THE INFLUENCE OF IMPERMEABLE CORES ON THE SEISMIC BEHAVIOUR OF EARTH DAMS

by

NORMAN JOHN SERFF

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ABSTRACT

The influence of an impermeable clay core on the static and dynamic behaviour of an earth dam is investigated. The cores used are of two types, central core and upstream sloping core. Recommendations are made on the suitability of each type of core for dams in areas subject to seismic activity.

The finite element method of analysis is used and the material is assumed to behave in a viscoelastic manner.

The sloping core dam is found to be less desirable than the central core dam for earthquake regions because of the unfavourable stress distributions in the upper part of the dam. Static tensile stresses develop in this region, which do not occur in the central core dam, and the extent of these stresses is increased when the dynamic stresses due to the earthquake are superimposed. The accelerations, which increase with elevation in the dam, indicating the necessity of using a variable seismic coefficient, are higher in the sloping core dam than in the central core dam.

It is found that the first mode, the only mode that approximates a shear mode, contributes the major share to the dynamic response of the dam.

The finite element method is shown to be sensitive to irregularities in the subdivision of the dam into finite elements.

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CHAPTER 1

INTRODUCTION

This thesis presents the results of a comparative study of the behaviour of three types of earth dam when subjected to the ground accelerations of the El Centro earthquake (Calif. - 1940). The dams studied were a homogeneous dam, a central core dam and a sloping core dam.

An earth dam is a three-dimensional continuum, constructed of material which is generally unhomogeneous, anisotropic and having non-linear stress-strain relations. Because of the complexity of the problem of analyzing the stresses and deformations in such a structure, a number of simplifying assumptions have to be made. The first of these is that the dam can be represented by a cross-section normal to the axis of the dam, thus reducing the analysis to a two-dimensional plane strain problem. Secondly, it is assumed that soil is linearly elastic with viscous damping. In this manner the problem is reduced to one which can be solved by using a standard finite element analysis computer program.

The dam studied in this thesis was 300 feet high, symmetrical about the center-line, of side slopes 1 in 3 and either homogeneous or with a core. Two types of core, central and sloping, were included in the study, and the comparative behaviour, static and dynamic, which may be of use to the engineer in selecting a type of core, is presented in the following chapters. Figure 1 shows the dimensions and properties of the dam and core.

The Young's modulus E_d = 81,300 p.s.i., the Poisson ratio μ = 0.45 and the unit weight γ = 130 p.c.f. were used so that comparison with previous work done in this field could be made; also, the properties used are associated with a shear wave velocity of 1000 f.p.s. which is typical of the material used in the shell of earth dams. Two values of elastic modulus for the impervious core material were used, E_c = 40,650 p.s.i. ($E_c = \frac{1}{2} E_d$) and E_c = 8,130 p.s.i. ($E_c = \frac{1}{10} E_d$) which, it was felt, represented the extreme values likely to be encountered in practice. The higher value is typical of a stiff clay of high strength and the lower of a soft clay with good self-healing properties in the case of cracking due to earthquake stresses.

The problem of determining the stresses, strains and displacements in a structure such as an earth dam requires that equilibrium and compatibility be satisfied within the region and that the stress-strain relations of the material be known. An analytical solution, even with the simplifying assumptions mentioned previously, would be very complex and the finite element method of analysis was used.

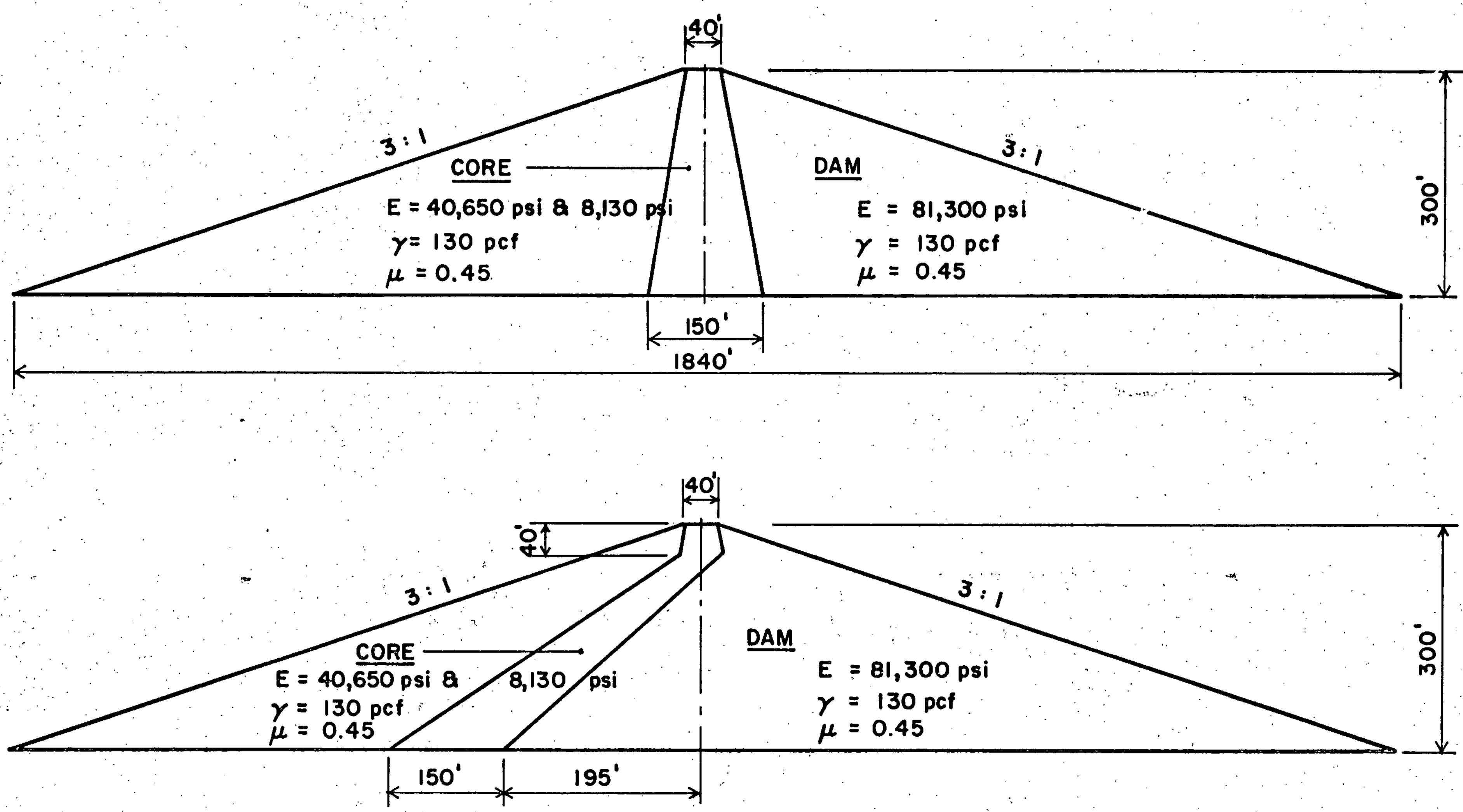

FIG. 1 DIMENSIONS AND PROPERTIES OF THE DAMS

This method of analysis has been described fully by a number of writers, e.g. Clough (1)* and a short description is included in Appendix I of this thesis. Briefly, the method consists of approximating the structure to be analyzed by an assemblage of elements and using an exact mathematical analysis of the approximation. Since the shape of the dam makes the use of triangular elements more convenient, this shape has been adopted here. The properties of the material used in the dam are retained in the individual elements and, since each element is defined separately, they may have different material properties, thus making possible the analysis of non-homogeneous systems. Fairly coarse networks can be used with good results, but, in areas of high stress gradients, the network should be finer. Computer storage limitations necessitate the use of a coarse subdivision except in areas of expected high stress gradients.

Once the behaviour of the dam under study has been determined, the static behaviour of geometrically similar dams can be determined by the use of appropriate scale factors. An increase in size will cause a proportional increase in stresses and strains and will cause an increase

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* Numbers in parentheses indicate reference number at the back of the thesis.

in displacements proportional to the square of the scale factor. Stresses in homogeneous structures are unaffected by changes in Young's modulus E, while strains and displacements vary in inverse proportion to E. Finally, strains and displacements all vary linearly with density.

Changes in the Poisson ratio cannot be accounted for by a scale factor, nor can a change in the side slopes; however, Clough and Woodward (2) have derived empirical relations which show how the stresses, strains and deformations vary with Poisson's ratio and the side slopes.

Transition zones and filters have not been included in the dam, though, in the case of earthquake design, Sherard (3) advocates the use of a wide transition zone of well-graded sand and gravel. The inclusion of such zones in the dam would present no difficulties in the finite element analysis.

The dams were subdivided into finite elements as shown in Figure 2. A finer subdivision was used in the region of the core in order to examine the effect of the core in modifying the stress patterns. The asymmetrical subdivision of the dam, which was necessary to include the cores, caused some inaccuracies in the results obtained. To examine the extent of these inaccuracies, a symmetrical subdivision was made of the case of the dam without a core and the errors due to asymmetrical subdivision were found to be small.

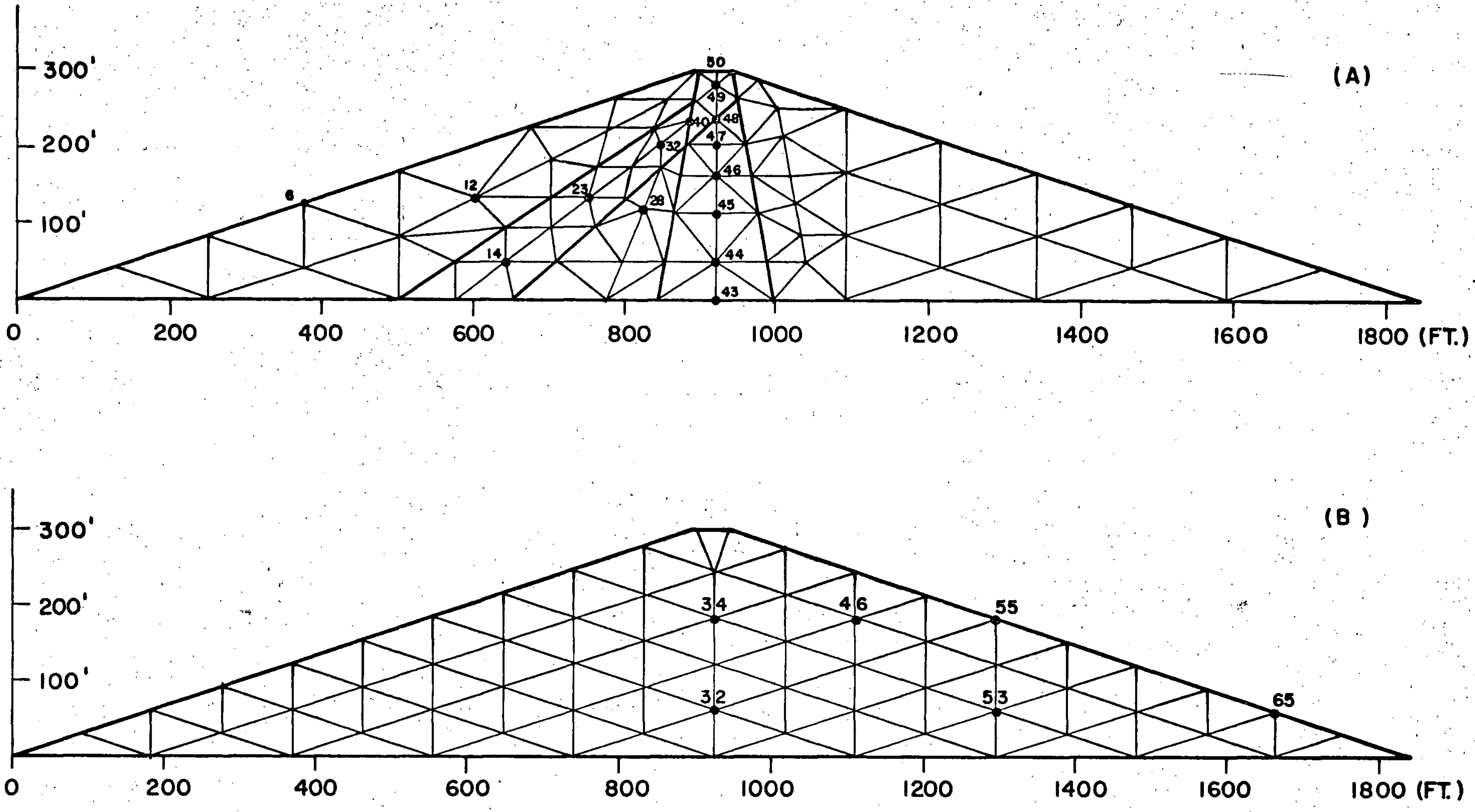


FIG. 2 DIVISION OF DAM INTO FINITE ELEMENTS SHOWING NODES USED IN PRESENTATION OF RESULTS
(A) DAM WITH CORE — CORE (CENTRAL OR SLOPING) SHOWN IN HEAVY OUTLINE
(B) DAM WITHOUT CORE (SYMMETRICAL SUBDIVISION)

The dam was divided into 129 elements with a total of 80 nodes. Nodes referred to in the discussion of results or in diagrams are shown numbered in Figure 2. Since the base of the dam was fixed, nodes in the base have no movement and the total number of degrees of freedom was 134, two for each node - horizontal and vertical. Gravity loading was introduced by lumping one-third of the weight of the surrounding elements at each node. Dynamic loading was applied to the dam by subjecting the base to the accelerations of the first 10 seconds of the North-South component of El Centro.

Chapter 2 presents the results of the static stress analysis. The normal stresses in the x-direction (horizontal) and the y-direction (vertical) together with the shear stress in the xy-plane were determined, along with the vertical and horizontal displacements due to the self-weight of the dam. These were determined for the three dams under study and modifications of the stress distribution and deformations due to the two types of cores were noted.

Chapter 3 deals with the dynamic behaviour of the dams. The modal frequencies and the mode shapes were first determined and the effects of the presence of the core was discussed. Next, the time histories of the stresses and accelerations at selected nodes in the dams were found with the purpose of determining how these were modified by the core. The power spectrum of the accelerations of El Centro was compared with those of the accelerations at the crest of the dams to determine the frequencies at which the energy of the earthquake was transmitted in the dams. Finally, the response of the dams in each mode was found and connections between the mode shape and the response in the mode were noted.

The last chapter presents the conclusions drawn from the study and suggestions for further research on this problem.

CHAPTER 2

STATIC STRESS ANALYSIS

2.1 Introduction

The determination of static stresses and displacements, due to the self-weight of the dam, is the initial step in the procedure of analyzing an earth dam subject to earthquake loading. The stresses and deformations due to the earthquake are then superimposed on the static stresses to give the stress distribution throughout the dam at any instant while the earthquake acts.

The gravity loading is applied by assuming one-third of the weight of the surrounding elements acts at each node. In the analysis herein it is assumed that the material of the dam is linearly elastic and isotropic and the effect of stored water is ignored.

In this analysis, the gravitational body forces are applied directly on the completed structure. In practice, the dam is built up by a succession of lifts. To check the validity of this practice, Clough and Woodward (2) investigated the effects on the stresses and displacement of direct application of gravity loading and compared them with those due to sequential loading. The dam used was 100 feet high and it was assumed to be built up of 10 layers of 10 feet in depth. There was very little difference in the stresses and the horizontal displacements between the two cases, though the vertical displacements obtained from direct load application were found to be inaccurate. In the case of direct loading, the maximum vertical deformation was at the top of the dam whereas, in the sequential loading case, the maximum vertical displacement occurred at the center of the dam and was approximately half the magnitude. It was, therefore, decided that the method of direct load application was adequate for this investigation.

It was found that the asymmetry of the irregular subdivision of the dam into finite elements necessitated by the sloping core caused some errors, though small, in the stresses and deformations obtained. Consequently, a symmetrical subdivision into finite elements was made of the dam for the homogeneous case to check this. Figure 2 shows the division into finite elements of the two dams.

2.2 Static Stresses

The stress distribution, horizontal normal stress and shear stress for the homogeneous dam, together with that of the sloping core and central core dams, expressed as a percentage of the stress in the homogeneous dam, is shown in Figures 3 and 4. Stresses are in pounds per square inch.

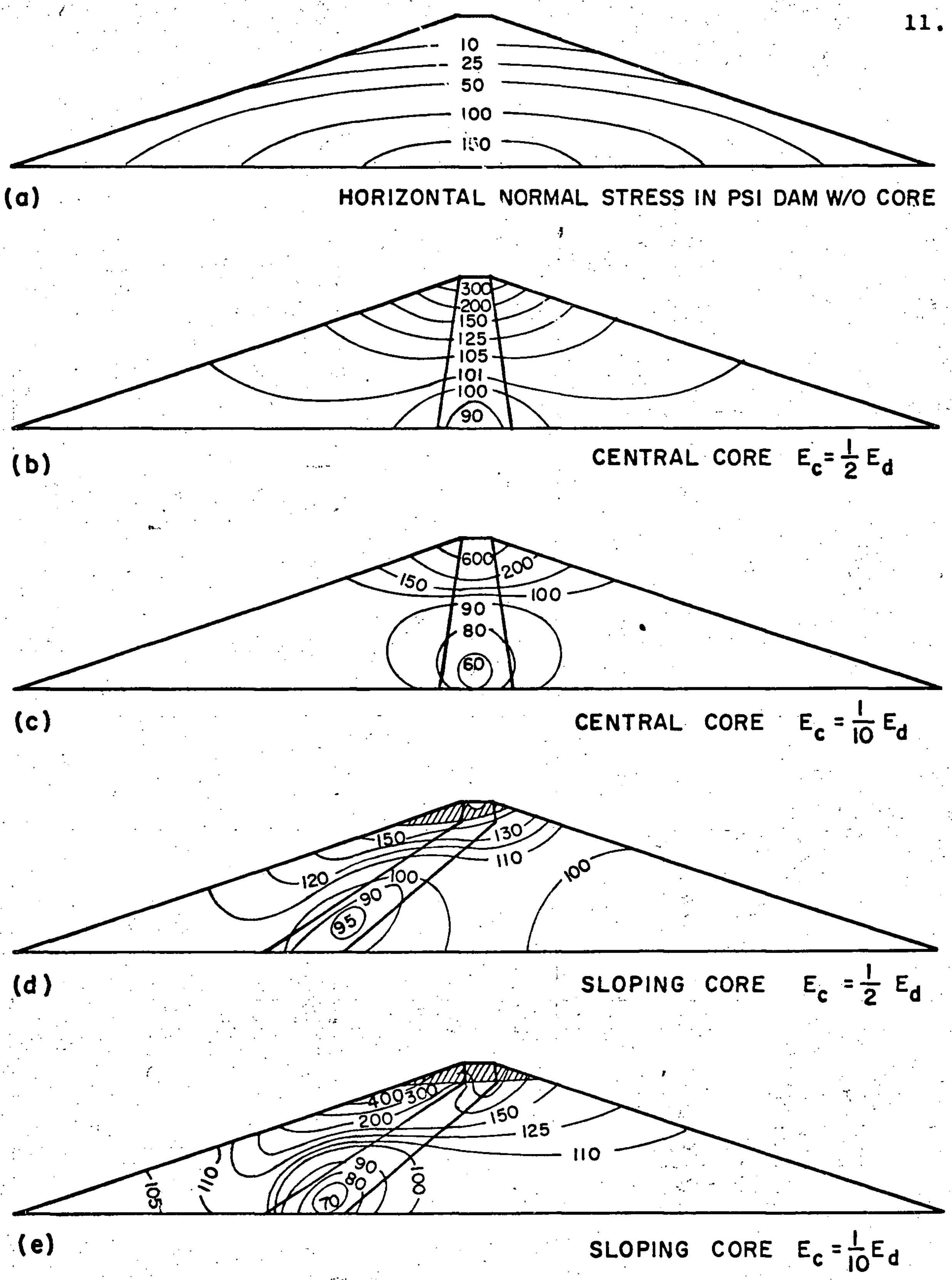


FIG. 3
HORIZONTAL STATIC STRESSES IN DAM WITH CORE EXPRESSED AS A PERCENTAGE OF THE STRESSES IN THE DAM WITHOUT A CORE (TOP DIAGRAM)

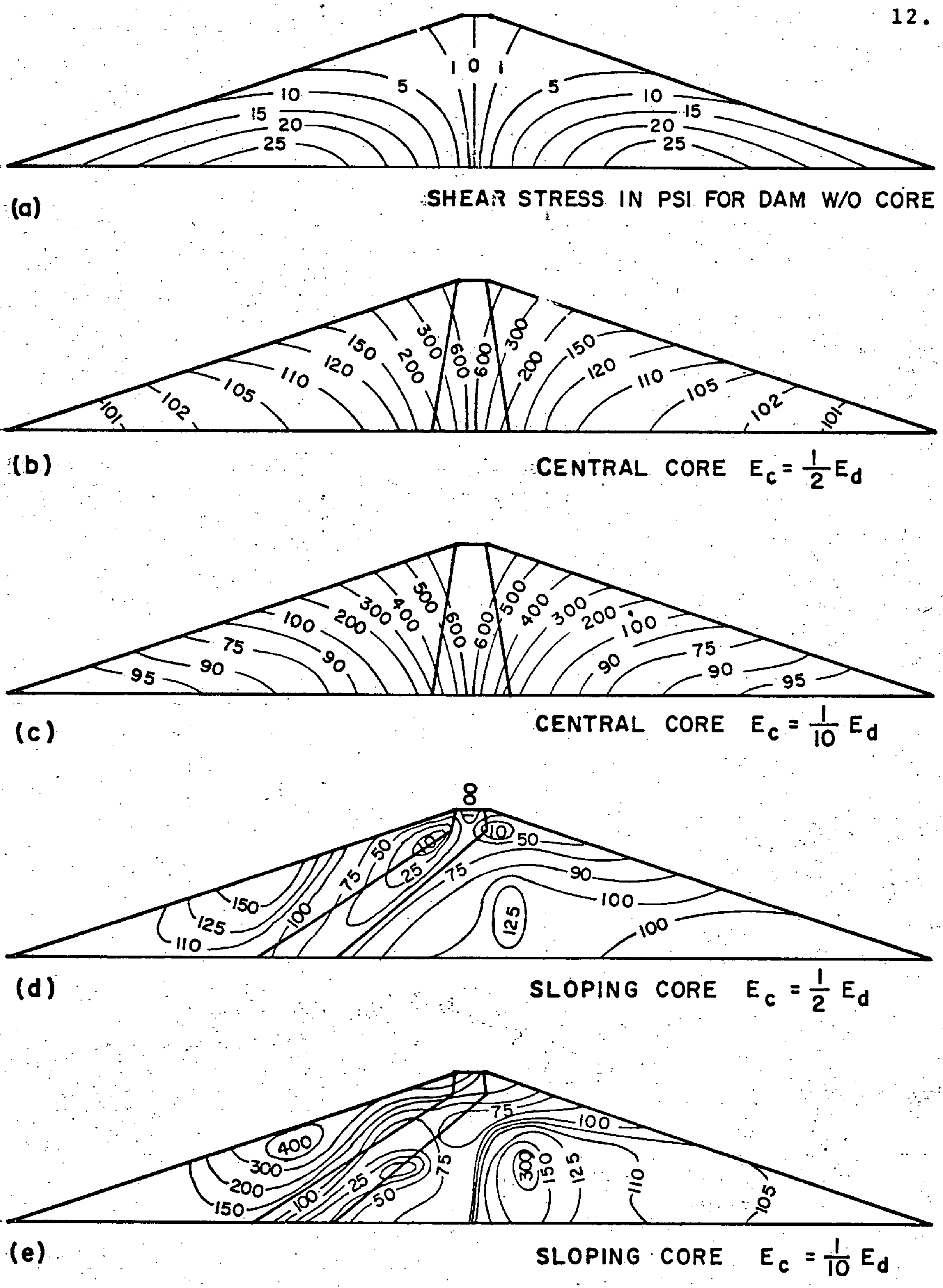


FIG. 4

STATIC SHEAR STRESSES IN DAM WITH CORE EXPRESSED AS A PERCENTAGE OF THE STRESSES IN THE DAM WITHOUT A CORE (TOP DIAGRAM)

Figure 3(a) shows the horizontal normal stresses for the homogeneous dam. The maximum stress is approximately 175 p.s.i. at the base of the dam, on the center line. In the central core dam, for the more rigid core, Figure 3(b), the changes in stress due to the core are confined mainly to the upper third of the dam and to the core itself. Stresses are increased by a maximum of 300 per cent near the crest and are reduced to about 90 per cent at the base of the core. Stress changes are small in the other parts of the embankment. For the dam with the more flexible central core, Figure 3(c), the pattern of stress change is similar though the variations are greater, ranging from 600 per cent at the crest to 60 per cent at the base of the core. Stresses are decreased slightly, about 5 - 10 per cent, in the lower two-thirds of the shells.

Changes in the horizontal normal stress in the sloping core dams are not as great as in the dam with a central core. For the dam with a more rigid sloping core, Figure 3(d), the stress changes are also greatest in the upper part of the dam. The stress decreases slightly at the base of the core and the changes are greater in the shell on the upstream side of the core. As the sloping core is made more flexible, Figure 3(e), the stress changes in the dam are greater and reach a maximum of 400 per cent towards the top of the upstream shoulder of the dam, and drop to

70 per cent at the base of the core. This high stress increase at the upstream shoulder of the sloping core dam, the part of the dam most liable to failure, makes this form of core less desirable than the central core.

In the upper zone of the homogeneous dam, from elevation 250 feet to the crest at 300 feet, there are some localized areas of horizontal tensile stresses. These areas of tension, which could cause a fissure to open and thus initiate a failure surface, do not exist in the central core dam, probably due to the inward and downward movement of the shoulder as the softer core is compressed, thus relieving the tensile stresses in the embankment. The development of horizontal tensile stresses is much more serious in the dam with a sloping core. These tensions, shown by the shaded area in Figure 3(d) and 3(e), have also been noted by Finn and Khanna (5). As the core is made more flexible, the tensile stresses increase and extend over a slightly larger area, reaching their maximum extent for a medium-soft core. Further softening of the core increases the tensile stresses slightly but causes them to become more localized.

During an earthquake, horizontal tensile stresses will be superimposed periodically on the existing tensions in the dam, giving a very unfavourable stress distribution in the top of the sloping core dam and making it more liable to failure than the dam with a central core - an important point to be considered in selecting a type of core for an earth dam. The softest core material, with its self-healing properties in the event of cracking, and the more localized area of tensile stresses, would be the better choice, if other considerations make it necessary to use a sloping core dam in an area subject to earthquakes.

The vertical stresses show little variation for the different dams shown here, being mainly dependent on the weight of the overlying material, and are not investigated further.

The static shear stress distribution in the xy-plane for the homogeneous dam is shown in Figure 4(a). The maximum shear stress of approximately 30 p.s.i. occurs on the base of the dam, midway between the center-line and the toe. The effect of a central core is to increase the shear stresses, Figure 4(b), the minimum increase being at the toe of the dam and becoming progressively larger toward the center, where the shear stresses are six times as great as in the homogeneous case. However, since the stresses in the homogeneous dam are at a minimum near the center, this large increase has little effect on the level of the shear stresses there. For the more flexible central core, Figure 4(c), the shear stresses are also unchanged at the toe. About one-third of the way in from the toe, the stresses decrease to about 75 per cent and increase again towards the center where the increase is higher than with the stiffer core.

For the sloping core dam, the pattern of stress change is more complicated. In the more rigid core, Figure 4(d), there is a general decrease in shear stress throughout the core, falling to about 10 per cent of the original stress towards the top. Stresses in the upstream side of the shell are increased slightly; in the downstream side they are generally lower than in the homogeneous dam. The stress change pattern is similar for the case of the soft core (Figure 4(e)), though the increase in shear stress in the upstream shell is higher, up to 400 per cent greater than the homogeneous case, and the shear stress in the top of the core also increases.

Thus, though the percentage increase in shear stress due to the presence of a core is higher for the central core dam, these large percentage increases occur in areas of very low shear stress and the resulting shear stresses are not high. The percentage increase in the sloping core dam, though not as great, occurs in the upstream shoulder, a region of high shear stress in the homogeneous dam, and also the most likely region for the initiation of a failure surface. This is especially so since the horizontal normal stress distribution is also most unfavourable in this region, with tensile stresses occurring toward the top of the shell. Hence, it would seem that, from static stress considerations, the central core dam is the safer type.

2.3 Static Displacements

A study of the displacements in the three dams shows that both the horizontal and vertical displacements are generally greater in the dam with a core. The horizontal displacements in the sloping core dam are very similar to those of the homogeneous dam in the area between the downstream face and the edge of the core. The difference is greatest along the center line of the core and decreases towards the upstream face of the dam. Maximum horizontal displacement for the homogeneous dam is 0.97 feet, occurring at the 150 foot level close to the face of the shell. The maximum for the sloping core dam is 1.1 feet at the same point in the dam. The differences in the vertical displacements follow a similar pattern. On the downstream side of the core, differences are negligible. Maximum differences occur at the upper edge of the core and increase with height in the dam. Maximum vertical displacement for the homogeneous dam is 2.3 feet, and 2.7 feet for the sloping core dam.

The horizontal displacements in the core of the central core dam are less than at corresponding points in the homogeneous dam. At a short distance from the core, the displacements are similar and, towards the face of the dam, are greater, though the difference decreases and is small at the face of the dam. Maximum displacement is 1 foot, occurring at the same point as in the homogeneous dam. The vertical displacements in the core and the surrounding shell are greater than at corresponding points in the homogeneous dam, though in the rest of the shell they are similar. Maximum vertical displacement is 3 feet at the crest.

Though the central core dam has smaller horizontal displacements than the sloping core dam, it is subject to greater vertical displacements. Neither type has any clear advantage where static deformation is concerned.

CHAPTER 3

DYNAMIC RESPONSE ANALYSIS

3.1 Introduction

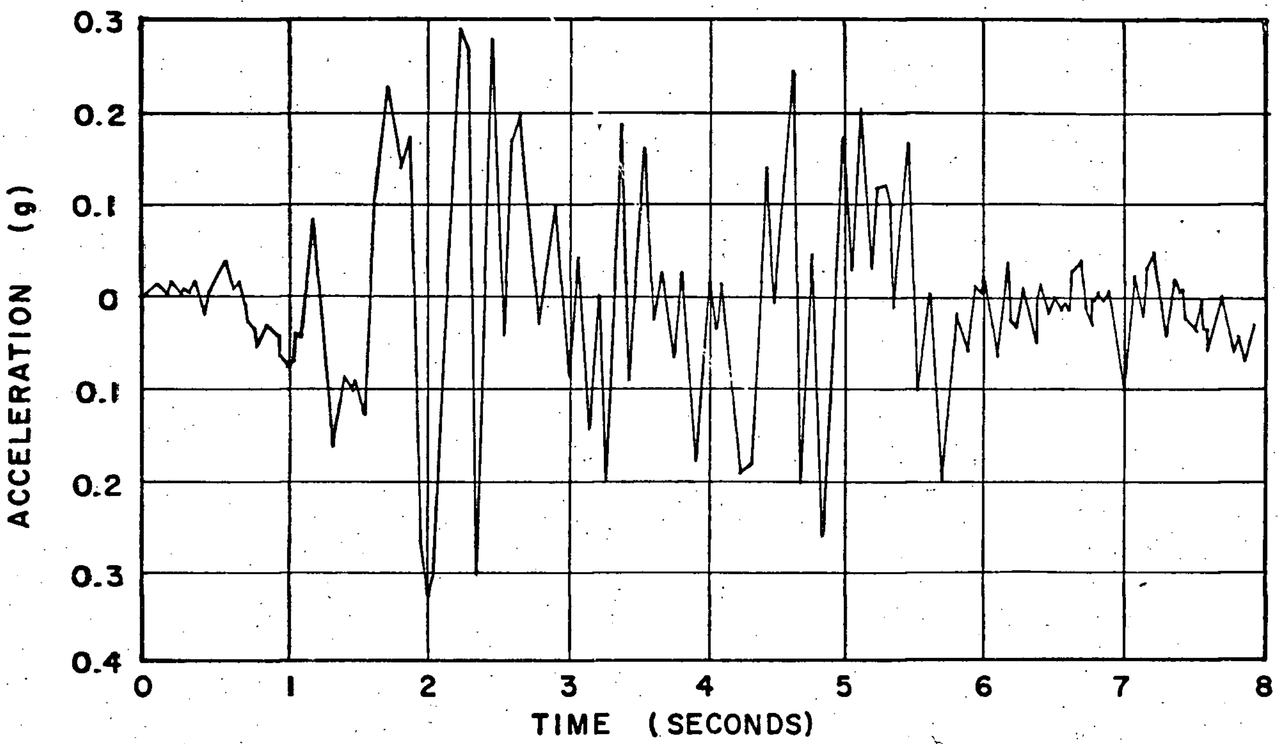
In the first investigations of the dynamic response of earth dams by Monobe (6), Hatanaka (7), Ambraseys (8) and others, the dam was represented by a wedge-shaped vertical shear beam. This method is restricted to homogeneous, symmetrical cross-sections and allows only one-dimensional displacements to occur. Ishizaki and Hatakeyama (9) have shown that the vertical shear beam approach is accurate only at the center-line of the dam. Errors occur everywhere else and reach a maximum at the face. Since the performance of a dam during an earthquake will be determined largely by the stress conditions at the face, it can be seen that the shear beam analysis may lead to significant errors in design.

The problem was next extended to allow two-dimensional displacements to occur, by Ishizaki and Hatakeyama (9), using the finite difference method. Difficulties are encountered in using this method when a non-homogeneous dam is studied, though this presents no problem in the finite element method used here. The finite element method was first applied to the seismic analysis of earth dams by Clough and Chopra (10) in 1966. This method takes into account non-homogeneity and simple cases of anisotropy, but is limited at present to linear elasticity for dynamic analysis.

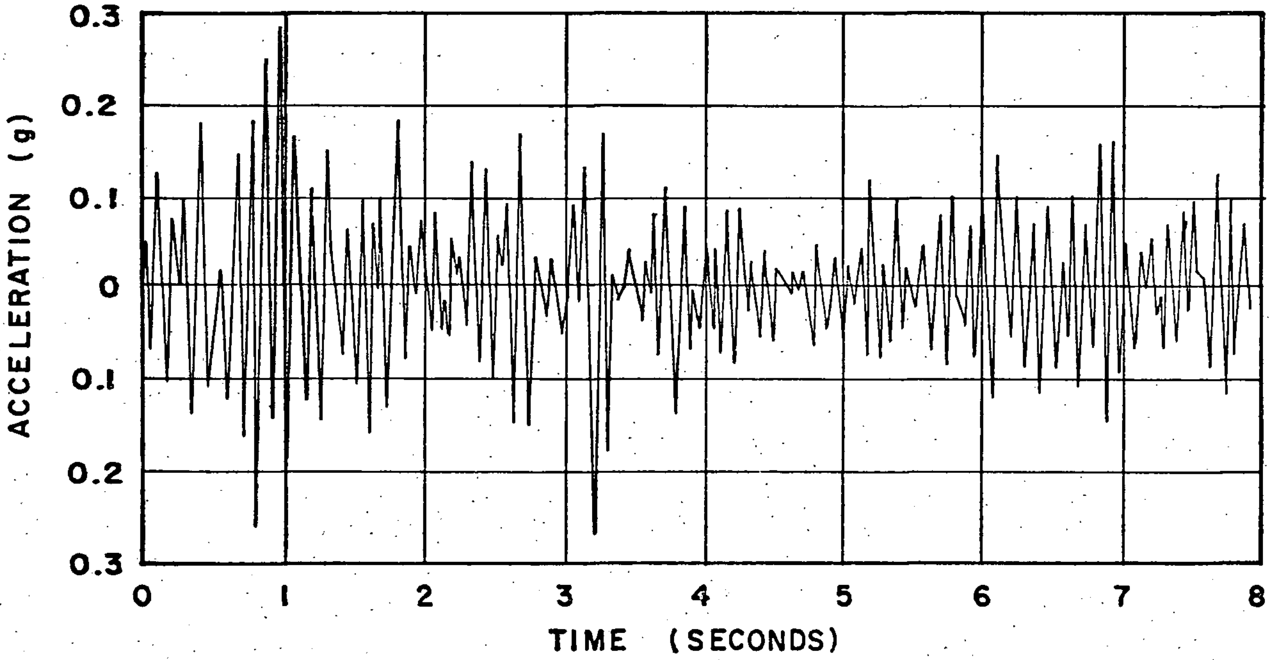
The disturbing force applied to the dam was the first 10 seconds of the North-South component of the El Centro earthquake, as shown in Figure 5(a). The accelerations of the earthquake were scaled to give a maximum value of 0.28g so that comparisons could be made with previous work done at the University of British Columbia by Finn (4) and Finn and Khanna (5). The vertical component of El Centro, which is not used in this analysis, is shown in Figure 5(b). A comparatively high value of damping of twenty per cent of critical in each mode was used to take into account the large absorption of energy due to inelastic deformation.

3.2 Mode Shapes and Frequencies

The dynamic response of a structure comprised of viscoelastic material to earthquake vibration can be analyzed as the summation of the responses in a number of individual modes. Thus, where a dam is disturbed from equilibrium by an earthquake, it can be considered as vibrating in its different modes simultaneously and the actual vibration may be obtained by superposition of the responses in the individual modes. The mode shapes and frequencies depend on the geometry and elastic properties of the material and not on the disturbing force. Thus the first step in the dynamic



(a) NORTH-SOUTH COMPONENT



(b) VERTICAL COMPONENT

FIG. 5 EL CENTRO EARTHQUAKE – CALIFORNIA – MAY 18, 1940 analysis is the determination of these mode shapes and frequencies.

The first ten mode shapes (scale exaggerated) for each dam are shown in Appendix II, Figures 22 to 30, and the natural periods for each are given. For the dams with cores, the natural period for the case of the more rigid core is given first, followed by that of the softer core. The mode shapes of the homogeneous dam, Figures 22 to 24, are of two basic types, symmetric about the center-line, and asymmetric. The symmetric modes, which create symmetric stress distributions, will not be excited by horizontal ground motion and this can be seen when the response of the dam is determined in each mode individually. Similarly, the asymmetric modes, which cause asymmetric stress distributions, will not be excited by vertical ground accelerations. This type of uncoupling exists for symmetrical cross-sections only and it can be seen from Figures 25 to 30 that the mode shapes for the dams with cores are all asymmetric. In general, any mode will be excited by horizontal and vertical ground accelerations.

It is interesting to note, in comparison with the shear wedge method which accounts only for shear stresses, that only the first mode represents pure shear distortion. Chopra (11) has shown that the difference in response between the center-line of the dam and the face is greater for flatter slopes, the results approaching the shear wedge solution as the slopes are made steeper. This difference is a maximum at about one-third the height of the dam. The mode shapes and modal frequencies do not depend on the side slopes of the dam when the shear wedge method is used, though they are very dependent on them in the finite element analysis.

The mode shape diagrams represent the deflected shape of the dam in that mode and can be used as maximum deflection diagrams by using the appropriate scale.

The modal period increases as the core is made more flexible, the difference between the more rigid and the more flexible core being about ten per cent. The fundamental period is lowest for the homogeneous dam, increases for the sloping core dam and is highest for the central core dam. The more rigid the dam, the lower the fundamental period will be.

3.3 Power Spectral Density Estimates

An earthquake causes accelerations at the base of a structure inducing inertia forces and consequent stresses and strains within the structure. These base accelerations, though of a random nature, can be considered as the superposition of a number of sinusoidal vibrations. A power spectral density analysis decomposes the accelerations into their basis frequencies and in this way a measure of the frequency at which most of the energy is being transmitted can be obtained.

Studies of damage caused by earthquakes to earth structures carried out by Ambraseys (8) indicate that most damage occurs when the fundamental frequency of the structure corresponds to the frequency at which most of the energy of the earthquake is being transmitted. Thus, the structure will resonate when subjected to certain ground motions.

Power spectral density analysis on a number of strong motion earthquakes indicate that most of the energy of the earthquakes is transmitted in a frequency range of 0.25 to 5 cycles per second. Therefore, structures with a fundamental frequency greater than about 10 cycles per second can be expected to respond in a rigid body manner and a dynamic analysis is not required. Structures with a fundamental frequency in a range of approximately 0.25 to 5 cycles per second can be expected to have a large response to strong motion earthquakes and a dynamic analysis is necessary.

The power spectral density (P.S.D.) analysis of the horizontal component of El Centro earthquake and of the accelerations at the crests of the three dams is shown in Figure 6. From Figure 6(a) it can be seen that El Centro is a high frequency earthquake, the energy transmitted being concentrated at a frequency of 2 cycles per second (c.p.s.). Since the fundamental frequency of the dam is approximately 1 c.p.s. for all cases, it can be inferred that the response to El Centro will be large. Figures 6(b), (c) and (d) show the P.S.D. of the crest accelerations of the homogeneous dam,

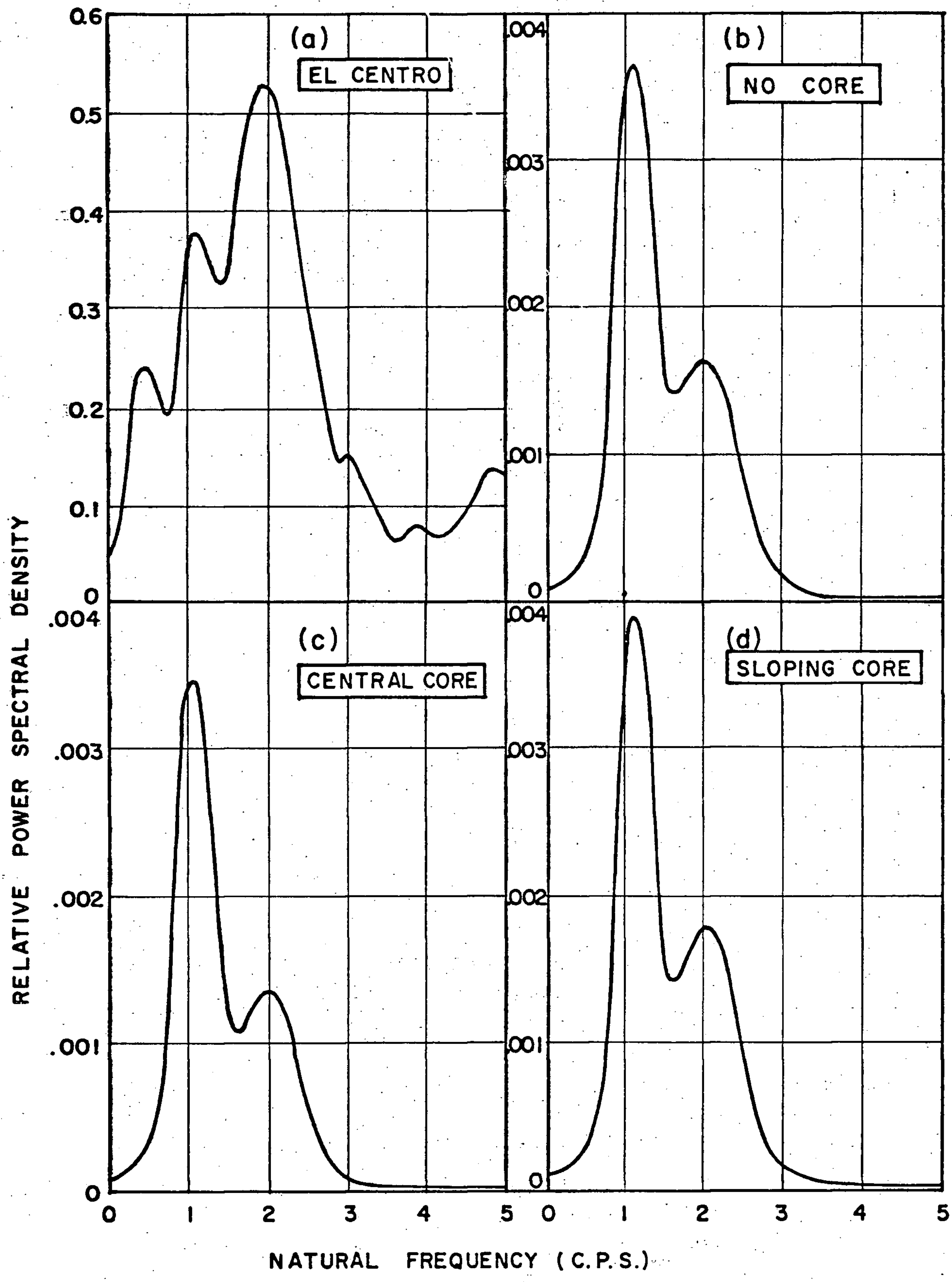


FIG. 6 COMPARISON OF P.S.D. OF EL CENTRO WITH P.S.D. OF ACCELERATION OF CREST OF DAM.

central core dam and sloping core dam respectively. There is a peak at 1 c.p.s. in each case, the fundamental frequency of the dam, showing that most of the response of the dam is in the first mode. This was confirmed, as will be seen later, when an analysis of the response in each separate mode was performed. The secondary peak at 2 c.p.s. corresponds quite closely to the natural frequency of the third mode for each dam.

3.4 Preliminary Investigation of Dynamic Response

An earthquake will apply both horizontal and vertical accelerations at the base of a structure. The dynamic analysis computer program used in the analysis could apply either a horizontal base acceleration alone, or vertical and horizontal base accelerations with the vertical acceleration equal to the horizontal. The program could have been modified to incorporate an independent vertical acceleration, but this was not done.

A check on the effect of vertical base acceleration was performed by comparing results using:

1) a horizontal base acceleration only

2) the same horizontal and vertical base accelerations.

The comparison is shown in Figure 7 where the horizontal normal stress, the shear stress in the x-y plane and the vertical normal stress were determined for a point near the

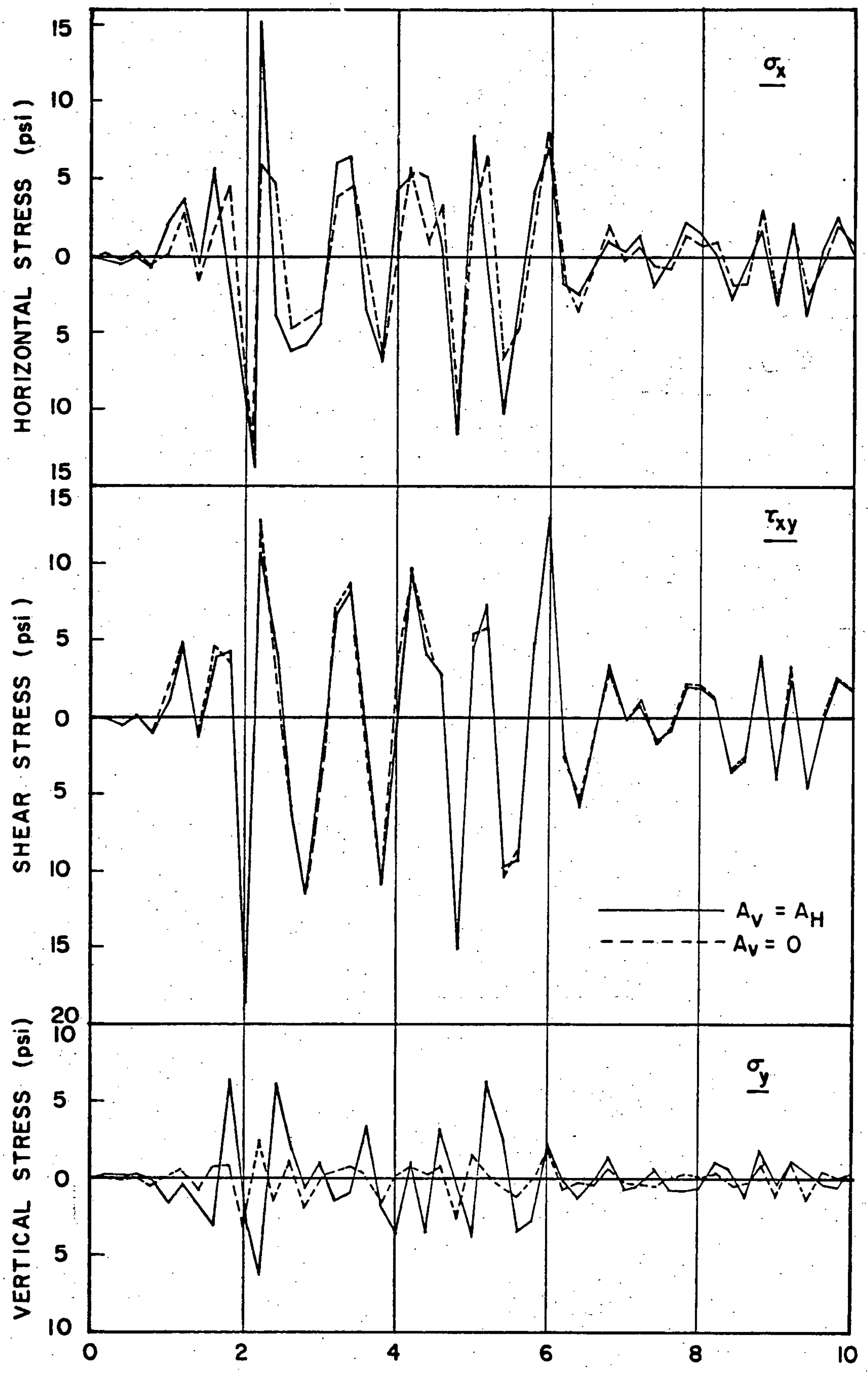


FIG. 7 DIFFERENCE IN DYNAMIC STRESSES DUE TO USE OF EITHER HORIZONTAL ACCELERATION ONLY OR EQUAL VERTICAL AND HORIZONTAL ACCELERATION (node 32 - dam without core)

center-line at approximately two-thirds the height of the dam. With the exception of the peak at 2.2 seconds, the horizontal normal stress was only slightly affected, the shear stress was affected even less, but the change in the vertical normal stress was in excess of 100 per cent at the peak values. The response of the normal stresses under vertical and horizontal acceleration is out of phase with that under horizontal acceleration only. The horizontal normal stress under combined acceleration components leads that under horizontal acceleration only, while the vertical noraml stresses lag. This method of applying vertical motion to the dam can give only an approximate indication of how the response will be affected by the actual vertical component of El Centro. It can be seen from Figure 5(b) that the vertical component of El Centro has a much higher frequency of vibration than the approximation used, though the actual value of the maximum acceleration is similar. Since the natural frequencies of the modes most affected by vertical motion, i.e. modes 2 and 4, are approximately equal to the frequency at which the horizontal component of El Centro transmits most of its energy, a resonance effect is to be expected when a vertical component of acceleration equal to the horizontal component of El Centro is used. Hence it was felt that the actual high frequency, vertical component would have less effect on the vertical stress than shown in Figure 7. Thus, it was felt that the use of the horizontal component only of El Centro gives rise to only slight error in the results. Idriss and Seed (15) have also shown that the vertical component of earthquake acceleration has little effect on the horizontal component of response of the dam or on the shear stress, but that it may influence the vertical response considerable.

From Figure 7, it can be seen that the peak in the horizontal stress at 2.2 seconds has been truncated for the case of no vertical acceleration component. This is because, due to time limitations on the computer, print-out of results was at 0.2 second intervals, though the response was calculated at 0.01 second intervals. To examine the extent of the error due to this limitation, the response at a point near the crest of the dam was printed out at the normal 0.2 second interval and at a 0.05 second interval. The result is shown in Figure 8 and it can be seen that the error in the response can be large. However, since this study is a comparative one, if the errors involved in the plotted response of each dam are similar, the conclusions should be valid.

Before the investigation could proceed further, it was necessary to decide on the number of modes which should be included in the integration to obtain the total response of the dam. Figure 9 shows a comparison of the horizontal normal stress at a point in the embankment using 5 modes and 15 modes in the integration. From this result, it was

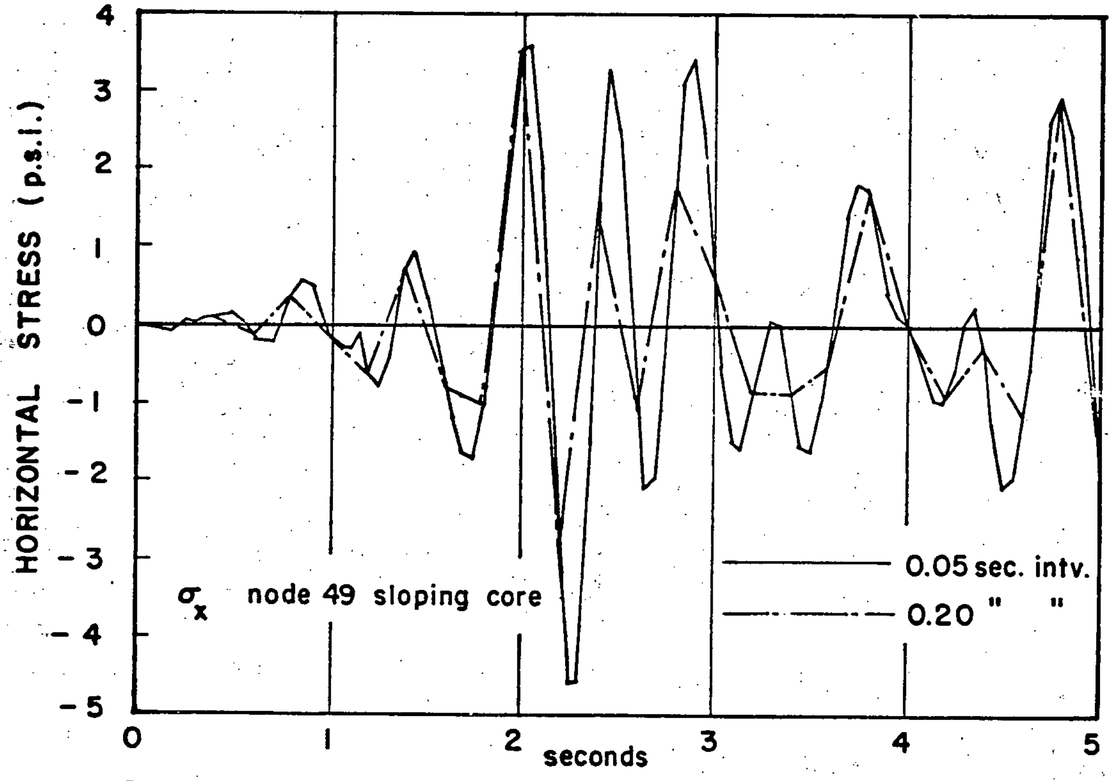


FIG. 8 DIFFERENCE IN RESPONSE DUE TO USE OF PRINTOUT INTERVALS OF 0.05 SECONDS AND 0.20 SECONDS

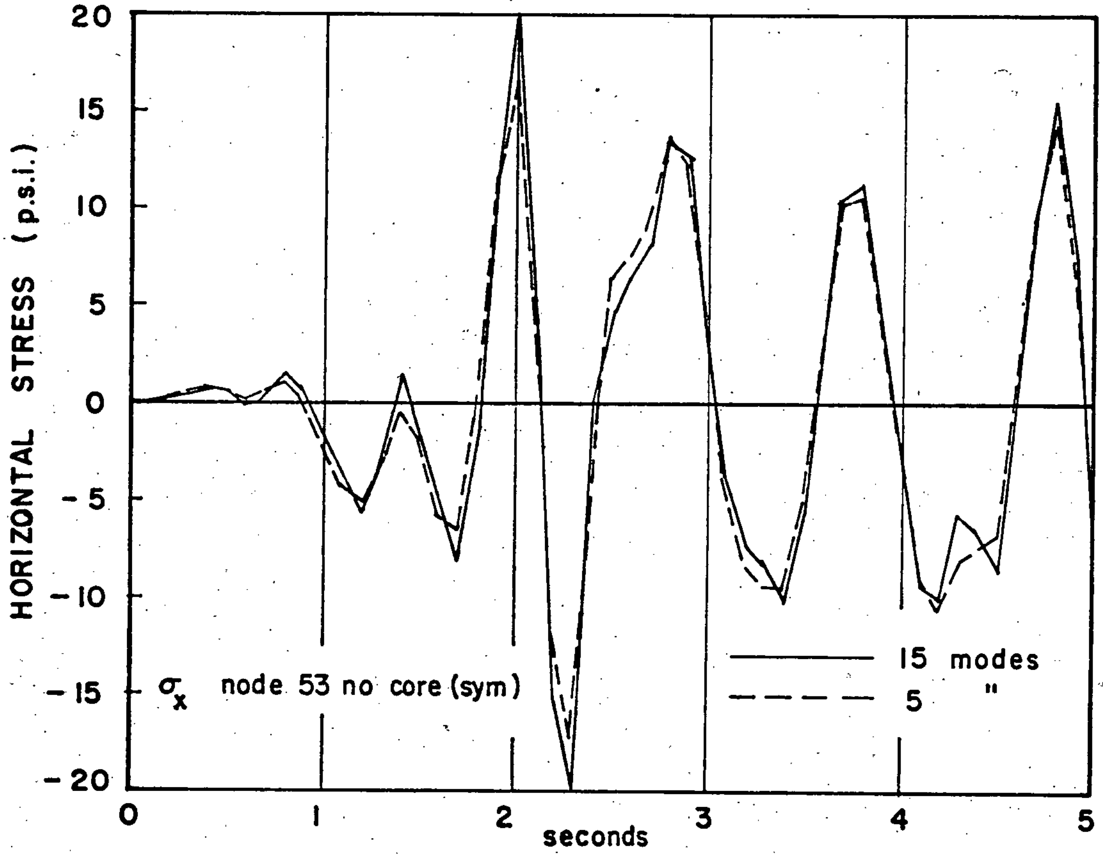


FIG. 9 DIFFERENCE IN RESPONSE BETWEEN USE OF 5 MODES AND 15 MODES

decided that, for economy of computer time, the use of the first 5 modes was sufficient.

3.5 Accelerations in Dam

The dynamic force induced in an element of a structure during an earthquake is the product of the mass times the absolute acceleration. If the acceleration history of the structure during the earthquake can be obtained, then the variation of dynamic forces with time can be determined. Thus, the structure will be subjected to a transient force system in addition to static forces. With known dynamic forces, Seed (12) has outlined a procedure for the design of an earth dam. Hence, the first step in this type of analysis is the determination of absolute accelerations.

The finite element method used in the dynamic analysis assumes that the material behaves in a viscoelastic manner. It is realized that plastic deformations will occur during strong motion earthquakes and to account for this a relatively high value of the percentage of critical damping was used.

Seismographs placed in dams show that, during earthquakes, the acceleration increases with height (Ambraseys (8)) and is in good agreement with viscoelastic analysis. Thus, it would appear that a viscoelastic response analysis provides a reasonable method for assessing the dynamic forces induced in an embankment by an earthquake. This increase with height is demonstrated in Figure 10 which compares the horizontal acceleration, in terms of g, of El Centro, at the base of the dam, with that of the crest. The acceleration peaks of the crest are nearly double those of the base, though they are generally out of phase with them.

Field tests on dams approximately 100 feet high, using forced vibration by large machines (Seed (13)) showed that peak accelerations are developed at certain characteristic frequencies, those of the free vibration modes (resonance effects), and also that the response was in agreement with that predicted by viscoelastic analysis.

The response of the dam under a different earthquake is shown in Figure 11 where the crest accelerations are compared for the El Centro and Alameda Park (Mexico) earthquakes. (Both earthquakes were scaled to give a maximum acceleration of 0.28g.) Alameda Park earthquake is a lower frequency earthquake than El Centro with a period of two seconds compared to El Centro's half second. Since the fundamental period of the dam is one second, resonance effects should be similar to those when El Centro acts, though peak accelerations will occur at different times. Figure 11 shows that the maximum accelerations are similar for both earthquakes, though the frequency of the response when the Alameda Park earthquake acts is slightly lower, and the maximum accelerations are reached later.

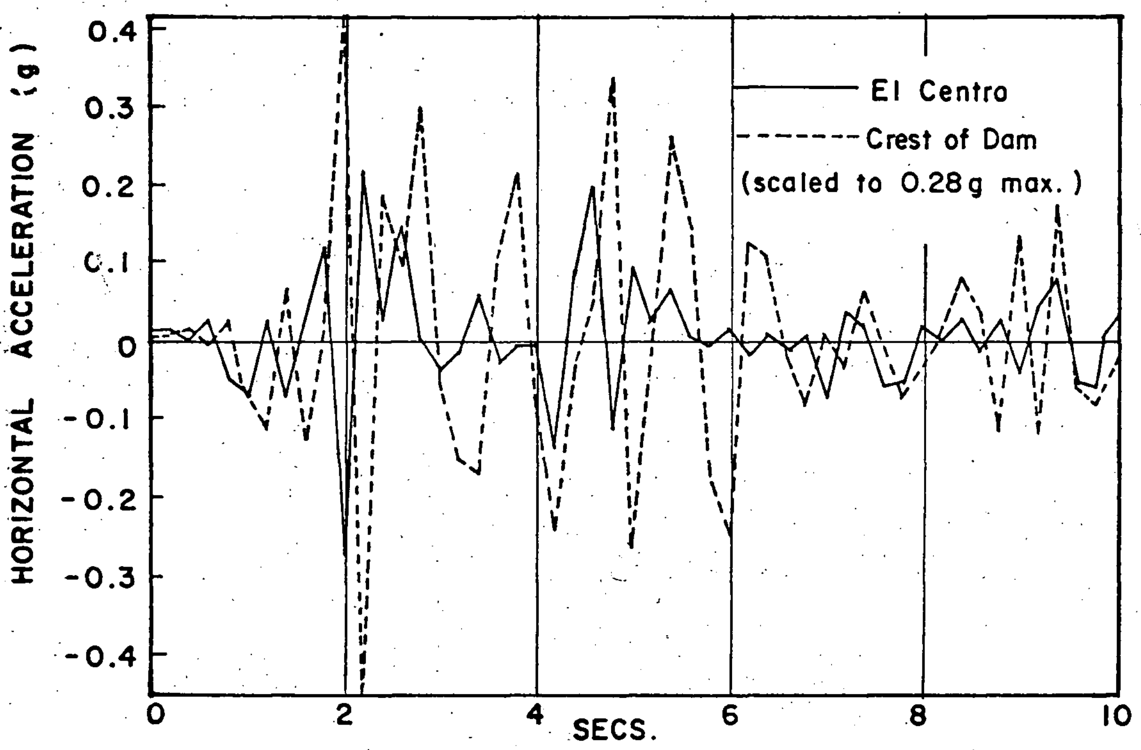


FIG. 10 COMPARISON OF ABSOLUTE HORIZONTAL ACCELERATION OF CREST OF DAM WITHOUT A CORE WITH HORIZONTAL COMPONENT OF ACCELERATION OF EL CENTRO

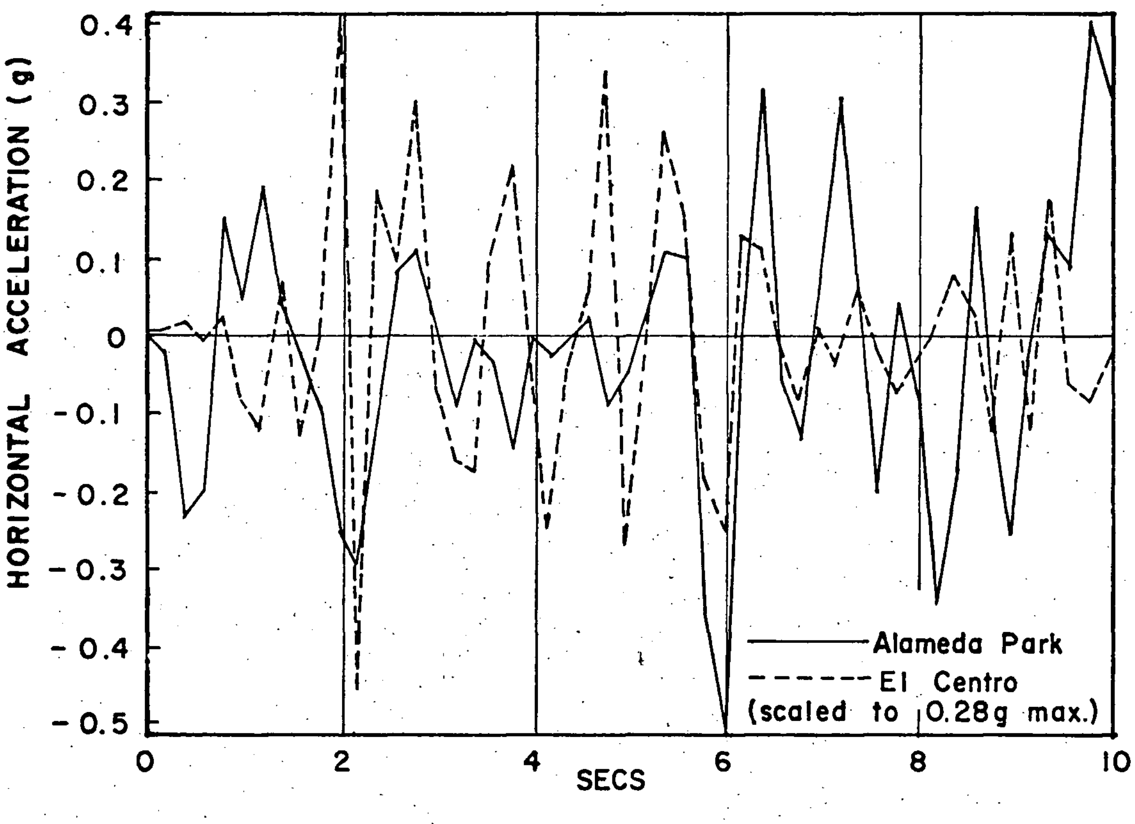


FIG. 11 COMPARISON OF ABSOLUTE HORIZONTAL ACCELERATION OF CREST OF DAM WITHOUT A CORE WHEN SUBJECTED TO EL CENTRO AND ALAMEDA PARK EARTHQUAKES

The variation of response with height within the dam is shown in Figure 12, where the accelerations at various heights along the center-line of the homogeneous dam are plotted. The response at node 43 at the base of the dam is the horizontal component of El Centro, since the foundation is rigid, and that at node 50 is the response at the crest, as shown in Figure 10. The acceleration, in the early stages of vibration, decreases to a minimum at a height of approximately 50 feet above the base and then increases with height, reaching a maximum at the crest. However, after the first two seconds, there is a gradual increase in response from base to crest. The oscillations at the various heights are generally out of phase and, at times, act in opposite directions.

The effect of the central core is to decrease accelerations at points within the core by 10 per cent for the more rigid core and by 30 per cent for the more flexible core. In the sloping core, the approximate average decrease in acceleration is 5 per cent for the more rigid core and 15 per cent for the more flexible core. Accelerations at points outside the cores are affected only very slightly.

The usual current practice in the seismic design of earth dams is to compute the factor of safety along a potential failure surface when a static horizontal force, acting on the sliding block, is included in the analysis. The problem is treated as a static analysis and the horizontal

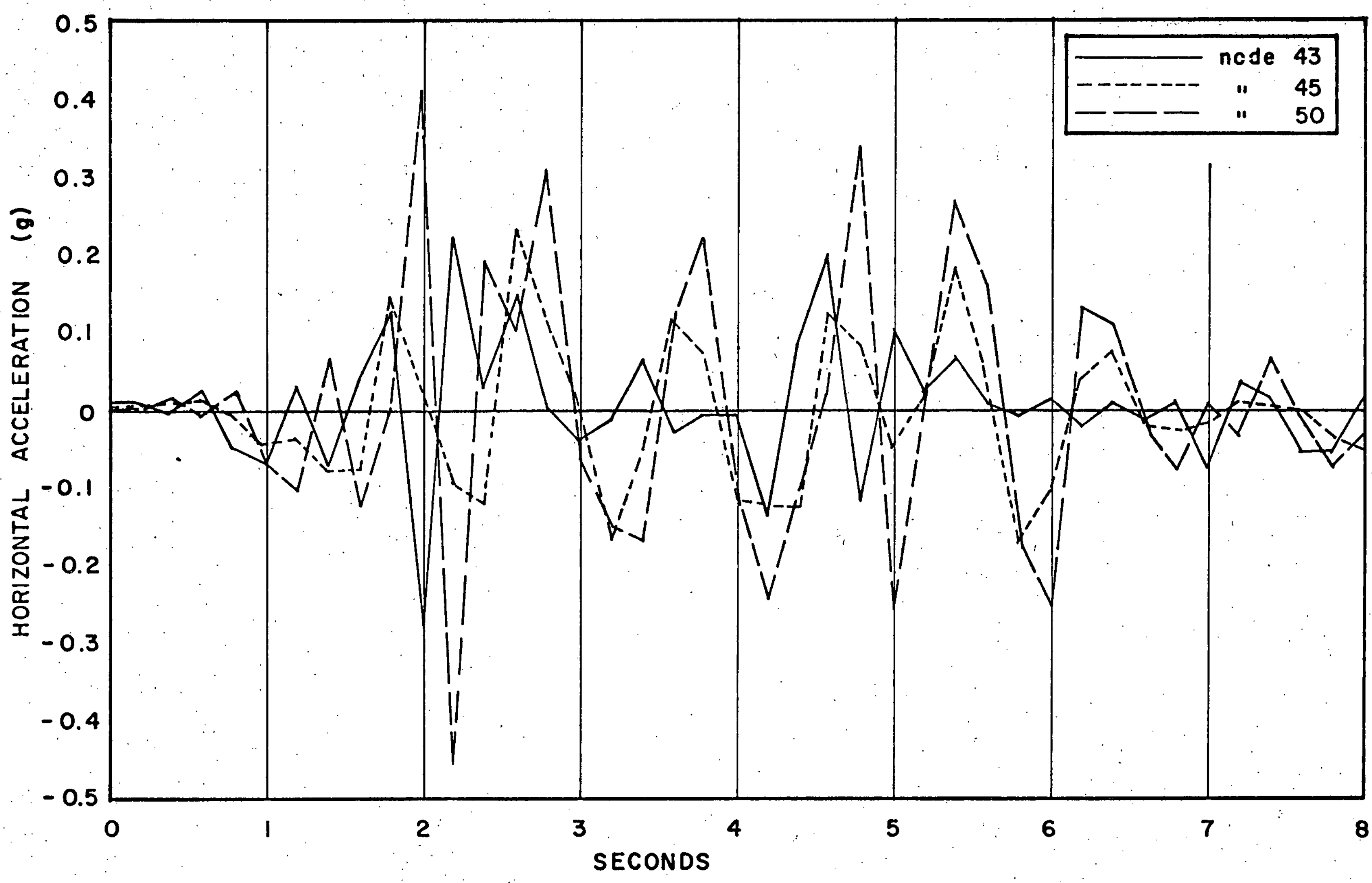


FIG. 12 VARIATION OF ABSOLUTE HORIZONTAL ACCELERATION WITH HEIGHT ALONG ℄ OF DAM WITHOUT A CORE

force is expressed as the product of the weight of the sliding mass and a coefficient called the seismic coefficient. The determination of the seismic coefficient is not included in this study; however, a method of determining the required horizontal force, from the results of the finite element analysis, is given by Chopra (11). The limitations of this pseudo-static approach are discussed by Seed and Martin (14) and a method is proposed where the seismic coefficient varies with time and with height within the dam.

The increase in response with elevation in the dam shows that the seismic coefficient should be increased with elevation. Codes do not recognize the non-rigid nature of earth dams subject to seismic loading and specify a constant seismic coefficient, with the exception of the Russian code, referred to by Ambraseys (8), where the seismic coefficient varies with height and depends on the damping properties of the embankment material and the seismic intensity of the region. The maximum acceleration is developed in a dam for only a short period of time, hence the resulting deformations may be small and, though other deformations will occur from other peaks during the earthquake, it does not seem reasonable to assume that the total deformations will be as great as if a static inertia force corresponding to the maximum acceleration were applied for an unlimited time.

3.6 Dynamic Stress Response

Figures 13 and 14 show the variation in dynamic stresses along a horizontal plane at elevations 60 feet and 180 feet respectively in the homogeneous dam. Since the structure is symmetrical, the horizontal and vertical normal dynamic stresses are zero at the center-line of the dam and the embankment material is in a state of pure shear. The horizontal normal stress reaches a maximum approximately midway between the center-line and the embankment face at the 60 foot elevation and then falls off slightly towards the face. It also increases with elevation, the peak value at the 180 foot level being approximately 100 per cent greater than that at the 60 foot level. At the 180 foot elevation, horizontal normal stress is a maximum at the face. The response is out of phase at the various points between the center-line and the face.

The vertical normal stresses are included for purpose of comparison only as it has already been shown that the use of just the horizontal component of El Centro makes these values questionable. The variation in the vertical normal stress follows the same pattern as that of the horizontal plane, but the trend is reversed with elevation, the vertical stresses decreasing with height.

At the 60 foot level, the variation of shear stress along a horizontal plane is considerable, decreasing from a maximum at the center-line to approximately 30 per cent of that

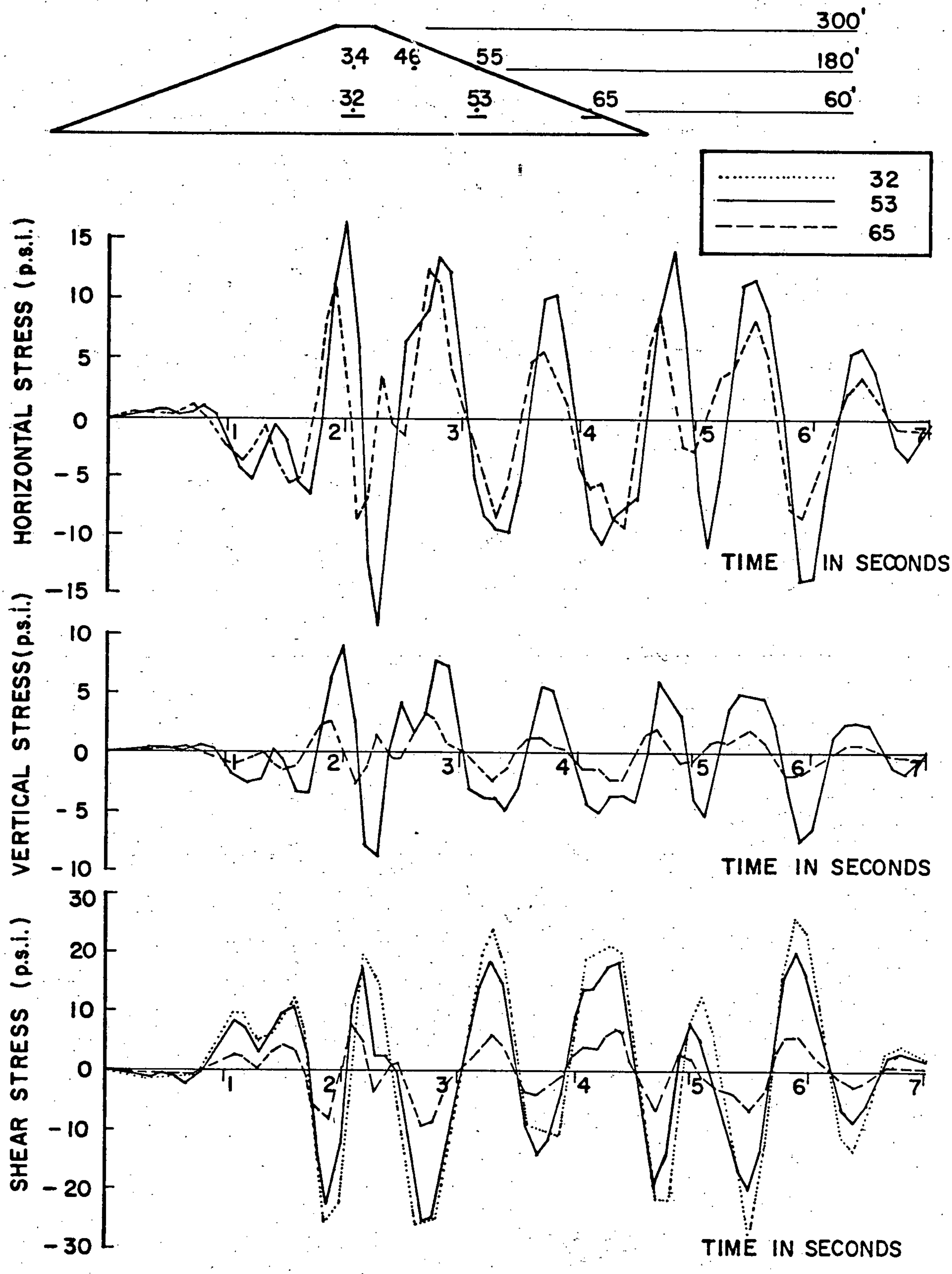


FIG. 13 VARIATION OF HORIZONTAL, VERTICAL AND SHEAR STRESS ALONG THE 60 FT. LEVEL OF DAM WITHOUT CORE

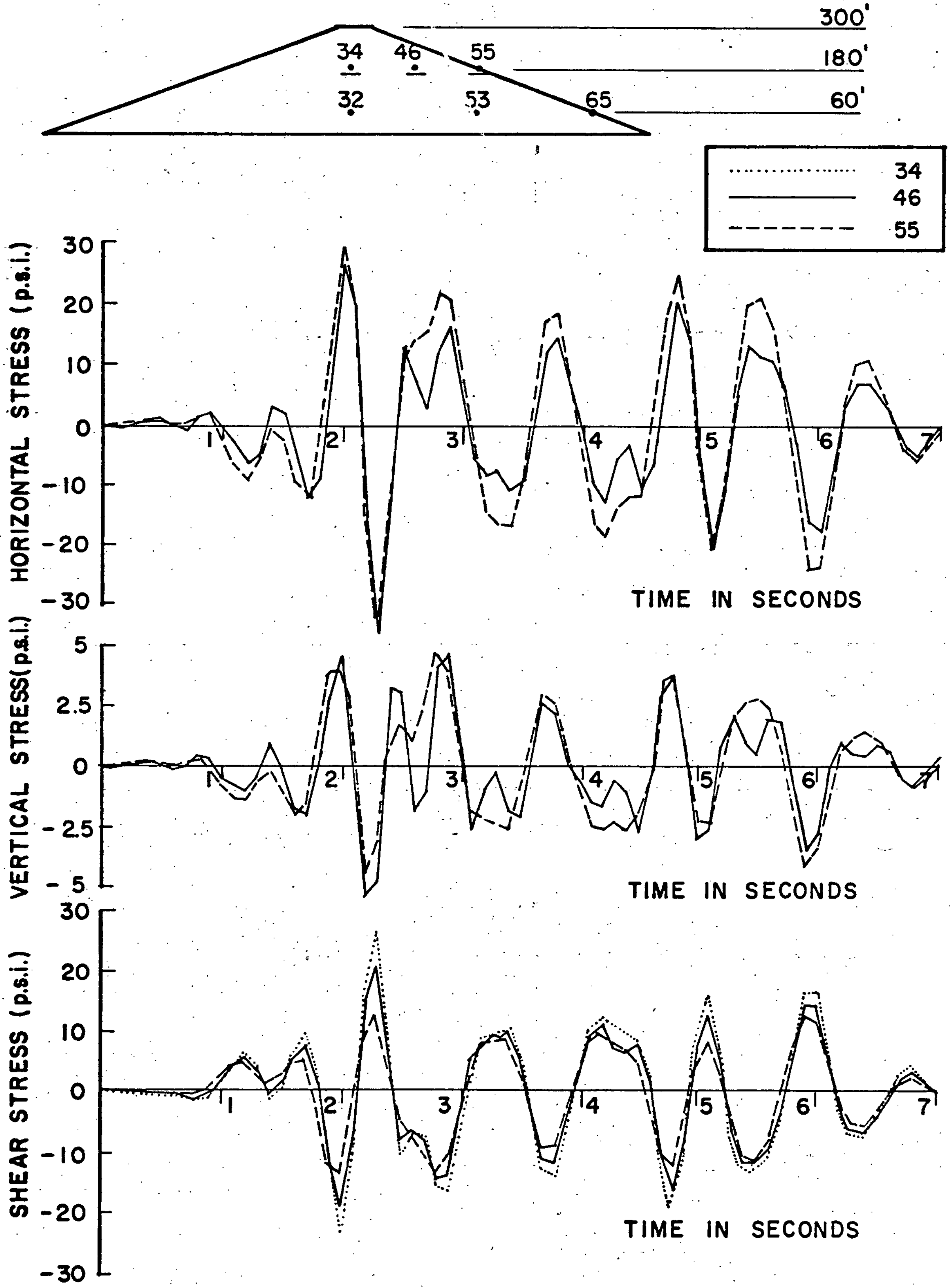


FIG. 14 VARIATION OF HORIZONTAL, VERTICAL AND SHEAR STRESS ALONG THE 180 FT. LEVEL OF DAM WITHOUT CORE

value at the face. In the shear wedge solution, the shear stress is constant from the center-line to the face. The shear stress is slightly lower at the 180 foot level and the variation between the center-line of the dam and the face is not as great.

The variation of the dynamic shear stress along a horizontal plane at elevation 125 feet in the three dams is shown in Figure 15. In all three cases, the shear stress increases from the face of the dam to the center-line (node 6 on face of dam, node 46 on center-line of dam). Of particular interest, however, is that the shear stresses are only affected to any degree by the presence of a core at points within the core. At other points in the embankment, the shear stresses are virtually unchanged. Within the more flexible material of the core, the shear stresses are lower, being approximately two-thirds of the stresses at corresponding points in the homogeneous dam. The data presented in Figure 15 is for the more rigid cores with elastic modulus equal to one-half that of the embankment material. The results for the more flexible cores, with elastic modulus one-tenth that of the embankment material, is similar though the effect on the shear stresses at points in the embankment near the core will be slightly greater and the shear stresses in the core will be smaller.

It can be seen from Figures 13 to 15 that the shear stresses along a horizontal plane in the dam are in phase, while the vertical and horizontal stresses are not. This is

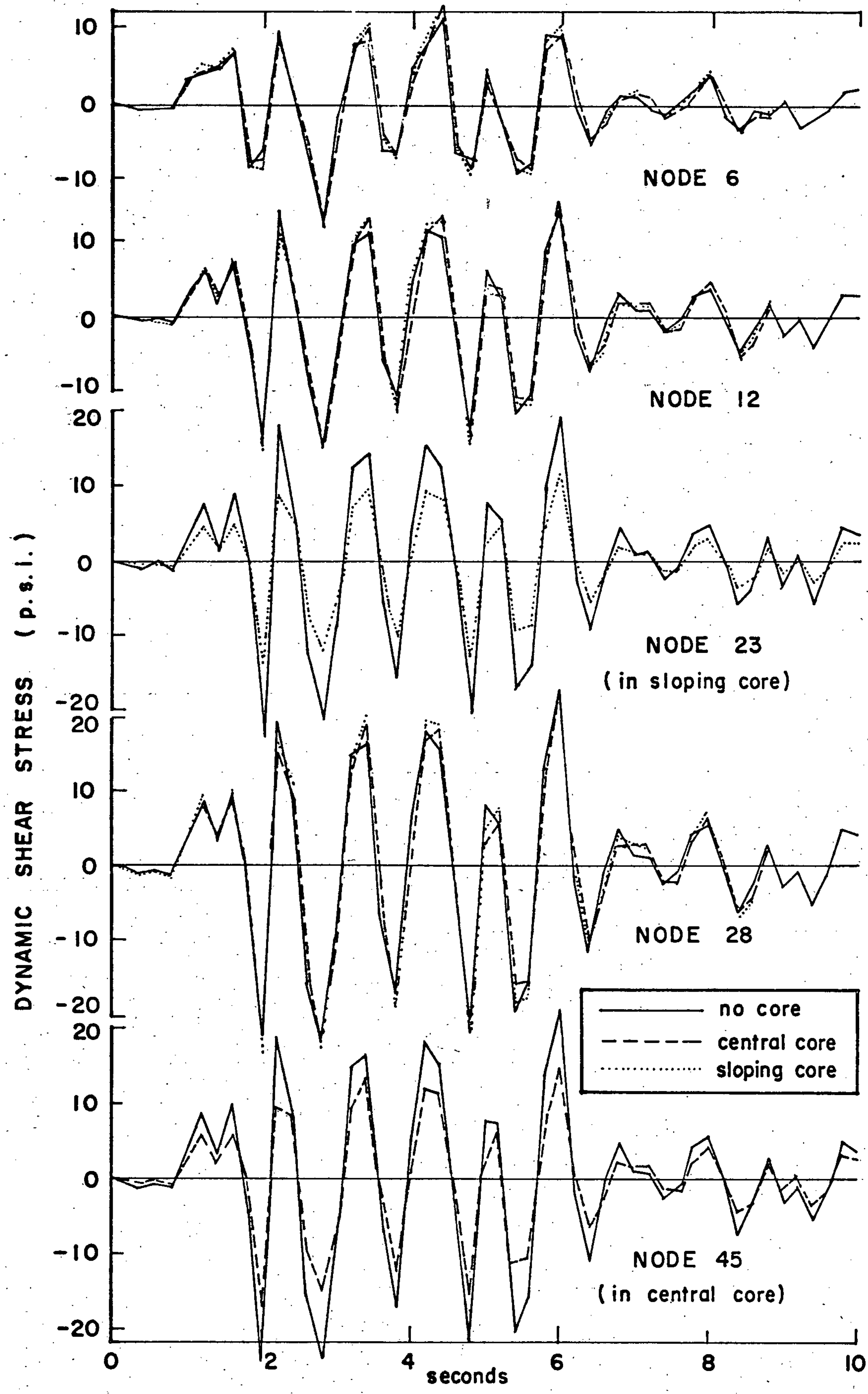


FIG. 15 VARIATION OF DYNAMIC SHEAR STRESS ALONG THE 125' LEVEL
(See Figure 2(A) For Location of Nodes)

because the response of the dam is mainly in the first mode, predominantly a shear mode, with little differential movement between the face of the embankment and the center-line.

Figure 16 presents the horizontal dynamic stress at various heights along the center of the sloping core. The stress is seen to decrease with elevation along the sloping core, though Figures 13 and 14 show an increase with elevation in the dam. The reason for the decrease shown in Figure 16 is that points higher in the core are also closer to the center-line of the dam where horizontal dynamic stresses are zero and this decrease with proximity to the center-line more than compensates for the increase with elevation in the dam. From the base to half the height of the dam, the horizontal stress in the core is greater for the softer core, just slightly below the stress at corresponding points in the homogeneous dam. The stress for the softer core is also opposite in sign to the other cases up to this level. From half the height of the dam to the crest, the stress is a maximum for the softer core, followed by the homogeneous dam and then the more rigid core.

Comparing the response of the central and sloping core dams, the horizontal dynamic stress in the core is very low for the central core dam. The high horizontal stress in the sloping core dam will induce higher pore water pressures in the impermeable core material during the earthquake and,

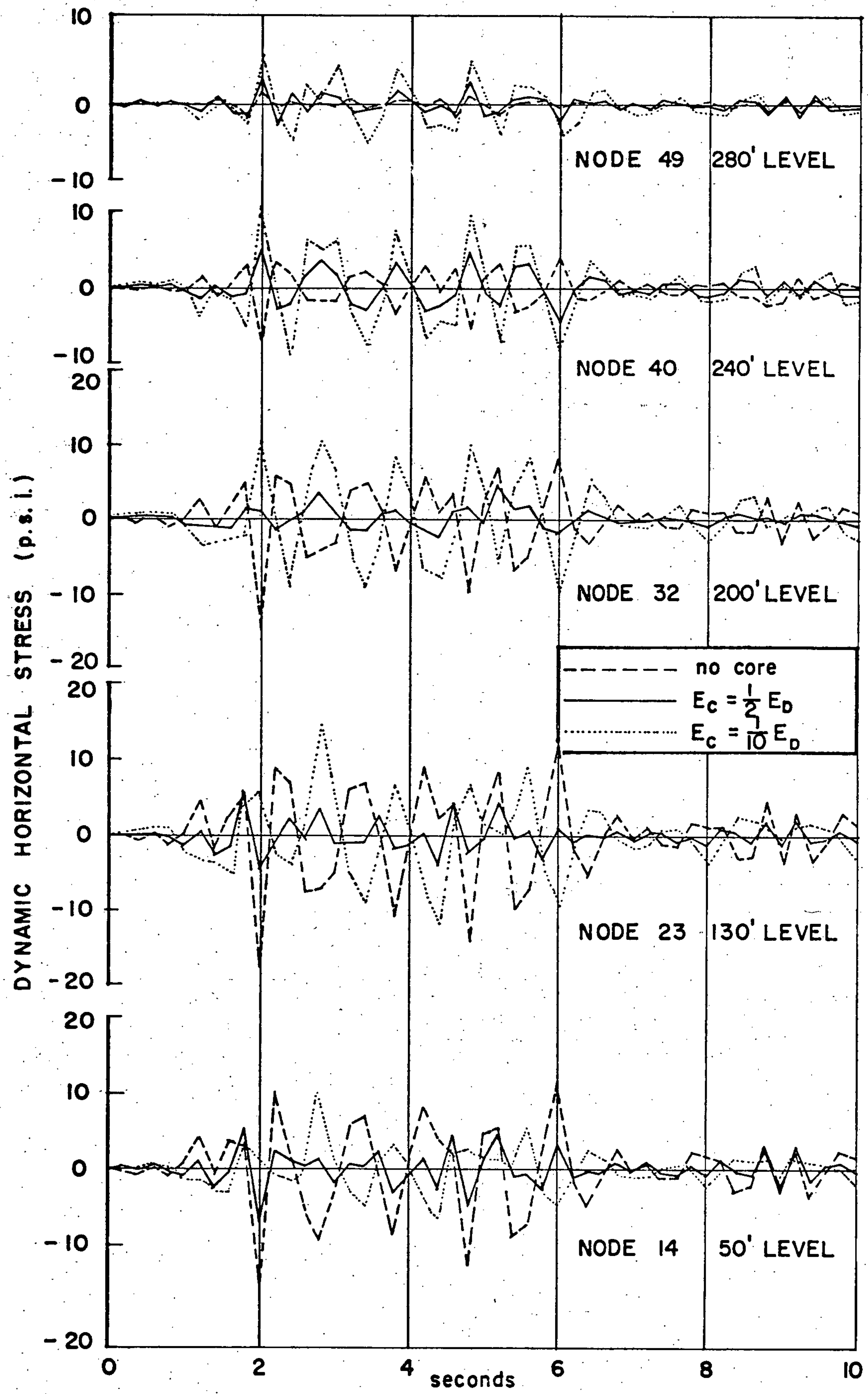


FIG. 16 VARIATION OF DYNAMIC HORIZONTAL STRESS WITH HEIGHT ALONG THE CENTER-LINE OF THE SLOPING CORE

hence, create a less desirable condition.

The effect of the core on the dynamic shear stress at points within the core is shown in Figures 17 and 18 where the shear stress in the core of the dam is compared with that at corresponding points in the homogeneous dam. Figure 17 shows the variation of the dynamic shear stress with height along the center-line of the central core dam, for both soft and rigid cores, compared with that of the homogeneous dam. It can be seen, for the lower part of the dam, that the shear stresses induced by the earthquake in the homogeneous dam are nearly twice as high as in the dam with the rigid core and about five times as high as in the dam with the soft core. Higher up in the dam, the differences are less marked until, in the upper fifth of the dam, the shear stresses are slightly greater in the core than in the homogeneous dam. Shear stresses at points outside the core are affected very little by the presence of the core. It is interesting to note that, while the shear stress decreases with height along the center-line of the homogeneous dam, it is nearly constant along the height of the central core, decreasing slightly near the top for the rigid core and actually increasing with height along the center-line of the flexible core.

The variation of dynamic shear stress with height along the center-line of the core in the sloping core dam is shown in Figure 18. The shear stress is compared at points in the dam with a rigid core, the dam with a flexible core,

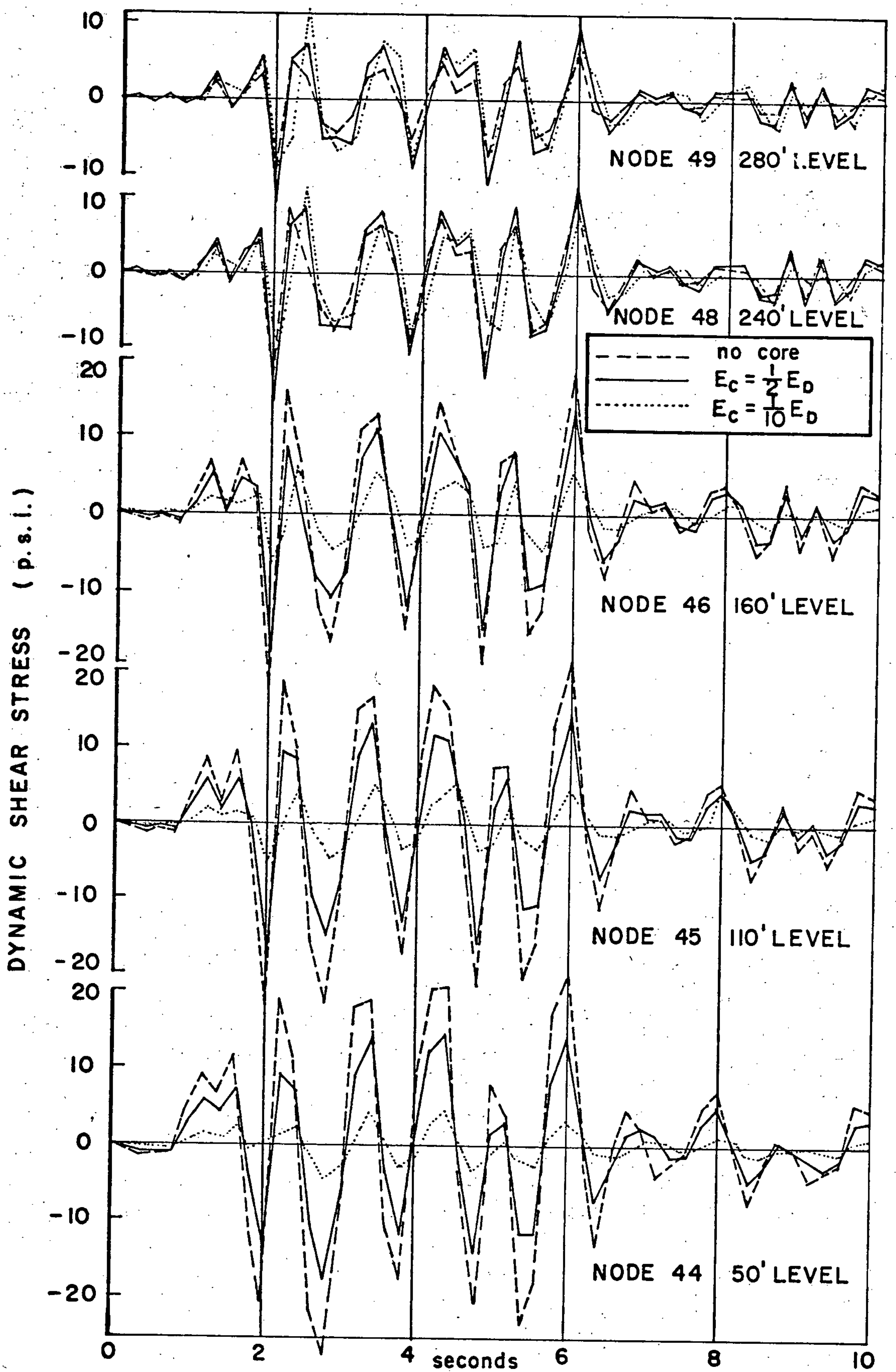


FIG. 17 VARIATION OF DYNAMIC SHEAR STRESS WITH HEIGHT ALONG CENTER-LINE OF CENTRAL CORE

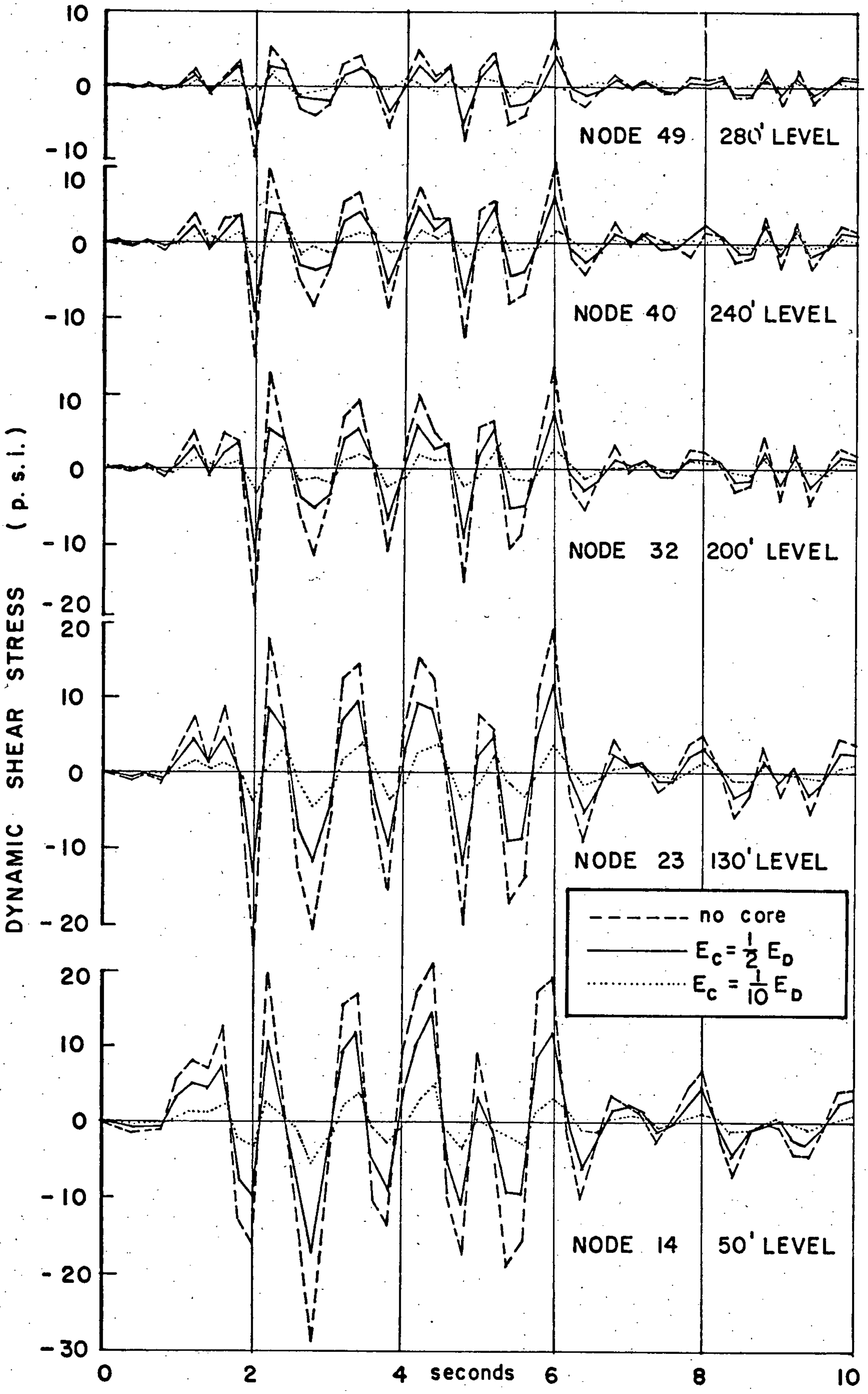


FIG. 18 VARIATION OF DYNAMIC SHEAR STRESS WITH HEIGHT ALONG CENTER-LINE OF SLOPING CORE

and at corresponding points in the homogeneous dam. The shear stress is highest for the homogeneous dam, approximately twice as high as for the rigid core dam, and five times as high as for the dam with the more flexible core. Unlike the central core dam, this ratio is maintained throughout the length of the core, the shear stress decreasing with height.

A common point to both cores, node 49, occurs 20 feet below the crest of the dam. From Figures 17 and 18, it is seen from the response at this common point and from the response at nodes 40 and 48, which are at the same elevation, that the shear stress is considerably higher in the upper part of the central core and is generally slightly higher at all levels than in the sloping core.

3.7 Responses by Modes

When the response of the homogeneous dam is computed for each mode individually, it is found that modes 2, 4, 7, 8 and 10 contribute nothing to the total response. On referring to Figures 22, 23 and 24 in Appendix II, it is seen that these modes are symmetrical about the central axis - they are vertical motion modes and are not excited by horizontal ground motion. Since only the horizontal component of El Centro was used, there is no response in these modes. Figure 19 shows the shear stress in each of the first ten modes for three points at the same elevation in the homogeneous dam, Figure 2(b). The response generally decreased as mode

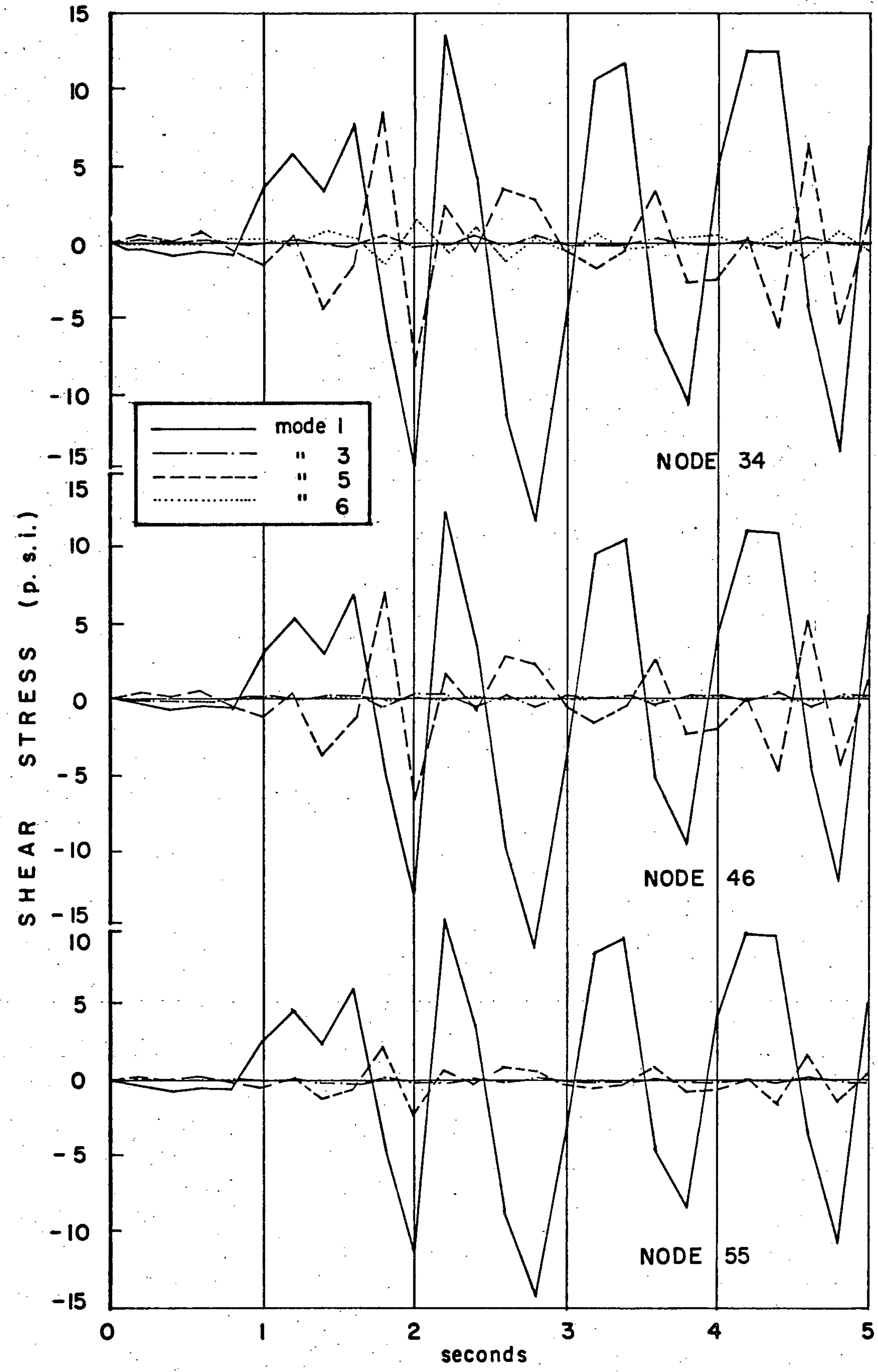


FIG. 19 DYNAMIC SHEAR STRESS IN FIRST TEN MODES OF DAM WITHOUT CORE - HORIZONTAL COMPONENT OF EL CENTRO ONLY

number increased and the response in mode 9 was too small to plot. The first mode, which is a shear mode, contributes the major part to the shear stress. The next largest contributor to the shear stress is not mode 3, as would be expected, but mode 5. The reason for this may be seen from an examination of the mode shapes in Figures 22 and 23 in Appendix II. Mode 5 is seen to resemble fairly closely a shear mode, with bending motion at a minimum, while mode 3 is predominantly a bending mode, hence the shear stress in mode 5 will be greater. The frequency of mode 5 is close to that of El Centro and hence a resonance effect is probably involved. The fact that mode 1, a shear mode, contributes the major part to the response of the dam explains why the shear wedge analysis can be used effectively to determine the seismic response of the earth dam.

The shear stress contributed by the individual modes to the total shear stress in the dam with a sloping core is presented in Figure 20. The figure shows the shear stresses in the first 5 modes at a point in the sloping core at half the height of the dam. As with the homogeneous dam, the major part of the response is in the first mode. Due to the asymmetrical nature of the dam, there are no symmetrical mode shapes and, consequently, each mode contributes to the response of the dam. As before, the fifth mode contributes the second largest share to the response, but the difference between modes 3 and 5 is not as pronounced as in the homogeneous dam. The first mode contributes a proportionally greater

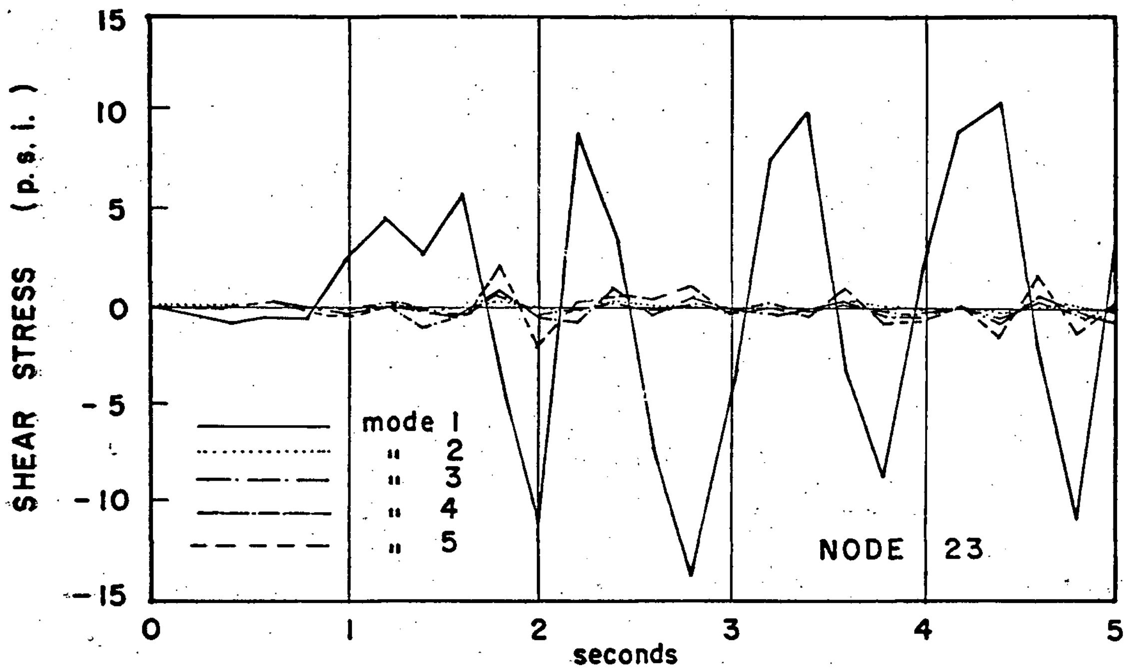


FIG. 20 DYNAMIC SHEAR STRESS IN FIRST FIVE MODES OF DAM WITH SLOPING CORE – HORIZONTAL COMPONENT OF EL CENTRO ONLY

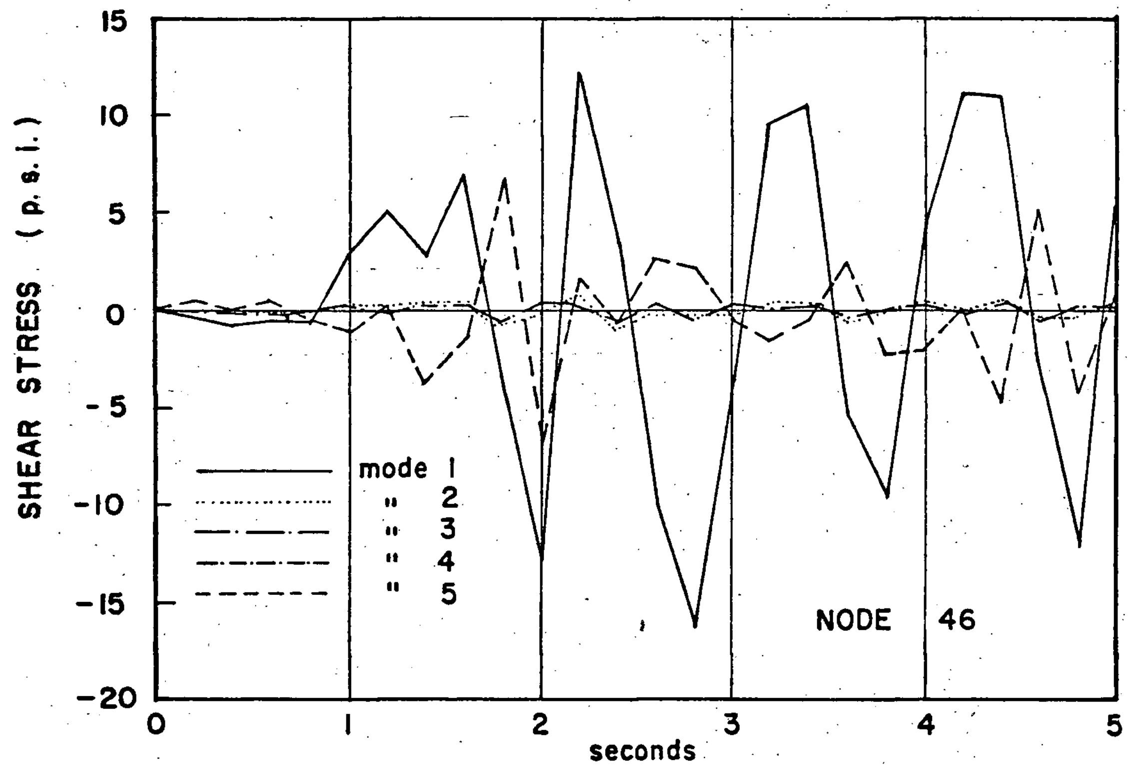


FIG. 21 DYNAMIC SHEAR STRESS IN FIRST FIVE MODES OF DAM WITHOUT CORE – VERTICAL AND HORIZONTAL ACCELERATION USED

share of the response than in the homogeneous dam and the error would be small if this mode alone were used to determine the shear stress.

In Figure 21, the response in the first 5 modes at a point in the homogeneous dam is presented assuming that a vertical component equal to the horizontal acceleration component of the earthquake is acting. When compared with the same point in Figure 19, the response due to the horizontal component of the earthquake only, it is seen that modes which are represented in Figure 19 are unchanged here. The effect of the vertical component of earthquake acceleration is to excite response in the symmetrical modes only, leaving the other modes unchanged.

3.8 Modal Participation Factor

The response in each mode can also be analyzed in terms of the modal participation factor (M.P.F.). The M.P.F. for any mode gives a measure of the proportion of the total response occurring in that mode. It depends only on the mode shape and direction of the earthquake base acceleration. For horizontal base acceleration alone, only mode shapes with horizontal displacements contribute to the response, while for vertical base acceleration alone, only mode shapes with vertical displacements contribute. The applicable M.P.F. for each case is given by equations 12 and 13 of Appendix I. For both horizontal and vertical base accelerations all modes will contribute to the responses and the M.P.F. is given by equation 14 of Appendix I. Since M.P.F.'s depend only on the mode shapes and direction of the earthquake base acceleration, they can be determined prior to integrating the equations of motion and, hence, the contribution of each mode is predetermined.

The M.P.F. for each type of dam, for modes 1 to 5, for horizontal earthquake acceleration only and for combined horizontal and vertical acceleration, is shown in Table 1. The M.P.F. for modes 2 and 4 of the symmetrical homogeneous dam, modes in which there was no response, as shown in Figure 19, is zero. When a vertical acceleration component is included in the earthquake, the M.P.F. for these two modes is now non-zero, the M.P.F. for the other modes remains unchanged. This uncoupling applies only to the symmetrical dam, though the central core dam is less affected than the more asymmetric sloping core dam. In the response of the more flexible core dams, the introduction of a vertical acceleration component decreases the response in some modes, notably mode 5 of the central core dam. Examination of Figures 19, 20 and 21 confirms that the modes with the highest M.P.F. contribute most to the response of the dam. Table 1 also shows that, for the dams with the more flexible cores, the response contributed by each mode is quite different than for the more rigid core. Mode 4 is the second largest contributor to the response of the flexible core dams, a mode which contributes nothing to the response of the homogeneous dam, though mode 1

TABLE 1

MODAL PARTICIPATION FACTOR

MODE	HOMOGENEOUS DAM		CENTRAL CORE DAM				SLOPING CORE DAM			
			Rigid Core		Flexible Core		Rigid Core		Flexible Core	
	Horizontal Component only	Horizontal & Vertical	H	H & V	H	H & V	H	H & V	H	H & V
1	1.54	1.54	-1.54	-1.54	-1.65	-1.63	1.58	1.56	1.61	1.56
2	0	0.45	-0.02	-0.63	0.06	0.91	0.05	0.58	-0.29	-0.88
3	-0.33	-0.33	0.44	0.45	0.60	1.44	-0.36	-0.40	0.42	0.10
4	0	-0.002	0.20	0.20	0.95	0.61	-0.10	-0.06	-0.66	-1.00
5	1.00	1.00	0.89	0.81	0.22	-0.01	-0.97	-1.09	0.25	0.86

remains the largest contributor in every case. The effect of including a vertical acceleration component in the modal response distribution is more pronounced in the case of the softer core.

Thus, it is shown that the M.P.F. is very useful in determining the modes that are the major contributors to the response before the dynamic analysis is run and, hence, an accurate estimate of the number of modes needed for the required accuracy in the integration can be made.

3.9 Application of Results

The finite element method yields the state of stress and strain in the dam for static loading cases and, the stress and absolute acceleration history of the dam for dynamic loading cases such as earthquakes. However, due to the high value of damping used in each mode to take into account the inelastic deformation, the actual displacement and strain history of the real dam is not known from such a viscoelastic finite element analysis. To be of use to the designer, it must be possible to relate the available results to the actual performance of the material used in the dam and predict the ultimate displacements of the dam due to the earthquake. It is necessary to recognize the importance of basing design on displacements produced, rather than on factor of safety, since a factor of safety of less than unity can exist for a short period of time without excessive displacements before the stress cycle is reversed (Newmark (16)).

Since the viscoelastic finite element analysis does not yield an adequate displacement history, assessment of the actual deformation of the dam will require the use of either the stress or absolute acceleration history. Finn (4) suggests using cyclic loading tests on representative samples to determine strains within the dam. First, the static finite element analysis is used to determine the anisotropic consolidation stresses to be used, and then the cyclic stress history is taken from the viscoelastic finite element solution for the dynamic loading. From the strains obtained in such cyclic tests, the deformation of the dam can be obtained. On the other hand, Seed (12) uses the absolute acceleration history to determine the inertia forces to introduce into a slip-circle analysis to determine the state of stress on the potential failure surface. These stresses are then used in cyclic loading tests and the strains obtained.

CHAPTER 4

CONCLUSIONS

That a case for the improved seismic design of earth dams is evident is shown by Ambraseys (17) who states that in an investigation of dams, levees and embankments subjected to strong earthquakes, most dams were severely damaged while all levees and embankments were destroyed. In recent years, there has been no failure of a major earth dam due to earthquake action, but this is probably because no large dam has been subjected to a major earthquake in that time.

The following conclusions are drawn from the results of the investigation presented in the preceding chapters:

1. The higher dynamic horizontal stresses induced in the upper part of the sloping core, coupled with the existing static tensile stresses in this region, combine to make this type of core less desirable for earth dams in areas subject to seismic activity. The dynamic horizontal stresses induced in the upper core by the earthquake increase as the core is made more flexible.

2. There are no tensile static stresses in the central core dam and the dynamic horizontal stresses in the core are very

low. However, the dynamic shear stresses are considerably higher in the central core than in the sloping core, particularly in the upper levels of the core.

3. The dynamic shear stress is lower in the core, both central and sloping, than at corresponding points in the homogeneous dam. It is affected only to a very small degree at points outside the core, depending on the flexibility of the core.

4. The variation of the accelerations with height in the dam indicates that, in the seismic coefficient method of design, a coefficient which increases with elevation should be used.

5. The accelerations are higher in the sloping core dam and, hence, a higher seismic coefficient would be required than for the central core dam.

6. The finite element method is seen to be sensitive to irregularities in the subdivision of the dam into finite elements. A symmetrical subdivision should be used when possible.

7. In a comparison between the finite element method and the shear wedge analysis, it is seen that only the first mode approximates to a shear mode. A state of pure shear exists only at the center-line of the dam, and the shear stress is

not uniform on a horizontal plane through the dam, an assumption made by the shear wedge approach.

8. The vertical earthquake acceleration modifies only the vertical dynamic stress and has a negligible effect on the dynamic shear stress and horizontal dynamic stress.

9. The Modal Participation Factor, which can be easily determined before the dynamic analysis is run, is shown to be a very useful guide in selecting the number of modes which should be used to achieve the desired degree of accuracy.

10. From an appraisal of the findings of this investigation, it is concluded that the use of a central type core in areas of earthquake activity would lead to safer seismic design of earth dams.

Further research on this topic is necessary. The effect of varying side slopes, height of dam, core placing and core dimensions, as well as the response of the dam when underlain by various types of foundations, should be studied. The dynamic response of the dam under earthquakes of the highest and lowest frequencies to be expected should be investigated, with particular emphasis on resonance effects.

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APPENDIX I

DESCRIPTION OF FINITE ELEMENT METHOD

The finite element method of stress analysis is a powerful extension of matrix structural analysis procedures for obtaining digital computer solutions to problems of the continuum. A general description of the method has been given by Clough (1).

In practice, the continuum may be comprized of non-homogeneous, anisotropic, and non-elastic materials. Non-homogeneity and simple forms of anisotropy introduce no difficulty into the finite element method of analysis; non-elasticity is approximated by multi-linear elasticity for static analysis. However, dynamic analysis is still restricted to linear elastic behaviour.

The actual continuum is idealized as an assemblage of discrete elements or segments connected at the nodes. Any shape of element may be used provided a stiffness matrix, giving the relationship between the nodal forces and nodal displacements, is available for the element. Rectangular and triangular elements are commonly used. The boundaries of a dam are most easily followed by triangular elements which are used herein.

Generally, the more refined the discretization of the region the more accurate the results. Because of limitations of computer storage and problems of maintaining accuracy in the solution of large sets of equations, in practice, the fineness of subdivision varies with anticipated stress gradients. In regions of high stress gradient a relatively finer subdivision is used; in regions of low stress gradients, a coarser subdivision.

If {F} are the nodal forces on each element and {r} the nodal displacements then the element stiffness matrix, [k], is defined by the equation

$$\{F\} = [k]\{r\} \qquad (1)$$

The element stiffness matrix used herein is determined on the assumption of a linear variation of displacements over the element. This assumption ensures compatibility of displacements along the edges of contiguous elements. The stiffness matrix [K], for the entire structure is obtained by superimposing the appropriate stiffness coefficients of the individual elements surrounding each node.

The nodal force-deformation relations are then given by

$$\{R\} = [K]\{r\} \qquad (2)$$

in which {R} is the matrix of the nodal forces. Nodal forces due to gravity are obtained by lumping one-third of each element weight at the nodes of the elements. For distributed applied loads, such as surface loads, statically equivalent concentrated loads are applied at the appropriate nodes.

The matrix [K] is a symmetric band matrix and, for the order of equations used herein, equations (2) are most conveniently solved by the Cholesky method (Faddeeva (18)). For systems of very high order an iterative method such as that described by Wilson (19) is desirable to reduce round-off errors.

The stresses, σ, in the elements are obtained by

$$\{\sigma\} = [S]\{r\} \tag{3}$$

in which [S], the stress transformation matrix, is determined by the assumed displacement pattern and the material properties. Nodal stresses are obtained by averaging the stresses in elements around each node. This procedure loses accuracy at the boundaries and for more accurate results the extrapolation procedure suggested by Wilson (19) may be used.

SEISMIC ANALYSIS

The seismic behaviour of a dam subjected to a base acceleration, $a(t)$, may be studied by considering the base to be at rest and the dam to be acted upon by inertia forces, $R_i(t)$, given by

$$R_i(t) = - M_i a(t) \tag{4}$$

The mass M_i is obtained by lumping at node i one-third of the masses of all elements surrounding node i. Letting the displacement of node i be r_i the equation of motion for node i becomes

$$M_i\ddot{r}_i + C_i\dot{r}_i + K_i r_i = R_i(t) \tag{5}$$

in which C_i is the viscous damping, K_i the appropriate stiffness and the dots indicate differentiation with respect to time. In matrix form, the equations of motion for the structure are

$$[M]\{\ddot{r}\} + [C]\{\dot{r}\} + [K]\{r\} = \{R(t)\} \tag{6}$$

in which [M] is the mass matrix [C] the viscous damping matrix, [K] the stiffness matrix, and R(t) the load matrix. In expanded form, R(t) is given by

$$R(t) = - \begin{Bmatrix} M_1 \\ 0 \\ M_2 \\ \vdots \\ M_n \\ 0 \end{Bmatrix} a_h(t) - \begin{Bmatrix} 0 \\ M_1 \\ 0 \\ M_2 \\ 0 \\ \vdots \\ M_n \end{Bmatrix} a_v(t) \tag{7}$$

in which $a_h(t)$ and $a_v(t)$ are the horizontal and vertical acceleration components of the earthquake.

The undamped free vibration mode shapes $[\phi]$ and the corresponding natural frequencies of vibration$\{\omega_n\}$ are first determined by solution of the usual characteristic value problem

$$-\omega_n^2[M]\{\phi_n\} + [K]\{\phi_n\} = 0 \tag{8}$$

The normal coordinates of the system, Y, may then by related to the nodal coordinates by

$$\{r\} = [\phi]\{Y\} \tag{9}$$

The equations of motion may now be reduced to n normal mode equations

$$\ddot{Y}_n + 2\lambda_n\omega_n\dot{Y}_n + \omega^2 Y_n = \frac{P_n^*(t)}{M_n^*} \tag{10}$$

in which $M_n^* = \{\phi_n\}^T[M]\{\phi_n\}$, $P_n^* = \{\phi_n\}^T\{R(t)\}$

and λ_n = % of critical damping in the nth mode. It is assumed that the damping is such that the damping matrix has the orthogonality property

$$\{\phi_m\}[C]\{\phi_n\} = 0 \quad m \neq n \tag{11}$$

The normal equations (10) are solved for Y_n using step-by-step matrix analysis method of integration (Wilson and Clough (20)). Then using equations (9) the dynamic displacements {r} are determined at discrete intervals of time. Applying equations (3) the dynamic element stresses and finally the dynamic nodal stresses are determined.

MODAL PARTICIPATION FACTOR

The modal participation factor (M.P.F.) discussed Chapter 3.8 were calculated as follows

1. For horizontal base motion only, the modal participation factor in the nth mode is given by

$$M.P.F. = \frac{1}{M_n^*} \begin{Bmatrix} \phi_n \end{Bmatrix}^T \begin{Bmatrix} M_1 \\ 0 \\ M_2 \\ \vdots \\ 0 \\ M_n \end{Bmatrix} \qquad (12)$$

2. For vertical base motion only

$$M.P.F. = \frac{1}{M_n^*} \begin{Bmatrix} \phi_n \end{Bmatrix}^T \begin{Bmatrix} 0 \\ M_1 \\ 0 \\ \vdots \\ M_n \\ 0 \end{Bmatrix} \qquad (13)$$

3. For combined horizontal and vertical base motion

$$M.P.F. = \frac{1}{M_n^*} \begin{Bmatrix} \phi_n \end{Bmatrix}^T \begin{Bmatrix} M_1 \\ M_1 \\ M_2 \\ \vdots \\ M_n \\ M_n \end{Bmatrix} \qquad (14)$$

A P P E N D I X II

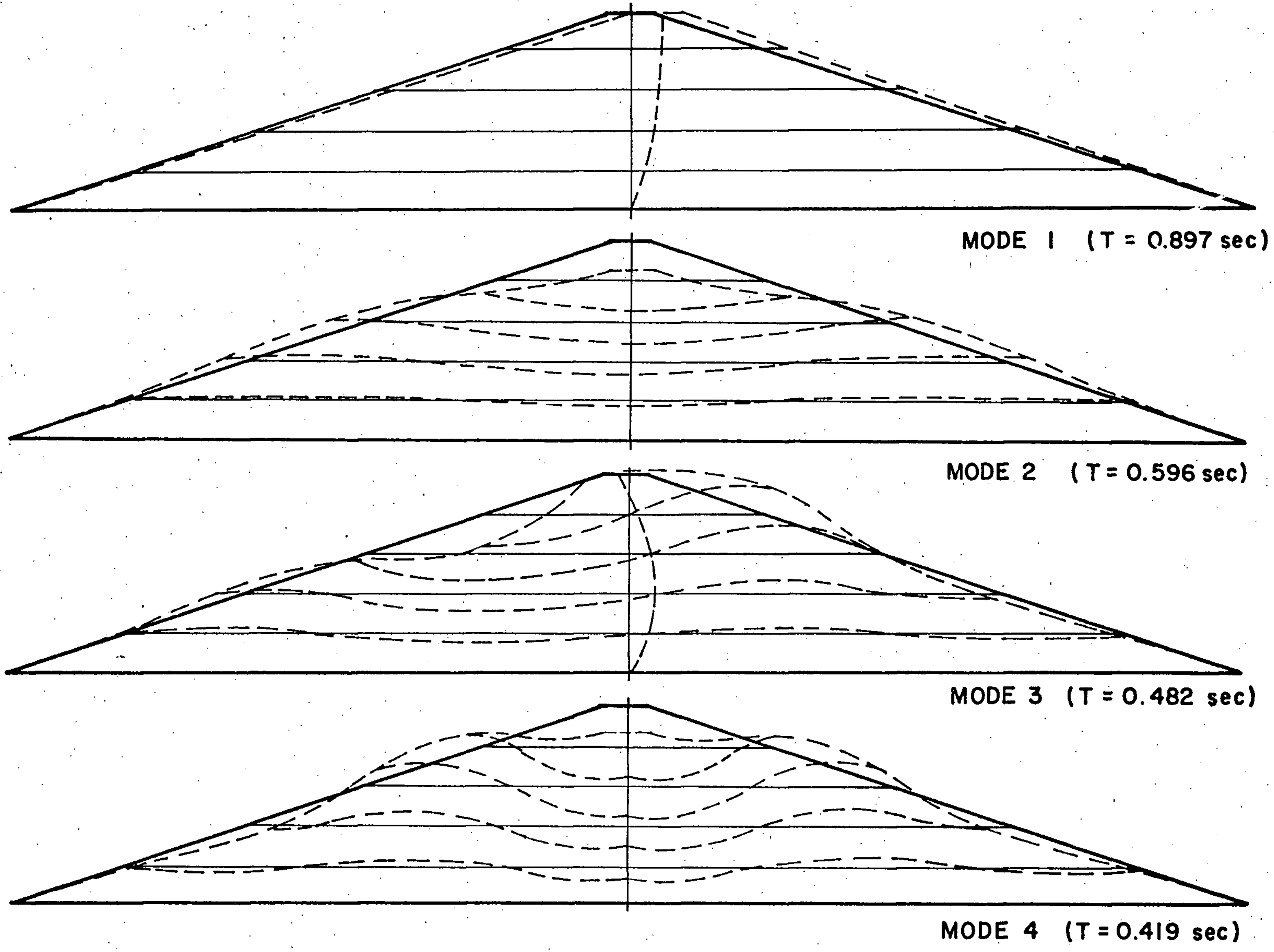


FIG. 22 MODE SHAPES 1 TO 4 - DAM WITHOUT CORE

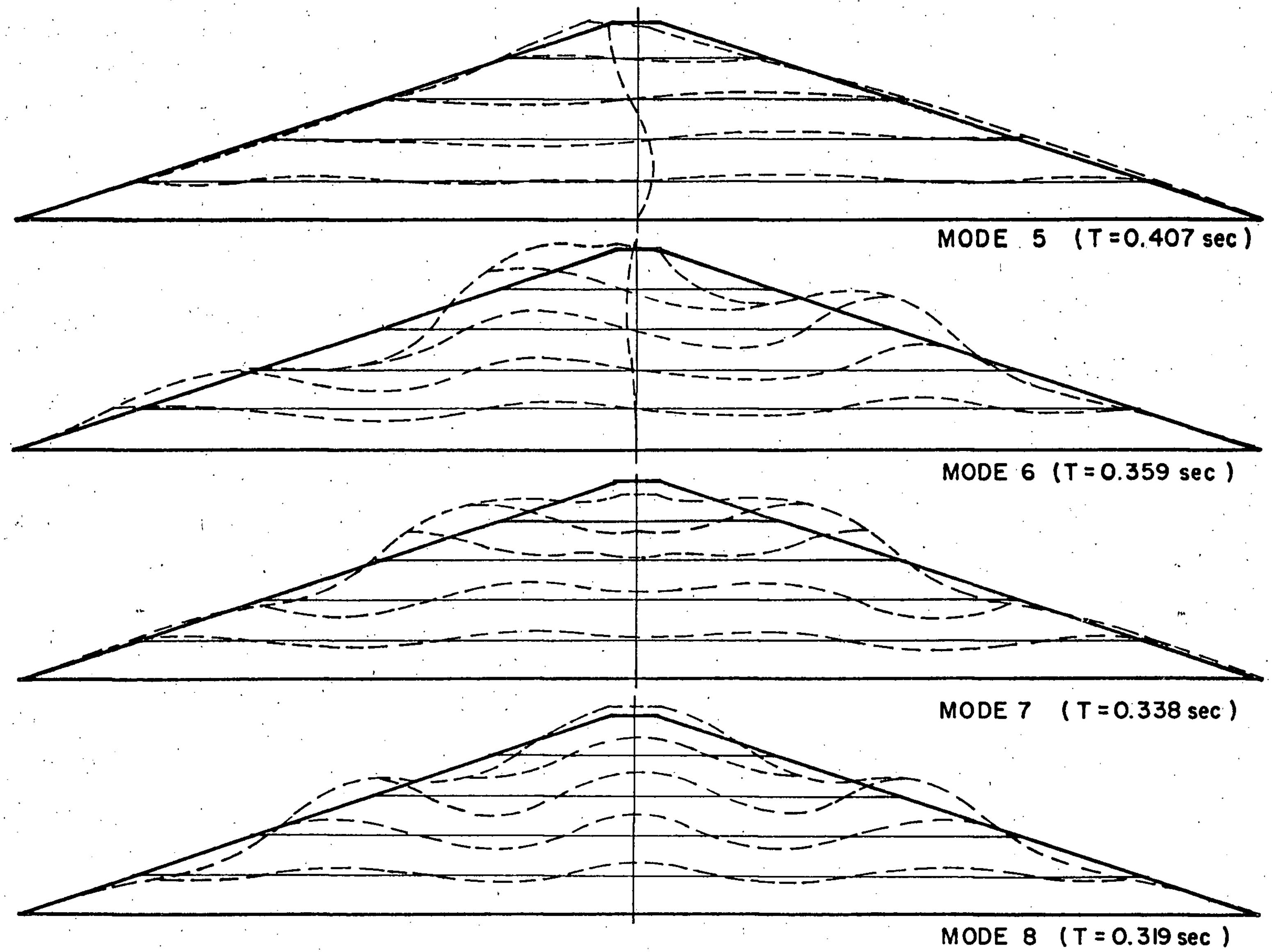
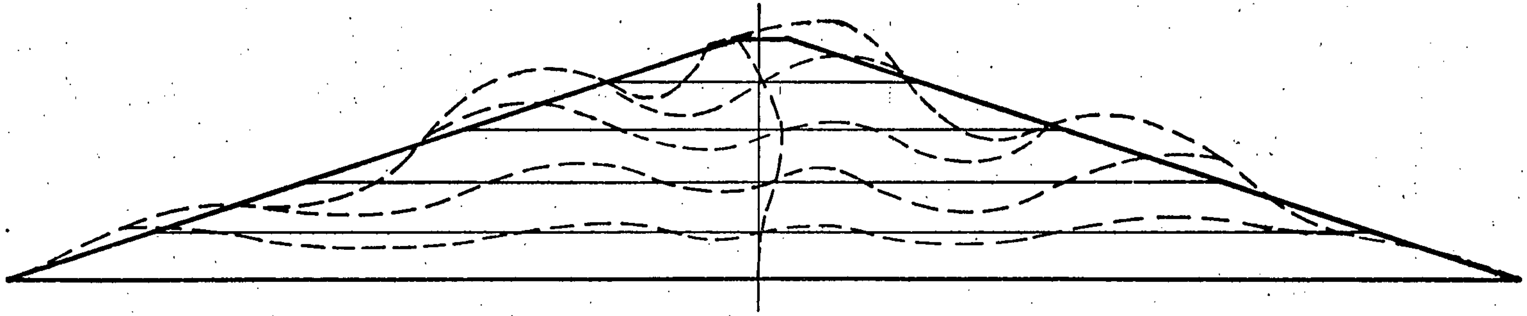
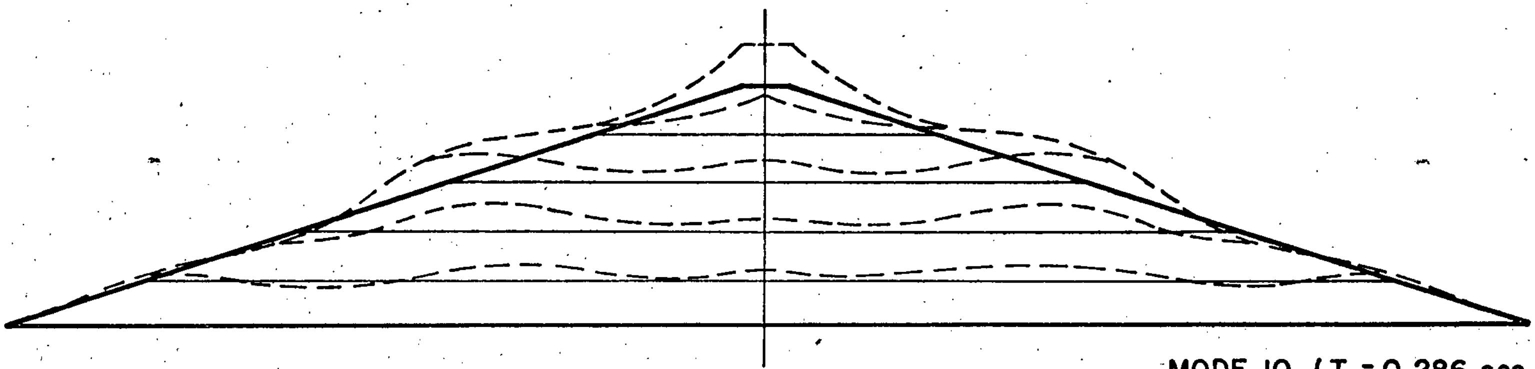


FIG. 23 MODE SHAPES 5 TO 8 — DAM WITHOUT CORE



MODE 9 (T = 0.293 sec)



MODE 10 (T = 0.286 sec)

FIG. 24 MODE SHAPES 9 AND 10 — DAM WITHOUT CORE

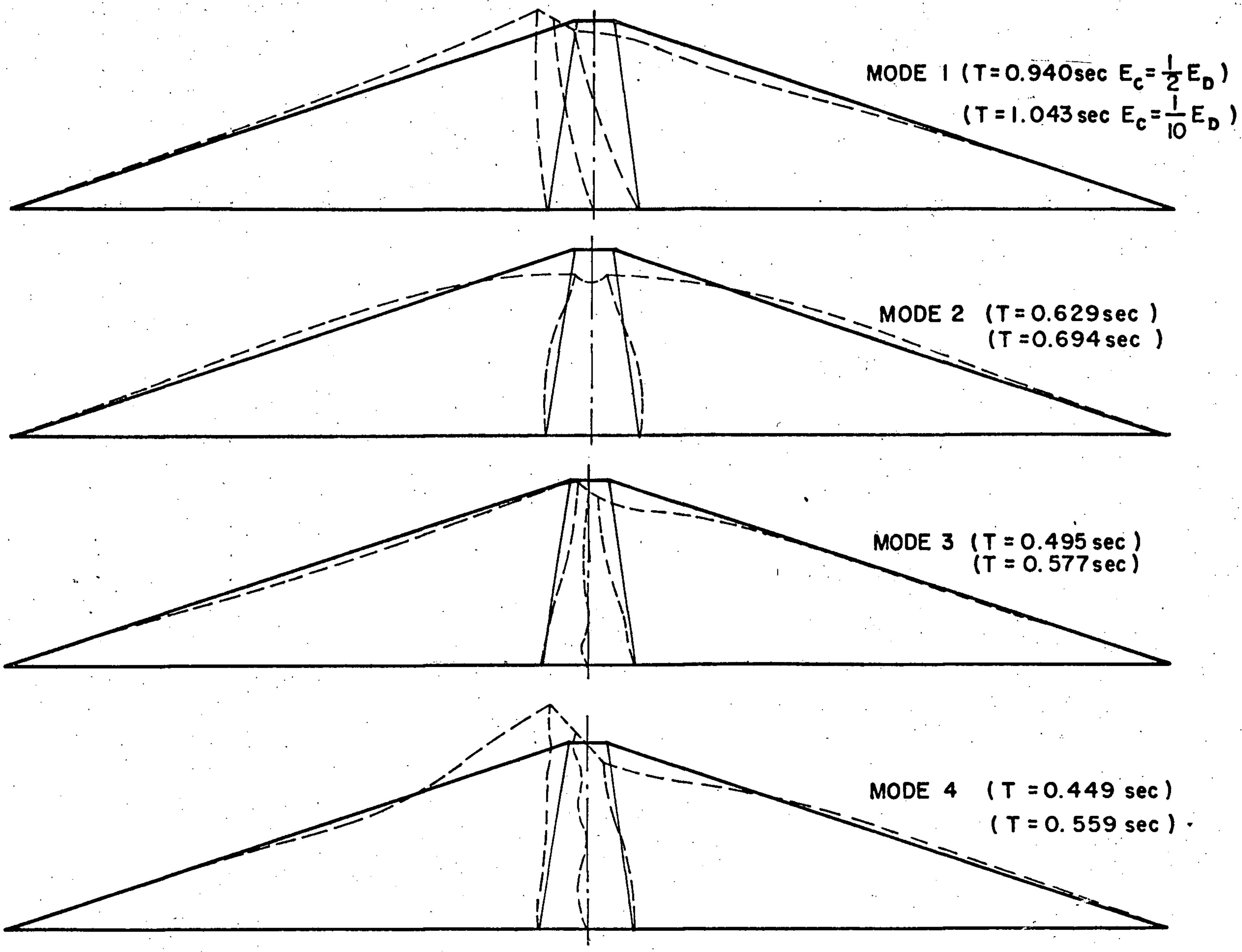


FIG. 25 MODES 1 TO 4 — DAM WITH CENTRAL CORE

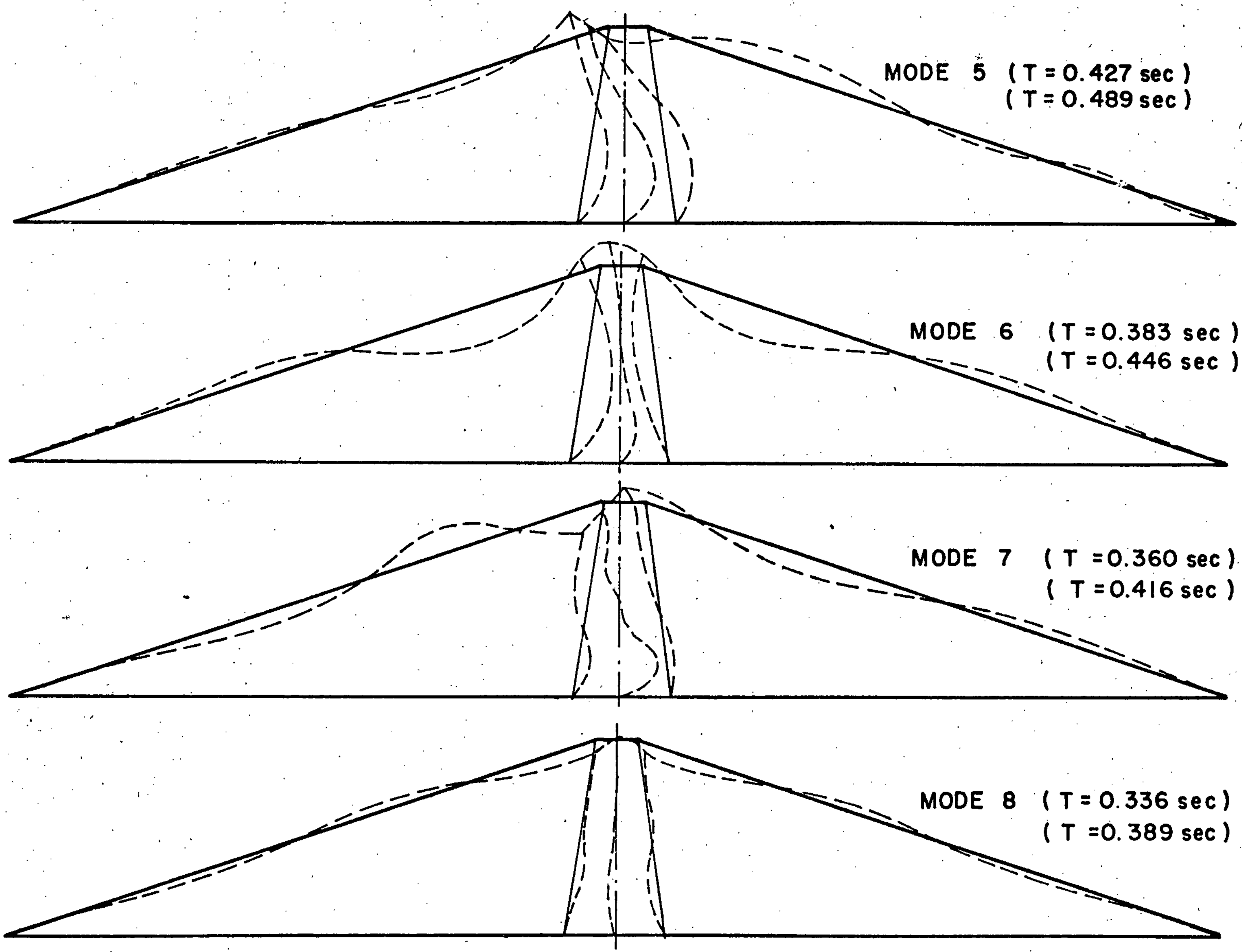


FIG. 26 MODES 5 TO 8 — DAM WITH CENTRAL CORE

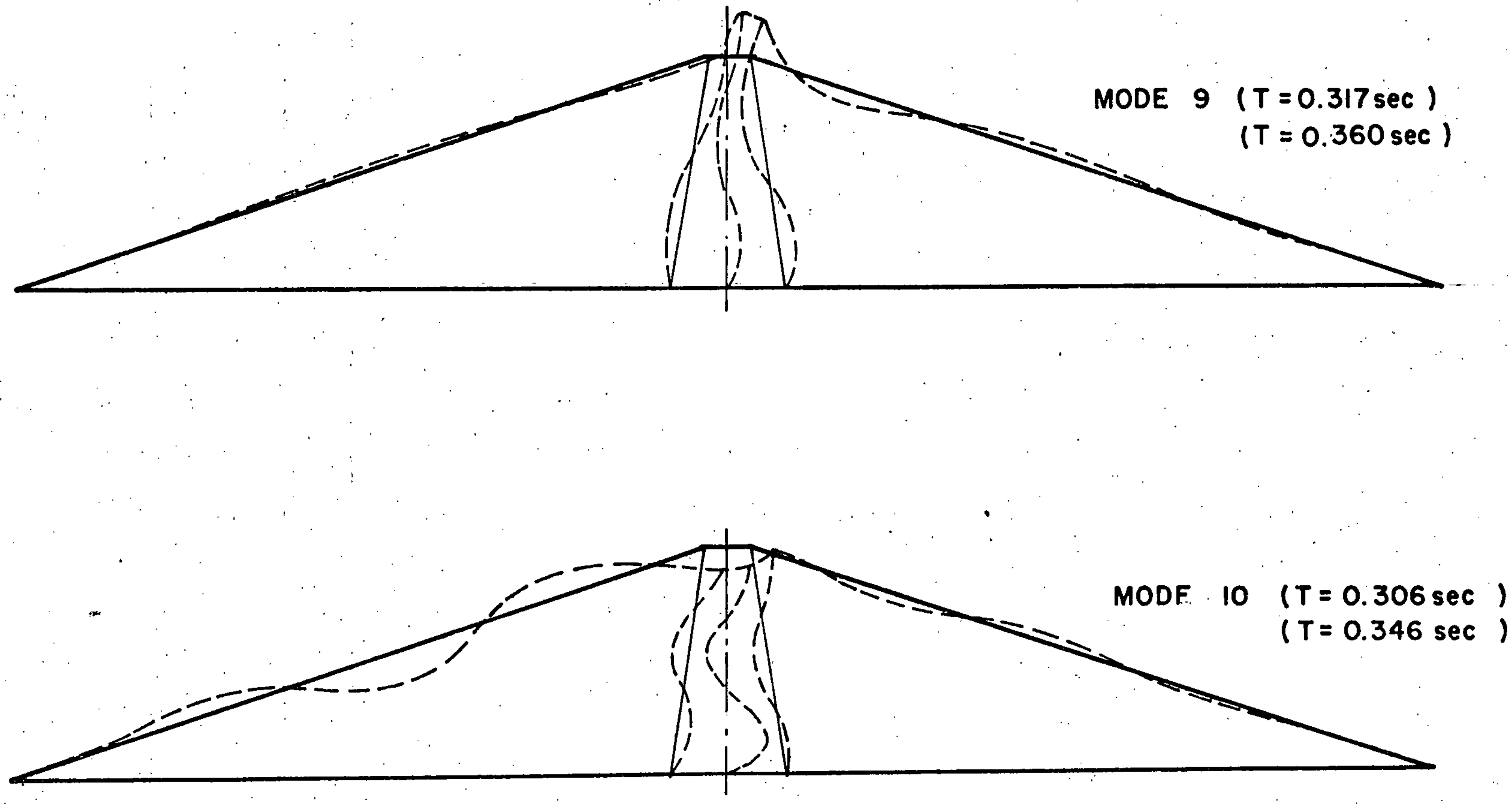


FIG. 27 MODES 9 AND 10 — DAM WITH CENTRAL CORE

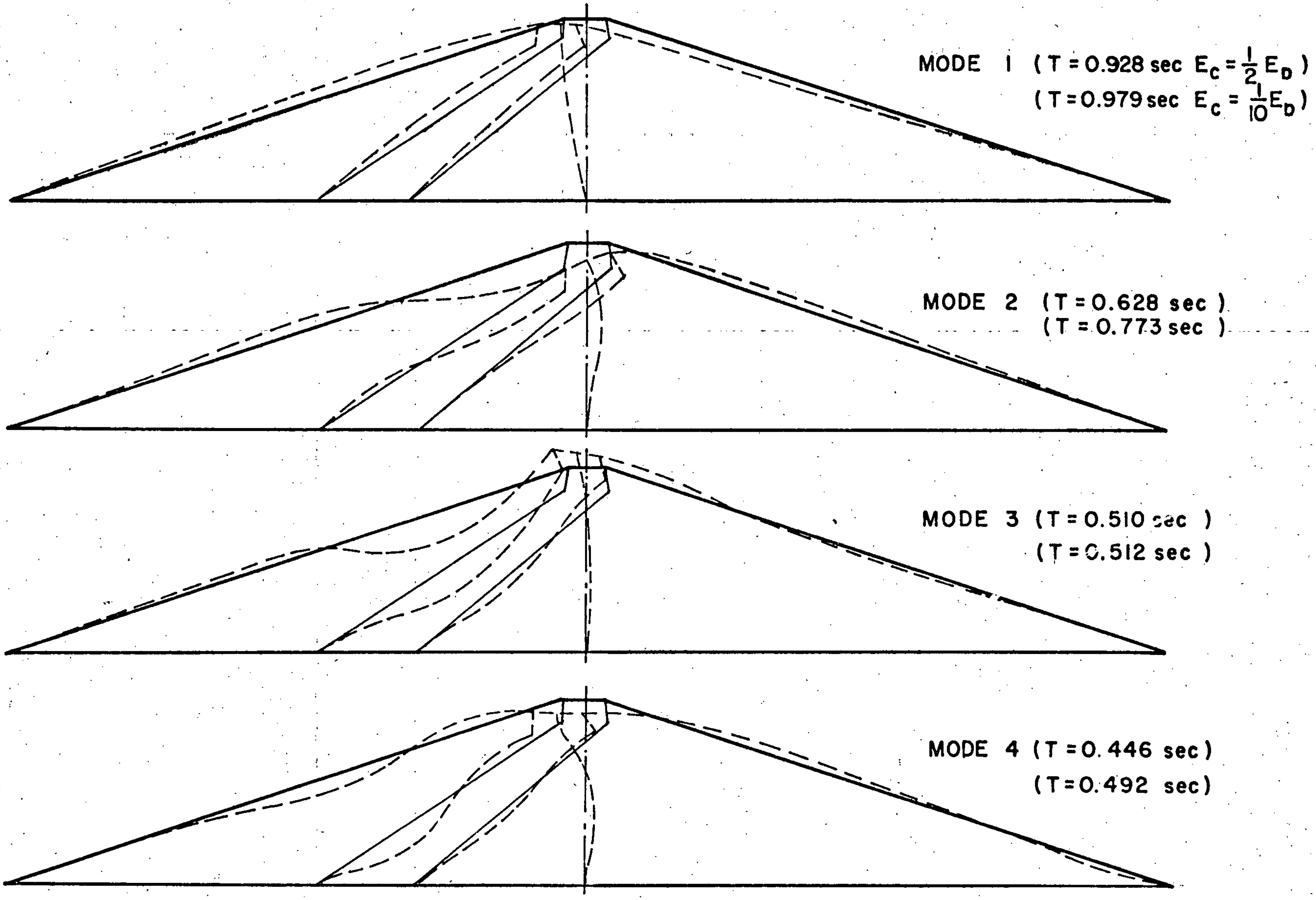


FIG. 28 MODES 1 TO 4 — DAM WITH SLOPING CORE

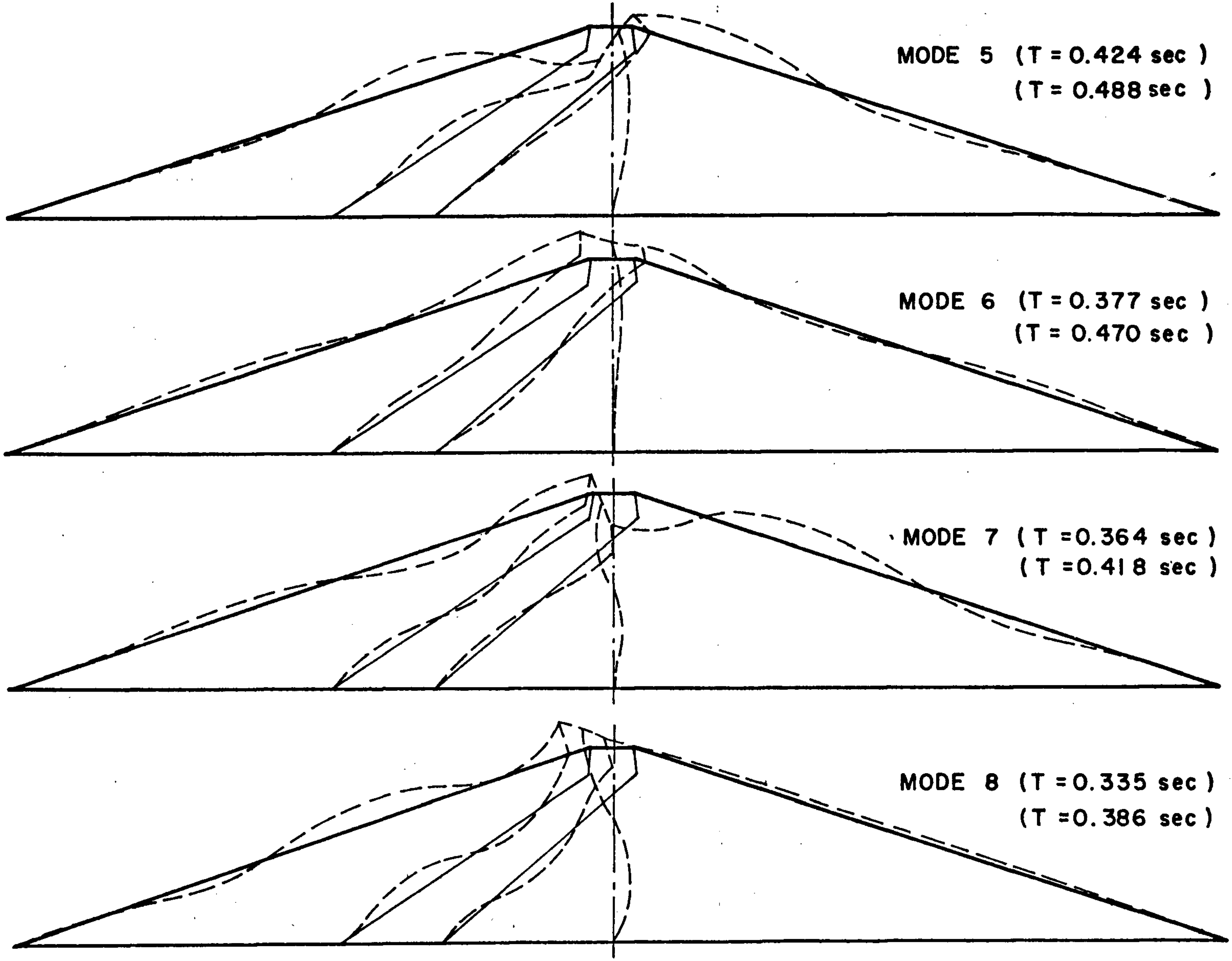


FIG. 29 MODES 5 TO 8 — DAM WITH SLOPING CORE

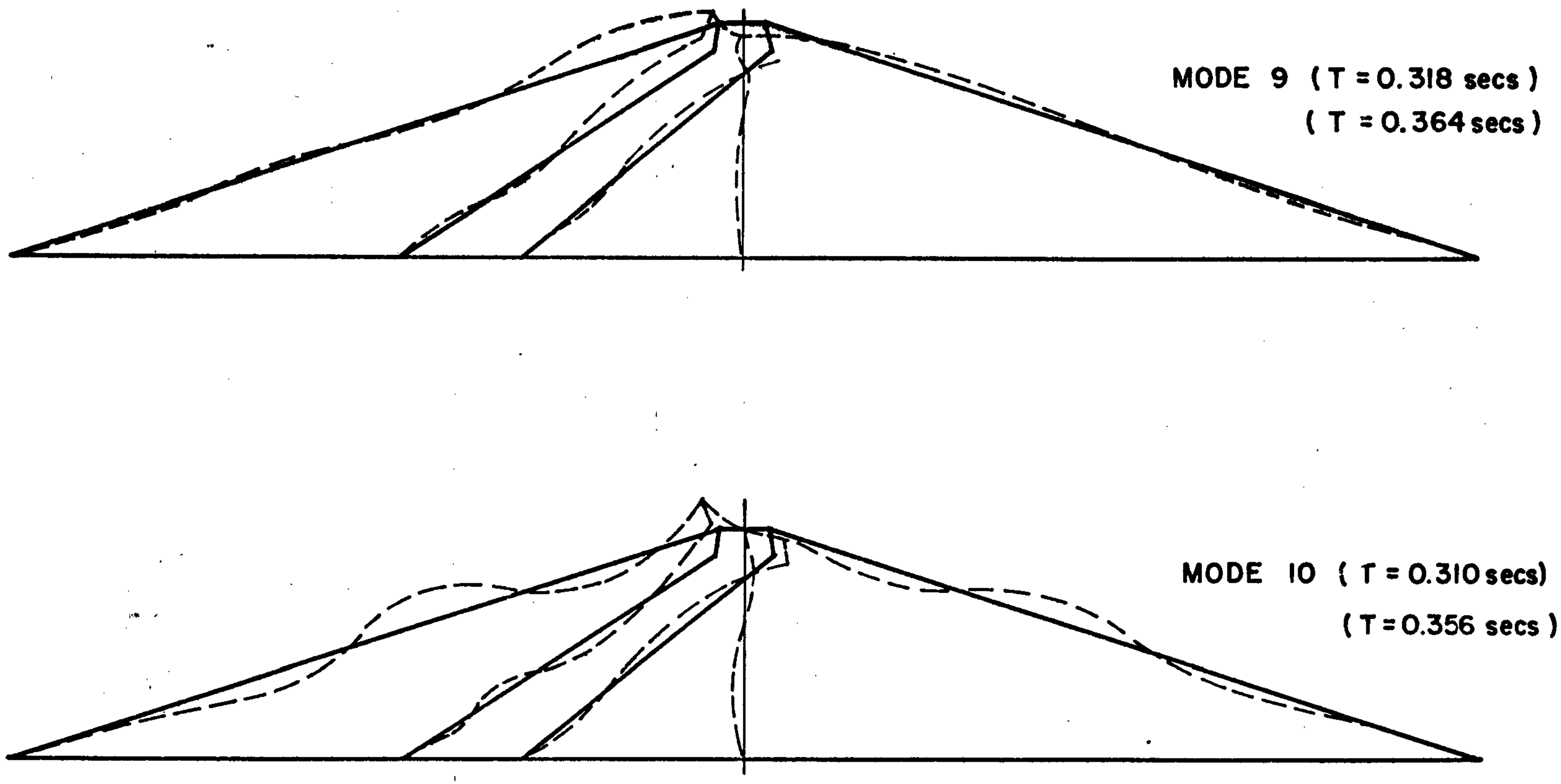


FIG. 30 MODES 9 AND 10 — DAM WITH SLOPING CORE