INFLUENCE OF LARGE OSCILLATIONS ON STABILITY OF SIMPLE SURGE TANK

by

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ABSTRACT

This study investigates the stability of a simple surge tank operating in conjunction with a turbine. The turbine is assumed to be governed for constant power output or for constant gate opening, depending on whether the instantaneous head acting on the turbine is greater or smaller than the rated head.

Early investigations on the stability of oscillations assumed that the governor on a turbine ensured constant power output. In practice, the governor on a turbine can maintain constant power output only when the oscillations remain above the rated head and the full-gate position is not reached. For this case, a computer study showed that results of the investigation are in close agreement with the results of several investigators. A revised form of the Thoma's formula for surge tank stability with oscillations of large amplitude is proposed.

When the head during the maximum downsurge falls below the rated head of the turbine the constant gate opening condition must be considered. A computer program was developed to study the stability of oscillations of large amplitude under constant gate opening condition. The transient conditions for three cases of load demand with initial steady state discharge of zero, one quarter and one half of the final steady state discharge were studied. The graphical form of the results allows the prediction of surge tank stability for oscillations of large amplitude for a simple surge tank. The graphs for all three conditions show that an increase in the rated head has a stabilizing influence on surge tank oscillations.

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NOTATION

		UNITS
a	Acceleration of flow in conduit	ft/sec^2
F	Cross-sectional area of surge tank	ft^2
f	Cross-sectional area of conduit	ft^2
F_{Th}	Thoma's critical area of surge tank	ft^2
h	Sum of velocity head and friction loss in conduit	ft
g	Acceleration of gravity	ft/sec^2
H_G	Gross head	ft
H_n	Net head acting on turbine during surge condition	ft
H_o	Net head acting on turbine in final steady state condition	ft
h_o	Head loss in conduit in final steady state condition	ft
H_r	Rated head of turbine	ft
h_r	Relative rated head, i.e., ratio of rated head to gross head	-
L	Length of conduit	ft
N	Ratio of surge tank cross-sectional area to Thoma's critical area	-
n	Relative initial discharge, Q_{in}/Q_o	-
q	Momentary turbine discharge	ft^3/sec
Q	Momentary conduit discharge	ft^3/sec
Q_{in}, Q_o	Initial and final steady state values of conduit discharge	ft^3/sec
q_r	Full gate turbine discharge under rated head	ft^3/sec
t	Time in seconds	sec

		UNITS
V	Velocity in conduit in surge condition	ft/sec
V_o	Final steady state velocity in conduit	ft/sec
w	Unit weight of water	lb/ft^3
X	Dimensionless parameter, $\dfrac{h_o}{\sqrt{\dfrac{L \cdot f}{g \cdot F}} \cdot V_o}$	-
Y	Dimensionless parameter, $\dfrac{h_o}{H_o}$	-
y	Height of water level in surge tank, measured from reservoir level (positive downward)	ft
z	Height of water level in surge tank, measured from final steady state water level in surge tank	ft
Z_*	Amplitude of oscillation of frictionless flow resulting from sudden total rejection, $V_o \cdot \sqrt{\dfrac{L \cdot f}{g \cdot F}}$	ft
ε	Vogt's dimensionless parameter, $\dfrac{L \cdot f \cdot V_o^2}{g \cdot F \cdot h_o^2}$	-
β	Vogt's dimensionless parameter, $\dfrac{h_o}{H_G}$	-

The subscripts, i, a, e, refer to initial, average and end values respectively of the variables y, h, Q, H_n and q.

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INTRODUCTION

Surge tank stability is affected by sudden load changes and unpredictable interruptions of water power delivery which cause oscillations of large amplitude in the surge tank. Undamped oscillations in the surge tank result in constant hunting of the governor. For satisfactory load regulation of a turbo-generator it is necessary that the oscillations in the surge tank be damped within a few minutes.

This study discusses the stability of a simple surge tank in conjunction with a turbine operating under various heads. First, oscillations of small amplitude are considered. From the equation of motion, the continuity equation, and the equation of constant power output, a general differential equation representing oscillations in a simple surge tank is obtained (equation (13) in Chapter I). If the amplitudes of oscillation are small, the nonlinear differential term and the terms equal to and higher than second degree may be neglected. The general differential equation will be simplified to a linear differential equation of the second order as follows (equation (16) in Chapter I) :

$$\frac{d^2z}{dt^2} + 2a \cdot \frac{dz}{dt} + b \cdot z = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where a and b are constants. The first term involves acceleration and the second term involves velocity of the water in surge tank and the last term involves displacement of water surface.

The solution of equation (1) with a = 0 for the case of oscillations of constant amplitude leads to the Thoma formula

$$F_{Th} = \frac{L \cdot f \cdot V_o^{\,2}}{2g \cdot H_o \cdot h_o} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

where

F_{Th} = Thoma's critical area of surge tank

L = length of conduit

f = cross-sectional area of conduit

V_o = final steady state velocity in conduit

H_o = net head acting on turbine at final steady state condition

h_o = head loss in conduit in final steady state condition.

In a surge tank with cross-sectional area F_{Th}, the oscillations have constant amplitude.

Several terms are neglected in the solution of the general differential equation by Thoma and this limits the validity of the solution to the case when the oscillations in the surge tank are small. Hence this criterion for the stability of a simple surge tank is applicable only to the case of oscillations of small amplitude. For the case of oscillations of large amplitude, Thoma's criterion leads to instability. Therefore, Jaeger,* Frank,* and others suggested that Thoma's criterion had to be refined for

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* see reference (3)

the case of oscillations of large amplitude. Correction for Thoma's criterion suggested by Jaeger is

$$N = \frac{F}{F_{Th}} = (1 + 0.482 \cdot \frac{Z_*}{H_G + h_o}) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where

F = cross-sectional area of surge tank

H_G = gross head

Z_* = $V_o \cdot \sqrt{\frac{L \cdot f}{g \cdot F}}$

All corrections for Thoma's criterion suggested by previous investigators are given in the section 4 in Chapter II in dimensionless form.

When oscillations of large amplitude are considered, the neglected terms are no longer negligible and must be included in the solution. No solution by direct integration is possible and numerical integration by digital computer is, therefore, used for the solution of the equation.

In dealing with the stability of oscillations of large amplitude, previous investigators have assumed that during the transient state the governor keeps the power output constant; thus

$$q \cdot H_n = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where

q = momentary turbine discharge

H_n = momentary net head.

In practice, the governor action results either in constant power output or in constant gate opening. The latter case occurs when the full-gate opening is reached. The head-discharge relationship for constant gate opening condition is approximately

$$q\sqrt{H_n} = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For an instantaneous head greater than the rated head of the turbine, the governor keeps the generator power output constant; for an instantaneous head less than the rated head, the governor maintains the gate opening constant.

Sudden load increases from steady state conditions corresponding to following relative discharges were considered :

(1) $n = Q_{in}/Q_o = 0$

(2) $n = Q_{in}/Q_o = 0.25$

(3) $n = Q_{in}/Q_o = 0.50$

where

Q_{in} = initial steady state discharge

Q_o = final steady state discharge.

This study shows that the results of the stability analyses derived by Jaeger, Frank, and others based on the constant power output criterion, are applicable if the net head on the turbine remains greater than the rated head during the oscillations. For oscillations of large amplitude, Thoma's criterion must necessarily result in instability if the constant horsepower output criterion is imposed.

CHAPTER I STABILITY OF SMALL OSCILLATIONS

1. Turbine Performance Characteristics

It has long been realized that the transient behaviour of a surge tank in a water power system is primarily influenced by the action of the turbine. Hence it is appropriate here to introduce first the performance characteristics of a turbine.

The rated head, H_r, of a turbine is the head at which the full-gate output is equal to the rated generator power output. With a net head greater than the rated head, unit output is limited by the capacity of the generator. With the net head less than the rated head, turbine power output is less than the rated output of the generator and the gates are kept at the fully open position. Therefore, while the turbine is operating under varying heads during the surge condition, the governor keeps the power output constant as long as the instantaneous head exceeds the rated head.

As the turbine head decreases, during the down surge while the net head is still greater than the rated head, the discharge is increased to maintain constant power output until the full-gate position is reached. At this point, although the head still continues to decrease, the discharge also decreases according to the full-gate opening discharge curve and constant power output is no longer maintained. In this thesis the full-gate discharge of a turbine, considered as the discharge through an orifice, is

assumed to be independent of the speed, and to vary as the square root of the net head. The full-gate head-discharge relationship can be expressed as

$$\frac{H_n}{H_r} = K \left(\frac{q}{q_r}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where K is a constant. This equation is plotted on head-discharge diagram in Figure 1 as a parabolic curve which passes through the point (1,1). K = 1 for equation (1).

It follows that with the instantaneous head of the turbine above the rated head the power output is limited by the capacity of the generators, but with the instantaneous head below the rated head, the full-gate capacity of the turbine limits the output of the plant.

As an example, the predicted performance of Shasta Power Plant operating over a wide range of head is shown in Figure 2. The regions of constant power output and constant gate opening are indicated in this figure.

2. Equation of Governor

For large oscillations, the governor equation $q \cdot H_n$ = constant is valid for the instantaneous head greater than the rated head and $q / \sqrt{H_n}$ = constant for the instantaneous head less than the rated head as shown in Figure 3.

For small oscillations, the governor will operate entirely in accordance with either (1) the equation $q \cdot H_n$ = constant, or (2) the equation $q / \sqrt{H_n}$ = constant. The former case is indicated by the heavy line AB in Figure 3.

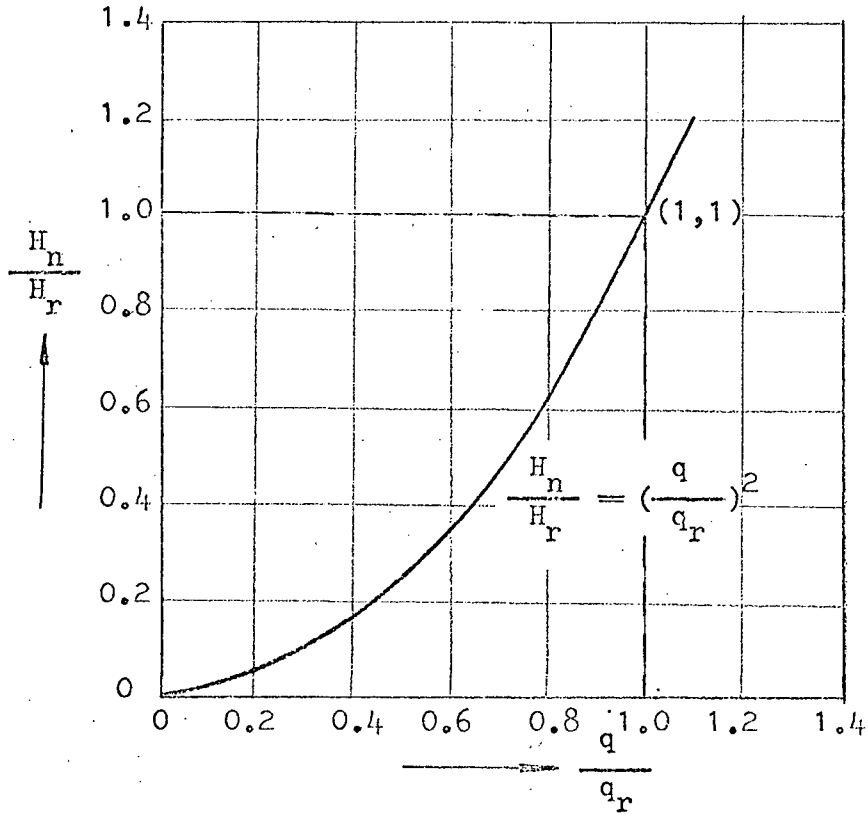

FIGURE 1 TURBINE FULL-GATE DISCHARGE

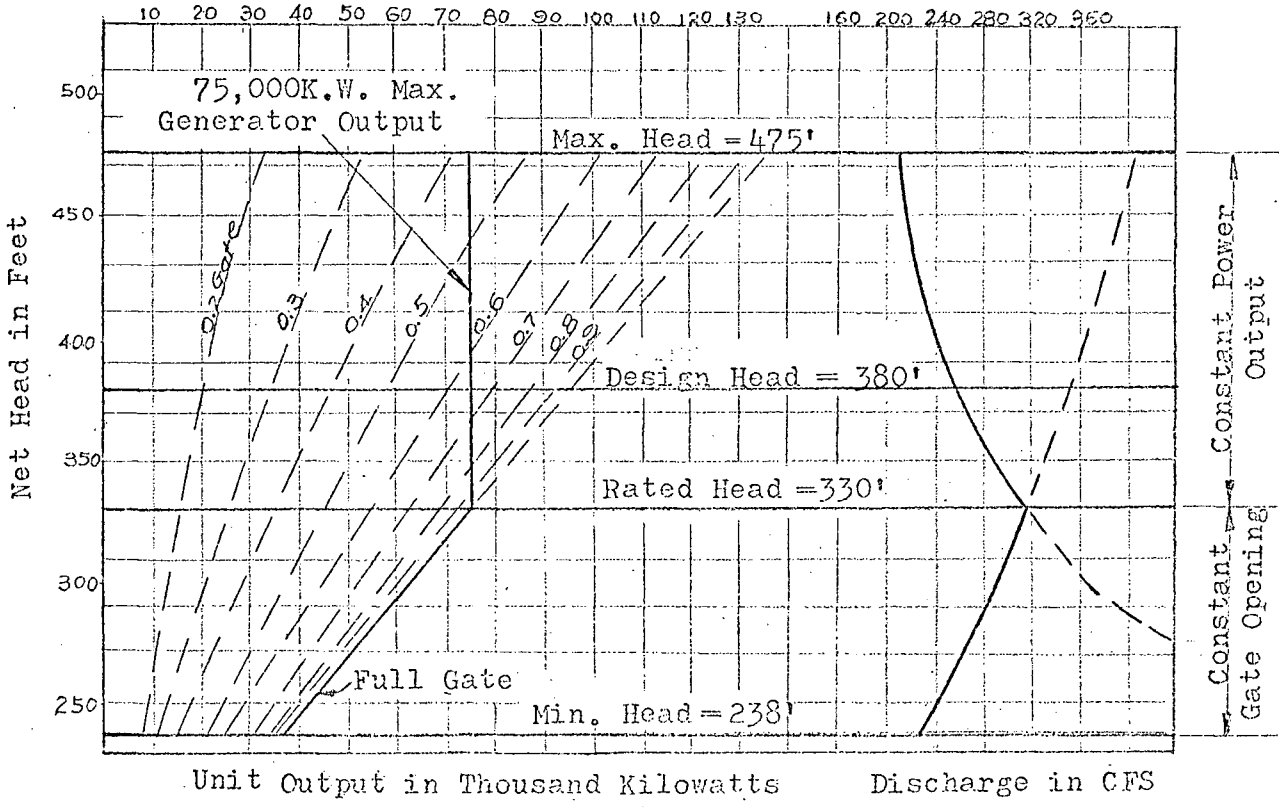

FIGURE 2 PERFORMANCE CURVES-SHASTA POWER PLANT

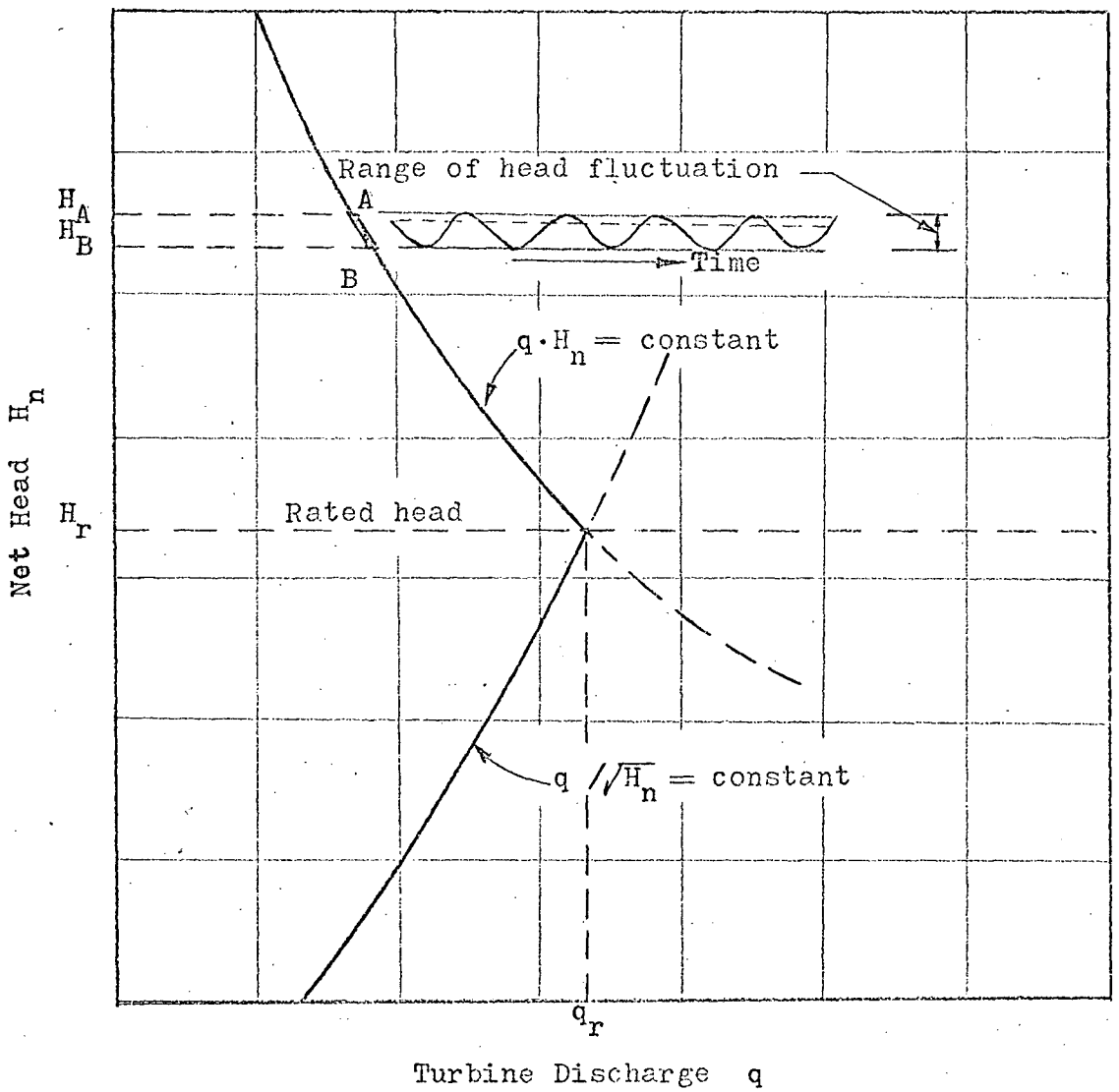

FIGURE 3 TURBINE HEAD — DISCHARGE RELATIONSHIP
SMALL OSCILLATIONS

For case (1) points A and B will lie on the curve $q \cdot H_n$ = constant and both H_A and H_B are higher than H_r. For case (2), points A and B will lie on the curve $q / \sqrt{H_n}$ = constant with both H_A and H_B less than H_r.

If the transient condition falls within the range of the constant gate condition, as in case (2), all surges are damped independently of the discharge, surge tank area and length and area of the conduit. Conversely for case (1), as the head decreases the discharge is increased to maintain constant power output. It is the discharge increase that induces the problem of stability.

3. Basic Assumptions

The investigation of oscillations of small amplitude is based on the following simplifying assumptions :

(1) Both conduit walls and water are incompressible.

(2) Oscillations are small.

(3) Turbine governor maintains power output constant.

(4) Turbine efficiency is constant.

(5) Cross-section of surge tank is constant.

(6) Penstock friction and accelerating head are neglected.

(7) Conduit velocity head $V^2 / 2g$ is negligible.

(8) Acceleration of water in surge tank neglected.

(9) Governor is highly sensitive.

(10) Isolated system.

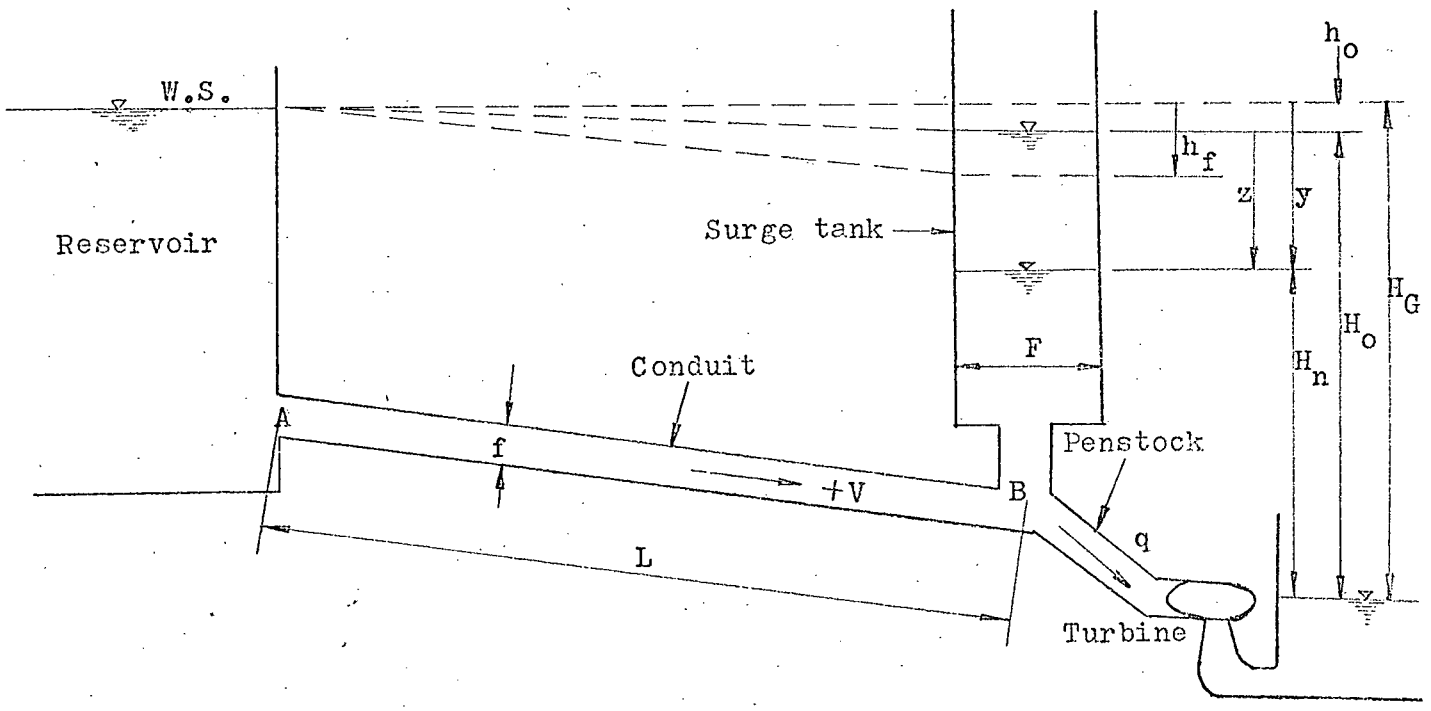

FIGURE 4 GENERAL INSTALLATION OF SURGE TANK

4. Fundamental Equations and their Solutions

The general installation of simple surge tank is illustrated in Figure 4.

(a) The Equation of Motion

In Figure 4, the unbalanced pressure force between the intake A and surge tank entrance B in the conduit is

$$P_1 = w \cdot f \cdot (z + h_o - h) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which

w = unit weight of water

f = cross-sectional area of conduit

z = height of water level in surge tank, measured from final steady state water level in surge tank

h_o = friction loss in conduit at final steady state condition

h = instantaneous head loss in conduit.

The acceleration of water column in the conduit is

$$a = \frac{dV}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where

V = velocity in the conduit.

Hence the required unbalanced force to accelerate the water column AB in the conduit in the direction of motion is

$$P_2 = L \cdot \frac{w}{g} \cdot f \cdot \frac{dV}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From the Newtonian laws of motion it follows that the unbalanced pressure force accelerates the water column in the conduit during the surge condition. We obtain the equation of motion by equating P_1 and P_2 as follows :

$$\frac{w}{g} \cdot f \cdot L \cdot \frac{dV}{dt} = w \cdot f \ (z + h_o - h) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or

$$\frac{L}{g} \cdot \frac{dV}{dt} = z + h_o - C \cdot V \cdot |V| \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where $h = CV \cdot |V|$. The arrow in Figure 4 defines the positive direction of V.

(b) <u>The Continuity Equation</u>

Consider the continuity at point B. The discharge in the penstock equals the discharges from conduit and the surge tank. Hence

$$V \cdot f + F \cdot \frac{dz}{dt} = q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

in which

F = cross-sectional area of surge tank.

(c) <u>The Governor Equation</u>

The governor maintains power output constant. Therefore

$$q \ (H_G - h_o - z) = Q_o \cdot H_o = f \cdot V_o \cdot H_o \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

in which

H_G = gross head of the plant

Q_o = final steady state discharge

$H_o = H_G - h_o$ = final steady state net head on turbine

V_o = velocity in conduit at final steady state condition.

By eliminating q from equation (7) and (8), we obtain

$$\frac{f \cdot V_o \cdot H_o}{H_G - h_o - z} = V \cdot f + F \cdot \frac{dz}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

or

$$V = \frac{V_o \cdot H_o}{H_G - h_o - z} - \frac{F}{f} \cdot \frac{dz}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Then

$$V^2 = \frac{F^2}{f^2} \left(\frac{dz}{dt}\right)^2 - \frac{2F \cdot V_o \cdot H_o}{f \cdot (H_G - h_o - z)} \cdot \frac{dz}{dt} + \frac{V_o^{\,2} \cdot H_o^{\,2}}{(H_G - h_o - z)^2} \quad . \quad . \quad . \quad (11)$$

and

$$\frac{dV}{dt} = \frac{V_o \cdot H_o}{(H_G - h_o - z)^2} \cdot \frac{dz}{dt} - \frac{F}{f} \cdot \frac{d^2 z}{dt^2} \quad . \quad . \quad . \quad . \quad (12)$$

Substituting equations (11) and (12) into equation (6), we obtain

$$\frac{d^2 z}{dt^2} - \frac{C \cdot g \cdot F}{L \cdot f} \left(\frac{dz}{dt}\right)^2 + \left(\frac{2g \cdot C \cdot V_o \cdot H_o}{L \cdot (H_o - z)} - \frac{f \cdot V_o \cdot H_o}{F \cdot (H_o - z)^2} \right) \frac{dz}{dt} + \frac{g \cdot f}{L \cdot F} \cdot z$$

$$+ \frac{g \cdot f \cdot h_o}{L \cdot F} - \frac{g \cdot f \cdot h_o \cdot H_o^{\,2}}{L \cdot F \cdot (H_o - z)^2} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The binomial terms in equation (13) expand into a series as follows:

$$\frac{1}{H_o - z} = \frac{1}{H_o} + \frac{z}{H_o^{\,2}} + \frac{z^2}{H_o^{\,3}} + \quad . \quad . \quad . \quad . \quad . \quad .$$

$$\frac{1}{(H_o-z)^2} = \frac{1}{H_o^{\,2}} + \frac{2z}{H_o^{\,3}} + \frac{3z^2}{H_o^{\,4}} + \cdots\cdots$$

The last equation becomes

$$\frac{d^2z}{dt^2} - \frac{C\cdot g\cdot F}{L\cdot f}\left(\frac{dz}{dt}\right)^2 + \left[\frac{2g\cdot C\cdot V_o}{L}\left(1 + \frac{z}{H_o} + \frac{z^2}{H_o^{\,2}} + \ldots\right) - \frac{f\cdot V_o}{F}\left(\frac{1}{H_o} + \frac{2z}{H_o^{\,2}} + \frac{3z^2}{H_o^{\,3}} + \ldots\ldots\right)\right]\frac{dz}{dt} + \frac{g\cdot f}{L\cdot F}\cdot z - \frac{g\cdot f\cdot h_o}{L\cdot F}\left[\frac{2z}{H_o} + \frac{3z^2}{H_o^{\,2}} + \ldots\ldots\right] = 0$$

For small oscillations, we neglect (1) the product of first degree terms $\frac{z}{H_o}$ and $\frac{dz}{dt}$, (2) the second degree terms of $\left(\frac{z}{H_o}\right)^2$ and the non-linear term of $\left(\frac{dz}{dt}\right)^2$, and (3) higher degree terms. Then

$$\frac{d^2z}{dt^2} + \left[\frac{2g\cdot C\cdot V_o}{L} - \frac{f\cdot V_o}{F\cdot H_o}\right]\frac{dz}{dt} + \left(\frac{g\cdot f}{L\cdot F} - \frac{2g\cdot f\cdot h_o}{L\cdot F\cdot H_o}\right)z = 0 \quad \cdot\cdot \quad (14)$$

or, on rearrangement

$$\frac{d^2z}{dt^2} + \left[\frac{2g\cdot C\cdot V_o}{L} - \frac{f\cdot V_o}{F\cdot H_o}\right]\frac{dz}{dt} + \frac{g\cdot f}{L\cdot F}\left(\frac{H_G-3h_o}{H_G-h_o}\right)z = 0 \quad \cdot\cdot\cdot \quad (15)$$

Equation (15) is valid for oscillations of small amplitude.

By rewriting equation (15), we obtain

$$\frac{d^2z}{dt^2} + 2a\cdot\frac{dz}{dt} + b\cdot z = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where

$$a = \frac{g\cdot C\cdot V_o}{L} - \frac{f\cdot V_o}{2F\cdot H_o}$$

$$b = \frac{f\cdot g}{L\cdot F}\cdot\frac{H_G - 3h_o}{H_o}$$

If we put $z = u\cdot e^{-at}$, we obtain for u an equation of the form

$$\frac{d^2u}{dt^2} + (b - a^2)\, u = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Let $K^2 = b - a^2$. Then the solution of equation (17) is

$$u = A\cdot\text{Cos } Kt + B\cdot\text{Sin } Kt$$

The general solution of equation (16) is

$$z = u\cdot e^{-at} = e^{-at}\,(A\cdot\text{Cos } Kt + B\cdot\text{Sin } Kt) \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The term "a" in the factor e^{-at} is called the "modulus of oscillation."

The oscillations of small amplitude can be classified as follows:

(1) For $a > 0$, the oscillations are damped since as t increases, the factor e^{-at} decreases.

(2) For $a = 0$, all oscillations are of constant amplitude and

$$z = A\cdot\text{Cos } Kt + B\cdot\text{Sin } Kt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

This condition leads to Thoma's criterion for surge tank area

$$a = \frac{g \cdot C \cdot V_o}{L} - \frac{f \cdot V_o}{2F \cdot H_o} = 0$$

$$F_{Th} = \frac{V_o^{\,2}}{2g} \cdot \frac{L \cdot f}{H_o \cdot z_o} = \frac{L \cdot Q_o^{\,2}}{2g \cdot f \cdot H_o \cdot h_o} \quad . \quad . \quad . \quad . \quad (20)$$

(3) For $a \lessgtr 0$, the oscillations are increased since as t increases, the factor e^{-at} also increases.

The critical area of a surge tank for oscillations derived by Thoma is given in equation (20). In the tank which has its area equal to Thoma's critical area, surges are neither damped by friction nor amplified by governor action.

If the surge tank area in a system is smaller than Thoma's critical area, it is possible for an initially small oscillation to increase in amplitude. Small surges, instead of being damped by friction, are amplified by the interaction of the surge tank and the governor.

It is noted that the required Thoma's critical area is inversely proportional to the net head. Therefore, the largest required Thoma's critical area occurs at the lowest net head where a constant power output can still be maintained, that is, at the rated head.

CHAPTER II STABILITY OF LARGE OSCILLATIONS

1. Basic Equations of Large Oscillations

For analysis of oscillations with large amplitude in a simple surge tank it will be convenient to let "y" equal the height of water level in surge tank measured from reservoir level, so that $y = z + h_o$, (see Figure 4).

The equation of motion of the water in the conduit AB, and the equation of hydraulic continuity at B are respectively :

$$\frac{L}{g} \cdot \frac{dV}{dt} = y - CV \cdot |V| \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$V \cdot f + \frac{dy}{dt} \cdot F = q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

As for the case of small oscillations, the governor equation is used as a boundary condition at the outlet of surge tank. This boundary condition depends not only on the head variations caused by oscillations of large amplitude but also on the ratio of rated head to design head. Owing to the wide range of net head fluctuation for oscillations of large amplitude, the range of operation will usually include both the constant power output and constant gate opening cases as shown in Figure 5, where points A and B represent two extreme points of fluctuation. Depending on the different values of rated head, the equation representing constant horsepower output or constant gate opening must be used as the governor criterion.

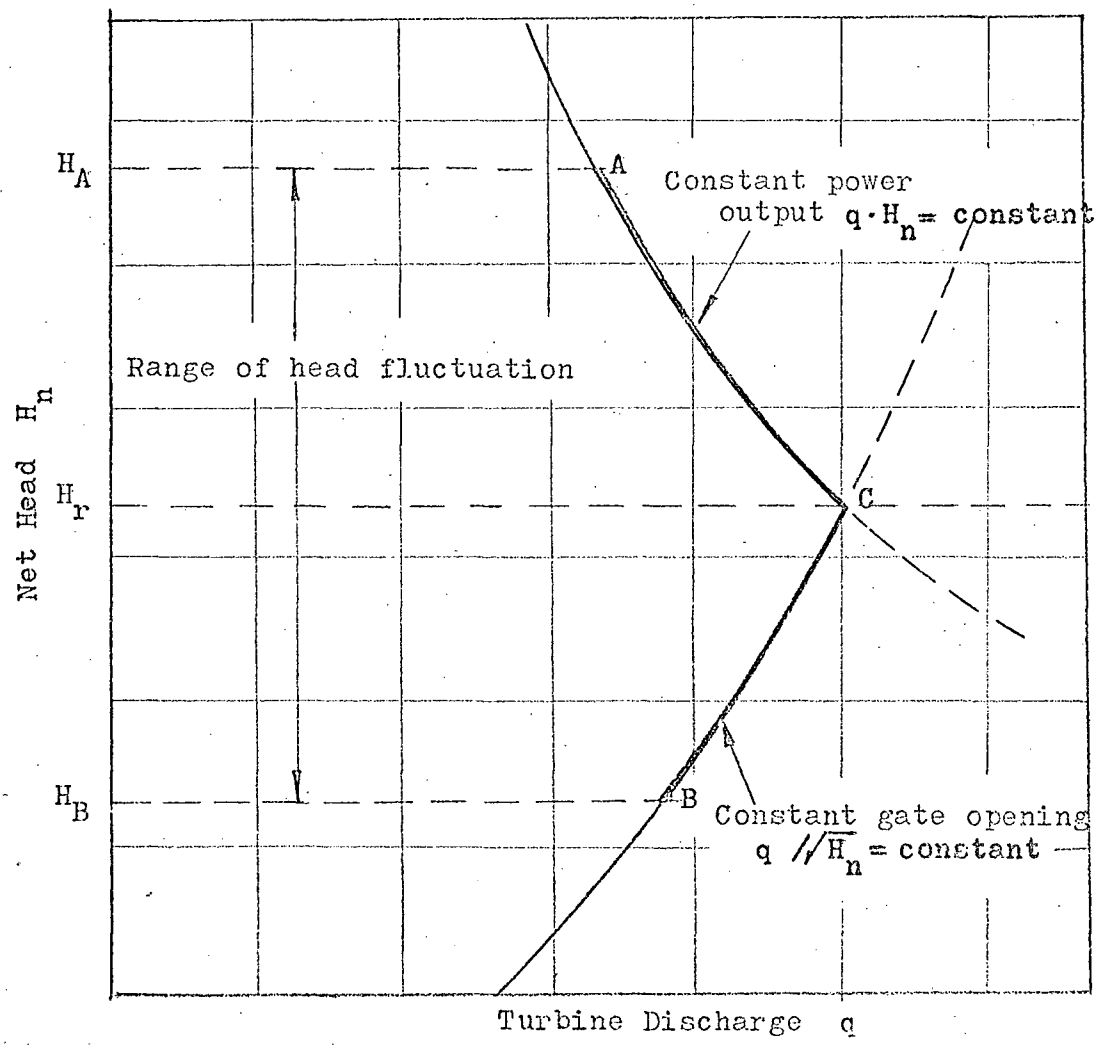

FIGURE 5 TURBINE HEAD-DISCHARGE RELATIONSHIP
LARGE OSCILLATIONS

For constant horsepower output

$$q\ (H_G - y) = q \cdot H_n = Q_o \cdot H_o = \text{constant} \quad . \quad . \quad . \quad . \quad (3)$$

and for constant gate opening

$$\frac{H_G - y}{H_r} = \frac{H_n}{H_r} = \left(\frac{q}{q_r}\right)^2$$

or

$$\frac{q}{\sqrt{H_n}} = \frac{q_r}{\sqrt{H_r}} = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

When the final steady state head is much higher than the rated head of turbine, the governor might maintain constant power output without reaching the condition of constant gate opening, i.e., the whole range of head fluctuation is above the rated head. This case leads to the criteria of limiting oscillations of large amplitude resulting from constant power output. The result of this special case in this study and its comparison with the work of previous investigators are given in section 4 of this chapter.

In order to simplify analysis by numerical procedures, all assumptions for large oscillations are as stated in the preceding chapter, except as follows :

(1) The governor maintains constant power output at heads above the rated head and constant gate opening at heads below the rated head.

(2) The conduit velocity head $V^2/2g$ is considered in numerical solutions.

2. Computer Solutions of Differential Equations

Differential equations for which no direct solution is possible, can be solved by step-by-step methods of integration. For an arithmetical integration, the derivatives dV/dt and dy/dt in the basic differential equations

$$\frac{dV}{dt} = \frac{g}{L}(y - h) \quad \ldots \quad (5)$$

$$\frac{dy}{dt} = \frac{q - V \cdot f}{F} \quad \ldots \quad (6)$$

and

$$q \cdot H_n = Q_o \cdot H_o \quad \ldots \quad (7)$$

or

$$\frac{q}{\sqrt{H_n}} = \frac{q_r}{\sqrt{H_r}} \quad \ldots \quad (8)$$

are replaced by the small finite difference quotients $\Delta V/\Delta t$ and $\Delta y/\Delta t$, where

$$\frac{\Delta V}{\Delta t} = \frac{g}{L}(y_a - h_a) \quad \ldots \quad (9)$$

$$\frac{\Delta y}{\Delta t} = \frac{q_a - V_a \cdot f}{F} \quad \ldots \quad (10)$$

$$q_a \cdot H_{na} = Q_o \cdot H_o \quad \ldots \quad (11)$$

$$\frac{q_a}{\sqrt{H_{na}}} = \frac{q_r}{\sqrt{H_r}} \quad \ldots \quad (12)$$

From (9), (10) we obtain

$$\Delta Q = \Delta V \cdot f = \frac{g}{L} \cdot f \cdot \Delta t \cdot (y_a - h_a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$\Delta y = \frac{\Delta t}{F} (q_a - Q_a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The resulting four equations, (11), (12), (13), and (14) are then solved by assigning values for variables, y_a, h_a, q_a, and Q_a, and calculating alternatively ΔQ and Δy. For a given time interval,

$$y_e = y_i + \Delta y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$Q_e = Q_i + \Delta Q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$y_a = y_i + \Delta y/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

$$Q_a = Q_i + \Delta Q/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$h_a = C \cdot V_a^{\,2} = C \cdot \frac{Q_a^{\,2}}{f^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

In these difference equations (11) to (19), the subscripts, i, a, e, refer to initial, average and end values respectively of the variables y, h, Q, H_n and q for time interval under consideration. Q_o, q_o, and H_o represent final steady conduit discharge, discharge through turbine and net head on turbine respectively.

The average height of the water level in the surge tank or the average discharge in the conduit during the time interval under consideration must first be estimated. It is preferable to estimate the average conduit discharge. A sufficiently accurate method for estimating the average conduit discharge assumes that the discharge at the beginning of interval is

increased by one half of the increment ΔQ, already found for the previous interval. For satisfactory accuracy of the solutions, more than a hundred time intervals in a quartercycle of a surge were utilized in the computer programs.

The numerical analyses for surge tank stability were carried out in dimensional form with the use of the dimensionless parameters ε and β of Vogt and for various values of relative rated head h_r, where

$$\beta = \frac{h_o}{H_G}$$

$$\varepsilon = \frac{L \cdot f \cdot V_o^2}{g \cdot F \cdot h_o^2}$$

$$h_r = \frac{H_r}{H_G}$$

To establish the criteria for critical oscillations, a value of β was selected on a trial and error basis to satisfy a given ε and H_r. From given values of ε and H_r and trial value of β and chosen constant values of H_G, f, L, and F, values for the variables h_o and V_o were calculated.

These values formed the input data for the computer program and were used to calculate one and one half cycles of the transient surge condition. The computer compared the first and second computer downsurge and automatically revised the trial value of β until both downsurges were of equal magnitude. The trial values of β were revised according to the following expression :

$$\beta_1 = \beta_o - \frac{D}{R}$$

where

β_o = first trial value of β

β_1 = second trial value of β

D = computed second maximum downsurge minus computed first maximum downsurge (see Figure 6a)

R = values varied from 100 to 1500, depending on the values of ε, for larger value of ε the greater value was used and vice versa.

For the given specified initial sudden increase in turbine discharge q and with the help of the procedure described above for estimating the average conduit discharge, and with the initial values of the primary variables H_n, and z, the differences of these variables can be determined from the above difference equations. The values of the primary variables at the end of the time interval are then obtained.

If $H_{na} > H_r$, the computer obeys the requirement of constant power output, that is, it solves equations (9), (10), and (11) simultaneously. On the other hand, the constant gate opening requirement is followed when $H_{na} < H_r$, i.e., the computer solves equations (9), (10), and (12) simultaneously.

Damped oscillation is described as "stable" and is represented by the decreasing spiral on Q-y plane shown as curve 1 in Figure 6b. Curve 2 represents the limiting condition when the surge is neither damped nor amplified but is indefinitely maintained. Oscillation in this case is referred to as "critical". Surge which increases in amplitude with time is represented by curve 3 as an increasing spiral and this oscillating condition is referred to as "unstable".

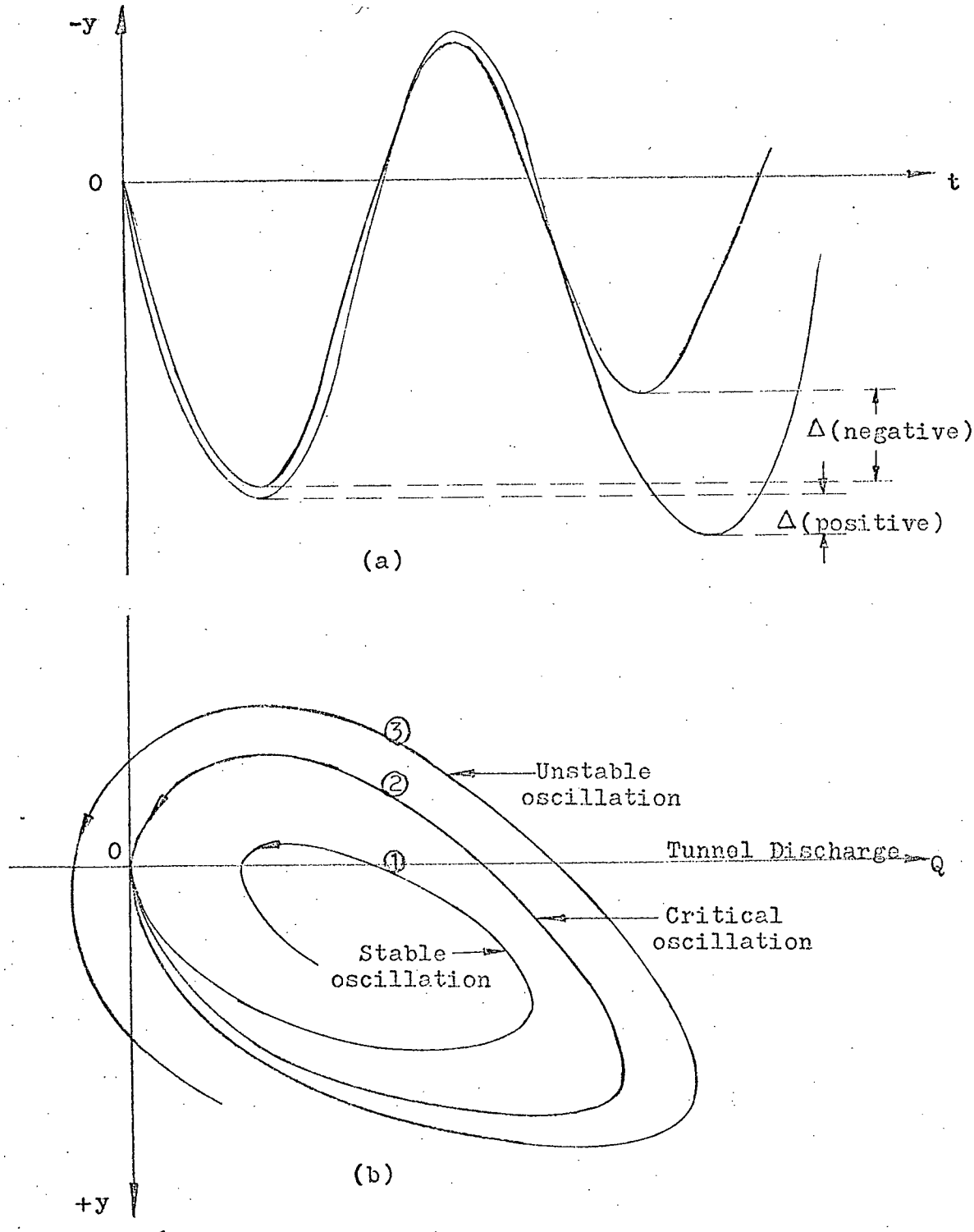

FIGURE 6 DETERMINATION OF STABLE, CRITICAL, AND UNSTABLE OSCILLATIONS

To obtain the critical condition for a surge tank, it is necessary that the first downsurge and second downsurge are equal, i.e., the oscillations have constant amplitude. An additional verification of the computer results was obtained by ensuring that the conduit discharge equals the initial conduit discharge, when the water elevation y in the tank, (z in computer programs) equals its initial steady state elevation. For example, in case of full-load demand, the critical oscillation is verified by ensuring Q equals zero when y = 0, as for curve 2 in Figure 6b, where the initial conduit discharge is zero.

For computer programs see Appendixes I(A), I(B), and I(C).

Figure 7 shows a plot of water surface elevation y with respect to conduit discharge Q for the case n = 0 as calculated by the computer. Curve A is based on constant power output only, while curve B is based on both constant power output and constant gate opening. Both curves for the same data are identical between the initial static head and the rated head (i.e., between a and b). Curves A and B between points b and c show the dampening effect of the constant gate output criterion.

3. Stability of Surges of Large Amplitude

The extent of the effect of changes in turbine discharge on the transient behaviour of a surge tank depends on the relative values of initial steady state turbine discharge and final steady state turbine discharge. These relative values are designated as "n" where $n = Q_{initial} / Q_{final}$, and range from 0 to 1.0. For example, full-load demand is represented by

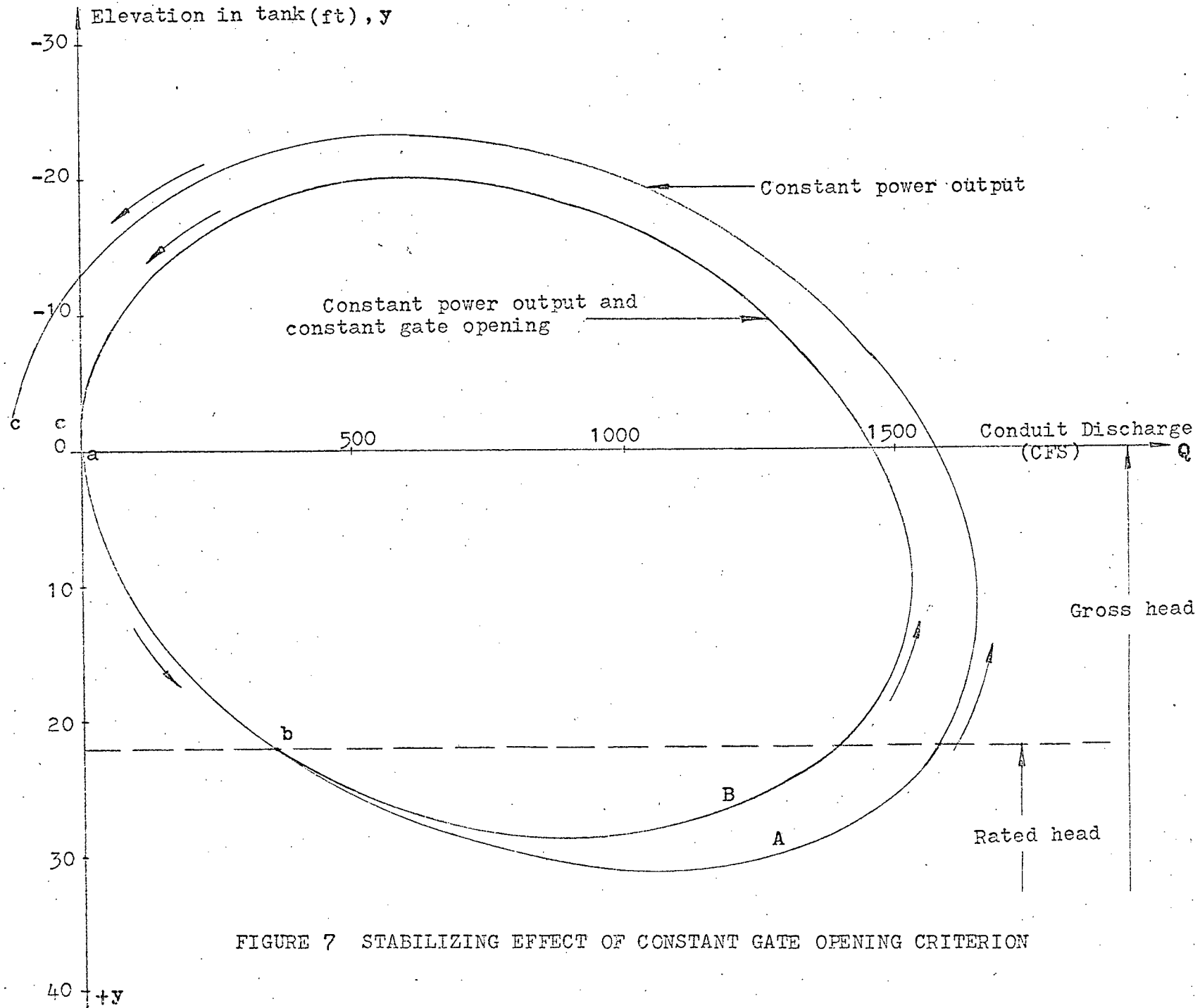

FIGURE 7 STABILIZING EFFECT OF CONSTANT GATE OPENING CRITERION

$n = 0$. Limiting oscillations for three different relative initial discharge $n = 0$, $n = 0.25$, and $n = 0.50$, and for more widely varying values of relative rated head were studied.

Results of analyses are shown on a dimensionless plane with ε as abscissa and β as ordinates and h_r as a parameter in Figures 8, 9, and 10 for the cases $n = 0$, $n = 0.25$, and $n = 0.50$ respectively. The stability criteria in this dimensionless $\varepsilon - \beta$ plane is divided into two main areas separated by an enveloping curve, or "curve of constant power output". Previous investigators, ignored the influence of constant gate opening on surge tank stability, and considered all oscillations in the region above the enveloping curve to be unstable.

In reality, whether the turbine accepts a load demand from zero to full load or from partial to full load, the turbine discharge can obviously not be increased further once the full-gate position has been reached, and therefore the generator can no longer maintain its rated output while the head on the turbine is less than the rated head. The effect of constant gate opening has to be introduced into the stability criteria for oscillations of large amplitude.

Hence the region above the enveloping curve is divided into two zones by the curves of parameter, h_r, as in Figures 8, 9, and 10. For the region to the left of the curve for a given rated head, the criteria of stable oscillations is still met.

In the zone to the right of the curve for a given rated head, oscillations are unstable.

In the region below the enveloping curve, oscillations of large amplitude are stable.

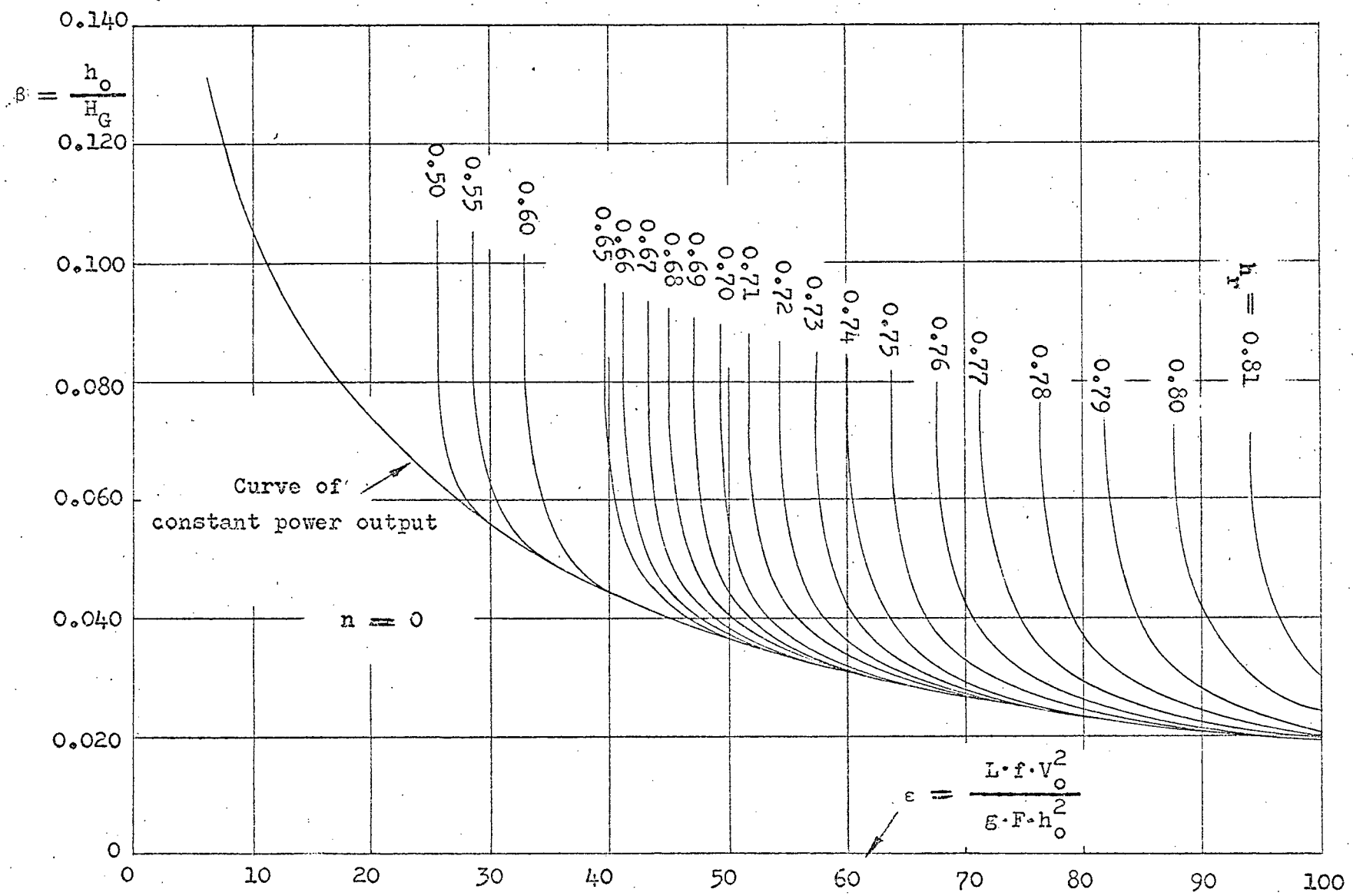

FIGURE 8 CRITERIA OF STABILITY FOR OSCILLATIONS OF LARGE AMPLITUDE (n = 0)

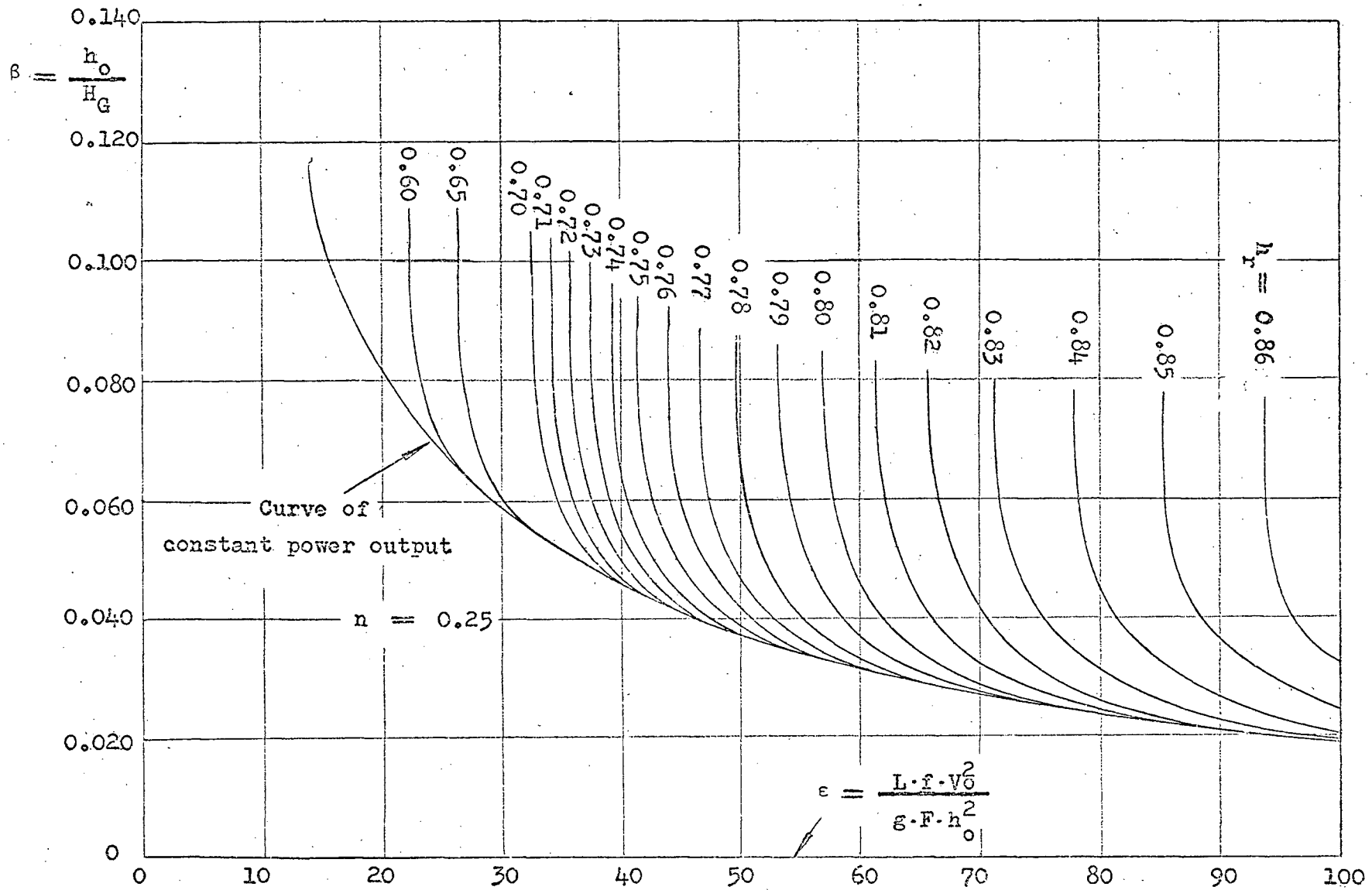

FIGURE 9 CRITERIA OF STABILITY FOR OSCILLATIONS OF LARGE AMPLITUDE (n = 0.25)

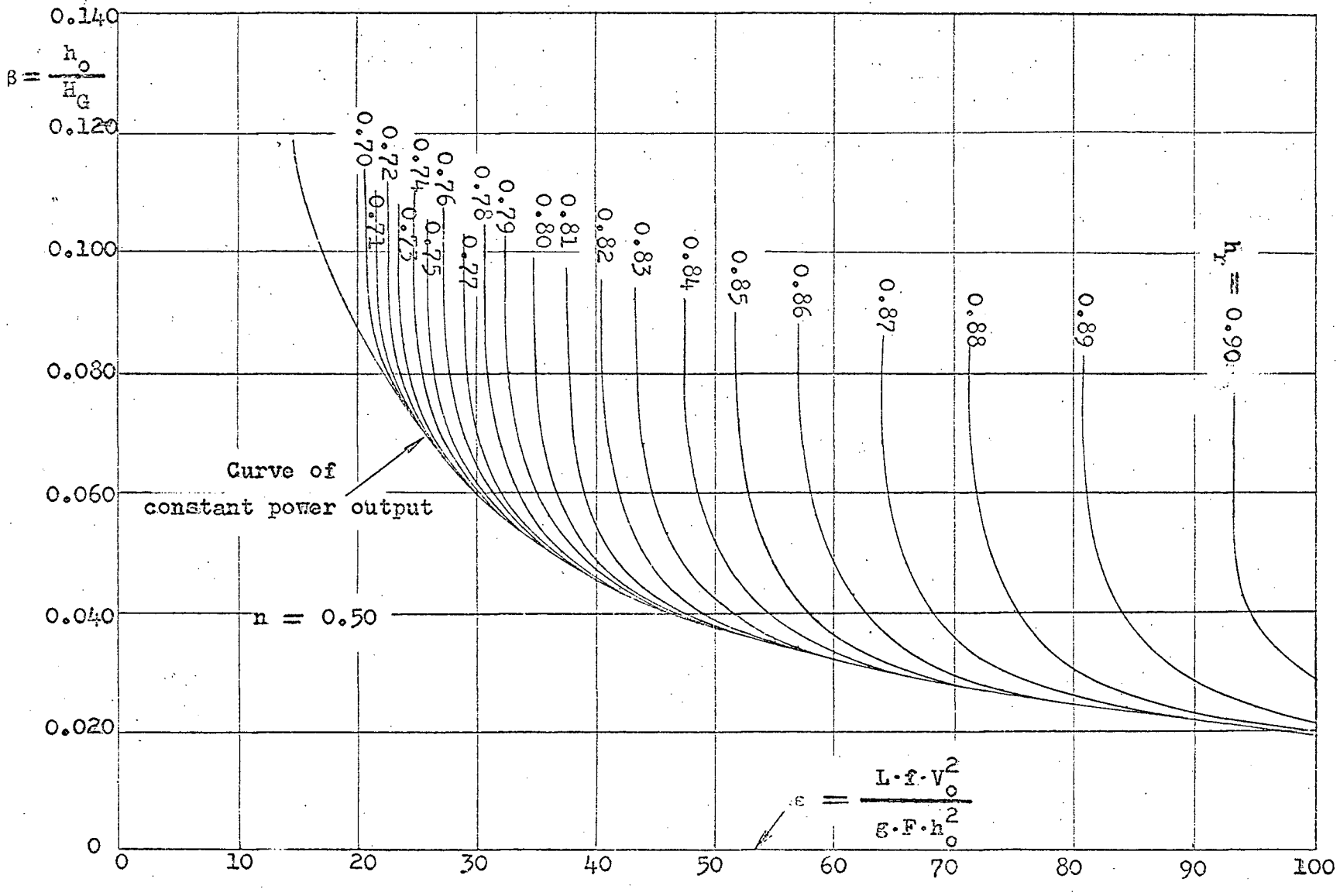

FIGURE 10 CRITERIA OF STABILITY FOR OSCILLATIONS OF LARGE AMPLITUDE (n = 0.50)

Figures 8, 9, and 10 show that all curves of parameter h_r are tangent to the enveloping curve. All points on the segment of enveloping curve to the right of the intersection of the particular h_r curve represent the condition of oscillations of constant amplitude with the whole range of head fluctuation above the rated head. This condition was studied by the previous investigators who considered that governor ensures only constant power output.

A comparison of the previous investigators' findings and the result of the study for n = 0 is given in the section 4 in this chapter.

Three different cases which were studied and plotted on a dimensionless plane are described as follows :

(A) Sudden load demand with the resulting discharge increase from zero to final steady state discharge Q_o (n = 0).

The stability criteria of the developed $\varepsilon - \beta - h_r$ relationship is illustrated in Figure 8. The relative rated head ranging from 0.50 to 0.80 is plotted on the figure as a parameter. The curves of limiting oscillations are tangent to the enveloping curve of constant power output.

(B) Sudden load demand with the resulting discharge increase from one-quarter discharge to final steady state discharge Q_o (n = 0.25)

Figure 9 shows a plot similar to that in Figure 8 with a slight change in the range of the rated head and enveloping curve.

(C) Sudden load demand with the resulting discharge increase from one-half discharge to final steady state discharge Q_o (n = 0.50)

The results are shown in Figure 10.

For cases other than the three indicated above, the method of interpolation can be used. Stability criteria for cases $n > 0.50$ can be interpolated between the enveloping curve in Figure 10 and Thoma's curve in Figure 11.

4. Limiting Curves Resulting from Constant Power Output

Introducing Vogt's parameters

$$\beta = \frac{h_o}{H_G} \qquad \text{and} \qquad \varepsilon = \frac{L \cdot f \cdot V_o^{\,2}}{g \cdot F \cdot h_o^{\,2}}$$

into Thoma's equation,

$$F_{Th} = \frac{V_o^{\,2}}{2g} \cdot \frac{L \cdot f}{H_o \cdot h_o}$$

we obtain

$$\varepsilon_{Th} = \frac{2(1 - \beta_{Th})}{\beta_{Th}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (20)$$

Jaeger* gives as the result of his work

$$\frac{\beta_J}{1 - \beta_J} + 0.482\sqrt{\varepsilon_J}\left(\frac{\beta_J}{1 - \beta_J}\right)^2 - \frac{2}{\varepsilon_J} = 0 \quad . \quad . \quad . \qquad (21)$$

In Figure 11, β_c # is the curve derived by the writer, using a digital computer. It is developed for the condition of constant power output and for the condition of oscillations of large amplitude in the case

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* see reference (3)

\# β_c is the enveloping curve in Figure 8 ($n = 0$)

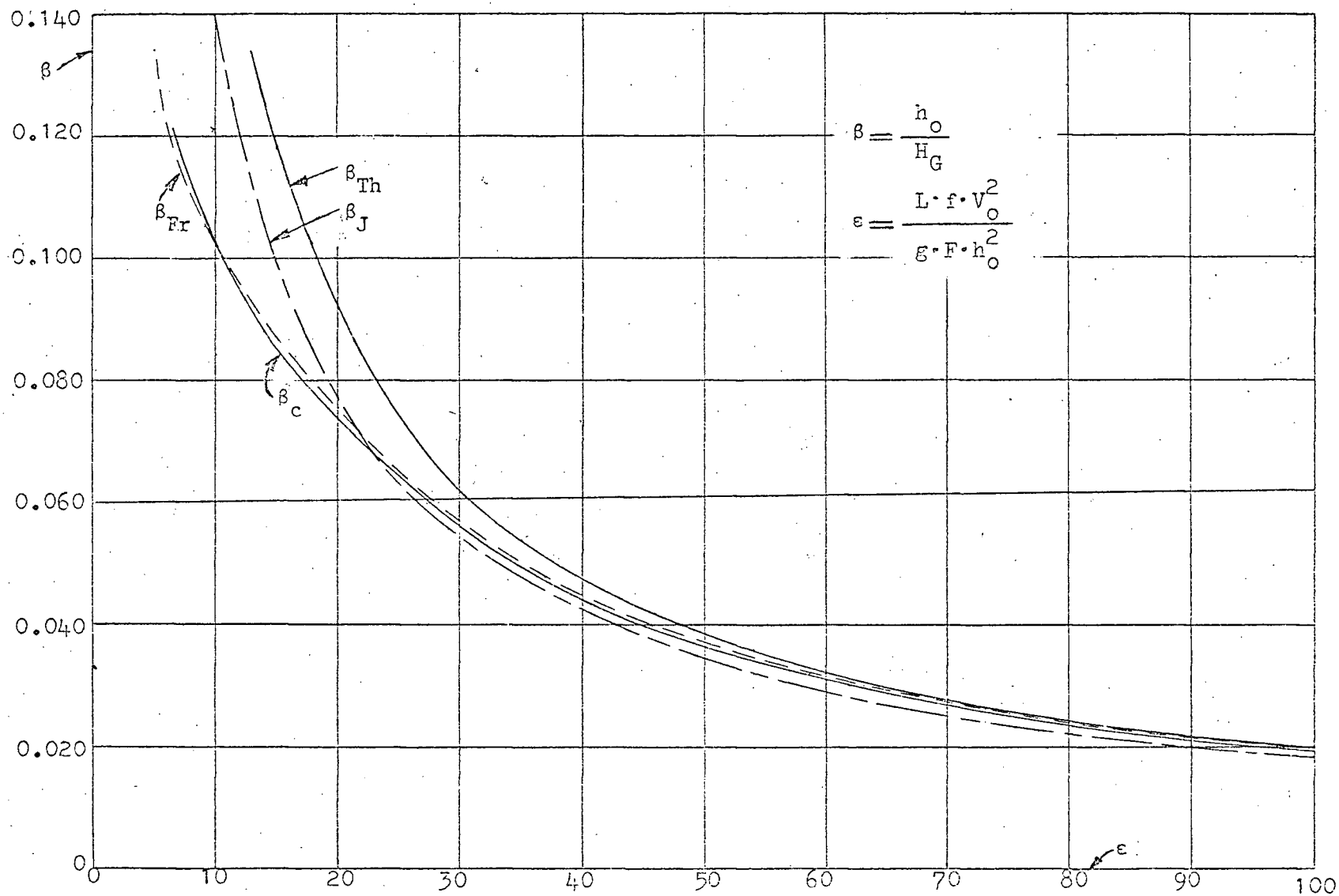

FIGURE 11 LIMITING CURVES OF CONSTANT POWER OUTPUT IN $\varepsilon - \beta$ PLANE

of sudden load acceptance from zero to full-load. This curve is compared with the curves of equations (20) and (21) above, together with the curve for Frank's β_{Fr}* which was obtained by direct graphical methods. All curves are plotted in Figure 11.

Values of Thoma's β_{Th}, of Frank's β_{Fr}, of Jaeger's β_J, and of writer's β_c developed by computer solution for n = 0 and for given ε are shown in Table I and II.

Table I

β_c for n = 0

ε	100	90	80	70	60	50	40	30	20	10
β_c	0.0191	0.0211	0.0236	0.0266	0.0308	0.0363	0.0442	0.0561	0.0741	0.1048

Table II

β_{Th}, β_{Fr}, and β_J for n = 0*

ε	100	50	40	30	20	10
β_{Th}	0.0196	0.0385	0.0476	0.0620	0.0910	0.1670
β_{Fr}	0.0196	0.0368	0.0440	0.0566	0.0750	0.1045
β_J	0.0181	0.0344	0.0424	0.0545	0.0776	0.1380

* see reference (3)

There is good agreement between Frank's and the writer's criteria for surge tank stability with the governor maintaining constant power output for oscillations of large amplitude (n = 0).

For further comparison with some other investigators' results, the above variables are further transformed by introducing the following dimensionless parameters :

$$X = \frac{1}{\sqrt{\varepsilon}} = \frac{h_o}{\sqrt{\frac{L \cdot f}{g \cdot F}} \cdot V_o}$$

$$Y = \frac{\beta}{1 - \beta} = \frac{h_o}{H_o}$$

Substitution of X and Y into equation (20) results in

$$Y = 2X^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and is shown as curve (1) in Figure 12.

Following some previous work by Paynter,* Marris* deals with the problem of the stability of oscillations of large amplitude in a way different from other investigators. His final formula is

$$X^2 = Y \left(\frac{(2 - Y) + \left[(2 - Y)^2 + 24Y \right]^{\frac{1}{2}}}{8} \right) \quad . \quad . \quad . \quad . \quad (23)$$

whereas Paynter's derivation yields

$$Y = 2(1 - Y)^2 \cdot X^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

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* see reference (2)

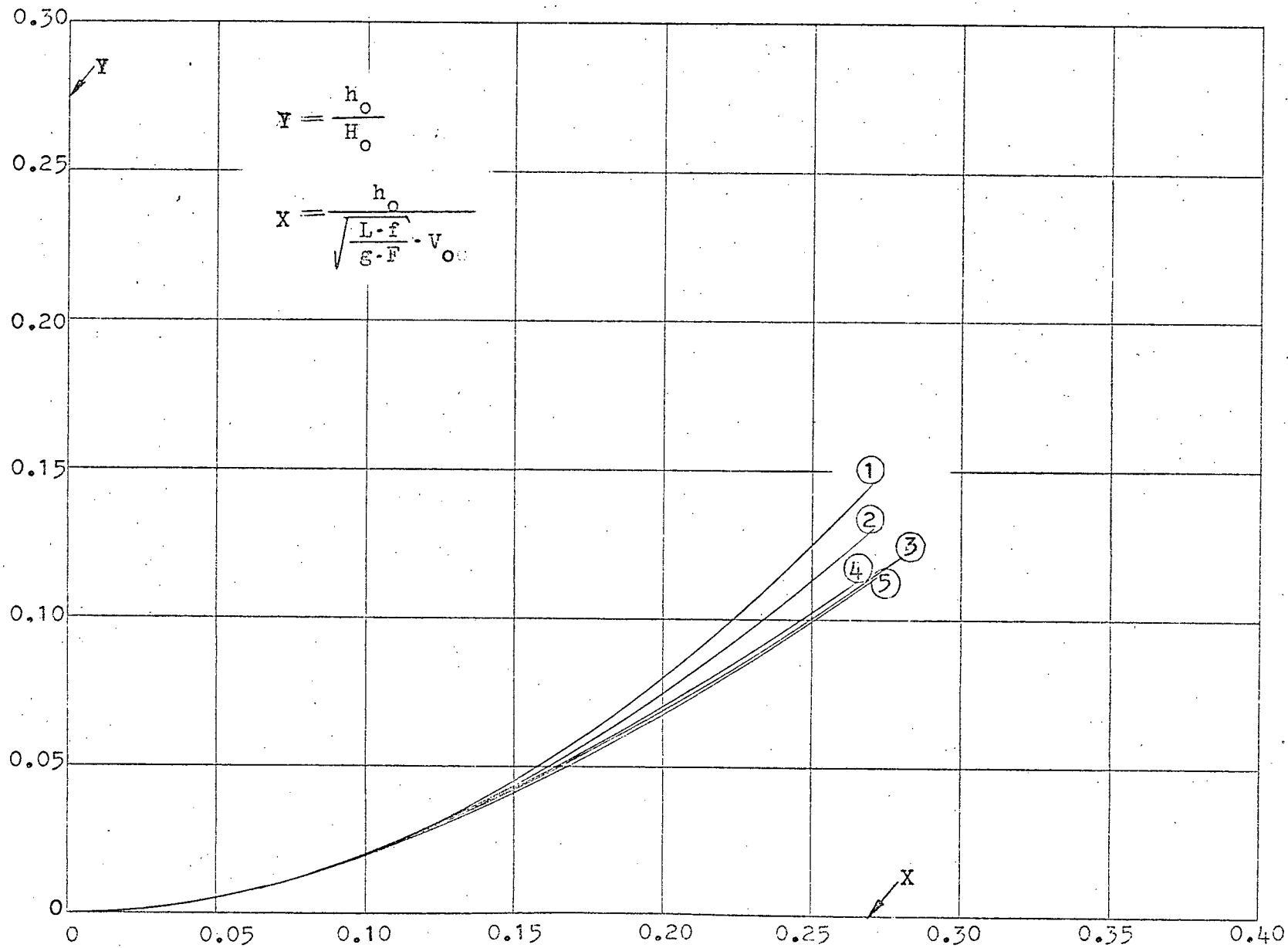

FIGURE 12 LIMITING CURVES OF CONSTANT POWER OUTPUT IN X - Y PLANE

Two curves (2) and (3), drawn on Figure 12 represent equations (23) and (24) respectively.

Jaeger's derivation, equation (21), can be rewritten

$$Y + 0.482 \frac{Y^2}{X} - \frac{2}{X^2} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

denoted by curve (4) in Figure 12.

The results for oscillations of large amplitude calculated with the computer program are drawn on Figure 12, as curve (5). The following equations were obtained by the least squares method of curve fitting.

$$\underline{Y = -0.013 + 0.232\ X + 0.886\ X^2} \quad . \quad . \quad . \quad . \quad . \quad (26)$$

or

$$\underline{Y = 0.003 + 1.66\ X^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

The curve fitting computer programs used are given in Appendix II.

To allow for oscillations of large amplitude, the Thoma's critical area must be increased. Formula (27) is the simplest and most direct one which can be used. As a modified Thoma's criterion for oscillations of large amplitude, equation (27) could be used for the correction of Thoma's criterion.

Transformation of equation (27) in dimensional form gives

$$\frac{h_o}{H_o} = 0.003 + 1.66 \frac{h_o^{\ 2}}{\frac{L \cdot f}{g \cdot F} \cdot V_o^{\ 2}} \quad . \quad . \quad . \quad . \quad . \quad (28)$$

from which we obtain

$$F = \frac{L \cdot f \cdot V_o^{\,2}}{1.66g \cdot h_o^{\,2}}\left(\frac{h_o}{H_o} - 0.003\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

It follows that

$$N = \frac{F}{F_{Th}} = 1.2\left[1 - 0.003\left(\frac{H_o}{h_o}\right)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

or

$$F = 1.2\left[1 - 0.003\left(\frac{H_o}{h_o}\right)\right]F_{Th} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

CHAPTER III CONCLUSIONS

The primary objective of this study was to demonstrate the influence of gate area restriction on the stability of oscillations of large amplitude of the simple surge tank. When the combined effects of constant power output and constant gate opening on turbine governing are considered, the rated head of the turbine is of great significance.

Previous investigators dealing with surge tank stability for oscillations of large amplitude assumed that the governor ensures constant power output. In reality, however, the governor can maintain a constant power output during surge conditions only when the whole range of head fluctuation is above the rated head of the turbine. Therefore, in general, the constant power output criterion is not a complete basis to analyse the problem of oscillations of large amplitude in a surge tank.

The portion of this investigation for oscillations of large amplitude based on the constant power output is in close agreement with the findings of several investigators, especially with the results of research by Frank who used a graphical method of solution.

Rated head has a stabilizing influence on oscillations in surge tank. Figures 8, 9, and 10 demonstrate this effect. Conditions corresponding to a point located above the enveloping curve, and to the left of the curve designating a given rated head, will lead to instability for the constant power output criterion; however for the constant power output and constant gate opening criterion combined, the resulting surge tank oscillations will be stable.

Figure 11 shows the comparison of the limiting curves for the constant power output obtained by previous investigators and by the writer. From this figure it can be concluded that Thoma's stability criterion must necessarily give instability for oscillations of large amplitude.

The writer, for the case of a surge tank subjected to oscillations of large amplitude, has derived the following expression for the case of constant power output, (equation (31) in previous chapter)

$$F = 1.2 \left[1 - 0.003\left(\frac{H_o}{h_o}\right) \right] \cdot F_{Th} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This equation gives the required minimum area for surge tank stability for $\varepsilon > 20$. For $\varepsilon < 20$ the writer suggests the equation

$$F = 1.2\ F_{Th} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In practice the turbine gates will move to the full open position when the head drops below the rated head under the surge conditions. During the constant gate opening condition, the oscillations are therefore reduced because of the limitation of gate opening on power output. If the range of head fluctuation for oscillations of large amplitude is above the rated head, the constant power output is maintained, the Figure 11 is then applicable for the case. The Figures 8, 9, and 10 are used to predict the surge tank stability for the case of oscillations of large amplitude with the range of operation including constant power output and constant gate opening.

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9. McCracken & Dorn. Numerical Methods and Fortran Programming, Wiley & Sons, New York, 1964.

APPENDIX I(A)

```
$FORTRAN
C
C  N = 0
C  PROGRAM OF SURGE ANALYSIS BY USING Q*H=CONST AND H/HR=(Q/Q0)**2
C
    6 READ(5,8)BETA, H0, HR, FACT, F, SF, CO, CE, K, I, CKT
    8 FORMAT(F8.4, 7F8.2, 2I4, F8.2)
      L = 1
      M = 1
      N = 0
      CKDEZ = 0.
   10 T = 0.
      QA = 0.
      ZE = 0.
      QE = 0.
      HN = 100.
      WRITE(6,12) BETA, K, HR
  120FORMAT(////30X, 7HBETA = , F8.4, 2X, 10HEPSILON = , I4, 2X, 5HHR =
     1 , F5.0, //12X, 4HTIME, 45X, 2HZE, 13X, 17HQA AT ZE APPRO. 0)
      V0 = CO * H0
      C = H0 / V0**2
      SQA = V0 * SF - 50.
      Q0 = V0 * SF
      Q0H0 = Q0 * (100. - H0)
      QR = Q0H0 / HR
      CONSTB = QR / SQRT(HR)
   15 DEZ = (SQA - QA) / (2. * F )
      ZAX = ZE + 0.5 * DEZ
      ZEX = ZE + DEZ
      HA = C * QA**2 / SF**2
      DEQ = CE * (ZAX - HA)
      HN = 100. - ZAX
      IF(HN .LT. HR) GO TO 20
      CONST = SQA * HN
      IF(ABS(CONST-Q0H0) .LT. 10.) GO TO 25
      SQA = Q0H0 / HN
      GO TO 15
   20 CONST = SQA / SQRT(HN)
      IF(ABS(CONST-CONSTB) .LT. 0.3) GO TO 25
      SQA = CONSTB * SQRT(HN)
      GO TO 15
   25 IF(DEZ .GT. 0.) GO TO 28
```

```
      IF(CKDEZ .LE. 0.) GO TO 28
      WRITE(6,45) T, ZEX
   45 FORMAT(10X, F8.2, 20X, F20.2)
      IF(M .GE. 2) GO TO 55
      M = M + 1
      CRZE = ZEX
      GO TO 50
   28 IF(T .LT. CKT) GO TO 50
      ZA = ZAX
      ZE = ZEX
      IF(ZE .LT. 0.) GO TO 50
      N = N + 1
      IF(N .GT. 1) GO TO 50
      WRITE(6,35) T, ZE, QA
   35 FORMAT(10X, F8.2, 20X, F20.2, 10X, F20.2)
   50 CKDEZ = DEZ
      ZA = ZAX
      ZE = ZEX
      QE = QE + DEQ
      QA = QE + 0.5 * DEQ
      SQA = (QOHO - 10000.) / (HN - DEZ)
      T = T + 0.5
      GO TO 15
   55 CKZE = ZEX -CRZE
      IF(ABS(CKZE) .LE. 0.02) GO TO 60
      BETA = BETA - CKZE / FACT
      HO = BETA * 100.
      IF(L .GT. 4) GO TO 60
      L = L + 1
      CKFACT = CKZE / FACT
      IF(ABS(CKFACT) .LT. 0.00002) GO TO 60
      M = 1
      N = 0
      CKDEZ = 0.
      GO TO 10
   60 IF(I .LT. 1) GO TO 6
      STOP
      END
$ENTRY


      DATA
```

APPENDIX I(B)

```
$FORTRAN
C
C   N = 0.25
C   PROGRAM OF SURGE ANALYSIS BY USING Q*H=CONST AND H/HR=(Q/Q0)**2
C
    6 READ(5,8)BETA, H0, HR, FACT, F, SF, CO, CE, K, I, CKT
    8 FORMAT(F8.4, 7F8.2, 2I4, F8.2)
      L = 1
      M = 1
      N = 0
      CKDEZ = 0.
   10 T = 0.
      HN = 100.
      WRITE(6,12) BETA, K, HR
  120FORMAT(////30X, 7HBETA = , F8.4, 2X, 10HEPSILON = , I4, 2X, 5HHR =
     1 , F5.0, //12X, 4HTIME, 45X, 2HZE, 13X, 17HQA AT ZE APPRO. 0)
      V0 = CO * H0
      C = H0 / V0**2
      V1 = V0 / 4.
      QA = V1 * SF
      ZE = C * V1**2
      QE = QA
      SQA = V0 * SF - 50.
      Q0 = V0 * SF
      Q0H0 = Q0 * (100. - H0)
      QR = Q0H0 / HR
      CONSTB = QR / SQRT(HR)
   15 DEZ = (SQA - QA) / (2. * F )
      ZAX = ZE + 0.5 * DEZ
      ZEX = ZE + DEZ
      HA = C * QA**2 / SF**2
      DEQ = CE * (ZAX - HA)
      HN = 100. - ZAX
      IF(HN .LT. HR) GO TO 20
      CONST = SQA * HN
      IF(ABS(CONST-Q0H0) .LT. 10.) GO TO 25
      SQA = Q0H0 / HN
      GO TO 15
   20 CONST = SQA / SQRT(HN)
      IF(ABS(CONST-CONSTB) .LT. 0.3) GO TO 25
      SQA = CONSTB * SQRT(HN)
      GO TO 15
```

```
   25 IF(DEZ .GT. 0.) GO TO 28
      IF(CKDEZ .LE. 0.) GO TO 28
      WRITE(6,45) T, ZEX
   45 FORMAT(10X, F8.2, 20X, F20.2)
      IF(M .GE. 2) GO TO 55
      M = M + 1
      CRZE = ZEX
      GO TO 50
   28 IF(T .LT. CKT) GO TO 50
      ZA = ZAX
      ZE = ZEX
      IF(ZE .LT. 0.) GO TO 50
      N = N + 1
      IF(N .GT. 1) GO TO 50
      WRITE(6,35) T, ZE, QA
   35 FORMAT(10X, F8.2, 20X, F20.2, 10X, F20.2)
   50 CKDEZ = DEZ
      ZA = ZAX
      ZE = ZEX
      QE = QE + DEQ
      QA = QE + 0.5 * DEQ
      SQA = (Q0H0 - 10000.) / (HN - DEZ)
      T = T + 0.5
      GO TO 15
   55 CKZE = ZEX -CRZE
      IF(ABS(CKZE) .LE. 0.02) GO TO 60
      BETA = BETA - CKZE / FACT
      H0 = BETA * 100.
      IF(L .GT. 4) GO TO 60
      L = L + 1
      CKFACT = CKZE / FACT
      IF(ABS(CKFACT) .LT. 0.00002) GO TO 60
      M = 1
      N = 0
      CKDEZ = 0.
      GO TO 10
   60 IF(I .LT. 1) GO TO 6
      STOP
      END
$ENTRY

      DATA
```

APPENDIX I(C)

```
$FORTRAN
C
C  N = 0.50
C  PROGRAM OF SURGE ANALYSIS BY USING Q*H=CONST AND H/HR=(Q/Q0)**2
C
    6 READ(5,8)BETA, H0, HR, FACT, F, SF, CO, CE, K, I, CKT
    8 FORMAT(F8.4, 7F8.2, 2I4, F8.2)
      L = 1
      M = 1
      N = 0
      CKDEZ = 0.
   10 T = 0.
      HN = 100.
      WRITE(6,12) BETA, K, HR
  120FORMAT(////30X, 7HBETA = , F8.4, 2X, 10HEPSILON = , I4, 2X, 5HHR =
     1 , F5.0, //12X, 4HTIME, 45X, 2HZE, 13X, 17HQA AT ZE APPRO. 0)
      V0 = CO * H0
      C = H0 / V0**2
      V1 = V0 / 2.
      QA = V1 * SF
      ZE = C * V1**2
      QE = QA
      SQA = V0 * SF - 50.
      Q0 = V0 * SF
      Q0H0 = Q0 * (100. - H0)
      QR = Q0H0 / HR
      CONSTB = QR / SQRT(HR)
   15 DEZ = (SQA - QA) / (2. * F )
      ZAX = ZE + 0.5 * DEZ
      ZEX = ZE + DEZ
      HA = C * QA**2 / SF**2
      DEQ = CE * (ZAX - HA)
      HN = 100. - ZAX
      IF(HN .LT. HR) GO TO 20
      CONST = SQA * HN
      IF(ABS(CONST-Q0H0) .LT. 10.) GO TO 25
      SQA = Q0H0 / HN
      GO TO 15
   20 CONST = SQA / SQRT(HN)
      IF(ABS(CONST-CONSTB) .LT. 0.3) GO TO 25
      SQA = CONSTB * SQRT(HN)
      GO TO 15
```

```
   25 IF(DEZ .GT. 0.) GO TO 28
      IF(CKDEZ .LE. 0.) GO TO 28
      WRITE(6,45) T, ZEX
   45 FORMAT(10X, F8.2, 20X, F20.2)
      IF(M .GE. 2) GO TO 55
      M = M + 1
      CRZE = ZEX
      GO TO 50
   28 IF(T .LT. CKT) GO TO 50
      ZA = ZAX
      ZE = ZEX
      IF(ZE .LT. 0.) GO TO 50
      N = N + 1
      IF(N .GT. 1) GO TO 50
      WRITE(6,35) T, ZE, QA
   35 FORMAT(10X, F8.2, 20X, F20.2, 10X, F20.2)
   50 CKDEZ = DEZ
      ZA = ZAX
      ZE = ZEX
      QE = QE + DEQ
      QA = QE + 0.5 * DEQ
      SQA = (Q0H0 - 10000.) / (HN - DEZ)
      T = T + 0.5
      GO TO 15
   55 CKZE = ZEX -CRZE
      IF(ABS(CKZE) .LE. 0.02) GO TO 60
      BETA = BETA - CKZE / FACT
      H0 = BETA * 100.
      IF(L .GT. 4) GO TO 60
      L = L + 1
      CKFACT = CKZE / FACT
      IF(ABS(CKFACT) .LT. 0.00002) GO TO 60
      M = 1
      N = 0
      CKDEZ = 0.
      GO TO 10
   60 IF(I .LT. 1) GO TO 6
      STOP
      END
$ENTRY

      DATA
```

APPENDIX II(A)

```
$FORTRAN
C
C   FINDIMG EXPERIMENTAL FORMULA FROM DATA BY USING LEAST SQUARES CURVE FITTING
C   Y = A + B*X + C*X**2
C
      DIMENSION X(200), Y(200),C(3,3), Z(3), P(4), D(3)
      WRITE(6,10)
   10 FORMAT(20X, 12HCOEFFICIENTS//)
      READ(5,15) M
   15 FORMAT(I2)
      DO 25 I = 1,200
      READ(5,20) X(I), Y(I)
   20 FORMAT(2F20.5)
      IF(X(I))25, 30, 25
   25 CONTINUE
      STOP
   30 NUMB = I-1
      MX2 = M * 2
      DO 35 I = 1,MX2
      P(I) = 0.0
      DO 35 J = 1,NUMB
   35 P(I) = P(I) + X(J)**I
      N = M + 1
      DO 50 I = 1,N
      DO 50 J = 1,N
      K = I + J - 2
      IF(K) 45, 45, 40
   40 C(I,J) = P(K)
      GO TO 50
   45 C(1,1) = NUMB
   50 CONTINUE
      D(1) = 0.0
      DO 55 I = 1,NUMB
   55 D(1) = D(1) + Y(I)
      DO 60 J = 2,N
      D(J) = 0.0
      DO 60 I = 1,NUMB
   60 D(J) = D(J) + Y(I) * X(I)**(J-1)
      CALL SOLVE(C, 3, D, Z)
   65 FORMAT(12X, 4HZ(1), 16X, 4HZ(2) // 2F20.5 ////)
      WRITE(6,65) (Z(I), I=1,3)
      STOP
```

```
      END
      SUBROUTINE SOLVE(A, N, B, T)
      DIMENSION A(3,3), B(3), T(3)
      NM = N - 1
      DO 85 K = 1,NM
      R = 1. / A(K,K)
      DO 75 J = K,N
   75 A(K,J) = R * A(K,J)
      B(K) = B(K) * R
      KP = K + 1
      DO 85 I = KP,N
      S = A(I,K)
      DO 80 J = K,N
   80 A(I,J) = A(I,J) - S * A(K,J)
   85 B(I) = B(I) - S * B(K)
      T(N) = B(N) / A(N,N)
      DO 99 K = 1,NM
      I = N - K
      KP = I + 1
      S = B(I)
      DO 95 J = KP,N
   95 S = S - A(I,J) * T(J)
   99 T(I) = S
      RETURN
      END
$ENTRY
 2
            0.10000            0.01950
            0.10530            0.02160
            0.11170            0.02410
            0.11540            0.02575
            0.11950            0.02730
            0.12400            0.02945
            0.14150            0.03770
            0.15800            0.04620
            0.18250            0.05950
            0.20000            0.06880
            0.00000
```

APPENDIX II(B)

```
$FORTRAN
C
C   FINDING EXPERIMENTAL FORMULA FROM DATA BY USING LEAST SQUARES CURVE FITTING
C   ASSUME Y=A + B*X**2
C
      DIMENSION X(200), Y(200), C(2,2), Z(2), P(4), D(2)
      WRITE(6,10)
   10 FORMAT(20X, 12HCOEFFICIENTS//)
      READ(5,15) M
   15 FORMAT(I2)
      DO 25 I = 1,200
      READ(5,20) X(I), Y(I)
   20 FORMAT(2F20.5)
      IF(X(I))25, 30, 25
   25 CONTINUE
      STOP
   30 NUMB = I-1
      MX2 = M * 2
      DO 35 I=2, MX2, 2
      P(I) = 0.0
      DO 35 J = 1,NUMB
   35 P(I) = P(I) + X(J)**I
      N = M
      DO 50 I = 1,N
      DO 50 J = 1,N
      K = I + J - 2
      L=2*K
      IF(L) 45, 45, 40
   40 C(I,J) = P(L)
      GO TO 50
   45 C(1,1) = NUMB
   50 CONTINUE
      D(1) = 0.0
      DO 55 I = 1,NUMB
   55 D(1) = D(1) + Y(I)
      D(2)=0.0
      DO 60 I = 1,NUMB
   60 D(2)=D(2) + Y(I) *X(I)**2
      CALL SOLVE(C, 2, D, Z)
   65 FORMAT(12X, 4HZ(1), 16X, 4HZ(2) // 2F20.5 ////)
      WRITE(6,65) (Z(I), I=1,2)
      STOP
      END
```

```
      SUBROUTINE SOLVE(A, N, B, T)
      DIMENSION A(2,2), B(2), T(2)
      NM = N - 1
      DO 85 K = 1,NM
      R = 1. / A(K,K)
      DO 75 J = K,N
   75 A(K,J) = R * A(K,J)
      B(K) = B(K) * R
      KP = K + 1
      DO 85 I = KP,N
      S = A(I,K)
      DO 80 J = K,N
   80 A(I,J) = A(I,J) - S * A(K,J)
   85 B(I) = B(I) - S * B(K)
      T(N) = B(N) / A(N,N)
      DO 99 K = 1,NM
      I = N - K
      KP = I + 1
      S = B(I)
      DO 95 J = KP,N
   95 S = S - A(I,J) * T(J)
   99 T(I) = S
      RETURN
      END
$ENTRY
  2
            0.10000          0.01950
            0.10530          0.02160
            0.11170          0.02410
            0.11540          0.02575
            0.11950          0.02730
            0.12400          0.02945
            0.14150          0.03770
            0.15800          0.04620
            0.18250          0.05950
            0.20000          0.06880
            0.00000
```