LATERAL STABILITY OF CONTINUOUS GLULAM BEAMS

BY

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ABSTRACT

This thesis presents the results of a theoretical and experimental study of the lateral buckling of straight beams of rectangular cross-section resting on columns, a type of structure commonly found in the roofing system of multi-bayed buildings. The structure is analyzed as a simply supported beam, uniformly loaded, restrained at one end against longitudinal torsion and resting near the other end on a flexible column which may provide various torsional and lateral restraints. Beyond the column is a cantilevered projection of various lengths and loads. The entire top edge of the beam is considered as fastened to a continuous decking which restrains it against horizontal displacement but permits free rotation about this edge.

The method of solution of the theoretical lateral buckling load is by using a computer program to calculate the structure stiffness matrix's determinant at increasing load levels, and a plot of the determinant versus load level yields the critical load (at determinant equals zero). This theoretical approach is verified by model experiments in the laboratory.

Design curves and equations are produced incorporating the usual flexural beam and axially loaded column strength concepts, with lateral buckling considerations. Recommended design code procedures are forwarded based on these curves which would permit more economical use of deep beams.

Included in the thesis is the computer program listing used in the solution technique.

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NOTATION

TEXT	PROGRAM	DEFINITION
	A	Length of cantilever projection
d	DB	Depth of beam
E_b	E	Modulus of elasticity of beam
E	EL	Modulus of elasticity of column
e	EC(I)	Eccentricity of top flange from centroid of beam segment I
f_b	2.1 ksi	Approximate flexural stress in beam
f_i		Forces at degrees of freedom
G	G	Shear modulus of beam
I	B(I)	Moment of inertia of beam segment I about y-y axis
I	BC	Moment of inertia of column about z-z axis
J	T(I)	Torsion constant
KT,KB	KT,KB	Fixity factor for top and bottom of column 0:pin, 1:fix
k		Basic buckling stiffness matrix of segment
$\bar{k}$		k transformed to top flange
$\bar{\bar{k}}$	SM(I,J)	$\bar{k}$ transformed for no horizontal displacement of top flange
L	CL	Length of column
l	X(I)	Length of beam segment I
L_b	L	Length of beam
M	BM(I)	Primary moment of midpoint of segment I
m		Torsional moment
n		Dimensionless factor for column
NF	NF	Rotational degree of freedom at which column acts
NI	NI	Number of load factors α input
NM	NM	Number of beam segments

TEXT	PROGRAM	DEFINITION
NP(I,J)	NP(I,J)	Displacement numbers for a beam segment
NRS	NRS	Structure number
NU	NU	Total number of degrees of freedom
P	P	Concentrated load on end of cantilever
Q		Translational force
r		Superscript denoting rotation
R	R	Primary axial column force
SK	SK	Spring constant replacing column properties
S	S(I,J)	Structure stiffness matrix
S_i	STB_i	Stability functions for column
t		Superscript denoting translation
T, T_1		Transformation matrices
V	V(I)	Primary shear in segment I
w	1.	Primary load intensity on beam
W		External load on joint
Z		Section Modulus
α	F(K)	Load multiplying factor. Also eigen value
β	1.	Rotational angle change about top flange
γ	P/L	Factor relating P to wL_b
δ_i	1.	Deflections at degrees of freedom of segment
η		Ratio of cantilever projection to L_b
λ	XL	Stability factor
σ_{CR}		Beam bending stress at critical buckling load
ϕ		Dimensionless factor
ρ		Abscissa of graphs

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INTRODUCTION

In the design of roof systems composed of continuous glulam beams and columns, economy favors deep and narrow sections. Since the top edge of such beams is usually supported against lateral translation by a roof deck, the lower edge is left free. In continuous systems, this lower edge is partly in compression from bending and loaded by a column. Such a system can become unstable and buckle sideways in a torsion-flexural mode about the constrained centre of rotation at the top. The object of this article is to investigate this phenomenon for some common conditions of support and continuity.

A stiffness matrix developed by B.A. Zavitz [1][a] and modified herein, has been chosen for the theoretical study. This is checked against several timber models to establish its validity, and an electronic computer is used to produce a variety of design charts. Finally, summary recommendations are made to ensure safe design of such structures.

The stiffness method is chosen as it offers the only practical way to include any load and boundary condition, together with column restraint and buckling. Other systems of analysis are available and in 1960 G. C. Lee [2] compiled a complete set of references for known solutions of various beams and loadings. However little research has been done on the buckling of beams as they might be commonly used in structures. Subsequent to Lee's work, L. A. Bell [3] presented a numerical solution to the governing differential equations but his system, although accurate, was too cumbersome to produce the design charts required, and it did not include column buckling.

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a. Numbers in square parenthesis refer to the bibliography.

DERIVATION OF STIFFNESS MATRICES

The general system under consideration, as shown in Fig.(1), consists of a continuous beam which may or may not be hinged to produce determinancy. It can be supported on a number of columns which are fixed or pinned to their base and to the beam. Diaphragms can be provided at various locations, such as at end walls or over the columns, to prevent rotation. The top edge is prevented from lateral translation at all points by the roof deck but no torsional restraint is provided on this edge.

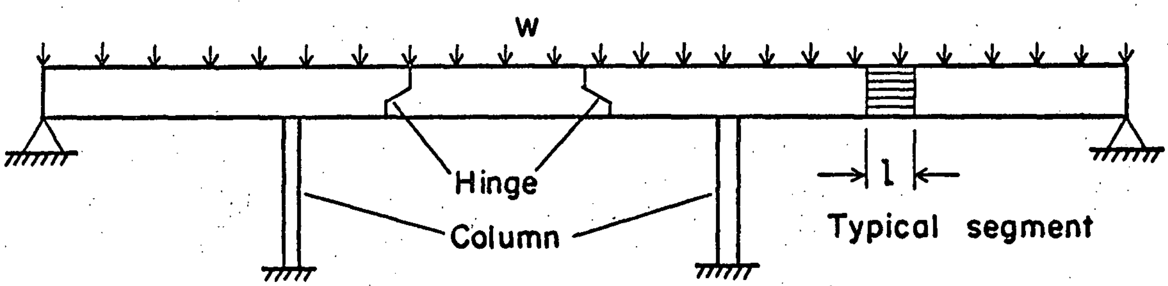


FIGURE I. GENERAL SYSTEM

2.1 ASSUMPTIONS

It is assumed that the beam is composed of a number of small segments of length l, loaded and constrained only at the joints between segments. The structure stiffness matrix is then generated from the stiffness matrices of these segments and the stiffness of the column. The critical load w is reached when the structure stiffness matrix becomes singular.

In the derivation of the member stiffness matrix and in the following theory, it is assumed that:

(a) The material remains elastic.

(b) Deflections are small.

(c) Plane sections remain plane.

(d) Vertical forces remain vertical during buckling.

(e) Beam is straight, horizontal and of rectangular cross-section.

(f) Column is straight, vertical and of constant moment of inertia.

The following derivations will set up the stiffness matrices of the various elements of the system.

2.2 STIFFNESS MATRIX OF BEAM SEGMENT

The matrix presented by Zavitz links the six components of force and deflection shown in Fig.(2) by

$$k\delta = f \tag{1}$$

where $\delta = (\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6)^*$

$f = (f_1, f_2, f_3, f_4, f_5, f_6)^*$

and k =

$$\begin{array}{|c|c|c|c|c|c|}
\hline
12\,\frac{EI}{l^3} & \frac{M}{l} - \frac{V}{2} & -6\,\frac{EI}{l^2} & -12\,\frac{EI}{l^3} & -\frac{M}{l} - \frac{V}{2} & -6\,\frac{EI}{l^2} \\
\hline
\frac{M}{l} - \frac{V}{2} & \frac{GJ}{l} & -\frac{Vl}{6} & -\frac{M}{l} + \frac{V}{2} & -\frac{GJ}{l} & \frac{Vl}{6} \\
\hline
-6\,\frac{EI}{l^2} & -M + \frac{Vl}{3} & 4\,\frac{EI}{l} & 6\,\frac{EI}{l^2} & \frac{Vl}{6} & 2\,\frac{EI}{l} \\
\hline
-12\,\frac{EI}{l^3} & -\frac{M}{l} + \frac{V}{2} & 6\,\frac{EI}{l^2} & 12\,\frac{EI}{l^3} & \frac{M}{l} + \frac{V}{2} & 6\,\frac{EI}{l^2} \\
\hline
-\frac{M}{l} - \frac{V}{2} & -\frac{GJ}{l} & \frac{Vl}{6} & \frac{M}{l} + \frac{V}{2} & \frac{GJ}{l} & -\frac{Vl}{6} \\
\hline
-6\,\frac{EI}{l^2} & \frac{Vl}{6} & 2\,\frac{EI}{l} & 6\,\frac{EI}{l^2} & M + \frac{Vl}{3} & 4\,\frac{EI}{l} \\
\hline
\end{array}$$

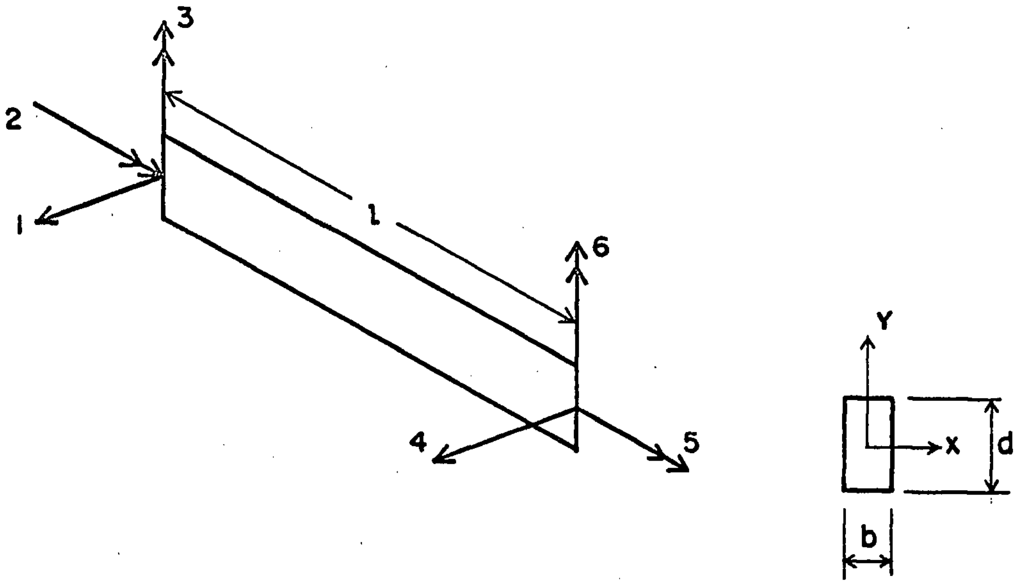


FIGURE 2. SEGMENT DEGREES OF FREEDOM FOR k

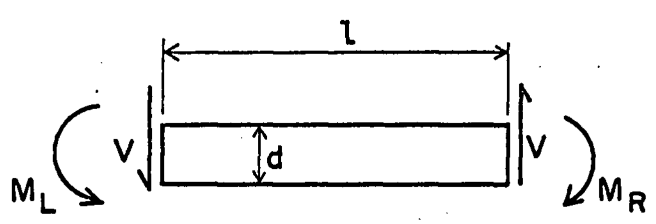
In the matrix k, the moment of inertia about the weak axis is:

$I = b^3d/12$

the torsion constant is:

$J = (b^3d/3)\ (1 - .63\ b/d)$

and the segment length is l. The symbols V and M represent the primary shear and moment about the strong axis induced by vertical loading as defined in Fig.(3).



$M = M_L + Vl/2 = ℄$ bending moment

FIGURE 3. SEGMENT SHEARS AND MOMENTS

2.3 TRANSFORMATIONS FOR RESTRAINTS

The Zavitz matrix k allows free and independent lateral translation and rotation of the cross sections. In the problem to be studied, the lateral motion of the top edge is prevented, but free torsional displacement is allowed. Fig.(4a) shows a cross section at the left end of the segment with degrees of freedom δ and f at the gravity axis. Fig.(4b) shows the same section with degrees of freedom $\bar{\delta}$ and $\bar{f}$ so placed to allow the locking of $\bar{\delta}_1$.

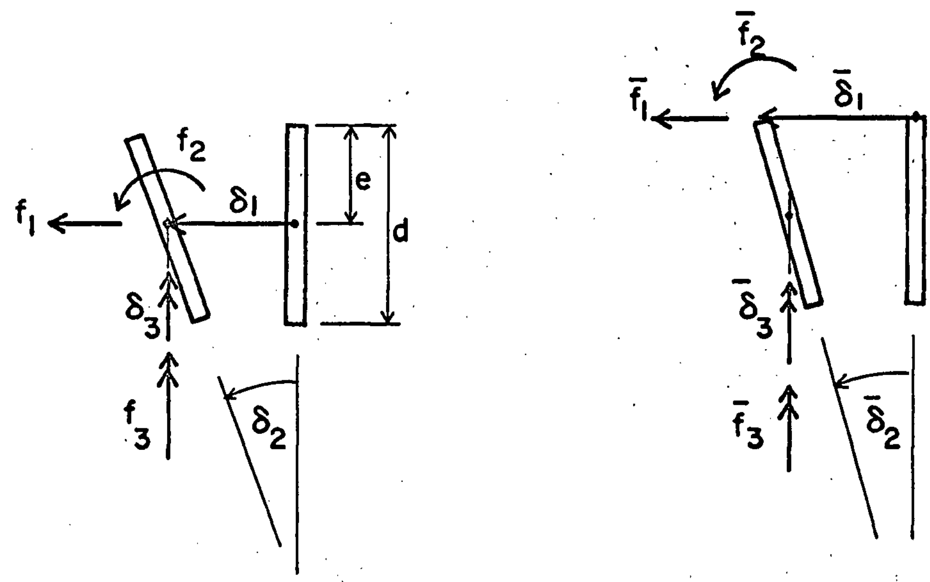


FIG. 4a FIG. 4b

FIGURE 4. LEFT END OF BEAM SEGMENT

The following equations will transform δ to $\bar{\delta}$:

$$
\left.
\begin{aligned}
\bar{\delta}_1 &= \delta_1 + e\delta_2 && \text{(a)}\\
\bar{\delta}_2 &= \delta_2 && \text{(b)}\\
\bar{\delta}_3 &= \delta_3 && \text{(c)}\\
\bar{\delta}_4 &= \delta_4 + e\delta_5 && \text{(d)}\\
\bar{\delta}_5 &= \delta_5 && \text{(e)}\\
\bar{\delta}_6 &= \delta_6 && \text{(f)}
\end{aligned}
\right\} \quad (2)
$$

that is,

$$\bar{\delta} = T\delta \tag{3}$$

where
$$T = \begin{pmatrix} 1 & e & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & e & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The forces f are transformed to $\bar{f}$ by

$$\bar{f} = T_1 f \tag{4}$$

where
$$T_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -e & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -e & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Eqns. (1), (3) and (4) yield:

$$(T_1 k T^{-1})\,\bar{\delta} = \bar{f} \tag{5}$$

Since $T^{-1} = T_1{}^*$

$$(T_1 k T_1{}^*)\,\bar{\delta} = \bar{f} \tag{6}$$

or $\bar{k} = T_1 k T_1{}^*$ (7)

where $\bar{k}$ links the deflections and forces of Fig.(4b).

Since the deck prevents $\bar{\delta}_1$ and $\bar{\delta}_4$, $\bar{k}$ can be reduced to a 4 x 4 matrix $\bar{\bar{k}}$ by removing rows and columns 1 and 4 from $\bar{k}$ to give:

$$\bar{\bar{k}} = \begin{bmatrix} 12\frac{EIe^2}{l^3} + \frac{JG}{l} - 2e\left(\frac{M}{l} - \frac{V}{2}\right) & 6\frac{EIe}{l^2} - \frac{Vl}{6} & -12\frac{EIe^2}{l^3} - \frac{JG}{l} + 2\frac{Me}{l} & 6\frac{EIe}{l^2} + \frac{Vl}{6} \\ 6\frac{EIe}{l^2} - M + \frac{Vl}{3} & 4\frac{EI}{l} & -6\frac{EI}{l^2} + \frac{Vl}{6} & 2\frac{EI}{l} \\ -12\frac{EIe}{l^3} - \frac{JG}{l} + 2\frac{Me}{6} & -6\frac{EIe}{l^2} + \frac{Vl}{6} & 12\frac{EIe^2}{l^3} + \frac{JG}{l} - 2e\left(\frac{M}{l} + \frac{V}{2}\right) & -6\frac{EIe}{l^2} - \frac{Vl}{6} \\ 6\frac{EIe}{l^2} + \frac{Vl}{6} & 2\frac{EI}{l} & -6\frac{EIe}{l^2} + M + \frac{Vl}{3} & 4\frac{EI}{l} \end{bmatrix}$$

Here $\bar{\bar{k}}\,\bar{\bar{\delta}} = \bar{\bar{f}}$ (8)

where $\bar{\bar{\delta}} = (\bar{\delta}_2, \bar{\delta}_3, \bar{\delta}_5, \bar{\delta}_6,)^* = (\bar{\bar{\delta}}_1, \bar{\bar{\delta}}_2, \bar{\bar{\delta}}_3, \bar{\bar{\delta}}_4,)^*$

$\bar{\bar{f}} = (\bar{f}_2, \bar{f}_3, \bar{f}_5, \bar{f}_6,)^* \quad (\bar{\bar{f}}_1, \bar{\bar{f}}_2, \bar{\bar{f}}_3, \bar{\bar{f}}_4,)^*$

where $\bar{\bar{\delta}}$ and $\bar{\bar{f}}$ are defined in Fig.(5).

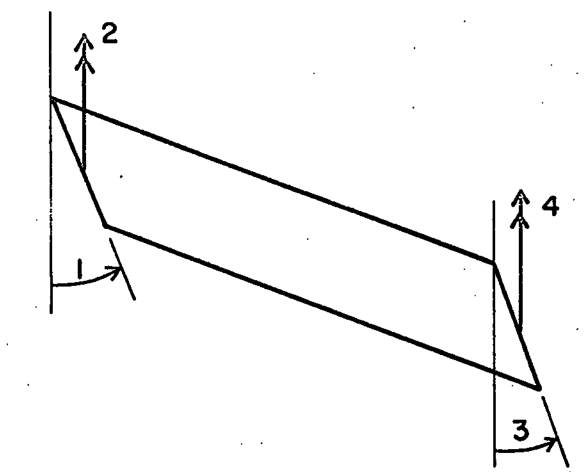


FIGURE 5. DEGREES OF FREEDOM FOR $\bar{\bar{k}}$

2.4 COLUMN STIFFNESS

As the beam rotates torsionally at the point of attachment of a column, the forces induced in the column top will create a torsional moment which will either increase or decrease this rotation. The column effect can then be replaced by a torsional spring constant SK which in turn is added to the diagonal term of the structure stiffness matrix S or subtracted from the load vector.

As the beam deflects at the column point, the rotation about the vertical axis could be restrained by torsional stress in the column. This effect is neglected.

In all cases it is assumed that none of the primary moment M from w goes into the column.

The effect of the column on the stiffness of the structure will depend upon the following:

1. The fixity of the ends of the column. There are five possible cases: pin-pin, pin-fix, fix-pin, fix-fix, and diaphragm where SK approaches infinity and no rotational degree of freedom exists at the top of the column.
2. Column properties: length L, moment of inertia, I_{zz} and modulus E.
3. Axial load R in the column.

(a) <u>Fix-Fix Case</u>

Here it is assumed that the column is fixed both at the foundation and to the beam so that a continuous deflection curve exists in the buckled state as shown in Fig.(6a).

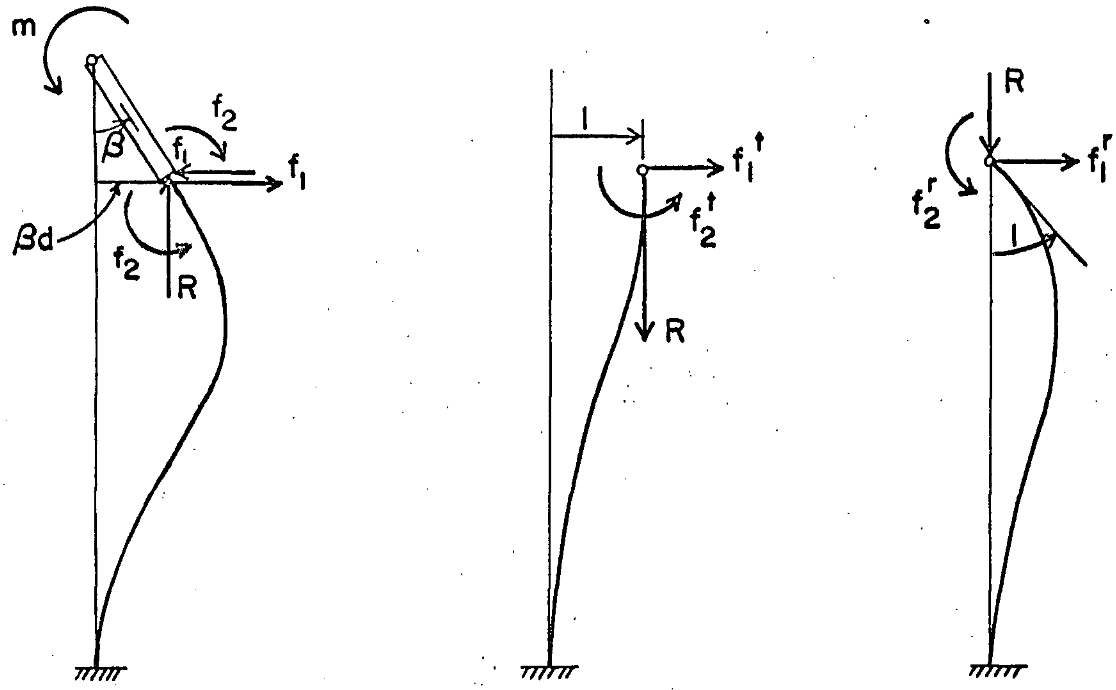


FIG. 6a FIG. 6b FIG. 6c

FIGURE 6. FIX – FIX COLUMN

Rather than solve for the forces f_1 and f_2 of Fig.(6a), due to rotation β and translation βd of the column top, it is more convenient to consider the two unit cases of Figs.(6b) and (6c). The forces at the column top of Figs. (6b) and (6c) are given in closed form by J. M. Gere and W. Weaver [4] as:

$$
\left.
\begin{aligned}
f_1^{\,t} &= 12\,\frac{EI}{L^3}\,\frac{(\lambda L)^3 \sin\lambda L}{12\,(2-2\cos\lambda L - \lambda L \sin\lambda L)} = 12\,\frac{EI}{L^3}\,S_1 && \text{(a)}\\
f_2^{\,t} &= 6\,\frac{EI}{L^2}\,\frac{(\lambda L)^2\,(1-\cos\lambda L)}{6\,(2-2\cos\lambda L - \lambda L \sin\lambda L)} = 6\,\frac{EI}{L^2}\,S_2 && \text{(b)}\\
f_1^{\,r} &= 6\,\frac{EI}{L^2}\,S_2 && \text{(c)}\\
f_2^{\,r} &= 4\,\frac{EI}{L}\,\frac{\lambda L\,(\sin\lambda L - \lambda L\cos\lambda L)}{4\,(2-2\cos\lambda L - \lambda L\sin\lambda L)} = 4\,\frac{EI}{L}\,S_3 && \text{(d)}
\end{aligned}
\right\}\quad (9)
$$

where $\lambda = \sqrt{R/EI}$

The final forces at the top are then:

$$f_1 = f_1^t \beta d + f_1^r \beta \tag{10}$$

$$f_2 = f_2^t \beta d + f_2^r \beta \tag{11}$$

Fig.(7) shows a more detailed view of the joint between two segments of beam to which the column is attached.

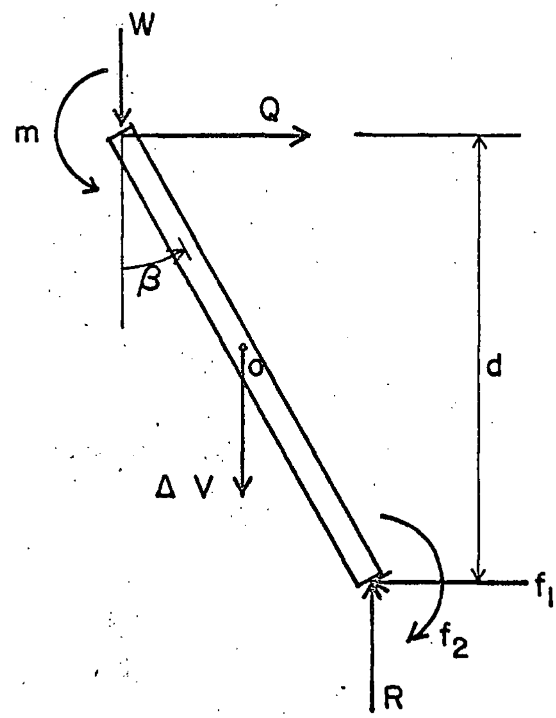


FIGURE 7. BEAM JOINT AT COLUMN

It should be noted that the force Q is shown as the locking force required to prevent top flange translation. Force W is an external joint load replacing the distributed load w on the segments. The final force ΔV represents the difference in primary shears V acting at the shear centre. Equilibrium of the joint requires that:

$$\Sigma V = 0 \qquad \Delta V = R - W \tag{12}$$

$$\Sigma H = 0 \qquad Q = f_1 \tag{13}$$

$$\Sigma Mo = 0 \qquad m = Qd/2 - W\beta d/2 + f_1 d/2 + f_2 - R\beta d/2 \tag{14}$$

or the torsional moment m equals:

$$m = f_1 d + f_2 - R\beta d/2 - W\beta d/2 \tag{15}$$

The term $W\beta d/2$ will be considered later so that:

$$m = (f_1{}^t \beta d + f_1{}^r \beta)d + (f_2{}^t \beta d + f_2{}^r \beta) - R\beta d/2 \tag{16}$$

The spring constant SK is m for $\beta = 1$ so that:

$$SK = 12\ EI\ S_1 d^2/L^3 + 2(6\ EI\ S_2 d/L^2) + 4\ EI\ S_3/L - Rd/2 \tag{17}$$

(b) Pin-Fix Case

Here the base is assumed fixed while the column top is pinned at the beam as shown in Fig.(8a).

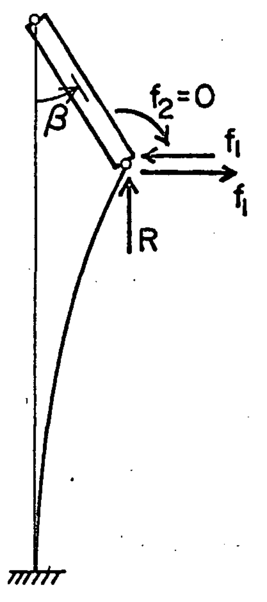


FIG. 8a

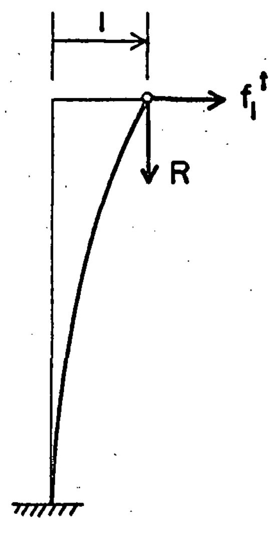


FIG. 8b

FIGURE 8. PIN — FIX COLUMN

As shown in Fig.(8b), the top of the column has no moment capacity and only the force f_1^t need be considered.

By superimposing the forces shown in Figs.(6b) and (6c), it can be shown that this force equals:

$$f_1^t = [f_1^t - f_1^r\ (f_2^t/f_2^r)] \tag{18}$$

where f_i on the RHS of Eqn.(18) are defined in Eqns.(9).

$$\text{Thus:}\quad SK = 3\ \frac{EI}{L^3}\ d^2\ \frac{(\lambda L)^3}{3(\tan\lambda L - \lambda L)} - Rd/2 \tag{19}$$

$$= 3\ EI\ S_5 d^2/L^3 - Rd/2 \tag{20}$$

$$\text{where } S_5 = (\lambda L)^3/3\ (\tan\lambda L - \lambda L) \tag{21}$$

(c) Fix-Pin Case

Here the base is assumed pinned while the top is fixed to the beam, Fig.(9a).

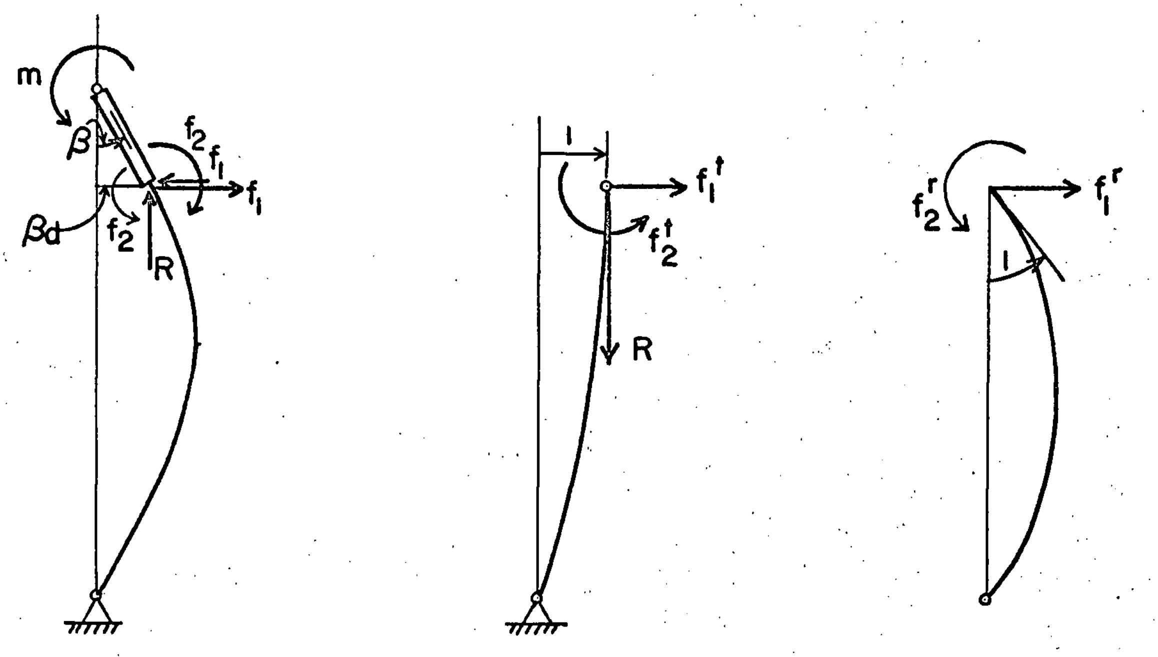


FIG. 9a FIG. 9b FIG. 9c

FIGURE 9. FIX — PIN COLUMN

The column top forces of Figs.(9b) and (9c) are found from a linear combination of the cases of Figs.(6b) and (6c) together with the function S_4 of Gere and Weaver. This gives:

$$SK = 3\ EI\ S_6 d^2/L^3 + 2\ (3\ EI\ S_7 d/L^2) + 3\ EI\ S_9/L - Rd/2 \qquad (22)$$

$$\text{where } S_6 = (\lambda L)^3/3\ (\tan \lambda L - \lambda L) \qquad (23)$$

$$S_7 = (\lambda L)^2 \tan \lambda L/3\ (\tan \lambda L - \lambda L) \qquad (24)$$

$$S_9 = S_7 \qquad (25)$$

(d) Pin-Pin Case

In this case it is assumed that the column is pinned at both the base and at the beam as shown in Fig.(10a).

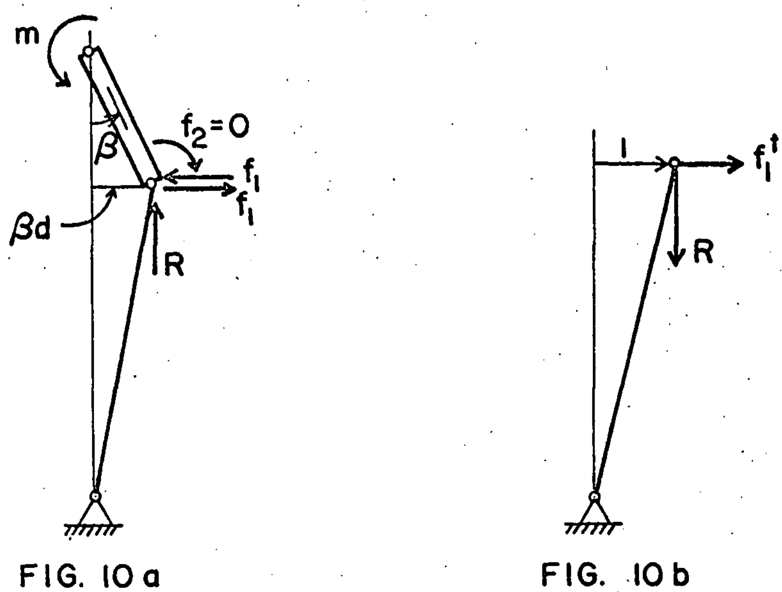


FIGURE 10. PIN — PIN COLUMN

From the statics of Fig.(10b), it is seen that: (26)

$$f_1^{\,t} = -\ R/L$$

$$f_1^{\,r} = f_2^{\,r} = f_2^{\,t} = 0 \qquad (27)$$

The spring constant equals: $SK = -\ Rd^2/L - Rd/2$ (28)

(e) Diaphragm Case

This case assumes that the lower edge of the beam and/or the top point of the column is so constrained that no possible rotation of the beam can occur at that point. This implies an infinitely stiff spring and the torsional degree of freedom is therefore set equal to zero.

2.5 EFFECT OF TOP EDGE LOADING

In the analysis of the forces acting at segment joints, Fig.(7), it was noted that a rotational moment m was produced due to the external joint load W. This final effect must be calculated at each joint in the structure and added to the diagonal elements of $\bar{\bar{k}}$. As previously derived in Eqn.(15) this moment is:

$$m = -W\beta d/2$$

Considering only one segment, $W = wl/2$ for a distributed load w on the segment. The additional buckling effect, SJL, as a spring constant is found by setting $\beta = 1$. Thus:

$$SJL = -wld/4 \tag{29}$$

must be added to elements 1,1 and 3,3 of $\bar{\bar{k}}$.

A concentrated joint load of P will produce a spring constant

$$SJL = -Pd/2 \tag{30}$$

which is added to element 1,1 of $\bar{\bar{k}}$ if the load is at the least joint number or element 3,3 if at the other end.

SOLUTION TECHNIQUE

The stiffness matrices presented in the previous chapters contain two types of parameters. Firstly are the system constants such as EI, L_b, J, etc., and secondly, the quantities M, V and R which depend on the external loads w and/or P. It is convenient to define M, V and R as primary internal forces due to basic input load of w and/or P. Since these internal forces vary linearly with external loads, the internal forces can be expressed as αM, αV and αR due to an external load of αw and/or αP. In other words the factor α defines a load level above the primary input level of loads.

The segment stiffness matrix now becomes at the load level α:

$$\bar{\bar{k}} = \bar{\bar{k}}_o + \alpha\bar{\bar{k}}_1 + \alpha\bar{\bar{k}}_2 \tag{31}$$

where
$$\bar{\bar{k}}_o = \begin{bmatrix} 12\dfrac{EIe^2}{l^3} + \dfrac{JG}{l} & 6\dfrac{EIe}{l^2} & -12\dfrac{EIe^2}{l^3} - \dfrac{JG}{l} & 6\dfrac{EIe}{l^2} \\ 6\dfrac{EIe}{l^2} & 4\dfrac{EI}{l} & -6\dfrac{EI}{l^2} & 2\dfrac{EI}{l} \\ -12\dfrac{EIe^2}{l^3} - \dfrac{JG}{l} & -6\dfrac{EIe}{l^2} & 12\dfrac{EIe^2}{l^3} + \dfrac{JG}{l} & -6\dfrac{EIe}{l^2} \\ 6\dfrac{EIe}{l^2} & 2\dfrac{EI}{l} & -6\dfrac{EIe}{l^2} & 4\dfrac{EI}{l} \end{bmatrix}$$

$$\bar{\bar{k}}_1 = \begin{bmatrix} -2\dfrac{Me}{l} & 0 & +2\dfrac{Me}{l} & 0 \\ -M & 0 & 0 & 0 \\ +2\dfrac{Me}{l} & 0 & -2\dfrac{Me}{l} & 0 \\ 0 & 0 & M & 0 \end{bmatrix}$$

$$\bar{\bar{k}}_2 = \begin{bmatrix} Ve & -\dfrac{Vl}{6} & 0 & +\dfrac{Vl}{6} \\ \dfrac{Vl}{3} & 0 & \dfrac{Vl}{6} & 0 \\ 0 & \dfrac{Vl}{6} & -Ve & -\dfrac{Vl}{6} \\ \dfrac{Vl}{6} & 0 & \dfrac{Vl}{3} & 0 \end{bmatrix}$$

The column spring constants are more difficult to consider as they contain αR inside the transcendental function S_i, which cannot be factored out as was done in segment matrices. However the column springs $SK(\alpha)$ can be written as:

$$\alpha \frac{[SK(\alpha)]}{[\ \alpha\ ]}$$

The structure stiffness matrix S can now be expressed as:

$$S = K_o + \alpha[K_1 + K_2 + SK(\alpha)/\alpha + SJL] \tag{32}$$

where K_o is the linear portion of the structure matrix generated by the appropriate addition of $\bar{\bar{k}}_o$ for each segment; K_1 and K_2 are the appropriate sums of $\bar{\bar{k}}_1$ and $\bar{\bar{k}}_2$ for each segment. The terms $SK(\alpha)/\alpha$ and SJL are added to the appropriate diagonal terms of $K_1 + K_2$. The terms in the square bracket of Eqn.(32) may be replaced by $K(\alpha)$ to give:

$$S = K_o + \alpha\ K(\alpha) \tag{33}$$

The square matrix $K(\alpha)$ is unfortunately a function of the load level α since it contains the transcendental function S_i.

The structure stiffness equation now becomes:

$$[K_o + \alpha\ K(\alpha)]\Delta = F = 0 \tag{34}$$

The problem now is to find a value of α for which S becomes singular. One way of doing this is to rewrite Eqn.(34) as

$$K_o\Delta = -\ \alpha\ K(\alpha)\Delta \tag{35}$$

and iterate as follows:

$$K_o\Delta_{n+1} = -\ \alpha\ K(\alpha)\Delta_n \tag{36}$$

A Δ_{n+1} is computed during one cycle and placed back in the right hand side as Δ_n for the next cycle. If this system is used with a Choleski routine, it has the advantage that only one conversion of the symmetric positive definite matrix K_o is required to lower triangular form while each iteration compares to an extra vector load. Further, the unsymmetric matrix $K(\alpha)$ is only needed to operate on Δ_n and no conversion or inversion is required. The disadvantage is that the system converges to the lowest absolute value of α and in some structures this may yield a negative α, indicating that the external load must be in the opposite direction to those assumed in calculating M, V and R. While this is mathematically precise, it is physically impractical as in the majority of practical cases, including the ones considered here, the loads are gravity loads so that a negative α indicates that the structure must be turned upside down. The system would have indicated which sense the wind must blow on an unsymmetric structure to produce minimum buckling load.

The iteration system was then rejected in favour of calculating the determinant of S as a function of α. The first time $|S|$ becomes zero gives a critical value of α from which all internal stresses at buckling can be calculated. Further, the graph of $|S|$ versus α can be started at any value of α to pick up positive or negative critical α as desired.

A computer program was written to output $|S|$ for various α for any type of column, and diaphragms. Input consists of the primary M and V for each segment together with primary external loads w and P and the column axial load R. A description of this program is given in Appendix I.

EXPERIMENTAL MODEL TESTS

4.1 INTRODUCTION

The three span beam with hinges in the middle span, Fig.(1), is a common type of structure where buckling is a problem. For this reason it was decided to concentrate on this example. Rather than model the whole structure, it was decided to use only the left portion as shown in Fig.(11) as the simply supported centre portion has its unsupported lower edge all in tension.

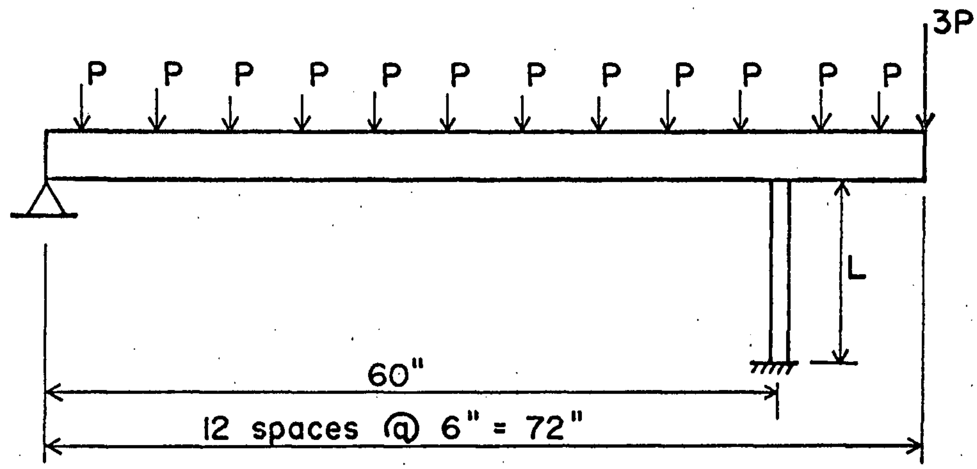


FIGURE 11. TEST BEAM AND LOADING

4.2 APPARATUS

The photo in Fig.(12) shows the test frame as a stiff bench supporting the left end and the column. The plywood wall behind was rigidly fastened to the bench and served to laterally brace the top edge.

The left support permitted free rotation about the principal beam axes but prevented all other motion. Aluminum bars, with a small hole in each end, slipped over finishing nails driven into the top edge of the beam and into a cleat on the wall. These bars, spaced at 3" on centres, prevented lateral motion of the top edge but were loose enough to allow free rotation.

Loads were applied to the top edge through scale pans below the table. The photo of Fig.(13) shows how the load is applied to the top edge so that no torsional restraint is provided. The bolts in compression were turned to a sharp point in a lathe to fit into a conical hole drilled into the steel plate on the top edge of the beam. The photo of Fig.(13) also shows the pin-pin column. This was also a metal to metal contact of a point in a conical hole to provide no rotational restraint.

The columns fixed at the base and pinned at the top were also made of steel. These were clamped to the bench and provided with a conical top end as was the pin-pin case.

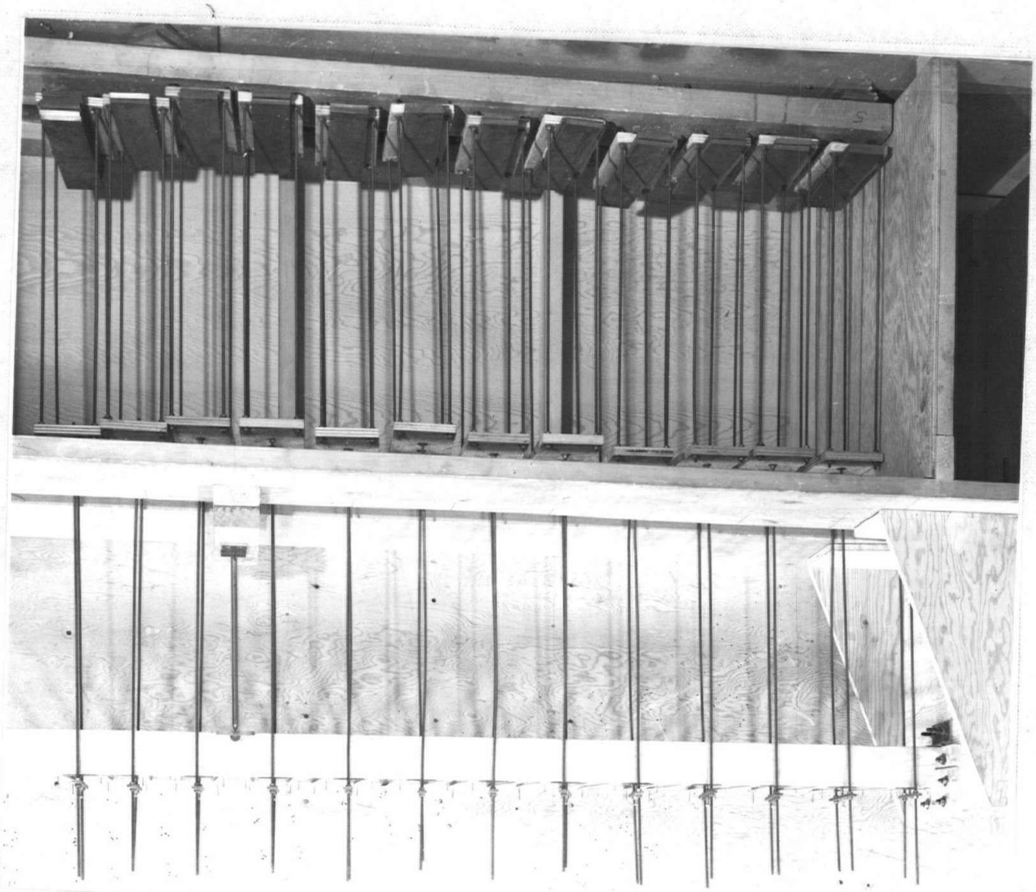


FIGURE 12. TEST FRAME

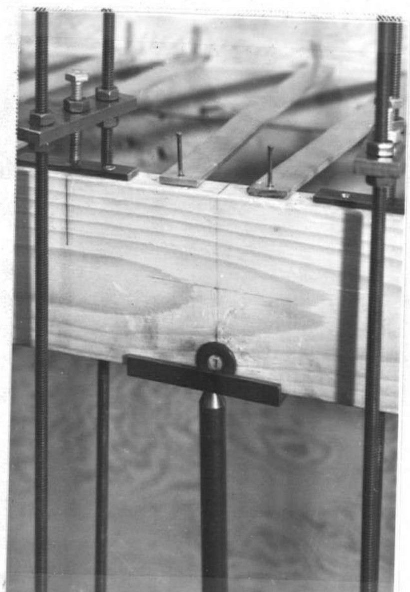


FIGURE 13. TOP FLANGE LOADING, PIN-PIN COLUMN

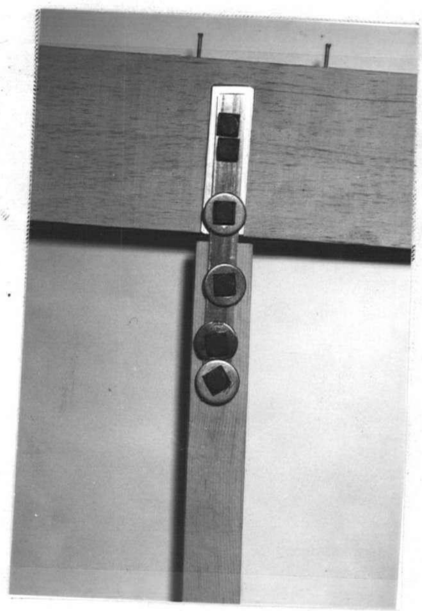


FIGURE 14. FIX-FIX COLUMN

The photo of Fig.(14) shows a fix-fix wood column tested and the method of connection to the beam. The bottom was clamped to the bench.

One of the bars used to restrain the top edge was placed at the column top to prevent lateral deflection of that point. This prevented torsional rotation and acted as a diaphragm.

4.3 PROCEDURE

The model beams used were clear, dry Douglas Fir. The value of E_b was measured by bending the sample about its weak axis over its full length with a concentrated force at the midpoint. The value of G was found by twisting the sample over its full length with equal end torques. The values of E_b and G together with measured dimensions of the piece were used in computer runs to predict the critical load. The critical load was found experimentally by means of a Southwell plot - (or Lundquist plot). The rotation of the beam about its longitudinal axis was measured at increasing load levels by means of a mirror taped to the side of the beam. The plot used this rotation with the load level.

4.4 EXPERIMENTAL RESULTS

Table II summarizes the results of eight tests. The spans were always as shown in Fig.(11). The loading was as in Fig.(11) for the first seven tests, but test #8 has a load P at the column point only.

TABLE I - EXPERIMENTAL RESULTS

NRS & Beam XS	Beam E & G (ksi)	Test #	Column Case and Parameters	Model P_{CR} (lb)	Theory P_{CR} (lb)	σ_{CR} at Column (psi)
102 3.52" .761"	2312. 82.7	1	pin-pin L = 15"	14.63	14.24	446.
		2	pin-fix 3/8" ϕ L = 13.5"	23.46	22.27	716.
103 3.034" .314"	1964. 148.9	3	diaphragm	47.73	54.58	4760.
		4	fix-fix .425" 1.015" L = 13.5" z	49.72	54.31	4960.
104 3.334" .365"	2029. 125.7	5	pin-fix .5" .5" L = 13.5"	5.53	5.29	390.
		6	pin-fix 3/8" ϕ L = 13.5"	12.42	11.02	877.
		7	diaphragm	77.72	78.82	5480.
105 102 with only load above column	2312. 82.7	8	pin-pin L = 15"	149.	148.	

4.5 DISCUSSION

The results were considered quite good considering the complexity of the structure and the number of variable parameters. They were consistent with expected results in that, due to friction true pin ends could not be expected, and the experimental results are thus slightly higher than theoretical. Conversely total restraint at the column could not be provided by the experimental diaphragm and fix-fix cases and therefore these cases are slightly below the theoretical.

In setting up the experiment, it was noted that sharpness of the pins of the loading apparatus and the sharpness of the pin-ended columns significantly affected the results. Both the pin ends and corresponding conical holes in the metal plates had to be carefully machined. The initial eccentricities of the loads and column reactions will not affect the final buckling moment. This is analgous to lateral loads on columns not affecting the Euler load. This fact can be used to advantage by offsetting the loads slightly to overcome initial deformations of the sample and keep the rotations small. A better Southwell plot can thus be obtained, i.e. one having small rotations right up to incipient buckling.

The experiments carried out verified that the computer program prepared in accordance with the presented theory was in fact in close agreement with the actual physical conditions. Therefore no further testing was deemed advisable and the program could be used to check various beam and column structures to produce design criteria.

DESIGN CHARTS

5.1 STRUCTURE PARAMETERS

It is impractical to construct design charts for general structures of the complexity considered here because the multitude of system parameters require a prohibitive number of computer runs and graphs. For this reason, it was decided to construct design graphs for the structure of Fig.(1) with two hinges in the centre span. Such information will help in understanding and predicting the behavior of similar structures. As was the case with the model study, only a portion of the structure was analyzed, Fig.(15), as the middle portion is assumed not to buckle. Fig.(15) shows the two load cases studied. Both load cases produce the same moment at the support and create compression in the unsupported bottom edge, but the half load of Fig.(15b) produces compression over the lower edge of the whole system, whereas that of Fig.(15a) has part of this edge in tension.

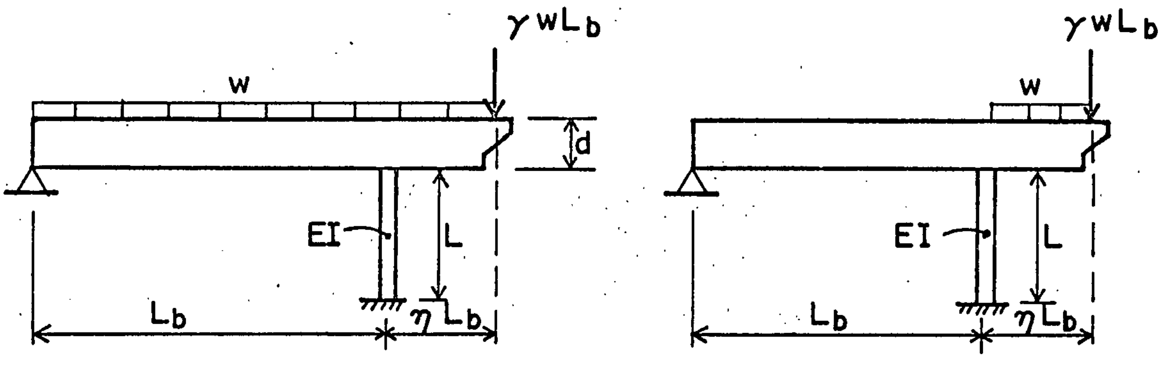


FIG. 15 a
FULL LOAD CASE

FIG. 15 b
HALF LOAD CASE

FIGURE 15. STRUCTURE AND LOAD CASES ANALYZED

The critical stress, σ_{CR}, must be a function of the following parameters for a specific type of loading:

$$\sigma_{CR} = f\ [b,\ d,\ L_b,\ E_b,\ G_b,\ \eta,\ \gamma,\ EI,\ L] \tag{37}$$

Herein:

E_b is the beam modulus of elasticity

G_b is the beam shear modulus

b is the beam width

and the remainder are defined in Fig.(15).

These ten items contain two dimensions,so that the following eight dimensionless parameters govern the problem:

$$\frac{\sigma_{CR}}{E} = f\ \left[\frac{b}{d},\ \frac{d}{L_b},\ \frac{E_b}{G_b},\ \eta,\ \gamma,\ \frac{L}{d},\ \frac{\phi EI\ L_b}{bd^2L^2E_b}\right] \tag{38}$$

where ϕ is a dimensionless number.

The final two ratios will not affect σ_{CR} if a diaphragm between adjacent beams is placed at the column top, and the final ratio is not required for the pin-pin column case.

In all cases the top edge is free to rotate torsionally but restrained from lateral motion. The left support prevents all motion except rotation about the principal axes. The right end is free to rotate about all three axes.

The final dimensionless ratio merits more discussion and is defined as:

$$n = \phi\ EI\ L_b/bd^2L^2E_b \tag{39}$$

where ϕ is a dimensionless constant and may be given any value without affecting the result. It is convenient to define ϕ as:

$$\phi = 6\pi^2\ \left(\frac{E_b}{f_b}\right)\ \left(\frac{Mm}{RL_b}\right) \tag{40}$$

where: Mm is the maximum moment in the beam at full load condition.

R is the column reaction at full load condition.

f_b is a reasonable allowable bending stress. This was set at 2.1 ksi for the whole study and in no way is it connected to the actual bending stress which may be present in any real beam.

The ratio E_b/f_b was set at 1600/2.1. Since Mm is proportional to RL_b, then ϕ depends on η and γ. This is permissible as η and γ have been included as ratios. With this definition of ϕ, the ratio n becomes:

$$n = \left[\frac{(\alpha_1 \, Mm)/Z}{f_b}\right] \left[\frac{\pi^2 \, EI/L^2}{(\alpha_1 R)}\right] \tag{41}$$

The load level α_1 as included does not affect n but affords the following interpretation for n. If α_1 is chosen to give the bending design load on the system, then α_1 Mm/Z is the maximum bending stress in the beam. The left hand ratio is then the ratio of this stress to an allowable stress and will be near unity. The term $\alpha_1 R$ then represents the column load at this condition so that the right hand term is the ratio of the critical load of a pin ended Euler column to the actual load in the column. Thus n is approximately equal to the factor of safety against column buckling and will vary from about two to five. It is not exactly the safety factor because f_b may not be 2.1 ksi and the column may not be pin-pin. However, this serves to define an order of magnitude of n. In all cases, n is to be calculated from the preceding formula, Eqn.(41), using f_b = 2.1 ksi, even if the column is not pin-pin.

Example

Determine n for a structure with

L_b = 40 ft. $\qquad$ η = .25

L = 255 in. $\qquad$ γ = .3

d = 50 in. $\qquad$ b = 5 in.

EI = 960,000 k in^2 (any support condition, any material)

Solution

If w is the uniform load (kips/ft.)

$$Mm = [w\ .3(40)\ .25(40) + w\ (.25 \times 40)^2/2]\ 12 = 2040\ w \text{ kip in.}$$

$$Z = 5(50)^2/6 = 2083 \text{ in}^3$$

$$\text{Full Load } R = [(40 + 10)^2 w/2 + .3(40)\ w(50)]/40 = 46.25\ w \text{ kips}$$

From Eqn.(41)

$$n = \frac{[\ 2040w\ ]}{[2.1(2083)]} \frac{[9.87(960,000)]}{[65025(46.25w)]} = 1.47$$

5.2 VALUES OF PARAMETERS

Results are presented for five separate cases of column type, namely:

(1) Pin-pin

(2) Pin-fix (fix at bottom)

(3) Fix-pin (fix at top)

(4) Fix-fix

(5) Diaphragm at column top

For all column types the following values of η and γ were used:

Case	η	γ
A	.1	.4
B	.2	.3
C	.2	.5
D	.22	.28
E	.3	.2

These values are for three equal bays, except case C which has the centre bay (column to column) equal to 1.4 L_b. Case D places the hinge such that the maximum positive and negative moments are equal.

The parameter L/d was constant at 5.1 to ease the plotting problem.

The factor n was set at 2.0, 3.0 and 5.0.

The ratio E_b/G_b is taken at 16 as the material under consideration is glulaminated timber.

The ratio L_b/d was set at 6.0, 15.0 and 37.5.

The depth to breadth ratio of the beams was taken as 6.0, 8.0 and 10.0.

Results of the above parameters are given in tabular form in Tables II to VI, and plotted on graphs, Fig.(16) to (30).

In all of the following graphs, the ordinate is:

$$1000\ \sigma_{CR}/E_b$$

where σ_{CR} represents the maximum bending stress at buckling whether it be from positive or negative moment, except in the diaphragm case where it represents the bending stress at the column. The abscissa used in all graphs is:

$$\rho = 1000\ (b/d)^2\ \sqrt{d/L_c}$$

where L_c is the length of the lower edge which is actually under compression. Many plots of critical stress were made against various parameters before ρ was chosen as the most convenient. This definition of ρ seems to bring the curves of each graph close together so they may be approximated with one straight line either horizontal or through the origin.

$\frac{L_b}{d}$ / LOAD	$\frac{d}{b}$	A ρ	A ORD.	B ρ	B ORD.	C ρ	C ORD.	D ρ	D ORD.	E ρ	E ORD.
6. FULL	6	26.05	15.67	18.90	10.45	17.12	9.11	18.15	9.90	15.89	9.07
	8	14.62	8.99	10.62	6.00	9.61	5.25	10.18	5.71	8.92	5.23
	10	9.36	5.89	6.80	3.89	6.16	3.64	6.52	3.70	5.71	3.39
6. HALF	6	10.82	6.64	10.35	6.91	10.35	6.56	10.28	6.64	9.95	6.62
	8	6.08	3.83	5.81	3.99	5.82	3.79	5.77	3.82	5.58	3.82
	10	3.90	2.48	3.73	2.58	3.73	2.46	3.70	-	3.58	2.48
15. FULL	6	16.45	8.07	11.93	6.54	10.81	6.00	11.47	6.29	10.03	5.79
	8	9.25	4.60	6.72	3.79	6.08	3.46	6.45	3.63	5.65	3.36
	10	5.92	3.02	4.29	2.43	3.89	2.25	4.12	2.35	3.61	2.18
15. HALF	6	6.84	4.82	6.54	4.93	6.55	4.73	6.49	4.72	6.28	4.65
	8	3.85	2.77	3.68	2.86	3.68	2.73	3.66	2.74	3.54	2.70
	10	2.46	1.83	2.36	1.86	2.36	1.78	2.34	1.78	2.26	1.75
37.5 FULL	6	10.42	5.42	7.56	4.69	6.85	4.44	7.26	4.59	6.36	4.34
	8	5.64	-	4.09	2.73	3.71	2.57	3.94	2.63	3.45	2.52
	10	3.74	2.07	2.72	1.77	2.46	1.67	2.61	1.71	2.28	1.64
37.5 HALF	6	4.33	3.95	4.14	4.01	4.14	3.84	4.11	3.84	3.98	3.83
	8	2.35	2.25	2.25	2.37	2.25	2.25	2.23	2.27	2.16	2.21
	10	1.56	1.48	1.49	1.52	1.49	1.46	1.48	1.45	1.43	1.44

TABLE II - DIAPHRAGM CASE RESULTS

L_b/d LOAD		d/b	A ρ	A ORD.	B ρ	B ORD.	C ρ	C ORD.	D ρ	D ORD.	E ρ	E ORD.
6.	FULL	6	26.05	.125	18.90	.225	17.12	.260	18.15	.231	15.89	.276
		8	14.62	.073	10.62	.131	9.61	.151	10.18	.134	8.92	.163
		10	9.36	.048	6.80	.085	6.16	.098	6.52	.087	5.71	.105
	HALF	6	10.82	.220	10.35	.382	10.35	.388	10.28	.389	9.95	.459
		8	6.08	.128	5.81	.222	5.82	.226	5.77	.226	5.58	.266
		10	3.90	.084	3.73	.144	3.73	.147	3.70	.147	3.58	.173
15.	FULL	6	16.45	.121	11.93	.214	10.81	.248	11.47	.219	10.03	.262
		8	9.25	.071	6.72	.125	6.08	.144	6.45	.127	5.65	.152
		10	5.92	.046	4.29	.081	3.89	.094	4.12	.083	3.61	.099
	HALF	6	6.84	.214	6.54	.368	6.55	.370	6.49	.370	6.28	.434
		8	3.85	-	3.68	-	3.68	-	3.66	-	3.54	-
		10	2.46	-	2.36	-	2.36	-	2.34	-	2.26	-
37.5	FULL	6	10.42	.119	7.56	.209	6.85	.242	7.26	.214	6.36	.255
		8	5.64	.069	4.09	.122	3.71	.141	3.94	.124	3.45	.148
		10	3.74	.045	2.72	.079	2.46	.092	2.61	-	2.28	.097
	HALF	6	4.33	.209	4.14	-	4.14	.361	4.11	.361	3.98	.424
		8	2.35	-	2.25	-	2.25	-	2.23	-	2.16	-
		10	1.56	-	1.49	-	1.49	-	1.48	-	1.43	-

TABLE III - PIN-PIN CASE RESULTS

CASE →		A				B				C				D				E			
	n →		2	3	5		2	3	5		2	3	5		2	3	5		2	3	5
$\frac{L_b}{d}$ LOAD	$\frac{d}{b}$	ρ	ORDINATE			ρ	ORDINATE			ρ	ORDINATE			ρ	ORDINATE			ρ	ORDINATE		
6. FULL	(6	26.05	.411	.478	.612	18.90	.368	.435	.569	17.12	.381	.446	.575	18.15	.359	.424	.552	15.89	.398	.463	.594
	(8	14.62	.295	.365	.496	10.62	.270	.337	.471	9.61	.276	.341	.470	10.18	.261	.327	.459	8.92	.286	.352	.482
	(10	9.36	.239	.305	.439	6.80	.222	.289	.423	6.16	.225	.289	.419	6.52	.216	.282	.413	5.71	.232	.297	.427
6. HALF	(6	10.82	.449	.567	.803	10.35	.604	.721	.955	10.35	.569	.666	.860	10.28	.600	.712	.934	9.95	.662	.771	.988
	(8	6.08	.360	.478	.714	5.81	.449	.566	.799	5.82	.412	.509	.702	5.77	.442	.554	.776	5.58	.476	.585	.801
	(10	3.90	.317	.435	.670	3.73	.374	.490	.726	3.73	.336	.432	.625	3.70	.366	.477	.698	3.58	.385	.494	.710
15. FULL	(6	16.45	.403	.470	.604	11.93	.356	.423	.557	10.81	.369	.434	.563	11.47	.343	.409	.540	10.03	.383	.448	.579
	(8	9.25	.291	.357	.491	6.72	.264	.330	.464	6.08	.269	.333	.462	6.45	.255	.320	.451	5.65	-	.343	.473
	(10	5.92	.236	.302	.436	4.29	.218	.285	.418	3.89	.220	.285	.414	4.12	.212	.277	.408	3.61	.226	.291	.421
15. HALF	(6	6.84	.445	.561	.797	6.54	.590	.703	.937	6.55	.552	.649	.842	6.49	.581	.693	.915	6.28	.637	.746	.963
	(8	3.85	-	-	-	3.68	-	-	-	3.68	-	-	-	3.66	-	-	-	3.54	-	-	-
	(10	2.46	-	-	-	2.36	-	-	-	2.36	-	-	-	2.34	-	-	-	2.26	-	-	-
37.5 FULL	(6	10.42	.397	.464	.597	7.56	.351	.417	.551	6.85	.363	.428	.556	7.26	.338	.403	.534	6.36	.376	.441	.572
	(8	5.64	.287	.353	.487	4.09	.260	.327	.460	3.71	.265	.330	.458	3.94	.251	.317	.448	3.45	.273	.338	.468
	(10	3.74	.233	.300	.433	2.72	.216	.282	.415	2.46	.218	.282	.410	2.61	.209	-	.405	2.28	.223	.288	.417
37.5 HALF	(6	4.33	.439	.557	.793	4.14	-	-	-	4.14	.543	.640	.833	4.11	.573	.684	.906	3.98	.627	.736	.952
	(8	2.35	-	-	-	2.25	-	-	-	2.25	-	-	-	2.23	-	-	-	2.16	-	-	-
	(10	1.56	-	-	-	1.49	-	-	-	1.49	-	-	-	1.48	-	-	-	1.43	-	-	-

TABLE IV - PIN-FIX CASE RESULTS

CASE →		A				B				C				D				E			
n →			2	3	5		2	3	5		2	3	5		2	3	5		2	3	5
$\frac{L_b}{d}$ LOAD	$\frac{d}{b}$	ρ	ORDINATE			ρ	ORDINATE			ρ	ORDINATE			ρ	ORDINATE			ρ	ORDINATE		
6. FULL	(6	26.05	2.01	2.97	4.89	18.90	2.02	2.99	4.92	17.12	1.98	2.93	4.82	18.15	1.98	2.94	4.84	15.84	1.99	2.95	4.84
	(8	14.62	1.97	2.93	4.84	10.62	1.99	2.95	4.84	9.61	1.95	2.89	4.66	10.18	1.95	2.90	4.73	8.92	1.96	2.90	4.67
	(10	9.36	1.95	2.91	4.81	6.80	1.96	2.91	4.18	6.16	1.92	2.83	3.69	6.52	1.93	2.86	3.65	5.71	1.90	2.84	3.36
6. HALF	(6	10.82	3.76	5.42	6.60	10.35	3.73	5.60	6.82	10.35	3.11	4.59	6.46	10.28	3.57	5.25	6.57	9.95	3.52	5.17	6.57
	(8	6.08	3.55	3.81	3.82	5.81	3.61	3.95	3.97	5.82	3.04	3.74	3.78	5.77	3.45	3.79	3.81	5.58	3.40	3.79	3.81
	(10	3.90	2.47	2.48	2.48	3.73	2.56	2.57	2.57	3.73	2.45	2.45	2.46	3.70	2.47	2.47	2.47	3.58	2.46	2.47	2.47
15. FULL	(6	16.45	2.00	2.97	4.88	11.93	2.01	2.98	4.90	10.81	1.98	2.93	4.79	11.47	1.98	2.93	4.80	10.03	1.99	2.94	4.79
	(8	9.25	1.96	2.93	4.83	6.72	1.98	2.93	4.15	6.08	1.94	2.87	3.45	6.45	1.95	2.79	3.61	5.65	1.95	2.87	3.35
	(10	5.92	1.95	2.90	4.80	4.29	-	2.65	2.70	3.89	1.91	2.23	2.25	4.12	1.92	2.34	2.35	3.61	1.91	2.17	2.18
15. HALF	(6	6.84	3.65	4.76	4.78	6.54	3.71	4.90	4.93	6.55	3.12	4.50	4.71	6.49	3.55	4.68	4.71	6.28	3.51	4.61	4.64
	(8	3.85	-	2.77	2.77	3.68	-	2.86	2.86	3.68	-	2.73	2.73	3.66	2.74	2.74	2.74	3.54	-	2.70	2.70
	(10	2.46	1.83	1.83	1.83	2.36	1.86	1.86	1.86	2.36	1.78	1.78	1.78	2.34	1.78	1.78	1.78	2.26	1.75	1.75	1.75
37.5 FULL	(6	10.42	2.00	2.96	4.87	7.56	2.01	2.97	4.84	6.85	1.97	2.92	4.39	7.26	1.97	2.92	4.48	6.36	1.98	2.93	4.31
	(8	5.64	1.96	2.92	4.82	4.09	1.97	2.87	3.00	3.71	1.94	2.55	2.57	3.94	1.94	2.61	2.63	3.45	1.94	2.50	2.51
	(10	3.74	1.94	2.87	4.63	2.72	1.89	1.95	1.95	2.46	1.66	1.67	1.67	2.61	1.70	1.71	1.71	2.28	1.63	1.63	1.64
37.5 HALF	(6	4.33	3.67	3.94	3.94	4.14	-	4.01	4.01	4.14	3.09	3.84	3.85	4.11	3.35	3.84	3.84	3.98	3.50	3.80	3.80
	(8	2.35	-	2.25	2.25	2.25	-	2.37	2.37	2.25	-	2.25	2.25	2.23	2.27	2.27	2.27	2.16	-	2.21	2.21
	(10	1.56	1.48	1.48	1.48	1.49	1.52	1.52	1.52	1.49	1.46	1.46	1.46	1.48	1.45	1.45	1.45	1.43	1.44	1.44	1.44

TABLE V - FIX-PIN CASE RESULTS

CASE →			A				B				C				D				E			
n →				2	3	5		2	3	5		2	3	5		2	3	5		2	3	5
$\frac{L_b}{d}$	LOAD	$\frac{d}{b}$	ρ	ORDINATE			ρ	ORDINATE			ρ	ORDINATE			ρ	ORDINATE			ρ	ORDINATE		
6.	FULL	(6	26.05	4.17	6.19	10.2	18.90	4.18	6.20	9.95	17.12	4.10	6.04	8.78	18.15	4.11	6.08	9.32	15.84	4.12	6.07	8.77
		(8	14.62	4.11	6.12	10.1	10.62	4.11	5.91	6.61	9.61	3.99	5.10	5.22	10.18	4.03	5.45	5.67	8.92	4.01	5.10	5.20
		(10	9.36	4.08	6.07	9.93	6.80	3.89	4.23	4.27	6.16	3.32	3.38	3.39	6.52	3.56	3.67	3.68	5.71	3.34	3.37	3.38
	HALF	(6	10.82	6.53	6.61	6.62	10.35	6.73	6.85	6.88	10.35	6.10	6.50	6.54	10.28	6.45	6.58	6.60	9.95	6.41	6.57	6.61
		(8	6.08	3.81	3.82	3.82	5.81	3.96	3.97	3.97	5.82	3.76	3.78	3.78	5.77	3.80	3.81	3.82	5.58	3.80	3.81	3.81
		(10	3.90	2.48	2.48	2.48	3.73	2.57	2.57	2.58	3.73	2.45	2.45	2.46	3.70	2.48	2.48	2.48	3.58	2.47	2.47	2.47
15.	FULL	(6	16.45	4.17	6.18	10.2	11.93	4.16	6.10	7.17	10.81	4.07	5.72	5.97	11.47	4.09	5.86	6.26	10.03	4.09	5.64	5.80
		(8	9.25	4.07	6.10	9.79	6.72	3.93	4.11	4.16	6.08	3.41	3.45	3.46	6.45	3.56	3.62	3.63	5.65	3.33	3.35	3.36
		(10	5.92	4.06	5.96	6.91	4.29	-	2.70	2.70	3.89	2.24	2.25	2.25	4.12	2.35	2.35	2.35	3.61	2.18	2.18	2.18
	HALF	(6	6.84	4.78	4.82	4.78	6.54	4.91	4.92	4.93	6.55	4.70	4.71	4.72	6.49	4.70	4.71	4.72	6.28	4.63	4.64	4.64
		(8	3.85	2.77	2.77	2.77	3.68	2.86	2.86	2.86	3.68	2.73	2.73	2.73	3.66	2.74	2.74	2.74	3.54	2.70	2.70	2.70
		(10	2.46	1.83	1.83	1.83	2.36	1.86	1.86	1.86	2.36	1.78	1.78	1.78	2.34	1.78	1.78	1.78	2.26	1.75	1.75	1.75
37.5	FULL	(6	10.42	4.16	6.16	10.0	7.56	4.14	5.14	5.17	6.85	4.02	4.42	4.43	7.26	4.04	4.54	4.54	6.36	4.02	4.33	4.34
		(8	5.64	4.09	6.05	-	4.09	2.99	3.00	3.00	3.71	2.57	2.57	2.57	3.94	2.63	2.63	2.63	3.45	2.51	2.51	2.51
		(10	3.74	4.01	4.73	4.76	2.72	1.95	1.95	1.95	2.46	1.67	1.67	1.67	2.61	1.71	1.71	1.71	2.28	1.64	1.64	1.64
	HALF	(6	4.33	3.94	3.94	3.94	4.14	4.01	4.01	4.01	4.14	3.84	3.85	3.85	4.11	3.84	3.84	3.84	3.98	3.80	3.80	3.80
		(8	2.35	2.25	2.25	2.25	2.25	2.37	2.37	2.37	2.25	2.25	2.25	2.25	2.23	2.27	2.27	2.27	2.16	2.21	2.21	2.21
		(10	1.56	1.48	1.48	1.48	1.49	1.52	1.52	1.52	1.49	1.46	1.46	1.46	1.48	1.45	1.45	1.45	1.43	1.44	1.44	1.44

TABLE VI - FIX-FIX CASE RESULTS

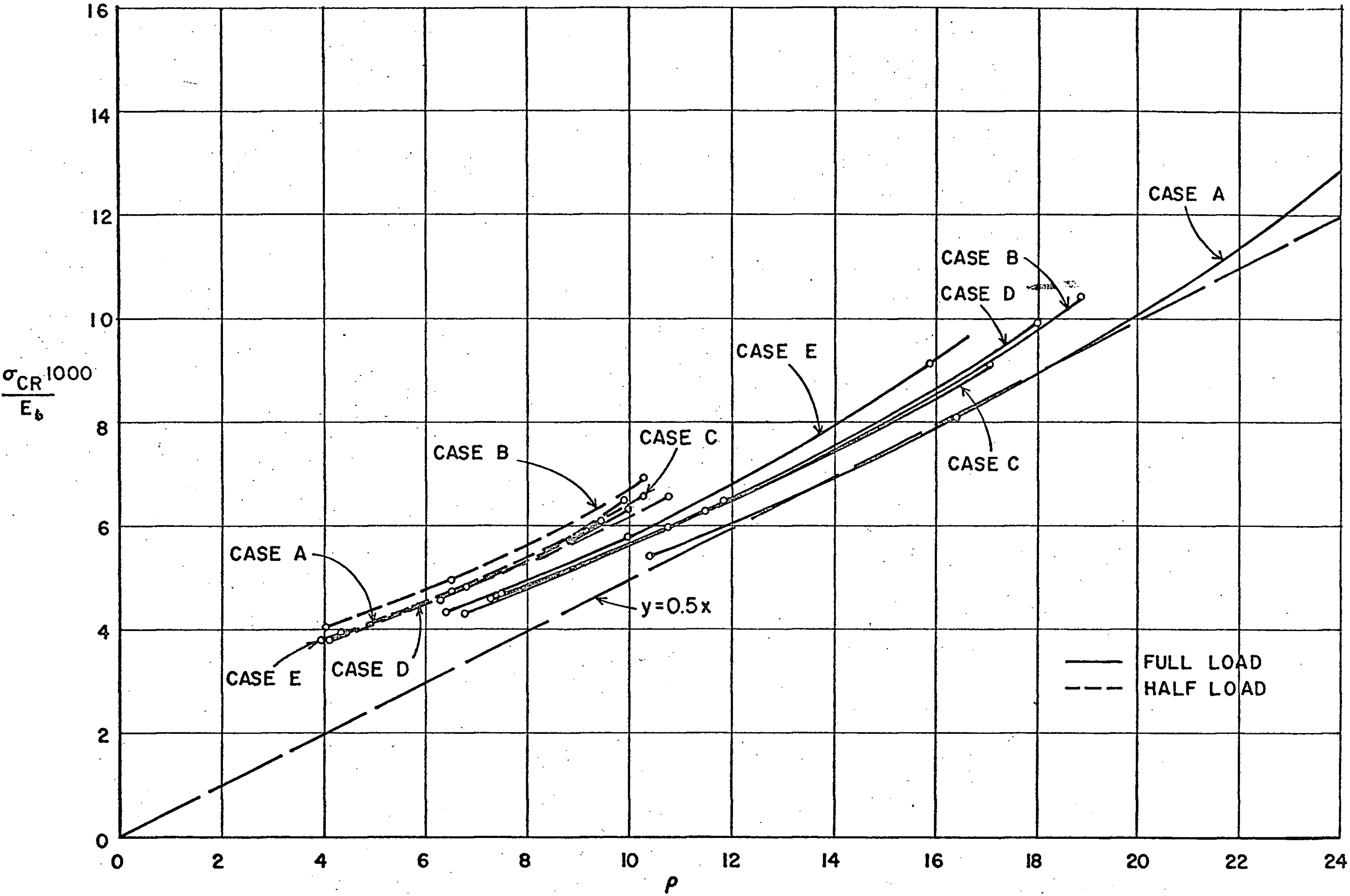


FIGURE 16. DIAPHRAGM CASES d/b = 6

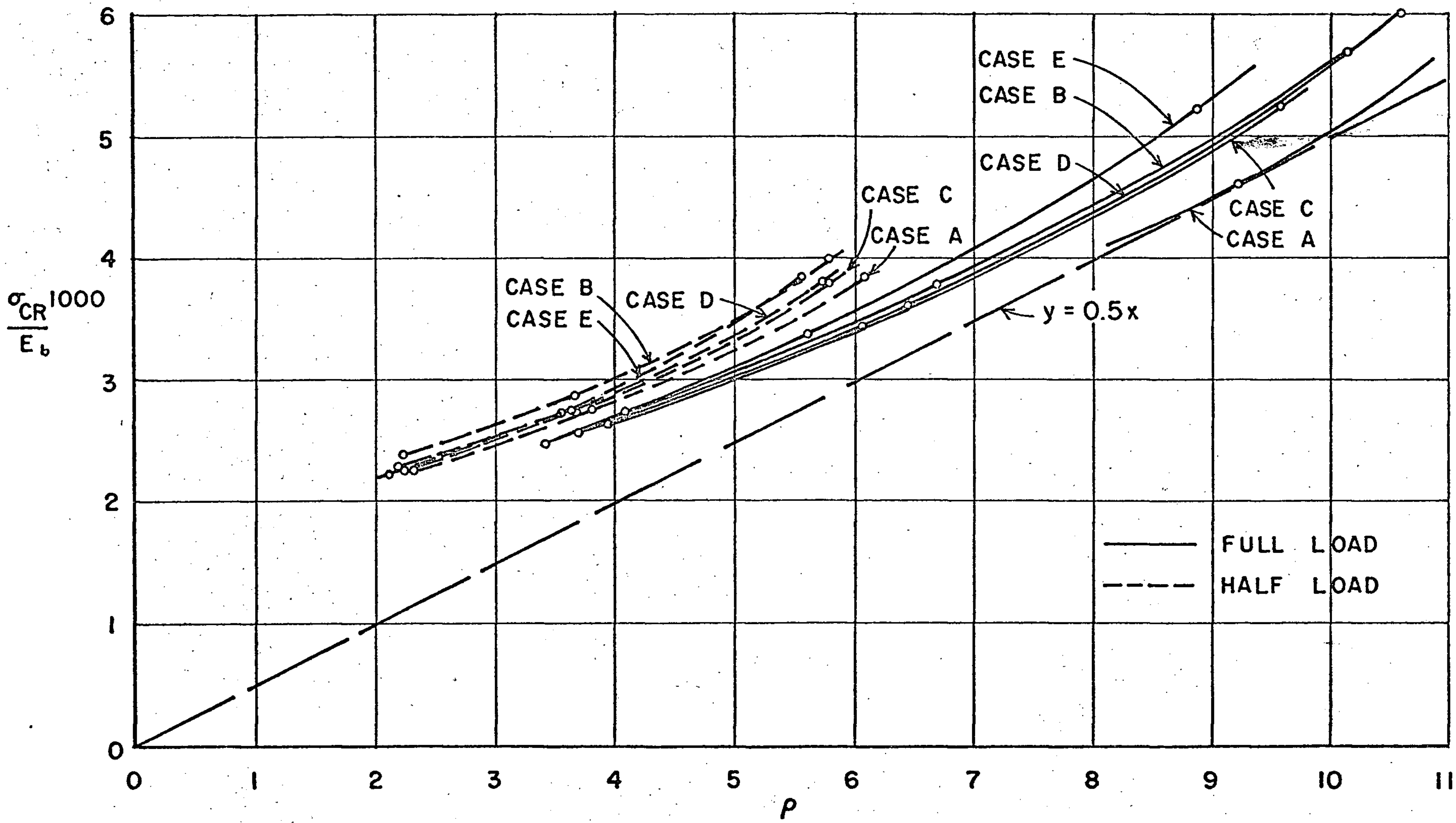


FIGURE 17. DIAPHRAGM CASES d/b = 8

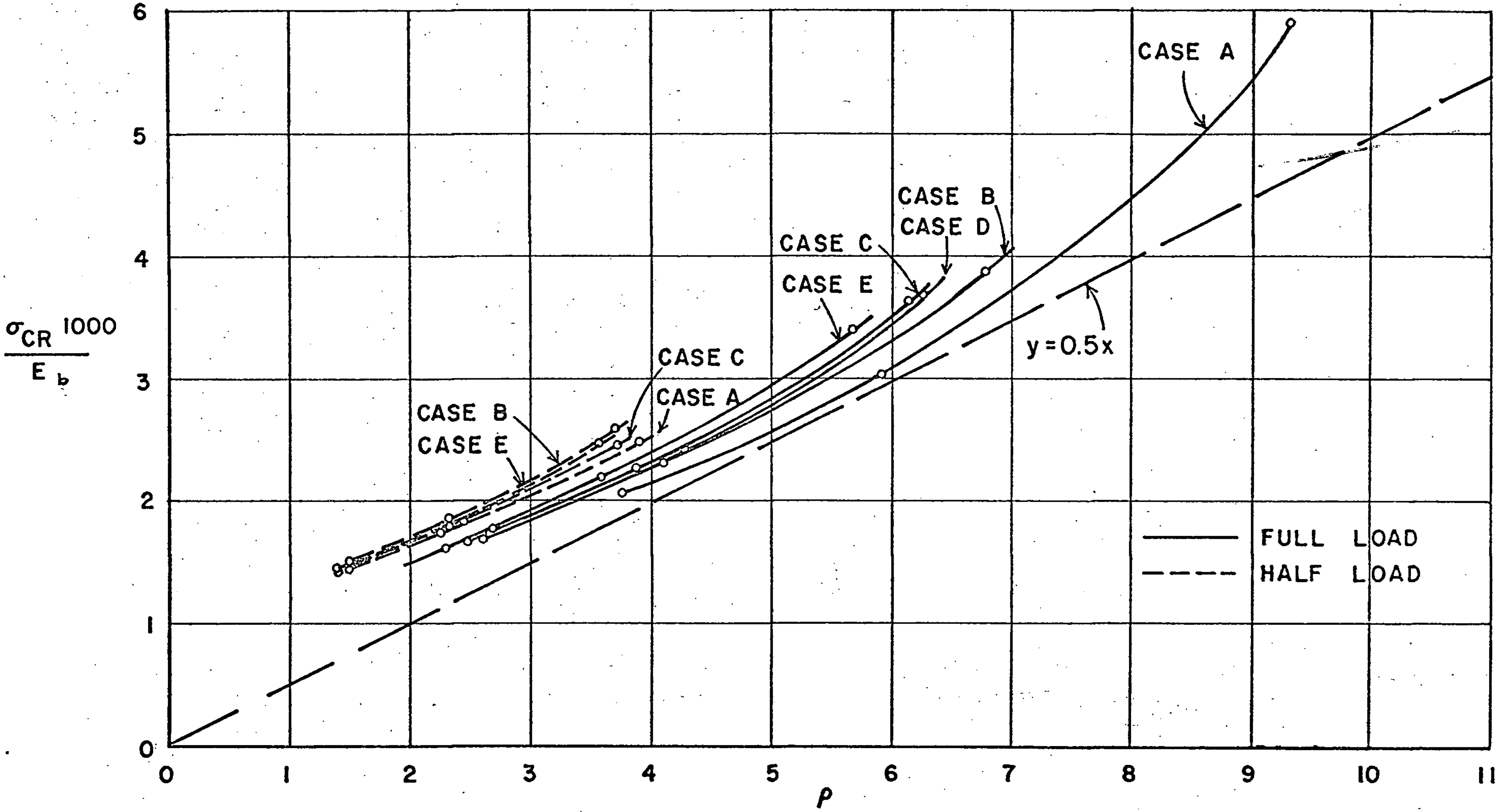


FIGURE 18. DIAPHRAGM CASES d/b = 10

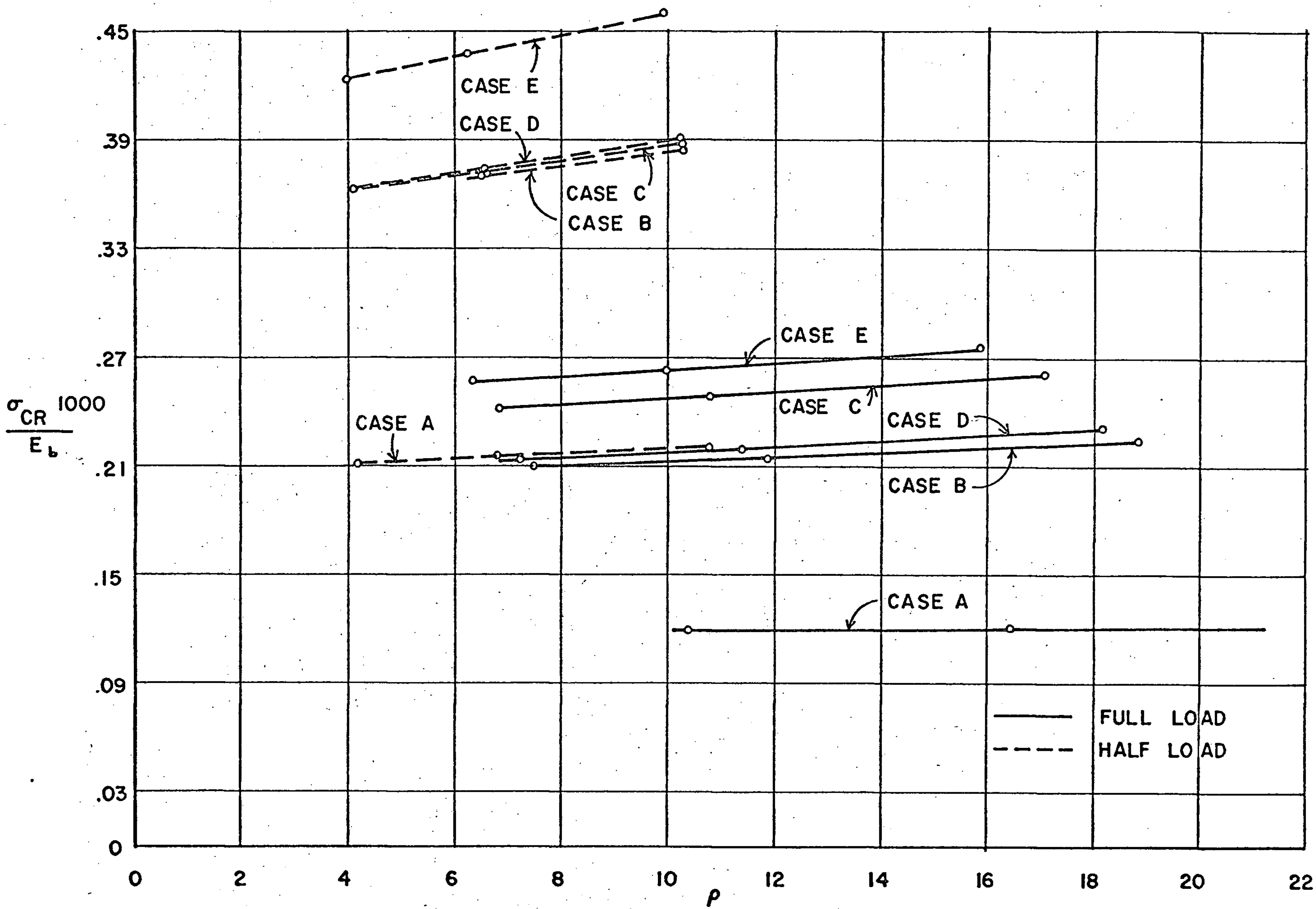


FIGURE 19. PIN-PIN CASES d/b = 6

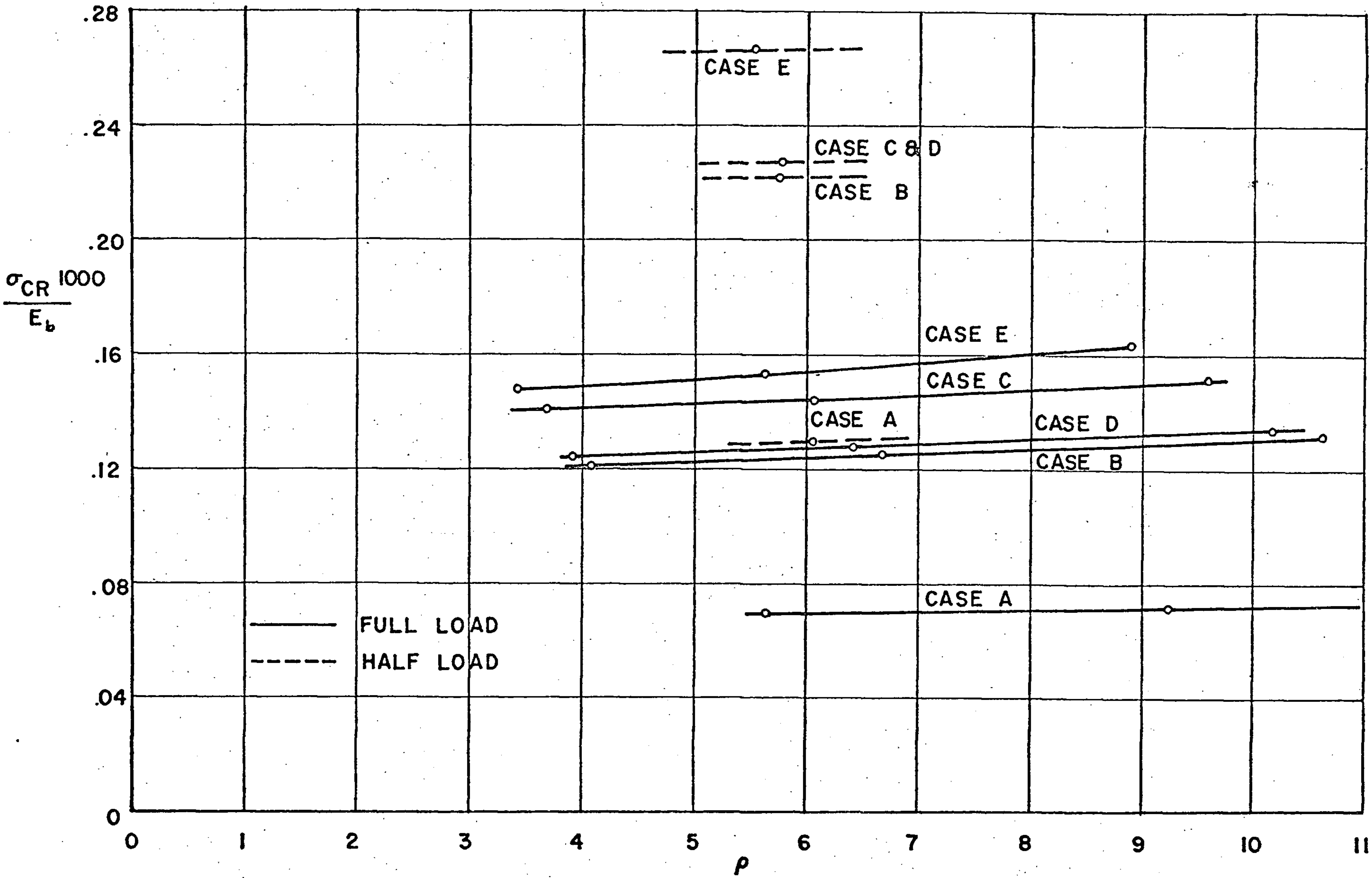


FIGURE 20. PIN—PIN CASES d/b = 8

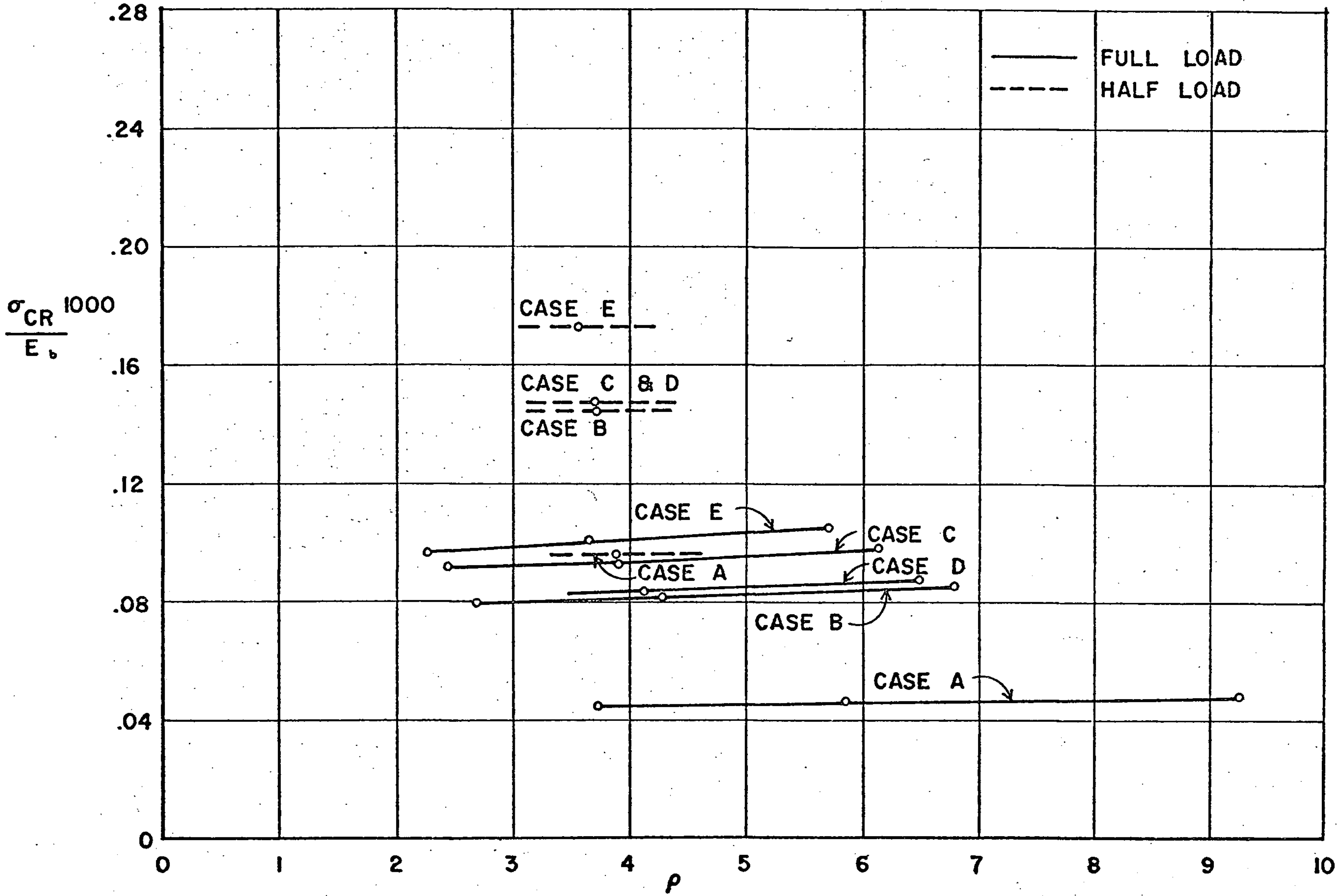


FIGURE 21. PIN – PIN CASES d/b = 10

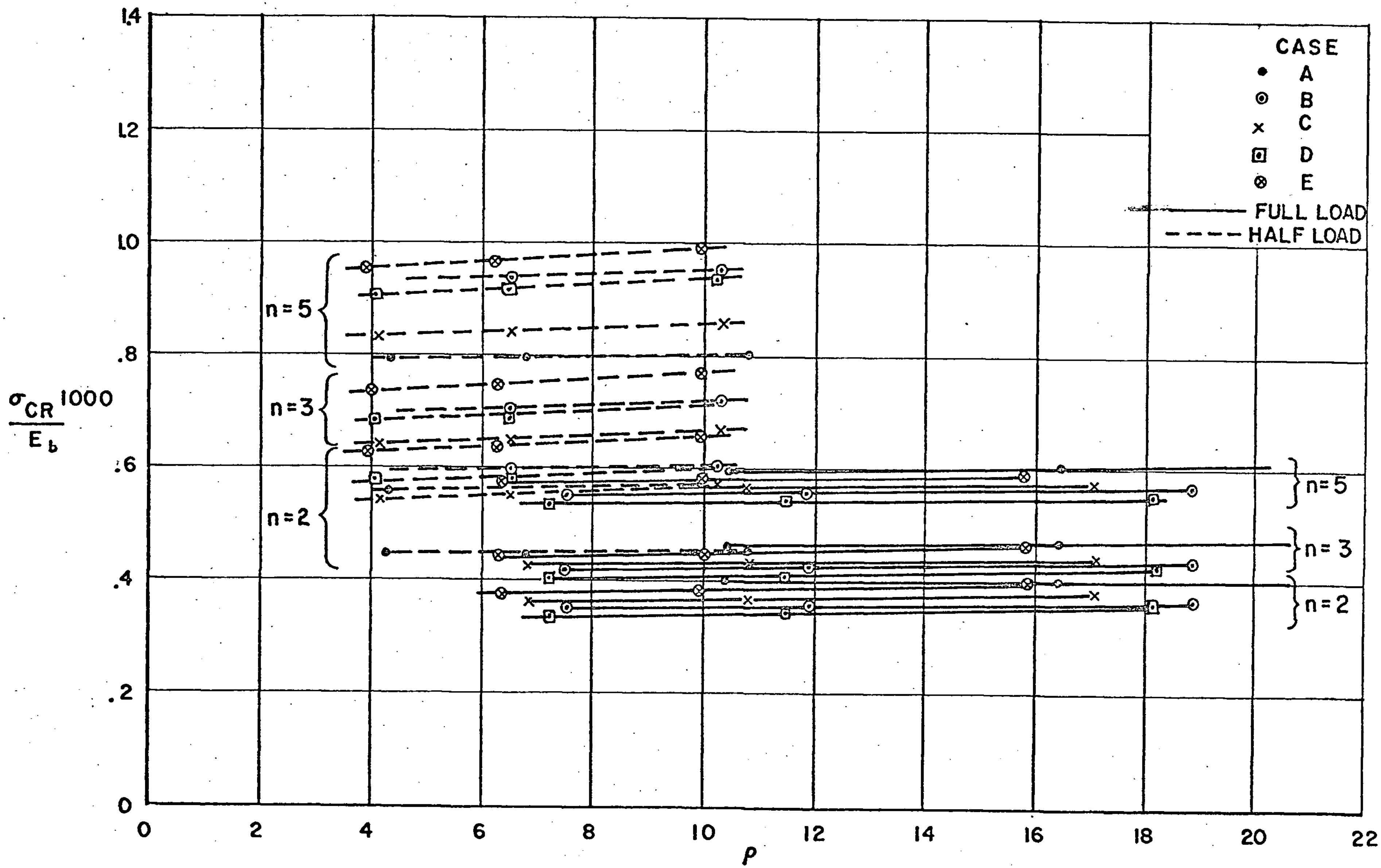


FIGURE 22. PIN — FIX CASES d/b = 6

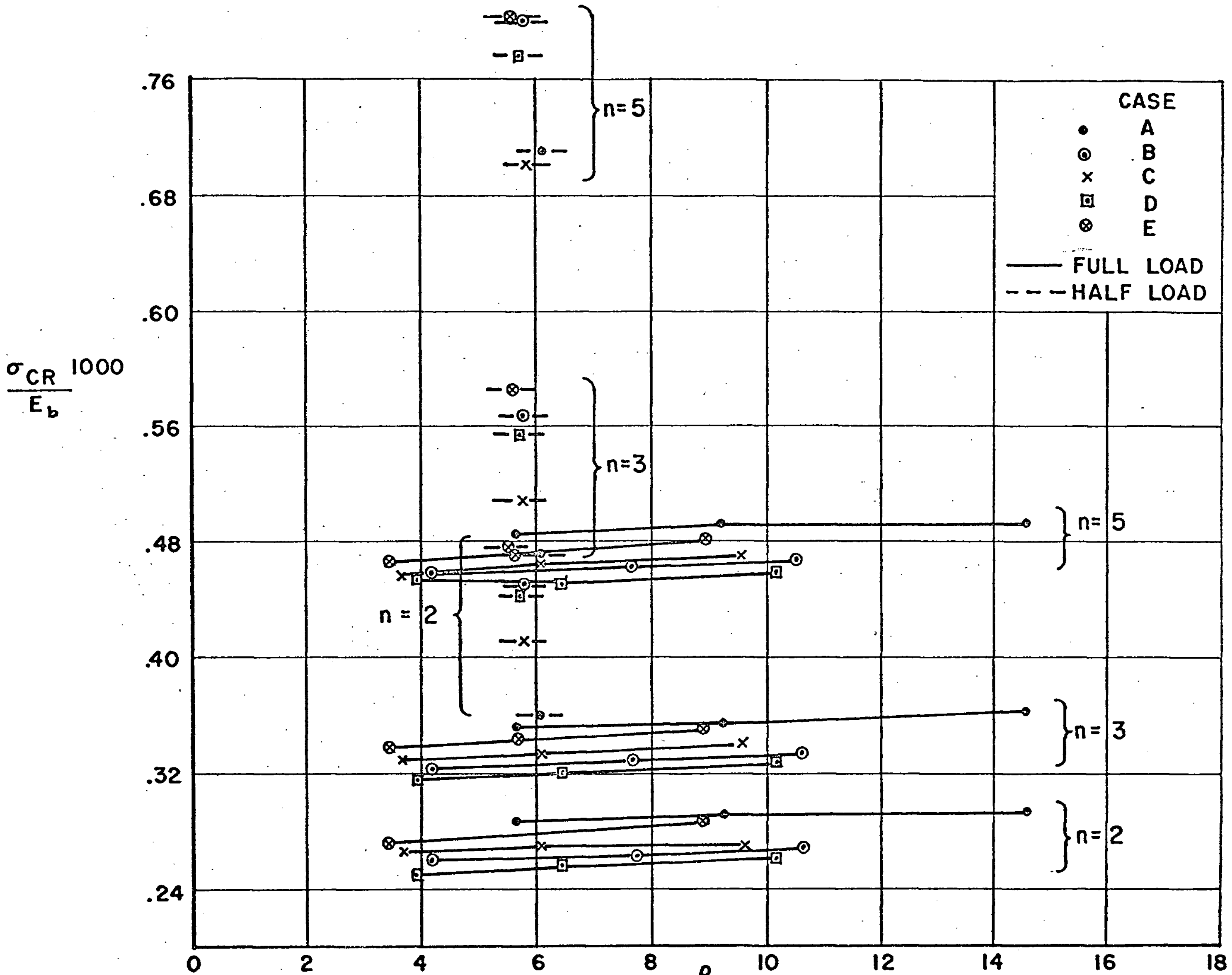


FIGURE 23. PIN — FIX CASES d/b = 8

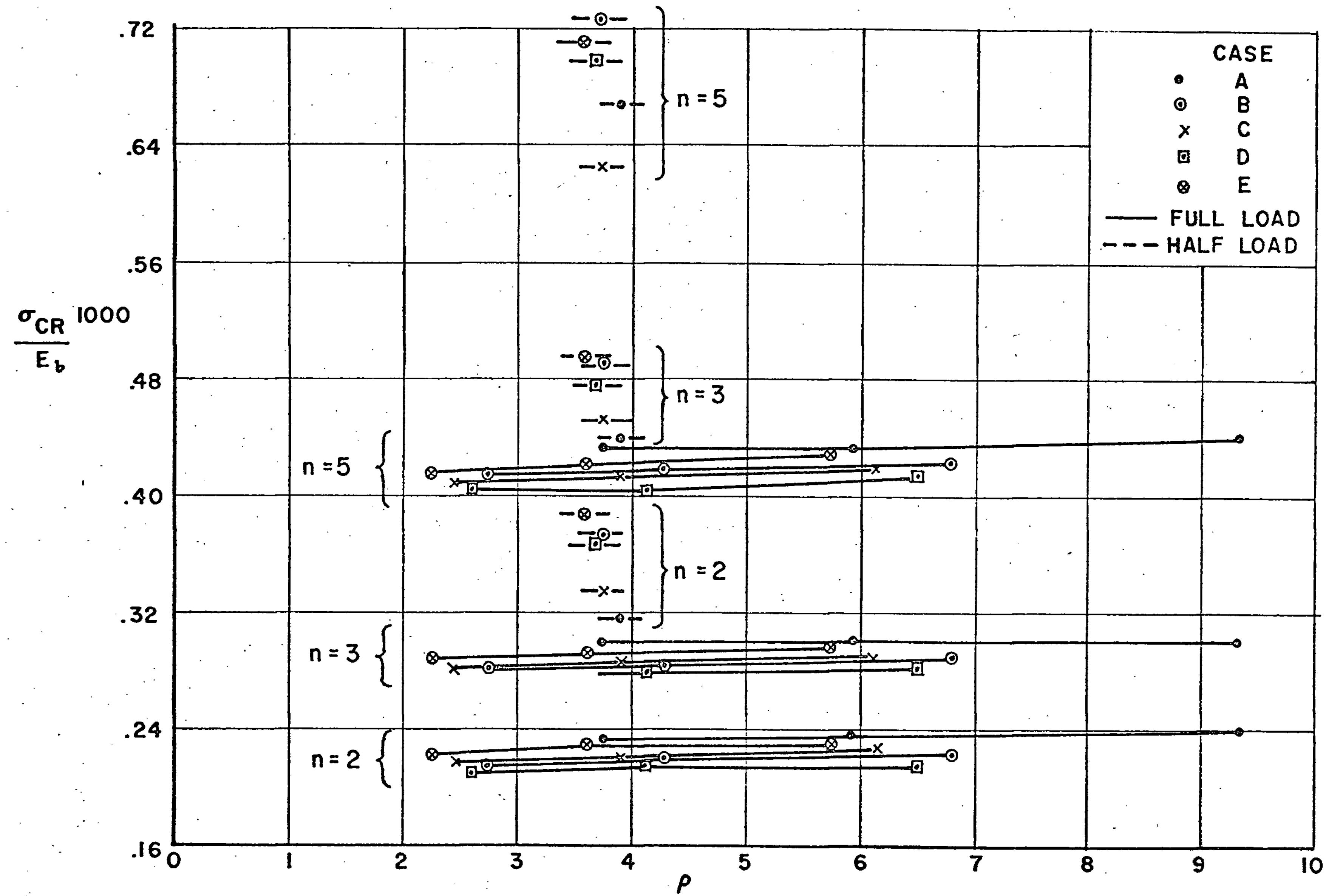


FIGURE 24. PIN — FIX CASES d/b = 10

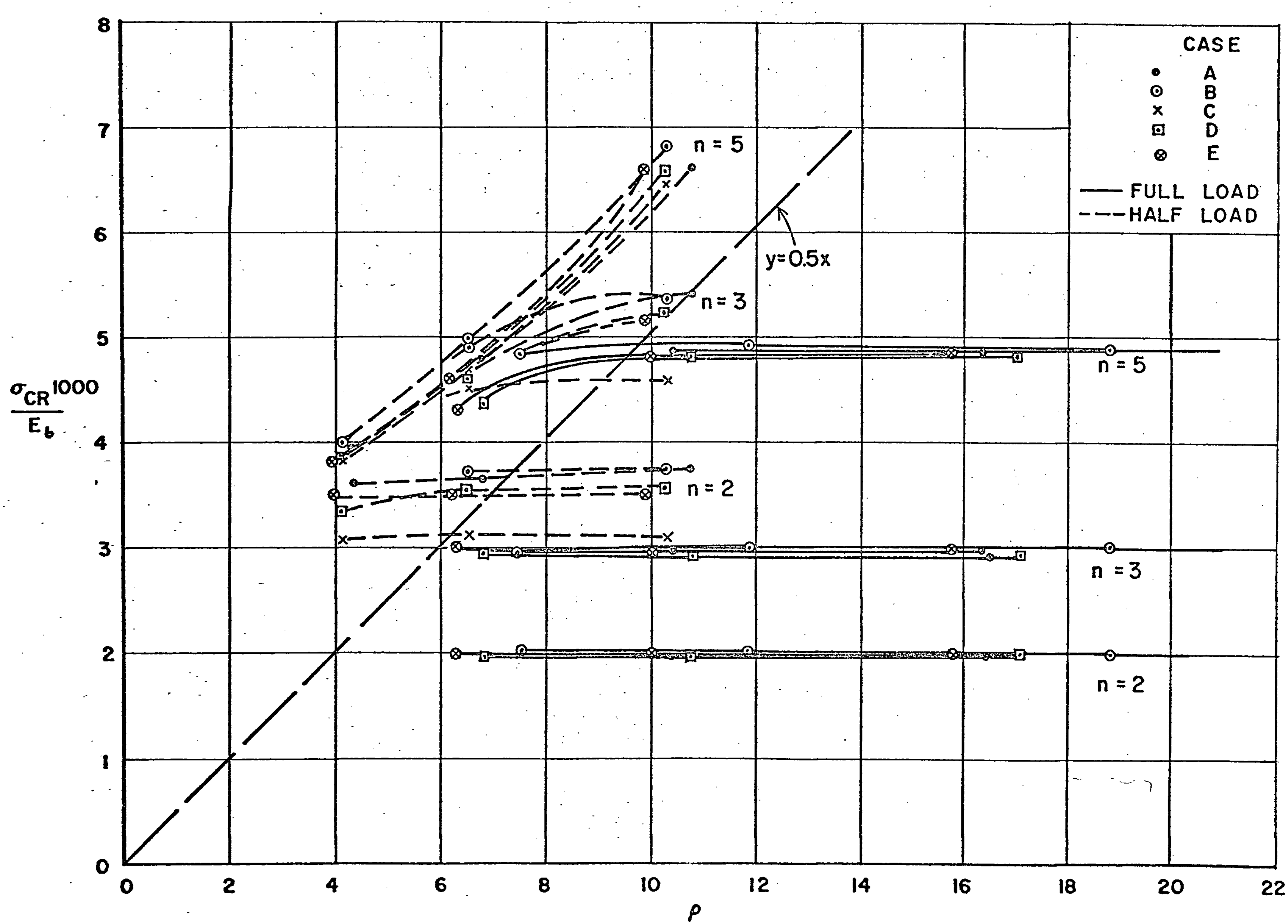


FIGURE 25. FIX - PIN CASES d/b = 6

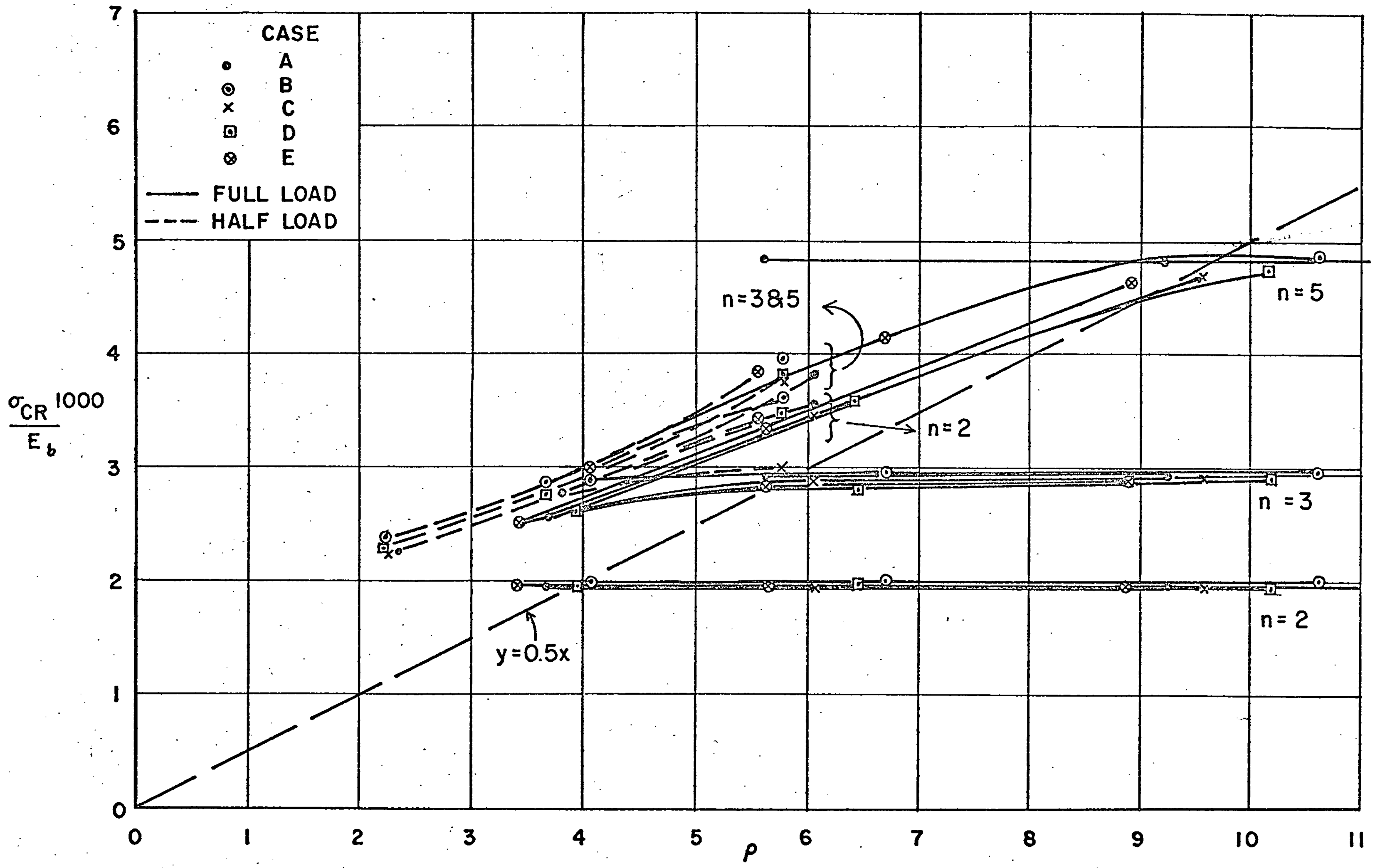


FIGURE 26. FIX — PIN CASES d/b = 8

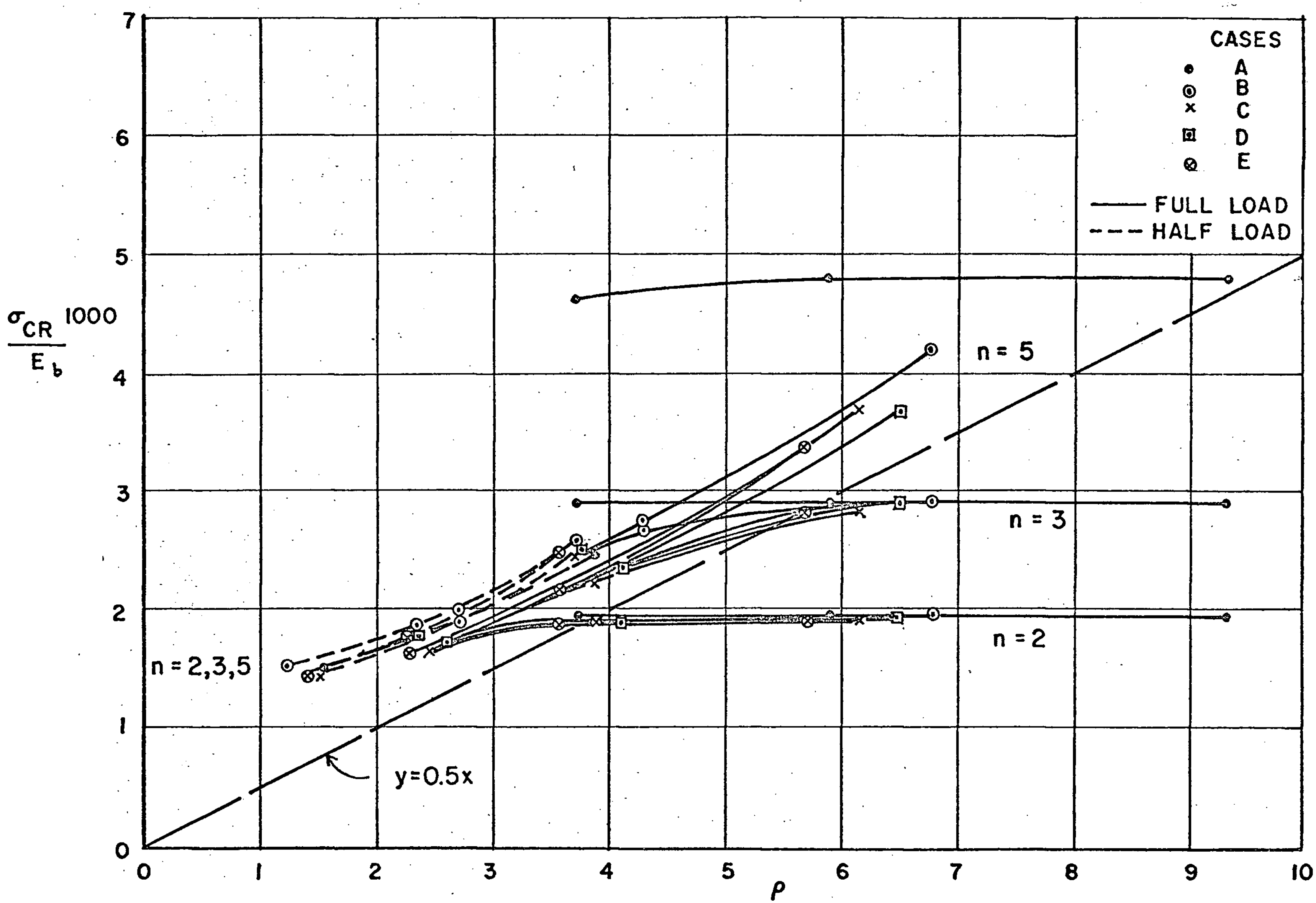


FIGURE 27. FIX - PIN CASES d/b = 10

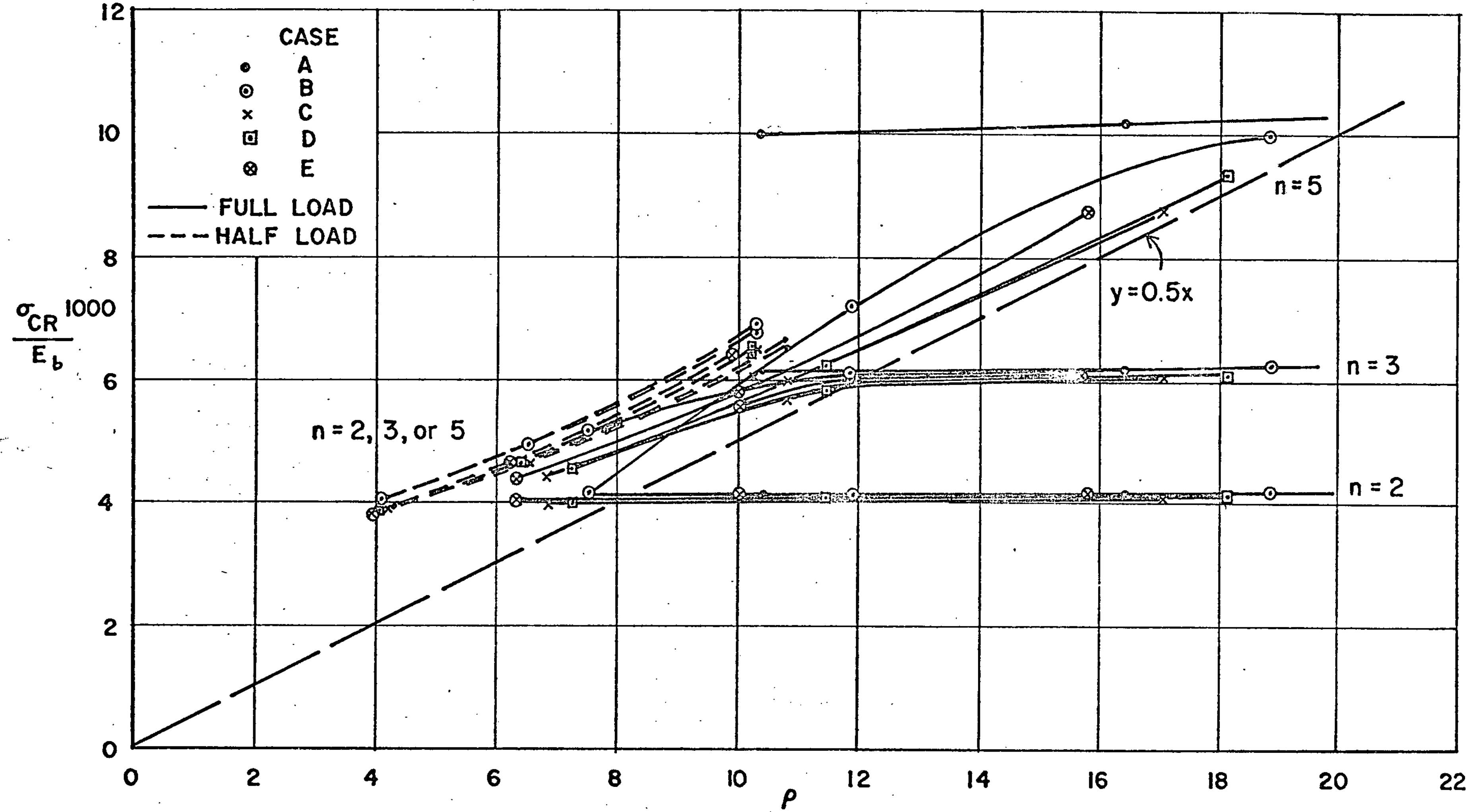


FIGURE 28. FIX — FIX CASES d/b = 6

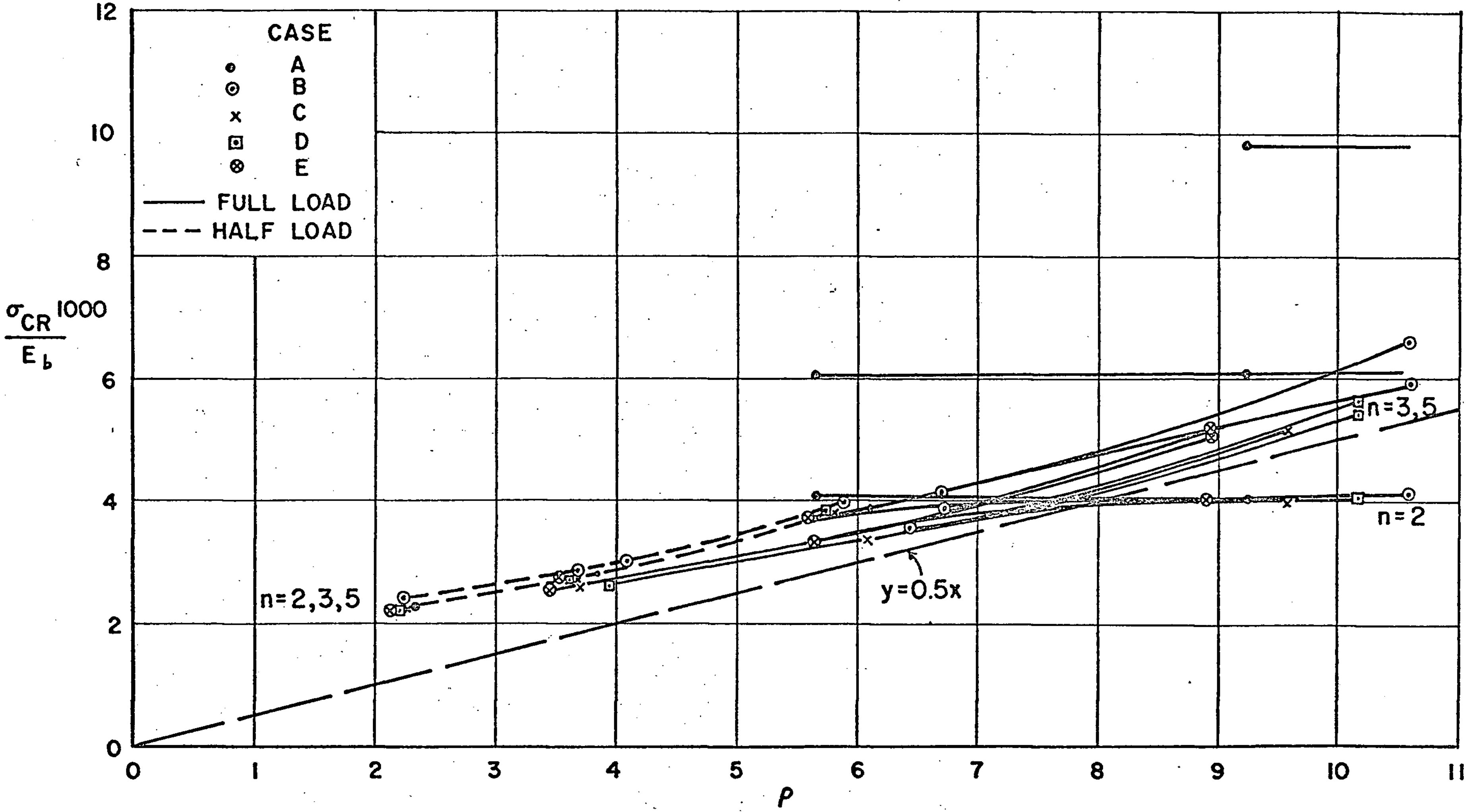


FIGURE 29. FIX — FIX CASES d/b = 8

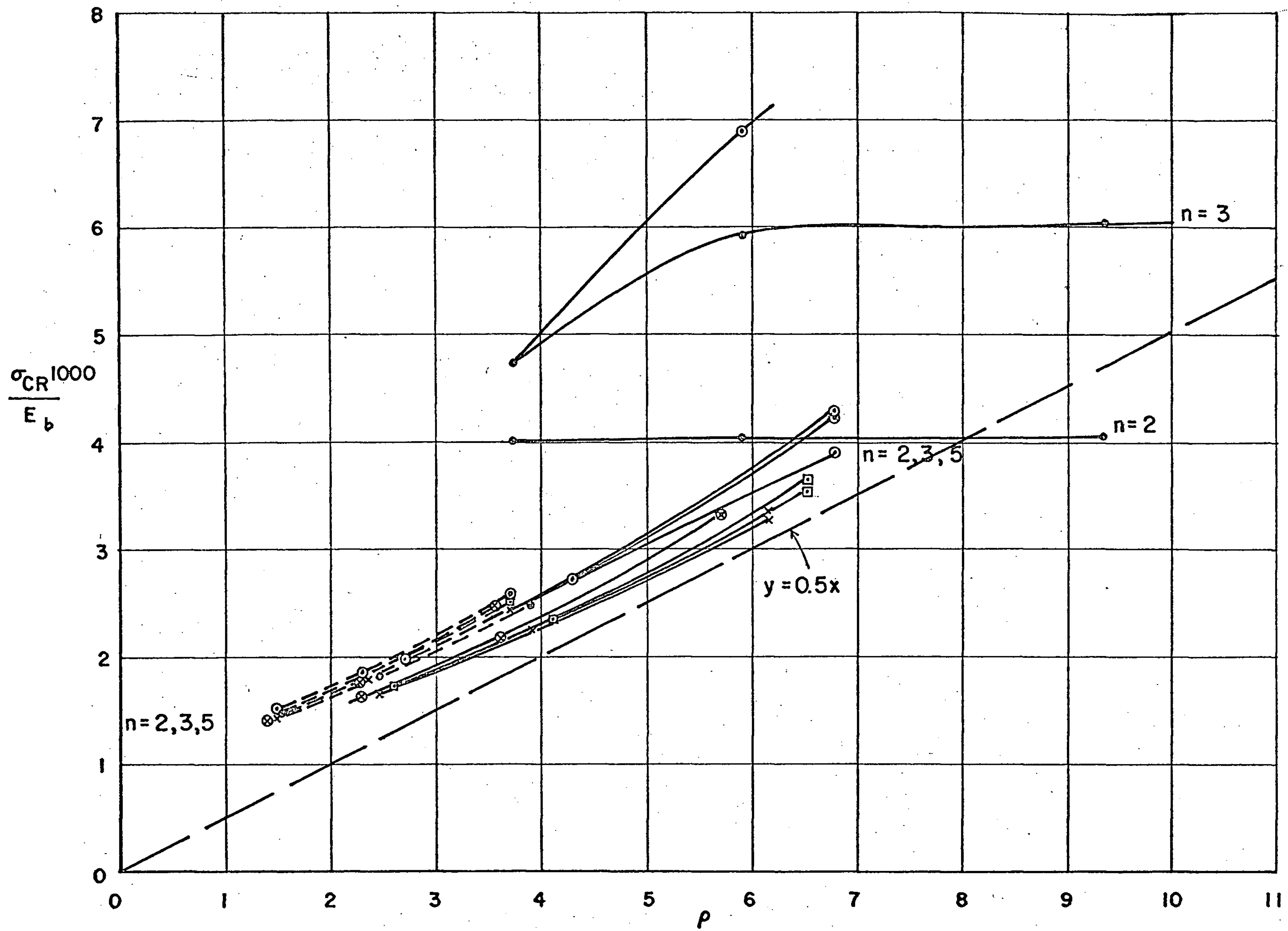


FIGURE 30. FIX — FIX CASES d/b = 10

5.3 DIAPHRAGM RESULTS

A study of Figs.(16), (17) and (18) shows that it is conservative to state that:

$$\sigma_{CR}/E = .5(b/d)^2 \sqrt{d/L_c} \tag{42}$$

for both full and half load cases.

A buckling factor of safety of 3 on the usual bending stress of 2. ksi gives:

$$3(2.)/1600. = .5(b/d)^2 \sqrt{d/L_c} \tag{43}$$

or

$$(d/b)^2 \sqrt{L_c/d} = 133 \tag{44}$$

Values of this ratio greater than 133 will require a reduction in the usual allowable bending stress, so that for economic design:

$$(d/b)^2 \sqrt{L_c/d} \lessapprox 133 \tag{45}$$

This is a general conclusion from the conservative approximations of the graphs and the allowable bending stress, but more accuracy can be obtained for individual cases from graphs or tables.

It should be specifically noted that the moment used for σ_{CR} in the diaphragm cases is the moment at the column, which may or may not be the maximum moment in the structure.

5.4 PIN-PIN AND PIN-FIX COLUMN RESULTS

Perusal of the graphs for these cases, Figs.(19) to (24), shows that the maximum ordinate is given by approximately:

$$1000\ \sigma_{CR}/E_b = 1.0 \tag{46}$$

with many cases less than half this value.

For E_b = 1600 ksi, this yields σ_{CR} equal to 1600 psi. With the usual factor of safety of 3 for buckling, this yields an allowable stress of 530 psi. This allowable stress is so low as to show that this system of support is uneconomical

omical. For this reason, no approximate formulae are presented for design. Instead, it is recommended that this support system not be used.

5.5 FIX-PIN COLUMN RESULTS

In this case, with the column fixed at the beam and pinned at the base, an entirely different type of behavior is observed from the graphs, Figs.(25) to (27). Herein, the column is strong enough to start restraining the beam from buckling. The left hand part of this plot can be approximated by a straight line through the origin as was done for the diaphragm case. The right hand part can be approximated by a horizontal line. The left hand linear part shows that the column is stiff enough to restrain the beam as would a diaphragm. This is then a beam buckling region with the column almost motionless. However, when the plot is horizontal, the beam is stiffer so this is essentially a column buckling portion with the beam slightly changing the critical load on the column.

Now that the phenomenon is understood, the ordinate at the horizontal part of these curves can be derived almost exactly. To do this, it is assumed that the critical load of the column is:

$$R_{CRIT} = \pi^2 EI/(L+d)^2 \tag{47}$$

This column load can be replaced by Mm to give, for the full load case,

$$\frac{Mm_{CRIT}\ \pi^2\ EI}{Zf_b nL^2} = \frac{\pi^2\ EI}{(L+d)^2} \tag{48}$$

from which:

$$\frac{1000\ \sigma_{CR}}{E_b} = \frac{n}{(1+d/L)^2}\ \frac{f_b}{E_b}\ 1000 \tag{49}$$

$$= \frac{1.32\ n}{(1+d/L)^2} \tag{50}$$

For $L/d = 5.1$ on these graphs

$$1000\ \sigma_{CR}/E_b = 0.92n \tag{51}$$

The graphs show this factor to average about 0.96n on full loading cases, as would be expected as the approximation for R_{CRIT} in Eqn.(47) is conservative.

For the half load case, the left side of Eqn.(48) must be multiplied by the ratio of half load reaction to full load reaction. For Case B this ratio is 0.539. The expression for critical stress on the horizontal line then becomes, for Case B:

$$\frac{1000\ \sigma_{CR}}{E_b} = 0.92n/.539 \tag{52}$$

$$= 1.71n \tag{53}$$

The average value for the half load graph is approximately 1.7n for $d/b = 6$.

It is noted from the graphs that as the beam gets stiffer the critical stress increases slowly, that is, the plot is not quite horizontal. This increase is due to the beam restraining the column from buckling. In the limit, with an infinitely stiff beam (zero length, for example), the top of the column is fixed not only to the beam, but fixed for rotation. At this point, the critical column load will be about twice the Euler load, so that the factor, .92n will increase to a maximum of 1.84n.

In summary then, it is recommended that the critical bending stress be taken from the smaller of the following two expressions:

$$\left.\begin{aligned} \sigma_{CR}/E_b &= .92n/1000 \qquad \text{(full load)} \qquad \text{(a)} \\ &= 1.7n/1000 \qquad \text{(half load)} \qquad \text{(b)} \end{aligned}\right\} \tag{54}$$

or

$$\sigma_{CR}/E_b = .5(b/d)^2\ \sqrt{d/L_c} \tag{55}$$

It is emphasized here that this is the maximum buckling stress in the beam wherever it occurs. More accuracy is obtainable from graphs for individual cases.

5.6 FIX-FIX COLUMN RESULTS

The curves here, Figs.(28) to (30) are similar to those for fix-pin columns except that the horizontal portions have ordinates of 2.0n instead of .96n for the full load case. This is to be expected as a fixed base will double the critical load of the column.

It is recommended then, that the critical bending stress be taken from the smaller of the following two expressions:

$$\left.\begin{aligned} \sigma_{CR}/E_b &= 1.84n/1000 \quad \text{(full load)} \quad \text{(a)} \\ &= 3.4\ n/1000 \quad \text{(half load)} \quad \text{(b)} \end{aligned}\right\} \tag{56}$$

or

$$\sigma_{CR}/E_b = .5(b/d)^2 \sqrt{d/L_c} \tag{57}$$

where the stress is the maximum bending stress in the beam. Such expressions are conservative but greater accuracy can be obtained for individual cases from the graphs.

DESIGN CODE RECOMMENDATIONS

The following design specification is given as a summary of the preceding work and as an aid in extension to other load and boundary conditions. It is conservative for the system considered in previous chapters and would be reasonably close for the cases not too far removed from those considered.

1.0 In a continuous beam and column system of glulam, wherein the top edge is continuously supported against lateral motion by suitable bracing or roof system, and the lower edge of these beams is not so supported but is in compression due to negative moment, the allowable bending stress shall be calculated in accordance to clause 1.1 below. In such a system the ends of the beams must be prevented from rotation torsionally.

1.1.1 If a diaphragm is provided between beams at the column and is so designed as to prevent torsional rotation of the beams, then the critical bending stress is given by:

$$\sigma_{CR} = .5E\,(b/d)^2 \sqrt{d/L_c}$$

where L_c is the length of the bottom edge in compression and σ_{CR} is from the moment at the column.

1.1.2 If a diaphragm is not provided as in 1.1.1 then:

(a) the column to beam connection must be square and rigid so that as the beam rotates torsionally, the column top undergoes the same rotation.

(b) the column to beam connection must be designed to resist the axial column load, together with a moment due to accidental eccentricity of magnitude RL/50, where R is the column force and L its length.

In such cases the critical bending stress is given by:

$$\begin{aligned} \sigma_{CR} &= 1.8\ n\ E_b/1000 && \text{fix base, full load} \\ &= 3.4\ n\ E_b/1000 && \text{fix base, half load} \\ &= 0.9\ n\ E_b/1000 && \text{pin base, full load} \\ &= 1.7\ n\ E_b/1000 && \text{pin base, half load} \end{aligned}$$

but in no case shall σ_{CR} exceed that stress given by:

$$\sigma_{CR} = .5\ E_b(b/d)^2 \sqrt{d/L_c}$$

where:

σ_{CR} is the maximum critical bending stress in the beam

E_b is the beam flexural modulus

and n is defined as follows:

$$n = (Mm/Zf_b)\ (\pi^2\ EI/L^2R)$$

where;

Mm is the absolute maximum moment in the beam

Z is the section modulus

f_b is 2.1 ksi

EI is the flexural rigidity of column.

1.1.3 **The allowable bending stress shall be calculated from the standard column formulae for glulam using F_b instead of F_a and using a slenderness ratio given by the following:**

slenderness ratio = $\pi \sqrt{E/12\ \sigma_{CR}}$

where σ_{CR} is given in clauses 1.1.1 and 1.1.2 [5].

Herein F_a is the basic allowable compressive stress parallel to grain if no axial buckling is possible and F_b is the basic allowable bending stress if no lateral buckling is possible.

1.1.4 Columns without diaphragms at the top, but pinned at the top and fixed at the base or pinned at both ends offer so little restraint to the beam that the critical stress is excessively low. If such systems are used, special test or analysis shall be made.

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2 Lee, G.C., A Survey of Literature on the Lateral Instability of Beams, Bulletin No.63, Welding Research, Council of the Engineering Foundation, New York, Aug.1960.

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4 Gere, J.M. and Weaver, W., Analysis of Framed Structures, D. Van Nostrand Co., Princeton, N.J., 1965, pp 428-431.

5 Hooley, R.F. and Madsen, B., "Lateral Stability of Glued Laminated Beams," Journal of the Structural Division, ASCE, ST3, June 1964.

APPENDIX I - PROGRAM FLOW CHART

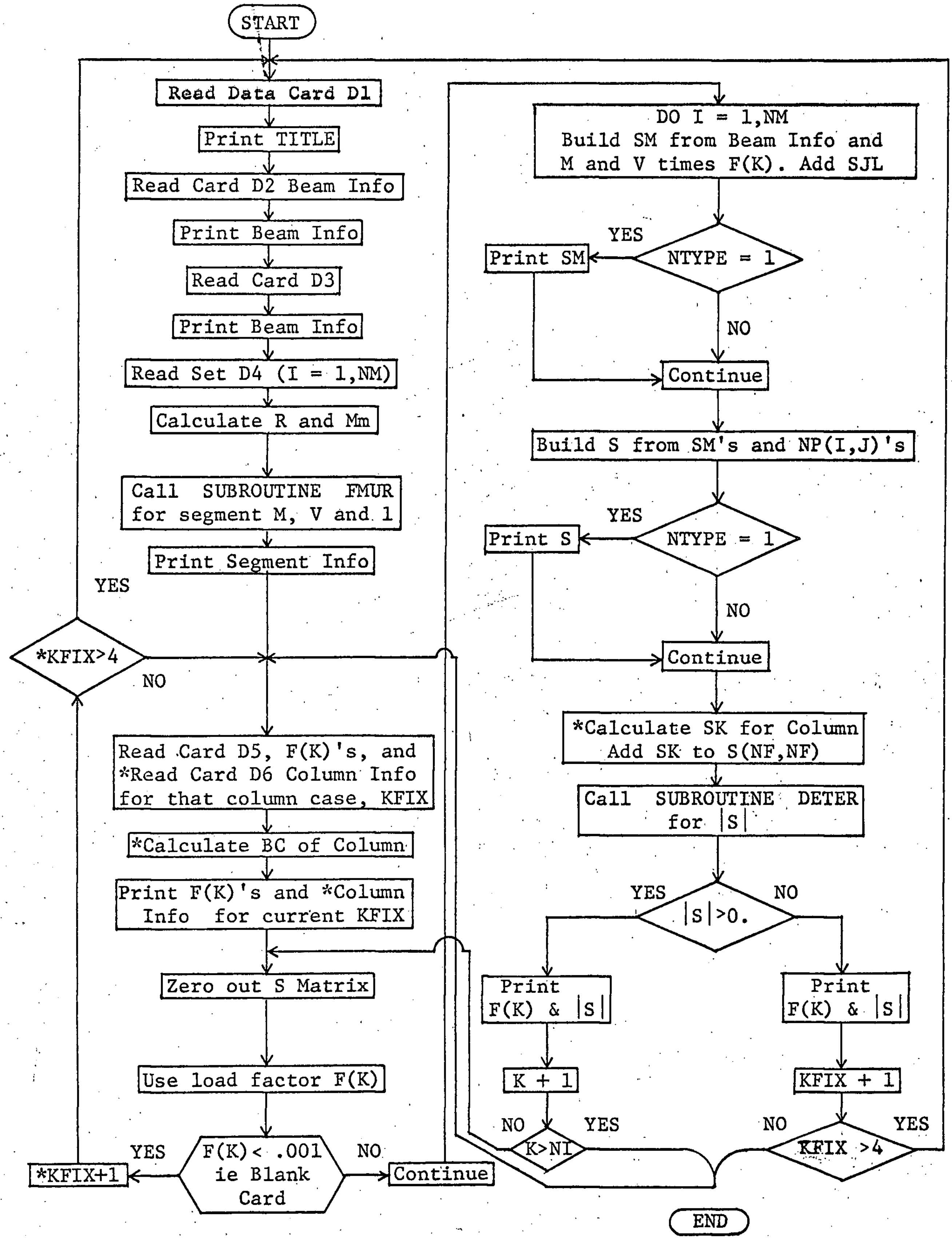
On the following page is the flow chart of the computer program used in the production runs for the column cases, full load. It is set up for the calculation of the determinant of S at NI load levels for each of the four column fixity cases. A negative $|S|$ ends that column case and causes the next column case to be considered. To bypass a column case it is only necessary to put a blank data card in for the F(K) values. The routine will be repeated for each set of structure cards input.

For runs to check the experimental findings, it is necessary to exclude the section of the source deck which calculates the moment of inertia of the column based on the strength of the beam. The actual BC of the column used in the experiment is input along with other column data.

As the column does not affect the diaphragm case, all sections dealing uniquely with the columns (those marked with *) can be deleted, and a single diaphragm case run for each structure.

On an IBM 7044, the time required for one structure considering four column cases, 18 beam segments and four load levels, was approximately 1 minute.

FLOW CHART - FULL LOAD



APPENDIX II - DATA INPUT - OUTPUT

Data information requiring units is input for the particular data cards as follows:

D1 - no units

D2 - ksi for E and G

D3 - feet for lengths and kips for P

D4 - in^4 for B and T, inches for EC

D5 - no units

D6 - inches for CL, ksi for EL

Uniform load of 1 kip/ft is assumed as basic load.

Table VII outlines the sequence and format details of data cards required in one complete structure analysis.

TABLE VII - DATA INPUT SEQUENCE

CARD	DATA INPUT	FORMAT	DETAILS
D1	TITLE	13A6	Alphanumerics only
D2	NRS, NM, NU, NI, E, G, NTYPE	4I5, 2F10., I10	One card
D3[1]	NRS, L, A, P, XL, XA, NL, NA	I10, 5F10., 2I5	One card
D4 set	NP(I,J), B(I), T(I), EC(I)	4I10, 3F10.	NM cards I = 1 to NM J = 1 to 4
D5[2]	F(K)	7F10.	K = 1 to NI
* D6[2]	CL, EL, KT, KB, NF	2F10., 3I10	One card

NOTES:

1 NL = Number of segments in L_b

NA = Number of segments in A

NL + NA = NM

XL = L/2NL

XA = A/2NA

2 Cards D5 and D6 are input four times, once for each column case.

* For diaphragm case, D6 is omitted and D5 input once only. NP(I,J) will be zero at NF.

Output prints out the following:

- all data input
- calculated M, V and 1 for each segment plus R and maximum moment
- values of S_i, f_1 and f_2 at column top, and SK
- values of F(K) and the corresponding determinants of S
- if NTYPE = 1, matrices SM and S.

APPENDIX III - COMPUTER PROGRAM LISTING

The complete program listing for production runs for column cases, full load is outlined on the following pages.

The listing is set up for n = 3. For any other value of n, place the following card at location marked "a" in listing.

BC = 7./3.*BC if n = 7.

For half load runs remove the seven cards marked "h" and replace with the following seven cards:

```
R = P*(L+A)/L + A*(L+.5*A)/L
RFL = .5*(L+A)**2/L + P/L*(L+A)
CENTM = 0.
COLR  = RFL*FLOAD
R1 = - P*A/L - A*A /(2.*L)
VX = - R1
M  = - R1*(Y*L/S - XL)
```

In addition, the following two cards go at location h_1 and h_2, as marked:

```
      IF (I.LE. NL) GO TO 500        (at h1)
500   CONTINUE                       (at h2)
```

```
$JOB    17087     E.A. EVEREST
$FORTRAN
C   ***********************************************************************@*********
C    LATERAL STABILITY OF BEAM ON COLUMN WITH TOP EDGE PINNED,COLUMN CASES
C   *********************************M********************************************
          DIMENSION TITLE(13)
100       READ(5,1001)TITLE
1001      FORMAT(13A6)
          WRITE(6,1002)TITLE
1002      FORMAT(1X,13A6)
C              --------------------------------------------------------
          DIMENSION B(30),T(30),X(30),EC(30),NP(30,4),BM(30),V(30),F(30)
          DIMENSION S(70,70),SM(4,4)
          COMMON/MATEXP/IEXP
          DET = 0.
          IEXP = 0
C    INPUT STRUCTURE INFO - STRUCTURE NO.,NUMBER OF MEMBERS,DEGREES OF
C    FREEDOM, NO. OF LOAD INCREMENTS, MODULUS OF ELASTICITY, SHEAR MODULUS
C    NTYPE = 1,PRINTS MATRICES
          READ 1,NRS,NM,NU,NI,E,G,NTYPE
1         FORMAT (4I5,2F10.0,I10)
          PRINT  99,NRS,NM,NU,NI,E,G,NTYPE
99        FORMAT(//1X,4HNRS=,I5,4X,3HNM=,I4,4X,3HNU=,I4,4X,3HNI=,I4,4X,2HE=,
         1F12.2,4X,2HG=,F10.2,4X,I4)
C    SUBROUTINE TO DETERMINE MOMENTS, SHEARS AND LENGTHS OF SEGMENTS
          REAL L,M
          VX = 1.
          SEG = 1.
          M = 1.
C    INPUT BEAM LENTHS--L IS BEAM LENGTH, A IS CANTILEVER OVERHANG,P IS CONC
C   LOAD ON END OF CANTILEVER,NL AND NA ARE NUMBER OF SEGMENTS INTO WHICH
C    L AND A  ARE RESP DIVIDED AND AND XL AND XA ARE DISTANCES TO CENTRE
C    OF FIRST MEMBERS IN L AND A RESP
          READ 1000,NRX,L,A,P,XL,XA,NL,NA
1000       FORMAT(I10,5F10.0,2I5)
          PRINT 2000,NRX,L,A,P,XL,XA,NL,NA
2000       FORMAT(1X,I9,5F10.3,2I5)
          PRINT 3
3         FORMAT(50X,2HBM,13X,1HV,14X,1HI,14X,1HJ,14X,2HEC,14X,1HX)
C    INPUT STRUCTURE MEMBER INFO--FOUR POSITION NUMBERS,MOM OF INERTIA,
C    TORSION CONSTANT, AND ECCENTRICITY OF TOP OF MEMBER
          DO 2 I=1,NM
```

```
          READ 102,NP(I,1),NP(I,2),NP(I,3)TNP(I,4),B(I),T(I),EC(I)
    102   FORMAT (4I10,3F10.0)
          CALL FMVR(NRX,L,A,P,XL,XA,NL,NA,VX,SEG,M,I)
          BM(I) = M
          V(I) = VX
           X(I) = SEG
          X(I) = 12.*X(I)
          PRINT 103,NP(I,1),NP(I,2),NP(I,3),NP(I,4),BM(I),V(I),B(I),T(I),EC(
         1I),X(I)
    103    FORMAT(1X,4I10,6F15.5)
          DB = 2.*EC(I)
          BSI = B(I)
    2     CONTINUE
          DO 14 I = 1,NM
    14    BM(I) = 12.*BM(I)
h ------  R = .5*(L+A)**2/L + P/L*(L+A)
h ------  R1 = (.5*(L*L-A*A)-P*A)/L
          COLM = A*A/2. + P*A
h ------  CENTM = (L*L-A*L+A*A)**2/(8.*L*L)
          PRINT 1003,R,COLM,CENTM
    1003  FORMAT(5H R = ,F10.3,18H POS MOM AT COL = ,F12.3,14H MAX NEG MOM =
         1 ,F12.3)
    C   PROGRAM TO BE DONE FOUR TIMES ,ONCE FOR EACH TYPE OF COLUMN FIXIDITY
          DO 47 KFIX =1,4
    C   INPUT LOAD FACTORS,TOTAL OF NI
           READ 5, (F(I),I=1,NI)
    5     FORMAT (7F10.0)
           PRINT 108 , (F(I),I=1,NI)
    108   FORMAT(///1X,7F12.5)
    C   INPUT COLUMN INFO -- LENGTH ,MODULUS OF ELASTICITY, FIXIDITY OF TOP
    C   AND BOTTOM,KX=0 - PIN,KX = 1 - FIX, AND DEGREE OF FREEDOM ,NF , WHERE
    C    SPRING CONSTANT IS APPLIED
    C   COLUMN FORCE IS ALREADY FOUND AND MOMENT OF INERTIA OF COLUMN IS DETERMINED
    C   FROM ALLOWABLE FLEXURAL LOAD LEVEL OF BEAM
          READ 15,CL, EL, KT, KB, NF
    15    FORMAT(2F10.0,3I10)
          CENTM = ABS(CENTM)
          CRITM = AMAX1(COLM,CENTM)
    C   ALLOWABLE MOMENT = F ALLOWABLE (2.1 KSI USED) X Z
          ALLM = .8013*BSI**(1./3.)*DB**(5./3.)
    C  DETERMINE ALLOWABLE LOAD FACTOR
          CRITM = 12.*CRITM
```

```
          FLOAD = ALLM/CRITM
h ------- COLR = R*FLOAD
      C   COLUMN MOM OF INERTIA ,FOR FACTOR OF SAFETY OF 3 USED
a ------- BC = COLR*CL*CL/(EL*3.27)
          PRINT 16,CL, EL, R,BC,KT,KB, NF
      16  FORMAT(11H COL DATA =,4F10.3,3I10)
          K = 1
      C   ZERO OUT STRUCTURE STIFFNESS MATRIX S
      8   DO 6 I=1,NU
          DO 6 J =1,NU
      6   S(I,J)=0.0
          IF(F(K).LT..001) GO TO 46
      C   BUILD MEMBER STIFFNESS MATRIX,SM,4*4,(ROW,COL)
          DO 9 I=1,NM
          SL = X(I)
          STC = E*B(I)/SL
          SE6 = 6.*STC/SL*EC(I)
          SM(1,1) = 2.*SE6/SL*EC(I) + T(I)*G/SL - 2.*EC(I)*(BM(I)/SL -
         1 V(I)/2.)*F(K)
          SM(2,1) = SE6 - (BM(I) - V(I)*SL/3.)*F(K)
          SM(3,1) =-2.*SE6/SL*EC(I) - T(I)*G/SL + 2.*BM(I)*EC(I)/SL*F(K)
          SM(4,1) = SE6 + V(I)*SL/6.*F(K)
          SM(1,2) = SE6 - V(I)*SL/6.*F(K)
          SM(2,2) = 4.*STC
          SM(3,2) =-SM(1,2)
          SM(4,2) = 2.*STC
          SM(1,3) = SM(3,1)
          SM(2,3) =-SM(1,2)
          SM(3,3) = 2.*SE6/SL*EC(I) + T(I)*G/SL - 2.*EC(I)*(BM(I)/SL+V(I)/2.
         1 )*F(K)
          SM(4,3) =-SE6 + (BM(I) + V(I)*SL/3.)*F(K)
          SM(1,4) = SM(4,1)
          SM(2,4) = SM(4,2)
          SM(3,4) =-SM(1,4)
          SM(4,4) = SM(2,2)
          IF(NTYPE.NE.1) GO TO 208
          PRINT 205,SM
      208 CONTINUE
      C   ADD SEGMENT JOINT LOAD,SJL,MOMENT INFLUENCE TO MATRIX
      C   SJL=1/2 THE LOAD ON THE SEGMENT FOR UDL OF 1 KIP/FT
h1 ------ SJL = -SL*F(K)/24.
          SM(1,1) = SM(1,1) + SJL*DB/2.
```

h_2

```
         SM(3,3) = SM(3,3) +SJL*DB/2.
         IF(I.EQ.NM) SM(3,3) = SM(3,3) -P*F(K)*DB/2.
         IF(NTYPE.NE.1) GO TO 204
         PRINT 205,SM
  205    FORMAT(1X,7H ***    ,4E15.7)
  204    CONTINUE
  C   BUILD STRUCTURE STIFFNESS MATRIX S,NU*NU FROM POSITION NUMBERS AND
  C   MEMBER STIFFNESS MATRICES
         DO 20 LL = 1,4
         N1 = NP(I,LL)
         IF(N1) 22,22,21
  21     DO 17 MM=1,4
         N2 = NP(I,MM)
         IF(N2) 19,19,18
  18     S(N1,N2) = S(N1,N2) +SM(LL,MM)
  19     CONTINUE
  17     CONTINUE
  22     CONTINUE
  20     CONTINUE
         DB = 2.*EC(I)
  9      CONTINUE
         IF(NTYPE.NE.1) GO TO 210
         CALL MATOUT(S,NU,70)
  210     CONTINUE
  C   DETERMINE CRITICAL LOADS FOR FIXIDITY
         CLS = CL*CL
         FB = EL*BC/CLS
         PE = 9.86*FB
         XL = CL*SQRT(R*F(K)/(EL*BC))
         IF(KT+KB-1)53,52,51
  C   CASE FIX - FIX
  51     PCR = 4.0*PE
         IF(K.EQ.1)PRINT 40
  40     FORMAT(10H *FIX-FIX*)
  C   MODIFY FOR NON-LINEARITY
         IF(R*F(K).LT..01*PE)GO TO 66
         DENOM = 2.-2.*COS(XL)-XL*SIN(XL)
         STB1 = XL**3*SIN(XL)/(12.*DENOM)
         STB2 = XL*XL*(1.-COS(XL))/(6.*DENOM)
         STB3 = XL*(SIN(XL)-XL*COS(XL)) /(4.*DENOM)
         GO TO 54
  66     STB1 = 1.
```

```
      STB2 = 1.
      STB3 = 1.
54    HC= 12.*FB/CL*STB1*DB + 6.*FB*STB2
      CM= 6.*FB*DB*STB2 + 4.*FB*CL*STB3
      PRINT 200
200   FORMAT(55H    FB        STB1        STB2        STB3        HC        CM)
      PRINT 201,FB,STB1,STB2,STB3,HC,CM
201   FORMAT(6F10.3)
      GO TO 65
52    PCR = 2.0*PE
      IF(KB-1)57,56,56
C   CASE PIN - FIX
C   MODIFY FOR NON-LINEARITY
56    DENOM = 3.*(TAN(XL)-XL)
      IF(K.EQ.1)PRINT 41
41    FORMAT(10H *PIN-FIX*)
      STB5 = XL**3/DENOM
      IF(R*F(K).LT..01*PE)STB5 =1.
58    HC=3.*FB/CL*DB*STB5
      CM=0.
      PRINT 206
206   FORMAT(16H   STB5        HC).
      PRINT 209,STB5,HC
209   FORMAT(2F10.3)
      GO TO 65
C   CASE FIX - PIN
C   MODIFY FOR NON-LINEARITY
57    DENOM = 3.*(TAN(XL)-XL)
      IF(K.EQ.1)PRINT 42
42    FORMAT(10H *FIX-PIN*)
      IF(R*F(K).LT..01*PE)GO TO 68
      STB6 = XL**3/DENOM
      STB8 = XL**2*TAN(XL)/DENOM
      STB7 = STB8
      STB9 = STB7
      GO TO 61
68    STB6 = 1.
      STB7 = 1.
      STB8 = 1.
      STB9 = 1.
61    HC= 3.*FB*(DB/CL*STB6 + STB8)
      CM= 3.*FB*(DB*STB7 + CL*STB9)
```

```
      PRINT 202
202   FORMAT(55H    FB        STB6        STB7        STB8        HC        CM)
      PRINT 203,FB,STB6,STB7,STB8,HC,CM
203   FORMAT(6F10.3)
      GO TO 65
C   CASE PIN - PIN
53    PCR = PE
      IF(K.EQ.1)PRINT 43
43    FORMAT(10H *PIN-PIN*)
C   MODIFY FOR NON-LINEARITY
      HC = -R*F(K)*DB/CL
      CM= 0.
C   DETERMINE SPRING CONSTANT SK
65    SK = CM + HC*DB - R*F(K)*DB/2.
97    PRINT 69,SK
69    FORMAT(6H SK = ,E15.7)
      S(NF,NF) = S(NF,NF) + SK
      IF(NTYPE.NE.1) GO TO 207
      PRINT 211,S(NF,NF)
211   FORMAT(40H DIAG ELEMENT S(NF,NF) AFTER SK ADDED = ,E15.7)
207   CONTINUE
      CALL  DETER ( S, NU, 3, 70, DET )
      PRINT 23,F(K),DET,IEXP
23    FORMAT(15H LOAD FACTOR = ,F9.5,7H DET = ,E15.7,I10)
      IF(DET.LE.0.) GO TO 47
      K=K+1
      IF(K-NI)8,8,47
46    CONTINUE
47    CONTINUE
      GO TO 100
      END
$IBFTC DET
C     DETERMINANT OF BAND MATRICES WIDTH 2M+1
      SUBROUTINE  DETER ( A, N, M, IA, DET )
      DIMENSION  A(IA,1)
      INTEGER  S, T
      COMMON/MATEXP/IEXP
      IEXP=0
      PROD=1.0
      N1=N-1
      DO 5 K=1,N1
      IEND=M+K
```

```
      IF (IEND .GE. N )  IEND=N
      K1=K+1
      DO 10 S=K1,IEND
      IF (A(K,K) .EQ. 0.0)  GO TO 99
      TEMP=A(S,K)/A(K,K)
      DO 11 T=K1,N
      A(S,T)=A(S,T)-TEMP*A(K,T)
11    CONTINUE
10    CONTINUE
      IF (ABS(PROD) .LT. 1.0E+15 )  GO TO 14
      PROD=PROD*1.0E-15
      IEXP=IEXP+15
14    IF (ABS(PROD) .GT. 1.0E-15 )  GO TO 15
      PROD=PROD*1.0E+15
      IEXP=IEXP-15
15     PROD=PROD*A(K,K)
5     CONTINUE
      IF (ABS(PROD) .LT. 1.0E+15 )  GO TO 16
      PROD=PROD*1.0E-15
      IEXP=IEXP+15
16    IF (ABS(PROD) .GT. 1.0E-15 )  GO TO 17
      PROD=PROD*1.0E+15
      IEXP=IEXP-15
17     PROD=PROD*A(N,N)
      DET=PROD
      RETURN
99    DET=0.0
      PRINT 1
1     FORMAT(33H DET = 0.0, ZERO TERM ON DIAGONAL)
      RETURN
      END
      SUBROUTINE FMVR(NRX,L,A,P,XL,XA,NL,NA,VX,SEG,M,I)
      REAL L,M
      S = NL
      T = NA
      Y = I
      V1 = P*A/L
h     R1 = .5*(L*L-A*A)/L
      IF(I.GT.NL) GO TO 1
h     VX = V1-R1-XL+2.*Y*XL
h     M = -.5*(L*L-A*A)/L*(2.*Y*XL-XL)+(2.*Y*XL-XL)**2/2.+P*A/L*(2.*Y*XL
     1-XL)
```

```
      SEG = L/S
      GO TO 2
    1 VX = -A -XA + 2.*(Y-S)*XA - P
      M = .5*(A+XA-2.*(Y-S)*XA)**2+ P*(A+XA-2.*(Y-S)*XA)
       SEG = A/T
    2 CONTINUE
      RETURN
      END
$ENTRY
```