STRUCTURAL DYNAMIC PROPERTIES FROM AMBIENT VIBRATIONS

by

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ABSTRACT

Ambient vibrations of a reinforced concrete tower structure were recordedand analyzed to obtain the natural frequencies, the associated mode shapes and an estimate of the equivalent viscous damping.

The structure investigated consists of four concrete wall panels,rigidly connected at various levels and contains a light precast concrete stairwell. It is similar to typical components of larger structures,such as stairwells and elevator shafts or cores. The given information should be useful in offering details of the dynamic behaviour of this type of structural elements.

The experimental results are compared with the theoretical results obtained from two- and three-dimensional dynamic analyses using matrix methods applied to linear elastic systems with lumped masses. An efficient computer program to find the eigenvalues and eigenvectors for this type of mathematical model is described.

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The architectural firm of Thompson,Berwick,Pratt and Partners kindly supplied the structural plans and design calculations for the clocktower. The writer is very much indebted to Mr. R. Ian Miller for his readiness to offer advice during the development of the computer programs and their often frustrating debugging procedure.

The research was made possible through a grant from the National Research Council of Canada. It enabled the writer not only to deepen his knowledge of structural engineering but also to learn to love the beautiful Province of British Columbia.

1. GENERAL

1.1. INTRODUCTION

This thesis presents the results of a comparative experimental and theoretical analysis of a simple reinforced concrete tower structure. It is meant as a contribution to the knowledge of basic dynamic characteristics of structures and their idealization as a mathematical model.

The technique used for the experimental program is well established [1,2,3,8 +] and is used to determine the natural frequencies of vibration, mode shapes and the percentage of equivalent viscous damping of the structure. It involves field measurements of the ambient vibrations of the tower due to natural (wind, microtremors, etc.) and cultural (traffic, machine vibrations, etc.) input sources. The recorded data is then analyzed by finite Fourier transform methods [9] to yield the desired information. This approach requires a relatively constant power spectrum of the input, which is at least the case for most of the natural sources. If this can be assumed, the structure is excited in its natural modes and amplifies the resonant frequencies proportional to the relative modal displacements. Chapter 2. outlines the experimental program and the various techniques used to extract the desired data.

+ Numbers in brackets refer to bibliography numbers

The next chapter describes the matrix analysis of three-, two- and one-dimensional mathematical models. The fundamentals of the computer program used are given in matrix notation. All models are assumed to be linearly elastic and are represented by prismatic members, the weight being lumped at the nodes.

The results obtained from experiment and theory are found to be generally in good agreement, which confirms the correctness of the assumptions made for the mathematical models.

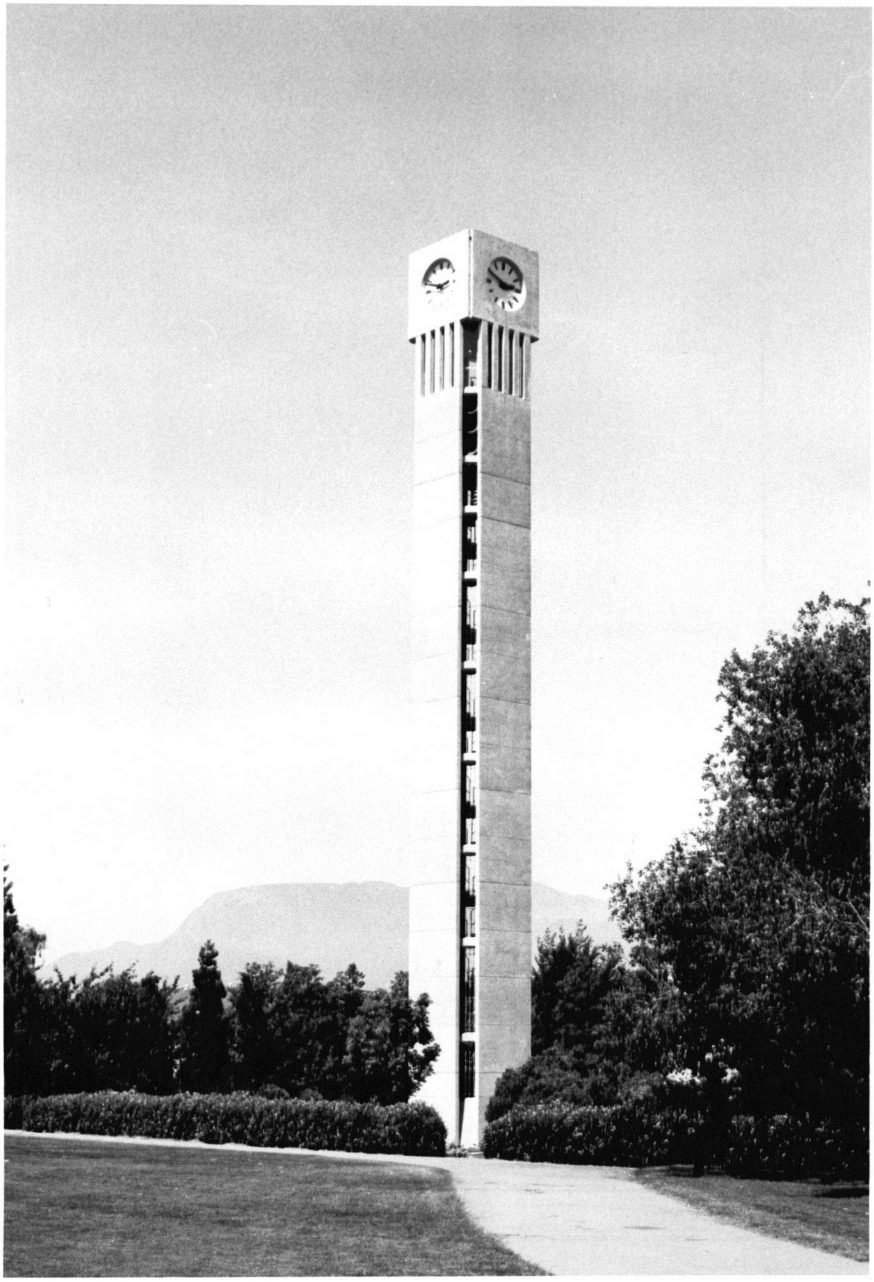


Fig.1 View of Ladner clock tower looking N-E

1.2. DESCRIPTION OF STRUCTURE AND SITE

The Ladner Tower at the University of British Columbia was designed by Thompson,Berwick,Pratt and Partners,Architects and built of reinforced concrete by Smith Brothers and Wilson Ltd.,Contractors in the summer of 1968 as a clocktower for the U. of B.C. campus. It is located in front of the Main Library on a paved plaza, surrounded by a park area.(Fig.1)

The structure rises 121.5 feet above the ground level (see Fig.2a) on a square plan of 13.5 x 13.5 ft.(see Fig.2b). It is founded 8 ft. below the ground level on a massive octagonal slab with a diameter of the circumscribed circle of 32.4 ft. The slab rests on a well graded sand deposit containing some silt and medium fine gravel; the standard penetration test value is approximately 100 blows/ft.

The wall panels were cast in situ and have the uniform cross-section shown in Fig.2b up to elevation 101.2 ft. Above that level the walls are broken up into small columns of 9.7 ft. height to accomodate an observation platform. The top story consists of a closed box section with circular holes of 7 ft. diameter for the clockdials on all four faces.

The stairs and stairlandings were prefabricated of concrete with the same 4000 psi. minimum 28-day strength as the structural parts of the tower. Heavily reinforced spandrels which incorporate the stairlandings connect and provide the necessary shear transfer between the wall panels.

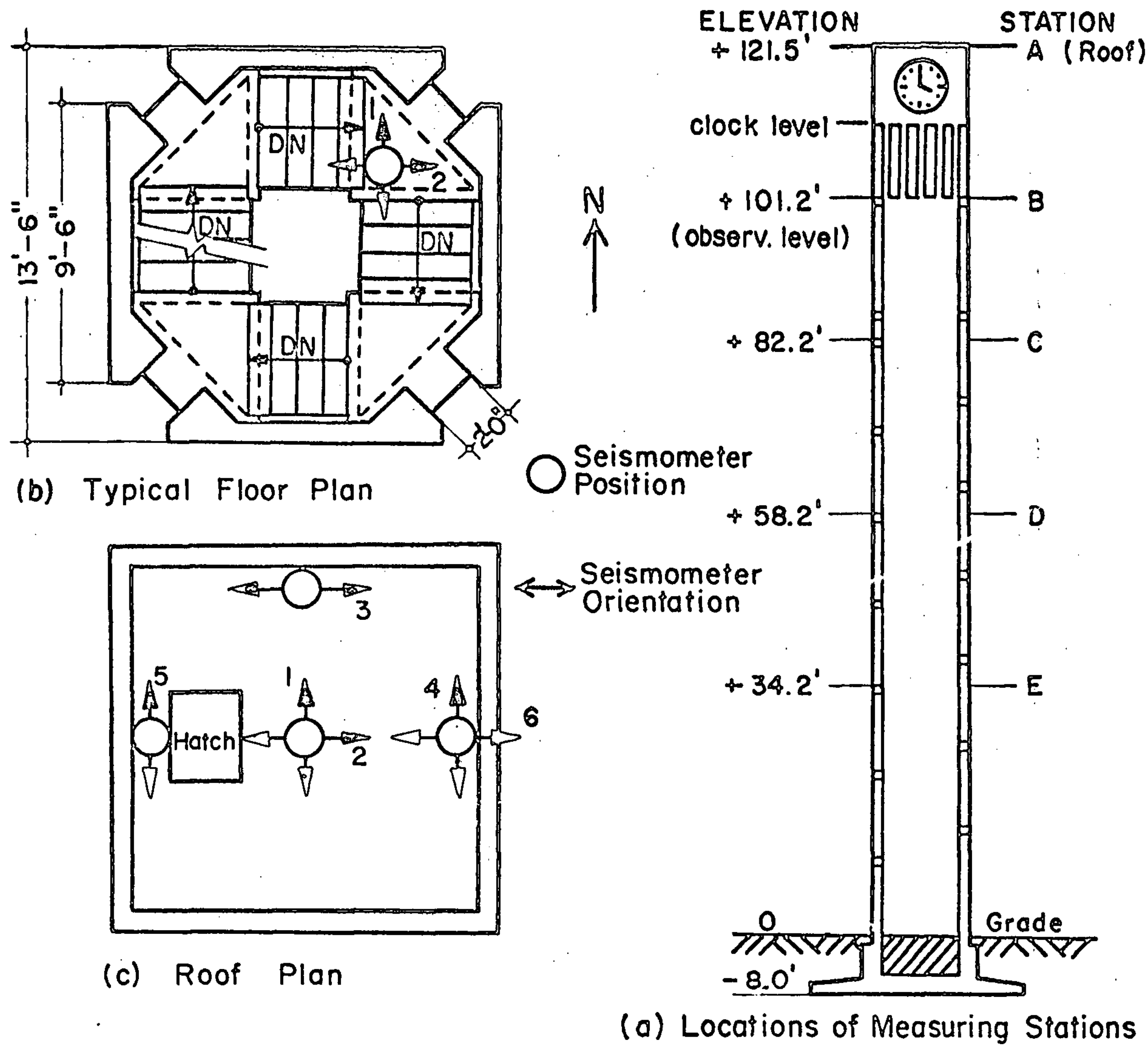


FIG. 2 LOCATIONS OF TRANSDUCERS

2. EXPERIMENTAL PROGRAM

2.1. INSTRUMENTATION

The sensing instruments used for measuring the ambient vibrations were two Willmore Mk. II seismometers with 130 ft. long shielded cables. A control panel for calibration and balancing of the system incorporated a Maxwell impedance bridge and Geotech solidstate amplifiers,Model AS-330, with high- and low-cut filters of 100 Hz. and 0.8 Hz respectively. For calibration and balancing procedures see Appendix I.

A Tektronix two-channel oscilloscope with inverting input and a Sanborn heated stylus single channel oscillograph were employed to monitor the tape input while recording. The analogue signal was recorded by a PI 7-track LP Monitoring Recorder, Model PI 5107, at a speed of 15/64 ips. A two way radio system proved very useful for communication between the recording crew on the ground and in the tower.

2.2 TEST PROCEDURE

The records used for the analysis were taken on May 4,1970, a day with light winds averaging 2-8 mph. Table 1 shows the hourly average windspeeds and directions as obtained from the Plant Science Field Laboratory, University of British Columbia.

Time (DST)	Direction	Average Speed (mph)	Record No.
13-14	SE	8	1,2,3
14-15	SE	8	4,5
15-16	S	5	6
16-17	SW	7	7,8,9
17-18	NW	3	10,11
18-19	S	2	12

TABLE 1: Hourly average wind speed and direction on test-day

The Willmores were set at a resonant frequency of about 1.2 cps. and damped to 55 percent of critical. To obtain a high signal to noise ratio, the tape input was kept close to the permissible level of 2 Volts peak-to-peak by adjusting the amplifier gain for each channel after observing the oscilloscope for some time before recording.

Three series of records were taken with the seismometers in different locations and orientations. In the first series of records,one seismometer was kept stationary as a reference in the centre of station A (roof) in N-S direction (Position 1, see Fig.2c). The other seimometer was then moved from station B to stations C,D and E successively. A record of both seismometer signals was taken for each setup. Assuming the base of the tower rigidly fixed, this procedure determines 6 ratios of relative amplitudes,which are sufficient for defining the first three mode shapes of translation.

The second series of records provided analogous information about the E-W direction.

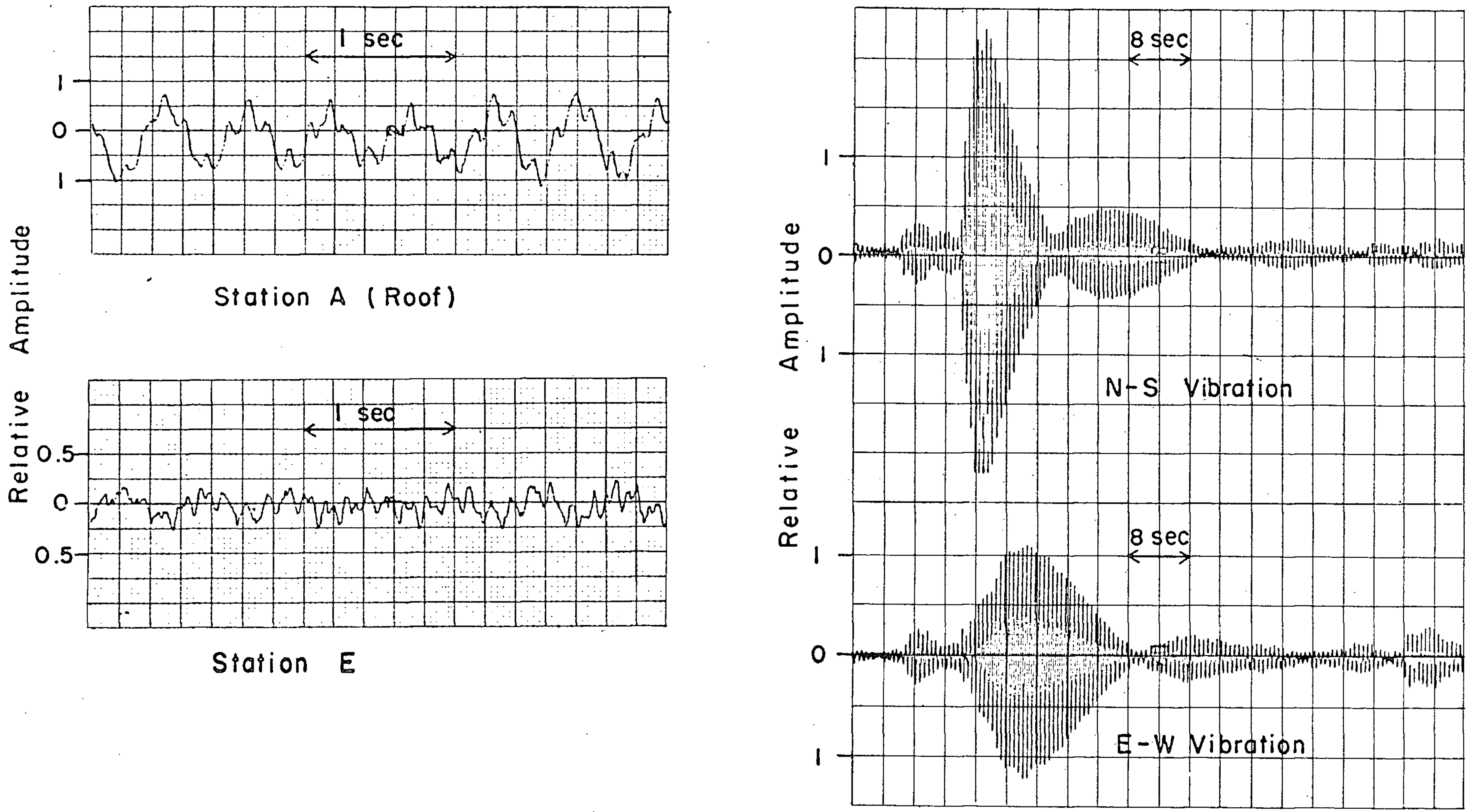
For the third series both seimometers were kept on the roof-level in different positions,yielding data to evaluate the torsional frequencies and the structural damping constants, as well as a relative calibration of the two seismometers.

To record torsional motions,the two seismometers were placed in parallel locations along opposite wall faces (locations 4 and 5,Fig.2c).

Man excitation was employed to obtain data for estimating the damping constants in the first translational mode from the logarithmic decrement. During this part of the tests,the two seismometers were in perpendicular positions (locations 3 and 5) The synchronization of the swaying of the crew in the tower with the fundamental frequency was achieved by giving commands through a walkie-talkie from the ground level where an oscilloscope was operated to monitor the vibration signal from both channels. An interesting beat-phenomenon was observed during these forced vibrations. After one direction had been excited, the vibration gradually decayed on one channel,returning periodically. By playing back both channels (see Fig.3b) on a two channel oscillograph,it could be seen that a rotation of the plane of vibration took place. A possible explanation for this may be related to the fact that the structure has a radial symmetry of stiffness. Thus the period of vibration in the excited fundamental mode is the same for any direction and a slight disturbance like a wind gust,or the influence of the stairwell which acts like a spiral inside the structure, may cause a rotation of the plane of vibration.

To reduce the effect of unwanted wind excitation during these recordings,the amplifiers were adjusted to low gain and an hour of the day was selected when the thermal wind was light. (see Table 1)

As suggested in [3] ,a relative calibration for the entire system was carried out. Both seismometers were set up in series (positions 2 and 6,Fig.2c) on the roof level. By taking the ratio of the amplitudes of the Fourier spectra at the different frequencies,relative calibration factors between the two seismometers could easily be obtained. It should be noted however,that these factors usually vary with the frequency. Thus it is not possible to apply one factor as a constant multiplier to the data or the spectra.



(a) Wind - Excited

(b) Man-Excited in N-S Direction

FIG. 3 TYPICAL RECORDS OF WIND AND MAN EXCITED VIBRATIONS

2.3. ANALYSIS OF DATA

The 7-track analogue tape which had been recorded at 15/64 ips. was played back at 15/16 ips. on a Hewlett-Packard Magnetic Data Recording System,3900 Series, and digitized at a rate of 130 samples/real sec./channel with one channel for each seismometer. 40,672 points/channel/record were digitized by an IBM 8092 digital computer and an analogue-to-digital converter. This represents 312.9 sec. of real recording time per record in digital form. To identify the digitized portion of the analogue record,a computer plot from each digital record was made by plotting every 10th point at a scale of 100 points/inch.

The rate of digitization had been chosen at 130 samples/sec. so that a folding frequency of 65 Hz. for the spectral analysis was available and any possible 60 Hz. electrical noise could be identified in the Fourier spectra.

The predominant frequencies of a vibration record can be visualized from the spikes of the Fourier spectrum. A Fourier spectrum is defined for a function $f(\tau)$,not equal to zero for $0<\tau<T$ as:

$$F(\omega) = \int_0^T f(\tau) \cdot e^{-i\omega\tau} \, d\tau$$

or,in terms of sines and cosines:

$$F(\omega) = \int_0^T f(\tau) \cdot \cos\omega\tau \, d\tau - i \int_0^T f(\tau) \cdot \sin\omega\tau \, d\tau.$$

The Fourier amplitude spectrum is then given by the square root of the sum of the squares of the real and imaginary parts:

$$|F(\omega)| = \sqrt{\left[\int_0^T f(\tau)\cdot\cos\omega\tau \, d\tau\right]^2 + \left[\int_0^T f(\tau)\cdot\sin\omega\tau \, d\tau\right]^2}$$

A convenient method to carry out this integration in a digital computer is the Cooley-Tukey algorithm [9] ,a fast Fourier transform. The program SPECTRA which was used in the data analysis is a standard program of the U. of B.C. Civil Engineering program library and is based on the Cooley-Tukey algorithm. To save computing time,SPECTRA analyzes the data in blocks and averages the real and imaginary parts separately for each block before normalizing the amplitudes. This is not quite exact,since the time shift is not accounted for in the subsequent blocks. To investigate the error introduced by this, the same portion of a record was analyzed by computing the real and imaginary parts and normalizing the amplitudes of each block separately before they were averaged. A maximum difference of only 3 percent was found,as compared to the method used in SPECTRA. The same result was also found when evaluating the amplitude ratios of two seismometers.

Several portions of a record with high-frequency content were analyzed with a bandwidth of 0.032 Hz. No significant peaks were evident on the Fourier spectra above 30 Hz. In order to save computer time and to reduce the amount of output from the spectral analysis,it was decided to lower the folding frequency to 32.5 Hz. This was accomplished by considering only every second point of the digital data and bringing the

the sampling rate to 65 samples/sec. The average computer time for the analysis of 4096 points at a bandwidth of 0.064 Hz. was approximately 10.7 sec. on an IBM 360/67.

To investigate the influence of the amplitude of the vibrations on the Fourier spectra, two portions of a record with high amplitudes induced by strong wind were analyzed and compared with the spectra of two low amplitude sections of the same record. No difference for the resonant frequencies and only little effect on the mode shape ratios was found. A significant difference, however, could be noted in the relative amplitudes of the spectral peaks between the fundamental and the higher modes. It was found that the stronger winds excite vibrations mainly in the fundamental mode whereas lighter winds seem to have an input spectrum which includes the frequency range of the higher modes. Earlier investigators [1] have also mentioned this result. It was further confirmed by the fact that the spectra of the recordings for the E-W direction, taken at wind speeds between 2-5 mph., quite clearly showed a peak for the third mode. In contrast, the spectra of the records for the N-S direction with stronger winds from 8-15 mph. showed no third mode distinguishable from the noise level.

Thus two different sections with a low average amplitude from each record were chosen for the spectra used to determine the mode shapes. The mode shape ratios were obtained by dividing the Fourier coefficients of the resonant frequency of the

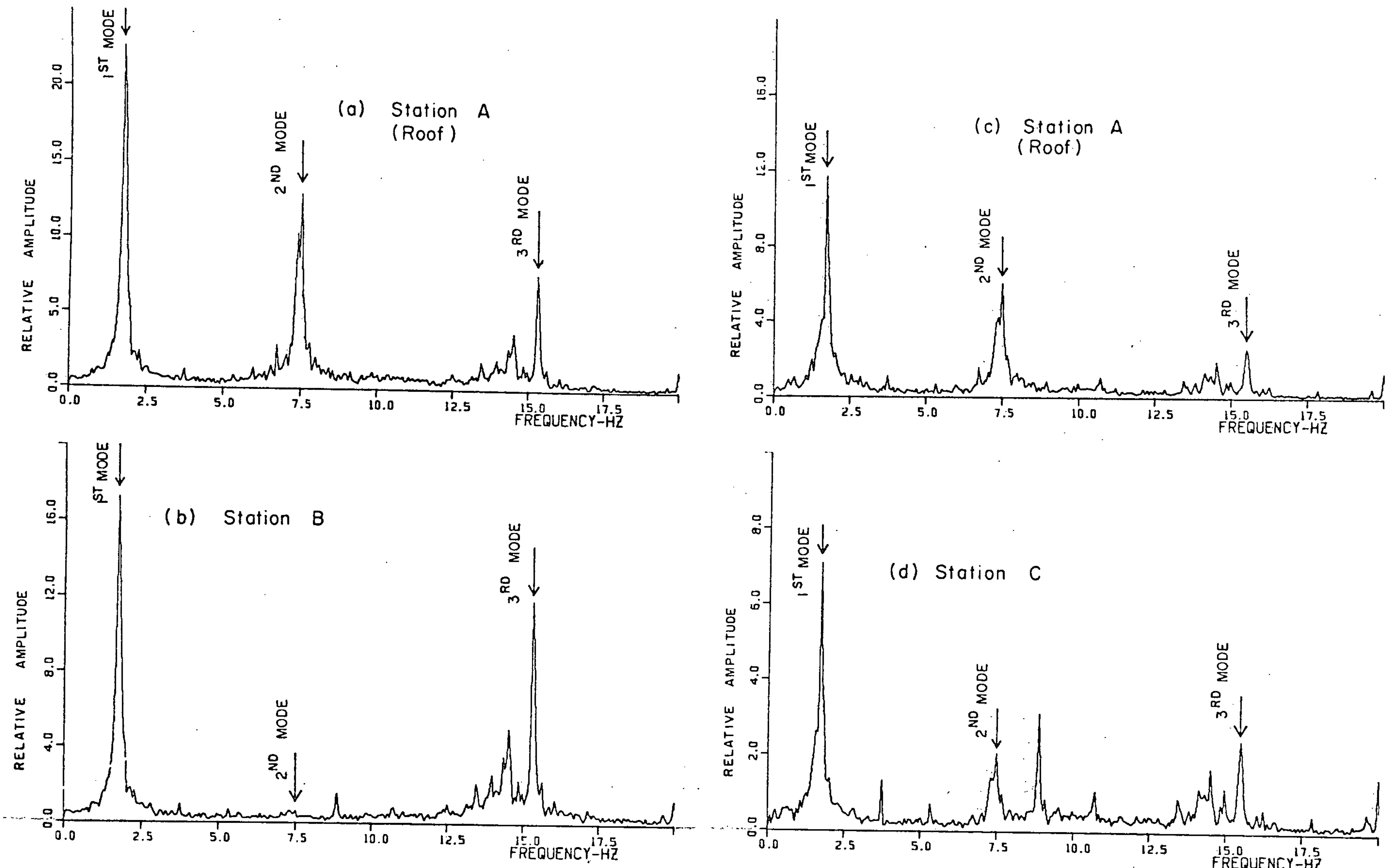


FIG.4 FOURIER SPECTRUM PAIRS FOR TRANSLATIONAL MODES (FROM SEISMOMETER POSITION 2)

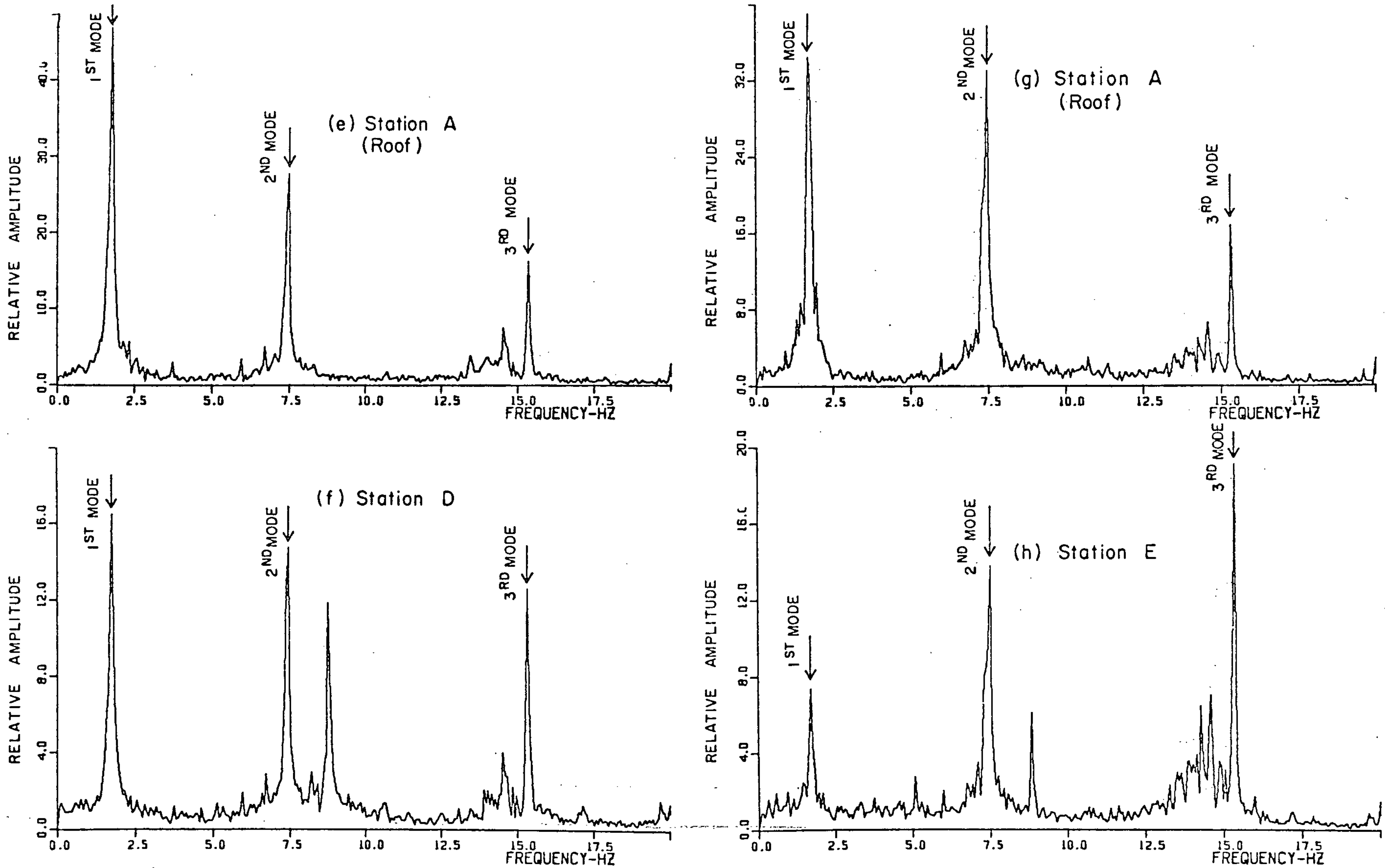


FIG. 4 (Cont'd) FOURIER SPECTRUM PAIRS FOR TRANSLATIONAL MODES (FROM SEISMOMETER POSITION 2)

level B,C,D and E by the Fourier coefficients of the reference level A on the roof. These ratios,together with a value of 1.0 for the top level and 0.0 for the base by assuming the building rigidly fixed at the ground level,were normalized with respect to the largest ratio and yielded the mode shapes at the resonant frequencies. It should be pointed out,that by always using the ratio of the same two seismometers,no calibration is necessary, because the ratio of the velocity sensitivities of the two seismometers is a constant for a given frequency. Typical spectra which were used for frequency and mode shape identification are shown in Fig.4.

To establish the phase between the two seismometers,the real parts of the respective Fourier coefficients are printed out and compared for their sign. If of the same sign,they are in phase,if they are of opposite sign,they are 180° out of phase, provided the seismometers were set up in the same direction. Another,more cumbersome way of establishing the phase is to form the sum and the difference of the digitized records of the two channels. If both are in phase,they must show a higher spectral value for the sum than for the difference. Conversely, if the difference yields a greater value,then they are 180° out of phase.

Similarly,to identify the torsional modes,the difference of the records of the two parallel seismometers on the top level in locations 4 and 5 was taken to eliminate the translational

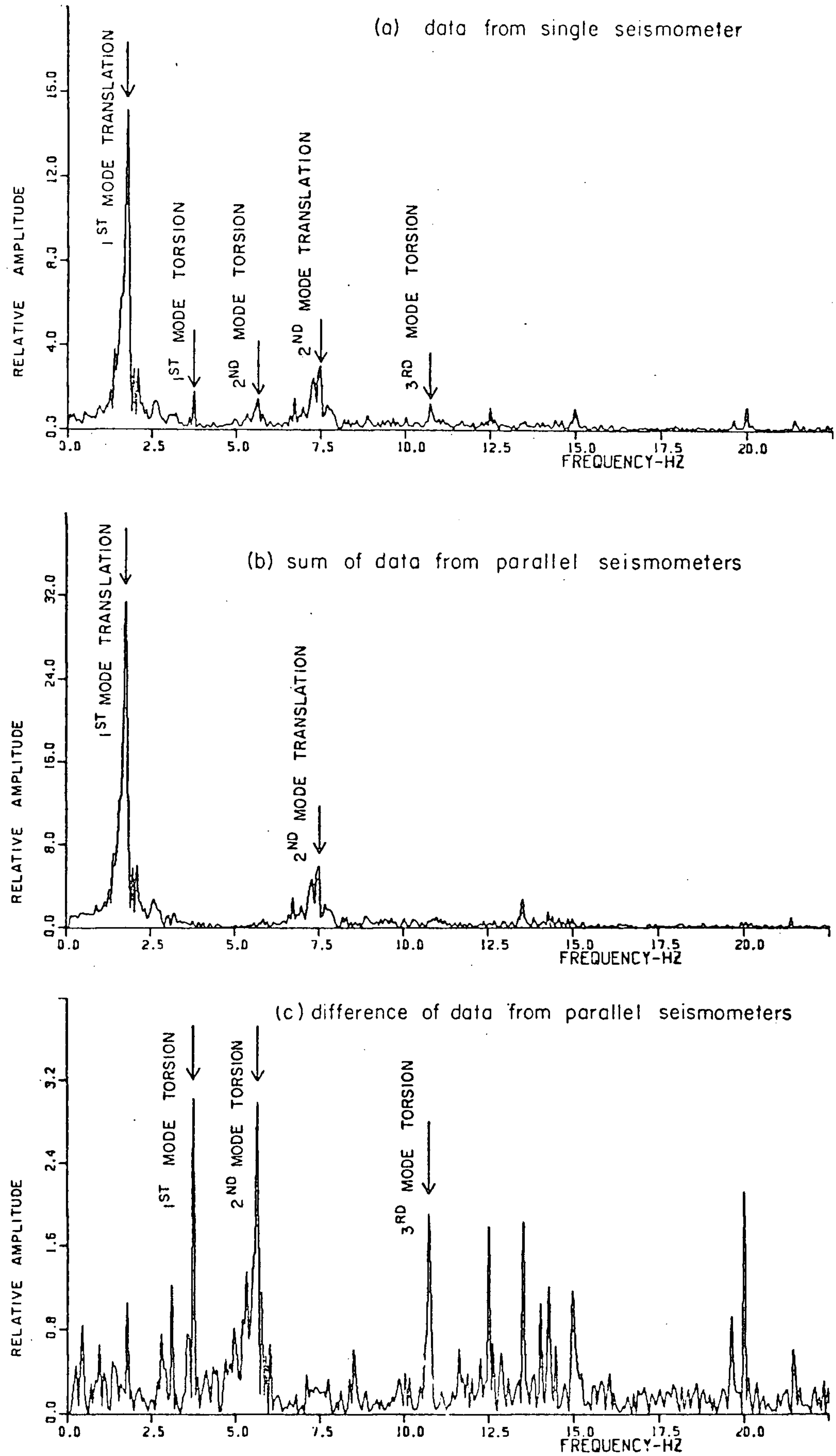


FIG. **5** FOURIER SPECTRA FOR TORSIONAL FREQUENCY IDENTIFICATION modes. As a check, the sum was used to subtract out the torsional modes. By comparing the two Fourier spectra thus obtained, the torsional frequencies can be readily identified as shown in Fig.5.(Note different scales on ordinates).

To obtain an estimate of the amount of equivalent viscous damping for the different modal resonances, two different methods were employed (see Ref. [5]).

The first is only feasible for modes of up to 2 cps. It consists of exciting a mode by letting a person push against the structure at a suitable elevation in the desired direction at the resonant frequency. The logarithmic decrement of the amplitude decay from the oscillograph record of the analogue signal then gives the damping ζ by the formula

$$\ln(A_n/A_{n+1}) = 2\pi\cdot(\zeta/\sqrt{1-\zeta^2})$$

where A_n and A_{n+1} are two successive amplitudes of vibration of the structure, after the exciting force has been removed. If $\zeta \leq 0.2$, then it can be found with sufficient accuracy from:

$$\ln(A_n/A_{n+1}) = 2\pi\cdot\zeta$$

Another method consists of measuring the bandwidth (see Fig.6) at the half power points of the Fourier spectral peaks at ω_0, giving the viscous damping as

$$\zeta = \Delta\omega/2\,\omega_0 .$$

For a derivation of the cited formulae above, see Ref.[5].

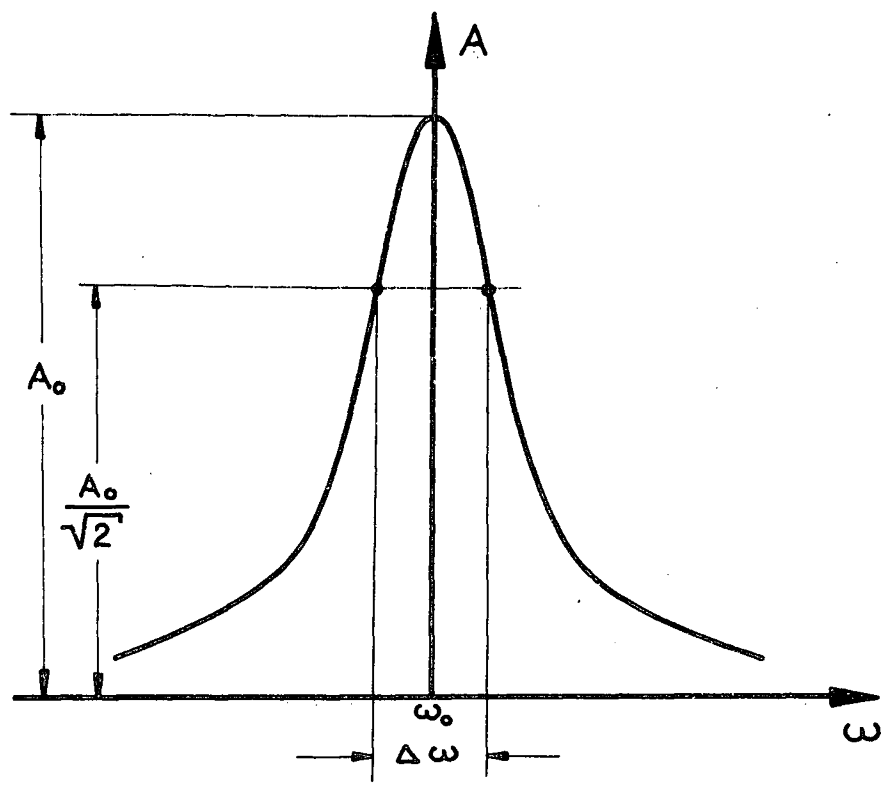


Fig. 6 Response of a S-D-F system to constant power excitation

3. THEORETICAL ANALYSIS

3.1. COMPUTER PROGRAMS AND THEORY OF MODAL ANALYSIS

A computer program was developed to find the eigenvalues and modeshapes of linearly elastic structures with prismatic members and lumped masses. There are two versions of the program, one for 2-dimensional structures with up to 3 degrees of freedom (d-o-f) per node, the other for 3-dimensional models with up to 6 d-o-f per node.

For a structure with n d-o-f the structure stiffness matrix $[K]$ is of the order n x n. Directly from this the m x m reduced flexibility matrix $[F^*]$ is found, retaining only those d-o-f which are associated with one of the m masses. This is done by solving m-times $[K]\{\delta\}=\{P\}$, where $\{P\}$ is a vector containing zeros except for a force of magnitude 1 in the row corresponding to one of the d-o-f to be retained. Thus the reduced flexibility matrix is generated column by column without inverting part of the matrix $[K]$. This procedure is of particular advantage if m is small compared to n, which is usually the case, when no rotational masses are being introduced. It is even more pronounced, if only one translation is associated with a mass.

By contrast, the conventional way of reducing a matrix by partitioning consists of inverting a matrix of the order (n-m) x (n-m), which can be very time consuming, if not impossible for large values of (n-m).

Knowing that $[F^*] = [K^*]^{-1}$, where $[K^*]$ is the reduced stiffness matrix, we can proceed as follows. If a matrix $[K^*]$ has the eigenvalues λ_i and eigenvectors $\{\delta_i\}$, then these satisfy the equation

$$\left[[K^*] - \lambda_i[M]\right]\{\delta_i\} = 0 \quad (1)$$

If we premultiply this equation by $-\frac{1}{\lambda_i}[K^*]^{-1}$ and postmultiply by $[M]^{-1}$, the inverse of the diagonal mass matrix, we obtain

$$\left[[K^*]^{-1} - \frac{1}{\lambda_i}[M]^{-1}\right]\{\delta_i\} = 0 \quad (2)$$

It follows, that the eigenvalues of equation (2) are the reciprocals of those of (1) while the eigenvectors remain the same. Thus the smallest eigenvalue of the original problem (1) can be found by taking the inverse of the largest eigenvalue of equation (2).

This way it is possible to solve the dynamic eigenvalue problem without inverting part of the usually large unreduced structure stiffness matrix and yet incorporate the exact structural behavior without restricting any d-o-f by assuming rigid girders for the mathematical model. Another advantage is that, with the subroutine used, the largest eigenvalues found from (2) are also more accurate than the smallest eigenvalues of (1).

For convenient solution, the unsymmetrical coefficient determinant of the frequency equation (2) is converted into the symmetrical form

$$\left[[M]^{\frac{1}{2}}[K^*]^{-1}[M]^{\frac{1}{2}} - \frac{1}{\lambda_i}[I]\right][M]^{-\frac{1}{2}}\{\delta_i\} = 0 \quad (3)$$

where $[I]$ is the identity matrix. The natural frequencies ω_i for each mode are then given as $\omega_i = \sqrt{1/\lambda_i}$.

3.2. MATHEMATICAL MODELS

Three entirely different mathematical models (see Fig. 7) were derived directly from the structural drawings to investigate various methods of idealizing a rather simple structure. One common assumption was made for all three models: that the structure is rigidly fixed 1 ft. below ground level. This seemed justified by the very rigid foundation walls below that point.

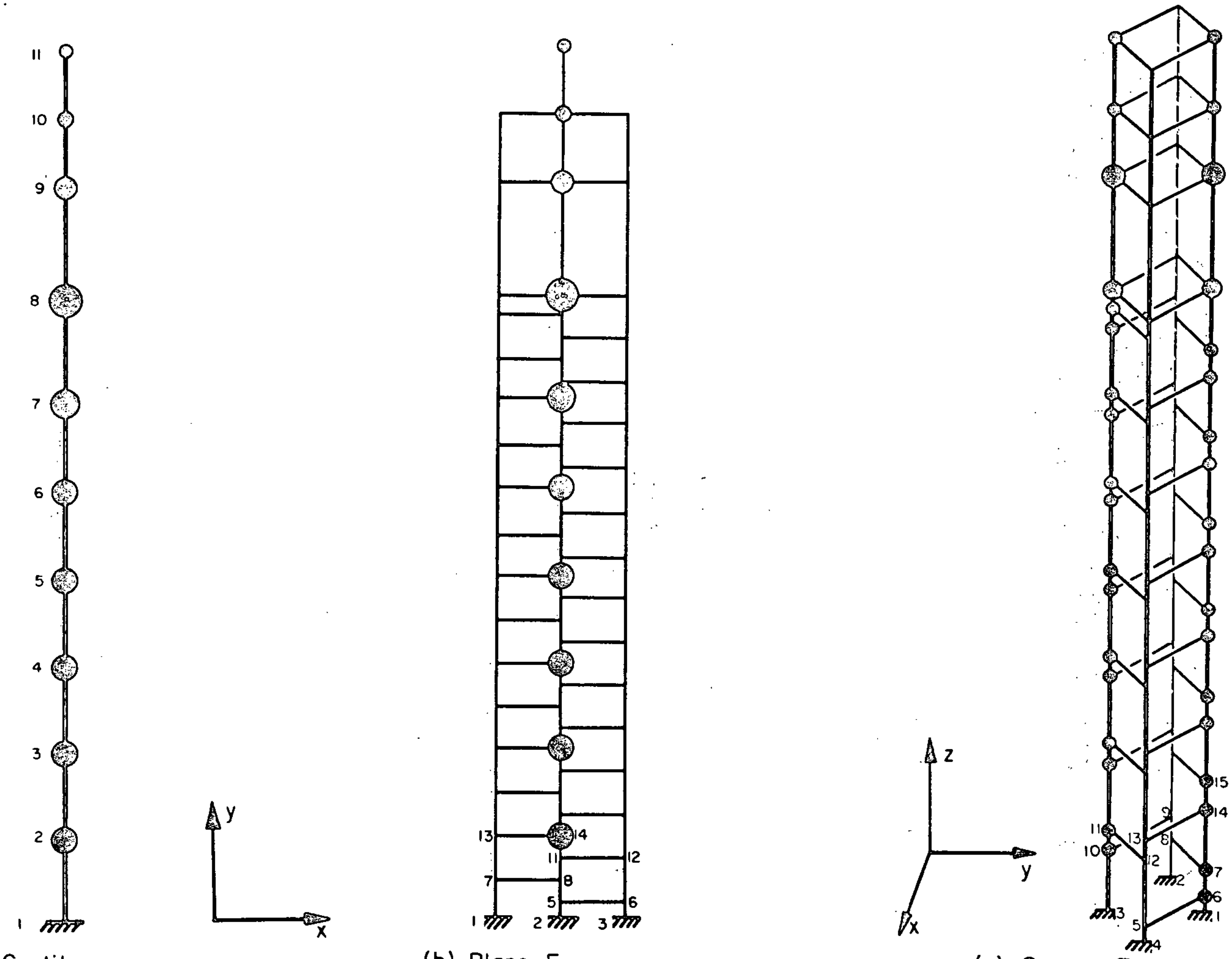
A uniform modulus of elasticity of 4000 ksi. was used for all models. It was calculated by using the formula

$$E_c = 57,000 \sqrt{f_c'}$$

where E_c is the modulus of elasticity of normal aggregate concrete and f_c' is the compressive strength (both in psi.). Shear deformations of the members were neglected.

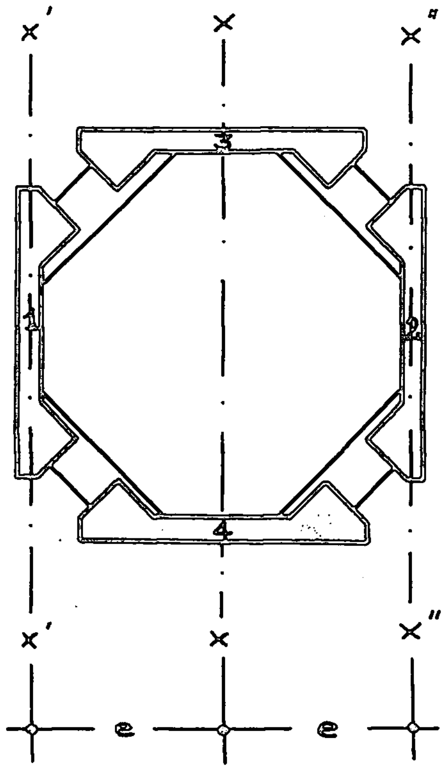
The first model (Fig.7a) is a cantilever with 10 masses and varying stiffness over the height. Since the degree of shear transfer between the wallpanels through the spandrels could not be estimated readily,two extreme cases were considered to allow modelling as a simple cantilever. To establish an upper bound on the stiffness,complete shear transfer was assumed. The lower bound would be only the sum of the individual moments of inertia of the four wall panels acting parallel without shear connection. This gives two different stiffness distributions for a simple cantilever in one plane.

The derivation of the member stiffnesses is shown in Fig.8 for



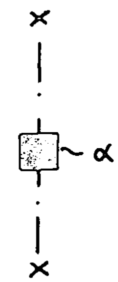
(a) Cantilever (b) Plane Frame (c) Space Frame

FIG. 7 MATHEMATICAL MODELS FOR DYNAMIC ANALYSIS



A = Area

CANTILEVER



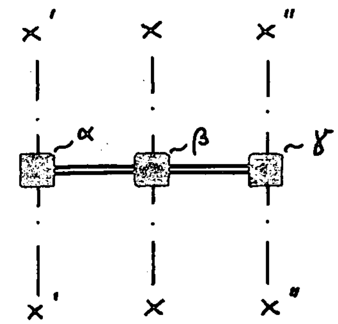
LOWER BOUND:

$$I_{\alpha xx} = I_{4xx} + I_{3xx} + I_{2x''x''} + I_{1x'x'}$$

UPPER BOUND:

$$I_{\alpha xx} = I_{4xx} + I_{3xx} + I_{2x''x''} + I_{1x'x'} + A_1 e^2 + A_2 e^2$$

PLANE FRAME

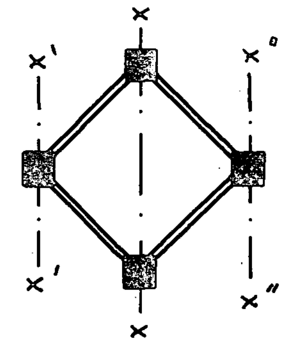


$$I_{\alpha x'x'} = I_{1x'x'}$$

$$I_{\beta xx} = I_{4xx} + I_{3xx}$$

$$I_{\gamma x''x''} = I_{2x''x''}$$

SPACE FRAME



All moments of inertia as for wall panels in original structure

FIG. 8 PLAN AND COLUMN STIFFNESSES OF MATHEMATICAL MODELS the cantilever as well as for the other two models. Appendix II lists the member properties as used in the computer analysis.

The next, more refined model, shown in Fig.7b is the plane frame which is modelled with the same 10 lumped masses used for the cantilever. The wallpanels and the shear transferring spandrels are represented in one plane as the columns and horizontal members of an unsymmetric frame. Since the columns do not have the width of the original walls, the modelled spandrels must be of greater length than in the real structure. To compensate for this and since the governing loadcase for these members is shear transfer, they are given a greater bending stiffness than the real elements. The moment of inertia is calculated so that under the action of a unit shear they deflect the same amount as would the shorter, real members. This leads to the formula

$$I_{new} = I_{old} \times (L_{new}/L_{old})^3$$

Similarly a corrected area can be found to yield the same axial stiffness of the spandrels in the model and in the real structure:

$$A_{new} = A_{old} \times (L_{new}/L_{old})$$

Both models, cantilever and plane frame, were solved by the plane frame version of the program described in section 3.1. Each joint was given 3 degrees of freedom (d-o-f). The reduced matrix was of a size 10 x 10 according to the horizontal d-o-f at the joints with a lumped mass.

The third model is a space frame,idealized as shown in Fig.7c. Various numbers of masses were used,each of which was acting in the two horizontal directions x and y. In the unreduced stiffness matrix,each node has 6 degrees of freedom,3 trans= lations and 3 rotations. Four columns represent the vertical wall panels. The horizontal members simulate the action of the spandrels,with corrected axial,bending and torsional stiffnesses as outlined on the foregoing page.

To be able to take advantage of the concept of reducing the structure stiffness matrix,the least possible number of masses should be used. The effect of decreasing the number of masses on the accuracy of the natural frequencies of the space frame was investigated. 72, 36 and 22 masses were used with the model which is shown with 36 masses in Fig.7c. The same member properties were used in each case. The effect of the number of masses on the execution time of the computer programs for an IBM 360/67 is demonstrated in Table 2.

	No. of masses	No. of members	No. of deg. of freedom	Matrix-bandwidth	CPU-Time (sec)
Space frame	72	116	432	48	299.1
	36	116	432	48	125.8
	22	116	432	48	84.7
Plane frame	10	100	198	18	15.1
Canti= lever	10	10	30	6	7.0

TABLE 2 : Execution times for modal analysis program

4. COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

As could be expected from the essentially symmetric structure, no significant difference was found between the E-W and N-S directions. Thus in the following tables only one direction is listed for the translational modes. These results are applicable for any axis through the centre of the tower plan because of the inherent radial symmetry.

4.1. NATURAL FREQUENCIES

Table 3 gives the natural frequencies of the various mathematical models as described under 3.2. and the results of the ambient vibration survey.

MODE		CANTILEVER		PLANE FRAME	SPACE FRAME			AMBIENT VIBRATION TEST
		Lower bound	Upper bound		22 masses	36 masses	72 masses	
TRANSLATION	1	0.91	1.93	2.07	1.98	1.95	1.95	1.78
	2	3.32	3.92	9.17	8.55	8.47	8.47	7.52
	3	7.45	15.62	17.55	16.95	16.95	16.95	15.38
TORSION	1	-	-	-	3.91	3.89	3.89	3.76
	2	-	-	-	9.53	9.52	9.52	5.65
	3	-	-	-	15.87	15.87	15.87	10.57

TABLE 3 : Natural frequencies in Hz.

The results of Table 3 are normalized as frequency ratios in Table 4 on the following page.

MODE		CANTILEVER		PLANE FRAME	SPACE FRAME			AMBIENT VIBRATION TEST
		Lower bound	Upper bound		22 masses	36 masses	72 masses	
TRANSLATION	1	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	2	3.67	2.04	4.38	4.32	4.33	4.33	4.22
	3	8.10	8.11	8.37	8.56	8.66	8.66	8.66
TORSION	1	-	-	-	1.00	1.00	1.00	1.00
	2	-	-	-	2.44	2.44	2.44	1.50
	3	-	-	-	4.06	4.08	4.08	2.86

TABLE 4 : Ratios of natural frequencies

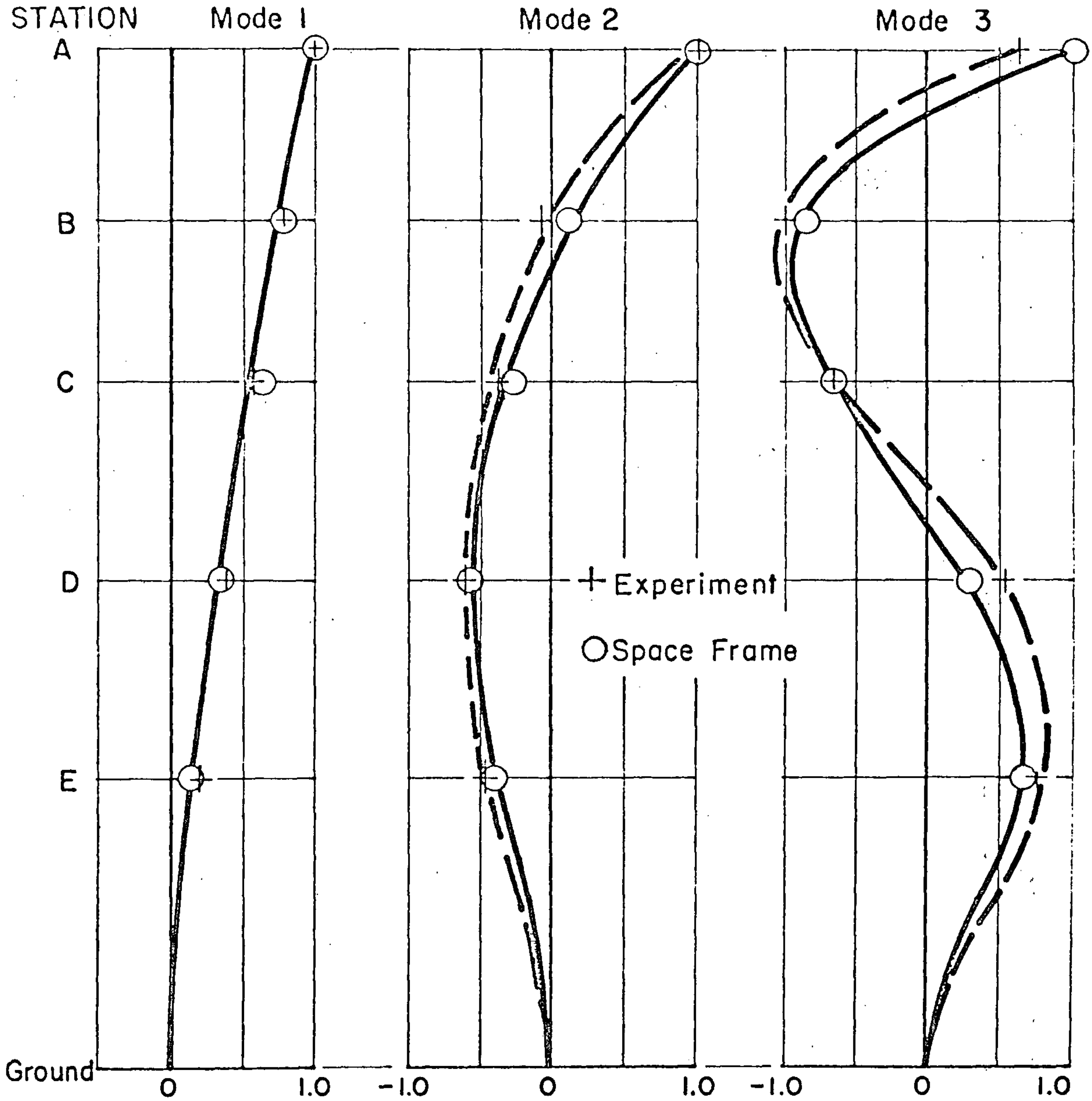
TRANSLATION: It can be seen that all models,except the lower bound on the cantilever give results which are in good agreement with the fundamental frequency observed in the ambient vibration test.

Table 4 shows that the ratios of the frequencies of the more accurate plane frame and space frame models are also in very good agreement with the experimental results. This means that almost any difference for all the frequencies could have been eliminated by using a lower modulus of elasticity E_c in the computer models.

In the analysis presented,a value of E_c=4000 ksi. was used. Since only data on the 28day strength of the concrete were available,the actual strength at the date of the test had to be extrapolated,thus involving some uncertainty. It would be desirable for future investigations to carry out non-destructive tests to give exact values for a dynamic modulus to be used for the analytical models.

It should be noted that for all models the transformed member sections (i.e including reinforcing steel) were used. A preliminary analysis using plain concrete sections produced frequencies which were up to 5 percent lower than those listed in Table 3.

TORSION: The fundamental torsional frequency of the space frame models is only 3.8 percent higher than the value from the ambient vibration tests,but for the higher modes differences of up to 41 percent are evident. Hence the ratios of the theoretical torsional frequencies do not compare favourably with the test results. No apparent reason could be found in the analytical model for this discrepancy. The space frame model is believed to be quite accurate,as can be seen from the translational modes and frequencies and the first torsional frequency.



MODAL COEFFICIENTS						
	1st MODE		2nd MODE		3rd MODE	
Station	Space Frame	Experiment	Space Frame	Experiment	Space Frame	Experiment
A	1.00	1.00	1.00	1.00	1.00	0.64
B	0.78	0.77	0.13	-0.07	-0.81	-1.00
C	0.62	0.58	-0.22	-0.37	-0.61	-0.61
D	0.36	0.38	-0.52	-0.58	0.28	0.51
E	0.15	0.20	-0.38	-0.43	0.65	0.73
Ground	0.00	—	0.00	—	0.00	—

FIG. 9 TRANSLATIONAL MODE SHAPES

4.2. MODE SHAPES

The comparison of the normalized mode shapes in Fig.8 shows excellent agreement for the first translational mode. The second and third mode shapes show some deviations at certain levels,but generally the analytical results are fairly consistent with the experiment. The table below the plotted mode shapes lists the modal amplitudes of the plot.

4.3. DAMPING

Table 6 lists the percentage of equivalent viscous damping obtained from man induced vibrations for the first mode and for the first three modes as evaluated from the Fourier spectral peaks. The torsional damping constants were only derived from the bandwidth of the modal resonances.

		1 st Mode	2 nd Mode	3 rd Mode
TRANSLATION	FOURIER SPECTRA	2.7	0.3	0.2
	MAN EXCITATION	3.2	-	-
TORSION FROM FOURIER SPECTRA		0.5	0.7	0.4

TABLE 5: Damping in percent of critical

The result for the first translational mode is in good agreement for the two methods,although it seems somewhat high for a structure of the type under consideration. The maximum displacements associated with the damping constants of Table 5 for wind and man excitation were 0.1 and 0.3 mm respectively.

5. CONCLUSIONS

The ambient vibration survey represents a simple and inexpensive method to obtain the resonant frequencies,mode shapes and damping coefficients of structures at a low stress level. To determine the higher modes it seems necessary to work with light winds as exciting force.

For future projects,if no absolute values are desired,the use of accelerometers as transducers should be considered. They offer the advantage of a more compact design,although they are more susceptible to accidental damage. Due to an incompatibility between the recorder input and the output of the available Brush carrier amplifiers,existing accelerometers could not be used with the PI magnetic tape recorder. A high speed computer is almost essential to evaluate the amount of data necessary for a meaningful sample size.

Because of the simplicity of the structure it was possible to construct reliable mathematical models for the analysis with digital computers. The concept of the reduced flexibility matrix allowed modelling with a high degree of accuracy. It is shown that very good results can be obtained even with a small number of lumped masses.

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APPENDIX I :

SEISMOMETER CALIBRATION AND BALANCING

It can be shown that a seismometer may be represented by an equivalent circuit (see Fig.A below and Ref. [4]).

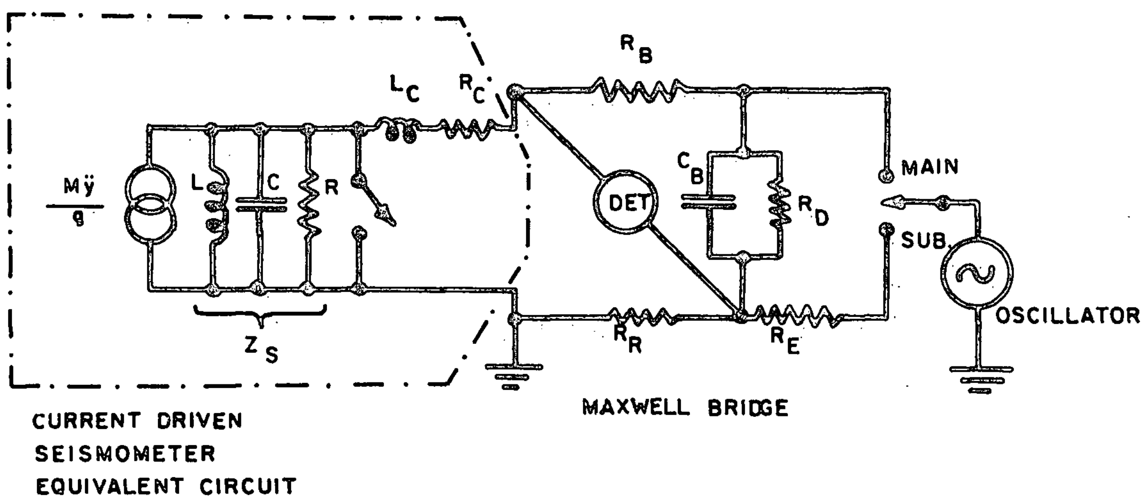

FIG. A SEISMOMETER CALIBRATION CIRCUIT

The values of the various components of the equivalent circuit are related to the seismometer constants, spring constant U, damping constant D, mass M, transducer constant g and to the ground acceleration $\ddot{y}$. R_c and L_c are the resistance and inductance of the coil and the switch is analogous to a clamp used to prevent the mass from swinging. As Kollar and Russell [4] observe, the electromagnetic seismometer and the equivalent circuit are indistinguishable by measurements made at the output terminals.

The calibration of the seismometers involves the determination of its response to sinusoidal ground motions in the desired frequency range. For the calibration the clamped seismometer is placed in the 'unknown' position of the Maxwell impedance bridge and the bridge is balanced in the usual manner for 'MAIN' input. The balance condition is independent of the frequency and gives the values of R_c and L_c.

Kollar and Russell have shown that with the seismometer un=clamped, the ratio of detector outputs for 'MAIN' and 'SUBSTITUTION' inputs is $(R_E \cdot Z_s)/(R_R \cdot R_B)$ from which Z_s may be determined as a function of ω. The positions of the resonant peak and the asymptotes of a logarithmic plot of Z_s (Fig.C) against ω, together with the known suspended mass M, determine the values of U, D and g. They also show that a potential v applied to the 'MAIN' input of the bridge produces the same result as a current generator v/R_B in parallel with Z_s, which, comparing with the equivalent circuit, is equivalent to a ground acceleration $(g \cdot v)/(M \cdot R_B)$. Fig.B shows a sketch of the seismic control panel used. S 1 , S 2 and S 3 represent switches; A and B denote the two seismometers.

BRIDGE BALANCE PROCEDURE :

Clamp Seismometer

S 1 = K

S 2 = K

S 3 = A or B

Connect 1 cps. 30 V p.p. sine wave to 'MAIN' terminals of bridge and adjust R_B to get a minimum deflection on the oscilloscope which is connected to the 'SANBORN' WHITE and

GREEN contacts through input 1 and inverted input 2.

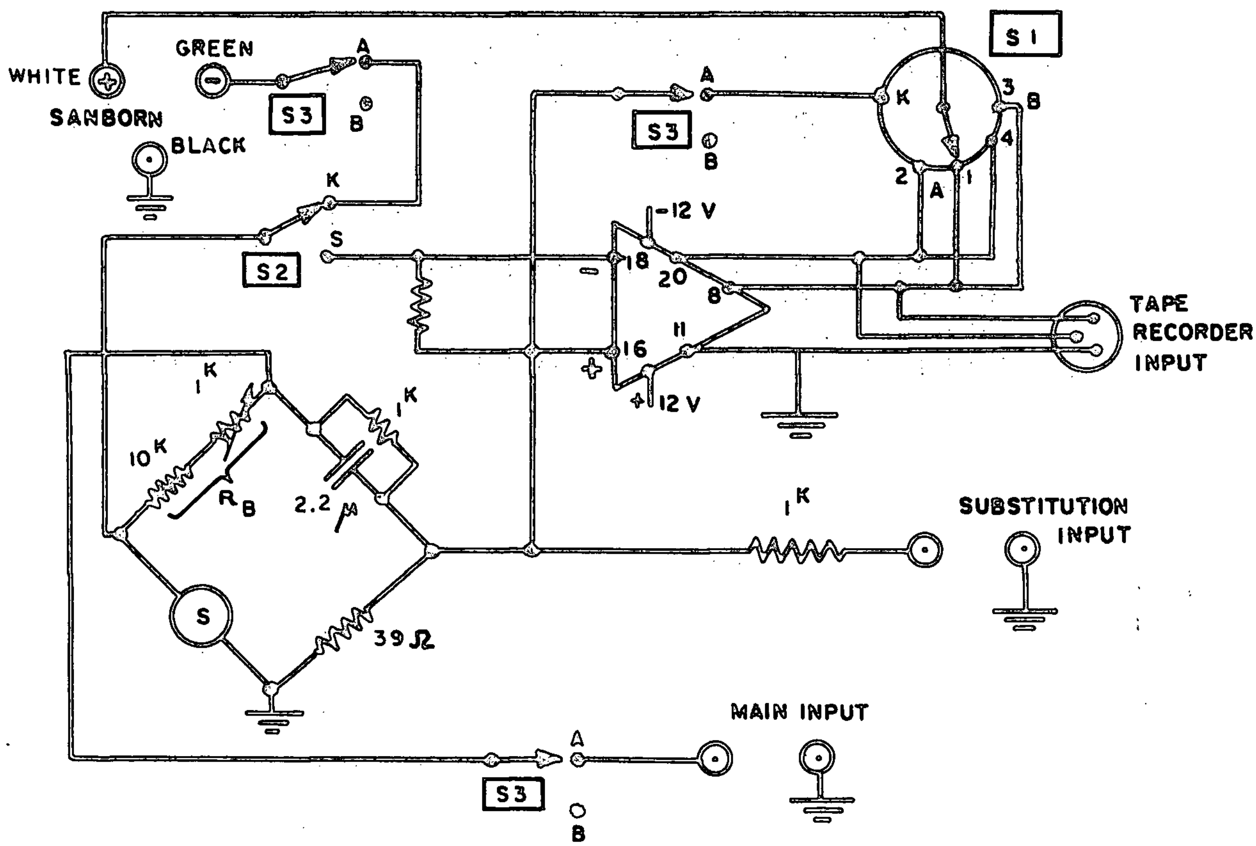

FIG. B CONTROL PANEL WIRING SCHEME

DETERMINATION OF SEISMOMETER CONSTANTS :

S 1 = K

S 2 = K

S 3 = A or B

Connect oscillator alternatingly to 'MAIN' and 'SUBSTITUTION' input.

'SANBORN' : WHITE = Inverted input 2 } Oscilloscope
GREEN = Input 1

Then unclamp seismometer,set period adjuster to 3 and level the instrument which preferably should be set up in a location with a low noise level. The attenuation of the oscillator should be between 2 and 5 V p.p. according to the noise level. If a high noise level is present, no clear readings can be taken at a high gain setting of the oscilloscope which will be required with a small input signal.

Starting with an oscillator frequency of 0.1 Hz. and increasing the frequency in steps suitable for a log-scale, the differential output on the oscilloscope should be read alternatively for 'SUBSTITUTION' and 'MAIN' input. The calibration curves for seismometer A and B are shown in Fig.C. From Fig.A the transfer function of the equivalent circuit is found as:

$$Z_s = \frac{1}{(1/R)+(1/j\omega L)+(j\omega C)} \qquad \text{with} \qquad R = g^2/D, \quad L = g^2/U, \quad C = M/g^2$$

Kollar and Russel [5] have shown that

$$V_{MAIN}/V_{SUB} = Z_s \cdot \frac{R_E}{R_R \cdot R_B} \tag{1}$$

For high frequencies ($\omega \gg \omega_n$) :

$$Z_s \rightarrow \frac{1}{j\omega C} \quad \text{and} \quad |Z_s| = \frac{1}{\omega C} = \frac{g_2^2}{\omega M} \tag{2}$$

At resonance ($\omega = \omega_n$) :

$$Z_s \rightarrow R \quad \text{and} \quad |Z_s| = R = \frac{g^2}{D} \tag{3}$$

and also $\omega_n = \sqrt{U/M}$ (4)

For low frequencies ($\omega \ll \omega_n$) :

$$Z_s \rightarrow j\omega L \quad \text{and} \quad |Z_s| = \omega L = \frac{\omega g_5^2}{U} \tag{5}$$

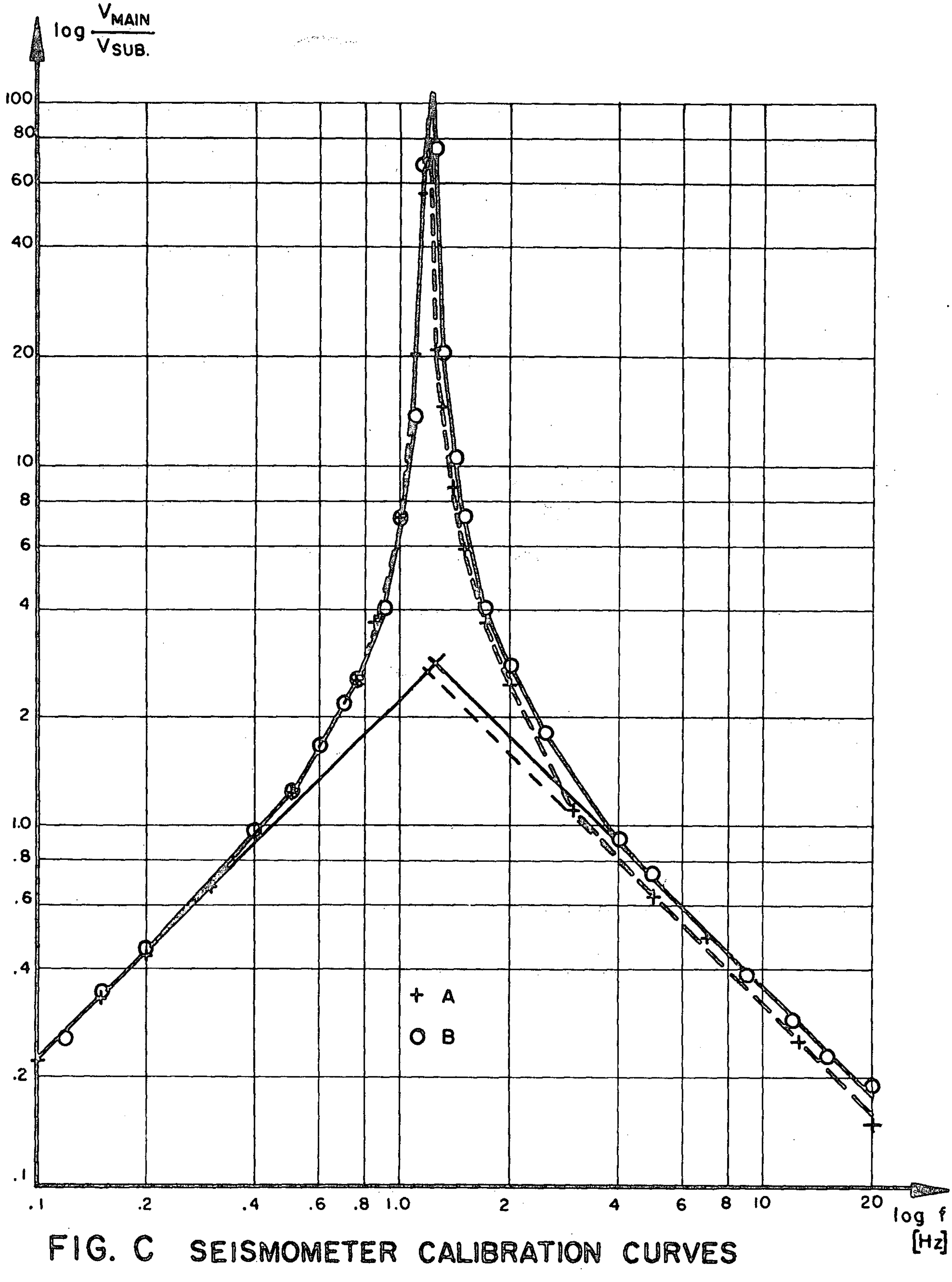

FIG. C SEISMOMETER CALIBRATION CURVES

With equations (1) through (5) on page 36 and M =4.75 kg the following constants for the seismometers were determined from Fig.C :

Seismometer	A	B
$(R_R \cdot R_B)/R_E$	401	390
ω_0 (rad/sec)	7.29	7.92
U (Newtons/m)	252	298
g_2 (Volt/m/sec)	188	206
g_5 (Volt/m/sec)	196	222
D (Newton/m/sec)	1.15	1.30

TABLE A : Seismometer constants

DETERMINATION OF VELOCITY RESPONSE OF COMPLETE SYSTEM.

S 1 = 1,2,3,4

S 2 = S

S 3 = A or B

Oscillator connected to 'MAIN' input. Oscillograph connected to oscillator to measure V_{input}; is later connected to PLAY-BACK-OUTPUT of taperecorder after signal is on tape and can be played back to measure output.

Kollar and Russell [5] have also shown that the velocity response of a system is given by:

$$(V_{out}/V_{in}) \cdot (\omega \cdot M \cdot R_B)/g = (\text{VOLTS/m/sec})$$

The velocity response of the two seismometers, including the amplifiers is plotted in Fig. D.

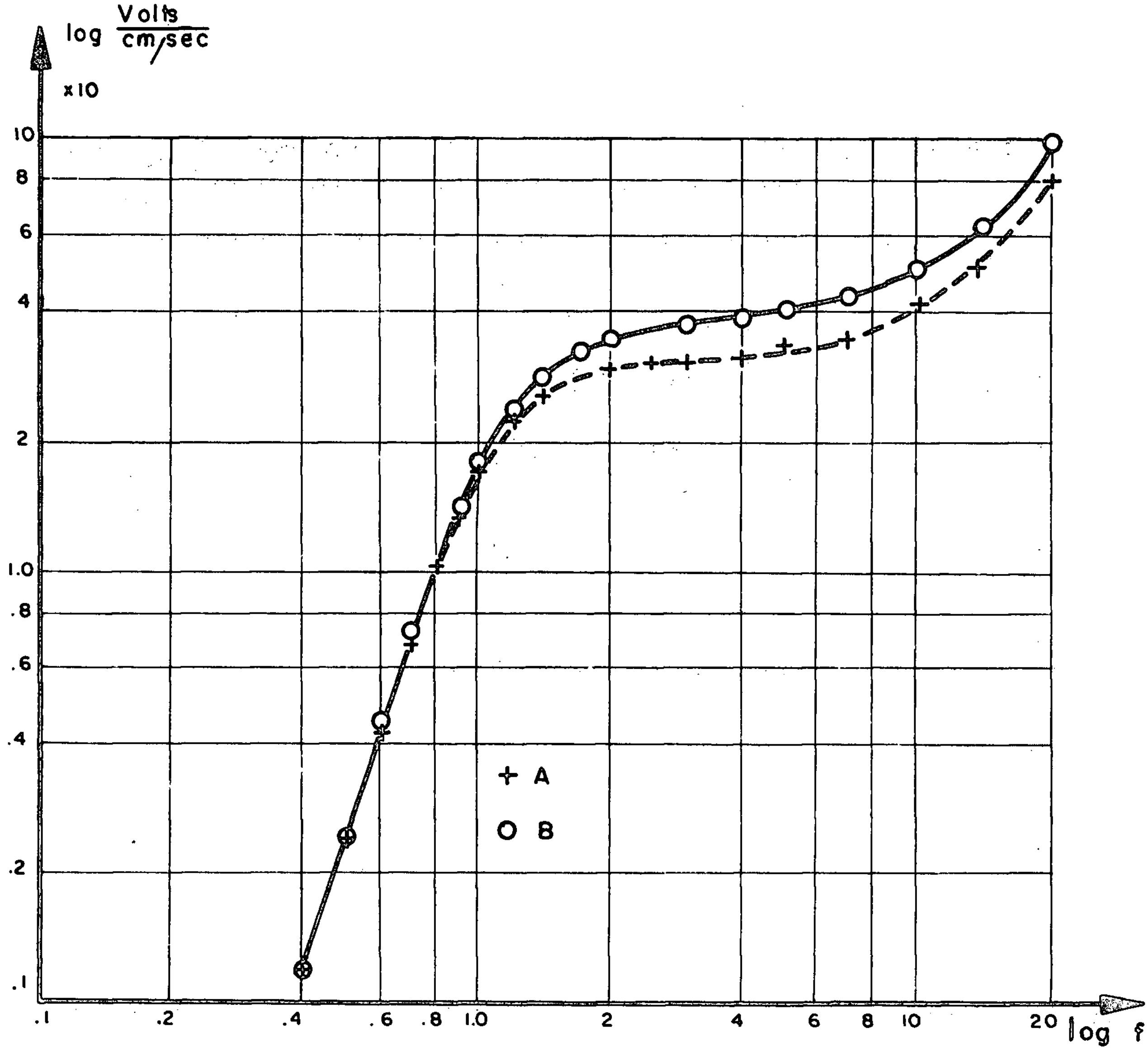

FIG. D VELOCITY RESPONSE

APPENDIX II: Output from Computer Analysis of Plane Frame Model and Listing of Masses for 3-dimensional Models.

```
RES NO. 770818                        UNIVERSITY OF B C COMPUTING CENTRE   MTS(AN120)                    23:26:25     07-22-70

******************* THIS JOB SUBMITTED THROUGH FRONT DESK READER *****************
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**LAST SIGNON WAS: 23:26:21   07-22-70
 USER "TOPF" SIGNED ON AT  23:26:26 ON 07-22-70
$LIS DATA

           JOINT NOS. AND MASSES FOR 36 MASS-SPACE FRAME

           2  36
           1   2
           6 20.      7 18.5    10 17.     11 17.     14 17.     15 17.25   18 17.5    19 17.5
          22 17.5    23 17.5    26 17.5    27 17.5    30 17.5    31 17.5    34 17.5    35 17.5
          38 17.5    39 17.5    42 17.5    43 17.5    46 17.5    47 18.5    50 19.5    51 19.5
          54 19.5    55 18.55   58 17.6    59 17.6    62 24.     64 24.     66 31.     68 31.
          70 19.     72 19.     74 16.     76 16.



           JOINT NOS. AND MASSES FOR 72 MASS-SPACE FRAME

           2  72
           1   2
           5 10.      6 10.      7 10.      8  8.5     9  8.5    10  8.5    11  8.5    12  8.5
          13  8.5    14  8.5    15  8.5    16  8.75   17  8.75   18  8.75   19  8.75   20  8.75
          21  8.75   22  8.75   23  8.75   24  8.75   25  8.75   26  8.75   27  8.75   28  8.75
          29  8.75   30  8.75   31  8.75   32  8.75   33  8.75   34  8.75   35  8.75   36  8.75
          37  8.75   38  8.75   39  8.75   40  8.75   41  8.75   42  8.75   43  8.75   44  8.75
          45  8.75   46  8.75   47  8.75   48  9.75   49  9.75   50  9.75   51  9.75   52  9.75
          53  9.75   54  9.75   55  9.75   56   8.8   57   8.8   58   8.8   59   8.8   60   8.8
          61  12.    62  12.    63  12.    64  12.    65  15.5   66  15.5   67  15.5   68  15.5
          69    9.5  70    9.5  71    9.5  72    9.5  73     8.  74     8.  75     8.  76     8.
END OF FILE


$SIG
```

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RFS NO. 770967                    UNIVERSITY OF B C COMPUTING CENTRE    MTS(AN120)                 17:27:53     07-27-70

**REMINDER--ALL TAPE MOUNTS MUST HAVE RACK NUMBERS**
******************* THIS JOB SUBMITTED THROUGH FRONT DESK READER *******************
$SIG TOPF PRIO=V P=500 T=200 COPIES=5
**LAST SIGNON WAS: 10:33:41   07-27-70
 USER "TOPF" SIGNED ON AT  17:27:54 ON 07-27-70
$LIS 2DRES
      1         $RUN OSEISMO 7=*DUMMY*
      2        EXECUTION BEGINS
      3        ******************************************************************
      4         CLOCK TOWER - PLANE FRAME ANALYSIS AS A FRAME-MODEL E=4000 KSI
      5        ******************************************************************
      6
      8             MODULUS OF ELASTICITY      4000.(KSI)
      9         IGRID =  1       NO. OF MEMBERS100
     10         NO. OF D-O-F OF REDUCED MATRIX 10
     11         NO. OF ELASTIC SUPPORTS  0
     12         IDIM= 0      NFIX= 1
     13         NBAY=  2      NSTOR= 32
     14         NO. OF JOINTS CONSTRAINED= 33
     16
     17
     20         INPUT JOINT DATA
     21               JOINT       HOR        VERT        ROT     X (FT)     Y (FT)
     22
     23                 1          0          0          0      0.0        0.0
     24                 2          0          0          0      6.167      0.0
     25                 3          0          0          0     12.334      0.0
     26                 4          0          0          0      0.0        2.167
     27                 5          1          1          1      6.167      2.167
     28                 6          1          1          1     12.334      2.167
     29                 7          1          1          1      0.0        5.167
     30                 8          1          1          1      6.167      5.167
     31                 9          0          0          0     12.334      5.167
     32                10          0          0          0      0.0        8.167
     33                11          1          1          1      6.167      8.167
     34                12          1          1          1     12.334      8.167
     35                13          1          1          1      0.0       11.167
     36                14          1          1          1      6.167     11.167
     37                15          0          0          0     12.334     11.167
     38                16          0          0          0      0.0       14.167
     39                17          1          1          1      6.167     14.167
     40                18          1          1          1     12.334     14.167
     41                19          1          1          1      0.0       17.167
     42                20          1          1          1      6.167     17.167
     43                21          0          0          0     12.334     17.167
     44                22          0          0          0      0.0       20.167
     45                23          1          1          1      6.167     20.167
     46                24          1          1          1     12.334     20.167
     47                25          1          1          1      0.0       23.167
     48                26          1          1          1      6.167     23.167
     49                27          0          0          0     12.334     23.167
     50                28          0          0          0      0.0       26.167
     51                29          1          1          1      6.167     26.167
     52                30          1          1          1     12.334     26.167
     53                31          1          1          1      0.0       29.167
     54                32          1          1          1      6.167     29.167
     55                33          0          0          0     12.334     29.167
```

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62	40	0	0	0	0.0	38.167
63	41	1	1	1	6.167	38.167
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65	43	1	1	1	0.0	41.167
66	44	1	1	1	6.167	41.167
67	45	0	0	0	12.334	41.167
68	46	0	0	0	0.0	44.167
69	47	1	1	1	6.167	44.167
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72	50	1	1	1	6.167	47.167
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82	60	1	1	1	12.334	56.167
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84	62	1	1	1	6.167	59.167
85	63	0	0	0	12.334	59.167
86	64	0	0	0	0.0	62.167
87	65	1	1	1	6.167	62.167
88	66	1	1	1	12.334	62.167
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90	68	1	1	1	6.167	65.167
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92	70	0	0	0	0.0	68.167
93	71	1	1	1	6.167	68.167
94	72	1	1	1	12.334	68.167
95	73	1	1	1	0.0	71.167
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97	75	0	0	0	12.334	71.167
98	76	0	0	0	0.0	74.167
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100	78	1	1	1	12.334	74.167
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102	80	1	1	1	6.167	77.167
103	81	0	0	0	12.334	77.167
104	82	0	0	0	0.0	80.167
105	83	1	1	1	6.167	80.167
106	84	1	1	1	12.334	80.167
107	85	1	1	1	0.0	83.167
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113	91	1	1	1	0.0	102.167
114	92	1	1	1	6.167	102.167
115	93	1	1	1	12.334	102.167

94	1	1	1	0.0	111.834
95	1	1	1	6.167	111.834
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OUTPUT JOINT DATA NU=198

(CODING NUMBERS)

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3	0	0	0
4	0	0	0
5	1	2	3
6	4	5	6
7	7	8	9
8	10	11	12
9	0	0	0
10	0	0	0
11	13	14	15
12	16	17	18
13	19	20	21
14	22	23	24
15	0	0	0
16	0	0	0
17	25	26	27
18	28	29	30
19	31	32	33
20	34	35	36
21	0	0	0
22	0	0	0
23	37	38	39
24	40	41	42
25	43	44	45
26	46	47	48
27	0	0	0
28	0	0	0
29	49	50	51
30	52	53	54
31	55	56	57
32	58	59	60
33	0	0	0
34	0	0	0
35	61	62	63
36	64	65	66
37	67	68	69
38	70	71	72
39	0	0	0
40	0	0	0
41	73	74	75
42	76	77	78
43	79	80	81
44	82	83	84
45	0	0	0
46	0	0	0
47	85	86	87
48	88	89	90
49	91	92	93

50	94	95	96
51	0	0	0
52	0	0	0
53	97	98	99
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55	103	104	105
56	106	107	108
57	0	0	0
58	0	0	0
59	109	110	111
60	112	113	114
61	115	116	117
62	118	119	120
63	0	0	0
64	0	0	0
65	121	122	123
66	124	125	126
67	127	128	129
68	130	131	132
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70	0	0	0
71	133	134	135
72	136	137	138
73	139	140	141
74	142	143	144
75	0	0	0
76	0	0	0
77	145	146	147
78	148	149	150
79	151	152	153
80	154	155	156
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82	0	0	0
83	157	158	159
84	160	161	162
85	163	164	165
86	166	167	168
87	0	0	0
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89	172	173	174
90	175	176	177
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93	184	185	186
94	187	188	189
95	190	191	192
96	193	194	195
97	0	0	0
98	196	197	198
99	0	0	0

INPUT MEMBER DATA

MEMBER	JNL	JNG	KL	KG	AREA (IN2)	I (IN4)
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2	7	13	1	1	1477.00	33370.00
3	13	19	1	1	1477.00	33370.00
4	19	25	1	1	1477.00	33370.00
5	25	31	1	1	1435.00	32560.00
6	31	37	1	1	1435.00	32560.00

237	7	37	43	1	1	1435.00	32560.00
238	8	43	49	1	1	1435.00	32560.00
239	9	49	55	1	1	1421.00	31830.00
240	10	55	61	1	1	1421.00	31830.00
241	11	61	67	1	1	1400.00	31220.00
242	12	67	73	1	1	1400.00	31220.00
243	13	73	79	1	1	1400.00	31220.00
244	14	79	85	1	1	1400.00	31220.00
245	15	85	88	1	1	1400.00	31220.00
246	16	88	91	1	1	1400.00	31220.00
247	17	91	94	1	1	1030.00	7780.00
248	18	3	6	1	1	1477.00	33370.00
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252	22	24	30	1	1	1435.00	32560.00
253	23	30	36	1	1	1435.00	32560.00
254	24	36	42	1	1	1435.00	32560.00
255	25	42	48	1	1	1435.00	32560.00
256	26	48	54	1	1	1421.00	31830.00
257	27	54	60	1	1	1421.00	31830.00
258	28	60	66	1	1	1400.00	31220.00
259	29	66	72	1	1	1400.00	31220.00
260	30	72	78	1	1	1400.00	31220.00
261	31	78	84	1	1	1400.00	31220.00
262	32	84	90	1	1	1400.00	31220.00
263	33	90	93	1	1	1400.00	31220.00
264	34	93	96	1	1	1030.00	7780.00
265	35	2	5	1	1	2954.00	3808000.00
266	36	5	8	1	1	2954.00	3808000.00
267	37	8	11	1	1	2954.00	3808000.00
268	38	11	14	1	1	2954.00	3808000.00
269	39	14	17	1	1	2954.00	3808000.00
270	40	17	20	1	1	2954.00	3808000.00
271	41	20	23	1	1	2954.00	3808000.00
272	42	23	26	1	1	2954.00	3808000.00
273	43	26	29	1	1	2870.00	3722000.00
274	44	29	32	1	1	2870.00	3722000.00
275	45	32	35	1	1	2870.00	3722000.00
276	46	35	38	1	1	2870.00	3722000.00
277	47	38	41	1	1	2870.00	3722000.00
278	48	41	44	1	1	2870.00	3722000.00
279	49	44	47	1	1	2870.00	3722000.00
280	50	47	50	1	1	2870.00	3722000.00
281	51	50	53	1	1	2842.00	3644000.00
282	52	53	56	1	1	2842.00	3644000.00
283	53	56	59	1	1	2842.00	3644000.00
284	54	59	62	1	1	2842.00	3644000.00
285	55	62	65	1	1	2800.00	3578000.00
286	56	65	68	1	1	2800.00	3578000.00
287	57	68	71	1	1	2800.00	3578000.00
288	58	71	74	1	1	2800.00	3578000.00
289	59	74	77	1	1	2800.00	3578000.00
290	60	77	80	1	1	2800.00	3578000.00
291	61	80	83	1	1	2800.00	3578000.00
292	62	83	86	1	1	2800.00	3578000.00
293	63	86	89	1	1	2800.00	3578000.00
294	64	89	92	1	1	2800.00	3578000.00
295	65	92	95	1	1	2060.00	15540.00
296	66	95	98	1	1	2860.00	9340000.00

297	67	5	6	1	1	1000.00	150000.00
298	68	7	8	1	1	1000.00	150000.00
299	69	11	12	1	1	1000.00	150000.00
300	70	13	14	1	1	1000.00	150000.00
301	71	17	18	1	1	663.00	91000.00
302	72	19	20	1	1	663.00	91000.00
303	73	23	24	1	1	663.00	91000.00
304	74	25	26	1	1	663.00	91000.00
305	75	29	30	1	1	663.00	91000.00
306	76	31	32	1	1	663.00	91000.00
307	77	35	36	1	1	663.00	91000.00
308	78	37	38	1	1	663.00	91000.00
309	79	41	42	1	1	663.00	91000.00
310	80	43	44	1	1	663.00	91000.00
311	81	47	48	1	1	663.00	91000.00
312	82	49	50	1	1	663.00	91000.00
313	83	53	54	1	1	663.00	91000.00
314	84	55	56	1	1	663.00	91000.00
315	85	59	60	1	1	663.00	91000.00
316	86	61	62	1	1	663.00	91000.00
317	87	65	66	1	1	663.00	91000.00
318	88	67	68	1	1	663.00	91000.00
319	89	71	72	1	1	663.00	91000.00
320	90	73	74	1	1	663.00	91000.00
321	91	77	78	1	1	663.00	91000.00
322	92	79	80	1	1	663.00	91000.00
323	93	83	84	1	1	663.00	91000.00
324	94	85	86	1	1	663.00	91000.00
325	95	88	89	1	1	850.00	120000.00
326	96	89	90	1	1	850.00	120000.00
327	97	91	92	1	1	850.00	120000.00
328	98	92	93	1	1	850.00	120000.00
329	99	94	95	1	1	850.00	120000.00
330	100	95	96	1	1	850.00	120000.00

331

332 OUTPUT MEMBER DATA

333 (CODING NUMBERS)

335	MEMBER	HL	VL	RL	HG	VG	RG
336							
337	1	0	0	0	7	8	9
338	2	7	8	9	19	20	21
339	3	19	20	21	31	32	33
340	4	31	32	33	43	44	45
341	5	43	44	45	55	56	57
342	6	55	56	57	67	68	69
343	7	67	68	69	79	80	81
344	8	79	80	81	91	92	93
345	9	91	92	93	103	104	105
346	10	103	104	105	115	116	117
347	11	115	116	117	127	128	129
348	12	127	128	129	139	140	141
349	13	139	140	141	151	152	153
350	14	151	152	153	163	164	165
351	15	163	164	165	169	170	171
352	16	169	170	171	178	179	180
353	17	178	179	180	187	188	189
354	18	0	0	0	4	5	6
355	19	4	5	6	16	17	18
356	20	16	17	18	28	29	30
357	21	28	29	30	40	41	42

358	22	40	41	42	52	53	54
359	23	52	53	54	64	65	66
360	24	64	65	66	76	77	78
361	25	76	77	78	88	89	90
362	26	88	89	90	100	101	102
363	27	100	101	102	112	113	114
364	28	112	113	114	124	125	126
365	29	124	125	126	136	137	138
366	30	136	137	138	148	149	150
367	31	148	149	150	160	161	162
368	32	160	161	162	175	176	177
369	33	175	176	177	184	185	186
370	34	184	185	186	193	194	195
371	35	0	0	0	1	2	3
372	36	1	2	3	10	11	12
373	37	10	11	12	13	14	15
374	38	13	14	15	22	23	24
375	39	22	23	24	25	26	27
376	40	25	26	27	34	35	36
377	41	34	35	36	37	38	39
378	42	37	38	39	46	47	48
379	43	46	47	48	49	50	51
380	44	49	50	51	58	59	60
381	45	58	59	60	61	62	63
382	46	61	62	63	70	71	72
383	47	70	71	72	73	74	75
384	48	73	74	75	82	83	84
385	49	82	83	84	85	86	87
386	50	85	86	87	94	95	96
387	51	94	95	96	97	98	99
388	52	97	98	99	106	107	108
389	53	106	107	108	109	110	111
390	54	109	110	111	118	119	120
391	55	118	119	120	121	122	123
392	56	121	122	123	130	131	132
393	57	130	131	132	133	134	135
394	58	133	134	135	142	143	144
395	59	142	143	144	145	146	147
396	60	145	146	147	154	155	156
397	61	154	155	156	157	158	159
398	62	157	158	159	166	167	168
399	63	166	167	168	172	173	174
400	64	172	173	174	181	182	183
401	65	181	182	183	190	191	192
402	66	190	191	192	196	197	198
403	67	1	2	3	4	5	6
404	68	7	8	9	10	11	12
405	69	13	14	15	16	17	18
406	70	19	20	21	22	23	24
407	71	25	26	27	28	29	30
408	72	31	32	33	34	35	36
409	73	37	38	39	40	41	42
410	74	43	44	45	46	47	48
411	75	49	50	51	52	53	54
412	76	55	56	57	58	59	60
413	77	61	62	63	64	65	66
414	78	67	68	69	70	71	72
415	79	73	74	75	76	77	78
416	80	79	80	81	82	83	84
417	81	85	86	87	88	89	90

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          82        91   92   93        94   95   96
          83        97   98   99       100  101  102
          84       103  104  105       106  107  108
          85       109  110  111       112  113  114
          86       115  116  117       118  119  120
          87       121  122  123       124  125  126
          88       127  128  129       130  131  132
          89       133  134  135       136  137  138
          90       139  140  141       142  143  144
          91       145  146  147       148  149  150
          92       151  152  153       154  155  156
          93       157  158  159       160  161  162
          94       163  164  165       166  167  168
          95       169  170  171       172  173  174
          96       172  173  174       175  176  177
          97       178  179  180       181  182  183
          98       181  182  183       184  185  186
          99       187  188  189       190  191  192
         100       190  191  192       193  194  195

     HALF BAND WIDTH NB= 18

     RETAINED DEGREES OF FREEDOM
          I  ND(I) JNO(I)    N
          1     1     14    22
          2     1     26    46
          3     1     38    70
          4     1     50    94
          5     1     62   118
          6     1     74   142
          7     1     89   172
          8     1     92   181
          9     1     95   190
         10     1     98   196
     GRAVITY=      32.20    NO. OF SPECTRUM CASES(NDISP)=  1    DAMPING PERCENTAGE=  0.0
     VERTICAL VIBRATION=    0     NMASS= 10
     LUMPED WEIGHTS
           1    68.000        2    70.000        3    70.000        4    70.000        5    70.000        6    78.000
           7    92.000        8    62.000        9    38.000       10    28.000
     EIGEN VALUES
          41403.135  28368.513  15263.330   7345.513   3250.492   1716.552   1185.738    372.853    103.410      5.396
     PERIODS(SEC.)
              0.005      0.007      0.009      0.013      0.019      0.027      0.032      0.057      0.109      0.477
     EIGEN VECTORS
        1    -0.951      1.000     -0.995     -0.913      0.548      0.024      0.287     -0.143     -0.057      0.016
        2     1.000     -0.557     -0.199     -0.962      1.000      0.056      0.721     -0.428     -0.189      0.063
        3    -0.983     -0.138      1.000      0.703      0.314      0.050      0.748     -0.635     -0.332      0.133
        4     0.838      0.767     -0.531      0.970     -0.743      0.007      0.271     -0.638     -0.441      0.222
        5    -0.598     -0.965     -0.625     -0.674     -0.822     -0.031     -0.381     -0.413     -0.485      0.325
        6     0.280      0.601      0.941     -0.946      0.177     -0.024     -0.725     -0.021     -0.447      0.439
        7    -0.077     -0.196     -0.459      1.000      0.921      0.045     -0.317      0.520     -0.282      0.590
        8     0.021      0.059      0.169     -0.509     -0.794      0.082      1.000      1.000      0.017      0.759
        9    -0.001     -0.003     -0.010      0.047      0.146     -0.998      0.012     -0.607      0.701      0.887
       10     0.001      0.002      0.007     -0.022     -0.044      1.000     -0.409     -0.957      1.000      1.000
     PARTICIPATION  FACTORS
            -21.279     17.526    -14.253     -7.659     14.209     47.351      8.285     -4.307    -19.521      2.255
     SPECTRAL DISPLACEMENTS(INCH OR CM)
              0.000      0.000      0.000      0.001      0.001      0.002      0.003      0.042      0.153      2.932
    MODAL FORCES                                                                                                MAX. PROBA
        1   453.403    392.881    317.822    156.744    174.638     25.414     53.322     55.160     98.996      3.243    729
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479	2	-491.028	-225.349	65.505	170.069	327.889	61.047	137.818	170.176	339.769	13.066	774
480	3	482.885	-55.611	-328.905	-124.238	102.892	54.329	143.000	252.316	598.008	27.738	903
481	4	-411.296	310.058	174.563	-171.413	-243.514	7.845	51.894	253.635	794.422	46.272	1041
482	5	293.537	-390.388	205.696	119.195	-269.503	-34.055	-72.922	164.069	873.610	67.744	1081
483	6	-153.123	270.848	-344.989	186.207	64.837	-28.958	-154.534	9.381	897.010	101.776	1046
484	7	49.663	-104.089	198.258	-232.287	396.730	64.860	-79.586	-271.793	666.692	161.349	904
485	8	-9.288	21.212	-49.272	79.643	-230.544	79.444	169.343	-352.155	-27.712	139.936	491
486	9	0.234	-0.609	1.842	-4.553	26.058	-591.881	1.297	131.105	-686.072	100.236	921
487	10	-0.156	0.366	-0.910	1.557	-5.814	437.073	-31.305	152.211	-720.738	83.257	861
489	MODAL SHEARS										MAX.	PROBA
490	1	453.403	392.881	317.822	156.744	174.638	25.414	53.322	55.160	98.996	3.243	729
491	2	-37.624	167.532	383.327	326.813	502.526	86.461	191.139	225.337	438.764	16.309	907
492	3	445.261	111.922	54.422	202.575	605.418	140.790	334.139	477.652	1036.772	44.047	1434
493	4	33.965	421.980	228.985	31.161	361.905	148.635	386.033	731.288	1831.195	90.319	2104
494	5	327.502	31.592	434.681	150.357	92.402	114.581	313.110	895.357	2704.804	158.063	2929
495	6	174.379	302.440	89.692	336.564	157.239	85.622	158.577	904.739	3601.815	259.839	3762
496	7	224.042	198.351	287.950	104.277	553.969	150.483	78.991	632.946	4268.507	421.188	4395
497	8	214.754	219.563	238.678	183.920	323.425	229.926	248.334	280.791	4240.795	561.125	4333
498	9	214.989	218.954	240.520	179.367	349.483	-361.954	249.631	411.896	3554.723	661.361	3707
499	10	214.832	219.320	239.610	180.924	343.669	75.119	218.326	564.107	2833.986	744.618	3043

500 EXECUTION TERMINATED
501 $SINK PREVIOUS
END OF FILE

$SIG