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ABSTRACT

This thesis investigates the response of cracked reinforced concrete subjected to reverse cyclic loading with particular emphasize on the contribution from tensile stresses in cracked concrete. An iterative strain compatibility approach using a nonlinear bond stress-slip relationship was implemented into a computer program. The program indicated that monotonic tension stiffening does not decay with increasing strain. Analyses also demonstrated that the variation of slip along a reinforcing bar is approximately linear, which allowed the development of a simple transparent tension stiffening model that could be presented in equation form. This model indicated that in addition to being proportional to the square root of the concrete strength, the reinforcement ratio, and inversely proportional to the bar diameter, tension stiffening is directly proportional to the crack spacing.

To better understand tension stiffening under reverse cyclic loading, the bond model was enhanced by adding a reverse cyclic bond stress-slip relationship. An empirical reverse cyclic tension stiffening model was developed, based on the data obtained from an experimental program. This involved testing 5 large-scale reinforced concrete elements under reverse cyclic axial load. The parameters which were investigated were the amount of concrete surrounding the reinforcement, as well as the load history.

A model for reinforcing steel, which was selected from the literature, was modified in order to generalize the model for different types of steel. A simple model for the concrete in compression was also selected from the literature. The analytical models were combined into a computer program which can predict the complete reverse cyclic axial response of reinforced concrete.
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Luciano Fronteddu

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Montréal, Québec
CHAPTER 1

Introduction

Many bridges in British Columbia are thought to be in desperate need of significant seismic retrofitting. One reason for this is that reinforced concrete bridge columns constructed before about 1975 have a reasonable amount of longitudinal reinforcement but very little transverse reinforcement. The concern is that if significant diagonal cracking occurs during the initial cycles of an earthquake, the columns will have very little shear resistance during subsequent loading cycles.

Recent experimental research (Bhide and Collins, 1989, Adebar, 1989) has demonstrated that concrete members without any transverse reinforcement may have significant post-cracking shear resistance if there is sufficient longitudinal reinforcement. The mechanism by which shear is resisted without transverse reinforcement (i.e. without stirrups) involves shear and compression stresses on the crack interface and tensile stresses in the concrete between the cracks. It is important to note that without tensile stresses, cracked uniaxially reinforced concrete would not be capable of resisting shear. An accurate assessment of tensile stresses in cracked concrete is also very important for predicting the deformations (stiffness) of cracked reinforced concrete members.

The objective of this thesis was to develop a better understanding of the response of cracked reinforced concrete subjected to reverse cyclic loading with particular emphasize on the contribution from tensile stresses in cracked concrete.

Before the more complex problem of reverse cyclic loading could be addressed, a complete understanding of the monotonic response of cracked concrete was required. Therefore a considerable portion of this thesis is dedicated towards this end. The response of cracked
reinforced concrete is controlled primarily by bond between reinforcing steel and concrete. In order to study the bond phenomenon, a computer program was developed which is based on an iterative strain compatibility approach and uses an assumed nonlinear bond stress-slip relationship. This computer model indicated that the tension stiffening phenomenon (i.e., contribution from tensile stresses in cracked concrete) was considerably different than the empirical models had indicated. For example, the computer model indicated that tension stiffening does not decay with increasing strain as suggested by the empirical models. Comparisons with experimental results confirmed the computer model predictions.

The computer model also indicated that the slip along a reinforcing bar is approximately linear. This allowed the development of a simple transparent tension stiffening model which could be presented in equation form. Comparisons with experimental results showed that this rational model gave better predictions than the previously proposed empirical models.

To better understand tension stiffening under reverse cyclic loading, the bond computer model was enhanced by adding a reverse cyclic bond stress-slip relationship and by modifications which allow the program to solve a complete load cycle. However, because of the complexity of reverse cyclic bond and the uncertainty of the parameters, a tension stiffening model based on the reverse cyclic bond relationship could not be developed as was done for monotonic tension stiffening. Thus an empirical approach was used.

To collect the data needed to develop an empirical reverse cyclic tension stiffening model, an experimental program was undertaken. This involved testing 5 large-scale reinforced concrete elements under reverse cyclic axial load. The parameters which were investigated were the amount of concrete surrounding the reinforcement, as well as the load history. Based on the experimental results, an analytical model for reverse cyclic tension stiffening is proposed.
CHAPTER 1 Introduction

In order to develop a computer program which could make use of the newly developed tension stiffening model to predict the load-deformation response of reinforced concrete subjected to reverse cyclic loading, analytical models were also required for reinforcing steel in tension or compression, and concrete in compression. A model for reinforcing steel, which was selected from the literature, was modified in order to generalize the model for different types of steel. A simple model for the concrete in compression was also selected from the literature. The computer program was found to accurately predict the response of the five specimens tested as part of this study.

The analytical model for reinforcing steel is presented in Chapter 2, while the analytical models for concrete are presented in Chapter 3. The experimental program is described in Chapter 4 and the experimental results are summarized in Chapter 5. Chapter 6 compares the analytical models with the present experimental results as well as some previous experimental results. The thesis concludes with Chapter 7 which summarizes the developments and gives recommendations for further study.
CHAPTER 2

Cyclic Response of Bare Reinforcing Bars

In this chapter an analytical model for the cyclic response of reinforcing bars is presented. A summary of the original model is followed by a description of the modifications suggested in order to generalize the model for different types of reinforcing steel. A comparison with test results and other analytical models conclude the chapter.

2.1 Introduction

Stress-strain relationships for reinforcing steel under monotonic loading are relatively simple. For example, Fig. 2.1 shows typical stress-strain curves for a normal strength reinforcing bar and for a high strength bar.

The increase in strength due to strain hardening is usually ignored when the steel has a well defined yield plateau. Thus the stress-strain relationship for a normal strength reinforcing bar is expressed quite simply as:

\[ f_s = E_s \varepsilon_s \leq f_{sy} \]  \hspace{1cm} (2.1)

For high strength bars a somewhat more complicated formulation is usually required. For example, the modified Ramberg-Osgood formulation can be used:

\[ f_p = E_p \epsilon_p \left\{ A + \frac{1 - A}{\left[ 1 + (B \epsilon_p)^c \right]^{1/c}} \right\} \leq f_{pu} \]  \hspace{1cm} (2.2)

where the parameters \( A \), \( B \) and \( C \), are adjusted to give the best fit. Further details are given by Collins and Mitchell (1987).

The stress-strain relationship for a normal strength reinforcing bar subjected to cyclic
Fig. 2.1  *Monotonic stress-strain response of typical reinforcing bars*

loading is complicated by the fact that the response depends on the previous load history. Response to cyclic loading is often idealized as shown in Fig. 2.2(a). A more accurate representation of the behaviour of a reinforcing bar under cyclic loading is shown in Fig. 2.2(b). Note that the response of a normal strength reinforcing bar with a well defined yield plateau becomes nonlinear at stresses much lower than the initial yield strength if it has yielded on a previous load cycle. This phenomenon, which is known as the Bauschinger effect, depends mainly on the initial properties of the steel and the load history (Stanton and McNiven, 1979).

### 2.2 Pinto Model for Cyclic Response

Giuffré and Pinto (1970) proposed a mathematical expression to reproduce the hysteretic response of reinforcing steel. It was later implemented into a computer program for predicting
the deflections of a frame under both monotonic and cyclic excitations (Menegotto and Pinto, 1973). In the literature, the model is often referred to as the Menegotto-Pinto Model. However, since the model was initially developed by Giuffré and Pinto it will be referred to as the Pinto et al. Model.

In the model, two points are required to draw a loading path (see Fig. 2.3): the previous load reversal point (C) and the current load reversal point (A). Giuffré and Pinto proposed the following Ramberg-Osgood formulation to represent the loading path:

\[ f^* = \epsilon^* \frac{E_1}{E_o} + \left(1 - \frac{E_1}{E_o}\right) \frac{\epsilon^*}{(1 + \epsilon^*)^{1/\gamma}} \]  

(2.3)

The curve has an initial slope of \(E_o\) and a final slope of \(E_1\) as shown in Fig. 2.4. Since load reversals can occur at any state of stress and strain, Eq. (2.3) is expressed in terms of normalized stress and strain \((f^* \text{ and } \epsilon^*)\).

Several definitions for \(\epsilon^* \text{ and } f^*\) have been proposed since the initial formulation by

---

**Fig. 2.2**

a) Idealized cyclic response of a normal strength reinforcing bar  
b) Actual response of a normal strength bar
Giuffré and Pinto, Fillipou, Popov and Bertero, (1983) suggest that the stresses and strains be normalized with respect to the previous point of load reversal \((\varepsilon_r, f_r)\) and the point of intersection of the two asymptotes \((\varepsilon_0, f_0)\). That is,

\[
\varepsilon^* = \frac{\varepsilon - \varepsilon_r}{\varepsilon_0 - \varepsilon_r} \quad f^* = \frac{f - f_r}{f_0 - f_r}
\]  

(2.4)

The parameter \(R\) in Eq. (2.3) controls the Bauschinger effect. It dictates the sharpness of the transition curve between the two slopes: the bigger the \(R\) value the sharper the transition curve. For example, before yielding of the steel, no Bauschinger effect is involved and therefore \(R\) is taken as 20. Once the steel has yielded, the Bauschinger effect becomes apparent and \(R\) is decreased to values ranging between 1 and 5.

Giuffré and Pinto noticed that the shape of the curve between points A and B (see Fig. 2.3) is essentially a function of the strain distance between points A and C. They suggested the following expression for parameter \(R\):
Fig. 2.4 Pinto curve

\[ R = R_o - \frac{A_1 \xi}{A_2 + \xi} \]  \hspace{1cm} (2.5)

where \( \xi \), the normalized strain distance between the point of intersection (\( \varepsilon_o \)) and the previous load reversal (\( \varepsilon_{pr} \)), is given by

\[ \xi = \left| \frac{\varepsilon_o - \varepsilon_{pr}}{\varepsilon_y} \right| \]  \hspace{1cm} (2.6)

Fillipou et al. (1983) suggest that the remaining parameters in Eq. (2.5), namely \( R_o \), \( A_1 \) and \( A_2 \), be taken as 20, 18.5 and 0.15, respectively.
2.3 Modifications to the Pinto et al. Model

The initial and final slopes of the loading path are important parameters in the model. The initial slope, $E_o$, decreases as the material is cycled into the plastic range. This phenomenon, called stiffness degradation, has been experimentally observed and introduced into stress-strain models by Stanton and McNiven (1979) and Seckin (1981). Since in most models the loading curve has essentially no linear portion, the initial slope, $E_o$, is simply the initial tangent modulus. Therefore ignoring the stiffness degradation by assuming a constant $E_o$ introduces very little error. This approach has been used by Singh et al. (1965), Kent and Park (1973), Fillipou et al. (1983), Stevens et al. (1987). It was also used in the original Pinto et al. Model and is maintained here.

The final slope of a loading path, $E_f$, is considerably more complicated. Fillipou et al. (1983) have suggested that all load cycles be asymptotic to a fixed line. Such a process can be called "fixed envelope" approach. Experimental results indicate that using a fixed envelope is not appropriate, especially for steel showing pronounced strain hardening (Aktan et al., 1973) where the cyclic envelope varies according to the amount of plastic straining. Filippou et al. (1983) referred to the concept of "isotropic strain hardening", which allowed them to lower the position of the compression asymptote depending on the amount of plastic straining in tension. Although they obtained reasonable agreement with the experimental results of Ma et al. (1976), their empirical approach requires certain parameters which would need to be optimized for each different type of reinforcing steel.

The approach used herein is somewhat different. It is assumed that the monotonic stress-strain curve contains all the information necessary to accurately model the behaviour of the given steel under cyclic loading. This was done since the monotonic response is usually well known.
or can be determined very easily. Therefore in this study a set of rational rules were developed to relate the monotonic stress-strain relationship to the cyclic envelope.

Firstly, the envelope is shifted according to the amount of plastic straining in tension and compression. The shift as described in Fig. 2.5 is applied to the boundary strains (maximum strain in tension and minimum strain in compression) reached during the load history. As a result, the tension envelope moves toward the compression side with increasing plastic straining in compression, while the compression envelope moves toward the tensile side due to plastic straining in tension. Note that envelopes move in one direction only.

With the shifting of the envelopes, the yield plateau disappears from the response once the reinforcing bar has been strained up to the point of strain hardening. The yield plateau is thus said to be unrecoverable. This phenomenon has been confirmed by the experimental results of Aktan et al. (1973).

Since the monotonic response will be used to represent the cyclic envelope, one can define the Bauschinger effect as the transition between the current point of reversal and the monotonic envelope. As explained earlier, Giuffré and Pinto realized that the distance between
the current point of reversal and the previous point of reversal has a strong influence on the Bauschinger effect. Its magnitude depends on the absolute maximum stress reached whether in tension or in compression, and the strain distance between the current point of reversal and the maximum strain reached in tension (when reloading) or the minimum strain reached in compression (when unloading). The variable \( \xi \) in Eq. (2.5), which shapes the Bauschinger effect, is determined as shown in Fig. 2.6.

The asymptote slope \( E_I \) (see Fig. 2.4), also called plastic modulus, is an important parameter. Filippou’s method of using a single asymptote turns out to give poor results when trying to represent the experimental results of Aktan et al. (1973). That kind of load history requires additional features such as a cut off on the stress. In the present study different kinds of envelopes and rules for shifting the envelopes were tried. The most appropriate one was found to be the four slope envelope shown in Fig. 2.7.
CHAPTER 2 Cyclic Response of Bare Reinforcing Bars

Fig. 2.7 Steel monotonic envelope

The curve can be described as follows: $E_s$ is the initial elastic slope, $b_1 E_s$ is the slope of the yield plateau, $b_2 E_s$ is the first strain hardening slope, $b_3 E_s$ is the second strain hardening slope. In addition the stress is limited to a max of $f_{ult}$.

2.4 Comparison with Test Results

The Modified Pinto et al. Model has been implemented into a computer program. Predictions from the program were compared to test results from Aktan, Karlsson and Sozen, (1973), and from Seckin, (1981). Aktan et al. submitted the rebars to significant plastic straining. In their tests the elongation ranged from -60 to 100 parts per thousand which is a total range of 16% strain. Seckin, on the other hand, subjected reinforcing bars to a range of only 15 parts per thousand.

Predictions from the Modified Pinto et al. Model are compared with the results of Aktan et al. (1973) in Fig. 2.8 and in Fig. 2.9. In Fig. 2.8 the reinforcing bar is subjected to symmetrical strain cycles (i.e. equal tension and compression strains). In Fig. 2.9, test #5 shows a rebar that was submitted to the largest strain in the first cycle. Test #6 shows a rebar first yielded in tension before being cycled a number of times in compression and then in
tension. These three tests show clearly that the yield plateau is unrecoverable. If the specimen is cycled several times between two given strains, the loop is stable (it does not degrade). When unloading in a region where no plastic straining has occurred, like in the last two cycles of test #6, the response is almost identical. This confirms the proposed rules for shifting the envelope.

Predictions from the model are also compared with the results of Seckin (1981) in Fig. 2.10 and Fig. 2.11. In Fig. 2.10, specimen BR01 was submitted to two large cycles before being unloaded and then cycled one last time. Specimen BR02 was first loaded to yield in compression and then cycled in tension and in compression. In Fig. 2.11, specimen BR07 is shown to have been loaded in cycles to progressively larger tensile strains. The last specimen, BR13, was also subjected to reverse cyclic loading in the tensile strain region.

![Graph of stress-strain relationship for Test #3](image)

**Fig. 2.8** Comparison with experimental results from Aktan, Karlsson and Sozen (1973)
Fig. 2.9 Comparison with experimental results from Aktan, Karlsson and Sozen (1973)
Fig. 2.10 Comparison with experimental results from Seckin (1981)
Fig. 2.11 Comparison with experimental results from Seckin (1981)
2.5 Discussion of Other Models

Several models for the hysteretic behaviour of reinforcing steel have been proposed. Among them are: Singh, Gerstle and Tulin (1965); Kent and Park (1973); Aktan, Karlsson and Sozen (1973); Ma, Bertero and Popov (1976); Seckin (1981); and Stevens, Uzumeri and Collins (1987).

The choice of an appropriate model depends on the particular application. In the present application the model will be used to predict the response of reinforced concrete elements using a strain controlled iterative approach. The most appropriate model in such a case gives the stress as a function of the strain.

In addition, a model should be easy to modify and ready to use with any type of reinforcing steel. Therefore models involving extensive regressions and optimization for one particular type of steel were discarded (e.g. the models of Kent et al., Aktan et al., Ma et al.). Seckin’s model and Stevens et al. model were discarded because they generate the tangent modulus. Such models are meant primarily for finite element implementation.

The above researchers have however made several observations on the behaviour of reinforcing steel under cyclic loading and have proposed different simplifications which have helped in the development of the rational modifications to the Pinto et al. Model.

In particular, the Stevens et al. procedure of generating the Bauschinger effect was adopted herein. In addition, the Stevens et al. model is bounded by the monotonic envelope, a feature which has been added to the Pinto et al. Model. The idea of shifting the envelope can be found in the work of Ma et al. (1976), although there only the strain hardening portion of the envelope was shifted.
2.6 Summary

The behaviour of reinforcing steel under monotonic and cyclic loading was studied. The Pinto et al. Model, which was chosen from the literature, was described along with the modifications proposed to generalize the model for any reinforcing steel. Comparisons with test results showed good agreement. The Modified Pinto et al. Model is a powerful mathematical model that can be implemented easily and simulates the cyclic behaviour of reinforcing steel with impressive accuracy.
CHAPTER 3

Concrete Model

This chapter presents analytical models for the cyclic response of concrete. For completeness the chapter begins with the description of a model for concrete under cyclic compression. The main focus of this chapter is on an analytical description of concrete in tension, which is presented in four parts. A description of the monotonic response of concrete is followed by an explanation of the interaction between concrete and steel (bond), the derivation of a tension stiffening model based on a bond model, and finally the generalization of the monotonic tension stiffening model for reverse cyclic loading.

3.1 Concrete in Cyclic Compression

A number of previous studies have focused almost entirely on the cyclic behaviour of concrete and the parameters which influence it. The purpose of these studies has been for example, predicting the cyclic response of short columns. The long term objective of the present investigation is to predict the cyclic shear response of reinforced concrete with little or no transverse reinforcement. In this case the concrete is subjected to relatively small compressive stresses and the accuracy of the prediction is not strongly dependant on the concrete compression model. Of more importance are issues such as the tensile response of concrete and bond behaviour. However, for completeness a brief discussion of concrete in cyclic compression is presented below.
3.1.1 Review of Previous Work

In one of the first studies conducted on the cyclic response of plain concrete, Sinha, Tulin and Gerstle (1964a) concluded that the monotonic envelope is also the cyclic envelope. Karsan and Jirsa (1969) came to a similar conclusion. They quantified the effect of subsequent load cycles on the loops defining a stability limit for the point of intersection between the unloading and reloading paths, and concluded that the stress-strain relationship was not unique but depended on the stress at peak strain reached during the previous loading cycle. Yankelevsky and Reinhardt (1987) gathered the experimental results of several previous investigators. They used the stability limit defined by Karsan and Jirsa in 1969 and included the concept of degrading stiffness at unloading and reloading.

Much work has been done on reinforced concrete subjected to axial compression and bending, particularly in assessing the earthquake resistance of beam-column joints. Park, Kent and Sampson (1972) used a modified plain concrete envelope where the post peak response is represented by a straight line and its slope is a function of confinement, i.e. the amount of transverse reinforcement. The loops were parallelograms of constant shape with no degradation involved. It is important to mention that the joint response is dominated by the yielding of the steel, therefore Park et al. (1972) used an elaborate steel model and simplified their concrete model. Blakeley and Park (1973) upgraded the loops in the previous model including degradation of the unloading and reloading stiffnesses using two straight lines for unloading and one line for reloading.

Seckin (1981) in a similar study proposed a polynomial for the unloading branch varying its degree according to the peak. He used a straight line for the reloading branch. Fillipou, Bertero and Popov, (1983) simplified the loop to a single line of degrading stiffness along with a confinement envelope. They assumed that the nonlinearity of the joint response is primarily
due to nonlinear behaviour of the reinforcing steel after yielding and due to bond slippage.

Some work has been done on the cyclic shear response of concrete applied to the resistance of shear walls. Darwin and Pecknold (1976) used a uniaxial stress-strain relationship for concrete formed by a parabola connected to a straight line post-peak. The relationship is generalized to the biaxial state of stress by having the maximum compressive stresses in the two principal directions satisfy the well known yield condition of Kupfer and Gerstle (1973). Four straight lines form a loop: three on unloading and one on reloading. Shirai and Sato (1981a and 1981b) analyzed the behaviour of a shear wall using a simple concrete model along with a yield criteria. They neglected the hysteresis by using a simple straight line as the unloading-reloading loop. The nonlinearities were captured by including bond slippage and dowel action.

Stevens et al. (1987) in predicting the response of membrane elements subjected to shear suggested a sophisticated model. The envelope is a parabola before peak and a cubic post peak. Its magnitude is modified according to the amount of confinement, the strain in the perpendicular direction and the damage accumulated. The loop is an assembly of one hyperbola for unloading and two parabolas for reloading. To position each parabola it is required to solve at least three equations per parabola. The advantage of Stevens’ model is that it ties explicitly the concrete compressive and tensile response together.

Izumo et al. (1989), in their formulation of the shear problem, used a model for concrete in compression with a model for shear stress-deformation along the crack. The uncracked concrete envelope is lowered for cracked concrete as a function of the maximum tensile strain reached during the load history. The unloading branch of the loop is a circular arc that has an infinite tangential stiffness at the start and that passes through a residual strain point at completion of unloading. The reloading branch is a straight line of degrading slope connecting the residual strain point to the last point on the envelope. As a result the loop is constructed in
reverse. First the stiffness of the reloading branch is computed taking into account the degradation, then the residual strain point is positioned, and finally the equation of the unloading arc is determined.

3.1.2 Description of Concrete Compression Model

The choice of a model for the behaviour of concrete in a reinforced concrete member depends on several factors such as: the type of member being modeled (e.g. beam, beam-column joint, shear wall), the predominant force to be resisted (e.g. axial load, bending moment or shear), the solution technique (e.g. finite element) and the required accuracy.

The analytical model chosen for concrete in compression is summarized in Fig. 3.1 and is described below.

If the concrete is not confined, the envelope is the well known simple parabola

\[ \frac{f_c}{f_{c_{\text{max}}}} = 2 \epsilon_{cn} - \epsilon_{cn}^2 \quad (3.1) \]

\[ \epsilon_{cn} = \frac{\epsilon_c}{\epsilon_{co}} \]

where \( f_{c_{\text{max}}} \) and \( \epsilon_{co} \) are the peak compressive stress and the strain at peak stress.

When there is significant confinement, a cubic function, given by Eq. (3.2), is used for the descending branch as proposed by Stevens et al. (1987).

\[ \frac{f_c}{K_c f_{c_{\text{max}}}} = 2(1 - K_r)\epsilon_{ccn}^3 - 3(1 - K_r)\epsilon_{ccn}^2 + 1 \quad (3.2) \]

\[ \epsilon_{ccn} = \frac{\epsilon_{cn} - 1}{\epsilon_{ccr} - 1} \]

\( K_c \) is a confinement factor, \( K_r \) is a residual resistant stress factor, \( \epsilon_{ccr} \) is the strain at beginning
of residual resistant stress plateau. In their formulation, Stevens et al. suggested a value of 0.3 for $K_r$, while in this study $K_r$ is left as a variable in order to keep the model as general as possible and enable the use of any confinement model.

As stated by Karsan and Jirsa (1969), the stress-strain relationship of concrete is not unique but depends on the load history. This is captured analytically by using the maximum stress-strain point reached in compression and the corresponding point reached in tension to shape the loops.

The response at unloading is represented by the parabola

$$f_c = A \varepsilon_c^2 + B \varepsilon_c + C$$  \hspace{1cm} (3.3)

The parameters in Eq. (3.3) are given by the following equations where $(\varepsilon_{un}, f_{un})$ is the starting point of unloading and $E_{cp}$ is the modulus at the residual plastic strain point $(\varepsilon_{cp}, f_{cp}=0)$. 

**Fig. 3.1** *Analytical model for plain concrete in compression.*
The plastic modulus \( E_{cp} \) can be evaluated using a formula proposed by Seckin and rearranged by Stevens to give

\[
E_{cp} = E_{ct} \left[ .98 - .7 \left( \frac{\varepsilon_{un}}{\varepsilon_{co}} \right)^{4.1} \right]
\]  \hspace{1cm} (3.5)

\[
E_{cp} \geq .05E_{ct}
\]

The point of residual plastic strain is positioned using the well known Karsan and Jirsa (1969) equation:

\[
\frac{\varepsilon_{cp}}{\varepsilon_{co}} = .127 \frac{\varepsilon_{un}}{\varepsilon_{co}} + .145 \left( \frac{\varepsilon_{un}}{\varepsilon_{co}} \right)^2
\]  \hspace{1cm} (3.6)

\[
\frac{\varepsilon_{cp}}{\varepsilon_{co}} \leq 3.02
\]

The condition on \( \varepsilon_{cp}/\varepsilon_{co} \) was not suggested by Karsan and Jirsa. It was added to the Stevens et al. Model and is maintained here in order to avoid the decrease in strain difference \( (\varepsilon_{un} - \varepsilon_{cp}) \) after a certain strain level. Such large strain levels were not reached by Karsan and Jirsa since they tested plain concrete. Confinement enables concrete to resist significant stress at large compressive strains.

The reloading curve is assumed to be a straight line that connects the point of residual plastic strain to the maximum point reached on the envelope during the load history.
3.2 Plain Concrete in Cyclic Tension

There have been only a few studies on the behaviour of plain concrete under axial tension. This is partly due to the difficulty in experimentally measuring the post-cracking response of concrete, and also it is only recently that the post-cracking response has been thought to be significant.

Gopalaratnam and Shah (1985) tested a series of prisms of plain concrete in monotonic tension. They observed that after cracking, the strain increased at the crack location, but decreased away from the crack location. They concluded that there is no unique stress-strain relationship for cracked plain concrete, and decided to represent the response by an average stress-average crack width relationship given by

\[
\frac{f_c}{f_{cr}} = e^{-kw^{1.01}}
\]

(3.7)

where \(w\) is the crack width in \(\mu\text{m}\) and \(k\) is a coefficient taken equal to 0.039. Gopalaratnam and Shah also tested a few specimens under cyclic axial tension. They suggested two equations to predict the average residual displacement of their test specimens:

\[
\delta_r = \frac{E_{ct} - \frac{f_{cr}}{\delta_{cr}}}{\delta_{cr}} \delta_u \quad \delta_u \leq \delta_{cr}
\]

(3.8)

\[
\delta_r = \delta_u - \frac{f_{cr}}{\delta_{cr}} \quad \delta_u \geq \delta_{cr}
\]

where \(\delta_u, \delta_r\) are the displacement at the start of unloading and the residual displacement. Note that the displacement is similar to the crack width.

Reinhardt, Cornelissen and Hordijk (1986) conducted a study on the tensile behaviour of plain concrete. Prior to cracking, an irreversible strain formed at a stress above \(0.6f_{cr}\). The
post-cracking response can be modelled with the following equation for normal-weight concrete:

\[
\frac{f_c}{f_{cr}} = \left[ 1 + 27 \left( \frac{\delta_c}{\delta_o} \right)^3 \right] \exp \left( -6.93 \frac{\delta_c}{\delta_o} \right) - 0.0274 \frac{\delta_c}{\delta_o}
\]

where \(\delta_c\) is the crack width, and \(\delta_o\) is the displacement at which concrete no longer sustains any tensile stresses (taken as 160 \(\mu\)m for normal-weight concrete). They also looked at cyclic tension, suggesting a set of rules for modelling the loops. They found that the stress drop at reloading to the envelope is greater when the concrete is cycled in compression. In a later study, Yankelevsky and Reinhardt (1989) refined the modelling of the loops with a complex set of rules based on a focal point approach.

Duda and König (1991) proposed a rheological model for the stress-crack width relation. Stresses are transferred over a crack by friction forces. The monotonic response was represented by the following equation:

\[
\frac{f_c}{f_{cr}} = 0.5 \left( e^{-\frac{w}{w_a}} + e^{-\frac{w}{w_b}} \right)
\]

where \(w_a = 14.4\mu\)m and \(w_b = 63.1\mu\)m. For the cyclic response, the model requires a complex set of rules involving 14 parameters.

A comparison of the monotonic responses predicted by the different models is presented in Fig. 3.2. It can be seen that the different models are reasonably similar except at large crack widths. It is interesting to note that the different investigators agree on the following points: the resistance of cracked plain concrete is controlled by the resistance mechanisms at the crack; the elongation is concentrated at the crack; the monotonic post-cracking response can be divided into two parts, an initial steep decrease followed by a moderate decrease; the monotonic response can be taken as the cyclic loading envelope, and; additional research is necessary.
Monotonic response of plain concrete according to three different authors.

3.3 Reinforced Concrete in Monotonic Tension

The response of a reinforced concrete element, such as the one shown in Fig. 3.3, is very different than the response of plain concrete. The difference being due to the presence of reinforcing steel at the crack location.

In order to understand the mechanisms involved in cracked reinforced concrete, assume that a force $N$ is applied on the element shown in Fig. 3.3 by pulling on the reinforcing bar at both ends. At the free end, the steel is the only material resisting the force. Away from the crack, the steel shares the force with the concrete. Since the steel is strained at the free face while the concrete is not, the steel slides against the concrete developing friction stresses known as bond stresses.

Bond stresses exist on a certain length up to the point where compatibility is reached. With increasing force, the transfer length increases and the stress in the concrete increases up to the cracking stress. At this point the compatibility zone can be referred to as a zone of potential cracking. A crack will form, not exactly in the middle, but somewhere near the middle depending on imperfections in the concrete and stress concentrations due to the presence of
transverse reinforcement.

After a new crack fully develops, it can be assumed that there are now two elements where the scenario described above is repeated. New cracks will continue to form until the crack spacing becomes too short for sufficient stress to be transferred to the concrete between cracks. At this point, the reinforced concrete is said to have reached a stable crack spacing.

Because the concrete between the cracks helps the reinforcing steel to resist the tensile force, the average deformation of the reinforcing bar is reduced. This phenomenon is known as "tension stiffening". This concept can be better visualized by looking at the experimentally measured load-deformation response of a typical reinforced concrete element shown in Fig. 3.4. The straight line plotted in Fig. 3.4 is the predicted response of the bare reinforcing steel. Note that since the specimen was tested under load-control, each time a crack occurred there was a sudden increase in deformation.

Tension stiffening can be accounted for by either modifying the stress-strain relationship of the reinforcing steel, or by modifying the stress-strain relationship of the concrete. The first approach is more common. For example this approach was adopted in the CEB-FIP Model Code (1978). The CEB-FIP Model Code tension stiffening expression was first proposed by Leonhardt (1977). The steel stress-strain relationship becomes a reinforced concrete stress-strain
relationship as follows:

\[ \epsilon = \epsilon_{cr} + \frac{F}{A_s E_s} \left[ 1 - \left( \frac{F_{cr}}{F} \right)^2 \right] \]  

(3.11)

where \( F \) is the applied load, \( F_{cr} \) is the cracking load and \( \epsilon_{cr} \) is the cracking strain of plain concrete.

Based on a series of panels subjected to shear, Vecchio and Collins (1986) suggested the following equation for the average tensile stress in cracked reinforced concrete:

\[ \frac{f_c}{f_{cr}} = \frac{1}{1 + \sqrt{200\epsilon_c}} \]  

(3.12)

In order to account for the apparent reduced stiffening effect in members subjected to uniaxial tension, Collins and Mitchell (1987) suggested the following modified version of the Vecchio and Collins expression:
where $\alpha_1$ is a factor to account for bond characteristics of the reinforcement ($\alpha_1=1.0$ for deformed bars, $\alpha_1=0.7$ for plain bars and $\alpha_1=0$ for unbonded bars) and $\alpha_2$ is a factor to account for the kind of loading ($\alpha_2=1.0$ for short-term monotonic and $\alpha_2=0.7$ for long-term and cyclic loading).

Hwang (1983) tested 34 specimens and proposed the following exponential function:

$$\frac{f_c}{f_{cr}} = e^{-1000(e_c - \epsilon_c)}$$

Equation (3.13) and Eq. (3.14) have something in common. In both equations the magnitude of the average tensile stress is dependent on essentially one single variable, namely the concrete tensile strength. Such formulations hide the real nature of tension stiffening. Tension stiffening is the result of an interaction between steel and concrete, therefore the relevant bond parameters should be included in the equation.

The CEB-FIP Model Code expression, Eq. (3.11), can be rearranged to solve for the concrete stress:

$$\bar{\epsilon}_c = \frac{E_s \rho}{2} \left[ \sqrt{(\epsilon_x - \epsilon_cr)^2 + 4 \left( \frac{f_{cr}}{E_s \rho} + \epsilon_c \right)^2} - (\epsilon_x + \epsilon_{cr}) \right]$$

Günther (1991b) proposed an enhancement to the CEB-FIP equation:

$$\epsilon = \epsilon_{cr} + \frac{F}{A_s E_s} \sqrt{1 - \left( \frac{F_{cr}}{F} \right)^2}$$

This equation can be rearranged as:
Eq. (3.15) and Eq. (3.17) are intimately related to the steel response. These equations include one geometrical property of the specimen, which is the concrete to steel ratio, but the area available for bond is missing.

Stevens et al. (1991) in a work developing a tension stiffening model for finite element application proposed:

\[
\frac{f_c}{f_{cr}} = (1 - \alpha) e^{-\lambda_t (\epsilon - \epsilon_c)} + \alpha
\]  

(3.18)

where

\[
\alpha = C_i \frac{\rho}{d_b} \quad \quad \quad C_i = 75mm
\]  

\[
\lambda_t = \frac{270}{\sqrt{\alpha}} \leq 1000
\]  

(3.19)

Stevens included two of the bond parameters, but did not include the crack spacing. As explained earlier, each time a crack forms there is a sudden increase in deformation which results in a sudden decrease in tension stiffening. In other words, the stiffening effect decreases with crack spacing which shows that tension stiffening is directly proportional to crack spacing.
3.4 Analytical Model for Bond

In order to develop an appropriate tension stiffening relationship for reverse cyclic loading it will first be necessary to fully understand bond in direct tension because it is the main phenomenon involved in monotonic tension stiffening.

3.4.1 Bond Stresses under Monotonic Loading

Bond in direct tension, unlike pull-out bond, is difficult to study experimentally because the bond stress and the slip are local phenomena that are difficult to measure. The relationship between bond stress and bond slip is not unique but varies according to the position relative to the crack (Nilson, 1972). The mechanisms involved in direct tension bond are similar to pull-out bond (bond mechanisms are thoroughly described in Eligehausen et al., 1983) but they are different in magnitude. In pull-outs there is always compression or shear applied to the concrete which produces significant compressive stresses, while in direct tension, the tensile forces applied at each end equilibrate each other. Any compressive stresses which are induced in the member are much smaller than the tensile stresses. This difference is best indicated by the peak bond stress which may be 4.9 MPa in direct tension but could reach as high as 13.5 MPa for a well confined pull-out specimen (for $f'_c = 30$ MPa).

Numerous authors have developed different techniques and managed to provide a better understanding of bond in direct tension, for example Nilson (1972), Mirza and Houde (1979), Somayaji and Shah (1981), Jiang, Shah and Andonian (1984), Scott and Gill (1987), and Günther (1991a). Several authors tried using these results to come up with analytical predictions for the behaviour of a simple uniaxial element: Somayaji and Shah (1981), Jiang et al. (1984), Yang and Chen (1988), Russo and Romano (1992).
The relationship adopted in this study is shown in Fig. 3.5. This relationship is similar to that used by Yannopoulos and Tassios (1991) and by Russo and Romano (1992). However, the relationship was modified to correspond more closely to the experimental results of Nilson (1972), Shirai and Sato (1981b) and Günther (1991a).

The relationship adopted in this study has two distinct parts: an ascending part and a plateau. Previous authors agree on the shape of the ascending part of the relationship, and on the magnitude of the slip at peak stress, $u_p$, which is typically assumed to be between 0.03 mm and 0.05 mm.

The initial portion of the relationship is described by the following relationship which is based on the work of Eligehausen

$$
\tau = \tau_{\text{max}} \left( \frac{u}{u_p} \right)^{\beta}
$$

where, $\beta$ is usually assumed to be 0.4. Although this equation was first proposed for pull-out failures, it is used here because of its simplicity. Other authors (Günther, 1991a, Nilson, 1972, Mirza and Houde, 1979) have suggested polynomials based on a regression analysis of their
experimental results.

There is no agreement in the literature on the second portion of the curve. Several authors have assumed a plateau: Shipman and Gerstle (1979), Somayaji and Shah (1981), Yannopoulos and Tassios (1991), while others such as Günther (1991a) suggested a descending curve.

In addition to relating the bond stress to the local slip, the local degradation of the bond properties near the crack must be accounted for. The length along which this degradation takes place depends on several factors such as the concrete cover (Yannopoulos et al., 1991) and the diameter of the bar (Günther, 1991a).

There is not enough data to enable the development of rational expressions which include the relevant parameters. However the relevant parameters are all included in the CEB-FIP crack spacing expression, and the bond degradation length is a characteristic of the specimen which is closely related to the crack spacing.

Tassios and Koroneos (1984) found that for points away from the primary crack face, at a distance of at least 0.125s from a primary crack face (where s is the distance between the two free faces of the element), a common bond stress-slip curve may be assumed with reasonable accuracy. For points closer to the primary crack face (i.e. at distances smaller than 0.125s from a crack face) much lower bond stress-slip curves are found which gradually tend to zero bond stress for finite slip values. Based on these experimental observations, it was decided to assign a value of 0.15s to the linear cut off. As a result, 30% of the total crack spacing is dependent on the linear cut off and 70% on the average bond stress-slip relationship.

Now that a bond stress-slip relationship has been established, it is possible to determine the stress distribution along the length of the bar. To do so, an iterative numerical technique is necessary. The algorithm used herein was proposed by Tassios and Yannopoulos (1981). The
procedure can be summarized as follows:

The element of length $s/2$ (from crack face to midway between cracks) is subdivided into $n$ sections of length $\Delta x$, assumed to have uniform bond stress and slip.

1. Assume a slip at the free end,

2. At each section $i$, compute

   the steel stress $f_s$ from,

   $$f_{si} = f_{si-1} - \frac{4\tau}{d_b} \Delta x$$

   (3.21)

   the concrete stress from,

   $$f_{ci} = f_{ci-1} + \frac{A_s}{A_c} \frac{4\tau}{d_b} \Delta x$$

   (3.22)

   the steel and concrete strain from the adopted monotonic relationships, and the slip $u_i$,

   $$u_i = u_{i-1} - 0.5 (\varepsilon_{si} + \varepsilon_{si-1}) \Delta x + 0.5 (\varepsilon_{ci} + \varepsilon_{ci-1}) \Delta x$$

   (3.23)

3. Calculate the slip at the mid-section.

   If the value of the slip at the mid-section is not close enough to zero return to step (1) and choose a new value for the slip at the crack.

This algorithm was implemented into a computer program called BOND. Program BOND computes the stress distribution in an element of given length. Some typical results obtained from the program are shown in Fig. 3.6.

3.4.2 Bond Stresses under Reverse Cyclic Loading

Reverse cyclic bond behaviour has been extensively studied for pull-out because such mechanisms prevail in beam-column joints.
Fig. 3.6 *Predicted stress variations using program BOND.*

The most commonly used reverse cyclic bond model is due to Eligehausen et al. (1983) (see Fig. 3.7). In this model the monotonic response is considered to be the envelope of the cyclic response. The monotonic response was described in the previous section, therefore here the focus will be on the intermediate loops and on how they connect to the envelope.
A typical loop can be divided into six elements:

1) unloading curve of a given slope,

2) frictional bond resistance,

3) connection to the envelope in the opposite direction (reloading curve),

4) unloading curve,

5) frictional bond resistance, and

6) connection to the envelope.

The unloading curve has a constant slope. The frictional bond resistance depends on the energy dissipated during the load history. A simple straight line stiffness equal to the unloading stiffness connects the friction resistance to the previous maximum point reached on that envelope. Fillipou et al. (1983) upgraded the connection procedure by using a fourth order polynomial instead of two straight lines.

Tassios has spent considerable effort trying to extend the knowledge gained in cyclic pull-out tests to cyclic direct tension. He published an exhaustive paper (Tassios, 1979) where he
gathers the experimental results of several investigators along with his own results, to try to explain the bond phenomenon. Yannopoulos and Tassios (1991) summarized their work in an attempt to predict the response of uniaxially reinforced elements under axial cyclic tension.

Cyclic bond models have been proposed or modified to be used in a few applications particularly for shear walls (e.g. Shipman and Gerstle, 1979) who based their model on work done by Nilson (1972) and by Lutz and Gergely (1967). Shirai and Sato (1981b) used a model similar to Tassios. Stevens et al. (1987) used the Eligehausen’s model.

The reverse cyclic bond stress-slip relationship presented in this study is shown in Fig. 3.8. It was implemented into the computer program BOND, but it was not used to predict the response of the test specimens because there was no way to determine the parameters involved. The cyclic bond model served as a basis for the choice of parameters included in the empirically developed cyclic tension stiffening model.

The six elements employed in the Eligehausen Model and Tassios Model are included in the assumed model, as well as a few additional features which have been added to enhance the loops.

The unloading modulus ($E_{un}$) is assumed to be constant if unloading occurs before $u_p$, and then to degrade with the increase in maximum slip:

$$E_{un} = E_{in} \quad \text{if } u_{un} \leq u_p$$

$$E_{un} = \alpha_{un} F_{in} \frac{u_{un} - u_p}{k_{un}} \quad \text{if } u_p < u_{un} \leq k_{un} u_p$$

where $E_{in}$ is assumed equal to 600 MPa/mm as proposed by Tassios, $\alpha_{un}$ is taken as 0.75 and $k_{un}$ is set to 3.

The unloading mechanism goes from stiff unloading to frictional resistance represented
by a plateau at a stress level dependent on the previous maximum stress-slip level reached in the load history. The connection to the envelope in the opposite direction depends on the maximum slip reached in that direction. For the first cycle, the frictional bond resistance connects to the monotonic envelope. For the subsequent cycles the frictional bond resistance picks up a stiffness of $E_{re}$ at slip $u_{re}$:

$$u_{re} = k_{re} u_{max}$$

$$E_{re} = \frac{\beta_{re} \tau_{max} - \beta_{un} \tau_{un}}{(1 - k_{re}) u_{max}} \quad E_{re} \leq E_{un}$$

where $u_{max}$ and $\tau_{max}$ are the maximum bond slip and stress reached in the load history, $\beta_{re}$ is set equal to 0.7, $k_{re}$ equals 0.7, and the frictional bond resistance factor ($\beta_{un}$) is assumed constant (independent on the number of cycles) and equal to a lower bound of 0.25.
3.5 Derivation of a Tension Stiffening Relationship Based on an Assumed Bond Stress-Slip Relationship

This derivation is better understood by first looking at what happens when the bond stress-slip relationship is assumed to be rigid-plastic. In that case there is no need to consider the compatibility of the reinforcing bar and concrete strains. The bond stress is equal to $\tau_{\text{max}}$ on the first half of the element and to $(-\tau_{\text{max}})$ on the second half.

At any section from the free face, the force transferred to the concrete is equal to the bond stress ($\tau_{\text{max}}$) multiplied by the area available for bond transfer, which is equal to the perimeter of the bar times the distance from the face ($x$).

$$F_c(x) = \tau_{\text{max}} \pi d_b x$$

(3.26)

The concrete force distribution along the length will be triangular with a maximum at the mid-section of:

$$F_c(\frac{S}{2}) = \tau_{\text{max}} \pi d_b \frac{S}{2}$$

(3.27)

where $s$ the stable crack spacing. The average concrete force can be computed easily as:

$$\bar{F}_c = \frac{F_c(s/2)}{2} = \frac{\tau_{\text{max}} \pi d_b s}{4}$$

(3.28)

and the average concrete stress will be:

$$\bar{f}_c = \frac{\tau_{\text{max}} \pi d_b s}{4 A_c} = \frac{\tau_{\text{max}} \pi d_b s}{4 A_c} \frac{4 A_s}{\pi d_b^2}$$

(3.29)

$$\bar{f}_c = \frac{\tau_{\text{max}} \rho s}{d_b} \text{ where } \rho = \frac{A_s}{A_c}$$

This simple derivation is sufficient to gather the important parameters involved in tension
stiffening. These parameters are discussed below: \( d_b \) and \( \rho \): bar diameter and ratio of steel area to concrete area. These are geometrical properties of the specimen. \( \tau_{\text{max}} \) and \( u_p \) are the peak bond stress and the corresponding slip at the peak bond stress. \( u_p \) can be varied between 0.03 mm and 0.08 mm. It affects the shape of the relationship, but not its magnitude. \( \tau_{\text{max}} \) is one of the two most important parameters. It reflects the quality of the bond transfer, and thus determines the amount of tension stiffening. This parameter should be chosen with great care. \( s \) is the crack spacing, which is a property of the specimen. This is the second most important parameter.

In reality, the bond stress-slip relationship is not rigid plastic, and therefore a strain compatibility solution is required. While running computer program BOND, it was observed that the slip distribution along the length of the element is relatively linear (see Fig. 3.9). Thus it was decided to develop a tension stiffening relationship assuming linear slip. This is presented below.

Assuming that no tensile stresses are transferred directly across the crack, the force in the concrete at any point \( x \) equals the force transferred by bond up to that point. That is,

\[
F_c(x) = f_c(x) A_c = \int_0^x \tau(u)dA_{\text{bond}} = \pi d_b \int_0^x \tau(u)dx
\]  

(3.30)

Solving for the stress in the concrete at any point gives

\[
f_c(x) = \frac{4\rho}{d_b} \int_0^x \tau(u)dx
\]  

(3.31)

The average concrete tensile stress in an element of length \( s \) is given by
Computing the average concrete tensile stress by discretizing the element into $n$ sections and performing the integration by summation, Equation (3.32) becomes

$$
\bar{f}_c = \frac{1}{s} \int_0^s f_c(x) \, dx
$$

The equilibrium expression of Eq. (3.31), the non-linear bond stress — slip relationship shown in Fig. 3.5, as well as the strain compatibility assumption that the slip varies linearly, are substituted into Eq. (3.33). The discontinuous nature of the bond stress — slip relationship and the reduction in bond stress close to the crack surface results in a discontinuous relationship between the average concrete tensile stress and the maximum slip.
CHAPTER 3 Concrete Model

For \( w \leq 2u_p \):

\[
\overline{f}_c = \frac{\rho \tau_{\text{max}}}{d_b} \left( \frac{2}{(2 + \beta)} \frac{w}{2u_p} \right)^\beta
\]

\[
- \frac{\rho \tau_{\text{max}}^2}{d_b} \left( \frac{w}{2u_p} \right)^\beta \left[ \frac{2}{(2 + \beta)} - \frac{1}{(2 + \beta)(3 + \beta)} \frac{s}{l_r} \left( 1 - \left( 1 - \frac{2l_r}{s} \right)^{3+\beta} \right) \right]
\]

(3.34)

for \( 2u_p < w \leq 2u_p/(1-2l_r/s) \):

\[
\overline{f}_c = \frac{\rho \tau_{\text{max}}}{d_b} \left[ 1 - \frac{\beta}{(2 + \beta)} \left( \frac{2u_p}{w} \right)^2 \right]
\]

\[
- \frac{\rho \tau_{\text{max}}^2}{d_b} \left[ 1 - \frac{s}{6l_r} - \frac{\beta}{(2 + \beta)} \left( \frac{2u_p}{w} \right)^2 \left( 1 - \frac{s}{2l_r} \right) \right]
\]

(3.35)

for \( w \geq 2u_p/(1-2l_r/s) \):

\[
\overline{f}_c = \frac{\rho \tau_{\text{max}}}{d_b} \left[ 1 - \frac{\beta}{(2 + \beta)} \left( \frac{2u_p}{w} \right)^2 - \frac{2l_r}{s} + \frac{4(l_r)^2}{3(s)^2} \right]
\]

(3.36)

where \( w \) is the crack width.

Replacing the variables by the values as discussed earlier results in the following expressions:
for $w \leq 2u_p$:

$$\bar{f}_c = 0.574 \frac{\rho \tau_{\text{max}}}{d_b} \left( \frac{w}{2u} \right)^{0.4}$$  \hspace{1cm} (3.37)

for $2u_p < w \leq (20/7)u_p$:

$$\bar{f}_c = \frac{\rho \tau_{\text{max}}}{d_b} \left[ \frac{10}{9} - \frac{5}{9} \left( \frac{2u_p}{w} \right)^2 \right]$$

$$- \frac{\rho \tau_{\text{max}}}{d_b} \left\{ \left( \frac{2u_p}{w} \right)^3 \left[ \frac{40}{153} - \frac{125}{153} \left( \frac{w}{2u_p} \right)^{3.4} \left( \frac{7}{10} \right)^{3.4} \right] \right\}$$  \hspace{1cm} (3.38)

for $w \geq (20/7)u_p$:

$$\bar{f}_c = \frac{\rho \tau_{\text{max}}}{d_b} \left[ 0.73 - \left( \frac{2u_p}{w} \right)^2 \right]$$  \hspace{1cm} (3.39)

where $w$ is the crack width.

Based on the experimental data of Nilson (1972), Shirai and Sato (1981b), and Günther (1991a) the following expression for the peak bond stress was adopted:

$$\tau_{\text{max}} = 0.9 \sqrt{\bar{f}_c}$$  \hspace{1cm} (3.40)

where all units are in MPa. This proposed expression varies as a function of the square root of the concrete strength, similar to the tension strength of concrete. It is consistent with the nature of bond transfer which involves the tensile resistance of concrete. A slip of 0.04 mm at the peak bond stress was adopted as suggested by Günther (1991a). But Eq. (3.40) involves only one characteristic of the specimen, namely the concrete strength. It would be appropriate to include the quality of the adherence between the bar and the concrete and the distribution of the bars over the cross section. However there is insufficient data to introduce appropriate expressions.
The crack spacing $s$ is a very important parameter. In this study, the formula proposed in the CEB-FIP model (1978) based on the work done by Leonhardt (1977) was adopted. This expression is

$$s = 2\left(c + \frac{a}{10}\right) + k_1 k_2 \frac{d_b}{\rho_{ef}}$$

(3.41)

where $c$ is the clear cover, $a$ is the spacing between the bars, $\rho_{ef}$ is the effective steel to concrete area ratio, and $k_1$ is a bond factor (0.4 for good bond properties, 0.8 for plain bars). $k_2$ is a strain gradient factor for sections subjected to bending. The value of $k_2$ depends on the maximum strain ($\epsilon_1$) and minimum strain ($\epsilon_2$) in the effective zone, and is given by

$$k_2 = .25 \frac{\epsilon_1 + \epsilon_2}{2\epsilon_1}$$

(3.42)

Equation (3.41) is meant to calculate the crack spacing at one particular location on the cross section which has a cover $c$. In this study it is the average crack spacing which is of interest, therefore the following formula is proposed for the average cover:

$$c = \frac{c_1^2 + c_2^2}{2(c_1 + c_2)}$$

(3.43)

where $c_1$ is the maximum distance from a point in the concrete to a reinforcing bar and $c_2$ is the minimum cover to a reinforcing bar. How to determine $c_1$ and $c_2$ for two different bar arrangements is demonstrated in Fig. 3.10.

A typical average tensile stress — crack width relationship determined from the equations presented above is shown in Fig. 3.11(a). Generally the equations which are based on an assumed linear slip give slightly higher stresses than the "exact" solution generated by the computer program BOND.
Fig. 3.10 Parameters $c_1$ and $c_2$ used to determine the average cover.

It should be noted that for cracked concrete the average tensile stress (i.e. tension stiffening) was found to increase with increasing deformation until a certain level at which it remains relatively constant. This is quite different than the decaying relationships which have been proposed for tension stiffening (for example: Leonhardt, 1977, Hwang, 1983, Vecchio and Collins, 1986, Günther, 1991b). The difference comes from the fact that in the present study the concrete is considered to be fully cracked, while the previous work included the concrete tensile stresses prior to the development of a stable crack pattern and the concrete tensile stresses which are transferred directly across the cracks.

As explained in Section 3.2, tensile stresses are transmitted across a crack because cracks do not propagate instantly through the concrete. The relationship suggested by Reinhardt et al. (1986) is shown in Fig. 3.2. The total average tensile stress in concrete is obtained by combining the tensile stresses which are transmitted directly across the crack with the tensile stresses transmitted by bond (tension stiffening). See Fig. 3.11(b).
3.6 Tension Stiffening Relationship for Monotonic Loading

The equations resulting from the analytical model described in Section 3.5 are rather complex. One of three equations must be selected depending on the slip at the crack face. One simpler equation which covers the entire range of possible crack widths would be preferable.

The equations presented earlier predict that for large crack width values, the average tensile stress reaches a plateau (see Fig. 3.11a). The height of the plateau is given by

$$\bar{f}_c = 0.73 \frac{\rho \tau_{\text{max}}}{d_b}$$  \hspace{1cm} (3.44)  

Substituting $\tau_{\text{max}}$ from Eq. (3.40), gives the following simple expression:
Thus what is required is a simple function to model the transition from the origin to the plateau. One possibility is the following simple exponential function:

\[
\frac{f_c}{f_c^*} = 1 - e^{-\lambda_c \varepsilon}
\]  

(3.46)

where \(\lambda_c\) is a parameter that can be adjusted to fit the simple function to the more complex relationship. A value of 3000 gave the best overall fit. Equation (3.46) has the advantage that it is simple and it can be easily inverted to express the strain as a function of stress:

\[
\varepsilon = \frac{1}{\lambda} \ln \left( 1 - \frac{f_c}{f_c^*} \right)
\]  

(3.47)

Unfortunately, Eq. (3.46) does not correspond well with the complex expressions at small crack widths. A better fit was obtained using the polynomial

\[
\frac{f_c}{f_c^*} = 1 - \frac{1}{1 + (k_1 \varepsilon)^a + (k_2 \varepsilon)^b}
\]  

(3.48)

where the best fit is obtained by setting \(k_1\) and \(k_2\) to 3000, \(a\) to 0.5 and \(b\) to 2. Equation (3.48) gives a much better fit of both the rising part and the plateau. The predictions from Eq. (3.46) and Eq. (3.48) and the more complex expressions from Section 3.5 are compared in Fig. 3.12.

It is important to realize that since the tension stiffening relationship is to be added to the contribution from tensile stresses transmitted directly across the crack at small crack width values, it is not necessary to have an exact fit.

A simple expression can also be developed for combined tension stiffening and residual...
average concrete stress (MPa)

average strain = (w/s)

Fig. 3.12 Numerical monotonic tension stiffening solution versus adopted expressions.

tensile stresses transmitted across the crack. For example the following exponential function can be used:

\[
\frac{f_c}{\bar{f}_c} = \left( \frac{f_{cr}}{\bar{f}_c} - 1 \right) e^{-\lambda_u (\varepsilon - \varepsilon_{cr})} + 1
\]  

(3.49)

where \( \lambda_u \) is the parameter determining the rate of decay (varies between 1500 and 3000). This function can be easily inverted to express the strain as a function of the stress,

\[
\varepsilon = \varepsilon_{cr} + \frac{1}{\lambda_u} \ln \left( \frac{f_{cr} - \bar{f}_c}{f - \bar{f}_c} \right)
\]  

(3.50)

Another formulation which is possible is to modify the Collins and Mitchell equation. A new \( K_{cm} \) factor is introduced to enforce the response to reach the residual tensile stress (\( \bar{f}_c \)) at a strain of 0.002:
\[
\frac{f_c}{f_{cr}} = \frac{1}{1 + K_{cm} \sqrt{500\epsilon_c}}
\]

(3.51)

\[
K_{cm} = \frac{f_{cr}}{f_c} - 1
\]

Note that in the original Collins and Mitchell equation, \( K_{cm} \) is always assumed to be equal to 1.0, which means that the residual tensile stress is equal to half the cracking stress \( (\overline{f}_c = 0.5f_{cr}) \).

If the cracking stress \( (f_{cr}) \) in Eq. (3.51) is replaced by

\[
f_{cr} = .33\overline{f}_c
\]

(3.52)

and the residual tensile stress \( (\overline{f}_c) \) is replaced by Eq. (3.45) the following expression results

\[
K_{cm} = \frac{d_b}{2\rho s} - 1
\]

(3.53)

The relationship chosen to be implemented in this study is the exponential function, Eq. (3.49). The modification to the Collins and Mitchell equation demonstrates that the model presented in here can be employed to refine existing tension stiffening models.

3.7 Tension Stiffening Relationship for Reverse Cyclic Loading

3.7.1 Review of Previous Work

A number of authors have tried to generalize tension stiffening relationships for cyclic loading. Stevens et al. (1987) suggested a model which is intimately connected to the cyclic compression response. The envelope is considered to be the monotonic relationship. The origin of the envelope is positioned at the residual plastic strain reached in compression as determined using the equation suggested by Karsan and Jirsa (1969), Eq. (3.6). The unloading curve is formulated in a finite element fashion, but instead of calculating a tangent modulus at each point,
an exponential equation is set to project the next point. The unloading modulus at the envelope is equal to:

\[
E_{mn} = \frac{4.5 \alpha}{1.3 + 1000 \Delta \varepsilon_{cr}} \tag{3.54}
\]

where \( \alpha \) is a factor to account for the amount of reinforcement and \( \Delta \varepsilon_{cr} \) is the strain difference between the strain at unloading and the repositioned cracking strain. The reloading curve is part of the hyperbola that serves as the compressive unloading curve. A straight line connecting the last point on the envelope in tension and the point of residual plastic strain in compression, constitutes the asymptote of the hyperbola.

Izumo et al. (1989) use a unique envelope. The unloading loop is the result of two actions: stresses produced by bond and stresses produced by the closing of the crack. The concrete stresses produced by bond are represented by a quadratic curve connecting the point at the start of unloading to a vertex point at zero strain where the stress, \( f_{cbo} \), is defined as:

\[
f_{cbo} = -0.0016E_{ct} \varepsilon_{x_{max}} \tag{3.55}
\]

\( \varepsilon_{x_{max}} \) is the maximum strain reached during the load history. The stresses produced by the closing of the cracks are added to the bond stresses when the unloading curve reaches zero stress. This stress is represented by a straight line which starts at \( \varepsilon_{co} = 0.00015 \) with a stiffness of \( 0.3E_{ct} \). To take into account shear deformation at the crack (in the case a biaxial stresses), \( \varepsilon_{co} \) is modified to:

\[
\varepsilon_{co} = 0.00015 + 0.1 |\gamma_{max}| \tag{3.56}
\]

where \( \gamma_{max} \) is the maximum shear strain reached during the load history. The reloading curve is a straight line which connects the point of zero load in compression to the last point reached on the envelope in tension.
Gunther (1991b) proposed a model for the cyclic response of reinforced concrete in tension where the tension stiffening is included in the stress-strain relationship of the steel. The monotonic envelope according to Gunther is:

$$\epsilon_m = \frac{F}{A_s E_s} \sqrt{1 - \left(\frac{F_{cr}}{F}\right)^2} \quad (3.57)$$

The effect of cycling is introduced by adding to the monotonic envelope (for a given load) an increment in strain:

$$\Delta \epsilon_m = \frac{F}{A_s E_s} \left[\sqrt{1 - \left(\frac{F_{cr}}{F}\right)^5} - \sqrt{1 - \left(\frac{F_{cr}}{F}\right)^2}\right] \quad (3.58)$$

where $F$ and $F_{cr}$ are the applied load and the cracking load of the reinforced element. Adding the increment to the envelope and rearranging to isolate $f_c$ gives

$$\left(E_s \rho \epsilon + f_c\right)^5 - \left(E_s \rho \epsilon + f_c\right)^3[(\epsilon - \epsilon_{cr})E_s \rho]^2 - \left(E_s \rho \epsilon_{cr} + f_{cr}\right)^5 = 0 \quad (3.59)$$

which is a 5th degree polynomial that must be solved iteratively. The loading and unloading curve are simple straight lines connecting the point ($\epsilon=0.0002, f_c=0$) to the point positioned with Equations (3.57) and (3.58).

### 3.7.2 Description of Cyclic Tension Stiffening Model

It was decided in the present study not to derive a cyclic tension stiffening model based on the cyclic bond model. The model has essentially been derived from the experimental results obtained from the tests conducted as part of this study. Although the cyclic bond model was not used to develop the tension stiffening model, it was used to better understand the mechanisms involved.
In a study on uniaxially reinforced concrete prisms, Bresler and Bertero (1968) found that the stress transfer between reinforcing steel and concrete at any given load is influenced by the previous stress history. The maximum peak stress level in the reinforcement was found to be an indicator of the stress transfer effectiveness at lower stresses in subsequent cycles. In this study the variable chosen to represent the load history is the maximum strain reached in tension.

A cyclic tension stiffening loop is divided into five parts: three cubic functions at unloading from tension and a parabola connected to a cubic at reloading in tension. A cubic function is required to connect two points if the slope is specified at each point. A typical loop is shown in Fig. 3.13.

The slope at the point of unloading \((E_{un})\) is:

\[
E_{un} = \frac{E_{ct}}{11000 \varepsilon_{un}} \tag{3.60}
\]

where \(E_{ct}\) is the tangent modulus of concrete and \(\varepsilon_{un}\) is the strain at unloading from the tensile envelope. The unloading slope is affected by the number of cycles. If the specimen is cycled in tension only the computed unloading modulus should be multiplied by factor \(\alpha_{un}\):

\[
\alpha_{un} = \frac{1}{1 + (0.05 (n_t - 1))^{0.5}} \tag{3.61}
\]

where \(n_t\) is the number of tensile cycles. If the specimen is cycled in tension and compression then the factor \(\alpha_{un}\) becomes:

\[
\alpha_{un} = \frac{1}{1 + (0.5 (n_t - 1))^{0.5}} \tag{3.62}
\]

where \(n_c\) is the number of tension - compression cycles.

At zero load the concrete will be subjected to residual stress due to the fact that the crack does not close completely. The residual strain \(\varepsilon_{rd}\) resulting from the crack not closing is
Fig. 3.13 Cyclic tension stiffening model.

influenced by the type of loading. If the member is loaded only in tension, the residual strain becomes:

\[ \varepsilon_{rd} = 0.000500 - \frac{0.000500}{1 + 1500 (\varepsilon_{un} - \varepsilon_{cr})} \]  

(3.63)

where \( \varepsilon_{cr} \) is the cracking strain. If the member is loaded in compression as well as in tension the residual strain becomes:

\[ \varepsilon_{rd} = 0.000250 - \frac{0.000250}{1 + 1500 (\varepsilon_{un} - \varepsilon_{cr})} \]  

(3.64)

The residual strain is not affected by cycling. The stress at residual strain \( f_{rd} \) is:

\[ f_{rd} = f_{un} - \Delta f_{rd} \]  

(3.65)

\[ \Delta f_{rd} = f_{cr} (1 - 700\varepsilon_{un} + (775\varepsilon_{un})^2 - (575\varepsilon_{un})^3) \]

where \( f_{cr} \) is the cracking stress. The residual stress is not affected by tensile cycles but is
affected by compressive cycles. If compressive cycles occur, \( \Delta f_{rd} \) should be multiplied by the factor \( k_{rd} \) defined as:

\[
k_{rd} = 1 - 0.107 (n_c - 1) \geq 0.465
\]

where \( n_c \) is the number of compressive cycles.

Between the unloading point and the residual point, the response becomes flatter, i.e. the element goes into friction bond resistance. An additional point \((\epsilon_{pl}, f_{pl})\) is positioned between the previous two points and marks the beginning of the friction bond resistance. The subscript \( pl \) stands for "plateau". The strain at the plateau is defined as:

\[
\epsilon_{pl} = \epsilon_{un} - \Delta \epsilon_{pl}
\]

\[
\Delta \epsilon_{pl} = 0.00044 \left[ 1 + (630 \epsilon_{un})^3 \right]
\]

and the stress at the plateau is:

\[
f_{pl} = f_{un} - k_{pl} \Delta f_{rd}
\]

\[
k_{pl} = 1 - 200 \epsilon_{un}
\]

The slope at the plateau \((E_{pl})\) is:

\[
E_{pl} = \frac{E_c}{1 + (3500 \epsilon_{un})^{2.5}}
\]

The slope \( E_{pl} \) should be multiplied by factor \( \alpha_{pl} \) to account for the effect of cycling. For tensile cycles only \( \alpha_{pl} \) becomes:

\[
\alpha_{pl} = \frac{1}{1 + [0.5(n_t - 1)]^{0.7}}
\]

where \( n_t \) is the number of tensile cycles. If the element is also loaded in compression \( \alpha_{pl} \)
becomes:

\[ \alpha_{pl} = 1 - [0.1(n_c - 1)]^{0.2} \]  \hspace{1cm} (3.71)

where \( n_c \) is the number of compressive cycles.

Getting to the point of zero load (residual point) the crack faces meet again and as a result the response becomes stiffer. There does not seem to be any direct relationship between the maximum strain or stress reached in the load history and the crack closing slope, although applying compression on the specimen affected the crack closing slope. It was decided to set the crack closing slope \( (E_{rd}) \) to an average of:

\[ E_{rd} = 0.055 E_{ct} \]  \hspace{1cm} (3.72)

for tensile cycles only and to an average of:

\[ E_{rd} = 0.080 E_{ct} \]  \hspace{1cm} (3.73)

if compression is applied on the specimen.

The connection to the compressive envelope depends on the load history in compression. If the specimen has never been loaded in compression then (according to the present test results) the point of connection to the envelope \( (\epsilon_{cmp}, f_{cmp}) \) is positioned at \((0, -2.5\text{MPa})\). The slope at connection is \( (E_{cmp}) \) set to:

\[ E_{cmp} = 0.50 E_{ct} \]  \hspace{1cm} (3.74)

This connection results in an offset of the compressive envelope \( (\epsilon_{off}) \) of:

\[ \epsilon_{off} = \epsilon_o \left[ 1 - \sqrt{1 - \frac{f_{cmp}}{f_{c}^{'} \epsilon_o}} \right] \]  \hspace{1cm} (3.75)

Once the element has been loaded in compression, the connection will be made at the maximum
strain reached in compression at a slope equal to the slope of the line connecting the maximum stress-strain point to the point of residual plastic strain positioned using Karsan and Jirsa’s equation, Eq. (3.6).

The unloading curve from compression is a parabola that follows the rules defined in Section 3.1.2 if the specimen is unloaded in the compressive strain range. If the specimen is unloaded before reaching the compressive range a simple straight line of slope $E_{ro}$ is employed up to zero stress or $f_{pl}$ if $f_{pl}$ is greater than zero.

$$E_{ro} = \frac{E_{ct}}{540 (\epsilon_{un} - \epsilon_{rd})^{0.6}}$$  \hspace{1cm} (3.76)

The slope at the residual plastic strain ($E_{cp}$) is computed using Seckin’s equation, Eq. (3.5). But the slope $E_{cp}$ is constrained by the following criteria to insure numerical stability of the model:

$$E_{cp} \leq \frac{f_{un}}{2 (\epsilon_{un} - \epsilon_{cp})}$$  \hspace{1cm} (3.77)

The plastic strain is connected to the point at tensile peak load ($\epsilon_{tpl}, f_{tpl}$). The point at tensile peak load is positioned in relation to the stress-strain state at the peak load when the peak load was reached for the first time ($\epsilon_{tp1} f_{tp1}$). There is an increase in deformation when reaching a given peak load for the second time that results in drifting of the stress-strain state. The strain at tensile peak load is calculated as follows:

$$\epsilon_{tpl} = \epsilon_{tplf} + \Delta \epsilon_{tpl}$$  \hspace{1cm} (3.78)

$$\Delta \epsilon_{tpl} = 0.05 \epsilon_{tplf} - 0.00002 \geq 0$$

If the specimen is cycled several times at a given peak load level, $\Delta \epsilon_{tpl}$ should be multiplied by
CHAPTER 3 Concrete Model

factor $k_{qpl}$ for tensile cycles only:

$$k_{qpl} = 0.7 n_t + 0.3 \leq 3.1 \quad (3.79)$$

and for tensile - compressive cycles $k_{qpl}$ becomes:

$$k_{qpl} = 0.585 n_c + 0.415 \leq 3.35 \quad (3.80)$$

If the specimen is loaded in tension only, the stress at previous peak load is computed using:

$$f_{qpl} = f_{qpl} - \Delta f_{qpl} \quad (3.81)$$

$$\Delta f_{qpl} = 55.8 \epsilon_{qpl} f_{qpl}$$

and if the specimen is cycled several times at the same tensile load level $\Delta f_{qpl}$ should be multiplied by $\beta_{qpl}$:

$$\beta_{qpl} = 0.833 n_t + 0.167 \leq 3.5 \quad (3.82)$$

If the specimen is loaded in tension and in compression, the stress at previous peak load is computed using:

$$f_{qpl} = f_{qpl} - \Delta f_{qpl} \quad (3.83)$$

$$\Delta f_{qpl} = 139.8 \epsilon_{qpl} f_{qpl}$$

and if the specimen is cycled several times at the same tensile load level $\Delta f_{qpl}$ should be multiplied by $\beta_{qpl}$:

$$\beta_{qpl} = 0.4 n_c + 0.6 \leq 2.6 \quad (3.84)$$

An alternative to the drift in stress at peak is to use the ratio of the reloading delta stress to the unloading delta stress ($\gamma_{qpl}$):

$$f_{qpl} = f_{rd} + \gamma_{qpl} \Delta f_{rd} \quad (3.85)$$
For tensile cycles $\gamma_{pl}$ is evaluated using:

$$\gamma_{pl} = 1 - 60 \left( e_{un} - e_{rd} \right)$$ \hspace{1cm} (3.86)

which is multiplied by the following $\beta_{pl}$ factor if cycled:

$$\beta_{pl} = \frac{1}{1 + 0.075 (n_t - 1)^{0.4}}$$ \hspace{1cm} (3.87)

For tensile-compressive cycles $\gamma_{pl}$ is evaluated using:

$$\gamma_{pl} = 1 - 115 \left( e_{un} - e_{rd} \right)$$ \hspace{1cm} (3.88)

which is multiplied by the following $\beta_{pl}$ factor if cycled:

$$\beta_{pl} = \frac{1}{1 + 0.20 (n_c - 1)^{0.45}}$$ \hspace{1cm} (3.89)

The slope at the previous peak load point ($E_{pl}$) is calculated using:

$$E_{pl} = \frac{E_{ct}}{650000 \epsilon_{pl}^{1.5}}$$ \hspace{1cm} (3.90)

and is independent of the number of cycles.

The third degree polynomial used in the model to connect two points of given slope say $(\epsilon_1, f_1, E_1)$ and $(\epsilon_2, f_2, E_2)$ is of the following form:

$$f = A \epsilon^3 + B \epsilon^2 + C \epsilon + D$$ \hspace{1cm} (3.91)

where the parameters would be:
The reverse tension stiffening model is considered to be valid only before the yielding of the steel. Once the steel has yielded, the tension stress in concrete is reduced to zero.

The reverse cyclic tension stiffening model has been implemented into a computer program called TS. Comparison with experimental results are given in Chapter 6. The equations that have been proposed in this section are shown in graphics form in Appendix B.

### 3.8 Summary

This chapter presented the analytical models for concrete and their theoretical and experimental background. The compressive model is a collection of procedures proposed by previous researchers. The monotonic bond model has been rationally derived from information given by previous researchers. The reverse cyclic bond model uses the skeleton of previously suggested models but it has been enhanced based on analytical studies conducted with computer program BOND. This model should be considered preliminary until a thorough calibration study has been undertaken to validate the model. Considerable progress has been made towards the development of a rational monotonic tension stiffening model and an empirical reverse cyclic tension stiffening model has been proposed.
CHAPTER 4

Experimental Programme

In this chapter, the test specimens are described and the material properties are given. Also, the UBC Beam Element Tester, which was used to load the specimens, and the instrumentation used during the testing are described. Finally, the testing procedure is explained.

4.1 Test Specimens

To study the effect of cyclic loading on tension stiffening, five symmetrically reinforced concrete members were axially loaded. These members can be considered full scale. Two variables were investigated in this study, namely:

• the reinforcement ratio (ratio of steel area to concrete area), and
• the load history.

All other variables were kept constant. For example, all specimens were cast from the same batch of concrete and all reinforcing steel were No. 20 bars which were intended to come from a single heat. All specimens were 1500 mm long and had 8- No. 20 reinforcing bars. Note the specimens did not have any transverse reinforcement. To vary the reinforcement ratio, the cross sectional dimensions of the specimens were changed as well as the cover and the spacing between the bars in order to keep the spacing between the bars equal to twice the cover. This was done in order to keep the effective concrete area around each bar perfectly squared for each specimen.

Figure 4.1 summarizes the first series of specimens which had different reinforcement ratios but were subjected to similar load histories. Specimen UC1 was 200mm × 400mm in
Fig. 4.1 Series 1, specimens with different reinforcement ratios subjected to similar load histories.
dimension and thus had a reinforcement ratio of \( \frac{2400 \text{ mm}^2}{80000 \text{ mm}^2} = 0.030 = 3.0\% \). The eight reinforcing bars were spaced at 100 mm in both direction and thus the cover to the centre of the bars was 50 mm. Specimen UC2 was 250mm \( \times \) 500mm in dimension and thus had a reinforcement ratio of 2.0\%. The reinforcing bars were spaced at 125 mm in both direction and the cover to the centre of the bars was 62.5 mm. Specimen UC5 was 350mm \( \times \) 700mm in dimension and thus had a reinforcement ratio of 1.0\%. The reinforcing bars were spaced at 250 mm in both direction and the cover to the centre of the bars was 125 mm.

Figure 4.2 summarizes the second series of specimens which all had the same cross sectional dimensions as UC5. The specimens were however subjected to different load histories. Specimen UC3 was subjected to ten cycles of tension only to an axial load level of 900 kN which is approximately 90\% of the yield load. The specimen was then subjected to 5 cycles of tension equal to the yield load. Specimen UC4 was cycled 10 times in tension and compression to a load level of 900 kN. The specimen was yielded and subjected to 5 cycles of tension - compression at twice the yield displacement in tension, 5 cycles to three times the yield displacement and 5 cycles to four times the yield displacement. The pre-yield load history of specimen UC5 consisted of cycles of tension only. In each cycle the specimen was subjected to 100 kN more than the previous cycle. The post-yield loading was similar to specimen UC4 except that the specimen was not loaded in compression. More details on loading procedure are given in Section 4.5.
**Fig. 4.2** Series 2, specimens with similar cross sections subjected to different load histories.
4.2 Material Properties

The concrete used to construct the specimens was supplied by a local ready-mix company. The specified compressive strength was 30 MPa, and the maximum aggregate size was \( \frac{3}{4} \)". Standard cylinder tests \((6" \times 12")\) were performed to determine the concrete strength. The age of the concrete at the time the specimens were tested varied between 45 and 72 days. The standard cylinder tests were performed at the end of the experimental sequence when the concrete was 75 days old. The average strength of the cylinders was found to be 33.5 MPa.

Weldable 20M reinforcing bars were used as longitudinal reinforcement. Table 4.1 summarizes the results from 6 coupon tests. The average yield strength was calculated to be 435 MPa and the elastic modulus was found to be 190700 MPa. The considerable variation in the measured yield strengths indicates the bars were probably not from the same heat as was intended.

Table 4.1 Reinforcing bar coupon test results.

<table>
<thead>
<tr>
<th>Coupon</th>
<th>( f_y ) (MPa)</th>
<th>( f_{ult} ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>460</td>
<td>667</td>
</tr>
<tr>
<td>B</td>
<td>426</td>
<td>600</td>
</tr>
<tr>
<td>C</td>
<td>460</td>
<td>667</td>
</tr>
<tr>
<td>D</td>
<td>386</td>
<td>573</td>
</tr>
<tr>
<td>E1</td>
<td>445</td>
<td>623</td>
</tr>
<tr>
<td>E2</td>
<td>445</td>
<td>623</td>
</tr>
<tr>
<td>Average</td>
<td>435</td>
<td>626</td>
</tr>
</tbody>
</table>
4.3 Loading Apparatus

The specimens were tested using the UBC Beam Element Tester (see Fig. 4.3). This tester is a versatile machine that can be used to apply any combination of axial load, bending moment and shear to reinforced concrete specimens. This study was the first time the tester had been used.

The machine consists of three jacks and three rigid links. Each jack has a capacity of 800 kN in tension and 1000 kN in compression. As a result, the machine capacity in pure axial tension is 1600 kN and 2000 kN in pure axial compression. The jacks and links are connected by pins to the yokes to which the specimen is bolted. The yokes rest on rollers to support the dead load of the specimen and the tester.
4.4 Construction of Specimens

The longitudinal reinforcement was welded to 19 mm thick plates at each end of the specimen (see Fig. 4.4). In order to prevent a crack from occurring at the interface between the steel plate and the concrete, additional 150 mm long reinforcing bars were welded to the plate. This procedure turned out to be successful since no crack occurred close to the end plates during the tests.

In order to bolt the specimen to the yokes of the testing machine, nuts were welded to the inside of the plates before casting. The steel cages were placed in wooden forms, and all specimens were cast at same time in approximately 3 lifts, vibrating each lift. The top of the specimens was finished with a steel trowel, were covered with plastic, and then were left to cure undisturbed. The forms were stripped after 7 days. UC1 was tested after 72 days, UC2 after 70 days, UC3 after 48 days, UC4 after 58 days and UC5 after 65 days.

Fig. 4.4 Steel cage.
4.5 Instrumentation and Testing Procedure

The measured parameters include the applied load, the elongation of the specimen, the crack widths and the crack spacings. The applied load was measured directly by pressure transducers on each jack and confirmed from the rigid links which are strain gauged and hence act as load cells. The elongation was measured with an L.V.D.T. displacement transducer on the top and the bottom face. The crack widths were measured manually using a Carton microscope with has a resolution of .0005 mm. The crack spacings were measured to the closest mm using a measuring tape.

The testing procedure (load history) was summarized in Fig. 4.1 and in Fig. 4.2. The sequential development of the crack widths and spacings was recorded with photographs taken after each load stage and at the peak load for each cycle. The cracks were numbered in the order of their appearance. For specimens UC1, UC2, UC3 and UC5, a load cycle is defined as loading from zero load up to a certain predetermined value and then unloading to zero load. For specimen UC4 a load cycle is defined as loading from zero to the peak value in tension, unloading to zero, reloading in compression (to a load equal to the peak load in tension) and then unloading to zero. All specimens were tested under load control (to a certain predetermined load value) prior to yield, and after yielding were tested under displacement control (to predetermined displacement ductilities). To avoid creep at the peak load levels (especially before reaching a stable crack pattern), the load was slightly reduced until the deformations stabilized.

The first two specimens, UC1 and UC2, were loaded similarly with 7 cycles prior to yielding, to predetermined peak load values of 300, 400, 500, 600, 700, 800 and 900 kN. Crack widths were measured at the peak load and at zero load. At the eighth cycle, the
specimen was loaded up to yield and deformed to twice the yield elongation. The subsequent cycles went up to the yield load incrementing the elongation by approximately 1 mm at each cycle.

For the first cycle, specimen UC3 was loaded up to 900 kN with 4 load stages at 500, 600, 700 and 800 kN. For the second cycle it was loaded again to 900 kN but this time with only 2 load stages at 300 and 600 kN. During the next 8 cycles the specimen was loaded to 900 kN without any intermediate load stage. The specimen was then loaded up to yield load (1020 kN) and cycled 5 times at that load level. Cycle 16 was up to twice the yield displacement, cycle 17 was up to three times the yield displacement and cycles 18 to 20 were up to four times the yield displacement. Crack widths were measured at each load stage and at the peak load levels before yielding.

For the first ten cycles, specimen UC4 was loaded in exactly the same way as UC3, except between each cycle it was loaded to a compression load equal to the peak tension load. For cycles 11 to 15, UC4 was cycled to twice the yield displacement in tension and to the tensile yield load (-1000 kN) in compression. For cycles 16 to 20, it was cycled to three times the yield displacement in tension and to the tensile yield load in compression. Finally, for cycles 21 to 25, it was cycled to four times the yield displacement in tension and to the tensile yield load in compression.

UC5 was loaded similar to UC1 and UC2 with 5 cycles before yielding. The peak load levels were at 500, 600, 700, 800 and 900 kN. At the sixth cycle, the specimen was loaded up to yield load and then deformed to twice the yield elongation. It was then cycled 5 times to twice the yield displacement. For cycles 11 to 15, it was cycled to three times the yield displacement, and finally, for cycles 16 to 20, the specimen was cycled to four times the yield displacement.
CHAPTER 5
Experimental Results and Observations

This chapter presents the experimental results and observations for the five specimens. After examining the test results, it was determined that shrinkage had a significant effect on the tension stiffening. The effect of shrinkage has been analytically determined in Appendix A and is included in the results employed in the comparisons of Chapter 6. However, because of the uncertainties associated with the shrinkage, no attempt was made to modify the data presented in this chapter.

5.1 Behaviour of Specimen UC1

UC1 is the 200mm × 400mm specimen (\(\rho = 3.0\%\)) which was subjected to uni-directional (tension only) loading cycles with 7 cycles to incremental peak load level before yield.

The complete response and a close up of the response before yield is shown in Fig. 5.1. Cracking occurred on the bottom face at a load of 121 kN, while the top face cracked at a load of 195 kN. The elongations of the top and the bottom face of the specimen were different which indicates that the specimen was bending under the applied load. This bending may have come from two sources: misalignment of the jacks or from the dead load of the specimen and the tester. A sectional analysis indicates that a misalignment of about 7 mm or a force equal to the weight of the yoke applied at each end of the specimen is required to get the measured deformations.

A combination of misalignment and dead load was likely responsible for the bending effect along with geometrical imperfections in the specimen. However bending has little effect on tension stiffening because it does not significantly influence the average elongation of the
member (very small bending moments). However since the crack widths were measured on the top face of the specimen, the following correction factor was used to determine the average crack widths from the crack widths on the top surface

\[ w_{av} = k_w w_{top} \]

\[ k_w = \frac{\epsilon_{top} + \epsilon_{bot}}{2 \epsilon_{top}} \]  \hspace{1cm} (5.1)

where \( w_{top} \) is the crack width measured on the top surface, \( w_{av} \) is the average crack width, \( k_w \) is a correction factor, \( \epsilon_{top} \) and \( \epsilon_{bot} \) are respectively the average strain measured on the top and bottom surfaces of the specimen (from the LVDT reading). The correction factors, the measured and the corrected average crack width are given for each specimen in Appendix C.

The calculated average crack width measured at the peak load level and after unloading (zero load) are shown in Fig. 5.2(a).

The measured average crack spacing versus the applied load is shown in Fig. 5.2(b). All primary cracks appeared in the first loading cycle (300 kN) but were not fully developed. At the fourth loading cycle, 94% of the cracks were fully developed on the top face. The stable crack spacing was 128 mm, which compares very well with the CEB-FIP prediction of 121 mm calculated using the modified cover formula suggested in Chapter 3.

The specimen yielded at a load of approximately 990 kN. Splitting cracks appeared on the eighth cycle. It is interesting to note that the strains due to yielding were not evenly distributed but were concentrated at a few cracks.
Fig. 5.1  UC1  Measured response.
CHAPTER 5 Experimental Results and Observations

Fig. 5.2  UC1  
a) Load at peak versus normalized average crack width  
b) Load level versus average crack spacing
5.2 Behaviour of Specimen UC2

UC2 is the 250mm x 500mm specimen ($\rho = 2.0\%$) which was subjected to uni-directional (tension only) loading cycles with 7 cycles to incremental peak load level before yield.

The complete response and a close up of the response before yielding is shown in Fig. 5.3. The bottom face cracked at a load of 198 kN, while the top face first cracked at 213 kN. The measured deformations were smaller on the top than on the bottom face indicating some bending occurred similar to what was observed in specimen UC1.

The normalized average crack width measured at the peak load and at unloading from peak are shown in Fig. 5.4(a). The normalized crack widths for the first load cycle (300 kN) was disregarded. The correction factor in this case was inappropriate due to the fact that the top and bottom strain were very different (the top cracks were not completely formed).

The average crack spacing versus the applied load is shown in Fig. 5.4(b). Four primary cracks appeared in the first loading cycle (300 kN) but one was not yet fully developed. At the second loading cycle (400 kN), the fourth crack fully developed and two additional cracks appeared bringing the total to six, while on the third loading cycle (500 kN), a seventh crack appeared, on the fourth cycle (600 kN), an eighth crack appeared, and on the fifth cycle (700 kN), a ninth crack appeared but only extended over half the section. The last crack eventually connected to another existing crack in the seventh cycle (900 kN). The stable crack spacing was 167 mm, which again compares very well with the CEB-FIP prediction of 171 mm calculated using the modified cover formula suggested in Chapter 3.

It is interesting to note that the older cracks opened more than the newer cracks at peak load levels and that the older cracks also did not close as much as the new cracks at zero load.
Specimen UC2 yielded at a load of approximately 980 kN. Again, the reinforcing steel was not yielding at all the cracks. On the ninth cycle the steel was yielding through the section at one single crack. On the tenth cycle the steel yielded at additional cracks, but at some cracks only half the steel was yielding, i.e. one half of the crack opened more than the other. With an increase in elongation, the number of cracks which yielded increased.
Fig. 5.3   UC2  Measured response.
Fig. 5.4  
UC2  
a) Load at peak versus normalized average crack width  
b) Load level versus average crack spacing
5.3 Behaviour of Specimen UC3

UC3 is the 350mm × 700mm specimen (ρ = 1.0%) which was subjected to uni-directional (tension only) loading cycles with 10 cycles at 900 kN and 5 cycles at approximately yield.

The complete response of specimen UC3 and a close up of the response before yielding is shown in Fig. 5.5. The specimen first cracked at a load of 388 kN.

The normalized average crack width are shown versus the load level for the first and second cycle in Fig. 5.6(a). The normalized average crack width at the peak load (900 kN) are plotted against the number of cycles in Fig. 5.6(b). The crack width at peak becomes constant after 4 cycles, while the residual crack width (crack width at zero load) seems to stabilize even earlier.

The average crack spacing versus the applied load is shown in Fig. 5.7. The first crack appeared at 388 kN and the second crack appeared at 458 kN just before stopping for a load stage at 500 kN. The third crack formed at 570 kN close to the north end of the specimen. The fourth crack occurred close to the south end at a load of 700 kN. A fifth crack developed in the middle of the specimen at a load of 780 kN and a sixth crack spread from a crack that was somewhat diagonal. The stable crack spacing was 296 mm, which again compares fairly well with the CEB-FIP prediction of 304 mm calculated using the modified cover formula.

The specimen was then unloaded and cycled 9 additional times to a load of 900 kN in tension only. No additional cracks formed.

The specimen yielded at a load of approximately 1020 kN. Again the steel was not yielding at all cracks. The first cracks to yield were the ones closest to the support. The sudden jump in load-deformation response at a strain of .0095, is due to slipping of one test frame support.
Fig. 5.5  UC3 Measured response.
Fig. 5.6  
UC3  
(a) Load at peak versus normalized average crack width
(b) Normalized average crack width versus number of cycles
5.4 Behaviour of Specimen UC4

UC4 is the 350mm × 700mm specimen (ρ = 1.0%) which was subjected to reverse cyclic loading (tension - compression cycles) with 10 cycles to a given load level before yield.

The complete response of specimen UC4 and a close up of the response before yielding is shown in Fig. 5.8. The specimen first cracked at a load of 380 kN.

The normalized average crack width is shown versus the load for the first and second cycle in Fig. 5.9(a). The normalized average crack widths at the peak load (900 kN) are plotted against the number of cycles in Fig. 5.9(b). The crack width at the peak load became constant after 5 cycles. Applying compression on the specimen reduced the residual crack width for the first few cycles, but had virtually no effect during the fourth and subsequent cycles.

The average crack spacing versus the applied load is shown in Fig. 5.10. The first crack appeared at 380 kN and the second crack appeared at 410 kN. At 500 kN, the third and fourth
crack formed. The fifth crack appeared on half the section close to the south end during the load stage at 700 kN. Between 700 kN and 800 kN the fifth crack fully developed. At 900 kN, just after stopping for the load stage, a sixth crack developed at the north end of the specimen. Just before that sixth crack appeared the average crack spacing was 274 mm, after it formed the average crack spacing was 219 mm which does not compare very well with the CEB-FIP crack spacing prediction of 304 mm.

The specimen was then unloaded to zero and loaded in compression to 900 kN. It was loaded 9 additional times in reverse cyclic tension - compression. While loading back to tension during the second loading cycle, a crack appeared on the bottom face but did not spread to the top face. Splitting cracks close at the north end of the specimen were noticed in the seventh cycle. These splitting cracks tended to open more under compression than under tension.

On the eleventh cycle the specimen was brought up to yield in tension at a load of approximately 980 kN. The specimen was cycled 4 additional times at twice the yield displacement in tension and at the tensile yield load in compression. Again the steel was not yielding at all the cracks. The first cracks to yield were the ones closest to the end of the specimen. During cycles 14 and 15 more longitudinal splitting cracks formed. At the sixteenth cycle the specimen was loaded up to three times the yield displacement and to the tensile yield load in compression. While loading to the high compression, one of the supports slipped causing a sudden impact. As a result, a splitting crack appeared from one end to the other. The specimen was loaded 4 other times up to three times the yield displacement and was then cycled 5 times at four times the yield displacement.
Fig. 5.8  UC4 Measured response.
Fig. 5.9  

UC4  a) Load at peak versus normalized average crack width  
b) Normalized average crack width versus number of cycles
Fig. 5.10  \textit{UC4}  \textit{Load level versus average crack spacing.}

5.5 Behaviour of Specimen UC5

UC5 is the 350mm × 700mm specimen (ρ = 1.0%) which was subjected to uni-directional (tension only) loading cycles with 5 cycles to incremental peak load level before yield.

The complete response and a close up of the response before yielding is shown in Fig. 5.11. The specimen first cracked at a load of 408 kN.

The normalized average crack width measured at the peak load and after unloading are shown in Fig. 5.12(a). The normalized crack width after unloading from the first cycle is believed to be incorrect due to the "bending" effect described in Section 5.1.

The average crack spacing versus the applied load is shown in Fig. 5.12(b). Two primary cracks appeared in the first loading cycle (500 kN). The first one formed at a load of 408 kN and the second one formed at 472 kN. During the second loading cycle (600 kN), two additional cracks appeared bringing the total to four cracks. The third crack formed at a load
of 524 kN and the fourth one formed at 600 kN. The crack pattern remained constant for the following five cycles (700 kN, 800 kN, 900 kN). The stable crack spacing was 296 mm. It compares fairly well with the CEB-FIP prediction of 304 mm calculated using the modified cover formula.

On the sixth loading cycle, the specimen was loaded up to yield (load of 1020 kN) and strained to twice the yield strain. Again, the steel was not yielding at all the cracks. The two cracks next to the supports were the first to yield. An additional crack spread over half the section, midway between the supports. The specimen was cycled 4 additional times at twice the yield strain. The specimen was then strained up to three times the yield strain. An additional crack formed on half the section close to the north support. The specimen was then subjected to four additional cycles at three times the yield strain and then five cycles at four times the yield strain.
Fig. 5.11  UC5  Measured response.
CHAPTER 5 Experimental Results and Observations

Fig. 5.12  

(a) Load at peak versus normalized average crack width
(b) Load level versus average crack spacing


5.6 Influence of Test Variables

In this section the effect of the two test variables, the reinforcement ratio and the load history, on the crack spacings and crack widths is investigated. Although the cover and the bar spacing were also varied with the reinforcement ratio, they were not considered to be test variables. It is obvious that everywhere the reinforcement ratio is involved the cover and the bar spacing are involved too, but in this experimental program they were determined in order to let the reinforcement ratio to be dominant.

5.6.1 Reinforcement Ratio

The reinforcement ratio affects the cracks in two ways. The crack spacing is strongly influenced by the reinforcement ratio ($\rho$) as shown in Fig. 5.13. The number of cracks decreases for lower reinforcement ratio. This effect is well captured by the CEB-FIP Model Code (1983) equation, in which the crack spacing is an inverse function of $\rho$. The number of cracks influences the average crack width ($w$). Specimen UC1 with $\rho = 3\%$ reached $w = 0.24$ mm, while UC2 with 2% reached $w = 0.33$ mm, and UC5 with 1% had a maximum crack width of 0.50 mm. The residual crack width when unloading from tension is also very much influenced by the reinforcement ratio. It ranged from 0.050 mm for UC1, 0.065 mm for UC2, and up to 0.160 mm for UC5. The larger residual crack width was probably due to the larger maximum crack widths as well as the larger concrete area which has a greater probability of having rougher discontinuities at the crack.
Fig. 5.13  Cracked specimens UC1, UC2 and UC5.
CHAPTER 5  Experimental Results and Observations

5.6.2 Load History

The effect of the load history is best seen by comparing specimens UC3, UC4 and UC5. A comparison of Fig. 5.6(a), Fig. 5.9(a) and Fig. 5.12(a) indicates that UC3 was the stiffest specimen followed by UC4 and UC5, although these three specimens were exactly the same. This may be explained by the fact that UC5 was tested two weeks after UC3 and might have been affected to a greater extent by shrinkage (see Appendix A). The residual crack width in specimen UC5 was twice as large as measured in specimen UC3.

It is difficult to determine if loading in compression had a definite effect on the crack width at peak. At the first peak there was a difference of 0.058 mm between UC3 and UC4. From one cycle to another the difference varied between 0.071 mm and 0.016 mm. At the tenth cycle the difference was 0.064 mm.

Loading in compression had an undoubtable effect on the residual crack width (crack width at zero load). After three complete cycles, the residual crack width unloading from tension and unloading from compression are the same. On average the residual crack width after cycling in compression (UC4) was 25% smaller than the crack width obtained when cycling only in tension (UC3).
CHAPTER 6
Comparative Study

This chapter presents the comparison between experimental results and the analytical models proposed in Chapter 2 and Chapter 3. The chapter begins with a comparison of the monotonic tension stiffening model with previous experimental results and other existing models. It is followed by a comparison between the cyclic tension stiffening model and the experimental results. Finally the concrete model and the steel model are put together to predict the complete cyclic response of the five elements tested in this study.

6.1 Monotonic Tension Stiffening

As explained in Chapter 3, the average stress in the concrete is directly proportional to the crack spacing. The CEB-FIP Model Code (1983) expression for calculating the average crack spacing was chosen and an expression for evaluating the average cover was proposed in Section 3.6. The comparison between the modified CEB-FIP expression and the measured crack spacing is shown in Fig. 6.1. The prediction is very accurate for specimens without any transverse reinforcement (UC Series, Bhide, Bischoff). The modified CEB-FIP expression overestimates the crack spacing for the specimens tested by Hwang, which is probably due to the presence of transverse reinforcement in Hwang’s specimens.

The monotonic tension stiffening model predictions are compared with the experimental results from two elements tested by Adebar (1989) in Fig. 6.2. For these elements the proposed relationships somewhat overestimate the tension stiffening, with the likely explanation for this being that the longitudinal reinforcement in these elements was concentrated near the top and bottom of the elements and was not well distributed as idealized in the model.
Fig. 6.1 *Measured versus predicted average crack spacing.*

The most important observation from the experimental results is that after the initial decrease in concrete tensile stresses during the formation of a stable crack pattern, the tension stiffening remains constant while the deformations increase by more than a factor of two.

The average concrete tensile stresses measured in a specimen tested by Bischoff (1983) is shown in Fig. 6.3. In this case the longitudinal reinforcement was well distributed and hence the tension stiffening was somewhat more than predicted. The experimental results again demonstrate that the tension stiffening does not decay rapidly. As a final comparison, Fig. 6.4 shows the results from eight specimens tested by Hwang (1983).

Considering the discrepancy between the predictions from the model and the experimental results of Adebar and Bischoff it was decided to enhance the prediction by introducing additional variables in the model.

Several geometrical characteristics of the specimens were examined to determine if they
would improve the model. The bond interaction between a reinforcing bar and the concrete causes a stress field to develop in the concrete surrounding the bar. For closely spaced bars these stress fields will overlap causing an increase in stress in the overlapped zones that may result in a splitting crack and a degradation of the bond effectiveness. To capture the effect of
Fig. 6.3  *Comparison of concrete tensile stresses and the experimental results of Bischoff* (1983).

overlapping it was decided to use the ratio of the smallest distance between reinforcing bars in a particular row \( (s_{\text{min}}) \) to the bar diameter. The ratio is normalized by 15 and limited to a maximum of 1.0. This is consistent with the CEB-FIP Model Code limit on the area of concrete to be considered as contributing to the stiffening of a bar, which is a maximum distance of 7.5 \( d_b \) from the bar.

Adebar's and Bischoff's experimental results show that the steel distribution in the section has an effect on the effective area ratio \( (\rho_e) \). To account for this effect, the ratio of the smallest distance between reinforcing bars of a row \( (s_{\text{min}}) \) to the maximum distance between two rows of rebars \( (s_{\text{max}}) \), shown in Fig. 6.5, was introduced in the calculation of the residual tensile stress. The factor \( K_{\text{cor}} \) combines the two ratios described above:

\[
K_{\text{cor}} = k_{\text{reg}} \left( \frac{s_{\text{min}}}{s_{\text{max}}} \right)^{0.05} \left( \frac{s_{\text{min}}}{15 d_b} \right) \tag{6.1}
\]

where \( k_{\text{reg}} = 2.35 \) was obtained from a linear regression on the results from Adebar (1989), Bischoff (1983), Bhide and Collins (1989), Hwang (1983), and the UC series. The linear
Fig. 6.4 Comparison of predicted concrete tensile stresses and the experimental results of Hwang (1983).
Fig. 6.5 *Maximum and minimum distance between rows of rebars, s_{max} and s_{min}.*

regression is shown in Fig. 6.6. The residual tensile stress can now be calculated as:

\[
\bar{f}_c = K_{cor} \sqrt[2]{\rho_s} \frac{s}{d_b}
\]  

(6.2)

Predictions performed using Eq (6.2) are compared to experimental results in Fig. 6.2, Fig. 6.3 and Fig. 6.4, and is labelled the "Corrected Model".

Fig. 6.6 *Residual tensile stress from different tests and determination of K_{cor}.*
6.2 Cyclic Tension Stiffening

Five specimens were tested in this study. In all comparisons the measured cracking stress \( f' \) was used. The comparison between the predicted tension stiffening response and the experimental response of specimen UC1 is shown in Fig. 6.7. The prediction was performed using the calculated crack spacing (121 mm) and the corrected residual tensile stress. The prediction is peak strain controlled, i.e. the strain at zero load is computed using the equation suggested in Chapter 3.

The comparison for specimen UC2 is shown in Fig. 6.8. Again the calculated crack spacing was employed (168 mm), the corrected residual tensile stress and the prediction is peak strain controlled. The prediction is fairly good up to unloading at the fourth cycle except the predicted loop at the second cycle is pinched unlike the experimental results. This is due to the modelling of the unloading curve which requires a position for the plateau. This approach is good at high unloading strain levels but needs to be upgraded at lower unloading strain levels. At the fourth cycle in the test an additional crack appeared at the bottom face and dramatically lowered the response by approximately 0.4 MPa. That is approximately the difference between the predicted stress at zero load and the experimental one.

The comparison for specimen UC3 is shown in Fig. 6.9. The calculated crack spacing (304 mm) was used but the \( K_{cor} \) factor was not used because it would penalise the cyclic part of the model. This way penalises the monotonic part. The prediction was peak strain controlled for the first cycle, the eleventh and so on. The peak strain was computed using the equation for the strain at previous load level when cycling to a constant load level.

The comparison for specimen UC4 is shown in Fig. 6.10. Again the calculated crack spacing (304 mm) was used, but the correction factor \( (K_{cor}) \) was not applied so as not to penalise
the cyclic model. The prediction was strain controlled for the first cycle, the eleventh cycle and so on. The peak strain was computed using the equation for the strain at previous load level when cycling to a constant load level, while the peak strain in compression was fixed by the user (from the experimental results).

The comparison for specimen UC5 is shown in Fig. 6.11. The calculated crack spacing (304 mm) was used and the correction factor \( K_{\text{cor}} \) was applied. The prediction was peak strain controlled. The model worked fairly well although the second loop is overly pinched for the same reasons given for specimen UC2.

For all specimens the exponent \( \lambda_u \) of the tension stiffening envelope has been set to 1500, unlike the predictions of Section 6.1 where \( \lambda_u \) was set equal to 3000. This exponent does not affect the magnitude of the average concrete stress at large strains but it represents the transition between first cracking and stabilized cracking. This transition varies from one specimen to another.
Fig. 6.7  Comparison between cyclic tension stiffening model and calculated experimental response from specimen UC1.
Fig. 6.8  Comparison between cyclic tension stiffening model and calculated experimental response from specimen UC2.
Fig. 6.9 Comparison between cyclic tension stiffening model and calculated experimental response from specimen UC3.
Comparison between cyclic tension stiffening model and calculated experimental response from specimen UC4.

Fig. 6.10
Fig. 6.11  Comparison between cyclic tension stiffening model and calculated experimental response from specimen UC5.
6.3 Complete Response of Reinforced Concrete Elements

The models proposed in Chapter 2 for reinforcing steel and in Chapter 3 for concrete have been implemented in a computer program called TS. The steel model and the model for concrete in compression are always active in the program regardless of the load history. The tension stiffening model is deactivated once the steel has yielded because the concrete contribution once the steel yielded is considered to be negligible.

These predictions are particularly interesting for the post yield behaviour of the model. In the post yield range, the load history of specimens UC1, UC2, UC3 and UC5 (shown in Fig. 6.12, Fig. 6.13, Fig. 6.14 and Fig. 6.16 respectively) are similar in the sense that none of the specimens were loaded in compression. It can be observed that the predicted area of the loop increases with the peak strain a little faster than the measured area. For specimen UC5 the prediction failed to reach zero load at the given strains.

Specimen UC4 shown in Fig. 6.15 was submitted to compressive loads. It can be observed that the area of the loops generated by the model is generally bigger than the measured ones. In addition the experimental response gets stiffer at a load of approximately 900 kN, while the prediction does not. This is believed to be due to the absence of a closing crack criteria in the model.

Except for the minor differences described above, the model seem to give very good predictions for the cyclic response of reinforced concrete.
Fig. 6.12 Comparison between complete reinforced concrete model and experimental result from specimen UC1.
Fig. 6.13  Comparison between complete reinforced concrete model and experimental result from specimen UC2.
Fig. 6.14  Comparison between complete reinforced concrete model and experimental result from specimen UC3.
Fig. 6.15  *Comparison between complete reinforced concrete model and experimental result from specimen UC4.*
Fig. 6.16  Comparison between reinforced concrete model prediction and experimental result from specimen UC5.
6.4 Summary

This chapter presented the comparison between experimental results and the analytical models proposed in Chapter 2 and Chapter 3. The comparison of the monotonic tension stiffening model to the experimental results showed good agreement. Unlike the other models, the proposed model does not predict a continuous decay but rather a plateau similar to the experimental results. The comparison between the cyclic tension stiffening model and the experimental results also showed good agreement. Finally, the concrete model and the steel model were combined in order to predict the complete cyclic response of the five elements tested in this study.
CHAPTER 7
Conclusions

The purpose of this study was to develop a better understanding of the response of cracked reinforced concrete subjected to reverse cyclic loading with particular emphasize on the contribution of tensile stresses in cracked concrete.

A model reproducing the behaviour of reinforcing steel under reverse cyclic loading was selected from the literature. The Pinto et al. Model was simplified and enhanced. It showed good agreement with several experimental results.

The behaviour of reinforced concrete under axial tension was studied. The average tensile stress which is transmitted across a crack decreases rapidly as the average crack width increases. On the other hand, the average tensile stress which is transferred from the reinforcement to the concrete between cracks through bond stresses (ie. tension stiffening) increases, or remains relatively uniform, as the crack width increases. In order to make an accurate prediction of the response of cracked concrete, a distinction needs to be made between these two types of concrete tensile stresses. A set of three tension stiffening equations were derived from an assumed bond stress-slip relationship assuming a linear variation of slip. This set of equations was then replaced by one simple exponential function since at relatively large strains, the average concrete tensile stress reaches a plateau. A simple expression was also proposed for the combined tensile stress from tension stiffening and the stresses transmitted directly across the crack by plain concrete.

The exponential equation used to represent bond transfer and the decaying curve including plain concrete make use of one simple equation for evaluating the plateau which includes all relevant bond parameters. Comparisons with experimental results indicated that this
simple expression gives reasonable predictions. A refinement of this equation was possible by considering the distribution of the longitudinal reinforcement.

Tension stiffening was found to be directly proportional to the crack spacing. The CEB-FIP Model Code (1983) average crack spacing formula was adopted and enhanced. An expression for evaluating the cover is introduced in order to capture the effect of the distribution of the steel on the cross section. Comparison with experimental results showed good agreement.

A reverse cyclic bond stress-slip relationship is described but a cyclic tension stiffening model could not be derived from the cyclic bond stress-slip model because at the present state of knowledge there is too many uncertainties. Thus an empirical approach was adopted and an experimental program was undertaken to develop the cyclic tension stiffening model.

Five specimens were tested involving only two test variables: the load history and the reinforcement ratio \( (\rho) \). The experiments confirmed that the reinforcement ratio and the resulting cover have a definite effect on the crack spacing. The number of cracks decreases with the reinforcement ratio and that is well captured by the CEB-FIP Model Code (1983) equation. As a result the reinforcement ratio also influences the average crack width: the more cracks the smaller the crack width.

The load history affects the crack width. The crack width at a given peak load level increases after subsequent loading cycles but stabilizes after about 5 cycles. The residual crack width is not influenced by cycling only in tension, while cycling in compression reduced the residual crack width by about 25% compared to a specimen cycled only in tension.

A model for concrete in compression gathering different expressions proposed in the literature was combined with the reverse cyclic tension stiffening model. The analytical models for steel and concrete were implemented into a computer program to predict the complete
response of uniaxially reinforced concrete elements subjected to cyclic axial loading. The comparisons between the predictions and the experimental results showed good agreement.

Further research is required to increase the accuracy of the analytical models for reinforced concrete under reverse cyclic loading. More tests are needed to determine the effect of cycling on the tension stiffening envelope. A test series where the specimens would be cycled five times at a strain of 0.0005, five times at .0010 and five times at .0015 should provide this information. The connection between the tensile response and the compressive response needs to be investigated in order to determine if the envelopes are fixed on the strain scale or if they shift similar to the steel envelopes. A well established crack closing criterion would enhance the predictions especially after yielding of the reinforcing steel in tension.

The most promising research route for cyclic tension stiffening is toward reverse cyclic bond. The success obtained with the monotonic solution proved the relevance of using a bond stress-slip relationship to derive a tension stiffening model. Indeed, bond is the heart of tension stiffening.
REFERENCES


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REFERENCES


REFERENCES


APPENDIX A

Effect of Shrinkage and Creep

It was obvious by looking at the data that shrinkage affected the response of specimen UC1. The specimen was deforming more than an equal amount of bare reinforcing steel. Ignoring shrinkage when computing tension stiffening resulted in negative tension stiffening (see Fig. A.1). Günther (1991b) observed the same phenomenon in similar tests.

Concrete, upon drying, tends to contract. Plain concrete shrinks and does not develop any stresses. As explained by Collins and Mitchell (1987), a "stress free" shrinkage strain ($\epsilon_{sh}$) can be computed for moist cured concrete using the following formula:

$$
\epsilon_{sh} = -k_s k_h \left( \frac{t}{35 + t} \right) .00051
$$

where $k_s$ and $k_h$ are two factors to account for size and relative humidity, and $t$ is the time in days which the concrete has been exposed to drying.

But in reinforced concrete, because of bond, the reinforcement restrains the concrete from shrinking causing tensile stresses in the concrete. The member will reduce in length up to the point where the elastic force in the steel will balance the force in the concrete due to the restrained shrinkage strain. Assuming uniform shrinkage strain over the entire cross section leads to the following concrete stress ($f_{cl}$):

$$
f_{cl} = \epsilon_i E_s \rho = (\epsilon_i - \epsilon_{sh}) E_c
$$

The sustained nature of these tensile stresses together with the young age of the concrete causes significant "tensile creep" (Bischoff, 1983). Following the method described in Collins and Mitchell (1987), a creep coefficient is computed ($\phi(t,t_i)$):
Fig. A.1 Neglecting shrinkage in calculation of tension stiffening for specimen UC1.

\[ \phi(t, t_i) = 3.5 k_s \left( \frac{1.58 - \frac{H}{120}}{t_i - 0.118} \right) \frac{(t - t_i)^6}{10 + (t - t_i)^6} \]  

(A.3)

where \( H \) is the relative humidity in percent, and \( t_i \) is the age of concrete in days at initial loading.

Using the creep coefficient the effective tangent modulus is computed \( (E_{c,eff}) \):

\[ E_{c,eff} = \frac{E_c}{1 + \chi \phi(t, t_i)} \]  

(A.4)

where \( \chi \) is a relaxation coefficient to account for the fact that not all the stress due to shrinkage is applied at the initial time; \( \chi \) is taken as 0.8. By substituting the effective tangent modulus into equation (A.2) and rearranging to isolate \( \epsilon_i \), (the strain prior to testing), we get:
APPENDIX A Effect of Shrinkage and Creep

\[
\epsilon_i = \frac{\epsilon_{sh}}{1 + \frac{E_s}{E_{c,eff}} \rho}
\]  

(A.5)

When subtracting the steel force from the applied force to get the force resisted by the concrete, the average strain used to calculate the steel force will be the sum of the measured strain (\(\epsilon_{mea}\)) and the initial strain:

\[
\epsilon_s = \epsilon_{mea} + \epsilon_i
\]  

(A.6)

while the concrete strain due to stress will be:

\[
\epsilon_{cf} = \epsilon_{mea} + \epsilon_i - \epsilon_{sh}
\]  

(A.7)

where the shrinkage strain is subtracted from the average strain.

Table A.1 Shrinkage and creep parameters.

<table>
<thead>
<tr>
<th>specimen</th>
<th>UC1</th>
<th>UC2</th>
<th>UC3</th>
<th>UC4</th>
<th>UC5</th>
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<tr>
<td>(\rho)</td>
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<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
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<tr>
<td>test time (day)</td>
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<td>70</td>
<td>48</td>
<td>58</td>
<td>65</td>
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<td>Humidity</td>
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<td>0.70</td>
<td>0.70</td>
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<tr>
<td>(k_h)</td>
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<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>Vol/Area (mm)</td>
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<td>83</td>
<td>117</td>
<td>117</td>
<td>117</td>
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<tr>
<td>(k_s)</td>
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<td>0.599</td>
<td>0.395</td>
<td>0.419</td>
<td>0.424</td>
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<tr>
<td>(\phi(t,t_i))</td>
<td>1.38</td>
<td>1.13</td>
<td>0.66</td>
<td>0.74</td>
<td>0.78</td>
</tr>
<tr>
<td>(E_{c,eff}) (MPa)</td>
<td>15027</td>
<td>16608</td>
<td>20649</td>
<td>19813</td>
<td>19487</td>
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<tr>
<td>(\epsilon_{sh})</td>
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<td>-0.000191</td>
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<td>-0.000130</td>
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<tr>
<td>(\epsilon_i)</td>
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<td>-0.000099</td>
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<td>-0.000119</td>
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APPENDIX B

Cyclic Tension Stiffening Model Equations

This appendix presents, in graphical form, the analytical expressions chosen for each parameter employed in the cyclic tension stiffening model. Shrinkage and creep were taken into account in the calculation of the stresses. The slopes were determined to represent the trend at each given zone (e.g. unloading, plateau, crack closing, reloading from zero load and previous peak).
APPENDIX B  Cyclic Tension Stiffening Model Equations

UC SERIES unloading slope at peak strain

UC SERIES effect of cycling on unloading slope

Fig. B.1  a) Unloading slope at peak strain
          b) Effect of cycling on unloading slope at peak strain
APPENDIX B  Cyclic Tension Stiffening Model Equations

UC SERIES slope at plateau

UC SERIES effect of cycling on slope at plateau

Fig. B.2  

a) Slope at plateau

b) Effect of cycling on slope at plateau
APPENDIX B  Cyclic Tension Stiffening Model Equations  B4

UC SERIES stress position of plateau slope

UC SERIES strain position of plateau slope

Fig. B.3  

a) Stress position of plateau slope  
b) Strain position of plateau slope
APPENDIX B Cyclic Tension Stiffening Model Equations

Fig. B.4  
(a) Crack closing slope  
(b) Effect of cycling on crack closing slope
APPENDIX B  Cyclic Tension Stiffening Model Equations

Fig. B.5  
a) Residual strain  
b) Slope reloading from zero load
APPENDIX B  Cyclic Tension Stiffening Model Equations  B7

UC SERIES delta stress at unloading

\[ f_{cr}(1 - 700\varepsilon_{un}) + (775\varepsilon_{un})^2 - (575\varepsilon_{un})^3 \]

peak strain

delta stress (MPa)

UC SERIES effect of cycling on delta stress unl.

1 - 0.107(n - 1) \geq 0.465

ratio of current to first delta stress

cycle

Fig. B.6  
a) Delta stress at unloading  
b) Effect of cycling on delta stress at unloading
APPENDIX B  Cyclic Tension Stiffening Model Equations

Fig. B.7  a) Drift in strain at previous peak
b) Effect of cycling on drift in strain at previous peak
Fig. B.8  

a) Drift in stress at previous peak  
b) Effect of cycling on drift in stress at previous peak
APPENDIX B  Cyclic Tension Stiffening Model Equations B10

Fig. B.9  
(a) Reloading delta stress ratio  
(b) Effect of cycling on reloading delta stress ratio
Fig. B.10  Slope at previous peak load.
APPENDIX C

Crack Width Measurements

Table C.1 UC1 Crack width measurements.

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<th>cycle</th>
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<th>$w_{top}$ (mm)</th>
<th>micro readings</th>
<th>$k_w$</th>
<th>$w_w$ (mm)</th>
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Table C.2 *UC2 Crack width measurements.*

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<th>micro readings</th>
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<th>$w_{av}$ (mm)</th>
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Table C.3 *UC3 Crack width measurements.*

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<th>micro readings</th>
<th>( k_w )</th>
<th>( w_{av} ) (mm)</th>
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### Table C.4 UC4 Crack width measurements (part 1).

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Table C.5  *UC4 Crack width measurements (part 2).*

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Table C.6  *UC5 Crack width measurements.*

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APPENDIX D

Computer Programs User's Manual

D.1 Computer Program TS

The reverse cyclic tension stiffening model proposed in this study was implemented into the computer program TS.

Four different types of analyses can be conducted using TS. The first type is the strain controlled history analysis where the bounding strain of each loading path is defined by the user.

The second type is the peak strain controlled history analysis where only the peak strain reached while loading in the tensile direction is defined. The program computes the strain at zero load before yielding of the reinforcing steel. After yielding, the strain reached while unloading from tension must be defined. This kind of analysis was used to predict the response of specimens UC1, UC2 and UC5.

The third type is the peak strain controlled repeated cycle analysis where the peak strain reached while loading in the tensile direction for the first cycle at a given tensile load level is defined. As a result, the user defines the strain at the given load level and the number of times the specimen is cycled at that load level. The program computes the strain at zero load before yielding of the reinforcing steel and the strain and stress at reloading to the previous tensile load level. This kind of analysis was used to predict the response of specimen UC3.

The fourth type is the strain controlled repeated cycle analysis where the peak strain reached while loading in the tensile direction for the first cycle at a given load level and the strain reached while loading in the compressive direction for each load cycle are defined. As a result, the user defines the strain at the given tensile load level and the number of times the specimen is cycled at that load level. The program computes the strain and stress at reloading.
to the previous tensile load level. This kind of analysis was used to predict the response of specimen UC4.

The program works with a unique input file format, the strain controlled format, which is the most stringent one. The strains bounding each loading path must be defined by the user. The program disregards the unneeded strains, as for example in performing repeated cycles analyses. To assist the user in preparing an input file a subroutine is provided. The user is prompted to give the following steel parameters: bar diameter \(d_b\), total number of bars in the specimen, the area of steel \(A_s\) which is computed but can be changed by the user), the modulus of elasticity \(E_s\), the yield stress \(f_y\), the yield plateau slope factor \(b_1\), the strain at beginning of hardening \(\epsilon_{sh1}\), the strain hardening slope factor \(b_2\), the strain at end of hardening \(\epsilon_{sh2}\), the ultimate slope factor \(b_3\) and the ultimate stress \(f_u\); and the following concrete parameters: the section area \(A_c\), the compressive strength \(f'_c\), the tangent modulus \(E_{ct}\) which is computed using the 5500 \(f'_c\) but can be changed), the tensile strength \(f'_t\) which is computed using the .33 \(f'_c\) but can be changed), the creep factor \(\phi\), the initial strain \(\epsilon_{in}\), and the shrinkage strain \(\epsilon_{sh}\).

The load cycles are limited to 30, giving 60 loading paths. For each loading path the program will ask for an end strain and a number of cycles. If the specimen is cycled at that given load level the total number of cycles \(x\) should be entered and for the next \(2x\) loading paths the user will be asked for an end strain only. If the specimen is not cycled at that given load level one \(1\) should be entered for the number of cycles. At the end, the program calculates the smallest strain increment it can handle for the given load history because of computer memory constraints when using the plotting feature. However a version of TS called
TSEXP was written to enable the use of very fine strain increments for long load histories. Although TSEXP does not include the plotting feature.

While running, the program needs for 3 informations: the name of the input file, the name of the output file and the crack width in mm. If the output file already exists it will be overwritten.

**D.2 Computer Program BOND**

The reverse cyclic bond model proposed in this study was implemented into the computer program BOND. Computer program BOND can be used to predict the full response of an element under one complete load cycle and also to visualize the distribution of stresses at any load level during the cycle.

To help the user in preparing an input file a subroutine was written. The user is prompted to give the following steel parameters: bar diameter ($d_b$), total number of bars in the specimen, the area of steel ($A_s$ which is computed but can be changed by the user), the modulus of elasticity ($E_s$), the yield stress ($f_y$); and the following concrete parameters are: the section area ($A_c$), the compressive strength ($f' c$), the tangent modulus ($E_{ct}$ which is computed using the $5500f' c$ but can be changed), the tensile strength ($f' c$ which is computed using the $.33f' c$ but can be changed). The monotonic bond relationship is similar to Eligehausen's relationship. It was implemented because it enables the use of various models besides the one that is proposed in this study. The monotonic bond parameters are: the peak bond stress ($\tau_{max}$), the slip at peak stress ($u_p$), the slip at the end of the plateau ($u_e$), the residual bond stress ($\tau_{res}$), the slip at the end of the plateau ($u_e$), the exponent of the rising portion ($\beta$). Two sets of monotonic bond parameters are necessary to represent the behaviour of confined concrete and unconfined
concrete. The confined concrete relationship becomes effective at a certain distance from the crack face entered as a percentage of the crack spacing. The unconfined concrete relationship is effective on a certain percentage of the length starting at the crack face. In the zone between the end of the unconfined concrete and the start of the confined concrete all the parameters are varied linearly (this is done automatically by the program). The solving technique is iterative, therefore a tolerance on the slip is required (0.00001 mm is usually adequate). The required cyclic bond parameters are: the unloading modulus \( (E_{un}) \), the minimum unloading modulus \( (\alpha_{un}E_{un}) \), the reloading modulus (which is used for initialisation), the reloading stress degradation factor \( (\beta_{re}) \), the origin of the reloading slope \( (k_{re}) \), the friction degradation factor \( (\beta_{fr}) \), the start of degradation for unloading modulus, the end of degradation for unloading modulus \( (k_{un}) \).

Computer program BOND can be used to predict the load versus crack width response of an element under monotonic loading and after two reversals of the load. For monotonic loading, the user is prompted to give the name of the input and output files, the maximum load, the number of load steps (maximum of 100), the crack spacing and the number of sections (30 sections are usually adequate). For a complete load cycle, instead of giving the maximum load the user is prompted to give the load at first reversal, at second reversal and at the end of the cycle. At the end of calculations the user can visualize the results on the screen in three different plots: force versus crack width, average concrete stress versus average strain, and average concrete stress versus crack width.

Computer program BOND can also be used to visualize the distribution of stresses at any load level during a cycle. For monotonic loading, the user is prompted for the crack spacing, the number of sections, the applied load and the initial slip to begin the iterative process (0 for the first load and [enter] for the following ones is usually adequate). For one reversal of load, the user is prompted first for the load at reversal then for the applied load. For two reversals
of load, the user is prompted for the load at first reversal then for load at second reversal and finally for the applied load. After the calculation of the distributions, the user can visualize on the screen the steel stress distribution along the length of the element, the steel strain, the concrete stress, the concrete strain, the bond stress and the bond slip. These informations are written in the output file.