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# OPTIMIZATION OF RECTANGULAR PLANE FRAMES
# USING
# BOX'S COMPLEX METHOD

by

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# ABSTRACT

This study is concerned with the minimum cost design of a multi-storey building. The building consists of rectangular steel plane frames which are evenly spaced and support identical floors and a roof. The frame spacing, the column positions within the frame, and the number of intermediate beams spanning between frames are optimized using Box's Complex method and the optimum solution verified by an exhaustive search procedure. Member sizes for the frame and floor system are determined by a fully-stressed design criterion, for the AISC code, within the limits of a discrete set of member properties.

The optimum design for several frames with various widths and heights is determined and the influence of the above variables, and the effect of cost parameters on the optimum solution is illustrated and the results discussed.

# TABLE OF CONTENTS

TABLE OF CONTENTS (cont'd)

# LIST OF TABLES

# LIST OF FIGURES

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# Chapter 1

## INTRODUCTION

Structural optimization has grown rapidly in the last 15 years. Two avenues of advancement have emerged:

(1) analytical optimization;

(2) mathematical programming techniques

Automated analysis-design schemes utilizing stress ratio methods and energy criterion were used in analytical optimization. In the second method algorithms and numerical optimization methods were adapted to problems of structural synthesis.

The purpose of this study is to examine rectangular plane frames. A design is defined by a set of parameters, some of which are preassigned while others are to be determined. Many designs are possible which will satisfy the constraints. The design giving the minimum objective function, while satisfying the constraints, is the design sought.

The automation of a scheme to determine a good solution is demonstrated, though no guarantee of finding a local or global minimum is given.

### 1.1 Brief Review of Structural Optimization

Papers by Wasiutynski and Brandt (1), and Sheu and Prager (2) made a comprehensive survey of the state of the art up to 1968. References 3, 4 also provide excellent reviews.

At the beginning of the century Michell dealt with the development of an optimum configuration for "trusses" under certain loads (5). The optimum configuration consisted of an orthogonal network of tension and compression members.

Later Cox (6) and Chan (7) applied the concept of "Michell Structures" to many problems of design optimization.

Using a mode of simultaneous failure a degree of success was achieved in the optimization of structural components. Shanley (8) and Gerard (9) and other pioneers of structural optimization almost always formulated the problem in terms of equations. This implies that certain constraints will be critical at the optimum design. The fully stresses design is an extension of the simultaneous failure approach (10), and assumes that in the optimum structure every component is subject to maximum allowable stress under at least one load condition.

Klein (11) showed that some minimum weight structural design problems could be viewed as mathematical programming problems. Instead of equations, inequalities could be used directly. Schmit (12) demonstrated the feasibility of using together the methods of mathematical programming and matrix structural analysis to provide a continuous automatic process for minimum weight design. He (12) defined structural synthesis "as the rational directed evolution of a structural system which, in terms of a defined objective function, efficiently performs a set of specified functional purposes".

A general class of structural synthesis problems can be defined in proper mathematical form:

Given the preassigned parameters and a set of distinct load conditions, find the vector of design variables $(\overline{x})$, such that the objective function $f(\overline{x})$ is minimized (or maximized) subject to a collection of inequality constraints on the design variables

$$h_j(\overline{x}) \leq 0 \quad ; \quad j = 1, 2, \ldots, m.$$

where the functions $h_j(\overline{x})$ are such that unsatisfactory behaviour of the structure is precluded.

A large class of structural design optimization problems take the form of nonlinear mathematical problems. Moses (13) introduced the idea of suc-

cessive linearizations. A wide variety of techniques are now being used including gradient projection methods, steepest descent methods and direct search methods. These methods have and are being successfully applied to a wide variety of practical structures under different load conditions.

In recent years there has been a renewed interest in the analytical approach to structural optimization. The derivation of optimality criterion for a variety of design conditions has been dealt with extensively (14,15,16).

Sheu and Prager (2) point out that while mathematical programming methods have been successful, the optimal design of complex structures taxes the capacity of present digital computers. On the other hand the analytical treatment of simple problems provides an insight into the analytical nature of optimality criteria.

## 1.2 Summary of Related Work

In 1965 Brown and Ang (17) did an extensive study of the minimum weight design of planar truss and frame structures. The problem was formulated in a standard form as a nonlinear programming problem.

The design variables considered were member section properties and were assumed to be continuous variables. After the optimization, standard structural shapes were selected. The objective function was taken as total weight of the structure. The A.I.S.C. code set limits on stress and displacement for a multiplicity of service load conditions. No configuration variables were considered. They showed that the minimum weight design for a statically indeterminate structure under several load conditions did not necessarily yield a fully stressed design; and suggested the design space was not necessarily convex.

Nakamura and Cornell (18) used a method of successive linearizations to solve the problem of design optimization of rectangular plane frames. Member section properties and configuration were considered as variables. Total struc-

ture weight was taken as the objective function. Constraints were typical A.I.S.C. stress limits. A single load condition only appears to have been used. The solution for the geometry variable converged to almost equal spacing of columns. A conclusion of the study was that the design space was convex.

Cornell (4) pointed out that the accurate elastic analysis methods used by Nakamura and Cornell was time consuming and thus not wholly satisfactory.

Soosaar (19) in his investigation of the cost optimization of topology and geometry for planar frames, used approximate frame design methods such as portal and cantilever methods for wind load, and an inflection point assumption for gravity loads. The frame was thus reduced to a set of statically determinate structures and so minimum weight members were fully stressed. A continuous set of member sizes were assumed, as well as functional relationships among member properties. Constraints on stress followed the A.I.S.C. code. Soosaar's objective function was total cost. The cost function included the cost of member existence (topology cost) and cost due to member weight. The design space was found to be irregular and proved to be sensitive to design changes near the optimum.

A two stage solution technique was used in accordance with an established hierarchy of variables. The topology solution was found first by classical optimization followed by a quadratic programming algorithm to determine the geometry. For any value of topology and geometry variables, the corresponding member sizes were also determined.

Soosaar did not include frame spacing as a variable and ignored the problem of floor system design.

## 1.3 Plan of Development

The structure studied is examined in detail in Chapter 2. Variables relating to the problem are identified and the constraints and objective func-

tion are specified. Outlines of an automated solution and design criteria are presented.

Chapter 3 indicates the various methods of solution of nonlinear constrained optimization. The algorithm to be used is presented and modifications outlined. The studied problem is set up in mathematical programming terms.

The results of the optimization procedure are presented in Chapter 4, and compared to an exhaustive search solution.

Chapter 5 examines the results of design parameter data.

Finally, Chapter 6 presents conclusions and recommendations relating to further research and development.

# Chapter 2

# THE DESIGN PROBLEM

## 2.1 Problem Description

The problem studied is the design for minimum cost of a multi-storey building consisting of a number of evenly spaced rectangular plane frames which support identical floors and a roof. The frames are to be composed of wide flange steel beam and column sections. The same grade of steel is to be used throughout. The floor and roof systems consist of reinforced concrete slabs supported either directly on the frames, or on intermediate beams running between frames.

The structure can be subjected to any one or several of the following load cases:

(i) full live load and full dead load;

(ii) checkerboard live load and full dead load;

(iii) alternate checkerboard live load and full dead load;

(iv) full deal load;

(v) (i) + wind load;

(vi) (ii) + wind load;

(vii) (iii) + wind load;

(viii) (iv) + wind load.

The members are designed by elastic theory in accordance with a slightly modified version of the A.I.S.C. 1969 code.

The floor slabs are designed for maximum span and the slab depth is constant throughout the floor.

## 2.2 Indentification of Variables

The parameters necessary to define and design the structure must be either

(1) preassigned, or (2) determined.

(1) Preassigned Parameters

(i) Structure Configuration

$H$, the total height of the structure;

$N_s$, the number of storeys;

$h_i$, the individual storey heights where $i = 1,\ldots N_s$;

$N_b$, the number of bays in the frame;

$L$, the distance between outer columns in the frame;

(ii) Material Properties

$F_y$, yield stress of steel;

$E$, the elastic modulus of the steel;

(iii) Load conditions

(iv) Unit cost parameter values.

(2) Variables to be Determined

(i) $x_1$, frame spacing;

(ii) $x_2$, the number of intermediate beams; one intermediate beam is placed at each column, and the others are evenly spaced between these;

(iii) $x_3,\ldots x_j$, column positions within the frame;

(iv) $x_j+1$, slab thickness;

(v) $x_j+2,\ldots x_k$, member sizes.

## 2.3 Constraints on the Variables

There are two types of constraints to be considered: geometric constraints and behavioural design constraints.

Variables $x_1,\ldots x_j$ are subject to geometric constraints, entered as upper and lower bounds. The remaining variables are governed by behavioural design constraints. These constraints - stress limits - are implicit, and cannot be

written as explicit functions of the design variables for this problem.

## 2.4 Objective Function

The objective function represents a basis for choice between acceptable alternative designs, and is a measure of the optimality of the structure. Since weight is easily quantified it is often taken as the objective function of structural design optimization. In general, cost and not weight, is the measure of a good civil engineering design.

The total weight or cost of the frame itself is often taken as the objective function (17,18,19). This type of objective function does not provide for a satisfactory comparison of structures with different configuration. A unit weight or unit cost as objective function does, and so the design of the floor system becomes a part of the problem.

The minimization of the cost per square foot of the total structural system is the objective of this study. The cost per square foot is given by

$$C = \frac{1}{Lx_1N_s}\{C_1W_1 + (C_2+C_3)N_c + (C_4+C_9)W_2 + C_4W_3 + 2C_5(N_g+x_2) + 2C_6Hx_1 + C_7LN_sx_1 + C_8Lx_1D + C_{10}(N_b+1)\} \qquad (2.1)$$

The cost parameters included in the objective function are:

$C_1$, cost per lb. of column material;

$C_2$, cost per column for preparation;

$C_3$, cost per column for splicing;

$C_4$, cost per lb. of beam material;

$C_5$, cost per end for beam preparation;

$C_6$, cost per sq. ft. of wall cladding;

$C_7$, cost per sq. ft. for formwork;

$C_8$, cost per sq. ft. per inch of thickness for in-situ reinforced concrete floors;

$C_9$, cost per lb. of beam material for beam joints;

$C_{10}$, cost per column footing.

The other parameters included in the objective function are:

$W_1$, weight of column material per frame;

$W_2$, weight of beam material per frame;

$W_3$, weight of beam material in floor system per bay.

$N_c$, the number of columns per frame;

$N_g$, the number of beams per frame;

D, the summation of slab depth over all floors per bay.

The cost model chosen for this study reflects the most important cost items in a design, but the model is easily expanded to include other costs. It is not intended to be a model of present pricing methods. Cost functions including material costs and fabrication costs are difficult to obtain. The costs associated with design and construction are only part of the overall cost which would usually include maintenance costs, insurance costs and many others.

Estimating costs for design and construction is difficult. The cost of labour, job location and market conditions are but a few of the variables involved. Even with a final total cost for a structure it is difficult to separate purely structural costs from electrical, mechanical and other costs.

C.I.S.C. (20) in a recent publication points out that the structural frame of a building costs between 10-20% of the total construction costs; the mechanical and electrical sections vary between 20-50%; and so if 10% more on the frame saves 10% of the mechanical the building is cheaper.

## 2.5 Automated Solution

The solution is divided into two parts;

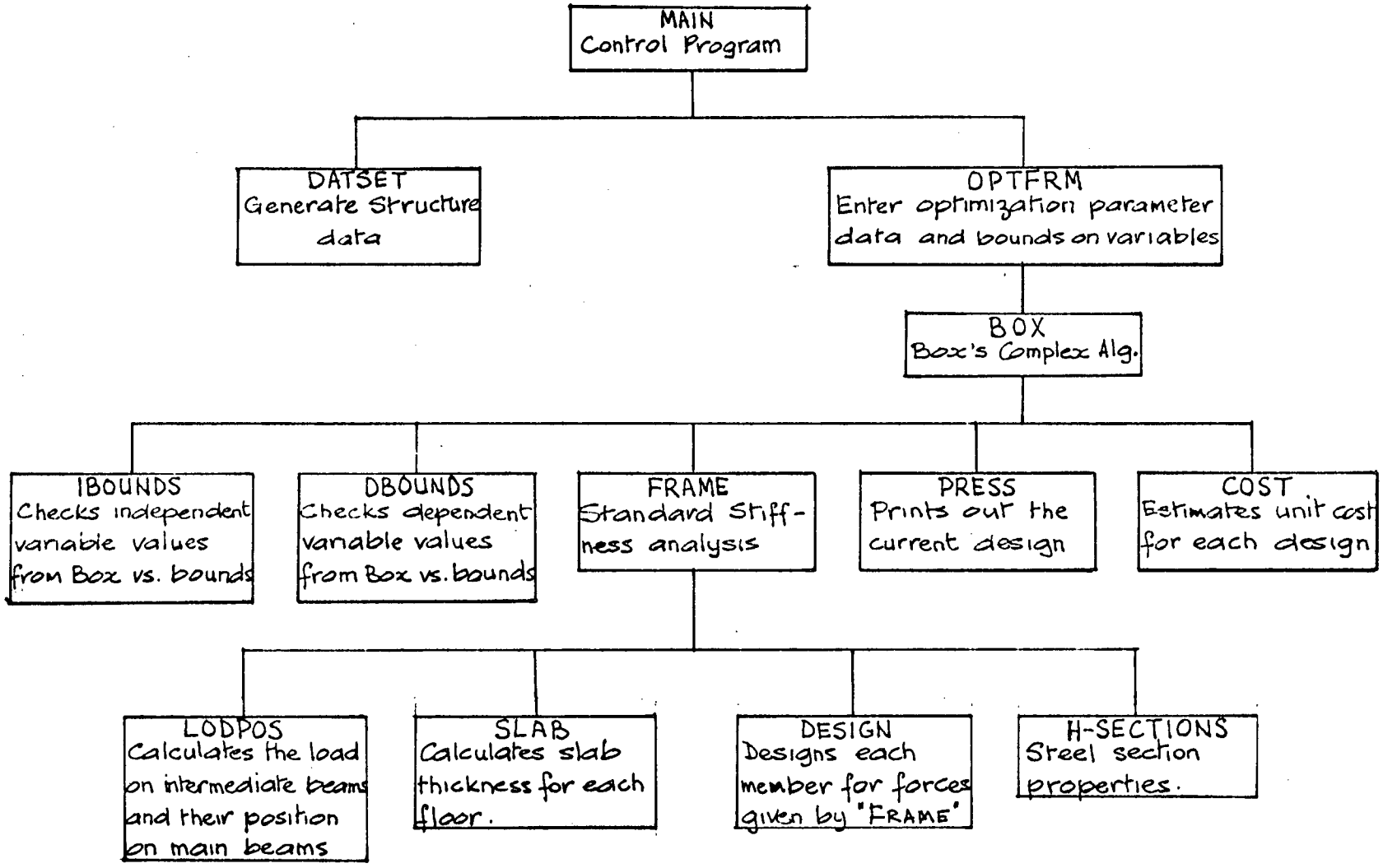



Figure 2.1 Modular Components of Program

(1) variables relating to geometry and topology are optimized using Box's Complex method;

(2) variables relating to the frame and floor system are determined by a fully stressed design criterion, within the limits of a discrete set of member properties.

A computer programme for the solution of the problem was developed. It was written in modular form and so any particular subroutine can be easily replaced by another to solve a slightly different problem. The modular form also facilitates the solution of analysis-design problems.

Figure 2.1 illustrates the main components of the programme.

The number of independent geometry variables may be reduced by specifying geometric symmetry of the structure. Column positions to the left of centre are then included as independent variables while those to the right are mirrored in automatically.

The analysis of a statically indeterminate structure requires that member properties - area, and moment of inertia - be known. In the automated scheme an initial structure is set up with all section properties for all members being equal - areas are arbitrarily set equal to 10 sq. in. and moments of inertia to 30 $in^4$.

The geometry and topology variables, determined in Box's Complex method, define a configuration for the structure. The structure is then analysed and designed for a fully stressed condition. Finally the procedure will yield a combination of geometry, topology, slab depths and member sizes which give an optimum cost.

To evaluate the objective function a complete analysis and design is made. This is by far the most time consuming portion of the programme and so it is desirable to reduce the number of function evaluations. This will be discussed in Chapter 4 in relation to the optimum number of analysis-design cycles.

## 2.6 Design Criteria

The design philosophy adopted is a deterministic one. Although used in common practice, Schmit (4) points out that in view of the uncertainties with respect to load levels and yield strengths, it would be more rational to treat these quantities as random variables. This implies the use of a probability based design philosophy.

The reinforced concrete slabs are designed using ultimate strength design methods for full dead and live loads. Each floor is assumed to consist of a continuous one way slab of constant depth. A yield stress of 60,000 psi is used for the reinforcement and the cylinder strength of concrete at 28 days is taken as 3,000 psi. The percentage of main steel reinforcement is arbitrarily chosen as 0.67%.

The frame members are designed by elastic analysis methods in accordance with the A.I.S.C. 1969 code, with the exception of the interaction formula for members subject to bending and compression. To simplify the automatic design process the interaction formula was taken as

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0 \qquad (2.2)$$

where, $f_a$ = axial compressive stress in member;

$F_a$ = allowable compressive stress;

$f_b$ = bending stress in member;

$F_b$ = allowable bending stress.

Equation (2.2) is used in the code only when the ratio of axial to allowable compressive stress is less than, or equal to 0.15. When this ratio is exceeded the section chosen for the member must satisfy similar formulae, one of which includes an amplification factor.

When the slab is supported directly by the frame intermediate beams are

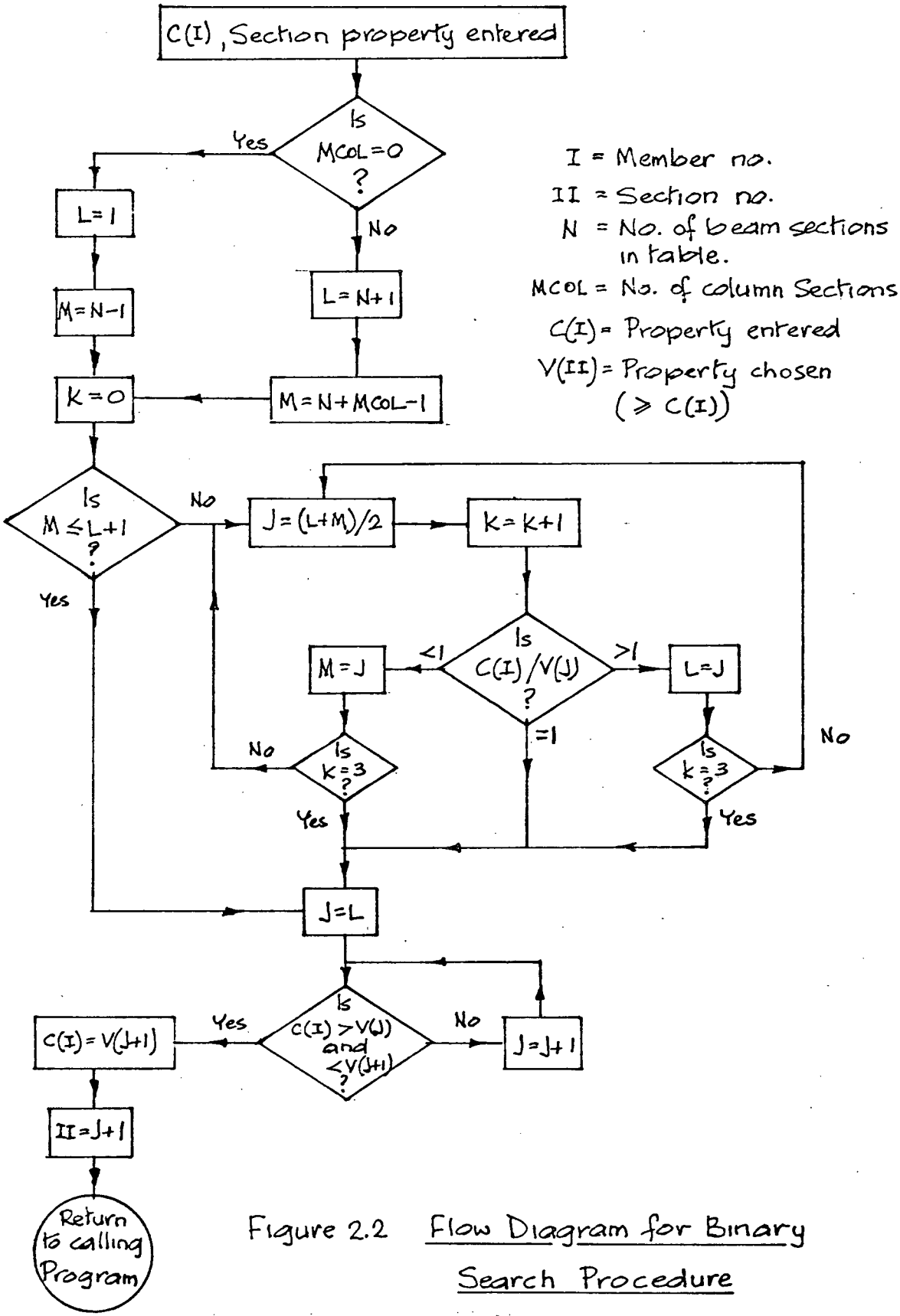



Figure 2.2 Flow Diagram for Binary Search Procedure

acting as tie beams between the frames and are provided only at the columns. Their moments of inertia about the major axis are made equal to the moments of inertia about the minor axis of the columns immediately below them. When the slab is carried on the intermediate beams they are designed as simply supported bending members.

The design is automated to handle members subject to any of the following:

(i) tension;

(ii) compression;

(iii) bending;

(iv) tension and bending;

(v) compression and bending.

The flow diagram for each of these conditions is given in Appendix A.

For each of these conditions a member property is calculated and a section is chosen from a table.

The table consists of wide flanged beam and column sections. A list of sections included is provided in Appendix B.

The section property chosen will be equal to or greater than the property required. Because of this the tables are arranged in ascending order of values for area, section modulus and as far as possible moment of inertia. It follows from this that the tables are arranged in ascending order of weight also. Therefore the section whose property is equal to or greater than the calculated property is the section of least weight in the table capable of sustaining the forces applied.

A binary search procedure is facilitated by this arrangement of the table. Figure 2.2 shows the flow diagram for the binary search method. Three iterations are permitted thus reducing by a factor of eight the number of sections to be examined.

It should be noted that member weight is not considered in the analysis.

# Chapter 3

# OPTIMIZATION PROBLEM

## 3.1 Optimization Solution Technique

The type of optimization problem that presents itself in structural engineering is a nonlinear one. By nonlinear is meantthat the objective function or the constraints are nonlinear. Generally both are so in structures.

Objective functions and constraints that are both linear give rise to what are called linear programming problems. A first approximation to a nonlinear problem might be to apply successive linearizations. Kelley (21) and Moses (13) used this technique. Linear programming methods, which are well developed in operations research, can then be used. However in problems with pronounced nonlinearities these methods may or may not work well.

Most numerical optimization techniques may be considered broadly as iterative methods. Such methods require an initial point to be specified.

There are two kinds of iterative methods - iterative direct search methods and gradient methods.

Iterative direct search methods do not require partial derivatives of the objective function with respect to the variables, but depend on previous values of the objective function and information from earlier iterations. There are two classes: (i) sequential; (ii) linear. Sequential methods use a set number of specified or random points in the space of the independent variables to locate an improved point. Box's Complex method falls into this category (22,23,24). Direction vectors are used in the linear method throughout the search. Having set out in one direction the results obtained will govern which way to go next (25,26).

The other type of iterative method was the gradient method. This method

requires partial derivatives of the objective function with respect to the variables to be calculated. The partial derivatives are usually first order but may be of higher order if desired.

Box, Davies and Swann (22) point out that the use of numerical differentiation in the gradient method will result in the selection of a poorer direction than would be obtained using analytical differentiation; that gradient methods will require extra function evaluations near the current point; and they further suggest that such methods are inferior to the best direct search methods, with the exception of the method of Stewart, 1967 (27). Stewart used a modified steepest descent method with step sizes for differencing chosen to approximately balance off the effects of truncation error in the differential approximation and error due to cancellation of significant figures.

There are also tabular direct search methods which divide the region between constraints into a grid and evaluate the objective function at each node. The coordinates of the node giving the least value of the objective function are then considered to be the variables necessary to produce a minimum.

Instead of dividing the region into a grid, a random search could be used to examine a large number of points and the point which gives the least value is said to be the minimum (22).

## 3.2 Box's Complex Method

Spendley, Hext and Himsworth devised a "Simplex" method for unconstrained minimization in 1962(23). In 1965 M.J. Box modified the method to find constrained minima. The modified method was called the "Complex" method.

An illustration of how the simplex method works follows. Figures 3.1 show how three equidistant initial points in a two variable space can, by replacing the point with the worst value of the objective function by a point with a better value, eventually approach the minimum point "A". It should be noted that the three points in the complex are equidistant throughout the

procedure; thus the last three points 7, 9, 10 will rotate about "A", never actually hitting it. This method indicates how an approach is made in a direct search procedure.

The procedure for the complex method may be viewed as:

(i) choosing an initial point;

(ii) generating a complex of points;

(iii) successively replacing the worst point in the complex with a better point;

(iv) applying a convergence criterion or criteria.

A feasible initial point must be provided with n coordinates, where n is the number of explicit variables.

Constraints are also provided and are inequalities of the form

$$\ell_i \leq x_i \leq u_i, \qquad i = 1,2...m$$

where m is the total number of variables, explicit and implicit; and $\ell_i$ and $u_i$, the lower and upper bounds respectively for the ith variable, are either constants or functions of the explicit functions; and where the implicit variables

$$x_{n+1}, x_{n+2}, \ldots x_m$$

are functions of the explicit variables

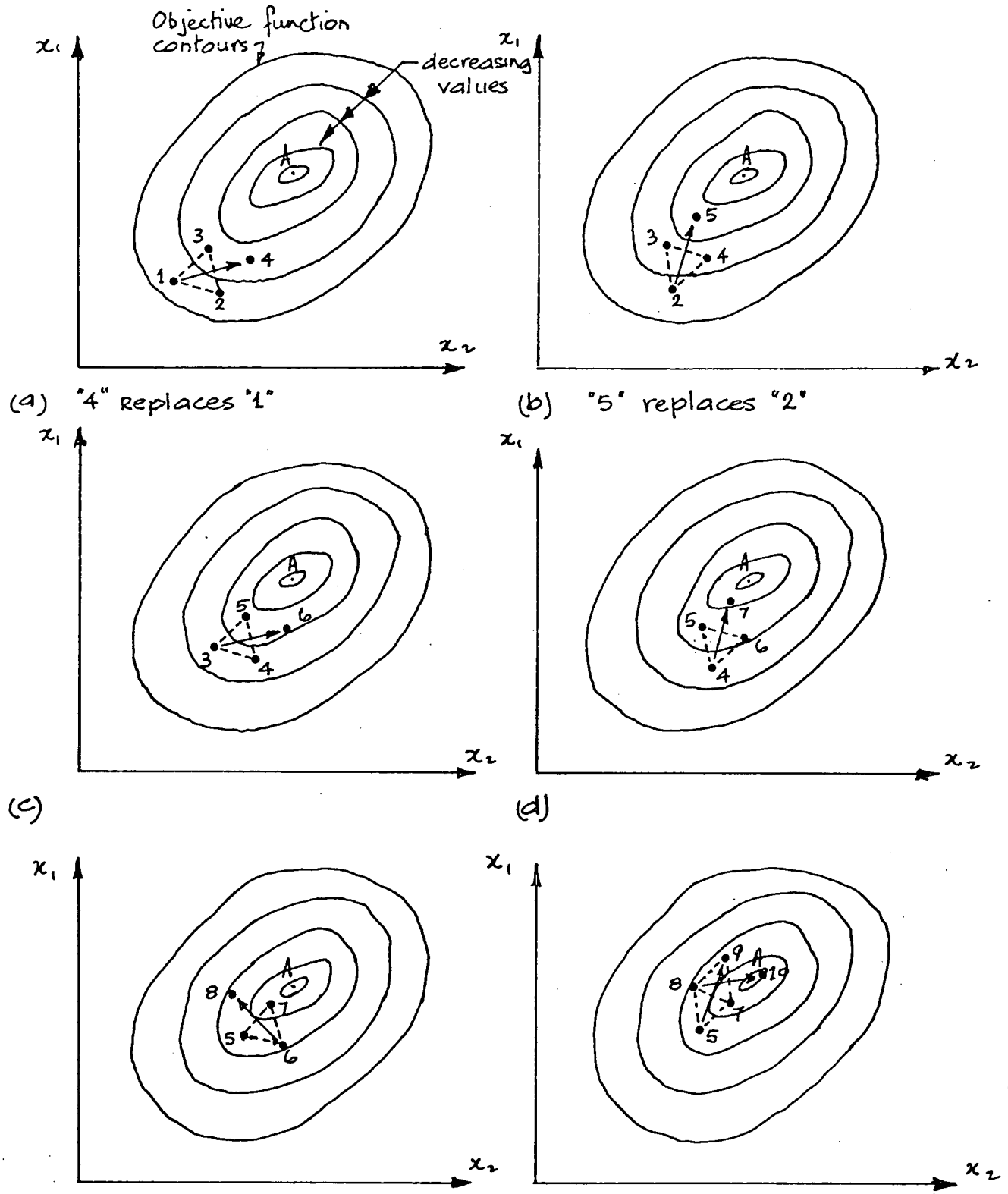
$$x_1, x_2, \ldots x_n.$$

The complex consist of k points where $k \geq n+1$. The initial point is provided and the remaining (k-1) points in the complex are generated so that each point has coordinates

$$x_i = \ell_i + r_i(u_i - \ell_i), \quad i = 1,2,...n$$

where $r_i$ are psuedo-random deviates rectangularly distributed in the interval (0,1).

This point must necessarily satisfy the explicit constraints but may not satisfy the implicit constraints. If an implicit constraint is violated, the centroid of all points generated so far is located and the point violating the




(a) "4" replaces "1"

(b) "5" replaces "2"

(c)

(d)

(e) "8" replaces "6"; "8" is now greatest, but cannot be replaced

(f) So next greatest, "5", is replaced instead.

Figure 3.1 Simplex Method with Two Variables

implicit constraint is moved halfway towards the centroid. By repeating this process a point is found inside all bounds.

Finally all k points in the complex will be located. The objective function is evaluated for each point and the point with the greatest value identified. The centroid of all other points is located and the worst point (one with greatest value) is replaced by a point which is on the produced line joining the centroid and the worst point, and which is $\alpha$ times as far from the centroid as the worst point.

If the new point satisfies all the constraints then the objective function is evaluated there. However if the new point does not satisfy an explicit constraint, it is placed inside the violated bound and the objective function evaluated. If an implicit constraint is violated or if the value of the objective function at the new point is not an improvement on the value at the replaced point, then the point is moved halfway to the centroid and the process repeated.

This process of improving the complex continues, with the worst point being replaced by a better point each time. Eventually the method converges to an area of the complex in which further improvement is not possible. The method is then considered to have found a minimum - at least a local one.

Box has suggested (28) that appropriate values for $\alpha$ and k might be 1.3 and 2n respectively. When $k \geq n+1$, it ensures the complex will not collapse prematurely into a subspace giving a local minimum. The over-reflection factor $\alpha$ greater than one provides the method with a system of expansion, while the move halfway towards the centroid allows for the complex to contract.

The method for finding the initial complex ensures that the problem is scaled to the order of each variable, and the use of the psuedo-random numbers gives a set of points which are sufficiently different from each other.

## 3.3 Modifications of the Complex Method

The automated solution substantially follows the method proposed by Box,

with a few minor modifications.

(i) The choice of an alternate method of finding an initial complex is included. It is a non-random method proposed by Mitchell and Kaplan (29). The point $P_{2j}$ is put equal to the initial point $P_1$, except for the $j^{th}$ coordinate which is put equal to the lower bound; likewise the point $P_{2j+1}$ is equal to $P_1$ except at the $j^{th}$ coordinate which equals the upper bound.

If any point thus generated fails to satisfy the implicit constraints then the $j^{th}$ coordinate is relocated halfway towards the initial coordinate. This process is repeated until ultimately the initial complex is formed.

This method for generating an initial complex ensures a variety of points in different parts of the complex. They suggest that "a good initial point leads to a good initial complex" thus improving the rate of convergence to a minimum.

It should be observed that the facility of changing the initial complex by changing the psuedo-random number initializer is removed and a good new initial point must be found in order to check out previous minima.

For the problem studied the non-random method gave slightly worse results than the random method.

(ii) The other modification involves the reflected point. If this point does not satisfy all the implicit constraints or if the value of the objective function has not improved then the point will be moved halfway towards the centroid three times, if necessary. If the point is still unsatisfactory it is moved halfway to the centroid on the other side. This is done twice. Should the point still be unsatisfactory the centroid is used as the point. If this fails, the best point is used as an initial point and a new complex generated, repeating the complete procedure.

This allows the method to continue even though the complex may be stuck in a subspace. By keeping the best point as the initial point in the new com-

plex, the possibility of improvement is greatly enhanced and the complex is once more well distributed throughout the design space.

An upper and lower limit is specified for the ratio of standard deviation to the best point. If the ratio falls between these bounds a restart is possible; however if a specified number of iteration has been completed there will be no new complex generated and the method will continue until some convergence criterion is reached.

## 3.4 Convergence Criteria

There are five suggested criteria by which the complex method may terminate. Convergence is assumed if:

(i) the coordinates of the centroid do not change by some specified amount, five iterations in succession; or,

(ii) the function value of the centroid does not vary by some specified amount, five iterations in succession; or,

(iii) the function value of the best point does not change by a specified percentage of the previous best point value, five iterations in a row; or,

(iv) the standard deviation of the function values of all the points in the complex divided by the best value is less than or equal to a specified amount.

(v) The method is arranged to terminate if the maximum permitted number of iterations is reached.

Box (22) recommended use of criterion (iv).

The first criterion generally governs when the curvature of the objective function is slight near the minimum. The design space studied in this problem is generally very flat and so this terminating criterion is likely.

When the function value at the centroid does not change, the space considered has, in general, pronounced curvature near the minimum.

The third criterion - failure to improve the function value of the best point - will occur when the design is locked into a subspace containing either a global or local minimum. A flat space will also keep this criterion active, while a highly irregular one is not likely to, except in the case of the method becoming trapped in a subspace.

When the standard deviation criterion governs, the complex has contracted into one area of the space and is approaching a minimum, either local or global.

The final criterion is the safety valve of the system, ensuring termination when the method shows no sign of converging.

The fourth criterion relating to the standard deviation is the most useful in that it clearly indicates the rate of convergence and the regularity of the objective function.

A different criterion will govern for different problems and there is no guarantee that any particular criterion will govern in any particular problem. All five criteria are possible terminating criteria for any problem, though the probability of any one terminating the procedure can be decreased by specifying a low value for the corresponding control parameter.

# Chapter 4

# RESULTS

## 4.1 Presentation of Results of Optimization Scheme

Several problems were considered in this study to examine the following factors in the optimization scheme: (i) the influence of the number of analysis-design cycles; (ii) the influence of the number and type of variables; (iii) the uniqueness and convergence rate of the optimization solution; (iv) the effect of variation in the cost model; (v) the capabilities of the algorithm.

Sixteen basic structures were used and are described in Table I. Figure 4.1 shows the basic structure parameters. Optimization results are shown in Tables II and III.

There were 49 optimization computer runs. All convergence criteria were included. The standard deviation criterion was quite severe for structures 1-15 (0.0005), but was relaxed to 0.005 for structure 16. Convergence was achieved 7 times - 5 for the standard deviation criterion, and twice because of failure to improve the objective function of the best point. It should be noted that 16 other runs would have "converged" if the standard deviation criterion for structure 16 had been applied throughout.

The solution given by the complex method for structure 1 is compared in Figure 4.2 to that found from an exhaustive search conducted in the design space with a coarse grid.

Frame spacing variable, $x_1$, was increased by an increment of 10 ft. from a lower bound of 10 ft. to an upper bound of 40 ft. The number of intermediate beams, $x_2$, was chosen for the grid so that beams were 5, 10 and 15 ft. apart. Later beams at 6 and 12 ft. centres were included, and a further set of values found for $x_1$ = 25 ft. When the search indicated an area in which the minimum was located, a design was evaluated there for $x_1$ = 25 ft. and $x_2$ = 8.

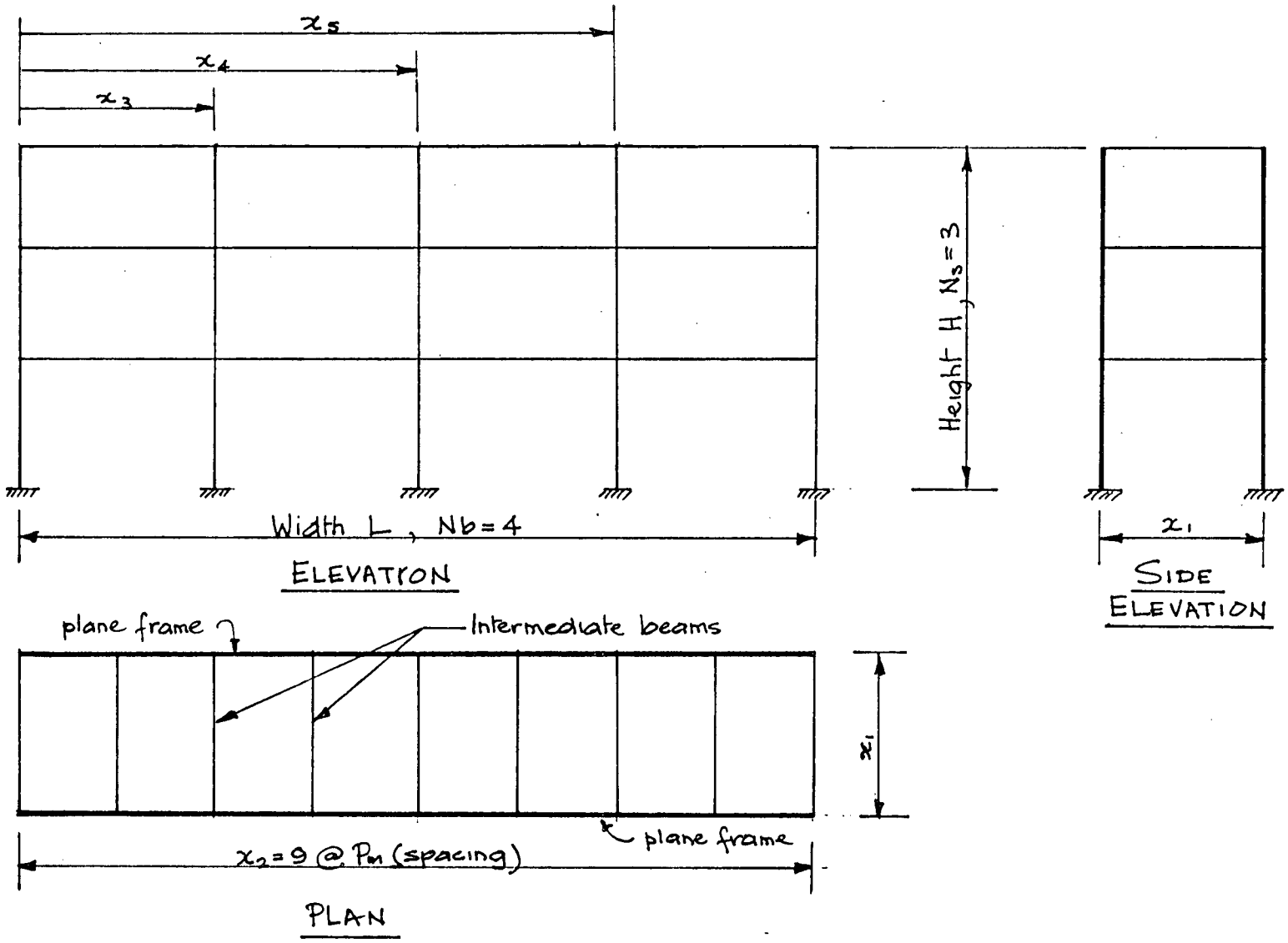



Figure 4.1 Basic Structure Showing Variables

TABLE I

Problem Parameters

| Str. No. | Width | $N_b$ | Sym | Ht. | $N_s$ | $x_1$ | $x_2$ | NLC | Load Types | Floor L.L | Floor D.L | Roof L.L | Roof D.L | Yield Stress | $N_d$ | ACTIVE VARIABLES $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | REMARKS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 60. | 1 | ✓ | 12. | 1 | - | - | 1 | 1 | 0.100 | 0.025 | — | | 36 | 2 | ✓ | ✓ | - | - | - | |
| 2 | 60. | 1 | ✓ | 24. | 2 | - | - | 1 | 1 | 0.100 | 0.025 | 0.100 | 0.025 | 36 | 2 | ✓ | ✓ | - | - | - | |
| 3 | 60. | 2 | ✓ | 24. | 2 | - | - | 1 | 1 | 0.040 | 0.030 | 0.040 | 0.030 | 33 | 1 | ✓ | ✓ | ✓ | - | - | |
| 4 | 60. | 3 | ✓ | 12. | 1 | - | - | 2 | 1,2 | 0.100 | 0.025 | — | | 36 | - | ✓ | ✓ | ✓ | - | - | |
| 5 | 60. | 2 | ✓ | 12. | 1 | - | - | 2 | 1,2 | 0.060 | 0.030 | — | | 36 | 3 | ✓ | ✓ | - | - | - | |
| 6 | 60. | 3 | ✓ | 30. | 2 | - | 7 | 3 | 1,2,3 | 0.060 | 0.030 | 0.060 | 0.030 | 36 | 3 | ✓ | - | ✓ | - | - | |
| 7 | 60. | - | ✓ | 210. | 17 | - | - | 3 | 1,2,8 | 0.060 | 0.030 | 0.030 | 0.020 | 36 | 1 | ✓ | ✓ | ✓ | ✓ | ✓ | $N_b$ = 1,2,3 |
| 8 | 90. | 3 | ✓ | 24. | 2 | - | - | 2 | 1,5 | 0.075 | 0.040 | 0.040 | 0.020 | 33 | 1 | ✓ | ✓ | ✓ | - | - | |
| 9 | 90. | 3 | - | 24. | 2 | - | - | 1 | 1 | 0.075 | 0.040 | 0.040 | 0.020 | 33 | 1 | ✓ | ✓ | - | - | - | |
| 10 | 90. | 3 | - | 24. | 2 | - | 4 | 1 | 5 | 0.150 | 0.060 | 0.100 | 0.050 | 33 | 1 | ✓ | - | ✓ | ✓ | - | |
| 11 | 90. | 3 | ✓ | 12. | 1 | - | - | 2 | 1,2 | 0.100 | 0.025 | — | | 36 | 2 | ✓ | ✓ | ✓ | - | - | |
| 12 | 80. | - | ✓ | 42. | 3 | - | - | 3 | 1,2,8 | 0.060 | 0.030 | 0.030 | 0.020 | 36 | 1 | ✓ | ✓ | ✓ | ✓ | - | $N_b$ = 2,3,4 |
| 13 | 80. | 3 | ✓ | 42. | 3 | - | - | 3 | 1,2,8 | 0.060 | 0.030 | 0.030 | 0.020 | 36 | 1 | ✓ | ✓ | ✓ | - | - | |
| 14 | 108. | - | ✓ | 78. | 6 | - | - | 3 | 1,2,8 | 0.060 | 0.030 | 0.030 | 0.020 | 36 | 1 | ✓ | ✓ | ✓ | ✓ | ✓ | $N_b$ = 3,4,5,...9 |
| 15 | 60. | 1 | ✓ | 12. | 1 | - | - | 1 | 1 | 0.060 | 0.030 | — | | 36 | 3 | ✓ | ✓ | - | - | - | |
| 16 | 150. | 5 | ✓ | 15. | 1 | 20 | 21 | 3 | 1,2,3 | 0.060 | 0.030 | — | | 36 | 2 | - | - | ✓ | ✓ | - | |

NOTE: $N_b$ = No. of bays in frame; $N_s$ = No. of storeys; NLC = No. of load conditions; $N_d$ = No. of analysis-design cycles; L.L = live load; D.L. = dead load; Sym = symmetry.

TABLE II

Parameters for Optimum Problems

(see bottom of table for definition of terms)

| Str. No. | No. Variables | No. Points in Complex | BOUNDS ON VARIABLES: Frame Spacing, $x_1$ L | Frame Spacing, $x_1$ u | No. of Interm. Beams, $x_2$ L | No. of Interm. Beams, $x_2$ u | Column Posn., $x_3$ L | Column Posn., $x_3$ u | Column Posn., $x_4$ L | Column Posn., $x_4$ u | Objective Function Type | REMARKS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 a | 2 | 4 | 10. | 40. | 4 | 11 | - | - | - | - | 1 | R=2 |
| b | 1 | 2 | 10. | 40. | - | - | - | - | - | - | 1 | R=2 |
| 2 a | 2 | 4 | 10. | 40. | 4 | 11 | - | - | - | - | 1 | R=2 |
| 3 a | 3 | 6 | 10. | 30. | 7 | 13 | 15. | 45. | - | - | 1 | R=2 |
| 4 a | 3 | 6 | 10. | 40. | 4 | 11 | 10. | 25. | - | - | 1 | R=2, $N_d$=2 |
| b | 3 | 6 | 10. | 40. | 4 | 11 | 10. | 25. | - | - | 1 | R=8, $N_d$=2 |
| c | 3 | 6 | 10. | 40. | 4 | 11 | 10. | 25. | - | - | 1 | R=2, $N_d$=1 |
| d | 3 | 6 | 10. | 40. | 4 | 11 | 10. | 25. | - | - | 1 | R=2, $N_d$=2 |
| e | 3 | 6 | 10. | 40. | 4 | 11 | 10. | 25. | - | - | 1 | R=2, $N_d$=3 |
| 5 a | 2 | 4 | 8. | 50. | 3 | 16 | - | - | - | - | 1 | R=2 |
| b | 2 | 4 | 8. | 50. | 3 | 16 | - | - | - | - | 1 | Non-Random |
| 6 a | 2 | 4 | 8. | 50. | - | - | 5. | 27.5 | - | - | 1 | Cost |
| b | 2 | 4 | 8. | 50. | - | - | 5. | 27.5 | - | - | 2 | Material Costs |
| 7 a | 3 | 6 | 10. | 40. | 5 | 13 | 10. | 25. | - | - | 1 | $N_b$=3 |
| b | 2 | 4 | 10. | 40. | 5 | 13 | - | - | - | - | 1 | $N_b$=2 |
| c | 2 | 4 | 10. | 40. | 5 | 13 | - | - | - | - | 1 | $N_b$=1 |
| 8 a | 3 | 6 | 10. | 25. | 12 | 23 | 15. | 37.5 | - | - | 1 | |
| 9 a | 4 | 8 | 10. | 25. | 12 | 23 | 15. | | | 75. | 1 | |



cont'd

TABLE II (cont'd)

| Str. No. | No. Vari-ables | No. Points in Complex | BOUNDS ON VARIABLES | | | | | | | | Objective Function Type | REMARKS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Frame Spacing, $x_1$ | | No. of Interm. Beams, $x_2$ | | Column Posn., $x_3$ | | Column Posn., $x_4$ | | | |
| | | | L | u | L | u | L | u | L | u | | |
| 10 a | 3 | 6 | 10. | 25. | - | - | 15. | | | 75. | 3 | 4 interm. beams |
| 11 a | 3 | 6 | 10. | 40. | 6 | 16 | 10. | 40. | - | - | 4 | High Labour Costs |
| b | 3 | 6 | 10. | 40. | 6 | 16 | 10. | 40. | - | - | 5 | Low Labour Costs |
| c | 3 | 6 | 10. | 40. | 6 | 16 | 10. | 40. | - | - | 6 | Weight |
| d | 3 | 6 | 10. | 40. | 6 | 16 | 10. | 40. | - | - | 1 | R=2, $N_d$=2 |
| e | 3 | 6 | 10. | 40. | 6 | 16 | 10. | 40. | - | - | 1 | R=8, $N_d$=2 |
| f | 3 | 6 | 10. | 40. | 6 | 16 | 10. | 40. | - | - | 1 | R=2, $N_d$=2 |
| g | 3 | 6 | 10. | 40. | 6 | 16 | 10. | 40. | - | - | 1 | R=2, $N_d$=1 |
| h | 3 | 6 | 10. | 40. | 6 | 16 | 10. | 40. | - | - | 1 | R=2, $N_d$=3 |
| 12 a | 3 | 6 | 10. | 40. | 6 | 21 | 10. | 30. | - | - | 1 | $N_b$=4, $N_d$=1 |
| b | 3 | 6 | 10. | 40. | 6 | 21 | 10. | 35. | - | - | 1 | $N_b$=3, $N_d$=1 |
| c | 2 | 4 | 10. | 40. | 6 | 21 | - | - | - | - | 1 | $N_b$=2, $N_d$=1 |
| 13 a | 1 | 4 | - | - | - | - | 10. | 35. | - | - | 1 | $x_1$=10, $x_2$=10 |
| b | 3 | 6 | 10. | 40. | 6 | 21 | 10. | 35. | - | - | 1 | |
| c | 2 | 4 | - | - | 6 | 21 | 10. | 35. | - | - | 1 | $x_1$=20 |
| d | 2 | 4 | 10. | 40. | - | - | 10. | 35. | - | - | 1 | $x_2$=10 |
| e | 1 | 2 | - | - | - | - | 10. | 35. | - | - | 1 | $x_1$=20, $x_2$=10 |

.....cont'd

TABLE II (cont'd)

| Str. No. | No. Vari-ables | No. Points in Complex | BOUNDS ON VARIABLES | | | | | | | | Objective Function Type | REMARKS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Frame Spacing, $x_1$ | | No. of Interm. Beams, $x_2$ | | Column Posn., $x_3$ | | Column Posn., $x_4$ | | | |
| | | | L | u | L | u | L | u | L | u | | |
| 14 a | 6 | 12 | 10. | 40. | 8 | 28 | | | | | 1 | $N_b$=9, $N_d$=1 |
| b | 5 | 10 | 10. | 40. | 8 | 28 | | | | | 1 | $N_b$=8, $N_d$=1 |
| c | 5 | 10 | 10. | 40. | 8 | 28 | | | | | 1 | $N_b$=7, $N_d$=1 |
| d | 4 | 8 | 10. | 40. | 8 | 28 | | | | | 1 | $N_b$=6, $N_d$=1 |
| e | 4 | 8 | 10. | 40. | 8 | 28 | | | | | 1 | $N_b$=5, $N_d$=1 |
| f | 3 | 6 | 10. | 40. | 8 | 28 | | | | | 1 | $N_b$=4, $N_d$=1 |
| g | 3 | 6 | 10. | 40. | 8 | 28 | | | | | 1 | $N_b$=3, $N_d$=1 |
| 15 a | 2 | 4 | 8. | 50. | 3 | 16 | - | - | - | - | 1 | $N_d$=3 |
| b | 2 | 4 | 8. | 50. | 3 | 16 | - | - | - | - | 2 | $N_d$=3, 6 restarts |
| 16 a | 2 | 4 | - | - | - | - | | | | | 1 | 2 restarts |
| b | 2 | 4 | - | - | - | - | | | | | 1 | 3 restarts |
| c | 2 | 4 | - | - | - | - | | | | | 1 | 2 restarts |
| d | 2 | 4 | - | - | - | - | | | | | 1 | 1 restart |
| e | 2 | 4 | - | - | - | - | | | | | 1 | 3 restarts |

Note: R = Random no. generator (=2, unless otherwise stated);

$N_d$ = no. of analysis-design cycles (=2, unless otherwise stated).

L = lower bound; u = upper bound.

TABLE III

Results of Optimization Problems

(see bottom of table for definition of terms)

| Str. No. | INITIAL DESIGN | | | | | | FINAL DESIGN | | | | | | Time (sec) | NI | NE | S.D. Best Value | Terminat$^{\underline{n}}$ Criterion |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | Obj.Fun. | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | Obj.Fun. | | | | | |
| 1 a | 11. | 7 | - | - | - | 5.3266 | 22.086 | 9 | - | - | - | 4.5247 | 85 | 26 | 56 | ≤0.0005 | ← |
| b | 11. | - | - | - | - | 5.3266 | 22.162 | - | - | - | - | 4.7411 | 142 | 26 | 112 | " | ← |
| 2 a | 11. | 7 | - | - | - | 5.0255 | 33.23 | 10 | - | - | - | 4.2220 | 149 | 20 | 50 | " | ← |
| 3 a | 25. | 9 | 25. | - | - | 3.6933 | 23.110 | 9 | 24.666 | - | - | 3.6200 | 100 | 11 | 32 | 0.0030 | GTE |
| 4 a | 39.9 | 5 | 15. | - | - | 4.5240 | 37.666 | 9 | 22.280 | - | - | 3.8816 | 300 | 14 | - | 0.00157 | GTE |
| b | 11. | 7 | 20. | - | - | 4.4969 | 30.770 | 8 | 16.962 | - | - | 3.8957 | 300 | 18 | 27 | 0.0228 | GTE |
| c | 11. | 7 | 20. | - | - | 4.4969 | 26.600 | 9 | 22.600 | - | - | 3.8090 | 300 | 46 | 96 | 0.000777 | GTE |
| d | 11. | 7 | 20. | - | - | 4.4969 | 24.998 | 9 | 22.638 | - | - | 3.8535 | 300 | 18 | 26 | 0.0015 | GTE |
| e | 11. | 7 | 20. | - | - | 4.4969 | 26.170 | 9 | 23.318 | - | - | 3.8689 | 300 | 14 | - | 0.0036 | GTE |
| 5 a | 10. | 15 | - | - | - | 4.7431 | 21.637 | 9 | - | - | - | 3.7960 | 181 | 13 | 35 | ≤0.0005 | ← |
| b | 10. | 15 | - | - | - | 4.7431 | 26.277 | 14 | - | - | - | 3.8677 | 300 | 18 | 27 | 0.00161 | GTE |
| 6 a | 11. | - | 10. | | | 4.8085 | 49.995 | - | 17.759 | - | - | 3.8038 | 500 | 11 | - | 0.011 | GTE |
| b | 11. | - | 10. | | | 0.6834 | 13.781 | - | 19.137 | - | - | 0.4501 | 500 | 8 | - | 0.074 | GTE |
| 7 a | 20. | 7 | 20. | | | 4.0633 | 28.200 | 9 | 14.567 | - | - | 4.0021 | 1200 | 5 | 13 | 0.0071 | GTE |
| b | 20. | 7 | - | | | 3.7226 | 21.035 | 8 | - | - | - | 3.6712 | 1200 | 9 | 11 | 0.0032 | GTE |
| c | 20. | 8 | - | | | 4.4586 | 19.264 | 9 | - | - | - | 4.4425 | 1200 | 11 | 34 | 0.0048 | GTE |
| 8 a | 15. | 14 | 35. | | | 3.4525 | 18.377 | 13 | 29.690 | - | - | 3.2785 | 750 | 17 | 26 | 0.0055 | GTE |

.....cont'd

TABLE III (cont'd)

| Str. No. | INITIAL DESIGN | | | | | | FINAL DESIGN | | | | | | Time (sec) | NI | NE | S.D. / Best Value | Terminat$^{n}$ Criterion |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | Obj.Fun. | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | Obj.Fun. | | | | | |
| 9 a | 15. | 15 | 35. | 55. | | 3.4547 | 23.152 | 11 | 21.246 | 51.797 | - | 3.2761 | 450 | 19 | 19 | 0.0067 | GTE |
| 10 a | 15. | - | 20. | 40. | - | 1.3540 | 21.480 | - | 29.081 | 59.810 | - | 1.1675 | 380 | 23 | 53 | - | O.F. constant for 5 iter. |
| 11 a | 11. | 8 | 25. | - | - | 7.0774 | 39.997 | 14 | 28.347 | - | - | 5.2768 | 300 | 15 | 31 | 0.002 | GTE |
| b | 11. | 8 | 25. | - | - | 2.9206 | 22.476 | 14 | 28.722 | - | - | 2.5088 | 300 | 14 | - | 0.001 | GTE |
| c | 11. | 8 | 25. | - | - | 7.9012 | 11.0934 | 8 | 25.515 | - | - | 7.4355 | 300 | 15 | 31 | 0.0074 | GTE |
| d | 11. | 8 | 25. | - | - | 4.3029 | 24.554 | 13 | 29.896 | - | - | 3.4649 | 200 | 15 | 28 | 0.0107 | GTE |
| e | 11. | 8 | 25. | - | - | 4.3029 | 28.996 | 14 | 26.980 | - | - | 3.4863 | 300 | 22 | 26 | 0.0099 | GTE |
| f | 39.9 | 16 | 30. | - | - | 3.6617 | 39.904 | 13 | 29.783 | - | - | 3.5595 | 300 | 19 | 36 | 0.0064 | GTE |
| g | 20. | 10 | 40. | - | - | 3.9048 | 22.073 | 13 | 30.014 | - | - | 3.4312 | 300 | 40 | 52 | 0.0026 | GTE |
| h | 11. | 8 | 25. | - | - | 4.3029 | 21.831 | 13 | 29.420 | - | - | 3.4590 | 300 | 15 | 30 | 0.0111 | GTE |
| 12 a | 20. | 10 | 20. | - | - | 3.4333 | 31.219 | 19 | 14.594 | - | - | 3.3847 | 1200 | 23 | 47 | 0.00067 | GTE |
| b | 20. | 10 | 30. | - | - | 3.5331 | 30.205 | 13 | 27.381 | - | - | 3.3752 | 1200 | 32 | 60 | 0.0008 | GTE |
| c | 20. | 10 | - | - | - | 3.4780 | 39.999 | 15 | - | - | - | 3.4199 | 750 | 17 | 76 | $\leq$0.0005 | ← |
| 13 a | - | - | 15. | - | - | 4.8329 | - | - | 24.354 | - | - | 3.8824 | 300 | 12 | - | 0.0063 | GTE |
| b | 15. | 10 | 15. | - | - | 3.8509 | 27.122 | 18 | 24.034 | - | - | 3.4335 | 300 | 11 | - | 0.0052 | GTE |
| c | - | 10 | 15. | - | - | 3.6727 | - | 15 | 23.087 | - | - | 3.4447 | 300 | 11 | - | 0.0036 | GTE |
| d | 15. | - | 15. | - | - | 4.4695 | 13.173 | - | 23.824 | - | - | 3.6567 | 300 | 14 | - | 0.0096 | GTE |
| e | - | - | 15. | - | - | 4.3091 | - | - | 15. | - | - | 4.3091 | 290 | 7 | 23 | - | O.F. constant for 5 iter. |

.....cont'd

TABLE III (cont'd)

| Str. No. | INITIAL DESIGN | | | | | | FINAL DESIGN | | | | | | Time (sec) | NI | NE | S.D. / Best Value | Terminat$^{\underline{n}}$ Criterion |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | Obj.Fun. | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | Obj.Fun. | | | | | |
| 14 a | 20. | 19 | 13. | 23. | 37.<br>47. | 3.1615 | 22.505 | 20 | 12.927 | 26.022 | 34.886<br>45.149 | 3.1402 | 1200 | 5 | - | 0.0211 | GTE |
| b | 20. | 17 | 15. | 28. | 40. | 3.0844 | 20. | 17 | 15 | 28. | 40. | 3.0844 | 1200 | 5 | 16 | 0.019 | GTE |
| c | 20. | 19 | 13. | 30. | 42. | 3.0973 | 27.057 | 17 | 13.959 | 27.309 | 46.076 | 3.0810 | 1200 | 7 | 16 | 0.013 | GTE |
| d | 20. | 19 | 23. | 35. | - | 3.0783 | 31.793 | 17 | 12.377 | 31.974 | - | 3.0522 | 1200 | 6 | 18 | 0.010 | GTE |
| e | 20. | 17 | 28. | 48. | - | 3.0207 | 20. | 17 | 28. | 48. | - | 3.0207 | 1200 | 5 | 17 | 0.035 | GTE |
| f | 20. | 17 | 34. | - | - | 3.0570 | 28.200 | 15 | 24.134 | - | - | 3.0191 | 1200 | 10 | 15 | 0.0196 | GTE |
| g | 20. | 17 | 40. | - | - | 3.1067 | 24.296 | 19 | 35.868 | - | - | 3.0787 | 1200 | 13 | 27 | 0.00054 | GTE |
| 15 a | 20. | 7 | - | - | - | 4.5259 | 32.310 | 9 | - | - | - | 4.2779 | 114 | 18 | 45 | $\leq$0.0005 | ← |
| b | 20. | 7 | - | - | - | 1.1300 | 21.131 | 9 | - | - | - | 1.0728 | 300 | 41 | 116 | 0.00109 | GTE |
| 16 a | - | - | 30. | 60. | - | 3.1356 | - | - | 30. | 60. | - | 3.1356 | 300 | 5 | - | - | GTE |
| b | - | - | 22.5 | 60. | - | 3.1871 | - | - | 27.239 | 60.301 | - | 3.1817 | 300 | 11 | - | 0.00917 | GTE |
| c | - | - | 15. | 60. | - | 3.2994 | - | - | 27.239 | 60.301 | - | 3.1777 | 300 | 9 | - | 0.01794 | GTE |
| d | - | - | 37.5 | 52.5 | - | 3.3061 | - | - | 23.344 | 52.06 | - | 3.2183 | 300 | 6 | - | 0.05126 | GTE |
| e | - | - | 45. | 67.5 | - | 3.3018 | - | - | 22.171 | 56.528 | - | 3.1934 | 300 | 9 | - | 0.00801 | GTE |

NOTE: O.F. = Objective function;
NI = No. of iterations by Box routine;
NE = No. of evaluations of objective function;
GTE = Global time exceeded on computer;
S.D.= Standard deviation;
BEST= Best O.F. value in complex.

The standard deviation convergence criterion parameter equals 0.0005 for problems 1 to 15; and equals 0.005 for problem 16.

The algorithm solution is verified by the exhaustive search for structure 1(a) and 1(b).

## 4.2 Influence of Analysis-Design Cycles

Results from the analysis and design of several multi-storey frames under a multiplicity of loading conditions indicate that the design was almost stable after 3 or 4 analysis-design cycles. In some cases one or two member sizes alternated between subsequent cycles which meant that stress constraints were not satisfied. This is not serious and in engineering design practice is acceptable. When one analysis-design cycle is used the design may stabilize after several iterations in Box since one design only is stored. The analysis-design portion of the programme is the most time consuming and so reduction of the number of function evaluations, or the number of analysis-design cycles would be desirable.

By increasing the number of analysis-design cycles, stability of the design is enhanced; but for a given computer time the number of function evaluations (and the number of iterations in Box) is reduced. This decreases the possibility of finding a lower value of the objective function.

Structures 4(c)-(e) and 11(f)-(h) show the effect of the number of analysis-design cycles. For each structure one analysis-design cycle gave the lowest optimum value of the objective function with approximately three times as many iterations in each case, and four times as many function evaluations for structure 4 and approximately twice as many for structure 11.

It is interesting to note that for structure 4(c)-(e) the method converged in each case to similar configurations; while for 11(g) and (h) the configurations are almost the same, but different for 11(f).

## 4.3 The Number and Type of Variables

For structure 1 the method converged faster with two variables than with one variable. This is shown in Figure 4.3. The large number of function evalu-

ations required for the one-variable problem indicates that the rate of improvement of the best value was slow. An initial complex was regenerated 6 times without improvement of the best point which implies the design space is quite flat near the optimum.

The influence of the number and type of variables was examined in structure 13. Comparison of 13(a)-(e) shows that the inclusion of all variables did not significantly improve the best value of the objective function over 13(c) and 13(d). In the case of 13(e) the two point complex collapsed onto the initial point and completed 7 iterations before "converging". The complex "converged" because the number of analysis-design cycles was one; the design did not stabilize and though both points in the complex had the same coordinates two distinct structures were designed with correspondingly different objective function values. For structure 13(a) the number of points in the complex was 4 and in 12 iterations the function value was reduced by approximately 25%. The conclusions to be drawn from this is that a problem with one variable should have a minimum complex size of 3 or 4, otherwise the method may break down. In 13(a)-(d) the optimum column positions were almost the same, which indicates that the objective function value variation is caused principally by $x_1$ and $x_2$.

Column Positions: The results of problem 11 and 16 show that an arrangement of columns with equal or near equal spacing gives a good design cost. This is in agreement with Nakamura (18) and Soosaar (19). Soosaar showed that equal spacing of columns gave a cost within 5% of the optimum, regardless of the number of storeys.

For the two tall structures considered, 7 and 14, with $N_b = 3$, the column positions at the termination of the optimization scheme were quite different. In structure 14 the columns were almost equally spaced, while in the taller structure, 7, the columns moved towards both ends. The influence of the wind loading causes this column concentration at the edges for structure 7, which has

a height to width ratio of 3.5. In structure 14 the almost equal spacing of columns reduces the bending moments due to vertical load, while the low height to width ratio of 0.72 provides adequate resistance to wind loads.

Number of Intermediate Beams: If the structures are grouped according to width, the optimum number of intermediate beams per floor within each group is almost equal. The average spacing of intermediate beams within each group is shown below in Table IV.

| Width | Average $x_2$ | Average Spacing |
|---|---|---|
| 60' | 9.35 | 7.20' |
| 80' | 12.85 | 6.75' |
| 90' | 12.60 | 7.75' |
| 108' | 17.35 | 6.60' |

TABLE IV : Average Intermediate Beam Spacing

These results show good agreement with the exhaustive search done for structure 1, where the optimum intermediate beam spacing is approximately 7 ft. (regardless of frame spacing), which corresponds to the minimum slab thickness permitted.

Frame Spacing: The final design in several cases (4a, 6a, 11a, 11f, 12c) where cost was the criterion, yielded a frame spacing close to the upper bound. Where weight was the criterion of a good design (11c) the frame spacing for the final design was close to the lower bound. This is verified by results of the exhaustive search (see Figure 5.3).

Only in one case however (12c) of the 7 runs that converged, did the algorithm converge to the upper bound or lower bound for frame spacing.

Frame spacing influences the objective function more than the other variables,

TABLE V

Cost Function Parameters

| Objective Function Type | $C_1$ ($ per lb.) | $C_2$ ($) | $C_3$ ($) | $C_4$ ($ per lb.) | $C_5$ ($) | $C_6$ ($ per sq.ft.) | $C_7$ ($ per sq.ft.) | $C_8$ ($ per sq. ft. per in.) | $C_9$ ($ per lb.) | $C_{10}$ ($) | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.100 | 50.000 | 20.000 | 0.090 | 10.000 | 3.000 | 0.750 | 0.155 | 0.030 | 100.00 | Normal costs |
| 2 | 0.100 | - | - | 0.090 | - | - | - | - | - | - | Material costs |
| 3 | 0.100 | 50.000 | 20.000 | 0.090 | 10.000 | 3.000 | - | - | 25.000 * | 100.00 | |
| 4 | 0.100 | 100.000 | 40.000 | 0.090 | 20.000 | 3.000 | 1.500 | 0.311 | 0.060 | 200.00 | High labour |
| 5 | 0.100 | 25.000 | 10.000 | 0.090 | 5.000 | 3.000 | 0.380 | 0.078 | 0.015 | 50.000 | Low labour |
| 6 | 1.000 | - | - | 1.000 | - | - | - | - | - | - | Weight |

* $C_9$ is the cost per lb. per end for beam joints except in type 3 where it is the cost per beam joint.

the degree of influence however seems to depend on the objective function chosen.

## 4.4 Uniqueness and Rate of Convergence

When a minimum has been found the only way of checking whether the minimum is local or global is to run the problem again with a new, and if possible, completely different initial point. If the method converges each of several times to the same point then it is inferred that this is the global minimum.

Such uniqueness of the optimum solution was checked for structures 4 and 11(d-h). A unique solution did not emerge though maximum deviations from the best solutions obtained were 2.28% and 3.44% for 4 and 11, respectively. For 4c the complex was rapidly contracting - shown by the small value for standard deviation divided by the least value of the objective function in the complex - and was near convergence.

Comparison of results for 12b and 13b (with $N_b = 3$, for each) shows that with four times more running time the optimum solution is improved by only 1.73%. The complex, at termination, has contracted about the minimum almost within the standard deviation convergence criterion limit.

It can be concluded that any minimum cannot be verified as a global minimum; and the general slow rate of convergence shown throughout the problems indicates a flat design space which does not agree with Soosaar's conclusion of pronounced curvature of the objective function near the minimum.

## 4.5 Cost Model Variation

Six different objective functions were formed by varying the cost parameters as in Table V. The influence of the type of objective function used can be seen in the results for 11(a-f).

The frame spacing shows the most sensitivity to the objective function

type. The results for optimum frame spacing in structure 11 (a-f), concur with the trend established in the exhaustive search for structure 1 shown in Figures 4.4, 4.5 and 4.6. High labour costs move frame spacing to the upper bound; medium labour costs show little sensitivity to frame spacing away from the lower bound; and low labour costs tend to move frame spacing slightly towards the lower bound. In Figure 4.4 this is shown by the reversal of the position of $x_1$ = 20' relative to $x_1$ = 30' as the costs change.

When weight or material cost is the objective function as in 6b, 11c and 15b, frame spacing moves to the lower bound. As the frame spacing decreases the weight per sq. ft. of frame increases slightly while the floor system weight reduces so much that the total weight per sq. ft. for the whole structure is decreased. This will be shown to be true in Chapter 5.

Objective functions involving labour and topology indicate that frame spacing is a more critical variable than either the number of intermediate beams or the column positions. When weight is the objective function the number of intermediate beams becomes more important than the frame spacing, which tends to the lower bound for an optimum design.

## 4.6 Algorithm Performance and Capabilities

Box's complex method in general performed quite adequately in spite of the severe standard deviation convergence criterion applied to most problems. However the overall optimization procedure became quite slow when large structures (7,12,14) were optimized. The major share of the time was spent in the analysis-design portion of the scheme. The number of iterations in the complex method was reduced significantly rending the method ineffective. For structure 12 the complex was close to convergence on termination in all cases; structure 14 was far from convergence except in the case of 14g.

This clearly indicates that the use of the optimization scheme is limited to small structures. The amount of time used in the design-analysis portion of

the program for structures 7, 12 and 14 would justify consideration of approximate method of analysis for larger structures.

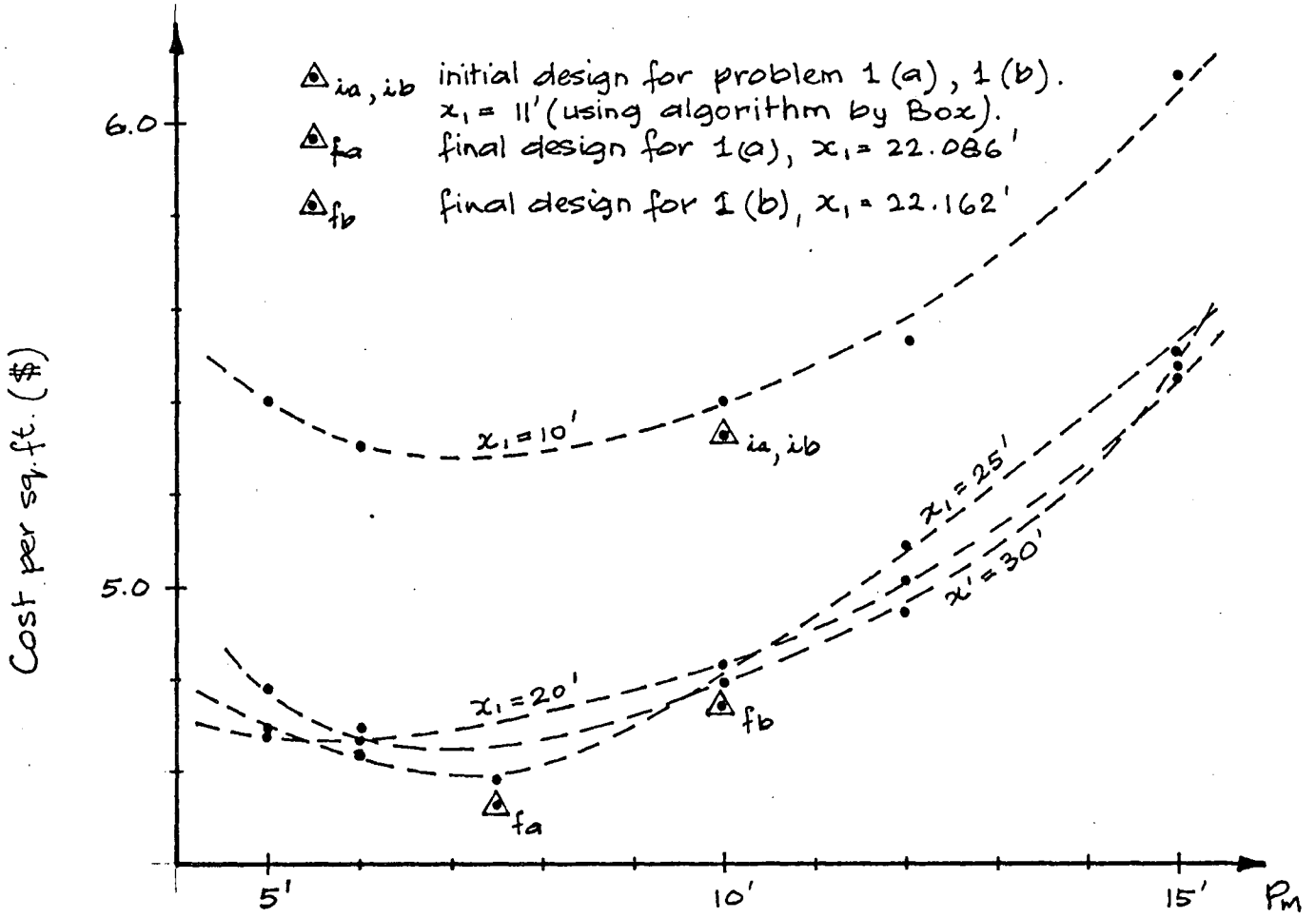



Figure 4.2
Comparison of Exhaustive Search
and Complex Solutions
for
Structure No. 1

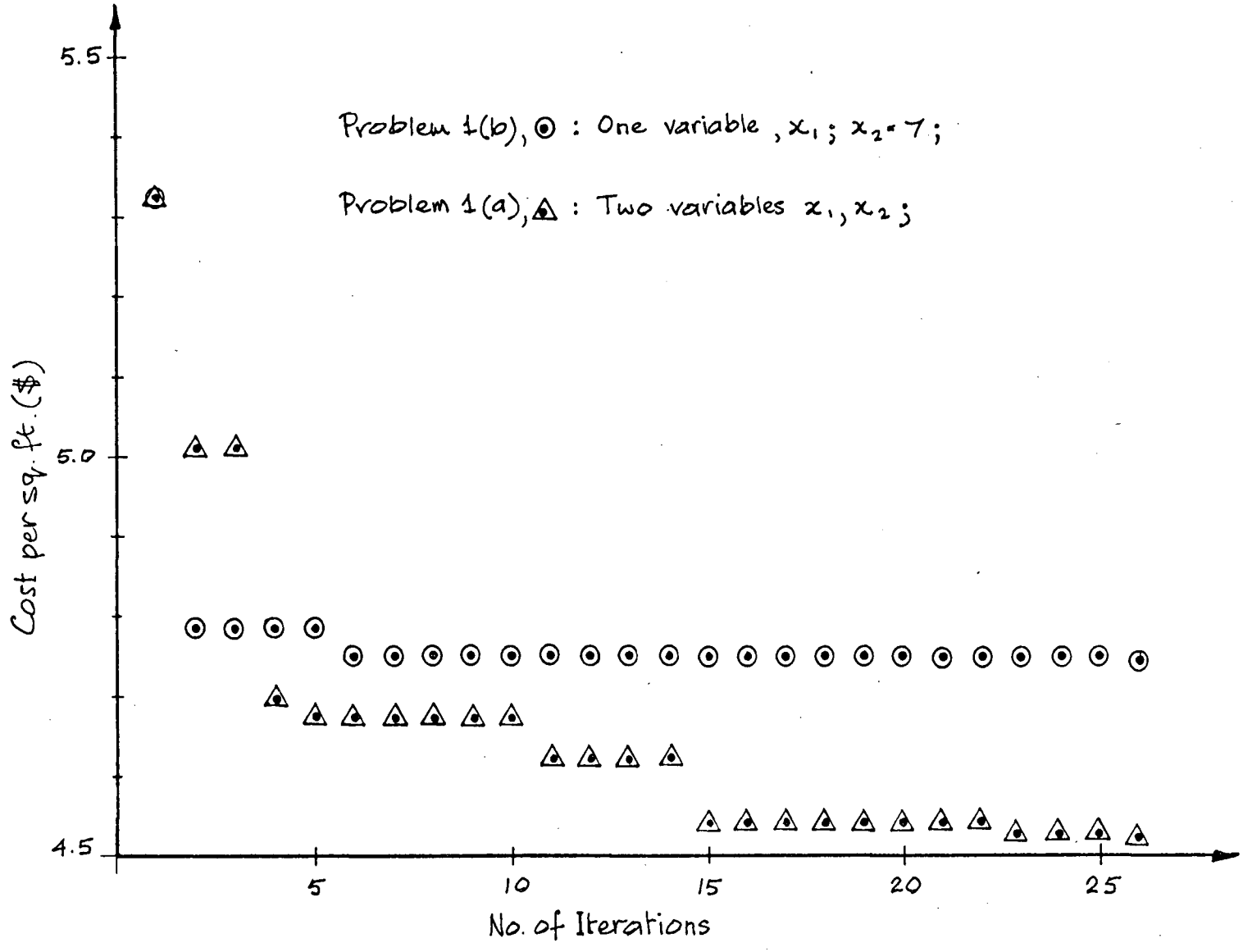



Figure 4.3 Convergence rates for Structure No. 1.

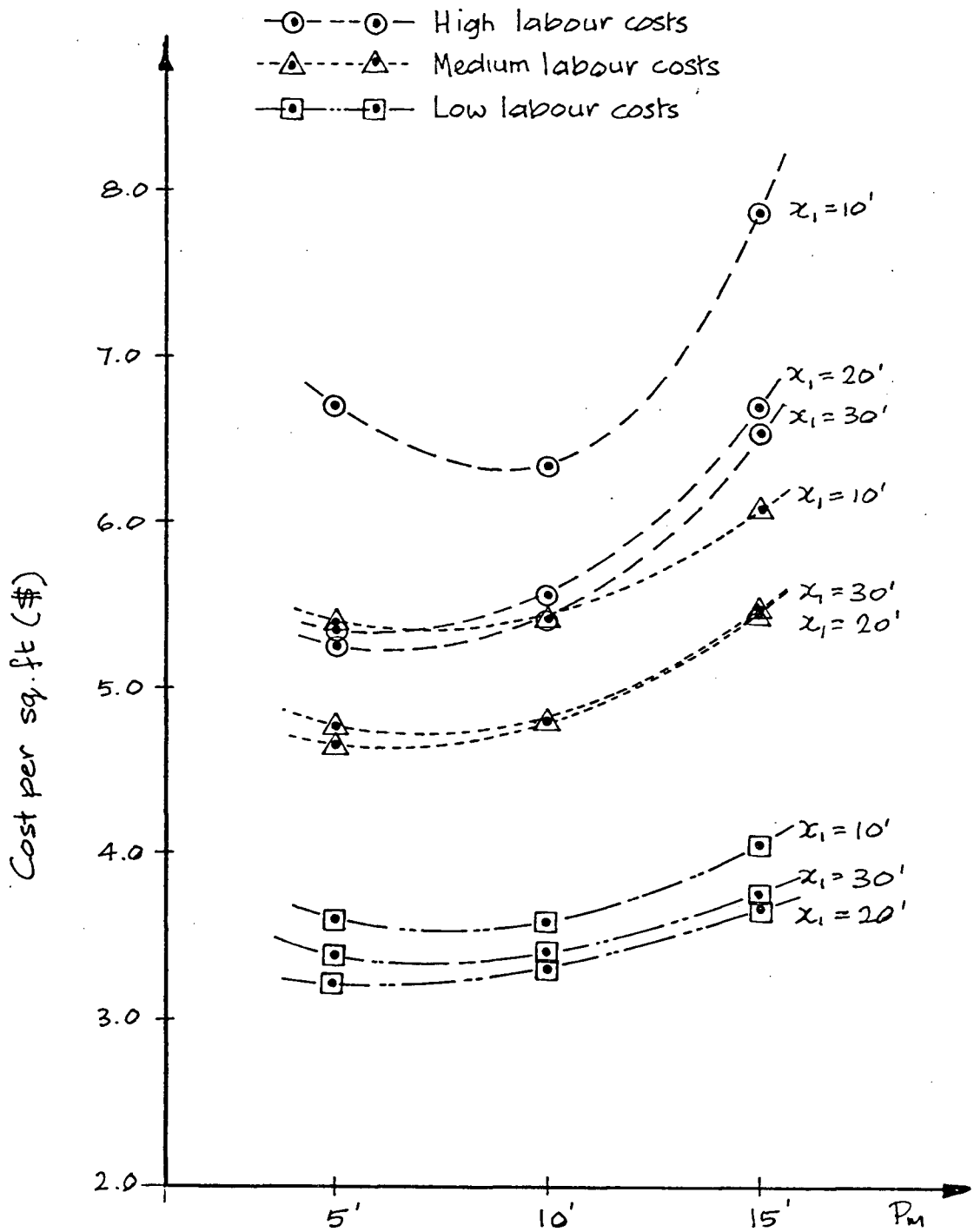



Figure 4.4

High, Medium and Low Labour Costs for Structure No. 1

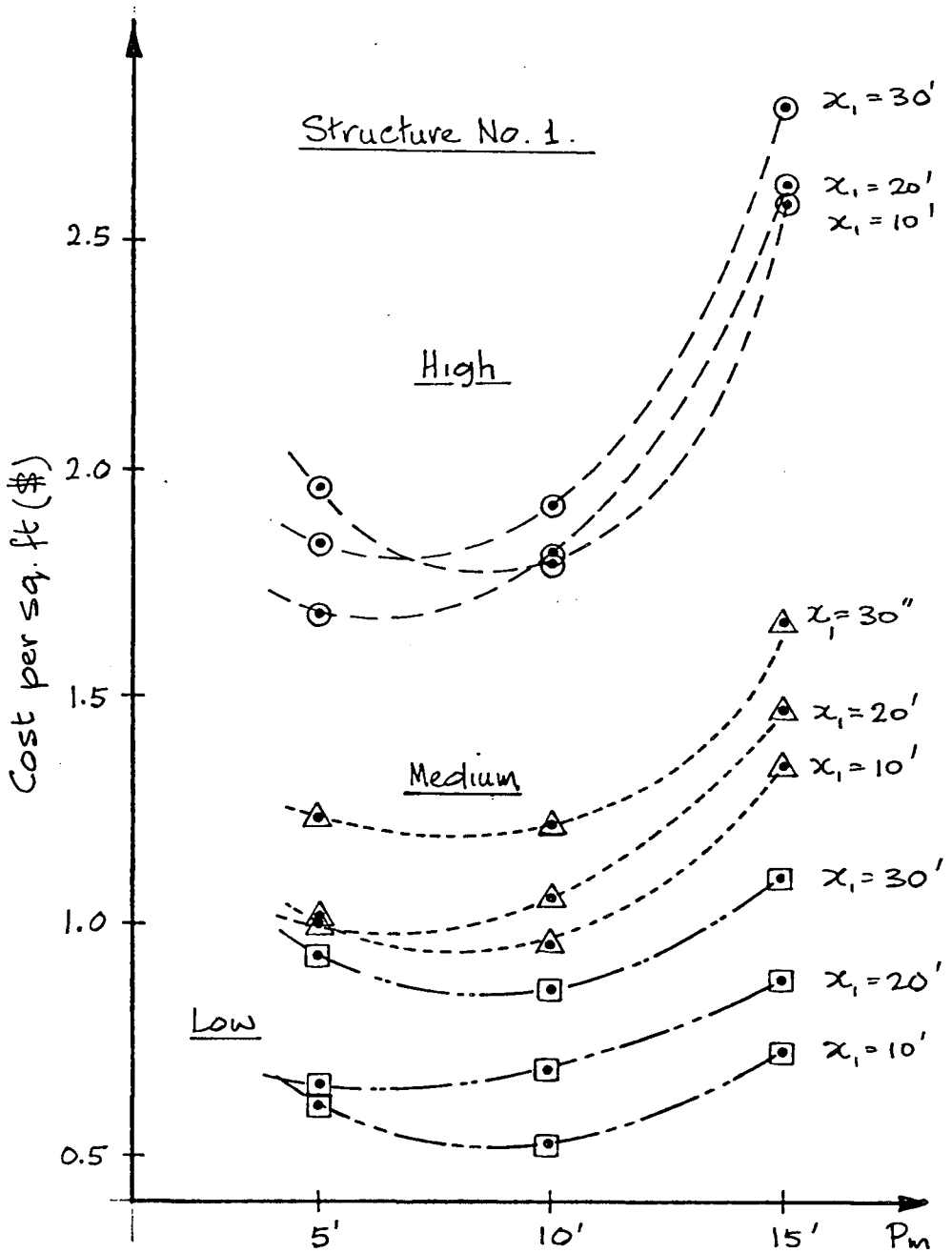



Figure 4.5
Roof System Costs
for
High, Low and Medium
Labour Costs

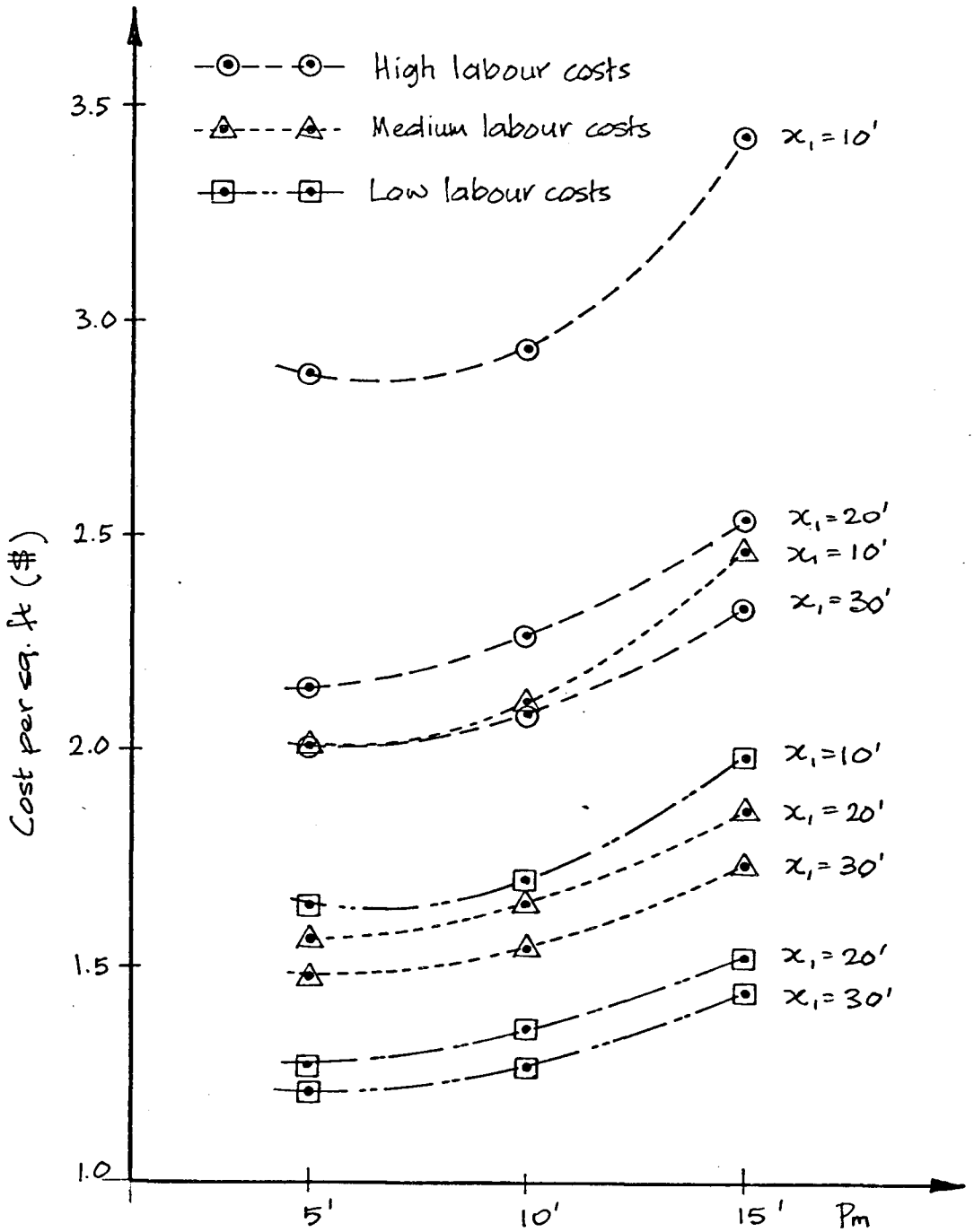



Figure 4.6
Costs for Frame Only
for
High, Low and Medium
Labour Costs
(Structure No. 1)

# Chapter 5

## PARAMETER STUDY

The influence of the following parameters on the cost and weight per sq. ft. of the structure is examined: (i) frame spacing; (ii) the number of intermediate beams; (iii) the number of bays in the frame; (iv) the number of storeys; and (v) the roof system.

An exhaustive search procedure was conducted for structure 1 for this purpose, and results from the optimization of structures 12 and 14 are also included.

### 5.1 Influence of Frame Spacing ($x_1$) and Number of Intermediate Beams ($x_2$)

Figure 4.2 shows that for a unit cost as objective function the optimum spacing of intermediate beams is approximately 7 ft. This span allows the slabs to be the minimum thickness permitted. Intermediate beam spacing below this results in the same slab thickness while the increased number of intermediate beams causes the objective function to increase.

With weight per sq. ft. as the objective function the optimum spacing of intermediate beams decreases to approximately 5 ft. when $x_1$ = 10'. The weight per sq. ft. of intermediate beams increases slightly in this case, but the frame weight is significantly reduced because the load applied by the intermediate beams to the frame is approaching a uniformly distributed load pattern which gives reduced bending moments in comparison to point loads.

For a constant number of intermediate beams the unit cost in Figure 4.2 shows very little sensitivity to variation in the frame spacing away from extreme values. However the unit weight as shown in Figure 5.1 decreases significantly with decreased spacing. It should be noted that the difference in unit weight between $x_1$ = 30' and $x_1$ = 10' is most pronounced for small inter-

mediate beam spacing.

## 5.2 Influence of Topology on Objective Functions

Number of bays: Figure 5.2 shows the effect of the number of bays and frame spacing on a unit cost objective function, when the intermediate beam spacing is constant at 10 ft. The objective function is insensitive to frame spacing away from the lower bound. A unit weight criterion showed in Figure 5.3 that the minimum frame spacing was the optimum, and increased rapidly with increasing frame spacing.

When the number of bays was increased from 1 to 3, the axial force and bending moment per column and the moments per frame beam were reduced. This gave an 18% and 14% reduction in unit cost for $x_1$ = 10' and $x_1$ = 40', respectively. With unit weight criterion the corresponding reductions were 65.6% and 41.6%.

The final design costs from the optimization scheme for structures 12 and 14 are plotted against the number of bays in Figures 5.4 and 5.5. The influence of the number of bays is not significant. Maximum difference in cost is approximately 0.60% and 0.76% for structures 12 and 14 respectively. No figures are available for the influence of the number of bays on unit weight in this case.

Number of Storeys: Structure 1, with $N_b$ = 3, was used as the basic structure to study the influence of the number of storeys. Figures 5.6 and 5.7 show the influence of frame spacing and the number of bays on the objective functions, types 1 and 6. The optimum design tends to move to the upper and lower bounds for cost and weight criterion respectively.

## 5.3 Influence of Roof System

A breakdown of the costs and weight of the exhaustive search for structure 1 is shown in Figures 5.8 and 5.9. The influence of the roof system is

particularly noticable when weight is the objective function. It causes the weight per sq. ft. of the total structure (see Fig. 5.1) to increase with increased spacing of frames.

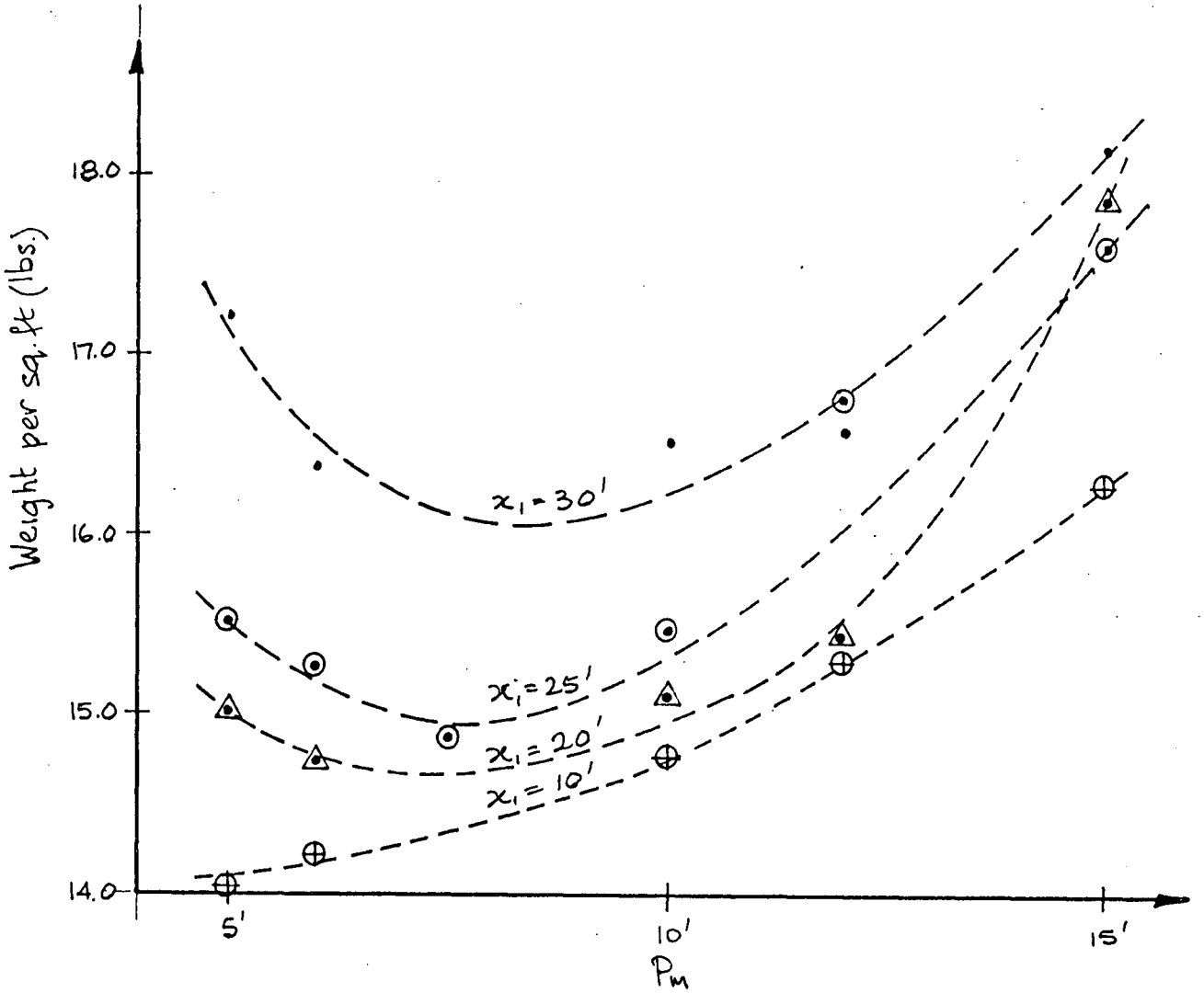



Figure 5.1
Exhaustive Search for
Structure No. 1
Weight Criterion Used

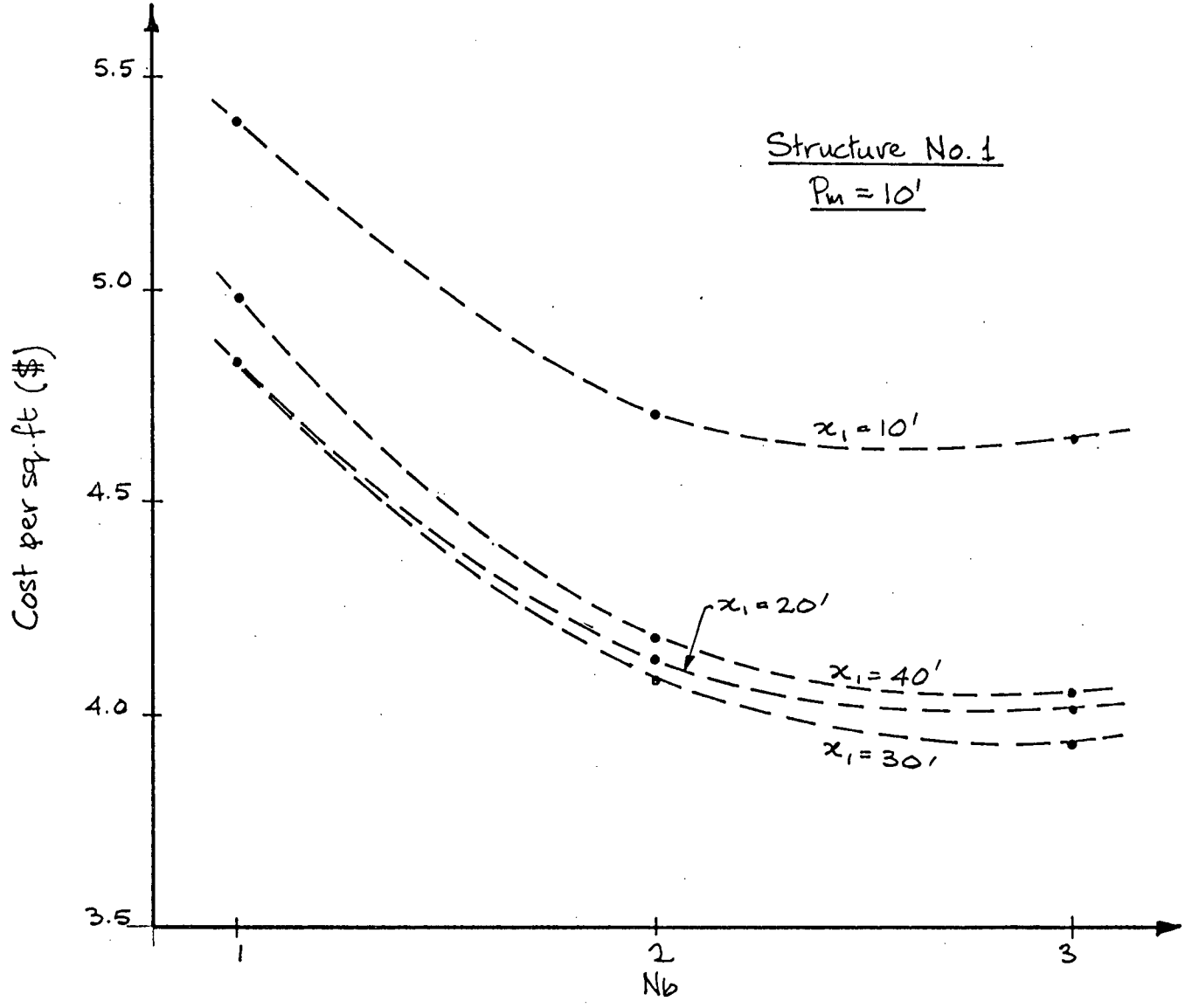



Figure 5.2
Variation of Cost with Frame Spacing and No. of Bays

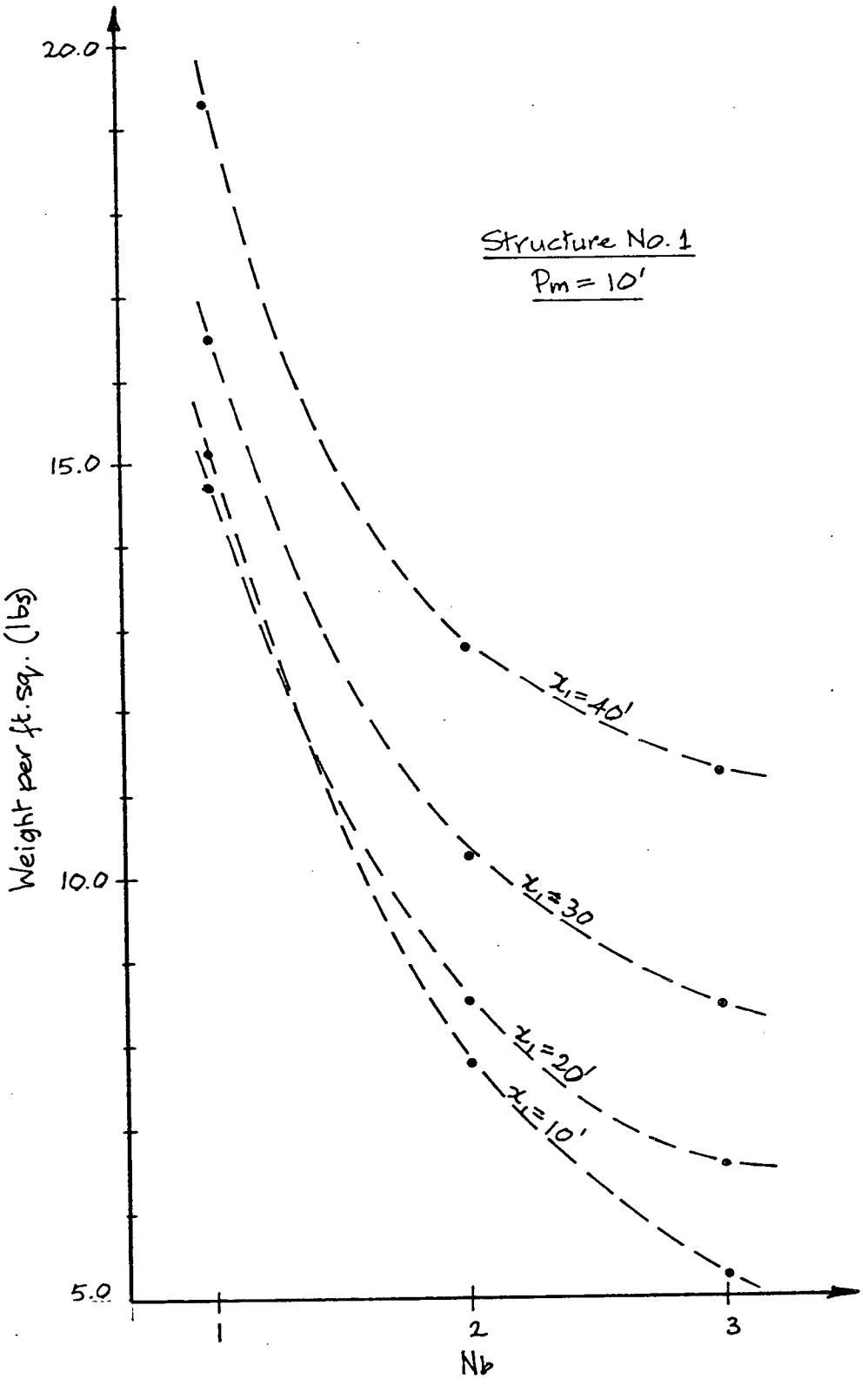



Figure 5.3
Variation of Weight with Frame Spacing and No. of Bays

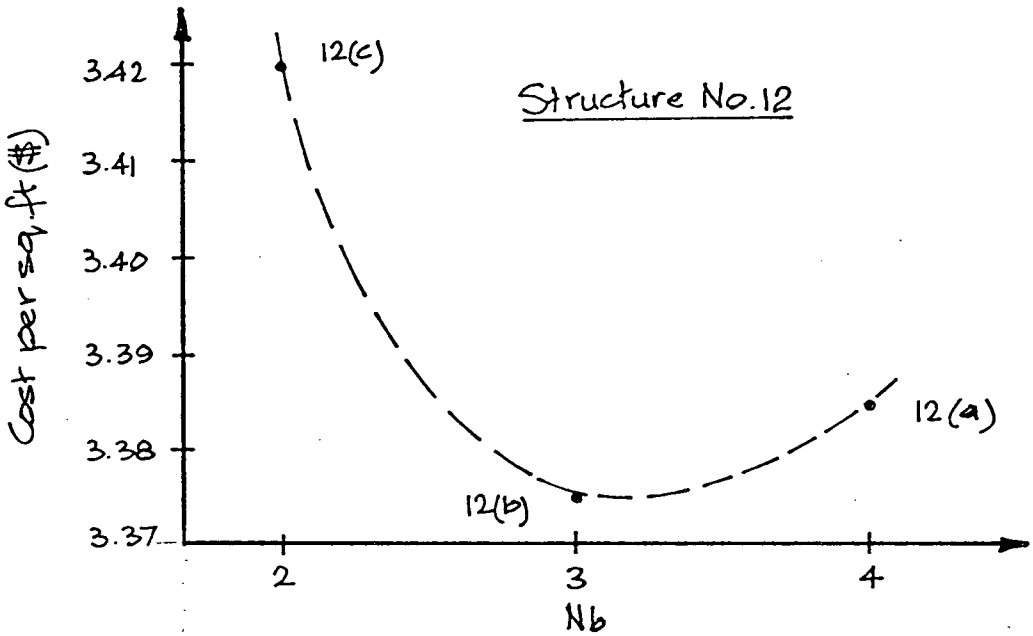



Figure 5.4 Optimum Design Costs vs. No. of Bays

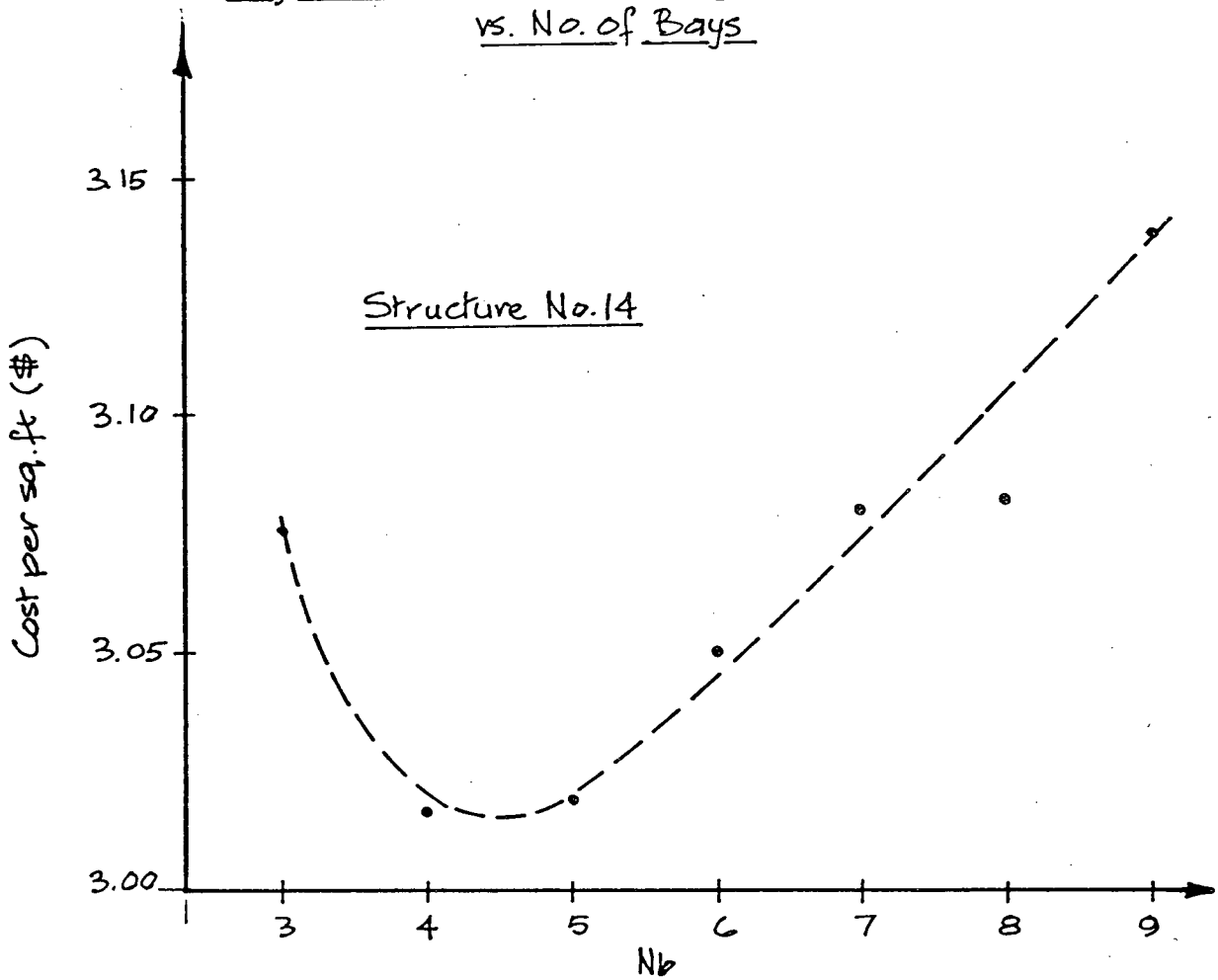



Figure 5.5 Optimum Design Costs vs. No. of Bays

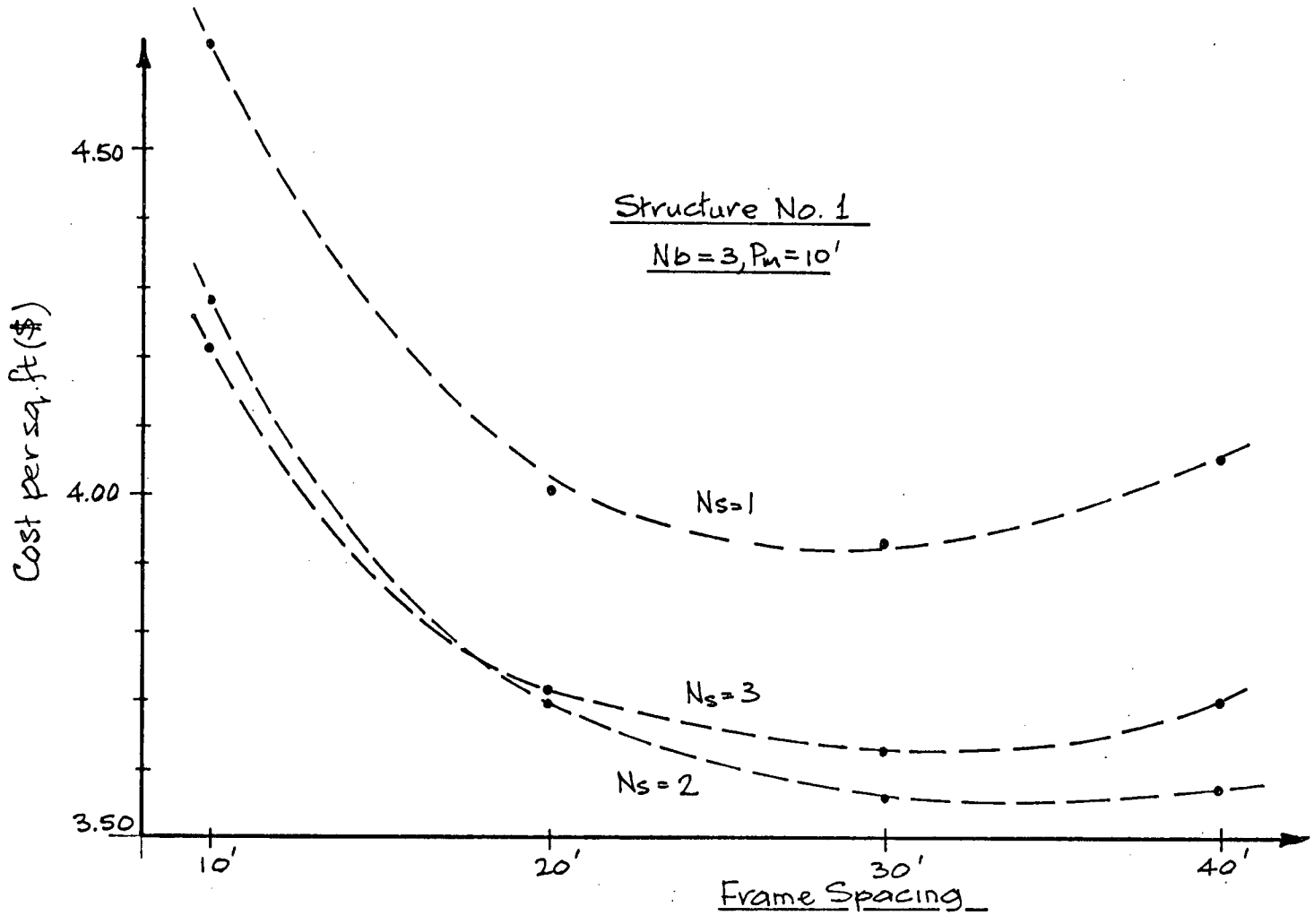



Figure 5.6 Variation of Cost with Frame Spacing and No. of Storeys

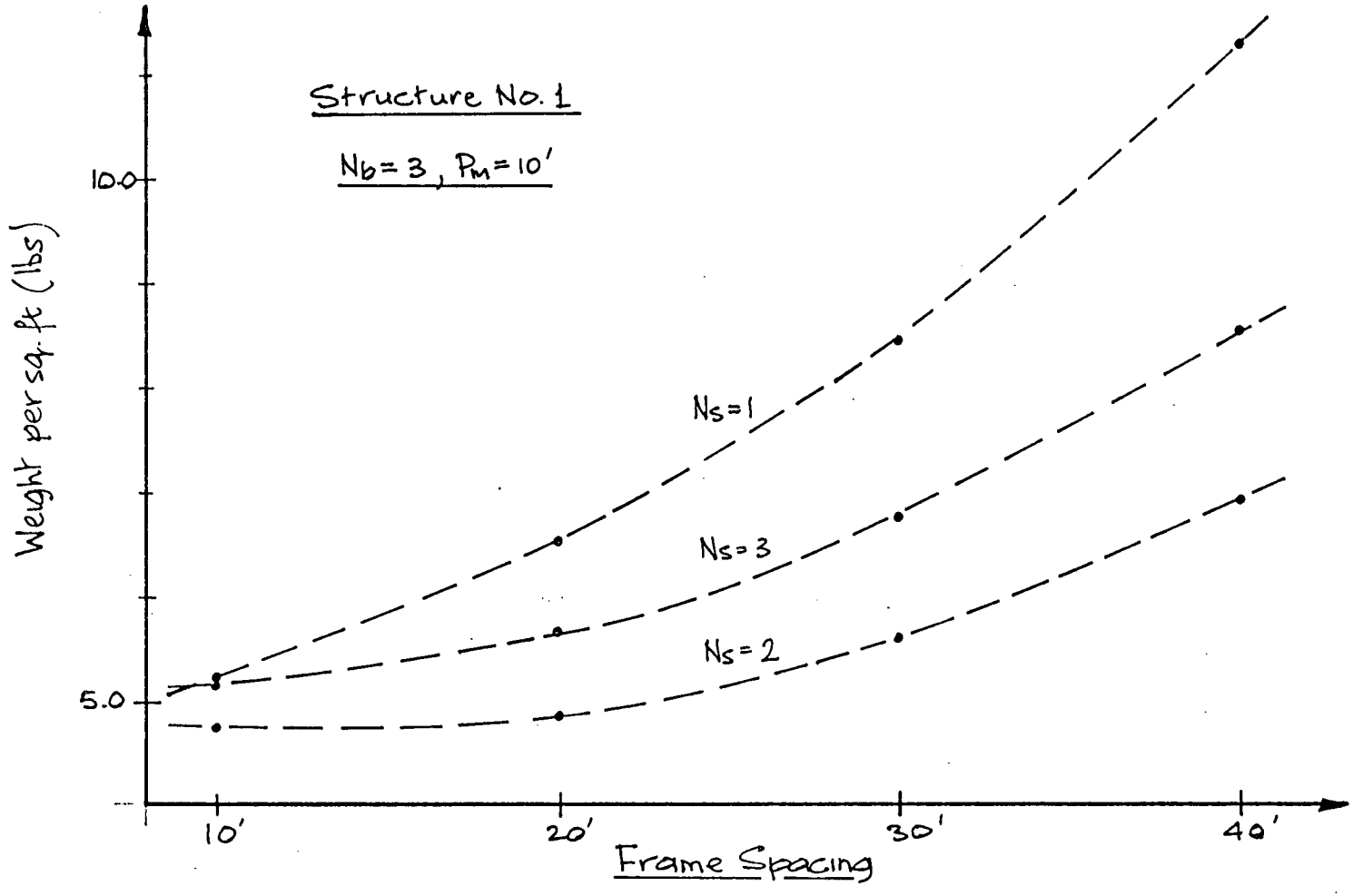
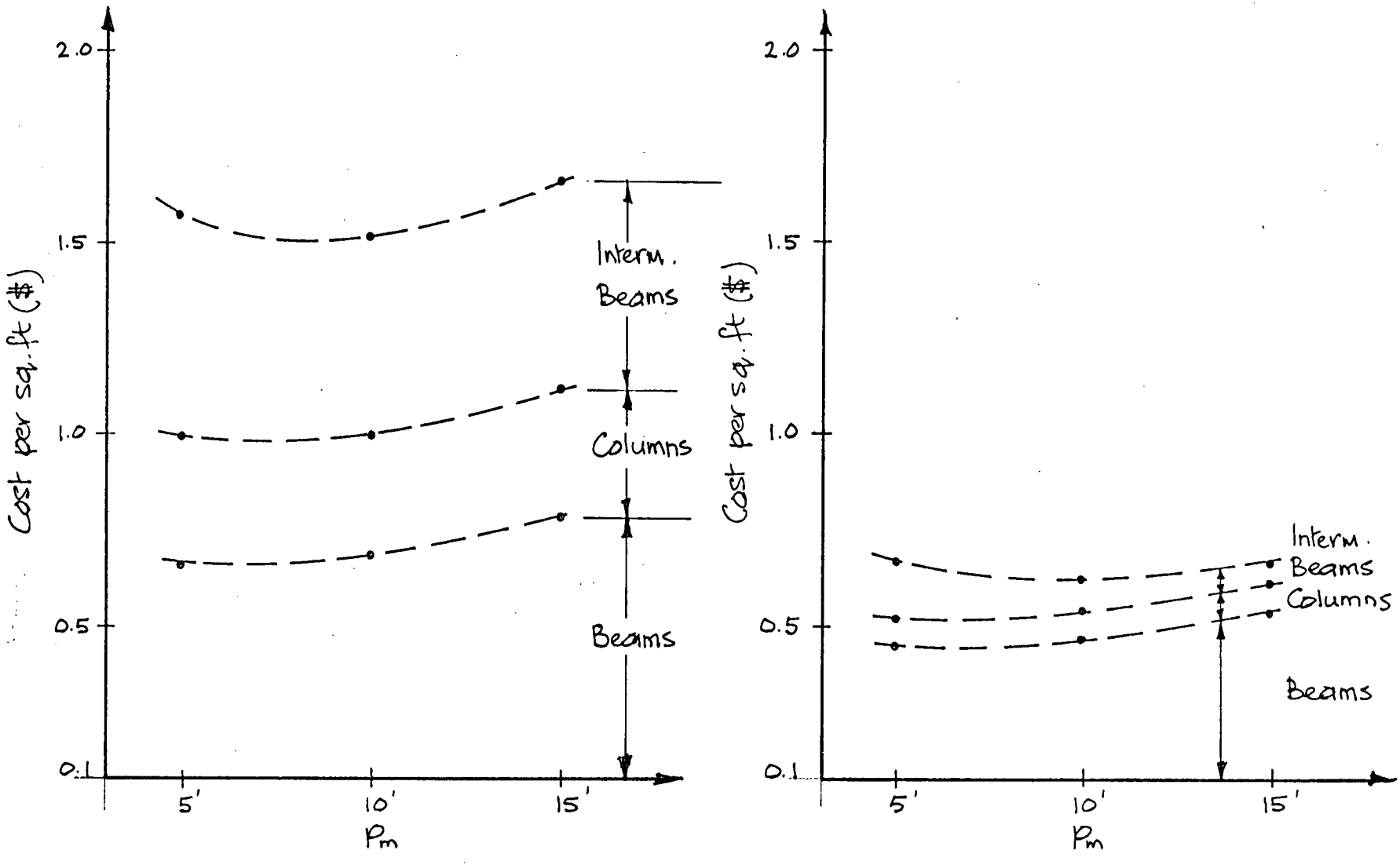



Figure 5.7

Variation of Weight with Frame Spacing and No. of Storeys

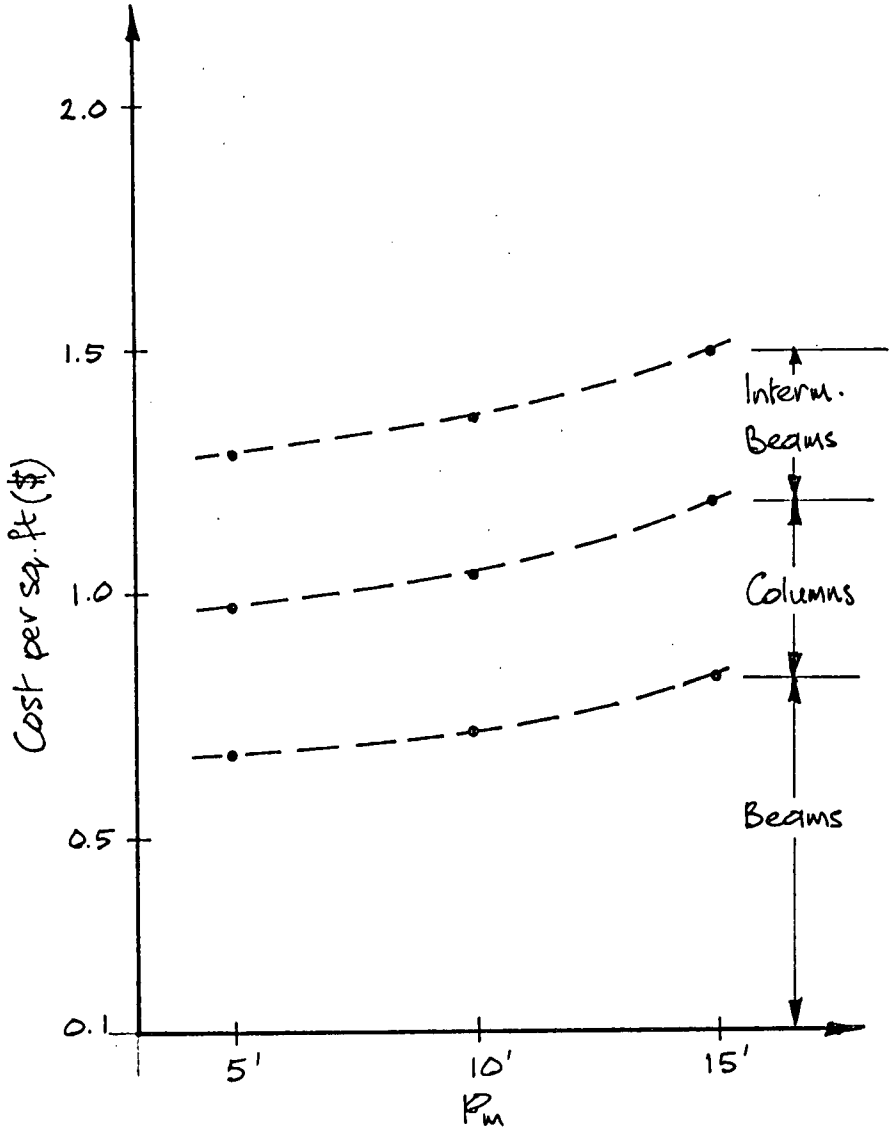



Material Cost Breakdown for Structure No. 1

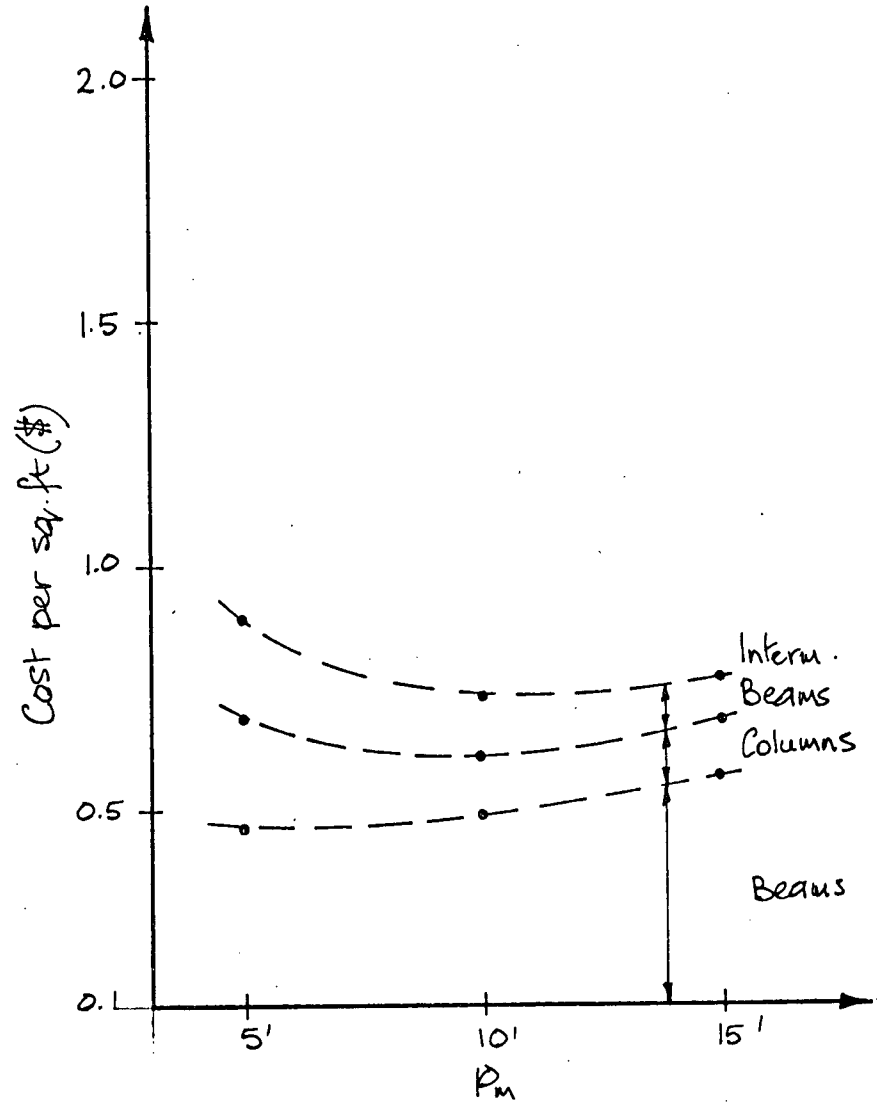
Labour Cost Breakdown for Structure No. 1

Figure 5.8

(a) Frame Spacing = 30'

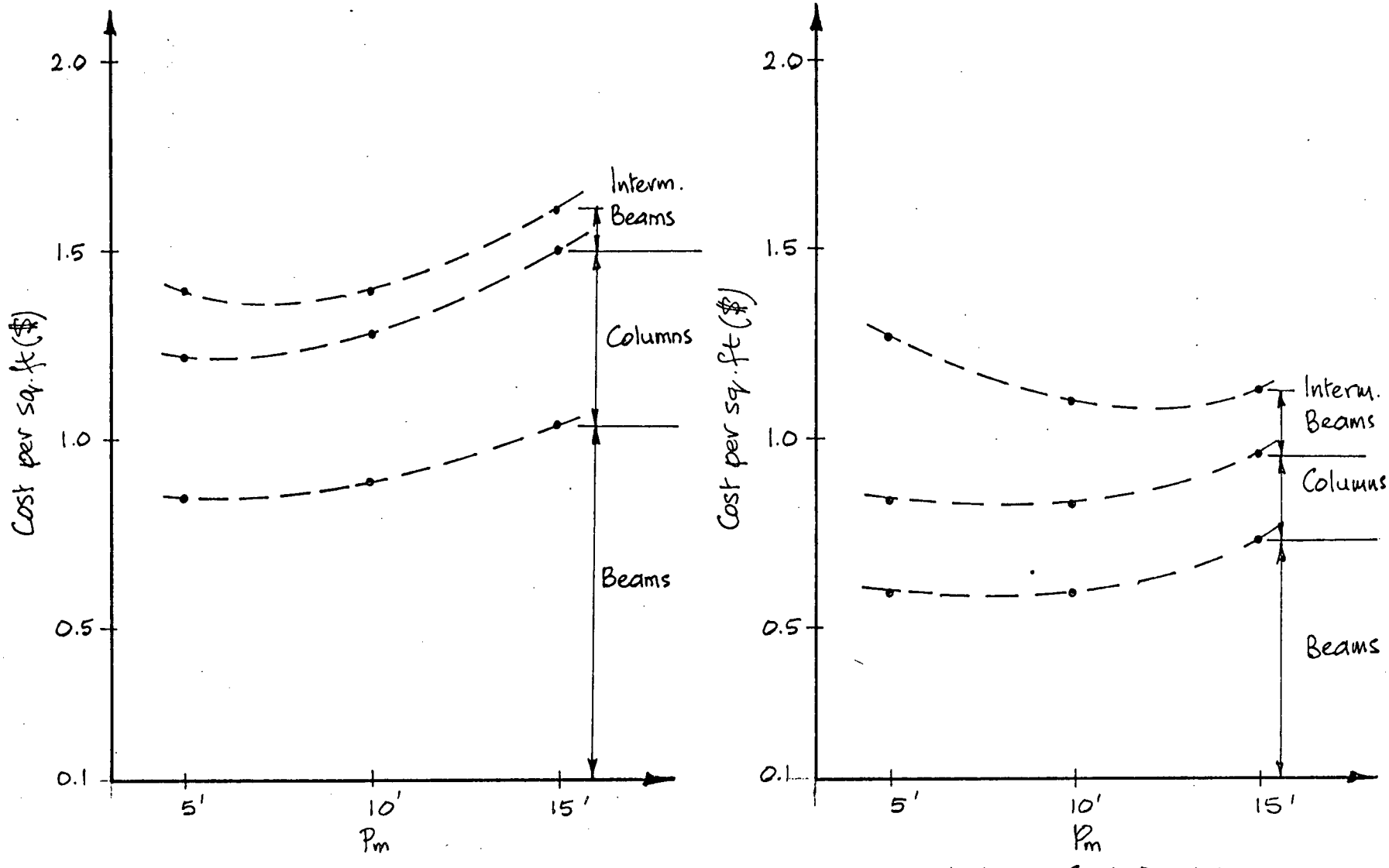



Material Cost Breakdown for Structure No.1




Labour Cost Breakdown for Structure No. 1

Figure 5.8
(b) Frame Spacing = 20'




Material Cost Breakdown for Structure No.1

Labour Cost Breakdown for Structure No.1

Figure 5.8
(c) Frame Spacing = 10'

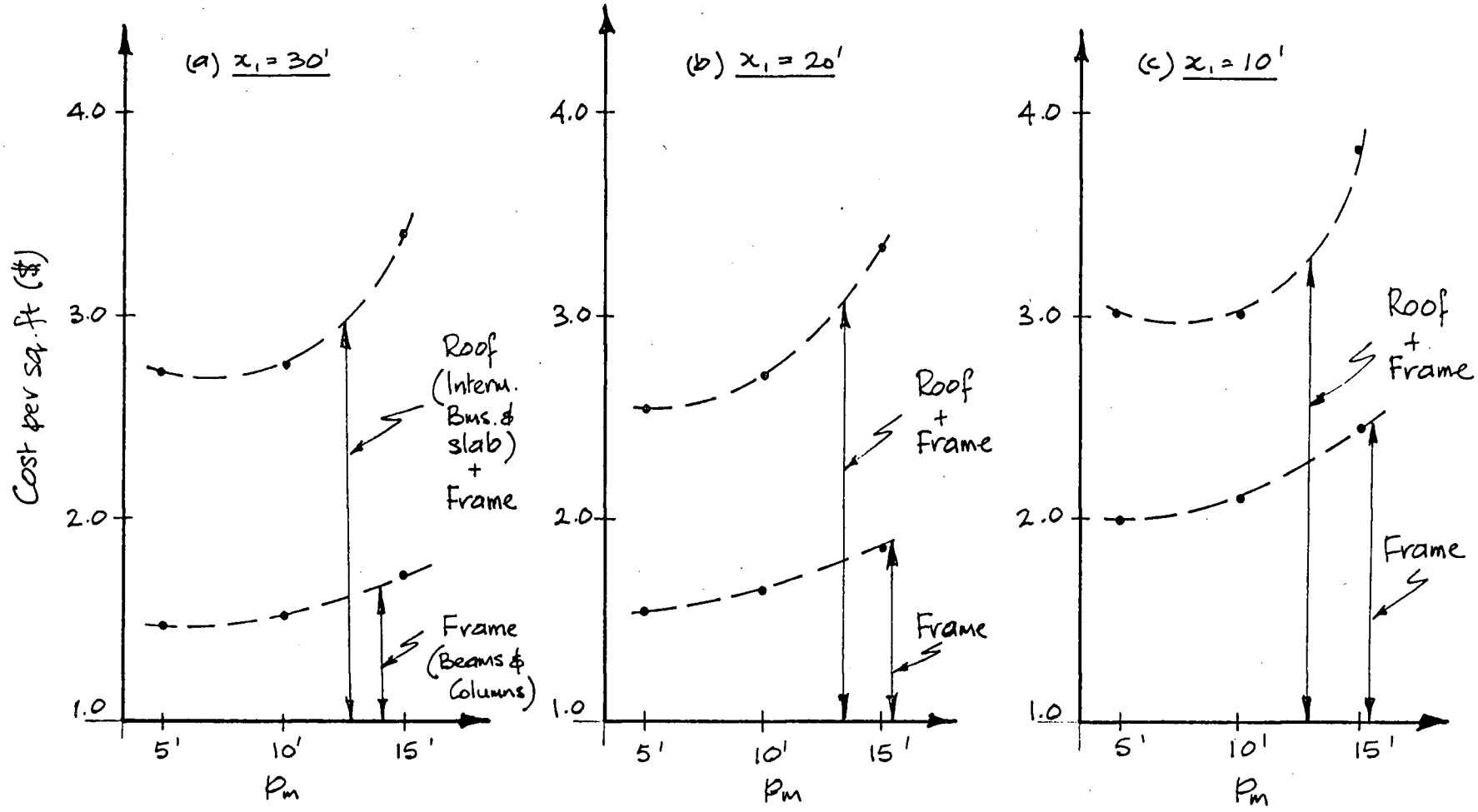



Figure 5.9 (a)-(c)
Frame and Roof System Costs
for Structure No. 1

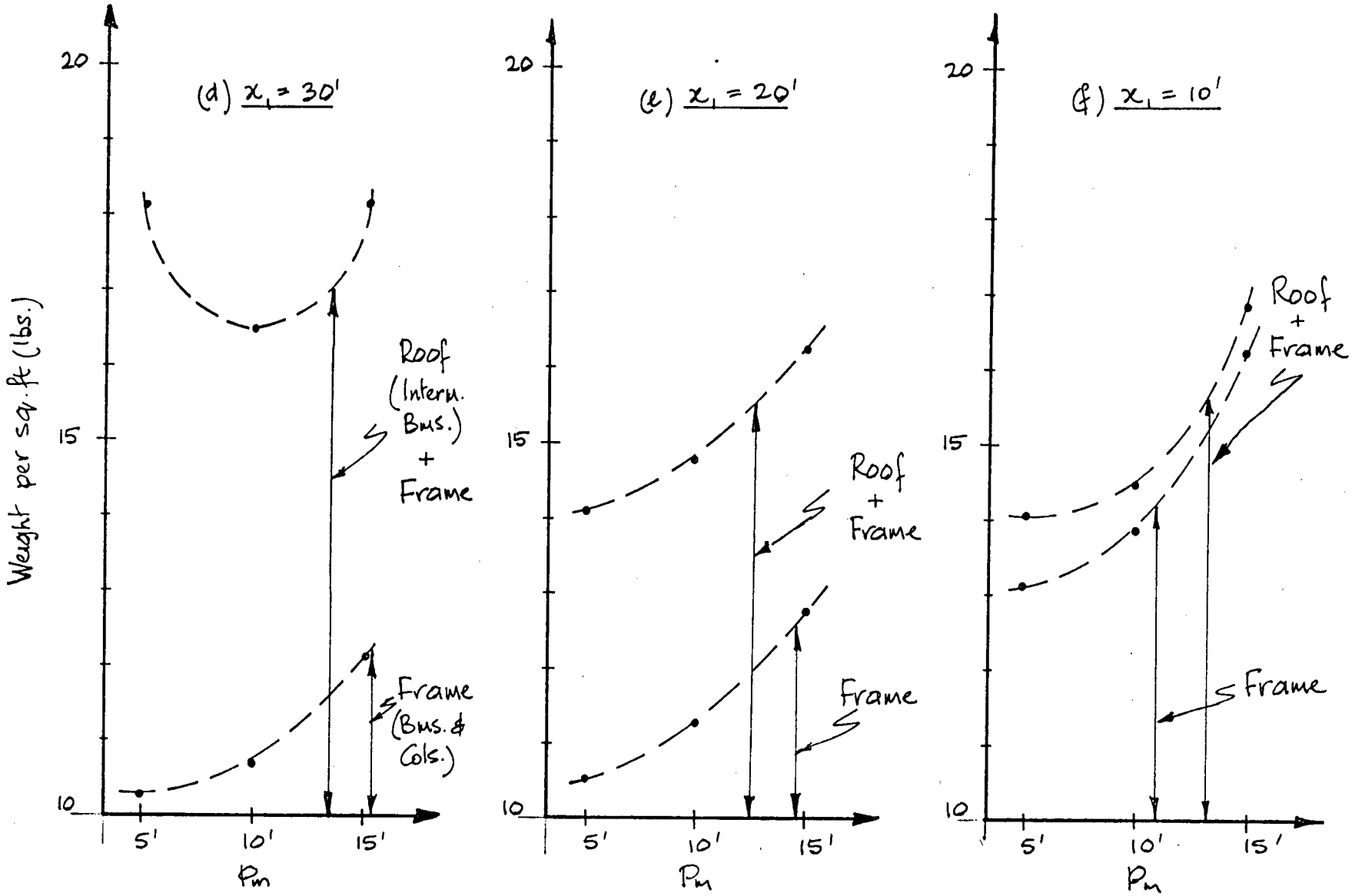



Figure 5.9 (d)-(f)
Frame and Roof System Weight for Structure No. 1

# Chapter 6

## SUMMARY OF CONCLUSIONS AND RECOMMENDATIONS

(i) The feasibility of an automated scheme for the optimum design of structures with rectangular plane frames, intermediate floor beams and concrete floors has been demonstrated. The method uses exact methods of analysis coupled with a modified AISC code for design and considers discrete member sizes.

(ii) The capabilities of the optimization scheme is limited by the amount of computer time available and computer storage capacity.

(iii) It cannot be proven that the optimum design found by the complex method is a local or a global minimum, however exhaustive search techniques did not disclose any points with a better design.

(iv) The floor system is an integral part of the structure and should be included in the optimization scheme. It has been shown by an exhaustive search procedure that the optimum frame spacing based on a unit weight criterion is significantly different when the floor system is included in the problem.

(v) Regardless of the objective function used the design space near the minimum appears to be flat. Soosaar (30) found that the design space had a more pronounced curvature near the minimum for a cost objective function than for a weight criterion. However Soosaar considered only the frame and not the floor system and this may account for the difference in results.

(vi) For structures with low height to width ratios equal column spacing gives objective function values very close to those for the optimum column position. For taller structures with a height to width ratio greater than 2 the interior columns tended to move towards the edges at optimum spacing. This shows agreement with the results of Nakamura (18).

(vii) The number of intermediate beams did not greatly effect the objective

function. In general it was found that intermediate beams spaced to give minimum slab thickness gave optimum results.

(viii) The type of objective function used is important. It was demonstrated that a weight and a cost criterion lead to widely different optimum designs with the major difference occurring in frame spacing. For a weight value system the optimum frame spacing occurred at the lower bound; while a high labour cost objective function gave the upper bound as the optimum frame spacing.

For future work in this area the following points are recommended:

(i) To reduce computer run time, approximate methods of analysis and design checked by exact analysis-design procedures every 20th iteration say, or at termination, should be included.

(ii) The geometry variable relating to column positions should be replaced by a topology variable related to the number of columns in the structure.

(iii) The number of intermediate beams could be treated in a separate sub-optimization scheme and when optimized entered into the main optimization procedure.

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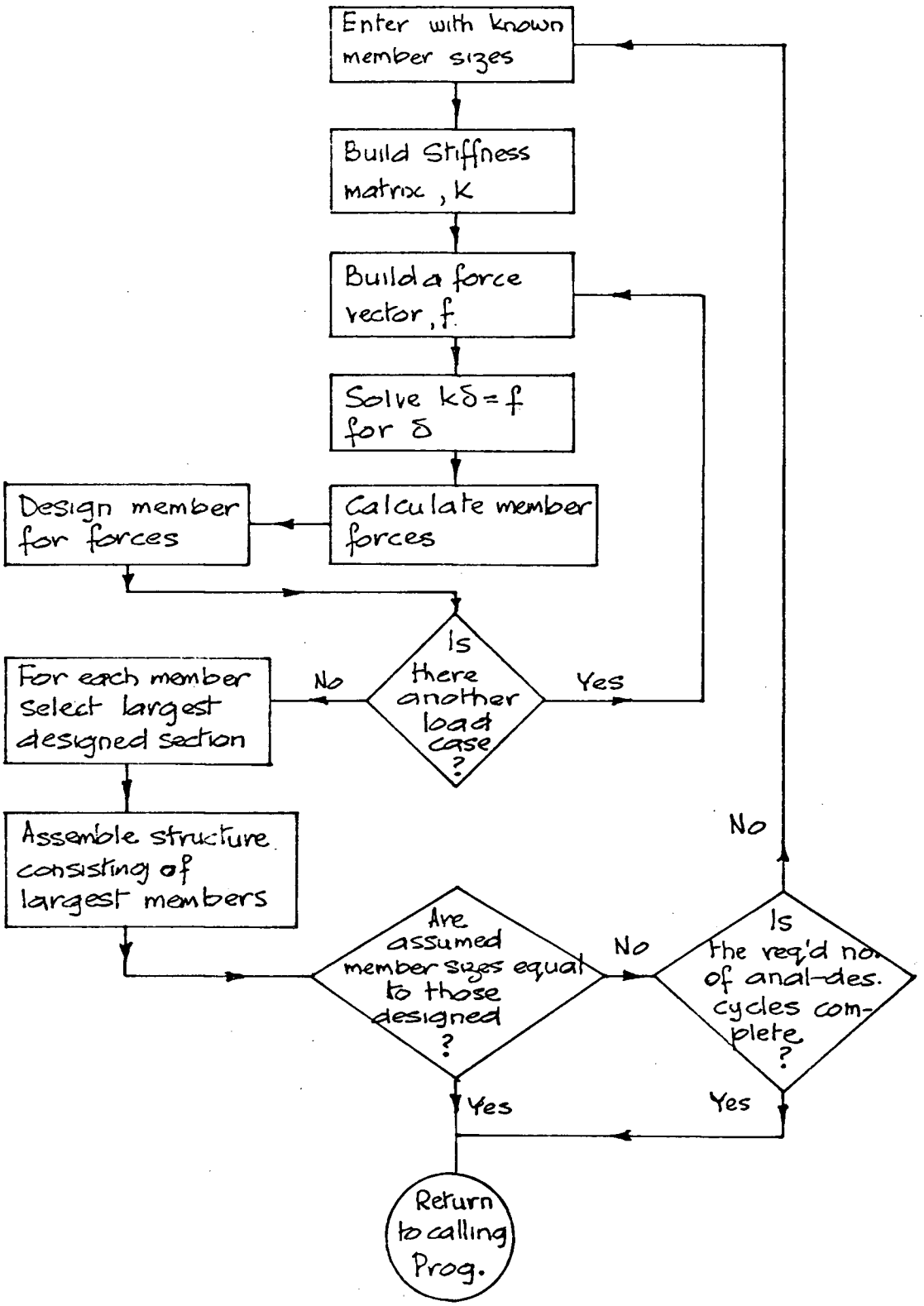
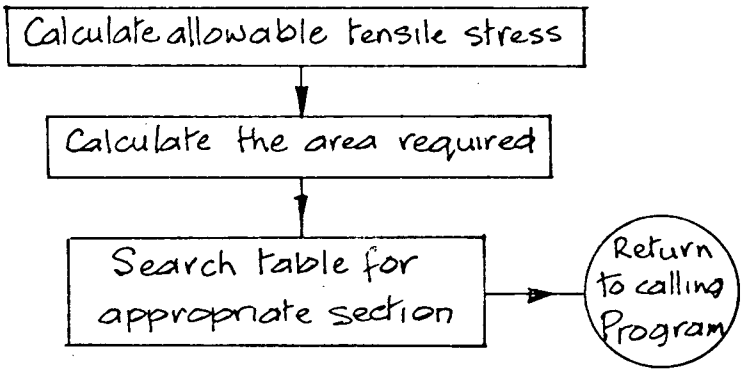
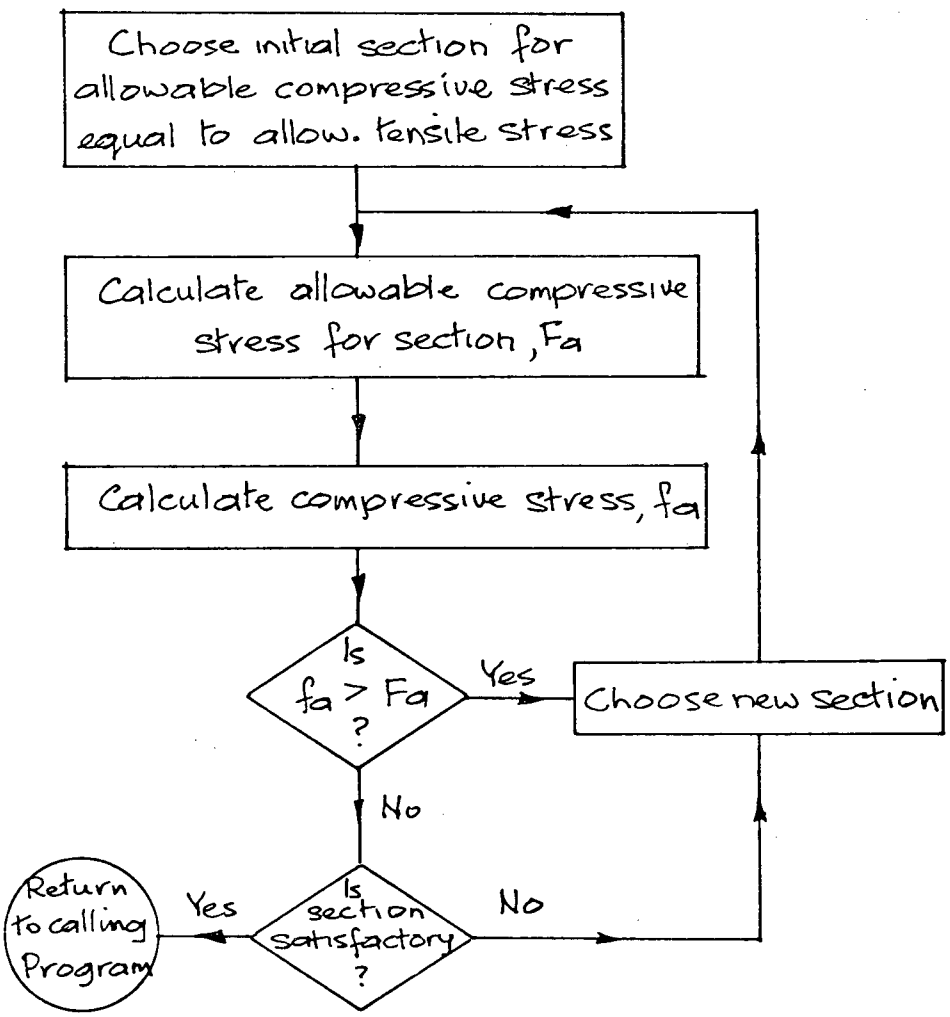
# APPENDIX A



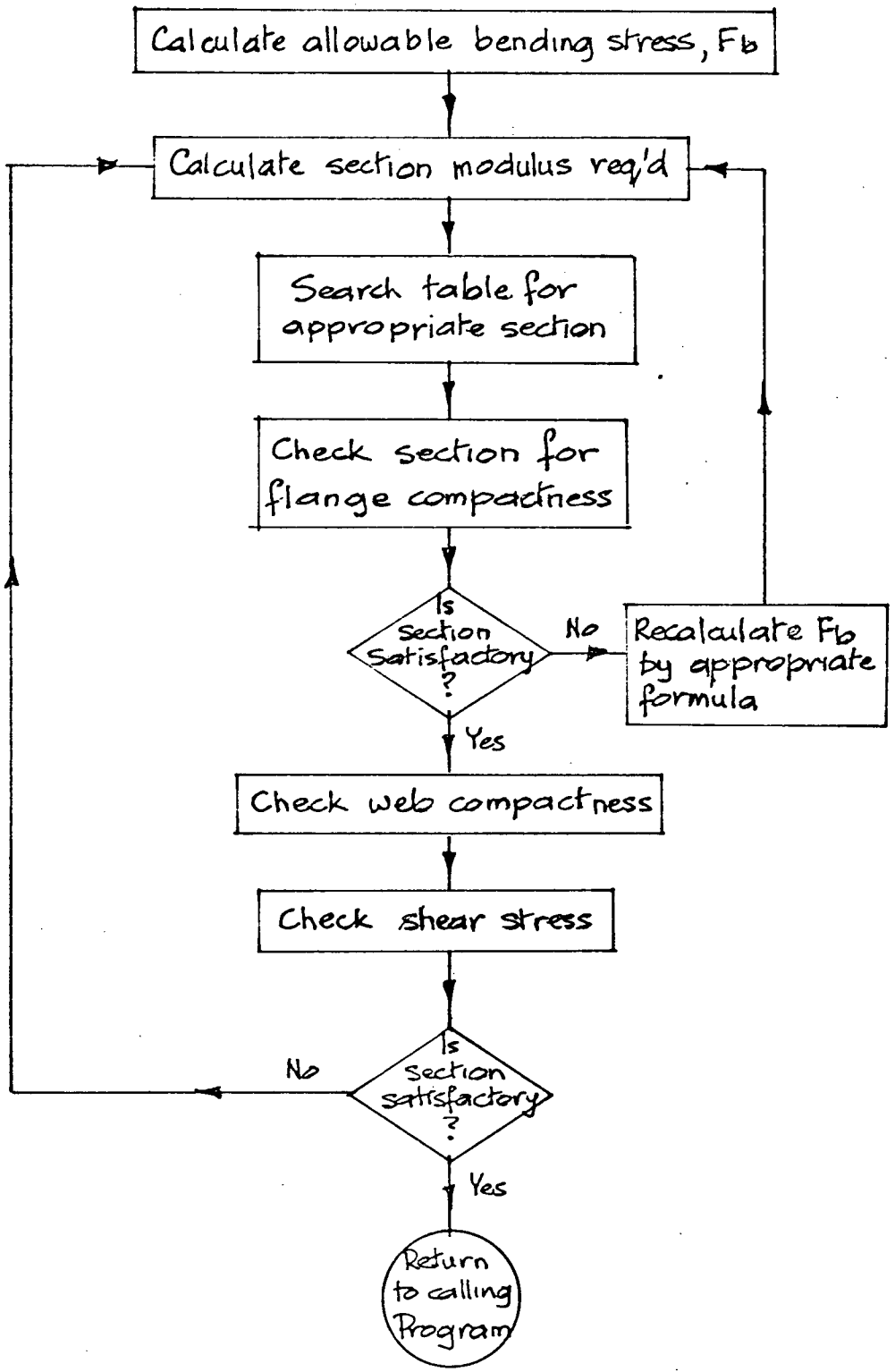

Flow Diagram for Analysis-Design Procedure



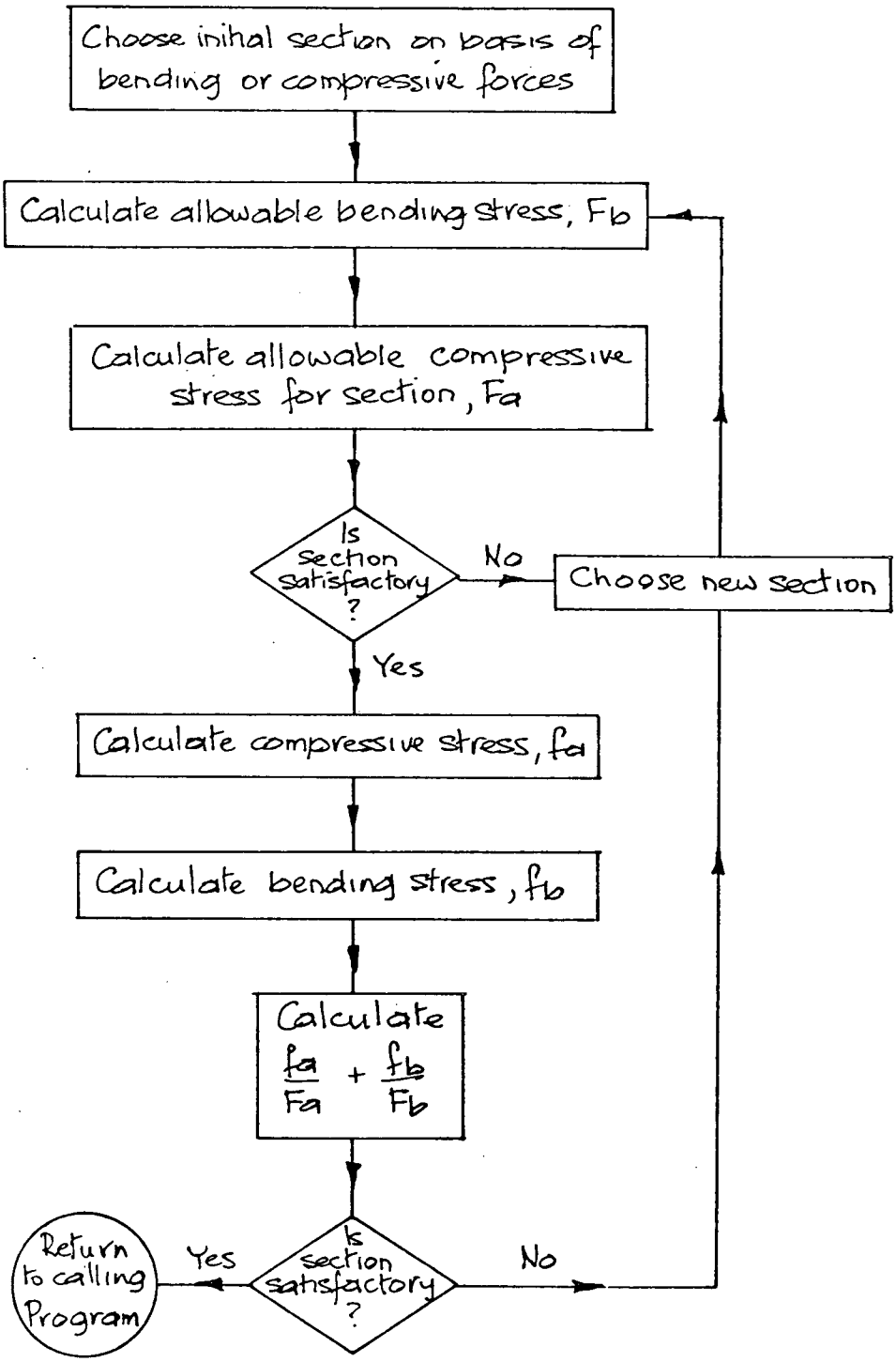

Flow Diagram for Design of Tension Member




Flow Diagram for Design of Compression Member




Flow Diagram for Design of Member Subject to Pure Bending




Flow Diagram for Design of Member Subject to Bending and Compression

# APPENDIX B

**********

TABLE OF SECTIONS

**********

WF BEAM SECTIONS

**********

| SIZE | AREA | DEPTH | FLANGE WIDTH | FLANGE THICK | WEB THICK | X-AXIS IX | X-AXIS SX | X-AXIS RX | Y-AXIS IY | Y-AXIS SY | Y-AXIS RY | WT/FT |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SMALL | 0.11 | 0.11 | 0.111 | 0.111 | 0.111 | 0.1 | 0.1 | 0.11 | 0.1 | 0.1 | 0.11 | 0.1 |
| 8JR6.5 | 1.95 | 8.00 | 2.281 | 0.189 | 0.135 | 18.7 | 4.7 | 3.12 | 0.3 | 0.3 | 0.42 | 6.5 |
| 10JR9 | 2.64 | 10.00 | 2.688 | 0.206 | 0.155 | 39.0 | 7.8 | 3.85 | 0.6 | 0.5 | 0.48 | 9.0 |
| 10B11.5 | 3.39 | 9.87 | 3.950 | 0.204 | 0.180 | 51.9 | 10.5 | 3.92 | 2.0 | 1.0 | 0.77 | 11.5 |
| 12JR11 | 3.45 | 12.00 | 3.063 | 0.225 | 0.175 | 72.2 | 12.0 | 4.57 | 1.0 | 0.6 | 0.53 | 11.8 |
| 12B14 | 4.14 | 11.91 | 3.970 | 0.224 | 0.200 | 88.2 | 14.8 | 4.61 | 2.3 | 1.1 | 0.74 | 14.0 |
| 12B16.5 | 4.86 | 12.00 | 4.000 | 0.269 | 0.230 | 105.3 | 17.5 | 4.65 | 2.8 | 1.4 | 0.76 | 16.5 |
| 14B17.2 | 5.05 | 14.00 | 4.000 | 0.272 | 0.210 | 147.3 | 21.0 | 5.40 | 2.7 | 1.3 | 0.72 | 17.2 |
| 12B19 | 5.62 | 12.16 | 4.010 | 0.349 | 0.240 | 130.1 | 21.4 | 4.81 | 3.7 | 1.8 | 0.81 | 19.0 |
| 10WF21 | 6.19 | 9.90 | 5.750 | 0.340 | 0.240 | 106.3 | 21.5 | 4.14 | 9.7 | 3.4 | 1.25 | 21.0 |
| 12WF22 | 6.47 | 12.31 | 4.030 | 0.424 | 0.260 | 156.0 | 25.3 | 4.91 | 4.6 | 2.3 | 0.85 | 22.0 |
| 14B22 | 6.48 | 13.72 | 5.000 | 0.335 | 0.230 | 197.4 | 28.8 | 5.52 | 6.4 | 2.6 | 0.99 | 22.0 |
| 14B26 | 7.65 | 13.89 | 5.025 | 0.418 | 0.255 | 242.6 | 34.9 | 5.63 | 8.3 | 3.3 | 1.04 | 26.0 |
| 16B26 | 7.66 | 15.65 | 5.500 | 0.345 | 0.250 | 298.1 | 38.1 | 6.24 | 8.7 | 3.2 | 1.07 | 26.0 |
| 14WF30 | 8.81 | 13.86 | 6.733 | 0.383 | 0.270 | 289.6 | 41.8 | 5.73 | 17.5 | 5.2 | 1.41 | 30.0 |
| 14WF34 | 10.00 | 14.00 | 6.750 | 0.453 | 0.287 | 339.2 | 48.5 | 5.83 | 21.3 | 6.3 | 1.46 | 34.0 |
| 16WF36 | 10.59 | 15.85 | 6.992 | 0.428 | 0.299 | 446.3 | 56.3 | 6.49 | 22.1 | 6.3 | 1.45 | 36.0 |
| 16WF40 | 11.77 | 16.00 | 7.000 | 0.503 | 0.307 | 515.5 | 64.4 | 6.62 | 26.5 | 7.6 | 1.50 | 40.0 |
| 16WF45 | 13.24 | 16.12 | 7.039 | 0.563 | 0.346 | 583.3 | 72.4 | 6.64 | 30.5 | 8.7 | 1.52 | 45.0 |
| 18WF45 | 13.25 | 17.86 | 7.477 | 0.499 | 0.355 | 704.5 | 78.9 | 7.30 | 31.9 | 8.5 | 1.55 | 45.0 |
| 16WF50 | 14.70 | 16.25 | 7.073 | 0.628 | 0.380 | 655.4 | 80.7 | 6.68 | 34.8 | 9.8 | 1.54 | 50.0 |
| 18WF50 | 14.71 | 18.00 | 7.500 | 0.570 | 0.358 | 800.6 | 89.0 | 7.38 | 37.2 | 9.9 | 1.59 | 50.0 |
| 18WF55 | 16.18 | 18.12 | 7.532 | 0.630 | 0.390 | 889.9 | 98.2 | 7.41 | 42.0 | 11.1 | 1.61 | 55.0 |
| 21WF55 | 16.19 | 20.80 | 8.215 | 0.522 | 0.375 | 1140.7 | 109.7 | 8.40 | 44.0 | 10.7 | 1.65 | 55.0 |
| 21WF62 | 18.23 | 20.99 | 8.240 | 0.615 | 0.400 | 1326.3 | 126.4 | 8.53 | 53.1 | 12.9 | 1.71 | 62.0 |
| 21WF68 | 20.00 | 21.13 | 8.270 | 0.680 | 0.430 | 1478.3 | 139.9 | 8.59 | 60.4 | 14.6 | 1.74 | 68.0 |
| 24WF68 | 20.01 | 23.71 | 8.961 | 0.582 | 0.416 | 1814.5 | 153.1 | 9.53 | 63.8 | 14.2 | 1.79 | 68.0 |

CONT.
WF BEAM SECTIONS
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| SIZE | AREA | DEPTH | FLANGE WIDTH | FLANGE THICK | WEB THICK | X-AXIS IX | X-AXIS SX | X-AXIS RX | Y-AXIS IY | Y-AXIS SY | Y-AXIS RY | WT/FT |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 24WF76 | 22.37 | 23.91 | 8.985 | 0.682 | 0.440 | 2096.4 | 175.4 | 9.68 | 76.5 | 17.0 | 1.85 | 76.0 |
| 24WF84 | 24.71 | 24.09 | 9.015 | 0.772 | 0.470 | 2364.3 | 196.3 | 9.78 | 88.3 | 19.6 | 1.89 | 84.0 |
| 27WF84 | 24.72 | 26.69 | 9.963 | 0.636 | 0.436 | 2824.8 | 211.7 | 10.69 | 95.7 | 19.2 | 1.97 | 84.0 |
| 24WF94 | 27.63 | 24.29 | 9.061 | 0.872 | 0.516 | 2683.0 | 220.9 | 9.85 | 102.2 | 22.6 | 1.92 | 94.0 |
| 27WF94 | 27.65 | 26.91 | 9.990 | 0.747 | 0.490 | 3266.7 | 242.8 | 10.87 | 115.1 | 23.0 | 2.04 | 94.0 |
| 30WF99 | 29.11 | 29.64 | 10.458 | 0.670 | 0.522 | 3988.6 | 269.1 | 11.70 | 116.9 | 22.4 | 2.00 | 99.0 |
| 30WF108 | 31.77 | 29.82 | 10.484 | 0.760 | 0.548 | 4461.0 | 299.2 | 11.85 | 135.1 | 25.8 | 2.06 | 108.0 |
| 30WF116 | 34.13 | 30.00 | 10.500 | 0.850 | 0.564 | 4919.1 | 327.9 | 12.00 | 153.2 | 29.2 | 2.12 | 116.0 |
| 33WF118 | 34.71 | 32.86 | 11.484 | 0.735 | 0.554 | 5886.9 | 358.3 | 13.02 | 170.3 | 29.7 | 2.22 | 118.0 |
| 33WF130 | 38.26 | 33.10 | 11.510 | 0.855 | 0.580 | 6699.0 | 404.8 | 13.23 | 201.4 | 35.0 | 2.29 | 130.0 |
| 36WF135 | 39.70 | 35.55 | 11.945 | 0.794 | 0.598 | 7796.1 | 438.6 | 14.01 | 207.1 | 34.7 | 2.28 | 135.9 |
| 33WF141 | 41.51 | 33.31 | 11.535 | 0.960 | 0.605 | 7442.2 | 446.8 | 13.39 | 229.7 | 39.8 | 2.35 | 141.0 |
| 36WF150 | 44.16 | 35.84 | 11.974 | 0.940 | 0.625 | 9012.1 | 502.9 | 14.29 | 250.4 | 41.8 | 2.38 | 150.0 |
| 36WF160 | 47.09 | 36.00 | 12.000 | 1.020 | 0.653 | 9738.8 | 541.0 | 14.38 | 275.4 | 45.9 | 2.42 | 160.0 |
| 36WF170 | 49.98 | 36.16 | 12.027 | 1.100 | 0.680 | 10470.0 | 579.1 | 14.47 | 300.6 | 50.0 | 2.45 | 170.0 |
| 36WF182 | 53.54 | 36.32 | 12.072 | 1.180 | 0.725 | 11281.5 | 621.2 | 14.52 | 327.7 | 54.3 | 2.47 | 182.0 |
| 36WF194 | 57.11 | 36.48 | 12.117 | 1.260 | 0.770 | 12103.4 | 663.6 | 14.88 | 355.4 | 58.7 | 2.49 | 194.0 |
| 33WF200 | 58.79 | 33.00 | 15.750 | 1.150 | 0.715 | 11048.2 | 669.6 | 13.71 | 691.7 | 87.8 | 3.43 | 200.0 |
| 33WF220 | 64.73 | 33.25 | 15.810 | 1.275 | 0.775 | 12312.1 | 740.6 | 13.79 | 782.4 | 99.0 | 3.48 | 220.0 |
| 36WF230 | 67.73 | 35.88 | 16.475 | 1.260 | 0.765 | 14988.4 | 835.5 | 14.88 | 870.9 | 105.7 | 3.59 | 230.0 |
| 36WF245 | 72.03 | 36.06 | 16.512 | 1.350 | 0.802 | 16092.2 | 892.5 | 14.95 | 944.7 | 114.4 | 3.62 | 245.0 |
| 36WF260 | 76.56 | 36.24 | 16.555 | 1.440 | 0.845 | 17233.8 | 951.1 | 15.00 | 1020.6 | 123.3 | 3.65 | 260.0 |
| 36WF280 | 82.32 | 36.50 | 16.595 | 1.570 | 0.885 | 18819.3 | 1031.2 | 15.12 | 1127.5 | 135.9 | 3.70 | 280.0 |
| 36WF300 | 88.17 | 36.72 | 16.655 | 1.680 | 0.945 | 20290.2 | 1105.1 | 15.17 | 1225.2 | 147.1 | 3.73 | 300.0 |
| TOO BIG | 99.99 | 36.99 | 20.000 | 1.999 | 0.999 | 99999.9 | 9999.9 | 20.00 | 3000.0 | 300.0 | 9.99 | 800.0 |

WF COLUMN SECTIONS
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| SIZE | AREA | DEPTH | FLANGE WIDTH | FLANGE THICK | WEB THICK | X-AXIS IX | X-AXIS SX | X-AXIS RX | Y-AXIS IY | Y-AXIS SY | Y-AXIS RY | WT/FT |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SMALL | 0.11 | 0.11 | 0.111 | 0.111 | 0.111 | 0.1 | 0.1 | 0.11 | 0.1 | 0.1 | 0.11 | 0.1 |
| 6W12 | 3.54 | 6.00 | 4.000 | 0.279 | 0.230 | 21.7 | 7.3 | 2.48 | 2.9 | 1.5 | 0.92 | 12.0 |
| 4W13 | 3.82 | 4.16 | 4.060 | 0.345 | 0.280 | 11.3 | 5.5 | 1.72 | 3.8 | 1.9 | 0.99 | 13.0 |
| 8W17 | 5.01 | 8.00 | 5.250 | 0.308 | 0.230 | 56.5 | 14.1 | 3.36 | 7.4 | 2.8 | 1.22 | 17.0 |
| 5W18.5 | 5.43 | 5.12 | 5.025 | 0.420 | 0.265 | 25.4 | 9.9 | 2.16 | 8.9 | 3.5 | 1.28 | 18.5 |
| 6W20 | 5.88 | 6.20 | 6.018 | 0.367 | 0.258 | 41.5 | 13.4 | 2.66 | 13.3 | 4.4 | 1.51 | 20.0 |
| 8W24 | 7.06 | 7.93 | 6.500 | 0.398 | 0.245 | 82.5 | 20.8 | 3.42 | 18.2 | 5.6 | 1.61 | 24.0 |
| 8W28 | 8.23 | 8.06 | 6.540 | 0.463 | 0.285 | 97.8 | 24.3 | 3.45 | 21.6 | 6.6 | 1.62 | 28.0 |
| 8M32.6 | 9.56 | 8.00 | 7.940 | 0.459 | 0.315 | 114.0 | 28.4 | 3.44 | 34.1 | 8.6 | 1.89 | 32.6 |
| 8W35 | 10.30 | 8.12 | 8.027 | 0.493 | 0.315 | 126.0 | 31.1 | 3.50 | 42.5 | 10.6 | 2.03 | 35.0 |
| 8W40 | 11.80 | 8.25 | 8.077 | 0.558 | 0.365 | 146.0 | 35.5 | 3.53 | 49.0 | 12.1 | 2.04 | 40.0 |
| 8W48 | 14.10 | 8.50 | 8.117 | 0.683 | 0.405 | 184.0 | 43.2 | 3.61 | 60.9 | 15.0 | 2.03 | 48.0 |
| 12W53 | 15.60 | 12.06 | 10.000 | 0.576 | 0.345 | 426.0 | 70.7 | 5.23 | 96.1 | 19.2 | 2.48 | 53.0 |
| 10W54 | 15.90 | 10.12 | 10.028 | 0.618 | 0.368 | 306.0 | 60.4 | 4.39 | 104.0 | 20.7 | 2.56 | 54.0 |
| 10W60 | 17.70 | 10.25 | 10.075 | 0.683 | 0.415 | 344.0 | 67.1 | 4.41 | 116.0 | 23.1 | 2.57 | 60.0 |
| 12W65 | 19.10 | 12.12 | 12.000 | 0.606 | 0.390 | 533.0 | 88.0 | 5.28 | 175.0 | 29.1 | 3.01 | 65.0 |
| 12W72 | 21.20 | 12.25 | 12.040 | 0.671 | 0.430 | 597.0 | 97.5 | 5.31 | 195.0 | 32.4 | 3.04 | 72.0 |
| 14W78 | 22.90 | 14.06 | 12.000 | 0.718 | 0.428 | 851.0 | 121.0 | 6.09 | 207.0 | 34.5 | 3.00 | 78.0 |
| 12W79 | 23.20 | 12.38 | 12.080 | 0.736 | 0.470 | 663.0 | 107.0 | 5.34 | 216.0 | 35.8 | 3.05 | 79.0 |
| 12W85 | 25.00 | 12.50 | 12.105 | 0.796 | 0.495 | 723.0 | 116.0 | 5.38 | 235.0 | 38.9 | 3.07 | 85.0 |
| 14W87 | 25.60 | 14.00 | 14.500 | 0.688 | 0.420 | 967.0 | 138.0 | 6.15 | 350.0 | 48.2 | 3.70 | 87.0 |
| 14W95 | 27.90 | 14.12 | 14.545 | 0.748 | 0.465 | 1060.0 | 151.0 | 6.17 | 384.0 | 52.8 | 3.71 | 95.0 |
| 14W103 | 30.30 | 14.25 | 14.575 | 0.813 | 0.495 | 1170.0 | 164.0 | 6.21 | 420.0 | 57.6 | 3.72 | 103.0 |
| 14W111 | 32.70 | 14.37 | 14.620 | 0.873 | 0.540 | 1270.0 | 176.0 | 6.23 | 455.0 | 62.2 | 3.73 | 111.0 |
| 14W119 | 35.00 | 14.50 | 14.650 | 0.938 | 0.570 | 1370.0 | 189.0 | 6.26 | 492.0 | 67.1 | 3.75 | 119.0 |

CONT.
WF COLUMN SECTIONS
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| SIZE | AREA | DEPTH | FLANGE WIDTH | FLANGE THICK | WEB THICK | X-AXIS IX | X-AXIS SX | X-AXIS RX | Y-AXIS IY | Y-AXIS SY | Y-AXIS RY | WT/FT |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 14W127 | 37.30 | 14.62 | 14.690 | 0.998 | 0.610 | 1480.0 | 202.0 | 6.29 | 528.0 | 71.8 | 3.76 | 127.0 |
| 14W136 | 40.00 | 14.75 | 14.740 | 1.063 | 0.660 | 1590.0 | 216.0 | 6.31 | 568.0 | 77.0 | 3.77 | 136.0 |
| 14W142 | 41.80 | 14.75 | 15.500 | 1.063 | 0.680 | 1670.0 | 227.0 | 6.32 | 660.0 | 85.2 | 3.97 | 142.0 |
| 14W150 | 44.10 | 14.88 | 15.515 | 1.128 | 0.695 | 1790.0 | 240.0 | 6.37 | 703.0 | 90.6 | 3.99 | 150.0 |
| 14W158 | 46.50 | 15.00 | 15.550 | 1.188 | 0.730 | 1900.0 | 253.0 | 6.40 | 745.0 | 95.8 | 4.00 | 158.0 |
| 14W167 | 49.10 | 15.12 | 15.600 | 1.248 | 0.780 | 2020.0 | 267.0 | 6.42 | 790.0 | 101.0 | 4.01 | 167.0 |
| 14W176 | 51.70 | 15.25 | 15.640 | 1.313 | 0.820 | 2150.0 | 282.0 | 6.45 | 838.0 | 107.0 | 4.02 | 176.0 |
| 14W184 | 54.10 | 15.38 | 15.660 | 1.378 | 0.840 | 2270.0 | 296.0 | 6.49 | 883.0 | 113.0 | 4.04 | 184.0 |
| 14W193 | 56.70 | 15.50 | 15.710 | 1.438 | 0.890 | 2400.0 | 310.0 | 6.51 | 930.0 | 118.0 | 4.05 | 193.0 |
| 14W202 | 59.40 | 15.63 | 15.750 | 1.503 | 0.930 | 2540.0 | 325.0 | 6.54 | 980.0 | 124.0 | 4.06 | 202.0 |
| 14W211 | 62.10 | 15.75 | 15.800 | 1.563 | 0.980 | 2670.0 | 339.0 | 6.56 | 1030.0 | 130.0 | 4.07 | 211.0 |
| 14W219 | 64.40 | 15.87 | 15.825 | 1.623 | 1.030 | 2800.0 | 353.0 | 6.59 | 1070.0 | 136.0 | 4.08 | 219.0 |
| 14W228 | 67.10 | 16.00 | 15.865 | 1.688 | 1.040 | 2940.0 | 368.0 | 6.62 | 1120.0 | 142.0 | 4.10 | 228.0 |
| 14W237 | 69.70 | 16.12 | 15.910 | 1.748 | 1.090 | 3080.0 | 382.0 | 6.65 | 1170.0 | 148.0 | 4.11 | 237.0 |
| 14W246 | 72.30 | 16.25 | 15.945 | 1.813 | 1.130 | 3230.0 | 392.0 | 6.68 | 1230.0 | 154.0 | 4.12 | 246.0 |
| 14W264 | 77.60 | 16.50 | 16.025 | 1.938 | 1.210 | 3530.0 | 427.0 | 6.74 | 1330.0 | 166.0 | 4.14 | 264.0 |
| 14W287 | 84.40 | 16.81 | 16.130 | 2.093 | 1.310 | 3910.0 | 465.0 | 6.81 | 1470.0 | 182.0 | 4.17 | 287.0 |
| 14W314 | 92.30 | 17.19 | 16.235 | 2.283 | 1.420 | 4400.0 | 512.0 | 6.90 | 1630.0 | 201.0 | 4.20 | 314.0 |
| 14W320 | 94.10 | 16.81 | 16.710 | 2.093 | 1.890 | 4140.0 | 493.0 | 6.53 | 1640.0 | 196.0 | 4.17 | 320.0 |
| 14W342 | 101.00 | 17.56 | 16.365 | 2.468 | 1.550 | 4910.0 | 559.0 | 6.99 | 1810.0 | 221.0 | 4.24 | 342.0 |
| 14W370 | 109.00 | 17.94 | 16.475 | 2.658 | 1.660 | 5450.0 | 608.0 | 7.08 | 1990.0 | 241.0 | 4.27 | 370.0 |
| 14W398 | 117.00 | 18.31 | 16.590 | 2.843 | 1.770 | 6010.0 | 657.0 | 7.17 | 2170.0 | 262.0 | 4.31 | 398.0 |
| 14W426 | 125.00 | 18.69 | 16.695 | 3.033 | 1.880 | 6610.0 | 707.0 | 7.25 | 2360.0 | 283.0 | 4.34 | 426.0 |
| 14W455 | 134.00 | 19.05 | 16.828 | 3.213 | 2.010 | 7220.0 | 758.0 | 7.35 | 2560.0 | 304.0 | 4.37 | 455.0 |
| 14W500 | 147.00 | 19.63 | 17.008 | 3.501 | 2.190 | 8250.0 | 840.0 | 7.49 | 2880.0 | 339.0 | 4.43 | 500.0 |
| 14W550 | 162.00 | 20.26 | 17.206 | 3.818 | 2.390 | 9450.0 | 933.0 | 7.64 | 3260.0 | 378.0 | 4.49 | 550.0 |
| 14W605 | 178.00 | 20.94 | 17.418 | 4.157 | 2.600 | 10900.0 | 1040.0 | 7.81 | 3680.0 | 423.0 | 4.55 | 605.0 |
| 14W655 | 196.00 | 21.67 | 17.646 | 4.522 | 2.830 | 12500.0 | 1150.0 | 7.99 | 4170.0 | 472.0 | 4.62 | 655.0 |
| 14W730 | 215.00 | 22.44 | 17.889 | 4.910 | 3.070 | 14400.0 | 1280.0 | 8.18 | 4720.0 | 527.0 | 4.69 | 730.0 |
| TOO BIG | 990.90 | 36.99 | 20.000 | 5.999 | 0.500 | 99999.9 | 9999.9 | 20.00 | 9000.0 | 900.0 | 9.99 | 800.0 |