COST OPTIMIZATION
OF A
STRUCTURAL ROOF SYSTEM

by

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ABSTRACT

The object of this investigation is to develop a procedure for the optimization of a roof system to yield minimum cost. The roof system is composed of parallel chord, steel trusses, with purlins and decking. The variables considered are geometry, topology, and member size.

The optimal values of the geometry and topology variables are determined by the application of Box's Complex Method. A fully stressed design criterion is used in selecting the member sizes.

It is shown that the optimization scheme always converges to a low cost region, although not necessarily to the global optimum.

A cost model is developed that includes member-related costs and weight-related costs. Parameter studies demonstrate the importance of selecting the proper ratios of fixed and weight-related costs.

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NOMENCLATURE

A	= area of cross section
A*	= $0.6F_y + mC_o$
A ₁	= C ₄ + C ₅
A ₂	= C ₆ NCORD + C ₇ NSPLICE
A ₃	= C ₈
A ₄	= C ₉ + C ₁₀
B*	= mL/r
c _j	= cost per pound of steel for jth member
C	= total cost per square foot
C _o	= 20 for $F_y < 50$
C ₁	= web material cost per pound
C ₂	= compression chord material cost per pound
C ₃	= tension chord material cost per pound
C ₄	= cost of preparation per web member
C ₅	= cost of web joints per web member
C ₆	= cost of preparation per chord member
C ₇	= cost per splice for chord member
C ₈	= cost per square foot of wall cladding
C ₉	= roof cost per square foot based on truss spacing
C ₉ S	= roof weight per square foot based on truss spacing
C ₁₀	= roof cost per square foot based on purlin spacing
C ₁₀ S	= roof weight per square foot based on purlin spacing
C _p	= $535/\sqrt{F_y - 13}$
d	= depth of truss
D/S	= depth to span ratio

NPAN	= number of panels
NPOINT	= number of points in complex (k)
NSPLICE	= number of chord splices
P.L.	= panel length
PLBS	= purlin lower bound spacing
RATIO	= depth to span ratio (D/S)
r	= radius of gyration
r_i	= stiffness coefficient
r_i	= pseudo random deviate
RNB	= random number base
S	= truss span
S_i	= axial force in ith panel
SPACE	= truss spacing
SPAN	= truss span
TOL	= maximum difference between worst and best point in complex
TLBS	= lower bound on truss spacing
TUBS	= upper bound on truss spacing
W^*	= material cost of truss per square foot
WBC	= weight of tension chord material in pounds
WN	= number of web members
WTC	= weight of compression chord material in pounds
WWEB	= weight of web material in pounds
w	= distributed load
$\underline{x}^*$	= optimum vector ($x_1^*, \dots, x_n^*$)
x_1	= number of panels

DLBR	= D/S lower bound
DUBR	= D/S upper bound
F_{Ai}	= allowable axial compressive stress in ith member
f_{ai}	= axial compressive stress in ith member
F	= nxm matrix of generalized forces
F_B	= allowable bending stress
f_{bi}	= bending stress in ith member
$f(x)$	= objective function
FY	= yield point of web members
FYC	= yield point of compression chord
FYT	= yield point of tension chord
F_y	= yield point
$g_i(x)$	= constraint function
$G_{ij}^L(x)$	= lower bound on constraint function
$G_{ij}^U(x)$	= upper bound on constraint function
k	= number of points in complex
k_i	= ith panel spacing coefficient
K	= stiffness matrix (nxn)
K_1	= weight cost coefficient
K_2	= $2x_1A_1+A_1+A_2$
K_3	= $2A_3x_2$
L	= truss span
m	= $(6.77+0.079F_y)/(C_p-C_o)$
M_i	= moment in ith member
NCORD	= number of chord pieces
NIT	= maximum number of points in space to be evaluated

x_2	= depth to span ratio
x_3	= truss spacing
x_4	= purlin spacing
$x_5 \dots x_L$	= panel spacing coefficients
$x_{L+1} \dots x_n$	= member sizes
Z	= section modulus
z	= objective function ($f(x)$)
α	= over reflection factor
β	= relation between section modulus and area
Δ	= nxm matrix of generalized displacements
γ	= $wL/2$
ℓ_j	= length of jth member
ρ	= weight per cubic volume

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CHAPTER 1

OUTLINE OF DESIGN PROBLEM

Introduction

During the last decade greater emphasis has been placed on the development of rational design for a large variety of structures using automated design-analysis schemes. Heretofore, automated methods had been applied to the analysis of only those structural configurations with given member properties. The impetus for automated design has come from the aircraft industry where the most common governing criterion for optimal design is weight, subject to the satisfaction of certain functional requirements and relevant constraints. Research in the design field is now being actively pursued and applied to a great variety of civil engineering structures.

Almost invariably, the measure of a good civil engineering structure is its cost and not its weight. The design of the structure is characterized by a set of parameters which specify the configuration of the structure and the properties of the structural members. Site conditions, code requirements, aesthetic considerations and other relevant factors determine the selection of the structure parameters. The design of the structure, satisfying the above considerations and yielding minimum cost, can be obtained only if it is possible to state the requirements explicitly in mathematical form.

The purpose of this investigation is to examine a simple, common, engineering structure. The structure selected is a roof structural system which employs a truss as the main structural

element, with purlins and/or decking. The parameters necessary to describe the system are stated and divided into two categories; parameters set by the designer, and those which can be determined by analytical means. The feasibility of automating the analytic decisions is demonstrated, and a technique is devised whereby a good solution can always be obtained, although it should be noted that the solution may not necessarily be the global nor even the local minimum. The optimal solution is one for which a lower cost cannot be obtained satisfying the governing constraints, which means that the optimal solution is the global minimum of the objective function.

Brief Resume of Past Work

Discussion will be limited to work which has used a discrete element structural idealization. According to Gellatly et al (1), and Melosh and Luik (2), this work can be examined under the three following headings:

"(i) Processes which are primarily concerned with the optimization of individual components for a single loading case.

(ii) Studies of optimization of realistic structures but with conditions and methods which are restricted to very special conditions.

(iii) Analyses which recognize the importance of factors such as multiplicity of loading and which directly or indirectly advance the state-of-the-art."

The types of search procedures utilized in optimal design can be classified as direct, semi-direct and indirect. The

direct method generally encompasses categories (i) and (ii) above. Basically, this procedure consists of explicitly relating each of the structural elements to the limiting conditions (constraints). If the design variables can be selected to simultaneously satisfy the upper and lower limits on the variables, then the corresponding value of the objective function is assumed to be the optimal solution. A pictorial representation of the direct search procedure is presented in figure (1-1).

The semi-direct and indirect search procedures are used in category (iii) and are represented in figures (1-2) and (1-3). Category (iii) may be further divided into three sections.

(1) Implementation of existing design procedures (semi-direct search).

This method involves the use of building codes, engineering experience, and iterative analysis and design of the structure. Engineering experience is used to determine the configuration of the structure and the type of members, etc., to be used, based on previous experience with structures performing the same functional requirements. A design criterion is substituted for the objective function and an iterative solution technique is used. The substitution of a design criterion involves a tacit assumption of the behavior characteristic of the optimum structure, such as the optimum structure being fully stressed. The validity of this type of approach is dependent upon the degree of correlation between the design criterion and the objective function. Under this category is a paper by Gray (3) which is closely

related in scope to the present study. Gray examined the minimum weight solution of a Pratt roof truss system considering depth to span ratio, number of panels, and truss spacing, as variables. Due to the small computer size at that time (1958), several simplifying assumptions were made which limited the general usefulness of his approach.

(2) Structural synthesis (indirect search method).

One of the first and most concise formulations of the problem of optimal structural design was presented by Schmit (4). He defined the structural synthesis problem as:

"Given a set of load conditions and side conditions for a structure, find, by systematic means, a structure which will support the loads, satisfy the side conditions and maximize the merit criterion by which the structure is to be evaluated."

A list of notable contributions to this field is presented in a recent paper by Schmit (5). The structural synthesis approach employs a mathematical programming technique dependent on the type of the objective function and constraints being considered. Basically, the search procedure depends on an initial feasible point and a distance and direction of travel based on the criterion of minimizing the objective function.

(3) Application of knowledge particular to the structural field in developing algorithms for optimum design (semi-direct plus indirect search procedure).

Examples include Gellatly, Gallagher and Lubracki (1),

The Design-Decision Process

The most general problem which could be solved would be first to specify the functional requirements of the structure (i.e., loads, aesthetic requirements, etc.) and the constraints (codes, site conditions); then to examine all types of structures which satisfy the requirements and constraints, in order to determine the best structure for a given criterion (cost, weight, stiffness, etc.). Such a problem would be almost impossible to solve, and would be computationally so expensive as to make any savings in structure costs insignificant compared to the cost of achieving them. The design process as viewed by Schmit (5) is presented in figure (1-4). The function of this process is to enable the designer to formulate explicitly those areas where engineering judgment is required, and those areas where an analytical determination of structure parameters can be implemented. Engineering judgment is used to reduce the general problem to a tractable one. It applies past experience with structures that were designed to satisfy the functional requirements, and chooses the type of structure to be used, based on a non-quantifiable type of optimization.

At present, only a small number of the parameters necessary to define a complete structure can be determined analytically by an optimization routine. The input parameters will be termed external parameters for this study, and the variables determined analytically by the optimization routine will be called internal variables. The blocks on figure (1-4) are described below and labelled internal or external, and the decisions relevant to this study are underlined.

FIG. 1-4 DESIGN PROCESS BLOCK DIAGRAM

NEEDS

Needs state the functional purpose of the structure.

The function considered herein is a large warehouse type structure or any building where the roof structure employs trusses, with purlins and/or decking (external).

CRITERIA

The criterion is stated in the objective function (merit function, cost function). The objective function provides the basis of choice between alternate feasible designs. A feasible design is one which satisfies both the functional requirements and the constraints. The objective function may be formulated on the basis of cost, weight, stiffness, etc., (external).

CONCEPTS

Concepts embody the experience, judgment, and creativity of the engineer. Concepts are divided into two interrelated sections.

- (a) Form and type of structure - examples include beams, domes, trusses. The type of structure used is a bottom chord bearing, parallel chord, truss. (external)
- (b) Levels of choice based on (a).
 - (i) Material choice - wood, concrete, steel (external).
 - (ii) Framing - rigid, pinned, semi-rigid (external).
 - (iii) Behavior - elastic analysis, plastic analysis (external).
 - (iv) Topology) Configuration - number of panels, depth
Geometry) to span ratio, panel
spacing (internal).
 - (v) Type of member cross section - WF, tubes, tees, double
angles (external).
 - (vi) Member longitudinal properties - haunches, tapered,
constant (external).

DEVELOP MODELS

A linear elastic analysis is used. (external)

ANALYSIS

The stiffness matrix of the structure is formed and the solution of $K\Delta = F$ is carried out (K = structure stiffness matrix, $n \times n$, Δ represents an $n \times m$ matrix of generalized displacements, and F is an $n \times m$ matrix of generalized forces corresponding to Δ where n is the number of degrees of freedom and m is the number of load cases.) (internal)

EVALUATE QUANTITATIVELY

The acceptability of the design in terms of the behavior constraints is determined. This step includes the redesign of the structure to satisfy the constraints. (internal)

OPTIMIZE

The objective function for the current design is evaluated in this step. Based on the value of the objective function, a direction and distance of travel is chosen to provide a new set of structure variables to yield an improved value of the objective function. (internal)

The other blocks in figure (1-4) are self-explanatory and will not be elaborated upon here.

The dotted portion of figure (1-4) represents the portion of the design process automated for this study. A more detailed examination of the automated portion of the design process is shown in figure (1-5). The following relation between analysis and design, as stated by Klotz (6) is used.

"Analysis is considered to be the determination of the forces acting in a member in a structural framework subjected to a specified set of internal and/or external loads. Design is considered to be the selection of a member's cross sectional geometry and material properties to resist the member forces in accordance with the applicable stress and deflection criteria. The validity of a design involves determining if an assumed member satisfies the stress and deflection specifications for the forces determined by analysis, which was in turn based on such a member existing in the framework."

This process, coupled with the objective function evaluation, is presented in figure (1-5).

FIG. 1-5 DESIGN-ANALYSIS-OPTIMIZATION
BLOCK DIAGRAM

Organization of Presentation

A detailed examination of the system studied is presented in Chapter 2. The variables considered are stated, and the objective function and cost model are presented. The modular nature of the program developed is also outlined.

In Chapter 3, the general mathematical programming problem

is presented. A brief discussion of optimization techniques is included to clarify the problem. The problem is formulated as a mathematical programming problem, and the detailed reasons are given as to why no all-inclusive algorithm exists for its solution. The problem is then restated so a solution may be obtained.

Chapter 4 presents the algorithm used to arrive at the best solution, although no claim is made that it represents the optimal solution. An examination of the results is also included.

Data obtained from an exhaustive search procedure are presented in Chapter 5. Parameter studies are presented along with a determination of the relative sensitivities of the variables and the uniqueness of the optimum. A discussion of the cost model is also included.

Chapter 6 examines the feasibility of developing a sub-optimization scheme for the panel spacing coefficients. Results of the study are provided.

Finally, Chapter 7 presents the conclusions, recommendations for future changes in the computer program developed, and possible extensions of the study for future investigation.

CHAPTER 2

THE DESIGN PROBLEM STUDIED

Problem Statement

The problem selected is the designing of a truss to yield the minimum total cost of the entire structural roof system. The roof system is composed of parallel chord steel trusses with the roof being applied to the upper chord. The types of member to be used are Tee sections for the chords and Double Angle sections for the webs. All joints are welded. The load cases to be considered are uniform live load on the entire span and uniform live load on half the span, plus dead load for all cases. The web configurations that may be used are the Pratt system or the crossed diagonal system. The roof may be applied to the truss at the panel points or anywhere between the panel points by purlins, or directly to the top chord, the only restriction being that the purlin placement must be symmetrical about the center of the truss. For all cases, continuity of the chords must be considered. The truss used is bottom chord bearing. The design of the truss is to be made in accordance with CSA Spec. S16-1965. The chords of the truss are to be of constant area for the entire span.

The example chosen for the problem used a span of 120'-0", and the magnitude of the live load was as specified for snow load in the National Building Code.

Structural parameters to be chosen by the designer (external).

(1) Material properties

- FY - yield point of web members
- FYC - yield point of compression chord
- FYT - yield point of tension chord
- (2) Web configuration - Pratt or crossed diagonal.
- (3) Roof system
 - (i) Panel point loading
 - (ii) Purlin loading anywhere on the top chord
 - (iii) Roof deck applied directly to the top chord
- (4) Number of chord pieces and chord splices.
- (5) Values of cost parameters.
- (6) Roof model parameters.

Variables to be provided by automated scheme (internal).

- (1) All member sizes.
- (2) Depth to span ratio of truss.
- (3) Number of panels of truss.
- (4) Panel spacing coefficients (optional).
- (5) Truss spacing.
- (6) Purlin spacing if considered as a variable.

Objective Function

The cost figure considered is the cost C per square foot. C represents the cost of the structural items of the roof system, consisting of the truss, purlins (if any), deck, and wall cladding above the bottom chord. C is computed as

$$C = \text{cost/ft}^2 = \frac{1}{\text{SPAN} \cdot \text{SPACE}} \left\{ C_1 \cdot W_{\text{WEB}} + C_2 \cdot W_{\text{TC}} + C_3 \cdot W_{\text{BC}} \right. \\ \left. + (C_4 + C_5) \cdot W_{\text{N}} + C_6 \cdot N_{\text{CORD}} + C_7 \cdot N_{\text{SPLICE}} + 2 \cdot C_8 \cdot \text{RATIO} \cdot \right. \\ \left. \text{SPAN} \cdot \text{SPACE} + (C_9 + C_{10}) \cdot \text{SPAN} \cdot \text{SPACE} \right\}$$

The cost parameters C_i are defined as follows:

C_1 = web material cost per pound

C_2 = compression chord material cost per pound

C_3 = tension chord material cost per pound

C_4 = cost of preparation per web member

C_5 = cost of web joints per web member (cost of welding both ends to the chords)

C_6 = cost of preparation per chord member

C_7 = cost per splice for chord member

C_8 = cost per square foot of wall cladding

C_9 = roof cost per square foot based on truss spacing

(When purlins are used, C_9 represents the cost of purlins - when the deck is applied directly to the chord, C_9 represents the deck cost.)

C_{10} = roof cost, per square foot, based on purlin spacing or panel point distance. When the panel spacing is a variable, the roof cost C_{10} is assumed to be governed by the largest panel length if purlins are not used between the panel points.

The remaining terms in the objective function are defined as follows:

WWEB = weight of web material in pounds

WTC = weight of compression chord material in pounds

WBC = weight of tension chord material in pounds

NPAN = number of panels

WN = number of web members

= $2(NPAN) + 1$ for Pratt web configuration

= $3(NPAN) + 1$ for crossed diagonal system

NCORD = number of chord pieces
NSPLICE = number of chord splices
RATIO = depth to span ratio of truss
SPACE = truss spacing
SPAN = truss span

Cost Model

An attempt was made to provide a realistic means of establishing the cost of the roof structural system. It is doubtful, however, that every fabricator would actually break his costs down in this fashion. In this study, labour and fabrication charges were specified independently from material costs. This was done to examine whether or not a topology cost would effect a change in the optimum configuration. The topology cost was represented by C_4 and C_5 , the costs of preparing and joining, respectively, the web member.

The breakdown in material costs, C_1 , C_2 , and C_3 was incorporated to reflect the different member types (Tees and Double Angles) and to allow a variation in the strength properties. C_6 and C_7 specified the costs associated with splitting the WF sections to produce Tees, and for splicing the Tees. C_4 , C_5 , C_6 and C_7 represented the fixed costs for a given topology, and their effect on the optimum structure is examined in Chapter 4 and Chapter 5. C_8 , the cost of wall cladding per square foot, will have an effect on the best depth to span ratio. The impact of C_8 will depend on the relative magnitude of the material and wall cladding costs.

Rather than include a design routine for the purlins and

an optimization scheme to give the best solution, are exceedingly costly to develop, and generally are expensive to run. In order for a program to be economically practical, it must at least be competitive with existing design costs, or the savings gained by the application of such a program must outweigh the execution costs. Unfortunately, it is difficult to estimate the cost of development and the efficiency of a method and, more important, the magnitude of the savings gained over current design techniques.

The present program, written in Fortran IV and run on an IBM 360/67, was developed to determine the feasibility of an automated design approach purporting to yield the optimal design of a structural system.

The largest part of the time spent in running a structural optimization program is in the analysis and design phase. Every time the objective function is evaluated, a complete design must be made. Hence it is necessary to minimize the number of evaluations of the objective function, and to make the design-analysis cycle as efficient as possible.

In order to incorporate versatility into the program, it was written in modular form. Each module performs a specific function, all modules being interdependent for values used in their execution, but the execution of the module being independent of and separable from, the rest of the program. For example, if a different web configuration is to be used, a new CONFIG module is substituted and no other changes in the program are necessary. Figure (2-2) depicts the modules used. An explanation of the function of each module follows:

FIG. 2-2 PROGRAM MODULES

MODULE FUNCTIONS

- BOX contains the optimization algorithm which is Box's Complex Method. This module controls the input parameters, and determines the variables to be used in the design and analysis routines.
- ROBOT performs all the functions necessary to achieve a design specified by the variables and parameters transferred from BOX. All subsequent modules are controlled by ROBOT. ROBOT performs the necessary bookkeeping to link the modules, and provides for the output of all the design information.
- CONFIG specifies the topology of the structure. It provides the coding information for the members in order to facilitate the building of the structure stiffness matrix.

GEOM takes the data from CONFIG and ROBOT and computes the structure coordinates and the geometry matrices for the load routine. GEOM also calls PSPACE and SOLTN.

PSPACE contains the panel spacing algorithm developed in Chapter 6.

SOLTN solves the equations generated in PSPACE.

LOAD builds the load matrix for the load cases. The present load cases include uniform load on full span, uniform load on full span and on half span, uniform load on full span and a single point load at any panel point, and uniform load placed on an integer number of panels, starting from 1 to NPAN. In all cases, dead load is included and is always updated to account for change in structure weight. These load routines are fully automated and include the cases where purlins are placed randomly on the top chord. External control is provided, such that any combination of load cases may be obtained or any pattern of loading achieved.

STRUCT builds the member stiffness matrices and the structure stiffness matrix. A plane frame structural element is used. A linear elastic analysis is employed to determine the member forces.

BAND solves the equation $K \Delta = F$, using the Choleski scheme.

SYMM chooses the governing forces for the web members.

DESIGN provides the member cross-sectional properties satisfying the code requirements. Only half of the web members are designed, since only a symmetrical truss is considered.

WEB The properties of the undesigned web members are set equal to their symmetrical counterparts.

CHORD equates the chord size in each panel with the maximum chord size (constant area constraint). The allowable bending and axial stress for each panel are then found.

PRICE evaluates the objective function.

TEE contains a table of Tee and Double Angle section properties, extracted from the AISC handbook.

A constant area criterion for the chords was used for two reasons. Firstly, it provides a fabrication simplification and hence a cost saving. Secondly, in order to determine the number of different chord sizes which should be used, an optimization scheme would have to be devised in order to determine the optimal balance between chord material saving and additional splice cost. Based on computational experience, it was found that the deflection constraint was never exceeded. For that reason, the present program does not include a redesign scheme to limit deflection.

CHAPTER 3

THE MATHEMATICAL PROGRAMMING PROBLEM

Classical optimization problem

An optimization problem is one for which it is desired to maximize or minimize a function of one or more variables, subject to certain restrictions or constraints, on the variables themselves, and on functions of the variables. In order to formulate the optimization problem it is necessary to know, accurately and quantitatively, how the system variables interact; to have a measure of system effectiveness (objective function) expressible in terms of the variables; and to establish a decision apparatus whereby the best solution is chosen based on the objective function.

A general form of the optimization problem is stated as: determine a vector $x^*=(x_1^*, \dots, x_n^*)$ which maximizes or minimizes

$$\left. \begin{array}{ll} z = f(x_1, \dots, x_n) & (3-1) \\ \text{subject to } g_i(x_1, \dots, x_n) \{ \leq, =, \geq \} b_i, i=1, m & (3-2) \\ \text{and } x_j \geq 0, j=1, n & (3-3) \end{array} \right\} \text{PROBLEM A}$$

Equation (3-1) represents the objective function, equations (3-2) represent constraint conditions on x , and equations (3-3) represent non-negativity conditions on x . The first solution of PROBLEM A was by the calculus and imposed four restrictions on equations (3-1) to (3-3), namely, no inequalities appeared in equations (3-2), no non-negativity or discreteness restrictions were permitted on the variables, $m < n$, and the functions $g_i(x_1, \dots, x_n)$ and $f(x_1, \dots, x_n)$ were continuous and possessed

partial derivatives at least through second order. A problem satisfying the above restrictions is termed a classical optimization problem. Generally, however, some or all of the above restrictions are violated and other techniques must be employed to render a solution to PROBLEM A.

PROBLEM A constitutes the general statement of the mathematical programming problem. To facilitate a meaningful discussion of the non-linear programming problem, the following definitions and theorem are presented along with a brief examination of the properties of the linear programming problem.

Definition A set of points T is a convex set if every convex combination of points in T is also in T . That is, for every $\lambda, 0 \leq \lambda \leq 1$, and any two points $x_1, x_2 \in T$, $\{\lambda x_1 + (1-\lambda)x_2\} \in T$.

Definition A function f is a convex function of x in a nonempty convex set S if for every two points $x_1 \in S$ and $x_2 \in S$, and every λ , where $0 \leq \lambda \leq 1$, $f\{\lambda x_1 + (1-\lambda)x_2\} \leq \lambda f(x_1) + (1-\lambda)f(x_2)$.

Definition A function $g(x)$ is concave in a convex set S if $-g(x)$ is a convex function in S .

Definition The convex programming problem is written as follows: minimize $z = f(x)$ subject to $g_i(x) \geq 0, i=1, \dots, m$ where $f(x)$ is a convex function and each $g_i(x)$ is a concave function.

Theorem (Local-Global Convexity Property)

Every local minimum x^* of the convex programming problem is a global minimum (7).

Linear Programming Problem

Find x^* such that the constraints

$$g_i(x_1, \dots, x_n) = \sum_{j=1}^n a_{ij}x_j \{ \leq, =, \geq \} b_i, \quad i=1, m$$

$$x_j \geq 0, \quad j=1, n$$

are satisfied and the objective function

$$z = f(x_1, \dots, x_n) = \sum_{j=1}^n c_j x_j$$

is maximized or minimized, where a_{ij} , b_i and c_j are known

constants. Note that no other restrictions may be specified on the variables. For example, if it is required that all or some of the x_i be integers, PROBLEM B would then become a non-linear problem.

PROBLEM B

Four important properties of the linear programming problem are:

- (a) The set of feasible solutions which satisfy the constraints and the non-negativity restrictions is a convex set which has a finite number of corners called extreme points.
- (b) The set of all n -tuples $(x_1 \dots x_n)$ which yield a specified value of the objective function is a hyperplane. The hyperplanes corresponding to different values of the objective function are parallel.
- (c) A local maximum or minimum is also the global maximum or minimum.
- (d) If the optimal value of the objective function is bounded, at least one of the extreme points of the convex set of feasible solutions will be an optimal solution.

The linear programming problem is readily solved by the use of the simplex method, which is an exceedingly efficient algorithm.

Non-linear Programming Problem

If any of the functions are non-linear, or if integer constraints apply to any of the variables of PROBLEM A, then PROBLEM A is termed a non-linear mathematical programming problem. Terms such as convex, concave, separable, quadratic, and factorable apply to special cases of PROBLEM A. Some or all of the properties which characterize a linear programming problem may be violated in the non-linear problem. If the set of feasible solutions is convex, the optimal solution may be at an interior point of the feasible set, at an extreme point, or tangent to a constraint condition. The problem of local optima which are not the global optimum causes considerable difficulty. Only in the case of the convex programming problem is there any assurance that a local optimum is the global optimum. When the set of feasible solutions does not form a convex set or the objective function is non-linear, then the solution to the problem becomes exceedingly difficult.

Pictorial representations for the solution spaces for a linear programming problem (convex linear solution space), a non-linear problem with a convex solution space (non-linear constraints), a non-convex solution space, and a convex solution space with non-linear objective function, are shown in figures (3-1) to (3-4).

FIG. 3-1 LINEAR PROGRAMMING PROBLEM

FIG. 3-2 NON-LINEAR PROGRAMMING PROBLEM

FIG. 3-3 LOCAL AND GLOBAL OPTIMA

FIG. 3-4 NON-LINEAR OBJECTIVE FUNCTION

Solution of the Non-linear Programming Problem

At present there are three basic approaches to the solution of the non-linear problem. Depending on the form of the objective function and constraints, an iterative scheme using linear programming as the solution algorithm, can be applied (Hadley (8),

Cornell et al (9)). Application of this technique, however, does not necessarily lead to the global optimum. Another group of solution algorithms utilizes some type of gradient approach. The third method is the exhaustive search method which involves the evaluation of all possible combinations of the variables. The exhaustive search method is only used when no other algorithm exists for the solution of the problem. Basically, all optimization schemes exploit the structure of the problem in order to arrive at a solution algorithm, and hence, no general all-inclusive solution technique exists for the non-linear programming problem.

Four basic steps are required for a search procedure:

(1) An initial feasible point, (2) A direction of travel, (3) A distance of travel, and (4) A termination criterion for the search. The following outline provides a brief resume of the decisions to be made and possible ways of making them for the solution of the non-linear problem. Detailed explanations of algorithms employed in solving the non-linear problem may be found in references (8), (10), and (11).

General problem

Minimize $f(x)$ where $x = (x_1 \dots x_n) \in R^n$

General solution technique

Select an initial point x^0

For $i = 0, 1, \dots$, point $x^i = (x_1^i, x_2^i, \dots, x_n^i)$ is known.

Select a direction $s^i = (s_1^i, \dots, s_n^i)$.

Select a distance α_i (scalar).

Then, a new point is given by

$$x^{i+1} = x^i + \alpha_i s^i \text{ or, in general, } x_k^{i+1} = x_k^i + \alpha_i s_k^i$$

for $k = 1, \dots$

Check the termination criterion.

Choice of direction, s^i

Based on derivatives

(i) First order methods - steepest ascent or descent (gradient methods).

(ii) Second order methods. In the vicinity of the minimum x^* , the first partials vanish and the Taylor series expansion gives (to second order terms)

$$f(x) = f(x^*) + \frac{1}{2}(x-x^*)^T H_f(x^*) (x-x^*)$$
 where $H_f(x^*)$ is the matrix of second partials of f (Hessian of f) evaluated at x^* .

Methods to be employed include Fletcher-Powell, conjugate gradient, Powell's method.

Choice of distance α_i

Choose α to make $g(\alpha) = f(x^{i+1}) = f(x^i + \alpha s^i)$ a minimum.

Set the derivative equal to zero.

$$\frac{dg}{d\alpha} = \sum_{k=1}^n \frac{\partial f(x^{i+1})}{\partial x_k^{i+1}} \frac{dx_k^{i+1}}{d\alpha} = \sum_{k=1}^n \frac{\partial f(x^{i+1})}{\partial x_k^{i+1}} s_k^i = 0$$

or, in vector form, $\nabla f(x^{i+1})^T s^i = 0$

An alternative method for choosing α_i would be to fit a quadratic or cubic to $f(x^{i+1})$ and minimize the resulting fitted curve.

Termination criteria

(1) Based on change in slope.

$$(i) \max_k \left| \frac{\partial f}{\partial x_k} \right| < \epsilon$$

$$(ii) \sum_{k=1}^n \left(\frac{\partial f}{\partial x_k} \right)^2 < \epsilon$$

(2) Based on change in function value

$$f(x^i) - f(x^{i+1}) < \epsilon$$

(3) Based on change in x

$$\max_k |x_k^{i+1} - x_k^i| < \epsilon$$

The Problem of Structural Optimization

The structural optimization problem may be stated as: choose that set of topological, geometrical and member variables which will minimize the structure merit function subject to code requirements, continuity requirements, site conditions, aesthetic considerations and other specified behavioural requirements.

Let the design variables be contained in the vector x . The design variables are subject to two types of constraints, namely, side constraints and behaviour constraints. Side constraints are those that specify upper and lower bounds on each of the design variables, whereas behaviour constraints are functions of the design variables. Let X_i^L and X_i^U be the lower and upper limits on the design variables. Let $g_{ij}(x)$ be the i th behaviour function for the j th load condition. Further, let G_{ij}^L and G_{ij}^U be the lower and upper limits respectively on the behaviour function $g_{ij}(x)$. In general, G_{ij}^L and G_{ij}^U will depend on the values of the design variables (e.g. allowable stress in compression member). Hence, the bounds on the behavioural constraints can only be stated implicitly.

A design x , which satisfies both the side constraints and the behaviour constraints, is classed as a feasible design point in the solution space. The choice between alternate acceptable (feasible) designs is accomplished by means of the objective function $f(x)$. Using the above definitions, the problem of structural optimization has been stated in matrix form by Schmit (5) as:

"Given the preassigned parameters and the design load system (generally includes a multiplicity of distinct load conditions) as well as the design limitations X^L , X^U , $G^L(x)$, $G^U(x)$

Find x^* subject to $X^L \leq x^* \leq X^U$

and $G^L(x^*) \leq g_{ij}(x^*) \leq G^U(x^*)$ such that $f(x^*)$ takes on a minimum value."

The preceding definition constitutes the formal statement of the structural synthesis problem and is invariably a non-linear programming problem. Much work has been done for the case where $f(x)$ is a linear function and $g_{ij}(x)$ are non-linear functions. This has usually meant that only member sizes have been considered, with the configuration and material of the structure treated as constant. Even in this case, there is no guarantee that any optimum found is a global optimum.

A simple design space for a two bar truss is shown in figure (3-5). Note that all the functions are considered to be linear and that X^L is not active.

S_{ij} equals the axial force in the i th member for the j th load case.

$$S_{11} = \frac{-P_1 \sin \alpha_1 + S_{21} \cos \beta_2}{\cos \beta_1}$$

$$S_{12} = \frac{P_2 \sin \alpha_2 + S_{22} \cos \beta_2}{\cos \beta_1}$$

$$S_{21} = \frac{P_1 (\cos \alpha_1 + \sin \alpha_1 \tan \beta_1)}{\sin \beta_2 + \cos \beta_2 \tan \beta_1}$$

$$S_{22} = \frac{P_2 (\cos \alpha_2 - \sin \alpha_2 \tan \beta_1)}{\sin \beta_2 + \cos \beta_2 \tan \beta_1}$$

$$f(x) = d\rho \left(\frac{x_1}{\sin \beta_1} + \frac{x_2}{\sin \beta_2} \right) \quad (\text{minimum weight})$$

$$X^L = \{0, 0\} \quad X^U = \{x_1^u, x_2^u\} \quad G^L = \{-\sigma, -\sigma\} \quad G^U = \{\sigma, \sigma\}$$

(compression considered positive)

$$\text{Let } \beta_1 = 60^\circ, \beta_2 = 45^\circ, \alpha_1 = 20^\circ, \alpha_2 = 45^\circ$$

$$\text{Then, } S_{11} = .436P_1, S_{21} = .793P_1, S_{12} = 1.032P_2, S_{22} = -.268P_2$$

Further, let $P_1 = P_2 = P = 20$ kips and let $\sigma = 20$ ksi.

$$\text{Also, define } f'(x) = \frac{f(x)}{d\rho} = 1.15x_1 + 1.41x_2$$

The design space is as shown below.

FIG. 3-5 DESIGN SPACE FOR TWO BAR TRUSS

Formulation of the thesis problem as a mathematical programming problem.

Let x_1 = number of panels	}	TOPOLOGY
x_2 = depth to span ratio		
x_3 = truss spacing	}	GEOMETRY
x_4 = purlin spacing		
$x_5, \dots, x_L$ = panel spacing coefficients		
$x_{L+1}, \dots, x_n$ = member sizes		

Minimize $f(x_1, \dots, x_n)$

$$= \frac{1}{Sx_3} \left\{ \sum_{j=L+1}^{j=n} \rho l_j x_j c_j + 2x_1 A_1 + A_1 + A_2 + 2A_3 Sx_2 x_3 + A_4 Sx_3 \right\}$$

where $S = \text{span}$

$\rho = \text{weight per cubic volume}$

$c_j = \text{cost per pound of steel; one of } C_1, C_2 \text{ or } C_3.$

$l_j = \text{length of } j\text{th member.}$

$A_1 = C_4 + C_5$

$A_2 = C_6 \text{NCORD} + C_7 \text{NSPLICE}$

$A_3 = C_8$

$A_4 = C_9 + C_{10} = g(x_3, x_4)$

(C_i is defined in Chapter 2)

subject to

$6 \leq x_1 \leq 16, x_1 \text{ an even integer}$

$\text{DLBR} \leq x_2 \leq \text{DUBR}$

$\text{TLBS} \leq x_3 \leq \text{TUBS}$

$\text{PLBS} \leq x_4 \leq S/x_1$

$x_5, \dots, x_L > 0$

$x_{L+1}, \dots, x_n \geq \text{minimum area in member table}$

$\sum_{j=5}^{j=L} x_j - 0.5 = 0$

(DLBR, DUBR, TLBS, TUBS, and PLBS are defined in Chapter 4.)

$x_{L+1}, \dots, x_n$ must satisfy all code requirements and are chosen from a discrete member spectrum.

The problem as stated, is a non-linear, non-convex mathematical programming problem for which no all-inclusive optimization algorithm exists.

Investigation of related work

The cost optimization of topology and geometry for structural frames has been investigated by Soosaar (13). Soosaar considered the cost to be composed of two major components: the fixed component which represents the cost of including a member, and the weight related costs. His work treated the case of rectangular multi-storey frames and is summarized as follows (12):

"...It is assumed, and later demonstrated, that for the purposes of configurational optimization, a determinate or similar simplified treatment is sufficient as well as necessary to model the actual complex elastic behaviour of the frame. The determinate portal and cantilever methods are employed for wind behaviour; gravity loads are treated by a combination of inflection-point assumption and a one or two-cycle moment distribution. Since the structure is then "determinate", the optimum member-level solution is fully stressed. A continuous spectrum of member sizes is assumed as well as functional relationships among the member properties. Member selection is by design sub-optimization based on the AISC specification.

These above assumptions result in major simplifications in the formulation in which the cost objective function contains only the topological (member existence) and geometrical (span length) variables, and in which the constraints are linear. A solution to the mathematical programming problem is obtained by a two-stage process using classical optimization to determine the topology level solution, followed by a quadratic programming algorithm to determine the geometry...."

Two important conclusions stated by Soosaar regarding the optimum configuration are:

"(1) The optimum configuration of the frame is nearly wholly dependent on the relative proportions of the weight costs and fixed costs. High fixed member costs lead to fewer columns.

(2) Unlike the objective functions in most structural optimization problems, the cost vs. topology curves for frames are far from flat when fixed costs are included; data obtained from steel fabricators indicate that the fixed costs are high. A minimum weight optimization (i.e., zero fixed costs) can result in a structure actually more costly by as much as 70%."

It is noted that Soosaar treated the storey height and frame spacing as input parameters. His approach to the configuration problem was not used for this study because of the following reasons:

(1) For a multiplicity of load cases, the explicit formulation of member forces in terms of geometric variables is not practical.

(2) Soosaar demonstrated the hierarchy of variables to be topology, geometry, and member sizes. Based on this division he was able to show that the optimization could proceed on these three levels sequentially without altering the previous optimization level (e.g., the geometry optimization did not affect the optimal topology). The division, however, of the configuration problem into a two-level solution procedure is not valid when the geometrical variables, truss depth and spacing, are considered. (The division is valid when the geometry level consists only of the panel spacing.) The effect on the objective function, of a

change in truss spacing or depth to span ratio, is of much greater magnitude than a change in topology (see Chapter 5). This implies a different hierarchy of variables than assumed by Soosaar.

(3) The formulation of the member sizes in terms of geometrical variables, using a fully stressed design criterion, is not possible if constraints are put on member sizes (e.g., chords of constant area).

In almost all work to date, the problem of member level optimization has been considered using a continuous member spectrum, and assuming functional relationships between the member properties. Gellatly et al (1), Gellatly (14), and Cornell et al (9), have made notable recent contributions to this field and have also included geometry as a variable. In all cases, weight has formed the basis of the objective function. The problem of a discrete member spectrum has been treated by Toakley (15). As of yet, no efficient algorithm exists for the solution of large integer programming problems, and further, considerable difficulty has been encountered in assuring convergence of the algorithm used.

In order for an all-inclusive algorithm to be applicable to the problem at hand, it would have to be capable of handling a mixed integer-continuous programming problem with a non-linear objective function, subject to non-linear constraints. At present, no such algorithm exists, and even if the problem is subdivided based on a hierarchy of variables, efficient algorithms providing the optimal solution for each subdivision do not exist.

The problem investigated herein was divided as follows in order to achieve a solution, although not an optimal one.

(1) The variables $x_1, \dots, x_4$ are considered in a separate optimization scheme. ($x_5, \dots, x_L$ were also included but later deleted.) (Chapter 4)

(2) $x_5, \dots, x_L$ are generated in a sub-optimization scheme. (Chapter 6)

(3) A fully stressed design for $x_{L+1}, \dots, x_n$ was performed, subject to code requirements and constant area constraints.

A fully stressed design is considered to be one where every member is stressed to the full allowable limit under at least one load condition. The constraint of constant area for the chords meant that only one section would govern the design of the chords. The algorithm developed for panel spacing in Chapter 6 used this fact to try and stress the compression chord in each panel to the full allowable limit, to determine whether such a procedure would lead to a reduction in the chord weight.

CHAPTER 4

SOLUTION OF OPTIMIZATION PROBLEM

Minimization algorithm (Box's Complex Method)

The algorithm used to determine the optimal values of x^* is examined, and the validity of the fully stressed design as the minimum weight design for x^* , is discussed.

The algorithm employed in the program was developed by M.J. Box (16). The method, called the Complex Method, was devised for finding the maximum or minimum of a general non-linear function of several variables within a constrained region. This constrained optimum is characterized by some of the variables lying at the limits of their permissible ranges. Box points out that if the problem studied does not have a constrained optimum, then a method with no provision for bounding the variables will produce the same result. Although the variables used in this study do not, in general, lie at the extremities of their ranges, Box's method is of such a general nature that it can be adapted to the present problem.

Box's Complex Method searches for the maximum value of a function $f(x_1, \dots, x_n)$ subject to m constraints of the form $g_k \leq x_k \leq h_k$, $k=1, m$ where $x_{n+1}, \dots, x_m$ are functions of $x_1, \dots, x_n$, and the lower and upper constraints g_k and h_k are either constants or functions of $x_1, \dots, x_n$. (For a minimization problem, $-f$ is maximized.) Assume that the set of feasible solutions is not a null set. Then, an initial point $x^0 = \{x_1^0 \dots x_n^0\}$ which satisfies the m constraints, must exist. For the search procedure, $k > n+1$ points form the complex, of which one is the

initial point. The $k-1$ points used with the initial feasible point to form the starting complex, are obtained independently by the use of pseudo random numbers, x_i being computed as $x_i = g_i + r_i(h_i - g_i)$ where r_i is a pseudo random deviate rectangularly distributed over the interval $(0,1)$. According to Box, a point so selected must satisfy the explicit constraints but not the implicit constraints. (The implicit constraints are $x_{n+1} \dots x_m$.) For the program developed, an implicit constraint existed only for the panel spacing coefficients, and it was always satisfied before evaluation of the objective function.

Following the generation of the $k-1$ points, the objective function is evaluated at each point. The point corresponding to least function value, x^J , is replaced by a new point, x^M . x^M is located $\alpha \geq 1$ times as far from the centroid of the remaining points (x^J excluded) as the reflection of the worst point in the centroid. x^M is collinear with the rejected point and the centroid of the retained points. Note that the centroid of the objective function cannot be used, as the value of the corresponding variables cannot be determined. If the new point is no improvement over the worst point, α is halved. This procedure is repeated until a constraint is violated or no improvement in the objective function can be made.

If a trial vector x does not satisfy an explicit constraint on a variable x_i , the variable is set equal to a value ϵ inside the appropriate limit.

"The use of over-reflection by a factor $\alpha > 1$, tends to cause

a continual enlargement of the complex and thus to compensate for the moves halfway toward the centroid. Furthermore, it enables rapid progress to be made when the initial point is remote from the optimum. It is also an aid towards maintaining the full dimensionality of the complex. So, too, is the use of $k > n+1$ points, since with $k = n+1$ points only, the complex is liable to collapse into a subspace. In particular, it tends to flatten itself against the first located constraint and thus be unable to move along an additional constraint when a corner is reached." (16)

The values of α and k suggested by Box are 1.3 and $2n$, respectively, with the algorithm not being prejudiced adversely by using alternative values. The effect of varying α and k is studied in the results presented.

The type of optimum (local or global) derived from the algorithm can be checked by initiating the search from a different starting point and from a different random number base. It is of interest to note that Box assumed that the set of feasible solutions formed a convex set.

TOPOLOGY x_1	D/S RATIO x_2	TRUSS SP. x_3		
TOPOLOGY x_1	D/S RATIO x_2	TRUSS SP. x_3	PURLIN SP. x_4	
TOPOLOGY x_1	D/S RATIO x_2	TRUSS SP. x_3	PANEL SP. x_5, x_L	
TOPOLOGY x_1	D/S RATIO x_2	TRUSS SP. x_3	PURLIN SP. x_4	PANEL SP. x_5, x_L

TABLE 4-1

GROUPS OF VARIABLES TREATED BY ALGORITHM

Program Discussion

The groups of variables which the optimization algorithm was set up to handle are summarized in Table (4-1). The parameters input to the algorithm are listed and described below.

NPOINT	Number of points in the complex (k)
NIT	Maximum number of points in the space to be considered (includes initial complex). NIT was incorporated to limit the computational cost, and hence it acts as a termination criterion.
TOL	TOL is the maximum percentage difference between the worst point and the best point of the complex. (Termination criterion)
ALPHA (α)	over-reflection factor
TLBS,TUBS	lower and upper bounds respectively on the truss spacing.
DLBR,DUBR	lower and upper bounds respectively on the truss depth to span ratio.
PLBS	lower bound on purlin spacing. The upper bound, PUBS, was considered to be the panel length.
RNB	random number base.

The topology constraints were built into the program.

Modifications to Box's Method implemented in the Program

When more than one explicit constraint was violated, α was halved, and the objective function of this new point was evaluated only if the explicit constraints were satisfied by the new point. If the constraints were still not satisfied, the

centroid was used as the new point, as it must always be a feasible point. If the objective function of a new point was worse than the previous worst point of the complex, α was again halved. If no improvement was made, the centroid was used. If still no improvement resulted, a point midway between the best point of the complex and the centroid was used. If no improvement resulted, the search was terminated. For the case when the region surrounding the optimum was very flat, these modifications managed to keep the search in progress. For the examples run, the region surrounding the optimum was found to be very flat, and hence, many alternate designs having the same cost were possible.

Because a non-optimal topology solution could contain a geometry optimum, adjacent topologies were examined. Although this had the effect of making the search more expensive, it greatly aided the convergence of the complex to the area of the optimum. A flow chart of the optimization process is displayed in Appendix A.

Presentation of Results

A number of computer runs were made to study the effect of varying the number of points in the complex, the value of α , the starting point for the optimization, and the random number base. For this purpose the cost parameters for the model outlined in Chapter 2 are as follows:

$$\begin{aligned} C_1 = C_2 = C_3 &= \$0.08 & C_4 + C_5 &= \$13.00 \\ C_6 &= \$15.00 & C_7 &= \$5.00 & C_8 &= \$0.20 \end{aligned}$$

	TRUSS SPACING IN FEET				
	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25
C_9 (\$/FT ²)	.06	.10	.18	.24	.32
C_9S (#/FT ²)	.50	1.00	1.60	2.10	4.00

	PURLIN SPACING IN FEET				
	0 - 6	6 - 10	10 - 17	17 - 24	24 - 29
C_{10} (\$/FT ²)	.25	.50	.75	1.00	1.50
$C_{10}S$ (#/FT ²)	3.00	6.00	9.00	12.00	18.00

For run 17, C_8 was set at \$2.00. For run 18, $C_4 + C_5$ was set at \$40.00. Except for runs 15 and 16 where the live load was 60 psf, the live load used was 40 psf. For run 19 the upper limit on the truss spacing was 40 feet and the cost associated with truss spacing was set equal to zero. A36 steel was used for all designs.

Table (4-2) lists the input data for each of the runs made, and Table (4-3) lists the results. Appendix B contains a typical computer output. A comparison of the algorithm answer and the results from an exhaustive search procedure is shown in figures (4-1) to (4-5). For the panel point loading case, the second best point of the final complex was used in figure (4-2), since the best result from the exhaustive search was employed as an initial point and could not be improved upon. Table (4-4) lists the initial and final complex for run 6.

(a) $k = 1.5n$ $\alpha = 1.3$ (runs 1, 2, 3)

In all cases, a good initial point was used. For run 3,

the best point of run 2 was used as the initial point. Because the complex was small, the worst points were removed quickly and convergence to a low cost, feasible region, resulted. It was noted in run 1, however, that the complex became locked in a subspace and no improvement could be made. The major advantage of a small complex is its replacement by a better complex with the minimum number of objective function evaluations. The major disadvantage of a small complex is that a uniform distribution over the solution space cannot be obtained with a small number of points for the initial complex, and hence the tendency to get locked in a subspace is more prevalent.

(b) $k = 2n$ $\alpha = 1.3$ (runs 4, 5, 6, 9)

The same initial point was used for all runs except 6. The only variation permitted was in the random number base. The use of a larger complex allowed a more uniform distribution of starting points. The initial point of run 6 was purposely chosen to be a poor starting point in order to judge the bias created by a good initial point. Apparently, the effect of the choice of the initial point does not appreciably alter the final result.

(c) $k = 2.5n$ $\alpha = 1.3$ (runs 11 & 12)

The effect of a large complex is to make the convergence of the entire complex to a low cost region very slow. It was noted, that while in parts (a) and (b) above, the maximum percentage difference between the best and worst point of the final complex was of the order of $\frac{1}{2}$ to 3 per cent, the difference for $k = 2.5n$ was approximately between 3 to 6 per cent.

(d) Effect of α (runs 7, 8, 9, 10)

The initial point and the random number base were held constant for all runs. Except for run 7, the choice of α does not appear to be critical. For run 7, the complex became locked in a subspace, and while small improvements could be made, the small value of α precluded the opportunity to enlarge the region of the complex sufficiently to allow movement of the complex from the subspace.

RUN	NPOINT	NIT	α	RNB	D/S RATIO		TRUSS SP.		PURLIN SP.	
					DLBR	DUBR	TLBS	TUBS	PLBS	PUBS
1	6	40	1.3	.45	.07	.14	12.	25.	4.	P.L.
2	6	20	1.3	.45	.07	.12	15.	25.	4.	4.
3	6	40	1.3	.75	.07	.12	15.	25.	4.	P.L.
4	8	30	1.3	.90	.07	.12	15.	25.	4.	P.L.
5	8	40	1.3	.10	.07	.12	15.	25.	4.	P.L.
6	8	30	1.3	.45	.07	.12	15.	25.	4.	P.L.
7	8	40	1.1	.45	.07	.12	15.	25.	4.	P.L.
8	8	40	1.2	.45	.07	.12	15.	25.	4.	P.L.
9	8	30	1.3	.45	.07	.12	15.	25.	4.	P.L.
10	8	30	1.4	.45	.07	.12	15.	25.	4.	P.L.
11	10	30	1.3	.45	.07	.12	15.	25.	4.	P.L.
12	10	30	1.3	.80	.07	.12	15.	25.	4.	P.L.
13	8	30	1.3	.45	.07	.12	15.	25.	P.L.	
14	8	30	1.3	.20	.07	.12	15.	25.	P.L.	
15	8	30	1.3	.45	.07	.12	15.	25.	4.	P.L.
16	8	30	1.3	.10	.07	.12	15.	25.	4.	P.L.
17	8	30	1.3	.60	.07	.12	15.	25.	4.	P.L.
18	8	30	1.3	.70	.07	.12	15.	25.	4.	P.L.
19	8	30	1.3	.80	.07	.12	15.	40.	4.	P.L.

NOTE: For all runs, TOL equalled 0.005, and the lower and upper bounds on the number of panels were 6 and 16, respectively.

P.L. designates panel point loading.

TABLE 4-2 INPUT DATA

(e) Runs 13 to 19

Runs 13 to 19 investigate trusses subjected to panel point loading, the effect of increased load, and changes in the parameter values of the cost model. (See figures (4-2), (4-3), (4-4), (4-5).)

From figures (4-1) to (4-5), it is noted that the correspondence of the best algorithm point to the best exhaustive search point is not always exact. Two reasons are put forward to account for this discrepancy. Firstly, the exhaustive search procedure considered only a very limited number of points, and therefore it is possible that a better answer exists, although any improvement in the best point would not be greater than about 1 per cent. Secondly, the truss spacing variable in the optimization routine was treated as a continuous variable. In fact, it should have been treated as a discrete variable because of the assumed form of the cost model. The following study verifies the latter conclusion.

RUN	I N I T I A L P O I N T S						F I N A L P O I N T S						SEARCH TERM- INATION CRITERION
	x_1	x_2	x_3	x_4	WEIGHT	$f(x)$	x_1^*	x_2^*	x_3^*	x_4^*	WEIGHT	$f(x^*)$	
1	10	.100	15.	6.	9231.	1.0708	10	.1000	15.	6.	9231.	1.0708	NO IMPROVEMENT
2	10	.100	15.	4.	9034.	1.0621	8	.0839	19.15	4.	11375.	1.0463	NIT
3	8	.084	19.2	4.	11376.	1.0449	8	.0834	19.35	4.	11389.	1.0410	NO IMPROVEMENT
4	10	.100	15.	6.	9231.	1.0708	10	.0950	15.	4.615	8986.	1.0508	NO IMPROVEMENT
5	10	.100	15.	6.	9231.	1.0708	8	.0829	19.38	4.	11379.	1.0397	NIT
6	12	.120	15.	10.	9233.	1.3578	8	.0932	19.89	4.286	11533.	1.0357	NIT
7	10	.100	15.	6.	9231.	1.0708	10	.097	19.01	5.		1.0565	TIME TERM
8	10	.100	15.	6.	9231.	1.0708	8	.0992	19.95	4.	11615.	1.0394	TOL
9	10	.100	15.	6.	9231.	1.0708	10	.0972	19.79	4.	11029.	1.0448	NIT
10	10	.100	15.	6.	9231.	1.0708	12	.098	19.85	4.	10358.	1.0430	NO IMPROVEMENT
11	10	.100	15.	6.	9231.	1.0708	10	.0801	15.	5.	9175.	1.0604	NO IMPROVEMENT
12	12	.120	15.	6.	9725.	1.1297	10	.0971	19.44	4.615	11017.	1.0537	NIT
13	10	.100	15.	P.L.	9379.	1.5774	12	.0964	15.	P.L.	8608.	1.3206	NO IMPROVEMENT
14	12	.100	20.	P.L.	10866.	1.3068	12	.1000	20.	P.L.	10866.	1.3068	NIT
15	10	.100	20.	6.	14826.	1.1671	10	.0962	18.49	4.	13195.	1.1587	NO IMPROVEMENT
16	12	.120	15.	10.	11565.	1.4614	10	.1063	17.82	4.615	13110.	1.1832	NIT
17	10	.100	20.	6.	11302.	1.4096	10	.0700	18.47	4.286	11569.	1.3425	NIT
18	12	.100	20.	6.	10994.	1.3423	8	.0894	19.71	4.615	12657.	1.2708	NO IMPROVEMENT
19	12	.120	15.	6.	9725.	0.9497	10	.1102	4.	6.	19467.	0.6700	NIT

TABLE 4-3 RESULTS OF OPTIMIZATION

I N I T I A L C O M P L E X					
x_1	x_2	x_3	x_4	WEIGHT	$f(x)$
12	.120	15.	10.	9233.	1.3578
14	.086	18.50	7.5	11637.	1.3949
6	.075	18.95	20.	15629.	1.9249
8	.105	17.07	10.	13243.	1.4416
16	.098	22.43	6.	12628.	1.1698
16	.084	22.63	5.	12976.	1.1698
12	.073	16.87	4.29	10092.	1.1130
6	.078	19.42	15.	20520.	1.8284
F I N A L C O M P L E X					
x_1	x_2	x_3	x_4	WEIGHT	$f(x)$
8	.0879	19.34	4.	11462.	1.0456
10	.1078	19.79	4.615	11319.	1.0588
12	.0985	19.12	4.286	10147.	1.0553
8	.0961	18.62	4.615	11343.	1.0648
8	.0809	17.95	4.	11007.	1.0661
8	.0969	19.59	4.615	11623.	1.0481
8*	.0932*	19.89*	4.286*	11533.*	1.0357*
12	.0990	19.81	4.	10396.	1.0456
* denotes optimum					

TABLE 4-4

INITIAL AND FINAL COMPLEXES FOR RUN 6

FIG. 4-1 COMPARISON OF OPTIMIZATION RESULTS WITH EXHAUSTIVE SEARCH

FIG. 4-2 COMPARISON OF OPTIMIZATION RESULTS WITH EXHAUSTIVE SEARCH

FIG. 4-3 COMPARISON OF OPTIMIZATION RESULTS WITH EXHAUSTIVE SEARCH

FIG. 4-4 COMPARISON OF OPTIMIZATION RESULTS WITH EXHAUSTIVE SEARCH

FIG. 4-5 COMPARISON OF OPTIMIZATION RESULTS WITH EXHAUSTIVE SEARCH

The objective function may be written as

$$\begin{aligned}
 f(x) &= \frac{1}{Sx_3} (K_1 W(x_1 \dots x_n, S, F) + 2x_1 A_1 + A_1 + A_2 + 2A_3 x_2 x_3 S \\
 &\quad + A_4 x_3 S) \\
 &= \frac{W^*(x_1 \dots x_n, S, F) + 2x_1 A_1 + A_1 + A_2 + 2A_3 x_2 + A_4}{Sx_3} \quad (4-1).
 \end{aligned}$$

where $W^*(x_1 \dots x_n, S, F)$ represents the material cost of the truss per square foot,

S is the span of the truss,

F represents the magnitude of the loading,

$x_1 \dots x_n$ are defined in Chapter 3.

Let $K_2 = 2x_1 A_1 + A_1 + A_2$ equal the topology cost plus the chord labour cost. Hence, K_2 represents the fixed cost per truss and is independent of the area supported by the truss.

Let $K_3 =$ wall cladding cost $= 2A_3 x_2$. Also, $A_4 = C_9(x_3) + C_{10}(x_4)$.

Hence, f becomes

$$f(x) = \frac{W^*(x_1 \dots x_n, S, F) + K_2 + K_3 + C_9(x_3) + C_{10}(x_4)}{x_3 S} \quad (4-2)$$

Consider S and x_4 to be constant. The change of f with respect to x_3 is given as

$$\frac{\partial f}{\partial x_3} = \frac{\partial W^*}{\partial x_3} - \frac{K_2}{x_3^2 S} + \frac{\partial C_9(x_3)}{\partial x_3} \quad (4-3)$$

Consider a region of the cost function, $x_3^0 \leq x_3 \leq x_3^1$, where C_9 is constant. Then,

$$\frac{\partial f}{\partial x_3} = \frac{\partial W^*}{\partial x_3} - \frac{K_2}{x_3^2 S}, \quad (4-4)$$

and the value of the objective function at a point x_3 in the interval is given by,

$$f = f_0 + \int_{x_3^0}^{x_3^1} \frac{\partial f}{\partial x_3} dx_3 \quad (4-5)$$

Two possibilities arise for the given region.

- (1) If the slope $\frac{\partial f}{\partial x_3}$ is of constant sign in the region considered, then f should be evaluated at one of the end points, depending on the sign of $\frac{\partial f}{\partial x_3}$.
- (2) If $\frac{\partial f}{\partial x_3}$ changes sign in the interval, values of f between the end points should be examined.

From computational experience, $\frac{\partial W^*}{\partial x_3}$ was found to be negative, and hence, $f(x)$ should be evaluated at the upper limit of constant roof cost with respect to truss spacing. An alteration in the program reflecting the above conclusion was implemented for a second set of cost parameters in Chapter 5. The effect of evaluating the objective function between the limits of a region of constant C_9 cost is particularly pronounced for runs 15, 16, and 17. If the objective function had been evaluated at the upper limit of the constant cost region for run 16, approximately 3 to 4 cents per square foot would have been saved, (based on slope of $\frac{\partial f}{\partial x_3}$), thereby making the algorithm answer comparable to the best exhaustive search answer.

Note in Table (4-2) that approximately 50 per cent of the runs were terminated by the iteration counter NIT which was employed to control the computation cost. Probably the best way to use the program would be to make an initial run with wide constraints and then make a second run, tightening the constraint limits, and changing the random number base.

Validity of the fully stressed design

A claim is made, that given $x_1 \dots x_4$, a unique design results from the fully stressed design procedure, and this design corresponds to the minimum weight design. A fully stressed design is considered to be a design where, for a multiplicity of loading conditions, the governing section of each member is stressed to the full allowable stress under at least one loading condition. A minimum weight design, on the other hand, is an arrangement of structural elements where all the design requirements such as stresses, deflections, and geometric constraints are satisfied and the total structural weight is minimized. The minimum weight design problem is formalized by stating it as a mathematical programming problem.

The relation of the minimum weight design to the fully stressed design is discussed by Razani (17), who considered the minimum weight design of trusses not subject to buckling, and Soosaar (13), who extended Razani's work to include a combined stress state. The criterion proposed uses the Kuhn Tucker optimality conditions and indicates whether or not the fully stressed design is a local optimum, but does not guarantee that the local optimum is the global optimum in a non-convex space. The method developed by Razani is not used herein.

Two basic questions regarding the fully stressed design must be answered. First: Does the method of fully stressed design converge to a unique answer regardless of the initial starting point? Second: Is the fully stressed design the minimum weight design?

The design of the members of the trusses considered in this study was always governed by stress limitations. At no time was the deflection constraint active. Note that in the case of active deflection constraints, the minimum weight design is not a fully stressed design.

For a statically determinate structure, the fully stressed design is always unique since the member forces are independent of the member properties. Razani demonstrated that the minimum weight problem for a determinate structure is a linear programming problem, and as discussed in Chapter 3, any local optimum is also a global optimum, and further, the optimum must occur at one of the extreme points. Because a fully stressed design occurs at an extreme point, the minimum weight design corresponds to the fully stressed design.

The uniqueness of the fully stressed design for this study for a given truss configuration was determined by a computational method. The initial design was started at different extreme points, and in all cases, converged to the same design.

If the structure can be reduced to a determinate structure, then the argument relating the fully stressed design to the minimum weight design may be employed. Assume the geometry and topology of the truss is fixed. Consider the truss web members. Neglecting the shear carried by the continuous chords, the web member forces are determined by the laws of statics and hence the forces are independent of the cross-sectional properties of the web members. (Single diagonal system considered.) Applying the constraint of constant area to the chords in effect reduces

them to determinate members. The distribution of axial force due to gravity loads is independent of the chord cross-sectional properties. For constant panel spacing, the bending stiffness I/L of the compression chord of each panel is uniquely set, and hence the moment pattern is established. Therefore, the same section of the chord will always govern the design of the chord. Based on the argument that the force distribution is independent of the member properties, the fully stressed design is the minimum weight design.

In Chapter 6 it is demonstrated that the preceding argument does not hold if the panel lengths are allowed to vary during the design cycle. The study in Chapter 6 attempts to stress the compression chord to the full allowable limit at more than one section by maintaining the constant area criterion but varying the panel lengths. It is shown that fully stressing the compression chord at least at one section in every panel can lead to a heavier design than stressing the chord to the allowable limit in only one panel.

CHAPTER 5

PARAMETER STUDIES

Results of optimization

Extensive results are presented in order to: (i) Investigate the effect of variations in the cost parameters; (ii) To determine the sensitivity of the objective function to changes in the structure variables; and, (iii) To examine the uniqueness of the optimum. The effect of varying the magnitude of the gravity load is also briefly discussed. A second set of cost parameters was used, in addition to that in Chapter 4. Both sets are listed in Table (5-1), and are designated as "low costs" and "high costs". In the first set (used previously) the labor costs for preparing and welding all members are substantially lower than in the second. Similarly, the wall cladding cost (C_8) is much lower in the first set. The differences in all other items are fairly small.

The total cost per square foot, C , is given as:

$$C = \frac{1}{\text{SPAN} \cdot \text{SPACE}} (C_1 \cdot \text{WWEB} + C_2 \cdot \text{WTC} + C_3 \cdot \text{WBC} + (C_4 + C_5) \cdot \text{WN} + C_6 \cdot \text{NCORD} \\ + C_7 \cdot \text{NSPLICE} + 2C_8 \cdot \text{RATIO} \cdot \text{SPAN} \cdot \text{SPACE} \\ + (C_9 + C_{10}) \cdot \text{SPAN} \cdot \text{SPACE})$$

(See Chapter 2 for definitions)

A span of 120'-0" was used throughout. All members were designed on the basis of 36 k.s.i steel. A live load of 40 psf was used, except when the effect of the magnitude of live load was examined. Only even panel spacing was considered. A very coarse grid of structure variables was evaluated to obtain the exhaus-

tive search results.

The exhaustive search grids were made as follows:

ITEM	"LOW COSTS"		"HIGH COSTS"	
C ₁	\$ 0.08		\$ 0.08	
C ₂ =C ₃	\$ 0.08		\$ 0.09	
C ₄ +C ₅	\$13.00		\$31.00	
C ₆	\$15.00		\$15.00	
C ₇	\$ 5.00		\$40.00	
C ₈	\$ 0.20		\$ 1.50	
NCORD	4		6	
NSPLICE	2		4	
C ₉	0-5 ft	\$0.06	0-17ft	\$0.17
	5-10	.10	17-24	.25
	10-15	.18	24-31	.34
	15-20	.24	31-38	.36
	20-25	.32	38-45	.42
C ₉ S	0-5 ft	.50 lbs	0-17ft	.566 lbs
	5-10	1.00	17-24	.830
	10-15	1.60	24-31	1.141
	15-20	2.10	31-38	1.503
	20-25	4.00	38-45	1.742
C ₁₀	0-6 ft	\$0.25	0-7 ft	\$0.40
	6-10	.50	7-8	.50
	10-17	.75	8-11.5	.70
	17-24	1.00	11.5-12.5	.80
	24-29	1.50	12.5-20	1.00
C ₁₀ ^S	0-6 ft	3.00 lbs	0-7 ft	2.020 lbs
	6-10	6.00	7-8	2.400
	10-17	9.00	8-11.5	2.500
	17-24	12.00	11.5-12.5	3.000
	24-29	18.00	12.5-20	3.664

TABLE (5-1)

COST PARAMETERS

VARIABLE	LOWER BOUND	UPPER BOUND	INCREMENT	
TRUSS SPACING	15.00	25.00	2.5	
D/S RATIO	0.08	0.12	.02	"LOW
NUMBER OF PANELS	8	16	2	COSTS"
PURLIN SPACING	4.	4.		
TRUSS SPACING	17.00	45.00	3.5	
D/S RATIO	0.07	0.11	.01	"HIGH
NUMBER OF PANELS	8	14	2	COSTS"
PURLIN SPACING	6.	6.		

For the "low costs", depth to span ratios of 0.07, 0.09, and 0.11 were evaluated for the 20'-0" truss spacing for all topologies.

For the "high costs", the roof deck was based on a wood system. Although the cost figures C_9 and C_{10} are not exact, the ratios between the steps of the roof model are of the correct magnitude. The numerical values of C_1 to C_8 are not intended to represent actual, current, fabrication charges. Results for the exhaustive search are presented in figures (5-1) to (5-6).

The "high costs" were based on a roof system consisting of steel joists for purlins (light weight joists to 31'-0" (\$0.30/#), long span joists for truss spacings greater than 31'-0" (\$0.24/#)), and steel Q decking. Two steel fabricators were consulted regarding costs, and average values of the cost parameters provided were assumed. Results of the exhaustive search are presented in figures (5-7) to (5-12).

It was found that the most economical trusses resulted when the purlin spacing was within the lowest constant cost region for C_{10} . For panel point loading, the chords of the truss are designed for axial load plus bending due to the secondary moments. When purlins are placed between panel points, the compression

chord must be designed for axial load plus primary bending moments caused by member loading and secondary moments. Hence for a given topology and dead plus live load, the panel point loaded truss results in less weight for the truss, and therefore less cost. This cost saving, however, is offset by the increased dead load due to the longer span of the decking necessary for panel point loading, and by the increased cost of the deck because of increased weight. For the cost models employed, the increased cost due to heavier deck type was always substantially greater than the material cost saving of panel point loading over member loading.

"Secondary moments" as used above, and in the balance of the thesis, refers to the bending moments in the top chord occasioned by the fact that the chord is continuous and the truss deflects under load.

A comparison of the minimum cost solution and the minimum weight solution is presented in figures (5-1) and (5-2) for the "low costs" and figures (5-7) and (5-8) for the "high costs". The discontinuities in the total cost vs truss spacing curves are the result of two factors. Firstly, the form of the cost model C_9 , and, secondly, the form of the dead weight model C_9S . The latter changes the material cost. Hence, a discontinuity should be shown in the minimum weight curves, but in order to show this effect, it would be necessary to evaluate, for adjoining constant cost steps, the upper limit of the lower cost region and the lower limit of the higher cost region. This was not done, and only a design for the upper limit of the lower cost region was performed.

The exhaustive search figures illustrate the conclusion of Chapter 4, namely, that the objective function should be evaluated only at the upper limit of a constant C_9 cost region.

EXHAUSTIVE SEARCH FOR MINIMUM COST CONFIGURATION ("LOW COSTS")

FIG. 5-1(a)

FIG. 5-1(b)

FIG. 5-1(c)

FIG. 5-1(d)

EXHAUSTIVE SEARCH FOR MINIMUM WEIGHT CONFIGURATION ("LOW COSTS")

FIG. 5-2(a)

FIG. 5-2(b)

FIG. 5-2(c)

FIG. 5-2(d)

TRUSS SPACING 20'-0" PURLIN SPACING 4'-0" LIVE LOAD 40 PSF ("LOW COSTS")

MINIMUM COST D/S RATIO FOR
DIFFERENT TOPOLOGIES

WEB WEIGHT/TOTAL WEIGHT VS D/S
RATIO FOR DIFFERENT TOPOLOGIES

EXHAUSTIVE SEARCH FOR MINIMUM WEIGHT CONFIGURATION ("HIGH COSTS")
(RESULTS PLOTTED FOR DIFFERENT D/S RATIOS)

EXHAUSTIVE SEARCH FOR MINIMUM WEIGHT CONFIGURATION ("HIGH COSTS")
 (RESULTS PLOTTED FOR DIFFERENT D/S RATIOS)

TOTAL COST/FT²

MATERIAL COST/FT²

WEB WEIGHT/TOTAL WEIGHT

MATERIAL COST/TOTAL COST

Observations from the Exhaustive Search

A "Low Costs"

- (1) For constant purlin spacing, the total cost C is most sensitive to a variation in truss spacing.
- (2) The maximum variation in results was 23.2 per cent for all the depth to span ratios, topologies and truss spacings considered.
- (3) The best truss spacing was always 20'-0" for minimum total cost.
- (4) Although the configuration for minimum weight was not sought, results from the exhaustive search indicate that the truss spacing for minimum weight occurs outside of the region considered, and is substantially higher than the spacing for the minimum cost solution.
- (5) The total cost is not affected greatly by a variation in the D/S ratio. The total variation in cost, in the range $.07 < D/S < .12$, is about 7 per cent of the minimum value in the case of the 8 panel truss, and about 3 per cent in the case of the 10 panel truss.
- (6) The maximum difference in total cost caused by a variation in topology occurred for a depth to span ratio of 0.12 and was 8.48 per cent.
- (7) For minimum weight, all designs are very close together except for the 8 panel truss.
- (8) No significant difference is observed between the minimum total cost depth to span ratio and the minimum weight depth to span ratio, for a given truss spacing. This similarity arises from the fact that a very low wall cladding cost was used (\$0.20).

B "High Costs"

(1) For the exhaustive search (6'-0" purlin spacing), the most expensive truss had a total cost of \$1.9620 per square foot and the minimum was \$1.5755 per square foot. This represents a total variation of 24.5 per cent.

(2) Of the 180 complete designs made for the exhaustive search, 145 were within 10 per cent of the minimum cost of \$1.5755 per square foot. This indicates that for a constant purlin spacing, there are many alternate feasible designs with substantial variations in configuration.

(3) Two distinct regions were found to exist for minimum cost based on truss spacing. These were the region surrounding the 24 foot spacing and that surrounding the 38 foot spacing. The 38 foot spacing, however, was the better of the two.

(4) For the minimum search, except in the case of the 8 panel topology, the minimum weight configuration occurred for a truss spacing between 38 and 45 feet, with the slope of the curve in this region being almost zero.

(5) For a truss spacing of 38'-0" and purlin spacing of 6'-0", the following conclusions can be made, based on figures (5-9) to (5-12).

(a) A realistic wall cladding cost substantially separates the minimum total cost depth to span ratio from the minimum weight depth to span ratio. The minimum cost D/S ratio was 0.07 except in the case of the 14 panel topology, while the minimum weight D/S ratio occurred at a value outside the range of the exhaustive search.

(b) The maximum spread in cost for all topologies and depth

to span ratio was 7.05 per cent.

(c) From figure (5-12) it is seen that minimizing the ratio of material cost to total cost does not lead to minimum cost.

The Importance of a Wall Cladding Cost and a Topology Cost

(1) Wall Cladding Cost

Figures (5-13(a)) and (b) depict the impact of a wall cladding cost on different topologies for the "low costs", and figures (5-15(a)) to (c) represent the same for the "high costs". By increasing the depth to span ratio of a truss, the axial force in the chords decreases and hence the chord area required decreases, whereas the web weight increases. Therefore there is, at some stage, a depth to span ratio yielding minimum weight. This minimum weight depth to span ratio is very dependent on the magnitude of the load carried. Associated with the height of a truss is the area to be covered with wall cladding. As this area increases, so does the cost of covering it. By including a wall cladding cost, a balance is achieved between high walls, minimizing truss weight, and low walls, minimizing wall cost, hence yielding an optimal cost solution.

When no cost is associated with the truss height (minimum weight case for D/S ratio), the maximum effect of the D/S ratio on the total cost is 8.5 per cent for the "low costs", and 6.2 per cent for the "high costs", within the range of D/S ratio considered.

For realistic wall costs (\$1.20 to \$2.00), the effect of the cladding cost is to lower the optimum D/S ratio. The maximum variation in total cost was 14.4 per cent and 7 per cent for the

"low costs" and "high costs", respectively. For small variations (0.01) from the optimum D/S ratio, the cost variation from the optimal value is approximately 2 to 5 per cent.

(2) Topology Cost

The topology cost (represented by $C_4 + C_5$) influences the structural configuration in the following two ways:

(i) It represents a fixed cost item and hence, directly affects the best truss spacing.

(ii) For a given truss spacing and depth to span ratio, varying the topology affects the truss weight. The greater the number of panels, the shorter the panel length and hence the less the reduction in allowable stress for the compression chord. Also, the primary moments, due to transverse loading on the compression chord decrease with an increasing number of panels. Therefore, the compression chord weight tends to decrease with increasing number of panels.

Only the impact of part (ii) is considered herein. Figures (5-14(a)) and (b) and figures (5-16(a)) to (c) illustrate the effect of a topology cost on the design with low and high costs, respectively. The topology cost is considered to be the cost of adding a web member to the truss. For zero topology cost, the maximum variation in cost was 3.5 per cent for the "low costs" and 3.8 per cent for the "high costs", demonstrating that the topology vs cost curve is flat for zero topology cost. For a high topology cost (\$48.00), the curves are not flat, and the maximum variation for the "low" and "high costs" are 12.9 and 6.7 per cent, respectively. For topologies adjacent to the optimal topology, the cost variation is less than 5 per cent.

STUDY OF THE EFFECT OF A WALL CLADDING COST ON THE BEST D/S RATIO ("LOW COSTS")

STUDY OF THE EFFECT OF A TOPOLOGY COST ON THE BEST CONFIGURATION ("LOW COSTS")

STUDY OF THE EFFECT OF A WALL CLADDING COST ON THE
BEST D/S RATIO ("HIGH COSTS")

TRUSS SPACING 38'-0"

PURLIN SPACING 6'-0"

LIVE LOAD 40 PSF

STUDY OF THE EFFECT OF A TOPOLOGY COST ON THE
BEST CONFIGURATION ("HIGH COSTS")

FIG. 5-16(a)

FIG. 5-16(b)

FIG. 5-16(c)

FIG. 5-16(d)

TRUSS SPACING 38'-0"

PURLIN SPACING 6'-0"

LIVE LOAD 40 PSF

Influence of Gravity Load

An examination of the influence of gravity load on the structure variables was made with loads of 20, 40, 60, and 80 psf. The "low" cost model was employed, along with a truss spacing of 20'-0" and a purlin spacing of 4'-0". Varying the magnitude of the gravity load results in the following conclusions:

- (1) For light loadings, the design is inefficient as minimum member areas govern. For compression members, the small member sizes result in low allowable stresses, and therefore, a low depth to span ratio governs.
- (2) The best depth to span ratio and number of panels increases with increasing load.
- (3) In general, the minimum weight geometry and topology do not correspond with the minimum cost geometry and topology.
- (4) The ratio of material cost to total cost increases substantially with increasing gravity load.

Sensitivity and Uniqueness of the Best Solution

The sensitivity of the best solution to variations in the structure variables, is directly dependent on the ratios of the cost parameters. For high fixed costs and system costs (chord costs, topology cost, wall and roof costs), the hierarchy of structure variables is purlin spacing, truss spacing, depth to span ratio and number of panels. For low topology cost, the best cost answer is not sensitive to small variations in the number of panels. With the truss spacing and purlin spacing held constant, many possible solutions exist which come within 5 per cent

of one another for small changes in depth to span ratio and topology. Results obtained from Box's Complex Method indicate that the region surrounding the optimum is very flat, and many possible alternate good designs exist for the solution of the problem.

Results obtained from the optimization runs for the "high costs" are presented in Tables (5-3) to (5-8). The initial and final complexes are shown to demonstrate the many alternate, good feasible designs possible. Note that for the 6 runs made, the maximum deviation from the best answer of 1.5749 is only 1.155 per cent. As stated in Chapter 4, only the values at the upper limits of the constant C_9 region were considered, instead of treating the truss spacing as a continuous variable. Table (5-2) defines the input variables.

RUN	NPOINT	NIT	ALPHA	RNB	D/S RATIO		TRUSS SP.		PURLIN SP.	
					DLBR	DUBR	TLBS	TUBS	PLBS	PUBS
1	8	30	1.3	.05	.06	.11	17.	45.	6.	P.L.
2	8	30	1.3	.45	.06	.12	17.	45.	6.	P.L.
3	8	30	1.3	.90	.06	.11	17.	45.	6.	P.L.
4	10	30	1.3	.05	.06	.11	17.	45.	6.	P.L.
5	10	30	1.3	.45	.06	.12	17.	45.	6.	P.L.
6	10	30	1.3	.90	.06	.11	17.	45.	6.	P.L.

Note: $C_9S(1) = 16.99$ feet.

TABLE 5-2 INPUT VARIABLES FOR "HIGH COSTS" OPTIMIZATION RUNS

I N I T I A L C O M P L E X					
x ₁	x ₂	x ₃	x ₄	WEIGHT	f(x)
10	.100	24.	6.	11885.	1.6211
14	.060	24.	5.455	15333.	1.6989
14	.096	38.	5.455	18279.	1.6470
16	.090	31.	6.667	15544.	1.7151
10	.073	31.	12.	17256.	2.0094
14	.061	45.	6.667	28625.	1.6855
8	.098	45.	5.455	22403.	1.6211
16	.068	24.	6.667	14168.	1.7266
F I N A L C O M P L E X					
x ₁	x ₂	x ₃	x ₄	WEIGHT	f(x)
8	.0721	45.	5.	25451.	1.5957
10	.0753	38.	5.	21201.	1.5929
8	.0750	45.	5.4545	24938.	1.5957
8	.0747	45.	5.4545	24927.	1.5945
8	.0862	38.	5.	20797.	1.5897
8	.0716	45.	5.	25376.	1.5928
8*	.0744*	38.*	5.4545*	22168.*	1.5821*
8	.0750	45.	5.4545	24938.	1.5957

* denotes optimum

TABLE 5-3 OPTIMIZATION RUN 1

I N I T I A L C O M P L E X					
x ₁	x ₂	x ₃	x ₄	WEIGHT	f(x)
10	.100	24.	6.	11885.	1.6211
14	.079	31.	7.5	16948.	1.7841
6	.066	31.	20.	23862.	2.2796
8	.103	24.	10.	14900.	1.9794
16	.093	38.	6.667	18977.	1.6796
16	.077	45.	6.	24292.	1.6813
12	.063	24.	5.	15017.	1.6553
6	.070	31.	15.	26259.	2.3487
F I N A L C O M P L E X					
x ₁	x ₂	x ₃	x ₄	WEIGHT	f(x)
8	.0896	45.	6.667	23862.	1.6206
8	.0909	38.	6.667	21190.	1.6112
12	.0866	38.	6.667	19180.	1.6126
12	.0843	38.	6.667	19930.	1.6209
8	.0912	38.	6.667	21141.	1.6112
10	.0905	38.	6.667	19812.	1.6097
10*	.0917*	38.*	6.*	18659.*	1.5906*
8	.0919	38.	6.667	20444.	1.5996

TABLE 5-4 OPTIMIZATION RUN 2

INITIAL COMPLEX					
x ₁	x ₂	x ₃	x ₄	WEIGHT	f(x)
10	.100	24.	6.	11885.	1.6211
14	.104	24.	5.455	12547.	1.7359
16	.061	31.	6.667	19078.	1.7165
16	.094	31.	5.455	15618.	1.7284
8	.093	31.	8.571	18030.	1.9527
8	.108	45.	8.571	23005.	1.9602
12	.068	24.	7.5	14540.	1.7539
8	.076	38.	12.	23131.	2.0070
FINAL COMPLEX					
x ₁	x ₂	x ₃	x ₄	WEIGHT	f(x)
8	.0960	38.	6.667	20249.	1.6077
8*	.0815*	38.*	6.667*	21665.*	1.5931*
8	.0950	38.	6.667	20215.	1.6039
8	.0930	38.	6.667	20923.	1.6123
8	.0945	38.	6.667	20552.	1.6093
8	.0949	38.	6.667	20211.	1.6035
10	.0934	38.	6.667	19571.	1.6136
8	.0950	38.	6.667	20215.	1.6039

TABLE 5-5 OPTIMIZATION RUN 3

INITIAL COMPLEX					
x ₁	x ₂	x ₃	x ₄	WEIGHT	f(x)
10	.100	24.	6.	11885.	1.6211
14	.096	38.	7.5	18813.	1.7571
14	.090	31.	7.5	16020.	1.7943
16	.073	31.	6.	17686.	1.7185
10	.061	45.	7.5	27818.	1.7262
14	.098	45.	7.5	22526.	1.7894
8	.068	24.	10.	16553.	1.9300
16	.064	24.	7.5	14394.	1.8242
6	.098	45.	12.	27359.	2.0797
8	.085	24.	12.	15669.	2.0544
FINAL COMPLEX					
x ₁	x ₂	x ₃	x ₄	WEIGHT	f(x)
10	.1000	24.	6.	11885.	1.6211
10	.0600	38.	5.4545	24223.	1.6073
10	.0794	45.	6.	23124.	1.6012
10	.0600	38.	6.	24223.	1.6073
10	.0711	45.	6.667	25383.	1.6148
10	.0600	38.	5.4545	24223.	1.6073
8*	.0773*	45.*	6.667*	24335.*	1.5923*
8	.0736	31.	5.4545	18951.	1.6190
8	.0831	45.	6.	24068.	1.6051
8	.0802	45.	6.667	24377.	1.6016

TABLE 5-6 OPTIMIZATION RUN 4

I N I T I A L C O M P L E X					
x ₁	x ₂	x ₃	x ₄	WEIGHT	f(x)
10	.100	24.	6.	11885.	1.6211
14	.066	31.	7.5	18341.	1.7809
6	.103	24.	20.	14613.	2.2271
8	.093	38.	10.	21254.	1.9189
16	.077	45.	6.667	24292.	1.6813
16	.063	24.	6.667	14892.	1.7364
12	.070	31.	5.455	17737.	1.6444
6	.104	45.	20.	22940.	2.2233
8	.093	38.	12.	22349.	2.0411
10	.078	24.	10.	15177.	1.9620
F I N A L C O M P L E X					
x ₁	x ₂	x ₃	x ₄	WEIGHT	f(x)
8	.0970	38.	5.4545	20362.	1.6129
10	.0801	45.	5.	23260.	1.6056
8	.0878	38.	6.667	21581.	1.6097
8	.0812	38.	5.	20978.	1.5786
8	.0810	38.	5.4545	21693.	1.5922
10	.0740	38.	6.	20811.	1.5813
8	.0828	45.	6.667	24070.	1.6042
8	.0747	38.	6.	22175.	1.5829
8	.0777	38.	5.4545	22008.	1.5886
8	.0884	38.	6.667	21418.	1.6083

TABLE 5-7 OPTIMIZATION RUN 5

I N I T I A L C O M P L E X					
x ₁	x ₂	x ₃	x ₄	WEIGHT	f(x)
10	.100	24.	6.	11885.	1.6211
14	.061	31.	7.5	19738.	1.7993
16	.094	31.	5.455	15618.	1.7284
16	.093	31.	7.5	15827.	1.8295
8	.108	45.	8.571	23005.	1.9602
8	.068	24.	12.	18208.	2.0826
12	.076	38.	10.	19837.	1.8960
8	.067	45.	15.	28761.	2.2340
16	.067	31.	6.667	17763.	1.7040
6	.080	31.	20.	20949.	2.2498
F I N A L C O M P L E X					
x ₁	x ₂	x ₃	x ₄	WEIGHT	f(x)
10	.0968	38.	6.	18105.	1.5944
10	.0871	38.	5.4545	20040.	1.6044
8	.0699	38.	6.	22493.	1.5749
8	.0687	38.	5.	22979.	1.5810
8	.0686	38.	6.667	23455.	1.5900
8	.0833	45.	6.667	24031.	1.6050
8	.0716	45.	6.667	26664.	1.6142
10	.0885	38.	6.	19229.	1.5925
8	.0929	38.	5.	20497.	1.6033
8	.0797	45.	6.667	24475.	1.6019

TABLE 5-8 OPTIMIZATION RUN 6

Discussion of Cost Parameters and Cost Model Form

A model which purports to represent the cost of a structural system must include cost of materials, design, detailing, fabrication, transportation, and erection. Some of these costs can be represented by a lump sum which is almost independent of the size and shape of the truss used. This figure could include design and detailing costs, as well as some "fixed" costs associated with fabrication and erection. Since, however, such costs do not affect the optimizing procedure, they have not been included. The cost parameters included in the model employed are examined below.

C_1, C_2, C_3 These parameters represent the material cost. The material costs arise from two distinct areas, which are, the cost of the material delivered from the mill to the fabricator, and unit costs which arise from fabrication and erection. Examples of the latter costs are painting, and transportation from the fabricator to the site.

C_4, C_5, C_6, C_7 These parameters are intended to represent labor cost of fabrication. These include the labor costs involved in cutting, punching, fitting, and welding, and are rather insensitive to weight, within limits. Other costs related to individual members as opposed to weight related costs, may be included.

In the model used, no variation is considered in these parameters. In fact, however, these variables are dependent on the magnitude of the forces carried (function of geometry, topology, and code requirements), and on the member size. Generally, the variation in $C_4 \dots C_7$ with respect to geometry, topology, and force level, is relatively small, and an average value may be

used without significant error. Depending on the magnitude of the parameters C_4 and C_5 , the cost vs topology curve may be flat, or have a definite slope, thereby establishing alternate topologies as the optimal solution, or a unique topology as the optimum.

From information extracted from fabricators, it was noted that a large variation existed for $C_4 \dots C_7$. The importance of choosing the correct values for these parameters is shown in figures (5-18(a)) and (b) for the low and high costs. The curves depict the effect of the topology cost and chord costs distributed over the area supported by one truss. Coupled with the effect of decreasing fixed cost per unit area with increased truss spacing, is the decrease of weight with increased spacing. This effect is not included in the curves. The cost model steps of C_9 are shown above the curves. When the combined decrease in cost, due to fixed cost and weight related costs, is less than the increase in roof cost due to a new constant cost region, the best truss spacing is defined.

C_8 No variation in the wall cost, with respect to truss spacing, is possible with the representation used. For reasonable truss spacings, a constant cost parameter is sufficient. The variation of wall cladding cost with spacing is very dependent on the type of material used. It is conceivable that for large truss spacings a variation in C_8 , with respect to spacing, would have to be considered in order to permit a valid cost representation.

THE DISTRIBUTION OF FIXED COSTS WITH TRUSS SPACING

RESULTS PLOTTED FOR DIFFERENT TOPOLOGIES

FIG. 5-17(a) "LOW COSTS"

FIG. 5-17(b) "HIGH COSTS"

Roof Cost C_9 and C_{10}

C_{10} , the cost of the decking, is represented exactly by the model used. C_9 , the purlin cost, is a function of truss spacing and purlin spacing. Only that variation related to truss spacing is modelled. The effect of purlin spacing has been ignored, and an average value used.

CHAPTER 6

THE EFFECT OF VARIABLE PANEL SPACING

Variable Panel Spacing

The design of the chords is always governed by the application of a uniform load to the entire span. The two chords constitute 40 to 80 per cent of the truss weight, while 55 to 75 per cent of the chord weight is comprised of the compression chord. In the discussion of previous chapters, a fully stressed design of the chords was defined as a design where at least one cross section of the chord was stressed to the full allowable limit. Using this definition, and varying the panel spacing, there is possible an infinite number of fully stressed designs. There is, however, one unique design which fully stresses to the allowable limit at least one cross section of the compression chord in each panel. Employing the criterion of constant chord area, the configuration which stresses each panel to the allowable limit is sought. The assumption implicit in this method is that the minimum compression chord weight is obtained by stressing each panel to the limit. The algorithm developed herein may be classified as a sub-optimization scheme applied to a subordinate element, namely, the compression chord, with the merit criterion being weight. The validity of the substitution of a different criterion for the selection of the best truss is questionable and must be rigorously examined. Note that the algorithm developed is dependent on the fact that the allowable compression stress is a variable, governed by the panel lengths.

The cases examined are: A pin connected truss with panel point loading: A continuous chord truss with panel point loading: A continuous chord truss with transverse loading on the compression chord. For panel point loading, the roof type is governed by the longest panel length. A detailed development is presented for the case of an eight panel truss, and results are included for different topologies.

Cases (i) and (ii) Pinned and continuous chord studies with panel point loading.

The algorithm was developed for the pinned chord study and was also employed when the chords were treated as continuous members, with panel point loading. The results in the latter case were affected, however, by the existence of the secondary moments.

FIGURE 6-1 PANEL SPACING CONFIGURATION

The axial forces S in the compression chord are defined as:

$$S_1 = S_4 - \frac{wL^2}{2d} (k_2 + k_3 + k_4)^2$$

$$S_2 = S_4 - \frac{wL^2}{2d} (k_3 + k_4)^2$$

$$S_3 = S_4 - \frac{wL}{2d} k_4^2 \quad (6-1)$$

$$S_4 = \frac{wL^2}{8d}$$

The formula for the allowable stress F_A in the intermediate range is assumed to apply. F_A , the allowable compressive stress is given by

$$F_A = 0.6F_y - m\left(\frac{k_i L}{r} - C_o\right) \quad (6-2)$$

where F_y = yield stress in k.s.i.

$$m = \frac{6.77 + 0.079F_y}{C_p - C_o} \quad C_p = \frac{535}{\sqrt{(F_y - 13)}}$$

$$C_o = 20 \text{ when } F_y \leq 50$$

r = radius of gyration

F_A may be written as follows:

$$F_A = A^* - B^*k_i$$

$$\text{where } A^* = 0.6F_y + mC_o = \text{constant} \quad (6-3)$$

$$B^* = \frac{mL}{r}$$

For a design to satisfy the code requirements, $f_{ai}/F_{Ai} \leq 1$, where f_{ai} is the actual axial stress in the i th member. The fully stressed design requires that the inequality be removed and $f_{ai}/F_{Ai} = 1$. Hence, the area A can be written as $A = S_i/F_{Ai}$. If a continuous member spectrum was used, and a functional relationship between the radius of gyration r and area A assumed, it would always be possible to make $f_{ai}/F_{Ai} = 1$. When a discrete member spectrum is used, f_{ai}/F_{Ai} can only approximate the right-hand side. Applying the constraint of constant area for the compression chord and letting N equal the number of panels in

half the truss, the following set of non-linear simultaneous algebraic equations results.

$$\sum_{i=1}^N k_i - 0.5 = 0 \quad (\text{constraint equation}) \quad (6-4)$$

$$S_i / (A^* - B \cdot k_i) - S_{i+1} / (A^* - B \cdot k_{i+1}) = 0$$

An iterative scheme must be employed to solve equations (6-4), and, moreover, a span slenderness ratio L/r must be assumed. Note that, for panel point loading, r is the smaller of r_x and r_y . Hence it will be necessary to solve the equations for each design made.

(iii) Continuous chord with uniform transverse load

In this case the compression chord is subject to large primary bending moments, and these must be formulated in an explicit form in terms of the panel spacing coefficients. In order to obtain these moments the chord was idealized as a continuous beam on rigid supports subjected to a uniform transverse load of intensity w , and a two pass moment distribution was performed. Advantage was taken of the fact that the area and the moment of inertia of the chord are constant for the entire span. No attempt was made to model the secondary moments arising from deflection of the truss. The moment expressions are stated in equations (6-5).

FIGURE 6-2 CONTINUOUS BEAM IDEALIZATION

$$\begin{aligned}
 M_1 &= \frac{wL^2}{12} (1.5k_1^2 + r_1(k_2^2 - 1.5k_1^2) - 0.5r_1r_3(k_3^2 - k_2^2)) \\
 M_2 &= \frac{wL^2}{12} (k_2^2 + r_3(k_3^2 - k_2^2) + 0.5r_2(k_2^2 - 1.5k_1^2) + 0.5r_3(r_5(k_4^2 - k_3^2) \\
 &\quad - r_2(k_2^2 - 1.5k_1^2))) \\
 M_3 &= \frac{wL^2}{12} (k_3^2 + r_5(k_4^2 - k_3^2) + 0.5r_4(1 - r_5)(k_3^2 - k_2^2)) \\
 M_4 &= \frac{wL^2}{12} (k_4^2 + 0.5r_6(k_4^2 - k_3^2) - 0.5r_4r_6(k_3^2 - k_2^2))
 \end{aligned} \tag{6-5}$$

$$r_1 = 3k_2 / (3k_2 + 4k_1) \quad r_2 = 1 - r_1$$

$$r_3 = k_3 / (k_2 + k_3) \quad r_4 = 1 - r_3$$

$$r_5 = k_4 / (k_3 + k_4) \quad r_6 = 1 - r_5$$

The axial force expressions were assumed to be the same as for the pin connected case. The code requirement for a combined stress case is presented in the form of an interaction formula,

$$f_{ai}/F_{Ai} + f_{bi}/F_B \leq 1.$$

where f_{ai} = actual axial stress = S_i/A

F_{Ai} = allowable axial stress = $A^* - B \cdot k_i$

f_{bi} = actual bending stress = M/Z

F_B = allowable bending stress = constant

A simple functional relationship of the form $Z = \beta A$ was assumed between the section modulus Z and area A . For a fully stressed design,

$$f_{ai}/F_{Ai} + f_{bi}/F_B = 1.$$

Applying the stress definitions, the interaction formula can be written as:

$$S_i/(A^* - B*k_i) + M_j/\beta F_B = A \quad (6-6)$$

Applying the constraint of constant chord area, the following set of equations was formed.

$$\sum_{i=1}^N k_i - 0.5 = 0 \quad (\text{constraint equation})$$

$$S_i/(A^* - B*k_i) - S_{i+1}/(A^* - B*k_{i+1}) + (M_j - M_{j+1})/\beta F_B = 0 \quad (6-7)$$

Note that the governing member moments in equations (6-7) can arise from three locations in the member. These are the largest of the two negative end moments, or the maximum positive moment between ends. Corresponding to the sign of the moments, different β values are applicable. Since the compression chord is continuously supported out of plane, r corresponds to r_x .

The solutions for equations (6-4) and (6-7) are summarized in Table (6-1) and figures (6-3) for an 8 panel truss. They were obtained by a generalized Newton-Raphson scheme. The truss span was 120'-0", the truss spacing 20'-0", and the live load 40 psf. The same cost model as in chapter 4 and 5 was used. For the case of transverse load, the deck was applied directly to the chord. Results are presented in Tables (6-2) to (6-6) for the following cases.

TABLE	CHORD FIXITY	PANEL SPACING	LOAD TYPE
6-2	PINNED	EVEN	PANEL POINT
6-3	PINNED	VARIABLE	PANEL POINT
6-4	CONTINUOUS	EVEN	CONTINUOUS
6-5	CONTINUOUS	VARIABLE	CONTINUOUS
6-6	CONTINUOUS	VARIABLE	CONTINUOUS

L/r	k_1	k_2	k_3	k_4
300	.2441	.1170	.0750	.638
400	.2233	.1207	.0832	.728
500	.2073	.1229	.0896	.0801
600	.1945	.1243	.0949	.0863
700	.1839	.1252	.0993	.0916
800	.1749	.1258	.1031	.0962
900	.1671	.1261	.1064	.1004
1000	.1603	.1263	.1094	.1041
1100	.1542	.1263	.1120	.1075
1200	.1488	.1263	.1144	.1106
1300	.1440	.1261	.1165	.1134
1400	.1395	.1260	.1185	.1160

SOLUTION TO EQUATIONS (6-4) FOR 8 PANEL TRUSS
(INDEPENDENT OF D/S)

L/r	k_1	k_2	k_3	k_4
400	.1611	.0564	.1554	.1271
500	.1592	.0681	.1467	.1260
600	.1574	.0789	.1397	.1240
700	.1551	.0879	.1345	.1224
800	.1528	.0957	.1305	.1210
900	.1504	.1025	.1274	.1198
1000	.1479	.1083	.1249	.1188
1100	.1455	.1132	.1231	.1182
1200	.1430	.1171	.1219	.1180
1300	.1404	.1203	.1212	.1181

SOLUTION TO EQUATIONS (6-7) FOR 8 PANEL TRUSS (D/S 0.10)

TABLE 6-1

Discussion of Results

In examining and interpreting the results, the following facts must be borne in mind.

(1) The chords carry load most efficiently when the depth of the truss is large. This minimizes the chord area.

(2) The allowable stress in the compression chord is highest when the area of the chord is large.

(3) Webs carry the shear most efficiently when they are short and the angle between the horizontal and the web member is large.

(4) For panel point loading, the shear carried by the webs increases with an increasing number of panels.

(5) The truss stiffness is proportional to the square of the depth. Hence, the deeper the truss, the smaller the secondary moments, and the more accurate is the representation of the compression chord as a continuous beam on rigid supports.

(6) The maximum slenderness ratio for tension members can govern the size of the diagonals.

(7) The allowable bending stress and axial stress are constants for the tension chord. The smaller the length of the center panel, the larger the force in the tension chord.

(8) The axial force in the compression chord for the center panel is a constant, independent of the panel spacing.

In the following discussion, all comparisons refer to the even panel spacing case.

(1) Pin connected case - the same load for even and variable spacing.

All members are subjected to a uniaxial state of stress. No deck dead load was applied, so that both the even and the variable panel spacing cases would be subject to the same loads. From Tables (6-2) and (6-3) it is seen that:

(i) The bottom chord weight remained the same in almost all cases.

(ii) No significant gain or loss was made in the web weight.

(iii) In almost all designs, a saving was made in the top chord weight, which was of sufficient magnitude to offset any increase in web weight or bottom chord weight. Although not identically equal to 1.0, the stress factors for the compression chord for each panel were almost equal to one another, and as close to 1.0 as allowed by the use of a discrete member spectrum.

(iv) The weight-saving ranged from -2.14 per cent to 6.95 per cent with the average saving between approximately 3 and 4 per cent.

(v) The distribution of panel coefficients is parabolic for constant L/r .

(vi) The roof deck cost is governed by the longest panel length. Hence, the cost of the variable panel spacing is adversely affected by the form of the cost model.

Several cases were run with the effect of roof dead load being included. For these cases, the truss weight for the variable panel spacing was substantially higher than for the even panel case, reflecting the change in deck weight due to the long end panel.

Continuous chord case - secondary moments included.

The above distribution of panel spacing coefficients was not valid. To satisfy code requirements, the chord was designed on the basis of an interaction formula. Because of the length of the end panel, a reduction in allowable bending stress had to be made. For long panel lengths, the moment term dominated the interaction formula because of the low allowable bending stress. Hence no weight saving was affected, and further, the total cost was substantially higher due to the long end panel length governing the deck cost.

Because the distribution of the panel spacing coefficients is parabolic, for the pin connected truss, a test was made using a parabolic distribution of the form $y = a_0 + a_1x^2$, with the ratio of the outside panel to center panel set at 1.5. A small weight saving was made in several instances, and the roof cost was maintained at the same level as for the even panel case.

(2) Continuous chord with transverse load.

In order to minimize the number of approximations made, a uniform load was used even though the main concern was with a purlin loading. This meant that only two approximations remained, i.e., that a two pass moment distribution was sufficient and that the secondary moments could be neglected. The validity of the two pass moment distribution for large variations in panel spacing was not rigorously examined. It was found that the secondary moments had a significant effect on the attempt to achieve fully stressed sections in all panels, by the use of the above equations.

NPAN	D/S	TR WT	WEB WT	TC WT	BC WT	L/r		COST/FT ²	MAT C/FT ²
						USED	ACTUAL		
8	.08	10907	2622	5100	3185	EVEN SPACING	720	1.5068	.364
8	.10	9861	3380	4080	2401		770	1.4800	.329
8	.12	9808	4287	3480	2042		765	1.4862	.327
10	.08	10348	2543	4620	3185		726	1.5099	.345
10	.10	9503	3291	3660	2552		800	1.4897	.317
10	.12	10039	4776	3180	2082		955	1.5155	.335
12	.08	10005	2740	4080	3185		770	1.2701	.334
12	.10	9612	3580	3480	2552		765	1.2650	.320
12	.12	9810	4847	2880	2082		765	1.2796	.327

TABLE 6-2 PINNED CHORD - EVEN SPACING

NPAN	D/S	TR WT	WEB WT	TC WT	BC WT	L/r		COST/FT ²	MAT C/FT ²
						USED	ACTUAL		
8	.08	10344	2539	4620	3185	726	726	1.7380	.345
8	.10	9488	3096	3840	2552	710	710	1.7175	.316
8	.12	10032	4469	3480	2082	765	765	1.7436	.334
10	.08	9636	2611	3840	3185	710	710	1.7361	.321
10	.10	9257	3225	3480	2552	765	765	1.7315	.309
10	.12	9340	4378	2880	2082	765	765	1.7423	.311
12	.08	9866	2841	3840	3185	710	710	1.7654	.329
12	.10	9302	3570	3180	2552	955	955	1.7547	.310
12	.12	9604	4921	2580	2103	783	783	1.7727	.320

TABLE 6-3 PINNED CHORD - VARIABLE SPACING

NPAN	D/S	TR WT	WEB WT	TC WT	BC WT	L/r		COST/FT ²	MAT C/FT ²
						USED	ACTUAL		
8	.08	14957	2953	7920	4083	EVEN SPACING	358	1.6518	.498
8	.10	13844	3819	6840	3185		389	1.6227	.461
8	.12	13580	4641	6354	2585		395	1.6219	.453
10	.08	13653	2918	6354	4381		395	1.6300	.455
10	.10	12386	3801	5400	3185		379	1.5958	.413
10	.12	12947	5144	5100	2703		466	1.6225	.432
12	.08	12730	3133	5400	4198		379	1.6209	.424
12	.10	11676	3991	4500	3185		454	1.5938	.389
12	.12	12570	5367	4500	2703		454	1.6316	.419

TABLE 6-4 CONTINUOUS CHORD - EVEN SPACING

Note: TR WT = truss weight
 WEB WT = web weight
 TC WT = top chord weight
 BC WT = bottom chord weight
 MAT C = material cost
 NPAN = number of panels

NPAN	D/S	TR WT	WEB WT	TC WT	BC WT	L/r		COST/FT ²	MAT C/FT ²
						USED	ACTUAL		
8	.08	16519	3138	9000	4381	354	354	1.7039	.550
8	.10	16099	3914	9000	3185	354	354	1.6979	.536
8	.12	15756	5132	7920	2703	358	358	1.6944	.525
10	.08	15181	3178	7920	4083	358	358	1.6810	.505
10	.10	14524	3899	7440	3185	378	378	1.6671	.484
10	.12	15534	5029	7920	2585	358	358	1.7087	.518
12	.08	14674	3151	7440	4083	378	378	1.6857	.489
12	.10	12713	4128	5400	3185	379	379	1.6283	.423
12	.12	13547	5444	5400	2703	379	379	1.6642	.451

TABLE 6-5 CONTINUOUS CHORD - VARIABLE SPACING

NPAN	D/S	TR WT	WEB WT	TC WT	BC WT	L/r		COST/FT ²	MAT C/FT ²
						USED	ACTUAL		
8	.08	14961	2958	7920	4083	1500	358	1.6519	.498
8	.10	14424	3799	7440	3185	1500	378	1.6420	.480
8	.12	14036	4611	6840	2585	1500	389	1.6371	.468
10	.08	13628	2893	6354	4381	1500	395	1.6292	.454
10	.10	12407	3822	5400	3185	1500	379	1.5965	.414
10	.12	13047	4944	5400	2703	1500	379	1.6258	.436
12	.08	12694	3097	5400	4198	1500	379	1.6197	.423
12	.10	12269	3984	5100	3185	1500	466	1.6135	.409
12	.12	12497	5294	4500	2703	1500	454	1.6291	.416

TABLE 6-6 CONTINUOUS CHORD - VARIABLE SPACING

NPAN	D/S	f ₁	f ₂	f ₃	f ₄	f ₅	f ₆
8	.08	.682	.899	.839	.880		
8	.10	.802	1.016	.895	1.000		
8	.12	.772	.956	.830	.933		
10	.08	.580	.790	.822	.907	.872	
10	.10	.576	.772	.777	.855	.880	
10	.12	.720	.916	.879	.961	1.001	
12	.08	.464	.668	.722	.834	.901	.932
12	.10	.579	.788	.842	.957	.964	.987
12	.12	.541	.717	.749	.845	.842	.880

TABLE 6-7 STRESS FACTORS FOR CONTINUOUS CHORD & EVEN SPACING

NPAN	D/S	f ₁	f ₂	f ₃	f ₄	f ₅	f ₆
8	.08	.997	.911	.947	.861		
8	.10	.963	.892	.903	.872		
8	.12	.970	.906	.925	.896		
10	.08	.892	.846	.802	.779	.785	
10	.10	.992	.931	.886	.841	.837	
10	.12	.988	.945	.799	.767	.745	
12	.08	.992	.933	.858	.821	.746	.710
12	.10	.977	.941	.912	.889	.783	.732
12	.12	1.006	.970	.889	.862	.720	.718

TABLE 6-8 STRESS FACTORS FOR CONTINUOUS CHORD & VARIABLE SPACING

NPAN	D/S	f ₁	f ₂	f ₃	f ₄	f ₅	f ₆
8	.08	.752	.950	.825	.842		
8	.10	.821	.998	.800	.872		
8	.12	.847	1.003	.769	.851		
10	.08	.754	.928	.774	.833	.814	
10	.10	.758	.918	.758	.815	.796	
10	.12	.733	.863	.667	.715	.701	
12	.08	.696	.862	.795	.866	.888	.896
12	.10	.836	.971	.813	.872	.781	.785
12	.12	.856	.978	.792	.849	.747	.763

TABLE 6-9 STRESS FACTORS FOR CONTINUOUS CHORD & VARIABLE SPACING

NPAN	D/S	k ₁ L	k ₂ L	k ₃ L	k ₄ L	k ₅ L	k ₆ L
8	.08	236.55	76.22	226.47	180.76		
8	.10	233.05	73.67	230.04	183.24		
8	.12	230.07	75.38	229.54	185.01		
10	.08	201.91	58.90	189.22	118.42	151.55	
10	.10	197.52	53.77	194.15	118.01	156.55	
10	.12	194.17	51.78	196.17	118.52	159.36	
12	.08	179.27	43.52	165.90	83.81	133.13	114.37
12	.10	172.76	49.28	160.53	90.41	131.05	115.97
12	.12	168.50	45.53	162.49	88.99	135.40	119.09

TABLE 6-10 PANEL LENGTHS IN INCHES FOR TABLE 6-5

NPAN	D/S	$k_1 L$	$k_2 L$	$k_3 L$	$k_4 L$	$k_5 L$	$k_6 L$
8	.08	194.16	179.04	174.67	172.12		
8	.10	193.90	178.58	175.03	172.49		
8	.12	193.64	178.14	175.37	172.85		
10	.08	172.33	143.78	141.02	132.18	130.69	
10	.10	171.34	142.12	141.83	132.95	131.75	
10	.12	170.39	140.52	142.51	133.58	133.00	
12	.08	157.79	121.88	122.24	109.08	105.43	103.57
12	.10	156.00	119.14	123.13	109.78	106.99	104.96
12	.12	154.41	116.73	123.88	110.36	108.45	106.18

TABLE 6-11 PANEL LENGTHS IN INCHES FOR TABLE 6-6

Tables (6-4) to (6-6), (6-7) to (6-9), and (6-10) to (6-11) illustrate cost and weight figures, stress factors, and panel lengths respectively, for the runs made. The columns headed L/r indicate the span slenderness ratio employed to obtain the panel spacing coefficients, and the actual values for the chords designed on this basis. Although the loads carried by the even and variable panel spaced trusses are the same, the variable panel spacing configuration resulted in substantially heavier compression chords. In Table (6-8), the stress factors for the compression chord increase from the center outward. This distribution was caused by the secondary moments which were not accounted for in the moment expressions used for the calculation of panel spacing coefficients. The secondary moments have an effect opposite to that of the primary moments. Considering the effect of a discrete member spectrum and the secondary moments, the variable spacing scheme resulted in a design more closely related to a fully stressed design than the even panel spacing, but with heavier chords.

By using a high span slenderness ratio, a distribution similar to the distribution due to the true span slenderness ratio, was achieved. A span slenderness ratio of 1500 was used to obtain the results presented in Table (6-6). This ratio was employed in order to investigate what occurs when a small variation from even panel spacing is permitted, while using a technique which should lead to a fully stressed design. For this study, the compression chord size was not increased greatly, and in some instances remained the same as for the even panel configuration. A reduction in the web weight was occasionally achieved, but no over-all net saving was made.

In order to determine the form of the minimum weight solution for the compression chord, the problem should be treated as a mathematical programming problem, which could be stated as:

$$\text{Minimize } A = S_m / (A^* - B * k_m) + M_m / \beta F_B$$

subject to

$$S_i / (A^* - B * k_i) + M_i / \beta F_B \leq A, \quad i = 1 \dots N, i \neq m$$

$$\sum_{i=1}^N k_i - 0.5 = 0$$

where S_i , $i = 1 \dots N$ and M_i are defined in equations (6-1) and (6-5).

At least one panel (mth panel) would be stressed to the full allowable limit for the minimum weight solution, but which panel this would be is not known beforehand. Therefore, the problem would have to be solved N times, with m varying from 1 to N . Before such a procedure should be attempted, a method of modeling the secondary moments has to be derived. Although it is

perhaps possible to find a distribution of panel spacing coefficients yielding lower structure weight than for even panel spacing, the total cost saving would likely be less than about 1 per cent or 2 per cent. Further, the substitution of a purlin loading for a continuous loading, which would be an accurate modelling of the real roof system, invalidates the moment expressions developed, unless the purlin spacing is small (2 to 3 feet), and the number of panels is small. A more lucrative search area would be to abandon the constant area criterion for long spans, and find the optimal number of chord size changes to be made, based on weight savings and splice cost.

CHAPTER 7

CONCLUSIONS

- (1) The feasibility of automating the analysis and design of a structural system to yield low cost, has been demonstrated.
- (2) The use of an optimization scheme employing Box's Complex Method is more economical than an exhaustive search procedure.
- (3) No claim is made that Box's Method yields a global minimum in the cost space
- (4) The cost model examined provides a reasonable representation of the cost of the structural system considered, and allows the inclusion of any costs which are dependent either on the weight, or on the numbers of members in the truss.
- (5) High labor costs, associated with numbers of members, lead to larger truss spacing.
- (6) Small deviations in the structural variables from the optimal configuration result in an increase in cost of less than 5 per cent.
- (7) The minimum weight structural configuration is not a minimum cost configuration for the cost model studied.
- (8) The hierarchy of structural variables for the system examined is purlin spacing, truss spacing, depth to span ratio, and number of panels (topology).
- (9) The optimal structural configuration is directly dependent on the ratios between the cost parameters.

POSSIBLE FUTURE EXTENSIONS

- (1) A preliminary design routine should be written so that only

approximate designs are made until the optimization algorithm converges to a low cost region. Such a change would substantially reduce the design - analysis cycle cost.

(2) A greater number of possible web configurations could be included in the program, as well as the design of trusses with sloping chords.

(3) The use of tables of members, other than Tees and Double Angles, could be implemented.

(4) A design scheme for deflection control could be developed.

(5) The structural system could be extended to include the columns, thus allowing the design of a complete bent.

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APPENDIX B

-108-

 \$SIGNON ADR T=10.0M P=50

**LAST_SIGNON WAS: 12:36:15 06-03-69

USER "ADR." SIGNED ON AT 13:23:22 ON 06-03-69

\$RUN BOX+ROBOT+PRATT+SUBR+DEC2 1=TEE

EXECUTION BEGINS

DESIGN FOR PARALLEL CHORD TRUSSES USING TEE SECTIONS FOR CHORD MEMBERS AND DOUBLE ANGLE SECTIONS FOR WEB MEMBERS.

YIELD STRESS FOR WEB MEMBERS EQUALS 36.0 KSI.

YIELD STRESS FOR COMPRESSION CHORD EQUALS 36.0 KSI.

YIELD STRESS FOR TENSION CHORD EQUALS 36.0 KSI.

DEFLECTION CONSTRAINT DELTA/SPAN EQUALS 0.00417.

UPPER BOUNDS ON TRUSS SPACING INCREMENTS ARE AS FOLLOWS-

16.99	24.00	31.00	38.00	45.00
DEAD LOAD OF ROOF PER SQUARE FOOT FOR EACH INCREMENT IS AS FOLLOWS-				
0.57	0.83	1.14	1.50	1.74

UPPER BOUNDS ON PURLIN SPACING INCREMENTS ARE AS FOLLOWS-

7.00	8.00	11.50	12.50	20.00
DEAD LOAD OF ROOF PER SQUARE FOOT FOR EACH INCREMENT IS AS FOLLOWS-				
2.02	2.40	2.50	3.00	3.66

LIVE LOADS AND ROOF DEAD LOAD APPLIED ON CHORDS BY PURLINS. TRUSS DEAD LOAD APPLIED AT PANEL POINTS.

EVEN PANEL SPACING CONSIDERED.

 CHORDS OF TRUSS CONSIDERED TO BE CONTINUOUS

TOTAL COST PER SQUARE FOOT EQUALS \$						1.6821
WORST POINT PARAMETERS						
15176.84	10.000	0.078	24.000	10.000	1.962	
CENTROID VARIABLE VALUES						
19179.38	12.889	0.074	33.333	6.365	1.712	
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
0.0	16.644	0.068	45.467	1.640		
CONSTRAINT EXCEEDED. KKKK EQUALS						3
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
0.0	14.767	0.071	39.400	4.003		
CONSTRAINT EXCEEDED. KKKK EQUALS						1
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6527
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6904
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6305
WORST POINT PARAMETERS						
21253.83	8.000	0.093	38.000	10.000	1.919	
CENTROID VARIABLE VALUES						
19593.25	13.333	0.072	34.111	5.810	1.680	
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
0.0	20.267	0.043	29.056	0.362		
CONSTRAINT EXCEEDED. KKKK EQUALS						3
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
0.0	16.800	0.057	31.583	3.086		
CONSTRAINT EXCEEDED. KKKK EQUALS						3
CENTROID USED AS NEW POINT						
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6349
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6620
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6154
WORST POINT PARAMETERS						
18340.84	14.000	0.066	31.000	7.500	1.781	
CENTROID VARIABLE VALUES						
19948.98	13.111	0.072	34.889	5.643	1.662	
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
0.0	11.956	0.080	39.944	3.229		
CONSTRAINT EXCEEDED. KKKK EQUALS						1
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6172
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6560
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6056
WORST POINT PARAMETERS						
14891.72	16.000	0.063	24.000	6.667	1.736	
CENTROID VARIABLE VALUES						
20878.80	12.444	0.074	37.222	5.458	1.647	
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
0.0	7.822	0.088	54.411	3.887		
CONSTRAINT EXCEEDED. KKKK EQUALS						2
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
0.0	10.133	0.081	45.817	4.672		
CONSTRAINT EXCEEDED. KKKK EQUALS						2
CENTROID USED AS NEW POINT						
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6183
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6317
TOTAL COST PER SQUARE FOOT EQUALS \$						1.5813
WORST POINT PARAMETERS						
19297.49	14.000	0.063	31.000	5.000	1.695	
CENTROID VARIABLE VALUES						
21046.93	12.000	0.075	38.000	5.569	1.635	
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
0.0	9.400	0.091	47.100	6.309		
CONSTRAINT EXCEEDED. KKKK EQUALS						1
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6300

TOTAL COST PER SQUARE FOOT EQUALS \$						1.6336
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6241
WORST POINT PARAMETERS						
18606.34	14.000	0.064	31.000	5.000		1.682
CENTROID VARIABLE VALUES						
21619.30	11.333	0.078	39.556	5.754		1.628
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
0.0	7.867	0.097	50.678			6.735
CONSTRAINT EXCEEDED. KKKK EQUALS 1						
TOTAL COST PER SQUARE FOOT EQUALS \$						1.7176
TOTAL COST PER SQUARE FOOT EQUALS \$						1.7317
TOTAL COST PER SQUARE FOOT EQUALS \$						1.7757
NEW POINT IS NO IMPROVEMENT OVER WORST POINT. HALVE ALPHA.						
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
22399.14	9.600	0.088	45.117			6.244
CONSTRAINT EXCEEDED. KKKK EQUALS 1						
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6278
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6385
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6149
WORST POINT PARAMETERS						
24291.70	16.000	0.077	45.000	6.667		1.681
CENTROID VARIABLE VALUES						
21572.57	10.444	0.079	39.556	5.754		1.621
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
0.0	3.222	0.083	32.478			4.568
CONSTRAINT EXCEEDED. KKKK EQUALS 2						
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
0.0	6.833	0.081	36.017			5.161
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6496
TOTAL COST PER SQUARE FOOT EQUALS \$						1.5922
WORST POINT PARAMETERS						
26310.64	12.000	0.070	45.000	5.000		1.649
CENTROID VARIABLE VALUES						
21059.48	10.000	0.081	38.778	5.805		1.614
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
0.0	7.400	0.095	30.689			6.851
TOTAL COST PER SQUARE FOOT EQUALS \$						1.7195
TOTAL COST PER SQUARE FOOT EQUALS \$						1.7500
TOTAL COST PER SQUARE FOOT EQUALS \$						1.7831
NEW POINT IS NO IMPROVEMENT OVER WORST POINT. HALVE ALPHA.						
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
16413.15	8.700	0.088	34.733			6.328
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6097
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6123
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6485
WORST POINT PARAMETERS						
17737.20	12.000	0.070	31.000	5.455		1.644
CENTROID VARIABLE VALUES						
21486.57	9.556	0.083	39.556	5.939		1.611
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
0.0	6.378	0.100	50.678			6.570
CONSTRAINT EXCEEDED. KKKK EQUALS 1						
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6841
TOTAL COST PER SQUARE FOOT EQUALS \$						1.6333
WORST POINT PARAMETERS						
22851.59	8.000	0.100	45.000	6.667		1.633
CENTROID VARIABLE VALUES						
21486.57	9.556	0.083	39.556	5.939		1.611
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS						
0.0	11.578	0.061	32.478			4.994

CONSTRAINT EXCEEDED. KKKK EQUALS 1					
TOTAL COST PER SQUARE FOOT EQUALS \$	1.6217				
TOTAL COST PER SQUARE FOOT EQUALS \$	1.6559				
TOTAL COST PER SQUARE FOOT EQUALS \$	1.6218				
WORST POINT PARAMETERS					
24978.64	12.000	0.071	45.000	5.000	1.630
CENTROID VARIABLE VALUES					
21318.69	9.556	0.082	38.778	5.939	1.610
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS					
0.0	6.378	0.095	30.689	7.161	
TOTAL COST PER SQUARE FOOT EQUALS \$	1.7814				
TOTAL COST PER SQUARE FOOT EQUALS \$	1.7209				
NEW POINT IS NO IMPROVEMENT OVER WORST POINT. HALVE ALPHA.					
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS					
16431.81	7.967	0.088	34.733	6.550	
TOTAL COST PER SQUARE FOOT EQUALS \$	1.6083				
TOTAL COST PER SQUARE FOOT EQUALS \$	1.6145				
TOTAL COST PER SQUARE FOOT EQUALS \$	1.6499				
WORST POINT PARAMETERS					
23757.64	8.000	0.091	45.000	6.667	1.624
CENTROID VARIABLE VALUES					
21058.72	9.556	0.081	38.000	5.939	1.608
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS					
0.0	11.578	0.068	28.900	4.994	
CONSTRAINT EXCEEDED. KKKK EQUALS 1					
TOTAL COST PER SQUARE FOOT EQUALS \$	1.6470				
TOTAL COST PER SQUARE FOOT EQUALS \$	1.6793				
TOTAL COST PER SQUARE FOOT EQUALS \$	1.6345				
NEW POINT IS NO IMPROVEMENT OVER WORST POINT. HALVE ALPHA.					
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS					
18891.01	10.567	0.075	33.450	5.467	
TOTAL COST PER SQUARE FOOT EQUALS \$	1.5901				
TOTAL COST PER SQUARE FOOT EQUALS \$	1.6135				
TOTAL COST PER SQUARE FOOT EQUALS \$	1.5829				
WORST POINT PARAMETERS					
23467.75	12.000	0.061	38.000	5.000	1.622
CENTROID VARIABLE VALUES					
20915.05	9.111	0.083	38.000	6.051	1.603
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS					
0.0	5.356	0.111	38.000	7.416	
CONSTRAINT EXCEEDED. KKKK EQUALS 1					
TOTAL COST PER SQUARE FOOT EQUALS \$	1.8183				
TOTAL COST PER SQUARE FOOT EQUALS \$	1.7522				
NEW POINT IS NO IMPROVEMENT OVER WORST POINT. HALVE ALPHA.					
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS					
21905.18	7.233	0.097	38.000	6.733	
TOTAL COST PER SQUARE FOOT EQUALS \$	1.7218				
TOTAL COST PER SQUARE FOOT EQUALS \$	1.7328				
TOTAL COST PER SQUARE FOOT EQUALS \$	1.7790				
CENTROID USED AS NEW POINT					
TOTAL COST PER SQUARE FOOT EQUALS \$	1.6224				
TOTAL COST PER SQUARE FOOT EQUALS \$	1.6325				
TOTAL COST PER SQUARE FOOT EQUALS \$	1.6042				
WORST POINT PARAMETERS					
11884.92	10.000	0.100	24.000	6.000	1.621
CENTROID VARIABLE VALUES					
22268.89	8.889	0.081	40.333	6.125	1.602
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS					
0.0	7.444	0.056	61.567	6.287	
CONSTRAINT EXCEEDED. KKKK EQUALS 2					

NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS					
0.0	8.167	0.068	50.950	6.206	
CONSTRAINT EXCEEDED. KKKK EQUALS				1	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6202	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6210	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6572	
WORST POINT PARAMETERS					
27594.12	8.000	0.068	45.000	6.667	1.620
CENTROID VARIABLE VALUES					
22268.89	8.889	0.081	40.333	6.125	1.602
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS					
0.0	10.044	0.097	34.267	5.420	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6134	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6304	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6129	
WORST POINT PARAMETERS					
21542.39	12.000	0.072	38.000	6.000	1.615
CENTROID VARIABLE VALUES					
22137.76	8.444	0.084	40.333	6.064	1.601
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS					
0.0	3.822	0.100	43.367	6.147	
CONSTRAINT EXCEEDED. KKKK EQUALS				1	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6839	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6330	
NEW POINT IS NO IMPROVEMENT OVER WORST POINT. HALVE ALPHA.					
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS					
22848.51	6.133	0.092	41.850	6.106	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6667	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6254	
CENTROID USED AS NEW POINT					
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6129	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6194	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6562	
WORST POINT PARAMETERS					
23871.18	8.000	0.088	45.000	6.667	1.615
CENTROID VARIABLE VALUES					
22199.98	8.444	0.083	40.333	6.064	1.601
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS					
0.0	9.022	0.078	34.267	5.280	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.5913	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6060	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.5886	
WORST POINT PARAMETERS					
24431.12	8.000	0.084	45.000	6.667	1.613
CENTROID VARIABLE VALUES					
21930.71	8.444	0.083	39.556	5.929	1.598
NEW POINT PARAMETERS BEFORE SATISFACTION OF INTEGER CONSTRAINTS					
0.0	9.022	0.081	32.478	4.971	
CONSTRAINT EXCEEDED. KKKK EQUALS				1	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.5974	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.6160	
TOTAL COST PER SQUARE FOOT EQUALS \$				1.5786	
SEARCH TERMINATED. MAXIMUM NO. OF ITERATIONS REACHED.					

DESIGN OF STATICALLY INDETERMINATE TRUSS

SPAN IN INCHES EQUALS 1440.0
DEPTH TO SPAN RATIO EQUALS 0.0812
NUMBER OF PANELS EQUALS 8
NUMBER OF JOINTS EQUALS 18
NUMBER OF MEMBERS EQUALS 33

NUMBER OF DEGREES OF FREEDOM EQUALS 51
HALF BAND WIDTH EQUALS 12

PURLIN SPACING IN INCHES EQUALS 60.00
TRUSS SPACING IN FEET EQUALS 38.00 . ROOF DEAD LOAD IN KIPS PER FOOT OF SPAN EQUALS 0.1339

INTERNAL LOAD ROUTINES ARE USED.

THE FOLLOWING LOAD CASES ARE CONSIDERED

UNIFORM LIVE LOAD ON FULL SPAN AND UNIFORM LIVE LOAD ON HALF SPAN
ROOF DEAD LOAD AND TRUSS DEAD LOAD ADDED IN FOR ALL CASES

THE MAGNITUDE OF THE UNIFORMLY DISTRIBUTED LOAD IS 1.52 KIPS PER FOOT.
THE MAGNITUDE OF THE POINT LOAD IS .0.0 KIPS.

TOTAL NUMBER OF LOAD CASES CONSIDERED IS 2

WEIGHT USED FOR REDESIGN EQUALS 20508.62
 WEIGHT USED FOR REDESIGN EQUALS 20500.43
 WEIGHT USED FOR REDESIGN EQUALS 20814.84
 PANEL DEPTH TO SPAN RATIO EQUALS 0.6495

MEMBER INFORMATION FOR DESIGN - COMPRESSION NEGATIVE - FORCE IN KIPS - AREA SQUARE INCHES

DESIGN OF WEB MEMBERS

MEMBER NUMBER	MAX. COMPRESSION LOAD	MAX. TENSION LOAD	DESIGN AREA	LENGTH	STRESS FACTOR	IDENTIFICATION		
1	-109.281	0.0	7.060	116.91	1.008	5.00	3.50	0.438 DOUBLE ANGLE
3	0.0	179.057	8.360	214.63	0.992	5.00	5.00	0.438 DOUBLE ANGLE
5	-97.727	0.0	7.000	116.91	1.018	4.00	3.50	0.500 DOUBLE ANGLE
7	0.0	124.378	5.720	214.63	1.007	4.00	4.00	0.375 DOUBLE ANGLE
9	-67.266	0.0	4.960	116.91	0.972	4.00	3.00	0.375 DOUBLE ANGLE
11	0.0	72.890	3.380	214.63	0.998	3.50	3.50	0.250 DOUBLE ANGLE
13	-40.388	0.0	3.560	116.91	0.905	3.50	2.50	0.313 DOUBLE ANGLE
15	-17.425	48.179	4.600	214.63	0.986	3.50	3.00	0.375 DOUBLE ANGLE
17	-28.306	0.0	3.240	116.91	0.907	3.00	2.50	0.313 DOUBLE ANGLE
19	-17.425	48.179	4.600	214.63	0.986	3.50	3.00	0.375 DOUBLE ANGLE
21	-40.388	0.0	3.560	116.91	0.905	3.50	2.50	0.313 DOUBLE ANGLE
23	0.0	72.890	3.380	214.63	0.998	3.50	3.50	0.250 DOUBLE ANGLE
25	-67.266	0.0	4.960	116.91	0.972	4.00	3.00	0.375 DOUBLE ANGLE
27	0.0	124.378	5.720	214.63	1.007	4.00	4.00	0.375 DOUBLE ANGLE
29	-97.727	0.0	7.000	116.91	1.018	4.00	3.50	0.500 DOUBLE ANGLE
31	0.0	179.057	8.360	214.63	0.992	5.00	5.00	0.438 DOUBLE ANGLE
33	-109.281	0.0	7.060	116.91	1.008	5.00	3.50	0.438 DOUBLE ANGLE

DESIGN OF CHORD MEMBERS

MEMBER NUMBER	DESIGN AREA	INERTIA	LENGTH	STRESS FACTOR	IDENTIFICATION		
2	15.89	349.52	180.00	0.017	15	54.00	TEE
4	22.50	440.74	180.00	0.744	18	85.00	TEE
6	15.89	349.52	180.00	0.455	15	54.00	TEE
8	22.50	440.74	180.00	1.006	18	85.00	TEE
10	15.89	349.52	180.00	0.801	15	54.00	TEE
12	22.50	440.74	180.00	1.009	18	85.00	TEE
14	15.89	349.52	180.00	0.979	15	54.00	TEE
16	22.50	440.74	180.00	0.972	18	85.00	TEE
18	15.89	349.52	180.00	0.979	15	54.00	TEE
20	22.50	440.74	180.00	0.973	18	85.00	TEE
22	15.89	349.52	180.00	0.801	15	54.00	TEE
24	22.50	440.74	180.00	1.009	18	85.00	TEE
26	15.89	349.52	180.00	0.455	15	54.00	TEE
28	22.50	440.74	180.00	1.006	18	85.00	TEE
30	15.89	349.52	180.00	0.017	15	54.00	TEE
32	22.50	440.74	180.00	0.744	18	85.00	TEE

CHORD FORCES - AXIAL FORCE IN KIPS - BENDING MOMENT IN KIP INCHES					
MEMBER NUMBER	LOAD CASE	AXIAL FORCE	BMJNL	BMJNG	
2	1	-0.004	0.0	34.834	
	2	-0.002	0.0	32.298	
4	1	-150.161	0.0	-348.407	
	2	-115.133	0.0	-354.188	
6	1	150.159	-34.833	34.927	
	2	115.133	-32.296	40.847	
8	1	-254.468	348.414	-227.458	
	2	-183.992	354.193	-200.991	
10	1	254.465	-34.924	122.087	
	2	183.991	-40.845	92.728	
12	1	-315.595	227.471	-72.065	
	2	-211.535	201.000	-187.452	
14	1	315.593	-122.084	86.097	
	2	211.534	-92.725	50.235	
16	1	-337.693	72.075	-156.594	
	2	-196.921	187.467	-77.250	
18	1	315.595	-86.095	122.089	
	2	156.516	-50.230	49.655	
20	1	-337.692	156.612	-72.052	
	2	-196.921	77.265	132.091	
22	1	254.467	-122.091	34.920	
	2	112.738	-49.653	0.0	
24	1	-315.596	72.064	-227.470	
	2	-156.516	-132.086	-37.709	
26	1	150.162	-34.922	34.836	
	2	59.804	0.0	8.070	
28	1	-254.467	227.481	-348.404	
	2	-112.738	37.717	-17.128	
30	1	-0.000	-34.837	0.0	
	2	0.000	-8.070	0.0	
32	1	-150.162	348.411	0.0	
	2	-59.804	17.127	0.0	
TOTAL WEIGHT OF TRUSS EQUALS		20978.176			
WEIGHT PER FOOT OF SPAN FOR ALL MEMBERS EQUALS		174.82			
TOTAL WEIGHT OF TOP CHORD EQUALS		10200.00			
TOTAL WEIGHT OF BOTTOM CHORD EQUALS		6488.41			
TOTAL LENGTH OF TRUSS MEMBERS IN FEET EQUALS		470.77			
TOTAL LENGTH OF TRUSS WEB MEMBERS EQUALS		230.77			
WEIGHT OF WEB MEMBERS EQUALS		4289.762			
RATIO OF WEB WEIGHT TO TOTAL TRUSS WEIGHT EQUALS		0.2045			
WEIGHT PER FOOT OF SPAN OF CHORD MEMBERS EQUALS		139.07			
WEIGHT PER FOOT OF SPAN OF WEB MEMBERS EQUALS		35.75			
COST COEFFICIENTS USED ARE AS FOLLOWS -					
WEB MATERIAL COST PER POUND EQUALS \$		0.08			
TOP CHORD MATERIAL COST PER POUND EQUALS \$		0.09			
BOTTOM CHORD MATERIAL COST PER POUND EQUALS \$		0.09			
COST OF PREPARATION PER WEB MEMBER EQUALS \$		6.00			
COST OF WEB JOINTS PER MEMBER EQUALS \$		25.00			
COST OF PREPARATION PER CHORD MEMBER EQUALS \$		15.00			
COST PER CHORD SPLICE EQUALS \$		40.00			
COST PER SQUARE FOOT OF WALL CLADDING EQUALS \$		1.50			
ROOF COST PER SQUARE FOOT BASED ON TRUSS SPACING					
UPPER BOUNDS OF SPACING INCREMENTS ARE AS FOLLOWS-					
	16.99	24.00	31.00	38.00	45.00
COST PER SQUARE FOOT OF ROOF FOR EACH INCREMENT IS AS FOLLOWS -					

0.17	0.25	0.34	0.36	0.42
ROOF COST PER SQUARE FOOT BASED ON PANEL LENGTHS				
UPPER BOUNDS ON PANEL LENGTHS ARE AS FOLLOWS -				
7.00	8.00	11.50	12.50	20.00
COST PER SQUARE FOOT OF ROOF FOR EACH PANEL INCREMENT IS AS FOLLOWS -				
0.40	0.50	0.70	0.80	1.00

NUMBER OF CHORD PIECES ASSUMED EQUALS	6
NUMBER OF CHORD SPLICES ASSUMED EQUALS	4
COST OF WEB MATERIAL EQUALS \$	343.18
COST OF TOP CHORD MATERIAL EQUALS \$	918.00
COST OF BOTTOM CHORD MATERIAL EQUALS \$	583.96
WEB LABOUR COST EQUALS \$	102.00
COST FOR WEB MEMBER JOINTS EQUALS \$	425.00
CHORD LABOUR COST EQUALS \$	90.00
CHORD SPLICE COST EQUALS \$	160.00
COST OF WALL CLADDING FOR ONE BAY EQUALS \$	1110.64
COST OF ROOFING DUE TO TRUSS SPACING EQUALS \$	1641.60
COST OF ROOFING DUE TO AVERAGE PANEL SPACING EQUALS \$	1824.00
TOTAL COST EQUALS \$	7198.38
TOTAL COST PER SQUARE FOOT EQUALS \$	1.5786

PRINT OUT OF PROP MATRIX AND COST VECTOR

20362.33	8.0000	0.0970	38.0000	5.4545	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6129
23260.07	10.0000	0.0801	45.0000	5.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6056
21581.02	8.0000	0.0878	38.0000	6.6667	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6097
20978.18	8.0000	0.0812	38.0000	5.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.5786
21692.81	8.0000	0.0810	38.0000	5.4545	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.5922
20810.65	10.0000	0.0740	38.0000	6.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.5813
24069.54	8.0000	0.0828	45.0000	6.6667	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6042
22174.62	8.0000	0.0747	38.0000	6.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.5829
22007.64	8.0000	0.0777	38.0000	5.4545	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.5886
21417.90	8.0000	0.0884	38.0000	6.6667	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6083
14613.02	6.0000	0.1025	24.0000	20.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.2271
22940.43	6.0000	0.1036	45.0000	20.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.2233
22348.63	8.0000	0.0933	38.0000	12.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.0411
15176.84	10.0000	0.0784	24.0000	10.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.9620
21253.83	8.0000	0.0931	38.0000	10.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.9189
18340.84	14.0000	0.0660	31.0000	7.5000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.7809
14891.72	16.0000	0.0631	24.0000	6.6667	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.7364
19297.49	14.0000	0.0629	31.0000	5.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6954
18606.34	14.0000	0.0640	31.0000	5.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6821
24291.70	16.0000	0.0771	45.0000	6.6667	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6813
26310.64	12.0000	0.0698	45.0000	5.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6490
17737.20	12.0000	0.0697	31.0000	5.4545	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6444
22851.59	8.0000	0.0996	45.0000	6.6667	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6333
24978.64	12.0000	0.0711	45.0000	5.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6305
23757.64	8.0000	0.0913	45.0000	6.6667	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6241
23467.75	12.0000	0.0607	38.0000	5.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6217
11884.92	10.0000	0.1000	24.0000	6.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6211
27594.12	8.0000	0.0684	45.0000	6.6667	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6202
21542.39	12.0000	0.0715	38.0000	6.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6154
23871.18	8.0000	0.0876	45.0000	6.6667	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6149
24431.12	8.0000	0.0837	45.0000	6.6667	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6129

STOP
EXECUTION TERMINATED

\$\$SIGNOFF