48).

Similarly, for the flow in the negative direction, one obtains

\[ V_{P,j,n+1} = \frac{1}{2} \left[ \left( C_{4,j} + \frac{A_j}{A_{j+1}} C_{4,j+1} \right) \right. \\
- \sqrt{ \left( C_{4,j} + \frac{A_j}{A_{j+1}} C_{4,j+1} \right)^2 - 4\left( C_{3,j} C_{4,j} + C\_{1,j+1} C_{4,j+1} \right) } \right]. 
\]

(2.52)

If \( C_{4,j} C_{3,j} + C_{4,j+1} C_{1,j} \) is greater than +1, the flow is in the normal direction and Eq. (2.51) is used; otherwise Eq. (2.52) is used.

**SPECIAL CASE:** Valve or orifice at an intermediate section of a pipe or at the
(e) VALVE AT JUNCTION OF TWO PIPES HAVING SAME DIAMETER

(f) ORIFICE AT JUNCTION OF TWO PIPES HAVING SAME DIAMETER

FIG. 2.6
junction of two pipes having the same diameter, wall thickness and wall material (Fig. 2.6-e):

In this case,

\[ A_j = A_{j+1}, \quad C_{2j} = C_{2j+1}, \quad \text{and} \quad C_{4j} = C_{4j+1}. \]

Hence, Eq. (2.51) takes the form

\[ V_{P_{j,n+1}} = -C_{4j} + \sqrt{(C_{4j})^2 + C_{4j}(C_{3j} + C_{1j+1})}. \] (2.53)

Similarly, for the flow in the reverse direction, Eq. (2.52) reduces to

\[ V_{P_{j,n+1}} = C_{4j} - \sqrt{(C_{4j})^2 - C_{4j}(C_{3j} + C_{1j+1})}. \] (2.54)

**CASE III. VALVE AT THE UPSTREAM END OF A PIPE. THE VALVE DISCHARGES INTO ATMOSPHERE** (Fig. 2.6-g):

The boundary conditions for a valve at the upstream end of a pipe can be derived by solving the following two equations simultaneously:

The negative characteristic equation for section \((j,1)\):

\[ V_{P_{j,1}} = C_{1j} + C_{2j} H_{P_{j,1}}. \]

The gate equation for the valve:

\[ V_{P_{j,1}} = \frac{V_{oj}}{\sqrt{H_{oj,1}}} \frac{\sqrt{H_{P_{j,1}}}}{\sqrt{H_{oj,1}}} . \] (2.55)

Squaring both sides of Eq. (2.55) and making the substitution

\[ C_{4j} = \frac{\frac{V_{oj}^2}{H_{oj,1}}}{C_{2j}} , \]
one obtains

\[ VP_{j,1}^2 = C_2j C_4j HP_{j,1} \]

Substitution of the value of \( HP_{j,1} \) into the above equation yields

\[ VP_{j,1}^2 - C_4j VP_{j,1} + C_1j C_4j = 0 \]

Solving the above equation for \( VP_{j,1} \) and neglecting the negative sign with the radical term, one obtains

\[ VP_{j,1} = \frac{1}{2} \left( C_4j + \sqrt{C_4j^2 - 4 \cdot C_1j C_4j} \right) \]  \hspace{1cm} (2.56)

Now, the value of \( HP_{j,1} \) can be determined from Eq. (2.55).

**CASE IV. VALVE AT A RESERVOIR** (Fig. 2.6-h):

The following two equations are solved to develop the boundary conditions for the valve at the reservoir:

The negative characteristic equation for section \((j,l)\):

\[ VP_{j,1} = C_1j + C_2j HP_{j,1} \]

The gate equation for the valve:

\[ VP_{j,1} = \frac{\tau V_{oj}}{\sqrt{H_{oj,1}}} \sqrt{H_{res}} - HP_{j,1} \]

Solving the above two equations simultaneously, making the substitution

\[ C_4j = \frac{\tau^2 v_{oj}^2}{H_{oj,1} C_2j} \]
Check valve fully closed on flow reversal

Pipe J

(j,1) steady flow

(j,2)

surge relief valve or surge anticipator

\[ \text{FIG. 2.6} \]

(g) VALVE AT AN UPSTREAM END OF A PIPE

\[ \text{FIG. 2.6} \]

(h) VALVE AT A RESERVOIR
and neglecting the negative sign with the radical term, one obtains

\[ \frac{VP_{j,1}}{C_4} = \frac{1}{2} \left\{ -C_4 + \sqrt{C_4^2 + 4C_4(C_2H_{res} + C_1)} \right\} . \]  

(2.57)

Proceeding in a similar manner, for flow in the reverse direction,

\[ \frac{VP_{j,1}}{C_4} = \frac{1}{2} \left\{ C_4 + \sqrt{C_4^2 - 4C_4(C_2H_{res} + C_1)} \right\} . \]  

(2.58)

2.7 THE SURGE TANKS:

The boundary conditions for an orifice surge tank are developed in this section. The results can, however, be used for a simple surge tank by taking the orifice resistance, \( H_{orf} \), equal to zero.

It is assumed that the time steps are small and the velocity of water in the pipe changes at the end of each time step, as shown in Fig. 2.7-a.

The following two cases are considered:

CASE I. TANK OF LARGE CROSS-SECTIONAL AREA (Fig. 2.7-b):

The cross-sectional area of the tank is assumed to be very large as compared to that of the discharge pipe. The effects of the waterhammer waves in the tank can, therefore, be neglected.

The discharge, \( q \), through the orifice of the tank is considered positive when water discharges out of the tank. By designating the cross-sectional
area of the surge tank by \( A_s \) and other variables as shown in Fig. 2.7-b, for the tank, the following equations can be written:

\[
A_s (Z - Z_P) = q \Delta t. \tag{2.59}
\]

\[
H_{P,j,n+1} = Z_P - H_{orf}. \tag{2.60}
\]

\[
H_{orf} = \frac{H_{orf}}{q_o^2} q^2, \tag{2.61}
\]

in which \( H_{orf} \) is the throttling loss in feet corresponding to a discharge of \( q_o \) cu. ft/sec. and \( H_{orf} \) is the throttling loss in feet corresponding to a discharge of \( q \) cu. ft/sec.

For the transient state conditions at the junction of the tank and the discharge pipe, the following equations can be written:

The equation for common pressure head:

\[
H_{P,j+1,1} = H_{P,j,n+1}. \tag{2.62}
\]

The continuity equation:

\[
V_{j,n+1} A_j + q = V_{j+1,1} A_{j+1}. \tag{2.63}
\]

The positive characteristic equation for section \((j,n+1)\):

\[
V_{P,j,n+1} = C_3_j - C_2_j H_{P,j,n+1}. \tag{2.64}
\]

The negative characteristic equation for section \((j+1,1)\):

\[
V_{P,j+1,1} = C_1_{j+1} + C_2_{j+1} H_{P,j+1,1}. \tag{2.65}
\]

It follows from Eqs. (2.61) and (2.63) that

\[
H_{orf} = \frac{H_{orf}}{q_o^2} (V_{j+1,1} A_{j+1} - V_{j,n+1} A_j) \left| V_{j+1,1} A_{j+1} - V_{j,n+1} A_j \right|. \tag{2.66}
\]
(a) VELOCITY CHANGES IN THE DISCHARGE PIPE

(b) SURGE TANK AT JUNCTION OF TWO PIPES

FIG. 2.7
Elimination of \( q \) from Eqs. (2.59) and (2.53) yields

\[
Z_P = Z - \frac{\Delta t}{A_s} (V_{j+1,1} A_{j+1} - V_{j,n+1} A_j).
\]  \hspace{1cm} (2.67)

Substituting these values of \( H_{orf} \) and \( Z_P \) into Eq. (2.60) one obtains

\[
H_{P,j,n+1} = Z - \frac{\Delta t}{A_s} (V_{j+1,1} A_{j+1} - V_{j,n+1} A_j) - \frac{H_{orf}}{2 q_o} (V_{j+1,1} A_{j+1} - V_{j,n+1} A_j) \left[ V_{j+1,1} A_{j+1} - V_{j,n+1} A_j \right].
\]  \hspace{1cm} (2.68)

Let

\[
C_5 = \frac{H_{orf}}{2 q_o},
\]

\[
C_6 = \frac{\Delta t}{A_s},
\]

and

\[
C_7 = V_{j+1,1} A_{j+1} - V_{j,n+1} A_j.
\]

Then,

\[
Z_P = Z - C_6 C_7
\]  \hspace{1cm} (2.69)

and

\[
H_{P,j} = Z_P - C_5 C_6 |C_7|.
\]  \hspace{1cm} (2.70)

The values of \( H_{P,j+1,1} \), \( V_{P,j,n+1} \) and \( V_{P,j+1,1} \) can now be determined from Eqs. (2.62), (2.64) and (2.65).
SPECIAL CASE: The surge tank located adjacent to the pump. The check valve closes simultaneously with pump failure;

The check valve closes instantly when the flow in the discharge pipe reverses. The assumption that the check valve closes simultaneously with the pump failure is fully justified for a pump of small moment of inertia. In this case, upon pump failure, the flow reverses after a short interval of time. With this assumption, the pump characteristics are eliminated from the water-hammer computations.

In this case,

\[ V_{j,n+1} = 0 \quad \text{and} \quad A_j = 0. \]

Hence, Eqs. (2.67) and (2.68) become

\[ Z_P = Z - \frac{\Delta t}{A_s} V_{j+1,1} A_{j+1}, \]

and

\[ H_{P j+1,1} = Z - \frac{\Delta t}{A_s} V_{j+1,1} A_{j+1} - \frac{H_{orfo}}{q_o^2} A_{j+1}^2 V_{j+1,1} \left| V_{j+1,1} \right|. \] (2.71)

Let

\[ C_8 = \frac{\Delta t}{A_s} A_{j+1}, \]

and

\[ C_9 = \frac{H_{orfo}}{q_o^2} A_{j+1}^2. \]

By using these relations, the equations for \( Z_P \) and \( H_{P j+1,1} \) can be written as

\[ Z_P = Z - C_8 V_{j+1,1}, \quad (2.72) \]
and

\[ HP_{j+1,1} = ZP - C_9 v_{j+1,1} \left| v_{j+1,1} \right| \]  \hspace{1cm} (2.73)

Now the value of \( VP_{j+1,1} \) can be determined from Eq. (2.65).

**CASE II. A SURGE TANK OF SMALL CROSS-SECTIONAL AREA** (Fig. 2.7-d)

Because of the smaller cross-sectional area of the tank as compared to that of the discharge pipe, the waterhammer waves in the tank cannot be neglected. The surge tank is, therefore, treated as a pipe having a free water surface. Considering the flow directions shown in Fig. 2.7-d as positive, the following equations can be written for the transient state conditions at the junction of the tank and the discharge pipe:

The continuity equation:

\[ VP_{j,n+1} \cdot A_j = VP_{j+1,1} \cdot A_{j+1} + VP_{j+2,1} \cdot A_{j+2} \]  \hspace{1cm} (2.74)

The equation for the common pressure head:

\[ HP_{j,n+1} = HP_{j+1,1} = HP_{j+2,1} \]  \hspace{1cm} (2.75), (2.76)

The positive characteristics equation for section \((j,n+1)\):

\[ VP_{j,n+1} = C_3 \cdot HP_{j,n+1} \]  \hspace{1cm} (2.77)

The negative characteristic equation for section \((j+1,1)\):

\[ VP_{j+1,1} = C_1 \cdot v_{j+1} + C_2 \cdot HP_{j+1,1} \]  \hspace{1cm} (2.78)

The negative characteristic equation for section \((j+2,1)\):

\[ VP_{j+2,1} = C_1 \cdot v_{j+2} + C_2 \cdot HP_{j+2,1} \]  \hspace{1cm} (2.79)
It follows from Eqs. (2.74) to (2.79) that

\[
(C_3_j - C_2_j H_p_{j,n+1}) A_j = (C_1_{j+1} + C_2_{j+1} H_p_{j,n+1}) A_{j+1} \\
+ (C_1_{j+2} + C_2_{j+2} H_p_{j,n+1}) A_{j+2}.
\]

Simplification and rearrangement of the terms of the above equation yields

\[
(C_2_j A_j + C_2_{j+1} A_{j+1} + C_2_{j+2} A_{j+2}) H_p_{j,n+1} = C_3_j A_j - C_1_{j+1} A_{j+1} - C_1_{j+2} A_{j+2},
\]

so that

\[
H_p_{j,n+1} = \frac{(C_3_j A_j - C_1_{j+1} A_{j+1} - C_1_{j+2} A_{j+2})}{(C_2_j A_j + C_2_{j+1} A_{j+1} + C_2_{j+2} A_{j+2})}.
\] (2.80)

Now the values of \(H_p_{j+1,1}, H_p_{j+2,1}, V_p_{j,n+1}, V_p_{j+1,1}\) and \(V_p_{j+2,1}\) can be determined from Eqs. (2.75) to (2.79).

Moreover, at the free water surface of the tank

\[
H_p_{j+2,n+1} = Z_p,
\] (2.81)

\[
V_p_{j+2,n+1} = C_3_j A_{j+2} - C_2_{j+2} H_p_{j+2,n+1}.
\] (2.82)

and

\[
(Z_p - Z) A_{j+2} = (V_p_{j,n+1} A_j - V_p_{j+1,1} A_{j+1}) \Delta t.
\]

Hence

\[
Z_p = Z + \frac{\Delta t}{A_{j+2}} (V_p_{j,n+1} A_j - V_p_{j+1,1} A_{j+1}).
\] (2.83)

The values of \(H_p_{j+2,n+1}\) and \(V_p_{j+2,n+1}\) can be determined from Eqs. (2.81) and (2.82).
Steady state water surface

Transient state W.S. at the beginning of time step

Transient state W.S. at the end of time step

(c) SURGE TANK NEAR THE PUMP

Surge tank of small cross-sectional area

(d) SURGE TANK OF SMALL CROSS-SECTIONAL AREA

FIG. 2.7
SPECIAL CASE. A surge tank of variable cross-sectional area (Fig. 2.7-e):

In this case Eqs. (2.81) and (2.83, become

\[ H_{j+2, n+1} = L_{j+2} + Z_P \]  

and

\[ Z_P = Z + V_P j+2, n+1 A_{j+2} \frac{\Delta t}{A_s} \]

(2.85)

in which \( L_{j+2} \) is the length of pipe \( j+2 \). The rest of the equations are the same as for Case II.

2.8 THE AIR CHAMBER:

The pressure and volume of air in the chamber follow the law

\[ H^* \cdot v^{m}_{air} = \text{constant}, \]

in which \( H^* \) and \( v^{m}_{air} \) are the absolute pressure and volume of air in the chamber. For the adiabatic and isothermal expansion of air, the values of \( m \) are equal to 1.4 and 1.0 respectively. The orifice in the chamber may be simple or of the differential type. The differential type of orifice throttles the reverse flow of water from the discharge pipe into the chamber while very little throttling is provided for the flow out of the chamber. If there is no orifice in the chamber, the throttling loss is taken equal to zero.

It is assumed that the time intervals are small and the velocity of water in the discharge pipe changes at the end of each time interval, as shown in Fig. 2.7-a.
Transient state W.S. at the end of time interval

Transient state W.S. at the beginning of time interval

Steady state W.S.

(e) SURGE TANK OF VARIABLE CROSS-SECTIONAL AREA

FIG. 2.7
The flow out of the chamber is considered positive.

The following equations can be written for the volume of air in the chamber:

\[ v_{\text{air}}^{P_\ell} = (Z_a - Z_P) A_a , \]  

(2.86)

in which \( v_{\text{air}}^{P_\ell} \) is the volume of air at the end of the time interval, \( Z_a \) and \( A_a \) represent the height and the cross-sectional area of the chamber, and \( Z_P \) denotes the height of water surface in the chamber above the centreline of the discharge pipe.

\[ \left[ HP_{j,n+1} - Z_P + 34 + H_{\text{or}} \right] v_{\text{air}}^m = C_{10} , \]  

(2.87)

in which \( H_{\text{or}} \) is the orifice friction loss (in ft.) corresponding to a discharge of \( q \) cu. ft/sec. and \( C_{10} \) is a constant given by

\[ C_{10} = H^*_o \left( v_{\text{air}} \right)^m , \]

in which \( H^*_o \) and \( v_{\text{air}} \) denote the initial steady state absolute pressure head and volume of air in the chamber.

For the transient state conditions at the junction of the chamber and the discharge pipe, the following equations can be written:

The continuity equation:

\[ V_{j+1,l} A_{j+1} \Delta t = V_{j,n+1} A_j \Delta t + (v_{\text{air}}^{P_\ell} - v_{\text{air}}) . \]

so that

\[ v_{\text{air}}^{P_\ell} = v_{\text{air}} + C_{11} \Delta t , \]  

(2.88)
in which
\[ C_{11} = V_{j+1,1} A_{j+1} - V_{j,n+1} A_j \]
The positive characteristic equation for section \((j,n+1)\):
\[ V_{P,j,n+1} = C3_j - C2_j H_{P,j,n+1} \] \hspace{1cm} (2.89)
The negative characteristic equation for section \((j+1,1)\):
\[ V_{P,j+1,1} = C1_{j+1} + C2_{j+1} H_{P,j+1,1} \] \hspace{1cm} (2.90)

Moreover, the pressure heads at the junction of the chamber and discharge pipe are equal, that is
\[ H_{P,j,n+1} = H_{P,j+1,1} \] \hspace{1cm} (2.91)
The orifice friction loss is given by
\[ H_{orf} = C_{orf} \frac{H_{orf}}{q_o} q^2 \]
in which \( C_{orf} \) is the orifice coefficient. For a simple orifice, \( C_{orf} = 1.0 \).

For a differential orifice: \( C_{orf} = 1.0 \) when water flows out of the chamber i.e. when
\[ V_{j+1,1} A_{j+1} - V_{j,n+1} A_j > 0 ; \]
\( C_{orf} = k_1 \) when water flows into the chamber i.e. when
\[ V_{j+1,1} A_{j+1} - V_{j,n+1} A_j < 0 , \]
where \( k_1 \) depends upon the amount of throttling provided by the orifice.

Substituting the value of \( q \) in the above equation, one obtains
\[ H_{orf} = C_{orf} \frac{H_{orf}}{q_o} \frac{1}{2} (V_{j+1,1} A_{j+1} - V_{j,n+1} A_j) \left| V_{j+1,1} A_{j+1} - V_{j,n+1} A_j \right| \]
so that
\[
H_{orf} = C_{orf} C_f C_{11} \left| C_{11} \right| ,
\]
(2.92)
in which
\[
C_f = \frac{H_{orf}}{2 q o} .
\]

Elimination of \( v_P_{air} \) from Eqs. (2.86) and (2.88) yields
\[
(Z_a - ZP) A_a = V_{air} + C_{11} \Delta t ,
\]
so that
\[
ZP = Z_a - \frac{V_{air}}{A_a} - \frac{C_{11} \Delta t}{A_a} .
\]
(2.93)

Substituting the values of \( ZP, v_P_{air} \) and \( H_{orf} \) into Eq. (2.87), one obtains
\[
\{H_{j,n+1} - Z_a + \frac{V_{air}}{A_a} + \frac{C_{11} \Delta t}{A_a} + 34 \}
\]
\[
+ C_{orf} C_f C_{11} \left| C_{11} \right| \} \{v_{air} + C_{11} \Delta t\}^m = C_{10} .
\]

Simplification and rearrangement of the terms of the above equation yields
\[
H_{j,n+1} = Z_a - \frac{V_{air} + C_{11} \Delta t}{A_a} - 34 - C_{orf} C_f C_{11} \left| C_{11} \right|
\]
\[
+ \frac{C_{10}}{(v_{air} + C_{11} \Delta t)^m} ,
\]
so that
\[
H_{j,n+1} = \frac{C_{10}}{(v_{air} + C_{11} \Delta t)^m} - \left\{ \frac{V_{air} + C_{11} \Delta t}{A_a} + (34 - Z_a) \right\}
\]
\[
+ C_{orf} C_f C_{11} \left| C_{11} \right| \}
\]
(2.94)
(a) AIR CHAMBER AT JUNCTION OF TWO PIPES

(b) AIR CHAMBER NEAR THE PUMP

FIG. 2.8
Let
\[ C_{ch} = 34 - Z_a \]  \hspace{1cm} (2.95)
and
\[ C_{air} = v_{air} + c_{ll} \Delta t \]  \hspace{1cm} (2.96)

Then
\[ HP_{j,n+1} = \frac{C_{10}}{(C_{air})^m} - \left( \frac{C_{air}}{A_a} + C_{ch} + C_{orf} \right) \left( C_{ll} \right) \]  \hspace{1cm} (2.97)

Now the values of \( VP_{j,n+1}, VP_{j+1,1} \) and \( HP_{j+1,1} \) can be determined from Eqs. (2.89) to (2.91). The value of \( v_{P_{air}} \) is computed from the equation:
\[ v_{P_{air}} = v_{air} + \frac{1}{2} \left( A_{j+1} (V_{j+1,1} + VP_{j+1,1}) - A_j (V_{j,n+1} + VP_{j,n+1}) \right) \Delta t \]  \hspace{1cm} (2.98)

**SPECIAL CASE:** The air chamber is near the pump. The check valve closes simultaneously with pump failure (Fig. 2.8-b):

Because of the assumption that the check valve closes simultaneously with the pump failure, all the flow in the discharge pipe is either from or into the chamber. This assumption eliminates the pump characteristics from the waterhammer computations.

In this case, \( V_{j,n+1} = 0 \) and \( A_j = 0 \). Hence, Eq. (2.97) reduces to
\[ HP_{j+1,1} = \frac{C_{10}}{(C_{air})^m} - \left( C_{orf} \right) \left( C_{ll} \right) + C_{ch} + \frac{C_{air}}{A_a} \]  \hspace{1cm} (2.99)
in which

\[ C_{11} = V_{j+1} A_{j+1} \]

Now the values of \( V_{P_{j+1,1}} \) and \( v_{P_{air}} \) can be determined from Eqs. (2.90) and (2.98).

2.9 THE CENTRIFUGAL PUMP:

In this section the dimensionless homologous pump characteristic are introduced. An iteration technique for computing the transient state rotational speed, discharge and pumping head of the pump, upon pump failure, is presented. A pump failure results from any of the following causes:

i) Power outage or voltage dip.

ii) Pump motor overload.

iii) Emergency stop.

The following parameters are used to store the pump characteristics in the computer memory:

For the head:

\[ \frac{h}{\alpha} \sim \frac{v}{\alpha} , \quad \frac{h}{v^2} \sim \frac{\alpha}{v} \]

and for the torque:

\[ \frac{\beta}{\alpha^2} \sim \frac{v}{\alpha} \quad \text{and} \quad \frac{\beta}{v^2} \sim \frac{\alpha}{v} , \]

in which

\[ h = \frac{H}{H_R} , \quad v = \frac{Q}{Q_R} , \quad \alpha = \frac{N}{N_R} \quad \text{and} \quad \beta = \frac{T}{T_R} , \]
in which H, Q, N and T are the transient state head, discharge, rotational speed and torque of the pump. The rated values are denoted by the subscript R.

The values of the ratios $R_1$ (equal to $\frac{v}{v}$) and $R_2$ (equal to $\frac{\alpha}{\alpha}$) range between +1 and -1. The values of $\frac{h}{\alpha^2}$, $\frac{h}{v^2}$, $\frac{\beta}{\alpha^2}$ and $\frac{\beta}{v^2}$ are stored in the computer memory. For any value of $\alpha$ and $v$, values of $h$ and $\beta$ are determined by parabolic interpolation. If the sign convention of Table No. 1 for $\alpha$ and $v$ is kept in mind and the ratio ($R_1$ or $R_2$) which has a value less than or equal to unity is used, only one characteristic curve and one point on it are found.

**TABLE NO. 1**

<table>
<thead>
<tr>
<th>Zone of Operation</th>
<th>Sign of $\alpha$</th>
<th>Sign of $v$</th>
<th>$\frac{v}{\alpha}$</th>
<th>$\frac{\alpha}{v}$</th>
<th>Curve to be used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>+</td>
<td>+</td>
<td>x</td>
<td>x</td>
<td>HAN**, BAN</td>
</tr>
<tr>
<td></td>
<td>+</td>
<td>+</td>
<td></td>
<td>x</td>
<td>HVN, BVN</td>
</tr>
<tr>
<td>Energy dissipation</td>
<td>+</td>
<td>-</td>
<td>x</td>
<td></td>
<td>HAD, BAD</td>
</tr>
<tr>
<td></td>
<td>+</td>
<td>-</td>
<td></td>
<td>x</td>
<td>HVD, BVD</td>
</tr>
<tr>
<td>Turbine operation</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td></td>
<td>HAT, BAT</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td></td>
<td>x</td>
<td>HVT, BVT</td>
</tr>
</tbody>
</table>

** In the notation$^{21}$, H refers to a head ratio, A means division by $\alpha^2$, and N represents the normal zone. V means division by $v^2$, D is the zone of energy dissipation, and T is the zone of turbine operation.
a) The rational speed of the pump:

Upon pump failure, an unbalanced torque, \( T \), that depends upon speed, \( \omega \), and the discharge ratio, \( v \), is applied to the rotating parts. If \( \frac{d\omega}{dt} \) is the rate of speed change, then

\[
T = -I \frac{d\omega}{dt},
\]

in which \( I \) is the moment of inertia of the moving parts. Eq. (2.100) can be written in the difference form as

\[
\Delta\omega = - \frac{T \Delta t}{I},
\]

in which \( \Delta\omega \) is the change in the rotational speed in radians/sec during a time interval of \( \Delta t \) seconds.

The values of the average discharge ratio, \( \bar{v}_i \), and the average speed ratio, \( \bar{a}_i \), for the \( i \)th time step may be estimated by the following equations:

\[
\bar{v}_i^{(0)} = \bar{v}_{i-1} + \frac{1}{2} \Delta v_{i-1}
\]

(2.101)

and

\[
\bar{a}_i^{(0)} = \bar{a}_{i-1} + \frac{1}{2} \Delta a_{i-1}
\]

(2.102)

in which \( \Delta v_{i-1} \) and \( \Delta a_{i-1} \) are the changes in \( v \) and \( a \) during the \( (i-1) \) time step. The superscript refers to the number of iterations performed. For example, \( \bar{v}_i^{(0)} \) denotes the initial estimated discharge ratio for the \( i \)th time interval. The average discharge and speed ratios for the \( i \)th time step after \( n \) iterations are represented by \( \bar{v}_i^{(n)} \) and \( \bar{a}_i^{(n)} \).
As outlined above, for these values of $\nu_i^{(0)}$ and $a_i^{(0)}$, torque ratio, $\beta_i^{(0)}$, is determined using parabolic interpolation. Now the change in the rotational speed of the pump, $\Delta a_i^{(0)}$, can be computed from the formula

$$\Delta a_i^{(0)} = - \frac{30 T_R \Delta t}{I N_R H} \beta_i^{(0)},$$

so that

$$\Delta a_i^{(0)} = C_p \beta_i^{(0)},$$

in which

$$C_p = - \frac{30 T_R \Delta t}{I N_R H}.$$  \hspace{1cm} (2.104)

Hence

$$a_i^{(1)} = a_{i-1}^{(0)} + \frac{1}{2} \Delta a_i^{(0)},$$

in which $a_{i-1}^{(0)}$ is the rotational speed at the end of $(i-1)$ time interval. If

$$\left| a_i^{(1)} - a_i^{(0)} \right| > \epsilon_1,$$

in which $\epsilon_1$ is the specified tolerance, then the above process is repeated considering $a_i^{(1)}$ as the initial estimated value. Suppose that after $n$ iterations

$$\left| a_i^{(n)} - a_i^{(n-1)} \right| \leq \epsilon_1.$$  \hspace{1cm} (2.105)

Then the speed ratio, $a_i$, at the end of the $i^{th}$ time interval is given by the equation

$$a_i = a_i^{(n)}.$$
\[ \alpha_i = \alpha_{i-1} + \Delta \alpha_i^{(n)} \tag{2.105} \]

b) **The Pumping Head and Discharge:**

The velocity, \( V_{P_i}^{(0)} \), in the discharge pipe at the end of the \( i \)th time step is estimated by the equation

\[ V_{P_i}^{(0)} = V_{i-1} + \Delta V_{i-1} \]

in which \( V_{i-1} \) is the velocity at the end of \((i-1)\) time interval and \( \Delta V_{i-1} \) denotes the change in the velocity during the \((i-1)\) time interval.

The discharge ratio, \( v_i^{(0)} \), is given by

\[ v_i^{(0)} = \frac{V_{P_i} A}{Q_R} \]

in which \( A \) is the cross-sectional area of the discharge pipe. By substituting \( C_{pm} = \frac{A}{Q_R} \) in the above equation, one obtains

\[ v_i^{(0)} = C_{pm} V_{P_i} \]

For this value of \( v_i^{(0)} \) and for \( \alpha_i \) given by Eq. (2.105), \( h_i^{(0)} \) is interpolated from the pump characteristic curves. The value of \( h_i^{(0)} \) now being known, the value of \( H_{P_i}^{(0)} \) is computed by the equation

\[ H_{P_i}^{(0)} = h_i^{(0)} H_R \]

in which \( H_R \) is the rated head of the pump. The negative characteristic equation for the upstream end of discharge pipe is now used to determine \( V_{P_i}^{(1)} \), i.e.
\[ V_{P_i}^{(1)} = C_1 + C_2 \cdot H_{P_i}^{(0)} \]  

(2.106)

in which \( C_1 \) and \( C_2 \) are constants defined by Eqs. (1.45) and (1.46).

If

\[ \left| V_{P_i}^{(1)} - V_{P_i}^{(0)} \right| > \epsilon_2, \]

in which \( \epsilon_2 \) is the specified tolerance, then considering \( V_{P_i}^{(1)} \) as the initial estimated value, the above process is repeated. If after the \( n^{th} \) iteration,

\[ \left| V_{P_i}^{(n)} - V_{P_i}^{(n-1)} \right| < \epsilon_2, \]

then

\[ V_{P_i} = V_{P_i}^{(n)}. \]  

(2.107)

The flow chart, given in Fig. 2.9-b, illustrates the procedure for computing the transient state conditions caused by a pump failure.

The transient state conditions for the following cases are considered:

(i) **Check valve closes instantly upon flow reversal:**

The flow in the discharge pipe decreases as the speed of the pump decreases upon pump failure. However, the speed of the pump reduces rapidly to a point where no water can be delivered against the existing head. If water is pumped to a reservoir, the flow reverses although the pump may still be rotating in the normal direction. The check valve closes instantly upon flow reversal. This prevents reverse flow through the pump. Thus, the transient state can be divided into two phases:
(a) The flow in the discharge pipe adjacent to the pump is in the positive direction. During this period, the pump works in the normal zone. The pump speed, head and discharge are determined as outlined above.

(b) The flow reverses in the discharge pipe and the check valve closes instantly. Hence

$$V_{Pj,1} = 0 . \quad (2.108)$$

The pressure head is determined from the following negative characteristic equation for the section \((j,1)\):

$$V_{Pj,1} = C_{1j} + C_{2j} \cdot H_{Pj,1} .$$

By substituting \(V_{Pj,1} = 0\), one obtains

$$H_{Pj,1} = -\frac{C_{1j}}{C_{2j}} . \quad (2.109)$$

The procedure for computing the transient state conditions is illustrated by means of a flow chart (See Fig. 2.9-a)

(ii) No check valve. Flow reverses through the pump:

This is similar to case (i) except that reverse flow through the pump is allowed. Thus, the pump operates in all the three zones of operation, i.e. normal, energy dissipation and turbine operation. The flow chart given in Fig. 2.9-b illustrates the procedure for determining the transient state conditions.
START

\[ \dot{v}_i = \frac{v_{i-1} + \frac{1}{2} \Delta v_{i-1}}{a_i} \]

\[ \ddot{a}_i = a_{i-1} + \frac{1}{2} \Delta a_{i-1} \]

\[ R = \frac{\dot{v}_i}{a_i} \]

\[ R > 1.0 \]

Determine \( R \) from BAN

\[ a_i = a_{i-1} \]

\[ \Delta a_i = C_p \Delta \beta_i \]

\[ a_i = a_{i-1} + \frac{1}{2} \Delta a_i \]

\[ |\ddot{a}_i - \dddot{a}_i| \leq \text{TOLER} \]

\[ \dot{a}_i = a_{i-1} + \Delta \dddot{a}_i \]

FLOW CHART FOR PUMP FAILURE, CHECK VALVE CLOSES UPON FLOW REVERSAL

FIG. 2.9 (a)
FLOW CHART FOR PUMP FAILURE, FLOW REVERSES THROUGH THE PUMP

FIG 2.9 (b)
(iii) Upon flow reversal the check valve closes instantly and the relief valve opens gradually (Fig. 2.6-g)

Up to the instant of flow reversal, computations are performed similar to case (i). After flow reversal, due to the sudden closure of the check valve, the pump characteristics are eliminated from the waterhammer computations. Now the relief valve starts to open and the upstream end is analyzed as a valve. For this purpose, the boundary conditions developed in section 2.6, case III are used.

(iv) Upon flow reversal, check valve fails to close, the relief valve opens at a rapid rate but closes slowly afterwards (Fig. 2.9-c):

The velocity and discharge are considered positive as shown in Fig. 2.9-c. For the transient state conditions at the junction the following equations can be written:

The continuity equation:

$$Q_{pump} = Q_{valve} + Q_{pipe}$$  \hspace{1cm} (2.110)

in which $Q_{pump}$, $Q_{valve}$ and $Q_{pipe}$ are the discharges through the pump, relief valve and discharge pipe.

The negative characteristic equation for section $(j,1)$:

$$V_{P,1} = C_{1j} + C_{2j}HP_{j,1}$$  \hspace{1cm} (2.111)

The gate equation for the relief valve:
(c) CHECK VALVE FAILS TO CLOSE UPON FLOW REVERSAL; RELIEF VALVE OPENS GRADUALLY

FIG. 2.9
so that

\[ Q_{P_{valve}} = C_v \tau \sqrt{H_P_{j+1,1}}, \]  

(2.112)

in which

\[ C_v = \frac{Q_o}{\sqrt{H_o}} \]

and \( Q_o \) = discharge through the valve under a pressure head of \( H_o \). The equation for the common pressure head is

\[ H_{P_{j,1}} = H_{P_{j+1,1}} \]  

(2.113)

Furthermore,

\[ Q_{P_{pipe}} = V_{P_{j,1}} A_j, \]

in which \( A_j \) is the cross-sectional area of the discharge pipe. On the basis of Eqs. (2.111) and (2.113), this equation takes the form

\[ Q_{P_{pipe}} = (C_{1,j} + C_{2,j} H_{P_{j+1,1}}) A_j \]  

(2.114)

From Eqs. (2.110), (2.112) and (2.114), it follows that

\[ Q_{P_{pump}} = C_v \tau \sqrt{H_{P_{j+1,1}}} + (C_{1,j} + C_{2,j} H_{P_{j+1,1}}) A_j \]  

(2.116)

Eq. (2.116) cannot be solved explicitly because \( Q_{P_{pump}} \) and \( H_{P_{j+1,1}} \) depend upon the pump characteristics and \( \tau \) depends upon \( \tau \sim t \) relationship. This is solved by an iterative technique, which is illustrated by means of a flow chart (See Fig. 2.9-d).
**Flow Chart Description:**

1. **FROM MAIN PROG**
   - Compute \( V, a, C \)
   - \( \alpha = a_t \cdot \frac{a}{a} \)

2. **FROM SUB PUMP**
   - \( N = [R] + 1 \)
   - If \( N = 2 \), go to \( N = 1 \)
   - Call MVAL

3. **SUBROUTINE MVAL**
   - \( KKK = M/2 \)
   - If \( R < 1 \), go to \( R = 1 \)
   - Call PARABH

4. **RETURN**

5. **SUBROUTINE PARABH**
   - \( HPI = h \cdot H \cdot \alpha \cdot \frac{a}{a} \)
   - \( QPI = (C_1 + C_2 \cdot HPI) \cdot \alpha \cdot \frac{a}{a} \)

6. **PRINT ITERATIONS FAILED**

7. **RETURN**

**Notes:**
1. Output from subr MVAL is value of \( M \)
2. Output from subr PARABH is \( BP \).
3. Output from subr PARABH is \( h \).
4. For expressions for \( C, B, C_v \) see section 2.9.

**Diagram:**

- **Fig 2.9**

---

**Flow Chart for Pump Failure:** Relief valve opens gradually, check valve fails to close.
3.1 DESCRIPTION OF THE IRRIGATION PIPING SYSTEM:

An irrigation piping system consists of the main, the branches, the laterals and the distributors (Fig. 3.2). Water is pumped from the main into the branches, from the branches into the laterals and from the laterals into the distributors. The sprinklers are located on the distributors. In the piping system under consideration, all the laterals and distributors have the same diameter and the distances between any two consecutive sprinklers on a distributor are equal. The laterals and the distributors are assumed to be horizontal. The branch pipes may be inclined.

An air chamber or a surge tank may be provided to keep the water-hammer pressures, caused by pump failure, within an allowable range. A number of valves are installed to cut off the water supply to a certain portion of the system, if so desired. Surge suppressors, relief valves, surge anticipators, by-pass valves and pressure reducers may be present in the system.

3.2 DESIGNATION OF PIPES:

A new method is devised to designate the pipes and pipe sections by
three dimensional and two dimensional subscripted variables. This method is not only helpful in designating the pipes and storing the input data in the computer, but also makes it possible to analyze all the boundary points of the same type by a do-loop.

Three dimensional subscripted variables (maximum allowed on the IBM 7040/44) are used to represent the steady state and transient state conditions at different pipe sections. The junctions of the branch and laterals, and of the laterals and distributors, are numbered in an ascending order in the steady state flow direction (Fig. 3.2-a). If two or more pipes of the same type meet at a joint, they are numbered in the anti-clockwise direction (see Fig. 2.4-a).

The upstream end (according to the steady state flow direction) of a pipe is designated as section 1.

The letters 'B', 'L' and 'D' at the end of the names of the variables refer to the branch, the lateral and the distributor. For example, DB, DL and DD represent the diameter of the branch, lateral and distributor.

If an appurtenance, to be considered as a boundary during the transient state conditions, is located at an intermediate section of a pipe, then the section on each side of the boundary is designated separately. For example, if the section on the upstream side of the appurtenance is designated by m, then the section on the downstream side is designated by m+1 (see Fig. 2.5-b).

The procedure for designation of different pipes is as follows:

(i) Branches:

Two dimensional and three dimensional subscripted variables are used
(a) DESIGNATION OF BRANCH PIPES

(b) DESIGNATION OF LATERALS AND DISTRIBUTORS

Note: Numbers in circles represent the number of junction.

FIG. 3.2
to designate a branch pipe and a section on it. For example, DB(I,J) denotes the diameter of the Jth pipe of the Ith branch, and VB(I,J,K) represents the velocity at the Kth section of the Jth pipe of the Ith branch. In other words, I is the number of the junction of the main and the branch and J is the number of the next junction of the branch and the lateral (Fig. 3.2-a).

(ii) **Laterals:**

Because all the laterals in the system have the same diameter and wall thickness, it is not necessary to specify separately pipe constants for each lateral. The following example illustrates how to designate the conditions at a section of a lateral.

The pressures at the sections on the upstream and on the downstream side of the Kth junction of the Jth lateral of the Ith branch is designated by HL(I,J,2K) and HL(I,J,2K+1) respectively (Fig. 3.2-b).

(iii) **Distributors:**

Since every distributor in the system has the same diameter, wall thickness and sprinkler spacing, only the pipe sections on the distributors are to be specified. The following example illustrates how to designate the transient conditions at different sections of a distributor. The velocities at the sections upstream and downstream of the Mth sprinkler on the Kth distributor of the Jth lateral of the ith branch is denoted by VDi(J,K,2*M) and VDi(J,K,2*M+1) respectively (Fig. 3.2-b). If a piping system of only one branch is to be analyzed the subscript i may be deleted.
CHAPTER IV

DESCRIPTION OF THE PROGRAMME

The computer programme comprises of the main programme and a number of subroutines. It is described by discussing the major functions of the main programme and the individual subroutines. A FORTRAN IV programme to study the transient response of the irrigation pipe system shown in Fig. 8-2 (a) is presented in Appendix C; the flow chart for the programme is given in Fig. 4.1-b. A number of subroutines used to study the transients at different appurtenances in a pipe system, are presented in Appendix D.

4.1 THE MAIN PROGRAMME:

The main programme serves primarily as a device for linking the various subroutines. It performs certain book-keeping and test computations at the beginning and at the end of each time interval. Its principal functions may be listed as follows:

(i) Specification of the storage locations for the subscripted variables (Dimension statement).

(ii) Allocation of the same storage locations for the variables common to the main programme and to the subroutines (Common statement).

(iii) Printing the steady state values.
Notes:
(i) The subscript k denotes the section k on the pipe i,j
(ii) MM = no. of time steps after which values are to be printed
(iii) $T_{last}$ is the time up to which transient computations are to be performed

(a) FLOW CHART FOR A PIPING SYSTEM

FIG. 4.1
(iv) Computation of the value of constants.
(v) Calling the subroutines.
(vi) Printing the transient state conditions.

The flow chart given in Fig. 4.1-a illustrates the sequence of operations in a main programme.

4.2 SUBROUTINE STEADY:

Purpose: This subroutine is used to determine the steady state velocities and pressures in the system.

Description: The steady state pressure is usually known at a junction of the branch pipe and the lateral. The sprinkler discharge is a function of the pressure at the sprinkler. Initially neither the sprinkler discharge nor the pressure is known. These are determined by the iterative procedure given below:

Assuming the pressure head at all the sprinklers on the lateral equal to its known value at the junction, the sprinkler discharge is computed. Hence the velocities and the friction losses between different sections of the distributors and lateral are computed. Having determined the friction losses, the pressures at the sprinklers are corrected and the discharges of the sprinklers corresponding to these pressures are determined. Now, the difference between the initial guessed value and the corrected value of pressure for every sprinkler is computed. If for any sprinkler, the numerical value of this difference is more than the specified tolerance, the process is repeated taking the corrected values of pressure as the initial estimated values. This iterative procedure is continued until results of required accuracy are obtained.
START

Read Data

Compute $\Delta t, N_j$

Compute coefficients and constants for pipes

Call sub "STEADY"

Compute constants for pump & chamber

$T = 0.0$

Print $T, V_{i,j,k}$, $H_{i,j,k}, \alpha, ZP$, and $V_{air}$

$M = 0$

$I$

$T = T + DT$

$M = M + 1$

STOP

$T \leq T_{last}$

Call "INTER"

Call sub. "BR1"

Call sub."BR2"

Call sub."PUMP"

Call sub."CHAM2"

Call sub."RESERVD"

$D\alpha = \alpha_P - \alpha$

$DV = V_{hi,j} - V_{hi,j}$

$V_{i,j,k} = V_{P_{hi,j,k}}$

$H_{i,j,k} = H_{P_{hi,j,k}}$

$V_{air} = v_{P_{air}}$

$\alpha = \alpha_P$

$M = M + M$

$I$

(b) FLOW CHART FOR AN IRRIGATION PIPE SYSTEM

FIG. 4.1 (b)
Now, the discharge and friction loss in the branch pipe are computed and hence the pressure at the junction of the next lateral and the branch pipe is determined. The pressures and discharges of the sprinklers on this lateral are found as outlined above. By proceeding in this manner, the steady state velocities and pressures in the pipe system are determined.

**How to Use:** The subroutine is called by

```
CALL STEADY (NBR,NDIST,NSPR)
```

in which NBR is the number of the branch, NDIST is the number of distributors on a lateral, and NSPR is the number of sprinklers on a distributor.

### 4.3 SUBROUTINE INTER:

**Purpose:** This subroutine is used to determine the velocities and pressures at the intermediate sections of a pipe.

**Description:** Eqs. (1.42) and (1.43) are used to determine the transient state velocities and pressures at the intermediate sections of a pipe. All sections on a pipe except the boundary points are called intermediate sections.

**How to Use:** The subroutine is called by

```
CALL INTER (I,J,K1,K2)
```

in which I is the number of the branch, and K1 and K2 are the first and the last intermediate section on pipe J.

### 4.4 SUBROUTINES BR1 AND BR2:

**Purpose:** These subroutines are used to determine the transient state velocity
and pressure at a junction. BR1 is used for connection of a branch pipe and a lateral and BR2 for the connection of a lateral and a distributor.

**Description:** Boundary conditions developed in Section 2.4 are used to determine the transient state conditions at a branch connection.

**How to Use:** These subroutines are called by

\[
\begin{align*}
\text{CALL BR1} & (I, J, K) \\
\text{CALL BR2} & (I, \text{NLAT, NDIST})
\end{align*}
\]

in which \(I\) is the branch number, \(J\) and \(K\) are the pipe and its last section, \(\text{NLAT}\) is the number of laterals on the branch and \(\text{NDIST}\) is the number of distributors on a lateral.

### 4.5 **SUBROUTINE CHAM1 AND CHAM2:**

**Purpose:** These subroutines compute the transient state conditions at an air chamber. CHAM1 is used when the chamber is situated near the pump and the check valve closes instantly upon pump failure. CHAM2 is used when the chamber is located at some intermediate point in the pipe system.

**Description:** Boundary conditions developed in Section 2.8 are used in these subroutines. The values of the constants for the air chamber are determined in the main programme and are provided in the subroutines through a COMMON statement.

**How to Use:** The subroutines are called by

\[
\begin{align*}
\text{CALL CHAM1} & (1, 1, 1) \\
\text{CALL CHAM2} & (I, J, K, M, N)
\end{align*}
\]

in which \(I\) is the branch number. The chamber is located at the junction of
pipe J and M. The first section on pipe M and the last section on pipe J are represented by N and K respectively.

4.6 SUBROUTINES SURGE1 AND SURGE2:

Purpose: These subroutines are used to determine the transient state conditions at an orifice surge tank. SURGE1 is used when the waterhammer waves in the tank are neglected; SURGE2 is used if the waves in the tank are to be considered.

Description: The boundary conditions developed in Section 2.7 are used to determine the transient state conditions. The constants for the tank are calculated in the main programme, and are provided in the subroutine by a COMMON statement. For a simple surge tank, orifice throttling loss is taken equal to zero.

How to Use: For a surge tank, situated at the junction of pipe J and M, the subroutines are called by

\[
\text{CALL SURGE1 (I, J, K, M, N)} \\
\text{CALL SURGE2 (I, J, K, M, N)}
\]

in which I is the number of the branch, and N and K are the first and the last section on pipe M and J.

4.7 SUBROUTINES RESERU, RESERD AND RESERI:

Purpose: These subroutines are used to determine the transient state conditions
at a reservoir of constant water surface elevation. RESERU, RESERD and RESERI are used when the reservoir is at the upstream end, at the downstream end and at some intermediate point of the system.

Description: The boundary conditions developed in Section 2.1 are used in these subroutines.

How to Use: The subroutines RESERU and RESERD are called by

```fortran
CALL RESERU (I, J, K)
CALL RESERD (I, J, K)
```
in which I is the number of the branch, J is the last pipe in the branch, and K is the last section on the pipe J.

For a reservoir at the junction of pipe J and M, subroutine RESERI is called by

```fortran
CALL RESERI (I, J, K, M, N)
```
in which I is the number of the branch, K is the last section on pipe J and N is the first section of pipe M.

4.8 **SUBROUTINE VALVE:**

**Purpose:** This subroutine is used to determine the transient state conditions caused by opening or closing a valve.

**Description:** The boundary conditions developed in Section 2.6 are used in this subroutine. The effective gate-opening ($\tau$) vs time (t) curve is stored in the computer by storing the values of $\tau$. For any intermediate value of t, the value of $\tau$ is determined by parabolic interpolation. For this, subroutine PARAB is called.
Note that when the valve is fully closed, boundary conditions for a dead end, developed in Section 2.2, are used to determine the transient state conditions at the valve.

**How to Use:** The subroutine is called by

```plaintext
CALL VALVE (I, J, K)
```

in which I is the number of the branch, J is the pipe and K is the last section on pipe J.

### 4.9 SUBROUTINES PUMP1, PUMP2 AND PUMP3:

**Purpose:** These subroutines are used to determine the transient state conditions at a centrifugal pump caused by a pump-failure. Subroutine PUMP1 is used when there is a check valve near the pump which closes upon flow reversal; PUMP2 is used when reverse flow through the pump is allowed. Subroutine PUMP3 is used when there is a check valve which closes instantly and a relief valve which opens gradually upon flow reversal.

**Description:** An iterative procedure, described in section 2.9, is used to determine the transient state conditions. Flow charts given in Fig. 2.9 (a), (b) and (c) illustrate the sequence of computations in the subroutines PUMP1, PUMP2 and PUMP3.

The pump characteristics are stored in the computer as outlined in section 2.9. By calling the subroutine MVAL, the appropriate characteristic curve is selected. The subroutines PARABH and PARABB are called to determine the intermediate values of the pressure head ratio, \( h \), and torque ratio, \( \beta \), by parabolic interpolation.
How to Use: The subroutines are called by the following statements:

\[
\text{CALL PUMP1 } (I, J, K) \\
\text{CALL PUMP2 } (I, J, K) \\
\text{CALL PUMP3 } (I, J, K)
\]

in which I is the number of the branch, J is the pipe and K is the first section on pipe J.

4.10 SUBROUTINE SPRINK:

**Purpose:** This subroutine is used to determine the transient state conditions at a sprinkler.

**Description:** The boundary conditions developed in Section 2.5 are used in this subroutine. The constants for the sprinkler are computed in the main programme and are linked with the subroutine through a COMMON statement.

**How to Use:** This subroutine is called by the statement

\[
\text{CALL SPRINK } (\text{NDIST, NSPR})
\]

in which NDIST is the number of distributors on a lateral and NSPR is the number of sprinklers on a distributor.

4.11 SUBROUTINE MVAL:

**Purpose:** This subroutine is used to select the appropriate pump characteristic curve.

**Description:** A number of conditional statements are used to select the appropriate curve. Flow chart of Fig. 2.9-d illustrates the sequence of
computations.

How to Use: This subroutine is called by

\texttt{CALL MVAL (A, B, C, M)}

in which \(A\) is the discharge ratio \(v\), \(B\) is equal to \(v/\alpha\), \(C\) is the speed ratio \(\alpha\), and \(M\) is the output from the subroutine. The value of \(M\) defines the pump characteristic curve.

4.12 \texttt{SUBROUTINE PARAB:}

\textbf{Purpose:} This subroutine determines the intermediate values of a dependent variable by parabolic interpolation. The values of the dependent variable at equal increments in the independent variable are stored in the computer.

\textbf{Description:} The values of the dependent variable and the value of the interval of the independent variable are given in the subroutine by a COMMON statement.

The initial value of the independent variable should be zero or made equal to zero. For example, the effective gate-opening, \(T\), values for a valve are given at a time interval of \(z\) seconds. If the valve starts to close at time \(t = t_0 = 10.0\) seconds, then the subroutine is called by the statement

\texttt{CALL PARAB (T - 10.0, TAU)}

to determine the value of \(\text{TAU}\) at time \(t = T\).

\textbf{How to Use:} The subroutine is called by

\texttt{CALL PARAB (X, Y)}
in which $X$ is the independent variable, and $Y$ is the dependent variable.

4.13 **FUNCTIONS VEL AND HEAD:**

**Purpose:** If the values of pressure and velocity are known at two points, then their values at any intermediate point are determined by linear interpolation by these function subprogrammes.

**Description:** Eq. (1.38) to (1.41) are used in these subprogrammes.

**How to Use:** The function subprogrammes are called by

VEL($I, J, K, L, M$)

HEAD($I, J, K, L, M$)

in which $I$ is the number of the branch, $K$ and $L$ are the sections of pipe $J$ at which the values of pressure and velocity are known, and $M$ is the type of pipe $J$. For the branch pipe, lateral and distributor, the value of $M$ is equal to 1, 2 and 3 respectively.
5.1 **NUMERICAL EXAMPLES:**

A number of examples are solved to illustrate how to use the boundary conditions. The description of each problem and the results obtained on the digital computer by the method of characteristics and by the graphical method, are presented in Appendix A. The programme-listing of the subroutines used in solving these examples is given in Appendix D.

5.2 **PIPE SYSTEM STUDIES:**

The transient studies, listed below, are made for the irrigation pipe system shown in Fig. B-2(a). The pipes are assumed horizontal and the friction losses are neglected. Transient conditions are caused by a pressure wave whose magnitude remains constant at the upstream end for $t \geq 0$. This is equivalent to a sudden rise in the water surface elevation in reservoir. The following studies are made:

a) Comparison of a transient pressure at different points of the system, when it is analyzed assuming the sprinklers,

1) on the distributors,
ii) on the laterals
and iii) on the branch pipes.

b) The effects of changes in the sprinkler discharge on the magnitude of the waterhammer wave-front. In this case, it is assumed that the sprinklers are located on the branch pipe.

The results for each case are presented in Appendix B.

5.3 COMPUTER PROGRAMME FOR AN IRRIGATION PIPE SYSTEM

The listing of the computer programme to study the transient conditions in the irrigation pipe system shown in Fig. B-2(a) is presented in Appendix C. The transient conditions are caused by a pump-failure. Flow chart for the programme is given in Fig. 4.1-b. In the programme, the slopes of the branch pipes and friction losses in the pipes are considered.
CONCLUSIONS

(1) Equations representing the boundary conditions as derived in this thesis can be used to analyze transient flow conditions in any pipe system.

(2) The validity of these boundary conditions is demonstrated by comparing the results obtained by the method of characteristics with those obtained by the graphical method.

(3) To analyze the transient state conditions in a complex system with large friction losses, the method of characteristics is recommended instead of the graphical method because the former method is faster, more accurate and versatile. However, for a simple system with small friction losses, (up to about 8% of total head), the graphical method is preferable for its clarity, ease of construction and because lumping the losses at certain points does not cause large errors.

(4) In the method of characteristics, like all other available methods, the number of computations is considerably increased if the distance between two boundary points is short.

(5) If friction losses are neglected, the solutions obtained by the method of characteristics closely agree with those obtained by the graphical method.
(6) The method developed to designate the pipes and pipe sections in an irrigation pipe system is helpful in storing the input data in the computer. Also, by this method it is possible to analyze similar types of boundary points by means of a do-loop.

(7) A pressure wave increases in magnitude as it traverses a pipe with step-wise decrease in diameter (see Fig. B-1 for \(Q_{sp} = 0.0\)). A similar surface wave phenomenon experienced in open channels is called funnelling effect.

(8) The magnitude of a pressure wave is reduced when it is reflected or transmitted at a sprinkler. The magnitude of the transmitted or reflected wave decreases as the sprinkler discharge increases.

(9) If the laterals and distributors are deleted and the sprinklers are considered as lumped on the branch pipe, waterhammer pressures are underestimated. The irrigation pipe system shown in Fig. B-2 was analyzed by making this assumption. It resulted in an underestimation of the waterhammer pressure by 34%. The system was analyzed by deleting the distributors and considering the sprinklers as lumped on the laterals. This analysis underestimated the pressure by only 4%. 
BIBLIOGRAPHY


APPENDIX A

NUMERICAL EXAMPLES

A-1. A pipeline with a reservoir at the upstream end. A valve closes at the downstream end.

A-2. Pipes in series with a reservoir at the upstream end. A valve closes at the downstream end.


A-4. Air chamber.

A-5. Pump failure:
   i) Check valve closes instantly upon flow reversal.
   ii) Reverse flow through the pump allowed.
   iii) Check valve closes instantly upon flow reversal. Relief valve opens rapidly and then closes slowly.
PROBLEM:

Determine the transient state pressures and velocities at the points A, B, C, D, E, and F. The transient conditions are caused by closing a valve at the point A. Effective gate opening-time relationship is given by the equation

\[ \tau = (1 - \frac{t}{t_c}) \]

in which \( t_c \) is the time of closure.

DATA:

Length of the pipeline (L) \( = 4253.5 \text{ ft} \).
Static head \( (H_0) \) = 300.0 ft.

Velocity of water in pipeline \( (V_0) \) = 3.5 ft/sec.

Inside diameter of the pipeline \( (D) \) = 3.0 ft.

Friction factor \((f)\) = 0.019

Waterhammer wave velocity \((a)\) = 3963. ft/sec.

Time of closure \((t_c)\) = 5.9 sec.

**CHECK:**

The computed values agree with those obtained by Streeter on the digital computer (Reference 21, page 28).
**PROBLEM:**

Determine the transient state conditions caused by closing a valve at the point A.

**DATA:**

<table>
<thead>
<tr>
<th></th>
<th>Pipe 1</th>
<th>Pipe 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the pipe, in ft.</td>
<td>1500</td>
<td>1800</td>
</tr>
<tr>
<td>Velocity of the waterhammer wave, in ft/sec.</td>
<td>3000</td>
<td>3600</td>
</tr>
<tr>
<td>Inside diameter of the pipe, in ft.</td>
<td>12.91</td>
<td>10.0</td>
</tr>
<tr>
<td>Initial steady state velocity of water, in ft/sec</td>
<td>5.37</td>
<td>8.94</td>
</tr>
<tr>
<td>Static head ( (H_o) ) (in ft)</td>
<td>500</td>
<td></td>
</tr>
</tbody>
</table>
Friction factor = 0.0
Gate closure time relationship is shown above.

CHECK:

Results determined graphically by Parmakian\textsuperscript{12} (page 55) are in close agreement with those obtained on the digital computer, using the method of characteristics (see Fig. A-2).
Graphical method (Parmakian\textsuperscript{12}, pp 55)

Method of characteristics

\[ h \quad v \quad h_A \quad h_B \]

\[ \text{TIME (SEC)} \]

TRANIENT STATE PRESSURES AND VELOCITIES IN THEPIPES IN SERIES
(Reservoir at the upstream end. A valve closes at the downstream end)

FIG. A-2
Determine the transient state conditions caused by closing a valve at the point E. Consider the changes in the elevation of the water surface in the tank.

DATA:

<table>
<thead>
<tr>
<th></th>
<th>Pipe 1</th>
<th>Pipe 2</th>
<th>Surge Tank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the pipe, in ft.</td>
<td>6000.0</td>
<td>1500</td>
<td>-</td>
</tr>
<tr>
<td>Inside diameter of the pipe, in ft.</td>
<td>10.0</td>
<td>5.0</td>
<td>11.55</td>
</tr>
</tbody>
</table>
Waterhammer wave velocity, in ft/sec.  
3000.0  3000.0  2000.0

Initial steady state velocity of water, in ft/sec  
5.37  21.466  0.0

Neglect the friction losses. The relationship for the valve is given above.

CHECK:

1) Considering the waterhammer waves in the tank:

Results obtained by Parmakian\(^\text{12}\) (page 108) by the graphical method agree with those obtained on the digital computer, using the method of characteristics (See Fig. A-3 (a))

ii) Neglecting waterhammer waves in the surge tank:

No check is available. Results obtained are presented in Fig. A-3 (b).
TRANSIENT STATE PRESSURES AND VELOCITIES
(Water-hammer waves in the surge tank considered)

FIG. A-3 (a)
TRANSIENT STATE PRESSURES AND VELOCITIES
(Water-hammer waves in the surge tank neglected)

FIG. A-3(b)
APPENDIX A-4

AIR CHAMBER

PROBLEM:

Determine the transient state pressures and velocities, volume of air and water surface elevation in the air chamber. The transient conditions are caused by pump failure.

DATA:

Check valve closes immediately upon pump failure.
Length of the pipeline \( L \) = 16080 ft.
Cross-sectional area of the pipe \( (A) = 4.0 \text{ ft}^2 \)

Cross-sectional area of the chamber \( (A_a) = 20.0 \text{ ft}^2 \)

Height of the chamber \( (Z_a) = 12.0 \text{ ft} \).

Atmospheric pressure at \( C = 34.0 \text{ ft. of water} \)

Initial steady state water surface elevation in the air chamber \( (Z_o) = 10.0 \text{ ft.} \)

Waterhammer wave velocity \( (a) = 3216 \text{ ft/sec.} \)

Initial steady state velocity of water in the pipeline \( (V_o) = 4.0 \text{ ft/sec.} \)

Orifice throttling loss = 0.0 ft.

Air expansion in the air chamber is isothermal, i.e.

\[
p^* \cdot v_{air} = \text{constant},
\]

in which \( p^* \) and \( v_{air} \) are the absolute pressure and volume of air in the chamber. Neglect the friction losses.

CHECK:

Results obtained on the digital computer using the method of characteristics are close to those obtained by Angus² (page 661) by the graphical method (See Fig. A-4).
FIG. A-4

TRANSIENT STATE CONDITIONS
(Air chamber near the pump. Check valve closes upon pump failure)
APPENDIX A-

PUMP FAILURE

PROBLEM:

Determine the transient state velocity and pressure in the discharge pipe and rotational speed of the pump for the following three cases:

i) Centrifugal pump with a check valve.

ii) Centrifugal pump without a check valve.

iii) Centrifugal pump with a check valve and a relief valve.

The transient conditions are caused by pump failure.

DATA:

Length of the pipeline (L) = 3940 ft.

Inside diameter of the pipe (D) = 32.0 in.

Waterhammer wave velocity (a) = 2820 ft/sec.

Initial steady state velocity in the pipe ($V_o$) = 5.81 ft/sec (for 3 pumps)

Static head ($H_o$) = 220 ft.
WR² of each pump and motor = 384.9 lb/ft².

Rated pump speed (NR) = 1760 p.m.

Pump efficiency (n_R) at rated values = 84.7%

Neglect the friction losses.

Pump characteristic curves* are given by the following table:

<table>
<thead>
<tr>
<th>v/α or α/v **</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>HAN</td>
<td>1.080</td>
<td>1.081</td>
<td>1.082</td>
<td>1.085</td>
<td>1.090</td>
<td>1.095</td>
<td>1.10</td>
<td>1.097</td>
<td>1.080</td>
<td>1.050</td>
<td>1.000</td>
</tr>
<tr>
<td>HVN</td>
<td>- ***</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.112</td>
<td>0.328</td>
<td>0.560</td>
<td>0.761</td>
</tr>
<tr>
<td>HAD</td>
<td>1.080</td>
<td>1.085</td>
<td>1.100</td>
<td>1.120</td>
<td>1.150</td>
<td>1.210</td>
<td>1.310</td>
<td>1.450</td>
<td>1.580</td>
<td>1.730</td>
<td>1.890</td>
</tr>
<tr>
<td>HVD</td>
<td>0.780</td>
<td>0.830</td>
<td>0.900</td>
<td>0.960</td>
<td>1.050</td>
<td>1.120</td>
<td>1.280</td>
<td>1.420</td>
<td>1.560</td>
<td>1.750</td>
<td>1.940</td>
</tr>
<tr>
<td>HAT</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.660</td>
<td>0.700</td>
<td>0.750</td>
<td>0.800</td>
</tr>
<tr>
<td>HVT</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>0.050</td>
<td>0.160</td>
<td>0.290</td>
<td>0.410</td>
</tr>
<tr>
<td>BAN</td>
<td>0.480</td>
<td>0.520</td>
<td>0.550</td>
<td>0.590</td>
<td>0.640</td>
<td>0.700</td>
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<td>0.820</td>
<td>0.880</td>
<td>0.940</td>
<td>1.000</td>
</tr>
<tr>
<td>BVN</td>
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<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>0.585</td>
<td>0.745</td>
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</tr>
<tr>
<td>BAN</td>
<td>0.480</td>
<td>0.460</td>
<td>0.430</td>
<td>0.425</td>
<td>0.445</td>
<td>0.490</td>
<td>0.600</td>
<td>0.770</td>
<td>0.988</td>
<td>1.170</td>
<td>1.420</td>
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<tr>
<td>BVD</td>
<td>1.130</td>
<td>1.180</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.400</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.899</td>
<td>0.880</td>
<td>0.875</td>
<td>0.820</td>
</tr>
</tbody>
</table>

* In Reference 12, (page 79-81) these curves are plotted as h v.

** For terminology, refer to Section 2.9. For the zone of energy dissipation v/α and α/v are negative.

*** Not available.
CHECK:

Case (i) and case (ii) are solved by the graphical method in Reference 12 (page 83). Graphical solution of case (iii) is given in Fig. A-5 (e). The results obtained on the digital computer using the method of characteristics and by the graphical method are presented in Fig. A-5 (a), (b), (c), and (d). The results obtained by the two methods are in close agreement.
(a) TRANSIENT STATE CONDITIONS
(Pump failure, check valve closes when flow reverses)

FIG. A-5
Method of characteristics

Graphical method (Parmakian, pp 83)

(b) TRANSIENT STATE CONDITIONS
(Pump failure, flow reverses through the pump)

FIG. A-5
Method of characteristics

Graphical method (See Fig A-5(f))

τ-t curve for the relief valve

Check valve closes instantly and relief valve opens gradually upon flow reversal

(c) TRANSIENT STATE CONDITIONS
(Pump failure, check valve closes instantly and relief valve opens gradually upon flow reversal)

FIG. A-5
(d) TRANSIENT STATE CONDITIONS

(Pump failure, Check valve closes instantly and relief valve opens gradually upon flow reversal)

FIG. A–5
Linear \( t - \tau \) relationship (Fig. A-5 (d))

Non-linear \( t - \tau \) relationship (Fig. A-5 (c))

(Graphical construction up to time 4.54 sec. has been done by Parmakian\(^2\) page 83)

(e). GRAPHICAL SOLUTION
(Pump failure, check valve closes instantly and relief valve opens gradually upon flow reversal)

FIG. A-5
APPENDIX B

PIPE SYSTEM STUDIES

B-1. Effect of the change in the sprinkler discharge on the waterhammer wave-front.

B-2. Transient state pressures for different locations of the sprinklers.
EFFECT OF THE CHANGE IN THE SPRINKLER DISCHARGE ON
THE WATERHAMMER WAVE-FRONT

PROBLEM:

Assuming the sprinklers on the branch pipes determine the
magnitude of the waterhammer wave-front as it traverses the branch pipes
shown in Fig. B-1(a).

DATA:

Lengths and diameters of the branch pipes are shown in Fig. B-1(a).
Initial steady state elevation of the water surface in the reservoir = 100 ft.
At time t = 0.0, the elevation of the water surface in the reservoir suddenly
rises to 150.0 ft.
Velocity of the waterhammer wave = 4000 ft/sec.
The sprinkler discharge, \( Q_{sp} \), in cu. ft. /sec. is:

i) 0.384  ii) 0.192  iii) 0.096  iv) 0.0
REMARKS:

The results are presented in Fig. B-2 (b). It is clear from the figure that the magnitude of a pressure wave is reduced when it is reflected or transmitted at a sprinkler. The magnitude of the transmitted or reflected wave decreases as the sprinkler discharge increases.
Reservoir

FIG. B-1 (a)

(b) EFFECT OF THE SPRINKLER DISCHARGE ON THE WAVE FRONT

FIG. B-1
APPENDIX B-2

TRANSIENT STATE PRESSURES FOR DIFFERENT LOCATION OF SPRINKLERS

**PROBLEM:**

Compare the transient state pressures at different points of the system shown in Fig. B-2 (a), considering the sprinklers on the:

i) distributors,
ii) laterals,
iii) branch pipes.

Transient conditions are caused by raising the elevation of the water surface in the reservoir by 50 ft.

**DATA:**

Lengths and diameters of the pipes are shown in Fig. B-2 (a).

Velocity of the waterhammer wave = 4000 ft/sec.

Discharge of a sprinkler when it is on the:

i) distributor = 0.005 cu. ft/sec
ii) lateral = 0.05 cu. ft/sec.
iii) branch pipe = 0.50 cu. ft/sec.

Diameter of the laterals = 4"
Notes:
(1) Numbers in the circles represent the number of the junction.
(2) All the laterals and distributors are horizontal.
(3) Reservoir has constant water level.
(4) Numbers in the brackets are the heights of the junctions above the datum line.
Diameter of the distributors = 2"

All pipes are horizontal. Neglect the friction losses.

REMARKS:

The results are presented in Fig. B-2(b) to (f). It is clear that transient pressures obtained by considering the sprinklers on the distributors are close to those obtained by deleting the distributors and assuming the sprinklers on the laterals. However, the analysis of the system by deleting the laterals and distributors and considering the sprinklers on the branch pipes gave totally different results.
(b) TRANSIENT STATE PRESSURES AT JUNCTION NO. 1

FIG. B-2
(c) TRANSIENT STATE PRESSURES AT JUNCTION NO. 3

FIG. B-2
(d) TRANSIENT STATE PRESSURE AT JUNCTION NO. 5

FIG. B-2
(e) TRANSIENT STATE PRESSURES AT JUNCTION NO. 7

FIG. B-2
(f) Transient State Pressures at Junction No. 9

Fig. B-2
APPENDIX C

COMPUTER PROGRAMME FOR A SAMPLE

IRRIGATION PIPE SYSTEM
APPENDIX C

PROBLEM:

Determine the transient state pressures and velocities in the system, rotational speed of the pump, and volume of air and elevation of the water surface in the chamber. The pipe system is shown in Fig. B-2(a). The transient conditions are caused by pump failure.

DATA:

Centrifugal pump:
Rated head of the pump = 177.8'
Rated discharge of the pump = 6.07 cu. ft/sec.
WR$^2$ of the pump and motor = 250 lb-ft$^2$
Rated speed of the pump = 1400 r.p.m.
Pump efficiency at rated speed and head = 0.84
Values for the pump characteristic curves are given in Appendix A-5.

Air chamber:
Diameter of the air chamber = 5.0 ft.
Height of the air chamber = 15.0 ft.
Initial air volume = 100 cu. ft.
Orifice throttling loss corresponding to a discharge of 5 cu. ft/sec

= 25.0 ft.

Throttling coefficient when water flows out of chamber = 1.0

Throttling coefficient when water flows into the chamber = 1.5

Air expansion follows the law $p^x \cdot v_{air}^{1.2}$ = constant.

Branch pipes:

Lengths, diameters and elevations are shown in Fig. B-2 (a).

Friction factor (f) = 0.01

Waterhammer wave velocity = 4000 ft/sec.

Laterals:

Diameter = 4"

Friction factor = 0.01

Waterhammer wave velocity = 4000 ft/sec.

All laterals are horizontal.

Distributors:

Diameter = 2"

Friction factor = 0.01

Waterhammer wave velocity = 4000 ft/sec.

All distributors are horizontal.
Sprinklers:
Sprinkler discharge corresponding to a head of 100 ft. = 0.005 cu. ft/sec.

REMARKS:

The results are presented in Fig. C-(1). When check valve is closed, it becomes a dead end. The pressure head fluctuation for the dead ends are clear from the graph.

The IBM 7044 takes about 27.5 minutes to compute transient state conditions for 10 seconds.
TRANSIENT STATE CONDITIONS AT THE PUMP, CHECK VALVE AND AIR CHAMBER

FIG. C-1
APPENDIX C

$JOB 17057 C.M.HANIF
$TIME 30
$PAGE 400

$IFPTC MAIN

C ANALYSIS OF WATERHAMMER IN THE IRRIGATION PIPE SYSTEM SHOWN IN
C FIGURE B-2.
C
C TRANSIENTS CAUSED BY PUMP FAILURE. CHECK VALVE CLOSES INSTANTLY
C UPON FLOW REVERSAL.
REAL LB, LL, LD, LBS, NRAT

DIMENSION HB(2,10,15), VB(2,10,15), HPB(2,10,15), VPB(2,10,15),
1 LB(2,10), DB(2,10), ELB(2,10), F(2,10), FFB(2,10),
2 AB(2,10), ARB(2,10), THB(2,10), C2B(2,10), HFB(2,10),
3 QOB(2,10), AN(2,10), LBS(2,10), NK(2,10), CINC(2,10),
4 HL(2,10,20), VL(2,10,20), HPL(2,10,20), VPL(2,10,20),
5 HD(10,10,20), VD(10,10,20), HPD(10,10,20), VPD(10,10,20),

COMMON /CM1/VB, HB, AB, THB, /CM2/VPB, HPB, C2B, FFB, ARB
1 /CM3/VL, HL, AL, THL, /CM4/VPL, HPL, C2L, FFL, ARL
3 /CM7/HSPEC, ELB, FB, FL, FD, NK, SINA, LBS
4 /CM8/CSP, CINC, NLAT, HRES
5 /CM9/10, CF, CORF, CH, ZA, AA, EXP, VPAIR, VAIR, ZP, DT
6 /CM10/ALPHA, ALPHAP, CP, HRAT, QRAT, DV, DALPHA
7 /CM11/DX, BAN, BVN, HAN, HVN

DATA FOR THE PIPES.
DATA LL, DL, AL, LD, DD, AD, 100, 0, 40, 4000, 0, 100, 0, 2, 0, 4000, 0/,
1 FD, FL, NLAT, NSPR/2*01, 3*10/, HSPEC, QSPEC/100...005/.

DATA FOR THE AIR CHAMBER.
2 VOAIR, EXP, ZA, DA, 00, CORF, HF0/100, 1.2, 15, 5, 00, 5, 1.5, 25/.

DATA FOR THE CENTRIFUGAL PUMP.
3 WR2, NRAT, EFFR/7250, 1400, 847, DX, TLAST/1, 10/.

THE PUMP CHARACTERISTICS ARE STORED IN THE COMPUTER BY THE
FOLLOWING READ STATEMENT.

READ 8, HVN, HAN, BVN, SAN
8 FORMAT(I11F6.0)

DT=LL/AL
$Z = 3.1416/4.0$

DO 11 J = 1, 10

C

READ 9, DB(I, J), ELB(I, J), LB(I, J), AB(I, J), FB(I, J)

C

ELB = ELEVATION OF THE DOWNSTREAM END OF THE PIPE ABOVE THE DATUM.

9

FORMAT (5F10.0)

C

DB(I, J) = DB(I, J) / 12.0

C

COMPUTATION OF THE PIPE CONSTANTS.

ARB(I, J) = Z * DB(I, J) ** 2

C

C2B(I, J) = 32.2 * AB(I, J)

FFB(I, J) = FB(I, J) * DT / (2.0 * DB(I, J)).

NK(I, J) = LB(I, J) / (DT * AB(I, J))

C

NK(I, J) = NUMBER OF REACHES INTO WHICH PIPE J IS DIVIDED.

AN(I, J) = NK(I, J)

THB(I, J) = AN(I, J) * DT / LB(I, J)

LBS(I, J) = LB(I, J) / AN(I, J)

NK(I, J) = NK(I, J) + 1

FB(I, J) = (FB(I, J) * LBS(I, J)) / (64.4 * DB(I, J))

IF(J.EQ.1) GO TO 10

SINA(I, J) = (ELB(I, J-1) - ELB(I, J)) / LB(I, J)

CINC(I, J) = SINA(I, J) * C2B(I, J) * DT

GO TO 11

10

SINA(I, J) = ELB(I, J) / LB(I, J)

CINC(I, J) = SINA(I, J) * C2B(I, J) * DT

NK(I, J) = NK(I, J) + 1

CONTINUE

C NSPR = NUMBER OF SPRINKLERS ON A DISTRIBUTOR.

KM = 2 * NSPR

C

NDIST = NUMBER OF DISTRIBUTORS ON A LATERAL.

KL = 2 * NDIST

C

COMPUTATION OF THE PIPE CONSTANTS FOR THE LATERALS AND DISTRIBUTORS.

DD = DD / 12.0

DL = DL / 12.0

ARL = Z * DL ** 2

ARD = Z * DD ** 2

C2L = 32.2 / AL

C2D = 32.2 / AD

THL = DT / LL

THD = DT / LD

FFL = FL * DT / (2.0 * DL)

FFD = FD * DT / (2.0 * DD)

FL = (FL * LL) / (64.4 * DL)
FD=(FD*LD)/(64.4*DD)
CSP=QSPEC/SQRT(HSPEC)
SUBROUTINE STEADY COMPUTES THE STEADY STATE PRESSURES AND
VELOCITIES IN THE PIPING SYSTEM.
CALL STEADY(1,10,10)
HRES=HB(1,10,11)
COMPUTATION OF CONSTANTS FOR THE AIR CHAMBER.
FOR DESIGNATION OF THE VARIABLES SEE SECTION 2.8.
AA=Z*A**2
CCH=34.-Z*A
CF=HF0/(Q0**2)
ZP=Z*A-VOAIR/AA
HOAIR=HB(1,1,2)-ZP+34.
VAIR=VOAIR
C10=HOAIR*(VOAIR**EXP)
COMPUTATION OF CONSTANTS FOR THE SUBROUTINE PUMP.
FOR DESIGNATION OF THE VARIABLES, SEE SECTION 2.9.
HRAT=HB(1,1,1)
QRAT=VB(1,1,1)*ARB(1,1)
CPM=ARB(1,1)/QRAT
TR=(60.*62.4*HRAT*QRAT)/(2.*3.1416*NRAT*EFFR)
CP=-(30.*32.*TR*DT)/(3.1416*NRAT*WR2)
ALPHA=1.
VPB(1,1,1)=VB(1,1,1)
ALPHAP=ALPHA
DV=0.0
DALPHA=0.0
T=0.0
WRITE(6,13) T,ALPHA,VAIR,ZP
FORMAT(1X,5HTIME=,F8.3/5X,6HALPHA=,F8.3/5X,5HVAIR=,F8.3/
15X,3HZP=,F8.3/2X,11HBRANCH PIPE /)
NLAT=NUMBER OF LATERALS ON THE BRANCH PIPE.
DO 19 J=1,NLAT
WRITE(6,14) J,(HB(I,J,K),K=1,11),(VB(I,J,K),K=1,11)
19 FORMAT(3X,14,11(4X,F7.2)/7X,11(4X,F7.2))
CONTINUE
IF(T.GT.1.) GO TO 22
WRITE(6,16)
FORMAT(4X,7HLATERAL 7)
DO 18 J=1,NLAT
18 FORMAT(4X,7HLATERAL 7)
DO 18 J=1,NLAT
KK=2*NDIST
GO TO 13
WRITE(6,17) (HL(I,J,K),K=1,KK,2),(VL(I,J,K),K=1,KK,2)
FORMAT(3X,14,10(4X,F7.2)/7X,10(4X,F7.2))
CONTINUE

DO 21 J=1,NLAT
WRITE(6,19) J
FORMAT(/4X,8HLATERAL=,I4/4X,11HDISTRIBUTOR/)
DO 21 K=1,NDIST
KK=2*NSPR
WRITE(6,20) K,(HD(J,K,M),M=1,KK,2),(VD(J,K,M),M=1,KK,2)
FORMAT(4X,14,10(4X,F7.2)/8X,10(4X,F7.2))
CONTINUE

T=T+DT
C TLAST=TIME UP TO WHICH TRANSIENT CONDITIONS ARE TO BE DETERMINED.
IF(T.GT.TLAST) GO TO 27
MM=MM+1

C PUMP IS SITUATED AT THE FIRST SECTION OF THE FIRST PIPE OF THE FIRST BRANCH.
CALL PUMP1(1,1,1)

C AIR CHAMBER IS SITUATED AT THE SECOND SECTION OF THE FIRST PIPE
OF THE FIRST BRANCH.
CALL CHAM2(1,1,2,1,3)

DO 23 J=1,NLAT
KK=NK(I,J)
M=2
IF(J.EQ.1)M=4
CALL INTER(I,J,M,KK-1)
CALL BR1(I,J,KK)
CONTINUE

CALL RESERD(1,10,11)
CALL BR2(I,NLAT,NDIST)
CALL SPRINK(NDIST,NSPR)
DV=VPB(1,1,1)-VB(1,1,1)
DALPHA=ALPHAP-ALPHA

DO 26 J=1,NLAT
KK=NK(I,J)
DO 24 K=1,KK

VB(I,J,K)=VPB(I,J,K)
HB(I,J,K)=HPB(I,J,K)
CONTINUE

DO 25 K=1,KL
VL(I,J,K)=VPL(I,J,K)
HL(I,J,K)=HPL(I,J,K)
25 CONTINUE
DO 26 K=1,NDIST
DO 26 KS=1,KM
VDT(J,K,KS)=VPD(J,K,KS)
HD(J,K,KS)=HPD(J,K,KS)
26 CONTINUE

CONTINUE
ALPHA=ALPHAP
VAIR=VPAIR
IF(MM.LT.4) GO TO 22

GO TO 12

STOP

END

SIBFTC STEADY
SUBROUTINE STEADY(NBR,NDIST,NSPR)
REAL LBS

DIMENSION HB(2,10,15),VB(2,10,15),QOB(2,11),HFR(2,10),ARB(2,10),
1 HL(2,10,20),VL(2,10,20),HPL(2,10,20),QOL(10,11),HFL(10,10),
2 HD(10,10,20),VD(10,10,20),HPD(10,10,20),QOD(10,11),HFD(10,10),
3 FB(2,10),ELB(2,10),LBS(2,10),QOSP(10,1,10,20),NK(2,10),
4 DB(2,10),AN(2,10),SINA(2,10),LB(2,10),AB(2,10),VPB(2,10,15),
5 THB(2,10),CB(2,10),FB(2,10),THPB(2,10,15),CINC(2,10),
6 VH(2,10,10),VBD(2,10,10),VPL(2,10,10)

COMMON /CM1/VB,HB,AB,THB /CM2/VPB,HPB,C2B,FFB,ARB
1 /CM3/VL,HL,AL,THL /CM4/VPL,HPL,C2L,FFL,ARL
3 /CM7/ HSPEC,ELB,FB,FL,FD,NK,SINA,LBS
4 /CM8/CSP,CINC,NLAT,HRES

DATA TOLER/0.01/
PRESSURE HEAD SPECIFIED AT THE DOWNSTREAM END OF THE BRANCH PIPE.
HSPEC=HSPEC+ELB(NBR,NLAT)

DO 15 I=1,NLAT
I=NLAT-I+1
MM=0
15 KMAX=2*NSPR
DO 5 J=1,NDIST
DO 5 K=2,KMAX,2
HD(I,J,K)=HSPEC
5 CONTINUE

C COMPUTATION OF THE VELOCITIES AND FRICTION LOSSES IN THE
C DISTRIBUTORS AND THE SPRINKLER DISCHARGE.
C
6 DO 7 J=1,NDIST
7 DO 7 K=1,NSPR


\[ \begin{align*}
\text{CONTINUE} \\
\text{DO } & 8 \quad J = 1 \text{NDIST} \\
\text{JJ} = & \text{NDIST} - J + 1 \\
\text{IF} (JJ, EQ, NDIST) & QOL(I, JJ+1) = 0.0 \\
QOL(I, JJ) = & QOL(I, JJ+1) + QOD(JJ, 1) \\
VL(NBR, I, 2*JJ) = & QOL(I, JJ)/ARL \\
VL(NBR, I, 2*JJ-1) = & VL(NBR, I, 2*JJ) \\
HFL(I, JJ) = & FL*VL(NBR, I, 2*JJ)**2 \\
\text{CONTINUE} \\
\text{HPL(NBR, I, 1)} = & HSPEC \\
\text{CORRECTION OF Pressures IN THE LATERAL AND DISTRIBUTORS.} \\
\text{DO } & 10 \quad J = 1 \text{NDIST} \\
\text{HPL(NBR, I, 2*J)} = & HPL(NBR, I, 2*J-1) - HFL(I, J) \\
\text{HPD(I, J+1)} = & HPL(NBR, I, 2*J) \\
\text{IF} (J, EQ, NDIST) \quad & \text{GO TO 9} \\
\text{HPL(NBR, I, 2*J+1)} = & HPL(NBR, I, 2*J) \\
\text{DO } & 10 \quad K = 2 \text{KMAX}, 2 \\
\text{KB2} = & K/2 \\
\text{HPD(I, J, K)} = & HPD(I, J, K-1) - HPD(J, KB2) \\
\text{IF} (K, EQ, KMAX) \quad & \text{GO TO 10} \\
\text{HPD(I, J, K+1)} = & HPD(I, J, K) \\
\text{CONTINUE} \\
\text{DO } & 11 \quad J = 1 \text{NDIST} \\
\text{DO } & 11 \quad K = 1 \text{KMAX} \\
\text{XX = HDI(I, J, K)} = & \text{HPD(I, J, K)} \\
\text{IF} (ABS(XX) GT TOLER) \quad & \text{GO TO 100} \\
\text{CONTINUE} \\
\text{GO TO } & 13 \\
\text{DO } & 12 \quad J = 1 \text{NDIST} \\
\text{HL(NBR, I, 2*J)} = & HPL(NBR, I, 2*J) \\
\text{HL(NBR, I, 2*J-1)} = & HPL(NBR, I, 2*J-1) \\
\text{DO } & 12 \quad K = 1 \text{KMAX} \\
\text{HD(I, J, K)} = & \text{HPD(I, J, K)}
\end{align*} \]
$IBFTC\ INTER$

$SIBFTC\ RESERD$

$SIBFTC\ INTER$

$SIBFTC\ RESERD$
C RESERVOIR IS SITUATED AT THE DOWNSTREAM END OF THE SYSTEM.
DIMENSION VP(2,10,15),HP(2,10,15),C2(2,10),FF(2,10),AR(2,10),
  CINC(2,10)

COMMON/CM2/VP, HP, C2, FF, AR
1 /CM8/CSP, CINC, NLAT, HRES
HP(I,J,K)=HRES
HR=HEAD(I,J,K,K-1,1)
VR=VEL(I,J,K,K-1,1)
C3=VR + C2(I,J)*HR-CINC(I,J)*VR-FF(I,J)*VR*ABS(VR)

VP(I,J,K)=C3-C2(I,J)*HRES
RETURN
END

$IBFTC CHAM2

SUBROUTINE CHAM2(I,J,K,M,N)

C AIR EXPANSION FOLLOWS THE LAW H*VAIR**1.2=CONSTANT
DIMENSION VPB(2,10,15),HPB(2,10,15),VR(2,10,15),HB(2,10,15),
  C2B(2,10),FFB(2,10),ARB(2,10),THB(2,10),AB(2,10),
  CINC(2,10)

COMMON /CM1/VB,HB,AB,THB,CM2/VPB, HPB, C2B, FFB, ARB
1 /CM8/CSP, CINC, NLAT, HRES
2 /CM9/ C10,CF,CORF,CCH,ZA,AA,EXP,VPAIR,VAIR,ZP,DT

VS=VEL(I,M,N+1,1)
HS=HEAD(I,M,N+1,1)
VR=VEL(I,J,K,K-1,1)
HR=HEAD(I,J,K,K-1,1)
C1=VS-C2B(I,M)*HS-FFB(I,M)*VS*ABS(VS)+CINC(I,M)*VS
C3=VR+C2B(I,J)*HR-FFB(I,J)*VR*ABS(VR)-CINC(I,J)*VR

C11=VB(I,M,N)*ARB(I,M,N)-VB(I,J,K)*ARB(I,J)
CAIR=(VAIR+C11*DT)
HPB(I,J,K)=C10/(CAIR**EXP)-(CORF*CF*C11*ABS(C11)+CCH+CAIR/AA)

HPB(I,M,N)=HPB(I,J,K)
VPB(I,J,K)=C3-C2B(I,J)*HPB(I,J,K)
VPB(I,M,N)=C1+C2B(I,M)*HPB(I,M,N)
VPAIR=VAIR+0.5*DT*(TVB(I,M,N)+VPB(I,M,N))*ARB(I,M)-
1 *(VB(I,J,K)+VPB(I,J,K))*ARB(I,J))
ZP=ZA-VPAIR/AA
RETURN
END

$IBFTC BR1

C SUBROUTINE BR1(I,J,K)

SUBROUTINE FOR CONNECTION OF THE BRANCH PIPE AND LATERAL.
DIMENSION HPB(2,10,15),VPB(2,10,15),C2B(2,10),FFB(2,10),
COMMON /CM2/VPL,HPL,CINC /CM4/VPL,HPL,C2L,FFL,ARL
COMMON /CM8/CSP,CINC,NLAT,ARL

VRB=VEL(I,J,K,K-1,2)
HRB=HEAD(I,J,K,K-1,2)
VSL=VEL(I,J+1,1,2)
HSL=HEAD(I,J+1,1,2)
C3B=VRB+C2B(I,J)*HRB-FFB(I,J)*VRB*ABS(VRB)-CINC(I,J)*VRB
C1L=VSL-C2L*HSL-FFL*VSL*ABS(VSL)

IF (J.LT.NLAT) GO TO 15
C AS RESERVOIR OF CONSTANT WATER SURFACE ELEVATION IS SITUATED AT
C THE DOWNSTREAM END OF THE 10TH BRANCH PIPE, SPECIAL BOUNDARY
C CONDITIONS ARE REQUIRED FOR THIS SECTION.
HPB(I,J,K)=HRES
GO TO 16

15 VSB=VEL(I,J+1,J+1,2)
HSB=HEAD(I,J+1,1,2)
C1B=VSB-C2B(I,J+1)*HSB-FFB(I,J+1)*VSB*ABS(VSB)-CINC(I,J+1)*VSB
HPB(I,J,K)=(C3B*ARL(I,J)-C1B*ARL(I,J+1)+C1L*ARL(I,J+1) + C2B(I,J)*ARL(I,J+1) + C2L*ARL(I,J+1))/
1 HPB(I,J+1,K)=HPB(I,J,K)

16 VPL(I,J+1)=C1B+C2B(I,J+1)*HPB(I,J+1,1)
VPL(I,J)=C1L+C2L*HPB(I,J,1)
RETURN
END

SUBROUTINE BR2
C SUBROUTINE FOR CONNECTION OF THE LATERAL AND DISTRIBUTORS.
DIMENSION VPL(2,10,20),HPB(2,10,20),VPD(10,10,20),HPD(I0,10,20)
COMMON /CM4/VPL,HPL,C2L,FFL,ARL /CM6/VPD,HPD,C2D,FFD,ARD
DO 18 J=1,NLAT
18 DO 18 L=1,NDIST
K=2*L
VRL=VEL(I,J,K,K-1,2)
HRL=HEAD(I,J,K,K-1,2)
VSD=VEL(J,L,1,2,3)
HSD=HEAD(J,L,1,2,3)
C3L=VRL+C2L*HRL-FFL*VRL*ABS(VRL)
C1D=VSD-C2D*HSD-FFD*VSD*ABS(VSD)
IF (L.LT.NDIST) GO TO 16

RETURN
END
HPL(I,J,K) = (C3L*ARL - C1D*ARD) / (C2L*ARL + C2D*ARD)

GO TO 17

16  VSL = VEL(I+J,K+1,K+2,2)

HSL = HEAD(I+J,K+1,K+2,2)

C1L = VSL - C2L*HSL - FLD * VSL * ABS(VSL)

HPL(I,J,K) = (C3L*ARL - C1L*ARD) / (C2L*ARL + C2D*ARD)

HPL(I,J,K+1) = HPL(I,J,K)

VPL(I,J,K+1) = C1L + C2L*HPL(I,J,K+1)

HPD(J,L+1) = HPL(I,J,K)

VPL(I,J,K) = C3L - C2L*HPL(I,J,K).

17  CONTINUE

RETURN

END

SIBFTC SPRINK

SUBROUTINE SPRINK(NDIST,NSPR)

DIMENSION VPD(10,10,20),HPD(10,10,20),CINC(2,10)

COMMON/CM6/VPD,HPD,C2D,FFD,ARD,CM8/CSP, CINC, NLAT, HRES

CK=C2D*ARD/CSP

KK=2*NSPR

DO 13 I=1,NLAT

DO 13 J=1,NDIST

DO 13 K=2,KK+2

VR=VEL(I,J+K-1,3)

HR = HEAD(I,J+K-1,3)

C3 = VR + C2D*HR - FFD*VR*ABS(VR)

C SPECIAL BOUNDARY CONDITIONS ARE REQUIRED FOR THE LAST SPRINKLER.

IF(K<KK) GO TO 11

HSQ = (-1. + SQRT(1. + A.*CK*C3*ARD/CSP))/(4.*CK)

HPD(I,J,K) = HSQ*ABS(HSQ)

GO TO 12

11  VS = VEL(I,J,K+1,K+2,3)

HS = HEAD(I,J,K+1,K+2,3)

C1 = VS - C2D*HS - FFD*VS*ABS(VS)

IF(C3.GT.C1) GO TO 100

PRINT 16, C3,C1,HR,HS,VR,VS,FFD,I,J,K

100 CONTINUE

HSQ = (-1. + SQRT(1. + B.*CK*(C3-C1)*ARD/CSP))/(4.*CK)

HPD(I,J,K) = HSQ*ABS(HSQ)

HPD(I,J,K+1) = HPD(I,J,K)

VPL(I,J,K+1) = C1 + C2D*HPD(I,J,K+1)

16  CONTINUE

RETURN

END
12 \[ VPD(I,J,K) = C3 - C2D \times HPD(I,J,K) \]
13 CONTINUE
14 RETURN
15 END

$IBFCTC$ VEL

FUNCTION VEL(I,J,K,KK,N)

C THE VALUE OF N DEFINES THE TYPE OF PIPE. FOR BRANCH PIPE, LATERAL
AND DISTRIBUTOR, N IS EQUAL TO 1, 2 AND 3 RESPECTIVELY.

DIMENSION V(2,10,15), H(2,10,15), TH(2,10), A(2,10), VL(2,10,20),
1 HL(2,10,20), VL(10,10,20), THD(10,10,20)
GO TO (15,16,17), N

15 \[ VEL = V(1+J,K) - TH(1+J) * A(I,J) * (V(1+J,K) - V(I,J,KK)) \]
RETURN
16 \[ VEL = VL(1+J,K) - THL(AL*(VL(1+J,K) - VL(I,J,KK)) \]
RETURN
17 \[ VEL = VD(1+J,K) - THD*AD*(VD(1+J,K) - VD(I,J,KK)) \]
RETURN
END

$IBFCTC$ HEAD

FUNCTION HEAD(I,J,K,KK,N)

C THE VALUE OF N DEFINES THE TYPE OF PIPE.

DIMENSION V(2,10,15), H(2,10,15), TH(2,10), A(2,10), VL(2,10,20),
1 HL(2,10,20), VL(10,10,20), THD(10,10,20)
GO TO (15,16,17), N

15 \[ HEAD = H(I,J,K) - TH(I,J) * A(I,J) * (H(I,J,K) - H(I,J,KK)) \]
RETURN
16 \[ HEAD = HL(I,J,K) - THL(AL*(HL(I,J,K) - HL(I,J,KK)) \]
RETURN
17 \[ HEAD = HD(I,J,K) - THD*AD*(HD(I,J,K) - HD(I,J,KK)) \]
RETURN
END

$IBFCTC$ PUMP1

SUBROUTINE PUMP1(I,J,K)

C CHECK VALVE CLOSES INSTANTLY UPON FLOW REVERSAL.

DIMENSION V(2,10,15), H(2,10,15), VP(2,10,15), HP(2,10,15), C2(2,10),
1 FF(2,10), AR(2,10), A(2,10), TH(2,10), CINC(2,10)
1 CM8/CS3,CINC,NLAT,HRES
2 CM10/ALPHA,ALPHAP,CP,HRAT,QRAT,DV,DALPHA
DATA TOLER1,TOLER2 / .01,0.04/
VS = VEL(I, J, K, K+1, I)
HS = HEAD(I, J, K, K+1, I)
C1 = VS - C2(I, J)*HS - F(I, J)*VS*ABS(VS) + CINC(I, J)*VS
IF(V(I, J, K) GT 0.0) GO TO 8
VP(I, J, K) = 0.0
HP(I, J, K) = -C1/C2(I, J)
RETURN
8
VPM = V(I, J, K) + 0.5* DV
ALPHA1 = ALPHA + 0.5*DALPHA
VRM = VPM*AR(I, J)/QRAT
RATIO = VRM / ALPHA1
IF(RATIO LE 1.0) GO TO 10
RATIO = 1.0 / RATIO
M = 2
GO TO 11
9
M = 1
10
CALL PARAB(RATIO, M, BP)
IF(M EQ 1) BETTA = BP*ALPHA1**2
IF(M EQ 2) BETTA = BP*ALPHA1**2/RATIO**2
DALPHA = CP*BETTA
ALPHA2 = ALPHA + 0.5*DALPHA
IF(ABS(ALPHA2 - ALPHA1) LE TOLER1) GO TO 12
ALPHA1 = ALPHA2
GO TO 9
11
ALPHAP = ALPHA + DALPHA
VPP = V(I, J, K) + DV
12
VRP = VPP*AR(I, J)/QRAT
RATIO = VRP/ALPHAP
IF(RATIO LE 1.0) GO TO 14
RATIO = 1.0 / RATIO
M = 4
GO TO 15
13
M = 3
14
CALL PARAB(RATIO, M, HRP)
IF(M EQ 3) HP1 = HRP*HRAT*(ALPHAP**2)
IF(M EQ 4) HP1 = HRP*HRAT*(ALPHAP**2)/(RATIO**2)
VP1 = C1 + C2(I, J)*HP1
IF(ABS(VP1 - VPP) LE TOLER2) GO TO 16
VPP = VP1
GO TO 13
15
M = 2
16
IF(VP1 LT 0.0) VP1 = 0.0
VP(I, J, K) = VP1
SUBROUTINE PARAB(X, MM, Y)
DIMENSION YBAN(11), YBVN(11), YHAN(11), YHVN(11)
COMMON/C'M1/DX, YBAN, YBVN, YHAN, YHVN
M = X/DX
AM = M

THETA = (X-AM*DX) /DX
IF(M*EQ.0) THETA = THETA-1.
M=M+1

IF(M*LT.2) M = 2
GO TO (6,7,8,9), MM

6 Y = YBAN(M) + 0.5*THETA*(YBAN(M+1) - YBAN(M-1) + THETA*(YBAN(M+1) +
     YBAN(M-1) - 2*YBAN(M))
RETURN

7 Y = YBVN(M) + 0.5*THETA*(YBVN(M+1) - YBVN(M-1) + THETA*(YBVN(M+1) +
     YBVN(M-1) - 2*YBVN(M))
RETURN

8 Y = YHAN(M) + 0.5*THETA*(YHAN(M+1) - YHAN(M-1) + THETA*(YHAN(M+1) +
     YHAN(M-1) - 2*YHAN(M))
RETURN

9 Y = YHVN(M) + 0.5*THETA*(YHVN(M+1) - YHVN(M-1) + THETA*(YHVN(M+1) +
     YHVN(M-1) - 2*YHVN(M))
RETURN
END

1.08  1.081 1.082  1.085  1.09  1.095  1.1  1.2  328  56  761  1
1.48  .52  .55  .59  .64  .70  .755  .82  .88  .94  1

10 11
12
14* 0  1000  4000  .01
14* 20* 1000  4000  .01
14* 30* 1000  4000  .01
12* 18* 1000  4000  .01
12* 40* 1000  4000  .01
12* 20* 1000  4000  .01
10* 23* 1000  4000  .01
10* 23* 1000  4000  .01
12* 23* 1000  4000  .01
10* 23* 1000  4000  .01
14* 20* 1000  4000  .01
APPENDIX D

SUBROUTINES

D-1. For reservoir (RESERU, RESERI)
D-2. For Air Chamber (CHAM1)
D-3. For Surge Tank (SURGE1, SURGE2)
D-4. For Valve (VALVE, PARAB)
D-5. For Relief Valve (RELIEF, PARAB)
D-6. For Pump (PUMP2, PARABB, PARABH)
SUBROUTINE RESERU(I,J,K)
C THIS SUBROUTINE IS FOR THE RESERVOIR AT THE UPSTREAM END OF THE
C SYSTEM.
DIMENSION VP(2,10,15), HP(2,10,15), C2(2,10), FF(2,10), AR(2,10),
1 CINC(2,10)
COMMON /CM2/VP, HP, C2, FF, AR
1 /CM8/CSP, CINC, NLAT, HRES
HS = HEAD(I,J,K,K+1,1)
VS = VEL(I,J,K,K+1,1)
HP(I,J,K) = HRES
C1 = VS - C2(I,J) * HS - FF(I,J) * VS * ABS(VS) + CINC(I,J) * VS
VP(I,J,K) = C1 + C2(I,J) * HP(I,J,K)
RETURN
END

SUBROUTINE RESER(I,J,K,M,N)
C THIS SUBROUTINE IS FOR THE RESERVOIR ANYWHERE IN THE SYSTEM.
C SECTION(I,J,K) IS JUST UPSTREAM AND SECTION (I,M,N) JUST
C DOWNSTREAM OF THE RESERVOIR.
DIMENSION VP(2,10,15), HP(2,10,15), C2(2,10), FF(2,10), AR(2,10),
1 CINC(2,10)
COMMON /CM2/VP, HP, C2, FF, AR
1 /CM8/CSP, CINC, NLAT, HRES
HS = HEAD(I,M,N,N+1,1)
VS = VEL(I,M,N,N+1,1)
VR = VEL(I,J,K,K+1,1)
HR = HEAD(I,J,K,K-1,1)
C3 = VR + C2(I,J) * HR - FF(I,J) * VR * ABS(VR) - CINC(I,J) * VR
C1 = VS - C2(I,M) * HS - FF(I,M) * VS * ABS(VS) + CINC(I,M) * VS
HP(I,J,K) = HRES
HP(I,M,N) = HRES
VP(I,J,K) = C3 - C2(I,J) * HP(I,J,K)
VP(I,M,N) = C3 - C2(I,M) * HP(I,M,N)
RETURN
END
APPENDIX D-2

SIBFTC CHAM1

SUBROUTINE CHAM1(I,J,K)
C     CHAMBER IS CLOSE TO PUMP, CHECK VALVE CLOSES SIMULTANEOUSLY
C     WITH POWER FAILURE
C     AIR EXPANSION IS ISOTHERMAL
DIMENSION VP(2:10,15), HP(2:10,15), C2(2:10), FF(2:10), AR(2:10),
1     CINC(2:10), V(2:10,15), H(2:10,15), A(2:10), TH(2,10)

COMMON /CM1/V*,H*,A*,TH
1   /CM2/VP,HP,C2,FF,AR /CM12/VPAIR,VAIR,ZP
2   /CM13/C5,CP,CF,CCH,ZA,AA
3   /CM87/CSP,CINC,NLAT,HRES

VS=VEL(I,J,K,K+1,1)
HS=HEAD(I,J,K,K+1,1)

C1= VS - C2(I,J)*HS - FF(I,J)*VS*ABS(VS) + CINC(I,J)*VS
VALUES OF THE COSTANTS C5, CP, CF, CCH ARE COMPUTED IN THE MAIN
PROGRAMME, FOR EXPRESSIONS FOR THE CONSTANTS, REFER TO SECTION 2.8.
CV= VAIR + CP* V(I,J,K)
VP(I,J,K) = C1 + C2(I,J)* HP(I,J,K)
VPAIR = VAIR + 0.5*(V(I,J,K) + VP(I,J,K))*CP

ZA=HEIGHT OF THE AIR CHAMBER.
AA=CROSS-SECTIONAL AREA OF THE CHAMBER.
ZP = ZA - CV/AA
RETURN
END
SUBROUTINE SERGE

C THIS SUBROUTINE IS FOR THE SURGE TANK AT THE JUNCTION OF
C PIPE J, AND PIPE J+1. K = LAST SECTION OF PIPE J, KK = SECTION NUMBER
C OF THE FREE SURFACE IN THE SURGE TANK. SURGE TANK OF ORIFICE TYPE.
C WATERHAMMER WAVES IN THE SURGE TANK ARE CONSIDERED.
C
DIMENSION VP(2*10,15), HP(2*10,15), C2(2*10), FF(2*10), AR(2*10),
C
CINC(2*10), V(2*10,15), H(2*10,15), A(2*10), TH(2*10)
C
COMMON/CM1/V,H,A,TH/CM2/VP,HP,C2,FF,AR
C
1 /CM15/ ZP,Z,CS,CSU
2 /CM8/CSP,CINC,NLAT,HRES

VR=VEL(I,J,K,K-1,1)
HR=HEAD(I,J,K,K-1,1)
VS=VEL(I,J+1,1,2,1)
HS=HEAD(I,J+1,1,2,1)
VSS=VEL(I,J+2,1,2,1)
HSS=HEAD(I,J+2,1,2,1)
C3=VR+C2(I,J)*HR-FF(I,J)*VR*ABS(VR)-CINC(I,J)*VR
C1=VS-C2(I,J+1)*HS-FF(I,J+1)*VS*ABS(VS)+CINC(I,J)*VS
C1S=VSS-C2(I,J+2)*HSS-FF(I,J+2)*VSS*ABS(VSS)
C
VALUES OF THE CONSTANTS CS AND CSU ARE DETERMINED IN THE MAIN
C
C PROGRAMME FOR EXPRESSIONS FOR THESE CONSTANTS SEE SECTION 2.7.
HP(I,J,K)=(C3*AR(I,J)-C1*AR(I,J+1)-C1S*AR(I,J+2))/CSU
HP(I,J+1,1)=HP(I,J,K)
HP(I,J+2,1)=HP(I,J,K)

12 VP(I,J,K)=C3-C2(I,J)*HP(I,J,K)
VP(I,J+1,1)=C1+C2(I,J+1)*HP(I,J+1,1)

11 VP(I,J+2,1)=C1S+C2(I,J+2)*HP(I,J+2,1)

10 ZP=Z+CS*(VP(I,J,K)*AR(I,J)-VP(I,J+1,1)*AR(I,J+1))

8 HP(I,J+2,KK)=ZP
9
VP(I,J+2,KK)=C3S-C2(I,J+2)*HP(I,J+2,KK)

7 HRS=HEAD(I,J+2,KK,KK-1,1)
C3S= VRS+C2(I,J+2)*HRS-FF(I,J+2)*VRS*ABS(VRS)

5 RETURN
END
SUBROUTINE SURGE2(I,J,K)

SUBROUTINE FOR THE SURGE TANK AT THE JUNCTION OF PIPE J AND J+1.

SURGE TANK OF ORIFICE TYPE. WTERHAMMER WAVES IN THE TANK NEGLECTED.

DIMENSION VP(2,10,15),HP(2,10,15),C2(2,10),FF(2,10),AR(2,10),
CINC(2,10),V(2,10,15),H(2,10,15),A(2,10),TH(2,10)

COMMON/CM1/V,H,A,TH /CM2/VP,HP,C2,FF,AR

1 /CM14/ ZP,Z,C12,C13,C14
2 /CM8/CSP,CINC,NLAT,HRES

C14=V(I,J+1,1)*AR(I,J+1) - V(I,J,K)*AR(I,J)

VALUE OF THE CONSTANTS C11,C13 COMPUTED IN THE MAIN PROGRAMME.

FOR EXPRESSIONS FOR THE CONSTANTS *SEE SECTION 2.7.*

ZP=Z-C13*C14

HP(I,J,K)=ZP-C12*C14*ABS(C14)

HP(I,J+1,1) = HP(I,J,K)

VR=VEL(I,J,K,K-1)

HR=HEAD(I,J,K,K-1)

VS=VEL(I,J+1,1,2)

HS=HEAD(I,J+1,1,2)

C3=VR + C2(I,J)*HR - FF(I,J)*VR*ABS(VR)

C1=VS - C2(I,J+1)*HS - FF(I,J+1)*VS*ABS(VS)

VP(I,J,K) = C3 - C2(I,J)*HP(I,J,K)

VP(I,J+1,1)=C1+C2(I,J+1)*HP(I,J+1,1)

RETURN

END
**TSFTC VALVE**

```fortran
SUBROUTINE VALVE(I,J,K)

C THIS SUBROUTINE IS FOR A VALVE AT THE DOWNSTREAM END OF A PIPE.
C THE VALVE DISCHARGES INTO ATMOSPHERIC PRESSURE.
DIMENSION VP(2,10,15), HP(2,10,15), C2(2,10), FF(2,10), AR(2,10),
C INC(2,10)
COMMON /CM2/VP, HP, C2, FF, AR
/CM16/T, TC, CV, TAU
/CM8/CINC, NLAT, HRES
VR=VEL(I,J,K,K-1,1)
HR=HEAD(I,J,K,K-1,1)
C3=VR + C2(I,J)*HR - FF(I,J)*VR*ABS(VR) - CINC(I,J)*VR
C WHEN THE VALVE IS FULLY CLOSED, IT BECOMES A DEAD END AND IS
C ANALYSED AS SUCH.
IF(T.GE.TC) GO TO 6
C SUBROUTINE PARAB DETERMINES THE VALUE OF TAU BY PARABOLIC INTER-
C POLATION.
CALL PARAB(T, TAU)
C VALUE OF CV IS COMPUTED IN THE MAIN PROGRAMME. FOR EXPRESSION FOR
C THIS CONSTANT, SEE SECTION 2.5.
C4 = CV* TAU**2
VP(I,J,K) = 0.5*C4*(-1.0 + SORT(1.0 + 4.0*C3/C4))
HP(I,J,K) = (C3 - VP(I,J,K))/C2(I,J)
RETURN
6
VP(I,J,K) = 0.0
TAU=0.0
HP(I,J,K) = C3/C2(I,J)
RETURN
END
```

**TSFTC PARAB**

```fortran
SUBROUTINE PARAB(X,YY)

DIMENSION Y(10)
COMMON /CM11/DX, Y
M =X/DX
AM=M
THETA = (X-AM*DX)/DX
IF(M.EQ.0.0) THETA= THETA-1.
```
M = M + 1
IF(M .LT. 2) M = 2
YY = Y(M) + 0.5*THETA*(Y(M+1) - Y(M-1) + THETA * (Y(M+1) + Y(M-1) -
1 / 2 * Y(M)))
RETURN
END
APPENDIX D-5

SUBROUTINE RELIEF(I,J,K)
C RELIEF VALVE ADJACENT TO THE PUMP UPON FLOW REVERSAL CHECK VALVE
C CLOSSES INSTANTLY RELIEF VALVE OPENS GRADUALLY.
C TIME-DEE Curve STORED IN THE COMPUTER. INTERMEDIATE VALUES
C INTERPOLATED PARABOLICALLY BY SUBR. PARAB.
DIMENSION VP(2,10,15),HP(2,10,15),C2(2,10),FF(7,10),AR(2,10),
1 CINC(2,10)
COMMON /CM2/ VP,HP,C2,FF,AR /CM 17/T,T1,CV,TAU
1 TCM,TCM
DATA T2,T3,T4/4.,10.,30./
HS=HEAD(I,J,K,K+1,1)
VS=VEL(I,J,K,K+1,1)
C1=VS-C2(I,J)*HS-FF(I,J)*VS*ABS(VS)+CINC(I,J)*VS
C T1=TIME WHEN FLOW REVERSES.
C T2=TIME REQUIRED FOR FULL OPENING.
C T3-T2=VALVE REMAINS OPEN FOR THIS PERIOD.
C T4-T3=TIME REQUIRED FOR FULL CLOSURE OF THE VALVE.
T1=T-T1
IF(TT.GT.T2.AND.TT.LT.T3) GO TO 7
IF(TT.GE.T3) GO TO 6
A - AR
GO TO 8
6 IF(TT.GE.T4) GO TO 9
GO TO 8
7 TAU=1.
C VALUE OF CV IS COMPUTED IN THE MAIN PROGRAMME. FOR EXPRESSION FOR
C CV, SEE SECTION 2.5.
8 C4=CV*TAU**2
9 VP(I,J,K)=0.5*C4*(1.-SQRT(1.-4.*C1/C4))
HP(I,J,K)=(VP(I,J,K)-C1)/C2(I,J)
RETURN
C WHEN THE VALVE IS FULLY CLOSED, IT IS ANALYSED AS A DEAD END.
9 TAU=0.0
VP(I,J,K)=0.0
SUBROUTINE PARAB
C THIS SUBROUTINE DETERMINES, BY PARABOLIC INTERPOLATION, INTERMEDIATE
C VALUES THE PUMP CHARACTERISTIC CURVES. THE CURVES ARE
C STORED IN THE COMPUTER BY GIVING VALUES OF Y AT UNIFORM
C INTERVALS OF X.

SUBROUTINE PARAB(X, MM, DX, Y)

DIMENSION YBAN(11), YBVN(11), YHAN(11), YHVN(11), TAUO(11), TAUC(11)

COMMON / CM18/YBAN, YBVN, YHAN, YHVN, TAUO, TAUC

M = X / DX

AM = M

THETA = (X - AM * DX) / DX

IF(M .EQ. 0) THETA = THETA - 1.

M = M + 1

IF(M .LT. 2) M = 2

GO TO (6, 7, 8, 9, 10, 11), MM

6 Y = YBAN(M) + 0.5 * THETA * (YBAN(M + 1) - YBAN(M - 1) - THETA * (YBAN(M + 1) + YBAN(M - 1) - 2 * YBAN(M)))

RETURN

7 Y = YBVN(M) + 0.5 * THETA * (YBVN(M + 1) - YBVN(M - 1) - THETA * (YBVN(M + 1) + YBVN(M - 1) - 2 * YBVN(M)))

RETURN

8 Y = YHAN(M) + 0.5 * THETA * (YHAN(M + 1) - YHAN(M - 1) - THETA * (YHAN(M + 1) + YHAN(M - 1) - 2 * YHAN(M)))

RETURN

9 Y = YHVN(M) + 0.5 * THETA * (YHVN(M + 1) - YHVN(M - 1) - THETA * (YHVN(M + 1) + YHVN(M - 1) - 2 * YHVN(M)))

RETURN

10 Y = TAUO(M) + 0.5 * THETA * (TAUO(M + 1) - TAUO(M - 1) - THETA * (TAUO(M + 1) + TAUO(M - 1) - 2 * TAUO(M)))

RETURN

11 Y = TAUC(M) + 0.5 * THETA * (TAUC(M + 1) - TAUC(M - 1) - THETA * (TAUC(M + 1) + TAUC(M - 1) - 2 * TAUC(M)))

RETURN

END
SUBROUTINE PUMP2(I,J,K)

THIS SUBROUTINE IS FOR THE PUMP OPERATING IN THE THREE ZONES OF

OPERATION. PUMP CHARACTERISTICS ARE STORED IN THE COMPUTER.
THE INTERMEDIATE VALUES ARE INTERPOLATED PARABOLICALLY.
DIMENSION VP(2,10,15),HP(2,10,15),C2(2,10),FF(2,10),AR(2,10),

COMMON /CM1/V,H,A,TH /CM2/VP,HP,C2,FF,AR

COMMON /CM8/CSP,ALPHAP,CP,HRAT,QRAT,DV,DALPHA,CPM

DATA TOLER1,TOLER2 / 0.025,0.2/

MM=0
NN=0
VS=VEL(I,J,K,K+1,1)
HS=HEAD(I,J,K,K+1,1)

C1=VS-C2(I,J)*HS-FF(I,J)*VS*ABS(VS)+CINC(I,J)*VS

COMPUTATION OF THE TRANSIENT SPEED OF THE PUMP.
VPM = V(I,J,K) + 0.5* DV

ALPHA1= ALPHA + 0.5* DALPHA
VRM=VPM*CPM

RATIO=VRM / ALPHA1

IF(NN.GT.25) GO TO 20

SUBROUTINE MVAL GIVES THE VALUE OF M, WHICH DEFINES THE CHARACTER.

CALL MVAL(VRM,RATIO,ALPHA1,M)

KKK=M/2

IF(M.EQ.2*KKK) GO TO 13

CALL PARABB(RATIO,M,BP)
BETTA=BP*ALPHA1**2

GO TO 14

RATIO=1./RATIO
CALL PARABB(RATIO,M,BP)

BETTA=BP*ALPHA1**2/RATIO**2

DALPHA= CP* BETTA

ALPHA2= ALPHA + 0.5* DALPHA
IF(ABS(ALPHA2 - ALPHA1) .LE. TOLER1) GO TO 15
ALPHA1 = ALPHA2
GO TO 6

20 WRITE(6,21) ALPHA, ALPHA2
21 FORMAT(5X,16HITERATION FAILED,2F12.3)
15 ALPHAP = ALPHA + D ALPHA

C COMPUTATION OF THE TRANSIENT STATE PUMPING HEAD.
VPP = V(I,J,K) + DV
16 VRP = VPP * CPM

MM = MM + 1
IF(MM .GT. 25) GO TO 22
RATIO = VRP / ALPHAP
CALL MVAL(VRP, RATIO, ALPHAP, M)
KKK = M / 2
IF(M .EQ. 2 * KKK) GO TO 17
CALL PARABH(RATIO, M, HRP)
HP1 = HRP * HRAT * ALPHAP ** 2
GO TO 18

17 RATIO = 1. / RATIO
CALL PARABH(RATIO, M, HRP)
HP1 = HRP * HRAT * ALPHAP ** 2 / RATIO ** 2

18 VPI = C1 + C2(I,J) * HP1
IF(ABS(VPI - VPP) .LE. TOLER2) GO TO 19
VPP = VPI
GO TO 16

22 WRITE(6,21) V(1,1,1), VPP
19 VP(I,J,K) = VPI
HP(I,J,K) = HP1
RETURN
END

SUBROUTINE PARABH(X, JJ, Y)
DIMENSION HAN(11), HVN(11), HAD(11), HVD(11), HAT(11), HVT(11)
COMMON /CM20/ HAN, HVN, HAD, HVD, HAT, HVT
DATA DX/.1/
M = X / DX
AM = M
THETA = (X - AM * DX) / DX
IF(M .EQ. 0) THETA = THETA - 1.
M = M + 1
$\text{SIBFTC PARABB}$

SUBROUTINE PARABB(X,JJ,Y)

THIS SUBROUTINE DETERMINES, BY PARABOLIC INTERPOLATION, INTERMEDIATE
TORQUE-RATIOS FROM THE PUMP CHARACTERISTIC CURVES. THE CURVES ARE
STORED IN THE COMPUTER BY GIVING VALUES OF Y AT UNIFORM
INTERVALS OF X.

DIMENSION BAN(11), BVN(11), BAD, BVD(11), BAT(11), BVT(11)

COMMON /CM217/ BAN, BVN, BAD, BVD, BAT, BVT

DATA DX/1/

M = X/DX

AM = M

THETA = (X - AM*DX) / DX

IF(M.EQ.0) THETA = THETA - 1.

IF(M.LT.2) M = 2

GO TO (6, 7, 8, 9, 10, 11, JJ)

Y = HAN(M) + 5*THETA*(HAN(M+1) - HAN(M-1)) + THETA*(HAN(M+1) + HAN(M-1) - 2.

RETURN

Y = HVN(M) + 5*THETA*(HVN(M+1) - HVN(M-1)) + THETA*(HVN(M+1) + HVN(M-1) - 2.

RETURN

Y = HAD(M) + 5*THETA*(HAD(M+1) - HAD(M-1)) + THETA*(HAD(M+1) + HAD(M-1) - 2.

RETURN

Y = HVD(M) + 5*THETA*(HVD(M+1) - HVD(M-1)) + THETA*(HVD(M+1) + HVD(M-1) - 2.

RETURN

Y = HAT(M) + 5*THETA*(HAT(M+1) - HAT(M-1)) + THETA*(HAT(M+1) + HAT(M-1) - 2.

RETURN

Y = HVT(M) + 5*THETA*(HVT(M+1) - HVT(M-1)) + THETA*(HVT(M+1) + HVT(M-1) - 2.

RETURN

END