cl

LARGE DEFLECTION ANALYSIS OF SHALLOW FRAMED STRUCTURES

by

BRIAN A. RADOMSKE

B.A.Sc. University of British Columbia, 1968

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

in the Department

of

Civil Engineering

We accept this thesis as conforming to the required standard:

THE UNIVERSITY OF BRITISH COLUMBIA

August 1972

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the Head of my Department or by his representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Civil Engineering

The University of British Columbia
Vancouver 8, Canada

Date September 11, 1972.

ABSTRACT

Elastic structures exhibit instabilities which arise through the occurrence of finite displacements even when constitutive properties remain linear. A non-linear analysis which recognizes rotations in the strain displacement relationship is formulated for analyzing three-dimensional framed structures.

A finite element method is used whereby the rotations within each element are restricted in size by use of a local element reference frame attached to the element. Two such coordinate systems are developed. Then an incremental solution technique based on an instantaneous linearization of a Taylor series expansion of the forces about the displacement configuration at the beginning of each increment is developed.

The snap-through buckling of shallow frames, arches and domes is studied with a view to documenting the effect on the equilibrium paths of the type of moving coordinate frame, the number of elements, and the size of the increment step.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGEMENTS

I am grateful for all the help and guidance given to me by my supervisor, Dr. N.D. Nathan.

I also would like to thank the National Research Council of Canada for financial assistance.

CHAPTER I

INTRODUCTION

Classical structural analysis implies a unique solution to every structural problem since the theory is based on infinitesimal linear displacements. In fact, real structures exhibit instabilities associated with non-unique solutions and, in order to detect these, it is necessary to introduce non-linear analysis.

Instabilities may arise through non-linear material properties or through the occurence of finite displacements even when the constitutive properties remain elastic. We confine our attention to the latter, the elastic instabilities associated with finite displacements.

Elastic instabilities can be of three kinds:

1. Bifurcation,
2. Snap-through,
3. Finite Disturbance.

The first may be detected by the solution of the classical eigenvalue problem which has been formulated to include rotations of infinitesimal elements in the equilibrium equations of elasticity. Recognition of the second and third requires a solution technique capable of tracing the equilibrium path after the advent of finite displacements. This necessitates the recognition of rotations in the strain displacement relationships. Frame structures exhibit either bifurcation or snap-through buckling, and we limit ourselves here to the study of such structures.

In the finite element analysis of frame structures, large rotations may be dealt with in one of two methods:

1. The member properties may be deduced, with

respect to a reference frame fixed in space, on the basis of the full non-linear strain-displacement relations.

2. The member properties may be deduced, with respect to a moving reference frame attached to the members in question. Then, by subdividing the structure into a sufficient number of members or "elements", we may restrict the rotations within each element relative to its own reference frame to any desired extent.

The latter approach is used here. Member properties are based on the assumption of small rotations and strains which are small compared to the rotations. The assumptions are adequate for the detection of bifurcation points in the equilibrium path derived from classical elasticity, but when finite displacements are studied, the moving coordinate systems as described above are required. Non-linear problems are then solved by an incremental procedure based on an instantaneous linearization within each increment of a Taylor series expansion of the force vector about the displacements at the beginning of the increment.

Early work on the theoretical analysis of elastic post-buckling was performed by Koiter (2). His work, using a continuum mechanics approach, centered on the investigation of imperfection sensitive structures. Britvec and Chilvers (5) developed matrix methods based on a potential energy formulation to analyze the initial post-buckling curves of rigidly-jointed plane frames. Martin (6), Supple (7), and Roorda (8), (9), have also done research into initial slopes of post-buckling paths with and without imperfections.

Snap-through buckling of shallow arches and plane frames have been studied by Argyris (1), Jennings (4) and Williams (10). Ebner and Ucciferro (12) compared several finite element methods and their applications to geometrically nonlinear structural problems. Their work was confined to planar structures only. A complete summary of the finite element analysis of nonlinear structures is given by Mallett and Marcal (3). The present work takes element stiffness matrices from Nathan (11).

Two-dimensional structures are studied with a view to documenting computational experience. Snap-through buckling of frames and arches is investigated with respect to factors such as element size and number, increment size, and choice of element moving reference system.

The necessary relationships are developed for extending the work of previous investigators into three-dimensions. A simple three-dimensional space dome element is studied and results compared to theoretical work by Wright (13). The snap-through buckling of a large ring dome is then studied. No attempt is made to study bifurcation buckling of the structures presented. Such modes of instabilities are prevented from occurring.

CHAPTER II

COORDINATE SYSTEMS

1. Configuration Space

We restrict the class of structures being studied to plane and space frames, i.e. where the length of each member predominates over its width and thickness. The structure can then be subdivided into any number of line elements connected to each other at "nodes". Displacements of all points within each element are completely determined by the displacements of the nodes. Thus, if there are a total of n nodal displacements or "degrees of freedom" defined by the vector $\underline{r}$, the configuration of the structure will be completely determined by the position of the point with coordinates $\underline{r}$ in an n-space, the so-called "configuration space".

We are at liberty to choose the locations of our nodes and thus of the number and sizes of our elements. Such choice as this represents the essential step in the mathematical idealization of the structure. Obviously there must be sufficient constraints on the degrees of freedom to provide for global equilibrium of the structure as a whole.

We begin by defining the necessary coordinate systems in which we measure displacements and forces and then we relate these reference frames to each other.

We require a global cartesian reference frame (X) with base vectors $\hat{\underline{X}}$ whose origin and axes orientations are quite arbitrarily fixed in space. We measure all nodal coordinates and global forces in this fixed frame.

Relative to the (X) frame we envision an arbitrary member JK , connecting nodes j to k; at each of these two nodes are fixed local global reference frames. These two frames, termed the (x^j) and (x^k) frames with base vectors $\underline{\hat{x}}^j$ and $\underline{\hat{x}}^k$ respectively, have origins fixed, with reference to the $(\underline{X})$ frame, by position vectors $\underline{P}^j$ and $\underline{P}^k$ respectively. The position vectors describe the distance from the $(\underline{X})$ frame origin to the initial position of the nodes j and k respectively. The (x^j) and (x^k) frame axes are fixed parallel to the global reference frame $(\underline{X})$ axes. Within these local global frames we measure the nodal deformations of the structure $\underline{r}$. The rotations $\underline{r}^r$ are taken about the original $(\underline{X})$ axes in a defined order and the translations $\underline{r}^t$ are in the global axes directions where:

$$\underline{r}^t = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_7 \\ r_8 \\ r_9 \end{bmatrix}$$

and

$$\underline{r}^r = \begin{bmatrix} r_4 \\ r_5 \\ r_6 \\ r_{10} \\ r_{11} \\ r_{12} \end{bmatrix}$$

At node j of member JK we define an initial local reference frame named (y°) with base vectors $\hat{\underline{y}}^\circ$. The coordinate axes of this frame are defined by the orientation of the undeformed member itself; the base vectors $\hat{\underline{x}}^j$ of the (x^j) frame are related to the $\hat{\underline{y}}^\circ$ base vectors of (y°) by an orthonormal transformation $\underline{\psi}^\circ$:

$$\hat{\underline{y}}^\circ = \underline{\psi}^\circ \; \hat{\underline{x}}^j$$

where $\underline{\psi}^\circ$ is a matrix function of the constant angular rotations $(r_4^\circ \;\; r_5^\circ \;\; r_6^\circ)^T$ defining the initial orientation of the member. The derivation of the $\underline{\psi}^\circ$ transformation and the determination of the initial angular rotations $(r_4^\circ \;\; r_5^\circ \;\; r_6^\circ)^T$ will be covered in Chapter III.

Now we must relate the position of the deformed member JK to the local global reference frames (x^j) and (x^k) by two moving coordinate systems (y^j) and (y^k) with base vectors $\hat{\underline{y}}^j$ and $\hat{\underline{y}}^k$ affixed to nodes j and k respectively. The base vectors $\hat{\underline{x}}^j$ are related to the base vectors $\hat{\underline{y}}^j$ by an orthogonal transformation $\underline{\psi}^j$ whose derivation follows in Chapter III:

$$\hat{\underline{y}}^j = \underline{\psi}^j \; \hat{\underline{x}}^j$$

where $\underline{\psi}^j$ is a matrix function of the large global rotations $(r_4^* \;\; r_5^* \;\; r_6^*)^T$ which correspond to $(r_4^\circ \;\; r_5^\circ \;\; r_6^\circ)^T$ and which define the current orientation of the member tangent at node j. Similarly, at node k we have

$$\hat{\underline{y}}^k = \underline{\psi}^k \; \hat{\underline{x}}^k$$

where $\underline{\psi}^k$ is a matrix function of large rotations $(r_{10}^* \;\; r_{11}^* \;\; r_{12}^*)^T$.

In order to evaluate the $\underline{r}^*$ we require to add the effect of deformations to the angles defining the initial orientation of the member. However, $\underline{r}^\circ$ and $\underline{r}^r$ are not directly additive and therefore we introduce intermediate measures of deformations $\bar{\underline{r}}$. These are peculiar to a given member and are given by $\bar{\underline{r}} = \underline{r}^* - \underline{r}^\circ$, where

$$\bar{\underline{r}} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4^* - r_4^\circ \\ r_5^* - r_5^\circ \\ r_6^* - r_6^\circ \\ r_7 \\ r_8 \\ r_9 \\ r_{10}^* - r_4^\circ \\ r_{11}^* - r_5^\circ \\ r_{12}^* - r_6^\circ \end{bmatrix}$$

The $\bar{\underline{r}}$ are related to the structure nodal deformations $\underline{r}$ by an incremental relationship

$$\delta \bar{\underline{r}} = \underline{b} \; \delta \underline{r}$$

where

$$\delta \underline{r} = \begin{Bmatrix} \delta r_1 \\ \delta r_2 \\ \delta r_3 \\ \delta r_4 \\ \delta r_5 \\ \delta r_6 \\ \delta r_7 \\ \delta r_8 \\ \delta r_9 \\ \delta r_{10} \\ \delta r_{11} \\ \delta r_{12} \end{Bmatrix}$$

The global translations $\bar{\underline{r}}^t$ for the element are equivalent to the structure nodal translations of $\underline{r}$ but the element rotations $\bar{\underline{r}}^r$ can be related to the structure nodal rotations of $\underline{r}$ only by the incremental equation given above, where

$$\bar{\underline{r}}^t = \begin{Bmatrix} \bar{r}_1 \\ \bar{r}_2 \\ \bar{r}_3 \\ \bar{r}_7 \\ \bar{r}_8 \\ \bar{r}_9 \end{Bmatrix}$$

and

$$\underline{\bar{r}}^{r} = \begin{bmatrix} \bar{r}_4 \\ \bar{r}_5 \\ \bar{r}_6 \\ \bar{r}_{10} \\ \bar{r}_{11} \\ \bar{r}_{12} \end{bmatrix}$$

Each member has associated with it a local moving coordinate system in which element displacements are measured and in which the stiffness matrix is formulated. The local moving frame base vectors are functions of the global deformations $\underline{r}$.

The element stiffness matrix can be transformed to the $(\underline{X})$ system and thus, ultimately, the response of the member in global coordinates can be related to the response of the structure in local element coordinates.

2. The Global Coordinate Systems

A right-handed cartesian reference frame called the structure global coordinate system $(\underline{X})$ is shown in Figure 2.1. This arbitrary reference frame remains fixed in space and all geometry ultimately relates to this coordinate system. Let $\underline{P}^j$ and $\underline{P}^k$ be vectors defining the position of nodes j and k of member JK respectively where:

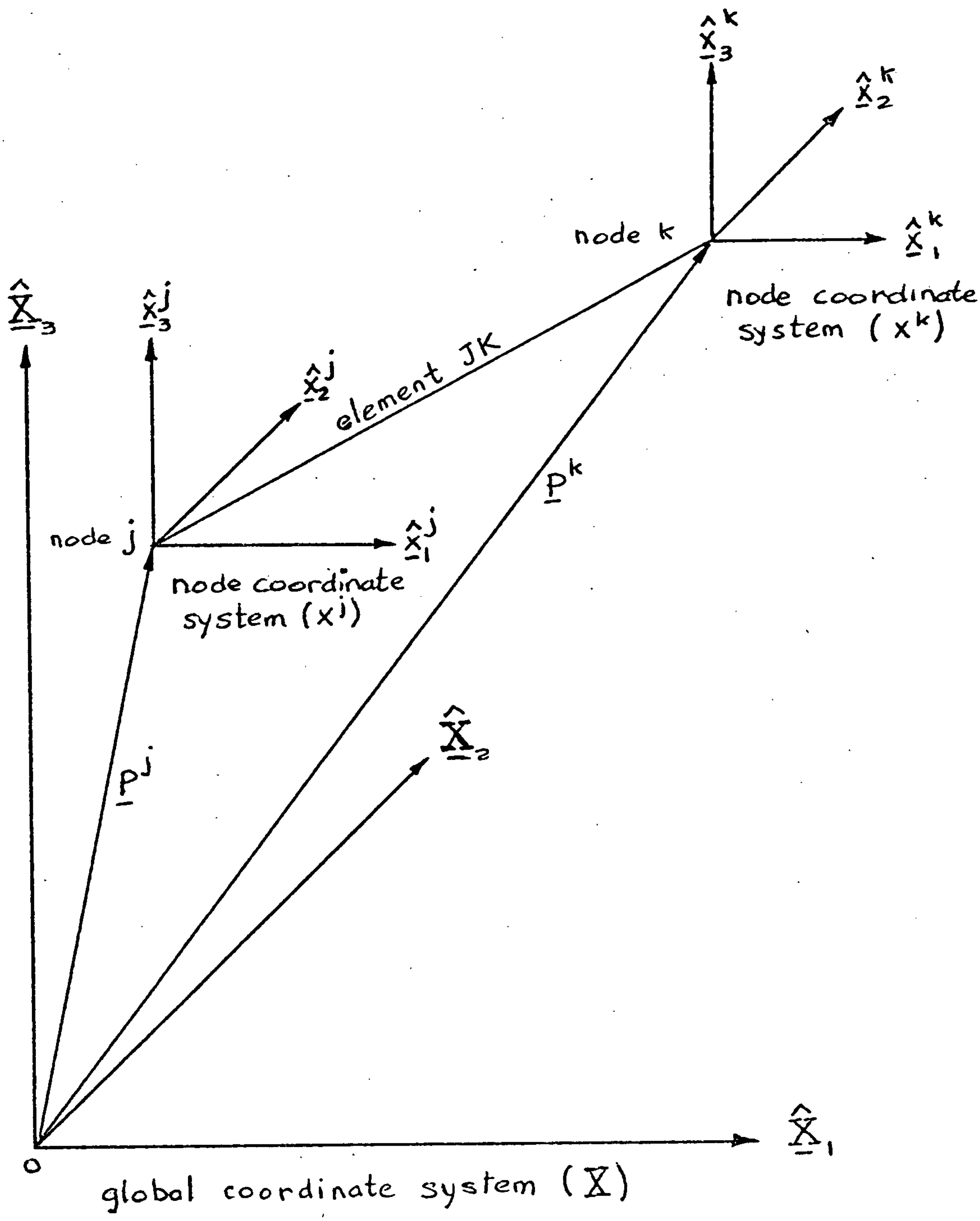


FIGURE 2.1 GLOBAL COORDINATE SYSTEMS

$$\underline{P}^j = \begin{pmatrix} P_1^j \\ P_2^j \\ P_3^j \end{pmatrix}$$

and (2.1)

$$\underline{P}^k = \begin{pmatrix} P_1^k \\ P_2^k \\ P_3^k \end{pmatrix}$$

If $\underline{P}^j$ and $\underline{P}^k$ are initial position vectors, then the straight line element JK has vector direction $\underline{P}^k - \underline{P}^j$. Node coordinate systems (x^j) and (x^k) defined at nodes j and k respectively have base vectors $\hat{\underline{X}}^j$ and $\hat{\underline{X}}^k$ where:

$$\hat{\underline{X}}^j = \begin{pmatrix} \hat{\underline{X}}_1^j \\ \hat{\underline{X}}_2^j \\ \hat{\underline{X}}_3^j \end{pmatrix}$$

and (2.2)

$$\hat{\underline{X}}^k = \begin{pmatrix} \hat{\underline{X}}_1^k \\ \hat{\underline{X}}_2^k \\ \hat{\underline{X}}_3^k \end{pmatrix}$$

The components of the $\hat{\underline{X}}^j$ and $\hat{\underline{X}}^k$ are themselves vectors, they define the vector direction of each of the coordinate axes. We assert that the base vectors $\hat{\underline{X}}$, $\hat{\underline{x}}^j$, $\hat{\underline{x}}^k$ are the same since the $\hat{\underline{x}}^j$ and $\hat{\underline{x}}^k$ base vectors remained fixed in orientation throughout all and any load displacement history of the structure. The initial position vectors $\underline{\rho}^j$ and $\underline{\rho}^k$ fix the origins of the (x^j) and (x^k) systems. We measure the nodal degrees of freedom $\underline{r}$ or structure (global) generalized displacement coordinates in these node coordinate systems.

3. Element Coordinate Systems

Structural problems involving finite displacements can be solved, as previously stated, by one of the following procedures. Firstly, we can include the effects of finite rotations within the element boundaries when deriving the element stiffness. Secondly, we can subdivide the structure into many elements so as to reduce the rotations and translations of the element to within small acceptable bounds. A less refined element stiffness is used and the problem of the large rotations is handled by the use of moving member coordinate systems fixed to the elements in question.

The latter procedure is employed in this work. We use an incremental load and displacement technique to follow the load-displacement behaviour of the structure. At each increment step we recalculate the stiffness of each element and reassemble the global stiffness, an instantaneous stiffness tangent to the real load-displacement surface. We define a moving member coordinate system which is fixed to each element, and which moves through the global displacements $\underline{r}$. In fact, two such element coordinate systems - the first called the tangential reference frame, and the second called the secant reference frame are studied herein.

The tangential reference frame (y^t) has unit base vectors given by $\hat{\underline{y}}^t$ where:

$$\hat{\underline{y}}^t = \begin{bmatrix} \hat{\underline{y}}_1^t \\ \hat{\underline{y}}_2^t \\ \hat{\underline{y}}_3^t \end{bmatrix} \tag{2.3}$$

The (y^t) frame has its origin at node j of the element JK. The $\hat{\underline{y}}_1^t$ base vector is tangent to the centroidal axis of the element at node j. The $\hat{\underline{y}}_2^t$ base vector is coincident with the major principal axis of the element cross section at node j while the $\hat{\underline{y}}_3^t$ base vector is coincident with the minor principal axes. Stiffness matrices are presented for rectangular cross sections only. Figure 2.2 illustrates this reference frame. All element deformations in this system will be defined by the displacements relative to these reference axes.

The secant reference frame (y^s) has unit base vectors given by $\hat{\underline{y}}^s$ where:

$$\hat{\underline{y}}^s = \begin{bmatrix} \hat{\underline{y}}_1^s \\ \hat{\underline{y}}_2^s \\ \hat{\underline{y}}_3^s \end{bmatrix} \tag{2.4}$$

In this frame the $\hat{\underline{y}}_1^s$ base vector is defined by the vector joining node j to node k. The principal axes of the cross section at node j are not in general normal to the y_1^s axis. Arbitrary definitions are therefore

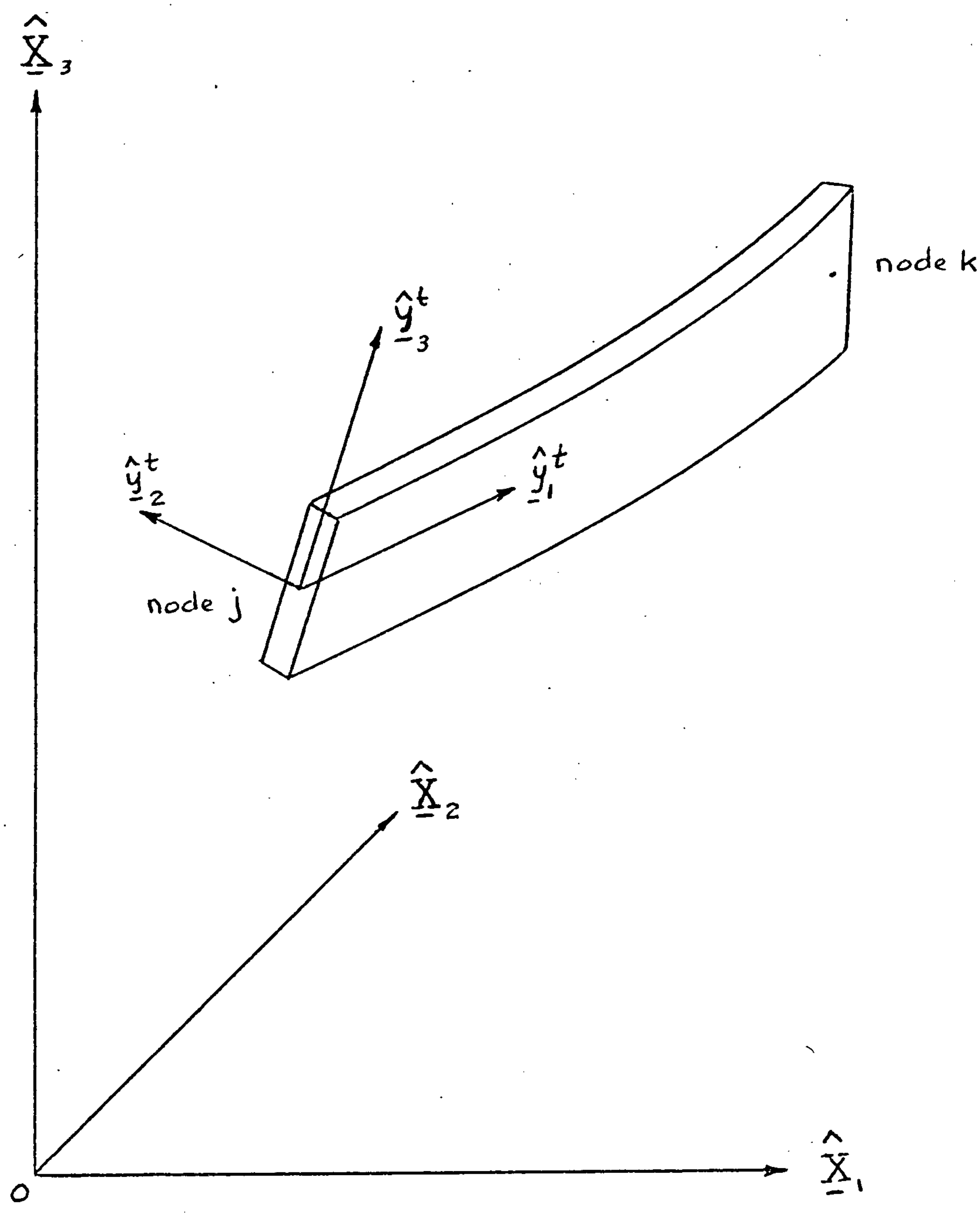


FIGURE 2.2 TANGENTIAL ELEMENT COORDINATE SYSTEM

given below for the directions of the coordinate axes y_2^s and y_3^s. After given displacements at nodes j and k we can calculate the base vector as:

$$\hat{\underline{y}}_1^s = \frac{(\bar{a}_1 + r_7 - r_1 \quad \bar{a}_2 + r_8 - r_2 \quad \bar{a}_3 + r_9 - r_3)^T}{\sqrt{(\bar{a}_1 + r_7 - r_1)^2 + (\bar{a}_2 + r_8 - r_2)^2 + (\bar{a}_3 + r_9 - r_3)^2}} \tag{2.5}$$

where $\bar{\underline{a}} = (\bar{a}_1 \ \bar{a}_2 \ \bar{a}_3)^T$ are the projections on the global axes of the initial member JK. Figure 2.3 shows the secant coordinate frame. We define the other base vectors by Equations 2.6 and 2.7.

$$\hat{\underline{y}}_2^s = \frac{\hat{\underline{y}}_3^t \times \hat{\underline{y}}_1^s}{\sqrt{(\hat{\underline{y}}_3^t \times \hat{\underline{y}}_1^s) \cdot (\hat{\underline{y}}_3^t \times \hat{\underline{y}}_1^s)}} \tag{2.6}$$

$$\hat{\underline{y}}_3^s = \hat{\underline{y}}_1^s \times \hat{\underline{y}}_2^s \tag{2.7}$$

In this coordinate system the element deformations are described by displacements measured at both nodes of the element.

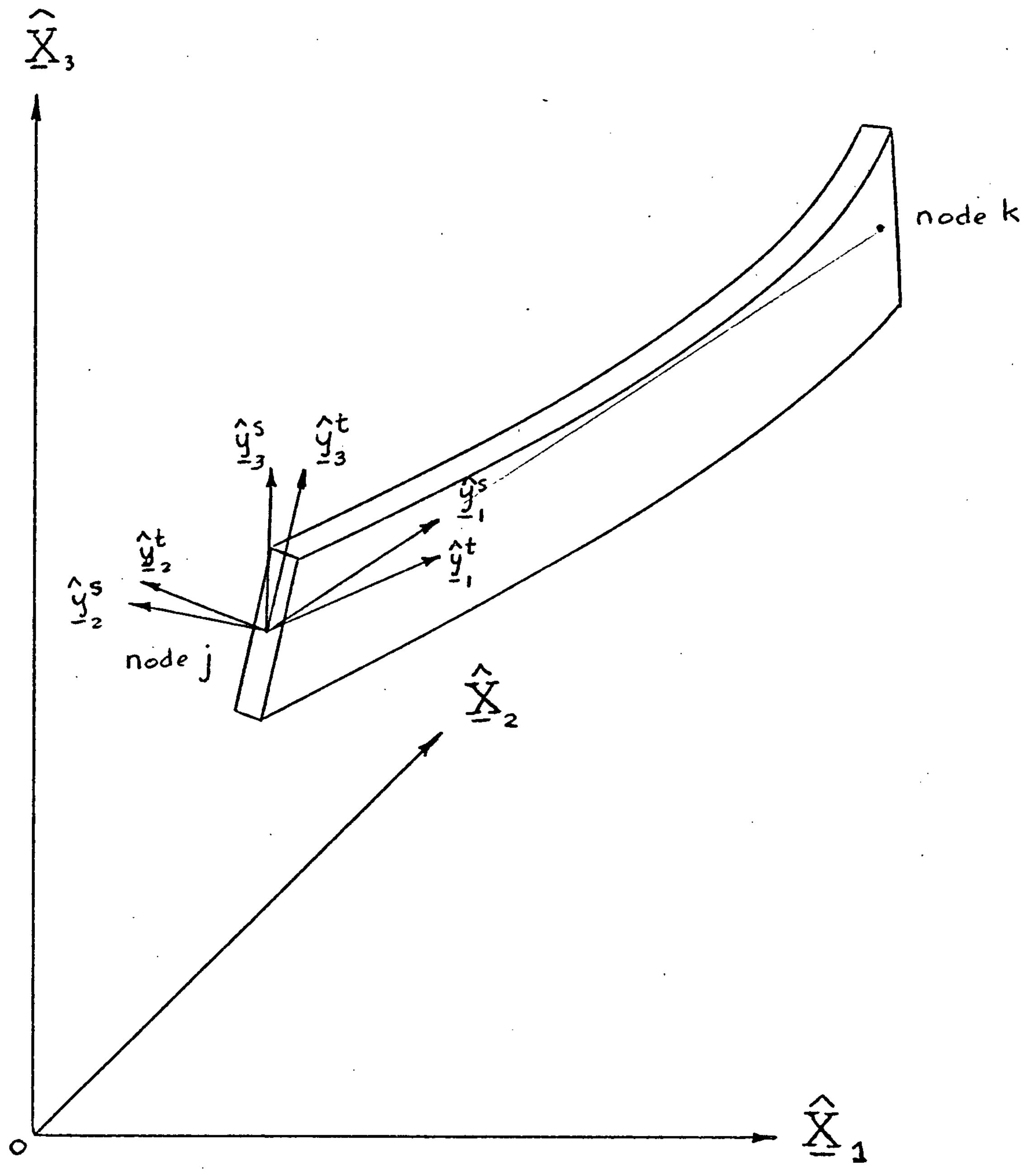


FIGURE 2.3 SECANT ELEMENT COORDINATE SYSTEM

CHAPTER III

DISPLACEMENT RELATIONSHIPS

1. Global Degrees of Freedom for an Element

Each element has twelve global degrees of freedom, six at each node, which are defined by the corresponding structure degrees of freedom $\underline{r}$ at that node. At each node there are three translational degrees of freedom acting along the (x) coordinate system axes and three rotational degrees of freedom whose axes of rotation depend upon the orientation of the member in space. Since the intention is to deal with finite rotations, the definitions of these axes of rotation is rather complex. Figure 3.1 shows the six degrees of freedom for node j of an element JK where single arrows denote translations and double arrows denote rotations.

We call the six element translational degrees of freedom $\bar{\underline{r}}^t$ where $\bar{\underline{r}}^t$ is defined in Chapter II as

$$\bar{\underline{r}}^t = \begin{Bmatrix} \bar{r}_1 \\ \bar{r}_2 \\ \bar{r}_3 \\ \bar{r}_7 \\ \bar{r}_8 \\ \bar{r}_9 \end{Bmatrix}$$

These translations can be related to the element or local displacements of the element by a vector transformation. However, the finite global rotations $\bar{\underline{r}}^r$ where $\bar{\underline{r}}^r$ is defined in Chapter II as

$$\bar{\underline{r}}^r = \begin{Bmatrix} \bar{r}_4 \\ \bar{r}_5 \\ \bar{r}_6 \\ \bar{r}_{10} \\ \bar{r}_{11} \\ \bar{r}_{12} \end{Bmatrix}$$

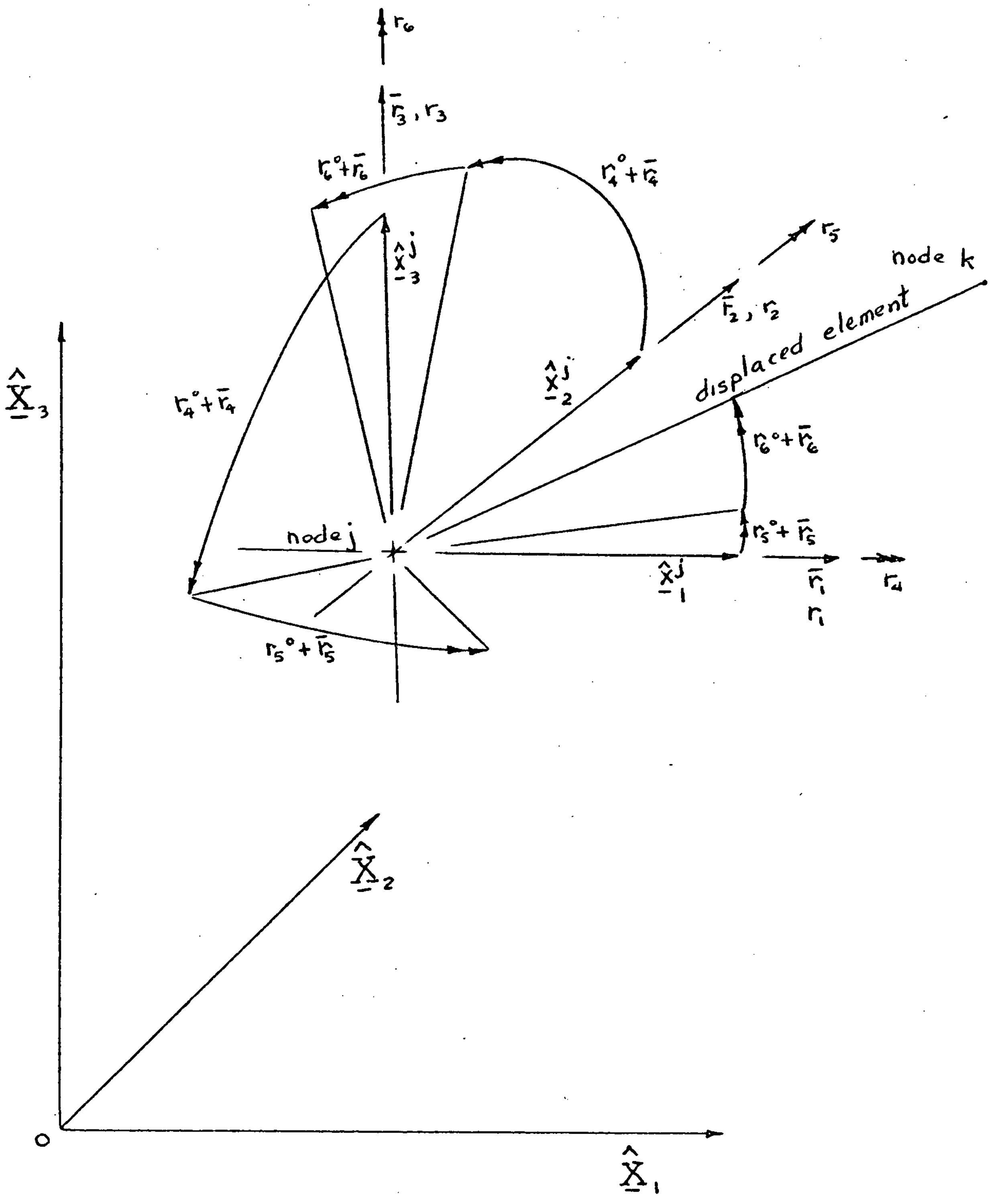


FIGURE 3.1 GLOBAL DEGREES OF FREEDOM AT NODE j

do not transform as vectors. Problems of this type can be handled by specifying a particular order for the rotations or by the use of Euler angles. However, since there is no single system of Euler angles covering a whole sphere without ambiguity, the former method will be used here. It is to be noted that three finite rotations taken in an order different from that specified will lead to a different orientation in space.

Therefore, the rotations at each node of our element will be defined as having occurred in a specific order and this order will be adhered to throughout the development of this thesis. It is asserted that this will lead to no insurmountable difficulties provided care is used in problem input and interpretation of results. Ambiguities can thus be avoided.

2. Transformation Matrices $\underline{\psi}^j$ and $\underline{\psi}^k$

The $\underline{\psi}^j$ matrix was introduced in Chapter II as a matrix function of rotations relating the base vectors of two coordinate systems, the $\underline{\hat{x}}^j$ of the (x^j) frame and an arbitrary (y^j) frame with base vectors $\underline{\hat{y}}^j$. The $\underline{\psi}^j$ will be the transformation of the $\underline{\hat{x}}^j$ vectors into the $\underline{\hat{y}}^j$ vectors at node j of element JK.

$$\underline{\hat{y}}^j = \underline{\psi}^j \, \underline{\hat{x}}^j \tag{3.1}$$

We have defined $\underline{\psi}^j$ previously as a matrix function of finite rotations $(r_4{}^* \; r_5{}^* \; r_6{}^*)^T$ where:

$$\begin{bmatrix} r_4{}^* \\ r_5{}^* \\ r_6{}^* \end{bmatrix} = \begin{bmatrix} r_4^\circ + \bar{r}_4 \\ r_5^\circ + \bar{r}_5 \\ r_6^\circ + \bar{r}_6 \end{bmatrix}$$

The $(r_4^\circ \; r_5^\circ \; r_6^\circ)^T$ define the initial member orientation in space. We propose to order the rotations as follows: first r_4^*, then r_5^*, and, finally, r_6^*.

First of all, the r_4^* rotation transforms from the base vectors $\hat{\underline{X}}^j$ to a set of instantaneous base vectors $\hat{\underline{X}}^I$ of an intermediate coordinate frame by Equation 3.2:

$$\hat{\underline{X}}^I = \underline{\psi}^I \; \hat{\underline{X}}^j \tag{3.2}$$

$\underline{\psi}^I$ is a single rotation as shown by Figure 3.2 and $\underline{\psi}^I$ is given by:

$$\underline{\psi}^I = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos(r_4^*) & \sin(r_4^*) \\ 0 & -\sin(r_4^*) & \cos(r_4^*) \end{vmatrix}$$

and

$$\hat{\underline{X}}^I = \begin{pmatrix} \hat{\underline{X}}^I_1 \\ \hat{\underline{X}}^I_2 \\ \hat{\underline{X}}^I_3 \end{pmatrix}$$

Secondly, the r_5^* rotation transforms from a known set of base vectors $\hat{\underline{X}}^I$ to the instantaneous coordinate frame base vectors $\hat{\underline{X}}^{II}$ of an intermediate frame (x^{II}) by Equation 3.3:

$$\hat{\underline{X}}^{II} = \underline{\psi}^{II} \; \hat{\underline{X}}^I \tag{3.3}$$

where

$$\underline{\psi}^{II} = \begin{vmatrix} \cos(r_5^*) & 0 & -\sin(r_5^*) \\ 0 & 1 & 0 \\ \sin(r_5^*) & 0 & \cos(r_5^*) \end{vmatrix}$$

and

$$\hat{\underline{x}}^{II} = \begin{Bmatrix} \hat{\underline{x}}_1^{II} \\ \hat{\underline{x}}_2^{II} \\ \hat{\underline{x}}_3^{II} \end{Bmatrix}$$

The r_5^* rotation was about an instantaneous axis defined by the $\hat{\underline{x}}_2^{I}$ base vector. Finally, the r_6^* rotation, which is about the instantaneous $\hat{\underline{x}}_3^{II}$ base vector, transforms to the base vectors $\hat{\underline{y}}^j$ of the (y^j) frame from the known base vectors $\hat{\underline{x}}^{II}$ of the (x^{II}) frame by Equation 3.4:

$$\hat{\underline{y}}^j = \underline{\psi}^{III}\, \hat{\underline{x}}^{II} \tag{3.4}$$

where

$$\underline{\psi}^{III} = \begin{vmatrix} \cos(r_6^*) & \sin(r_6^*) & 0 \\ -\sin(r_6^*) & \cos(r_6^*) & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

The rotations r_5^* and r_6^* are shown in Figures 3.3 and 3.4. From Equations 3.2, 3.3 and 3.4 we derive Equation 3.5:

$$\begin{aligned} \hat{\underline{y}}^j &= \underline{\psi}^{III}\, \underline{\psi}^{II}\, \underline{\psi}^{I}\, \hat{\underline{x}}^j \\ \hat{\underline{y}}^j &= \underline{\psi}^j\, \hat{\underline{x}}^j \end{aligned} \tag{3.5}$$

where

$$\underline{\psi}^j = \underline{\psi}^{III}\, \underline{\psi}^{II}\, \underline{\psi}^{I}$$

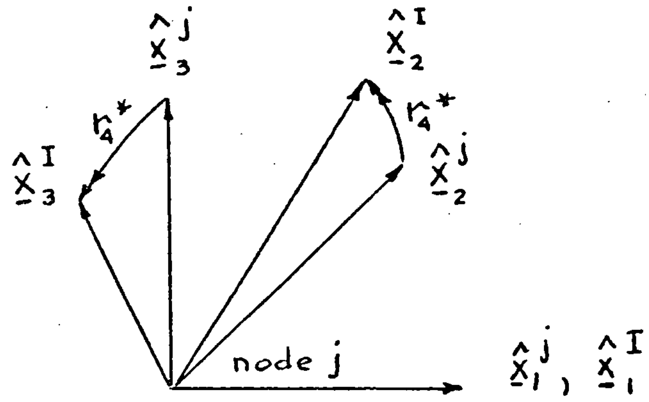


FIGURE 3.2 $\underline{\psi}^{I}$ TRANSFORMATION

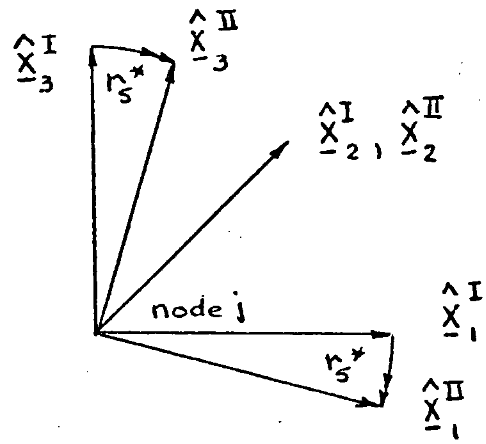


FIGURE 3.3 $\underline{\psi}^{II}$ TRANSFORMATION

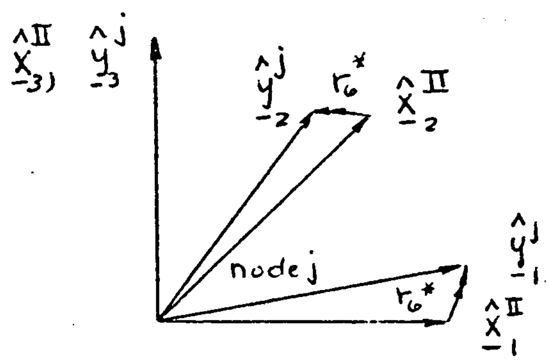


FIGURE 3.4 $\underline{\psi}^{III}$ TRANSFORMATION

and

$$\underline{\Psi}^j = \begin{bmatrix} \cos(r_5^*)\cos(r_6^*) & \cos(r_4^*)\sin(r_6^*) + \sin(r_4^*)\sin(r_5^*)\cos(r_6^*) & -\cos(r_4^*)\sin(r_5^*)\cos(r_6^*) + \sin(r_4^*)\sin(r_6^*) \\ -\cos(r_5^*)\sin(r_6^*) & \cos(r_4^*)\cos(r_6^*) - \sin(r_4^*)\sin(r_5^*)\sin(r_6^*) & \cos(r_4^*)\sin(r_5^*)\sin(r_6^*) + \sin(r_4^*)\cos(r_6^*) \\ \sin(r_5^*) & -\sin(r_4^*)\cos(r_5^*) & \cos(r_4^*)\cos(r_5^*) \end{bmatrix}$$

where the order of rotations is, again, r_4^*, r_5^*, r_6^*.

It can be easily shown that $\underline{\Psi}^j$ is an orthonormal transformation since it has the following properties

$$[\underline{\Psi}^j]^{-1} = [\underline{\Psi}^j]^T$$

and

$$[\underline{\Psi}^j]^T [\underline{\Psi}^j] = [I]$$

where superscript T denotes the transpose of the matrix and $[I]$ is the identity matrix. Thus

$$\hat{\underline{x}}^j = [\underline{\Psi}^j]^T \hat{\underline{y}}^j$$

An arbitrary vector $\underline{v}$ in the (x^j) frame with components

$$\underline{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

will have components in the (y^j) frame given by $\underline{w}$ in Equation 3.7:

$$\underline{w} = \underline{\Psi}^j \underline{v} \tag{3.7}$$

where

$$\underline{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

are the components of $\underline{\omega}$, which are the components of $\underline{v}$ measured in the (y^j) frame.

At node k of element JK we have a transformation matrix Ψ^k which was introduced in Chapter II as a matrix of rotations relating the base vectors of the (x^k) frame and the base vectors of an arbitrary (y^k) frame. The $\underline{\Psi}^k$ will be the transformation of the $\underline{\hat{x}}^k$ vectors into the $\underline{\hat{y}}^k$ vectors by Equation 3.8:

$$\underline{\hat{y}}^k = \underline{\Psi}^k \, \underline{\hat{x}}^k \tag{3.8}$$

The $\underline{\Psi}^k$ transformation is composed of large angle rotations $(r_{10}^* \; r_{11}^* \; r_{12}^*)^T$ in that specific order and is similar in form to $\underline{\Psi}^j$.

$$\underline{\Psi}^k = \begin{bmatrix} \cos(r_{11}^*)\cos(r_{12}^*) & \begin{matrix}\cos(r_{10}^*)\sin(r_{12}^*) \\ +\sin(r_{10}^*)\sin(r_{11}^*)\cos(r_{12}^*)\end{matrix} & \begin{matrix}-\cos(r_{10}^*)\sin(r_{11}^*)\cos(r_{12}^*) \\ +\sin(r_{10}^*)\sin(r_{12}^*)\end{matrix} \\ -\cos(r_{11}^*)\sin(r_{12}^*) & \begin{matrix}\cos(r_{10}^*)\cos(r_{12}^*) \\ -\sin(r_{10}^*)\sin(r_{11}^*)\sin(r_{12}^*)\end{matrix} & \begin{matrix}\cos(r_{10}^*)\sin(r_{11}^*)\sin(r_{12}^*) \\ +\sin(r_{10}^*)\cos(r_{12}^*)\end{matrix} \\ \sin(r_{11}^*) & -\sin(r_{10}^*)\cos(r_{11}^*) & \cos(r_{10}^*)\cos(r_{11}^*) \end{bmatrix}$$

3. <u>The Initial Coordinate Frame (y^0)</u>

In Equations 3.6 and 3.8 we defined the relationships between the base vectors of the node global coordinate systems and the arbitrary coordinate systems (y^j) and (y^k) at the j and k nodes of member JK respectively. If we give the base vectors $\hat{\underline{x}}^j$ and $\hat{\underline{x}}^k$ the following values:

$$\hat{\underline{x}}^j = (\underline{\epsilon}_1 \quad \underline{\epsilon}_2 \quad \underline{\epsilon}_3)^T$$
$$\hat{\underline{x}}^k = (\underline{\epsilon}_1 \quad \underline{\epsilon}_2 \quad \underline{\epsilon}_3)^T \tag{3.9}$$

where

$$\underline{\epsilon}_1 = (1 \quad 0 \quad 0)^T$$
$$\underline{\epsilon}_2 = (0 \quad 1 \quad 0)^T$$
$$\underline{\epsilon}_3 = (0 \quad 0 \quad 1)^T$$

then the base vectors $\hat{\underline{y}}^j$ will be the rows of $\underline{\psi}^j$ and the base vectors $\hat{\underline{y}}^k$ will be the rows of $\underline{\psi}^k$, i.e.:

$$\hat{y}_i^j = (\psi^j(i,1) \quad \psi^j(i,2) \quad \psi^j(i,3))^T$$
$$\hat{y}_i^k = (\psi^k(i,1) \quad \psi^k(i,2) \quad \psi^k(i,3))^T \qquad i = 1, 2, 3 \tag{3.10}$$

where $\hat{\underline{y}}^j = (\hat{y}_1^j \quad \hat{y}_2^j \quad \hat{y}_3^j)^T$

and $\hat{\underline{y}}^k = (\hat{y}_1^k \quad \hat{y}_2^k \quad \hat{y}_3^k)^T$

Initially, before any deformations have been applied to the structure, the elements will be straight and the (y^j) and (y^k) frames will be aligned in space. The transformation matrices $\underline{\psi}^j$ and $\underline{\psi}^k$ reduce to an initial transformation matrix $\underline{\psi}^o$ which relates the initial nodal coordinate frame (y^o) base vectors and the $\hat{\underline{x}}^j$ or $\hat{\underline{x}}^k$ base vectors. We can calculate the initial angles, defined as $(r_4^o \; r_5^o \; r_6^o)^T$, knowing the initial geometry of each member. This requires that we know the coordinates of all the nodes as well as a point which defines the initial orientation of the member cross section in space. Figure 3.5 illustrates the initial coordinate system base vectors in relation to the node global coordinate

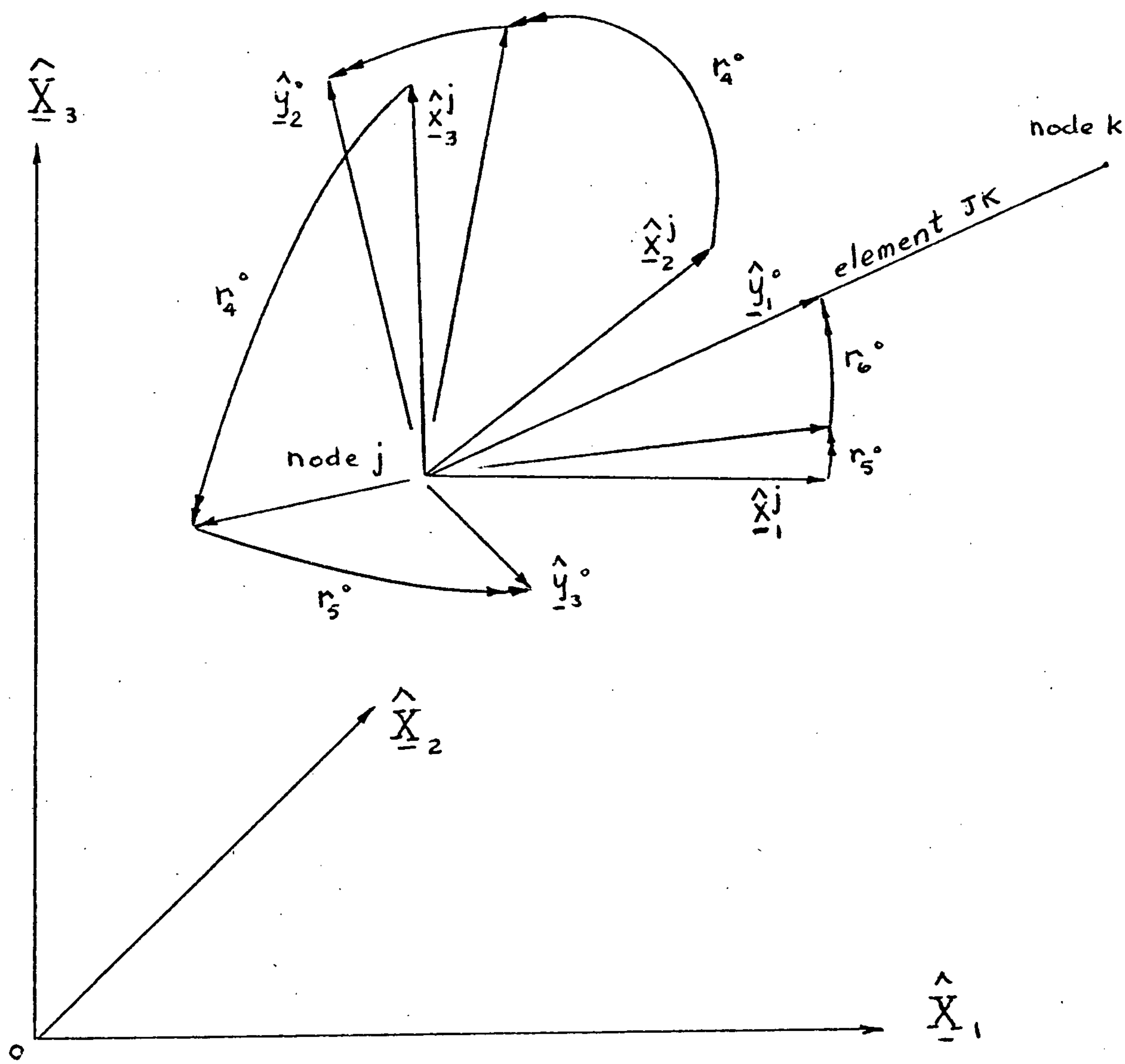


FIGURE 3.5 INITIAL COORDINATE FRAME

system base vectors.

The (y°) frame has base vectors $\hat{\underline{y}}^\circ$ where:

$$\hat{\underline{y}}^\circ = \left(\hat{\underline{y}}_1^\circ \quad \hat{\underline{y}}_2^\circ \quad \hat{\underline{y}}_3^\circ \right)^T \tag{3.11}$$

The y_3° axis is defined as the minor principal axis for flexure of the undeformed element cross section. The y_2° axis is defined as the major principal axis and y_1° is the centroidal axis of the element.

Now, if we write:

$$\hat{\underline{x}}^j = [\underline{\psi}^\circ]^T \hat{\underline{y}}^\circ \tag{3.12}$$

with

$$\hat{\underline{y}}_1^\circ = \left(y_{11}^\circ \quad y_{12}^\circ \quad y_{13}^\circ \right)^T$$

(3.13)

and

$$\hat{\underline{y}}_3^\circ = \left(y_{31}^\circ \quad y_{32}^\circ \quad y_{33}^\circ \right)^T$$

then the components of $\hat{\underline{y}}_1^\circ$ and $\hat{\underline{y}}_3^\circ$, y_{11}°, y_{12}°, etc., are easily determined from the coordinates of the nodes. ($\hat{\underline{y}}_2^\circ$ is dependent upon $\hat{\underline{y}}_1^\circ$ and $\hat{\underline{y}}_3^\circ$ and yields no new information) But, from Equation 3.10, we know the components of $\hat{\underline{y}}_1^\circ$ and $\hat{\underline{y}}_3^\circ$ in terms of r_4°, r_5°, and r_6°, viz. Equation 3.14:

$$\hat{\underline{y}}_1^\circ = \begin{pmatrix} \cos(r_5^\circ)\cos(r_6^\circ) & \begin{matrix} \sin(r_4^\circ)\sin(r_5^\circ)\cos(r_6^\circ) \\ +\cos(r_4^\circ)\sin(r_6^\circ) \end{matrix} & \begin{matrix} -\cos(r_4^\circ)\sin(r_5^\circ)\cos(r_6^\circ) \\ +\sin(r_4^\circ)\sin(r_6^\circ) \end{matrix} \end{pmatrix}^T$$

(3.14)

$$\hat{\underline{y}}_3^\circ = \left(\sin(r_5^\circ) \quad -\sin(r_4^\circ)\cos(r_5^\circ) \quad \cos(r_4^\circ)\cos(r_5^\circ) \right)^T$$

Thus we write the following equations which will be solved for the three angles, r_4°, r_5°, and r_6° :

$$
\begin{aligned}
y_{11}^{\circ} &= \cos(r_5^{\circ})\cos(r_6^{\circ})\\
y_{12}^{\circ} &= \cos(r_4^{\circ})\sin(r_6^{\circ}) + \sin(r_4^{\circ})\sin(r_5^{\circ})\cos(r_6^{\circ})\\
y_{13}^{\circ} &= \sin(r_4^{\circ})\sin(r_6^{\circ}) - \cos(r_4^{\circ})\sin(r_5^{\circ})\cos(r_6^{\circ})\\
y_{31}^{\circ} &= \sin(r_5^{\circ})\\
y_{32}^{\circ} &= -\sin(r_4^{\circ})\cos(r_5^{\circ})\\
y_{33}^{\circ} &= \cos(r_4^{\circ})\cos(r_5^{\circ})
\end{aligned}
\tag{3.15}
$$

Solving Equation 3.15 yields 3.16:

$$
\begin{aligned}
&\sin(r_5^{\circ}) = y_{31}^{\circ}\\
&\cos(r_5^{\circ}) = \sqrt{(y_{32}^{\circ})^2 + (y_{33}^{\circ})^2}\\
&\sin(r_4^{\circ}) = \frac{-y_{32}^{\circ}}{\cos(r_5^{\circ})} \qquad \text{provided } \cos(r_5^{\circ}) \neq 0\\
&\cos(r_4^{\circ}) = \frac{y_{33}^{\circ}}{\cos(r_5^{\circ})} \qquad \text{provided } \cos(r_5^{\circ}) \neq 0\\
&\text{(a)}\quad \sin(r_6^{\circ}) = \frac{y_{12}^{\circ}\cos^2(r_5^{\circ}) + y_{32}^{\circ}\, y_{31}^{\circ}\, y_{11}^{\circ}}{y_{33}^{\circ}\cos(r_5^{\circ})}\\
&\qquad \text{provided } y_{33}^{\circ} \neq 0 \text{ and } \cos(r_5^{\circ}) \neq 0\\
&\qquad \cos(r_6^{\circ}) = \frac{y_{11}^{\circ}}{\cos(r_5^{\circ})}\\
&\text{(b)}\quad \text{or}\quad \sin(r_6^{\circ}) = \frac{y_{13}^{\circ}\cos^2(r_5^{\circ}) + y_{33}^{\circ}\, y_{31}^{\circ}\, y_{11}^{\circ}}{-y_{32}^{\circ}\cos(r_5^{\circ})}\\
&\qquad \text{provided } y_{32}^{\circ} \neq 0 \text{ and } \cos(r_5^{\circ}) \neq 0
\end{aligned}
\tag{3.16}
$$

If $y_{33}^{\circ} = 0$ choose Equation (b), but if y_{32}° also = 0, then use the following procedure.

There are two singular points for which the above equations are not valid and that is when $y_{31}^{\circ} = \pm 1$; it follows then that $y_{32}^{\circ} = 0$ and $y_{33}^{\circ} = 0$ and the $\cos(r_5^{\circ}) = 0$. The (y°) frame would have its y_3° axis along the $\hat{\underline{x}}_1$ direction as shown in Figure 3.6. The $\hat{\underline{y}}_1^{\circ}$ base vector

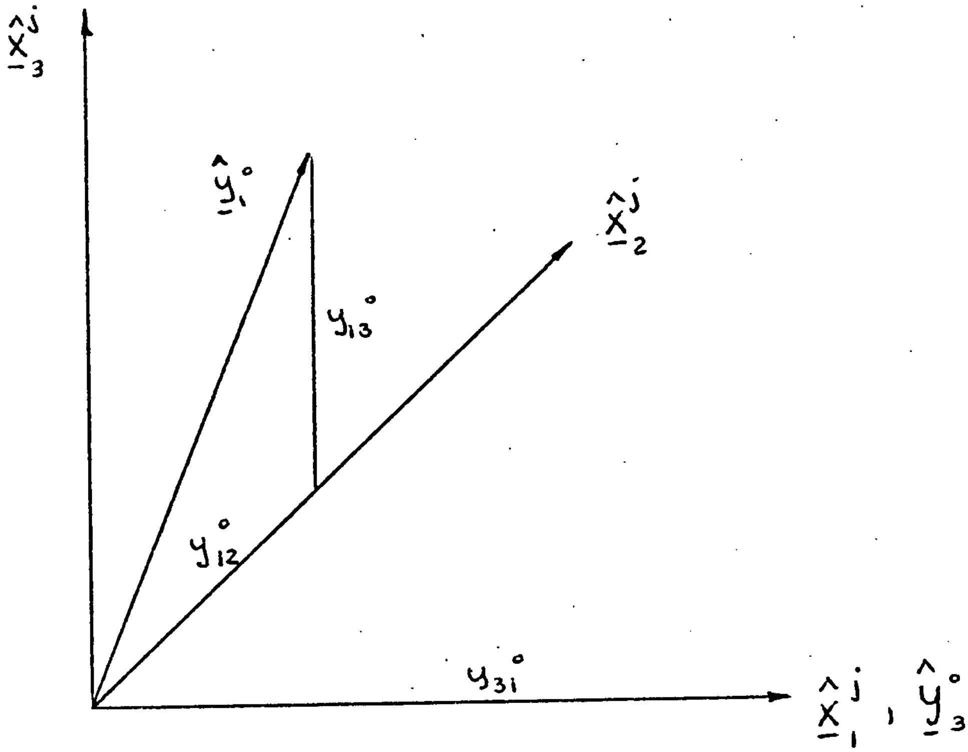


FIGURE 3.6 SINGULARITY OF INITIAL COORDINATE FRAME

will lie in the $X_2 X_3$ plane and thus the y_{11}° component will be zero. Therefore:

$$\begin{aligned} \cos(r_4^{\circ}) &= 1 \\ \sin(r_4^{\circ}) &= 0 \\ \cos(r_5^{\circ}) &= 0 \\ \sin(r_5^{\circ}) &= \pm 1 \\ \cos(r_6^{\circ}) &= \mp y_{13}^{\circ} \\ \sin(r_6^{\circ}) &= y_{12}^{\circ} \end{aligned} \tag{3.17}$$

4. <u>The Incremental Structure Deformations $\delta \underline{r}$</u>

The relationship between the intermediate incremental deformations $\delta \underline{\bar{r}}$ and the structure incremental nodal degrees of freedom $\delta \underline{r}$ follows using equations 3.2 and 3.3. It will be recalled that $\delta \underline{\bar{r}}^r$ are the increments in the rotations $\underline{r}^*$ arising from increments $\delta \underline{r}^r$ in the structure degrees of freedom $\underline{r}$. Thus r_4^* , $\delta \bar{r}_4$, and δr_4 are all rotations about the X_1^j axis; but r_5^* and $\delta \bar{r}_5$ are rotations about the X_2^{I} axis while δr_5 is about the X_2^j axis; and r_6^* and $\delta \bar{r}_6$ are rotations about the X_3^{II} axis while δr_6 is about the X_3^j axis. At node k similar observations can be made.

Therefore the transformation from $\delta \underline{r}$ to $\delta \underline{\bar{r}}$ is given by

$$\delta \underline{\bar{r}} = \underline{b} \; \delta \underline{r} \tag{3.18}$$

where:

$$\underline{b} = \left|\begin{array}{c|c|c|c} \underline{I} & \underline{O} & \underline{O} & \underline{O} \\ \hline \underline{O} & \underline{b}_1 & \underline{O} & \underline{O} \\ \hline \underline{O} & \underline{O} & \underline{I} & \underline{O} \\ \hline \underline{O} & \underline{O} & \underline{O} & \underline{b}_2 \end{array}\right|$$

$$\underline{b}_1 = \left|\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & \cos(r_4^*) & \sin(r_4^*) \\ \hline \sin(r_5^*) & -\sin(r_4^*)\cos(r_5^*) & \cos(r_4^*)\cos(r_5^*) \end{array}\right|$$

$$\underline{b}_2 = \left|\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & \cos(r_{10}^*) & \sin(r_{10}^*) \\ \hline \sin(r_{11}^*) & -\sin(r_{10}^*)\cos(r_{11}^*) & \cos(r_{10}^*)\cos(r_{11}^*) \end{array}\right|$$

$$\underline{I} = \left|\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 1 \end{array}\right|$$

and

$$\underline{O} = \left|\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right|$$

Figure 3.7 shows the incremental rotations for node j of element JK.

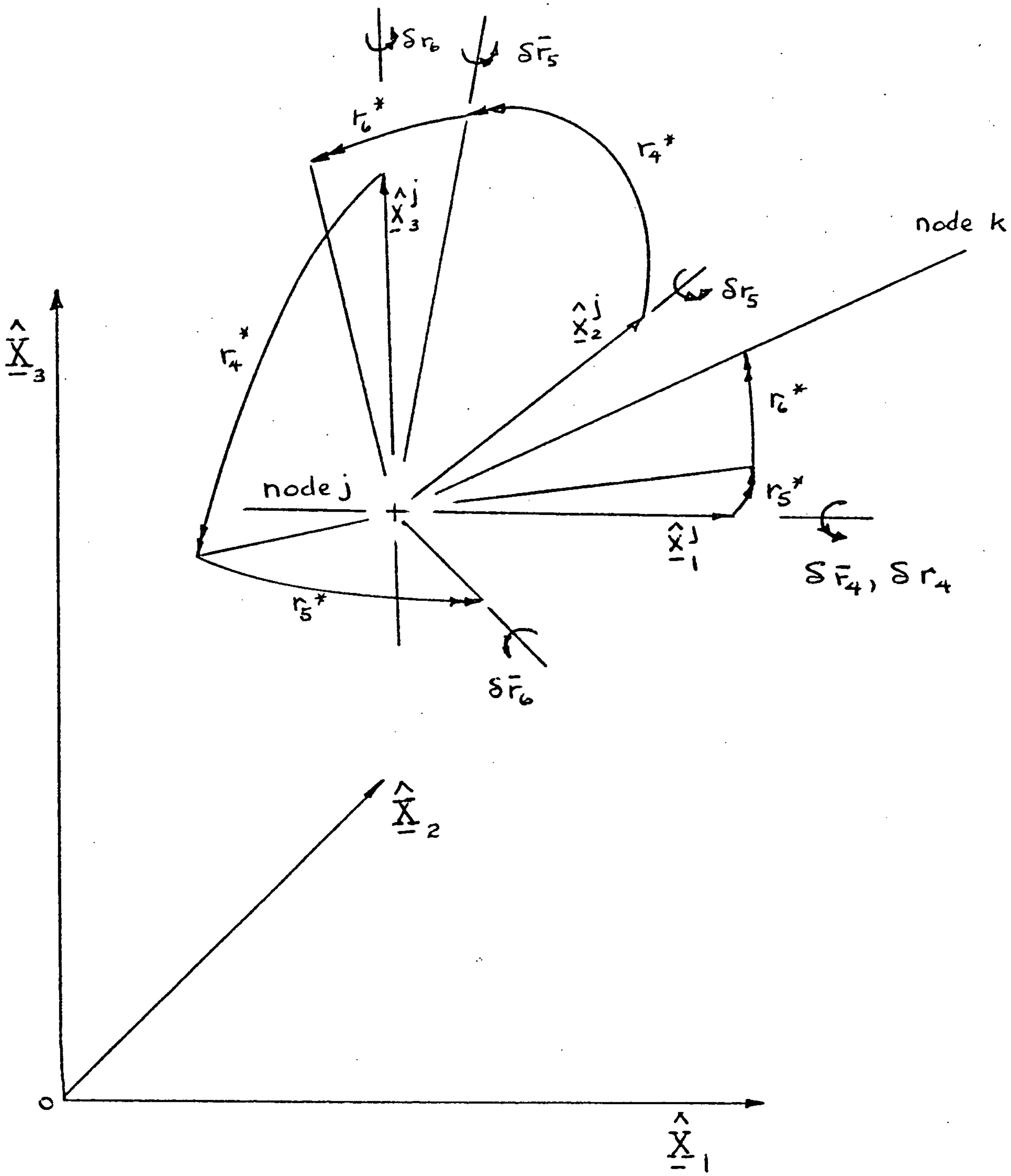


FIGURE 3.7 INCREMENTAL ROTATIONS AT NODE j

CHAPTER IV

TANGENTIAL REFERENCE FRAME

1. Definition of the Element Degrees of Freedom $\underset{\sim}{s}$

The displacements of the nodes of an element with reference to the local coordinate system, the 'local degrees of freedom' $\underset{\sim}{s}$ are functions of the structure degrees of freedom $\underset{\sim}{r}$ defined previously in Chapter III. For the tangential reference frame there are six degrees of freedom per element, all defined at one node as shown in Figure 4.1. The $\underset{\sim}{s}$ are kept small with respect to the length of the element by suitably increasing the number of elements required to model a given structure under a large rotation or translation.

The translations $(s_1 \; s_2 \; s_3)^T$ act along the tangential reference frame base vectors $\hat{\underset{\sim}{y}}^t$ respectively and the rotations $(s_4 \; s_5 \; s_6)^T$ are rotations about these same base vectors. Actually the $(s_4 \; s_5 \; s_6)^T$ are only approximately about the $\hat{\underset{\sim}{y}}^t$ base vectors, as will be shown in the following sections, but the approximations will be found acceptable. The (y^t) frame coincides with the (y^j) frame defined in Equation 3.5, i.e.

$$\hat{\underset{\sim}{y}}^t = \underset{\sim}{\psi}^j \, \hat{\underset{\sim}{x}}^j \tag{4.1}$$

2. Derivation of the Translational $\underset{\sim}{s}$

Figure 4.1 shows the element in the tangential reference frame before and after the finite displacements $\underset{\sim}{r}$, in which the element deformations are exaggerated for clarity.

Consider the element JK with j at the origin of the (x^j) frame

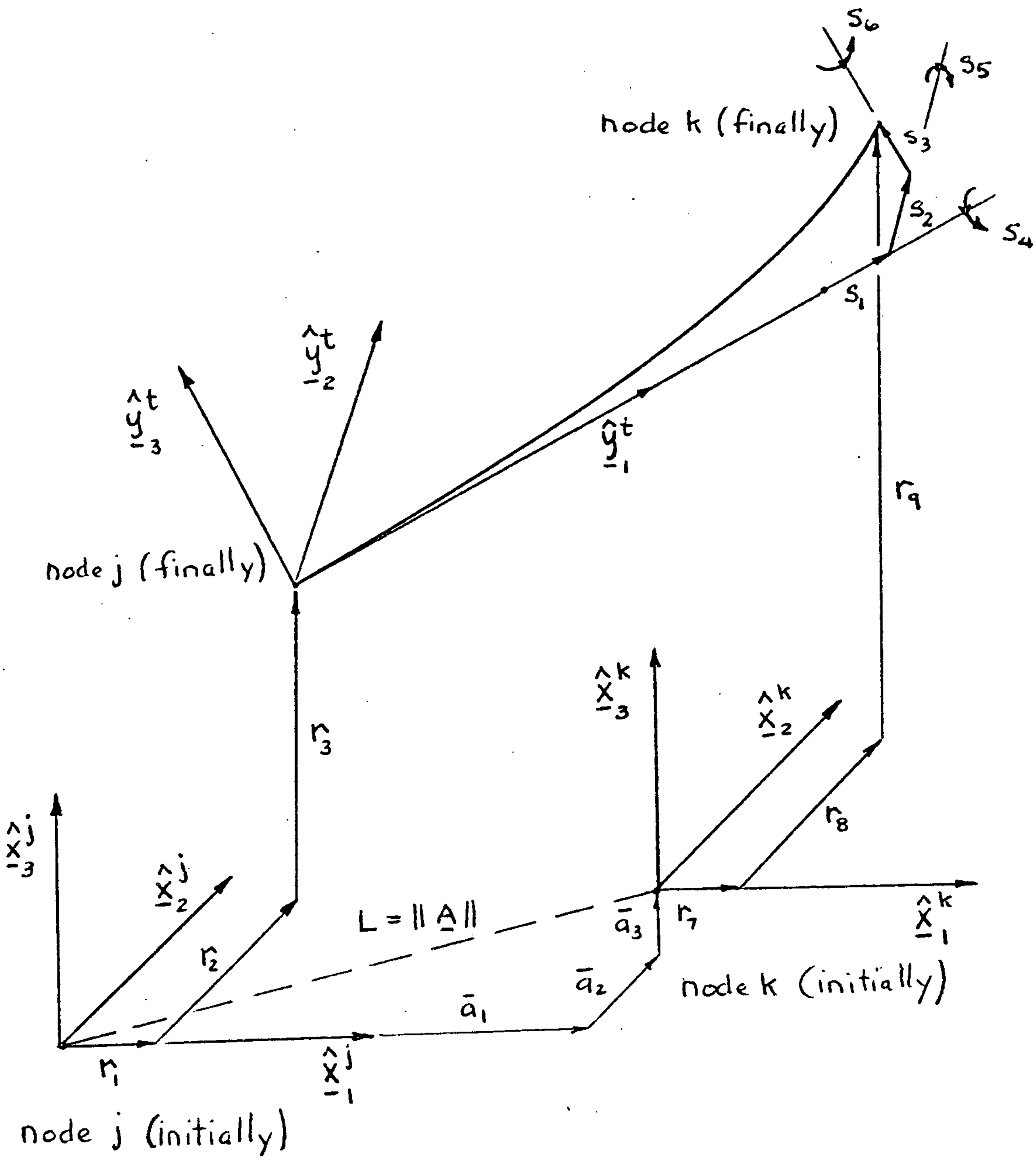


FIGURE 4.1 DEFORMATIONS $\underline{s}$ FOR TANGENTIAL FRAME

initially and having length L. Before the displacements, the nodal coordinates of j in (x^j) are $(0 \;\; 0 \;\; 0)^T$ and of k are given by the components of $\underline{A}$ where:

$$\underline{A} = (\bar{a}_1 \;\; \bar{a}_2 \;\; \bar{a}_3)^T \qquad \text{and} \qquad L = \sqrt{\underline{A} \cdot \underline{A}} \tag{4.2}$$

After the displacements $\underline{r}^t$ where $\underline{r}^t$ is defined in Chapter II, node j has (x^j) frame coordinates of $\begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix}$ and node k has coordinates $\begin{bmatrix} \bar{a}_1 + r_7 \\ \bar{a}_2 + r_8 \\ \bar{a}_3 + r_9 \end{bmatrix}$.

In the (y^t) frame, the coordinates of j and k before displacements are $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix}$ respectively. If the displacement were a rigid body motion, these coordinates would remain unchanged; node k would not move in the (y^t) frame. The difference between the actual motion of node k and that corresponding to a rigid body motion is the deformation vector $(S_1 \;\; S_2 \;\; S_3)^T$. Therefore, the deformations $(S_1 \;\; S_2 \;\; S_3)^T$, seen as components of displacement of k with respect to j in the (x^j) frame, are given by Equation 4.3 and called $\underline{S}^*$:

$$\underline{S}^* = \begin{bmatrix} \bar{a}_1 \\ \bar{a}_2 \\ \bar{a}_3 \end{bmatrix} + \begin{bmatrix} r_7 \\ r_8 \\ r_9 \end{bmatrix} - \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} - L \begin{bmatrix} \psi^j(1,1) \\ \psi^j(1,2) \\ \psi^j(1,3) \end{bmatrix} \tag{4.3}$$

where $L \begin{bmatrix} \psi^j(1,1) \\ \psi^j(1,2) \\ \psi^j(1,3) \end{bmatrix}$ are the x^j-components of the length of the element which is oriented along the $\underline{\hat{y}}_1^t$ direction. In the (y^t) frame, the local coordinate system, the vector $\underline{S}^t = (S_1 \;\; S_2 \;\; S_3)^T$ is the $\underline{S}^*$ transformed by Equation 4.4.

$$\underline{S}^t = \underline{\Psi}^j \; \underline{S}^* \tag{4.4}$$

$\underline{\Psi}^j$ has been derived in Chapter III and is composed of rotations given as in Chapter II:

$$\begin{bmatrix} r_4^* \\ r_5^* \\ r_6^* \\ r_{10}^* \\ r_{11}^* \\ r_{12}^* \end{bmatrix} = \begin{bmatrix} r_4^\circ + \bar{r}_4 \\ r_5^\circ + \bar{r}_5 \\ r_6^\circ + \bar{r}_6 \\ r_4^\circ + \bar{r}_{10} \\ r_5^\circ + \bar{r}_{11} \\ r_6^\circ + \bar{r}_{12} \end{bmatrix}$$

3. <u>Derivation of the Rotational Components of $\underline{S}$</u>

We have the following transformations from Equations 3.5 and 3.8:

$$\hat{\underline{y}}^j = \underline{\Psi}^j \; \hat{\underline{x}}^j$$

$$\hat{\underline{y}}^k = \underline{\Psi}^k \; \hat{\underline{x}}^k$$

We can relate base vectors at nodes j and k from Equations 3.5 and 3.8 by noting that the base vectors $\hat{\underline{x}}^j$ and $\hat{\underline{x}}^k$ are identical. Thus:

$$\left[\underline{\Psi}^j\right]^T \hat{\underline{y}}^j = \left[\underline{\Psi}^k\right]^T \hat{\underline{y}}^k$$

and $$\hat{\underline{y}}^j = \left[\underline{\Psi}^j\right]\left[\underline{\Psi}^k\right]^T \hat{\underline{y}}^k \tag{4.5}$$

We define the transformation $\underline{\gamma} = \left[\underline{\Psi}^j\right]\left[\underline{\Psi}^k\right]^T$ which relates base vectors $\hat{\underline{y}}^k$ and $\hat{\underline{y}}^j$. This transformation matrix, composed of three rotations $(\theta_1 \; \theta_2 \; \theta_3)^T$, will have the following form:

$$\underline{\gamma} = \begin{bmatrix} \cos(\theta_2)\cos(\theta_3) & \cos(\theta_1)\sin(\theta_3) + \sin(\theta_1)\sin(\theta_2)\cos(\theta_3) & -\cos(\theta_1)\sin(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_3) \\ -\cos(\theta_2)\sin(\theta_3) & \cos(\theta_1)\cos(\theta_3) - \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) & \cos(\theta_1)\sin(\theta_2)\sin(\theta_3) + \sin(\theta_1)\cos(\theta_3) \\ \sin(\theta_2) & -\sin(\theta_1)\cos(\theta_2) & \cos(\theta_1)\cos(\theta_2) \end{bmatrix} \tag{4.6}$$

Although this transformation matrix implies that θ_1 is about the y_1^t axis, θ_2 is about some intermediate axis between y_2^t and y_2^k, and θ_3 is about the y_3^k axis, the rotations are small and the axes of rotation can be considered to be the y^t axes. If we assume that the sine of the angle is the angle itself and the cosine of the angle is unity, $\underline{\gamma}$ reduces to:

$$\underline{\gamma} = \begin{bmatrix} 1.0 & \theta_3 & -\theta_2 \\ -\theta_3 & 1.0 & \theta_1 \\ \theta_2 & -\theta_1 & 1.0 \end{bmatrix} \tag{4.7}$$

We make the assumption that $(\, s_4 \;\; s_5 \;\; s_6 \,)^T$ are the rotations $(\, \theta_1 \;\; \theta_2 \;\; \theta_3 \,)^T$ respectively. It is felt that these approximations will not affect the required derivatives of $\underline{s}$ with respect to $\underline{r}$. From Equations 4.5 and 4.7 we obtain Equations 4.8:

$$\begin{aligned} s_4 &= -\sum_{i=1}^{3} \psi^k(3,i) \cdot \psi^j(2,i) \\ s_5 &= \sum_{i=1}^{3} \psi^k(3,i) \cdot \psi^j(1,i) \end{aligned} \tag{4.8}$$

$$S_6 = -\sum_{i=1}^{3} \psi^{k}(2,i) \cdot \psi^{j}(1,i) \qquad \text{(4.8 cont.)}$$

In summary, for the tangential coordinate system, we have:

$$S_1 = (a_1 + r_7 - r_1)\,\psi^{j}(1,1) + (a_2 + r_8 - r_2)\,\psi^{j}(1,2) + (a_3 + r_9 - r_3)\,\psi^{j}(1,3) - L$$

$$S_2 = (a_1 + r_7 - r_1)\,\psi^{j}(2,1) + (a_2 + r_8 - r_2)\,\psi^{j}(2,2) + (a_3 + r_9 - r_3)\,\psi^{j}(2,3)$$

$$S_3 = (a_1 + r_7 - r_1)\,\psi^{j}(3,1) + (a_2 + r_8 - r_2)\,\psi^{j}(3,2) + (a_3 + r_9 - r_3)\,\psi^{j}(3,3)$$

$$S_4 = -\psi^{k}(3,1)\,\psi^{j}(2,1) - \psi^{k}(3,2)\,\psi^{j}(2,2) - \psi^{k}(3,3)\,\psi^{j}(2,3)$$

$$S_5 = \psi^{k}(3,1)\,\psi^{j}(1,1) + \psi^{k}(3,2)\,\psi^{j}(1,2) + \psi^{k}(3,3)\,\psi^{j}(1,3)$$

$$S_6 = -\psi^{k}(2,1)\,\psi^{j}(1,1) - \psi^{k}(2,2)\,\psi^{j}(1,2) - \psi^{k}(2,3)\,\psi^{j}(1,3)$$

(4.9)

CHAPTER V

SECANT REFERENCE FRAME

1. The Element Degrees of Freedom $\underset{\sim}{S}$

In the secant reference frame, like the tangential reference frame, the element will have six local degrees of freedom $\underset{\sim}{S}$. This number results after subtracting the six rigid body motions from the twelve global degrees of freedom for the element. The six retained degrees of freedom consist of an elongation, four bending rotations, and a torsional rotation; two bending rotations are at node j and the other four are at node k. Figure 5.1 shows the member before and after finite displacements $\underset{\sim}{r}$, where the member displacements have been exaggerated for clarity. The figure shows the base vectors of the (x^j) system transformed through $(r_4^* \; r_5^* \; r_6^*)^T$ to the (y^j) system at node j and through $(r_{10}^* \; r_{11}^* \; r_{12}^*)^T$ to the (y^k) system at node k by Equations 3.5 and 3.8 respectively.

The secant reference frame base vectors $\hat{\underset{\sim}{y}}^s$ have been defined by Equations 2.5, 2.6 and 2.7. Equation 2.5 eliminates translational rigid body motions. Equation 2.6 sets $\hat{\underset{\sim}{y}}_2^s$ orthogonal to $\hat{\underset{\sim}{y}}_1^s$ and the tangent base vector $\hat{\underset{\sim}{y}}_3^t$ (i.e. orthogonal to the plane containing $\hat{\underset{\sim}{y}}_1^s$ and the minor principal axis of the cross section in its deformed position at j). Equation 2.7 completes the right-handed orthogonal triad.

We will also need a local coordinate system at node k aligned with the $\hat{\underset{\sim}{y}}_1^s$ base vector in order to define the element degrees of freedom adequately. We call this coordinate frame the (z^s) frame with base vectors $\hat{\underset{\sim}{z}}^s$ which are given in Equations 5.1, 5.2 and 5.3.

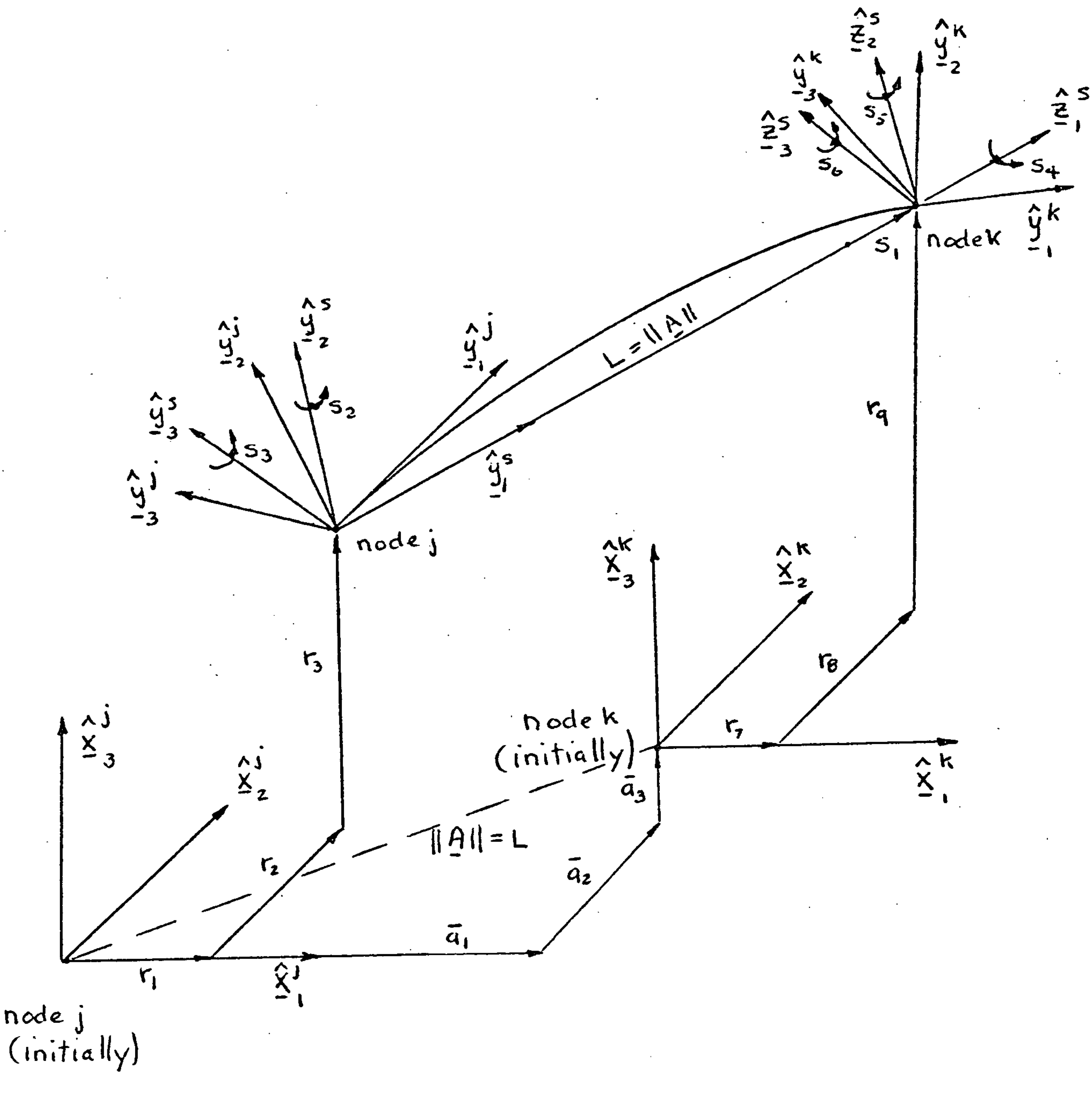


FIGURE 5.1 DEFORMATIONS $\underline{S}$ FOR SECANT FRAME

$$\hat{\underline{z}}_1^s = \hat{\underline{y}}_1^s \tag{5.1}$$

$$\hat{\underline{z}}_2^s = \frac{\hat{\underline{y}}_3^k \times \hat{\underline{z}}_1^s}{\left\| \hat{\underline{y}}_3^k \times \hat{\underline{z}}_1^s \right\|} \tag{5.2}$$

$$\hat{\underline{z}}_3^s = \hat{\underline{z}}_1^s \times \hat{\underline{z}}_2^s \tag{5.3}$$

2. Derivation of $\underline{s}$ as a Function of $\bar{\underline{r}}$

We begin by noting that since the element deformations are required to be small, then the (y^j) coordinate system and the (y^s) coordinate systems will be almost coincident. Likewise, the (y^k) coordinate frame and (z^s) coordinate frame are almost coincident and therefore the rotation transformations relating their base vectors will be composed of small angles. We can say, then, that the following simplifications to the scalar products hold:

$$\hat{\underline{y}}_1^j \cdot \hat{\underline{y}}_1^s = \cos(\alpha_1) \cong 1.0$$

since α_1 is considered small

$$\hat{\underline{y}}_1^j \cdot \hat{\underline{y}}_2^s = \cos(90° \pm \alpha_2) \cong \mp \alpha_2$$

$$\hat{\underline{y}}_1^j \cdot \hat{\underline{y}}_3^s = \cos(90° \pm \alpha_3) \cong \mp \alpha_3$$

α_2 , α_3 are also small angles

The elongation of the element s_1 is determined by the extension of the line joining node j to node k :

$$s_1 = \sqrt{(\bar{a}_1 + r_7 - r_1)^2 + (\bar{a}_2 + r_8 - r_2)^2 + (\bar{a}_3 + r_9 - r_3)^2} - L \tag{5.4}$$

where L defines the length of the element, $L = \sqrt{\bar{a}_1^2 + \bar{a}_2^2 + \bar{a}_3^2}$ and $(\bar{a}_1\ \bar{a}_2\ \bar{a}_3)^T$ are the initial components of the length vector.

From Figure 5.2 it can be seen that:

$$s_2 = -\hat{\underline{y}}_1^{j} \cdot \hat{\underline{y}}_3^{s} \tag{5.5}$$

$$s_3 = \hat{\underline{y}}_1^{j} \cdot \hat{\underline{y}}_2^{s} \tag{5.6}$$

At node k we obtain similar expressions for the s_5 and s_6 rotations:

$$s_5 = -\hat{\underline{y}}_1^{k} \cdot \hat{\underline{z}}_3^{s} \tag{5.7}$$

$$s_6 = \hat{\underline{y}}_1^{k} \cdot \hat{\underline{z}}_2^{s} \tag{5.8}$$

The final degree of freedom, the angle of twist of the element, can be approximated by the scalar product of vectors $\hat{\underline{y}}_3^{s}$ and $\hat{\underline{z}}_2^{s}$.

$$s_4 = \hat{\underline{y}}_3^{s} \cdot \hat{\underline{z}}_2^{s} \tag{5.9}$$

In summary, for the secant element coordinate system, we have the following degrees of freedom :

$$\begin{aligned} s_1 &= \sqrt{(\bar{a}_1 + r_7 - r_1)^2 + (\bar{a}_2 + r_8 - r_2)^2 + (\bar{a}_3 + r_9 - r_3)^2} - L \\ s_2 &= -\hat{\underline{y}}_1^{j} \cdot \hat{\underline{y}}_3^{s} \\ s_3 &= \hat{\underline{y}}_1^{j} \cdot \hat{\underline{y}}_2^{s} \end{aligned} \tag{5.10}$$

$$S_4 = \underline{\hat{y}}_3^{s} \cdot \underline{\hat{z}}_2^{s}$$

$$S_5 = -\underline{\hat{y}}_1^{k} \cdot \underline{\hat{z}}_3^{s} \qquad \text{(5.10 cont'd)}$$

$$S_6 = \underline{\hat{y}}_1^{k} \cdot \underline{\hat{z}}_2^{s}$$

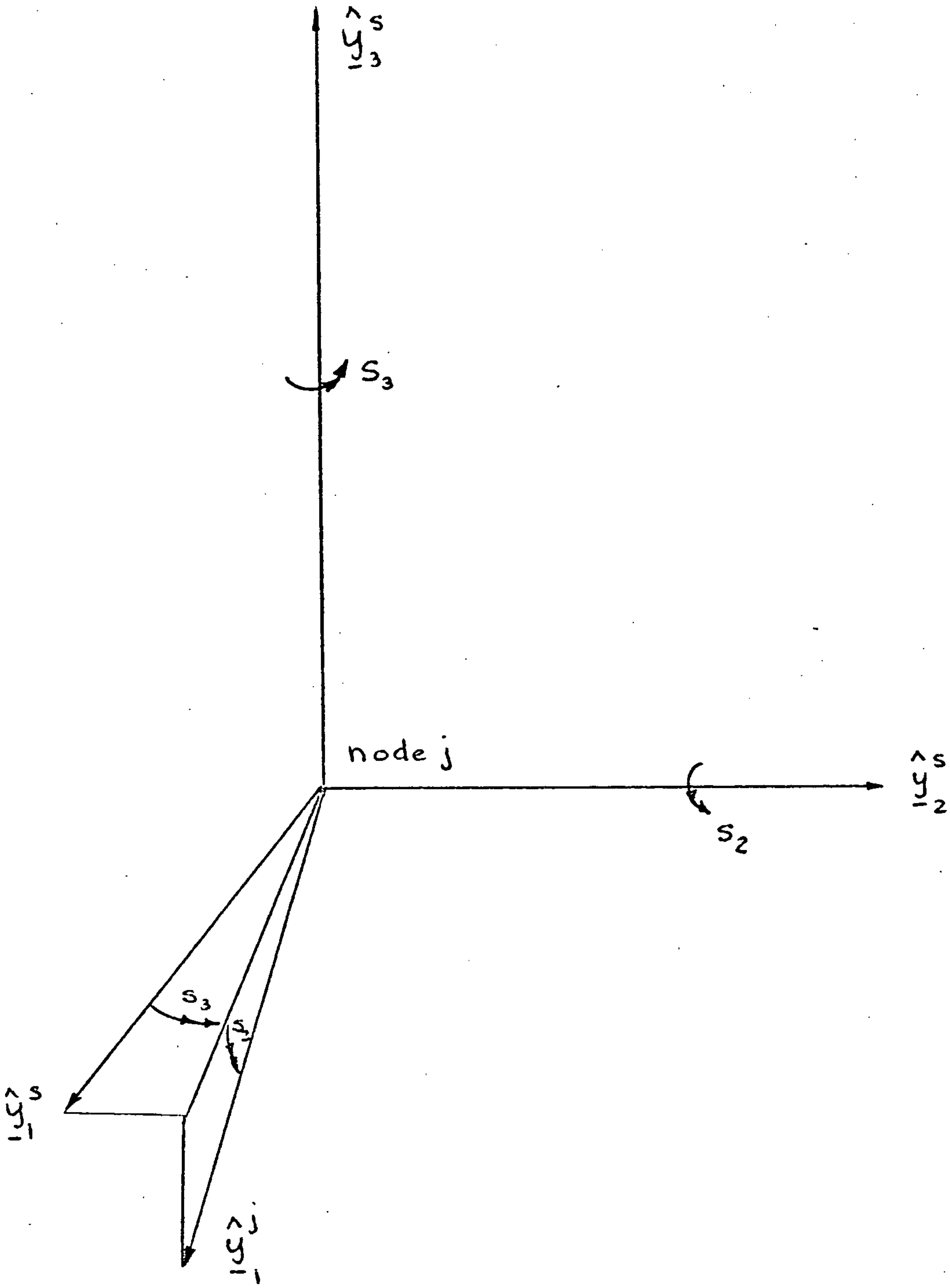


FIGURE 5.2 ROTATIONAL DEGREES OF FREEDOM S_2 AND S_3

CHAPTER VI

FORCE-DISPLACEMENT RELATIONSHIPS

1. Element Stiffness

The element stiffness matrices were taken from Nathan (11). They were developed for a doubly-symmetric thin prismatic element with eleven degrees of freedom: three translational, four rotational, two torsional and two rate of change of twist angle. Nathan's stiffness matrices were reduced from eleven degrees of freedom to six by eliminating those which are restrained in the present case. The matrices were also transformed to conform to the present coordinate system definitions. We present here only the stiffness matrices for the cantilever element since the secant element stiffness can be derived from the given matrices.

In reference (11) the element forces $\mathcal{S}_i$ are first determined as non-linear functions of the displacements $\underline{s}$: $\mathcal{S}_i = \mathcal{S}_i(\underline{s})$. These equations are then linearized by an expansion of the element forces in a Taylor series about some instantaneous element displacements $\underline{s}^\circ$. By making the displacement increments $\Delta\underline{s}$ suitably small, terms past the first can be discarded from Equation 6.1.

$$\mathcal{S}_i(\underline{s}) - \mathcal{S}_i(\underline{s}^\circ) = \mathcal{S}_i - \mathcal{S}_i^\circ = \Delta\mathcal{S}_i$$

$$\Delta\mathcal{S}_i = \sum_j \Delta s_j \left.\frac{\partial \mathcal{S}_i(\underline{s})}{\partial s_j}\right|_{\underline{s}^\circ} + \frac{1}{2}\sum_j\sum_k \Delta s_j \, \Delta s_k \left.\frac{\partial^2 \mathcal{S}_i(\underline{s})}{\partial s_j \, \partial s_k}\right|_{\underline{s}^\circ} + \cdots \quad (6.1)$$

$$\Delta\mathcal{S}_i = \sum_j \Delta s_j \left.\frac{\partial \mathcal{S}_i(\underline{s})}{\partial s_j}\right|_{\underline{s}^\circ}$$

$$\Delta \underset{\sim}{S} = \underset{\sim}{k} \; \Delta \underset{\sim}{s} \tag{6.2}$$

where $$\underset{\sim}{k} = \left. \frac{\partial S_i(\underset{\sim}{s})}{\partial s_j} \right|_{\underset{\sim}{s}^\circ}$$

The matrix $\underset{\sim}{k}$ can be written $\underset{\sim}{k} = \underset{\sim}{k}_o + \underset{\sim}{k}_1$, where the stiffness matrix $\underset{\sim}{k}_o$ is independent of $\underset{\sim}{S}^\circ$ and the matrix $\underset{\sim}{k}_1$ is linear in $\underset{\sim}{S}^\circ$. Use of the matrices given below requires that elongations, shear displacements, and rotations remain small. Displacements must be small compared to the element dimensions and the rotations must be much less than unity.

Nathan's transformed element stiffnesses follow:

$$\underset{\sim}{k}_o = \begin{bmatrix} \frac{AE}{L} & & & & & \\ 0 & \frac{12EI_3}{L^3} & & & \text{Sym} & \\ 0 & 0 & \frac{12EI_2}{L^3} & & & \\ 0 & 0 & 0 & \frac{GJ}{L} & & \\ 0 & 0 & \frac{6EI_2}{L^2} & 0 & \frac{4EI_2}{L} & \\ 0 & -\frac{6EI_3}{L^2} & 0 & 0 & 0 & \frac{4EI_3}{L} \end{bmatrix} \tag{6.3}$$

where I_3 is the moment of inertia about the cantilever coordinate system y_3^t axis and I_2 is about the y_2^t axis. I_2 is greater than I_3.

J = torsional moment of inertia

A = element area

L = element length

E = Young's modulus of elasticity

G = Torsional modulus

$$
\underline{k}_1(s_1^\circ) = s_1^\circ \begin{bmatrix} 0 & & & & & \\ 0 & 0 & & & \text{sym} & \\ 0 & 0 & \frac{6AE}{5L^2} & & & \\ 0 & 0 & 0 & \frac{EI_2}{L^2} & & \\ 0 & 0 & \frac{AE}{10L} & 0 & \frac{2}{15}AE & \\ 0 & -\frac{AE}{10L} & 0 & 0 & 0 & \frac{2}{15}AE \end{bmatrix} \tag{6.4}
$$

$$
\underline{k}_1(s_2^\circ) = s_2^\circ \begin{bmatrix} 0 & & & & & \\ \frac{6AE}{5L^2} & 0 & & & \text{sym} & \\ 0 & 0 & 0 & & & \\ 0 & 0 & -\frac{6EI_2}{L^3} & 0 & & \\ 0 & 0 & 0 & -\frac{21}{5}\frac{EI_2}{L^2} & 0 & \\ -\frac{AE}{10L} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6.5}
$$

$$
\underline{k}_1(s_3^\circ) = s_3^\circ \begin{bmatrix} 0 & & & & & \\ 0 & 0 & & & \text{sym} & \\ \frac{6AE}{5L^2} & 0 & 0 & & & \\ 0 & -\frac{6EI_2}{L^3} & 0 & 0 & & \\ \frac{AE}{10L} & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & \frac{21EI_2}{5L^2} & 0 & 0 \end{bmatrix} \tag{6.6}
$$

$$\underset{\sim}{k}_1(s_5^\circ) = s_5^\circ \begin{bmatrix} 0 & & & & & \\ 0 & 0 & & & \text{Sym} & \\ \frac{AE}{10L} & 0 & 0 & & & \\ 0 & -\frac{21}{5}\frac{EI_2}{L^2} & 0 & 0 & & \\ \frac{2}{15}AE & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & \frac{16}{5}\frac{EI_2}{L} & 0 & 0 \end{bmatrix} \tag{6.7}$$

$$\underset{\sim}{k}_1(s_6^\circ) = s_6^\circ \begin{bmatrix} 0 & & & & & \\ -\frac{AE}{10L} & 0 & & & & \\ 0 & 0 & 0 & & \text{Sym} & \\ 0 & 0 & \frac{21}{5}\frac{EI_2}{L^2} & 0 & & \\ 0 & 0 & 0 & \frac{16}{5}\frac{EI_2}{L} & 0 & \\ \frac{2AE}{15} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6.8}$$

2. <u>Global System Equilibrium</u>

We turn now to the problem of deducing the system response in global coordinates from the element equilibrium equations.

We suppose that there are n independent global degrees of freedom $\underset{\sim}{r}$. The element or local degrees of freedom $\underset{\sim}{s}$ have been expressed in previous chapters as functions of the coordinates $\bar{\underset{\sim}{r}}$.

$$s_i = f(\bar{\underset{\sim}{r}}) \qquad i = 1, 2, \cdots m \tag{6.9}$$

The element incremental displacements follow as:

$$\delta s_i = \frac{\partial s_i}{\partial \bar{r}_j} \, \delta \bar{r}_j$$

$$= a_{ij}^{*}(\bar{\underline{r}}) \, b_{jk}(\bar{\underline{r}}) \, \delta r_k \tag{6.10}$$

or expressed in matrix notation:

$$\begin{aligned} \delta \underline{s} &= \underline{a}^{*} \, \underline{b} \, \delta \underline{r} \\ &= \underline{a} \, \delta \underline{r} \end{aligned} \tag{6.11}$$

The transformation matrix $\underline{a}$ is a function of the global coordinates $\bar{\underline{r}}$. By the principle of virtual work, the external work of the $\underline{R}$ forces moving through the corresponding virtual displacements $\delta \underline{r}$ must be equal to the internal work of the element forces $\underline{\mathcal{S}}$ moving through the compatible virtual displacements $\delta \underline{s}$. Thus,

$$\delta \underline{r}^{T} \underline{R} = \delta \underline{s}^{T} \underline{\mathcal{S}} \tag{6.12}$$

Using the compatibility Equation 6.11, we have

$$\delta \underline{r}^{T} \underline{R} = \delta \underline{r}^{T} \underline{a}^{T} \underline{\mathcal{S}} \tag{6.13}$$

If 6.13 is true for an arbitrary virtual displacement $\delta \underline{r}$, we may conclude that:

$$\underline{R} = \underline{a}^{T} \underline{\mathcal{S}} \tag{6.14}$$

We expand the R_i in a Taylor series about an initial global displacement $\underline{r}^{\circ}$, on the assumption that there are no singularities. The domain of $\underline{r}$ is suitably restricted to ensure a one-to-one correspondence between load and displacement.

$$\Delta R_i = \sum_j \frac{\partial R_i(\underline{r})}{\partial r_j} \Delta r_j \bigg|_{\underline{r}^\circ} + \frac{1}{2} \sum_j \sum_k \frac{\partial^2 R_i(\underline{r})}{\partial r_j \partial r_k} \Delta r_j \Delta r_k \bigg|_{\underline{r}^\circ} + \cdots \tag{6.15}$$

If we restrict the increments $\Delta \underline{r}$ such that the second and subsequent terms of the expansion can be neglected, we obtain:

$$\Delta R_i = \sum_j \frac{\partial R_i(\underline{r})}{\partial r_j} \bigg|_{\underline{r}^\circ} \Delta r_j \tag{6.16}$$

or in matrix form:

$$\Delta \underline{R} = \underline{K} \; \Delta \underline{r} \tag{6.17}$$

where $\underline{K} = \left[\dfrac{\partial R_i(\underline{r})}{\partial r_j} \right]_{\underline{r}^\circ}$

The j th column of $\underline{K}$ is given by

$$\frac{\partial \underline{R}(\underline{r})}{\partial r_j} \bigg|_{\underline{r}^\circ} = \frac{\partial}{\partial r_j} \left[\underline{a}^T(\bar{\underline{r}}) \; \underline{S}(\underline{r}) \right] \bigg|_{\underline{r}^\circ}$$

$$= \underline{a}^T(\bar{\underline{r}}) \frac{\partial \underline{S}(\underline{r})}{\partial r_j} + \frac{\partial \underline{a}^T(\bar{\underline{r}})}{\partial r_j} \underline{S}(\underline{r}) \bigg|_{\underline{r}^\circ}$$

$$= \underline{a}^T \frac{\partial \underline{S}}{\partial \underline{s}} \frac{\partial \underline{s}}{\partial \underline{r}} \frac{\partial \underline{r}}{\partial r_j} + \left[\underline{b}^T \frac{\partial \underline{a}^{*T}}{\partial \bar{\underline{r}}} \frac{\partial \bar{\underline{r}}}{\partial \underline{r}} \frac{\partial \underline{r}}{\partial r_j} + \frac{\partial \underline{b}^T}{\partial \bar{\underline{r}}} \frac{\partial \bar{\underline{r}}}{\partial \underline{r}} \frac{\partial \underline{r}}{\partial r_j} \underline{a}^{*T} \right] \underline{S}$$

$$= \underline{a}^T \frac{\partial \underline{S}}{\partial \underline{s}} \underline{a} \, \underline{\alpha}_j + \left[\underline{b}^T \frac{\partial \underline{a}^{*T}}{\partial \bar{\underline{r}}} \underline{b} \, \underline{\alpha}_j + \frac{\partial \underline{b}^T}{\partial \bar{\underline{r}}} \underline{b} \, \underline{\alpha}_j \, \underline{a}^{*T} \right] \underline{S}$$

$$= \underline{a}^T \left[\underline{k}_\circ + \underline{k}_1 \right] \underline{a} \, \underline{\alpha}_j + \left[\sum_{i=1}^{m} \left(\underline{V}_i^* \underline{b} + \underline{a}^* \underline{c}_i \right)^T b_{ij} \right] \underline{S} \tag{6.18}$$

where $\underline{\alpha}_j$ is a column vector of zeros with a one in the j th place,

and,
$$\underline{V}_i^* = \frac{\partial^2 \underline{S}}{\partial \underline{\bar{r}}\ \partial \bar{r}_i}$$

$$\underline{c}_i = \frac{\partial \underline{b}}{\partial \bar{r}_i}$$

Thus the global stiffness $\underline{K}$ is given by

$$\underline{K} = \underline{K}_0 + \underline{K}_1 + \underline{K}_2 \qquad (6.19)$$

where
$$\underline{K}_0 = \underline{a}^T \underline{k}_0 \underline{a}$$

$$\underline{K}_1 = \underline{a}^T \underline{k}_1 (\underline{S}^0) \underline{a}$$

and the columns of $\underline{K}_2 = \left[\sum_{i=1}^{m} (\underline{V}_i^* \underline{b} + \underline{a}^* \underline{c}_i)^T \underline{b}_j \right] \underline{S}$

The element stiffnesses $\underline{k}_0$ and $\underline{k}_1$ are found in Equations 6.3 to 6.8.

3. Derivation of the $\underline{a}^*$ and $\underline{V}^*$ Matrices

Having outlined the equilibrium equations required to solve the large displacement structural problem, we next consider the $\underline{a}^*$, $\underline{V}^*$ and $\underline{c}$ matrices necessary for the computation of the instantaneous stiffness $\underline{K}$ of Equation 6.17. Then, given the global displacements $\underline{r}$ and $\underline{\bar{r}}$ and the element forces $\underline{S}$ at any stage, we shall be able to calculate the global instantaneous or tangent stiffness of the structure $\underline{K}$. This stiffness defines the tangent plane to the load-displacement surface at the instantaneous displacement configuration.

The loads $\underline{R}$ are assumed to be a continuous twice differentiable single-valued function of the displacements $\underline{r}$. This assumption is, in fact, violated in the case of bifurcation buckling, which will be discussed separately below.

The differentiation of Equations 4.9 and 5.10 for the secant or tangent element stiffness systems is a mechanical process; however, the results are extremely lengthy and have been included directly in the computer program and there seems little point in reproducing these results here.

4. Incremental Solution Technique

When the loads are single-valued functions of displacements, an incremental displacement method can be used. This method is most useful in following the complete load displacement history of snap-through buckling problems where the same total load occurs at different displacements. Otherwise an incremental load method may be used. No attempt is made here to analyze structures whose load-displacement curves involve bifurcations of the buckling paths beyond the bifurcation point.

The following illustrates the procedures involved during one increment of the solution technique.

At the beginning of an increment, the total global displacements $\underline{r}$ at the end of the previous increment are used to calculate the $\underline{a}^*$, $\underline{b}$ and $\underline{V}^*$ matrices defined by Equations 6.10, 3.18 and 6.18. The element displacements $\underline{S}$ at the end of the previous increment are used in the stiffness matrix $\underline{k}_1(\underline{s})$ and then the matrix products of Equation 6.19 are calculated to produce the instantaneous tangent stiffness matrix $\underline{K}$ of Equation 6.17.

The stiffness matrix $\underline{K}$ is to be inverted by a band inversion routine. For maximum solution efficiency the band width of the matrix should be a minimum. Both positive-definite and positive semi-definite matrices are allowed.

A one parameter (λ) load system is applied to the structure and is assumed to vary linearly with displacement during the increment. For the incremental load solution method, the load increment vector which is supplied by the analyst is used in equation 6.17 to produce the incremental deflection vector. However, for the displacement increment method, a unit load vector $\underline{\beta}$ is applied to produce the deflection vector $\underline{\eta}$

$$\underline{K}\,\underline{\eta} = \underline{\beta} \tag{6.20}$$

The load increment λ is calculated by linearly proportioning the deflections ρ and ξ.

$$\lambda = \frac{\rho}{\xi} \tag{6.21}$$

where ρ is the displacement increment specified by the analyst and ξ is the displacement at the same degree of freedom as ρ under the unit load vector $\underline{\beta}$.

Thus we have:

$$\Delta \underline{R} = \lambda \underline{\beta} \tag{6.22}$$

and

$$\Delta \underline{r} = \lambda \underline{\eta} \tag{6.23}$$

These increments of global force and displacement are added to the total force and displacement vectors $\underline{R}$ and $\underline{r}$ respectively. $\delta \underline{s}$ is then calculated by Equation 6.11 and $\delta \underline{S}$ by Equation 6.2 before proceeding to the next increment of the solution.

CHAPTER VII

BIFURCATION BUCKLING

Many structures exhibit a branching of their equilibrium paths that is associated with some mode of structural instability. For most such structures, the determination of this 'critical value' involves using the linear stiffness based on the initial geometry plus a non linear contribution based on some small perturbation from the initial position. Thus the stiffness matrices $\underline{K}_0$, $\underline{K}_1$, and $\underline{K}_2$ should be useful for determination of the bifurcation point. The nature of the equilibrium paths in the immediate post buckling range is of interest to the analyst, but an investigation of this problem will not be attempted here.

Unfortunately, the solution of the eigenvalue problem associated with the determination of the bifurcation point involves the inversion of a full matrix and this puts a constraint on the size of the problem which can be handled. But if only a few of the smaller eigenvalues are required and the stiffness matrices are well banded, then an iterative algorithm is possible. The method is efficient for finding a few of the lowest eigenvalues of a positive definite matrix $\underline{K}_0$. No attempt should be made to determine all of the eigenvalues by this iterative scheme. A problem in convergence does arise when two eigenvalues are the same or almost equal but we assume that this case does not occur often and that, when it does, it can be solved by an alternate procedure.

The procedure for determining the lowest critical value only will be outlined here, since the interest of the analyst is usually confined to this one. An iterative algorithm has the advantage that the solution can be obtained to any desired accuracy consistent with the inherent roundoff errors of the computer.

We restrict the load configuration on the structure to a single parameter system where the loads increase uniformly according to a factor λ. The deflection vector $\underline{\delta r}'$ corresponding to a normalized force vector $\underline{\delta R}'$ is calculated using the linear stiffness $\underline{K}_o$. $\underline{K}_1$ and $\underline{K}_2$ are then built using this deflection vector to yield:

$$\underline{K}_P = \underline{K}_1 \left(\underline{\delta r}' \right) + \underline{K}_2 \left(\underline{\delta r}' \right) \tag{7.1}$$

We form the total stiffness of Equation 7.2 assuming that $\underline{K}_P$ is linear up to some critical value of λ, λ_{cr}.

$$\left(\underline{K}_o + \lambda \underline{K}_P \right) \underline{\delta r} = \underline{\delta R} \tag{7.2}$$

The classical eigenvalue problem results when the right-hand side of Equation 7.2 vanishes. On rearrangement we obtain:

$$\frac{1}{\lambda_{cr}} \underline{K}_o \underline{\delta r} = - \underline{K}_P \underline{\delta r} \tag{7.3}$$

where $\underline{\delta r}$ corresponds to the eigenvector and, again, λ_{cr} is the eigenvalue.

In the iterative scheme, a trial vector $\underline{\delta r}''$ is inserted in the right-hand side of Equation 7.3, and the left-hand side is used to solve for an improved eigenvector $\underline{\delta r}'''$. The length of the new vector is a first approximation of the inverse of the eigenvalue λ_{cr}. We normalize $\underline{\delta r}'''$ and repeat this procedure until the critical value λ_{cr} stabilizes according to some convergence criterion or until the number of iterative cycles exceeds some given number. The rate of convergence to $\underline{\lambda_{cr}}$ and $\underline{\delta r}$, the eigenvector, and the time required depends on the size of the problem as well as on the relative difference between the first and second eigenvalues.

Derivation of this procedure is given in Wilkinson (16).

CHAPTER VIII

NUMERICAL EXAMPLES

1. Williams' Toggle

In 1964 Williams (10) extensively studied the snap-through buckling phenomena of a shallow planar frame illustrated in Figure 8.1. The geometric and elastic properties of this frame, called a 'toggle' by Williams, are the following:

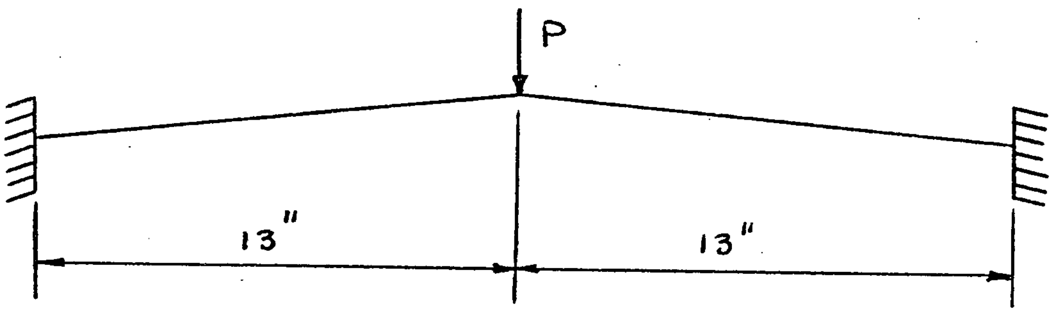
$$EI = 9.27 \times 10^3 \text{ lb. inch}^2$$
$$AE = 1.885 \times 10^6 \text{ lb.}$$
$$L = 26 \text{ inch}$$
$$h = 0.32 \text{ inch}$$

The large deflection behaviour of this structure under a vertical applied load on the centerline is characterized by a softening region followed by a hardening region.

Ebner and Ucciferro (12) in a recent paper reviewed and compared five particular methods and their applications to geometric non-linear planar problems. These five formulations are:

1. Argyris (1964), (13)
2. Martin (1965), (6)
3. Jennings (1968), (4)
4. Mallet and Marcal (1968), (3)
5. Powell (1969), (14).

In all of these methods the non-linear force displacement relationships



$EI = 9.27 \times 10^3$ lb inch^2

$AE = 1.885 \times 10^6$ lb

rise at ℄ .32 inch

FIGURE 8.1 WILLIAMS' TOGGLE WITH CENTRAL LOAD have been formulated in terms of stiffness matrices. The solution techniques incorporated were:

1. direct solution,
2. load incrementation,
3. displacement incrementation.

Williams' toggle was used by Ebner and Ucciferro for testing the five methods. The solutions for the toggle under a constant centerline vertical load of eighty pounds are given in Table 1. In view of symmetry, only half of the structure was modelled and only eight elements were used.

The results, as can be expected, varied but Jennings' procedures give excellent agreement with Williams' solution for all three solution techniques. The 'secant' and 'tangential' element solutions for various displacement increment sizes and numbers of elements (modelling half of the toggle) are shown in Table 2 and Table 3. For this example, the 'secant' element solutions converged monotonically to a lower centerline deflection than the 'tangential' element solution with the number of elements constant at ten per half span. However, the difference of 0.002 inches in 0.6159 or 0.3% is negligible. When the displacement increment size was kept constant at 0.01 inch and the number of elements used to model the structure were varied, the 'secant' elements performed extremely well even when using only one element. The 'tangential' element solutions showed a more rapid convergence although no solutions are available for using one or two elements per half span. This element behaves poorly when the element displacements become too large. It is probable that the 'tangential' element gives a poor reflection of the axial deformation when element

TABLE 1

SOLUTIONS FOR WILLIAMS' TOGGLE

(Under 80 lb. load and 8 elements per half)

	Formulation	Centerline Deflection (inches)
Jennings	Direct	0.611
Powell	Direct	0.611
Mallet-Marcal	Direct	0.600
Williams	Exact	0.611
Jennings	1 lb. load increments	0.639
Powell	1 lb. load increments	0.639
Martin	1 lb. load increments	0.640
Jennings	0.007 inch. displ. increments	0.616
Martin	0.007 inch. displ. increments	0.621
'Tangential' element	0.007 inch. displ. increments	.6203
'Secant' element	0.007 inch. displ. increments	.6161

TABLE 2

WILLIAMS' TOGGLE

0.01 inch displacement increments and 80 lb. load

	'Tangential' Elements Centerline Deflection (inches)	'Secant' Elements Centerline Deflection (inches)
1 element per half	-	0.6312
2 elements per half	-	0.6308
5 elements per half	0.6550	0.6191
10 elements per half	0.6200	0.6179
20 elements per half	0.6180	0.6179

TABLE 3

WILLIAM'S TOGGLE

10 elements per half and 80 lb. load

	'Tangential' Elements Centerline Deflection (inches)	'Secant' Elements Centerline Deflection (inches)
0.007 inches per increment	0.6178	0.6159
0.010 inches per increment	0.6200	0.6179
0.020 inches per increment	0.6262	0.6246
0.040 inches per increment	0.6406	0.6387

deflections are large. This would be important in the structure under discussion.

Jennings (4) derived two element stiffnesses including varying degrees of geometric non-linearity, and was able to obtain good agreement with Williams' results. He noted that where geometric changes were significant, many more of his less sophisticated elements were required than of his sophisticated non-linear elements to accurately model the load deformation behaviour of the structure. Although Jennings does not specify what displacement increment sizes he used, the 'tangential' element gave similar results to Jennings' less sophisticated element solutions. The 'secant' element solutions appear to agree with the results by Jennings' better element.

In general the 'secant' elements performed better than the 'tangential' elements. However, the time required for the 'secant' element solution was greater than for the corresponding 'tangential' element solution. Also, the difference in the results between using twenty elements per half span and ten elements per half span is small, indicating that the ultra-fineness of solution is not warranted. The reduced increment size seems more important than increasing the number of elements in the structure.

Some of the load deflection paths for this structure are shown in Figure 8.2.

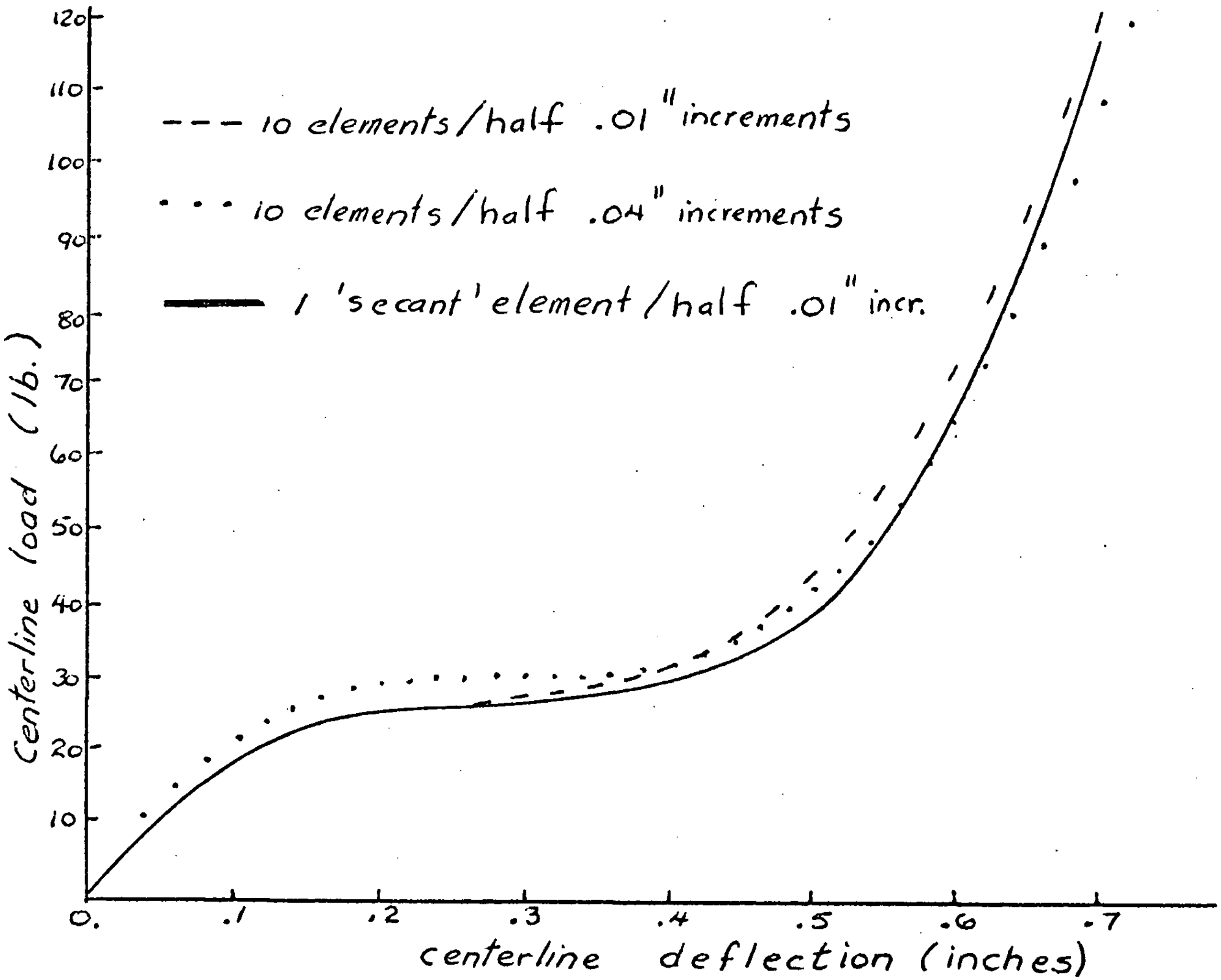


FIGURE 8.2 INCREMENTAL DEFLECTION SOLUTIONS FOR WILLIAMS' TOGGLE

2. Argyris' Arch

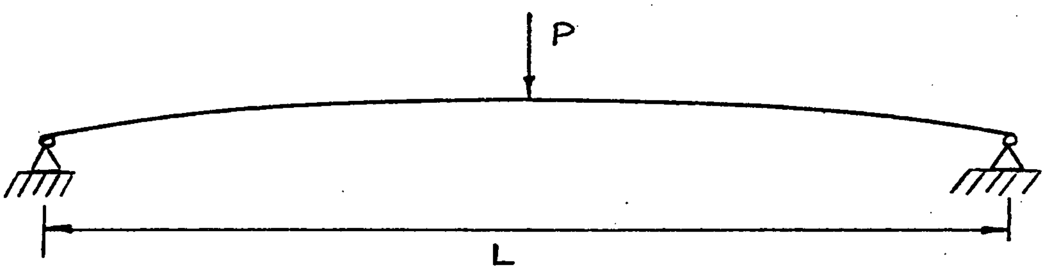
Ebner and Ucciferro (12) also used a plane arch, first tested by Argyris (1), to compare the five various formulations cited previously. The results of these formulations are given in Table 4 along with the 'tangential' and 'secant' element solutions for the same number of elements per half span and size of displacement increment.

The elastic and geometric properties of the arch shown in Figure 8.3 are as follows:

EA	=	10^7 lb.
EI	=	10^7 lb. inches^2
rise	=	3.14 inches
radius of curvature	=	400 inches
length	=	100 inches

The arch is pinned at the ends and the load is applied vertically at the centerline. Since straight line elements are used, at least ten elements are required to model the entire arch as a polygonal arc. The fewer elements used, the poorer the mathematical model of the intended structure.

For this problem, there was little difference in the results between the 'secant' element and 'tangential' element solutions. Agreement with the displacement incrementation solutions of Martin, Jennings and Argyris is good. Definitely, smaller deflection increment sizes and more increments produced 'better' results for the arch of a reasonable number of elements than for the same structure composed of many elements but a larger increment size. In the snap-through buckling of this arch, the true solution curve can be expected to fall below the upper snap value that the



$EA = 10^7$ lb

$EI = 10^7$ lb inch.^{2}

rise = 3.14 inch.

radius of curvature = 400 inch.

L = 100 inch.

FIGURE 8.3 ARGYRIS' ARCH

TABLE 4

ARGYRIS' ARCH

Formulation	Number of elements	Increment	Upper Snap lb.	Lower Snap lb.
Martin Load Incr.	5/half	25 lbs.	2300	-
Powell Load Incr.	5/half	25 lbs.	2300	-
Jennings Load Incr.	5/half	25 lbs.	2300	-
Martin Displ. Incr.	5/half	0.07 inch	2344	814
	10/half	0.07 inch	2360	828
Jennings Displ. Incr.	5/half	0.07 inch	2360	836
	10/half	0.07 inch	2358	837
Argyris	10/half	0.157 inch	2450	800
'Secant' Elements	5/half	0.07 inch	2328	849.6
	10/half	0.07 inch	2356	838.2
	10/half	0.157 inch	2460	846.1
'Tangential' Elements	5/half	0.07 inch	2325	824.9
	10/half	0.07 inch	2355	835.9
	10/half	0.157 inch	2460	843.8

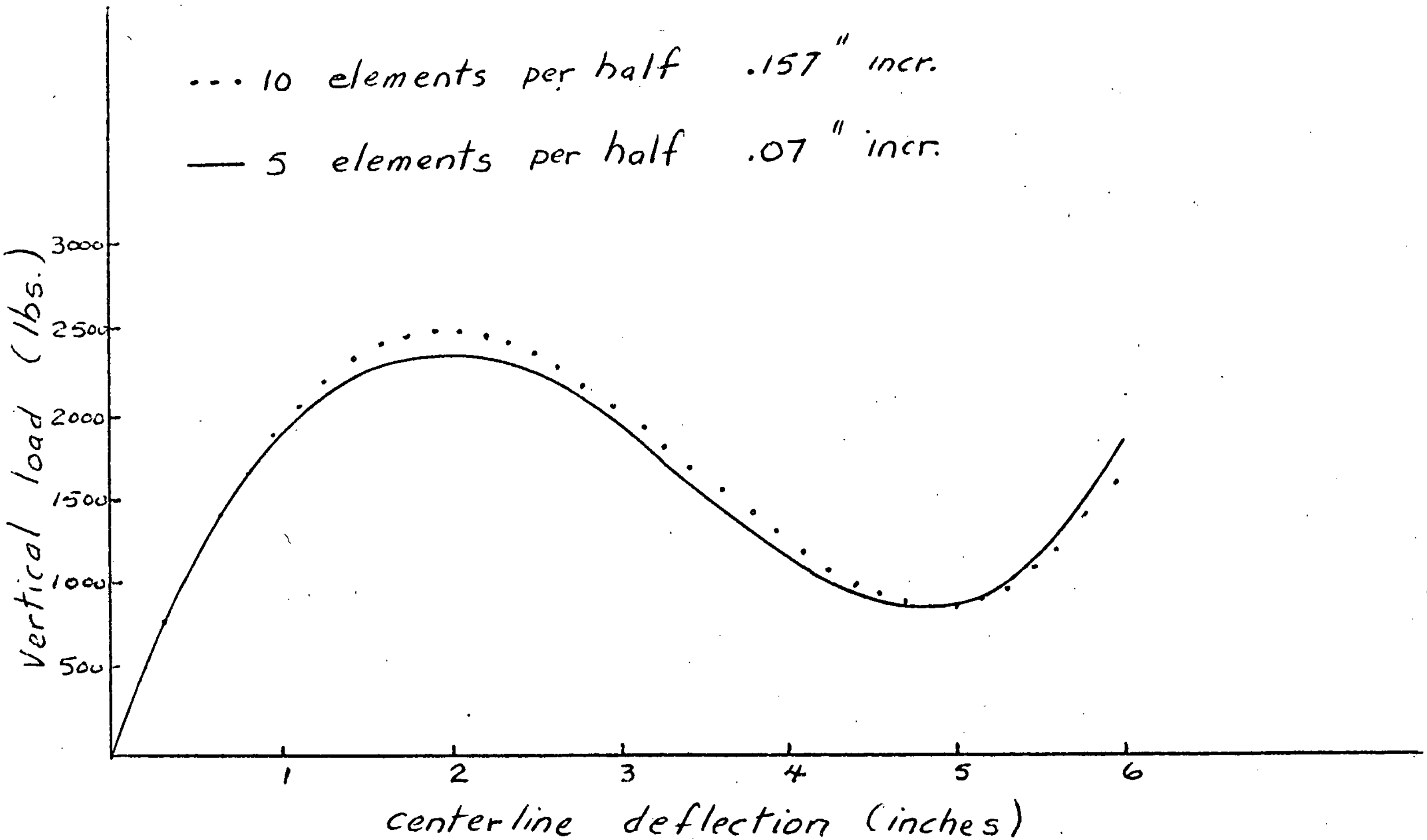


FIGURE 8.4 LOAD DEFLECTION CURVE FOR ARGYRIS' ARCH

'secant' and 'tangential' element solutions predict. Nevertheless the solutions for the arch modelled by five elements per half span predict lower upper snap values than the corresponding solution value for the ten elements per half span arch. Quite likely the polygonal structures are sufficiently different for these two cases to account for this anomaly.

One advantage possessed by the displacement incrementation solution technique, compared with the load incrementation solution method, is the ability to follow the complete load deflection curve of a snap-through buckling problem. Several such curves for the arch for various increment sizes and numbers of elements are shown in Figure 8.4.

3. Wright's Reticulated Shell Segment

D.T. Wright (13) in 1965, in a paper concerning the design and stability of reticular domed structures, gave some theoretical calculations for snap-through buckling of a particular spherical shell segment shown in Figure 8.5. He considered the behaviour of this segment under a vertical load P at node A . In the derivation of the load deflection curve he assumed that the shell was shallow and that the following ratios held:

$$\frac{L}{L_r} \approx \sin\frac{L}{L_r} \approx \tan\frac{L}{L_r}$$

where L_r = the radius of curvature of the dome

L = the length of the element shown in Figure 8.5.

When he considered the nodes B to be pinned and ignored members BB which do not contribute any moment resistance, he arrived at an upper bound solution for snap-through buckling of the shell. His theoretical solution was:

$$P = \frac{3AE}{L^3} h'(h^2 - h'^2) + 44 \frac{EI}{L^3}(h - h') \tag{8.1}$$

where h and h' are defined in Figure 8.5.

Differentiating Equation 8.1 with respect to h' he found that snap through buckling would only occur when the following condition held:

$$r_g \leqslant .263\, h \tag{8.2}$$

where $r_g = \sqrt{\frac{I}{A}}$, the radius of gyration of the members

h = the rise of the element.

When he considered the case where the joints B are allowed to translate and the members BB are extensible, he arrived at the lower bound solution for the load-deflection curve of the shell given by Equation 8.3.

$$P = \frac{3}{2} \frac{AE}{L^3} h'(h^2 - h'^2) + 44 \frac{EI}{L^3}(h - h') \tag{8.3}$$

The critical values for this equation upon differentiating with respect to h' occurs when:

$$r_g \leqslant .185\, h \tag{8.4}$$

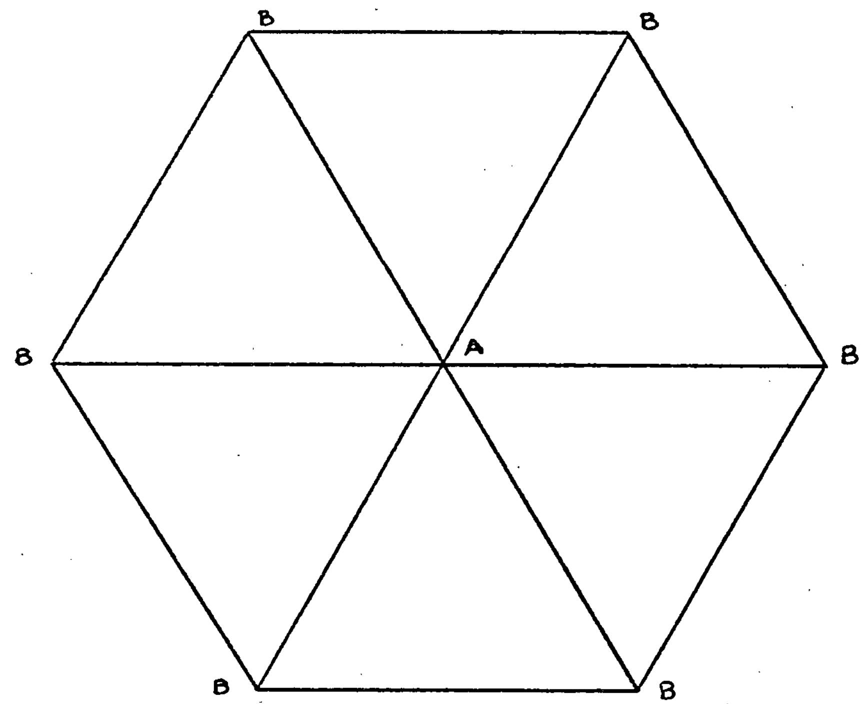
To verify Equations 8.1 and 8.3 the material properties of the toggle of Figure 8.1 were substituted into these equations.

The radius of gyration, rg, for this structure is .072 inches. Thus both equations 8.2 and 8.4 are satisfied since

$r_g \leq .1052$ for equation 8.2

$r_g \leq .074$ for equation 8.4

Therefore snap through buckling will occur for the pinned structure and for the structure which allows members BB to extend. Buckling out of plane has been prevented as we are solely interested in verifying Wright's calculations.



all members have equal properties

$A = .0628$ inches^2

$I = .000309$ inches^4

$E = 3.0 \times 10^7$ psi

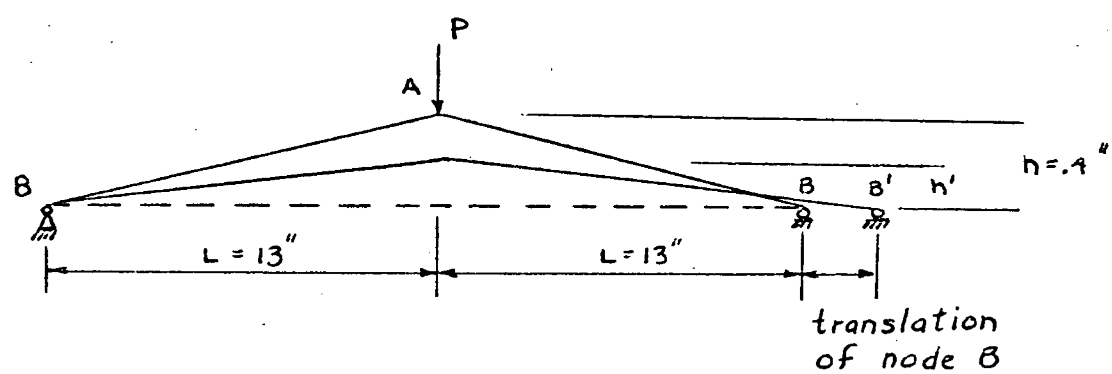


FIGURE 8.5 WRIGHT'S SHELL SEGMENT

TABLE 5

WRIGHT'S SHELL SEGMENT

	Upper Snap	Lower Snap
BB Inextensible	lb.	lb.
Wright's formulation	99.9	48.5
12 'Secant' elements 0.01" deflection increments	96.8	59.4
12 'Tangential' elements 0.01" deflection increments	96.1	49.0
BB Extensible		
Wright's formulation	75.2	73.4
12 'Secant' elements 0.01" deflection increments	*	*
12 'Tangential' elements 0.01" deflection increments	71.8	71.6

* Solution does not exhibit maximum or minimum

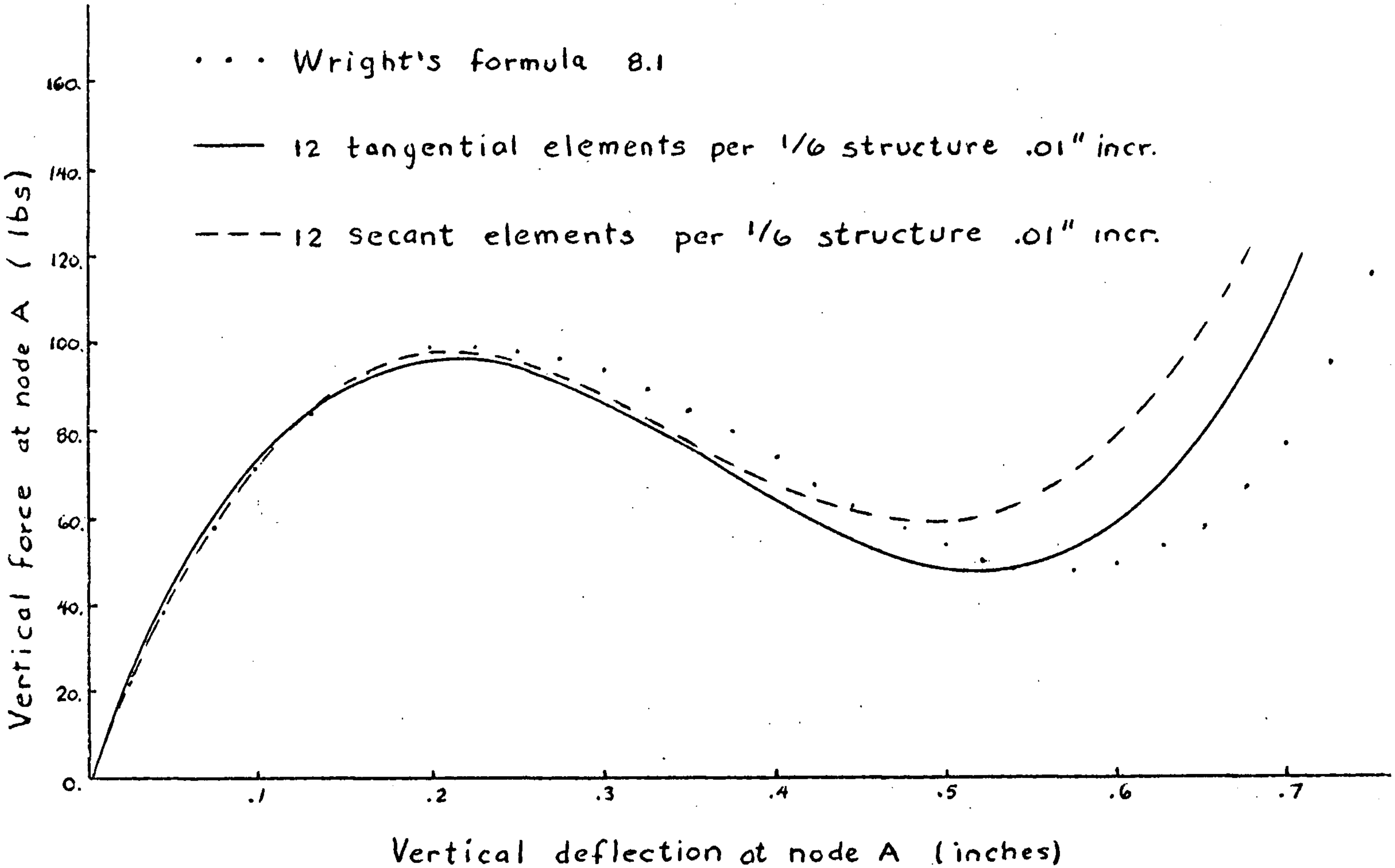


FIGURE 8.6 LOAD DEFLECTION CURVE OF WRIGHT'S SEGMENT WITH POINTS B FIXED AGAINST TRANSLATION

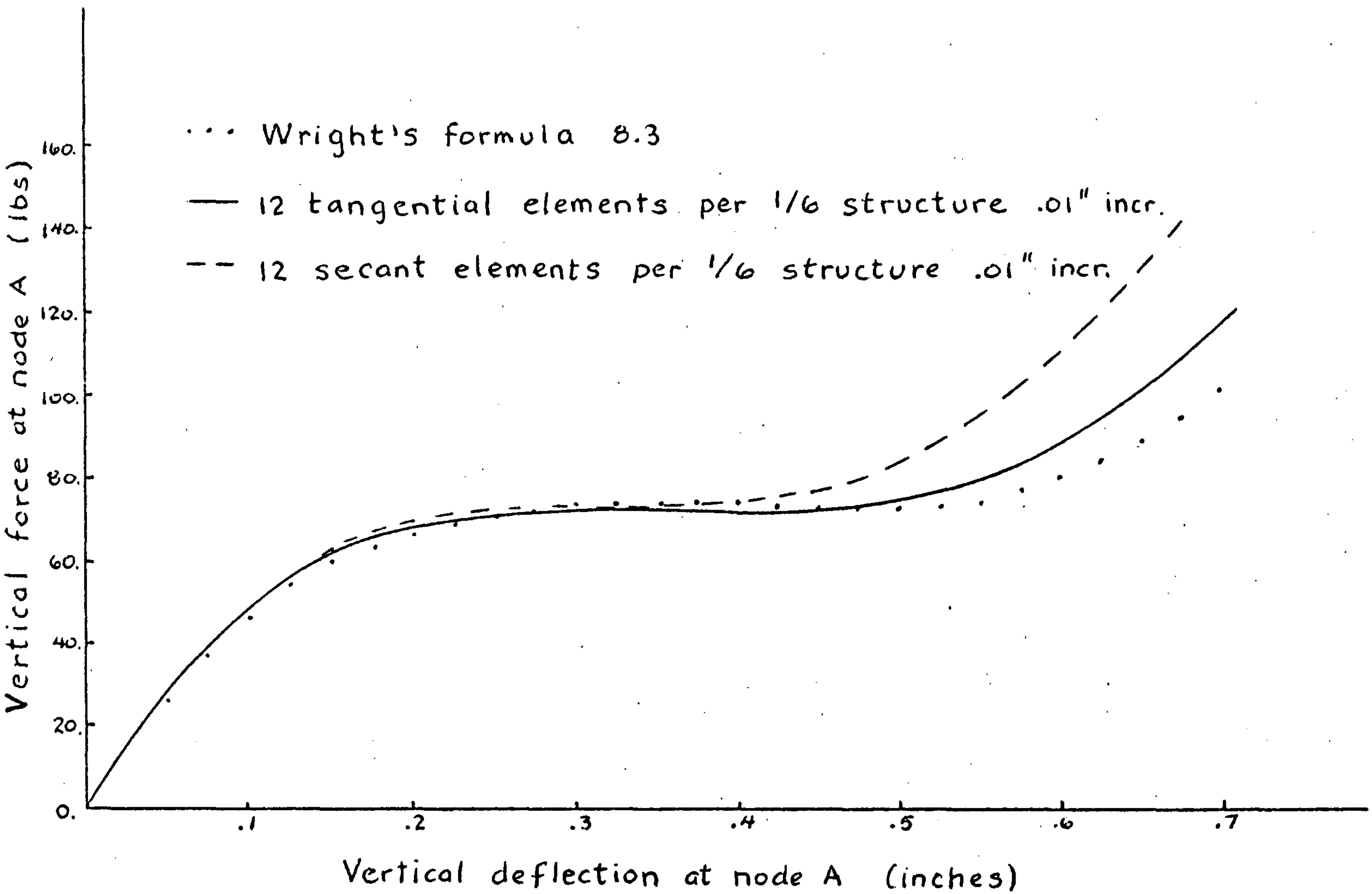


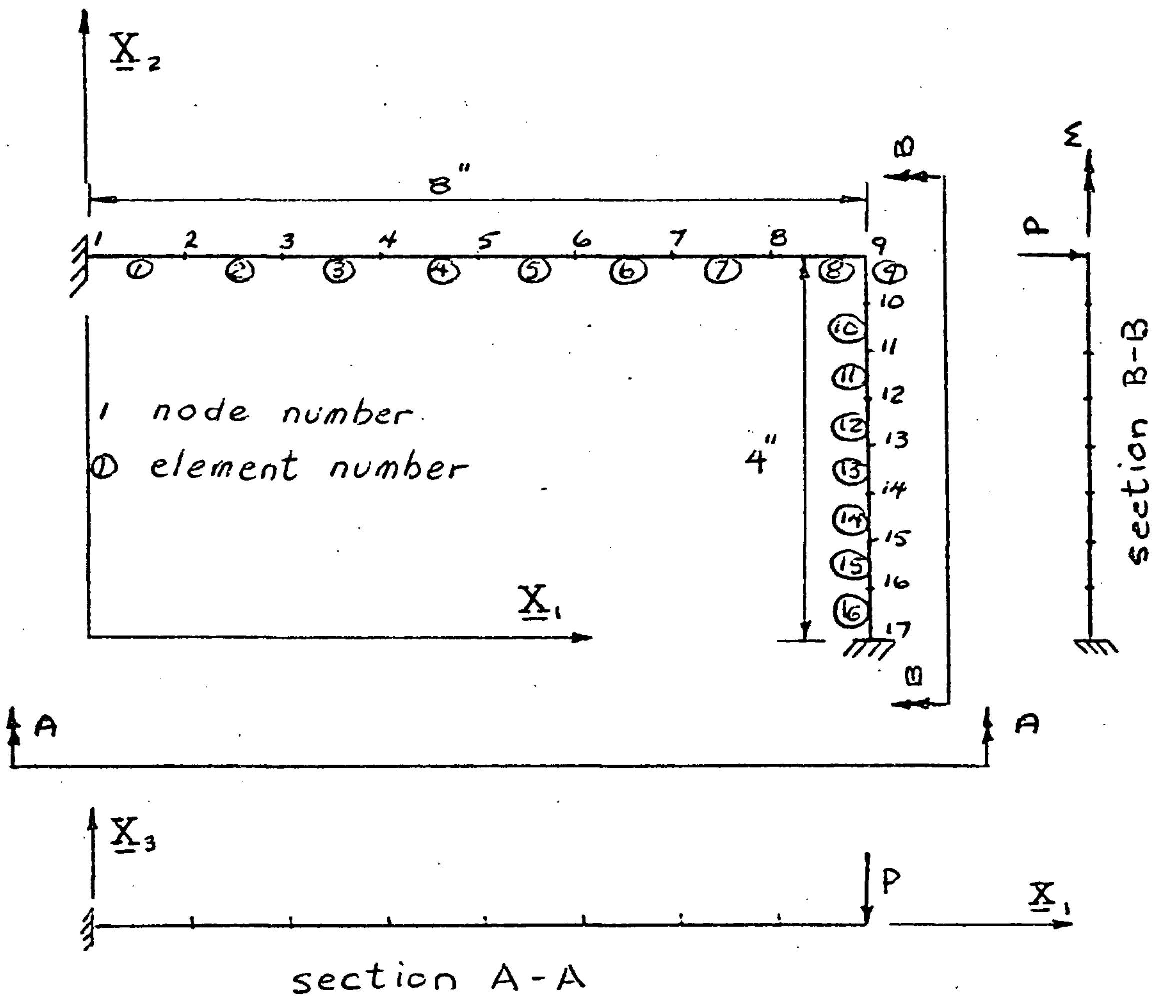
FIGURE 8.7 LOAD DEFLECTION CURVE OF WRIGHT'S SEGMENT WITH POINTS FREE TO TRANSLATE

By symmetry, we need consider only one sixth of the shell segment. Twelve elements were used to model this structure and deflection increments of 0.01 inches were used. Agreement with the formulas 8.1 and 8.3 was fair as shown in Table 5. The theoretical load-deflection curves and the experimental results are shown in Figures 8.6 and 8.7.

4. Three Dimensional Elbow

In order to compare the accuracy of the 'tangential' and 'secant' elements, the structure shown in Figure 8.8, an elbow fixed at both ends was subjected to an applied force increment. The actual applied forces were compared with those required to equilibrate calculated internal forces (Equation 6.14: $\underline{R} = \underline{a}^T \underline{S}$).

Firstly, the elbow was modelled by sixteen elements and twenty vertical force increments of -30 were applied at node 9. Secondly, twenty increments of moment about the X_2 axis of 60 were applied at node 9. The results given by Table 6 compare the known total applied force with the forces calculated by equation 6.14 for various increment steps. Also shown are the deflections under the applied force and the vector of calculated forces at node 9 of the structure for increment step 20. All but the applied force component of this vector should be zero but the magnitude of the error when compared with the applied force is small. Considering the size of the deflection, 0.9 in a length of 4, and of the rotation, 0.265 radians in only twenty increment steps, the results are considered reasonable.



$I_1 = 1.0\ in^4$ torsion

$I_2 = 1.0\ in^4$ strong bending plane

$I_3 = 1.0\ in^4$ weak bending plane

$A = 1.0\ in^2$

$E = 10000.$ ksi

$G = 3000.$ ksi

FIGURE 8.8 THREE-DIMENSIONAL ELBOW

TABLE 6

THREE DIMENSIONAL ELBOW

Applied Vertical Force Increments

Increment Step	Total Applied Force	Force Calculated by 'Tangential' Elements: Force	'Tangential' Elements: Deflection	'Secant' Elements: Force	'Secant' Elements: Deflection
5	- 150	- 150.03	- 0.2481	- 149.95	- 0.2492
10	- 300	- 300.18	- 0.4849	- 299.92	- 0.4953
15	- 450	- 450.32	- 0.7023	- 449.90	- 0.7343
20	- 600	- 600.69	- 0.8982	- 599.93	- 0.9633

$\underline{R} = \underline{a}^T \underline{S}$ for node nine and increment step 20

'Tangential' (- 4.44, - 9.53, - 600.69, 7.48, 3.79, - 1.18)

'Secant' (- 4.51, - 10.20, - 599.93, 0.21, - 0.21, 0.08)

Node forces $(F_{x_1}, F_{x_2}, F_{x_3}, M_{x_1}, M_{x_2}, M_{x_3})$

Applied Moment Increments

Increment Step	Total Applied Moment	Moment Calculated by 'Tangential' Elements: Moment	'Tangential' Elements: Rotation	'Secant' Elements: Moment	'Secant' Elements: Rotation
5	300	299.95	0.0655	299.97	0.0655
10	600	599.85	0.1314	599.87	0.1312
15	900	899.74	0.1974	899.64	0.1977
20	1200	1199.60	0.2646	1199.15	0.2654

$\underline{R} = \underline{a}^T \underline{S}$ for node nine

'Tangential' (- 0.79, - 0.12, - 0.02, 0.97, 1199.60, - 83.04)

'Secant' (- 0.23, 0.32, - 0.08, 0.32, 1199.15, - 83.39)

5. Ring Dome

The large ring dome shown in Figure 8.9 represents a practical structure. By symmetry only half of the dome was modelled. This helped reduce the band width of the problem immensely and meant that subdividing the structure into more elements would not seriously increase this band width.

The elastic and geometric properties of the ring dome are as follows:

All members:

I_1 = 144 in.^{4} (torsional moment of inertia)

I_2 = 720 in.^{4} (strong bending plane)

I_3 = 48.0 in.^{4} (weak bending plane)

Area = 4.0 in.^{2}

Diameter of dome = 80 ft.

Rise at Centre = 8 ft.

Radius of Curvature = 104 ft.

E = 3000 ksi

G = 1200 ksi

The vertical plane was made the strong bending plane and the exterior nodes were fixed.

The structure was divided into 108 'cantilever' elements resulting in 504 degrees of freedom and a band width of only 42. The 'tangential' elements were chosen primarily because of cost. The principal intent was to show the load deflection behaviour of the dome. For this, the upper ring was subjected to 16 vertical deflection increments of 0.25 feet under a uniform vertical load on the upper ring. The load deflection curve and the configuration of a radius of the structure after 16 increments are shown in Figure 8.10 and figure 8.11. However no attempt has been made to thoroughly analyze all the instabilities of this structure. Had another mode of failure occured other than that shown, then the computer program would have at least indicated this when the stiffness matrix went negative definite. Although there is no solution with which to compare these results, they are self-consistent and there is every indication that the method was able to handle this comparatively large problem successfully.

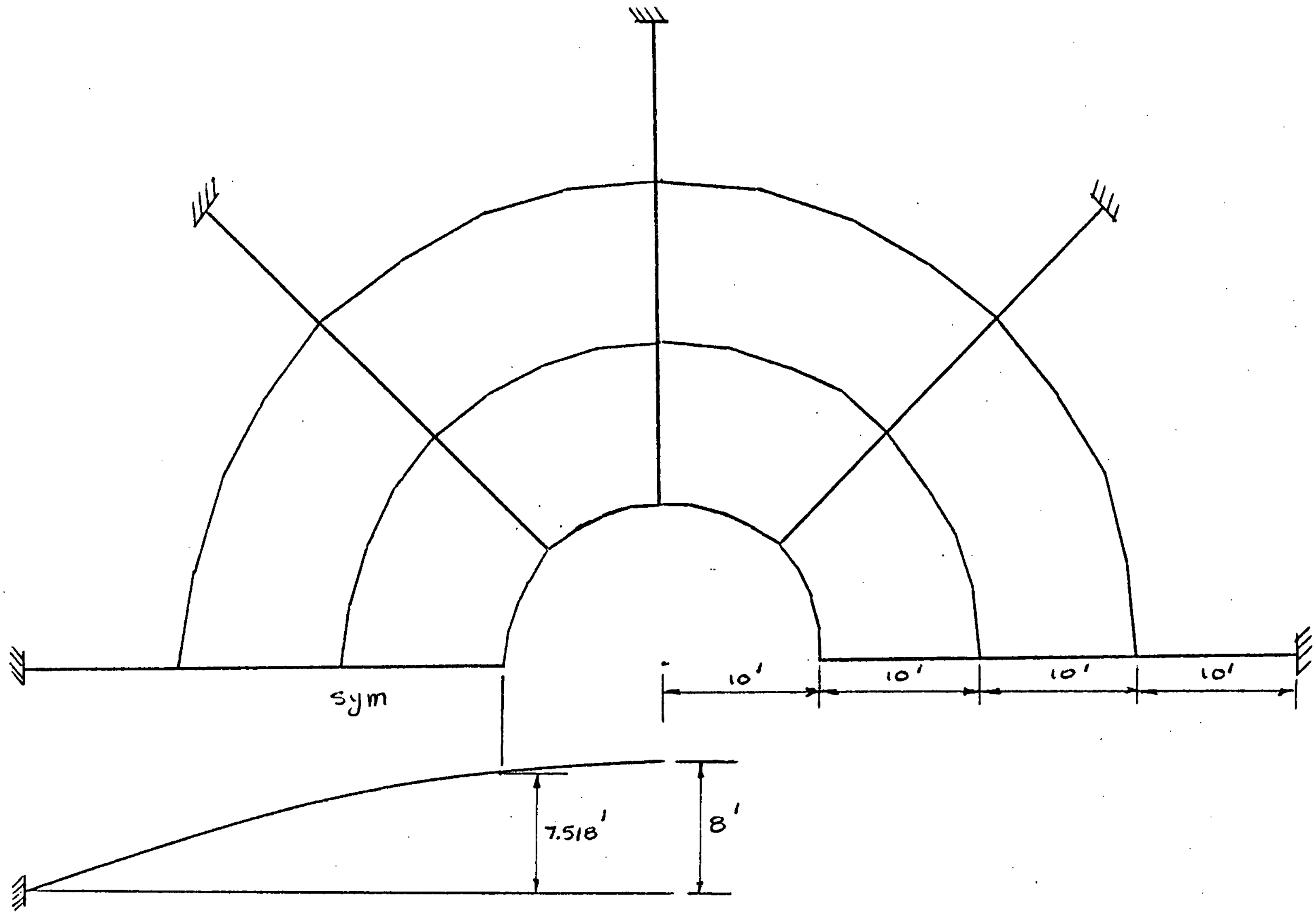


FIGURE 8.9 RING DOME

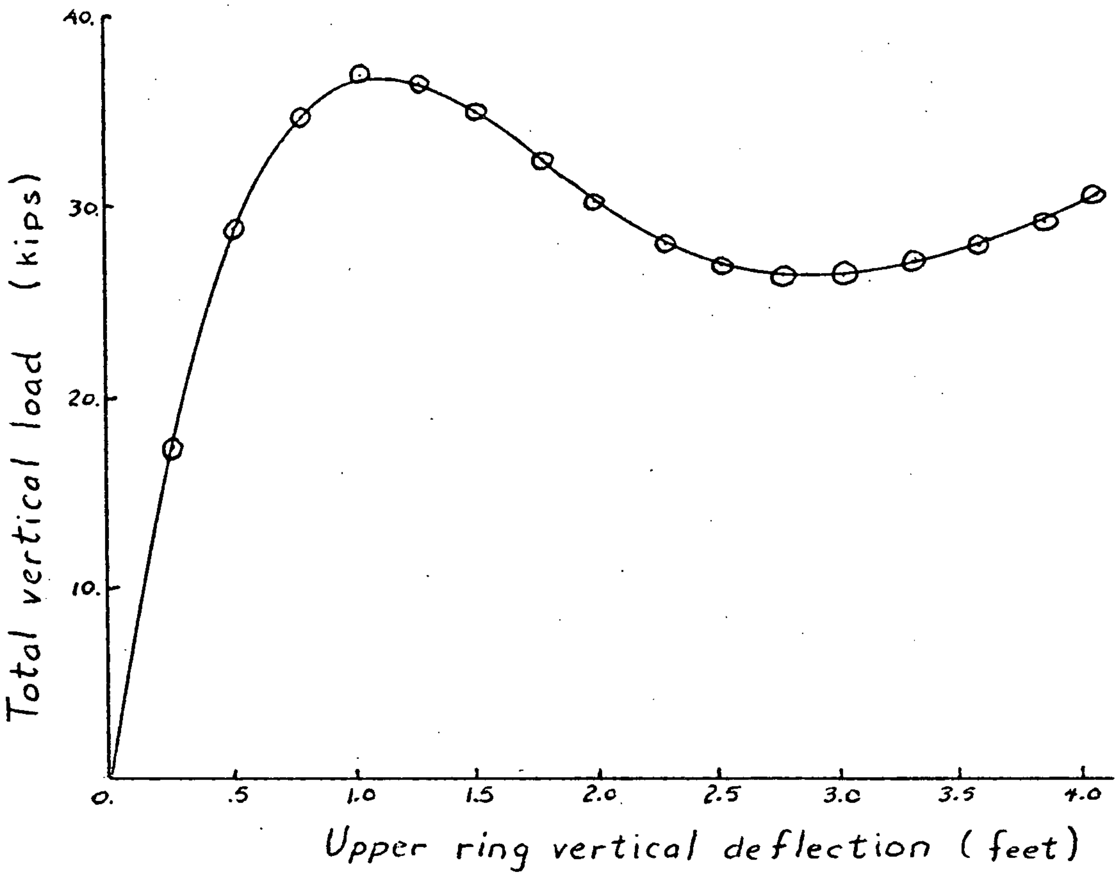


FIGURE 8.10 LOAD DEFLECTION CURVE FOR RING DOME

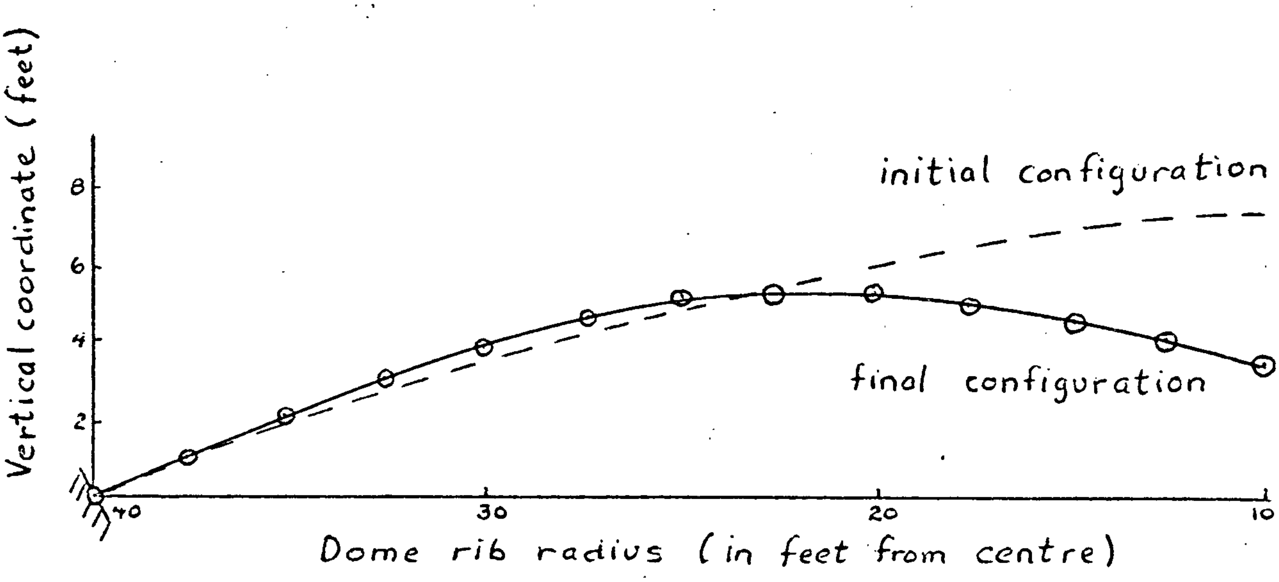


FIGURE 8.11 DEFLECTED CONFIGURATION OF A DOME RIB

CHAPTER IX

CONCLUSIONS

A procedure was developed, using non-linear stiffness matrices from Nathan (11), to follow the load deflection paths of shallow framed structures. The necessary transformation matrices and geometrical relationships were formulated for extending previous two-dimensional work to three dimensions.

An incremental solution technique was used and shown to be practical. Two moving element coordinate systems and elements, the 'secant' and 'tangential' systems, were developed and related to a fixed global system.

The snap-through buckling paths of plane frames, arches and space frames were studied. The 'secant' element was found to be more efficient for some large deflection problems in that fewer elements were required to reach a suitable solution. However, the 'tangential' solution required about one half the computer time of the corresponding 'secant' solution. Reducing the increment step size was found more effective than increasing the number of elements used to model the structure.

BIBLIOGRAPHY

1. Argyris, J.H., "Continua and Discontinua", *Proceedings of the Conference on Matrix Methods in Structural Mechanics*, Wright Patterson Air Force Base, October 1965, Air Force Flight Dynamics Laboratory, Dayton, Ohio, AFFDL-TR-66-20.

2. Koiter, W.T., "Elastic Stability and Post-Buckling Behaviour", *Non-Linear Problems*, Edited by R.E. Langer, University of Wisconsin Press, Madison, Wisconsin, 1963.

3. Mallett, R.H. and Marcal, P.V., "Finite Element Analysis of Nonlinear Structures", *Journal of the Structural Division*, Proceedings of the A.S.C.E., Vol. 94, No. ST9, September 1968.

4. Jennings, A., "Frame Analysis including Change of Geometry", *Journal of the Structural Division*, Proceedings of the A.S.C.E., Vol. 94, No. ST3, March 1968.

5. Britvec, S.J. and Chilver, A.H., "Elastic Buckling of Rigidly-Jointed Braced Frames", *Journal of the Engineering Mechanics Division*, Proceedings of the A.S.C.E., Vol. 89, No. EM6, December 1963.

6. Martin, H.C., "On the Derivation of Stiffness Matrices for the Analysis of Large Deflection and Stability Problems", *Proceedings of Conference on Matrix Methods in Structural Mechanics*, Wright Patterson Air Force Base, October 1965, Air Force Flight Dynamics Laboratory, Dayton, Ohio, pp. 697-715.

7. Supple, W.J., "Initial Post-Buckling Behaviour of a Class of Elastic Structural Systems", *International Journal of Non-Linear Mechanics*, Vol. 4, pp. 23-36, 1969.

8. Roorda, J., "Stability of Structures with Small Imperfections", *Journal of the Engineering Mechanics Division*, Proceedings of the A.S.C.E., Vol. 91, No. EM1, February 1965.

9. Roorda, J., "The Buckling Behaviour of Imperfect Structural Systems", *Journal of the Mechanics of Physics and Solids*, Vol. 13, pp. 267-280, 1965.

10. Williams, F.W., "An Approach to the Nonlinear Behaviour of the Members of a Rigidly Jointed Plane Framework with Finite Deflections", *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. 17, 1964, pp. 451-469.

11. Nathan, N.D., "Finite Element Formulation of Geometrically Non-Linear Problems," *Ph.D. Dissertation*, University of Washington, Seattle, Washington, March 1969.

12. Ebner, A.M. and Ucciferro, J.J., "A Theoretical and Numerical Comparison of Elastic Nonlinear Finite Element Methods", *National Symposium on Computerized Structural Analysis and Design*, George Washington University, Washington D.C., March 1972.

13. Wright, D.T., "Membrane Forces and Buckling in Reticulated Shells", *Journal of the Structural Division*, Proceedings of the A.S.C.E., Vol. 91, No. ST1, February 1965.

14. Powell, G.H., "Theory of Nonlinear Elastic Structures", *Journal of the Structural Division*, Proceedings of the A.S.C.E., Vol. 95, No. ST12, pp. 2687-2701, 1969.

15. Argyris, J.H., "Recent Advances in Matrix Methods of Structural Analysis", *Progress in Aeronautical Sciences*, Vol. 4, MacMillan Co., New York, 1964.

16. Wilkinson, J.H., *The Algebraic Eigenvalue Problem*, Oxford (1965).

```
T SNAPS
      C    PROGRAM TO FOLLOW THE LOAD DEFLECTION HISTORY OF FRAMED STRUCTURES
      C    BOTH CANTILEVER AND SECANT TYPE ELEMENTS
           IMPLICIT REAL*8(A-H,O-Z)
           REAL*4 TITLE
      C    DIMENSION BLOCK ONE AND TWO
           COMMON /BLK1/ DM,P1,P2,SMKO,NP
           COMMON /BLK2/ C4,C5,S4,S5,C10,C11,S10,S11,SMPREV,A
      C    DIMENSION JOINT AMOUNTS
           DIMENSION X(150),Y(150),Z(150),JN(150),ND(150,6)
      C    DIMENSION MEMBER AMOUNTS
          *,JNL(150),JNG(150),NTYPE(150),MLN(150)
          *,RR(150,12),SFF(150,6),SS(150,6),JLN(150),S40(150),C40(150)
          *,S50(150),C50(150),S60(150),C60(150),XORIG(150),YORIG(150)
          *,ZORIG(150),TR(150,12)
      C    DIMENSION LOAD AMOUNTS
          *,EV(600),DR(600),DCL(150),PCRIT(150)
      C    DIMENSION OTHERS
          *,TITLE(20),R(6),AAA(15),EX(15),EY(15),EZ(15),EEE(15),GGG(15),ES(6)
          *,FCONS(6),TOTAL(25),LDEG(25),NP(12),SMKO(6,6),DEFL(3)
          *,EDR(12),ER(12),SMPREV(6,6),EDS(6),EDSF(6),A(6,12),NDL(3)
          *,V(12,6,12),SV(12,12),SMK1(6,6),SSKO(12,12),SSK1(12,12),ATO(12,6)
          *,AT1(12,6),TEV(25),BLKA(45),BLKB(116),NINCR(20),SINCOS(6),XYZO(3)
          *,SKO(25000),SK1(25000)
           EQUIVALENCE (BLKA(1),DM),(BLKB(1),C4)
           DATA NINCR/1,2,3,6,11,21,31,41,51,61,71,81,91,101,111,121,151,
          *176,201,1001/
      C**
      1    FORMAT(20A4)
      2    FORMAT(20I4)
      3    FORMAT(7I4,3F10.3)
      4    FORMAT(6F10.3)
      23   FORMAT(2F10.3,2I4)
      5    FORMAT(1H1)
      6    FORMAT(/20X,' STRUCTURE NUMBER',I4,' HAS',I4,' JOINTS AND',I4,' ME
          *MBERS'/20X,' THERE ARE',I4,' INCREMENTS AND',I4,' ELEMENT TYPES'/
          *20X,' NBIF IS',I4,' N7 IS',I4,' AND N8 IS',I4,' AND NELMT IS',I4
          *,/20X,' (NELMT IS 0 MEANS CANTILEVER ELEMENTS, NON ZERO MEANS SECA
          *NT ELEMENTS)')
      7    FORMAT(/' JOINT NO. THE 6 DEGREES OF FREEDOM     THE INITIAL JOINT
          * COORDINATES'/11X,'   X    Y    Z   MX   MY   MZ',10X,'X',9X,'Y',9X,'Z'
          *)
      8    FORMAT(6X,I4,1X,6I4,4X,3F10.3)
      9    FORMAT(/' MEMBER  JNL   JNG    JOINT IN X1-X3 PLANE  MEMBER TYPE
          *          XP              YP              ZP')
      10   FORMAT(1X,I4,2X,I4,2X,I4,5X,I4,20X,I4,9X,3D12.4)
      11   FORMAT(/' THE DIFFERENT ELEMENT TYPES:  NUMBER    AREA         I(X1)
          *         I(X2)          I(X3)          E               G')
      12   FORMAT(31X,I4,6D12.4)
      13   FORMAT(/ ' DEFLECTION PARAMETER DIVIDED BY LENGTH OF',F10.3,/' THE
          * PORTION OF THE EIGENVECTOR USED AS A  DEFLECTION INCREMENT IS',F1
          *0.4,' (THIS IS USED ONLY IF NBIF .NE. 0)')
      14   FORMAT(/50X,'JOINT  DEGREE OF FREEDOM NUMBERS')
      15   FORMAT(49X,I4,4X,6I4)
      16   FORMAT(/' THE TOTAL NUMBER OF DEGREES OF FREEDOM =',I4)
      17   FORMAT(I4,6F8.0)
```

```
   18 FORMAT(2I4,3F8.0)
   19 FORMAT(' THE MAXIMUM HALF BAND WIDTH INCLUDING DIAGONAL',I4/
     *' TOTAL NUMBER OF LOCATIONS IN STIFFNESS NUXNB',I6)
   20 FORMAT(/54X,'INCREMENT NUMBER ',I5)
   21 FORMAT(/91X,'MEMBER DATA'/50X,' MN JNL JNG     IXX          IYY
     *    IZZ          E          G           AREA')
   22 FORMAT(50X,I3,2I4,3E12.4,3E11.3)
   24 FORMAT(' ELEMENT TOTAL DISPLACEMENTS R',10X,6F14.5/,40X,6F14.5)
   25 FORMAT(10X,' ELEMENT TOTAL FORCES S',6X,6F14.4)
   26 FORMAT(' RATIO DET =',D14.5)
   27 FORMAT(41X,'DEGREE OF FREEDOM   TOTAL FORCE'/50(45X,I4,7X,D14.5/))
   28 FORMAT('*',/2I10/20A4)
   29 FORMAT(6F12.7)
   30 FORMAT(' (THESE FORCES ARE NORMALIZED TO A UNIT FORCE VECTOR IN ',
     *'THE APPLIED DEFLECTION INCREMENT METHOD)')
   31 FORMAT(20X,' APPLIED FORCE INCREMENT METHOD USED')
   32 FORMAT(20X,' APPLIED DEFLECTION INCREMENT METHOD USED')
  700 FORMAT('  JOINT   APPLIED FORCES'/2X,I4,2X,6F12.3)
  701 FORMAT(' JOINT  DOF  NDEFL       DEFL',2I4,3(I4,G12.3),G12.3/)
  702 FORMAT(20X,' OBSERVED MEMBERS',4X,20I4)
  703 FORMAT(' POSITN  NUMBERS FOR MEMBER',I4,/1X,12I4)
  801 FORMAT(20X,' E G AR ST1 ST2 ST3 DM',1X,7G12.3,/20X,' XM YM ZM P1
     *P2 P3 P4',2X,7G12.3)
  802 FORMAT(30X' ELEMENT SMKO',/6(1X,6G12.3/))
  704 FORMAT(20X,' OBSERVED JOINTS',5X,20I4)
  705 FORMAT(30X,'STRUCTURE DATA'//' JOINT   DEGREES OF FREEDOM          X
     *                Y              Z')
  706 FORMAT(7I4,3X,3D12.4)
  707 FORMAT(80X,' MEMBER NUMBER',I5)
  708 FORMAT( ' ELEMENT DELTA R DISPL.',12F9.5)
  709 FORMAT(10X,' ELEMENT DELTA S DISPL.',7X,6F14.5,/10X,' TOTAL S '
     *'DISPL.',15X,6F14.5)
  804 FORMAT(/50X,'A MATRIX',/ 6(11G11.3,G10.3/))
  805 FORMAT(//50X,'TWELVE V MATRICES',// 12(6(11G11.3,G10.3/)///))
  710 FORMAT(10X,' ELEMENT INCREMENTAL FORCES',2X,6F14.4)
  711 FORMAT(40X,'V*SFF MATRIX'/,12(11G11.3,G10.3/))
  712 FORMAT(50X,' K1 STIFFNESS'/6(20X,6G13.5/))
  806 FORMAT(50X,' ELEMENT STIFFNESS KO+K1'/ 6(20X,6G13.5/))
  713 FORMAT(40X,' AT*KO*A',/12(1X,12(1PE10.3)/))
  714 FORMAT(40X,' AT*K1*A',/12(1X,12(1PE10.3)/))
  809 FORMAT(' SKO'1X,11E11.3)
  810 FORMAT(' SK1',1X,11E11.3)
  715 FORMAT(' DR INCR.',1X,10(1PD12.3)/(10X,10(1PD12.3)))
 1001 READ(5,1,END=1000) TITLE
      WRITE(6,5)
      WRITE(6,1) TITLE
      READ(5,2)NRS,NJ,NM,NINCRT,NELMT,ITYPE,NBIF,N7,N8,NWRT1,NWRT2,LODI
      WRITE(6,6) NRS,NJ,NM,NINCRT,ITYPE,NBIF,N7,N8,NELMT
      IF(LODI.NE.0)WRITE(6,31)
      IF(LODI.EQ.0)WRITE(6,32)
C** NRS IS THE STRUCTURE NUMBER
C** NJ IS THE NUMBER OF JOINTS
C** NM IS THE NUMBER OF MEMBERS
C** NINCRT IS THE NUMBER OF INCREMENTS
C** ITYPE IS THE NUMBER OF DIFFERENT  ELEMENT TYPES
C** NBIF = 0 DO NOT FIND LOWEST BUCKLING LOAD
C** NBIF NE 0   CALCULATE LOWEST BUCKLING LOAD
C** N7 .NE.0  OUTPUT ON UNIT 7   N8.NE.0 OUTPUT ON UNIT 8
C** NELMT = ZERO FOR CANTILEVER ELEMENTS
C**         = NON ZERO FOR SECANT ELEMENTS
```

```
      C** NWRT1 IS THE INCREMENT NUMBER AT WHICH OUTPUT ON UNITS 7 AND 8
      C** STARTS
      C** NWRT2 IS THE INCREMENT NUMBER AT WHICH THIS OUTPUT STOPS
      C** LCDI = 0 FOR APPLIED DEFLECTION INCREMENT METHOD
      C** LCDI = NON 0  FOR APPLIED FORCE INCREMENT METHOD
      C** READ IN THE JOINT INFORMATION
            DO 100 I=1,NJ
            READ(5,3) JN(I),(ND(I,K),K=1,6),X(I),Y(I),Z(I)
      100   CONTINUE
            WRITE(6,7)
            WRITE(6,8) (JN(I),(ND(I,K),K=1,6),X(I),Y(I),Z(I),I=1,NJ)
      C** READ IN INITIAL MEMBER DATA: JNL JNG JNP NTYPE
      C** JNP IS A JOINT EXISTING IN THE X1-X3 PLANE NECESSARY TO DEFINE X3 AXIS
            WRITE(6,9)
      C**
            DO 101 I=1,NM
            READ(5,2) JNL(I),JNG(I),JNP,NTYPE(I)
            R(1)=X(JNP)-X(JNL(I))
            R(2)=Y(JNP)-Y(JNL(I))
            R(3)=Z(JNP)-Z(JNL(I))
            R(4)=X(JNG(I))-X(JNL(I))
            R(5)=Y(JNG(I))-Y(JNL(I))
            R(6)=Z(JNG(I))-Z(JNL(I))
            CALL CROSS(R(4),R(1),ER)
            CALL CROSS(ER,R(4),R(1))
            XP=R(1)+X(JNL(I))
            YP=R(2)+Y(JNL(I))
            ZP=R(3)+Z(JNL(I))
            WRITE(6,10) I,JNL(I),JNG(I),JNP,NTYPE(I),XP,YP,ZP
            XORIG(I)=X(JNG(I))-X(JNL(I))
            YORIG(I)=Y(JNG(I))-Y(JNL(I))
            ZORIG(I)=Z(JNG(I))-Z(JNL(I))
            XPO=XP-X(JNL(I))
            YPO=YP-Y(JNL(I))
            ZPO=ZP-Z(JNL(I))
            DLZ=DSQRT(XPO**2+YPO**2+ZPO**2)
            X1=DSQRT(XORIG(I)**2+YORIG(I)**2+ZORIG(I)**2)
            XO1=XORIG(I)/X1
            YO1=YORIG(I)/X1
            ZO1=ZORIG(I)/X1
            XO2=XPO/DLZ
            YO2=YPO/DLZ
            ZO2=ZPO/DLZ
            T1= DSQRT(1.-XO2**2)
            IF(DABS(T1).LT.1.D-14)GOTO145
            C4O(I)= ZO2/T1
            S4O(I)=-YO2/T1
            C5O(I)= T1
            S5O(I)= XO2
            C6O(I)=XO1/T1
            IF(DABS(ZO2).LT.1.D-12)GOTO226
            S6O(I)=(YO1*T1*T1+YO2*XO2*XO1)/(ZO2*T1)
            GOTO101
      226   IF(DABS(YO2).LT.1.D-12)GOTO227
            S6O(I)=-(ZO1*T1*T1+ZO2*XO1*XO2)/(T1*YO2)
            GOTO101
      227   S6O(I)=YO1
            GOTO101
      145   C4O(I)=1.D0
            S4O(I)=0.D0
```

```
            C5C(I)=0.D0
            S5C(I)=X02
            C6C(I)=-X02 *Z01
            S6C(I)= Y01
    101     CONTINUE
            WRITE(6,11)
            DO 102 I=1,ITYPE
            READ(5,4) AAA(I),EX(I),EY(I),EZ(I),EEE(I),GGG(I)
            WRITE(6,12) I,AAA(I),EX(I),EY(I),EZ(I),EEE(I),GGG(I)
    102     CONTINUE
C** WRITE OUT INITIAL DATA
            READ(5,23) PLCTL,EIGFAC,JOINT,NDOF
            WRITE(6,13) PLCTL,EIGFAC
C** READ IN THE DEFLECTION PARAMETER AND LOAD CONFIGURATION
C** CARE SHOULD BE EXERCISED IN CHOOSING THE DEFLECTION PARAMETER
C** NLPTS IS THE NUMBER OF JOINTS WHERE LOADS ARE APPLIED
C**
            READ(5,2) NLPTS
            WRITE(6,14)
            IF(NBIF.NE.0)NBIF=1
            NU=1
            DO 104 J=1,NJ
            DO 105 I=1,6
            IF(ND(J,I)-1 ) 200,201,202
    200     ND(J,I)=0
            GO TO 105
    201     ND(J,I)=NU
            NU=NU+1
            GO TO 105
    202     NN=ND(J,I)
            ND(J,I)=ND(NN,I)
    105     CONTINUE
            WRITE(6,15) JN(J),(ND(J,K),K=1,6)
    104     CONTINUE
            NU=NU-1
            WRITE(6,16) NU
            INDEX=0
            IF(JOINT.NE.0.AND.NDOF.NE.0)INDEX=ND(JOINT,NDOF)
            N=1
            NINCRT=NINCRT+1
            TF=0.D0
            NGONE=0
            NN=0
            DO 103 J=1,NLPTS
            READ(5,17)JOINT,(FCONS(K),K=1,6)
            DO 137 K=1,6
            M=ND(JOINT,K)
            IF(M.EQ.0)GOTO137
            IF(FCONS(K).EQ.0.D0)GOTO137
            NN=NN+1
            LDEG(NN)=M
            IF(INDEX.EQ.0)INDEX=LDEG(1)
            TFV(NN)=FCONS(K)
            TOTAL(NN)=0.D0
            TF=TF+TFV(NN)**2
    137     CONTINUE
            WRITE(6,700)JOINT,(FCONS(K),K=1,6)
    103     CONTINUE
            NLPTS=NN
            IF(LODI.NE.0)GOTO144
```

```
          TF=DSQRT(TF)
          DO 138 J=1,NLPTS
          TFV(J)=TFV(J)/TF
  138     CONTINUE
          WRITE(6,30)
  144     READ(5,18) JOINT,NDOF,(DEFL(K),K=1,3)
C** NDOF = 1 IF THE DEFLECTIONS APPLIED ARE TRANSLATIONS
C** NDOF = 4 IF THE DEFLECTIONS APPILED ARE ROTATIONS
          K=0
  224     JNT=ND(JOINT,NDOF+K)
          IF(JNT.EQ.0)GOTO223
          NDL(K+1)=JNT
          K=K+1
          IF(K.EQ.3)GOTO225
          GOTO224
  223     NDL(K+1)=1
          DEFL(K+1)=0.0
          K=K+1
          IF(K.GE.3)GOTO225
          GOTO224
  225     DEFLD=DEFL(1)**2+DEFL(2)**2+DEFL(3)**2
          WRITE(6,701)JOINT,NDOF,(NDL(K),DEFL(K),K=1,3)
  204     N19=N+19
          READ(5,2) (MLN(J),J=N,N19)
C    READ IN THOSE MEMBERS THAT YOU WISH TO SEE A COMPLETE LOAD-DEFLECTION
C    HISTORY
          IF(MLN(N).EQ.0) GO TO 203
          IF(MLN(N).NE.0.AND.N7.NE.0)WRITE(7,702)(MLN(J),J=N,N19)
          N=N+20
          GO TO 204
  203     CONTINUE
C    READ IN THOSE JOINTS THAT YOU WISH TO FOLLOW THROUGH THE LOAD DEFLECTI
C    HISTORY
          N=1
  206     N19=N+19
          READ(5,2) (JLN(J),J=N,N19)
          IF(JLN(N).EQ.0) GO TO 205
          IF(JLN(N).NE.0.AND.N7.NE.0)WRITE(7,704)(JLN(J),J=N,N19)
          N=N+20
          GO TO 206
  205     CONTINUE
          REWIND 1
          REWIND 2
          REWIND 3
          NGONE=0
          NB=0
          PCR=DABS(TFV(1))/TFV(1)
          IF(N8.EQ.0)WRITE(6,21)
C    CALCULATE POSITION NUMBERS NP
          DO 106 K=1,NM
          I=JNL(K)
          J=JNG(K)
          NP(1)=ND(I,1)
          NP(2)=ND(I,2)
          NP(3)=ND(I,3)
          NP(4)=ND(I,4)
          NP(5)=ND(I,5)
          NP(6)=ND(I,6)
          NP(7)=ND(J,1)
          NP(8)=ND(J,2)
```

```
      NP(9)=ND(J,3)
      NP(10)=ND(J,4)
      NP(11)=ND(J,5)
      NP(12)=ND(J,6)
      L=NTYPE(K)
      IF(N7.NE.0)WRITE(7,703) K,NP
      E=EEE(L)
      G=GGG(L)
      AR=AAA(L)
      ST1=EX(L)
      ST2=EY(L)
      ST3=EZ(L)
      IF(G.EQ.0.)G=E/2.6
      IF(ST1.EQ.0.) ST1=ST2+ST3
      XM=X(J)-X(I)
      YM=Y(J)-Y(I)
      ZM=Z(J)-Z(I)
      DM=DSQRT(XM**2+YM**2+ZM**2)
      P1=AR*E/DM
      P2=12.*E*ST3/(DM**3)
      P3=12.*E*ST2/(DM**3)
      P4=G*ST1/DM
      CALL ELMSTO(P1,P2,P3,P4,SMKO,DM,NELMT)
      WRITE(1) BLKA
C    CALCULATE CLASSICAL STIFFNESS FOR THE ELEMENT
C    CALCULATE MEMBER BAND WIDTH
      I1=0
      J1=1000
      DO 107 L=1,12
      N=NP(L)
      IF(N.EQ.0) GO TO 107
      IF(I1.LT.N)I1=N
      IF(J1.GT.N) J1=N
  107 CONTINUE
      NB1=I1-J1+1
      IF(NB.LT.NB1)NB=NB1
      IF(N8.NE.0)WRITE(8,801) E,G,AR,ST1,ST2,ST3,DM,XM,YM,ZM,P1,P2,P3,P4
      IF(N8.NE.0)WRITE(8,802) SMKO
      IF(N8.EQ.0)WRITE(6,22) K,I,J,ST1,ST2,ST3,E,G,AR
  106 CONTINUE
      NTOTAL=NL*NB
      WRITE(6,19) NB,NTOTAL
      DO 108 K=1,NL
      DR(K)=0.
  108 CONTINUE
      IUNIT=2
      JUNIT=3
      NSEVEN=N7
      NEIGHT=N8
      IF(NWRT2.EQ.0)NWRT2=NINCRT
      DO 146 J=1,6
      ER(J)=0.
      ER(J+6)=0.
      ES(J)=0.D0
   146 CONTINUE
      DO 110 J=1,12
      DO 110 K=1,NM
      TR(K,J)=0.D0
      RR(K,J)=0.D0
      IF(J.GT.6) GO TO 110
```

```
          SFF(K,J)=0.D0
          SS(K,J)=0.D0
  110     CONTINUE
          DO 109 JK=1,NINCRT
          JKM=JK-1
          N8=0
          N7=0
          IF(NWRT1.LE.JKM.AND.NWRT2.GE.JKM)N7=NSEVEN
          IF(NWRT1.LE.JKM.AND.NWRT2.GE.JKM)N8=NEIGHT
          IF(JKM.EQ.0.AND.N7.EQ.0)GOTO229
          WRITE(6,20) JKM
  229     ITEMP=IUNIT
          IUNIT=JUNIT
          JUNIT=ITEMP
          NINC=0
          DO 111 L=1,20
          IF(NINCR(L).EQ.JK)NINC=1
          IF(NINC.EQ.1) GO TO 207
  111     CONTINUE
  207     IF(N7.NE.0)WRITE(6,705)
          IF(N7.NE.0)GOTO231
          IF(JKM.NE.0.AND.NINC.EQ.1)WRITE(6,705)
          IF(JKM.NE.0.AND.NINC.EQ.0.AND.JLN(1).NE.0)WRITE(6,705)
  231     DO 112 JL=1,NJ
          DO 113 J=1,3
          NN=ND(JL,J)
          IF(NN.EQ.0) GO TO 113
          GO TO (208,209,210),J
  208     X(JL)=X(JL)+DR(NN)
          GO TO 113
  209     Y(JL)=Y(JL)+DR(NN)
          GO TO 113
  210     Z(JL)=Z(JL)+DR(NN)
  113     CONTINUE
          IJT=0
          IF(JKM.EQ.0.AND.N7.EQ.0)GOTO112
          DO 114 L=1,500
          IF(JLN(L).EQ.0)GO TO 211
          IF(JLN(L).EQ.JL)IJT=1
          IF(IJT.EQ.1)GO TO 211
  114     CONTINUE
  211     IF(IJT.EQ.1.OR.N7.NE.0.OR.NINC.EQ.1)WRITE(6,706) JN(JL),
         *(ND(JL,L),L=1,6),X(JL),Y(JL),Z(JL)
  112     CONTINUE
          DO 115 I=1,NTOTAL
          SKC(I)=0.
          SKI(I)=0.
  115     CONTINUE
          REWIND 1
          REWIND 2
          REWIND 3
          DO 116 ML=1,NM
          READ(1) ELKA
          IMN=0
          DO 117 L=1,500
          IF(MLN(L).EQ.0)GO TO 212
          IF(MLN(L).EQ.ML)IMN=1
          IF(IMN.EQ.1)GO TO 212
  117     CONTINUE
  212     IF(JKM.EQ.0.AND.N7.EQ.0)GOTO228
```

```
      IF(IMN.EQ.1.OR.N7.NE.0.OR.NINC.NE.0)WRITE(6,707)ML
  228 DO 120 J=1,6
      EDR(J)=0.0
      EDR(J+6)=0.0
      EDS(J)=0.
      EDSF(J)=0.
  120 CONTINUE
      DO 118 J=1,12
      NS=NP(J)
      IF(NS.LE.0)GOTO118
      EDR(J)=DR(NS)
      TR(ML,J)=TR(ML,J)+EDR(J)
  118 CONTINUE
      IF(JKM.EQ.0)GOTO215
      IF(N7.NE.0.OR.IMN.NE.0)WRITE(6,708)(EDR(J),J=1,12)
      READ(IUNIT) BLKB
      DO 119 K=1,6
      DO 119 J=1,12
      EDS(K)=EDS(K)+A(K,J)*EDR(J)
  119 CONTINUE
      DO 121 J=1,6
      SS(ML,J)=SS(ML,J)+EDS(J)
      ES(J)=SS(ML,J)
  121 CONTINUE
      T5=EDR(5)
      EDR(5)=T5*C4+EDR(6)*S4
      EDR(6)=EDR(4)*S5-T5*S4*C5+EDR(6)*C4*C5
      T5=EDR(11)
      EDR(11)=T5*C10+EDR(12)*S10
      EDR(12)=EDR(10)*S11-T5*S10*C11+EDR(12)*C10*C11
      DO 122 J=1,12
      RR(ML,J)=RR(ML,J)+EDR(J)
      ER(J)=RR(ML,J)
  122 CONTINUE
      IF(N7.NE.0.OR.IMN.NE.0.OR.NINC.NE.0)WRITE(6,709) EDS,ES
      IF(N7.NE.0.OR.IMN.EQ.1.OR.NINC.NE.0)WRITE(6,24)(TR(ML,JI),JI=1,12)
      DO 123 J=1,6
      DO 124 K=1,6
  124 EDSF(J)=SMPREV(J,K)*EDS(K)+EDSF(J)
      SFF(ML,J)=SFF(ML,J)+EDSF(J)
  123 CONTINUE
      IF(IMN.EQ.1.OR.N7.NE.0.OR.NINC.NE.0)WRITE(6,25)(SFF(ML,J),J=1,6)
      IF(N7.NE.0) WRITE(6,710) EDSF
  215 XYZO(1)=XORIG(ML)
      XYZO(2)=YORIG(ML)
      XYZO(3)=ZORIG(ML)
      SINCOS(1)=S4C(ML)
      SINCOS(2)=C4C(ML)
      SINCOS(3)=S5C(ML)
      SINCOS(4)=C5C(ML)
      SINCOS(5)=S6C(ML)
      SINCOS(6)=C6C(ML)
      IF(NELMT.EQ.0)CALL AVMATC(V,ER,XYZO,SINCOS)
      IF(NELMT.NE.0)CALL AVMATS(V,ER,XYZO,SINCOS)
      CALL TRANS(V)
      IF(N7.NE.0) WRITE(7,804)((A(J,K),K=1,12),J=1,6)
      IF(N8.NE.0)WRITE(8,805)(((V(K,J,I),K=1,12),J=1,6),I=1,12)
      DO 125 K=1,12
      DO 126 J=1,12
      SV(J,K)=0.0
```

```
      DO 127 I=1,6
      SV(J,K)= SV(J,K) +V(K,I,J)*SFF(ML,I)
  127 CONTINUE
  126 CONTINUE
  125 CONTINUE
      IF(N7.NE.0)WRITE(7,711)((SV(J,K),K=1,12),J=1,12)
      CALL ELMST1(P1,P2,DM,ES,SMK1,NELMT)
      DO 128 I=1,6
      DO 128 J=1,6
      SMPREV(I,J)=SMK0(I,J)+SMK1(I,J)
  128 CONTINUE
      IF(N7.NE.0)WRITE(7,712)((SMK1(I,J),I=1,6),J=1,6)
      IF(N8.NE.0)WRITE(8,806)((SMPREV(I,J),I=1,6),J=1,6)
      WRITE(JUNIT) BLKB
      DO 129 L=1,12
      DO 129 K=1,12
      SSK0(K,L)=0.
      SSK1(K,L)=0.
      IF(L-6)130,130,129
  130 AT0(K,L)=0.
      AT1(K,L)=0.
  129 CONTINUE
      DO 131 K=1,12
      DO 131 J=1,6
      DO 132 I=1,6
      AT0(K,J)=AT0(K,J)+A(I,K)*SMK0(I,J)
      AT1(K,J)=AT1(K,J)+A(I,K)*SMK1(I,J)
  132 CONTINUE
  131 CONTINUE
      DO 133 K=1,12
      DO 133 J=1,12
      DO 134 I=1,6
      SSK0(K,J)=SSK0(K,J)+AT0(K,I)*A(I,J)
      SSK1(K,J)=SSK1(K,J)+AT1(K,I)*A(I,J)
  134 CONTINUE
  133 CONTINUE
      IF(N7.NE.0)WRITE(7,713) ((SSK0(K,J),K=1,12),J=1,12)
      IF(N7.NE.0)WRITE(7,714) ((SSK1(K,J),K=1,12),J=1,12)
      DO 135 L=1,12
      DO 136 M=1,12
      M1=NP(L)
      M2=NP(M)
      IF(M1.EQ.0)GO TO 135
      IF(M2.EQ.0)GO TO 136
      IF(M1.GT.M2)GO TO 136
      K= (M1-1)*(NE-1)+M2
      SK0(K)= SK0(K)+SSK0(L,M)
      SK1(K)= SK1(K)-SSK1(L,M)-SV(L,M)
  136 CONTINUE
  135 CONTINUE
  116 CONTINUE
      IF(N8.NE.0)WRITE(8,809)  (SK0(JI),JI=1,NTOTAL)
      IF(N8.NE.0)WRITE(8,810)  (SK1(JI),JI=1,NTOTAL)
      IF(NBIF.NE.0.AND.JK.LE.3)CALL CRITLD(SK0,SK1,NU,NB,JK,DR,
     *EV,EIGFAC,PCR)
      IF(JK.GT.1.AND.NBIF.NE.0.AND.JK.LE.3)GO TO 216
      DO 141 J=1,NTOTAL
      SK0(J)=SK0(J)-SK1(J)
      SK1(J)=SK0(J)
  141 CONTINUE
```

```
 219   DO 139 J=1,NU
       DR(J)=0.
 139   CONTINUE
       DO 140 J=1,NLPTS
       DR(LDEG(J))= TFV(J)
 140   CONTINUE
       DET=1.D-14
       NSCALE=1
       IF(NGONE.EQ.0)CALL DFBAND(SK0,DR,NU,NB,1,DET,AET,JET,NSCALE)
       IF(NGONE.EQ.0.AND.DET.LT.1.D-14)GO TO 217
       IF(NGONE) 220,220,218
 217   NGONE=1
       GO TO 219
 218   CALL DBANDI(SK1,DR,NU,NB,1,DET)
 220   IF(JKM.EQ.0.AND.N7.EQ.0)GOTO230
       WRITE(6,26)DET
 230   DIV=(DR(NDL(1))*DEFL(1)+DR(NDL(2))*DEFL(2)+DR(NDL(3))
      **DEFL(3))/DEFLD
       DIV=1./DIV
       IF(NBIF.NE.0.AND.JK.EQ.1)DIV=1.
       IF(LODI.NE.0)DIV=1.0
       DO 142 J=1,NU
       DR(J)=DR(J)*DIV
 142   CONTINUE
 222   DO 143 J=1,NLPTS
       TOTAL(J)=TOTAL(J) +TFV(J)*DIV
 143   CONTINUE
       IF(JKM.EQ.0)GOTO232
       JKL=JK-NBIF
       JKR=JKL-1
       IF(JKR.EQ.0)JKR=1
       PCRIT(JKL)=TOTAL(1)/PCR
       DOL(JKL)=DABS(DR(INDEX)/PLOTL)+DOL(JKR)
       GOTO233
 232   PCRIT(1)=TOTAL(1)/PCR
       DOL(1)=DABS(DR(INDEX)/PLOTL)
 233   WRITE(6,27) (LDEG(J),TOTAL(J),J=1,NLPTS)
 221   IF(N7.NE.0)WRITE(7,715) (DR(JI),JI=1,NU)
       GO TO 109
 216   DIV=PCR
       IF(JK.EQ.3)DIV=0.
       GO TO 222
 109   CONTINUE
       WRITE(0,28) NRS,JKL,TITLE
       WRITE(0,29)(PCRIT(I),DOL(I),I=1,JKL)
       GO TO 1001
 1000  STOP
       END
       SUBROUTINE TRANS(V)
       IMPLICIT REAL*8(A-H,O-Z)
       COMMON /BLK2/C4,C5,S4,S5,C10,C11,S10,S11,SM,A
       DIMENSION SM(6,6),A(6,12),V(12,6,12),G(12,12)
      *,B1(3,3),C(3,3),T(6,12),L(6,12),B(6,12),TM(3,3)
       DO 1 J=1,12
       DO 2 K=1,12
       G(K,J)=0.D0
       IF(K.EQ.J)G(K,J)=1.D0
       IF(K.GT.6)GOTO2
       B(K,J)=A(K,J)
       A(K,J)=0.D0
```

```
 2    CONTINUE
 1    CONTINUE
      G(5,5)=C4
      G(6,6)=C4*C5
      G(5,6)=S4
      G(6,5)=-S4*C5
      G(6,4)=S5
      G(11,11)=C10
      G(12,12)=C11*C10
      G(11,12)=S10
      G(12,11)=-S10*C11
      G(12,10)=S11
      DO 106 N=1,7,6
      DO 200 J=1,3
      DO 201 K=1,3
      B1(K,J)=G(K+2+N,J+2+N)
 201  CONTINUE
 200  CONTINUE
      DO 105 M=1,4,3
      DO 102 I=1,12
      DO 100 J=1,3
      DO 101 K=1,3
      TM(K,J)=V(I,K+M-1,J+N+2)
 101  CONTINUE
 100  CONTINUE
      DO 150 J=1,3
      DO 151 K=1,3
      C(K,J)=0.D0
      DO 152 L=1,3
      C(K,J)=C(K,J)+TM(K,L)*B1(L,J)
 152  CONTINUE
 151  CONTINUE
 150  CONTINUE
      DO 103 J=1,3
      DO 104 K=1,3
 104  V(I,K+M-1,J+N+2)=C(K,J)
 103  CONTINUE
 102  CONTINUE
 105  CONTINUE
 106  CONTINUE
      NN=1
      N1=4
      N2=6
 99   DO 40 J=1,3
      DO 41 K=1,3
 41   TM(K,J)=0.D0
 40   CONTINUE
      GOTO(50,60,70,80),NN
 50   TM(2,2)=-S4
      TM(2,3)=C4
      TM(3,2)=-C4*C5
      TM(3,3)=-S4*C5
      M=4
      GOTO81
 60   TM(3,1)=C5
      TM(3,2)=S4*S5
      TM(3,3)=-C4*S5
      M=5
      GOTO81
 70   TM(2,2)=-S10
```

```
      TM(2,3)=C10
      TM(3,2)=-C10*C11
      TM(3,3)=-S10*C11
      M=10
      N1=10
      N2=12
      GOTO81
   80 TM(3,1)=C11
      TM(3,2)=S10*S11
      TM(3,3)=-C10*S11
      M=11
   81 DO 90 J=1,6
      DO 92 K=N1,N2
      DO 91 L=N1,N2
   91 A(J,K)=A(J,K)+B(J,L)*TM(L-N1+1,K-N1+1)
   92 CONTINUE
   90 CONTINUE
      DO 93 K=N1,N2
      DO 94 J=1,6
      V(M,J,K)=V(M,J,K)+A(J,K)
      A(J,K)=0.D0
   94 CONTINUE
   93 CONTINUE
      NN=NN+1
      IF(NN.LE.4)GOTO99
      DO 250 K=1,12
      DO 251 J=1,6
      T(J,K)=V(5,J,K)
      U(J,K)=V(11,J,K)
  251 CONTINUE
  250 CONTINUE
      DO 3 J=1,6
      DO 4 K=1,12
      V(4,J,K)=V(4,J,K)+V(6,J,K)*S5
      V(5,J,K)=T(J,K)*C4-V(6,J,K)*S4*C5
      V(6,J,K)=T(J,K)*S4+V(6,J,K)*C4*C5
      V(10,J,K)=V(10,J,K)+V(12,J,K)*S11
      V(11,J,K)=U(J,K)*C10-V(12,J,K)*S10*C11
      V(12,J,K)=U(J,K)*S10+V(12,J,K)*C10*C11
      DO 5 L=1,12
      A(J,K)=A(J,K)+B(J,L)*G(L,K)
    5 CONTINUE
    4 CONTINUE
    3 CONTINUE
      RETURN
      END
      SUBROUTINE ELMSTC(A,B,C,D,K,L,NN)
      REAL*8 A,B,C,D,L,K(6,6)
      DO 1 I=1,6
      DO 2 J=1,6
      K(J,I)=0.
    2 CONTINUE
    1 CONTINUE
      IF(NN.NE.0)GOTO10
      K(1,1)=A
      K(2,2)=C
      K(2,6)=-.5*C*L
      K(3,3)=B
      K(3,5)=.5*B*L
      K(4,4)=D
```

```
      K(5,5)=B*L*L/3.
      K(6,6)=C*L*L/3.
      GOTO11
 10   K(1,1)=A
      K(2,2)=B*L*L/3.
      K(2,5)=K(2,2)/2.
      K(3,3)=C*L*L/3.
      K(3,6)=K(3,3)/2.
      K(4,4)=D
      K(5,5)=B*L*L/3.
      K(6,6)=C*L*L/3.
 11   DO 3 I=1,6
      DO 3 J=I,6
      K(J,I)=K(I,J)
 3    CONTINUE
      RETURN
      END
      SUBROUTINE ELMST1(P1,P2,DM,S,SMK1,NN)
      REAL*8 P1,P2,DM,SC,S(6),SMK1(6,6)
      DO 2 I=1,6
      DO 2 J=1,6
 2    SMK1(I,J)=0.
      IF(NN.NE.0)GOTO10
      SMK1(2,6)= -.1*P1*S(1)
      SMK1(3,3)= 1.2*P1/DM*S(1)
      SMK1(3,5)= .1*P1*S(1)
      SMK1(4,4)= P2*DM/12.*S(1)
      SMK1(5,5)= 2./15.*P1*DM*S(1)
      SMK1(6,6)= 2./15.*P1*DM*S(1)
      SMK1(2,2)= 1.2*P1/DM*S(1)
      SMK1(1,3)= +.1*P1*S(5) +1.2*P1/DM*S(3)
      SMK1(1,5)= +2./15.*P1*DM*S(5) + .1*P1*S(3)
      SMK1(2,4)=-4.2/12.*P2*DM*S(5)- .5*P2*S(3)
      SMK1(4,6)= +3.2/12.*P2*DM*DM*S(5) + 4.2/12.*P2*DM*S(3)
      SMK1(1,2)=-.1*P1*S(6) +1.2*P1/DM*S(2)
      SMK1(1,6)=-.1*P1*S(2) + 2./15.*P1*DM*S(6)
      SMK1(3,4)= 4.2/12.*P2*DM*S(6)- .5*P2*S(2)
      SMK1(4,5)= 3.2/12.*P2*DM*DM*S(6)- 4.2/12.*P2*DM*S(2)
      GOTO200
 10   SMK1(2,2)= 2./15.*P1*DM*S(1)
      SMK1(3,3)= SMK1(2,2)
      SMK1(2,5)= -P1*DM*S(1)/30.
      SMK1(3,6)= SMK1(2,5)
      SMK1(4,4)= P2*DM/12.*S(1)
      SMK1(5,5)= 2./15.*P1*DM*S(1)
      SMK1(6,6)= 2./15.*P1*DM*S(1)
      SMK1(3,4)= .8/12.*P2*DM*DM*S(2)+P2*DM*DM/12.*S(5)
      SMK1(4,6)=3.2/12.*P2*DM*DM*S(5)+P2*DM*DM/12.*S(2)
      SMK1(1,3)= 2./15.*P1*DM*S(3)-P1*DM/30.*S(6)
      SMK1(1,6)= 2./15.*P1*DM*S(6)-P1*DM/30.*S(3)
      SMK1(2,4)= .8/12.*P2*DM*DM*S(3)+P2*DM*DM/12.*S(6)
      SMK1(4,5)=3.2/12.*P2*DM*DM*S(6)+P2*DM*DM/12.*S(3)
      SMK1(1,2)= 2./15.*P1*DM*S(2)- P1*DM/30.*S(5)
      SMK1(1,5)= 2./15.*P1*DM*S(5)- P1*DM/30.*S(2)
 200  DO 3 I=1,6
      DO 3 J=1,6
      SMK1(J,I)=SMK1(I,J)
 3    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CRITLD(SKO,SK1,N,M,JK,DR,EV,EIGFAC,PCR)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION DR(1), SKO(1),SK1(1),EV(1),ST(2500)
      IF(JK.EQ.1)GO TO 1
      IF(JK.EQ.3)GO TO 10
      EPS=1.D-6
      EPSV=1.D-4
      IT=100
      CALL DVPOWR(SKO,SK1,N,M,M,EV,N,EVALI,1,EPS,EPSV,IT,ST,COND)
      PCR=EVALI
      WRITE(6,4) EVALI
    4 FORMAT(' CRITICAL LOAD IS',E14.5)
      DIV=0.0
      DO 16 J=1,N
      EVR=DABS(EV(J))
   16 DIV=DMAX1(DIV,EVR)
      DO 2 J=1,N
    2 EV(J)=EV(J)/DIV
      WRITE(6,3) (EV(JI),JI=1,N)
    3 FORMAT(//' EIGENVECTOR'// 100(1X,12G10.3/))
      DO 5 J=1,N
    5 DR(J)=DR(J)*(PCR-1.)
    1 RETURN
   10 DO 15 J=1,N
   15 DR(J)=EV(J)*EIGFAC
      RETURN
      END
      SUBROUTINE CROSS(A,B,C)
      REAL*8 A,B,C
      DIMENSION A(1),B(1),C(1)
      C(1)=A(2)*B(3)-A(3)*B(2)
      C(2)=A(3)*B(1)-A(1)*B(3)
      C(3)=A(1)*B(2)-A(2)*B(1)
      RETURN
      END
      SUBROUTINE DEANCI(A,B,N,M,LT,DET)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /ZDET/ DE,NCN
      COMMON /ZCON/ COND
      DIMENSION A(1),B(1)
      DOUBLE PRECISION DSQRT,DABS,DSIGN
      IF(M.EQ.1) GO TO 100
      MM=M-1
      NM=N*M
      NM1=NM-MM
      IF (LT.NE.1) GO TO 55
      COND=1.0
      NCN=0
      DE=0.
      MP=M+1
      KK=2
      FAC=DET
      NROW=1
      IF(A(1).EQ.0.) GO TO 60
      SUM=0.D0
      DO 77 I=1,M
   77 SUM=SUM+A(I)*A(I)
      S=1.0/DSQRT(DABS(A(1)))
      DE=A(1)
      A(1)=S
```

```
      IF(A(1).LT.0.) A(1)=-S
      COND=COND/(A(1)*A(1)*DSQRT(SUM))
      SUM=A(2)*A(2)+A(MP)*A(MP)
      M2=2*M
      BIGL=DABS(A(1))
      SML=BIGL
      MPP=MP+1
      DO 88 I=MPP,M2
   88 SUM=SUM+A(I)*A(I)
      A(2)=A(2)*DABS(A(1))
      S=A(MP)-A(2)*A(2)*DSIGN(1.D0,A(1))
      IF(S.NE.0)   GO TO 16
      NROW=2
      GO TO 60
   16 A(MP)=1.0/DSQRT(DABS(S))
      DE=DE*S
      COND=COND/(A(MP)*A(MP)*DSQRT(SUM))
      IF(S.LT.0.)   A(MP)=-A(MP)
      AAA=DABS(A(MP))
      IF(AAA.GT.BIGL) BIGL=AAA
      IF(AAA.LT.SML) SML=AAA
      IF(N.EQ.2) GO TO 53
      MP=MP+M
      DO 62 J=MP,NM1,M
      JP=J-MM
      MZC=0
      IF(KK.GE.M) GO TO 1
      KK=KK+1
      II=1
      JC=1
      GO TO 2
1     KK=KK+M
      II=KK-MM
      JC=KK-MM
2     DO 65 I=KK,JP,MM
      IF(A(I).EQ.0.)GO TO 64
      GO TO 66
64    JC=JC+M
65    MZC=MZC+1
      ASUM1=0.
      GO TO 61
66    MMZC=MM*MZC
      II=II+MZC
      KM=KK+MMZC
      SUM=A(KM)*A(KM)
      A(KM)=A(KM)*DABS(A(JC))
      ASUM1=A(KM)*A(KM)*DSIGN(1.D0,A(JC))
      IF(KM.GE.JP)GO TO 6
      KJ=KM+MM
      JJ=JC
      DO 5 I=KJ,JP,MM
      SUM=SUM+A(I)*A(I)
      JJ=JJ+M
      ASUM2=0.
      IM=I-MM
      II=II+1
      KI=II+MMZC
      KZ=JC
      DO 7 K=KM,IM,MM
      ASUM2=ASUM2+A(KI)*A(K)*DSIGN(1.D0,A(KZ))
```

```
      KZ=KZ+M
    7 KI=KI+MM
      A(I)=(A(I)-ASUM2)*DABS(A(KI))
                ASUM1=ASUM1+A(I)*A(I)*DSIGN(1.D0,A(JJ))
    5 CONTINUE
    6 CONTINUE
      JMM=J+MM
      DO 3 I=J,JMM
    3 SUM=SUM+A(I)*A(I)
   61 S=A(J)-ASUM1
      IF(S.NE.0.)   GO TO 63
      NROW=(J+MM)/M
      GO TO 60
   63 A(J)=1.0/DSQRT(DABS(S))
      COND=COND/(A(J)*A(J)*DSQRT(SUM))
      DE=DE*S
      IF(DABS(DE).GT.1.E-15) GO TO 144
      DE=DE*1.E+15
      NCN=NCN-15
      GO TO 145
  144 IF(DABS(DE).LT.1.E+15) GO TO 145
      DE=DE*1.E-15
      NCN=NCN+15
  145 CONTINUE
      IF(S.LT.0.0)   A(J)=-A(J)
      AAA=DABS(A(J))
      IF(AAA.GT.BIGL) BIGL=AAA
      IF(AAA.LT.SML) SML=AAA
   62 CONTINUE
      GO TO 53
   60 WRITE(6,99) NROW
   54 DET=0.
   99 FORMAT(35H0ERROR CONDITION ENCOUNTERED IN ROW,I6)
      RETURN
   53 DET=SML/BIGL
   55 B(1)=B(1)*DABS(A(1))
      KK=1
      K1=1
      J=1
      L4=1
      DO 8 L=2,N
      BSUM1=0.
      LM=L-1
      J=J+M
      IF(KK.GE.M)GO TO 12
      KK=KK+1
      GO TO 13
   12 KK=KK+M
      K1=K1+L
      L4=L4+M
   13 JK=KK
      L5=L4
      DO 9 K=K1,LM
      BSUM1=BSUM1+A(JK)*B(K)*DSIGN(1.D0,A(L5))
      JK=JK+MM
      L5=L5+M
    9 CONTINUE
    8 B(L)=(B(L)-BSUM1)*DABS(A(J)   )
      B(N)=B(N)*A(NM1)
      NMM=NM1
```

```
            NN=N-1
            ND=N
            DO 10 L=1,NN
            BSUM2=0.
            NL=N-L
            NL1=N-L+1
            NMM=NMM-M
            NJ1=NMM
            IF(L.GE.M)ND=ND-1
            DO 11 K=NL1,ND
            NJ1=NJ1+1
            BSUM2=BSUM2+A(NJ1)*B(K)
   11       CONTINUE
   10       B(NL)=(B(NL)-BSUM2)*A(NMM)
            RETURN
   100      DO 101 I=1,N
            IF(A(I).EQ.0.0) GO TO 60
   101      B(I)=B(I)/A(I)
            RETURN
            END
            SUBROUTINE AVMATC(V,R,XX,SN)
            IMPLICIT REAL*8(A-H,O-Z)
            COMMON /BLK2/C4,C5,S4,S5,C10,C11,S10,S11,SMPREV,A
            DIMENSION WI(3,3),WJ(3,3),AA(3),T(5),P(5),Q(6),DWI(3,3,3),DWJ(3,3,
           *3),DT(5,6),DP(5,6),DQ(6,6),DDWI(3,3,3,3),DDWJ(3,3,3,3)
           *,A(6,12),V(12,6,12),XX(3),SN(6),R(12),SMPREV(6,6)
            DO 1 J=1,12
            DO 1 I=1,6
            DO 1 K=1,12
            A(I,J)= 0.D0
            V(K,I,J)=0.D0
    1       CONTINUE
    6       C4=DCOS(R(4))*SN(2)-DSIN(R(4))*SN(1)
            S4=DCOS(R(4))*SN(1)+DSIN(R(4))*SN(2)
            C5=DCOS(R(5))*SN(4)-DSIN(R(5))*SN(3)
            S5=DCOS(R(5))*SN(3)+DSIN(R(5))*SN(4)
            C6=DCOS(R(6))*SN(6)-DSIN(R(6))*SN(5)
            S6=DCOS(R(6))*SN(5)+DSIN(R(6))*SN(6)
            C11=DCOS(R(11))*SN(4)-DSIN(R(11))*SN(3)
            S11=DCOS(R(11))*SN(3)+DSIN(R(11))*SN(4)
            C10=DCOS(R(10))*SN(2)-DSIN(R(10))*SN(1)
            S10=DCOS(R(10))*SN(1)+DSIN(R(10))*SN(2)
            S12=DCOS(R(12))*SN(5)+DSIN(R(12))*SN(6)
            C12=DCOS(R(12))*SN(6)-DSIN(R(12))*SN(5)
            WI(1,1)=C5*C6
            WI(1,2)=-C5*S6
            WI(1,3)= S5
            WI(2,1)= C4*S6+S4*S5*C6
            WI(2,2)= C4*C6-S4*S5*S6
            WI(2,3)= -S4*C5
            WI(3,1)= S4*S6-C4*S5*C6
            WI(3,2)= S4*C6+C4*S5*S6
            WI(3,3)= C4*C5
            WJ(1,1)=C11*C12
            WJ(1,2)=-C11*S12
            WJ(1,3)=  S11
            WJ(2,1)= C10*S12+S10*S11*C12
            WJ(2,2)= C10*C12-S10*S11*S12
            WJ(2,3)=-S10*C11
            WJ(3,1)= S10*S12-C10*S11*C12
```

```
      WJ(3,2)= S10*C12+C10*S11*S12
      WJ(3,3)= C10*C11
      AA(1)=XX(1)+ R(7)-R(1)
      AA(2)=XX(2)+ R(8)-R(2)
      AA(3)=XX(3)+ R(9)-R(3)
      DO 20 K=1,3
      DO 20 I=1,3
      DO 20 J=1,3
      GO TO(21,20,23),K
21    IF(I.EQ.1)DWI(K,I,J)=0.D0
      IF(I.EQ.2)DWI(K,I,J)=-WI(3,J)
      IF(I.EQ.3)DWI(K,I,J)= WI(2,J)
      IF(I.EQ.1)DWJ(K,I,J)=0.D0
      IF(I.EQ.2)DWJ(K,I,J)=-WJ(3,J)
      IF(I.EQ.3)DWJ(K,I,J)= WJ(2,J)
      GOTO20
23    IF(J.EQ.1)DWI(K,I,J)=WI(I,2)
      IF(J.EQ.1)DWJ(K,I,J)=WJ(I,2)
      IF(J.EQ.2)DWI(K,I,J)=-WI(I,1)
      IF(J.EQ.2)DWJ(K,I,J)=-WJ(I,1)
      IF(J.EQ.3)DWI(K,I,J)=0.D0
      IF(J.EQ.3)DWJ(K,I,J)=0.D0
20    CONTINUE
      DWI(2,1,1)=-S5*C6
      DWI(2,1,2)= S5*S6
      DWI(2,1,3)= C5
      DWI(2,2,1)= S4*C5*C6
      DWI(2,2,2)=-S4*C5*S6
      DWI(2,2,3)= S4*S5
      DWI(2,3,1)=-C4*C5*C6
      DWI(2,3,2)= C4*C5*S6
      DWI(2,3,3)=-C4*S5
      DWJ(2,1,1)=-S11*C12
      DWJ(2,1,2)= S11*S12
      DWJ(2,1,3)= C11
      DWJ(2,2,1)= S10*C11*C12
      DWJ(2,2,2)=-S10*C11*S12
      DWJ(2,2,3)= S10*S11
      DWJ(2,3,1)=-C10*C11*C12
      DWJ(2,3,2)= C10*C11*S12
      DWJ(2,3,3)=-C10*S11
      T(1)=-WJ(2,3)*WI(3,1)+WJ(3,3)*WI(2,1)
      T(2)=-WJ(1,3)*S5*C6 +WJ(2,3)*S4*C5*C6-WJ(3,3)*C4*C5*C6
      T(3)= WJ(1,3)*WI(1,2)+WJ(2,3)*WI(2,2)+WJ(3,3)*WI(3,2)
      T(4)=-T(1)
      T(5)= C11*WI(1,1) +S10*S11*WI(2,1)-C10*S11*WI(3,1)
      P(1)= -WJ(2,3)*WI(3,2) +WJ(3,3)*WI(2,2)
      P(2)= WJ(1,3)*S5*S6-WJ(2,3)*S4*C5*S6+WJ(3,3)*C4*C5*S6
      P(3)=-WJ(1,3)*WI(1,1)-WJ(2,3)*WI(2,1)-WJ(3,3)*WI(3,1)
      P(4)= -P(1)
      P(5)= C11*WI(1,2)+S10*S11*WI(2,2)-C10*S11*WI(3,2)
      Q(1)=-WJ(2,2)*WI(3,1)+WJ(3,2)*WI(2,1)
      Q(2)=-WJ(1,2)*S5*C6 +WJ(2,2)*S4*C5*C6-WJ(3,2)*C4*C5*C6
      Q(3)= WJ(1,2)*WI(1,2)+WJ(2,2)*WI(2,2)+WJ(3,2)*WI(3,2)
      Q(4)= -Q(1)
      Q(5)= S11*S12*WI(1,1)-S10*C11*S12*WI(2,1) +C10*C11*S12*WI(3,1)
      Q(6)=-WJ(1,1)*WI(1,1)-WJ(2,1)*WI(2,1)-WJ(3,1)*WI(3,1)
      DO 15 J=1,3
      DO 16 K=1,3
      A(J,K)= -WI(K,J)
```

```
      A(J,K+6)=-A(J,K)
      A(J,5)= AA(K)*CWI(2,K,J)+A(J,5)
16    CCNTINUE
      A(J,4)=-AA(2)*WI(3,J)+AA(3)*WI(2,J)
      A(1,6)= AA(J)*WI(J,2)+A(1,6)
      A(2,6)=-AA(J)*WI(J,1)+A(2,6)
15    CCNTINUE
      DC 12 K=1,3
      A(5,K+3)= T(K)
      A(4,K+3)=-P(K)
      A(6,K+3)=-C(K)
      IF(K.EQ.3)GCTC12
      A(5,K+9)= T(K+3)
      A(4,K+9)=-P(K+3)
      A(6,K+9)=-C(K+3)
12    CONTINUE
      A(6,12)= -C(6)
      DC 84 N=1,3
      DC 84 K=1,3
      DO 84 I=1,3
      DC 84 J=1,3
      GOTO(85,84,86),K
85    IF(I.EQ.1)CDWI(N,K,I,J)=0.DO
      IF(I.EQ.2)CDWI(N,K,I,J)=-DWI(N,3,J)
      IF(I.EQ.3)CDWI(N,K,I,J)= DWI(N,2,J)
      IF(I.EQ.1)CDWJ(N,K,I,J)=0.DO
      IF(I.EQ.2)CDWJ(N,K,I,J)=-DWJ(N,3,J)
      IF(I.EQ.3)CDWJ(N,K,I,J)= DWJ(N,2,J)
      GCTC84
86    IF(J.EQ.1)CDWI(N,K,I,J)= DWI(N,I,2)
      IF(J.EQ.1)CDWJ(N,K,I,J)= DWJ(N,I,2)
      IF(J.EQ.2)DDWI(N,K,I,J)=-DWI(N,I,1)
      IF(J.EQ.2)CDWJ(N,K,I,J)=-DWJ(N,I,1)
      IF(J.EQ.3)CDWI(N,K,I,J)=0.CO
      IF(J.EQ.3)CDWJ(N,K,I,J)=0.CO
84    CONTINUE
      DC 87 J=1,3
      CDWI(1,2,1,J)=0.DC
      CDWJ(1,2,1,J)=C.DC
      CDWI(3,2,J,3)=0.DO
      CDWJ(3,2,J,3)=C.DO
      CDWI(3,2,J,1)=DWI(2,J,2)
      CDWJ(3,2,J,1)=DWJ(2,J,2)
      CDWI(3,2,J,2)=-DWI(2,J,1)
      CDWJ(3,2,J,2)=-DWJ(2,J,1)
      CDWI(2,2,1,J)=-WI(1,J)
      CDWJ(2,2,1,J)=-WJ(1,J)
      CDWI(1,2,2,J)=-DWI(2,3,J)
      CDWJ(1,2,2,J)=-DWJ(2,3,J)
      CDWI(1,2,3,J)= DWI(2,2,J)
      CDWJ(1,2,3,J)= DWJ(2,2,J)
87    CCNTINUE
      CDWI(2,2,3,1)= C4*S5*C6
      CDWJ(2,2,3,1)= C10*S11*C12
      CDWI(2,2,3,2)=-C4*S5*S6
      CDWJ(2,2,3,2)=-C10*S11*S12
      CDWI(2,2,3,3)=-C4*C5
      CDWJ(2,2,3,3)=-C1C*C11
      CDWI(2,2,2,1)=-S4*S5*C6
      CDWJ(2,2,2,1)=-S10*S11*C12
```

```
      CCWI(2,2,2,2)= S4*S5*S6
      CCWJ(2,2,2,2)= S10*S11*S12
      CCWI(2,2,2,3)= S4*C5
      CCWJ(2,2,2,3)= C11*S10
      V(5,1,1)=S5*C6
      V(6,1,1)=-WI(1,2)
      V(4,1,2)= WI(3,1)
      V(5,1,2)=-S4*C5*C6
      V(6,1,2)=-WI(2,2)
      V(4,1,3)=-WI(2,1)
      V(5,1,3)= C4*C5*C6
      V(6,1,3)=-WI(3,2)
      V(4,1,4)=-AA(2)*WI(2,1)-AA(3)*WI(3,1)
      V(5,1,4)= AA(2)*C4*C5*C6 +AA(3)*S4*C5*C6
      V(6,1,4)=-AA(2)*WI(3,2) +AA(3)*WI(2,2)
      V(8,1,4)=-WI(3,1)
      V(9,1,4)= WI(2,1)
      V(5,1,5)=-AA(1)*C5*C6-AA(2)*S4*S5*C6+AA(3)*C4*S5*C6
      V(6,1,5)= AA(1)*S5*S6-AA(2)*S4*C5*S6+AA(3)*C4*C5*S6
      V(7,1,5)= -S5*C6
      V(8,1,5)= S4*C5*C6
      V(9,1,5)=-C4*C5*C6
      V(6,1,6)=-AA(1)*WI(1,1)-AA(2)*WI(2,1) -AA(3)*WI(3,1)
      V(7,1,6)= WI(1,2)
      V(8,1,6)= WI(2,2)
      V(9,1,6)= WI(3,2)
      V(5,2,1)=-S5*S6
      V(6,2,1)= WI(1,1)
      V(4,2,2)= WI(3,2)
      V(5,2,2)= S4*C5*S6
      V(6,2,2)= WI(2,1)
      V(4,2,3)=-WI(2,2)
      V(5,2,3)=-C4*C5*S6
      V(6,2,3)= WI(3,1)
      V(4,2,4)=-AA(2)*WI(2,2)-AA(3)*WI(3,2)
      V(5,2,4)=-AA(2)*C4*C5*S6 -AA(3)*S4*C5*S6
      V(6,2,4)= AA(2)*WI(3,1) -AA(3)*WI(2,1)
      V(8,2,4)=-WI(3,2)
      V(9,2,4)= WI(2,2)
      V(5,2,5)= AA(1)*C5*S6+AA(2)*S4*S5*S6-AA(3)*C4*S5*S6
      V(6,2,5)= AA(1)*S5*C6-AA(2)*S4*C5*C6+AA(3)*C4*C5*C6
      V(7,2,5)= S5*S6
      V(8,2,5)=-S4*C5*S6
      V(9,2,5)= C4*C5*S6
      V(6,2,6)= -AA(1)*WI(1,2)-AA(2)*WI(2,2)-AA(3)*WI(3,2)
      V(7,2,6)=-WI(1,1)
      V(8,2,6)=-WI(2,1)
      V(9,2,6)=-WI(3,1)
      V(5,3,1)= -C5
      V(4,3,2)=WI(3,3)
      V(5,3,2)=-S4*S5
      V(4,3,3)= -WI(2,3)
      V(5,3,3)= C4*S5
      V(4,3,4)= -AA(2)*WI(2,3)+AA(3)*CWI(1,2,3)
      V(8,3,4)= -WI(3,3)
      V(9,3,4)=  WI(2,3)
      V(5,3,5)=-AA(1)*S5 +AA(2)*S4*C5 -AA(3)*C4*C5
      V(7,3,5)= C5
      V(8,3,5)= S4*S5
      V(9,3,5)=-C4*S5
```

```
      DO 120 J=1,3
      DT(1,J)=-WJ(2,3)*DWI(J,3,1)+WJ(3,3)*DWI(J,2,1)
      DT(1,J+3)=-DWJ(J,2,3)*WI(3,1)-DWJ(J,3,3)*WI(2,1)
      DT(2,J)=+WJ(1,3)*DDWI(J,2,1,1)+WJ(2,3)*DDWI(J,2,2,1)+WJ(3,3)*DDWI(
     *J,2,3,1)
      DT(2,J+3)=+DWJ(J,1,3)*DWI(2,1,1)+DWJ(J,2,3)*DWI(2,2,1)+DWJ(J,3,3)*
     *DWI(2,3,1)
      DT(3,J)= WJ(1,3)*DWI(J,1,2)+WJ(2,3)*DWI(J,2,2)+WJ(3,3)*DWI(J,3,2)
      DT(3,J+3)=DWJ(J,1,3)*WI(1,2)+DWJ(J,2,3)*WI(2,2)+DWJ(J,3,3)*WI(3,2)
      DT(4,J)=-DT(1,J)
      DT(4,J+3)=-DT(1,J+3)
      DT(5,J)= DWJ(2,1,3)*DWI(J,1,1)+DWJ(2,2,3)*DWI(J,2,1)+DWJ(2,3,3)*
     *DWI(J,3,1)
      DT(5,J+3)=DDWJ(J,2,1,3)*WI(1,1)+DDWJ(J,2,2,3)*WI(2,1)+DDWJ(J,2,3,3
     *)*WI(3,1)
      DP(1,J)=-WJ(2,3)*DWI(J,3,2)+WJ(3,3)*DWI(J,2,2)
      DP(1,J+3)=-DWJ(J,2,3)*WI(3,2)+DWJ(J,3,3)*WI(2,2)
      DP(2,J)=WJ(1,3)*DDWI(J,2,1,2)+WJ(2,3)*DDWI(J,2,2,2)+WJ(3,3)*DDWI(
     *J,2,3,2)
      DP(2,J+3)=DWJ(J,1,3)*DWI(2,1,2)+DWJ(J,2,3)*DWI(2,2,2)+DWJ(J,3,3)*
     *DWI(2,3,2)
      DP(3,J)=-WJ(1,3)*DWI(J,1,1)-WJ(2,3)*DWI(J,2,1)-WJ(3,3)*DWI(J,3,1)
      DP(3,J+3)=-DWJ(J,1,3)*WI(1,1)-DWJ(J,2,3)*WI(2,1)-DWJ(J,3,3)*WI(3,1
     *)
      DP(4,J)=-DP(1,J)
      DP(4,J+3)=-DP(1,J+3)
      DP(5,J)=DWJ(2,1,3)*DWI(J,1,2)+DWJ(2,2,3)*DWI(J,2,2)+DWJ(2,3,3)*
     *DWI(J,3,2)
      DP(5,J+3)=DDWJ(J,2,1,3)*WI(1,2)+DDWJ(J,2,2,3)*WI(2,2)+DDWJ(J,2,3,3
     *)*WI(3,2)
      DQ(1,J)=-WJ(2,2)*DWI(J,3,1)+WJ(3,2)*DWI(J,2,1)
      DQ(1,J+3)=-DWJ(J,2,2)*WI(3,1)+DWJ(J,3,2)*WI(2,1)
      DQ(2,J)=WJ(1,2)*DDWI(J,2,1,1)+WJ(2,2)*DDWI(J,2,2,1)+WJ(3,2)*DDWI(J
     *,2,3,1)
      DQ(2,J+3)=DWJ(J,1,2)*DWI(2,1,1)+DWJ(J,2,2)*DWI(2,2,1)+DWJ(J,3,2)*
     *DWI(2,3,1)
      DQ(3,J)=WJ(1,2)*DWI(J,1,2)+WJ(2,2)*DWI(J,2,2)+WJ(3,2)*DWI(J,3,2)
      DQ(3,J+3)=DWJ(J,1,2)*WI(1,2)+DWJ(J,2,2)*WI(2,2)+DWJ(J,3,2)*WI(3,2)
      DQ(4,J)= -DQ(1,J)
      DQ(4,J+3)=-DQ(1,J+3)
      DQ(5,J)=DWJ(2,1,2)*DWI(J,1,1)+DWJ(2,2,2)*DWI(J,2,1)+DWJ(2,3,2)*
     *DWI(J,3,1)
      DQ(5,J+3)=DDWJ(J,2,1,2)*WI(1,1)+DDWJ(J,2,2,2)*WI(2,1)+DDWJ(J,2,3,2
     *)*WI(3,1)
      DQ(6,J)=-WJ(1,1)*DWI(J,1,1)-WJ(2,1)*DWI(J,2,1)-WJ(3,1)*DWI(J,3,1)
      DQ(6,J+3)=-DWJ(J,1,1)*WI(1,1)-DWJ(J,2,1)*WI(2,1)-DWJ(J,3,1)*WI(3,1
     *)
  120  CONTINUE
      V(4,5,4)=DT(1,1)
      V(5,5,4)=DT(1,2)
      V(6,5,4)=DT(1,3)
      V(10,5,4)=DT(1,4)
      V(11,5,4)=DT(1,5)
      V(5,5,5)= DT(2,2)
      V(6,5,5)= DT(2,3)
      V(10,5,5)=DT(2,4)
      V(11,5,5)=DT(2,5)
      V(6,5,6)= DT(3,3)
      V(10,5,6)=DT(3,4)
      V(11,5,6)=DT(3,5)
```

```
      V(10,5,10)=DT(4,4)
      V(11,5,10)=DT(4,5)
      V(11,5,11)=DT(5,5)
      DO 31 J=1,3
      DO 32 K=1,3
      V(K+3,4,J+3)=-DP(J,K)
      IF(K.GE.3)GOTO 32
      V(K+9,4,J+3)=-DP(J,K+3)
      IF(J.GE.3)GOTO32
      V(K+9,4,J+9)=-DP(J+3,K+3)
 32   CONTINUE
 31   CONTINUE
      DO 41 J=1,3
      DO 42 K=1,3
      V(K+3,6,J+3)=-DQ(J,K)
      V(K+9,6,J+3)=-DQ(J,K+3)
      V(K+9,6,J+9)=-DQ(J+3,K+3)
 42   CONTINUE
 41   CONTINUE
      DO 51 K=1,12
      DO 52 J=1,6
      DO 53 I=I,12
      V(I,J,K)= V(K,J,I)
 53   CONTINUE
 52   CONTINUE
 51   CONTINUE
      RETURN
      END
      SUBROUTINE AVMATS(V,R,XX,SN)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLK2/ C4,C5,S4,S5,C10,C11,S10,S11,SMPREV,A
      DIMENSION A(6,12),V(12,6,12),SMPREV(6,6),XX(3),SN(6),R(12)
      DIMENSION WII(3,3),WJJ(3,3),AA(3),YSI(3),BS(3),YSJ(3),YSK(3),
     *ZSI(3),CS(3),ZSJ(3),ZSK(3),DWI(3,3,3),DWJ(3,3,3),DEL(9),DYSI(3,9),
     *DBS(3,9),DCS(3,9),DYESJ(9),DZESJ(9),DYSJ(3,9),DZSJ(3,9),DYSK(3,9),
     *DZSK(3,9),DYISI(9),DZISI(9),DYISJ(9),DZISJ(9),DYISK(9),DZISK(9)
      DIMENSION DDEL(9,9),DDYSI(9,3,9),DDWI(3,3,3,3),DDWJ(3,3,3,3),
     *DDBS(9,3,9),DDCS(9,3,9),DDYESJ(9,9),DDZESJ(9,9),DDYSJ(9,3,9),
     *DDZSJ(9,3,9),DDYSK(9,3,9),DDZSK(9,3,9),DDYISI(9,9),DDZISI(9,9),
     *DDYISJ(9,9),DDZISJ(9,9),DDYISK(9,9),DDZISK(9,9)
      DO 1 J=1,12
      DO 1 I=1,6
      DO 1 K=1,12
      A(I,J)= 0.D0
      V(K,I,J)=0.D0
 1    CONTINUE
 6    C4=DCOS(R(4))*SN(2)-DSIN(R(4))*SN(1)
      S4=DCOS(R(4))*SN(1)+DSIN(R(4))*SN(2)
      C5=DCOS(R(5))*SN(4)-DSIN(R(5))*SN(3)
      S5=DCOS(R(5))*SN(3)+DSIN(R(5))*SN(4)
      C6=DCOS(R(6))*SN(6)-DSIN(R(6))*SN(5)
      S6=DCOS(R(6))*SN(5)+DSIN(R(6))*SN(6)
      C11=DCOS(R(11))*SN(4)-DSIN(R(11))*SN(3)
      S11=DCOS(R(11))*SN(3)+DSIN(R(11))*SN(4)
      C10=DCOS(R(10))*SN(2)-DSIN(R(10))*SN(1)
      S10=DCOS(R(10))*SN(1)+DSIN(R(10))*SN(2)
      S12=DCOS(R(12))*SN(5)+DSIN(R(12))*SN(6)
      C12=DCOS(R(12))*SN(6)-DSIN(R(12))*SN(5)
      WII(1,1)=C5*C6
      WII(1,2)=-C5*S6
```

```
      WII(1,3)= S5
      WII(2,1)= C4*S6+S4*S5*C6
      WII(2,2)= C4*C6-S4*S5*S6
      WII(2,3)= -S4*C5
      WII(3,1)= S4*S6-C4*S5*C6
      WII(3,2)= S4*C6+C4*S5*S6
      WII(3,3)= C4*C5
      WJJ(1,1)=C11*C12
      WJJ(1,2)=-C11*S12
      WJJ(1,3)=  S11
      WJJ(2,1)= C10*S12+S10*S11*C12
      WJJ(2,2)= C10*C12-S10*S11*S12
      WJJ(2,3)=-S10*C11
      WJJ(3,1)= S10*S12-C10*S11*C12
      WJJ(3,2)= S10*C12+C10*S11*S12
      WJJ(3,3)= C10*C11
      AA(1)=XX(1)+ R(7)-R(1)
      AA(2)=XX(2)+ R(8)-R(2)
      AA(3)=XX(3)+ R(9)-R(3)
      EL=DSQRT(AA(1)**2+AA(2)**2+AA(3)**2)
      YSI(1)= AA(1)/EL
      YSI(2)= AA(2)/EL
      YSI(3)= AA(3)/EL
      BS(1)= WII(2,3)*YSI(3)-WII(3,3)*YSI(2)
      BS(2)= WII(3,3)*YSI(1)-WII(1,3)*YSI(3)
      BS(3)= WII(1,3)*YSI(2)-WII(2,3)*YSI(1)
      YESJ=DSQRT(BS(1)**2+BS(2)**2+BS(3)**2)
      YSJ(1)= BS(1)/YESJ
      YSJ(2)= BS(2)/YESJ
      YSJ(3)= BS(3)/YESJ
      CALL CROSS(YSI,YSJ,YSK)
      ZSI(1)=YSI(1)
      ZSI(2)=YSI(2)
      ZSI(3)=YSI(3)
      CS(1)= WJJ(2,3)*YSI(3)-WJJ(3,3)*YSI(2)
      CS(2)= WJJ(3,3)*YSI(1)-WJJ(1,3)*YSI(3)
      CS(3)= WJJ(1,3)*YSI(2)-WJJ(2,3)*YSI(1)
      ZESJ=DSQRT(CS(1)**2+CS(2)**2+CS(3)**2)
      ZSJ(1)=CS(1)/ZESJ
      ZSJ(2)=CS(2)/ZESJ
      ZSJ(3)=CS(3)/ZESJ
      CALL CROSS(ZSI,ZSJ,ZSK)
      YISI=WII(1,1)*YSI(1)+WII(2,1)*YSI(2)+WII(3,1)*YSI(3)
      ZISI= WJJ(1,1)*YSI(1)+WJJ(2,1)*YSI(2)+WJJ(3,1)*YSI(3)
      YISK=WII(1,1)*YSK(1)+WII(2,1)*YSK(2)+WII(3,1)*YSK(3)
      YISJ=WII(1,1)*YSJ(1)+WII(2,1)*YSJ(2)+WII(3,1)*YSJ(3)
      ZISJ=WJJ(1,1)*ZSJ(1)+WJJ(2,1)*ZSJ(2)+WJJ(3,1)*ZSJ(3)
      ZISK=WJJ(1,1)*ZSK(1)+WJJ(2,1)*ZSK(2)+WJJ(3,1)*ZSK(3)
      DO 20 K=1,3
      DO 20 I=1,3
      DO 20 J=1,3
      GO TO(21,20,23),K
21    IF(I.EQ.1)DWI(K,I,J)=0.D0
      IF(I.EQ.2)DWI(K,I,J)=-WII(3,J)
      IF(I.EQ.3)DWI(K,I,J)= WII(2,J)
      IF(I.EQ.1)DWJ(K,I,J)=0.D0
      IF(I.EQ.2)DWJ(K,I,J)=-WJJ(3,J)
      IF(I.EQ.3)DWJ(K,I,J)= WJJ(2,J)
      GOTO20
23    IF(J.EQ.1)DWI(K,I,J)=WII(I,2)
```

```
      IF(J.EQ.1)DWJ(K,I,J)=WJJ(I,2)
      IF(J.EQ.2)DWI(K,I,J)=-WII(I,1)
      IF(J.EQ.2)DWJ(K,I,J)=-WJJ(I,1)
      IF(J.EQ.3)DWI(K,I,J)=0.D0
      IF(J.EQ.3)DWJ(K,I,J)=0.D0
 20   CONTINUE
      DWI(2,1,1)=-S5*C6
      DWI(2,1,2)= S5*S6
      DWI(2,1,3)= C5
      DWI(2,2,1)= S4*C5*C6
      DWI(2,2,2)=-S4*C5*S6
      DWI(2,2,3)= S4*S5
      DWI(2,3,1)=-C4*C5*C6
      DWI(2,3,2)= C4*C5*S6
      DWI(2,3,3)=-C4*S5
      DWJ(2,1,1)=-S11*C12
      DWJ(2,1,2)= S11*S12
      DWJ(2,1,3)= C11
      DWJ(2,2,1)= S10*C11*C12
      DWJ(2,2,2)=-S10*C11*S12
      DWJ(2,2,3)= S10*S11
      DWJ(2,3,1)=-C10*C11*C12
      DWJ(2,3,2)= C10*C11*S12
      DWJ(2,3,3)=-C10*S11
      DO  30 J=1,3
      DEL(J)= -YSI(J)
      DEL(J+3)=0.0
      DEL(J+6)=-DEL(J)
      DO 31 K=1,3
      DEL(K)=-YSI(K)
      DEL(K+6)=-DEL(K)
      DYSI(J,K)=+DEL(J)*DEL(K)/EL
      IF(J.EQ.K)DYSI(J,K)=DYSI(J,K)-1./EL
      DYSI(J,K+6)=-DYSI(J,K)
      DYSI(J,K+3)=0.0
 31   CONTINUE
 30   CONTINUE
      DO 40 J=1,3
      DO 40 K=1,3
      M=J+1
      IF(M.EQ.4)M=1
      N=M+1
      IF(N.EQ.4)N=1
      DBS(J,K)= WII(M,3)*DYSI(N,K)-WII(N,3)*DYSI(M,K)
      DCS(J,K)= WJJ(M,3)*DYSI(N,K)-WJJ(N,3)*DYSI(M,K)
      DBS(J,K+6)=-DBS(J,K)
      DCS(J,K+6)=-DCS(J,K)
      DBS(J,K+3)= DWI(K,M,3)*YSI(N)-DWI(K,N,3)*YSI(M)
      DCS(J,K+3)= DWJ(K,M,3)*YSI(N)-DWJ(K,N,3)*YSI(M)
 40   CONTINUE
      DO 50 K=1,9
      DYESJ(K)=(DBS(1,K)*BS(1)+DBS(2,K)*BS(2)+DBS(3,K)*BS(3))/YESJ
      DZESJ(K)=(DCS(1,K)*CS(1)+DCS(2,K)*CS(2)+DCS(3,K)*CS(3))/ZESJ
      DO 51 J=1,3
      DYSJ(J,K)=(DBS(J,K)-DYESJ(K)*YSJ(J))/YESJ
 51   DZSJ(J,K)=(DCS(J,K)-DZESJ(K)*ZSJ(J))/ZESJ
 50   CONTINUE
      DO 150 K=1,9
      DO 52 J=1,3
      M=J+1
```

```
      IF(M.EQ.4)M=1
      N=M+1
      IF(N.EQ.4)N=1
      DYSK(J,K)= YSI(M)*DYSJ(N,K)+DYSI(M,K)*YSJ(N)-YSI(N)*DYSJ(M,K)
     1 -DYSI(N,K)*YSJ(M)
      DZSK(J,K)= YSI(M)*DZSJ(N,K)+DYSI(M,K)*ZSJ(N)-YSI(N)*DZSJ(M,K)
     1 -DYSI(N,K)*ZSJ(M)
   52 CONTINUE
  150 CONTINUE
      DO 60 K=1,3
      DYISI(K)=WII(1,1)*DYSI(1,K)+WII(2,1)*DYSI(2,K)+WII(3,1)*DYSI(3,K)
      DZISI(K)=WJJ(1,1)*DYSI(1,K)+WJJ(2,1)*DYSI(2,K)+WJJ(3,1)*DYSI(3,K)
      DYISI(K+6)=-DYISI(K)
      DZISI(K+6)=-DZISI(K)
      DYISI(K+3)=DWI(K,1,1)*YSI(1)+DWI(K,2,1)*YSI(2)+ DWI(K,3,1)*YSI(3)
      DZISI(K+3)=DWJ(K,1,1)*YSI(1)+DWJ(K,2,1)*YSI(2)+ DWJ(K,3,1)*YSI(3)
      DYISJ(K)=WII(1,1)*DYSJ(1,K)+WII(2,1)*DYSJ(2,K)+WII(3,1)*DYSJ(3,K)
      DYISJ(K+6)=-DYISJ(K)
      DZISJ(K)=WJJ(1,1)*DZSJ(1,K)+WJJ(2,1)*DZSJ(2,K)+WJJ(3,1)*DZSJ(3,K)
      DZISJ(K+6)=-DZISJ(K)
      DYISJ(K+3)=WII(1,1)*DYSJ(1,K+3)+WII(2,1)*DYSJ(2,K+3)+WII(3,1)*DYSJ
     *(3,K+3)+DWI(K,1,1)*YSJ(1) +DWI(K,2,1)*YSJ(2) +DWI(K,3,1)*YSJ(3)
      DZISJ(K+3)=WJJ(1,1)*DZSJ(1,K+3)+WJJ(2,1)*DZSJ(2,K+3)+WJJ(3,1)*DZSJ
     *(3,K+3)+DWJ(K,1,1)*ZSJ(1) +DWJ(K,2,1)*ZSJ(2) +DWJ(K,3,1)*ZSJ(3)
      DYISK(K)=WII(1,1)*DYSK(1,K)+WII(2,1)*DYSK(2,K)+WII(3,1)*DYSK(3,K)
      DZISK(K)=WJJ(1,1)*DZSK(1,K)+WJJ(2,1)*DZSK(2,K)+WJJ(3,1)*DZSK(3,K)
      DYISK(K+6)=-DYISK(K)
      DZISK(K+6)=-DZISK(K)
      DYISK(K+3)= WII(1,1)*DYSK(1,K+3)+WII(2,1)*DYSK(2,K+3)+WII(3,1)*DYS
     *K(3,K+3)+DWI(K,1,1)*YSK(1)+DWI(K,2,1)*YSK(2) +DWI(K,3,1)*YSK(3)
      DZISK(K+3)= WJJ(1,1)*DZSK(1,K+3)+WJJ(2,1)*DZSK(2,K+3)+WJJ(3,1)*DZS
     *K(3,K+3)+DWJ(K,1,1)*ZSK(1)+DWJ(K,2,1)*ZSK(2) +DWJ(K,3,1)*ZSK(3)
   60 CONTINUE
      DO 70 K=1,9
   70 A(1,K)= DEL(K)
      DO 71 K=1,9
      A(2,K)=(-YISI*DYISK(K)+YISK*DYISI(K))/YISI**2
   71 A(3,K)=(YISI*DYISJ(K)-YISJ*DYISI(K))/YISI**2
      M=1
      N=3
   73 DO 72 K=M,N
      A(5,K)=(-ZISI*DZISK(K)+ZISK*DZISI(K))/ZISI**2
      A(6,K)=(+ZISI*DZISJ(K)-ZISJ*DZISI(K))/ZISI**2
      A(4,K)= DYSK(1,K)*ZSJ(1)+DYSK(2,K)*ZSJ(2)+DYSK(3,K)*ZSJ(3) +DZSJ(1
     1,K)*YSK(1)+ DZSJ(2,K)*YSK(2) +DZSJ(3,K)*YSK(3)
   72 CONTINUE
      IF(M.EQ.1)GOTO74
      GOTO75
   74 M=7
      N=9
      GOTO73
   75 DO 76 K=4,6
      A(5,K+6)=(-ZISI*DZISK(K)+ZISK*DZISI(K))/ZISI**2
      A(6,K+6)=(ZISI*DZISJ(K)-ZISJ*DZISI(K))/ZISI**2
      A(4,K)= DYSK(1,K)*ZSJ(1)+DYSK(2,K)*ZSJ(2) +DYSK(3,K)*ZSJ(3)
      A(4,K+6)= DZSJ(1,K)*YSK(1)+ DZSJ(2,K)*YSK(2)+ DZSJ(3,K)*YSK(3)
   76 CONTINUE
      DO 80 J=1,3
      DO 80 K=1,9
      DDEL(J,K)=-DYSI(J,K)
```

```
      DDEL(J+3,K)=0.0
      DDEL(J+6,K)=-DDEL(J,K)
80    CONTINUE
      DO  81 I=1,9
      DO  81 J=1,3
      DO  81 K=1,3
      DDYSI(I,J,K)=+DDEL(J,I)*DEL(K)/EL+DDEL(K,I)*DEL(J)/EL
     *-DEL(J)*DEL(K)*DEL(I)/EL**2
      IF(J.EQ.K) DDYSI(I,J,K)=DDYSI(I,J,K)+DEL(I)/(EL*EL)
      DDYSI(I,J,K+6)=-DDYSI(I,J,K)
      DDYSI(I,J,K+3)=0.0
81    CONTINUE
      DO 84 N=1,3
      DO 84 K=1,3
      DO 84 I=1,3
      DO 84 J=1,3
      GOTO(85,84,86),K
85    IF(I.EQ.1)DDWI(N,K,I,J)=0.D0
      IF(I.EQ.2)DDWI(N,K,I,J)=-DWI(N,3,J)
      IF(I.EQ.3)DDWI(N,K,I,J)= DWI(N,2,J)
      IF(I.EQ.1)DDWJ(N,K,I,J)=0.D0
      IF(I.EQ.2)DDWJ(N,K,I,J)=-DWJ(N,3,J)
      IF(I.EQ.3)DDWJ(N,K,I,J)= DWJ(N,2,J)
      GOTO84
86    IF(J.EQ.1)DDWI(N,K,I,J)= DWI(N,I,2)
      IF(J.EQ.1)DDWJ(N,K,I,J)= DWJ(N,I,2)
      IF(J.EQ.2)DDWI(N,K,I,J)=-DWI(N,I,1)
      IF(J.EQ.2)DDWJ(N,K,I,J)=-DWJ(N,I,1)
      IF(J.EQ.3)DDWI(N,K,I,J)=0.D0
      IF(J.EQ.3)DDWJ(N,K,I,J)=0.D0
84    CONTINUE
      DO 87 J=1,3
      DDWI(1,2,1,J)=0.D0
      DDWJ(1,2,1,J)=0.D0
      DDWI(3,2,J,3)=0.D0
      DDWJ(3,2,J,3)=0.D0
      DDWI(3,2,J,1)=DWI(2,J,2)
      DDWJ(3,2,J,1)=DWJ(2,J,2)
      DDWI(3,2,J,2)=-DWI(2,J,1)
      DDWJ(3,2,J,2)=-DWJ(2,J,1)
      DDWI(2,2,1,J)=-WII(1,J)
      DDWJ(2,2,1,J)=-WJJ(1,J)
      DDWI(1,2,2,J)=-DWI(2,3,J)
      DDWJ(1,2,2,J)=-DWJ(2,3,J)
      DDWI(1,2,3,J)= DWI(2,2,J)
      DDWJ(1,2,3,J)= DWJ(2,2,J)
87    CONTINUE
      DDWI(2,2,3,1)= C4*S5*C6
      DDWJ(2,2,3,1)= C10*S11*C12
      DDWI(2,2,3,2)=-C4*S5*S6
      DDWJ(2,2,3,2)=-C10*S11*S12
      DDWI(2,2,3,3)=-C4*C5
      DDWJ(2,2,3,3)=-C10*C11
      DDWI(2,2,2,1)=-S4*S5*C6
      DDWJ(2,2,2,1)=-S10*S11*C12
      DDWI(2,2,2,2)= S4*S5*S6
      DDWJ(2,2,2,2)= S10*S11*S12
      DDWI(2,2,2,3)= S4*C5
      DDWJ(2,2,2,3)= C11*S10
      DO 82 I=1,3
```

```
      DO 82 J=1,3
      DO 82 K=1,3
      M=J+1
      IF(M.EQ.4)M=1
      N=M+1
      IF(N.EQ.4)N=1
      DDBS(I,J,K)=WII(M,3)*DDYSI(I,N,K) -WII(N,3)*DDYSI(I,M,K)
      DDCS(I,J,K)=WJJ(M,3)*DDYSI(I,N,K)-WJJ(N,3)*DDYSI(I,M,K)
      DDBS(I,J,K+6)=-DDBS(I,J,K)
      DDCS(I,J,K+6)=-DDCS(I,J,K)
      DDBS(I,J,K+3)= DWI(K,M,3)*DYSI(N,I) -DWI(K,N,3)*DYSI(M,I)
      DDCS(I,J,K+3)= DWJ(K,M,3)*DYSI(N,I) -DWJ(K,N,3)*DYSI(M, I)
      DDBS(I+6,J,K)= WII(M,3)*DDYSI(I+6,N,K)-WII(N,3)*DDYSI(I+6,M,K)
      DDCS(I+6,J,K)= WJJ(M,3)*DDYSI(I+6,N,K)-WJJ(N,3)*DDYSI(I+6,M,K)
      DDBS(I+6,J,K+6)= -DDBS(I+6,J,K)
      DDCS(I+6,J,K+6)= -DDCS(I+6,J,K)
      DDBS(I+6,J,K+3)= DWI(K,M,3)*DYSI(N,I+6) -DWI(K,N,3)*DYSI(M,I+6)
      DDCS(I+6,J,K+3)= DWJ(K,M,3)*DYSI(N,I+6) -DWJ(K,N,3)*DYSI(M,I+6)
      DDBS(I+3,J,K)= DWI(I,M,3)*DYSI(N,K)-DWI(I,N,3)*DYSI(M,K)
      DDCS(I+3,J,K)= DWJ(I,M,3)*DYSI(N,K)-DWJ(I,N,3)*DYSI(M,K)
      DDBS(I+3,J,K+3)= DDWI(I,K,M,3)*YSI(N)-DDWI(I,K,N,3)*YSI(M)
      DDCS(I+3,J,K+3)=DDWJ(I,K,M,3)*YSI(N)-DDWJ(I,K,N,3)*YSI(M)
      DDBS(I+3,J,K+6)= -DDBS(I+3,J,K)
      DDCS(I+3,J,K+6)= -DDCS(I+3,J,K)
   82 CONTINUE
      DO 90 I=1,9
      DO 91 K=1,9
      DDYESJ(K,I)=(DDBS(I,1,K)*BS(1)+DBS(1,K)*DBS(1,I)+DDBS(I,2,K)*BS
     *(2)+DBS(2,K)*DBS(2,I)+DDBS(I,3,K)*BS(3)+DBS(3,K)*DBS(3,I))/YESJ
     *-DYESJ(K)*DYESJ(I)/YESJ
      DDZESJ(K,I)=(DDCS(I,1,K)*CS(1)+DCS(1,K)*DCS(1,I)+DDCS(I,2,K)*CS
     *(2)+DCS(2,K)*DCS(2,I)+DDCS(I,3,K)*CS(3)+DCS(3,K)*DCS(3,I))/ZESJ
     *-DZESJ(K)*DZESJ(I)/ZESJ
      DO 92 J=1,3
      DDYSJ(I,J,K)=(DDBS(I,J,K)-DDYESJ(K,I)*YSJ(J)-DYESJ(K)*DYSJ(J,I))/
     *YESJ -DYSJ(J,K)*DYESJ(I)/YESJ
      DDZSJ(I,J,K)=(DDCS(I,J,K)-DDZESJ(K,I)*ZSJ(J)-DZESJ(K)*DZSJ(J,I))/
     *ZESJ- DZSJ(J,K)*DZESJ(I)/ZESJ
   92 CONTINUE
   91 CONTINUE
   90 CONTINUE
      DO 191 I=1,9
      DO 190 K=1,9
      DO 93 J=1,3
      M=J+1
      IF(M.EQ.4)M=1
      N=M+1
      IF(N.EQ.4)N=1
      DDYSK(I,J,K)= DYSI(M,I)*DYSJ(N,K)+YSI(M)*DDYSJ(I,N,K)+DDYSI(I,M,K)
     **YSJ(N)+DYSI(M,K)*DYSJ(N,I)-DYSI(N,I)*DYSJ(M,K)-YSI(N)*DDYSJ(I,M,K
     *)-DDYSI(I,N,K)*YSJ(M)-DYSI(N,K)*DYSJ(M,I)
      DDZSK(I,J,K)=DYSI(M,I)*DZSJ(N,K)+YSI(M)*DDZSJ(I,N,K)+DDYSI(I,M,K)*
     *ZSJ(N)+DYSI(M,K)*DZSJ(N,I)-DYSI(N,I)*DZSJ(M,K)-YSI(N)*DDZSJ(I,M,K)
     *-DDYSI(I,N,K)*ZSJ(M)-DYSI(N,K)*DZSJ(M,I)
   93 CONTINUE
  190 CONTINUE
  191 CONTINUE
      DO 94 K=1,3
      DO 95 I=1,9
      DDYISI(K,I)=WII(1,1)*DDYSI(I,1,K)+WII(2,1)*DDYSI(I,2,K)+WII(3,1)*
```

```
      *DDYSI(I,3,K)
       DDZISI(K,I)=WJJ(1,1)*DDYSI(I,1,K)+WJJ(2,1)*DDYSI(I,2,K)+WJJ(3,1)*
      *DDYSI(I,3,K)
       DDYISJ(K,I)=WII(1,1)*DDYSJ(I,1,K)+WII(2,1)*DDYSJ(I,2,K)+WII(3,1)*
      *DDYSJ(I,3,K)
       DDZISJ(K,I)=WJJ(1,1)*DDZSJ(I,1,K)+WJJ(2,1)*DDZSJ(I,2,K)+WJJ(3,1)*
      *DDZSJ(I,3,K)
       DDYISK(K,I)=WII(1,1)*DDYSK(I,1,K)+WII(2,1)*DDYSK(I,2,K)+WII(3,1)*
      *DDYSK(I,3,K)
       DDZISK(K,I)=WJJ(1,1)*DDZSK(I,1,K)+WJJ(2,1)*DDZSK(I,2,K)+WJJ(3,1)*
      *DDZSK(I,3,K)
 95    CONTINUE
 94    CONTINUE
       DO 195 K=1,3
       DO 194 I=4,6
       N=I-3
       DDYISI(K,I)=DWI(N,1,1)*DYSI(1,K)+DWI(N,2,1)*DYSI(2,K)+DWI(N,
      *3,1)*DYSI(3,K)
       DDZISI(K,I)=DWJ(N,1,1)*DYSI(1,K)+DWJ(N,2,1)*DYSI(2,K)+DWJ(N,
      *3,1)*DYSI(3,K)
       DDYISJ(K,I)=DDYISJ(K,I)+DWI(N,1,1)*DYSJ(1,K)+DWI(N,2,1)*DYSJ(2
      *,K)+DWI(N,3,1)*DYSJ(3,K)
       DDZISJ(K,I)=DDZISJ(K,I)+DWJ(N,1,1)*DZSJ(1,K)+DWJ(N,2,1)*DZSJ(2,K)+
      *DWJ(N,3,1)*DZSJ(3,K)
       DDYISK(K,I)=DDYISK(K,I)+DWI(N,1,1)*DYSK(1,K)+DWI(N,2,1)*DYSK(2,K)+
      *DWI(N,3,1)*DYSK(3,K)
       DDZISK(K,I)=DDZISK(K,I)+DWJ(N,1,1)*DZSK(1,K)+DWJ(N,2,1)*DZSK(2,K)+
      *DWJ(N,3,1)*DZSK(3,K)
 194   CONTINUE
 195   CONTINUE
       DO 196 K=1,3
       DO 197 I=1,9
 96    DDYISI(K+6,I)=-DDYISI(K,I)
       DDZISI(K+6,I)=-DDZISI(K,I)
       DDYISJ(K+6,I)=-DDYISJ(K,I)
       DDZISJ(K+6,I)=-DDZISJ(K,I)
       DDYISK(K+6,I)=-DDYISK(K,I)
       DDZISK(K+6,I)=-DDZISK(K,I)
       M=K+3
       DDYISI(M,I)=DWI(K,1,1)*DYSI(1,I)+DWI(K,2,1)*DYSI(2,I)+DWI(K,3,1)*
      *DYSI(3,I)
       DDZISI(M,I)=DWJ(K,1,1)*DYSI(1,I)+DWJ(K,2,1)*DYSI(2,I)+DWJ(K,3,1)*
      *DYSI(3,I)
       DDYISJ(M,I)=WII(1,1)*DDYSJ(I,1,M)+WII(2,1)*DDYSJ(I,2,M)+WII(3,1)*
      *DDYSJ(I,3,M)+DWI(K,1,1)*DYSJ(1,I)+DWI(K,2,1)*DYSJ(2,I)+DWI(K,3,1)*
      *DYSJ(3,I)
       DDZISJ(M,I)=WJJ(1,1)*DDZSJ(I,1,M)+WJJ(2,1)*DDZSJ(I,2,M)+WJJ(3,1)*
      *DDZSJ(I,3,M)+DWJ(K,1,1)*DZSJ(1,I)+DWJ(K,2,1)*DZSJ(2,I)+DWJ(K,3,1)*
      *DZSJ(3,I)
       DDYISK(M,I)=WII(1,1)*DDYSK(I,1,M)+WII(2,1)*DDYSK(I,2,M)+WII(3,1)*
      *DDYSK(I,3,M)+DWI(K,1,1)*DYSK(1,I)+DWI(K,2,1)*DYSK(2,I)+DWI(K,3,1)*
      *DYSK(3,I)
       DDZISK(M,I)=WJJ(1,1)*DDZSK(I,1,M)+WJJ(2,1)*DDZSK(I,2,M)+WJJ(3,1)*
      *DDZSK(I,3,M)+DWJ(K,1,1)*DZSK(1,I)+DWJ(K,2,1)*DZSK(2,I)+DWJ(K,3,1)*
      *DZSK(3,I)
 197   CONTINUE
 196   CONTINUE
       DO 198 K=1,3
       DO 199 I=4,6
       M=K+3
```

```
      N=I-3
      DDYISI(M,I)=DDWI(N,K,1,1)*YSI(1)+DDWI(N,K,2,1)*YSI(2)+DDWI(N,K,3,1
     *)*YSI(3)
      DDZISI(M,I)=DDWJ(N,K,1,1)*YSI(1)+DDWJ(N,K,2,1)*YSI(2)+DDWJ(N,K,3,1
     *)*YSI(3)
      DDYISJ(M,I)=DDYISJ(M,I)+DWI(N,1,1)*DYSJ(1,M)+DWI(N,2,1)*DYSJ(2,M)+
     *DWI(N,3,1)*DYSJ(3,M)+DDWI(N,K,1,1)*YSJ(1)+DDWI(N,K,2,1)*YSJ(2)+
     *DDWI(N,K,3,1)*YSJ(3)
      DDZISJ(M,I)=DDZISJ(M,I)+DWJ(N,1,1)*DZSJ(1,M)+DWJ(N,2,1)*DZSJ(2,M)+
     *DWJ(N,3,1)*DZSJ(3,M)+DDWJ(N,K,1,1)*ZSJ(1)+DDWJ(N,K,2,1)*ZSJ(2)+
     *DDWJ(N,K,3,1)*ZSJ(3)
      DDYISK(M,I)=DDYISK(M,I)+DWI(N,1,1)*DYSK(1,M)+DWI(N,2,1)*DYSK(2,M)+
     *DWI(N,3,1)*DYSK(3,M)+DDWI(N,K,1,1)*YSK(1)+DDWI(N,K,2,1)*YSK(2)+
     *DDWI(N,K,3,1)*YSK(3)
      DDZISK(M,I)=DDZISK(M,I)+DWJ(N,1,1)*DZSK(1,M)+DWJ(N,2,1)*DZSK(2,M)+
     *DWJ(N,3,1)*DZSK(3,M)+DDWJ(N,K,1,1)*ZSK(1)+DDWJ(N,K,2,1)*ZSK(2)+
     *DDWJ(N,K,3,1)*ZSK(3)
  199 CONTINUE
  198 CONTINUE
      DO 110 I=1,9
      DO 111 K=1,9
      V(I,1,K)= DDEL(I,K)
  111 CONTINUE
  110 CONTINUE
      DO 113 I=1,9
      DO 114 K=1,9
      V(I,2,K)=(-YISI*DDYISK(K,I)-DYISI(I)*DYISK(K)+YISK*DDYISI(K,I)+
     *DYISK(I)*DYISI(K))/(YISI**2) -2.*A(2,K)*DYISI(I)/YISI
      V(I,3,K)=(YISI*DDYISJ(K,I)+DYISI(I)*DYISJ(K)-YISJ*DDYISI(K,I)-
     *DYISJ(I)*DYISI(K))/YISI**2-2.*A(3,K)*DYISI(I)/YISI
      M=I
      N=K
      IF(I.GE.4.AND.I.LE.6)M=I+6
      IF(K.GE.4.AND.K.LE.6)N=K+6
      V(M,5,N)=(-ZISI*DDZISK(K,I)-DZISI(I)*DZISK(K)+ZISK*DDZISI(K,I)+
     *DZISK(I)*DZISI(K))/ZISI**2-2.*A(5,N)*DZISI(I)/ZISI
      V(M,6,N)=(ZISI*DDZISJ(K,I)+DZISI(I)*DZISJ(K)-ZISJ*DDZISI(K,I)-
     *DZISJ(I)*DZISI(K))/ZISI**2-2.*A(6,N)*DZISI(I)/ZISI
  114 CONTINUE
  113 CONTINUE
      M=1
      N=3
      M1=1
      N1=3
  119 DO 115 I=M,N
      DO 116 K=M1,N1
      V(I,4,K)= DDYSK(I,1,K)*ZSJ(1)+DYSK(1,K)*DZSJ(1,I) +DDYSK(I,2,K)*ZS
     *J(2) +DYSK(2,K)*DZSJ(2,I) +DDYSK(I,3,K)*ZSJ(3)+DYSK(3,K)*DZSJ(3,I)
     * +DDZSJ(I,1,K)*YSK(1)+DZSJ(1,K)*DYSK(1,I)+DDZSJ(I,2,K)*YSK(2)+DZSJ
     *(2,K)*DYSK(2,I)+DDZSJ(I,3,K)*YSK(3)+DZSJ(3,K)*DYSK(3,I)
  116 CONTINUE
  115 CONTINUE
      IF(M.EQ.1)GOTO117
      GOTO 118
  117 M=7
      N=9
      GOTO119
  118 M=1
      N=3
      IF(M1.EQ.1)GOTO130
```

```
      M1=1
      N1=3
      GCTC 124
 13C  M1=7
      N1=9
      GCTO119
 124  DC 120 I=4,6
      DC 121 K=M1,N1
      V(I,4,K)= CCYSK(I,1,K)*ZSJ(1)+CDYSK(I,2,K)*ZSJ(2)+DDYSK(I,3,K)*ZSJ
     *(3) +CZSJ(1,K)*CYSK(1,I)+DZSJ(2,K)*CYSK(2,I)+CZSJ(3,K)*DYSK(3,I)
 121  CCNTINUE
 12C  CCNTINUE
      IF(M1.EQ.1)GCTC122
      GCTC123
 122  M1=7
      N1=9
      GCTC124
 123  DC 126 I=4,6
      DO 126 K=M1,N1
      V(I+6,4,K)= CYSK(1,K)*DZSJ(1,I)+DYSK(2,K)*DZSJ(2,I)+DYSK(3,K)*DZSJ
     *(3,I)+DDZSJ(I,1,K)*YSK(1)+CDZSJ(I,2,K)*YSK(2)+CCZSJ(I,3,K)*YSK(3)
 12C  CONTINUE
      IF(M1.EQ.7)GCTC131
      GCTC132
 131  M1=1
      N1=3
      GCTC123
 132  DC 133 I=M1,N1
      DO 134 K=4,6
      V(I,4,K)= CCYSK(I,1,K)*ZSJ(1)+CYSK(1,K)*DZSJ(1,I)+DDYSK(I,2,K)*ZSJ
     *(2)+DYSK(2,K)*CZSJ(2,I)+DDYSK(I,3,K)*ZSJ(3)+CYSK(3,K)*DZSJ(3,I)
      V(I,4,K+6)=CCZSJ(I,1,K)*YSK(1)+DZSJ(1,K)*DYSK(1,I)+DDZSJ(I,2,K)*YS
     *K(2)+DZSJ(2,K)*CYSK(2,I)+CDZSJ(I,3,K)*YSK(3)+DZSJ(3,K)*DYSK(3,I)
 134  CCNTINUE
 133  CCNTINUE
      IF(M1.EQ.1)GCTO135
      GCTO136
 135  M1=7
      N1=9
      GCTO132
 13C  DC 137 I=4,6
      DC 138 K=4,6
      V(I,4,K)= CCYSK(I,1,K)*ZSJ(1)+CDYSK(I,2,K)*ZSJ(2)+DDYSK(I,3,K)*ZSJ
     *(3)
      V(I+6,4,K)=CYSK(1,K)*CZSJ(1,I)+CYSK(2,K)*DZSJ(2,I)+DYSK(3,K)*CZSJ(
     *3,I)
      V(I,4,K+6)= CZSJ(1,K)*CYSK(1,I)+DZSJ(2,K)*DYSK(2,I)+DZSJ(3,K)*DYSK
     *(3,I)
      V(I+6,4,K+6)=CCZSJ(I,1,K)*YSK(1)+DCZSJ(I,2,K)*YSK(2)+DDZSJ(I,3,K)*
     *YSK(3)
 13E  CCNTINUE
 137  CCNTINUE
      RETURN
      ENC
CF FILE
```

IG