STRAIN RATE EFFECTS IN ONE-DIMENSIONAL CONSOLIDATION OF PEAT

by

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ABSTRACT

A series of one-dimensional strain rate controlled consolidation tests were performed on remolded peat to establish the void ratio - effective stress - strain rate relationship for this soil. The relationship was found to be independent of the size of sample. In addition, the void ratio - permeability relationship was also determined and was found to be independent of both the strain rate and the size of the sample.

From these relationships the behaviour of any size sample subjected to incremental loading was predicted in terms of the time - settlement and the time - pore pressure curves. These predictions included both 'primary' and 'secondary' settlement. The comparison of observed and predicted time - settlement and time - pore pressure curves was found to be in close agreement.

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LIST OF SYMBOLS

English Letters

a	coefficient of linear compressibility
a_v	Terzaghi's coefficient of compressibility
b	constant depending on the variation in void ratio with depth
$\frac{b}{r}$	a dimensionless ratio indicating the variation in void ratio with depth
CRS	Constant Rate of Strain
$C = \frac{k(1+e_1)}{\gamma_\omega}$	parameter related to permeability
c_v	Terzaghi's coefficient of consolidation
Dz	thickness of an element of soil at any time
Dz_o	initial thickness of element of soil
e	void ratio
e_1	initial void ratio
e_2	final void ratio
e_3	void ratio at drainage boundary
$\acute{e} = e - e_2$	
$f_b(e)$	$e - p$ curve at $\dot{\varepsilon} = 0$ or $t = \infty$
$F_1(p)$	function of effective stress
$F_2(\dot{\varepsilon})$	function of strain rate
$f(\dot{\varepsilon})$	function of strain rate
H	thickness of sample
i	time dimension

j	space dimension
k	coefficient of permeability
k_1	initial value of k
k_2	final value of k
p	effective stress
p_1	initial value of p
p_2	final value of p
p_{av}	average value of p over depth
p_b	bond stress at end of primary
p_v	viscous resistance to compression
$p_p = p_b + p_v$	plastic resistance to compression
r	rate of change of average void ratio
$R = \frac{T}{Ts}$	dimensionless ratio relating the two time factors associated with diffusion and strain rate effects
t	true time
$T = \frac{C.t}{a.H^2}$	dimensionless time factor associated with diffusion process
$T_s = \frac{t.(\Delta\sigma)^{\beta-1}}{a.\alpha^{\beta}}$	dimensionless time factor associated with strain rate effects
u	pore pressure excess
u_{av}	average pore pressure excess over depth
u_b	base pore pressure excess
$u' = \frac{u}{\Delta\sigma}$	dimensionless pore pressure variable
v	volume of sample
$\dot{v} = \frac{dv}{dt}$	rate of change of volume
$x = \frac{z}{H}$	dimensionless length variable

z	space variable

Greek Letters

α	coefficient related to strain rate vs void ratio relationship
α_{av}	value of α averaged over an increment of load
β	index in power law relating strain rates to the function of strain rate
γ_ω	unit weight of water
$\Delta f_b(e)$	component of total stress increment due to the basic e - p relationship
$\frac{\Delta \dot{v}}{v}$	volumetric strain rate
$\Delta\sigma$	increment of total stress
$\dot{\varepsilon}$	strain rate
$\eta = \frac{\alpha_{av}}{(1+e_1)\frac{1}{\beta}}$	coefficient of non-linear function of strain rate
$\lambda = \frac{\acute{e}}{a.\Delta\sigma}$	dimensionless
$\mu = \frac{e_1 - e}{e_1 - e_2}$	degree of consolidation
σ	total stress
ρ	settlement

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CHAPTER ONE

INTRODUCTION

A general problem of current interest in soil mechanics, both from an academic and engineering point of view, concerns the theoretical predictions of one dimensional consolidation of soils.

The usual method for estimating both the rate and amount of settlement is to perform a consolidation test in which the vertical displacements are measured at appropriate time intervals. Each increment is such that the ratio of the increment to load ($\frac{\Delta\sigma}{\sigma}$) equals unity. Each increment is left in place for 24 hours. The data recorded allows the determination of the coefficient of consolidation, c_v, based on Terzaghi theory for a particular increment. The c_v values thus obtained are used to predict the rates of settlement in the field. The displacement corresponding to the one day reading is used to plot the void ratio vs. effective stress relationship which is then used to estimate the ultimate settlement for the field conditions.

However, both laboratory and field tests on soils indicate that the rate of settlement may deviate considerably from the Terzaghi prediction.

Taylor (1940, 1942) thoroughly investigated this lack of agreement between the Terzaghi theory and the behaviour of samples in the laboratory. The Terzaghi theory for the prediction of the rate of settlement is based on the rate of dissipation of excess pore water pressure. However, Taylor found that appreciable settlements still occured when the excess pore pressure was essentially zero and to take account of this Taylor proposed his Theory A.

Taylor also found that there were appreciable deviations from the Terzaghi theory during the dissipation of the excess pore water pressure and to account for this he proposed his Theory B.

Both of these theories are essentially saying that the void ratio versus effective stress relationship assumed by Terzaghi should be replaced by a more realistic relationship which takes into account the fact that void ratio is also affected by strain rate.

This void ratio - effective stress - strain rate relationship can be obtained by running constant rate of strain (called CRS, hereafter) consolidation tests on a soil over a range of strain rates. If the base pore pressures are measured during the same tests the void ratio - permeability relationship can be obtained by using a recently proposed theory (Byrne and Aoki (1969), Smith and Wahls (1969)).

It is generally believed that there are two components of strain, one due to pore pressure dissipation and the other due to the time or strain rate effect. The void ratio - effective stress - strain rate and void ratio - permeability relationships obtained from the strain rate controlled tests can be used in the equation of continuity of mass to predict the settlement behaviour of a sample of any size under load controlled conditions. The consolidation due to the dissipation of excess pore pressure and that due to the strain rate effect occur concurrently. Therefore, the strain rate effect is considered during the dissipation of excess pore pressure. However, after a time the excess pore water pressure drops to a very small value and the additional time lag in the settlement curve is due wholly to the strain rate effect. This time lag can then be obtained by integration of the basic void ratio - effective stress - strain rate relationship at constant effective stress.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter is divided into three sections. The first section deals with the equation governing the flow of water through a porous medium and the work of the different investigators to solve this flow problem. The literature review of a recently (1969) proposed theory of strain rate controlled consolidation is presented in the second section. The underlying theory of the strain rate controlled consolidation test is presented in the third section.

2.2 One-Dimensional Consolidation of Soils

Terzaghi (1923) developed a consolidation equation

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \qquad \ldots (2.1)$$

where

z = space variable,

t = time variable,

u = excess pore water pressure,

and $c_v = \dfrac{k(1 + e)}{\gamma_\omega \, a_v}$ and is called coefficient of consolidation, where e = void ratio

k = coefficient of permeability,

γ_ω = unit weight of water,

and $a_v = -\frac{de}{dp}$ and is called coefficient of compressibility,

based on the following assumptions,

i) the soil is homogenous and saturated,

ii) the water and solids are incompressible as compared to the soil skeleton,

iii) Darcy's law is valid,

iv) the strains and drainage occurs only in one direction (usually vertical),

v) coefficient of permeability, k, is a constant,

vi) the total stress, throughout the depth, during consolidation is constant,

vii) coefficient of consolidation defined above is constant for a particular load increment involving small changes in void ratio, and

viii) the relationship between void ratio and effective stress is linear and independent of time.

Deviations from the Terzaghi theory were observed both in the field and the laboratory. In the laboratory it was observed that considerable settlement occured after the excess pore water pressure had essentially dissipated. This additional settlement was termed "secondary" settlement by many researchers. Deviations from the Terzaghi theory were also observed during the dissipation of the excess pore water pressure or "primary" consolidation.

Taylor (1942) put forward Theory B and Theory A to account for the deviations from Terzaghi's theory observed in the laboratory testing. In his Theory B Taylor proposed that the intergranular pressure at any time and at any point within the consolidating sample may be expressed

$$p = f_b(e) + P_p \qquad \ldots (2.2)$$

where p = intergranular pressure

f_b (e) = the basic equilibrium curve

The equilibrium curve is the curve representing points where void ratio has reached equilibrium under that particular pressure. This curve could be considered the zero strain rate or $t = \infty$ curve and

P_p = plastic resistance to compression

Taylor further showed that P_p is a function of strain rate and hence from Equation (2.2) the void ratio is a function of the effective stress and the strain rate, i.e.

$$e = F_1(p) + F_2(\dot{\varepsilon}) \qquad \ldots (2.3)$$

where $\dot{\varepsilon}$ = rate of strain

Taylor simplified the problem by dividing P_p into two components P_b, the bond at the end of primary and P_v, the viscous resistance to compression acting during primary.

He ignored P_b in his Theory B.

To consider the secondary compression Taylor proposed Theory A. He desired to set up a theory which could properly account for both primary and secondary compression. However, due to the complexity of the problem, he treated them in two different theories. He noted that the combined use of ideas from two such theories must be the ultimate procedure for obtaining the best understanding of the consolidation process.

Bjerrum (1967) divided the volume changes into two components:

(a) an 'instant compression' which occured simultaneously with the increase in effective stress and caused a reduction in void ratio until an equilibrium value was reached at which the structure effectively supported the overburden pressure;

(b) a 'delayed compression' representing reduction in volume at constant effective stress.

His concept of having a system of e - log p curves for the compressibility characteristics of a clay showing delayed consolidation is similar to that by Taylor (1942) and Crawford (1965).

Gibson and Lo (1961) proposed a one-dimensional theory in which the compressibility of the soil skeleton is represented by a rhelogical model comprising a Hookean spring in series with a Kelvin body. This theory is identical to Taylor's Theory A.

In general, the recent approach used to predict the behaviour of an element of soil is to choose a rhelogical model and compare the observed and predicted behaviour. The parameters of the model are revised as necessary until realistic predictions can be made without it being necessary to alter the rhelogical parameters at each change of test conditions (Barden 1965, 1968, 1969, Berry 1969, Ishii 1951, and others).

2.3 Strain Rate Controlled Consolidation

2.3.a Literature Review

Hamilton and Crawford (1959) made one of the earliest mentions of a constant rate of strain consolidation (CRS) test. They used this type of test as a rapid means of determining both the preconsolidation pressure and the void ratio - effective stress relationship. When compared with the conventional tests, the CRS tests showed lower compressibility, but the general shapes of the curves were somewhat similar. Hamilton and Crawford mentioned that the pore pressures developed would have caused this apparent decrease in compressibility but they did not measure the pore pressures. The range of strain rates in this series varied from 0.15% per min. to 0.005% per min., but even this large difference did not greatly affect the data. The data for the slower strain rates did show a tendency towards closer agreement with the conventional tests. Observation of the test specimen after drying indicated that side friction apparently caused more serious stress variations through the specimens in the standard test than in the CRS test.

In a second paper Crawford (1964) reported more data from the CRS tests. He presented the effect of strain rate as a factor that had been too long ignored in settlement analysis. Crawford noted that laboratory consolidation rates are frequently several million times larger than those experienced in the field. Data from the CRS tests appeared quite similar to that obtained from the standard tests using different load durations. During this series of tests excess pore pressures were measured at the base of the sample. The maximum excess pore pressure in the CRS test was approximately 5% of the applied pressure. Because of the low excess pore water pressures Crawford concluded that the compression in the CRS test is totally secondary compression. The strain rates in this series were approximately 4% per hour to the preconsolidation load, and then 7% to 14% per hour, depending on specimen thickness.

In a discussion of this paper, Schmertmann (1965), however, indicated a preference for a controlled rate of loading test in which values of c_v and k could be determined as well as the preconsolidation pressure and the e - p relationship. To Schmertmann the great advantage of this testing method was the short time required to obtain the desired information.

Crawford (1965) reported a further investigation in which the rates of strain varied from 0.133% per min. to 0.0027% per min., and the maximum excess pore pressure measured at the base of the sample was equal to 15% of the applied pressure. Pore pressures were measured at the base of the samples throughout the duration of the test and the average effective stress on the sample was calculated by substracting one half of the excess pore pressure at the base from the total vertical stress. Standard incremental consolidation tests were conducted for comparison purposes.

From these tests, Crawford concluded that the soil structure had an important time - dependent resistance to compression. Test data showed that the higher the rate of strain, the lower the compressibility or the greater the plastic resistance. He also suggested that the final void ratio for a particular load is mainly dependent on the average rate of compression and not the method by which the load is applied. This was indicated by the marked similarity of the results of the incremental and the CRS tests.

In general agreement with the work done by Crawford was the study conducted on remolded samples and reported by Wahls and DeGodoy (1965). The strain rates in this series of tests varied from 0.23% per min. to 0.053% per min., and pore pressures were measured at the base of the sample. Values of $\frac{u_b}{\sigma}$ varied from 25% for the slowest to 75% for the fastest test. These maximum values resulted from exponential type increases with increasing strain, and were much larger than the values recorded for the major portion of the test. As was the case in Crawford's work, there was an increase in compressibility with decreasing strain rates. When compared with standard test results Wahls and DeGodoy observed that for the range of strain rates used all of the CRS test results showed more compressibility than the standard test. This is in contrast to Crawford's work, where the opposite was true. It has been suggested that this difference may be caused by the increment duration for the standard tests or the different soils used in the test studies.

2.3.b Theory of Strain Rate Controlled Consolidation

Byrne and Aoki (1969) presented a theory of strain controlled consolidation. Based on assumptions (i) to (iv) Section 2.2 inclusive the basic equation of continuity of mass is

$$\frac{\partial}{\partial z}\left[\frac{k(e)}{\gamma_\omega}\frac{\partial u}{\partial z}\right] = \frac{1}{1+e}\frac{\partial e}{\partial t} \qquad \ldots (2.4)$$

Next introducing the assumption (v) the equation reduces to

$$\frac{k}{\gamma_\omega} \frac{\partial^2 u}{\partial z^2} = \frac{1}{1 + e} \frac{\partial e}{\partial t} \qquad \ldots (2.5)$$

They further assumed that the strain rate should be sufficiently slow such that the void ratio can be assumed to be uniform throughout the sample at all times, then the right hand side of Equation (2.5) can be integrated quite simply.

For the boundary conditions

$u = o$ at $z = o$ for all t

and $\frac{\partial u}{\partial z} = o$ at $z = H$ for all t

the following expression for the excess pore pressure at any time is obtained:

$$u(z) = - \frac{1}{1 + e} \frac{\partial e}{\partial t} \cdot \frac{\gamma_\omega}{k} \cdot (Hz - \frac{z^2}{2}) \qquad \ldots (2.6)$$

the excess pore pressure at the base ($z = H$) is given by

$$u_b = - \frac{1}{1 + e} \frac{\partial e}{\partial t} \cdot \frac{\gamma_\omega}{k} \frac{H^2}{2} \qquad \ldots (2.7)$$

and since the excess pore pressure is parabolic with z, the average pore pressure throughout the sample is given by

$$u_{av} = \frac{2}{3} \cdot u_b = -\frac{2}{3} (\frac{1}{1 + e} \frac{\partial e}{\partial t}) \frac{\gamma_\omega}{k} \cdot \frac{H^2}{2} \qquad \ldots (2.8)$$

$$u_{av} = -\frac{1}{3} (\frac{1}{1 + e}) \frac{\partial e}{\partial t} \cdot \frac{\gamma_\omega}{k} \cdot \frac{H^2}{2} \qquad \ldots (2.9)$$

and $$p_{av} = \sigma - u_{av} \qquad \text{... (2.10)}$$

The term $(\frac{1}{1+e} \cdot \frac{\partial e}{\partial t})$ is the volumetric strain rate and will be negative for all cases where the volume of the sample is decreasing with time and hence a positive excess pore pressure will result. Also

$$\frac{1}{1 + e} \ \frac{\partial e}{\partial t} = \frac{\Delta \dot{v}}{v} \qquad \text{... (2.11)}$$

where $\frac{\Delta \dot{v}}{v}$ = volumetric strain rate.

For one dimensional consolidation

$$\frac{1}{1 + e} \cdot \frac{\partial e}{\partial t} = \frac{\Delta \dot{v}}{v} = \frac{1}{H} \frac{\Delta H}{\Delta t} \qquad \text{... (2.12)}$$

where H = height of the sample.

Substitution of (2.11) and (2.12) in (2.7) gives

$$u_b = - \frac{\Delta H}{\Delta t} \cdot \frac{1}{H} \cdot \frac{\gamma_\omega}{k} \frac{H^2}{2} \qquad \text{... (2.13)}$$

$$u_b = - \frac{\Delta H}{\Delta t} \cdot \frac{\gamma_\omega}{k} \cdot \frac{H}{2} \qquad \text{... (2.14)}$$

When a constant deformation rate is applied to a sample with base pore pressure measured, Equation (2.14) allows the coefficient of permeability to be calculated:

$$k = - \frac{H}{2} \frac{\gamma_\omega}{u_b} \cdot \frac{\Delta H}{\Delta t} \qquad \text{... (2.15)}$$

It should be noted that this k is function of void ratio of the sample. It is further assumed in the integration of Equation (2.5) that k is only a function of time and not a function of z.

Smith and Wahls (1969) also developed a strain rate controlled consolidation mathematical model. Instead of assuming constant void ratio throughout the depth of a sample they assumed a linear variation

$$e(z,t) = e_o - rt \left[1 - \frac{b}{r} \left(\frac{z - 0.5H}{H}\right)\right] \qquad \ldots (2.16)$$

in which $r = - \frac{d\bar{e}}{dt}$ is rate of change of average void ratio,

b = a constant that depends on the variation in void ratio with depth and time.

The dimensionless ratio, $\frac{b}{r}$ indicates the variation in void ratio with depth. Smith and Wahls substituted Equation (2.16) in Equation (2.5) and integrated to give an expression for the pore pressure. It can be shown that Byrne and Aoki's theory is mathematically a special case of Smith and Wahls' theory with $\frac{b}{r} = 0$. Smith and Wahls, however, mentioned the working limits of $\frac{b}{r}$ between 0 and 2. They recommended use of $\frac{b}{r} = 1$, because this gave closest agreement, when comparing c_v values from standard and CRS tests.

Byrne (1970) in a discussion of Smith and Wahls' (1969) paper pointed out that there are reasons other than the $\frac{b}{r}$ ratio why the results of conventional and strain controlled tests would yield different c_v values. He also showed that $\frac{b}{r}$ values will depend upon strain rate and can be calculated by

$$\frac{b}{r} = \frac{a_v}{t} \cdot \frac{u_b}{r} \qquad \ldots (2.17)$$

In this thesis the strain controlled consolidation tests will be used to determine the void ratio - permeability and the void ratio - effective stress - strain rate relationships. These relationships and Equation (2.4) are then used to predict the behaviour of a conventional test. The solution technique for this purpose is discussed in the next chapter.

CHAPTER THREE

PREDICTION OF CONVENTIONAL TEST FROM CRS DATA

3.1 Introduction

In a conventional test an increment of total stress is added to a sample and the time - settlement and time - pore pressure curves are observed. The intent of this thesis is to show that these curves can be predicted from data obtained from CRS tests. The data obtained from the CRS tests will be in terms of void ratio - effective stress - strain rate and void ratio - permeability relationships.

3.2 The Void Ratio - Effective Stress - Strain Rate Relationship

It is desired to replace Assumption (viii) Section 2.2, stating "the relationship between void ratio and effective stress is linear and independent of time", by a more realistic relationship considering void ratio as a function of both effective stress and strain rate.

To obtain this relationship, the strain rate effect is first eliminated by imposing a constant strain rate. If the base pore pressure is measured, effective stress - void ratio relationships can be plotted, for the particular strain rate. If now on another sample of the same material a different strain rate is applied, it will give a different e - p plot. The difference in these two curves will depend upon the nature of material. In this fashion a series of curves can be obtained running tests at different strain rates. Theoretical limits of strain rates will be $\dot{\varepsilon} = 0$ to ∞. Figure 3.1 shows that for different strain rates, the effective stress will be different for a given void ratio.

It is assumed that before placing a stress increment, $\Delta\sigma$, on a sample the system is in equilibrium with all the stresses transferred to the grain bond. This means that if the void ratio is e_1 the effective stress will be p_1 and the point lies on $\dot{\varepsilon} = 0$ curve. Let the final void ratio be e_2. As soon as the load is applied the sample starts straining. The strain rates will be higher near the drainage boundary and lower near the impermeable boundary. After a very small time interval void ratio at a point on the drainage boundary will be e_3 and the effective stress will be p_2. This point will then move down vertically until it reaches the final void ratio e_2.

At a point somewhat removed from the drainage boundary the void ratio will be e and the effective stress will be p depending on $\dot{\varepsilon}$ at some time t. As time goes on this point moves on different strain rate lines until

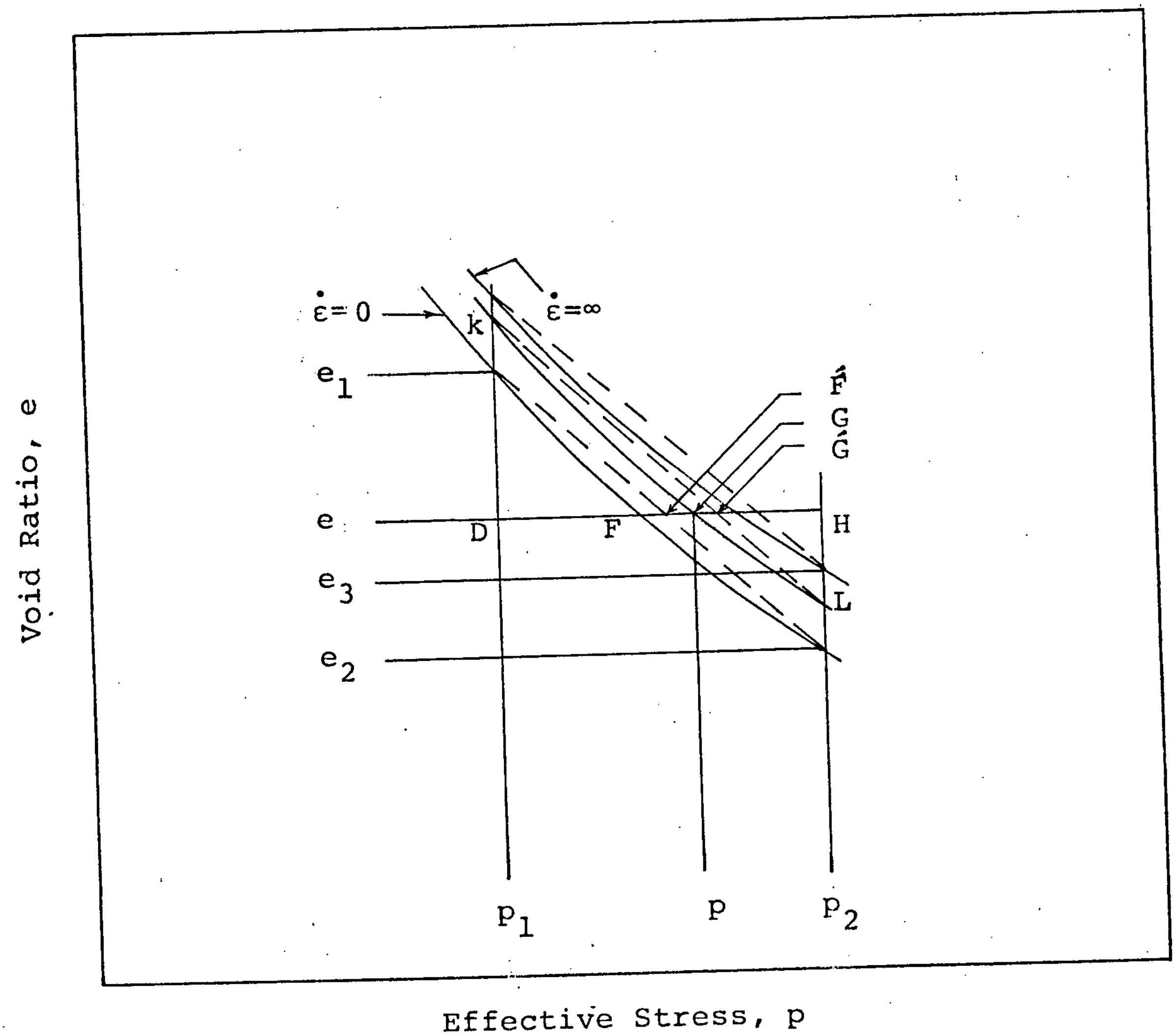

Figure 3.1 Natural Scale Plot of $\dot{\varepsilon} = 0$ and $\dot{\varepsilon} = \infty$ for an Increment

finally it also reaches the zero strain rate curve or the point (p_2, e_2).

From Figure 3.1 equilibrium requires that for every point within the sample

$$\Delta\bar{\sigma} = \Delta f_b(e) + f(\dot{\varepsilon}) + u \qquad \ldots (3.1)$$

or $$DH = DF + FG + GH \qquad \ldots (3.2)$$

Making the simplifying assumption that $f_b(e)$ curve can be replaced by the straight line AB, and similarly approximating the curve KGL by straight line $KG'L$. It is obvious that the use of this assumption will involve much less approximation in the case of smaller load increment ratios. This assumption is similar to the one made by Taylor (1942) in his Theory B.

The Equation (3.2) reduces to

$$DH = DF' + F'G' + G'H \qquad \ldots (3.3)$$

where $DH = \Delta\sigma$ = total stress increment $= \dfrac{e_1 - e_2}{a}$

'a' being slope of line AFB.

$DF' = \Delta f_b(e)$ = component of total stress due to the function of basic e - p relationship and

$= \dfrac{e_1 - e}{a}$

$F'G' = f(\dot{\varepsilon})$ = function of strain rate

$G'H = u$ = excess pore pressure

This concept of three phase system is in essential agreement with Terzaghi (1941), Taylor (1942), Barden (1965) and others. Equation (3.1) can be rewritten

$$\frac{e - e_2}{a} = u + f(\dot{\varepsilon}) \qquad \ldots (3.4)$$

Equation (3.4) is similar in form to Equation (2.3). Taylor referred to the term $f(\dot{\varepsilon})$ as the plastic resistance to compression. This term can be determined from CRS data and substituted in Equation (3.4) to give the desired void ratio as a function of effective stress and strain rates.

3.3 The Void Ratio - Permeability Relationship

In Equation (2.4) the permeability, k, is a function of the void ratio, e. If pore pressures are measured during a CRS test, Equation (2.15) allows the determination of the e - k relationship. The coefficient of permeability may or may not be a function of strain rate. Byrne and Aoki's (1969) data on undisturbed samples of a sensitive clay shows that e - log k relationship is essentially a straight line and that no trend with strain rate was discernible. Smith and Wahls (1969) did not show e - k plots. It will be assumed here that the coefficient of permeability is not a function of strain rate. If the data obtained for a particular soil shows that the e - k relationship is dependent on strain rates, then the equations may be changed accordingly.

In the Terzaghi theory, k is assumed constant during an increment. Hansbo's (1960) treatment of permeability variation, in incremental loading test, indicated that for

$$\frac{k_1}{k_2} < 3$$

where k_1 = initial permeability and

k_2 = final permeability for a particular load increment, the difference in the time - settlement and time - pore pressure relationships is not appreciably different from the Terzaghi theory.

3.4 Governing Equation of Consolidation

The major sources of non-linear behaviour for clay and peat soils are:

i) Finite strain - leading to changes in the length of drainage path,

ii) Varying permeability within the sample,

iii) Varying compressibility, and

iv) Void ratio - effective stress relationships which are dependent on time or strain rates.

Although all of these non-linearities can be included in a single general treatment the presentation is not simple (Berry (1969)). The usual analysis includes only those non-linearities which are relevant to the soil under consideration.

A consolidation equation is presented in the following pages to account for all the non-linearities mentioned above, i.e. finite strain, varying permeability, varying compressibility and the void ratio - effective stress relationships which are dependent on time or strain rates.

i) Finite Strain

The following represents a brief summary of the development of the continuity equation for finite strain.

Figure 3.2.a shows a soil layer having an initial thickness H_o which is subjected to an instantaneous increase in pressure $\Delta\sigma$. The width of the soil layer and the extent of loaded area are considered to be infinite, so that the problem is essentially one dimensional. The soil rests on an impermeable boundary and is free to drain to its upper surface.

Consider an element of soil at a distance z_o from the impermeable boundary having initial thickness dz_o (Figure 3.2.a). During consolidation the thickness of the element decreases and its position in space moves vertically downwards. That is, the element undergoes finite strain, and the subsequent development of the problem requires the mathematical treatment of a moving drainage boundary. Let dz denote the new height of the element and z its distance from the boundary at some time t after the start of consolidation, as shown in Figure 3.2.b, then comparing the

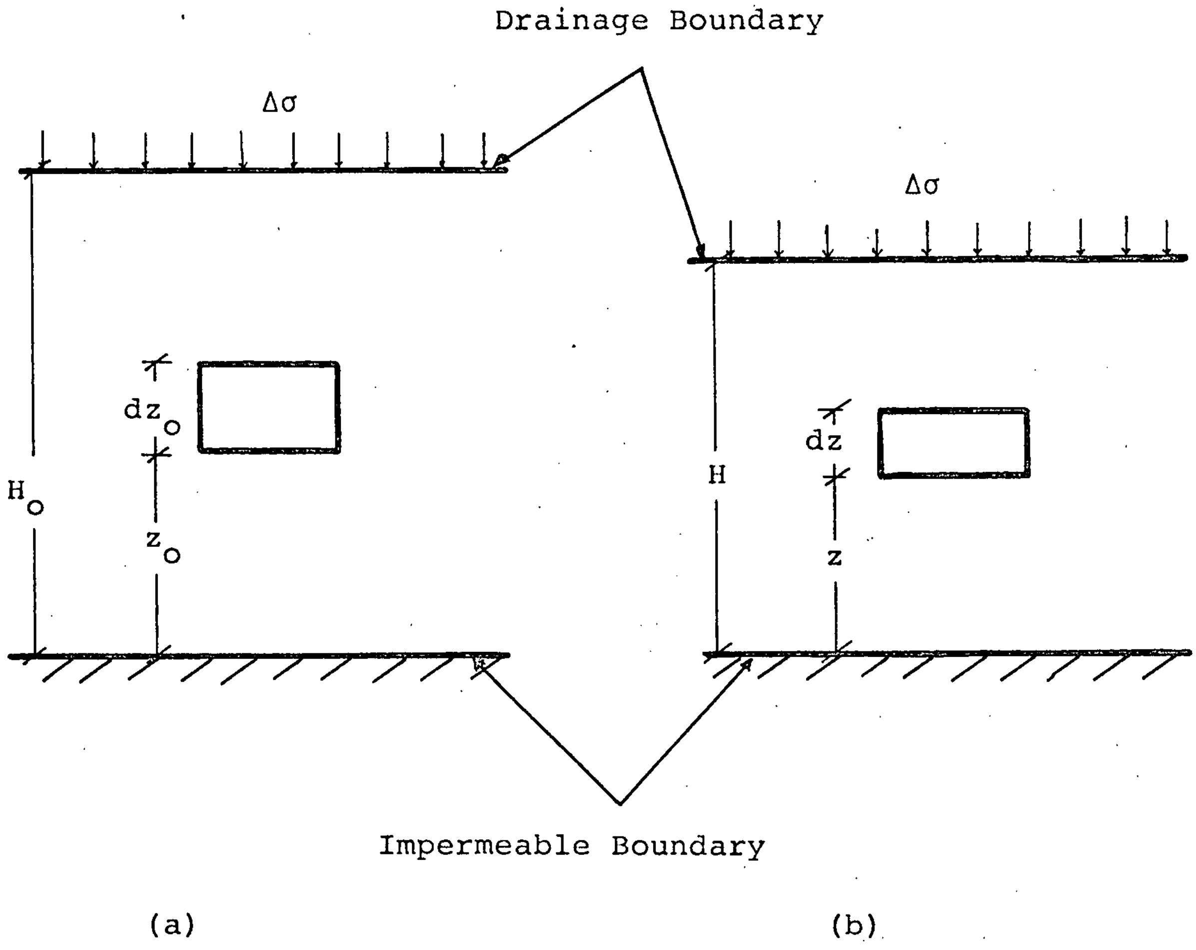

Figure 3.2 Moving Drainage Boundary

thickness of the elements shown in Figures 3.2(a) and 3.2(b)

$$\frac{dz}{dz_o} = \frac{1 + e}{1 + e_1} \qquad \text{... (3.4.a)}$$

Substituting in Equation (2.4)

$$\frac{(1+e_1)^2}{1+e} \frac{\partial}{\partial z_o} \left(\frac{k(e)}{\gamma_\omega (1+e)} \frac{\partial u}{\partial z_o}\right) = \frac{1}{1+e} \frac{\partial e}{\partial t} \qquad \text{... (3.5)}$$

Equation (3.5) defines the consolidation with respect to the initial soil conditions and accounts for finite strain or change in the length of the drainage path during consolidation.

ii) Variation in Permeability

Differentiating Equation (3.5):

$$\frac{(1+e_1)}{\gamma_\omega} \left[\frac{\partial}{\partial z_o} \left(\frac{k(e)}{1+e}\right) \frac{\partial u}{\partial z_o} + \frac{k(e)}{1+e} \frac{\partial^2 u}{\partial z_o^2}\right] = \frac{1}{1+e_1} \frac{\partial e}{\partial t} \qquad \text{... (3.6)}$$

After the e - k relationship is established from CRS tests and substituted above, Equation (3.6) will account for both finite strain and varying permeability.

iii) Varying Compressibility

This non-linearity can be introduced into the consolidation equation, when relating e, u and $\dot{\varepsilon}$.

iv) Void Ratio, a Function of Effective Stress and Strain Rates

Equation (3.4) represents this relationship, which can be found by running strain controlled consolidation tests on the given soil, over a range of strain rates.

Substitution of Equation (3.4) into Equation (3.6) will give the governing consolidation equation. The resulting equation will be highly non-linear and recourse has to be made to a numerical solution procedure. For that purpose it is more convenient to work with the two Equations (3.4) and (3.6) simultaneously.

3.5 A Desired Solution Procedure

Consider the grid shown in Figure 3.3. Assuming all of the e and u values along the depth at any time i are known, then application of explicit central finite difference approximation gives

$$\left(\frac{\partial^2 u}{\partial z_o^2}\right)_{ij} = \frac{1}{Dz_o^2}\left[u_{i,j-1} - 2u_{i,j} + u_{i,j+1}\right] \qquad \ldots (3.7)$$

$$\left(\frac{\partial u}{\partial z}\right)_{ij} = \frac{1}{2Dz_o}\left[u_{i,j-1} - u_{i,j+1}\right] \qquad \ldots (3.8)$$

$$\left(\frac{\partial e}{\partial t}\right)_{i,i+1} = \frac{1}{Dt}\left[e_{i+1,j} - e_{i,j}\right] \qquad \ldots (3.9)$$

The term $\frac{1}{1+e}$ on the left hand side of Equation (3.6) may be approximated by $\frac{1}{1+e_{i,j}}$, for the particular time

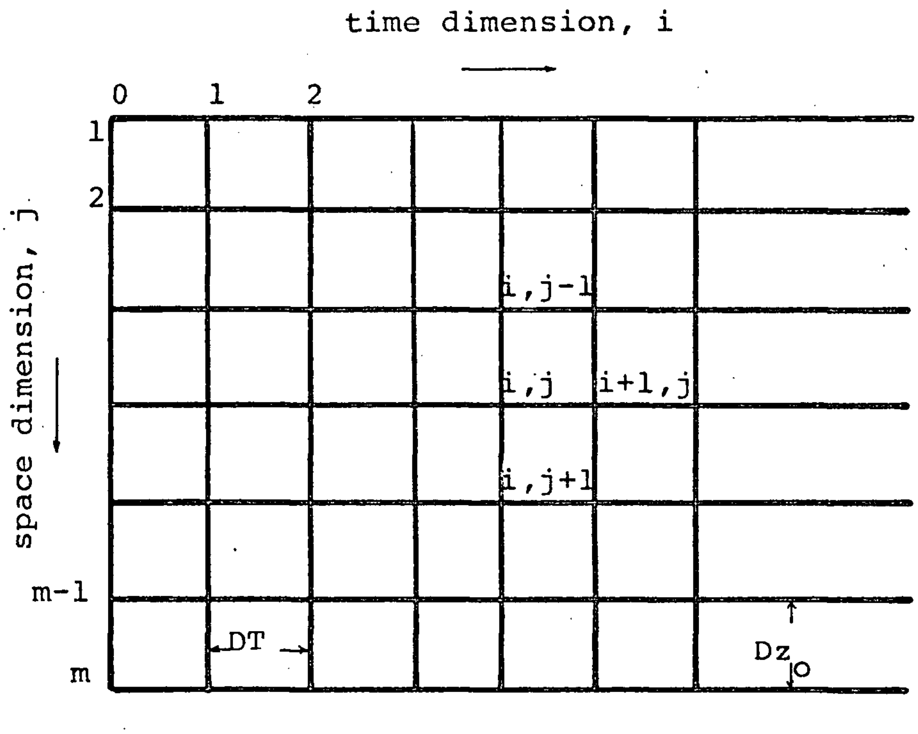

Figure 3.3 Finite Difference Space-Time Grid

interval, without much error, then

$$(\frac{\partial}{\partial z_o} \frac{k(e)}{1 + e})_{ij} = \frac{1}{2Dz_o} [\frac{k_{i,j-1} - k_{i,j+1}}{1 + e_{i,j}}] \qquad \ldots (3.10)$$

Use of Equations (3.7) to (3.10) inclusive in Equation (3.7) enables the determination of $e_{i+1,j}$, from all the previously known values of e and u.

The boundary condition $u(1,0) = 0$ can be imposed directly and the second boundary condition, $(\frac{\partial u}{\partial z})_{im} = 0$, can be satisfied by considering a fictitious point outside the boundary such that $u_{i,m+1} = u_{i,m-1}$ and hence there is no pore pressure gradient across the impermeable boundary.

All of the e values along the depth at the next time can, therefore, be predicted and the strain rate at a particular time and for any given point can then be computed as follows:

$$\dot{\varepsilon}_{i,j+1} = \frac{1}{1 + e_1} \cdot \frac{e_{i+1,j} - e_{i,j}}{Dt} \qquad \ldots (3.11)$$

Substitution of Equation (3.11) into Equation (3.4) will give the $u_{i,j+1}$. Hence all the pore pressures at the next time can be computed. New permeability values can also be evaluated for each point in the grid corresponding to each void ratio. The solution, therefore, can proceed to the next time interval.

Although the proposed method seems simple to apply, a stable solution was not obtained and hence the following alternate procedure was adopted in this thesis.

3.6 The Adopted Solution Procedure

Starting from Equation (2.4), an average k is assumed for an increment, the error involved depends on the final and initial values of permeability. Hansbo (1960), has shown, however, that the difference in the time - settlement and time - pore pressure values is small if the ratio $\frac{k_1}{k_2} < 3$. The equation, therefore, reduces to

$$\frac{k_{av}}{\gamma_\omega} \frac{\partial^2 u}{\partial z^2} = \frac{1}{1 + e} \frac{\partial e}{\partial t} \qquad \text{... (3.12)}$$

Let $\frac{1}{1 + e}$ equal $\frac{1}{1 + e_1}$

then $$C \frac{\partial^2 u}{\partial z^2} = \frac{\partial e}{\partial t} \qquad \text{... (3.13)}$$

where $$C = \frac{k_{av} (1 + e_1)}{\gamma_\omega}$$

Writing Equation (3.4) as

$$e - e_2 = a[u + f \{\frac{1}{1 + e_1} (-\frac{\partial e}{\partial t})\}] \qquad \text{... (3.14)}$$

the -ve sign shows decrease in e with time.

Equations (3.13) and (3.14) are then solved for the usual boundary conditions.

Barden (1965) proposed two methods to solve simultaneous partial differential equations.

Poskitt (1967) devised numerical procedures which take about one tenth of the computing time required by the methods originally proposed by Barden (1965). The Poskitt methods are based on the explicit method (MCM, 1962) and the Crank - Nicolson method (MCM, 1962). Both of these methods are extremely stable and give answers in close agreement with each other (Poskitt (1967)). The explicit method is used in this analysis (see Appendix 1).

When $u \to 0$, Equation (3.14) can be directly integrated to give the part of the compression which occurs at constant effective stress.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction

The peat used in this investigation was brought from a peat farm on the intersection of 499 Highway and No. 5 Road in Richmond, about ten miles south of Vancouver, B.C.

The data presented in this investigation concerns the properties of remolded peat. The use of remolded soils in basic research has important advantages, especially with regard to the uniformity of test specimens, Hvorslev (1960). Many relations concerning the physical properties of clays were first determined by means of tests on remolded specimens. The results of tests on remolded soils, however, cannot be directly used for solution of practical problems involving undisturbed soils.

Different batches of peat were remolded under vacuum at 1,175% moisture content (natural water content = 600 to 700%) with two mechanical stirrers for a period of 48 hours. All of these batches were then combined and mixed thoroughly in a larger container. This container was covered and kept in a humid room.

The peat in the container was periodically stirred and was thoroughly mixed before a sample was taken.

Identification tests were performed to determine some of the properties of the soil used before starting the main testing programme and the results are shown in Table 1.

TABLE 1

SOME PROPERTIES OF PEAT USED

Specific Gravity, G_s	1.4
Natural Water Content	600 - 700%
Liquid Limit	1100 - 1200%
pH	3.5 - 4.0
Nature	Amorphous granular

The procedure for the incremental tests was in accordance with that outlined in Appendix 2. Further references were made to Lamb (1962) and Bishop and Henkel (1964). The testing procedure for the CRS tests is also given in Appendix 2.

A computer programme was used to reduce the data and to calculate e - p and e - k for different tests using

Equations (2.10) and (2.15) respectively.

4.2 General Discussion

Byrne and Aoki's (1969) theory of strain rate controlled consolidation is based on the assumption that the void ratio is uniform throughout the depth of the sample. The assumption is acceptable when the strain rates are sufficiently slow and the ratio of $\frac{u_b}{\sigma}$ are small. The validity of the assumption becomes seriously questionable at high strain rates.

Smith and Wahls (1969) in their development, which was discussed in Chapter Two, assumed that the void ratio varied linearly with depth, Equation (2.16), and dimensionless ratio, $\frac{b}{r}$, indicates the variation in void ratio with depth. The limits of this ratio are from 0 to 2. If, for example, in a particular test $\frac{b}{r}$ equals 2, the use of $\frac{b}{r}$ equal to 0 will introduce an error of $0.08\left(\frac{u_b}{\sigma}\right)$ in the value of effective stress, which can be neglected without loss of accuracy. To compare the $\frac{b}{r}$ ratio for different samples, it was necessary to use this ratio per unit initial thickness of the sample, because of the different thicknesses of the samples. Figure 4.1 shows the $\frac{b}{rH_o}$ vs time plot for different strain rates. The maximum value of $\frac{b}{rH_o}$ from this plot is 0.03 and for this test $\frac{b}{r}$ equals about 0.02, the effective stress calculated by using $\frac{b}{r}$ equal to zero will be $0.05 \frac{u_b}{\sigma}$ % greater than the one which will result by the use of $\frac{b}{r}$

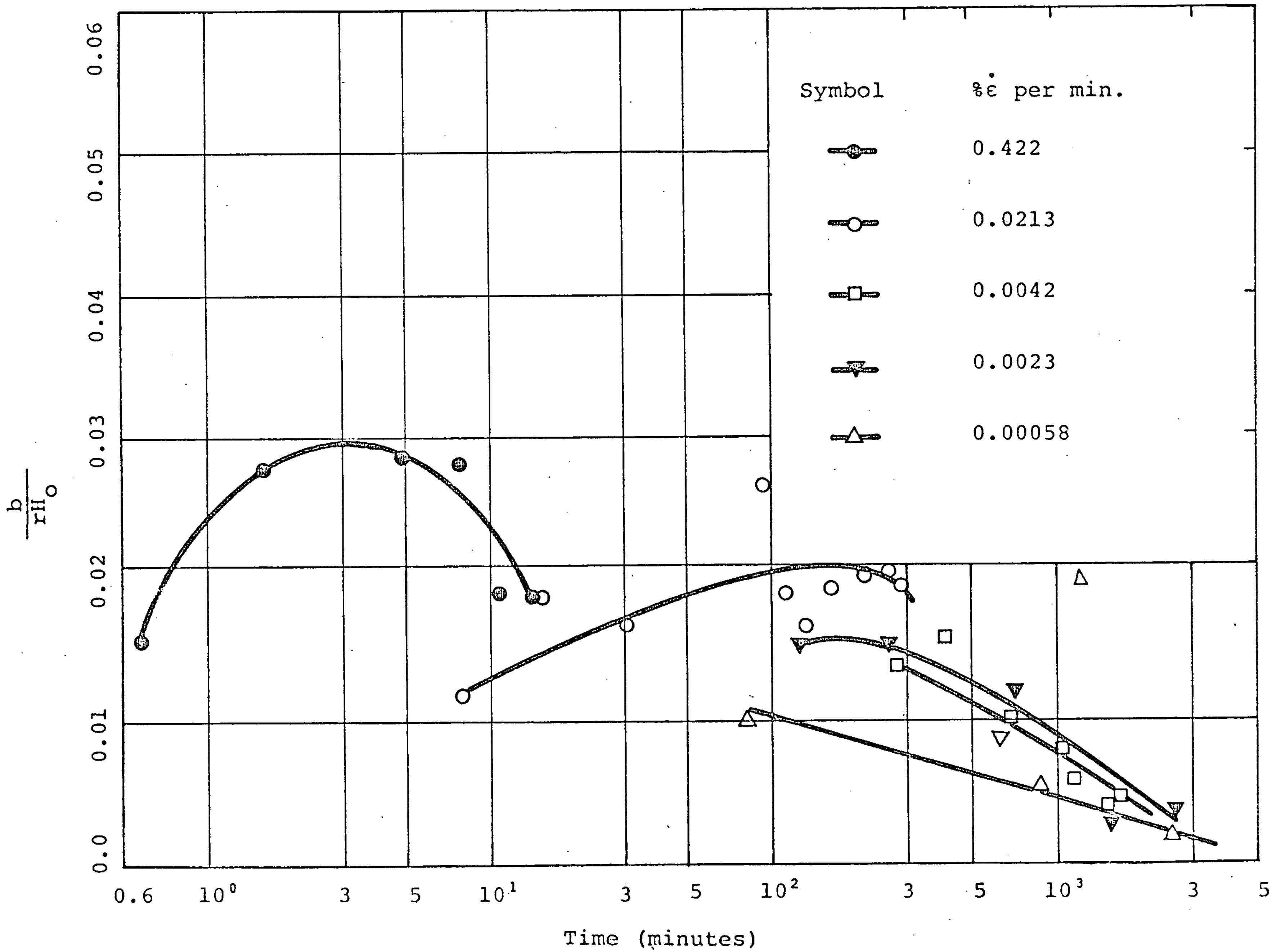

Figure 4.1 $\frac{b}{rH_o}$ vs Time

equal to 0.02. On this basis it was decided to use $\frac{b}{r}$ equal to zero.

Figure 4.2 shows a $\frac{u_b}{\sigma}$ vs t plot which is another indication of the non-uniformity of void ratio throughout the depth. It shows that the higher the strain rate the higher the $\frac{u_b}{\sigma}$ ratio and, therefore, the larger the variation in void ratio through the depth of a sample. In tests A-1 to A-4 inclusive the $\frac{u_b}{\sigma}$ ratio is fairly constant throughout the duration of tests. In test A-5 the ratio is initially 70% and then drops to a minimum of about 40%. The initial part of this test on void ratio - effective stress plot was markedly different from the others. This difference could not be accounted for by considering a linear variation of void ratio. On this basis it was decided that the $\frac{u_b}{\sigma}$ ratio of 50% will be accepted as reasonable. It may be mentioned that Smith and Wahls decided on the limiting value of $\frac{u_b}{\sigma}$ as 50% on the basis of the comparison of c_v values with the incremental loading test. This comparison of c_v values is, however, not attempted here because a more basic comparison of incremental and CRS test is presented.

4.3 e - p - $\dot{\varepsilon}$ Relationship from CRS Tests

Although the remolded samples are uniform enough to carry out basic research it was thought necessary to first check the accuracy of reproducibility of results.

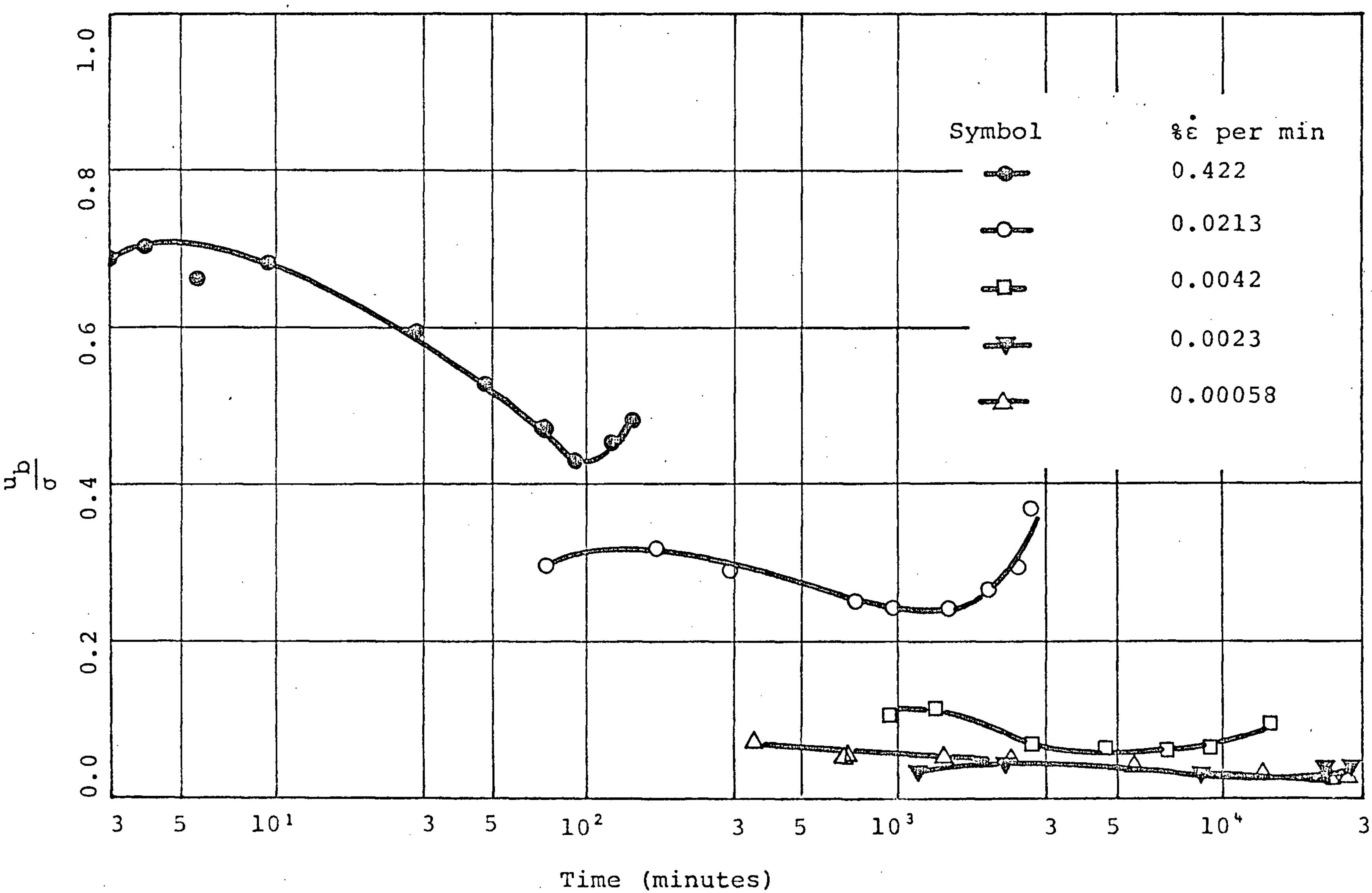

Figure 4.2 $\frac{u_b}{\sigma}$ vs Time

Three (A-4, RA-4, R-4) CRS tests on samples 3 in. dia. and 2 in. in height were performed with the same initial conditions and the same strain rate (0.0023% per min.). The e vs p relationships obtained are plotted for these three tests and lie essentially on the same curve, Figure 4.3. The result is typical of a normally consolidated or remolded soil sample.

It was necessary to check the effect of sample size on the e - p - $\dot{\varepsilon}$ relationship, because the data is to be used for the prediction of incremental loading tests on samples of any size.

A test (C-1) on 5.57" high and 8.26" diameter sample was performed and e - log p points for this test are also plotted in Figure 4.3. It may be seen that all points lie on the same line so that for any one strain rate there is only one e vs p relationship regardless of the size of the test specimen.

Four more tests (A-1, A-2, A-3, and A-5) were performed on different sizes of samples with varying strain rates and the results are plotted in Figure 4.4. These curves are essentially parallel straight lines on the semi log plot in the region of interest, except test A-5, which has been mentioned earlier.

These results are in general agreement with the data obtained by Crawford (1964, 1965), Byrne and Aoki (1969) and Smith and Wahls (1969) and show that for a given void

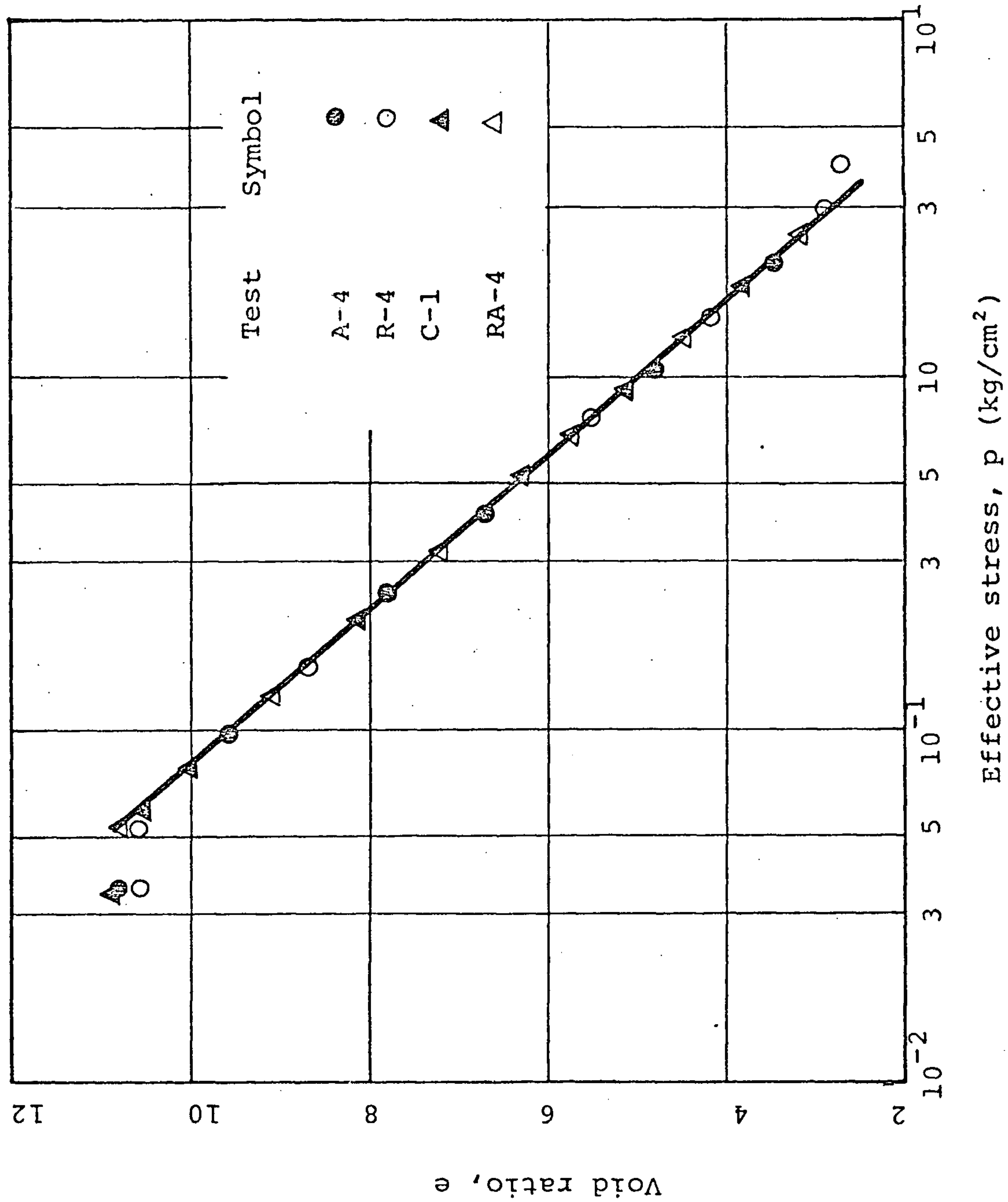

Figure 4.3 Void Ratio vs Effective Stress at $\dot{\varepsilon}$ = 0.0213% per min.

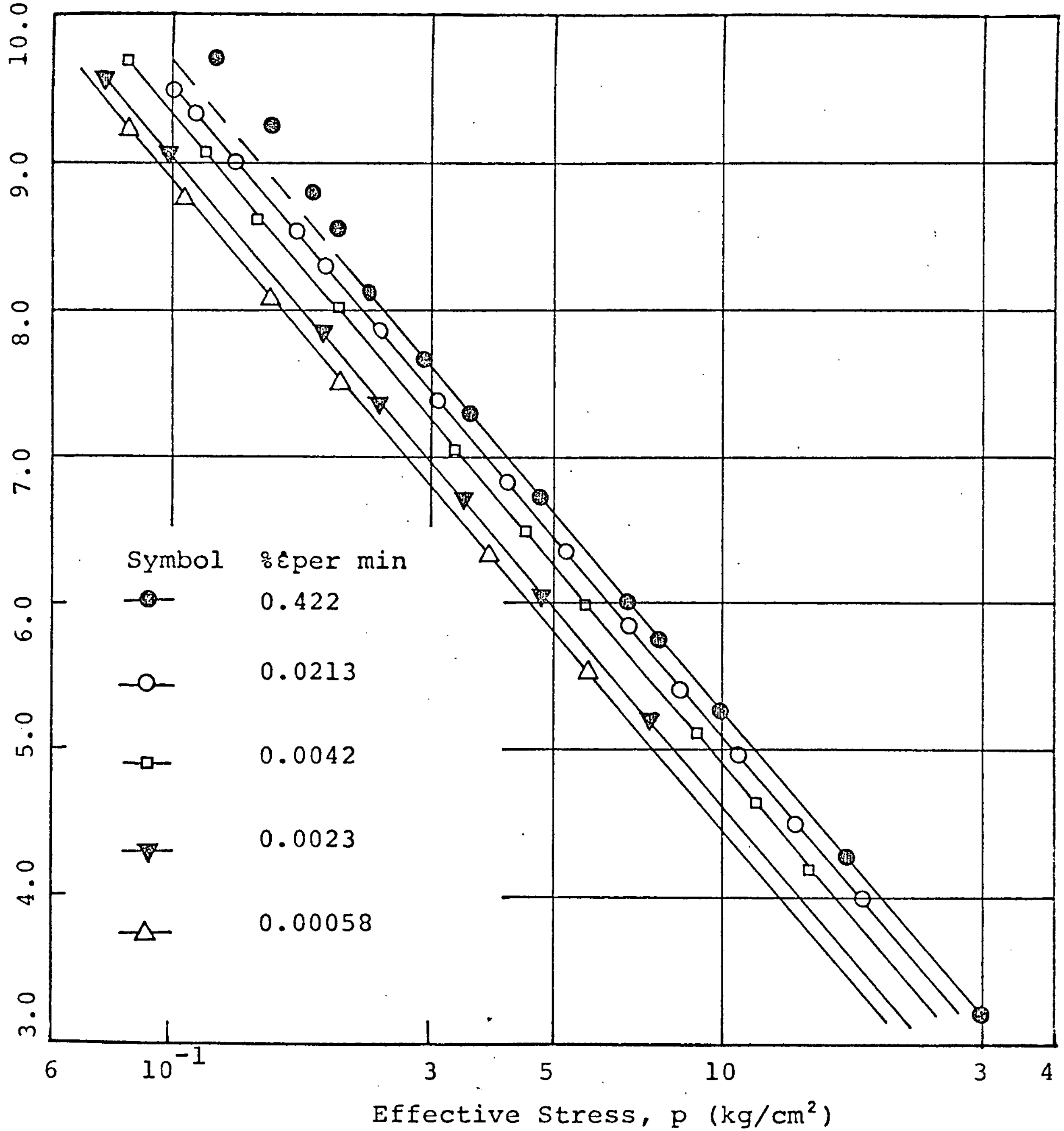

Figure 4.4 e - p - $\dot{\varepsilon}$ relationship from CRS Test

ratio the compressibility is lower for higher strain rates. This behaviour is in agreement with the concept of soil as a viscoelastic material.

The void ratio vs strain rate relationship at constant effective stress can be directly obtained from Figure 4.4 and is shown in Figure 4.5 for an effective stress, p, of 0.8 kg/cm^2. Because Figure 4.4 is on a semi log plot, this e vs $\dot{\varepsilon}$ curve for any other effective stress will be similar in slope but shifted above or below depending upon whether the stress level is higher or lower than this arbitrarily chosen value of 0.8 kg/cm^2.

This curve indicates that marked changes of void ratio with strain rate takes place for strain rates in the range 10^{-1} to 10^{-4} percent per min. On both the high and low side of this range the variation in the void ratio at constant effective stress is relatively small for this soil.

Figure 4.6 shows the general void ratio - effective stress - strain rate relationship plotted by extrapolating the values of void ratio for different strain rates from Figure 4.5 at constant effective stress.

In this figure the $\dot{\varepsilon} = 0$ line has been assumed, but it is realized that probably the actual position of this line will not be much different, as suggested by Figure 4.5.

Figure 4.6 represents Equation (2.3) for the soil tested, i.e. void ratio as function of effective stress and strain rate.

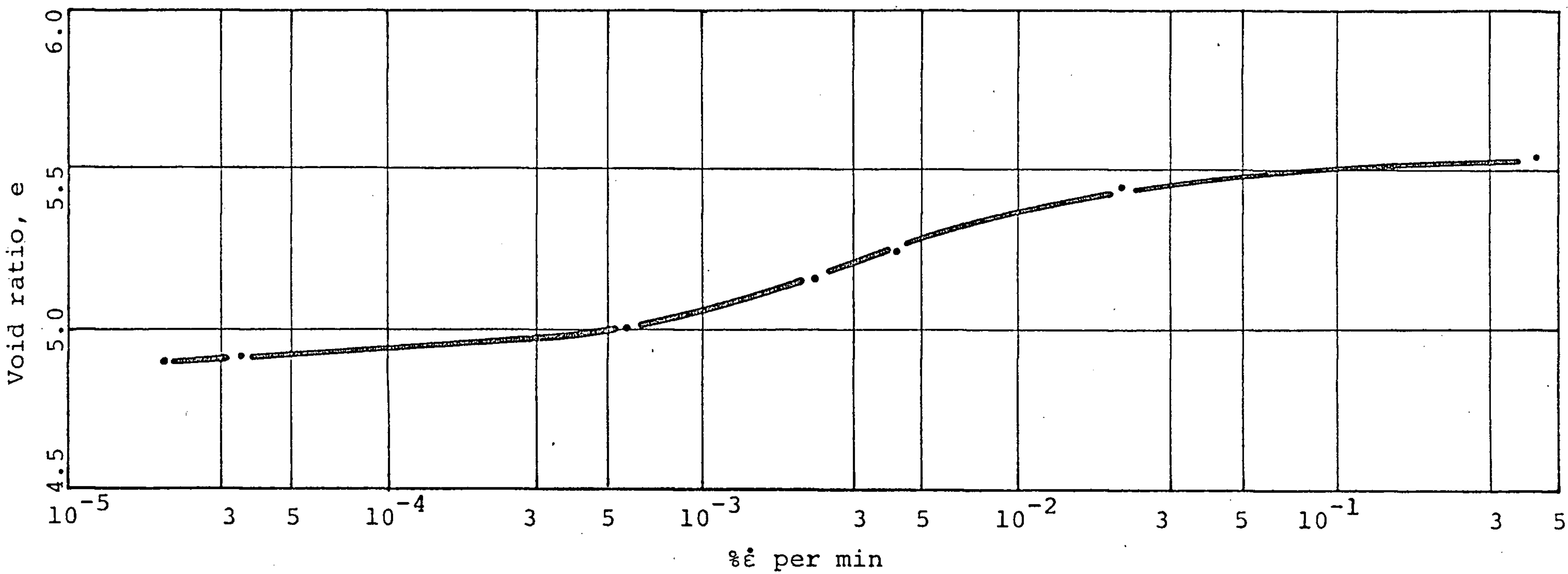

Figure 4.5 e vs $\dot{\varepsilon}$ at $p = 0.8$ kg/cm^2

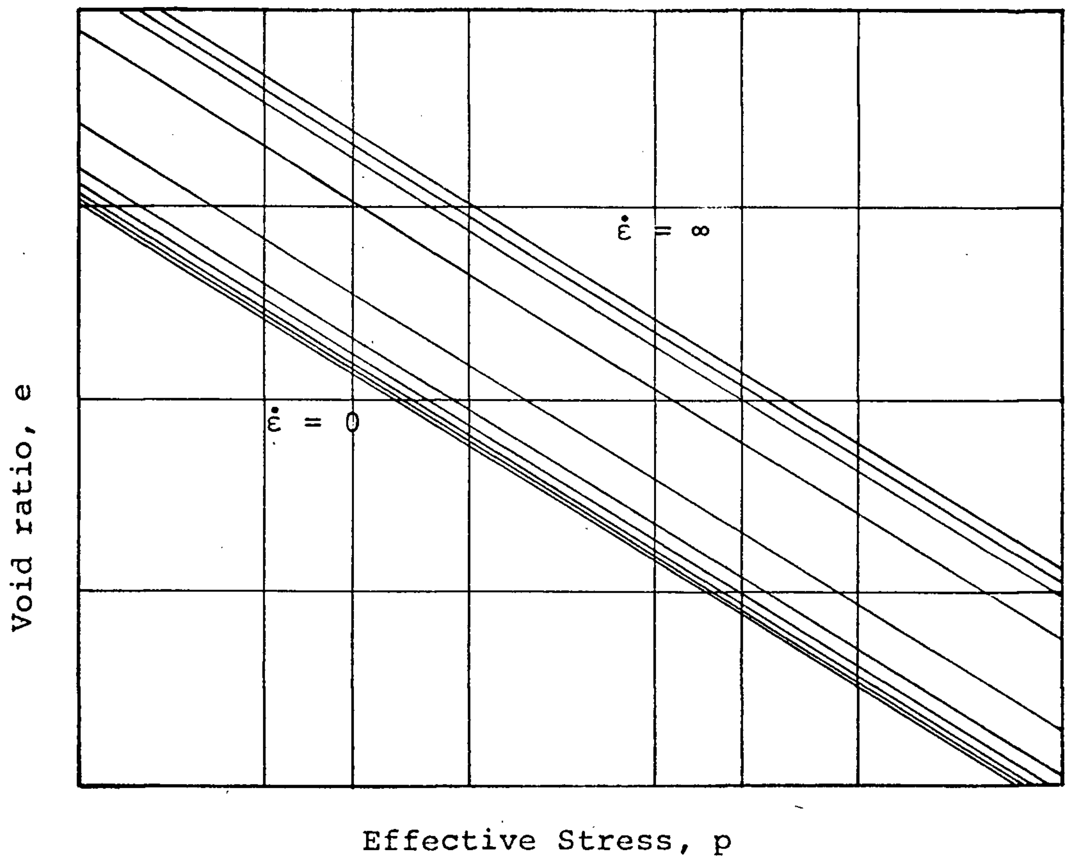

Figure 4.6 General e - p - $\dot{\varepsilon}$ Relationship for Peat Tested

Equation (3.14) can be obtained from Figure 4.6 as explained in Section 3.2. The values of $f(\dot{\varepsilon})$ are obtained at a particular void ratio and are plotted in Figure 4.7 against corresponding strain rates, both on the logarithmic scale. This relationship can be expressed by

$$f(\dot{\varepsilon}) = \alpha(\dot{\varepsilon})^{\frac{1}{\beta}} \qquad \ldots (4.1)$$

which is similar to Ostwald's empirical power law, relating shear stress and shear strain rate. Barden (1965) used Ostwald's law in a rhelogical model to represent the behaviour of an element of a soil.

It can be shown that, because the void ratio vs effective stress curves for different strain rates are parallel lines on semi log plot, the strain rate vs f (strain rate) curves will be parallel lines on the logarithmic plot for different effective stress values. Hence, β, in Equation (4.1) will remain constant for all the effective stresses, but α will vary as

$$\frac{\alpha_1}{\alpha_2} = \frac{p_1}{p_2} \qquad \ldots (4.2)$$

For the necessary simplification, an average value of α will be used for a particular load increment.

Hence

$$e - e_2 = a[u + \eta \ (\frac{\partial e}{\partial t})^{\frac{1}{\beta}} \qquad \ldots (4.3)$$

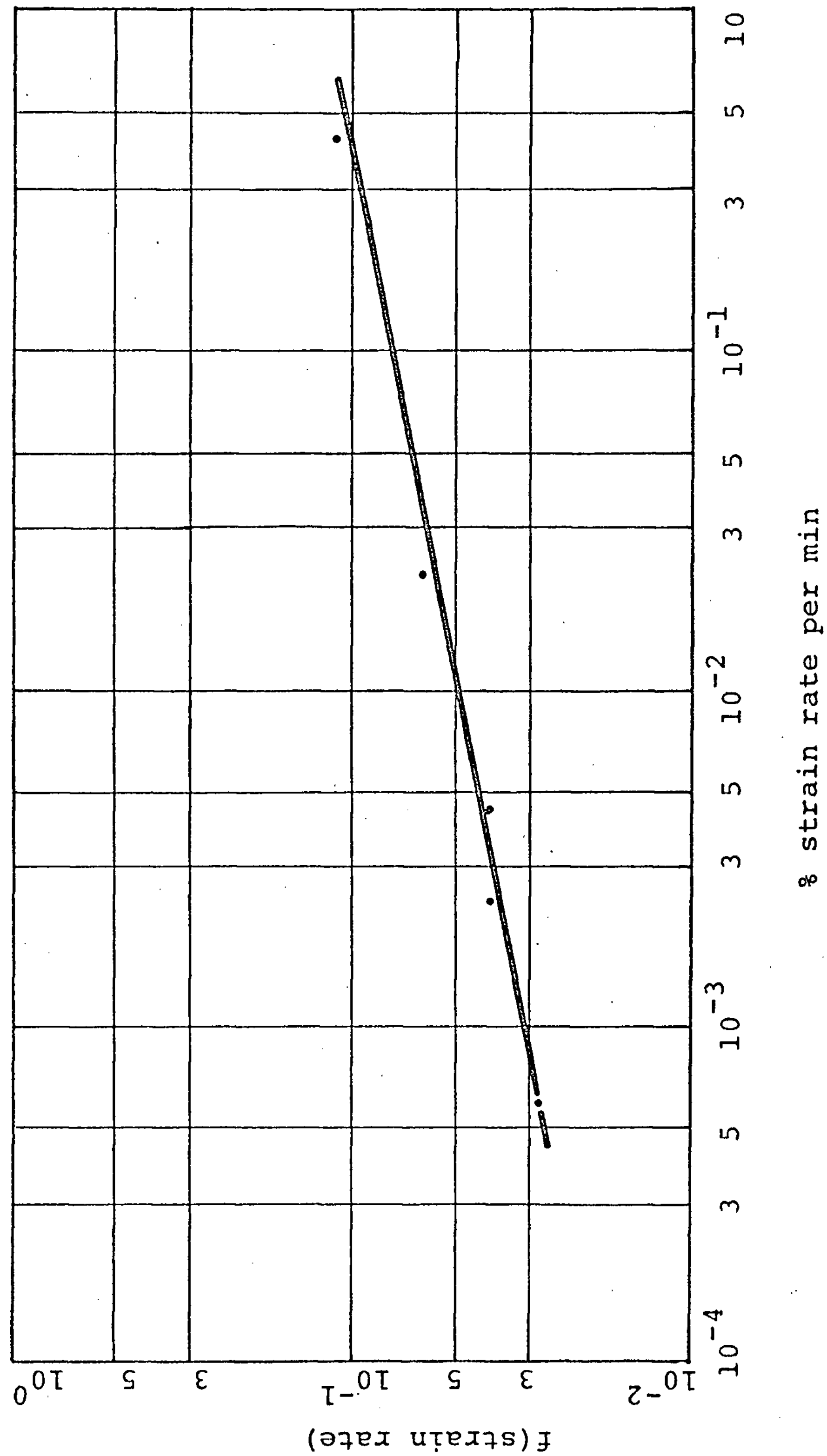

Figure 4.7 f(Strain Rate) vs Strain Rate Relationship at p = 0.8 kg/cm^2

where $\eta = \frac{\alpha_{av}}{(1 + e_1)^{\frac{1}{\beta}}}$

4.4 Void Ratio - Permeability Relationship

Using Equation (2.15) the coefficient of permeability, k, can be determined at any time for that void ratio.

Figure 4.8 shows void ratio vs log (permeability) plotted from different CRS tests. The points are essentially located on the same curve and show that the permeability of this soil is a function of the void ratio and independent of strain rates, as was assumed in Section 3.2. Hence,

$$k = F(e) \qquad \ldots (4.4)$$

4.5 Comparison of Predicted and Observed Behaviour of Incremental Loading Tests

A computer programme was developed to predict the behaviour of incremental tests using the CRS tests data. The programme was tested using the assumptions of Taylor's Theory B and the results were identical to the closed form solution obtained by Taylor.

Two incremental loading tests were performed to compare the observed and the predicted behaviour of samples. The two tests are described in Table 2.

Figures 4.9 and 4.10 show the time vs pore pressure

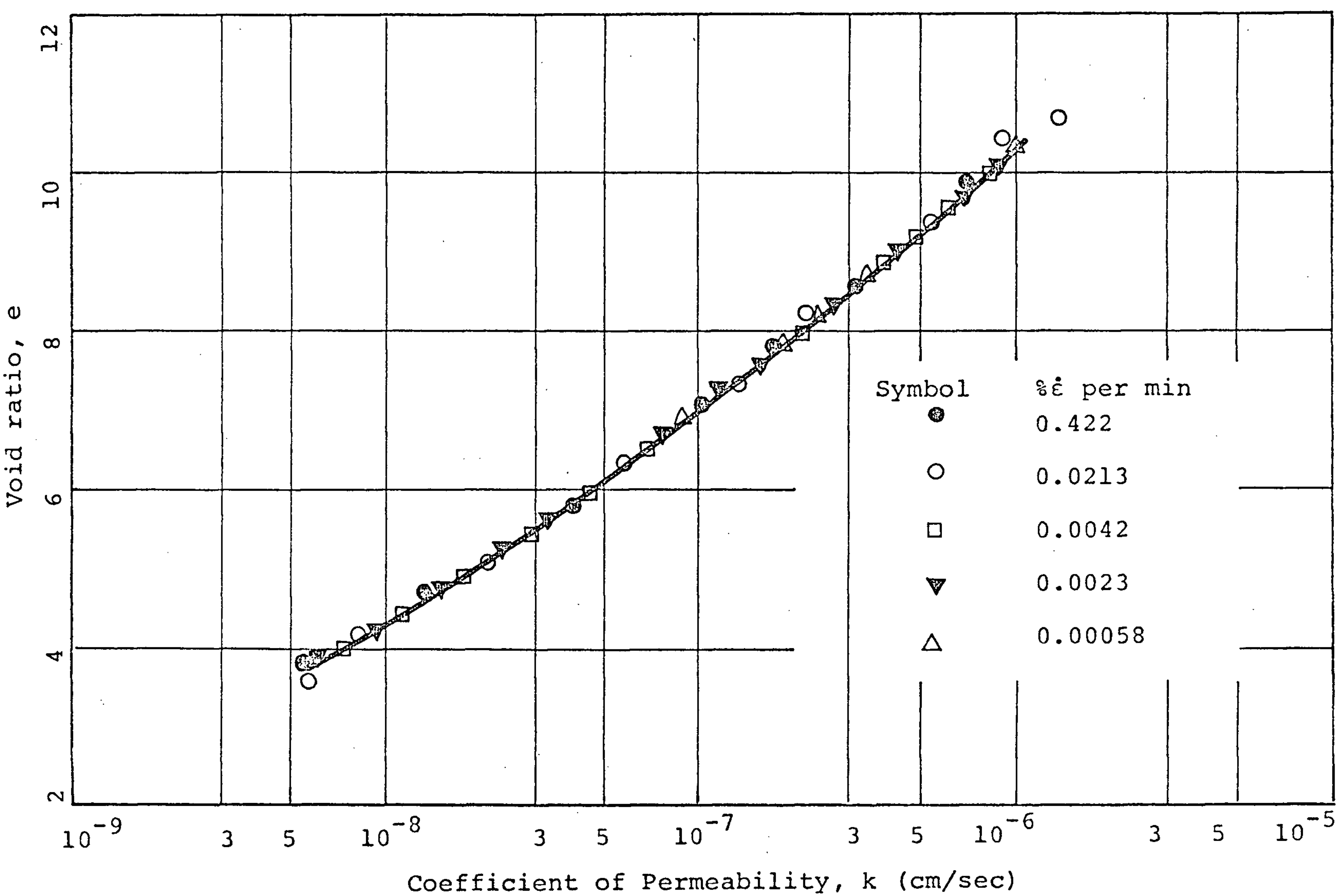

Figure 4.8 e - k - $\dot{\varepsilon}$ Relationship for Peat Tested

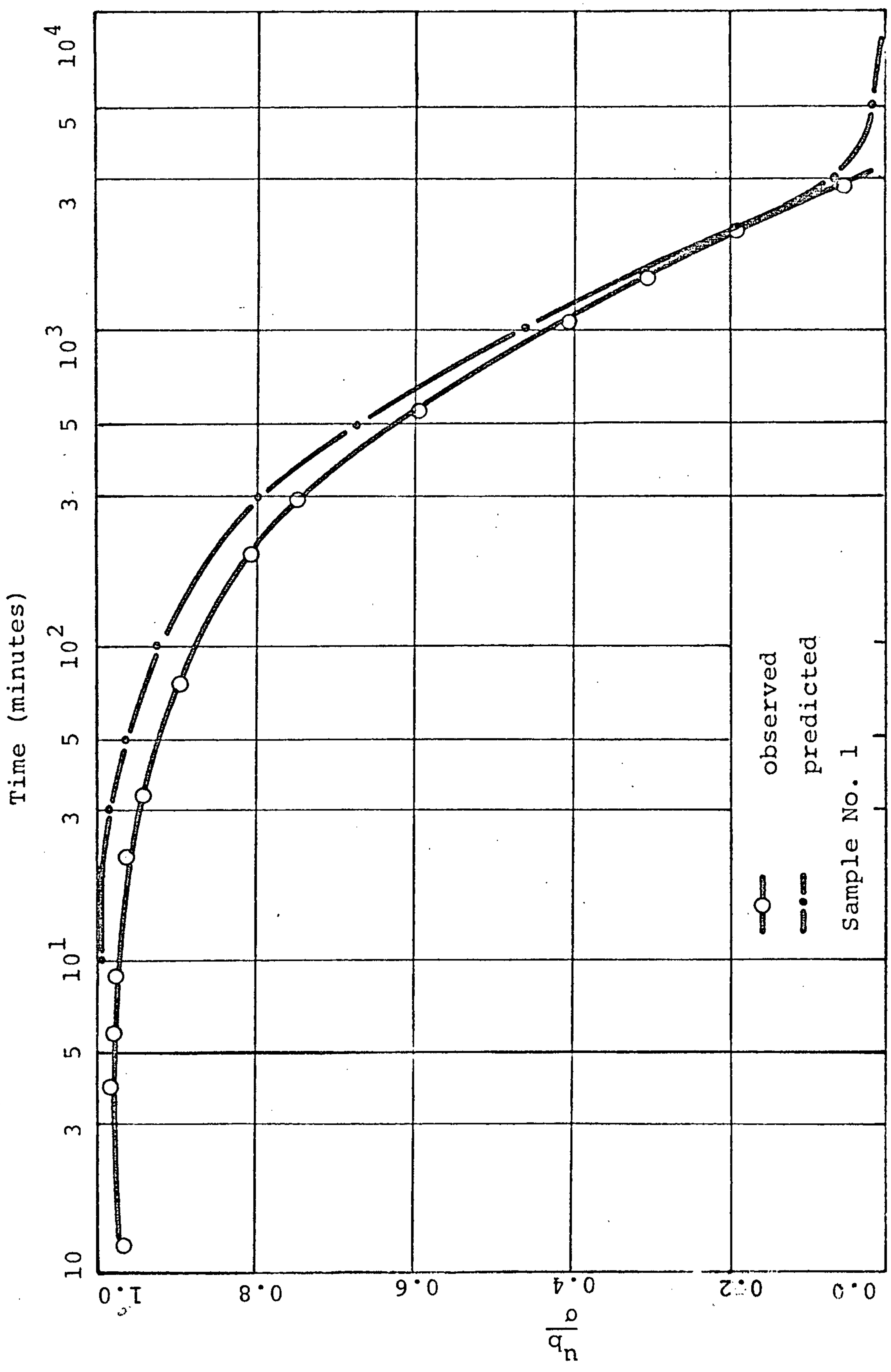

Figure 4.9 Comparison of Time - Pore Pressure Curves for Sample No. 1

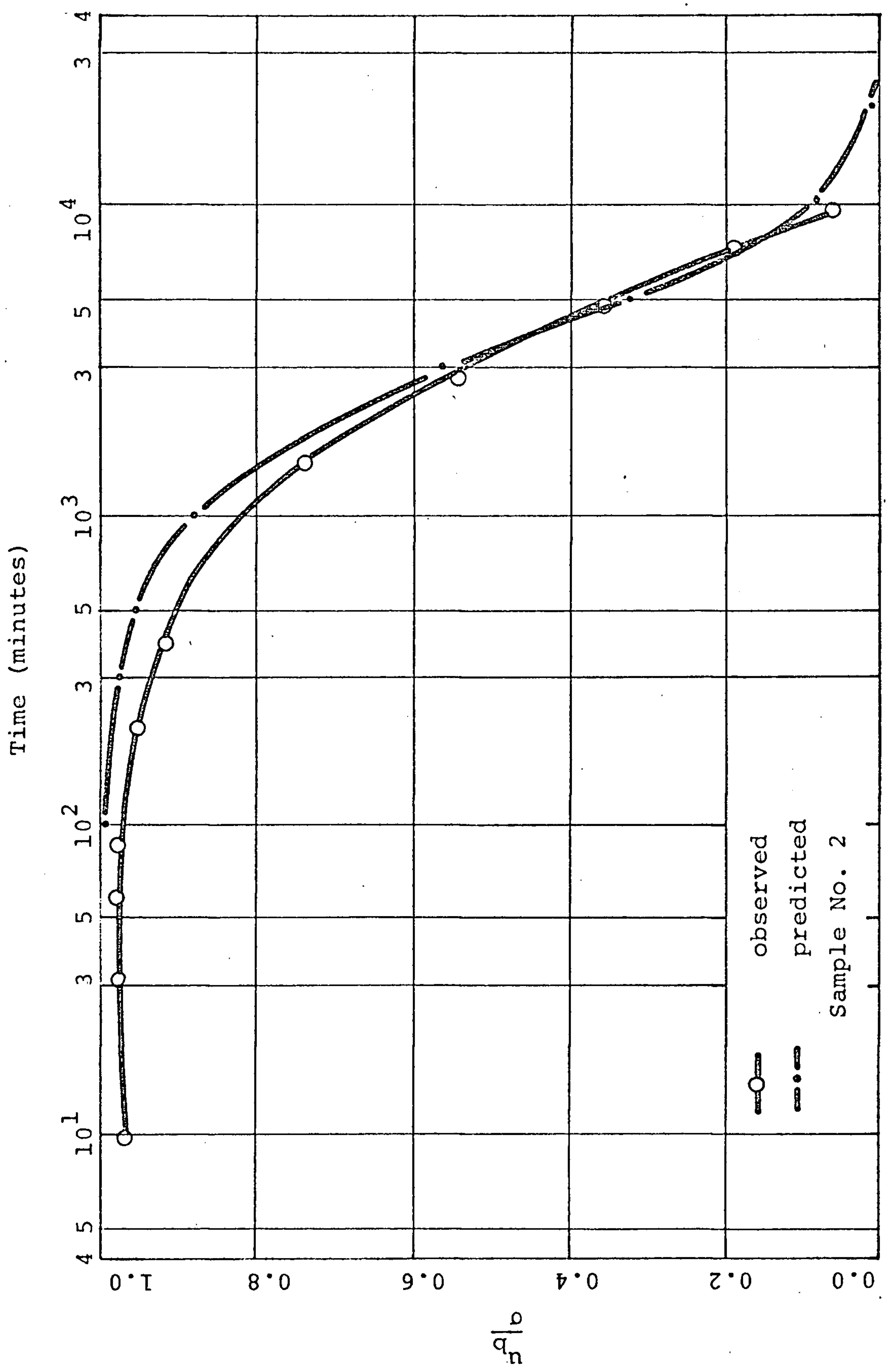

Figure 4.10 Comparison of Time - Pore Pressure Curves for Sample No. 2

comparison for the two tests. The predicted pore pressures are higher in the earlier and lower in the later part of the curve when compared with the observed pore pressures. This behaviour may be because of the use of an average permeability for the complete duration of the tests. If the variation in permeability is accounted for, as suggested in the desired solution procedure, Section 3.5, this discrepancy is likely to be smaller. However, the general agreement is close.

TABLE 2

DESCRIPTION OF TWO INCREMENTAL TESTS

Sample No.	Load Increment kg/cm^2	Duration Days	Initial Height cms	Diameter cms
1	0.1 - 0.2	21	8.70	20.98
2	0.4 - 0.8	13	2.65	7.5

The time - settlement curves are presented in Figures 4.11 and 4.12, the predicted and observed curves being almost identical.

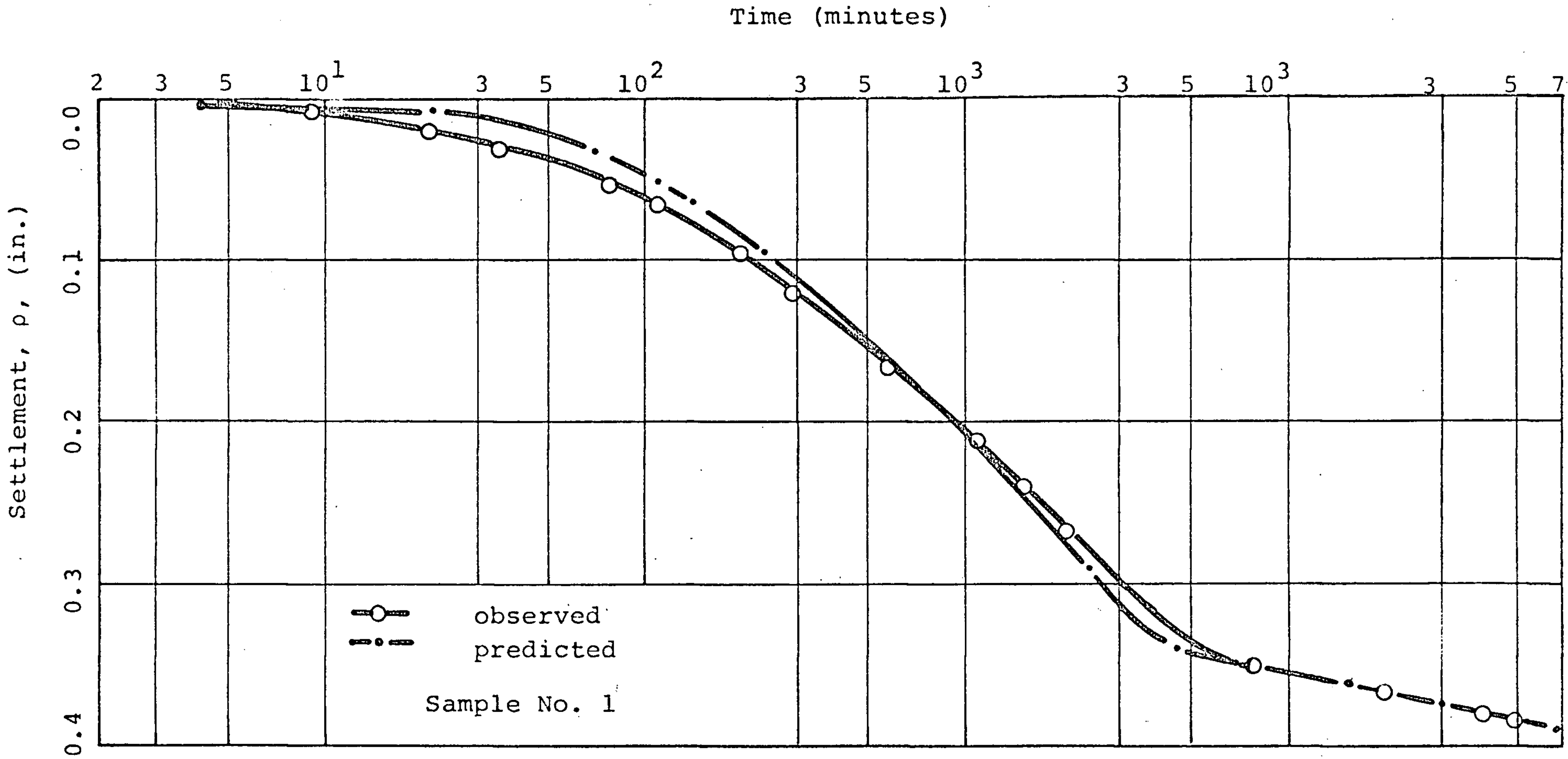

Figure 4.11 Comparison of Time - Settlement Curves for Sample No.1

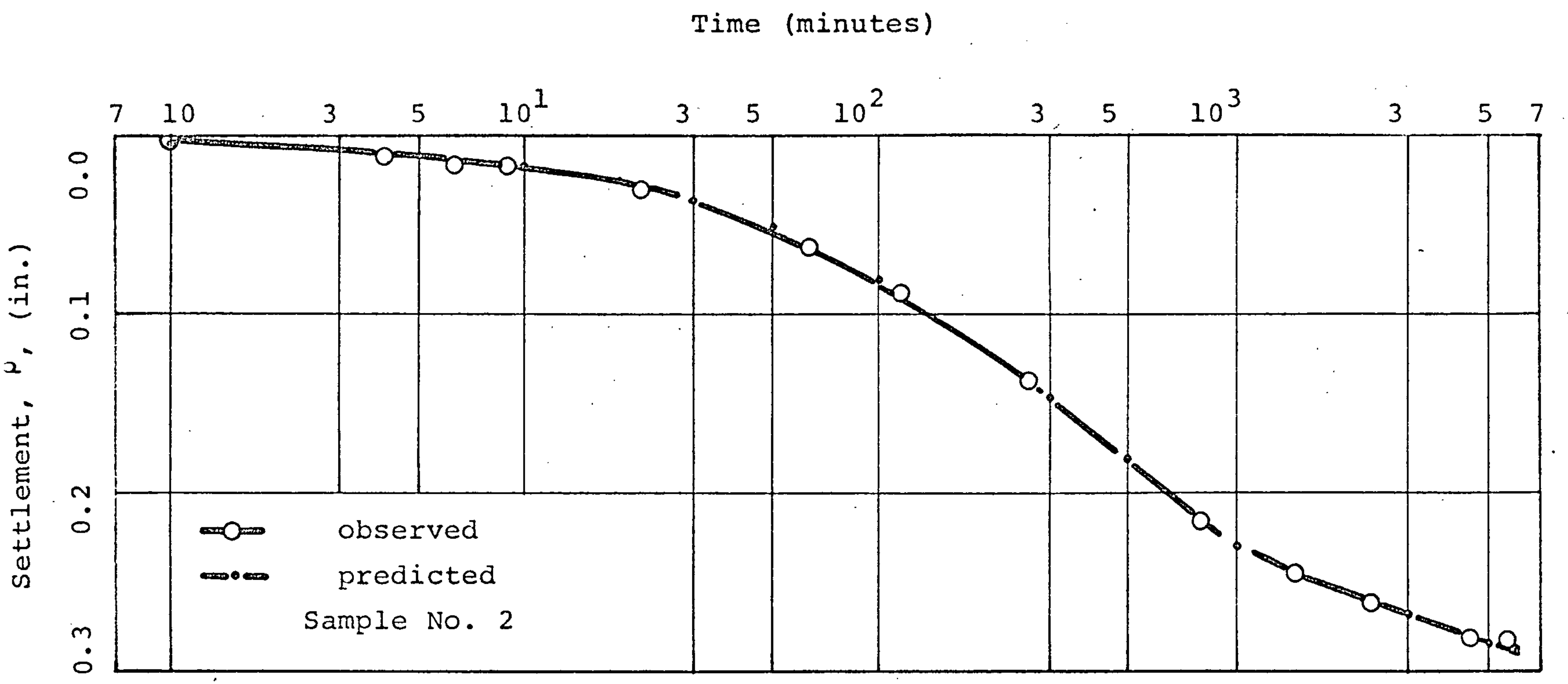

Figure 4.12 Comparison of Time - Settlement Curves for Sample No. 2

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

5.1 Summary

A number of one-dimensional constant rate of strain consolidation tests were performed on remolded peat to establish the e - p - $\dot{\varepsilon}$ and the e - k relationship for this soil. The void ratio - permeability relationship was found to be fairly linear on a semi log plot and independent of both the strain rate and the size of sample. The void ratio - effective stress - strain rate relationship was found to be independent of the size of sample and was of the form

$$e - e_2 = a\left(u + \eta\left(\frac{\partial e}{\partial t}\right)^{\frac{1}{\beta}}\right) \qquad \ldots (5.1)$$

η being the coefficient of non-linear strain rate effects on the void ratio - effective stress relationship and β being the index in the power law relating the strain rates to the void ratio.

The value of β found from CRS tests was 4.92 and was constant for all values of effective stress. To predict the behaviour of an incremental loading test under an increment of 0.4 - 0.8 kg/cm^2 the CRS tests gave the value of η equal to 0.1284 and it was found to vary directly with effective stress, p.

The behaviour of samples subjected to incremental loading was predicted by substituting the expression for the void ratio given in Equation (5.1) into the equation

$$C \cdot \frac{\partial^2 u}{\partial z^2} = \frac{\partial e}{\partial t} \qquad \ldots (5.2)$$

The resulting equation was solved by a numerical integration procedure, Poskitt (1967), and is described in Appendix 1.

The predicted and observed time settlement and time - pore pressure curves were compared for two incremental loading tests on different size of samples.

The predicted and the observed behaviour was found to be in close agreement.

5.2 Conclusions

It is concluded that the void ratio - effective stress - strain rate relationship for the peat tested can be established by running one-dimensional CRS tests over a range of strain rates. This relationship was found to be independent of the size of the sample. The void ratio - permeability relationship can also be obtained and for this soil was independent of both the strain rates and size of the sample.

The relationships obtained from CRS tests allow the prediction of the time - settlement and the time - pore pressure behaviour of the material under incremental loading conditions. These predictions include both 'primary' and 'secondary' settlement. The results suggest that the behaviour of soil under incremental loading conditions can be predicted from CRS tests data.

5.3 Suggestions for Further Research

It is suggested that the proposed method of prediction should be checked for some other types of tests, eg. constant rate of loading and controlled gradient tests.

The application of the method to predict the field behaviour of soil should be studied.

The soil used in the present investigation was a highly compressible organic material. It is suggested that the method should be applied to other soils which are not as compressible as peat.

It is further suggested that a numerical technique should be developed to obtain a stable solution of the "Desired Solution Procedure", Section 3.5, which does not include the approximations introduced in the "Adopted Solution Procedure", Section 3.6.

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APPENDIX 1

THE NUMERICAL SOLUTION OF THE GOVERNING CONSOLIDATION EQUATIONS

The equations to be solved are

$$C \cdot \frac{\partial^2 u}{\partial z^2} = \frac{\partial e}{\partial t} \qquad \ldots \text{(A.1)}$$

and

$$e - e_2 = a\left(u + \eta \left(\frac{\partial e}{\partial t}\right)^{\frac{1}{\beta}}\right) \qquad \ldots \text{(A.2)}$$

where

$C = \frac{k(1 + e_1)}{\gamma_\omega}$	parameter related to permeability
u	the excess pore pressure
z	the space variable
e	void ratio
t	time variable
e_2	final value of void ratio
a	coefficient of linear compressibility
$\eta = \frac{\alpha_{av}}{(1 + e_1)^{\frac{1}{\beta}}}$	coefficient of non-linear function of strain rate
β	index in power law relating strain rates to the function of strain rate
e_1	initial value of void ratio

Putting $\acute{e} = e - e_2$ and $\frac{\partial \acute{e}}{\partial t} = \frac{\partial e}{\partial t}$

$x = \frac{z}{H}$, where H is the length of drainage path

$\acute{u} = \frac{u}{\Delta\sigma}$

$\mu = \frac{(e_1 - e)}{(e_1 - e_2)}$, the degree of consolidation

$\lambda = 1 - \mu = \frac{\acute{e}}{a(\Delta\sigma)}$

Eliminating u from Equations (A.1) and (A.2)

$$C. \frac{\partial^2}{\partial(xH)^2} \left[\frac{\acute{e}}{a} - \eta \left(\frac{\partial\acute{e}}{\partial t}\right)^{\frac{1}{\beta}} \right] = \frac{\partial\acute{e}}{\partial t} \qquad \text{... (A.3)}$$

Dividing $\Delta\sigma$ and rearranging

$$\frac{\partial^2}{\partial x^2} \left[\lambda - \left(a.\eta^{\beta} (\Delta\sigma)^{1-\beta}. \frac{\partial\lambda}{\partial t}\right)^{\frac{1}{\beta}} \right] =$$

$$\frac{aH^2}{C} \frac{\partial\lambda}{\partial t} \qquad \text{... (A.4)}$$

Putting $\frac{\partial\lambda}{\partial T} = \frac{aH^2}{C} . \frac{\partial\lambda}{\partial t}$,

the dimensionless time factor governing the diffusion process is:

$$T = \frac{C}{a} . \frac{t}{H^2} \qquad \text{... (A.5)}$$

Putting $\left(a\eta^{\beta}(\Delta\sigma)^{1-\beta} \frac{\partial\lambda}{\partial t}\right)^{\frac{1}{\beta}} = \left(\frac{\partial\gamma}{\partial T_s}\right)^{\frac{1}{\beta}}$ ... (A.6)

the dimensionless time factor associated with the strain rate effects is T_s where:

$$T_s = \frac{t \; . \; (\Delta\sigma)^{\beta-1}}{a \; . \; \eta^{\beta}} \qquad \ldots \text{ (A.7)}$$

T_s is best expressed by means of the dimensionless ratio:

$$R = \frac{T}{T_s} = \frac{C \; . \; \eta^{\beta}}{H^2 \, (\Delta\sigma)^{\beta-1}} \qquad \ldots \text{ (A.8)}$$

Since Equation (A.4) is non-linear, Barden (1965) found it convenient to work with the following two simultaneous equations:

$$\frac{\partial\lambda}{\partial T} = \frac{\partial^2 \acute{u}}{\partial x^2} \qquad \ldots \text{ (A.9)}$$

$$\frac{\partial\lambda}{\partial T} = - \frac{1}{R} \; (\lambda - \acute{u})^{\beta} \qquad \ldots \text{ (A.10)}$$

Poskitt (1967), however, used the following formulation which is more stable and easier to apply.

From Equations (A.9) and (A.10), the following equation may be found

$$\lambda = \acute{u} + (-R \frac{\partial^2 \acute{u}}{\partial x^2}) \qquad \ldots \text{ (A.11)}$$

Substitution of Equation (A.11) in Equation (A.9) gives

$$\frac{\partial}{\partial T} \left[\acute{u} + (-R \frac{\partial^2 \acute{u}}{\partial x^2})^{\frac{1}{\beta}} \right] = \frac{\partial^2 \acute{u}}{\partial x^2} \qquad \ldots \text{ (A.12)}$$

or differentiating

$$\frac{\partial \acute{u}}{\partial T} - \frac{R^{\frac{1}{\beta}}}{\beta} \left[\left(\frac{\partial^2 \acute{u}}{\partial x^2}\right)^2 \right]^{\frac{(1-\beta)}{2\beta}} \frac{\partial^3 \acute{u}}{\partial x^2 \partial T} = \frac{\partial^2 \acute{u}}{\partial x^2} \qquad \ldots \text{(A.13)}$$

Boundary and Initial Conditions

Boundary Conditions:

$$\frac{\partial \acute{u}}{\partial x} = 0 \quad \text{at } x = 0 \quad \text{for any } T.$$

This implies no flow across the impermeable boundary, and

$$\acute{u} = 0 \quad \text{at } x = 1 \text{ and any } T$$

that is free drainage to the top of the sample.

Initial Condition:

The initial condition of the sample is that the void ratio is everywhere equal to e, so that

$$\lambda = 1 \text{ at } T = 0 \text{ and any } x.$$

Substituting this value into Equation (A.10), the initial excess pore water pressure distribution is given by

$$\frac{\partial^2 \acute{u}}{\partial x^2} = - \frac{1}{R} (1 - \acute{u})^{\beta} \qquad \ldots \text{(A.14)}$$

In the Terzaghi theory the initial excess pore water pressure is uniform and equal to the applied load, that is, $\acute{u}(x,0) = 1$. However, as pointed out by Taylor

(1942) and as shown by Equation (A.14), this is not quite so, due to the strain rate effects.

Equation (A.14) must be solved subject to the boundary conditions.

$$\frac{d\acute{u}}{dx} = 0 \text{ at } x = 0; \quad \acute{u} = 0 \text{ at } x = 1$$

As R approaches zero, however, the initial pore pressure distribution approaches the Terzaghi value, i.e., $\acute{u}(x,0) = 1$. Referring to Equation (A.13) and writing it in finite difference form for the ith point along the x - axis (see Figure A.1) and the jth point along the T - axis, then the solution may be forwarded in time steps ΔT by the formula

$$\acute{u}_{j+1} = \acute{u}_j + (A_j^{-1} P_j) \Delta T \qquad \ldots \text{(A.15)}$$

where

$$u = \begin{Bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_i \\ \cdot \\ \cdot \\ \cdot \\ u_N \end{Bmatrix} , \qquad P = \begin{Bmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ \cdot \\ P_i \\ \cdot \\ \cdot \\ \cdot \\ P_N \end{Bmatrix}$$

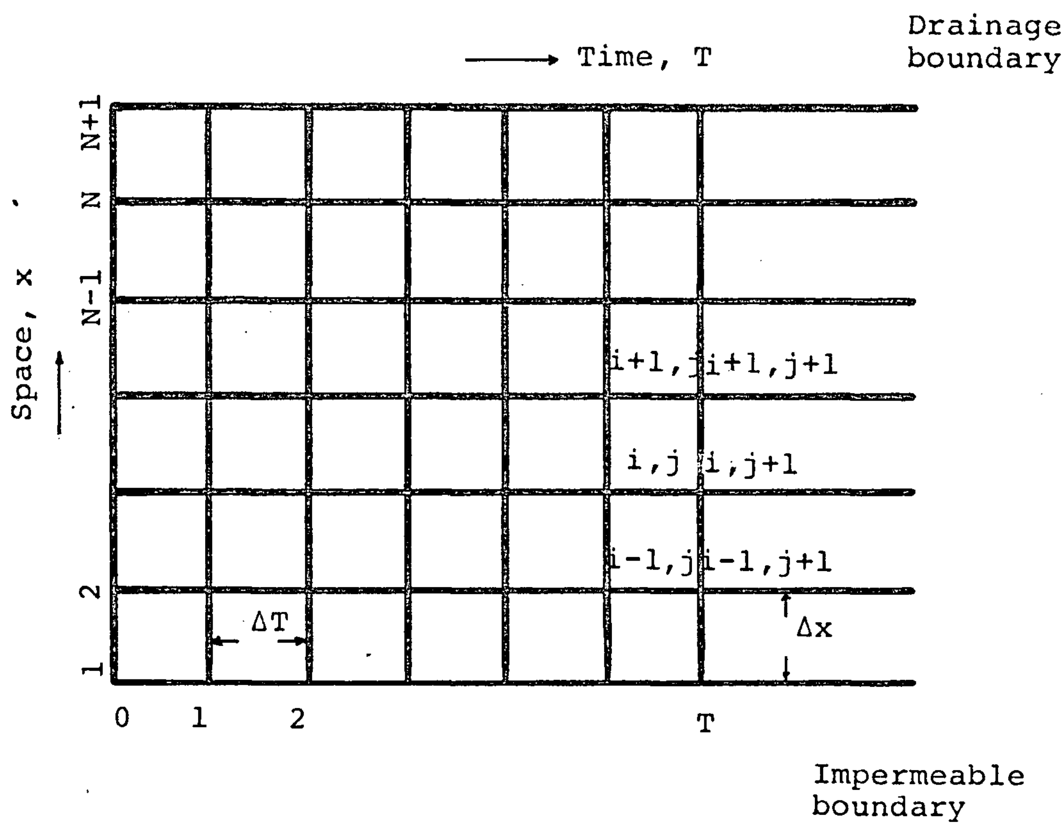

Figure A.1 Finite Difference Space - Time Grid

$$P_i = \frac{u_{i-1} - 2u_i + u_{i+1})}{(\Delta x)^2}$$

$$A = \begin{bmatrix} (1 + 2Q_1) & -2Q_1 & & & \\ -Q_2 & (1 + 2Q_2) & -Q_2 & & \\ & -Q_3 & \ddots & \ddots & \\ & & & -Q_N & (1 + 2Q_N \end{bmatrix}$$

and

$$Q_i = \frac{R^{\frac{1}{\beta}}}{\beta} \cdot \frac{[(P_i)^2]^{(\frac{1-\beta}{2\beta})}}{\beta \cdot (\Delta x)^2}$$

When solving the tridiagonal equations $(A_j^{-1} P_j)$ use was made of the subroutines available on the U.B.C. Computer library.

APPENDIX 2

ONE DIMENSIONAL CONSOLIDATION TESTS

A schematic diagram of the CRS testing set-up is shown in Figure A.2. Two power driven constant deformation rate machines were available with a wide range of speeds. To exclude the temperature effects, all tests were performed in a temperature controlled room which was kept constant at 20^{0}C (± 0.5^{0}C). To ensure accurate measurement of the base pore pressure, great care was taken to saturate the sample. The base porous stone was boiled for fifteen minutes to remove air and then cooled to room temperature. To reduce friction between the sample and the ring, a thin film of silicon lubricant was applied to the ring. The peat was poured in the consolidation ring. The base of the sample was sealed preventing drainage and the base pore pressures were measured. All samples were consolidated under a stress of 0.044 kg/cm^2 for 24 hours. A guiding rod was used to avoid tipping of the top porous stone due to the low strength associated with the high initial water content of the samples. This rod was replaced by a ball before the samples were subjected to the desired strain rate. For an incremental test, however, increments of load were added at desired intervals of time.

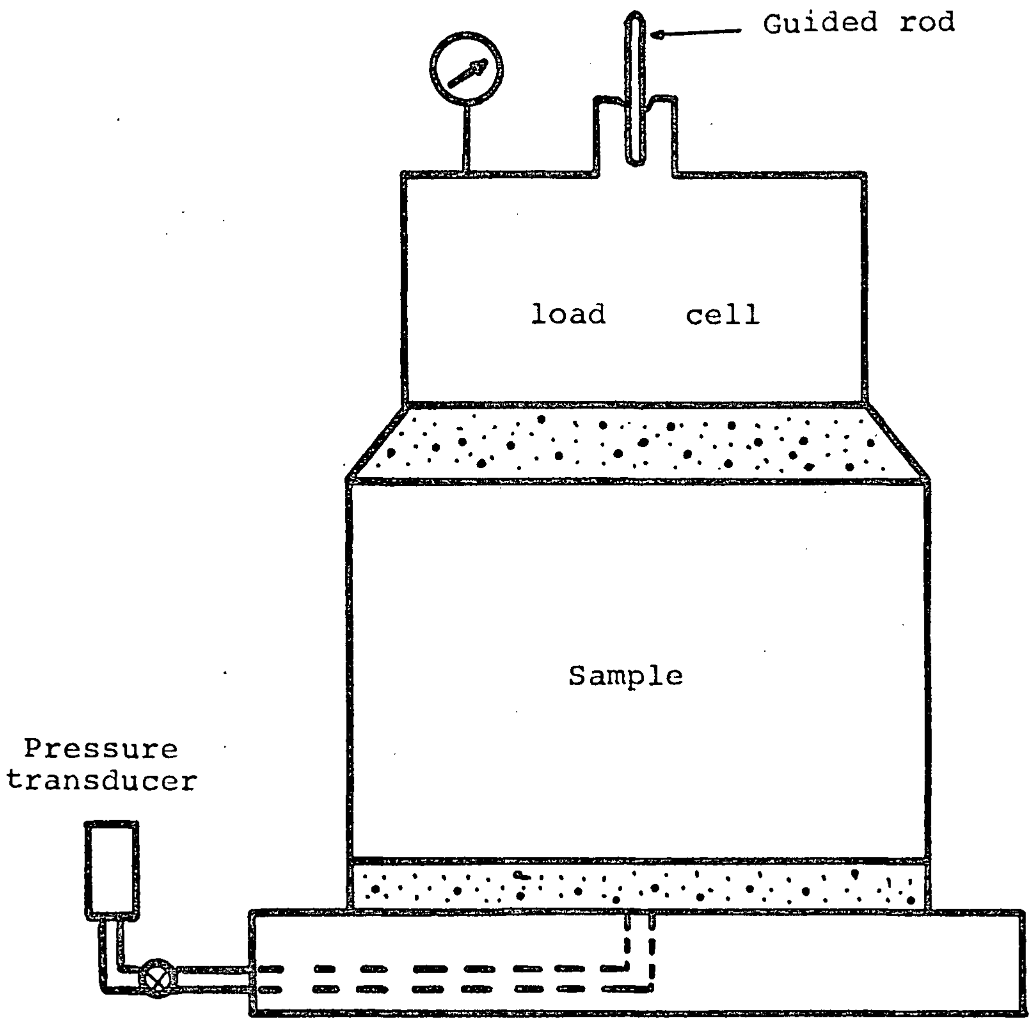

Figure A.2 Schematic Diagram of Testing Set-Up

Periodical readings of the vertical displacement base pore pressure and total load were recorded by vertical dial gauge, pore pressure transducer and a load cell respectively. Barometric readings were also taken to apply corrections to the pore pressure readings.