A COMPARATIVE STUDY OF THE SIMULATION OF DAILY STREAMFLOW SEQUENCES

by

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ABSTRACT

Using three years of daily streamflow and meteorological data from the Similkameen watershed at Princeton, B.C., the model parameters of the existing deterministic UBC Budget Model are evaluated. With these model parameters and the available meteorological data, the synthetic streamflow sequences are generated for the other seven years for the Similkameen watershed. These are subsequently compared with the actual flows.

A separate statistical stochastic model is developed by using the spectral analysis, and the three years of the same daily flows are decomposed into 30 sub-harmonics or Fourier coefficients. By interpolating the Fourier coefficients and by estimating the anticipated mean annual flows from the snowpack data at Blackwall Peak, the synthetic traces of the daily streamflow sequences are simulated for the other seven years. A first order Markovian model is used to explain the random component. The comparative study is then carried out between the actual daily streamflow sequences and those generated by the deterministic UBC Budget Model and the stochastic spectral model.

In comparison with the stochastic spectral model, good fits are obtained with the fixed model parameters of the UBC Budget Model for the sequence of peaks for the simulated hydrographs of the intervening years. Since the winter melt factor

in the UBC Budget Model was assumed to be a constant for this analysis, some errors occur between the actual and the generated cumulative volumes. With the deterministic periodic component of the spectral model, the reconciliation between the cumulative volumes is fairly well maintained. Since the role of operational hydrology is not concerned with the prediction of actual flows, the stochastic spectral model should be judged on its ability in presenting the designer with a series of synthetic traces that are likely to occur during the lifetime of a particular project.

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CHAPTER I

INTRODUCTION

I.1 OBJECTIVES OF RESEARCH

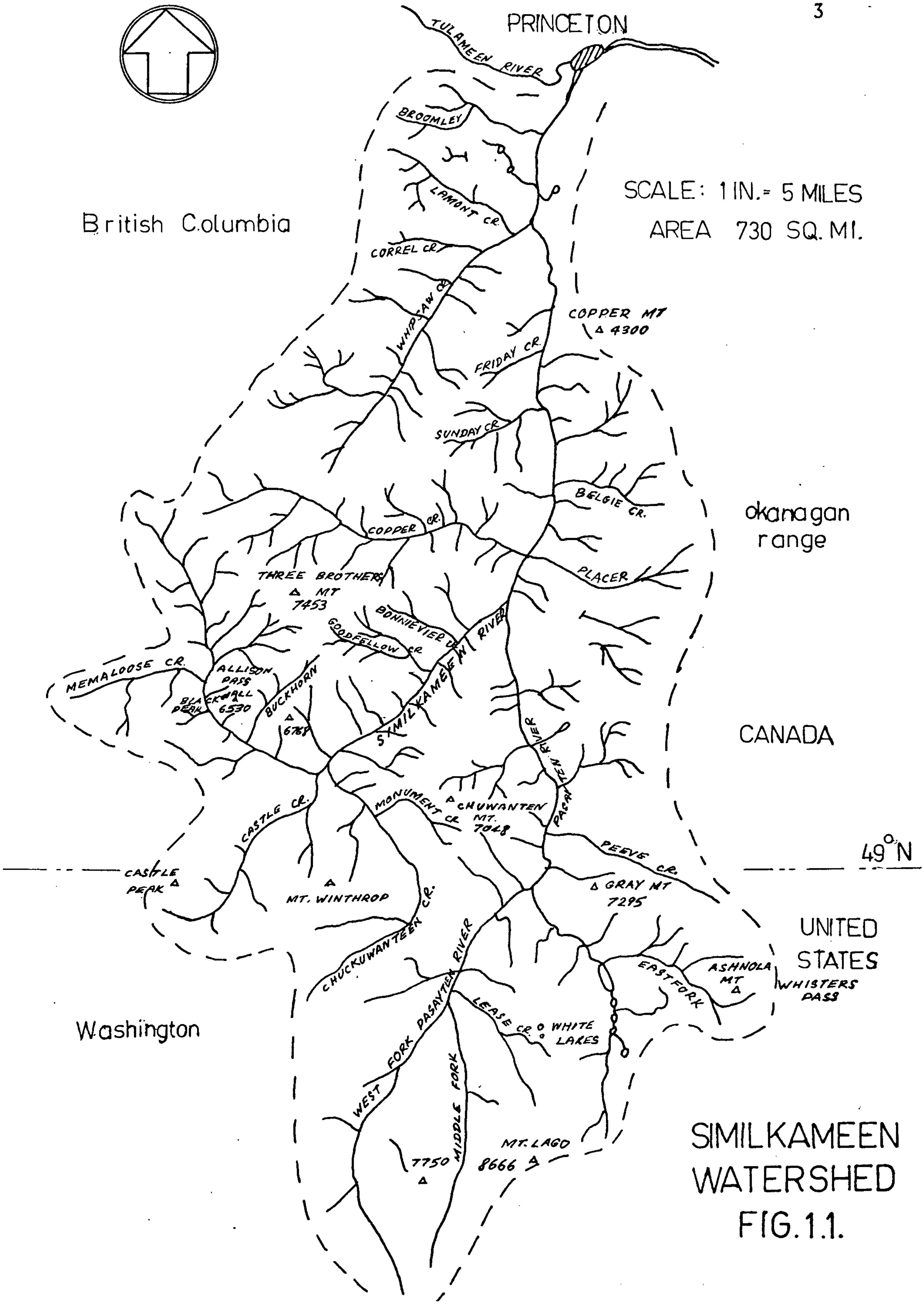
The research is concerned with the shortage of streamflow records for some watersheds and it is involved with the possibility of generating daily synthetic streamflow sequences from the available records. Two separate approaches are taken in analysing and solving this problem. The first approach is concerned with the use of the longer meteorological records and the existing UBC Budget Model. The second approach is concerned with the spectral analysis of the available streamflow records and the use of the estimated mean annual flows from the meteorological data.

In estimating the synthetic sequences of daily streamflows based on the interpolated values of the sub-harmonics of the available streamflow records and the estimated mean annual flows, it should be noted that the generated sequences of the spectral model are not the duplication of the actual flow sequences of the past years. If it is possible to evaluate accurate parametric values for the UBC Budget Model, then the generated daily streamflow sequences should resemble the actual streamflow series of the past years. This raises the question on the evaluation of the techniques or methods used for the generation of the daily streamflow sequences. But this would evidently reflect on the particular objectives of the hydrologist and on the availability of adequate meteorological records.

From the few years of available streamflow data and from the meteorological data, the realistic model parameters for the UBC Budget Model are fixed and improved on a trial and error basis until a good fit is obtained for the actual and the reconstituted hydrographs. The reconciliation of the cumulative flows for the actual and reconstituted hydrographs is the other requirement.

For the Similkameen River at Princeton, with a natural watershed area of 730 square miles, three years of streamflow data for 1963, 1964 and 1970 are concurrently used with the meteorological and snowpack data for fixing the model parameters. Having accomplished this, the streamflow sequences for the intervening years from 1961 to 1970 are generated by using the relevant meteorological and snowpack data. The generated streamflow sequences are subsequently compared with the actual streamflow series.

Using the alternative statistical spectral model, the daily streamflow series for the years 1963, 1964 and 1970 are decomposed respectively to its sub-harmonics or Fourier coefficients. The available snowpack data at Blackwall Peak (6350 feet) is used to estimate the mean annual flows for the intervening years, and by interpolating the values of the Fourier coefficients to estimate the spectrum for the relevant years, hydrographs are generated and compared with the actual hydrographs and with the hydro-



SIMILKAMEEN WATERSHED

FIG. 1.1.

graphs obtained from the UBC Budget Model. The statistical tests that could be applied to both the UBC Budget Model and the stochastic spectral model are the X^2 and the F tests.

Initially the research was intended to be applied to a practical situation like the Trout Creek watershed, which is adjacent to the Similkameen watershed. This is a regulated watershed and difficulty was experienced in fitting the UBC Budget Model to this watershed. Hence the Similkameen watershed is selected for investigating the possibility of generating streamflow sequences and for carrying out a comparative study on the advantages and limitations of the spectral model.

The UBC Budget Model is a deterministic model, and since it takes into consideration the mean daily temperature, precipitation, pan evaporation, areal distribution of the snowpack water equivalent and the band switch times, more realistic hydrographs are obtained. However, it is quite difficult to estimate the areal distribution of the snowpack since the number of gauging stations are limited. The reconstituted hydrograph covers a period of 182 days, from the 2nd of March to the 30th of August.

The statistical spectral model or stochastic model is used for generating synthetic traces of daily streamflow sequences and it should not be considered as an analytical solution. It is a stochastic model since it is the combination of a deterministic periodic component and a random component. The random component explains the residuals in the form of a first order autoregressive or Markovian model.

I.2 INTERACTION OF WEATHER SYSTEMS AND HYDROLOGIC PROCESSES

During the preliminary analysis, feasibility studies on the possibility of incorporating a random component in the flow model for the migratory low and high pressure systems, which control the weather and precipitation, were carried out. Attempts were also made to find a relationship between precipitation and the weather patterns.

Better definitions of the interaction of the various geophysical phenomena should enable the hydrologist of the future to evaluate more realistic parameters for deterministic and stochastic models. With the increase in the time scale, it has been found to be difficult to prove the existence of deterministic and periodic trends in the geophysical phenomena. Nevertheless, hydrologists are able to evaluate realistic model parameters related to time series in the order of hours, days and months.

The characteristic approach in synoptic meteorology is involved with the use of weather maps showing the geographical distribution of temperature, pressure, wind, humidity and weather patterns. These are issued by the weather stations at several agreed times during the day. The subsequent task is to analyze the ever-changing distribution of the weather system and to predict forthcoming changes by empirical extrapolation.

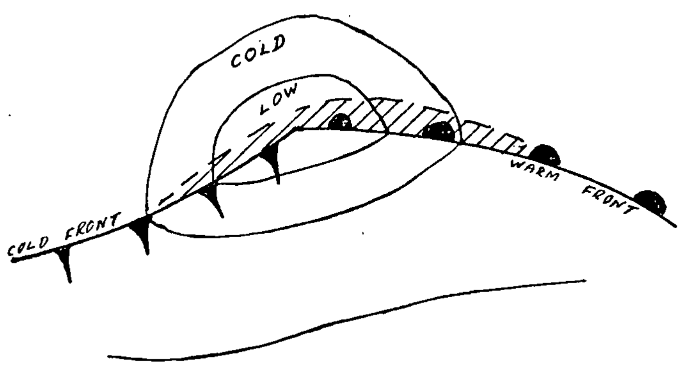
The most important factors that control the wind and the weather are the travelling depressions or cyclones and the anticyclone systems. These develop as entities within a matter of hours and go through a typical life cycle of maturity, decay and dissolution within a few days. Cyclones are areas of low barometric pressure and anticyclones are areas of high barometric pressure. The number of cyclones and anticyclones vary from time to time. The continuous sequence of migratory highs and lows move more or less eastwards in the middle and higher latitudes of the northern hemisphere. The intensity, duration and speed of the migratory cells vary from day to day. They tend to be more numerous, stronger and move at a greater speed during winter than in summer. The changes of the geostropic winds with elevation is of vital importance in understanding the structure and mechanism of the cyclonic and anticyclonic disturbances. This is of particular significance in weather forecasting. Abnormal distribution of temperature also results in the changes of the geotrophic wind with elevation.

The seat of the weather is considered as the middle or upper troposphere and for levels where the pressure is 300 to 500 millibars. This region is in the heart of the upper winds. Stagnation of the upper winds generally causes stagnation in the progression of weather.

The front, which is normally represented by a single boundary line, indicates the narrow bands of transition between the warm and cool air masses. The continuation of the boundary through the atmosphere results in a frontal surface, and this causes the warmer air mass to lie above the cooler air mass.

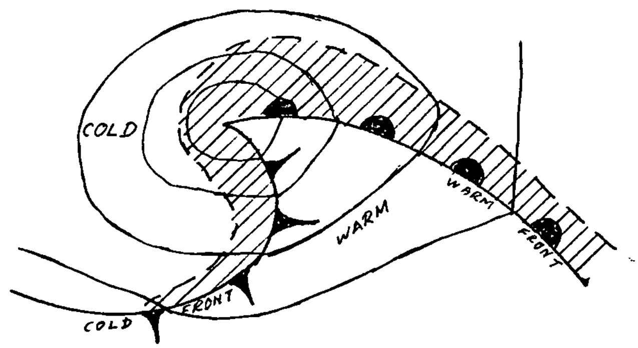
The upsliding at this surface results in the formation of clouds and rain. The dynamical theory has been applied to explain the cycle of development, maturity and decay, but so far no convincing explanation pertaining to the true characteristic behaviour of the fronts has been made available.

Bjerknes (17) explains the cyclonic development according to the Bergen school of meteorologists and this is shown in Figure 1.2. Considering the northern hemisphere, a typical depression is diagnosed as a disturbance going through a life cycle of formation, maturity and decay. In the initial stages the warm and cool air masses lie alongside each other in a quiescent state and there is a relatively slow movement. The warm air mass moving from west to east eventually shears past the relatively slow moving cooler air mass. The characteristic gradual fall in pressure over an area, generally several hundred square miles, with the subsequent circulation of winds and the wave-like distortion of the front, indicates the formation of a depression. The front is carried along by the winds and the area ahead of the centre is called the warm front. This warm front will replace the cold air by the warm air. To the west of the centre, the existing circulation replaces the warm air by the cold air at the cold front. The warm sector is the area between the warm and the following cold front. The above process will be maintained by a continuous fall in pressure or the deepening of the depression, a resulting increase in the speed of the wind, often reaching to the strength



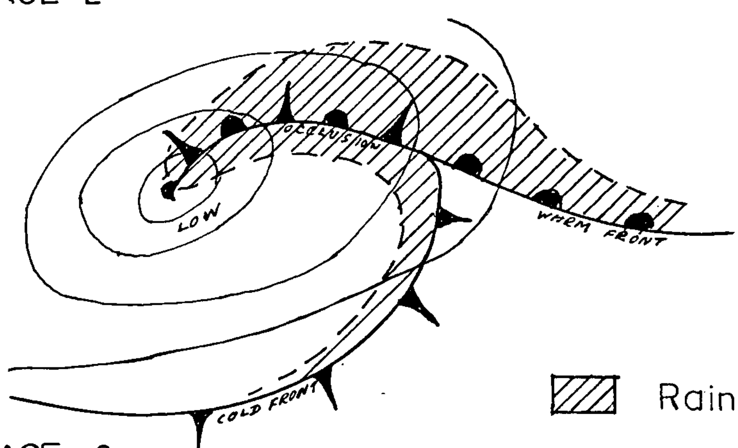
Initiation: shallow depression and wave like distortion of the polar front.

STAGE 1



Rapid development: deepening depression and large warm sector

STAGE 2



Fully developed: deep occluded depression

Rain Areas

STAGE 3

CYCLONIC DEVELOPMENT ACCORDING TO BERGEN SCHOOL (after Bjerknes)

FIG. 1.2.

of gale forces and by a continuous distortion of the fronts. This process is terminated with the formation of an 'occlusion' when the cold front gradually overtakes the warm front. Cirrus clouds appear ahead of the moving depression and this is followed by altostratus and nimbostratus clouds, accompanied with falling rain. The whole system may cover several hundred square miles and extend outwards in a radial direction from the centre of the depression.

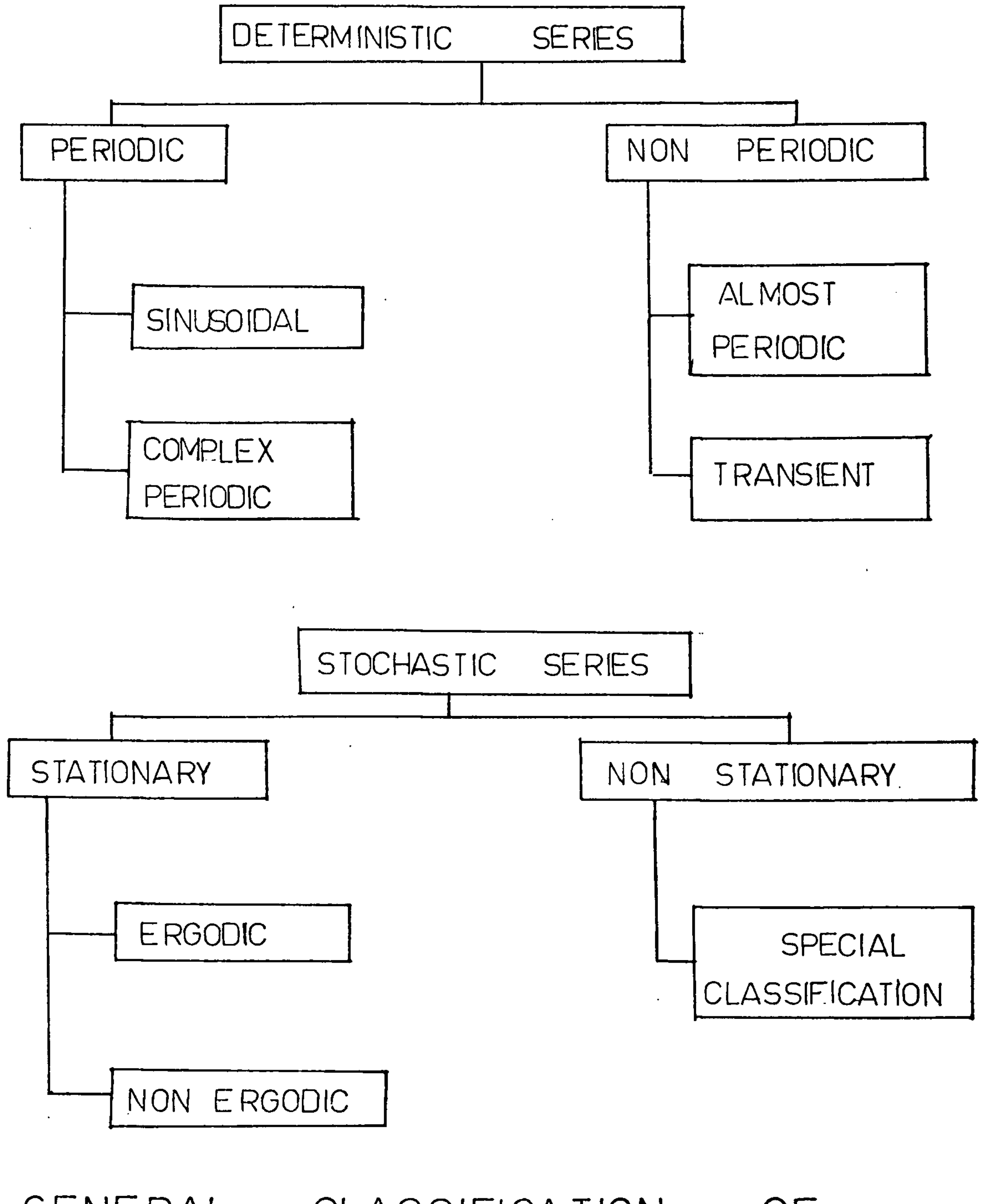
In hydrometeorology, long range studies will remain empirical until some working solution is discovered. To verify the existence of some periodic variation in the weather, a characteristic rhythm must be recognized. Seeking correlations without a good physical reasoning is a hazardous operation. Subsequently it may be proved to be an accidental coincidence without any predictive value. The drawbacks in synoptic meteorology are related to the limited scope for the prediction of the mean daily temperature and the total precipitation for the coming month.

However the theoretical explanation of the interaction of the various geophysical phenomena will still remain a formidable task. Studies have been made with some reasonable success in correlating precipitation with the phase of the moon, and the mean annual discharge with the sunspot cycle. The sunspot cycle occurs once in 11¼ years.

All geophysical phenomena, such as the precipitation, runoff, evaporation, groundwater levels, lake levels, sediment transport and river water quality, have deterministic components, which are basically periodic series. The main cyclic frequency for monthly values is 1/12 and the cyclic frequency for daily values is 1/365. A complex periodic component is present in the daily discharges affected by the melting of ice and snow, and in the daily cycle of radiation, temperature and evaporation. These complex periodic components are also associated and combined with the stochastic components of the greater and smaller effects on the series.

I.3 METHODS OF INVESTIGATING HYDROLOGIC PROCESSES

Deterministic models are generally used for short-range analysis and these are concerned with the input-output type of modelling. Stochastic models are used for long-range studies, and the main objective is to produce a set of traces sufficiently long and statistically identical to the historical record. However, most models are influenced by the particular aims and interests and the background of the hydrologist. The role of operational hydrology is not concerned with the prediction of flows, but the emphasis is rather in presenting the designer with a spectrum of synthetic traces, each of which is equally likely to be the actual sequence during the life of a particular project. Stochastic models are frequently used in reservoir yield studies, and the statistics of interests are related to runs and run sums. These models are gradually becoming more accepted as tools for hydrologic planning and design.



GENERAL CLASSIFICATION OF SEQUENCES IN HYDROLOGY

(after Yevjevich)

FIG. 1.3.

I.3.1 Deterministic Techniques. The watershed is treated as a hydrologic system with an input consisting of precipitation and accumulated winter snowpack, and an output consisting of runoff. The formulation of the hydrologic model is evolved on the basis of the principle of the conservation of mass, and comprises the component processes involving soil moisture deficit, sub-surface runoff, fast runoff, slow runoff, pan evaporation and storage in the watershed.

I.3.2 Stochastic Techniques. The particular mathematical techniques developed for investigating stochastic hydrologic processes are:

(a) Moving Average Model. This is expressed as:

$$\mu_t = a_1\varepsilon_t + a_2\varepsilon_{t-1} + \ . \ . \ . \ + a_m\varepsilon_{t-m+1} \qquad (I.1)$$

where

$a_1, a_2, . . . a_m$ are the weights;

m is the number of significant moving averages;

ε is the random variable.

For example, a relationship could be developed between the annual runoff, μ and the annual effective precipitation ε. The weights a_1, a_2, . . ., a_m must be positive and equal to unity, and m is the extent of the carry-over related to the water retardation characteristics of the particular watershed.

(b) Method of Autocorrelation Coefficients. The use of autoregressive techniques in the modelling of time series is a well-developed statistical procedure. This model may be expressed as:

$$\mu_t = f(\mu_{t-1}, \mu_{t-2}, \ldots, \mu_{t-k}) + \varepsilon_t \tag{I.2}$$

where

f( ) is a mathematical function;

k is an integer;

ε_t is a random variable.

The typical form of the linear autoregression model of the m^{th} order is expressed as:

$$\mu_t = a_1\mu_{t-1} + a_2\mu_{t-2} + \ldots + a_m\mu_{t-m} + \varepsilon_t \tag{I.3}$$

where

$a_1, a_2, \ldots a_m$ are the regression coefficients;

ε_t is a random variable.

For m = 1, the linear autoregression model is transformed into the first order Markovian process, and this is expressed as:

$$\mu_t = a\mu_{t-1} + \varepsilon_t \tag{I.4}$$

where

a is the Markovian process coefficient;

ε_t is a random variable.

(c) Method of Spectral Densities. For a discrete sample series, the model may be expressed as:

$$\mu_t = \mu_x + \sum_{j=1}^{m} (A_j \cos \frac{2\pi jt}{T} + B_j \sin \frac{2\pi jt}{T}) + \varepsilon_t \quad (I.5)$$

where

μ_x is the mean of the series;

A_j and B_j are the amplitudes or Fourier coefficients;

$\frac{2\pi jt}{T}$ is the period of cyclicity with $j = 1, 2, \ldots, m$;

ε_t is a random variable.

The above model is characterized by an oscillatory or regular form of variation, which is present in the diurnal, seasonal and secular changes of the hydrologic phenomena.

(d) Method of Mean Ranges. This is defined as the maximum difference of the sum of deviations from the mean for the values of a sub-series 1, 2, . . ., n. The expected value is dependent on the size of the sub-series, $ER_n = f(n)$. This technique is used for investigating hydrologic processes and is related to water storage problems, and water surplus and deficit problems.

(e) Method of Mean Runs. The run is defined as the sum of deviations above or below a given value of the series, known as the crossing level. The expected value of the run-length or the expected value of the run-sum may be related to the crossing level x_o, or to the probability of occurrence q of values smaller than or equal to x_o. This technique is used for investigating the dependence of hydrologic sequences, and this is closely related to the various problems of extremes such as droughts, floods, water surplus and deficits.

Deterministic and probabilistic models do not conform closely with the natural phenomena, and subsequently lead to the overdesigning or underdesigning of hydrologic projects. However the actual choice of a mathematical model would inevitably reflect on the particular aims of the hydrologist.

CHAPTER II

UBC BUDGET MODEL

II.1 APPLICATION OF THE UBC BUDGET MODEL

The UBC Budget Model consists of a main program and eight separate sub-routines. Using three years of streamflow and meteorological data from the Similkameen watershed for the years 1963, 1964 and 1970, the Model parameters are evaluated on the basis of the reconstitution of the actual hydrograph. The winter melt is a factor which varies from year to year and it affects the final volume of the estimated runoff. For this particular research project the winter melt factor is assumed to be a constant, so that the feasibility studies for generating and comparing the synthetic daily streamflow sequences for the other years using only meteorological and snowpack data could be carried out.

The UBC Budget Model is expressed in terms of inputs, outputs and transformations. The term 'black box' refers to the system that transforms the inputs into outputs, and this form of simulation could be considered as an indirect investigation of the response or behaviour of the watershed. The input consists of data for temperature, precipitation, snowpack water equivalent and pan evaporation. This data is available from the *Monthly Record of Meteorological Observations*, *Canada* and from the British

Columbia *Snow Survey Bulletin*. Daily streamflow values for the Similkameen watershed at Princeton are available from the *Surface Water Data*, British Columbia and from the *Water Resources Paper - Surface Water Data for Pacific Drainage*. Princeton is selected for the temperature and precipitation data since no other meteorological station is located in the interior of the watershed. Blackwell Peak (6,350 feet) provides 13 years of snowpack data from 1960 to 1972, and this station is selected since the data from Copper Mountain (4,300 feet) did not produce representative hydrographs. Pan evaporation data is taken from Summerland, and this is the only available station in the interior of British Columbia. The areal distribution of snowpack water equivalent on the different elevation bands is approximated by using a semi-logarithmic plot similar to that used in the Carrs Landing Watershed (9). The other relevant input data are the arbitrary soil moisture deficits, land areas, elevation bands, elevations of the meteorological stations, and the control parameters for the graphical output. The temperature is lapsed over the different elevation bands with the International Standard Atmosphere value of $3.5^{\circ}F/1{,}000$ feet.

The other sub-routines are concerned with the division of runoff into fast or surface flow, medium or interflow and slow or groundwater recharge. The daily streamflow is eventually routed to a channel system. The amount and form of the daily precipitation is evaluated from the precipitation elevation relationship. The potential evapotranspiration at each mid-band elevation is estimated from the monthly pan evaporation data and the maximum temperature data. The computation of the areal recession of the snowpack and the depletion of the snowmelt runoff potential are carried out. The daily water budget at each elevation band is evaluated from the daily runoff excesses from the snowmelt and the rainfall inputs after having given consideration to the soil moisture deficiencies, losses to the groundwater recharge and to the daily evapotranspiration. Having given allowance for the impermeability factor, the effective daily basin input is computed from the cumulative band excesses. Finally, the daily runoff is generated on the basis of the unit hydrograph convolution.

II.2 EVALUATION OF MODEL PARAMETERS FOR THE UBC BUDGET MODEL

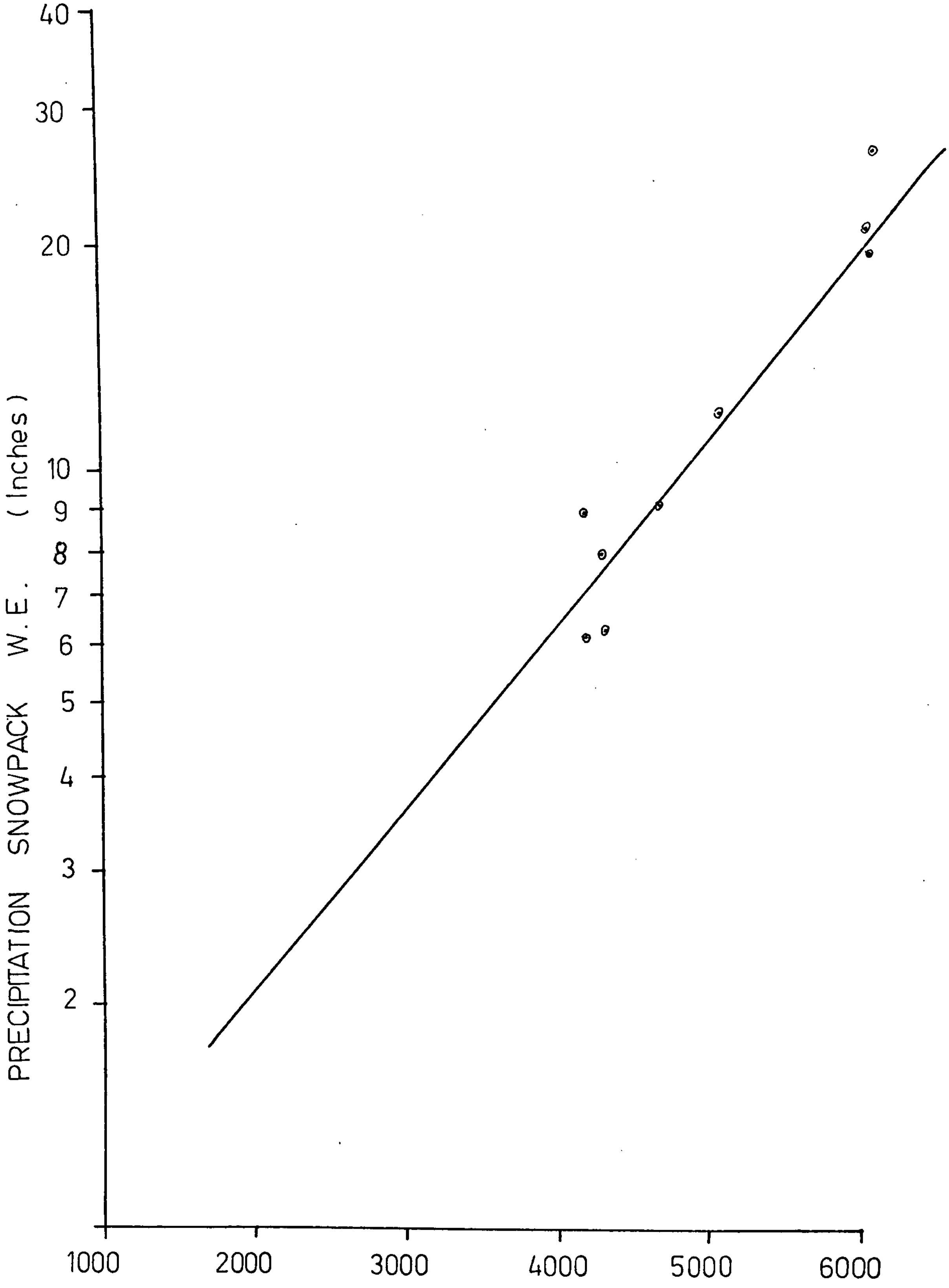
The consistent model parameters for the Similkameen watershed for the years 1963, 1964 and 1970 are evaluated from the *Monthly Record of Meteorological Observations, Canada,* the British Columbia *Snow Survey Bulletin, Surface Water Data, British Columbia* and from the *Surface Water Data for Pacific Drainage.* The lapse rate is taken as 3.5°F/1,000 feet, the value of the winter melt is assumed to be a constant, and the elevation constants for both the snowfall elevation relationship and the rainfall elevation relationship are assumed to be 30,000. The constant for the baseflow is taken as 100 cfs or 200 acre feet, the upland area is not subjected to the upland storage regulation, and the distribution of the snowpack water equivalent on the different elevation bands are evaluated according to the semi-logarithmic plot as shown in Figure 2.1.

TABLE II.1

AREAS AND ELEVATIONS OF BANDS FOR UBC BUDGET MODEL

BAND NO	FROM (feet)	TO (feet)	ELEVATION OF MID-BAND (feet)	AREA (sq. miles)
1	2,000	3,000	2,500	12
2	3,000	4,000	3,500	44
3	4,000	5,000	4,500	251
4	5,000	6,000	5,500	278
5	6,000	7,000	6,500	124
6	7,000	8,000	7,500	20
7	8,000	9,000	8,500	3

The emphasis on the vital significance of the distribution of the initial snowpack water equivalent on each of the band areas should not be overlooked. This is the crucial aspect, which is further accentuated by the scarcity of representative snowpack gauging stations in the Similkameen watershed. Hence the evaluated distribution of the snowpack water equivalent may not be



SNOW ELEVATION RELATIONSHIP
(after Pipes)

FIG. 2.1.

representative of the actual areal distribution. Other factors pertaining to the non-homogeneity of soils and vegetation could affect the distribution of the snowpack.

The following variables of the model parameters are adjusted to fit the reconstituted hydrographs of the Similkameen watershed for the years 1963, 1964 and 1970:

(a) the maximum fraction of impermeability;

(b) the unit hydrograph shape parameter;

(c) the additional shape parameter for the interflow;

(d) the constant for the interflow storage reservoir.

The visual inspection of the actual and simulated hydrographs and the reconciliation of the actual cumulative volumes and the simulated cumulative volumes are the basis of evaluating the above variables through a series of repetitive computer runs.

For the Similkameen watershed the values for the following variables are considered as most appropriate:

(a) the maximum fraction of impermeability - 0.60;

(b) the unit hydrograph shape parameter - 1.80;

(c) the additional shape parameter for the interflow - 0.00;

(d) the constant for the interflow storage reservoir - 0.10.

II.3 SIMULATION OF STREAMFLOW SEQUENCES USING THE UBC BUDGET MODEL

Having ascertained the relevant model parameters for the Similkameen watershed for the years 1963, 1964 and 1970, hydrographs are simulated for the intervening seven years from 1961 to 1970 using merely data from the *Monthly Record of Meteorological Observations, Canada* and the British Columbia *Snow Survey Bulletin.* These hydrographs are then visually compared with the sequence of the actual hydrographs, available from the *Surface Water Data, British Columbia,* and from the *Surface Water Data for Pacific Drainage.*

The initial runs generated with the snowpack water equivalent data from the Copper Mountain (4300 feet) did not produce representative hydrographs. Hence Blackwall Peak (6350 feet) is selected for evaluating the distribution of the snowpack water equivalent. However this snowpack data is only available for the period from 1960 to 1972.

The monthly pan evaporation data is taken from Summerland and this gauging station is not in the vicinity of the Similkameen watershed. Further data is not available for some of the intervening months and these values are approximated from the gauging station at the University of British Columbia. The monthly lake evaporation data is not available for 1961, and for the relevant run the values are assumed to be similar to the values of 1962.

CHAPTER III

STOCHASTIC SPECTRAL MODEL

III.1 APPLICATION OF THE STATISTICAL SPECTRAL MODEL

The spectral analysis and the autocorrelation techniques are the alternative approaches for the investigation of hydrologic processes. These techniques have not been well established in water resources economics and hydrologic planning, and research is still being carried out on the application in these fields. The spectral analysis techniques are well established in the study of wave phenomena, vibration analysis, earthquakes and structural response. There is some degree of uncertainty in its application to the complex setting of the hydrologic phenomena.

Stochastic models are used for the following cases:

1. For the practical and economic planning of water supply and irrigation projects.

2. Most frequently used in reservoir yield studies; and the statistical analysis is related to the runs and run sums.

3. In preserving certain estimates of the statistical properties from a given record, and these estimates are used for generating many equally likely sequences. These sequences are used in place of the historical record or for the anticipated life of a particular project and for the development of optimal designs.

Rodriguez (13) has emphasized that stochastic models for streamflow generation are becoming more accepted for planning studies since the concept of deterministic hydrologic cycles could be questioned from the physical and statistical point of view. The daily and monthly hydrologic data tend to indicate a strong cyclic feature. Rodriguez (13) also asserts that statistical models should not be judged by the degree in which the lower moments, namely the mean, variance and sometimes skewness are preserved, but rather by their ability in preserving the characteristics that are of significance, e.g., reservoir yield.

Solar radiation is the main controlling factor of the terrestrial weather. The existence of the annual cycle in both the daily and monthly hydrologic data is attributed to the effect from the solar radiation. Solar activity is not a deterministic periodic process and random filters such as the atmosphere and clouds through which the solar radiation acts, should be considered.

The basic principles for the statistical modelling of the Similkameen watershed could be summarized as follows.

There are two different processes, which when added up describe the original hydrologic series. The first is the deterministic component and this is due to the average annual periodicity in the daily streamflow sequences. The daily and monthly periodicities in the hydrologic time series may not be distinct or remain constant from year to year. The second process accounts for the random variations in the atmosphere with the passing of time. The random component or residuals can be approximately explained by a first order autoregressive or Markovian model. The explanation of the random component by this technique is still subject to controversy.

The stochastic model used for simulating the daily sequences of streamflow for the Similkameen watershed is expressed as

$$X_\tau = \mu_\tau + \sigma_x \varepsilon_\tau \qquad \text{(III.1)}$$

where

$\tau = 1, 2, \ldots, w$;

w = basic period;

σ_x = standard deviation of x_τ

ε_τ = stochastic component.

The deterministic component μ_τ is expressed mathematically as:

$$\mu_\tau = \mu_x + \sum_{j=1}^{m} (A_j \cos 2\pi f_j \tau + B_j \sin 2\pi f_j \tau) \qquad \text{(III.2)}$$

where

μ_x = mean of the series;

A_j and B_j are the Fourier coefficients;

m = number of significant harmonics.

The Fourier coefficients for the relevant series are evaluated as:

$$A_j = \frac{2}{w} \sum_{\tau=1}^{w} (x_\tau - \mu_x) \cos \frac{2\pi j \tau}{w} \qquad \text{(III.3)}$$

$$B_j = \frac{2}{w} \sum_{\tau=1}^{w} (x_\tau - \mu_x) \sin \frac{2\pi j \tau}{w} \qquad \text{(III.4)}$$

where

$$f_j = \frac{j}{w}$$

It must be emphasized that the statistical spectral analysis cannot be used easily or directly in water resources economics and planning at the present state or level of these techniques. But it provides a better insight into the critical sequences of flow, which historical data alone may not be able to yield.

III.2 EVALUATION OF THE SUB-HARMONICS FOR THE SIMILKAMEEN WATERSHED USING THE SPECTRAL MODEL

Using equations III.3 and III.4, the daily streamflow series for the Similkameen watershed for the years 1963, 1964 and 1970 are respectively decomposed to the first 35 sub-harmonics or Fourier coefficients. Through a series of trial and error techniques, the first 30 sub-harmonics appear to give a reasonably good visual fit for the reconstituted hydrograph using equation III.2.

Having fitted the periodic or deterministic component with the sub-harmonics for the relevant years, the unexplained variance or residuals are analyzed by the multiple regression technique. The residuals are lagged by one day, and a first order autoregressive or Markovian model is fitted for the respective years. This is expressed as:

$$\varepsilon_\tau = r_{1},\varepsilon_{\tau-1} + \eta_\tau \qquad \text{(III.5)}$$

where

ε_τ = residuals;

r_1 = autocorrelation coefficient;

η_τ = independent random variable with mean zero and a given variance.

A random number generator FRANDN is used for generating the realistic positive values for the stochastic component. This is expressed as

$$\eta_\tau = \text{FRANDN } (0.0)\ \sigma\ (1-R^2) \tag{III.6}$$

where

η_τ = independent random variable;

σ = standard deviation;

R^2 = multiple correlation coefficient.

III.3 SIMULATION OF STREAMFLOW SEQUENCES USING SPECTRAL MODEL

For the intervening years from 1961 to 1970, the mean annual flows at Princeton are estimated linearly from the snowpack water equivalent data at Blackwall Peak (6350 feet), available from the British Columbia *Snow Survey Bulletin* and from the known mean annual flows for 1963, 1964 and 1970. This is shown in Figure 3.11.

The 30 Fourier coefficients for the relevant intervening years and for the estimated mean annual flows of the Similkameen watershed are evaluated by interpolating the values of the Fourier coefficients for the years 1963, 1964 and 1970. These interpolated values are shown in Tables III.1 and III.2. The stochastic component is selected on the basis of the assumptionthat the random component for the year 1970 is representative of the respective estimated mean annual flows over the period under consideration.

The synthetic stochastic hydrographs for the intervening seven years are then generated from the interpolated values of the Fourier coefficients and from the estimated mean annual flows. Equation III.1 is used for generating the synthetic streamflow sequences. The stochastic hydrographs are eventually compared with the deterministic UBC hydrographs and with the actual hydrographs for the respective years. The Calcomp plot is used for the comparative study.

TABLE III.1

EVALUATED AND INTERPOLATED A_i VALUES
OF FOURIER COEFFICIENTS
FOR THE SIMILKAMEEN WATERSHED

1966 & 1970 557 & 528 cfs.	1969* 649 cfs.	1962 & 1965* 711 & 759 cfs.	1963 794 cfs.	1961 & 1968* 898 & 901 cfs.	1967* 940 cfs	1964 1120 cfs.
-723	-705	-693	-675	-877	-973	-1482
430	418	409	396	572	659	1097
-134	-118	-106	- 89	-234	-306	- 667
-125	-159	-181	-214	- 82	- 16	315
276	273	271	276	199	165	- 7
-289	-262	-244	-217	-204	-197	- 163
212	179	157	123	150	164	230
-113	- 60	- 24	29	- 34	- 65	- 222
18	- 43	- 85	-143	- 64	- 25	174
40	88	120	168	152	144	102
- 54	20	24	- 35	- 11	1	59
43	28	18	2	- 3	- 6	- 18
- 33	13	45	90	69	58	3
5	- 62	-106	-173	-120	- 94	39
20	73	108	160	112	88	- 33
- 23	- 45	- 59	- 80	- 51	- 37	34
16	15	15	13	5	1	- 18
- 14	8	24	46	29	21	- 20
25	- 25	- 57	-107	- 60	- 37	80
- 50	- 2	31	79	32	9	- 108
78	43	20	- 16	11	25	93
- 90	- 53	- 28	10	- 9	- 19	- 66
82	60	45	22	20	20	15
- 48	- 45	- 43	- 40	- 23	- 14	30
- 18	- 16	14	- 11	- 25	- 32	- 67
80	61	48	28	43	50	85
-108	- 78	- 58	- 28	- 38	- 43	- 68
104	73	52	20	22	23	27
- 76	- 63	- 54	- 41	- 24	- 15	28
42	16	- 2	- 28	- 42	- 49	- 82

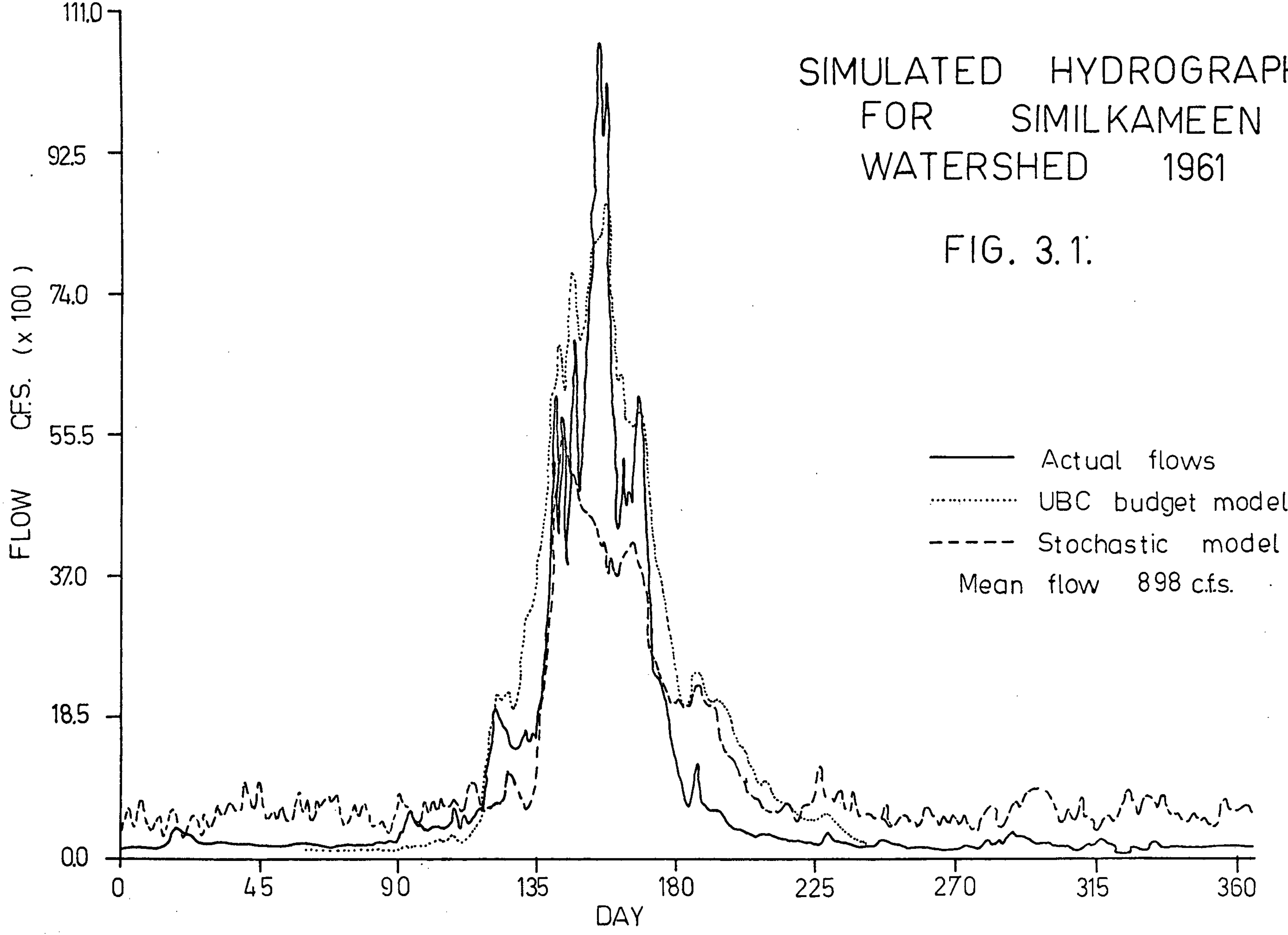
*Interpolated Values.
Evaluated for 1963, 1964 and 1970.

TABLE III.2

EVALUATED AND INTERPOLATED B_i VALUES
OF FOURIER COEFFICIENTS†
FOR THE SIMILKAMEEN WATERSHED

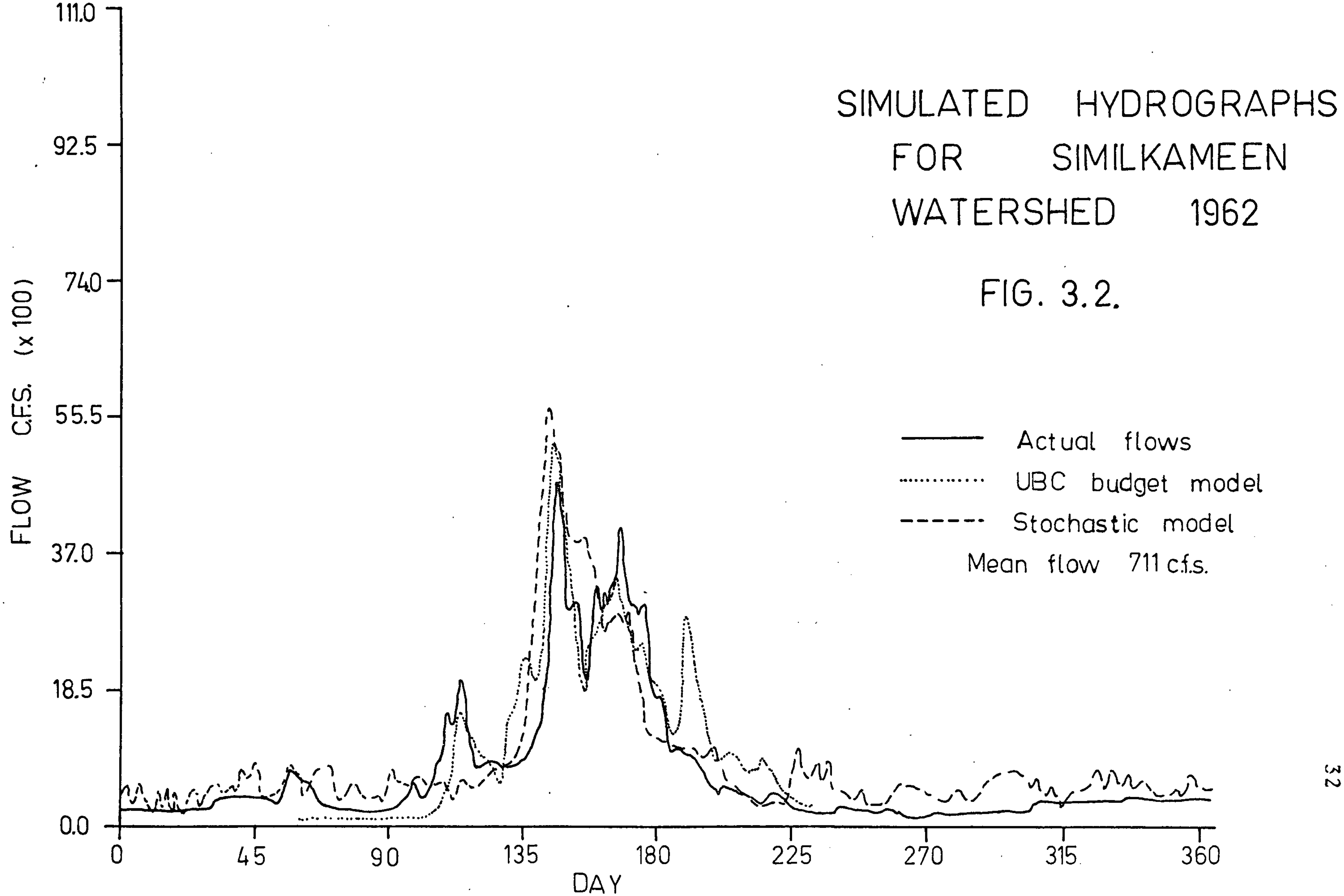
1966 & 1970 557 & 528 cfs.	1969* 649 cfs.	1962 & 1965* 711 & 759 cfs.	1963 794 cfs.	1961 & 1968* 898 & 901 cfs.	1967* 940 cfs.	1964 1120 cfs.
327	317	310	300	320	330	381
-553	-546	-540	-533	-562	-577	-649
585	533	499	447	537	582	806
-482	-433	-400	-350	-420	-454	-627
272	218	182	128	230	280	534
- 80	- 41	- 15	25	- 62	-106	-323
- 60	-129	-175	-244	-149	-101	138
117	153	177	213	178	160	73
-111	-114	-116	-119	-117	-115	-109
66	57	52	44	78	95	180
- 26	19	49	94	28	- 5	-170
- 8	- 76	-120	-188	- 86	- 35	222
33	61	79	107	40	7	-161
- 42	- 44	- 46	- 49	1	26	151
37	- 1	- 26	- 62	- 70	- 74	- 93
- 29	34	70	137	118	109	60
20	61	-115	-196	-150	-127	- 10
- 20	20	48	88	67	56	2
31	- 1	- 22	- 53	- 40	- 34	- 2
- 38	- 27	- 19	- 8	1	5	26
21	39	50	67	37	22	- 54
18	- 16	- 38	- 71	- 28	- 7	102
- 73	- 49	- 32	- 6	- 33	- 47	-115
113	77	52	15	39	50	109
-124	- 91	- 68	- 33	- 37	- 38	- 48
96	72	55	30	23	19	1
- 43	- 36	- 32	- 24	- 2	9	63
- 12	- 23	30	- 41	- 54	60	- 91
49	42	37	31	49	58	101
- 69	- 54	- 44	- 28	- 37	- 41	- 63

*Interpolated values.
†Evaluated for 1963, 1964 and 1970



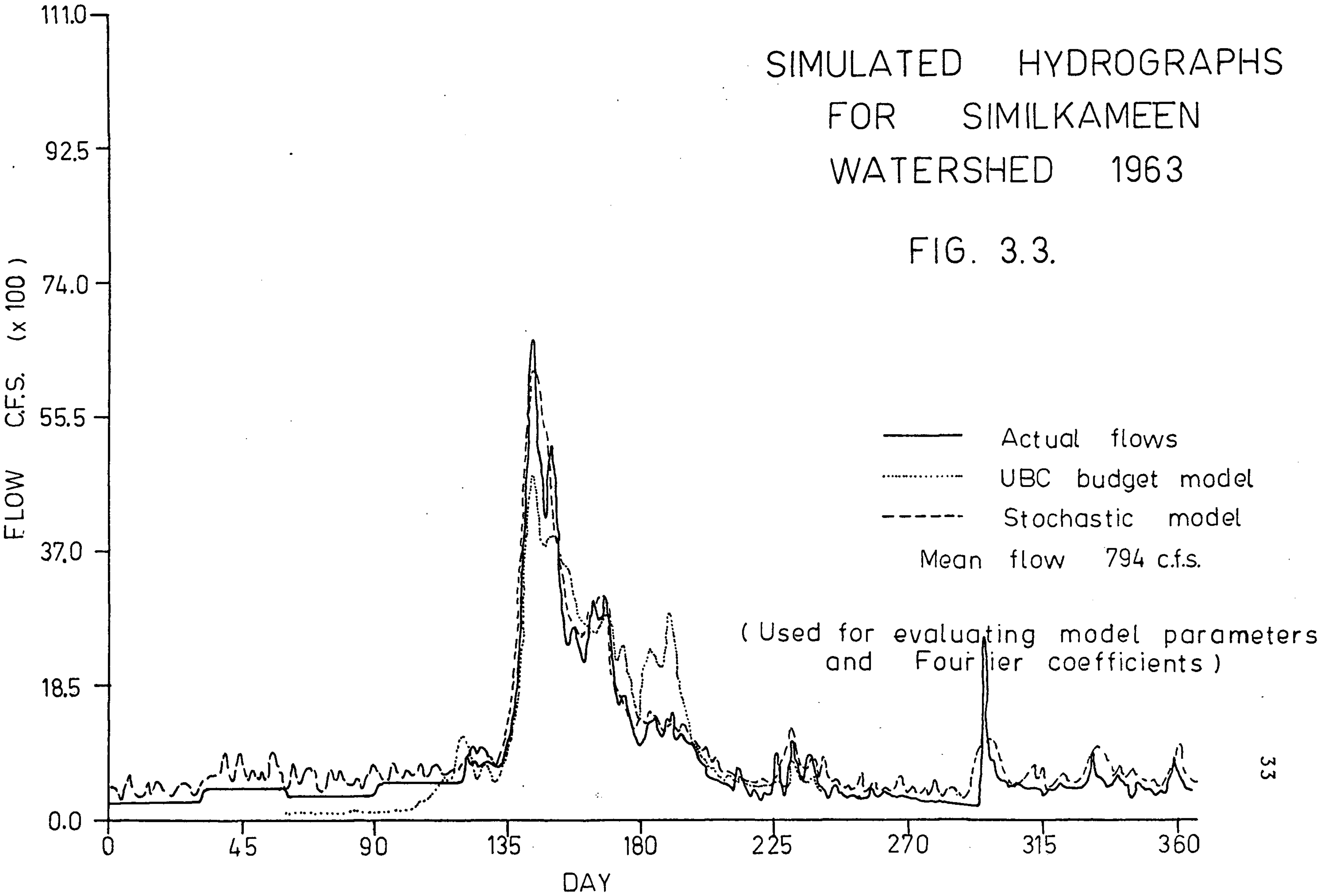
SIMULATED HYDROGRAPHS FOR SIMILKAMEEN WATERSHED 1961

FIG. 3.1.



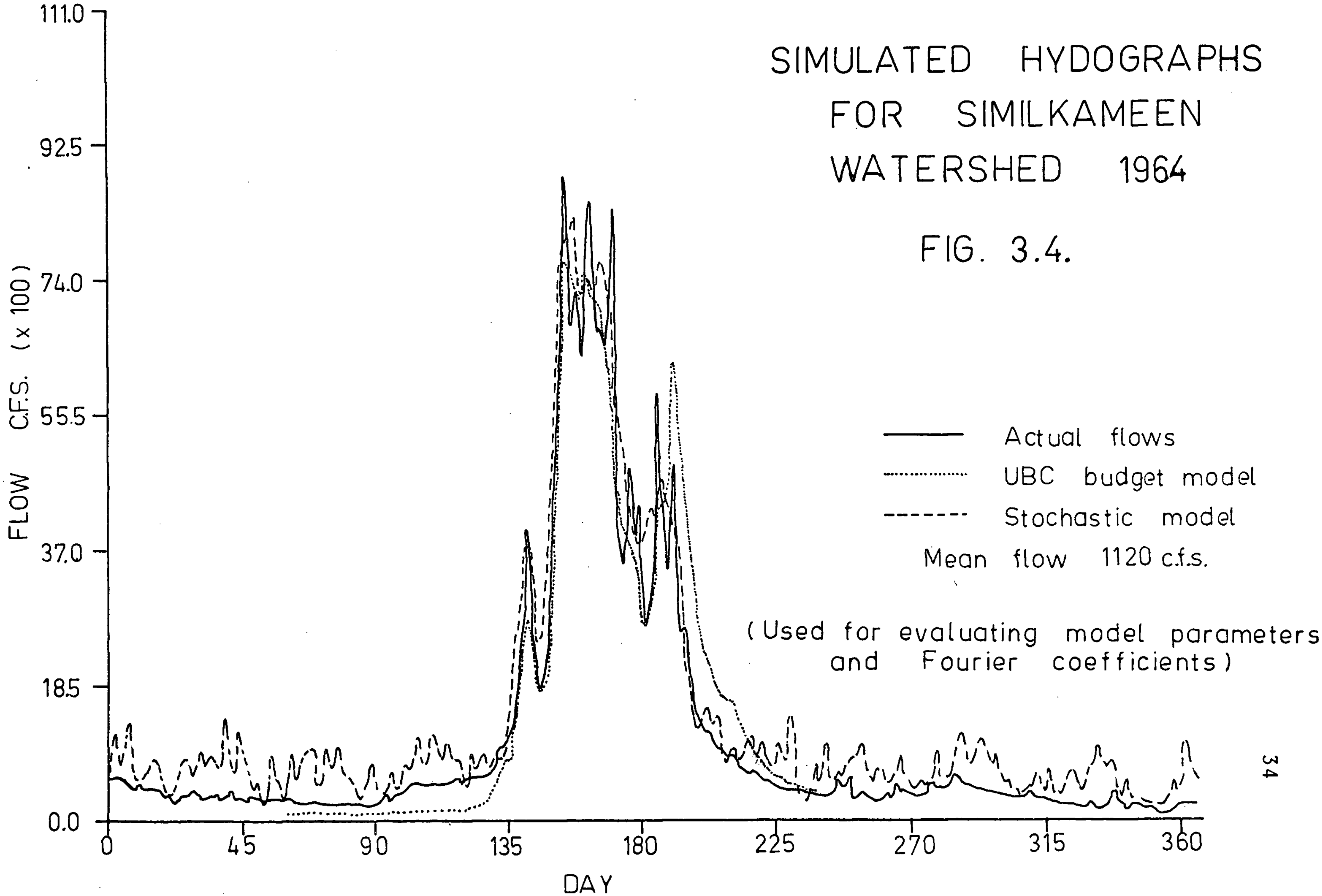
SIMULATED HYDROGRAPHS FOR SIMILKAMEEN WATERSHED 1962

FIG. 3.2.



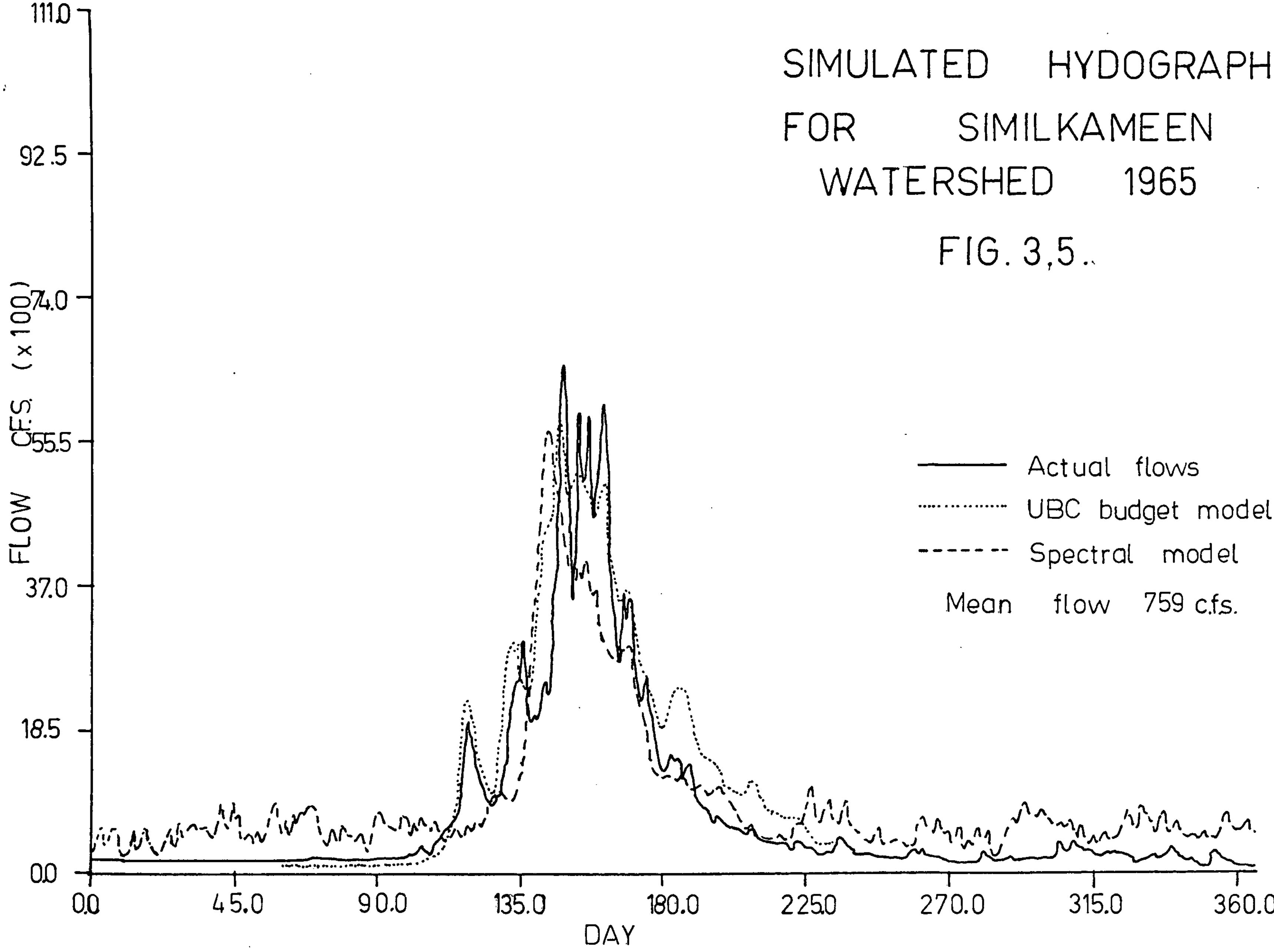
SIMULATED HYDROGRAPHS FOR SIMILKAMEEN WATERSHED 1963

FIG. 3.3.



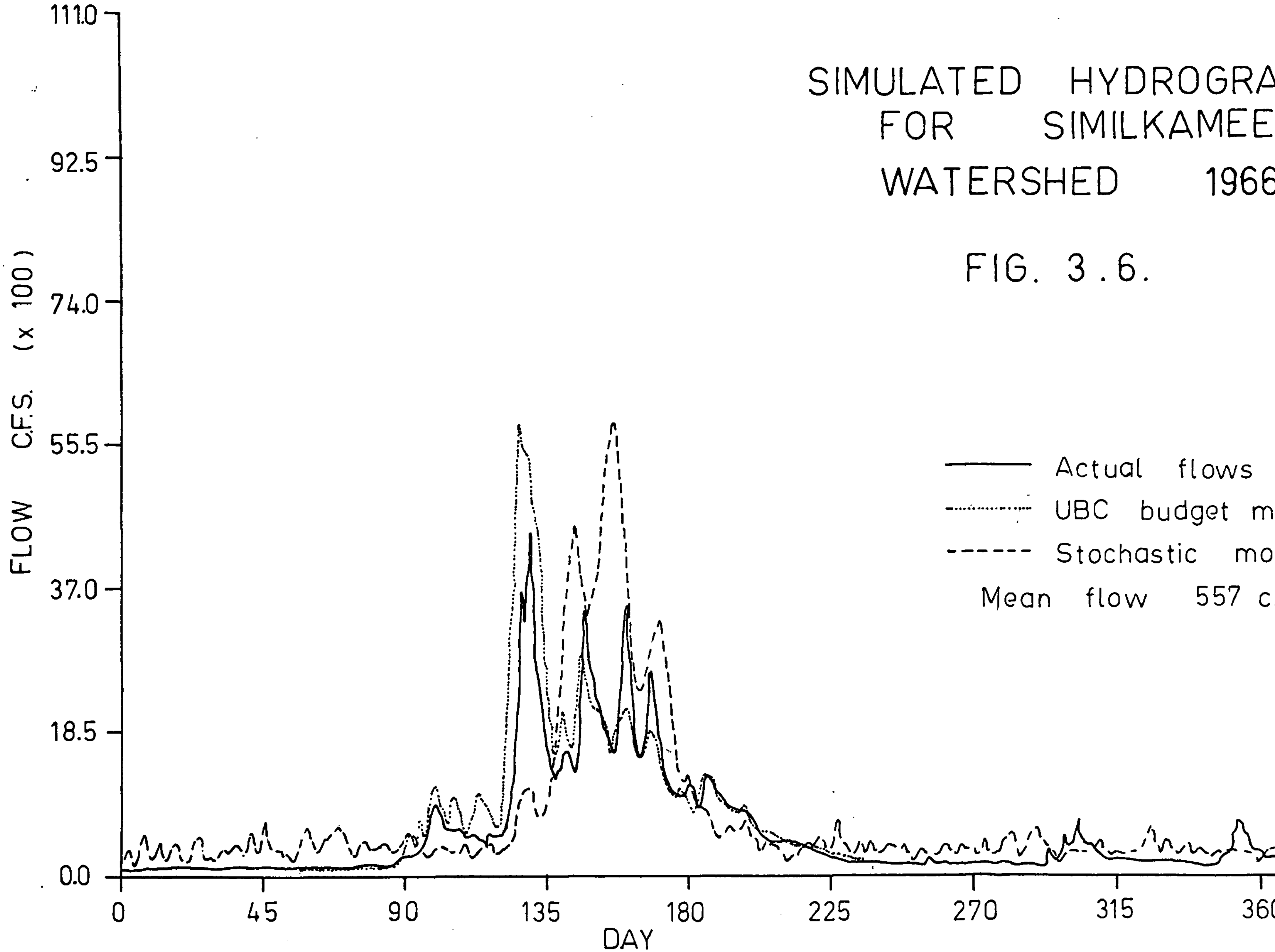
SIMULATED HYDOGRAPHS FOR SIMILKAMEEN WATERSHED 1964

FIG. 3.4.



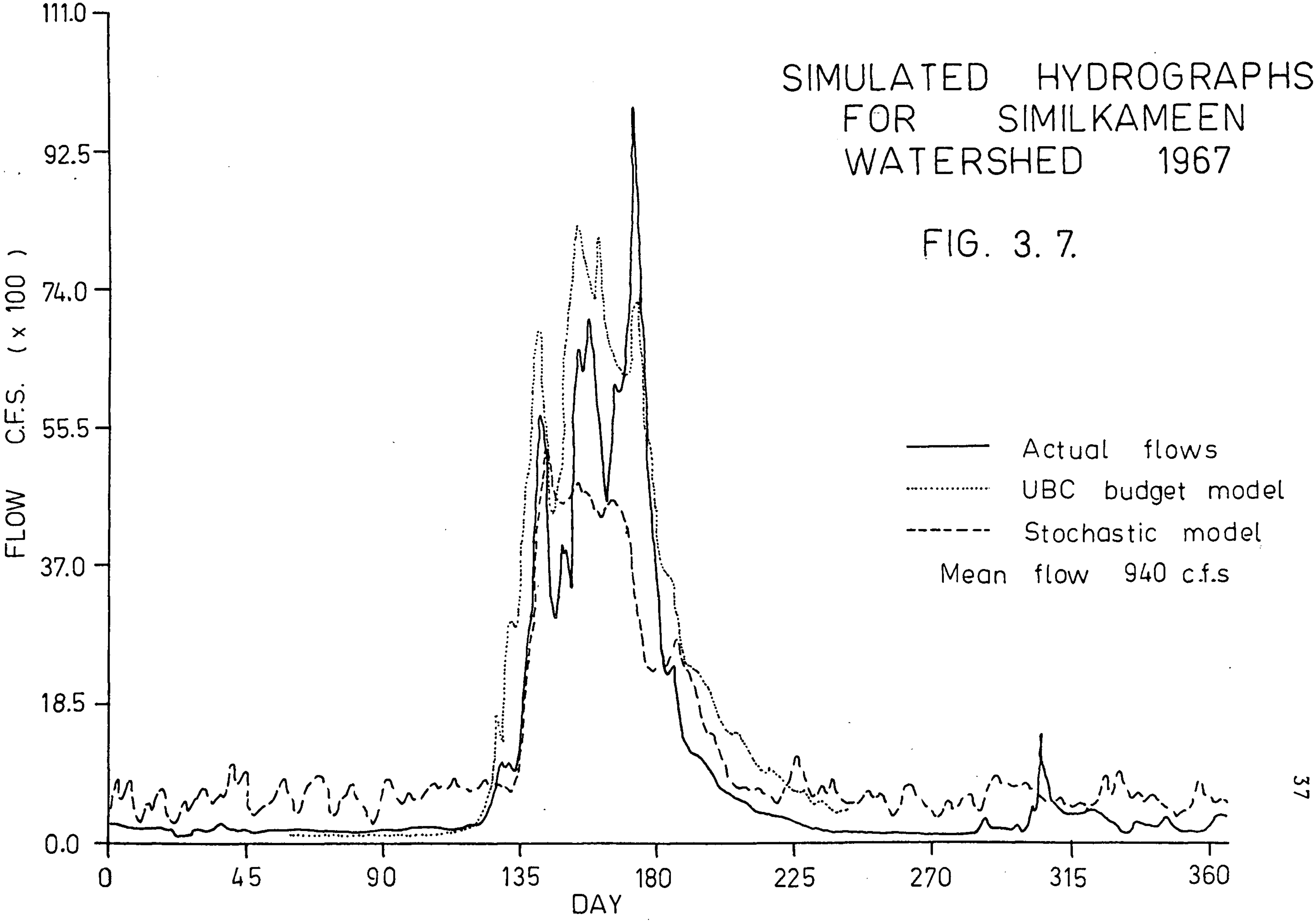
SIMULATED HYDOGRAPHS FOR SIMILKAMEEN WATERSHED 1965

FIG. 3,5..

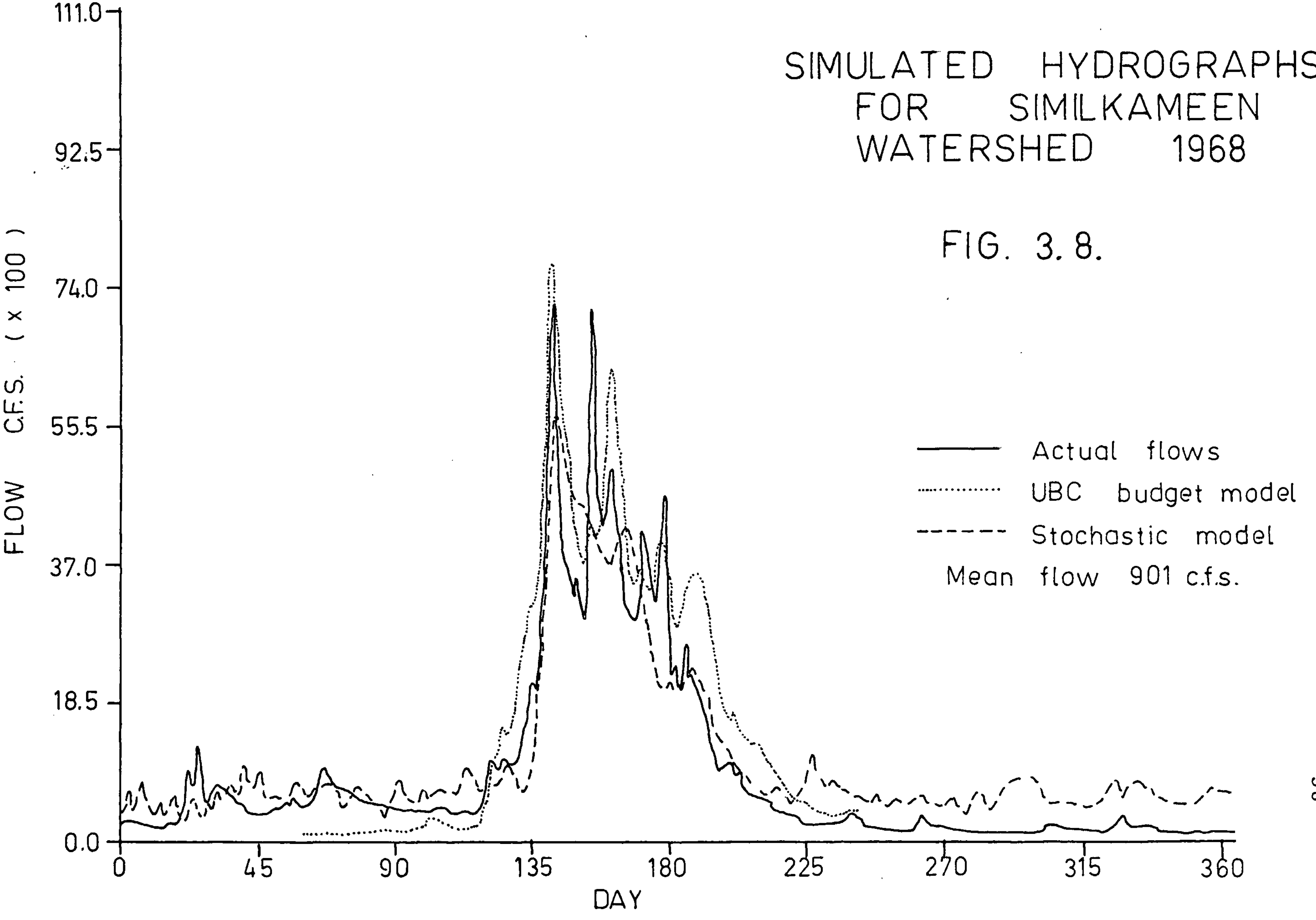


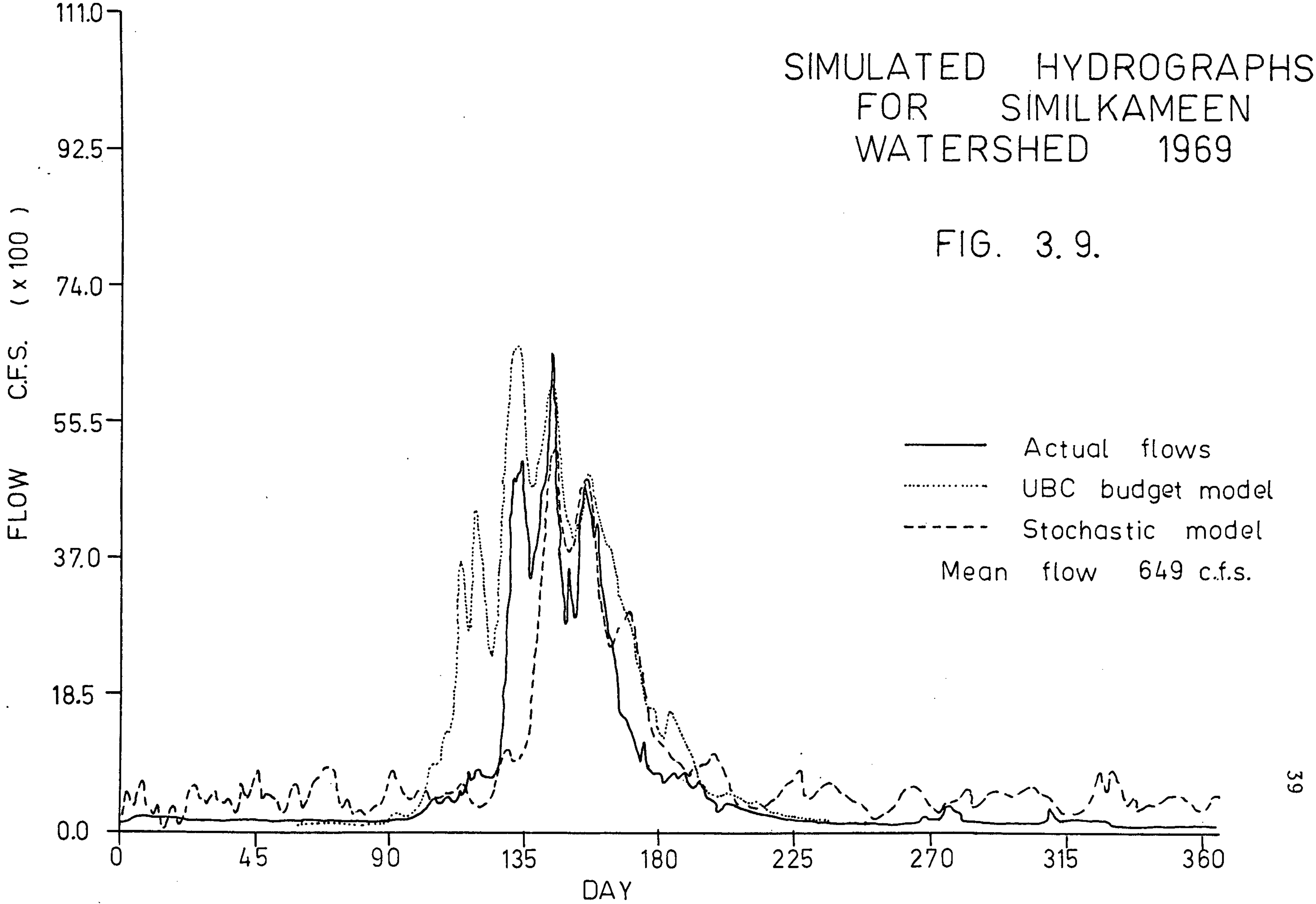
SIMULATED HYDROGRAPHS FOR SIMILKAMEEN WATERSHED 1966

FIG. 3.6.



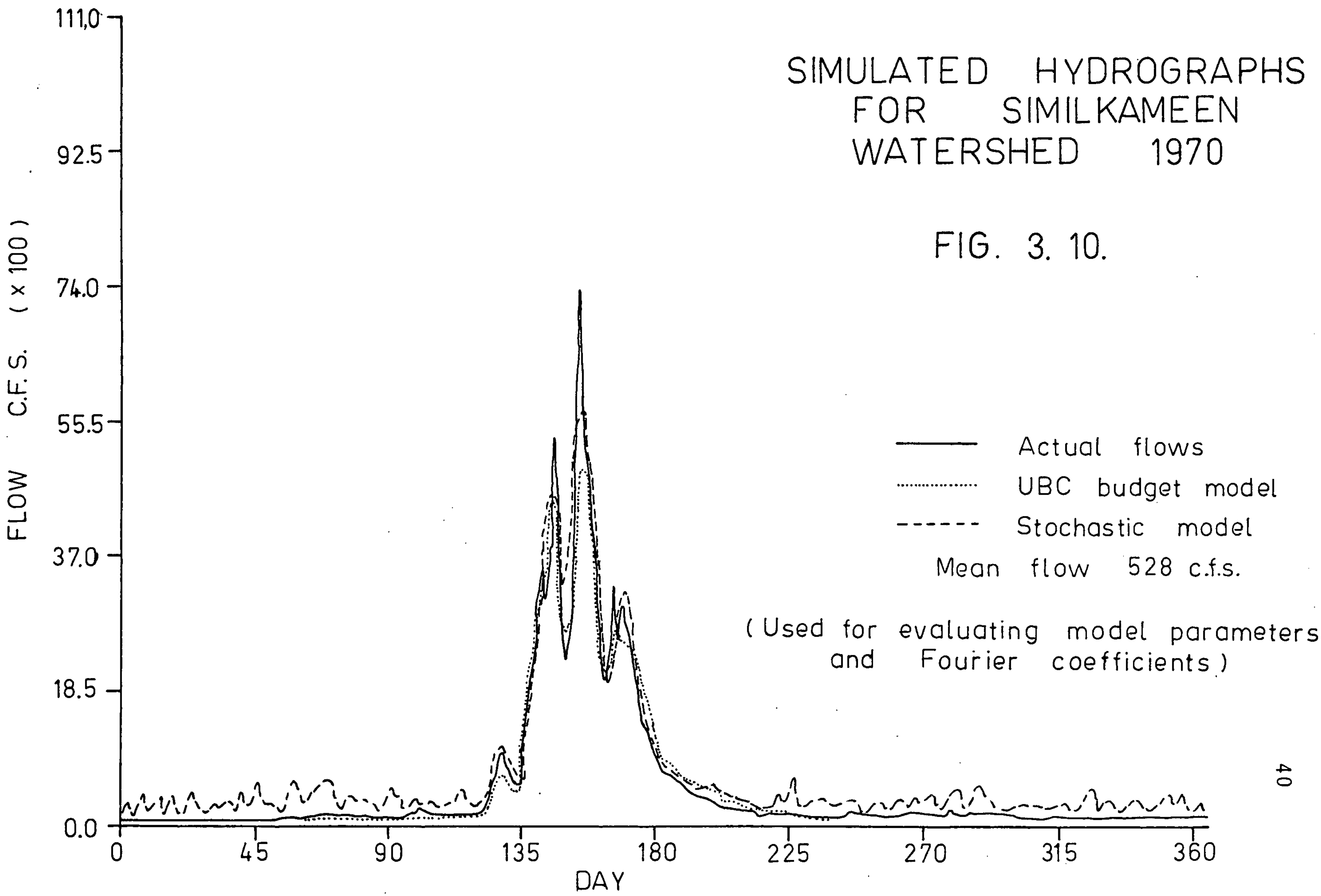
SIMULATED HYDROGRAPHS FOR SIMILKAMEEN WATERSHED 1967
FIG. 3. 7.
Actual flows
UBC budget model
Stochastic model
Mean flow 940 c.f.s
FLOW C.F.S. (x 100)
111.0
92.5
74.0
55.5
37.0
18.5
0.0
0
45
90
135
180
225
270
315
360
DAY


111.0
92.5
74.0
55.5
37.0
18.5
0.0
FLOW C.F.S. (x 100)
0
45
90
135
180
225
270
315
360
DAY
SIMULATED HYDROGRAPHS
FOR SIMILKAMEEN
WATERSHED 1968
FIG. 3.8.
Actual flows
UBC budget model
Stochastic model
Mean flow 901 c.f.s.



SIMULATED HYDROGRAPHS FOR SIMILKAMEEN WATERSHED 1969

FIG. 3. 9.


SIMULATED HYDROGRAPHS
FOR SIMILKAMEEN
WATERSHED 1970
FIG. 3. 10.
Actual flows
UBC budget model
Stochastic model
Mean flow 528 c.f.s.
(Used for evaluating model parameters
and Fourier coefficients)
FLOW C.F.S. (x 100)
111,0
92.5
74.0
55.5
37.0
18.5
0.0
0
45
90
135
180
225
270
315
360
DAY

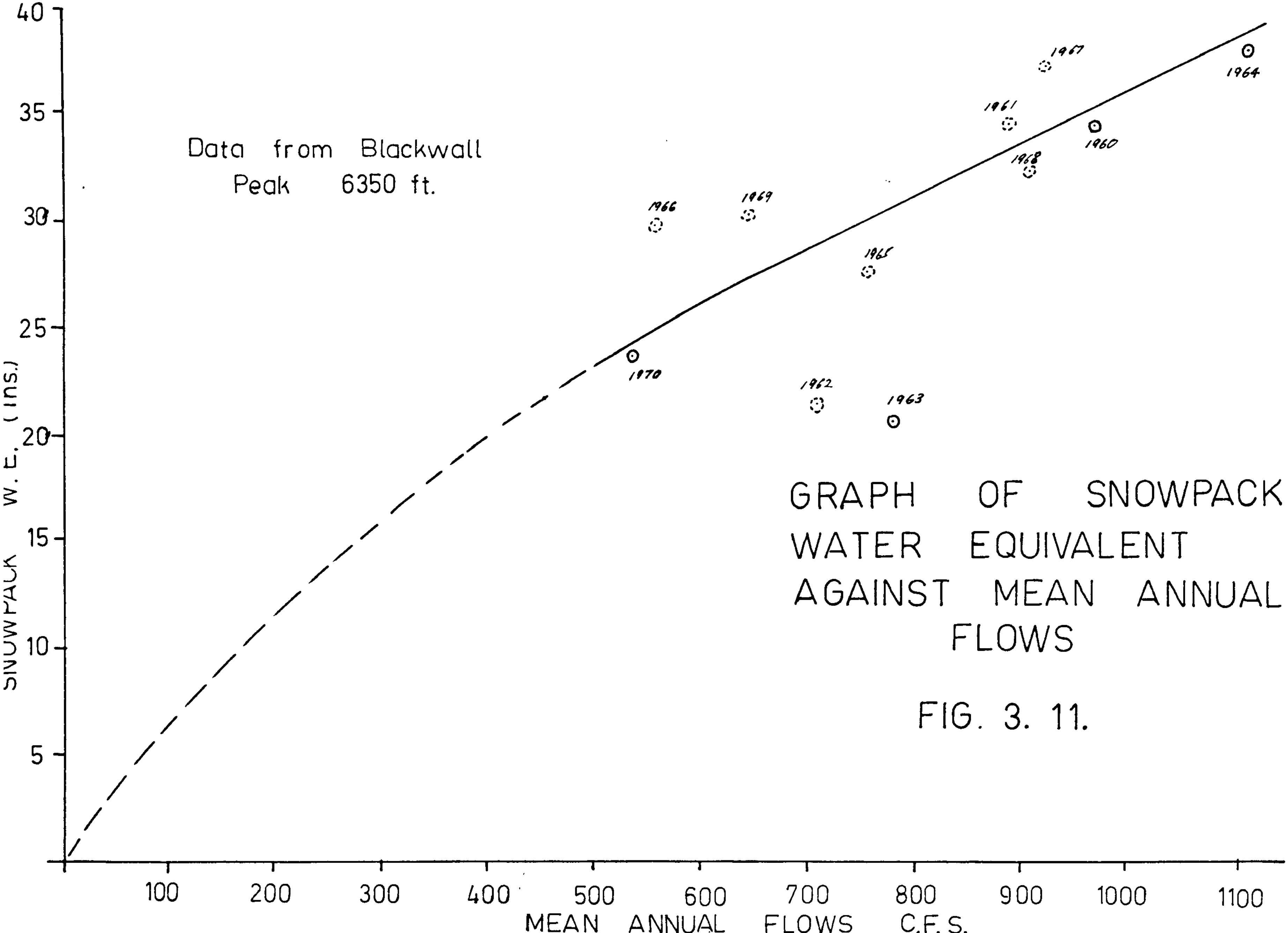

Data from Blackwall
Peak 6350 ft.
SNOWPACK W. E. (ins.)
MEAN ANNUAL FLOWS C.F.S.
1960
1961
1962
1963
1964
1965
1966
1967
1968
1969
1970
GRAPH OF SNOWPACK WATER EQUIVALENT AGAINST MEAN ANNUAL FLOWS

FIG. 3. 11.

III.4 SPECTRAL MODEL FOR THE SIMILKAMEEN WATERSHED AND THE ASSOCIATED SUB-ROUTINES

The computer program for the above model is expressed as:

```
$RUN * WATFIV      3 = -P
$ COMPILE
              REAL Q1 (30,500), QGS (500), CUMQG, A(40), B(40),
              QT (500), RES(500), QGSR(5,100)
              READ, (A(I), B(I), I = 1,30)
              Z = RANDN (7.7)
              RES(1) = 7.6
              DO 7  I = 1, 365
              SI = I
              J = I + 1
              N = I

        8     RES(J) = 0.562750 * RES(N) + FRANDN (0.0)*182.843
              IF (RES (J) . LT . 0.0) N = J
              IF (RES (J) . LT . 0.0) GO TO 8

        7     CONTINUE
              CUMQG = 0.0
              DO 4  L = 1, 365
              SL = L
              QT(L) = 0.0
              DO 5    K = 1, 30
              Q1 (K, L) = A(K) * COS ((44.0 * K * L)/2555.0) +
              B(K) * SIN((44.0 * K * L)/2555.0)
              QT(L) = QT(L) + Q1 (K,L)

        5     CONTINUE
              QGS(L) = 528.290 + QT(L)
              CUMQG = CUMQG + QGS(L)

        4     CONTINUE
              DO 9   M = 1, 365
              SM = M
              QGSR(M) = QGS(M) + RES(M)
              WRITE (3,103)  SM, QGSR(M)

      103     FORMAT (2F10.0)

        9     CONTINUE
              RETURN
              END
```

```
$DATA

                (Values of A_j and B_j of the Fourier coefficients)

$STOP

$RUN * FORTRAN
                DIMENSION Q(500)
                READ (4,100) (Q(I), I = 1, 365)

       100      FORMAT (F 10.0)
                DO 2   I = 1, 365
                SI = I
                WRITE (2,105)  SI, Q(I)

       105      FORMAT (2F10.0)

         2      CONTINUE
                RETURN
                END



$ENDFILE
$RUN  -  LOAD# 4 = SIMILKMEEN70   2 = -S


$RUN  * FORTRAN

                DIMENSION QG(200)
                READ (2,101) (QG(I),  I = 1,182)

       101      FORMAT (F 10.0)
                DO 3  I = 1,182
                SI = I
                WRITE (1,106) SI, QG(I)

       106      FORMAT (2F10.0)

         3      CONTINUE
                RETURN
                END

$ENDFILE
$RUN  -  LOAD # 2 = SIMILKGEN70   1 = -T
```

```
$RUN    * FORTRAN
                DIMENSION X(1100), Y(1100)

          4     N = 0
                DO 3    I = 1,365
                READ (2, 101, END = 200) X(I), Y(I)

          3     N = I

        200     IF (N.EQ.0) GO TO 300
                CALL SUMFUN (X, Y, N)
                GO TO 4

        300     CONTINUE

        101     FORMAT (2F 10.0)

          7     M = 0
                DO 6    J = 1,365
                READ (3, 102, END = 400) X(J), Y(J)

          6     M = J

        400     IF (M.EQ.0) GO TO 500
                CALL GENFUN (X, Y, M)
                GO TO 7

        500     CONTINUE

        102     FORMAT (2F 10.0)

          9     L = 0
                DO 8    K = 1, 182
                READ (1, 103, END = 600) X(K), Y(K)

          8     L = K

        600     IF (L.EQ.0) GO TO 700
                CALL UBCFUN (X, Y, L)
                GO TO 9

        700     CONTINUE

        103     FORMAT (2F 10.0)
                CALL PLOTND
                STOP
                END

                SUBROUTINE SUMFUN (X, Y, N)
                DIMENSION X(N), Y(N)
                DO 5    I = 1,N
                X(I) = X(I)/45
                Y(I) = Y(I)/1850
```

```
    5     CONTINUE
          CALL AXIS (0., 0., 'DAY', -3, 8., 0., 0., 45.)
          CALL AXIS (0., 0., 'FLOW CFS', 8, 6., 90., 0., 1850.)
          CALL LINE (X, Y, N, 1)
          CALL SYMBOL (2.0, 6.0, 0.5, 'SIMULATED HYDROGRAPH
           FOR SIMILKAMEEN WATERSHED 1970', 0., 51)
          CALL PLOT (0., 0., 3)
          RETURN
          END


          SUBROUTINE GENFUN (X, Y, M)
          DIMENSION X(M), Y(M)
          DO 2   J = 1, M
          X(J) = X(J)/45
          Y(J) = Y(J)/1850

    2     CONTINUE
          CALL DASHLN (0.06, 0.06, 0.06, 0.06)
          DO 10   I = 1, M
          CALL PLOT (X(I), Y(I),4)

   10     CONTINUE
          CALL PLOT (0., 0., 3)
          RETURN
          END


          SUBROUTINE UBCFUN (X, Y, L)
          DIMENSION X(L), Y(K)
          DO 11   K = 1, L
          X(K) = 1.311 + X(K)/45
          Y(K) = Y(K)/1850

   11     CONTINUE
          CALL DASHLN (0.5, 0.05, 0.05, 0.05)
          CALL PLOT (X(1), Y(1),3)
          DO 12   J = 2, L
          CALL PLOT (X(J), Y(J), 4)

   12     CONTINUE
          RETURN
          END

$ENDFILE
$RUN   -   LOAD #   2 = -S   3 = -P   1 = -T
$ENDFILE

$RUN PLOT: Q      PAR = LINED
$ENDFILE
```

CHAPTER IV

RESULTS AND CONCLUSIONS

IV.1 COMPARISON OF THE UBC BUDGET MODEL AND THE SPECTRAL MODEL

During the initial investigations with the UBC Budget Model for the appropriate model parameters of the Similkameen watershed for the years 1963, 1964 and 1970, consistent parametric values are obtained for the respective years. These values can be considered as reasonably good since the data for the monthly record of meteorological observations is taken from Princeton, which is at the northern extremity of the watershed. Further the snowpack data is measured only from one station at Blackwall Peak (6350 feet), and the pan evaporation data is taken from Summerland and this station is not in the vicinity of the Similkameen watershed.

While generating the synthetic hydrographs with the fixed model parameters of the UBC Budget Model for the intervening years, surprisingly good fits for the sequence of peaks are obtained with the actual hydrographs. Some errors occur in the cumulative volumes generated by the UBC Budget Model. This may be attributed to the snowpack data at Blackwall Peak, which may not be representative of the actual snowpack distribution over the entire watershed. The precipitation data for the UBC Budget Model is taken from Princeton, and this gauging station is not in the vicinity of Blackwall Peak. For the purpose of this study the winter melt factor has been assumed to be a constant, and this largely accounts for some errors in the cumulative volumes. In real time operation the winter melt factor is evaluated by the feedback of the early season runoff response. More recent work has indicated that this winter melt factor can be evaluated by running the model right through the previous year. The UBC Budget Model would function as a useful tool for the investigation of hydrologic processes in research watersheds which are well instrumented for pan evaporation, snowpack and meteorological observations.

The application of the statistical spectral model should not be judged from the basis of providing an analytical solution, but in its ability in providing a sequence of synthetic traces, which are likely to occur during the life of a particular project. There is a fair degree of reconciliation between the generated and the actual cumulative volumes for the synthetic hydrographs generated by the deterministic or periodic component of the spectral model. But the sequence of the reconstituted peaks is not comparable with that obtained from the UBC Budget Model.

For the comparative study, the measures of central location and dispersion are evaluated from 2nd March to 30th August for the actual streamflow series and for the respective models. These parameters, shown in Table IV.1, provide a better perspective

TABLE IV.1

MEASURES OF CENTRAL LOCATION AND DISPERSION
FROM 2nd MARCH to 30th AUGUST
SIMILKAMEEN WATERSHED
(730 sq. miles)

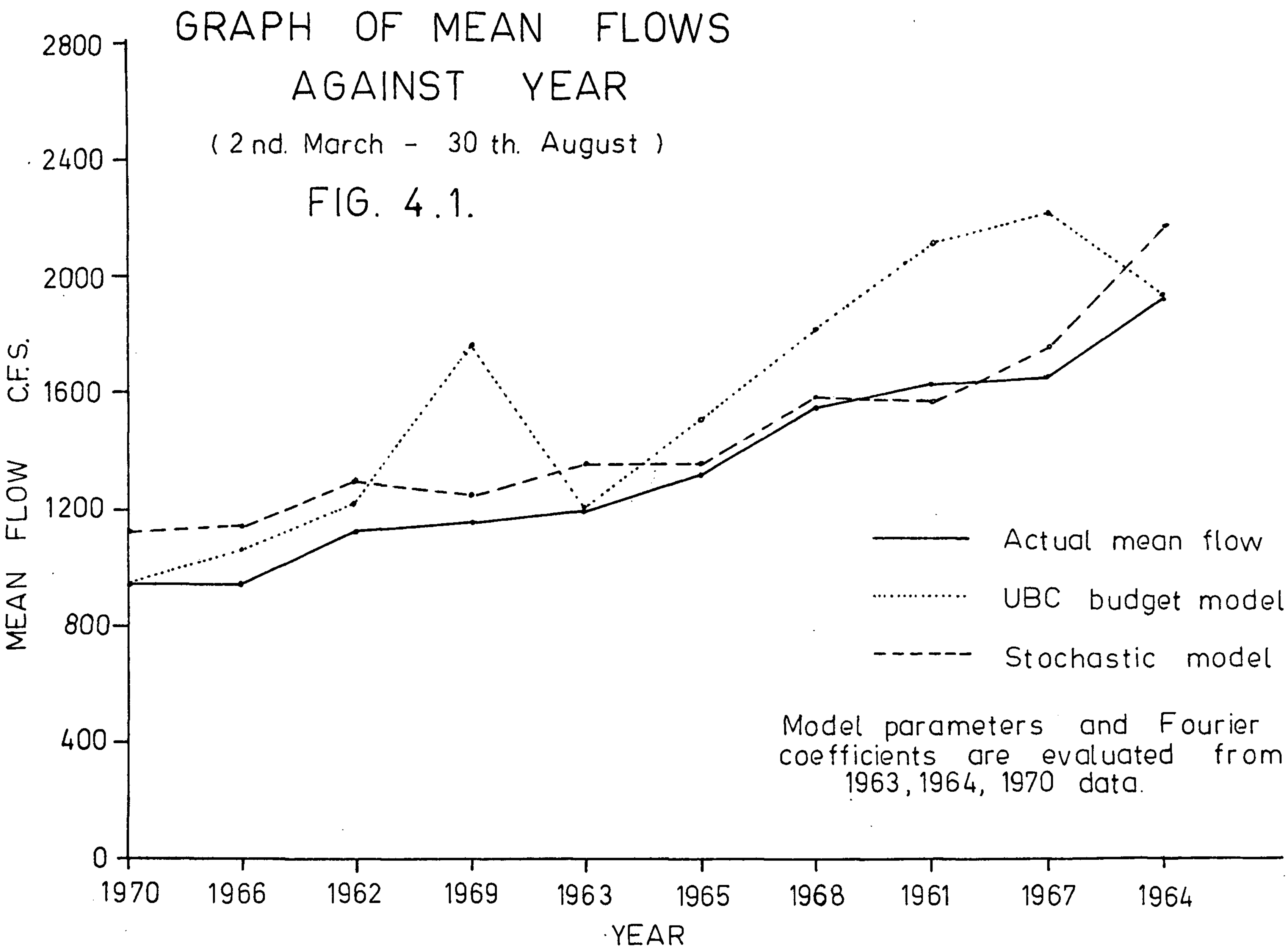
	MEAN FLOW cfs			STANDARD DEVIATION cfs		
YEAR	FOR ACTUAL FLOWS	UBC BUDGET MODEL	STOCHASTIC SPECTRAL MODEL	FOR ACTUAL FLOWS	UBC BUDGET MODEL	STOCHASTIC SPECTRAL MODEL
1961	1629	2121	1578	2319	2446	1391
1962	1128	1234	1300	1107	1210	1261
1963*	1195	1214	1352	1281	1221	1272
1964*	1913	1916	2196	2332	2314	2256
1965	1315	1513	1348	1525	1615	1261
1966	944	1072	1152	890	1159	1370
1967	1645	2235	1667	2244	2582	1490
1968	1551	1822	1581	1595	1900	1391
1969	1157	1772	1244	1532	1949	1284
1970*	942	943	1123	1398	1290	1370

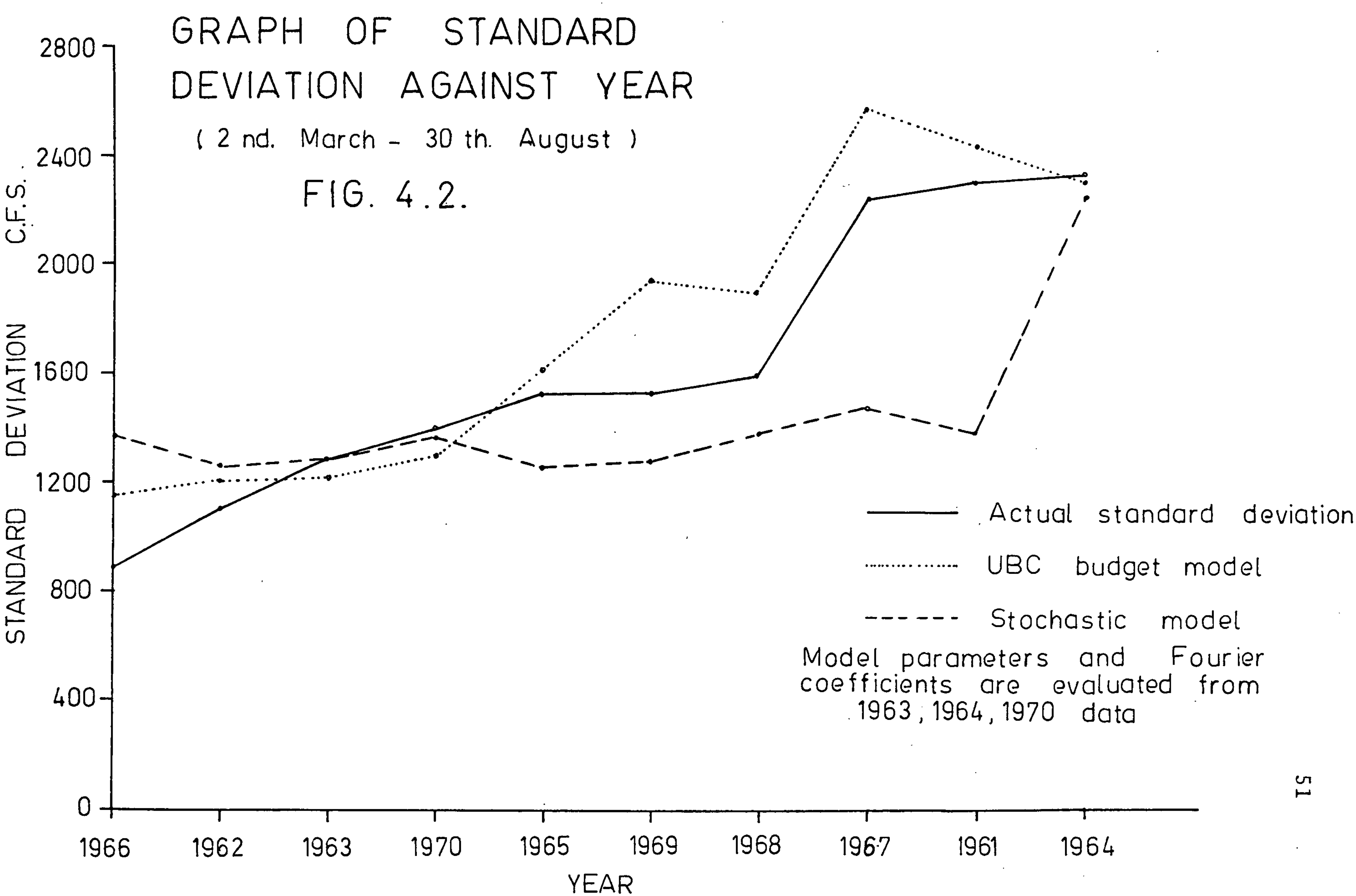
*Used for evaluating Fourier Coefficients and UBC Model Parameters.

on the significance of the fitted models, in addition to the visual inspection of the simulated hydrographs as shown in Figures 3.1 to 3.10. The variance is by far the most important measure of spread. Normally it is desirable to supplement a statement of the location of a distribution on how closely the data is concentrated about the mean. In Figures 4.1 and 4.2, the means and the standard deviations are monotonically plotted for the actual streamflow series. The respective statistical parameters for the UBC Budget Model and the stochastic spectral model are subsequently plotted against these values.

The UBC Budget Model produces consistent measures of spread and preserves the values of the means for 1962, 1963, 1964, 1965, 1966, 1968 and 1970. There are some fluctuations on the high side for the means of 1961, 1967 and 1969.

For the generation of the daily synthetic streamflow sequences with the stochastic spectral model, using Equation III.1, the mean annual flows for the intervening years are estimated from the graph in Figure 3.11 from the known values of the snowpack water equivalent of Blackwall Peak. As shown in Figure 4.1 there are some errors in the values of the means from 2nd March to 30th August for the stochastic spectral model. In Figure 4.2 the standard deviations tend to fluctuate on the low side for 1961 and 1967. This is evidenced and confirmed by the visual inspection of the simulated hydrographs in Figures 3.1 and 3.7. For 1961 and 1967, the UBC Budget Model preserves the measure of dispersion, while the stochastic spectral model fails in preserving this important statistic.


GRAPH OF MEAN FLOWS
AGAINST YEAR
(2 nd. March - 30 th. August)
FIG. 4.1.
MEAN FLOW C.F.S.
2800
2400
2000
1600
1200
800
400
0
1970
1966
1962
1969
1963
1965
1968
1961
1967
1964
YEAR
Actual mean flow
UBC budget model
Stochastic model
Model parameters and Fourier
coefficients are evaluated from
1963, 1964, 1970 data.



GRAPH OF STANDARD DEVIATION AGAINST YEAR
(2 nd. March - 30 th. August)
FIG. 4.2.
STANDARD DEVIATION C.F.S.
2800
2400
2000
1600
1200
800
400
0
1966
1962
1963
1970
1965
1969
1968
1967
1961
1964
YEAR
Actual standard deviation
UBC budget model
Stochastic model
Model parameters and Fourier coefficients are evaluated from 1963, 1964, 1970 data

IV.2 LIMITATIONS OF THE SPECTRAL MODEL

Chow (3) asserts that the behaviour of the hydrologic phenomena changes with time according to the law of probability as well as with the sequential relationship between the occurrence of the phenomena. This confirms the stochastic nature of the hydrologic phenomena. But there is some degree of uncertainty in stochastic models, and this has resulted in the gradual acceptance of these models by hydrologists.

Stochastic models are employed in preserving certain estimates of the statistical properties of a given record, and these results are then used for generating many equally likely sequences. These synthetic sequences could be used in place of the historical records, and a spectrum of equally likely optimal designs could be developed.

With the introduction of computer technology, techniques which have been largely untapped due to the voluminous calculations are now being rapidly explored. This has evidently contributed to the increasing popularity and use of the statistical stochastic models. But at the present level of these spectral analysis techniques, these models cannot be easily or directly used for solving problems related to water resources economics and planning.

IV.3 CONCLUSIONS

The two approaches used for this particular research problem are quite different and are not aimed at producing identical results.

The UBC Budget Model is classified as a deterministic or parametric model and the land phase of the hydrologic cycle is broken into several components. The major components are infiltration, evapotranspiration, aquifer response and the routing of the streamflow. Empirical approximations are used for defining the processes that control these components. The main emphasis is towards a better understanding of the physical laws governing the components and in the attempt to make the empirical model approximate more closely to the underlying physical situation. The UBC Budget Model has been developed to help in the solution of very practical operational problems. Its usefulness has been demonstrated in many different types of situations. Some of the applications of parameteric models are concerned with the design capacities of small reservoirs, the determination of the effect of channel improvements upon the flood frequency characteristics of a catchment, the effect of urbanization on flood peaks, the extension of streamflow records for small basins on the basis of rainfall records as well as for the solution of problems pertaining to aquifer response.

Stochastic modelling is concerned with the statistical simulation of a measured response of a system. Having derived the parameters, the statistical model is used for generating many equally likely time series. These are used in the design of water resources systems. Stochastic models are widely used for systems analysis and synthesis, particularly for reservoir planning studies and for the simulation of inputs to complex systems. It has widespread use in the analysis of the response of complex systems.

For this particular research problem, the UBC Budget Model appears to be more versatile since adequate meteorological and snowpack data are available. Generally, it is able to preserve the means and the standard deviations of the streamflow series of the relevant years. The stochastic spectral model may prove to be applicable to watersheds which do not have adequate meteorological data.

As mentioned earlier, spectral analysis has been successfully applied to certain problems in dynamics of systems such as earthquake analysis, electrical and sound theory, and many types of wave study. In all these situations there are clear-cut cause and effect relationships. The stochastic spectral model used for this analysis does not take into consideration the cause and effect relationship for the daily streamflow sequences. It therefore appears that as currently presented the spectral analysis techniques do not go very far in the solution of problems pertaining to water resources economics and planning. Perhaps with further research and recognizing the cause and effect relationship, the techniques may be capable of useful development, but this is uncertain.

Mathematical models are playing a vital role in hydrology, and future progress would inevitably depend on the development and innovation of these techniques. Dawdy (4) asserts that the division of mathematical modelling into stochastic, parametric and systems studies is quite arbitrary, and in reality all approaches blend together. Many problems in hydrology require computers in order to handle large quantities of data and to realize any level of accuracy. The technique used for arriving at a solution of a particular problem may prove to be more important than the type of mathematical model used. This emphasizes the urgency and need of being aware of mathematical advances in order to keep abreast of the latest techniques used by modern hydrologists for the solution of problems.

R E F E R E N C E S

(1) British Columbia *Snow Survey Bulletin*, Water Investigations Branch, Water Resources Service, Department of Lands, Forests and Water Resources, Victoria, B.C.

(2) Carlson, R. F., A. J. A. McCormack and D. G. Watts. "Application of Linear Random Models to Four Annual Streamflow Series," *Water Resources Research*, Vol. 6, No. 4 (1970), pp. 113-130.

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APPENDIX 1

LIST OF VARIABLES FOR THE SPECTRAL MODEL AND THE CALCOMP PLOT

A(K)	A_j values of the Fourier coefficients	
B(K)	B_j values of the Fourier coefficients	
CUMQG	Cumulative volumes of the generated streamflow sequences	cfs days
FRANDN	Random number generator	
Q1 (K,L)	Value of the periodic component for each subharmonic of the array	cfs
Q(I)	Actual streamflow series	cfs
QG(I)	Daily streamflow sequences generated by the UBC Budget Model	cfs
QGS(L)	Periodic or deterministic component for the daily streamflow sequences generated by the spectral model	cfs
QGSR(M)	Daily synthetic streamflow sequences generated by the stochastic spectral model	cfs
QT(L)	Cumulative values of the subharmonics of the periodic component	cfs
RES(J)	Stochastic component of the daily streamflow sequences generated by the random number generator FRANDN	cfs