RELIABILITY ANALYSIS OF STRUCTURAL CONCRETE ELEMENTS

by

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Abstract

The reliability of reinforced concrete elements and design iceberg impact loads for offshore structures were studied. The reliability program RELAN was used to perform FORM reliability analysis for reinforced beams subjected to bending, and for a reinforced concrete wall from the Hibernia offshore structure subjected to complex loading. RELAN was also used to establish the probabilistic distribution of ice impact loads.

To study the reliability of concrete beams accounting for the variability of the intervening variables, and in order to determine the theoretical flexural capacity of concrete beam, computer program TIN was developed. TIN uses a strain compatibility approach accounting for the non-linear stress-strain relationships of concrete and reinforcing steel. As a pilot study on the reliability of concrete elements, a beam designed according to the Canadian concrete code was analyzed with the objective of evaluating the effect of different spans and reinforcing steel ratios on the reliability of the beam.

To study the reliability of more complex elements, an element from the icewall of the Hibernia offshore structure was used. The theoretical strength of the wall element was evaluated with program SHELL474. In order to link SHELL474 to RELAN for the reliability study, the main subroutine in SHELL474 was modified. Since one of the major factors in reliability studies of concrete offshore structures are the uncertainties associated with extreme environmental load conditions, the statistics for ice impact loads for the Hibernia structure were derived using RELAN and applications of energy conservation principles.
Abstract

For the purpose of deriving the ice load distributions to evaluate the reliability of the Hibernia icewall element, the program PROB, which is a product of the reliability program RELAN and the energy conservation principles, was developed.
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CHAPTER 1

Introduction

1.1. Introduction

Engineering analysis and design normally requires resolution of uncertainties. These imply that an absolute assurance of safety and performance of a design is not practically obtainable, which basically means that some risk is invariably involved. In order to deal with the uncertainties in design, building codes introduce factors of safety, or load factors. However, the variability of the intervening variables and their significance for structural safety and performance can be analyzed systematically through methods of probability theory.

Reliability methods have been developed to assist engineers in making design decisions in the presence of uncertainties, where the conceptual basis is the probability of failure, $P_f$. In general, failure denotes the exceedence of a limit state but does not necessarily imply collapse. The probability $P_f$ may be calculated from the probability distributions of the random resistance and load variables, if these distributions are known.
1.2. Reliability Concepts

The main problem in general design is the determination of a structural capacity to assure adequate safety and performance of the system. Failure of a system, which generally means the realization of a specified limit state, including collapse, can be defined as the event \( R \leq D \), where \( R \) is the resistance and \( D \) is the demand or the applied load. Thus, as shown in Figure 1.1, the probability \( P_f \) corresponds to the probability of the event \( [G = R - D] < 0 \).

The application of analytical methods to compute \( P_f \) is mainly limited to rather simple systems which involve few random variables. Because usually many random variables are involved, approximate methods like the standard Monte Carlo simulation [8], and the variance reduction technique, often known as the Iterative Fast Monte Carlo (IFM) [48] procedure, have been developed to estimate the probability of failure. These methods have proven effective for the analysis of complex systems. However, in order to make the probability calculation more accessible, the First and Second Order Methods (FORM/SORM) [26] have been developed. The FORM/SORM procedures are based on the calculation of the reliability index \( \beta \), which enables engineers to evaluate the probability of failure by using the standard normal probability distribution function (see Figure 1.1). The estimation of the probability of failure in this manner will only be exact if all the intervening variables are normally distributed and uncorrelated, and if they combine linearly in the performance function \( G \).
Figure 1.1: Definition of the Safety Index $\beta$

Since generally the variables are non-Normal and correlated, the FORM/SORM procedures introduce an appropriate transformation to convert all variables to normals and to eliminate correlations if present.

A geometric interpretation of the reliability index $\beta$ permits the development of iterative computer algorithms [26] as will be discussed in Chapter 2. Using FORM/SORM procedures, a program entitled RELAN, or RELiability ANalysis, was developed by Foschi, Folz and Yao [19]. RELAN, has been applied extensively to study the reliability of wood structures and the calibration of corresponding reliability-based design guidelines in Canada.
1.3. Concrete Elements and Variability

The basic information required to study the reliability of reinforced concrete elements is the probability distribution of each load and resistance variable, including as a minimum, estimates of their means and standard deviations or coefficients of variation.

In order to express the ultimate strength for specific values of the random variables, a theoretical deterministic calculation procedure needs to be established. This procedure can then be linked to RELAN and called for each realization of the random variables.

To study the moment capacity of a simple beam, program TIN was developed. TIN calculates the ultimate flexural capacity using a general strain compatibility solution and a layer approach, using assumed non-linear stress-strain relationships for the concrete and the reinforcing steel. As a pilot study, and in order to gain a better understanding of the reliability of more complex concrete elements, the influence of different spans and different reinforcing steel ratios were studied.

In case of reliability of offshore structure elements, program SHELL474, developed by Adebar and Collins [1], was used to perform the capacity calculation for an element from the Hibernia offshore structure. Because SHELL474 was written to analyze only one set of variables for each run, the main subroutine in SHELL474 had to be modified so the variables could be brought in through RELAN as random variables. A reliability calculation was then performed for one-hundred-year eccentric and concentric ice impact, as well as to determine the probability density function (PDF) for the element strength.
By performing reliability calculations, the designer can gain a better understanding of the behavior of the element and the influence of the many random variables involved, and is therefore more capable of economical design and preventing drastic failures.

1.4. Ice Impact and Offshore Structures

Icebergs or floes present one of the most severe threats to offshore installations in sub-polar regions, particularly off the coast of Newfoundland. Various methods are available for calculating the maximum impact load experienced during a collision. For icebergs impacting a large structure, the limit momentum approach can be used to equate the kinetic energy of the impacting icebergs with the energy dissipated during the collision and to predict the maximum impact load (see Chapter 5). Because of the nature of these loads, they are normally predicted with probabilistic procedures such as Monte Carlo simulation or FORM/SORM procedures. However, obtaining the required statistical data can sometimes be difficult, mainly because quite often the data do not exist or are not available, and therefore it is hard and sometimes even not possible to develop unambiguous characterizations of uncertainties.

A major problem in establishing the impacting force is due to the difficulty of relating small-scale ice properties to a prediction of how ice behaves on a large scale. Because of this, no generally accepted methods for predicting ice loads on offshore structures exist.
When the probability of failure for the offshore elements was calculated in this thesis, the intervening ice parameters, i.e. the thickness, the diameter, the velocity, and the ice compressive strength, where used as random variables. But since the impacting force would change with time due to increasing contact area, so will the stresses in the element. It is apparent that an updating finite element analysis is also needed to reevaluate the stresses in the element, when RELAN would change the ice parameters in order to find the gradient to the failure surface. However, if the joint probability function for the ice impact would be known, the randomness can be represented by only one random variable with its mean and standard deviation. Based on the theoretical model derived in Chapter 5, a program was develop in order to derive the PDF function for one hundred year eccentric and concentric iceberg.

1.5. Outline of Thesis

Chapter 2 discusses the reliability concepts in more detail and introduces different methods available, including the basic principles of First and Second Order Methods and their implementation in the reliability program RELAN. The variability of the intervening variables, such as the material strength and geometric properties, are reviewed in Chapter 3, including discussion on stress-strain relationships for concrete and reinforcing steel. Chapter 4 presents a reliability calculation for simple beams, of different spans and reinforcing ratios, and explains how the reliability program RELAN was linked to the beam design program TIN, which was developed for this study.
The reliability of offshore structure elements and the extreme ice loads are discussed in Chapter 5, including the energy conservation methods used in the analysis of the impacting icebergs. Chapter 6 presents the study of the offshore structure element from the Hibernia offshore structure, making use of program SHELL474. Finally, Chapter 7 includes conclusions and discusses recommendation for further study.
CHAPTER 2

Reliability Concepts

2.1. Introduction

One of the principal aims of engineering analysis and design is the assurance of system performance within the constraint of economy. This means quite often that satisfactory failure rates for different limit states corresponds to a "trade-off" between human safety or serviceability on the one hand, and economy, including expected losses due to failures, on the other hand. In practice, satisfactory failure rates are achieved through competent structural engineering, manufacturing, and erection, and by the use of safety and serviceability criteria.

Most planning and design of engineering systems must be done on a basis which requires the resolution of uncertainties because of incomplete information, e.g. the actual lifetime maximum load and the actual capacity of the structure. In view of these uncertainties, risk is unavoidable and the way it is dealt with traditionally is to apply factors or margins of safety and adopt conservative assumptions in the process of design. This is done by ascertaining that a "worst," or minimum, resistance conditions will remain adequate under a "worst," or maximum demand conditions. These conditions are often defined on the basis of subjective judgment and similarly the adequacy or inadequacy of the applied "margins" may be evaluated or calibrated only in terms of past experience with similar systems.
The proper way to deal with uncertainties and their significance on structural safety and performance is to systematically analyze the structure through methods of probability. As before, the available resistance and actual demand cannot be determined precisely, but in order to represent or reflect the significance of uncertainty they may be modeled as "random variables". The probability of system non-performance may be evaluated by using either Monte Carlo simulation or FORM/SORM procedures.

Monte Carlo simulation is a powerful engineering tool which enables one to perform a statistical analysis of the uncertainty in structural engineering problems, being particularly useful for complex problems where numerous random variables are related through nonlinear equations. The fundamental step in a Monte Carlo analysis is the development of a set of random numbers by simulating the samples randomly over the entire range of each variable. The only disadvantage in using Monte Carlo simulation is the cost of execution where estimating low probability of failure may take a long time. A variance reduction technique, often known as the Iterative Fast Monte Carlo (IFM) procedure, can be most advantageously applied for such cases. Instead of simulating the samples randomly over the entire range of each variable, it is concentrated at the important regions, i.e. at those regions where most of the contributions to the failure probability is expected.

The significant role of probability in structural engineering lies in providing a logical framework for uncertainty analysis and a quantitative basis for risk and safety assessment. Therefore it is important that probability of failure as a measurement for uncertainty will be accepted, even if only as a relative measure of safety and performance.
2.2. Reliability Based Design

In the design of most engineering systems multiple variables are involved and they may influence either the resistance or the demand. It is necessary to formulate the performance of the system in terms of basic design variables, to be able to predict the probability of non-performance.

For the purpose of a generalized formulation the performance function is described mathematically as follows [19]:

\[ G(X) = G(x_1, x_2, \ldots, x_N) \]  

where \( X = (x_1, x_2, \ldots, x_N) \) is a N-dimensional vector of design variables such as the concrete compressive strength and the steel yield strength. Because most of these variables, some of which represent the mechanical and geometric properties of the system, while others characterize the load demands, are uncertain or random they need to be described statistically. In some cases this cannot be done without tests or surveys which would provide statistical information on each variable.

The performance function, \( G \), can be written in terms of the resistance, \( R \), and the demand, \( D \), as following:

\[ G = R - D \]  

where the failure of the system will occur when the demand exceeds the resistance, i.e. \( G < 0 \) and the system will survive when \( G > 0 \).
The situation when \( G = 0 \) is usually known as the limit state between survival and failure, where all variable combinations satisfying \( G = 0 \) are said to belong to this limit state.

For simple cases it is possible to obtain the probability of failure relatively easily with analytical methods, but since there are usually many random variables involved, the calculation requires the PDF-function or the joint probability density function and multiple integration over the failure region. Since the required joint probability is rarely known and difficult to obtain, approximate methods such as Monte Carlo simulation or the FORM/SORM procedures have been developed to estimate the probability of failure. FORM/SORM procedures enable engineers to evaluate the probability of failure by using the standard normal probability distribution function \( \Phi \) [19]:

\[
p_r = \Phi(-\beta)
\]

In order to make use of the standard normal distribution, the FORM/SORM calculation procedure defines a new set of variables \( x_i \) by transforming the original \( X_i \) according to:

\[
x_i = \frac{X_i - \overline{X}_i}{\sigma_{x_i}} \quad i = 1, \ldots, N
\]

where \( \overline{X}_i \) is the mean of \( X_i \) and \( \sigma_{x_i} \) its standard deviation, the origin \( x = 0 \) corresponds to the mean value of \( X \).
The reliability index $\beta$ is the minimum distance between the origin and the limit state surface $G(x) = 0$, and the corresponding point on the limit state surface is known as the "Design Point", as illustrated in Figure 2.1, for the case of two variables $x_1$ and $x_2$.

![Geometric Representation of FORM/SORM Reliability Calculation](image)

*Figure 2.1: Geometric Representation of FORM/SORM Reliability Calculation [19]*

If FORM/SORM procedures are used to calculate the probability of failure the estimation will be exact if all the intervening variables are normally distributed and uncorrelated, and if they combine linearly in the performance function $G$. Generally the variables are not normal and uncorrelated, and the performance function is non-linear. Since the FORM/SORM procedures introduce an appropriate transformation to convert all variables to normals and to eliminate correlations if present, the approximation of the probability of failure, $P_f$, is influenced solely by the non-linearity in $G$. 
The difference between FORM and SORM methods is that FORM assumes the limit state surface \( G(x) \) can be approximated by the tangent plane at the design point, where SORM on the other hand, assumes that the true limit state can be approximated by a quadratic surface (see Figure 2.1).

### 2.3. Calculation of the Reliability Index using the Program RELAN

As mentioned earlier the geometric interpretation of \( \beta \) permits the development of iterative computer algorithms as illustrated in Figure 2.2.

RELAN, which is a general FORM/SORM FORTRAN program, must be supplemented by a description of the performance function and its gradient with respect to the intervening random variables [20]. When the performance function is linear the FORM method gives a good estimate but in case of non-linearity a more approximate estimate can be made by the SORM method. For SORM calculations, the matrix of second order derivatives of \( G \) is also needed. It is sufficient to describe the function \( G \) for each specific problem, since first and second order derivatives can be obtained numerically by RELAN.

The algorithm in RELAN adjusts for the case where the random variables are not normally distributed, and also where there are correlations between the variables.
CHAPTER 2 Reliability Concepts

Figure 2.2: Flow Chart for System Reliability Calculation
In order to estimate the importance of each variable to the system reliability, RELAN calculates the sensitivity coefficients which indicate the relative influence of each variables uncertainty in the reliability index $\beta$. This can be very useful for designers in cases where the system behavior is too complex and many different modes of failure might influence the system performance.

To perform calculations by RELAN the user has to provide four FORTRAN subroutines, i.e. DETERM, GFUN, DFUN and D2FUN. The subroutine DETERM defines all the deterministic variables which then are shared with GFUN, DFUN and D2FUN through a common block$^1$. When the deterministic values have been defined, and the random variables brought through with an array$^1$, GFUN calculates the value of the performance function and returns that value to RELAN main subroutine. As mentioned before, the first and second order derivatives can be obtained numerically by RELAN. The user can also provide DFUN and D2FUN, which return respectively the gradient and the second derivatives of the performance function.

RELAN can be used to determine the probability of failure for problems in virtually any field of study in civil engineering, i.e. structures, soil mechanics, seismic risk, construction, transportation and hydraulics.

$^1$Communication Option in FORTRAN.
3.1. Introduction

A theoretical deterministic calculation procedure needs to be established to express the ultimate strength for specific values of the random variables. The values must be selected from the corresponding statistical distributions. Data on both structural resistance, i.e. strength and geometric properties, and load variables are required in order to conduct probability-based design calculations. The basic information required is the probability distribution of each load and resistance variable and estimates of its mean and standard deviation or coefficient of variation. The mean and coefficient of variations of these basic variables should be representative of values that would be expected in actual structures in situ. The normal way to develop statistics for the material properties is through testing. While frequently there are sufficient data to obtain a reasonable estimate of the probability distribution, in many other cases this must be assumed on the basis of physical argument or for convenience. This is because different techniques, instruments and the human factor, can create different results for the same specimen.

The variables which most affect the strength variability of reinforced elements are the compressive strength of concrete and the yield strength of steel as well as the geometric properties.
3.2. Compressive Strength of Concrete

The compressive strength of concrete is one of its most important properties. Other important properties, which influence the strength and stiffness, can be approximately correlated to the compressive strength. Thus, in probabilistic calculations for concrete elements the compressive strength needs to be studied carefully.

The response of concrete to uniaxial compression is usually determined by loading a standard cylinder, 150 mm in diameter and 300 mm long. The standard loading rate is such that the maximum stress, $f'_c$, is reached in 2 to 3 minutes. Even though the actual shape of the stress-strain relation for concrete is not unique and depends on several factors, such as the cylinder strength, density, rate and duration of loading, the relationship between axial stress, $f_c$, and the axial strain caused by this stress, $\varepsilon_{ef}$, is reasonably accurately represented by the equation [13]:

$$\frac{f_c}{f'_c} = 2\frac{\varepsilon_{ef}}{\varepsilon_c} - \left(\frac{\varepsilon_{ef}}{\varepsilon_c}\right)^2$$

(3.1)

which is a parabola shown in Figure 3.1. This parabola, which is widely used, describes the rising portion and the immediate post-peak response reasonably well but somewhat over estimates the rate of which the stress drops off at larger strains especially in cases where elements have a high degree of confinement. To capture this effect Kent and Park [29] proposed another curve for the stress-strain relationship to capture this over estimation, which consists of a second order parabola up to the maximum stress $f'_c$ at a strain $\varepsilon_0$ and then a linear falling branch.
By having a linear falling branch, this curve does not have the same problem in over estimating the compression strength at least not after it reaches the peak.

3.2.1. Variability in the Compressive Strength

An essential component in the development of probabilistic-based design for concrete elements is to introduce how various factors effect the concrete compression strength.
3.2.1.1. In-situ versus Cylinders Tests

Tests have shown that the strength of concrete in a structure tends to be lower than its specified design strength and may not be uniform throughout the structure. The major sources of variations in concrete strength are due to one or all of the following:

i. Variations in material properties and proportions of the concrete mix.
ii. Variations in mixing.
iii. Transporting.
iv. Placing and curing methods.
v. Variations in testing procedures and the rate of loading.
vi. Size effects.

The reduction in the in-situ strength of concrete is partially offset by the requirement that the average cylinder strength must be about 700-900 psi (4.8-6.2 MPa) greater than the specified strength to meet the existing design codes [31].

Based on this observation and on equations and data, it has been suggested [31] that the 28-day strength of concrete in a structure for minimum acceptable curing can be expressed as:

$$\bar{f}_{\text{c35}} = 0.675 f'_c + 1,100 \text{ (psi)}$$

(3.2)

where $\bar{f}_{\text{c35}}$ is less than or equal to $1.15 f'_c$. 


3.2.1.2. **Size Effects**

The phenomenon of "size effects", which is a change in indicated unit strength due to a change in specimen size, has been noted by many researchers while investigating the properties of concrete and other materials.

The effect of size on properties of concrete is particularly important if small scale models are to be used to predict the behavior of prototype structures. It can be concluded that since there are a smaller number of flaws in a smaller specimen the strength of the small specimens is on the average larger than that of the larger specimens. Despite the fact that the mean strengths are significantly affected by volume, the influence of size on the minimum strength seems to be quite small [38]. Since in reliability study of concrete elements the lower strength tail is most important it might be acceptable to neglect the effect of volume in probabilistic studies involving strength.

3.2.1.3. **Influence of Rate of Loading**

It has been observed by testing cylinders with different loading rates, that fast loading increases the strength by about 20% while slow loading reduces it by about 20% (see Figure 3.2) [13]. However, in design the decrease in strength caused by long term loading is usually neglected because of the fact that the concrete will typically gain 20 to 40% in strength due to the hydration that occurs after 28-days. It is usually assumed that these two effects will compensate each other, resulting in a conservative assumption.
In the work done by Mirza, Hatzinikolas and MacGregor [38] the mean value for the in-situ compressive strength of concrete at a given rate of loading $R$ (psi/sec) was given by:

$$f_{cstr} = f_{c35}[0.89(1+0.08\log R)] \text{ (psi)} \quad (3.3)$$

where $f_{c35}$ is given by Equation 3.2 and the normal rate of loading for standard cylinder test is approximately 35 psi/sec.

The majority of researchers have represented the distribution of concrete compressive strengths with a normal distribution.
A review of literature indicates that the coefficient of variation of field-cast laboratory-cured specimens is in many cases between 15% and 20%, which suggests that 20% is a reasonable maximum value for average controls. However the standard deviation and the coefficient of variation are not constant for different strength levels so it appears that the average coefficient of variation can be taken as roughly constant at 10%, 15% and 20% for strength levels below 4,000 psi (27.6 MPa) for excellent, average, and poor control, respectively [38]. Based on this result, MacGregor [31] suggested that the coefficient of variation for concrete in a structure should be taken as 0.18.

### 3.3. Initial Tangent Modulus

Although several equations are available in the literature to estimate the static modulus of elasticity of concrete, the available data on the variability of this parameter is limited. If the parabolic stress-strain relationship is used then the initial slope, \( E_{ct} \), is given by:

\[
E_{ct} = 2 \frac{f_c'}{\varepsilon_c'}
\]  
(3.4)

If only the cylinder crushing strength of the concrete is known, then the initial tangent modulus, \( E_{ct} \), can be estimated from following approximate expression [13]:

\[
E_{ct} = 5500 \sqrt{f_c'} \quad \text{(MPa)}
\]  
(3.5)
CHAPTER 3 Review of Statistical Definitions

After determining $E_{ct}$, $\varepsilon_c^*$ can be found and the parabolic equation can then be used. In the same manner the direct cracking strength can be found by [10]:

$$f_{cr} = 0.33 \lambda \sqrt{f_c} \quad \text{(MPa)}$$  \hspace{1cm} (3.6)

where $\lambda$ is a factor accounting for the density of the concrete.

An analysis of test data from the University of Illinois for 139 tests of standard cylinders of normal weight concrete indicated that a high degree of correlation, or approximately 0.9, existed between initial tangent modulus and compressive strength. In the same study, a statistical analysis of the ratios of observed to calculated modulus showed the distribution of the initial tangent modulus of concrete relative to its calculated value can be approximated by a normal distribution. Based on the same data the following relationship for initial tangent modulus was obtained [38]:

$$E_{ct} = 60400 \sqrt{f_c} \quad \text{(psi)}$$  \hspace{1cm} (3.7)

and when the mean value of the initial tangent modulus is estimated from Equation 3.7 the variability relative to the calculated value should be taken as 0.08.

Like the compressive strength, the mean value and dispersion of the modulus of elasticity of concrete are subject to a rate of loading effect. In order to express this effect the following equation can be used [38]:

$$\overline{E}_{ct} = (1.16 - 0.08 \log t) \overline{E}_{c35}$$  \hspace{1cm} (3.8)

where $t$ is the load duration in seconds.
3.4. Reinforcing Steel

Since concrete is very weak in tension it has to be "reinforced" with material which is stronger in tension, like steel bars, wires or welded wire fabric. Because steel is much stronger than concrete even in compression, it can also be used to carry compressive stresses if it is desired to reduce the dimensions of the concrete section.

The reinforcing steel generally in use are hot-rolled and deformed bars and cold-drawn wires. Deformed bars are classified into three grades based on minimum specified yield strength: 300, 350 and 400 MPa where grade 400 bars are the most frequently used type of reinforcement in Canada.

3.4.1. Stress-Strain Response of Reinforcement

Typical stress-strain curves for steel show that the initial stiffness is essentially the same even though the strength will differ a lot especially in case of prestressing steel. The stress-strain relationship for steel bars are normally assumed to be bilinear (see Figure 3.3) where the stress, $f_s$, and the strain caused by this stress, $\varepsilon_{sf}$, can be expressed by following:

$$f_s = E_s \varepsilon_{sf}$$  \hspace{1cm} (3.9)

where $f_s$ is less or equal to $f_y$, and the modulus of elasticity $E_s$ is equal to 200000 MPa. This relationship is assumed to be valid for both tension and compression as illustrated in Figure 3.3 [13].
For strands and wire the response can be approximated by a bilinear relationship, but a more accurate representation of the stress-strain response of prestressing strands can be obtained by using the modified Ramberg-Osgood function [13]. Most frequently used strand is the low-relaxation strand. The response of this type of strand, which has a peak stress $f_{pu} = 1860$ MPa can be described as following [13]:

$$f_p = 200000 \varepsilon_{pf} \left\{ 0.025 + \frac{0.975}{[1+(118\varepsilon_{pf})^{10}]^{1/10}} \right\} \leq 1860 \text{ (MPa)} \quad (3.10)$$
3.4.2. Variability of Mechanical Properties of Reinforcement

To understand the effects of the variability of the strength and geometrical properties of reinforcing steel on the strength of reinforced concrete members, the variability of reinforcing steel needs to be studied.

3.4.2.1. The Yield Strength of Steel

The variability of yield strength depends on the source and nature of the data. The variation in strength within a single bar or strand is relatively small, while for the in-batch variation in a given heat is slightly larger.
CHAPTER 3 Review of Statistical Definitions

When the samples are derived from different batches, from one mill or especially from different mills, there will be significantly more variation [36]. This is expected since rolling practices and quality measures vary for different manufacturers and different bar and strand sizes.

Different values can be obtained for the yield strength depending on how it is defined. The most common definition for yield strength is the static yield strength which is based on the nominal area. The static yield strength seems to be desirable because the strain rate in tests is similar to what is expected in a structure and because designers use the nominal areas in their calculation [36].

The statistics for bars, stirrups and prestressing strands are mostly documented in References 31, 37 or 39. The mean values were found by calculating the ratios between the tabulated specified and the tabulated actual values, which then are used to scale the current design values.

The probability distribution for the yield strength of steel bars and stirrups is assumed to be log-normal [31] but a number of investigators have recommended the use of a normal distribution for higher strength steel such as prestressing strands [39].

3.4.2.2. The Modulus of Elasticity

The modulus of elasticity of steel has been found to have a small dispersion and to be more or less insensitive to the rate of loading or the bar size.

The probabilistic distribution of the modulus of elasticity for reinforcing bars or strands can be considered normal with the actual mean value equal to the specified value and a coefficient of variation 3.3% [39].
3.4.2.3. Variations in Steel Area of Cross Section

The actual areas of reinforcing bars or strands tend to deviate from the nominal areas due to the manufacturing process. Most researchers have indicated that the probability distribution of steel area should be taken as normal. The mean value for the ratio between the measured and nominal areas should be 0.99 with a coefficient of variations 2.4% [36]. However, if the effect of variability in the steel area is considered negligible, Mirza and MacGregor have suggested [36] that a single value of 0.97 could be used. According to MacGregor [31] the ratio between the measured and nominal areas could also be taken to have a mean of 1.0 with a coefficient of variations 6.0%. It should be noted, that the statistics for yield strength are assumed here to reflect the nominal steel area, although in the calculations the actual area was included as a random variable. This was mainly because in the references used it was not clear whether the actual or the nominal area was applied.

3.5. Geometric Properties

Variations in dimensions or geometric imperfections can significantly affect the size and hence the strength of concrete members. These variations and imperfections are caused by deviations from the specified values of the cross-sectional shape and dimensions, the position of reinforcing bars and strands, ties and stirrups, and the grades and surfaces of the constructed structures [35]. There are many reasons for these variations but two of the most important ones are the construction process, e.g. size, shape, and quality of the used forms, and curing of the concrete.
The process of collecting data for statistical purposes has not been standardized yet [35], mainly because it is difficult to compare the results of measurements reported by various researchers, when the quality of construction technique and equipment are different between countries.

Most researchers have indicated that the probability distributions for the geometric properties should be taken as normal [39]. One should keep in mind that all suggestions for geometric properties are based on interpretation of available data and as such they should be considered preliminary [35].
CHAPTER 4

Reliability of Concrete Beams in Bending

4.1. Introduction

To predict the resistance of a concrete beam, the strains in the concrete and reinforcement are assumed to vary linearly such that plane sections remain plane. The compressive stresses in the concrete can be calculated using an appropriate stress-strain relationship, often assumed parabolic as was discussed in Chapter 3. If now the concrete stresses are integrated over the section, an equilibrium can be used to obtain the sectional moment and the axial force. To evaluate the moment-curvature response two methods are most often used:

i. The Layer-by-Layer Approach

ii. The Stress Block Factor Method

were the first one is designed for microcomputers because it involves numerical integration of the stress-strain curves, while the second is more appropriate for programmable calculators or hand calculations.
A beam program, which was developed, uses the Layer-by-Layer approach and a general strain compatibility with the assumed stress-strain relationships, to find the corresponding moment for each curvature step until a decrease in moment with curvature is obtained. The program uses the values before the drop but decreases the curvature step and tries to find a higher solution than the current maximum value by changing the sign of the curvature. The program keeps iterating until it drops again, then it changes the step and the sign of the curvature again. This process is repeated until the program cannot find a higher solution.

In the reliability calculation, the accuracy in evaluating the moment curvature response must be sufficient to permit an accurate calculation of gradient of the performance function. When the program RELAN was linked to the moment-curvature subroutines and the concrete reliability program TIN was created, a major problem was detected in a few runs. This problem indicated that the accuracy in evaluating the moment-curvature response must be sufficient for the program to detect changes in capacity when RELAN was changing the values in order to find the gradient to the failure surface for each variable.

4.2. Flexural Strength of Reinforced Concrete Beams

The assumption that plane section remains plane makes it possible to define the concrete strain with only two variables, i.e. the strain at the top and the bottom face. To define the linear strain distribution for the section the strain at the centroid, $\varepsilon_{cen}$, and the curvature, $\phi$, are used (see Figure 4.1). The curvature is equal to the change in slope per unit length along the section and also the strain gradient over the depth.
If the strain distribution across the section is known, then the assumed stress-strain relationship can be used to find the distribution of stresses across the section and the moment acting at the section can be determined from the equilibrium equations [13].

![Diagram of a cross section and concrete strains](image)

**Figure 4.1: Definition of Sectional Parameters [13]**

**Compatibility conditions:**

The concrete strain at any level \( y \) can be found by:

\[
\varepsilon_c = \varepsilon_{cen} - \phi y
\]  

\( (4.1) \)

The strain in the bars at any level \( y \) is equal to the strain in the surrounding concrete:

\[
\varepsilon_s = \varepsilon_{cen} - \phi y
\]  

\( (4.2) \)
CHAPTER 4 Reliability of Concrete Beams in Bending

The strain in the prestressing tendons at any level $y$ is equal to the strain in the surrounding concrete plus the strain difference, $\Delta \varepsilon_p$, at this level:

$$\varepsilon_p = \varepsilon_{con} - \phi y + \Delta \varepsilon_p$$  

(4.3)

**Equilibrium condition:**

At any section the stresses, when integrated over the section, must add up to the required sectional moment $M$ and the sectional force $N$:

$$\int f_c dA_c + \int f_s dA_s + \int f_p dA_p = N$$  

(4.4)

$$\int f_c y dA_c + \int f_s y dA_s + \int f_p y dA_p = -M$$  

(4.5)

In the equilibrium equations, it has been assumed that tensile strains and stresses are positive and compressive strains and stresses are negative. The axial load, $N$, reacting at the section is taken positive in tension and negative in compression. The curvature, $\phi$, like the moment, $M$, is positive if the section develops tensile stresses at the bottom [13].
4.2.1. Beam Program

What makes it difficult to evaluate the response of a flexural member, is the varying of stresses and strains over the depth of the section. To perform the integrals of Equations 4.4 and 4.5, they can be simplified by assuming that the reinforcing bars and the prestressing tendons consist of a number of discrete elements and their contributions can be replaced by summations:

\[ \int f_c y dA_c + \sum f_y z A_z + \sum f_p y A_p = -M \]  

The force in each bar or tendon is assumed to be equal to the stress at its center times the area. In order to evaluate stresses in the concrete it is also convenient to idealize the cross-section as a series of rectangular layers (see Figure 4.2) and assume that the strain in each layer is uniform and equal to the actual strain at the center of the layer. If the strain is uniform over the layer then the concrete stress will also be uniform over the layer. The force in each layer can now be found by multiplying the stress in the layer by the area of the layer, while the moment contribution can be found by multiplying the layer force by the distance between the middle of the layer and the reference axis [13].
4.2.2. The Stress Block Factor Method

The layer-by-layer approach is a good procedure as an algorithm for microcomputers. However, in cases where cross sections have essentially constant widths, the concrete stress integrals can be efficiently evaluated by using stress-block factors. Instead of using the nonlinear stress distributions, equivalent uniform stress distributions are applied (see Figure 4.3).

For a given compressive stress distribution, the stress-block factors $\alpha_i$ and $\beta_i$ are determined so that the magnitude and location of the resultant force are the same in equivalent uniform stress distribution as in the actual distribution. The requirement that the magnitude of the resultant force remains the same can be described as following [13]:

$$\int_0^c f_c b dy = \alpha_i f_c \beta_i cb$$  \hspace{1cm} (4.7)
Even though the stress block may be imagined to have any convenient shape, the requirement that the location of the resultant force remains the same [13]:

\[
\int_{0}^{c} f_c b dy = c - 0.5\beta_1 c
\]

where \( y \) in this case is measured from the neutral axis (see Figure 4.3).

\[\text{(4.8)}\]

![Figure 4.3: Stress-Block Factor Method [13]](image)

Many researchers have tried to develop an expression to represent the compressive stress-strain response of concrete and a simple parabola has been found to describe it reasonably well.
For such case, i.e. a parabolic stress-strain curve and a constant width, $b$, Equations 4.7 and 4.8 can be reduced to simple expressions listed in Reference [13] by Collins and Mitchell.

The Simplified Stress Block Factor Method or the Code Method, recommends that the actual concrete stress distribution should be taken as equivalent to rectangular concrete stress distribution and the strains in the steel and the concrete are assumed to vary linearly with distance from neutral axis with the maximum compressive strain in the concrete limited to 0.003. The steel stress is taken as $\phi_s f_s = \phi_s E_s \varepsilon_s \leq \phi_s f_y$ and tensile strength of concrete is neglected.

The uniform stress and the depth of stress block recommended is essentially the same as those determined experimentally, where $\alpha_i$ is taken as 0.85. Therefore the uniform stress is taken as $0.85 \phi_c f'_c$ over a depth $a = \beta_i c$ (see Figure 4.3), where $\beta_i$ is taken as 0.85 for concrete strengths $f'_c$ up to and including 30 MPa and, beyond this, it is reduced continuously at a rate of 0.08 for each additional 10 MPa of strength, but with a minimum value for $\beta_i$ of 0.65.

In order to calculate the moment capacity for concrete beam by using the simplified stress block factor method the equilibrium for the section is expressed in terms of two forces i.e. the concrete force, $C$, from the compression zone and the steel force, $T$, in the bars (see Figure 4.3). Those two forces have to be equal so the equilibrium for the section can easily be written as $C=T$ or in terms intervening variables [43]:

$$\alpha_i \phi_c f'_c b \beta_i c = A_s \phi_s f_y$$

\[ (4.9) \]
Based on this the critical moment can be written as:

\[ M_{cr} = A \phi_y f_y \left( d - \frac{a}{2} \right) \]  \hspace{1cm} (4.10)

where \( d \) is the effective depth, \( a = \beta c \) and \( c \) can be found by the formula:

\[ c = \frac{A \phi_y f_y}{\alpha \beta \phi_y f_y b} \]  \hspace{1cm} (4.11)

### 4.3. Design of Concrete Beams in Bending

As a first example, a study of a simple beam is used to explore the general problem and gain an understanding of reliability based design of concrete elements.

#### 4.3.1. Design According to the CSA Code

Let us assume that we are to design a roof of a department store in downtown Ottawa with a parking lot on top. The roof structure, which has a 17 m span, is carried by 500 x 1300 mm simply supported concrete beams with 5 m spacing, and 210 mm thick slab (see Figure 4.4).
In addition to the dead load, the building has to be designed, for 5.0 kN/m² service load and 2.5 kN/m² snow load. If we now design the beam in the roof structure, the applied load can be calculated with the values listed in Table 4.1, where the factored load, $q_f$, is given by:

$$ q_f = 1.25D_n + 1.5(L_n + S_n) $$  \hspace{1cm} (4.12)  

and the maximum moment, $M_{\text{max}}$, is:

$$ M_{\text{max}} = \frac{1}{8} q_f L^2 $$  \hspace{1cm} (4.13)  

Figure 4.4: Design of Concrete Beam in Bending
Table 4.1 Material Factors and Nominal Values

<table>
<thead>
<tr>
<th>Definition of Variables</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_c$ - Material Factor for Concrete</td>
<td>0.6</td>
</tr>
<tr>
<td>$\phi_s$ - Material Factor for Steel</td>
<td>0.85</td>
</tr>
<tr>
<td>$\alpha_1$ - Concrete Stress Block Factor</td>
<td>0.85</td>
</tr>
<tr>
<td>$\beta_1$ - Concrete Stress Block Factor</td>
<td>0.816</td>
</tr>
<tr>
<td>$f'_c$ - Concrete Compressive Strength (MPa)</td>
<td>35.0</td>
</tr>
<tr>
<td>$f_s$ - Steel Yield Strength (MPa)</td>
<td>400.0</td>
</tr>
<tr>
<td>$b$ - Beam width (mm)</td>
<td>500.0</td>
</tr>
<tr>
<td>$h$ - Beam height (mm)</td>
<td>1300.0</td>
</tr>
<tr>
<td>$L$ - Beam Span (m)</td>
<td>17.0</td>
</tr>
<tr>
<td>$D_n$ - Dead Load (kN/m)</td>
<td>42.5</td>
</tr>
<tr>
<td>$L_n$ - Service Load (kN/m)</td>
<td>25.0</td>
</tr>
<tr>
<td>$S_n$ - Snow Load for Ottawa (kN/m)</td>
<td>12.5</td>
</tr>
</tbody>
</table>

By following the design procedure step by step, and the preceding equations and design values used, the amount of reinforcement needed for the beam is 12 No. 35 bars (see Appendix B).

4.3.2. Development of the User Subroutines for RELAN

As mentioned earlier, to perform a reliability calculation by RELAN, it must be supplemented by a description of the performance function and its gradient with respect to the intervening random variables.
In order to do so, the user has to provide four subroutines, i.e. DETERM, GFUN, DFUN and D2FUN. Despite the importance of all the subroutines, GFUN can be accounted to be the most important, as GFUN contains the performance function which is the core of the reliability calculation. If the first and the second order derivatives are computed numerically by RELAN, DFUN and D2FUN are not needed.

Let us now look at the designed beam in Section 4.3.1 and Appendix B, to establish the performance function. The actual applied moment, $M_{act}$, which the simply supported beam has to support can be written:

$$M_{act} = \frac{1}{8} (D_{act} + S_{act} + L_{act}) L^2$$  (4.14)

which can also be written in terms of ratios between the nominal and the actual loads:

$$M_{act} = \frac{S_n L^2}{8} \left[ \left( \frac{D_{act}}{D_n} \right) \left( \frac{S_n + L_n}{S_n} \right) + \left( \frac{S_{act}}{S_n} \right) + \left( \frac{L_{act}}{L_n} \right) \left( \frac{L_n}{S_n} \right) \right]$$  (4.15)

or in a simplified way:

$$M_{act} = \frac{S_n L^2}{8} (D_r \gamma \delta + S_r + L_r \varepsilon)$$  (4.16)

where:

$$D_r = \frac{D_{act}}{D_n} \quad S_r = \frac{S_{act}}{S_n} \quad L_r = \frac{L_{act}}{L_n} \quad \gamma = \frac{D_n}{S_n + L_n} \quad \delta = \frac{S_n - L_n}{S_n} \quad \varepsilon = \frac{L_n}{S_n}$$
CHAPTER 4 Reliability of Concrete Beams in Bending

If we now recall the fundamental formulation in Chapter 2 of the performance function, i.e. \( G = R - D \), we can write the performance function for the beam case as following:

\[
G = M_{\text{cap}} - \frac{S_{n}L^{2}}{8} (D_{r} \gamma \delta + S_{r} + L_{r} \varepsilon)
\]  
(4.17)

where \( M_{\text{cap}} \) is the calculated theoretical strength of the beam.

According to the CSA code, the maximum moment which the beam can sustain is:

\[
M_{\text{max}} = \frac{S_{n}L^{2}}{8} (1.25 \gamma \delta - 1.5 \varepsilon)
\]  
(4.18)

and the critical moment, \( M_{\text{cr}} \), in Equation 4.10 can also be written as:

\[
M_{\text{cr}} = \alpha \phi f_{c} b \beta c \left( d - \frac{a}{2} \right)
\]  
(4.19)

If the critical and the maximum moment are set equal, the performance function becomes:

\[
G = M_{\text{cap}} - \frac{\alpha \phi f_{c} b \beta c \left( d - \frac{a}{2} \right)}{(1.25 \gamma \delta + 1.5 \varepsilon)} (D_{r} \gamma \delta + S_{r} + L_{r} \varepsilon)
\]  
(4.20)
4.3.3. Variability of Intervening Variables

One of the most important factors in the reliability calculation, is the variability of the intervening variables. The statistical data, which will be reviewed in following sections, is more or less based on definitions in Chapter 3.

4.3.3.1. Material Statistics

The statistical data used for the beam example are listed in Table 4.2. It should be noted that because of lack of information, the mean value of the concrete compression strength is only adjusted for the rate of loading and the gain in strength that occurs with time has been ignored (see Table 4.2).

<table>
<thead>
<tr>
<th>Definition of Variables / Units</th>
<th>Mean</th>
<th>COV</th>
<th>Distribution/Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c$ - Compression Strength (MPa)</td>
<td>27.77</td>
<td>0.18</td>
<td>Normal / [38]</td>
</tr>
<tr>
<td>$E_{ct}$ - Concrete Stiffness (MPa)</td>
<td>24529.78</td>
<td>0.08</td>
<td>Normal / [38]</td>
</tr>
<tr>
<td>$f_y$ - Steel Yield Strength (MPa)</td>
<td>445.34</td>
<td>0.093</td>
<td>Log-Normal / [37]</td>
</tr>
<tr>
<td>$E_s$ - Steel Modulus of Elasticity (MPa)</td>
<td>200000.0</td>
<td>0.033</td>
<td>Normal / [37]</td>
</tr>
<tr>
<td>$\phi_{st}$ - Diameter of Stirrups (mm)</td>
<td>11.19</td>
<td>0.024</td>
<td>Normal / [36]</td>
</tr>
<tr>
<td>$\phi_{bar}$ - Diameter of Steel Bars (mm)</td>
<td>35.34</td>
<td>0.024</td>
<td>Normal / [36]</td>
</tr>
<tr>
<td>$b$ - Beam Width (mm)</td>
<td>501.52</td>
<td>0.013</td>
<td>Normal / [37]</td>
</tr>
<tr>
<td>$h$ - Beam Height (mm)</td>
<td>1301.52</td>
<td>0.005</td>
<td>Normal / [37]</td>
</tr>
<tr>
<td>$C_c$ - Concrete Cover (mm)</td>
<td>48.38</td>
<td>0.087</td>
<td>Normal / [37]</td>
</tr>
<tr>
<td>$S_c$ - Spacing between Layers (mm)</td>
<td>25.0</td>
<td>0.050</td>
<td>Normal</td>
</tr>
</tbody>
</table>
4.3.3.2. Load and Fitted Distributions

All statistical data for the loads are listed in Table 4.4, where the dead load, $D$, is described with normal distribution, while the maximum annual snow load, $S$, and the maximum annual service load, $L$, is described with Gumbel extreme type I distribution.

According to the CSA code, the 30 years return snow load is used in design, but because the probability of getting 30 years snow and service load at the same time is rather low, the max annual service load is used with the 30 years snow load. The basic Gumbel extreme type I distribution is written as following:

$$F(x) = \exp\left\{-\exp[-A(x-B)]\right\} \quad (4.21)$$

and by rearranging the formula and solve for $x$ we get:

$$x = B + \frac{\ln\left\{\ln\left(\ln\left(F(x)\right)\right)\right\}}{A} \quad (4.22)$$

where:

$$A = \frac{\pi}{\sqrt{6}\sigma} \quad \text{and} \quad B = \bar{x} - \left(\frac{0.577}{A}\right)$$

To be able to describe the distribution of maximum load in N-years, we need to expand Equation 4.22:

$$x_N = B + \frac{\ln N - \ln\left\{\ln\left(F(x)\right)\right\}}{A} \quad (4.23)$$
The 30 years return snow load, will correspond to a probability of non-performance of \( F(x) = 29/30 \) in Equation 4.22. The coefficients \( A \) and \( B \) are given in Table 4.3. Using Equation 4.23 for \( N = 30 \), the ratio \( S_r \) can then be expressed as:

\[
S_r = B^* + \frac{\ln(\ln(F(x)))}{A^*}
\]

where:

\[
A^* = AB + 3.3843 \quad \text{and} \quad B^* = \frac{AB + \ln N}{AB + 3.3843}
\]

If we use this now, to find the distribution of maximum load for Ottawa and Vancouver we get following values:

Table 4.3 Extreme Parameters

<table>
<thead>
<tr>
<th></th>
<th>Mean kN/m²</th>
<th>COV</th>
<th>A</th>
<th>B</th>
<th>A*</th>
<th>B*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ottawa</td>
<td>1.255</td>
<td>0.452</td>
<td>2.260</td>
<td>1.00</td>
<td>5.644</td>
<td>1.003</td>
</tr>
<tr>
<td>Vancouver</td>
<td>0.523</td>
<td>1.202</td>
<td>2.039</td>
<td>0.24</td>
<td>3.874</td>
<td>1.004</td>
</tr>
</tbody>
</table>

This can now be used to find the mean and the corresponding standard deviation for \( S_r \), which is the ratio between the actual and nominal value, where in case of the distribution of maximum load in N-years, the standard deviation can be found by:

\[
\sigma = \frac{\pi}{\sqrt{6A^*}}
\]

(4.25)
By using corresponding values for Equation 4.24 and Equation 4.25, and the relationship $\sigma = \bar{x}COV$, we get the mean and the covariance values listed in Table 4.4.

The parameters $A$ and $B$ for the maximum annual service load, $L$, can be found by using the following relationship from Equation 4.22, assuming that the design load $L_n$ is also a 30 year return value:

$$5.0 = B + \frac{\left\{ \ln\left( -\ln\left(\frac{29}{30}\right) \right) \right\}}{A}$$

Also, assuming that the COV of $L$ is 0.25 (see Table 4.4), we can write:

$$COV_x B + \frac{COV_x 0.5772}{A} = \frac{\pi}{\sqrt{6A}} \Rightarrow \frac{\sqrt{V}}{A} = 0.2196B$$

and therefore the extreme parameters $A$ and $B$ become:

$$B = 2.868, A = 1.588$$

which gives us the equation for maximum annual service load, i.e.:

$$L = 2.868 + \frac{\left( -\ln\left( -\ln\left( F(x) \right) \right) \right)}{1.588}$$
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This equation can then be expressed in terms of the ratio between the actual and nominal load as:

\[ L_v = 0.5736 + \frac{\left(-\ln(-\ln(F(x)))\right)}{7.936} \]

<table>
<thead>
<tr>
<th>Table 4.4 Intervening Load Random Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definition of Variables / Units</td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td>( D_r )-Actual-Nominal Dead load Ratio</td>
</tr>
<tr>
<td>( S_r )-Actual-Nominal Snow load Ratio</td>
</tr>
<tr>
<td>( L_r )-Actual-Nominal Live load Ratio</td>
</tr>
</tbody>
</table>

4.3.4. Example Runs

By performing a FORM calculation with TIN, which is a product of the theoretical Beam subroutines and RELAN, and by formulating the performance function as in Equation 4.20 with 0.9 correlation between the concrete compression strength and the initial tangent modulus, we get following results from RELAN:

<table>
<thead>
<tr>
<th>Table 4.5 RELAN Results: Code Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>( \beta )-Reliability Index (FORM)</td>
</tr>
<tr>
<td>( P_f )-Probability of Failure (FORM)</td>
</tr>
</tbody>
</table>
Instead of using Equation 4.26 to describe the performance function, we can write it as function of the span or as in Equation 4.23:

\[ G = M_{cap} - \frac{S_n L^2}{8} (D_r \gamma \delta + S_r + L, \varepsilon) \]

which should give us a slightly higher reliability index, because the maximum moment, \( M_{max} \), is now the exact value where in the other case we use the design values, which are always conservative. Now by running TIN, where the performance is a function of the exact maximum moment, we get following results from RELAN:

<table>
<thead>
<tr>
<th>Product</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )-Reliability Index (FORM)</td>
<td>4.270</td>
</tr>
<tr>
<td>( P_f )-Probability of Failure (FORM)</td>
<td>0.977E-5</td>
</tr>
</tbody>
</table>

One very important product of RELAN are the sensitivity factors. They can indicate what mode of failure might be expected at each time, and for that reason they play a major roll in the reliability design.
The two preceding examples give nearly the same sensitivity factors, which was expected, because there was only a slight difference in the reliability index. If now the sensitivity factors in Table 4.7, which are good representatives of both the runs, are studied and the load sensitivity factors are excluded, it can be seen that the compression strength of the concrete influences the reliability of the beam the most. This means that if the actual load exceeds the design load we might expect a compression failure.

<table>
<thead>
<tr>
<th>Definition of Variables / Units</th>
<th>Mean</th>
<th>Sensitivity factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c'$ - Compression Strength (MPa)</td>
<td>27.77</td>
<td>0.515</td>
</tr>
<tr>
<td>$E_o$ - Concrete Stiffness (MPa)</td>
<td>24529.78</td>
<td>0.111</td>
</tr>
<tr>
<td>$f_y$ - Steel Yield Strength (MPa)</td>
<td>445.34</td>
<td>0.641E-5</td>
</tr>
<tr>
<td>$E_s$ - Steel Modulus of Elasticity (MPa)</td>
<td>200000.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$\phi_u$ - Diameter of Stirrups (mm)</td>
<td>11.19</td>
<td>0.366E-2</td>
</tr>
<tr>
<td>$\phi_{bar}$ - Diameter of Steel Bars (mm)</td>
<td>35.34</td>
<td>0.974E-1</td>
</tr>
<tr>
<td>$b$ - Beam Width (mm)</td>
<td>501.52</td>
<td>0.410E-1</td>
</tr>
<tr>
<td>$h$ - Beam Height (mm)</td>
<td>1301.52</td>
<td>0.863E-1</td>
</tr>
<tr>
<td>$c$ - Concrete Cover (mm)</td>
<td>48.38</td>
<td>0.595E-1</td>
</tr>
<tr>
<td>$D_r$ - Actual-Nominal Dead load Ratio</td>
<td>1.0</td>
<td>0.309</td>
</tr>
<tr>
<td>$S_r$ - Actual-Nominal Snow load Ratio</td>
<td>1.105</td>
<td>0.286</td>
</tr>
<tr>
<td>$L_r$ - Actual-Nominal Live load Ratio</td>
<td>0.647</td>
<td>0.706</td>
</tr>
</tbody>
</table>
4.4. Effect of Span on Beam Reliability

The program RELAN can be used to analyze the same cross-section, under the same loads, for different spans. Let us now use Equation 4.17 as performance function and run the program for different spans.

Figure 4.5: Changes in Safety for Different Spans

On the left side of the original design span we get concrete compression failure, but by increasing the span the sensitivity of the concrete becomes less and less important until we get a combination of tension and compression failure after 18 m.
Chapter 4 Reliability of Concrete Beams in Bending

The shift in the failure mode could be for the reason that the section is close to the balance point. For 14 m span, the sensitivity to compression strength was 0.965 while for steel yield strength is was 0.0. On the other hand, at 20 m span, the corresponding sensitivities were 0.402 and 0.504. It can be concluded from Figure 4.5, that by adding steel to the tension part of the section, when the span is less than 18 m, the reliability of the beam is not going to be substantially affected. By performing this type of calculation, it is possible to make an economical design, and also prevent certain failures, which can be drastic like the compression failure, and danger to human lives.

The way this is normally dealt with in design, as explained earlier, is to have criteria for maximum and minimum reinforcement.

From this it can be seen, that reliability calculation cannot only be used for risk assessments, but also as a tool for engineers to understand how different conditions can affect the overall behavior of the section.

4.5. Effect of Steel Ratio on Beam Reliability

Most concrete codes have criteria for minimum and maximum reinforcement in concrete members in order to obtain a ductile failure. The reason for minimum reinforcement ratio is to avoid a sudden tension failure of an element. The specified minimum ratio, $\rho_{\text{min}}$, according to CSA A23.3-M84 (10.5) for a member subjected to bending is given by following ratio [43]:

$$\rho_{\text{min}} = \frac{14}{f_y}$$

(4.26)
This gives roughly the steel area required to have a strength equal to the cracking moment of an identical plain concrete section.

The maximum reinforcement ratio on the other hand is to ensure that the beam reinforcement will yield prior to the concrete crushing. When the ratio is close to the upper limit the sections tend to have to small effective depth and therefore a problem with deflections. Too high reinforcement ratio may also result in a compression failure. The specified maximum ratio according to CSA A23.3-M84 (10.3.3) is given by following [43]:

\[
\frac{c}{d} \leq \frac{600}{600 + f_y}
\]  

(4.27)

Limitations of deflections, convenience in placement of reinforcement and economy in design generally dictate larger overall beam dimensions with correspondingly lower reinforcement ratios, usually in the range of 30-40% of the maximum limit. These lower reinforcement ratios result in further improvement in the ductility of beams.

Let us now look at the same beam but change the reinforcement ratio, in order to study the different failure modes which occur, and also to see how effective the code limits are.
By running TIN, with both Vancouver and Ottawa snow load, for different steel ratios, we get the following:

**Table 4.8 RELAN Results: Different Steel Ratios**

<table>
<thead>
<tr>
<th>Layer/No. bars</th>
<th>As (mm²)</th>
<th>Def (mm)</th>
<th>Ottawa β</th>
<th>Ottawa Pf</th>
<th>Vancouver β</th>
<th>Vancouver Pf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/3</td>
<td>3000</td>
<td>1230.85</td>
<td>3.129</td>
<td>0.878E-3</td>
<td>3.034</td>
<td>0.121E-2</td>
</tr>
<tr>
<td>1/6</td>
<td>6000</td>
<td>1230.85</td>
<td>3.831</td>
<td>0.637E-4</td>
<td>3.750</td>
<td>0.885E-4</td>
</tr>
<tr>
<td>2/1</td>
<td>1000</td>
<td>1222.18</td>
<td>4.074</td>
<td>0.231E-4</td>
<td>3.926</td>
<td>0.432E-4</td>
</tr>
<tr>
<td>2/2</td>
<td>2000</td>
<td>1215.67</td>
<td>4.168</td>
<td>0.154E-4</td>
<td>4.060</td>
<td>0.245E-4</td>
</tr>
<tr>
<td>2/3</td>
<td>3000</td>
<td>1210.62</td>
<td>4.276</td>
<td>0.949E-5</td>
<td>4.172</td>
<td>0.151E-4</td>
</tr>
<tr>
<td>2/4</td>
<td>4000</td>
<td>1206.57</td>
<td>4.375</td>
<td>0.608E-5</td>
<td>4.287</td>
<td>0.905E-5</td>
</tr>
<tr>
<td>2/5</td>
<td>5000</td>
<td>1203.26</td>
<td>4.369</td>
<td>0.624E-5</td>
<td>4.247</td>
<td>0.108E-4</td>
</tr>
<tr>
<td>2/6</td>
<td>6000</td>
<td>1200.5</td>
<td>4.247</td>
<td>0.108E-4</td>
<td>4.164</td>
<td>0.156E-4</td>
</tr>
<tr>
<td>3/1</td>
<td>1000</td>
<td>1193.5</td>
<td>3.943</td>
<td>0.402E-4</td>
<td>3.928</td>
<td>0.429E-4</td>
</tr>
<tr>
<td>3/2</td>
<td>2000</td>
<td>1187.49</td>
<td>3.798</td>
<td>0.731E-4</td>
<td>3.779</td>
<td>0.787E-4</td>
</tr>
<tr>
<td>3/3</td>
<td>3000</td>
<td>1182.29</td>
<td>3.672</td>
<td>0.120E-3</td>
<td>3.653</td>
<td>0.130E-3</td>
</tr>
<tr>
<td>3/4</td>
<td>4000</td>
<td>1177.74</td>
<td>3.547</td>
<td>0.195E-3</td>
<td>3.518</td>
<td>0.218E-3</td>
</tr>
<tr>
<td>3/5</td>
<td>5000</td>
<td>1173.72</td>
<td>3.442</td>
<td>0.288E-3</td>
<td>3.427</td>
<td>0.305E-3</td>
</tr>
<tr>
<td>3/6</td>
<td>6000</td>
<td>1170.15</td>
<td>3.330</td>
<td>0.434E-3</td>
<td>3.321</td>
<td>0.448E-3</td>
</tr>
<tr>
<td>4/3</td>
<td>3000</td>
<td>1152.81</td>
<td>3.100</td>
<td>0.966E-3</td>
<td>3.084</td>
<td>0.102E-2</td>
</tr>
<tr>
<td>4/6</td>
<td>6000</td>
<td>1139.8</td>
<td>2.940</td>
<td>0.164E-2</td>
<td>2.900</td>
<td>0.187E-2</td>
</tr>
</tbody>
</table>

The results listed in Table 4.8, which are also illustrated in Figure 4.6, show that there is a drastic change in the reliability at certain points, which basically means that we are observing different failure modes.
However, because the variability of intervening variables such as the compression strength affects the failure pattern, and the code does not involve the probability directly, the criteria for maximum and minimum reinforcement may not serve their purpose.

**Figure 4.6: Changes in Safety for Different Steel Ratios**

The cumulative distribution curve can also be plotted from the data in Table 4.8 (see Figure 4.7).
By fitting some known distribution through the data in Figure 4.7 and finding the derivative of the function, the probability density curve can be established.

By looking at the sensitivity factors, which are listed in Table 4.9 and illustrated in Figures 4.8 and Figure 4.9, it can be seen that there is a shift from tension failure at point, f, to combination of both tension and compression failure at point, g, to pure compression failure at point, i. The limit for the maximum reinforcement in the code (see Figure 4.6), is to the right of point, i, which means that the code does not prevent compression failure in this case.
CHAPTER 4 Reliability of Concrete Beams in Bending

Table 4.9 Sensitivity Factors: Different Steel Ratios

<table>
<thead>
<tr>
<th>Definition of Variables / Units</th>
<th>Points on Curve (see figure 4.6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>f</td>
</tr>
<tr>
<td>$f_c'$-Compression Strength (MPa)</td>
<td>0.309E-2</td>
</tr>
<tr>
<td>$E_c$-Concrete Stiffness (MPa)</td>
<td>0.495E-3</td>
</tr>
<tr>
<td>$f_y$-Steel Yield Strength (MPa)</td>
<td>0.616</td>
</tr>
<tr>
<td>$E_s$-Steel Modulus of Elasticity (MPa)</td>
<td>0.225E-4</td>
</tr>
<tr>
<td>$\phi_s$-Diameter of Stirrups (mm)</td>
<td>0.166E-2</td>
</tr>
<tr>
<td>$\phi_{bar}$-Diameter of Steel Bars 1.st layer (mm)</td>
<td>0.194</td>
</tr>
<tr>
<td>$\phi_{bar}$-Diameter of Steel Bars 2.nd layer (mm)</td>
<td>0.124</td>
</tr>
<tr>
<td>$\phi_{bar}$-Diameter of Steel Bars 3.rd layer (mm)</td>
<td>0.0</td>
</tr>
<tr>
<td>$b$-Beam Width (mm)</td>
<td>0.126E-3</td>
</tr>
<tr>
<td>$h$-Beam Height (mm)</td>
<td>0.331E-1</td>
</tr>
<tr>
<td>$C_c$-Concrete Cover (mm)</td>
<td>0.269E-1</td>
</tr>
<tr>
<td>$D_r$-Actual-Nominal Dead load Ratio</td>
<td>0.283</td>
</tr>
<tr>
<td>$S_r$-Actual-Nominal Snow load Ratio</td>
<td>0.256</td>
</tr>
<tr>
<td>$L_r$-Actual-Nominal Live load Ratio</td>
<td>0.648</td>
</tr>
</tbody>
</table>

One of the explanations why the code gives an unconservative estimation for the limit of tension-compression failure in this case, could be the code assumes that the actual compression strength which we normally get is higher than the specified value, while according to Mirza, Hatzinikolas and MacGregor [38] the actual mean value for compression strength (see Table 4.2) is assumed to be lower than the specified value (see Table 4.1).
It is interesting to see in Figure 4.8 and Figure 4.9, how sudden the shift is in the variable sensitivity, and also how clear difference is between different modes, i.e. tension and compression failure.

![Figure 4.8: Changes in Material Sensitivity](image)

The cause of the relatively high sensitivity of the concrete compression strength and this sudden shift between failure modes could be, as was mentioned earlier, because by using Equation 3.3 and ignoring the increase due to hydration we might get a conservative estimation of the mean value for concrete compression strength.
Further study on concrete compression strength seems to be needed in order to develop appropriate reliability based limits for maximum reinforcement and prevent this premature failure mode.

Figure 4.9: Changes in Load Sensitivity
5.1. Introduction

One of the major factors in reliability study of concrete offshore structures, are the uncertainties associated with extreme environmental load conditions, such as iceberg impact, imposed on the offshore structure. Because ice is not really an isotropic material, even though it could appear so, the issue of predicting ice forces on structures essentially reduces to what at first sight appears to be a relatively simple problem, that of understanding how ice deforms and fails under stress. The deformation and strength properties of ice are affected by two major characteristics i.e. temperature and brittleness. For the reason ice is an extremely brittle material it cannot resist tension very well which makes it ease for cracks to propagate. At the same time ice is a solid close to its melting point, and therefore it exhibits creep and its compressive strength is temperature dependent. This explains the various shapes of offshore structures because what designers are interested in is reducing the impact load as much as possible by failing the ice where it is weakest. The failure which the ice undergoes during an impact is crushing-, tension- and flexural failure (see Figure 5.1).
During the past two decades our understanding of these aspects of ice mechanics has advanced considerably. But despite extensive research and well quantified laboratory tests of response of ice to stress, problems remain in extrapolating from the knowledge of small-scale ice properties to a prediction of how ice behaves on a large scale. Because of this difficulty and lacking full scale tests there is no generally accepted methods for predicting ice loads on structures.

One way to predict ice impacts on offshore structures is to use probabilistic programs. Most of them approach the ice impact by using either simulation process like Monte Carlo (see Figure 5.2) or FORM/SORM procedures.
ICE ENVIRONMENT FOR THE DEVELOPMENT LOCATION

STRUCTURAL DESIGN CONCEPT

ICE LOADING SCENARIOS
\[ i = 1, \ldots, n \]

SELECT SCENARIO \( i \)

LOAD PROCESS CONSIDERING UNCERTAINTY IN CONCENTRATION

\[ \text{LOADING EVENTS} \]

\[ \text{TIME} \]

\( i = i + 1 \)

\[ \text{ICE FEATURE SIZE MORPHOLOGY} \]
\[ \text{ENVIRONMENTAL DRIVING FORCES} \]
\[ \text{MECHANICAL PROPERTIES OF ICE} \]

PROBABILITY
\[ \text{e.g. ICE THICKNESS} \]

PROBABILITY
\[ \text{e.g. ICE VELOCITY} \]

PROBABILITY
\[ \text{e.g. AVE. ICE PRESSURE} \]

\[ i < n \]

EMPLOY EXTREMAL ANALYSIS TO ASSIGN PROBABILITY DISTRIBUTIONS FOR ICE LOADINGS ON THE STRUCTURE

PROBABILITY / ANNUAL MAXIMUM ICE LOAD

PROVIDE ICE LOADS FOR DESIGN TO COMPARE WITH STRUCTURAL RESISTANCE

Figure 5.2: Flow Chart for Probabilistic Approach for Ice Load
5.2. Icebergs and Multi-Year Floes

Icebergs are not frozen sea water as one might think but are composed of freshwater ice from land-based glaciers flowing off the land into the sea. Glacier ice develops from successive snowfalls of pure freshwater snow which compress under their own weight until they become solid ice (see Figure 5.3).

The term "Multi-year ice", which stands basically for frozen sea water, is normally defined to be ice which has survived at least two summer seasons and is formed from second-year ice by continuing dynamic action and by melting and refreezing (see Figure 5.3).

![Diagram of Icebergs and Multi-Year Floes]

*Figure 5.3: Icebergs and Multi-Year Floes [47]*
5.3. Dynamic Impact from Icebergs and Floes

No full-scale measurements have ever been made of forces exerted during impact of an iceberg with a structure. However, the problem would seem not to be very substantially different from that of impact of very thick multi-year floe with a structure. In both cases we expect progressive failure of ice over a steadily growing contact zone, and we expect the ice mass to come to rest when all its energy has been dissipated by the failure process. The principal differences between iceberg impact and ice floe impact are following:

i. An iceberg is generally free to rotate about all three axes during impact, though depending on the size of the iceberg, while an ice floe is typically able to rotate significantly only in horizontal plane, about the vertical axis. Because there are so few data on which to base analysis Sanderson [47] proposed that all rotational components of motion should be neglected.

ii. The contact zone of an iceberg with a structure typically grows progressively in two dimensions rather than just one, i.e. the contact width and the contact depth both grow as penetration proceeds.

iii. The failure modes of the ice may be rather different. In most cases full-thickness flexural failure is unlikely to occur during iceberg impact and we might also expect a higher degree triaxial confinement during crushing of an impacting iceberg.
5.4. Evaluation of the Ice Impact with Energy Principles

When a multi-year floe or iceberg impacts with a structure it continues moving until all its kinetic energy is dissipated. If the floe or the iceberg has a mass, $m$, and initial velocity, $V$, then it will come to rest at total penetration, $x$, when [11]:

$$\sqrt{2m(1+C_m)V^2} = \int_0^x p(x)dx$$  \hspace{1cm} (5.1)

This simple energy model, based on the formulation given by Johnson and Nevel (1985), assumes that the total kinetic energy of the ice feature is observed in the progressive crushing of the ice contact zone. The kinetic energy, $E_k$, of the ice feature is given by the equation:

$$E_k = (1+C_m)mV^2 / 2$$  \hspace{1cm} (5.2)

where $C_m$ is added mass factor obtained by Croasdale and Marcellus (1981):

$$C_m = 0.9h / (2z - 0.9h)$$  \hspace{1cm} (5.3)

and $z$ is the water depth.

Also given by Rothrock and Thorndike (1984) the area of typical floe is related to mean caliper diameter, $D$, by the approximate formula [47]:

$$A = 0.66D^2$$  \hspace{1cm} (5.4)
From this we can say that the mass, \( m \), is:

\[
m = 0.66D^2 h \rho_i \tag{5.5}
\]

where \( h \) is the ice thickness and \( \rho_i \) the density. Now we can write following expression:

\[
\sqrt{\frac{1}{2} 0.66D^2 h \rho_i (1 + C_m) V^2} = \int_0^x p(x) \, dx \tag{5.6}
\]

The impact load, \( p(x) \), is defined as:

\[
p(x) = A(x) \sigma_c \tag{5.7}
\]

where \( A(x) \) is the contact area and \( \sigma_c \) is the unconfined compressive strength of ice. Now we can say that the energy dissipated during crushing based on above will then become [11]:

\[
E_c = \int_0^x p(x) \, dx \tag{5.8}
\]
Through energy principles we know that internal work is the same as the external work i.e.:

\[ E_k = E_c \]  \hspace{1cm} (5.9)

and from that we can get the maximum penetration and the maximum impact load.

5.4.1. Calculation of the Ice Contact Area

To calculate the ice impact and the contact area for different velocity, diameter, thickness or compressive strength we need to come up with an equation where all these variables are introduced. The only complications are how we formulate the changes in the contact area for a particular ice feature and for a particular structure because as stated earlier the contact width and the contact depth both grow as penetration proceeds.

Let's now look at two different structures (see Figure 5.4), i.e. a cylindrical structure and a structure with multiple wedge-shaped indentors assuming a wedge-shaped ice feature, as an example to see how we can establish the impact force. The mathematical expressions for those two cases, i.e. the cylindrical and the multiple wedge-shaped structure, can now be established easily from Figure 5.4.
CHAPTER 5 Offshore Structure Ice Impact

Cylindrical structure:

\[ A_c(x) = 2LR \sin \phi + \tan \alpha + \tan \beta \left( \phi \sin \phi \cos \phi \right) R^2 \]  \hspace{1cm} (5.10)

and:

\[ \cos \phi = \frac{(R - x)}{R} \]  \hspace{1cm} (5.11)

where \( R \) is the radius of the structure, \( L \) the thickness of the ice and \( x \) the penetration (see Figure 5.4).
Multiple wedge-shaped indentors:

\[ A_n(x) = (2L + x \tan \alpha + x \tan \beta)(x / \cos \gamma) \]  

(5.12)

where L the thickness of the ice and x the penetration (see Figure 5.4).

It should be noted that the angles at the top and the bottom of the ice feature could be kept as variables but in order to simplify calculation later they will be assumed to be constants.

5.4.2. The Ice Compressive Strength

Observation by Sanderson [47] concluded that the upper bound of data collected at Tarsiut P-45 in 1984-1985 appeared to depend on inverse square root of area, \( 1/\sqrt{A} \), and the fact that theory would lead us to suppose that indeed it should do so, suggests that a normalization of these data can be carried out. This means that we can normalize all pressure measurements by the inverse square root of contact area, \( A(x) \), and express then relative to a single "reference contact area", \( A_0 \). This means that for any measurement of stress, \( \sigma \), over an area, \( A \), a normalized stress, \( \sigma^* \), can be calculated over the reference area using the assumption of inverse square root dependence on area. The expression for the normalized stress is as following:

\[ \sigma^* = \sqrt{\frac{A(x)}{A_0}} \sigma_c \]  

(5.13)
Based on the same data it was concluded that a mean normalized indentation stress, $\sigma_m^*$, is equal to 0.92 MPa with a standard deviation of, $\sigma_x^*$, equal to 0.45 MPa (see Figure 5.5).

\[ \sigma_c = \sigma_m^* \sqrt{\frac{A}{A(x)}} \lambda_m \]  

(5.14)

Figure 5.5: Indentation Pressure at Peak Load [47]

As mentioned before the observation was based on upper bound data and because normally the mean stress represents the actual compressive strength better Sanderson suggested that factor, $\lambda_m$, would be used in order to express the average ratio of mean load to peak load. Based on this the compressive strength becomes:
5.5. Ice Impact Force for the Hibernia Structure

As an example let's look at the case which was investigated for the Hibernia Development Project in offshore Newfoundland by applying the energy theory [11]. By assuming that the unconfined compressive strength of the ice is constant the energy dissipation formula becomes:

\[
E_c = \lambda_m \sigma_m^* \sqrt{A_0 \int_0^{x_m} (2L + x \tan \alpha + x \tan \beta)(x / \cos \gamma) dx}
\] (5.15)

This leads then to the final equation by using energy principles:

\[
E_k = \lambda_m \sigma_m^* \sqrt{A_0 \int_0^{x_m} (2L + x \tan \alpha + x \tan \beta)(x / \cos \gamma) dx}
\] (5.16)

where:

\[
E_k = \sqrt{0.66D^2h\rho_l(1+C_m)V^2}
\] (5.17)

and the by carrying out the integration we can find the \(x_m\), which is the maximum penetration and then find the contact area, \(A(x)\), which leads us to the maximum impact load, \(p(x)\) [47]:

\[
p(x) = \sqrt{A(x)A_0 \sigma_m^* \lambda_m}
\] (5.18)
For the Hibernia structure the angle, $\gamma$, is $45^\circ$ so the energy formula can be written as following:

$$\sqrt{2} 0.66D^2h \rho_i (1 + C_m)V^2 = \lambda_m \sigma_m^* \sqrt{A_0} \int_0^{x_m} \sqrt{2(2Lx + (\tan \alpha + \tan \beta)x^2)} dx$$  \hspace{1cm} (5.19)

### 5.6. Reliability Based Formulation of Ice Impact

The next step, in order to estimate the probability distribution of ice load, is to evaluate the integral in the energy equation by using numerical methods. The objective is to fix the basic variables, i.e. the diameter, the thickness, the velocity and the compressive strength of the ice feature, to compute the corresponding maximum ice load and use RELAN to establish the corresponding Cumulative Distribution Function.

By using Gauss integration to solve $x_m$ we need to change the coordinate system from $x$ to $\eta$:

$$x = \frac{x_m}{2} (1 + \eta) \Rightarrow dx = \frac{x_m}{2} d\eta \begin{cases} \eta = -1, x = 0 \\ \eta = +1, x = 0 \end{cases}$$

if we introduce this now to the original equation and solve for $y = 0$:

$$y = \frac{\lambda_m \sigma_m^* \sqrt{A_0} x_m^{3/2}}{2} \int_{-1}^{1} \sqrt{2(L(1 + \eta) + (\tan \alpha + \tan \beta)x^2)} \left(1 + \eta \right)^2 d\eta - E_k$$ \hspace{1cm} (5.21)
In terms of programming procedure:

\[
y^* = \frac{\lambda_m \sigma_m \sqrt{A_0 x_m} \sqrt{2}}{2} \left\{ \sum_{i=1}^{NG} \sqrt{2} \left( (1 + \eta_i) + (\tan \alpha + \tan \beta) \frac{x_m}{4} (1 + \eta_i^2) \right) w_i \right\} - E_k \quad (5.22)
\]

where \(NG\) is the number of Gauss points used, \(\eta_i\) location of point, and \(w_i\) the weight at the point. To evaluate the integral we can write a simple FORTRAN Do-loop i.e.:

\[
\text{sum} = 0.0 \\
\text{Do 10 I = 1, NG} \\
\text{sum} = \text{sum} + \sqrt{2} \left( (1 + \eta_i) + (\tan \alpha + \tan \beta) \frac{x_m}{4} (1 + \eta_i^2) \right) w_i \\
10 \text{ continue}
\]

and then iterate \(x_m\) for solution which gives us then the contact area, \(A(x)\).

After the penetration has been found for one set of random variables we can find the impacting force and express it in terms of probability. Now in order to construct the CDF-Curve for the ice impact we can write following:

\[
P_f = P(F_{\text{max}} < F_0) \quad (5.23)
\]

which is the probability of that the maximum impact force, \(F_{\text{max}}\), will be less than certain impact force \(F_0\). The performance function in RELAN will therefore become:

\[
G = \lambda_m \sigma_m \sqrt{A_0 \sqrt{A(x)}} - F_0 \quad (5.24)
\]
CHAPTER 6

Reliability of Concrete Offshore Structures

6.1. Introduction

The design of a complex concrete offshore structure, which is exposed to extreme environmental loads such as icebergs and waves, involves determining the sectional forces at various locations of the structure by using a linear elastic analysis. The response due to the eight sectional forces, i.e. two normal forces $N_x$ and $N_y$, a membrane shear force $N_{xy}$, two flexural bending moments $M_x$ and $M_y$, a torsional bending moment $M_{xy}$ and two transverse shear forces $V_x$ and $V_y$ (see Figure 6.1), can be predicted using a generalization of the strain compatibility approach used for beams (see Chapter 4). While the case of a beam subjected to bending involves uniaxial strains and stresses, the case of eight sectional forces involves triaxial strains and triaxial stresses.

The program SHELL474, which is based on a 3-D strain compatibility approach, was used to account for the influence of the intervening variables, and to evaluate the theoretical capacity for reliability calculation of offshore structure wall elements. SHELL474 was developed by Adebar and Collins [1] as verification of the new Canadian concrete offshore structure code (CSA S474). SHELL474 calculates the factored sectional resistance of an element for given concrete and reinforcement dimensions, material grades and loading ratios.
6.2. Sectional Strength of Concrete Wall Elements

The following is a brief summary of the theoretical procedure used by SHELL474, but a more detailed description is given by Adebá and Collins [1]. An introduction into the strain compatibility approach for reinforced concrete in bending can also be found in Chapter 4.

The three membrane forces and the three bending moments (see Figure 6.1), which a wall element is subjected to, is predicted by assuming that the three biaxial strains \( \varepsilon_x, \varepsilon_y \) and \( \gamma_{xy} \), vary linearly over the thickness of the element.

![Figure 6.1: Sectional Forces at Complex Concrete Structure](image)

Thus, the complete biaxial strain state can be described by six variables, i.e. three strains at the top surface and three strains at the bottom surface. For a given set of the six strain variables, the stresses in the concrete and the reinforcement can be determined from biaxial stress-strain relationships.
By integrating the stresses over the thickness of the element, the six corresponding stress resultants $N_x$, $N_y$, $N_{xy}$, $M_x$, $M_y$ and $M_{xy}$ can be found.

When a wall element is subjected to transverse shears $V_x$ and $V_y$ (see Figure 6.1), the out-of-plane strains $\varepsilon_z$, $\gamma_{zx}$ and $\gamma_{zy}$ cannot be ignored, hence the problem involves triaxial strains and stresses. While the biaxial strains are considered over the thickness of the section, the triaxial strains are only evaluated at one location in SHELL474, e.g. at the mid-plane of the section or at the centroid of the flexural tension reinforcement.

### 6.3. Design of Hibernia Offshore Wall Element

The concrete offshore structure chosen for the reliability study was the Hibernia Gravity Base Structure, which will stand in 80 meters of water on the Grand Banks of Newfoundland. The Hibernia GBS (1986 update design) structure has a 1.4 meters thick icewall with 30 gear teeth in order to reduce the ice impact forces. The overall diameter of the structure, from tip to tip of the teeth, is 104 meters.

In the design for the ultimate limit states of Hibernia GBS the sectional strength, system ductility, and the fatigue were considered, while in the case of serviceability limit states, crack control and control of local damage were considered.

According to a recent study [5], the most critical load case of all the various limit states for the design of the reinforcement in the icewall is the 100 year eccentric ice impact. The wall element used for the reliability study (see Figure 6.2), was designed for local damage with 100 year ice impact.
Figure 6.2: Details of Hibernia GBS Icewall Design Adapted from Reference [5]
6.3.1. Modified SHELL474 and Subroutines for RELAN

To perform reliability calculation for offshore structure wall element, the main subroutine in SHELL474, entitled S1.FOR, was modified. Instead of finding a solution for only one set of variables, SHELL474 is now able to calculate the capacity for many sets of random variables coming from RELAN.

As mentioned earlier, the performance function is defined in the subroutine GFUN. In order to describe the performance function for the offshore structure wall element, an eight-dimensional ultimate capacity vector \( \{N_x, N_y, M_{xy}, M_x, M_y, V_x, V_y\} = \{\sigma_d\} \) needs to be evaluated.

Since the applied forces and the corresponding ultimate capacity vector change with time, a linear finite element program is needed to re-evaluate the applied resultant vector at each time during the impact. Here, as an approximation, it is assumed that the applied resultant components \( \sigma_{Ai} \), corresponding to the random applied load \( P \), are constant over time and given by:

\[
\sigma_{Ai} = \left( \frac{P}{P_o} \right) \sigma_{Ai_o}
\]  

(6.1)

where \( \sigma_{Ai_o} \) are the resultant components obtained from the finite element analysis for the load \( P_o \). The performance function used for the reliability calculation of offshore structure wall elements can be written in terms of vectors or as following:

\[
G = |OQ| - |OP|
\]  

(6.2)

where the vectors \(|OQ|\) and \(|OP|\) are illustrated in Figure 6.3.
In detail, $|OP|$ is the norm of the applied resultants, and $|OQ|$ represents the norm of the capacity vector for a loading path in the direction OP. Failure, i.e. $G < 0$, occurs when $|OP| > |OQ|$.

![Figure 6.3: Performance Function for Wall Elements](image)

In terms of the eight dimensional vectors the performance function can be written as follows:

$$ G = \sqrt{\sigma_{c1}^2 + \ldots + \sigma_{c8}^2} - p \sqrt{\sigma_{A1}^2 + \ldots + \sigma_{A8}^2} $$

(6.3)

where $p$ is the ratio between the random applied load $P$ and the applied load $P_o$ used for the finite element analysis. The performance function can also be written in terms of the ratio between the norms of the resistance and the applied load vectors, as follows:

$$ G = \frac{\sqrt{\sigma_{c1}^2 + \ldots + \sigma_{c8}^2}}{\sqrt{\sigma_{A1}^2 + \ldots + \sigma_{A8}^2}} - p $$

(6.4)
It should be noted that the components, which are combined in the resultant vector, have different units. To make the influence of each component equal to its real effect on the resultant direction, a scale factor was used. A scale factor is applied to the performance function by dividing the two bending moments and the torsional moment by the thickness of the wall.

6.3.2. Comparison of Beam Program and Modified SHELL474

To compare and test the modified version of SHELL474, both the beam program TIN and the modified SHELL474 were used to perform reliability calculation for the beam designed in Appendix B (see Figure 6.4).

![Comparison between Beam vs. Shell Program](image)

*Figure 6.4: Comparison between TIN and modified SHELL474*
The performance function was written in terms of the theoretical capacity for the resistance and for a constant applied load. However, it should be noted that the results presented in Figure 6.4 are only for comparison and to illustrate the reliability of the modification done to SHELL474. The variability of the load is ignored and therefore the results should not be taken out of context.

### 6.3.3. Variability of Intervening Variables

The material statistics listed in Table 6.1 and used for the concrete offshore structure wall elements, are explained in Chapter 3. It should be noted, that the rate of loading was ignored for high strength concrete compression strength unlike what was done for the beam case. The reason was simply, that the actual concrete compression strength is normally expected to be 10-15% higher than the specified value, and the effect from the rate of loading and the increase in strength due to additional hydration are expected to approximately cancel each other out. However, due to other factors discussed in Chapter 3, Equation 3.2 was used for high strength concrete.
## Table 6.1 Intervening Material Random Variables

<table>
<thead>
<tr>
<th>Definition of Variables / Units</th>
<th>Mean</th>
<th>COV</th>
<th>Distribution/Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'_c$ - Compression Strength (MPa)</td>
<td>41.33</td>
<td>0.10</td>
<td>Normal / [38]</td>
</tr>
<tr>
<td>$h$ - Sectional Thickness (mm)</td>
<td>1401.52</td>
<td>0.0045</td>
<td>Normal / [37]</td>
</tr>
<tr>
<td>$f_{yx}$ - Yield Strength of Stirrups (MPa)</td>
<td>445.34</td>
<td>0.093</td>
<td>Log-Normal / [37]</td>
</tr>
<tr>
<td>$A_{x1}$ - Area of Steel in X-dir. mm$^2$</td>
<td>702.0</td>
<td>0.06</td>
<td>Normal / [31]</td>
</tr>
<tr>
<td>$A_{x2}$ - Area of Steel in X-dir. mm$^2$</td>
<td>2000.0</td>
<td>0.06</td>
<td>Normal / [31]</td>
</tr>
<tr>
<td>$S_{x1}$ - Spacing of Bars in X-dir. mm</td>
<td>125.0</td>
<td>0.05</td>
<td>Normal</td>
</tr>
<tr>
<td>$S_{x2}$ - Spacing of Bars in X-dir. mm</td>
<td>125.0</td>
<td>0.05</td>
<td>Normal</td>
</tr>
<tr>
<td>$Z_{x1}$ - Location of X-Bars in Z-dir. mm</td>
<td>621.52</td>
<td>0.01</td>
<td>Normal / [37]</td>
</tr>
<tr>
<td>$Z_{x2}$ - Location of X-Bars in Z-dir. mm</td>
<td>-616.52</td>
<td>0.01</td>
<td>Normal / [37]</td>
</tr>
<tr>
<td>$f_{x1}$ - Steel Yield Strength (MPa)</td>
<td>445.34</td>
<td>0.093</td>
<td>Log-Normal / [37]</td>
</tr>
<tr>
<td>$f_{x2}$ - Steel Yield Strength (MPa)</td>
<td>445.34</td>
<td>0.093</td>
<td>Log-Normal / [37]</td>
</tr>
<tr>
<td>$A_{y1}$ - Area of Steel in Y-dir. mm$^2$</td>
<td>1000.0</td>
<td>0.06</td>
<td>Normal / [31]</td>
</tr>
<tr>
<td>$A_{y2}$ - Area of Steel in Y-dir. mm$^2$</td>
<td>1000.0</td>
<td>0.06</td>
<td>Normal / [31]</td>
</tr>
<tr>
<td>$A_{y3}$ - Area of Steel in Y-dir. mm$^2$</td>
<td>990.0</td>
<td>0.02</td>
<td>Normal / [31]</td>
</tr>
<tr>
<td>$A_{y4}$ - Area of Steel in Y-dir. mm$^2$</td>
<td>990.0</td>
<td>0.02</td>
<td>Normal / [31]</td>
</tr>
<tr>
<td>$S_{y1}$ - Spacing of Bars in Y-dir. mm</td>
<td>325.0</td>
<td>0.05</td>
<td>Normal</td>
</tr>
<tr>
<td>$S_{y2}$ - Spacing of Bars in Y-dir. mm</td>
<td>235.0</td>
<td>0.05</td>
<td>Normal</td>
</tr>
<tr>
<td>$S_{y3}$ - Spacing of Bars in Y-dir. mm</td>
<td>470.0</td>
<td>0.05</td>
<td>Normal</td>
</tr>
<tr>
<td>$S_{y4}$ - Spacing of Bars in Y-dir. mm</td>
<td>470.0</td>
<td>0.05</td>
<td>Normal</td>
</tr>
<tr>
<td>$Z_{y1}$ - Location of Y-Bars in Z-dir. mm</td>
<td>586.52</td>
<td>0.011</td>
<td>Normal / [37]</td>
</tr>
<tr>
<td>$Z_{y2}$ - Location of Y-Bars in Z-dir. mm</td>
<td>-581.52</td>
<td>0.011</td>
<td>Normal / [37]</td>
</tr>
<tr>
<td>$Z_{y3}$ - Location of Y-Bars in Z-dir. mm</td>
<td>561.52</td>
<td>0.011</td>
<td>Normal / [37]</td>
</tr>
<tr>
<td>$Z_{y4}$ - Location of Y-Bars in Z-dir. mm</td>
<td>-561.52</td>
<td>0.011</td>
<td>Normal / [37]</td>
</tr>
<tr>
<td>$f_{y1}$ - Steel Yield Strength (MPa)</td>
<td>445.34</td>
<td>0.093</td>
<td>Log-Normal / [37]</td>
</tr>
<tr>
<td>$f_{y2}$ - Steel Yield Strength (MPa)</td>
<td>445.34</td>
<td>0.093</td>
<td>Log-Normal / [37]</td>
</tr>
<tr>
<td>$f_{y3}$ - Strand Yield Strength (MPa)</td>
<td>1742.54</td>
<td>0.025</td>
<td>Normal / [39]</td>
</tr>
<tr>
<td>$f_{y4}$ - Strand Yield Strength (MPa)</td>
<td>1742.54</td>
<td>0.025</td>
<td>Normal / [39]</td>
</tr>
</tbody>
</table>
6.3.4. Ice Impact Prediction Using the PROB Program

By running the program PROB, which is based on the theoretical model derived through energy principles in Chapter 5, the CDF-Curves for concentric and eccentric ice impact can be derived (see Figure 6.5). To find the CDF-Function for the eccentric load, the concentric load was multiplied by the angle of the eccentricity or $1/\sqrt{2}$. The nominal concentric ice impact used for the finite element analysis was 555 MN and therefore the nominal eccentric ice impact became 392.4 MN. To fit a distribution to the program results, the Gumbel distribution was used. All the statistical data for the CDF-Curves can be found in Table 6.2.

![CDF-CURVES FOR 100 YEAR ICE-IMPACT](image)

*Figure 6.5: CDF-Curves for 100 Year Ice-Impact*
Even though no data are available, confirming a correlation between the intervening parameters effecting the ice impact, i.e. the velocity, the thickness, the diameter and the ice compression strength, a correlation factor of 0.6 was used between the velocity and the thickness for the sake of comparison. Figure 6.5 shows results for no correlation and for the correlated case.

Table 6.2 Extreme Parameters for Ice-Impact Loads

<table>
<thead>
<tr>
<th>Type of Ice-Impact</th>
<th>( A )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>555 MN Concentric/uncorrelated</td>
<td>0.0104</td>
<td>-8.4326</td>
</tr>
<tr>
<td>555 MN Concentric/correlated</td>
<td>0.0079</td>
<td>-15.4286</td>
</tr>
<tr>
<td>392.4 MN Eccentric/uncorrelated</td>
<td>0.0146</td>
<td>-5.9628</td>
</tr>
<tr>
<td>392.4 MN Eccentric/correlated</td>
<td>0.0112</td>
<td>-10.9097</td>
</tr>
</tbody>
</table>

While the CDF-Function for ice impact can be easily derived by using the procedure in Chapter 5 if enough available data exist, the effect on the applied load resultant components remains unknown. Instead of re-evaluating the changes at every time due to the increasing contact area with a finite element analysis, the 8-dimensional applied load resultant vector was scaled accordingly (see Equation 6.1) and assumed to be constant over time.
6.3.5. Example Runs

The applied load resultant components used for the example runs and listed in Table 6.3, as part of preliminary study by Allyn, Yee and Adebar [5], are results from the finite element analysis program COSMOS. It should be noted that the gravity and the ballast loads have been subtracted from the resultants listed in Table 6.3.

The element used for reliability study, was originally designed for load combination No. 34 according to CSA S474. Load combination No. 34, which is used for an evaluation of local damage, is a combination of gravity loads, solid ballast load and 100 year eccentric ice impact where the load factors are set to 1.0.

Table 6.3 Resultant Load Components from Linear Finite Element Analysis

<table>
<thead>
<tr>
<th>Load Vectors for Wall Elements</th>
<th>Concentric</th>
<th>Eccentric</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_x$-Normal Force in X-dir. (kN/m)</td>
<td>446.0</td>
<td>170.0</td>
</tr>
<tr>
<td>$N_y$-Normal Force in Y-dir. (kN/m)</td>
<td>-165.0</td>
<td>-1835.0</td>
</tr>
<tr>
<td>$N_{xy}$-Membrane Shear Force (kN/m)</td>
<td>250.0</td>
<td>-384.0</td>
</tr>
<tr>
<td>$M_x$-Bending Moment in X-dir. (kNm/m)</td>
<td>-559.0</td>
<td>4760.0</td>
</tr>
<tr>
<td>$M_y$-Bending Moment in Y-dir. (kNm/m)</td>
<td>-33.0</td>
<td>1690.0</td>
</tr>
<tr>
<td>$M_{xy}$-Torsional Bending Moment (kNm/m)</td>
<td>12.0</td>
<td>-12.0</td>
</tr>
<tr>
<td>$V_x$-Transverse Shear in X-dir. (kN/m)</td>
<td>-78.0</td>
<td>2640.0</td>
</tr>
<tr>
<td>$V_y$-Transverse Shear in Y-dir. (kN/m)</td>
<td>-7.0</td>
<td>6.0</td>
</tr>
</tbody>
</table>
By running the SHELL474 program for the element, the load factor, which is the ratio between the element strength resultant vector and the applied load resultant vector, was only 1.027 (see Appendix C). Since the load factor is so low for this load case, the safety index $\beta$ was expected to be low.

While the statistics for the ice impact parameters are quiet often not available and therefore assumed, and also because of the approximation for the direction of the resultant load vector, the probability of failure will not be realistic. However, because the eccentric load case is rather extreme, and for the sake of comparison, components from concentric load were also applied.

The results from the reliability analysis for the example runs are listed in Table 6.4 and Table 6.5. The difference between the concentric and the eccentric load case is rather large, which was partly expected because of the size difference between the resultant load vectors.

Table 6.4 RELAN Results: Wall Element

<table>
<thead>
<tr>
<th>Type of Ice-Impact</th>
<th>$\beta$</th>
<th>Pf</th>
</tr>
</thead>
<tbody>
<tr>
<td>555 MN Concentric/uncorrelated</td>
<td>5.378</td>
<td>0.376E-7</td>
</tr>
<tr>
<td>555 MN Concentric/correlated</td>
<td>4.441</td>
<td>0.448E-5</td>
</tr>
<tr>
<td>392.4 MN Eccentric/uncorrelated</td>
<td>1.395</td>
<td>0.815E-1</td>
</tr>
<tr>
<td>392.4 MN Eccentric/correlated</td>
<td>0.467</td>
<td>0.320</td>
</tr>
</tbody>
</table>

It should also be noted as mentioned earlier, because the change in direction of the ultimate resultant components is totally ignored and an assumption was made for the ice impact, the reliability results should not be taken out of context.
### Table 6.5 Sensitivity Factors: Wall Element

<table>
<thead>
<tr>
<th>Definition of Variables / Units</th>
<th>The Main Sensitivity Factors from Each Run</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Con./unc.</td>
</tr>
<tr>
<td>$f_c'$-Compression Strength</td>
<td>0.346E-1</td>
</tr>
<tr>
<td>$f_y$-Yield Strength of Stirrups</td>
<td>0.0</td>
</tr>
<tr>
<td>$A_{s1}$-Area of Steel in X-dir.</td>
<td>0.183</td>
</tr>
<tr>
<td>$A_{s2}$-Area of Steel in X-dir.</td>
<td>0.543E-3</td>
</tr>
<tr>
<td>$S_{s1}$-Spacing of Bars in X-dir.</td>
<td>0.137</td>
</tr>
<tr>
<td>$S_{s2}$-Spacing of Bars in X-dir.</td>
<td>0.275E-3</td>
</tr>
<tr>
<td>$Z_{s1}$-Location of X-Bars in Z-dir.</td>
<td>0.184E-1</td>
</tr>
<tr>
<td>$Z_{s2}$-Location of X-Bars in Z-dir.</td>
<td>0.288E-2</td>
</tr>
<tr>
<td>$f_{s1}$-Steel Yield Strength</td>
<td>0.209</td>
</tr>
<tr>
<td>$f_{s2}$-Steel Yield Strength</td>
<td>0.0</td>
</tr>
<tr>
<td>Ice-Ice Impact</td>
<td>0.950</td>
</tr>
</tbody>
</table>
6.4. Development of PDF-Functions for Load and Resistance

In order to represent the probability of failure with geometric representation, the PDF-Functions can be derived from the Gumbel extreme distribution described in Equation 4.21. The corresponding probability density function, which is basically the derivative of Equation 4.21, can be written as following:

\[ f(x) = A \exp\left[-A(x - B)\right] \exp\left[-\exp\left[-A(x - B)\right]\right] \]  

(6.5)

Figure 6.6: PDF-Curves for Annual Concentric Ice-Impact
To plot the PDF-Curves (see Figure 6.6 and 6.7) for annual concentric and eccentric ice impact, Equation 6.5 and the corresponding A and B listed in Table 6.2, respectively, the location and the scale parameters, can be used.

\[ f(x) = NA \exp\{-A(x - B)\} \exp\{-N \exp\{-A(x - B)\}\} \]  

(6.6)

Figure 6.7: PDF-Curves for Annual Eccentric Ice-Impact

If the annual PDF-Function is now expanded for N-years we get the following:

\[ f(x) = NA \exp\{-A(x - B)\} \exp\{-N \exp\{-A(x - B)\}\} \]  

(6.6)
Now the PDF-Curves for 100 year concentric and eccentric ice impact can be plotted as shown in Figure 6.8 and 6.9.

**Figure 6.8: PDF-Curves for 100 Year Concentric Ice-Impact**
In order to derive the CDF-Function for element resistance, the performance function for the modified SHELL474 can be written in the same manner as Equation 5.24 or in terms of load ratio versus certain load constant:

$$G = \frac{\sqrt{\sigma_{c1}^2 + \ldots + \sigma_{c3}^2}}{\sqrt{\sigma_{d1}^2 + \ldots + \sigma_{d8}^2}} - L_o$$  \hspace{1cm} (6.7)$$

If the probability of failure is calculated for number of load constants, the CDF-Curves can be plotted.

Figure 6.9: PDF-Curves for 100 Year Eccentric Ice-Impact
CHAPTER 6 Reliability of Concrete Offshore Structures

Using the Least Square Method to fit to the data results, the location and scale parameters $A$ and $B$ can be derived.

If now the PDF-Curves for element resistance and 100 year uncorrelated/correlated eccentric ice impact are plotted (see Figure 6.10 and 6.11), the corresponding reliability calculation results can be found in Table 6.4.

![PDF-Curves for Resistance of Wall Element and 100 Year Eccentric Ice-Impact](image)

*Figure 6.10: Element Resistance and 100 Year Uncorrelated Eccentric Ice-Impact*
6.5. System Performance Using the Joint PDF-Functions

If the joint probability functions are known for the element resistance and the applied load, or can be derived as in Section 6.4, the performance function can be written in terms of two random variables instead of 28 as for the example runs:

\[ G = R - D \] (6.8)
In this case, $R$ represents the derived joint probability distribution for the resistance of the element with its mean and standard deviation, and $D$ represents the derived joint probability distribution for the applied load with its mean and standard deviation.

With joint probability distributions known, the evaluation of the reliability for a certain element with a certain load can be simplified somewhat, but since the joint probability distributions are rarely known programs like TIN and the modified SHELLA74 are still needed.
Probabilistic methods provide a logical framework for uncertainty analysis and safety evaluation in structural engineering. Through reliability calculations, designers are provided with a powerful tool, which can help them understand the most complex behavior of structural systems. One very important by-product of reliability calculations, are the sensitivity factors. They help designers to understand the importance of different design variables and various modes of failure, which the system might undergo when considering different strength and geometric properties.

Linking concrete design programs such as TIN and the state-of-the-art program SHELL474 to reliability evaluation programs like RELAN, makes it possible to deal with the influence of the variability of all intervening variables on the theoretical strength.

In order to conduct probability-based design calculations, basic information on each random variable, such as the probability distribution and estimates of the mean and standard deviation are needed. While frequently there are sufficient data to obtain reasonable estimates of the probability distributions, physical argument and convenience must be used in many other cases.
CHAPTER 7 Concluding Remarks and Further Study

Further study seems to be needed in case of the concrete compression strength, because in the work by Mirza, Hatzinikolas and MacGregor [38] it is suggested that the mean value should be adjusted for rate of loading, when at the same time the increase due to additional hydration is ignored, and the actual concrete compression strength is usually 10-15% higher than the specified value. In the study how different spans and steel ratios effect the reliability of a beam, the suggestions by Mirza, Hatzinikolas and MacGregor were used. The results indicate that the code limits for maximum reinforcement ratio to prevent premature concrete compression failure is too high when the variability of intervening variables is taken into account.

While the statistical data for applied load, i.e. gravity, snow and service load, seems to be well established, and easily applied through the code equation for the beam case, following needs to be considered to establish the probabilistic ice impact in order to use it as a load on wall elements:

i. The ice impact is influenced by the variability of its thickness, diameter, velocity and compression strength, all affecting the contact area.

ii. It can be assumed that there is a correlation between the ice parameters even though there is no data which confirms it.

iii. Based on test results, the ice compression strength depends on contact area and decreases with increase in contact area [47].
Based on this, it can be seen that increasing penetration means increasing contact area and therefore changes in the eight sectional forces, i.e. the two normal forces $N_x$ and $N_y$, a membrane shear force $N_y$, two flexural bending moments $M_x$ and $M_y$, a torsional bending moment $M_{xy}$, as well as two transverse shear forces $V_x$ and $V_y$. The runs made in this study did not account for the change in element sectional forces. The ratios of the eight sectional forces (stress resultants), which were calculated using finite element program COSMOS, were assumed to be constant as the ice impact and the penetration progressed. As a recommendation for further study, a finite element program such as COSMOS needs to be linked to both the modified SHELL474 and RELAN in order to take the variability of ice impact and other loads on reliability calculation of wall elements into account.
Bibliography


40. Mirza, S. M., (1967) "An Investigation of Combined Stresses in Reinforced Concrete Beams", thesis presented to McGill University, at Montreal, Canada, in partial fulfillment of the requirements for the degree of Ph.D.


47. Sanderson, T. J. O., (1988) "Ice Mechanics Risks to Offshore Structures", Published by Graham and Trotman Inc.


Appendix A1

Beam Program Subroutines
$debug
SUBROUTINE MAIN

MAIN.FOR

CFY - YIELD STRENGTH OF STEEL (N/mm²)
CPF - YIELD STRENGTH OF LOW RELAXATION STEEL (N/mm²)
FCPP - PEAK STRESS OBTAINED FROM A CYLINDER TEST (N/mm²)
ECT - INITIAL TANGENT MODULUS (N/mm²)
FCR - LONGITUDINAL STRESS (N/mm²)
DEP - SRAIN DIFFERENCE
ES - MODULUS OF ELASTICITY FOR STEEL (N/mm²)
EP - MODULUS OF ELASTICITY FOR LOW RELAXATION STEEL (N/mm²)
NLAY - NUMBER OF LAYERS
TOL - SPECIFIED TOLERANCE ON AXIAL LOAD (N)
A(i) - CROSS SECTIONAL AREA OF CONCRETE (mm²)
A(i) - CROSS SECTIONAL AREA OF STEEL (mm²)
A(i) - CROSS SECTIONAL AREA OF TENDON (mm²)
Z(i) - LOCATION OF THE FORCE (mm)
MTYP(i) - TYPE OF MATERIAL
ICURV - INITIAL CURVATURE (rad/mm)
CSTEP - CURVATURE STEP
CMAX - MAXIMUM ALLOWABLE CURVATURE
AXL - AXIAL FORCE (N)
MCAP - MOMENT CAPACITY OF THE SECTION (Nm)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 MCAP, ICURV, ISTRN, MISM
INTEGER COUNT, COUNT2
COMMON/LAYER/A(50), Z(50), MTYP(50)
COMMON/B/NLAY
COMMON/B2/PHIC, PHIS, PHIP
COMMON/B3/MCAP
COMMON/B4/AXL
COMMON/B7/COUNT, COUNT2
COMMON/B10/BETA1, FPCN, FYN, BN, ASN, DEFFN

TOLERANCE FOR MAXIMUM MOMENT CAPACITY
MISM = 0.001

INITIALIZE COUNTER AND BENDING MOMENTS
I = 0
BEND1 = 0.0D0
MCAP = 0.0D0

INITIALIZE STRAIN
ISTRN = 0.0D0
STRN = ISTRN

INITIALIZE CURVATURE
ICURV = 0.0D0
CURV = ICURV

CURVATURE STEP INITIALIZED
CSTEP = 1.0

MAXIMUM CURVATURE FOR THE SECTION
EC = 0.003, ES = 0.02
THEN CMAX IS MULTIPLIED WITH 10 TO MAKE SURE THE PROGRAM IS
GOING TO RUN FOR HIGH ENOUGH CURVATURE
CMAX = (0.023/(DEFFN))*1E7
IF(AXL.NE.0.0)THEN
  TOL = 0.01*AXL
ELSE
  TOL = 0.001*FPCN*BN*DEFFN
ENDIF

2 CURV = CURV + CSTEP
CALL ITER(CURV,STRN,AXL,BEND,TOL,NN)

5 IF(BEND.GT.0.0D0)GOTO 5
CSTEP = CSTEP/2
CURV = 0.0D0
GOTO 2

10 IF(CURV.LE.CMAX)THEN
  CURV = CURV + CSTEP
  CALL ITER(CURV,STRN,AXL,BEND,TOL,NN)
  BEND2 = BEND
  I = I + 1
  IF(I.GE.2)GOTO 15
  SLOPE1 = (BEND2-BEND1)/CSTEP
  BEND1 = BEND2
  GOTO 10

15 SLOPE2 = (BEND2-BEND1)/CSTEP
  P = SLOPE1*SLOPE2
  IF(P.LE.0.0D0)GOTO 50
  SLOPE1 = SLOPE2
  BEND1 = BEND2
  GOTO 10

50 IF(DABS(BEND1-MCAP).LE.(MISM*MCAP))GOTO 100
  MCAP = BEND1
  CURV = CURV - 2.0D0*CSTEP
  CSTEP = CSTEP/2.0D0
  CALL ITER(CURV,STRN,AXL,BEND,TOL,NN)
  BEND1 = BEND
  I = 0
  GOTO 10

100 CURV = CURV - CSTEP

IF(COUNT2.LE.43)THEN
  OPEN(UNIT=4,FILE='gis.out',ACCESS='APPEND',STATUS='UNKNOWN')
  WRITE(4,110)CURV,MCAP
  110 FORMAT(',2X,F15.5,SX,F25.5)
  COUNT2 = COUNT2 + 1
  CLOSE(4)
ENDIF
ELSE
OPEN (UNIT=4, FILE='gis.out', ACCESS='APPEND', STATUS='UNKNOWN')
WRITE (4, 120) 'CURVATURE EXCEEDED CMAX'
ENDIF
RETURN
END

C-----------------------------
C STRESS.FOR
C-----------------------------
C SUBROUTINE
C WHICH CALCULATES THE STRESS FOR A GIVEN STRAIN
SUBROUTINE STRES(STRN,MTYP,STRS)
IMPLICIT REAL*8(A-H,O-Z)
COMMON/B1/FCPP,FY,FP,ECT,FCR,ES,EP
COMMON/B6/DEP
C FCP IS NEGATIVE
FCP = (-1.0)*(DABS(FCPP))
C CONCRETE, STEEL, PRESTRESSED STEEL
C CONCRETE WITH TENSION STIFFENING
IF(MTYP.EQ.1)THEN
ECR = FCR/ECT
ECP = 2.0*FCP/ECT
ECF = 2.0*ECP
IF (STRN.LE.ECF) THEN
STRS = 0.0
ELSEIF (STRN.GT.ECF.AND.STRN.LE.0.0) THEN
STRS = FCP*(((2.0*STRN/ECP)-(STRN/ECP)**2)
ELSEIF (STRN.GT.0.0.AND.STRN.LE.ECR) THEN
STRS = STRN*ECT
ELSEIF (STRN.GT.ECR) THEN
ENDIF
C CONCRETE WITHOUT TENSION STIFFENING
ELSEIF (MTYP.EQ.2) THEN
ECR = FCR/ECT
ECP = 2.0*FCP/ECT
ECF = 2.0*ECP
IF (STRN.LE.ECF) THEN
STRS = 0.0
ELSEIF (STRN.GT.ECF.AND.STRN.LE.0.0) THEN
STRS = FCP*(((2.0*STRN/ECP)-(STRN/ECP)**2)
ELSEIF (STRN.GT.0.0.AND.STRN.LE.ECR) THEN
STRS = STRN*ECT
ENDIF
C REINFORCING STEEL
ELSEIF(MTYP.EQ.3)THEN
  IF(STRN.LE.-0.002)THEN
    STRS = -FY
  ELSEIF(STRN.GT.-0.002.AND.STRN.LE.0.002)THEN
    STRS = ES*STRN
  ELSEIF(STRN.GT.0.002)THEN
    STRS = FY
  ENDIF
C PRESTRESSED STEEL
ELSEIF(MTYP.EQ.4)THEN
  STRNP = STRN+DEP
  IF(STRNP.LE.-0.008)THEN
    STRS = -FP
  ELSEIF(STRNP.GT.-0.008.AND.STRNP.LE.0.0)THEN
    STRS = EP*STRNP
  ELSEIF(STRNP.GT.0.0)THEN
    STRS = STRNP*EP*(0.025+0.975/(1+(118*STRNP)**10)**0.10)
  ENDIF
  IF(STRS.GT.FP)THEN
    STRS = 0.0
  ENDIF
ENDIF
RETURN
END

C ITER.FOR

C******************************************************************************
C Subroutine ITER
C******************************************************************************
C Version 1.10 Written by P.E. Adebar February 10, 1990
*C Description: ITERates for the strain at the centroid which results in the
*   required axial load at a specified curvature. Calls subroutine FORMOM to calculate
*   the axial load and bending moment associated with a given curvature and strain at the centroid.
*C Input Variables:
*C    CURV ......... specified CURVature (rad/mm)
*C    STRN ......... STRaiN at the section centroid
*C    AXL ......... required AXial load (N)
*C    TOL ......... specified TOlerance on the axial load
*C Output Variables:
*C    STRN ......... STRaiN at the centroid
*C    BEND ......... BENDing moment associated with the specified curvature and axial load
*C    NN ........... Number of iterations

SUBROUTINE ITER(CURV,STRN,AXL,BEND,TOL,NN)
IMPLICIT REAL*8(A-H,O-Z)

106
C Set strain increment
   DSTRN = 0.0005
C Set limit on maximum iterations
   MAXNN = 250
C Check that tolerance is not zero
   TOL = DABS(TOL)
   IF(TOL .LT. 1.0E-10) THEN
      WRITE(*,*)'**** Specified Tolerance Too Small ****'
      STOP
   ENDIF
   NN = 0
C First try to bound solution
   IF(NN .GT. MAXNN) THEN
      WRITE(*,*)'**** No Solution Found ****1'
      BEND = 0.0
      STRN = 0.0
      GOTO 1000
   ENDIF
   CALL FORMOM(CURV,STRN,FOR,BEND)
   PDIF = DIF
   DIF = AXL-FOR
C Check tolerance just in case
   IF(DABS(DIF) .LE. TOL) GOTO 1000
C Decide on direction to increment strain
   IF(FOR .LT. AXL) THEN
      KK = +1
   ELSE
      KK = -1
   ENDIF
C If first iteration step
   IF(NN .EQ. 1) THEN
      PSTRN = STRN
      STRN = STRN + KK*DSTRN
      GOTO 10
   ENDIF
C Check if solution bounded
   IF(PDIF*DIF) 20,21,22
   WRITE(*,*)'**** No Solution Found ****2'
   BEND = 0.0
   STRN = 0.0
   GOTO 1000
21   PSTRN = STRN
   STRN = STRN + KK*DSTRN
   GOTO 10
22   X1 = PSTRN
   X2 = STRN
   F1 = PDIF
   F2 = DIF
   NN = NN+1
C WRITE(*,*)'**** No Solution Found ****3'
IF(NN .GT. MAXNN)THEN
  BEND = 0.0
  STRN = 0.0
  GOTO 1000
ENDIF

C Linearly interpolate for new guess
  X3 = X2 - ((X2 - X1) * F2 / (F2 - F1))
  STRN = X3

  CALL FORMOM(CURV, STRN, FOR, BEND)

  F3 = AXL - FOR

C Check tolerance
IF(DABS(F3) .LT. TOL)GOTO 1000

  F13 = F1 * F3
IF(F13 .LT. 0)THEN
  X2 = X3
  F2 = F3
ELSE
  X1 = X3
  F1 = F3
ENDIF
GOTO 30

1000 RETURN
END

C-------------------------------------------------------------------------------------
C FORMOM.FOR
C-------------------------------------------------------------------------------------
C SUBROUTINE WHICH CALCULATES THE FORCE AND MOMENT
SUBROUTINE FORMOM(CURV, STRAIN, AXL, BEND)
IMPLICIT REAL*8(A-H, O-Z)
COMMON/LAYER/A(50), ZC(50), MTYP(50)
COMMON/B/NLAY
STRN = 0.0
AXL = 0.0
BEND = 0.0
DO 10 I = 1, NLAY
IF(CURV.EQ.0.0)THEN
  STRN = STRAIN
ELSE
  STRN = STRAIN-CURV*Z(I)/1E6
ENDIF
CALL STRES(STRN, MTYP(I), STRS)

  AXL = A(I) * STRS + AXL
  BEND = (-1.0) * Z(I) * A(I) * STRS + BEND

10 CONTINUE
RETURN
END
SUBROUTINE DETERM (IMODE)

AXL-AXIAL LOAD

GAMMA-RATIO BETWEEN NOMINAL DEAD AND LIVE LOAD

PHIC-MATERIAL FACTOR FOR CONCRETE

PHIS-MATERIAL FACTOR FOR STEEL

BETA1-STRESS BLOCK FACTOR

DEP-STRAIN DIFFERENCE BETWEEN CONCRETE AND TENDONS

FPCN-NOMINAL COMPRESSION STRENGTH OF CONCRETE

FYN-NOMINAL YIELD STRENGTH OF STEEL

BN-NOMINAL SECTION WIDTH

ASN-NOMINAL TOTAL STEEL AREA

DEFFN-NOMINAL EFFECTIVE DEPTH

IMPLICIT REAL*8 (A - H, O - Z)

REAL*8 LN,N1,N2,N3,N4

INTEGER COUNT,COUNT2

CHARACTER*5 PASS

COMMON/B/NLAY

COMMON/B2/PHIC,PHIS,PHIP

COMMON/B4/AXL

COMMON/B5/DN,SN,N1,N2,N3,N4

COMMON/B6/DEP

COMMON/B7/COUNT,COUNT2

COMMON/B10/BETA1,FPCN,FYN,BN,ASN,DEFFN

COUNT = 1

COUNT2 = 1

DO 20 TALA = 1,3

CALL CLS

CALL MOCUR(3,0)

WRITE(*,*),'****************************************************'

WRITE(*,*),'PLEASE ENTER YOUR PASSWORD'

WRITE(*,*),'****************************************************'

READ(*,'(A5))PASS

IF((PASS.EQ.'SIGGA').OR.(PASS.EQ.'sigga'))GOTO 30

WRITE(*,*)CHAR(7)

WRITE(*,*)CHAR(7)

WRITE(*,*)CHAR(7)

20 CONTINUE

WRITE(*,*), '### ACCESS DENIED ###'

STOP

30 CALL CLS

CALL MOCUR(3,0)

OPEN(UNIT=3,FILE='gisli.in,STATUS='OLD')

OPEN(UNIT=4,FILE='gis.out',STATUS='UNKNOWN')

WRITE(4,*)'****************************************************'

WRITE(4,*)'MCAP MCODE GXP'

CLOSE(4)

OPEN(UNIT=7,FILE='gis.con',STATUS='UNKNOWN')

WRITE(7,*)'****************************************************'

WRITE(7,*)'MCAP MCODE GXP'

CLOSE(7)

READ(3,*),DN,LN,SN,N1,N2,N3,N4

READ(3,*),PHIC,PHIS,PHIP,BETA1

READ(3,*),NLAY,DEP,AXL

READ(3,*),FPCN,FYN,BN,ASN,DEFFN
SUBROUTINE GFUN (X, N, IMODE, GXP)

C B-BEAM WIDTH
C H-BEAM HEIGHT
C C-CONCRETE COVER
C PHI1-DIAMETER OF STIRRUP
C PHI2-DIAMETER OF STEEL IN THE 1ST LAYER
C PHI3-DIAMETER OF STEEL IN THE 2ND LAYER
C E1-DISTANCE BETWEEN 1ST AND 2ND LAYER
C DR-RATIO BETWEEN ACTUAL DEAD LOAD AND NOMINAL
C LR-RATIO BETWEEN ACTUAL LIVE LOAD AND NOMINAL
C CATOT-TOTAL AREA
C WY-Y-SECTION MODULUS FOR THE SECTION
C YO-CENTER OF GRAVITY FOR THE SECTION
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 LR, LN, MCAP, MCODE, MULT, N1, N2, N3, N4
INTEGER COUNT, COUNT2
DIMENSION X(N)
COMMON/LAYER/A(50), Z(50), MTYPE(50)
COMMON/B/NLAYS
COMMON/B1/FCPP, FY, FP, ECT, FCR, ES, EP
COMMON/B2/PHI1, PHI2, PHI3
COMMON/B3/MCAP
COMMON/B4/AAX
COMMON/B5/DN, LN, SN, N1, N2, N3, N4
COMMON/B7/COUNT, COUNT2
COMMON/B10/BETA, FPCN, FYN, BN, ASN, DEEFFN
FCPP = X(1)
FY = X(2)
FP = X(3)
ECT = X(4)
FCR = X(5)
ES = X(6)
EP = X(7)
B = X(8)
H = X(9)
C = X(10)
PHI1 = X(11)
PHI2 = X(12)
PHI3 = X(13)
PHI4 = X(14)
PHI5 = X(15)
E1 = X(16)
E2 = X(17)
E3 = X(18)
DR = X(19)
SR = X(20)
LR = X(21)
Appendix A1

\begin{align*}
A1 &= N1*(22/7)*X(12)^2/4 \\
A2 &= N2*(22/7)*X(13)^2/4 \\
A3 &= N3*(22/7)*X(14)^2/4 \\
A4 &= N4*(22/7)*X(15)^2/4 \\
\text{ATOT} &= X(8)*X(9)+A1+A2+A3+A4 \\
WY &= A1*(X(10)+X(11)+X(12)/2) \\
WY &= WY*A2*(X(10)+X(11)+X(12)+X(16)+X(13)/2) \\
WY &= WY+A3*(X(10)+X(11)+X(12)+X(13)+X(16)+X(17)+X(14)/2) \\
WY &= WY+A4*(X(10)+X(11)+X(12)+X(13)+X(14)+X(16)+X(17)+X(18)+X(15)/2) \\
& + X(15)/2)+X(8)*X(9)^2/2 \\
YO &= WY/ATOT \\
Z(1) &= X(10)+X(11)+X(12)/2-Y0 \\
A(1) &= A1 \\
\text{MTYP}(1) &= 3 \\
Z(2) &= X(10)+X(11)+X(12)+X(16)+X(13)/2-Y0 \\
A(2) &= A2 \\
\text{MTYP}(2) &= 3 \\
Z(3) &= X(10)+X(11)+X(12)+X(13)+X(16)+X(17)+X(14)/2-Y0 \\
A(3) &= A3 \\
\text{MTYP}(3) &= 3 \\
Z(4) &= X(10)+X(11)+X(12)+X(13)+X(14)+X(16)+X(17)+X(18)+X(15)/2-Y0 \\
A(4) &= A4 \\
\text{MTYP}(4) &= 3 \\
Z(5) &= -Y0/4 \\
A(5) &= X(8)*Y0/2 \\
\text{MTYP}(5) &= 2 \\
Z(6) &= -3*Y0/4 \\
A(6) &= X(8)*Y0/2 \\
\text{MTYP}(6) &= 2 \\
Z(7) &= X(9)-Y0)/4 \\
A(7) &= X(8)*(X(9)-Y0)/2 \\
\text{MTYP}(7) &= 2 \\
Z(8) &= 3*(X(9)-Y0)/4 \\
A(8) &= X(8)*(X(9)-Y0)/2 \\
\text{MTYP}(8) &= 2 \\
\text{MC1} &= ((BETA1*ASN*PHIS*FYN)/(0.85*BETA1*PHIC*FPCN*BN)) \\
\text{CODE} &= (0.85*PHIC*FPCN*BN*MC1*(DEFFN-MC1/2.0)* \\
& 1 ((X(19)*(DN/(SN+LN)))*((SN+LN)/SN)+X(20)+(LN/SN)*X(21))/ \\
& 2 (1.25*(DN/(SN+LN)))*((SN+LN)/SN)+1.5*(1+LN/SN)) \\
\text{MCODE} &= \text{CODE}/1E6 \\
\text{CALL MAIN} \\
\text{MULT} &= \text{MCAP}/1E6 \\
\text{GXP} &= \text{MULT}-\text{MCODE} \\
\text{IF(COUNT.LE.43)THEN} \text{OPEN(UNIT=7,FILE='gis.con',ACCESS='APPEND',STATUS='UNKNOWN')} \\
\text{WRITE}(7,210)MULT,MCODE,GXP \\
\text{210 FORMAT(' ',F15.4,3X,F15.4,3X,F15.4)} \\
\text{COUNT} &= \text{COUNT}+1 \\
\text{CLOSE(7)} \\
\text{ENDIF} \\
\text{RETURN} \\
\text{END}
\end{align*}
SUBROUTINE DFUN (X, N, IMODE, DELTA)

IMPLICIT REAL*8 (A - H, O - Z)
DIMENSION X(N), DELTA(N)
RETURN
END

SUBROUTINE D2FUN (X, N, IMODE, D2, N2)

IMPLICIT REAL*8 (A - H, O - Z)
DIMENSION X(N), D2(N2,N2)
RETURN
END
Appendix A2

Modified Main Subroutine in SHELL474
SUBROUTINE SKEL(FC, FCRK, EC, ECP, AGG, THICK, DV,
  1 DIAx, ASX, SPX, ZSX, FYX, DEPSX,
  2 DIAy, ASy, SPy, ZSy, FYY, DEPSY,
  3 RHOSz, ROWSz, DIAz, NASx, NASy, SPz, SPzY, FYz,
  4 RNxCON, RNyCON, RNxyCON, RMxCON, RMyCON, RMxyCON, RVxCON, RVyCON,
  5 RNx, RNy, RNMx, RMY, RMXY, RVx, RVY,
  6 PHIC, PHIS, PHIP,
  7 IPRINT,
  8 RESULT)

REAL*4 FC, FCRK, EC, ECP, AGG, THICK, DV,
  . DIAx(10), ASx(10), SPx(10), ZSx(10), FYx(10), DEPSx(10),
  . DIAy(10), ASy(10), SPy(10), ZSy(10), FYY(10), DEPSy(10),
  . RHOSz, ROWSz, DIAz, SPz, SPzY, FYz,
  . RNxCON, RNyCON, RNxyCON, RMxCON, RMyCON, RMxyCON, RVxCON, RVyCON,
  . RNx, RNy, RNMx, RMY, RMXY, RVx, RVY,
  . PHIC, PHIS, PHIP,
  . RESULT(8)

INTEGER NASx, NASy, IPRINT
CHARACTER*35 SPNAME

C Local variable declaration
C Common blocks
REAL t, epsc0, fcp, epscr, fcr, Eec
COMMON /sepcon/ t, epsc0, fcp, epscr, fcr, Eec

INTEGER Nnlay, nip
REAL zc(21), dz(20)
COMMON /seplay/ Nnlay, nip, zc, dz

INTEGER nas(3)
REAL deps(3,10), as(3,10), es(3,10), fy(3,10), zs(3,10)
COMMON /sepete/ nas, deps, as, es, fy, zs

REAL rhos(2), rhoc(2), sm(2), smdc, fcrack, magg, ffc
COMMON /fcl/ rhos, rhoc, sm, smdc, fcrack, magg, ffc

REAL RROWcx, RROWcy, RROWcz, SSmx, SSmy, SSsz
  . RROWsx, RROWSy, RROWSz, FFyZ,
  . FFFC, SSMODC, EECP, FFCR, AAGG
COMMON /SPC/ RROWcx, RROWcy, RROWcz, SSmx, SSmy, SSsz
  . RROWsx, RROWSy, RROWSz, FFyZ,
  . FFFC, SSMODC, EECP, FFCR, AAGG

REAL DDV
COMMON /CHK/ DDV
INTEGER*2 ikey
COMMON /key/ ikey

REAL*4 GAMC, ISS(9), RNN(8), BSS(9), BNN(8), ACC,
  . SPN(8), RNNMAX(8), DNN(8), ISSMAX(9),
  . STORN(8,1000), STORS(9,1000), DATA(28),
  . SPNCON(8), RNNP(7)
LOGICAL*2 GOTS8
INTEGER IN(5),MAXIT,NDB

CALL REZERO()

C Program version number
vernum=4.00

DDV=DV

IF(IPRINT.EQ.1)THEN
  OPEN(11,FILE='SHELL.OUT',ACCESS='APPEND',STATUS='UNKNOWN')
ENDIF

20  NTYPE=1

C Apply material resistance factors
FC=FC*PHIC
DO 42 I=1,NASX
  IF(DEPSX(I).EQ.0.)THEN
    FYX(I)=FYX(I)*PHIS
  ELSE
    FYX(I)=FYX(I)*PHIP
  ENDIF
42 CONTINUE

DO 43 I=1,NASY
  IF(DEPSY(I).EQ.0.)THEN
    FYY(I)=FYY(I)*PHIS
  ELSE
    FYY(I)=FYY(I)*PHIP
  ENDIF
43 CONTINUE

FYZ=FYZ*PHIS

CALL PARA(NASX,DIAX,ASX,SPX,ZSX,
  .  NASY,DIAY,ASY,SPY,ZSY,
  .  ROWSZ,DIAZ,SPZX,SPZY,THICK,
  .  ROWSX,ROWSY,ROWCX,ROWCY,ROWCZ,
  .  SMX,SMY,SMZ,
  .  SMXTOP,SMXBOT,SMYTOP,SMYBOT)

Output data
DATA(1)=FC
DATA(2)=FCRK
DATA(3)=EC
DATA(4)=ECP
DATA(5)=AGG
DATA(6)=THICK
DATA(7)=ROWSZ
DATA(8)=DIAZ
DATA(9)=SPZX
DATA(10)=SPZY
DATA(11)=FYZ
DATA(12)=DV
DATA(13)=ROWCX
DATA(14)=ROWCY
DATA(15)=ROWCZ
DATA(16)=SMX
DATA(17)=SMY
DATA(18) = SMZ
DATA(19) = SMXTOP
DATA(20) = SMXBOT
DATA(21) = SMYTOP
DATA(22) = SMYBOT
DATA(23) = DELFYX
DATA(24) = DELFYX
DATA(25) = DELFYZ
DATA(26) = ROWSX
DATA(27) = ROWSY

SPNAME = 'RELIABILITY OF CONCRETE ELEMENTS'

IF(IPRINT.EQ.1) THEN
  CALL OUTPUT(SPNAME, NASX, NASY, DATA,
               DIAX, ASX, SPX, ZSX, FYX, DEPSX,
               DIAY, ASY, SPY, ZSY, FYY, DEPSY)
ENDIF

C Set switches
NDB = 0
ACC = .01
MAXIT = 10

C Common /sepcon/
  T = THICK
  FCP = FC
  FCR = FCRK

C Common /nlay/
  IF(RMX .NE. 0. .OR.
     . RMY .NE. 0. .OR.
     . RMXY .NE. 0. .OR.
     . RMXCON .NE. 0. .OR.
     . RMYCON .NE. 0. .OR.
     . RMXYCON .NE. 0. ) THEN
    NNLAY = 9
  ELSE
    NNLAY = 1
  ENDIF

C Common /sepste/
  NAS(1) = NASX
  NAS(2) = NASY
  NAS(3) = 0.
  DO 10 I = 1, NASX
    AS(1,I) = ASX(I)/SPX(I)
    FY(1,I) = FYX(I)
    ZS(1,I) = ZSX(I)
    DEPS(1,I) = DEPSX(I)
    ES(1,I) = 200000.
  10 CONTINUE
  DO 11 I = 1, NASY
    AS(2,I) = ASY(I)/SPY(I)
    FY(2,I) = FYY(I)
    ZS(2,I) = ZSY(I)
    DEPS(2,I) = DEPSY(I)
    ES(2,I) = 200000.
  11 CONTINUE
C Common /fc1/
  RHOS(1)=ROWSX
  RHOS(2)=ROWSY
  RHOC(1)=ROWCX
  RHOC(2)=ROWCY
  SM(1)=SMX
  SM(2)=SMY
  SMODC=EC
  FCRACK=FCRK
  MAGG=AGG
  FFC=FC

C Common /SPC/
  RROWCX=ROWCX
  RROWCY=ROWCY
  RROWCZ=ROWCZ
  SSMX=SMX
  SSMY=SMY
  SSMZ=SMZ
  RROWSX=ROWSX
  RROWSY=ROWSY
  RROWSZ=ROWSZ
  FXYZ=FYZ
  FFFC=FC
  SSMODC=EC
  EECP=ECP
  FFCR=FCRK
  AAGG=AGG

CALL SEPINI( )

C Common /sepcon/
  EPSCO=ECP
  EEC=EC

C Convert to notation and units of loading used by subprogram SEP
  SPNN(1)=RNX
  SPNN(2)=RNY
  SPNN(3)=RNXY
  SPNN(4)=-RMX*1000.
  SPNN(5)=-RMY*1000.
  SPNN(6)=-RMXY*1000.
  SPNN(7)=RVX
  SPNN(8)=RVY
  SPNCON(1)=RNXCON
  SPNCON(2)=RNYCON
  SPNCON(3)=RNXYCON
  SPNCON(4)=-RMXCON*1000.
  SPNCON(5)=-RMYCON*1000.
  SPNCON(6)=-RMXYCON*1000.
  SPNCON(7)=RVXCON
  SPNCON(8)=RVYCON

1000 RLF=0.
  NCON=0
  NUMIT=0
  DO 50 I=1,8
  RNNMAX(I)=SPNCON(I)
  ISSMAX(I)=0.
50 CONTINUE
ISSMAX(9)=0.
RK=0.5
DO 100 JJ=1,11
RK=RK*2.
DLF=1./RK
PLF=RLF
DO 51 I=1,8
DNN(I)=SPNN(I)/RK
RNN(I)=RNNMAX(I)
ISS(I)=ISSMAX(I)
CONTINUE
ISS(9)=ISSMAX(9)
200 NUMIT=NUMIT+1
PLF=PLF+DLF
DO 52 I=1,8
RNN(I)=RNN(I)+DNN(I)
CONTINUE
CALL SHELL(ISS,RNN,ACC,MAXIT,NDB,DV,
BSS,BNN,GOTS8)
IF(GOTS8)THEN
RLF=PLF
ENDIF
C Store load deformation data
NCON=NCON + 1
DO 57 I=1,3
STORN(I,NCON)=BNN(I)
STORS(I,NCON)=BSS(I)
CONTINUE
DO 58 I=4,6
STORN(I,NCON)=-0.001*BNN(I)
STORS(I,NCON)=-1000.*BSS(I)
CONTINUE
DO 59 I=7,8
STORN(I,NCON)=BNN(I)
STORS(I,NCON)=BSS(I)
CONTINUE
STORS(9,NCON)=BSS(9)
C Store data to iterate to the peak
DO 53 I=1,8
RNNMAX(I)=RNN(I)
ISSMAX(I)=BSS(I)
CONTINUE
ISSMAX(9)=BSS(9)
GOTO 200
ENDIF
100 CONTINUE
C Create result file by reading data from STORN(I)
DO 70 I=1,8
RESULT(I)=STORN(I,NCON)
CONTINUE
C Print load deformation data for ULS analysis

IF(IPRINT.EQ.1)THEN

WRITE(11,*) '******* RESULTS FROM ULS ANALYSIS *******'
WRITE(11,522)' ',
'Membrane Forces (kN/m)', 'Membrane Strains (mm/mm)'
WRITE(11,524)' ',
'LS', 'Nx', 'Ny', 'Vxy', 'Ex,o', 'Ey,o', 'Gxy,o'
DO 61 I=1,NCON
WRITE(11,530)I,STORN(1,I),STORN(2,I),STORN(3,I),
STORS(1,I),STORS(2,I),STORS(3,I)
61 CONTINUE
WRITE(11,523)' ',
'Bending Moments (kNm/m)', 'Curvatures (rad/m)'
WRITE(11,524)' ',
'LS', 'Mx', 'My', 'Txy', 'phix', 'phiy', 'phixy'
DO 62 I=1,NCON
WRITE(11,530)I,STORN(4,I),STORN(5,I),STORN(6,I),
STORS(4,I),STORS(5,I),STORS(6,I)
62 CONTINUE
WRITE(11,525)' ',
'Transverse Shears (kN/m)', 'Transverse Strains (mm/mm)'
WRITE(11,526)' ',
'LS', 'Vxz', 'Vyz', 'Gxz', 'Gyz', 'Ez'
DO 63 I=1,NCON
WRITE(11,532)I,STORN(7,I),STORN(8,I),
STORS(7,I),STORS(8,I),STORS(9,I)
63 CONTINUE

write(11,*) '
write(11,*) '
write(11,534) ' Maximum Load Factor = ',RLF

534 format(A,F8.3)
if(ikey .eq. 27)then
write(11,*) '
write(11,*) '
write(11,*) ' >> Note Analysis Terminated by User <<'
endif
CLOSE(11)

522 FORMAT(/,A,9X,A,12X,A,/)  
523 FORMAT(/,A,9X,A,15X,A,/)  
524 FORMAT(A,A,7X,A,10X,A,9X,A,1X,A,9X,A,8X,A,/)  
525 FORMAT(/,A,7X,A,13X,A,/)  
530 FORMAT(' ',I3,2X,F7.0,2(5X,F7.0),3(5X,F8.6))  
532 FORMAT(' ',I3,5X,F7.0,7X,F7.0,6X,3(5X,F8.6))  
536 FORMAT(4X,A,7X,A,6X,A,7X,A,6X,A,6X,A,6X,A,/)  
538 FORMAT(F7.0,7(2X,F7.0))

ENDIF
RETURN
END
Appendix A3

Subroutines to Link RELAN and SHELL474
SUBROUTINE DETERM (IMODE)

C IPRINT - OPTION FOR OUTPUT DATA
C PHIC - MATERIAL FACTOR FOR CONCRETE
C PHIS - MATERIAL FACTOR FOR STEEL
C PHIP - MATERIAL FACTOR FOR TENDONS
C AGG - MAXIMUM SIZE OF AGGREGATE
C NASX AND NASY - NUMBER OF LAYERS IN X AND Y-DIRECTION
C DEPSX AND DEPSY - STRAIN DIFFERENCE IN X AND Y-DIRECTION
C RHOSZ (ROWSZ) - TRANSVERSE REINFORCEMENT RATIO IN Z-DIRECTION
C RNX AND RNY - AXIAL FORCES (kN/m)
C RNX AND RNY - BENDING MOMENTS (kNm/m)
C RMX AND RMY - BENDING MOMENTS (kNm/m)
C RVX AND RVY - TRANSVERSE SHEAR (kN/m)
C DEMAND - VECTOR WHICH MEASURES THE DEMAND FOR THE STRUCTURE (kN/m)

REAL*4 AGG, RHOSZ, ROWSZ, DEPSX(10), DEPSY(10), DV, SPZX, SPZY,
  RNXCON, RNYCON, RNXYCON, RMXCON, RMYCON, RMXYCON, RVXCON, RVYCON,
  PHIC, PHIS, PHIP,
  RNX, RNY, RNXY, RMX, RMY, RMXY, RVX, RVY,
  SCALE, DEMAND

INTEGER IMODE, IPRINT, IPRINT2, ICOUNT, NASX, NASY, I

COMMON/B/IPRINT, IPRINT2, PHIC, PHIS, PHIP
COMMON/B1/AGG, NASX, NASY, DV, SPZX, SPZY, RHOSZ, ROWSZ
COMMON/B2/DEPSX, DEPSY
COMMON/B3/RNXCON, RNYCON, RNXYCON, RMXCON, RMYCON, RMXYCON
1 RVXCON, RVYCON
COMMON/B4/RNX, RNY, RNX, RMY, RMX, RNY, RMXY, RVX, RVY
COMMON/B5/DEMAND
COMMON/B10/ICOUNT, SCALE
ICOUNT = 0
IF(IMODE.GT.1)GOTO 250

READ IN SECTIONAL INFORMATION
OPEN(4,FILE='ALAG.DAT',STATUS='OLD')
OPEN(7,FILE='INN.DAT',STATUS='OLD')
READ(7,*) IPRINT, IPRINT2, PHIC, PHIS, PHIP
READ(7,*) AGG, DV, SPZX, SPZY
READ(7,*) RHOSZ, NASX, NASY, SCALE
DO 38 I = 1, NASX
    READ(7,*) DEPSX(I)
38 CONTINUE
DO 39 I = 1, NASY
    READ(7,*) DEPSY(I)
39 CONTINUE
CLOSE(7)

ROWSZ = RHOSZ / 1000000.0

AXIAL FORCES: Nx, Ny (kN/m), SHEAR FORCE: Vxy (kN/m)
BENDING MOMENTS: Mx, My (kNm/m), TORSION: Txy (kNm/m)
TRANSVERSE SHEAR FORCE: Vxz, Vyz (kN/m)

INPUT CONSTANT COMPONENT OF SECTIONAL FORCES
See CSA S474 - Fig. 8.2 for sign convention
READ(4,*) RNXCON, RNYCON, RNXYCON
READ(4,*) RMXCON, RMYCON, RMXYCON
READ(4,*) RVXCON, RVYCON
Appendix A3

C INPUT SECTIONAL FORCES TO BE INCREASED PROPORTIONALLY
C See CSA S474 Fig. 8.2 for sign convention
READ(4,*)RNX,RNY,RNXY
READ(4,*)RMX,RMY,RMXY
READ(4,*)RVX,RVY

C INPUT CONSTANT LOAD VECTOR

READ(4,*)DEMAND

IF(IMODE.EQ.20)CLOSE(4)

CALL CLS
CALL MOVCUR(3,0)
WRITE (*,100)
100 FORMAT (17X,'**********************************************************************/
   . 17X,** University of British Columbia 1991-1992 */
   . 17X,** Part of M.A.Sc Thesis by Gisli Jonsson */
   . 17X,** In The Department of Civil Engineering */
   . 17X,** Reliability Analysis of Concrete Elements */
   . 17X,** Iceberg Impact Based on Energy Principles */
   . 17X,** Capacity calculation: SHELL 474 P.E.ADEBAR */
   . 17X,** Reliability calculation: RELAN R.O.FOSCHI */
   . 17X,**
   . 17X,**
   . 17X,**
RETURN
END

SUBROUTINE GFUN (X, N, IMODE, GXP)
C
C IPRINT - OPTION FOR OUTPUT DATA
C AGG - MAXIMUM SIZE OF AGGREGATE
C NASX AND NASY - NUMBER OF LAYERS IN X AND Y-DIRECTION
C FC - CYLINDER COMPRESSION STRENGTH (MPa)
C FCK - TENSILE STRENGTH FOR CONCRETE (MPa)
C EC - SECANT MODULUS (MPa)
C ECP - STRAIN AT PEAK COMPRESSION STRESS
C THICK - THICKNESS OF SECTION (mm)
C DV - SHEAR DEPTH OF SECTION (mm)
C DIAx,DIAY OR DIAZ - DIAMETER OF BARS AND STIRRUPS (mm)
C ASX OR ASY - AREA OF BARS IN X AND Y-DIRECTION (mm2)
C SPX OR SPY - SPACING OF BARS IN X AND Y-DIRECTION (mm)
C SPZX OR SPZY - SPACING BETWEEN STIRRUPS IN X AND Y-DIRECTION (mm)
C ZSX OR ZSY - LOCATION OF BARS IN Z-DIRECTION FROM MIDPLANE (mm)
C Fyx,Fyy OR Fyz - YIELD STRENGTH OF BARS AND STIRRUPS (MPa)
C DEPSX AND DEPSY - STRAIN DIFFERENCE IN X AND Y-DIRECTION
C RHOSZ (ROWSZ) - TRANSVERSE REINFORCEMENT RATIO IN Z-DIRECTION
C RNX AND RNY - AXIAL FORCES (kN/m)
C RNXY - SHEAR FORCE (kN/m)
C RMX AND RMY - BENDING MOMENTS (kNm/m)
C RMXY - TORSION (kNm/m)
C RVX AND RVY - TRANSVERSE SHEAR (kN/m)
C RESIST - VECTOR WHICH MEASURES THE STRUCTURAL RESISTANCE (kN/m)
C DEMAND - VECTOR WHICH MEASURES THE DEMAND FOR THE STRUCTURE (kN/m)
REAL*4 FC,FCRK,EC,ECP,THICK,
  . ASX(10),SPX(10),ZSX(10),FYX(10),DIAZ(10),
  . ASY(10),SPY(10),ZSY(10),FYY(10),DIAY(10),
  . DEPSX(10),DEPSY(10),DIAZ,FYZ,
  . AGG,DV,SPZX,SPZY,RHOSZ,ROWSZ,PHIC,PHIS,PHIP,
  . RNXCON,RNYCON,RNXYCON,RMXCON,RMYCON,RMXYCON,
  . RNX,RNY,RNXY,RMX,RMY,RMXY,RVX,RVY,
  . RESULT(8),
  . RATIO1,RATIO2,RATIO3,RATIO4,RATIO5,RATIO6,RATIO7,RATIO8,
  . SCALE,RESIST,LOAD,CAPACITY,DEMAD
REAL*8GXP,X(N)
INTEGERIMODE,IPRINT,IPRINT2,ICOUNT,ISTART,NASX,NASY,I
INTRINSICSQRT
COMMON/B/IPRINT,IPRINT2,PHIC,PHIS,PHIP
COMMON/B1/AGG,NASX,NASY,DV,SPZX,SPZY,RHOSZ,ROWSZ
COMMON/B2/DEPSX,DEPSY
COMMON/B3/RNXCON,RNYCON,RNXYCON,RMXCON,RMYCON,RMXYCON
COMMON/B4/RNX,RNY,RNXY,RMX,RMY,RMXY,RVX,RVY
COMMON/B5/DEMAND
COMMON/B10/ICOUNT,SCALE
ICOUNT=ICOUNT+1
FC=SNGL(X(1))
THICK=SNGL(X(2))
FYX=SNGL(X(3))

ISTART = 3
DO 10 I=1,NASX
  ASX(I)=SNGL(X(ISTART+I))
CONTINUE

ISTART = 3 + NASX
DO 12 I=1,NASX
  SPX(I)=SNGL(X(ISTART+I))
CONTINUE

ISTART = 3 + (2*NASX)
DO 14 I=1,NASX
  ZSX(I)=SNGL(X(ISTART+I))
CONTINUE

ISTART = 3 + (3*NASX)
DO 16 I=1,NASX
  FYX(I)=SNGL(X(ISTART+I))
CONTINUE

ISTART = 3 + (4*NASX)
DO 18 I=1,NASY
  ASY(I)=SNGL(X(ISTART+I))
CONTINUE

ISTART = 3 + (4*NASX) + NASY
DO 20 I=1,NASY
  SPY(I)=SNGL(X(ISTART+I))
CONTINUE

ISTART = 3 + (4*NASX) + (2*NASY)
DO 22 I=1,NASX
  ZSY(I)=SNGL(X(ISTART+I))
CONTINUE

ISTART = 3 + (4*NASX) + (3*NASY)
DO 24 I=1,NASY
  FY(I)=SNGL(X(ISTART+I))
CONTINUE
DO 38 I=1,NASX
DIAx(I)=SQRT(1.2732*ASx(I))
38 CONTINUE

DO 39 I=1,NASY
DIAy(I)=SQRT(1.2732*ASY(I))
39 CONTINUE

EC=5000.0*SQRT(FC)
ECP=-1.7*FC/EC
FCRK=0.33*SQRT(FC)
DIAZ=SQRT(1.2732*ROWSZ*SPZX*SPZY)

CALL SKEL(FC,FCRK,EC,ECP,AGG,THICK,DV,
1 DIAx,ASx,SPx,ZSx,FYx,DEPSx,
2 DIAy,ASY,SPY,ZSY,FYY,DEPSY,
3 RHOSZ,ROWSZ,DIAZ,NASX,NASY,SPZX,SPZY,FYZ,
4 RNXCON,RNYCON,RNXYCON,RMXCON,RMYCON,RVXCON,RYCON,
5 RNx,RNY,RNX,rmx,Rmy,rmxy,RVX,RVY,
6 PHIC,PHIS,PHIP,
7 IPRTN,
8 RESULT)

C CAPACITY OF THE ELEMENT WHERE THE MOMENTS ARE SCALED DOWN
C BECAUSE OF DIFFERENT UNITS

RESIST = SQRT((RESULT(1))**2+(RESULT(2))**2+(RESULT(3))**2
1 +(RESULT(4)/SCALE)**2+(RESULT(5)/SCALE)**2+(RESULT(6)/SCALE)**2+
2 +(RESULT(7))**2+(RESULT(8))**2)

C APPLIED LOAD ON THE ELEMENT

LOAD = SQRT((RNX)**2+(RNY)**2+(RNXY)**2+(RMX/SCALE)**2
1 +(RMY/SCALE)**2+(RMXY/SCALE)**2+(RVX)**2+(RVY)**2)

C PERFORMANCE FUNCTION FOR THE ELEMENT WHERE GXP<0 MEANS FAILURE

CAPACITY = RESIST/LOAD

GXP = DBLE(CAPACITY) - DBLE(DEMAND)

IF(IPRINT2.EQ.1)THEN
WRITE OUT SECTIONAL INFORMATION TO DATA FILE
OPEN(8,FILE='UT.DAT',ACCESS='APPEND',STATUS='UNKNOWN')
WRITE(8,*)'********************************************************************
WRITE(8,532)'SECTIONAL INFORMATION'
532 FORMAT(10X,A)
WRITE(8,533) 'ITERATION #',ICOUNT
533 FORMAT(/,10X,A,14)
WRITE(8,*)'********************************************************************
WRITE(8,*),IPRTN,IPRTN2,NASX,NASY,PHIC,PHIS,PHIP,AGG
WRITE(8,*),FC,THICK,FCRK,EC,ECP
WRITE(8,*),FYx,SPZX,SPZY,DV,DIAZ,RHOSZ

DO 48 I=1,NASX
WRITE(8,*),DIAx(I),ASx(I),SPx(I),ZSx(I),FYx(I),DEPSx(I)
48 CONTINUE
DO 49 I=1,NASY
WRITE(8,*)(DIAY(I),ASY(I),SPY(I),ZSY(I),FYY(I),DEPSY(I)
49 CONTINUE
WRITE(8,550) 'RNX', 'RNY', 'RNXY', 'RMX', 'RMY', 'RMXY',
. 
. 'RVX', 'RVY'
WRITE(8,551) RNX,RNY,RNXY, RMX,RMY,RMXY, RVX, RVY
551 FORMAT(F7.0,7(2X,F7.0))
WRITE(8,536) 'NX', 'NY', 'VXY', 'MX', 'MY', 'MXY',
. 'VXZ', 'VYZ'
WRITE(8,538) RESULT(1),RESULT(2),RESULT(3),RESULT(4),
. RESULT(5),RESULT(6),RESULT(7),RESULT(8)
538 FORMAT(F7.0,7(2X,F7.0))
WRITE(8,542) 'RESIST=', RESIST, 'LOAD=', LOAD
WRITE(8,543) 'CAPACITY=', CAPACITY, 'DEMAND=', DEMAND, 'GXP=', GXP
542 FORMAT(/,4X,A,F10.4,3X,A,F10.4,3X,A,F10.4)
543 FORMAT(/,4X,A,F10.4,3X,A,F10.4,3X,A,F10.4)
CLOSE(8)
ENDIF
RETURN
END

C-----------------------------------------------
SUBROUTINE DFUN (X, N, IMODE, DELTA)
C-----------------------------------------------
IMPLICIT REAL*8 (A - H, O - Z)
DIMENSION X(N), DELTA(N)
RETURN
END

C-----------------------------------------------
SUBROUTINE D2FUN (X, N, IMODE, D2, N2)
C-----------------------------------------------
IMPLICIT REAL*8 (A - H, O - Z)
DIMENSION X(N), D2(N2,N2)
RETURN
END

C-----------------------------------------------
Appendix A4

Subroutines for Probabilistic Evaluation of Ice-Impact
$DEBUG
SUBROUTINE DETERM(IMODE)
C
N - NUMBER OF GAUSS POINTS
P - THE ICEBERG MASS DENSITY (kg/m3)
Z - THE WATER DEPTH (m)
AO - REFERENCE AREA (100 m2)
A - THE ICEBERG ANGLE AT THE TOP (DEGREES)
B - THE ICEBERG ANGLE AT THE BOTTOM (DEGREES)
F0 - APPLIED LOAD (MN)
E(I) - LOCATION OF POINT
H(I) - WEIGHT OF POINT
REAL*8 Pi,Z,ao,a,b,fo,E(32),h(32)
INTEGER IMODE,N
COMMON/G1/N,Pi,Z
COMMON/G2/AO,A,B
COMMON/G3/FO
COMMON/G4/E,H
IF(IMODE.GT.1)GOTO 10
C
READ SECTIONAL INFORMATION
OPEN(4,FILE='PROB.DAT',STATUS='OLD')
OPEN(5,FILE='NID.DAT',STATUS='UNKNOWN')
WRITE(5,*)'***********************************************************'
WRITE(5,*)'2CMAXF0'
WRITE(5,*)'***********************************************************'
READ(4,*)N
READ(4,*)Pi,Z,ao,a,b
CALL GAUSS(N,E,H,IERR)
10 READ(4,*)FO
IF(IMODE.EQ.20)CLOSE(4)
RETURN
END

SUBROUTINE GFUN(X,N,IMODE,GXP)
C
SM - COMPRESSIVE STRENGTH OF THE ICEBERG (MPa)
VI - THE VELOCITY OF THE ICEBERG (m/s)
LI - THE WIDTH OF THE ICEBERG (m)
HI - THE ICEBERG THICKNESS (m)
AO - REFERENCE AREA (100 m2)
A - THE ICEBERG ANGLE AT THE TOP (DEGREES)
B - THE ICEBERG ANGLE AT THE BOTTOM (DEGREES)
LM - FACTOR TO EXPRESS THE AVERAGE RATIO OF MEAN TO PEAK LOAD
XM - MAXIMUM PENETRATION (m)
AX - CONTACT AREA BASED ON XM (m2)
FX - MAXIMUM ICEBERG IMPACT LOAD (MN)
F0 - APPLIED LOAD (MN)
GXP - PERFORMANCE FUNCTION
REAL*8 SM,VI,LI,HI,
AO,A,B,FO,
LM,XM,AX,FX,GXP,
X(N)
INTEGER IMODE
INTRINSIC DTAN
COMMON/G2/AO,A,B
COMMON/G3/FO
SM=X(1)
VI=X(2)
LI=X(3)
HI=X(4)
LM = 2.0/3.0

CALL DELTA(XM,SM,VI,LI,HI,LM)

AX = DSQRT(2.0)*(2.0*LI*XM+(DTAN(A*22.0/7.0*1.0/180.0)
1 +DTAN(B*22.0/7.0*1.0/180.0))*(XM**2.0))
FX = (LM*SM*DSQRT(AO)*DSQRT(AX))
GXP = FX-FO
WRITE(5,200)XM,AX,FX
200 FORMAT(15.4,3X,F15.4,3X,F15.4)
RETURN
END

C-----------------------------------------------
SUBROUTINE DFUN (X, N, IMODE, DELTA)
C-----------------------------------------------
IMPLICIT REAL*8 (A - H, O - Z)
DIMENSION X(N), DELTA(N)
RETURN
END

C-----------------------------------------------
SUBROUTINE D2FUN(X, N, IMODE, D2, N2)
C-----------------------------------------------
IMPLICIT REAL*8 (A - H, O - Z)
DIMENSION X(N), D2(N2,N2)
RETURN
END

C-----------------------------------------------
SUBROUTINE DELTA(XM,SM,VI,LI,HI,LM)
C-----------------------------------------------
C VI - THE ICEBERG VELOCITY (m/s)
C HI - THE ICEBERG THICKNESS (m)
C LI - THE WIDTH OF THE ICE (m)
C SM - COMPRESSIVE STRENGTH OF THE ICE (MPa)
C Z - WATER DEPTH (m)
C CM - THE ADDED MASS FACTOR
C Pi - THE ICEBERG MASS DENSITY (kg/m3)
C LM - FACTOR TO EXPRESS THE AVERAGE RATIO OF MEAN TO PEAK LOAD
C XM - MAXIMUM PENETRATION (m)
C AO - REFERENCE AREA (100 m2)
C A - THE ICEBERG ANGLE AT THE TOP (DEGREES)
C B - THE ICEBERG ANGLE AT THE BOTTOM (DEGREES)
C E(I) - LOCATION OF POINT
C H(I) - WEIGHT OF POINT

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REAL*8 XM, D, DD, E(32), H(32), SUM,
   A, B, AO, CM, Z, PI, KE, Y, YP, F,
   LM, SM, LI, HI, VI
INTEGER I, N, NINT
INTRINSIC DTAN, DSQRT, DABS
COMMON/G1/N, PI, Z
COMMON/G2/AO, A, B
COMMON/G4/E, H
D = 0.0
DD = 0.01
NINT = 0

1 SUM = 0.0
DO 10 I = 1, N
   SUM = SUM + DSQRT(DSQRT(2.0)*(HI*(1.0+E(I))
   1 + (DTAN(A*22.0/7.0*1.0/180.0) + DTAN(B*22.0/7.0*1.0/180.0))
   2 *(D/4.0)*((1.O+E(I))**2))*H(I)
10 CONTINUE
CM = (0.9*HI)/((2.0*Z)-(0.9*HI))
KE = (0.5*0.66*(LI**2.0)*HI*PI*(1.0+CM)*(VI**2.0))/1.0E6
Y = (((D**(3.0/2.0))*LM*SM*DSQRT(AO))/2.0)*SUM - KE
IF(NINT.EQ.0)GOTO 5
F = Y*YP
IF(F.LE.0.0)GOTO 20
5 YP = Y
D = D + DD
NINT = NINT + 1
GOTO 1
20 D = (D- DD)+DD*DABS(YP)/(DABS(YP)+DABS(Y))
XM = D
RETURN
END

SUBROUTINE GAUSS(N, E, H, IERR)
C---------------------------------------------------------------------
REAL*8 E(32), H(32)
M = (N-2)*(N-3)*(N-4)*(N-5)*(N-6)*(N-7)*(N-8)
   M = M*(N-9)*(N-10)*(N-11)*(N-12)*(N-15)*(N-16)*(N-32)
IF (M.NE.0) GO TO 50
IERR = 0
IF (N.EQ.32) GO TO 40
IF (N.EQ.16) GO TO 30
IF (N.EQ.15) GO TO 29
IF (N.EQ.12) GO TO 28
IF (N.EQ.11) GO TO 27
IF (N.EQ.10) GO TO 26
IF (N.EQ.9) GO TO 25
IF (N.EQ.8) GO TO 23
IF (N.EQ.7) GO TO 20
IF (N.EQ.6) GO TO 18
IF (N.EQ.5) GO TO 15
IF (N.EQ.4) GO TO 13

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Appendix A4

IF (N.EQ.3) GO TO 12
E(1) = 0.577350269189626D0
E(2) = -E(1)
H(1) = 1.0D0
H(2) = H(1)
RETURN

12 E(1) = 0.774596669241483D0
E(2) = 0.0D0
E(3) = -E(1)
H(1) = 0.555555555555556D0
H(2) = 0.888888888888889D0
H(3) = H(1)
RETURN

13 E(1) = 0.861136311594053D0
E(2) = 0.339981043584856D0
H(1) = 0.347854845137454D0
H(2) = 0.652145154862546D0
DO 1 I = 1,2
E(5-I) = -E(I)
H(5-I) = H(I)
1 RETURN

15 E(1) = 0.906179845938664D0
E(2) = 0.538469310105683D0
E(3) = 0.0D0
H(1) = 0.236926885056189D0
H(2) = 0.478628670499366D0
H(3) = 0.568888888888889D0
DO 2 I = 1,2
E(6-I) = -E(I)
H(6-I) = H(I)
2 RETURN

18 E(1) = 0.932469514203152D0
E(2) = 0.661209386466265D0
E(3) = 0.238619186083197D0
H(1) = 0.171324492379170D0
H(2) = 0.360761573048139D0
H(3) = 0.467913934572691D0
DO 3 I = 1,3
E(7-I) = -E(I)
H(7-I) = H(I)
3 RETURN

20 E(1) = 0.949107912342759D0
E(2) = 0.741531185599394D0
E(3) = 0.405845151377397D0
E(4) = 0.0D0
H(1) = 0.129484966168870D0
H(2) = 0.279705391489277D0
H(3) = 0.381830050505119D0
H(4) = 0.417959183673469D0
DO 4 I = 1,3
E(8-I) = -E(I)
H(8-I) = H(I)
4 RETURN

23 E(1) = 0.960289856497536D0
E(2) = 0.796666477413627D0
E(3) = 0.525532409916329D0
E(4) = 0.183434642495650D0
H(1) = 0.101228536290376D0
H(2) = 0.222381034453374D0
Appendix A4

\[ H(3) = 0.313706645877887D0 \]
\[ H(4) = 0.362683783378362D0 \]
\[ \text{DO 5 I = 1, 4} \]
\[ E(9-I) = -E(I) \]
\[ H(9-I) = H(I) \]
\[ \text{RETURN} \]

\[ E(1) = 0.968160239507626D0 \]
\[ E(2) = 0.836031107326636D0 \]
\[ E(3) = 0.613371432700590D0 \]
\[ E(4) = 0.324253423403809D0 \]
\[ E(5) = 0.00D0 \]
\[ H(1) = 0.081274388361574D0 \]
\[ H(2) = 0.180648160694857D0 \]
\[ H(3) = 0.260610696402935D0 \]
\[ H(4) = 0.312347077040003D0 \]
\[ H(5) = 0.330239355001260D0 \]
\[ \text{DO 5 I = 1, 4} \]
\[ E(10-I) = -E(I) \]
\[ H(10-I) = H(I) \]
\[ \text{RETURN} \]

\[ E(1) = 0.973906528517172D0 \]
\[ E(2) = 0.865063666889855D0 \]
\[ E(3) = 0.679409568299024D0 \]
\[ E(4) = 0.433395394129247D0 \]
\[ E(5) = 0.148874338981631D0 \]
\[ H(1) = 0.066671344308688D0 \]
\[ H(2) = 0.149451349150581D0 \]
\[ H(3) = 0.219086362515982D0 \]
\[ H(4) = 0.269266719309996D0 \]
\[ H(5) = 0.2955242224714753D0 \]
\[ \text{DO 7 I = 1, 5} \]
\[ E(11-I) = -E(I) \]
\[ H(11-I) = H(I) \]
\[ \text{RETURN} \]

\[ E(1) = 0.978228658146057D0 \]
\[ E(2) = 0.887062599768095D0 \]
\[ E(3) = 0.730152005574049D0 \]
\[ E(4) = 0.519096129206812D0 \]
\[ E(5) = 0.269543155952345D0 \]
\[ E(6) = 0.00D0 \]
\[ H(1) = 0.055668567116174D0 \]
\[ H(2) = 0.125580369464905D0 \]
\[ H(3) = 0.186290210927734D0 \]
\[ H(4) = 0.233193764591990D0 \]
\[ H(5) = 0.26280454510247D0 \]
\[ H(6) = 0.272925086777901D0 \]
\[ \text{DO 77 I = 1, 5} \]
\[ E(12-I) = -E(I) \]
\[ H(12-I) = H(I) \]
\[ \text{RETURN} \]

\[ E(1) = 0.981560634246719D0 \]
\[ E(2) = 0.904117256370475D0 \]
\[ E(3) = 0.769902674194305D0 \]
\[ E(4) = 0.587317954286617D0 \]
\[ E(5) = 0.367831498998180D0 \]
\[ E(6) = 0.125233408511469D0 \]
\[ H(1) = 0.047175336386512D0 \]
\[ H(2) = 0.106939325995318D0 \]
\[ H(3) = 0.160078328543464D0 \]
Appendix A4

\[
H(4) = 0.203167426723066 \times 10^0
\]
\[
H(5) = 0.23349253638355 \times 10^0
\]
\[
H(6) = 0.24914704581340 \times 10^3
\]
\[
\text{DO } 8 \ I = 1,6
8
\]
\[
E(13-I) = -E(I)
H(13-I) = H(I)
\]
\[
\text{RETURN}
\]

29

\[
E(1) = 0.987992518020485 \times 10^0
\]
\[
E(2) = 0.937273392400706 \times 10^0
\]
\[
E(3) = 0.848206583410427 \times 10^0
\]
\[
E(4) = 0.724417731360170 \times 10^0
\]
\[
E(5) = 0.570972172608539 \times 10^0
\]
\[
E(6) = 0.394151347077563 \times 10^0
\]
\[
E(7) = 0.201194093997435 \times 10^0
\]
\[
E(8) = 0.0 \times 10^0
\]
\[
H(1) = 0.030753241966117 \times 10^0
\]
\[
H(2) = 0.070366047488108 \times 10^0
\]
\[
H(3) = 0.107159220467172 \times 10^0
\]
\[
H(4) = 0.139570677926154 \times 10^0
\]
\[
H(5) = 0.166269205816994 \times 10^0
\]
\[
H(6) = 0.186161000015562 \times 10^0
\]
\[
H(7) = 0.198431485327112 \times 10^0
\]
\[
H(8) = 0.202578241925561 \times 10^0
\]
\[
\text{DO } 88 \ I = 1,7
88
\]
\[
E(16-I) = -E(I)
H(16-I) = H(I)
\]
\[
\text{RETURN}
\]

30

\[
E(1) = 0.98940093499165 \times 10^0
\]
\[
E(2) = 0.944575023073233 \times 10^0
\]
\[
E(3) = 0.865631202387832 \times 10^0
\]
\[
E(4) = 0.755404408355003 \times 10^0
\]
\[
E(5) = 0.617862444026444 \times 10^0
\]
\[
E(6) = 0.45801677657227 \times 10^0
\]
\[
E(7) = 0.28160355077925 \times 10^0
\]
\[
E(8) = 0.09501259837637 \times 10^0
\]
\[
H(1) = 0.027152459411754 \times 10^4
\]
\[
H(2) = 0.06225323938648 \times 10^0
\]
\[
H(3) = 0.09515851168249 \times 10^3
\]
\[
H(4) = 0.124628971255534 \times 10^3
\]
\[
H(5) = 0.14995988816577 \times 10^0
\]
\[
H(6) = 0.169156519395003 \times 10^0
\]
\[
H(7) = 0.182603415044924 \times 10^0
\]
\[
H(8) = 0.189450610455068 \times 10^0
\]
\[
\text{DO } 9 \ I = 1,8
9
\]
\[
E(17-I) = -E(I)
H(17-I) = H(I)
\]
\[
\text{RETURN}
\]

40

\[
E(1) = 0.99726386184948 \times 10^2
\]
\[
E(2) = 0.985611511545268 \times 10^0
\]
\[
E(3) = 0.964762255587506 \times 10^0
\]
\[
E(4) = 0.93490607593774 \times 10^0
\]
\[
E(5) = 0.896321155766052 \times 10^0
\]
\[
E(6) = 0.84936761373257 \times 10^0
\]
\[
E(7) = 0.79448379596794 \times 10^0
\]
\[
E(8) = 0.73218211874029 \times 10^0
\]
\[
E(9) = 0.663044266930215 \times 10^0
\]
\[
E(10) = 0.587715757240762 \times 10^0
\]
\[
E(11) = 0.506899908932229 \times 10^0
\]
\[
E(12) = 0.421351276130635 \times 10^0
\]
E(13) = 0.331868602282128D0
E(14) = 0.239287362252137D0
E(15) = 0.144471961582796D0
E(16) = 0.048307665687738D0
H(1) = 0.007018610009471D0
H(2) = 0.016274394730906D0
H(3) = 0.025392065309262D0
H(4) = 0.034273862913021D0
H(5) = 0.042835898022227D0
H(6) = 0.050998059262376D0
H(7) = 0.058684093478536D0
H(8) = 0.06582222776362D0
H(9) = 0.072345794108849D0
H(10) = 0.078193895787070D0
H(11) = 0.083311924226947D0
H(12) = 0.087652093004404D0
H(13) = 0.091173878695764D0
H(14) = 0.093844399080805D0
H(15) = 0.095638720079275D0
H(16) = 0.096540088514728D0
DO 10 I = 1, 16
E(33-I) = -E(I)
10
H(33-I) = H(I)
RETURN
50 WRITE(*,1000)
1000 FORMAT(' WRONG CHOICE FOR NUMBER OF GAUSS INTEGRATION POINTS'/)
IERR = 1
RETURN
END

C-----------------------------------------------

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Appendix B

Design of Concrete Beam According to the CSA
In design of the beam section we have to make sure the applied moment will not exceed the critical moment, and that the minimum and maximum reinforcement criteria is met.

From Equations 4.11 and 4.27 we know that the maximum reinforcement can be written as:

\[
\rho_{\text{max}} = \left( \frac{\alpha_1 \beta_1 \phi_s f_y}{\phi_s f_y} \right) \left( \frac{600}{600 + f_y} \right)
\]  \hspace{1cm} (B.1)

and also from Equation 4.10 and 4.11 the critical moment can be written as:

\[
M_{cr} = K_r b d^2
\]  \hspace{1cm} (B.2)

where \( K_r \) is:

\[
K_r = \rho \phi_s f_y \left( 1 - \frac{\rho \phi_s f_y}{2 \alpha \phi_s f_c} \right)
\]  \hspace{1cm} (B.3)

and the reinforcement ratio \( \rho \):

\[
\rho = \frac{A_s}{bd}
\]  \hspace{1cm} (B.4)
In order to design a beam section, we need to assume certain number of steel layers. As an initial assumption, the beam will be assumed to need one layer of steel bars, with spacing according to the CSA code (see Figure B.1).

\[ S_c = \text{Clear Spacing} = (db, 25 \text{ mm}, 1.33 \text{ MAX. AGGR. Size}) \]
\[ d_c = 50 \text{ mm} + db/2 \text{ For No. 10 Stirrups} \]
\[ d_o = 40 \text{ mm} + \text{DIA. of Stirrup} + \text{Inside RAD. of Stirrup Bend}, \text{For Larger than No. 10} \]
\[ S = \text{MIN. Spacing} = S_c + db \]
\[ \text{MAX. Number of Bars} = \left(\frac{b - 2dc}{S}\right) + 1 \]

**Figure B.1: Code Requirements for Reinforcement Placing**

First of all we need to find the applied load, which the simply supported beam has to resist. By using the values from Table 4.1, Equation 4.13 gives us:

\[ M_{\text{max}} = \frac{1}{8}(1.25 \times 42.5 + 1.5 \times (25.0 + 12.5))17^2 = 3951.2 \text{ (kNm)} \]

Then maximum reinforcement ratio according to Equation B.1 is:

\[ \rho_{\text{max}} = \left(\frac{0.85 \times 0.816 \times 0.6 \times 35}{0.85 \times 400}\right) \left(\frac{600}{600 + 400}\right) = 0.0257 \]
By introducing $\rho_{\text{max}}$ as an initial value into Equation B.3 we get:

$$K_r = 0.0257 \times 0.85 \times 400 \left(1 - \frac{0.0257 \times 0.85 \times 400}{1.7 \times 0.6 \times 35}\right) = 6.5993 \text{ (MPa)}$$

Assuming that the beam will have only one layer, we can find the required effective depth from Equation B.2:

$$d = \sqrt[3]{\frac{3951.2 \times 10^{-6}}{6.5993 \times 500}} = 1094 \text{ (mm)}$$

but assuming only one layer of No.35 steel bars and beam dimensions 500 x 1300 mm the effective depth becomes:

$$d = 1300 - \left(40 + 11.3 + \frac{35.7}{2}\right) = 1231 \text{ (mm)}$$

Now the new $K_r$ can be found from Equation B.2:

$$K_r = \frac{3951.2 \times 10^{-6}}{500 \times 1231^2} = 5.2149 \text{ (MPa)}$$

and by rearranging Equation B.3 we get the actual reinforcement ratio:

$$\rho = \frac{0.85 \times 0.6 \times 35}{0.85 \times 400} \left(1 - \sqrt{1 - \frac{2 \times 5.2149}{0.85 \times 0.6 \times 35}}\right) = 0.0187$$
The required steel area for the beam can now be calculated from Equation B.4:

\[ A_s = 0.0187 \times 500 \times 1231 = 11510 \text{ (mm}^2) \]

which is approximately 12 No.35 bars. The minimum spacing criteria between bars in the CSA code, forces us to have two steel layers (see Figure B.1):

\[ d_e = 50 + \frac{35.7}{2} = 67.85 \text{ (mm)} \quad \text{,} \quad S = 35.7 + 35.7 = 71.4 \text{ (mm)} \]

and therefore the maximum numbers of bars in each layer is:

\[ \text{Max}_{\text{bars}} = \frac{500 - 2 \times 67.85}{71.4} + 1 \approx 6 \]

Because we have now two layers instead of one, the calculated effective depth changes according to that:

\[ d = 1300 - \left( \frac{6 \times 69.15 + 6 \times 129.85}{12} \right) = 1200.5 \text{ (mm)} \]

and in the same manner \( K_r \):

\[ K_r = \frac{3951.2 \times 10^{-6}}{500 \times 1200.5^2} = 5.4832 \text{ (MPa)} \]
The actual reinforcement ratio is then:

$$\rho = \frac{0.85 \times 0.6 \times 35}{0.85 \times 400} \left(1 - \sqrt{1 - \frac{2 \times 5.4832}{0.85 \times 0.6 \times 35}}\right) = 0.0199 < \rho_{\text{max}}$$

and the required steel area for the beam consequently becomes:

$$A_s = 0.0199 \times 500 \times 1200.5 = 11945 \text{ (mm}^2)$$

which is less than 12 No.35 steel bars so this section seems okay.
Appendix C

Results from SHELL474
Appendix C

Program SHELL474 version 4.00

************************************************************************
************************ECCENTRIC LOAD*******************************
************************************************************************

Concrete Properties

- Fac. Cylinder Comp. Strength = 50.00 MPa
- Strain at peak stress x 1000 = -2.40
- Cracking stress = 2.33 MPa
- Maximum aggregate size = 20.00 mm
- Secant modulus of elasticity = 35355 MPa

Section Thickness = 1400.00 mm

Shear Depth = 1200.00 mm

In-Plane Reinforcement

<table>
<thead>
<tr>
<th>Dir.</th>
<th>Bar dia. (mm)</th>
<th>Area per bar (sq mm)</th>
<th>Spacing of bars (mm)</th>
<th>Z Coord. of layer (mm)</th>
<th>Fac. yld. stress (MPa)</th>
<th>Prestrain (x 1000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>29.9</td>
<td>702.</td>
<td>125.</td>
<td>620.</td>
<td>400.</td>
<td>.00</td>
</tr>
<tr>
<td>X</td>
<td>50.5</td>
<td>2000.</td>
<td>125.</td>
<td>-615.</td>
<td>400.</td>
<td>.00</td>
</tr>
<tr>
<td>Y</td>
<td>35.7</td>
<td>1000.</td>
<td>235.</td>
<td>585.</td>
<td>400.</td>
<td>.00</td>
</tr>
<tr>
<td>Y</td>
<td>35.7</td>
<td>1000.</td>
<td>235.</td>
<td>-580.</td>
<td>400.</td>
<td>.00</td>
</tr>
<tr>
<td>Y</td>
<td>35.5</td>
<td>990.</td>
<td>470.</td>
<td>560.</td>
<td>1675.</td>
<td>6.00</td>
</tr>
<tr>
<td>Y</td>
<td>35.5</td>
<td>990.</td>
<td>470.</td>
<td>-560.</td>
<td>1675.</td>
<td>6.00</td>
</tr>
</tbody>
</table>

Transverse Reinforcement

<table>
<thead>
<tr>
<th>Amount (sq mm/sq m)</th>
<th>Bar Dia. (mm)</th>
<th>Spacing X (mm)</th>
<th>Spacing Y (mm)</th>
<th>Fac. yld. stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3414.0</td>
<td>11.3</td>
<td>235.0</td>
<td>125.0</td>
<td>400.0</td>
</tr>
</tbody>
</table>

Reinforcement Ratios

- X direction = 1.544%
- Y direction = 0.909%
- Z direction = 0.341%
### Effective Concrete Area Ratios

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X direction</strong></td>
<td>55. %</td>
<td></td>
</tr>
<tr>
<td><strong>Y direction</strong></td>
<td>58. %</td>
<td></td>
</tr>
<tr>
<td><strong>Z direction</strong></td>
<td>72. %</td>
<td></td>
</tr>
</tbody>
</table>

### Average Crack Spacings at Mid-Depth

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X direction</strong></td>
<td>1420. mm</td>
<td></td>
</tr>
<tr>
<td><strong>Y direction</strong></td>
<td>1410. mm</td>
<td></td>
</tr>
<tr>
<td><strong>Z direction</strong></td>
<td>558. mm</td>
<td></td>
</tr>
</tbody>
</table>

#### RESULTS FROM ULS ANALYSIS

<table>
<thead>
<tr>
<th>Membrane Forces (kN/m)</th>
<th>Membrane Strains (mm/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>Nx</td>
</tr>
<tr>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>1</td>
<td>156.</td>
</tr>
<tr>
<td>2</td>
<td>224.</td>
</tr>
<tr>
<td>3</td>
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<table>
<thead>
<tr>
<th>Bending Moments (kNm/m)</th>
<th>Curvatures (rad/m)</th>
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<tbody>
<tr>
<td>LS</td>
<td>Mx</td>
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<tr>
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<td>4766.</td>
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<tr>
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</tr>
<tr>
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<table>
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<tr>
<th>Transverse Shears (kN/m)</th>
<th>Transverse Strains (mm/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>Vxz</td>
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<tr>
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<td>4</td>
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</table>

- Maximum Load Factor = 1.027
- Minimum Force Ratio = 1.000
- Strain control index = 4