OPERATIONAL DECISION MAKING FOR A MULTI-PURPOSE RESERVOIR WITH TOTAL SEASONAL INFLOW FORECAST
by

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## A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF APPLIED SCIENCE

in the Department of
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We accept this thesis as conforming to the required standard

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## ABSTRACT

This study investigates the operational decision process for Okanagan Lake, a natural lake regulated by a dam at the outlet for flood control, irrigation and water supply purposes. In addition, the Lake supports a substantial tourist industry. The Lake is principally supplied by snowmelt and a forecast of total inflow volume during the critical runoff season is available to assist the operator.

The operational decision process was found to differ from the sequential decision basis of many Operations Research techniques and the absence of information on costs and benefits precluded the use of conventional optimization procedures. The importance of making the best use of the inflow forecast to achieve the operational goals was recognized and was used as the basis of the decision analysis developed.

The method developed assesses possible immediate operational decisions by evaluating the effectiveness of future discharges to correct for past decision errors. The evaluation is made in terms of the probabilities of exceeding operational constraints and of achieving operational goals. The method involves simulation of sets of monthly inflows for the remainder of the runoff season given an inflow volume forecast and knowledge of the probable accuracy of the forecast; computation of water levels which would occur with various operating procedures; frequency analysis of the resulting levels; interpretation of the frequencies as probabilities; and presentation of the resulting information describing the operational
situation in readily assimilable form.

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## CHAPTER I

## INTRODUCTION

Operational decision policies for water resources projects have been obtained by a number of methods, emphasis currently being placed on simulation and optimization techniques. Generally a decision policy is obtained by considering a time period covering a large number of successive decisions and treating all aspects in a deterministic fashion. The analysis and resulting policy are related to this theoretical context but do not consider the predicament faced by the operator at the time he makes an individual decision. Unless a "rigid rule" philosophy is being imposed on the operator, the decision sequence which will unfold in practice will consist of a series of short term commitments, each one representing the operator's best effort to resolve the predicament facing him at the time of making the commitment. Thus it follows that, in order to achieve the best decision sequence in actual practice, each individual decision must be the best under the prevailing circumstances. "Best" in this context need not necessarily imply the maximum economic return, but, because of the operator's hazy view of the future, a decision which will permit the greatest future corrective action may be regarded as the most desirable.

This study specifically considers the position of the operator when making an individual decision. The operational situation of an existing multi-purpose natural reservoir, Okanagan Lake, British Columbia, was
chosen as the basis of the study. A major factor in the operation of this Lake is that it is fed principally from snowmelt, with the result that the inflow volume can be predicted by direct measurement of the snowpack. This prediction is not precise as many other factors influence the amount of water which will ultimately reach the Lake. It does, however, provide valuable information on the probable future inflow volume during the run-off season. The importance of using this information to its best effect was recognized and was a major factor in prompting this study.

The operational requirements of Okanagan Lake involve providing adequate storage capacity prior to peak inflows to prevent flooding and ensuring that the Lake is full at the end of the run-off season to meet consumptive demands throughout the summer. With present consumptive demands the latter will also provide over-year storage and hence some protection against very low inflows in the following year. The extent to which these two requirements conflict is dependent upon many factors, one of which is the accuracy with which one can predict not only future total inflow volume but also the rates at which this inflow will occur. The available volume forecast provides no information on inflow rates and a major portion of this study is concerned with overcoming this deficiency.

It was anticipated that an existing Operations Research technique would form the basis of the decision evaluation. However, it was found that most of the available techniques required accurate and comprehensive data on the economic value of consumed water and costs associated with extreme Lake levels. This information was not available for Okanagan

Lake. Furthermore, because of the intangible nature of many of the benefits (and costs) associated with the operation of the Lake, it was apparent that this information was essentially unobtainable with any degree of accuracy. This precluded the application of many optimization/decision making methods and led to the development of the method described in this thesis.

The method proposed does not attempt to determine the optimal decision but combines historic inflow data, Lake operating constraints, and the forecast information, together with an estimate of the probable forecast accuracy, to provide an indication of the operational situation. This situation is described in terms of the probable consequences of various possible courses of action on Lake level with respect to limits and storage goals. The actual decision would then have to be made by the operator in the light of this situation information and his own experience and judgement.

The method is described in the context of the Okanagan Lake problem, but would require little modification for many typical multi-purpose reservoir situations. Chapter VIII discusses some possible extensions of the method.

GENERAL DESCRIPTION OF OKANAGAN LAKE AND ITS OPERATION

## Description

Okanagan Lake is located in a semi-arid region in the interior of the Province of British Columbia. It is fed principally by snowmelt from the surrounding hills with most runoff occurring during the late spring months.

The Lake has a surface area of 84,200 acres (131.5 sq. mls.) and it is controlled at its outlet at Penticton over a normal operating range of 4.0 ft . This provides a live storage volume of 337,000 acre ft. Discharge flow from the Lake is limited by the capacity of the channel downstream of the Penticton control structure. On occasions the discharge capacity must be further curtailed to avoid flooding at a downstream junction with the Similkameen River. The maximum discharge capacity was taken as 1800 c.f.s. which is equivalent to 108,000 acre $f t$. per month. Variations in available discharge capacity at certain times or under certain conditions were not considered in this study but could be readily accommodated by the method developed (see Chapter VIII).

Estimates of net monthly inflows into the Okanagan Lake have been determined from the net inflows to the Lake (computed from Lake elevation changes and outflows) plus an allowance for the volume of water intercepted and consumed before entry into the Lake. This infor-
mation [1] was provided for a period of 48 years by the British Columbia Water Resources Service, the British Columbia Department of Lands, Forests and Water Resources, Victoria. These computed net inflows were used as basic data in this study and intercepted volumes (in upstream storage reservoirs) were accounted for by including them in the demand volumes. The total annual net inflow varies between 96,000 acre ft. and 796,000 acre ft. with an average of 401,000 acre ft . The live storage is thus about 84 per cent of the average annual net inflow and 42 per cent of the maximum recorded inflow.

Approximately 90 per cent of the annual net inflow occurs during the April to July period with the highest inflow usually occurring in May. The average annual peak monthly inflow is 198,500 acre ft . and the highest recorded inflow for one month is 402,000 acre ft.

The computed net monthly inflows for 48 years are shown in Appendix 1 and monthly averages and standard deviations are given in Table VII.1. Evaporation losses from the Lake are large and may be as high as 50,000 acre ft. per month during the summer months. As net inflows are computed from Lake elevation changes, evaporation losses are automatically incorporated so that when evaporation losses exceed inflow in a particular month, then a negative inflow will be recorded for that month.

Okanagan Lake serves three major purposes:
(1) Flood control;
(2) Storage for irrigation requirements;
(3) Recreation and tourism.

Benefits from the Lake are secondary or indirect and also predominantly of
an intangible nature. Typically for a natural reservoir in an area which has been developing over a long period of time, there is little available quantitative data relating to these benefits and, consequently, methods for conversion of intangible benefits to monetary units could not be applied. Realistically, it could be assumed that information of this type is unobtainable unless inordinate effect and funds were directed towards its collection. This fact had a significant bearing on the effectiveness of the Operations Research techniques which were considered and on the method which was subsequently developed.

The total consumptive demand on the Okanagan Lake Basin, based upon 1966 figures [2] is 216,000 acre ft., a figure which allows for return flows to the Lake. This quantity meets the requirements of domestic water supply, irrigation, and minimum flows in the Okanagan River which carries the discharge from the Lake. Table II. 1 gives a breakdown of the annual total water demand and estimated demands for the months April, May, June and July. These monthly demands (and to some extent the annual demand) will vary from year to year depending on climatic conditions, etc., but it is assumed that a conservative estimate of immediate future seasonal demands will always be possible with reasonable accuracy.

TABLE II. 1

OKANAGAN LAKE WATER DEMANDS

|  |  |  | MONTHLY DEMANDS |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| USE | TOTAL DEMAND | APRIL | MAY | JUNE | JULY |
| Irrigation | 100,000 | Neglig. | 10,000 | 10,000 | 25,000 |
| Domestic and Waterworks | 8,000 | 1,000 | 1,000 | 1,000 | 1,000 |
| Minimum Flow Okanagan River | 108,000 | 8,000 | 8,000 | 8,000 | 8,000 |
| TOTAL | 216,000 | 9,000 | 19,000 | 34,000 | 34,000 |

## Lake Operation

The Lake level is controlled by operating the gates in the control dam at Penticton in response to discharge decisions made at intervals varying from one week to one month. The discharge decisions are based upon current inflow forecasts, inflows to date, downstream conditions on the Okanagan River, and past operational experience.

The major operational constraints are the maximum and minimum Lake levels, set at elevations 102.5 ft . and 98.5 ft . respectively, and
the maximum discharge capacity. Flood control requirements during the runoff period and storage requirements to meet the irrigation needs must be met by operation within these limits. Recreational and tourism requirements are met by maintaining the Lake between the same limits. Operation of the Lake is made difficult by the limited accuracy of the total volume forecast for the runoff season and the difficulty is compounded by the inability to predict the timing and rate of runoff.

## OKANAGAN LAKE INFLOW FORECASTING

Most of the inflow to Okanagan Lake originates from snowmelt and thus measurement of the snowpack within the Okanagan Lake Basin, which completely melts each year, provides a basis for forecasting the total inflow volume.

A variety of other factors influence the total inflow and major discrepancies can occur between estimates based solely on snowpack measurements and the inflow which subsequently occurs. Retention of runoff in the soil, groundwater recharge, evaporation, and precipitation during the runoff season are some of the factors involved. Investigations into the runoff process of the Okanagan Lake Basin are expected to provide better estimates of runoff contributing to groundwater recharge but factors such as precipitation and evaporation during the snowmelt season depend upon short term meteorological effects and are not predictable more than a few hours in advance.

Forecasting of the inflow rates during weekly or month1y periods, while highly desirable from an operational standpoint, is not attempted because these are also dependent on short term meteorological effects. Volume forecasts are based on prediction equations which are obtained by applying conventional multiple regression methods to historic records of snowpack measurements, antecedent precipitation, and subsequent Lake inflows.

Commencing with receipt of the first snowpack data in late February, a forecast of the total inflow which will occur by the end of July is made. As time elapses the forecast is revised for the April to July, May to July, and June to July periods. These forecasts, as would be expected, becoming progressively more accurate as the season advances. Although forecasts could be made commencing November or December of the previous year since some of the antecedent factors are known at that time, they would be of very low accuracy. The significance of early forecasts is considered in conjunction with the method developed.

A value of the Standard Error of Estimate for the forecast is given by the multiple regression analysis and is used as an indication of forecast accuracy in this study. The assumptions involved are discussed in Chapter V.of this thesis.

## CHAPTER IV

THE OPERATIONAL DECISION PROCESS: ANALYSIS AND ASSESSMENT OF A DECISION

## The Decision Process

Available decision analysis techniques in Operations Research concentrate on determining an optimal sequence of decisions in order to maximize explicitly defined returns while not violating any of the system constraints.

Stochastic optimization techniques, which recognize the unpredictability of future events, offer the most realistic simulation of operational decision situations but become highly involved when applied to real life situations. The return or objective function must still be explicitly stated but in this instance it is the expected return that is maximized (i.e., the sum of all possible returns, each multiplied by its probability of occurrence); thus introducing a further degree of artificiality to the analysis. Even after considerable simplification stochastic optimization techniques place extreme demands on computing facilities and the process of simplification may have to be carried out beyond the point where the results remain meaningful.

Deterministic optimization techniques rely upon the maximization of the return function over a long simulation run and reduce the computational load to a more manageable level. This approach introduces the perspective of a long range planner to operational decision making which may
not be appropriate. An optimal long run operating policy may conceivably allow occasional minor flooding in order to realize more than compensating benefits in future months or years. However, in reality, it is difficult to envisage an operator, with dubious knowledge of the future, actually permitting flooding to any degree if it is within his power to prevent it. As stated previously, the lack of suitable information on economic returns from operation of the Lake precludes developing an objective function and further disqualifies the above techniques from application to this particular problem.

Since standard techniques appeared to be inapplicable, an examination of the Okanagan Lake operational decision situation was made in order to define the actual decision process involved and seek a basis for its analysis. Although operational decisions may be made at intervals from one week to one month it was found desirable for study purposes to base the analysis on a monthly decision period. This coincides with the recorded monthly inflow data and the monthly updating of the inflow forecast for the Lake.

At the time of making a discharge decision for the forthcoming month, the operator has exact information on the present Lake level and its position with respect to the upper and lower level limits. The maximum discharge capacity at his disposal will also be known together with the storage goal at the end of July. His view of the future is confined to the current updated total inflow forecast. In the absence of forecasts or foresight extending a number of years into the future, the operator cannot determine a strategy which will result in an optimal return over a period
beyond the immediate summer ahead. Achievement of full storage at the end of the current period, provided this is accomplished without flooding, will however be synonymous with a long term optimal operation. On this basis it was concluded that an operational decision method which enabled the lake to be brought to a full storage condition in July each year, or at least maximized the probability of achieving this goal, would meet the long term optimization objective. (It is not suggested that this will necessarily be the general case in all reservoirs, but it specifically applies to Okanagan Lake).

A perfect forecast of the total inflow and inflow rates will always enable the operator to achieve the desired storage level without flooding provided the inflow in any month does not exceed the available storage volume plus maximum discharge in one month. In the practical situation, with imperfect volume forecast and no inflow rate forecast, the ability of the operator to achieve these goals diminishes and conflicts arise in meeting the operational requirements. It is apparent, though, that the best possible use of the available forecast information is essential to determining the best operating decision. Significantly, little work has been carried out on decision methods incorporating forecast information except perhaps in Game Theory which confines itself mainly to single decisions and simply stated returns.

The factors which determine if the operational goals can be met during the runoff season, and at the same time determine a "correct" operational decision are:
(a) past decisions;
(b) actual inflows to the present time;
(c) lake level at the present time;
(d) future monthly inflows;
(e) future decisions and decision errors.

Both (a) and (b) are embodied in the Lake level at the time of making the new decision and these factors can be reduced to (c) alone without loss of relevant information.

The only information available on future inflows is contained in the current total inflow forecast, but this does not provide a direct estimate of inflow by months. This prevents estimation of what future decisions might be and consequently provides no basis for the simulation of a future decision sequence. The remaining "dynamic" factors which are known and upon which an operational decision must be based are now:
(a) present lake level;
(b) current total inflow forecast;
together with the constraint and goal factors:
(a) upper and lower limits of Lake level;
(b) maximum discharge capacity;
(c) desired terminal level at the end of July.

The analysis of the operational decision process to this point reveals that it consists of a sequence of virtually self-contained single decision problems. This contrasts with the strongly interdependent decision sequence usually found in dynamic decision models. Furthermore, the criteria for the "best" decision must now be defined in a context which is somewhat different
from the conventional decision sequence analysis. As the runoff season advances the operator acquires more and more accurate information on the future inflow situation and this enables him to make improved corrections to the Lake level. At the same time his capacity to make these corrections is progressively reduced by the reduction in time available for the corrective discharges. The operator must therefore attempt to make current decisions which will ensure that he will be able to achieve the operational requirements without requiring corrective discharges in excess of those at his disposal at any time during the runoff season. In other words, the "best" current decision will maximize the effectiveness of future corrective capacity. This is the basis of assessment adopted in this study, but before describing the method developed, the terms corrective action and corrective capacity must be defined.

Corrective action would be a low or minimum discharge following an error of too large a discharge, and a high or maximum discharge following an error of too small a discharge.

Corrective capacity for a period extending one month beyond the current decision month would be the range of volumes, from minimum to maximum, which could be discharged during the second month. For a period extending two months beyond the current decision month, the corrective capacity becomes greater. Maximum and minimum discharges can now be sustained for two months following the current month. It
should be noted that the correction is being applied solely to the potential error in the current decision, this being the only commitment under consideration.

As the period is lengthened the corrective capacity increases but at the same time the inflow volume for the period will also increase and will, to some extent, offset the corrective capacity.

While the above is a convenient way to develop the concept of corrective capacity, in reality, the period, which ends at a fixed point in time (in the case of this study, 31 July), will progressively shorten and the corrective capacity reduces. For purposes of comparison of various decisions it is necessary to obtain some quantitative measure of the effectiveness of the corrective capacity which may be subsequently applied. This is accomplished by simulating the Lake level response with a mathematical model and subjecting this model to probable seasonal inflow situations, a current discharge decision, and maximum appropriate corrective discharges. The resulting Lake levels (maximum, minimum, etc., from each simulation) can then be represented in the form of a histogram, a single point on the histogram indicating the probability of a certain Lake level being exceeded. The histogram provides the required quantitative measure of effectiveness of the correctivesapacity.

Decision Analysis

The various steps in the method of decision analysis are outlined below.

1. A decision at a particular point in time is considered and is considered to be firm for a period of one month.
2. The full range of possible current discharge decisions is assessed in conjunction with only extreme (and therefore known) corrective discharges in the future. Corrective action will not however be the same for avoiding violation of the upper and lower level. constraints and it will be necessary to separately analyze each decision with respect to maximum and minimum Lake levels, and terminal levels.

- maximum corrective action associated with the peak or maximum level criterion will be a full discharge during the remaining months in the runoff period (following the decision month);
- the maximum corrective action associated with the minimum level criterion will be a minimum (i.e., zero voluntary) discharge during the remaining months;
- corrective action for the terminal level criterion will be the same as for minimum level.

3. The current value of the total future inflow forecast and an estimate of its accuracy are used as a basis for generating a synthetic population of "actual" future inflows which might occur following this forecast; in turn these "actual" inflows are used as a basis for the generation of sets of synthetic monthly inflows, each set representing one runoff season. The generation procedure adopted is described in the following chapter.
4. Maximum, minimum and terminal levels are computed from the Lake level response model for each set of monthly inflows, a range of current discharge decisions, and the appropriate corrective actions. A large number of simulations are run and a frequency analysis made of the resulting maximum, minimum and terminal levels for each current discharge decision. In practice it was found only necessary to test the largest and smallest possible current decisions and one or two intermediate decisions for calibration purposes.
5. The frequency results obtained are interpreted as probabilities of exceeding, or falling below, various levels and are presented in the form of cumulative probability curves for each current decision analyzed. When plotted on normal probability graph paper, the cumulative probability data closely approximates straight lines. Figure 4.1 shows a typical set of probability lines for a single possible current discharge decision $D_{1}$. Critical Lake levels are superimposed to aid in the interpretation. If the discharge decision $D_{1}$ were taken in the current month, and maximum appropriate corrective action was taken in subsequent months, the probabilities that the upper and lower Lake level limits would be exceeded are indicated by points (a) and (b) in Figure 4.1. Similarly, point (c) indicates the probability that the desired terminal level will be achieved.

The comparison of the two decision alternatives is shown in Figure 4.2. In this case decision $D_{2}$ is the larger discharge decision and will result in reduced probability of exceeding the upper level limit but will also reduce the probability of achieving the desired terminal level. At the same time the probability of falling below the lower limit is reduced. If $D_{1}$ and $D_{2}$ represented the minimum and maximum discharge decision possibilities then all intermediate decision lines will fall between these pairs of probability lines.

The reservoir operator may now assess the extent to which a present decision will restrict the influence of future corrective action on achieving the primary operational requirements and goals in probabilistic terms.

The decision probability lines also provide additional information. The slope of the lines is an indication of the variance of the possible outcomes. As the accuracy of the forecast improves this variance is reduced and the slope of the lines is reduced. A horizontal line represents a forecast with zero error. The horizontal separation of a pair of decision lines is an indication of the effect which the variation in discharge for the current decision month can have on future consequences.


Figure $4 \cdot 1$


Figure $4 \cdot 2$

COMPARISON OF DECISION ALTERNATIVES.

## CHAPTER V

gENERATION AND SIMULATION METHODS

There are no universal generating techniques available for producing appropriate synthetic hydrologic events following a forecast. The outcomes following a forecast and their probabilities are dependent upon the nature of the quantity being forecasted, the type of predictive model, measurement errors in the model variables, and numerous other factors. The generation method developed for the purposes of this study is considered satisfactory. It could provide the basis for a more precise method but a more rigorous approach was beyond the scope of this study and in any event not warranted by the accuracy of the original data.

## Monthly Inflow Generation

In order to assess an operational decision it is subjected to a large number of seasonal sets of monthly inflows which might occur following a forecast of given accuracy. These are generated from the forecast information in two stages.

Generation of a synthetic "actual" total inflow. The forecasting method for Okanagan Lake is currently based on a predictive equation obtained by multiple linear regression [3]. This yields a total inflow prediction and a value of the Standard Error of Estimate. It is assumed at this point that the predictive model is of the correct form and that the forecast error is normally distributed with mean zero and standard devia-
tion equal to the standard error of estimate [6]. These assumptions are difficult to justify in the absence of a long historic record of the forecast performance but are the best which can be made with available information. Since there will undoubtedly be significant measurement error and interdependency in the "independent" variables of the multiple linear regression, there is no readily available technique to establish the true nature and probable magnitude of the forecast error.

On the basis of these assumptions a series of synthetic "actual" inflows which might follow a forecast of given value and standard error of estimate are generated from the following equation:

$$
\begin{equation*}
T_{i}=T_{F}+s_{F} \cdot t_{i} \tag{5.1}
\end{equation*}
$$

where

$$
\begin{aligned}
\mathrm{T}_{\mathrm{i}}= & \text { an "actual" total inflow and } i=1, \dot{n} \text { is the total number of "actual" inflows to be }
\end{aligned} \begin{aligned}
& \text { generated. }
\end{aligned}
$$

Both the forecast total inflow and the synthetic "actual" total inflows are for the period commencing in the current decision month and terminating at the end of the runoff period.

Generation of a Set of Monthly Inflows From a Synthetic "Actual" Total Inflow. The generation procedure adopted is similar to that origi-
nally proposed by Thomas and Fiering [4], but the simple serial correlation component is replaced by a simple linear regression component relating the inflow of a particular month to the total inflow from the beginning of that month through to the end of the runoff period.

The Thomas and Fiering generation equation can be stated in general terms as:

$$
\begin{equation*}
y_{s}=\bar{Y}+b(x-\bar{X})+\operatorname{s.t}\left(1-R^{2}\right) \tag{5.3}
\end{equation*}
$$

together with the simple linear regression relationship:

$$
\begin{equation*}
\mathrm{y}=\overline{\mathrm{Y}}+\mathrm{b}(\mathrm{x}-\overline{\mathrm{X}}) \tag{5.4}
\end{equation*}
$$

based upon historic values of the dependent variable $y$ and independent variable $x$, where

$$
\begin{aligned}
& y_{S}=a \text { synthetic value of } y \\
& \bar{Y}=\text { mean of the recorded } y \\
& b=\text { regression coefficient } \\
& R=\text { correlation coefficient from the simple linear regression } \\
& \bar{X}=\text { mean of the recorded } x \text { values } \\
& s=\text { standard deviation of the recorded } y \text { values } \\
& t=a \text { unit random normal deviate }
\end{aligned}
$$

Regardless of the quantities represented by $x$ and $y$, it can be shown that the above generation equation will produce a series of synthetic $y$ values with the same mean and variance as the recorded y .

From Equation (5.3)

$$
E\left(y_{s}\right)=E(\bar{Y})+E[b(x-\bar{X})]+E\left[\operatorname{s.t}\left(1-R^{2}\right)^{1 / 2}\right]
$$

where $E()$ is the expected value.
But $\overline{\mathrm{Y}}, \mathrm{b}, \mathrm{s}$, and R are all constants; also:

$$
\begin{aligned}
& E(x-\bar{X})=0 \\
& E(t)=0
\end{aligned}
$$

therefore

$$
E\left(y_{s}\right)=\bar{Y}
$$

Equation (5.3) is a linear combination of the two independent random variables $x$ and $t[\operatorname{cov}(x, t)=0]$, hence:

$$
V\left(y_{s}\right)=b^{2} V(x-\bar{x})+s^{2}\left(1-R^{2}\right) \bar{V}(t)
$$

where $V()$ is the variance.
But $t$ is a unit random normal deviate; therefore $V(t)=1.0$

$$
\begin{equation*}
V\left(y_{s}\right)=b^{2} V(x-\bar{x})+s^{2}\left(1-R^{2}\right) \tag{5.5}
\end{equation*}
$$

Let $S_{x}=$ corrected sum of squares of recorded $x$ values. Then:

$$
\mathrm{V}(\mathrm{x}-\overline{\mathrm{X}})=\frac{\mathrm{SS}_{\mathrm{x}}}{\mathrm{n}}
$$

where $n=$ number of recorded values.
Let $S S_{y}=$ corrected sum of squares of recorded $y$ values. Then:

$$
\mathrm{s}^{2}=\frac{\mathrm{SS}}{\mathrm{n}} \mathrm{y}
$$

From simple linear regression theory:

$$
R^{2}=\frac{b^{2} S_{x}}{S S_{y}}
$$

Therefore, from Equation (5.5),

$$
\begin{array}{rlrl}
V\left(y_{S}\right) & =b^{2} \frac{S S_{x}}{n}+\frac{S S}{n} & \left(1-b \frac{2}{S S} \frac{x}{S S}\right) \\
& =\frac{y}{n} & \\
& =s^{2} & \begin{array}{l}
\text { i.e., the variance of the recorded } \\
y
\end{array} & \begin{array}{l}
y_{s}=s .
\end{array}
\end{array}
$$

Thus the mean and standard deviation of the historic $y$ values will be preserved in a generation of the form of Equation (5.3) [5] provided that the standard deviation of the independent (x) variable in the generation is the same as the standard deviation of the historic values of $x$ used in obtaining the regression and correlation coefficients.

Thomas and Fiering used Equation (5.3) to produce a series of consecutive synthetic monthly values. The independent variable of the regression relationship Equation (5.4) was the previous month's historic inflow and in the generation was the previous month's synthetic inflow. The cylical nature of the continuous generation provides the requisite
historic standard deviation in the independent variable of the generation equation and hence reproduces the historic standard deviation in the dependent or synthesized variable.

This study required realistic sets of synthetic monthly inflows with the sum of these inflows specified by an 'actual' total. The form of the monthly inflow generation equation based upon total "actual" inflow is:

$$
\begin{equation*}
I_{j}^{\prime}=\bar{I}_{j}+b_{j k}\left[\left(T_{a}-\sum_{n=1}^{j-1} I_{n}^{\prime}\right)-\bar{T}_{j k}\right]+s_{j} \cdot t\left(1-R_{j k}^{2}\right)^{\frac{1}{2}} \tag{5.7}
\end{equation*}
$$

Subscript i indicates the current decision month; $k$ indicates the last month of the runoff period; $j=1,$. . ., k

$$
\begin{aligned}
& I_{j}^{\prime}=\text { synthetic inflow for the } j \text { th month } \\
& \overline{\mathrm{I}}_{\mathrm{j}}=\text { mean historic inflow for the } j \text { th month }
\end{aligned}
$$

$$
\begin{aligned}
& R_{j k}=\underset{\text { correlation coefficient }}{ } \quad \mathrm{j} \text { th month to total inflow } \\
& T_{a}=\text { total "actual" inflow from ith to } k \text { th month } \\
& \overline{\mathrm{T}}_{\mathrm{jk}}=\text { mean historic total inflow from } j \text { th to } k \text { th month } \\
& s_{j}=\text { historic standard deviation of } j \text { th month } \\
& \text { t = unit random normal deviate }
\end{aligned}
$$

A value for $\mathrm{T}_{\mathrm{a}}$ is obtained from Equation (5.1). The constraint that the sum of the generated monthly inflows should be equal to $\mathrm{T}_{\mathrm{a}}$ is
met as the regression and correlation coefficients for the last (kth) month, $b_{k k}$ and $R_{k k}$, are both 1.0. Thus for the last month the generation equation becomes:

$$
I_{k}^{\prime}=\bar{I}_{k}+\left(T_{a}-\sum_{n=i}^{k-1} I_{n}^{\prime}\right)-\bar{T}_{k k}
$$

But the mean historic total inflow from the kth to the kth month $\overline{\mathrm{T}}_{\mathrm{kk}}$ is equal to the mean historic inflow for the $k$ th month $\overline{\mathrm{I}}_{\mathrm{k}}$; therefore:

$$
I_{k}^{\prime}=T_{a}-\sum_{n=i}^{k-1} I_{n}^{\prime}
$$

Hence:

$$
T_{a}=\sum_{n=1}^{k-1} I_{n}^{\prime}+I_{k}^{\prime}
$$

$$
=\sum_{n=i}^{k} I_{n}^{\prime} \quad \text { i.e., the sum of the month1y inflows. }
$$

A requirement of the generation based on total inflow is that the monthly inflows generated over a specified period from the complete set of historic inflows for the same period should have the same mean and standard deviation as the historic monthly values. Equation (5.5) can be used to demonstrate that this is always the case for the first month of the period being generated:

$$
V\left(y_{s 1}\right)=b_{1}^{2} V(x-\bar{x})+s_{1}^{2}\left(1-R_{1}^{2}\right)
$$

can be rewritten:

$$
V\left(y_{s 1}\right)=b_{1}^{2} V(x)+\left(1-R_{1}^{2}\right) V\left(Y_{1}\right)
$$

but from simple linear regression theory:

$$
R^{2}=\frac{b^{2} S S_{x}}{S S_{y}}=\frac{b^{2} V(X)}{V(Y)}
$$

therefore:

$$
V\left(y_{s 1}\right)=V\left(Y_{1}\right)+b_{1}^{2}[V(x)-V(X)]
$$

When $V(x)=V(X)$, i.e., when the variance of the independent variable of the generation is equal to its historic variance, then the second term goes to zero and:

$$
V\left(y_{s 1}\right)=V\left(Y_{1}\right)
$$

hence, the standard deviations of $y_{s 1}$ and $Y_{1}$ are the same.
The expression for the second month's variance reveals that interdependence exists between successive monthly generations.

$$
V\left(y_{s 2}\right)=V\left(Y_{2}\right)+b_{2}^{2}\left[V\left(x-y_{s 1}\right)-V\left(X-Y_{1}\right)\right]
$$

and

$$
V\left(y_{s 3}\right)=V\left(Y_{3}\right)+b_{3}^{2}\left[V\left(x-y_{s 2}\right)-V\left(X-Y_{1}-Y_{2}\right)\right] \text {, etc. }
$$

As a result of the dependency of the $x$ and $y_{s 1}, y_{s 2}$, . . . etc. expansion of the variance terms will introduce covariances. While it is possible to express these covariances in terms of the various coefficients and statistical parameters for the generated variables these would have to be equated with values of covariance for the historic variables $X, Y_{1}, Y_{2}$, etc. As an alternative to this involved procedure a series of generation runs on the computer for various periods based upon the historic period totals confirmed that the historic mean and standard deviation were being maintained in the generated monthly values.

In the proposed application of the generation equation, the mean and standard deviation of the total inflows would be determined by the forecast rather than the historic data. The effect of this change on the mean and variance of the generated values will be:

$$
\begin{aligned}
E\left(y_{s}\right) & =\bar{Y}+b \cdot E(x-\bar{X}) \\
& =\bar{Y}+b(\bar{x}-\bar{X})
\end{aligned}
$$

and

$$
V\left(y_{s}\right)=V(Y)+b^{2}[V(x)-V(X)]
$$

Thus the mean will be shifted by an amount which is dependent upon the deviation of the forecast total inflow value from the historic mean and the linear dependency of the monthly inflow on the total inflow. Similarly, the variance will be reduced by an amount dependent on the reduction in variance of the total inflows following the forecast from the historic variance and the linear dependency of the monthly inflow on total inflow.

## Simulation of Lake Level Response

A simple continuity model was used to determine the effect of inflows, compulsory demands, and discharge decisions, on the Lake leve1.

$$
E_{j 1}=E_{j 0}+\frac{1}{A}\left(I_{j}^{\prime}-C_{j}-D_{j}\right)
$$

where:

$$
\begin{aligned}
& E_{j 1}=\text { Lake elevation at end of } j \text { th month } \\
& E_{j 0}=\text { Lake elevation at beginning of } j \text { th month } \\
& A=\text { Surface area of Lake } \\
& I_{j}^{\prime}=\text { Synthetic inflow for } j \text { th month } \\
& C_{j}=\text { Compulsory demand for } j \text { th month } \\
& D_{j}=\text { Discharge decision during } j \text { th month. }
\end{aligned}
$$

This model assumes that the Lake area remains constant over the range of
levels considered. This is virtually true for Okanagan Lake over the relatively narrow operating range of 4 ft .

COMPUTATIONAL PROCEDURE FOR DECISION ASSESSMENT

## Data Analysis for Synthetic Inflow Generation

Historic records of net monthly inflows for Okanagan Lake extending over a period of 48 years were analyzed to obtain:
(a) Mean and standard deviation for each month's inflow;
(b) Total inflows for periods commencing in each month and terminating at the end of the runoff season, i.e., July 31;
(c) Simple linear regression and correlation coefficients for each month's inflow related to the total inflow values obtained in (b).

The above, and following computational steps are identified in the computer program shown in Appendix II.

Generation of Sets of Monthly Inflows from the Forecast
The monthly inflow sets generated commence in the decision month and terminate in the last month of the runoff season.
(a) Given values of the inflow forecast and standard error of estimate are inserted in Equation (5.1) to obtain a synthetic "actual" inflow total. A value of the random normal deviate is obtained from a standard computer sub-program.
(b) The synthetic "actual" inflow total, appropriate statistical parameters, and coefficients are used in Equation (5.7) to obtain a synthetic value for the inflow during the current decision month. This value is stored.
(c) The above generated inflow is subtracted from the total "actual" inflow to obtain a revised total inflow which is then used to generate an inflow for the second month, again using Equation (5.7) and the appropriate parameters and coefficients for the second month.
(d) The above procedure (c) is repeated until monthly inflow values have been generated for each month in the runoff period.

Procedure (a), (b), (c) above will produce one set of synthetic monthly inflows. When this is repeated (a) will produce a different value of synthetic "actual" total inflow and hence a new set of monthly inflow values.

## Decision Evaluation

The Lake elevation at the beginning of the decision month is taken as datum level $\left(\mathrm{E}_{\mathrm{j} 0}=0.0\right)$.
a) Maximum level evaluation. The values of the synthetic inflow, demand and a discharge decision for the first month of the period are inserted in Equation (5.8) to obtain the Lake level at the end of the first month. The level at the end of the first month becomes the level at the beginning of the second month of the period and the process is repeated but with the decision discharge equal to maximum discharge (i.e., maximum corrective discharge with respect to maximum levels) for all months remaining to the end of the period. The maximum level of the period is stored.
b) Minimum level evaluation. The computational procedure is similar to (a) with the exception that a minimum (zero) discharge decision is applied to the months following the decision month. The minimum level
occurring over the period is stored.
c) Terminal level evaluation. The Lake level at the end of the runoff period obtained in (b) above is stored as terminal level.

This completes the analysis of one set of monthly inflows. The program returns to step (a) [under the Generation of Sets of Monthly Inflows from the Forecast section] and the procedure is repeated for a number of seasonal periods with the same current month discharge decision.

## Output

The maximum, minimum, and terminal level values obtained from each runoff period simulation are each analyzed to produce cumulative frequency values at intervals of 0.5 ft . over the range of Lake levels 5 ft . above and below the datum level. This data is then plotted in the form of cumulative distribution curves for maximum, minimum and terminal levels. These results being associated with a single discharge decision.

## Evaluation of all Current Discharge Decision Possibilities

The decision evaluation described in the preceding sections is repeated with other possible discharge decisions in the current month. In practice it is only necessary to perform the decision evaluation for the maximum, minimum, and one or two intermediate discharge decisions to provide adequate information on the full range of operational possibilities.

In the computer program shown in Appendix II, two current discharge decisions (one of which is a zero discharge) and the three level values are analyzed concurrently.

## CHAPTER VII

## RESULTS AND INTERPRETATION

## Analysis of Data

A certain amount of data analysis was necessary to establish the basic statistical parameters and regression coefficients required in the generation. Additional information on simple serial correlation (one month lag) was obtained for comparative purposes.

Table VII. 1 gives:

- means and standard deviations of historic monthly inflows;
- means of historic inflow totals for each month through to the end of July;
- regression and correlation coefficients for the simple linear regression relating the inflow of each month to the inflow total to the end of July;
- correlation coefficients for the simple serial correlation with one month lag.

Figures 7.1, 7.2, 7.3 and 7.4 show:

- cumulative distributions of April, May, June and July historic inflows with normal cumulative distribution of the same mean and standard deviation superimposed.

Figure 7.5 shows:

- cumulative distribution of historic inflow totals for the period April to July with a normal cumulative distribution of the same mean and standard deviation superimposed.


## TABLE VII. 1

HISTORIC MONTHLY INFLOW ANALYSIS

|  |  | AUG | SEPT | OCT | NOV | DEC | JAN | FEB | MAR | APR | MAY | JUNE | JULY |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bar{I}_{j}$ | Average Inflow | $-4.4$ | -10.2 | $-1.3$ | 4.3 | 7.5 | 6.4 | 7.3 | 14.7 | 56.3 | 193.5 | $\therefore 113.4$ | 13.6 |
| $S_{j}$ | Standard Deviation | 17.2 | 16.1 | 14.8 | 11.8 | 10.8 | 10.3 | 8.4 | 8.5 | 34.8 | 81.1 | 56.8 | 23.4 |
| $\overline{\mathrm{I}}_{\mathrm{jk}}$ | Average Inflow to July |  |  |  |  |  | 405.2 | 398.8 | 391.5 | 376.8 | 320.5 | 127.0 | 13.6 |
| $b_{j k}$ | Simple Regression Coefficient |  |  |  |  |  | . 01 | . 02 | . 02 | . 09 | . 54 | . 75 | 1.00 |
| $\mathrm{R}_{\mathrm{jk}}$ | Correlation Coefficient |  |  |  |  |  | . 21 | . 40 | . 32 | . 34 | . 87 | . 96 | 1.00 |
| $R_{j, j-1}$ | Simple Serial <br> Correlation <br> Coefficient <br> (One month lag) |  |  |  |  |  | . 04 | -. 19 | . 06 | . 42 | . 29 | . 43 | . 57 |

Figure 7.6 shows:

- cumulative distributions of synthetic May inflows generated from typical low, medium and high forecast inflows for the period April to July. The cumulative distribution for the historic May inflows is superimposed.

Figures $7.1,7.2,7.3,7.4$ and 7.5 show that the historic monthly and total inflows over the critical runoff period have distributions which may be considered to approximate to normal distributions with the same mean and standard deviation. Thus the generation methods used, which produce normally distributed values, will produce synthetic values with cumulative distributions similar to the historic records provided the means and standard deviations are maintained.

Figure 7.6 shows that synthetic inflows generated for a particular month, in this case May, on the basis of a range of forecast total inflows with typical standard error of estimates, does not produce monthly inflow values which are significantly outside the historic range.


Figure 7.I
CUMULATIVE DISTRIBUTION OF HISTORIC MONTHLY INFLOWS FOR APRIL.


Figure 7.2
CUMULATIVE DISTRIBUTION OF HISTORIC MONTHLY INFLOWS FOR MAY.


Figure 7.3

CUMULATIVE DISTRIBUTION OF HISTORIC MONTHLY INFLOWS FOR JUNE.


Figure 7.4
CUMULATIVE DISTRIBUTION OF HISTORIC MONTHLY INFLOWS FOR JULY.


Figure 7-5
CUMULATIVE DISTRIBUTION OF HISTORIC
TOTAL INFLOWS FROM APRIL TO JULY.


Figure 7. 6
CUMULATIVE DISTRIBUTION OF SYNTHETIC MAY INFLOWS FOR VARIOUS FORECASTED TOTAL INFLOWS FROM APRIL TO JULY.

Decision Assessment

The decision assessment method was applied to a number of hypothetical operational situations and subsequently to the actual conditions experienced during the 1970 runoff season. The results obtained from three of the hypothetical situations are used to demonstrate the effect of time and forecast accuracy on the decision making situation.

The basic data described in Chapter II was used and it was assumed. that the Lake level was 2 ft . below the upper limit at the beginning of the decision month. Figure 7.7 shows the situation for the decision month of February with a forecast of 400,000 acre $f t$. and a forecast standard error of estimate of 160,000 acre ft. This low forecast accuracy is chosen to reflect the difficulty in obtaining a good forecast in midwinter before accurate snowpack data is available. Decision A represents the minimum discharge decision in February and Decision $B$ the maximum. The upper and lower level limits and terminal level goal have been superimposed to facilitate interpretation.

Low probabilities $a, a^{\prime}, b, b^{\prime}$ are indicated for both maximum and minimum decisions and the commitment risk associated with either of these extreme decisions is low with respect to the upper and lower limits. The probability of achieving the desired terminal level is shown to be significantly reduced from 81 per cent to 61 per cent if a maximum discharge decision is made in February. This probability exists in spite of taking full corrective action in subsequent months. Under these circumstances there is little or no conflict between the level limit and terminal goal criteria and a minimum discharge decision is appropriate.

Figure 7.8 indicates the situation in April with no change in the forecast inflow; this will often occur in practice when no significant runoff occurs in February or March, but with the standard error of estimate reduced to 80,000 acre ft. reflecting the improved forecasting accuracy as the season progresses. The probability of falling below the lower Lake level is negligible for both extreme decisions (and therefore all intermediate decisions) and attention may be concentrated on the maximum and terminal level criteria.

Decision A is the most desirable for achieving the terminal level goal, but Figure 7.8 shows that it would result in a relatively high (30 per cent) probability that the upper level limit would be exceeded, this again in spite of full corrective maximum discharge in all subsequent months. An intermediate decision must now be considered. While increasing discharge decisions in April will result in a reduction in probability of exceeding the upper level limit it will at the same time reduce the probability of achieving the desired terminal level. Assessment of intermediate decisions is facilitated by calibration between the extreme decision lines; however, this was found to be essentially linear until the decision month of June.

The decision make must now make his final decision on the basis of a compromise and may introduce other factors such as past operational experience, information not included in this analysis, and so on.

Figure 7.9 shows the effect of a further improvement in forecast accuracy in April. The increased horizontal separation of the pairs of decision lines for maximum and minimum April discharges indicates that the
range of decisions has a greater apparent effect on the outcome probabilities and the reduced slope reflects the reduced range of possible Lake level outcomes. The more accurate forecast essentially increases the constraint on possible future inflows and there is consequently less uncertainty about the consequences of the various possible discharges. The above results are shown in tabular form in Table VII.2.


DECISION ASSESSMENT OF FEBRUARY.



TABLE VII. 2

TYPICAL RESULTS

DISCHARGE DECISION
MAXIMUM
MINIMUM

Fig. 7.7
Decision Month February
Forecast 400,000 acre ft.
Std. Error 160,000 acre ft.
Probability of:

| exceeding upper limit | 0.6 | $\%$ | 3.0 |
| :--- | ---: | ---: | ---: |
| $\%$ |  |  |  |
| exceeding lower limit | 2.5 | $\%$ | 0.4 |
| $\%$ |  |  |  |
| achieving terminal level | 61.0 | $\%$ | 81.0 |

Fig. 7.8
Decision Month April
Forecast 400,000 acre ft.
Std. Error 160,000 acre ft.

Probability of:

| exceeding upper limit | $1.0 \%$ | $30.0 \%$ |
| :--- | :---: | :---: |
| exceeding lower limit | $.09 \%$ | $<.01 \%$ |
| achieving terminal level | $67.0 \%$ | $96.0 \%$ |

Fig. 7.9
Decision Month April
Forecast 400,000 acre ft.
Std. Error 40,000 acre ft.
Probability of:

```
exceeding upper limit
exceeding lower limit
achieving terminal leve1
```

| $.05 \%$ | $20.0 \%$ |
| ---: | ---: |
| < $01 \%$ | $<.01 \%$ |
| $97.0 \%$ | $>99.99 \%$ |

## CHAPTER VIII

## SUMMARY

The problem of operating a multi-purpose reservoir with the assistance of a total future inflow forecast has been considered. Standard Operations Research techniques, and particularly optimization methods, were not found to be applicable to the operational problem on an existing reservoir due to the lack of suitable data.

Investigation of the decision making process indicated that it consisted of a series of essentially self-contained individual decisions which ideally gave the operator the greatest ability to effect future corrective action and hence achieve future goals. A method of decision assessment which overcame the data deficiency and related closely to the actual decision process was developed. The assessment is presented in the form of probabilities of exceeding Lake leve1 limits and achieving Lake storage goals at the end of the runoff season for all possible current decisions. The analysis considers forecast inflow and accuracy and corrective actions which may be taken in the future to overcome forecast and decision errors. The assessment does not yield an explicit optimal decision but provides information on the current operational situation. The operator can then proceed to make his decision in the light of this information, his past experience, and any other considerations he may feel applicable. The structure of the proposed method is flexible and could be adapted to many operational situations where some kind of forecast is involved. The
incorporation of forecast information into the simulation and analysis of operational situations also makes it possible to demonstrate the value of the forecast and its accuracy.
[1] Present, Future and Ultimate Water Requirements in the South Thompson Watershed and their Effect in Combination With the Shuswap River-Okanagan Lake Water Supply Canal Division (Scheme 3). Victoria, B.C.: Water Resources Service, Department of Lands, Forests and Water Resources, Ju1y 1968.
[2] Present (2966) and Future Water Requirements in Okanagan and North Okanagon. Victoria, B.C.: Water Resources Service, Department of Lands, Forests and Water Resources.
[3] Private correspondence with Mr. H. I. Hunter, Chief Hydrologist, B.C. Water Resources Service, Department of Lands, Forests and Water Resources, Victoria, B.C.
[4] Mass, Arthur, Maynard M. Hufschmidt, Robert Dorfman, Harold A. Thomas, Jr., Stephen A. Marglin, Gordon Maskew Fair. Design of Water Resources Systems. Cambridge, Mass.: Harvard University Press, 1962.
[5] Fiering, M. B. Streconflow Synthesis. London: Macmillan, 1967.
[6] Draper, N. R. and H. Smith. Applied Regression Analysis. New York: John Wiley and Sons, 1966.

APPENDIX I
TABLE OF MONTHLY VIRGIN INFLOWS IN THOUSAND ACRE FEET
OKANAGAN LAKE BASIN PERIOD: CLIMATIC YEARS 1921-1968
(48 YEARS)

| CL. <br> YEAR | APRIL | MAY | JUNE | JULY | AUG | SEPT | OCT | NOV | DEC | JAN | FEB | MAR | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1921 | 31.8 | 226.8 | 150.5 |  | -12.0 | -22.9 | $-7.4$ | 1.3 | 3.9 | 8.9 | - 5.4 | 13.4 | 396.0 |
| 1922 | 29.9 | 165.1 | 128.7 | -10.0 | -8.6 | - 3.3 | :0.4 | 4.4 | 4.1 | 6.6 | - 1.0 | 25.3 | 332.8 |
| 1923 | 54.4 | 187.9 | 181.8 | 19.5 | - 6.1 | -23.9 | - 5.9 | - 5.4 | 11.4 | -19.7 | 14.8 | 15.9 | 424.4 |
| 1924 | 16.1 | 137.2 | 31.2 | -20.3 | -16.3 | -28.3 | - 2.9 | 5.3 | 3.6 | 11.2 | 10.3 | 8.0 | 155.2 |
| 1925 | 63.0 | 190.3 | 65.3 | -17.2 | -26.8 | -30.8 | - 8.5 | -10.2 | 10.0 | 21.0 | - 3.4 | 11.1 | 263.8 |
| 1926 | 57.2 | 97.3 | 5.7 | -15.4 | -22.2 | -22.3 | - 1.1 | 1.9 | - 2.3 | 10.3 | - 7.3 | - 1.2 | 100.7 |
| 1927 | 24.1 | 124.6 | 132.9 | 3.4 | - 5.4 | 41.2 | 59.4 | 24.7 | 27.2 | 12.9 | 8.7 | 25.0 | 478.7 |
| 1928 | 83.2 | 402.2 | 140.9 | 56.6 | -17.0 | -21.2 | -8.9 | 31.3 | -21.7 | 0.2 | - 7.4 | 14.1 | 652.4 |
| 1929 | 16.3 | 52.8 | 62.8 | -16.8 | 7.5 | 22.1 | -10.6 | -12.4 | - 9.0 | - 0.6 | 3.6 | 7.9 | 123.6 |
| 1930 | 51.6 | 55.3 | 52.7 | - 9.2 | - 6.9 | -15.0 | -19.8 | - 0.2 | -17.4 | 3.6 | - 4.8 | 7.9 | 98.2 |
| 1931 | 23.1 | 84.1 | 26.6 | 1.9 | -27.8 | -18.3 | -14.8 | 1.1 | 3.6 | - 7.3 | - 6.5 | 30.7 | 96.2 |
| 1932 | 68.9 | 209.2 | 113.9 | - 0.7 | 1.3 | -14.8 | -10.0 | 21.9 | 0.0 | 4.2 | 4.2 | 14.0 | 412.2 |
| 1933 | 48.9 | 201.3 | 180.8 | 27.3 | - 9.4 | 3.6 | 20.2 | 14.3 | 33.7 | 25.2 | 9.2 | 36.6 | 591.6 |
| 1934 | 223.4 | 148.1 | 29.6 | 3.0 | - 0.9 | -12.5 | - 6.9 | 22.7 | 16.0 | 13.5 | 14.3 | 29.9 | 480.0 |
| 1935 | 34.9 | 216.9 | 138.6 | 92.4 | 22.1 | - 5.9 | - 7.8 | 2.4 | 6.6 | 26.0 | - 5.2 | 13.0 | 534.0 |
| 1936 | 93.0 | 208.5 | 100.0 | 12.5 | -10.7 | -16.1 | -11.2 | -22.0 | 15.9 | - 6.0 | 15.0 | 13.8 | 392.8 |
| 1937 | 37.9 | 174.3 | 162.2 | 11.2 | -14.6 | -12.8 | - 3.0 | 21.7 | 8.2 | 6.5 | 9.0 | 18.0 | 418.5 |
| 1938 | 66.5 | 204.7 | 67.1 | - 3.0 | -14.4 | - 7.8 | -11.4 | -12.4 | 6.9 | 11.1 | - 6.6 | 13.2 | 313.9 |
| 1939 | 50.5 | 136.6 | 63.7 | 0.4 | -26.0 | -18.6 | -15.7 | - 1.1 | 11.5 | - 0.3 | 1.4 | 24.5 | 226.7 |
| 1940 | 55.5 | 132.3 | 30.7 | -13.9 | -27.7 | -12.3 | -11.9 | - 6.8 | 1.6 | 11.7 | - 0.1 | 12.8 | 171.9 |
| 1941 | 82.0 | 98.7 | 69.4 | 10.7 | - 5.0 | 16.0 | 30.2 | 17.6 | 23.5 | - 4.3 | 11.7 | 2.9 | 353.5 |
| 1942 | 84.0 | 235.3 | 167.8 | 49.8 | -11.5 | -23.3 | -17.6 | - 3.8 | 15.6 | - 2.1 | 11.4 | 2.7 | 508.5 |
| 1943 | 58.8 | 128.8 | 104.0 | 7.1 | -14.4 | -28.4 | 2.5 | $-8.9$ | - 9.7 | 2.1 | 14.7 | -4.4 | 252.1 |
| 1944 | 33.0 | 117.8 | 107.1 | 3.0 | -8.3 | - 6.2 | 4.5 | 12.7 | 3.5 | 12.7 | 13.6 | 6.4 | 300.1 |
| 1945 | 27.1 | 269.0 | 139.7 | 0.4 | -13.8 | -18.5 | 15.2 | 2.1 | 13.1 | 15.6 | 6.3 | 16.1 | 472.2 |
| 1946 | 91.9 | 346.6 | 163.4 | 9.6 | - 2.7 | -22.7 | -15.2 | 7.2 | - 4.6 | 0.4 | 8.8 | 18.6 | 601.2 |
| 1947 | 50.7 | 104.3 | 58.2 | - 1.9 | - 9.8 | -10.3 | 6.1 | 8.9 | - 0.4 | 5.5 | 3.0 | 3.3 | 217.6 |
| 1948 | 56.8 | 368.0 | 208.2 | 38.4 | 66.4 | 6.7 | 15.1 | - 5.5 | 6.3 | - 4.6 | 17.3 | 23.7 | 796.5 |

APPENDIX I (continued)

| CL. <br> YEAR | APRIL | MAY | JUNE | JULY | AUG | SEPT | OCT | NOV | DEC | JAN | FEB | MAR | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1949 | 101.4 | 272.1 | 61.0 | 18.3 | 9.1 | -16.9 | -11.0 | 3.7 | 27.3 | -13.1 | 8.7 | 21.1 | 481.9 |
| 1950 | 39.9 | 208.9 | 216.8 | 17.4 | 11.7 | -25.3 | 5.0 | 8.7 | 16.3 | - 4.7 | 30.6 | 22.0 | 547.5 |
| 1951 | 99.7 | 346.2 | 114.0 | 28.4 | -15.3 | - 5.1 | 10.0 | 3.7 | 14.6 | - 3.9 | 18.8 | 14.4 | 625.7 |
| 1952 | 106.4 | 279.7 | 90.5 | 25.0 | -10.2 | -16.9 | -11.0 | -10.6 | 1.2 | 23.1 | 10.4 | 4.3 | 492.1 |
| 1953 | 31.2 | 163.5 | 139.5 | 18.0 | 16.5 | -25.1 | - 0.8 | 11.4 | 11.4 | 6.3 | 11.4 | 11.9 | 395.2 |
| 1954 | 20.5 | 245.1 | 162.9 | 67.1 | 26.0 | 19.3 | 5.0 | 29.8 | 10.4 | 8.7 | 7.9 | 14.4 | 617.3 |
| 1955 | 33.2 | 130.6 | 216.8 | 68.8 | - 4.3 | - 5.9 | 1.6 | 0.3 | 20.5 | 17.2 | 1.2 | 26.2 | 506.4 |
| 1956 | 93.0 | 296.5 | 145.2 | 20.8 | 2.4 | -16.0 | - 3.5 | - 1.4 | 10.4 | 1.2 | 12.1 | 15.2 | 576.1 |
| 1957 | 42.4 | 304.1 | 71.1 | 3.9 | 20.1 | - 4.3 | - 4.3 | 2.0 | 6.2 | 20.5 | 21.4 | 21.1 | 504.4 |
| 1958 | 54.2 | 241.7 | 68.5 | 1.3 | -22.0 | -15.2 | 6.6 | 4.6 | 17.3 | 18.1 | 7.2 | 20.3 | 402.3 |
| 1959 | 52.7 | 287.0 | 204.5 | 20.0 | -10.9 | 28.5 | 23.9 | 18.8 | 17.4 | 13.2 | 19.4 | 10.5 | 685.0 |
| 1960 | 71.2 | 178.4 | 107.3 | - 1.1 | -14.2 | - 7.9 | -10.3 | 5.4 | - 2.2 | 5.0 | 9.9 | 12.6 | 353.8 |
| 1961 | 34.4 | 227.9 | 99.2 | 6.1 | -13.6 | -43.1 | -12.9 | -12.0 | 13.5 | 3.9 | 12.7 | 6.2 | 322.3 |
| 1962 | 54.8 | 135.4 | 99.5 | 12.8 | 0.3 | -22.5 | 10.3 | 2.2 | -1.6 | 0.6 | 3.5 | 18.4 | 313.6 |
| 1963 | 34.8 | 118.0 | 52.2 | 22.5 | 2.4 | - 5.1 | -17.1 | 6.1 | 8.0 | 8.8 | 0.9 | 12.6 | 244.0 |
| 1964 | 25.6 | 158.5 | 220.5 | 41.4 | 20.8 | 10.7 | 18.3 | 19.7 | - 5.2 | 29.8 | 10.0 | 9.7 | 559.8 |
| 1965 | 84.0 | 205.9 | 120.2 | 8.9 | 12.6 | - 9.2 | - 1.2 | 3.7 | 1.4 | 0.4 | 3.5 | 9.1 | 439.3 |
| 1966 | 36.7 | 120.6 | 58.2 | 16.7 | -12.4 | -10.7 | -19.2 | 4.3 | 11.4 | 8.8 | 4.3 | 5.9 | 224.7 |
| 1967 | 24.1 | 166.5 | 171.6 | - 8.1 | -27.0 | -23.1 | -11.5 | -8.7 | 8.7 | - 6.9 | 14.8 | 23.5 | 324.1 |
| 1968 | 21.4 | 210.0 | 177.7 | 12.4 | 12.0 | - 2.3 | - 7.3 | 7.1 | 10.0 | 8.0 | 1.4 | 20.9 | 471.3 |

## COMPUTER PROGRAM FOR OPERATIONAL DECISION ASSESSMENT (See Chapter VI)

\$COMPILE
INTEGER M, Y, MM1, MMM1, YY, YYY
INTEGER YX, MX
DIMENSION $\mathrm{Q}(12,26), \mathrm{QAVE}(12), \mathrm{B}(12), \mathrm{R}(12), \mathrm{S}(12)$
DIMENSION EL(4), EMAX (4), EMIN(4), ETER(4), $\operatorname{SMAX}(4,20)$
DIMENSION $\operatorname{SMIN}(4,20), \operatorname{STER}(4,20), \operatorname{DEM}(12)$
DIMENSION QQ $(12,501), \operatorname{QSAV}(12)$
DIMENSION $\operatorname{SSMAX}(4,20), \operatorname{SSMIN}(4,20), \operatorname{SSTER}(4,20)$
WRITE $(6,4)$
4 FORMAT (40X, 'OKANAGAN LAKE - SITUATION Ist MAY, $1970^{\prime}$ )
C SPECIFY DECISION MAKING MONTH
MONTH $=5$
WRITE $(6,10)$ MONTH
10 FORMAT (25X, 'DECISION MONTH', 1X, 13)
C SPECIFY FORECAST TOTAL INFLOW TO JULY 31st (TE)
$\mathrm{TF}=297.0$
WRITE $(6,11) \mathrm{TF}$
11 FORMAT (15X, 'FORECAST TOTAL INFLOW TO JULY', F10.1)
C SPECIFY FORECAST STD ERROR AS A FRACTION OF TF
$\mathrm{FSERR}=0.2 * \mathrm{TF}$
WRITE $(6,12)$ FSERR
12 FORMAT ('FORECAST STD. ERROR', F10.1, 'THOU. A.FT.')
C SPECIFY MAXIMUM DISCHARGE DECISION DURING DECISION MONTH
DECIS $=45.0$
WRITE $(6,13)$ DECIS.
13 FORMAT ('MAXIMUM DISCH. DECISION DURING DECISION MONTH' F10.I)
C SPECIFY MAXIMUM DISCHARGE CAPACITY IN THOU. A.FT./MONTH
SPILMAX $=90.0$
WRITE $(6,14)$ SPILMAX
14 FORMAT ('MAXIMUM DISCHARGE CAPACITY', F10.1)
C SPECIFY COMPULSORY DEMANDS FOR EACH MONTH
$\operatorname{DEM}(3)=9.48$
$\operatorname{DEM}(4)=9.60$
$\operatorname{DEM}(5)=15.00$
$\operatorname{DEM}(6)=18.9$
$\operatorname{DEM}(7)=18.96$
$\mathrm{AREA}=84.2$
(See Chapter VI, Section: Data Analysis for Synthetic Inflow
$5 \operatorname{READ}(5,6)((Q(M, Y), M=1,12), Y=1,26) \quad$ Generation)
6 FORMAT (12F9.1)
$D=25.0$

## APPENDIX II (Continued)

```
20 DO 25 M = 1,12
    QAVE(M) = 0.0
    DO 24 Y = 1,25
    QAVE(M) = AQVE(M) + Q(M,Y)/D
24 CONTINUE
25 CONTINUE
30 DO 49 M = 1,12
    B1 = 0.0
    B3 = 0.0
    B5 = 0.0
    B6 = 0.0
    B7 = 0.0
31 DO 35 Y = 1,25
    YY = Y
    QS = 0.0
    QSAVE = 0.0
    MM = M
    C = FLOAT (8-MO
    IF (M.GT.7) C = FLOAT (20-M)
    IF (MM.GT.12)YY = Y + 1
    IF (MM.GT.12)MM = I
    QS = QS + Q(MM,YY)/C
    QSAVE = QSAVE + QAVE(MM)/C
    MM = MM + I
    IF (MM.N.E.*) GO to 33
    B1 = B1 + Q(M,Y)*QS
    B2 = D*ASAVE*QAVE(M)
    B3 = B3 + QS*QS
    B4 = D*QSAVE*QSAVE
    B5 = B5 + Q (M,Y)*Q (M,Y)
    B6 = D*QAVE (M)*QAVE(M)
    B7 = B7 + (Q(M,Y) - QAVE (M))*(Q(M,U) - QAVE(M))
35 CONTINUE
    B (M) = (B1-B2)/(B3-B4)
    R(M) = (B1-B2)/(SQRT((B3-B4)*(B5-B6)))
    S(M) = SQRT(B7/D)
    QSAV(M) + QSAVE
(See Chopter VI, Section: Generation of Sets of Monthly Inflows From the Forecast)
GENERATE MONTHLY INPUTS FOR A GIVEN FORECAST
T = RANDN(10.)
DO 215 N = 1,4
DO 214 1 = 1,20
```

```
    \(\operatorname{SMAX}(\mathrm{N}, \mathrm{L})=0.0\)
    \(\operatorname{SMLN}(N, 1)=0.0\)
    \(\operatorname{STER}(N, 1)=0.0\)
214 CONTINUE
215 CONTINUE
    DO 279 YY \(=1,500\)
    \(Y X=Y Y\)
    GENERATE AN ACTUAL TOTAL INPUT TO JULY 31st
    \(T=T F+\operatorname{RANDN}(0) * F S E R\).
    GENERATE MONTHLY INPUTS
    MX = MONTH
120 IF (MX.GT.12) YY \(=Y X=1\)
    IF (MX.GT.12) MX = 1
    \(\mathrm{C}=\mathrm{FLOAT}(8-\mathrm{MX})\)
    IF (MX.GT.7) C + FLOAT (20 - MX)
    \(\mathrm{QQ}(\mathrm{MX}, \mathrm{YX})=\mathrm{QAVE}(\mathrm{MX})+\mathrm{B}(\mathrm{MX}) *(\mathrm{~T} / \mathrm{C}-\mathrm{QSAV}(\mathrm{MX}))+\)
    1 RANDN (0.) \(\div \mathrm{S}(M X) \div \operatorname{SQRT}(1-R(M X) \div R(M X))\)
    \(T=T-Q Q(M X, Y X)\)
    \(M X=M X+1\)
    IF (MX.NE.8) GO to 120
    (See Chapter VI, Section: Decision Evaluation)
    \(\mathrm{Y}=\mathrm{YY}\)
    DO \(222 \mathrm{~N}=1,4\)
    \(\mathrm{EL}(\mathrm{N})=0.0\)
    \(\operatorname{EMAX}(N)=0.0\)
    \(\operatorname{EMIN}(\mathrm{N})=0.0\)
    \(\operatorname{ETER}(N)=0.0\)
222 CONTINUE
    \(\mathrm{K}=\mathrm{MONTH}\)
    \(\mathrm{M}=\mathrm{MONTH}\)
\(225 \operatorname{IF}(\mathrm{M} . \mathrm{GT} .12) \mathrm{Y}=\mathrm{Y}+1\)
    IF (M.GT.12) \(M=1\)
    SPIL = SPILMAX
    IF(M.EQ.K) SPIL = DECIS
    \(\operatorname{EL}(1)=\operatorname{EL}(1)+(\mathrm{QQ}(\mathrm{M}, \mathrm{Y})=\mathrm{DEM}(\mathrm{M})) / \mathrm{AREA}\)
    \(I F(M . E Q \cdot K) E L(2)=E L(1)\)
    \(\operatorname{IF}(\mathrm{M} . \mathrm{NE} . \mathrm{K}) \mathrm{EL}(2)=\mathrm{EL}(2)+(\mathrm{QQ}(\mathrm{M}, \mathrm{Y})-\mathrm{DEM}(\mathrm{M})-\mathrm{SPIL}) / A R E A\)
    \(\operatorname{EL}(4)=\operatorname{EL}(4)+(\mathrm{QQ}(\mathrm{M}, \mathrm{Y})-\operatorname{DEM}(\mathrm{M})-\mathrm{SPIL}) / \mathrm{AREA}\)
    \(I F(M, E Q . K) E L(3)=E L(4)\)
    \(\operatorname{IF}(\mathrm{M} . \mathrm{NE} . \mathrm{K}) \mathrm{EL}(3)=\mathrm{EL}(3)+(\mathrm{QQ}(\mathrm{M}, \mathrm{Y})-\mathrm{DEM}(\mathrm{M})) / \mathrm{AREA}\)
    DO \(227 \mathrm{~N}=1,4\)
    \(\operatorname{IF}(\operatorname{EL}(N) \cdot \operatorname{GT} \cdot \operatorname{EMAX}(N)) \operatorname{EMAX}(N)=\operatorname{EL}(N)\)
    \(\operatorname{IF}(E L(N) \cdot \operatorname{LT} \cdot \operatorname{EMIN}(N)) \operatorname{EMIN}(N)=\operatorname{EL}(N)\)
    \(\operatorname{IF}(\mathrm{M} . \mathrm{EQ} .7) \operatorname{ETER}(\mathrm{N})=\mathrm{EL}(\mathrm{N})\)
227 CONTINUE
    \(M=M+1\)
    IF (M.NE. 8) GO to 225
```

(See Chapter VI, Section: Output)
SORT
250 D0 $259 \mathrm{~N}=1,4$
251 DO 258 : $=1,20$
$\mathrm{J}=1-10$
$\mathrm{XL}=\operatorname{FLOAT}(\mathrm{J}) / 2.0$
$\operatorname{IF}(\operatorname{EMAX}(N) . \operatorname{GT} . X L) \operatorname{SMAX}(N, 1)=\operatorname{SMAX}(N, 1)+1.0$
$\operatorname{IF}(\operatorname{EMIN}(N) . \operatorname{GT} \cdot X L) \operatorname{SMIN}(N, 1)=\operatorname{SMIN}(N, 1)+1.0$
$\operatorname{IF}(\operatorname{ETER}(\mathrm{N}) . \operatorname{GT} . \mathrm{XL}) \operatorname{STER}(\mathrm{N}, 1) \doteqdot \operatorname{STER}(\mathrm{N}, 1)+1.0$
258 CONTINUE
259 CONTINUE
IF(YY.NE.500) GO to 279
WRITE $(6,325)$ YY
325 FORMAT('GENERATION PERIOD',3X,14,3X,'YEARS')
WRITE $(6,330)$
330 FORMAT ( ${ }^{\prime}-4.5-4.0-3.5-3.0-2.5-2.0-1.5-1.0-0.50 .00 .5$
11.01 .52 .02 .53 .03 .54 .04 .5 5.0')

DO $350 \mathrm{~N}=1,4$
DO $3511=1,20$
$\operatorname{SSMAX}(N, 1)=\operatorname{SMAX}(N, 1) * 100.0 / F L O A T(Y Y)$
$\operatorname{SSMIN}(\mathrm{N}, 1)=\operatorname{SMIN}(\mathrm{N}, 1) * 100.0 / \operatorname{FLOAT}(\mathrm{YY})$
$\operatorname{SSTER}(\mathrm{N}, 1)=\operatorname{STER}(\mathrm{N}, 1) * 100.0 / \operatorname{FLOAT}(\mathrm{YY})$
351 CONTINUE
350 CONTINUE
280 DO $300 \mathrm{~N}=1,4$
WRITE $(6,285) \mathrm{N}$
285 FORMAT(//'DECISION', 3X, 12, /)
$290 \operatorname{WRITE}(6,291)(\operatorname{SSMAX}(\mathrm{N}, 1), 1=1,20)$
291 FORMAT (20F5.1)
$292 \operatorname{WRITE}(6,293)(\operatorname{SSMIN}(\mathrm{N}, 1), 1=1,20)$
293 FORMAT (20F5.1)
$294 \operatorname{WRITE}(6,295)(\operatorname{SSTER}(N, 1), 1=1,20)$
295 FORMAT (20F5.1)
300 CONTINUE
279 CONTINUE
STOP
END

