INVESTIGATION OF CONTINUITY IN JOINTS BETWEEN PRECAST AND "CAST IN PLACE" REINFORCED CONCRETE MEMBERS

by

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ABSTRACT

The investigation dealt mainly with the shear transfer capacity of a joint between a precast concrete column and a cast-in-place concrete beam. Four reinforced concrete frames, each consisting of two columns and two beams, were cast, assembled and tested in a special loading frame. To obtain a general pattern of failure mechanisms, a series of loads consisting of different ratios of moments, shears and axial forces were imposed on these frames. All recording of test data was done electronically in the form of punched tape to facilitate computer analysis.

The investigation showed clearly that high values of shear transfer can be reached even under the most adverse load conditions.
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LIST OF SYMBOLS

a = shear arm, i.e. distance of vertical load from column face (inches)
A_s = area of steel in tension zone (square inches)
A'_s = area of steel in compression zone (square inches)
A_v = total area of web reinforcement within a distance "s" measured in a direction parallel to the longitudinal reinforcement (square inches)
b = width of the beam (inches)
c = column width
C_C = compression force in concrete (kips)
C_s = compression force in steel (kips)
d = effective depth of flexural sections measured from the compression face to the centroid of the steel (inches)
d' = distance from the centroid of the compression reinforcement to the compression face (inches)
d" = distance from centroid of tension steel to the tension face of a flexural member (inches)
E_C = modulus of elasticity of concrete (Ksi)
E_s = modulus of elasticity of steel (29,000 Ksi)
f_C = concrete stress (psi)
f_h = hoop stress due to transverse reinforcement
f'_c = compressive strength of concrete (psi)
f_s = stress in tension steel (psi)
f'_s = stress in compression steel (psi)
\( f_{cu} \) = ultimate stress reached by concrete during cylinder strength test

\( f_y \) = yield strength of reinforcing steel (psi)

\( f'_t \) = modulus of rupture of concrete (psi)

\( h \) = height of column from bottom pin to center of beam (ft.)

\( H \) = axial tension in beam (kips)

\( HL \) = equal horizontal load applied to both sides of the reinforcing concrete frame

\( k \) = ratio of distance between extreme compressive fibre and neutral axis to the depth "d" of a flexural section

\( k_y \) = ratio of distance between extreme compressive fibre and neutral axis to the depth "d" of a flexural section at the point when tension steel yields

\( k_u \) = ratio of distance between extreme compressive fibre and neutral axis to the depth "d" of a flexural section when the concrete reaches its ultimate strain in the extreme fibre

\( k_1 \) = ratio of distances between resultant compressive concrete stress and the neutral axis to kd

\( L \) = Span length between column faces (inches)

\( l \) = distance over which beam rotation is measured (inches)

\( M \) = bending moment at the column face (kip-inches)

\( M_u \) = ultimate bending moment at the column face (kip-inches)

\( M_y \) = bending moment at the column face when tension steel yields (kip-inches)

\( N \) = axial compression in column (kips)

\( n \) = modular ratio for steel and concrete \( E_s/E_c \)
\[ P \] = concentrated point load on beam (kips)
\[ p \] = steel ratio \( A_s/\text{bd} \)
\[ p' \] = steel ratio \( A'_s/\text{bd} \)
\[ r \] = web reinforcing steel ratio \( (A_v/\text{bd}) \times \text{d/s} \)
\[ S \] = horizontal sway force (kips)
\[ s \] = spacing of web reinforcement (inches)
\[ t \] = overall depth of reinforced concrete beam (inches)
\[ v \] = shear stress \( V/\text{bd} \) (psi)
\[ v_c \] = concrete shear stress (psi)
\[ v_{cu} \] = ultimate shear stress carried by concrete (psi)
\[ v_u \] = ultimate total shear stress of a reinforced concrete section \( V_u/\text{bd} \) (psi)
\[ V \] = shear force at column face (kips)
\[ V_c \] = shear force carried by the concrete (kips)
\[ V_{cr} \] = critical shear, i.e. shear at first concrete cracking
\[ V_{cu} \] = shear force carried by the concrete at ultimate (kips)
\[ V_u \] = total shear force carried by a reinforced concrete section (kips)
\[ \gamma \] = coefficient used in shear expression
\[ \Delta \] = concrete compression force of element \( dx \)
\[ \varepsilon_c \] = unit strain in concrete
\[ \varepsilon_{ct} \] = concrete strain in outer compression fibre
\[ \varepsilon_s \] = unit strain in tension steel
\[ \varepsilon'_s \] = unit strain in compression steel
\[ \varepsilon_{sy} \] = unit strain in tension steel at its yield point
\[ \varepsilon_{cy} = \text{unit strain in concrete at yield point of tension steel} \]

\[ \varepsilon_{cu} = \text{ultimate strain for concrete} \]

\[ \varepsilon_{o} = \text{unit concrete strain corresponding to maximum concrete stress} \]

\[ \varepsilon_{su} = \text{strain in tension steel at ultimate strength of the concrete} \]

\[ \theta_{p} = \text{plastic rotation of a concrete section} \]

\[ \theta_{u} = \text{ultimate rotation of a concrete section} \]

\[ \phi = \text{capacity reduction factor} \]

\[ \phi_{p} = \text{plastic curvature of a concrete section} \]

\[ \phi_{u} = \text{ultimate curvature of a concrete section} \]

\[ \phi_{y} = \text{yield curvature of a concrete section} \]
ACKNOWLEDGEMENTS

The author expresses his grateful appreciation to Professor S. Lipson of the Civil Engineering Department, University of British Columbia, for his constructive criticism during the investigation and writing of this thesis.

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Vancouver, B.C.
1.0 INTRODUCTION

1.1 Purpose

The object of this investigation was the study of shear transfer capacity of a joint between a precast column and a cast-in-place concrete beam subjected to various combinations of shear, bending moments, and axial tension.

1.2 Synopsis of preceding investigations

Shear transfer has been one of the most widely investigated and discussed topics in the field of reinforced concrete technology. Various expressions for calculating shear capacity have been proposed and these involve different parameters. Some of these expressions are discussed below and tabulated on page 8.

Washa (1) and Badoux and Hulsbos (2) deal with horizontal shear between precast beams and cast-in-place slabs. This is of a similar nature to vertical shear at a beam-column interface; therefore, the expressions derived from these experimental results were of interest. Badoux and Hulsbos proposed further that a different coefficient be used for rough, intermediate, and smooth interface of joint.

Kriz and Raths (3) describe a project directed towards development of design criteria for reinforced concrete corbels. As these are essentially short cantilever beams of a/d ratios up to 1.5, they fall within the range of the
tests made in this investigation. Of special interest is the effect of direct tensile load "H" on the shear transfer strength as was clearly shown by the test results, and the resulting inclusion of "H" into the ultimate shear expression.

P.W. and H.W. Birkeland (4) and R.F. Mast (5) investigated the shear friction hypothesis; that is, the increase of shear strength with compression across the section. The influence of joint finish (rough or smooth) was shown by the former, while the latter further developed the shear expressions to include joint tension. These expressions are straight lines fitted to experimental data.

Krefeld and Thurston (6,7) investigated the contribution of longitudinal steel to the shear resistance of a simply supported beam. This shear resistance is in the form of dowel action of the steel. Distinction was made between cases with and without stirrups. It was found that in cases without stirrups the dowel action resulted in tensile splitting of the concrete along the longitudinal steel. The addition of stirrups provides forces which restrict the relative displacement of the segments in a diagonally cracked zone after the inclined crack has formed and thus retards the crack propagation. The derived shear expressions were based on the above conclusions and fitted to their results. Since the expressions were derived for simply supported beams, parts of them will not apply to a beam-column joint which was used in this investigation.
Hanson and Connor (8) performed tests on a monolithically cast frame which included a joint similar to that in this investigation. The type of loading is of another nature, being repeated loading to determine seismic resistance. Special attention was given to confinement of concrete by hoops or spirals and to splices at the critical sections. The shear and design expressions were taken out of the ACI code and Earthquake Manual. Although an attempt was not made to determine the quantitative effect of joint confinement, it was established that the tests with confined joints proved stronger and more ductile.

Hofbeck, Ibrahim and Mattock (9) present a test series to study shear transfer in reinforced concrete. The stress state in all tests was constant; that is, constant axial stress and near constant shear stress across the joint. Although their test results show close agreement with the shear friction hypothesis and very good agreement with the Zia Envelope to Mohr Circles, no attempt was made at a more practical case with varying stresses across the joint. The following conclusions were presented:

1) a pre-existing crack along the shear plane will both reduce ultimate shear transfer and increase slip at all levels of loading.

2) Shear transfer strength is a function of "pfy".  

3) Dowel action of reinforcing bars across the shear plane is insignificant in initially uncracked concrete, but is substantial in concrete with a pre-existing crack along
the shear plane.

4) Shear-friction theory gives a reasonably conservative estimate of shear transfer if \( \tan \phi = 1.4 \), provided \( pf_y < 15f'_c \) or < 600psi.

5) The Zia Failure Envelope closely agrees with observed failures.

Smith (10) presents a summary of tests made by known research centres. He divides shear failure into three groups depending on \( a/d \) ratio as follows:

- Diagonal tension failure for \( a/d > 2.4 \)
- Shear compression failure for \( 1.0 < a/d < 2.4 \)
- Shear proper failure for \( a/d < 1.0 \)

For \( a/d > 2.4 \), the Paduart equation is applicable; while for \( a/d < 2.4 \), the Laupa equation is applicable. The Laupa and Paduart curves intersect in a series of points depending on "p", which corresponds to the transition from Diagonal Shear to Shear Compression failure.

The matter of confinement of concrete in the joint zone can influence the failure mechanism considerably as demonstrated by Newmark (11). He states:

"It is seen that both the strength and ductility of the concrete increases as the lateral pressure is increased. For a lateral pressure of 4,090psi, the concrete (unconfined cylinder strength 3,660psi) attains a maximum stress of 19,000psi at a strain of 0.05, the latter value being about 25 times what would be expected for unconfined concrete at maximum stress."
For the confined concrete core of a rectangular column the following expression was derived:

\[ \sigma_a = 0.85f'_c + \frac{4.1A_fy}{s(d-d')} \]

Newmark also developed an expression for shear strength of a reinforced concrete section subjected to a tensile force "H".

The ACI (318-63) standards show expressions to be used in shear transfer calculations (12).

T.C. Zsutty (13) applied the techniques of dimensional analysis and statistical regression analysis to data of beam-shear tests performed at major research centres. By the method of least squares he shows that the value of \( a/d = 2.5 \) is the transition between beam action and arch action. The small percentage error shows the good fit of the resulting expressions which were derived both for cracking and ultimate load.

Burton, Corley and Hognestad (14) developed expressions to evaluate the shrinkage forces developed in beam members of a multi-story reinforced concrete frame. The tests showed the increase of shrinkage forces developed by partially and fully-restrained beams over a time interval.

Combined with Mattock's report (15) regarding the influence of shape factor on the shrinkage and creep as recorded from reinforced concrete test beams, the above
information provided a means to determine the tensile force which was applied to the test beams in this investigation.

Cohen (16) developed a series of rotation expressions for reinforced concrete members in the plastic range. Some of these expressions were applied in the evaluation of limiting conditions. These are given below:

\[ \theta_p = \text{plastic rotation} = \int_0^1 \phi_p \, dx \]

\[ \phi_p = \phi_u - \phi_y \]

Where:

\[ \phi_u = \frac{\epsilon_{cu}}{k_u} \]

\[ \phi_y = \frac{\epsilon_{cy}}{k_u} \]

For constant stress conditions over the plastic zone:

\[ \theta_u = \phi_u \cdot 1 = \frac{\epsilon_{cu}}{k_u} \cdot d \quad ...1.2-1 \]

\[ \theta_y = \phi_y \cdot 1 = \frac{\epsilon_{cy}}{k_y} \cdot d \quad ...1.2-2 \]

Where: "1" is the distance over which rotation is measured.

The "Ultimate Flexural Analysis Based On Stress-Strain Curves of Cylinders" by Young and Smith (17) proved useful in the theoretical correlation and evaluation of concrete strength tests performed on a number of concrete cylinders of the same batch as the test beams.

Table I shows a summary of the shear transfer expressions as derived by the above mentioned authors.

Table II shows these expressions as applied to the joint tested in this investigation. Figure 1 shows a plot of these expressions. The widest variation was found for low a/d ratios,
while towards the generally accepted transition point of $a/d=2.5$ the agreement was better.
<table>
<thead>
<tr>
<th>INVESTIGATOR</th>
<th>ULTIMATE SHEAR EXPRESSION</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAEMANN and WASHA</td>
<td>$v_{cu} = \frac{2700 + 30,000p}{a/d+5} \frac{33-a/d}{(a/d)^2+6a/d+5}$</td>
<td>for $f'_c = 3000$psi</td>
</tr>
<tr>
<td>BADOUX and HULSBOS</td>
<td>$v_{cu} = \frac{3500}{11+a/d} + 20,000p$</td>
<td>rough finish</td>
</tr>
<tr>
<td></td>
<td>$v_{cu} = \frac{2000}{11+a/d} + 20,000p$</td>
<td>intermediate finish</td>
</tr>
<tr>
<td>KRIZ and RATHS</td>
<td>$v_{cu} = 0.85[6.5\sqrt{f'_c(1-0.5d/a)} \times (1/3+0.4H/V)]$</td>
<td>restrictions on reinforcement: $A_s &lt; 0.013, A_v &lt; 0.5A_s$ bd</td>
</tr>
<tr>
<td>BIRKELAND, BIRKELAND and MAST</td>
<td>$v_{cu} = \frac{(A_s f_y - H u) \tan \phi}{bd}$</td>
<td>tan $\phi = 0.7$ to $1.4$ according to joint roughness</td>
</tr>
<tr>
<td>KREFELD and THURSTON</td>
<td>$v_{cu} = [\sqrt{f'_c(1.3+d/d')} + 18,000p] \times \frac{d'/d}{\sqrt{a/d}}$</td>
<td>limited by: $v_u = v_{cu}$ for $r_f &lt; 30$</td>
</tr>
<tr>
<td></td>
<td>$v_{cu} = 1.8\sqrt{f'_c} + \frac{2600p}{(M/Vd)x}$</td>
<td>$v_u = v_{cu} + 1.5r_f - 44$ for $30 &lt; r_f &lt; 90$</td>
</tr>
<tr>
<td></td>
<td>$v_u = v_{cu} + \frac{yd^2}{tr}r_f$</td>
<td>$v_u = v_{cu} + r_f$ for $90 &lt; r_f$</td>
</tr>
<tr>
<td>ACI(318-1963) HANSON and CONNOR</td>
<td>$v_{cu} = \phi[1.9\sqrt{f'_c} + \frac{2500pVd}{M-N(4t-d)/8}]$</td>
<td>$M/Vd &lt; 1, \phi = 1$</td>
</tr>
</tbody>
</table>

Table I
TABULATION OF SHEAR STRENGTH THEORIES
<table>
<thead>
<tr>
<th>INVESTIGATOR</th>
<th>ULTIMATE SHEAR EXPRESSION</th>
<th>REMARKS</th>
</tr>
</thead>
</table>
| SMITH        | \( v_{cu} = 1.9 \sqrt{f'_{c}(1+750p)} \times [0.186+.00157(9-a/d)^2] \)                                                                                          \( v_{cu} = 1.9 \sqrt{f'_{c}(\sqrt{100p-17p})d/a} \) for \( a/d > 2.4 \)  
\( 1 < a/d < 2.4 \) |                                                                                                                                                                                                                                                   |                               |
| NEWMARK      | \( v_{cu} = (1-H/8A_c \sqrt{f'_{c}})1.9 \sqrt{f'_{c}} \)  
\( v_u = v_{cu} + r_f y \)                                                                                                                                                                                                                           |                               |
| ZSUTTY       | \( v_{cr} = 59(f'_{c}pd/a)^{1/3} \)  
\( v_{cu} = 63.4(f'_{c}pd/a)^{1/3} \)                                                                                                                                                                                                                  |                               |

Table I (continued)
<table>
<thead>
<tr>
<th>INVESTIGATOR</th>
<th>$M=112.5K$ in $a/d=0.5$</th>
<th>$M=225K$ in $a/d=1$</th>
<th>$M=450K$ in $a/d=2$</th>
<th>SHEAR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SAEMANN and WASHA</strong></td>
<td>22+72=94</td>
<td>20+49=69</td>
<td>17+27=44</td>
<td>$V_u$</td>
</tr>
<tr>
<td><strong>BADOUX and HULSBOS</strong></td>
<td>14+12=26</td>
<td>13+12=25</td>
<td>12+12=24</td>
<td>$V_u$</td>
</tr>
<tr>
<td></td>
<td>8+12=20</td>
<td>7.5+12=19.5</td>
<td>7+12=19</td>
<td>$V_u$</td>
</tr>
<tr>
<td><strong>KRIZ and RATHS</strong></td>
<td>25</td>
<td>17</td>
<td>10</td>
<td>$V_u$</td>
</tr>
<tr>
<td><strong>BIRKELAND, BIRKELAND and MAST</strong></td>
<td>35.6</td>
<td>35.6</td>
<td>35.6</td>
<td>$V_u$</td>
</tr>
<tr>
<td><strong>KREFELD and THURSTON</strong></td>
<td>4.4+3.1=7.5</td>
<td>3.1+2.2=5.3</td>
<td>2.2+1.6=3.8</td>
<td>$V_{cu}$</td>
</tr>
<tr>
<td></td>
<td>4.5+3.2=7.7</td>
<td>4.5+1.6=6.1</td>
<td>4.5+0.8=5.3</td>
<td>$V_{cu}$</td>
</tr>
<tr>
<td></td>
<td>7.7+33=40.7</td>
<td>6.1+33=39.1</td>
<td>5.3+33=38.3</td>
<td>$V_u$</td>
</tr>
<tr>
<td><strong>ACI(318-1963) HANSON and CONNOR</strong></td>
<td>4.8+2.9=7.7</td>
<td>4.8+1.4=6.2</td>
<td>4.8+0.7=5.5</td>
<td>$V_{cu}$</td>
</tr>
<tr>
<td></td>
<td>7.7+33=40.7</td>
<td>6.2+33=39.2</td>
<td>5.5+33=38.5</td>
<td>$V_u$</td>
</tr>
<tr>
<td><strong>SMITH</strong></td>
<td>9.0</td>
<td>4.5</td>
<td>2.25</td>
<td>$V_{cu}$</td>
</tr>
<tr>
<td><strong>NEWMARK</strong></td>
<td>2.9</td>
<td>2.9</td>
<td>2.9</td>
<td>$V_{cu}$</td>
</tr>
<tr>
<td></td>
<td>2.9+33=35.9</td>
<td>2.9+33=35.9</td>
<td>2.9+33=35.9</td>
<td>$V_u$</td>
</tr>
<tr>
<td><strong>ZSUTTY</strong></td>
<td>11.5</td>
<td>9.1</td>
<td>7.2</td>
<td>$V_{cr}$</td>
</tr>
<tr>
<td></td>
<td>12.4</td>
<td>9.8</td>
<td>7.8</td>
<td>$V_{cu}$</td>
</tr>
</tbody>
</table>

**Table II**  
SHEAR CAPACITY OF JOINT
Fig. 1 Shear capacity of the joint
1.3 Scope of tests

The investigation involved the testing of two groups of three-pinned reinforced concrete frames. Each group consisted of two identical frames which were loaded to failure by a combination of point loads, sway loads and axial loads. The only difference between the two groups was the arrangement of longitudinal reinforcement - the first having continuous longitudinal reinforcement, and the second having lapped longitudinal reinforcement at the beam-column joint. In both cases lap and anchorage lengths were adequate to prevent bond failure.

The tests were performed within the following limitations applied to loading:

1) the maximum moments were to be smaller than the ultimate moment in order to restrict the failure to shear failures at the beam-column joint.

2) the joint tension induced was to simulate shrinkage forces developed in restrained members.

3) sway was applied to produce different failure conditions in the otherwise identical two joints of the symmetrical test frames.

1.4 Description of test frames

Figure 2 shows one half section of the reinforced concrete frame.

The columns were cast first with the side of joint-interface against steel forms. The weaker beam concrete was cast six weeks later against the smooth column surface
Fig. 2 Reinforced concrete frame
to represent actual construction conditions between precast columns and cast-in-place beams. In the last two frames the longitudinal steel was lapped as shown in dotted lines in Figure 2. The lap lengths according to ACI code were:

- 18 in. for #5 bars
- 12 in. for #3 bars

1.5 Measurement of Specimen Properties

For each test frame six four-inch diameter concrete test cylinders were loaded to failure in direct compression. Three of the cylinders were of the same batch as the column concrete, and the other three were of the same batch as the beam concrete. The cylinders were crushed on the same day as the respective frame test was performed. Stress and strain measurements were taken up to failure to aid with the theory developed in sections 3.1 and 3.2. These are represented as stress-strain curves in the Appendix A. Several reinforcing steel specimens of both the #5 and #3 steel used in the beams were loaded to failure in tension. The stress-strain readings taken are shown in the Appendix.
2.0 APPARATUS

2.1 Loading frame

Figure 3, plate 1 and plate 2 show the loading frame. Three hydraulic load cylinders provided the required load connections between the loading frame and the test frame.

The two horizontal jacks were bolted to the loading frame in line with the centre-line of the beams and connected to the ends of the test frame by connecting rods and ball joints, to insure the horizontal position of the load, as shown on plate 3.

The accumulator connected in front of the jack on the south side facilitated a horizontal movement of the frame, after shutting off the south supply valve, with a negligible change in jack pressure. Thus a constant force can be maintained at one side of the frame while the force on the other side can be further increased for side-sway.

The vertical load cylinder is situated inside the "loading triangle" which keeps it in a vertical position during all stages of frame side-sway. The vertical load cylinder is connected to the "loading beam" by two pin-jointed connection plates. The "loading beam" is resting on top of the test frame on two adjustable load points.

2.2 Load and deformation measurement

The two horizontal loads were measured by "Strainsert" bolts connecting the horizontal load jacks through a ball joint to the loading frame.
Fig. 3 Loading frame
Plate 1. Frame setup

Plate 2. Frame setup (closeup)
The central load was measured by a load cell placed in series with the central load cylinder.

All deformations were measured by direct current displacement transducers (DCDT's). The vertical beam slip relative to the column was measured by a 7DCDT-500 with a displacement range of $+0.5\text{in}$. The transducers were attached to the column, one above and one below the beam. The core extensions rested on a horizontal plate glued on the beam, the bottom one being held in place by a rubber sling. Beam and column rotations were measured by a setup as shown in Figure 4 and Plate 4. The transducers used were 7DCDT-100 with a displacement of $+0.1\text{in}$.

The rotations were measured as follows:

Since the pendulum always assumes the vertical position due to gravity, any rotation of the frame will move the transducer core, which is connected to the pendulum through a bearing (Fig. 4). For small rotations the horizontal core movement is proportional to the pendulum rotation. In order to overcome the frictional resistance of the core sliding in the transducer and in the bearings, a buzzer oscillating at a high frequency, was attached to the transducer bracket. An adjustment device was attached between the mounting plate and the concrete (Fig. 4 in dotted lines). Thus the transducer readings could be calibrated against a known angle change.

Figure 3 shows the positions of the load and deformation measurement devices. All measurements were recorded
Plate 3. Detail of horizontal loading setup

Plate 4. Rotation measuring device
Fig. 4 Rotation measuring device
in the form of voltages by a Vidar digital voltmeter and punched out on paper tape. This served as input for computer programs which performed the numerical analysis of the test data.
3.0 APPLIED THEORY

3.1 Mathematical representation of concrete stress-strain curve

In order to determine the moment of resistance of a reinforced concrete beam section at a stress state below ultimate where neither the linear concrete stress variation (modular ratio) method nor the rectangular (Whitneys) stress block applies, it was attempted to represent the actual stress distribution by a mathematical expression. By evaluating resisting moments corresponding to specific concrete or steel strains and the respective position of the neutral axis, and applying these values to the expressions as given by Cohen (16) in Chapter 1.2, a correlation is possible between the recorded displacement and load data.

Four expressions were used to find the best fitting theoretical representation of the stress-strain curves obtained experimentally. The first expression was derived by Young and Smith (17):

\[ f_c = f'_c G e^{(1-G)} \]  \hspace{1cm} \ldots 3.1-1

Where \( G = \frac{\varepsilon_c}{\varepsilon_0} \)

The same expression was used with the following corrective term as derived by the author:

\[ f_c = f'_c G [1+0.1 \cos(\pi f'_c/8000)(1-G)G]e^{(1-G)} \]  \hspace{1cm} \ldots 3.1-2

The next expression was taken from Saenz (18):

\[ f_c = E_i \frac{\varepsilon_c}{[1+(R+R_E-2)G-(2R-1)G^2+RG^3]} \]

Where \( R = \frac{R_E (R_f-1)}{(R_e-1)^2 - 1/R_e} \)
and: \[ R_E = \frac{E_c}{E_{SE}} \]
\[ R_e = \frac{E_{cu}}{E_o} \]
\[ R_f = \frac{f'_c}{f_{cu}} \]
\[ E_{SE} = \text{Secant Modulus} = \frac{f'_c}{\varepsilon_o} \]
\[ E_i = \text{Initial Tangent Modulus} = E_{SE} \cdot e \]

The fourth expression based on parabolic stress-strain relationship was derived by the author:
\[ f_c = 2f'_c G(1-G/2) \quad \ldots 3.1-4 \]

In the Appendix these expressions were superimposed on the experimental stress-strain data found by the tests. Two plots were done, each containing the information obtained from the six test cylinders belonging to one batch.

From the comparison of all these plots the following can be said:

i) expression 3.1-1 shows close agreement with the experimental values, but is sometimes slightly on the high side.

ii) expression 3.1-4 also shows close agreement with the experimental values and is more conservative in most cases.

Both of these expressions were applied to the reinforced concrete theory in section 3.2.
Fig. 5 Stress and strain distribution by stress function
3.2 Reinforced Concrete Theory

The following analysis is similar to the one presented by Young and Smith (17).

From the geometry of the strain diagram we have:

\[ \varepsilon_{ct} = \frac{\varepsilon_s}{1-k} \quad \ldots 3.2-1 \]

\[ \varepsilon'_s = \varepsilon_s \frac{k - d'/d}{1-k} \quad \ldots 3.2-2 \]

(a) exponential stress-strain relationship given by 3.1-1:

\[ f_c = f'_c e^{(1-G)} \]

\[ \Delta_c = \int f'_c b \, dx \]

Where \( G = \frac{\varepsilon_c}{\varepsilon_0} \)

Therefore

\[ C_c = \int_0^{\rho kd} f'_c b \, dx \]

and

\[ \varepsilon_c = \frac{\varepsilon_{ct} x}{kd} \]

Hence by evaluating the integrals

\[ C_c = \int_0^{\rho kd} f'_c b \, dx \]

and

\[ k_1 kd = \frac{1}{C_c} \int_0^{\rho kd} (f'_c b \, dx) x \]

the expressions for the concrete compression force and its leverarm coefficient are obtained:

\[ C_c = kbd f'_c [1 - (J+1)e^{-J}]e^{-J} \quad \ldots 3.2-3 \]

and

\[ k_1 = \frac{2 - [J + 2(1+J)]e^{-J}}{J[1 - (J+1)e^{-J}]} \quad \ldots 3.2-4 \]

Where \( J = \frac{\varepsilon_{ct}}{\varepsilon_0} \)
This can also be written
\[ C = kbdf_a \]
Where:
\[ f_a = \text{average concrete stress} \]
\[ = f_c' \left[ 1 - (J+1)e^{-J} \right] e/J \]  
\[ \ldots 3.2-5 \]

This is used in the horizontal equilibrium equation:
\[ H = T - C_c - C_s \]

In the following analysis either \( \varepsilon_{ct} \) is known; that is, \( \varepsilon_{ct} = \varepsilon_{cu} \) at ultimate flexural strength, or \( \varepsilon_s \) is known; i.e. \( \varepsilon_s = \varepsilon_{sy} \) at onset of yielding of tension steel. Thus either the steel strains are given in terms of the concrete strain, or the concrete strain and compression steel strain are given in terms of the tension steel strain to evaluate the moment of resistance of the section.

For \( \varepsilon_{ct} = \varepsilon_{cu} \):
\[ H = A_s f_y - k_u b df_a - A_s' E_s (1-k_u d'/d) \varepsilon_{cu} \]

This produces a quadratic expression in \( k_u \) which is solved to produce:
\[ k_u = \frac{\sqrt{(A_s f_y + A_s' E_s \varepsilon_{cu} + H)^2 + 4b df_a A_s' E_s \varepsilon_{cu} d'/d} - (-A_s f_y + A_s' E_s \varepsilon_{cu} + H))}{2b df_a} \]  
\[ \ldots 3.2-6 \]

Hence:
\[ M_u = A_s f_y (t/2 - d') + k_u b d^2 f_a \left[ .5t/d - k_u (1-k_1) \right] + \]
\[ A_s' f'(t/2 - d') \]  
\[ \ldots 3.2-7 \]

Where \( f_s' = E_s (1-k_d' /d) \varepsilon_{cu} \leq f_y \)
For $\varepsilon_s = \varepsilon_{sy}$:

$$H = A_s f_y - k_y bdf_a - A'_s f_y \left(\frac{k_y - d'/d}{1-k_y}\right)$$

but $f_a = f_c'[1-(F+1)e^{-F}]e/F$

where $F = \frac{\varepsilon_{sy} k_y}{\varepsilon_0 (1-k_y)}$

This can be solved for $k_y$ by approximate methods, but it is too cumbersome for practical calculations. Therefore, $k_y$ is found by the parabolic stress-strain distribution as shown in section (b).

(b) Parabolic stress-strain relationship given by 3.1-4:

$$f_c = 2f'_c(1-G/2)G$$

Where $G = \varepsilon_c/\varepsilon_0$

Using the same approach as above we get:

$$C_c = kbd f'_c (1-J/3)J$$

or $C_c = kbd f_a$

Where:

$$f_a = f'_c (1-J/3)J \quad \ldots 3.2-8$$

Also:

$$k_1 = \frac{2-3J/4}{3-J} \quad \ldots 3.2-9$$

Where $J = \varepsilon_{ct}/\varepsilon_0$

Equations 3.2-6 and 3.2-7, as derived before in the case where $\varepsilon_{ct} = \varepsilon_{cu}$, still apply if the above expressions for $f_a$ and $k_1$ are used.
For $\varepsilon_s = \varepsilon_{sy}$:

Substituting into the horizontal equilibrium equation we get:

$$H = A_f \frac{f}{s_y} - k b d f a - A' \frac{k_y \cdot d'/d}{1 - k_y} f_y$$

Where:

$$f_a = f'F[1-F/3]$$

Substituting and rearranging terms produces the following equation in ",","k_y":

$$k_y^3 b d f \varepsilon (1+\varepsilon_r/3) + k_y^2 (A_s + A'_s - b d f \varepsilon_r H/f_y) - k_y(2A_s + A'_s(1+d'/d) - 2H/f_y) + A_s + A'_s d'/d - H/f_y = 0$$

Where:

$$f_r = f'/f$$

$$\varepsilon_r = \varepsilon_{sy}/\varepsilon_o$$

This can be solved by the approximation technique:

$$(k_y)_{n+1} = (k_y)_n - f(k_y)_n/f'(k_y)_n$$

The moment equation is:

$$M = A_f \frac{f}{y \cdot -d') + k b d^2 f a \left[ \frac{t}{2d} - k_y (1-k) \right] + A'_f \frac{f}{s_y \cdot -d')}{2}$$

Where:

$$f'_s = \frac{k_y \cdot d'/d}{1 - k_y} \varepsilon_{sy} E_s \leq f_y$$
In order to verify the validity of the theory developed in this chapter, a comparison was done with the corresponding expressions as derived from the rectangular stress block at ultimate stress state, applied to the beam used in this investigation.

(i) Ultimate moment from rectangular stress block:
Assuming axial tension in beam as 5Kips, it was found that:

\[ k_u = 0.220 \]
\[ M_u = 165.0K \text{ inches} \]

(ii) Ultimate moment from exponential stress distribution:
axial tension = 5Kips
\[ k_u = 0.212 \]
\[ M_u = 166.2K \text{ inches} \]

(iii) Ultimate moment from parabolic stress distribution:
axial tension = 5Kips
\[ k_u = 0.218 \]
\[ M_u = 166.8K \text{ inches} \]

(iv) For the same axial tension, using the parabolic stress distribution, it was found that at yielding of the tension steel:
\[ k_y = 0.378 \]
\[ M_y = 138.0K \text{ inches} \]

(v) To show the effect of axial tension, the ultimate moment as found from the rectangular stress block for zero axial tension is:
\[ k_u = 0.248 \]
\[ M_u = 180.5K \text{ inches} \]
The ultimate moments found in sections (i), (ii) and (iii) show close agreement. Therefore, the expressions 3.2-6, 3.2-7, 3.2-10 and 3.2-11 were used in the following chapter to determine the bending moments corresponding to any specific stress level in the concrete or reinforcing steel.
4.0 TEST PROCEDURE AND EVALUATION

4.1 Description of tests

For all test frames, except the last, three loading cycles were performed. They consisted of the following sequence: The first cycle was performed with the vertical load position at 5 inches from the beam-column joint, from zero load up to working stress range and back to zero load. The second cycle was performed with the vertical load position at 22 inches from the beam-column joint. Working stress here refers to a shear stress level corresponding to cracking shear. The third cycle was performed with the vertical load position at 5 inches from the beam-column joint up to failure. The last frame was loaded to failure during the first cycle with the vertical load position at 5 inches from the beam-column joint.

Failure was assumed to have occurred when the load increase became erratic, or dropped off, or when the displacements and cracking became severe to the point where continued loading could cause damage to the equipment.

The axial tension was applied to the beams first, by loading the two horizontal jacks until they registered about 5Kips. Then the valve was closed on the south jack to produce an approximately constant force on this side during the subsequent alternate vertical and sway loading. The sway and vertical loading were increased alternately in such a way that the moment at the south joint remained
low while the moment at the north joint kept increasing. Thus two different stress states were created in the two joints. The south joint reached a shear stress slightly lower than the north joint, while the moment on the south joint was kept considerably lower than at the north joint, at the same axial tension force.

The slip was recorded only by the bottom DCDT's since the wedge of concrete underneath the top DCDT's did not move relative to the column. All rotations recorded below are the rotation of a section on the beam 2.5 inches away from the column, with respect to the column. In the following paragraphs the magnitudes of vertical loads are as recorded on each side of the horizontal frame member, or half the force in the jack. Behavior of the frames during the loading to failure was as follows:

**Frame Number 1:**

The frame was loaded horizontally at both ends until a load of 5.5K was reached. No visual observations were made.

The sway-load was then applied up to 2.75K. At the end of this loading stage, tensile cracks became visible at the bottom of the beam under the south vertical load point. This signifies the low moment capacity of the beams to positive moment due to the weaker longitudinal reinforcement on the bottom of the beam.

Vertical load was then applied up to 5.5K on each beam. The cracking did not increase as moments in both joints increased in the negative sense and no other observations were made.
As the vertical load was increased to 6K a crack could be seen on the bottom south interface between column and beam. The crack ran vertically up to about 2/3 of the beam height, then 2 inches toward the vertical load point. At increased vertical loading this diagonal crack lengthened and more diagonal cracks appeared parallel to it. No cracks had yet appeared on the north joint, although it recorded a much higher moment.

With further increase of vertical loading, the north joint also started showing the same pattern of shear cracking. These cracks developed rapidly and both sides showed about 1/2" slip. At this stage the rate of increase of slip was considered extensive and further loading was terminated. Failure was assumed to have taken place.

Frame Number 2:

Both horizontal jacks were loaded to 4K. Vertical load was applied up to 5K. No visual observations were made.

Sway-load was applied up to 2.25K. Tensile cracks opened as in the first frame.

Vertical load was increased again and the south joint started cracking in the same way as the previous frame. Unfortunately the last five observation cycles were lost due to faulty recording. At a vertical load of 23K (2K after recording terminated) the bottom-slip transformers were removed as damage from the slipping beam was expected. At this stage the north joint had also cracked extensively and slip
occurred.

At a vertical load of 24.5K loading was terminated as the load increase became erratic and the slip severe. This test run was remarkably similar to cracking and slip behaviour in the first frame.

Frame Number 3:

This frame, and the next, differed from the first two by having lapped instead of continuous longitudinal reinforcement through the joint.

Both horizontal jacks were loaded to 5.2K. Vertical load was applied up to 5K. No visual observations were made.

Sway-load was applied up to 2.75K. A slight opening of the tensile cracks was observed under the south vertical load point.

The vertical load was again increased, and slight shear cracking was observed at 10K on the south joint.

Figure 6 shows the extent of these cracks at the following load stages:

1. At a vertical load of 11.3K
2. At a vertical load of 17K
3. At a vertical load of 21K

At the vertical load of 21K the first cracks had opened up considerably and visual slip of the south beam relative to its column was noticed. The north joint also started to show slight shear cracking similar to #1 above.

The south joint then showed the following increased cracks:
Plate 5: North crack pattern of Frame 3

Fig. 6 South crack pattern of Frame 3
Scale: 1/4 inch = 1 foot
At a vertical load of 22K

At a vertical load of 23K

The north side also showed some visual slip. The sway load had started to drop at this stage and was down to 2K.

The vertical load was increased further until 29.5K at which stage an apparent maximum had been reached and failure was assumed to have taken place.

**Frame Number 4:**

Both horizontal jacks were loaded to 5.5K.

Sway-load was increased up to 1.5K. No observations were made.

Vertical load was increased up to 15K. At 8K the first shear cracks appeared on the south side along the bottom of the joint. This increased in a similar way as for Frame 3.

Sway-load was increased to 4.75K. This produced the tensile cracking at the bottom of the south beam under the vertical load point, that was observed in previous frames.

The vertical load was increased again. The sway displacement increased to an extent where the oil pressure in the right jack (producing sway-load) dropped off and could not be recovered as it drained the hydraulic hand pump.

At a vertical load of 16.5K the shear cracks on the south joint became clearly defined.

As the vertical load was increased to 21K and the sway-load dropped to zero, the south side cracked extensively and
Fig. 7a South Crack Pattern
Scale: 1/4 inch = 1 foot

Fig. 7b North Crack Pattern
Scale: 1/4 inch = 1 foot

Crack Patterns For Frame 4
visual slip took place while the north side also started cracking.

At 22.5K the rotational deformations had become severe and the top slip transformers were removed to prevent damage. At 24K the cracking of the north side (under negative sway force) had developed to a stage where the beam pendulum plate started to come off due to cracks extending under it. Other cracks appeared at the bottom of the north beam at the load point.

Apparent maximum loads were reached at 29.7K and further loading was terminated. At this stage the cracking had reached the stage shown in Figure 7a & b for the south and north.

Generally the south joint cracked at a lower load and showed larger slips than the north side, although the shear was slightly less and the bending moment was considerably less than the north side. The first two frames with the continuous longitudinal reinforcement showed cracking and slip at a lower load than the last two frames with lapped longitudinal reinforcement, and the final slip values were bigger, and were recorded at smaller ultimate loads. Table III shows the loads and displacements as recorded for each frame at failure.
<table>
<thead>
<tr>
<th>FRAME NUMBER</th>
<th>SHEAR KIPS</th>
<th>MOMENT KIP INCHES</th>
<th>SLIP INCHES</th>
<th>ROTATION DEGREES</th>
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<td>-</td>
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<td>.44</td>
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Table III

JOINT FORCES AND DISPLACEMENTS AT FAILURE
4.2 Correlation and discussion of test results

The plotted test data for the loading cycle to failure of the four frames are compared in this section. In order to explain the irregularities observed in the displacement plots and to correlate theoretical and observed data, the concrete theory as developed in chapter 3.0 was applied. The relevant theoretical expressions were plotted against joint tension in Fig. 8.

To investigate the effect which the yielding of the tension steel has on the displacement observations, the yield moment and corresponding beam rotation between column face and point at which rotation was measured, were evaluated for each frame. The theoretical ultimate moment and the corresponding beam rotation were also evaluated in order to complete the theoretical moment rotation curve.

The rotation calculations are based on constant conditions over the 2.5 inches over which the rotations were measured. This is close enough for comparison for the north side, but can not be applied to the south side since the moment variation is too large, sometimes resulting in curvature reversal inside the 2.5 inch test length. Therefore only the plot of the north side can serve as a comparison with the theoretical results.

The loads and corresponding joint tension, joint shear and joint bending moment are as follows:

\[ P = \text{vertical load on each beam} \]
Fig. 8 Theoretical bending moment and beam rotation curves
HL = horizontal load applied at each end of the frame
S = horizontal sway-load applied at the north side of
    the frame only
H = joint tension = HL + S/2 - P x (a+c/2)/h
V = joint shear = P - S x h/l for south side
    = joint shear = P + S x h/l for north side
M = joint bending moment P x a - S(1-c)h/(2x1) for south side
    = joint bending moment P x a + S(1-c)h/(2x1) for north side

Frame Number 1:

Fig.9 shows the applied loading curve as plotted
against the observation numbers.

Fig.10, the SLIP-SHEAR plot, shows a rapid increase in
slip after reaching a shear of 13 Kips for the south side
and a similar increase in slip after reaching a shear of
18 Kips for the north side. The curves are parallel over
the region where most of the slip takes place, ending with
almost the same failure shear.

Fig.11, the SLIP-MOMENT plot, shows the rapid increase
of slip for the south side occurs at a point where the
bending moment is quite small (35 to 50 Kip inches). On
the north side the rapid increase of slip coincides with the
moment at which the tension steel starts to yield
(My = 138K inches at a joint tension of 5.5K) and progresses
rapidly even after the moment has fallen off slightly.

Fig.12 shows the ROTATION-MOMENT plot. The south side
shows an erratic response due to the moment reversal and
Fig. 9 Loading curves for Frame 1

LOADING IN KIPS

OBSERVATION NUMBERS

Fig. 9 Loading curves for Frame 1
Fig. 10 SLIP-SHEAR plot for Frame 1
Fig. 11 SLIP-MOMENT plot for Frame 1

- Slip-Moment plot
- Vertical Slip (inches)
- Bending Moment (kip in.)
- Sign convention: +SLIP for south, -SLIP for north
Fig. 12 ROTATION-MOMENT plot for Frame 1
the summing of positive and negative curvature mentioned above. The north side shows a somewhat flatter curve for the recorded data than expected from the theoretical plot.

Frame Number 2:

Fig.13 shows the applied loading curve as plotted against the observation numbers.

Fig.14, the SLIP-SHEAR plot, and Fig.15, the SLIP-MOMENT plot, show a rapid increase in slip for the south side after reaching a shear of 18 Kips and a moment of 50 Kip inches. The north side shows a rapid increase in slip only after reaching a shear of 25 Kips and a moment of over 180 Kip inches. The last part of the records were lost due to faulty equipment so that the data corresponding to the observed visual cracking of the north side could not be plotted.

The MOMENT-ROTATION curves, Fig.16, are very similar to the curves observed from Frame 1.

Frame Number 3:

Fig.17 shows the applied loading curve as plotted against the observation numbers.

Fig.18, the SLIP-SHEAR plot, and Fig.19, the SLIP-MOMENT plot, show a rapid increase in slip for the south side after reaching a shear of 13 Kips and a moment of 25 Kip inches. The north side shows a rapid increase in slip only after reaching a shear of 24 Kips and a moment of 163 Kip inches. The tension steel starts to yield at a moment of
Fig. 13 Loading curves for Frame 2
Fig. 14 SLIP-SHEAR plot for Frame 2
Fig. 15 SLIP-MOMENT plot for Frame 2
Fig. 16 ROTATION-MOMENT plot for Frame 2
Fig. 17 Loading curves for Frame 3
Fig. 18 SLIP-SHEAR plot for Frame 3
Fig. 19 SLIP-MOMENT plot for Frame 3
Fig. 20 ROTATION-MOMENT plot for Frame 3
143K inches for a joint tension of 2.4K. The final shear carried by the two joints is again almost the same. Fig.20 shows that the ROTATION-MOMENT plot is again almost identical with the response obtained from the first two frames.

Frame Number 4:

Fig.21 shows the applied loading curve. During the last stage of loading the sway-load was reversed and the joint tension increased up to 10K. This resulted in a different displacement response in the final load stage.

Fig.22, the SLIP-SHEAR plot, and Fig.23, the SLIP-MOMENT plot, again show a rapid increase in slip for the south side after reaching a shear of 14K and a moment of 40K inches. The north side shows a rapid increase in slip only after reaching a shear of 20K and a moment of 170K inches. The tension steel yield moment is 136K inches for a joint tension of 6K.

Fig.24, the ROTATION-MOMENT plot, shows a response as before for the north side up to the point where the sway-load was reversed. The reversal of sway-load (and thus sway-moment) resulted in a decrease of rotation on the north side and an increase of rotation on the south side during the final stages of loading. The moment reversal did not affect the slip response, as the final slip and shear is again almost the same for both joints. Since the failure shear and slip results of this frame, under a much larger joint tension, were of the same magnitude as for the first three frames, it must be concluded that the joint tension has no
Fig. 21 Loading curves for Frame 4
Fig. 22 SLIP-SHEAR plot for Frame 4
Fig. 23 SLIP-MOMENT plot for Frame 4
Fig. 24 ROTATION-MOMENT plot for Frame 4
major effect on the ultimate shear transfer capacity. A joint tension of 10K corresponds to an average concrete stress of 185psi or a steel stress of 12000psi in the test beam. Since the modulus of rupture is about 200psi for the beam concrete, 10 Kips constitutes an upper limit of joint tension which should not be exceeded under normal action of shrinkage and temperature variation.

To relate the main observations made with respect to critical shear, an interaction plot was attempted, Fig. 25, which clearly shows critical shear differences between low and high moment-shear failures. Critical shear in this case was defined as the shear which initiates slip and cracking of the joint. Table IV shows the loads and displacements as observed at critical shear for each frame. In order to explain the two apparent shear failure modes, the physical components which transfer the shear have to be investigated in more detail.

The shear is transferred across the joint by the following three structural components:
1) beam tension steel
2) beam compression steel
3) beam concrete

Without more detailed experimental tests no magnitudes of shear can be allocated to these components. Since the mechanism of the dowel action of the tension (top) steel is different from the dowel action of the compression (bottom) steel, no generalization is possible. The test results show that the shear transfer capacity of the joints
is higher in the presence of a large bending moment than for a small bending moment. Since an axial tension was induced in the beam, enough compression was not developed in the concrete to transfer a significant amount of shear (shear friction hypothesis (4), (5)). But under the couple of a large bending moment, the compression block in the beam concrete, enhanced due to the confining forces of top and bottom steel and stirrups (axial compression capacity in the presence of lateral confinement (11)), could transfer a significantly higher amount of shear to the column. This tendency was observed in all four frames, especially during the initial stages of joint cracking. Once the joint concrete has cracked to a higher degree, as shown in the previous chapter, only the dowel action of the steel remained and in all cases the shear transferred at ultimate approached the same value of about 30 Kips irrespective of moment.
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<th>FRAME NUMBER</th>
<th>SHEAR KIPS</th>
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<th>SLIP INCHES</th>
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<td>+.12</td>
</tr>
<tr>
<td>4-North</td>
<td>20.0</td>
<td>170.0</td>
<td>.010</td>
<td>-.24</td>
</tr>
</tbody>
</table>

**Table IV**

JOINT FORCES AND DISPLACEMENTS AT CRITICAL SHEAR
Fig. 25 Moment-shear failure interaction plot
4.3 Conclusion

The joint interface was purposely made smooth to obtain uniform test conditions and to produce the worst shear-transfer condition with regard to the concrete. This joint still showed a considerable shear-transfer capacity, even under adverse loading combinations. To calculate the ultimate shear transfer capacity of such a joint, taking into account all the factors contributing to it, is not possible from the limited results obtained from this investigation. It is felt, however, that the major observations described in the previous chapters clearly indicate the following:

1) The type of joint investigated can transfer the full beam shear to an adjacent column.

2) Shear failure in the form of combined diagonal tension cracking and vertical joint slip occurs at two levels of shear-moment action. One develops under the action of a low moment and fairly low shear force. The second develops under the action of a much higher moment, which apparently coincides with the yielding of the tension steel, and a considerable shear force. The ultimate shear, however, reaches approximately the same value in both cases.

3) Lapping of the longitudinal reinforcement has no detrimental effect on the joint capacity. It was even observed that the first two frames showed slightly higher slip deflections at lower loads than the last two frames which were reinforced with lapped longitudinal reinforcement.
4) The shear transfer capacity of the joint is not appreciably affected by the presence of a joint tension.

More detailed tests are required to produce a failure envelope under the action of the type of loading which was applied in this investigation. More tests are also required to find the contribution of the reinforcing steel and the concrete to the shear capacity of the joint.
References

1. SAEMANN, J.C. and WASHA, G.W.  
   "Horizontal Shear Connections Between Precast Beams and Cast-In-Place Slabs", ACI, November 1964.

2. BADOUX, J.C. and HULSBOS, C.L.  

3. KRIZ, L.B. and RATHS, C.H.  
   "Connections in Precast Concrete Structures-Strength of Corbels", PCI, February 1965.

4. BIRKELAND, P.W. and BIRKELAND, H.W.  

5. MAST, R.F.  
   "Auxiliary Reinforcing in Concrete Connections", ASCE, June 1968.

6. KREFELD and THURSTON.  

7. KREFELD and THURSTON.  
   "Shear and Diagonal Tension Strength of Simply Supported Reinforced Concrete Beams", ACI, March 1966.

8. HANSON, N.W. and CONNOR, H.W.  

9. HOFBECK, J.A., IBRAHIM, I.O. and MATTOCK, A.H.  
   "Shear Transfer in Reinforced Concrete", ACI, February 1969.

10. SMITH, R.B.C.  
    "Interaction of Moment and Shear on the Failure of Reinforced Concrete Beams Without Web Reinforcement", CIVIL ENGINEERING & PUBLIC WORKS REVIEW (LONDON), June, July and August, 1966.

11. PORTLAND CEMENT ASSOCIATION.  
    DESIGN OF MULTI-STORY REINFORCED CONCRETE BUILDINGS FOR EARTHQUAKE MOTIONS, 1961.
12. AMERICAN CONCRETE INSTITUTE.  
   BUILDING CODE REQUIREMENTS FOR REINFORCED CONCRETE,  
   ACI (318-1963)  

13. ZSUTTY, T.C.  
   "Beam Shear Strength Predictions by Analysis of  
   Existing Data", ACI, November 1968.  

14. BURTON, CORLEY and HOGNESTAD.  
   "Connections in Precast Concrete Structures-  
   Effects of Restrained Creep and Shrinkage",  
   PCI, April 1967.  

15. MATTOCK, A.H.  
   "Creep and Shrinkage Studies", PCA RESEARCH &  
   DEVELOPMENT LABORATORY, May 1961.  

16. COHEN, M.Z.  
   "Rotation Compatibility in the Limit Design of  
   Reinforced Concrete Continuous Beams",  
   REINFORCED CONCRETE SYMPOSIUM.  

17. YOUNG, L.E. and SMITH, G.M.  
   "Ultimate Flexural Analysis Based on Stress-Strain  
   Curves of Cylinders", ACI, December 1956.  

18. SAENZ, L.P.  
   Discussion of "Equation for the Stress-Strain  
   Curve of Concrete by P. Desayi and Krishnan",  
   ACI, September 1964.
APPENDIX: STRESS-STRAIN CURVES OF TEST SPECIMENS
Stress-strain diagram for 3/8 inch diameter reinforcing steel

- ult. 73.6 ksi (turned down to Ø = 0.29 in.)
- ult. 72.9 ksi (turned down to Ø = 0.253 in.)
- ult. 72.8 ksi (Ø = 3/8 in. nominal)
Stress-strain diagram for 5/8 inch diameter reinforcing steel

Ultimate stress values:
- 87.2 ksi (Ø = 5/8 in. nominal)
- 78.5 ksi (turned down to Ø = 0.423 in.)
- 72.4 ksi (turned down to Ø = 0.42 in.)
- 72.8 ksi (turned down to Ø = 0.42 in.)
- 73.9 ksi (turned down to Ø = 0.447 in.)
Stress-strain plot of 6 test cylinders taken from beam-concrete and superimposed theoretical curves
Stress-strain plot of 6 test cylinders taken from column-concrete and superimposed theoretical curves